

Cookbook for Rheological Models – Asphalt Binders

FINAL REPORT
May 2016

Submitted by:

Offei A. Adarkwa
PhD in Civil Engineering

Nii Attoh-Okine
Professor

Pamela Cook
Unidel Professor

University of Delaware
Newark, DE 19716

External Project Manager
Karl Zipf

In cooperation with
Rutgers, The State University of New Jersey
And
State of Delaware
Department of Transportation
And
U.S. Department of Transportation
Federal Highway Administration

Disclaimer Statement

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein. This document is disseminated under the sponsorship of the Department of Transportation, University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.

The Center for Advanced Infrastructure and Transportation (CAIT) is a National UTC Consortium led by Rutgers, The State University. Members of the consortium are the University of Delaware, Utah State University, Columbia University, New Jersey Institute of Technology, Princeton University, University of Texas at El Paso, Virginia Polytechnic Institute, and University of South Florida. The Center is funded by the U.S. Department of Transportation.

1. Report No. CAIT-UTC-062		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle Cookbook for Rheological Models – Asphalt Binders				5. Report Date May 2016	
				6. Performing Organization Code CAIT/University of Delaware	
7. Author(s) Offei A. Adarkwa Nii Attoh-Okine Pamela Cook				8. Performing Organization Report No. CAIT-UTC-062	
9. Performing Organization Name and Address University of Delaware Newark, DE 19716				10. Work Unit No.	
				11. Contract or Grant No. DTRT12-G-UTC16	
Center for Advanced Infrastructure and Transportation Rutgers, The State University of New Jersey 100 Brett Road Piscataway, NJ 08854				13. Type of Report and Period Covered Final Report 7/1/15 – 12/31/15	
				14. Sponsoring Agency Code	
12. Sponsoring Agency Name and Address					
15. Supplementary Notes U.S. Department of Transportation/OST-R 1200 New Jersey Avenue, SE Washington, DC 20590-0001					
16. Abstract Rheology is defined as the science of the deformation and flow of matter (Hackley and Ferraris, 2001). The measurement of rheological properties of matter has become very important in various fields, especially the construction industry, where prevailing external conditions have a great impact on the behavior of construction materials. A vast amount of literature in the past addressed the application of various rheological models of binders (including polymer binders). Several models provide information about the stability, elasticity, thermal susceptibility, chemical composition, and additives of materials. These models are not easy to apply to real world conditions, for example, materials in the presence of heating and cooling. Furthermore, considerable studies using various models to construct better master curves have still proved elusive in providing reasonable simulation fits to the data. This guidebook attempts to provide guidance for the selection (in some cases) of appropriate models in given conditions.					
17. Key Words Asphalt binders; Rheological models; Pavements				18. Distribution Statement	
19. Security Classification (of this report) Unclassified		20. Security Classification (of this page) Unclassified		21. No. of Pages 26	22. Price

Acknowledgments

DELDOT Materials Lab – Karl Zipf – Asphalt Engineer

Table of Contents

1.	DESCRIPTION OF THE PROBLEM	1
1.1	BACKGROUND	1
1.1.1	MEASURING RHEOLOGICAL PROPERTIES.....	2
1.1.2	DYNAMIC SHEAR RHEOMETER (DSR) TESTS	3
1.1.3	BENDING BEAM RHEOMETER (BBR) TESTS.....	5
1.2	RESEARCH OBJECTIVE	6
2.	APPROACH.....	6
3.	FINDINGS.....	7
3.1	MECHANICAL ELEMENT MODELS.....	7
3.1.1	LINEAR ELASTIC SPRING	7
3.1.2	LINEAR VISCOUS DASH-POT.....	8
3.1.3	MAXWELL MODEL.....	8
3.1.4	KELVIN MODEL	9
3.1.5	HUET MODEL	10
3.1.6	HUET-SAYEGH MODEL	11
3.1.7	DI BENEDETTO AND NEIFAR (DBN) MODEL.....	12
3.1.8	THE 2S2P1D MODEL.....	13
3.1.9	THREE-ELEMENT MODELS.....	13
3.1.10	GENERALIZED MODELS.....	15
3.2	EMPIRICAL ALGEBRAIC EQUATION MODELS	16
3.2.1	JONGEPIER AND KUILMAN'S MODEL	16
3.2.2	CHRISTENSEN AND ANDERSON (CA) MODEL.....	17
3.2.3	CHRISTENSEN ANDERSON AND MARASTEANU (CAM) MODEL	18
3.2.4	POLYNOMIAL MODEL.....	18
3.2.5	SIGMOIDAL MODEL	19
4.	CONCLUDING REMARKS	19

List of Figures

Figure 1. Phase Angle for Elastic solids, Viscous liquids, and Viscoelastic Materials	4
Figure 2. Elastic Body	7
Figure 3. Linear Viscous Dash-pot	8
Figure 4. Maxwell Model	9
Figure 5. Kelvin Model	9
Figure 6. Huet Model.....	10
Figure 7. Huet-Sayegh Model	11
Figure 8. Di Benedetto and Neifar (DBN) Model.....	12
Figure 9. The 2S2P1D Model.....	13
Figure 10. Three-Element Model.....	14
Figure 11. Generalized Maxwell Model	15
Figure 12. Generalized Kelvin Chain.....	16
Figure 13. Definition of Parameters from the Christensen and Anderson Model.....	18

List of Tables

Table 1. Classification of Creep	5
Table 2. Classification of Relaxation	6

1. DESCRIPTION OF THE PROBLEM

Rheology has been a powerful tool for characterizing and quantifying asphalt binder properties. Furthermore, it has been established that the properties of the asphalt binder have a major influence on pavement performance and on the rate of deterioration under different load and environmental conditions.

