Review of Land Use Models: Theory and Application

Kazem Oryani, URS Greiner, Inc. and Britton Harris, University of Pennsylvania

Abstract

In this paper we will discuss our methodology in reviewing land use models and identifying desired attributes for recommending a model for application by the Delaware Valley Planning Commission (DVRPC). The need for land-use transportation interaction is explored, followed by a synthesis of inventory of land use models for agency use. This is followed by an overview of three operational land use models.

Details of three operational models — DRAM-EMPAL (S. H. Putman), MEPLAN (Marcial Echenique) and METROSIM (Alex Anas) — are explored in terms of model formulation and special features. In order to draw on the main factors in implementation, we conducted detailed telephone interviews with five major Metropolitan Planning Organizations (MPOs) who use land use models. These interviews were performed as part of our work for the enhancement of DVRPC's travel simulation models in 1996.

A two-step model selection and implementation process is proposed. The recommendation is that limited versions of the above three modes be acquired for prototype use, policy analysis and impact assessment. The final selection will be based on objective performance of the models using a similar battery of tests on the same data sets.

The Need for Land Use Models

This paper is based on our report for the Delaware Valley Regional Planning Commission (DVRPC) to review land use models as part of the DVRPC's on-going effort to enhance their travel simulation models. DVRPC's model enhancement effort has twelve tasks. The land use model review is the task for which we had direct responsibility.

DVRPC is the MPO serving Bucks, Chester, Delaware and Montgomery counties as well as the City of Philadelphia in the State of Pennsylvania plus Burlington, Camden, Gloucester and Mercer counties in the State of New Jersey. In total, DVRPC covers eight counties in two states and the City of Philadelphia. The commission's predecessor agency was the Penn-jersey Transportation Study during 1959-1964. DVRPC was created in 1965.

Our report for DVRPC contains Appendix B, a parallel paper written by Britton Harris for the study, which traces development of the land use model from the historical perspective with respect to theories underpinning these models.

What is the function of land use models or more precisely what does the Transportation-Land Use Interaction model do? Land use models deal with describing activities of land consuming actors and their competition for land in an urban setting. These actors are households, firms and retail establishments, each with particular requirements for space and access to jobs, schools and markets. Describing the spatial distribution of these activities at present and projecting future land uses are the main two aspect of these models. These models also consider interaction among these activities through the transportation network.

Land use transportation interaction models overcome the deficiencies in the existing traditional

four-step models. Consider the addition of a new facility in a metropolitan area. As the result of the addition of this facility, there will be some route changes, even possibly some mode switching and possibly some destination changes where travelers can satisfy, say, their shopping trip needs. What the existing transportation models are not capable of capturing is the projection of relative changes in household location and employment location of the land consuming actors.

The reason for the deficiencies of the existing traditional four-step transportation models is that land use activities considered in the trip generation phase have a fixed spatial pattern. It is known that improving transportation facilities or even anticipation of a new transportation facility creates a secondary effect. These changes in population and employment location are due to the fact that some of these zones in the study area become more accessible and therefore households and firms start to relocate to take advantage of the new facility, even anticipating these changes. These secondary effects are not considered in the traditional four-step transportation models.

In traditional land use planning, the future transportation system is also assumed to be fixed, while the increase in population or activity in zones might require further facility enhancement. However, often these are not considered in the assumed transportation network which is being used for land use projection.

Over time, this disjointed planning framework creates imbalances between transportation and land use. The imbalances show themselves as congestion, overloaded networks in some part, and under-utilized facilities on the other. Famous examples of these imbalances are the overloaded Shirley Highway in Washington, DC or the London Orbital Highway in England. These facilities became prematurely overloaded few years after their opening. Ordinarily these highway were assumed to have a 30-year service life-span.

Not all imbalances show themselves in congested facilities. Under-utilized facilities such as the Sawgrass Expressway built by the Broward County Expressway Authority in Florida is the other side of the imbalance. The Sawgrass Expressway when opened in 1986, realized only a portion of the traffic projected on the facility.

These and many other examples show that there is a need for feedback between land use and transportation models.

As Harris explains in appendix B of the DVRPC report, modern research on housing choices, and other aspects of urban form theory, began with Wingo (1961) and Alonso (1964) books in which they explain that people in different income classes compete for residential land, and considering a monocentric employment city, locate in concentric rings as densities decline going away from the employment center. Locators are trading longer commutes and higher transportation costs for added space and amenities. In this formulation, all members of a household class behave identically, which in reality is not the case.

