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Production of Synthetic Winds for the Global Reference Atmosphere Model (GRAM)

December 15, 2010

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Volpe National Transportation Systems Center
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14. ABSTRACT

The Aerospace Corporation was tasked by the Volpe National Transportation Systems Center to provide technical support to the Federal Aviation Administration, Office of Commercial Space Transportation (FAA/AST), in developing a method based on Principal Component Analysis (PCA) to generate synthetic winds that may be used for studies of aerodynamic loads and the dispersion of rocket trajectories. A notable feature of the analysis is that it is performed on winds separated in time by certain time deltas (30, 60, 120 minutes, etc). This approach produces synthetic winds that preserve relevant statistics on the observed winds. These include the variance, spatial spectra, correlations between wind components, and temporal decorrelation. This is different from synthetic winds computed with the Global Reference Atmosphere Model (GRAM) for example. The GRAM output has two main components: a deterministic monthly mean component $\bar{\psi}$, where ψ may be any GRAM output variable, and a stochastic perturbation component ψ'. The two components are combined to give a realization that can be used for trajectory simulations. A difficulty with the present GRAM formulation for calculating temporal effects is that each realization of ψ' is uncorrelated with any other realization. In addition, the present GRAM predicts that the north-south and east-west components of the wind, u' and v', respectively, are uncorrelated and that the power spectral density of the wind components falls off less rapidly with decreasing wavelength than observed. We have developed an approach for calculating ψ' based on PCA that resolves all of these difficulties and that may be used with the deterministic part of GRAM to produce more realistic synthetic winds. The methodology and first results are presented herein.

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Abstract

The Aerospace Corporation was tasked by the Volpe National Transportation Systems Center to provide technical support to the Federal Aviation Administration, Office of Commercial Space Transportation (FAA/AST), in developing a method based on Principal Component Analysis (PCA) to generate synthetic winds that may be used for studies of aerodynamic loads and the dispersion of rocket trajectories. A notable feature of the analysis is that it is performed on winds separated in time by certain time deltas (30, 60, 120 minutes, etc.). This approach produces synthetic winds that preserve relevant statistics on the observed winds. These include the variance, spatial spectra, correlations between wind components, and temporal decorrelation. This is different from synthetic winds computed with the Global Reference Atmosphere Model (GRAM) for example. The GRAM output has two main components: a deterministic monthly mean component $\bar{\psi}$, where ψ may be any GRAM output variable, and a stochastic perturbation component ψ'. The two components are combined to give a realization that can be used for trajectory simulations. A difficulty with the present GRAM formulation for calculating temporal effects is that each realization of ψ' is uncorrelated with any other realization. In addition, the present GRAM predicts that the north-south and east-west components of the wind, u' and v', respectively, are uncorrelated and that the power spectral density of the wind components falls off less rapidly with decreasing wavelength than observed. We have developed an approach for calculating ψ' based on PCA that resolves all of these difficulties and that may be used with the deterministic part of GRAM to produce more realistic synthetic winds. The methodology and first results are presented herein.

Acknowledgments

The authors are grateful to the Office of the Associate Administrator for Commercial Space Transportation, Federal Aviation Administration (FAA/AST), for the opportunity to work on this project. Specifically, gratitude is extended to Mr. Steven Millard at FAA/AST for valuable guidance on AST regulatory and licensing needs as well as insights on wind effects on rocket launches. The authors also wish to thank Ms. Ruth A. Hunter, DOT Volpe National Transportation Systems Center, who served as the contracting officer's technical representative and provided valuable insight into relevant technical issues and government needs and regulations. The authors are also indebted to Mr. Robert W. Seibold, who served as The Aerospace Corporation program manager.

Contents

reduced set of PCVs ($R < N$). One orders the set of singular values from largest to smallest ($\sigma_1 > \sigma_2 > \cdots > \sigma_N$) and truncates the PCV set at σ_R such the accuracy in the reconstruction is greater than the required accuracy. The set $\{\boldsymbol{a}_r\}$ contain the expansion coefficients for $\boldsymbol{X}$ in terms of the set $\{\boldsymbol{v}_r\}$. The data may be synthesized in terms of the eigenvectors by means of the superposition

$$\boldsymbol{X} = \sum_{r=1}^{R} \sigma_r \boldsymbol{a}_r \boldsymbol{v}_r^T \tag{6}$$

2.2 Procedure for Generating Synthetic Winds

A powerful advantage of the PCA approach is that the $\boldsymbol{a}_r$ are uncorrelated. The process for creating synthetic pairs involves a means for calculating $\boldsymbol{u}' = (u', v')$ based on a randomization of the $\boldsymbol{a}_r$. To create synthetic winds one interprets $\boldsymbol{a}_r$ as a random variable α_r for which we have N samples. By construction the $\boldsymbol{a}_r$ are orthonormal and therefore the α_r are bounded as $-1 < \alpha_r < 1$. Calculations for the Eastern Range winds give an expected value $E[\alpha_r] \approx 0$ and variance $Var[\alpha_r] \approx 1/N$. Consider the hyperbolic transformation of α_r that maps $(-1,1) \rightarrow (-\infty, \infty)$

$$t_r = \log\left(\frac{1+\alpha_r}{1-\alpha_r}\right) \tag{7}$$

or

$$\alpha_r = \frac{e^{t_r}-1}{e^{t_r}+1} = \tanh\left(\frac{t_r}{2}\right) \tag{8}$$

calculations show that the random variables t_r can be modeled by uncorrelated normal random variables

$$t_r \in N(0, \sigma^2) \tag{9}$$

To show how σ is related to N consider the transformation (8) whence the variance of $\boldsymbol{\alpha}$ is given by

$$\frac{1}{N} = Var[\boldsymbol{\alpha}] = \int_{-1}^{1} u^2 f_{\boldsymbol{\alpha}}(u) du = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \tanh^2\left(\frac{t}{2}\right) e^{-\frac{t^2}{2\sigma^2}}\, dt \tag{10}$$

