Use of Simple Models to Determine Wake Vortex Categories for New Aircraft

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The paper describes how to use simple models and, if needed, sensitivity analyses to determine the wake vortex categories for new aircraft. The methodology provides a tool for the regulators to assess the relative risk of introducing new aircraft into the current fleet.

Nomenclature

b	=	wingspan, ft
b_0	=	separation of the two rolled up vortices, ft
C_l	=	roll moment coefficient
C_L	=	lift coefficient
$C_{L\alpha}$	=	lift curve slope
DOT	=	Department of Transportation
FAA	=	Federal Aviation Administration
ICAO	=	International Civil Aviation Organization
IFR	=	Instrument Flight Rules
L	=	lift
MLW	=	Maximum Landing Weight, lb
MTOW	=	Maximum Certificated Gross Takeoff Weight, lb
Ν	=	number of non-dimensional time units for vortex strength to go to zero
NM	=	nautical miles
NTSB	=	National Transportation Safety Board
q	=	dynamic pressure, kg/(m sec ²)
r	=	radius of vortex, ft
RECAT	=	Recategorization
RMC	=	roll moment coefficient
S	=	spanwise loading coefficient
S	=	wing area, ft ²
sec	=	second(s)
Т	=	non-dimensional time or torque

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U	=	final approach speed, ft/sec
U_∞	=	aircraft speed, ft/sec
US	=	United States
V_{v}	=	tangential velocity of a vortex, ft/sec
W	=	weight, lb
у	=	distance along a wing from the center of the fuselage, ft
α	=	angle induced on a section of a wing due to a vortex, radians
Γ	=	vortex circulation strength, ft ² /sec
Γ_0	=	initial vortex circulation strength, ft ² /sec

 ρ = air density, slug/ft³

I. Introduction

For the sole purpose of mitigation of wake turbulence encounter risk, additional separation is required between some in-trail aircraft pairs beyond the minimum radar separation required for collision risk mitigation. Existing procedures governing the application of wake turbulence separation in Instrument Flight Rules (IFR) operations are dependent on the weight classes of the two in-trail aircraft. Current aircraft weight class definitions in the United States [1] include Small aircraft with Maximum Certificated Gross Takeoff Weight (MTOW) of 41,000 lb or less, Large aircraft with MTOW more than 41,000 lb but less than 300,000 lb, and Heavy aircraft with MTOW of 300,000 lb or more. As such, MTOW has served as a surrogate metric for both the wake turbulence severity generated by the lead aircraft and the wake encounter vulnerability of the trailing aircraft. Historically, new aircraft have been automatically assigned to a weight class based on their MTOW, and thus have been assigned wake turbulence separation in front and behind, with no additional assessments.

In 1994, the National Transportation Safety Board (NTSB) issued Recommendation A-94-056 which states:

"Require manufacturers of turbojet, transport category airplanes to determine by flight test or other suitable means, the characteristics of the airplanes' wake vortices during certification environments."

While sharing the Board's overall concern, the Federal Aviation Administration (FAA) recognized that the wake turbulence separation is not a certification issue, but an operational issue that includes procedures and airspace design that, when combined with separation minima, provide for an acceptably safe system. The FAA's Wake Turbulence Research Office has worked with the FAA's regulatory agency to assess new aircraft. In addition, the Air Traffic Organization and Aviation Safety within the FAA are pursuing a new wake turbulence categorization that includes aircraft characteristics, in addition to weight, that more completely define the wake turbulence risk as a generator and encounter aircraft (Recategorization [2], RECAT I, RECAT 1.5 [3] and RECAT II).

