## Projecting Future Landside Congestion Delays at the BWI Airport

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This study examines the access to the BWI Airport in detail. Complex density functions are utilized to determine the congestion curves faced by passengers arriving at the airport. It is found that all lots have congestion during peak periods and that this congestion approaches levels where marginal entry access times are greatly accelerating. The lots that are most susceptible to this phenomenon are the satellite lots and the terminal roadway for drop-off passengers. The results highlight the importance of using figures other than the average access time in determining the impact of congestion on terminal access times. The passengers susceptible to congestion arrive at the terminal much more slowly than the difference in the average access times would indicate.

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The purpose of this report is to identify congestion curves for access to the BWI airport. The methodology is not unique to this facility, but the data and conclusions are. This report builds upon the results in "BWI Terminal Accessibility Study" which can be obtained from the National Transportation Center at Morgan State University. Rather than replicate this study, many sections of this report reference the data and results in that report. Particularly, the methods of data collection and summary statistics concerning the data are not included in this report.

One of the key goals of this project is to determine if it is possible to build congestion curves that are useful in analyzing facility access. This is a new field that requires much exploration since there are no current means of building congestion curves for this type of data. Much of the report is focused on building the results from the most basic form up toward a usable model that allows congestion curves to be built. While full complexity is not reached in this report, a great improvement in the ability to forecast congestion is achieved.

This report is organized into seven sections. Section I contains some basic background on congestion and the theory that underlies the estimations and results presented in this report. Section II contains results for the simple density function estimations for all lots. Section III contains results for complex density function estimations for all lots. Section IV contains results for the congestion analysis at the facility level. Section V contains results for the congestion analysis by time of day and day of week. Section VI contains the results for the congestion analysis by variable. Section VII contains some concluding remarks. A data appendix contains some more complex equations and data descriptions.

Because all of the lots are handled in nearly identical fashion, the estimation methodology and other material that is constant for all lots are presented in Section I. The tables and figures are presented in the text of this report for ease of reference. At the end of each section a brief summary of the results of that section are presented. For those not interested in the details of the methodology, data and analysis this summary should suffice to highlight the results of the congestion analysis.

## I. Background and Theory for Congestion and Estimations

The two principal components of this study are the idea of congestion and the concept of density functions and distributions. This paper uses the latter to identify and quantify the former. This section begins with a description of congestion, and then moves on to identify the properties of density functions and how they can be used to quantify congestion. Finally, estimation procedures and methodology are presented in this section.

## I.A. Congestion

For most people, congestion is something that happens to roads when too many other people are trying to go to the same place. This is a correct definition of congestion, but not complete. Formally, congestion is the result of increasing marginal costs to all users, as more users attempt to use a shared resource. Less formally, this means that as more and more people use a particular resource, these potential users "get in the way" of each other. In fact, each person adds more to the congestion than the last person. Eventually, if enough people try to use a resource
simultaneously, the costs rise to infinity. In the case of congestion on roads (or at parking facilities), rising costs are the time that it takes users to complete the activity.

This increasing cost of using the facility as more users attempt to use the same resource is a common occurrence in our modern society. We wait at traffic signals, we wait to checkout at grocery stores, we queue to enter a freeway, we sit in slowed or stopped traffic during rush hour, and we wait to enter sporting or theatrical events. And, we are slowed when entering the airport to catch a flight. For the most part, we don't even notice small amounts of congestion delay for most activities. However, even if congestion delays are minimal, they can be measured and identified. This is extremely important in instances when the delays may grow over time (as usage of a fixed shared resource increases) and mitigation is possible.

The goal of this study is to measure congestion for accessing a parking facility (particularly at an airport where high volume is maintained over long periods of time) and determine the root cause of congestion. While it seems trivial to identify the cause of congestion (too many people trying to use the same facility), this is not the case. A simple example is a single lot at the airport. There are several factors that may contribute to the length of time it takes to complete the activity at the facility (parking and then accessing the airport). The primary cause of elevated access times could be any combination of three (to keep things simple) variables. The most obvious variable is the number of vehicles entering the facility during a certain time period. However, there are two other variables that may matter just as much or more: 1) the overall activity at the airport at this time (people doing things other than using the parking facility may impede those using the facility), and 2) the excess capacity of the facility at that time (in this case the number of empty parking spaces). The results below will show that these variables are important in determining the total access time. In fact, they are sometimes more important than the volume of usage in determining access times.

## I.B. Probability Functions

In this study, probability functions (also called density functions) are used to estimate congestion levels. A brief description of density functions, and the reason they are employed, is presented.

It is obvious that not every person that uses a facility uses it in exactly the same amount of time. While this is true in almost every circumstance - it may be exaggerated in a parking facility where there is a random element to exactly when the vehicles arrive and where the empty parking spaces are among other variables. Therefore, there is not an access time but, rather, a distribution of access times. For some purposes, the average access time might be the appropriate measure of congestion. However, this is not true in general. For this reason, the distribution of access times is estimated.

This allows the use of whatever access benchmark is appropriate. For example, an airport planner might well be interested in the average access time and then normal regression type estimations would be sufficient. But, another planner might be interested in the $90^{\text {th }}$ percentile access time and yet another might be interested in the $99^{\text {th }}$ percentile access time. With normal estimation procedures, these figures are difficult to obtain and require a separate estimation for each. By estimating the density function, every benchmark is immediately available from the
same results. This is of particular importance when the distribution is "spread" due to congestion.

It is intuitively obvious that as more people try to use the parking facility, it will take longer to use the facility. For most uses it would be sufficient to state that the average access time rises. It is most likely true that all benchmark times will rise as well (it is very likely that the $90^{\text {th }}$ percentile access time will rise when the average access time rises). However, it is possible that the higher percentile access times rise by more than the average access time. This means that normal estimation procedures might underestimate the increase in congestion because they usually report means (or equivalent measures).

It seems likely that the variance of access times increases as congestion increases. Most distributions, and the data in this study, follow this pattern. This implies that high percentile access times will increase by more than the mean increases during congested periods. The estimation of density functions allows this phenomenon to be captured. As an example, perhaps the average access time increases by 4 minutes during peak usage. It is possible that some other benchmark, such as the $90^{\text {th }}$ percentile access time, will increase by 7 minutes. In order to capture this impact of congestion, density functions are estimated directly rather than standard procedures, which only estimate a benchmark. ${ }^{1}$

## I.C. Density Function Representation

There is no theory that dictates the distribution of the data concerning facility access time. Therefore, multiple density functions are estimated and presented. The density functions employed in this study are the Poisson, Normal, and Chi-Squared density functions. The Poisson is a discrete distribution that is often used for arrival data. The Normal is a central distribution that often describes the distribution of random variables. The Chi-Squared is a non-central distribution that appears to describe the data well in simple forms.

The appendix contains the exact forms of these distributions. Of importance for estimation is that the Poisson and the Chi-Squared are single parameter distributions and the Normal is a twoparameter distribution. In both the Poisson and the Chi-Squared distribution, the variance automatically increases as the mean increases. In the Normal distribution, it is possible to move the mean and variance independently.

The first step in the process of identifying the distribution of the data is to estimate a simple representation of each distribution for the access times for each lot. This is done by maximizing the likelihood that the observed data comes from a distribution of one particular type and finding the corresponding coefficient(s). After this is done, a complex estimation is performed for each distribution. This entails allowing each parameter of the distribution to be a function of the variables that might influence congestion.

[^0]The included variables are the percentage of parking spaces filled in the lot, the number of vehicles entering the facility during the hour, the number of vehicles leaving the facility during the hour, and a series of departing capacity variables. The departing capacity variables are the number of available seats on flights departing the airport in the next 30, 60, 90, 120 and 180 minutes. Various combinations of these departing capacity variables are employed to capture the impact of this variable.

The amount of time it takes to access the airport from any facility is thus made to be a function of the airside activity of the airport (capacity), the excess capacity in the facility (percentage of spaces filled), and the amount of traffic in the facility (number of vehicles entering and leaving the facility). In some estimations second order and cross-terms are employed as well.

Because the estimations are of the density functions rather than the access times, all of the included variables potentially affect the spread of the distribution as well as the level of the distribution. In other words, it is possible that a variable not only raises the amount of time it takes to access the airport but it might also affect the variance of the distribution. This allows a much more flexible determination of the various percentile access times that might be of interest.

The estimations are carried out using maximum likelihood techniques in GAUSS. The nonparametric distributions are then built from the data and the density functions estimated. These are graphed and various points in the distribution are examined as potential benchmarks for congestion. In addition, the impact of the different variables are studied and compared to determine which have the highest correlation with congestion. For each of the access facilities the same analysis is carried out. The results are presented in the individual facility sections. A data appendix contains the forms of each of the estimations.

## II. Simple Density Function Estimations

The simple density functions assume that all data for a facility can be described as coming from the same distribution. This is an assumption that is obviously simplistic. Even a casual observer could determine that the distribution of access times looks different during peak hours than off-peak hours. However, this gives us a base case to work from and allows demonstration of the methodology with a small number of parameters. It also provides insight into the basic "shape" of the distributions.

There are three estimations that are carried out. They are represented here

$$
\begin{aligned}
& a t=f_{P}(\lambda) \\
& a t=f_{\chi^{2}}(n) \\
& a t=f_{n}(\mu, \sigma)
\end{aligned}
$$

where at represents the access time, $f$ the density function, $p$ indexes the Poisson distribution, $\chi^{2}$ indexes the Chi-Squared distribution, $n$ indexes the normal distribution, $\lambda$ is the parameter of the Poisson distribution, $n$ is the parameter of the Chi-Squared distribution, and $\mu$ and $\sigma$ are the parameters of the Normal distribution. The parameters $\lambda, n$, and $\mu$ represent the means of their respective distributions.

These estimations are carried out for each lot. The resulting parameters and a graphical representation of the distributions are presented for each lot below.

## II.A. ESP Facility

The ESP lot at BWI has the lowest volume of any of the facilities studied. There were approximately 220 observations for this lot compared with 1400 for the smallest lot other than ESP. This means that the results for the ESP lot have a very high standard error and the results are the least robust of those presented. The simple density functions were estimated and the results are presented in Table 1 and Figure 1.

| Table 1-Simple Density Function Parameters for the ESP Lot |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Poisson Distribution |  | Chi-Squared Distribution |  | Normal Distribution |  |
| $\lambda$ | 9.6698 | n | 10.0582 | $\mu$ | 9.6698 |
|  |  |  |  | 3.3822 |  |

Note that the Poisson and the Normal distributions identify the mean as 9.6698 which is the actual mean while the Chi-Squared identifies the mean as slightly higher. This does not necessarily mean that the Chi-Squared is incorrect. The estimated density functions are found to maximize the fit to all observations. When there are outlying observations, they can influence the mean slightly.


Figure 1
The distribution of the actual data is presented as the dashed red line. Note that all of the distributions seem to be centered roughly near the peak with the exception of the Chi-Squared, which is slightly shifted to the left of the peak. The Chi-Squared, however, picks up the few outlier observations with access times in the area of 26 minutes that the other distributions fail to predict. Visual inspection might lead one to select the Poisson as the distribution that best fits these data. While this could be misleading, there are no nested tests of this type that are valid on these data. More complex estimations are carried out using the other variables.

## II.B Garage Facility

The estimation of the parameters for the garage lot are presented in Table 2 and the distributions are graphically displayed in Figure 2.

| Table 2 - Simple Density Function Parameter Estimates for Garage Lot |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Poisson Distribution | Chi-Squared Distribution |  | Normal Distribution |  |  |
| $\lambda$ | 7.9479 | n | 8.1604 | $\mu$ | 7.9479 |
|  |  |  |  | 3.7560 |  |

Again we see that the Poisson and the Normal distributions identify the mean as 7.9479 which is the actual mean while the Chi-Squared identifies the mean as slightly higher.


Figure 2
The data here are not easily described by any of the distributions. The Normal distribution is shifted right of the data and not peaked enough. The Poisson is peaked enough but is shifted right at the peak and left in the high access time tail. The Chi-Squared is not peaked enough (but peaks at the right value) but does fit the far right tail well. Visually one could choose either the Poisson or the Chi-Squared as the distribution that fits the data best.
II.C Satellite Facility

Blue Lot

| Table 3-Simple Density Function Parameter Estimates for Blue Lot |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Poisson Distribution | Chi-Squared Distribution |  | Normal Distribution |  |  |
| $\lambda$ | 19.2051 | n | 19.1376 | $\mu$ | 19.2051 |
|  |  |  |  | 6.4058 |  |

Again we note that the Poisson and the Normal distribution find the mean precisely and in this case the Chi-Squared slightly understates the mean.