A change of variables and symmetry lead to

$$N = \left(\frac{\sqrt{2}}{\pi} \int_{0}^{\infty} \tanh^2\left(\frac{\sigma}{\sqrt{2}}\tau\right) e^{-r^2} d\tau\,)\right)^{-1} \tag{11}$$

The quantity N was calculated by evaluating the integral numerically for $0.02 \leq \sigma \leq 0.5$. A least squares fit and a simple approximation to the numerical results is given by $\sigma = 2.13114529/\sqrt{N}$.

For the 120-minute wind pairs $N = 943$ and $\sigma = 0.065239$.

To generate synthetic winds using R PCVs

1. Generate R normal mode random variables $t_1, t_2, \ldots.., t_R$ from $N(0, \sigma^2)$.
2. Transform t_r toα_r using (8).
3. Calculate an augmented synthetic win $\widehat{\boldsymbol{w}}$d by linearly combining the PC shapes

$$\widehat{\boldsymbol{w}}^T = \sum_{r=1}^{R} \alpha_r (\sigma_r \boldsymbol{v}_r^T) + \overline{\boldsymbol{w}}^T = [\widehat{\boldsymbol{u}}_A^T : \widehat{\boldsymbol{v}}_A^T : \widehat{\boldsymbol{u}}_B^T : \widehat{\boldsymbol{v}}_B^T] \tag{12}$$

4. Extract the u' and v' components of the A and B wind.

2.3 Results

Figure 1 shows a sample set of observed winds constructed (synthesized) from a superposition of the first 400 principle components based on 943 120-minute pairs for the Eastern Range. The u and v components are shown for A and B winds. There is very little error for $r = 400$. Figure 2 shows the same synthesis, but for synthetic winds. In qualitative terms the persistence over 120 minutes is similar for the observed and synthetic pairs.

Figure 3 shows the means and standard deviations for the transformed coefficients, t_r, for the various principal component vectors (PCVs) for the same data described Figure 1. The indices are ordered by the magnitude of the eigenvalues (singular values). The smaller the singular value the greater the variance explained by the corresponding PCV, so that the PCVs associated with smaller indices account for most of the variance. The means differ only slightly from zero except for approximately the 10 highest indices. Similarly, the standard deviations are nearly equal to the mean standard deviation ($\sigma = 0.065239$) except for approximately the 20 largest indices, which are significantly larger than the mean value. However, the indices associated with the anomalous means and standard deviations account for a very small fraction of the total variance and are not used in our reconstructions. The construction of synthetic winds is done for $R = 400$ and for $r \leq R$. The assumption of zero mean and constant σ is well justified.

Figure 4 shows the covariance of the coefficient vectors $\boldsymbol{t_r}$ for the first 100 PCVs based on the same data for Figure 1. The covariance is essentially zero except along the diagonal (variance). This demonstrates the independence (orthogonality) of the coefficient vectors.

Figures 5-7 show *Q-Q* plots for the same data as for Figure 1 for the coefficients of the first 12 principal components (Q denotes "quantile"). Four components are shown per figure starting with the coefficients for PC 1-4 in Figure 5. Each figure has four panels; each panel shows the *Q-Q* plot for a single principal component. These plots compare the probability distribution of t_r for the observed winds to the normal distribution, $N(0, \sigma^2), used$ for the synthetic winds. The two distributions are similar if the plotted points lie along the diagonal, from the lower left to upper right. If the points lie above the diagonal the distribution for the synthetic winds is more dispersed than for the observed winds; if the plot lies below the line the reverse is true. For all of the plots, the points lie on or close to the diagonal over the main part of the distribution. For about half the plots the points fall on or near the diagonal over essentially the entire distribution.

Figure 8 shows the mean PSD of u and v for both observed and synthetic winds. Only A-wind PSDs are shown. The agreement between the synthetic and observed winds is excellent. Also shown is the –2 slope given by the GRAM perturbation method for large wavenumbers. The –2 slope is significantly shallower than the observed slope for wavenumbers larger than 10^{-4} $cycles\ ft^{-1}$. The observed slope is close to –3 up to wavenumbers near 10^{-4} $cycles\ ft^{-1}$, and somewhat greater (steeper) thereafter.

