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**ADAPTIVE CRUISE CONTROL  
SYSTEM DESIGN AND ITS  
IMPACT ON TRAFFIC FLOW**

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# **Adaptive Cruise Control System Design And Its Impact on Traffic Flow**

**Final Report**

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## Table of Contents

Chapter 1	Development of an Evaluation Framework for ACC Vehicles.....	1
Chapter 2	Should ACC Systems be Designed to Maintain a Constant Time-Gap Between Vehicles?.....	6
Chapter 3	Overall Evaluation of the Constant Time-Gap Spacing Policy.....	30
Chapter 4	Addressing the Trade-Off Between Safety and Traffic Flow.....	32
Chapter 5	Development of an Ideal Spacing Policy.....	43
Chapter 6	Simulation and Analysis of Mixed ACC/ Manual Traffic.....	52
References.....		117
Appendix A.....		A-1
Appendix B.....		B-1
Appendix C.....		C-1

## Notation

A summary of major symbols used in the report

$x_i$  inertial longitudinal position of  $i$  th vehicle

$\ell_i$  length of  $i$  th vehicle

$\varepsilon_i$  spacing error for  $i$  th vehicle

$\delta_i$  spacing error for  $i$  th vehicle

$L_{des}$  desired inter-vehicle spacing at zero speed

$\tau$  time constant for 1<sup>st</sup> order lag model of acceleration tracking by vehicle

$h$  time-gap

$\lambda$  control gain used in CTG, VTG and other control laws

$Q$  traffic flow volume rate

$\rho$  traffic density in vehicles/ meter

$v_f$  speed parameter used in VTG spacing policy

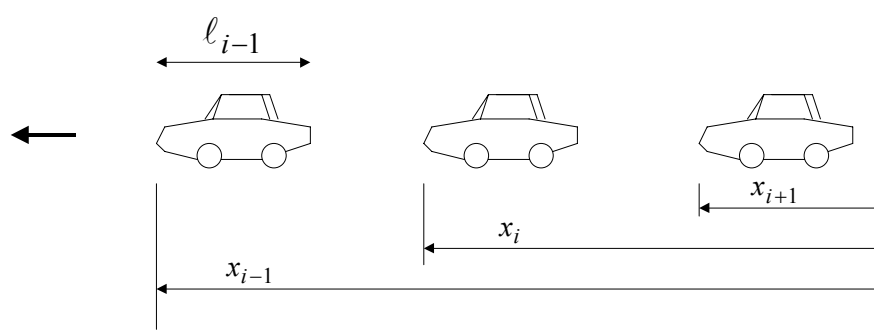
$\rho_m$  density parameter used in VTG spacing policy

$g(\dot{x}_i)$  desired spacing as a function of vehicle velocity

## Executive Summary

### 1. SHOULD ACC SYSTEMS BE DESIGNED TO MAINTAIN A CONSTANT TIME-GAP BETWEEN VEHICLES ?

The desired spacing that an ACC vehicle attempts to maintain with respect to the preceding vehicle is called the spacing policy. In Fig. 1, the desired spacing is the desired value of  $x_{i-1} - x_i - \ell_{i-1}$ . The desired distance is typically a function of the ACC vehicle velocity [1] but could also be a constant or a function of other variables such as the relative velocity.



**Fig. 1 String of adaptive cruise control vehicles**

The spacing policy is important because it determines vehicle safety, traffic flow as well as user-acceptance of the ACC system.

The most common spacing policy used in ACC systems by researchers as well as automotive manufacturers is the constant time-gap spacing policy. The constant time-gap spacing policy is given by

$$\delta_i = \varepsilon_i + h\dot{x}_i + L \tag{1}$$

where the inter-vehicle spacing is

$$\varepsilon_i = x_i - x_{i-1} \tag{2}$$

Is it a good idea for an ACC system to be designed so as to maintain a constant time-gap ? There has significant controversy about the implication of the constant time-gap policy for traffic flow. [1], [2].

A recent result by Swaroop, et. al. [1] stated that the traffic flow obtained on a highway is unstable when all vehicles on the highway use the constant time-gap policy. Traffic flow instability here refers to the unattenuated upstream propagation of disturbances that occurs when a density perturbation is introduced into the traffic flow [1]. In the proof of this result, Swaroop considers an open stretch of highway where all vehicles use the constant time-gap policy and there is a (small) constant inflow of vehicles from a ramp.

The result by Perry, et. al. [2] appears to contradict that of Swaroop [1]. Li's paper considers a circular highway with no inlets or exits for vehicles to enter or leave the highway. It shows that the consequent traffic flow obtained with the constant time-gap spacing policy is stable.

The results contained in the present report resolve the above controversy. Further, the report objectively evaluates the performance of the constant time-gap spacing policy in terms of safety and traffic flow. The questions we seek to answer in the report are

- 1) Is the traffic flow obtained with the constant time-gap policy stable ?
- 2) What does traffic flow instability imply from a practical point of view ?
- 3) How does stability depend on operating conditions/ boundary conditions ? Can any spacing policy be stable for all operating conditions ?
- 4) If we choose an alternate spacing policy (other than constant time-gap), what traffic flow and safety benefits can we obtain ?

The major results obtained with respect to the constant time-gap policy are

- 1) The results of both Swaroop [1] and Li [2] are found to be mathematically accurate with no contradiction.



- a) Stability of the traffic flow, in the case of the constant time-gap policy, is found to depend on the boundary conditions. The flow is stable for some boundary conditions, unstable for others.
  - b) If a spacing policy could be designed which resulted in a steady-state flow-density curve with a positive slope  $\left(\frac{\partial Q}{\partial \rho} > 0\right)$ , then the traffic flow would be stable for all boundary conditions.
- 2) The practical implications of the mathematical stability results are
- a) In the case of the constant time-gap policy, inflow from a ramp can be accommodated only when there is slack in the highway, i.e. when the mainline flow decreases to a level where the vehicles switch from spacing control to speed control.
  - b) In the absence of a slack in the highway, inflow from an inlet ramp will eventually cause traffic to come to a stop.
  - c) The use of a ramp meter to allow vehicles to enter from a ramp only when there is slack on the mainline would be valuable.

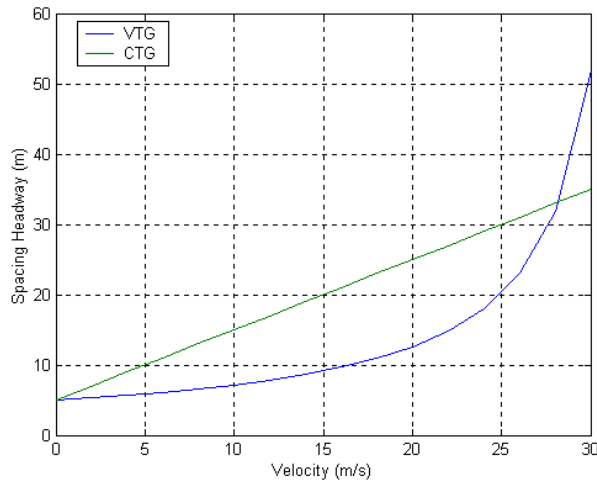
The answer to the question “Should ACC systems be designed to maintain a constant time gap between vehicles?” is NO. It is easy to find alternate spacing policies with better stability properties.

## 2. ALTERNATE SPACING POLICIES

Having shown that the CTG policy does not satisfy the stability condition  $\left(\frac{\partial Q}{\partial \rho} > 0\right)$ , we look at alternate spacing policies. Alternate spacing policies can be developed in which  $\left(\frac{\partial Q}{\partial \rho} > 0\right)$  can be ensured over a range of operating densities. In such alternate spacing policies, the inter-vehicle spacing would be a nonlinear function of the ACC vehicle velocity. One such alternate spacing policy is:

$$S(\dot{x}_i) = \frac{1}{\rho_m (1 - \dot{x}_i / v_f)}$$

where  $\rho_m$  is a density parameter and  $v_f$  is a speed parameter. In **Fig.2**, the inter-vehicle spacing of the CTG and VTG policies are compared as a function of velocity. Values of  $\rho_m = 1/L$ ,  $L = 5$  and  $v_f = 75\text{mph}$  are used. One can see that spacing increases with velocity in the case of the VTG policy, but not proportionally.



**Fig. 2 Desired spacing as a function of ACC vehicle velocity**

The resulting traffic flow from the above spacing policy is shown to be stable for a wide range of operating densities. Significantly higher traffic capacity can also be obtained.

While new spacing policies can be designed with superior stability properties (such as the one above), it has been found that there are some fundamental constraints one will encounter :

- a) No matter what spacing policy is chosen, there will be a certain critical density beyond which the traffic flow will be unstable.
- b) The critical parameters that can be determined by design of the spacing policy are the value of the critical density and the value of the traffic flow that can be achieved at the critical density.
- c) In case of the constant time-gap policy, the critical density turns out to be the same as the density at which spacing control is initiated. Hence the ACC control

system is initiated only after the capacity of the system has already been exceeded.

- d) There are safety Vs traffic flow trade-offs in choosing the values of critical density and maximum traffic flow.

Detailed simulation results are presented in the report comparing the safety and traffic flow performance of the new spacing policy with that of the CTG policy.

### **3. ADDRESSING THE TRADE-OFF BETWEEN SAFETY AND TRAFFIC FLOW**

New spacing policies which are nonlinear functions of the ACC vehicle velocity can improve traffic capacity and ensure traffic flow stability. However, as can be deduced from Fig. 2, they inherently have a trade-off in safety. Is it at all possible to achieve traffic flow improvements without any deterioration in safety ? Results show that if the spacing policy is explicitly made a function of relative velocity, then significant improvements in safety can be obtained without any change in steady state traffic flow characteristics. The nonlinear spacing policy developed in section 2 can be modified to take relative velocity into account. The steady state traffic flow characteristics then remain the same. Safety is analyzed analytically and through a number of simulation scenarios, including

- A vehicle merging at short range into the path of the ACC vehicle.
- The ACC vehicle closing-in on a significantly slower moving vehicle.
- The leading vehicle in a string of ACC vehicle decelerates suddenly to a lower speed or to a complete stop.

#### **4. ANALYSIS AND SIMULATION OF MIXED TRAFFIC (ACC AND MANUAL VEHICLES)**

ACC vehicles will coexist with manually driven vehicles on the existing roadway system long before they become universal. Simulation results of various mixed fleet scenarios are presented in the report. The analysis of the effect of mixing on capacity and stability of traffic are based on these simulation results. It has been found that throughput increases with the proportion of ACC vehicles under below capacity conditions. But above capacity, speed variability increases and speed drops with the CTG system compared to human drivers.

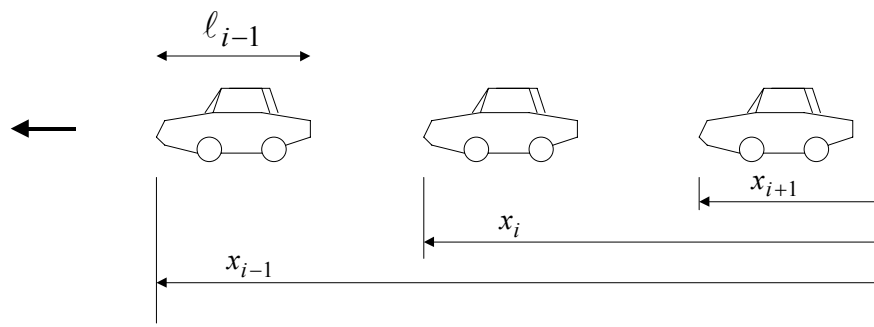
# 1. Development of an Evaluation Framework for ACC Vehicles

## 1. INTRODUCTION TO ADAPTIVE CRUISE CONTROL

Adaptive cruise control (ACC) systems are currently being developed by automotive manufacturers for highway vehicle automation [9],[10]. An ACC system enhances regular cruise control by using an on-board radar to maintain a desired spacing from a preceding vehicle that has been detected in the same lane on the highway.

First-generation ACC systems are being developed primarily from the point of view of increased driving comfort with some additional potential for an increase in safety. The long-term impact of ACC systems on highway traffic has been inadequately studied [4]. Under a futuristic scenario where a large number of highway vehicles operate under ACC, the impact of the ACC control algorithm on highway traffic flow dynamics and highway safety needs to be carefully analyzed.

This report focuses on the design and analysis of the inter-vehicle spacing policies and control laws used by ACC systems. The spacing policy refers to the desired distance  $x_{i-1} - x_i - \ell_{i-1}$  (see Fig. 1) that the ACC system attempts to maintain from the preceding vehicle. The desired distance is typically a function of the ACC vehicle velocity [1] but could also be a constant or a function of other variables such as the relative velocity. A variety of different spacing policies have been developed by researchers [5], [6], [8].



**Fig. 1 String of adaptive cruise control vehicles**

This chapter develops a framework for the design and evaluation of spacing policies for adaptive cruise control.

The following framework is proposed to evaluate the spacing policy and the associated control law

- a) The spacing policy and associated control law should guarantee stability of the individual vehicle.
- b) The spacing policy and associated control law should guarantee string stability in a string of vehicles that possess the ACC system [3].
- c) The spacing policy should yield stable traffic flow [4].
- d) The control effort required by the control law should be within practical vehicle limitations.
- e) Spacing policies that satisfy (a), (b) and (c) should be compared based on the traffic capacity they yield at highway speeds.

The following paragraphs define and describe the terms individual vehicle stability, string stability and traffic flow stability.

The spacing error for the  $i$ th vehicle (the ACC vehicle under consideration) is defined as  $\delta_i = x_i - x_{i-1} + L_{des}$  (please see Fig. 1). Here  $L_{des}$  is the desired spacing and includes the preceding vehicle length  $\ell_{i-1}$ . The desired spacing  $L_{des}$  could be chosen as a function of

variables such as the vehicle speed  $\dot{x}_i$ . The ACC control law is said to provide individual vehicle stability if the following condition is satisfied

$$\ddot{x}_{i-1} \rightarrow 0 \quad \Rightarrow \quad \delta_i \rightarrow 0 \quad (1)$$

In other words, the spacing error of the ACC vehicle should converge to zero if the preceding vehicle is operating at constant velocity. If the preceding vehicle is accelerating or decelerating, then the spacing error is expected to be non-zero.

Since the spacing error is expected to be non-zero during acceleration/ deceleration of the preceding vehicle, it is important to describe how the spacing error would propagate from vehicle to vehicle in a string of ACC vehicles that use the same spacing policy and control law. The string stability of a string of ACC vehicles refers to a property in which spacing errors are guaranteed not to amplify as they propagate towards the tail of the string ([3], [8]). For example, string stability ensures that any errors in spacing between the 2<sup>nd</sup> and 3<sup>rd</sup> cars does not amplify into an extremely large spacing error between cars 7 and 8 further down in the string of vehicles. In this paper, the following condition is used to determine if the system is string stable :

$$\|\hat{H}(s)\|_{\infty} \leq 1 \quad (2a)$$

where  $\hat{H}(s)$  is the transfer function relating the spacing errors of consecutive vehicles

$$\hat{H}(s) = \frac{\delta_i}{\delta_{i-1}}. \quad (2b)$$

In addition to (2a), a condition that the impulse response function  $h(t)$  corresponding to  $\hat{H}(s)$  does not change sign is sometimes considered desirable ([4], [5]). The reader is referred to [4] for details.

In designing the controller to achieve individual vehicle stability and string stability, the following plant model is utilized

$$\ddot{x}_i = u \quad (3)$$

Thus, the acceleration of the car is assumed to be the control input. However, due to the finite bandwidth associated with the engine, engine controller, brake controller, etc., each car is actually expected to track its desired acceleration imperfectly. The performance specification is

therefore re-stated as that of meeting vehicle stability and string stability robustly in the presence of a first-order lag in tracking the desired acceleration:

$$\ddot{x}_i = \frac{1}{\tau s + 1} \ddot{x}_{i-des} = \frac{1}{\tau s + 1} u_i \quad (4)$$

Equation (3) is thus assumed to be the nominal plant model while the performance specifications have to be met even if the actual plant model were given by equation (4).

This report assumes a lag of  $\tau = 0.5$  sec for analysis and simulation. The maximum acceleration and deceleration possible are assumed to be 0.5g and  $-0.5g$  respectively.

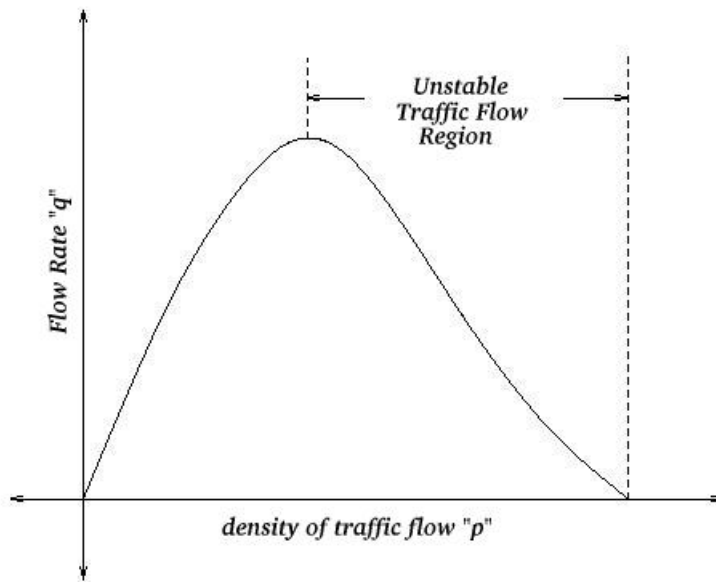
The traffic flow stability of a spacing policy refers to a macroscopic property associated with the traffic flow that would be obtained on a highway if all the vehicles on the highway adopted this particular spacing policy. For purposes of this paper, we will consider a one-lane highway with all the vehicles on the highway being ACC vehicles that follow the same spacing policy. We will then define traffic flow to be stable if the gradient of the traffic flow volume with respect to highway vehicle density is positive i.e.

$$\partial Q / \partial \rho > 0 \quad (5)$$

Here  $Q$  is the traffic flow volume on the highway described in units such as vehicles/hour and  $\rho$  is the traffic density described in units such as vehicles/km. Once the spacing policy has been defined, the steady state relation between  $Q$  and  $\rho$  for the highway can be determined, as will be shown in section 3. The traffic flow stability of the spacing policy can then be evaluated.

The significance of the above traffic flow stability condition can be understood from the  $Q - \rho$  characteristic shown in Fig. 2. This is a typical  $Q - \rho$  characteristic that is obtained on today's highways with manually driven vehicles. The traffic flow first increases with increasing vehicle density i.e. with the entry of more vehicles into the highway. However, after a critical density, the gradient  $\partial Q / \partial \rho$  becomes negative and the traffic flow is said to be unstable in this region. In the unstable region, as more vehicles enter the highway (as density increases), the traffic flow actually decreases.





**Fig. 2 Typical  $Q - \rho$  curve**

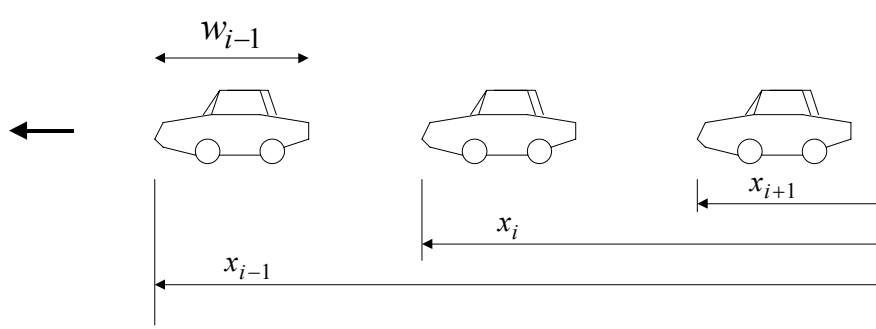
Traffic engineers have known for many years that shock waves occur in the region where  $\partial Q / \partial \rho < 0$  [5]. Swaroop [4] has also presented results that show that when  $\partial Q / \partial \rho < 0$ , density and velocity disturbances that occur in the steady state flow propagate without attenuation upstream from the source. However, there is also some controversy related to this result. The result by Perry, et. al. [12] appears to contradict that of Swaroop [4]. Li's paper considers a circular highway with no inlets or exits for vehicles to enter or leave the highway. It shows that for this circular highway, the consequent traffic flow obtained with the constant time-gap spacing policy is stable. Chapter 3 of this report resolves the above controversy and shows that the condition  $\partial Q / \partial \rho < 0$  guarantees "unconditional" traffic flow stability in which traffic flow is stable independent of the boundary conditions at the inlets and outlets of the highway. Thus the condition  $\partial Q / \partial \rho < 0$  is a very desirable condition to satisfy. A better understanding of traffic flow stability/ instability can be obtained from the description and results in Chapter 3.

## 2. Should Adaptive Cruise Control (ACC) Systems Be Designed to Maintain a Constant Time-Gap Between Vehicles?

This chapter addresses the stability of traffic flow on a highway when the vehicles operate under an adaptive cruise control (ACC) system. ACC systems are commonly designed to maintain a constant time-gap between vehicles during vehicle following. Previous researchers in literature have produced contradictory results on whether the traffic flow is stable when the constant time gap spacing policy is used. This chapter resolves the contradiction and shows that the boundary conditions used at the inlets and exits influence traffic flow stability in the case of the constant time-gap policy. Further, the chapter shows that it is possible to design an unconditionally stable spacing policy, i.e. a spacing policy, which guarantees traffic stability under all boundary conditions. The practical implications of instability are shown through traffic simulation results. The advantages of an unconditionally stable spacing policy over the constant time-gap policy are demonstrated. The answer to the question “Should ACC systems be designed to maintain a constant time gap between vehicles?” is NO. It is quite easy to develop alternate spacing policies with superior stability properties.

### 1. INTRODUCTION

As described earlier, the desired spacing that an ACC vehicle attempts to maintain with respect to the preceding vehicle is called the spacing policy. In **Fig. 1**, the desired spacing is the desired value of  $x_{i-1} - x_i - \ell_{i-1}$ . The desired distance is typically a function of the ACC vehicle velocity [6] but could also be a constant or a function of other variables such as the relative velocity.



**Fig. 1 String of adaptive cruise control vehicles**

The spacing policy is important because it determines:

- 1) Vehicle safety: The inter-vehicle distance determines the time available to brake and the time available for the driver to take manual control of the vehicle.
- 2) Traffic flow on the highway: Smaller inter-vehicle distances (without a corresponding decrease in speed) can lead to higher traffic flow utilization of the highway.
- 3) User-acceptance: A spacing that is too large or too small is likely to make the driver uncomfortable. A large spacing can lead to “cut-ins” from other vehicles making the driver question the value of the ACC system. A small spacing can make the driver “feel” unsafe.

This chapter deals with the first two issues of vehicle safety and highway traffic flow.

The most common spacing policy used in ACC systems by researchers as well as automotive manufacturers is the constant time-gap (CTG) spacing policy. In the CTG spacing policy, the desired spacing of the  $i$ -th vehicle is  $h\dot{x}_i + L$  where  $L$  is a constant that includes the vehicle length  $w_{i-1}$  of the preceding vehicle.

The spacing error under the CTG spacing policy is given by

$$\delta_i = \varepsilon_i + h\dot{x}_i + L \tag{1}$$

where the inter-vehicle spacing is

$$\varepsilon_i = x_i - x_{i-1} \tag{2}$$

A control law that ensures that the spacing error  $\delta_i$  converges to zero is given by [6]

$$\ddot{x}_{ides} = -\frac{1}{h}(\dot{x}_i - \dot{x}_{i-1} + \lambda\delta_i) = -\frac{1}{h}(\dot{\varepsilon}_i + \lambda\delta_i) \quad (3)$$

Is it a good idea for an ACC system to be designed so as to maintain a constant time-gap? There is significant controversy about the implication of the constant time-gap policy for traffic flow. ([3], [6]).

A recent result by Swaroop, et. al. [6] states that the traffic flow obtained on a highway is unstable when all vehicles on the highway use the constant time-gap policy. Traffic flow instability here refers to the unattenuated upstream propagation of disturbances that occurs when a density perturbation is introduced into the traffic flow [6].

In the proof of the above result, Swaroop, et al consider an open stretch of highway where all vehicles use the constant time-gap policy. The open stretch highway has inlets and exits for vehicle to enter and leave the highway. For any given inlet flow conditions, there are corresponding equilibrium conditions on the highway that are achieved at steady state. The stability about these equilibrium conditions is analyzed. Swaroop, et al [6] show mathematically that the entry of additional vehicles from a ramp results in perturbations due to which the traffic flow conditions are disturbed from the equilibrium conditions and never come back to equilibrium.

The result by Li, et. al. [3] appears to contradict that of Swaroop, et al. [6]. Li's paper considers a circular highway with no inlets or exits for vehicles to enter or leave the highway. It shows that the consequent traffic flow obtained with the constant time-gap spacing policy is stable i.e. density perturbations attenuate with time and the traffic flow returns to equilibrium.

The present paper aims to resolve the above mathematical controversy. The paper shows that different boundary conditions can lead to different conclusions on traffic flow stability. This explains the strikingly different conclusions about the CTG policy in the results [3] and [6]. Further, the paper shows that it is possible to design a spacing policy such that it leads to

stable traffic flow for all boundary conditions. Based on this result, a new spacing policy that leads to stable traffic flow is developed.

The paper also discusses the practical implications of instability. It presents simulation results to explain how an unstable policy can cause traffic flow to come to a stop in the absence of a “slack” (or relief) in demand. The new spacing policy developed in the paper allows traffic to continue flowing smoothly even when there is no slack in demand.

## 2. BOUNDARY CONDITIONS INFLUENCE TRAFFIC FLOW STABILITY

Consider the spatially discrete model of a pipeline highway shown in Fig. 2. The highway is partitioned into  $N$  sections of equal length. Let  $\ell$  be the length of each section. The density in the  $i$ -th section is denoted by  $\rho_i$  and equals the number of vehicles in the section divided by the length of the section.

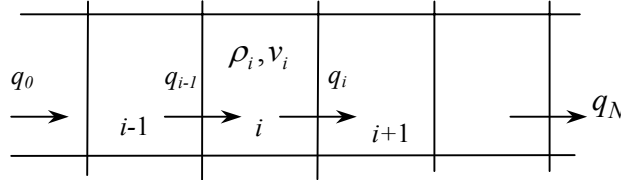


Fig. 2: Traffic flow in a sectioned pipeline highway

The dynamics of each section can be described by the following equation.

$$\dot{\rho}_i = \frac{1}{\ell} [q_{i-1} - q_i] \quad i=1, \dots, N \quad (4)$$

where  $q_i$  is the flow rate out of section  $i$ . Each  $q_i$  in equation (4) is defined to be a convex combination of the two ideal upstream and downstream flow conditions [3,6]:

$$q_i = \alpha_i \rho_i v_i + (1 - \alpha_i) \rho_{i+1} v_{i+1} \quad (5)$$

In equation (5),  $0 \leq \alpha_i \leq 1$ ,  $i=1, \dots, N$  are the mixing coefficients that include the influence of the upstream and downstream flow conditions. Assuming all the vehicles on the highway use the constant time-gap policy

$$v_i = \frac{1}{h} \left[ \frac{1}{\rho_i} - L \right] \quad (6)$$

where  $h$  is headway time and  $L$  includes the length of the preceding vehicle.

From equation (4) and (5) we get

$$\dot{\rho}_i = \frac{1}{\ell} [\rho_i v_i (-\alpha_i - \alpha_{i-1} + 1) + \rho_{i+1} v_{i+1} (-1 + \alpha_i) + \rho_{i-1} v_{i-1} (\alpha_{i-1})] \quad (7)$$

Substituting from (6) into (7), we can obtain dynamic equations for  $\rho_i$ . The stability of the system can then be analyzed. Consider three different boundary conditions as discussed in Cases 1, 2 and 3 below. The 3 different cases lead to different conclusions on stability.

### Case 1: Traffic flow is stable for a circular highway

In the circular highway case of investigation of [3], the effect of boundary conditions was eliminated by assuming a circular highway with no inlets and exits. The traffic dynamics for the circular highway can be written as

$$\dot{\rho} = A\rho \quad (8)$$

where  $\rho = [\rho_1, \dots, \rho_N]^T$  and

$$A = \frac{L}{\ell h} \begin{bmatrix} -(1-\alpha_N - \alpha_1) & (1-\alpha_1) & 0 & \dots & \dots & -\alpha_N \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & -\alpha_{N-1} & (\alpha_i + \alpha_{i-1} - 1) & 1-\alpha_i & 0 & \dots \\ 0 & 0 & \vdots & \vdots & \vdots & 1-\alpha_{N-1} \\ (1-\alpha_N) & 0 & \dots & \dots & \dots & (\alpha_{N-1} + \alpha_N - 1) \end{bmatrix} \quad (9)$$

It turns out that the above system is stable when  $\alpha_i < 0.5$  for  $i=0, \dots, N$  [3], and in steady state we obtain

$$\rho_{iss} = \frac{n}{N\ell} \quad (10)$$

where  $n$  is the total number of vehicles on the whole highway and  $N$  is the number of sections in the highway.

**Case 2: Traffic flow is stable on an open-stretch highway for these boundary conditions**

The dummy  $N + 1$  th section is assumed to be empty, i.e.  $\rho_{N+1} = 0$ . In the investigation in [3], the following boundary conditions were suggested

$$q_0 = (1 - \alpha_0)\rho_1 v_1 + r_0 \quad (11)$$

$$q_N = \alpha_N \rho_N v_N \quad (12)$$

where  $r_0$  is some exogenous input signal representing the traffic demand[3]. Then the traffic flow dynamics for the  $N$  -sectioned highway can be written as:

$$\dot{\rho} = A\rho + Bu \quad (13)$$

where  $u = \frac{1}{\ell h}$ ,

$$A = \frac{L}{\ell h} \begin{bmatrix} (\alpha_0 + \alpha_1 - 1) & (1 - \alpha_1) & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & -\alpha_{i-1} & (\alpha_i + \alpha_{i-1} - 1) & 1 - \alpha_i & 0 & \dots \\ 0 & 0 & \vdots & \vdots & \vdots & 1 - \alpha_{N-1} \\ 0 & 0 & \dots & \dots & -\alpha_{N-1} & (\alpha_{N-1} + \alpha_N - 1) \end{bmatrix} \quad \text{and } B = \begin{bmatrix} -\alpha_0 \\ 0 \\ 0 \\ 0 \\ 1 - \alpha_N \end{bmatrix} \quad (14)$$

When  $\alpha_i < 0.5$  for  $i=0, \dots, N$ , the real parts of all the eigenvalues of matrix  $A$  in equation (14) are negative and the system is stable. The section densities converge to equilibrium values [3].

One should note that the above boundary conditions result in an outflow from the  $N$  th section of  $q_N = \alpha_N \rho_N v_N$  and an inflow into the  $N$  th section of  $q_{N-1} = (1 - \alpha_{N-1})\rho_N v_N + \alpha_{N-1}\rho_{N-1}v_{N-1}$ . The fact that  $q_N < q_{N-1}$  for  $\alpha_{N-1}, \alpha_N < 0.5$  seems to indicate that these boundary conditions are unreasonable. There will be an accumulation of vehicles in the  $N$ -th section of the highway because the outlet flow of that section is always less than its inlet flow.

**Case 3: Traffic flow is unstable on an open stretch highway for these boundary conditions**

In the investigation in [6], the boundary conditions were selected as  $q_0 = \text{constant}$ ,

$q_N = \rho_N v_N$  (i.e.  $\alpha_0 = \alpha_N = 1$ ). Then we obtain  $\dot{\rho} = A\rho + Bu$  where  $u = \frac{1}{\ell h}$

$$A = \frac{L}{\ell h} \begin{bmatrix} \alpha_1 & (1-\alpha_1) & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & -\alpha_{i-1} & (\alpha_i + \alpha_{i-1} - 1) & 1-\alpha_i & 0 & \dots \\ 0 & 0 & \vdots & \vdots & \vdots & 1-\alpha_{N-1} \\ 0 & 0 & \dots & \dots & -\alpha_{N-1} & \alpha_{N-1} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

The system is unstable because some real parts of the eigenvalues of matrix  $A$  in equation (15) are positive [6].

Conclusion

From the results in this section, we see that stability of the traffic flow depends on the boundary conditions. Even for the same spacing policy (CTG), we get different conclusions about stability depending on how the boundary conditions are chosen.

**3. UNCONDITIONALLY STABLE TRAFFIC FLOW**

From the previous section, we concluded that stability results can vary with boundary conditions. However, as we shall now show, if the following condition is satisfied at equilibrium :

$$\frac{\partial Q}{\partial \rho} > 0$$

then the traffic flow is stable for all of the boundary conditions considered. This will be called unconditional traffic flow stability.



**Theorem :**

Consider an open stretch of highway, divided into  $N$  sections of equal length, as in Fig. 2. For simplicity, we will consider no ramp inflows or outflows from any section. The inflow and outflow occur only into the 1st section and from the  $N$ th section respectively. (These results are, however, extendable to the case where there are intermediate ramp inflows and outflows). Let all the vehicles on the highway follow a spacing policy  $v_i = g(\rho_i)$  and let the equilibrium conditions for the traffic flow be  $\rho_i^*, v_i^*, q_i^*$  for the  $i$ th section. The equilibrium point  $(\rho_i^*, q_i^*)$  is locally stable for all the boundary conditions considered in Cases (1), (2) and (3) if

$$\left. \frac{\partial}{\partial \rho_i} (\rho_i g(\rho_i)) \right|_{\rho_i = \rho_i^*} > 0.$$

**Proof :**

For each section

$$\dot{\rho}_i = \frac{q_{i-1} - q_i}{\ell} \quad (16)$$

where  $\ell$  is the length of each section. As before, the flow through section boundary will be assumed to be

$$q_i = \alpha \rho_i v_i + (1 - \alpha) \rho_{i+1} v_{i+1} \quad (17)$$

Let the equilibrium conditions be denoted by  $\rho_i^*, v_i^*, q_i^*$  for the  $i$ th section. For the boundary conditions described in Case 3 of the previous section, we find that the equilibrium conditions will satisfy the following equations

1) For each section  $q_{i-1}^* = q_i^* \Rightarrow$

$$\alpha \rho_{i-1}^* v_{i-1}^* + (1 - 2\alpha) \rho_i^* v_i^* - (1 - \alpha) \rho_{i+1}^* v_{i+1}^* = 0 \quad (18)$$

2) At the inlet

$$\alpha \rho_1^* v_1^* + (1 - \alpha) \rho_2^* v_2^* = q_0 \quad (19)$$

3) At the outlet

$$\alpha \rho_{N-1}^* v_{N-1}^* + (1 - \alpha) \rho_N^* v_N^* - \rho_N^* v_N^* = 0 \quad (20)$$

Let the spacing policy be given by

$$v_i = g(\rho_i) \quad (21)$$

and let

$$d_i = \frac{\partial}{\partial \rho_i}(\rho_i v_i) = \frac{\partial}{\partial \rho_i}(\rho_i g(\rho_i)) \text{ i.e.}$$

$$d_i = g(\rho_i) + \rho_i \frac{\partial}{\partial \rho_i} g(\rho_i) \quad (22)$$

For equilibrium conditions

$$d_i^* = g(\rho_i^*) + \rho_i^* \frac{\partial}{\partial \rho_i} g(\rho_i^*) > 0 \quad (23)$$

Consider a linearization of the system about the equilibrium conditions.

$$\text{Let } \tilde{\rho}_i = \rho_i - \rho_i^*$$

The nonlinearity  $\rho_i g(\rho_i)$  can be linearized using the Taylor series expansion as

$$\rho_i g(\rho_i) \cong \rho_i^* g(\rho_i^*) + \left. \frac{\partial}{\partial \rho_i} [\rho_i g(\rho_i)] \right|_{\rho_i = \rho_i^*} (\rho_i - \rho_i^*)$$

$$\rho_i g(\rho_i) \approx \rho_i^* g(\rho_i^*) + g(\rho_i^*) + \rho_i^* \frac{\partial g(\rho_i^*)}{\partial \rho_i} \tilde{\rho}_i$$

or

$$\rho_i g(\rho_i) \approx \rho_i^* g(\rho_i^*) + d_i^* \tilde{\rho}_i \quad (24)$$

Then

$$\begin{aligned} \dot{\tilde{\rho}}_i = \dot{\rho}_i - 0 &= \frac{\alpha_i \rho_{i-1} v_{i-1} + (1-\alpha) \rho_i v_i - \alpha \rho_i v_i - (1-\alpha) \rho_{i+1} v_{i+1}}{\ell} \\ &= \frac{\alpha_i \rho_{i-1} v_{i-1} + (1-2\alpha) \rho_i v_i - (1-\alpha) \rho_{i+1} v_{i+1}}{\ell} \end{aligned}$$

or

$$\begin{aligned} \dot{\tilde{\rho}}_i &= \frac{\alpha \rho_{i-1}^* h(\rho_{i-1}^*) + \alpha d_{i-1}^* \tilde{\rho}_{i-1} + (1-2\alpha) \rho_i^* h(\rho_i^*) + (1-2\alpha) d_i^* \tilde{\rho}_i - (1-\alpha) \rho_{i+1}^* h(\rho_{i+1}^*) - (1-\alpha) d_{i+1}^* \tilde{\rho}_{i+1}}{\ell} \\ &= \frac{\alpha d_{i-1}^* \tilde{\rho}_{i-1} + (1-2\alpha) d_i^* \tilde{\rho}_i - (1-\alpha) d_{i+1}^* \tilde{\rho}_{i+1}}{\ell} \end{aligned}$$

$$+ \frac{1}{\ell} [\alpha \rho_{i-1}^* h(\rho_{i-1}^*) + (1-2\alpha) \rho_i^* h(\rho_i^*) - (1-\alpha) \rho_{i+1}^* h(\rho_{i+1}^*)]$$

From equation (18), the second set of terms of the above equation equals zero. Hence, we obtain the linearized equations as

$$\dot{\tilde{\rho}}_i = \frac{\alpha d_{i-1}^* \tilde{\rho}_{i-1} + (1-2\alpha) d_i^* \tilde{\rho}_i - (1-\alpha) d_{i+1}^* \tilde{\rho}_{i+1}}{\ell} \quad (25)$$

At the inlet

$$\begin{aligned} \dot{\tilde{\rho}}_1 &= \dot{\rho}_1 - 0 \\ &= \frac{q_0 - q_1}{\ell} \\ &= \frac{q_0}{\ell} - \frac{1}{\ell} [\alpha \rho_1 v_1 + (1-\alpha) \rho_2 v_2] \\ &= \frac{q_0}{\ell} - \frac{1}{\ell} [\alpha \rho_1^* v_1^* + \alpha d_1^* \tilde{\rho}_1 + (1-\alpha) \rho_2^* v_2^* + (1-\alpha) d_2^* \tilde{\rho}_2] \\ &= \frac{1}{\ell} [q_0 - \alpha \rho_1^* v_1^* - (1-\alpha) \rho_2^* v_2^*] - \frac{\alpha}{\ell} d_1^* \tilde{\rho}_1 - \frac{1-\alpha}{\ell} d_2^* \tilde{\rho}_2 \end{aligned}$$

The first set of terms is zero from equation (19). Hence

$$\dot{\tilde{\rho}}_1 = -\frac{\alpha}{\ell} d_1^* \tilde{\rho}_1 - \frac{(1-\alpha)}{\ell} d_2^* \tilde{\rho}_2 \quad (26)$$

At the outlet

$$\begin{aligned} \dot{\tilde{\rho}}_N &= \dot{\rho}_N - 0 \\ &= \frac{q_{N-1} - q_N}{\ell} \\ &= \frac{\alpha \rho_{N-1} v_{N-1} + (1-\alpha) \rho_N v_N - \alpha \rho_N v_N - (1-\alpha) \rho_{N+1} v_{N+1}}{\ell} \quad (\rho_{N+1} = 0) \end{aligned}$$

$$\begin{aligned}
&= \frac{\alpha \rho_{N-1} v_{N-1} + (1-2\alpha) \rho_N v_N}{\ell} \\
&= \frac{\alpha \rho_{N-1}^* v_{N-1}^* + \alpha d_{N-1}^* \tilde{\rho}_{N-1} + (1-2\alpha) \rho_N^* v_N^* + (1-2\alpha) d_N^* \tilde{\rho}_N}{\ell} \\
&= \frac{\alpha \rho_{N-1}^* v_{N-1}^* + (1-2\alpha) \rho_N^* v_N^*}{\ell} + \frac{1}{\ell} [\alpha d_{N-1}^* \tilde{\rho}_{N-1} + (1-2\alpha) d_N^* \tilde{\rho}_N]
\end{aligned}$$

The first set of terms are again zero at the equilibrium point. Hence we find that the equations about the equilibrium point are given by

$$\dot{\tilde{\rho}}_1 = \frac{1}{\ell} [-\alpha d_1^* \tilde{\rho}_1 - (1-\alpha) d_2^* \tilde{\rho}_2] \quad (27a)$$

$$\dot{\tilde{\rho}}_i = \frac{1}{\ell} [\alpha d_{i-1}^* \tilde{\rho}_{i-1} + (1-2\alpha) d_i^* \tilde{\rho}_i - (1-\alpha) d_{i+1}^* \tilde{\rho}_{i+1}] \quad (27b)$$

$$\dot{\tilde{\rho}}_N = \frac{1}{\ell} [\alpha d_{N-1}^* \tilde{\rho}_{N-1} + (-\alpha) d_N^* \tilde{\rho}_N] \quad (27c)$$

In matrix form, it turns out that the stability of the following matrix determines stability of the overall system

$$\begin{bmatrix}
(-\alpha) d_1^* & (\alpha-1) d_2^* & 0 & \cdots & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \alpha d_{i-1}^* & (1-2\alpha) d_i^* & (\alpha-1) d_{i+1}^* & 0 & \cdots \\
0 & 0 & \vdots & \vdots & \vdots & \cdots \\
0 & 0 & \cdots & \cdots & \alpha d_{N-1}^* & (-\alpha) d_N^*
\end{bmatrix}$$

This matrix can be written as

$$\begin{bmatrix}
-\alpha & \alpha-1 & 0 & \cdots & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \alpha & 1-2\alpha & \alpha-1 & 0 & \cdots \\
0 & 0 & \vdots & \vdots & \vdots & \cdots \\
0 & 0 & \cdots & \cdots & \alpha & -\alpha
\end{bmatrix}
\begin{bmatrix}
d_1^* & & & & & \\
& d_2^* & & & & \\
& & \ddots & & & \\
& & & d_{N-1}^* & & \\
& & & & d_N^* &
\end{bmatrix} \quad (28)$$

and the stability of the first matrix in the product then determines stability of the overall system.

