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16. Abstract

Comparing published NAVD 88 Helmert orthometric heights of First-Order bench marks against GPS-determined orthometric heights showed that GEOID03 and GEOID09 perform at their reported accuracy in Connecticut. GPS-determined orthometric heights were determined by subtracting geoid undulations from ellipsoid heights obtained from a network least-squares adjustment of GPS occupations in 2007 and 2008. A total of 73 markers were occupied in these stability classes: 25 class A, 11 class B, 12 class C, 2 class D bench marks, and 23 temporary marks with transferred elevations. Adjusted ellipsoid heights were compared against OPUS as a check. We found that: the GPS-determined orthometric heights of stability class A markers and the transfers are statistically lower than their published values but just barely; stability class B, C and D markers are also statistically lower in a manner consistent with subsidence or settling; GEOID09 does not exhibit a statistically significant residual trend across Connecticut; and GEOID09 out-performed GEOID03. A "correction surface" is not recommended in spite of the geoid models being statistically different than the NAVD 88 heights because the uncertainties involved dominate the discrepancies. Instead, it is recommended that the vertical control network be re-observed.

We tested networks in which no phase-center variations were applied and in which all vectors (including "substandard" vectors) were included. We found that PCV cannot be ignored and that including substandard vectors did not affect the results.

17. Key Words Orthometric heights; GPS; geoid modeling; OPUS; GEOID03; GEOID09; hybrid geoid models; NAVD 88	18. Distribution Statement No restrictions. This document is available to the public through the National Technical Information Service Springfield, Virginia 22161

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SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
°F	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	°C
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
°C	Celsius	1.8C+32	Fahrenheit	°F
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

*SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380.
(Revised March 2003)

Table of Contents

List of Figures

List of Tables

Symbols

H	orthometric height (m)
H_{88}	NAVD 88 orthometric height (m)
H_G	GPS-determined orthometric height
H_{G03}	H_G determined with GEOID03
H_{G09}	H_G determined with GEOID09
h	ellipsoid height (m)
p	p-value, the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true
N	geoid height; geoid undulation (m)

Introduction

The U.S. National Geodetic Survey (NGS) is responsible for the definition, creation, and maintenance of the National Spatial Reference System (NSRS). In its 10-year plan (NGS 2008), NGS announced an intention to replace the existing North American Datum of 1983 (NAD 83) and the North American Vertical Datum of 1988 (NAVD 88) with a new conventional terrestrial reference frame for geometric positions and an Earth gravity model (EGM) to provide dynamic positions (i.e., heights referred to Earth's gravity field). While research is underway to realize a 1-cm geoid model (Roman and Smith 2001), NGS is transitioning away from horizontal and vertical passive survey control markers to active Continuously Operating Reference Stations (CORS) for geometric positioning control and CORS-plus-hybrid gravimetric geoid models for vertical control. U.S. hybrid geoid models are created from an EGM ("pure" gravimetric) that has been modified so that its level surface (the equipotential surface deemed its reference surface) fits the NAVD 88 level surface as well as possible.

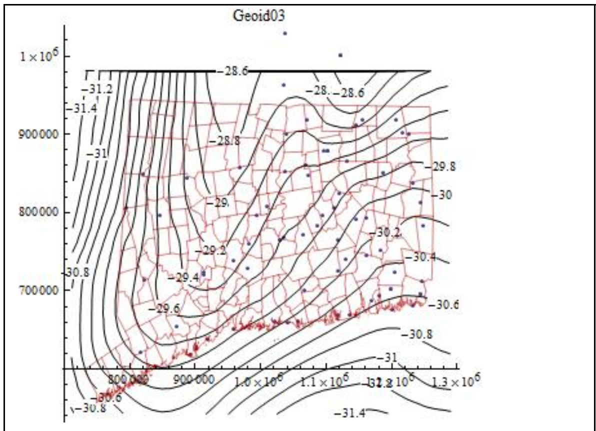

Figure 1. GEOID03 geoid heights (m). Dots indicate observed stations.

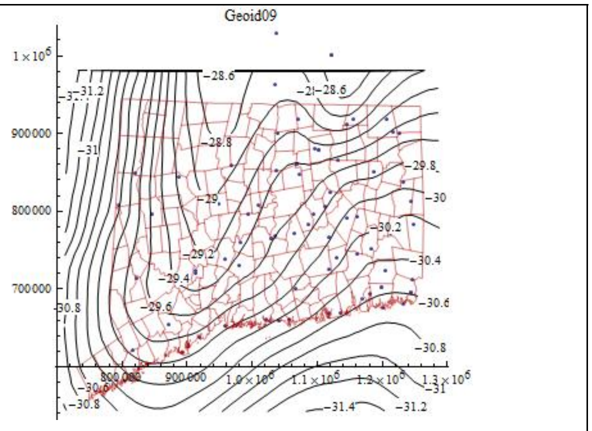

Figure 2. GEOID09 geoid heights (m). Dots indicate observed stations.

GEOID03 and GEOID09 are the two most recent NGS hybrid geoid models (Figs 1 and 2), which are derivatives of the Earth Gravity Model of 1996 (EGM96) (Lemoine et al. 1998) and the USGG2003 for GEOID03, and the Earth Gravity Model of 2008 (EGM08) and the USGG2009 for GEOID09. The EGM96, USGG2003, USGG2009, and EGM08 geoids are consistent with the Department of Defense (DoD) World Geodetic System 1984 (WGS 84) geocenters, which are known to be offset by roughly two meters from the NAD 83 geocenters (Soler and Snay 2004); therefore, the hybrid geoid models are crafted to reflect this difference in geocentricity. Also, the NAVD 88 level surface is thought to be a small but unknown distance from "the geoid" in North America (Zilkoski et al. 1992), and is known to have discernable systematic departures from the actual equipotential surface it is based on (Roman et al. 2004). The NGS hybrid geoid models have their level surfaces altered to match the NAVD 88 level surface as well as possible, which makes it possible to transform geometric positions referred to, say, WGS 84(G1150) into horizontal geometric coordinates referred to NAD 83(CORS96) and a vertical dynamic coordinate referred to NAVD 88.

