A Framework for Validating Traffic Simulation Models at the Vehicle Trajectory Level

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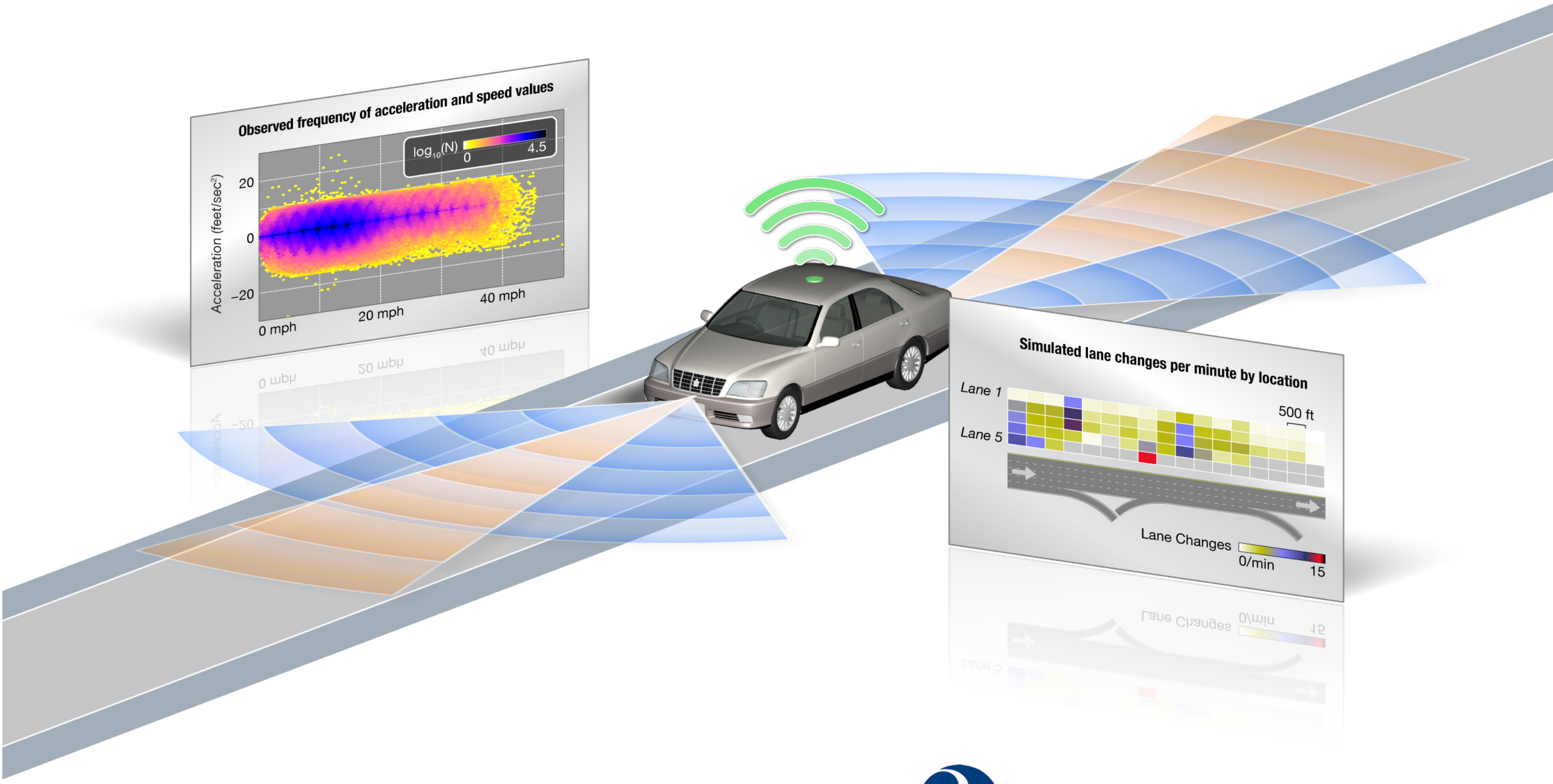

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Michalis Xyntarakis, Vassili Alexiadis, Vincenzo Punzo, Robert Campbell, Alex Skabardonis, Erin Flanigan

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100 Cambridge Park Drive, Suite 400
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16. Abstract

Based on current practices, traffic simulation models are calibrated and validated using macroscopic measures such as 15-minute averages of traffic counts or average point-to-point travel times. For an emerging number of applications, including connected vehicles, the realism of simulated driver dynamics at the second-by-second or sub-second trajectory level plays an important role. A framework to validate the realism of simulated vehicle dynamics at the trajectory level is presented in this report. Trajectory measures related to safety, comfort, vehicle kinematics, and traffic flow are presented. Example validation measures include time to collision, lane change urgency and rate, acceleration range, jerk, and root mean square of acceleration. Insights on the distribution and characteristics of each of these measures have been assembled from naturalistic driving studies, the American Association of State Highway and Transportation Officials (AASHTO), the International Organization for Standardization (ISO), the Federal Motor Carrier Safety Administration (FMCSA), the National Highway Traffic Safety Administration (NHTSA), and the analysis of trajectory data collected in this project. Practitioners can use the validation framework to assess the realism of the simulated vehicle dynamics in a model. Realistic vehicle dynamics at the sub-second level are required in the modeling of many emerging and technologically advanced applications that involve autonomous vehicles or vehicle-to-vehicle communications. In more traditional modeling applications, realistic vehicle dynamics result in realistic traffic properties at the aggregate level and the ability to reliably emulate a wide range of traffic phenomena. The proposed framework can be used by practitioners, researchers and software developers to document and improve the capabilities of traffic microsimulation models with respect to vehicle dynamics.

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APPROXIMATE CONVERSIONS TO SI UNITS

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
°F	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	°C
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
		LENGTH		
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m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
°C	Celsius	1.8C+32	Fahrenheit	°F
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

*SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380. (Revised March 2003)

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Executive Summary

Traffic microsimulation models are designed to emulate vehicle dynamics at the microscopic sub-second level. Realistic vehicle dynamics are a prerequisite for the successful application of simulation models in a variety of existing and emerging fields. Modeling the benefits of safety and mobility of emerging transportation technologies, such as connected vehicles, is not possible unless the simulation models are able to realistically capture how drivers accelerate, decelerate, select speeds and change lanes.

This report presents a framework of microscopic and macroscopic tests to validate simulated vehicle dynamics in terms of safety, comfort, feasibility, and compatibility with known traffic flow properties. For each validation test, insights from naturalistic and other trajectory datasets are provided as references. If on-site trajectory data exist, the validation tests can be used to statistically test observed and simulated distributions such as the time to collision. If on-site trajectory data do not exist, the framework includes reference information that can be used to qualitatively assess the realism of simulated vehicle dynamics.

The validation tests presented in this report have been categorized into three major application areas: a) safety-related that quantify driver aggressiveness and collision risk; b) acceleration-based tests associated with mechanical feasibility and driver comfort; and c) traffic flow modeling tests linked to microscopic and macroscopic properties of traffic flow. Taken in total, the validation tests have been designed to cover all aspects of simulated vehicle dynamics including longitudinal (car-following) and lateral (lane-changing).

Guidance is provided on the microscopic trajectory variables necessary for validation and on the calculation of each microscopic or macroscopic validation measure. The trajectory computational engine developed as part of this project was used to analyze simulated and observed trajectory datasets and compute the validation tests. Instrumented vehicle data, collected as part of this project, supplement the insights obtained from naturalistic, connected-vehicle and other trajectory datasets.

Chapter 1. Introduction

Traffic simulation models are designed to emulate vehicle dynamics or the behavior of individual vehicles in traffic at a sub-second level. Specifically, microsimulation models emulate how vehicles accelerate, decelerate, and change lanes in response to travel and driving goals, and to surrounding traffic. Operational needs related to connected, autonomous, and active transportation and demand management (ATDM) applications rely on the realism of the simulated traffic dynamics to provide robust policy recommendations. In ATDM applications, for example, realistic vehicle dynamics are required for model calibration. In connected and automated vehicle (CAV) applications, realistic emulation of vehicle dynamics is a prerequisite for measuring the impacts of human-assistive technologies such as collision warning systems. More generally, to meaningfully evaluate a technology or scenario, the analyst should ensure realistic driving dynamics with and without the technology being tested.

Current methodologies to develop, calibrate and validate simulation tools are based on approaches that are using macroscopic or aggregate-level field data such as 15-minute averages of volumes and speeds. When using aggregate data only, the model calibration and validation space becomes an arbitrary multi-parametric field that often provides too many options to achieve the same aggregate goodness-of-fit. For example, to improve the modeled travel time on a link, one can change the desired speed, driver aggressiveness for overtaking, or even the travel demand, all of which may yield a better macroscopic goodness-of-fit. However, more often than not, these changes, if they are not done systematically, result in over-fitting the model and reducing its descriptive power at both the macroscopic and microscopic level.

Analysis and validation of simulated vehicle dynamics at the sub-second trajectory level is not frequently performed in practice for two main reasons. First, there is no conceptual framework that defines validation tests at the microscopic trajectory level. This is in part due to the fact that a limited number of trajectory datasets exist to base vehicle dynamics measurements. Second, usually there are no on-site trajectory data for the driver population to be modeled. Trajectory data, as sequences of time-stamped positions, have limited transferability and vary significantly on a day-to-day basis even for the same driver and the same location. Even though a holistic picture of driver behavior under different traffic, weather, incident, and information provision situations may still be elusive, the increasing number of trajectory datasets becoming available can provide further insights into the patterns and norms of driver behavior.

This report presents a framework of microscopic and macroscopic validation tests that assess simulated vehicle dynamics in terms of safety, comfort, feasibility, and compatibility with known traffic flow properties. Insights from naturalistic and other trajectory datasets are provided as guidance for each validation test proposed. Since driver dynamics can vary significantly, the guidance for each test should be treated as reference and not as strict limits of compliance.

1.1 Motivation

Simulating real-world traffic phenomena including stop and go waves, instabilities, capacity drops and weaving sections has been the objective of traffic flow modeling for many decades. The focus has primarily been on creating more advanced mathematical models or better calibration techniques that improve how simulation models replicate what is being observed in the field at the macroscopic or aggregate level. To this end, traffic simulation models regardless of resolution (micro vs meso) capture the interrelationship of flow, density, and speed as demonstrated in the traffic fundamental diagram. Introspection on the simulated vehicle dynamics at the sub-second level is limited even though many emerging vehicle technologies and driver assistance systems depend on this kind of detail and resolution.

Microscopic traffic simulation models emulate how drivers accelerate, decelerate and change lanes at the sub-second level in response to travel and driving goals, and to surrounding traffic. For microscopic models, the properties of macroscopic flow as revealed in the fundamental diagram are not exogenously defined but they are an emerging property of the collective vehicle dynamics. Vehicle dynamics are considered as the interplay of acceleration, speed, relative speed, and distance among vehicles. Realistic vehicle dynamics result in realistic traffic flow properties at the macroscopic level and are prerequisite for any type of policy analysis.

Technological advancements in the fields of CAVs and driver aids increase the application areas of simulated models but at the same time place greater demands on the level of realism embedded in the models. This is because significant emphasis is now placed on realistically representing vehicle interactions at the sub-second level in addition to faithfully replicating traffic flow properties at the macroscopic level. Connected vehicles for example are expected to communicate information about vehicle kinematics many times during a second. Based on the communicated information the driver may be warned or a decision will be made by an algorithm. Therefore, to simulate the impacts of CAVs on traffic flow it is necessary that vehicle dynamics or the interplay among acceleration, relative distance, and speed are realistically represented in simulation analysis.

1.2 Framework Overview and Usage

The framework is divided into three major application areas or groups: safety, vehicle limits and driver comfort, and traffic flow. These three areas correspond to existing and future major application areas for simulation models. Applications concerned predominantly with one area, such as traffic flow, may give less emphasis to tests related to other areas. The tests in all three areas are related to each other. For example, unsafe distances at high speeds that are revealed by calculating the measures and executing the associated tests in the safety section, such as time to collision, may also be responsible for a higher than usual number of hard decelerations captured in the second area that focuses on acceleration limits and driver comfort levels.

The level of realism in vehicle dynamics may or may not be related with the model’s goodness-of-fit and how close the model matches traffic counts and travel times. Validation tests included in this framework may or may not convey information on how well the model reproduces the traffic counts, queue lengths, and corridor travel times used for calibration. In other words, assessing the realism of vehicle dynamics is a necessary but not sufficient condition for effective policy analysis through simulation.

The framework presented in this document should be used in conjunction with any pre-existing guidance on microsimulation model calibration and validation. It is particularly applicable for analyzing scenarios that modify driver behavior or vehicle dynamics such as those related to the introduction of new driver aids made possible through CAVs. It is recommended that a simulation model that has been built to test the impact of different collision warning or autonomous vehicle algorithms validates vehicle dynamics in the presence and absence of the new technologies.

Overall, the analyst is always limited by the level of realism the simulation engine is able to achieve. If assumptions and limitations of each car-following and lane changing algorithm are previously known, the appropriate model can be chosen based on the type of analysis requested. In any particular model the level of realism in vehicle dynamics depends on the:

- Car-following, lane-changing and other parameters modified by the analyst to calibrate the model.
- Hard-coded equations and parameters that are part of the simulation engine and are not accessible to the analyst.

It is therefore recommended that the analyst understands the physical interpretation of the input parameters and their range of impact on vehicle dynamics. The framework and the associated tests can assist the analyst in producing better models of driver behavior and traffic flow at the microscopic and macroscopic levels.

Practitioners can use this validation framework to assess the realism of the resulting vehicle dynamics of any model or scenario. As it was stated previously, realistic vehicle dynamics is a necessary condition in the modeling of many CAV related applications that require modeling of vehicle interactions at the sub-second level. In other applications, concerned only with the macroscopic properties of traffic flow, realistic vehicle dynamics will result in realistic traffic properties at the macroscopic level, enhancing the range of phenomena models can be used to emulate.

Researchers can use the framework to document, analyze, and assess the properties of different microsimulation models. The mathematical equations of the models may not always make their properties apparent. For example, the speed versus acceleration relationship, or how aggressively vehicles are likely to accelerate at different speeds may not be easily discernible by reading the mathematical equations. Calculating and then visualizing this model property may become more transparent to the researcher. For any given model, the application of the framework can show deficiencies in certain areas and strengths in others. Given the increasing demands CAVs place on simulation models and the range of behaviors that need to be emulated, this framework can provide transparency and structure in evaluating current and future models.

Software developers can use the framework to assess and document the properties of their software packages for different types of applications. Although previously published research usually forms the basis of each of the core models used by traffic microsimulators, software developers, as required, apply various modifications and improvements. These modifications are not always known and may not be transparent to practitioners or researchers. By adopting elements of this framework and providing documentation based on the proposed tests, software developers can assist the transportation community in using vehicle simulation models for an increasing range of technologically advanced scenarios.

As stated previously, validation tests have been categorized into three major application areas: a) safety-related tests that quantify driver aggressiveness and collision risk; b) acceleration tests associated with mechanical feasibility and driver comfort; and, c) traffic flow modeling tests linked to the microscopic and macroscopic properties of traffic flow. The validation tests have been designed to cover all aspects of vehicle dynamics including both longitudinal (car-following) and lateral (lane-changing) properties. Although all tests are related to each other, each of them highlights a different aspect of vehicle dynamics. An overview of the validation tests is provided in Table 1-1.

A validation measure is a microscopic or macroscopic performance measure that is used to perform the **validation test** with the same name. Each validation test includes insights and reference information from naturalistic and other trajectory datasets that can serve as guidance. Guidance also includes statistical tests that can be used to compare distributions of observed and simulated measures from the same site.

Table 1-1. Overview of validation measures and tests.

