GPS/GLONASS RAIM Augmentation to WAAS for CAT I Precision Approach

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Biographies

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Abstract

This paper deals with the potential use of Receiver Autonomous Integrity Monitoring (RAIM) to supplement the FAA's Wide Area Augmentation System (WAAS). Integrity refers to the capability of a navigation or landing system to provide a timely warning when the system no longer meets specifications and should not be used. The focus of this paper is on Category I precision approach where WAAS is the primary source for the integrity function and RAIM, if used, will serve as an augmentation. RAIM can protect against local error sources (tropospheric and ionospheric effects, and possible interference) not observable to the WAAS. RAIM will not, however, be required for an approach to be made. If RAIM is available and indicates that there is an integrity problem, the approach will not be made.

Since RAIM serves as a back-up to WAAS, the missed alert probability requirement can be relaxed significantly. The analysis in this paper is an extension of work presented earlier this year [1] which demonstrated that RAIM is available for CAT I precision approach less than 80% of the time with a constellation of 24 GPS satellites and three geostationary satellites. This study examines the improvement in RAIM availability for CAT I if GLONASS satellites are incorporated into the solution in addition to GPS and WAAS satellites. RAIM detection availability results are presented as a function of missed alert rate for various combinations of GPS, GLONASS and WAAS geostationary ranging satellites.

Previous RAIM studies have been based upon the explicitly stated assumption that only one satellite will fail at any given time. Hence RAIM need only contend with single-satellite failures when operating outside of WAAS coverage. When RAIM is applied to differentially-corrected pseudoranges, however, the single-failure assumption is no longer valid. In this case, the notion of a failing satellite gives way to the notion of the inability of the WAAS corrections to suitably correct locally-measured pseudoranges. Reasons for this include the possible presence of local propagation effects not observable to WAAS e.g. local tropospheric or ionospheric effects or multipath. This paper addresses the situation where multiple differentially-corrected pseudoranges are corrupted.

1. Introduction

RAIM may be used as a backup to WAAS during normal operations. In this application, RAIM performs a consistency check on the WAAS differentially-corrected pseudoranges. RAIM can protect against local error sources (tropospheric and ionospheric effects, and possible interference), not observable to the WAAS. RAIM will not, however, be required for an approach to be made. If RAIM is available and indicates that there is an integrity problem, the approach will not be made. A paper presented earlier this year [1] demonstrated that RAIM availability at a 10^{-3} missed alert level only reaches about 80% for CAT I precision approach with 24 operational GPS satellites, even when ranging measurements from three Inmarsat geostationary satellites are included.

The purpose of this study is to examine the improvement in RAIM availability if the Russian GLONASS satellites also are incorporated. This analysis makes the assumption that the WAAS reference sites include GPS/GLONASS receivers and the geostationary satellites broadcast differential corrections for both GPS and GLONASS satellites. The authors acknowledge this concept currently is not planned for inclusion in the WAAS architecture.

The availability of RAIM as a backup to WAAS can be expressed in two different ways. In the conventional method, one sets the probability of false alert and the probability of missed alert at fixed values, allowing the protection level to vary with geometry. This approach is consistent with previous work on RAIM. It answers the question: how often can RAIM backup WAAS by providing a specific missed alert probability (e.g. 10^{-3} or better) for a specified alert limit? Moreover, because the horizontal and vertical protection levels vary as a function of satellite geometry, they tell us how far down the tunnel an aircraft can proceed with a specified level of added integrity when the tunnel concept is used.

A second method of computing availability also is of interest. In this case, the false alert rate and the protection level are held fixed, and the probability of missed alert is allowed to vary with geometry. This answers a somewhat different question, namely how much additional RAIM integrity can be provided? This is useful information from an operational standpoint if RAIM is to serve as a back-up to WAAS.

The results contained in this paper use the second method where the missed alert rate varies with geometry for a fixed vertical alert limit (VAL) of 18.3 meters. The 18.3 meter VAL corresponds to a decision height of 200 feet and the full-scale sensitivity of 50 feet given in the WAAS MOPS [2].

The second part of this paper addresses the potential inability of the WAAS corrections to suitably correct locally-measured pseudoranges. Almost all of the previously reported work on RAIM is based upon the explicitly stated assumption of a single satellite failure. That does not imply that no other satellites will be out of service, only that no more than one soft failure (e.g. excessive satellite clock drift), will occur at any given time. The difficulty here, is that with two or more failures, errors may tend to cancel in the test statistic, while at the same time, combining in the solution space to produce a large navigation error. This problem has yet to be addressed by conventional RAIM algorithms.

2. RAIM Algorithm

Unlike the nonprecision approach case where all satellites are assumed to have the same measurement uncertainty (33 meters), WAAS differentially-corrected signals have much smaller measurement uncertainties that are unique to each satellite. Also, local anomalies (which RAIM is intended to detect), may affect more than one satellite at the same time.

The problem of unequal measurement uncertainties is handled in the RAIM algorithm through the use of a weighting matrix. Satellites with large measurement uncertainties (typically the lower elevation satellites) are given less weight, while satellites with small measurement uncertainties receive greater weight. Measurement weighting is described in [3] and in Appendix E of the WAAS MOPS [2].

WAAS corrected pseudoranges are assumed to be weighted in accordance with Appendix E of the WAAS MOPS. Hence, the post-weighting noise variance for each satellite has a value of unity. A slowly developing bias error is placed on the most-difficult-to-detect satellite which corresponds to the maximum slope in the (horizontal or vertical) position error vs. test statistic diagram, as shown in Figure 1.