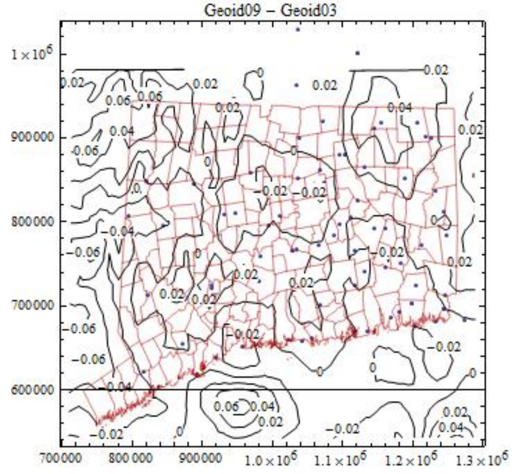

Figure 3. GEOID09 geoid heights – GEOID03 geoid heights (m)

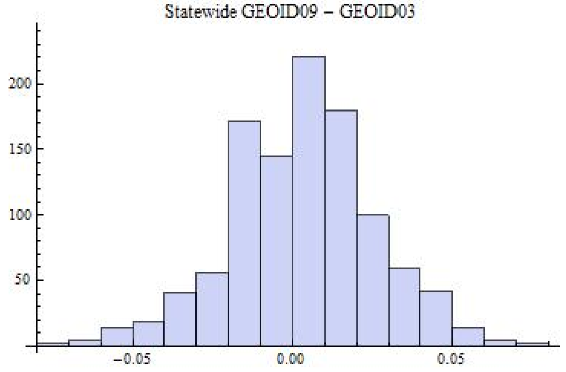

Figure 4. Histogram (frequency) of GEOID09 geoid heights – GEOID03 geoid heights (m)

The newer geoid models have finer spatial resolutions than the older models and are based upon more data, notably gravity observed from the GRACE satellite program (Tapley et al. 2004); gravity observed from aircraft through the GRAV-D program and additional satellite observations from the GOCE mission (Rebhan et al. 2000) will be added to future models (Pers. comm., Daniel Roman). Nevertheless, GEOID03 and GEOID09 look very similar in Connecticut, so the differences are highlighted in Fig. 3, which shows GEOID03 geoid undulations subtracted from GEOID09 geoid undulations. This figure provides a preconception of how different GPS-determined heights using the different models could be, and is summarized in Fig. 4 as a histogram of geoid undulation differences on our observation stations. NGS accuracy assessments report GEOID03 to be uncertain at the 2.4 cm (1-s) and 1.3 cm (1-s) levels for the national average and for Connecticut, respectively (Roman et al. 2004), and for GEOID09 uncertainties are at 1.5 cm (1-s) and 1.6 cm (1-s) levels for the national average and for Connecticut, respectively (Roman et al. 2009).

NGS recommends using the hybrid geoid models (GEOID09 supersedes GEOID03) for orthometric heighting in the U.S., so it is prudent to independently verify their accuracy. A preliminary study showed the GEOID03 was a little more than one centimeter too low over a very small part of Connecticut (Tranes et al. 2007), so this study expanded on the previous work to examine the entire State.

As stated in prose above, the equation relating orthometric height (H) to ellipsoid height (h) and geoid height (N) (also called geoid undulation) is

$$H = h - N$$

This equation is, in fact, not exact but it is close enough for all practical purpose (Jekeli 2000). Careful attention must be paid to the surfaces to which these heights refer. In this study, H is a NAVD 88 Helmert orthometric height denoted by H_{88}. Here ellipsoid heights refer to GRS 80 as placed by NAD 83(CORS96). Hereafter, H_G denotes any GPS-determined orthometric height, and H_{G03} denotes H_G determined with GEOID03, similarly H_{G09} for GEOID09.

Geoid-model accuracy can be assessed by testing whether H_G is statistically indistinguishable from a First-Order, differentially-leveled orthometric height (Ananga and Sakurai 1996; Featherstone 2000; Martin et al. 2005; You 2006; Nahavandchi and Soltanpour 2006; Daho and Fairhead 2007). This comparison is meaningful only if the level surface of the geoid model is the same as, or very close to, the level surface of the vertical datum's level surface. Since no other geoid model reflects the NAD 83/GRS 80 nongeocentricity and no other geoid model is fit to NAVD 88, no other EGMs were considered.

Methods

This study was a cooperative effort between the University of Connecticut (UConn) and the Connecticut Department of Transportation (CTDOT). Connecticut is a relatively small New England state (roughly 100 miles x 60 miles) whose southern border is in the Long Island Sound. Hartford is the largest urban center and is located roughly in the center of the State. Other urban centers line the southern coast; the rest of Connecticut is largely rural. CTDOT built, operates, and maintains a network of 9 CORS; almost nowhere in Connecticut is further than 30 km from a CORS either in Connecticut, Massachusetts, or Rhode Island (see Table 1).

Table 1. NAD 83(CORS96) control coordinates for CT CORS antenna reference points (ARPs). Planimetric coordinates in SPCS83 (m). Ellipsoid height (h) is in meters.

