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Impact of Truck Loading on Design and Analysis of Asphaltic Pavement Structure - Phase IV: Mixedmode Fracture Characterization

Yong-Rak Kim, Ph.D. Associate Professor

Department of Civil Engineering University of Nebraska-Lincoln

Soohyok Im, Ph.D. Assistant Transportation Researcher Texas A&M Transportation Institute

Hoki Ban, Ph.D. Assistant Professor Department of Civil Engineering Kangwon National University

Francisco Thiago Sacramento Aragão, Ph.D.

Assistant Professor Department of Civil Engineering Universidade Federal do Rio de Janeiro/COPPE



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Yong-Rak Kim, Ph.D. Associate Professor Department of Civil Engineering University of Nebraska-Lincoln

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Abstract

Cracking in asphalt concrete pavements causes primary failure in the pavement structure. It is considered one of the key issues to be addressed when selecting paving materials and designing sustainable pavement structures. Due to the diverse nature of truck loads and pavement geometry, the asphalt mixture in the pavement is subjected to complex cracking behavior, such as mixed-mode fracture, which is a combination of opening mode and shearing mode of fracture. For a better understanding of asphalt fracture and a more accurate design of pavement structure, mode-dependent fracture behavior needs to be characterized. This study presents integrated experimental-numerical efforts to characterize the mixed-mode fracture of a fine aggregate matrix (FAM), which is the primary phase of cracks around stiffer coarse aggregates when typical asphalt concrete mixtures are subjected to heavy truck loads at intermediate service temperatures. Experimentally, semicircular bend (SCB) fracture tests were conducted by varying the geometric-loading configurations with different initial notch inclination angles and supporting spans to achieve different fracture modes (opening, shearing, and mixed). The SCB fracture test results were then integrated with the extended finite element modeling, which is also incorporated with mode-dependent cohesive zone fracture to properly identify the modedependent fracture properties. The test and model simulation results indicated that the cohesive zone fracture toughness of mode-II (shearing) is quite different from mode-I (opening) fracture toughness. The critical fracture energy was related to the mixed-mode ratio, which presented a power relationship between the total fracture toughness and involvement of mode-II fracture in the total. Findings and observations from this study, although they are limited at this stage, imply that the mixed-mode fracture characteristics are significant and need to be considered in the structural design of asphalt pavements with which multi-axial cracking is usually associated.

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Chapter 1 Introduction

Various asphalt pavement distresses are related to fracture, including fatigue cracking (both top-down and bottom-up), thermal (transverse) cracking, and reflective cracking of the asphalt layer. Cracking in asphalt concrete pavements causes primary failure in the pavement structure and leads to long-term durability issues that are often related to moisture damage. The fracture resistance and characteristics of asphalt materials significantly affect the service life of asphalt pavements and, consequently, the maintenance and management of the pavement network.

For a better understanding of the fracture behavior and cracking mechanisms in asphaltic materials, recent studies have attempted various fracture tests and corresponding analysis methods. They include the single-edge notched beam (SEB) test [1-3], the disk-shaped compact tension (DCT) test [4, 5], the double-edged notched tension (DENT) test [6], and the semicircular bend (SCB) test [7-11]. However, the tests in most studies cited above considered only pure opening (i.e., mode-I) fracture. But mode-I cracking does not occur solely in asphalt pavements because of multiaxial stresses due to traffic loads and the geometric complexity of pavement structures. The limited effort of many studies is mainly due to technical challenges involved both in performing shearing mode (i.e., mode-II) or mixed-mode fracture tests and in the resulting data analyses. Very limited efforts have been attempted in the asphalt pavement community, to the authors' best knowledge, to characterize the mode-II fracture behavior of asphalt mixtures. In 2008, Braham [12] conducted the SEB test with an offset notch to analyze the mixed-mode fracture characteristics of different asphalt concrete mixtures. He clearly showed that fracture is mixture-specific and that the fracture work increases as the level of

mode-II increases. However, the physical identification and quantification of the mode-II fracture toughness in the mixed-mode test were not fully estimated.

Perhaps the characterization of a complicated fracture behavior such as the mixed-mode fracture targeted in this study can be better achieved with the aid of well-defined numerical techniques. Along with experimental efforts (i.e., laboratory fracture tests), the finite element method (FEM) has been used to solve various crack problems in many studies. More recently, the FEM has been incorporated into a rigorous fracture model, for example, the cohesive zone concept, to simulate crack initiation and propagation of asphaltic media, as presented in several studies [13-16]. The cohesive zone approach in asphalt materials and flexible pavements was first employed by Jeng and co-workers [17, 18] to model crack resistance and propagation in asphalt pavement overlays. Since then, it has received increasing attention from the asphalt mechanics community because it can overcome the drawbacks of traditional approaches of linear elastic fracture mechanics. Moreover, it provides great versatility and efficiency with various computational methods, such as the FEM, for the modeling of both brittle and ductile failures, which are typically observed in asphalt mixtures due to the wide range of service temperatures and loading rates to which asphalt is subjected.