II. Simple Models and Data

One of the methods for assessing the wake turbulence separation for new aircraft is the use of simple models -they are easy to apply and they contain sufficient physics to describe the problem at hand. These models are applied to determine the relative risk of introducing the new aircraft into the fleet of current aircraft. This paper describes the conditions when simple modeling is used and the conditions when the use of simple modeling requires additional sensitivity analyses to provide safe separation minima. It also addresses the range of sensitivity analyses necessary under both conditions. It addresses the required aircraft parameters to use in the simple models and where the parameters can be obtained. And, finally, the simple models are described. Although this paper addresses the use of simple models when introducing new aircraft, the concepts are extended to many Heavy and Large aircraft in the new FAA wake turbulence categorization projects mentioned above (RECAT).

A Heavy aircraft has a MTOW in excess of 300,000 lb (136,000 kg). Using simple modeling, the Airbus A380-800 (A388) was determined to require an additional 2 NM in separation relative to the Boeing B747-400 (B744). Similarly, the B747-8 (B748) was assessed using simple modeling and the resulting required separation was 0.1 NM more than for the B744. Both manufacturers chose to perform so-called back-to-back flight tests with dedicated wake turbulence measurements under controlled conditions as described in Ref. [4]. Based on those flight tests, the required separation for the A388 did not change from that established with the simple modeling and the separation for the B748 was established to be no greater than required for other Heavy aircraft. Figure 1 shows the large extent of the Heavy/Large/Small aircraft, both in MTOW and wingspan. If the new aircraft MTOW and wingspan place the aircraft within the cluster of current Heavy/Large/Small aircraft, the new aircraft is likely to be appropriately characterized as a Heavy/Large/Small aircraft. Because the models used herein are "simple", some aircraft

performance characteristics are not finalized until high lift flight tests are performed and one characteristic in particular -- the separation of the two rolled up vortices, denoted by b_0 -- is not well known without lidar testing, the use of these simple models needs to be coupled with sufficient sensitivity analyses to describe the wake vortex effects near and beyond the region of Heavy/Large/Small MTOWs and wingspans to ensure conservative separation minima.



Figure 1. Wingspan vs MTOW for the Current US Wake Turbulence Classes.

The simple models require the following parameters for each make/model/series of the new aircraft:

- Wingspan, b.
- Maximum landing weight, MLW. The maximum landing weight is required to characterize the wake turbulence for landing operations.
- Final approach speed, U. The final approach speed is required to characterize the wake turbulence for landing operations.

If the analysis is to be conducted early in the development of the new aircraft, the manufacturer is likely to have only bounds on the parameters needed, and thus the separation determination will need to include a sensitivity analysis using the appropriate maxima and minima of the parameters. The sensitivity analyses will include the manufacturer's bounds and may include additional bounds drawn from other aircraft in operation. It is preferred to use the production version of the aircraft parameters. If the analysis is conducted close to the time of initial flight tests, then a good manufacturer-prepared document containing the parameters is the airport planning document, as it contains values for the many variants of the new aircraft that will be offered.

III. Assumptions

Two assumptions have been fundamental in developing a safety risk assessment of proposed wake vortex categories of new aircraft. These assumptions are: (A) current operations are safe and (B) aircraft performance characteristics provide for a more complete wake vortex risk assessment than does MTOW alone. This paper addresses separations for new Heavy aircraft.

The current wake turbulence separations as applied to Heavy aircraft, which at the lower end of the weight category is represented by the A300-600 (A306) aircraft and at the upper end by the B747-400 (B744) aircraft, are safe. [The heavier B747-8 is not used herein as it is a relatively new aircraft and has not yet accumulated sufficient operational experience.] This assumption has been validated through more than 20 years of National Airspace System operations with the current weight-based category system. The lower weight boundary for Heavy aircraft was raised in 2010 from 255,000 lb to the International Civil Aviation Organization (ICAO) 300,000 lb boundary as a result of the FAA bringing the three variants of the B757 into the Large category (with additional separations required behind the B757s) [5]. Thus, nothing in the assessment of today's operations or previous wake turbulence analyses contradict the assumption that today's operations are safe.