Name	PID	x (m)	y (m)	h (m)
CTBR	DH5825	249003.97	226342.13	53.307
CTDA	DH5827	241156.16	178502.04	-13.265
CTEG	DH5829	309043.04	273469.23	30.283
CTGR	DH5831	363418.46	208383.58	-18.346
CTGU	DH5833	311677.65	203050.08	-18.111
CTMA	DH5835	349660.41	252275.48	55.182
CTNE	DH7113	307755.55	245712.82	41.743
CTPU	DH5837	376234.81	271187.13	57.100
CTWI	DH5839	278269.71	270664.99	192.080

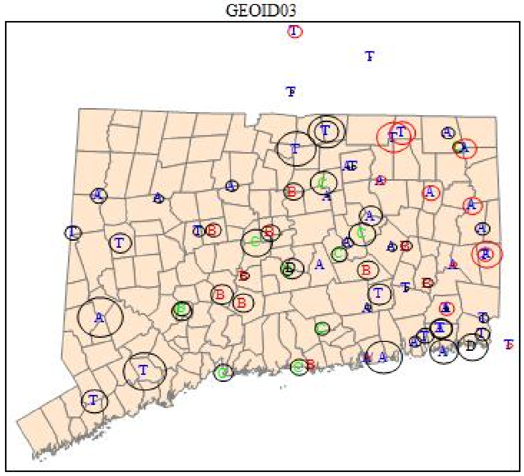

Figure 5. Circles indicate 1-s uncertainty of H_{G03}. Red circles are too high, black are too low. Letters indicate marker stability class.

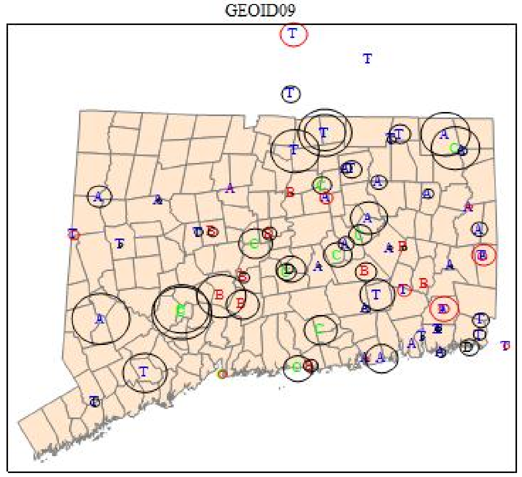

Figure 6. Circles indicate 1-s uncertainty of H_{G09}. Red circles are too high, black are too low. Letters indicate marker stability class.

Exposed bedrock ledge is frequent in Connecticut; therefore much of the First-Order bench mark network is still intact: markers were often set in ledge above the roadway to be safe from snow plows, and the marks in the undeveloped areas have been spared destruction from bulldozers. We recovered and occupied only First-Order bench marks with 25 in stability class A, 11 B class, 12 C class, and 2 D class (Table 2 and Figs. 5 and 6). Undeveloped areas of Connecticut are naturally overgrown with broadleaf hardwoods that often occlude enough of the sky so as to make the bench marks beneath them unsuitable for GPS observations (Meyer et al. 2002). UConn constructed 14 temporary bench marks (TBM) from 70-cm lengths of rebar whose tops were ground round and dimpled with a 1-mm divot to seat a range pole. CTDOT constructed 11 TBMs using square stock, brass bolts in concrete, and survey nails depending on the conditions at each transfer location. Elevations were transferred to the TBMs using Second-Order protocols. Transfer lines were double run twice: once when the mark was set and once after it was occupied for the last time to verify it had not settled. UConn transfer lines were observed with a TOPCON AT301 three-wire automatic level fit with a micrometer observing INVAR rods. After adjustment, all lines closed to better than 0.5 mm per square root km run. CTDOT transfer lines were observed with a Zeiss NI-2 three-wire automatic level and with a TOPCON DL-101 digital level, also with INVAR rods.

Low ridges (less than 725 m elevation) line Connecticut's western border with New York, the rest of the State's topography consists of low rolling hills incised by six river systems that empty into Long Island Sound. The Connecticut River divides the State east-to-west. UConn was largely responsible for observations east of the Connecticut River, and CTDOT was largely responsible for observations west of the River.

GPS observations followed NGS guidelines (Zilkoski et al. 1997; Zilkoski et al. 2000) and were conducted in 2007-2008. CTDOT observed with Trimble 5700 receivers and Trimble Zephyr geodetic antennas with ground planes. UConn observed with TOPCON HiPer Lite+ receivers with internal antennas, Javad Odyssey receivers with internal antennas, and Javad Legacy receivers having LegAnt antennas. These are dualfrequency, C/A-Code, P-Code (codeless), and L1/L2-phase observing instruments. All antennas were set atop fixed height, 2-m range poles set in tripods. Cap-divot depth was not recorded, but was assumed to be 1 mm for all stations. Observation sessions had four simultaneously observing receivers by design and lasted four hours. All stations were re-observed at least once.

GPS observations were processed using TOPCON Pinnacle. Observations were processed with IGS precise ephemerides to produce double-differenced base line vectors between receiver phase centers. NGS antenna calibrations for phase center offsets and for phase center variation were input into Pinnacle’s antenna database. Pinnacle expects the instrument height to be the vertical separation between the mark and the antenna reference point, so all instrument heights (*hi*’s) were set to 2.000 m.

Vectors were assembled into networks and adjusted using least squares to produce NAD 83(CORS96) ellipsoid heights. Following NGS guidelines, vectors whose RMS exceeded 2.0 cm were deleted from the network. Ellipsoid heights were compared against those produced by PAGES through its OPUS interface to check consistency across different processing kernels; otherwise, it would be impossible to discern whether any discrepancies in orthometric height were due to the geoid model or from the GPS processing software. The vectors were also checked geometrically by inversing between CORS coordinates and comparing with Pinnacle-processed baseline vectors: all vectors were statistically indistinguishable (95%) from the geometric vector coordinates produced by inversing.

This data set is suitable for testing the central objective but it is also suitable for exploring other ideas. The central hypotheses are that at each station H_{G03} equals H_{88} and that H_{G09} equals H_{88}. We also constructed networks (i) using all vectors – not just those meeting NGS guidelines – to test whether substandard vectors would corrupt the results (663 in-specification vs. 1659 substandard), and (ii) without applying PCV corrections to test whether these corrections impacted the outcome.