Rheological testing and models of asphalt binder are widely used to describe and evaluate the behavior of binder. The majority of these rheological models are based on time derivatives of integer order.

The cookbook attempts to present the assumptions and models applied to asphalt binder for pavement design. The models are examined presented in the particular conditions under which experiments are carried out on the binders, particularly in shear flow and, for a solid as opposed to liquid state, a bending beam rheometer).

1.1 BACKGROUND

Rheology is defined as the study of the flow and deformation of material. In flow, elements of the liquids are deforming, and adjacent points in the liquids are moving relative to one another. The two basic flows are:

- a) Shear flow – when liquid elements flow over or past each other
- b) Extensional flow – adjacent elements flow toward or away from each other

A significant number of pavement distresses such as ruts, cracks, and fatigue failure are related to the rheological properties of asphalt binder, mixture load, and environmental conditions. It is important to note that most of the data used to determine the Performance Grade (PG) of pavement materials are obtained from rheological tests (Carret et al., 2015).

1.1.1 MEASURING RHEOLOGICAL PROPERTIES

The rheological behavior of materials is typically measured using viscometers and rheometers. A viscometer principally measures the viscosity of fluids, and a rheometer measures a host of rheological properties over an extended range of conditions (Hackley and Ferraris, 2001). The underlying principle behind most (shearing) rheometric measuring devices is rotation. Basically, the test fluid is sheared between two surfaces, of which either one or both are rotating. Often the imposed strain (motion of the moving surface) is small amplitude and oscillatory, with the resulting stress tracked and the in phase and out of phase components identified in this linear deformation, or consists of a step strain with the resulting relaxation (stress) of the material measured. These testing protocols confine information to the linear regime. In order to understand the nonlinear properties, the material a continuous shearing (shear rate control) or a fixed torque is applied with the resulting stress, respectively shear rate measured.

Thus, rotational rheometers can characterize rheological properties of a material under controlled-stress or controlled shear-rate conditions. The rheometer system is made up of these four basic parts:

- (i) Measurement tool with a well-defined geometry;
- (ii) Device for applying constant torque or rotational speed;
- (iii) Device for measuring shear stress and shear rate response; and
- (iv) Temperature-controlling device.

In measuring rheological properties of binders, the success of the performance grade (PG) system can be attributed to the following:

- a) The application of fundamental and mathematical concepts to asphalt binder characterization
- b) Detailed and comprehensive test methods and analyses

The data required to determine the PG of a binder are obtained from two experimental testing procedures:

- Frequency sweep performance dynamic shear rheometer (DSR) (AASHTO 315, 2012) at high and intermediate temperatures
- Bending creep tests performed with a bending rheometer (BBR) (AASHTO T 313, 2012) at low temperatures

In a frequency sweep test, the structure of the material is investigated at a strain below the critical strain. This is achieved by measurements made at a constant oscillation amplitude, and temperature over a range of oscillation frequencies. For an elastic solid, the storage modulus, G' , is independent of the frequency at low strains, and so if the material's G' is dependent on the frequency in a frequency sweep test, the material exhibits fluid-like or gel-like behavior. In such materials, the loss modulus, G'' , may dominate at low frequencies indicating a fluid-like behavior. The presence of both viscous and elastic properties suggests a system with an internal network structure which can be disturbed with ease. Using the storage, G' , and loss moduli, G'' , the complex modulus of the material can be calculated.

1.1.2 DYNAMIC SHEAR RHEOMETER (DSR) TESTS

The parameters obtained in DSR are:

- a) $|G^*|$ represents the absolute value of the shear complex modulus.

In testing a shearing strain of the form $\gamma_0 \sin(\omega t)$ is applied and the resultant stress is tracked, $\sigma = \text{Re}(G^* \omega * e^{i\omega t})$. The complex shear modulus is defined mathematically as:

$$|G^*| = \sqrt{G'^2 + G''^2} \quad (1)$$

Or

$$G^* = G' + iG'', \quad (2)$$

where G' and G'' are the in phase component amplitude (Imaginary part of G^*), the storage modulus, and the out of phase component amplitude, the loss modulus. Here $i = \sqrt{-1}$.

- b) $\delta(\omega)$ represents the phase angle.

The phase angle refers to the phase shift between the deformation and the measured stress on a material in a rheological test (Somwangthanaroj, 2010). For elastic solids, the stress is proportional to the strain. The stress is always in phase with the sinusoidal strain deformation. For viscous liquids, the stress is proportional to the rate of strain deformation.

The applied strain and stress are out of phase with a phase angle of $\pi/2$. The proportionality constant for the solids is the elastic modulus, and the proportionality constant for the liquids represents the viscosity.

For viscoelastic materials, the stress response has contributions which are in and out of phase with the strain. This reveals the extent of solid-like and liquid like behavior in the material. Therefore, the phase angle is $0 < \delta < \pi/2$ (Wyss et al., 2007). See Figure 1 for an illustration of phase angle.