A missed alert occurs when two independent events happen simultaneously. They are: a) that the test statistic, which is formed by the avionics, is less than the detection threshold, and b) that the navigation error (not observable to the avionics) is larger than the protection level. Recall that RTCA requires not only that the bias error be placed on the worst-case satellite, but also that its magnitude maximize the miss rate (worst-case bias). Protection levels depend upon the worst-case bias which varies with satellite geometry. The avionics must therefore have a computationally efficient method for determining the worst-case bias and the corresponding protection level.

The following probabilities (*on a per sample basis*) are of interest:

(a) Pr1 ≡ Pr{test statistic < detection threshold}

(b) Pr2 ≡ Pr{estimation position error > trial protection level}

(c) $Pr_{MISSED\ ALERT}$ ≡ Pr{'test statistic < detection threshold' and 'position error > trial protection level'}

Probabilities Pr1 and Pr2 describe events which take place in different mathematical spaces. Pr1 is most easily described in parity space (observation space) where the parity vector is a measure of the consistency of the measurements. Pr2, on the other hand, is described in the solution or navigation space. This space involves the navigation error which is not observable to the receiver.

The probability of a missed alert per sample is the joint probability of events (a) and (b) happening together. This is denoted by the cross-hatched region in the upper left quadrant shown in Figure 1.

$$Pr_{MISSED\ ALERT} = Pr1 \bullet Pr2$$

R.G. Brown [4] has shown that for certain conditions (likely to apply to the RAIM problem) the equation ($Pr_{MISSED\ ALERT}$ = Pr1•Pr2) is exactly true. His proof also holds for multiple failures.

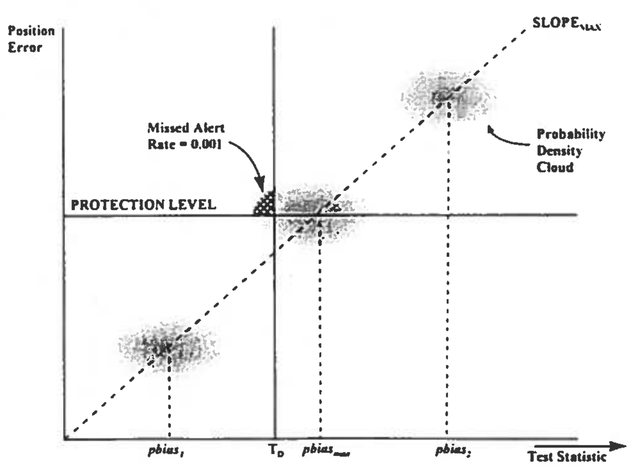

Figure 1 Position Estimation Error vs. Test Statistic

2.1 Parity Space

Parity space concepts have been applied to RAIM for detection and isolation/exclusion of a faulty satellite [5]. The basic idea of parity space is as follows. For the overdetermined case where there are more measurements than unknowns, it possible to transform the redundant information contained in the extra measurements into a vector contained in an subspace of N-4 dimensions. Figure 2 shows an example for a six-satellite (N=6) geometry. Because only four measurements are required to solve for the four unknowns (x,y,z,t), the corresponding parity space has two dimensions (i.e. the problem has two degrees of freedom).

The transformation of the measurement vector into the parity vector has a unique property. If the measurement noise components are independent zero-mean Gaussian random variables $N(0,\sigma^2)$ (the usual assumption) then the noise in parity space will be spherically symmetric with independent components $N(0,\sigma^2)$. This is shown in Figure 2. Note that as the deterministic component of the parity vector P_{DET} increases, the fraction of the data points which fall within the decision circle decreases. Consequently, the probability of no detection is a monotonically decreasing function of *pbias* as shown in Figure 3.

Another property of the parity vector is that its magnitude is equal to the square root of the sum of the squares of the measurement residuals, a measure often used for RAIM detection. The term *pbias* (Figure 1) denotes the deterministic (no-noise) biased component of the detection test statistic. As the range bias (*rbias)* and concurrently *pbias* increase in time, the Pr{no detection} decreases and the Pr{position error > protection level} increases. The product $Pr_{MISSED\ ALERT}$ increases to a peak value and then decreases. See Figures 3 and 4.

The problem remaining is to develop a suitable method by which the two probabilities Pr1 and Pr2 can be calculated efficiently to the desired accuracy. This is discussed in the following paragraphs.

The probability of a missed alert reaches a maximum for a critical value called $pbias_{max}$. Because we do not have a simple analytical expression for the curve shown in Figure 4, it is necessary to compute $pbias_{max}$ for a specific missed alert probability (e.g.10^{-3}) through an iterative procedure. A limited study at the Volpe Center using a modified binary search routine obtained convergence within two to three iterations. Computation of the detection threshold T_D and the critical bias value $pbias_B$ is discussed in Section 2.2.

