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# Stress-Intensity Factors Along Three-Dimensional Elliptical Crack Fronts

May 1998

Final Report

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16. Abstract

The objective of the present investigation is to determine the mode I stress-intensity factors along two symmetric surface cracks emanating from a centrally located hole in a rectangular plate (the so-called Round Robin Problem) using the domain integral method. In order to validate the present three-dimensional domain integral implementation, two comparisons were made with benchmark solutions. We first considered the problem of an elliptical crack embedded in a rectangular plate. For plate dimensions much greater than the largest characteristic dimension of the elliptical crack, we compared the present finite element results with the solution of Irwin (1962) for an elliptical crack embedded in an infinitely extended solid. Next, we considered the problem of a quarter-elliptical corner crack in a rectangular plate and compared the results with those of Newman and Raju (1983). Excellent agreement was obtained for both benchmark comparisons.

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## EXECUTIVE SUMMARY

The objective of the present investigation is to determine the mode I stress-intensity factors along two symmetric surface cracks emanating from a centrally located hole in a rectangular plate (the so-called Round Robin Problem) using the domain integral method. In order to validate the present three-dimensional domain integral implementation, two comparisons were made with benchmark solutions. We first considered the problem of an elliptical crack embedded in a rectangular plate. For plate dimensions much greater than the largest characteristic dimension of the elliptical crack, we compared the present finite element results with the solution of Irwin (1962) for an elliptical crack embedded in an infinitely extended solid. Next, we considered the problem of a quarter-elliptical corner crack in a rectangular plate and compared the results with those of Newman and Raju (1983). Excellent agreement was obtained for both benchmark comparisons.

## INTRODUCTION

A particularly useful method for evaluating fracture parameters is the domain integral method in which the crack tip integral is recast as an integral over a finite domain surrounding the crack tip. The calculation of the crack tip parameters of interest can then be carried out in a straightforward post processing step in the finite element method. The domain integral method has been employed by Shih et al. (1986) to evaluate the energy release rate along a three-dimensional crack front in a thermally stressed body and has been used by Nikishkov and Atluri (1987) to evaluate the mixed-mode stress-intensity factors along an arbitrary three-dimensional crack. Although the implementation of the domain integral method with the finite element method is straightforward, the computation of fracture parameters along arbitrary three-dimensional crack fronts is expensive in terms of the time required to generate a mesh, in-core storage requirements for large three-dimensional calculations, and solution time. Mesh generation is particularly time consuming due to the difficulties associated with constructing a mesh which accurately captures the singular nature of the stress field in the vicinity of the crack front and near stress concentrations. Nevertheless, fracture parameters can be obtained very accurately with the method for arbitrary three-dimensional geometries.

The objective of the present investigation is to determine the mode I stress-intensity factors along two symmetric surface cracks emanating from a centrally located hole in a rectangular plate (a round robin problem) using the domain integral method. In order to validate the present three-dimensional domain integral implementation, two comparisons were carried out. We first considered the problem of an elliptical crack embedded in a rectangular plate. For plate dimensions much greater than the largest characteristic dimension of the elliptical crack, we compared the present finite element results with the solution of Irwin (1962) for an elliptical crack embedded in an infinitely extended solid. Next, we considered the problem of a quarter-elliptical corner crack in a rectangular plate and compared the results with those of Newman and Raju (1983). Excellent agreement was obtained for both comparisons.

Results were then obtained for the round robin problem. Several meshes were constructed having between 12,000 to 20,000 degrees of freedom. The finite element calculations were performed on a Silicon Graphics R4000 workstation equipped with 192 megabytes of random access memory (RAM). The in-core storage requirement for the most refined mesh used was approximately 112 megabytes. Successive mesh refinement was carried out until the results converged.

In all of the numerical calculations, the pointwise energy release rates $G(s)$ along the crack front were obtained directly by the domain integral method, and the mode I stress-intensity factors $K_I(s)$ at each point along the crack front were obtained using the plane strain relation

$$K_I(s) = \left\{ \frac{EG(s)}{1-\nu^2} \right\}^{1/2} \tag{1}$$

where E is Young's modulus and $\nu$ is Poisson's ratio. Although we recognize that the asymptotic field has a lower order singularity than $1/\sqrt{r}$ near intersections of the crack front and free surfaces, the extent of the boundary layer is known to be small and thus equation 1 was used throughout for the computation of $K_I$.