Aircraft characteristics such as weight, wingspan, and approach speed and additional wake parameters such as circulation and roll moment coefficient (RMC) provide for a more complete wake turbulence risk assessment (as a leader or wake-generator aircraft and as a follower or wake-encounter aircraft) than does MTOW alone. Wake strength, as characterized by circulation, is a function of the aircraft weight, wingspan and approach speed and therefore, the use of these additional factors provides for a more robust risk assessment for the aircraft as a leader than does weight (MTOW) alone. The roll moment coefficient is a more direct characterization that relates to aircraft vulnerability in a wake encounter than MTOW. Roll moment coefficient includes the wingspan and approach speed of the follower aircraft and the torque applied to the encountering aircraft from the wake of the generating aircraft. These factors provide a more robust assessment of the risk for the aircraft as a trailing aircraft than MTOW provides alone.

IV. Comparison Method

Wake turbulence separations are determined by characterizing the wake vortices generated by the new aircraft as the lead aircraft and ensuring that the response of a following aircraft to the wake turbulence is safe. In the examples discussed below, the aircraft parameters were obtained from publically available sources such as Jane's, manufacturer websites and type certificates or calculated using publically available parameters (e.g., using stall speed to determine approach speed).

A. New Aircraft as the Lead Aircraft

From a wake vortex standpoint the safety critical region is on final approach where aircraft are slowing down, are flying within a narrow corridor and are close to the ground with limited maneuvering space. Separation standards are established to ensure that the wake vortices from a lead aircraft have an adequate time to move away from the approach zone and/or decay to an acceptably safe level.

To estimate the circulation of the wake vortices of the new aircraft which a following landing aircraft could experience, two models are extensively used: a model for the initial strength of the vortices and a model for the decay of the vortices. The initial strength of a vortex (denoted by Γ_0) is given by:

$$\Gamma_0 = W/s\rho Ub,\tag{1}$$

where Γ_0 is the initial strength (just behind the aircraft after wake rollup), *W* is the weight of the aircraft, *s* is the span wise loading coefficient, ρ is the air density, *U* is the final approach speed and *b* is the wingspan. *W* is the landing weight which is set at the MLW as it will give the strongest vortex strength. For simplicity, *s* is usually assumed to be $\pi/4$ as a starting point for the analysis. The simple modeling includes the possibility of deviations of *s* from the $\pi/4$ assumption which will lead to conservative results.

Extensive measurements of the decay of wake vortices have shown that the strength of wake vortices indicate a nearly linear behavior over much of their lifetime when vortices are in ground effect and can be bounded by a linear function when expressed in terms of non-dimensional quantities. One unit of time represents the time for vortices to descend a distance equal to the initial spacing between the two vortices $(\pi^3 b^2/8\Gamma_0)$; for a B744 at MLW and $s=\pi/4$, the vortices take 28.4 seconds to descend a distance equal to the initial spacing between the vortices. So for the B744 one unit of time is 28.4 seconds, two units of time are 56.8 seconds and so on. However, the strength of

vortices eventually goes to zero and the number of non-dimensional time units required for the vortices to go to zero is denoted by N. Values of N in the literature vary from about 6 to 8 and N=8 will be used herein as it represents longer-lasting vortices (for the B744, N=8 indicates a time of 8x28.4 or 227.2 seconds); the N=8 value was derived using the data reported in Ref. [6] and used in the 1994 analyses which set the separation standards for the B757. The strength of a vortex as a function of non-dimensional time is bounded by:

$$\Gamma = \Gamma_0 \left(1 - T/N \right),\tag{2}$$

where Γ_0 is the initial strength of the vortices as noted above and T is the non-dimensional time. For example, a B744 vortex at T = 4 non-dimensional time units (4x28.4 or 113.6 seconds) will have decayed to $\frac{1}{2}$ its initial strength.

