

## Technical Report Documentation Page

echnical Report Document			
1. Report No.:	2. Government Accession No.:	3. Recipient's Catalog No.:	
FHWA-GA-12-1022			
4. Title and Subtitle:		5. Report Date:	
Freeway Travel-time Estimat	ion and Forecasting	September 2012	
5	8	6. Performing Organization	Code:
		of refiniting organization	0040.
7. Author(s):		8. Performing Organization	Report No.
Angshuman Guin, Ph.D., Jorg	e Laval Ph D Bhargava R	10-22	1
Chilukuri	5° 20° 00, 1 102 °, 2000 gu ° 0 10	10 ==	
Chinakan			
0 Deufenneine Oneeniertien Neu		10 W	
9. Performing Organization Nan		10. Work Unit No.:	
Georgia Tech Research Corpo			
School of Civil and Environ.	Engineering	11. Contract or Grant No.:	
790 Atlantic Drive			
Atlanta, GA 30332-0355			
12. Sponsoring Organization Na	me and Address:	13. Type of Report & Period	l Covered:
Georgia Department of Trans	portation, Office of Materials	Final; August 2010 – Sep	tember
and Research; 15 Kennedy D		2012	
2534		14. Sponsoring Agency Cod	e.
		14. Sponsoring Agency Cou	с.
15. Supplementary Notes:			
	the US Department of Transporta	tion, Federal Highway Adm	inistration
1 1	1 1		
16. Abstract:			
			<b>a i</b>
	simulation-based framework for g		
time on freeway corridors. The	e microsimulation model that is d	leveloped (GTsim), replicate	es freeway
capacity drop and relaxation p	phenomena critical for modeling r	on-steady state conditions.	This
	e on a real-world freeway corrido		
	ansportation's video detection sys		
	ravel time data; the comparison de		
	c forecasts. The study results sho		
	e and 10-minute forecasts and that	it outperforms state-of-the-	practice
methods such as instantaneou	s travel time.		
17. Key Words		18. Distribution Statement	
Travel-time forecasting.			
19. Security Class (This	20. Security Class (This Page)	21. No of Pages	22. Price
Report)	Unclassified	77	
Unclassified	Uliciassilieu	//	

 Unclassified
 One of the form and completed page is authorized

 Form DOT F 1700.7 (8/72) Reproduction of form and completed page is authorized

GDOT Research Project No. RP 10-22 Final Report

#### FREEWAY TRAVEL-TIME ESTIMATION AND FORECASTING

By

Angshuman Guin, Ph.D., Principal Investigator Jorge Laval, Ph.D., co-Principal Investigator Bhargava R. Chilukuri, Graduate Research Assistant

School of Civil and Environmental Engineering Georgia Institute of Technology

Contract with

Georgia Department of Transportation

In cooperation with

US Department of Transportation Federal Highway Administration

September 2012

The contents of this report reflect the views of the authors who are responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Georgia Department of Transportation or the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

# **Table of Contents**

1	Inti	<i>coduction</i>
	1.1	Objective
2	Lite	erature Review
	2.1	Travel-time Estimation Algorithms
	2.1.	1 Direct Methods
	2.1.	2 Indirect Methods
	2.2	Travel-time Forecast Algorithms
	2.2.	1 Statistical Methods 12
	2.2.	2 Simulation Methods
3	Pre	diction Framework
4	Dat	a Collection and Preliminary Analysis
	4.1	Traffic Volume Data Collection
	4.2	Bluetooth Data Collection
	4.3	Velocity Data Collection
5	Tra	ffic Flow Simulation Application
	5.1	Process Flow
	5.2	Critical Modules
6	Me	thodology

9	Refe	prences	62
8	Con	clusion	60
	7.2	Analysis of Simulation Results	52
	7.1	Analysis of Bluetooth Data	49
7	Rest	ults and Discussion	49
	6.2	Calibration and Validation	45
	6.1	Origin Destination Matrix Estimation Algorithm	43

# **List of Figures**

Figure 1. Popular Travel-time Estimation and Prediction methods	4
Figure 2. Real-time Travel-time estimation and prediction framework	20
Figure 3: Study Corridor (background: http://maps.google.com)	25
Figure 4: Bluetooth Data Collection Locations (background: http://maps.google.com)	27
Figure 5: Actual Travel-time Data collected by Bluetooth devices	29
Figure 6: Time space speed plot of the study corridor on March 7, 2012	30
Figure 7: Time Series of Speed of the NaviGAtor station #2850032	31
Figure 8: Process Flow for the Simulation Application	35
Figure 9: Transportation Network background for the Application	36
Figure 10: Sample Network	37
Figure 11: Sample Animation snapshot	38
Figure 12: ITT, RTT, ATT and Bluetooth data on the corridor	51
Figure 13: Simulation results vs. Bluetooth Data on the corridor	54
Figure 14: Travel-time Predictions for Different Time Horizons on the Corridor	54
Figure 15: Comparison of Travel-time Predictions for Different Horizons on the Corridor	55
Figure 16: Y-Y Plot of 1 <sup>st</sup> Prediction TT and ITT against Bluetooth Data	57
Figure 17: Comparison of variability in 1 <sup>st</sup> Prediction, 2 <sup>nd</sup> prediction, 3 <sup>rd</sup> prediction and ITT	58

# List of Tables

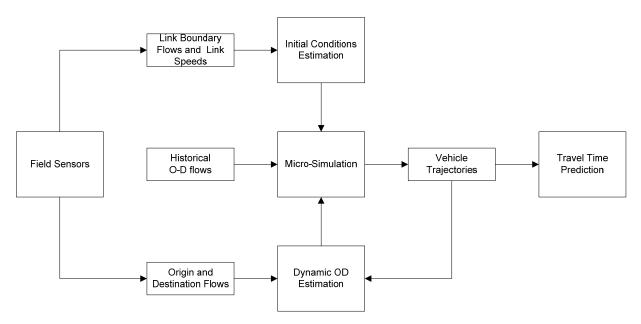
Table 1.	Calibrated Parameters	6
Table 2.	RMSE and MAPE	59

## **Executive Summary**

Real-time traffic information provided by GDOT has proven invaluable for commuters in the Georgia freeway network. The increasing number of Variable Message Signs, addition of services such as My-NaviGAtor, NaviGAtor-to-go etc. and the advancement of the 511 traffic information system will require the Traffic Management Center to provide more detailed and accurate traffic information to an increasing number of users. In this context, the ability to forecast traffic conditions (both in space and time) would augment the services provided by NaviGAtor by allowing users to plan ahead for their trip. Forecasts built into the estimation model will make the travel-time estimates more accurate by reducing the use of stale data. Additionally, spatial forecast can help GDOT provide reliable information in areas with temporary outages in coverage; e.g. outages due to detector or cameras malfunction.

The vast majority of real-time travel-time estimation algorithms proposed in the literature are based on data mining techniques. Unfortunately, this approach is unable to produce reliable forecasts because it does not take into account traffic dynamics (e.g., via a simulation model). In Germany, a simulation-based forecast system is in place at most metropolitan areas, with very favorable user impacts. Although successful, the German example is based on a type of simulation model (a Cellular Automata model) that has critical drawbacks: difficulty of calibration, inability to incorporate different user classes (e.g., cars and trucks), and inadequate capability of replicating detailed traffic dynamics on freeways. The model proposed in this study overcomes these drawbacks by incorporating the latest advances in traffic flow theory and simulation.

The proposed prediction framework for real-time travel-time estimation and prediction is shown in Figure 2. The two main components of this framework are the micro-simulation and traffic sensor infrastructure on the freeways. The micro-simulation runs faster than real-time using the data reported by the traffic sensors to give travel-time estimates and predictions.



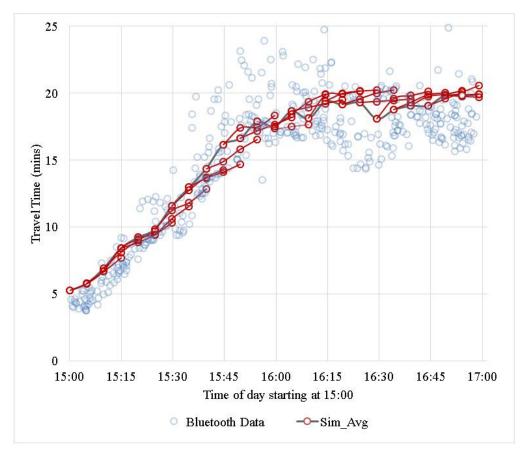
Real-time Travel-time estimation and prediction framework

The traffic sensors provide time series of traffic volumes and speeds at both the boundaries of the network and also on mid-segments. The traffic volumes at the network boundaries are used for estimating dynamic origin-destination matrices. The traffic volumes and speeds on the mid-sections are used to determine initial queues used for the simulation. The vehicle trajectories produced by the simulation are used by a dynamic OD estimation module to generate OD matrices for the next simulation run. The dynamic OD matrices are validated with the historic OD matrices before being used as input in the next simulation run. The vehicle trajectories generated by the simulation are used to generate travel-time forecasts.

The study was performed by simulating a 6.5 miles long EB/SB I-285 corridor between GA-400 and I-85 for 2 hours during the evening peak period (from 15:00 to 17:00). The volume data was manually extracted from videos recorded using GDOT's traffic monitoring cameras. Bluetooth

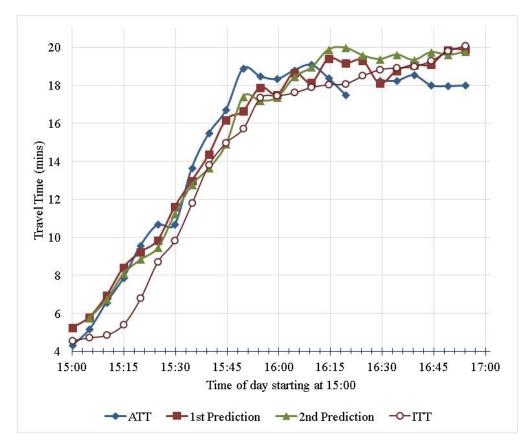
data was collected from select overpasses in the corridor in the same day as the traffic volume data collection to serve as travel-time ground truth. The OD matrix was changed every 5 minutes and a new simulation was initiated every time with new initial queue and new OD flows. Six travel-time forecasts were made for every simulation run. The predictions made every 5 minutes were compared with the Bluetooth travel-time data collected in the field.

The following plot of travel-time predictions for different time horizons on the corridor show that the predictions made during each simulation run follow the pattern observed with the Bluetooth data. During congestion buildup, the forecasts show an increase in travel-time for prediction horizons. Eventually, after the corridor gets congested, the forecasted travel-times flatten out.



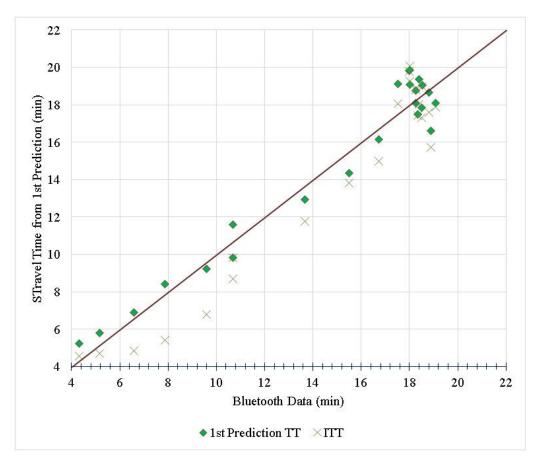
Travel-time Predictions for Different Time Horizons on the Corridor

The following figure shows a comparative plot of the probe travel-times or actual travel-times (ATT) with the simulated and instantaneous speed travel-times (ITT). The instantaneous travel-time is based on the speeds across the corridor at a single instant and is a close approximation of the methodology typically used by Transportation Management Centers for travel-times posted on variable message signs.



Comparison of Travel-time Predictions for Different Horizons on the Corridor

The following Y-Y plot of ATT vs ITT and 1st prediction (simulated) travel-times shows that the 1st prediction results are closer to the actual travel-times than the ITT estimates. The 1st prediction travel-time data points shown in red are the instances when the sample sizes from Bluetooth data are less than 10. Apart from the outlier points, it can be seen that the performance of the simulation is superior to the ITT method under dynamic conditions, whereas the performance becomes comparable under heavy, stable congestion.



Y-Y Plot of 1st Prediction TT and ITT against Bluetooth Data

This study demonstrated the use of a simulation based framework to make short-term travel-time predictions in real-time. The results show that sufficiently accurate 5-minute and 10-minute predictions can be made using this framework. The lessons learned from the study stresses that it is critical to adequately calibrate the simulation model and for this purpose it is essential to accurately calibrate the vehicle detection sensors. Currently, the simulation is manually initiated each time a new OD matrix becomes available. For a seamless implementation, the initiation process needs to be automated. In future studies the researcher would like to automate the simulation to run continuously by getting sufficient predictions from a run, pausing the simulation until the next OD update is available, and updating the OD flows and initial queues. When incidents occur, the corresponding lane blockage can be incorporated in the simulation before predictions are made.

# Acknowledgments

The authors of this report wish to thank the Georgia Department of Transportation for its support and assistance throughout this effort, in particular the efforts of Binh Bui, David Jared, Mark Demidovich and W. Grant Waldrop. In addition the authors wish to thank the USDOT for the support from a companion project provided through the University Transportation Center program.

### **1** Introduction

Real-time traffic information provided by GDOT has proven invaluable for commuters in the Georgia freeway network. The increasing number of Variable Message Signs, addition of services such as My-NaviGAtor, NaviGAtor-to-go etc. and the advancement of the 511 traffic information system will require the Traffic Management Center to provide more detailed and accurate traffic information to an increasing number of users. In this context, the ability to forecast traffic conditions (both in space and time) would augment the services provided by NaviGAtor by allowing users to plan ahead for their trip. Forecasts built into the estimation model will make the travel-time estimates more accurate by reducing the use of stale data. Additionally, spatial forecast can help GDOT provide reliable information in areas with temporary outages in coverage; e.g. outages due to detector or cameras malfunction.

The vast majority of real-time travel-time estimation algorithms proposed in the literature are based on data mining techniques [1-5]. Unfortunately, this approach is unable to produce reliable forecasts because it does not take into account traffic dynamics (e.g., via a simulation model). In Germany, a simulation-based forecast system [6] is already in place at most metropolitan areas, with very favorable user impacts. Although successful, the German example is based on a type of simulation model (a Cellular Automata model) that has critical drawbacks: difficult to calibrate, unable to incorporate different user classes (e.g., cars and trucks), and not proven to replicate detailed traffic dynamics on freeways. The model proposed in this project [7-9] overcomes these drawbacks by incorporating the latest advances in traffic flow theory and simulation.