Validation Measures and Tests	Description
Safety	
Time to collision (TTC)	Quantifies risk of collision and driver behavior in terms of aggressiveness. TTC is a widely used safety surrogate measure.
Number of rear-end safety events	Events include rear-end crashes, near-misses, and warning messages. The number of such events per mile quantifies driver safety and aggressiveness.
Time Gap	The time until the subject vehicle reaches the current position of the principal other vehicle.
Lane Change Urgency (LCU)	Quantifies lane changing risk from the perspective of the subject vehicle. It quantifies tailgating risk.
Lane Change Severity (LCS)	Quantifies lane changing risk from the perspective of the vehicle that can be cut off on the adjacent lane.
Vehicle Limits and Driver Comfort Levels	
Acceleration range (AR)	Determines feasible vehicle dynamics using ranges. Comfortable limits are also defined.
Acceleration jerk (AJ)	Determines feasible vehicle dynamics and characterizes traveler discomfort.
Acceleration root mean square (ARMS)	Determines driver comfort based on International Organization for Standardization (ISO) guidelines.
Traffic Flow	
Lane type (LT)	Mandatory vs discretionary lane changes
Lane changes per vehicle mile (LCVM)	Driver propensity to change lanes
Lane change rate (LCR)	Reveals lane changing intensity over time and space.
Fundamental Diagram (FD)	Identifies macroscopic properties of flow and dispersion around ideal traffic flow theory assumptions.

Source: Cambridge Systematics, Inc.

A major influence of driver behavior is on safety. In terms of microscopic vehicle trajectory variables, safety translates into different distance gaps based on relative speed, cruising speed, and driver characteristics such as reaction time. The most widely used safety surrogate measure in advanced driver assistance systems is the time to collision (TTC). This measure alone or in combination with acceleration and time- or distance- gap is behind most of the safety-related validation tests. For rear-end collisions, driver aggressiveness and safety risk can be assessed by calculating the number of safety events per driver mile. For assessing the risk of lane-changing maneuvers, two measures are introduced. Lane change urgency (LCU) classifies collision risk from the perspective of the vehicle executing the maneuver. Lane change severity (LCS) classifies risk for the vehicle in the adjacent lane that may have to brake abruptly to avoid a collision. From a traffic flow perspective, the chosen safety distances and time gaps define collectively the properties of the Fundamental Diagram (FD) that is addressed in the third application area.

Unsafe or overly aggressive driving results in high values of acceleration or frequent fluctuations in speed and acceleration. The second application area named “vehicle limits and driver comfort levels” provides acceleration limits based on vehicle capabilities. For example, a deceleration of approximately one g is the maximum deceleration a modern vehicle can achieve under ideal conditions. When safety is not an issue, it is primarily driving comfort that determines the amount of acceleration or deceleration drivers experience at different speed levels. Acceleration jerk and especially the acceleration root mean square measure can be also used to assess how fluctuations in acceleration impact driver comfort.

The validation tests under the “traffic flow characteristics” section provide insights on how collective vehicle dynamics determine the macroscopic properties of the simulated traffic flow. To validate traffic flow properties, the fundamental diagram (FD) can be used to estimate the resulting roadway capacity or shockwave speed. When the diagram is constructed using microscopic instead of macroscopic (aggregate) quantities, microscopic phenomena related to safety or flow are revealed. Lane changes can be considered as flow at the lateral direction of travel. It is well documented that the amount of lane changing has a direct and significant impact on traffic flow and on the properties of the FD. For these reasons lane change validation tests are grouped together with the FD. An example macroscopic test of lateral dynamics includes the calculation of the lane change rate (LCR) and its visualization in a heat map. The LCR heat map can provide insights on the time, location, and intensity, of lane changes that is hard to obtain otherwise. Finally, calculating the number of lane changes by vehicle mile or the percentage of non-mandatory lane changes can be used to validate against excessive levels of lane changing in the model that impact flow characteristics.

Proper application of the validation tests requires that the microscopic or macroscopic validation measures are consistently defined in the simulated and field data. Furthermore, the calculation time step should be also consistent and appropriate for the particular validation test. The chosen time step should take into account the simulation time step and measurement noise in the field data.

1.3 Definitions and Basic Microscopic Trajectory Variables

This report is primarily concerned with a number of microscopic trajectory validation measures and tests such as time to collision that are defined based on specific variables in trajectory datasets. Different trajectory datasets, simulated or observed, may contain varying amounts of information.

Table 1-2 presents the microscopic measures at the trajectory level that are necessary for the computation of the validation tests in this report. Some validation tests, such as those related to acceleration, may use trajectory acceleration and speed as reported in the simulated or field dataset without any further computations provided their definitions are consistent. Other validation measures combine more than one variables from Table 1-2. It should be noted that before any computations or validation tests are performed, the accuracy of each variable in the trajectory dataset should be assessed. In simulated datasets, this not a serious issue since there is no measurement error. In observed datasets, however, measurement error can result in misleading conclusions unless the accuracy level is known and accounted for.

A macroscopic or aggregate measure or statistic summarizes a collection of microscopic measures over space, time or both. Examples include, flow, density, average speed, and number of lane changes per vehicle mile.

Vehicle trajectory is a sequence of time-stamped points that correspond to a fixed position on the frame of a moving vehicle represented by a unique vehicle id. In addition to position, simulated or observed trajectory data are often accompanied by microscopic measures such as speed, acceleration, lane position and other variables related to vehicle dynamics.

Table 1-2. Basic microscopic trajectory variables.

$x_{a+1}(t)$	Longitudinal position of the leader vehicle
$x_a(t)$	Longitudinal position of the follower vehicle
$d_a(\mathrm{t})$	Distance gap (feet) between vehicles a and $a+1$
$g_a(t)$	Time gap (sec) between vehicles a and $a+1$
$h_a(t)$	Time Headway (sec) between vehicles a and $a+1$
$s_a(t)$	Distance headway (feet) between vehicles a and $a+1$
$v_a(t)$	Vehicle speed (feet per second)
$a_a(t)$	Vehicle acceleration (feet per second squared)
$lpos_a(t)$	Lane number or position

Source: Cambridge Systematics, Inc.

Figure 1-1 clarifies the relationship between the spatial and temporal trajectory variables shown in Table 1-2 and used throughout this document. It depicts two vehicle trajectories in a typical time-space diagram in which time is on the horizontal axis and space is on the vertical. The trajectories of two vehicles are drawn as two straight lines. The net vertical separation between the two lines that does not include vehicle length will be referred as the distance gap d_a and the horizontal separation as the time gap g_a. The reader should note that distance gap d_a is the distance from the rear bumper of the leading vehicle to the front bumper of the following vehicle. Distance headway s_a is the vehicle length which is represented in gray Figure 1-1 plus the distance gap d_a.

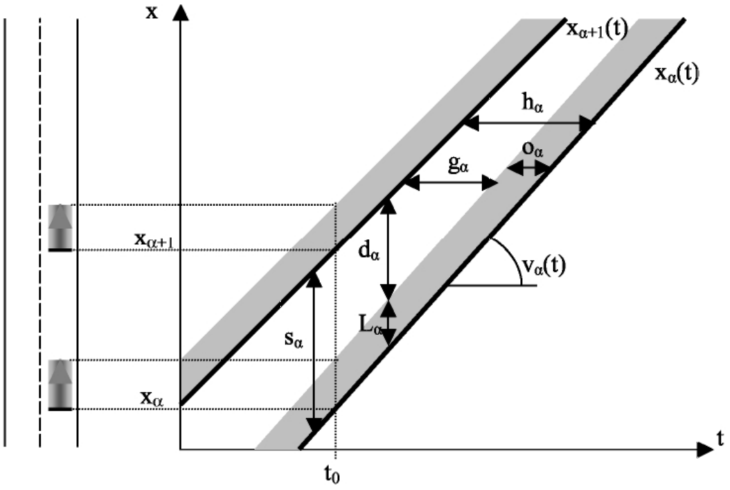

Figure 1-1. Graph. Microscopic trajectory variables.

(Source: Maerivoet, Sven, and Bart De Moor. “Traffic flow theory.” arXiv preprint physics/0507126 (2005).)

Connected vehicles can exchange information such as position, speed, and acceleration. Regardless of data acquiring method, positional measurements contain measurement error that should be taken into account in any subsequent analysis. When positional measurements are differentiated to obtain speed or acceleration, measurement error can be amplified prohibitively based on the time interval used. Appendix B contains a section that analyzes the impact of positional measurement error on derivative calculations such as speed and accelerations.

In analyzing interactions between vehicles, the **Subject Vehicle** (SV) is the vehicle under focus whose behavior is examined as a function of the rest of the vehicles and the environment. When two vehicles travel on the same lane and direction as a pair of leader and follower, SV is equivalent to the follower vehicle. **Principal Other Vehicle** (POV) is the primary vehicle that influences or constrains driver action or the vehicle involved in a safety event with the SV. The leader vehicle in a car-following situation is equivalent to the POV.

In this report, the terminology Subject Vehicle (SV) and Principal Other Vehicle (POV) is frequently used. This terminology can help analyze cases where the two vehicles that interact with each other are on the same or different lanes. In the simple case of the subject vehicle always corresponds to the follower vehicle and POV corresponds to the leader. In more complex geometries such as the ones shown in Figure 1-2, POV can be on an adjacent lane or an opposing lane. For example, in the bottom right corner of Figure 1-2, the subject vehicle is the one represented by the blue rectangle that enters the acceleration lane to merge to the freeway. The POV can be any other vehicle on the same or adjacent lanes that influences the SV’s behavior and whose interactions with the SV are the focus of the analysis. In this particular case it can be the vehicle in front on the acceleration lane or the one behind it on the rightmost lane of the freeway. Time to collision measures are typically calculated between the SV and the POV.

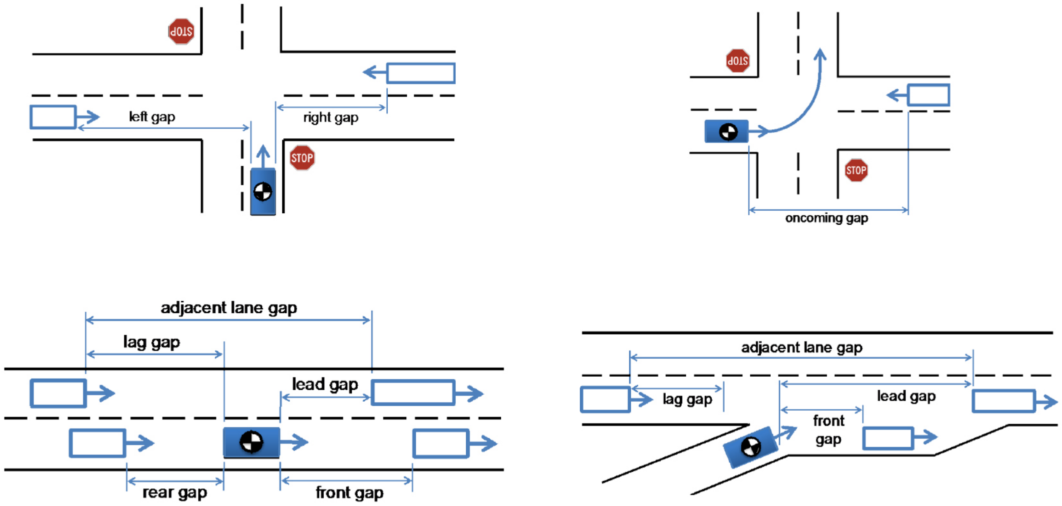

Figure 1-2. Simulation. Distance gap definition for arterials and freeways.

(Source: Society of Automotive Engineers, Surface Vehicle Recommended Practice, 2015.)

Chapter 2. Safety Validation Measures and Tests

This chapter introduces a set of measures and tests intended to characterize and quantitatively describe collision risk and safety. The primary sources of guidance behind this chapter are naturalistic driving studies and other relevant reports that have been conducted for the National Highway Traffic Safety Administration (NHTSA). The primary safety surrogate measure is time to collision (TTC) which is part of most of the validation tests presented in the framework. Validation tests for both car-following and lane changing are included. Unsafe or very aggressive driver behavior is closely related to extreme vehicle deceleration or to significant fluctuations in acceleration. However, acceleration measures alone cannot be used to properly classify and distinguish safety events that depend on the relative speeds and distances between vehicles. As it is reported in Table 1-1, this chapter defines the following validation measures and tests related to safety:

- Time to collision.
- Time gap.
- Number of rear-end safety events.
- Lane change severity.
- Lane change urgency.

In naturalistic studies, collision risk is measured by the frequency of rear-end safety events such as crashes, near-misses, and collision warning messages. In the framework, the definition of a safety event combines TTC and acceleration thresholds related to a near-miss maneuver. In addition to the number of safety events, the distribution of TTC can also be used to assess safety improvements and compare their impact between two scenarios. However, it should be noted that only small TTC values (close to 2 seconds) correspond to risk. It is recommended that both the distribution of TTC and the number of safety events per driver mile are used to evaluate collision risk.

Based on the Surrogate Safety Assessment Model (SSAM) conducted in 2008 virtual crashes occur in some microsimulation models. (https://www.fhwa.dot.gov/publications/research/safety/08049/) Based on the findings of SSAM, the virtual crashes were due to imperfections in the algorithmic logic and not an attempt to model the frequency and severity of crashes in the field. In some simulation models the frequency of the virtual crashes diminished significantly under certain parameter ranges. Also, in some of the same simulation models, unrealistic accelerations and decelerations accompanied a virtual crash. Clearly, a simulator that does not model collision risk or collision frequency cannot be used in assessing the impact of technologies that prevent collisions. Some of the currently available traffic simulation algorithms are collision free while others are not. (Kesting, Arne, Martin Treiber, and Dirk Helbing. "Enhanced intelligent driver model to access the impact of driving strategies on traffic capacity. "Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 368.1928 (2010): 4585-4605.) A collision-free algorithm will not result in a virtual collision regardless of user-specified car-following and lane-changing parameters. Other algorithms may result in collisions under certain parameter ranges that correspond to realistic assumptions of driver behavior.

Since TTC is the basis for most of the validation measures discussed in this chapter, it is addressed in a separate section first. Although it has been used in the past for evaluating safety improvements related to advanced driver assistance systems, time gap is not a good proxy for safety. Nevertheless, it is included in this chapter for those cases where TTC cannot be computed reliably. Classification of lane change maneuvers is based on two measures: lane change urgency captures the risk associated with the vehicle executing the maneuver, and lane change severity quantifies the risk of the maneuver on the future follower vehicle on the adjacent lane that may need to brake abruptly.

2.1 Time to Collision

One of the primary measures of driver collision risk in the literature is the time to collision, the time it takes two vehicles to collide if they continue on their current path with their present kinematic characteristics held constant. If the TTC is small, an immediate action taken by any of the drivers including a change in course is needed to avoid a collision. As an indication, collision warning messages are typically issued when TTC becomes approximately 2 seconds, although there is significant variation in the literature and industry applications. In a mixed flow that consists of autonomous and connected vehicles, TTC may be lower for the autonomous or connected cars without adversely impacting safety.

Definition

In general, time to collision is defined as the time required for two vehicles to collide if they continue traveling on their path with their present kinematic characteristics. The kinematic characteristics of the vehicles can be described using speed only or with both speed and acceleration. In the literature, both ways to calculate TTC are used indistinguishably. To distinguish between the two methods in this report, TTCs, where "s" at the end stands for speed, will be used if accelerations are not taken into account in the calculations. The corresponding equation is shown in Table 2-1. However, in a car-following situation in which the leader vehicle brakes abruptly, taking into account the sudden increase in deceleration yields a more accurate TTC measure. Connected vehicles, by communicating acceleration values near instantaneously can improve safety by enabling for more accurate TTC calculations. When both speeds and accelerations enter into the equation of TTC, the TTCa acronym will be used in this report. In general TTCa should be preferred. However, when a significant margin of error is associated with the calculation of the POV or SV acceleration, TTCa can fluctuate significantly and may provide an unwarranted high number of false positive warning messages. In this case, TTCs should be preferred. Regardless, TTC should be defined consistently in the observed and simulated data in all validation tests. In this document, time to collision (TTC) corresponds to the measure in general regardless of computation method.
Table 2-1 provides specific details on the equations and the data involved.