To check that the new aircraft fits within the current aircraft fleet, Eq. (2) is used to compare the new aircraft with current aircraft. The equation is plotted for the non-dimensional equivalent times of 60, 90 and 120 seconds (corresponding approximately to 3 NM, 4 NM and 5 NM separations behind a Heavy aircraft) for each aircraft. Figure 2 shows the results of these calculations comparing the B787-8 (B788) and B787-9 (B789) aircraft with other current Heavy aircraft.

It is noted that the B788 and B789 values for the initial strength Γ_0 and the strength at separations relevant to following aircraft are all bracketed by the values for the current Heavy aircraft. Therefore, this simple model can be used to check whether a new aircraft appropriately fits into the current Heavy category. Note that, if the model shows that the values for the new aircraft exceed the values for the existing Heavy aircraft, additional sensitivity analyses are required. In addition, if the vortex spacing (b_0) for the new aircraft is expected to vary significantly from $\pi/4$ times the wingspan, the analysis should be repeated with values of vortex spacing which bracket the expected values. This can arise, for example, when different manufacturers have different design philosophies or different mission requirements.

As an additional case, consider the application of the described framework to the new A350-900. Using Eq. (1) and (2) and assuming $\pi/4$ for the spanwise loading coefficient, the initial circulation and the circulation at wake ages of 60, 90 and 120 seconds are shown in Fig. 3. The A350-900 circulation is less than the B744, A346, B773 and B772 at 60 seconds; less than the B744, A346 and B773 at 90 seconds; and less than the B744 and equal to the A346 at 120 seconds. Thus, as a wake generator, the A350-900 belongs in the Heavy category.



Heavy Category Aircraft Circulation Decay

Figure 2. Initial Strength at MLW and Strength at 60/90/120 seconds comparing B787 with Heavy Aircraft.



Figure 3: Initial Circulation at MLW and Circulation at 60/90/120 seconds comparing A350-900 with Heavy Aircraft.

B. New Aircraft as the Follower Aircraft

The previous section examined whether the new aircraft can be appropriately placed in the Heavy category as the wake vortex generator. Now consider the new aircraft as a wake vortex encountering aircraft.

Using standard aerodynamic methods, it can be shown that the non-dimensional vortex-induced rolling moment coefficient (Torque/*qSb*, where *q* is the dynamic pressure and *S* is the wing area) on an aircraft encountering a wake vortex of strength Γ is proportional to the encountered strength divided by the product of the aircraft speed and wingspan (Γ/Ub). [See Appendix.] The vortex strength Γ is for the vortex-generating aircraft and the approach speed and wingspan are for the vortex-encountering aircraft, the new aircraft in this discussion. It represents a maximum roll moment coefficient when it is assumed that the vortex is a potential vortex (zero vortex core size) and the wing section lift curve slope is constant across the span (no wing tip loss of efficiency).

To determine if the new aircraft appropriately fits in the Heavy category as a following aircraft, or requires additional separation as an in-trail aircraft, the vortex-induced rolling moments of current Heavy aircraft landing at their MLW are compared to the vortex-induced rolling moment of the new aircraft landing at its MLW. Again, in this analysis, only aircraft for which there is significant operational experience are used. A relative comparison is made for the current Heavy aircraft and the new aircraft following the heaviest aircraft in the current Heavy category. The strengths are calculated using the equations developed above and at a non-dimensional time of 90 seconds as that corresponds to a separation distance of about 4 NM at typical approach speeds, the approximate separation for a Heavy aircraft following a Heavy aircraft. The values of Γ/Ub of the current Heavy aircraft and the ratio generation of the B744 as the circulation of the leader, as the B744 is currently the largest Heavy with sufficient operational history. To make the results easier to visualize, the ratios were all divided by the ratio for the A300B4-600 (A306) which is the smallest of the Heavy aircraft. Figure 4 shows the normalized roll moment coefficient ratios at 90 seconds with the B744 as the lead aircraft.



Figure 4. Normalized Roll Moment Coefficient Ratio at 90 seconds comparing B787 with Heavy Aircraft.