Table 2. Occupied First-Order bench marks and TBMs. Marks named “Trans *PID*” and “TBMxxxx” are temporary bench marks with transferred elevations. Marks whose stability class has an asterisk are tidal bench marks. BM 3080 was reset by CTDOT.

Name	PID	stability	H (m)
13RM01	LX0901	B*	2.196
846 4336 E		C*	5.8445
BM 1554	LX1634	B	22.588
BM 1559	LX1640	B	47.693
BM 1692	LX1650	C	25.762
BM 2152	LW1675	C	110.649
BM 2468	LX1955	A	162.76
BM 2595	LX2362	A	154.561
BM 2608	LX2376	C	56.856
BM 2710	LX2723	B	56.343
BM 2728	LX2699	A	87.288
BM 2792	LX2629	A	305.39
BM 2803	LX2792	B	116.116
BM 2892	LX3397	B*	9.155
BM 2952	LX2892	B	16.037
BM 3080	LX3292	B reset	91.109
BM 3086	LX3312	B	134.537
BM 3109	LX3162	B	37.844
BM 3118	LX3066	D	9.434
BM 3138	LX2948	C	60.398
BM 3178	LX3266	C	79.747
BM 640	LX0198	A	1.857
BM 697	LX0452	C	2.147
BM 827	LX0466	B	4.8
BM 960	LW0733	D	1.914
CGS 5921	LX2511	C	56.902
CGS 5922	LX2509	C	44.859
LIGHT	LX7598	C*	6.006
LW1668	LW1668	A	161.533
LW1677	LW1677	A	98.666
LW1700	LW1700	A	82.905
LX2170	LX2170	A	127.391
LX2912	LX2912	A	258.323
LX3028	LX3028	A	200.566
LX3081	LX3081	A	157.836
LX3101	LX3101	A	112.571
LX3206	LX3206	A	165.737
LX3298	LX3298	A	93.419
LX3353	LX3353	A	76.182
LX3431	LX3431	A	37.423
LX3438	LX3438	A	64.35
LX3467	LX3467	A	12.064
Trans LW0739		T	11.896
Trans LW0821		T	18.81831
Trans LW1745		T	30.574
Trans LX2069		T	56.39
Trans LX2097		T	285.919
Trans LX2914		T	201.311
Trans LX3320		T	110.647
Trans LX3371		T	93.04
Trans LX3418		T	8.289
Trans LX3431		T	37.1396
Trans LX3438		T	63.143
Trans MZ1098		T	19.241
Trans MZ1125		T	139.761
Trans MZ1159		T	37.175
SKYLINE RM4	LX3109	C	229.104
TBM1400		T	126.0192
TBM2248		T	55.5444
TBM2267		T	282.1454
TBM2392		T	40.248
TBM2490		T	109.174

LW1717	LW1717	A	76.766		TBM2575		T	131.3569
LW1808	LW1808	A	159.942		TBM2656		T	151.056
LW1818	LW1818	A	118.955		TBM2807		T	186.3837
LX0112	LX0112	A	4.27		TBM2941		T	25.04577
LX0121	LX0121	A	7.147		ZIEMBA	LX2642	C	121.737
LX2150	LX2150	A	88.636					

Results

Pinnacle Validation

Figure 7 shows Pinnacle ellipsoid heights subtracted from OPUS ellipsoid heights to assess correlation. The plot was sorted by difference, so the abscissa indicates a station's place in the sorting order and the ordinate is ellipsoid height difference in meters. The letters indicate the stability class of the marker, and the different colors merely indicate stability class making it easier to see the class members. Figure 8 shows the same data plotted with error bars. The two end points indicate extreme differences. These stations had the most sky occlusion from tree canopies, and OPUS produced ellipsoid heights with around a meter of uncertainty. Only the two extreme stations were statistically different from the mean (99%). These stations had the worst sky occlusion from tree canopy and OPUS returned very large uncertainty for these stations.

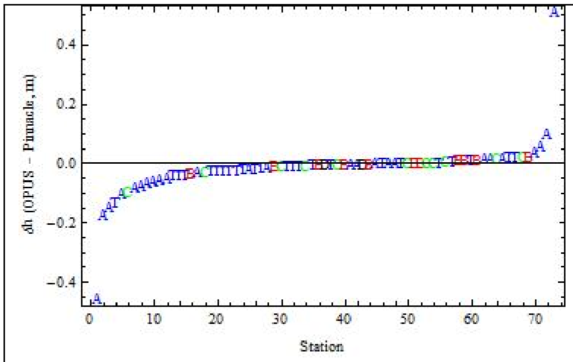

Figure 7. Pinnacle ellipsoid heights subtracted from OPUS ellipsoid heights. Letters indicate stability class. Color groups stability classes visually.

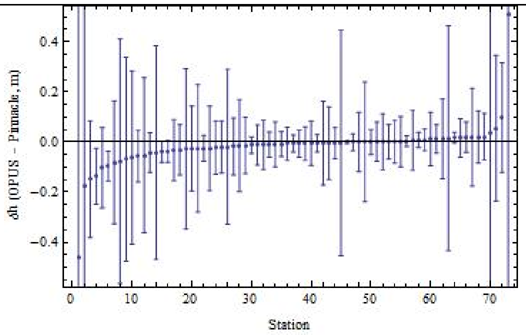

Figure 8. Error bar plot of Fig. 7.

Figure 9 shows the results after removing the highest and lowest stations to focus on the more representative results, and Fig. 10 is a histogram of these differences. The population is zero-centered and appears to be slightly left-tailed, meaning Pinnacle's ellipsoid heights appear to be slightly lower than OPUS's. Based on these results, there is no statistical evidence that Pinnacle and PAGES produce significantly different ellipsoid heights (p = 0.0001).

