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16. Abstract

This report documents the concept design and analysis of intermodal freight systems. The primary objective of this project was to quantify the various tradeoffs and relationships between fundamental system design parameters and operating strategies, as they impact costs and performance. The report is in two volumes:

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PREFACE

This report documents the concept design and analysis of intermodal freight systems. The work was performed for the Office of Systems Engineering, U.S. Department of Transportation (DOT) under contract DOT-OST-77-031. The project technical monitor was Dr. S. C. Chu of DOT. This report consists of the following two volumes:

- Volume I: Executive Summary
- Volume II: Methodology and Results.

The study was conducted by the Transportation and Industrial Systems Center at SRI International. Dr. P. J. Wong was the project leader and directed a team consisting of:

- Mr. R. M. Corbett--developed cost models
- Mr. A. R. Grant--responsible for simulation modeling and analysis
- Ms. M. A. Hackworth--performed analysis of simulation data
- Mr. A. E. Moon--responsible for costing methodology
- Dr. M. Sakasita--responsible for hand-analytical investigations.

The author would like to acknowledge the active technical participation of Dr. S. C. Chu, the DOT project technical monitor who contributed substantially to the technical direction and content of this project. Also, appreciation is expressed to the following for their comments and technical advice during the research effort: J. Ward of the Office of Science and Technology, and R. Favout and G. Watros both of the Transportation Systems Center.

I INTRODUCTION

A. Background

The economics of transporting certain commodities requires the combination of two or more modes—one to accomplish local pickup or delivery of the commodity and another to perform the linehaul movement of the commodity over significant distances to the destination of the shipment, where the local service mode may be used again. The vehicles needed for the local service and the linehaul modes, the packaging techniques and equipment that facilitate transfer of the loads, and the terminal facilities that perform the transfer constitute an intermodal system. Examples of intermodal systems in current use are the truck—rail trailer— and container—on—flatcar (TOFC/COFC) service and the marine container system.

The three major components of our intermodal system are the local service, linehaul, and terminal systems. With only a relatively small capacity, the local service component serves distributed shippers or consignees. Examples are trucks that pick up and deliver shipments from shipper or consignee loading docks, and the carload railroad service that provides cars to be switched to customer sidings. The linehaul component usually has higher capacity and may combine shipments carried in several local service vehicles into a single movement. Examples are the overthe-road part of a truck or freight train journey. At terminals shipments are transferred between local service and linehaul vehicles or between linehaul vehicles. Because rates of arriving and departing shipments are frequently different, area for storage and information systems for rapid retrieval of store shipments must also be provided in the terminal. The sophistication and degree of automation in the terminal system are governed by its scale of operations.

A freight system is economical when each component is designed to perform so that the total cost of the shipment, in the framework of its distribution or production system, is minimized. Numerous tradeoffs are possible; for example, the radius of local service for a terminal will set the volume of terminal use and thus the degree of automation that is economically justified at the terminal. The greater speed of the linehaul will increase overall performance and vehicle productivity at the expense of more sophisticated and costly vehicles and added energy consumption.

The truck-rail intermodal services now available have gained a relatively small share of the intercity freight tonnage, largely due to a lack of integration of the system components. Highway vehicles are poorly proportioned to take advantage of the carrying capacity of railroad cars; thus, the ratio of net to gross weight is low and cost to the rail mode is high. These economic factors, combined with a fragmented railroad ownership and poor financial resources, have resulted in modest use of the service.

Research and development is under way to improve the current truck-rail intermodal system by designing new container systems, new railroad cars, and new terminal design and transfer equipment. However, a longer-range approach is needed to understand and achieve the full economics inherent in an intermodal system. A new economically successful intermodal surface mode is possible only if the local service, linehaul, and terminal components are harmonized and fully integrated. This harmonizing of all the freight shipment components requires an understanding of tradeoffs between fundamental system design parameters. Currently, the knowledge concerning fundamental design and integration issues is incomplete.

B. Objectives and Scope

There has been a lack in the fundamental understanding of the intrinsic interrelationships/properties of the various components necessary to have an economical freight system which provides effective levels of service. This lack of understanding has manifested itself in attempting to optimize each component of the system, hoping thereby to optimize the entire system. Unfortunately, this approach rarely leads to an optimized total system. The development of a fundamental understanding of the quantitative interrelationship between major elements of a freight system would:

- Enhance the design of freight systems under existing technology.
- Guide future technologic innovations where they will contribute the most system impact.
- Provide a systematic basis for transportation planning/policy decisions.

The primary objective of this project was to quantify the various tradeoffs and relationships between fundamental system design parameters and operating strategies, as they impact costs and performance. The purpose was to determine the directions in which the greatest payoff lay and, therefore, the type of research and development (R&D) that needs to be further pursued. Thus, the outputs of this study laid a firm foundation and understanding for the concept design of intermodal freight systems.

The framework of the study assumed an intermodal system consisting of a basic grade-separated dedicated right-of-way network for linehaul vehicles. Intraregional collection and distribution was to be performed by pickup/delivery vehicles on highways. For such a system it was assumed that the freight is containerized and that transfer between pickup/delivery vehicles and linehaul vehicles occurs at terminals.

The emphasis in the study was not to evaluate or analyze specific realizations or implementations of advanced intermodal concepts, because to do so would provide a narrow knowledge base. Rather, the emphasis

was on the study of a generic intermodal system which is independent of a specific implementation. In this manner, understanding was developed along a continuous spectrum of freight system characterizations, and thereby was not restricted to specific alternatives. Therefore, a quantitative data base was developed to evaluate and analyze all system alternatives.

Because of the generic nature of the study, the important tradeoff questions were among fundamental design parameters or technology variables at an aggregate level, rather than on a detailed "micro" design level. Examples of the tradeoff questions and issues of interest included:

- There is a broad spectrum of operating strategies available ranging from nonstop, origin-to-destination train movements to trains with several intermediate stops. For each intermediate stop, time is spent for container transfer and for container accumulation, thus affecting equipment utilization. The economies that can be achieved by aggregating containers at intermediate nodes and the decreased utilization of equipment due to the time spent waiting at these nodes is an area to be analyzed.
- Vehicle and crew productivity increases with increasing speed, on the other hand, so do the costs of maintenance, energy and investments. Thus, each system design will have an optimal operating speed range for the given demand that the system has to serve. The relationship of "optimal" design speed to the number and size of trains needs to be analyzed.
- A container can spend a substantial amount of time both in the linehaul and the terminal portions of the system. The optimum match between terminal processing capability and linehaul speed requires investigation.

Note that not only are the quantitative data derived from this study important, but the development of a systematic set of procedures, methods, and tools to study freight system concepts is equally important. Thus, it is expected that this project will enhance the procedures available to systematically analyze freight systems.

C. Organization of the Report

The structure of this report is relatively straightforward. Section II details the complexity of studying a complete generic freight system, and motivates the selection of a simple five node linear network as the case study for concentration during this project.

Two separate but interrelated analysis procedures were developed to study the simple linear system. The first is based on a detailed computer simulation called LINET. The LINET model, associated analysis, and tradeoff results are described in Section III. The second procedure is based on developing simple closed form analytical equations;

this line of investigation is called "hand analysis." The hand analysis procedures and results are documented in Section IV.

The appendices document the costing methods used in the study as well as other supporting analyses and results.

II ANALYZING A GENERIC FREIGHT SYSTEM

A. Complexity of System Characterization

A generic intermodal freight system model is a characterization which is independent of a particular hardware implementation. In such a model, the essential elements, processes, and resources are described by parameters and variables that specify major systems effects or performance levels. For example, it may be sufficient to specify a terminal in terms of its ability to process trains and containers, without going into the detail of the terminal design or the type of container transfer equipment. The analysis of a generic system provides the ability to study many system representations, demand levels, and operating cost formulations by suitable adjustment of parameters or variables. Furthermore, systems can be studied in which the hardware and technology currently do not exist. Hypothetical cost formulations can be studied corresponding to alternative technology and financing, and varying demand patterns can be evaluated. Thus, the study of a generic system model allows the systematic exploration of the inherent structure and tradeoffs of a freight system free from the encumbrances of dealing with a specific hardware implementation.

Table 1 displays some of the critical elements of a generic intermodal freight system organized into the following categories:

- Network structure
- Demand
- Container system
- Pickup/delivery system
- Terminal system
- Linehaul system
- Network operations strategy
- Cost.

Under each category the parameters/characterization of the generic elements are listed. Furthermore, where appropriate, example questions/tradeoffs are displayed.

The categories listed in Table 1 are arbitrary; other ways of organizing the elements of a generic system could be developed. Displaying the information in Table 1 illustrates that a complete model of a generic intermodal freight system is complex with many variables and

PARAMETERS/CHARACTERIZATION

QUESTIONS/TRADEOFFS

Links

Number Lengths Capacity

Terminals Number Location

Interconnection matrix

Network Structure

Sparse LH network with large PD region, or dense LH network with small PD region

Demand

Average shipment Weight Cubes or density

O-D demand matrix Containers or tons

Multiple commodity classes

Effect of varying demand level of service/cost Effect of varying demand distribution on service/

Container

Volume of container

Tare weight of container Maximum allowed weight of load

Number of customers in service area Radius of service area Number of PDVs

Number of containers per PDV Speed of PDV

Average time to load/unload PDV at customer dock

Average transfer time between PDV and terminal (function of number of containers per PDV)

Average transfer time between LHV and terminal (function of number of containers per LHV)

Maximum container storage capability of terminal Number of PDV docks Number of LHV docks

Number of LHV Maximum speed of LHV Container carrying capacity of LHV

Minimum allowed headway between LHV

LHV stopping policy Direct service Intermediate stops

LHV schedule Fixed Demand responsive

Container sorting Number of intermediate sortings Final sorting near origin Final sorting near destination

Empty container distribution

Effect of weight limited container on service/cost Effect of cube limited container on service/cost

Pickup/Delivery System

Service/cost vs. PDV speed

Service/cost vs. number of PDVs as a function of radius of service area and number of customers Service/cost vs. container carrying capacity of PDV

Service/cost vs. transfer time at customer dock

Terminal System

Effect of LHV and PDV schedules on terminal accumulation time

Required storage capability of terminal vs. processing rates

Effect of processing rates on terminal detention

Linehaul System

Service/cost vs. speed of LHV Service/cost vs. size of LHV

Network Operational Strategy

What are tradeoffs between direct service and intermediate stops?

What are tradeoffs between small high-frequency and larger low-frequency LHV operations?

What are the tradeoffs in requiring containers to to be transferred between LHV?

Which technologic changes would be most costeffective? How do various forms of financing affect freight system?

Terminal costs LHV and PDV fleet costs Container costs

Operating costs

Guideway costs

^{*}PD = pickup and delivery, PDV = pickup and delivery vehicle, LH = linehaul, LHV = linehaul vehicle, O-D = origin-destination.

degrees of interaction between various parts. Thus, its analysis and study would require a large, long-term research effort.

Because of the inherent complexity of a total intermodal freight system representation, it was decided to concentrate this research effort on only a portion of the intermodal system. In particular, it was decided to focus attention on the interaction and tradeoffs associated with terminals and the linehaul system (the pickup and delivery system was not included). To facilitate this analysis, a simple representation of a linear corridor system was extensively investigated.

B. <u>Simplifying Assumptions</u>

Because a complete intermodal system characterization is complex with many variables and degrees of interaction, and there exists very little research on the systematic understanding of freight shipment tradeoffs, it was decided to focus attention on the simple linehaul system represented by the five node linear network shown in Figure 1.

The study of such a simple linear system has real world analogs in the numerous heavy volume freight "corridors" which exist in the United States. Furthermore, the study of a more complex two-dimensional network can be conceptually decomposed into the study of a sequence of linear segments with container transfers between linear segments. As demonstrated in the remainder of this report, although the linear network is simple, it provides an abundance and richness of insights which are necessary before one can systematically cope with a more complex two-dimensional system.

For this simple linear system, the key simplifying assumptions are:

- The demand for the system is characterized by the number of containers going from each origin to each destination.
- The linehaul vehicle is called a train; a train moves at a constant speed over the links; a train has a fixed maximum capacity for carrying containers.
- A terminal is characterized by a single processing time which is a combined time to both load and unload containers from a train. The number of terminal platforms (or berths) determines the number of trains which can be simultaneously processed. All terminals have the same characteristics.

C. Representation of Simple Canonical Operating Strategies

The specification of the simple linear system is incomplete until the train operating strategy on this linear network is specified. It was decided that the most insight can be gained by restricting the study of train operations to a small set of simple "canonical" strategies. Canonical strategies represent fundamental strategies; all other train

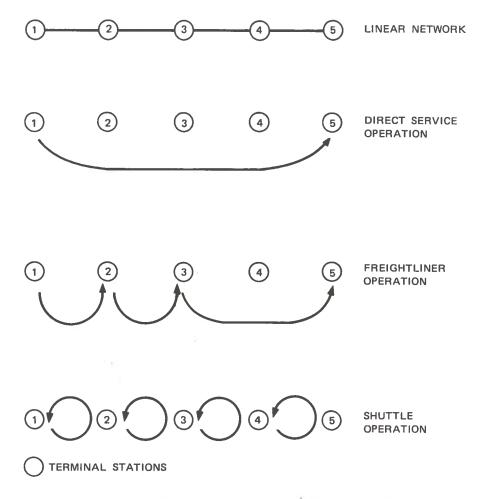


FIGURE 1 LINEAR NETWORK AND TRAIN OPERATIONS

strategies can be considered as a hybrid combination of these canonical strategies.

Operating strategies can be specified in terms of a routing policy, a departure policy, and a stopping policy. The routing policy specifies the routing of the trains along the network. As shown in Figure 2, there are three basic alternatives for the routing policy—direct service, local service, and shuttle service:

- (1) <u>Direct</u> service in which a train takes containers directly from origin to final destination without intermediate stops.
- (2) Local service in which a train makes intermediate stops between origin and final destination to pick up and set out containers.
- (3) Shuttle service in which trains only shuttle back and forth between adjacent terminals; containers going beyond an adjacent terminal are required to transfer to as many connecting shuttles as appropriate.

The departure policy specifies the criteria with which trains depart from a terminal. The two basic alternatives are a fixed or a flexible policy:

- (1) Fixed departure schedule in which the train leaves a terminal based on a scheduled time.
- (2) Flexible departure schedule in which the train leaves the terminal when it is filled.

The stopping policy specifies the criteria with which trains stop at a terminal. Similar to the departure policy, the two basic alternatives are a fixed or a flexible policy:

- (1) Fixed stopping policy in which the trains always make scheduled stops at terminals.
- (2) Flexible stopping policy in which trains stop at a terminal only if a container is to be set out or picked up.

Figure 3 displays all the possible strategies derived by choosing the various alternatives for routing policy, departure policy, and stopping policy depicted in Figure 2. Note that all the strategies are not practical; only those with asterisks (*) are viable. A variation of a local strategy in which a train stops at intermediate terminals only when necessary (i.e., skips stops) is called "freightliner" (L-2) and has many desirable properties.

Figure 4 illustrates that direct service, local service, and shuttle service are in some sense canonical strategies. For example, if we conceive the set of operating strategies as a triangle with the "fixed version" of direct service, local service, and shuttle service as the "pure" strategies at the vertices of the triangle, the other strategies

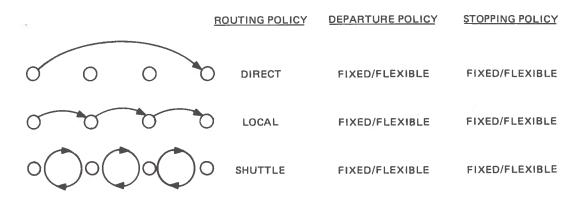


FIGURE 2 SPECIFYING OPERATING STRATEGY

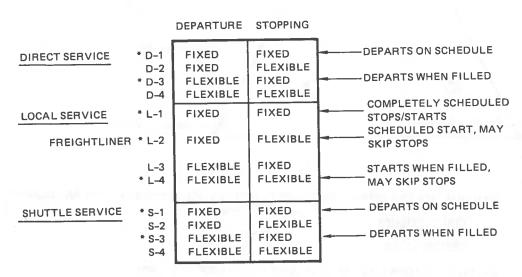
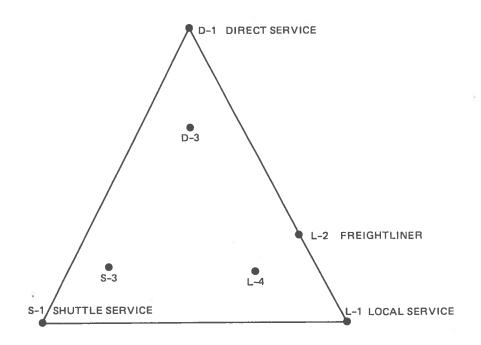


FIGURE 3 VIABLE OPERATING STRATEGIES



(1) FREIGHTLINER CAN BEHAVE MORE LIKE DIRECT SERVICE IF ENOUGH STOPS ARE SKIPPED

DIRECT SERVICE 0 0 0 0 0 FREIGHTLINER

(2) SHUTTLE CAN LOOK LIKE LOCAL IF TERMINAL TRANSFER IS RAPID

SHUTTLE OOOOO

(3) TRANSFER BETWEEN TWO LOCALS HAS SHUTTLE CHARACTER



FIGURE 4 CANONICAL OPERATING STRATEGIES

can be considered as hybrid strategies occupying points inside the triangle; nearness to a pure strategy indicates the degree to which a strategy behaves like a pure strategy.

It was decided that an operating strategy should be selected from each of the three categories of direct service, local service, and shuttle service, and that they should be evaluated on the basis of the simple linear system. The three canonical strategies chosen are direct service (D-1), shuttle service (S-1), and a modification to local service which we call "freightliner" (L-2) (see Figure 3). It was concluded that a local service which made fixed stops even when not necessary was not as effective nor as interesting from an analysis viewpoint as a local service which could skip stops if appropriate.

D. Case-Study Specifications of Linear Corridor System

For the simple linear corridor system which was studied, the demand is characterized by the number of containers going from each origin to each destination. A number of trains move over the linehaul segments carrying containers between terminals; a train moves at a constant speed over the linehaul segments and has a fixed maximum capacity (or size) for carrying containers. All terminals are identical and are characterized by a single processing time which is the combined time to both load and unload containers from a train; the number of terminal platforms (or berths) determines the number of trains which can be simultaneously processed. Thus, the five main engineering system design parameters whose interrelationships and tradeoffs were studied are displayed in Table 2.

Table 2

ENGINEERING SYSTEM DESIGN PARAMETERS

Number of trains (fleet size): N

Train speed (mph): V

Train capacity (capacity in containers): C

Terminal processing time (loading/unloading a train): P

Number of terminal platforms (train berths for loading/unloading):* P

L

^{*}In this analysis, the number of terminal platforms refers to the number of platforms in a terminal for one direction only; we assume that terminals are symmetric.