Similarly, equilibrium conditions for the boundary conditions in Case 2 can be shown to satisfy

$$(1-2\alpha)\rho_1^*v_1^* - (1-\alpha)\rho_2^*v_2^* = 0 \quad (29)$$

$$\alpha\rho_{N-1}^*v_{N-1}^* + (1-2\alpha)\rho_N^*v_N^* = 0 \quad (30)$$

and

$$\alpha\rho_{i-1}^*v_{i-1}^* + (1-2\alpha)\rho_i^*v_i^* - (1-\alpha)\rho_{i+1}^*v_{i+1}^* = 0 \quad (31)$$

Equations about the above equilibrium points for Case 2 turn out to be

$$\dot{\tilde{\rho}}_1 = \frac{1}{\ell}[(1-2\alpha)d_1^*\tilde{\rho}_1 - (1-\alpha)d_2^*\tilde{\rho}_2]$$

$$\dot{\tilde{\rho}}_i = \frac{1}{\ell}[\alpha d_{i-1}^*\tilde{\rho}_{i-1} + (1-2\alpha)d_i^*\tilde{\rho}_i - (1-\alpha)d_{i+1}^*\tilde{\rho}_{i+1}]$$

$$\dot{\tilde{\rho}}_N = \frac{1}{\ell}[\alpha d_{N-1}^*\tilde{\rho}_{N-1} + (1-2\alpha)d_N^*\tilde{\rho}_N]$$

In matrix form, the stability of the following matrix

$$\begin{aligned} & \begin{bmatrix} (1-2\alpha)d_1^* & (\alpha-1)d_2^* & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \alpha d_{i-1}^* & (1-2\alpha)d_i^* & (\alpha-1)d_{i+1}^* & 0 & \cdots \\ 0 & 0 & \vdots & \vdots & \vdots & \cdots \\ 0 & 0 & \cdots & \cdots & \alpha d_{N-1}^* & (1-2\alpha)d_N^* \end{bmatrix} \\ & = \begin{bmatrix} 1-2\alpha & \alpha-1 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \alpha & 1-2\alpha & \alpha-1 & 0 & \cdots \\ 0 & 0 & \vdots & \vdots & \vdots & \cdots \\ 0 & 0 & \cdots & \cdots & \alpha & 1-2\alpha \end{bmatrix} \begin{bmatrix} d_1^* \\ d_2^* \\ \ddots \\ d_{N-1}^* \\ d_N^* \end{bmatrix} \end{aligned} \quad (32)$$

determines stability of the overall system. Again the stability of the first matrix in the product determines overall stability.

In the case of the circular highway (Case 1), the equations about the equilibrium point can be represented by the matrix equation  $\dot{\tilde{\rho}} = A\tilde{\rho}$  where the matrix

$$\begin{aligned}
 A &= \begin{bmatrix} (1-2\alpha)d_1^* & (\alpha-1)d_2^* & 0 & \cdots & \cdots & \alpha d_N^* \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \alpha d_{i-1}^* & (1-2\alpha)d_i^* & (\alpha-1)d_{i+1}^* & 0 & \cdots \\ 0 & 0 & \vdots & \vdots & \vdots & \cdots \\ (\alpha-1)d_1^* & 0 & \cdots & \cdots & \alpha d_{N-1}^* & (1-2\alpha)d_N^* \end{bmatrix} \\
 &= \begin{bmatrix} 1-2\alpha & \alpha-1 & 0 & \cdots & \cdots & \alpha \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \alpha & 1-2\alpha & \alpha-1 & 0 & \cdots \\ 0 & 0 & \vdots & \vdots & \vdots & \cdots \\ \alpha-1 & 0 & \cdots & \cdots & \alpha & 1-2\alpha \end{bmatrix} \begin{bmatrix} d_1^* \\ d_2^* \\ \vdots \\ d_{N-1}^* \\ d_N^* \end{bmatrix} \\
 &\quad (33)
 \end{aligned}$$

It can be shown that the matrices in equations (28) and (32) are asymptotically stable as long as  $\alpha > 0.5$ . Similarly, for  $\alpha > 0.5$ , the matrix in equation (33) has one eigen value at the origin with all other eigen values being in the open left-half plane. The eigenvector corresponding to the eigen value at the origin is  $[1 \ 1 \ \cdots \ 1]^T$ . Hence, in this case the system converges to equal densities in all sections.

Values of  $\alpha$  greater than 0.5 make good physical sense and are reasonable to use when traffic flow is not congested. Previous researchers who developed and experimentally validated traffic simulation models have shown that  $\alpha$  ranges from 0.5 to 1 ([4], [5], [9] and [10]) when traffic flow is un-congested (i.e.  $\frac{\partial Q}{\partial \rho} > 0$ )

## End of proof

### Stability of the CTG policy

For the CTG spacing policy, the steady-state spacing is given by  $\delta = L + hv$ , where  $v$  is aggregate highway velocity. This leads to a steady-state density of

$$\rho = \frac{1}{L + hv}$$

Solving for  $v$  in terms of  $\rho$ , one can get

$$v = \frac{1}{\rho h}(1 - \rho L)$$

and the traffic flow is

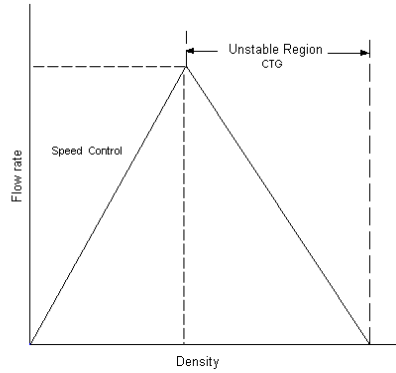
$$Q = \frac{1}{h}(1 - \rho L) \tag{34}$$

The gradient  $\frac{\partial Q}{\partial \rho}$  is thus always negative which means the CTG policy is not unconditionally stable.

## 4. THE PRACTICAL IMPLICATIONS OF TRAFFIC FLOW INSTABILITY

The theoretical results of the previous section showed that when all the vehicles on a highway use the CTG policy, a density perturbation would cause the traffic flow to move away from equilibrium under realistic boundary conditions. In this section, we verify the same result through microscopic simulations. The simulations will also illustrate the impact of this instability from a practical point of view.

First, note that in a practical system, spacing control is initiated by an ACC system only in the presence of a preceding vehicle. In the absence of a preceding vehicle, the ACC vehicle is expected to maintain a constant speed equal to the speed limit or a user-set value. In terms of steady state traffic flow, the switching between speed and spacing control can be simplified into the following representation. A critical density can be obtained above which spacing control is initiated.



**Fig. 3 CTG  $Q - \rho$  curve**

Below the critical density, vehicles will be assumed to operate under speed control. If the speed limit is denoted by  $v_m$ , then for the CTG policy, the critical density equals  $\frac{1}{L + hv_m}$ .

For operation below critical density, speed control results in a flow rate of  $Q = \rho v_m$ . This results in a positive slope for the  $Q - \rho$  as shown in Fig. 3. The CTG spacing control is initiated after the critical density and in this region the  $Q - \rho$  curve has a negative slope. This means the CTG policy always works in the unstable region of traffic flow !

In the following simulations the traffic behavior of the CTG ACC system operating in the unstable regime (at a density beyond the critical density) is illustrated. A single lane freeway with an inlet ramp is considered, as shown in Fig. 4. All the vehicles in the simulation operate under the CTG ACC algorithm. The ramp will be used to introduce density perturbations into the mainline flow. A time gap  $h$  of 1 sec and a control gain  $\lambda = 0.4$  are selected for this simulation. A lag of  $\tau = 0.1$  sec for vehicles to track the desired acceleration is assumed. The vehicles' maximum acceleration and deceleration limits are assumed to be  $0.3g$  and  $-0.5g$  respectively. The length of each vehicle ( $L$ ) is uniformly set to be  $5m$  in this simulation. A vehicle merging into the automated traffic from the on-ramp is placed halfway between the vehicles closest to the ramp and has an initial speed equal to the average velocity of the closest vehicles. This simulation is set up such that the lead vehicle in the freeway always attempts to maintain the speed limit ( $65$  mph).



Vehicles entering from the main lane enter at the speed limit of 65 *mph*. We assume the traffic has reached steady state before entering the simulation pipeline. This means the inflow rate is calculated from the ACC spacing law at equilibrium i.e.

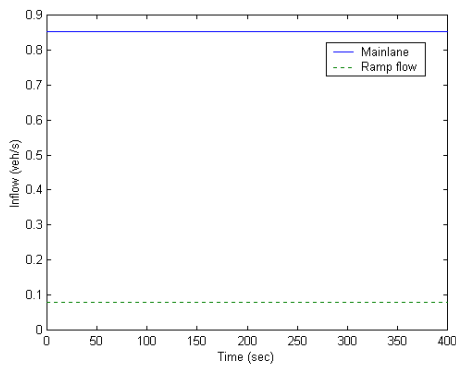
$$\rho^* = \frac{1}{L + hv_i}$$

or

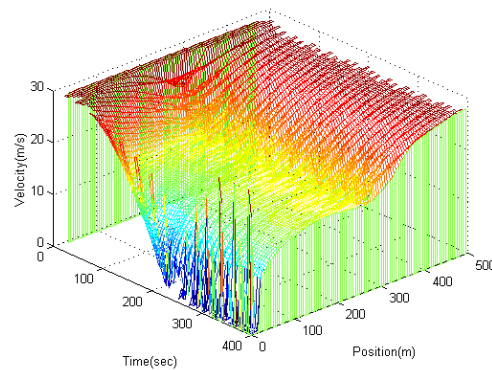
$$Q_{\text{inflow}} = \rho^* v_i = v_i / (L + hv_i) \quad (35)$$

where  $Q_{\text{inflow}}$  is the inflow rate at the main inlet,  $\rho^*$  is steady state or equilibrium density,  $v_i$  is initial velocity equal to the speed limit,  $h$  is time gap and  $L$  is vehicle length.

Several scenarios with different inflow rates on the on-ramp were run in the simulations. The simulation results shown in Fig. 5 indicate that a small vehicle inflow rate from the ramp (equal to 0.08 vehicles / sec) results in all the vehicles on the highway eventually coming to a stop upstream. The 3-D plot shows the vehicles' speed values in the space (whole pipeline) – time (whole simulation period) domain.



**Fig. 5a Traffic inflow from main lane and ramp (without inflow relief)**



**Fig. 5b Simulation result (without relief)**

The minimum amount of ramp inflow that triggers traffic to come to a stop depends on the downstream length. As the downstream length is increased, the minimum ramp inflow

required to trigger a stop decreases. In the limit, as the downstream length is made infinite, even an infinitesimally small ramp inflow is enough to trigger the traffic to come to a stop. This is in agreement with the theory – the traffic flow conditions are not unconditionally stable under these conditions and there exist boundary conditions that will cause traffic to come to a stop.

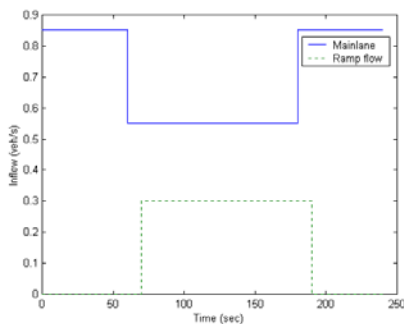
The existence of a finite upstream length provides relief, since vehicles can exit at the speed limit. Simulations showed that the traffic flow is stable under the following conditions

- a) The downstream length is finite, providing a relief boundary condition
- b) The ramp inflow is below a threshold, the value of the threshold being determined by the downstream length.

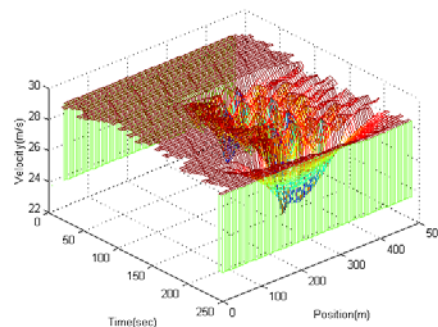
Thus the simulation results verify the theoretical result of instability under certain boundary conditions when the unconditional stability condition is not satisfied.

To avoid traffic coming to a stop in the main lane, the inflow from the ramp can be metered so that inflow is only allowed when there is a slack in the highway, i.e. when the mainline flow decreases to a level where the vehicles switch from spacing control to speed control. This is shown in Fig. 6a. In Fig. 6b, vehicles do not come to a stop in spite of an inflow from the ramp that exceeds the critical threshold.

Hence, the introduction of a ramp meter to control the vehicles enter from a ramp is an extremely good idea if vehicles were to operate under the CTG ACC policy.



**Fig. 6a Traffic inflow from main lane and ramp (with inflow relief)**



**Fig.6b Simulation result (with relief)**

## 5. CAN WE FIND A SPACING POLICY BETTER THAN THE CONSTANT TIME-GAP POLICY?

Is it possible to find a spacing policy that leads to unconditionally stable traffic flow? We propose the following ACC spacing policy (adapted from an idea suggested in [6]). We shall term the new spacing policy a variable time-gap (VTG) policy. The VTG policy is a nonlinear function of vehicle velocity.

The desired spacing under the proposed new policy is given by  $\frac{1}{\rho_m(1-\dot{x}_i/v_f)}$  and the

spacing error is given by

$$\delta_i = \varepsilon_i + \frac{1}{\rho_m(1-\dot{x}_i/v_f)} \quad (36)$$

where  $\rho_m$  is a density parameter and  $v_f$  is a speed parameter. In Fig.7, the inter-vehicle spacing of the CTG and VTG policies are compared as a function of velocity. Values of  $\rho_m = 1/L$ ,  $L = 5$  and  $v_f = 65\text{mph}$  were used. One can see that spacing increases with velocity in the case of the VTG policy, but not proportionally. At low speeds, the desired spacing of the VTG policy is less than that of the CTG's. And at high speeds, the spacing of the VTG policy is larger than that of CTG's.

Setting  $\dot{\delta}_i = -\lambda\delta_i$  by differentiating equation (36), the desired acceleration with the VTG policy can be obtained as

$$\ddot{x}_{ides} = -\rho_m(v_f - \dot{x}_i)(1 - \dot{x}_i/v_f)(\dot{\varepsilon}_i + \lambda\delta_i) \quad (37)$$

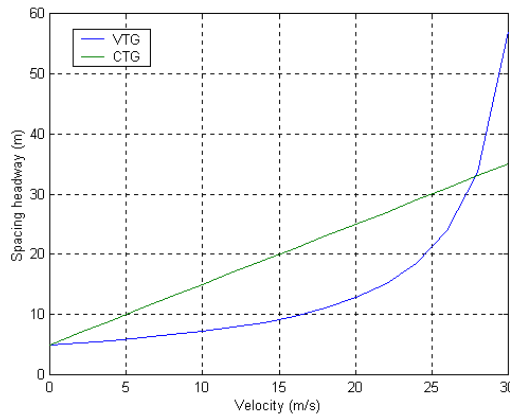
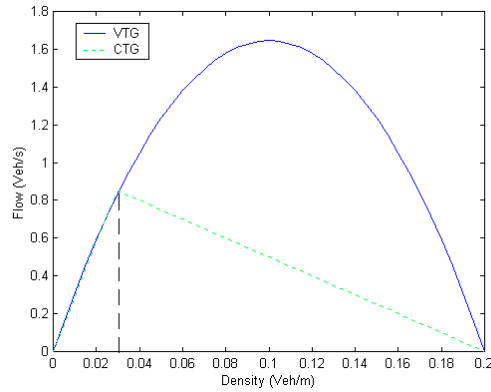


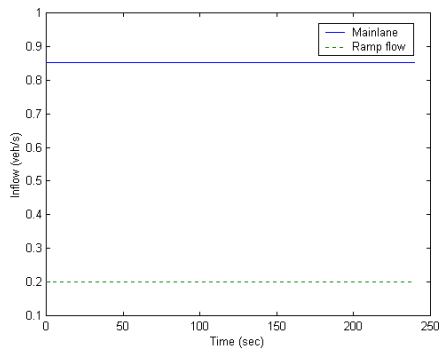
Fig. 7 Inter-vehicle spacing as a function of velocity for the CTG and VTG policies

For the VTG policy, the density at steady state is given by  $\rho = \rho_m(1 - \frac{v}{v_f})$ . One can then see that the aggregate velocity in terms of density would be given by  $v = v_f(1 - \frac{\rho}{\rho_m})$  and the traffic flow would be given by  $Q = v_f\rho(1 - \frac{\rho}{\rho_m})$ . Fig. 8 illustrates the traffic flow and density characteristic curves of both the CTG and VTG systems. We see that the VTG ACC system is stable ( $\frac{\partial Q}{\partial \rho} > 0$ ) when the spacing control is initiated at a density of 0.03 vehicles per meter. It only becomes unstable at a density beyond  $\rho_m/2$  ( $= 0.1$ , a significantly higher density). The CTG policy on the other hand has  $\frac{\partial Q}{\partial \rho} < 0$  right from the initiation density of 0.03 vehicles per meter.

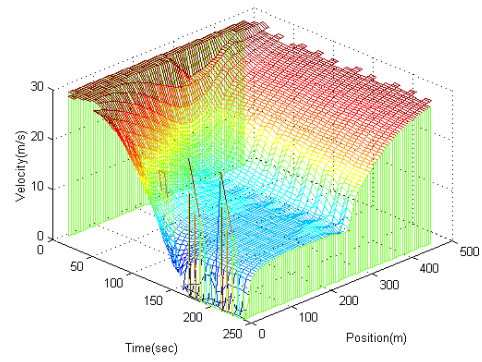


**Fig. 8 Q-  $\rho$  curve of CTG and VTG**

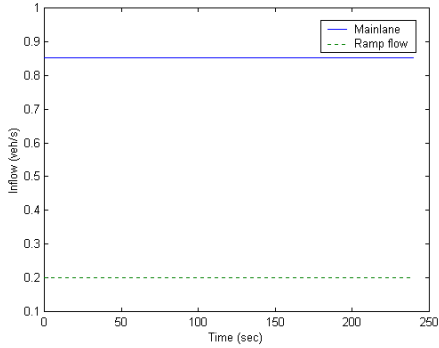
From the comparison of the CTG and VTG simulation results in Fig. 9 and Fig. 10 respectively, it is easy to see that the VTG gives better traffic behavior than the CTG does. Not only do no vehicles stop in the main lane in the case of the VTG system, but also the main lane traffic remains at a level equal to (mainline + ramp) inflow even as the ramp flow persistently remains at 0.2 *vehicle/sec* for a long time.



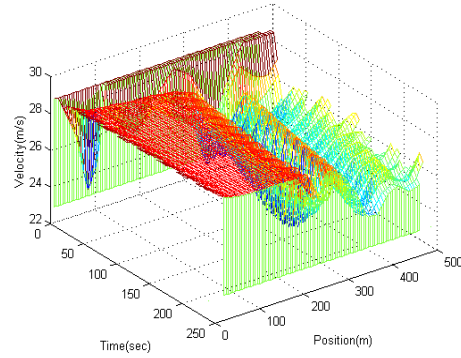
**Fig. 9a Traffic inflow from main lane and ramp (CTG)**



**Fig. 9b Simulation result (CTG)**



**Fig. 10a Traffic inflow from main lane and ramp (VTG)**



**Fig. 10b Simulation result (VTG)**

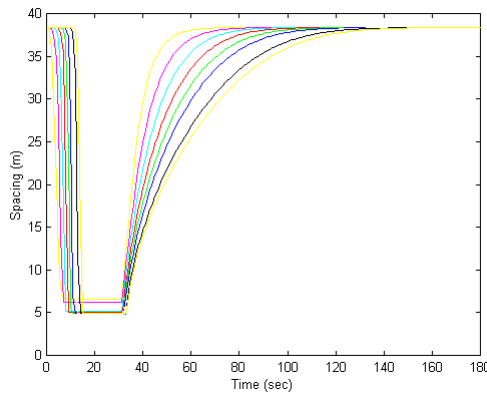
In Fig. 10a and Fig. 10b, it can also be seen that the density disturbance introduced by the ramp at the middle of the pipeline gets attenuated as it propagates upstream.

The total travel (TT) and total travel time (TTT) of the two ACC systems are listed in Table 1. It is apparent that the VTG ACC system provides far better traffic mobility than the CTG system.

**Table 1 Mobility comparison of two ACC system**

	Constant Time Gap policy (1.0s)	Variant Time Gap policy
Total Travel (km*vehicle)	104.59	121.72
Total Travel Time (h*vehicle)	2.266	1.287
System Speed (km/h)	46.17	94.54

An additional important consideration in an ACC system is safety. It is important to ensure that the traffic flow improvement is not obtained at the expense of safety. While safety is not rigorously analyzed here, the following simulation verifies that the VTG system is able to handle hard-braking by the lead car. A string of 9 cars operating under the new control algorithm was simulated. In the simulation, all the cars are initially moving at steady state at the speed limit (65mph). The lead car performs a hard brake maneuver with a deceleration of (-0.5g) to come to a stop. Then, after all the cars stop, the leading car slowly accelerates back to the speed limit. Fig.11 shows that there is no car collision during the hard deceleration and acceleration of the lead car.



**Fig. 11 Inter-vehicle spacing with the VTG system in the safety simulation study**

## 6. STRING STABILITY

In ACC systems, it is important to describe how the spacing error would propagate from vehicle to vehicle if a string of ACC vehicles used the same spacing policy and control law. The string stability of a string of ACC vehicles refers to a property in which spacing errors are guaranteed not to amplify as they propagate towards the tail of the string ([4], [9]). For example, string stability ensures that any errors in spacing between the 2<sup>nd</sup> and 3<sup>rd</sup> cars does not amplify into an extremely large spacing error between cars 7 and 8 further down in the string of vehicles. In this paper, the following condition is used to determine if the system is string stable :

$$\|\hat{H}(s)\|_{\infty} \leq 1 \quad (38a)$$

where  $\hat{H}(s)$  is the transfer function relating the spacing errors of consecutive vehicles

$$\hat{H}(s) = \frac{\delta_i}{\delta_{i-1}}. \quad (38b)$$

In addition to (38a), a condition that the impulse response function  $h(t)$  corresponding to  $\hat{H}(s)$  does not change sign is sometimes considered desirable ([9]). The reader is referred to [9] for details.

In designing the controller to achieve individual vehicle stability and string stability, the following plant model is utilized

$$\ddot{x}_i = u \quad (39)$$

Thus, the acceleration of the car is assumed to be the control input. However, due to the finite bandwidth associated with the engine, engine controller, brake controller, etc., each car is actually expected to track its desired acceleration imperfectly. The performance specification is therefore re-stated as that of meeting string stability robustly in the presence of a first-order lag in tracking the desired acceleration:

$$\ddot{x}_i = \frac{1}{\tau s + 1} \ddot{x}_{i-des} = \frac{1}{\tau s + 1} u_i \quad (40)$$

Equation (39) is thus assumed to be the nominal plant model while the performance specifications have to be met even if the actual plant model were given by equation (40).

This paper assumes a lag of  $\tau = 0.1$  sec for analysis and simulation. The maximum acceleration and deceleration possible are assumed to be  $0.5g$  and  $-0.5g$  respectively. It has also been shown [9] that the above controller guarantees string stability for  $h > 2\tau$ , where  $\tau$  is the lumped lag associated with the actuators, engine and drive-line and described in equation (40). Hence, this spacing policy (and controller) satisfies the string stability criterion as long as a sufficiently large headway time  $h$  is maintained.

The linearized representation of the VTG spacing policy error is given by:

$$\delta_i = \varepsilon_i + S(\dot{x}_{i0}) + \frac{\partial S}{\partial \dot{x}_i}(\dot{x}_i - \dot{x}_{i0}) \quad (41)$$

where  $\dot{x}_{i0}$ =equilibrium velocity. The variable  $\frac{\partial S}{\partial \dot{x}_i}$  is thus equivalent to the time-gap  $h$  in

the CTG spacing policy. Hence from a linearization point of view if  $\frac{\partial S}{\partial \dot{x}_i} > 2\tau$ , then the

spacing policy will have string stability [9]. For VTG spacing policy,

$$\frac{\partial S}{\partial \dot{x}_i} = \frac{v_f}{\rho_m (v_f - \dot{x}_i)^2} > 2\tau \text{ is required to guarantee string stability.}$$

In our case  $\rho_m = 0.2$ ,  $v_f = 75\text{mph}$  and  $\tau = 0.1\text{s}$ , so string stability is guaranteed for all speeds above 4.47m/s or 10.0mph.

## 7. CONCLUSIONS

The following points summarize the conclusions obtained from the analysis in the paper.

- a) Should ACC systems be designed to maintain a constant time-gap between vehicles ?  
Based on the results presented in this paper, the answer is clearly NO. While the CTG system can be stable under certain boundary conditions, it is not unconditionally stable. It requires relief at either the upstream inlet or at a downstream outlet in order to be able to accommodate inflow from a ramp.
- b) It is possible to develop alternate spacing policies which are unconditionally stable for a range of operating densities. Such policies do not require relief at either the upstream or downstream ends in order to accommodate ramp inflow.
- c) No matter what spacing policy is chosen, however, there will be a certain critical density beyond which the traffic flow will be unstable. This conclusion is derived from the assumption that the traffic flow has to fall to zero at maximum density (jam density). For



this reason, there will be some part of the  $(\rho, Q(\rho))$  curve where  $\frac{\partial Q}{\partial \rho} < 0$ . Hence, it is not possible to achieve traffic flow stability for all  $\rho$ .

- d) The critical parameters that can be determined by design of the spacing policy are the value of the critical density and the value of the traffic flow that can be achieved at the critical density. There are safety Vs traffic flow trade-offs in choosing the values of critical density and maximum traffic flow. However, a new spacing policy can be designed in which higher traffic capacity can be obtained while still maintaining safety by use of relative velocity in the spacing policy.

### 3. Overall Evaluation of the Constant Time-Gap Spacing Policy

As discussed previously, the most common spacing policy used in ACC systems by researchers as well as automotive manufacturers is the constant time-gap spacing policy. The constant time-gap spacing policy is given by

$$\delta_i = \varepsilon_i + h\dot{x}_i + L \quad (1)$$

where the inter-vehicle spacing is

$$\varepsilon_i = x_i - x_{i-1} \quad (2)$$

The corresponding longitudinal controller associated with the above spacing policy is described by

$$u_{i\_des} = -\frac{1}{h} \left( \dot{\varepsilon}_i + \lambda \delta_i \right) \quad (3)$$

The present chapter evaluates the constant time gap policy from the point of view of the evaluation framework proposed in Chapter 2.

It can be easily shown [1] that the above control provides individual vehicle stability i.e. if  $\ddot{x}_{i-1} \rightarrow 0$ , then  $\delta_i \rightarrow 0$ . It has also been shown [2] that the above controller guarantees string stability for  $h > 2\tau$ , where  $\tau$  is the lumped lag associated with the actuators, engine and drive-line and described in equation (4) of Chapter 2. Hence, this spacing policy (and controller) satisfies the required criterion (b) as long as a sufficiently large headway time  $h$  is maintained.

The traffic flow stability of the constant time-gap controller can be analyzed by evaluating the sign of the corresponding  $\partial Q / \partial \rho$ . Since the steady-state spacing is given by  $\delta = L + hv$ , where  $v = \dot{x}_i$  is aggregate highway velocity, this leads to a steady state density of

$$\rho = \frac{1}{L + hv} \quad (4)$$

By inverting equation (4), one can then see that the aggregate velocity in terms of density would be given by

$$v = \frac{1}{\rho h}(1 - \rho L) \quad (5)$$

and the traffic flow would be given by

$$q = \frac{1}{h}(1 - \rho L) \quad (6)$$

The variable  $\partial Q / \partial \rho$  is thus always negative with the adoption of the constant time-gap spacing policy and hence the traffic flow is always unstable for all traffic densities.

The constant time-gap spacing policy thus violates criterion (c) of the evaluation framework, although it satisfies criteria (a) and (b) in the same evaluation.

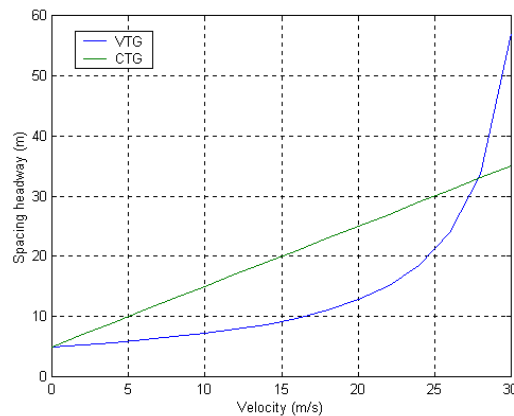
## 4. Addressing the Trade-Off Between Safety and Traffic Flow

This chapter addresses the trade-off between safety and traffic flow and deals with the design of new adaptive cruise control (ACC) systems that can improve traffic flow while at the same time ensuring safe operation on today's highways.

The new spacing policy, referred to as a variable time-gap (VTG) policy developed in chapter 3, is shown analytically to lead to better traffic flow and a higher highway capacity. Practical advantages of using the new spacing policy have been demonstrated through traffic simulations. However, a detailed analysis of safety shows that the traditional CTG policy is superior in several scenarios. The VTG policy is therefore modified by explicitly taking inter-vehicle relative velocity into account in the definition of desired spacing. The resulting new spacing policy is shown to retain the advantages of stable traffic flow and a higher capacity while providing the same level of safety as the CTG policy.

### 1. HIGHER CAPACITY AT THE COST OF SAFETY?

The traffic flow advantages of the new VTG spacing policy from Chapter 3 have been clearly demonstrated. The VTG policy provides a higher traffic capacity and the ability to accommodate ramp inflows at high operating densities. However, the inter-vehicle spacing as a function of speed is smaller than that of the CTG policy for a wide range of speeds, as seen from **Fig. 1**.



**Fig. 1** Inter-vehicle spacing as a function of velocity for the CTG and VTG policies

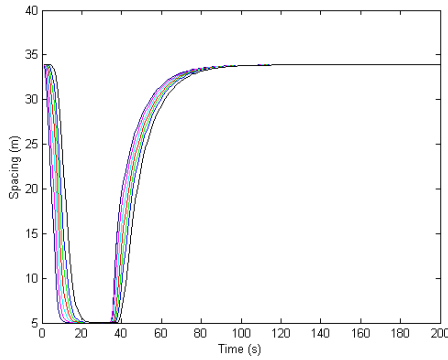
This leads to the question of the safety of operation of ACC systems using the VTG policy. In this section, the safety properties of the CTG and VTG policies are compared based on the following operational scenarios:

- The preceding vehicle decelerates suddenly to a lower speed or brakes to a stop.
- The ACC vehicle encounters a slower moving preceding vehicle.
- A vehicle cuts into the path of the ACC vehicle at short range.

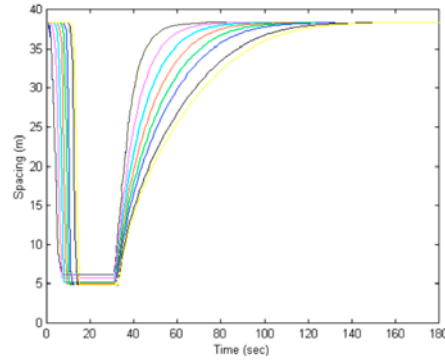
The above set of tests does not cover all the driving scenarios an ACC system is likely to encounter. This section is not intended to be an exhaustive study of safety but a qualitative study of the relative performances of the CTG and VTG systems. In our simulations we will consider not just one ACC vehicle but a string of ACC vehicles with the lead ACC vehicle encountering the scenarios described above.

### **1. 1 The Lead Vehicle in a String of ACC Vehicles Suddenly Brakes to a Stop**

A common safety-critical scenario encountered on the highway is one where the lead vehicle in a string of vehicles decelerates suddenly to a significantly lower speed or to a complete stop. The ACC system must guarantee that there will not be an upstream collision in such a scenario. Consider the case where there is a string of eight ACC vehicles all using the same spacing policy. Initially, all the vehicles are in steady state i.e. the initial speed of every vehicle is equal to the speed limit (65 *mph*) and the inter-vehicle spacing is equal to the desired steady state spacing. During the first 30 seconds of the simulation, the lead vehicle in the string performs a hard brake maneuver with a deceleration of 0.5 *g* to come to a complete stop. After all the vehicles in the platoon stop, the leading car accelerates back up again to the speed limit. **Figs. 2** and **Fig. 3** show how the vehicle headways (inter-vehicle spacing) change with time for the CTG and VTG systems respectively. Note the transient behavior during the braking of the lead vehicle and then during the subsequent gentle acceleration. The lengths of the vehicles are assumed uniformly to be 5 *m*.



**Fig. 2 Inter-vehicle spacing with CTG in scenario 1**



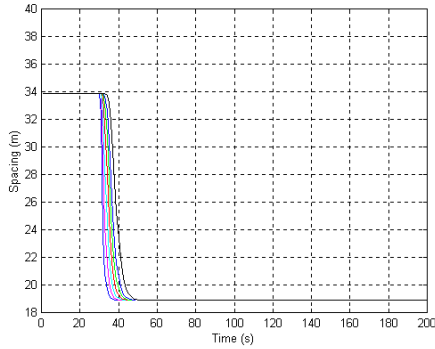
**Fig. 3 Inter-vehicle spacing with VTG in scenario 1**

Under the CTG policy, the vehicles come to a stop with the inter-vehicle spacing equal to 5 meters. The transient behavior of the VTG system is poorer than that of the CTG system. However, the inter-vehicle spacing when the vehicles come to a stop is only marginally less than that of the CTG system.

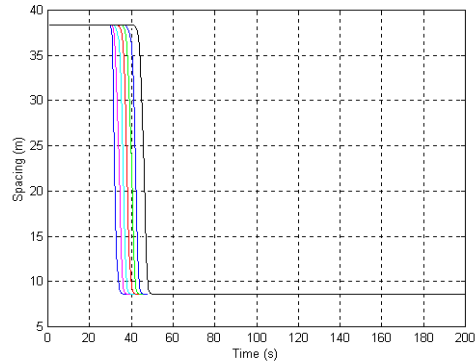
For this particular simulation scenario, the CTG system is only marginally better than the VTG system. This is in part expected because the vehicles in the string all start with the same speed and the same safe inter-vehicle spacing. The desired inter-vehicle spacing at a complete stop is equal for both the CTG and VTG systems, though it is different at the initial speed. Since both systems are able to track the desired spacing, they achieve the same final inter-vehicle spacing. The transient performance of the VTG system, however, is poorer.

## 1.2 The String of ACC Vehicles Encounters a Slower Moving Vehicle

This test is aimed at checking how the ACC vehicles respond when a significantly slower vehicle is encountered in the same lane, a likely scenario on the highway. The same string of 8 vehicles is used in the simulations. All the ACC vehicles are initially in steady state with desired inter-vehicle spacing. At a time  $t=30$  s, the leading ACC vehicle detects a preceding vehicle that is 15 m/s slower. **Figs. 4** and **Fig. 5** show the time histories of the inter-vehicle spacing.



**Fig. 4 Spacing of CTG in safety simulation**

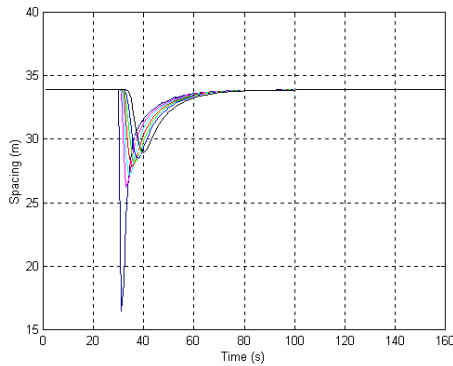


**Fig. 5 Spacing of VTG in safety simulation**

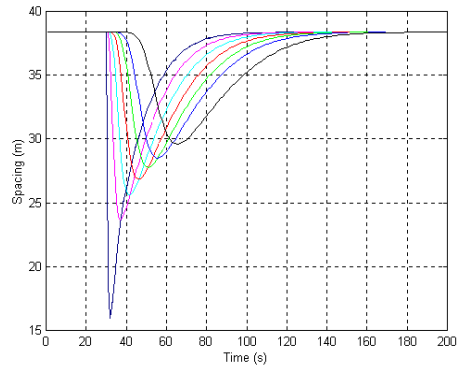
One can see that both the CTG and VTG systems make the ACC vehicles smoothly and safely reach a new steady state in which they have the same velocity as the slower moving preceding vehicle. The transient performance of the VTG system is slower (poorer) compared to that of the CTG system. This is because the desired inter-vehicle spacing changes more rapidly at the operating speed of 65 mph in the case of the VTG system compared to that of the CTG system.

### 1.3 A Vehicle Cuts into the Path of the ACC Vehicle at Short Range

An important scenario in which safety should be analyzed is one where a slower moving vehicle cuts into the path of an ACC vehicle at short range. This scenario occurs, for example, very commonly in the merging area of an inlet-ramp. A string of 8 ACC vehicles is considered in the simulations. At the beginning, the vehicles run at steady state, all the 8 vehicle at the speed limit (65 mph). At a time  $t=30$  s, a vehicle with a speed slower by 5m/s suddenly merges into the path of the second ACC vehicle of the string at a distance of 15 meters. The lengths of the vehicles are assumed uniformly to be 5m. Figs. 6 and Fig. 7 show the time histories of the inter-vehicle spacing for the CTG and VTG spacing policies respectively. Only the spacing plots of the vehicles behind the merging vehicle are shown.



**Fig. 6 Merging simulation of CTG**



**Fig. 7 Merging simulation of VTG**

Comparing **Fig. 6** and **Fig. 7**, one can see that the CTG system in general ensures larger spacing between vehicles during the transient period. For instance, vehicle 2 in the string has a minimum distance of 27 meters in the case of the CTG policy and 24 meters in the case of the VTG policy. Further, the transient performance is again poorer for the VTG system with a significantly longer time being taken to reach steady state.

## 2. MODIFIED VARIABLE TIME-GAP (MVTG) POLICY

The VTG system developed in chapter 3 achieves a significantly higher traffic capacity than the CTG system and ensures traffic flow stability over a wide range of operating densities. However, this improvement in traffic flow does come at the cost of decrease in safety, as described through the simulations in section 1. The smaller desired inter-vehicle spacing of the VTG system over most operating speeds, as shown in Fig. 1, is of course responsible.