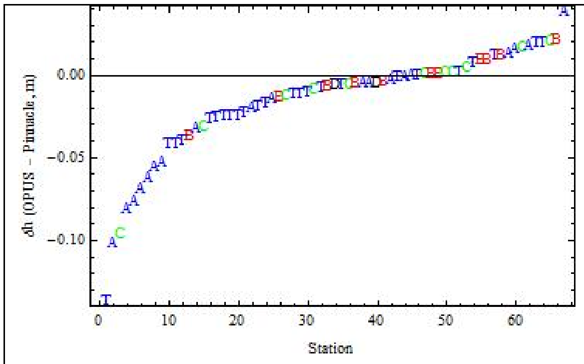

Figure 9. Pinnacle ellipsoid heights subtracted from OPUS ellipsoid heights after removing highest and lowest extreme stations. Letters indicate stability class. Color groups stability classes visually.

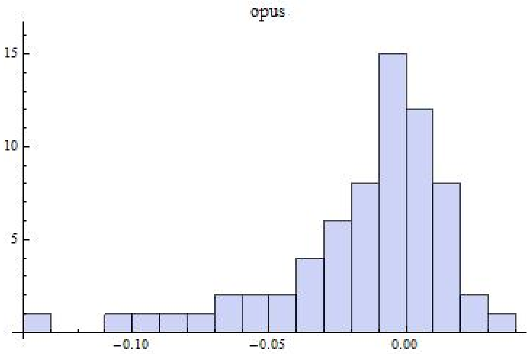

Figure 10. Frequency histogram of Pinnacle/OPUS ellipsoid height differences after removing outliers.

Accuracy of GEOID03 and GEOID09

Figure 11 shows $H_{G03} - H_{88}$ and $H_{G09} - H_{88}$ to assess correlations between Pinnacle and the geoid models and between the models themselves, GEOID03 results marked with a `3' and GEOID09 results marked with a `9'. The plot was sorted by difference, so the abscissa indicates a station's place in the sorting order and the ordinate is orthometric-height difference in meters. The most extreme differences were -5.8 cm and +3.3 cm. This compares well with the preconceived maximum differences above. The box-whisker plot in Fig. 12 indicates the top two GEOID03 results are outliers and none are outliers for GEOID09. Frequency histograms are shown in Fig. 13. Neither GEOID03 nor GEOID09 is zero-centered: for GEOID03 and GEOID09, the means are -0.011 m and -0.015 m and standard deviations are 0.016 m and 0.020 m, respectively. Both appeared skewed but, from inspecting Fig. 11, the two models track each other well.

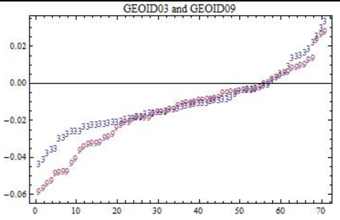

Figure 11. $H_{G03} - H_{88}$ plotted with "3" and $H_{G09} - H_{88}$ plotted with "9"

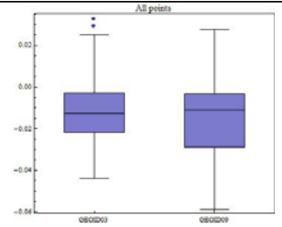

Figure 12. Box-whisker plots of *H* differences

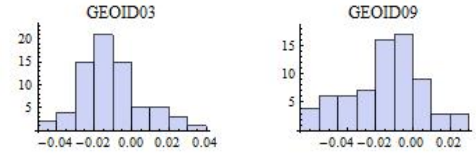

Figure 13. Frequency histograms of *H* differences

We wondered if marker stability played a role in these results. Figures 5 and 6 show the markers in their geographic context. The radii of the circles are proportional to the difference between the GPS-derived heights and leveling; red circles mean GPS-heights are too high and black means too low.

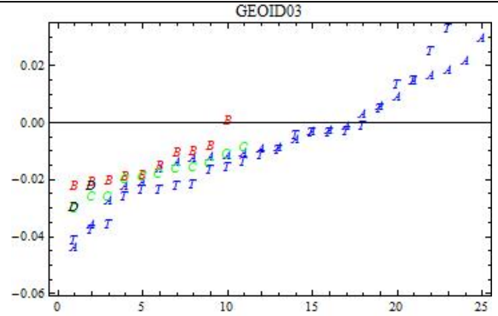

Figure 14. $H_{G03} - H_{88}$ **plotted by stability class**

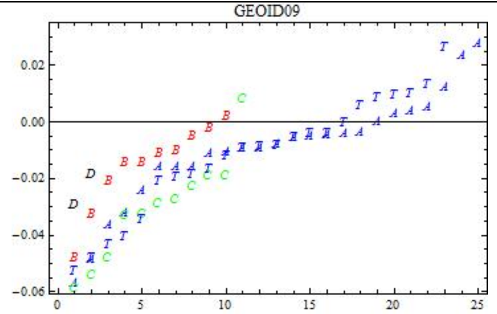

Figure 15. $H_{G09} - H_{88}$ **plotted by stability class**

No clear patterns seem to merge in Figs. 5 and 6, so we plotted differences by stability class, see Figs. 14 and 15. Stability A and T results show a slight negative average; they occur less often too high as too low, meaning the NAVD 88 published height is higher than the GPS-determined height more often than not. All the other stability classes are uniformly too low. We believe the results for class B-D indicate marker subsidence or settling (and possibly disturbance).