The specification of the simple linear corridor system is incomplete until the train operating strategies on this linear network are specified. It was decided that most insight can be gained by restricting the study of train operations to a small set of simple "canonical" strategies. Canonical strategies represent fundamental strategies; all other train strategies can be considered hybrid combinations of these canonical strategies. The three strategies (see Figure 1) are:

- (1) <u>Direct Service</u>——A train carries containers directly from origin to destination without intermediate stops.
- (2) Freightliner—A train leaves the initial terminal carrying all containers going in the same direction; a train will stop at an intermediate terminal only if it has containers to set out or containers to pick up above a specified amount. Once stopped, the train will pick up additional containers going in the same direction. (This is essentially a local train operation which can skip stops.)
- (3) Shuttle--Trains shuttle back and forth between adjacent terminals; containers desiring to go further than the next stop are required to transfer between shuttle trains.

To study the simple linear corridor system with the three operating strategies, two methodologies (tools) were developed. One approach to the analysis was based on a computer simulation model; this model is called LINET. This approach can consider a reasonably complex system description. The second approach is based on the development of a closed form set of analytical equations; this approach is called "hand analysis." The methods, procedures, and results for these two approaches are described in the next two sections. Note that the assumptions and problem formulation are slightly different for each approach; the hand analysis requires more simplifying assumptions to achieve closed form analytical results.

Although there is overlap in the area of investigation by the two methods, the LINET investigations have tended to focus on fundamental tradeoffs between system parameters, whereas the hand analysis has focused on a comparison of the three canonical operating strategies (direct service, freightliner, and shuttle). Often the understanding and interpretation of LINET results were aided by the insights gained in the hand analysis, because in the hand analysis the causal effects are explicitly displayed, whereas they may not be easily understood from the interpretation of a computer output. For these reasons the LINET investigations and hand analysis were mutually complementary.

III LINET INVESTIGATION

A. Introduction and Background

To determine the basic operational elements in the delivery of containers in a linehaul system and to estimate their interrelationships, the analysis of a five-node linear network with a variety of modes of operation (shown in Figure 1 and described in Section II-D) was studied using a GPSS* simulation model called LINET. Using LINET a wide range of linehaul speeds and terminal processing times were examined. The interrelationship with train capacity, linehaul link length, and number of terminal platforms were also analyzed.

Based on LINET a two-pronged approach was used. First, an analysis of the basic factors involved in the container shipment problem was performed to determine their interrelationships and estimate results; the interrelationships were encoded as simple analytical formulas. Second, the movement of containers and trains through the system was simulated in detail in the LINET model and operational data was accumulated in the process. These two approaches (analytical and simulated) were then combined by selecting pertinent data from the model results and refining the analytical formulas with the simulation results. The end result was a set of analytical formulas that could reproduce many of the simulation results, and in particular the principal measures of effectiveness. The formulas were then used to extend the results attained to areas not covered by the simulation runs.

B. LINET Model Description

The LINET model simulates the movement of trains and containers along a five node linear network in which the stations are approximately evenly spaced (see Figure 1). Two basic types of operating modes were built into the model: freightliner and direct service (see Section II-D). In the freightliner strategy, a train will stop at an intermediate terminal only if it has containers to set out or if there are containers to pick up greater than a specified threshold. Once a freightliner train stops, it drops off containers destined for the terminal and picks up containers going in the same direction as the train (up to the train capacity); priority for train pickup is given to containers going to the most distant station. In the direct service strategy, trains go directly from an origin terminal to a destination terminal. In both modes, trains

^{*}GPSS is the acronym for General Purpose Simulation Systems.

do not have scheduled departure times; trains depart immediately from a terminal after it has unloaded and loaded containers. In this manner trains are permitted to move through the system as rapidly as possible.

The principal measures of effectiveness were represented by the time in the system for a container, the variability of time in the system, and the container speed through the system for a feasible configuration. The principal measure of cost was assumed to be the total average daily cost associated with the system operation. Productivity is defined as the average speed of containers through the system divided by the total daily cost in millions, i.e.:

$\begin{array}{c} \text{Productivity} \, = \, \frac{\text{average effective container velocity}}{\text{total daily system cost}} \quad . \end{array}$

The LINET model applies link travel time and terminal processing time for each train and determines queuing delays dynamically. Time spent in the system by containers (from entry into the origin station to delivery at the destination station) is measured directly in the model. All operational parameters are varied stochastically about the mean parameter values.

The demand pattern for container movement is unbalanced, heaviest at one end of the line, i.e., the demand is the lightest at terminal 1, increasing to a maximum at terminal 5 (see Figure 1). The demand varies with time of day.

In most cases, data was taken from a simulation of 3 days of continuous operation of the system.

The basic system parameters in the LINET model can be placed into four categories: (1) exogenous system parameters, (2) operational parameters (both 1 and 2 are called variables of choice), (3) intermediate system parameters, and (4) system performance parameters. The exogenous system parameters describe the assumptions about the operating environment, i.e., network and demand characteristics. The operational system parameters describe the operating variables, i.e., train and terminal operating characteristics. Later in the discussion, we often refer to both the exogenous and operational system parameters as "variables of choice," because they are both model inputs and can be chosen or varied with each simulation run. The intermediate system parameters are those parameters which describe the characteristics of intermediate processes in the system. Finally, the system performance parameters describe the overall performance of the system. Following is a list of the four types of system parameters and their characteristics.

Exogenous System Parameters (Variables of Choice)

- Distance between stations (length of links in miles): (D)
- Demand level (in number of containers): (R)
- Average container trip length (in links): (A)

Operational System Parameters (Variables of Choice)

- Linehaul train speed (miles per hour): (V)
- Train capacity (in maximum number of containers): (C)
- Processing time at terminals (minutes or hours): (P)
- Number of trains: (N)
- ullet Number of loading/unloading platforms per station: (P_L)

The Intermediate System Parameters

- Train loading factor, or utilization (fraction of train capacity utilized): (U)
- Train lost time per link (train waiting time to enter/ leave link because of congestion): (Q)
- Loading/unloading applicability* (fraction of total terminal processing time utilized): (K)
- Container time waiting to be picked up at a station (minutes or hours): (W)

System Performance Parameters

- ullet Number of containers delivered during period of study: (C_D)
- Average time in the system for containers: (T_C)
- Daily cost of the system: (COST)
- \bullet Effective container speed through the system (total distance traveled divided by total time in the system): (V_E) \cdot

The purpose of the LINET simulation is to determine how the variables of choice (exogenous and operational system parameters) impact the system performance parameters, and, therefore, where the technical payoffs are to be found.

A wide variety exists for composite combinations of these parameters. Two which will be used widely in the following analysis are link transittime (D/V) and a composite measure of system performance which we call productivity ($V_{\rm p}/{\rm COST}$).

^{*}Because trains do not always need to load or unload, the full processing time does not always apply. The processing time is the sum of loading and unloading time.

The results attained are summarized briefly in the following pages. The subjects below are covered:

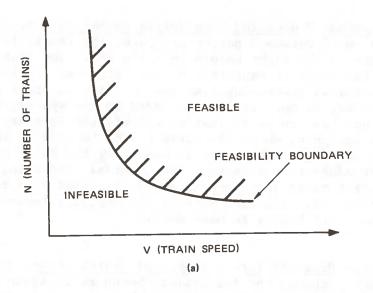
- Feasibility regions and boundaries
- Analytic formulations and approximations
- Time in the system along the feasibility boundary
- Estimated costs formulas
- Relation between feasibility curves and cost curves
- Productivity curves
- Time in the system curves
- Relation of time in the system and productivity curves
- Estimated effects of increasing system distances
- Estimated effects of changing train capacities
- Effects of changes in guideway costs
- Effects of one and two-terminal platforms.

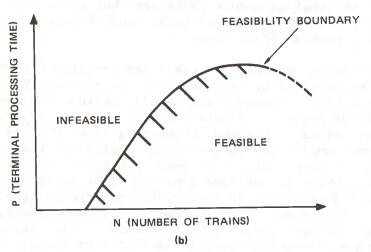
It should be emphasized that LINET was run against certain base case assumptions in the system parameters. Although we did not vary all system parameters, we hoped to discover fundamental trends in the interrelationships of the parameters we did vary.

C. Feasibility Regions and Boundaries

Using the LINET simulation model, it is possible to divide the system parameter space into two regions. In one part the system is capable of satisfying the given demand. In the other, it is not. The criterion for determining feasibility is found in the container backlog history obtained from the model (i.e., the accumulation of containers waiting to be picked up). If the backlog increases consistently throughout the period simulated, the system is considered infeasible. If the backlog does not build up, it is considered feasible. In many cases it is difficult to decide and the simulation run is considered marginal. Figure 5 shows examples of these feasibility regions in several two-dimensional parameter spaces. The curve which separates the feasible from the infeasible region is called the feasibility boundary. The feasibility boundary is in reality a surface in a multidimensional parameter space. Figure 5 shows only "slices" of this surface in two dimensions. The following three examples shown in Figure 5 are discussed below.

Case 1: Number of Trains Versus Train Speed (N versus V)—In the V versus N parameter space, the feasibility boundary is hyperbolic in shape as shown in Figure 5a. The vertical asymptote indicates that a minimum train speed is required to satisfy delivery of the containers. The horizontal asymptote indicates that a minimum number of trains is required.





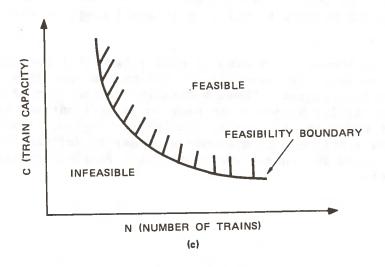


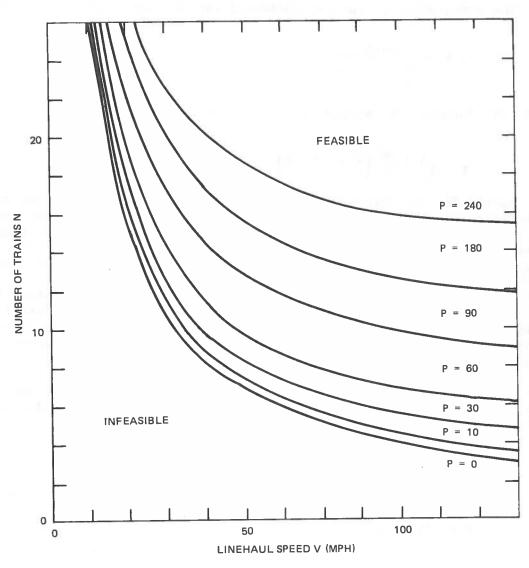
FIGURE 5 FEASIBILITY BOUNDARIES IN TWO DIMENSIONS

Case 2: Terminal Processing Time Versus Number of Trains
(P versus N)—In the P versus N parameter space, the feasibility boundary
rises with a slope to the right before leveling off. The initial rise
of the curve to the right is explained by the fact that initially with
few trains the terminal processing time must be fast in order to satisfy
the demand. However, as more trains are added to the system, the terminal
processing does not have to be as fast to satisfy the delivery of the
containers up to the point where the curve begins to bend to the right and
level off. This bending of the curve is caused by the fact that as additional trains are added to the system, the terminal processing time must
be sufficiently fast to prevent queuing delays for trains in the terminal
waiting to be processed. (In fact some of our analysis indicates that
the curve at some point begins to bend down.)

Case 3: Train Capacity Versus Number of Trains (C versus N)--In the C versus N parameter space, the feasibility boundary is again hyperbolic in shape. The vertical asymptote indicates that there is a minimum number of trains required to satisfy the demand; the horizontal asymptote indicates there is a minimum train capacity.

In the following, we describe LINET investigations that concentrated primarily on the feasibility boundaries in the N versus V parameter space (i.e., number of trains versus train speed). Figure 6 depicts in more detail a family of feasibility curves in the N versus V parameter space for the various values of terminal processing time (P). As the linehaul speed (V) becomes small, the curves become nearly vertical and closely spaced. Thus, the number of trains needed increases very rapidly as speed decreases, while at the same time the sensitivity to terminal processing time (P) decreases. At low speeds the transit time is large, diminishing the importance of processing time in determining feasibility. Conversely, as V becomes large, the curves are nearly horizontal and for practical purposes determine the minimum number of trains needed to operate the system successfully, regardless of speed. The curves here are widely spaced because travel time is small compared with processing time.

As will be shown, it is also generally true that operation on the feasibility boundary will generally be cheaper in cost than in any nearby feasibility region. Thus, a detailed examination of results along the feasibility boundary can provide significant and useful results without examining the entire feasibility region. Furthermore, each curve has a noticeable, although not sharply defined "knee." This knee will be found to contain the most cost-effective combination of system parameters.



NOTE: Feasibility boundaries for linehaul speed versus number of trains for various processing times (P) in minutes

D = 108 Miles
C = 100 Containers
1 Platform/Station
Freightliner Mode
Demand = 2300/Day

FIGURE 6 FEASIBILITY CURVE FAMILY IN N-V PLANE

D. Analytical Formulas and Approximations

Analytical formulas have been prepared to approximate the most important system performance parameters in terms of the variables of choice and other parameters. Among these are the following.*

The formula for containers delivered per day is:

$$C_{D} = \frac{24CNU}{A\left(\frac{D}{V} + KP + Q\right)},$$
(1)

while the formula for average time in the system is:

$$T_{C} = \left(A + \frac{8W}{N}\right) \left(\frac{D}{V} + KP + Q\right) \qquad . \tag{2}$$

Values for the intermediate parameters and the system performance parameters can be obtained from the simulation output. Each simulation run provides the intermediate system and system performance parameters corresponding to the input parameters used in the simulation. Each simulation run can be considered a single, although multivalued, data point.

It is desirable to eliminate the intermediate system parameters from the formulations for system performance parameters so that the latter may be expressed solely as functions of the variables of choice. In order to do this, it was necessary to find approximations of the intermediate system parameters in terms of the variables of choice.

Using the results of many base case simulation runs, linear approximations of intermediate system parameters in terms of the variables of choice were computed and found to be of sufficient accuracy for use in the analytical formulations. These linear approximations are:

$$Q = \left(.08 + .24P + .021P \frac{D}{V}\right) \left(1.6 - .006C\right)$$
 (3)

$$K = .74 - .025 \frac{D}{V}$$
 (4)

$$C_{D} = 2000 - 80 \frac{D}{V}$$
 (5)

$$U = .6 - .014 \frac{D}{V}$$
 (6)

$$W = 1.25(1.6 - .006C) (7)$$

 $^{^{\}star}$ The formulas are derived in Appendix A.

In obtaining these approximations, data from the simulations were plotted over a wide range of variables, and simple patterns were sought. When D/V was used as the independent variable, it was found that the patterns could be approximated by linear functions to a satisfactory degree. Regression methods then provided the mathematical functions shown. In the case of Q, it was found necessary to use three successive linear regressions.

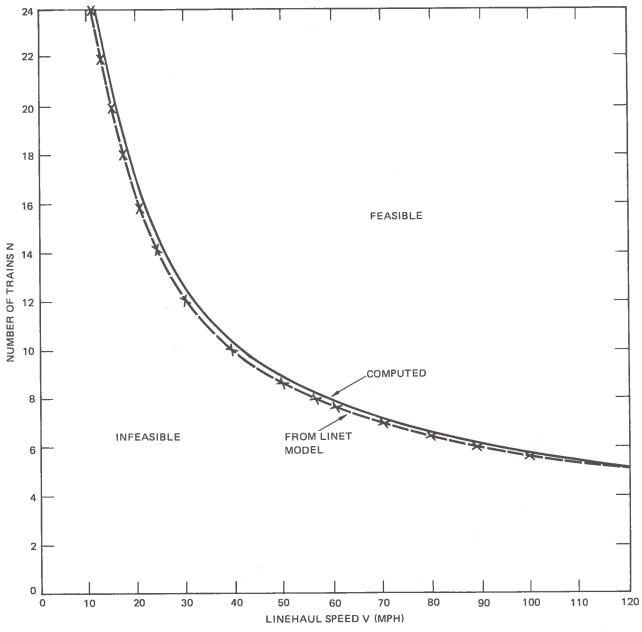
Obviously the above linear approximations are simplifications. For perfect accuracy we should expect that the intermediate system parameters are nonsimple functions of several variables of choice. However, based on LINET results, we find that some variables of choice have a very limited or highly inconsistent effect on the intermediate parameters. Therefore, they were omitted from the formulation. The criterion for the inclusion of a variable of choice was the extent to which inclusion was needed to reproduce LINET results. The above are the least complex formulations that do so.

Because the approximations are based on data taken from the LINET model result along the feasibility boundary, there is a value of N corresponding to each value of D/V, and a set of formulations can be worked out expressing the intermediate parameters in terms of N. However, in practice, it is more likely that D/V will be picked as a variable of choice rather than N, consequently, the above formulations in terms of D/V have been used in the subsequent analysis. When using them, the value of N can be computed from the basic formulas (Equations 1 and 2) using the approximations (Equations 3 through 7) in terms of D/V.

In the ensuing analysis, the linear approximations (Equations 3 through 7) for the intermediate parameters given above were inserted into the formulas for the system performance parameters (Equations 1 and 2), with the result that those formulas then depend solely on the variables of choice. When this is done, the values obtained closely approximate those provided by the simulation runs, with the additional advantage of the data being smoothed out and the "ideal" shape of the curve being revealed. Figure 7 shows typical feasibility curves determined from LINET results and as calculated by analytical approximations. It shows the fit is quite close, with the computed curve being slightly less "permissive" than the curve derived by eye from LINET results. It is clear that the basic shape of the curve has been preserved, particularly at the critical "knee."

E. Time in the System Along the Feasibility Boundary

Figure 8 shows a family of curves for time in the system plotted against linehaul speed for a specific combination of train capacity, demand, and interstation distance. The hyperbolic shape is clearly evident and the general shape is typical.