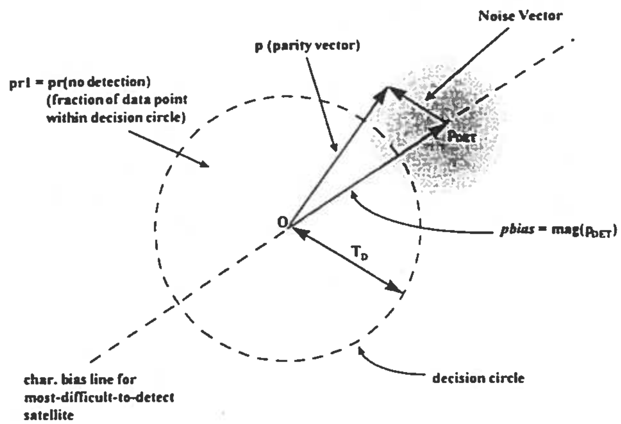

Figure 2 Illustration of Parity Vector Relations and Decision Circle – Example Parity Space for 2DOF

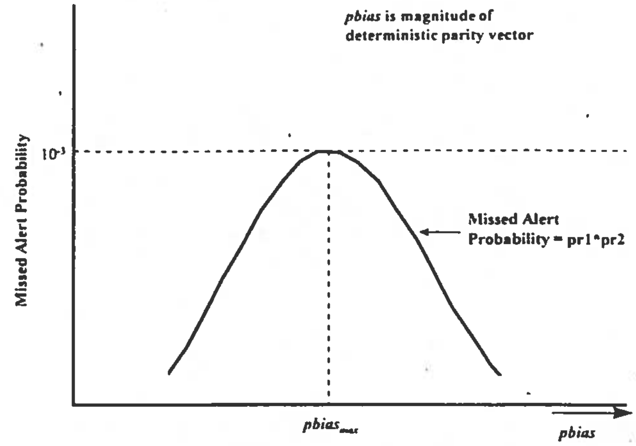

Figure 4 Missed Alert Probability vs. *pbias*

Accuracy of Computation of Pr1 and Pr2

Pr1 has a chi or chi-squared distribution depending on whether we use the magnitude of the parity vector or its square. Since there is a one-to-one relationship between the two, either can be used. As a matter of convenience, we choose to use the magnitude squared and the corresponding chi-squared distribution.

Pr2 is described by the bivariate Gaussian distribution for the two-dimensional (horizontal) case and by the univariate Gaussian distribution (error function) for the vertical case, in which case no approximation is involved. The error function was found to be quite accurate for the horizontal cases examined and is considered suitable for computation of Pr2.

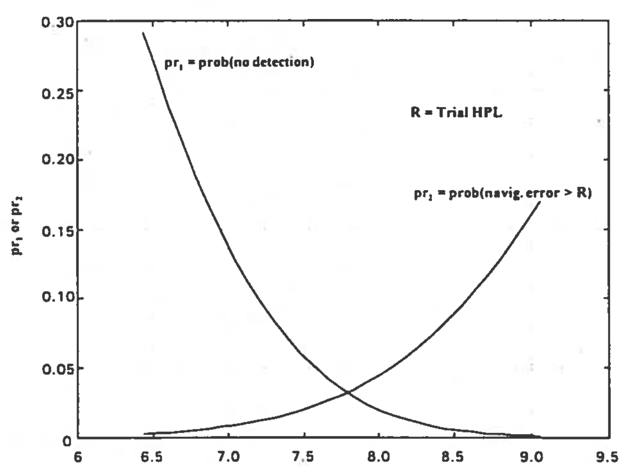

Figure 3 Probability of No Detection and Probability of Error Greater Than Trial HPL

The accuracy of Pr1 can be improved by using the chi-squared distribution. Unfortunately, the noncentral chi-squared density function cannot be written is closed form. It involves the Bessel function which is represented by an infinite series. R.G. Brown [6] has, however, developed an accurate 50-term approximation suitable for RAIM. This 50-term approximation is used in this paper.

2.2 Computation of Normalized Threshold T_D and Critical Bias $pbias_B$

A Category I precision approach is assumed to last for 150 seconds. A WAAS update rate of six seconds gives 25 statistically independent samples per approach. Using a false alert rate on $4.48 * 10^{-5}$ per approach gives a false alert rate of $1.76 * 10^{-6}$ per sample. The normalized detection threshold T_D is found by integrating the central chi-squared density function from zero to $(T_D)^2$. Because the test statistic depends only on the number of satellites in view and not on specific user-satellite geometry, the detection threshold T_D can be pre-computed and stored in a table.

Similarly, when the probability of no detection Pr1 (left half plane in Figure 1) is set to a specific value (e.g. 10^{-3}), the corresponding bias ($pbias_B$) is found by integrating the non-central chi-squared density function. Again, it can be shown that the critical bias $pbias_B$ (corresponding to a specific probability of no detection) is independent of satellite geometry and depends only on the number of satellites in view. Table 1 contains the normalized threshold T_D and $pbias_B$. Values are given for up to 24 satellites to account for a combined GPS/GLONASS constellation In general, the noncentrality parameter (λ) is equal to the normalized bias ($pbias$) squared [6]. For example, for six satellites in view, $\lambda_B = (8.1609)^2 = 66.60$.