However, the conventional FEM with cohesive zones has an inherent drawback in predicting randomly developed cracks, because cracks can only occur in a predefined mesh structure. Thus, finite element simulation results are affected by the size and orientation of cohesive zone elements. In order to properly capture the temporally and spatially dependent development of cracks, enough cohesive zone elements need to be placed in an optimized manner; otherwise, considerable accuracy can be lost. To overcome this limitation, Belytschko and Black [19] introduced the extended finite element method (XFEM) as an extension of the

conventional FEM in order to model arbitrary cracks in finite element meshes. The key feature of the XFEM, which is simulating arbitrary crack growth, can be efficiently integrated with the experimental efforts and the cohesive zone concept for a more realistic characterization of mixed-mode fracture in asphalt mixtures.

1.1 Research Objective and Scope

This research aims to explore a better understanding of the mode-dependent fracture behavior of asphalt mixtures and to use the resulting fracture characteristics to aid understanding of fundamental aspects of pavement design related to crack-associated distresses such as fatigue cracking and thermal cracking. Clearly, research efforts are necessary to seek an appropriate fracture test-analysis method that is fundamentally sound and practically efficient. Toward that end, this research attempted experimental and numerical efforts to find overall mode-dependent fracture characteristics (i.e., mode-I [opening], mode-II [shearing], and mixed-mode with different levels) of asphalt mixtures. For this characterization, a simple and efficient fracture test method was first explored, and the resulting test data were incorporated into relevant mechanical theories (such as the cohesive zone fracture concept) and a numerical technique (i.e., XFEM).

At the current stage of our research efforts, a limited scope was attempted: only one asphalt mixture was subjected to different fracture modes (opening, shearing, and mixed) with a representative loading rate of 10 mm/min. and at intermediate testing temperature of 21°C. The specific objectives of this research are:

- To explore a mechanically sound and reasonable fracture test method that can identify the mode-dependent fracture characteristics of asphalt mixtures;
- To develop an integrated experimental-numerical approach for the characterization of asphalt fracture in pavement structures; and

• To identify fracture characteristics that account for the variation in mode-dependent fracture resistance of asphaltic materials.

Chapter 2 Materials and Mixtures

The asphalt mixture used in this study is a mixture of a PG 64-28 asphalt binder and fine aggregates smaller than 1.19 mm. The mixture is denoted as the "fine aggregate matrix (FAM)" in this study. As presented in figure 2.1, FAM is one of the three primary phases, along with coarse aggregates and air voids, when the microstructure of a typical asphalt concrete mixture is considered. Although the FAM is not exactly the same as typical asphalt concrete mixtures implemented in practice, it is acceptable for the present research because the FAM surrounds much stiffer coarse aggregates and tends to crack when asphalt concrete mixtures are subjected to traffic loads at room temperature conditions and are not strongly associated with moisture damage. Thus, the FAM is the phase where fracture characteristics need to be properly examined. Furthermore, the testing of FAM is much more repeatable and efficient than the testing of highly heterogeneous asphalt concrete mixtures, particularly for the mixed-mode fracture condition pursued in this study.



Figure 2.1 A typical three-phase microstructure of asphalt concrete mixtures

Test results from FAM mixtures can also be directly employed in microstructure modeling and simulations [16]. Microstructure models can predict the properties and the

performance of asphalt concrete mixtures by accounting for the properties of the individual components such as the FAM and the microstructural heterogeneity of the mixtures.

The FAM was designed based on the volumetric mix design characteristics of its corresponding asphalt concrete mixture, which contains 4.0% air voids and 6.0% asphalt binder by weight of total mixture. For the mix design of FAM, the same binder was mixed with fine aggregates smaller than 1.19 mm (No.16 sieve). Because the FAM only contains particles passing sieve No.16, a mix design of the FAM was postulated by assuming that the coarse aggregates in the corresponding asphalt concrete mixture could be separated from the FAM by virtually picking them out of the compacted asphalt concrete mixture microstructure. In other words, algebraically, the required binder content to produce the FAM was proposed as what remained after excluding the binder absorbed by the coarse aggregates from the total binder in the asphalt concrete mixture. This approach resulted in a binder content of 8.0% by the total weight of aggregates in the FAM mixture. It should be noted that the FAM mix design and fabrication conducted in this study involves several technical assumptions that require further investigation to confirm; however, the FAM is considered sufficient to meet the specific objectives for this study. Table 2.1 shows the gradation, fine aggregate angularity (FAA), and binder content of the FAM mixture used in this study.

