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N INVESTIGATION OF THE EFFECTIVENESS OF XISTING BRIDGE DESIGN METHODOLOGY IN ROVIDING ADEQUATE STRUCTURAL RESISTANCE TO SEISMIC DISTURBANCES. Phase III: Analytical Investigations of Seismic Response of Short, Single, or Multiple-Span Highway Bridges.

M. Chen and J. Penzien

October 1974

Final Report

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FEDERAL HIGHWAY ADMINISTRATION

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16. Abstract

This report is the third in a series to result from the study, "An Investigation of the Effectiveness of Existing Bridge Design Methodology in Providing Adequate Structural Resistance to Seismic Disturbances," sponsored by the U. S. Department of Transportation, Federal Highway Administration. Descriptions are given to the analytical investigations of the seismic response of short, single or multiple-span highway bridges of the type where soil-structure interaction effects are important.

Six different mathematical model elements are incorporated into the computer program which possess the capability of performing linear or non-linear analyses. Finite element modeling is used for the backfill soils. Bridge deck, piers, and abutments are modeled using prismatic beam elements. A frictional element is used to model the discontinuous behavior at the interface of backfill soils and abutments. Discontinuous type expansion joint elements are also included. Linear spring elements provide flexibility at the vertical soil boundaries. The soil foundation flexibilities under columns are established using elastic half-space theory. In the non-linear mathematical model the effects of separation and impact at the interface between abutments and backfills, the yielding at concrete columns and backfill soils and slippage at the expansion joints are taken into consideration.

Parameter studies are first carried out considering a rigid wall backfill soil system. A short, stiff, three-span bridge is then investigated with full soil-structure interaction effects included. Finally, based on the analytical results, a general conclusion regarding the analyses capability is deduced.

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Memorandum

U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL HIGHWAY ADMINISTRATION

FEB 18 1975

TO : Individual Researchers

DATE:

In reply refer to: HRS-11

FROM : James D. Cooper, Project Manager, FCP Project 5A, Improved Protection Against Natural Hazards of Earthquake and Wind

SUBJECT: Transmittal of Research Report No. FHWA-RD-75-10 "Effectiveness of Existing Bridge Designs in Resisting Earthquakes, Phase III: Short, Single or Multiple-Span Highway Bridges"

Distributed with this memorandum is the subject report intended primarily for research audiences. This report will be of interest to structural researchers concerned with earthquake resistant highway bridges.

This recently issued report is the third in a series to result from research being conducted at the University of California, Berkeley, for the Federal Highway Administration. Analytical investigations of the seismic response of short, single or multiple-span highway bridge structures of the type which suffered heavy damage during the San Fernando earthquake of February 9, 1971, are described in the report.

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Attachment

PREFACE

The investigation with interpretation as described in this report was sponsored by the U. S. Department of Transportation, Federal Highway Administration, under Contract No. DOT-FH-11-7798 covering the period July 1, 1971 through September 30, 1974.

The general investigation called for in this contract is under the supervision and technical responsibility of Professors R. W. Clough, W. G. Godden, and J. Penzien. Professor Penzien acts as principal investigator.

ACKNOWLEDGEMENT

The authors wish to express their sincere appreciation to the California State Division of Highways, Department of Public Works, for providing the engineering data of the bridge structures studied in this investigation.

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I INTRODUCTION

A. STATEMENT OF PROBLEM

The seismic response of short, single or multiple span, highway bridges are greatly affected by the phenomenon of soil-structure interaction. The dynamic forces exerted by backfills on the abutments often add significantly to the maximum seismic forces developed in the overall structural system. Also, bridges of this type usually have relatively short and stiff columns which interact strongly with their supporting foundations. Neglecting these interaction effects can lead to large errors in predicting design loads. It is therefore the objective of this investigation to establish appropriate mathematical models which will yield realistic seismic response for certain soil-structure systems of this type. Further, computer programs are written to carry out time-history dynamic analyses as an aid to the design process.