The phase angle can be expressed in terms of the loss and storage moduli as:

$$\delta = \tan^{-1}(G''/G') \quad (3)$$

where the symbols have the same meanings defined previously.

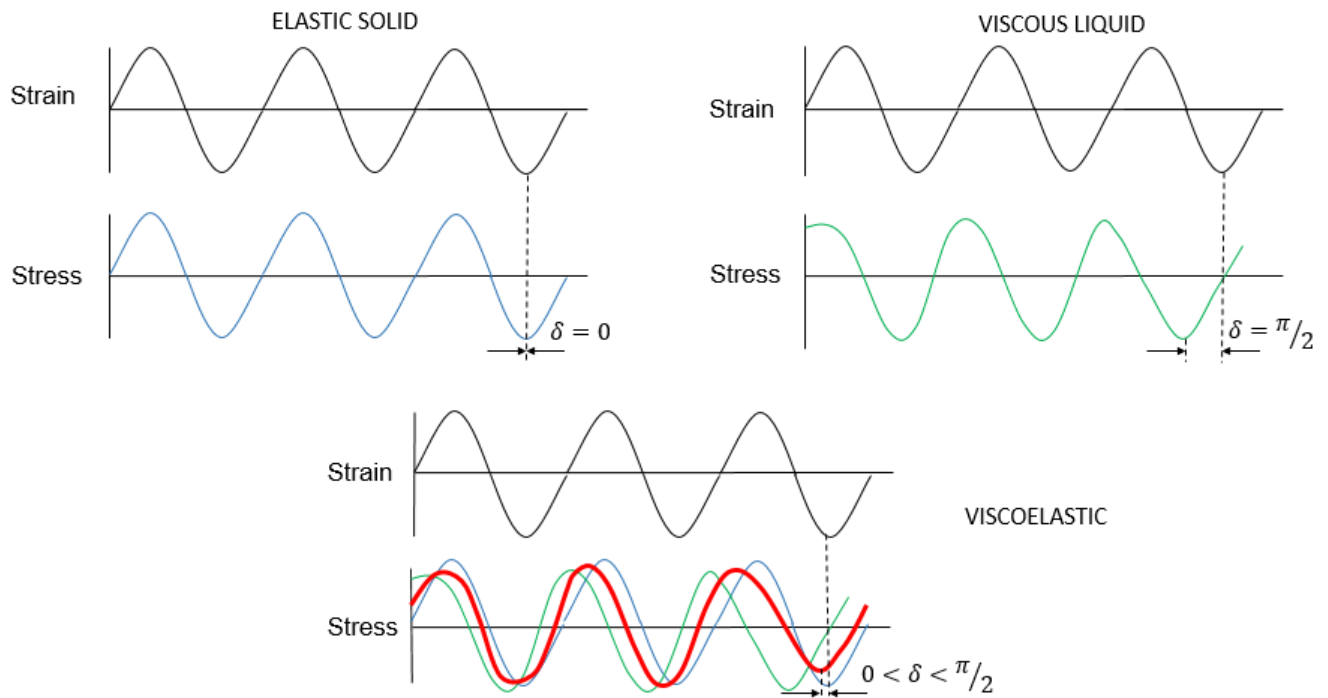


Figure 1. Phase Angle for Elastic solids, Viscous liquids, and Viscoelastic Materials

1.1.3 BENDING BEAM RHEOMETER (BBR) TESTS

Bending Beam Rheometer (BBR) tests are performed to provide information on the asphalt binder's ability to resist low temperature cracking. The parameters obtained from BBR are known as the creep stiffness, represented by $S(t)$, and the m-value.

a) Creep Stiffness, $S(t)$: Creep stiffness, expressed as a function of time, can be calculated using standard beam theory with the equation:

$$S(t) = \frac{PL^3}{4bh^3\delta(t)}, \quad (4)$$

where $S(t)$ is the binder stiffness at time t , P is the applied constant load, L is the distance between beam supports, b represents beam width, h is the beam thickness, and $\delta(t)$ is the deflection of the beam at time t . Creep refers to the slow continuous increase in strain after a sudden application of a constant stress (Moczó et al., 2006). Removal of stress yields recovery; classifications of creep and recovery are shown in Table 1 below.

Table 1. Classification of Creep

application of constant stress	→	creep - slow continuous increase of strain				
removal of the stress	→	recovery - gradual decrease of strain	complete	elastic creep		
			partial	elastic flow		
			no	flow	linear strain rate	viscous flow
					nonlinear strain rate	plastic flow

Table 2. Classification of Relaxation

application of constant strain	→	relaxation - gradual decrease of stress	in material characterized by elastic flow	relaxation to nonzero stress
			in material characterized by elastic creep	relaxation to zero stress

b) m-value: The m-value represents the slope of the master stiffness curve and is a measure of the rate at which the asphalt binder relieves stress through plastic flow (Pavement Interactive, 2011).

1.2 RESEARCH OBJECTIVE

The objective of this research is to develop a guidebook that will provide some guidelines for using the appropriate rheological models of asphalt binders. A sound knowledge of the rheological properties of asphalt binders can have a major impact on the workability and durability of asphalt during and after construction. For example, asphalt that deforms slowly will be susceptible to rutting and bleeding while asphalt that is too stiff can lead to fatigue and cracking (Yussof et al., 2011). The guidebook will present various rheological models from the Maxwell Model to non-linear models and will attempt to discuss when to use appropriate models, explaining the advantages and disadvantages of each model.