Sieve Number	#16	#30	#50	#100	#200
Sieve Size (mm)	(1.19)	(0.6)	(0.3)	(0.15)	(0.075)
Gradation (% passing)	100.0	66.7	47.6	33.3	16.7
FAA (%)	45.0				
Binder Type	PG 64-28				
Binder Content (%)			8.00		

Table 2.1 FAM mix design and properties of fine aggregates

Chapter 3 Mode-Dependent SCB Fracture Test

As mentioned earlier, one of the primary objectives of this study was to explore an appropriate (simple, scientifically reasonable, and easy to perform) test method for the characterization of mode-dependent (opening, shearing, and mixed) fracture behavior of asphalt materials. Among the various fracture test geometries, the semicircular bend (SCB) test was selected as it has several advantages compared to other fracture test methods and has been evaluated by many researchers in the asphaltic materials/pavements community [8,10,11,15]. The SCB test is attractive because it is relatively repeatable, is simple to perform, and allows one to prepare multiple testing specimens easily through a routine process of Superpave gyratory compacting and/or field coring from asphalt roadways. The SCB geometry is particularly attractive for mixed-mode fracture tests because several studies have attempted to characterize the mode-dependent fracture toughness of engineering materials, such as concrete and rocks [20-24].

Figure 3.1 shows a schematic representation of the SCB geometry-loading condition and some of actual testing specimens used in this study for mixed-mode fracture characterization. As illustrated, different fracture modes can be achieved depending on the geometric parameters used, such as the inclination angle α , notch length *a*, and supporting span *s* with the applied load. For pure mode-I (opening), Chong and Kuruppu [25] suggested a geometry with a span ratio (*s/r*) of 0.8 and $\alpha = 0^{\circ}$ to characterize the fracture behavior of rocks based on the concept of linear elastic fracture mechanics. Since then, their testing configuration for mode-I fracture has been selected for various materials, including asphalt mixtures. Lim et al. [20, 26] examined the most appropriate SCB testing configuration to study mode-II fracture behavior. They reported

that the SCB geometries for pure mode-II (shearing) have a span ratio (*s*/*r*) of 0.5 and a normalized notch length (*a*/*r*) of 0.35 ± 0.04 with an α less than 60°.



(a) schematic configuration of SCB testing



Figure 3.1 Geometry and loading conditions of the SCB specimen

Ayatollahi and Aliha [24] also investigated the SCB geometries that represent the mixedmode and the pure mode-II. Table 3.1 summarizes findings from the study by Ayatollahi and Aliha [24]. It presents the maximum span ratio (*s*/*r*) necessary to yield a mode-II fracture condition for a given normalized notch length (*a*/*r*) when the inclination angle α is less than 60°.

Table 3.1 Maximum span ratio (s/r) for mode-II fracture condition

a/r	0.3	0.4	0.5	0.6
s/r	≤0.47	≤ 0.565	≤0.65	≤0.725

On the basis of findings from the aforementioned studies for other engineering materials, this study attempted three different SCB specimen geometries: a constant normalized notch length (a/r) of 0.33 with three different notch inclination angles of 0°, 45°, and 50°. To simulate the mode-I fracture, the specimen with the 0° inclination angle was tested by placing metallic rollers separated by a distance of 122 mm (14 mm from the edges of the specimen), providing a span ratio (s/r) of 0.8. To achieve fracture other than the pure mode-I, span ratios (s/r) from 0.4 to 0.8 for two different inclination angles ($\alpha = 45^\circ$ and 50°) were attempted. It is important to note that the specimen geometries and testing configurations in this study were arbitrary based on previous studies [20-24] of concrete and rock materials. The testing configurations selected were evaluated as to whether they are truly appropriate to characterize the mode-I, mode-II, and mixed-mode fracture behavior of asphaltic materials, which is described in later sections of this report.