Table 1 Normalized Threshold for Probability(false alert) = 1.76e-6, and $pbias_B$ for Probability (no detection) = 0.001

No. of Visible Satellites	Degrees of Freedom	Normalized Threshold T_D (dimensionless) for Pr_{FA} = 1.76e-6 per sample	Chi-Square Non-Central Parameter λ_B (dimensionless) for Pr(no detection) = 0.001	Normalized $pbias_B$ (dimensionless) for Pr(no detection) = 0.001
5	1	4.779	NA	7.8600
6	2	5.148	66.60	8.1609
7	3	5.431	70.00	8.3666
8	4	5.672	73.00	8.5440
9	5	5.887	75.60	8.6948
10	6	6.083	78.00	8.8318
11	7	6.264	80.00	8.9443
12	8	6.434	82.00	9.0554
13	9	6.593	84.05	9.1679
14	10	6.746	85.85	9.2655
15	11	6.901	87.75	9.3675
16	12	7.031	89.23	9.4460
17	13	7.164	90.79	9.5285
18	14	7.300	92.43	9.6143
19	15	7.419	93.76	9.6829
20	16	7.540	95.16	9.7552
21	17	7.658	96.53	9.8252
22	18	7.772	97.86	9.8924
23	19	7.884	99.15	9.9574
24	20	7.992	100.41	10.0204

2.3 Computation of Protection Level

Computation of the protection level for the single-failure case (fixed VAL and variable missed alert rate) is discussed in [1,7]. It involves an iterative procedure wherein the missed alert probability is computed as the product of two constituents. The first is the probability that the test statistic is less than the detection threshold (a classical missed detection) and the second is that the navigation error exceeds the VAL. The procedure involves setting the value of the satellite bias error to a succession of values and computing the missed alert probability. The value which gives the largest probability of a missed alert is selected as the worst case bias. In the single-failure case, the bias is placed on the satellite which has the maximum slope in the navigation error vs. test statistic diagram.

Recent studies use analytic methods to set protection levels. These methods tend to be less sensitive to changes in parameter values and are less computationally demanding. The analytical method described in this paper is a refinement of that described in [8].

3. Availability Analysis Parameters

This section describes the analysis parameters such as the satellite constellations, mask angle, and coverage grid used to conduct this availability study

3.1 Satellite Constellations

RAIM availability is computed for GPS only (differential corrections, but no ranging on WAAS satellites) and for various combinations of WAAS, GPS and GLONASS satellites. This study uses the GPS Optimized 24 satellite constellation in Appendix B of the WAAS MOPS which consists of 24 GPS Block II satellites. GLONASS satellites are selected from a 24-satellite GLONASS constellation which has been used for analysis by RTCA and will be included in Appendix P in Change 2 to the WAAS MOPS. Up to three GPS and three GLONASS satellites are taken out of service to examine the impact on availability with a reduced satellite constellation. The GPS satellites taken out of service for GPS are those providing a worst-case PDOP, based on a study by the Aerospace Corporation [9]. It should be noted that these satellites are not necessarily the ones which will have the worst impact on integrity. Since a similar study did not exist for GLONASS, the satellites chosen were based on past studies performed at the Volpe Center [10].

The three Inmarsat-3 satellites visible from the CONUS are: Atlantic Ocean Region East (AORE) at longitude 15.5° W, Atlantic Ocean Region West (AORW) at longitude 55.5° West, and Pacific Ocean Region (POR) at longitude 180°. Two Inmarsat satellites are visible from the east and west coasts of CONUS. There is, however, a strip in the center of the country where only one Inmarsat satellite is visible and redundant WAAS coverage is lost. A fourth geosynchronous satellite is added at 100 degrees west to fill this gap. At this point in time only two of the Inmarsat-3 satellites are committed to the WAAS system (AORE will be part of the EGNOS system). RAIM availability is computed for various numbers of geosynchronous satellites to examine their impact on availability. A five-degree elevation mask is used for all satellites and satellite measurements are weighted according to elevation angle.

3.2 Satellite Selection

A decision needed to be made in this analysis concerning the number of channels to be devoted to GPS/WAAS and to GLONASS, and also the method of satellite selection when there were more visible satellites than available channels. The method chosen treats the GPS and WAAS satellites as one group whose function is to provide the best vertical accuracy under the constraint that two WAAS satellites are to be used if visible. GLONASS satellites are selected to provide for RAIM integrity. This was accomplished by choosing the GLONASS satellites as the ones closest to the GPS satellites, i.e. minimum angle between the ray paths from the user to the satellites. The number of channels in the receivers were varied to evaluate the impact on availability: 12, 14, 16, and 24 channel receivers were considered. The satellite selection for these receivers is as follows: 12 channels (8 GPS/WAAS and 4 GLONASS), 14 channels (8 GPS/WAAS and 6 GLONASS), 16 channels (8/WAAS GPS and 8 GLONASS), and 24 channels (12 GPS/WAAS and 12 GLONASS).

3.3 Analysis Grid

Computations are made for each space-time point in a CONUS grid extending from 25 to 50 degrees north latitude and from 65 to 125 degrees west longitude. A grid spacing of five degrees is used for both latitude and longitude. Data is computed for each grid point every 30 minutes over a 24 hour day. This gives a total of 3822 space-time points (geometries).

4. Availability Results

This section discusses with the availability of RAIM with WAAS and GLONASS satellites at a variable missed alert rate, a VAL of 18.3 meters, and an HAL of 48 meters [11]. This analysis is done for various combinations of GPS, GLONASS, and WAAS assuming an all-in-view receiver. This analysis was performed under a single-failure assumption and although this assumption cannot be justified for a worst-case analysis, the problem does not demand worst-case analysis. RAIM applied to the WAAS differentially-corrected pseudoranges only serves as a backup to WAAS.