Green Lot

| Table 4-Simple Density Function Parameter Estimates for Green Lot |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Poisson Distribution | Chi-Squared Distribution |  | Normal Distribution |  |  |
| $\lambda$ | 21.9085 | n | 21.8858 | $\mu$ | 21.9085 |
|  |  |  |  | 6.5766 |  |

We see that all three distributions report means very close to the actual mean of the data in this case. The Chi-Squared slightly understates the mean.

Amtrak Lot

| Table 5-Simple Density Function Parameter Estimates for Amtrak Lot |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Poisson Distribution | Chi-Squared Distribution |  | Normal Distribution |  |  |
| $\lambda$ | 18.3810 | n | 18.2601 | $\mu$ | 18.3810 |
|  |  |  |  | 6.5555 |  |

All three distributions closely represent the mean of the data. The Chi-Squared again understates the mean slightly.

Satellite Lot

| Table 6-Simple Density Function Parameter Estimates for Combined Satellite Lots |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Poisson Distribution | Chi-Squared Distribution |  | Normal Distribution |  |  |
| $\lambda$ | 20.5213 | n | 20.4238 | $\mu$ | 20.5213 |
|  |  |  |  | 6.6404 |  |

While the Chi-Squared distribution slightly understates the mean, the other two exactly predict the mean once again.


Figure 3
All three distributions actually fit the data from the Blue Satellite lot reasonably well. The Normal distribution is slightly right-shifted and the Poisson is slightly too peaked. The ChiSquared fits the data extremely well. It may be slightly less peaked than the actual data, but it fits both sides of the distribution well and predicts the peak as well as either of the other distributions.


Figure 4
As with the Blue lot data, all three distributions fit the data well. The Normal and Poisson distributions are slightly right-shifted at the peak but are still good representations of the data. The Chi-Squared fits both sides of the distribution well and predicts the peak relatively well. It is centered on the peak but just a bit less peaked than the data.


Figure 5
The Amtrak lot had very few observations. With the exception of the peak in the observed data at 13 minutes, all three distributions do a good job of predicting the data. The Poisson is slightly too peaked but the other two distributions fit quite well. Visually the Chi-Squared fits better at the peak and to the right of the peak and the Normal distribution fits better to the left of the peak.


Figure 6
Not surprisingly, the graph of the distributions for all of the Satellite lots looks similar to the individual lot graphs. The Poisson distribution is a bit too peaked and slightly right-shifted at the peak. The Normal distribution fits the data well, but is right-shifted at the peak. The ChiSquared distribution fits the data quite well except for the height of the peak, which is slightly under-predicted by this distribution. Visually, the Chi-Squared Distribution is the best fit for the Satellite lot data both in aggregate and on an individual lot basis.

## II.D Drop-Off Passengers

The drop-off passenger data are the most variant and susceptible to congestion of all the facilities. As such, they are the most difficult to estimate and, at the same time, the most interesting to describe.

| Table 7 - Simple Density Function Parameter Estimates for Drop-Off Passengers |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Poisson Distribution |  | Chi-Squared Distribution |  | Normal Distribution |  |
| $\lambda$ | 95.448614 | n | 81.1930 | $\mu$ | 95.4486 |
| $\lambda$ | 95.448614 | n | 81.1930 | $\sigma$ | 68.9542 |

Again, the Poisson and Normal distributions identify the mean exactly while the Chi-squared distribution identifies a mean less than the actual mean. This is different than the other cases where the means were quite close.

Because the drop-off data are obtained in seconds instead of minutes, some aggregation is necessary in order to graph the results meaningfully. The data are combined in 10 -second increments before being graphed. All analysis is carried out with the actual data until the graphing is done.

Simple Density Function Graphs vs. Observed Observations for the Drop Off Times


Figure 7
The increased variance and the spread of the distributions are both very evident in the graph. Note that all distributions are peaked to the right of the data. This is due to the fact that the observed data are peaked far to the left of the mean with a very large right-side tail (from congested periods it would appear). The Poisson distribution matches the data the best in this simple representation. The attempt of the distributions to capture the extremely large right-hand side tail is evident in the graphs of the density functions. The complex density function estimations below mitigate this problem.

## II.E Simple Density Function Analysis

It is apparent that the data, with the exception of the drop-off passenger data, are relatively well represented by the distributions. However, in order to meaningfully predict congestion, the best possible representation is desired. In particular, the right hand tails of the data need to be described well before it is possible to look at congestion. This is true because congestion implies that we are studying times when accessibility is hampered and this is the right hand tail of the distributions. In the following section, estimations with expanded variable lists and correspondingly better predictive powers are presented. Congestion results are viewed after the full presentation of the estimations

While it is not clear what it means to be a good fit at this point, the Chi-Squared distribution seems to be the best fit to the data overall. This is because the Chi-Squared distribution has a large right tail in its natural form. When complex density functions are utilized, all distributions will be able to lengthen their tails and this relationship may no longer hold.

The main reason for expanding the explanatory variable set is that there is a need for a large number of observations to estimate density functions. In some cases, difficulty in estimation is still obtained with several hundred observations. While it might not be possible to obtain several hundred observations for each set of conditions, it is feasible to obtain a few thousand observations overall and fully describe the data with complex density functions. This will allow analysis of each set of conditions without needing a full dataset for each of them. Thus, congestion analysis can be carried out for multiple conditions with complex density functions on a few thousand observations rather than hundreds, if not thousands, of extra observations for each period and separate estimations for each set of conditions. The success of this will be detailed below.

## III. Complex Density Function Estimations

Three sets of complex density functions were attempted for each lot and distribution. Within each set of density functions, various sets of variables were tried as the independent variables in the estimation. Unfortunately, density function analysis is subject to a much greater degree of sensitivity than most standard estimation methodologies. Due to this, a large number of the attempted estimations did not work properly. This was due to multicolinearity problems.

The first set of complex density functions are estimated using a small set of right-hand side variables. This set of variables has no second-order or cross-terms and thus introduces firstorder affects of variables on access time. However without the changes in slope allowed with second-order terms, the curvature of the access time graphs (congestion) cannot be captured.

The second set of complex density functions are estimated using the same small set of right-hand side variables but including second-order terms to allow access times to have non-linear properties. This allows discovery of true congestion points.

The last set of complex density functions includes all of the variables in the second estimation plus a large set of variables for days of week and times of day that allow even more precise
estimation. However, the colinearity problems arise and these estimations are very difficult to perform. In fact, these become almost impossible to analyze for a broad range of data and density functions. This is not surprising given the sensitivity of density function estimation.

The forms of each of the estimations remain the same as in the simple density function analysis with the exception that the density function parameters are more complex. The forms are given here and the results for each lot and distribution are given below.

## Poisson Distribution

The estimation of the Poisson Distribution has the following form

$$
\begin{aligned}
& a t=f_{P}(\lambda) \\
& \lambda=\alpha+\beta_{e} e+\beta_{x} x+\beta_{f} f+\beta_{c} c+\beta_{t} t
\end{aligned}
$$

where $\alpha$ is a constant in the equation for the parameter of the distribution, $\beta$ are the slope coefficients for the variables that might impact the parameter of the distribution, $e$ represents the number of cars entering the facility during the hour, $x$ represents the number of cars exiting the facility during the hour, $f$ represents the percentage of parking spaces filled in the facility, $c$ represents a vector of airside capacity variables, $t$ represents of vector of dummy variables for day of week and time of day. ${ }^{2}$ These last variables are only included in the full estimation.

## Chi-Squared Distribution

The estimation of the Chi-Squared Distribution ahs the following form

$$
\begin{aligned}
& a t=f_{\chi}(n) \\
& n=\alpha+\beta_{e} e+\beta_{x} x+\beta_{f} f+\beta_{c} c+\beta_{t} t
\end{aligned}
$$

where all symbols are the same except that $n$ is the parameter of the distribution.

## Normal Distribution

The estimation of the Normal Distribution has the following form

$$
\begin{aligned}
& a t=f_{n}(\mu, \sigma) \\
& \mu=\alpha_{\mu}+\beta_{\mu e} e+\beta_{\mu c} x+\beta_{\mu f} f+\beta_{\mu c} c+\beta_{\mu t} t \\
& \sigma=\alpha_{\sigma}+\beta_{\sigma e} e+\beta_{\sigma x} x+\beta_{\sigma f} f+\beta_{\sigma c} c+\beta_{\sigma t} t
\end{aligned}
$$

where all symbols are the same as in the last two distributions except that $\mu$ and $\sigma$ as subscripts index parameters that impact that parameter in the distribution.

[^1]
## III.A ESP Facility

The results for the first group of complex estimations for the ESP facility are presented in Table 8.

| Table 8 - Estimates for the Basic Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 11.1713 | 1.5076 | 0 |
| $\beta_{e}$ | -0.0053 | 0.0183 | 0.7731 |
| $\beta_{x}$ | 0.1217 | 0.1013 | 0.2294 |
| $\beta_{f}$ | -0.0242 | 0.0256 | 0.3455 |
| $\beta_{c 120}$ | 0.1458 | 1.0455 | 0.8891 |
| Chi-Squared Distribution |  |  |  |
| $\alpha$ | 10.342 | 8.0177 | 0.1971 |
| $\beta_{e}$ | 0.0097 | 0.0938 | 0.9173 |
| $\beta_{x}$ | 0.043 | 0.5798 | 0.9409 |
| $\beta_{f}$ | -0.0225 | 0.1384 | 0.8707 |
| $\beta_{c 120}$ | -1.2918 | 5.1397 | 0.8015 |
| $\alpha_{\sigma}$ | 3.394635 | -0.00373 |  |
| $\beta_{\sigma e}$ | -0.01475 | -0.01113 |  |
| $\beta_{\sigma x}$ | -0.01694 | 0.002774 |  |
| $\beta_{\sigma f}$ | -0.0092 | 0.001806 |  |
| $\beta_{\sigma c 120}$ | -0.00096 | 0.000901 |  |
| $\alpha_{\mu}$ | 11.30579 | 0.000046 |  |
| $\beta_{\mu e}$ | -0.00378 | 0.003238 |  |
| $\beta_{\mu x}$ | 0.134264 | -0.00103 |  |
| $\beta_{\mu f}$ | -0.01768 | 0.000378 |  |
| $\beta_{\mu c 120}$ | 0.293187 | 0.001015 |  |

It can be seen that all of the distributions are right-shifted from the simple distributions from examining the means. Each of the distributions has a base mean that is above the actual mean of the data. This is evidence that the complex distributions are needed to better describe the data (since changes imply that the simple density estimates are not sufficient to describe the data).

Because there are very few observations, the lack of significance in the coefficients is not very meaningful. However, it can be noted that the variables have the same impact on the Poisson and Normal distribution means but not on the Chi-Squared. Number of cars entering and percentage of spaces filled in the lot have negative impacts on the mean while the number of cars leaving and the departing capacity have positive impacts. In the normal distribution, the
variables with positive impacts are highly significant while those with negative impacts are not significant. This means that it is possible that these variables (those with negative impacts) are not estimated precisely with this small number of observations. From these estimations, it appears that each of the variables lowers the variance of the distribution.

Figure 8 depicts the new distributions against the observed data.

Nonparametric Density Function Graphs for Esp Lot


Figure 8
Note that the new distributions follow the same pattern as the simple distributions. The Poisson remains the best fit for both left of the peak and at the peak. The Chi-Squared is still the best distribution to the right of the peak. Overall, the Chi-Squared is most likely the best distribution for these data but the Poisson is also quite good and the Normal performs well also.