From a safety point of view, a larger inter-vehicle spacing is much more important during transients where there is a speed difference between successive vehicles than in the steady state where all the vehicles have the same speed. The larger the spacing that the following vehicle can keep from the preceding one during transients, the more the room the following vehicles have to deal with cut-ins as well as other unexpected events. To maintain the same steady state traffic flow characteristics as the VTG system and yet sacrifice no safety, the VTG spacing policy is modified to take into account the difference between velocities of the



current vehicle and its preceding one. The proposed desired spacing of the modified VTG policy is as follows:

$$S = \frac{1}{\rho_m (1 - \dot{x}_i / v_f)} + r \dot{\varepsilon}_i \quad (1)$$

with  $\rho_m$  and  $v_f$  being the same variables as those in Chapter 3,  $\dot{\varepsilon}_i = \dot{x}_i - \dot{x}_{i-1}$  being the relative velocity between  $i$ th vehicle and  $i-1$ th vehicle and  $r$  a (positive) coefficient which determines the weight of the relative velocity variable in the desired spacing. Under the modified VTG (MVTG) policy, the desired inter-vehicle spacing is the same as that of the VTG policy when there is no speed difference with respect to the target vehicle (as shown in **Fig. 1**). The desired spacing, however, will be larger when the ACC vehicle's speed is higher than that of the preceding vehicle. In the case where the ACC vehicle's speed is lower than that of the preceding vehicle (no safety needs to be considered), the desired spacing becomes smaller than that of the VTG system. The steady state traffic flow characteristics of the MVTG system are the same as that of the VTG system.

The spacing error of the modified VTG ACC spacing policy is given by:

$$\delta_i = \varepsilon_i + \frac{1}{\rho_m (1 - \dot{x}_i / v_f)} + r \dot{\varepsilon}_i \quad (2)$$

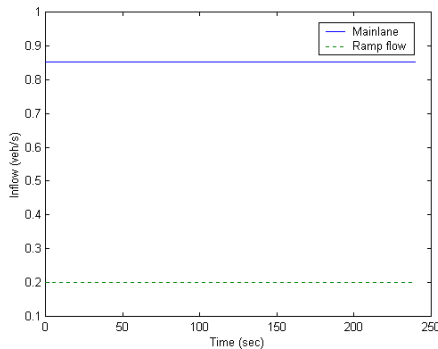
Setting  $\dot{\delta}_i = -\lambda \delta_i$ , differentiating equation (2), the desired acceleration is given by:

$$\ddot{x}_{ides} = -\rho_m (v_f - \dot{x}_i) (1 - \dot{x}_i / v_f) (\dot{\varepsilon}_i + r \ddot{\varepsilon}_i + \lambda \delta_i) \quad (3)$$

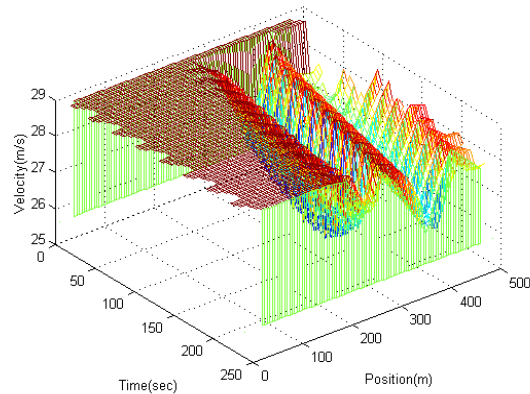
This is the control law for the MVTG system and will ensure that the desired MVTG spacing is accurately tracked by the ACC vehicle. To calculate the desired acceleration  $\ddot{x}_{ides}$ , an additional measurement (or estimate), relative acceleration  $\ddot{\varepsilon}_i$ , is needed.

## Implications of Traffic Flow Stability

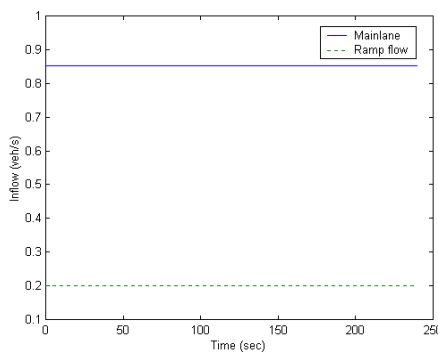
The same microscopic simulation setup as earlier is used to illustrate the traffic flow behavior of the MVTG system. Simulation results with relative velocity weight coefficients  $r$  chosen as 1.0 and 5.0 are shown in **Fig. 8a** and **Fig. 8b** respectively. It can be seen that although the density disturbances at the middle of the pipeline propagate a longer distance upstream compared to the VTG system, the disturbances are still strictly attenuated as they propagate upstream. Thus the MVTG system yields stable traffic flow. Ramp inflows can be accommodated over a wide range of operating densities during spacing control.



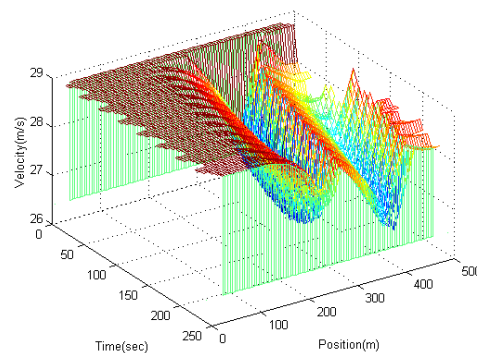
**Fig. 8a** Traffic inflow from main lane and ramp (relative)



**Fig. 8b** Simulation result ( $r = 1.0$ )



**Fig. 9a** Traffic inflow from main lane and ramp (relative)



**Fig. 9b** Simulation result ( $r = 5.0$ )

The total travel (TT) and total travel time (TTT) of each of the three ACC systems using the CTG, VTG and MVTG algorithms are listed in **Table 1**. It is readily seen that the VTG and

the MVTG systems have significantly better traffic mobility and productivity values than the CTG system.

**Table 1 Mobility and Productivity Comparison of Three ACC System**

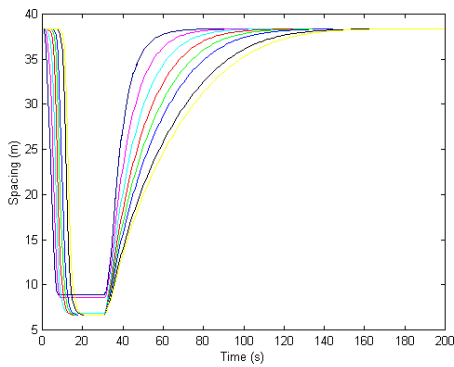
	Constant Time Gap Policy (1.0s)	Variable Time Gap Policy	Modified Variable Time Gap Policy ( $r=1.0 / 5.0$ )
Total Travel (km*vehicle)	104.59	121.72	121.51 / 120.43
Total Travel Time (h*vehicle)	2.266	1.287	1.396 / 1.388
System Speed (km/h)	46.17	94.54	87.04 / 86.77

### 3. SAFETY RECOVERED USING THE MVTG POLICY

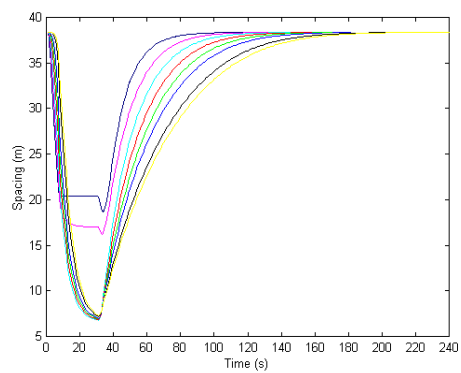
The safety performance of the control system using the MVTG policy is described for the same three simulation scenarios in this section.

#### 3.1 The Lead Vehicle Suddenly Brakes to a Stop

Again a string of 8 ACC vehicles are used in this simulation. The vehicles are initially in steady state, at a speed of 65 *mph* and at desired inter-vehicle spacing. During the first 30 *seconds* of the simulation, the leading vehicle performs a hard brake maneuver (-0.5 *g*) to come to a complete stop. After all the vehicles in the platoon stop, the leading car accelerates back again to the 65 $mph$  speed. **Fig. 10 – Fig. 11** show how the vehicle space headways change with time for the MVTG policy with  $r=1$  and 5 respectively.



**Fig. 10 MVTG Spacing in simulation ( $r=1.0$ )**

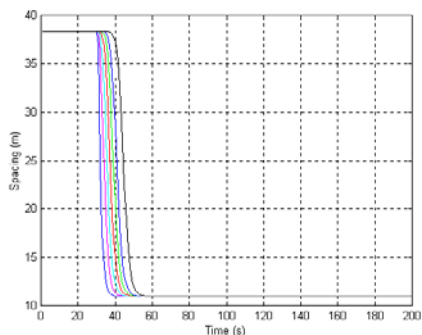


**Fig. 11 MVTG Spacing in simulation ( $r=5.0$ )**

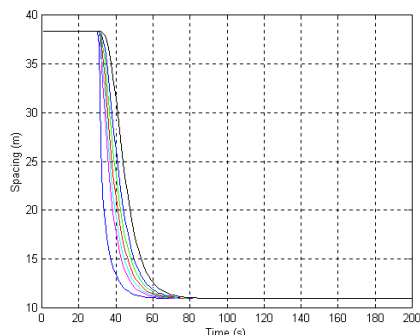
One can see that safety is improved by having introduced relative velocity into the VTG policy in the sense that the vehicles now maintain more inter-vehicle spacing as they stop compared with both the CTG and VTG policies. The bigger the value of  $r$ , the larger the spacing that is maintained between the vehicles as they stop, hence the safer system is in this scenario. The spacing at a complete stop is also larger. The transient performance appears to be poorer. This is only because the desired spacing is smaller for most intermediate operating speeds and then suddenly increases sharply as we approach the speed limit (see Fig. 1).

### 3.2 The String of ACC Vehicles Encounters a Slower Moving Vehicle

In this simulation scenario, the same string of 8 ACC cars is used. Initially all the ACC vehicles are at steady state. At a time  $t=30\text{ s}$ , the leading ACC vehicle detects a preceding vehicle that is  $15\text{ m/s}$  slower than the ACC platoon speed in the same lane on the highway. Following **Fig. 12 – Fig. 13** show how the vehicle space headways change with time for the modified VTG spacing policies with  $r=1$  and  $5$  respectively.



**Fig. 12 Spacing of relative simulation ( $r=1.0$ )**

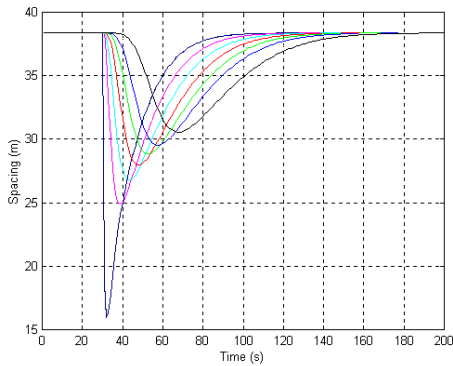


**Fig. 13 Spacing of relative in safety ( $r=5.0$ )**

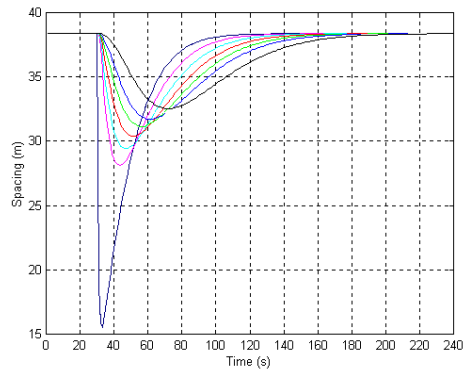
One can see that the MVTG policy has a performance similar to the other two systems. All the three spacing policies ensure that the ACC platoon reaches steady state with the same velocity as the preceding vehicle smoothly.

### 3.3 A Vehicle Cuts into the Path of the ACC Vehicle at Short Range

Again a string of 8 ACC vehicles is considered. At the beginning, the vehicles are at steady state, all the 8 vehicles with the same speed as the speed limit (65 mph). At  $t=30$  s, a vehicle with a speed 5m/s less than the lead vehicle speed suddenly merges into the path in front of the (now) second ACC vehicle of the platoon at a distance of 15 meters. The lengths of the vehicles are assumed uniformly to be 5m. The following figures **Fig. 14** – **Fig. 15** show how the vehicle space headways (spacing) change with time in the transient period after the cut-in for the modified VTG policy with  $r=1$  and 5 respectively. Only the space headways of the vehicles behind the merging vehicle are shown. (Every line in the plot indicates the spacing of a vehicle)



**Fig. 14** Merging simulation of relative ( $r=1.0$ )



**Fig. 15** Merging simulation of relative ( $r=5.0$ )

In **Fig. 14**, we see that the relative velocity weight coefficient  $r = 1$  allows the new policy to keep larger space headways compared to the VTG system. Furthermore, in **Fig. 15**, with the relative velocity weight coefficient  $r$  increased to 5, vehicles using this spacing policy can maintain much larger space headways than that of the VTG system and even larger than that of the CTG system. The transient performance seems poorer but is actually due to the fact

that the desired spacing is smaller for most of the operating speeds and increases sharply only towards the speed limit region.

#### **4. CONCLUSIONS**

This chapter analyzed the design of new adaptive cruise control (ACC) systems from the point of view of improving traffic flow while at the same time ensuring safe operation on today's highways. A new inter-vehicle spacing policy called a variable time-gap (VTG) policy that is a nonlinear function of vehicle speed was developed in Chapter 3. The new spacing policy led to stable traffic flow and a higher capacity. Practical advantages of using the new spacing policy were demonstrated through traffic simulations. However, an analysis of safety showed that the traditional CTG policy was superior in several scenarios. The VTG policy was then modified by taking inter-vehicle relative velocity into account. The resulting new spacing policy was shown to provide stable traffic flow, a higher capacity and the same level of safety as the CTG policy.

## 5. Development of an Ideal Spacing Policy

This chapter proposes an “ideal” spacing policy that evolves naturally from the evaluation framework proposed in Chapter 2. The proposed spacing policy is a nonlinear function of speed. It provides string stability and traffic flow stability as well as a higher traffic flow capacity compared to the standard time-gap controller. An associated result proved in the chapter is that traffic flow stability implies string stability for flow volumes up to a described maximum value.

### 1. STRUCTURE OF THE PROPOSED “IDEAL” SPACING POLICY

As a starting point, the structure of the proposed “ideal” spacing policy is assumed to be

$$\delta_i = \varepsilon_i + g(\dot{x}_i) \quad (1)$$

where  $g(\dot{x}_i)$  is a nonlinear function of vehicle speed that will be defined later so as to satisfy criteria (a), (b) and (c) of the evaluation framework. The associated controller is obtained using feedback linearization. Differentiating equation (1)

$$\dot{\delta}_i = \dot{\varepsilon}_i + \frac{\partial g}{\partial \dot{x}_i} \ddot{x}_i \quad (2)$$

Setting  $\dot{\delta}_i = -\lambda\delta_i$  the desired acceleration is given by

$$\ddot{x}_{i\_des} = -\frac{1}{\partial g / \partial \dot{x}_i} \left( \dot{\varepsilon}_i + \lambda\delta_i \right) \quad (3)$$

This control law automatically ensures individual vehicle stability, so that criterion (a) is satisfied.

### 2. STRING STABILITY

The proposed controller has the same structure as the constant time-gap controller but with the headway time constant replaced by a velocity dependent time headway. The corresponding linearized spacing policy is given by

$$\delta_i = \varepsilon_i + g(x_{i0}) + \frac{\partial g}{\partial x_i}(x_i - x_{i0}) \quad (4)$$

where  $x_{i0}$  = equilibrium velocity. From a linearization point of view if

$$\frac{\partial g}{\partial x_i} > 2\tau \quad (5)$$

for all points on the  $(x_i, g(x_i))$  curve, then the spacing policy will guarantee string stability. We assume that satisfying equation (5) is adequate to satisfy criterion (b) of the evaluation framework. Simulation results also indicate that satisfaction of equation (16) is adequate to ensure string stability.

Since traffic flow capacity and traffic flow stability are better studied on the  $(\rho, Q(\rho))$  curve, the criteria (5) on  $g(v)$  curve needs to be transformed into a condition on the  $Q(\rho)$  curve.

Let  $Q(\rho)$  be the steady state traffic flow obtained when the steady state density is  $\rho$ . Let  $v$  be the corresponding steady state velocity. Then

$$Q = \rho v \Rightarrow \frac{\partial Q}{\partial \rho} = v + \rho \frac{\partial v}{\partial \rho} \quad (6)$$

The steady state density is given by

$$\rho = \frac{1}{g(v)} \quad (7)$$

The derivative  $\frac{\partial g}{\partial x_i}$  can then be evaluated in terms of  $\rho$  and  $Q$  as follows :

$$\frac{\partial v}{\partial \rho} = -\frac{g(v)^2}{\partial g / \partial v} \quad (8)$$

$$\Rightarrow \frac{\partial Q}{\partial \rho} = v - \rho \frac{g(v)^2}{\partial g / \partial v} = v - \frac{1}{\rho \partial g / \partial v} \quad (9)$$

$$\frac{\partial g}{\partial v} = \frac{1}{\rho(v - \partial Q / \partial \rho)} \quad (10)$$

Imposing the condition (5) on  $\partial g / \partial v$ , we obtain



$$\frac{1}{\rho(v - \partial Q / \partial \rho)} \geq 2\tau \quad (11)$$

or

$$\frac{\partial Q}{\partial \rho} \geq v - \frac{1}{2\tau\rho} \quad (12)$$

$$\Rightarrow \frac{\partial Q}{\partial \rho} \geq \frac{Q}{\rho} - \frac{1}{2\tau\rho} \quad (13)$$

It can be readily seen that the constant time headway policy satisfies this inequality as an equality. If 'C' is the maximum traffic flow achieved, then (13) leads to

$$\rho \frac{\partial Q}{\partial \rho} \geq C - \frac{1}{2\tau} \quad (14)$$

If it is assumed that the traffic flow capacity drops to zero (velocity becomes zero) at maximum density then this condition implies that at very high densities the  $Q(\rho)$  curve for the given spacing policy must lie below the  $Q(\rho)$  curve of the constant time-gap policy.

### 3. TRAFFIC FLOW STABILITY

Traffic flow stability ensures that density disturbances do not propagate downstream without attenuation. Using conservation of mass principle and considering traffic velocity dynamics, Swaroop [4] has shown that small perturbations to the steady state density and velocity can magnify downstream without attenuation in spite of the spacing policy achieving string stability.

The conservation of mass equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (15)$$

and the traffic velocity dynamics is modeled by

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{1}{\tau} (h(\rho) - v) - \mu(\rho) \frac{\partial \rho}{\partial x} \quad (16)$$

where

$v = h(\rho)$  given by the spacing policy

$\mu > 0$  is a function of  $\rho$  alone.

Using density and velocity perturbations of the form

$$\rho_p = \tilde{\rho} e^{ikx+wt} \quad \text{and} \quad v_p = \tilde{v} e^{ikx+wt} \quad (17)$$

Swaroop [4 ] has shown that, for density perturbations about the steady state  $(\rho_o, v_o)$

$$w = -\frac{\partial Q(\rho_o) / \partial \rho}{2v_o \tau} \quad (18)$$

and  $w < 0$  for traffic flow stability.

The traffic flow is stable as long as  $\partial Q / \partial \rho > 0$ . As described earlier, in the case of the constant time-gap spacing policy,  $\partial Q / \partial \rho < 0$  for all  $\rho$  and hence the traffic flow is always unstable.

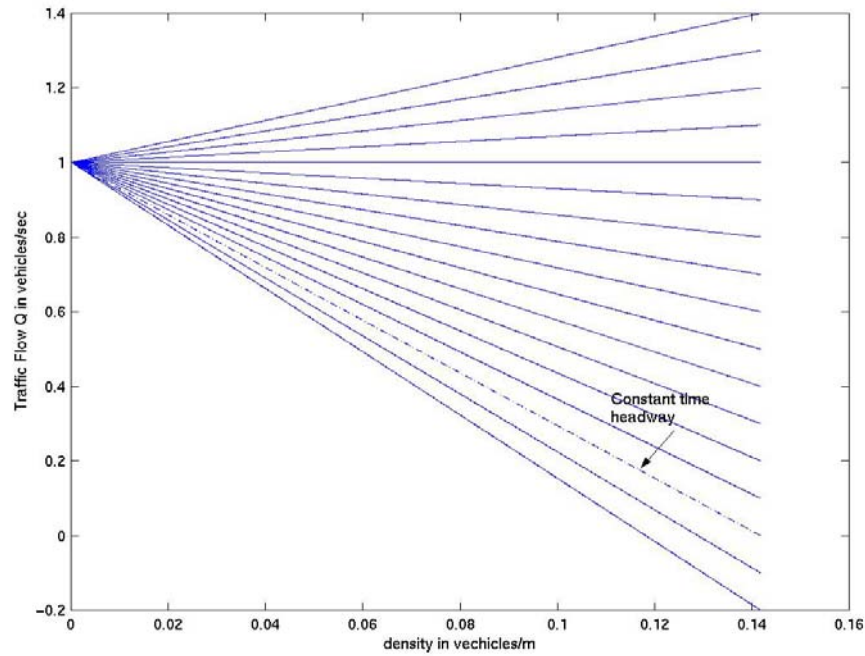
Since it is assumed that the traffic flow has to fall to zero at maximum density, there will be some part of the  $(\rho, Q(\rho))$  curve when  $\partial Q / \partial \rho < 0$ . Hence it is not possible to achieve traffic flow stability for all  $\rho$ . However, traffic flow stability can be achieved for all densities that are below a value close to the maximum density.

If the maximum traffic flow achieved,  $C_{is} \leq 1/2\tau$  (the maximum traffic flow achieved by constant time headway policy), then it can be seen that traffic flow stability ( $\frac{\partial Q}{\partial \rho} > 0$ ) implies string stability i.e., the corresponding inequality is satisfied.

#### 4. DESIGN OF THE IDEAL SPACING POLICY

Assuming  $\tau = 0.5$  sec in (13), we get

$$\rho \frac{\partial Q}{\partial \rho} - Q \geq -1 \quad (19)$$



**Fig. 1 Set of Q curves satisfying equation (19)**

Fig. 1 shows the set of  $(\rho, Q(\rho))$  curves that would satisfy (30) as an equality. The  $(\rho, Q(\rho))$  curve corresponding to the constant time headway policy is part of the set. A maximum density of 0.1429 Vehicles/m (corresponding to a minimum spacing of 7m) was assumed.

It can be seen that any spacing policy with the controller given by (3) and achieving greater traffic flow than the constant time headway policy will have non-zero traffic flow at maximum density. So in order to achieve more traffic flow and have zero traffic flow at maximum density (jam density), the spacing policy will have to be string unstable at some densities.

The following properties were assumed while designing the nonlinear spacing policy

- Assuming a speed limit of 65 MPH and 20-65 MPH to be the range of vehicle speeds observed on today's highways, the spacing policy should guarantee higher traffic flow than constant time headway policy.
- The spacing policy should guarantee string stability in the range of speeds 20-65 MPH.
- The spacing policy should guarantee traffic flow stability in this range.

A  $(\rho, Q(\rho))$  curve is designed to satisfy the constraints (18) and (19) in the speed range 20-65 MPH. A lookup table of  $v$  Vs  $g(v)$  can be formed from

$$\begin{aligned} v &= Q / \rho \\ g(v) &= 1 / \rho \end{aligned} \quad (20)$$

The function  $Q(\rho)$  is given by

$$Q(\rho) = Q_1(\rho) + Q_2(\rho) + Q_3(\rho) \quad (21)$$

where

$\rho$  = traffic density

$$\begin{aligned} Q_1(v) &= a_{11} + a_{12}\rho + a_{13}\rho^2 + a_{14}\rho^3 \text{ for } v > 65\text{MPH} \\ &= 0 \text{ elsewhere} \end{aligned}$$

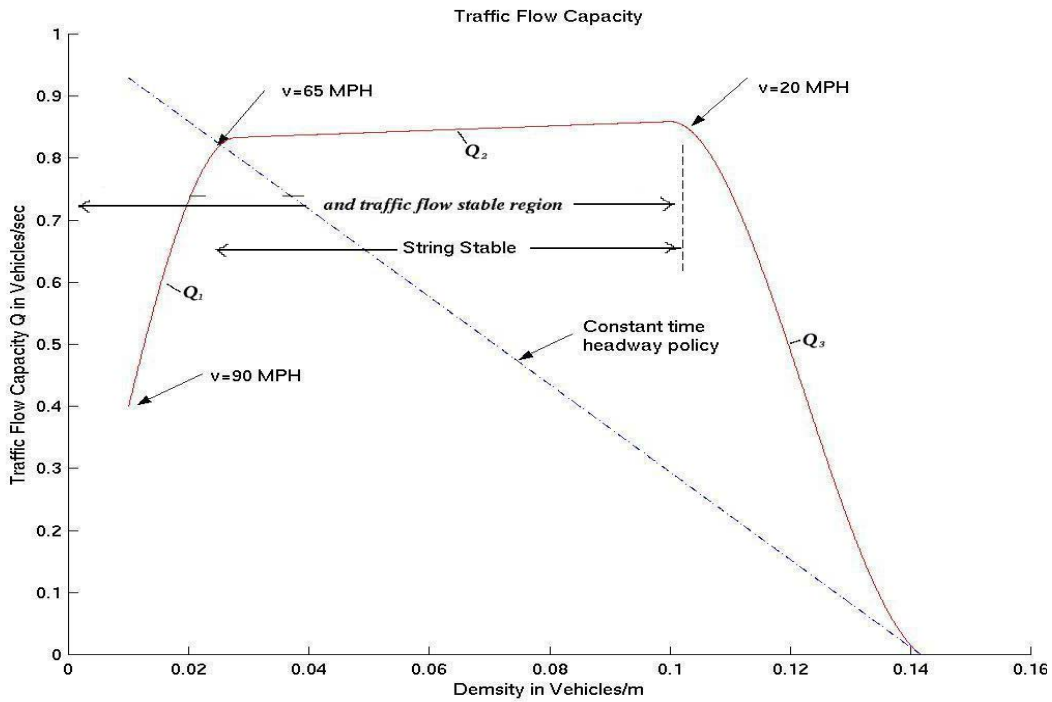
$$\begin{aligned} Q_2(v) &= a_{21} + a_{22}\rho + a_{23}\rho^2 + a_{24}\rho^3 \text{ for } 65\text{MPH} \geq v \geq 20\text{MPH} \\ &= 0 \text{ elsewhere} \end{aligned}$$

$$Q_3(v) = a_{31} + a_{32}\rho + a_{33}\rho^2 + a_{34}\rho^3 \text{ for } v < 20\text{MPH}$$

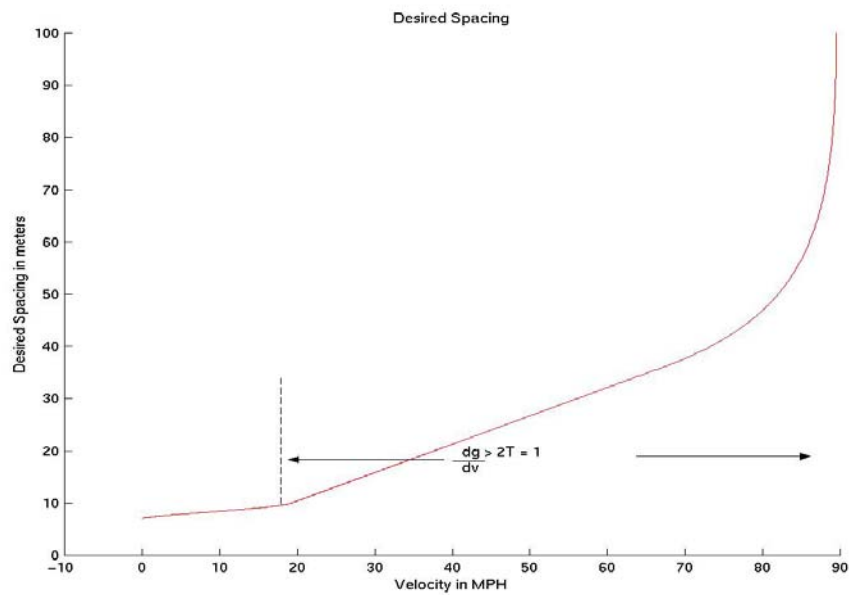
The above structure was adopted to reduce the complexity of  $Q(\rho)$  in satisfying all the stated requirements. Further more to ensure smooth transitions in the spacing function, the following properties are satisfied by proper choice of  $a_{ij}$ .

$$\begin{aligned} \frac{\partial Q_1}{\partial \rho} &= \frac{\partial Q_2}{\partial \rho} \text{ at } v = 65\text{MPH} \\ \frac{\partial Q_2}{\partial \rho} &= \frac{\partial Q_3}{\partial \rho} \text{ at } v = 20\text{MPH} \end{aligned} \quad (22)$$

Fig. 2 shows the designed  $(\rho, Q(\rho))$  curve and the corresponding desired spacing as a function of speed is shown in Fig. 3.



**Fig. 2 Designed Traffic Flow Capacity Curve**



**Fig. 3 Designed Desired Spacing Curve**

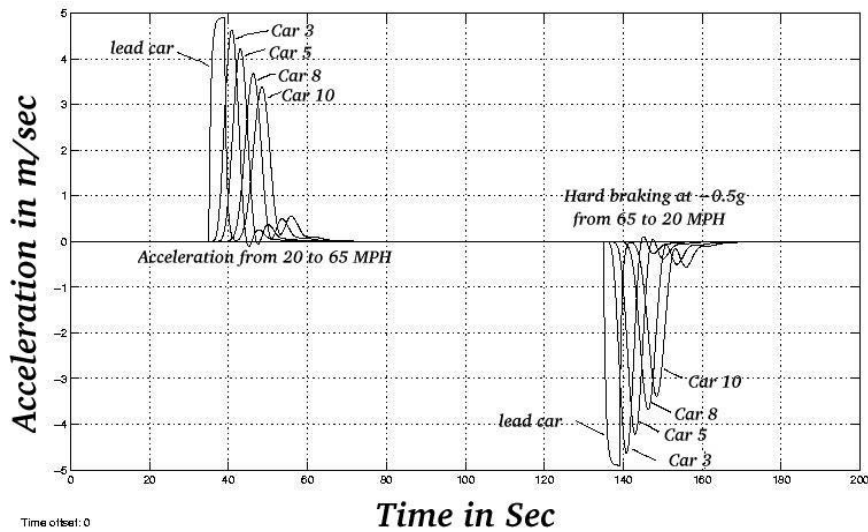
The designed curve guarantees string stability and traffic flow stability in the speed range 20-65 MPH while providing higher traffic flow capacity than the standard constant time headway

policy. Beyond 80 MPH (Fig. 5)  $\partial g / \partial \dot{x}_i \rightarrow \infty$  (changes rapidly) and so the control effort  $\rightarrow 0$  for any spacing error. Thus the string stability results will not hold in that region.

## 5. SIMULATION RESULTS

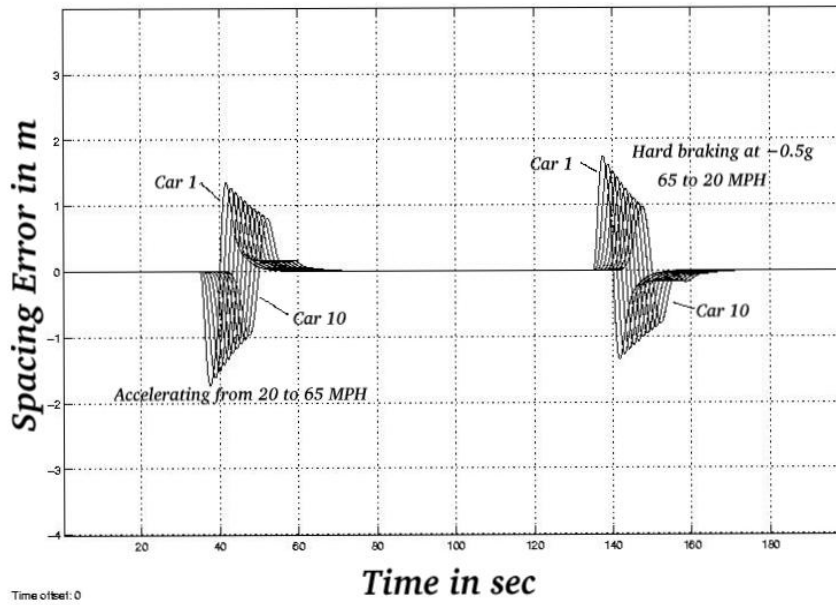
For simulation, the lead car performed the following maneuver. Starting from 20 MPH and correct spacing, the car accelerates to 65 MPH and then undergoes a hard brake maneuver (-0.5 g deceleration) to drop back to a speed of 20 MPH.  $\lambda = 0.4$  was used for the simulation.

Fig. 4 shows the control effort for the cars in a string of 10 cars following each other. The control effort reduces downstream. Given that the lead car can brake at the maximum deceleration, the following cars require lesser acceleration hence assuring that they will be capable of achieving the desired control effort during a hard brake scenario.



**Fig. 4 Control Efforts of Cars in a Platoon**

Fig. 5 shows the spacing errors during the acceleration and deceleration maneuvers. The errors in spacing decrease downstream. This ensures that if the first car doesn't collide with the lead car, then there will not be a collision downstream between the  $i^{th}$  and the  $(i+1)^{th}$  car ( $i > 1$ ).



**Fig. 5 Spacing Error in a Platoon**

## 6. CONCLUSIONS

A general framework for the design and evaluation of ACC spacing policies was developed in chapter 2 of this report. A specific nonlinear spacing policy that could be considered “ideal” evolved naturally from the framework. The spacing policy used autonomously available information. Analytical calculations showed that the spacing policy would guarantee string stability and traffic flow stability while maintaining smaller steady state spacing and hence providing larger traffic flow capacities in the speed range 20-65 MPH.

The simulation results in the paper confirmed the theoretical results. As opposed to non-autonomous spacing policies, this spacing policy can be readily implemented on ACC vehicles on today’s highways.

## **6. Simulation and Analysis of Mixed Adaptive Cruise Control / Manual Traffic**

Adaptive (Intelligent) Cruise Control (ACC) Vehicles will coexist with manually driven vehicles on the existing roadway system long before they become universal. This mixed fleet scenario creates new capacity and safety issues. In this chapter, measurement of traffic quality is discussed. A definition of traffic flow stability is proposed. Simulation results of various mixed fleet scenarios are presented. The analysis of the effect of mixing on capacity and stability of traffic system is based on these results. It is found that throughput increases with the proportion of ACC vehicles when flow is below capacity conditions. But above capacity, speed variability increases and speed drops with Constant Time Headway (CTH) control ACC compared with human drivers.

This chapter also addresses the impact of Adaptive Cruise Control laws on the traffic flow. Simulation results of various mixed fleet scenarios under different ACC laws are presented. Explicit comparison of two ACC laws, Constant Time Headway and Variable Time Headway (VTH), are based on these results. It is found that VTH has better performance in terms of capacity and stability of traffic. Throughput increases with the proportion of CTH vehicles when flow is below capacity conditions. But above capacity, speed variability increases and speed drops with the CTH traffic compared with manual traffic, while the VTH traffic always performs better than CTH.

### **1. LITERATURE REVIEW**

#### **1.1 DESIGN CONSIDERATIONS OF ADAPTIVE CRUISE CONTROL**

While vehicle manufacturers hope that ACC systems will improve the driver's comfort and safety, research on the properties of fully automated vehicle platoons has shown the potential benefits for capacity and safety (Van Arem et al. 1996; Broqua et al. 1991; Minderhoud and Bovy 1998). It seems an appealing scenario that all of the vehicles on a highway are automated. But, it is more reasonable to imagine that at the initial stage of deploying automated or semi-automated vehicles, they will coexist with conventional manually driven vehicles. For instance,



Tier One forecasts that by 2010, 20 percent of cars will be equipped with either ACC-Collision Warning or other headway control systems (Tier One, 2001b). This mixed control scenario raises complex capacity and safety issues on traffic flow that we must probe before ACC becomes reality. The impacts of the deployment of ACC on the traffic flow pattern and its control must be taken into consideration in the very early stage of ACC design. Up to now, important design considerations of ACC systems largely include: (1) Maintaining a safe distance between vehicles: the system may fully control the vehicle, and it must guarantee that the vehicle will not enter an unsafe state as a result of the control; (2) Characteristics of real-time response: the response time of the vehicle to control inputs must be short; (3) All-weather capability: Performs well in poor visibility conditions and should not be adversely affected by poor weather conditions; (4) Performs well during road turns, bumps and slopes; (5) Simplicity of use: A driver with no prior experience can use it correctly.

Patterson (1998) studied ACC's impacts on capacity and safety. His thesis compared ACC with conventional cruise control and manual driving at both the macroscopic and microscopic level. At the macroscopic level, it is found that ACC was used more in similar trips and the number of brake interventions in ACC vehicles is larger than that in CCC vehicles. At the microscopic level, it is found that manual driving results in larger headway. But ACC and CCC have similar speed-headway profiles.

Because of some advantages to fuzzy logic models, Wu et al. (1998) gave a complete description of the fuzzy sets for both car following and lane changing in FLOWSIM which offers a user defined update rate and applies accelerations. The fuzzy inference model for car following is based on the divergence of the ratio of vehicle distance to desired vehicle distance and the relative speed of two vehicles. Holve et al. (1995a) suggested that the ACC system has to meet the expectations of the human driver to a certain degree. They proposed an adaptive fuzzy logic controller that is flexible in different driving situations and comprehensible for the driver. Holve et al. (1995b) also proposed a scheme to generate fuzzy rules for the ACC controller, in which the driver is a component of the ACC control loop. Their Fuzzy-ACC has been tested in normal road traffic. Similar work can be found in Chakroborty et al. (1999), in which relative speed,

distance headway and acceleration/deceleration rate of leading vehicle are the inputs to a fuzzy logic model.

Marsden et al. (2001) employed Wu's car following model in simulation. An ACC algorithm based on a manufacturer prototype was also employed in which the acceleration rate of the ACC vehicle is related to the vehicle mass, the gap headway, the rate of change of gap headway and the velocity of the equipped vehicle. Their results showed that ACC could reduce the variation of acceleration compared to manual driving (Marsden et al., 2001). It should be noted that the authors also pointed out the limitations of microscopic simulation in modeling the impacts of ACC because of the lack of knowledge of the behavior and interaction between the driver and the ACC system in different traffic conditions (Marsden et al., 2001).

Ioannou et al. (1994) proposed their ACC scheme for constant time vehicle following. Their results showed that the scheme could maintain a steady state of inter-vehicle spacing without reducing the driver comfort.

## **1.2 Study In the Aspect of Traffic Flow**

The designers of ACC algorithms often consider the "string stability" as the primary criterion (Darbha and Rajagopal 1999, Fancher and Bareket 1995, Li and Shrivastava 2001). Liang and Peng (1999) presented a two-level ACC synthesis method which calculates desired acceleration rate and controls vehicles to achieve the desired rate accurately. They suggested that their method can guarantee string stability and yield minimum impact on vehicles nearby. Furthermore, they (Liang and Peng, 2000) suggested a framework to analyze string stability and defined a marginal index to give a quantitative measurement of ACC designs.

Many simulation studies have evaluated the impacts of ACC systems (Van Arem et al. 1996; Broqua et al. 1991; Minderhoud and Bovy 1998). However, the traffic flow characteristics that ACC will bring are difficult to quantify. And it is not possible to make direct comparison among these documents, because these studies have employed different ACC algorithms, different driver behavior models, and different driving environments. What we can do is to find some common trend and make some qualitative explanations. In their work, Broqua et al. (1991) estimated that gains in throughput are 13% with 40% of vehicles equipped with constant-space-headway ACC when the target time-gap of the system was 1 second. Van Arem et al. (1996) and

Minderhoud and Bovy (1998) have found a decrease in average speed caused by a collapse of speed in the fast lane for ACC target time-gaps of 1.4 s and above. Bose and Ioannou (2001) reported their studies on the mixed traffic of ACC vehicles and manual driven vehicles. Their results showed that 10% presence of ACC vehicles smoothed traffic flow in the case of rapid acceleration of the leading vehicle, which results in less fuel consumption and pollution levels than pure manual driven traffic.

Sponsored by the National Highway Traffic Safety Administration (NHTSA), an ACC system evaluation project (Koziol et al. 1999) was implemented by the University of Michigan Transportation Research Institute from July 1996 to September 1997. Based on data obtained from the experiments, the Volpe National Transportation Systems Center (Volpe Center) investigated ACC's impacts. They concluded that deployment of ACC results in safer driving (Koziol et al. 1999).

Fancher et al. (1998) reported the ICC Field Operational Test of NHTSA and UMTRI. They concentrated on the safety and comfort issues of ICC (ACC) system. They found that ACC is very attractive to most drivers and is used in many traffic conditions (Fancher et al., 1998). On the other hand, they found some issues indicate potential impacts on safety and traffic; the importance of human-centered design is also highlighted (Fancher et al., 1998). In the aspect of assessing impacts on traffic flow, they suggested that the current results are not enough for analysis (Fancher et al., 1998).