Caveats and Considerations

Although TTC has been widely used to classify collision risk, using a single measure for this purpose may not represent all possible situations and may have a significant percentage of false positives or negatives. This is because a single measure may have deficiencies or "blind spots" that do not allow it to be all inclusive. When the differential speed between leader and follower changes sign, TTC changes abruptly and by large amounts, while time gap generally remains more stable over time. For example, if a lead vehicle's speed is slightly higher than the following vehicle's speed, TTC becomes negative, because the two vehicles are not in a collision course. High values of TTC greater than 15 seconds should be excluded from calculating statistics based on the Society of Automotive Engineers. (Green, Paul. "Standard definitions for driving measures and statistics: overview and status of recommended practice J2944. *Proceedings of the 5th international conference on automotive user interfaces and interactive vehicular applications*. ACM, 2013.)

Estimation

Table 2-1 provides definitions for the TTCs and TTCa measures and the type of data that are required. Calculating TTCs is straightforward based on kinematic relationships from physics: the distance gap is divided by the differential speed of the two vehicles. The TTCa calculation is more involved since the acceleration of the principal other vehicle is also part of the equation. Time to collision does not take into account a driver's reaction time.

Time to collision can be estimated in simulated trajectory data provided that distances between vehicles have been already calculated. Typically, simulation models provide only the positions of the simulated vehicles and not the distances between them. In instrumented vehicle data distance gaps and relative speeds and accelerations can be calculated based on radar, LIDAR, or other sensor input that provides distance.

Table 2-1. Time to collision calculations.

Measure	Equation	Comments on Field Measurement
Subject Vehicle (SV) Speed	V_{SV}	A vehicle speedometer can be used. GPS does not have the accuracy unless differential corrections are applied
Principal Other Vehicle (POV) Speed	V_{POV}	A radar or a speedometer on a connected vehicle is required. Knowing the level of accuracy benefits calculations
Distance gap between SV and POV	d	A radar is required. A range (R) measurement can be used when both vehicles travel on a straight line.
POV Acceleration	a_{POV}	Acceleration sensor from connected vehicle provides more accurate measurements
Time gap	$g = \frac{d}{V_{SV}}$	When vehicles travel on a straight line, g is equivalent to Time gap, represented by the letter g, is equal to $g = \frac{R}{V_{SV}}$ where R is range
Time to collision (TTCs) using speeds	$TTCs = \frac{d}{V_{SV} - V_{POV}}$	When POV acceleration can be computed reliably TTCa should be preferred. Negative values should be dropped.
Time to collision (TTCa) using speed and acceleration	$\mathrm{TTCa} = \frac{V_{SV} - V_{POV} \pm \sqrt{(V_{POV} - V_{SV})^2 + 2(a_{POV} - a_{SV})d}}{a_{POV} - a_{SV}}$	

Source: Modified from McLaughlin, Shane B., et al. "Development of an FCW algorithm evaluation methodology with evaluation of three alert algorithms." *National Highway Traffic Safety Administration, Tech. Rep* (2009).

Validation Tests

The distribution of TTC values can be used to evaluate safety improvements and to compare observed field and simulated trajectories.

Due to the significant number of outliers, TTC values greater than 15 seconds are recommended to be excluded from the analysis. (Society of Automotive Engineers, Surface Vehicle Recommended Practice, 2015.) The Kolmogorov-Smirnov test presented in Appendix B can be used to statistically compare two distributions. As it was stated in the introduction of this chapter, safety events are associated with TTC values less than or close to 2 seconds.

The behavior of any driver can be evaluated with respect to safety by examining the distribution of TTC values. Distributions of TTC values that have been calculated in naturalistic driving studies that cover both freeways and arterials can be indicative. Figure 2-1 provides insights as to the number of seconds per hour drivers spend on TTC bins that are less than 10 seconds. On the y-axis of the figure is the number of seconds per hour. On the x-axis there is TTC ranging from zero to 10 seconds. Time to collision values that are 2 seconds or less occur one or two seconds per hour. In the figure, young drivers are 19-24 years old and old drivers 56 to 68 years of age. Young drivers are more comfortable with lower TTC values based on the data presented.

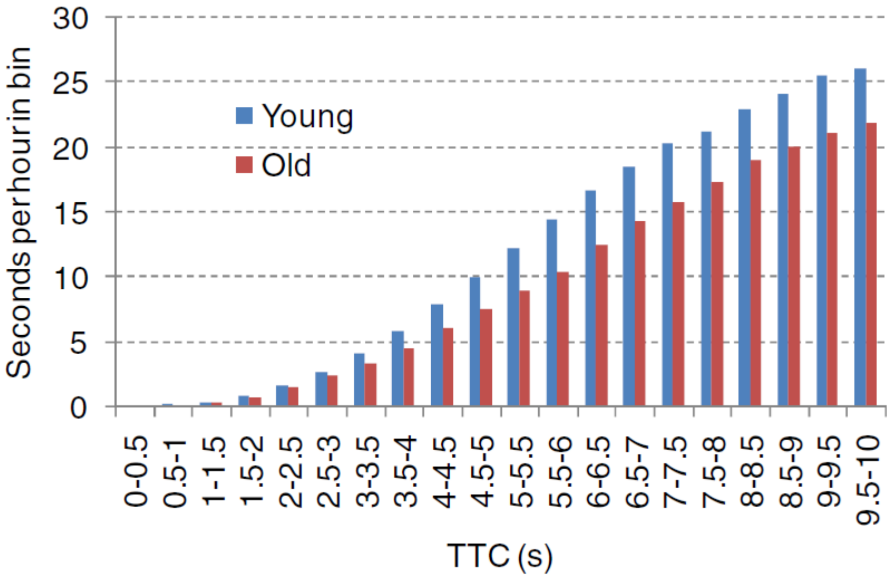

Figure 2-1. Bar graph. Distribution of time spent at TTCs values between 0 and 10 seconds in the 100-car naturalistic driving study.

(Source: McLaughlin, Shane, Jonathan Hankey, and Thomas Dingus. "Driver measurement: Methods and applications." International Conference on Engineering Psychology and Cognitive Ergonomics. Springer Berlin Heidelberg, 2009.)

2.2 Time Gap

The headway and time gap measure the time that elapses between the successive passage of two vehicles from the same location. Time gap is equal to the headway minus the occupancy time interval of the leading vehicle. Time gap refers to a specific location in space while distance gap to a specific point in time. Time gap has been used in the past in assessing safety impacts. However, it should be avoided when TTC can be computed.

Definition

Time gap at a specific location can be defined as the time that elapses between two successive vehicle passes when the first point in time is the tail of the leader vehicle and the second point in time is the head of the trailing vehicle (Figure 1-1). The most common statistic on time gap is the mean time gap. However, similar to the TTC distribution, when there are many large time gap values, the mean may be shifted significantly to the right. It is recommended that the time gap distribution is truncated below three seconds for scenario comparison. (Society of Automotive Engineers, Surface Vehicle Recommended Practice, 2015.)

Caveats and Considerations

In the absence of good measurements of POV speed and acceleration that preclude the use of TTC, the distribution of time gap has been used in the past to assess collision risk improvements. The time gap does not take into account the speed or acceleration of the principal other vehicle and as a result it cannot measure risk or safety. For example, the 100-car Naturalistic Study states that in 86 percent of the rear-end crashes, the time gap at the onset of the event was greater than 2 seconds. (Dingus, Thomas A., et al. The 100-car naturalistic driving study, Phase II results of the 100-car field experiment. No. HS-810 593. 2006.) A time gap of 2 seconds is typical in platooned traffic moving at free-flow speeds. Relevant research has shown that the time gap is always a lower bound to TTC based on the mathematics of the equations. (Vogel, Katja. “A comparison of headway and time to collision as safety indicators.” Accident analysis & prevention 35.3 (2003): 427-433.) However, time gap is not a good proxy of TTC and should not be used as such.

Estimation

Time gap at a specific location can be estimated from trajectory data using linear interpolation.

Validation Tests

In the absence of TTC data the distribution of time gaps can be used to evaluate safety improvements.

It is also recommended that time gaps greater than 3 seconds are discarded from the analysis. This is because time gaps greater than 3 seconds correspond to cases with limited interaction and influence between leader and follower as in freeway conditions that correspond to level of service B. (Society of Automotive Engineers, Surface Vehicle Recommended Practice, 2015.) The Kolmogorov-Smirnov test presented in Appendix B can be used to statistically compare two distributions.

Figure 2-2 presents the cumulative distribution of time gap in the Automotive Collision Avoidance System (ACAS) study in 2004. The cumulative distribution corresponds to all the drivers under all driving conditions including freeways and arterials. Time gaps between 0.3 and 3 seconds were analyzed for speeds greater than 25 mph. Gaps greater than 3 seconds were disregarded because drivers only engage in car following in relatively short distances. The red line represents the cumulative distribution of time gaps when none of the advanced collision avoidance systems were activated. Based on the red line, 30% of the time drivers had a time gap less than 1 second and approximately 80% of the time drivers chose a gap less than 2 seconds. Median is about 1.4 seconds and mode 1.0 s. When ACAS were activated one-second time gaps or smaller reduced by about 4 percent. However, the ACAS systems deployed in early 2000s provided many false positives that diluted driver attention. They were prototypes in development that do not have the reliability and accuracy of current systems.

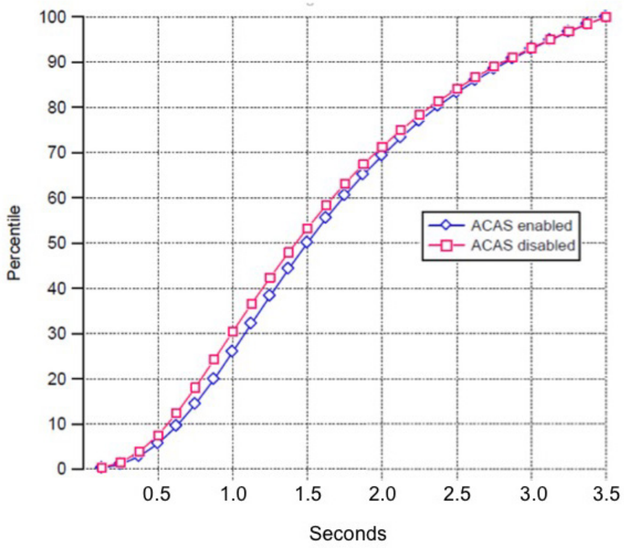

Figure 2-2. Graph. Cumulative distribution of time gap (ACAS Study).
(Source: Automotive Collision Avoidance System Field Operation Test.)

2.3 Number of Rear-End Safety Events per Vehicle Mile

Calculating the number of safety events per vehicle miles provides statistics that are easy to interpret and compare. The measures presented in this section are based on TTC in combination with acceleration. The bases for this section are safety-related studies conducted for NHTSA.

Definition

Safety events in this report constitute crashes, near-crashes, and forward collision warning messages. In the literature, the terms near-miss and near-crash are used interchangeably. Criteria for near-crashes and forward collisions warning events are defined quantitatively based on research conducted for NHTSA.

Crash is any contact with an object, moving or fixed, at any speed. In reality low impact physical contact between cars is the majority of vehicle crashes. More severe crashes can be easily detected in observed trajectory data when decelerations of multiple g are found.

Near-crash is any circumstance that requires a rapid evasive maneuver by the subject vehicle or any other vehicle, pedestrian, or cyclist to avoid a crash. A rapid evasive maneuver is defined as steering, braking, accelerating, or any combination of control inputs that approaches the vehicle capability limits. Quantitatively, a rapid maneuver is defined as longitudinal deceleration greater than 0.5 g. (Guo, Feng, et al. "Evaluating the relationship between near-crashes and crashes: Can near-crashes serve as a surrogate safety metric for crashes?." (2010).)

Forward collision warning (FCW) messages are transmitted to drivers based on a number of criteria, the most important of which is the TTC. In practice, the TTC threshold to issue a warning varies from 1.7 to 3.1 seconds depending on the speed of the leader vehicle. For simplicity, and based on past NHTSA research, this study considers a TTC equal to 2.4 seconds as the threshold for a FCW message.

The following table summarizes the quantitative criteria for the three types of safety events presented above.

Table 2-2. Definition of safety events.

Safety Event	Description	Criteria
Crash	Any physical contact between two vehicles.	Distance gap equal or less than zero
Near-Crash	Any circumstance that requires a rapid evasive maneuver by the subject vehicle to avoid a crash.	Acceleration > 0.5 g and TTC < 2 seconds
Forward Collision Warning	A potentially dangerous situation of vehicle conflict.	TTC < 2.4 seconds

Source: Cambridge Systematics, Inc.

Caveats and Considerations

As it was stated earlier, based on the findings of the SSAM project, virtual collisions occur in some simulation models that are an artifact of algorithm imperfections and not an attempt to model collision risk. Clearly, a simulator that does not model collision risk or collision frequency cannot be used in assessing the impact of technologies that prevent collisions.

A significant amount of research has been conducted in the last 15 years on Collision Avoidance Systems (CAS) by the NHTSA, the National Transportation Safety Board (NTSB), and other agencies in the U.S. and Europe. Currently, vehicle manufacturers perform their own testing and configure installed collision warning systems to their specifications. The Society of Automotive Engineers is in the process of developing standards for the timing of a warning, as well as other performance aspects of CAS. Table 2-3 shows the TTC values as measured by NHTSA on a small subset of vehicles equipped with CAS. In the first test scenario the test vehicle travels at 45 mph towards a stationary lead vehicle. In the second scenario, the test vehicle, traveling at 45 mph, approaches another vehicle that moves slowly at 20 mph. As seen in the table, the TTCs values vary significantly among manufacturers. According to NHTSA, the minimum time warning for a decelerating lead vehicle is 2.4 seconds and for a slower-moving lead vehicle 1.8 seconds.

Table 2-3. Timing of forward collision warning alerts in three test vehicles.

Vehicle	Stationary Lead Vehicle (TTC in Seconds)	Slow-Moving Lead Vehicle (TTC in Seconds)
2009 Acura RL	1.7	2.3
2009 Mercedes-Benz S600	2.3	2.3
2008 Volvo S80	2.4	3.1

Source: Forkenbrock, Garrick, et al. A test track protocol for assessing forward collision warning driver-vehicle interface effectiveness. No. HS-811 501. 2011.

Applying the definitions in Table 2-2 can yield a large number of false positives if the radar data technology used is older and proper filtering of the data is not applied. In the 100-car naturalistic driving study researchers identified safety events based on a large number of criteria and manually identified near-crashes by watching the associated video footage. Radar technology is known for detecting flickering targets that constitute false positives. However, technology has advanced considerably and as an indication, current collision warning systems offered by vehicle manufacturers are more reliable and give fewer unwarranted messages.

Estimation

Crashes, near-crashes, and collision warnings depend on correctly identifying the distance gaps between vehicles. Since simulation models only report vehicle position, it is the responsibility of the analyst to conduct the appropriate calculations at each time step. In general, vehicle collisions can be identified by locating negative distance gaps. However, this may not be straightforward. If vehicles travel on a roadway link, calculating the distance gap is relatively easy by taking into account vehicle position and size. At intersections or weaving junctions finding potential collisions is more complex. The analyst needs to computationally construct a polygon for each vehicle and check for negative distance gaps between vehicles. Figure 1-2 displays different types of distance gaps that are encountered in arterials and freeways. The Surrogate Safety Assessment Model contains a detailed methodology on how to calculate distance gaps from trajectory data. (https://www.fhwa.dot.gov/publications/research/safety/08049/) As it will be illustrated in the next chapter, decelerations greater than 2 g which sometimes occur in simulation models cannot be achieved by jamming on the brakes alone and may serve as a proxy for virtual collisions.

Identifying near-crashes or time and location of collision warning messages in a simulated environment is straightforward, provided that distance gaps have been already calculated. Time to collision should be monitored every simulation time step or 0.1 seconds and when it becomes lower than the threshold values presented in Table 2-2, a simulated warning message can be considered to have taken place. Currently, collision warning systems have different criteria for issuing messages. Therefore, the observed number of messages per vehicle mile can vary. This is an active area of research and standardization by ISO and SAE is currently under way. Additional guidance is also expected by NHTSA.