The enhanced travel-time estimation methodology developed in this research would substantially increase the accuracy of the estimates provided by GDOT to the commuters via the Changeable

Message Signs, the Georgia-NaviGAtor website, the \*DOT service and the 511 service. This will not only improve the credibility of the estimates, but will allow people to better schedule their commutes or take less congested alternative routes in order to avoid congestion.

### 1.1 Objective

The objectives of this project are:

Incorporate recent advances in traffic flow theory and simulation to build a framework able to predict the onset and propagation of congestion across the Metropolitan Atlanta freeway network

Demonstrate proof of this concept by generating realistic real time travel-time forecasts on a freeway corridor based on short-term forecasts of future congestion levels in the network obtained from simulation

## 2 Literature Review

Travel-time is one of the performance measures used by transportation agencies to evaluate the operation of freeway systems. The Federal Highway Administration (FHWA) issued a memorandum in 2004 that encouraged all the transportation agencies to display travel-time information on the dynamic message signs [10]. Travel-time on freeway sections can be generally classified into:

- Instantaneous Travel-time
- Reactive Travel-time
- Predictive Travel-time

*Instantaneous Travel-time* refers to the average travel-time observed on a small roadway segment along a corridor. The average total travel-time along the corridor is obtained by adding the instantaneous travel-time on each of its segments. The inherent assumption made is that the traffic conditions on each of the segments remain unaltered between when the vehicle enters and exits the freeway corridor. This assumption holds true during free-flow conditions but fails at the border of free-flow and congestion. This assumption results in underestimating travel-time at the onset of congestion and overestimating at the dissipation of congestion.

*Reactive Travel-time* refers to the travel-time of the vehicle that just exited the corridor. While this type of travel-time measurement is precise and is based on actual travel-time observed in the field, this measurement does not mean that a driver who just entering the corridor upstream will have the same travel-time. It has been shown that this is not a good measure for online travel-time prediction [11].

*Predictive Travel-time* refers to the forecasted travel-times that are expected to be experienced by drivers who enter the corridor at that time instant or in future. This is the most useful information from a driver's perspective, but the most difficult one to forecast accurately. Forecasting travel-time during congested conditions is particularly challenging since small instability in conditions can result in significantly varying spatial and temporal travel velocities.

In this chapter, the methods and algorithms used in the literature to measure instantaneous and reactive travel-times are grouped as Travel-time Estimation methods. Similarly, methods used for calculating predictive travel-time will be called Travel-time Prediction methods.

Figure 1 describes different methods used for Travel-time Estimation and Prediction. The rest of this chapter briefly describes each of these methods.

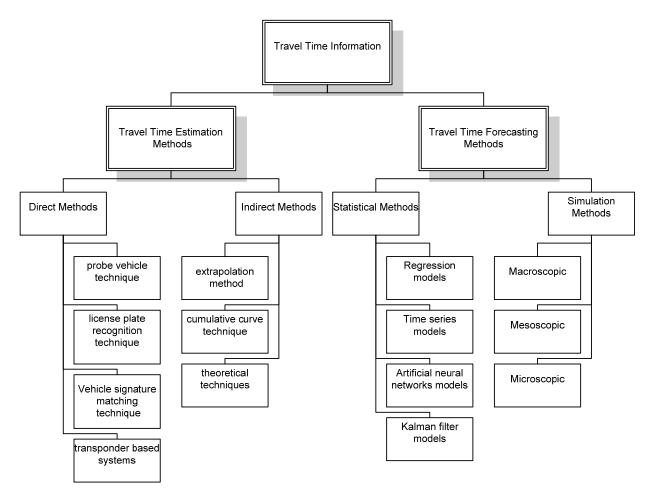


Figure 1. Popular Travel-time Estimation and Prediction methods

### 2.1 Travel-time Estimation Algorithms

The travel-time estimation methods provide information to understand the current traffic conditions on the roadways. This information can be estimated by directly measuring the travel-time (here after called Direct methods) or by measuring traffic variables such as velocity, occupancy, and flow and estimating travel-time information (here after called Indirect methods).

#### 2.1.1 Direct Methods

The following paragraphs briefly describe four popular methods; probe vehicle technique, license plate recognition technique, signature matching, and transponder based systems used to directly measure the travel-time in the field.

#### 2.1.1.1 Probe Vehicle Technique

Probe vehicle technique is a common method that consists of vehicle(s) specifically dispatched to drive with the traffic stream to collect travel-time data. The Travel-time Data Collection Handbook [11] defines the following three driving styles that the agencies can adopt for the probe vehicles:

- Average car vehicle travels according to the driver's judgment of the average velocity of the traffic stream
- Floating car driver "floats" with the traffic by attempting to safely pass as many vehicles as pass the probe vehicle
- Maximum car vehicle is driven at posted velocity limit unless impeded by actual traffic conditions or safety considerations.

Even though floating car driving style is the most commonly referenced, most agencies use a hybrid of the floating car and average car styles for their travel-time studies.

Travel-time data can be collected using electronic instruments such as Distance Measurement Instrument (DMI) or Global Positioning System (GPS) to automatically record vehicle location and velocity at preset checkpoints or time intervals.

#### 2.1.1.2 License Plate Matching

License plate matching (LPM) technique consists of measuring travel-time by matching license plate information at various checkpoints. The license plate characters and arrival times are collected at various checkpoints and the license plates are matched to compute the travel-times between the check points. The check points are typically spaced half a mile to two miles on arterials and one mile to five miles on the freeways. Even though there are equations to calculate the sample size under different conditions, generally a minimum sample size of 50 license plate matches is used to determine the average travel-time for a roadway segment [12]. One of the important requirements of this technique is synchronizing the clocks at various checkpoints. Since the vehicle velocities may vary significantly by lane during peak hours (and hence the travel-times), a representative sample of license plate should be collected from all through lanes. One of the advantages of the automated License Plate Recognition (LPR) technology is that, by

knowing the local Division of Motor Vehicles policy on the vehicle license plate syntax, one can program the system to exclude information from trucks and trolley vehicles that do not travel at regular traffic velocity. However, the accuracy of data from the LPR is sensitive to poor visibility conditions. [13, 14]. Moreover, collecting and storing license plate information can result in privacy issues. Therefore, transportation agencies have to ensure safe disposal of the license plate information after extracting the travel-time data.

#### 2.1.1.3 Signature Matching Method

In the signature matching method the travel-time is calculated by matching (correlating) unique vehicle signatures between sequential observation points. This is similar to the LPM techniques, but utilizes the widely spread existing detector infrastructure instead of installing new LPM technology. The signature matching method can be used with Inductance Loop Detectors (ILD), weigh-in-motion sensors, video cameras, and laser scanning detectors.

With the use of ILD, the travel-time can be calculated in multiple ways, on the basis of the reidentification of particular vehicles in consecutive loop detectors by means of characteristic length [15, 16] or particular inductive signature on the detector [17, 18, 19]. For video camera based detectors, visual vehicle signatures from wayside cameras [20, 21] and vehicle dimension matching for the laser based detectors [22] were developed.

Coifman and Cassidy [23] developed an improved signature matching algorithm by incorporating platoon matching. This algorithm estimates average travel-time by matching unique features of vehicle platoons such as the position and/or distribution of vehicle gaps or unique vehicles. However, this methods is heavily is reliant on proper functioning of sensors and will quickly deteriorate the quality of the travel-time information if the percent of sensor failure is moderately large.

#### 2.1.1.4 Transponder Based Systems

To overcome the limitations of earlier methods, transponder based systems are sometimes used by transportation agencies. These utilize electronic transponders or receivers installed in personal, public transit, or commercial vehicles in the traffic stream to collect travel-times. Signpost-based method, Automatic Vehicle Identification (AVI), and GPS based system are some of the traveltime collection applications of the transponder based systems.

*Signpost-based method* is a popular technique used by transit agencies where probe vehicles communicate with transmitters mounted on existing fixed signpost structures. Signpost transmitters are typically spaced 5 miles along bus routes with buses polling every 60 seconds for an odometer reading. While signpost-based method requires simple infrastructure, it has limited coverage area.

*AVI method* has vehicles equipped with electronic tags that communicate with roadside transceivers to identify unique vehicles and collect travel-times between transceivers. Tags are electronically encoded with unique identification (ID) numbers synonymous with the electronic registration number used to determine vehicle ownership in electronic toll collection. The transceivers emit radio frequency (RF) signals within a capture range across one or more freeway lanes. Examples of existing AVI systems include the TranStar system in Houston [24], the TransGuide system in San Antonio [25], and the Transmit system in the New York/New Jersey metropolitan area [26]. AVI technology is proved to provide high accuracy of travel-time data with minimal personnel requirement [27, 28, 29, 30, and 31]. However, the limitations of this method are high infrastructure dependency and clock drift problems.

In the *GPS method*, probe vehicles are equipped with GPS receivers and two-way communication to receive signals from earth-orbiting satellites. The positional information determined from the

GPS signals is transmitted to a control center to display real-time position of the probe vehicles from which travel-time information is extracted. There are many applications of automatic vehicle location that employ GPS for vehicle tracking in real-time. These include emergency service vehicles, rental cars, commercial fleets, taxis, and transit vehicles. While this method has low initial and operating cost, privacy issues and occasional loss of communication are some of the concerns with this method. Results obtained by Herrera [32] suggest that a 2–3% penetration of GPS-enabled cell phones in the drivers' population is enough to provide accurate measurements of the velocity of the traffic stream.

#### 2.1.2 Indirect Methods

Indirect methods involve calculating travel-time indirectly using the parameters such as velocity, flow, density, or occupancy from "point" vehicle detection equipment such as Inductive Loop Detectors (ILD), cameras or similar traffic monitoring technologies. Some of the important techniques include extrapolation method, cumulative curve methods and theoretical techniques.

#### 2.1.2.1 Extrapolation Method

In the extrapolation technique, the travel-time data is estimated with the use of spot velocities measured by ILD. The average velocity between ILD locations is extrapolated to determine the average travel-time along the segment. Velocity can be directly measured in case of dual loop detectors. However, in case of single loop detectors, velocity is often calculated on the basis of flow, occupancy, and average effective vehicle length. The last parameter is usually calibrated during light traffic conditions by imposing a value for free flow velocity. Hall and Persaud [33] suggested that effective length might be prone to a systematic bias with respect to occupancy.

The extrapolation method is simple and can be used for applications that do not require high levels of accuracy. One of the inherent assumptions of this technique is that velocity within a segment can be closely represented by the extrapolation method. Some of the common extrapolation methods include half-distance approach, average velocity approach, and minimum velocity approach, piece-wise constant, piece-wise linear approach, and piece-wise quadratic approach [34, 35].

Several researchers have evaluated various extrapolation techniques with several alternative techniques such as inverse velocity based trajectory method, Kalman filtering, etc. [36, 37, 38, 39, 40, 41, 42, and 43). The main disadvantage of the extrapolation methods is that the accuracy of the estimates decreases with increasing flow conditions.

#### 2.1.2.2 Cumulative Count Method

Another approach for travel-time estimation is based on the cumulative arrivals at successive detector stations assuming First In First Out (FIFO) mechanism [44, 45, 46, 47, and 48]. The cumulative number of vehicles to cross the two loop detectors located at either end of a link are measured to determine the link travel-time. If one were to plot the cumulative vehicle count vs. time of these detectors then the travel-time on the link is the distance between the two curves along the time axis.

Nam and Drew [47] developed two different equations for travel-time calculations, one for uncongested and one for congestion conditions. Rakha and Zhang [49] later corrected Nam and Drew's equations and showed that delay computations for shockwave analysis and queuing theory were consistent. Using rescaled cumulative curves, Yeon and Elefteriadou [50] examined the accuracy of these above methods comparing with the field-measured travel-time. Yeon and

Elefteriadou noted that all three approaches give similar results for freeway sections without entering/exiting ramps. The authors showed that with the presence of ramps between detectors, all the methods perform inadequately on certain segments.

However, this method has several practical problems. First, one needs to know the number of vehicles initially located between the two detectors before the algorithm begins (a quantity which is not readily available). Second, loop detectors are notorious for over- and under-counting vehicles. Hence the cumulative flow lines may systematically drift over time giving inaccurate estimates over time (the lines may even cross).

#### 2.1.2.3 Theoretical Methods

Dailey [51] used cross-correlation of the flow at the upstream and downstream detectors to estimate travel-time between two single-loop detectors placed 0.5 miles apart. While this method is robust, this method requires constant calibration during changes in traffic regimes to provide accurate results. Petty et al. [52] suggested a model for estimating travel-time directly from flow and occupancy data, based on the assumption that the vehicles that arrive at an upstream point during a given interval of time have a common probability distribution of travel-times to a downstream point.

All the methods described above estimate Instantaneous Travel-time and Reactive Travel-time. The following subsection describes travel-time forecasting methods used to calculate the Predictive Travel-time.

### 2.2 Travel-time Forecast Algorithms

As show in Figure 1, there are two major categories of methods for forecasting travel-time; statistical methods and simulation based methods. The statistical methods use statistical tools on the historic and real time data to derive models to predict travel-time. On the other hand, simulation methods use traffic flow models and congestion propagation phenomenon to forecast travel-time

#### 2.2.1 Statistical Methods

The statistical models can be broadly divided into four categories; Regression models, Time series models, Artificial Neural Networks models, and Kalman filter based models.

#### 2.2.1.1 Regression Models

These models predict travel-time using linear regression with a stepwise variable selection method [53, 54]. These models typically use historical and/or real-time flow and occupancy data from the detectors as explanatory variables to determine the response variable, travel-time. Kwon et al. [55] used linear regression and advanced statistical methods such as tree methods to develop models for predicting travel-time. Later, Rice and Van Zwet [56] and Zhang and Rice [57] proposed methods to predict freeway travel-time using a linear model in which the coefficients vary as smooth function of the departure time. Instead of using flow and occupancy, Chakraborty and Kikuchi [58] used bus travel-times and developed a simple linear equation using regression to predict automobile travel-time based on the bus travel-time.

Some researchers also fit the historic travel-time data to probabilistic distributions and used them to predict travel-time (53, 59, 60, and 61). Others have used historic velocity profile and location data to develop prediction models [6, 63, and 64].

However, these models are found to perform well during short-term prediction horizon under normal traffic condition but are less accurate during congestion onset and congested conditions. Also, there models perform poorly in the presence of incidents and other special traffic conditions.