C = 100 Containers

P = 60 Minutes

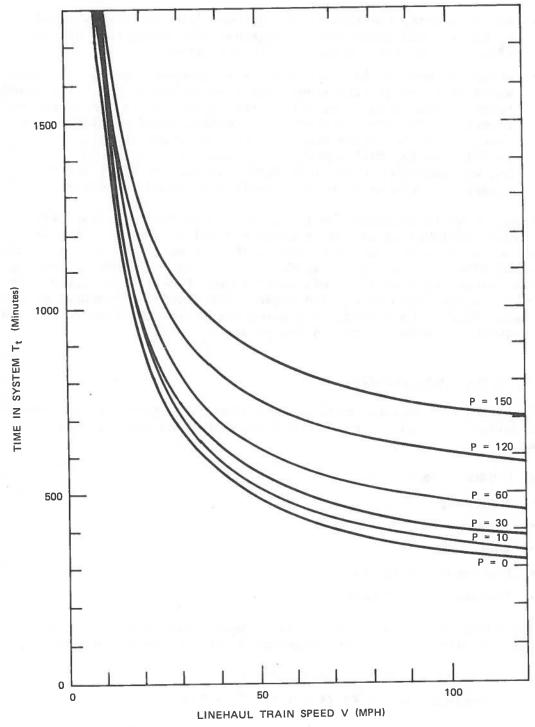
D = 108 Miles

Demand = 2300 Containers/Day

I = Terminal Platform

Freightliner Mode

FIGURE 7 COMPARISON OF FEASIBILITY CURVES FROM LINET AND ANALYTICALLY COMPUTED



NOTE: Time in the system versus linehaul speed along the feasibility boundary for various processing times (P) in minutes

FIGURE 8 TIME IN SYSTEM VERSUS LINEHAUL SPEED

A number of useful inferences can be made from this figure:

- At low speeds, the time in the system rises rapidly as speed decreases, and reductions in terminal processing time are not effective in reducing the time in the system.
- At speeds over 50 mph, the reverse is generally true. Increased speed does not greatly reduce the time in the system. Increased terminal processing time either increases time in the system or requires very large increases in linehaul speed if time in the system is to be maintained constant. At those speeds, the travel time is small compared with other time components (loading and unloading time, lost time, and waiting time) and the travel time component becomes smaller as speed increases.

Figure 8 would be useful in the initial selection of parameters for a system designed to provide a certain level of service. For instance, for an average container time in the system of 600 min, a terminal processing time of 60 min would allow a line speed of 55 mph. Reducing terminal processing time to 30 min would reduce the linehaul speed only to 45 mph. A zero processing time would still require a linehaul speed of 38 mph. On the other hand, increasing the processing time to 120 min would require a linehaul speed in excess of 100 mph.

F. Estimated Costs Formulas

Daily costs associated with given system characteristics in terms of the variables of choice have been estimated. The cost categories include:

- Guideway cost
- Terminal cost
- Crew cost
- Fuel cost
- Equipment capital cost
- Equipment maintenance cost.

The formulas for daily costs, in terms of variables of choice are given in the listing below (see Appendix B for cost development).

Guideway costs = D(395 + .304
$$V^2$$
)

Terminal costs = $CP_L(157 + 133/P) + 1370$

Crew costs = 949N, where N = $\frac{DV + PV^2 - 450}{V(.33V - .83)}$

Fuel costs = $\frac{C(16 + 20U)[3.8V + V^2(.0515 + .89/D)](train-miles)}{1,000,000}$, where train-miles = $\frac{2.18DCD}{CU}$

Equipment capital costs = $.003NV^2C$ Equipment maintenance costs = .02C(train-miles) + .6(fuel cost).

Note, that formulas for U and \mathbf{C}_{D} in terms of variables of choice were given earlier.

The cost formulas represent extrapolations of current rail technology. As will be seen later, other forms of guideway costs were employed to test the sensitivity of results.

G. Relation Between Feasibility Curves and Cost Curves

Figure 9 shows a typical feasibility curve and the associated equal cost contours taken from model results. It will be seen that the cost curves are somewhat similar in shape and orientation to the feasibility curves, but with less curvature and with cost increasing with distance from the origin. It is obvious that minimum costs will be found in the "knee" of the feasibility curve, in which area the feasibility curve is tangent to some cost curve. Thus, in the knee of the feasibility curve, costs are not only at minimum but fairly constant throughout the knee.

The knee of the feasibility curve is an area in which not only is the minimum cost achieved, but that minimum cost extends over a considerable range of parameters. As shown in Figure 9, the range of approximately equal costs extends from 11 trains at 50 mph to 16 trains at 20 mph, with perhaps the cheapest feasible solution using 13 trains at 30 mph.

It should be emphasized that this is not necessarily the most costeffective solution; it is merely the cheapest feasible solution. Points on the interior of the feasibility region may provide higher costeffectiveness even though at a higher cost.

H. Productivity Curves

One of the principal measures of productivity for a system configuration is taken as the effective speed with which the average container moves through the system compared with the total cost of achieving that speed. When we refer to productivity in this analysis, we refer to this measure—the effective velocity divided by cost, i.e.:

$$\label{eq:productivity} \text{Productivity} = \frac{\text{average effective container velocity}}{\text{total daily system cost}} = \frac{V_E}{\text{COST}} \; .$$

The average effective container velocity is the total distance traveled divided by the total time in the system; the total time in the system includes waiting time in terminals and over the linehaul (due to congestion).

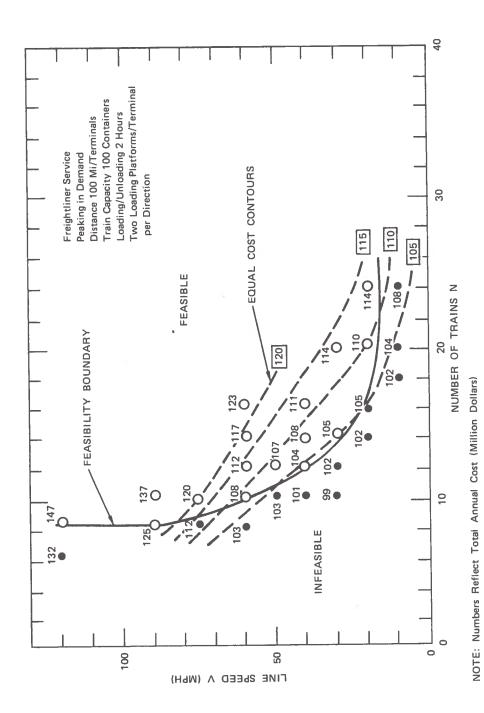


FIGURE 9 FEASIBILITY CURVE AND EQUAL COST CONTOURS IN V-N PLANE

Figure 10 presents curves of equal productivity plotted for values of travel time per link (D/V) and terminal processing time (P) for one selection of the variables of choice. Note that the horizontal axis is graduated in linehaul travel time per link in hours, while the vertical axis is in terminal processing time (P), also in hours. Thus, both axes are in equal time units.

The plot shows flattened oval patterns skewed to the right and up. These plots are typical.

An approximate maximum occurs where P=.7 and D/V=2.9. The corresponding linehaul speed is 37 mph. This speed is typical of the speeds used in existing operations, but the processing time, 42 min, is considerably shorter.

Much more can be obtained from this chart. Note that productivity is not sensitive to changes in the vicinity of the maximum in any direction. However, after a certain point, any changes toward the left (increasing speed) or downward (decreasing processing time) cause a very sudden and drastic reduction in productivity. This is due to the costs associated with achieving the higher speeds and lower processing times required.

On the other hand, the penalty associated with increases in processing time or decreases in speed is neither sudden nor drastic.

In general, we expect these patterns to hold for other combinations of system parameters. There will always be an optimum combination of speed and processing time, but considerable latitude around the optimum can be allowed, and the principal sensitivity will be to effect substantial changes upward in speed or downward in processing time.

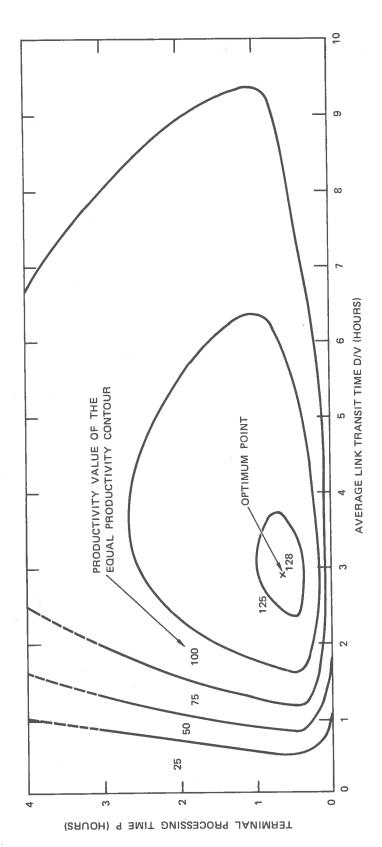
I. Time in the System Curves

It is informative to plot curves of equal container time in the system for various values of P and D/V. Figure 11 shows such curves for a specific combination of other variables of choice.

The lines in this figure are fairly straight and evenly spaced. This should not be surprising as P and D/V are combined linearly in computing time in the system and heavily influence the result.

Figure 11 illustrates a means of rapid tradeoff between $\ensuremath{\text{D/V}}$ and $\ensuremath{\text{P}}$ for any given level of service.

The sections of the curves between P=3 and P=4 are unsubstantiated by LINET runs and are therefore indicated with dashed lines.



FOR PARAMETER VALUES:

C = 100 Containers

D = 108 Miles/Station

Demand = 2300 Containers/Day 1 Terminal Platform

FIGURE 10 CONTOUR CURVES OF EQUAL PRODUCTIVITY

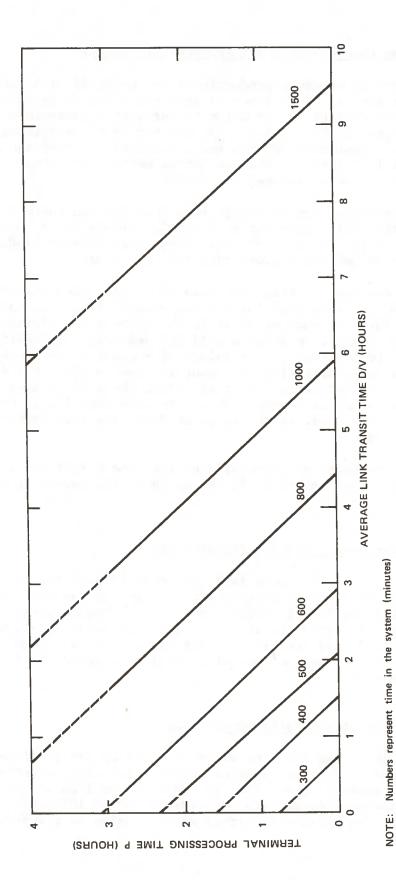


FIGURE 11 CURVES OF EQUAL TIME IN THE SYSTEM FOR P AND D/V

C = 100 Containers
D = 108 Miles/Station
Demand = 2300/Day
1 Platform

J. Relation of Time in the System and Productivity Curves

In Figure 12, curves of equal productivity are overlaid with curves of equal time in the system. Such a method provides a better means of comparing the relative effects of variation in particular parameters. For instance, operating at D/V = 2.5(V = 42) and P = .5 is associated with an average time in the system of 600 min and a productivity near the maximum. This same level of service could be maintained at many other combinations of D/V and P, but at reduced productivity.

Any attempt to achieve service levels less than 600 min sharply reduces the productivity, but a relaxation of service levels has a less drastic effect. A service level of 800 min allows near maximum productivity for speeds of 30 mph and a processing time of 50 min.

Another way of rearranging this same data is to plot productivity (V_E/COST) versus time in the system for various values of P. This has been done in Figure 13. For each value of P, the curve rises rapidly as time in the system increases, reaching a well-defined peak. Thereafter, each curve tails off to the right. For values of P equal to or greater than 1 hr, the peaks in productivity are about the same height, but displaced to the right as P increases. In these cases the peak is associated with a linehaul speed of about 50 mph. When P is less than 1 hr, the peak moves to the left and is lower, as the value of P becomes more important in the cost formulations.

Interestingly, the peaks are generally of the same height for a P of 1 hr or more. For these values of P, the speed is the governing element of cost.

K. The 95% Zone Surrounding Maximum Productivity

In the productivity plots, there is a considerable area around the maximum in which the productivity differs little from the maximum, but there is no sharp dividing line to suggest a cut-off. However, the line defining the area in which productivity is at least as great as 95% of the maximum encompasses a considerable breadth of parameters. We arbitrarily chose that zone for the purpose of comparing the productivity of various configurations.

L. Variations in Productivity with System Size

Most of the analysis was done for a system in which the stations were about 100 mi apart. However, limited analysis was also performed on larger systems. Figure 14 shows the 95% zones for configurations in which the average distance between stations (D) is set at 108, 200, and 500 mi. In each zone the maximum is marked and the value at the zone boundary is given.

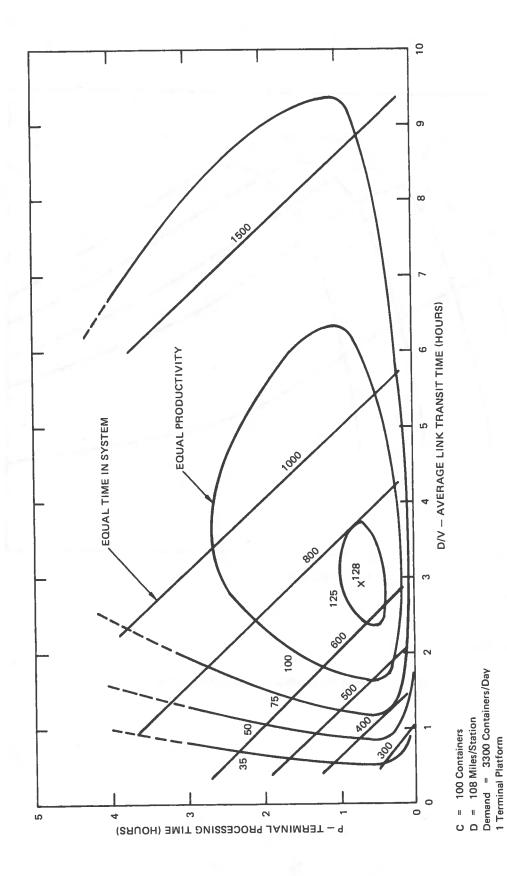
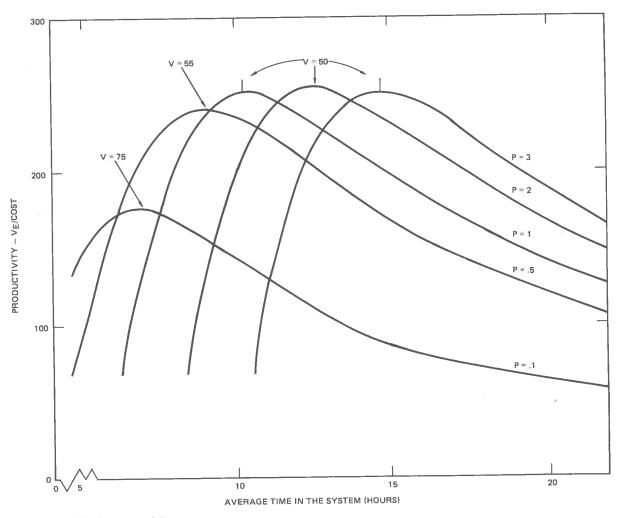


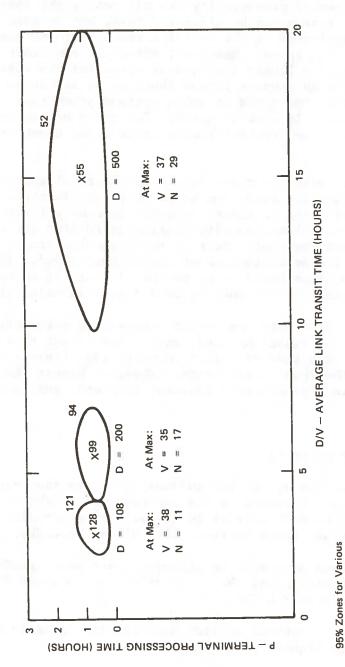
FIGURE 12 CURVES OF EQUAL PRODUCTIVITY AND EQUAL TIME IN SYSTEM



V/COST Versus Time in the System for Various Values of P

C = 100 Containers D = 108 Miles/Station Demand = 2300 Containers/Day 1 Terminal Platform

FIGURE 13 PRODUCTIVITY VERSUS TIME IN THE SYSTEM



95% Zones for Various
Distances Between
Stations
C = 100
X = Point of Maximum
Cost Effectiveness

FIGURE 14 PRODUCTIVITY MAXIMUMS AS A FUNCTION OF SYSTEM SIZE

Note that as the distance increases, the zones move to the right (transit time is increased) and slightly downward, and the size of the zone increases.

Perhaps the most surprising feature is that the linehaul speeds associated with the maximum of productivity are all nearly the same value—about 36 mph. No reason can be given for this, but we note that the cost equations are dominated by the linehaul speed (V), and cost increases quadratically with speed. Moreover, effective container velocity ($V_{\rm E}$) is in general a linear function of speed (within reasonable operating ranges) and an inverse linear function of distance. Thus, one would expect that the speed at which optimum productivity occurs would not change greatly with distance. One would not necessarily expect that the optimum linehaul speeds would be as close as is indicated.

On balance it seems to be a combination of mathematical and physical factors. The cost function works out to be very closely parabolic with speed, while the effective container velocity is nearly linear with speed. Combining these functions and differentiating to find the maximum productivity, we find that the coefficient of the V-square term in the cost equation dominates the determination of the optimum speed. This coefficient forces the optimum speed to be in the vicinity of 40 mph. The effect of the remaining coefficients is to reduce this value slightly.

We cannot, however, attribute the result entirely to mathematics. Physical factors govern the equations used, and to the extent that costs are truly quadratic and effective container velocity ($V_{\rm E}$) truly linear with speed, we believe the results are valid. Thus, we expect that the optimum speed will remain approximately constant even when system size varies greatly.

M. Variations in Guideway Costs

The cost formulas assign all of the guideway costs to the intermodal freight system operations. Inasmuch as the guideway may be shared with other activities, it may be more correct to use different formulas. Also, the assumption that guideway costs increase with ${\tt V}^2$ may be unduly severe.

Four different methods of computing guideway costs were examined. The different cost formulations and the associated optimum speed for the productivity maximum are shown in Table 3.

All of the alternative methods provide dramatic increases in maximum productivity although higher speeds are required.

Table 3
METHODS OF COMPUTING GUIDEWAY COSTS

Cost Function*	Productivity Maximum	Optimum Speed at Maximum
Basic cost formulation	128	37 mph
Guideway cost in basic cost formu- lation is multiplied by 1/10	271	52 mph
The velocity dependent terms in the basic cost formulation are multiplied by 1/10	223	83 mph
Guideway cost in basic cost formu- lation is replaced by twice the fuel costs [†]	266	60 mph
		oo mpii

^{*}The basic cost, guideway cost, and fuel cost formulations are shown in Section III-F.

[†]This cost formulation represents the financing of guideway costs through a user fuel tax or surcharge.

N. The Effects of Train Capacity on Productivity

Table 4 summarizes selected operational effects for five choices of train capacity. In general, if capacity is to be decreased, speed must be increased and processing time decreased, and the freedom of choice for operating parameters becomes smaller. The data presented in Table 4 apply only to the optimum points.