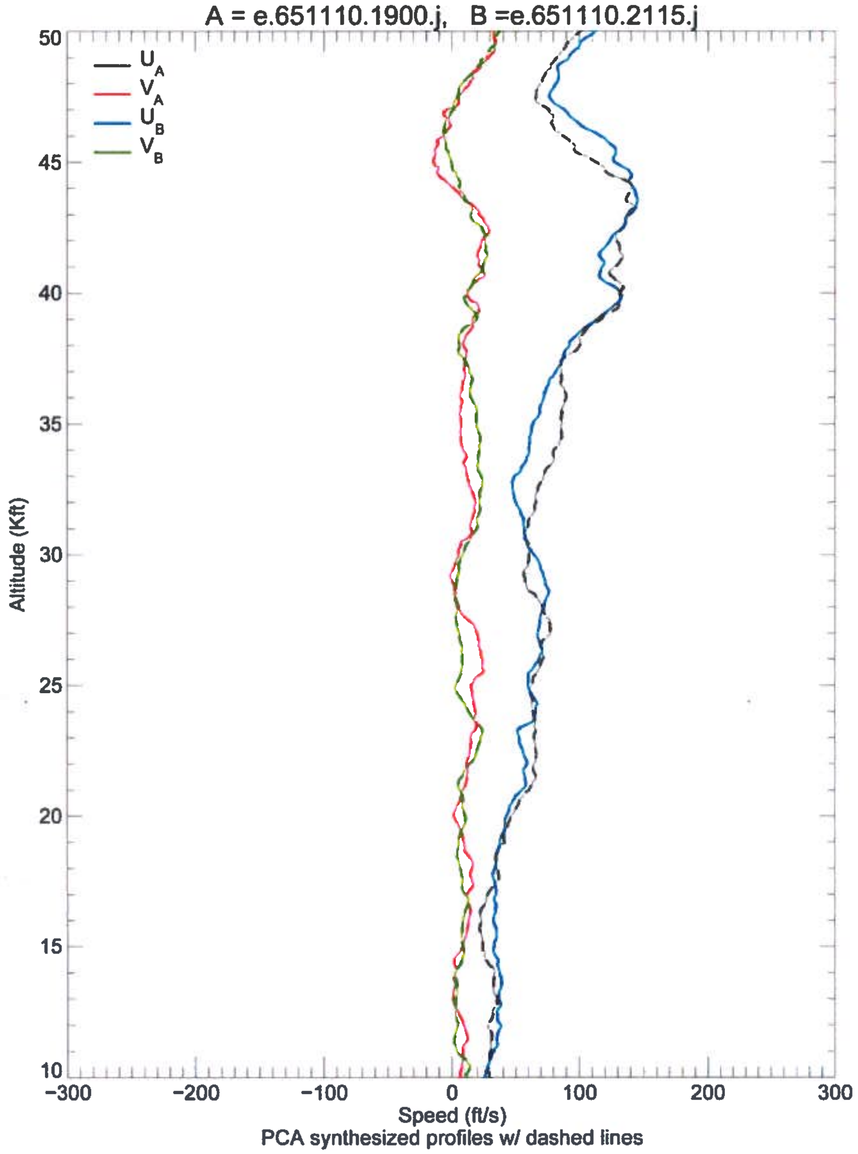


Figure 1. A sample set of observed winds constructed (synthesized) from a superposition of the first 400 principle components based on 943 120-minute pairs for the Eastern Range. The u and v components are shown for A and B winds.