#### 2.2.1.2 Time Series Models

These models utilize the time series information of velocity, flow, occupancy, and/or travel-time data to derive predictive models. Several researchers used flow and occupancy to derive auto regressive prediction model [65, 66]. D'Angelo et al. [67], Al-Deek [68], and Ishak and Al-Deek [4] later used nonlinear time series with multifractal analysis to develop prediction models. The univariate time series models have been found to provide superior performance in accurately predicting travel-times compared to many complex algorithmic and hybrid methods developed in recent times. However, to improve the accuracy of predictions obtained by the univariate models, Stathapoulos and Karlaftis [69] developed a multivariate time series model using traffic flow parameters. Kamarianakis and Prastacos [70] compared the forecasting performance of two univariate and two multivariate models and found that multivariate time series models performed better than univariate models for short term traffic predictions.

On the other hand, Williams and Hoel [71] used the Box and Jenkins technique, more specifically, Autoregressive Integrated Moving Average (ARIMA) technique to develop a one-step prediction model for traffic flow at smaller time steps. Later, Guin [72] developed an ARIMA model to predict travel-time and found that the models showed a strong weekly seasonality and produced reliable predictions on a short time scale. While the time series models are found to perform better under normal flow conditions, their performance was found to deteriorate rapidly with the onset of congestion, or under any unusual conditions.

#### 2.2.1.3 Artificial Neural Networks Models

Artificial Neural Network (ANN) is a non-linear statistical data model consisting of an interconnected group of nodes that processes information using a connectionist approach to computation. Cherrett et al. [73] reported the use of a feed-forward ANN model for the prediction of link journey time. Later, Ohba et al. [74] proposed a travel-time prediction model using a mixed structure type neural network system. Park and Rilett [75], Park et al. [76], Rilett and Park [77], and Kisgyorgy and Rilett [3], suggested modifications such as clustering techniques, modular neural network, expanded input nodes, and spectral basis neural network to ANN to account for nonlinear nature of travel-time data for prediction. Kisgyorgy and Rilett's models have indicated that the travel-time prediction error can be as small as approximately 4%.

Several researchers have used advanced methods in neural networks to develop travel-time prediction models. Matsui and Fujita [78] used fuzzy reasoning and You and Kim [79] used nonparametric regression and GIS technology to predict travel-time. Jiang and Zhang [8] used mix-structure neural network model that can predict travel-time of roadway segments without detectors, based on the data from segments with detectors. Similarly, Van Lint et al. [44] investigated using state space neural networks, Wei et. al. [81] used advanced neural network, and Dharia and Adeli [82] used counter-propagation neural networks, for the forecasting of freeway link travel-time. Though these ANN models produced accurate results, developing these models is complex and needs good calibration.

#### 2.2.1.4 Kalman Filter Models

The Kalman filter [83] is recursive estimator used for filtering measurements that are observed over time and contain noise and other inaccuracies to estimate true values of measurements and their associated calculated values. It also produces forecasts by predicting a value, estimating the uncertainty of the predicted value, and computing a weighted average of the predicted value and the measured value. Thus, this method enables the prediction of the state variable to be continually updated as new observation becomes available.

Szeto and Gazis [84], Okutani and Stephanedes [85] and Okutani [86] used Kalman filtering to estimate traffic volumes, density and trip distribution. Later Stephanedes and Kwon [87] used Kalman filtering to determine real-time demand diversion. Chien et al. [88] used Kalman filtering algorithm on a combination of historical and real-time data for short term prediction of travel-time. They found that predictions using link-based travel-times are more accurate than prediction using path-based travel-times. Kuchipudi et al. [89] developed a model in which both path-based data and link-based data are used to predict travel-times. The travel-times were chosen based on the prediction error obtained using those two data sources. Later researchers have developed adaptive least squares method, which was a special case of the Kalman filtering approach.

Thus, all the statistical methods can be categorized as data driven approaches that treat traffic dynamics as 'black boxes' and use statistical relations from past data (e.g. volumes, velocities, densities, travel-times, etc.) to infer future travel-times. The fundamental deficiency of these methods is that they cannot provide satisfactory prediction during non-recurrent congestion due to work zones, incidents, and special events, during which travel-time prediction is more important.

#### 2.2.2 Simulation Methods

On the other hand traffic simulation models use traffic flow knowledge to predict the traffic conditions on the corridor, and integrate mathematical algorithms to predict travel-time. There are three types of simulation models used for travel-time prediction, Macroscopic, Mesoscopic, and Microscopic simulation models. The following paragraphs describe METANET, DynaMIT, and Cellular Automata based models that belong to the above categories of traffic simulation models.

#### 2.2.2.1 Macroscopic Models

Macroscopic models treat traffic flow as fluid streams. The traffic flow characteristics such as density, flow, and mean velocity are the average values of the traffic stream. The most popular macroscopic model is the LWR model, a first order model developed by Lighthill and Whitham [90] and Richards [91]. Later higher order models were developed to overcome the limitations of the first order model. METANET is a macroscopic simulation model based on second order traffic flow model [92].

In addition to the traffic simulation, METANET also has the capability of taking control actions such as ramp metering and route guidance. METANET has two distinct modes of operation, with or without destination-oriented mode. Since this is a macroscopic model, variables such as density, velocity and flow, represent the average behavior of traffic at certain times and locations. The time and space arguments are discretized. See [93] for a detailed description of the model.

This simulation model is used to model A10 ring road motorway of the Amsterdam network in both directions [94]. The total length of the network is 143 km and engulfs Amsterdam. The total number of links that was used to model the motorway network is 654 and divided into a total of

291 segments. The length of each segment ranges from 400 to 800 m. This network is used to capture the congestion dynamics and develop control strategies and provide ATIS information to commuters. One of the drawbacks of macroscopic models is that these models only produces aggregate measures and do not guarantee replicating driver behavior at microscopic levels.

#### 2.2.2.2 Mesoscopic Models

While macroscopic modeling deals with average values of traffic parameters, microscopic models deal with individual car behavior. Mesoscopic models are intermediate and combine the properties of both microscopic and macroscopic simulation models. These models simulate individual vehicles, but describe their activities and interactions based on aggregate (macroscopic) relationships at the network link levels. DYNAMIT is a mesoscopic simulation model developed at MIT for ATIS applications.

DynaMIT is a simulation developed by Ben-Akiva et. al. [95] that is used for real-time traveltime estimation and forecasting. It is designed to estimate current state of transportation network and predicts future traffic conditions. This system combines real-time traffic data from surveillance system and historic data for estimation and prediction purposes. The system takes data from historical database and real-time surveillance system as an input. The demand simulator estimates and predicts time-dependent origin-destination matrices and driver decisions in terms of mode and route choices. The supply simulation emulates the interaction between demand and network. The state estimation is done through an iterative process of these two simulators (demand simulation and supply simulation) till it reproduces the real-time surveillance system data. Even though mesoscopic models are relatively more detailed compared to the macroscopic models, they still do not replicate driving behavior at microscopic levels. To overcome this drawback several researchers have used microscopic models.

#### 2.2.2.3 Microscopic Models

Microscopic simulation models are dynamic and model individual vehicle movements within a transportation network. Each vehicle is moved through the network according to the physical characteristics of the vehicle (length, maximum acceleration rate, etc.), the fundamental rules of motion (e.g. acceleration times time equals velocity, velocity times time equals distance) and rules of driver behavior (car following rules, lane changing rules, etc.). Cellular Automata (CA) models are popular microscopic simulation models used for traffic applications.

CA models were first proposed by Von Neumann in 1952 [96]. They were later used for transportation by Cremer and Ludwig in 1986 [97]. CA models have become popular in transportation applications after Nagel and Schreckenberg [98] proposed simple steps to reproduce traffic dynamics on transportation networks. Since then, CA models been widely used to simulate traffic networks [98, 99, 100, 101], freeways [102], intersections [103], roundabouts [104], and toll stations [105]. Due to their design using simple conditions, cellular automata models are very efficient for simulation of large-scale networks.

Lane changes are incorporated using a variable,  $l_n \in \{\text{left, right, straight which notes if the vehicle} n should change the lane during the actual time-step or not, as explained in detail in [106]. In other words, the lane changing logic works as follows: first, a vehicle checks if it is hindered by the predecessor on its own lane. Then it has to take into account the gap to the successor and to the predecessor on the target lane. If the gaps allow a safe change the vehicle makes a lane change. A systematic approach to lane-changing rules on multi-lane roads is given in [107, 108, 109, 110].$ 

CA based simulation is extensively used in Germany as a part of their autobahn information system [111]. The details of this CA based system can be found at www.autobahn.nrw.de, where the simulated actual traffic state on the freeway network in North Rhine Westphalia is presented. This estimation and forecasting system uses data input from more than 4,000 loop detectors that are installed on the autobahn and deliver minute by minute data. The traffic state information from the CA model is adjusted in accordance with measurements of the real traffic flow provided by the loop detectors. The transportation network comprised of 3,988 links, 830 on- and offramps, and 67 intersections. The overall length of the lanes is approximately 12,200 km, corresponding to more than 8 million cells. This CA model is currently used to graphically present real-time traffic state information and also a prediction time horizon of 30 minutes. CA based models are also developed to estimate traffic state and incident-related travel-time on freeways in US [112,113,114, 115 and 116] used CORSIM to predict travel-time for real world networks, but the latest advances in traffic flow models will be able to replicate the real-world conditions better than the traditional models that cannot reproduce the capacity drop phenomenon and relaxation phenomenon observed on the freeways.

## **3** Prediction Framework

The prediction framework for real-time travel-time estimation and prediction is shown in Figure 2. The two main components of this framework are the micro-simulation and traffic sensor infrastructure on the freeways. The micro-simulation will run at faster than real-time using the data reported by the traffic sensors to give travel-time estimates and predictions.

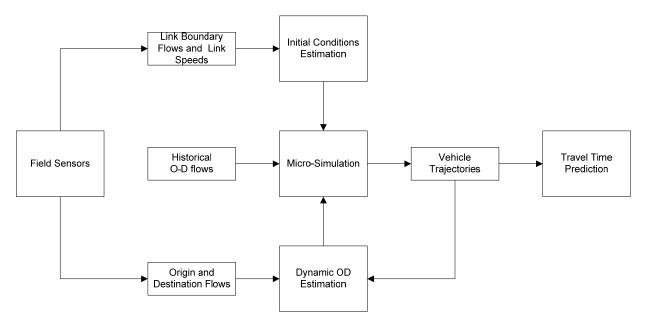


Figure 2. Real-time Travel-time estimation and prediction framework

The traffic sensors mentioned above could be inductive loops (IDL), video based systems or any other system that collect volume and spot speeds on the freeways. These sensors provide time series of traffic volumes and velocity at both the boundaries of the network and also on mid-segments.

The traffic volumes at the network boundaries are used for estimating dynamic origin-destination matrices and the traffic volumes and velocity on the mid-sections are used to determine initial queues used for the simulation. The vehicle trajectories produced by the simulation are used by the dynamic OD estimation module described in section 6.1 to generate OD matrices for the next simulation run. The dynamic OD matrices are validated with the historic OD matrices before

using as input in the next simulation run. Also, the vehicle trajectories are used to generate traveltime forecasts.

In general, the forecast horizon is an important parameter in any prediction framework. There are two kinds of forecast horizons, short-term and long-term. Short term prediction typically represents prediction of five to ten minutes into the future. Long-term prediction refers to longer range prediction, typically in the order of 30 minutes to few hours. However, during non-steady state traffic current traffic data quickly becomes less relevant to make long-term predictions and historical experience could be supplemented to improve the prediction quality.

This framework can be written as an algorithm as follows:

- Let a new simulation be initiated (with new OD flows) at time t = h, 2h, 3h,..ph...nh with prediction horizons are r, 2r, 3r,...mr where n, h, m, r and p are integers.
- Therefore, at any time t = ph, travel-time predictions are made for t = ph+r, ph+2r, ph+3r,...ph+mr
- 3. For each simulation run perform the following steps:
  - Calculate initial queue using queuing analysis on each link. The total number of arrivals and departures are continuously measured across boundaries to calculate the initial queue at the beginning of every simulation. The average speed for each link is estimated based on the speed sensor observations.
  - Calculate OD matrix using the travel-time predictions made for each origin and destination set in the previous simulation run to calculate the corresponding flows on the ramps. These flows are used in the OD estimation algorithm described in section 6.1 to calculate OD flows.

- 4. Derive vehicle trajectories to make travel-time predictions for different prediction horizons
- 5. Repeat step 3 for each simulation run

If the traffic conditions are stable, the prediction horizon and OD flows changeover period can be sufficiently large to make reliable predictions. However, during the periods of congestion buildup and dissipation, it is desirable to have small h (<<mr) since the operating conditions change rapidly making the long term predictions less reliable. Also, inaccurate travel-time predictions make new OD matrix calculations less reliable.

More details of how this framework was implemented in this study will be described in chapter 6.

# **4** Data Collection and Preliminary Analysis

This section describes different types of data needed for this study and data collection methodology used in this study.

After consulting with GaDOT personnel and based on the data analysis performed during the Ramp Metering project Phase I, a 6.5 miles long EB/SB I-285 segment between GA-400 and I-85 was selected for this study. The study corridor has the following five entry locations (called origins for the OD terminology) to feed traffic to the network:

- Upstream Freeway
- Peachtree Dunwoody Road
- Ashford Dunwoody Road
- North Peachtree Road
- Peachtree Industrial Pkwy

Similarly, the corridor has the following eight exit locations (called destinations) as shown in figure 3:

- Ashford Dunwoody Road
- Chamblee Dunwoody Road
- SB Peachtree Industrial Pkwy
- NB Peachtree Industrial Pkwy
- Buford Hwy
- SB I-85 connector

#### • NB I-85 connector

#### • Downstream Freeway

Based on the congestion characteristics analysis performed as a part of the Ramp Meteting Phase I project (TO 02-49; RSCH PROJ 07-22), this analyses focus on the congested evening peak period. Within the 6.5 -mile study corridor, there are 21 GDOT detection stations that collect velocity, volume and occupancy data. All the four on-ramps on the corridor were metered and the volume data at these ramps were polled out over the TACTICS framework. However, initial observation of the TACTICS data and the data obtained from GDOT's video detection system (hereafter referred to as the NaviGAtor data) revealed that these systems were recording unreasonable ramp volumes. To verify this, three ramps (two off-ramps and one on-ramp) from the test corridor were video recorded during evening peak period and the volumes were manually counted to compare against the NaviGAtor and TACTICS data.

