

Assessing Segment- and Corridor-Based Travel-Time Reliability on Urban Freeways

Final Report
September 2016



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16. Abstract <p>Travel time and its reliability are intuitive performance measures for freeway traffic operations. The objective of this project was to quantify segment-based and corridor-based travel time reliability measures on urban freeways. To achieve this objective, a travel-time estimation model and a travel-time reliability prediction framework were developed.</p> <p>The proposed travel-time estimation model considers spatially correlated traffic conditions. Segment-level and corridor-level travel-time distributions were estimated using travel time estimates and compared with estimates based on probe vehicle data. Corridor-level travel-time reliability measures were extracted from travel-time distributions. The proposed travel-time estimation model can well capture the temporal pattern of travel time and its distribution.</p> <p>For the corridor-level travel-time reliability prediction framework, travel time observations are classified based on weather conditions, segment travel-time distributions are estimated, and segment travel-time distributions are synthesized to corridor travel-time distributions. The segment travel-time distribution estimation model was found to capture the pattern of actual travel-time distributions and could adequately represent the short-term corridor-level travel-time distributions. The proposed travel-time reliability prediction framework provides a systematic way to estimate real-time and near-future corridor travel-time reliability by considering weather impact.</p> <p>A Vissim simulation calibrated to Iowa compared travel-time distribution based on simulated data to that based on probe vehicle data. The simulated travel-time distribution is similar to the travel-time distribution based on probe data.</p>			
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EXECUTIVE SUMMARY

Travel time and its reliability are intuitive system performance measures for freeway traffic operations. Moreover, precise prediction of travel time and its reliability is a key component for many advanced traveler information and traffic management systems. The objective of this project was to quantify travel-time reliability measures on urban freeways, including segment-based measures, which are concerned with the service quality at a bottleneck, and corridor-based measures, which capture the variation in travel times along a corridor. To achieve this objective, a travel-time estimation model and a travel-time reliability prediction framework were developed.

We propose a travel-time estimation model that considers spatially correlated traffic conditions. Segment-level and corridor-level travel-time distributions were estimated using these travel-time estimates and compared with the ones estimated based on probe vehicle data. The maximum likelihood estimation was used to estimate the parameters of Weibull, gamma, normal, and lognormal distributions. According to the log-likelihood values, lognormal distribution is the best fit among all the tested distributions. Corridor-level travel-time reliability measures were extracted from the travel-time distributions. The proposed travel-time estimation model can well capture the temporal pattern of travel time and its distribution.

Furthermore, a corridor-level travel-time reliability prediction framework is proposed. The framework contains four parts. First, travel-time observations are classified based on weather conditions using hierarchical cluster analysis. Second, segment travel times are predicted using the autoregressive integrated moving average (ARIMA) model. Third, travel-time distribution is estimated based on the predicted travel time. Fourth, segment travel-time distributions are synthesized to corridor travel-time distributions. Inspired by Winkler's (1981) consensus model, the correlated travel times on adjacent segments are treated as bivariate normally distributed random variables. The synthesizing method is extended to two-component mixture normal distribution. The performance of the travel-time distribution estimation method and the synthesizing method were evaluated and compared with the probe vehicle travel-time data. The results of 95th percentile travel-time analysis show that the segment travel-time distribution estimation model captures the pattern of actual travel-time distributions. The proposed synthesizing method could adequately represent the short-term corridor-level travel-time distributions. Therefore, the proposed short-term corridor-level travel-time reliability prediction framework provides a systematic way to estimate real-time and near-future corridor travel-time reliability by considering weather impact.

Moreover, by using the specific Iowa-based calibration factors of Vissim, the simulated corridor-level travel-time distribution was compared with the travel-time distribution based on INRIX data. The simulated travel-time distribution is similar to the travel-time distribution based on probe data.

INTRODUCTION

From a driver's perspective, travel time and its reliability are considered more intuitive measures of service quality than the levels of service defined in the Transportation Research Board's 2010 *Highway Capacity Manual*. Highly reliable travel times allow for arriving at work or other destinations on time in the context of personal travel and facilitate just-in-time logistics services in freight operations, while highly variable travel times lead to unpredictable trip times and a low quality of transportation services (Turochy and Smith 2002).

Background

Traffic congestion is a problem in cities all over the world. It results from increasing rates of car ownership and the limited supply capacity of the road system. This problem is causing traffic systems to be highly unreliable. Travel time and its reliability have been used as performance measures to evaluate traffic system conditions. Numerous studies have focused on the reliability and variability of travel time in the past decade.

Precise prediction of travel time and its reliability could help travelers decide individual departure times and route choices for pre-trip planning as well as route navigation. Travelers could choose the route with a good balance of both mean travel time and travel-time reliability.

Project Objectives

One of the primary objectives of this research project was to quantify segment- and corridor-based reliability measures on urban freeways. By analyzing urban freeway traffic data and probe vehicle data, a corridor-level travel-time reliability measure estimation model was developed. Because probe vehicles directly collect travel-time data, segment-level and corridor-level travel-time distributions can be easily estimated. In the absence of the direct measurement of travel times, point measurements of traffic conditions obtained from loop detectors or roadside sensors were used to estimate travel time and reliability measures along a stretch of urban freeway. In particular, the flow rates and speeds measured by roadside radar sensors on consecutive freeway segments were used to estimate segment travel-time distributions and correlation coefficients between segments. Accordingly, corridor-level travel-time reliability measures were developed.

Another objective of this research project was to predict corridor-level travel-time reliability on urban freeways. By analyzing probe vehicle data and weather data, a corridor-level travel-time reliability prediction framework considering the impacts of weather was developed. Moreover, the simulated travel-time distribution was compared to the travel-time distribution based on the probe data.

Report Organization

The remainder of this report is organized as follows. Chapter 2 provides a literature review on the methods of estimating and predicting travel time and its reliability. Chapter 3 provides a

summary of the data processing methods used in this study. Chapter 4 describes a travel-time estimation method based on radar sensor data. The travel-time reliability prediction framework is also discussed in this chapter. Chapter 5 describes the case study of this report. Chapter 6 presents the conclusions of this research and directions for future study.

LITERATURE REVIEW

Estimation Methods for Travel Time and Travel-Time Reliability

With the advances in sensing technology, a number of travel-time estimation methods have been proposed based on data collected from various sources (e.g., Soriguera and Robusté 2011a, Tam and Lam 2008). Reviews of the research efforts on travel-time estimation methods can be found in Mori et al. (2015) and Vlahogianni et al. (2014). In particular, loop detectors have been widely used to measure traffic conditions at specific locations. Segment travel times can be estimated by simply extending the point speed measurements to the entire segment (Van Lint and Van Der Zijpp 2003, Soriguera and Robusté 2011b, Bovy and Thijs 2000). Moreover, to capture the traffic dynamics along the segment, several methods have been proposed to estimate travel time based on kinematic wave theory and other traffic flow theories (e.g., Van Arem et al. 1997, Coifman 2002, Zhang 2006, Kesting and Treiber 2008, Deniz et al. 2013, Aksoy and Celikoglu 2012).

Some travel-time estimation methods are essentially based on flow conservation and propagation principles (e.g., Celikoglu 2007, Celikoglu 2013a,b). Castillo et al. (2014) proposed a method that considered both the probabilistic and physical consistency of traffic-related random variables to estimate segment- and route-level traffic flows and travel times. Moreover, a number of queuing-based travel-time models have been developed in the literature (e.g., Daganzo 1995, Nie and Zhang 2005, Lei et al. 2013). These queuing-based models used a vertical queue or point-queue to describe traffic dynamics at bottlenecks. The point-queue models assume that the length of the queue is zero and that the segment has unlimited storage capacity. As a result, point-queue-based models usually ignore the spillback from a downstream bottleneck.

In addition, various approaches have been developed to estimate travel-time reliability (e.g., Richardson 2003, Oh and Chung 2006, Kwon et al. 2011). One way to examine travel-time variation is to look at the distribution of travel times. Based on travel-time distributions, various reliability measures can be derived, including the standard deviation of travel times, buffer time, 90th or 95th percentile travel times, buffer index, planning time index, and the probability that a trip can be successfully completed within a specified time interval (Dong et al. 2006, Tu et al. 2007a, Higatani et al. 2009).

Different functional forms have been used to describe segment travel-time distributions. Van Lint and Van Zuylen (2005) and Susilawati et al. (2010) pointed out that travel-time distributions are skewed and have a long upper tail. Based on travel-time data collected using an automatic vehicle identification system, Li et al. (2006) suggested that a lognormal distribution best characterizes the distribution of travel time when a large time window (e.g., in excess of one hour) is under consideration and in the presence of congestion, and a normal distribution is more appropriate for departure time windows on the order of minutes. Moreover, after using Weibull, exponential, lognormal, and normal distributions to fit the travel-time data collected from dual-loop detectors, Emam and Al-Deek (2006) suggested that lognormal distribution is the best fit. Furthermore, to determine the probability of accomplishing a trip within a time window, a corridor-level reliability measure needs to capture the variability in the total travel times of

multiple roadway segments along the corridor. Although travel times can be easily integrated across time (with successive time frames constituting a trip) and space (with adjacent segments constituting a path), travel-time distributions are generally non-additive because of the spatial and temporal correlations. Considering the correlation among multiple bottlenecks along a freeway corridor, the travel time along a stretch of freeway can be computed as the sum of a set of correlated segment travel times. Accordingly, the corridor-level travel-time distributions, as well as various travel-time reliability measures, can be estimated.

This study presents methods to estimate corridor-level travel-time reliability measures based on roadside radar sensor and probe vehicle data. Because probe vehicles directly collect travel-time data, segment-level and corridor-level travel-time distributions can be easily estimated. In the absence of the direct measurement of travel times, point measurements of traffic conditions obtained from loop detectors or roadside sensors were used to estimate travel time and reliability measures along a stretch of urban freeway. In particular, the flow rates and speeds measured by roadside radar sensors on consecutive freeway segments were used to estimate segment travel-time distributions and correlation coefficients between segments. Accordingly, corridor-level travel-time reliability measures were developed.

Prediction Methods for Travel Time and Travel-Time Reliability

Over the past few decades, numerous segment travel time prediction approaches have been developed. These approaches can be categorized as parametric or non-parametric models. Parametric models, such as K nearest neighbor algorithms (Smith et al. 2002, Clark 2003, Bajwa et al. 2005) and neural network techniques (Park et al. 1999, Dia 2001, Van Lint 2006), try to determine the linkage between input and output factors. Non-parametric models, such as Kalman filtering models (Van Lint 2008, Chen and Chien 2001, Yang et al. 2004, Chien and Kuchipudi 2003, Nanthawicit et al. 2003) and autoregressive integrated moving average models (Oda 1990, Yang 2005, Wang et al. 2014), try to match the current system state with similar patterns observed in the past. It has been proved that both parametric and non-parametric approaches can predict segment travel time accurately. In contrast to predicting a single measure of travel time (such as the mean travel time), predicting travel-time distribution captures the variability of the dynamic traffic network in the future.

Despite the extensive literature on travel-time prediction, only a few studies have focused on travel-time distribution prediction. In particular, Van Lint and Van Zuylen (2005) proposed a method to predict long-term freeway travel-time reliability using the width and skew of the day-to-day travel-time distribution. Fei et al. (2011) proposed a Bayesian inference-based dynamic linear model to predict route travel time. The posterior route travel-time distribution was used to generate both single-value travel time and a confidence interval representing the uncertainty of travel-time prediction. Du et al. (2012) presented an adaptive information fusion model to predict short-term segment travel-time distributions, considering quality of information.

Moreover, weather has been shown to be an important factor impacting traffic conditions on freeways (Ibrahim and Hall 1994, Kim et al. 2010). Tu et al. (2007b) pointed out that adverse weather conditions can make travel time less reliable. However, only a few travel-time

prediction models have considered the effects of weather (Qiao et al. 2012, El Faouzi et al. 2010). To the best of the authors' knowledge, none of the existing travel-time distribution prediction models consider the impact of weather. Moreover, the existing travel-time prediction models are either inadequate or have limitations in terms of predicting the corridor-level travel-time distribution. To address these limitations, a corridor-level travel-time reliability prediction framework is proposed in this study.

DATA DESCRIPTION

Two independent data sources were used in this study to examine travel time and its reliability at the segment and corridor levels: probe vehicle data and radar sensor data. The speed and volume data collected by radar sensors at fixed locations were used to estimate travel time and its distribution. The probe vehicle travel-time data were used not only as the baseline to verify the accuracy of estimated travel times, but also to develop the corridor-level travel-time reliability prediction framework. Moreover, weather data and road surface condition data were used to predict travel time and its reliability.

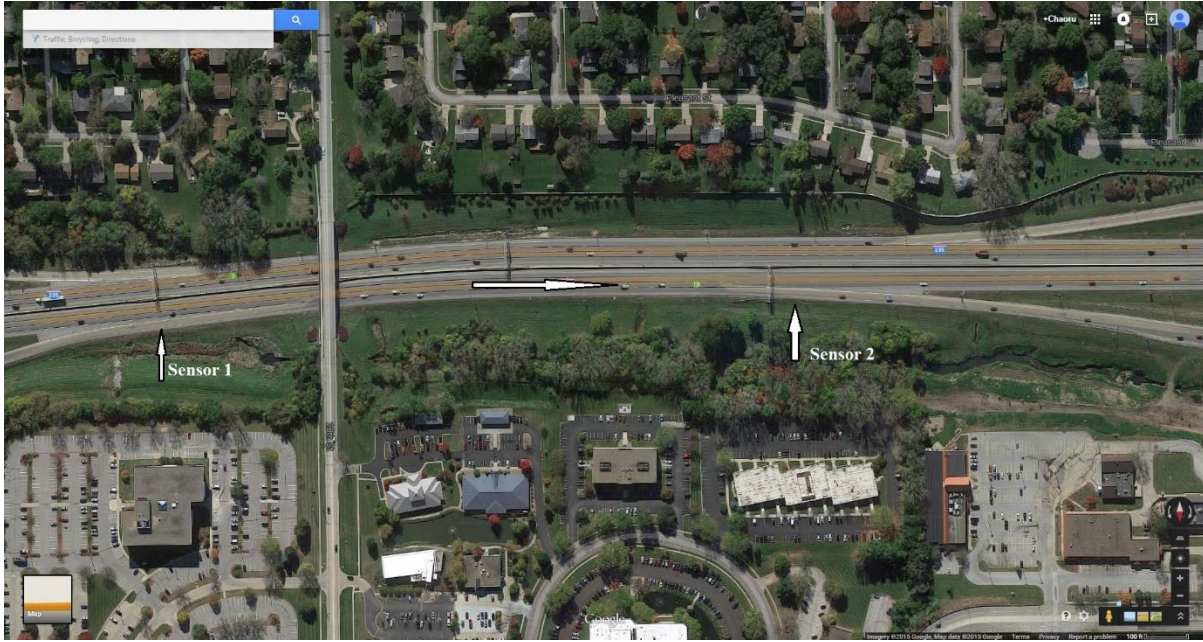
Probe Vehicle Data

The probe vehicle travel-time data used in this study were provided by INRIX, a commercial company that provides real-time traffic data collected from in-vehicle transponders on commercial vehicles and increasingly from cell phones in passenger cars. In the Des Moines metropolitan area, the INRIX probe vehicle network covers all of the first, second, and third class roads, as well as the highway network. In this research, the probe vehicle travel-time data were queried from Regional Integrated Transportation Information System (RITIS), which archives INRIX probe vehicle data at 1-minute aggregation intervals. This dataset provides time-stamped segment-based speeds, travel times, historical average speeds, free-flow speeds, and confidence scores. As stated in the INRIX Interface Guide (INRIX 2014), the data represent real-time data only when the confidence score equals 30; otherwise the value is estimated from historical data. Consequently, the travel times used in this study were those with a confidence score of 30.

Radar Sensor Data

In recent years, the Iowa Department of Transportation (DOT) has been placing Wavetronix radar sensors along Interstates and major highways in the state. The majority of sensors are in the major metropolitan areas and provide valuable information for the Iowa DOT in terms of incident management, traffic operations, and planning. The existing Iowa DOT Wavetronix sensors cover the highway network in the Des Moines metropolitan area. These sensors count vehicles by lane and classification and register vehicle speeds. The aggregated data were obtained through an online data portal maintained by TransSuite. The data can be aggregated at different time intervals: 20 seconds; 5, 15, 30, 60 minutes; and 24 hours. To make these data consistent with the travel-time data generated by INRIX, the 20-second data were aggregated into 1-minute data and used to estimate travel times. The aggregated data obtained from TransSuite included volume, average speed, and average occupancy, by lane. The volume can be broken down by vehicle class as well.

On-ramps and off-ramps are potential bottlenecks on freeways (Bertini and Malik 2004, Newell 1999, Liu and Danczyk 2009). As a result, roadway sensors are usually placed close to ramps, as illustrated in Figure 1. In such cases, both the ramp flow and the mainline flow can be monitored using a side-fired radar sensor, as can the space mean speed.



Imagery ©2015 Google. Map data ©2015 Google.

Figure 1. Sensor locations

The radar sensors sometimes report extreme values due to malfunction. Such abnormal data were identified and removed using the rules proposed by Vanajakshi (2005), as detailed in Table 1.

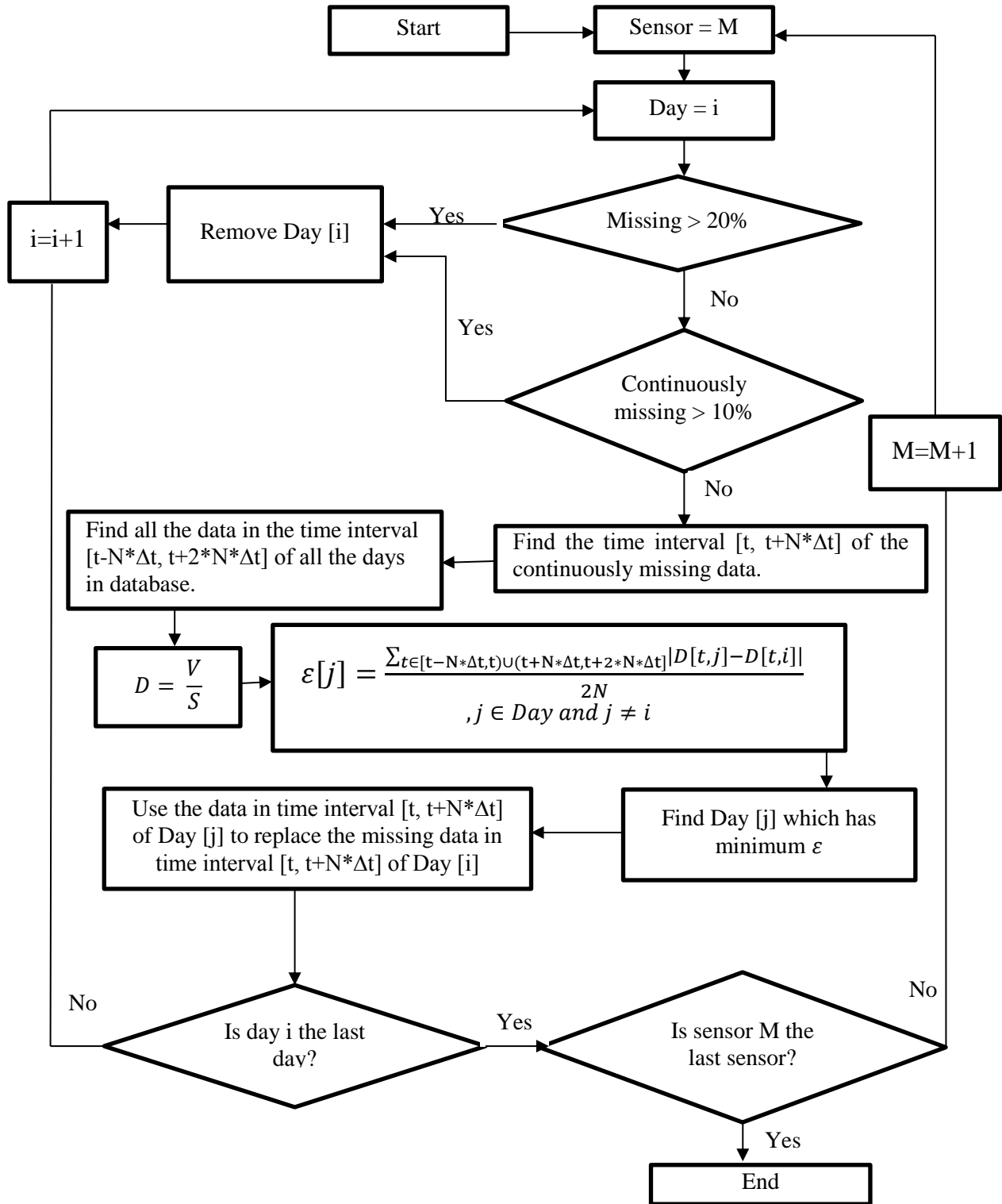
Table 1. Outlier identification rules for radar sensor data

Individual Rules	
1) $q > 50$	Error
2) $v > 100$	Error
3) $o > 90$	Error
Combination Rules	
4) $v = 0, q = 0, o > 0$	Error
5) $v = 0, q > 0, o > 0$	Error
6) $v = 0, q > 0, o = 0$	Error
7) $v > 0, q = 0, o = 0$	Error
8) $v > 0, q = 0, o > 0$	Error
9) $v > 0, q > 0, o = 0$	Error
10) $v = 0 - 100, q = 0 - 50, o = 0 - 90$	Accept

q = volume in vehicles per minute per lane,
 v = speed in mph, o = occupancy in percent

Source: Vanajakshi 2005

Because the proposed travel-time estimation method needs to use volume and speed data during each time interval, the missing data were handled by the procedure shown in Figure 2.



N = number of missing values in this interval, Δt = data aggregation level, D = density, V = volume, S = speed

Figure 2. Flow chart for replacing missing data

Basically, if a significant amount of data was missing on a certain day, that day was removed from the analysis. If data were missing only for a short time period, the data were imputed based on the data collected during the same time period on other days.

The 1-minute interval data from 7:00 a.m. to 9:30 a.m. on weekdays from December 1, 2013 to December 1, 2014 were used in this study. Figure 3 shows the available radar sensor data and real-time INRIX data after the outliers were removed and the selected missing data were replaced.

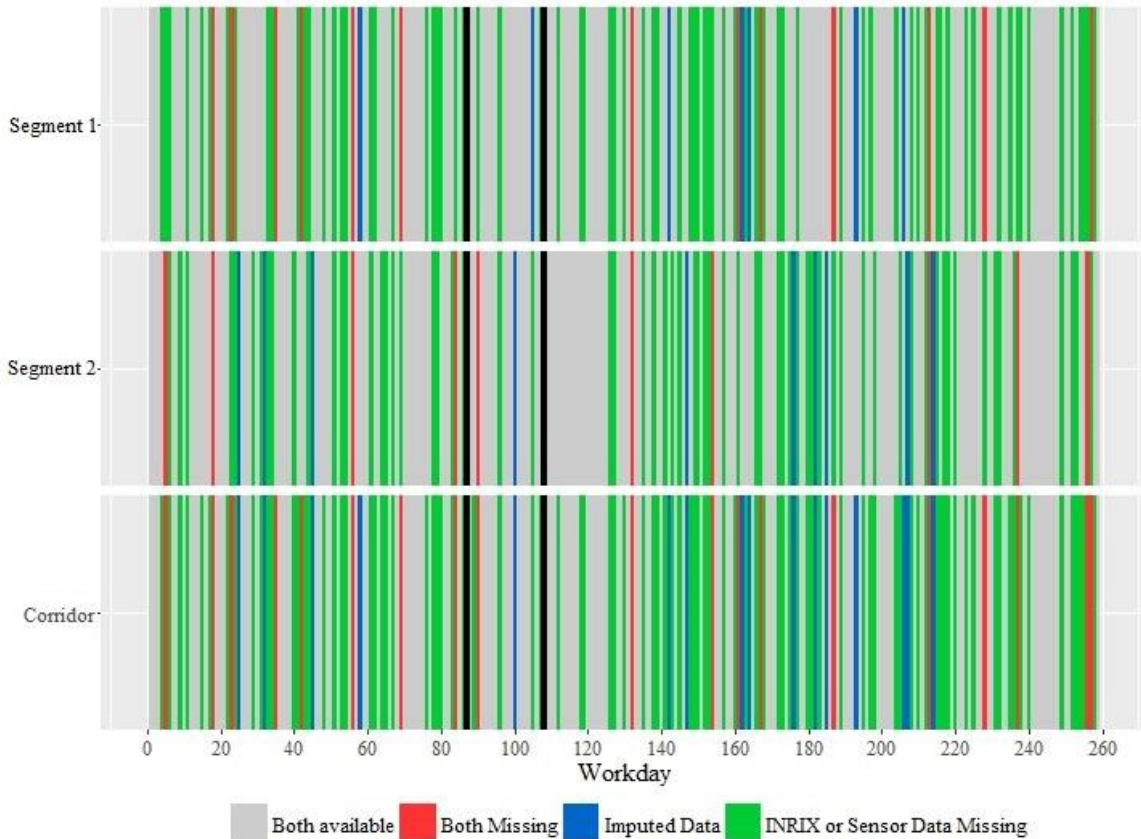


Figure 3. Available INRIX and sensor data

To validate the proposed model against INRIX travel times, data needed to be available from both sources. The plot between the black lines indicates the data for April 2014, when both the INRIX and sensor data are available on most days for all segments. Inconsistencies between missing INRIX data and sensor data can cause differences between the model-based travel time and INRIX travel-time reliability indices.