Table 4
OPERATIONAL EFFECTS FOR TRAIN CAPACITY

	Train Capacity				
Parameters	10	25	50	100	150
Productivity	1.75	171	153	128	119
Number of trains	59	31	19	11	7
Linehaul speed	60	46	40	37	36
System capacity	590	775	950	1100	1050
Container speed	48	34	25	21	22
Daily cost	.27	.20	.17	.16	.18
Terminal process-					
ing time	.1	. 2	.36	.6	. 85

Maximum productivity is seen to increase rapidly for a time as capacity decreases. This is due largely to increase in container speed and smaller increases in daily cost. However, the number of trains required also increases rapidly and queuing effects may negate the benefits of smaller trains if that trend is pursued further.

An examination of the results when capacity is reduced to 10 bears this out. Productivity for C=10 is only slightly better than for C=25. On the other hand, the speed must be greatly increased and processing time must be reduced to 6 min per train.

The conclusion we must draw here is that productivity is improved by smaller trains in conjunction with faster terminal processing and faster train speeds, but there may be technical engineering difficulties and the law of diminishing returns may apply in this operating regime.

O. Terminal Platform Analysis

The number of platforms at a terminal determines the number of trains which can be processed simultaneously arriving from the same direction. Note that by our definition a one-platform terminal actually has two platforms; one platform for each train direction.

A number of simulation runs were made in which the number of loading and unloading platforms at each terminal was increased to two. Figure 15 shows the two feasibility curves (one and two platform cases) which were obtained for the freightliner mode of operation; the characteristics were essentially the same for direct service. We see that adding an extra platform extends the feasibility region so that system operation with a larger number of smaller trains is possible. With only one platform, operation with a large number of smaller trains is likely to result in queuing congestions at the terminal caused by arriving trains waiting for an empty platform to be processed.

Additional one and two platform sensitivity analysis was performed in which average time in the system was plotted against train speed, and average time in the system was plotted against terminal processing time when operating along the feasibility boundary for the one platform case. In both cases there seems to be little difference between the one and two platform cases.

The above analysis results can be explained as follows. The extra platform expands the region in which system operation is feasible; however, once a system operation is feasible, the addition of an extra platform appears to have little system effect.

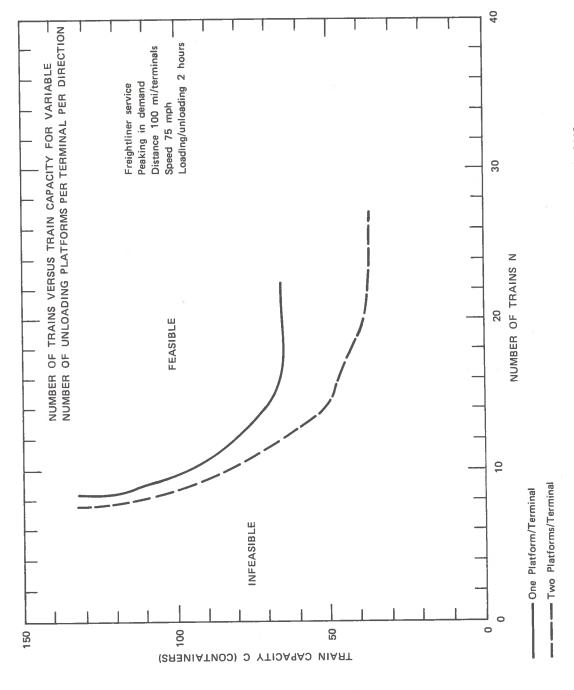


FIGURE 15 FEASIBILITY CURVES FOR ONE AND TWO PLATFORMS

IV HAND ANALYSIS INVESTIGATION

A. Introduction

This section discusses the hand analysis procedure and its results. Here we will try to replicate the freight system activities by means of manual analysis. We believe this method provides good insight into the underlying structure of the problem. We tried to formulate the hand analysis model as close as possible to the computer simulation model LINET. However, in many aspects the manual analysis model is much simpler than the LINET model and some assumptions adopted in the hand analysis are different from the assumptions in LINET. Because of these differences, there is not total agreement on the results of the two models.

This section consists of four parts. The first part gives the system descriptions, which includes a description of the three systems studied and major assumptions adopted in the study. The second part describes the measure of effectiveness and the system cost used in the hand analysis. The third part describes the formulation of the problem and the feasibility boundary for the three different operational strategies. The fourth part describes the analysis results obtained by using the hand analysis model.

B. System Description

Three operational strategies were studied. They are: The direct service strategy, the freghtliner strategy and the shuttle strategy. These operational strategies were applied to the five node linear network in which terminals are separated at equal distances (see Figure 1 in Section II).

The common assumptions for the three strategies are:

- The container arrival patterns at terminals follows Poisson distribution.
- The train arrivals at a terminal are random with a given arrival rate.

The notation of variables used in this analysis are:

- R = Rate of traffic generation between any origin and destination pair (containers per hour).
- V = Linehaul speed (miles per hour).
- V_{AVE} = Average train speed including stops (miles per hour).

D = Length of all links (miles).

k = Average load factor (ratio).

M = Mean load per train (containers).

P = Terminal processing time for loading/unloading activity (hours).

 $T_{R,i}$ = Round trip time of route i (hours).

 $T_{H,i} = Train headway on route i (hours).$

 C_{T}^{\dagger} = Train capacity (containers).

 $C_{R,i}$ = Route capacity for route i (containers/hour).

 $D_{R,i}$ = Route demand for route i (containers/hour).

 N_{i} = Number of trains on route i.

m = Number of loading/unloading platforms at a terminal.

 Δt_i = Terminal delay time on route i due to train queue.

 $T_{\rm HM}$ = Train headway at a loading platform of a terminal.

 $T_{A,F}$ = Average transit time of freightliner strategy.

 $T_{A,D}$ = Average transit time of direct service strategy.

 $T_{A,S}$ = Average transit time of shuttle strategy.

 $\Delta t_{\rm E}^{'}$ = Terminal delay time of freightliner strategy.

 $\Delta t_D^{}$ = Terminal delay time of direct service strategy.

 Δt_{S} = Terminal delay time of shuttle strategy.

Tha notation of variables used in hand analysis is identical to that used in LINET analysis with the exception of average load factor, train capacity, and the number of loading/unloading platforms at a terminal.

The three operational strategies are briefly described below:

Freightliner Strategy—The train leaves the initial terminal carrying all the containers which are going in the same direction. The train will stop at an intermediate terminal only if it is carrying containers to be set out at the terminal. If the train makes a stop at a terminal, it will take all the containers which have accumulated going in the same direction as the train (see Figure 1 in Section II).

Two routes are considered in the freightliner strategy. Route 1 starts from terminal 1 and terminates at terminal 5, and Route 2 starts from terminal 2 and terminates at terminal 4. Route 1 serves container traffic demands of all the O-D pairs. Route 2 serves container traffic demands of O-D pairs 2-3, 3-4 and 2-4 (O-D traffic for the reverse direction is also served by this route). In theory, the trains in the freightliner system have a certain probability of skipping stops at terminals. However, here it was assumed that the mean load per train is sufficiently large to assume that every train stops at every terminal. The derivation of the expected number of stops of a freightliner train is given in Appendix C.

Direct Service Strategy—A route is set up for each origin and destination pair. The train shuttles back and forth between the two terminals, carrying only those containers which have a common destination terminal. The departure interval of the train is random with a given rate of departures per unit time (see Figure 1 in Section II).

Shuttle Strategy—The shuttle strategy has four routes. Trains in the system travel back and forth between the adjacent terminals. At one terminal, the containers going in the same direction are all picked up by the same train, and those containers which are sent more than one terminal away from the origin terminal must be transferred to a train in the neighboring route at intermediate terminals (see Figure 1 in Section II).

C. Measure of Effectiveness and System Cost

1. Measure of Effectiveness

The measure of effectiveness adopted in this section is mean transit time of the all containers handled by the system. The transit time of a trip includes the time of waiting for a train arrival at an origin terminal, the loading time at the origin terminal, the linehaul transit time, the intermediate stopping time including the waiting time of the train in a terminal queue, the time for terminal queue, and the unloading time at the destination terminal.

It is assumed that there is sufficient train capacity at every terminal so that the extra waiting time for originating containers due to space unavailability does not have to be considered. The average transit times for the three strategies are given as follows:

Average transit time of the freightliner strategy, $T_{A,F}$:

$$T_{A,F} = 2P + \frac{2D}{V} + \frac{2}{5} \Delta t_F + \frac{9k C_T}{80R}$$
 (8)

Average transit time of the direct service strategy, $T_{A,D}$:

$$T_{A,D} = P + \frac{2D}{V} + \Delta t_D + \frac{k C_T}{2R}$$
 (9)

Average transit time of the shuttle strategy, $T_{A,S}$:

$$T_{A,S} = 2P + \frac{2D}{V} + \left(\frac{2}{5}\Delta t_1 + \frac{3}{5}\right)\Delta t_2 + \frac{9k C_T}{80R}$$
 (10)

In the above equations, the first term indicates the time spent by containers already in transit at the origin and intermediate terminals,

the second term indicates the linehaul transit time, the third term indicates the time spent at destination and intermediate terminals due to train queues, and the last term indicates the waiting time for originating container terminals.

The average time equation for the shuttle strategy does not take into account the possible queuing phenomena of containers at intermediate stops. However, without the container queuing delay at the intermediate stops, the shuttle strategy never has a shorter average transit time than the freightliner strategy. This is because the terminal delay of shuttle trains is always at least as large as that for the freightliner strategy.

The derivation of the above equations is given in Appendix D.

2. System Cost

The system cost consists of the guideway cost, the terminal cost, the crew, the fuel cost, the equipment capital cost, and the equipment maintenance cost. The cost function used in the hand analysis is a simplified version of the one given in Appendix B. The cost function (per day) of each category is given as follows:

Guideway costs (dollars) = D(395 + .304
$$V^2$$
) ,

Terminal costs (dollars) = $C_T m(157 + \frac{133}{P}) + 1370$,

Crew costs (dollars) = 949N ,

Fuel costs (dollars)
$$= \frac{C_T (16 + 2C_T k) \left[3.8V + V^2 (.0515 + \frac{.89}{D})\right] \text{ (train-miles)}}{1,000,000}$$
Equipment capital costs (dollars) = .003N V^2 C $_T$,

Equipment maintenance costs (dollars)
$$= .02C_T \text{ (train-miles)} + .6 \text{ (fuel cost)}$$

The guideway cost is the dominant subsystem cost. The linehaul speed (V) plays an important role; this is especially true when the speed is high (say over 100 mph). This cost function, when applied to the three strategies, shows that the shuttle strategy can never be less expensive than the freightliner strategy when all the parameters except C (train capacity) and N (number of trains in the system) are fixed to the same values for both strategies.

D. Formulation of the Problem and the Feasibility Boundary

We will assume a system for all three strategies in which trains are traveling their assigned routes continuously without any headway adjustments.

The system operation becomes feasible if the capacity of each route is larger than the demand on that route, and if the terminals have sufficient loading platforms to handle trains. Those two conditions are expressed using inequalities such that:

$$C_{R,i} \ge D_{R,i} \qquad (12)$$

$$T_{HM} \ge \frac{1}{m} P \qquad . \tag{13}$$

The capacity of each route is estimated as:

Capacity of a route = (train capacity) x (number of trains passing by a point per unit time)

= (train capacity)/(train headway) (14)

= (train capacity) x (number of trains in the system)/(round trip time).

Here, the train capacity and the number of trains in the system are given parameters. The round trip time is computed using parameters such as the terminal processing time, distance between terminals, linehaul speed, and the terminal queuing time.

The terminal processing time constraint expressed in (13) holds for any P values, because the average headway at a loading platform of a terminal, T_{HM} , is a function of P and the terminal delay, Δt , as well as other variables, and the large P value yields even larger T_{HM} value due to the large P and Δt values.

Here the constraint expressed in (13) merely says that one train can be served at a loading platform. There are no constraints in the number of trains that queue up at a terminal.

The terminal constraint can be expressed in various forms. For example, if the constraint is that only one train can occupy a loading platform and there is no space for trains to wait, then it will be expressed as:

$$T_{HM} \ge \frac{1}{m} (P + \Delta t) \qquad . \tag{15}$$

In this hand analysis we assume the constraint expressed in (13). This constraint was chosen because we felt it to be realistic and also because this same assumption was adopted for LINET.

The amount of delay is estimated by a conventional queuing model. We assume that each train is assigned a loading platform at each terminal in such a manner that exactly 1/m of total train traffic will arrive at a loading platform if the terminal has m loading platforms.

If we assume that at a loading platform of a terminal the trains arrive at random with an average headway of $T_{\rm HM}$ and that the service time at that terminal is a constant amount P, then a conventional queuing equation to estimate delay applies, and the amount of delay, Δt , is expressed as:

$$\Delta t = \frac{P^2}{2(T_{HM} - P)} \qquad . \tag{16}$$

The train headway at a terminal T_{HM} is given as a function of N, P, D, V, k, m, and Δt . However, the T_{HM} value becomes constant when the system is operated on the feasibility boundary or along the points where the inequality (12) becomes equality.

The formulation of models for the three operational strategies is presented below.

Freightliner Strategy

It is assumed that two routes are operated in the freightliner strategy. Route 1 starts at terminal 1 and terminates at terminal 5, and Route 2 starts at terminal 2 and terminates at terminal 4 (see Figure 16). The trains of routes 1 and 2 have a chance of being delayed at only two terminals: terminals 2 and 4. This is because these two terminals are the only point where the two routes "merge" into one stream of train flow. If we denote the amount of delay per train at one merge point by $\Delta t_{\rm F}$, then, the amount of delay per route is two times $\Delta t_{\rm F}$ on both routes.

By applying the general inequality shown in (12) to this system, we obtain:

$$P \le \frac{k C_T}{40R} (N-1) - \frac{D}{V} - \frac{3}{10} \Delta t_F$$
 (17)

The derivation of (17) and $\Delta t_{\overline{F}}$ is given in Appendix E.

Figure 17 shows the feasibility region of the freightliner system as as function of P (terminal processing time), V (linehaul speed), and m (number of loading platforms per terminal). This figure was constructed from (17). Fixed values were assumed for the other parameters. The figure

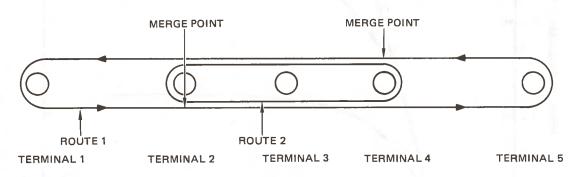
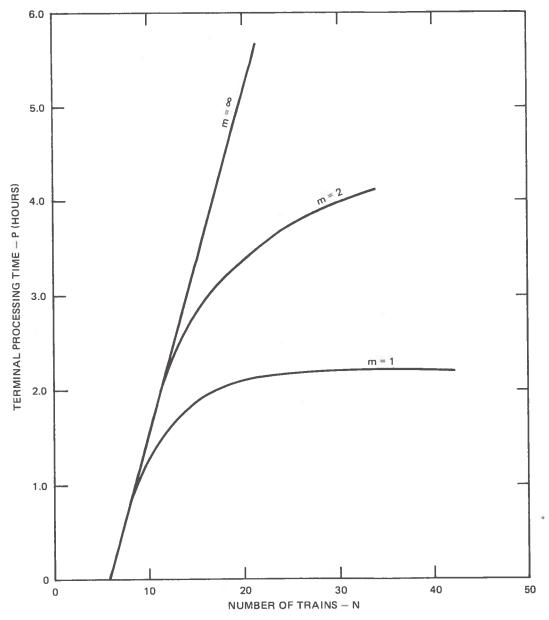


FIGURE 16 ROUTE CONFIGURATION OF FREIGHTLINER SYSTEM



k = .7

C_T = 100 Containers

R = 5 Containers/Hour

D = 100 Miles

V = 60 mph

FIGURE 17 FEASIBILITY REGION FOR THE FREIGHTLINER STRATEGY AS A FUNCTION OF TERMINAL PROCESSING TIME, P, NUMBER OF TRAINS, N, AND NUMBER OF LOADING PLATFORMS, m

shows that for m=1, the upper bound of P is more or less constant for N greater than 20. This is due to the progressive increase of terminal delay as the number of trains, N, increases. The feasibility boundary grows linearly as N grows if there are an infinite number of loading platforms (see curve with $m=\infty$).

Inequality (17) can be rewritten as:

$$C_{T} \ge \frac{40R}{K(N-1)} \left[P + \frac{D}{V} + \frac{3}{10} \Delta t_{F} \right]$$
 (18)

Inequality (18) shows that the feasibility region boundary becomes a hyperbolic function if the feasibility boundary is expressed by $C_{\rm T}$ (train capacity) and N (number of trains in the system) and all the other parameters are given a fixed value.

Figure 18 shows the feasibility regions constructed from (18). In the figure the variables are N (number of trains in the system), $C_{\rm T}$ (train capacity), R (container generation rate per O-D pair), and m (number of loading platforms per terminal).

2. Direct Service Strategy

The system has 10 routes connecting every 0-D pair directly as shown in Figure 19. The trains shuttle back and forth between a given pair of terminals.

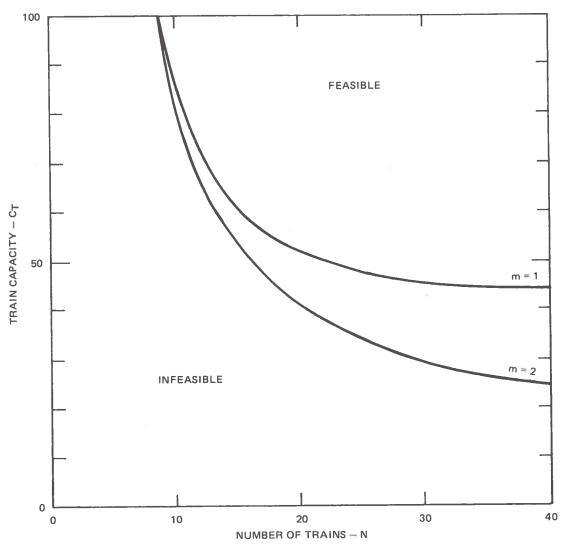
If we assume that the load factor on all the routes is identical, then the train headways of all the routes are also identical, because the container generation rates for all the origin-destination pairs are assumed to be equal. Furthermore, we assume that the amount of delay for any train is identical at any terminal, and for any route.

By applying the capacity/demand constraint to each route and summing them up, we obtain:

$$P \le \frac{kC_T}{20R} (N-5) - \frac{2D}{V} - \Delta t_D \qquad (19)$$

The derivation of inequality (19) and $\Delta t_{\rm D}^{}$ is given in Appendix E.