Modeling urban form, as represented by location (land use) models, was initiated by Lowry in his Model of Metropolis (1964). Lowry considered the City of Pittsburgh, where the location of steel mills and other large industries, because of their size requirement and distant markets, are independent of local employee location. These were considered as basic employment. The model is based on the assumption that, everything else being equal, the place of employment determines the place of residence. Place of work (basic employment location) implies the place of residence (population and dwelling units). The resident population requires "services", therefore, place of

service employment is determined by resident population. The service employees themselves require housing in relation to their place of work. This additional population requires further services which will be fulfilled by additional service employment. The new service employees require housing in relation to their place of work. This round of reasoning continues until there are very few service employees or households to be located.

Households and employment are constrained by regional employment and household totals. The heart of the model for placing households is a gravity model relating homes to employment using an impedance function of power form. One of the derivatives of Lowry's model is the Time Oriented Metropolitan Model (Crecine, 1994) which introduces an element of time in the model. The original Lowry formulation, as Lowry himself puts it, generates an "instant metropolis" (Lowry 1964).

Wilson (1967, 1970, 1971) introduced principles from information theory to estimate a typical trip table which is used to create a series of spatial interaction models. DRAM, a reformulation of the Projective Land Use Model (PLUM, Goldner 1968, in Putman, 1979), is based on the use of the explicit determination of a trip table using Wilson's maximum entropy formulation. A more detailed description of Putman's model and associated flowchart will follow in section III.

The MEPLAN model of Marcial Echenique and Partners (Planning and Design, 1994) introduces elements of relative rent for land (comparative prices) as a market clearing mechanism. The model uses an economic input-output method to describe the flow of activities over the transportation network. Relative cost of transportation (including congestion) is used in reallocation of land uses. A more detailed description of Echenique's model and associated flowchart will follow in section III.

Others efforts in modeling urban form have used optimization theory. These models assume the pattern of household and employment locations can be described as allocations of new land uses in such a way as to optimize an objective function which consists of transportation costs and activity establishment costs. The models have constraints intended to ensure that zones are not filled beyond capacity and that all activities are allocated. Technique for Optimum Placement of Activities into Zones (TOPAZ) uses a non-linear objective function (for more detail see Oryani 1987). It is one of a small number of optimizing models which have been used by planning agencies to define extremes of alternatives.

The Herbert-Stevens (1961) model attempted to simulate market conditions for redistributing locational choices. It based its formulation on the economic theory of trading time for lower densities and other amenities in suburban development. This model was extended by Harris (1963) and Wheaton (1974) to form a model in which a non-linear adjustment of bid rents cleared the market.

It was not until the NBER (Ingram 1972) model that multiple employment location was considered. The model considers housing preferences in locating the household, and deals with housing conversion and redevelopment. As Harris explains in Appendix B of the DVRPC report, in reality transportation planners and geographers did know that people locate themselves in such a way that similar households do indeed behave differently and thus require the use of some form of the gravity model.

Wilson's work on gravity models had pointed out that there were essentially three formal types —

unconstrained, singly constrained, and doubly constrained. Trip distributions are doubly constrained, so that trips and opportunities are balanced at zones of departure and arrival. It can be shown, although it is not widely recognized, that the "balancing factors" in this model have an economic significance with regard to locational advantage which is analogous to the dual variables in linear programming, as in the NBER model. The original Lowry model and most of its successors were, however, singly constrained: the trips originating at the place of employment were exactly distributed, but the arrivals at residential destinations were uncontrolled, and excess arrivals which could not be accommodated with available land were arbitrarily redistributed. Even when this model was doubly constrained, the economic significance of the constraints was not adequately recognized.

This difficulty began to be overcome in the early 1970s. Echenique (for a review of his work see the Journal of Planning and Design, 1994) working with the larger model systems discussed in the next section, recognized the need for constraints in the Lowry Model which he had been using, and made the key innovation of using land or housing rents as the constraint. It now seems obvious that well-located or well-designed residential precincts, which attract unusual numbers of residents, can charge higher prices or rents, and that it is precisely these user costs which prevent the areas from actually becoming overcrowded. This is exactly the way in which market-clearing models operate, but in this case the idea of rents was applied in a model which did not have uniform economic behavior, but rather the dispersed behavior of the gravity model. At about the same time, coming from the Wingo-Alonso-Mills school of economic models, Anas (1975, 1987) introduced discrete choice behavior into models with economically specified behavior and market clearing.