Validation Tests

The number of crashes, near-misses, and collision warning messages calculated per vehicle mile can be used to evaluate safety improvements.

Observed values for those events can be obtained from naturalistic studies or NHTSA reports. It is recommended that the number of safety events per simulation or simulation hour is also calculated to provide insights to the probability of traffic breaking down during model execution.

Table 2-4 provides rates for crashes and near-crash events derived from the 100-car naturalistic driving study. In addition to the average rate in the third column the last two columns provide insights to the distribution of safety events by showing the lower and upper 95% percentile values.

Table 2-4. Rates and confidence limits for safety events.

Safety Event	Count	Average Rate per Million Vehicle Miles Traveled (RMVMT)	Lower 95% percentile for (RMVMT)	Upper 95% percentile for (RMVMT)
Crash	16	9	5	13
Near-Crash	293	214	189	238

Source: Modified from Dingus, Thomas A., et al. The 100-car naturalistic driving study, Phase II—results of the 100-car field experiment. No. HS-810 593. 2006.

2.4 Lane Change Severity

For lateral dynamics, lane change severity (LCS) classifies the impact of the lane changing maneuver on the prospective follower vehicle on the adjacent lane. Aggressive lane changing that is likely to cause the future follower to brake abruptly to maintain a safe following distance is classified as a maneuver with high LCS in a rating scale from 1 to 7.

Definition

The lane change severity rating classifies the effect of the lane changing maneuver on the future follower vehicle, designated as principal other vehicle, on the target lane. This vehicle may be cut off if the lane changing maneuver is too aggressive. A seven-point severity rating is defined based on the POV's time to reach the end of the fast approach zone (T_r) shown in Figure 2-3 under constant speed measured at the start time of the lane changing maneuver. In simulation model lane changes happen in one time step, therefore both the start and the end of the maneuver are easy to identify. Table 2-5 presents the time gap thresholds that define the rating categories. For example, if there is no vehicle in the adjacent lane or if the time gap to reach the fast approach zone for the existing vehicle is more than 5 seconds the lane changing maneuver is categorized with rating one. The last two columns in Table 2-5 present observed frequencies for each of the categories in a naturalistic driving dataset.

Table 2-5. Lane change severity classification and frequency.

Rating	Description	Frequency	%
1	POV in the fast approach zone with time to reach closest end of zone, Tr, > 5.0 sec, including case where there is no vehicle in the adjacent lane.	8,241	95.1
2	POV in the fast approach zone with time to reach closest end of zone in the range 3.0 < Tr < 5.0 sec.	106	1.2
3	POV in the fast approach zone with time to reach closest end of zone in the range 1.0 < Tr < 3.0 sec.	14	0.2
4	POV in the fast approach zone with time to reach closest end of zone, Tr., < 1.0 sec.	2	<0.1
5	POV in the proximity zone.	299	3.4
6	A near-miss emergency action or unplanned sudden maneuver is required to avoid a collision with a vehicle in the adjacent lane into which the driver of the SV was attempting to move.	5	0.1
7	Crash of any sort.		

Source: Modified from Lee, Suzanne E., Erik CB Olsen, and Walter W. Wierwille. *A comprehensive examination of naturalistic lane-changes*. No. HS-809 702. 2004.

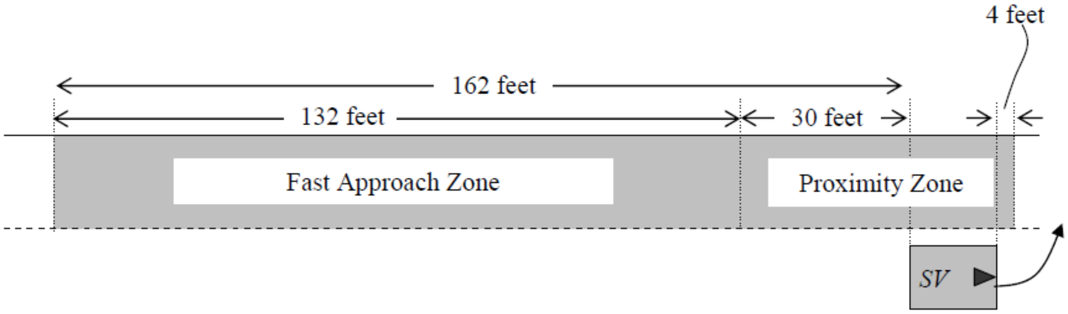

Figure 2-3. Diagram. Fast approach and proximity areas for lane change severity maneuvers.
(Source: Modified from Lee, Suzanne E., Erik CB Olsen, and Walter W. Wierwille. *A comprehensive examination of naturalistic lane-changes*. No. HS-809 702. 2004.)

Caveats and Considerations

Since LCS in Table 2-5 is computed for trajectories spanning the entire length of commute trips, the percentages defined in the last column of Table 2-5 may not apply to a short weaving section with intense lane changing.

Estimation

Calculating the distance gap between vehicles in observed or simulated data is a prerequisite step. The time to reach the closest end of the fast approach zone (Tr) in Table 2-5 is computed by dividing the distance to the end of the approach zone by the POV speed. The severity of the lane-changing maneuver should be calculated at the last time step prior to the lane-changing maneuver.

Validation Tests

It is recommended that the classification schema presented in Table 2-5 is applied to all lane changes in a simulation model.

The relative percentages found in the last column of Table 2-5 can serve as an indication. However, since these values have been computed for trajectories spanning the entire length of commute trips they may or may not apply to trajectories specific to short weaving sections.

2.5 Lane Change Urgency

Lane Change Urgency (LCU) takes into account the safety risks of driving too closely behind another vehicle from the perspective of the subject vehicle. Often drivers increase or decrease their speed on the present lane to be able to merge into the adjacent lane smoothly. Such a maneuver often results in unsafe situations for the vehicles on the same lane because it represents a deviation from safe car-following.

Lane change urgency classifies driver collision risk with respect to all the vehicles involved except the future follower for which the LCS measure and test is applicable. A possible case of a high-risk maneuver captured by LCU involves the driver braking abruptly on the target lane due to a relative small lead gap on the target lane. The LCU measure also takes into account high-risk rear gaps prior to the lane change such as those involved in tailgating.

Definition

Time to collision is used to classify LCU into four categories as defined by the thresholds shown in Table 2-6. If TTC between the SV and any of the vehicles involved except the future follower is more than 5.5 seconds, the maneuver is classified as non-urgent. The frequency column in Table 2-6 indicates that the vast majority of the lane changes have a TTC value 5.5 seconds or higher.

Caveats and Considerations

Lane change aggressiveness may vary substantially from location to location. The percentage values presented in Table 2-6 should serve as an indication. The lane changes included involve entire trips from origin to destination that involve both freeways and arterials. In a freeway section with a high percentage of weaving the percentage values may be different.

Table 2-6. Lane change urgency categories.

Category	Description	Frequency	Percentage
1	Non-urgent: TTC > 5.5 s to any vehicle but the future follower	8,303	95.1
2	Urgent: 3 s < TTC < 5.5 s to any vehicle but the future follower	341	3.9
3	Forced: TTC < 3 s to any vehicle but the future follower	23	0.3
4	Critical incident/crash: physical contact occurs or a sudden near-miss maneuver is required to avoid collision		

Source: Modified from Lee, Suzanne E., Erik CB Olsen, and Walter W. Wierwille. *A comprehensive examination of naturalistic lane-changes*. No. HS-809 702. 2004.

Estimation

Time to collision measures should be calculated at the beginning of the LC maneuver between the SV and the rest of the vehicles on the same and future lane except the future follower vehicle.

Validation Tests

It is recommended that the lane changes in a simulation model are categorized based on the four categories presented in Table 2-6.

The percentages shown at the last column of Table 2-6 can serve as a reference.

Chapter 3. Vehicle Limits and Driver Comfort Validation Measures and Tests

Safety and driver comfort are major influences in driver behavior. Modeling the physical capabilities of vehicles in terms of acceleration and deceleration is as important as modeling the acceleration and deceleration levels humans are comfortable with. When drivers do not engage in a safety maneuver, it is comfort that primarily influences the amount of acceleration and deceleration that they use. In this section, both the physical capabilities of vehicles and the levels of acceleration drivers are comfortable with are being addressed.

As it is reported in Table 1-1 this chapter defines the following validation measures & tests related to vehicle acceleration:

- Acceleration Range (AR) thresholds for acceleration and deceleration related to comfort and vehicle capabilities.
- Acceleration Jerk (AJ) values related to comfort and vehicle capabilities.
- Acceleration Root Mean Squared (ARMS) as a measure for comfort based on a relevant standard by the International Organization for Standardization.

ARMS can only convey driver comfort. Acceleration and acceleration jerk can convey both information of feasible vehicle limits and driver comfort. Although the validation tests presented in this section are related to each other, getting a realistic response in one of the tests does not ensure that the rest of the tests turn out the same way. For this reason, it is recommended that all tests are performed.

3.1 Acceleration and Deceleration Thresholds

Drivers rarely engage in maneuvers close to the physical limit of vehicle performance and, when they do so, it is a clear indication of unsafe and uncomfortable driving behavior. The vast majority of the time, drivers use a fraction of the acceleration and deceleration capabilities of their vehicle for a variety of reasons that include safety, comfort, and fuel consumption among others. Driver preferences statistically vary and the same driver can behave differently depending on the occasion. An aggressive driver may be characterized by more frequent fluctuations in speed and acceleration and by driving faster than others under the same conditions.

Definition

Acceleration is formally defined as the second time derivative of position, or the rate of change in speed. Common U.S. Customary units for acceleration are (feet per second) per second (fpss). An alternative popular unit is miles per hour per second (mph/s).

Caveats and Considerations

Table 3-1 presents acceleration and deceleration limits that can be used to evaluate vehicle trajectories in a simulation model. These limits have been compiled based on American Association of State Highway and Transportation Officials (AASHTO) guidance on acceleration comfort levels, on the minimum deceleration limit required by the Federal Motor Carrier Safety Administration (FMCSA), and by additional research conducted in this project.

Any of the limits presented in Table 3-1 should be seen probabilistically and not in absolute terms. In the table, the maximum acceleration is set to 18 fpss or 12 mph/s which corresponds approximately to the maximum acceleration for a car that reaches 60 mph in 8 seconds. The typical maximum deceleration is set to 1 g based on Figure 3-2. According to NCHRP Report 400 and AASHTO, the maximum comfortable deceleration level is 11.2 fpss for 90 percent of the population while 14.8 fpss is uncomfortable and can be related to a safety event in which the driver had to decelerate aggressively. (Fambro, D. B., K. Fitzpatrick, and R. J. Koppa. “NCHRP Report 400: Determination of Stopping Sight Distances. National Cooperative Highway Research Program.” Transportation Research Board, National Research Council. Washington, DC: National Academy Press (1997). And Transportation Depth Reference Manual for the Civil PE Exam, Norman R. Voigt, Professional Publications, 2011.; Hancock, Michael W., and Bud Wright. “A Policy on Geometric Design of Highways and Streets.” (2013).)

Table 3-1. Acceleration and deceleration threshold values.

Threshold	Value	Comment
Maximum acceleration	18 feet per second squared (fpss)	This corresponds to max acceleration for a vehicle that goes from 0 to 60 in 8 seconds
Max comfortable acceleration (hard acceleration)	12 fpss	This is based on the AASHTO recommendation of a max comfortable deceleration
Max comfortable deceleration	12 fpss	Based on AASHTO
Uncomfortable deceleration (hard deceleration)	15 fpss	Based on AASHTO
Min deceleration requirements by FMCSA	15 to 20 fpss depending on vehicle type	Most vehicles exceed the minimum requirements.
Typical maximum deceleration of current cars	32 fpss or 1 g	Modern vehicles can exceed this value. On a wet surface 0.5 g may be achieved
Deceleration at an accident	Varies by level of contact but can reach many g up to 100. (https://www.wired.com/2011/04/crashing-into-wall/.)	Decelerations greater than 2 g cannot be achieved without vehicle contact.

Source: Cambridge Systematics, Inc.

Maximum acceleration varies by speed significantly. The maximum vehicle acceleration can be achieved at relatively low speeds, usually less than 30 mph. As speed increases the available maximum acceleration diminishes. Figure 3-1 shows the maximum acceleration for a particular car by speed. The horizontal axis is miles per hour ranging from 0 to 75 mph. The vertical axis depicts acceleration measured in mph/s. The range of the acceleration axis in the figure is between 0 to 12 mph/s, which corresponds to 18 fpss or approximately 0.6 g. The maximum acceleration, equal to 0.6 g, is achieved at approximately 20 mph. After that peak, the maximum acceleration diminishes almost linearly by speed. At 75 mph the max possible acceleration is 5 mph/s (7.5 fpss or 0.23 g). The color in the figure corresponds to fuel consumption. Green is low fuel consumption, yellow is medium, and red is high.

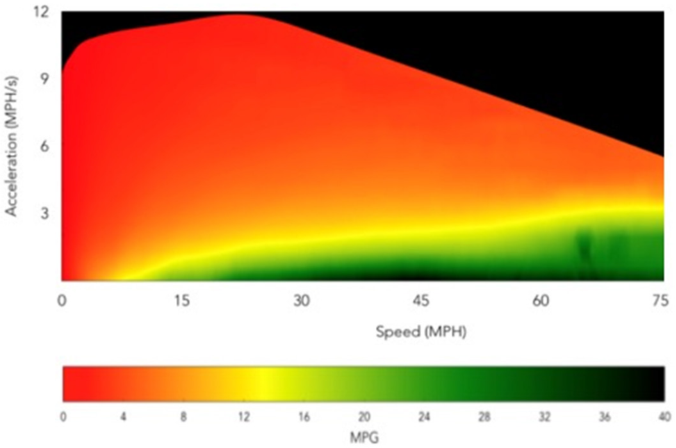

Figure 3-1. Heat map. Acceleration versus speed versus fuel consumption (BMW 328i).

(Source: Modified from: https://blog.automatic.com/the-hidden-costs-of-aggressive-driving-7828a9742fdc#.gmp2rohoc.)

Maximum deceleration values are higher than the acceleration ones. Deceleration can reach 1 g on a typical car by “jamming on the brakes” when the driver exerts as much force as he or she can muster to stop as quickly as possible. Figure 3-2 shows deceleration by time for such a maneuver in which the vehicle gets to a complete stop from 64 mph in less than 3 seconds. The horizontal axis in Figure 3-2 is time in seconds, from 0 to 7 s. Braking starts at 2.5 s and the vehicle is effectively stopped at second 5.0. In the vertical axis, the negative acceleration values correspond to deceleration ranging from zero to 1 g. It can be seen from the figure that reaching the maximum deceleration of 0.9 g is not instantaneous. It takes about 0.7 seconds to reach this value from second 2.5 until second 3.2 approximately. After the maximum breaking point is reached at 3.2 s, it is typical for deceleration to drop to a slightly lower value even though the driver exerts the same pressure on the brake pedal. Figure 3-2 corresponds to tests done in 2000 using a typical full-size passenger vehicle not equipped with and automated braking system (ABS). Maximum deceleration on more recent cars is likely going to reach or exceed 1 g. On a wet pavement, tests with the same car showed that the maximum deceleration was not greater than 0.4 g although the level of rainfall can play a significant role on vehicle performance. Figure 3-2 also shows the distance traversed by the vehicle as it brakes. The y-axis on the right hand side of the figure has units of distance in meters. The blue line that represents distance by time has a constant slope before braking starts at 2.5 seconds. After that point in time the slope which corresponds to speed becomes smaller and smaller until it is zero at the fifth second.