The results for the estimations with second-order terms are presented in Table 9.

| Table 9-Estimates for the Second Complex Density Functions |  |  |
| :---: | :---: | :---: |
| Poisson Distribution |  |  |
| Parameter | Estimates | Standard <br> Error |
| $\alpha$ | 8.873644 | 0.000242 |
| $\beta_{e}$ | -0.13645 | -0.000007 |
| $\beta_{x}$ | 0.807423 | -0.000003 |
| $\beta_{f}$ | -0.717872 | 0.000006 |
| $\beta_{e e}$ | 0.002229 | -0.000025 |
| $\beta_{e x}$ | -0.008896 | 0.000018 |
| $\beta_{e f}$ | 0.006925 | -0.000014 |
| $\beta_{x x}$ | 0.044922 | 0.000032 |
| $\beta_{x f}$ | -0.014782 | 0.00007 |
| $\beta_{f f}$ | -0.009444 | 0.000096 |
| $\beta_{c 120}$ | -0.007175 | 0.000194 |
| $\beta_{c 120 c 120}$ | -2.665009 | -0.000076 |

The results for the estimations with all the variables are presented in Table 10. Note that only certain estimations work in this case.

| Table 10 - Estimates for the Final Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 26.61 | 28.1424 | 0.3444 |
| $\beta_{\text {tue }}$ | 8.2094 | 7.084 | 0.2465 |
| $\beta_{\text {wed }}$ | 12.734 | 12.5074 | 0.3086 |
| $\beta_{\text {thur }}$ | 11.0776 | 15.6813 | 0.4799 |
| $\beta_{\text {fri }}$ | 10.1993 | 15.7283 | 0.5167 |
| $\beta_{\text {sat }}$ | -16.2237 | 14.9869 | 0.279 |
| $\beta_{\text {sun }}$ | 10.1644 | 5.3766 | 0.0587 |
| $\beta_{\text {amrush }}$ | -1.6394 | 1.3051 | 0.209 |
| $\beta_{\text {lateam }}$ | -0.6266 | 3.061 | 0.8378 |
| $\beta_{\text {midday }}$ | 1.0837 | 1.778 | 0.5422 |
| $\beta_{\text {earlypm }}$ | -9.9741 | 9.877 | 0.3126 |
| $\beta_{\text {pmrush }}$ | 0.0187 | 0.0107 | 0.0811 |
| $\beta_{e}$ | -0.0131 | 0.1148 | 0.9093 |
| $\beta_{x}$ | -0.003 | 0.0132 | 0.8232 |
| $\beta_{f}$ | -0.193 | 0.1865 | 0.3009 |
| $\beta_{\text {cl20 }}$ | -0.0981 | 0.2545 | 0.6999 |
| $\beta_{\text {ee }}$ | 0.0755 | 0.1027 | 0.4622 |
| $\beta_{\text {ex }}$ | -0.9293 | 0.7496 | 0.2151 |
| $\beta_{e f}$ | 0.0126 | 0.0281 | 0.6526 |
| $\beta_{\text {ecl20 }}$ | 0.2143 | 0.2564 | 0.4032 |
| $\beta_{x x}$ | 2.128 | 4.9139 | 0.665 |
| $\beta_{x f}$ | 26.61 | 28.1424 | 0.3444 |
| $\beta_{x c 120}$ | 8.2094 | 7.084 | 0.2465 |
| $\beta_{\text {ff }}$ | 12.734 | 12.5074 | 0.3086 |
| $\beta_{\text {fc120 }}$ | 11.0776 | 15.6813 | 0.4799 |
| $\beta_{c 120 c 120}$ | 10.1993 | 15.7283 | 0.5167 |
|  |  |  |  |

Figure 9 shows the various Poisson distributions against the observed data.
Nonparametric Poisson Density Function for Esp Lot


Figure 9
Note how each addition of right-hand side variables to the estimation improve the fit of the Poisson distribution to the upper tail of the distribution. This means that congestion levels are better predicted with the larger number of right-hand side variables. However, for this lot, the predictions of the peak values of the distribution are no better and in the last estimation are quite poor. However, this more complex estimation is better analytically - despite the lack of visual evidence.

Of importance are the relative changes in the distributions as the number of right-hand side variables is increased. Notice that the basic complex density function is only slightly shifted from the simple density function. The second complex density function shifts away from the first a bit more and the full complex density function shifts significantly. This is important because it points out the non-linear relationships for the access time. The variables added from the first to the second complex density function are the non-linear terms. The variables added from the second to the third (full) complex density function are the dummy variables for time of day and day of week. It is clear that these variables are important in describing the distribution.

Figure 10 shows the various Normal distributions against the observed data.


Figure 10
The ESP lot appears not to fit as well with the complex normal density function. Analytically, the complex density function is a better fit. The lack of marked improvement is due to the small number of observations. Also, because the two complex representations do not converge, the most significant improvements in the density function are not observed.

Figure 11 shows the various Chi-Squared distributions against the observed data.


Figure 11
The complex density function is a slightly better fit than the simple density function for the ChiSquared distribution and the ESP lot. The lack of improvement is not surprising given the good fit from the simple density function and the low number of observations. Also, as in the other distributions, the non-linear relationships add more to the description and those distributions do not converge for this distribution. The lack of convergence of these distributions is discussed in more detail in the congestion analysis.

## III.B Garage Facility

The results for the first group of complex estimations for the Garage are presented in Table 11.

| Table 11- Estimates for the Basic Complex Density Functions |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |  |  |  |
| Parameter | Estimates | Standard Error | Probability |  |  |  |
| $\alpha$ | 7.9423 | 0.0477 | 0 |  |  |  |
| $\beta_{e}$ | 0.0005 | 0.0003 | 0.1192 |  |  |  |
| $\beta_{x}$ | 0 | 0.0002 | 0.9972 |  |  |  |
| $\beta_{f}$ | 0.01 | 0.0019 | 0 |  |  |  |
| $\beta_{c 120}$ | -0.1301 | 0.0284 | 0 |  |  |  |
| Chi-Squared Distribution |  |  |  |  | 0.2636 | 0 |
| $\alpha$ | 7.4067 | 0.0016 | 0.8909 |  |  |  |
| $\beta_{e}$ | 0.0002 | 0.0013 | 0.9476 |  |  |  |
| $\beta_{x}$ | -0.0001 | 0.0101 | 0.2368 |  |  |  |
| $\beta_{f}$ | 0.012 | 0.153 | 0.8964 |  |  |  |
| $\beta_{c 120}$ | -0.0199 | Normal Distribution | 0.0 .044 |  |  |  |
| $\alpha_{\sigma}$ | 3.7337 | 0.0003 | 0.035 |  |  |  |
| $\beta_{\sigma e}$ | 0.0006 | 0.0002 | 0.1719 |  |  |  |
| $\beta_{\sigma x}$ | -0.0003 | 0.0017 | 0.9239 |  |  |  |
| $\beta_{\sigma f}$ | -0.0002 | 0.0243 | 0 |  |  |  |
| $\beta_{\sigma c 120}$ | -0.1629 | 0.0636 | 0 |  |  |  |
| $\alpha_{\mu}$ | 7.9425 | 0.0004 | 0.2848 |  |  |  |
| $\beta_{\mu e}$ | 0.0004 | 0.0003 | 0.8707 |  |  |  |
| $\beta_{\mu x}$ | 0.0001 | 0.0025 | 0 |  |  |  |
| $\beta_{\mu f}$ | 0.0101 | 0.0343 | 0.0002 |  |  |  |
| $\beta_{\mu c 120}$ | -0.1284 |  |  |  |  |  |

The estimates for this lot are barely changed from the simple estimations. No variables have significant impacts on the access time or variance. The most likely reason for this is that the variance in accessing the terminal from the garage is more individual based than congestion based. More will be said of this in the congestion results section.

Figure 12 depicts the new distributions against the observed data.


Figure 12
Like the simple density function graph, the Poisson appears to be the best fit to the data through the peak and well past. The Chi-Squared is the best fit far to the right of the peak. All three are slightly better than the simple estimates but the Garage lot has the smallest improvement in fit from the first complex density estimations (over the simple estimations).

The results for the estimations with second-order terms are presented in Table 12.

| Table 12 - Estimates for the Second Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 8.0054 | 0.0656 | 0 |
| $\beta_{e}$ | 0.0006 | 0.0006 | 0.3574 |
| $\beta_{x}$ | 0.0004 | 0.0005 | 0.4455 |
| $\beta_{f}$ | 0.037 | 0.0042 | 0 |
| $\beta_{e e}$ | 0 | 0 | 0.0227 |
| $\beta_{e x}$ | 0 | 0 | 0.0002 |
| $\beta_{e f}$ | -0.0002 | 0 | 0 |
| $\beta_{x x}$ | 0 | 0 | 0 |
| $\beta_{x f}$ | 0 | 0 | 0.1159 |
| $\beta_{f f}$ | 0 | 0.0001 | 0.8308 |
| $\beta_{c 120}$ | -0.1888 | 0.0648 | 0.0036 |
| $\beta_{c 120 c 120}$ | 0.0157 | 0.0166 | 0.3452 |

The results for the estimations with all the variables are presented in Table 13. Note that only certain estimations work in this case.

| Table 13 - Estimates for the Final Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 7.3113 | 0.4613 | 0 |
| $\beta_{\text {tue }}$ | 1.3025 | 0.2801 | 0 |
| $\beta_{\text {wed }}$ | 0.7616 | 0.4444 | 0.0866 |
| $\beta_{\text {thur }}$ | 0.8118 | 0.4568 | 0.0756 |
| $\beta_{\text {fri }}$ | 0.2256 | 0.4574 | 0.6219 |
| $\beta_{\text {sat }}$ | -0.4722 | 0.4223 | 0.2636 |
| $\beta_{\text {sun }}$ | -0.5122 | 0.298 | 0.0857 |
| $\beta_{\text {amrush }}$ | 0.055 | 0.2372 | 0.8166 |
| $\beta_{\text {lateam }}$ | 0.4741 | 0.2035 | 0.0198 |
| $\beta_{\text {midday }}$ | 0.5588 | 0.1716 | 0.0011 |
| $\beta_{\text {earlypm }}$ | 1.116 | 0.1735 | 0 |
| $\beta_{\text {pmrush }}$ | 0.9948 | 0.1593 | 0 |
| $\beta_{e}$ | -0.0003 | 0.0008 | 0.6914 |
| $\beta_{x}$ | 0.001 | 0.0008 | 0.2125 |
| $\beta_{f}$ | 0.0105 | 0.0068 | 0.1215 |
| $\beta_{\text {cl20 }}$ | -0.181 | 0.0978 | 0.0641 |
| $\beta_{\text {ee }}$ | 0 | 0 | 0.1809 |
| $\beta_{\text {ex }}$ | 0 | 0 | 0.2128 |
| $\beta_{e f}$ | -0.0001 | 0 | 0.0104 |
| $\beta_{\text {ecl20 }}$ | 0.0003 | 0.0003 | 0.3017 |
| $\beta_{x x}$ | 0 | 0 | 0.1288 |
| $\beta_{x f}$ | 0 | 0 | 0.8186 |
| $\beta_{x c 120}$ | 0.0001 | 0.0003 | 0.6674 |
| $\beta_{\text {ff }}$ | -0.0003 | 0.0003 | 0.3325 |
| $\beta_{\text {fcl20 }}$ | -0.0052 | 0.0044 | 0.2388 |
| $\beta_{\text {c120c120 }}$ | 0.0039 | 0.03 | 0.8969 |
|  |  |  |  |

Figure 13 shows the various Poisson distributions against the observed data.


Figure 13
The improvement in the fit of the distribution to the data from this lot is not very dramatic. There is an improvement in the fit both at the peak and at the right-side extremes in the distribution as variables are added. However, the improvements are small and hard to see visually.

Figure 14 shows the various Normal distributions against the observed data.


Figure 14
As with the Poisson distribution, there is little improvement in the fit for this lot with the complex density function.

Figure 15 shows the various Chi-Squared distributions against the observed data.


Figure 15
There is a slight improvement in the fit of the Chi-Squared distribution from the complex density function. The improvement in the fit of this distribution is enough that it is observable visually. The lack of convergence with the non-linear terms does not allow observation of how well this distribution might fit.

## III.C Satellite Facilities

Blue Lot

The results for the first group of complex estimations for the Blue lot satellite facility are presented in Table 14.

| Table 14 - Estimates for the Basic Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 21.5799 | 0.2932 | 0 |
| $\beta_{e}$ | 0.0077 | 0.0022 | 0.0005 |
| $\beta_{x}$ | 0.0526 | 0.0091 | 0 |
| $\beta_{f}$ | 0.0682 | 0.0081 | 0 |
| $\beta_{c 120}$ | 0.3286 | 0.2616 | 0.209 |
| Chi-Squared Distribution |  |  |  |
| $\alpha$ | 20.5546 | 2.6364 | 0 |
| $\beta_{e}$ | -0.0029 | 0.0181 | 0.8726 |
| $\beta_{x}$ | 0.0283 | 0.0828 | 0.733 |
| $\beta_{f}$ | 0.063 | 0.0656 | 0.3368 |
| $\beta_{c 120}$ | 0.3556 | 2.2352 | 0.8736 |
| $\alpha_{\sigma}$ | Normal Distribution |  |  |
| $\beta_{\sigma e}$ | 6.4125 | 0.2611 | 0 |
| $\beta_{\sigma x}$ | 0.0128 | 0.002 | 0 |
| $\beta_{\sigma f}$ | 0.0236 | 0.0078 | 0.0025 |
| $\beta_{\sigma c 120}$ | 0.0152 | 0.0077 | 0.0483 |
| $\alpha_{\mu}$ | 0.0016 | 0.1763 | 0.9929 |
| $\beta_{\mu e}$ | 21.5801 | 0.4094 | 0 |
| $\beta_{\mu x}$ | 0.0063 | 0.0031 | 0.0398 |
| $\beta_{\mu f}$ | 0.0532 | 0.0128 | 0 |
| $\beta_{\mu c 120}$ | 0.066 | 0.0109 | 0 |
|  | 0.3276 | 0.349 | 0.3479 |

The first result to note from this table is that all of the density functions shifted right to accommodate the data. This makes all of the distributions fit the data much better than in the simple density function cases. The impacts of the variables are almost uniformly positive. With the exception of the number of cars entering, which has no impact for the Chi-squared distribution, all variables included increase the amount of time needed to access the terminal. This means that as more cars are entering the lot, it takes longer to access the terminal; as more cars are exiting the lot, it takes longer to access the terminal; as fewer parking spaces are available in the lot, it takes longer to access the terminal; and, as the airside departing capacity
increases, it takes longer to access the terminal. These are all expected results. Also of interest is the fact that in the Normal distribution, the only distribution where variance is measured separately, each of these variables also increases variance. This will have a notable impact on the congestion curves.