2. APPROACH

The guidebook will present the mathematical equations of various rheological models without providing any derivation. The main focus will be the summary of the models in the existing literature for pavement engineering and other related topics. Rheological experiments generally consider shearing flows, or separately extensional flows of the materials. In pavement testing shearing flows are used for evaluation of sample fluidic properties and for an extensional

experiment the solid material is subject to bending beam testing. For this reason, we focus on modeling under these conditions.

3. FINDINGS

3.1 MECHANICAL ELEMENT MODELS

Mechanical Element Models provide a means of expressing viscoelastic material behavior mathematically by combining elements of Hooke's Model for elastic solids and Newton's Model for Newtonian fluids. An elastic solid that obeys Hooke's Law will have a constant relationship between stress and strain. A Newtonian fluid on the other hand, extends at a rate proportional to the applied stress. A mechanical model can be built by considering linear elastic springs and linear viscous dash-pots (Kelly, 2015). To understand mechanical element models in rheology, the basic mechanical models for the elastic spring and viscous dash-pot must be first explained.

3.1.1 LINEAR ELASTIC SPRING

For a linear elastic spring of stiffness, E , shown in Figure 2, the response can be modeled by:

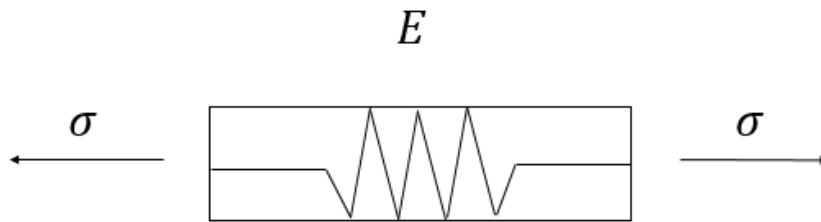


Figure 2. Elastic Body

$$\varepsilon = \frac{1}{E} \sigma \quad (5)$$

Where E = stiffness, ε = strain, and σ = applied stress.

The elastic material, illustrated by the spring, undergoes instantaneous strain after loading. It then undergoes instantaneous de-straining upon release of the load.

3.1.2 LINEAR VISCOUS DASH-POT

The linear viscous dash-pot can be imagined as a piston-cylinder arrangement filled with a viscous fluid, shown in Figure 3 (Kelly, 2015). The response relates the stress and the strain rate by the expression:

$$\dot{\epsilon} = \frac{1}{\eta} \sigma \quad (6)$$

Where $\dot{\epsilon}$ =strain rate, η = viscosity, and σ =applied stress.

The relationship between stress and strain can be expressed in terms of compliance, $J(t)$, as:

$$\epsilon(t) = \sigma_0 \cdot J(t) \quad (7)$$

Where $J(t) = \frac{t}{\eta}$ and σ_0 is an instantaneous load.

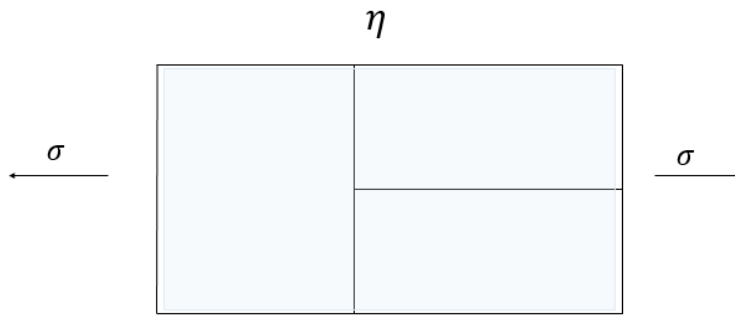


Figure 3. Linear Viscous Dash-pot

3.1.3 MAXWELL MODEL

The Maxwell Model consists of a spring element and a dash-pot in series, as shown in Figure 4 (Kelly, 2015). Total strain in the system is obtained from the spring and dash-pot and therefore expressed as:

$$\text{Total strain} = (\text{spring } \epsilon_1) + (\text{dash-pot } \epsilon_2) \quad (8)$$

The Maxwell Model can be expressed mathematically as:

$$\sigma + \frac{\eta}{E} \dot{\sigma} = \eta \dot{\epsilon} \quad (9)$$

All symbols have the meanings as defined previously.

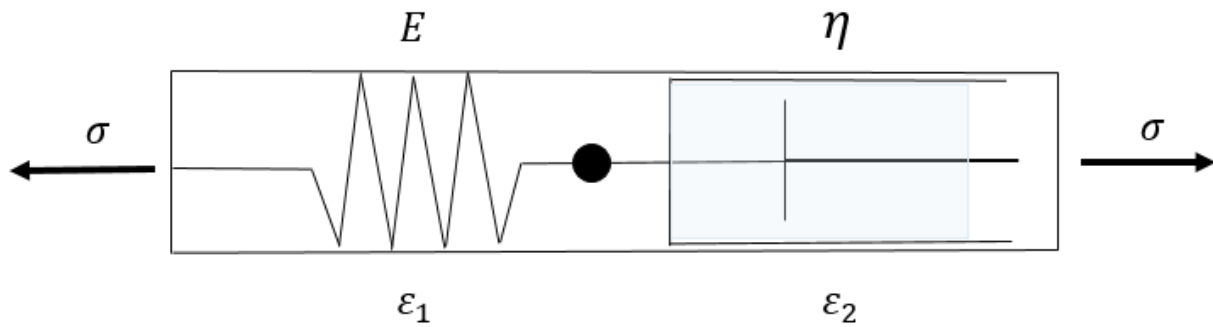


Figure 4. Maxwell Model

3.1.4 KELVIN MODEL

The Kelvin Model consists of an elastic spring and a dash-pot in parallel. With this arrangement, the strain in the spring is the same as the strain in the viscous dash-pot (Kelly, 2011). See Figure 5.