Weather Data

Weather data were collected from the Automated Surface Observing System (ASOS) station at the Des Moines International Airport. Road surface condition data were obtained from the road

weather information system (RWIS) at that location. Road surface condition data include six types of road surface states, namely dry, trace moisture, wet, chemically wet, ice watch, ice warning. One-minute interval data from 7:00 a.m. to 8:00 p.m. on weekdays from 2013 to 2014 were used in this study.

METHODOLOGY

Spatial Correlation of Segment Travel Times

In several studies (e.g., Park and Rilett 1999, Zou et al. 2014), researchers have pointed out that the correlation between two segments decreases with an increase in the distance between those two segments. In order to discover the extent to which this empirical trend applies to the area observed in this study, freeway travel-time data obtained from RITIS, which archives periodic travel-time data from the INRIX probe vehicle network, were investigated. The INRIX network provides travel-time data in 1-minute intervals for select routes. The system utilizes GPS probe data to estimate a segment's space mean speed using a sample of speeds collected during the reporting interval. The system then computes the travel time based on the integer space mean speed.

To examine the spatial correlations within the travel-time data, the correlation coefficient is used to represent the relationship between segment travel times. Equation (1) describes the cross-correlation of travel times between different segments:

$$\rho_{x,y} = \frac{\sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y} \quad (1)$$

where,

- $\rho_{x,y}$ is the correlation value between two variables x and y
- μ_x and μ_y are the means of variables x and y, respectively
- σ_x and σ_y are the standard deviations of variables x and y, respectively
- n is sample size

Three model specifications are considered here to determine the relationship between the correlation value and the distance between the two segments. The formulations of these three models can be written as follows:

Linear model:

$$\rho = \beta_1 + \beta_2 x + \varepsilon \quad (2)$$

Power model:

$$\ln \rho = \ln \beta_1 + \beta_2 \cdot \ln x + \varepsilon \quad (3)$$

Exponential model:

$$\ln \rho = \ln \beta_1 + \beta_2 \cdot x + \varepsilon \quad (4)$$

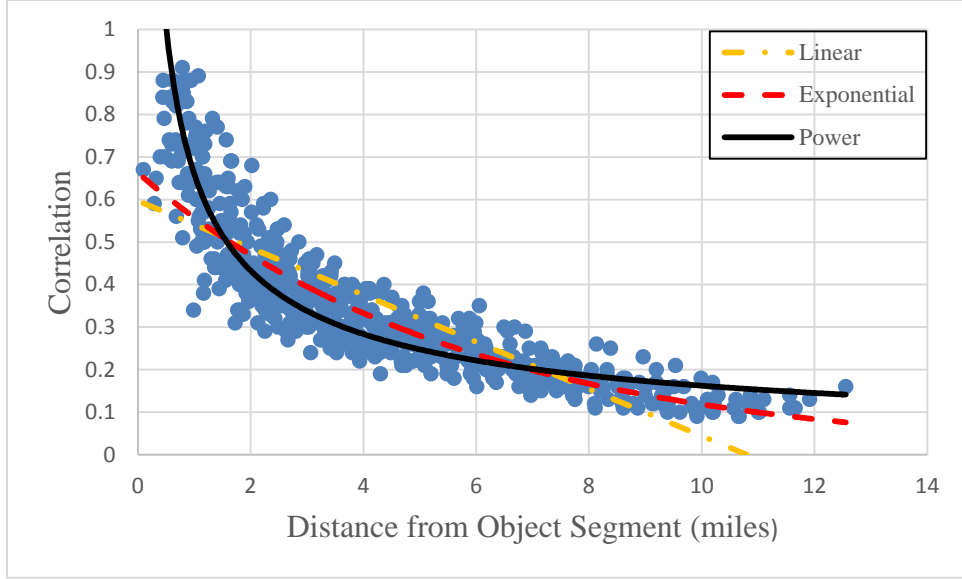
where,

- ρ is the correlation value
- x is the distance between two segments
- β_1 and β_2 are coefficients
- ε is random error

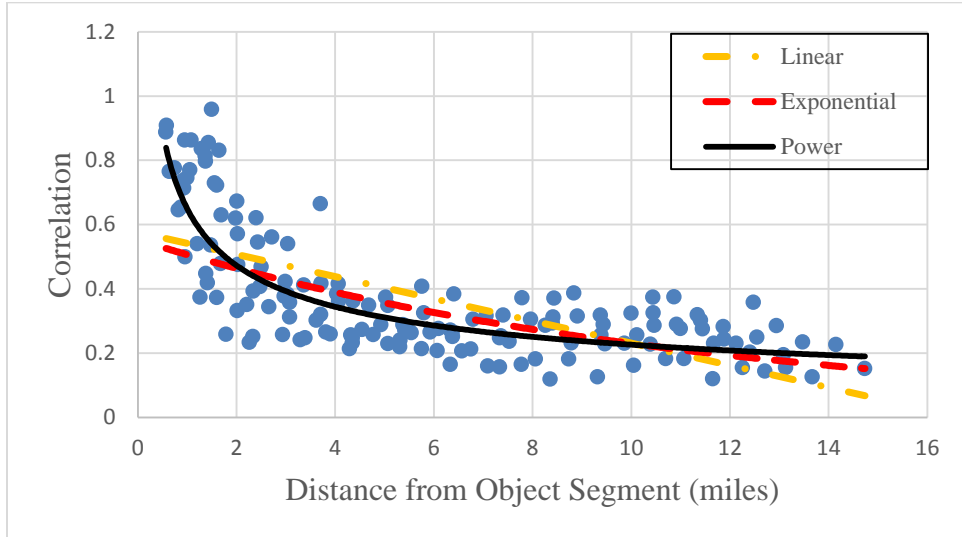
Historic travel-time data are also utilized to find the relationship between the correlation value and the distance between the two segments. The results of this analysis are shown in Table 2 and Figure 4.

Table 2. Calibration results of models

Route	Linear			Exponential			Power		
	β_1	β_2	R-square	β_1	β_2	R-square	β_1	β_2	R-square
I-235	0.597	-0.055	0.698	0.663	-0.172	0.807	0.6605	-0.609	0.820
I-35/I-80	0.567	-0.035	0.460	0.553	-0.088	0.499	0.6495	-0.458	0.633



I-235



I-35/I-80

Figure 4. Relationship between the correlation value and the distance between the two segments for Des Moines, Iowa freeway data from 7:00 a.m. to 8:00 p.m.

Based on the findings in the literature and the results from Table 2, we assume that a power relationship exists between the correlation value and the distance between the two segments:

$$\rho = \begin{cases} 1 & , 0 < x \leq e^{-\frac{\ln \beta_1}{\beta_2}} \\ \beta_1 \cdot x^{\beta_2} & , x > e^{-\frac{\ln \beta_1}{\beta_2}} \end{cases} \quad (5)$$

Travel-Time Reliability Based on Radar Sensor Data

Travel-Time Estimation

Consider a corridor with N potential bottlenecks. Assume that each bottleneck (i.e., sensor location) is a node and that the road segments between these nodes are represented by segments with homogeneous capacities. Denote node 1 as the starting point and node N as the last node. The segment between node M and node $M+1$ is denoted as segment M . Figure 5 illustrates the node-segment representation for part of the corridor, from node M to node $M+3$.

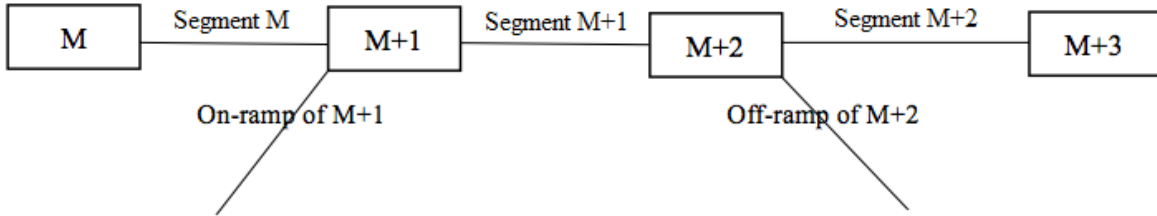


Figure 5. Node-segment representation of part of a corridor

An on-ramp or off-ramp might be connected to a node. The on-ramp or off-ramp is denoted as “On-ramp of M ” or “Off-ramp of M .” For example, in Figure 5 the on-ramp that is connected to node $M+1$ is denoted as “On-ramp of $M+1$.”

In order to construct a numerically tractable model for computing corridor-level travel time, the first-in, first-out property is assumed to ensure that any vehicles that enter the segment first would leave the segment first (Lei et al. 2013). In addition, traffic breakdowns can be detected when speeds drop significantly (e.g., by 10 mph) and the low speeds persist for a long period (e.g., 15 minutes) (Dong and Mahmassani 2009). Considering the spatial correlations between segments, three possible conditions might occur when estimating the travel time of segment M . For each condition, a travel-time calculation method is proposed.

The first condition is when no breakdown occurs on segment M and segment $M+1$. The travel time of segment M at time t can be estimated based on the length of the segment and the average of speeds measured at the two ends of the segment, as follows:

$$T_{c1}[M, t] = \frac{2 \cdot D[M]}{S[M, t] + S[M+1, t]} \quad (6)$$

where,

- $D[M]$ is length of segment M
- $S[M]$ and $S[M+1]$ are the speeds measured at node M and $M+1$ at time t , respectively

The second condition is when the breakdown occurs at bottleneck M+1, causing congestion on segment M. The travel time of segment M at time t is calculated as follows. Assuming that the vehicles in the platoon are traveling at the same speed, the spacing between two vehicles in the platoon on segment M can be calculated as follows:

$$Space[M, t] = d_0 + S[M + 1, t] * \tau \quad (7)$$

where,

- d_0 is the initial space between vehicles
- τ is the reaction time
- $S[M+1]$ is the speed measured at node M+1 at time t

The number of vehicles on segment M at time t can be computed as follows:

$$x[M, t] = x[M, t - 1] + (F[M, t - 1] - F[M + 1, t - 1] + R[M, t - 1] + R[M + 1, t - 1]) * t_{interval} \quad (8)$$

where,

- $t_{interval}$ is the length of the time intervals
- $x[M, t - 1]$ is the number of vehicles on segment M at time t-1
- $F[M, t-1]$ and $F[M+1, t-1]$ are the flow rates measured at nodes M and M+1 at time t-1, respectively
- $R[M, t-1]$ and $R[M+1, t-1]$ are the ramp flow rates measured at nodes M and M+1 at time t-1, respectively. The on-ramp flow rates are positive. The off-ramp flow rates are negative.

Assuming that the increment of vehicles during the period adds to the queue, the number of vehicles in the queue (or queue size) can be computed as follows:

$$Q[M, t] = (F[M, t] - F[M + 1, t] + R[M, t] + R[M + 1, t]) * t_1 + x[M, t] \quad (9)$$

where,

- t_1 is the free-flow travel time on segment M

The queue length is as follows:

$$L_Q = Q[M, t] * (L_V + Space[M, t]) \quad (10)$$

where,

- L_V is the average vehicle length

The deceleration distance can be calculated as follows for vehicles entering segment M at speed $S[M]$ and needing to decelerate before joining the slow moving traffic traveling at speed $S[M+1]$:

$$D_s = \frac{S^2[M,t] - S^2[M+1,t]}{2a} \quad (11)$$

where,

- a is the deceleration rate

The sum of free-flow travel distance, deceleration distance, and queue length equals the length of segment M; that is,

$$D[M] = D_s + S[M, t] * t_1 + L_Q \quad (12)$$

The free-flow travel time t_1 can be solved for as follows:

$$t_1 = \frac{D[M] - D_s - x[M,t] * (L_V + Space[M,t])}{(F[M,t] - F[M+1,t] + R[M,t] + R[M+1,t]) * (L_V + Space[M,t]) + S[M,t]} \quad (13)$$

As a result, the travel time of segment M at time t can be calculated as follows:

$$T_{c2}[M, t] = t_1 + \frac{L_Q}{S[M+1,t]} + \frac{S[M,t] - S[M+1,t]}{a} \quad (14)$$

The third condition is when the breakdown occurs at bottleneck M+2 at time t . Under this condition, if the queue spills back onto segment M, the travel time of segment M is impacted by the breakdown; otherwise, the travel time of segment M can be estimated in the same fashion as when no breakdown occurs.

Similar to the second condition, the average spacing between two vehicles in the platoon, number of vehicles, queue size, and deceleration distance on segment M+1 can be derived by changing M and M+1 in equation (8) through equation (11) to M+1 and M+2, respectively. Therefore, the following situations are taken into consideration.

When the queue length is longer than the length of segment M+1, the travel time is calculated as follows:

$$T_{c3}[M, t] = \frac{D[M] + D[M+1] - L_Q - D_s}{S[M,t]} + \frac{L_Q - D[M+1]}{S[M+2,t]} + \frac{S[M,t] - S[M+2,t]}{a} \quad (15)$$

When the queue length is shorter than the length of segment M+1, but the queue length plus the deceleration distance is longer than the length of segment M+1, the travel time can be calculated as follows:

$$T_{c3}[M, t] = \frac{D[M]+D[M+1]-L_Q-D_S}{S[M,t]} + \frac{S[M,t]-S[M+2,t]}{a} * \frac{D_S+L_Q-D[M+1]}{D_S} \quad (16)$$

If the sum of the queue length and deceleration distance is shorter than the length of segment M+1 (i.e., the breakdown at bottleneck M+2 has no impact on travel time on segment M), the travel-time estimation method for segment M is same as the method described under the first condition.

Furthermore, empirical studies have documented that flow breakdown does not necessarily occur at the same prevailing flow level, and therefore the pre-breakdown flow rate (i.e., the flow rate observed immediately before traffic breaks down) has been treated as a random variable in order to model the probabilistic nature of traffic breakdown (Brilon et al. 2005, Dong and Mahmassani 2009). This results in a probability of breakdown occurring at a given flow (demand) level. The probability distribution function of the pre-breakdown flow rates has been calibrated to follow the Weibull distribution based on data samples from freeway sections in California (Dong and Mahmassani 2009, Kim et al. 2010) and Germany (Brilon et al. 2005). The pre-breakdown flow distribution function expresses the probability that traffic breaks down in the next time interval (for a given time discretization):

$$P[M, t] = 1 - e^{-\left(\frac{F[M,t]}{\sigma}\right)^s} \quad (17)$$

where,

- P[M,t] is the pre-breakdown probability at node M at time t
- s is the shape parameter
- σ is the scale parameter
- F[M,t] is the flow rate measured at node M at time t

Thus, the expected travel time of segment M is as follows:

$$T_E[M, t] = [(1 - P[M, t])(1 - P[M + 1, t]) + (1 - P[M, t])P[M + 1, t]] * T_{c1}[M, t] + P[M, t](1 - P[M + 1, t]) * T_{c2}[M, t] + P[M, t]P[M + 1, t] * T_{c3}[M, t] \quad (18)$$

where,

- P[M,t] and P[M+1,t] are the pre-breakdown probabilities at nodes M and M+1 at time t, respectively

Consequently, a vehicle that departs from node M at time t would arrive at node M+1 at time $t+T_E[M]$. The travel-time estimation procedure presented above is repeated to estimate travel time on segment M+1 using measurements collected at time $t+T_E[M]$. The corridor-level travel time from bottleneck 1 to bottleneck N can be calculated as the sum of the time-dependent segment travel times:

$$T_{corridor} = \sum_{i=1}^N T_E[i] \quad (19)$$

The proposed model detects different spillback conditions and uses the queue length and deceleration distance to calculate the delay at the bottleneck with queue spillback. However, the proposed model has a limitation. If the breakdown occurs between two sensors and the queue does not propagate to a sensor located upstream of the bottleneck, the model cannot detect the breakdown.

In order to evaluate the performance of the proposed model, the travel-time estimation method proposed by Vanajakshi et al. (2009) was compared with the proposed method. In Vanajakshi et al. (2009), the travel time is calculated as follows:

$$T_E[M, t] = \begin{cases} \frac{D[M]}{2} \frac{K[M, t-1] + K[M, t]}{F[M+1, t]} & F[M+1, t] > 500 \text{ veh/hr/ln} \\ \frac{2 * D[M]}{S[M, t] + S[M+1, t]} & \text{otherwise} \end{cases} \quad (20)$$

where,

- $K[M, t-1]$ and $K[M, t]$ are the density measured at node M at times t-1 and t, respectively

In addition, a naïve approach was also tested to estimate segment travel time based solely on the point measurement of speeds, that is, using equation (6) to calculate segment travel time. In this approach, the corridor travel time is simply the summation of the segment travel times.

Travel-Time Distribution

Four statistical distributions are considered to fit the data, as shown in Table 3.

Table 3. Plausible function forms of travel-time distribution

	Probability density function	Parameters	Mean	Mode
Gamma	$p(x) = \frac{1}{\theta^k \cdot \Gamma(k)} x^{k-1} \cdot e^{-x/\theta}$	$k > 0$ – shape $\theta > 0$ – scale	$k\theta$	$(k-1)\theta$, for $k \geq 1$
Weibull	$p(x) = \frac{k}{\theta} \cdot \left(\frac{x}{\theta}\right)^{k-1} \cdot e^{-(x/\theta)^k}$	$k > 0$ – shape $\theta > 0$ – scale	$\theta \cdot \Gamma(1 + \frac{1}{k})$	$\theta \left(\frac{k-1}{k}\right)^{1/k}$, for $k > 1$
Lognormal	$p(x) = \frac{1}{x \cdot \sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}$	$\sigma^2 > 0$ – shape $\mu \in R$ – log scale	$e^{\mu + \sigma^2/2}$	$e^{\mu - \sigma^2}$
Normal	$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\sigma^2 > 0$ – variance $\mu \in R$ – mean	μ	μ

$\Gamma(\cdot)$ – Gamma function

Based on the travel-time distribution, various reliability measures can be derived, including the following:

- Planning time – The total travel time, which includes buffer time (i.e., calculated as the 95th percentile travel time)
- Planning time index – How much larger the total travel time is than the ideal or free-flow travel time (i.e., calculated as the ratio of the 95th percentile to the ideal)

$$\text{Planning time index} = \frac{\text{95th percentile travel time}}{\text{free flow travel time}} \quad (21)$$

- Buffer time – The extra time required (i.e., calculated as the difference between the 95th percentile travel time and the average travel time)

$$\text{Buffer time} = \text{95th percentile} - \text{average travel time} \quad (22)$$

- Buffer index – The size of the buffer as a percentage of the average (i.e., calculated as the 95th percentile travel time minus the average, divided by the average)

$$\text{Buffer index} = \frac{\text{95th percentile travel time} - \text{average travel time}}{\text{average travel time}} \quad (23)$$

Corridor-Level Travel-Time Reliability Prediction

The travel-time reliability prediction framework contains four parts. First, travel-time observations are classified based on weather conditions using hierarchical cluster analysis. The

travel times are categorized into three weather conditions: good weather, rain, and snow. Second, segment travel times are predicted using the autoregressive integrated moving average (ARIMA) model. Third, travel-time distribution is estimated based on the predicted travel time. A two-component mixture normal model is developed to estimate and synthesize travel-time distributions. Fourth, segment travel-time distributions are synthesized to corridor travel-time distributions. Inspired by Winkler’s (1981) consensus model, the correlated travel times on adjacent segments are treated as bivariate normally distributed random variables. The synthesizing method is extended to two-component mixture normal distribution.

Classify Travel-Time Observations Based on Weather Conditions

Hierarchical clustering, a widely utilized non-parametric method to determine the hierarchy of clusters, is used in this study to classify travel times based on weather conditions. The “hclust” function in R’s stats package, developed by the R Development Core Team (2011), is utilized in this study to classify travel-time observations based on weather conditions.

Predict Segment Travel Time

The ARIMA model has been widely used to forecast traffic flow and travel times in the literature (e.g., Van Der Voort et al. 1996, Williams and Hoel 2003, Billings and Yang 2006). In this study, the ARIMA model is used to predict segment travel time. The R package “forecast,” developed by Hyndman and Khandakar (2007), is used in this study to fit the ARIMA model and predict short-term travel time.

Estimate Segment Travel-Time Distribution Based on Mean Travel Time

Multistate Model

Multistate models have been proposed by some researchers to fit travel-time distribution (Park et al. 2011, Guo et al. 2010). Multistate travel-time reliability models tend to outperform unimodal distribution models, such as normal, gamma, and lognormal distributions, because multistate models capture the characteristics of the corresponding state and the probability of encountering a specific state (Guo et al. 2010). As in the study by Guo et al. (2010), a two-component normal model is used in this study. The first distribution, which has a smaller mean, corresponds to the free-flow state. The other distribution, which has larger mean, corresponds to the congested state. Consequently, the probability density function (PDF) of the two-component travel-time model is shown as follows:

$$p(x) = \alpha p_1(x) + (1 - \alpha) p_2(x) \tag{24}$$

where,

- $p_i(x)$ is normal density function

- α is the mixture proportion, $1 \geq \alpha \geq 0$

In this study, the R package “mixtools,” developed by Benaglia et al. (2009), is utilized to fit the multistate model.

Relationship between Mean and Standard Deviation of Travel Time per Unit Distance

Several studies (Jones et al. 1989, Mahmassani et al. 2013, Mahmassani et al. 2012, Kim and Mahmassani 2015) have claimed that the mean and standard deviation of distance-normalized travel time have a strong linear relationship:

$$\sigma_T = \theta_1 + \theta_2 \mu_T + \varepsilon \quad (25)$$

where,

- T is travel time per unit distance
- μ_T and σ_T are mean and standard deviation of T, respectively
- θ_1 and θ_2 are coefficient to be estimated
- ε is random error

Travel-Time Distribution Estimation

From the two-component model, which is composed of two component normal distributions, we can derive the mean and standard deviation of travel times as follows:

$$\mu = (1 - \lambda) * \mu_2 + \lambda * \mu_1 \quad (26)$$

$$\sigma^2 = \lambda[(\mu_1 - \mu)^2 + \sigma_1^2] + (1 - \lambda)[(\mu_2 - \mu)^2 + \sigma_2^2] \quad (27)$$

Guo et al. (2010) pointed out that the mean and standard deviation of travel times in the free-flow state show little fluctuation throughout the day. Therefore, we can assume the following:

$$\mu_1 = FFTT \quad (28)$$

$$\sigma_1 = \sigma_{FFTT} \quad (29)$$

Therefore, we have the following:

$$\mu = (1 - \lambda) * \mu_2 + \lambda * FFTT \quad (30)$$

$$\sigma^2 = \lambda[(FFTT - \mu)^2 + \sigma_1^2] + (1 - \lambda)[(\mu_2 - \mu)^2 + \sigma_2^2] \quad (31)$$

When the freeway is in free-flow condition, λ is 0. Alternatively, when the freeway is congested, λ is 1.