Swaroop and Huandra (1999) studied the design problem of the ACC algorithm. Based on analysis and numerical simulation, they demonstrated that a good ACC spacing policy must satisfy the condition that the slope of the corresponding fundamental traffic characteristics is always positive.

VanderWerf et al. (2001) studied the impacts of autonomous ACC (AACC) and cooperative ACC (CACC) on traffic based on their microscopic simulation. They found that AACC have very small impact on highway capacity. The capacity gain from 0% to 20% AAC penetration is greater than that from 20% to 40%. And there is no capacity increase with more AACC penetration (VanderWerf et al., 2001). Cooperative ACC, on the other hand, can potentially increase capacity quadratically along with CACC penetration (VanderWerf et al., 2001).

All of this research provided the estimates of impacts of ACC in some specified situations. Their results are meaningful for the traffic operators to outline the potential impacts of the ACC

system. Our research will begin with some simplified scenarios. Then more complex situations are simulated in our microscopic traffic simulation program. We will try to summarize the impacts of ACC from a large number of simulations in which some stochastic mechanisms make the results more realistic.

### **1.3 PURPOSE OF THIS RESEARCH**

The main purpose of our research includes: (1) To develop a framework to evaluate adaptive cruise control (ACC) algorithms; and (2) To develop new autonomous vehicle following algorithms that overcome the shortcomings of existing ACC algorithms. To achieve these goals, both theoretical analysis and simulation work are needed. The conventional methods in control theory, such as Laplace transform and frequency domain analysis can be used to investigate the properties of the vehicle platoons. However, these methods cannot be used in the analysis of mixed traffic in which the relationship between vehicles can't be described by a uniform differential equation or its derived forms.

The only way to provide an explicit observation of the mixed traffic's behavior is microscopic simulation in which each vehicle is controlled by its own control algorithm, either a car-following algorithm which emulates the human driver's behavior statistically, or an adaptive cruise control law. Most of our conclusions in this report are obtained from the simulation results. The steps of our work include: (1) To develop microscopic traffic simulation tools; (2) To evaluate the impacts of ACC algorithms on traffic flow; (3) To compare various ACC algorithms and find the optimal algorithm and the mode of operation that benefits the effectiveness of traffic flow.

In this report, we will first discuss methodology and criteria that can be used to evaluate the impacts of ACC algorithms on traffic flow. A number of cases with mixed ACC and manually controlled traffic are simulated and analyzed using a microscopic traffic simulation program. To simplify the analysis, a one-lane highway is studied. The semi-automated vehicles are equipped with an ACC algorithm that allows them to keep a constant time headway or variable time headway while following. The newly-developed variable time headway control algorithm is implemented in simulation and compared with the constant time headway algorithm. Gipps' model (Gipps, 1981) is used to simulate manually driven vehicles. The density and speed profiles as a function of the proportion of ACC vehicles are investigated, which show the potential

benefits of the semi-automated vehicles. Different vehicle following scenarios with sudden decelerations and accelerations are analyzed in order to study the effect of the response of ACC vehicles in mixed traffic.

In chapter 2, the evaluation issues of traffic flow, such as stability, robustness and safety, are discussed. Chapter 2 also describes the mixed traffic scenario that is investigated and the simulation program. Chapter 3 to Chapter 5 summarize the simulations on the different level of highway traffic as a function of the proportion of ACC vehicles. The stability and transient response of traffic flow in different mixed traffic situations are illustrated in the results. Chapter 3 presents the simulation results of traffic flow mixed by constant time headway ACC and manually driven cars. Chapter 4 compares the performance of variable time headway and constant time headway ACC. Chapter 5 shows the simulation result in which headways of ACC and manually driven vehicles are randomly distributed. Comparisons are made to show the impacts of random factors. Chapter 6 summarizes the simulation of mixed traffic based on AIMSUN and GETRAM Extension. Some concluding remarks in Chapter 7 complete the report.

## **2 EVALUATION OF ADAPTIVE CRUISE CONTROL**

### **2.1 INTRODUCTION**

The evaluation of the impacts of ACC on traffic flow is based on measurements of traffic quality. Traffic operators want (1) a high capacity of traffic flow in the current infrastructure; (2) stable traffic flow in the cases of high demand and incidents and (3) guaranteed safety for each driver.

High capacity can only be obtained when the average time headway is reduced while keeping the same speed. That means vehicles are closer to each other in the same speeds. ACC provides a very short response and may take the place of the driver in longitudinal control. However, this improvement will raise a safety problem if the response time of the driver-vehicle system cannot be reduced in the same scale. The tradeoff point in terms of capacity and safety can be chosen based on the safety requirements, which are determined by the mechanics of vehicle and

infrastructure, such as vehicle dynamics, road surface friction, efficiency of Anti-lock Brake System and even weather.

Smooth traffic is also desired by users and operators because it means less travel time and less potential incidents. The problem is how to reduce congestion and make traffic more stable. A clear definition of traffic flow stability is still unavailable in previous studies because of the nonlinear, dynamic and stochastic nature of traffic and the fuzzy aim of term defining. Stability, which is traditionally used to describe the internal characteristic of a system to maintain a bounded movement, is different from robustness, which is used to evaluate the external characteristic of system: the insensitivity of a system to input variation. In traffic studies, these two terms are often blurred and treated as the same quantity. But distinguishing these two quantities is significant when one considers the upstream traffic as the input of downstream traffic in traffic flow operation. Unfortunately, up to now, there are no explicit definitions of stability or robustness. A promising approach to investigate these characteristics is the study of traffic congestion in which many reasons cause oscillations in speed, density and flow rate. Traffic congestion is such a complex phenomenon that no up-to-date theories, such as car-following model (Rothery 2000), queuing theory (Newell. 1982), kinetic theory (Prigogine And Herman 1971), cellular automata (Nagel and Shreckenberg 1992), higher-order model (Kuhne and Michalopoulos 2001) and kinetic wave theory (Lighthill and Whitham 1955), can soundly describe it, though everyone experiences it daily.

In this research, the primary focus is to characterize the performance of ACC systems, driver behavior under normal driving conditions, and driver interaction with partially automated vehicles. Although some progress has been made in these areas, much work is still required to better understand the relationships between benefits, user acceptance, ACC system requirements, driver behavior, and the driving environment. Moreover, the impact of drivers or automatic control dynamics on system capability and benefits is not well understood. In a word, the criteria now being used are just suitable for an idealized traffic system. The criteria that quantify drivers and system benefits need further research.

## **2.2 MEASUREMENTS OF TRAFFIC QUALITY**

### **2.2.1 TRAFFIC FLOW STABILITY**

Many investigations have been conducted to study the stability issues of traffic flow. But most of them are concentrated on string stability. **String stability** is stability with respect to inter-vehicular spacing. It ensures the position and speed of each vehicle in a string change within small boundaries of error, and disturbances in vehicle speeds do not be amplified when they are propagated upstream in traffic. Normally, no vehicle enters or leaves the string in the study of string stability. It should be noted that guaranteeing string stability doesn't mean guaranteeing traffic flow stability. String stability might ensure that the space between vehicles in the string remain the same constant, but can not keep the speed of vehicles in the string from decreasing to a level that blocks a motorway section and thus causes an instable state of traffic flow.

At a macroscopic scale, traffic flow is the aggregation of strings and single vehicles in many sections of motorway. Traffic flow stability deals with the evolution of aggregate velocity and density in response to change in the flow rate. So vehicles enter and leave specified traffic in the study of flow stability. Darbha and Rajagopal (1999) proposed that, "Traffic flow stability can be guaranteed only if the velocity and density solutions of the coupled set of equations is stable, i.e., only if stability with respect to automatic vehicle following and stability with respect to density evolution is guaranteed." Their definition is in the sense of Lyapunov Stability. A formal definition of Lyapunov stability is (Murray et al. 1994):

*The equilibrium point  $x^*=0$  of  $\dot{x} = f(x,t)$  is stable at  $t=t_0$  if for any  $\varepsilon > 0$  there exists a  $\delta(t_0, \varepsilon) > 0$  such that:*

$$\|x(t_0)\| < \delta \quad \Rightarrow \quad \|x(t)\| < \varepsilon, \quad \forall t > t_0 \quad (1)$$

Furthermore, the **asymptotic stability** is defined as (Murray et al. 1994):

*The equilibrium point  $x^*=0$  of  $\dot{x} = f(x,t)$  is asymptotic stable at  $t = t_0$  if*

1.  $x^*=0$  is stable, and
2.  $x^*=0$  is locally attractive; i.e. there exists  $\delta(t_0)$  such that

$$\|x(t_0)\| < \delta \quad \Rightarrow \quad \lim_{t \rightarrow \infty} x(t) = 0 \quad (2)$$

Here a point  $x^*$  is an equilibrium point of  $\dot{x} = f(x,t)$  where  $f(x^*,t) \equiv 0$ .

Darbha and Rajagopal define **Traffic Flow Stability** for automated vehicle traffic as:

Let  $v_0(x,t)$ ,  $k_0(x,t)$  denote the nominal state of traffic. Let  $v_p(x,t)$ ,  $k_p(x,t)$  be the speed and density perturbations to the traffic, consistent with the boundary conditions and are such that  $v_p(x,t) \equiv 0$ ,  $k_p(x,t) \equiv 0 \quad \forall x \geq x_u$ . The traffic flow is stable, if

1. given  $\varepsilon > 0$ , there exists a  $\delta > 0$  such that

$$\sup_{x \leq x_u} \{ |v_p(x,0)| |k_p(x,0)| \} < \delta \quad \Rightarrow \quad \sup_{t \geq 0} \sup_{x \leq x_u} \{ |v_p(x,t)| |k_p(x,t)| \} < \varepsilon \quad (3)$$

and

$$2. \lim_{t \rightarrow \infty} \sup_{x \leq x_u} \{ |v_p(x,t)| |k_p(x,t)| \} = 0 \quad (4)$$

Where  $\sup(\cdot)$  is the abbreviation for supremum or least upper bound, meaning, for a given set of numbers S, the smallest element of a set U is the upper bound of S. This definition of stability can be described in Figure 1.  $(v_0, k_0)$  is a steady state of traffic. Here the traffic flow is defined to be stable when the disturbance  $(v_p, k_p)$  does not exceed a boundary if its initial value is within a limit, and, in the end, the disturbance becomes zero. As shown in the enlarged figure in Figure 1, if the initial state is within the boundary (the triangle point), it goes to the equilibrium point  $(v_0, k_0)$  in the end; but if the initial state is beyond the boundary (the rectangle point), it does not converge to the equilibrium point but goes to other indefinite states.

This definition is very strong in that the disturbance must be eliminated over time by its own movement. In real-world traffic, disturbances or unstable traffic usually are eliminated because of light demand inflows which happen from time to time. On the other hand, it's also a loose definition because it doesn't present the traffic state change from free flow to congestion in which the change of the "nominal state of traffic" itself is the source of traffic flow instability. Thus, a Lyapunov stability definition in terms of speed-density relation may not work well in describing traffic stability.



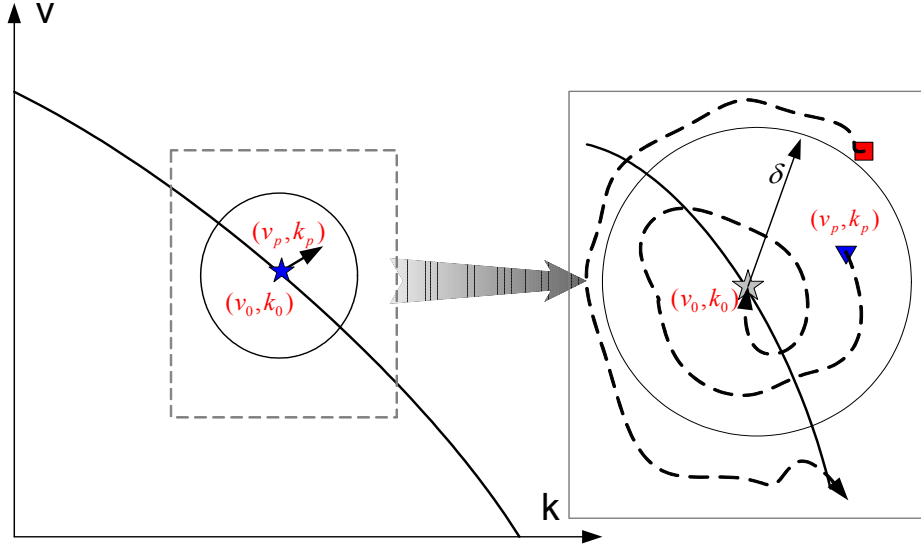


Figure 1. Stability in Term of Speed-Density Relation

In Zou and Levinson (2001), a new criterion function of the relation between density and flow rate is proposed to detect potential traffic breakdown.

(1) The convolution is defined as:

$$C(f, h) = \int_{-\infty}^{\infty} f(t)h(u - t)du \quad (5)$$

The moving-average of density is defined as:

$$\begin{cases} k_{low} = C(k(t), P(t)) \\ P(t) = \begin{cases} 1 & t_2 < t < t_1 \\ 0 & t < t_2, t > t_1 \end{cases} \end{cases} \quad (6)$$

(2) The filtered high frequency components  $k^*(t)$  are restored by subtracting low frequency components from the original signal, which is defined as **Density Dynamics**:

$$k^*(t) = k(t) - k_{low}(t) \quad (7)$$

(3) **Cross-Correlation** is always used to detect the diversity of measurements:

$$\langle f, h \rangle = \int_{-\infty}^{\infty} f(t)h(u + t)du \quad (8)$$

We conduct a template cross-correlation calculation of density dynamics and flow rate:

$$corr(k^*, q) = \max \langle \bar{k}^* \cdot \bar{q} \rangle \quad (10)$$

where  $\bar{k}^*$  and  $\bar{q}$  are templates of  $k^*$  and  $q$ , respectively, move simultaneously. Template means a consecutive portion of data in a series. The new criterion function is defined as:

$$z(t) = \frac{d}{dt}(\text{corr}(k^*, q)) \quad (11)$$

The computation results based on real world traffic data justified the effectiveness of this function. It's shown that there is a traffic breakdown only when  $z(t) = \frac{d}{dt}(\text{corr}(k^*, q)) > z_0$ , where  $z_0$  could be a threshold obtained from experience. If the changing rate of the cross-correlation exceeds the threshold, the transition is unreturnable. Another important property is that the criterion function has a singular peak in the onset of the phase transition. These results indicate that the criterion function might be a mathematical description of phase transition in traffic flow which presents the traffic flow moving from stable to unstable states.

In deriving the criterion function, we find concentrations of traffic states in both free flow and congested traffic by means of moving-average computation, as shown in Figure 2. This intuitively provides support for previous studies that suggest distinct phases in traffic flow. Also, the transitions that happened between relatively stable phases represent cases when the system loses or regains stability. Thus the transient condition of traffic flow we obtained sheds light on the conceptions of traffic stability and robustness.

If the traffic transferring from free flow to congestion is considered as an unstable state, we can provide stability and instability criteria as:

### ***Stability Criterion***

The traffic flow will remain stable if the changing rate of the cross-correlation between flow rate and density dynamics is always within a boundary, i.e.,  $z(t) = \frac{d}{dt}(\text{corr}(k^*, q)) \leq z_0$ .

### ***Instability Criterion***

The traffic flow will be unstable if the changing rate of the cross-correlation between flow rate and density dynamics exceed a boundary, i.e.,  $z(t) = \frac{d}{dt}(\text{corr}(k^*, q)) > z_0$ .

Figure 3 illustrates the basic movement of traffic flow in losing and regaining stability. Our study shows that: if the initial state is within the boundary of stable states cluster, and if the state transition satisfies the stability criterion, the traffic will remain stable. Otherwise, traffic will loss

stability and goes to congestion states. This is a kind of Lyapunov stability. By Lyapunov stability, we mean that the state disturbances, that satisfy the boundary conditions, remain bounded.

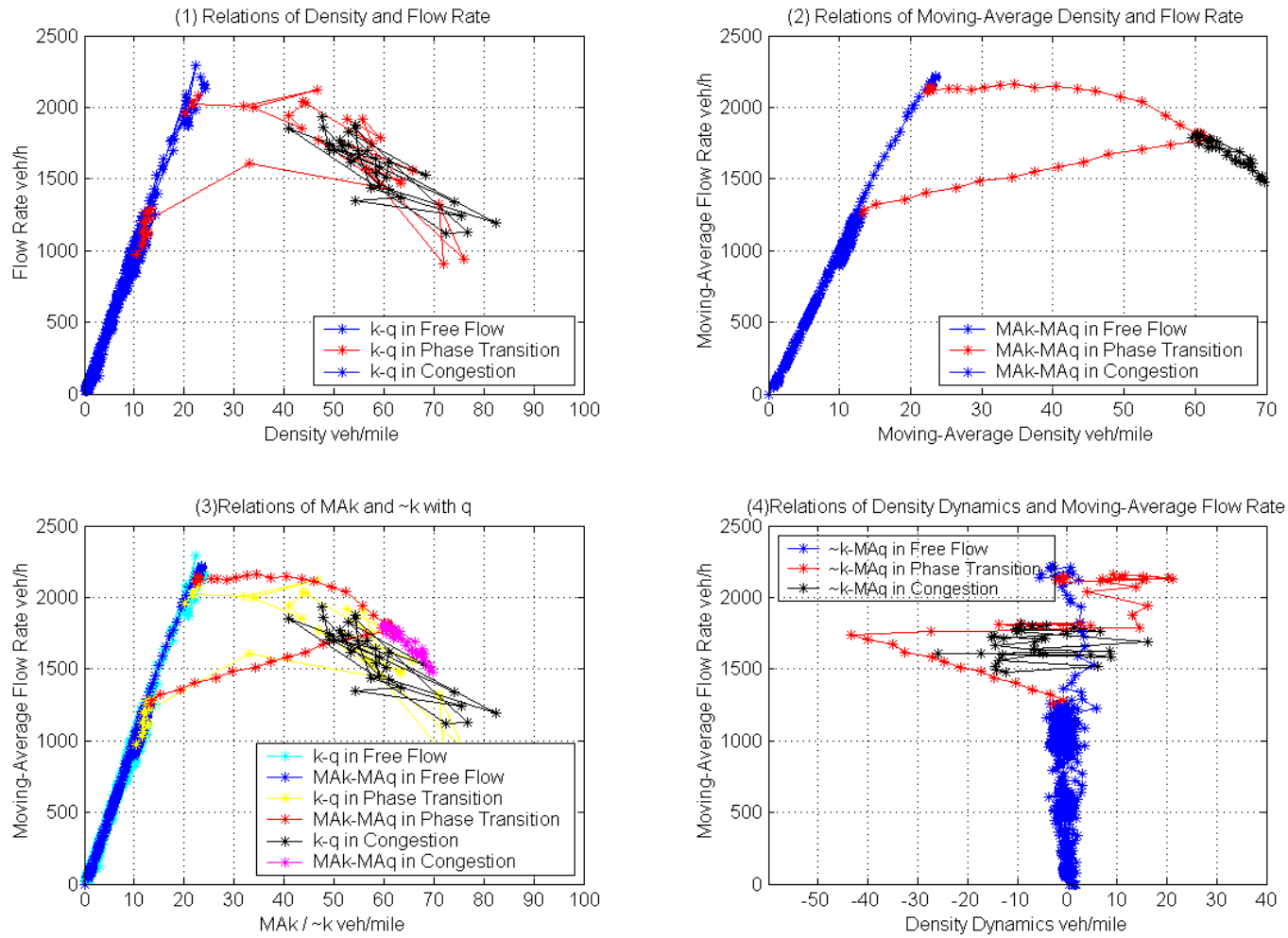


Figure 2 ( $\sim k$  is  $k^*$  in these graphs; MA means moving-average)

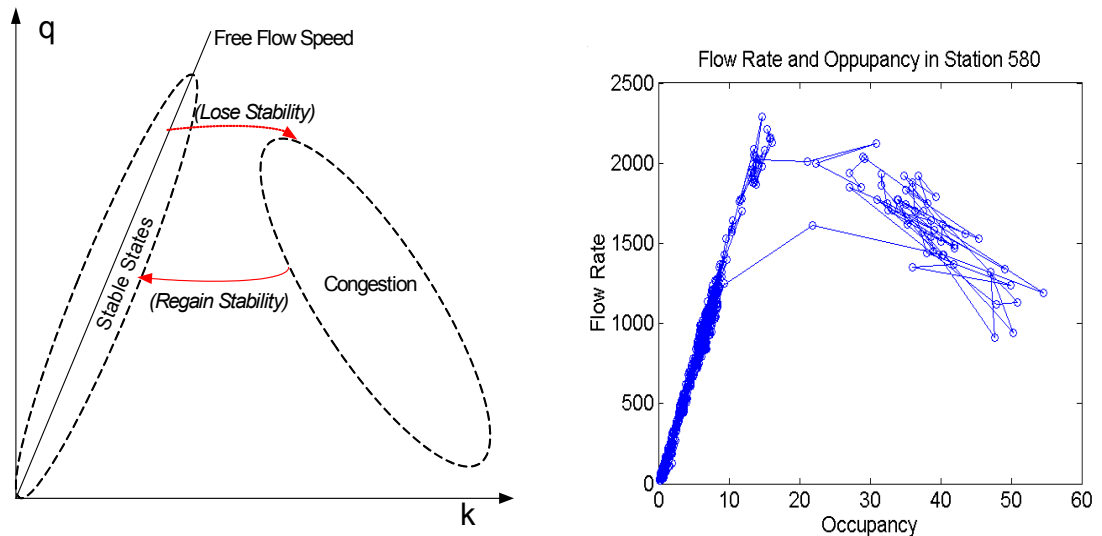


Figure 3. Stability Change of Traffic Flow

**Robustness of Traffic Flow** can also be defined. A robust system is a system that can restore its normal condition after being disturbed by internal or external noise or disturbances. Formally, a robust system is defined as a system that behaves in a controlled and expected manner when expected variations arise in its dominant parameters, but also in the face of unexpected variations (EASi GmbH, 2001). In traffic systems, typical variations include the acceleration noise of vehicles, internal disturbances such as the sudden braking of a vehicle in the string and external disturbances such as the change of demand at the entry of the road. We can qualitatively judge the robustness of the traffic system by observing the profiles of flow, speed, and density. Our results show that if the disturbances are within the boundary, the phase of traffic flow will not change. So the measurement of robustness of traffic flow is the boundary of the changing rate of the cross-correlation between flow rate and density dynamics. As one can see in Figure 4, the traffic experienced a disturbance at some time and run away from the free flow curve, but it didn't result in a congestion but went back to stable traffic (the circled portion). At another time, the boundary was exceeded and jam presented.

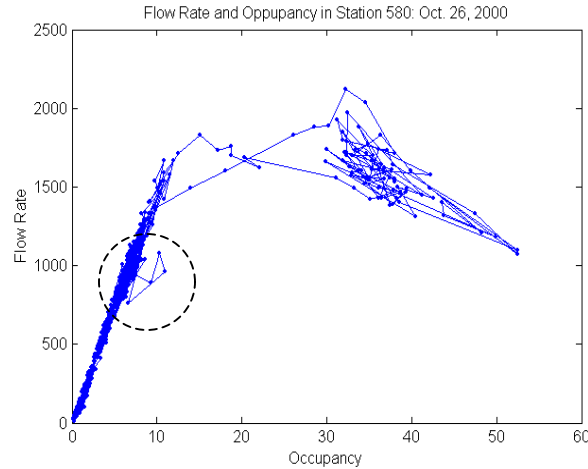


Figure 4.

Though this criterion function cannot be easily applied to the evaluation of ACC, it at least indicates that the density dynamics should not be too large in the case of high flow rate. Otherwise, traffic will be unstable. From the microscopic point of view, it means that the response time of vehicles should not be too small. It's a reasonable induction in that drivers responses to the density change ahead. If the change is drastic, drivers will respond more seriously, which causes big disturbances that affect upstream traffic adversely. The ACC designers should consider this influence of ACC's behavior in mixed traffic.

### 2.2.2 SAFETY

Traffic safety is a problem related with headway, response time of driver/automated system, vehicle mechanical delay and road surface condition. Normally safety can be guaranteed if the headway is greater than the summation of response time and delays.

Normally, the design of ACC algorithms is headway control design. While the headway control can be achieved effectively, there are still some practical problems to be solved. ACC headway design must consider all these factors. Such as:

(1) Who should be responsible for emergency braking? The maximum of deceleration of current ACC system is not enough for it. If it is the driver's responsibility, when should the driver be alerted? That is a liability problem. An ideal solution might be an ACC system combined with collision warning/avoiding system. In the latter one, ACC system is connected to ABS to apply braking commands.

(2) Should ACC change headway in the cases of special weather condition such as icy, sleeting and rainy days? The preset headway in slippery surface should be longer than those in dry surface in the same speeds. Should the driver be responsible for identifying the environment changes and changing the preset headway? How much should the driver change? The ACC designer must provide guidance for these situations.

(3) Should the ACC system provide headway options according to driver's response capability? Should it determine the lower headway limit for driver with longer response time? How is the liability if the driver uses the least headway? ACC designers may take conservative values of preset headway to avoid risks, but the efficiency of traffic might deteriorate. On the other hand, the system might test the response time of driver by recording their behavior to imminent vehicles approaching.

## **2.3 MICROSCOPIC SIMULATION SYSTEM CONFIGURATION**

The above discussion on traffic flow stability and safety is meaningful for the evaluation of ACC algorithms in that it provides some qualitative and quantitative measurements. But the evaluation of the impacts of ACC cannot be easily solved analytically, because the mixed traffic flow is the aggregation of vehicles with different control behavior and its properties cannot be obtained from the uniform mathematical model or differential equations. So we use simulation tools to observe traffic behavior and summarize ACC's impacts on the capacity, stability and safety of traffic before we get a breakthrough in the theoretical analysis.

### **2.3.1 SYSTEM CONFIGURATION: MIXED TRAFFIC SCENARIO**

When traffic is comprised of vehicles controlled by different kinds of controllers, adaptive cruise control or/and human drivers, we consider it to be "mixed". For this simulation, Constant Time Headway (CTH) control and Variable Time Headway (VTH) control ACC algorithms were selected. Although a number of driving simulator studies have been undertaken, these have focused on critical safety aspects of ACC use, such as Nilsson (Nilsson, 1995). Very few studies exist on how drivers incorporate the functionality of ACC into their driving cycle. For the purpose of this research, it has been assumed that if there is an ACC system equipped in the vehicle, it is used. However,

according to the US Field Operational Test trials, ACC was used for just over 50% of all miles driven at speeds of above 35 mph. In addition, usage rates for individuals varied between 20% and 100% (Fancher et al. 1998). Nevertheless, the purpose of the simulation is to define the range of traffic effects that could be found. So we don't stochastically change modes of control in our simplified simulations.

A simple scenario of a one-lane highway section, 3.2 km long, with one entry and one exit was established. No lane changing is considered in this simulation work. Significant inter-vehicle interaction is present throughout the simulation. The scenarios were designed to test whether or not ACC could generate a higher capacity while guaranteeing stable driving. ACC vehicles are allowed onto the current highway system used by manually driven vehicles. Metering is done at the entrance to guarantee enough initial headway on the highway, and while waiting to enter the highway, ACC vehicles are treated just like manual vehicles.

The role of the driver of the ACC vehicle is the same in these scenarios. On reaching the target lane, the driver engages the automated control system of the vehicle that takes over the longitudinal control of the vehicle. The driver is responsible for all driving functions as in a manually driven vehicle except for the longitudinal control. The driver disengages the headway control of the ACC vehicle and accelerates to a maximum speed if the highway is clear before him and at last exits the lane.

The maximum deceleration of the ACC equipped vehicle when under time headway control mode is limited to  $2 \text{ m/s}^2$  while the maximum acceleration under ACC is  $1.5 \text{ m/s}^2$ . Three typical scenarios are of most interest, these include:

- (a) No-ACC traffic: All vehicles on the road are controlled by Gipps' car-following model. This is the scenario to simulate the current manually controlled traffic.
- (b) Mixed traffic: ACC vehicles mix with Gipps' vehicles with certain penetration. We will highlight this scenario as the intermediary stage of ACC deployment. The safety and stability issues in this scenario are expected to be more complicated than others.
- (c) Pure ACC traffic: All vehicles are controlled by ACC. It can be called semi-automated in which each vehicle individually assigns desired headway and desired speed.



### 2.3.2 DYNAMIC MODELS OF THE COMPONENTS OF MIXED TRAFFIC

The main dynamic models used in the simulation are the vehicle dynamics, the ACC algorithm and the car-following model.

#### *Vehicle Dynamics*

The vehicle dynamics is simplified to a third-order differential equation:

$$\ddot{x}_i = \frac{1}{\tau}(\ddot{x}_{ides} - \ddot{x}_i) \quad (12)$$

where:  $\ddot{x}_i$  is the jerk of vehicle i;  $\ddot{x}_i$  is the acceleration of vehicle i;  $\ddot{x}_{ides}$  is the desired acceleration of vehicle i which is generated by the car-following model or ACC algorithm.

#### *Adaptive Cruise Control Policy*

The most conventional ACC algorithm is **Constant Space Headway** control, which is in form of:

$$\begin{cases} \ddot{x}_{ides} = -k_1 \varepsilon_i - k_2 \dot{\varepsilon}_i \\ \varepsilon_i = x_i - x_{i-1} + L \end{cases} \quad (13)$$

Though it takes advantages of the relative position and relative speed as the control input, it has been proven that this control law cannot guarantee string stability (Darbha and Rajagopal 1999). So we don't pursue this control law.

**Constant Time Headway** (CTH) control can be represented as

$$\begin{cases} \ddot{x}_{ides} = -\frac{1}{h} \left( \dot{\varepsilon}_i + \lambda \delta_i \right) \\ \delta_i = x_i - x_{i-1} + L + h \dot{x}_i \end{cases} \quad (14)$$

CTH takes advantage of the relative speed and contains an extra term to fulfill time headway control. It has been proven that this control law can guarantee string stability (Darbha and Rajagopal 1999) and thus becomes a promising alternative to the constant space gap law. In this report, we use CTH control law as the primary one to test the impacts of ACC on the traffic flow.

**Variable Time Headway** (VTH) control (Wang and Rajamani, 2001) takes the relative velocity into account in the desired spacing, which is given by as follows:

$$\begin{cases} \ddot{x}_{ides} = -\rho_m \left( v_f - \dot{x}_i \right) \left( 1 - \frac{\dot{x}_i}{v_f} \right) \left( \dot{\varepsilon}_i + b \ddot{\varepsilon}_i + \lambda \delta_i \right) \\ \delta_i = \varepsilon_i + \frac{1}{\rho_m \left( 1 - \frac{\dot{x}_i}{v_f} \right)} + b \dot{\varepsilon}_i \end{cases} \quad (15)$$

Where,  $\rho_m$  is the maximum density of the highway, at which point traffic will stop (we assume  $\rho_m = 1/L$ ,  $L$  is the uniform vehicle length);  $v_f$  is the free flow speed;  $\dot{\varepsilon}_i = \dot{x}_i - \dot{x}_{i-1}$  is the relative velocity between  $i$ th vehicle and  $i-1$ th vehicle;  $b$  is a positive coefficient which determine the how much the relative velocity contributes to the desired spacing.

### ***Car-following Model***

Many models are developed to emulate the human driver's driving behavior, such as the GM model, Greenshield's model, Drew's model and Gipps' model (Gipps 1981). In our simulation, we use Gipps' Model to represent the acceleration and deceleration of manually controlled vehicles. This model states that, the maximum speed to which a vehicle ( $n$ ) can accelerate during a time period ( $t, t+T$ ) is given by:

$$\dot{x}_a(n, t+T) = \dot{x}(n, t) + 2.5 \ddot{x}(n) T \left( 1 - \frac{\dot{x}(n, t)}{\dot{x}^*(n)} \right) \sqrt{0.025 + \frac{\dot{x}(n, t)}{\dot{x}^*(n)}} \quad (16)$$

where:

$\dot{x}(n, t)$  is the speed of vehicle  $n$  at time  $t$ ;

$\dot{x}^*(n)$  is the desired speed of the vehicle ( $n$ ) for the current section;

$\ddot{x}(n)$  is the maximum acceleration for vehicle  $n$ ;

$T$  is the reaction time = updating interval = simulation step.

On the other hand, the maximum speed that the same vehicle ( $n$ ) can reach during the same time interval ( $t, t+T$ ), according to its own characteristics and the limitations imposed by the presence of the leader vehicle is:

$$\dot{x}_b(n, t+T) = d(n)T + \sqrt{d(n)^2 T^2 - d(n) \left| 2\{x(n-1, t) - s(n-1, t) - x(n, t)\} - \dot{x}(n, t)T - \frac{\dot{x}(n-1, t)^2}{d'(n-1)} \right|} \quad (17)$$

where:

$d(n)$  ( $< 0$ ) is the maximum deceleration desired by vehicle  $n$ ;

$x(n, t)$  is position of vehicle  $n$  at time  $t$ ;

$x(n-1, t)$  is position of preceding vehicle  $(n-1)$  at time  $t$ ;

$s(n-1)$  is the effective length of vehicle  $(n-1)$ ;

$d'(n-1)$  is an estimation of vehicle  $(n-1)$  desired deceleration.

In any case, the definitive speed for vehicle  $n$  during time interval  $(t, t+T)$  is the minimum of those previously defined speeds:

$$\dot{x}(n, t+T) = \min \left\{ \dot{x}_a(n, t+T), \dot{x}_b(n, t+T) \right\} \quad (18)$$

Then, the position of vehicle  $n$  inside the current lane is updated taking this speed into the movement equation:

$$x(n, t+T) = x(n, t) + \dot{x}(n, t+T)T \quad (19)$$

### 2.3.3 GENERATION OF TRAFFIC

As we have mentioned before, traffic generation complies with given traffic demand profiles. Normally, we use a constant inflow rate or pulse inflow rate to test the system. Each vehicle entering the road is controlled by a mechanism that changes the initial headway according to the specific inflow rate at that time. The demand profile employed for the study was chosen to overload road capacity during the middle 150 seconds (from 200 to 350 second) of the simulation. The other issue is to control the proportion of ACC vehicles. In our simulation, ACC vehicles are in traffic flow following a uniform distribution.

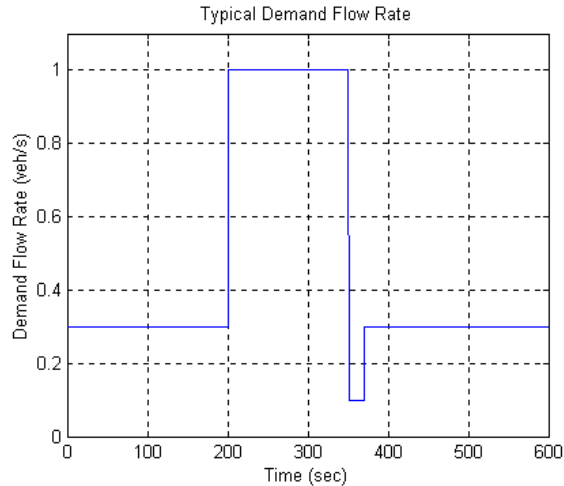


Figure 5. Typical Traffic Demand Profile

## 2.4 TRAFFIC SIMULATION PROGRAM

A microscopic simulation program is developed in the programming language C++. The flowchart of the program is shown in Figure 6. There is a main cycle of calculation in which the states of vehicles and the traffic flow are updated in a single sampling time duration. The sampling time is 0.1 second in our simulation, which is the response time of the ACC equipped vehicle. The main cycle includes:

- (a) Vehicle entry procedure that determines whether a new vehicle should enter the road. If so, it generates a new vehicle with a randomly selected control law, either Gipps' model or ACC algorithm;
- (b) Vehicle exit procedure that determines whether the leading vehicle should exit from the road. If so, it deletes the leading vehicle and modifies the second vehicle to be the leading vehicle. In our simulation, the leading vehicle will be free to accelerate until it reaches the maximum speed;
- (c) Vehicle state calculation calls the functions to update the states of each vehicle in current sampling duration. The car dynamics function will call the Runge Kutta algorithm (Press et al. 1992) that solves the differential equations. Either Gipps' car-following model or ACC algorithm will generate the desired acceleration for each individual vehicle;
- (d) Road state calculation procedure gets the instantaneous mean density, space mean speed, inflow rate etc. in the current sampling time.

The important parameters used in the simulation are summarized in Table 1 and Table 2.

Table. 1

<b>System Configuration</b>	
Road length	3212 meters
Maximum size of vehicles	4 meters
Initial speed of vehicles	17.79 m/s (40 mph)
Maximum Speed	28.9 m/s (65 mph)
<b>Parameters of operation</b>	
Sample time (calculation cycle)	0.1 second
Simulation time duration	600 ~900 seconds

Table 2.