These approaches, from opposite schools of residential modeling, effectively unified ideas of market clearing and dispersed behavior to provide for realistic modeling of the residential land and housing market.

Similar modeling of retail trade and service location (for example, Harris and Wilson, 1979) and industrial location have begun to solve somewhat less difficult problems. These activities taken together lay the basis for large-scale unified models of metropolitan growth and function.

Putman (1971, 1983) deserves recognition as the first to clearly emphasize in publications the importance of this final integration. His subsequent work has built on the Lowry model and has recently introduced new methods for dealing with industrial location. Echenique has continued to pursue his revision of the Lowry Model, and has for many years emphasized the importance of transport and transport modeling in his work. Anas has undertaken several modeling efforts dealing with all of these issues, from a more or less rigorous economic viewpoint, with transportation inputs. His models of industrial location are less complete than those of Echenique, and his transportation modeling is not at the level of most transportation planning agencies. Putman has only recently begun to introduce constraints and product differentiation in his housing models.

This discussion has emphasized the work done by only three individuals and their associates, since they have played key roles in the development of the field. Echenique has some students who have produced models on their own account, and Putman has a few practitioner-students using and developing his models in US agencies. These three individuals and a few of their students are the only sources for commercially available integrated models.

SHORT HISTORY OF URBAN MODELS

ECONOMIC MODELS	GRAVITY MODELS	CONTRIBUTORY WORK
		Transportation Tree Tracing Assignment Trip Distribution CATS 1958
Monocentric City Wingo 1961 Alonso 1964 Mills		
LP Not Monocentric Herbert-Stevens	**Model of Metropolis** LOWRY 1964	
Combined LP- Discrete Choice w. Home to Work NBER	**England** Structure Planning	**Entropy Maximizing** Wilson
	Harris-Wilson Retail Trade	**Equilibrium Congestion** Evans
ANAS- Add Gravity	**ITLUP** DRAM/EMPAL Putman	**Discrete Choice** McFadden
ANAS- Metrosim	**Echenique MEPLAN** ADD Rents	

Years marked on timeline: 1955, 1961, 1968, 1970, 1975

Inventory of Land Use Models

In a demonstration project to develop methodologies for evaluating alternative land use patterns for air quality implications by the Organization of 1,000 Friends of Oregon, the following fourteen land use models were identified:[1]

1.	TOPAZ	Australia
2.	MEP	U.K.
3.	ITLUP (DRAM-EMPAL)	U.S.A.
4.	LILT	U.K.
5.	AMERSFOORT	Netherlands
6.	CALUTAS	Japan
7.	IRPUD (Dortmund)	Germany
8.	OSAKA	Japan
9.	SASLOC	Sweden
10.	MEPLAN	U.K.
11.	TRANUS	Venezuela
12.	TRACKS	Australia
13.	TRANSTEP	Australia
14.	TOPMET	Australia

Most of the above models are not available commercially for agency use. Available models include TOPAZ, TOPAZ82, MEPLAN, ITLUP, TRANUS, TRACKS and TRANSTEP. Among the available models only ITLUP (DRAM-EMPAL), MEPLAN, and TRANUS have sufficient installation sites to enable users to share experiences to shorten the learning curve in modeling applications.

A survey of MPOs covering the twenty largest Metropolitan Statistical Areas in the United States, and two additional agencies known to be on the leading edge of model use was made by the same 1,000 Friends of Oregon study. Information about land use data and land use procedures in travel demand modeling was provided by nineteen of these twenty-two agencies.

The survey found that eight agencies use land use data in the traditional form in trip generation. None of these "traditional" agencies had a land use model for allocation of development activities to zones.

A second group of five agencies, called innovative by the Oregon study, used land use allocation models to provide input data to the trip generation phase of their transportation models. Except for one agency which used its own specific technique, the other four agencies utilized DRAM-EMPAL models.