The Kelvin Model is expressed as:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} \quad (10)$$

All symbols have the same meanings as previously defined.

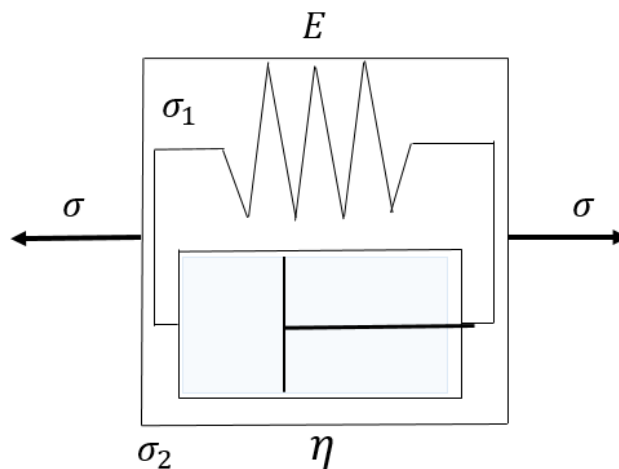


Figure 5. Kelvin Model

3.1.5 HUET MODEL

The Huét Model was developed by Christian Huét to model the rheological behavior of asphalt and bitumen (Yussof et al., 2011). The model is made up of a spring and two parabolic creep elements, k and h . This is illustrated in Figure 6.

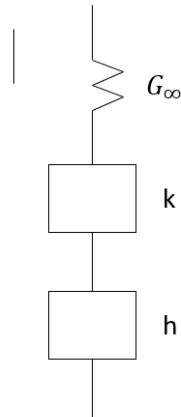


Figure 6. Huét Model

The parabolic element is an analogical model with a parabolic creep function. Creep compliance is expressed as:

$$J(t) = a(t/\tau)^h \quad (11)$$

and complex shear modulus, G^* , as:

$$G^* = \frac{(i\omega\tau)^h}{a\Gamma(h+1)} \quad (12)$$

Where $J(t)$ is the creep function, h is within the range $0 < h < 1$, a is a dimensionless constant, t is the loading time, Γ is the gamma function, $i = \sqrt{-1}$, ω is the angular frequency, and τ is the characteristic time.

An analytical expression for the complex modulus is:

$$G^* = \frac{G_\infty}{1 + o(i\omega\tau)^{-k} + (i\omega\tau)^{-h}} \quad (13)$$

Where G_∞ is the limit of the complex modulus, h and k are exponents with the range $0 < h < k < 1$, and o is a dimensionless constant. The other symbols represent the same factors defined above.

Two major drawbacks with this model are that there is no viscous element for simulating permanent deformation, and the model is unable to accurately model modified bitumen.

3.1.6 HUET-SAYEGH MODEL

This is a generalization of the Huet Model by Sayegh. It is similar to the Huet Model with the parallel addition of a small spring of relatively lower rigidity compared to G_∞ . This is illustrated in Figure 7.

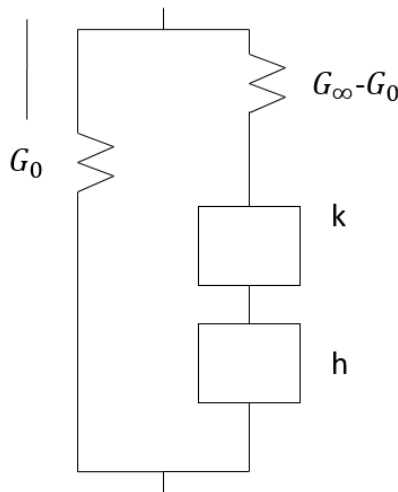


Figure 7. Huet-Sayegh Model

The complex modulus is defined for the Huet-Sayegh Model as:

$$G^* = \frac{G_\infty - G_0}{1 + o(i\omega\tau)^{-k} + (i\omega\tau)^{-h}} \quad (14)$$

Where G_0 is the elastic modulus and the remaining symbols represent the same factors defined for the Huet Model. The Huet-Sayegh Model also lacks any form of representation for the

permanent deformation characteristics of bitumen. The model does not perform well for bitumen at very low frequencies (Yussof et al., 2011).

3.1.7 DI BENEDETTO AND NEIFAR (DBN) MODEL

This model takes into account the linear viscoelastic behavior in the low strain regions and plastic behavior in the large strain regions (Yussof et al., 2011). The DBN model can simulate the behavior of any bituminous binder, mastic, and mixes (Carret et al., 2015). The DBN Model is illustrated in Figure 8 below. It consists of one spring and a number of elementary bodies in parallel with dashpots.