## COMPARISONS

In order to validate the present three-dimensional domain integral implementation, two comparisons were made. We first considered the problem of an embedded elliptical crack in a rectangular plate subjected to tensile loading as shown in figure 1(a). As shown in the figure, the dimensions of the elliptical crack are given by the parameters a and c and the thickness, height, and width of the plate are given by t, H, and W. The dimensionless ratios for the plate and elliptical crack were taken to be as follows:

H/W = 1.0
a/c = 0.4
a/t = 0.2
c/W = 0.1

and Poisson's ratio was taken to be $\nu = 0.3$. We note that the results are independent of Young's modulus, and the remote applied stress was taken to be unity. Defining the parametric angle $\phi$ as shown in figure 2, the calculated values for the mode I stress-intensity factors along the crack front are plotted versus parametric angle as shown in figure 3. The stress-intensity factors are normalized with respect to the crack depth $a$ and the parameter $Q$ which is the square of the complete elliptical integral of the second kind and was approximated by the formula

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \qquad \frac{a}{c} \leq 1 \tag{2}$$

Because the dimensions of the elliptical crack were taken to be small in comparison with the plate dimensions, the results were compared to the solution by Irwin (1962) for an elliptical crack embedded in an infinite solid. As shown in the figure, excellent agreement between the two results was obtained. The finite element model for this problem consisted of approximately 4500 eight-node brick elements, and we note that no singular crack tip elements were employed.

For the second comparison, the problem of a quarter-elliptical corner crack in a rectangular plate subjected to tensile loading was considered as shown in figure 1(b). The dimensionless ratios for the plate and elliptical crack were taken as