The third group included four agencies which are in transition from "traditional" to "innovative" approaches in land use data. Except for one agency, which is in the process of creating its own land use model, the three other agencies are in different stages of implementing DRAM-EMPAL as their land use model

---

1. Source: "Making the Land Use Transportation/Air Quality Connection", Volume 1, October 1991 prepared for the Organization of 1,000 Friends of Oregon, Cambridge Systematic, Inc., with Hague Consulting Group

The fourth group consists of two agencies. One uses a variant of DRAM-EMPAL models integrated into transportation modeling with necessary feedback mechanisms between the transportation and land use models. The other agency, the Association of Bay Area Governments, has created its own land use model, POLIS, which is described below.

A more recent inventory of operational models is made by Wegener (1994) entitled "Operational Urban Models: State of the Art". The following twelve models are identified as being operational. He made no judgements on the quality of the models, but the criteria of being applied to real cities and being operational had been satisfied:

1. POLIS: the Projective Optimization Land Use Information System developed by Prastacos for the Association of Bay Area Governments

2. CUFM: the California Urban Future Model developed at the Institute of Urban and Regional Development of the University of California at Berkeley

3. BOYCE: Combined models of location and travel choice developed by Boyce

4. KIM: the non-linear version of the urban equilibrium model developed earlier by Kim et al.

5. ITLUP: the DRAM-EMPAL Integrated Transportation and Land Use Package developed by Putman

6. HUDS: the Harvard Urban Development Simulation developed by Kain and Apgar

7. TRANUS: the transportation and land-use model developed by de la Barra

8. 5-LUT: The 5-Stage Land Use Transport Model developed by Martinez for Santiago de Chile

9. MEPLAN: the integrated modeling package developed by Marcial Echenique & Partners

10. LILT: the Leeds Integrated Land-Use/Transport Model developed by Mackett

11. IRPUD: the model of the Dortmund region developed by Wegener

12. RURBAN: the Ransom-Utility Urban Model developed by Miyamoto

In a later paper by Wegener (1995), the following model is added to the above list:

13. METROSIM: the new microeconomic land use transportation model by Anas

The most recent Survey of Land Use and Travel Data of the Metropolitan Planning Organizations of the 35 Largest U.S. Metropolitan Areas by Porter (1995) contains information about land use forecasting procedures and the use of land use models. According to this survey:

- Twelve MPOs are using DRAM-EMPAL models

- Five MPOs are using their own models (POLIS, PLUM, and three local models)

- One MPO is in the process creating its own model

- Two MPOs use the Delphi (exchange of expert opinion) Technique

Fifteen agencies do not use land use models but use qualitative procedures. This group allocates land use to TAZ on the basis of forecasts of population and employment.

Methodology for Selecting a Land Use Model

Many of the models mentioned were one time applications at a single city. We decided that the model should be commercially available, be operational, be used in multiple locations and be theoretically sound.

With these criteria, our list of models were reduced to three models:

DRAM-EMPAL (S. H. Putman Associates)

MEPLAN (Marcial Echenique Associates)

METROSIM (Alex Anas)

A brief description of each model and their associated flow-chart follows:

DRAM (Desegregated Residential Allocation Model) is a singly constrained Lowry-derivative model which forecasts household location in relation to employment and probability of work trips between zones.

The work trips probability has two components: Travel impedance and measure of attractiveness of the zone for household location. The attractiveness measure uses the following variables:

1- Vacant, build able land in origin zone

2- Percentage of buildable land which is not already built

3- Residential land

4- Percentage of household in the lowest income group

5- Percentage of household in the low middle income group

6- Percentage of household in the upper income group

7- Percentage of household in the upper income group

The travel function is a modified gamma function.

EMPAL is also is a singly-constrained model with lagged employment using an impedance cost matrix for projecting the location of new employment.

According to the author of the model, it has been applied in Atlanta, Chicago, Dallas, Houston, Los Angeles, Sacramento, Boston, Detroit, Kansas City, Phoenix, San Antonio, Seattle, and Orlando.

MEPLAN is another Lowry-derivative model which uses economic base theory in an input-output model framework with price function. An input-output model is applied to represent flows between activities in the form of demand for space. The coefficients of the input-output model are used to calculate prices in an elastic form to represent land allocation within zones. Random utility is used to represent an explicit spatial system where households and firms decide where to live and locate in a utility maximization or a cost minimization framework within specified constraints. This allows market land prices to be considered in the model explicitly. On the same basis, the price of transport might be formulated in terms of time penalties representing conges-

tion.