Figure 3.2 presents the fabrication process of the SCB test specimens and resulting specimens ready to be loaded. In the preparation of the SCB test specimens, a Superpave gyratory compactor was used to produce tall compacted FAM samples, which were 150 mm in diameter and 175 mm high. Five slices (each with a diameter of 150 mm and a height of 25 mm) were then obtained after removing the top and the bottom parts of each tall sample. Each slice was then cut into halves to yield SCB specimens with a notch 2.5 mm wide and 25 mm deep to yield the normalized notch length (a/r) of 0.33. Multiple SCB specimens were prepared to complete at least two or three replicates for each testing condition. Special care was taken when

the initial notch was produced because the geometry and the quality of the notch tip may significantly affect the fracture behavior of SCB specimens. Before testing, the individual SCB specimens were placed inside the environmental chamber of a mechanical testing machine for temperature equilibration, with the target test temperature 21°C. Following the temperature-conditioning step, the SCB test was performed with a monotonic displacement rate of 10 mm/min. applied to the top center line of the specimens. The reaction force at the loading point was monitored by the data acquisition system installed in the mechanical testing machine.



Figure 3.2 Illustration of specimen preparation and testing process

As shown in figure 3.2, in the present study, the digital image correlation (DIC) system was integrated with the fracture test to more accurately identify the mode-dependent fracture

characteristics. The DIC is an easy-to-use, noncontact technique. It can identify complex material behavior of a specimen, such as the time-varying local fracture process, which is the focus in this study. Two pairs of dot gauges were attached to the surface of the specimens to capture the opening displacements (denoted as notch tip opening displacements [NTODs]) and the shearing displacements (denoted as notch tip shearing displacements [NTSDs]) at the tip of the initial notch. These displacements are particularly important because they are used to compute the mode-specific fracture energies that are used to check whether the selected test configurations are appropriate to characterize the different modes of fracture (mode-I, mode-II, and mixed-mode). The DIC recognizes the surface structure of the specimen in digital video images and allocates coordinates to the image pixels. The first image represents the undeformed state, and further images are recorded during the deformation of the specimen. Then, the DIC compares the digital images and calculates the relative displacement between the dot gauges, yielding the NTODs and NTSDs. A one megapixel Photron camera was used to capture images and ARAMIS, a commercial package for the DIC analysis, was adopted in this study.

Chapter 4 SCB Fracture Test Results

Figure 4.1 presents the test results as the loading time increased for each test case. The test results among the replicates at the same test conditions were repeatable, without large discrepancies. Several observations can be clearly noted from the figure. For both notch angle inclinations, as the span ratio (*s*/*r*) decreased, the SCB specimens were more resistant to fracture due to the greater peak forces and the larger areas under the force-time curves. The fracture resistance was smallest when the specimen with a vertical notch ($\alpha = 0^{\circ}$) was subjected to the largest span ratio (*s*/*r* = 0.8). When the fracture resistance between the two notch inclination angles was compared, the specimens with a 45° angle were generally more resistant to cracking than those with a 50° angle under the same testing condition. Even before conducting any further scientific investigations, these observations indicate that asphaltic materials are more resistant to the shearing mode than to the opening fracture, and that mode-dependent fracture characteristics should be considered in the design process of pavement structures to predict crack-related distress, such as fatigue cracking and thermal cracking, more accurately.

As mentioned earlier, the SCB fracture test was integrated with the DIC system to ensure that the mode-dependent fracture behavior of the asphalt mixtures in each test configuration attempted was accurately estimated. Using the DIC test results, the behavior of the mode-specific fracture configurations was carefully investigated at the notch tip. Two displacements, NTODs and NTSDs, represent the opening mode fracture and the shearing mode fracture, respectively, at the tip of the initial notch. These displacements were measured using the DIC system as the loading time increased. The test data were then used to determine the mode-dependent fracture characteristics of the FAM mixture and to assess the potential stage at the moment of crack propagation when the load reached slightly after the peak.



Figure 4.1 SCB fracture test results: (a) s/r = 0.8, $\alpha = 0^{\circ}$; (b) s/r = 0.4 - 0.8, $\alpha = 45^{\circ}$; (c) s/r = 0.4

- 0.8, $\alpha = 50^{\circ}$

Figure 4.2 presents the two displacements (NTODs and NTSDs) and their corresponding loads monitored for each test case. As expected, the specimens with the 0° inclined notch showed the most significant changes in the NTODs at the notch tip with insignificant changes in the NTSDs, indicating that the opening displacements primarily occurred at the moment of crack development. However, when the notch was not vertical (see the example at 45° in fig. 4.2), NTSDs and NTODs appeared, and the magnitude of the NTSDs became greater as the span ratio (*s*/*r*) decreased. When the span ratio was the smallest (i.e., 0.4), the SCB specimens showed clear shearing displacements at the moment of crack propagation, whereas the opening displacements were insignificant. Another observation from the test results is that the NTODs began rising immediately only when $\alpha = 0^\circ$ case. When the notch inclination angle was 45°, both NTODs and NTSDs only appeared to rise when fracture is imminent.