B. REVIEW OF PERTINENT RESEARCH INVESTIGATIONS

Numerous analytical investigations have been carried out in the past to determine dynamic earth pressures acting on retaining walls. One of the earliest was carried out by Okabe and later a similar study was reported by Mononobe and Matsuo [80, 74]*. In each of these investigations an equivalent static earth pressure was determined as a function of acceleration with its resultant acting on the wall at a

---

*Numbers in square brackets refer to Bibliography numbers.

position corresponding to that of static fluid pressure, i.e. at a height one-third the distance from the base. Much later, Valera used the finite element method to predict the intensities of seismic earth pressures acting on rigid walls [106]. In this investigation, nonlinear soil behavior was considered. Seed and Whitman summarized 31 investigations carried out from 1926 to 1969 and made valuable suggestions for their application to design [97]. Later, Wood derived analytical expressions for earth pressures acting against rigid and rotating walls using linear soil properties [117]. All of these investigations were concerned with intensities of dynamic pressures but in each case they assumed the pressures to act independently of the dynamic response of the wall itself.

In recent years, the influence of soil conditions on the seismic response of structural systems has received considerable attention. Seed recently published a report covering two main aspects of this problem: (1) changes in seismic ground motions adjacent to buildings as a result of physical interaction effects, and (2) changes in seismic response of buildings as a result of the changes in ground motions due to different soil deposits. This investigation did not, however, include full dynamic soil-structure interaction effects for structures other than buildings [98].

Few analytical investigations have been reported in which dynamic soil-bridge interaction effects were treated rigorously. One such study was reported by Penzien in 1970 [82]. In this particular investigation, the soil foundation was represented by a series of lumped masses interconnected by bilinear hysteretic shear springs having properties which varied with soil depth and also interconnected with viscous linear dashpots. The bridge deck, supporting piers, and pile foundations were also modelled as lumped mass systems. The three dimensional effects of the foundation soils were determined using the Mindlin Theory of the elastic half-space. Equations of motion were developed which considered all dynamic interaction effects. Recently, Tseng and Penzien reported an investigation on the seismic response of long multiple span bridges in which the mathematical modelling included soil-structure interaction effects at the bases of columns; however, due to the long structural types considered, soil-abutment interaction effects were neglected [105].

As far as experimental results are concerned, most investigations have been carried out on simple wall-soil systems. Important contributions have been made by Matsuo, Ishii and Arai and Tsuchida, Matsuo and Ohara, Tschebotanoff, Ohara, and Niwa [68, 52, 69, 104, 79, 78].

Shepherd and Charleson, and Shepherd and Sidwell have conducted field experiments on an existing bridge in the small loading range [100, 101]. Their results show that the soil around the bridge displayed high energy absorption qualities, i.e. provided considerable damping to the overall soil-structure system. Broms and Ingelson have also conducted field experiments to determine the static earth pressures acting on abutment walls [11].

C. REVIEW OF DAMAGES CAUSED BY EARTHQUAKES

Iwasaki, Penzien, and Clough prepared an extensive summary of the damages caused to bridges during earthquakes for the period 1923 to 1971 [53]. Jennings and Wood, and Lew, Legendecker, and Dikkers have reported on the damages to bridges during the San Fernando earthquake of February 9, 1971 [59, 65]. Duke and Leeds reported similar damages caused by the Chilean earthquake of 1960, while Rose, Seed, and Migliaccio reported on the Alaskan earthquake of 1964 [29, 92]. As indicated in these reports, short bridges have suffered damages ranging from column failures to cracking, tilting, and even overturning of piers, abutments, and wing walls. Clearly, this evidence shows that large dynamic forces are induced in the overall structural system by backfill earth pressures.

D. SCOPE OF PRESENT INVESTIGATION

The present investigation is a study of the seismic response of short, single or multiple span, highway bridges having narrow abutment backfills as shown in Fig. 1.