<b>Parameters of ACC Algorithms</b>		<b>Parameters of Gipps' Model</b>	
Constant Time Headway		Desired Speed	28.9 m/s (65 mph)
$\lambda$	0.2	Maximum Acceleration	1.7 m/s <sup>2</sup>
Time Headway	1.0~1.2 seconds	Maximum Deceleration	-3.4 m/s <sup>2</sup>
Variable Time Headway		Time Headway	2 seconds
Free Flow Speed	31.78 m/s (71.5 mph)		
$\lambda$	0.2		

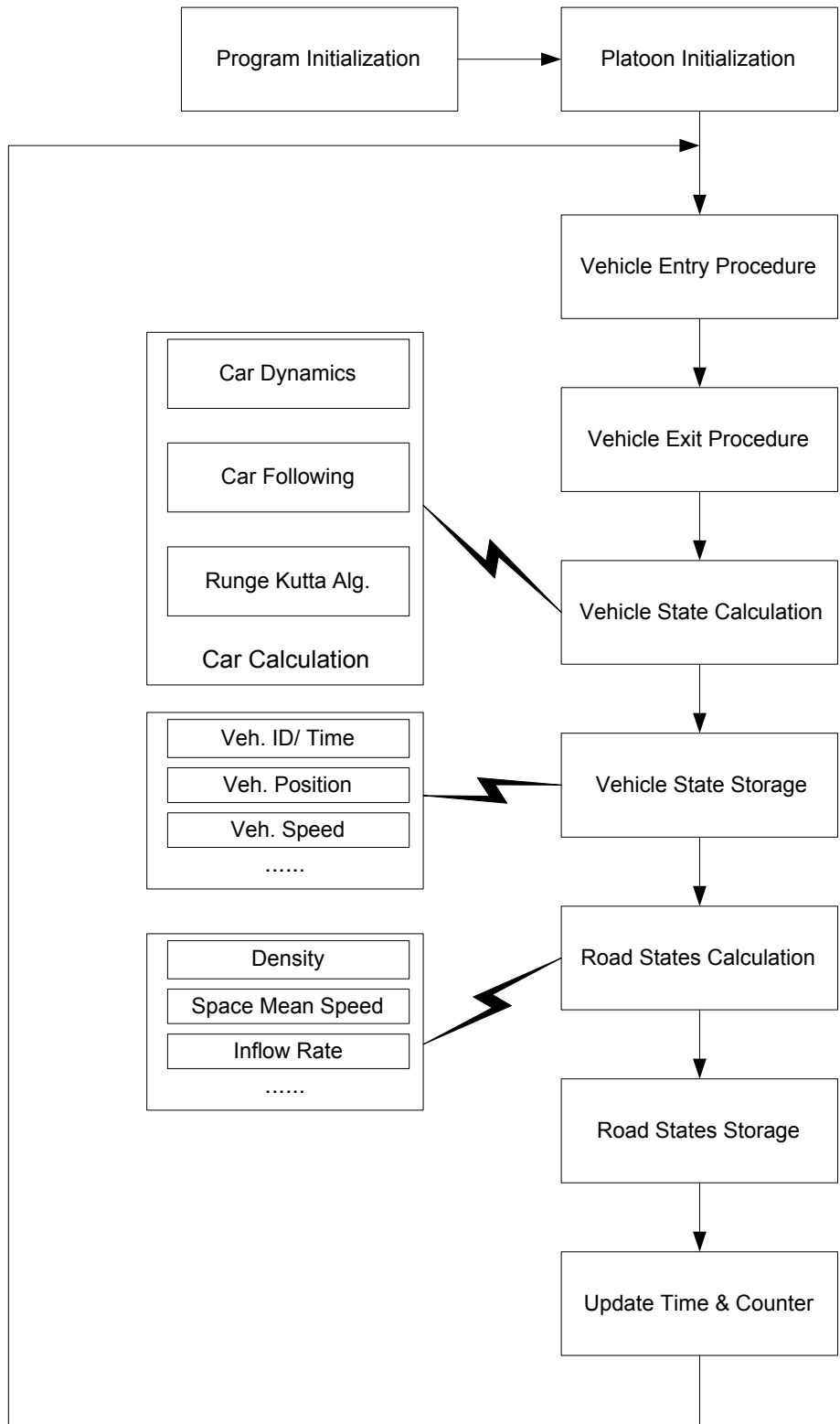
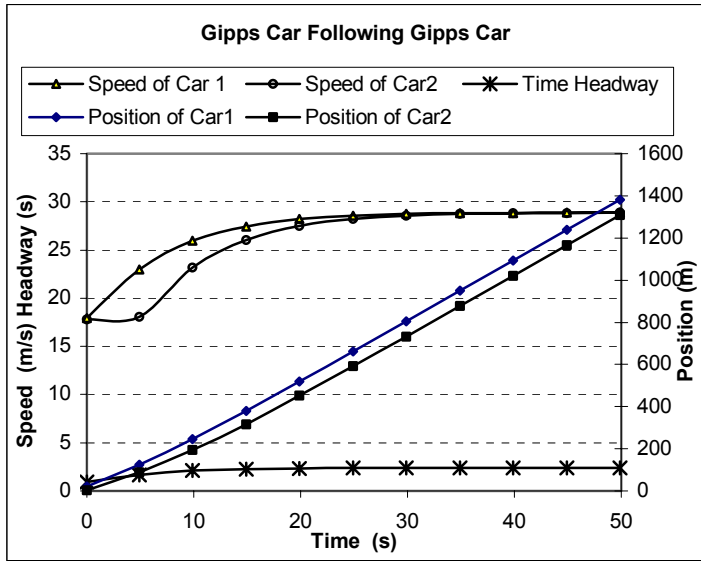


Figure 6 Flowchart of simulation Program

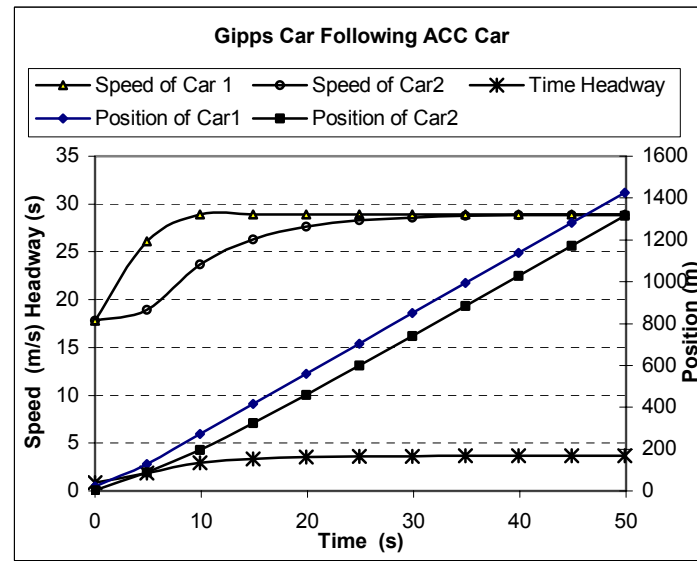
### **3 SIMULATION OF CONSTANT TIME HEADWAY AND MANUAL CARS**

#### **3.1 SINGLE VEHICLE FOLLOWING BEHAVIOR**

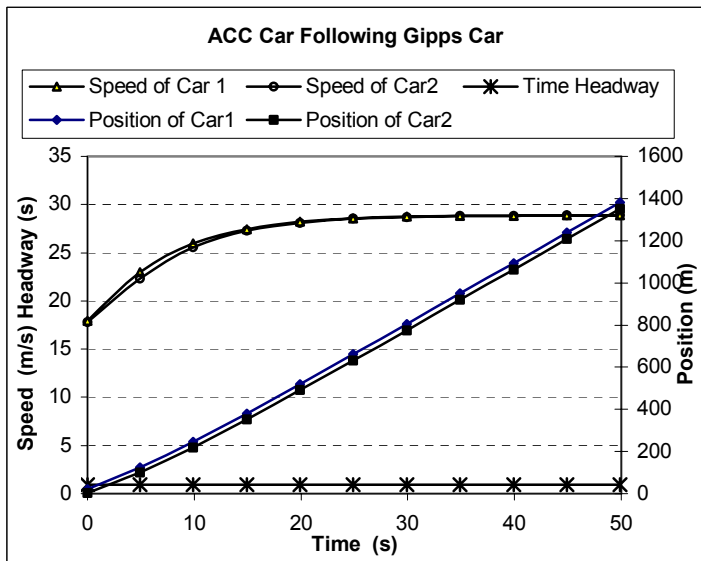
The single vehicle following behavior includes the behaviors of vehicles with various controls under different settings. Some typical results are shown in Figure 7. In these simulations, the preset time headway of ACC vehicle is 1 second, while that of Gipps' vehicle is 2 seconds. The two vehicles in the pair start up with the same initial speed and with a 20-meter distance. It is shown that ACC vehicles will have a quicker response and thus smaller transient time than manual driven vehicles. So in these scenarios, Gipps' vehicles cannot catch the leading vehicles because the leading vehicles are free to accelerate. In contradiction, ACC vehicles can always maintain the constant time-gap. This result highlights an important advantage of ACC compared to manual vehicles: small time headway is more easily achieved by ACC vehicles; thus ACC vehicles generate capacity.



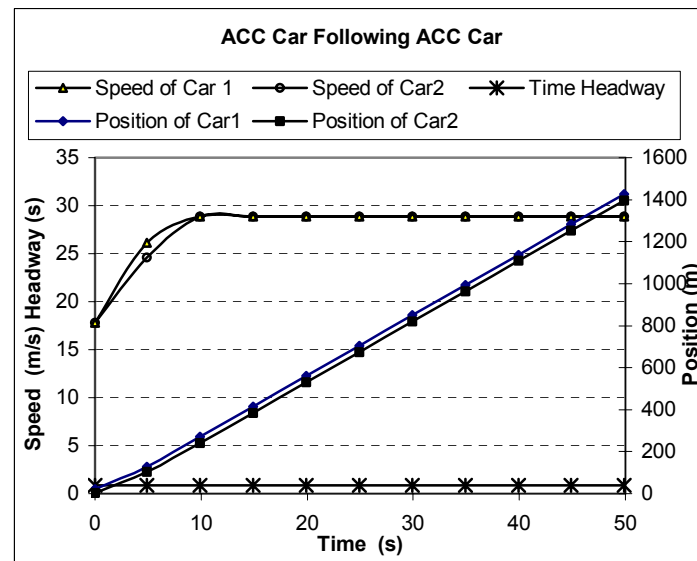
(a)



(b)



(c)



(d)

Figure 7. Single Vehicle's Following Behaviors

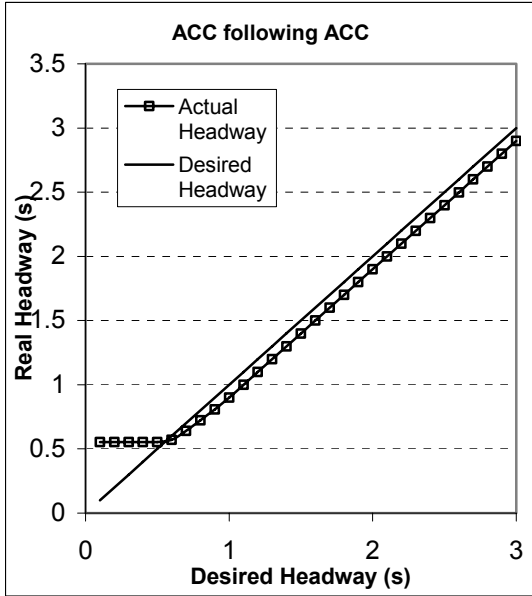


### 3.2 HEADWAY RESPONSE OF VEHICLES

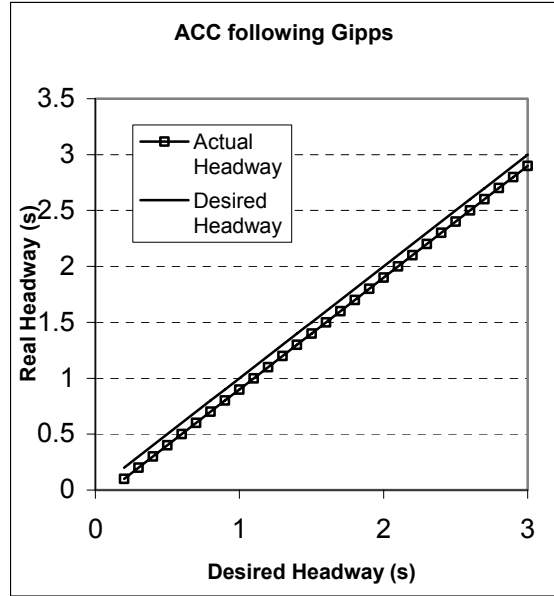
The headway response is the basic behavior that impacts the safety and capacity of the road. We experimented with the response of ACC vehicles and Gipps' vehicles to the preset headway under car-following scenarios. The basic conditions include: (a) Two vehicles have the same initial speed (17.7 m/s); (b) The leading vehicle's initial position is 20 meters from the entry, while the following vehicle is located in the entry point; (c) The maximum speed for both vehicles is 28.9 m/s. The original version of Gipps' model doesn't have a mechanism for achieving certain time headway. In the simulation, we added a time headway term that can affect the speed of vehicle to realize the headway control, i.e. {if (space headway)/(speed of following vehicle) < (desired time headway), then (the definitive speed) <= (current speed)}. This modified Gipps' model is more realistic with respect to the real condition that most drivers adjust their speeds according to estimated time headway (Koppa, 1998).

Figure 8 shows the time headway response of ACC vehicles and Gipps' vehicles. The setting (desired) time headway changes from 0.1 second to 3 seconds, which is represented by the solid curve in each graph. In fact, headways under 0.5 seconds are rarely used because most drivers will change to space control under such conditions. But these simulations are meaningful to show the responses of these models in emergency situations or in the case of congestion.

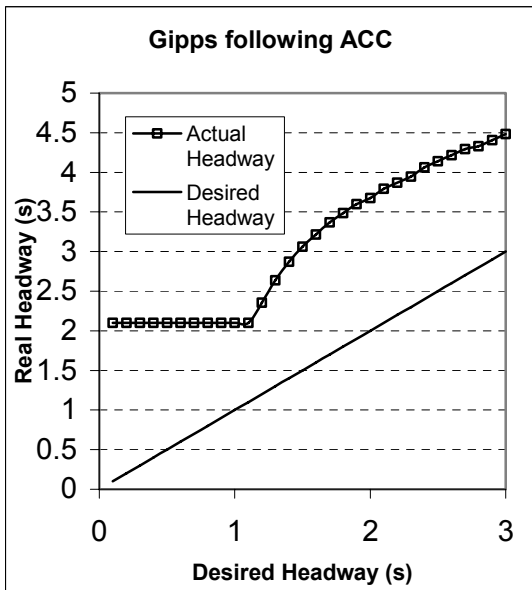
The real headway response is shown by the curve with a marker in each graph. It is shown that ACC vehicles can always achieve the setting headway with a small error. Changing the parameters of the algorithm cannot eliminate this error, which is largely from lack of an integral component in the controller. From (c) we can see, the Gipps' vehicle cannot catch the ACC vehicle. That is because the ACC vehicles can get to the maximum speed more quickly. On the other hand, as shown in (d), the headway errors for Gipps' vehicles are quite large compared to ACC vehicles. Though it is an innate disadvantage of Gipps' model, we expect the same amount of errors for human drivers.



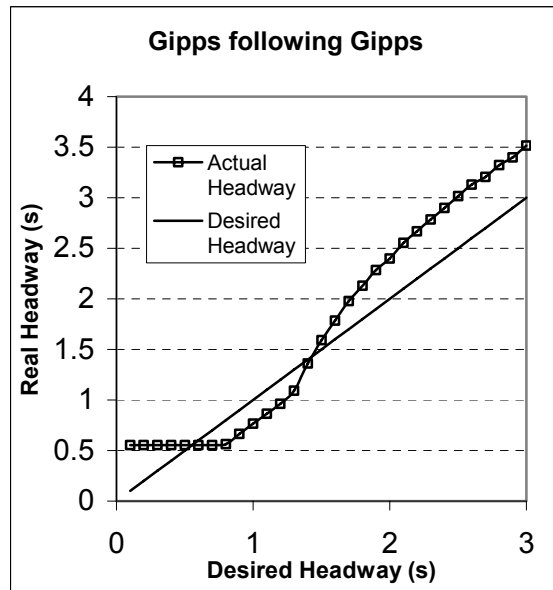
(a)



(b)



(c)



(d)

Figure 8. Headway Response Of Single Vehicle

By comparing these results, we conclude that the headway response of ACC vehicles can fulfill the requirement that will bring into potential of high capacity. Gipps' model can

emulate the human drivers' behavior to some extent. The traffic flow comprised by these two kinds of vehicles is a mixed flow with heterogeneous headway behaviors.

### 3.3 SYSTEM RESPONSE TO INTERNAL PULSE

The system response to an internal disturbance is shown in Figure 9. The internal disturbance is generated by a sudden braking of a vehicle in the string. After a while, the speed of that vehicle is restored to normal conditions. The vehicles behind the braking vehicle will be affected. As shown in Figure 9, which is the case of 100% ACC penetration, the speeds of some vehicle are reduced to maintain safe distance. After the speed of the leading vehicle is restored, the affected vehicles can return to normal speeds.

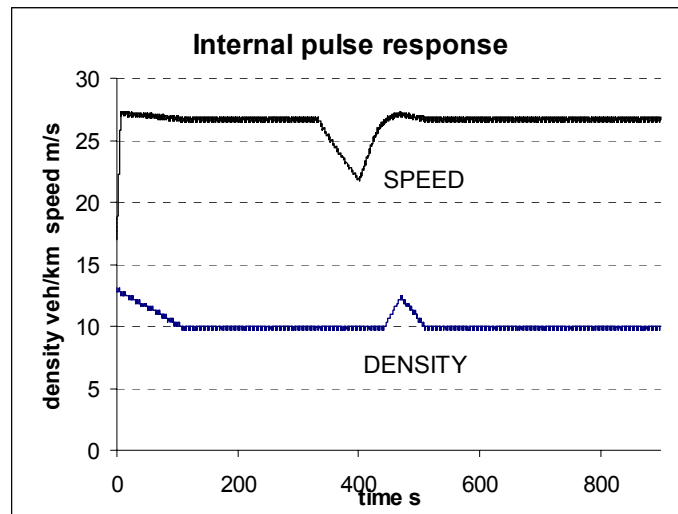


Figure 9. Density and Speed Profiles in an Internal Pulse

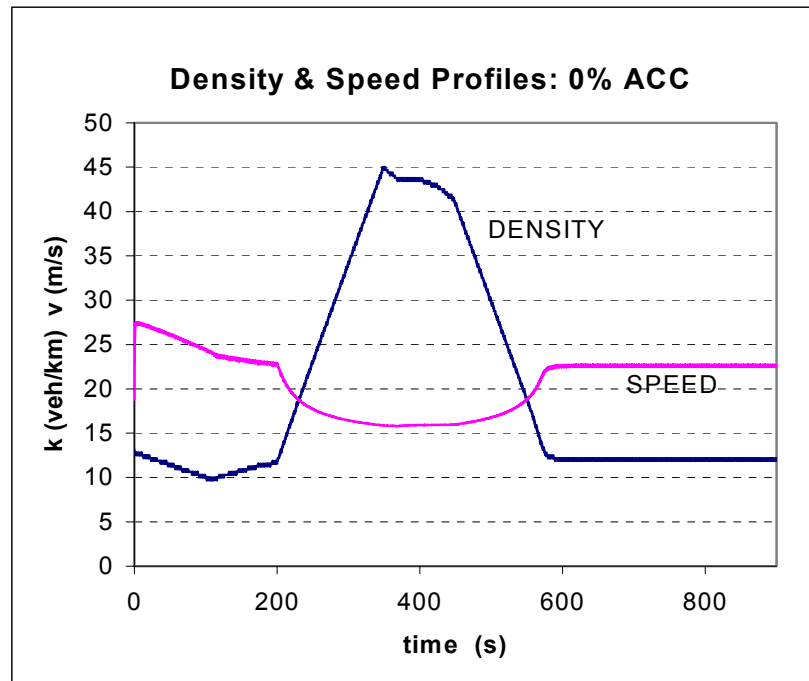
Furthermore, the restored platoon is running under a one-second time headway, which is smaller than that of normal condition. So there is a capacity gain that can compensate the loss caused by braking. It should be noted that this gain could only be obtained when the normal running of traffic is below the capacity of the system. This result shows that ACC has the potential to stabilize the traffic under small disturbances.

### 3.4 SYSTEM RESPONSE TO EXTERNAL PULSE

What we are most interested in is the response of the mixed traffic to the external disturbance that is generated by the pulse demand as shown in the Figure 5. This is because this kind of disturbance is a typical case in real traffic. The scenarios with

different penetration of ACC are simulated and the results are shown from Figure 10 to Figure 13. As we can see:

- (a) The densities and space mean speeds of the system in the disturbance are always within a boundary and can return to normal after the pulse.
- (b) The density-speed curves of these scenarios largely comply with inverse proportional relation, while high penetration of ACC can increase the system speed in the pulse.
- (c) The density-flow rate curves show a linear relationship in the un-congested region.
- (d) High penetration of ACC vehicles can reduce the system density and the speed drop during the pulse. This means there are potential capacity gains under high penetration of ACC vehicles.
- (e) The mixed traffic has larger speed oscillations than the cases of pure ACC traffic or pure manual traffic. The oscillations may come from the different acceleration behaviors among ACC vehicles and manual vehicles.



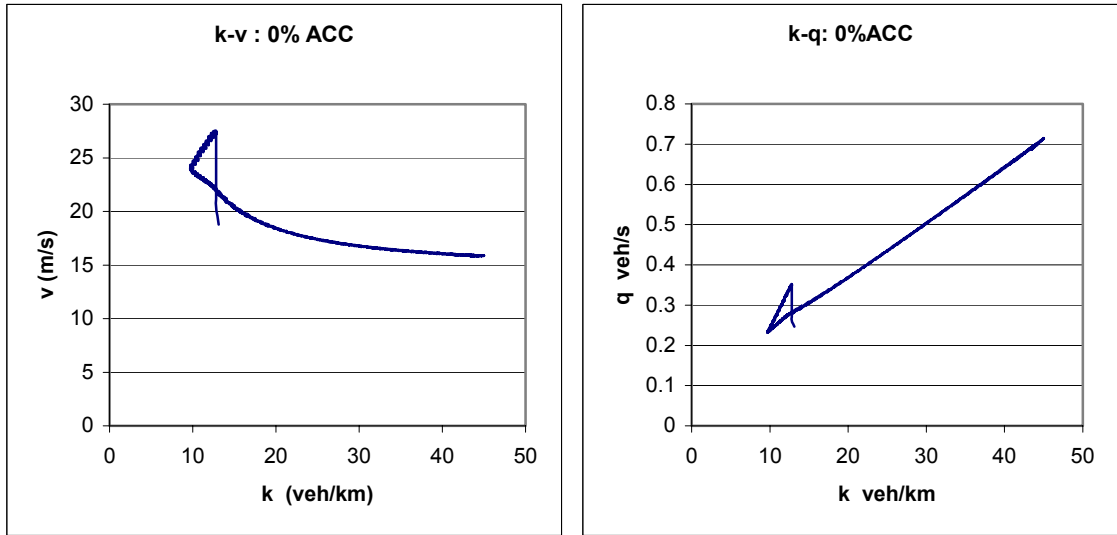


Figure 10. Response of 0%ACC system to External Impulse

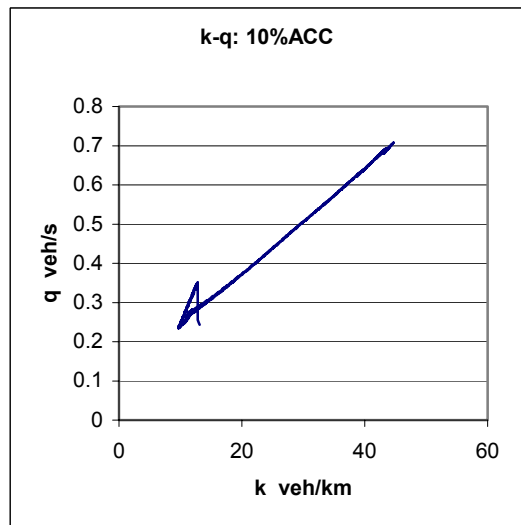
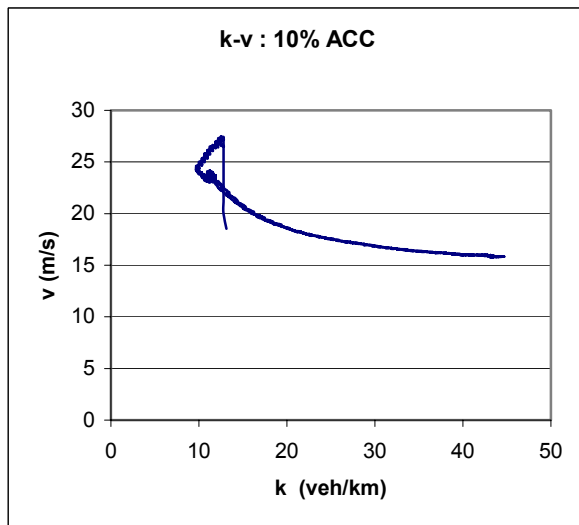
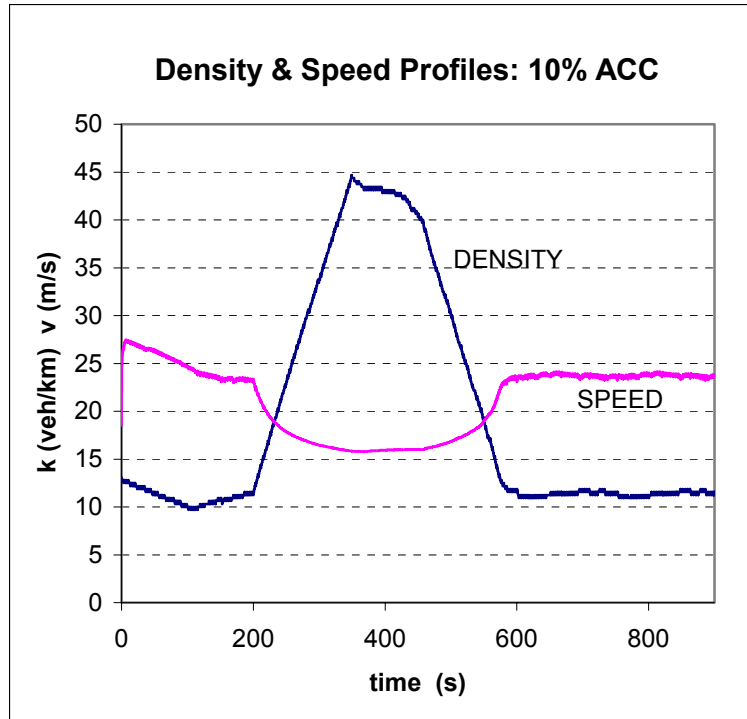


Figure 11. Response of 10%ACC system to External Impulse

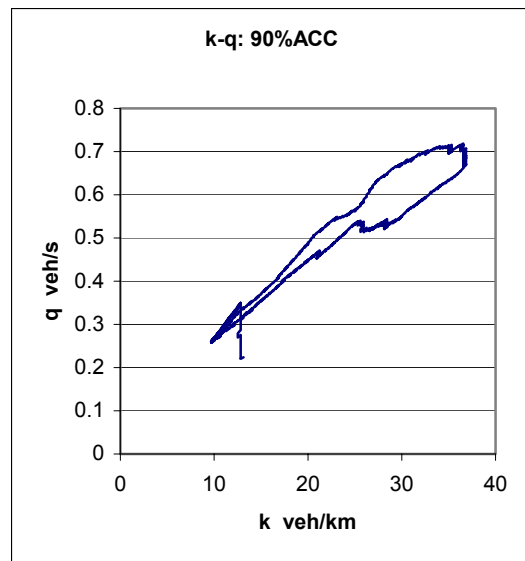
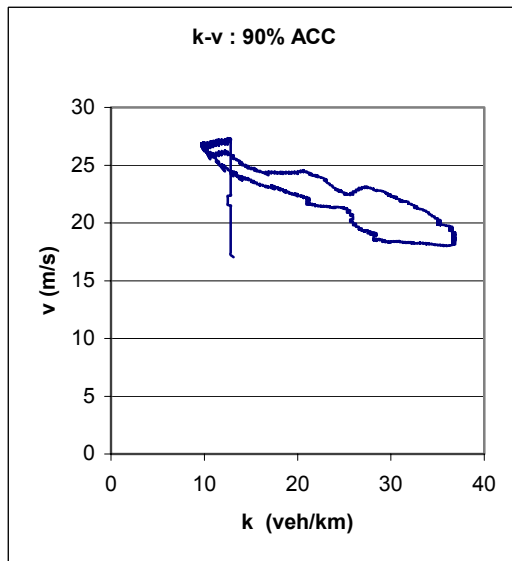
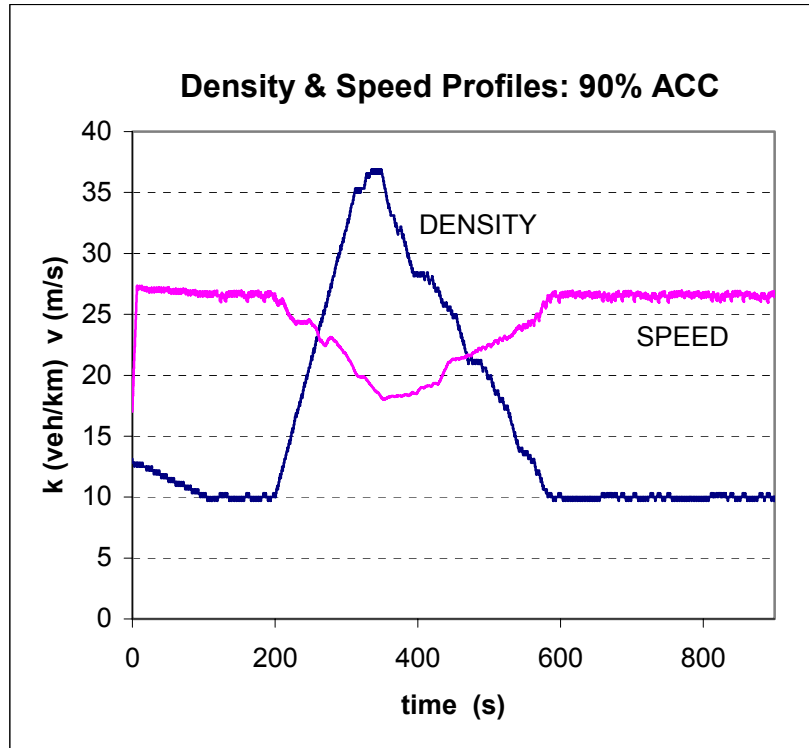


Figure 12. Response of 90%ACC system to External Impulse

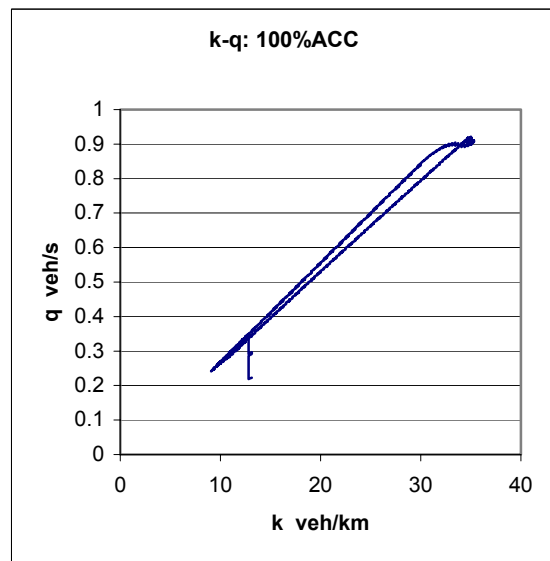
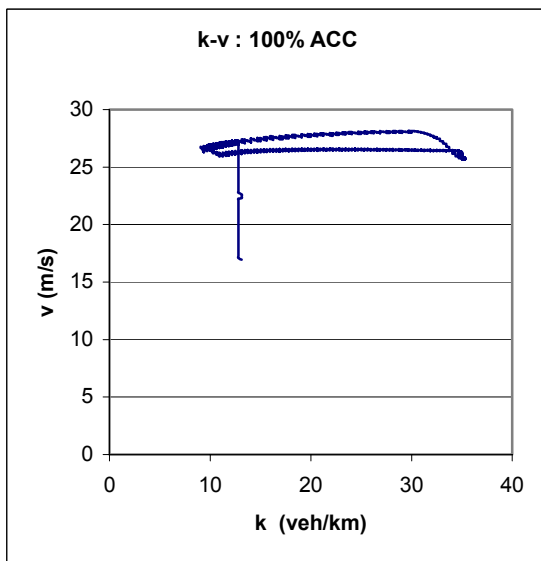
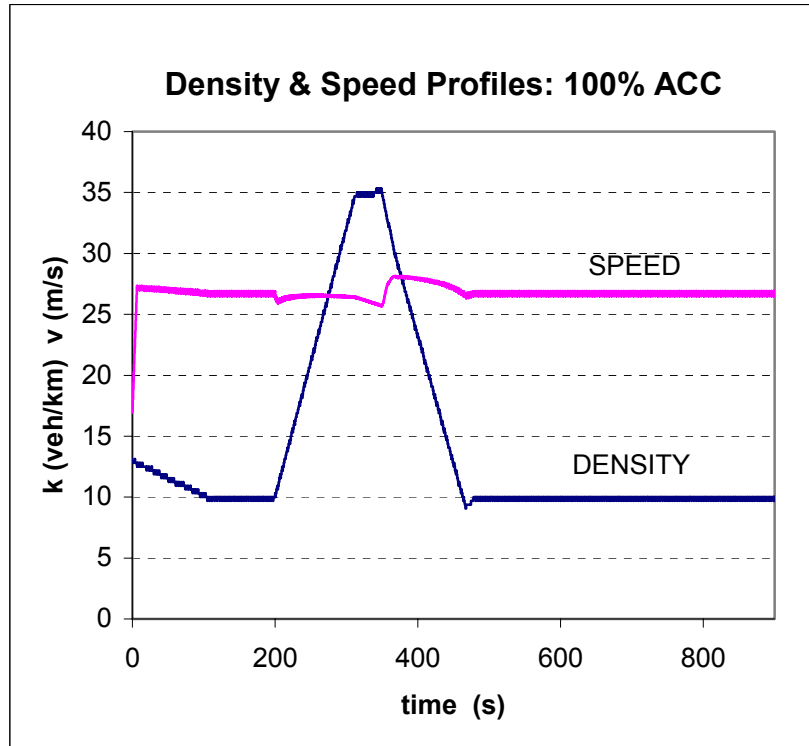


Figure 13. Response of 100%ACC system to External Impulse



### 3.5 SPEED PROFILES OF TRAFFIC FLOW WITH DIFFERENT ACC PENETRATION

After imposing the same disturbances in the system with different ACC vehicle penetrations we can compare the result speed profiles and get the impacts of ACC on the mixed traffic, as shown in Figure 14.

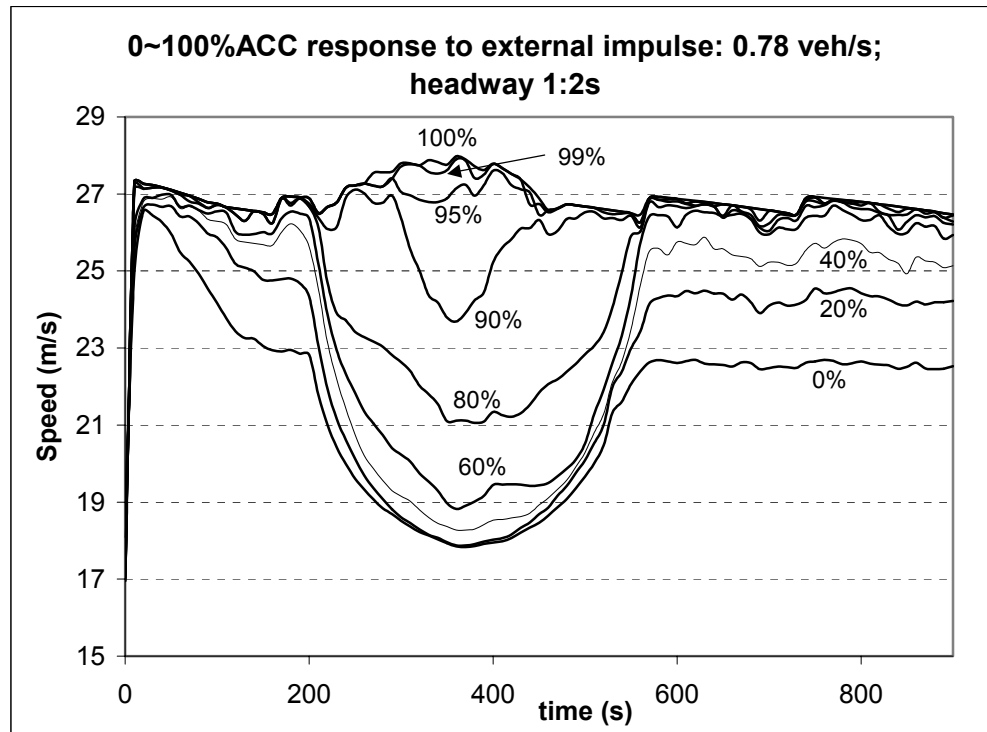


Figure 14. Speed Profiles under Different ACC Penetrations

As we can see, the penetration of ACC will significantly affect the speed profiles:

- The system uses less time to get to the normal running state with higher ACC penetration;
- The system with higher ACC penetration uses less time to restore to normal state after a disturbance by the external pulse;
- The reductions of speed drop in the pulse are not linear with ACC penetration. The most remarkable change is happened between 90% and 100% penetration. This means that high penetration of ACC reduce speed loss.
- A questionable result in this graph is the speed profile with 100% ACC penetration. The Space mean speed increases instead of decreasing in the pulse. A tentative explanation of this phenomenon is that because of the high inflow rate of the demand,

more vehicles on the road accelerate to the maximum speed than that under the normal case. In other words, a smaller portion of vehicles on the road are in the process of acceleration. As we can see in Figure 15, the proportion of low speed vehicles is zero in the peak of the speed profile. In the calculation, a vehicle with the speed lower than 20 m/s is called a low speed vehicle. Thus a higher mean speed is obtained in the peak where most vehicles are high speed. However, at the peak of the pulse, some vehicles cannot enter the system. They are queued at the bottleneck waiting to enter the system and are not counted.

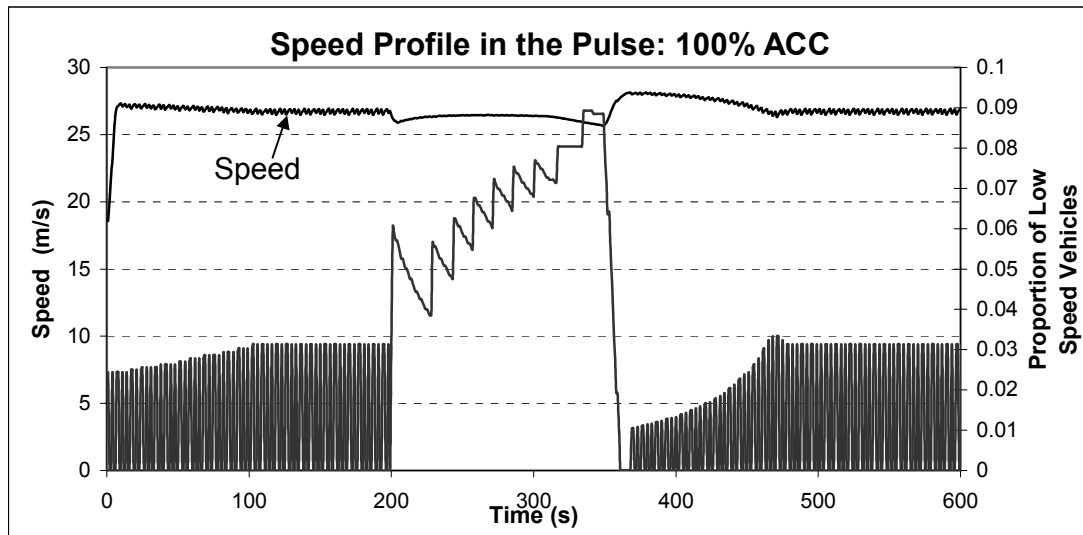


Figure 15. Speed Profile in Pulse

### 3.6 K-V AND K-Q RELATION IN MIXED TRAFFIC

The typical relationships among density, flow rate and space mean speeds are meaningful in analyzing the impacts of ACC on the traffic system. In our work, two types of these relations result from the simulation.

The first k-v and k-q relations are obtained from the dynamic process that the system encounters a saddle demand and is restored to normal state. Figure 16 shows the k-q and k-v curves for a 100% ACC system that encounters an over-capacity demand. It is shown that k-q curve is linear below capacity, and descends and ascends in the saddle demand part. In contrast, the k-v curve is nearly constant in under-capacity part. That means a pure ACC system can keep the free-flow speed before entering the congested region. Figure 17 compares the cases that 0% ACC system and 100% ACC system encounter at

near-capacity saddle demand. It is obvious that 100% ACC system has a higher speed and lower density than a 0% ACC system.