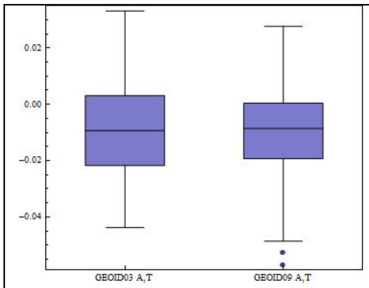

Figure 16. $H_G - H_{88}$ **box-whisker plots by stability class A&T**

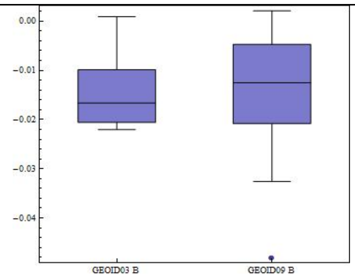

Figure 17. $H_G - H_{88}$ **box-whisker plots by stability class B**

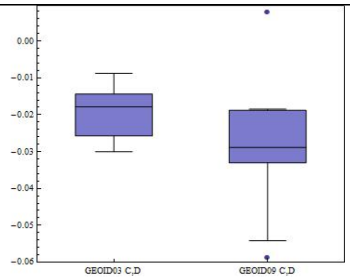

Figure 18. $H_G - H_{88}$ **box-whisker plots by stability class C&D**

Figures 16-18 show box-whisker plots for class A and T together, Class B, and classes C and D together. None are zero centered. After grouping Class A and Class T markers together, a statistical test (99%) rejected the null hypothesis that the mean was equal to zero for GEOID03 ($p = 0.0011$) and GEOID09 ($p < 0.0001$). Even so, Figs. 19 and 20 show that no GEOID03 markers (all classes) are statistically different (99%) than zero and that 16/70 = 23% GEOID09 markers (all classes) are statistically different (99%) than zero.

Fitting a linear surface to the differences gives these models:

for GEOID03 $dif = -0.011 + 0.007x + 0.005\,y$, and

for GEOID09 $dif = -0.015 + 0.006\,x + 0.0004\,y$,

where x and y are SPCS83(0600) eastings and northings standardized to have zero mean and a standard deviation of one exactly. These models seem practically the same, but although the constants of both models are significant, both of the GEOID03 trend terms are significant ($px < 0.0001$ and $py = 0.003$), but neither of the GEOID09 trend terms are significant ($px = 0.015$ and $py = 0.87$).

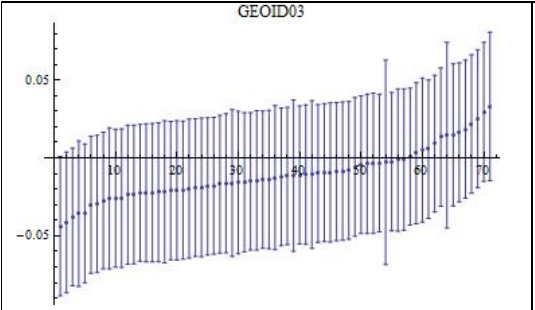

Figure 19. $H_{G03} - H_{88}$ **error bar plot for all stations**

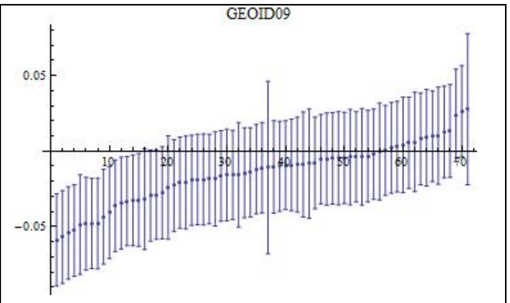

Figure 20. $H_{G09} - H_{88}$ **error bar plot for all stations**

The Affect of Substandard Vectors

We created a Pinnacle vector network containing all vectors, including those deemed substandard by NGS guidelines, to compare the heights produced including the substandard vectors against the heights produced having only the prescribed vectors. Figures 21 and 22 show scatter plots of H_{G03}-H_{88} and H_{G09}-H_{88} with the abscissa having H_G computed with the substandard vectors and the ordinate having been computed with the prescribed vectors. The straight lines show perfect correspondence; departure from the line indicates different performance.

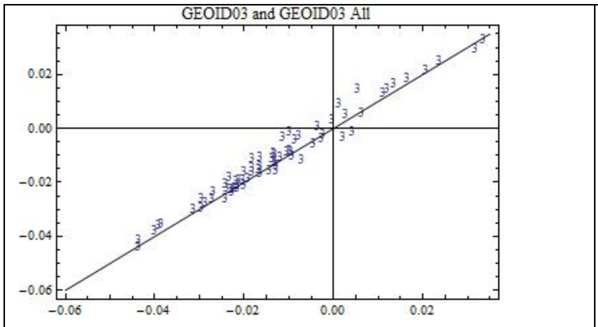

Figure 21. $H_{G03} - H_{88}$ **computed with all vectors against** $H_{G03} - H_{88}$ **computed with good vectors**

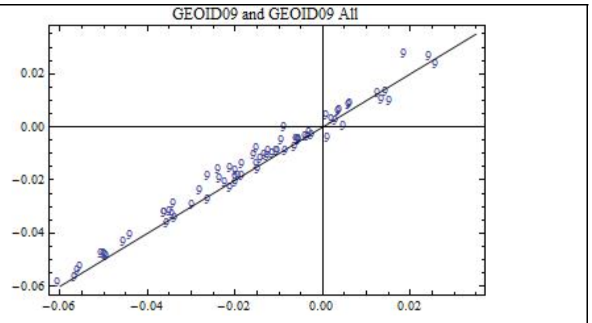

Figure 22. $H_{G09} - H_{88}$ **computed with all vectors against** $H_{G09} - H_{88}$ **computed with good vectors**

Testing whether the differences between the substandard versus prescribed models were different revealed a difference ($p = 0.00003$ and $p = 0.00004$) for GEOID03 and GEOID09, respectively, with the substandard networks being slightly closer to the published values. Even so, we conclude that including the substandard vectors had no practical effect, deleterious or beneficial, because the uncertainty in the results far exceeds the discrepancy.