The feasibility region for m=1 is shown in Figure 20 overlapped with the feasibility regions of the other strategies. Also, in this case, the upper limit of terminal processing time increases as the number of trains in the system increases. However, the shape of the feasible region under the direct service strategy is somewhat different from the one under the freightliner strategy. Here, the upper bound of terminal processing time increases as the number of trains in the system increases even when m=1.



k = .7

P = 1.0 Hour

R = 5 Containers/Hour

D = 100 Miles

V = 60 mph

FIGURE 18 FEASIBILITY REGION FOR FREIGHTLINER STRATEGY AS A FUNCTION OF TRAINS, N, TRAIN CAPACITY, C_T, AND NUMBER OF LOADING PLATFORM, m

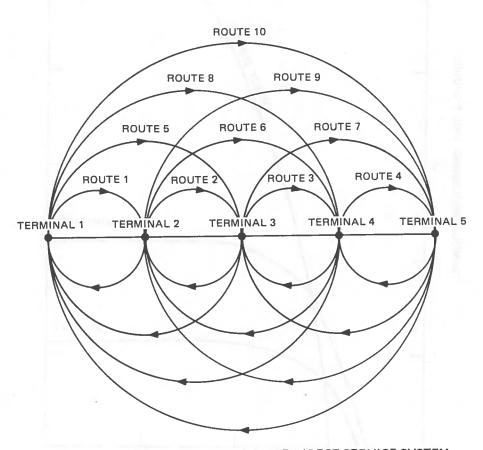
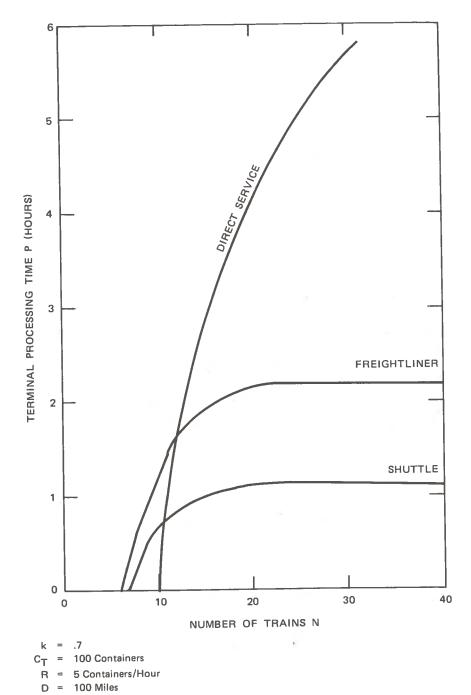


FIGURE 19 ROUTE CONFIGURATION OF DIRECT SERVICE SYSTEM



V = 60 mph m = 1

FIGURE 20 COMPARISON OF FEASIBILITY REGIONS ON THE N-P PLANE

Inequality (19) can be rewritten as:

$$C_{T} \ge \frac{20R}{k(N-5)} (P + \frac{2D}{V} + \Delta t_{D})$$
 (20)

Using (20) we define the feasibility region of the system (see Figure 21). Here, the variables are the train size, $C_{\rm T}$, the number of trains in the system, N, and the number of loading platforms per terminal, m. All other parameters are given a fixed value. The feasibility regions in the figure also look similar to the ones for the freightliner strategy.

Shuttle Strategy

The shuttle strategy has four routes. Trains in the system travel back and forth between the adjacent terminals. At one terminal, the containers going in the same direction are all picked up by the same train, and those containers which are sent more than one terminal away from the origin terminal must be transferred to a train in the neighboring route at intermediate terminals. Figure 22 shows the route configuration of the shuttle strategy.

Just as we did in the other two cases, we set up inequalities which define the feasibility conditions of the system; i.e., the capacity of each route is at least equal to or greater than the demand on that route. By summing up these feasibility inequalities we obtain, for the whole system:

$$P \le \frac{kC_{T}}{40R} (N-2) - \frac{D}{V} - \frac{1}{5} \Delta t_{1} - \frac{3}{10} \Delta t_{2}$$
 (21)

where t_1 = the amount of delay in routes 1 and 4

 t_2 = the amount of delay in routes 2 and 2.

The deviation of this inequality is given in Appendix E.

The feasibility region of the shuttle system as a function of the terminal processing time, P, the number of trains in the system, N, and the number of loading platforms for m=1 is given in Figure 20, overlapped with the feasibility regions of the other strategies. The figure shows that the upper bound of the feasibility regions are much smaller compared with those of the other systems.

Inequality (21) can be rewritten as:

$$C_T \ge \frac{40R}{k(N-2)} (P + \frac{D}{V} + \frac{1}{5} \Delta t_1 + \frac{3}{10} \Delta t_2)$$
 (22)

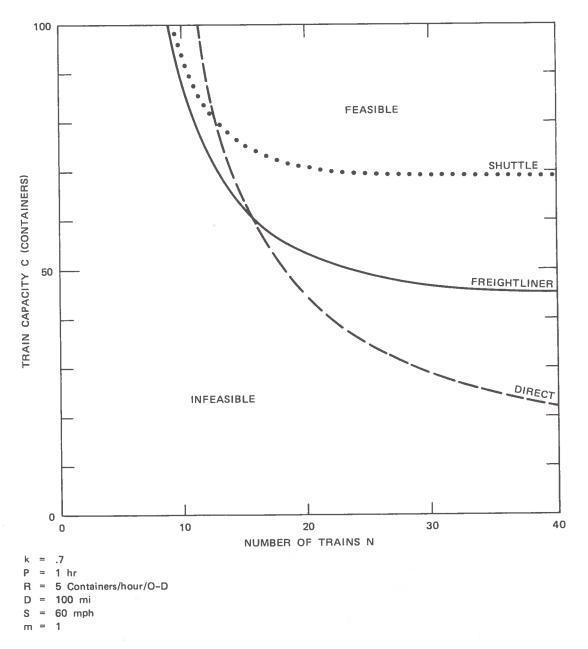


FIGURE 21 COMPARISON OF FEASIBILITY REGIONS ON THE N-CT PLANE

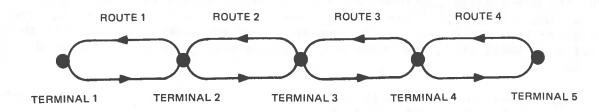


FIGURE 22 ROUTE CONFIGURATION OF THE SHUTTLE STRATEGY

From (22) we can define the feasibility region of the shuttle system as a function of the train capacity, C_T , the number of trains in the system, N, and the number of loading platforms in a terminal. This feasibility region is presented in Figure 21, overlapped with the feasibility of the other strategies. The general shapes of the feasibility regions are similar to those of the other two systems.

4. Comparison of Feasibility Boundaries

The upper bounds of terminal processing time for these strategies were derived in inequalities (17), (19), and (21).

Because the terminal delay, expressed as Δt_F for the freightliner strategy, Δt_D for the direct service strategy, and Δt_1 and Δt_2 for the shuttle strategy, contribute significantly to these equations, it is not simple to find how different these curves are without going through a numerical analysis. However, in all three cases, kC_T/R , and D/V appear as if they are a single parameter.

Figure 20 was obtained by overlaying the feasibility region curves for the one loading platform case for the three strategies. The figure shows that the freightliner strategy has a larger terminal processing time boundary than the direct service strategy when the number of trains in the system is small (N is less than 16), and just the opposite for larger N values.

The lower bounds of train capacity for the three strategies were given in inequalities (18), (20), and (22).

Figure 21 was drawn to compare the feasibility regions of three strategies on the N - $C_{\rm T}$ plane. Here again the shuttle system is always inferior to the freightliner strategy; here it always requires more train capacity than the freightliner strategy.

E. Selection of Parameter Values

In this part, we will describe an analysis conducted using the handanalysis model, which was discussed at the beginning of this section. The strategies dealt with in the analysis are the direct service and the freightliner strategies. The shuttle strategy was not pursued because it was found mostly to be inferior to the freightliner system.

The parameters which have been examined in this analysis include:

- Operational strategies--direct service/freightliner
- V = Linehaul speed
- P = Terminal processing time
- $C_{T} = Train capacity$

- N = Number of trains in the system
- R = Container generation rate
- D = Distance between terminal
- m = Number of platforms in a terminal.

The objectives of the study are multiple:

- To find the sets of parameter values in which a specific operational strategy is most efficient.
- To find a set of parameter values which are most cost-effective.
- To find the effect of each individual parameter on the average transit time and the system cost.

1. Freightliner Versus Direct Service

First, the transit times of the freightliner strategy and the direct service strategy are compared using the two equations shown in (8) and (9). The result of comparison is given in Figure 23. The figure shows the region in which one of those two operational strategies has a smaller transit time—when the terminal processing time, P, is constant regardless of the number of containers processed at a terminal. The figure indicates that the freightliner strategy yields less transit time than the direct service in the region in which the terminal processing time is short and the train capacity is large. The border line which indicates the preferred operational strategy moves as the rate of container generation and the number of loading/unloading platforms varies. The operational region of the freightliner strategy becomes wider as the container generation rate decreases. It also becomes wider as the number of loading/unloading platforms increases.

Note that the operational region in which one of the two strategies has a smaller transit time is not a function of linehaul speed.

Next, the system costs of the freightliner and the direct service strategies are compared. This is done by comparing the number of trains in the system, instead of by comparing the exact total system cost. In actuality, those two comparison methods should give the same results, because the only difference in cost when all the other parameters are fixed is in the number of trains. The required number of trains for the two operational strategies is given as:

For the freightliner strategy:

$$N \ge \frac{40R}{kC_T} (P + \frac{D}{V} + \frac{3}{10} \Delta t_F) + 1$$
 (23)

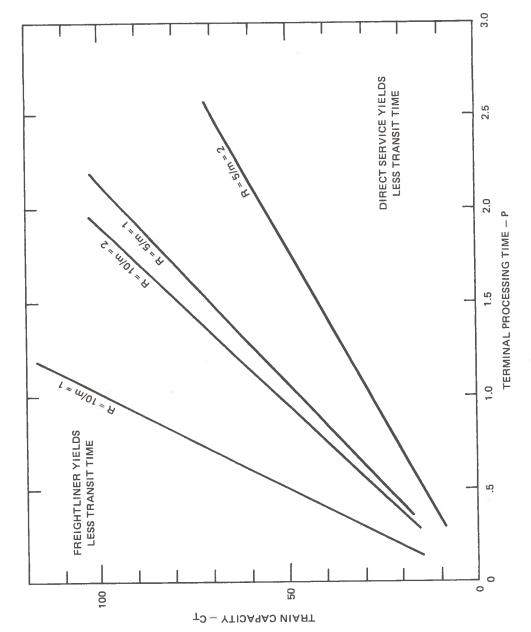


FIGURE 23 BEST OPERATIONAL REGION BASED ON TRANSIT TIME

For the direct service strategy:

$$N \ge \frac{20R}{kC_T} (P + \frac{2D}{V} + \Delta t_D) + 5$$
 (24)

The result is shown in Figure 24. The figure also shows that the freight-liner strategy requires less cost than the direct service strategy for a small terminal processing time and large train capacity. It indicates that the region in which the freightliner strategy requires less cost than the direct service strategy becomes wider as the container generation rate decreases and as the number of loading/unloading platforms increases. Note that the region in which one of the two strategies requires less cost is not a function of linehaul speed.

Thus, the freightliner strategy is superior to the direct service strategy when the terminal processing time is small and the train capacity is large. The region in which the freightliner strategy is superior to the direct service strategy becomes wider as the rate of container generation decreases, and also as the number of loading/unloading platforms increases.

2. Most Cost-Effective System

Freightliner Strategy—The objective here is to define a set of most cost—effective systems. The parameter types varied in the study are: P (terminal processing time), C_T (train size), V (linehaul speed), N (number of trains in the system), and the operational strategy. The work consists of two stages. The first stage is to find the cost and the average transit time for each combination of train size and terminal processing time. In this process the number of trains required in the system is determined using the feasibility boundary inequalities discussed previously in this section. The cost and the average transit time were obtained using equations 8 and 11.

A set of sample results from this stage is given in Figures 25 and 26. Figure 25 shows the total daily costs for various combinations of terminal processing time and train size. Figure 26 shows the average transit times in the system for the corresponding points. The same type of figures were constructed for six different speeds for each of the two operational strategies.

The second stage is to find the minimum system cost for each combination of linehaul speed and terminal processing time based on the first stage results. The corresponding average transit time for each parameter combination is also obtained in this stage. The resulting plot for the freightliner strategy is given in Figure 27. The figure shows the two types of contour maps: One is the daily total system cost contour map and the other is the average transit time contour map. The figure shows that the total system cost is minimum (\$.143 million/day) when the terminal processing time is 1.7 hr and the linehaul speed is 10 mph. The system

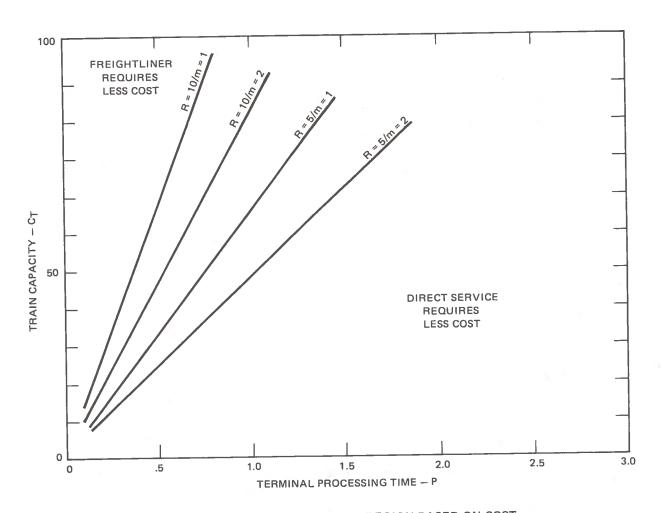
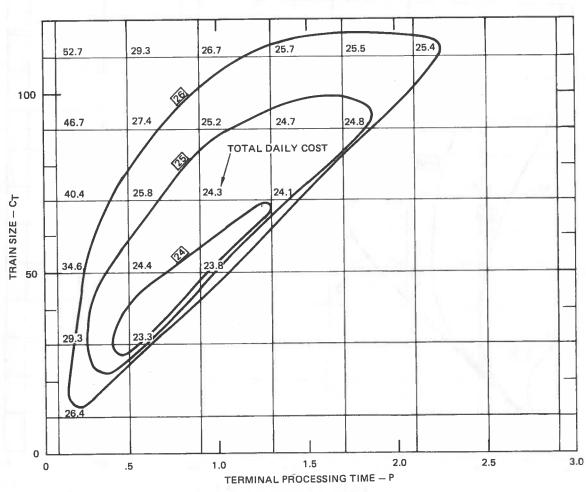


FIGURE 24 BEST OPERATIONAL REGION BASED ON COST



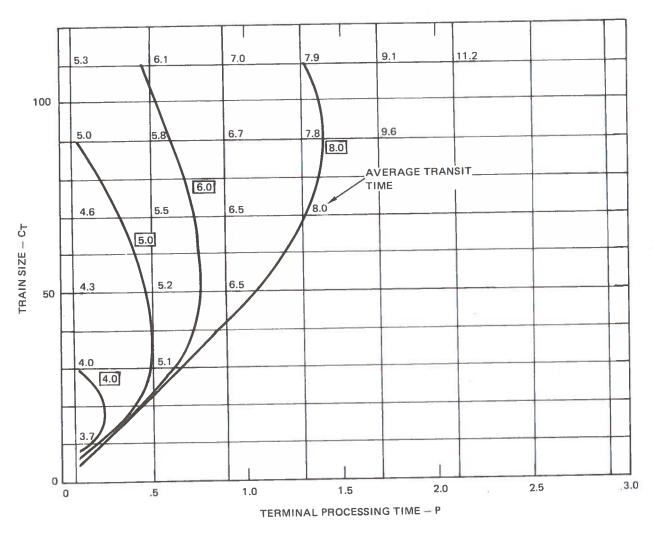
FREIGHTLINER SERVICE

V = 60 mph

R = 5 Containers/Hours/O-D

Cost in Units of \$10,000

FIGURE 25 EQUAL DAILY TOTAL COST CONTOURS (FREIGHTLINER)



FREIGHTLINER SERVICE

V = 60 mph

R = 5 Containers/Hour/O-D

Distance = 200 Miles

FIGURE 26 EQUAL AVERAGE TRANSIT TIME CONTOURS (FREIGHTLINER)

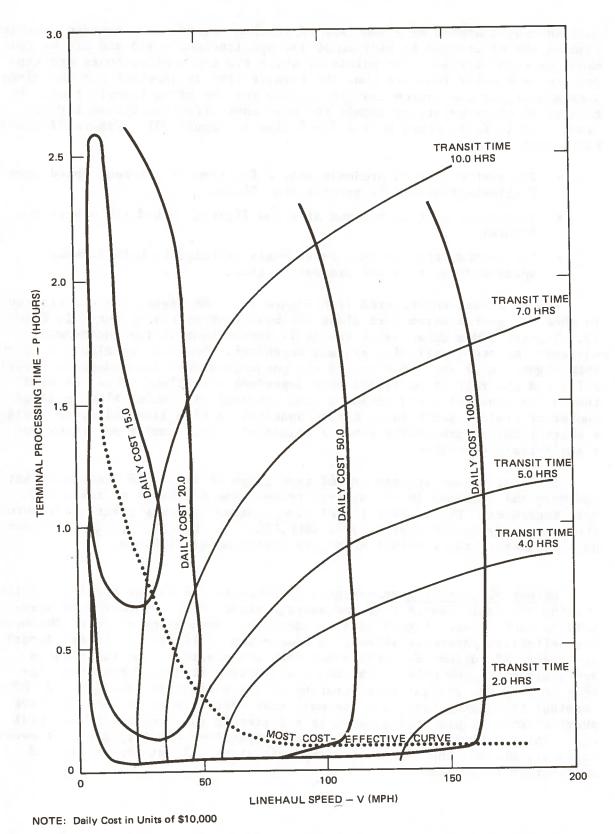


FIGURE 27 EQUAL DAILY TOTAL COST AND AVERAGE TRANSIT TIME CONTOURS (FREIGHTLINER)

cost increases almost as a function of linehaul speed. The average transit time on the other hand is minimum at 180 mph linehaul speed and .10 hr terminal processing time. The points at which two types of contours are tangent to each other indicate that the transit time is shortest for the given system cost, or the system cost is minimum for the given transit time. By connecting these points we obtain the most cost-effective system for any given cost (this is shown by the "---" line in Figure 27). Figure 27 indicates that:

- The system cost is predominantly a function of linehaul speed when V (linehaul speed) is greater than 50 mph.
- The system cost is minimum at a low linehaul speed (10 mph in the figure).
- The average transit time is strongly affected by both linehaul speed and the terminal processing time.

Figure 28 was constructed from Figure 27. The figure was constructed to show the system parameters along the most cost-effective curve in Figure 27. Figure 28 was drawn as if the daily system cost is the independent variable and other variables are all dependent. However, we also can treat this figure as if any other one of the parameters were the independent variable, and the rest of variables were dependent variables. Thus, we find that a low linehaul speed yields a long terminal processing time, a large number of trains, and a large train capacity. A high linehaul speed yields a short terminal processing time, a relatively small number of trains and a small train capacity.