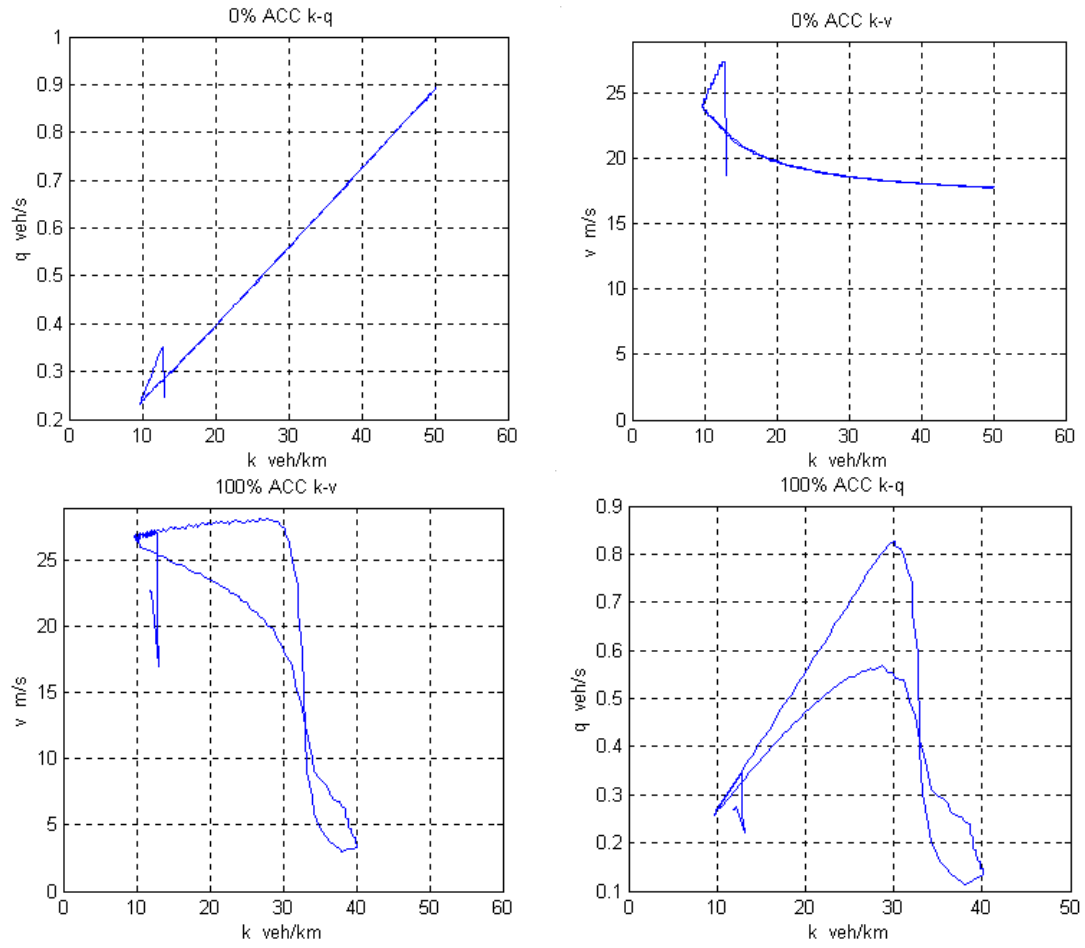


Figure 16. k-q & k-v Relation of Dynamic Process (Beyond Capacity)

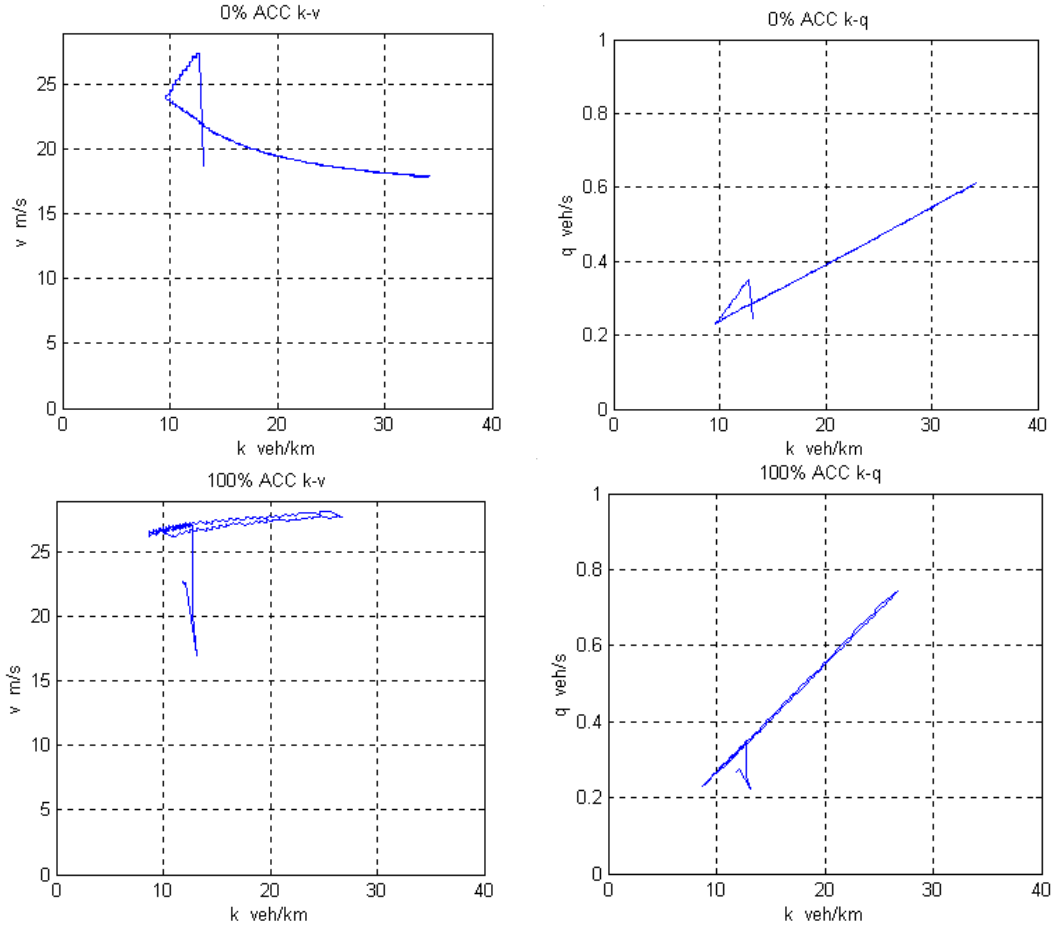


Figure 17. k-q & k-v Relation of Dynamic Process (Under Capacity)

### 3.7 INFLUENCE OF HEADWAY OF VEHICLES

Because we use constant time gap ACC in our simulation, the preset headway will determine the throughput of the system. Mean time headway can be computed as:

$$h_a = h_{acc}p + h_{man}(1 - p) \quad (18)$$

where:  $h_a$ : average headway

$h_{acc}$ : headway of ACC vehicles

$h_{man}$ : headway of manual vehicles

$p$ : proportion of ACC vehicles

Throughput for semi-automated vehicles with ACC can be obtained from:

$$q = 3600 / h_a \quad (19)$$

So we can increase the throughput by reducing the preset headway of ACC vehicles.

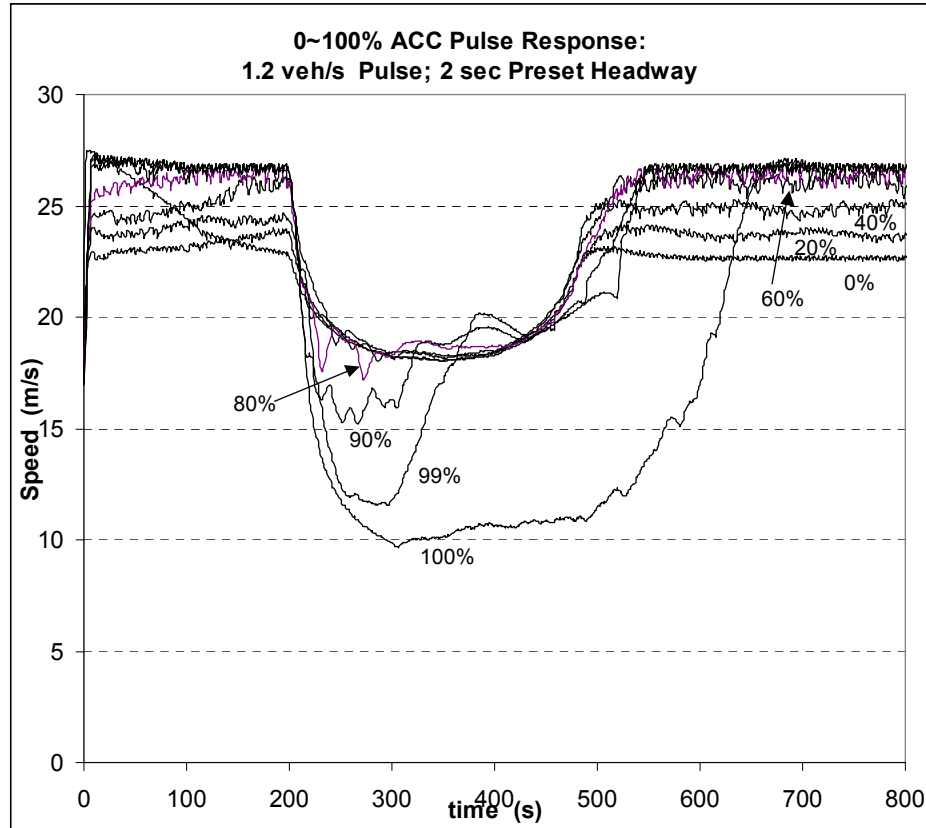


Figure 18. Speed Profiles under Different ACC Penetrations

On the other hand, there is a serious disadvantage of constant time headway control. If the demand flow rate is higher than the inverse of the preset headway of ACC vehicles, a rapid drop of speed will happen, as we can see in Figure 18. In this case, the differences of the system in pulse with various proportions of CTH vehicles are rather small and the benefits of high penetration of ACC are non-existent. There is an adverse effect if penetration of ACC is higher than a limit. This result shows CTH is not capable of reducing congestion in high demand condition.

### 3.8 THE VARIANCE OF SPEED IN THE EQUILIBRIUM STATE

The ripples in the pattern of the speed can be evaluated by the variance. The speed discussed here is the space mean speed in the equilibrium state. It seems that CTH ACC vehicles will generate more oscillations in the patterns of the speed, as shown in Figure 19. This effect is more serious if the proportion is very high (greater than 95%). In the stable range with low proportion of CTH vehicles, the variance is always small.

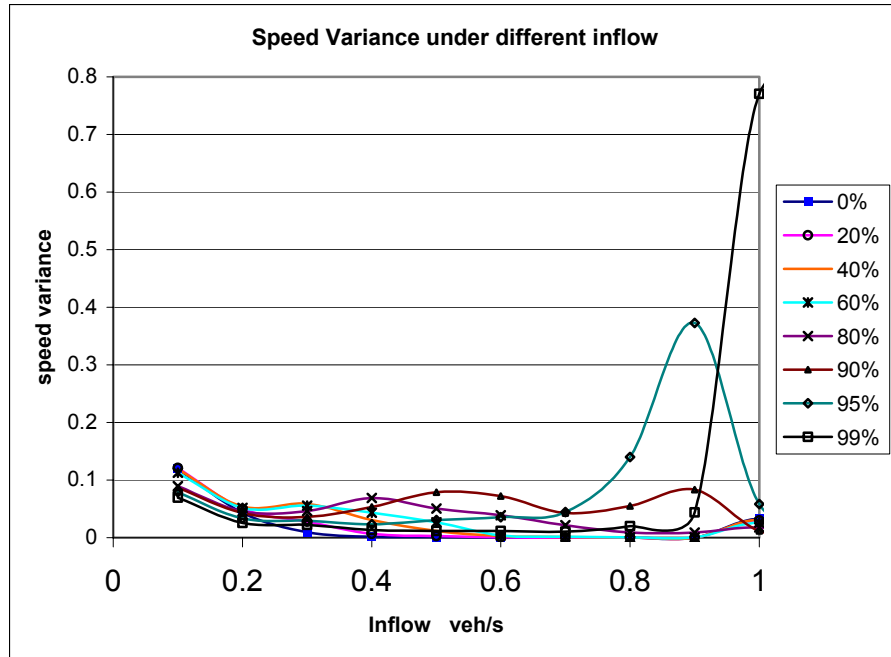


Figure 19. Speed Variance under Different Inflow and Different ACC Penetration

On the other hand, the speed variances discussed here may not accurately represent the real value, because the road is rather short (3.2 km) and the number of vehicles is small. We can expect smaller variance in a larger system.

### 3.9 QUEUE ON THE ROAD AND TRAVEL TIME

Another method to evaluate mixed traffic is to measure the length of queue and the travel time of vehicles. Figure 20 compares the number of vehicles on the road under different ACC penetrations. In this case, the pulse of the inflow rate, as shown in Figure 5, is just the upper limit of ACC capacity. As we can see, the numbers of vehicles on the road during high ACC penetration scenarios are smaller than that of pure manual traffic in the pulse. It means less congestion in this section of the road. On the other hand, the duration of congestion is shorter with high ACC penetrations than with pure manual traffic.

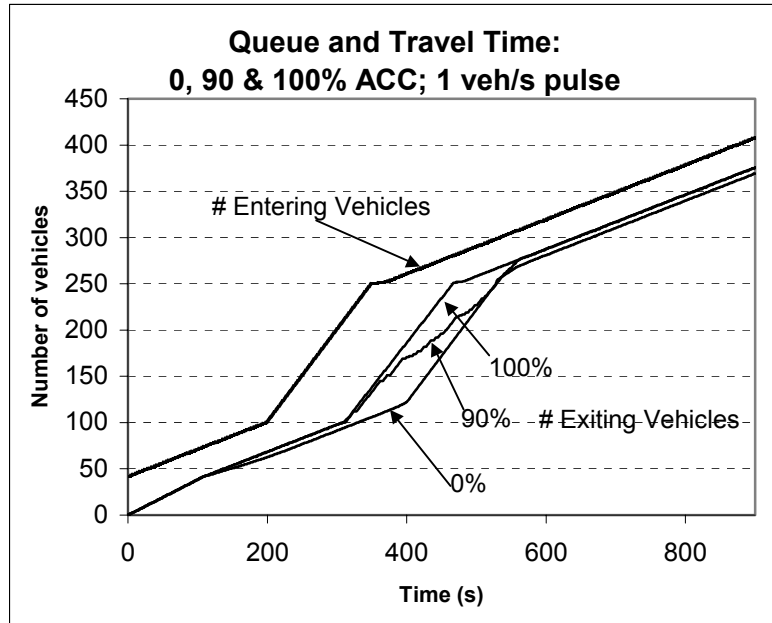


Figure 20.

As we discussed before, it is a property of CTH ACC system that it cannot cope with high demand flow rate beyond its capacity, which is determined by its preset headway. In the case that the inflow rate is higher than the capacity of ACC, we see a decrease of capacity. As shown in Figure 21, the numbers of vehicles under high ACC penetrations are larger than the former cases.

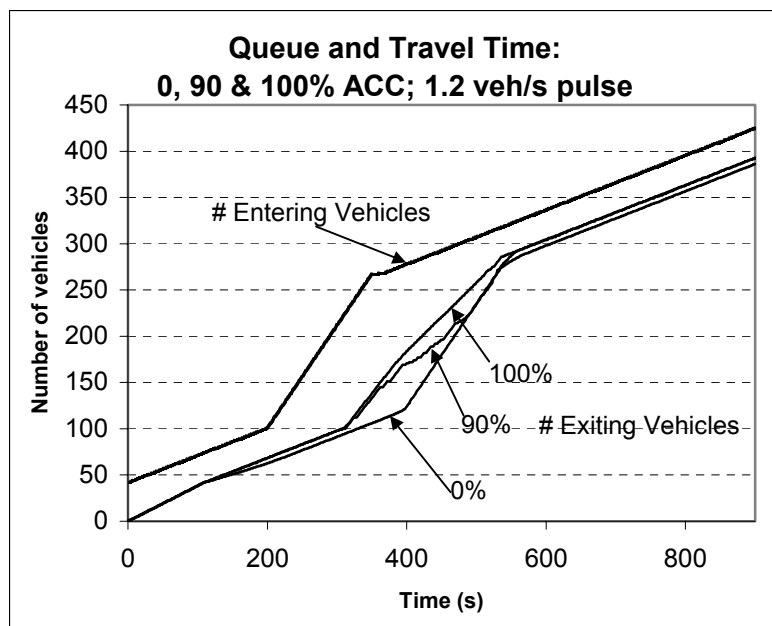


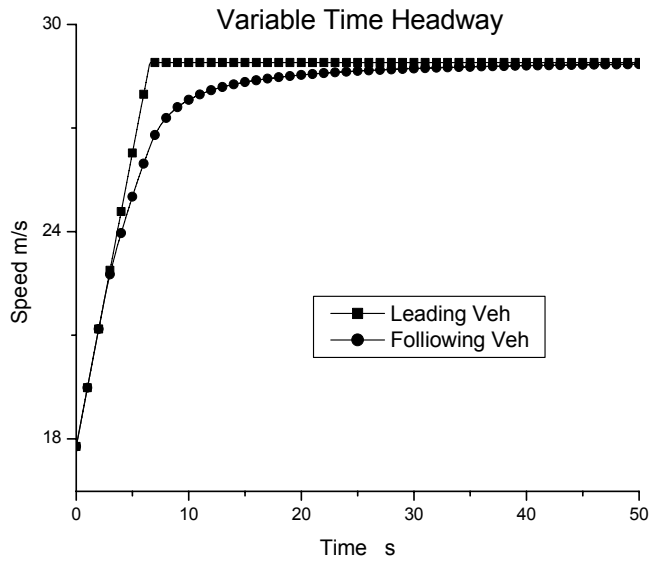
Figure 21.

## **4 COMPARISON OF VARIABLE TIME HEADWAY AND CONSTANT TIME HEADWAY**

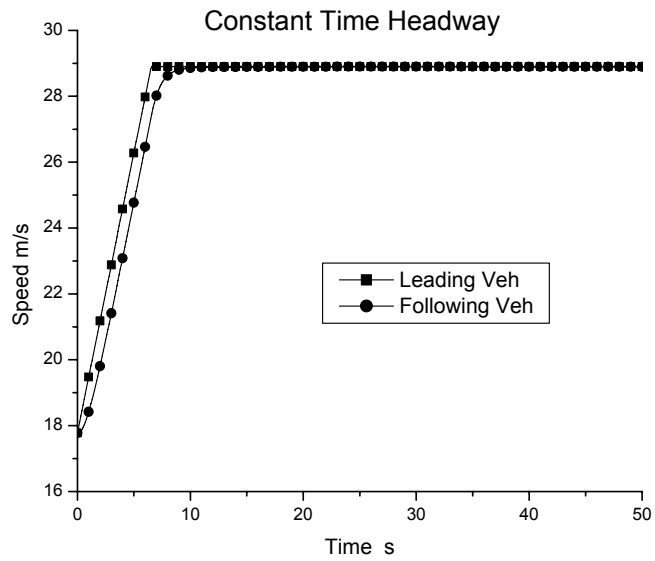
### **4.1 SINGLE VEHICLE FOLLOWING BEHAVIOR**

The typical single car following behaviors is shown in Figure 22. In these simulations, the preset time headway of CTH vehicle is 1 second. The two vehicles in the pair start up with the same initial speed and with a 20 meters distance. It is shown that vehicle controlled by VTH has slower response and takes a relatively long time to get to steady state. On the other hand, all vehicles can ultimately attain enough distance and maintain the constant time-gap. For CTH vehicles, it happens shortly after the arrival of the following vehicle. For VTH vehicles, it happens after two vehicles get to the maximum speed.





(a)



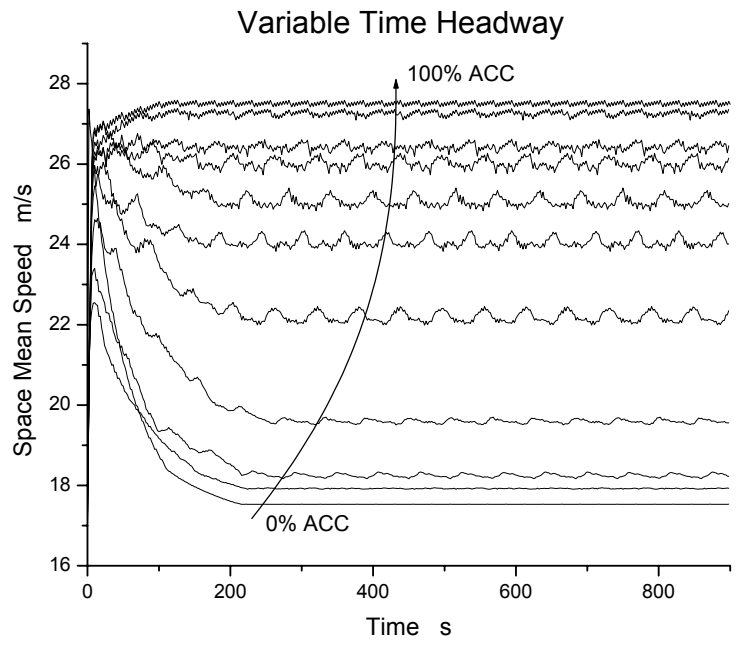
(b)

Figure 22. Single Vehicle's Following Behavior

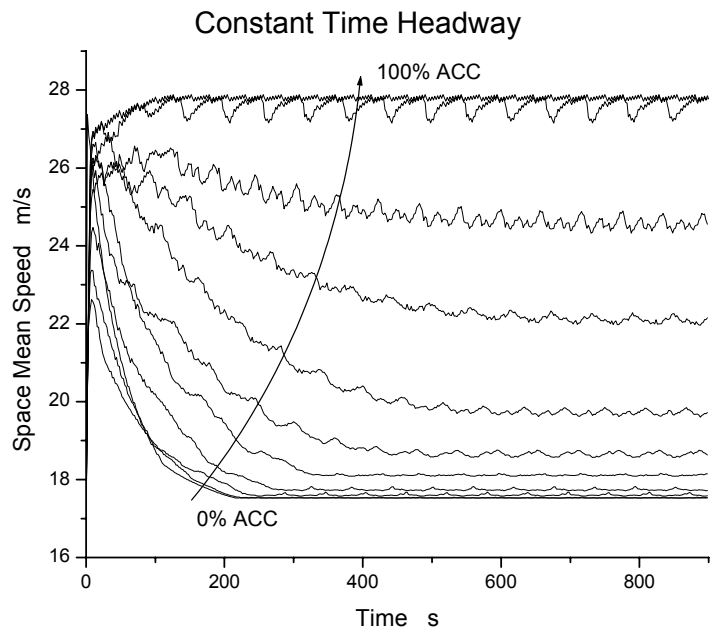
## 4.2 SPEED PROFILES OF TRAFFIC FLOW WITH DIFFERENT ACC PENETRATION

Under different constant demands, the mixed traffic of VTH cars performs better than those of CTH cars. As shown in Figure 23, under the same demand, the speed of VTH traffic is always higher than CTH traffic except the 100% case, and they always have shorter response time to reach the steady state.

The mixed traffic has larger speed oscillations than the cases of pure ACC traffic or pure manual traffic. The ripples in the pattern of the speed can be evaluated by the variance. The speed discussed here is the space mean speed in the equilibrium state. It seems that CTH control ACC vehicles will generate more oscillations in speed, as shown in Figure 24. This effect is more serious if the proportion is very high (greater than 95%). In the stable range with low proportion of CTH vehicles, the variance is always small. On the other hand, a little higher speed variance is found with VTH vehicles, as shown in Fig. 26. But this phenomenon is reversed in the cases of very high ACC penetration, such as 99% and 100%.



(a)



(b)

Figure 23.

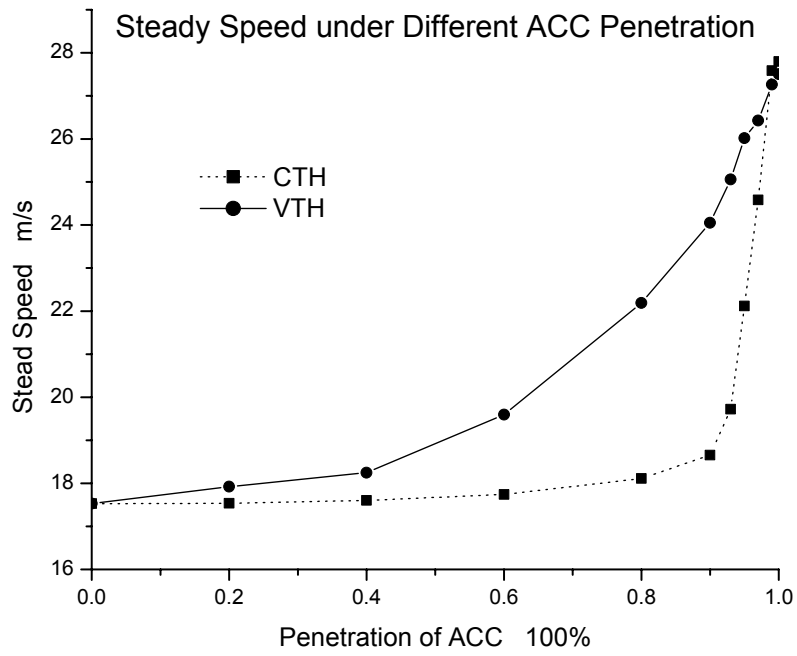


Figure 24.

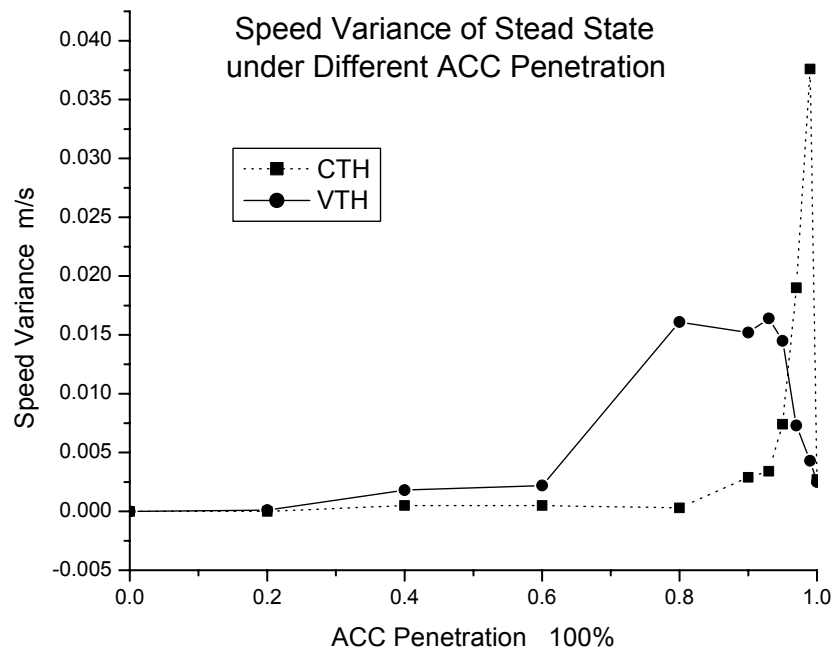


Figure 25.

### **4.3 SYSTEM RESPONSE TO EXTERNAL PULSE**

After exerting the same disturbances in the system with different ACC vehicle penetrations we can compare the speed profiles and get the impacts of ACC on the mixed traffic, as shown in Figure 26. As we can see, the penetration of ACC will significantly affect the speed profiles:

- (a) The system is restored to the normal state more quickly with higher VTH penetration than with CTH.
- (b) High penetration of VTH can reduce the system density and the speed drop during the pulse compared to a similar penetration of CTH cars. Under high demand, the drop of space mean speeds of the VTH traffic in the disturbance are always smaller than manual traffic and can easily return to normal after the pulse, while CTH traffic may experience serious speed drop that is even worse than that of pure manual traffic.

### **4.4 K-V AND K-Q RELATION IN MIXED TRAFFIC**

The first k-v and k-q relations are obtained from the dynamic process that the system encounters a saddle demand, which is comprised of a linearly increasing part (150 seconds) and a linearly decreasing part (150 seconds). Figures 27 and 28 show the k-q and k-v curves for a 100% ACC system that encounters an over-capacity demand. For CTH traffic, it is shown that k-q curve is linear below capacity, and descends and ascends in the saddle demand part. In contrast, the k-q curve is nearly linear for VTH traffic. That means a VTH system can keep the free-flow speed in a longer range. Figure 28 compares the k-v curves of VTH and CTH traffic encountering an over-capacity saddle demand. It is shown that VTH traffic has a higher speed and lower density than CTH traffic.

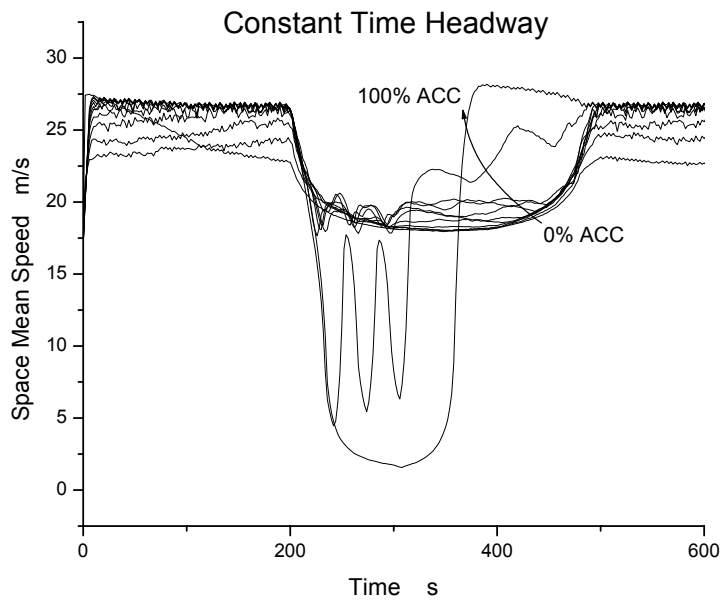
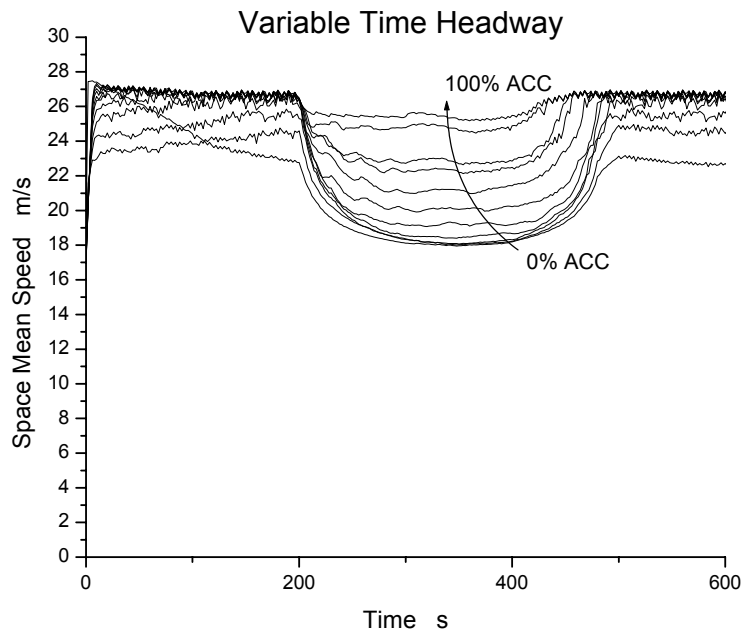


Figure 26. Speed Profiles of Traffic Flow with Different ACC Penetration

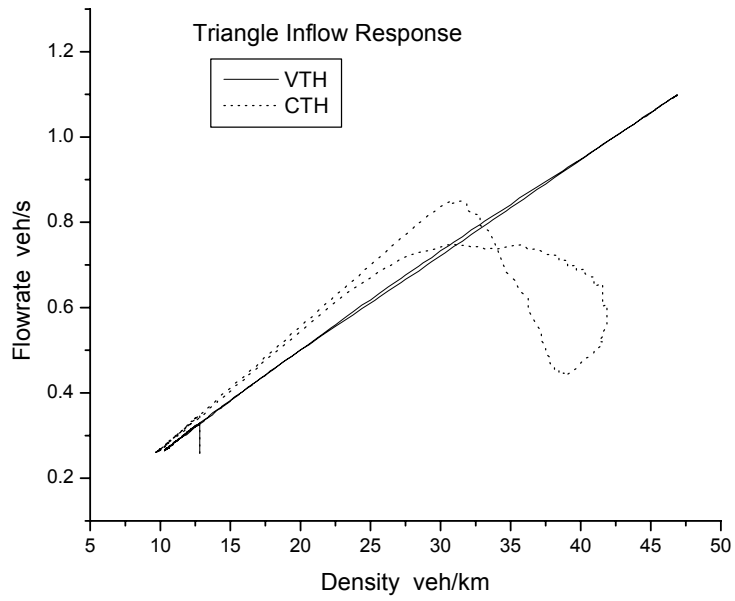


Figure 27.

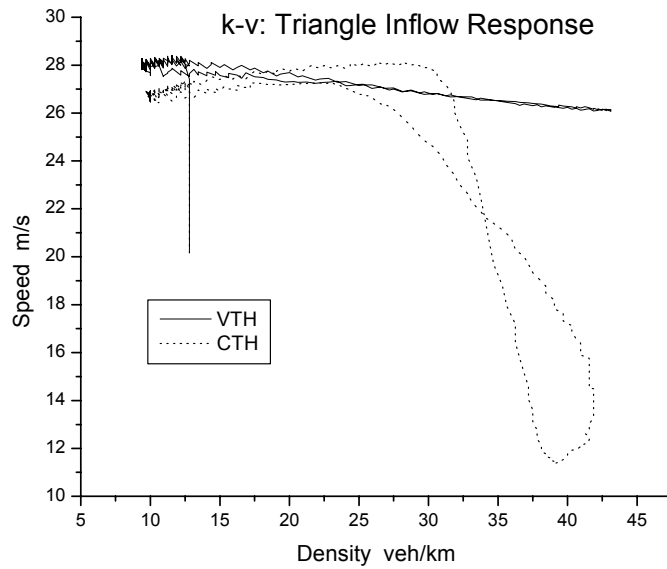


Figure 28.

Under the condition of very high demand inflow, VTH traffic decreases the speed and maintains the density until the demand is released, as shown in Fig. 30. The response process is shown in Fig. 31. As one can see, the system stops to accommodate more

vehicles after the speed gets to a low point. In this case, the inflow rate is not the indication of the demand but the reflection of system capacity.

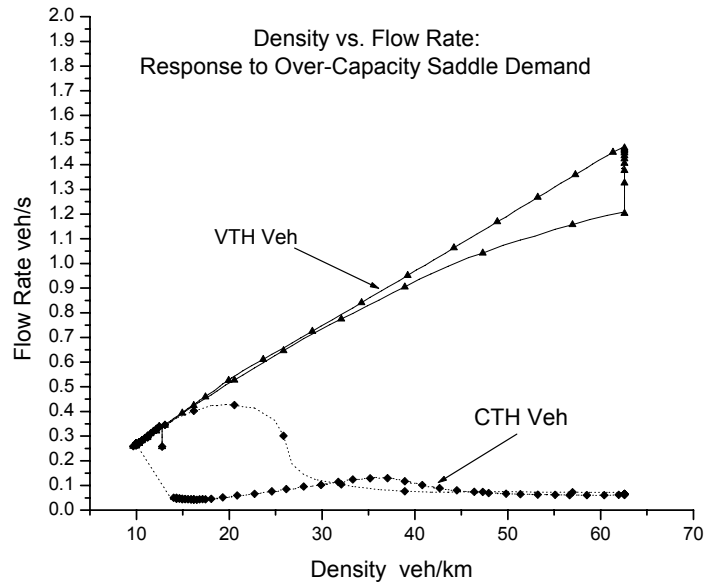


Figure 29.

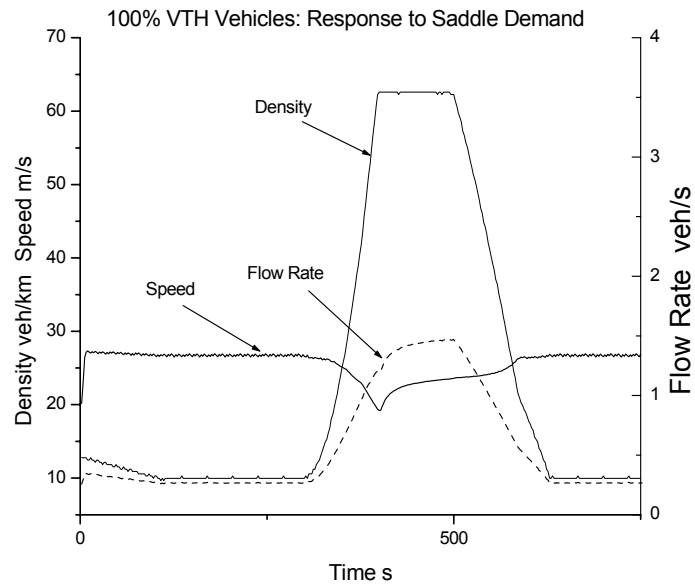


Figure 30.



## **5 SIMULATION WITH SOME RANDOM EFFECTS**

### **5.1 INTRODUCTION**

As a simplified analysis, the simulations in Chapter 3 do not include the variation of headways among ACC and manual driven vehicles. To make the simulation more realistic, a randomly chosen headway is implemented in simulation. It is assumed that each driver keeps his/her favorite headway all the time. A new attribute of vehicle class: vehicle.headway is preset in the vehicle generation procedure, which determines the headway choice of each vehicle and cannot be changed during simulation. Though it is still a simplified situation, its result is important in that it separates the impacts of drivers' personal headway choices. So we can compare them with the former results we obtained. The time headway of Gipps' vehicle is normally distributed with mean=2 sec and a given standard deviation and a 1-second minimum. Time headway of CTH vehicle is normally distributed with mean=1 sec and a given standard deviation and a 0.8-second minimum. A normally distributed random number is generated in function: float gasdev(long \*idum). Experimental results are summarized below:

### **5.2 MIXED TRAFFIC OF CTH AND MANUAL DRIVEN VEHICLES**

Five combinations of CTH and manual driven vehicles with different headway distributions are simulated, which include:

- (1) CTH=1 sec; Gipps= 2 sec, as shown in Figure 31;
- (2) CTH=1 sec; Gipps=  $\max(2+ N(0,1), 1)$  sec, as shown in Figure 32;
- (3) CTH=1 sec; Gipps=  $\max(2+ N(0,1)*2, 1)$  sec, as shown in Figure 33;
- (4) CTH=  $\max(1.0+ N(0,1)/2, 0.8)$  sec; Gipps=  $\max(2+ N(0,1)*2, 1)$  sec, as shown in Figure 34;
- (5) CTH=  $\max(1.0+ N(0,1)/4, 0.8)$  sec; Gipps=  $\max(2+ N(0,1)*2, 1)$  sec, as shown in Figure 35.

As one can see, the random headways of manual vehicles do not have much influence on the performance of traffic. In contrast, the random headways of CTH ACC vehicles greatly affect the traffic. Higher headway deviation of CTH vehicles will lead to higher speed drop and oscillations, especially when the ACC penetration is very high. In the

case of 100% CTH ACC penetration, higher headway deviation results in serious speed drop and longer time to recover.

Figure 36 presents the comparison of the average speed in the five CTH experiments; Figure 37 presents the comparison of the speed variance. As one can see, high penetration of CTH ACC increases the average speed in most of cases. But high headway deviation deteriorates this effect. On the other hand, the speed variance is not significant in most cases, except the case of 100% ACC penetration under high headway deviation. It can be concluded that these results can not provide support of the claim that CTH ACC will add traffic capacity.

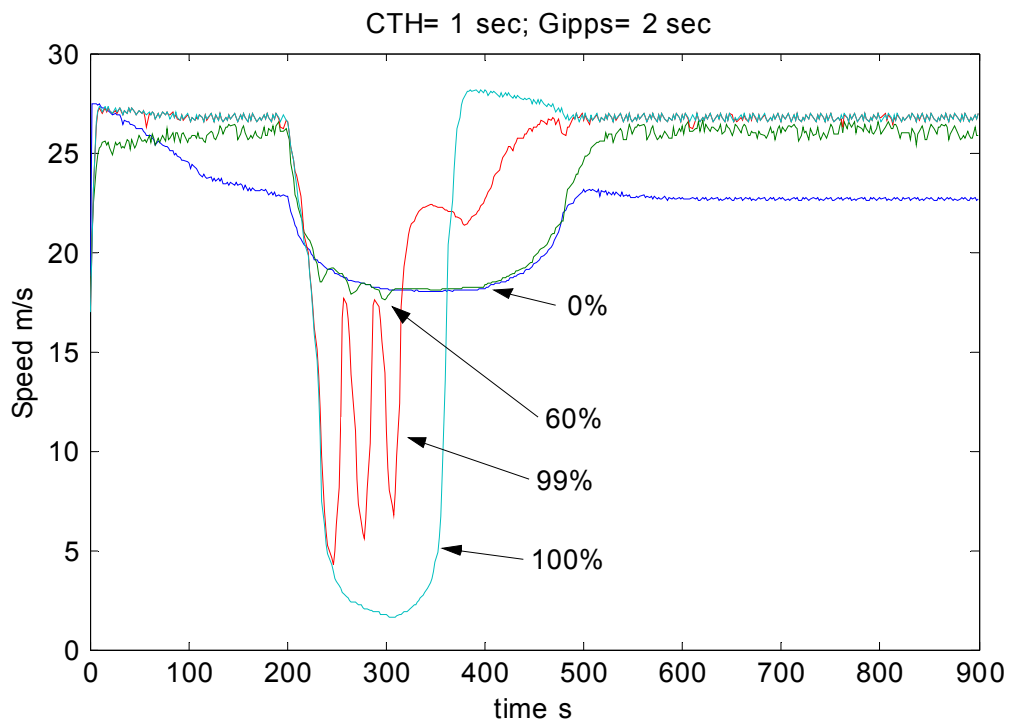


Figure 31.

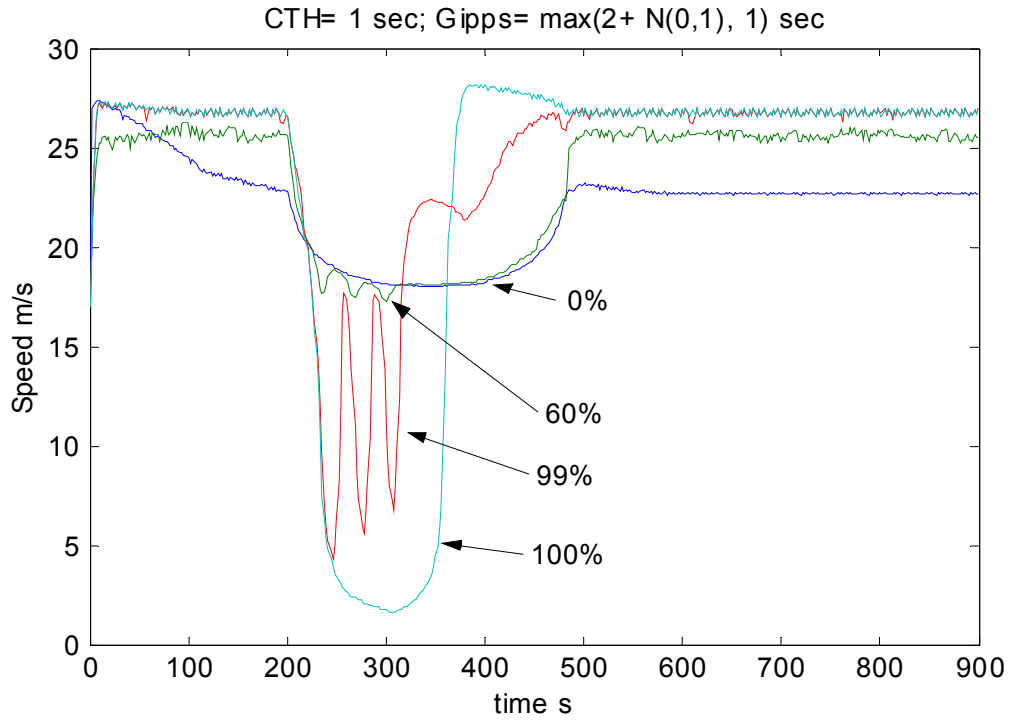


Figure 32.