The Affect of Phase Center Variation

We created a Pinnacle vector network containing only per-specification vectors but explicitly prevented Pinnacle from applying phase center variation corrections to examine whether these corrections would affect the results. Figures 23 and 24 show scatter plots of H_{G03}-H_{88} and H_{G09}-H_{88} with the abscissa having H_G computed without

PCV corrections and the ordinate having been computed with PCV corrections. The straight lines show perfect correspondence; departure from the line indicates different performance.

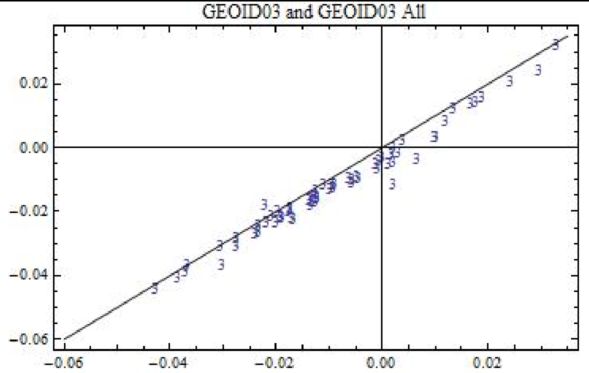

Figure 23. $H_{G03} - H_{88}$ **computed with no PCV correction against** $H_{G03} - H_{88}$ **computed with PCV correction**

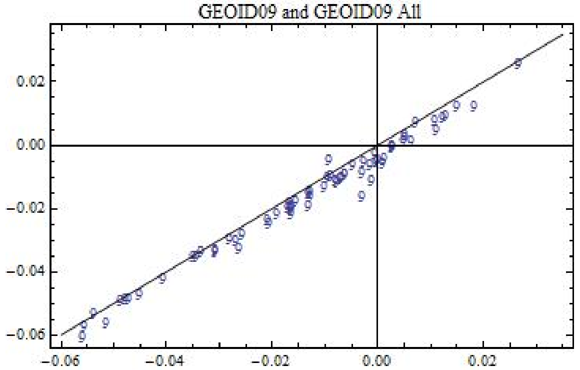

Figure 24. $H_{G09} - H_{88}$ **computed with no PCV correction against** $H_{G09} - H_{88}$ **computed with PCV correction**

Testing whether the differences between the two networks revealed significant differences ($p < 0.0001$) for GEOID03 and GEOID09. The average difference was 3.6 mm (2 mm 1-s). We conclude that PCV cannot be ignored.

Discussion

These results are consistent with those in Tranes et al. (2007) in that both geoid models provide orthometric heights roughly one centimeter too low on average. We note that GEOID09 was created using NAD 83(2007) ellipsoid heights and that we used NAD 83(CORS96) ellipsoid heights for control; no NAD 83(2007) control coordinates were available for the Connecticut CORS. It's possible that the new control values could significantly improve GEOID09's performance because one-centimeter differences would not be surprising (Daniel Roman, pers. comm).

GEOID09 appears to perform better than GEOID03 in light of the state-wide difference residuals showing no trend, whereas GEOID03 does show a trend. We believe the generally "too low" GPS-determined results are also in part due to marker subsidence or settling and, possibly, to a difference between Pinnacle and PAGES. It's also possible that the greater discrepancies in orthometric height differences between GEOID03 and GEOID09 are real and a reflection of GEOID09's better accuracy. The differences between GEOID09-determined heights and NAVD 88 heights are too small to recommend creating a correction surface for Connecticut, especially in the light of there being no geographic trend to the residuals and due to the uncertainty of the results exceeding the difference from zero. Therefore, we recommend that: (i) in Connecticut, GPS-determined heights use GEOID09 for the geoid undulations, (ii) no correction surface be created, and (iii) the First-Order vertical control network should be re-observed with GPS and densified with differential leveling for markers that are not GPS-able.

Application of Results Elsewhere

Our main result shows that GEOID03 and GEOID09 are suitable for GNSS heighting in Connecticut at the accuracy levels stated by NGS. This study's results probably would reflect those for other U.S. areas, possibly excepting mountainous areas. Gravity holdings in mountainous areas are relatively sparse (to non-existent) compared with low-lying areas; likewise First-Order bench marks. This paucity of gravity data, along with a dearth of First-Order bench marks on mountain peaks, presents a great challenge to accurate gravity modeling. It is known that GEOID03 does not perform according to specifications in Arizona (Fig. 25). Although NGS plans to collect additional gravity observations in mountainous regions through the GRAV-D project, it is still unclear how GEOID09 will perform in the mountains.