Since the earth pressures acting against the abutments can greatly affect the seismic response of the bridge, the mathematical modelling includes a two-dimensional soil element representation of the abutment backfills. This representation accounts for nonlinear soil properties and allows different vertical boundaries to be present as shown in Figs. 1D, 1E and 1F. The bridge deck, piers (or columns), and abutments are modelled using prismatic beam elements which may be permitted to have hysteretic yielding properties. A frictional element is used to model the nonlinear, discontinuous behavior at the interfaces of backfill soils and abutments and nonlinear, discontinuous type of expansion joint element is included. The soil foundation flexibilities under the columns are represented by equivalent columns. The mathematical model of the overall soil-structure system permits the study of interaction effects and yields the distribution of forces throughout the system.

Chapter II of this report describes the different elements used in the mathematical modelling and presents the derivations of elastic stiffnesses used for these elements. Chapter III discusses soil material properties used in the modelling with particular emphasis on nonlinear properties. Chapter IV develops the stiffnesses of nonlinear elements with emphasis on the derivation of the elasto-plastic force-displacement relations for the soil finite elements and the bridge column elements. Chapter V presents the numerical techniques used in the time-history dynamic analysis while Chapter VI presents the results of parameter studies. Chapter VII presents certain conclusions and recommendations. Finally, listings of the computer program are presented in Appendix A.