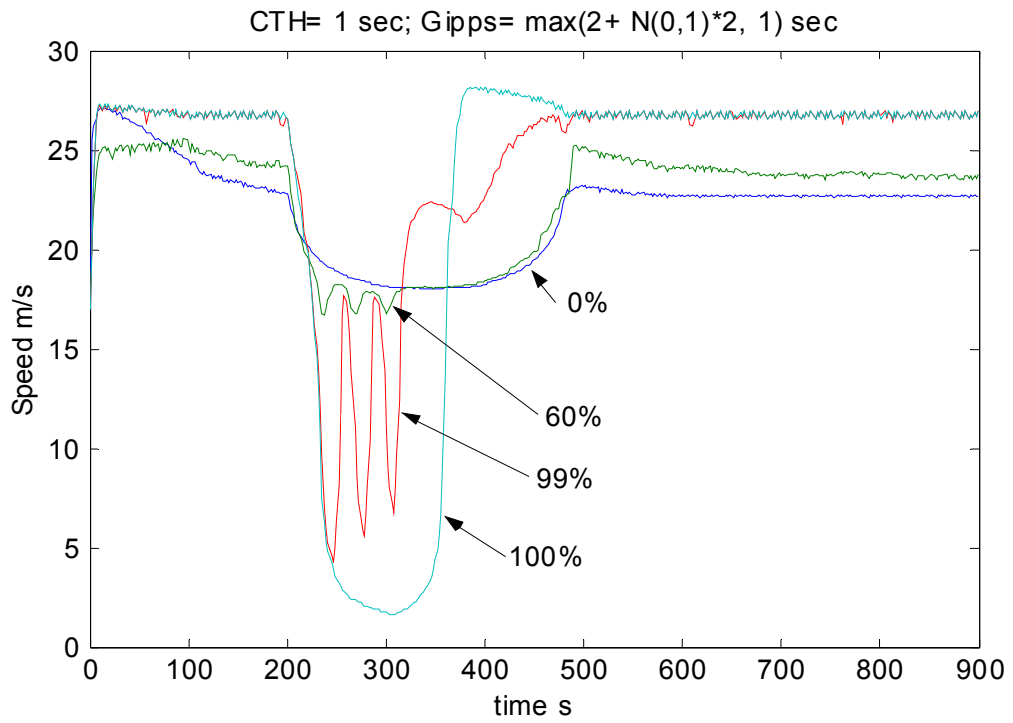


Figure 33.

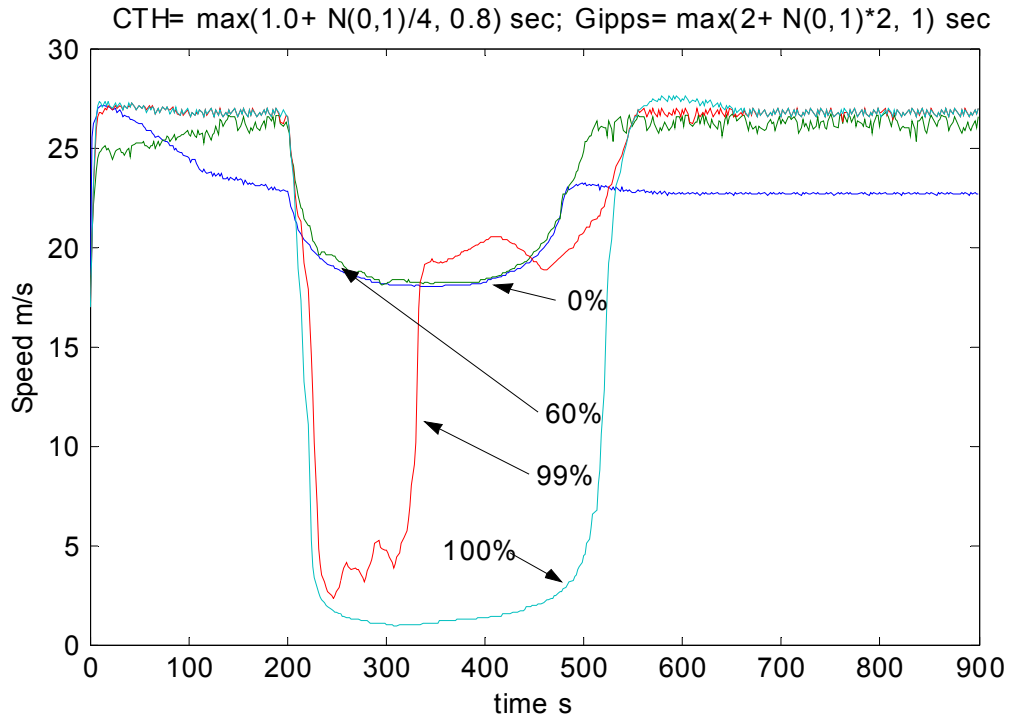


Figure 34.

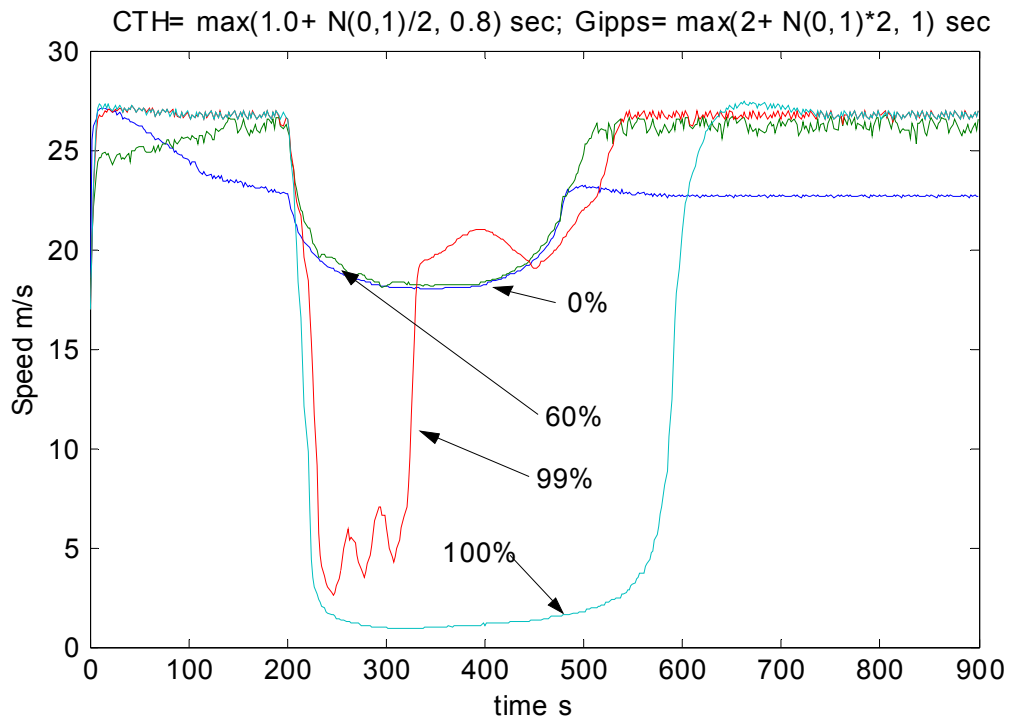


Figure 35.

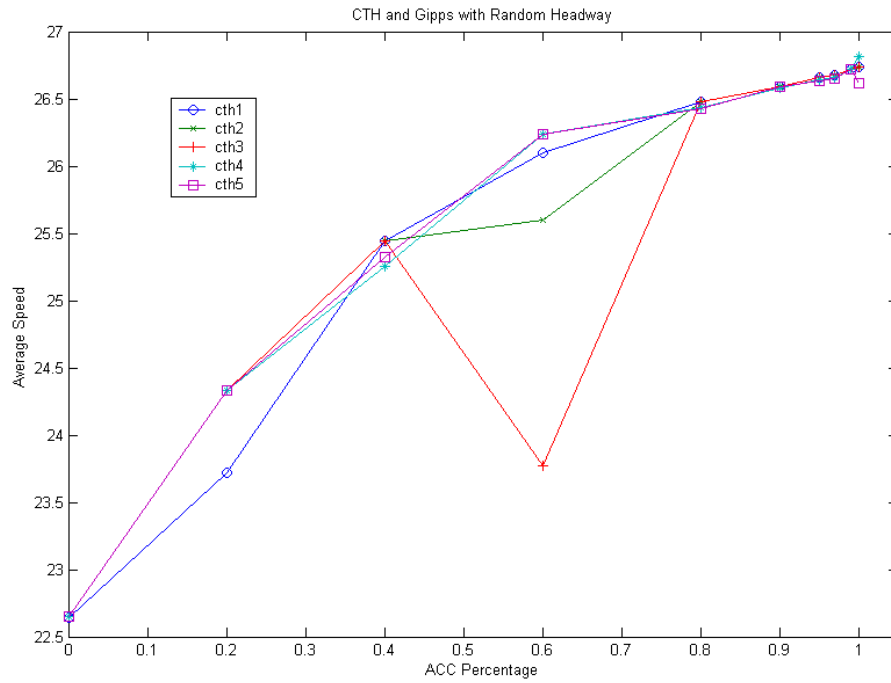


Figure 36.

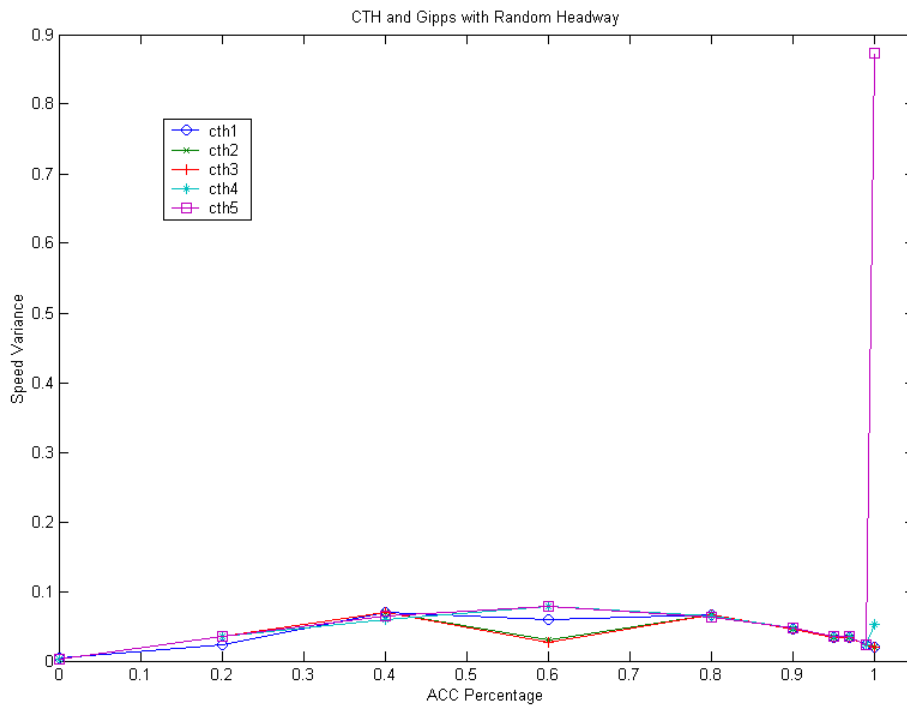


Figure 37.

### 5.3 MIXED TRAFFIC OF VTH AND MANUAL DRIVEN VEHICLES

Three combinations of VTH and manual vehicles with different headway distributions are simulated, which include:

- (1) VTH and Gipps = 2 sec, as shown in Figure 38;
- (2) VTH and Gipps =  $\max(2 + N(0,1), 1)$  sec, as shown in Figure 39;
- (3) VTH and Gipps =  $\max(2 + N(0,1)*2, 1)$  sec, as shown in Figure 40.

Because VTH does not have preset headway, its headway is always in changing. The only random factor here is the random headway of human drivers in manual driven vehicles. It is shown that VTH ACC always performs well facing different headway deviation of human driver.

Figure 41 presents the comparison of the average speed in the three VTH experiments; Figure 42 presents the comparison of the speed variance. There is no significant difference in terms of speed and speed variance. These results further justify the advantage of VTH ACC compared with CTH ACC.

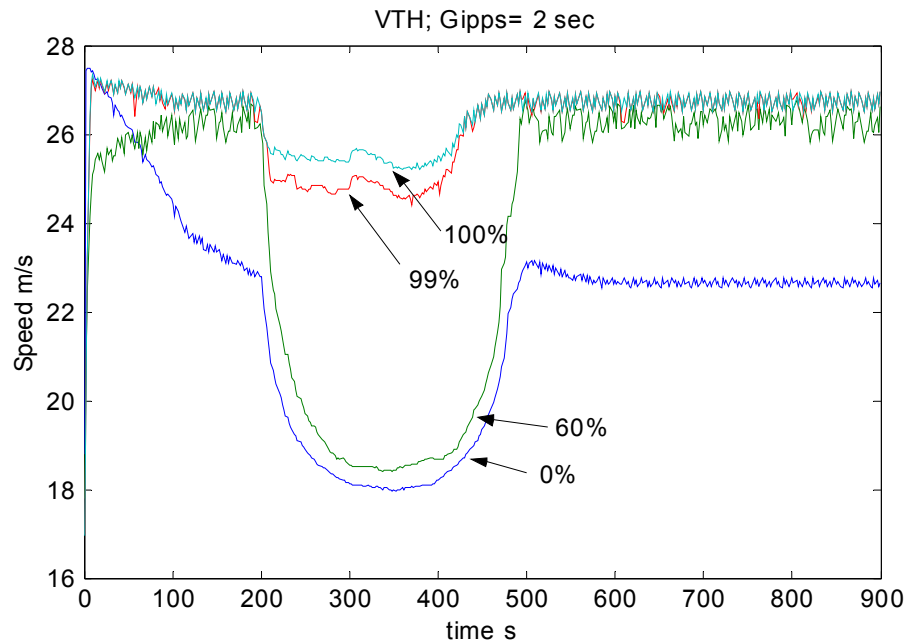


Figure 38.

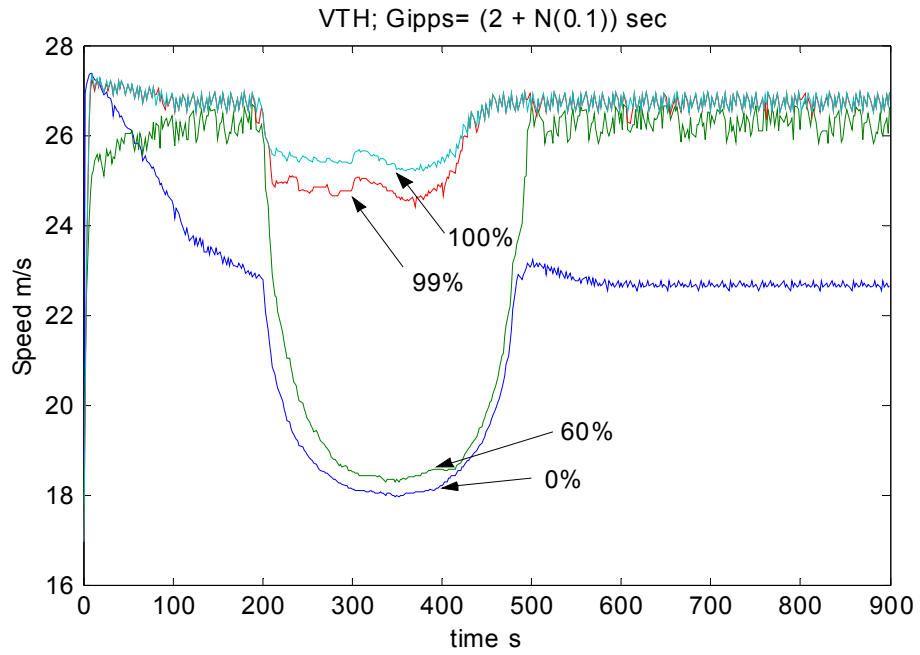


Figure 39.

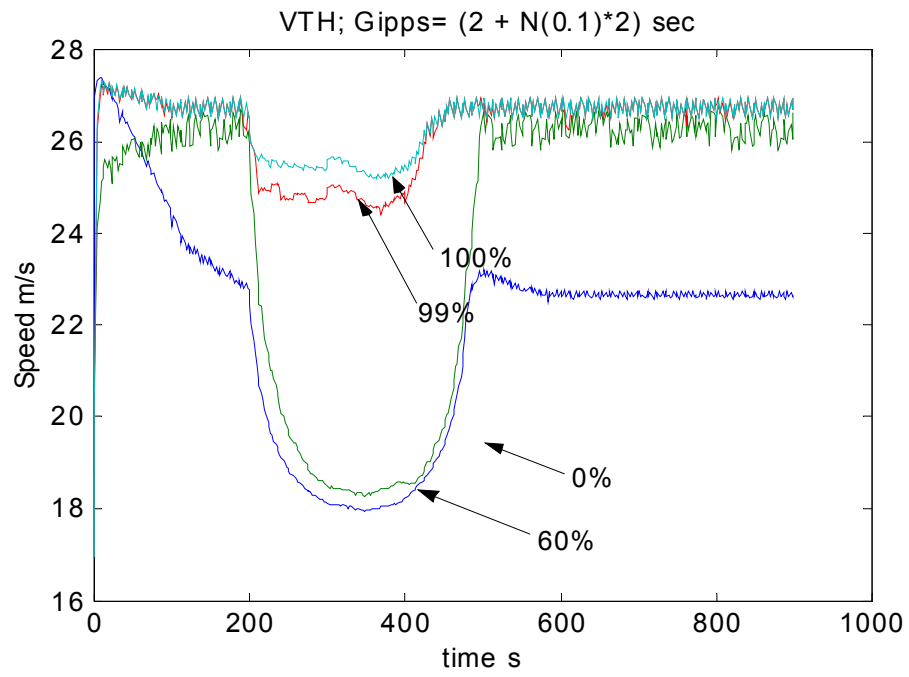


Figure 40.

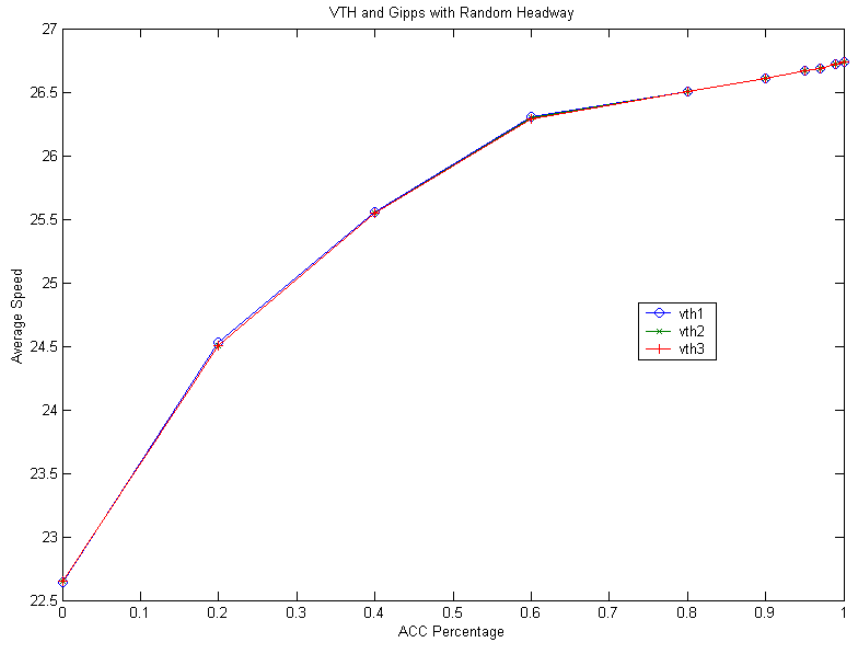


Figure 41.

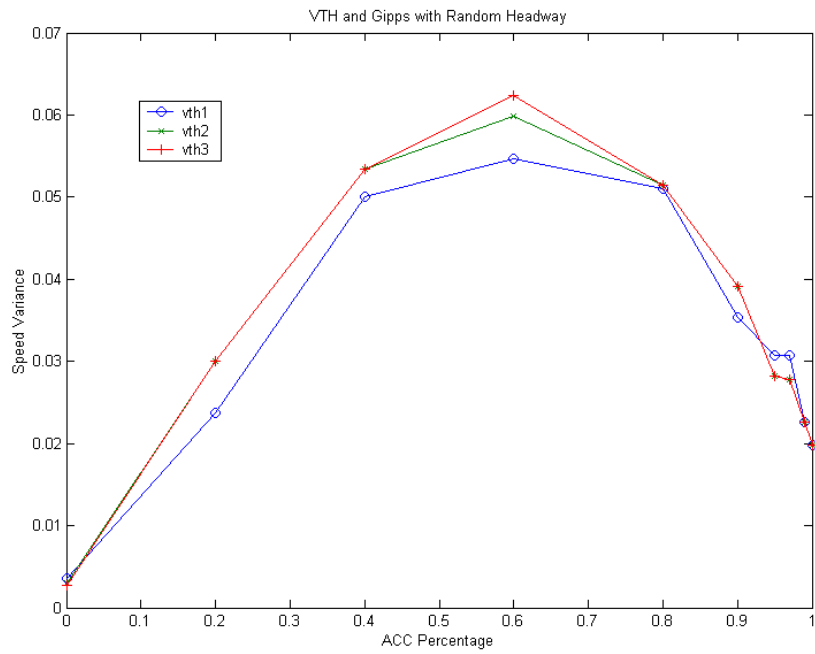


Figure 42.



## 6. SIMULATION OF MIXED TRAFFIC IN AIMSUN

Simulations of one-lane traffic shown above are some ideal scenarios. To evaluate impacts of ACC on traffic flow in a more realistic way, we need the observation of mixed traffic in multi-lane, on-ramp and off-ramp scenarios. These simulations cannot be easily realized based on the pipeline simulation program. On the other hand, commercial traffic simulation software such as AIMSUN provides a good base for complex traffic simulation (TSS 2001). GETRAM Extension provides interfaces to obtain information from AIMSUN and modify some information during running, as shown in Figure 43. In this research, we take advantage of this package to implement individual vehicle control. To realize ACC policies and control of traffic, some functions of GETRAM Extension are used.

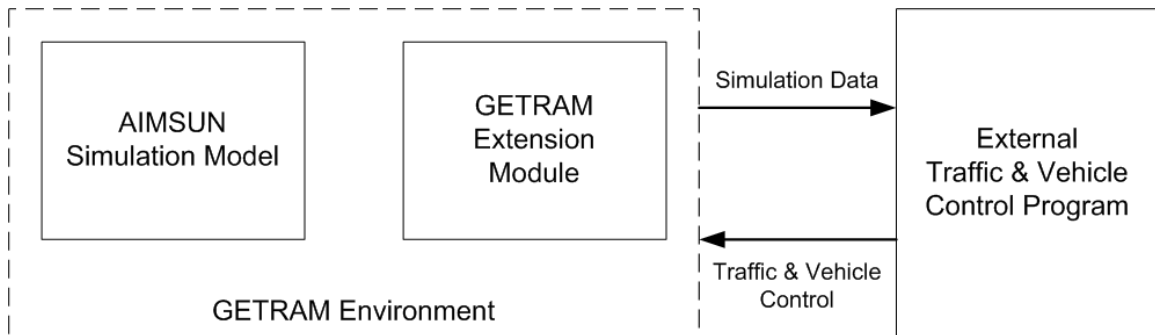


Figure 43. Function of GETRAM Extension

In the first simulation of mixed traffic in AIMSUN, a pipeline scenario is tested, with which we can compare our former simulation results. A problem with AIMSUN is that the simulation step which is also the reaction time of all vehicles is larger than 0.5 seconds. But normally the reaction time of ACC system is 0.1 second. And we must simulate the case in which vehicles are with different reaction times. To solve this problem, a modified version of AIMSUN was obtained from the developer (TSS). In this version, the simulation step ranges from 0.01 second to 1 second.

In each simulation step, the program reads the information of each vehicle on the road from AIMSUN, calculates its new states and updates them. A problem is that the only function can be used to update vehicle states is the function to modify vehicle speed. So

in this simulation, we cannot fully control vehicle behaviors. The shapes of simulation results are identical to those of former simulations, as shown in Figure 44 comparing to Figure 16(d).

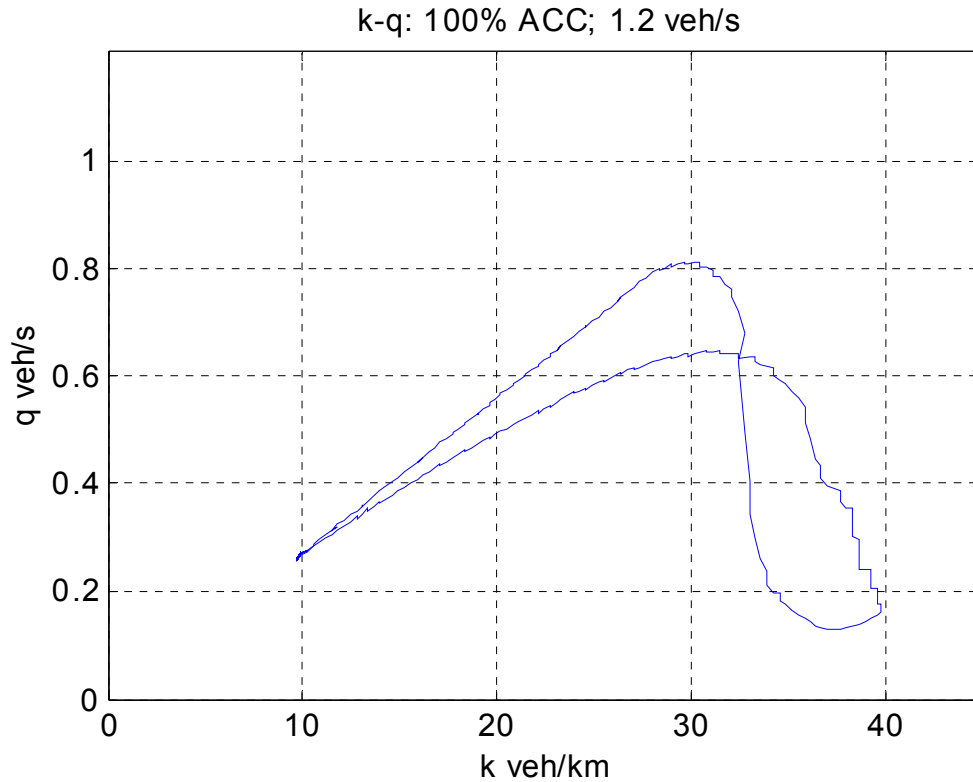


Figure 44. Simulation Results in AIMSUN: 100% CTH, 1.2 veh/s

In the second simulation, we built up a simple scenario, as shown in Figure 45, in which the characteristics of mixed traffic can be observed. In this scenario:

1. All vehicles are tracked all the time. Vehicles enter from the left side of section 1 and section 2 according to programmable demand curve;
2. If a vehicle exits from section 2, a new vehicle is “put” in section 1 at the intersection;
3. The speed of the new vehicle is the mean of speeds of vehicles before and after the initial position;
4. The vehicle put in the intersection keeps the same following mode (Gipps or ACC) as the one exits from section 2;

5. When a new vehicle is put in the intersection, its position is either 1351m (middle point) or 1343m (the left point), depending on whether there is a vehicle in the intersection;
6. If there is a vehicle in section 1 near the intersection, a “virtual” vehicle is put at the end of section 2; otherwise, the road ahead of the first vehicle in section 2 is clear. By this we means that vehicles in section 2 will be affected by the traffic condition in the intersection;
7. All vehicles adjust speeds according to states of itself and the leader. If a new vehicle is inserted in, the follower treats it as a leader in the next cycle.

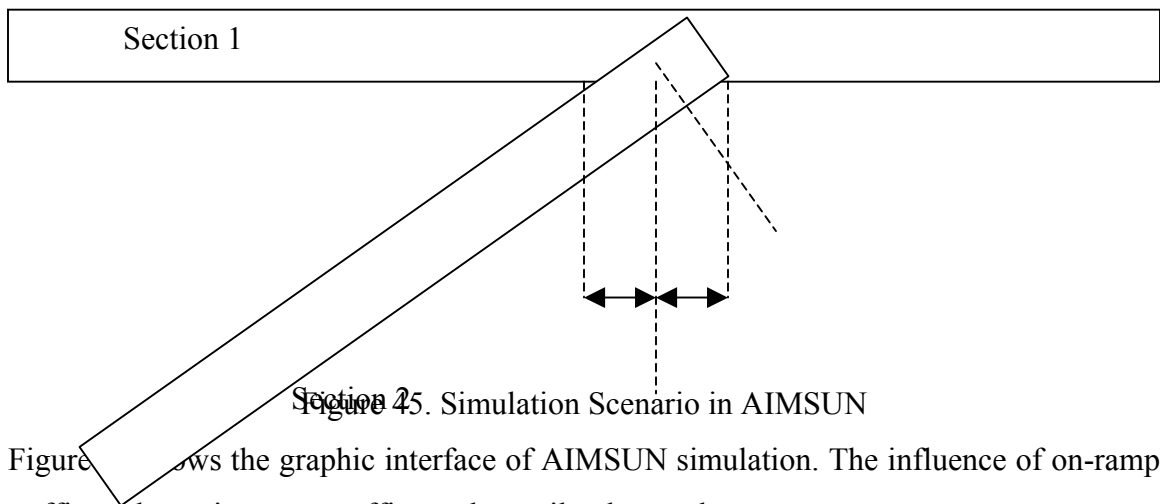


Figure 45 shows the graphic interface of AIMSUN simulation. The influence of on-ramp traffic to the mainstream traffic can be easily observed.

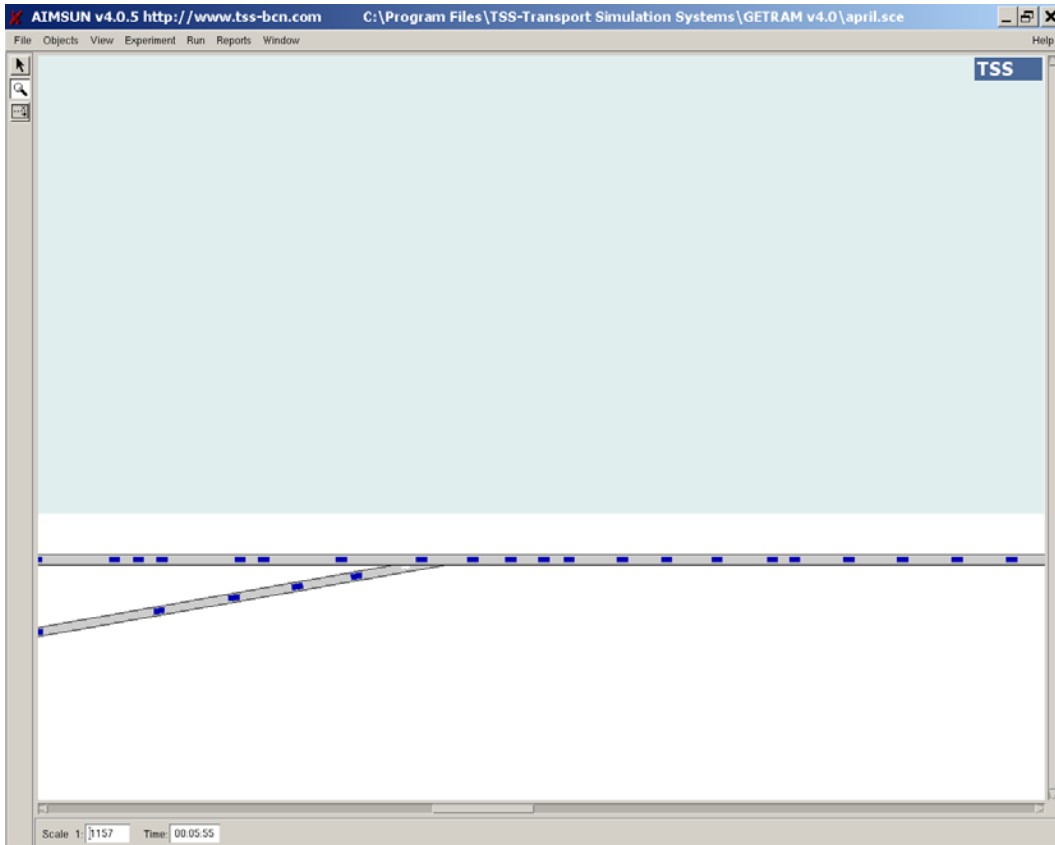
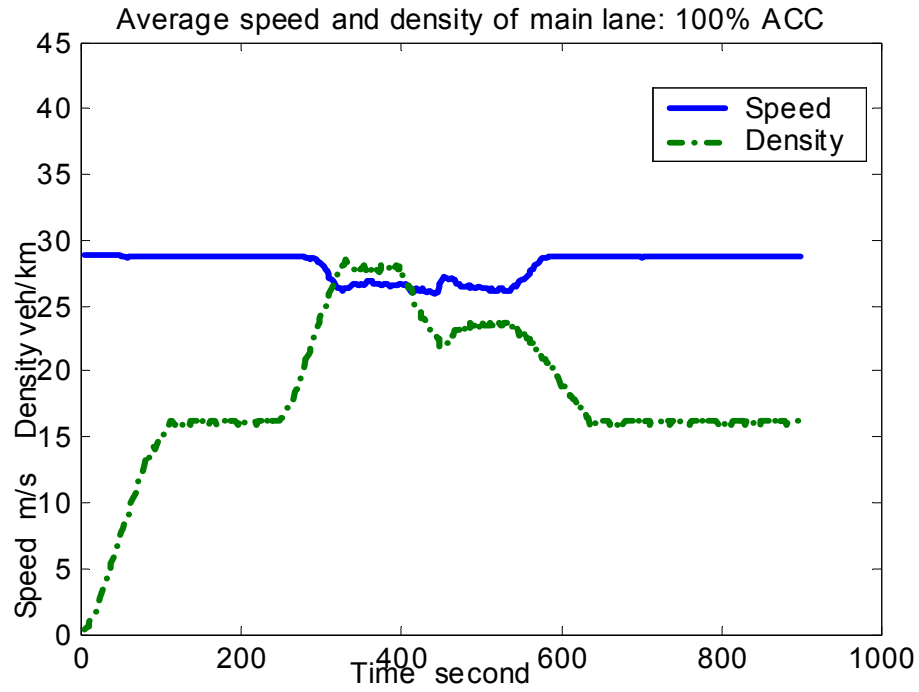
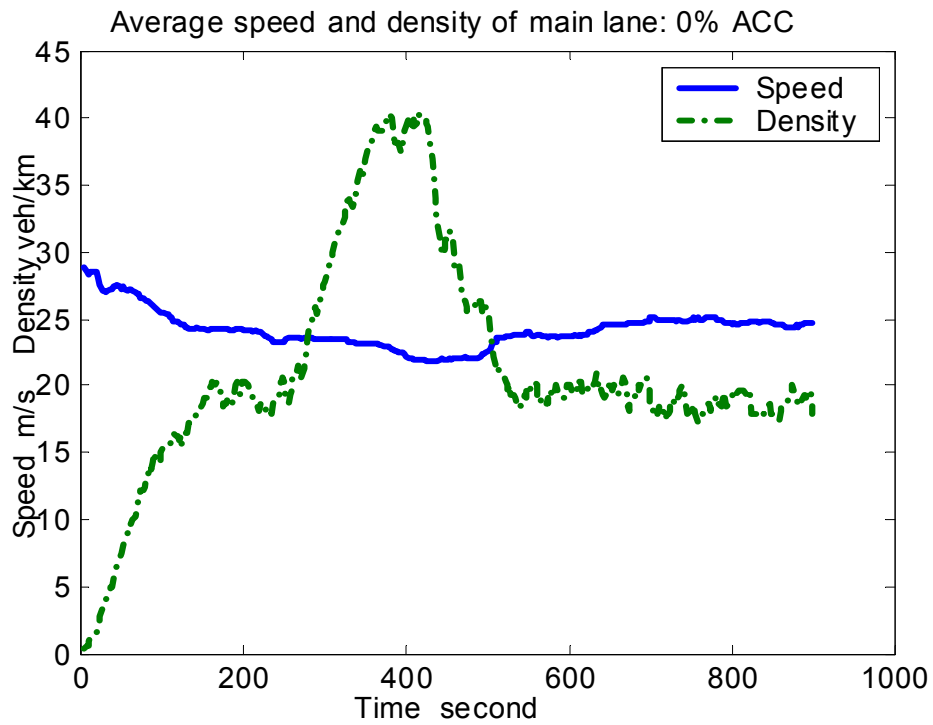


Figure 46. Simulation of Mixed Traffic AIMSUN (Screen Copy)

Figure 47 shows some examples of the AIMSUN simulation. Pulse demands are imposed in entrances both of sections, in which the normal demand is 0.3 veh/s, the demand in pulse is 0.6 veh/s. Figure 47(a) shows the average density and speed of the main lane with 100% ACC traffic while Figure 47(b) with 0% ACC traffic. These results present similar patterns as those in former simulation results. With 100% ACC traffic, the speed drop is not as serious as our former result. This is because the traffic entering the intersection from the ramp is affected by the main lane traffic. The traffic will be congested on the ramp so that the demand contributed by the ramp is limited.



(a)



(b)

Figure 47. Simulation Results from AIMSUN simulation

The realization of multi-lane mixed traffic is still difficult because of a lack of understanding of lane-changing behaviors of ACC vehicles. Actually it is still a topic for ACC system designers. One can assume the ACC vehicle turns off the cruise control while conducting lane-changing. But the simulation result will not be realistic and very similar to that of pipeline simulation. Due to the complex nature of lane-changing behavior of ACC vehicles, a descriptive model is expected to soundly represent it. However, the data about ACC vehicles' lane-changing behavior in the real world is not available. And it's impossible to build up the model just based on observations of manual driven vehicles. Further progress in this topic needs more profound research and field operation.

## 7 CONCLUSIONS

(1) To evaluate the impacts of ACC on the traffic flow and to find a better ACC algorithm, we designed a simulation environment to implement microscopic level simulation of mixed traffic. The performance of mixed traffic is simulated in every level of the traffic system, from a single car's following behavior to macroscopic traffic characteristics. These simulations provide a basis of evaluating safety, efficiency and cost/benefit of the system. It is observed that the presence of ACC vehicles helps increase the space-mean speed of the system, which is a mark of system efficiency, but CTH vehicles may lead to a speed drop in the case of high demand while VTH mixed traffic always performs well.

(2) It is observed that the presence of ACC vehicles helps increase the space-mean speed of the system, which is a mark of system efficiency, but may lead to oscillations that have negative fuel and environmental implications. For instance, if we use a constant time headway algorithm to achieve high speed, we find that a high (95 ~ 99%) penetration of CTH vehicles increase throughput. But it is at the expense of high speed oscillation at above capacity inflow rates. From a traffic flow perspective, constant time headway control is potentially worse under select conditions than no ACC at all. This requires additional research into alternative control laws that are not detrimental to traffic flow before ACC should be deployed.

(3) If we use VTH to achieve high speed, we find it is at the expense of higher speed oscillation at above capacity inflow rates. From a traffic flow perspective, CTH control is potentially worse under select conditions than no ACC at all. VTH is a promising alternative to CTH as it is not detrimental to traffic flow when high demand is present.

(4) All of the conclusions drawn above should be conditional and tentative because many assumptions are used to idealize the system to make it computationally feasible. We note that the headway errors of vehicles can be seen as a source of the disturbance generated in the traffic flow. After we simulate the situations in which ACC and manual driven vehicles have different distributions of preset headway, it is concluded that the headway deviation doesn't have much impacts on the traffic performance in most of situations.

(5) Another direction of the work is to find the theoretical tools that can be used to analyze mixed traffic and quantitatively define traffic flow stability. This report presents

a study based on the density-flow rate relation of real world traffic. The criterion function is effective in forecasting potential congestions. A definition of traffic flow stability is provided based on this criterion function. Further study in this direction is worthwhile.