It should be noted that figure 4.2 only presents the DIC results of the 45° case. However, the DIC analysis results for the 50° case were very similar to those shown in figure 4.2. The limited data and the results of the DIC analysis suggest that the SCB test configuration attempted in this study is generally appropriate for differentiating the mode-dependent fracture behavior of asphaltic materials. As previously verified for rock and concrete materials [20-24], SCB geometry with a normalized notch length (a/r) of 0.33, a notch inclination angle of 0°, and a span ratio of 0.80 is valid for characterizing mode-I fracture, and a SCB specimen with a normalized notch length (a/r) of 0.33 and notch inclination angles of 45-50° is primarily subjected to shearing fracture behavior when a span ratio of 0.40 is applied. Between the two cases, mixed-mode fracture occurs, and the level of each mode is controlled by loading configurations such as the span ratio.



Figure 4.2 DIC analysis results (NTODs and NTSDs from each test case)

Chapter 5 Fracture Property Characterization

Using the SCB test results, the fracture properties can be characterized. Among several methods [3, 4, 27-29] available to characterize the fracture properties, this study attempted two approaches: one is based on the use of force-displacement curves and the other is by modeling the SCB fracture tests with cohesive zone elements. The first approach makes it relatively simple to characterize the fracture resistance by merely calculating an area under the force-displacement curve up to peak force. The second approach, extended finite element modeling of the SCB tests with cohesive zones, were conducted to determine the fracture properties that are locally associated to initiate and propagate cracks through the specimens.

5.1 Fracture Characterization with Force-Displacement Curves

Figure 5.1 presents the SCB test results with the DIC measurements by plotting the average values of the reaction forces and the opening/shearing displacements. It should be noted that figure 5.1 does not present the results of the 45° case; however the analysis results for the 45° case were very similar to those for the 50° case shown in figure 4.2.

The figure confirms that the SCB testing attempted in this study by varying the orientation of the initial notch and the loading configuration can differentiate the mode-specific fracture characteristics of asphalt mixtures. The SCB geometry with the notch inclination angle of 0° was primarily subjected to the normal opening mode (mode-I) fracture; whereas the SCB geometry with the notch inclination angle of 45-50° was primarily subjected to shear-mode (mode-II) fracture when the span ratio of 0.40 was applied. Varying the span ratio from 0.40 to 0.80 with the notch inclination angle of 45-50° led to mixed-mode fracture, and the span ratio affected the level of individual fracture mode (i.e., as the span ratio decreased, the mode-II fracture was more dominant).



Figure 5.1 Force-NTOD and force-NTSD curves

To quantify the mode-dependent fracture resistance, the mechanical energy required for crack propagation (W_{cp}) was calculated for each test case. Among the several methods available in the literature to calculate the fracture resistance, this method adopted a simple way by calculating the area under the force-displacement curves (force-NTODs and force-NTSDs) up to peak force. It should be noted that the fracture resistance, W_{cp} , obtained from the approach described herein is limited to characterizing the true fracture properties of the material because it only represents materials' resistance to fracture before crack propagation. Moreover, forcedisplacement curves are global measurements, which are dependent on the choice of displacement measurements, the geometry of the test specimen, and the applied boundary conditions. In addition, the viscoelastic nature of the asphaltic material creates a further complication when identifying fracture properties at intermediate testing conditions. Therefore, fracture characteristics should be examined locally at the tip of fracture process zone, not by global force measurements. Regardless of the several technical issues that could somewhat mislead the true fracture properties, the mechanical energy required for crack propagation was utilized herein for practical reasons. It implies that the fracture characteristics herein were incorporated with the SCB-DIC results without relying on any further mechanical analyses and/or advanced modeling, such as the XFEM with cohesive zone fracture that is presented in the next section.

Table 5.1 presents W_{cp} value of each case. The coefficient of variation (COV) values of each case were also investigated and resulted in a range between 4.2% and 27.6%. The variability observed in this study seems satisfactory according to a study by Marasteanu et al. [31], where the variability of 25% was reported as repeatable. The mechanical energy, W_{cp-I} , indicates the mode-I fracture resistance obtained from the force-NTOD curve, and W_{cp-II}

indicates the mode-II fracture resistance obtained from the force-NTSD curve. As expected and clearly seen from the table, the fracture resistance values obtained from the force-NTSD curve increased as the span ratio decreased.