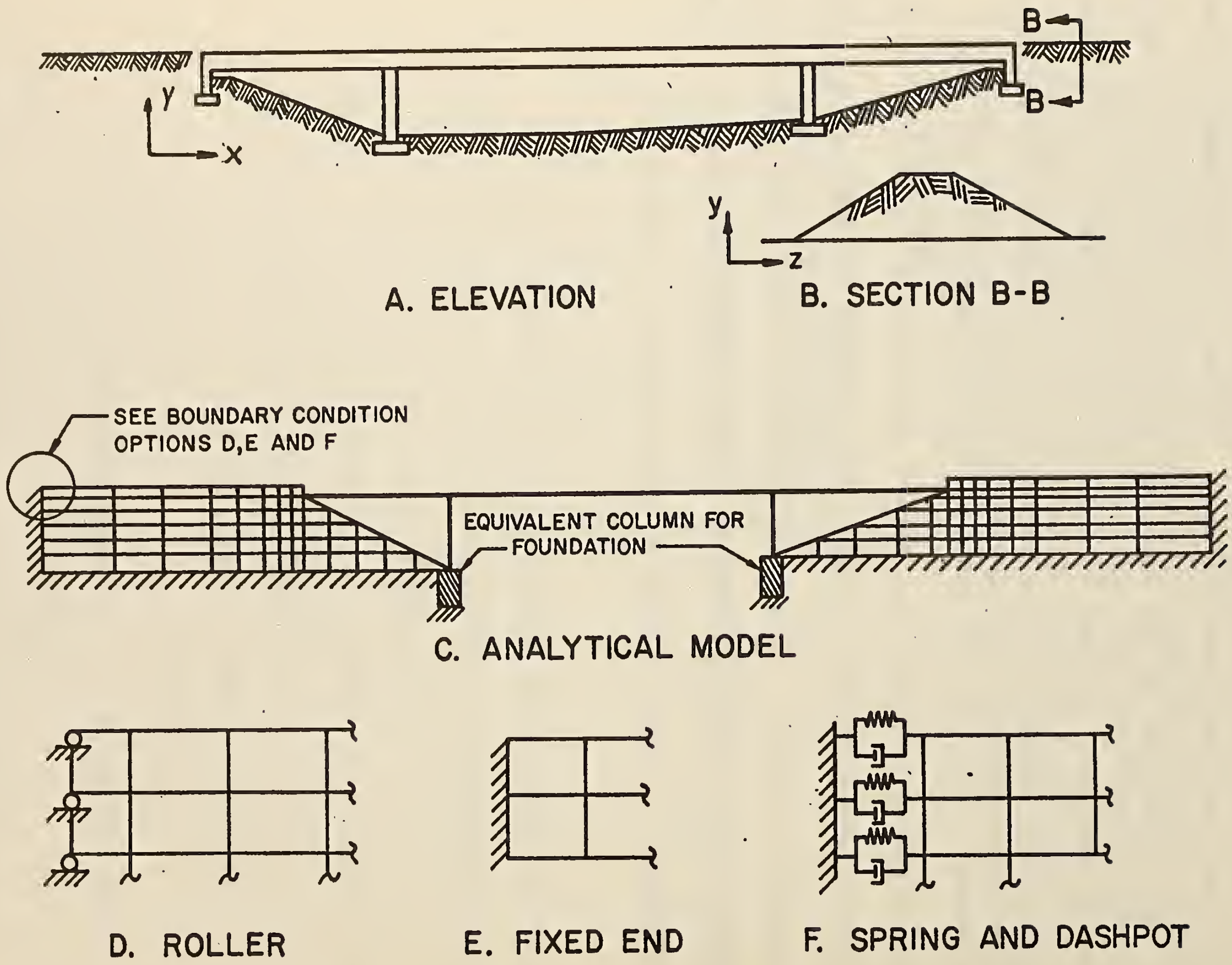


FIG. 1 BRIDGE STRUCTURE AND ANALYTICAL MODEL

II ELASTIC STIFFNESSES OF MODEL ELEMENTS

Six basic elements are used in the mathematical modelling of the bridge-soil system (1) soil finite element of a continuum, (2) soil boundary element, (3) frictional element at interface of soil and abutment wall, (4) prismatic beam element, (5) expansion joint element, and (6) equivalent column of foundation. It is the purpose of this chapter to describe these elements and to derive their elastic stiffnesses.

A. SOIL FINITE ELEMENT OF A CONTINUUM

The bridge-soil system shown in Fig. 1 is subjected to earthquake motions in the vertical direction and in one horizontal direction coinciding with the longitudinal axis of the bridge. Because of the conditions of symmetry, it is assumed that the soils adjacent to the abutments respond to these motions in a two-dimensional manner. This assumption allows these soils to be modelled using two-dimensional finite elements of a continuum interconnected at their nodal points. Material properties are defined individually for each element which may have an arbitrary quadrilateral or triangular shape.

The quadrilateral element used is the isoparametric element for which the geometry and displacements are described in terms of the same parameters of similar order. Using the natural coordinate system and interpolation displacement models, the isoparametric formulation has several advantages over the generalized coordinate method [25]. First, the nodal displacements provide a more direct visualization of actual structural deformations than do the generalized displacements. Second, it is computationally more efficient since no transformation from one coordinate system to another is required. Finally, the local coordinate system of the isoparametric approach coincides with the global coordinate system; thus, eliminating the need for transformation of loads and stiffnesses from one coordinate frame to the other.

The following derivation of the stiffness matrix for the two-dimensional quadrilateral isoparametric element is similar to that found in certain textbooks [25, 34, 118].

1. Coordinate System - The global Cartesian coordinates (x,y) and the local (or natural) coordinates (s,t) as shown in Fig. 2 are related through an interpolation function h_i by the relations

$$x = \sum_{i=1}^{4} h_i \, x_i \qquad\qquad (1)$$
$$y = \sum_{i=1}^{4} h_i \, y_i$$

where

$$h_1 = \tfrac{1}{4}\,(1-s)(1-t)$$
$$h_2 = \tfrac{1}{4}\,(1+s)(1-t) \qquad\qquad (2)$$
$$h_3 = \tfrac{1}{4}\,(1+s)(1+t)$$
$$h_4 = \tfrac{1}{4}\,(1-s)(1+t)$$

2. Strain-Displacement Equations - The displacements are approximated using the same interpolation function which can provide for both

flexible and rigid body modes. In this case, one can state

$$
\begin{aligned}
u_x(s,t) &= \sum_{i=1}^{4} h_i \, u_{x_i} \\
u_y(s,t) &= \sum_{i=1}^{4} h_i \, u_{y_i}
\end{aligned}
\tag{3}
$$