Results indicated that there are significant discrepancies in vehicle count observations and the data reported by these stations. The quality of the data was not suitable for use in a micro-simulation based travel-time prediction framework. Therefore, based on discussions with GDOT, the research team proceeded with manually collecting data for the rest of the study while GDOT worked towards improving the quality and availability of data from the TACTICS framework. GDOT's PTZ cameras were used to record all the entry and exit points on the corridor and the videos were processed manually to extract the time-series of traffic volumes on the study corridor. The next section describes the traffic volume data collection procedure in more detail.



Figure 3: Study Corridor (background: http://maps.google.com)

#### 4.1 Traffic Volume Data Collection

For this research thirteen PTZ cameras were used to record the five entry locations and eight exit locations from 14:00 to 20:00 on Wednesday, March 7, 2012. Time-series of traffic volumes were manually extracted from the videos using the tablet based traffic counting application developed at GaTech [117]. Oblique curves were plotted and extensively examined to verify the quality of the data. To minimize chances of data loss one of the researchers was stationed at the GDOT's Traffic Management Center during the data collection period to ensure that the PTZ cameras maintained the view required for the counts. While most of the cameras recorded for the 6 hours, one of the cameras only recorded from 15:00 to 17:00. Therefore, this study was performed for only the duration that all six required cameras were continuously available.

It was observed that during the study duration the corridor was initially free flowing and the starts to get congested (can be seen from the Bluetooth data in Figure 5) initially due to spillback from the NB I-85 off ramp and later due to weaving and merging at various locations. By the end of the study duration, the congestion on the freeway encompasses all the ramps in the study corridor. It was observed that the Peachtree Industrial Pkwy on-ramp dumped a lot of traffic on the freeway (2-lane ramp). However, due to its proximity to the spaghetti junction, the queue spill back from I-85 ramps disrupted the smooth merging of the ramp traffic and resulting in a queue downstream of the ramp meter that sometimes spilled to the ramp meter upstream. Moreover, huge inflows from the ramp and close proximity of the off-ramp downstream (Buford Hwy off is 0.5 miles downstream) significantly increased lane changing activity in this vicinity.

To calibrate and validate the simulation model, actual travel-time data is collected in the field during the traffic volume data collection period using Bluetooth sensors. The next section describes the Bluetooth data collection procedure.

## 4.2 Bluetooth Data Collection

Bluetooth travel-times were collected from three overpasses along I-285; Perimeter Center Parkway NE, Chamblee Dunwoody Rd, and New Peachtree Rd as shown in Figure 4. Bluetooth data was collected on the same day as the traffic volume data collection, Wednesday, March 7, 2012 from 13:45 to 20:00. Bluetooth data was collected simultaneously from all the three overpasses. At each location, the team deployed the Bluetooth equipment on the sidewalks of overpasses, with Bluetooth readers facing oncoming traffic. A total of six Bluetooth adapters, each mounted on a separate tripod, were used at each location Each tripod was setup at least 10 feet apart to capture all the lanes on the freeway. At each site, the equipment was setup in such a way that it minimizes equipment visibility and any distraction to drivers on I-285. During the Bluetooth data collection no personally identifiable information was collected by the field team.

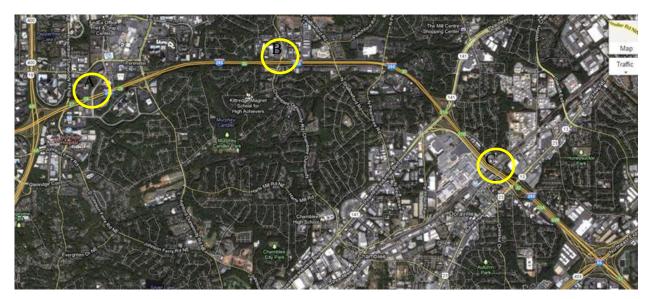


Figure 4: Bluetooth Data Collection Locations (background: http://maps.google.com)

Figure 5 shows actual travel-time data collected using Bluetooth technology. The data is plotted in three graphs, the travel-time from A to B, B to C, and A to C (see figure 4 for A, B, and C locations). Figure 5a shows that the segment from A to B does not get significantly congested till 15:30. However, the congestion continues to increase the travel-time on this segment till 16:07 and then stabilizes (indicating queue spilled upstream of A). However, unlike travel-times on A to B, travel-times on B to C and A to C get congested shortly after the 15:00. The travel-times increase till about 15:55 and then stabilizes. While the Bluetooth data looks reliable for calibration and validation purposes, a closer look at the data reveals that the capture rate of Bluetooth is very low. On each of the three segments (AB, BC, and AC), during any of the 5 minute intervals, there are anywhere between 8 to 35 vehicles that were captured by Bluetooth on a 5-lane freeway. Therefore, during some of the 5-minute analysis periods, the Bluetooth sample sizes were small enough to raise data reliability concerns.

It appears from the Bluetooth data for the corridor that around 16:25, traffic congestion alleviated and travel-time improved temporarily. A closer look at the Bluetooth data revealed that during that 5-minute period, only 8 vehicles (out of a population of almost 800 vehicles) were recorded indicating that the Bluetooth has captured insufficient sample size to reliably measure the average travel-time during that time period (during other surrounding time periods, at least 21 vehicles were recorded). To confirm any temporary alleviation in congestion during that time period, video recordings were observed. Visual observation of videos did not indicate any evidence to show temporary alleviation of congestion on the corridor. Thus, it is determined that the temporary drop that appeared in the blue tooth data is an artifact of insufficient data.

One of the drawbacks of the Bluetooth data collection methodology is the inability to identify lane information for the vehicles captured. Moreover, a higher propensity of Bluetooth sensors to capture slower vehicles that spend longer time in the Bluetooth detection zone also creates an appearance of higher variability in the data and biases the data towards longer travel-times.

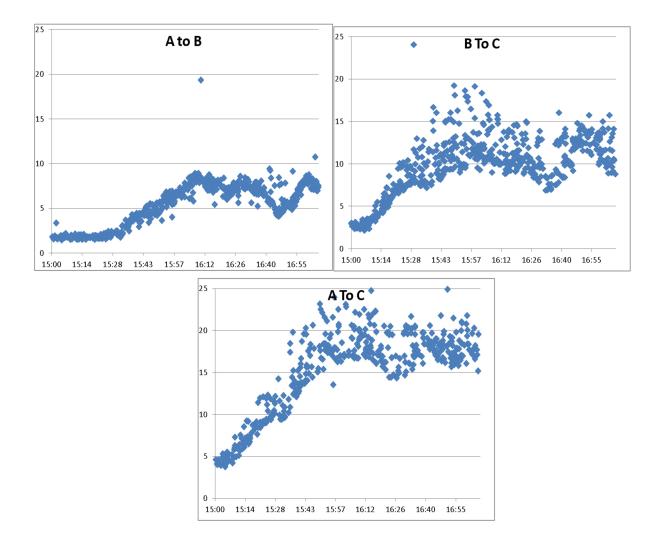


Figure 5: Actual Travel-time Data collected by Bluetooth devices

# 4.3 Velocity Data Collection

The traffic volume data for this study was manually collected from videos and no corresponding speed data could not be directly extracted. NaviGAtor stations automatically report both flow and speed velocity, albeit questionable quality [118]. The errors in the count data is more severe in the context of this study because the errors are cumulative, while the errors of the speeds can be

accounted for, to some extent, by using appropriate calibration factors. For lack of a better source, speed data obtained from the NaviGAtor system was used in the study. The time space speed plot of the corridor shown in Figure 6 supports the observations from Bluetooth data that the corridor was initially almost free flowing and congestion built up during the analysis period to the point where the entire corridor was congested by the end of the study period.

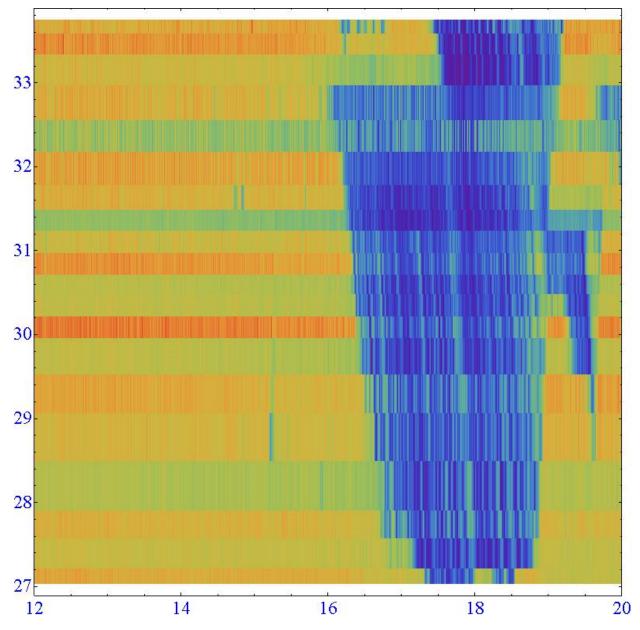


Figure 6: Time space speed plot of the study corridor on March 7, 2012

It should be seen in Figure 6 that the study corridor was an isolated bottleneck during the entire study period (15:00 - 17:00). The spillback from downstream freeway onto the study corridor happened around 17:30 (as seen in Figure 6) but data beyond 17:00 was not used in this study. To capture the impact of the downstream bottleneck and extend the study beyond 17:30, the extents of the simulation needs to be extended further downstream.

Figure 7 shows a sample time-series of the speed of a NaviGAtor station #2850032 during the study period. This station is located basic freeway section, not influenced by on-ramp and off-ramp. The time series plot indicates that the speed during the free flow was fairly constant, but once the congestion sets in the speeds fluctuate significantly indicating presence of traffic oscillations.

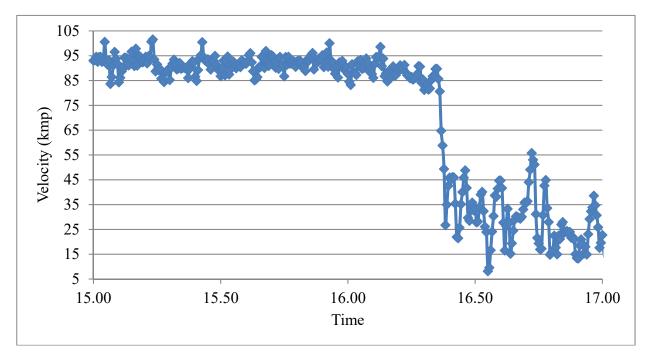


Figure 7: Time Series of Speed of the NaviGAtor station #2850032

The time series of velocity data at each of the 21 stations was utilized to derive average velocity on the mid-segments. More details on how this data was utilized in this study are described in Chapter 6.

Next chapter describes the modules of the micro-simulation developed for this study.

# 5 Traffic Flow Simulation Application

It is well known that the existing off-the-shelf traffic simulation softwares are deficient in simulating congested traffic dynamics on freeways. This is mainly due to insufficient inbuilt models and absence of flexibility to allow users to change the models.

Georgia Tech has developed a micro-simulation application, called hereafter **GTsim**, which has the flexibility to accommodate future improved models to accurately model real-world conditions. This application includes the latest advancements in lane changing models that are capable of explaining congestion dynamics such as capacity drop. It also has several driver behavioral models for mandatory lane changes to replicate real-world behavior during congested traffic conditions.

GTsim was developed for a Windows XP based operating system and is currently in the process of upgrading to newer Windows based operating system. The application has been written in JAVA using the NetBeans IDE 6.01 [119].

## 5.1 Process Flow

Figure 8 shows the process flow for the GTsim application. The classes shown on the left form the critical modules of the application. The methods shown in the center are some of the critical functionalities of the modules on the left. Lastly, the supporting classes provide some additional functionality for the modules.

The framework for GTsim is setup such that every time the simulation is initiated, it creates an instance of simulation which calls various modules during every time-step to update each vehicle's velocity, location, and animation. Thus this framework allows writing new methods for

any future models to replace the old models and facilitate easy implementation of new models in the simulation. The next section describes some of the details of the critical modules.

# 5.2 Critical Modules

The application includes the following critical classes that are interdependent and form the core of the application:

- Traffic Simulation Application
- Simulation
- Connection
- Lane
- Vehicle
- Intersection

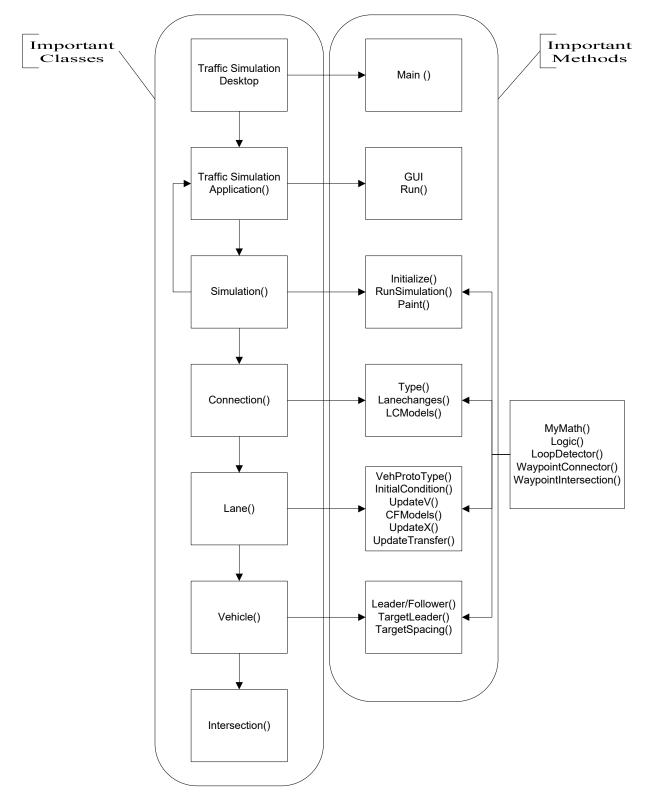


Figure 8: Process Flow for the Simulation Application

# Traffic Simulation Application

This module has all the application's user interface with background map as shown in Figure 9. The GUI has the provision to Navigate to any part of the map, zoom in/out capability, add/erase connections and intersections, start/pause/stop simulation, create new/open existing/save a network model, and several other features.