When $\mu = a\mu_T$ and $\sigma = a\sigma_T$, from equation (25), we could derive the following:

$$\frac{\sigma}{a} = \theta_1 + \theta_2 \frac{\mu}{a} + \varepsilon \quad (32)$$

$$\sigma = a\theta_1 + \theta_2\mu + a\varepsilon \quad (33)$$

where,

- a is the length of the segment or corridor

Then, we could assume the following:

$$\sigma_2 = a\theta_1 + \theta_2\mu_2 + a\varepsilon \quad (34)$$

Because ε is very small, it can be ignored. Moreover, from equation (26), we have the following:

$$\mu = \mu_2 + \lambda * (FFTT - \mu_2) \quad (35)$$

Substituting equations (29) and (34) into equation (31), we have the following:

$$\sigma^2 = \lambda * [(FFTT - \mu)^2 + \sigma_{FFTT}^2] + (1 - \lambda)[(\mu_2 - \mu)^2 + (a\theta_1 + \theta_2\mu_2)^2] \quad (36)$$

Based on equation (30), we have the following:

$$\lambda = \frac{\mu_2 - \mu}{\mu_2 - FFTT} \quad (37)$$

Substituting equation (14) into equation (13), we have the following:

$$\sigma^2 = \frac{\mu_2 - \mu}{\mu_2 - FFTT} * [(FFTT - \mu)^2 + \sigma_{FFTT}^2] + \frac{\mu - FFTT}{\mu_2 - FFTT} [(\mu_2 - \mu)^2 + (a\theta_1 + \theta_2\mu_2)^2] \quad (38)$$

Then, we have the following:

$$B * \mu_2^2 + C * \mu_2 + D = 0 \quad (39)$$

where,

- $B = (\mu - FFTT)(1 + \theta_1^2)$
- $C = 2(\mu - FFTT)(\theta_1\theta_2a - \mu) + (FFTT - \mu)^2 + \sigma_{FFTT}^2 - \sigma^2$
- $D = 2(\mu - FFTT)[(\mu - FFTT)(\mu^2 + \theta_1^2a^2) + FFTT * \sigma^2]$

The derivation of equation (39) is shown in the Appendix.

If we solve this equation, we have the following:

$$\mu_2 = \frac{-C \pm \sqrt{C^2 - 4BD}}{2B}, \mu_2 > FFTT \quad (40)$$

Therefore, we can derive λ and σ_2 . As a result, travel-time distribution is as follows:

$$PDF \sim (1 - \lambda) * N(\mu_2, \sigma_2^2) + \lambda * N(FFTT, \sigma_1^2) \quad (41)$$

Based on equation (41), reliability measures such as buffer time or 90th or 95th percentile travel times can be derived. Because most reliability measures depend on 95th percentile travel time, 95th percentile travel time is used to evaluate the performance of the travel-time distribution estimation model.

Options for Synthesizing

In this subsection, we examine methods for synthesizing and develop a method from there. First, the naïve method is a basic method to synthesize corridor-level travel-time distribution where the segment travel-time distributions are treated as independent distributions. Second, a consensus model (extended Winkler's method), which is widely used in management science to aggregate information, is used to synthesize corridor-level travel-time distribution. Third, a mathematical method is developed by assuming that the travel times on k segments are k-variate normal densities.

Naïve Method

Generally, travel-time distributions are treated as independent distributions. If the segment travel-time distributions are Gaussian mixture distributions, such as $f(x)$ and $g(y)$, the following is true:

$$f(x) = \sum_{i=1}^n \omega_i f_i(x) \text{ and } g(x) = \sum_{j=1}^n \theta_j g_j(y) \quad (42)$$

$$f_i(x) \sim N(\mu_{xi}, \sigma_{xi}^2) \text{ and } g_i(y) \sim N(\mu_{yi}, \sigma_{yi}^2) \quad (43)$$

Robbins and Pitman (1949) proposed the following:

$$PDF = \sum_{i=1}^n \sum_{j=1}^n \theta_j \omega_i \iint df_i(x) dg_j(y) \quad (44)$$

Then, the naïve method of travel-time distribution estimation is as follows:

$$PDF = \sum_{i=1}^n \sum_{j=1}^n \theta_j \omega_i \iint df_i(x) dg_j(y) \sim \sum_{i=1}^n \sum_{j=1}^n \theta_j \omega_i N(\mu_{xi} + \mu_{yj}, \sigma_{xi}^2 + \sigma_{yj}^2) \quad (45)$$

Extended Winkler's Method

Genest and Zidek (1986) reviewed several methods that are utilized to combine probability distributions. The primary focus of these methods includes obtaining information about consensus beliefs, expert use, and some relevant aspects of group decision making. Winkler (1981) proposed a consensus model that treats the information from k experts as k-variate normal densities. The mean of the consensus distribution is a linear combination of the probability distributions' means. For bivariate normal distribution, we have the following:

$$\mu^* = \frac{(\sigma_y^2 - \rho\sigma_x\sigma_y)\mu_x + (\sigma_x^2 - \rho\sigma_x\sigma_y)\mu_y}{\sigma_y^2 + \sigma_x^2 - 2\rho\sigma_x\sigma_y} \quad (46)$$

$$\sigma^{*2} = \frac{(1-\rho^2)\sigma_x^2\sigma_y^2}{\sigma_y^2 + \sigma_x^2 - 2\rho\sigma_x\sigma_y} \quad (47)$$

As a result, the probability density function is $T \sim N(\mu^*, \sigma^{*2})$.

We can extend this method to synthesize travel-time distribution. If the segment travel-time distributions are Gaussian mixture distributions, such as in equation (24), and because μ^* in Winkler's model does not represent the additive character of travel time, the model can be slightly modified as follows:

$$PDF \sim \sum_{i=1}^n \sum_{j=1}^n \theta_j \omega_i N\left(\mu_{xi} + \mu_{yj}, \frac{(1-\rho^2)\sigma_{xi}^2\sigma_{yj}^2}{\sigma_{xi}^2 + \sigma_{yj}^2 - 2\rho\sigma_{xi}\sigma_{yj}}\right) \quad (48)$$

If we assume that the correlation between synthesized route travel-time distribution and adjacent segment travel-time distribution can be calculated by equation (5), the route travel time PDFs can be synthesized by adding one segment repeatedly. As a result, the route travel-time distribution can be derived.

Proposed Method

Based on the Winkler's (1981) assumption, a mathematical method was proposed. For bivariate normal distribution, we have the following:

$$F(X, Y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(X-\mu_X)^2}{\sigma_X^2} + \frac{(Y-\mu_Y)^2}{\sigma_Y^2} - \frac{2\rho(X-\mu_X)(Y-\mu_Y)}{\sigma_X\sigma_Y}\right]\right) \quad (49)$$

Assume the following:

$$x = X - \mu_X \text{ and } y = Y - \mu_Y \quad (50)$$

Therefore, the corridor-level travel time can have following formulation:

$$T = X + Y = x + y + \mu_X + \mu_Y = t + \mu_T \quad (51)$$

As a result, we have the following:

$$F(x, y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_X^2} + \frac{y^2}{\sigma_Y^2} - \frac{2\rho xy}{\sigma_X\sigma_Y}\right]\right) \quad (52)$$

Assume the following:

$$A = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \quad (53)$$

$$B = -\frac{1}{2(1-\rho^2)\sigma_X^2} \quad (54)$$

$$C = -\frac{1}{2(1-\rho^2)\sigma_Y^2} \quad (55)$$

$$D = \frac{\rho}{(1-\rho^2)\sigma_X\sigma_Y} \quad (56)$$

By substituting equations (53) to (56) into equation (52), we obtain the following intermediate form:

$$F(x, y) = A \times \exp[Bx^2 + Cy^2 + Dxy] \quad (57)$$

In order to obtain corridor travel-time distribution, we have the following:

$$x + y = t, t > x \text{ and } t > y \quad (58)$$

Then, the integral of equation (52) is shown as follows:

$$\int_{-\infty}^{t-x} F(x, y) dy = A \times \exp[Bx^2] \int_{-\infty}^{t-x} \exp[Cy^2 + Dxy] dy \quad (59)$$

In order to simplify equation (59), we have the following:

$$y = t - x \quad (60)$$

Then, we can derive the integral of part of equation (59) as follows:

$$\begin{aligned} \int_{-\infty}^{t-x} \exp[Cy^2 + Dxy] dy &= \frac{\sqrt{\pi} \cdot \exp\left[-\frac{D^2 x^2}{4C}\right] \cdot \operatorname{erf}\left[\sqrt{-C} \cdot y + \frac{Dy}{2\sqrt{-C}}\right]}{2\sqrt{-C}} \Bigg|_{-\infty}^{t-x} \\ &= \frac{\sqrt{\pi} \cdot \exp\left[-\frac{D^2 x^2}{4C}\right]}{2\sqrt{-C}} \operatorname{erf}\left[\sqrt{-C} \cdot (t - x) + \frac{Dx}{2\sqrt{-C}}\right] \end{aligned} \quad (61)$$

As a result, the cumulative density function can be derived as follows:

$$CDF = \int_{-\infty}^t dx \int_{-\infty}^{t-x} F(x, y) dy = \int_{-\infty}^t \frac{A \cdot \exp[Bx^2] \cdot \sqrt{\pi} \cdot \exp\left[-\frac{D^2 x^2}{4C}\right]}{2\sqrt{-C}} \operatorname{erf}\left[\sqrt{-C} \cdot (t - x) + \frac{Dx}{2\sqrt{-C}}\right] dx \quad (62)$$

According to the Leibniz rule, which is a widely used method for differentiating an integral, we have the following:

$$\frac{d}{dt} \left(\int_{g(t)}^{h(t)} F(x, t) dx \right) = \{F[h(t), t] \dot{h}(t) - F[g(t), t] \dot{g}(t)\} + \int_{g(t)}^{h(t)} \frac{\partial F(x, t)}{\partial t} dx \quad (63)$$

Flanders (1973) pointed out that one of the frequent cases is as follows:

$$\frac{\partial}{\partial b} \left(\int_a^b f(x) dx \right) = f(b) \quad (64)$$

Then, we obtain the following form:

$$PDF = \frac{\partial(CDF)}{\partial T} = \frac{A \cdot \exp[Bt^2] \cdot \sqrt{\pi} \cdot \exp\left[-\frac{D^2 t^2}{4C}\right]}{2\sqrt{-C}} \operatorname{erf}\left[\frac{Dt}{2\sqrt{-C}}\right] \quad (65)$$

Based on equation (51), we have the following:

$$t = T - \mu_T \quad (66)$$

As a result, the probability density function of corridor-level travel time is shown as follows:

$$PDF = \frac{\partial(CDF)}{\partial T} = \frac{A \sqrt{\pi} \cdot \exp\left[B(T - \mu_T)^2 - \frac{D^2 (T - \mu_T)^2}{4C}\right]}{2\sqrt{-C}} \operatorname{erf}\left[\frac{D(T - \mu_T)}{2\sqrt{-C}}\right] \quad (67)$$

Winitzki (2003) provided a uniform approximation to error function with an error less than 2%. The function is described as follows:

$$\operatorname{erf}(x) = 1 - \frac{e^{-x^2}}{x\sqrt{\pi}} q(x) \quad (68)$$

$$q(x) \approx \frac{\sqrt{\pi} + (\pi - 2)x^2}{1 + x\sqrt{\pi} + (\pi - 2)x^2} \quad (69)$$

Consequently, the probability density function can be calculated.

Furthermore, this method can be extended to multivariate conditions and other distributions, such as lognormal, gamma, and exponential distributions. However, as the value of k increases, the number of dimensions of the covariance matrix increases rapidly, and it becomes difficult to derive the probability density function based on this method.

In order to simplify the model, we assume that only adjacent segments have correlations to each other and that the correlation between non-adjacent segments is zero. Consequently, travel-time distribution on every other segment is treated independently. If the travel-time distributions have a normal distribution, then we have the following:

$$X \sim N(\sum_{i=0}^n \mu_{2i+1}, \sum_{i=0}^n \sigma_{2i+1}^2) \quad (70)$$

$$Y \sim N(\sum_{i=0}^n \mu_{2i}, \sum_{i=0}^n \sigma_{2i}^2) \quad (71)$$

The correlation between X and Y can be calculated as follows:

$$\bar{\rho} = \frac{\sum_{i=1}^{n-1} \rho_i}{n-1} \quad (72)$$

where,

- ρ_i is the correlation between adjacent segments
- n is number of segments

Then, equation (67) is used to derive the corridor travel-time distribution.

If the segment travel-time distributions are independent mixture distributions, such as in equation (42), then we can use equation (45) to derive the density function. After this step, the bivariate model for the mixture distributions can be used to synthesize route travel time PDFs.

Travel-Time Reliability Prediction Framework

With all aspects of the travel-time reliability prediction framework having been described, the overall short-term travel-time reliability prediction framework can be presented as follows:

Step 1. Classify current and future travel times on each segment based on weather data.

Step 2. Use the ARIMA model to predict travel times based on current travel times.

Step 3. Use the travel-time distribution estimation method to estimate the short-term travel-time distribution of each segment.

Step 4. Use the synthesizing algorithm to derive the corridor-level travel-time distribution.

Step 5. Derive travel-time reliability measures from the corridor-level travel-time distribution.

Travel-Time Distribution Based on Vissim

The corridor-level travel time can also be obtained by simulation with PTV Vissim 7.00. Vissim is popular microscopic traffic simulation software that adopts the psycho-physical car-following model developed by Wiedemann (PTV AG 2014). Because Vissim can accurately simulate the behavior of individual vehicles and produce diverse evaluation parameters, it has been widely used in transportation engineering for modeling various traffic scenarios. There are two car following models available in Vissim, Wiedemann 74 and Wiedemann 99, which are used to model urban traffic and freeway traffic, respectively. In this study, the Wiedemann 99 car following model was used.

The driver behavior parameters were calibrated by Dong et al. (2015) before simulation. Three car-following model parameters, including standstill distance (CC0), headway time (CC1), and “following” variation (CC2), have been found to have a significant influence on traffic capacity. Therefore, the default values were replaced with the calibrated values, where CC0 is 4.92 ft, CC1 is 0.90 s, and CC2 is 13.12 ft. More details about the calibration can be found in Dong et al. (2015). The traffic volume on the study corridor was balanced based on the method proposed by Shaw and Noyce (2014).

In this study, the congested and uncongested conditions were simulated separately. The travel time-distributions were derived by resampling the travel times from the output from Vissim based on the percentage of the congested and uncongested conditions in the real world.

RESULTS

Travel-Time Reliability Based on Radar Sensor Data

The proposed methodology was applied to estimate travel time on part of I-235, as shown in Figure 6. This six-lane freeway section (three lanes in each direction) is one of the busiest freeways in West Des Moines, Iowa. The locations of the roadway sensors are shown in Figure 6. All of the sensors are located in the merging/diverging areas, where the sensors can collect data from the ramps and the main road.



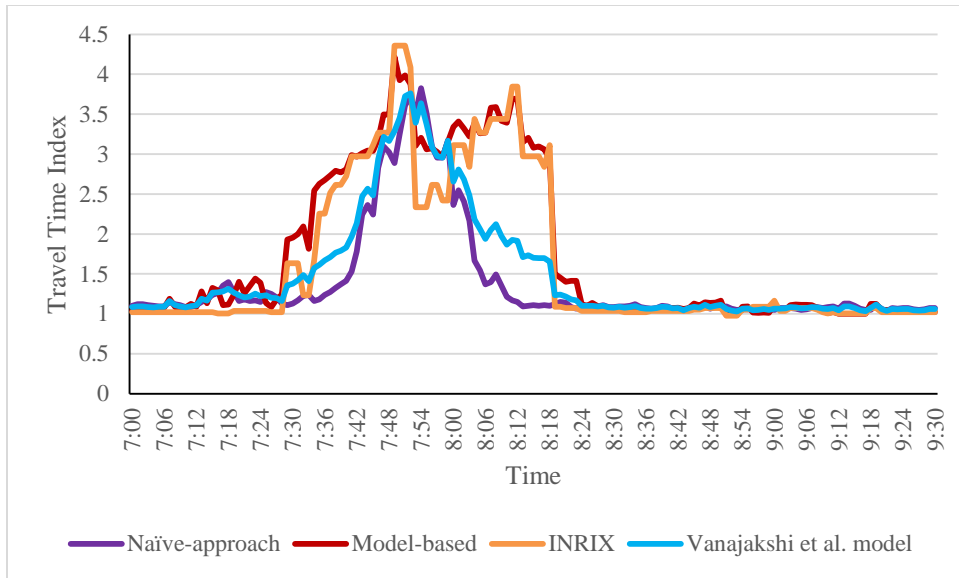
Map data ©2015 Google

Figure 6. Study corridor and sensor locations

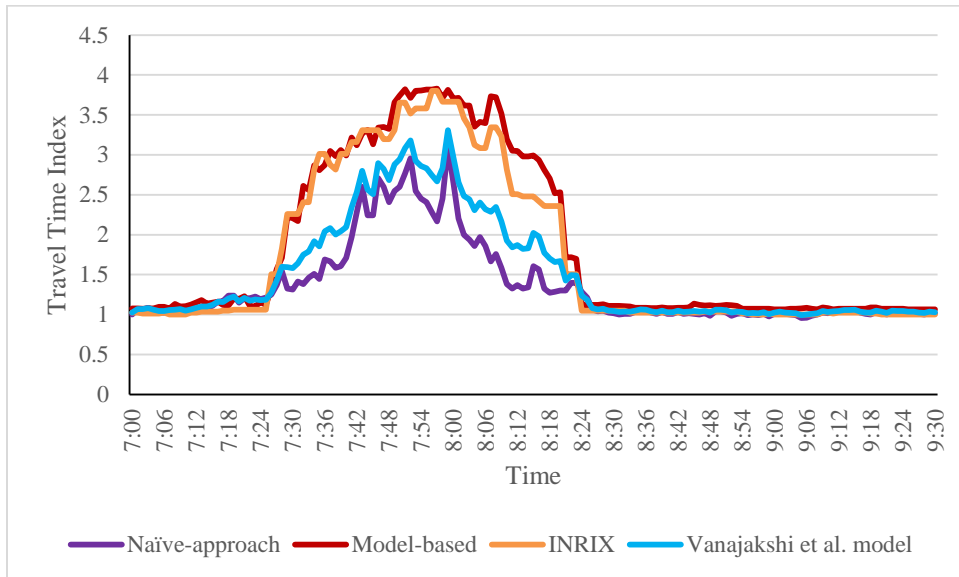
Because the INRIX travel times are provided segment by segment, a temporally stitched algorithm (Chase et al. 2012) was adopted to generate probe vehicles at 1-minute time intervals. The temporally stitched algorithm was intended to simulate the experienced travel time of a probe vehicle traveling along the corridor. In this paper, the probe vehicle travel times are used as the ground truth.

Travel-Time Calculation

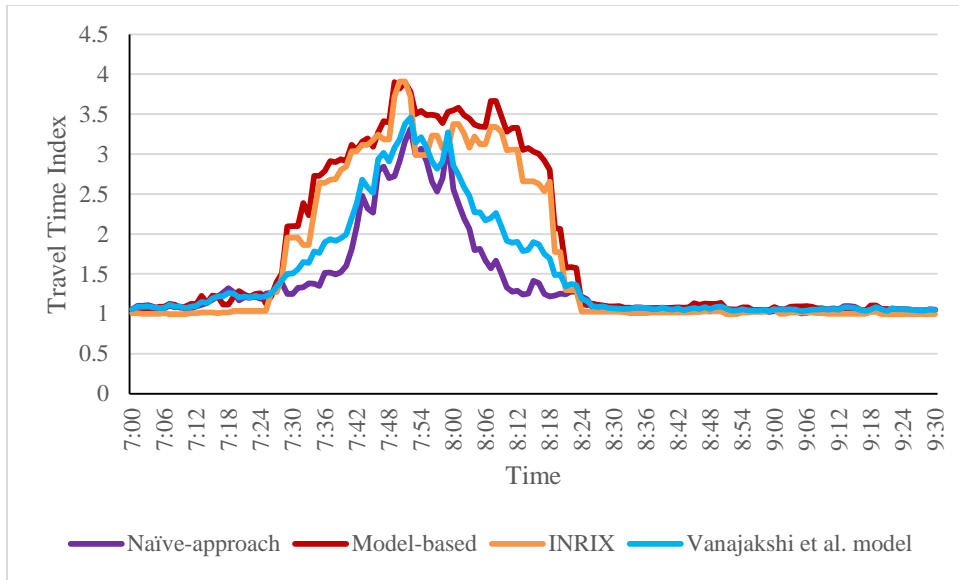
The model-based travel time (MTT), Vanajakshi et al. (2009) travel time, naïve approach-based travel time, and INRIX travel time (INRIX-TT), represented as travel-time index, are plotted in Figure 7. Because congestion generally occurred during the morning peak hours on weekdays at the study site, the travel times were estimated for each 1-minute interval from 7:00 a.m. to 9:30 a.m. using one month of data from April 2014. Figure 7 compares the time-dependent travel times estimated by different methods on an example day.



Segment 1



Segment 2



Corridor

Figure 7. Comparison of model-based travel-time index, Vanajakshi et al. (2009) travel-time index, naïve approach-based travel-time index, and INRIX travel-time index

Figure 7 shows that the model-based travel-time index estimation follows the pattern of the INRIX travel-time index well, at both the segment and corridor levels. The naïve approach and Vanajakshi et al. (2009) model, however, underestimate the delay in terms of congestion duration and severity. Similar patterns are observed for other days as well.

To show the spread of the breakdown, the speed contour during the congested period, from 7:20 a.m. to 8:30 a.m., is plotted in Figure 8. It can be seen that the decreases in speed begin at sensor 3 and propagate to sensor 1. At sensor 4 the traffic is free flowing.

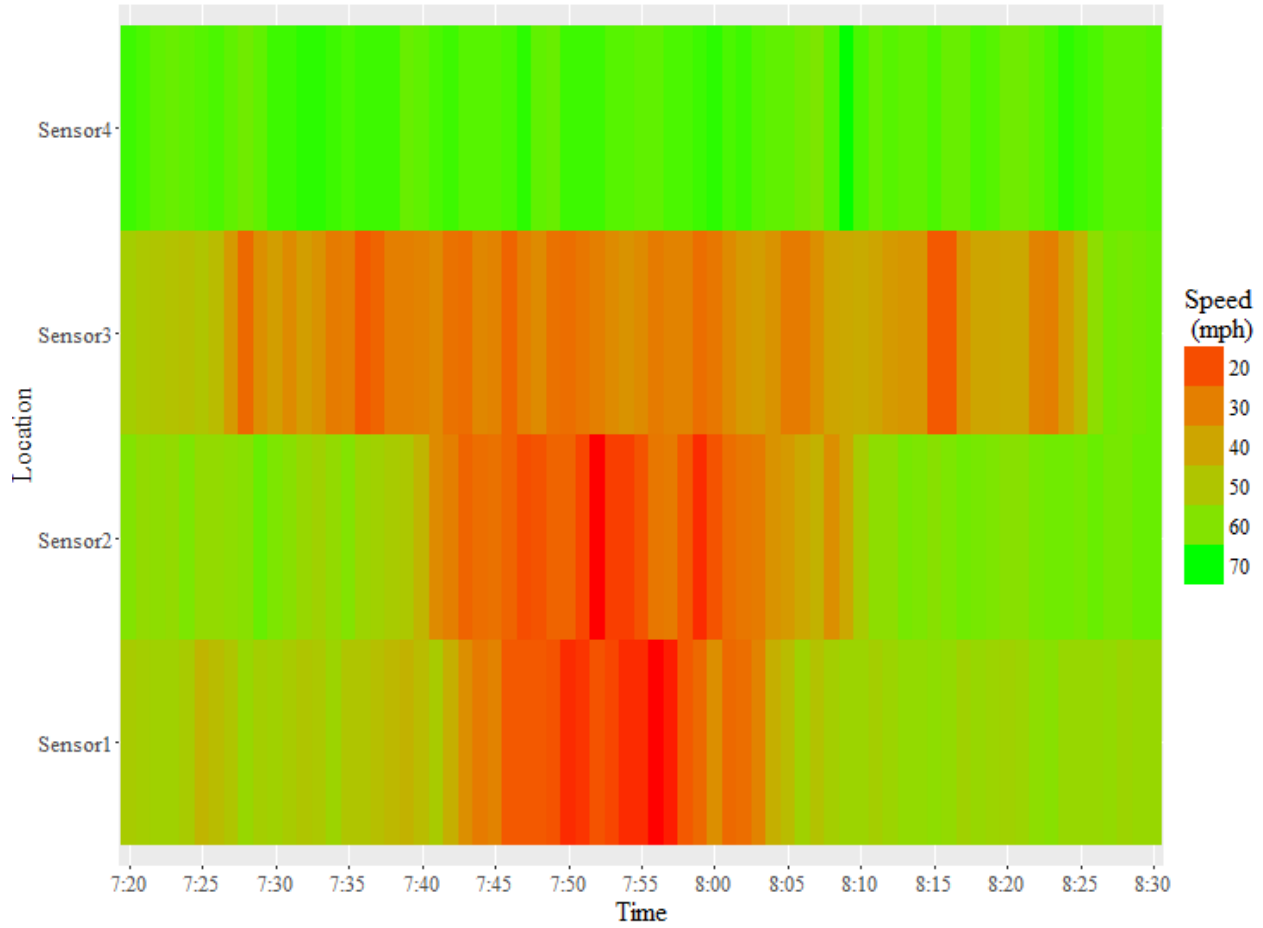


Figure 8. Speed contour of sensors

Performance measures, including mean square error (MSE) and mean absolute percentage error (MAPE), are calculated based on the one-month data as follows:

$$MSE = \frac{\sum(\text{estimated} - \text{actual})^2}{\text{number of observations}} \quad (73)$$

$$MAPE = \frac{\sum \frac{|\text{estimated} - \text{actual}|}{\text{actual}}}{\text{number of observations}} * 100\% \quad (74)$$

Table 4 compares the values of the performance measures for all of the methods at both the segment and corridor levels. It can be seen that the proposed method outperforms the other methods.