Figure 16 depicts the new distributions against the observed data.
Nonparametric Density Function Graphs for Blue Lot


Figure 16
Both the Poisson and Normal distributions fit the data much better than in the simple estimation case. However, the Chi-Squared is still the best distribution for describing the data. The addition of the right-hand side variables pulled the peak of the Poisson distribution closer to the peak of the actual data, but it still underestimates the far right portions of the data. The ChiSquared peak has been drawn upward and fits the data better and very closely describes the data right of the peak.

The results for the estimations with second-order terms are presented in Table 15.

| Table 15 - Estimates for the Second Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 23.7438 | 0.5441 | 0 |
| $\beta_{e}$ | -0.0302 | 0.0091 | 0.0009 |
| $\beta_{x}$ | 0.0769 | 0.0187 | 0 |
| $\beta_{f}$ | 0.1979 | 0.0276 | 0 |
| $\beta_{e e}$ | 0.0002 | 0 | 0 |
| $\beta_{e x}$ | -0.0007 | 0.0002 | 0.0002 |
| $\beta_{e f}$ | -0.0004 | 0.0002 | 0.1416 |
| $\beta_{x x}$ | -0.0023 | 0.0004 | 0 |
| $\beta_{x f}$ | 0.0023 | 0.0008 | 0.0043 |
| $\beta_{f f}$ | 0.0018 | 0.0007 | 0.0077 |
| $\beta_{c 120}$ | 0.4824 | 0.2971 | 0.1044 |
| $\beta_{c 120 c 120}$ | -0.5225 | 0.2017 | 0.0096 |

The results for the estimations with all the variables are presented in Table 16. Note that only certain estimations work in this case.

| Table 16 - Estimates for the Final Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 20.1765 | 1.3677 | 0 |
| $\beta_{\text {tue }}$ | 1.0123 | 1.1511 | 0.3792 |
| $\beta_{\text {wed }}$ | 1.7951 | 1.7532 | 0.3059 |
| $\beta_{\text {thur }}$ | 0.5129 | 1.1469 | 0.6547 |
| $\beta_{\text {fri }}$ | -0.1691 | 1.1613 | 0.8842 |
| $\beta_{\text {sat }}$ | 0.5997 | 1.8089 | 0.7403 |
| $\beta_{\text {sun }}$ | 1.8875 | 1.6487 | 0.2523 |
| $\beta_{\text {amrush }}$ | 2.1074 | 1.1023 | 0.0559 |
| $\beta_{\text {lateam }}$ | 2.5264 | 0.9225 | 0.0062 |
| $\beta_{\text {midday }}$ | 3.1538 | 0.8741 | 0.0003 |
| $\beta_{\text {earlypm }}$ | 4.9387 | 0.8553 | 0 |
| $\beta_{\text {pmrush }}$ | 4.3295 | 0.8426 | 0 |
| $\beta_{e}$ | -0.0308 | 0.0144 | 0.032 |
| $\beta_{x}$ | 0.0959 | 0.0268 | 0.0003 |
| $\beta_{f}$ | 0.1813 | 0.0355 | 0 |
| $\beta_{c 120}$ | 4.9183 | 1.1324 | 0 |
| $\beta_{e e}$ | 0.0002 | 0.0001 | 0.0315 |
| $\beta_{\text {ex }}$ | -0.0007 | 0.0003 | 0.0058 |
| $\beta_{e f}$ | -0.0002 | 0.0003 | 0.3675 |
| $\beta_{\text {ecl20 }}$ | 0.0128 | 0.0077 | 0.0982 |
| $\beta_{x x}$ | -0.0011 | 0.0005 | 0.0154 |
| $\beta_{x f}$ | 0.0041 | 0.001 | 0.0001 |
| $\beta_{\text {xcl20 }}$ | 0.1624 | 0.0424 | 0.0001 |
| $\beta_{\text {ff }}$ | 0.0002 | 0.0013 | 0.8644 |
| $\beta_{\text {fc120 }}$ | 0.0385 | 0.0197 | 0.0506 |
| $\beta_{\text {c120c120 }}$ | -0.4879 | 0.3312 | 0.1406 |
|  |  |  |  |

Figure 17 shows the various Poisson distributions against the observed data.


Figure 17
The Poisson distribution fits the data quite well once expanded to include all the variables. The improvement in fit at both the peak and the higher access time portions of the distribution is quite obvious. While this may not be the best distribution, there is no doubt that increasing the number of right-hand side variables greatly improves the fit of the distribution to the data. As in the ESP lot, note that the most drastic improvement is with the addition of non-linear and time/day variables on the right-hand side. Again, this shows that these terms are extremely important in congestion analysis.

Figure 18 shows the various Normal distributions against the observed data.


Figure 18
The complex normal fits the data better over the entire range of the distribution. Visually, this can be seen by noting that (with the exception of a few very small areas) the red line always lies between the green and blue lines. This means that the fit has been improved over the entire region. Again, the lack of convergence for the more full representations prevents a further significant improvement in fit.

Figure 19 shows the various Chi-Squared distributions against the observed data.


Figure 19
The Chi-Squared distribution fits the data better when moved to a complex density function as well. This is not as striking an improvement as in the Poisson or Normal distributions, but this is due to the fact that the simple density function already fit the data relatively well. The improvement lies mostly at the peak of the data and to the right of this point. While the ChiSquared distribution fits quite well, the lack of convergence with non-linear and time/day variables precludes finding an even better fit with this distribution.

## Green Lot

The results for the first group of complex estimations for the Green lot satellite facility are presented in Table 17.

| Table 17 - Estimates for the Basic Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 23.071 | 0.4406 | 0 |
| $\beta_{e}$ | 0.034 | 0.0172 | 0.048 |
| $\beta_{x}$ | 0.0034 | 0.0044 | 0.4496 |
| $\beta_{f}$ | -0.0319 | 0.0143 | 0.026 |
| $\beta_{c 120}$ | -1.0395 | 0.2412 | 0 |
| Chi-Squared Distribution |  |  |  |
| $\alpha$ | 22.8584 | 3.8223 | 0.0 .1486 |
| $\beta_{e}$ | 0.063 | 0.038 | 0.6716 |
| $\beta_{x}$ | -0.0009 | 0.121 | 0.9819 |
| $\beta_{f}$ | -0.02 | 2.0131 | 0.8686 |
| $\beta_{c 120}$ | -1.3345 | 0.5074 |  |
| $\alpha_{\sigma}$ | 6.5782 | 0.4384 | 0 |
| $\beta_{\sigma e}$ | -0.0051 | 0.0174 | 0.7688 |
| $\beta_{\sigma x}$ | 0.0083 | 0.0051 | 0.1066 |
| $\beta_{\sigma f}$ | -0.0074 | 0.0151 | 0.6249 |
| $\beta_{\sigma c 120}$ | -0.0011 | 0.2915 | 0.9971 |
| $\alpha_{\mu}$ | 23.0713 | 0.6145 | 0 |
| $\beta_{\mu e}$ | 0.0312 | 0.0242 | 0.1979 |
| $\beta_{\mu x}$ | 0.0043 | 0.0066 | 0.5159 |
| $\beta_{\mu f}$ | -0.0342 | 0.0204 | 0.0941 |
| $\beta_{\mu c 120}$ | -1.0394 | 0.3406 | 0.0023 |

The results for this lot are similar to those of the Blue above, with the exception that less precise estimates are obtained. Some of the variables have negative impacts on for some distributions. More will be said concerning this below as more variables are added to the estimations.

Figure 20 depicts the new distributions against the observed data.
Nonparametric Density Function Graphs for Green Lot


Figure 20
As in the graph for the Blue lot, the new distributions are improved from the simple density functions but the Chi-squared remains the best fit. The Normal distribution appears to fit the data best in the region far to the left of the peak, but the Poisson and Chi-Squared are nearly as good in this region. The Chi-Squared describes the peak of the distribution the best and is nearly perfect to the right of the peak. It is clear that the Chi-Squared remains the best distribution to describe this data.

The results for the estimations with second-order terms are presented in Table 18.

| Table 18 - Estimates for the Second Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 19.6589 | 1.855 | 0 |
| $\beta_{e}$ | -0.1998 | 0.0832 | 0.0164 |
| $\beta_{x}$ | 0.1355 | 0.0327 | 0 |
| $\beta_{f}$ | -0.3129 | 0.0862 | 0.0003 |
| $\beta_{e e}$ | -0.0029 | 0.0013 | 0.0278 |
| $\beta_{e x}$ | 0.0038 | 0.0012 | 0.0025 |
| $\beta_{e f}$ | -0.0126 | 0.0034 | 0.0002 |
| $\beta_{x x}$ | -0.0003 | 0.0002 | 0.0622 |
| $\beta_{x f}$ | 0.0017 | 0.0007 | 0.0152 |
| $\beta_{f f}$ | -0.0002 | 0.0016 | 0.9025 |
| $\beta_{c 120}$ | -0.2668 | 0.322 | 0.4074 |
| $\beta_{c 120 c 120}$ | -0.4114 | 0.2517 | 0.1022 |

The results for the estimations with all the variables are presented in Table 19. Note that only certain estimations work in this case.

| Table 19 - Estimates for the Second Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 8.6087 | 2.9626 | 0.0037 |
| $\beta_{\text {tue }}$ | 0.3691 | 0.4995 | 0.4599 |
| $\beta_{\text {wed }}$ | 3.889 | 1.4317 | 0.0066 |
| $\beta_{\text {thur }}$ | 10.0641 | 4.0632 | 0.0133 |
| $\beta_{\text {fri }}$ | 7.2919 | 3.2855 | 0.0265 |
| $\beta_{\text {sat }}$ | 0.7636 | 1.541 | 0.6202 |
| $\beta_{\text {sun }}$ | -1.0949 | 0.9511 | 0.2497 |
| $\beta_{\text {amrush }}$ | -2.1067 | 4.7166 | 0.6551 |
| $\beta_{\text {lateam }}$ | -0.1332 | 4.9474 | 0.9785 |
| $\beta_{\text {midday }}$ | 2.1437 | 4.5604 | 0.6383 |
| $\beta_{\text {earlypm }}$ | 2.8582 | 4.3449 | 0.5107 |
| $\beta_{\text {pmrush }}$ | 3.4373 | 3.8655 | 0.3739 |
| $\beta_{e}$ | -0.3186 | 0.128 | 0.0128 |
| $\beta_{x}$ | 0.2519 | 0.084 | 0.0027 |
| $\beta_{f}$ | -0.3156 | 0.1368 | 0.0211 |
| $\beta_{\text {cl20 }}$ | -0.9878 | 1.2196 | 0.418 |
| $\beta_{\text {ee }}$ | -0.003 | 0.0016 | 0.0675 |
| $\beta_{\text {ex }}$ | 0.0095 | 0.0034 | 0.0052 |
| $\beta_{e f}$ | -0.0267 | 0.0083 | 0.0013 |
| $\beta_{\text {ec120 }}$ | -0.0655 | 0.0383 | 0.0874 |
| $\beta_{x x}$ | -0.0001 | 0.0003 | 0.7709 |
| $\beta_{x f}$ | 0.0013 | 0.0011 | 0.2067 |
| $\beta_{\text {xcl20 }}$ | -0.0835 | 0.0436 | 0.0554 |
| $\beta_{\text {ff }}$ | 0.0075 | 0.0043 | 0.077 |
| $\beta_{\text {fc120 }}$ | -0.1835 | 0.0692 | 0.008 |
| $\beta_{\text {c120c120 }}$ | -0.4173 | 0.5514 | 0.4491 |
|  |  |  |  |

Figure 21 shows the various Poisson distributions against the observed data.


Figure 21
The Poisson distribution with all the right-hand side variables better describes the green lot data, but the difference is less dramatic than for the blue lot. The distribution fits the data better with each successive addition of descriptive variables. The fit never attains the level of precision of the Chi-Squared distribution however. As with the other lots, the most significant improvement in the fit comes when both the non-linear and time/day variables are included in the estimation of the density function.

Figure 22 shows the various Normal distributions against the observed data.


Figure 22
The Normal distribution changes only slightly when more descriptive variables are added. Again, this new distribution is an analytical improvement over the simple density function that is not visually obvious. It is possible that an even better fit could be obtained with the addition of non-linear and time/day variables.