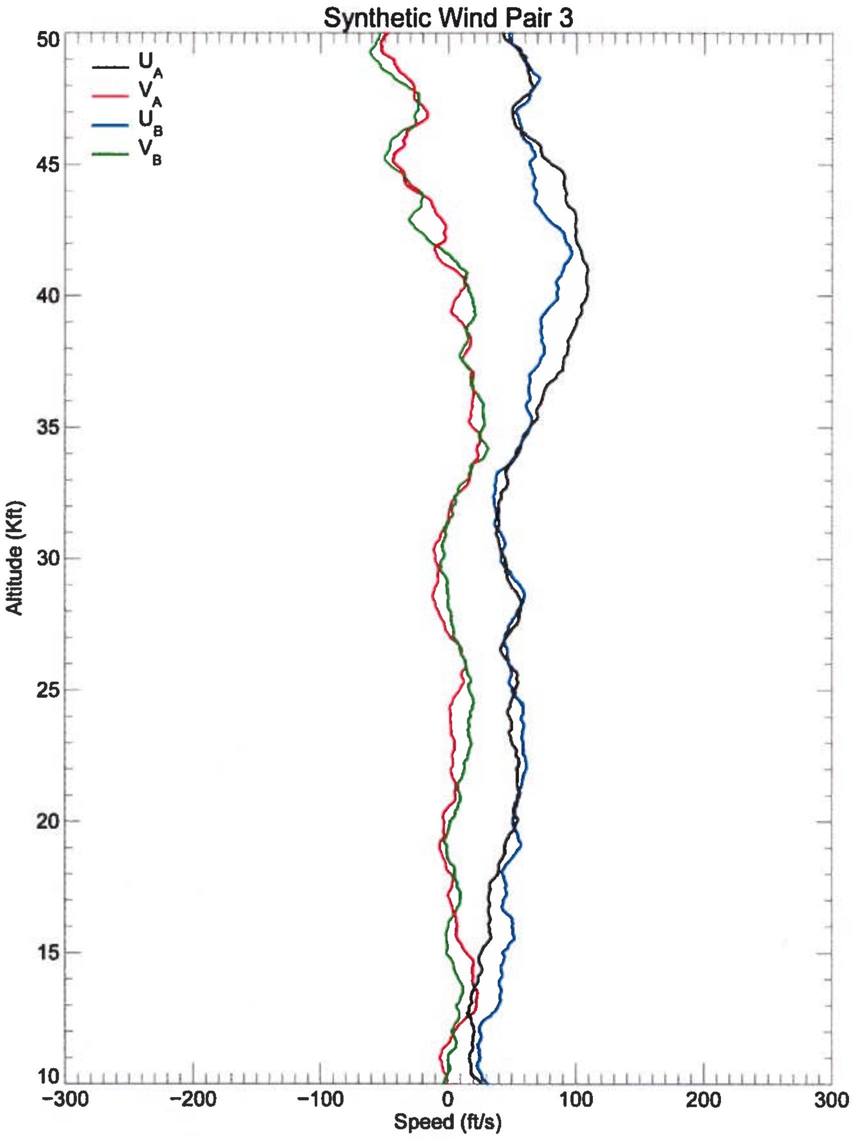


Figure 2. Same as Figure 1 but for synthesized winds.

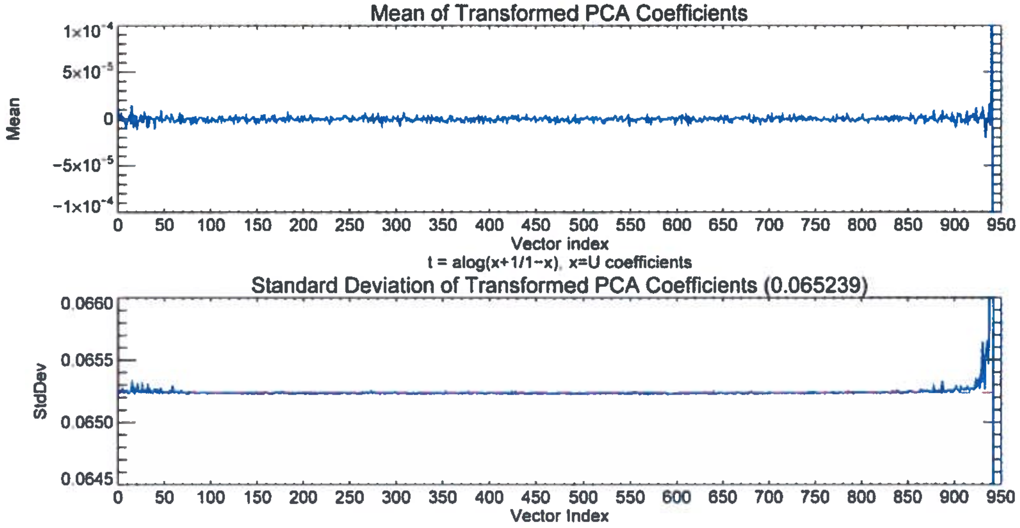


Figure 3. Means (upper panel) and standard deviations (lower panel) for the coefficients of the various principal components (PCVs) for the same data described Figure 1.

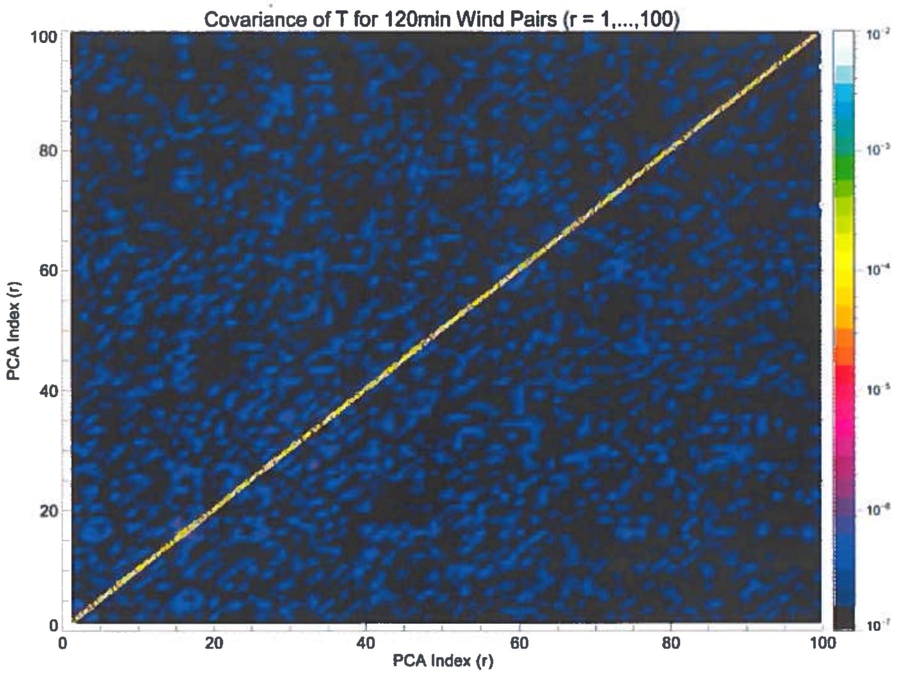


Figure 4. The covariance of the coefficient vectors t_r for the first 100 PCVs based on the same data used for Figure 1.