Table 4. Performance measures of different methods

	Corridor		Segment 1		Segment 2	
	MSE	MAPE	MSE	MAPE	MSE	MAPE
Model-based approach	0.029	0.661	0.017	0.541	0.087	1.557
Vanajakshi et al. (2009) model	0.188	7.698	0.243	12.742	0.233	9.191
Naïve approach	0.234	12.155	0.365	13.451	0.273	13.150

Additionally, Table 5 shows the impact of data aggregation on the performance of the proposed model.

When the aggregation level increases, the error of the proposed model increases. The differences in the errors of three methods become less noticeable at larger aggregation levels. For example, with 1-minute aggregation level data, the proposed model is significantly better than the other two; with 5-minute aggregation level data, the proposed model performs similarly to the Vanajakshi et al. (2009) model.

Table 5. Performance measures at different data aggregation levels

		Corridor			Segment 1			Segment 2		
		Model-based	Naïve	Vanajakshi et al. model	Model-based	Naïve	Vanajakshi et al. model	Model-based	Naïve	Vanajakshi et al. model
MSE	1 min	0.03	0.19	0.21	0.02	0.36	0.26	0.09	0.32	0.23
	5 min	0.18	0.22	0.20	0.14	0.24	0.12	0.23	0.26	0.23
	10 min	0.23	0.14	0.21	0.33	0.40	0.31	0.30	0.34	0.30
	15 min	0.24	0.25	0.25	0.23	0.24	0.22	0.30	0.31	0.35
MAPE	1 min	0.66	7.70	7.15	0.54	13.45	8.58	1.56	9.45	9.39
	5 min	9.32	11.01	9.45	7.73	12.71	6.71	11.13	13.96	9.24
	10 min	10.74	4.81	8.24	12.65	16.08	12.49	11.40	14.03	12.07
	15 min	17.42	18.39	18.39	11.30	12.78	11.44	14.27	15.14	15.47

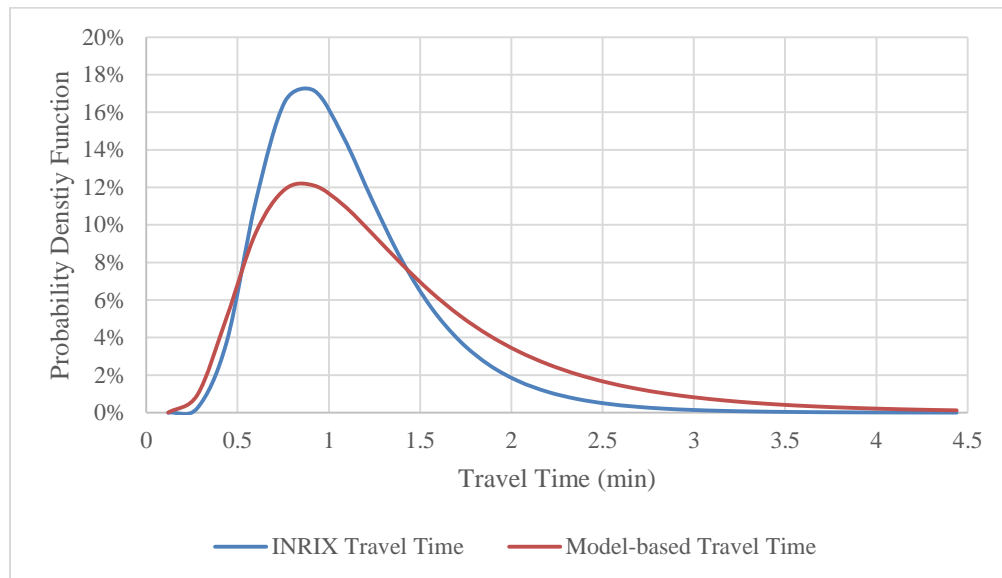
Travel-Time Distribution

The maximum likelihood estimation was used to fit the distributions. To demonstrate the goodness of fit, the log-likelihood value of each distribution is summarized in Table 6. Because the lognormal distribution has the smallest log-likelihood value, it was selected as the best distribution to fit the travel-time data.

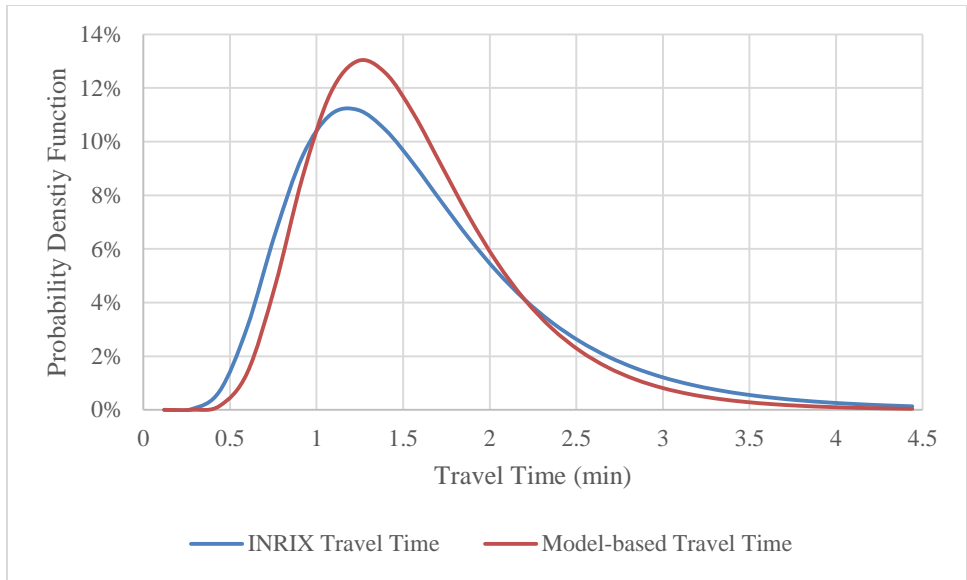
Table 6. Model selection based on log-likelihood

		Weibull	Gamma	Lognormal	Normal
INRIX-TT	Segment 1	-8992.789	-8329.302	-7743.998	-10314.31
	Segment 2	-7331.44	-6568.859	-5617.297	-9385.414
	Corridor	-13409.2	-12715.01	-12028.62	-14849.25
MTT	Segment 1	-1575.996	-1465.623	-1313.615	-1968.53
	Segment 2	-1309.031	-1163.501	-1093.47	-1393.158
	Corridor	-2460.338	-2320.344	-2206.8	-2684.203

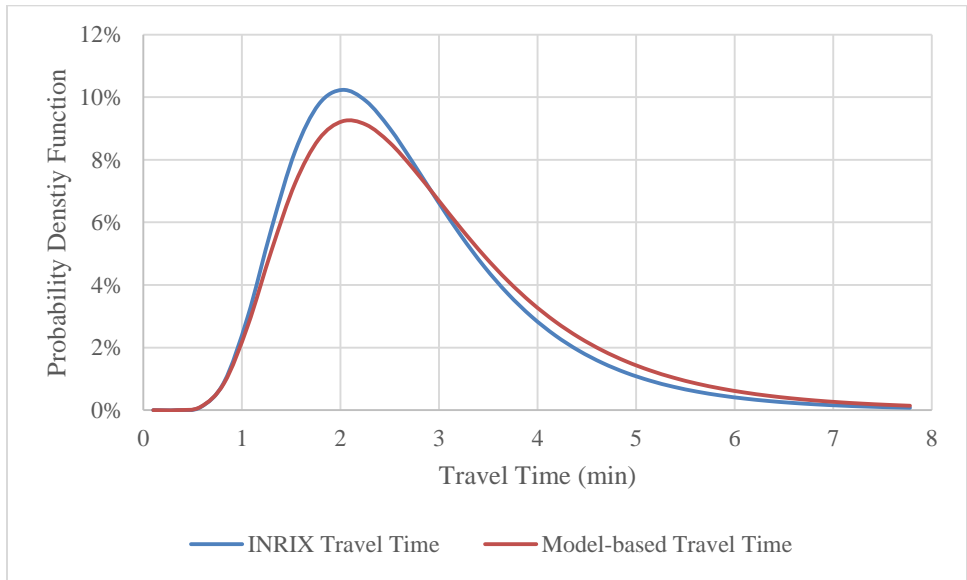
The weekday data for the peak 15-minute travel times (7:45 a.m. to 8:00 a.m.) from December 1, 2013 to December 1, 2014 were used to estimate the travel-time distribution. After removing the outliers, the correlation between segment 1 and segment 2 is 0.83 for the INRIX data and 0.97 for the model-based travel time. The proposed MTT method slightly overestimated the correlation. The travel-time distributions are shown in Figure 9. The MTT distribution captured the tendency of the INRIX travel-time distribution well.



Lognormal distribution of segment 1 travel times



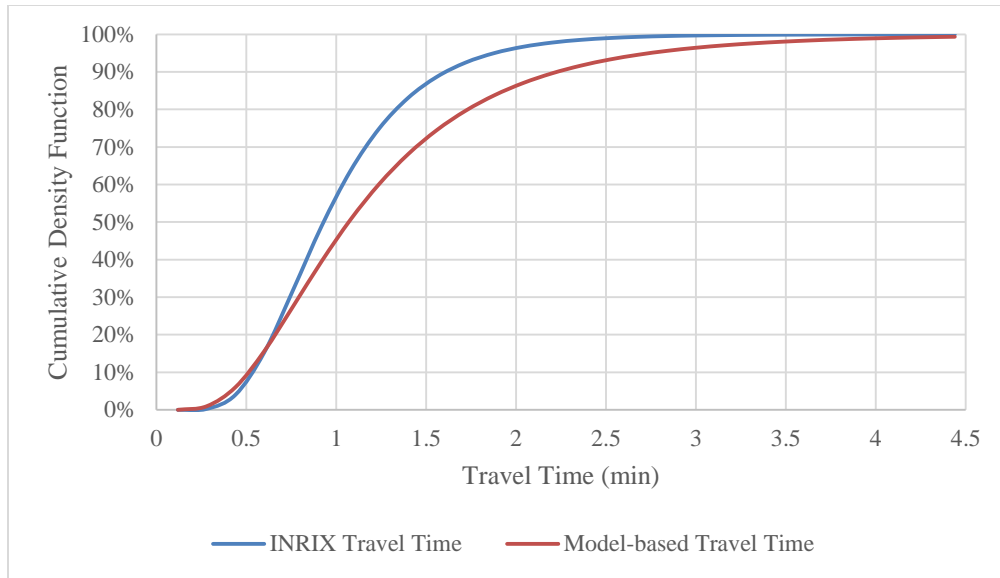
Lognormal distribution of segment 2 travel times



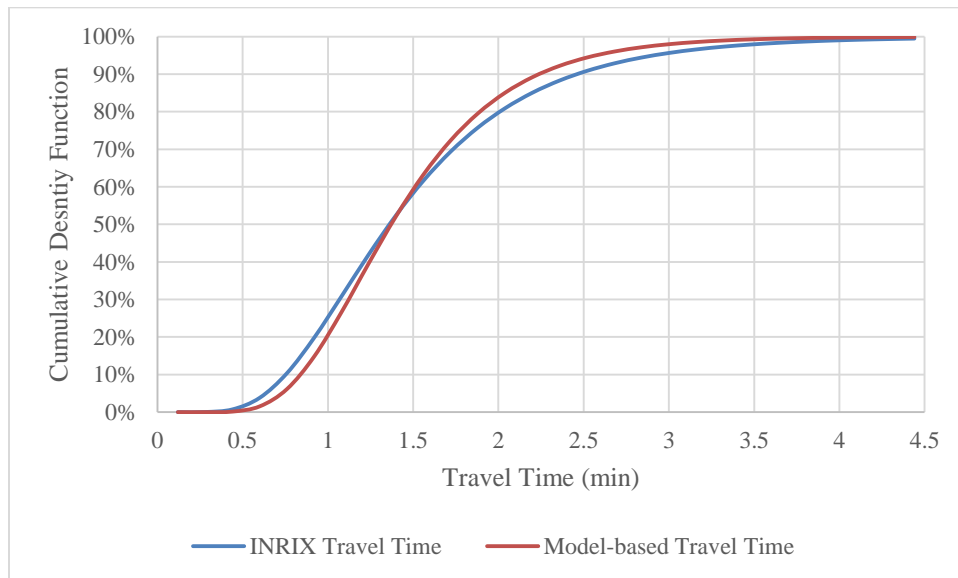
Lognormal distribution of corridor travel times

Figure 9. Probability density distributions of peak 15-minute travel times

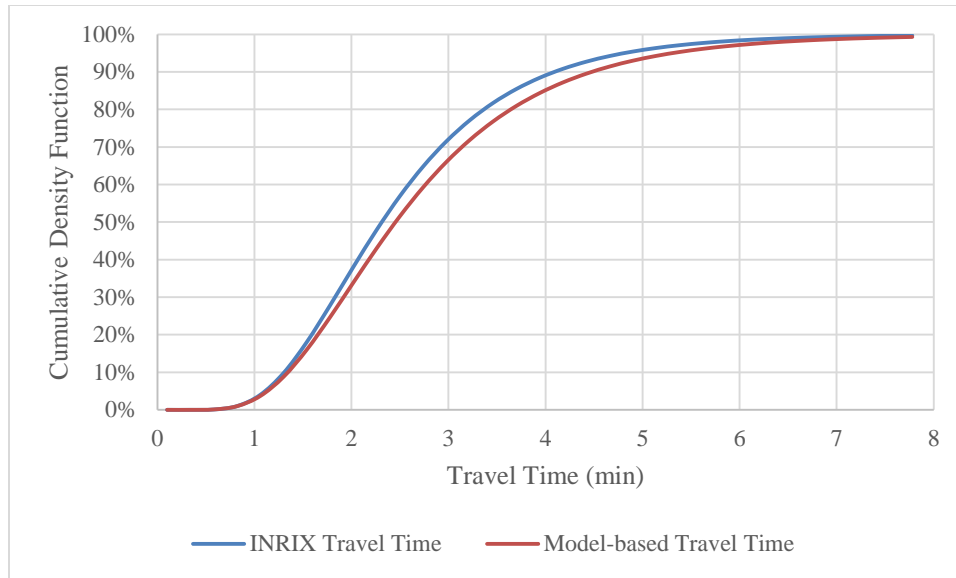
Figure 10 plots the cumulative distribution functions of the lognormal distributions estimated based on the INRIX travel time and model-based travel time.



Cumulative distribution of segment 1 travel times



Cumulative distribution of segment 2 travel times



Cumulative distribution of corridor travel times

Figure 10. Cumulative density distributions of peak 15-minute travel times

Table 7 compares the travel-time reliability indices of the model-based travel-time estimates and INRIX travel times.

Table 7. Variance of reliability indices of INRIX travel time and model-based travel time

		Mean (min)	Standard Deviation	95th Percentile (min)	Planning Time Index	Buffer Time (min)	Buffer Time Index
INRIX-TT	Segment 1	1.14	1.38	1.88	2.04	0.74	0.65
	Segment 2	1.51	1.35	2.56	1.95	1.05	0.69
	Corridor	2.55	1.37	4.90	2.21	2.35	0.92
MTT	Segment 1	1.26	1.56	2.52	2.74	1.26	1.00
	Segment 2	1.47	1.40	3.00	2.29	1.53	1.03
	Corridor	2.74	1.36	5.32	2.40	2.58	0.94

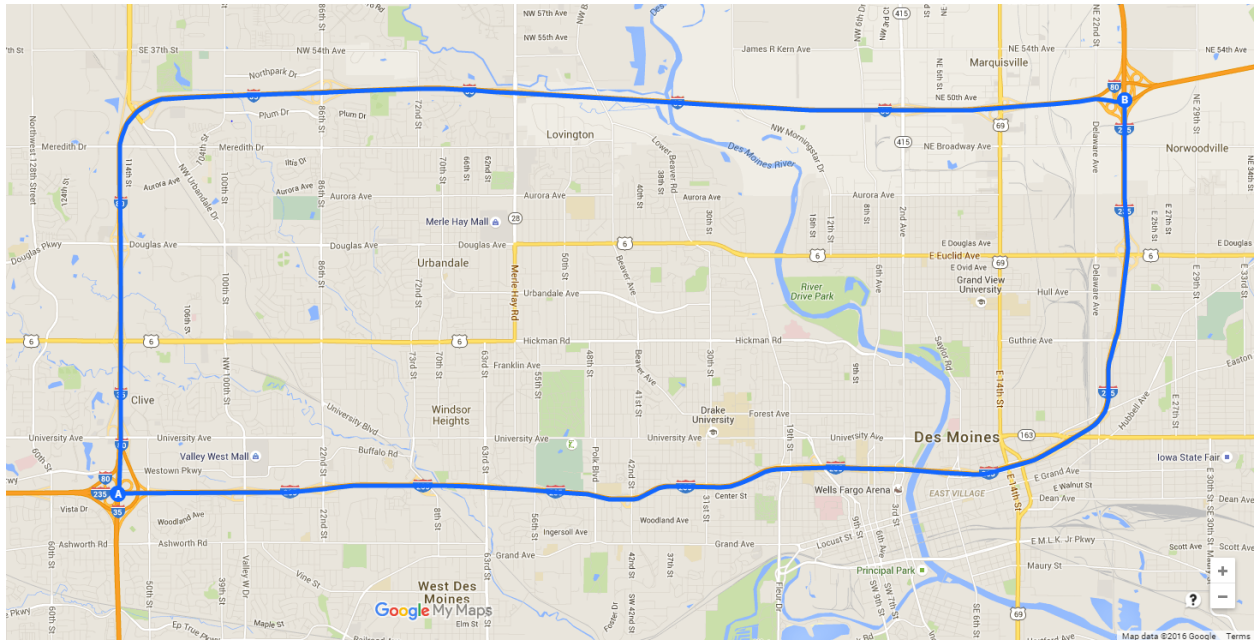
At the corridor level, all MTT reliability indices are within a 10% error range, in contrast to the indices calculated based on INRIX travel times. At the segment level, although the means and standard deviations of the MTT estimates are close to those of the INRIX travel times, the 95th percentile travel time, planning time index, buffer time, and buffer time index show fairly significant discrepancies, up to 70%. Note that the MTT estimation method tends to overestimate reliability indices in most cases.

The errors in the proposed travel-time estimation model could be attributed to several factors. First, the first-in, first-out assumption does not take lane change behavior into consideration in

the calculation. As a result, the model may underestimate or overestimate the number of vehicles approaching the bottleneck. Second, missing values from the radar sensor data might also cause errors in estimating travel times, which in turn may cause errors in computing travel-time reliability indices.

Corridor-Level Travel-Time Reliability Prediction

The study freeway network is located in Des Moines, Iowa. Figure 11 shows a map of the study area. The network contains two major freeways, I-80/35 and I-235. The distances of these two corridors are 16 and 15 miles, respectively.



Map data ©2015 Google

Figure 11. Study area

Based on hierarchical clustering, the weather and road surface conditions were classified into three conditions:

- Rain: rainy, wet, and trace moisture conditions
- Snow: snowy, chemically wet, ice watch, and ice warning conditions
- Good Weather: conditions other than rain and snow

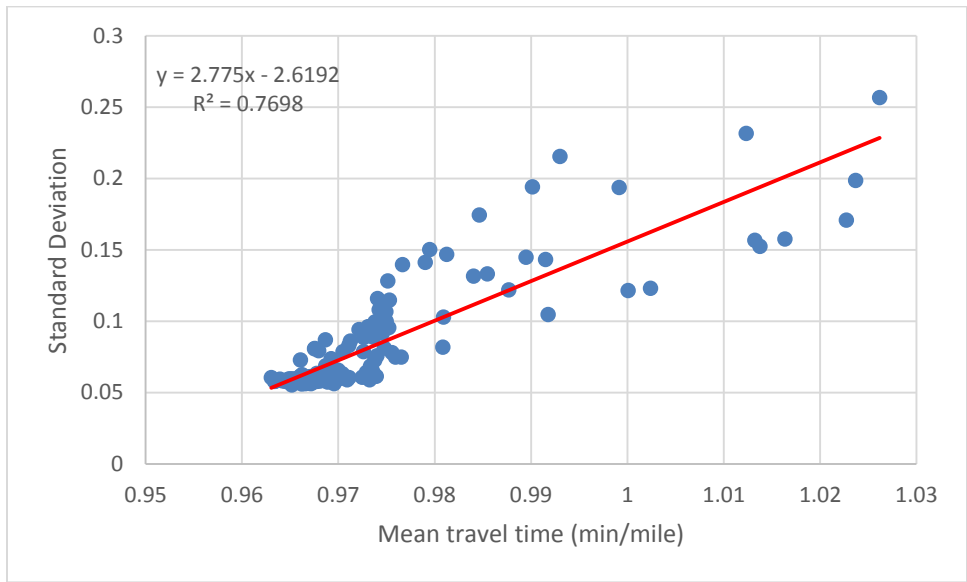
Validating Model Assumptions

The actual probe vehicle data described above were used to validate three major assumptions made to derive the models: (1) the relationship between the mean and standard deviation of the travel time per distance is linear, (2) the mean and standard deviation of the free-flow state of the

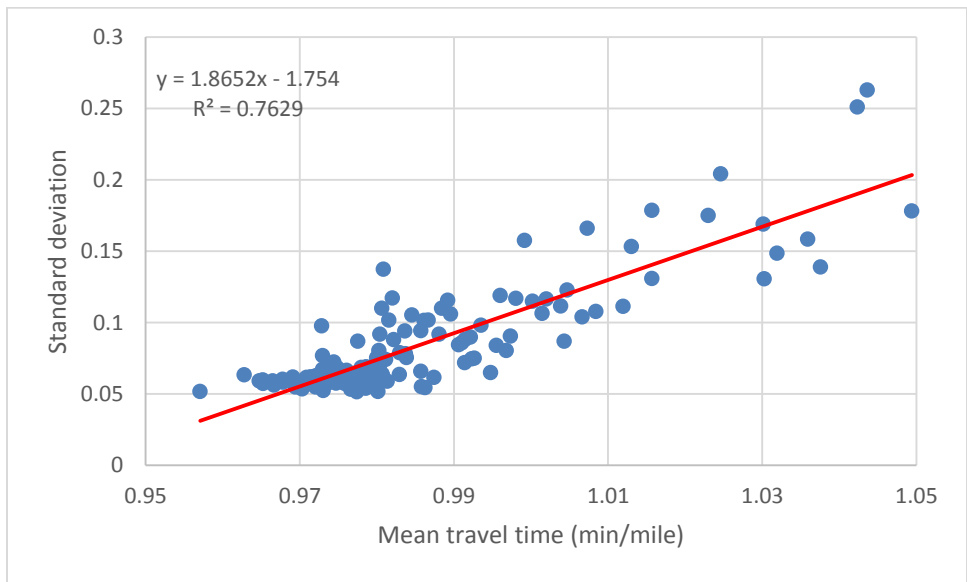
two-component normal distribution shows little fluctuation throughout the day, and (3) mean travel times vary with time during the peak hour.