Figure 23 shows the various Chi-Squared distributions against the observed data.


Figure 23
The Chi-Squared distribution has an improved fit with the added descriptive variables. The changes in the distribution are visually obvious and, similar to the blue lot, primarily at and to the right of the peak of the data. Again, this distribution fits the data quite well but even better representations might be possible with non-linear and time/day variables.

All lots combined

The results for the first group of complex estimations for the ESP facility are presented in Table 20.

| Table 20 - Estimates for the Basic Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 21.9641 | 0.2344 | 0 |
| $\beta_{e}$ | 0.003 | 0.0015 | 0.0415 |
| $\beta_{x}$ | 0.0573 | 0.0059 | 0 |
| $\beta_{f}$ | 0.0218 | 0.0057 | 0.0001 |
| $\beta_{c 120}$ | -0.018 | 0.1216 | 0.8825 |
| Chi-Squared Distribution |  |  |  |
| $\alpha$ | 20.8978 | 2.0363 | 0 |
| $\beta_{e}$ | -0.0036 | 0.0127 | 0.7776 |
| $\beta_{x}$ | 0.0429 | 0.0538 | 0.4253 |
| $\beta_{f}$ | 0.0179 | 0.0485 | 0.7113 |
| $\beta_{c 120}$ | -0.3264 | 1.1734 | 0.7809 |
| $\alpha_{\sigma}$ | Normal Distribution |  |  |
| $\beta_{\sigma e}$ | 7.7684 | 0.2589 | 0 |
| $\beta_{\sigma x}$ | 0.006 | 0.0015 | 0.0001 |
| $\beta_{\sigma f}$ | 0.025 | 0.006 | 0 |
| $\beta_{\sigma c 120}$ | 0.0296 | 0.0063 | 0 |
| $\alpha_{\mu}$ | -0.1101 | 0.1807 | 0.5425 |
| $\beta_{\mu e}$ | 21.966 | 0.3529 | 0 |
| $\beta_{\mu x}$ | 0.0022 | 0.0022 | 0.3246 |
| $\beta_{\mu f}$ | 0.0621 | 0.0089 | 0 |
| $\beta_{\mu c 120}$ | 0.0201 | 0.0084 | 0.0167 |
|  | -0.0075 | 0.1741 | 0.9655 |

The results for the combined Satellite lots are very similar to those for the Blue lot. This implies that the negative signs in the Green lot are a bit deceptive. As variables are added to the estimations more can be said on this. The departing capacity variable has no impact on the access time in all of the distributions. This is most likely because the other variables fully describe this impact. In other words, as departing capacity increases so do the number of cars entering and leaving the lot and the percentage of spaces filled in the lot. Thus, the impact of the departing capacity is already represented by variables that are more directly related to access times (volume at the facility being studied). All of the variables with positive impacts on access times also increase variance. This will be important in the congestion analysis later.

Figure 24 depicts the new distributions against the observed data.


Figure 24
The graph of the first set of complex density functions for all the satellite lots confirms the results of the two lots individually. The Chi-Squared describes the data extremely well throughout the range of the data. In the important region to the right of the peak, the ChiSquared is by far the best distribution.

The results for the estimations with second-order terms are presented in Table 21.

| Table 21 - Estimates for the Second Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 22.2139 | 0.4501 | 0 |
| $\beta_{e}$ | 0.0184 | 0.0071 | 0.0092 |
| $\beta_{x}$ | 0.0315 | 0.0163 | 0.0534 |
| $\beta_{f}$ | 0.0293 | 0.0256 | 0.2533 |
| $\beta_{e e}$ | 0 | 0 | 0.9982 |
| $\beta_{e x}$ | -0.0004 | 0.0001 | 0.0015 |
| $\beta_{e f}$ | 0.0006 | 0.0002 | 0.0001 |
| $\beta_{x x}$ | -0.0011 | 0.0003 | 0 |
| $\beta_{x f}$ | -0.0008 | 0.0005 | 0.1012 |
| $\beta_{f f}$ | 0.0003 | 0.0004 | 0.4232 |
| $\beta_{c 120}$ | 0.8169 | 0.2071 | 0.0001 |
| $\beta_{c 120 c 120}$ | -0.447 | 0.1093 | 0 |

The results for the estimations with all the variables are presented in Table 22. Note that only certain estimations work in this case.

| Table 22 - Estimates for the Second Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 21.1543 | 1.1908 | 0 |
| $\beta_{\text {tue }}$ | -0.7044 | 0.6221 | 0.2575 |
| $\beta_{\text {wed }}$ | -1.1777 | 1.0521 | 0.263 |
| $\beta_{\text {thur }}$ | -0.248 | 0.7867 | 0.7526 |
| $\beta_{\text {fri }}$ | -1.4128 | 0.8714 | 0.105 |
| $\beta_{\text {sat }}$ | -3.3808 | 1.0525 | 0.0013 |
| $\beta_{\text {sun }}$ | -1.9599 | 0.9035 | 0.0301 |
| $\beta_{\text {amrush }}$ | 1.3457 | 0.7479 | 0.072 |
| $\beta_{\text {lateam }}$ | 2.5105 | 0.6533 | 0.0001 |
| $\beta_{\text {midday }}$ | 3.7375 | 0.6245 | 0 |
| $\beta_{\text {earlypm }}$ | 5.3597 | 0.5752 | 0 |
| $\beta_{\text {pmrush }}$ | 4.599 | 0.5551 | 0 |
| $\beta_{e}$ | 0.0197 | 0.0117 | 0.0928 |
| $\beta_{x}$ | -0.0007 | 0.0275 | 0.9802 |
| $\beta_{f}$ | 0.1837 | 0.047 | 0.0001 |
| $\beta_{\text {cl20 }}$ | 2.9859 | 0.5104 | 0 |
| $\beta_{e e}$ | 0 | 0.0001 | 0.9665 |
| $\beta_{\text {ex }}$ | -0.0002 | 0.0002 | 0.1658 |
| $\beta_{e f}$ | 0.0005 | 0.0002 | 0.0137 |
| $\beta_{\text {ecl20 }}$ | 0.0039 | 0.0046 | 0.3947 |
| $\beta_{x x}$ | -0.0009 | 0.0003 | 0.003 |
| $\beta_{x f}$ | -0.0004 | 0.0007 | 0.5658 |
| $\beta_{\text {xc120 }}$ | 0.0161 | 0.0092 | 0.0817 |
| $\beta_{\text {ff }}$ | 0.003 | 0.0009 | 0.0011 |
| $\beta_{\text {fc120 }}$ | 0.0412 | 0.0122 | 0.0007 |
| $\beta_{\text {c120c120 }}$ | -1.1424 | 0.1834 | 0 |
|  |  |  |  |
|  |  |  | 0 |

Figure 25 shows the various Poisson distributions against the observed data.


Figure 25
The Poisson distribution for all the satellite lots is somewhere between the blue and green lot in its ability to predict the distribution of access times. The predictive power is increasing in the number of descriptive variables and the fit gets better at all locations on the distribution. However, the Poisson distribution with the full complement of descriptive variables never approaches the level of fit the Chi-Squared distribution displays with few descriptive variables. As in the individual satellite lots, the most obvious improvement is obtained with both non-linear and time/day variables included in the estimation.

Figure 26 shows the various Normal distributions against the observed data.


Figure 26
As with the other satellite lots, the analytical improvement in the fit of the normal distribution with the additional descriptive variables is barely noticeable visually. The peak has been drawn up and the right-hand side of the distribution is slightly more accurately predicted. As with all the other estimations of the Normal distribution, the variables that create the largest improvement in the fit of the function are not included due to lack of convergence.

Figure 27 shows the various Chi-Squared distributions against the observed data.


Figure 27
As with both the blue and green lots, the more complex density function leads to a better fit to the data at the peak and to the right of the peak. The Chi-Squared distribution seems to be the best fit to the data and thus only small improvements in fit are obtained with added descriptive variables. As with the individual satellite lots, the distribution is a very good fit but the nonlinear and time/day variable impacts are not obtained due to the lack of convergence.

## III.D Drop-Off Passengers

The data for drop-off passengers is also analyzed with complex density functions. However, a slight difference is necessitated by the lack of data concerning the use of the lot that was utilized for the other facilities. There is no data on the number of cars entering, leaving or physically located in the terminal roadway. Thus, the only right-hand side variables used to analyze congestion are the dummy variables for time of day and day of week along with the departing capacity. This means that all terms for the variables $e, x$ and $f$ are not included since they are not available for these data.

| Table 23 - Estimates for the Linear Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 97.6023 | .7345 | 0 |
| $\beta_{c 120}$ | 7.7772 | .8122 | 0 |
| $\alpha$ | 83.5833 | .3021 | 0 |
| $\beta_{c 120}$ | 7.1196 | .3364 | 0 |
| Normal Distribution |  |  |  |
| $\alpha_{\sigma}$ | 68.5322 | 1.1462 | 0 |
| $\beta_{\sigma c 120}$ | 4.6626 | 1.5505 | 0.003 |
| $\alpha_{\mu}$ | 97.6015 | 1.6080 | 0 |
| $\beta_{\mu c 120}$ | 7.7758 | 1.9085 | 0 |

The graphs of these density functions against the actual data are contained in Figure 28. Again, the data are aggregated in 10 -second intervals for presentation.


Figure 28
The Poisson distribution fits the data well with the exception of being slightly right shifted. The Chi-Squared distribution has difficulty with the distribution and does not capture the right-hand tail very well at all. The Normal distribution severely underestimates the peak but captures the far right end of the distribution. When examining the percentile passengers below, the poisson will be the best except for high percentile passengers in which case the normal may actually better describe the access time process.

The results for the estimates of the density functions that contain second-order terms are presented in Table 24.

| Table 24-Estimates for the Non-Linear Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 97.5640 |  |  |
| $\beta_{c 120}$ | 7.7704 |  |  |
| $\beta_{c 120 c 120}$ | .00797 |  | 0 |
| $\alpha$ | Chi-Squared Distribution |  |  |
| $\beta_{c 120}$ | 83.4687 | .3053 | 0 |
| $\beta_{c 120 c 120}$ | 5.5899 | .5849 | 0.002 |

The results from the full complex density function estimations are contained in Table 25.

| Table 25 - Estimates for the Second Complex Density Functions |  |  |  |
| :---: | :---: | :---: | :---: |
| Poisson Distribution |  |  |  |
| Parameter | Estimates | Standard Error | Probability |
| $\alpha$ | 109.1568 | 2.8209 | 0 |
| $\beta_{\text {tue }}$ | -1.1429 | 2.9463 | .6981 |
| $\beta_{\text {wed }}$ | -39.0385 | 2.9072 | 0 |
| $\beta_{\text {thur }}$ | -35.9897 | 2.5098 | 0 |
| $\beta_{\text {fri }}$ | -10.6754 | 2.8738 | 0 |
| $\beta_{\text {sat }}$ | -11.9001 | 3.0275 | 0 |
| $\beta_{\text {sun }}$ | -49.2604 | 3.5919 | 0 |
| $\beta_{\text {amrrsh }}$ | 0.9212 | 2.8479 | .7463 |
| $\beta_{\text {lateam }}$ | 10.5234 | 2.4029 | 0 |
| $\beta_{\text {midday }}$ | -1.8622 | 2.8824 | .5182 |
| $\beta_{\text {earlypm }}$ | 6.2619 | 2.8807 | .0297 |
| $\beta_{\text {pmrrush }}$ | 15.9967 | 2.8166 | 0 |
| $\beta_{\text {c120 }}$ | 20.1194 | 9.927 | 0 |
| $\beta_{\text {cl20c120 }}$ | -1.2701 | .5267 | .0159 |

The density functions are graphed against the actual....


Figure 29
As the complexity of the distribution is increased, the fit becomes better at all points in the data. The right hand tail of the distribution is never fully explained by the Poisson distribution.


Figure 30
While the linear density function is better than the simple density function, the Normal distribution is not good at describing the peak of the distribution.


## Figure 31

The Chi-Squared distribution does not work well for these data. Unlike the satellite lots, where it fit quite well, the drop-off passenger data are not amenable to use of this distribution.
III.E Summary of Complex Density Function Analysis

The increase in predictive power as terms are added to the density function is apparent and intuitive. This better fit will be put to use in the following sections to describe the access patterns from the various facilities. Given the difference in the goodness of fit from the various distributions, each facility will have a different distribution that is the best and thus discussed the most in the following sections.

## IV. Facility Level Congestion Analysis

The purpose of estimating the density functions presented in the last two sections was to create a statistical base from which to examine congestion curves. For all but the Poisson distribution, the congestion curves will not have curvature (this will become clear later) but will still show the impact of increasing volume on access time if only in a linear fashion.