The A300-600 (A306) was used to normalize the results and thus the normalized roll moment coefficient ratio of 1.0 corresponds to the most easily upset aircraft in the current Heavy category. A normalized roll moment coefficient ratio less than 1.0 indicates that, if the new aircraft encounters a vortex from a landing Heavy aircraft, the new potentially Heavy aircraft can tolerate a vortex stronger than the A306. Using this simple model a follower aircraft with a normalized roll moment coefficient ratio between the worst case (1.0 for the A306) and the best case (0.6 for the B744) indicates that the aircraft as a follower aircraft is in the Heavy category. Because the models are indeed simple, if the result of this analysis yields a normalized roll moment coefficient ratio for the new aircraft close to 1.0, it may indicate that an additional sensitivity analysis is warranted or another means for determining the separation must be used.

As another example, consider the new A350-900. The results are shown in Figure 5. As expected, the B744 is best able to resist a vortex encounter since it is the largest Heavy aircraft. The A350-900 normalized roll moment coefficient ratio falls within the bounds of the 11 Heavy aircraft as would be expected since its weight, wingspan and speed are similar to the other Heavy aircraft. Thus, the A350-900 is appropriate as a follower aircraft in the Heavy category.

C. Refinements to the Models

Refinements to the simple models based on improvements in understanding of wake and aircraft performance characteristics will allow for more robust analysis. For example, based on extensive data collection, it is noted that landing Heavy aircraft are typically at about 85% of MLW. Figure 6 shows the decay times associated with 4, 5 and 6 NM, the current separation standards for Heavy, Large and Small aircraft, respectively, at runway threshold for aircraft landing behind selected Heavy aircraft. The colored blocks displayed vertically provide a depiction of the relevant timeframes for the different wake category pairings. For Heavy aircraft pairings, Fig. 6 shows that the circulation strengths for the two new A350 aircraft are bounded by numerous Heavy aircraft, including the B744 at lower times and the B777-200LR (B772) at later times.



Figure 5. Normalized Roll Moment Coefficent Ratio at 90 Seconds comparing the A350-900 with Heavy Aircraft.

V. Summary

The use of simple models and supplemental sensitivity analyses to determine the wake turbulence separation requirements for new potential Heavy aircraft has been successfully used for the B748, B788 and B789 and the FAA has also applied this accepted means of compliance to the A350-900. It represents a tool for the regulators to assess the relative risk of introducing new Heavy aircraft into the current fleet of Heavy aircraft.



Figure 6. Circulation vs. Time for A350 and Comparable Aircraft.

Appendix Derivation of the Roll Moment Coefficient (RMC) Severity Metric

The potential severity of a wake encounter can be characterized by the wake strength, technically termed circulation, of the generating aircraft. It can also be characterized by the potential impact of the wake on the following aircraft, in particular the torque or rolling moment imposed on the following aircraft. It is standard aerodynamic practice to express the rolling moment in non-dimensional form as a rolling moment coefficient (RMC). This provides a means of comparing aircraft of different sizes, operating at different speeds, and with different atmospheric density. This is analogous to the use of lift coefficient to characterize the lift on different aircraft.

In addition to circulation and RMC, the literature also has documented the concept of using rolling moment which the control surfaces (ailerons and spoilers) can exert to oppose the wake-induced rolling moment can also be expressed in terms of the same rolling moment coefficient. This provides a method of direct comparison of the potential effect of the wake on the following aircraft and the ability of the following aircraft to counter that effect. A comparison of the wake-induced and counter-control rolling moment coefficients has historically been used as one of the additional metrics for determining the acceptability of a wake encounter. However, the control characteristics of modern aircraft are often proprietary and hard to determine. Therefore, it is difficult to accurately compare these two roll moment coefficients over a broad range of aircraft.