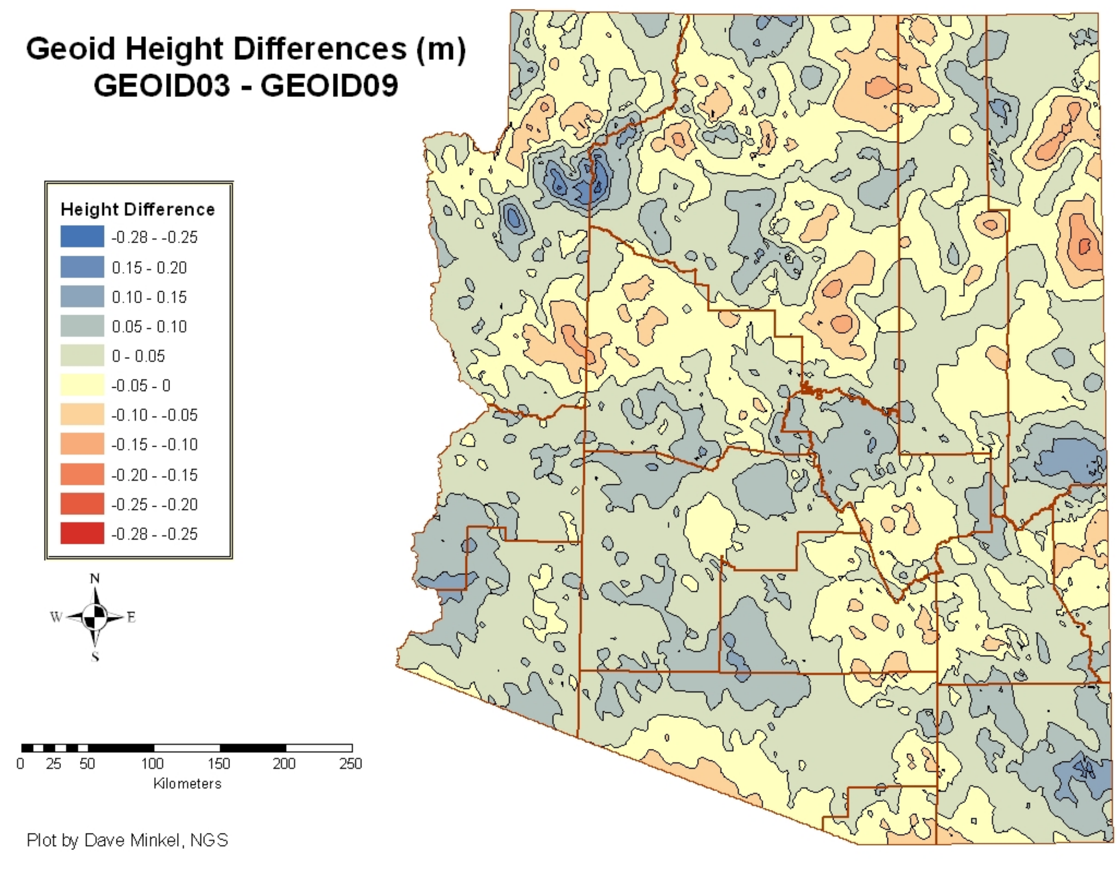

Figure 25. **Differences in GEOID03 and GEOID09 geoid heights in Arizona. (Image courtesy of Dave Minkel, NGS.)**

Our success is due in large part to the close spacing of the Connecticut CORS. NGS guidelines suggest local densification of control for projects, which was unnecessary for this study. We see the investment in a dense CORS network to be very valuable. Success can also be attributed to long, redundant observation sessions and the power of the least-squares network adjustment. Our methods should have produced conservative results: we observed in leaf-on conditions and we occupied no bench marks for control. Therefore, it is likely that practicing surveyors would see results similar to ours, or better.

Phase Center Variation

Our results show that phase center variation cannot be neglected when using GNSS positioning for heighting. The PCV patterns for our antennas are shown in Fig. 26, and we note that the variation of old antennas often will exceed the variability tolerance goals for the study.

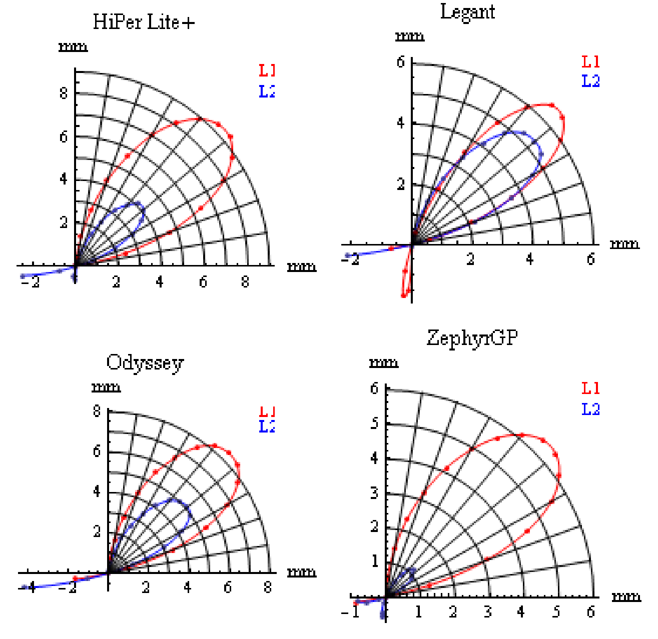

Figure 26. **Phase center variation diagrams for the GPS antennas. (Data from the NGS ANTCAL website.)**

Exactly-correct antenna heights and antenna models were indispensible for accurate results. Chasing down *hi* blunders in practice could be difficult because, of course, the results would mostly not have a published value to check against and a blunderous *hi* might not produce an obviously erroneous height. Given the importance of this aspect of GNSS surveying, we hope that software packages will help the surveyor find problems with and check *hi*.

Substandard Vectors

It is not surprising that the network with substandard vectors did not produce statistically different results than the network with only prescribed vectors: the least-squares adjustment of the substandard-vector network might down-weight the substandard vectors to the point that the two networks were practically identical. For a completely fair comparison, the two networks would have to be created from entirely separate data, which means a full duplication of the fieldwork, which was impractical. Even so, the result has some merit. A completely separate “good” network’s vectors would fully replicate those same vectors in the substandard network, and since the good vectors are repeatable, the results would likely be practically the same.

NGS presumably stipulated eliminating substandard vectors out of a concern that they would reduce the accuracy of the results. If substandard vectors are presumed to be inaccurate, then this is certainly true: even down-weighted data still contribute, so removing them also removes their bad influence. However, if the substandard vectors are accurate but imprecise, it does not follow that removing these vectors necessarily improves the accuracy of the result. Like all averaging approaches, least squares works best with large amounts of data, which was our situation. The impact of substandard vectors might be more troublesome with smaller networks, which would certainly be the normal in standard surveying practice. Since most land surveying networks are much more manageable in size than this one, the prudent surveyor should still remove substandard vectors as prescribed by NGS.

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