The impacts on access times are presented first for the full facility. These are the least informative but the easiest to understand and follow. After looking at the impact of various phenomena on the access times for an entire lot, the impacts will be examined for two different sub-sets. Congestion will be studied at particular points in time and the impact of various conditions on access time will be examined specifically.

The access times and congestion will be studied by looking at several figures. Primarily, the distribution of access times will be examined and various benchmarks will be used to discuss congestion. Table 26 below describes the measures used.

| Table 26 - Benchmark Measures for Describing Congestion |  |
| :---: | :---: |
| Name | Description |
| $\overline{a t}$ | $10^{\text {th }}$ Percentile of the Access Time Distribution ${ }^{3}$ |
| $10 \%$ tile | $25^{\text {th }}$ Percentile of the Access Time Distribution |
| $25 \%$ tile | $50^{\text {th }}$ Percentile of the Access Time Distribution |
| $50 \%$ tile | $75^{\text {th }}$ Percentile of the Access Time Distribution |
| $75 \%$ tile | $90^{\text {th }}$ Percentile of the Access Time Distribution |
| $90 \%$ tile | $95^{\text {th }}$ Percentile of the Access Time Distribution |
| $95 \%$ tile | $99^{\text {th }}$ Percentile of the Access Time Distribution |
| $99 \%$ tile | $99.9^{\text {th }}$ Percentile of the Access Time Distribution |
| $999 \%$ tile |  |

The mean of the access times in the distribution returns the average time it takes a passenger to access the terminal. There will be times that the mean is a relevant measure, but this is not likely to be the most important benchmark measure. The percentiles of the distribution are used because this is most likely the measure of importance in maintaining satisfactory service levels at the facilities. It is likely that airport officials desire to have "X percentage of passengers accessing the terminal within Y minutes". Most likely the percentiles of interest in this setting are between the $75^{\text {th }}$ and the $99^{\text {th }}$. The lower percentiles are given to show the impact of various conditions on the fastest users of the facility and the higher percentiles are given to demonstrate the true impact of heavy congestion on the "unlucky" passenger.

Unless otherwise specified, the Poisson distribution with full descriptive variables and the ChiSquared and Normal distributions with the basic complex density function are used in this analysis. This means that the only distribution that can display curvature to the access time or congestion levels is the Poisson.

[^2]
## IV.A ESP Facility

The ESP facility remains the most difficult lot to work with due to the lack of observations. Table 27 contains the benchmark levels for the three distributions.

| Table 27-Access Time and Congestion Benchmarks |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Poisson | Normal | Chi-Squared |
| $\overline{a t}$ | 12.2455 | 10.5863 | 10.7928 |
| $10 \%$ tile | 5.9559 | 8.3595 | 5.1677 |
| $25 \%$ tile | 8.4490 | 9.2710 | 7.0413 |
| $50 \%$ tile | 11.5565 | 10.1277 | 9.6384 |
| $75 \%$ tile | 14.7412 | 10.8968 | 12.8370 |
| $90 \%$ tile | 17.6714 | 11.7720 | 16.2789 |
| $95 \%$ tile | 19.5195 | 12.3059 | 18.6049 |
| $99 \%$ tile | 23.4735 | 13.3773 | 23.5098 |
| $999 \%$ tile | 29.3843 | 14.6814 | 29.8698 |

Figure 32 shows the benchmarks for the three distributions.

Percentile Access Times for Esp Lot


Figure 32
A few comments are possible from these figures. From Table 27 it is apparent that all three of the distributions are left-shifted from central ( $50^{\text {th }}$ percentile value less than the average). More will be said of this after the all the congestion results are presented. The Normal distribution provides unsatisfactory percentile results (this could have been corrected by using the simple instead of the basic complex density function for these results) but the other two distributions are similar.

The access times are steadily climbing through the percentiles at a relatively constant slope. This is by no means necessary (see graphs below for difference). The Poisson or the Chi-Squared distributions are the best for describing this data and so the benchmarks from those distributions can be discussed. While these are still aggregate figures, a few comments can be made. While the mean access time for this lot is around 10 minutes (according to actual data and the distributions estimated), the $90^{\text {th }}$ percentile access time is about 16-17 minutes. This means that it takes longer than this for $10 \%$ of the passengers to access the terminal from this facility. The $95^{\text {th }}$ percentile access time is about 19 minutes and the $99^{\text {th }}$ percentile access time is about 23.5 minutes. This situation is more strongly represented in the time of day and day of week congestion analysis below.

## IV.B Garage Facility

Table 28 contains the benchmark levels for the three distributions.

| Table 28 - Access Time and Congestion Benchmarks |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Poisson | Normal | Chi-Squared |
| $\overline{a t}$ | 8.9432 | 9.0997 | 8.3906 |
| $10 \%$ tile | 4.8258 | 3.8305 | 3.5330 |
| $25 \%$ tile | 6.3607 | 6.0256 | 5.0661 |
| $50 \%$ tile | 8.2559 | 8.5316 | 7.2446 |
| $75 \%$ tile | 10.3198 | 11.0732 | 10.0035 |
| $90 \%$ tile | 12.3172 | 13.4018 | 13.0596 |
| $95 \%$ tile | 13.5847 | 14.7777 | 15.1633 |
| $99 \%$ tile | 15.9934 | 17.3979 | 19.6674 |
| $999 \%$ tile | 18.9615 | 20.3184 | 25.6019 |

Figure 33 shows the benchmarks for the three distributions.
Percentile Access Times for Garage Lot


Figure 33

Again, all of these distributions are left-shifted from central. The Chi-Squared distribution did the best job of describing the data and so most attention is paid to that distribution. Note the rapid rise in access time at high percentiles in this facility. This is partially a function of the fact that there are no shuttle buses. When there are shuttle buses running from the parking facility to the airport terminal, people are more likely to make an effort to catch the next shuttle (since they don't know for sure how long it will be between shuttles). In the garage, no shuttle is necessary. If someone finds a parking space quickly and has plenty of time they may choose to rearrange their luggage (or whatever activity they choose) in the lot. They can do this because they are not dependent on an external schedule (shuttle) to get the rest of the way to the terminal.

The increasing slope of the percentile lines does imply one other thing about congestion in the garage facility. This is indicative of true congestion at high usage levels. While not conclusive proof, this is compatible with there being a small subset of people that experience highly elevated access times from this facility. This will be discussed in detail in the sections that examine congestion under particular conditions.
IV.C Satellite Facilities

Blue Lot
Table 29 contains the benchmark levels for the three distributions.

| Table 29 - Access Time and Congestion Benchmarks |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Poisson | Normal | Chi-Squared |
| $\overline{a t}$ | 20.9684 | 20.2153 | 19.3520 |
| $10 \%$ tile | 14.0599 | 11.8197 | 11.5056 |
| $25 \%$ tile | 16.6888 | 15.4692 | 14.4079 |
| $50 \%$ tile | 19.9138 | 19.5675 | 18.1782 |
| $75 \%$ tile | 23.5730 | 23.7848 | 22.5661 |
| $90 \%$ tile | 27.5027 | 27.7721 | 27.0475 |
| $95 \%$ tile | 30.3384 | 30.2827 | 29.9933 |
| $99 \%$ tile | 36.7518 | 35.3473 | 36.0746 |
| $999 \%$ tile | 44.3119 | 41.7966 | 43.7605 |

Figure 34 shows the benchmarks for the three distributions.
Percentile Access Times for Blue Lot


## Figure 34

As with all the lots, the distribution for the blue lot is left-shifted. The full complex Poisson and the Chi-Squared distributions are the best fit to the data and they will be the results discussed here. Note the increasing slope of the percentile curves at high percentile values. As with the garage lot, this is indicative (though not proof) of congestion for some periods in the sample. More will be said on this in later sections when specific factors leading to congestion are discussed.

The median passenger accesses the terminal in about 19 minutes. The rapid escalation of values occurs after about the $75^{\text {th }}$ percentile. The $99^{\text {th }}$ percentile access times are about double the median percentile access times. The underestimating of the percentile benchmarks by the Normal distribution becomes quite obvious at these higher percentiles. The curvature of the access time graph (congestion curve) will be looked at in detail below.

## Green Lot

Table 30 contains the benchmark levels for the three distributions.

| Table 30 - Access Time and Congestion Benchmarks |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Poisson | Normal | Chi-Squared |
| $\overline{a t}$ | 22.5111 | 22.9939 | 22.0628 |
| $10 \%$ tile | 15.4481 | 13.9881 | 13.6696 |
| $25 \%$ tile | 18.3218 | 17.9937 | 16.8305 |
| $50 \%$ tile | 21.7292 | 22.4598 | 20.8858 |
| $75 \%$ tile | 25.3788 | 26.9531 | 25.5659 |
| $90 \%$ tile | 28.8779 | 31.0307 | 30.3271 |
| $95 \%$ tile | 31.1074 | 33.5210 | 33.4405 |
| $99 \%$ tile | 35.7421 | 38.2008 | 39.8061 |
| $999 \%$ tile | 42.1541 | 43.6301 | 47.7850 |

Figure 35 shows the benchmarks for the three distributions.
Percentile Access Times for Green Lot


Figure 35

While the full complex Poisson distribution fits the data reasonably well, the Chi-Squared distribution is by far the best fit for this lot. The distribution for this lot has a different shape than the Blue lot. It is slightly less left-shifted. The percentile access times are closer to those from the blue lot early than late. This means that there are most likely a larger number of observations from the truly congested periods for this lot. This will be discussed in more detail in later sections.

## All Lots

Table 31contains the benchmark levels for the three distributions.

| Table 31 - Access Time and Congestion Benchmarks |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Poisson | Normal | Chi-Squared |
| $\overline{a t}$ | 21.8201 | 21.5169 | 20.5526 |
| $10 \%$ tile | 15.1275 | 12.6022 | 12.5028 |
| $25 \%$ tile | 17.8403 | 16.5299 | 15.5068 |
| $50 \%$ tile | 21.0706 | 20.9125 | 19.3876 |
| $75 \%$ tile | 24.5373 | 25.3783 | 23.8674 |
| $90 \%$ tile | 27.8391 | 29.5348 | 28.4611 |
| $95 \%$ tile | 29.9015 | 32.0933 | 31.4619 |
| $99 \%$ tile | 33.9435 | 37.1953 | 37.6238 |
| $999 \%$ tile | 38.7530 | 43.4613 | 45.3718 |

Figure 36 shows the benchmarks for the three distributions.


Figure 36
As for each of the other analyses, the total satellite results lie "between" the blue lot and green lot results. The strong increase in the slope of the percentile lines again implies the presence of strong congestion during at least some periods.

## IV.D Drop-Off Passengers

Table 32 contains the benchmark levels for the drop-off passengers.

| Table 32 - Access Time and Congestion Benchmarks |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Poisson | Normal | Chi-Squared |
| $\overline{a t}$ | 97.56 | 97.60 | 83.47 |
| $10 \%$ tile | 49.33 | 27.92 | 56.54 |
| $25 \%$ tile | 68.13 | 57.40 | 64.54 |
| $50 \%$ tile | 91.81 | 97.81 | 74.26 |
| $75 \%$ tile | 117.99 | 142.03 | 84.79 |
| $90 \%$ tile | 143.55 | 184.75 | 95.41 |
| $95 \%$ tile | 159.24 | 209.24 | 102.01 |
| $99 \%$ tile | 190.30 | 259.31 | 118.04 |
| $999 \%$ tile | 227.68 | 320.97 | 138.08 |

Figure 37 shows these benchmarks for each distribution.


Figure 37

## IV.E. Summary of Facility Level Congestion Analysis

The congestion curves show by the percentile access times are of interest for several reasons. First, they definitely imply that congestion is an issue for at least a small percentage of passengers. In most of the lots the $99^{\text {th }}$ percentile access time is approximately twice the median access time. For the drop-off passengers this increase is even greater - approaching 2.5 times the median access time.

This relationship shows that using the difference in average access time between various lots and conditions might strongly understate the congestion impact. If the airport is concerned with the access time of a high percentile passenger, the impact might be as much as 2.5 times higher than that found with an analysis of the means.

The strongly increasing slope of the percentile curves implies that the density functions are capturing at least some of the congestion that is present at high volume time periods. The next section will specifically address the congestion during these periods.

## V. Time of Day and Day of Week Congestion Analysis

In this section, the distributions are analyzed by time of day and day of week. Actual congestion analysis is saved for the next section, but the impact of increasing volume on the distributions is evident by examining the lots under differing conditions. For each lot, all three distributions are compared for each of four different time periods. In addition, each distribution is compared across the four dime periods. The graphs of the three distributions across time periods are presented mostlyto show how the different distributions react to the differing conditions. Commentary will be limited to discussion of how the individual distributions change over the time periods.