At the present time there is not an agreed-upon failure model for the WAAS differential corrections. Such a model will be heavily dependent upon the assumptions made for local effects including ionospheric and tropospheric gradients, multipath, and possible interference. For example, if the ray path for a single satellite were to pass through a portion of the ionosphere which were not well modeled by WAAS, then the single-failure analysis would apply. If, on the other hand, two or more ray paths experience propagation delays not well modeled by WAAS, then the single-failure assumption no longer applies. That subject is the discussed later on in this paper.

The availability of CAT I RAIM in the horizontal dimension (HPL<HAL) was found to be 100% with a combined GPS, GLONASS, and three geostationary satellite constellation at a 10^{-3} missed alert rate. The only exception to this was when three satellites were removed from the GPS and GLONASS constellations due to the fact that fewer than four GPS satellites were visible at some locations. The resulting availability in the horizontal dimension was then 99.76%.

The results for the vertical dimension (VPL<VAL) for all combinations of satellites considered are given in Table 2. The increase in the number of channels does improve availability, essentially providing 100% availability for the 24 and 23 GPS/GLONASS and three geostationary satellite cases. However, there was no improvement in availability for more than 16 channels. The sensitivity to which satellites are taken out of service (OTS) is apparent in this table. Note that the 21 GPS + 21 GLONASS case provides higher availability than 22 satellites in each constellation, which is due to the choice of the two GPS satellites to be taken out of service.

Table 2 Availability of GPS, GLONASS, and 3 Geostationary SVs (VPL<VAL) at Pr(miss)=10^{-3}

Satellite Constellation	12 Channels	14 Channels	16 Channels	24 Channels
24 GPS + 24 GLONASS	99.56%	99.97%	100%	100%
23 GPS (SV C1 OTS) + 23 GLONASS (SV 1 OTS)	98.82%	99.71%	99.90%	99.90%
23 GPS (SV C1 OTS) + 23 GLONASS (SV 15 OTS)	98.61%	99.84%	100%	100%
23 GPS (SV C1 OTS) + 23 GLONASS (SV 22 OTS)	98.67%	99.82%	100%	100%
22 GPS (SVs A2, D2 OTS) + 22 GLONASS (SVs 1, 15 OTS)	95.47%	99.08%	99.61%	99.61%
22 GPS (SVs A2, D2) + 22 GLONASS (SVs 1, 22 OTS)	95.68%	99.08%	99.58%	99.56%
22 GPS (SVs A2, D2 OTS) + 22 GLONASS (SVs 15, 22 OTS)	94.71%	99.14%	99.76%	99.76%
21 GPS (SVs B1, C2, C3 OTS) + 21 GLONASS (SVs 1, 15, 22 OTS)	97.17%	98.48%	99.29%	99.29%

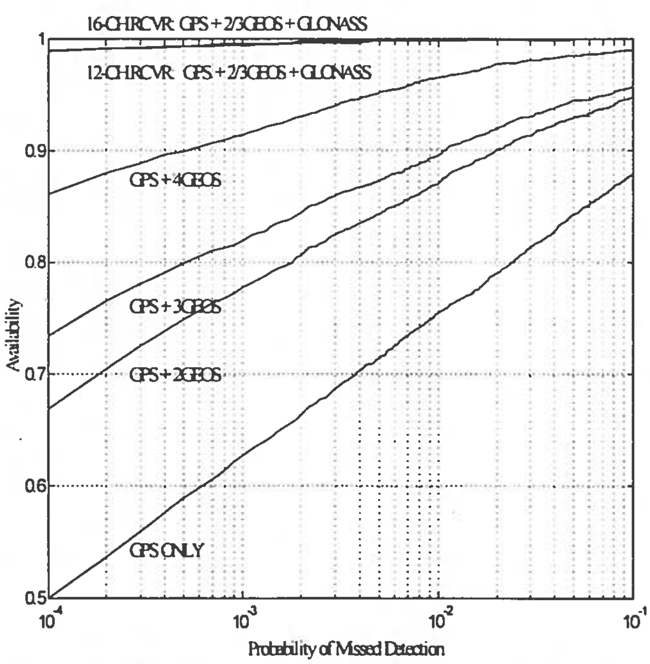

Figure 5 Availability of Protection at VAL=18.3 meters Over CONUS Grid

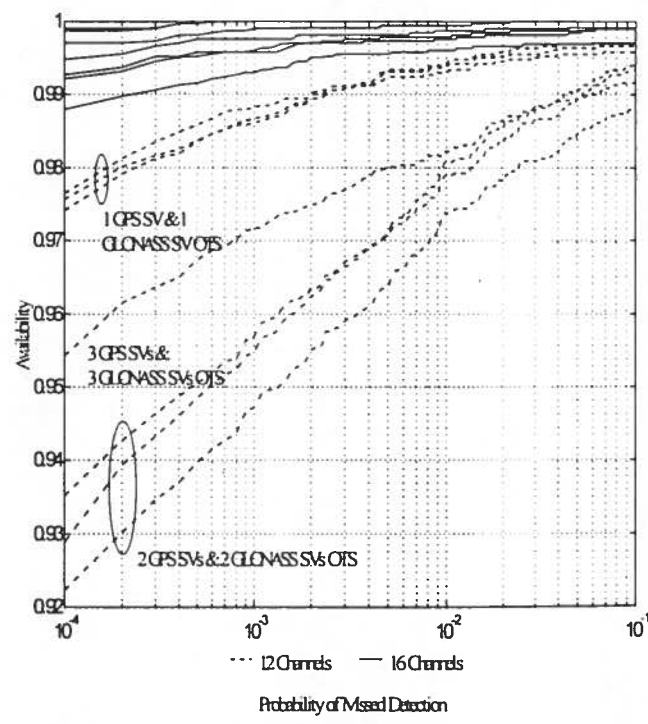

Figure 6 Availability of Protection at VAL=18.3 meters Over CONUS Grid

Figures 7 depict where the outages occur and their duration for CAT I RAIM with 24 GPS satellites and three geostationary satellites using a 12 channel receiver and 10^{-3} missed alert rate. Figure 8 is the same plot, but with GLONASS satellites incorporated as well.