First Assumption: Relationship between Mean and Standard Deviation of Travel Time per Distance Is Linear

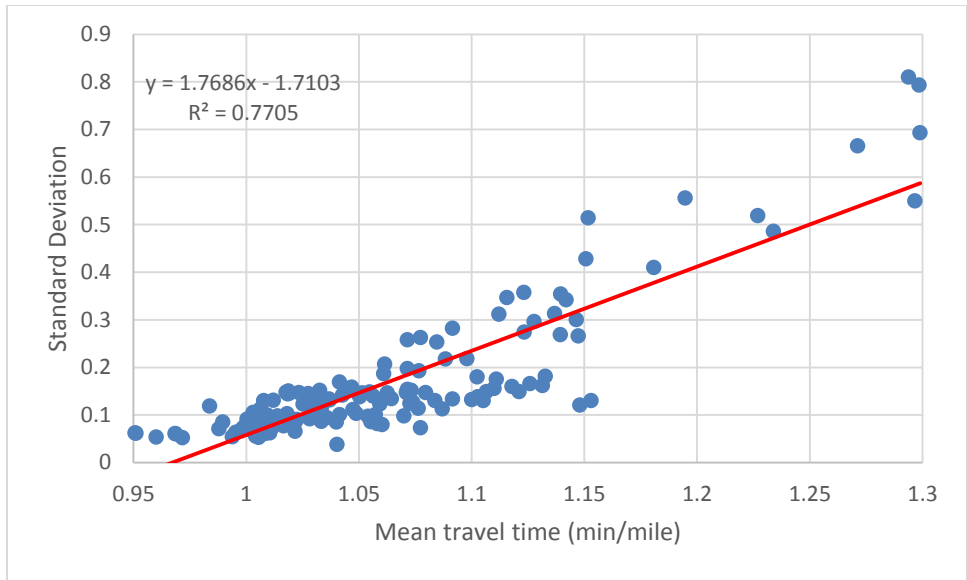
Figure 12 shows the linear regression results, including the formula and the coefficient of determination (R^2). For the regression results, R^2 values usually serve as the diagnostic measure. In this study, the general definition of R^2 was utilized to evaluate the goodness of fit. Figure 12 shows that the linear proportional model has a relatively high goodness of fit in all cases.



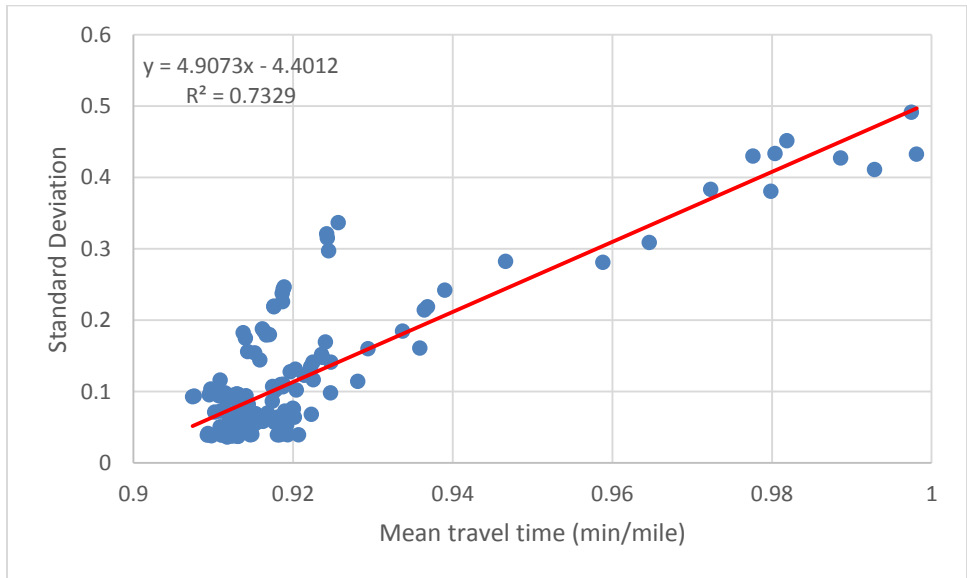
Good weather (I-235)



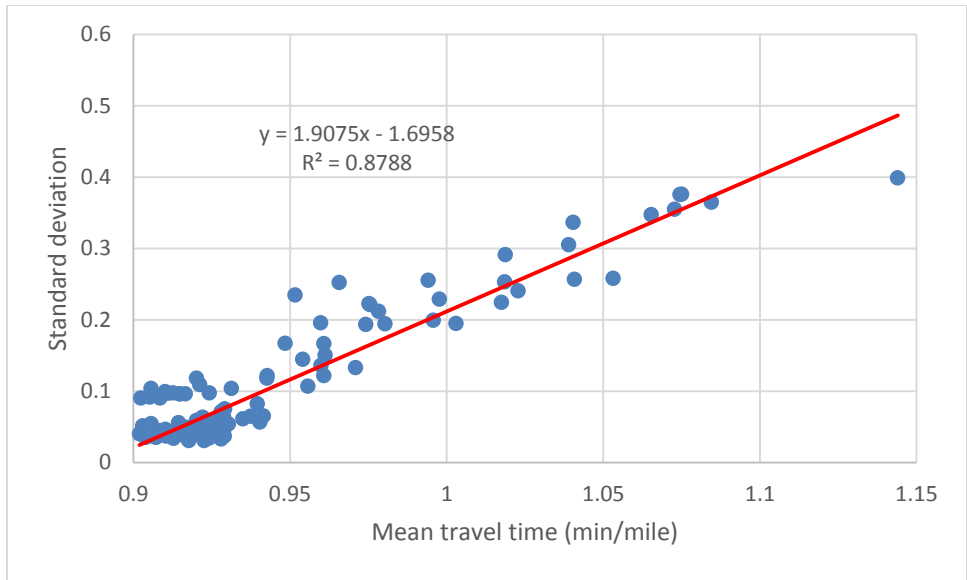
Rain (I-235)



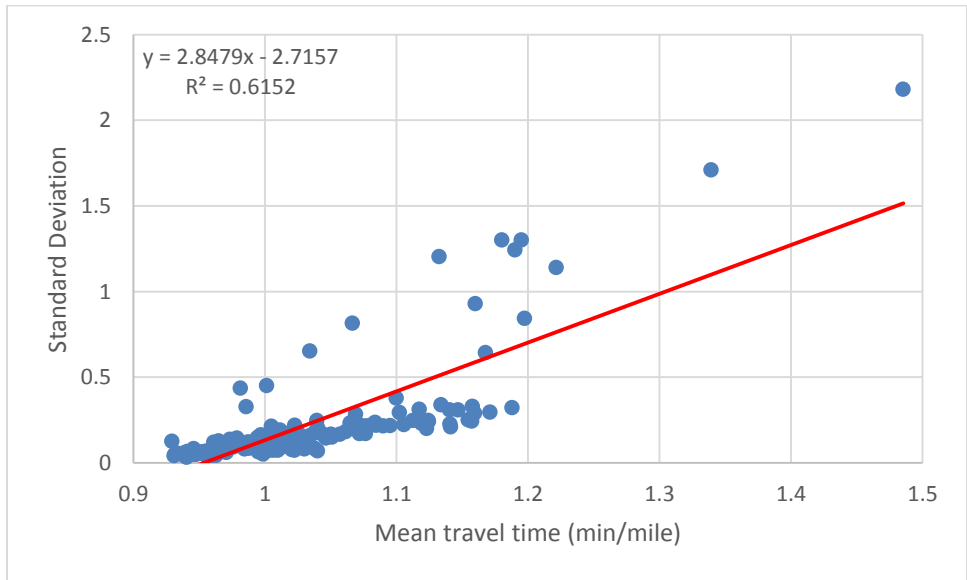
Snow (I-235)



Good weather (I-35/80)



Rain (I-35/80)

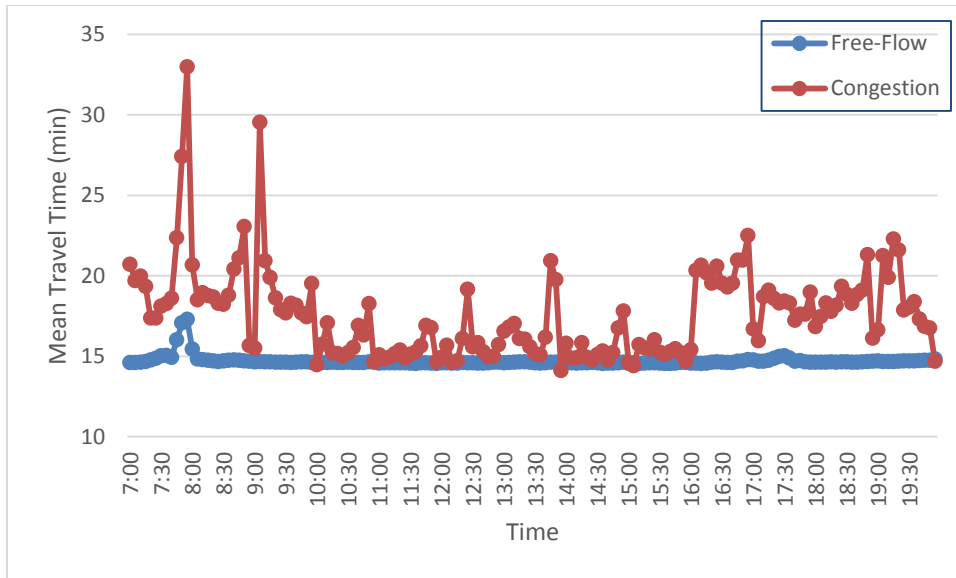


Snow (I-35/80)

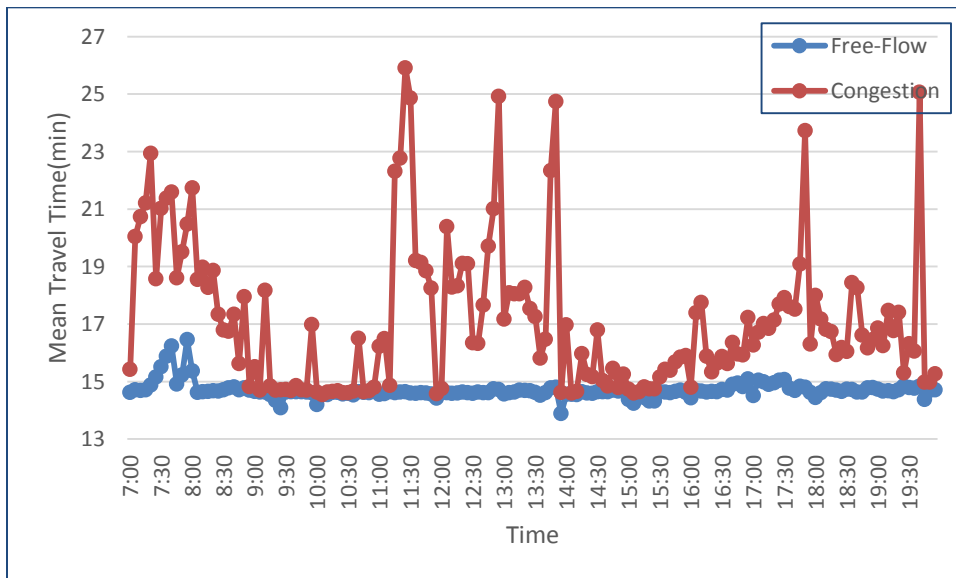
Figure 12. Relationship between mean and standard deviation of travel time per mile under different weather conditions for Des Moines, Iowa freeway data

Second Assumption: Mean and Standard Deviation of Free-flow State Shows Little Fluctuation throughout the Day

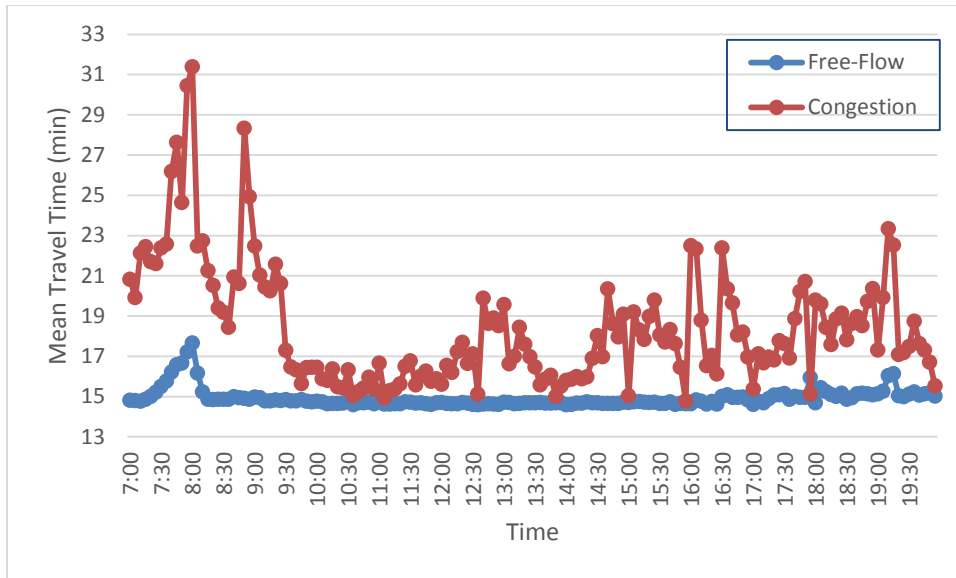
In order to validate the assumption about the component means and standard deviations of the two-component mixture normal model, the two-component mixture normal model was used to fit the travel-time distribution under different weather conditions and time intervals. The results are shown in Figure 13 and Figure 14.



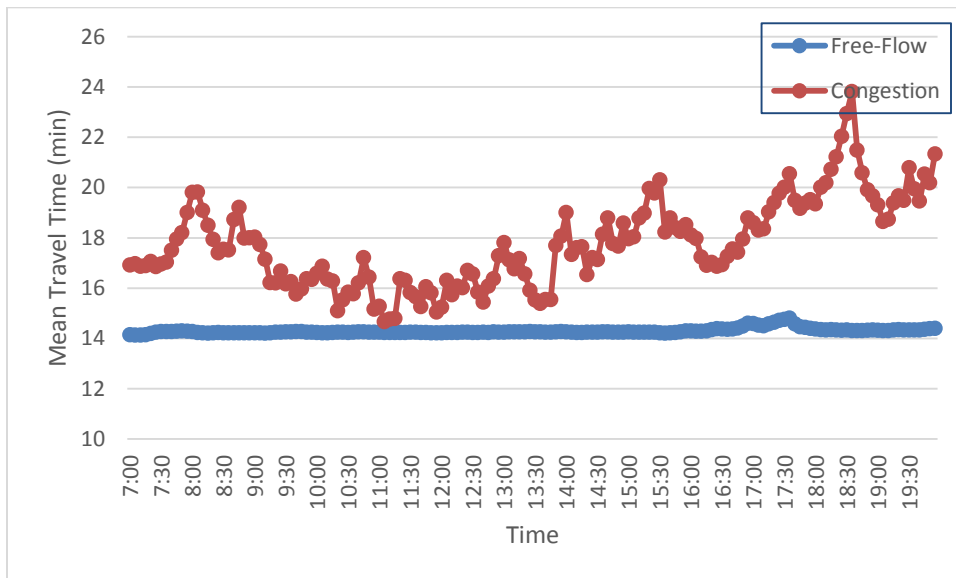
Good weather (I-235)



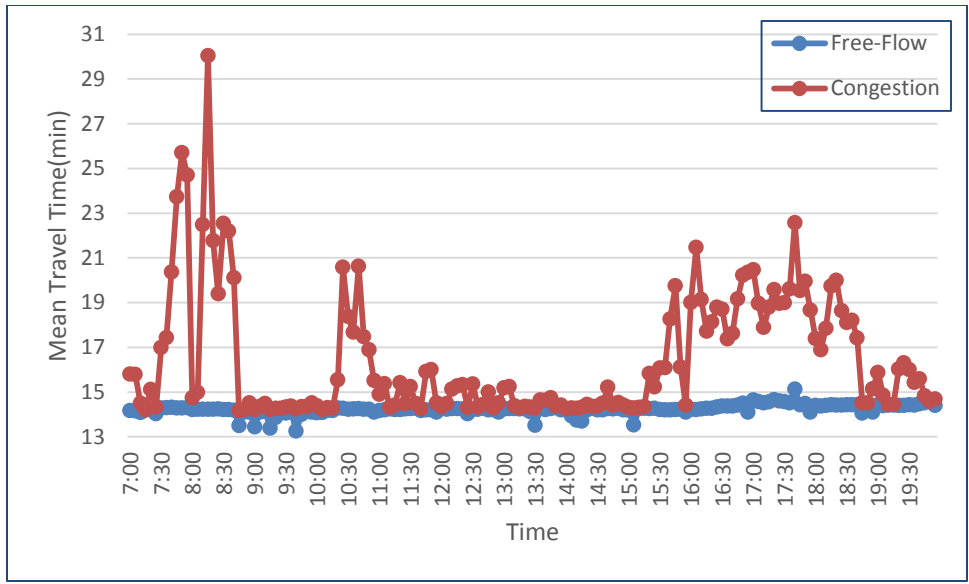
Rain (I-235)



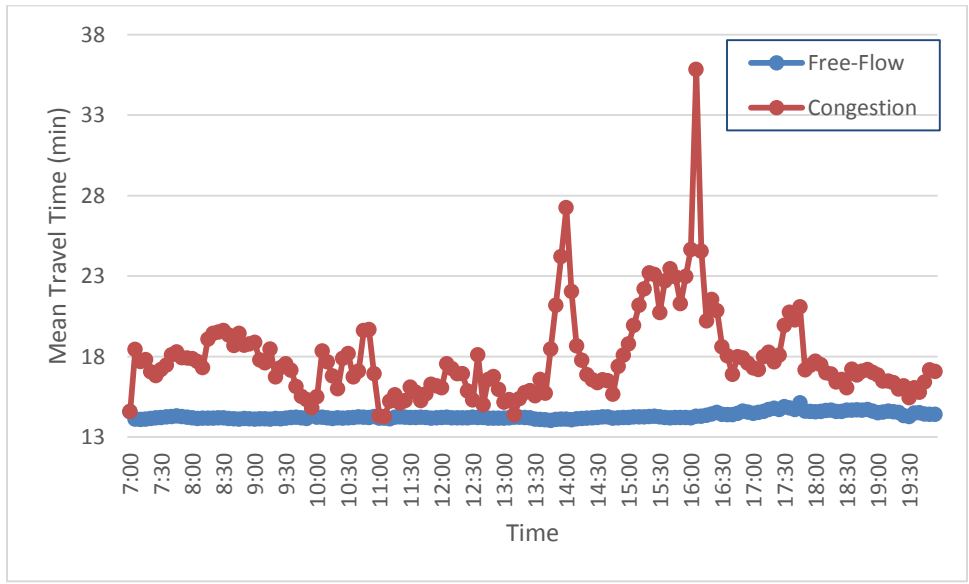
Snow (I-235)



Good weather (I-35/80)

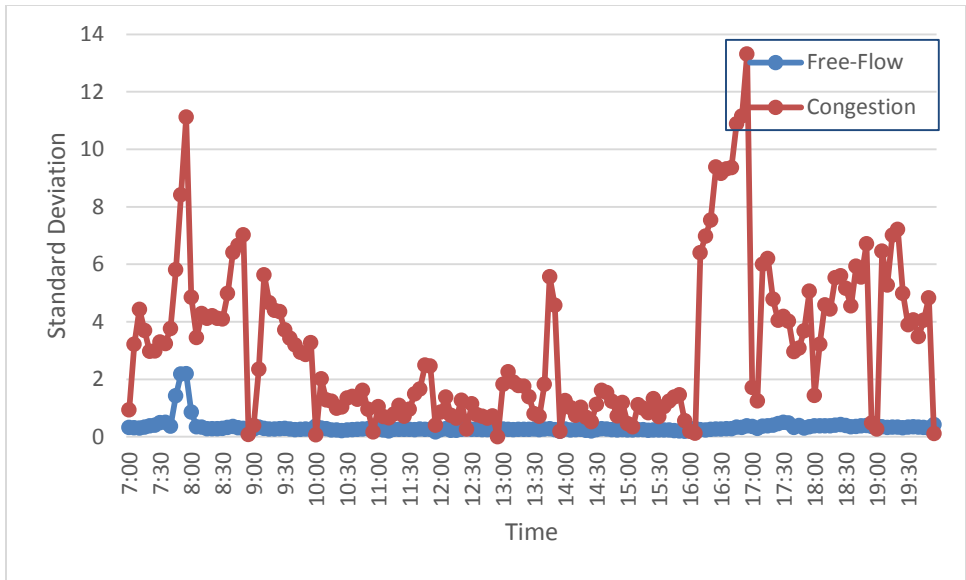


Rain (I-35/80)

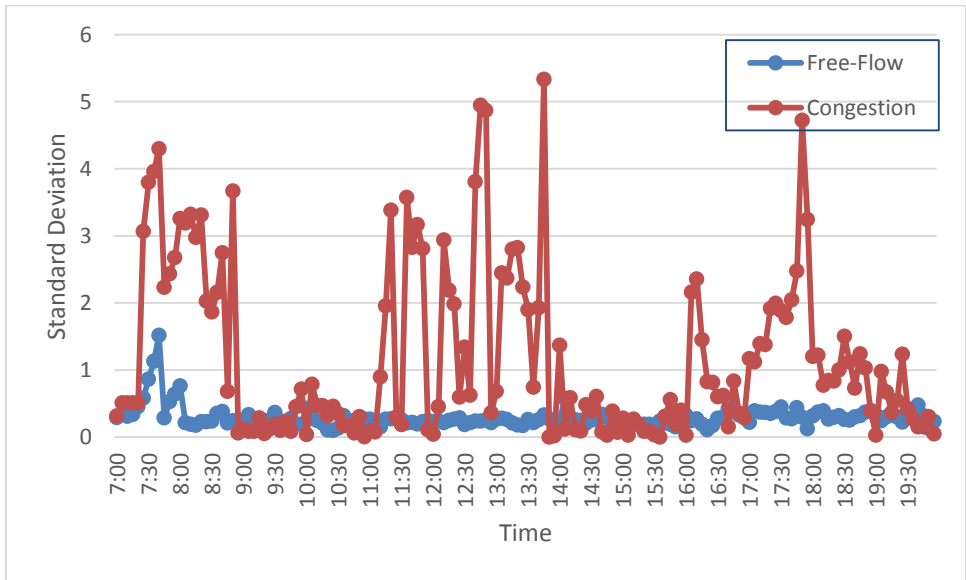


Snow (I-35/80)

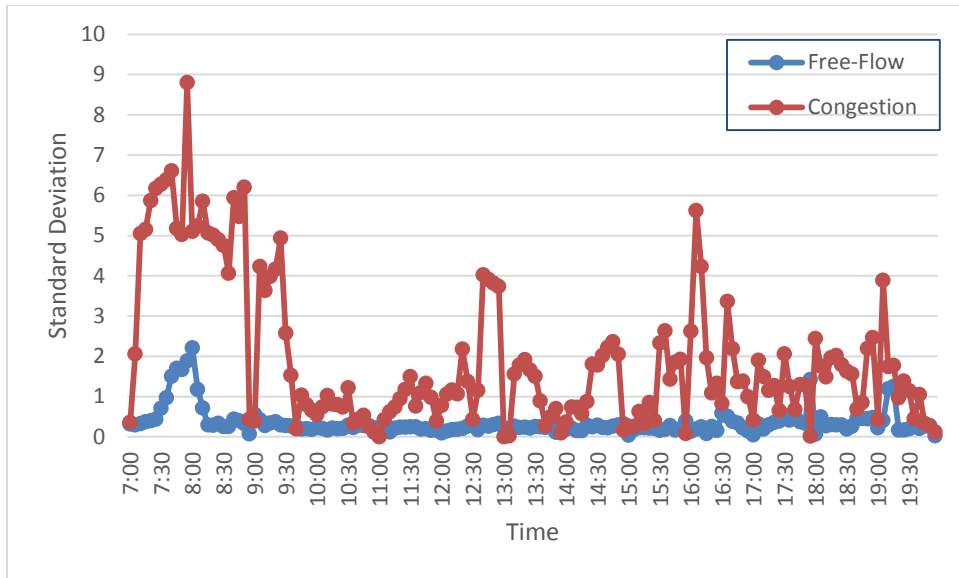
Figure 13. Mean travel times of two-component multistate model under different weather conditions



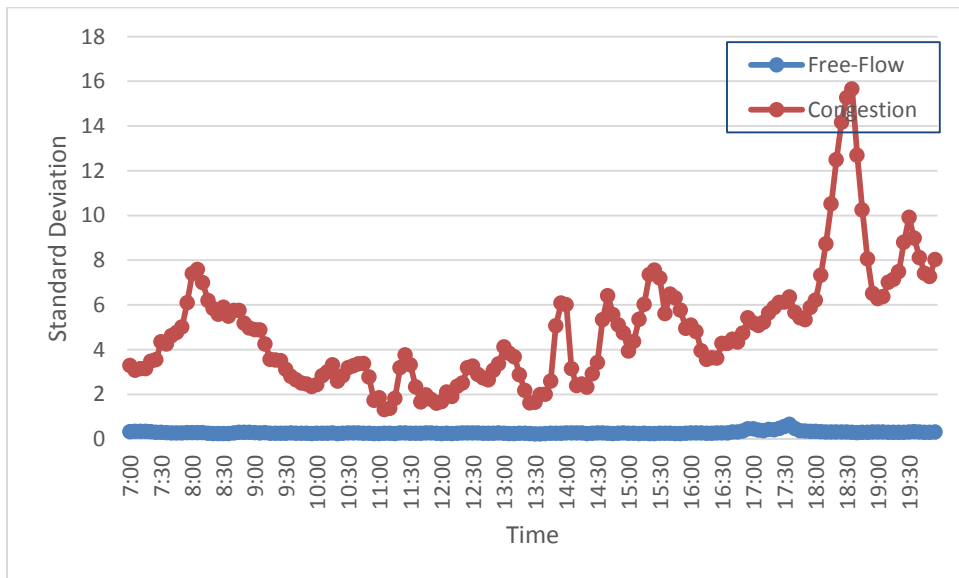
Good weather (I-235)