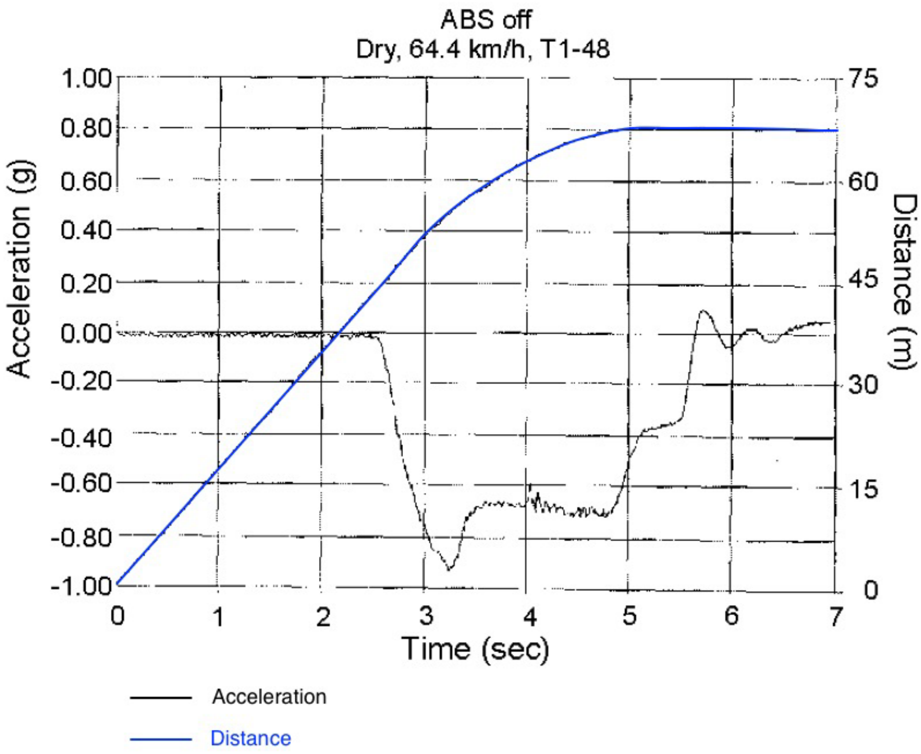

Figure 3-2. Graph. Typical acceleration profile in emergency braking on a dry surface.

(Source: Koppa, R.J. "Human Factors" Chapter 3 of: Gartner, N.H., Messer, C.J. and Rathi, A.K. (Eds) Traffic Flow Theory Transportation Research Board Monograph, National Research Council, Washington D.C. 2000 (on Web; hardcopy publication date, 2003).)

Typical acceleration and deceleration values vary by speed considerably especially for arterials. Based on this project's collected data, an acceleration or deceleration of 10 fpss or greater corresponds to one percent of the time at 10 mph but is much rarer at higher speeds. Figure 3-3 shows the distribution of acceleration at different speed levels for the collected instrumented vehicle data in April and May 2016 at Berkeley, California. The left plot corresponds to freeway driving on I-80 and the right plot is specific to arterials. The horizontal axis on both plots is mph while the vertical axis is fpss. Both axes have the same ranges, 0 to 80 mph for speed and -10 to 10 fpss for acceleration. In both plots the shaded gray areas correspond to the following percentiles, 75%, 95%, and 99%. For example, on freeways, 75% of the time acceleration or deceleration is less than 1 fpss, and 95% of the time less than 5 fpss. On the freeway, the bandwidth of the percentile curves designates that drivers tend to accelerate and decelerate more when speeds are lower than 40 mph. On arterials, in contrast, the acceleration pattern is significantly different. The maximum 99th percentile acceleration and deceleration is approximately 10 fpss, almost twice the corresponding value on the freeway section. On arterials, the triangular shape of the percentile curves shows that that drivers accelerate and decelerate more heavily at around 10 mph. After their peak at 10 mph the percentile acceleration and deceleration distribution curves drop almost linearly. At 30 mph the 99th percentile acceleration or deceleration becomes less than 5 fpss. It should be noted that the speed limit on the arterial is 25 mph. Speed values greater than 30 mph exist but they are fewer in number. It can be seen from the right figure that at speeds greater than 30 mph there is an increase in acceleration and deceleration possibly due to the increased alertness of the drivers.

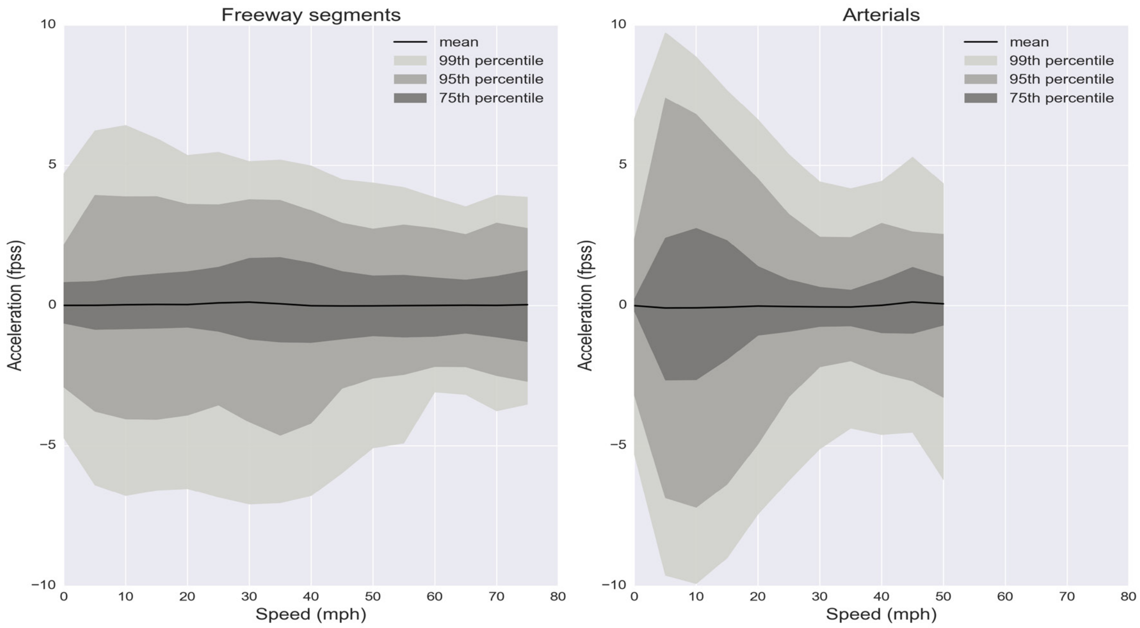

Figure 3-3. Graphs. Distribution of speed versus acceleration in the collected instrumented vehicle data.

(Source: Cambridge Systematics, Inc.)

Estimation

In a simulated environment, acceleration is typically reported at each time step by the software and can be readily analyzed. Acceleration measurements obtained from on-board vehicle sensors are best suited for trajectory analytics and have the lowest error. It is recommended that a time step up close to 0.1 second is used for all calculations in the simulation.

Validation Tests

It is recommended that the acceleration distribution in simulated trajectories is evaluated based on the thresholds presented in Table 3-1. The percentage of total driving time under comfortable and uncomfortable acceleration levels can be estimated and compared with corresponding values in connected vehicle datasets in the Research Data Exchange Web site. An example is provided in Figure 3-4 below. The following statistics are recommended:

- Maximum acceleration and deceleration.
- Number of events where acceleration is greater than the maximum and minimum thresholds of Table 3-1. The number of such events can also be divided by vehicle mile to calculate their frequency from a driver's perspective.
- Percentage of driving time acceleration or deceleration is uncomfortable, or the number of uncomfortable deceleration or acceleration events per vehicle mile.

When comparisons between observed and simulated distributions are made it should be noted that acceleration varies by speed as shown in Figure 3-3. Attention should be paid on the matching average speed and speed distribution of the observed and simulated datasets. If the two distributions vary considerably, it is recommended that the distribution of acceleration is computed for comparable speed bins in both datasets. The Kolmogorov-Smirnov statistical test presented in Appendix B can be used for this comparison.

In Figure 3-4, the distribution of acceleration and deceleration values in the collected instrumented vehicle data are shown in three separate plots. All three plots show the same data that include both the freeway and arterial section; however, the second and third plots use a logarithmic transformation of the y-axis to highlight outliers. The vertical dashed lines in the second and third plot correspond to the limits presented in Table 3-1.

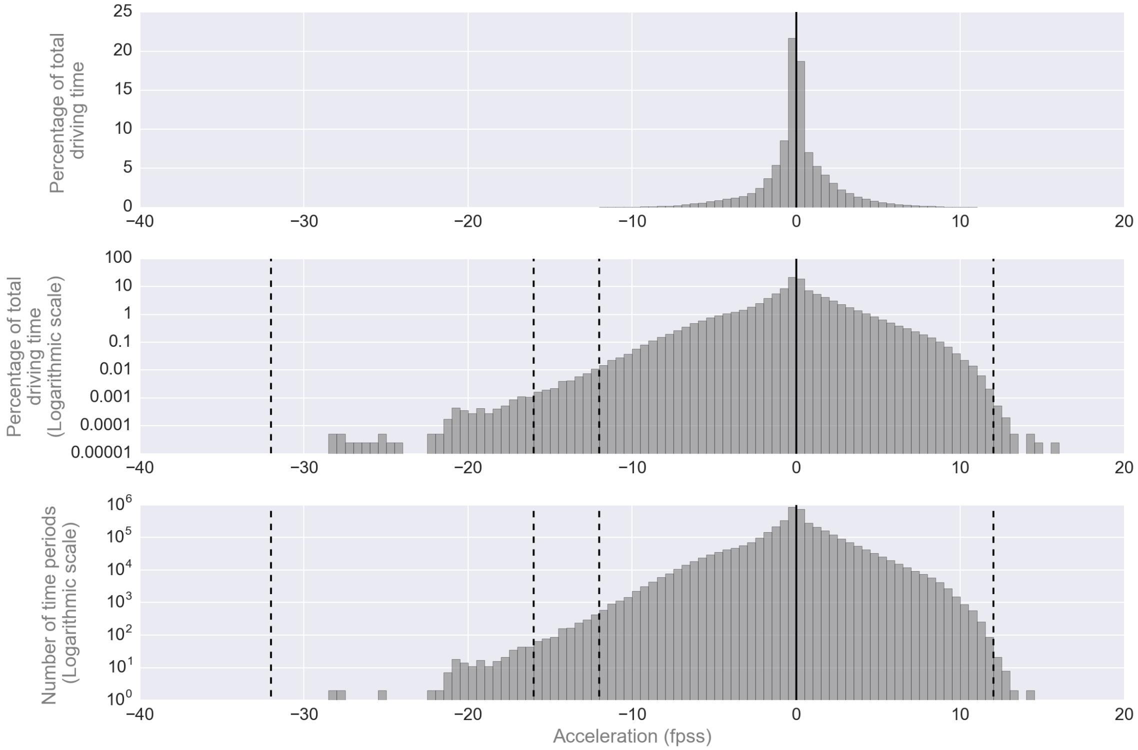

Figure 3-4.Graphs. Distribution of acceleration in the collected instrumented vehicle data.
(Source: Cambridge Systematics, Inc.)

In the top plot the distribution of acceleration as a percentage of driving time is shown after filtering out stopped time. The y-axis is the percentage of total driving time and each bar corresponds to an increment of 0.5 fpss. Acceleration and deceleration values that are 0.5 fpss from zero correspond to approximately 40% of the driving time. Outliers or acceleration values greater than 12 fpss or deceleration values greater than 15 fpss are not easily seen on the top chart because their percentage is close to zero. The second and the third plot are used to identify and quantify the outliers.

In the second plot the y-axis has been logarithmically transformed to highlight outlier values in Table 3-1. The length of each bar is no longer proportional to the percentage of driving time as in the first plot. Hard accelerations greater than 12 fpss are very infrequent in the dataset and correspond to a value close to

0.001 percent of the total time. Hard decelerations greater than 15 fpss are rare but more frequent. Since high values of deceleration can correspond to safety events or even accidents it is often important to identify them by their number and not just as a percentage of total time. This is achieved in the third plot.

In the third plot, the y-axis is logarithmically scaled but this time it shows the number of occurrences or time periods for which acceleration had a specific value. The numbers on the y-axis are 1, 10, 100, 1000, and so on. For example, it can be seen that deceleration close to -30 fpss (0.93 g) occurs close to 10^0 times. Since 10^0 is one, it should be deduced that acceleration reached close to one g in the dataset a few times that may be worthy of further investigation. It is recommended that the analyst calculates descriptive statistics before any distribution is plotted.

3.2 Acceleration Jerk

Acceleration jerk is used extensively in many fields to measure human comfort under motion. Researchers and practitioners of traffic simulation have occasionally examined the maximum values and distribution of jerk (https://en.wikipedia.org/wiki/Jerk_(physics)). In particular, Punzo et al. examined the distribution of jerk values in the raw NGSIM dataset and suggested that jerk values greater than 50 feet per second cubed (approximately 1.5 g per second) are mechanically infeasible. (Punzo, Vincenzo, Maria Teresa Borzacchiello, and Biagio Ciuffo. "Estimation of vehicle trajectories from observed discrete positions and Next-Generation Simulation Program (NGSIM) data." *TRB 2009 Annual Meeting*. 2009.) Similar to the acceleration validation tests presented above, jerk can be used to identify both mechanically infeasible and uncomfortable occasions.

Definition

Jerk is the time derivative of acceleration. It is positive if the acceleration increases and negative otherwise. Specifically, ISO 2041:2009 definition 1.5, defines jerk as "a vector that specifies the time-derivative of acceleration."

Caveats and Considerations

Threshold values for acceleration jerk are shown in Table 3-2. Comfortable jerk values are around 0.1 g/s (3 ft/s^3 or $1 m/s^3$). Uncomfortable deceleration values greater 15 ft/s^2 correspond to a jerk greater than 0.4 g/s. Values greater than 1.5 g per second (50 ft/s^3 or 15 m/s^3) cannot be justified by a vehicle's mechanical capabilities unless a crash has occurred.

Similar to acceleration and to the acceleration root mean square error presented in the next section, the distribution of jerk depends on speed. Outliers such as jerk events closer to 1 g/s that signify safety events can happen at any speed. Overall, the number of outliers in the jerk distribution is of primary importance since they reveal potentially unsafe events.

Table 3-2. Acceleration jerk threshold values.

Threshold	Comment
0.1 g/s (3 ft/s^3)	Typical values travelers feel comfortable with
> 0.4 g/s (15 ft/s^3)	Uncomfortable situations
> 1.5 g/s (50 ft/s^3)	Mechanically infeasible or indication of a crash

Source: Cambridge Systematics, Inc.

Estimation

Jerk can be estimated using acceleration data by dividing the change in the acceleration by the corresponding time step. A time step of one second is recommended. If acceleration is reported with significant accuracy errors, those errors will cascade when jerk is computed resulting to a high number of misleading outliers. The microscopic measure accuracy section in the Appendix can assist the analyst in identifying a time step that is consistent with the accuracy of the observed data. In simulated datasets acceleration accuracy is usually not an issue because there is no measurement error. However, when acceleration is not provided and is calculated from positional values that do not have enough significant digits the accuracy of the acceleration calculations should be taken in to account.

Validation Tests

In general, the analyst can evaluate the number of jerk-related events by vehicle mile, or vehicle hour.

The following tests are recommended:

- Maximum jerk in the dataset.
- Number of mechanically infeasible acceleration jerk events as an absolute number or divided by vehicle miles.
- Number of uncomfortable acceleration jerk events per vehicle mile.

3.3 Acceleration Root Mean Square

The root mean square of acceleration (ARMS) is frequently used in many fields including driving to measure human discomfort in an environment that involves vibrations. This measure captures the variability and fluctuations of acceleration values necessary to classify a vehicle ride as comfortable or not. Intensive or unrealistic car-following in which the driver frequently accelerates or decelerates based on perceived changes in speed and distance to the leader vehicle are captured by ARMS.

Definition

Mathematically, ARMS is similar to the root mean square error that is frequently calculated in the traffic simulation field. In the ISO equation presented in the estimation section, acceleration is defined more generally as a three-dimensional vector that represents longitudinal, lateral and vertical vibrations.