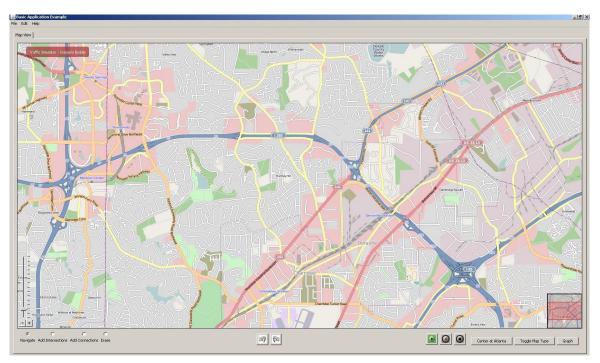
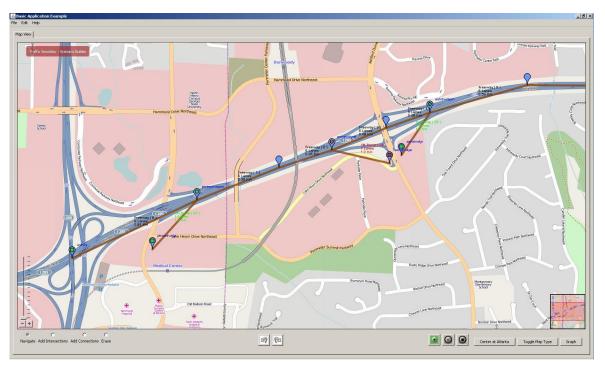


Figure 9: Transportation Network background for the Application



#### Figure 10: Sample Network

Every network is built with the help of intersection and connections (just like edges and vertices in a directed graph). The application allows eight different intersection types (on-ramp with lane merge, on-ramp with lane addition, exit with shared lane, exit with a lane drop, exit with two lanes drop, Origin, Destination, and Generic) and six connector types (on-ramp with lane merge, on-ramp with lane addition, exit with shared lane, exit with a lane drop, exit with two lanes drop, on-ramp with lane addition, exit with shared lane, exit with a lane drop, exit with two lanes drop, and Generic). For each connection and Intersection several geometric and operational parameters could be defined. Figure 10 shows a sample coded network.

After the network is coded/loaded, clicking the play button initializes the Origin-Destination (OD) matrix. The default OD matrix can be changed using the GUI or can be read from a text file. After the OD matrix is loaded, the simulation will display an animation window as shown in Figure 11. The animation of vehicles on any given link can be viewed by clicking on the desired link on the network.

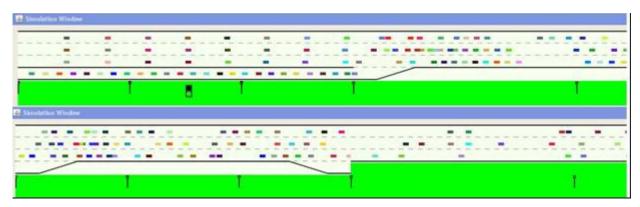


Figure 11: Sample Animation snapshot

#### Simulation

This class has methods to initialize the network connectors, intersections, loop detectors, and ramp meters. This class also calculates the shortest path (sp) for each pair of source and destination nodes. This class calls various updating functions on each connection in the network for each time period. Also this class contains methods for displaying the lanes and repainting the cars during every time step.

#### Connection

This class has the constructors to create each of the six connection types. The constructors also call for lane instances for each connection. Two important methods of this class are lane changing decisions and making the lane changes. The first method checks for both discretionary and mandatory lane changes.

The discretionary lane changes are done according to the lane change model that replicated capacity drop phenomenon [8]. In addition to this model, this class also has other lane changing models such as gap acceptance model 1, gap acceptance model 2, and relax model and allows the user to select the models for the simulation.

Since there is limited literature on mandatory lane changing behavior, the team developed several new models to replicate driver lane changing behavior observed in the real-world. Earlier, simpler models for the mandatory lane changes resulted in either some vehicles not able to exit in time

and resulting in "errors" or exiting vehicles unrealistically changing lanes and accumulating on a lane. To overcome these deficiencies, the following models were incorporated in the simulation that resulted in replicating realistic driver behavior:

*Threshold Model*: The location of the mandatory lane change is determined by the vehicle's distance from the exit and a lane-change-distance threshold. The threshold is dependent on the lateral distance of a lane from the exit. Thus, the threshold for left lanes is higher than the right lanes and varies as lanes are added (due to on-ramp) or dropped (due to exit) on the freeway. Moreover, since some of the exits on the study corridor were very close to each other (distance between SB Peachtree Industrial off ramp and NB Peachtree Industrial off ramp is less than 200 meters), different functions were used to determine the threshold values for different segments connecting serving the exits.

*Velocity Drop Model*: It was observed in the videos that when vehicles want to make a mandatory lane change, but cannot complete lane change due to traffic conditions on the destination lane, they tend to slow down to ensure they have enough time and distance to safely change lanes and reach the their destination exit. To replicate this behavior in the simulation, we incorporate this model by which whenever a mandatory lane change maneuver is unsuccessful, the vehicle reduces its velocity at a rate of a lane change need (between 0 to 1, 1 being high need) \* velocityfactor (parameter to be calibrated). So if the vehicle is very close to the exit and it cannot make a lane change, it reduces its velocity more than a vehicle wanting to make a lane change far from exit and was unsuccessful.

*Exit Lane Model*: Vehicles approaching an exit lane (not their exit) will gradually shift to the left lanes depending on its distance from the nearest exit.

*Long Haul Model*: If an entering vehicle is destined to exit at least 2 miles from its entry location, then it will tend to move to left lanes if the velocity differential between the velocity in the adjacent lane and existing lane d/s velocity is perceived to be over a threshold. Moreover, the attraction factor for each lane is different and decreases from right lanes to left lanes.

#### Lane

This class had provision to create four types lanes; normal lane, exit lane, auxiliary ramp lane or auxiliary exit lane. This class also has methods to assign vehicle properties and generate appropriate number of vehicles for each origin and destination combination. Moreover, this class reads a text file to load the initial queues on the network before the simulation begins. Three important functions this class performs are updating the velocity and location of all the vehicles on a lane and transfer them to appropriate lane across connections. The application currently incorporates the relaxation phenomenon [8] for calculating the velocity of the vehicles.

Moreover, the team has incorporated a new Friction Model to capture the influence of operating conditions in the adjacent lane velocity on vehicle's accelerating and deceleration behavior. To account for this behavior, we use a SpeedReductionFactor that will reduce the velocity of vehicles if the adjacent lane is congested (if velocity differential is more than a threshold).

The next section describes the methodology used in this study.

# 6 Methodology

The study was conducted for two hour period, from 15:00 to 17:00. Since the study duration encompassed congestion buildup and sustenance period, it was decided that the OD matrix will need to be changed frequently; every five minute. Therefore, in a two hour period, the OD matrix was changed 25 times and a new simulation was initiated each time. For each simulation run, the prediction horizons were decided to be 5 minutes, 10, minutes, 15, minutes, 20 minutes, 25 minutes, and 30 minutes into the future. Therefore, for example, a simulation initiated at time t = 15:45 will give travel-time predictions for t = 15:50, 15:55, 16:00, 16:05, 16:10, and 16:15.

Due to the constraints on the data availability and data quality, the study was performed in an offline fashion. The OD matrices and initial queues for all simulation runs were determined offline before the simulation runs were made to generate travel-time predictions. Moreover, a slightly different methodology was adopted to calculate initial queue and OD matrix estimation as described below:

### Initial Conditions

- Initial queue: Since the time series of the boundary flows were extracted from videos instead of data from the NaviGAtor system, the total number of vehicles on each connection at the beginning of each simulation run (initial condition) is not directly known. Therefore, queuing analysis was performed by shifting the cumulative arrival and departure curves across all the connections for the whole study duration based on the average travel-time during each of the 25 intervals to find the initial queue.
- Vehicle speed: The speed of the initial queue was obtained from NaviGAtor system's speed data.

• Origin and destinations: The origins and destinations are needed to be defined for each vehicle in the initial queue. This is done by iteratively estimating the cumulativeProtoProbTable (cPPT) for each connection in the network. cPPT stores the cumulative prototype-probability values to generate appropriate number of vehicles between any given origin and destination. cPPT is estimated for each connection starting from upstream freeway and moving to downstream connections. While crossing an on-ramp/off-ramp the cPPT is updated with new inflows from/to the ramp. The cPPT for a connection is assigned to all its lanes. Thus, once the initial queue is determined, the queue is distributed across all the lanes and their origins and destinations are defined based on its cPPT.

#### OD calculation

• The average travel-time between every origin and destination pair for every simulation interval is calculated using the NaviGAtor speed data. This information is used to determine the exit ramp flows for use in calculating the OD flows. For example, to determine the OD matrix for 15:45, the flow from 15:45 to 15:50 at the upstream freeway is used. The flow on the other entrance and exit points is determined as the flow observed on the corresponding ramp between the time the first and last vehicle from the upstream freeway reaches the corresponding entrance or exit. The entrance and exit flows are used in the algorithm mentioned in section 6.1 to calculate the OD flows. The same process is repeated for other time periods.

Using the corresponding initial queue and OD flows, simulations are run to derive travel-time prediction. The next section describes the OD estimation algorithm developed in this study.

# 6.1 Origin Destination Matrix Estimation Algorithm

Underlying the Origin Destination (OD) estimation problem is the following measurement equation:

 $\Delta q = x$ Where

q is an  $0 \times 1$  vector of travel demands to be estimated;

x is an  $m \times 1$  vector of measured traffic counts

 $\Delta$  is an m×o assignment matrix whose entry represents the proportion of trips

When congestion effects are unimportant, the assignment matrix  $\Delta$  may be exogenously specified. This is known as the proportional-assignment approach. Proportional-assignment methods assume that users' route choices are given and independent of the estimation process. This method is typically used for planning purposes and for short-term operational analyses, time-varying OD matrices estimation is extremely useful.

Time-varying or Dynamic OD estimation is typically done by either statistical methods or using Dynamic Traffic Assignment (DTA). While the former methods are computationally easier, the later incorporates driver behavior/response to traffic conditions. The usual procedure to obtain OD matrices is to estimate them indirectly from the traffic volumes they induce on the links of the network. The latter can be easily measured using standard surveillance equipment. Of course, the estimation procedure would also include any prior information that is available.

Dynamic OD matrices can be estimated in two ways. The first one involves estimation of a set of time-dependent O-D matrices given a time-series of link volumes (and other information such as travel-times, historical O-D flows, etc.). The second method involves O-D estimation in tandem

with a DTA within a real-time traffic management system. For the purposes of this study, the first method is used to calculate time-varying OD matrices for the corridor. The algorithm used is developed based on the least-squared error of observed and estimated flows. Details of this algorithm and limitations are discussed below.

Let the

Set of origins be  $i \forall i = 1,2,3 \dots n$ 

Set of destinations be  $j \forall j = 1,2,3 \dots m$ 

Set of links be  $k \forall k = 1,2,3 \dots p$ 

 $\alpha_{ijk} = \begin{cases} 1 & \text{ if link } k \text{ in the path from origin } i \text{ to destination } j \\ 0 & \text{ otherwise} \end{cases}$ 

Flow measured by traffic sensor on a link be  $q_k \forall k = 1,2,3 \dots p$ 

Flow estimated between an origin *i* and destination *j* be  $q_{ij} \forall i, j$ 

Flow estimated on a link  $q_k^*(t) = \sum_{i=1}^n \sum_{j=1}^m q_{ij} (t - \Delta_{ik}) \alpha_{ijk}$ 

 $\Delta$  is the travel-time between the origin *i* and the link *k* 

The optimization problem is to calculate  $q_{ij} \forall i, j$  so as to

Minimize 
$$\sum_{i=1}^{n} \sum_{j=1}^{m} (q_k - q_k^*)^2$$

subject to the constraints:

$$q_{ij} > 0 \quad \forall i, j$$
  
 $\sum_{j=1}^{m} q_{ij} \leq q_i \forall i$ 

$$\sum_{i=1}^n q_{ij} \leq q_j \,\forall j$$

It should be noted that while this algorithm is simple and intuitive, it is important to use the correct travel-time to estimate the flows. Incorrect flows will result in erroneous OD flows. Since the origin-destination problem allows multiple solutions for a given error threshold, field observed origin-destination flows cannot be directly obtained using this methodology. It requires a careful calibration process to determine accurate origin-destination flows. However, since the travel-times are more governed by traffic dynamics and not significantly affected by small variations in OD flows, this method is appropriate for this framework.

This algorithm is implemented in an excel spreadsheet to calculate origin-destination flows given field-measured flows on the on-ramps, off-ramps and mid-block section on the freeway.

## 6.2 Calibration and Validation

The network was built in GTsim using the data collected in section 4 which included information such as Number of lanes, distance between ramps, ramp configurations, and auxiliary lanes and their lengths, etc. A total of 35 intersections and 33 links were used to build the study corridor. The next step was to calibrate the model.

Several parameters were selected for calibrating the model. Some of them are related to capacity (Free flow speed, Jam density, and Wave Speed) and other related to lane changing (Friction factor, Velocity Reduction factor, distance From Exit Threshold, nearest Exit Threshold, tau, sd, epsilon, and zeta).

Due to the limited data availability (traffic volume data for one day only), the calibration was conducted based on the following criteria:

- Compliance of the traffic features observed in the video recordings and simulation animation
- Compliance of the travel-time estimates from Bluetooth data and the first prediction horizon.

One of the advantages of using offline calibration using videos is that visual inspection of videos was helpful in identifying traffic conditions and driver behavior not apparent in counts and floating car runs. Thus the goal for the first criterion was to match qualitative aspects of the freeway operation such as:

- location of the bottlenecks
- initiation times for bottleneck
- extent of the queues,
- onramp performance

The parameters were adjusted to ensure that the travel-times derived from the simulation comply with that observed from the Bluetooth data. The parameters calibrated for this study are shown in Table 1.

Calibrated Parameter	Parameter Value
Friction factor	20 kph
Velocity reduction factor	10 kph
Distance from exit threshold	2 km
Nearest exit threshold	1 km

## Table 1. Calibrated Parameters

Free flow speed	100 kph
Jam density	150 veh/km
Wave speed	20 kph
Tau	4 seconds
Sd	200 meters
Epsilon	2
Zeta	0.5
Distance from exit threshold factor	0.75*

\* Since the NB and SB Peachtree Industrial Blvd exits are less than 200 meters apart, distanceFromExitThreshold factor for these exits was calibrated separately (= 0.4).

In addition to calibration of parameters, the following customizations were made to reflect field conditions:

- It was observed the congestion on the study corridor was triggered by the spillback from the I-85 NB off-ramp. The spillback resulted in standing queues on the right two exit lanes while the three left lanes were initially free flowing. To replicate this behavior in the simulation the initial queue was distributed only on the two exit lanes and the free flow conditions were initialized on the other three lanes. Moreover, since these vehicles are supposed to stay there, the destination of all these vehicles were assigned as I-85 to make sure they stay in the exit lanes.
- As a part of automatic calibration process, the application includes steps to prevent abrupt changes in the initial queues and OD flows across successive time steps. If the initial queue and OD flows during any simulation run are less than 80% or more than 120% of the corresponding values in the previous time step, then the values are adjusted to 80% or 120% of the values in previous simulation run.