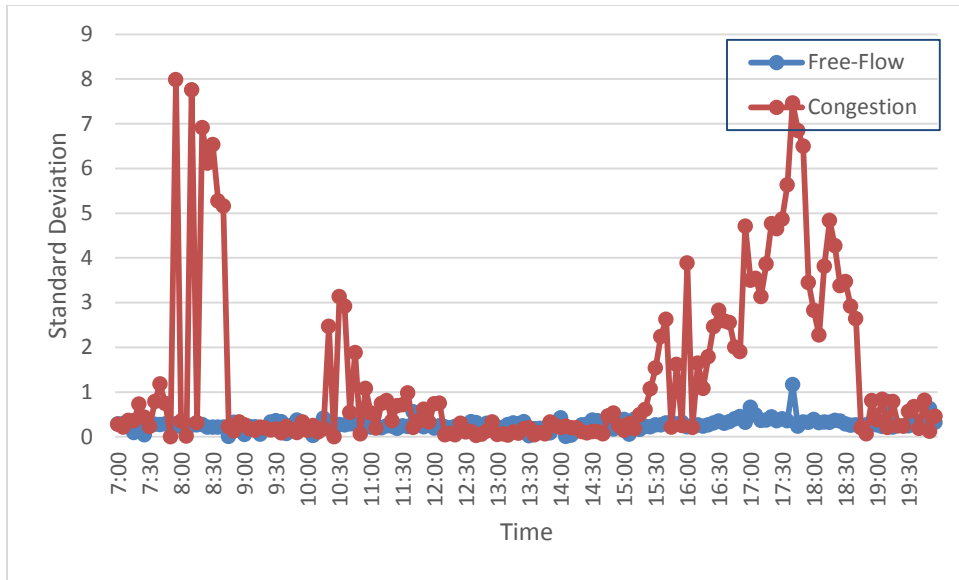
Rain (I-235)



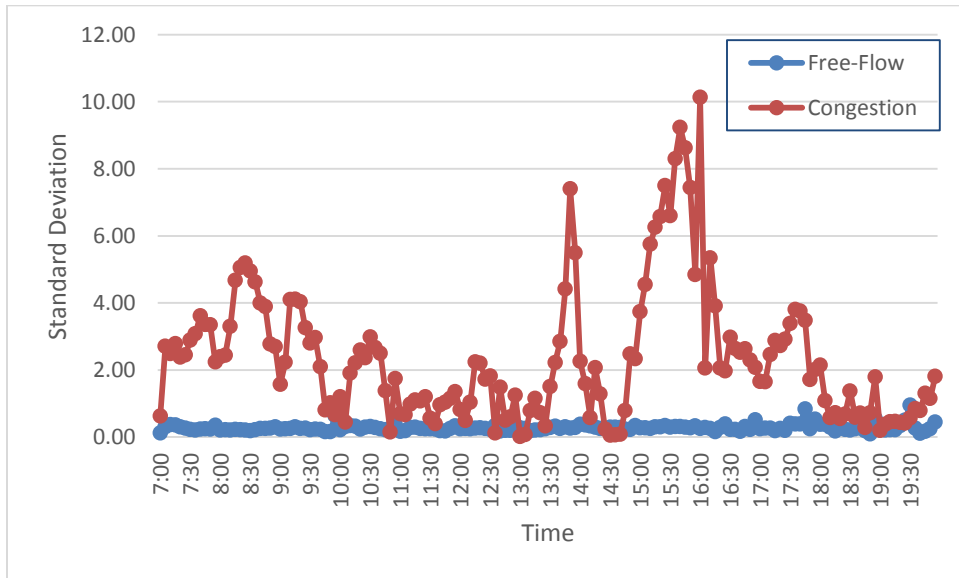
Snow (I-235)



Good weather (I-35/80)



Rain (I-35/80)



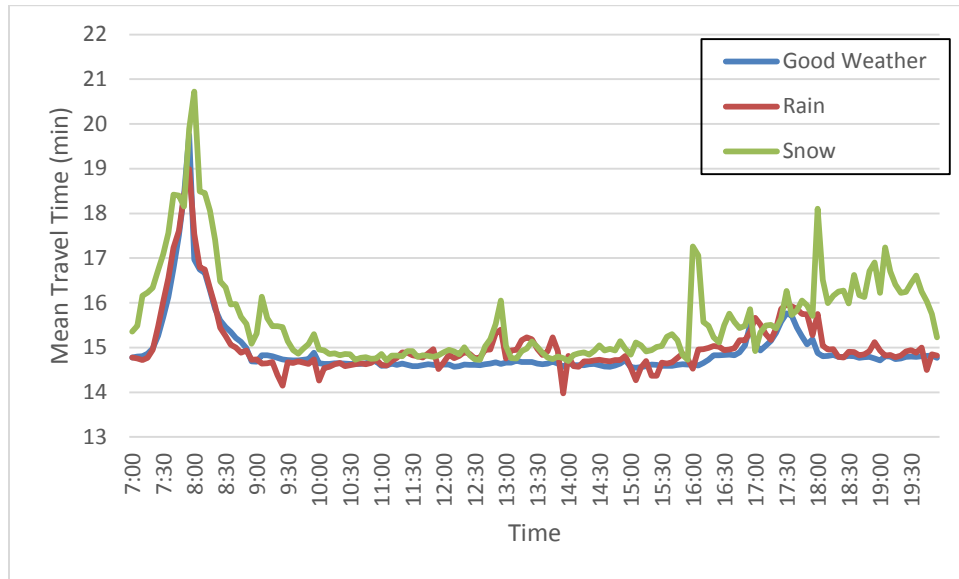
Snow (I-35/80)

Figure 14. Standard deviation of two-component multistate model under different weather conditions

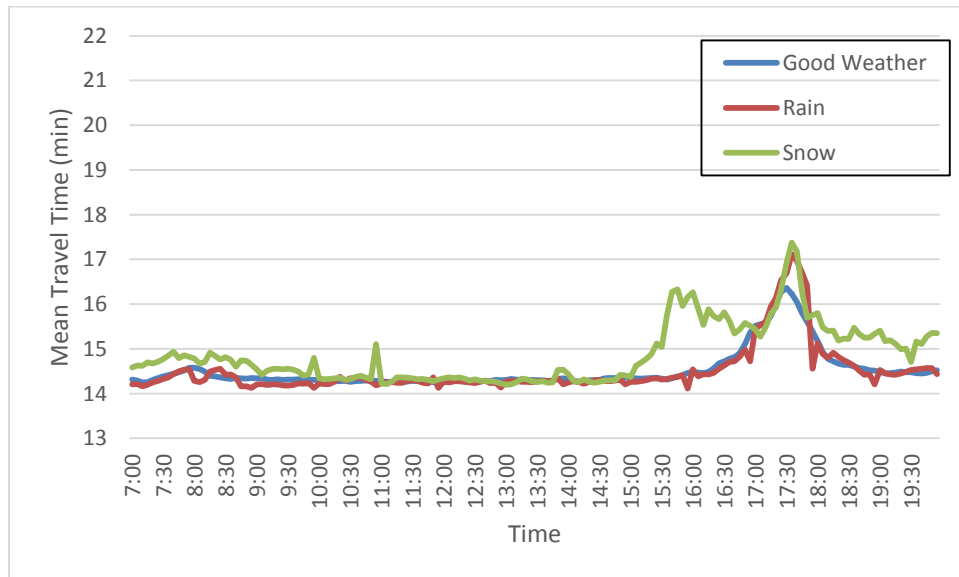
Figure 13 and Figure 14 show the means and standard deviations, respectively, of travel times in a free-flow state and a congested state. The figures show that the means and standard deviations of travel times in a free-flow state have little fluctuation throughout the day and that the means and standard deviations of travel times in a congested state vary with time. Roughly, the means and standard deviations of travel times in a congested state are higher during the morning peak and afternoon peak.

Mean Travel Times during Peak Hour

Figure 15 shows that the mean travel times vary with time. The mean travel times are higher during the morning peak on I-235 and the afternoon peak on I-35/80. Consequently, the ARIMA model was used to fit and predict the travel time during peak hours.



I-235

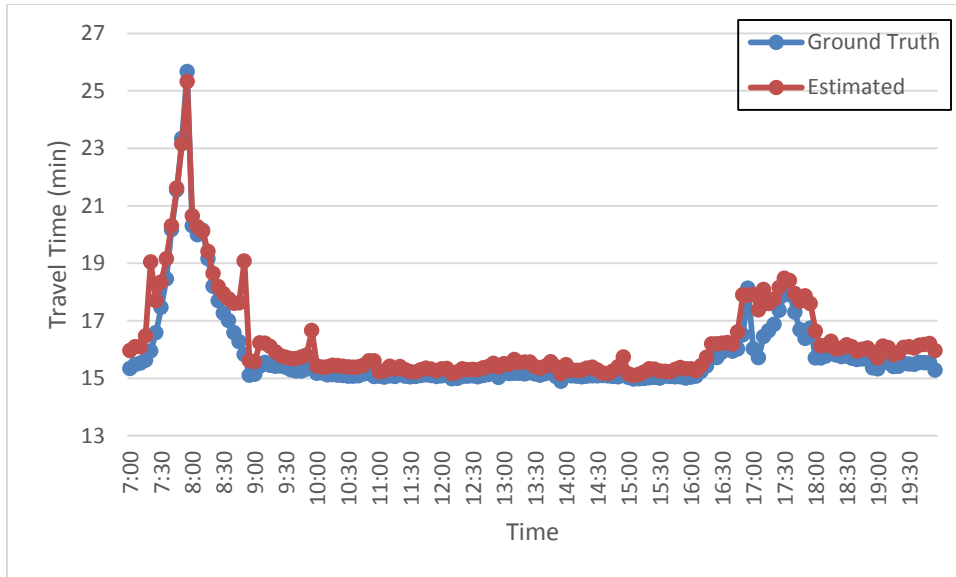


I-35/80

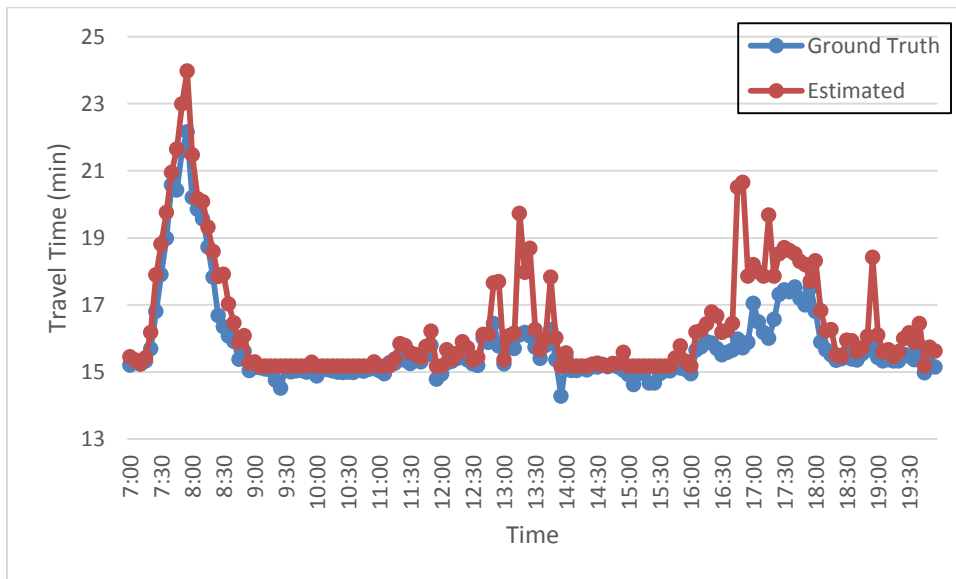
Figure 15. Historical mean travel times from January 2013 through December 2014

Evaluating Performance of Travel-Time Distribution Estimation Method

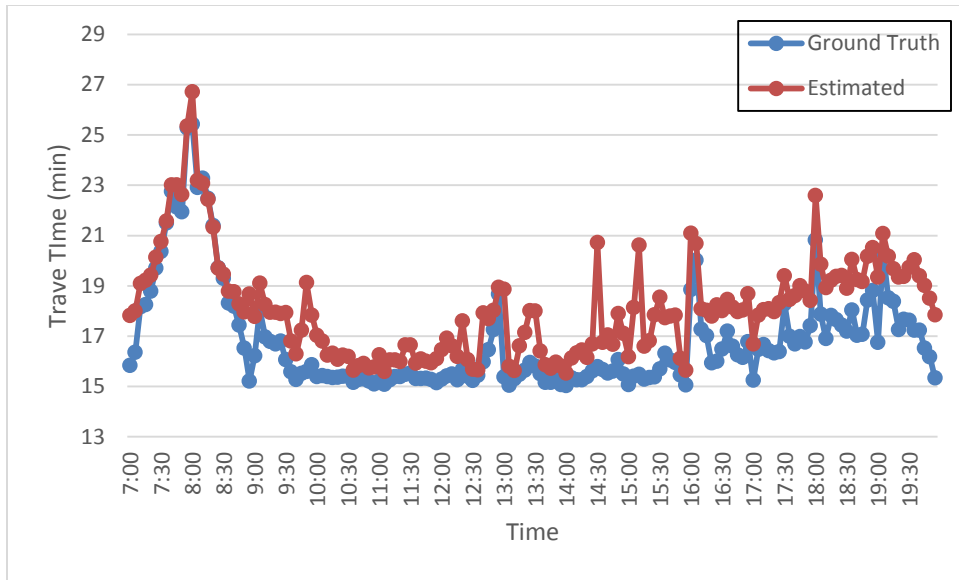
In this study, according to the historical free-flow travel-time distributions, the standard deviation of the free-flow state was estimated to be 0.3. Because the 95th percentile travel time is the key travel-time measure, it was used as the index to evaluate the performance of the travel-time distribution estimation framework.



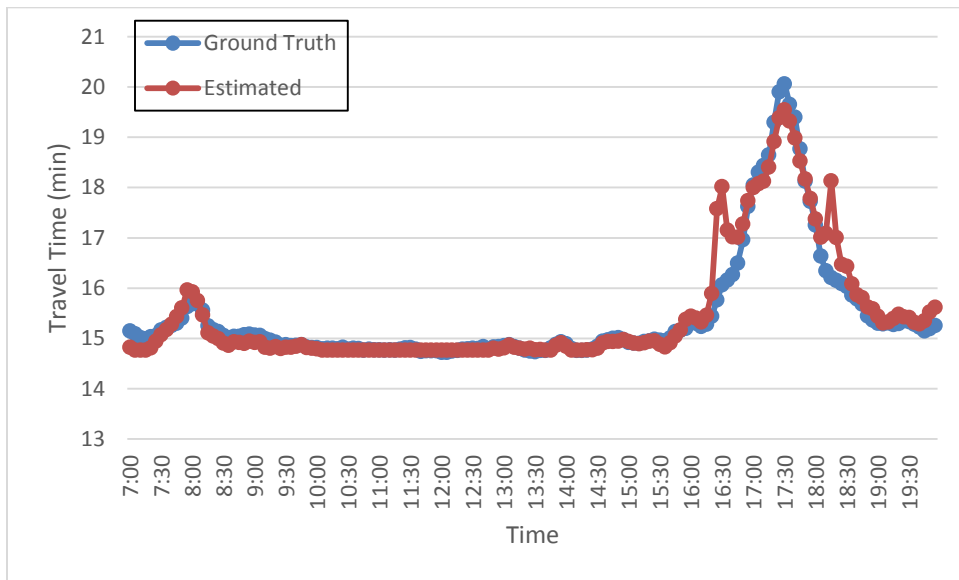
Good weather (I-235)



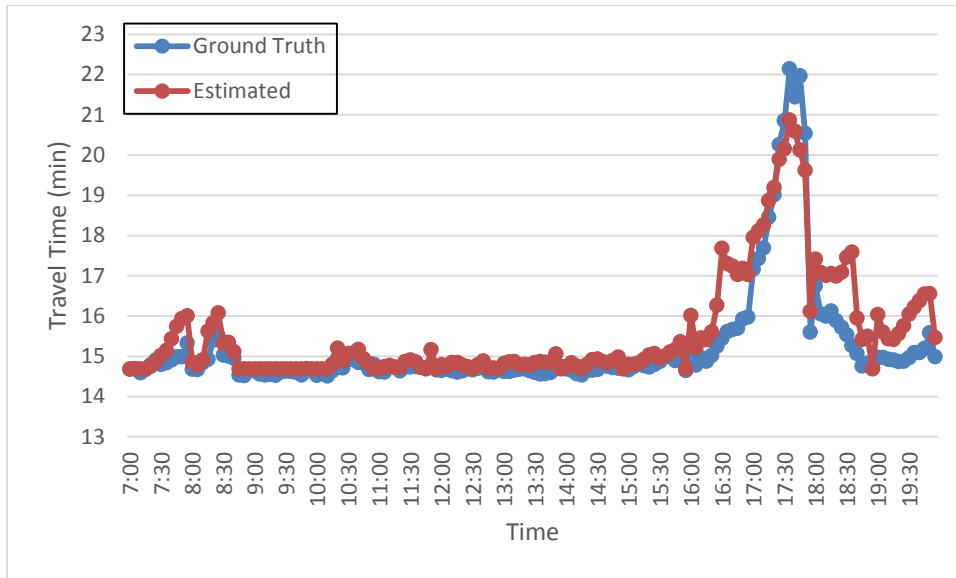
Rain (I-235)



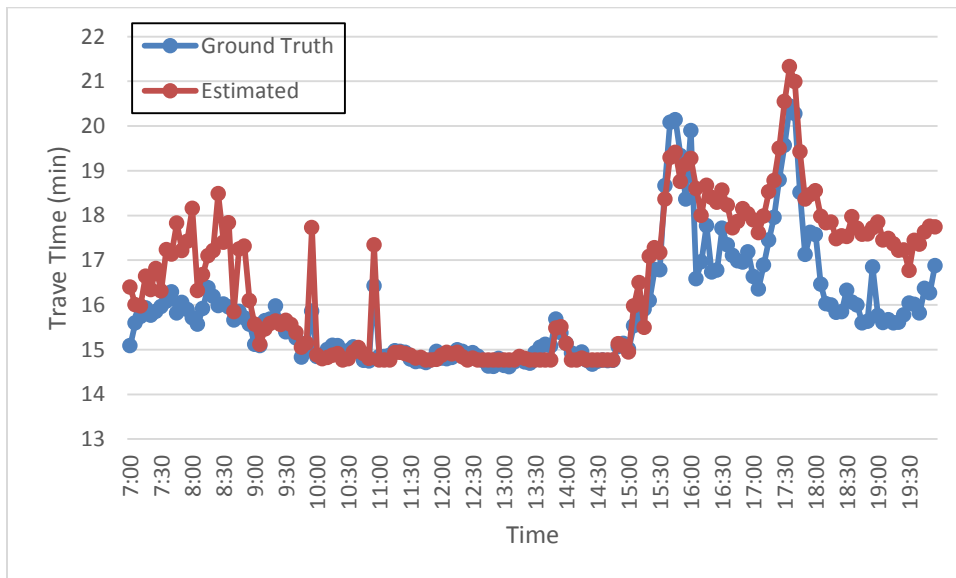
Snow (I-235)



Good weather (I-35/80)



Rain (I-35/80)



Snow (I-35/80)

Figure 16. Ground truth and estimated 95th percentile travel time under different weather conditions

From Figure 16, the estimated 95th percentile travel time well captured the pattern of ground truth 95th percentile travel time under different weather conditions. Under snowy conditions, the model shows slightly lower accuracy.

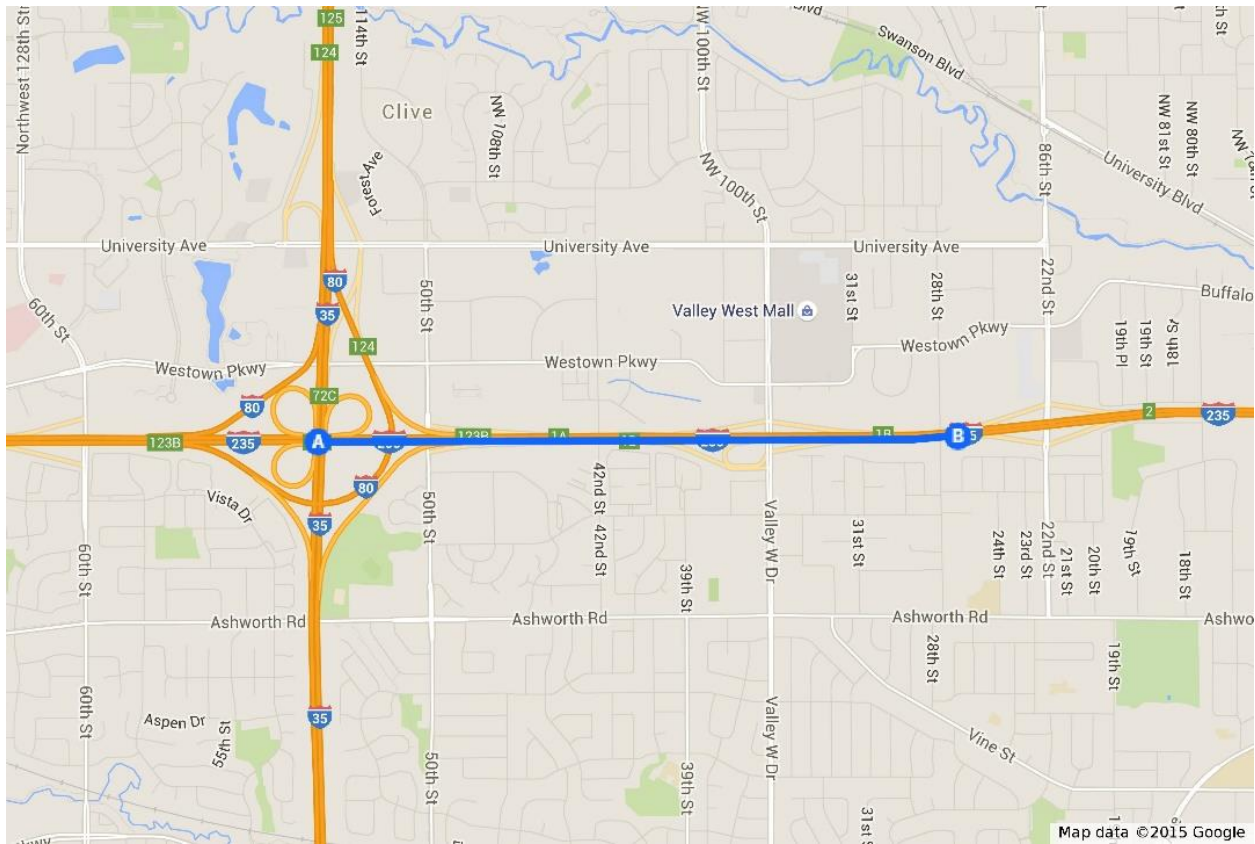
Evaluating the Performance of Synthesizing Methods

Because a two-component normal model is used in this study, the components of the corridor-level travel-time distribution would increase exponentially with an increase in the number of

segments. Consequently, the naïve method and extended Winkler’s method cannot be used in a large-size corridor that contains a large number of segments. The numbers of segments on I-235 and I-35/80 in the study area are 39 and 14, respectively. In this section of the report, the respective peak hours on I-235 and I-35/80 are used to evaluate the performance of synthesizing methods.