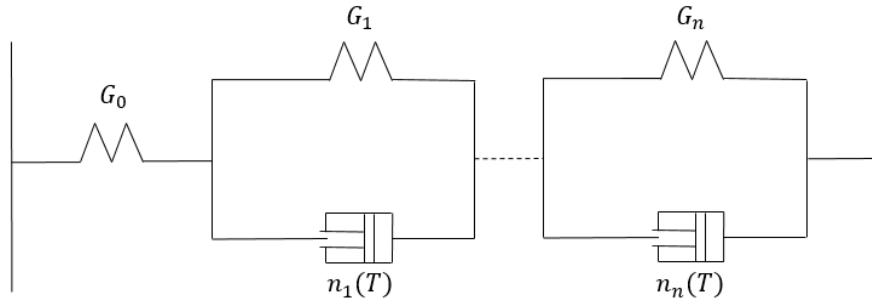


Figure 8. Di Benedetto and Neifar (DBN) Model

The complex modulus for the DBN Model is expressed as:

$$G^* = \left(\frac{1}{G_0} + \sum_{i=1}^n \frac{1}{G_i + i\omega\eta_i(T)} \right)^{-1} \quad (15)$$

Where G_0 is the elastic modulus of the spring and η_i is the viscosity function of the temperature T . The number of elementary bodies, n , can be arbitrarily chosen.

3.1.8 THE 2S2P1D MODEL

The 2-spring, 2-parabolic-1-linear dashpot (2S2P1D) Model developed by Di Benedetto and Olard is an improved version of the Huet Models (Woldekidan, 2011). The model is illustrated in Figure 9 below.

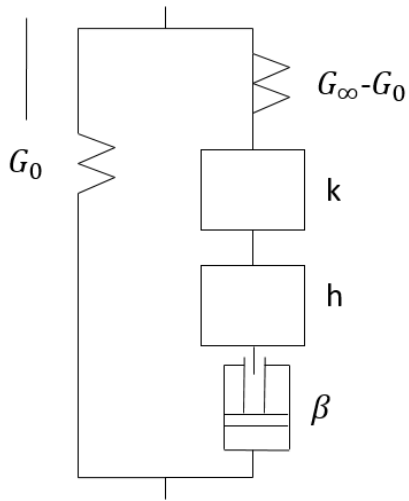


Figure 9. The 2S2P1D Model

The complex modulus, G^* is expressed as:

$$G^* = \frac{G_\infty - G_0}{1 + o(i\omega\tau)^{-k} + (i\omega\tau)^{-h} + (i\omega\tau\beta)^{-1}} \quad (16)$$

Where o and β are dimensionless constants, and all other symbols have the same meanings as previously defined. The model does not fit well when the phase angle is between 50° and 70° (Yussof et al., 2011).

3.1.9 THREE-ELEMENT MODELS

More realistic material responses can be modelled using more elements. Four possible three-element models are shown in Figure 10 below. Figure 10 (a) and (b) are solids while Figure 10 (c) and (d) are considered liquids. The constitutive equations are:

$$(a) \quad \sigma + \frac{\eta}{E_1 + E_2} \dot{\sigma} = \frac{E_1 E_2}{E_1 + E_2} \varepsilon + \frac{\eta E_1}{E_1 + E_2} \dot{\varepsilon} \quad (17)$$

$$(b) \quad \sigma + \frac{\eta}{E_2} \dot{\sigma} = E_1 \varepsilon + \frac{\eta(E_1 + E_2)}{E_2} \dot{\varepsilon} \quad (18)$$

$$(c) \quad \sigma + \frac{\eta}{E_2} \dot{\sigma} = (\eta_1 + \eta_2) \dot{\varepsilon} + \frac{\eta_1 \eta_2}{E} \ddot{\varepsilon} \quad (19)$$

$$(d) \quad \sigma + \frac{\eta_1 + \eta_2}{E} \dot{\sigma} = \eta_1 \dot{\varepsilon} + \frac{\eta_1 \eta_2}{E} \ddot{\varepsilon} \quad (20)$$

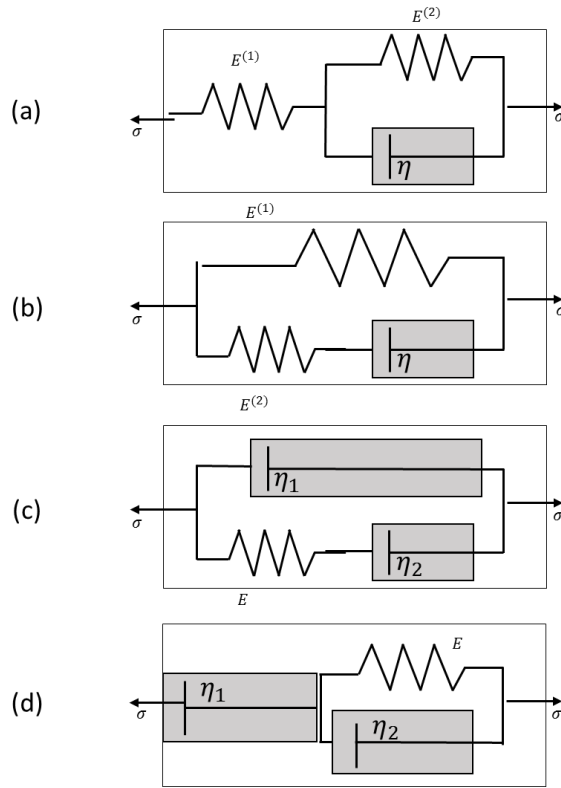


Figure 10. Three-Element Model

Of course adding enough parameters one can fit most any curve, so the real question is what is the basic relationship between the deformation and the stress.