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## APPENDIX A: Pipeline Simulation Program

```
// Pipeline Simulation Program
// Version 3.0 Beta June. 2001
//
// This program is used to simulate the traffic flow in a single pipeline.
//

#include <iostream.h>
#include <string.h>
#include <vector>
#include <algorithm>
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "vehicle.h"

const short maxVehicleNumber= 200;
const float RoadLength= 3200; // meters , 2 miles
const float simulateTime= 600; // seconds
const float initialSpeed= 17.78; //meter/s ; 40 mph
const float sampleTime = 0.1; // s
const float maxVehicleSize = 4; // meters
const float maxSpeed = 28.9; // m/s

vehicle vehicleCalculation(int counter, float time, vehicle veh1, vehicle veh2, long followingMode);
vehicle grkt1(float time,float sampletime,vehicle veh1, vehicle veh2);
vehicle gipps(int counter, float time, vehicle veh1, vehicle veh2);
void carDynamics(float time, vehicle curruntVehicle, vehicle leadingVehicle, float *det);
float constantTime(float time, vehicle veh1, vehicle veh2);
float inFlow(float time);
float rand01(float *r);

// Initialize the platoon
typedef vector<vehicle> Platoon;
Platoon vehiclePlatoon;
```

```

using namespace std;    //introduces namespace std
int main()
{

    int i, counter;
    long numberOfVehicles, numberOfACC, numberOfGipps;
    long firstVehicleID=1;
    long lastVehicleID;
    float time=0.0, timeHeadway, lastTimeHeadway, LeftTime, oldLeftTime;
    float randNumber, tmp, tmp1, tmp2;
    float lastEntryTime=0, ACC_proportion=0.00;
    float density[10000], speed[10000], inflow[10000], outflow[10000];
    FILE *fd1, *fd2;

    vehicle junkVehicle, junkVehicle1, junkVehicle2, endVehicle;

// Open the data file to store simulation results
    fd1=fopen("mixdata1.dat","w");
    fd2=fopen("mixdata2.dat","w");

// Calculate the initial number of vehicles
    timeHeadway= 2;
    numberOfACC=0; numberOfGipps=0;
    numberOfVehicles = RoadLength / (initialSpeed * timeHeadway*2 + maxVehicleSize);
    lastVehicleID = numberOfVehicles;
    //numberOfVehicles=2;

// Initialize the position, speed and acceleration of the platoon
    endVehicle.vehicleID= numberOfVehicles+1;
    endVehicle.vehiclePosition=RoadLength+20;
    endVehicle.vehicleSpeed=maxSpeed;
    endVehicle.vehicleAccel=0.0;
    endVehicle.desiredAccel=0.0;
    endVehicle.flowrate=1;
    for(i=0;i<=numberOfVehicles;i++) //
    {
        junkVehicle.vehicleID=i;

```

```

        junkVehicle.vehiclePosition=RoadLength - (initialSpeed * timeHeadway*2 +
maxVehicleSize) * i;
        junkVehicle.vehicleSpeed=initialSpeed;
        junkVehicle.vehicleAccel=0.0;
        junkVehicle.desiredAccel=0.0;
        junkVehicle.flowrate=1;
        //ACC_proportion=rand01(&randNumber);///100+0.99
        tmp=rand01(&randNumber);
        if(tmp>=ACC_proportion)
        {
            junkVehicle.followingMode=1;           // use Gipps model
            numberOfGipps++;
        }
        else
        {
            junkVehicle.followingMode=0;           // use contant Time
Headway control
            numberOfACC++;
        }
        vehiclePlatoon.push_back(junkVehicle);
    }

//
// The main loop: simulate until time=simulateTime
//
    counter=0;
    do{

        // (1)
        // The last vehicle's timeHeadway = lastVehicle's position / initialSpeed
        // If lastTimeHeadway > timeHeadway, then Generate New Vehicle
        timeHeadway= 1 / inFlow(time);
        junkVehicle=vehiclePlatoon[numberOfVehicles]; //back();
        //lastTimeHeadway = junkVehicle.vehiclePosition / initialSpeed;
        lastTimeHeadway=time-lastEntryTime;
        if(numberOfVehicles<=maxVehicleNumber)
            if(lastTimeHeadway >= timeHeadway)

```



```

    {
        numberOfVehicles++;
        lastVehicleID++;

        // Parameters of New Vehicle
        junkVehicle1.vehicleID=lastVehicleID;
        junkVehicle1.vehiclePosition = maxVehicleSize;
        junkVehicle1.vehicleSpeed=initialSpeed;
//junkVehicle.vehiclePosition/timeHeadway;
        junkVehicle1.oldSpeed=initialSpeed;
//junkVehicle.vehiclePosition /timeHeadway;
        junkVehicle1.vehicleAccel=0;
        junkVehicle1.desiredAccel=0;
        junkVehicle1.flowrate=inFlow(time);

        //ACC_proportion=rand01(&randNumber)*2;//100+0.99
        tmp=rand01(&randNumber);
        if(tmp>=ACC_proportion)
        {
            junkVehicle1.followingMode=1;           // use   Gipps
model
            numberOfGipps++;
        }
        else
        {
            junkVehicle1.followingMode=0;           // use   contant
Time Headway control
            numberOfACC++;
        }
        // Add new vehicle
        vehiclePlatoon.push_back(junkVehicle1);
        inflow[counter]=1/(time-lastEntryTime);
        lastEntryTime=time;
    }
    else
    {
        if(counter==0) inflow[counter]=0;
        else inflow[counter]=inflow[counter-1];
    }
}

```

```

    }

// (2)
// If vehiclePosition > RoadLength then delete first vehicle
junkVehicle=vehiclePlatoon[0]; //.front();
if(junkVehicle.vehiclePosition > RoadLength)
{
//delete Vehicle
    if(junkVehicle.followingMode==1) numberOfGipps--;
    else numberOfACC--;

    vehiclePlatoon.erase(vehiclePlatoon.begin());
    numberOfVehicles--;
    firstVehicleID++;
    LeftTime=time;
    outflow[counter]=1/(LeftTime-oldLeftTime);
    if (outflow[counter]/5>outflow[counter-1] && outflow[counter-1]>0.1)
        outflow[counter]=outflow[counter-1];
    oldLeftTime=LeftTime;
}
else
{
    if(counter==0) outflow[counter]=0;
    else outflow[counter]=outflow[counter-1];
}

// (3)
// Calculate the new states of each vehicle, not including the first vehicle
for(i=0;i<=numberOfVehicles;i++) //IDinPlatoon; IDinPlatoon!=vehiclePlatoon.begin();
IDinPlatoon--)
{
    if(i==0)
    {
        // If it's the first vehicle, accelerate until the maximum speed
        junkVehicle1 = vehiclePlatoon.at(i);
        junkVehicle2 = endVehicle;
    }
}

```

```

        junkVehicle1 = vehicleCalculation(counter, time, junkVehicle1,
junkVehicle2, junkVehicle1.followingMode);
        vehiclePlatoon[i]= junkVehicle1;
    }
    else
    {
        junkVehicle1 = vehiclePlatoon.at(i); // current vehicle
        junkVehicle2 = vehiclePlatoon.at(i-1); // the leading vehicle
        junkVehicle1 = vehicleCalculation(counter, time, junkVehicle1,
junkVehicle2, junkVehicle1.followingMode);
        vehiclePlatoon[i]= junkVehicle1;
    }
    // Calculate the new states of vehicles in time+sampleTime
}

//
// (4)
// Save the states of each vehicle
if(time>420)
{
for(i=0;i<=numberOfVehicles;i++)
{
    junkVehicle1 = vehiclePlatoon[i];
    //fprintf(fd1, "%8.4f\t", time);
    //fprintf(fd1, "%d\t", junkVehicle1.vehicleID);
    //fprintf(fd1, "%8.4f\n", junkVehicle1.vehiclePosition);
    //fprintf(fd1, "%8.4f\n", junkVehicle1.vehicleSpeed);
    //fprintf(fd1, "%8.4f\n", junkVehicle1.vehicleAccel);
}
}
// (5)
// Traffic Parameters Calculation: q, u, k
density[counter] = numberOfVehicles/(RoadLength/1000);
speed[counter]=0;
for(i=0;i<=numberOfVehicles;i++)
{
    junkVehicle1 = vehiclePlatoon[i];

```

```

        //speed[counter]=speed[counter]+junkVehicle1.vehicleSpeed;
        speed[counter]=speed[counter]+1/junkVehicle1.vehicleSpeed;
    }
    speed[counter]=(numberOfVehicles-1)/(speed[counter]);
    //speed[counter]=(speed[counter])/(numberOfVehicles-1);
    tmp=float(numberOfACC)/float (numberOfVehicles+1);
    tmp1=float(numberOfGipps)/float (numberOfVehicles+1 );
if(counter>1500)
{
    i=fmod(counter,10);
    if(i==0)
    {
        fprintf(fd2, "%8.4ft", time);
        //fprintf(fd2, "%8.4ft", density[counter]);
        //fprintf(fd2, "%8.4ft", speed[counter]);
        //fprintf(fd2, "%8.4ft", speed[counter]*density[counter]/1000);
        fprintf(fd2, "%4d\t", lastVehicleID-32);
        fprintf(fd2, "%4d\n", firstVehicleID-1); //lastVehicleID-numberOfVehicles
        //fprintf(fd2, "%8.4f\n", outflow[counter]);
        //fprintf(fd2, "%8d\t", firstVehicleID);
        //fprintf(fd2, "%8.4f\n", tmp);
        //fprintf(fd2, "%8.4f\n", tmp1);
    }
}

// (6)
// Update Time
time=time+sampleTime;
counter++;

} while(time<simulateTime);

tmp=0;
for(i=3000;i<=counter-1;i++) tmp=tmp+speed[i];
tmp=tmp/(counter-3000);
tmp1=3000;
for(i=3000;i<=counter-1;i++) tmp1=tmp1+(speed[i]-tmp)*(speed[i]-tmp);
tmp1=tmp1/(counter-3000);

```

```

printf("mean:\t %8.4f\t", tmp);
printf("variance\t %8.4f\n", tmp1);

fclose(fd1);
fclose(fd2);
return 0;
}

// Calculate the state of vehicle according to its control mode
vehicle vehicleCalculation(int counter, float time, vehicle veh1, vehicle veh2, long followingMode)
{
    // Data used in calculation:
    // The former positions and speeds of this vehicle and the leading vehicle
    // Data generated : the present position, speed and acceleration of this vehicle
    vehicle junkVehicle;
    int i=0;
    switch(followingMode)
    {
        case 0: // ACC Mode
            veh1=grkt1(time, sampleTime, veh1, veh2);
            break;

        case 1: // Gipps Mode
            veh1=gipps(counter, time, veh1, veh2);
            break;

        default:
            break;
            // veh1.vehiclePosition = veh1.vehiclePosition + veh1.vehicleSpeed *
            // sampleTime + 0.5 * veh1.vehicleAccel * sampleTime* sampleTime;
    } //end of switch

    return veh1;
}

vehicle grkt1(float time,float sampletime,vehicle veh1, vehicle veh2)
{
    //extern void carDynamics();

```

```

int i,j,l;
float a[4],tt,*b,*d;
int n=3;           // number of variables
    b=(float *) malloc(n*sizeof(double));
d=(float *) malloc(n*sizeof(double));
float h=0.02;     // length of integral step
    a[0]=h/2.0; a[1]=a[0];
a[2]=h; a[3]=h;

int k= sampleTime/h + 1; // number of integral step
float y[3];
float z[3*10];
    float oldSpeed;

y[0]=veh1.vehiclePosition;
y[1]=veh1.vehicleSpeed;
y[2]=veh1.vehicleAccel;
    oldSpeed=veh1.vehicleSpeed;
for (i=0; i<=n-1; i++) z[i*k]=y[i];
for (l=1; l<=k-1; l++)
{
    carDynamics(time, veh1, veh2, d);
for (i=0; i<=n-1; i++) b[i]=y[i];
for (j=0; j<=2; j++)
{
    for (i=0; i<=n-1; i++)
    {
        y[i]=z[i*k+1-1]+a[j]*d[i];
        b[i]=b[i]+a[j+1]*d[i]/3.0;
    }
}
tt=time+a[j];
carDynamics(tt, veh1, veh2, d);
}
for (i=0; i<=n-1; i++) y[i]=b[i]+h*d[i]/6.0;
for (i=0; i<=n-1; i++) z[i*k+1]=y[i];
time=time+h;
}

```

```

free(b); free(d);

veh1.vehiclePosition=z[k-1];
    if(z[3*k-1]>1.7) z[3*k-1]=1.7;
    if(z[3*k-1]<-2) z[3*k-1]=-2;
    if(z[2*k-1]>=maxSpeed) // 28.9 m/s -- 65 mph
    {
        z[2*k-1]=maxSpeed;
    }
    if(z[2*k-1]<0.5) z[2*k-1]=0.5;
    veh1.vehicleSpeed=z[2*k-1];
    veh1.vehicleAccel=z[3*k-1];
    if((veh2.vehiclePosition-veh1.vehiclePosition)<6)
    {
        veh1.vehicleSpeed=oldSpeed;
    }
    return veh1;
}

// State equations of a single vehicle
void carDynamics(float time, vehicle curruntVehicle, vehicle leadingVehicle, float *det)
{
    float tau=0.1;
    vehicle veh1,veh2;
    veh1=curruntVehicle;
    veh2=leadingVehicle;

    %%State equations
    curruntVehicle.desiredAccel=constantTime(time, veh1, veh2);
    det[0]=curruntVehicle.vehicleSpeed;
    det[1]=curruntVehicle.vehicleAccel;
    det[2]=(-1/tau) * curruntVehicle.vehicleAccel + (1/tau) *curruntVehicle.desiredAccel;
//xdes_dot_dot;
}

// Constant time headway control algorithm
float constantTime(float time, vehicle veh1, vehicle veh2)
{

```

```

float vehPosition1=veh1.vehiclePosition;
float vehSpeed1=veh1.vehicleSpeed;
float vehPosition2=veh2.vehiclePosition;
float vehSpeed2=veh2.vehicleSpeed;

float timeHeadway = 1; // / veh1.flowrate;
float lamda = 0.2;      // Control gain for constant time-gap control law
float gama = 1;
float desiredAccel = -((vehSpeed1 - vehSpeed2)* gama + lamda * (vehPosition1 - vehPosition2 +
maxVehicleSize + timeHeadway* vehSpeed1))/timeHeadway;
return desiredAccel;
}

// Gipps Car-following Model
vehicle gipps(int counter, float time, vehicle veh1, vehicle veh2)
{
    int i;
    float vehPosition1=veh1.vehiclePosition;
    float vehSpeed1=veh1.vehicleSpeed;
    float oldSpeed1=veh1.oldSpeed;
    float vehPosition2=veh2.vehiclePosition;
    float vehSpeed2=veh2.vehicleSpeed;

    float va, vb, junk, definitiveSpeed;
    float v_desired = 65 * 1.6* 1000 /(3600);
    float accl_max=1.7;
    float del_max=-2.0*accl_max;
    float del_max_est;
    float timeHeadway=2;

    vehicle junkVehicle;

    va=vehSpeed1+2.5*accl_max*sampleTime*15*(1-
vehSpeed1/v_desired)*sqrt(0.025+vehSpeed1/v_desired);

    del_max_est=(-2.0 < ((double)(del_max-2.0)/2)) ? (-2.0):((del_max-2.0)/2);

```



```

junk = 2 * (vehPosition2 - maxVehicleSize - vehPosition1) - vehSpeed1 * sampleTime*15 -
(oldSpeed1*oldSpeed1) / del_max_est;

```

```

vb=del_max*sampleTime*15+sqrt(del_max*del_max * sampleTime * sampleTime*225-
del_max * junk);

```

```

definitiveSpeed = (va<vb)? va:vb;

```

```

if(definitiveSpeed<=1) definitiveSpeed=1;

```

```

if((vehPosition2-vehPosition1)<4)

```

```

    definitiveSpeed=vehSpeed1;

```

```

if(((vehPosition2-vehPosition1)/vehSpeed1)<timeHeadway)

```

```

    definitiveSpeed=vehSpeed1;

```

```

i=fmod(counter,15);

```

```

if(i==0)

```

```

    veh1.vehicleSpeed=definitiveSpeed;

```

```

    veh1.vehiclePosition=veh1.vehiclePosition+sampleTime*veh1.vehicleSpeed;

```

```

    return veh1;

```

```

}

```

```

// Generation of random number between 0 and 1 with uniform distribution

```

```

float rand01(float *r)

```

```

{

```

```

    int m;

```

```

    float s,u,v,p;

```

```

    s=65536.0; u=2053.0; v=13849.0;

```

```

    m=(int)(*r/s); *r=*r-m*s;

```

```

    *r=u*(*r)+v; m=(int)(*r/s);

```

```

    *r=*r-m*s; p=-*r/s;

```

```

    return(p);

```

```

}

```

```

// The entering flow rate

```

```

float inFlow(float time)

```

```

{

```

```

    if(time>=200 && time<350) return 2; //time*0.005;

```

```
else if(time>=350 && time<370) return 0.1;
else    return 0.3;
//float randNumber;
//return rand01(&randNumber);
}
```

## APPENDIX B: HEAD FILE

```
// vehicle.h
// Define the class : vehicle and related functions

// Class of Vehicle
using namespace std;    //introduces namespace std
class vehicle
{
// Data members...
    private:

// Member functions...
    public:
        long vehicleID;
        float vehicleLength;
        float vehiclePosition;
        float vehicleSpeed;
        float oldSpeed;
        float vehicleAccel;
        float desiredAccel;
        float flowrate;
        long nextVehicle;
        long followingMode;

        vehicle();
        vehicle(long id, float vehiclePosition, float vehicleSpeed, float vehicleAccel);
        ~vehicle();

        long ID() const;
        float Length() const;
        float Position() const;
        float Speed() const;
        float Accel() const;
        long NextVehicle() const;
};
```

```

inline long vehicle::ID() const
{
    return vehicleID;
}
inline float vehicle::Length() const
{
    return vehicleLength;
}
inline float vehicle::Position() const
{
    return vehiclePosition;
}
inline float vehicle::Speed() const
{
    return vehicleSpeed;
}
inline float vehicle::Accel() const
{
    return vehicleAccel;
}
inline long vehicle::NextVehicle() const
{
    return nextVehicle;
}

vehicle::vehicle()
{
    vehicleID = 0;
    vehiclePosition= 0;
    vehicleSpeed = 0;
    vehicleAccel=0;
}
// Single Vehicle generation: initial position, speed and acceleration
vehicle::vehicle(long id, float vehPosition, float vehSpeed, float vehAccel)
{
    vehicleID = id;
    vehiclePosition= vehPosition; // initial Position
}

```

```
        vehicleSpeed = vehSpeed; //initial Speed;
        vehicleAccel=vehAccel;
//      cout << "Creating new vehicle #" << vehicleID << '\n';
}

// Single Vehicle Deletion
vehicle::~~vehicle()
{      }
```

## Appendix C: AIMSUN Simulation Program

```
#if __GNUG__ >= 2
# pragma implementation
#endif
```

```
#define GIPPS 1
#define ACC 0
```

```
#include <iostream.h>
#include <string.h>
#include <vector>
#include <algorithm>
#include <math.h>
```

```
#include "GetExt.h"
#include "GetExt_common.h"
#include "AKIProxie.h"
#include "CIProxie.h"
#include "mylib.h"
#include <stdio.h>
#include <queue>
#include <list>
#include "stdlib.h"
#include "time.h"
```

```
int i,j,k, manualCarTimeTag=10; // manualCarTimeTag determine the number of step manual car used to
response
int idVehFirst=0, counter=0, sectionID[4]; // counter is used to count time step of manual car before next
response
int idVehEnter, numberOfVeh[4], vehInSection[4], vehInPlatoon[4];
int numberOfSections, minPosition1, minPosition2, lastVehInLane1, lastVehInLane2, firstVehSect3;
int timeIndex=0;
FILE *fd1, *fd2;
char cadena[1024], res[1024];
float lastEnterTime[4]={0,0,0,0};
float enterFlowRate=0.78; // veh/s
```

```

float stepTime=0;
float speed[4096];
float ACC_proportion=0.5;
float maxAcceleration=2.8;
float maxDeceleration=4;
int vehIntersection, leaderID, leader, newCarIn, nbVehOverIntersection=0;
int carExit=0, exitCarID=-1, newCarFollowingMode, nextCarID;
float leaderSpeed, followerSpeed, newCarSpeed;
float leaderPosition, followerPosition, newCarPosition;

StructAkiEstadSection sect1, sect2;
vehicle junkVehInfo;

typedef std::queue < int, std::list<int> > vehPlatoon;
vehPlatoon Platoon[3];

int FollowingMode[4096];
float OldSpeed[4096];

//vehOldSpeed OldSpeed;

//typedef vector<vehicle> infoPlatoon;
//infoPlatoon vehInfoPlatoon;

// Aimsun Extension Functions
int GetExtInit()
{
    int i;
    //int sectionID[10];

// Open the data file to store simulation results
//    fd1=fopen("aimsun1.dat","w+");
//    fd2=fopen("aimsun.dat","w");

    if( InitGetExtSystem() )
    {
        // Init successful
        // read number of sections and Section ID

```

```

        numberOfSections = AKIInfNetNbSections();
        for(i=0;i<numberOfSections;i++)
        {
            sectionID[i] = AKIInfNetGetSectionId(i);
            sprintf(cadena,"Section: %d \n", sectionID[i]);
            AKIPrintString(cadena);
        }

        return 0;
    }
    else
    {
        // Init GetExt fails
        return -1;
    }
}

int GetExtManage(float time, float timeSta, float timTrans, float acicle)
{
    int speedModify, i,j, k, vehIndex, change;
    int numberOfVehSection, currentSection;
    int inttmp1, inttmp2, inttmp3;
    int nbSec1,nbSec2,nbSec3;

    InfVeh aimVeh1, aimVeh2;
    StaticInfVeh junkVeh1Stat, junkVeh2Stat; // static info
    InfVeh junkVeh1, junkVeh2; // temp info

    float vehPosition1, vehPosition2, vehSpeed1,vehSpeed2, newspeed[1024], oldLeadingSpeed;
    float desiredAccel, vehAccel1, vehAccel2;
    float timeHeadway = 1; // / veh1.flowrate;
    float b=1; // Control gain for constant time-gap control law
    float lamda=0.4, gama=1;
    float flowRate, tmp, randNumber, tmp1, tmp2;

    //time=AKIGetDurationTransTime();

```



```

nbSec1=AKIVehStateGetNbVehiclesSection(1);
nbSec2=AKIVehStateGetNbVehiclesSection(2);

for(currentSection=1;currentSection <=numberOfSections;currentSection++)// for1
{
    // vehicle generation in Section 1 and 2
    if(time<250) enterFlowRate=0.3;
    else if(time>=250 & time<400) enterFlowRate=0.6;
    else if(time>=400 & time<420) enterFlowRate=0.1;
    else if(time>=420) enterFlowRate=0.3;

    // Enter vehicle according to given flow rate
    if(currentSection==1)
    {
        if(time-lastEnterTime[currentSection]>(1/enterFlowRate))
        {
            idVehEnter= AKIEnterVehTrafficFlow(currentSection, "car", 2); //
enter a car in Section 1 and track it
            if(idVehEnter>0)
            {
                lastEnterTime[currentSection]=time;
                AKIVehTrackedModifySpeed(idVehEnter, 64); // Initial speed
is 40mph -- 64 km/h

                tmp=rand01(&randNumber);
                if(tmp>=ACC_proportion)
                {
                    FollowingMode[idVehEnter]=GIPPS;
                }
                else
                {
                    FollowingMode[idVehEnter]=ACC;
                }
                OldSpeed[idVehEnter]=64/3.6; //initial speed 40 mph --
64 km/h
            }
        }
    }
}

```

```

    }

    if(currentSection==2)
    {
        if(time-lastEnterTime[currentSection]>(1/enterFlowRate))
        {
            idVehEnter= AKIEnterVehTrafficFlow(currentSection, "car", 2); //
enter a car in Section 1 and track it
            if(idVehEnter>0)
            {
                lastEnterTime[currentSection]=time;
                AKIVehTrackedModifySpeed(idVehEnter, 64); // Initial speed
is 40mph -- 64 km/h

                tmp=rand01(&randNumber);
                if(tmp>=ACC_proportion)
                {
                    FollowingMode[idVehEnter]=GIPPS;
                }
                else
                {
                    FollowingMode[idVehEnter]=ACC;
                }
                OldSpeed[idVehEnter]=64/3.6; //initial speed 40 mph --
64 km/h
            }
        }
    }

    // Get reference of Front of the platoon
    k=AKIVehStateGetNbVehiclesSection(currentSection); //Platoon[currentSection].front();

    if (currentSection==1)
    {
        // Find the vehicles near the intersection (1343 -- 1359)in the section 1
        // If so, put a virtual car at the end of section 2
        // Otherwise, mike it clear

```

```

// Record the IDs of the two vehicles nearby
vehIntersection=0;
nbVehOverIntersection=0;
for (int m=0;m<nbSec1;m++)
{
    junkVeh1=AKIVehStateGetVehicleInfSection(1, m);
    if (junkVeh1.CurrentPos>1343 &
junkVeh1.CurrentPos<1359)
    {
        vehIntersection++;
        //sprintf(res, "veh %d in lane 1\n",junkVeh1.idVeh);
        //AKIPrintString(res);
    }
    if (junkVeh1.CurrentPos>1343)
    {
        nbVehOverIntersection++;
    }
}
//sprintf(res, "nbVehOverIntersection--- %d\n",
nbVehOverIntersection);

//AKIPrintString(res);

if(nbVehOverIntersection==0)
{
    // If no vehicle in section 1 goes over the intersection,
    // can put new vehicle in the intersection
    newCarIn=1;
    junkVeh1=AKIVehStateGetVehicleInfSection(2, 0);
    newCarSpeed=junkVeh1.CurrentSpeed;
    newCarPosition=1351;
}

// If there is no car in between, find the two car nearby
// The new car can be put in, the speed is the mean of the two car
if (vehIntersection<=0 && nbVehOverIntersection>0)
{
    leaderID=-1;
    leader=0;
}

```

```

for (int m=0;m<nbSec1;m++)
{
    junkVeh1=AKIVehStateGetVehicleInfSection(1, m);
    if (junkVeh1.CurrentPos>1359 && m>leader)
    {
        leaderID=junkVeh1.idVeh;
        leaderSpeed=junkVeh1.CurrentSpeed;
        leaderPosition=junkVeh1.CurrentPos;
        leader=m;
    }
}
// The follower ID
junkVeh1=AKIVehStateGetVehicleInfSection(1, leader+1);
followerSpeed=junkVeh1.CurrentSpeed;
followerPosition=junkVeh1.CurrentPos;
newCarSpeed=(leaderSpeed+followerSpeed)/2;
newCarPosition=1351; //(leaderPosition+followerPosition)/2;
//newCarPosition=(newCarPosition<1343)?
1343:newCarPosition;
//newCarPosition=(newCarPosition>1359)?
1359:newCarPosition;

newCarIn=1;
//sprintf(res, "leader %d --- follower %d\n", leader, leader+1);
//AKIPrintString(res);

}
else if (vehIntersection>0 && nbVehOverIntersection>0)
{
    // If there is a car, find it and the car follows it.
    // The new car cannot be put in, the virtual car is put at the end of
    // section 2 with speed 0 m/s
    for (int m=0;m<nbSec1;m++)
    {
        junkVeh1=AKIVehStateGetVehicleInfSection(1, m);
        if (junkVeh1.CurrentPos>1343 &&
junkVeh1.idVeh>leaderID)
    {

```



```

vehPosition1=junkVeh1.CurrentPos;
vehSpeed1=junkVeh1.CurrentSpeed/3.6; // m/s
vehAccel1=(vehSpeed1-OldSpeed[inttmp1])/sampleTime; //
m/s2

vehSpeed2=28.9;
vehPosition2=3400;
vehAccel2=12;

if (FollowingMode[inttmp1]==ACC)
{
    // If the current car is ACC car
    //tmp1=(vehSpeed1 - vehSpeed2)* gama;
    //tmp2=lamda * (vehPosition1 - vehPosition2 +
maxVehicleSize + accHeadway* vehSpeed1);
    //desiredAccel = -((vehSpeed1 - vehSpeed2)* gama +
lamda * (vehPosition1 - vehPosition2 + maxVehicleSize + accHeadway* vehSpeed1))/accHeadway;
    //newspeed[i]= grkt1(vehSpeed1, vehPosition1,
vehSpeed2, vehPosition2, vehAccel1, vehAccel2);
    //desiredAccel=constantTime(vehSpeed1,
vehPosition1, vehSpeed2, vehPosition2);

    float lamda=0.2, gama=1, timeHeadway=1.0;
    desiredAccel = -((vehSpeed1 - vehSpeed2)* gama +
lamda * (vehPosition1 - vehPosition2 + maxVehicleSize + timeHeadway* vehSpeed1))/timeHeadway;

    float vf= maxSpeed*1.1;
    float km= 1/maxVehicleSize;
    float b=1; // Control gain for constant time-
gap control law

    //float delta=(vehPosition1-
vehPosition2)+b*(vehSpeed1-vehSpeed2)+1/(km*(1-vehSpeed1/vf));
    //gama=(vehSpeed1-vehSpeed2)+lamda*delta +
b*(vehAccel1-vehAccel2);
    //desiredAccel = -(km/vf)*(vf-vehSpeed1)* (vf-
vehSpeed1)*gama;

```

```

tmp=(exp(-10)*vehAccel1+(1- exp(-
10))*desiredAccel);
newspeed[inttmp1]= vehSpeed1+ sampleTime*
(exp(-10)*vehAccel1+(1- exp(-10))*desiredAccel);
OldSpeed[inttmp1]= vehSpeed1;
oldLeadingSpeed=vehSpeed1;
}
else
{
newspeed[inttmp1]=gipps(time,
vehPosition1, vehSpeed1, vehPosition2, vehSpeed2);//junk Veh2.CurrentSpeed/3.6

oldLeadingSpeed=junkVeh1.CurrentSpeed/3.6;

OldSpeed[inttmp1]=junkVeh1.CurrentSpeed/3.6;
}
}

if(currentSection==2 && nbSec2>0)
{
// If it's Section 2, the first vehicle adjust speed according to the
condition of the intersection

junkVeh1= AKIVehStateGetVehicleInfSection(2, 0); //
Current car

inttmp1= junkVeh1.idVeh;
//sprintf(res, "lead veh in section 2: %d\n",inttmp1);
//AKIPrintString(res);

vehPosition1=junkVeh1.CurrentPos;
vehSpeed1=junkVeh1.CurrentSpeed/3.6; // m/s
vehAccel1=(vehSpeed1-OldSpeed[inttmp1])/sampleTime; //
m/s2

if(newCarIn==0)
{

```

```

        vehSpeed2=0;
        vehPosition2=460;
        vehAccel2=0;
    }
    else if(newCarIn==1)
    {
        vehSpeed2=28.9; // m/s
        vehPosition2=550;
        vehAccel2=12;
    }

    if (FollowingMode[inttmp1]==ACC)
    {
        // If the current car is ACC car
        //tmp1=(vehSpeed1 - vehSpeed2)* gama;
        //tmp2=lamda * (vehPosition1 - vehPosition2 +
maxVehicleSize + accHeadway* vehSpeed1);
        //desiredAccel = -((vehSpeed1 - vehSpeed2)* gama +
lamda * (vehPosition1 - vehPosition2 + maxVehicleSize + accHeadway* vehSpeed1))/accHeadway;
        //newspeed[i]= grkt1(vehSpeed1, vehPosition1,
vehSpeed2, vehPosition2, vehAccel1, vehAccel2);
        //desiredAccel=constantTime(vehSpeed1,
vehPosition1, vehSpeed2, vehPosition2);

        float lamda=0.2, gama=1, timeHeadway=1.0;
        desiredAccel = -((vehSpeed1 - vehSpeed2)* gama +
lamda * (vehPosition1 - vehPosition2 + maxVehicleSize + timeHeadway* vehSpeed1))/timeHeadway;

        float vf= maxSpeed*1.1;
        float km= 1/maxVehicleSize;
        float b=1; // Control gain for constant time-
gap control law

        //float delta=(vehPosition1-
vehPosition2)+b*(vehSpeed1-vehSpeed2)+1/(km*(1-vehSpeed1/vf));
        //gama=(vehSpeed1-vehSpeed2)+lamda*delta +
b*(vehAccel1-vehAccel2);

```



```

//desiredAccel =  $-(km/vf)*(vf-vehSpeed1)* (vf-$ 
vehSpeed1)*gama;

tmp=(exp(-10)*vehAccel1+(1- exp(-
10))*desiredAccel);

newspeed[inttmp1]= vehSpeed1+ sampleTime*
(exp(-10)*vehAccel1+(1- exp(-10))*desiredAccel);

OldSpeed[inttmp1]= vehSpeed1;
oldLeadingSpeed=vehSpeed1;
}
else
{
// If the current car is manual driven car
if (counter==20)
{ // Calculate new speed if time step > given
newspeed[inttmp1]=gipps(time,
vehPosition1, vehSpeed1, vehPosition2, vehSpeed2);//junkVeh2.CurrentSpeed/3.6

oldLeadingSpeed=junkVeh1.CurrentSpeed/3.6;

OldSpeed[inttmp1]=junkVeh1.CurrentSpeed/3.6;
}
else
{ // Otherwise, keep current speed

oldLeadingSpeed=junkVeh1.CurrentSpeed/3.6;

OldSpeed[inttmp1]=junkVeh1.CurrentSpeed/3.6;

newspeed[inttmp1]=junkVeh1.CurrentSpeed/3.6;
}
}
// OldSpeed[inttmp1]=((OldSpeed[inttmp1]<30)?
30:OldSpeed[inttmp1]);
// newspeed[inttmp1]=((OldSpeed[inttmp1]<30)?
30:OldSpeed[inttmp1]);
//}

```

```

    }
}
else if(i>=1)
{
    // If the current car is NOT the front of the queue: k+1, ... ,
k+vehInPlatoon

    // Modify its speed according to its state and the leading car
    // Get car pair information

    junkVeh1= AKIVehStateGetVehicleInfSection(currentSection, i); //
Current car

    junkVeh2= AKIVehStateGetVehicleInfSection(currentSection, i-1); //
the leading car

    vehPosition1=junkVeh1.CurrentPos;
    vehPosition2=junkVeh2.CurrentPos;
    inttmp1= junkVeh1.idVeh;
    inttmp2= junkVeh2.idVeh;
    vehSpeed1=junkVeh1.CurrentSpeed/3.6; // m/s
    vehSpeed2=junkVeh2.CurrentSpeed/3.6; // m/s
    vehAccel1=(vehSpeed1-OldSpeed[inttmp1])/sampleTime; // m/s2
    vehAccel2=(vehSpeed2-OldSpeed[inttmp2])/sampleTime; // m/s2

    if (FollowingMode[inttmp1]==ACC)
    {
        // If the current car is ACC car
        //tmp1=(vehSpeed1 - vehSpeed2)* gama;
        //tmp2=lamda * (vehPosition1 - vehPosition2 +
maxVehicleSize + accHeadway* vehSpeed1);
        //desiredAccel = -((vehSpeed1 - vehSpeed2)* gama + lamda *
(vehPosition1 - vehPosition2 + maxVehicleSize + accHeadway* vehSpeed1))/accHeadway;
        //newspeed[i]= grkt1(vehSpeed1, vehPosition1, vehSpeed2,
vehPosition2, vehAccel1, vehAccel2);
        //desiredAccel=constantTime(vehSpeed1, vehPosition1,
vehSpeed2, vehPosition2);

        float lamda=0.2, gama=1, timeHeadway=1.0;

```

```

desiredAccel = -((vehSpeed1 - vehSpeed2)* gama + lamda *
(vehPosition1 - vehPosition2 + maxVehicleSize + timeHeadway* vehSpeed1))/timeHeadway;

float vf= maxSpeed*1.1;
float km= 1/maxVehicleSize;
float b=1; // Control gain for constant time-gap control

law

//float delta=(vehPosition1-vehPosition2)+b*(vehSpeed1-
vehSpeed2)+1/(km*(1-vehSpeed1/vf));
//gama=(vehSpeed1-vehSpeed2)+lamda*delta +
b*(vehAccel1-vehAccel2);
//desiredAccel = -(km/vf)*(vf-vehSpeed1)* (vf-
vehSpeed1)*gama;

tmp=(exp(-10)*vehAccel1+(1- exp(-10))*desiredAccel);
newspeed[inttmp1]= vehSpeed1+ sampleTime* (exp(-
10)*vehAccel1+(1- exp(-10))*desiredAccel);
OldSpeed[inttmp1]= vehSpeed1;
if (time>250)
{
//sprintf(res, "veh %d: (%f %f)\t (%f %f)\t (%f %f : %f %f)\t
%f\n",i ,vehSpeed1, vehSpeed2, vehPosition1, vehPosition2, vehAccel1, vehAccel2, desiredAccel, tmp,
newspeed[i]);

//AKIPrintString(res);
//sprintf(res, "**** %f\n", (vehPosition1 - vehPosition2 +
maxVehicleSize + 1.2* vehSpeed1));

//AKIPrintString(res);
//sprintf(res, "**** %f\n", ((vehSpeed1 - vehSpeed2)* 1 + 0.2
* (vehPosition1 - vehPosition2 + maxVehicleSize + timeHeadway* vehSpeed1)));
//AKIPrintString(res);

}
oldLeadingSpeed=vehSpeed1;
}
else
{

```

```

// If the current car is manual driven car
if (counter==20)
{
    // Calculate new speed if time step > given
    newspeed[inttmp1]=gipps(time,      vehPosition1,
vehSpeed1, vehPosition2, vehSpeed2);//junkVeh2.CurrentSpeed/3.6
    oldLeadingSpeed=junkVeh1.CurrentSpeed/3.6;
    OldSpeed[inttmp1]=junkVeh1.CurrentSpeed/3.6;
}
else
{
    // Otherwise, keep current speed
    oldLeadingSpeed=junkVeh1.CurrentSpeed/3.6;
    OldSpeed[inttmp1]=junkVeh1.CurrentSpeed/3.6;
    newspeed[inttmp1]=junkVeh1.CurrentSpeed/3.6;
}
}
}
// Modify the speed of current car
newspeed[inttmp1]=(newspeed[inttmp1]<(104/3.6))?
newspeed[inttmp1):(104/3.6);
newspeed[inttmp1]=(newspeed[inttmp1]<0)? 0:newspeed[inttmp1];
AKIVehTrackedModifySpeed(inttmp1, newspeed[inttmp1]*3.6);

} // for2

// Find if a car exits from section 2
if (currentSection==2 && nbSec2>0)
{
    junkVeh1= AKIVehStateGetVehicleInfSection(2, 0); // Current car
    if(junkVeh1.idVeh > exitCarID && junkVeh1.CurrentPos > 456 &&
junkVeh1.CurrentPos < 460)
    {
        exitCarID=junkVeh1.idVeh;
        //nextCarID=junkVeh2.idVeh;
        newCarFollowingMode=FollowingMode[exitCarID];
        carExit=1;
        //sprintf(res, "%d is exiting: carExit= %d \n", exitCarID, carExit);
        //AKIPrintString(res);
    }
}

```



```

if (counter==19)
{
    counter=0;
    speed[timeIndex]=0;
    for (i=0;i<nbSec1;i++)
    {
        junkVeh1= AKIVehStateGetVehicleInfSection(1, i); // Current car
        vehSpeed1=junkVeh1.CurrentSpeed;
        speed[timeIndex]=speed[timeIndex] + vehSpeed1;
        //sprintf(res, "%f\t %d\t %f\t %d in %d\n", time, junkVeh1.idVeh, vehSpeed1, i,
nbSec1);

        //AKIPrintString(res);
    }
    speed[timeIndex]=speed[timeIndex] / nbSec1;
    if(time>4)
    {
        fprintf(fd2, "%f\t %f\t %d\n", time, speed[timeIndex]/3.6, nbSec1);
        //sect1=AKIEstGetParcialStatisticsSection(1, 1, NULL);
        //fprintf(fd2, "%f\t %f\t %f\t %f\n", time, sect1.Sa, sect1.Flow, sect1.Density);
    }
    timeIndex++;
}
else counter++;

return 0;
}

```

```

int GetExtPostManage(float time, float timeSta, float timTrans, float acicle)
{
    //sprintf(cadena,"POST MANAGE ....Time = %f TimeSta = %f TimeTrans = %f cycle = %f",
time, timeSta, timTrans, acicle);
    //AKIPrintString(cadena);
    //int nba = AKIInfNetNbSections();
    //int id = AKIInfNetGetSectionId(nba-1);
    //sprintf(res, "\t\t section %d\t id\t :%d \n",nba, id);
    //AKIPrintString(res);
}

```

```
        return 0;
    }

int GetExtFinish()
{
    fclose(fd2);
    return 0;
}
```