Notch Angle	s/r	W _{cp-I}	W _{cp-II}	W _{cp-T}	W _{cp-II} /W _{cp-T}
$\alpha = 0^{o}$	0.8	0.17	-	0.17	0.00
	0.8	0.29	0.24	0.53	0.45
a = 450	0.6	0.33	0.40	0.73	0.55
$\alpha = 45^{\circ}$	0.5	0.20	0.53	0.73	0.73
	0.4	-	0.84	0.84	1.00
$\alpha = 50^{\circ}$	0.8	0.26	0.30	0.56	0.54
	0.6	0.34	0.62	0.96	0.64
	0.5	0.20	0.72	0.92	0.78
	0.4	-	1.01	1.01	1.00

Table 5.1 Average W_{cp} value (J) of each test case

The total fracture resistance (W_{cp-T}) was then defined as the sum of W_{cp-I} and W_{cp-II} in this study. As shown in table 5.1, the W_{cp-T} values for the 45° inclined notch case were lower, but not significantly different compared to the W_{cp-T} values for the 50° inclined notch case. This implies that the two notch angles attempted in this study did not significantly influence the total fracture resistance, although the notch angles affected the load-displacement responses of the asphaltic materials including the peak force and the softening behavior, as presented in figure 4.1. However, more extensive testing and data analyses at different notch angles are needed to confirm this observation.

Based on the test results, the mode-dependent fracture characteristics account for the variation in the fracture resistance of asphaltic materials. Figure 5.2 presents the total mechanical

energy required for crack propagation as a function of the mixed-mode ratios (W_{cp-II}/W_{cp-T}) obtained from the SCB tests. The data were then fitted to a simple power relationship as follows:

$$W_{\rm cp-T} = 0.161 + 0.802 \left(W_{\rm cp-II} / W_{\rm cp-T} \right)^{0.71}$$
(5.1)



Figure 5.2 Mixed-mode fracture characteristics for asphalt mixture in this study

As illustrated in figure 5.2 and indicated from the equation 1, the total fracture resistance at either zero or the unity value of the mixed-mode ratio represents the pure mode-I and mode-II fracture condition, respectively. Consequently, the required mechanical energies to develop either the pure opening mode or the pure shearing mode fracture can be estimated from the power law by simply taking the two extreme values: 0.161 J for mode-I fracture condition and 0.963 J for mode-II fracture condition. The fracture resistance for the mode-II condition was approximately six times greater than the fracture resistance for the mode-I condition.

5.2 Fracture Characterization with XFEM Modeling

To determine the mode-specific fracture properties of FAM mixtures, the SCB test results were then integrated with numerical model simulations. The integrated approach between test results and numerical modeling can identify unknown fracture characteristics by targeting good agreements between laboratory test results and numerical simulations equipped with a fracture model, i.e., the mode-dependent cohesive zone model in this study. Several recent relevant studies [15, 30-33] have taken this approach to determine the fracture properties of asphaltic materials at different conditions. For this particular study, the XFEM with a cohesive zone was used for the fracture characterization. The XFEM is exceptionally feasible for modeling arbitrary crack growth, which is the case in this study where fracture paths are expected to be random in the semicircular specimen when the fracture is associated with mixed-mode. The use of conventional FEM with cohesive zones presents an inherent drawback in simulating random crack growths, because cracks can generally develop only along a predefined path in the finite element mesh. Therefore, the mode-II or mixed-mode fracture in the SCB geometry could be significantly misled by the mesh structure if the conventional FEM with cohesive zone fracture were to be adopted.

Figure 5.3 exemplifies XFEM meshes (vertical notch and inclined notch with $\alpha = 50^{\circ}$) used for this study after completing the mesh convergence analysis. The specimens were discretized with four-node bilinear plane stress elements (CPS4) implemented in a commercial finite element program, ABAQUS [34], which is the numerical tool for modeling the SCB tests in the present study. As also seen in figure 5.3, the graded mesh was constructed by refining only

around the potential crack growth region, whereas in the area of low stress gradients, large elements were used to reduce computational time.



(b) XFEM mesh with an inclined notch ($\alpha = 50^{\circ}$)

Figure 5.3 XFEM meshes constructed after mesh convergence analysis

Figure 5.4 illustrates the mixed-mode cohesive zone fracture model incorporated into the XFEM simulation. Currently, the mixture was modeled as an isotropic and elastic material subjected to mode-dependent fracture. Therefore, the two linear elastic properties (i.e., Young's

modulus *E* and Poisson's ratio v) and the three fracture properties (i.e., cohesive zone strength T_{max} and cohesive zone fracture energies Γ_{I} , Γ_{II}), represented by a linearly decaying cohesive zone model [35], were necessary as model inputs.