H/W = 1.0
a/c = 0.4
a/t = 0.2
c/W = 0.1

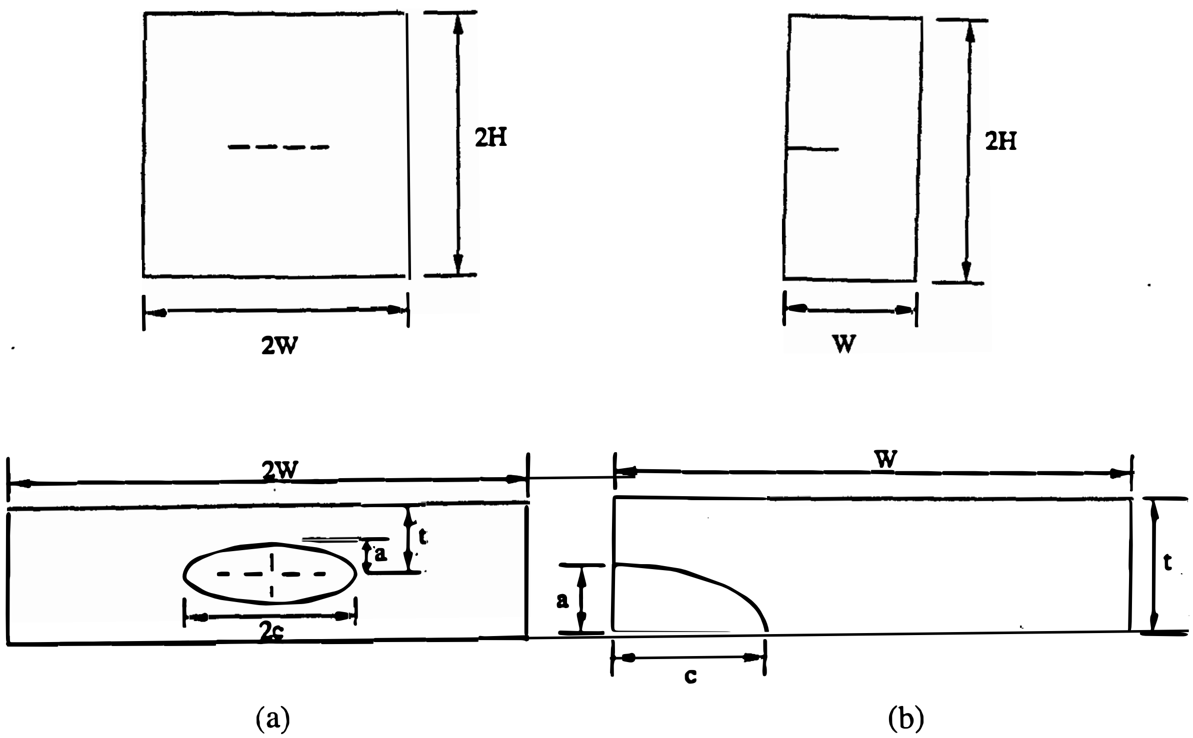



FIGURE 1. COMPARISON PROBLEMS—EMBEDDED ELLIPTICAL CRACK (a) AND QUARTER-ELLIPTICAL CORNER CRACK (b)

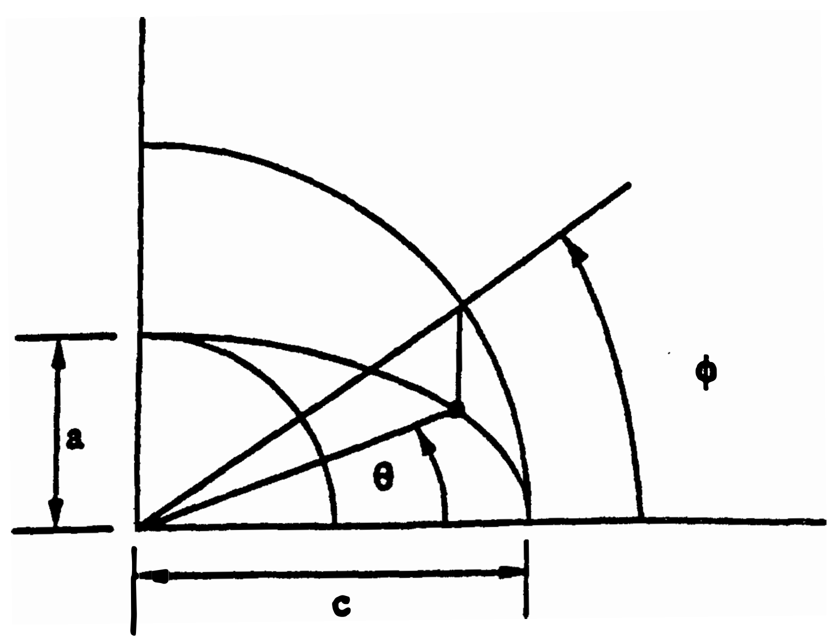



FIGURE 2. DEFINITION OF PARAMETRIC AND PHYSICAL ANGLES

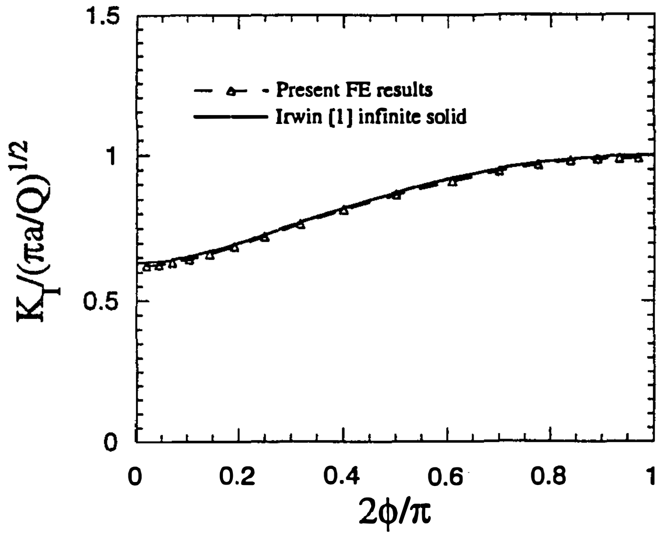



FIGURE 3. DISTRIBUTION OF NORMALIZED STRESS-INTENSITY FACTORS ALONG CRACK FRONT FOR AN EMBEDDED ELLIPTICAL CRACK (a/c = 0.4, a/t = 0.2, c/W = 0.1, H/W = 1.0)