According to the author of the model, it has been applied in Cambridge and Stevenage, U.K.; Santiago, Chile: Sao Paulo, Brazil; Tehran, Iran; Bilbao, Spain; Helsinki, Finland; Tokyo, Japan.

METROSIM takes an economic approach to modeling housing and land-use location. The model embodies the discrete choice method with economically-specified behavior and a market clearing mechanism. The model is formulated in three market equilibria: 1) labor market equilibrium and job assignment, 2) housing market equilibrium and 3) commercial space equilibrium. The model iterates between these markets and the transportation system for equilibrium of land-use and transportation flows. This model has evolved from applications in Chicago consisting of residential location-housing and mode choice sub-models. In the New York Region's implementation of the model, non-work travel choices and commercial real estate markets were added.

According to the author of the model, it has been applied in Chicago (CATLAS), New York (NYSIM), Chicago, Houston, Pittsburgh and San Diego (CPHMM) and New York (for NYMTC)
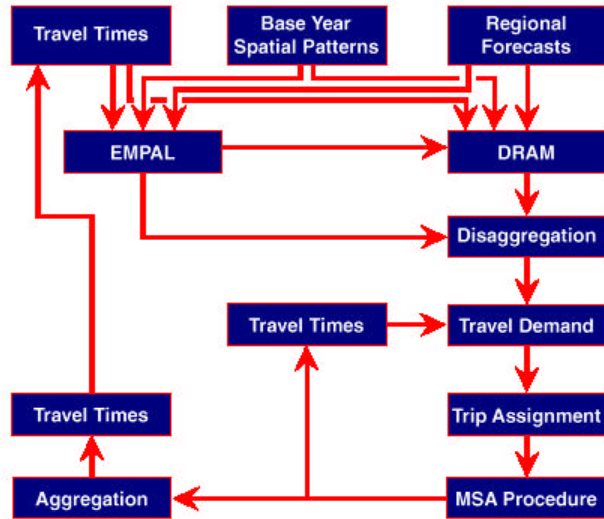
In terms of experiences of other MPOs with land-use models, a telephone interview was conducted with five MPOs using the DRAM-EMPAL model. These MPOs were:

1. Atlanta Regional Commission

2. Northeast Illinois Planning Commission (Chicago)

3. North Central Texas Council of Governments (Dallas)

4. Houston-Galveston Area Council of Governments (Houston)

5. Sacramento Area Council of Governments (Sacramento)

Questions were asked about calibration-forecast years, land-use zones and transportation zone systems, transportation software, household and employment categories and means of projecting control totals. Review processes of forecasts made and use of such forecasts were also questioned. The main finding of the telephone interview was that although there is a need for improvement in the DRAM-EMPAL model, the majority of users are satisfied with the model. However there is one MPO which is actively looking for a replacement. The satisfaction comes from the consensus that instead of starting a new model altogether, efforts should be made by the author of the model as well as the user community to enhance the model system. Those MPOs who are happy with the model attribute their success not only to the model system but also to their own efforts, especially in providing a sound employment location data base.
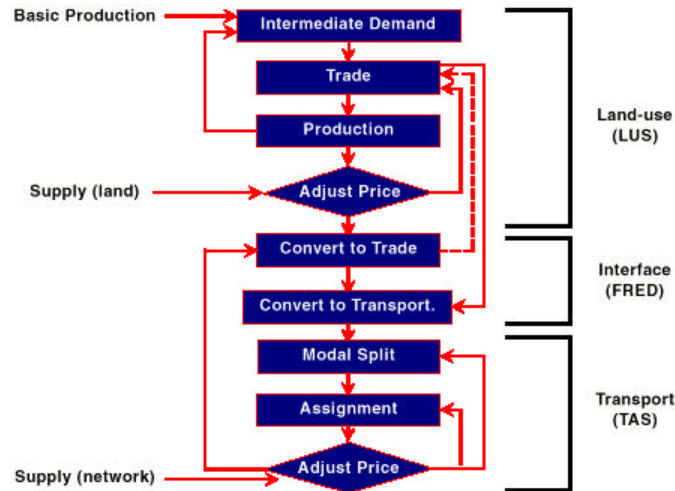
It was essential that the DVRPC model benefit from the experiences of other MPOs with existing operational land-use models. At the same time, improvements to the model system should be possible as the component modules become available. We proposed a two-step selection and implementation phase: short-term and long-term. In the short term we recommended that limited versions of the DRAM- EMPAL, MEPLAN and METROSIM models be acquired for competitive testing in prototype use, policy analysis and impact assessment. For long-term needs, the model system should be modular to allow the insertion of better component modules as they become available.