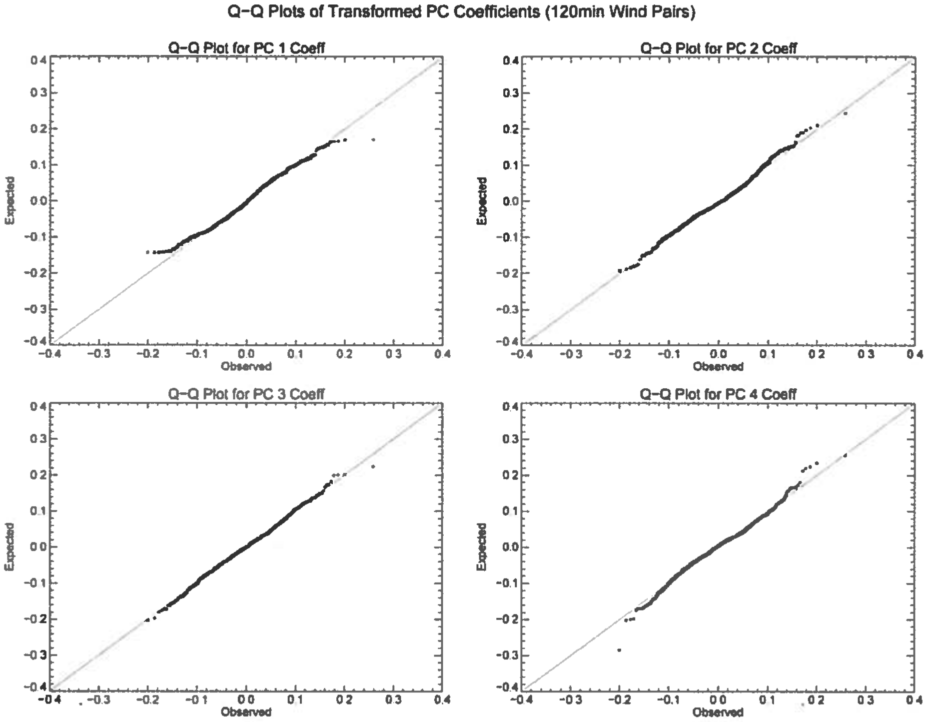


Figure 5. *Q-Q* plots for the coefficients of the first four principal components. Results are based on the same data as Figure 1.