the remote applied stress was taken to be unity, and Poisson's ratio was taken to be $\nu = 0.3$. The present finite element results for the quarter-elliptical corner crack were compared with the finite element results of Newman and Raju (1983) as shown in figure 4. The maximum difference between the two results is less than two percent. The finite element mesh for the corner crack was identical to the mesh used for the embedded crack—the solution to the corner crack problem was obtained through a modification of the boundary conditions.

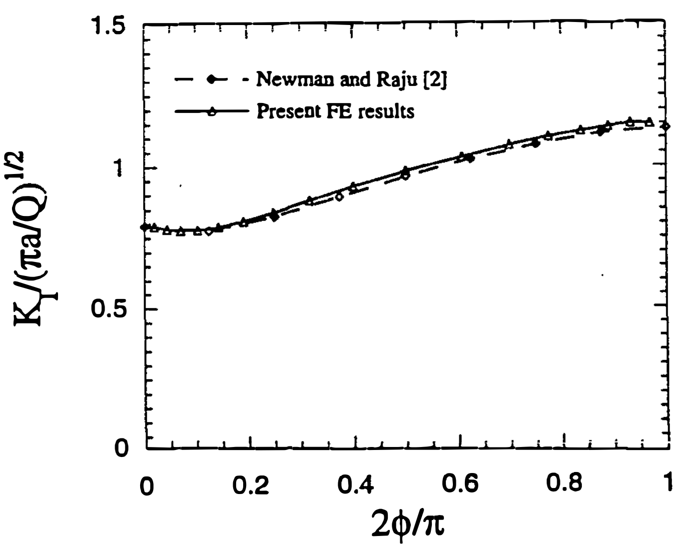



FIGURE 4. DISTRIBUTION OF NORMALIZED STRESS-INTENSITY FACTORS ALONG CRACK FRONT FOR A QUARTER-ELLIPTICAL CORNER CRACK (a/c = 0.4, a/t = 0.2, c/W = 0.1, H/W = 1.0)

ROUND ROBIN PROBLEM

The objective of the round robin problem was to determine the mode I stress-intensity factors along two symmetric elliptical cracks emanating from a centrally located hole in a rectangular plate subjected to uniform tensile loading. The geometry for the problem is depicted in figure 5. The dimensionless ratios for the crack and plate geometry were specified as