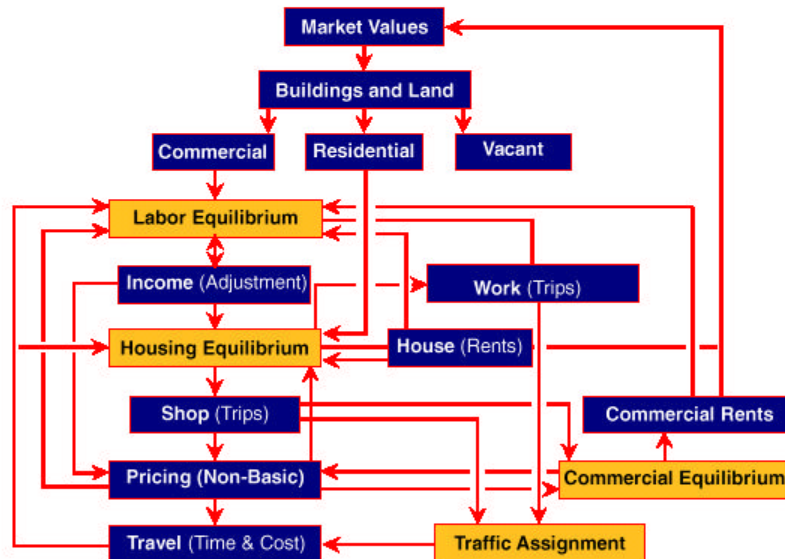# LINKED MODELS OF LAND USE -TRANSPORTATION: DRAM-EMPAL

Travel Times

Base Year Spatial Patterns

Regional Forecasts

EMPAL

DRAM

Disaggregation

Travel Times → Travel Demand

Travel Times

Trip Assignment

Travel Times

Aggregation ← MSA Procedure

Source: Equilibrium Condition In Land Use Travel Forecasting, March 1995, S.H. Putnam Associates, Inc.

MEPLAN MODEL

Basic Production → Intermediate Demand

Trade

Production

Supply (land) → Adjust Price

Land-use (LUS)

Convert to Trade

Convert to Transport.

Interface (FRED)

Modal Split

Assignment

Supply (network) → Adjust Price

Transport (TAS)

Source: "Urban And Regional Studies At The Martine Centre," M. Echenique, 1994

LINKS AND FEEDBACKS AMONG METROSIM MODULES

Market Values

Buildings and Land

Commercial   Residential   Vacant

Labor Equilibrium

Income (Adjustment)   Work (Trips)

Housing Equilibrium   House (Rents)

Shop (Trips)   Commercial Rents

Pricing (Non-Basic)   Commercial Equilibrium

Travel (Time & Cost) ← Traffic Assignment

Source: Alex Anas, Model Documentation

References

Review of Land Use Model and Recommended Model For DVRPC, Kazem Oryani and Britton Harris, Enhancement of DVRPC's Travel Simulation Models, Delaware Valley Regional Planning Commission, Publication No. 96008, 1996, Philadelphia, PA

Making the Land Use Transportation/Air Quality Connection, Volume 1, October 1991 prepared for the Organization of 1000 Friends of Oregon, Cambridge Systematic, Inc. with Hague Consulting Group

Short-Term Travel Model Improvements, Travel Model Improvement Program, October 1994, U.S. Department of Transportation, U.S. Environmental Protection Agency

Travel Model Improvement Program: Land Use Modeling Conference Proceedings, 1995, U.S. Department of Transportation, U.S. Environmental Protection Agency

Alonso, William, 1964, Location and Land Use, Harvard University Press, Cambridge, MA.

Anas, Alex, 1975, "The Empirical Calibration and Testing of a Simulation Model of Residential Location", Environment and Planning A, 7, 899-920.

Anas, Alex, 1987, Modeling in Urban and Regional Economics, Harwood Academic Publishers, Chur, Switzerland.

Crecine, J. P. 1964, A Time-Oriented Metropolitan Model For Spatial Location, in S. H. Putman, Urban Residential Location Models, Martinus Nijhoff Publishing 1979

Echenique, Marcial H., 1994, "Urban and Regional Studies at the Martine Centre: Its Origin, Its Present, Its Future", Environment and Planning B: Planning and Design 1994, Volume 21, pp. 157-533

Goldner, W., 1971, "The Lowry Model Heritage", Journal of the American Institute of Planners, **37**. 100-110.