Evaluating the Performance of Synthesizing Methods on Small-size Corridors

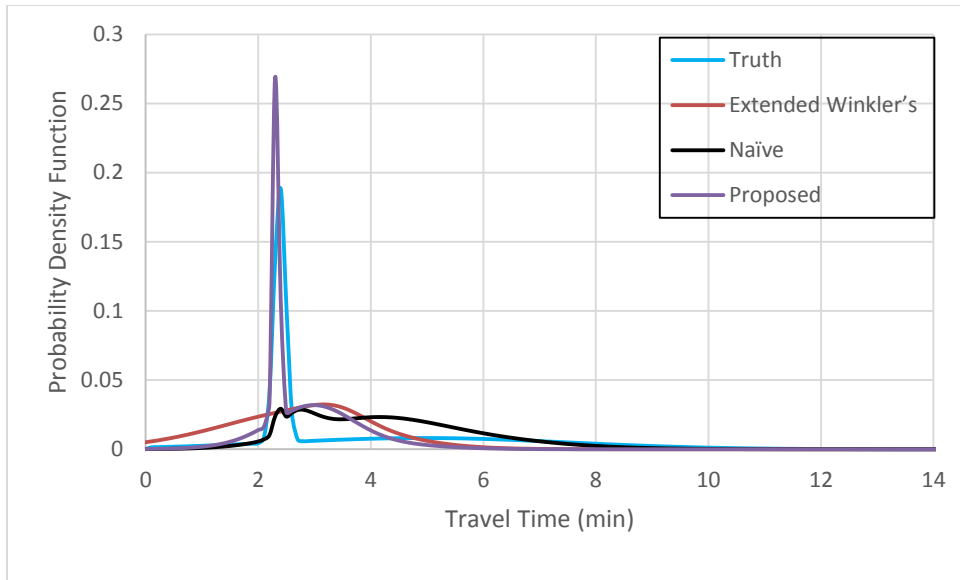
Part of I-235, shown in Figure 17, was used to evaluate the performance of all synthesizing methods.



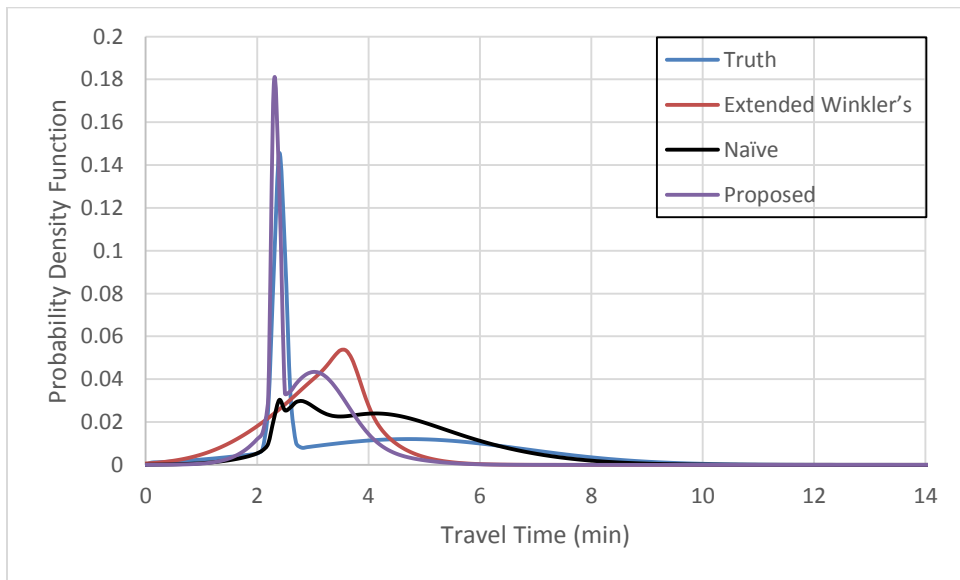
Map data ©2015 Google

Figure 17. Study corridor

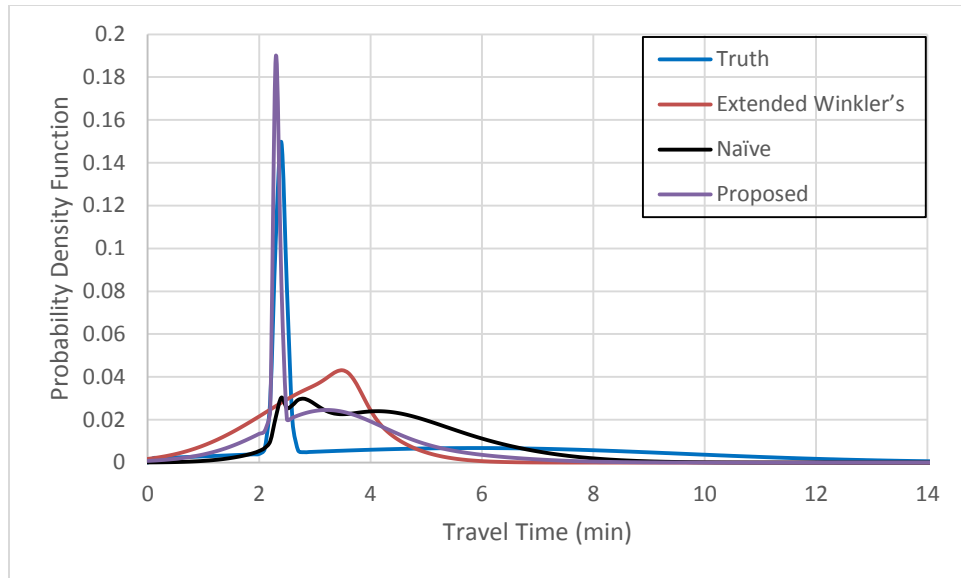
As shown in Figure 18, only the proposed method roughly captured the pattern of PDFs for the ground truth travel time. However, this method underestimated the long tail of the travel-time distribution.



Good weather



Rain



Snow

Figure 18. Performance of synthesizing methods under different weather conditions

The error in the proposed method may be also caused by a time-varying correlation between segments. Chandra and Al-Deek (2009) proposed that the correlations between segments vary with time. Consequently, it can be assumed that β_1 and β_2 are time-varying parameters.

Figure 19 provides an example to illustrate that β_1 and β_2 in equation (5) are time-varying parameters.

If we could identify a functional form or empirical trend with time series methods, such as the ARIMA model or the generalized autoregressive conditionally heteroscedastic (GRACH) model, then we would be able to apply the proposed model to capture time-dependent correlated traffic conditions in the statistical representation of travel-time distribution. However, such efforts would be part of a future study.

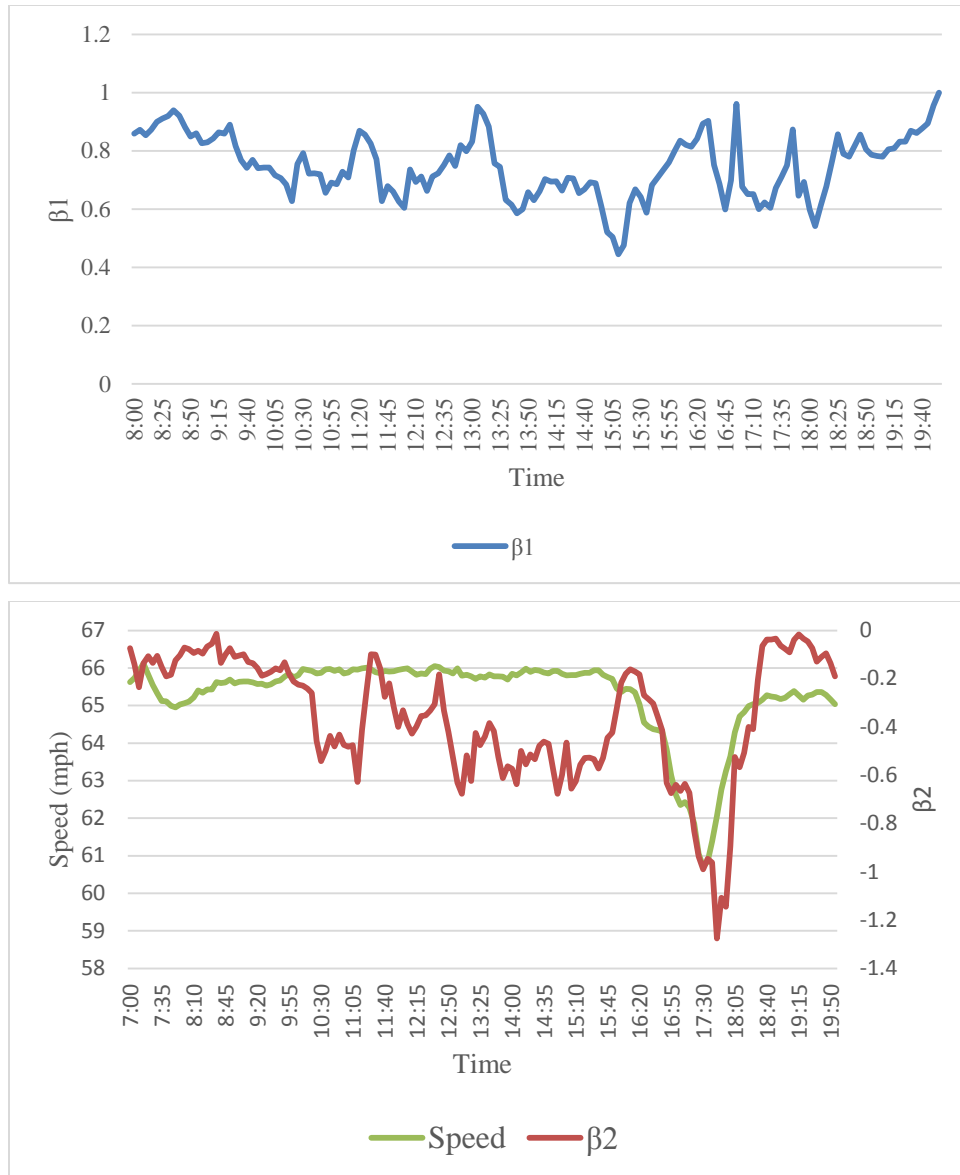


Figure 19. Time-varying parameters of I-35/80 (7:00 a.m. to 8:00 p.m.)

Evaluating the Performance of the Proposed Method on a Large-size Corridor

Because the proposed method would result a distribution similar to a two-component normal distribution, the framework proposed to solve the problem of large-size corridors has the form shown in Figure 20.

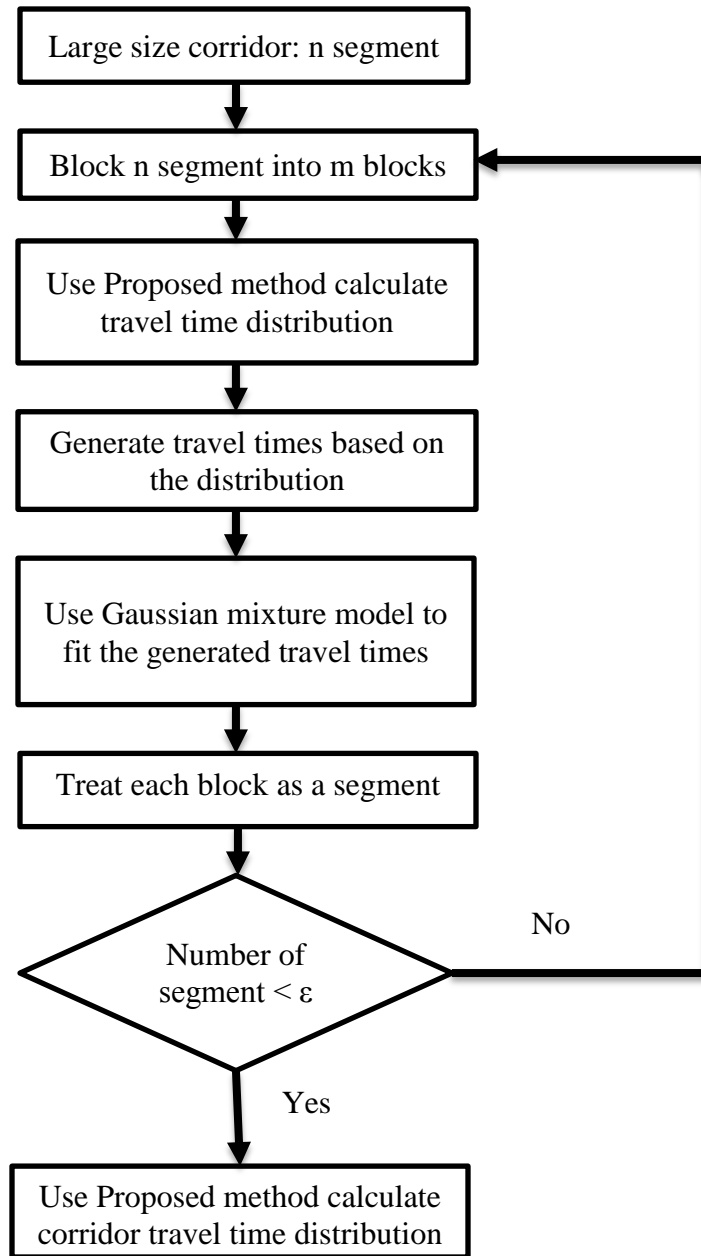
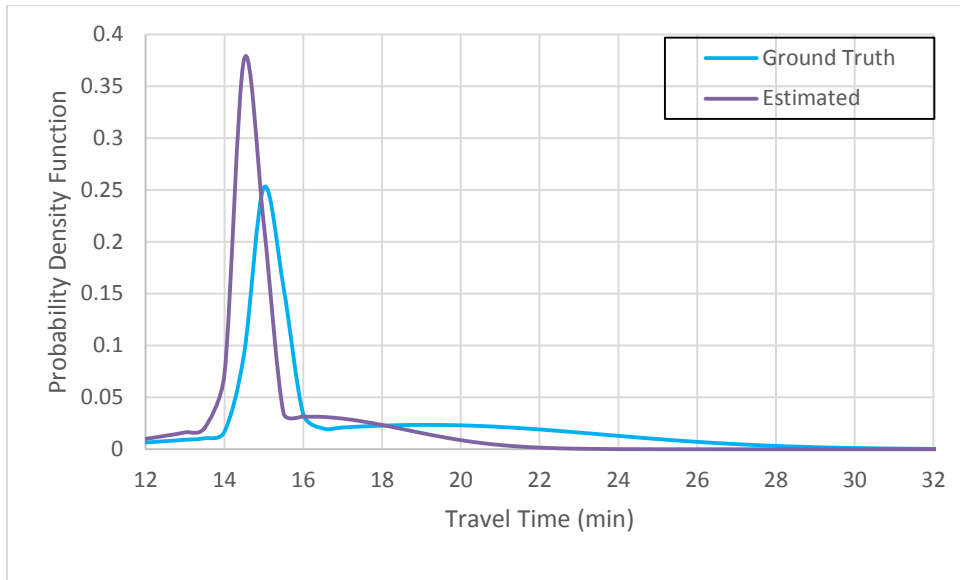
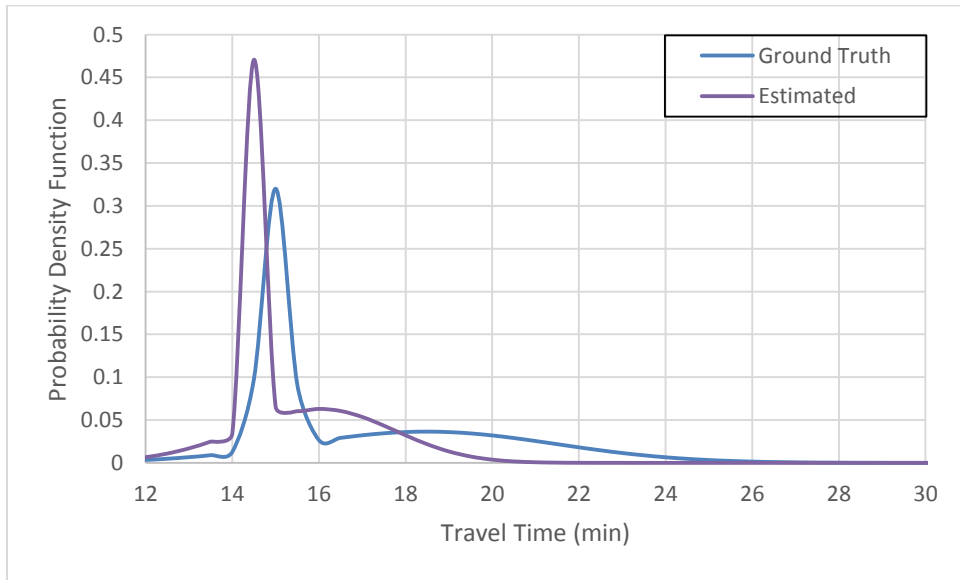


Figure 20. Corridor decomposition method

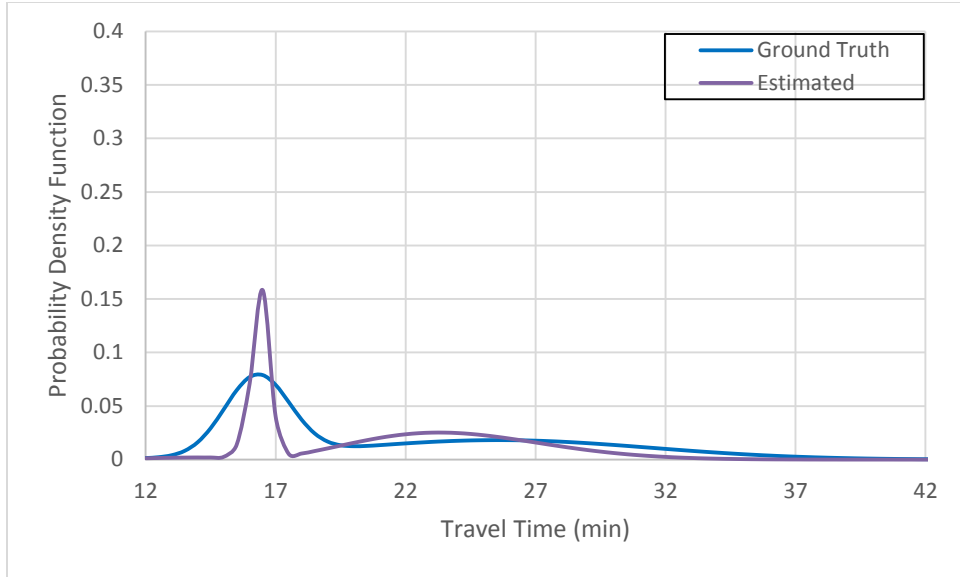
The corridor-level travel-time distribution was calculated based on this method. The results are shown in Figure 21.



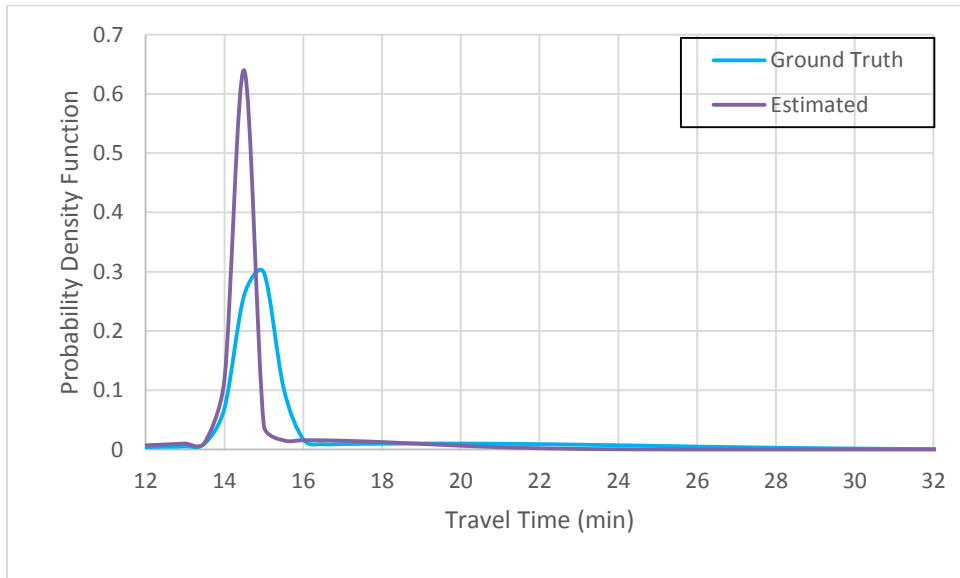
Good weather (I-235)



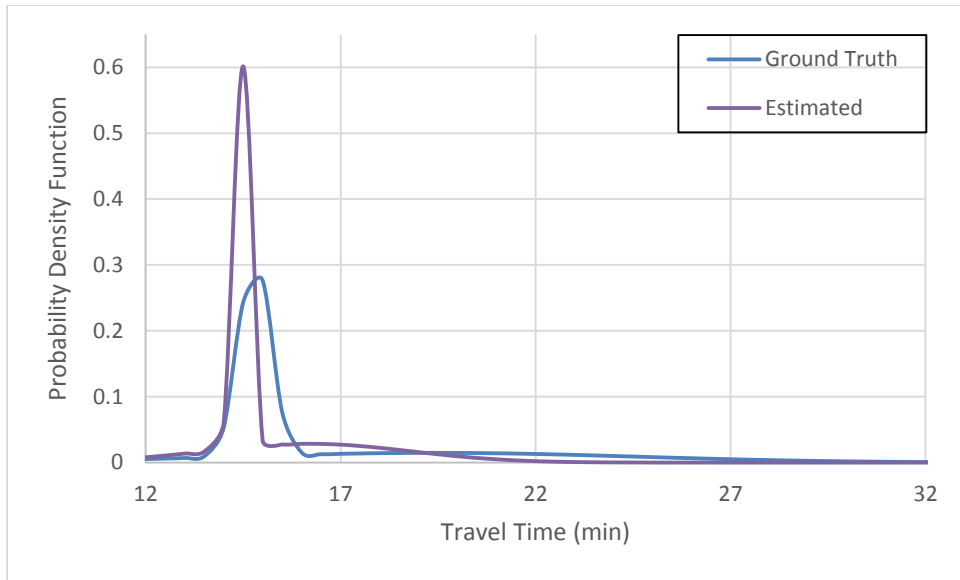
Rain (I-235)



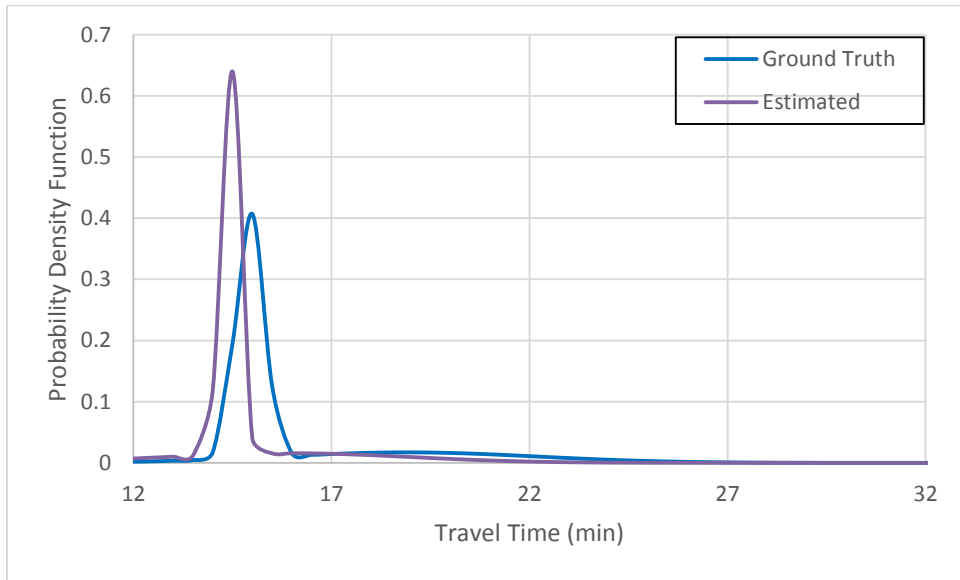
Snow (I-235)



Good weather (I-35/80)



Rain (I-35/80)



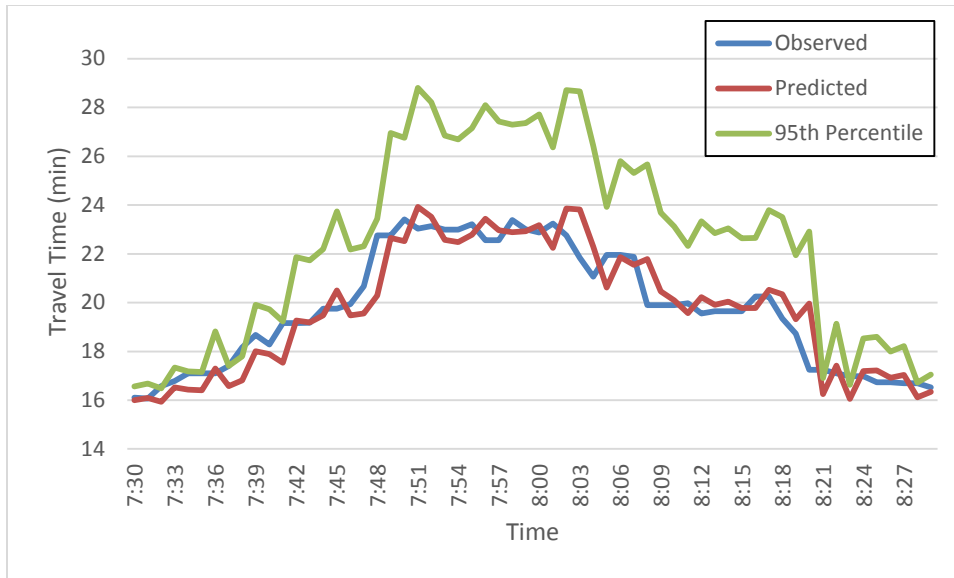
Snow (I-35/80)

Figure 21. Performance of synthesizing methods under different weather conditions

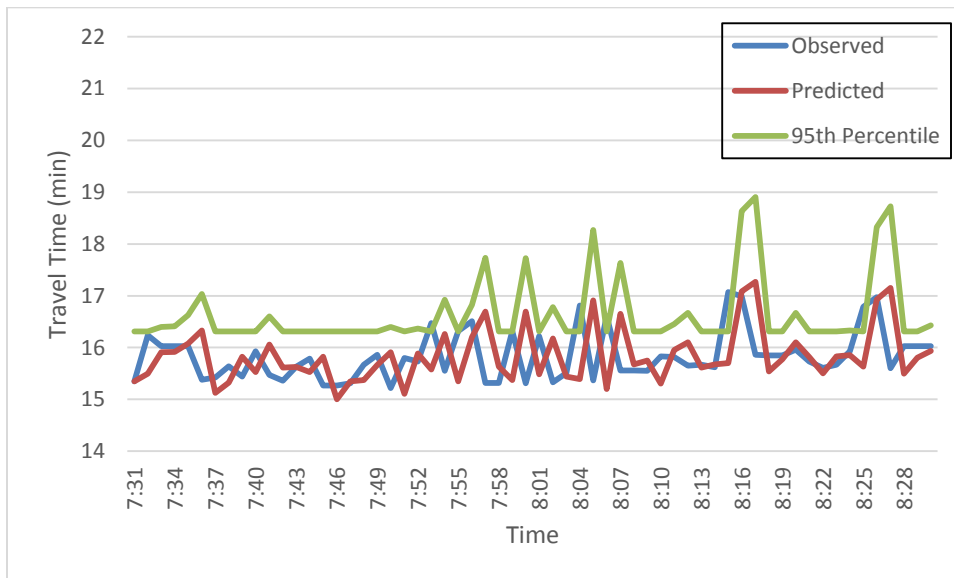
As shown in Figure 21, the proposed synthesizing method is able to capture the pattern of corridor-level travel-time distribution. Moreover, the proposed method captures the long tail of the travel-time distribution.