3.1.10 GENERALIZED MODELS

These are complex models in the form of a Generalized Maxwell Model or a Generalized Kelvin Chain (Kelly, 2015). Figure 11 illustrates the Generalized Maxwell Model, which consists of a series of Maxwell units in parallel. The absence of an isolated spring ensures a fluid-type response while the absence of a dash-pot ensures instantaneous response (Kelly, 2015).

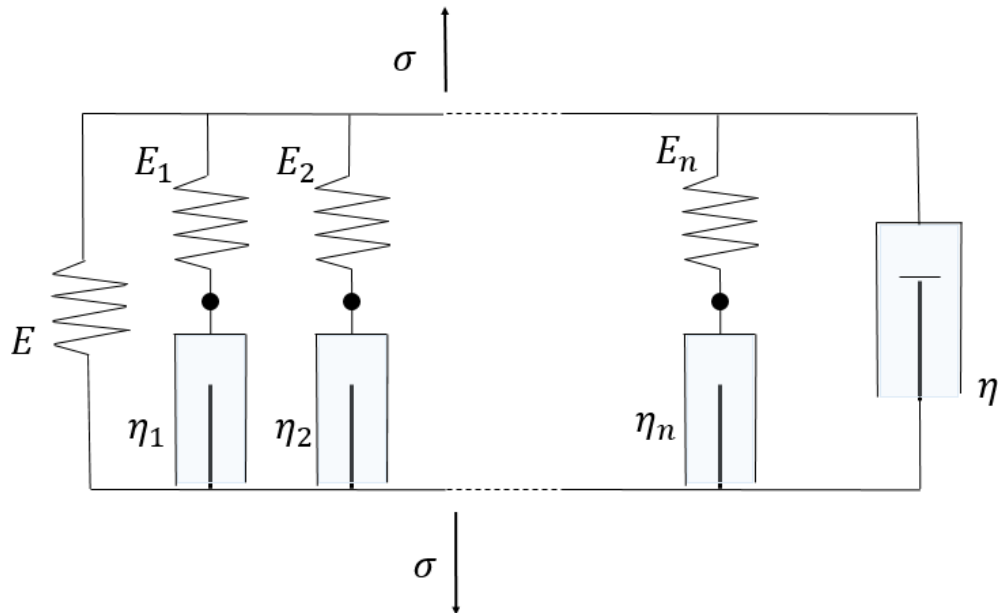


Figure 11. Generalized Maxwell Model

The Generalized Kelvin Chain consists of multiple Kelvin models as illustrated in Figure 12.

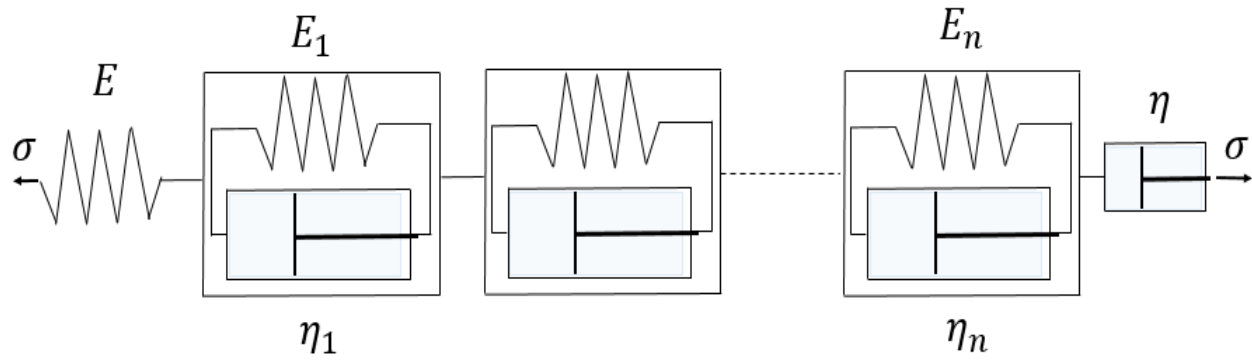


Figure 12. Generalized Kelvin Chain

3.2 EMPIRICAL ALGEBRAIC EQUATION MODELS

The complex moduli of bituminous binders have been studied and explored using different algebraic equations. In some literature, they are referred to as the functional forms of rheological models. The functions can extend results, providing better data for fitting mechanical models (Rowe et al., 2011). Examples are the Dobson's Model, the Christensen and Anderson (CA) Model, and many others. A few of them are explained below.

3.2.1 JONGEPIER AND KUILMAN'S MODEL

In 1969, Jongepeer and Kuilman developed rheological functions based on their suggestion that relaxation spectra for asphalt cement are approximately log normal in shape. The model is expressed using several equations. The first one involves the conversion of the reduced frequency into a dimensionless frequency parameter, ω_r , expressed as (Anderson et al., 1994):

$$\omega_r = \omega \eta_0 / G_g \quad (21)$$

Where ω_r is the reduced frequency, η_0 is the steady-state Newtonian Viscosity, and G_g is the glassy modulus.