- It was observed from the time series of traffic volume on the NB I-85 off-ramp that the ramp spills back onto the study corridor triggering congestion on it. Also, it was observed that when the spillback occurs, the flow on the two lane ramp drops gradually from over 3500 to nearly 900 vph. Therefore, to replicate this flow drop, the capacity and free-flow speed on these lanes was externally reduced during appropriate simulation runs.
- To obtain any benefit from this prediction framework, the simulation was performed faster than real-time.

Results presented in the next section indicate that the offline calibration process performed in this study using data collected from videos was adequate. However, when using NaviGAtor and TACTICS data for real-time travel-time prediction, calibration need to be performed again to account for input data from different sources.

# 7 Results and Discussion

The study was performed by simulating EB/SB I-285 corridor for 2 hours during the evening peak period (from 15:00 to 17:00). The OD matrix was changed every 5 minutes and a new simulation was initiated every 5 minutes with new initial queue and new OD flows. Six travel-time forecasts can be made for every simulation run. The predictions made every 5 minutes are compared with the Bluetooth travel-time data collected in the field. This chapter discusses the results of comparison of the Bluetooth data to the travel-times generated by the simulation analysis.

## 7.1 Analysis of Bluetooth Data

This section compares the instantaneous, reactive, and actual travel-times measured on the corridor. As defined in Chapter 2, instantaneous travel-time (ITT) refers to the average travel-time calculated based on the instantaneous speed measured on each of its segments. On the other hand, reactive travel-time (RTT) refers to the travel-time of the vehicle that just exited the corridor. Since the speed data was available from Navigator, average ITT was calculated for each 5-minute interval. Similarly, average value of RTT and actual travel-time (ATT) were calculated for every 5-minute interval from the Bluetooth data. Figure 12 shows plots of ITT, RTT, and ATT against the Bluetooth data for the corridor. Note that since it was determined (see section 4.2) that lower travel time observed at 16:25 is an artifact of insufficient sample size, it is not considered for the rest of the analyses.

Figure 12 shows that though ITT and RTT estimates follow the same pattern as ATT, they are smaller than the ATT for the most part. As the literature suggests, ITT estimates will be poor during non-steady state conditions such as congestion buildup and dissipation because the back

of the queue would have moved upstream by the time the vehicles travels downstream to the congested area. Thus the ITT estimates are always lower than ATT during congestion built up. It can be seen that the difference between the ITT and ATT values seem to relatively stabilize once the entire corridor is congested.

One should note that the travel time measurements presented in figure 12 are obtained from different sources; ITT measurements were calculated from NaviGAtor data and the RTT and ATT were made from Bluetooth data. Therefore, one of the factors contributing to the difference between the ITT and ATT could be the calibration of the NaviGAtor stations. It was observed in the NaviGAtor speed data that different stations reflected different free-flow speeds (varied between 54 mph and 72 mph) while the speed limit on the corridor was 55 mph throughout.

It should be noted that the greatest difference between ATT and ITT is only 2.8 minutes (at 15:20). This is relatively a small value because the rate of congestion propagation on this corridor is relatively slow (note that the travel time stabilizes around 16:00 indicating that it took about an hour for the entire 4.5 mile corridor to get congested). Moreover, since the corridor is only 4.5 miles and free flow travel time is less than 5 minutes, the travel time estimated based on ITT values do not change significantly by the time vehicles reach back of the queue.

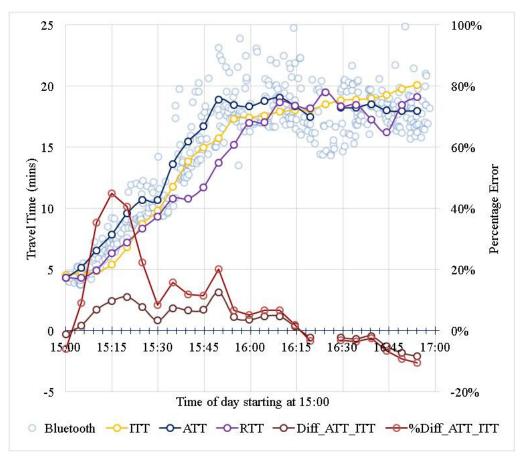


Figure 12: ITT, RTT, ATT and Bluetooth data on the corridor

Though the difference between ATT and ITT estimates are very close, (at most 2.44 minutes around 15:15), the percent difference is significant (almost 45%). However, one should note that the percent difference between ITT and RTT is dependent on several factors such as length of the corridor, number of lanes on the corridor between different ramps, and inflow and outflow at different ramps. If the corridor is long, then the ITT values will significantly lag the ATT traveltimes. However, since the travel-time is long, the percent difference is affected accordingly. Similarly, the speed growth of back of queue on any segment is dependent on outflow and inflow and inflow and the number of lanes on that segment. The speed of growth of back of queue significantly influences the difference between ATT and ITT. If the speed is higher (lower), the difference is higher (lower). In this study, since two out of 5 lanes are spilling back, the back of queue grew faster and the close proximity of Peachtree Industrial on-ramp (two-lane on ramp) that dumped

significant amount of traffic on the freeway increased the speed of back of the queue that may be the reason for the significant difference between the ATT and RTT values near 15:15.

# 7.2 Analysis of Simulation Results

For the purposes of this study a prediction was recorded if there were at least 100 vehicles that started within a given 5 minutes and exited the network by the time the simulation was terminated (0.5 hours of simulation duration). If there are fewer than 100 vehicles that completed their trip during any 5 minute interval, the simulation was extended for 45 minutes to ensure predictions are made for that interval. A 30 minute simulation always produced short term predictions such as 5 minutes and 10 minutes predictions under both congestion built up and sustained congestion scenarios. However, for longer term predictions longer simulation duration is dependent on corridor length and desired prediction horizons. For longer corridors and/or long term predictions, longer simulation durations are needed to obtain sufficient sample size for predictions during congested conditions.

As explained in section 3, the simulation is restarted every 5 minutes with data from field sensors as input. The simulation runs faster than real time to predict travel-times into the future. Figure 13 shows the travel-times estimated using simulation and the travel-times from the Bluetooth sensors on the corridor. The figure also shows the bands of one standard deviation from the mean. The simulation based travel-times shown in these figures are based on the travel-time data collected for the vehicles that crossed the upstream boundary and entered the corresponding segment during the first 5 minutes of the simulation. This is designated as the travel-time projection for the vehicles traveling the corridor in the next 5 minutes. The agreements between these results are used to confirm that the simulation is calibrated sufficiently to replicate the travel-

time pattern recorded by the Bluetooth data. This provides the evidence that the calibration process performed was adequate and forecasts made using the simulation can be expected to be reliable.

Figure 14 shows the travel-time predictions for different time horizons on the corridor. These results are extracted from the simulation by aggregating the travel-times of all the vehicles that entered the study corridor during the corresponding 5–minute time period. A prediction was made by averaging the travel-time of at least 100 vehicles that completed the trip during any given 5-minute period. If there are not enough vehicles to make a prediction, the simulation is extended up to 45 minutes to get enough samples. At least 4 predictions are made for every 5-minute interval; for example based on data available at time 15:35, travel-time predictions are made for 15:40, 15:45, 15:50 and 15:55.

It can be seen that the predictions made during each simulation run follow the pattern observed with the Bluetooth data. During congestion buildup, the forecasts show an increase in travel-time for prediction horizons. However, after the corridor gets congested, the forecasted travel-times seem to flatten out predicting conditions accurately.

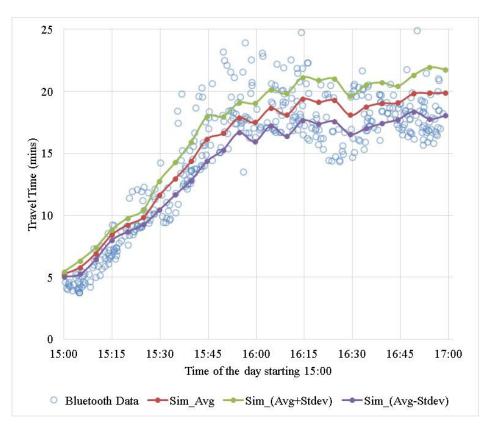


Figure 13: Simulation results vs. Bluetooth Data on the corridor

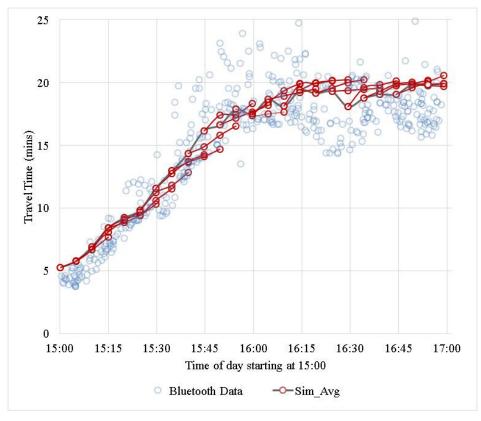


Figure 14: Travel-time Predictions for Different Time Horizons on the Corridor

Figure 15 shows the comparison of 1st prediction, 2nd prediction, and instantaneous travel-time against the actual travel-time data. 1st prediction is defined as the travel-time estimated based on the previous time interval using the simulation framework. This means that if a simulation is initiated at time 15:45 based on known conditions till that time, 1st prediction refers to estimated travel-time of all the vehicles that traversed the corridor between 15:45 and 15:50. Thus, 1st prediction is a 5 minute forecast. It can be readily observed that the simulation framework produced results that improve upon the instantaneous travel-time estimates during the congestion buildup and congested periods. However, note that even though the actual travel-time at 16:25 is significantly lower (as explained in section 4.2), the predictions sustain higher travel-times as expected during congestion as exhibited by the instantaneous travel-time.

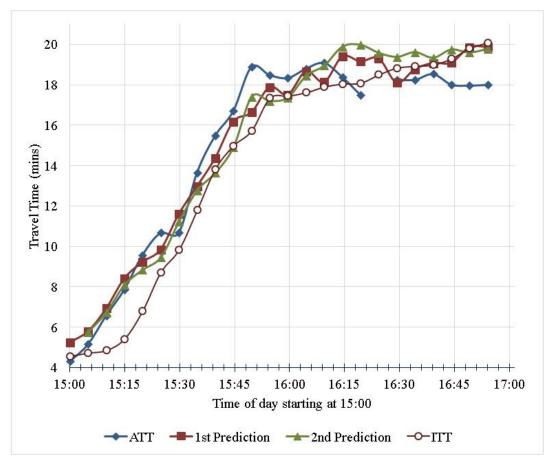


Figure 15: Comparison of Travel-time Predictions for Different Horizons on the Corridor

2nd prediction is defined as the prediction made for a given time period based on information obtained from two time periods earlier. This means that if a simulation is initiated at time 15:40 based on known conditions till that time, 2nd prediction refers to estimated travel-time of all the vehicles that traveled the corridor between 15:45 and 15:50. Thus, the 2nd prediction is a 10 minute forecast obtained using simulation. It was observed that the 2nd prediction results are slightly worse than 1st prediction. Even though it is expected that simulation based travel-times will closely replicate ITT during congested conditions, insufficient Bluetooth sample size at 16:25 resulted in led to large difference in predictions at 16:25. Recollect that the automatic calibration process implemented in the simulation which uses traffic flows from previous time period if the flows in the current period deviates by more than 20% compared to flows previous time period. This resulted in increased travel-times in the predictions. However, the results show that 2nd prediction results are slightly worse than the 1st prediction results even when the entire corridor is congested.

It should be noted that the quality of the 2nd predictions are dependent on the flow characteristics and prediction horizon length. Shorter prediction horizons produce better quality results compared to longer prediction horizons during frequently changing flow conditions. It was observed that quality of the 3rd prediction results were comparable to ITT estimates.

Figure 16 shows a Y-Y plot of ATT, ITT and 1st prediction Travel-times. The figure shows that the 1st prediction results are closer to the Y-Y line compared to the ITT values during the congestion buildup. However, once the entire corridor is congested, the ITT and 1<sup>st</sup> prediction values are almost uniformly spread on either side of the Y-Y line.

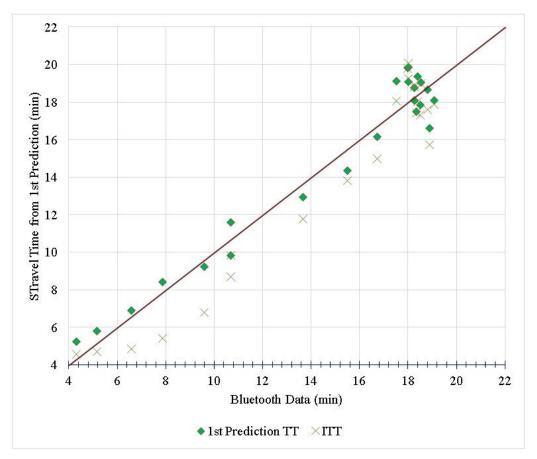


Figure 16: Y-Y Plot of 1st Prediction TT and ITT against Bluetooth Data

One should note that it is easy to induce bias in the way results may be are extracted from the simulation if the simulation duration is not sufficiently long compared to the prediction horizon. During the extraction of travel-time from simulation results, if the simulation duration is not long enough the travel-times of only the vehicles that completed their trip during the simulation will be represented in the results. It is possible that there are several vehicles that may not complete their trip during the simulation duration and were not captured in this study. Thus, simulation results are biased towards capturing fast moving vehicles. However, in our study the simulation duration duration was least 30 minutes to make 5-minute and 10-minute predictions and avoid any bias.

Figure 17 shows the comparison of variability in 1<sup>st</sup> prediction, 2<sup>nd</sup> prediction, 3<sup>rd</sup> prediction and ITT with ATT values. Note that the values shown in the figure are their difference with the ATT

values. The figure shows that the mean differences of the simulation based predictions are close to 0. However, the ITT is little over 0.5 minutes. This indicates that ITT generally underestimates the travel-time during the study period which covers the onset of congestion and a period of heavy congestion (as can be confirmed from figures 15 and 16). However, during the dissipation of congestion, the ITT will be expected to overestimate travel-times because of the inherent lag of the methodology. The 1-standard deviation bound widths of the 1<sup>st</sup> and 2<sup>nd</sup> predictions are much lower than the ITT bound width indicating that the simulation generates estimates with a tighter confidence bounds around the actual travel-time values. However, the width of the 3<sup>rd</sup> estimate bounds is comparable to the ITT bounds indicating that 3<sup>rd</sup> predictions perform poorly compared to the other two simulation based predictions. This is expected, since the accuracy level of the predictions are usually inversely proportional to the length of the time horizon of the predictions.