Evaluating Performance of Framework

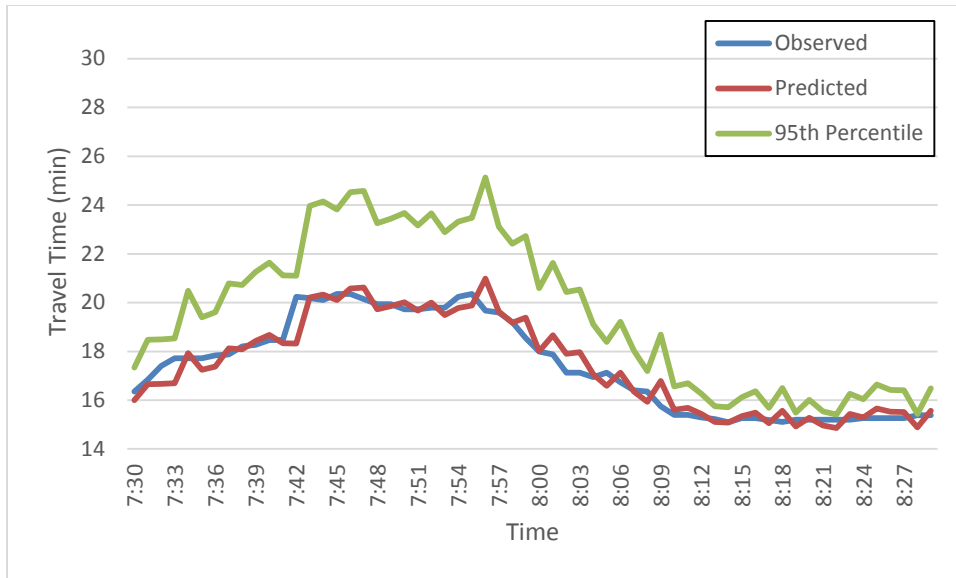
As shown in Figure 22, the predicted travel time follows the observed travel time with a slight hysteresis effect.



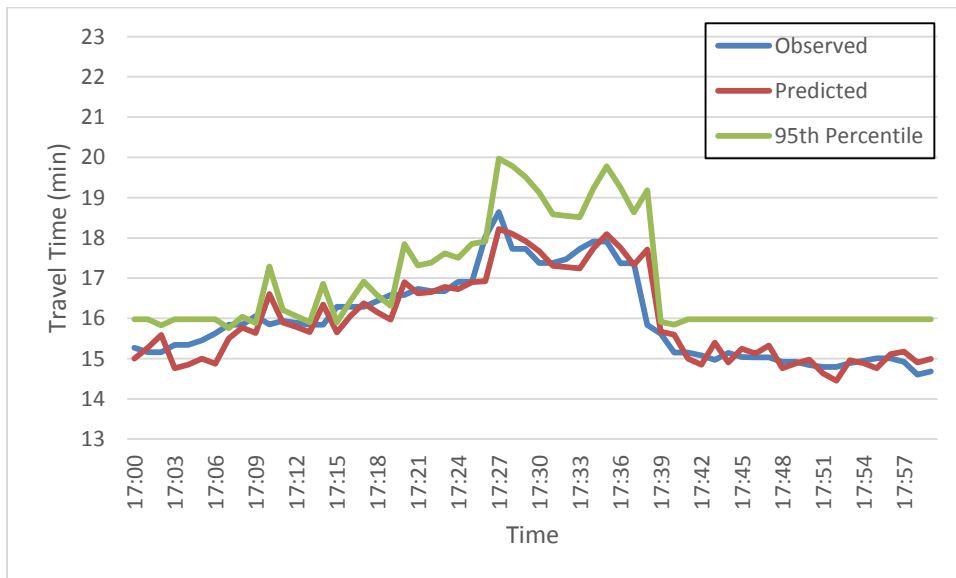
Good weather (I-235)



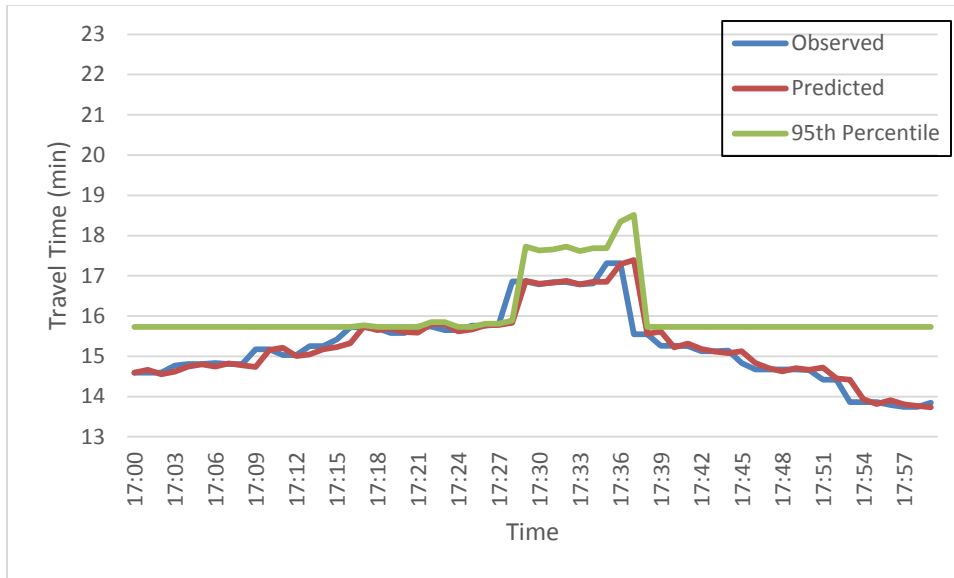
Rain (I-235)



Snow (I-235)



Good weather (I-35/80)



Rain (I-35/80)

Figure 22. Observed travel time, predicted travel time, and predicted 95th percentile travel time in different weather conditions

The observed time duration did not yield sufficient data points to provide a stable distribution. As a result, the accuracy of the predicted 95th percentile travel time could not be proved. However, based on the performance of the travel-time distribution estimation framework, we believe the result is accurate. Moreover, with a more accurate short-term travel-time prediction model, the accuracy of predictions of travel-time distribution would improve.

Travel-Time Distribution Based on Vissim

Congested and uncongested scenarios for eastbound traffic on I-235 from 42nd Street to I-80 in Des Moines, Iowa were simulated in Vissim. The target corridor, shown in Figure 23, was 13.05 miles long.

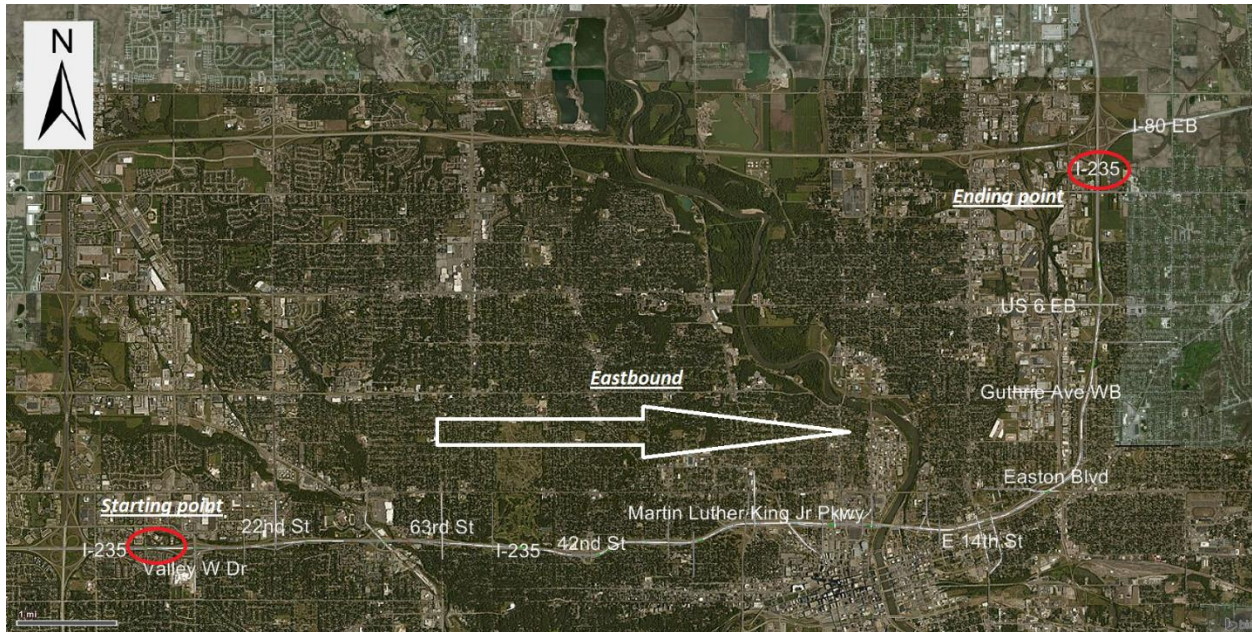


Figure 23. Test segment on I-235, Des Moines, IA

The traffic volumes of each segment/on-ramp/off-ramp for both congested and uncongested scenarios are shown in Table 8.

Table 8. Traffic volumes in congested and uncongested conditions

Order	Segment Description	Type	Volume (vph)	
			Congested	Uncongested
1	I-235 EB to VALLEY WEST-EB	Main	5327	1733
2	I-235 EB to VALLEY WEST-EB-R	Off-ramp	540	108
3	Ramp 1	On-ramp	285	438
4	I-235 EB from Vly West Dr-EB	Main	5072	2063
5	I-235 EB from Vly West Dr-EB-R	On-ramp	330	580
6	Ramp 2	Off-ramp	146	114
8	I-235 WB E of 22nd STREET-EB	Main	5256	2529
7	I-235 WB E of 22nd STREET-EB-R	On-ramp	874	308
9	Ramp 3	Off-ramp	470	455
10	I-235 EB @ 8th Street Loop-EB	Main	5660	2382
11	I-235 EB @ 8th Street Loop-EB-R	On-ramp	408	307
12	Ramp 4	Off-ramp	176	52
13	I-235 EB EAST OF 63RD-EB	Main	5892	2637
14	I-235 EB EAST OF 63RD-EB-R	On-ramp	644	360
15	Ramp 5	On-ramp	127	127
16	Ramp 6	Off-ramp	982	665
17	I-235 at 42nd STREET EB-EB	Main	5681	2459
18	Ramp 7	On-ramp	12	380

Order	Segment Description		Volume (vph)	
	Station	Type	Congested	Uncongested
19	Ramp 8	On-ramp	39	172
20	I-235 EB 28th STREET-EB	Main	5732	3011
21	Ramp 9	Off-ramp	751	385
22	Ramp 10	On-ramp	35	35
23	I-235 EB to MLK-EB	Main	5016	2661
24	Ramp 11	Off-ramp	278	346
25	Ramp 12	Off-ramp	171	214
26	Ramp 13	Off-ramp	310	387
27	Ramp 14	On-ramp	120	120
28	Ramp 15	On-ramp	15	15
29	I-235 WB WEST END of BRIDGE-EB	Main	4390	1848
30	Ramp 16	On-ramp	247	462
31	I-235 EB at WALKWAY-EB	Main	4637	2310
32	I-235 EB at WALKWAY-EB-R	Off-ramp	375	108
33	I-235 EB 9th STREET WALL-EB	Main	4262	1460
34	I-235 EB 9th STREET WALL-EB-R	Off-ramp	460	42
35	Ramp 17	On-ramp	46	524
36	Ramp 18	Off-ramp	373	273
37	Ramp 19	On-ramp	2	46
38	Ramp 20	Off-ramp	59	59
39	I-235 EB at E 21st St-EB	Main	3418	1656
40	I-235 WB at Washington-EB	Main	3418	1656
41	I-235 WB at Washington-EB-R	On-ramp	374	192
42	Ramp 21	Off-ramp	20	164
43	Ramp 22	On-ramp	57	107
44	Ramp 23	Off-ramp	642	317
45	I-235 NB EUCLID LOOP-EB	Main	3187	1474
46	I-235 NB EUCLID LOOP-EB-R	On-ramp	216	52
47	Ramp 24	On-ramp	201	10
48	I-235 NB from EUCLID-EB	Main	3604	1536
49	I-235 NB from EUCLID-EB-R	On-ramp	425	144

Each scenario was simulated for 99,999 seconds. The travel time of each vehicle traveling from the starting point to the ending point was collected. Considering that the traffic was not stable at the beginning of the simulation period, only the travel-time data after 1,800 seconds were used in the data analysis described below.

As shown in Figure 24, the simulated travel-time distribution is similar to the travel-time distribution based on probe vehicle data.

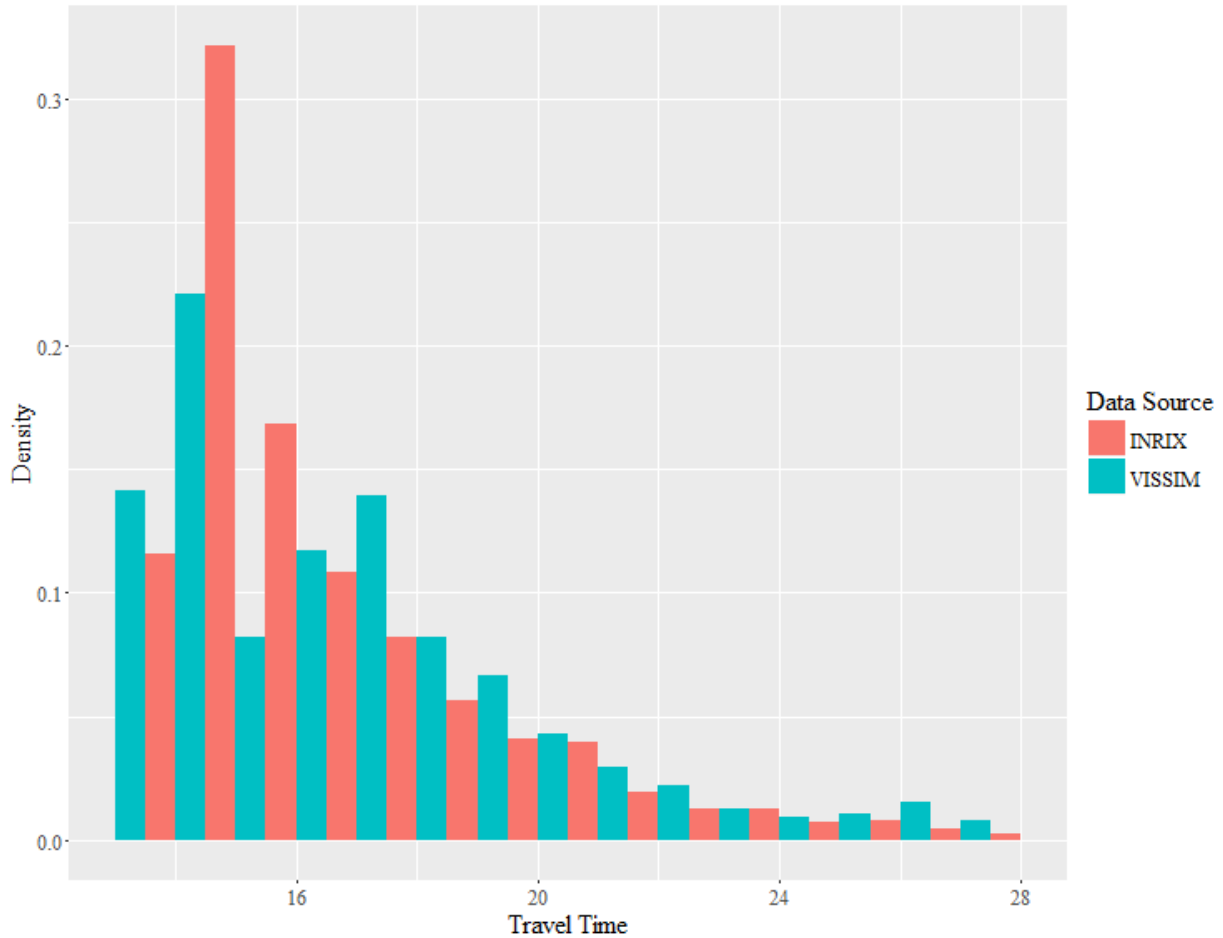


Figure 24. Travel-time distributions based on INRIX data and simulated data

The differences between these two distributions are caused by the following reasons:

- The parameters were manually calibrated and used some data collected at different freeway segments.
- The traffic volume on freeways is dynamic. However, deterministic values were used in the simulation.
- The percentages of congested and uncongested travel times were based on a Gaussian mixture model.

CONCLUSIONS

In this study, a travel-time estimation model and a travel-time reliability prediction framework were proposed.

The travel-time estimation model considers spatially correlated traffic conditions. Segment- and corridor-level travel-time distributions were estimated using probe vehicle data and roadside radar sensor data. Corridor-level travel-time reliability measures were extracted from the travel-time distributions. When compared to the probe vehicle data from INRIX, the proposed travel-time estimation model was found to capture the patterns of travel time and its distribution well. However, inconsistencies or gaps in the missing INRIX and sensor data can randomly cause the MTT method to overestimate/underestimate travel-time reliability.

The short-term travel-time reliability prediction framework was developed based on real-time traffic conditions. The predicted distributions are intended to help traffic management practice by providing valuable information about short-term traffic conditions and their variability. The multistate distribution can account for congested and uncongested travel times. From the distribution, travel-time reliability measures can be derived that can indicate uncertainty in corridor travel time for the corresponding time interval.

The travel-time reliability prediction framework contains four parts:

1. Travel-time observations are classified based on weather conditions using hierarchical cluster analysis. The travel times are categorized into three weather conditions: good weather, rain and snow.
2. The ARIMA model is used to predict travel time in the corridor. However, ARIMA is not the only option for this part of the framework. The more precise the travel-time predictions, the more accurate the travel-time distributions that can be derived.
3. Starting from the linear relationship between the mean and standard deviation of travel time per unit distance, a travel-time distribution estimation method is derived to derive a travel-time reliability measure, such as 95th percentile travel time.
4. According to the non-linear relationships between the correlation of different segments to each other and the distance of different segments from each other, a method for synthesizing route travel time is determined.

The proposed framework was tested on a freeway network in Des Moines, Iowa. The model parameters were estimated and the underlying assumption validated by using actual probe vehicle data. From the analysis results, the linear relationship between the mean and standard deviation of travel time per unit distance was validated. Moreover, the non-linear relationship between the correlation of different segments to each other and the distance of different segments from each other was adequately represented by a power model. The results of the travel-time distribution estimation method and the synthesizing method were evaluated and compared with the probe vehicle travel-time data. The results of a 95th percentile travel-time estimation show that the segment travel-time distribution estimation model captured the pattern of actual travel-time distributions. Furthermore, the proposed synthesizing method can adequately represent

short-term corridor-level travel-time distributions. Additionally, the Vissim-simulated travel-time distribution is similar to the travel-time distribution based on the probe data.

In conclusion, the following are some directions for future research. First, the method for estimating corridor-level travel time and its distribution relies on point measurements collected from side-fired radar sensors. In future research, the impacts of lane-changing behavior and the temporal correlation of this behavior to travel time can be incorporated into the model. Moreover, it is desirable to examine and consider the distinct car-following behavior of passenger cars and heavy vehicles in the travel-time estimation model.

Additionally, the short-term travel-time reliability prediction framework can be developed in future research. The framework provides a systematic way to estimate the reliability of real-time and near-future corridor travel time. In this framework, the synthesizing method, which is based on two-component mixture normal distribution, well captures the long tail of the ground truth travel-time distribution. The proposed framework also incorporates the impacts of weather on travel-time reliability. Based on the findings of this study and the performance of this framework, there are some potential future research directions. First, the proposed travel-time distribution estimation method can be extended to estimate the arterial travel-time distribution. Second, the synthesizing method can be extended to other distributions, such as lognormal and gamma distributions. Third, the proposed travel-time distribution estimation method can be extended to estimate travel-time distribution from limited probe samples.

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APPENDIX

Equation (39) is derived as follows:

$$\sigma^2 = \frac{\mu_2 - \mu}{\mu_2 - FFTT} * [(FFT T - \mu)^2 + \sigma_{FFT T}^2] + \frac{\mu - FFT T}{\mu_2 - FFT T} [(\mu_2 - \mu)^2 + (a\theta_1 + \theta_2\mu_2)^2]$$

$$(\mu_2 - FFT T) * \sigma^2 = (\mu_2 - \mu)[(FFT T - \mu)^2 + \sigma_{FFT T}^2] + (\mu - FFT T)[(\mu_2 - \mu)^2 + (a\theta_1 + \theta_2\mu_2)^2]$$

Assume $A = [(FFT T - \mu)^2 + \sigma_{FFT T}^2]$, we have

$$\mu_2 * \sigma^2 - FFT T * \sigma^2 = \mu_2 * A - \mu * A + (\mu - FFT T)[(1 + \theta_2^2)\mu_2^2 + 2(\theta_1\theta_2a - \mu)\mu_2 + (\mu^2 + \theta_1^2a^2)]$$

$$(\mu - FFT T)(1 + \theta_2^2)\mu_2^2 + [2(\mu - FFT T)(\theta_1\theta_2a - \mu) + A - \sigma^2]\mu_2 + [(\mu - FFT T)(\mu^2 + \theta_1^2a^2) + FFT T * \sigma^2] = 0$$

Assume,

$$B = (\mu - FFT T)(1 + \theta_1^2)$$

$$C = 2(\mu - FFT T)(\theta_1\theta_2a - \mu) + A - \sigma^2$$

$$D = 2(\mu - FFT T)[(\mu - FFT T)(\mu^2 + \theta_1^2a^2) + FFT T * \sigma^2]$$

Then, we have

$$B * \mu_2^2 + C * \mu_2 + D = 0$$