The relaxation spectrum, $H(\tau)$, is also expressed as:

$$H(\tau) = \frac{G_g}{\beta\sqrt{\pi}} e^{-\left\{\frac{\ln\tau/\tau_m}{\beta}\right\}^2} \quad (22)$$

G_g is the glassy modulus, τ is the relaxation time, τ_m is the exponential mean of the natural log of relaxation time, and β is the scale parameter for the log normal distribution.

The tangent of the phase shift is the ratio of the loss modulus to the storage modulus expressed by the following equations:

Loss modulus,

$$G' = \frac{G_g}{\beta\sqrt{\pi}} e^{-\left\{\frac{\beta(x-1/2)}{z}\right\}^2} x \int_0^\infty e^{-\left(\frac{u}{\beta}\right)^2 \frac{\cosh(x-1/2)}{\cosh u}} du \quad (23)$$

and

Storage modulus,

$$G'' = \frac{G_g}{\beta\sqrt{\pi}} e^{-\left\{\frac{\beta(x-1/2)}{z}\right\}^2} x \int_0^\infty e^{-\left(\frac{u}{\beta}\right)^2 \frac{\cosh(x+1/2)}{\cosh u}} du \quad (24)$$

Where all variables have the same meanings as previously defined and $x = \left(2/\beta^2\right) \ln\omega_r$ and $u = \ln\omega_r\tau$. This model did not perform well for asphalts with large β values. Another major disadvantage of the Jongepier and Kuilman's Model is its complexity, which makes it difficult to use for routine applications in paving technology (Anderson et al., 1994).

3.2.2 CHRISTENSEN AND ANDERSON (CA) MODEL

With this model, the primary parameters used to define the rheological behavior of materials were the glassy modulus (G_g), crossover frequency (ω_c), steady state viscosity (η_{ss}), and rheological index (R). The absolute complex modulus is expressed as (Yussof et al., 2010):

$$|G^*| = G_g \left[1 + (\omega_c/\omega)^{\log 2/R}\right]^{R/\log 2} \quad (25)$$

Over the years, research has shown that this model does not perform well at high temperatures, long loading times, or the combination of these two conditions. The parameters for the CA model are illustrated in Figure 13.

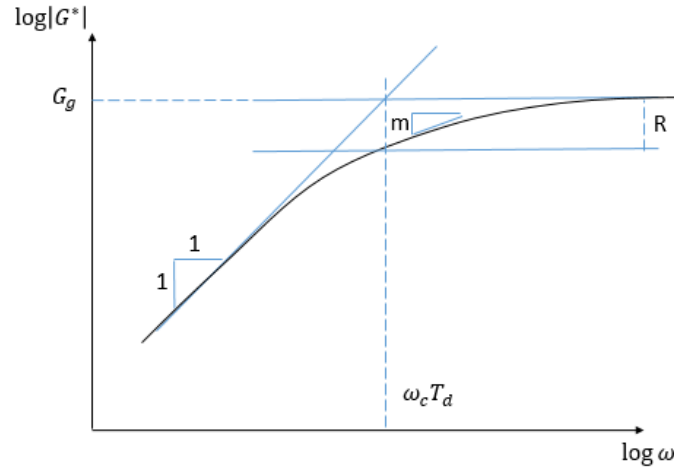


Figure 13. Definition of Parameters from the Christensen and Anderson Model

3.2.3 CHRISTENSEN ANDERSON AND MARASTEANU (CAM) MODEL

The CAM Model is a modified version of the Christensen Anderson Model. This model improves fitting in the low and high zone frequency range for bitumens. It is expressed as:

$$|G^*| = G_g [1 + (\omega_c/\omega)^v]^{-w/v} \quad (26)$$

Where the symbols have the same meanings as defined in the CA model, with v representing \log^2/R . The w parameter addresses the rate at which the absolute complex modulus converges at the two asymptotes. Subsequent research indicated that this model shows lack of fit at high temperatures (Yussof et al., 2011).

3.2.4 POLYNOMIAL MODEL

The Polynomial Model can be used to describe the complex modulus for both asphalt and bitumen. It can be expressed in this form:

$$\log|G^*| = A(\log f)^3 + B(\log f)^2 + C(\log f) + D \quad (27)$$

Where A , B , and C are shape parameters, f is the reduced frequency, and D is the scaling parameter. The only downside is that, as temperatures increase, the model leads to skewed curves. Also, the polynomial does not account for the phase shift (Yussof et al., 2011).

3.2.5 SIGMOIDAL MODEL

The Sigmoidal Model is expressed as:

$$\log|G^*| = \nu + \frac{\alpha}{1+e^{\beta+\gamma(\log(\omega))}} \quad (28)$$

Where $\log(\omega)$ is the reduced frequency, ν is the lower asymptote, and α is the difference between the lower and upper asymptote. β and γ represent the shape between the asymptotes and the inflection point (Yussof et al., 2011).

4. CONCLUDING REMARKS

The current rheological models of asphalt binders are based on differential equations of integer order. This does not give a true picture of the response. A more physically accurate description for asphalt binders may require the introduction of fractional differential equations.