Figure 5.4 Illustration of the SCB specimen with cohesive zone fracture

In this study, it was assumed that, among the listed mechanical properties, Poisson's ratio was not affected by fracture modes and therefore maintained a constant value of 0.3. The elastic modulus of the mixture was determined by taking only the elastic components in the generalized Maxwell model, which is a linear viscoelastic mechanical analog. The elastic components were used in the present study to reasonably estimate the Young's modulus of FAM, because the current XFEM in ABAQUS was not feasible for modeling viscoelastic materials with fracture. The other three cohesive zone fracture parameters (T_{max} , Γ_{I} , and Γ_{II}) were then determined for

each case via a calibration process by reaching a good agreement between the simulation and test results. The cohesive zone strength (T_{max}) could readily be determined by calibrating the peak force between the model simulation and testing, and the cohesive zone fracture energies were determined by calibrating the softening behavior of the force-time curves between the model and testing.

In order to account for the variation of fracture energy due to the level of fracture mode, the mixed-mode criterion proposed by Benzeggagh and Kenane [36] was used in this study (B-K criterion). This criterion is expressed as a function of mode-I and mode-II fracture energy and a parameter *n*:

$$\Gamma_{C} = \Gamma_{IC} + \left(\Gamma_{IIC} - \Gamma_{IC}\right) \left(\frac{\Gamma_{II}}{\Gamma_{T}}\right)^{n}$$
(5.2)

$$\Gamma_T = \Gamma_I + \Gamma_{II} \tag{5.3}$$

In the B-K fracture criterion, Γ_{IC} and Γ_{IIC} are fracture energies for mode-I and mode-II, respectively, and the parameter *n* determines the shape of the mixed-mode fracture trend. As figure 5.5 illustrates, the critical fracture energy at either zero or unity of the mixed-mode ratio (Γ_{II}/Γ_T) represents pure mode-I and mode-II, respectively. When *n*-value is greater than unity (for example n = 3 in the figure), the critical fracture energy grows at an increasing rate as the mixed-mode ratio increases; however, for *n* values smaller than unity (such as n = 0.35 in the figure), the critical fracture energy evolves with a decreasing rate as the level of mode-II fracture increases in the overall fracture process. At the sufficiently high and low *n*-value (10¹⁰ and 10⁻¹⁰

in the figure), the fracture behavior is mainly governed by pure mode-I and mode-II, respectively.



Figure 5.5 B-K mixed-mode fracture criterion

Model simulations with the B-K fracture criterion were first conducted for mode-I and mode-II cases. Cohesive zone fracture properties were determined through the calibration process until a good agreement was achieved between experimental results and numerical simulations of SCB testing. As mentioned earlier, extremely high and low *n*-value in the B-K fracture criterion was used to characterize fracture properties at the pure mode-I and mode-II, respectively. Figure 5.6 compares the SCB test results (averages of replicates) with the XFEM model simulations after the model calibration process was completed. For the mode-I case, almost identical results between the model simulation and testing were found, and relatively good agreements were obtained for the mode-II case with some mismatches that might be due to

the combined effects of the model limitations: linear elastic, isotropic, two-dimensional, etc. Resulting fracture properties (T_{max} , Γ_{IC} and Γ_{IIC}) are shown in the figure. It can be noted that the mode-II fracture energy is about three times greater than mode-I fracture energy, which implies that asphalt mixtures are approximately three times more resistant to the shearing mode fracture than to the opening mode failure.



Figure 5.6 SCB test results vs. XFEM model simulation results for mode-I and mode-II

After the material properties at mode-I and mode-II fracture condition were determined, the B-K model parameter *n* could be identified with mixed-mode fracture data. To this end, a series of simulations at the three other testing configuration (s/r = 0.5, 0.6 and 0.8) conducted with the two different notch inclination angles (45° and 50°) were repeated by changing the *n*value until deviations between the SCB test results and model simulations were minimized. To achieve the calibration process with a validation purpose, the *n*-value was first obtained from the data set at the inclination angle of 50° , and the *n*-value found was then applied to the test results at the inclination angle of 45°. Figure 5.7 illustrates comparison plots between test results (average of three replicates) and the XFEM model simulations at the two different notch angles.



(a) Model predictions vs. test results at the notch angle of 50°



(b) Model predictions vs. test results at the notch angle of 45°



Model simulations generally traced the test results with some mismatches particularly after peak loads. However, considering the aforementioned model limitations (such as linear elastic, isotropic, and two-dimensional) at the current stage of modeling efforts, the deviations between the model predictions and test results would be insignificant and can be reduced by incorporating other modeling features such as material viscoelasticity.