Harris, Britton, 1963, "Linear Programming and the Projection of Land Uses", Penn- Jersey Paper No. 20, Philadelphia, PA.

Harris, Britton, and Alan G. Wilson, 1979, "Equilibrium Values and Dynamics of Attractiveness Terms in Production-Constrained Spatial-Interaction Model, Environment and Planning A, **10**, 371-388.

Herbert, John S., and Benjamin H. Stevens, 1960, "A Model of the Distribution of Residential Activity in Urban Areas", Journal of Regional Science, **2**, 21-36.

Ingram, Gregory K., John F. Kain, and J. Royce Ginn, 1972, The Detroit Prototype of the NBER Urban Simulation Model, New York, National Bureau of Economic Research.

Lowry, I. S., 1964, A Model Of Metropolis, Rand Corporation, Santa Monica, California, in S. H. Putman, Urban Residential Location Models, Martinus Nijhoff Publishing 1979

Mills, Edwin S., 1972, Urban Economics, Scott Foresman, Glenview, IL.

PLANNING AND DESIGN, 1994, Special Issue on Urban and Regional Studies at the Martin Centre,21, 157-533.

Oryani, Kazem, 1987, Performance of Behavioral Land-Use Transportation Models and Optimi-

zation Land Use Models, Ph.D. Dissertation, Department of City and Regional Planning, University of Pennsylvania

Porter Chris, Melendy Laura, and Deakin Elizabeth, 1995, Land Use and Travel Survey Data: A Survey of the Metropolitan Planning Organizations of the 35 Largest U.S. Metropolitan Areas, Institute of Urban and Regional Development, University of California at Berkeley, Berkeley, CA

Putman, Stephen H., 1971, "The Effect of Changing Transport Facilities on Regional Economic Development", Papers and Proceedings of the Regional Science Association, **27**, 151-166.

Putman, S. H., 1979, Urban Residential Location Models, Martinus Nijhoff Publishing

Putman, S. H., 1983, Integrated Urban Models, Policy Analysis of Transportation and Land Use, Pion Limited, London

Putman S. H., 1991, Integrated Urban Models 2: New Research and Application of Optimization and Dynamics, Pion Limited, London

Putman, S. H., 1995, "EMPAL and DRAM Location and Land Use Models: A Technical Overview," Urban Simulation Laboratory, Department of City and Regional Planning, University of Pennsylvania, Land Use Modeling Conference Proceedings, Dallas, TX

Webster, F. Vernon, Philip H Bly and Neil J Pauley, eds, 1988, Urban Land-Use and Transport Interaction. Policies and Models. Report of the International Study Group on Land-Use/Transport Interaction (ISGLUTI), Aldershot, Avebury.

Webster, F Vernon and Mira Dasgupta, 1991, Land Use and Transport Interactions. Report of the ISGLUTI Study. Crowthorne, Transport and Road Research Laboratory.

Wegener, M., 1994, "Operational Urban Models: State of the Art," Journal of American Planning Association, Vol. 60, No.1, Winter 1994, pp 17-29

Wegener, M., 1995 "Current and Future Land Use Models," Travel Model Improvement Program: Land Use Modeling Conference Proceedings, pp. 13-40, U.S. Department of Transportation, U.S. Environmental Protection Agency

Wheaton, William C., 1974, "Linear Programming and Locational Equilibrium: The Herbert-Stevens Model Revisited", Journal of Urban Economics, **1**, 278- 287.

Williams, I. N., 1994, "A Model of London and The South East," Environment and Planning B: Planning and Design, Volume 21, pp. 535-553

Wilson, A. G., 1967, "A Statistical Theory of Spatial Distribution Models," Transportation Research, Vol. 1, pp. 253-269

Wilson, A. G., 1970, "Advance and Problems in Distribution Modeling," Transportation Research, Vol. 4, pp. 1-18

Wilson, A. G., 1971, "A Family of Spatial Interaction Models and Associated Developments," Environment and Planning, Vol. 3, pp. 1-32

Wingo, Lowdon Jr., 1961, Transportation and Urban Land Use, The Johns Hopkins Press, Baltimore, MD.