The dynamics of a wake encounter may be quite different for large and small aircraft. Large aircraft have both large wingspans and large roll inertias, both of which act to slow the dynamics of an encounter. The use of a static metric such as RMC is therefore most appropriate for the largest aircraft which have large roll inertias and thus tend to roll slowly in a wake encounter. In addition, the slower response of these larger aircraft allows pilots more time to use controls to oppose the wake-induced roll.

The coefficient of lift, C_L, is defined by:

$$Lift Coefficient = C_L = L/(qS)$$
(3)

where L is the lift, $q = \frac{1}{2}U_{\infty}^{2}$ is the dynamic pressure, ρ is the air density, U_{∞} is the aircraft speed, and the wing area, S, is the wingspan times a mean chord. The coefficient of lift is non-dimensional, allowing wings from different aircraft to be compared, even though the wings may be very different physically.

In a similar way, the rolling moment coefficient, C_l , is defined by:

Rolling Moment Coefficient =
$$C_l = T/(qSb)$$
 (4)

where *T* is rolling moment or torque from a vortex located at the fuselage. Figure 7 shows the vortex located at the center of the fuselage, with the forces from the vortex, shown at particular locations on the wing, as the red arrows.



Figure 7. Schematic view of an airplane in an encounter with a vortex. The vortex is located at the center of the fuselage.

It is noted that Eq. (3) is similar to Eq. (4). But, since torque is a force times a distance, to non-dimensionalize C_l it is necessary to add a length to the denominator of Eq. (4), which is the wingspan, b.

In defining the safety metric, assumptions producing the largest possible impact on the following aircraft were made. In particular, since the vortex core sizes for different aircraft are not generally known, a potential vortex or zero core size was assumed. In addition, since the wing section lift coefficient, c_L , is not generally known, its maximum theoretical value of 2π was used, and it was assumed to be constant across the wingspan.

To derive an expression for the torque, the angle induced on a section of the wing due to the vortex was evaluated (Figure 8).



Figure 8. The angle induced on a section of the wing due to a vortex located at the fuselage.

For a potential vortex located at the fuselage center, the circulation is given by $\Gamma = 2\pi r V_{\nu}$, where Γ is the vortex circulation, r is the radius from the core, and V_{ν} is the tangential velocity from the vortex. In Fig. 8, $\Gamma/2\pi y$ is the tangential velocity, V_{ν} , where y is the distance along the wing from the center of the fuselage. If V_{ν} is small compared to U, then the tangent of the angle is approximately equal to the angle (in radians) so $\Delta \alpha$ is approximately $\Gamma/(2\pi yU)$. The torque is then given by an integration over the half-span as:

$$T=2\int_{0}^{b/2} q c(y) c_{L\alpha}(y) \Delta \alpha(y) y dy$$
⁽⁵⁾

where $c_{L\alpha}$ is the lift curve slope, $dc_L/d\alpha$, $\Delta\alpha(y) = \frac{\Gamma(y)}{2\pi y U_{\infty}}$, and c is the local chord. In Eq. (5), $c_{L\alpha}$ is assumed

constant across the span (for the largest possible torque) and integrated to get:

$$T = \frac{q c_{L\alpha} S \Gamma}{2\pi U_{\infty}}.$$

Putting Eq. (6) into Eq. (4) gives

$$\mathbf{C}_{l} = \frac{\mathbf{c}_{\mathrm{L}\alpha} \Gamma}{2\pi \mathbf{U}_{\infty} \mathbf{b}} \,. \tag{7}$$

Since $c_{L\alpha}$ is assumed constant across the span and equal to 2π , the severity metric is:

$$RMC = \Gamma/Ub.$$
(8)

In Eq. (8), Γ is the circulation of the leading aircraft wake, while U and b are the speed and wingspan of the follower aircraft.

It is noted that the severity metric given in Eq. (8) agrees with the *a priori* notions of a reasonable severity metric. First, as the vortex strength (Γ) of the leader increases, the severity metric increases. Second, as the speed of the follower (U) increases, the severity metric decreases. Finally, as the wingspan of the follower (b) increases, the severity metric decreases.

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