Caveats and Considerations

Thresholds for ARMS according to the ISO standard 2631-1, which is titled Mechanical Vibration and Shock Evaluation of Human Exposure, are shown in Table 3-3. Passenger seats in a car are especially designed to dampen chassis vibrations. (International Organization for Standardization, ISO 2631, Mechanical vibration and shock—Evaluation of human exposure to whole-body vibration (1997).) As a result, ARMS values that are calculated using vehicle chassis accelerations are likely to overestimate driver discomfort. It is recommended that the thresholds in Table 3-3 be used indicatively unless damping factors are applied to both the empirical and simulated data. Regardless, measuring the relative magnitude of the observed and simulated ARMS provides insights as to the relative level of comfort and acceleration fluctuations in a simulation model. The majority of trajectory datasets in the Research Data Exchange Web site include acceleration values from on-board vehicle sensors that are attached to the chassis.

In Figure 3-5 the relationship between ARMS and speed in the collected trajectory dataset is presented. On the horizontal axis there is speed between 0 and 50 mph. On the vertical axis there is ARMS in the metric units of ISO 2631, namely m/s^2. Freeway driving is associated with nearly constant values of ARMS close to 0.6 m/s^2 regardless of speed. Driving on the signalized San Pablo Arterial is associated with much higher values of ARMS, closer to 1.2 m/s^2 at 10 mph. When drivers traveled close to the speed limit on San Pablo, which is 25 mph, ARMS drops to 0.4 m/s^2 possibly because drivers moderated their speed at or around the posted speed limit.

Table 3-3. Comfort reaction to vibration environments (ISO 2631-1 1997).

ARMS (m/s2)	Comfort Reaction
< 0.315	Not uncomfortable
0.315 ~ 0.63	A little uncomfortable
0.5 ~ 0.1	Fairly uncomfortable
0.8 ~ 1.6	Uncomfortable
1.25 ~ 2.5	Very uncomfortable
> 2	Extremely uncomfortable

Source: ISO 2631-1 1997.

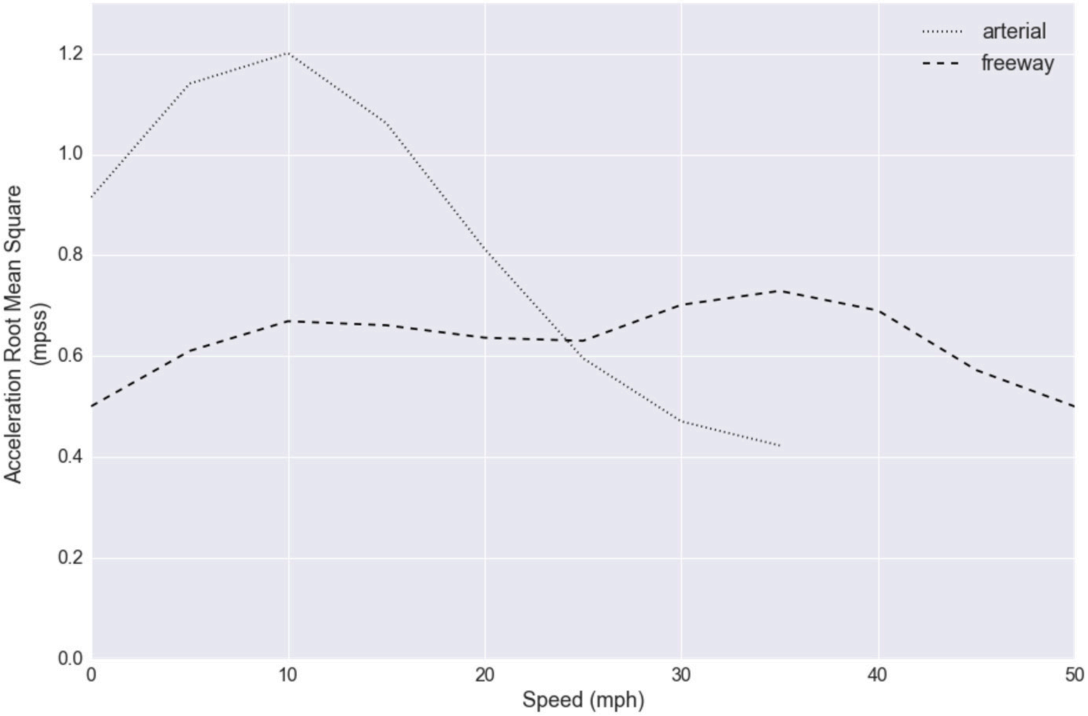

Figure 3-5. Line graph. Distribution of ARMS by speed.

(Source: Cambridge Systematics, Inc.)

Estimation

Most simulation models can only emulate the longitudinal component of the three-dimensional acceleration vector and omit the lateral and vertical dimensions. Including the lateral and vertical acceleration components increases ARMS because the square of acceleration is always positive. Even though all three components can be taken into account when using observed data, for comparison purposes, it is recommended that only the longitudinal acceleration values are entered in the equation.

$$ARMS = \sqrt{\frac{\int_{t_{start}}^{t_{end}} a(t)^2 dt}{t_{end} - t_{start}}} = \sqrt{\frac{\sum_{t=1}^{t=n} a^2}{n}}$$

Figure 3-6. Equation. Acceleration root mean square.

Validation Tests

It is recommended that ARMS values are calculated by speed and by roadway type such as freeway versus arterial.

Validation tests include:

- A single ARMS value for the entire simulation by speed bin and roadway type. This value can be compared with the range of values obtained from published connected vehicle datasets regardless of ISO thresholds.
- An ARMS value for each driver. The distribution of ARMS per driver and possible outliers are of importance. This is a more refined analysis that calculates the level of comfort at the trip level.
- If proper damping factors have been developed, it is recommended that the percentage of time driving in each of the categories of Table 3-3 is calculated for a simulation model. Otherwise the values in Table 3-3 should be taken indicatively.

In the collected trajectory dataset, the average ARMS value is 0.74 m/s^2. This value is relatively high and based on the ISO 2631 standard is classified as fairly uncomfortable (Table 3-3). However, as stated above this is not a fair comparison because the accelerometer in the observed data was not placed on a passenger seat as it is the case with relevant research but was attached firmly to the body of the car. (Nahvi, Hassan, Mohammad Hosseini Fouladi, and Mohd Jailani Mohd Nor. “Evaluation of whole-body vibration and ride comfort in a passenger car.” *International journal of acoustics and vibration* 14.3 (2009): 143-149.) The reader should also note that the calculated ARMS was insensitive to time interval selection for time intervals up to 1 second.

Chapter 4. Traffic Flow Validation Measures and Tests

Validating traffic flow properties is the focus of this chapter. The Fundamental Diagram (FD) is the primary method to express and document traffic flow properties for a specific section or link. Furthermore, it is the primary means to document changes in flow properties such as the impact of weather and connected vehicles. Lane changes, which can be considered as flow in the lateral direction of travel, do not appear in the FD as a quantity. Nevertheless, it is well documented that the amount of lane changing intensity has a direct and significant impact on the properties of the FD such as maximum capacity. Even though lane changing maneuvers are a major source of congestion, instability, and unsafe incidents, the rate and location of lane changing in simulation models are typically only visually verified. Three validation tests are presented in this chapter pertaining to lane change type, number of lane changes per vehicle mile and lane change rate.

As it is reported in Table 1-1, this chapter defines the following validation measures & tests related to traffic flow:

- Lane change type.
- Lane changes per vehicle mile.
- Lane change rate.
- Fundamental diagram.

Lane Change Type is presented first followed by the number of lane changes per vehicle mile and the lane change rate. The last portion of this section is devoted to the FD and particularly on its derivation from microscopic measures since there is an abundance of materials for the macroscopic version. All the tests in this section are related to each other but each of them validates a different aspect of traffic flow in a simulation model. For example, lane changing intensity which is revealed at the corridor level using the lane change rate test has an impact on the speed, capacity, and flow shown in the FD. A high number of lane changes may be due to higher than usual discretionary lane changes, something that can be revealed by looking at lane change type. Finally, the number of lane changes per vehicle mile can show differences in behavior at the driver level.

4.1 Lane Change Type

Modeling weaving sections is critical in any traffic simulation model. On freeways, weaving sections are potential bottleneck locations that often pose a significant calibration challenge. In general, the location and intensity of lane-changing is calibrated by modifying location-specific parameters such as the look-ahead distance, a parameter that controls how far upstream drivers position themselves favorably for a mandatory lane change. Discretionary lane changes on the other hand are often triggered in simulation models by the difference in speed between leader and follower.

Definition

A mandatory lane change is a necessary movement to position the vehicle on the appropriate lane for a turn or a highway exit. For each path from origin to destination the number of mandatory lane changes is the minimum number of lane changes that need to occur for the vehicle to complete its trip regardless of traffic. In simulation models every lane change other than mandatory is usually considered as discretionary. Discretionary lane changes are influenced by a number of factors such as the speed on the current and adjacent lanes.

Caveats and Considerations

Simulation models do not usually report if a lane change is mandatory or discretionary. As a result, the cause of a lane change maneuver and the intensity of discretionary lane changing may not be easily computed. In terms of modeling lane changing in general, the reader should note that in many traffic simulation models lane changes are discrete and not continuous events. Their duration is equal to the models' time step which is usually less than a second. This is because a two-dimensional roadway space with N lanes is modeled as n mono-dimensional lines i.e., the lane centerlines. Longitudinal interactions of vehicles along a line are governed by car-following models while vehicles change lanes instantaneously (in one simulation step) without any explicit lateral movement being modeled. As a result, neither lateral acceleration nor lane-change duration is modeled even though traffic visualizers present such movements as smooth and gradual ones.

Estimation

Identifying the type of lane change in collected data is not trivial. Watching the associated video footage is probably necessary. In simulated datasets the simulation engine should be configured to provide the lane change classification.

Validation Tests

It is recommended that the ratio of discretionary to mandatory lane changes in a simulation model is calculated provided this information is reported by the software or can be deduced.

Percentage values can be compared to results from naturalistic studies such as the ones included in Table 4-1. In addition, the percentage of discretionary lane changes can be computed per vehicle or weaving section.

Researchers at the Virginia Tech Transportation Institute watched recorded video and classified lane changes into the eleven categories shown in the following table. These categories are often a superset of what is modeled by simulation tools. Lane changes due to a slow lead vehicle were the majority in the naturalistic dataset. The next three categories, namely Exit/Prepare to Exit, Return, Enter, and Lane Drop are likely to be considered as mandatory lane changes by simulation tools. Figure 4-1 provides insights as to the relative frequency of mandatory and discretionary lane changes at a macroscopic level.

Knowing the percentage of mandatory and discretionary lane changes in a simulation model is insightful because it reveals how aggressively drivers are overtaking slower vehicles. Also, as it can be seen from the duration column in Table 4-1 lane changes have different characteristics based on their type. Specifically, a lane change to pass a slower vehicle lasts approximately 50% longer than a lane change associated with an Exit maneuver.

Table 4-1. Lane change type and frequency.

Maneuver Type	Freq	Percentage	Mean Duration (Second)	Description
Slow lead vehicle	3,228	37.2%	12.98	Lane change to pass a slower vehicle.
Exit/Prepare to Exit	2,018	23.3%	6.25	Lane change associated with exiting.
Return	1,549	17.9%	6.72	Lane change to return to preferred driving lane.
Enter	680	7.9%	6.89	Lane change to enter road (e.g., from on-ramp).
Tailgated	353	4.1%	6.08	Vehicle tailgating/approaching quickly.
Merging vehicle	226	2.6%	7.39	Vehicle entering roadway causing SV to change lanes.
Lane drop	201	2.3%	6.69	End of driver’s lane (e.g., road goes from 3 to 2 lanes).
Other	161	1.9%	10.82	Lane change for any other reason
Added lane	157	1.8%	5.98	Addition of a lane (e.g., road goes from 2 to 3 lanes).
Unintended	70	0.8%	13.67	Unintended lane deviation (e.g., distraction in car).
Obstacle avoidance	24	0.3%	8.73	Maneuver to avoid obstacle or rough road surface.
Grand Total or Mean	**8,677**	**100%**	**9.07**	

Source: Modified from Lee, Suzanne E., Erik CB Olsen, and Walter W. Wierwille. A comprehensive examination of naturalistic lane-changes. No. HS-809 702. 2004.

4.2 Lane Changes Per Vehicle Mile

The number of lane changes per vehicle mile (LCVM) is a macroscopic measure that can be used to assess the intensity of lane changing in a simulation. Lane changing requires a higher cognitive load of the driver compared to car-following. The driver needs to assess the distance gap and relative speeds of vehicles in the adjacent lane and mentally calculate the safety of the maneuver. This cognitive load increases with roadway density as vehicles travel closer together. As density increases, the driver may have to assess his or her chances several times before an appropriate gap is found and the maneuver is initiated.

Based on past VTTI research using naturalistic driving data, the minimum acceptable values that 95 percent of drivers feel comfortable with are listed below: (Lee, Suzanne E., Erik CB Olsen, and Walter W. Wierwille. A comprehensive examination of naturalistic lane-changes. No. HS-809 702. 2004.)

- TTC of between 4 to 6 seconds for the POVs ahead of the SV or approaching the fast approach zone of the destination lane (Figure 2-3).
- Relative velocity of less than 20 fps both forward and rearward.
- Distance gap 40 feet or higher forward or rearward.

These criteria can be used to assess how many lane changes per vehicle mile are conducted in comfortable conditions.

Definition

In a simulation model, the number of lane changes per vehicle mile is equal with the total number of lane changes divided by the total number of miles. This measure can also be calculated for each driver in addition to the entire simulation.

Caveats and Considerations

On a macroscopic scale the number of lane changes per mile may differ significantly between different locations or corridors. The data provided in this section pertain to trajectories that correspond to entire commute trips. Typically, simulation models only represent a portion of a freeway section and not an entire region. Therefore, calculating LCVM over a shorter section may yield much higher values than calculating the same ratio for the entire trip from beginning to end.

Estimation

Calculating LCVM in simulated datasets is straightforward. In observed trajectory datasets, a fairly accurate GPS device that can identify lane position very accurately is required. Otherwise, the number of lane changes will be overestimated. Identifying lane changes programmatically is not straightforward due to constant changes in roadway geometry. A digital map that has a centerline for each lane is required.

Validation Tests

It is recommended that the average LCVM is computed for all drivers in the simulation. Based on two naturalistic datasets, LCVM varies by traffic density. Average values of LCVM range between 0.25 and 0.36 (approximately one lane change every three miles).

Useful insights about LCVM can be obtained from naturalistic studies that involve a large number of drivers under different conditions. Table 4-2 provides the average LCVM per vehicle for all the drivers participating in a study conducted by Virginia Tech. The first column of Table 4-2 is the driver ID. The second column distinguishes between Interstate (I) and Highway (U.S.) routes. Lane changes per mile vary significantly by vehicle as it can be seen from the LCVM column with the minimum equal to 0.16, the maximum 0.50, and the average 0.36 lane changes per mile.

Table 4-2. Number of lane changes per vehicle mile.

Vehicle ID	Route	Vehicle Type	LCs	Miles	LCVM	Commutes	Miles per Commute
2	I	SedDrv	852	2,206.6	0.39	42	52.54
3	I	SedDrv	838	2,167.2	0.39	46	47.11
4	U.S.	SedDrv	263	876.2	0.30	40	21.91
5	U.S.	SedDrv	653	1,350.4	0.48	41	32.94
6	I	SUVDrv	458	2,209.1	0.21	48	46.02
7	I	SUVDrv	501	1,821.8	0.28	42	43.38
8	U.S.	SedDrv	451	910.9	0.50	40	22.77
9	I	SedDrv	438	1,334.9	0.33	40	33.37
10	U.S.	SUVDrv	634	1,280.6	0.50	40	32.02
11	U.S.	SUVDrv	336	956.8	0.35	38	25.18
12	I	SedDrv	873	1,825.1	0.48	40	45.63
13	U.S.	SUVDrv	100	613.7	0.16	40	15.34
14	U.S.	SUVDrv	452	1573	0.29	40	39.33
15	I	SUVDrv	537	1,407.8	0.38	40	35.2
16	I	SUVDrv	816	2,125.6	0.38	40	53.14
17	U.S.	SedDrv	465	1,289.4	0.36	41	31.45
Grand Total or Mean			**8,667**	**23,949.1**	**0.36**	**658**	**36.40**

(Source: U.S. Department of Transportation. "Integrated Vehicle-Based Safety Systems: Light-Vehicle Field Operational Test Key Findings Report." (2011). http://umtri.umich.edu/content/IVBSS_LV_Key_Findings.pdf)

Figure 4-1 shows the average number of lane changes per 100 miles in the Integrated Vehicle-Based Safety Systems Project conducted for NHTSA. (https://www.nhtsa.gov/research-data/crash-avoidance) Approximately 20 to 30 lane changes per 100 miles are typical across the categories presented in the chart. The average is about 25 lane changes which corresponds to 0.25 LCVM. As it can be deduced from the last three columns of Figure 4-1 the number of lane changes increases with traffic density.