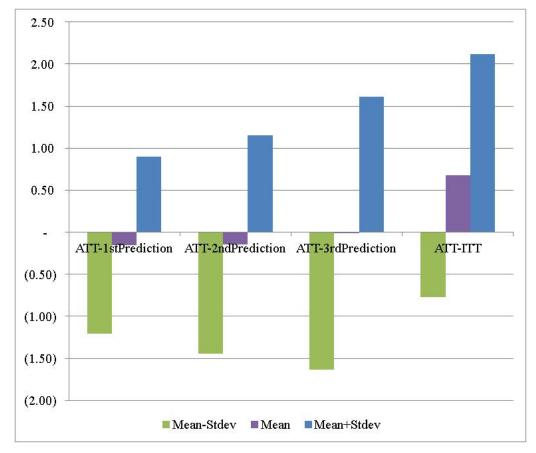


Figure 17: Comparison of variability in 1<sup>st</sup> Prediction, 2<sup>nd</sup> prediction, 3<sup>rd</sup> prediction and ITT

A direct statistical comparison of ITT and simulation based predictions against ATT are not performed since the travel times during each of the 5-minute intervals come from different distributions (different parameter values). Root Mean Square Error (RMSE) and Mean Absolute Percent Error (MAPE) are used to compare the error levels of the travel-time estimates from each method. Table 2 shows the RMSE and MAPE of different travel time estimates.

Travel time data source	RMSE (mins)	MAPE (%)
1st prediction	1.04	6.79%
2nd prediction	1.28	7.37%
3rd prediction	1.58	8.35%
ITT	1.56	10.64%

Table 2. RMSE and MAPE

Table 2 shows that the RMSE and MAPE values for 1<sup>st</sup> and 2<sup>nd</sup> predictions are better than tham the ITT values. However, 3rd predictions are comparable to the ITT values.

## 8 Conclusion

This project demonstrated the use of a simulation based framework to make short-term travel-time predictions in real-time. The study results show that sufficiently accurate 5-minute and 10-minute predictions can be made using this framework.

This study showed that while this framework can be successfully used to make quality predictions, it is critical to adequately calibrate the VDS stations, and simulation model. This study was performed with limited calibration data availability. However, it is hypothesized that with sufficient high quality data, accurate long-term predictions can be made using this framework.

While a definite improvement in the accuracy in the predictions has been observed in this test case, the real strength of this prediction framework will be highlighted in the case where the corridor length is long or when incidents decrease the capacity leading to increase in the speed of the congestion propagation.

Based on the experience gained during this project the following alternatives methods are currently being evaluated to streamline the process in the future:

Initial conditions: Instead of using the queuing analysis to find the queue buildup in any segment to calculate the initial conditions for the simulation, average velocities on a connector can be used to calculate the average density on a segment (based on a calibrated fundamental diagram for that segment) and thus the initial number of vehicles and their velocities on that segment. However, this methodology will require accurate calibration of fundamental diagram and VDS stations.

Automation: currently, the simulation has to be manually initiated each time a new OD matrix becomes available. The simulation can be automated to run continuously by getting sufficient predictions from a run, pausing the simulation until the next OD update is available, and updating the OD flows and initial queues. When incidents occur, the corresponding lane blockage can be incorporated in the simulation before predictions are made.

Uncertainty in the input data: It is important for future versions of the simulation to incorporate uncertainty in the count data and OD estimation. The authors are currently working on this problem, and preliminary results are forthcoming in Laval and Chilukuri [120].

## **9** References

- M. Bielli, G. Ambrosino and M. Boero (Eds.) "Artificial Intelligence Application to Traffic Engineering", VSP, Utrecht, ISBN 90-6764-171-5, 1994.
- Rose, G. and Paterson, D. (1999) Dynamic Travel Time Estimation on Instrumented Freeways. Proceedings of 6th World Congress on Intelligent Transport Systems, Toronto, Canada.
- Kisgyorgy, L. and Rilett, L.R. (2002) Travel Time Prediction by Advanced Neural Network. Periodica Polytechnica Civil Engineering, Vol. 46, No. 1, 15-32.
- Ishak, S. and Al-Deek, H. (2002) Performance evaluation of short-term time-series traffic prediction model, Journal of Transportation Engineering, Vol. 128, No. 6, 490-498.
- Chien, S.I-J. and Kuchipudi, C. M. (2003) Dynamic travel time prediction with real-time and historic data, Journal of Transportation Engineering, Vol. 129, No. 6, 608-616.
- 6. Traffic State in NRW. http://autobahn.nrw.de/, accessed February 25, 2013.
- Laval, J. A. and Leclercq, L. (2010) Mechanism to describe stop-and-go waves: A mechanism to describe the formation and propagation of stop-and-go waves in congested freeway traffic. Forthcoming in Proceedings of The Royal Society A.
- J A Laval and L Leclercq. (2008) Microscopic modeling of the relaxation phenomenon using a macroscopic lane-changing model. Transportation Research Part B, 42 (6):511-522.

- 9. L Leclercq, J Laval, and E Chevallier. (2007) The Lagrangian coordinate system and what it means for first order traffic flow models. In B Heydecker, M Bell, and R Allsop, editors, 17th International Symposium on Transportation and Traffic Theory, Elsevier, New York.
- 10. Federal Highway Administration, Information And Action Memorandum "Dynamic Message Sign (DMS) Recommended Practice and Guidance", July 16, 2004, Jeffery Paniati http://www.ops.fhwa.dot.gov/travelinfo/resources/cms/rept/travtime.htm
- Turner, S.M., Eisele, W.L., Benz, R.J., Holdener, D.J., (1998) Travel time data collection hand book. Research Report FHWA-PL-98-035. Federal Highway Administration, Office of Highway Information Management, Washington, DC.
- Taylor, M.A.P., Bonsall, P.W. and Young, W. (2000) Data on travel times. In, Understanding Traffic Systems: Data, Analysis and Presentation. Ashgate Publishing Ltd, 197-206.
- French, L.J., III, D.R. Martinelli, R.W. Eck, and J. Pascoli. (1998) "Specifications for Automated License Plate Reading Equipment for Transportation Planning." Paper presented at the Transportation Research Board's Annual Meeting, Washington, DC.
- Bertini,R.L., Lasky, M., Monsere, C.M. (2005), Validating Predicted Rural Corridor Travel Times from an Automated License Plate Recognition System: Oregon's frontier project, Proceedings of the 2005 IEEE Conference on Intelligent Transportation Systems, Sep. 13-15, 2005 pp. 296-301.

- 15. Coifman, B., Ergueta, E., (2003) "Improved Vehicle Reidentification and Travel Time Measurement on Congested Freeways", ASCE Journal of Transportation Engineering, Vol 129, No 5, pp 475-483.
- 16. Coifman, B., Krishnamurthy, S., Wang, X., (2003) "Lane Change Maneuvers Consuming Freeway Capacity", Proc. of the Traffic and Granular Flow 2003 Conference, Delft, Netherlands, pp 3-14.
- 17. Abdulhai, B., S. M. Tabib (2003) Spatio-temporal inductance-pattern recognition for vehicle reidentification. Transportation Research Part C, 11 (3-4), 223-239.
- Sun, C., Ritchie, S. G., and Jayakrishnan, R. (1999). "Use of vehicle signature analysis and lexicographic optimization for vehicle reidentification on freeways." Transportation Research, part c, Vol. 7, No. 4, 167-185.
- Pfannerstill, E. (1984) A pattern recognition system for the re-identification of motor vehicles, in: Proceedings of the 7th IEEE International Conference on Pattern Recognition, Montreal, New Jersey, 553–555.
- 20. Chitturi, M., J. C. Medina, and R. F Benekohal, (2007) Accuracy of Video Detection Systems for Traffic Counting. Proceedings of the 2007 ITE International Annual Meeting and Exhibit.
- 21. MacCarley, A., City of Anaheim/Caltrans/FHWA Advanced Traffic Control System Field Operational Test Evaluation: Task C Video Traffic Detection System. California Polytechnic State University, San Luis Obispo, CA, 1998.
- 22. Larson, J., Van Katwyk, K., Liu, C., Cheng, H., Shaw, B., Palen, J., (1998) A Real-Time Laser-Based Prototype Detection System for Measurement of Delineations

of Moving Vehicles. UCB-ITS-PWP-98-20, PATH, University of California, Berkeley, CA, 1998.

- B. Coifman, M. Cassidy, (2002)Vehicle reidentification and travel time measurement on congested freeways, Transportation Research: Part A, 36 (10), pp. 899–917
- 24. Houston TranStar, 2001. TranStar description. <http://traffic.tamu.edu/central2.html>. (Accessed 14. 01. 2011).
- 25. SwRI, 1998. Automatic vehicle identification model deployment initiative system design document. Report prepared for TransGuide, Texas Department of Transportation, Southwest Research Institute, San Antonio, TX
- 26. Mouskos, K.C., Niver, E., Pignataro, L.J., Lee, S., (1998) Transmit system evaluation. Final Report, Institute for Transportation, New Jersey Institute of Technology, Newark, NJ.
- 27. Levine, S., McCasland W., (1994) Monitoring Freeway Traffic Conditions with Automatic Vehicle Identification Systems, ITE Journal, Vol 64, No 3, pp 23-28
- 28. Balke, K., Ullman, G., McCasland, W., Mountain, C., Dudek, C., (1995) Benefits of Real-Time Travel Information in Houston, Texas, Southwest Region University Transportation Center, Texas Transportation Institute, College Station, TX.
- Cui, Y., Huang, Q., (1997) Character Extraction of License Plates from Video, Proc. 1997 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, IEEE, pp 502-507.

- 30. Tam, M.L. & Lam, W. H. K. (2009). Short-Term Travel Time Prediction for Congested Urban Road Networks, Proceedings of the 88th Transportation Research Board Annual Meeting, 11-15 January, Washington D.C., U.S.A., Paper no. 09-2313
- 31. Ma, X. and H. Koutsopoulos, (2010) The Camera based Spatial Travel Time Information: analysis platform and estimation approaches, proceeding of the 15th HKSTS international conference, Hong Kong, China.
- 32. Herrera, J.C., Work, D.B., Herring, R., Ban, X., Jacobson, Q., Bayen, A.M., (2010) Evaluation of traffic data obtained via GPS-enabled mobile phones: the Mobile Century field experiment. Transportation Research Part C. Volume 18, Issue 4, Pages 568-583
- 33. Hall, F L And Persaud, B N, (1989) Evaluation Of Velocity Estimates Made With Single-Detector Data From Freeway Traffic Management Systems, Transportation Research Board, Transportation Research Record No. 1232, P. 9-16,
- 34. L. D. Vanajakshi. (2004) Estimation and prediction of travel time from loop detector data for intelligent transportation systems applications, Ph.D Dissertation.
- 35. Van Lint J.W. C., (2010) Empirical Evaluation of New Robust Travel Time Estimation Algorithms, Transportation Research Record: Journal of the Transportation Research Board, No. 2160, Transportation Research Board of the National Academies, Washington, D.C., pp. 50–59

- 36. Sisiopiku, V.P., N.M. Rouphail and A. Santiago, (1994) "Analysis of correlation between arterial travel time and detector data from simulation and field studies" Transportation Research Board 1457, pp :166-173
- 37. Dailey, D. J. (1997), "Travel time estimates using a series of single-loop volume and occupancy measurements." Presented at the 76th Annual Meeting (CD-ROM), Transportation Research Board, Washington D.C.
- 38. Ferrier, P. J. (1999), "Comparison of vehicle travel times and measurement techniques along the I-35 corridor in San Antonio, Texas." Master's thesis, Department of Civil Engineering, Texas A&M University, College Station, Texas.
- 39. Lindveld, C. D. R., and Thijs, R. (1999), "On-line travel time estimation using inductive loop data: The effect of instrumentation peculiarities." 6th Annual World Conf. on Intelligent Transp. Systems (CD-ROM), Toronto.
- 40. M. Treiber and D. Helbing (2002) Reconstructing the spatio-temporal traffic dynamics from stationary detector data, Cooperative Transportation Dynamics 1 3.1-3.24.
- 41. Van Lint, J. W. C. and N. J. Van der Zijpp (2003). An Improved Travel-Time Estimation Algorithm Using Dual Loop Detectors. 82nd Transportation Research Board, Washington D.C., USA.
- 42. Van Hinsbergen, C. P. I. and Van Lint, J. W. C. (2008). Bayesian combination of travel time prediction models. Transportation Research Record, 2064, 73-80.
- 43. Hans van Lint, Serge P. Hoogendoorn, Henk J. van Zuylen: State Space Neural Networks for Freeway Travel Time Prediction. ICANN 2002: 1043-1048

- 44. Morales, J.M. (1989) Analytical procedures for estimating freeway traffic congestion. TRB Research Circular, 344, pp. 38-46
- 45. Westerman, M., Immers, L., (1992) A Method for Determining Real-Time Travel Times on Motorways, Road Transport Informatics/Intelligent Vehicle Highways Systems, ISATA, pp 221-228.
- 46. Westerman, M., Litjens, R., Linnartz, J., (1996) Integration of Probe Vehicle and Induction Loop Data- Estimation of Travel Times and Automatic Incident Detection. PATH, University of California at Berkeley.
- 47. Nam, D. and Drew, D. (1998). "Analyzing Freeway Traffic under Congestion: Traffic Dynamics Approach." J. Transp. Eng., 124(3), 208–212.
- 48. Oh, J., Jayakrishnan, R., and Recker, W. (2003), "Section travel time estimation from point detection data." Center for Traffic Simulation Studies, Paper VCI-ITS-TS-WP-02-15.
- 49. Rakha H. and Zhang W.\* (2005), Estimating Traffic Stream Space-mean Speed and Reliability from Dual and Single Loop Detectors, Transportation Research Record: Journal of the Transportation Research Board, No. 1925, pp. 38-47.
- 50. Yeon, J. and Elefteriadou, L. (2006) Comparison of Travel Time Estimation Using Three Previously Developed Methods to Field Data Along Freeways. 5th International Symposium on Highway Capacity and Quality of Service, pp. 229-238.
- Dailey, D. J. (1993). Travel-time estimation using cross-correlation techniques. Transportation Research, Part B, 27B(2):97{107.