The cost versus average travel time graph of Figure 28 indicates that the marginal decrease in the average travel time decreases as the system cost increases. The average travel time decrease becomes practically zero when the daily system cost is over \$600,000. At this point, the most dominant parameter, the linehaul speed, is approximately 120 mph.

Direct Service Strategy—Figure 29 presents the direct service results showing the daily system cost and average travel time countour maps over—laid on each other. Figure 29 also shows the curve which connects the most cost—effective parameter values. A comparison of Figure 27 (freightliner) and Figure 29 (direct service) shows that the shapes in the two figures are similar to each other. The major difference found in those two figures is that the freightliner strategy covers a slightly smaller range of terminal processing time, and its most cost—effective curve tends to have shorter terminal processing times in relation to the direct service strategy. This is because the train in the freightliner strategy stops at every terminal, whereas the direct service train stops only at the origin and destination terminals.

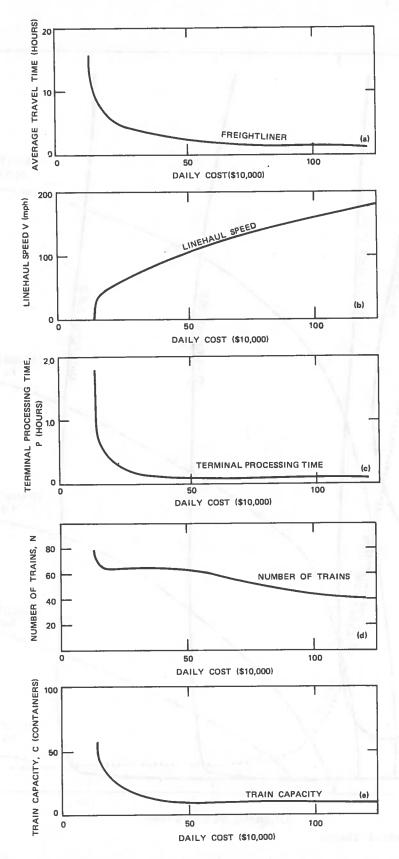
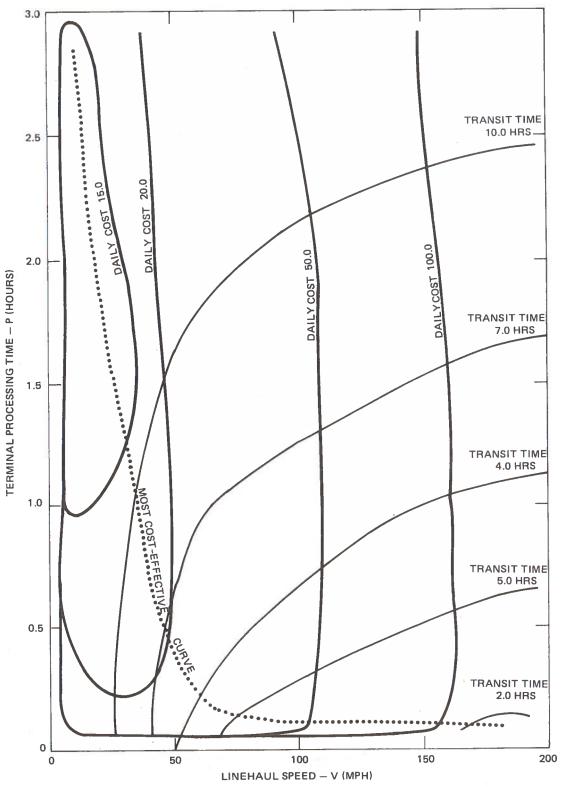


FIGURE 28 MOST COST-EFFECTIVE SYSTEM DESIGN PARAMETERS (FREIGHTLINER)



NOTE: Daily Cost in Units of \$10,000

FIGURE 29 EQUAL DAILY TOTAL COST AND AVERAGE TRANSIT TIME CONTOURS (DIRECT SERVICE)

3. Effect of Each Parameter on System Cost and Average Travel Time

Here, we will focus on three types of parameters and examine their effect on the system cost and average transit time. The three key parameters discussed are linehaul speed, terminal processing time, and train capacity. For each parameter type the parameter value is varied, and the system cost and the average transit time are computed. In that process, the other parameters are also varied in such a manner that not only the feasibility constraints are met, but the system has the minimum cost for that parameter value.

Linehaul Speed--Table 5 shows the daily total cost and average travel time variations as the linehaul speed changes. The table also shows that the daily total cost is a monotonically increasing function of linehaul speed when the linehaul speed is faster than 60 mph. However, when the linehaul speed is slower than 60 mph, the trend of the cost variation as a function of the linehaul speed varies depending on the terminal processing time value. The average transit time is a monotonically decreasing function of linehaul speed for all (terminal processing time) P values. Note that the rate of growth in cost becomes larger as the linehaul speed is increased, but the rate of decrease in the average travel time becomes smaller as the linehaul speed is increased.

Terminal Processing Time--Table 5 also shows how the daily total cost and average travel time vary as the terminal processing time changes. The table shows that the daily cost variations over different terminal processing time values are small.

The average travel time increases almost linearly as the terminal processing time increases. The rate of average transit time increase looks almost identical for all the five linehaul speeds.

Train Capacity—Table 6 shows how the daily total cost and average travel time vary as the train capacity and terminal processing time change. This table was constructed for a linehaul speed of 60 mph. The range of train capacity variation is defined by the terminal processing time; the short terminal processing time allows both many small—capacity trains and a few large—capacity trains, but the long terminal processing time allows only a few large capacity trains. For all the P values, the daily total cost is a monotonically increasing function of train capacity. The average travel time also is a monotonically increasing function of train capacity. For V = 60 mph, both the daily total cost and the average transit time become minimum at $C_{\rm T}$ = 10 containers.

Table 5

DAILY COST AND AVERAGE TRANSIT TIME VARIATIONS

AS A FUNCTION OF LINEHAUL SPEED

AND TERMINAL PROCESSING TIME

(Freightliner)*

	Dai	ily Total	Cost (x 1	0,000 dollar	s)
	V =	V =	V =	V =	V =
	10 mph	30 mph	60 mph	100 mph	140 mph
P = .1 hr	12	13	25	45	78
P = .9 hr	13	12	23	43	79
P = 1.7 hr	15	21	25	48	83
		Average	Transit	Time (hr)	
	V =	V =	V =	V =	V =
	10 mph	30 mph	60 mph	<u>100 mph</u>	140 mph
P = .1 hr P = .9 hr P = 1.7 hr	21 23 27	7.2 10 13	3.8 6.5 9.5	2.5 5.0 8.5	1.8 4.5 7.9

^{*} The other variables are kept constant, i.e., R=5 containers/ hr/O-D, D=100 mi, k=.7.

Table 6.

DAILY COST AND AVERAGE TRANSIT TIME VARIATIONS

AS A FUNCTION OF TRAIN CAPACITY

AND TERMINAL PROCESSING TIME

(Freightliner)*

		ly Total Co 0,000 doll	
		$C_T = 50$	
P = .1 hr P = .9 hr P = 1.7 hr	12	17 25 	23 26 25
		Transit Transit $C_T = 50$	
P = .1 hr P = .9 hr P = 1.7 hr	3.8	4.4 6.6 —	4.6 7.0 8.9

^{*}The other variables are kept constant, i.e., R = 5 containers/hr/O-D, D = 100 mi, k = .7, V = 60 mph.

F. Summary and Conclusions

In this section we presented the theoretical derivations and numerical results of the hand analysis study. The system operational strategies studied were the freightliner strategy, the direct service strategy, and the shuttle strategy. The feasibility boundaries of these strategies were established based on conditions such that the system has sufficient link capacity to carry the demand, and the system has sufficient capacity to handle the train traffic at the terminals. The assumption of terminal operation adopted was that at a terminal, a loading platform can handle only one train at a time, but any number of trains can wait without blocking bypassing trains.

Three different inequalities were derived—one for each strategy to describe the link capacity constraints. It was found that these bounds were expressed in terms of P, D/S, kC_T/R , N, and delay terms.

The study results under the given assumptions are:

- The freightliner strategy always yields a smaller number of trains and a smaller (or equal) transit time in comparison to the shuttle strategy.
- The freightliner strategy yields a smaller transit time and a smaller fleet size in comparison to the direct service strategy at short terminal processing times, and larger train capacities.
- The direct service strategy yields a smaller transit time and a smaller fleet size in comparison to the freightliner strategy at long terminal processing times and smaller train capacities.
- Both the freightliner strategy and the direct service strategy have similar cost and effectiveness characteristics.
- The system cost is predominantly a function of linehaul speed when y (linehaul speed) is greater than 50 mph.
- The average transit time is strongly affected by both linehaul speed and the terminal processing time.
- A high linehaul speed yields a short terminal processing time, a relatively small number of trains and small train capacity.
- The rate of growth in cost becomes larger as the linehaul speed is increased, but the rate of decrease in the average travel time becomes smaller as the linehaul speed is increased.
- The average travel time is almost a monotonically increasing linear function of terminal processing time.
- \bullet Both the daily total cost and the average travel time become minimum at a small train capacity (C $_{T}$ = 10 in the example) at the speed of 60 mph.

Appendix A

LINET ANALYSIS: DERIVATION OF ANALYTICAL FORMULAS

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Appendix A

LINET ANALYSIS: DERIVATION OF ANALYTICAL FORMULAS

In this appendix we shall derive the formula for "containers delivered per day (C_D) " and "average time in the system (T_C) " discussed in Section IIID.

The formula for containers delivered per day (Equation 1, Section III) was derived in the following manner. The average time per train link was taken as the travel time plus loading and unloading time, plus lost time per link, or

$$\frac{D}{V} + KP + Q \qquad . \tag{A-1}$$

Then, links per train per day can be given as:

$$\frac{24}{\frac{D}{V} + KP + Q} \qquad (A-2)$$

As the average train contents is given as CU, and the number of trains is N, the container links per train can be given as:

$$\frac{24\text{CNU}}{\frac{D}{V} + \text{KP} + Q} \qquad (A-3)$$

Finally, as the average number of links per container delivery, A, is known, the total container deliveries per day is:

$$C_{D} = \frac{24CNU}{A\left[\frac{D}{V} + KP + Q\right]}$$
 (A-4)

The formula for time in the system (Equation 2, Section III) is composed of two parts, one representing the active delivery time, and the other the time waiting to be picked up. The active delivery time is taken as the time per link times the number of links per delivery, or

$$A\left[\frac{D}{V} + KP + Q\right] \qquad . \tag{A-5}$$

The waiting time, obtained from the simulation, is expressed as a fraction of the headway, which is given as links per train times hours per link, or

$$H = \frac{8}{N} \left[\frac{D}{V} + KP + Q \right] \qquad . \tag{A-6}$$

Therefore, waiting time can be stated as:

$$\frac{8W}{N}\left[\frac{D}{C} + KP + Q\right] \qquad . \tag{A-7}$$

Adding this to the active delivery time, and consolidating, we obtain:

$$T_{C} = \left[A + \frac{8W}{N}\right] \left[\frac{D}{V} + KP + Q\right] \qquad (A-3)$$

Appendix B

COST FACTORS USED IN THE ANALYSIS

Appendix B

COST FACTORS USED IN THE ANALYSIS

The purpose of this appendix is to describe the cost elements that make up the cost of an intermodal system; to present baseline values of these costs, derived from existing railroad-highway intermodal systems; and to show how these cost elements might vary with performance over a wider range than is available with existing technology. The reader is cautioned that many simplifying assumptions have been made to obtain cost variability functions that could be used in the analysis. Where the analysis shows that decisions would be sensitive to the cost values that are used, more detailed analysis of the cost is required.

The appendix presents first the cost elements and values for railroad and terminal technology of the present state of the art, then presents a discussion of how these factors would vary with performance.

Railroad Cost Factors

The cost factor presentation below is largely based on SRI's long-run cost and energy model (B-1).* Cost factors from the model are updated to mid-1978 values through the use of indices compiled by the Association of American Railroads (B-2). The result of using these indices is that the 1975 prices are increased by the amounts listed below:

- Fuel, 25.8%
- Other materials, 21.0%
- Wages, 24.1%
- All nonfuel prices, 24.5%.

The components of cost that will be discussed are:

- Guideway cost
- Terminal cost
- Operating cost
- Equipment cost.

 $^{^{\}star}$ References for Appendix B are listed at the end of this appendix.

Guideway Cost

In the conventional railway technology, guideway is the roadbed, associated structures, track, ties, ballast, and associated signaling. Most of the cost is the capital recovery of the initial investment in these items. Maintenance cost, other than the replacement of track and ties, is relatively small, and, in general does not vary with traffic. For simplicity, we have assumed that the track and tie cost is a capital investment, and have assumed a relatively long operating life associated with the relatively light traffic densities that are found in the analysis.

Table B-1 presents the investment amounts and equivalent annual charges to recover these investments over the life indicated. The table is divided between "fixed" items, which are assumed independent of traffic levels, and "variable" items, which reflect traffic; the former amount to \$750,000 per road-mile, the latter \$165,000 per track-mile. Some of the fixed investments do not depreciate. Based on a 12% discount rate for all items and average lives as specified, total annual costs amount to \$90,000 per road-mile plus \$20,000 per track-mile.

Annual maintenance costs are shown in Table B-2: \$2,900 per road-mile plus \$160 per track-mile. These factors are so low that they should be incorporated into the capital levy. In summary, then, annual road costs are as follows:

Annual road costs = \$94,746 per road-mile + \$20,544 per track-mile

The major simplifications in this estimate indicate that:

- Grade separations are not assumed at rail/highway crossings.
- Track, tie, and surfacing facilities do not reflect annual tonnage implications.

Terminal Cost

Terminal cost is the capital recovery for the investment in facilities and equipment at the terminal, together with operating and maintenance cost of the terminal. This discussion presents terminal costs in a manner that links directly to the transportation model. It has been loosely adapted from a model prepared for the National Intermodal Network Feasibility Study (B-3).

The formulation assumes that terminal capacity is measured in terms of the terminal's ability to process units of 25 containers at one peak time. This unit of capacity corresponds exactly to that provided by one gantry crane or comparable facility operating at full potential with full backup support. Arriving trains are processed in units of 10 cars at a time, with total processing time a function of containers to be transferred and available capacity.

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Table B-1

INVESTMENT AND ANNUAL CAPITAL CHARGES FOR GUIDEWAY

Assumptions and Comments		100-ft right-of-way; \$12,450 per acre; \$37,350 for acquisition		Lives for rail, tie, and surfacing based on 20,000,000 net tons per year	
Annual Charge		\$22,589 37,350 10,428 2,241 13,334 5,893	\$91,835	\$ 2,241 705 6,091 11,242	\$20,279
Life		Infinite Infinite Infinite 20 years 20 years		Infinite 10 years 35 years 50 years	
Unit Cost		\$188,244 311,250 86,901 18,675 99,600 44,011	\$748,681	\$ 18,675 3,984 49,800 93,375	\$165,834
Cost Category	Costs per road-mile	Land Grading and preparation Roadway Basic communications Structures Protected grade crossings	Subtotal "fixed" costs Cost per track-mile	Incremental communications Surfacing Rails Ties	Subtotal "variable" costs

Source: SRI Long-Run Average Cost and Energy Model

Table B-2

ANNUAL MAINTENANCE COSTS FOR GUIDEWAY

Cost Category	Unit Cost
Guideway Maintenance Costs	
Weed control Basic communications Grade crossings	\$ 623/mile/year 165/mile/year 2,123/mile/year
Subtotal "fixed investment"	\$2,911/mile/year
Incremental communications	\$165 track-mile/year
Subtotal "variable investment"	\$165 track-mile/year

Source: SRI Long-Run Average Cost and Energy Model

Terminal capacity is defined for peak 1-hr activity. The formulation assumes that facilities operate for an average of 15 hrs per day and that peak hour activity is 24% of daily activity. For this reason, there are negligible economies of scale for increasing units of capacity.

Terminal costs are assumed to vary with peak handling requirements of 25 containers per hour, in units of this amount, as described above. The peak design factor is assumed equivalent to an average daily handling load of 108 containers.

Table B-3 identifies the capital cost of all terminal facilities for each handling set (or capacity unit) as \$917,000 equivalent on an annualized basis to \$127,000 per year. Table B-4 identifies the terminal maintenance costs. Total cost for the capital and maintenance components is \$143,000 a year for each handling set of 25 peak hour containers.

The operating cost elements of terminals are presented in Table B-5. These costs vary by containers handled, regardless of direction, because all containers undergo the same handling within the terminal area. Average daily costs amount to \$937 in labor plus \$119 in equipment operating costs. In addition, there are constant annual terminal management wages of \$99,900.

Terminal costs as described above do not include the costs of pickup and delivery for originating/terminating containers.

Table B-3

INVESTMENT AND ANNUAL CAPITAL CHARGES FOR TERMINALS (Per Capacity Unit of 25 Containers Handled)

Cost Category	Unit Cost	Life	Annual Charge
Land Site preparation Surfacing Lighting Gantry crane Hostling tractor Receiving track	\$224,100 182,268 130,850 14,940 305,025 12,450 47,061 \$916,694	Infinite Infinite 10 years 20 years 20 years 5 years 10 years	\$ 26,892 21,872 23,160 2,000 40,843 3,454 8,330 \$126,551

Source: Based on <u>National Intermodal Feasibility</u> Study, Reebie Associates, 1976

Table B-4

ANNUAL MAINTENANCE COST FOR TERMINALS
(Per Capacity Unit of 25 Containers Handled)

Cost Category	Annual Cost	
Surfacing Lighting Rail	\$ 7,289 8,416 448	
	\$16,153	

Source: Based on National Intermodal Feasibility Study, Reebie Associates, 1976

Table B-5

AVERAGE DAILY OPERATING COST FOR TERMINALS (Per Capacity Unit of 25 Containers Handled)

Cost Category	Daily Cost
Gantry operators Tractor drivers Office clerks	\$ 379 186 372
Subtotal operating labor	\$ 937
Gantry operating Tractor operating	95 24
Subtotal operating equipment	\$ 119
Average daily total	\$1,056

Source: Based on <u>National Intermodal Feasibility Study</u>, Reebie Associates,

Operating Cost

Transportation operating costs are assumed to consist of train crew and fuel costs. No other direct operating costs are assumed for train movements.

Crew costs, based on current rail operations, are estimated at \$39.54 per train-hour. This figure assumes three-man crews, includes benefits and payroll taxes, and estimates crew utilization at 88% of available hours.