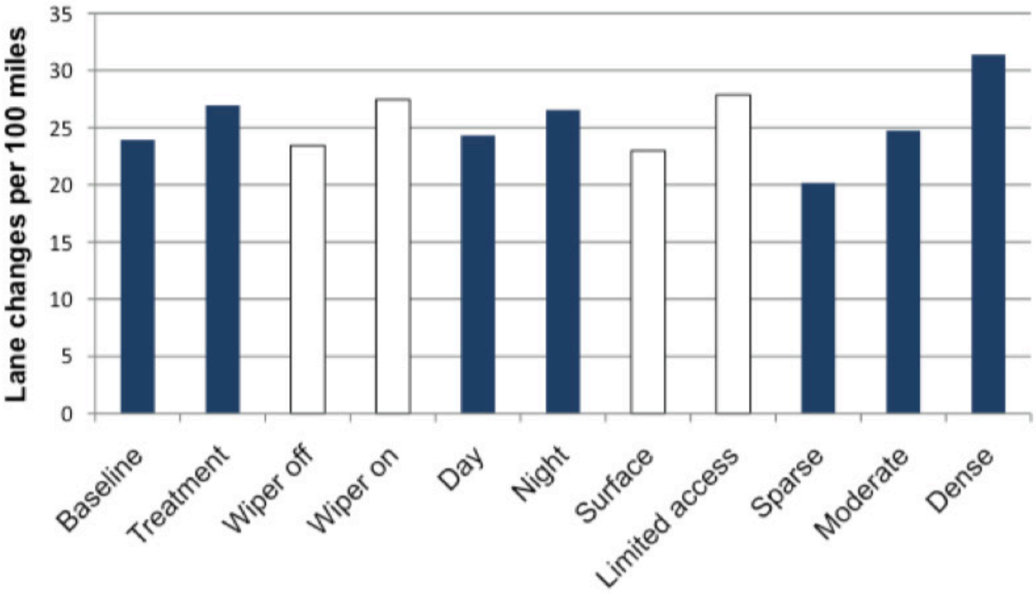

Figure 4-1. Bar graph. Number of lane changes per 100 miles.
(Source: Society of Automotive Engineers, Surface Vehicle Recommended Practice, 2015.)

4.3 Lane-Change Rate

The lane-change rate (LCR) provides an overview of the lane changing patterns in a corridor. Similar to a corridor speed heat map LCR can be visualized over space and time in a heat map to identify locations and times of intense lane changing. Heat maps of LCR values are complementary to LCVM tests or safety-related tests such as the LCU or LCS.

Definition

Lane-change rate is the total number of lane changes occurring in a specific location and time window by hour and mile. After deciding on the spatial bins and temporal time window, LCR is calculated as the ratio of the number of lane changes divided by the spatial bin and time window length. The lane-change rate can be calculated for all the lanes combined or for each lane separately.

Caveats and Considerations

Calculating the LCR heat map using simulated trajectories is relatively straightforward since the trajectories of all vehicles are known. In contrast, the LCR heat map has rarely been calculated using observed data because full trajectory datasets similar to NGSIM are not abundant. As a result, quantitative comparisons between simulated and observed LCR heat maps may be rare. Regardless, computing the simulated LCR and visualizing it in a heat map provides invaluable qualitative and quantitative insights on the spatiotemporal distribution of lane changing that cannot be obtained otherwise.

Estimation

Identifying lane changes in observed datasets is not straightforward and requires a combination of GIS and analytics skills. Instrumented vehicle datasets such as the one collected in this project may not be used as the basis for the LCR heat map because they do not correspond to full trajectories at a given location and time. As it was stated earlier, estimating the LCR in simulated trajectory datasets is straightforward.

Validation Tests

The LCR heat map should be constructed to explore lane changing activity in a corridor. This plot it is very similar to the widely used speed heat map. The LCR heat map can help the analyst analyze the following:

- The time-space distribution of lane changing and how far upstream a weaving section drivers change lanes.
- The maximum lane change rate. This value may be checked against theoretical values to validate against excessive lane changing.

Figure 4-2 shows the LCR heat map for the I-80 corridor between 5 and 5:30 PM based on the NGSIM data. (https://ops.fhwa.dot.gov/trafficanalysistools/ngsim.htm) Each rectangle from left to right represents a separate lane. The flow of traffic is from bottom to top. Each cell in each rectangle corresponds to the LCR value for a bin of 200 feet long and time window of 5 minutes. Values of LCR are mapped to a continuous color map that starts with green and transitions to blue, red, and finally black. The maximum lane change rate happens on the on-ramp (lane 7) and is equal to 72 lane changes per hour and mile (lcphpm). Lanes one and two have light green colors up to 10 lcphpm. More intense lane-changing equal to 20 lcphpm happens on lanes 5 and 6 that are adjacent to the on-ramp. The higher LCR on lanes 5 and 6 may be attributed to positioning for an imminent exit downstream or to drivers deciding to pick a different lane in anticipation of the heavier lane changing at the ramp.

4.4 Fundamental Diagram

The fundamental diagram (FD) of traffic flow is typically derived from the macroscopic quantities of flow (vehicles per hour), density (vehicles per mile), and space-mean speed. It is recommended that the analyst constructs the FD using macroscopic quantities. (https://www.fhwa.dot.gov/publications/research/operations/tft/chap2.pdf) For additional insight, FD can be constructed from microscopic quantities (MicroFD) without any aggregation. If so, outliers in driving behavior such as vehicles traveling too fast and too close together can be revealed. This section starts with a general description of the fundamental diagram. Focus is given on MicroFD since there is an abundance of documentation on how to construct FD with macroscopic quantities.

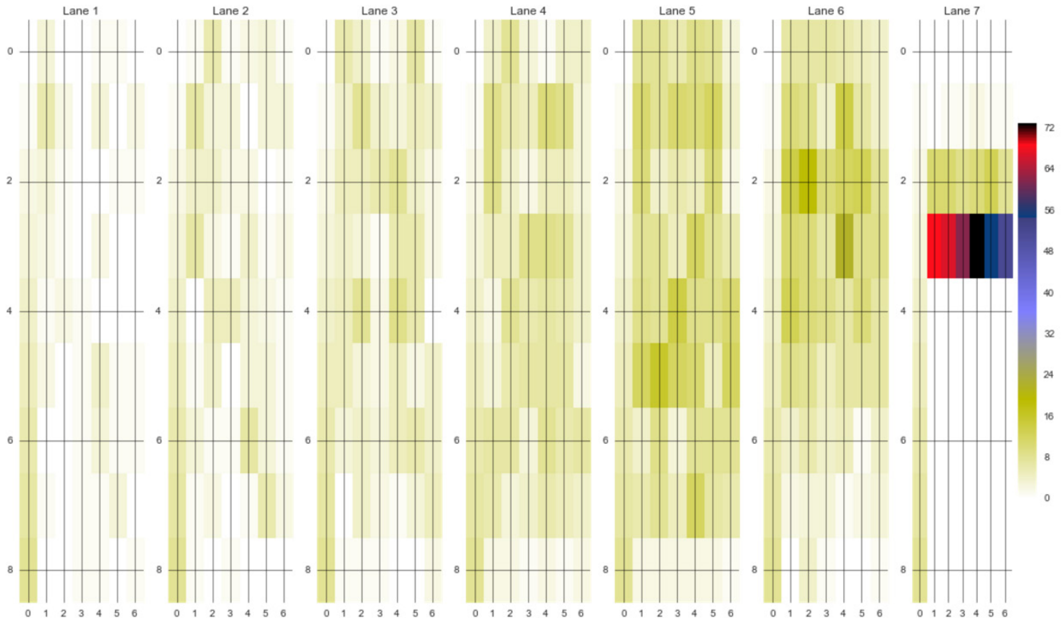

Figure 4-2. Graph. Distribution of lane-change rate by space and time in the I-80 NGSIM Dataset (5:00 to 5:30 p.m.).

(Source: Cambridge Systematics, Inc.)

Definition

The diagram has three equivalent forms: flow-density (concave), speed-density (monotone decreasing) and speed-flow with an upper and lower component. The FD is of critical importance for a number of active traffic management applications such as ramp metering and variable speed limit strategies. In mesoscopic and macroscopic simulation models the FD does not emanate from the collective behavior of individual vehicles as in simulation models but has to be defined exogenously by the analyst. The actual shape of the diagram is location or link specific. Figure 4-3 represents a simplified schematic of the FD and specifically the flow-density relationship. The uncongested state of traffic is represented by a straight line from 0 density and flow to maximum flow and breakdown density. The simplified straight line of the diagram that starts from zero assumes that drivers travel with constant speed v_h until traffic breaks down. The congested stop-and-go portion of the diagram is represented by another straight line from critical density and maximum flow at congested conditions to maximum density (ρ_J) and zero flow. Maximum density depends on the average length of the vehicles and the space drivers leave when they stop behind another car. The straight line of the congested portion of the diagram assumes that shockwave speed w is constant for different density levels. The vertical drop between maximum flow in uncongested conditions and maximum flow at congested conditions corresponds to the widely-observed phenomenon called “capacity drop.” The capacity drop phenomenon dictates that once a traffic jam happens, traffic flow has to fall substantially from the maximum value achieved to resolve the jam. Active traffic management applications try to avoid conditions resulting in a capacity drop because once the roadway section transitions to the congested state the capacity of the corridor further degrades exacerbating congestion.

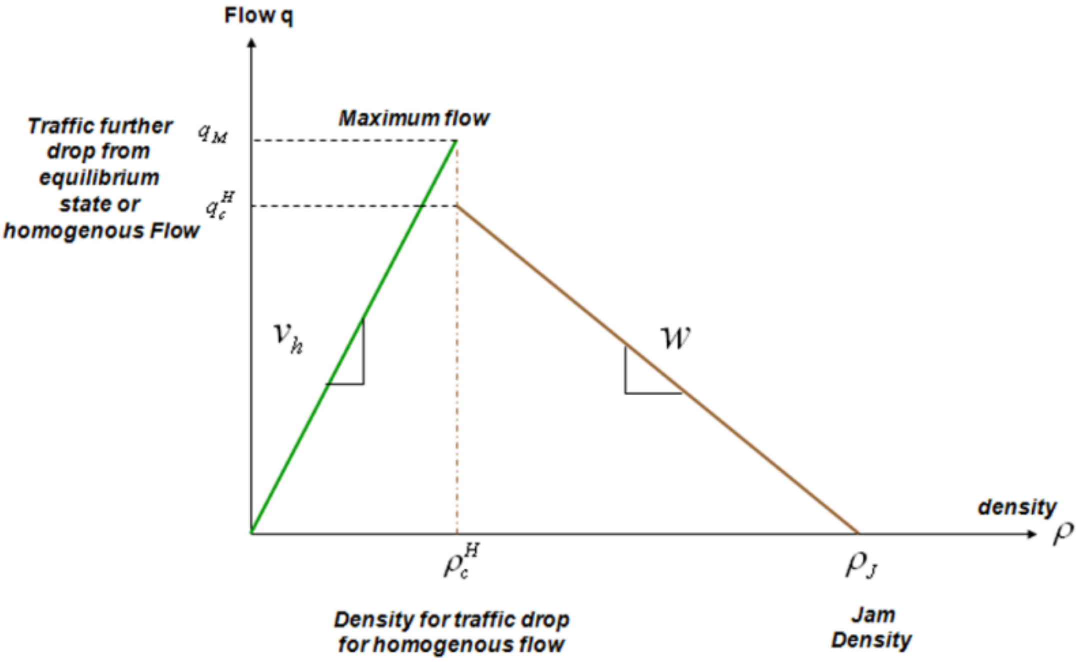

Figure 4-3. Diagram. Simplified schematic of the fundamental diagram.

(Source: Lu, Xiao-Yun, P. Varaiya, and Roberto Horowitz. "Fundamental diagram modeling and analysis based NGSIM data." IFAC Proceedings Volumes 42.15 (2009): 367-374.)

Caveats and Considerations

The problem with the FD is that some macroscopic variables (i.e., density and space-mean speed) are not measurable at a roadway cross-section (this measurement requires aerial observation).

On the contrary, if one has all the trajectories over a time-space domain the previous variables can be directly measured, without having to resort to approximations like exchanging space-mean speed with time-mean speed (or, better, with the harmonic mean of cross-sectional speeds).

When such data are available, one can use the generalized definitions by Edie to calculate the macroscopic variables from the trajectories. (Treiber, Martin, and Arne Kesting. "Traffic flow dynamics." Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg (2013).)

In many applications of the fundamental diagram, 15-minute or hourly intervals are used to calculate density, flow or mean speed. (Skabardonis, Alexander, and Richard Dowling. "Improved speed-flow relationships for planning applications." Transportation Research Record: Journal of the Transportation Research Board 1572 (1997): 18-23.) Using longer time intervals results in a static version of the diagram with less dispersion. For many planning applications this may be sufficient. For validating traffic flow at the microscopic level, using smaller temporal and spatial intervals can provide a more fine-grained picture of the traffic dynamics in the simulation model or the study area. For example, Lu, Varaiya, and Horowitz used a time interval of 10 seconds and a space interval of 170 meters for constructing the FD from NGSIM data. (Lu, Xiao-Yun, P. Varaiya, and Roberto Horowitz. "Fundamental diagram modeling and analysis based NGSIM data." IFAC Proceedings Volumes 42.15 (2009): 367-374.)The reader should note that the vast majority of applications use macroscopic quantities to derive the FD. Lack of datasets has resulted in limited insights on the microscopic version of the FD.

Estimation

In the analysis presented in this section the fundamental diagram is derived from microscopic quantities without any aggregation over time or space. The microscopic fundamental diagram, has each trajectory point represented as a point in the two-dimensional space of the diagram. The agglomeration of all points, typically millions, takes the form of the known fundamental diagram (Figure 4-4). Outliers can clearly identify cases with significant measurement errors or cases where the simulation engine behaves unrealistically.

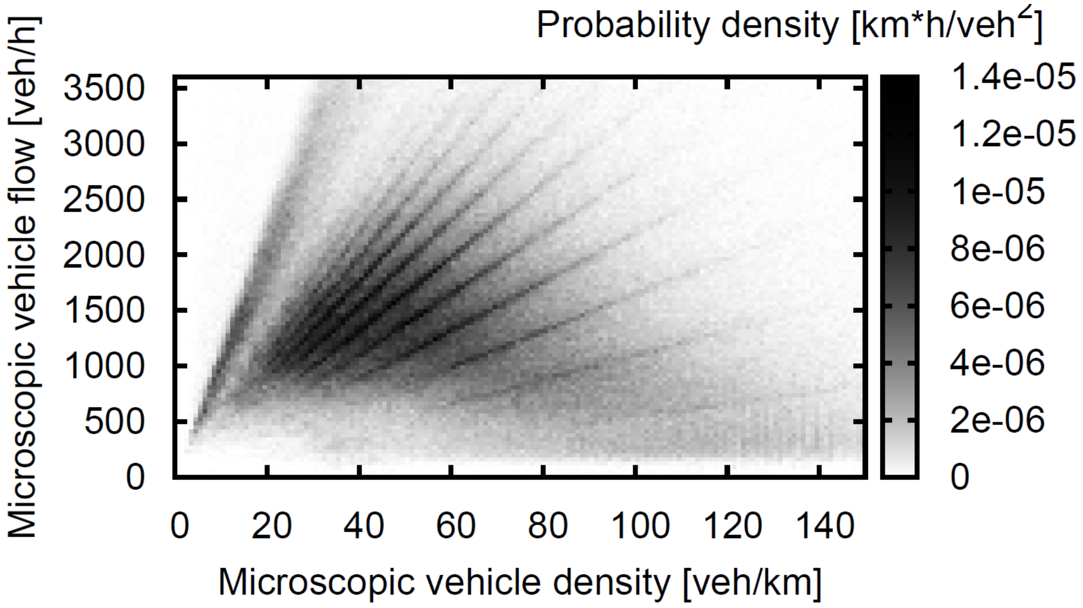

Figure 4-4. Diagram. Microscopic fundamental diagrams of the NGSIM datasets.