- 52. Petty, K., Bickel, P., Jiang, J., Ostland, M., Rice J., Ritov, Y., and Schoenberg, F. (1998). Accurate estimation of travel times from single-loop detectors. Transp. Res. A 32(1) 1–17.
- 53. Garib, A., Radwan, A.E. and Al-Deek, H. (1997) Estimating Magnitude and Duration of Incident Delays. Journal of Transportation Engineering, Vol.123, No.6, pp.459-466.
- 54. Ozbay, K. and Kachroo, P. Incident Management in Intelligent Transportation Systems. Artech House, 1999.
- 55. Kwon, J., Coifman, B., and Bickel, P. (2000), "Day-to-day travel time trends and travel time prediction from loop detector data." Transp. Res. Rec. 1717, Transportation Research Board, Washington, D.C., 120-129.
- 56. Rice, J., and van Zwet, E. (2002), "A simple and effective method for predicting travel times on freeways." IEEE Intelligent Transp. Systems Conf. Proc., Piscataway, New Jersey, 227-232.
- 57. Zhang, X., and Rice, J. (2003), "Short term travel time prediction." Transp. Research C, 11, 187-210.
- 58. Partha Chakroborty and Shinya Kikuchi., (2004) Estimating Travel Times on Urban Corridors using Bus Travel Time Data. Transportation Research Record, 1870, pp. 18-25.
- 59. Giuliano, G. (1989) Incident Characteristics, Frequency, and Duration on a High Volume Urban Freeway. Transportation Research – A, Vol.23A, No.5, pp.387-396.

- Sullivan, E.C. (1997) New Model for Predicting Incidents and Incident Delay.
   ASCE Journal of Transportation Engineering, Vol.123, pp.267-275.
- 61. Nam, D. and Mannering, F. (2000) An Exploratory Hazard-Based Analysis of Highway Incident Duration. Transportation Research – A, Vol.34A, No.2, pp.85-102.
- 62. Tarko, A., and Rouphail, N.M. (1993), "Travel time data fusion in ADVANCE." In Proc. of the 3rd Int. Conf. on Applications of Advanced Technologies in Transp. Engineering, ASCE, New York, 36-42.
- 63. Boyce, D., Rouphail, N., and Kirson, A. (1993), "Estimation and measurement of link travel times in the ADVANCE project." inProc. of the Vehicle Navigation and Information Systems Conf., IEEE, New York, 62-66.
- 64. Manfredi, S., Salem H. H., and Grol, H. J. M.(1998), "Development and application of coordinated control of corridors."
- 65. < ftp://ftp.cordis.europa.eu/pub/telematics/docs/tap\_transport/daccord\_d3.1.pdf > (Feb 25, 2013)
- 66. Oda, T., (1990) An algorithm for prediction of travel time using vehicle sensor data.In: Third International Conference On Road Traffic Control, Institute of Electrical Engineers, pp. 40–44.
- 67. Iwasaki, M., and Shirao, K. (1996), "A short term prediction of traffic fluctuations using pseudo traffic patterns." 3rd World Congress on Intelligent Transport Systems Conference (CD-ROM), Orlando, Florida.

- 68. D'Angelo, M. P., Al-Deek, H. M., and Wang, M. C. (1998), "Travel time prediction for freeway corridors." Transp. Res. Rec. 1676, Transportation Research Board, Washington, D.C., 184-191.
- 69. Al-Deek, H. M. (1998), "Travel time prediction with non-linear time series." Proc. of the 5th Int. Conf. on Applications of Advanced Technologies in Transportation Engineering (AATT-5): ASCE, Newport Beach, California, 317-324.
- 70. Stathopoulos, A and Karlaftis, M.G. (2003), A multivariate state space approach for urban traffic flow modeling and prediction, Transportation Research Part C, vol 11, 121–135
- 71. Kamarianakis, Y. and Prastakos, P. (2003) Forecasting traffic flow conditions in an urban network: comparison of multivariate and univariate approaches, Transportation Research Record, 1857, pp. 74-84.
- 72. Williams, B. M., & Hoel, L. A. (2003). Modeling and forecasting vehicular traffic flow as a seasonal ARIMA process: Theoretical basis and empirical results. Journal of Transportation Engineering, 129(6), 664-672.
- 73. Guin A., (2006) "Travel Time Prediction Using a Seasonal Autoregressive Integrated Moving Average Time Series Model," Proceedings of the 9th IEEE International Conference on intelligent Transportation system, pp. 493–498.
- 74. Cherrett, T. J., Bell, H. A., and Mc Donald, M. (1996), "The use of SCOOT type single-loop detectors to measure speed, journey time and queue status on non-SCOOT controlled links." 8th Int. Conf. on Road Traffic Monitoring and Control, 23-25.

- 75. Ohba, Y., Koyama, T., and Shimada, S. (1997), "Online learning type of traveling time prediction model in expressway." IEEE Conf. on Intelligent Transp. Systems, Boston, Massachusetts, 350-355.
- 76. Park, D. and L.R. Rilett (1999), Forecasting Freeway Link Travel Times with a Feedforward Multilayer Neural Networks, Computer-Aided Civil and Infrastructure Engineering, Vol. 14, pp. 357-367.
- 77. Park, D., L.R. Rilett, and G. Han (1999) Spectral Basis Neural Networks for Real-Time Travel Time Forecasting, ASCE Journal of Transportation Engineering, Vol 125, No.6, 515-523.
- 78. Rilett, L. R., and Park, D., (2001) "Direct Forecasting of Freeway Corridor Travel Times Using Spectral Basis Neural Networks", Transportation Research Record, 1752, Transportation Research Board, Washington, DC., pp. 140-147.
- 79. Matsui, H., and Fujita, M. (1998), "Travel time prediction for freeway traffic information by neural network driven fuzzy reasoning." in Neural networks in transp. applications, V. Himanen, P. Nijkamp, A. Reggiani, and J. Raitio, eds., Ashgate Publishers, Burlington, Vermont, 355-364.
- You, J., and Kim, T. J. (2000), "Development of hybrid travel time forecasting model." Transp. Research C, 8, 231-256.
- Jiang, G. and Zhang, R. (2001) Travel Time Prediction for Urban Arterial Road: A Case on China. Proceedings of Intelligent Transport System, IEEE, 255-260.
- Wei, C.-H., Lin, S.-C. and Li, Y. (2003) Empirical Validation of Freeway Bus Travel Time Forecasting., Transportation Planning Journal, Vol. 32, 651-679.

- Bharia, A. and Adeli, H. (2003) Neural network model for rapid forecasting of freeway link travel time. Engineering Applications of Artificial Intelligence, Vol. 16, 607-613.
- Kalman, R. (1960) A New Approach to Linear Filtering and Prediction Problems. Journal of Basic Engineering, 82D (1), pp.35-45.
- 85. Szeto M.W., Gazis D.C., (1972) Application of Kalman Filtering to the Surveillance and Control of Traffic Systems, Transportation Science, Vol. 6, No. 4, pp. 419-439
- 86. Okutani, I. and Y. Stephanedes. (1984) Dynamic Prediction of Traffic Volume through Kalman Filtering Theory. Transportation Research, Part B, Vol.18B, No.1, pp.1-11.
- 87. Okutani, I. (1987) The Kalman Filtering Approaches in Some Transportation and Traffic Problems. In Transportation and Traffic Theory, N. Gartner and N. Wilson (Ed.), Elsevier Science Publishing Co., Inc.
- 88. Stephanedes, Y. and E. Kwon. (1993) Adaptive Demand-Diversion Prediction for Integrated Control of Freeway Corridors. Transportation Research, Part C, Vol.1c, No.1, pp.23-42.
- 89. Chien, S., Liu, X., and Ozbay, K. (2003), "Predicting travel times for the South Jersey real-time motorist information system." Presented at the 82nd Annual Meeting (CD-ROM), Transportation Research Board, Washington D.C.

- 90. Kuchipudi, C. M., and Chien, S. I. J. (2003), "Development of a hybrid model for dynamic travel time prediction." Presented at the 82nd Annual Meeting (CD-ROM), Transportation Research Board, Washington D.C.
- 91. M J Lighthill and J B Whitham. (1955) On kinematic waves II: A theory of traffic flow on long crowded roads. Proceedings of the Royal Society A, 229:317 - 245.
- 92. P I Richards (1956). Shockwaves on the highway. Operations Research, 4:42-51.
- Messmer and M. Papageorgiou. (1990) METANET: A macroscopic simulation program for motorway networks. Traffic Engineering and Control, 31(9):466–470.
- 94. Kotsialos, M. Papageorgiou, C. Diakaki, Y. Pavlis and F. Middelham (2002), Traffic flow modeling of large-scale motorway networks using the macroscopic modeling tool METANET, IEEE Transactions on Intelligent Transportation Systems 3 (4), pp. 282–292
- 95. Ben-Akiva, M., M. Bierlaire, H. Koutsopoulos, and R. Mishalani. (1998)."DynaMIT: A Simulation-Based System for Traffic Prediction." In Proceedings of the DACCORD Short-Term Forecasting Workshop.
- 96. Ulam, S. (1952) Random Process and Transformations. Proceedings of the International Congress on Mathematics, Vol. 2, pp. 264-275.
- 97. Cremer, M. and Ludwig, J. (1986) A Fast Simulation Model for Traffic Flow on the Basis of Boolean Operations. Mathematics and Computers in Simulation, Vol. 28, No. 4, pp. 297-303.
- 98. Nagel, K. and Schreckenberg, M. (1992) A Cellular Automaton Model for Freeway Traffic. Journal De Physique I France, Vol. 2, pp. 2221-2229.

- 99. Larraga, M.E., del Rio, J.A. and Alvarez-Icaza, L. (2005) Cellular Automata for One-Lane Traffic Flow Modeling. Transportation Research Part C: Emerging Technologies, Vol. 13, No. 1, pp. 63-74.
- 100. Simon, P.M. and Gutowitz, H.A. (1998) Cellular Automaton Model for Bidirectional Traffic. Physical Review E (Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics), Vol. 57, No. 2, pp. 2441-4.
- 101. Fouladvand, M.E. and Lee, H.W. (1999) Exactly Solvable Two-Way Traffic Model with Ordered Sequential Update. Physical Review E (Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics), Vol. 60, No. 6, pp. 6465-79.
- 102. Hafstein, S.F., Chrobok, R., Pottmeier, A., Schreckenberg, M. and Mazur, F.C. (2004) A High Resolution Cellular Automata Traffic Simulation Model with Application in a Freeway Traffic Information System. Computer-Aided Civil and Infrastructure Engineering, Vol. 19, No. 5, pp. 338-350.
- 103. Brockfeld, E., Barlovic, R., Schadschneider, A. and Schreckenberg, M. (2001)
  Optimizing Traffic Lights in a Cellular Automaton Model for City Traffic. Physical
  Review E (Statistical, Nonlinear, and Soft Matter Physics), Vol. 64, No. 5, pp. 056132-1.
- 104. Fouladvand, M.E., Sadjadi, Z., Shaebani, M.R. (2004), Characteristics of vehicular traffic flow at a roundabout, Physical Review E Statistical, Nonlinear, and Soft Matter Physics, Volume 70, Issue 4 2, , Article number046132, Pages 046132-1-046132-8

- 105. Zhu, L.-H., Chen, S.-D., Kong, L.-J. and Liu, M.-R. (2007) The Influence of Tollbooths on Highway Traffic. Acta Physica Sinica, Vol. 56, No. 10, pp. 5674-8.
- 106. Knospe W. (2002) Synchronized traffic: Microscopic modeling and empirical observations. PhD thesis, Gerhard-Mercator-Universit<sup>a</sup>t Duisburg
- 107. Chowdhury D., Santen L., Schadschneider A. (2000) Statistical Physics of Vehicular Traffic and Some Related Systems. Physics Reports 329, 199–329
- 108. Helbing D., Herrmann H., Schreckenberg M., Wolf D. (Eds.) (2000) Traffic and Granular Flow '99. Springer, Heidelberg
- 109. Nagel K., Wolf D. E., Wagner P., Simon P. (1998) Two-lane traffic rules for cellular automata: A systematic approach. Phys. Rev. E 58, 1425–1437
- 110. Schreckenberg M., Wolf D. (Eds.) (1998) Traffic and Granular Flow '97. Springer, Singapore
- 111. S.F. Hafstein, R. Chrobok, A. Pottmeier, M. Schreckenberg, and F. Mazur. (2004)
  A High-Resolution Cellular Automata Traffic Simulation Model with Application
  in a Freeway Traffic Information System. Computer-Aided Civil and Infrastructure
  Engineering 19-5, pp. 338-350.
- 112. Dailey, D. and N. Taiyab. (2002) A Cellular Automata Model for Use with Real Freeway Data. WA-RD 537.1. Technical Report. Washington State Department of Transportation.
- 113. Jia, B., Jiang, R. and Wu, Q. (2003) The Traffic Bottleneck Effects Caused by the Lane Closing in the Cellular Automata Model. International Journal of Modern Physics C, Vol. 14, No. 10, pp. 1295-303.

- 114. Nassab, K., Schreckenberg, M., Boulmakoul, A. and Ouaskit, S. (2006) Effect of the Lane Reduction in the Cellular Automata Models Applied to the Two-Lane Traffic. Physica A: Statistical Mechanics and its Applications, Vol. 369, No. 2, pp. 841-852.
- 115. Wang, Z. and Murray-Tuite, P. (2010) A cellular automata approach to estimate incident-related travel time on interstate 66 in near real time. VTRC 10-CR4. Technical Report. Virginia Transportation Research Council..
- 116. Ying Liu, Pei-Wei Lin, Xiaorong Lai, Gang-Len Chang, and Alvin Marquess. (2006) Developments and Applications of Simulation-Based Online Travel Time Prediction System Traveling to Ocean City, Maryland. Transportation Research Record: Journal of the Transportation Research Board, No. 1959, Transportation Research Board of the National Academies, Washington, D.C., pp. 92–104.
- 117. Toth C., Suh W., Elango V., Sadana R, Guin A., Hunter M., Guensler R. (2013),Tablet-Based Traffic Counting Application Designed to Minimize Human Error,Transportation Research Board Annual Conference.
- 118.F Castrillon, A Guin, R Guensler, JA Laval. (2012) Imputation of Intelligent Transportation System VDS Data using Various Modeling Approaches. Forthcoming in Journal of the Transportation Research Board.
- 119. NetBeans.org: The User Interface Developer 's Corner. http://ui.netbeans.org
- 120. Laval, J. A., Chilukuri, B. R. The Distribution of Congestion on a Class of Stochastic Kinematic Wave Model. Forthcoming in Transportation Science, 2013.