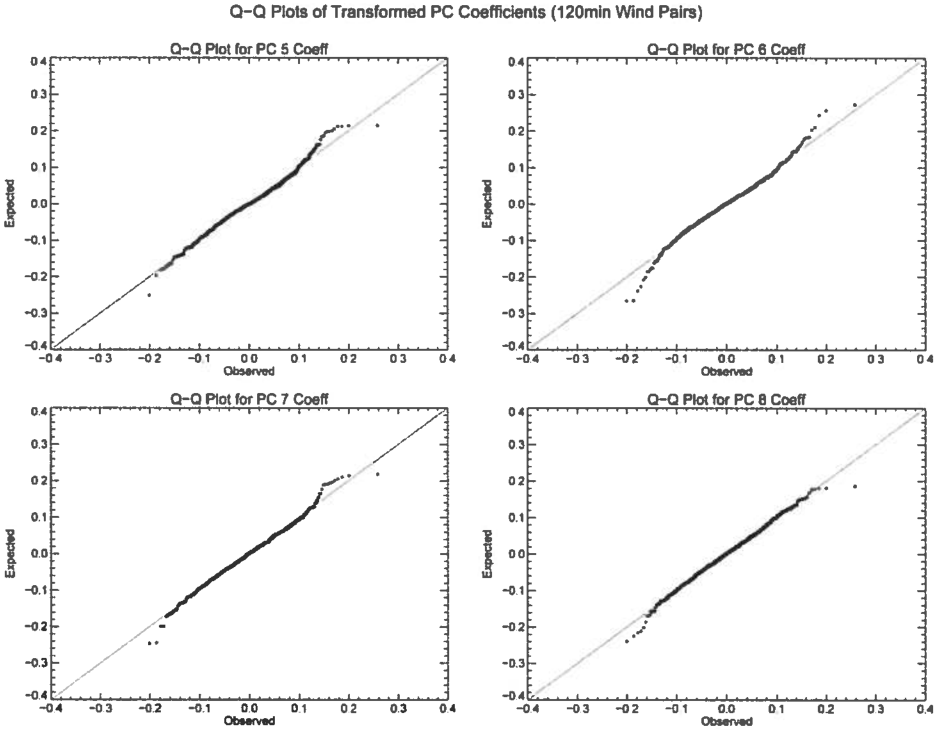
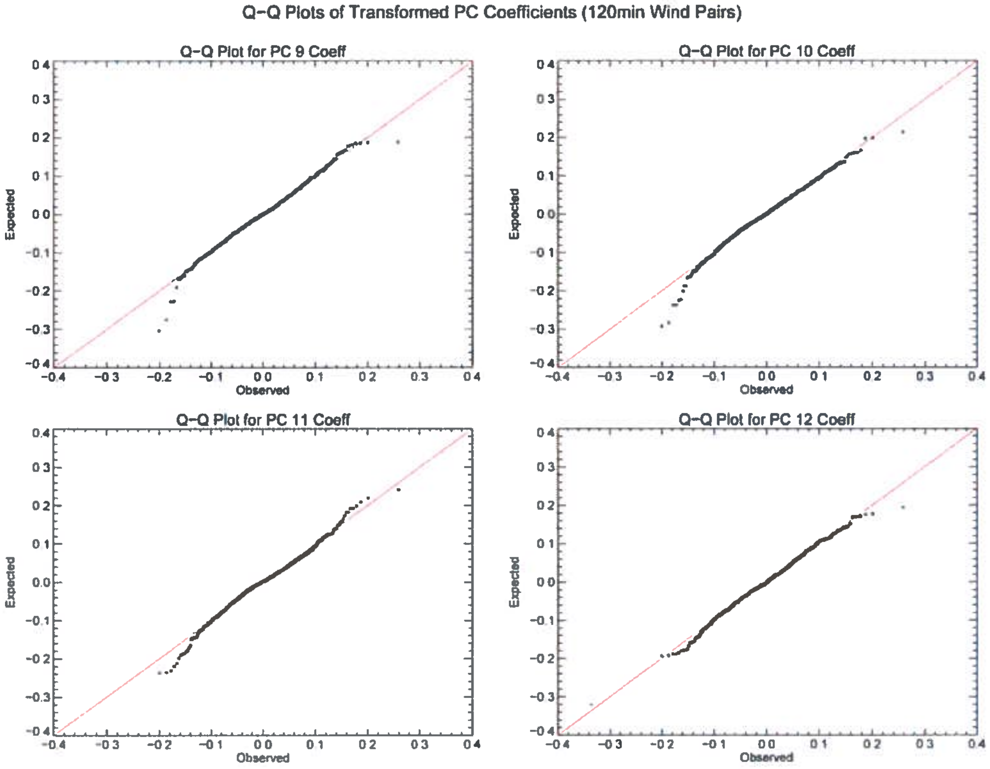
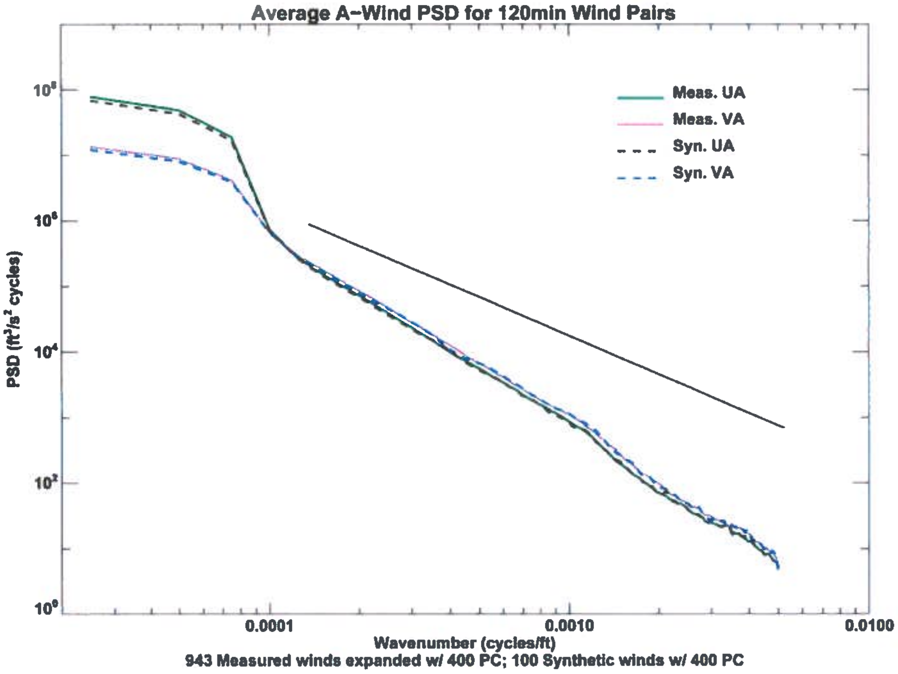


Figure 6. Same as Figure 5 except for components 5-8.



Q–Q Plots of Transformed PC Coefficients (120min Wind Pairs)

Figure 7. Same as Figure 5 except for components 9-12.



Average A–Wind PSD for 120min Wind Pairs

943 Measured winds expanded w/ 400 PC; 100 Synthetic winds w/ 400 PC

Figure 8. Mean PSD of *u* and *v* for both observed and synthetic winds. Results are based on the same data as Figure 1. The straight line is a "–2" slope for reference (see text).

Figure 9 shows the squared coherency K^2 for the A versus the B winds for observed and synthetic winds as a function of wavenumber. The complex amplitude of the A – B cross-spectrum was calculated and averaged over five frequencies with a running average. The squared modulus of the phase-averaged cross-spectra was normalized by the product of the amplitudes of the A and B autospectra to obtain K^2. The value of K^2 is a measure of the A – B correlations as a function of wavenumber. One expects a high degree of correlation for small wavenumbers since the longer-scale components change comparatively slowly in time. On the other hand, one expects the small-scale large wavenumber features to decorrelate rapidly. This is reflected in K^2. The synthetic winds calculated using PCA give values of K^2 that are in excellent agreement with the observed winds.