(Source: Thiemann, Christian, Martin Treiber, and Arne Kesting. "Estimating acceleration and lane-changing dynamics from next generation simulation trajectory data." Transportation Research Record: Journal of the Transportation Research Board 2088 (2008): 90-101.)

For each vehicle position in a trajectory dataset, it is necessary to derive the inverse of the space headway $(\Delta x_a)^{-1} = (x_{a+1} - x_a)^{-1}$ and the inverse of the time headway, $(h_a)^{-1}$. These quantities are more intuitively described as "microscopic density" and "microscopic flow." They have the units of density (vehicles per mile) and flow (vehicles per hour). An example of the MicroFD constructed using NGSIM data is presented in Figure 4-4. On the x-axis there is microscopic vehicle density in vehicles per kilometer from 0 to 140. On the y-axis there is microscopic vehicle flow in vehicles per hour from 0 to 3500. The shade of grey of each point in the diagram is mapped to levels of probability density that are shown on the right hand side in the figure. Measurement noise in the NGSIM data reveals significant dispersion around the shape of the FD. The stripes in the diagram correspond to preferred speeds produced by the NGSIM algorithm. MicroFD can also be constructed from the instrumented vehicle data collected.

In simulated datasets, MicroFD can be easily constructed. Instrumented vehicle data similar to the ones collected for the current project can also be used. However, when a small number of drivers or runs over the same location are used, the resulting MicroFD may not be representative to the entire population. The plot in Figure 4-4 does not show the average microscopic vehicle flow for each microscopic vehicle density level. As a result, the capacity cannot be estimated. Qualitatively, this plot is in agreement with the FD that was derived by Varaiya. (Lu, Xiao-Yun, P. Varaiya, and Roberto Horowitz. "Fundamental diagram modeling and analysis based NGSIM data." IFAC Proceedings Volumes 42.15 (2009): 367-374.)

Validation Tests

It is recommended that FD or the MicroFD is constructed from simulated trajectories for a specific location. Regardless of the existence of the observed data FD can help the analyst identify the following:

- The average macroscopic capacity of the segment.
- The average desired free flow speed and percentile values.
- The shockwave speed.
- The MicroFD can help locate outliers that reveal vehicles traveling for example too fast and too close together (top right quadrant in Figure 4-4).

Determining and validating the capacity of a roadway segment is often not part of the calibration and validation procedures in simulation modeling. Instead, focus is given in matching flows and travel times. However, in an Active Traffic Management context where a simulation model is used under different demand scenarios having validated the properties of the FD ensures that the model will respond realistically to different demand levels. When both observed and simulated values exist, the two diagrams can be compared quantitatively by computing the root mean square error after discretizing the x and y axes into bins.

Appendix A. List of Acronyms

AMS	Analysis, Modeling and Simulation
AASHTO	American Association of State Highway Transportation Officials
ABS	Anti-Lock Braking System
ACAS	Advanced Collision Avoidance Systems
ADAS	Advanced Driver Assistance Systems
AJ	Acceleration Jerk
AR	Acceleration Range
ARMS	Acceleration Root Mean Square
ATDM	Active Transportation and Demand Management
CAS	Collision Avoidance Systems
CAV	Connected Automated Vehicles
FCW	Forward Collision Warning
FD	Fundamental Diagram
FHWA	Federal Highway Administration
FMCSA	Federal Motor Carrier Safety Administration
fps	Frame per second
fpss	Frame per second squared
GPS	Global Positioning System
ISO	International Organization for Standardization
IVBSS	Integrated Vehicle-Based Safety Systems Project
K-S	Kolmogorov Smirnov
LC	Lane Change
LCVM	Lane Changes Per Mile
LCR	Lane Change Rate
LCS	Lane Change Severity
LCU	Lane Change Urgency
LIDAR	Light Detection And Ranging
MicroFD	Microscopic Fundamental Diagram
NDS	Naturalistic Driving Study
NGSIM	Next Generation Simulation
NHTSA	National Highway Traffic Safety Administration
NTSB	National Transportation Safety Board
POV	Principal Other Vehicle
RMVMT	Rate per Million Vehicle Miles Traveled
SAE	Society of Automotive Engineers
SSAM	Surrogate Safety Assessment Model
SV	Subject Vehicle
TRB	Transportation Research Board
TTC	Time to collision
TTCa	Time to collision (speed plus acceleration based)
TTCs	Time to collision (speed based)
U.S. DOT	U.S. Department of Transportation
VTTI	Virginia Tech Transportation Institute

Appendix B. Statistical Tools for Comparing Distributions

There is a rich body of literature for comparing probability distributions. Many tests are being used for comparing one-dimensional probability distributions. Among them the Kolmogorov-Smirnov (K-S) test is among the most frequently used. The statistical formulations and the steps taken to apply it are presented below. There are two variations of the test: one that compares empirical data against a fitted parametric theoretical distribution (e.g., for comparing a dataset to a theoretical distribution), and one that compares two empirical distributions (e.g., for comparing two datasets).

Although the K-S test has been extended to two or more dimensions by several authors, its application is not straightforward. For this reason, it is recommended that for multivariate data, the analyst perform a one-dimensional K-S test across each dimension individually instead of attempting to use a higher-dimension K-S test on the dataset directly.

One-Dimensional Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test can be used to test the hypothesis that a data sample comes from a population that follows a reference probability distribution (which can be either theoretical or empirical). (http://ocw.mit.edu/courses/mathematics/18-443-statistics-for-applications-fall-2006/lecture-notes/lecture14.pdf) A variant of the test, the Kolmogorov-Smirnov Test for Two Samples, can perform a similar evaluation for two empirical data sets. This two-sample version is relevant to trajectory validation, as it enables the direct comparison of a simulation data set and a field data set without any fitting to known parameterized distributions.

The K-S test works by examining the distance between the two empirical PDFs from the data sets, which will be denoted $F_m(x)$ and $G_n(x)$. This is simply a CDF that puts a mass of 1/*n* at each data point in the sample, where *n* is the total number of points in the sample. (Wasserman. *All of Statistics. A Concise Course in Statistical Inference.* Springer Texts in Statistics, Second Printing, 2005. Page 97.) Figure B-2 shows a sample empirical distribution function $F_n(x)$ in dark magenta. The Kolmogorov-Smirnov Test for Two Samples test statistic is given by:

$$D_{mn} = \left(\frac{mn}{m+n}\right)^{1/2} sup_x|F_m(x) - G_n(x)\,|$$

Figure B-1. Equation. Kolmogorov Smirnov Statistic.

The hypothesis regarding the distributional form is rejected if the test statistic is greater than the critical value obtained from a reference table.

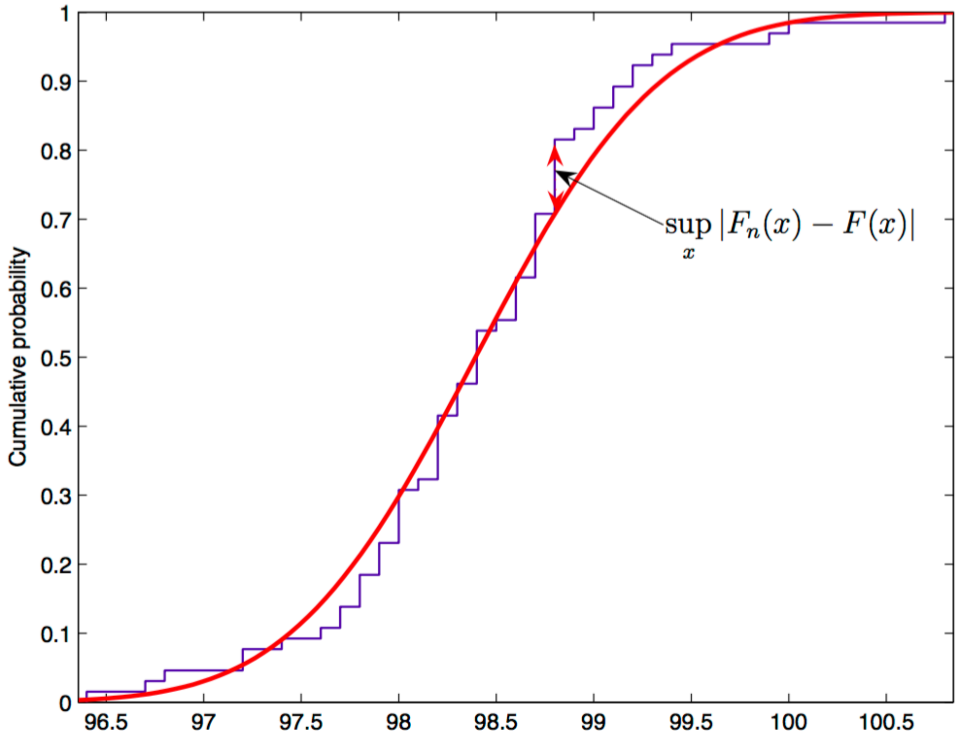

Figure B-2. Graph. Kolmogorov-Smirnov test statistic.

(Source: Cambridge Systematics, Inc.)

Steps required:

1. Calculate empirical distributions for both samples.
2. Calculate (D_{mn}), the test statistic for the Kolmogorov-Smirnov Test for Two Samples.
3. Evaluate the test statistic using a table of reference values for the desired level of significance.

Appendix C. Microscopic Measure Accuracy

For observed location data from the field, measurement error or uncertainty is always a factor (e.g., GPS data, radar measurements, and video data) that may have an effect on any derived quantities based on those data. In the case of velocity and acceleration, the first and second time derivatives of position, errors in position can be mitigated by using larger time intervals over which those differences are measured. However, the concepts presented here can also be applied to other variables for which the standard deviation of error can be measured or assessed beforehand.

Figures C-1 and C-2 below provide the definitions for average velocity and average acceleration at time t_i.

$$v(t_i) = \frac{\mathrm{x}(t_{i+1}) - \mathrm{x}(t_{i-1})}{t_{i+1} - t_{i-1}}$$

Figure C-1. Equation. Definition of speed.

$$a(t_i) = \frac{\mathrm{v}(t_{i+1}) - \mathrm{v}(t_{i-1})}{t_{i+1} - t_{i-1}}$$

Figure C-2. Equation. Definition of acceleration.

The first-order approximation for the errors in these calculations is given by equations in Figures C-3 and C-4 below, where $\sigma_x(t_i)$ is the standard deviation of error associated with the measured position at time (t_i), or simply the error associated with measured position data at any time if this error is assumed to be independent of the exact time of measurement.

$$\sigma_v(t_i) = \frac{\sqrt{[\sigma_x(t_{i+1})]^2 + [\sigma_x(t_{i-1})]^2}}{t_{i+1} - t_{i-1}}$$

Figure C-3. Equation. Definition of the standard deviation of speed error.

$$\sigma_a(t_i) = \frac{\sqrt{[\sigma_v(t_{i+1})]^2 + [\sigma_v(t_{i-1})]^2}}{t_{i+1} - t_{i-1}}$$

Figure C-4. Equation. Definition of the standard deviation of acceleration error.

The equations in Figures C-3 and C-4 reveal the reason that larger time steps reduce the overall error in the measured acceleration and velocity. Specifically, larger time steps make the denominator larger, and therefore make the entire evaluated function smaller. The example below shows how the analyst can test a given time step and examine its influence on calculated values of velocity and acceleration given a specified level of position measurement error.

Given a time step of ½ second and a positional error of 1 foot, the equation in Figure C-3 yields a velocity error of $\sqrt{2}$ *fps* and an acceleration error of 2 *fpss*. Given that most values of vehicle acceleration range between -10 and 10 *fpss*, it is clear that even a relatively small positional error (e.g., 1 foot) can result in significant data noise for the derived quantities of acceleration and velocity. Kalman filtering is a common method for smoothing time series data in an attempt to filter out errors.

For the data collection component of this project, a time step of 50 milliseconds is used for the field data, which can produce large errors in velocity and acceleration calculations without Kalman filtering. The radar measurements obtained from the instrumented vehicle have a positional accuracy of approximately 2 feet.

Experimental Example

A simple simulation experiment was constructed by adding Gaussian noise to the position of a vehicle moving along a line with speed given by the following equation:

$$u = 51 \sin\left(\frac{t}{5}\right) + 51 \quad [fps]$$

Figure C-5. Definition of test speed function.

In this example, the parameters of the speed equation were selected so that speed fluctuates periodically between 0 and 102 feet per second, corresponding to 0 and 70 mph. The true acceleration at any given time is theoretically defined as the time derivative of the equation shown in Figure C-6, as provided below.

$$a = 10.2 \cos\left(\frac{t}{5}\right) \quad [fpss]$$

Figure C-6. Equation. Definition of test acceleration function.

In this case, acceleration fluctuates between -10.2 and 10.2 feet per second squared (fpss), which aligns well with data from simulation and field tests.

By artificially introducing different levels of noise (σ_X) into position measurements, it is possible to obtain insight into the effect of these errors on the derived measures of velocity and acceleration. In Figure C-7, acceleration errors are shown for this example case with a time step of one second and a position error of $(\sigma_X = 1\,ft)$. As Figure C-7 reveals, this level of error produces larger errors in the calculated acceleration to the point where the underlying acceleration behaviors are fully obfuscated by the position noise.

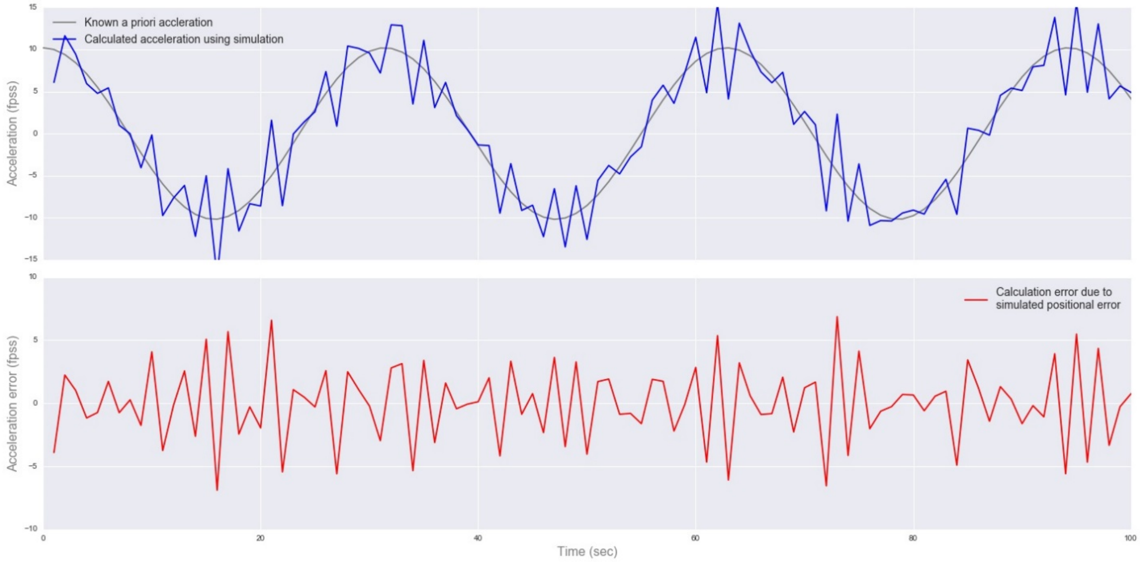

Figure C-7. Graphs. Acceleration error when time step is 1 second and standard deviation of positional noise is 1 foot.

(Source: Cambridge Systematics, Inc.)

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