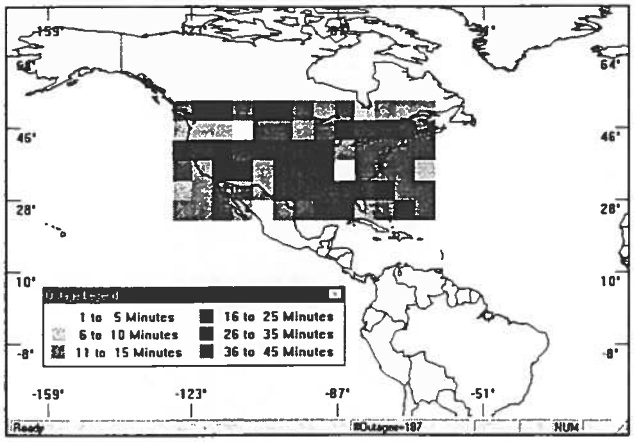

Figure 7 CAT I RAIM Availability with 24 GPS SVs and 3 Geos (12 Channels)

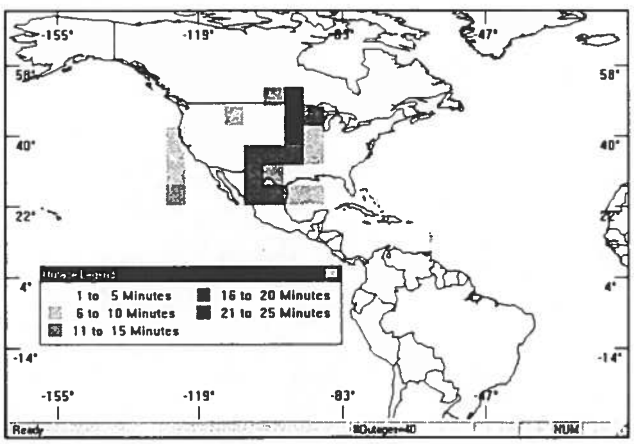

Figure 8 CAT I RAIM Availability with 24 GPS SVs, 24 GLONASS and 3 Geos (12 Channels)

5. Single and Multiple Satellite Failures

Previous RAIM studies are based upon the explicitly stated assumption that only one satellite will fail at any given time. Hence RAIM only needed to contend with single-satellite failures when operating outside of WAAS coverage. When RAIM is applied to differentially-corrected pseudoranges, however, the single-failure assumption is no longer valid. In this case, the notion of a failing satellite gives way to the notion of the inability of the WAAS corrections to suitably correct locally-measured pseudoranges. Reasons for this include the possible presence of local propagation effects not observable to WAAS e.g. local tropospheric or ionospheric effects or multipath.

In the single-failure case, if we neglect noise, there is a linear relationship between the bias on a single satellite and both the test statistic and the navigation position error. The position error varies linearly with the test statistic. In the multiple-failure case (again neglecting noise) the problem is still linear, but the relationship between the test statistic and the position error depends on which pseudoranges contain the errors, there relative magnitudes, and the relative signs of those errors. Certain situations lead to bias cancellation in the test statistic and enhancement (constructive interference) in the position error.

This paper covers two limiting cases of multiple failures. The first case is for two failures where the failed satellites and their associated biases are chosen to minimize coupling into the test statistic, while maximizing coupling into the position error. The second case includes failures on all satellites in view. In this case, the failures are modeled by an enhanced noise level on all of the satellites. Here, zero mean noise replaces the biases used in the two-failure case.

5.1 Multiple Failures

As previously discussed, the single-failure assumption cannot be strictly justified when RAIM is applied to WAAS-corrected pseudoranges. Consideration must be given to the possibility that the measurement biases will tend to cancel in the test statistic, while reinforcing in the position estimate, giving a high missed alert rate. This corresponds to a steep slope in the position error vs. test statistic diagram.

In the single-failure case, one is concerned with a measurement bias such as might be caused by a drifting satellite clock. Errors of this type are corrected by WAAS. The need for RAIM is based on the possibility of measurement errors resulting from local propagation effects not modeled by WAAS. For example, ionospheric and tropospheric propagation delays, multipath and possible interference may affect more than a single measurement. These effects are difficult to quantify, and the number that might be affecting the measurements at any given time is uncertain. Consideration of all possible combinations is inconsistent with the backup role of RAIM over WAAS. Therefore, the two-failure case and the all-in-view failure case have been selected for study. The two-failure case might represent a situation where ray paths to two satellites pass through an inhomogeneity in the ionosphere not fully modeled by WAAS. The all-in-view failure case is intended to represent a situation where the measurements to several satellites are corrupted. This situation is modeled by increasing the noise level on all of the measurements. This represents a practical solution to the problem when more than two measurements are faulty. Data are not yet available to support a more complex measurement error model.