## V.A ESP Facility

Figures 38 and 39 show the different distributions for the two time periods of relevance for the ESP lot.

```
    Tuesday AM Rush
Complex Density Function Graphs
    for the ESP Lot Access Times
```



Figure 38


Figure 39

Figures 40,41 , and 42 show how each of the distributions handle the changing conditions. Only the Normal distribution changes enough to be noticeable. This is likely a result of the small number of observations for this lot.


Figure 40


Figure 41


Figure 42

The Normal distribution shows a small change between the two time periods. A slightly more spread and rightward shifted distribution is evidenced for the Saturday time period. Not much en be read into this given the small number of observations and the small change evidenced in the distributions.

## V.B Garage Facility

Figures $43,44,45$, and 46 show the various distributions over the four time periods.

> Tuesday AM Rush
> Complex Density Function Graphs
> for the Garage Lot Access Times


Figure 43

```
Wednesday Early PN Complex Density Function Graphs
for the Garage Lot Access Times
```



Figure 44


Figure 45


Figure 46
The main characteristic of these graphs worth noting is that each of the distributions remains fairly constant relative to the others over all four time periods. In other words, the Normal distribution always lies below and to the right of the other two distributions while the Poisson distribution has a higher peak that descends more rapidly than the other two distributions in all four time periods. The Chi-Squared distribution always has a larger tail to the far right side of the distribution than the other two distributions. Thus, the characteristics of the fir for the entire lot remain the same in the individual time periods as well.

Figures 47,48 , and 49 contain the graphs of the individual distributions for each of the time periods.

Nonparametric Poisson Density Functions for Garage Lot over time


Figure 47
The Poisson distribution shifts to the right in more busy time periods. The least busy period is the Saturday morning time period and the busiest (for the garage) is the Wednesday early evening period (See Reed (2001) for details of garage and other lot activity). The shifts are quite small and this explains the small shift in the overall density function as parameters are added.


## Figure 48

Given the small change in the distribution for the entire garage facility, the small changes over time periods are not surprising for the Chi-Squared distribution. With more explanatory variables, perhaps this change would be greater. However, the garage data seems to be more susceptible to changes in variance than in average access time (see the Normal distribution below).


Figure 49
The Normal distribution shows a small change in the average access time (relative location of the peak) and also a small change in the "spread" of the distribution (variance). The changes are small and hard to see, but the distribution does "spread" during peak periods. This is evidence that at peak periods there is congestion for at least a portion of the passengers that use the facility.

## V.C Satellite Facilities

## Blue Lot

Figures 50, 51, 52, and 53 show the various distributions over the time periods.

```
    Tuesday AM Rush
Complex Density Function Graphs
    for the Blue Lot Access Times
```



Figure 50


Figure 51


Figure 52


Figure 53
In the satellite lots the value of the non-linear terms can be seen. While the Chi-Squared distribution and the Poisson distribution fit the overall data equally well, the Poisson moves with changing conditions more than the Chi-Squared. In each of the time periods, the Chi-Squared lies to the right of the Normal with a larger far right tail in the distribution. However, the relative location of the Poisson changes over the time periods.

In the Tuesday morning and Saturday morning periods, the Poisson lies to the left of the chiSquared and has a much smaller tail. In the Wednesday afternoon period, the Poisson distribution lies to the right of the Chi-Squared for much of the distribution with a very similar tail. In the Thursday mid-day period, the Poisson lies almost directly above the Chi-Squared but with a much smaller tail. While the Chi-Squared did a better job of describing the data for the whole distribution, the non-linear terms allowed a more micro-level analysis with the Poisson distribution.

Figures 54,55 , and 56 show the various times of day for the different distributions.


Figure 54
The Poisson distribution moved considerable to caption the differences in the distribution of access time over the different periods. The two higher volume periods (Wednesday early afternoon and Thursday mid-day) are represented with distributions that are right-shifted and have elongated right-hand side tails (this is less obvious than in the comparison to the simple density function in Section III). The two lower volume periods (Tuesday morning rush hour and Saturday morning) have peaks further to the left with smaller right-side tails. The non-linear and time variables moved the distribution significantly to match the data.


Figure 55
The Chi-Squared distribution shifts to the right to capture higher volume periods, but not nearly as far as the Poisson. The lack of non-linear and time variables precludes a larger shift in the distributions. The Chi-Squared distribution fits the data quite well, but is not as good for microlevel analysis as a distribution with more terms.


Figure 56
While the Normal distribution does not describe the data well in general, some interesting results are evident from these distributions. First, the peaks do move to the right in higher volume periods. Second, the distribution is both the furthest left and the narrowest of the distributions shown. The Thursday afternoon period is the furthest right and also the widest distribution. This means that as volumes increase, not only does it take longer for the average passenger to access the terminal; it also means an even stronger increase in higher percentile passengers access times (see below for this analysis).

## Green Lot

Figures $57,58,59$, and 60 show the different distributions over the various time periods.


Figure 57

```
Wednesday Early PM
Complex Density Function Graphs
for the Green Lot Access Times
```



Figure 58


Figure 59

```
Saturday Late AM
Complex Density Function Graphs
for the Green Lot Access Times
```



Figure 60
The impact of the different time of day on the green lot distributions is almost identical to that for the blue lot. Discussion of the percentile changes is carried out below.

Figures 61,62 , and 63 show each distribution and how it reacts to the differing conditions.


Figure 61
The results on the green lot are nearly identical to those on the blue lot with the exception of the exact value over which the distributions are centered.


Figure 62
The changes in the Chi-Squared distribution for the green lot are also nearly identical to those for the blue lot. There is a bit more differentiation between each of the time periods in the green lot, but this difference is not drastic.


Figure 63
The changes in the Normal distribution over the time periods is similar to that for the blue lot but perhaps a little less obvious. The distributions still move right and spread as the volume in the period rises. Again, both sat4ellite lots behave similarly in these respects.

## ALL lots

Figures $64,65,66$, and 67 contain the graphs of the distributions for each time period.

$$
\begin{aligned}
& \text { Tuesday AM Rush } \\
& \text { Complex Density Function Graphs } \\
& \text { for all Satellite Lots Access Times }
\end{aligned}
$$



Figure 64

```
                                    Wednesday Early PN
Complex Density Function Graphs
for all Satellite Lots Access Times
```



Figure 65


Figure 66


Figure 67
The graphs for the combined lots are very similar to each of the individual lot graphs above. The nearly identical pattern to the lots in this micro-level analysis implies that the differences in access time patterns between the blue and green lot are not specific to the lot but are a result of differing conditions when the data were collected from these lots.

Figures 68,69 , and 70 show the movement of the distributions over the time periods.


Figure 68
There is almost no difference in the graph for both satellite lots compared to the individual lots.


Figure 69
The Chi-Squared distribution shows a bit less flexibility for the combined lots than for the individual lot data. This could be a function of the lack of non-linear and time variables terms. The graphs are basically the same as for the individual lots with a bit less flexibility across the time periods evidenced.


Figure 70
The Normal distribution graphs look very similar to the individual lot graphs. As volume increases, the distributions shift to the right and spread. This shows that at least some passengers are meeting with congestion in the higher volume periods.

## V.D DROP-OFF PASSENGERS

This facility differs slightly from the others in that there is no lot usage data. Thus, all the variance in time of day and day of week is derived from the dummy variables and the departing capacity variable. Figures $71,72,73$, and 74 show the various distributions over the four time periods.


Figure 71


Figure 72
Thursday Mid PM
Complex Density Function Graphs
for the Drop Off Times


Figure 73


Figure 74
The lack of fit from the Chi-Squared and Normal distributions is evident in all of the time periods. The Normal distribution suffers from a lack of non-linear and time specific effects due to the lack of convergence of these models and the Chi-Squared suffers from an inability to stretch to fit the data. Unlike the satellite (and other) facilities, the Chi-Squared distribution is not satisfactory at any time period. The Poisson distribution fits the data fairly well and shifts to meet the different conditions.

Figures 75, 76 and 77 contain the graphs of the individual distributions for each of the time periods

Drop-off, Poisson Nonparametric Density Functions with C120


Figure 75

The Poisson distribution shifts to accommodate the congestion present in the morning periods.
Both of the morning periods showed congestion, much heavier on Tuesday, and the distribution shifts to the right to accommodate this. Because the Poisson does not have a parameter for the spread of the distribution, the full congestion impact is not registered in the graph.


## Figure 76

The Chi-Squared distribution shows a poor fit to all the time periods. This distribution is not appropriate for these data.


Figure 77
The Normal distribution only converged for the linear case and so there is little shifting of the distribution across congestion time periods. The spread of the distribution fits the upper reaches of the distribution nicely, but due to lack of convergence, the distribution does not represent the middle of the data well at all.
V.E. Summary of Time of Day and Week Congestion Analysis

The use of the complex density functions allows the discovery of congested time periods from the analysis. In addition, it allows a graphical analysis of how strong the congestion is during any particular period. Those distributions that converge for higher complexity show the greatest ability to reflect the congestion in the data.

It is clear that there are highly congested times and also that during these periods the right hand tail of the distribution is further right shifted than the peak. This means that examining just the average access time during different periods does not reveal the true cost of congestion. If the facility is interested in a high percentile passenger, the impact of congestion is much higher than the difference in average access time during these periods.

## VI. Individual Variable Level Congestion Analysis

Some of the most important results from this study are the relationships between access time and the individual variables that can be monitored directly by airport authorities. These variables are those included on the right-hand side in the estimations. Access times are studied relative to each of these variables. The variables examined are: 1) the number of cars entering the facility during the hour, 2 ) the number of cars exiting the facility during the hour, 3 ) the percentage of parking spaces already filled when the passenger arrives at the facility, and 4) the departing capacity at the airport in the next two hours.

As stated above, the departing capacity variable used is that for 120 minutes. This seemed to be the departing capacity variable that led to the best fit in the estimations. However, other time periods were nearly as good (from 60 minutes to 180 minutes) due to the high correlation in these variables.

For each of the four variables, three cases are examined: a low, middle, and high value case. This means that, for the first variable, the impact of the number of cars entering the facility is examined for low values of the other variables, average values of the other variables, and high values of the other variables. This is equivalent to looking at the impact of the variable in question during slow periods at the airport, during average periods at the airport and during peak periods at the airport.

Each lot is again examined separately for each variable and case. The values of the variables in question are allowed to range roughly from their minimum observed value to $125 \%$ of their maximum observed value. This allows projection of the impact of the variable into the future when usage rates may increase at the airport. The impact of each of the variables is studied on both the mean of the access time distribution and the percentile benchmarks for all three distributions. For each distribution, the most complex case estimated is used in the analysis. This means that the Poisson distribution is examined for the full complex case and the Normal and Chi-Squared distributions are examined for the basic complex case. While the Poisson distribution will allow an examination of the curvature of the relationships it must be kept in mind that the Chi-Squared distribution was a better fit for almost every facility. Sometimes the results of these two distributions must be "averaged" to obtain the best indication of true relationships.

## VI.A. ESP Facility

Figure 78 shows the relationship between mean access time and the number of cars entering the facility under medium volume conditions.


Figure 78
Note that both the Chi-squared and Normal distributions display slightly upward slopes to the line. As more cars enter the facility the access time rises slightly. The impact of this variable appears to be quite low however. In the case of the Poisson distribution, where there are nonlinear terms, note that at low levels of volume an increase in the number of cars entering actually lowers the access time. This seems to be a contradiction. However, this is correct and picking up the influence of shuttle frequency. As volume increases, the frequency of the shuttles increases and thus the access time is reduced. This is true until the point where cars entering the facility congest the facility and slow the access down more than the increased shuttle frequency speeds access. This appears to happen at around 35-40 entries per hour. This is at roughly 60 percent of the maximum observed in the data.

The other variables are not examined for this lot because the low number of observations led to deceptive results in some variables. The lack of precision with which the variables are measured shows quite clearly in this micro-level analysis (particularly for the full complex Poisson distribution which had many parameters to estimate).

Figures 79,80 , and 81 show the relationships between access time and the various conditions affecting access times.

The relationship between the number of cars entering the facility and the access time is shown here for high volume conditions (Tuesday morning rush hour for the Poisson distribution). Both the Chi-Squared and the Normal distributions show a positive slope to the line indicating that increased volume slows access time at this facility. The Poisson distribution line shows a decrease in the access time for high levels of volume. It should be recalled that the actual data do not contain a value above approximately 625 . Thus, the curve beyond this point is not part of the estimation.

While it is doubtful that the curve actually turns down (and this is in the region that the Poisson distribution had the most difficulty fitting the actual data), the concavity of the line in the Poisson distribution is strong evidence that this variable is not responsible for congestion at this facility. It appears that the impact of more entering cars does not grow after 400-500 cars are entering the lot.