Figure 10 shows K^2 based on the u - v cross-spectrum for the A and B winds separately. Again there is excellent agreement between the observed and synthetic values. The GRAM perturbations would give essentially zero correlation at all wavelengths.

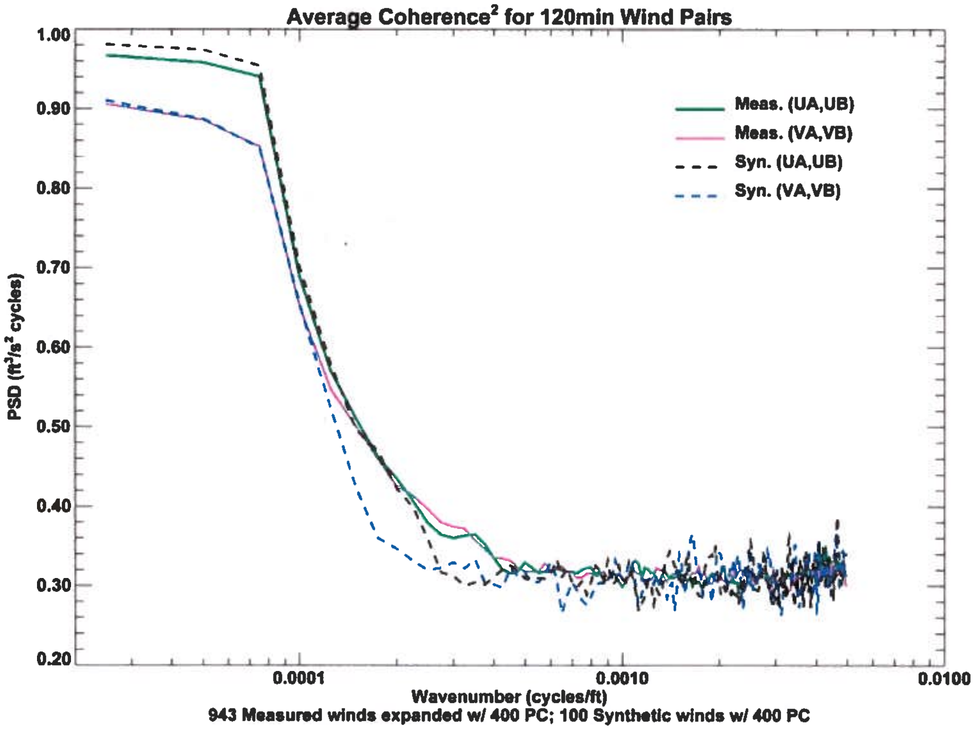


Figure 9. Squared coherency K^2 for the A versus the B winds for observed and synthetic winds as a function of wavenumber. Results are based on the same data as Figure 1.

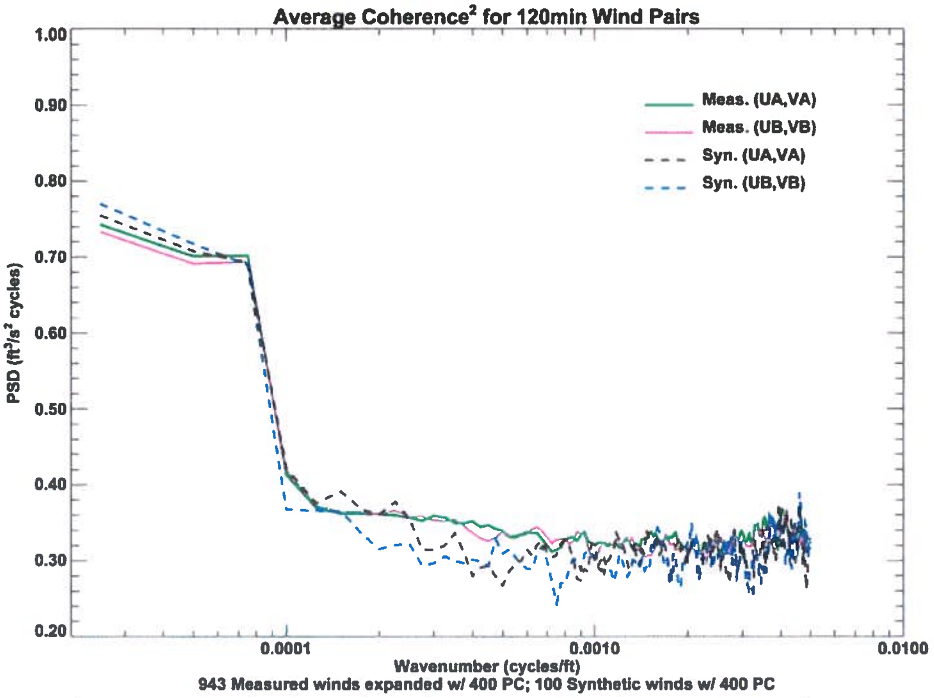


Figure 10. Squared coherency K^2 for the u versus v components for observed and synthetic winds as a function of wavenumber. Results are based on the same data as Figure 1.

3. Summary and Recommendations

The approach for calculating winds used in the GRAM cannot readily be adapted for the calculation of synthetic wind pairs when one desires realistic correlation between pairs and between wind components and when one desires realistic spectra (PSDs). We have explored an alternate approach based on PCA which preserves all relevant properties of wind pairs. Implementation in the GRAM requires a set of principal component vectors (shapes) for various time separations and a simple routine to calculate coefficients based on draws from a normal distribution.