$$R/t = 2.0$$
$$a/t = 0.2$$
$$a/c = 0.8$$
$$R/W = 0.2$$
$$H/W = 2.0$$

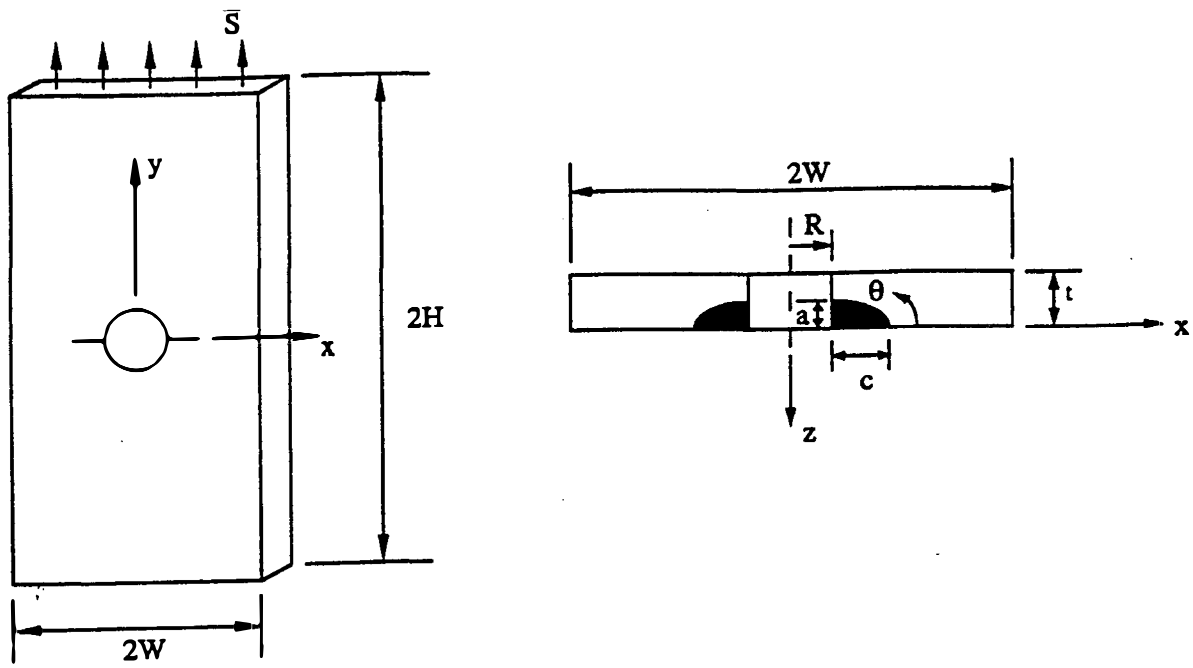
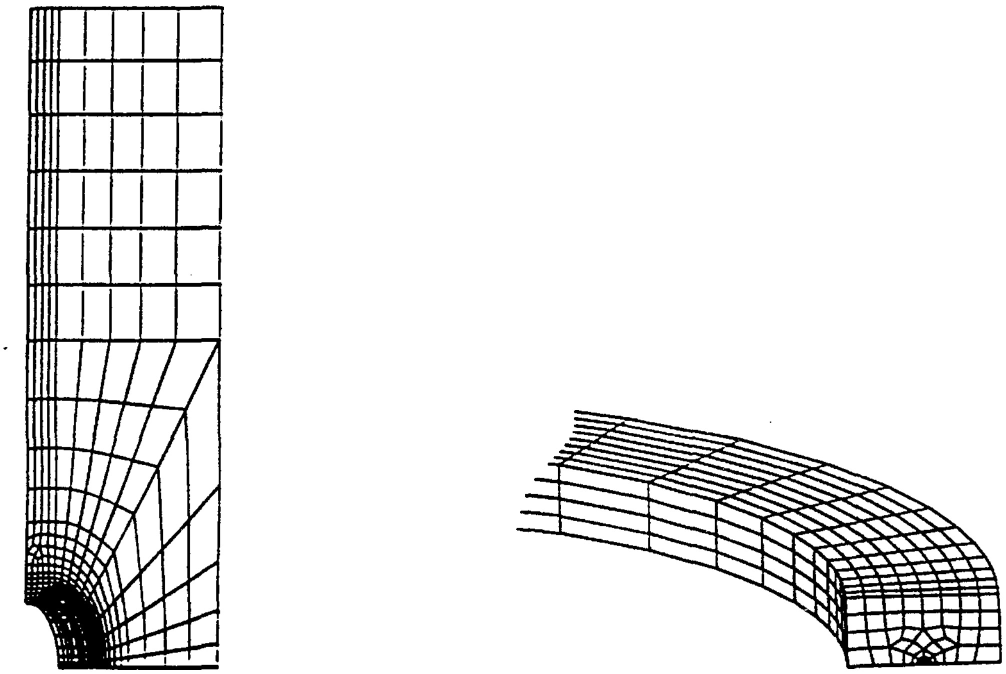



FIGURE 5. THE ROUND ROBIN PROBLEM—QUARTER-ELLIPTIC SURFACE CRACK AT THE CENTER OF A HOLE IN A PLATE SUBJECTED TO TENSION ($R/t = 2.0$, $a/t = 0.2$, $a/c = 0.8$, $R/W = 0.2$, $H/W = 2.0$, $t = 1.0$, $S = 1.0$, $E = 1.0$, $\nu = 0.3$)

In addition, the remote applied stress was prescribed to be unity $S = 1$, the thickness of the plate was given as $t = 1$, and Poisson's ratio $\nu = 0.3$.