Fuel costs are considerably more complex. The total resistance, in pounds, overcome for each train is assumed given. The work undertaken to overcome this resistance, measured in foot-tons, is defined as follows:

Work =
$$\frac{\text{resistance x distance x 5,280 ft}}{2,000 \text{ lb}}$$

= 2.64 x resistance x distance

Fuel consumption is a function of work plus a locomotive idling consumption. The gallons of fuel expended for work are computed as follows:

Gallons line-haul =
$$\frac{\text{specific fuel consumption x work}}{\text{engine efficiency x 990}}$$

= 0.059/0.8/990

 $= 0.0000744 \times work$

= 0.0001964 x resistance x distance

Idling fuel consumption is estimated to be:

Gallons idling = travel time x 0.0021 gallons per horsepower

x locomotive horsepower

These amounts are added and multiplied by \$0.34 per gallon for fuel costs.

Equipment Cost

Transportation equipment cost is the capital recovery of the investment in locomotives, cars, and containers and the annual maintenance cost of these items. Capital recovery is computed at a 12% rate for the investment amounts and service lives listed below.

Equipment	Unit Price	Estimated Life	Annual Charge
2,000 hp locomotive * Two-container flatcar Container	\$475,000	20 years	\$63,359
	30,000	20 years	4,002
	5,000	10 years	885

^{*}Average freight locomotive horsepower in 1976 was 2,071.

The annual maintenance costs of equipment are assumed to vary directly with mileage for cars and with fuel consumption (a proxy for the severity of use) for locomotives, as follows:

Car maintenance: \$0.039 per car-mile

Locomotive maintenance: \$0.249 per gallon of fuel

Containers are not assumed to require maintenance.

Variation of Cost with Performance

The analysis of generic systems and the exploration of ways to improve system performance requires a functional relationship between cost and performance. The cost analysis presented so far in this appendix is representative of what can routinely be achieved by current technology—linehaul speeds of 35 to 50 mph (depending on region and location) and processing times of 1 to 2 hrs in terminals. Better performance can be achieved but at a higher cost. This part describes in general the reasons why costs increase with speed or reductions in processing time, and draws on design studies of higher performance systems to derive performance versus cost relationships. Because the design studies that are used to derive the relationships are only partially applicable, and all were performed at earlier dates requiring extensive cost escalation, the results should be viewed as very approximate.

In this analysis of performance, the cost elements considered are guideway cost, equipment cost, fuel cost, terminal cost, maintenance, and crew costs.

Guideway Cost

Guideway cost increases with speed because the alignment is more critical as the loads increase. Whereas surface irregularities and curvature might be tolerable at 50 mph, because they are controllable by conventional suspension systems and design techniques, less and less curvature is permissible as speeds increase. Extensive tunneling and bridging are required for the new Tokaido line in Japan to achieve the alignment that is needed for the speeds reached.

Estimates of guideway cost as a function of speed have been obtained from two advanced systems: the northeast corridor improvement (B-4) and from studies of magnetically levitated vehicles (B-5). Figure B-1 shows the estimated cost per mile for these systems plotted against speed capability, together with cost estimates for conventional railroad and a line with a slope of speed squared. The line shows that the square function of speed approximately describes the cost relationship shown by the points in the figure.

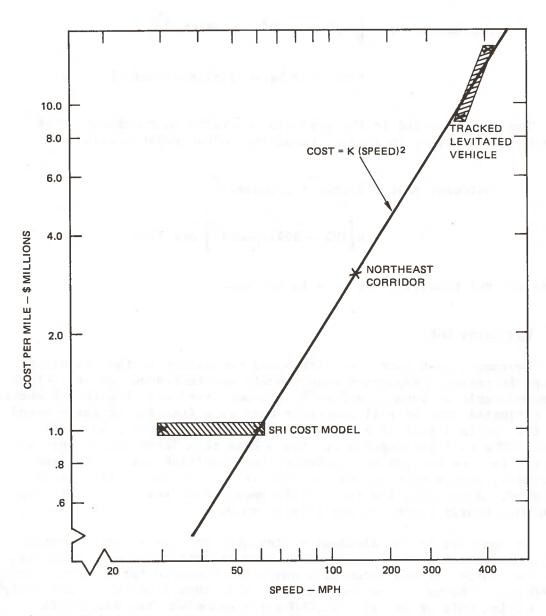


FIGURE B-1 GUIDEWAY COST FOR CONVENTIONAL AND ADVANCED SYSTEMS

Using the square function, all components of guideway cost, except right-of-way, communication, and grade crossings, were made a function of speed squared, while the land and other costs were assumed to remain constant. The resulting guideway cost function is:

Guideway cost =
$$\left[36,000 + 10^{5} \text{ (speed/60)}^{2}\right]$$

(days simulated/365)(road-miles)

The case analyzed in the analysis considers four segments that average 107.25 miles in length, resulting in the relationship:

Guideway cost =
$$42,500 + 33(\text{speed})^2$$

= $D[395 + 304(\text{speed})^2]$ per link

These two relationships are used in the text.

Equipment Cost

Equipment cost increases with speed capability as the propulsive effort increases, suspension requirements are increased, and reduction of aerodynamic resistance and weight become critical. Figure B-2 shows the estimated cost of rail passenger cars as a function of their speed and the combined cost of a freight car and the associated locomotive power. The rail passenger cars show a less than speed square increase in cost for the two points. However, both vehicles draw power from trackside, so the central station capacity or electrification is not included. Accounting for these differences shows that the square function more nearly describes the relationship.

In contrast to the discussion presented earlier in this appendix, the proportional cost of locomotive power is associated with each car. In other words, if two locomotive units are required for an 80-car train at 60 mph, 2/80ths of the locomotive cost is associated with each freight car. Using unit prices of \$750,000 for locomotives and \$40,000 for freight cars, and a discount rate of 12%, the factor of \$21.56 was obtained. The cost of the equipment was then scaled according to the square of the ratio of the expected linehaul speed to 60 mph, as follows:

Equipment cost = \$21.56(cars/train)(number of train sets)

(days simulated)
$$\left[(\text{speed})/60 \right]^2$$

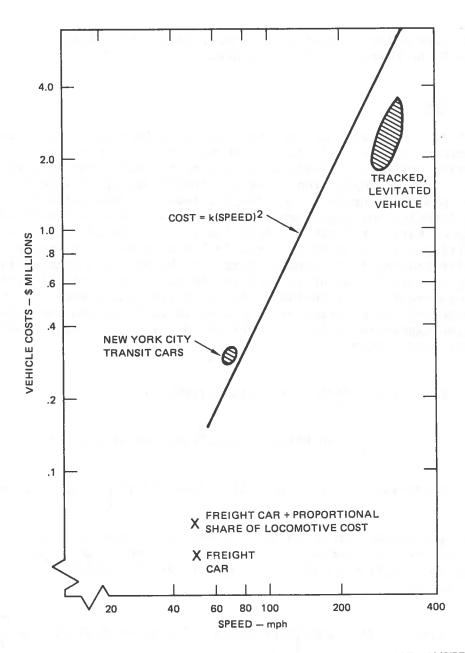


FIGURE B-2 EQUIPMENT COST FOR CONVENTIONAL AND ADVANCED SYSTEMS

With a car capacity of two containers, this relationship becomes:

This expression is used in the text, with C the capacity of the train and N the number of trains in use.

Fuel Cost

Fuel cost increases as the linehaul speed increases due to increased aerodynamic resistance. It is assumed that vehicles designed for higher speed service will incorporate streamlining to reduce the aerodynamic resistance. An approximation to such a train made up of streamlined cars is a passenger train. Accordingly, fuel cost was estimated by applying formulas for passenger train resistance to speeds well beyond the range of data for which the formulas were developed, then converting the resulting resistance to work and fuel consumption. A factor to account for braking to dissipate energy at the end of a segment is included, as this component is expected to be significant. If this braking component is not included, the peak linehaul speed would have to be increased, with attendant increases in guideway and equipment costs. The expression below is developed by fitting an approximation to the resistance curve:

Fuel cost = (\$70.7) (resistance) (WT) (D)
$$(10^{-6})$$

+ (0.89) (WT) (speed²) (number of stops) (10^{-6}) .
Resistance = 4.1 [(speed/75) + (speed/75)²] pounds per ton

Cars are assumed to weigh 32 tons and have a capacity of two containers per car, and loaded containers are assumed to weigh 20 tons. Inserting and simplifying these factors yields the following expression used in the text:

Fuel =
$$C(16 + 20U)$$
 [3.86 speed + (0.0515 + 0.89/D)speed²]

per million train-miles .

Terminal Cost

Over the range of loading and unloading times down to about 0.1 hr, conventional equipment can load and unload a train by providing extreme excess capacity, i.e., one gantry per container position on the train. Accordingly, the costs of the terminal were divided between functions associated with the gantry and others, with the result that about half the terminal costs will vary with the loading and unloading time requirement. The cost function is then:

Terminal cost = (\$785 + 665/processing time)(train capacity/25)

(number of platforms + \$271)(days simulated) .

With five terminals on the network, this expression becomes:

Terminal cost = $CP_L(157 + 133/P) + 1370$

Maintenance and Crew Costs

The algorithms for equipment maintenance and crew cost are not changed by the speed function. It is assumed that maintenance will continue to be a function of fuel consumed, and that crew members will continue to be paid on an hourly basis.

REFERENCES FOR APPENDIX B

- B-1 "Energy Study of Railroad Freight Transportation; Volume 2: Industry Description," SRI International, Menlo Park, CA, August 1977.
- B-2 "Yearbook of Railroad Facts: 1979 Edition," Economics and Finance Department, Association of American Railroads, Washington, D.C., May 1979.
- B-3 R. S. Reebie et al., "National Intermodal Network Feasibility Study, Appendices 1-4," Federal Railroad Administration Report No. FRA/ OPPD-76/2.11, U.S. Department of Transportation, Washington, D.C., May 1976.
- B-4 Letter from Gary Watros, U.S. Department of Transportation, Transportation Systems Center, to Peter Wong.
- B-5 "Technology Assessment of Future Intercity Passenger Transportation Systems; Volume 3: Technological Characteristics of Future Intercity Transportation Modes," p. V-33, March 1976.

Appendix C

HAND ANALYSIS: EXPECTED NUMBER OF INTERMEDIATE STOPS (Freightliner Strategy)

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Appendix C

HAND ANALYSIS: EXPECTED NUMBER OF INTERMEDIATE STOPS (Freightliner Strategy)

The lower bound of the expected number of stops per trip for a freightliner train is obtained by assigning O-D traffic volumes to one of the two routes. Here, we will assume that the traffic of O-D pairs 1-2, 2-5, 1-3, 3-5, 1-4, 4-5 are solely carried by freightliner route 1, or the trains which start from terminal 1 and terminate at terminal 5. The traffic of O-D pairs 2-3, 3-4, and 2-4 are solely carried by freightliner route 2, or the trains which start from terminal 2 and terminate at terminal 4.

The probability that a train stops at a terminal is identical to the probability that the train carries one or more containers whose destination is that terminal. Because we assume that the rate of traffic generation for any origin-destination pair is a constant R (containers/hour), the probability that a container on the train has the destination of a specific terminal is 0.25 for the 5 terminal linear network case. The probability that the train stops at a specified intermediate terminal when the train leaves terminal 1 carrying X containers is given as:

$$q_{X} = \sum_{n=1}^{X} {\binom{X}{n}} p^{n} (1-p)^{X-n}$$

$$= 1 - (1-p)^{X}$$
(C-1)

where

$$p = 0.25$$

If we assume that the mean train load is M containers, and that the number of containers carried by a train at terminal 1, X, follows a Poisson distribution, with parameter M, then, the probability that the train stops at a specific terminal is given as:

$$q = \sum_{X=0}^{\infty} \left[1 - (1-p)^{X}\right] \cdot \frac{M^{X}e^{-M}}{X} \qquad (C-2)$$

Note: The notation of variables used in this appendix follows that used in Section IV.

The probability that the train stops at n intermediate terminals on its way to the end terminal (or terminal 5) is given as a binomial distribution such as:

$$B(n;3,q) = {3 \choose n} q^{n} (1-q)^{3-n}$$
 (C-3)

where

$$n = 0, 1, 2, and 3 for X > 3$$

The probability that a train stops at a given intermediate terminal, q, is a function of the mean train load, and it increases as the mean train load increases. The q value as a function of mean train load is plotted on a graph shown in Figure C-1. The figure shows the rapid increase of probability of stopping at a terminal as the mean train load increases.

The expected number of intermediate stops of the freightliner train between terminal 1 and terminal 5 is also given as:

$$E(x) = 3xq (C-4)$$

where

E(x) = expected number of intermediate stops q = as defined in (C-2).

The E(x) values on different mean train loads are also plotted in the graph shown in Figure C-1. The graph shows that trains with mean loads of over 20 containers virtually stop at all three intermediate terminals.

The expected number of intermediate stops would be even higher than that shown in the graph, if no specific O-D traffic assignments are done for either one of the two routes.

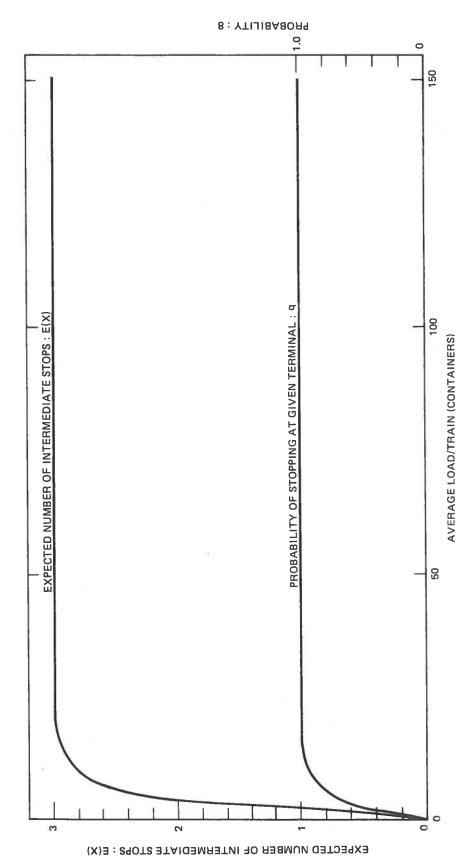


FIGURE C-1 FREIGHTLINER SYSTEM CHARACTERISTICS

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Appendix D

HAND ANALYSIS: AVERAGE TRANSIT TIME

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Appendix D

HAND ANALYSIS: AVERAGE TRANSIT TIME

To obtain the average transit time, first transit times for all the origin-destination combinations are computed. Next, those transit times are added together. The average transit time is given by dividing the total value by the number of origin-destination combinations. The exact procedure for each strategy is given below.

Freightliner Strategy

Transit time for each origin-destination combination is given as:

OD	Transit Time Including O/D Terminal Processing Times and Intermediate Stops	Waiting Time for a Train at Origin Terminal*		
	1163-234 TH-4-5 XX			
1-5	$4P + 4D/V + \Delta t_{F}$	kC _T /8R		
1-4	$3P + 3D/V + \Delta t_F$	kC _T /8R		
1-3	$2P + 2D/V + \Delta t_F$	kC _T /8R		
1-2	$P + D/V + \Delta t_F$	kC _T /8R		
2-5	3P + 3D/V	kC _T /8R		
2-4	2P + 2D/V	kC _T /12R		
2-3	P + D/V	kC _T /12R		
3-5	2P + 2D/V	kC _T /8R		
3-4	P + D/V	$kC_{\overline{T}}/12R$		
4-5	P + D/V	kC _T /8R		

Note: The notation of variables used in this appendix follows that used in Section IV.

^{*}The waiting time for a train at an origin terminal was assumed to be one-half of average train headway. This assumption also applies to the other two strategies.

Total transit time

=
$$20P + 2D/V + 4t_F + \frac{9kC_T}{8R}$$
.

Average transit time, $T_{A,F}$ is:

$$T_{A,F} = 2P + 2D/V + \frac{2}{5} \Delta t_F + \frac{9kC_T}{80R}$$

Direct Service Strategy

Transit time for each origin-destination combination is given as:

	Transit Time Including O/D	Waiting Time
	Terminal Processing Times	for a Train at
O-D	and Intermediate Stops	Origin Terminal
1-5	$P + 4D/V + \Delta t_D$	$kC_{\mathrm{T}}/2R$
1-4	$P + 3D/V + \Delta t_D$	$kC_{\mathrm{T}}^{\prime}/2R$
1-3	$P + 2D/V + \Delta t_D$	kC _T /2R
1-2	$P + D/V + \Delta t_D$	kC _T /2R
2-5	$P + 3D/V + \Delta t_D$	$kC_{\mathrm{T}}^{\prime}/2R$
2-4	$P + 2D/V + \Delta t_D$	kC _T /2R
2-3	$P + D/V + \Delta t_D$	kC _T /2R
3-5	$P + 2D/V + \Delta t_D$	$kC_{\mathrm{T}}/2R$
3-4	$P + D/V + \Delta t_D$	kC _T /2R
4-5	$P + D/V + \Delta t_D$	kC _T /2R

Total transit time

= 10P + 20D/V + 10
$$\Delta t_{D}$$
 + 5 kC_{T}/R

Average transit time, $T_{A,D}$, is:

$$T_{A,D} = P + 2D/V + \Delta t_D + kC_T/2R$$

Shuttle Strategy

Transit time for each origin-destination combination is given as:

<u>O-D</u>	Transit Time Including O/D Terminal Processing Times and Intermediate Stops	Waiting Time for a Train at Original Terminal
1-5 4F	$P + 4D/V + \Delta t_{S,2} + \Delta t_{S,3} + \Delta t_{S,4}$	kC _T /8R
1-4 3P	$P + 3D/V + \Delta t_{S,2} + \Delta t_{S,3} + \Delta t_{S,4}$	kC _T /8R
1-3 2F	$P + 2D/V + \Delta t_{S,2} + \Delta t_{S,3}$	kC _T /8R
1-2 P	$P + D/V + \Delta t_{S,2}$	kC _T /8R
	$P + 3D/V + \Delta t_{S,3} + \Delta t_{S,4}$	kC _T /8R
2-4 2P	$P + 2D/V + \Delta t_{S,3} + \Delta t_{S,4}$	kC _T /12R
2-3 P	$P + D/V + \Delta t_{S,3}$	kC _T /12R
	$P + 2D/V + \Delta t_{S,4}$	kC _T /8R
3-4 P	$P + D/V + \Delta t_{S,4}$	kC _T /12R
	P + D/V	kC _T /8R

Total transit time

=
$$20P + 20D/V + 4 \Delta t_{S,2} + 6 \Delta t_{S,3} + 6 \Delta t_{S,4} + \frac{9kC_T}{8R}$$

Average transit time, $T_{A,S}$, is:

$$T_{A,S} = 2P + 2D/V + \frac{1}{10} (4 \Delta t_{S,2} + 6 \Delta t_{S,3} + 6 \Delta t_{S,4}) + \frac{9kC_T}{80R}$$

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Appendix E

HAND ANALYSIS: DEFINING THE FEASIBILITY REGION

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Appendix E

HAND ANALYSIS: DEFINING THE FEASIBILITY REGION

Freightliner Strategy

Two routes are considered in the freightliner strategy. Route 1 starts from terminal 1 and terminates at terminal 5, and route 2 starts from terminal 2 and terminates at terminal 4. Route 1 serves container traffic demands of all 0-D pairs. Route 2 serves container traffic demands of 0-D pairs, 2-3, 3-4 and 2-4 (0-D traffic for the reverse direction is also served by this route). In theory the trains in the freightliner system have a certain probability of skipping stops at terminals. However, here it was assumed that the mean load per train is sufficiently large to assume that every train stops at every terminal. Then, the round trip time of route 1, $T_{R,1}$, is:

$$T_{R,1} = 8P + 8D/V + \Delta t_1$$
 (E-1)

The round trip time of route 2, $T_{R,2}$ is:

$$T_{R,2} = 4P + 4D/V + \Delta t_2$$
 (E-2)

The train headways of the two routes are:

$$T_{H,1} = \frac{1}{N_1} [8P + 8D/V + \Delta t_{J.}]$$
 (E-3)

$$T_{H,2} = \frac{1}{N_2} [4P + 4D/V + \Delta t_2]$$
 (E-4)

Note: The notation of variables used in this appendix follows that used in Section IV.