For a full implementation of this approach the following work should be done:

1. Determine the minimum set of PCVs required ($r = 400$, is probably much larger than necessary).

2. Perform a PCA for various time separations out to 24 hours.

3. Repeat the analysis for the Western Range and Wallops Island.

4. Perform simulations of the temporal effects on rocket-impact-point dispersion using synthetic winds and compare the results to results based on observed winds.

5. Implement the code in the GRAM model and perform sample simulations.

4. Nomenclature

a_r	expansion coefficients for X
$\bar{\psi}$	GRAM output, deterministic monthly mean component
ψ'	GRAM output, stochastic perturbation component
K	number of levels
N	number of profiles
Q	quantile
t_r	transformed coefficients
u'	north-south perturbation wind components
v'	east-west perturbation wind components
$\hat{\boldsymbol{w}}$	augmented synthetic wind
X	data matrix for perturbation segment

5. Abbreviations and Acronyms

COTR	Contracting Officer's Technical Representative
DOT	Department of Transportation
FAA	Federal Aviation Administration
FAA/AST	FAA Office of Commercial Space Transportation
GRAM	Global Reference Atmosphere Model
PCA	Principal Component Analysis
PCV	Principal Component Vector
PSD	Power Spectral Density

6. References

Jolliffe, I. T. (2002), *Principal Component Analysis*, second edition, Springer-Verlag, New York, 487 pp.

Justus, C. G., F. N. Alyea and D. M. Cunnold (1986), *Improvements in the Global Reference Atmosphere Model,* Final Report on USRA Project P5042-0AO, Government Prime: NAS8-36400/1, 49 pp.

Vandaele, W. (1983), *Applied Time Series and Box-Jenkins Models,* Academic Press, 417 pp.

Appendix. The Global Reference Atmosphere Reference Model (GRAM) Approach to Synthetic Winds

We review the generation of synthetic winds within the present GRAM framework [Justus et al. 1986]. Presently GRAM produces winds using a first order autoregressive technique as follows

$$u'(\boldsymbol{x}_b, t) = R_X u'(\boldsymbol{x}_a, t) + < u'^2 >^{1/2} (1 - R_X^2)^{1/2} r \tag{A1}$$

Where

$$R_X(\Delta \boldsymbol{x}_{ab}(t)) = \frac{<u'(\boldsymbol{x}_a, t) u'(\boldsymbol{x}_a + \Delta \boldsymbol{x}_{ab}, t)>}{<u'^2>} \tag{A2}$$

and where $\boldsymbol{x} = (x, y, z)$, u is the east-west scalar component of the wind (positive eastward), a and b denote successive values of $\boldsymbol{x}$ separate by $\Delta\boldsymbol{x}$, and t is time. The quantities $x, y, and\ z$ denote east-west, north-south and vertical coordinates, respectively. The quantity r is a random number drawn from a normal distribution with zero mean and unit standard deviation. The primes denote departures from the respective means (perturbations). Similar expressions apply for the north-south component of the wind v

Equation (1) represents a prediction of $u'(\boldsymbol{x}_b, t)$ in terms of a simple first-order autoregressive Markov model [Vandaele 1983]. A convenient formula for R_X, which gives predictions whose variances are independent of $\Delta\boldsymbol{x}$, is

$$R_Z = \exp\left(-\frac{|\Delta z|}{L_z}\right) \tag{A3}$$

where Δz has replaced $\Delta\boldsymbol{x}$ since the goal is to produce vertical profiles. The quantity L_z is the integral scale of R_Z and may be interpreted as a correlation scale. The GRAM model has two correlation scales. One is for large-scale perturbations attributable to synoptic scale disturbances or planetary waves. The other is for small-scale perturbations attributable to mesoscale disturbances, gravity waves, and turbulence. The former should have little effect on differences between profiles separated by a few hours.

Correlations: The GRAM approach gives wind profiles with the correct variances and means. However, because each realization is seeded differently and because the random variable r is different for each realization, each realization of $\boldsymbol{u}'$ is uncorrelated with another. Similarly, the wind components are each independent realizations and are uncorrelated, even for the same realization of $\boldsymbol{u}' = (u', v')$.

GRAM Spatial Spectrum: The vertical wave number power spectrum generated by (3) gives spectral slopes of –2 for larger wavenumbers. Observations give slopes of from ~ –2.5 to –3. Thus with the same variance the GRAM formulation puts too little power at larger wavenumbers.

We examined the possibility of adapting the approach given in (A1) to (A3) to satisfy all of the constraints relating to correlations and spectra. It is possible to generate a second profile from an initial realization by substituting time for space in (A1) – (A3), but this leaves the spatial spectra for the second profile relatively unconstrained. The lack of correlation between the u' and v' components for a given profile can be remedied by calculating the v' profile given u', say, using the joint probability of v' given u' but these joint probabilities (if known) are not necessarily consistent with (A1 – A3) and do not guarantee that the end result would have realistic spectra.

It was concluded that the present GRAM method for constructing wind perturbations cannot be adapted for constructing synthetic wind profiles with the desired properties.

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