Several finite element meshes were constructed having between 12,000 and 20,000 degrees of freedom. A typical finite element mesh employed in the analysis is shown in figure 6(a). The finite element domains along the elliptical crack front needed for the evaluation of the pointwise energy release rates were constructed by sweeping a two-dimensional crack tip mesh through an elliptical curve as shown in figure 6(b). The two-dimensional crack tip mesh employed for the round robin problem is identical to those used in the benchmark comparisons.



(a) (b)

FIGURE 6. A TYPICAL FINITE ELEMENT MESH EMPLOYED IN THE ANALYSIS (16743 DEGREES OF FREEDOM) (a). THE FINITE ELEMENT DOMAINS ALONG THE ELLIPTICAL CRACK FRONT WERE CONSTRUCTED BY SWEEPING A TWO-DIMENSIONAL CRACK TIP MESH THROUGH AN ELLIPTICAL CURVE (b).

The present finite element results for the round robin problem for three different meshes are plotted versus physical angle $\theta$ in figure 7. The physical angle $\theta$ around the elliptical crack front is defined in figure 2. Keeping the discretization for the two-dimensional crack tip mesh constant, the mesh was refined until the results no longer changed significantly. The final mesh had 19,179 degrees of freedom, and the tabulated values for the stress-intensity factors $K_I$ versus physical angle $\theta$ for this case are given in table 1.

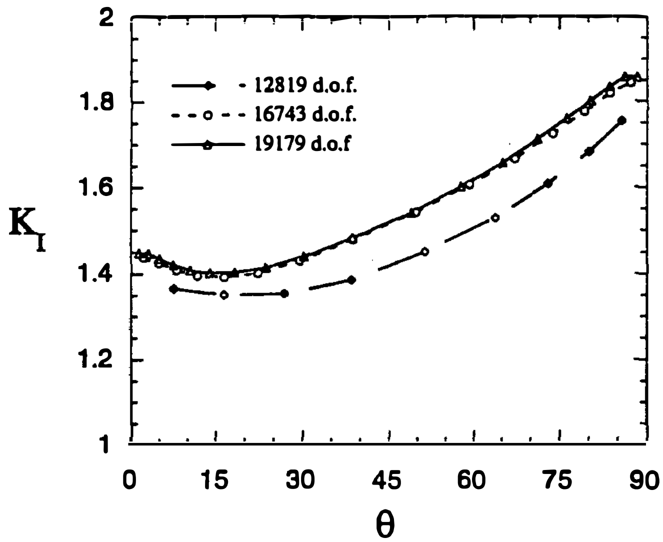



FIGURE 7. STRESS-INTENSITY FACTORS VERSUS PHYSICAL ANGLE FOR THE ROUND ROBIN PROBLEM

TABLE 1. STRESS-INTENSITY FACTORS VERSUS PHYSICAL ANGLE FOR ROUND ROBIN PROBLEM

| θ | $K_I$ |
|---|---|
| 1.39 | 1.448 |
| 3.05 | 1.448 |
| 5.05 | 1.436 |
| 7.47 | 1.422 |
| 10.38 | 1.410 |
| 13.92 | 1.403 |
| 18.24 | 1.403 |
| 23.57 | 1.414 |
| 30.22 | 1.440 |
| 38.66 | 1.483 |
| 48.89 | 1.539 |
| 57.67 | 1.600 |
| 65.07 | 1.658 |
| 71.20 | 1.712 |
| 76.21 | 1.760 |
| 80.28 | 1.801 |
| 83.56 | 1.834 |
| 86.19 | 1.857 |
| 88.31 | 1.857 |

We state as a final cautionary remark that the finite element results for this problem depend heavily on mesh design as well as the number of degrees of freedom in the model. It is extremely important to design a mesh which accurately captures the stress field along the crack front as well as the stress concentration around the hole. An inappropriate mesh along the crack front and/or a mesh which is not adequately focused around the hole can yield considerably lower values for $K_I$.

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