Then, the capacity of route 1 is:

$$C_{R,1} = \frac{kC_TN_1}{8[P + D/V] + \Delta t_1}$$
 (E-5)

The capacity of route 2 is:

$$C_{R,2} = \frac{kC_TN_2}{4[P + D/V] + \Delta t_2}$$
 (E-6)

The system operation becomes feasible if this capacity of each route is larger than the demand on that route, and if the terminals have sufficient loading platforms to handle trains. Those two conditions are expressed using inequalities such that:

$$\frac{kC_{T}N_{1}}{8[P + D/V] + \Delta t_{1}} \ge 4R . \tag{E-7}$$

$$\frac{kC_T^{N_2}}{4[P+D/V]+\Delta t_2} \ge 2R \qquad . \tag{E-8}$$

$$T_{HM} \ge \frac{1}{m} P \qquad . \tag{E-9}$$

This terminal processing time constraint expressed in (E-9) holds for any P values, because the average headway at a loading platform of a terminal, T_{HM} , is a function of P and the terminal delay, Δt , as well as other variables, and the large P value yields even larger T_H value due to the large P and Δt_F values.

Here the constraint expressed in (E-9) merely says that one train can be served at a loading platform. There are no constraints in the number of trains that queue up at a terminal.

The terminal constraint can be expressed in various forms. For example, if the constraint is that only one train can occupy a loading platform and there is no space for trains to wait, then it will be expressed as:

$$T_{HM} \ge \frac{1}{m} (P + \Delta t) \qquad (E-10)$$

In this hand analysis we assume the constraint expressed in (E-9). This constraint was chosen because we felt it to be realistic and also because this same assumption was adopted for LINET.

The total number of trains in the system, N, is the sum of the number of trains in the two routes, and is expressed as:

$$N = N_1 + N_2 \qquad . \tag{E-11}$$

Inequality (E-7) can be rewritten as:

$$N_1 \ge \frac{4R}{kC_T} [8P + 8D/V + \Delta t_1]^*$$

$$= \frac{4R}{kC_T} (8P + 8D/V + \Delta t_1) + \Delta N_1 \qquad (E-12)$$

In the same manner inequality (E-8) can be rewritten as:

$$N_2 \ge \frac{2R}{kC_m} (4P + 4D/V + \Delta t_2) + \Delta N_2$$
 (E-13)

The trains of routes 1 and 2 have a chance of being delayed at only two terminals: 2 and 4. This is because these two terminals are the only point where the two routes "merge" into one stream of train flow. If we denote the amount of delay per train at one merge point by Δt_F , then, the amount of delay per route is two times Δt_F on both routes. Furthermore, if we assume the expected fractional amount of trains on each route to be one-half, then:

$$N = N_1 + N_2 \ge \frac{R}{kC_T} (40P + 40D/V + 12\Delta t_F) + 1$$
 (E-14)

This can be rewritten as:

^{*}The symbol $\lceil \ \rceil$ indicates that any fractional amounts are due to be rounded up to the next highest integer.

$$P \le \frac{kC_T}{40R} (N-1) - D/V - \frac{3}{10} \Delta t_F$$
 (E-15)

To satisfy (E-7) and (E-8) with the same load factor (per train) on the two routes, the number of trains on route 1 must be four times as large as the number of trains on route 2, i.e.:

$$N_1 = 4N_2$$
 (E-16)

Then, from (E-3), (E-4), and (E-16) we obtain:

$$T_{H,1} = \frac{10P + 10D/V + \frac{5}{2}\Delta t_F}{N_1 + N_2} . \qquad (E-17)$$

$$T_{H,2} = \frac{20P + 20D/V + 10\Delta t_{F}}{N_{1} + N_{2}} \qquad (E-18)$$

The average headway of trains where both routes overlap (between terminals 2 and 4 in both directions), $T_{\rm H}$, is expressed as:

$$T_{H} = 1 / \left(\frac{1}{T_{H,1}} + \frac{1}{T_{H,2}}\right)$$
 (E-19)

From (E-17), (E-18), and (E-19) the train headway at a loading platform of a terminal, $T_{\rm HM}$, is:

$$T_{HM} = \frac{1}{(N_1 + N_2) - 1} \left(\frac{20}{3} P + \frac{20}{3} D/V + 2\Delta t_F \right)$$
 (E-20)

We assume that each train is assigned a loading platform at each terminal in such a manner that exactly $\frac{1}{m}$ of total train traffic will arrive at a loading platform if the terminal has m loading platforms. Then from Equation (E-20),

$$T_{HM} = \frac{m}{N-1} \left(\frac{20}{3} P + \frac{20}{3} D/V + 2\Delta t_F \right)$$
 (E-21)

If we assume that at a loading platform of terminals 2 and 4 the trains arrive at random with an average headway of T_{HM} and the service time at that terminal is a constant amount P, then a conventional queuing equation to estimate delay applies, and the amount of delay, Δt_F , is expressed as:

$$\Delta t_{\rm F} = T_{\rm HM} \frac{(P/T_{\rm HM})^2}{2(1-P/T_{\rm HM})}$$

$$= \frac{P^2}{2(T_{\rm IM} - P)} . \tag{E-22}$$

But we also know that the mean headway T_{HM} is expressed as a function of N, P, D, V, and Δt_F as shown in (E-21). From (E-21) and (E-22), we obtain:

$$\frac{P^2}{2\Delta t_F} + P = \frac{m}{N-1} \left(\frac{20}{3} P + \frac{20}{3} D/V + \Delta t_F \right) . \tag{E-23}$$

By solving (E-23) for $\Delta t_{\rm F}$, we obtain:

$$\Delta t_{F} = \frac{1}{2} \left\{ \left(\frac{(N-1)}{2m} P - \frac{10}{3} P - \frac{10}{3} D/V \right) + \sqrt{\frac{(N-1)}{2m} P - \frac{10}{3} P - \frac{10}{3} D/V + \left(\frac{N-1}{m} \right)} \right\}.$$
(E-24)

Thus Δt_F is given.

Inequality (E-15) can be rewritten as:

$$C_{\rm T} \ge \frac{40R}{k(N-1)} \left[P + D/V + \frac{3}{10} \Delta t_{\rm F} \right]$$
 (E-25)

Inequality (E-25) shows that the feasibility region boundary becomes a hyperbolic function if the feasibility boundary is expressed by $C_{\rm T}$ (train capacity) and N (number of trains in the system) and all the other parameters are given a fixed value.

Direct Service Strategy

The method used in defining the feasibility region of the direct service strategy is similar to that used in the freightliner strategy study.

The round trip times of routes 1, 2, 3, and 4 are expressed as:

$$T_{R,1} = T_{R,2} = T_{R,3} = T_{R,4}$$

$$= 2P + 2D/V + \Delta t_{i} \qquad i = 1,2,3 \text{ and } 4 \qquad (E-26)$$

The round trip times of routes 5, 6, and 7 are expressed as:

$$T_{R,5} = T_{R,6} = T_{R,7}$$

$$= 2P + 4D/V + \Delta t_{i} \qquad i = 5,6, \text{ and } 7 \qquad (E-27)$$

The round trip times of routes 8 and 9 are expressed as:

$$T_{R,8} = T_{R,9}$$

= $2P + 6D/V + \Delta t_{i}$ $i = 8 \text{ and } 9$. (E-28)

The round trip time of route 10 is expressed as:

$$T_{R,10} = 2P + 8D/V + \Delta t_{10}$$
 (E-29)

To make the system operations feasible, the capacity of each system must be at least equal to or greater than the demand on that route. This constraint is expressed as:

$$kC_T \frac{N_i}{T_{R,i}} \ge R$$
 for $i = 1,2,...,10$ (E-30)

or

$$N_i \ge \frac{R}{kC_T} T_{R,i}$$
 for $i = 1,2,...,10$ (E-31)

The total number of trains in the system, N, is the sum of the number of trains on each route:

$$N = \sum_{i=1}^{10} N_i . (E-32)$$

Thus, the total number of trains in the system is given as:

$$N \ge \frac{R}{kC_{T}} \left\{ \left[2P + 2D/V + \Delta t_{1} \right] + \dots + \left[2P + 2D/V + \Delta t_{4} \right] + \left[2P + 4D/V + \Delta t_{5} \right] + \dots + \left[2P + 4D/V + \Delta t_{7} \right] + \left[2P + 6D/V + \Delta t_{8} \right] + \left[2P + 6D/V + \Delta t_{9} \right] + \left[2P + 8D/V + \Delta t_{10} \right] \right\} . \tag{E-33}$$

Inequality (E-33) can be rewritten as:

$$N \ge \frac{R}{kC_T} (20P + 40D/V + \sum_{i=1}^{10} \Delta t_i) + \sum_{i=1}^{10} \Delta N_i$$
 (E-34)

where ΔN_i is the fractional amount of a train on route i. We assume that:

$$E\left\{\sum_{i=1}^{10} \Delta N_i\right\} = 5 \tag{E-35}$$

and furthermore we assume that the amount of delay for any train is identical at any terminal and for any route. If we denote the amount of delay at a terminal Δt_D , then:

$$N \ge \frac{R}{kC_T} (20P + 40D/V + 20\Delta t_D) + 5$$

$$= \frac{20R}{kC_{T}} (P + 2D/V + \Delta t_{D}) + 5$$
 (E-36)

or

$$P \le \frac{kC_T}{20R} (N-5) - 2D/V - \Delta t_D$$
 (E-37)

Thus, the capacity constraint is established.

The number of trains on route i is:

$$N_{i} = \frac{T_{R,i}}{T_{H,i}} \qquad (E-37)$$

If we assume that the load factor on all the routes is identical, then the train headways of all the routes are also identical, because the container generation rates for all the origin-destination pairs are assumed to be equal. Let

$$T_{H,i} = T_H^{\dagger} \tag{E-39}$$

then, the total number of trains in the system, N, is expressed as:

$$N = \sum_{i=1}^{10} \frac{T_{R,i}}{T_H^{'}} . \qquad (E-40)$$

From (E-26), (E-27), (E-28), (E-29), and (E-40), we obtain:

$$N = \frac{1}{T_{H}'} (20P + 40D/V + \sum_{i=1}^{10} \Delta t_{i}) + \sum_{i=1}^{10} \Delta N_{i} . \qquad (E-41)$$

The average train headway at any terminal, $\mathbf{T}_{\boldsymbol{H}},$ is one quarter of that of each individual route, or

$$T_{H} = \frac{1}{4} T_{H}'$$
 (E-42)

If we assume that:

$$\Delta t_{i} = 2\Delta t_{D} \text{ for } \forall i$$
 (E-43)

then, from (E-41), (E-42), and (E-43) we obtain:

$$T_{H} = \frac{1}{N-5} (5P + 10D/V + 5\Delta t_{D})$$
 (E-44)

If we assume that the trains arrive at a loading platform at random at the rate of $1/T_{\hbox{HM}}$ trains per hour, and the service time at the platform is P hours, then the amount of delay at a terminal per train is estimated as:

$$\Delta t_{D} = \frac{P^2}{2(T_{HM} - P)} \qquad (E-45)$$

The average incoming headway to a loading platform is expressed as:

$$T_{HM} = \frac{m}{N-5} (5P + 10D/V + 5\Delta t_D)$$
 (E-46)

From (E-45) and (E-46) we obtain:

$$P + \frac{P^2}{2\Delta t_D} = \frac{m}{N-5} (5P + 10D/V + 5\Delta t_D)$$
 (E-47)

By solving (E-47) for Δt_D , we obtain:

$$\Delta t_{D} = \frac{1}{2} \left\{ \frac{(N-5)}{5m} - P - 2D/V + \sqrt{\left(\frac{(N-5)}{5m} - P - 2D/V\right)^{2} + \frac{2(N-5)}{5m} P^{2}} \right\} .$$
(E-48)

Inequality (E-37) can be rewritten as:

$$C_{T} \ge \frac{20R}{k(N-5)} (P + 2D/V + \Delta t_{D})$$
 (E-49)

Shuttle Strategy

The shuttle strategy has four routes. Trains in the system travel back and forth between the adjacent terminals. At one terminal, the containers going in the same direction are all picked up by the same train, and those containers which are sent more than one terminal away from the origin terminal must be transferred to a train in the neighboring route at intermediate terminals.

The round trip time of each route is given as:

$$T_{R,i} = 2(P + D/V) + \Delta t_i$$
 for $i = 1,2,3$, and 4. (E-50)

The train headway of each route is expressed as:

$$T_{H,i} = \frac{1}{N_i} [2(P + D/V) + \Delta t_i]$$
 for $i = 1,2,3$, and 4. (E-51)

The route capacity for the i th route is:

$$C_{R,i} = \frac{kC_{T}}{T_{H,i}} \qquad (E-52)$$

From (E-51) and (E-52) we obtain:

$$C_{R,i} = \frac{kN_iC_T}{[2(P + D/V) + \Delta t_i]}$$
 (E-53)

Just as we did in the other two cases, we set up inequalities which define the feasibility conditions of the system. The set of inequalities states that the capacity of each route is at least equal to or greater than the demand on that route, expressed as:

$$\frac{kN_{i}C_{T}}{[2P + 2D/V + \Delta t_{i}]} \ge 4R \qquad \text{for } i = 1 \text{ and } 4$$

$$\frac{kN_{i}C_{T}}{[2P + 2D/V + \Delta t_{i}]} \ge 6R \qquad \text{for } i = 2 \text{ and } 3$$

$$(E-54)$$

The total number of trains in the system, N, is:

$$N = N_1 + N_2 + N_3 + N_4 . (E-55)$$

From (E-54) and (E-55) we obtain:

$$N \ge \left[\frac{40R}{kC_{T}} (P + D/V) + \frac{8R}{kC_{T}} \Delta t_{1} + \frac{12R}{kC_{T}} \Delta t_{2} \right]$$

$$\cong \frac{40R}{kC_{T}} (P + D/V) + \frac{8R}{kC_{T}} \Delta t_{1} + \frac{12R}{kC_{T}} \Delta t_{2} + 2 . \qquad (E-56)$$

The delay of the train on route i, Δt_i , is a sum of the delay at the origin terminal and the delay at the destination terminal of the train. The amount of delay suffered by a train differs from terminal to terminal in the shuttle strategy. If we denote the amount of delay per train at terminal j by $\Delta t_{S,j}$, then $\Delta t_{S,j}$ is estimated using a conventional M/D/l queuing system equation and is given as:

$$\Delta t_{S,j} = \frac{P^2}{2(T_{HM,j} - P)}$$
 (E-57)

where $T_{\mbox{HM},j}$ is the average headway of trains at a loading platform of terminal j.

The delay of trains for each route is expressed as:

$$\Delta t_{1} = \Delta t_{4}$$

$$= \Delta t_{S,2}$$

$$= \frac{P^{2}}{2(T_{HM,2} - P)}$$

$$\Delta t_{2} = \Delta t_{3}$$

$$= \Delta t_{S,2} + \Delta t_{S,3}$$

$$= \frac{P^{2}}{2(T_{HM,2} - P)} + \frac{P^{2}}{2(T_{HM,3} - P)} . \qquad (E-59)$$

We assume that the required number of trains on the four routes is proportional to the container demands on those routes. Then we have:

$$N_1 = N_4 = \frac{2}{3}N_2 = \frac{2}{3}N_3$$
 (E-60)

The basic premise of (E-60) is that the round trip time of the four routes can be considered equal. Then the total number of trains in the system, N, is expressed as:

$$N = 2(N_1 + N_2) . (E-61)$$

Inequality (E-36) is rewritten as:

$$P \le \frac{kC_T}{40R} (N-2) - D/V - \frac{1}{5}\Delta t_1 - \frac{3}{10}\Delta t_2$$
 (E-62)

From (E-53), (E-60), and (E-61) we obtain:

$$T_{H,1} = \frac{5}{N} (2P + 2D/V + \Delta t_1)$$

$$T_{H,2} = \frac{10}{3N} (P + 2D/V + \Delta t_2)$$
(E-63)

The average headway at a loading platform of terminal 2 is approximated by neglecting the difference in the delay terms of the two merging routes and is written as:

$$T_{HM}^{2} = \frac{2m}{(N-2)} (2P + 2D/V + \Delta t_{1}) \qquad (E-64)$$

We know that the average train headway at terminal 3 is one-half of the average headway of route 2. The average train headway at a loading platform of terminal 3 is expressed as:

$$T_{HM}^3 = \frac{5m}{3(N-2)} (2P + 2D/V + \Delta t_2)$$
 (E-65)

The values of Δt_1 and Δt_2 are obtained from (E-58), (E-59), (E-64), and (E-65) as:

$$\Delta t_{1} = \frac{1}{2} \left\{ \frac{1}{2} (N-2) 2P - D/V + \sqrt{\left(\frac{1}{2} (N-2) - 2P - 2D/V\right)^{2} - (n-2)^{2}} \right\}$$
(E-66)

and

$$\Delta t_2 = \frac{1}{2} \left(B + \sqrt{B^2 + 4C} \right)$$
 (E-67)

where

$$B = \frac{3}{5}(N-2) + \Delta t_1 - 2D/V - 2P$$

$$C = (2P + 2D/V)\Delta t_1 - \frac{3}{5}(N-2)\Delta t_1 + \frac{3}{10}(N-2)^2 .$$

Thus, the feasibility region of the shuttle system is defined.

Inequality (E-67) can be rewritten as:

$$C_{T} \ge \frac{40R}{k(N-2)} (P + D/V + \frac{1}{5}\Delta t_{1} + \frac{3}{10}\Delta t_{2})$$
 (E-68)