Figure 5.8 presents resulting mode-dependent fracture characteristics of the FAM mixture. The B-K criterion is also shown in the figure. The *n*-value determined herein was 0.35. It can be observed from the figure that the critical fracture energy of the FAM mixture tends to increase somewhat sharply at the beginning stage of mixed mode ratio and then gradually saturates. This implies that fracture resistance of asphalt mixtures is greater to and more sensitive with shearing.



Figure 5.8 Mixed mode fracture criterion of the FAM mixture

The observation in figure 5.8 is in fact quite similar to the finding in figure 5.2. In figure 5.2, the mixed-mode fracture with the SCB test results was experimentally characterized by simply calculating the area under the load-displacement curves up to peak force. The resulting fracture criterion could relate the total mechanical energy required for crack propagation as a function of the mixed-mode ratios in a simple power relationship as the B-K relation shown in figure 5.8. The *n*-value obtained from the experimental characterization was also less than unity.

The validity of the model simulation is further illustrated in figure 5.9. It compares crack paths at different mixed-mode conditions between laboratory tests and model simulations. The crack paths were reasonably affected by the test configurations, and the simulation results closely match the realistic crack pattern. A reasonable match between the test results and the numerical simulations implies that the resulting model parameters (i.e., cohesive zone fracture properties and the B-K mixed-mode fracture criterion) were properly defined.

Test-modeling results presented in this study are limited, and several technical limitations exist at this stage. As noted, only one asphalt mixture in the form of FAM was subjected to a loading rate of 10 mm/min. at an intermediate temperature condition of 21°C in this study. The numerical model assumed the FAM as elastic with damage only by the mode-dependent fracture. A wide range of testing temperatures and loading rates for different types of asphalt mixtures needs to be considered to confirm the observations and findings, because asphalt mixtures are temperature- and rate-dependent viscoelastic materials. Despite the limitations here and the challenges facing future studies, the integrated experimental-numerical approach in this study provided significant insights on the subject of asphalt fracture in pavements. As shown here, the fracture of asphalt mixtures is highly mode related, and the fracture resistance between the fracture modes is very different. The findings of this study clearly indicate that the modesensitive nature of the fracture characteristics needs to be considered when pursuing a more

accurate design of pavement structures with which multi-axial deformation is usually associated due to the complicated pavement structural geometry and the various traffic loading conditions.





(a) s/r = 0.5



(c) s/r = 0.8

Figure 5.9 Crack path comparison between SCB tests and XFEM simulations

Chapter 6 Summary and Conclusions

This study investigated mixed-mode fracture characteristics of asphalt materials. SCB fracture tests with different notch geometries and loading configurations were incorporated with the results of DIC analysis to characterize the mode-dependent fracture behavior of a FAM mixture subjected to a 10 mm/min. loading rate and an intermediate temperature condition of 21°C. SCB fracture tests were then integrated with the XFEM technique with the mode-dependent cohesive zone fracture concept. Based on the test results and the data analyses, the following conclusions can be drawn:

- The SCB fracture test with FAM mixtures presented reasonable and relatively repeatable results. In addition, by varying the notch geometries and loading configurations, the SCB test was generally appropriate for differentiating mode-I, mixed-mode, and mode-II fracture conditions of asphalt mixtures;
- The SCB geometry with the notch inclination angle of 0° and the span ratio of 0.80 was characterized by mode-I fracture, and the SCB geometry with the notch inclination angle of 45-50° was primarily subjected to mode-II fracture behavior when the span ratio of 0.40 was applied. Between the two cases, mixed-mode fracture occurred, and the level of each mode was controlled by the span ratio;
- The numerical modeling approach based on the XFEM technique incorporated with the cohesive zone fracture was successful for the characterization of mode-dependent asphaltic fracture by integrating the model predictions with the SCB test results;
- The test and model simulation results indicated that fracture toughness of mode-II is quite different from mode-I fracture toughness. Furthermore, the critical fracture

energy of the mixture tended to increase somewhat sharply at the beginning stage of mixed mode ratio and then gradually saturated;

- The findings and observations from this study provides significant insight into pavement design and asphaltic materials. The mixed-mode fracture phenomenon identified herein can potentially be used in the future in mechanistic pavement design with which multi-axial damage is usually associated in order to better address crack-associated distresses such as fatigue cracking; and
- The observations from this study may not be directly applied to other mixtures at different conditions, since asphalt mixtures are highly rate- and temperature-dependent viscoelastic. To confirm the results and findings obtained in this study, future studies should conduct additional tests and analyses at different testing temperatures and loading rates with various different mixtures.

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