Analysis shows that in the worst case whether there is zero detectability on the one hand or possibility of detection on the other hand depends on the number of failures (n_F) in relation to the number of satellites (n) that can be used for navigation. For the n-satellite geometries, the 1, 2, 3 up to (n-4) failures cases will always have a nonzero test statistic ($\mathbf{f}^T\mathbf{f}$ or equivalently $\mathbf{p}^T\mathbf{p}$) so that there is possibility of detection. Given n-satellite geometries, for the (n-3), (n-2), (n-1), to n failures cases, there are worst-case magnitudes of the range biases leading to a zero value test statistic; the geometric interpretation is that there are worst-case sets of range bias magnitudes such that the range error vector lies within the column space of the linear connection matrix **G**. For example, with eight satellites in-view, 8, 7, 6, 5 failures in the worst cases will leads to zero-detectability (that is, no possibility of detection) while 1, 2, 3, 4 failures will lead to possibility of detection.

5.2 Two Simultaneous Failures

Partial Cancellation of Characteristic Vectors in Parity Space

Deterministic Case with Noise Omitted

One considers failures occurring at the same time on two GPS satellites in the case of weighted Category I precision approach. Let there be a geometry with n satellites in view. In the corresponding parity space (of n-4 dimensions) there would be n characteristic lines and among these there are two particular characteristic lines corresponding to the two failed satellites. A poor situation exists when the angle between these two characteristic lines is small and the two component parity bias vectors are comparable in magnitude and directed in opposite directions. Consider the case with noise omitted. The magnitude of the resultant parity vector may be small and there may be no detection even though the two component bias vectors may be large. Denote by pbias the magnitude of the deterministic parity vector -- where noise is omitted. The receiver can actually only observe the resultant parity vector (and resultant pbias) and is not able to observe the component parity vectors along the characteristic bias lines. The component parity vectors along the characteristic bias lines, however, are presented in the theoretical analysis.

Two Failures Corresponding to the Minimum Angle Between Pairs of Characteristic Lines.

In the computer simulation, one obtains for each geometry the pair of characteristic lines with the smallest angle. For n satellites in view, there are n characteristic lines in a parity space of (n-4) dimensions. For two failures, there are K combinations: K=(n things taken two at a time)=n!/[(n-2)!2!]. The 3822 geometries are sorted in ascending order by the (minimum) angles. Twenty percent of the geometries with the smallest minimum angles are extracted. Then, fifty geometries from this subset are selected to obtain a representative range of angles.

The weighted range bias $rbias_{mI}$ is equal to the unweighted range bias $rbias_I$ divided by the noise standard deviation σ_I . For a given geometry, let the two faulted satellites with the smallest angle be labelled I and J. The weighted parity biases along the characteristic lines of the Ith and Jth satellites are assigned the value unity ($pbias_{m\,I} = pbias_{m\,J} = 1$). For a given geometry, one can obtain the corresponding range biases $rbias_{mI}$ and $rbias_{mJ}$. For example, the weighted range bias on satellite I is given by

$$rbias_{mI} = pbias_{mI} \,/\, \text{norm (Ith column of the Weighted Parity Transformation Matrix)}$$

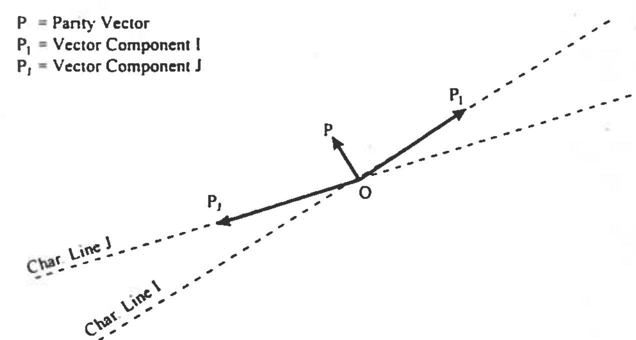

Figure 9 Characteristic Bias Vectors in Parity Space

Figure 9 Illustrates a situation where characteristic vectors in nearly opposite directions partially cancel each another so that the resultant parity vector is small in magnitude.

The data from a set of 50 satellite geometries used in previous work show that as the angle increases from 5.5 degrees to 31.6 degrees the magnitude of the resultant weighted parity vector (in n-4 dimensional parity space) increases from around 0.1 to about 0.4. Table 3 gives computed data for seven representive cases extracted from the 50 geometries. The trend of the data from these seven geometries illustrates the main trend of the larger 50 samples.

The parity matrix **P** is a function of the linear connection matrix **G**. From nx1 range error vector ε with the two range biases, one obtains the vertical position error and also the test statistic |**p**|. The parity vector **p** is obtained from the following equation.

$$\mathbf{p} = \mathbf{P}\,\varepsilon.$$

We define VSLOPE as the ratio of the vertical error to |**p**|. This is given in column 5 of Table 3. Note that the vertical slope VSLOPE decreases with the magnitude of the angle. The worst cases for detection (largest slopes) occur for the smallest angles.