Figure 79

The relationship with the number of cars leaving seems to be constant or slightly downward sloping. This was true for all conditions and is show for low volume conditions for this facility (late morning on Saturday for the Poisson distribution). The Normal distribution shows a slight upward trend and the Chi-Squared a slight downward trend. The Poisson distribution shows a stronger, increasing and downward trend as the number of cars exiting the lot increases. It does not appear that the volume of cars leaving the lot contributes to congestion. In fact, the impact seems to be to reduce congestion ceteris parabus.

Congestion by Leaving Vehicles, Low volume, Saturday Late AM Estimated Access Time for Garage Lot


Figure 80
The relationship between departing capacity and the access time is presented for two differing sets of conditions. In figure 79, the relationship is shown during a high volume period (Tuesday morning rush hour for the Poisson Distribution). The Normal distribution shows a downward slope to the relationship while the Chi-Squared distribution shows a nearly constant (but downward) relationship. The Poisson shows a slight upward relationship at a much higher level of access times. This shows the impact of the non-linear and time variables included in the complex Poisson distribution that are not included in the Chi-Squared and Normal distribution estimations.


Figure 81
Figure 81, shows the same relationship between departing capacity and access times but for a very low volume period (Saturday morning for the Poisson distribution). The Normal and ChiSquared distributions are the same as in the last figure but at a slightly lower access time level. The Poisson distribution now shows a slight downward relationship at a very low access time. Again, the difference in levels between the Poisson and other distributions is attributable to the extra terms included in the estimation.

The percentage of spaces filled in the facility is not examined because the Poisson distribution fits this variable very poorly. The Chi-Squared and Normal distribution both show a strong upward slope to the relationship indicating that the number of spaces filled is indicative of elevated access times.

## Blue Lot

Figures 82-86 show the relationships between access time and the various conditions.


Figure 82
The relationship between the number of cars entering and access time is shown for low volume conditions (Saturday morning for the Poisson distribution). The Normal distribution shows a very slight upward trend to this relationship while the Chi-Squared distribution shows an equally slight downward trend. The Poisson distribution, with the extra non-linear terms, shows the rapidly increasing slope associated with congestion. In this very low volume period, average access times increase from about 17.5 to about 18 minutes as the number of cars entering moves from 0 to 100 (roughly 60 percent of the maximum in the data). The slope begins to increase rapidly from this point. From 100 entering cars to 150 entering cars the access time increases from 18 to 20 minutes. From 150 to 200 cars entering the access time increases from about 20 to 24 minutes. This is indicative of congestion effects from this variable. The fact that is observable at low volume conditions shows how important this variable is to congestion.


Figure 83
The relationship between cars leaving the lot and access time is also shown for low volume conditions (Saturday morning for the Poisson distribution). The Normal and Chi-Squared distributions both show an upward trend in this relationship. The Poisson distribution shows a downward trend. The better fit of the complex Poisson distribution leads to the conclusion that the volume of cars exiting has a downward impact on access times. This means that the congestion impact of the volume is outweighed by the increased shuttle frequency brought by exiting passengers at low volumes.


Figure 84
The relationship between the percentage of spaces filled in the facility and access times is also shown for low volume conditions. All three distributions show an upward trend in this relationship. There is a slight "steepening" of the Poisson distribution curve, which implies congestion effects from the lot being full. It appears that this relationship is roughly linear and that an increase in the percentage of spaces filled increases access time. It should be pointed out that the satellite lots are closed when they are close to full and thus some of the congestion impact is avoided or transferred to outside the lot (see Reed (2001) for a discussion of this).


Figure 85
The relationship between departing capacity and access times is shown for two different sets of conditions. Figure 85 shows the relationship for a high volume set of conditions (Thursday early afternoon for the Poisson distribution). The Normal and Chi-Squared distributions show an upward trend to the relationship. The departing capacity figures extend to twice the maximum observed in the data and so the highest value in the data (about 2.5) is associated with a 25 to 27 minute access time for these distributions. The Poisson distribution shows a much steeper slope to the relationship. This distribution shows an increase in access time from about 13 minutes at the lowest observed departing capacity to about 48 minutes at the highest observed departing capacity. It shows an access time of over 60 minutes with as little as a $20 \%$ increase in airside volume.


Figure 86
Figure 86 shows the same relationship for medium volume conditions (Tuesday mid-day for the Poisson distribution). The relationships remain the same except that the levels are lower and the slope of the relationship is less steep for the Poisson distribution. The average access time rises from about 16 minutes for the minimum capacity to about 34 minutes at the maximum observed capacity. The decreasing slope is deceptive and some of this congestion is actually picked up by entering cars rather than this variable.

## Green Lot

Figures $87,88,89$, and 90 show the relationships between access time and the various conditions.


Figure 87
The relationship between the number of cars entering the lot and access time is shown for a medium volume set of conditions (Tuesday mid-day for the Poisson distribution). The Normal and Chi-Squared distributions both show an upward trend in the relationship. The Poisson shows this as well except for at high levels of entering vehicles. However, the portion of the Poisson curve that is downward sloping is all above the actual observed values in the data and thus the downward trend should not be taken too seriously. The decreasing slope of the line merely shows that congestion from entering cars peaks at around 30 cars entering the lot per hour.


Figure 88
The relationship between the number of vehicles leaving the lot and the access time is also shown for a medium volume period. The Normal and Chi-Squared distributions show a slight downward trend (basically constant) and the Poisson distribution shows a nearly linear upward trend. The curve for the Poisson is not convex and thus this variable does not seem to indicate congestion for these data either.


Figure 89
The relationship between the percentage of spaces filled and the access time is represented for a medium volume set of conditions. The Normal and Chi-Squared distributions both show a slight downward trend. The Poisson shows a downward relationship until the lot is half filled and then a very rapidly increasing function. It should be noted that this lot was never less than about half filled during the study and thus the right side of this graph is more important. The relationship for the Poisson distribution indicates that this variable may be indicative of the congestion for the facility.


Figure 90
The relationship between departing capacity and access time is displayed for a low volume period Saturday morning for the Poisson distribution). The Chi-Squared and Normal distributions both show a negative relationship. The Poisson distribution shows an increasing relationship with a decreasing slope. For this lot, it appears that either the percentage of parking spaces filled is the variable that most represents the congestion or it is a combination of variables and these two-dimensional representations don't pick up that relationship.

## All Lots

Figures 91, 92, and 93 show the relationships between access time and the various conditions.


## Figure 91

The relationship between the number of cars entering the facilities and the access time is shown for a medium volume set of conditions (Tuesday mid-day for the Poisson distribution). The ChiSquared distribution shows a slight downward relationship while the Normal and Poisson distributions show a slight upward trend. The Poisson distribution is linear and does not convey the appearance of congestion at high levels. This is consistent with the fact that cars entering the lot slow access but also increase frequency, which mitigates the congestion effect.


Figure 92
The relationship between the number of cars leaving the facility and access time is shown for a high volume period (Thursday afternoon for the Poisson distribution). The Normal and ChiSquared distributions show a slight upward trend while the Poisson shows a decreasing trend. This pattern is similar to the pattern in the individual lot relationships.


Figure 93
The relationship between the percentage of spaces filled and access time is shown for a low volume period. The Normal and Chi-Squared distributions show an upward relationship and the Poisson shows a strong upward relationship with convexity. The congestion is quite evident in this variable. Access times approach 55 minutes for lots that are full. Actual data points from full satellite lots are not inconsistent with this strong upward slope in the Poisson distribution for this condition.

## VI.D Drop-Off Passengers

The drop-off passenger data are compared to the departing capacity data. Figure 94 contains this relationship for a midweek time period.

Congestion by Departing Capacity for Drop-Off Passengers


Figure 94

The Chi-Squared and Poisson distribution both show strong upward trends to the average dropoff time as departing capacity increases. The normal distribution shows a linear congestion curve due to the fact that no non-linear terms converged for this distribution. The Chi-Squared distribution shows a convex shape to the curve and the Poisson distribution shows a concave shape. In this distribution, the Poisson distribution fit the data much better. The shape of this curve implies severe congestion early in the process and an asymptotic leveling of the distribution at very high levels of departing capacity. Of course, the portion of the curve that is concave lies outside the current data.

## VI.E Summary of Individual Variable Congestion Analysis

The individual variable analysis shows the benefits of utilizing complex density functions. Those distributions that converged for higher-order functions exhibit greater curvature in the predicted mean access times. This means that even the average access time exhibits non-linear changes over time and more complex functions better fit the data.

The curvature in the congestion curves has become more evident at each level of complexity. The steep curves for some of the distributions show that some of the variables are extremely important to defining the congestion and can have large impacts on the accessibility of the airport terminal.

## Concluding Remarks and Recommendations

Several results were obtained in this study. In addition to the various facility and condition specific results, a few more general results of import should be mentioned. First, the direct estimation of density functions was shown to be possible for a broad class of data and distributions. It proves difficult to obtain estimates for complex density functions for the more sensitive distributions (normal and chi-squared in this report). More work is being carried out to determine if these complex density functions can be estimated properly. However, even with the level of complexity utilized, improvements in congestion analysis are achieved.

The next general result of import is that the complex density function analysis that allows potential congestion related terms to enter in a non-linear fashion greatly increases the predictive power of the estimations. This was obvious in the Poisson and was even apparent in the Normal and Chi-Squared distributions where the non-linear and day/time dummy variables could not be included.

A third general result is that the estimation of the density function instead of estimation of mean benchmarks (such as a regression yields) is necessary for capturing the impact of congestion. The increasing difference in access times for high percentile passengers compared to the mean passenger is an important difference that planners need to understand. Estimates of mean values over various conditions do not tell the whole story concerning the impact of volume on access times (or any other distributional data subject to congestion conditions).

Specific results to the BWI airport and the facilities studied are left primarily to the sections above. However, a few main points are reiterated here.

The facilities at BWI, with the exception of the ESP lot, all showed definite signs of being subject to congestion during some peak periods. The evidence was stronger for some lots than for others. The strongest evidence was in the satellite lots where even in low volume periods, high average access times are possible under certain conditions. The percentage of spaces filled in the facility lot is a very important predictor of congestion, as is the number of cars entering the facility.

The departing capacity variable was not as important as it might seem it should be at a first glance. This is because the other variables (such as number of cars entering a facility) are highly correlated with this variable. One potential future study of interest would be the relationship between the departing capacity variable and the other variables used in this study. This is not necessary to understand the impact of congestion, but to understand how each lot will be affected by changes in air travel - a step that is outside the congestion analysis but obviously a part of the same planning process.

Another result that is apparent from this study is that the two satellite lots behave similarly. This is only an important point as a footnote to the "BWI Terminal Accessibility Study". The preliminary data, regression analysis and simple density function analysis, show differences between the two lots. Only in the complex density function analysis, over various periods, does it become obvious that the two lots behave the same and that the data just reflect a difference in the "average" conditions of the two lots during the study.

The drop-off passenger analysis provides an interesting set of results. These results are particularly interesting given BWI Airport's recent renovations of this area. The congestion impact of departing capacity shows that this area is strongly susceptible to congestion during peak times. The percentile graph on page 66 show that some passengers are forced to wait much longer than the average length of time to gain a spot at curbside. The analysis contained in this study suggests that this facility may be the most susceptible to problems arising from congestion.

Finally, it is worth pointing out what has not been found in this study. This study does not purport to analyze future conditions at the airport. The congestion variables are analyzed out to twenty five percent above the maximum observed in the data. This does not reflect any information about how these variables are likely to change over time. The study recommended above concerning the relationship between departing capacity and the other variables would help understand how these conditions might change as airport usage increased in the future.

The actual study of determinants of congestion is not very well established. This is because prior to the past few years computing power was not sufficient to allow the estimation of complex density functions. However, this study points to a methodology that can be used and highlights the variables that are of the greatest interest in such an analysis.

## References

DeGroot, Morris H. Probability and Statistics. New York: Addison Wesley, 1986.
Reed, Randal. "BWI Terminal Accessibility Study," Technical Report, National Transportation Center, December 2001, 82 pages.


[^0]:    ${ }^{1}$ The best example of this lack of sophistication in standard estimations is for a regression. A complex regression (even one of non-linear form) merely estimates average values. There is no regression technique that allows for estimation of the increases variance and thus the spread in the distribution as congestion increases.

[^1]:    ${ }^{2}$ The capacity variables included are the departing capacity in the next $30,60,90,120$ and 180 minutes. Some estimations are carried out with combinations of these capacity variables. Results are reported for the best-fit capacity variables rather than for all permutations tried. In general, it was not important which were used since they are highly correlated and all seemed to be good predictors of the "busyness" of the airport. Unless otherwise stated, the departing capacity over the next two hours (c120) will be used in the estimations.

[^2]:    ${ }^{3}$ The $X^{\text {th }}$ percentile is the value for which $\mathrm{X} \%$ of the passengers access the terminal in that time or less.