TABLE 3 Representative Cases Showing Slopes for Vertical Case After a Satellite with a Bias is Omitted from the Set Used For Navigation

1 Geometry Number	2 No. of Sats. In View (n)	3 Minimum Angle In Deg. Between Biased Char. Lines In Parity Space	4 Original n Sat. In View, Two Biases (On Sat. I & J)	5 Original n Sat. In View, Two Biases (On Sat. I & J) VSLOPE	6 Reduced (n -1) Sat. In View (Omit Sat. I, One Bias Remains) VSLOPE	7 Reduced (n-1) Sat. In View (Omit Sat. J, One Bias Remains) VSLOPE
1951	8	5.5	(I=1, J=2)	24.94	24.93	24.94
4215	8	8.8	(I=1, J=4)	21.76	21.69	21.72
4213	9	12.7	(I=1, J=4)	14.77	14.64	14.73
4661	8	18.4	(I=4, J=6)	9.68	9.70	9.41
1557	11	22.7	(I=7, J=9)	5.32	5.24	5.18
2201	8	27.3	(I=1, J=4)	4.47	4.47	4.59
1355	11	31.6	(I=7, J=9)	3.43	3.40	3.20

Omit One Biased Satellite: Reduced Set of n-1 Satellites (n-5 Dimensional Parity Space)

One might think that by removing one of the two biased satellite, one would detect the remaining faulted satellite. The data of the 50 samples show that one is not able to do so and that the resultant |**p**| values remain quite small. The vertical slope induced by the remaining failed satellite has almost the same value as that of the original two failure case. For example, upon examining Table 1, columns 5, 6, and 7, one finds very little change in the slopes when one omits one of the two biased satellites from the navigation set.

The normalized threshold decreases gradually with the reduction of the number of satellites. The normalized threshold TD is set for Pr(False Alert) = $1.76*10^{-6}$. The numerical values are given in the form of a row vector as follows:

Td = [0 0 0 0 4.779 5.148 5.431 5.672 5.887 6.083 6.264 6.434]

where for five satellites in view, TD = 4.779; and for six satellites in view, TD = 5.148.

Plot of Magnitude of Parity Vector Omitting One Biased Satellite.

Recall that pbias is the magnitude of the deterministic component of the parity vector. Figure 10 shows a plot of pbias vs. the angle between two characteristic bias lines in parity space for the fifty selected geometries. For each geometry, pbias of the original n satellites and the pbiases of n subsets of n-1 satellites are calculated and shown in the plot. For a given geometry there are n+1 pbias values: p_0 is for full set. p_1, p_2, p_3 · · · p_n are the cases where the subscripted number indicates the omitted satellite. A line in the plot connects the respective p_k points. Figure 10 shows that by omitting the Ith (or the Jth) biased satellite one is not able to substantially increase the resultant pbias (magnitude of the parity vector) in the reduced parity space so as to make a detection. (The normalized detection threshold decreases a little when the number of visible satellites is reduced by one.) The upper bound in the plot corresponds to p_0 (the magnitude of the parity vector) for the original full set of n satellites. One see that by omitting a satellite one is not able to obtain a resultant pbias better than that of the full set p_0.

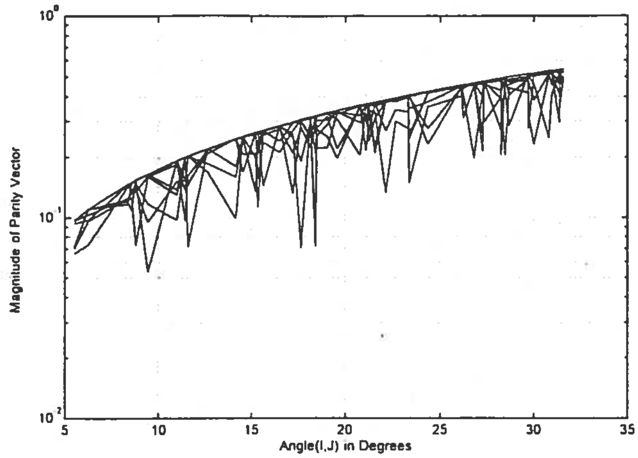

Figure 10 Magnitude of Parity Vector (No Noise) vs. Minimum Angle Between Pair of Characteristic Lines in Parity Space

Summary and Conclusions

Computation of the missed alert probability for a specific alert limit, e.g.VAL=18.3 meters, requires integration of two probability density functions. They are, the non-central chi-squared density and the uni- or bivariate normal density. Integration of these density functions may be computationally burdensome for the avionics. The chi-squared density can, however, be represented to a high accuracy by a 50-term approximation which can be integrated without too much difficulty. The bivariate normal density (needed for the horizontal case) can be represented to an adequate accuracy by the univariate normal density which is more easily integrated.

The availability of RAIM protection at a 10^{-3} missed alert rate for VAL=18.3 meters is quite low. The situation can be improved considerably by relaxing the missed alert rate to something closer to 10^{-2}. Such an increase in the missed alert rate may be tolerable because RAIM, when used in conjunction with WAAS, is only serving in a back-up role.

Inclusion of the GLONASS satellites greatly improves the availability of RAIM detection when used with WAAS. A 16-channel receiver provides essentially the same availability as does a 24-channel receiver when the missed alert rate is set to 10^{-3}. Even a 12-channel receiver provides more than 99.5% availability when full 24-satellite GPS and GLONASS constellations are used with three WAAS satellites. For all cases examined, 14 or more channels provide at least 98.4% availability with three GPS and three GLONASS satellites out of service.

Consideration should be given to the possibility of multiple failures when RAIM is used with WAAS. For example, with two failures, bias cancellation in the test statistic coupled with reenforcement in the position error can lead to very low (potentially zero) detection probabilities.

The results given in this paper are based on simplified error models. Improved models for the potential inability of WAAS to fully correct for local propagation anomalies are needed.

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