The two-dimensional strains are obtained by taking derivations of the displacements with respect to x and y in the following manner:

$$
\underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_x}{\partial x} \\ \dfrac{\partial u_y}{\partial y} \\ \dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{4} \dfrac{\partial h_i}{\partial x} u_{x_i} \\ \sum_{i=1}^{4} \dfrac{\partial h_i}{\partial y} u_{y_i} \\ \sum_{i=1}^{4} \left(\dfrac{\partial h_i}{\partial y} u_{x_i} + \dfrac{\partial h_i}{\partial x} u_{y_i} \right) \end{Bmatrix}
\tag{4}
$$

Since functions h_i (i=1,2,3,4) are expressed in terms of s and t, the chain rule of derivations can be applied using the equations

$$
\begin{aligned}
\frac{\partial h_i}{\partial x} &= \frac{\partial h_i}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial h_i}{\partial t}\frac{\partial t}{\partial x} \\
\frac{\partial h_i}{\partial y} &= \frac{\partial h_i}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial h_i}{\partial t}\frac{\partial t}{\partial y}
\end{aligned}
\tag{5}
$$

The chain rule is

$$
\begin{Bmatrix} \dfrac{\partial(\ )}{\partial s} \\ \dfrac{\partial(\ )}{\partial t} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial s} & \dfrac{\partial y}{\partial s} \\ \dfrac{\partial x}{\partial t} & \dfrac{\partial y}{\partial t} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial(\ )}{\partial x} \\ \dfrac{\partial(\ )}{\partial y} \end{Bmatrix}
\tag{6}
$$

which in its inverted form becomes

$$\begin{Bmatrix} \frac{\partial(\ )}{\partial x} \\ \\ \frac{\partial(\ )}{\partial y} \end{Bmatrix} = \frac{1}{J} \begin{bmatrix} \frac{\partial y}{\partial t} & -\frac{\partial y}{\partial s} \\ \\ -\frac{\partial x}{\partial t} & \frac{\partial x}{\partial s} \end{bmatrix} \begin{Bmatrix} \frac{\partial(\ )}{\partial s} \\ \\ \frac{\partial(\ )}{\partial t} \end{Bmatrix} \tag{7}$$

where the Jacobian J is defined by

$$J = \frac{\partial x}{\partial s} \frac{\partial y}{\partial t} - \frac{\partial x}{\partial t} \frac{\partial y}{\partial s} \tag{8}$$

Thus, the required derivatives are given by

$$\begin{bmatrix} \frac{\partial s}{\partial x} & \frac{\partial t}{\partial x} \\ \\ \frac{\partial s}{\partial y} & \frac{\partial t}{\partial y} \end{bmatrix} = \frac{1}{J} \begin{bmatrix} \frac{\partial y}{\partial t} & -\frac{\partial y}{\partial s} \\ \\ -\frac{\partial x}{\partial t} & \frac{\partial x}{\partial s} \end{bmatrix} \tag{9}$$

Taking derivatives of Eqs. (1), one can write

$$\begin{aligned} \frac{\partial x}{\partial s} &= \sum_{i=1}^{4} \frac{\partial h_i}{\partial s} x_i \\ \frac{\partial x}{\partial t} &= \sum_{i=1}^{4} \frac{\partial h_i}{\partial t} x_i \\ \frac{\partial y}{\partial s} &= \sum_{i=1}^{4} \frac{\partial h_i}{\partial s} y_i \\ \frac{\partial y}{\partial t} &= \sum_{i=1}^{4} \frac{\partial h_i}{\partial t} y_i \end{aligned} \tag{10}$$

Making use of Eqs. (2) and (10), the Jacobian, as defined by Eq. (8) becomes

$$J = \frac{1}{8}[s(x_3 - x_4)(y_1 - y_2) - (x_1 - x_2)(y_3 - y_4)$$
$$+ t(x_2 - x_3)(y_1 - y_4) - (x_1 - x_4)(y_2 - y_3) \quad (11)$$
$$+ (x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)]$$

and the strain vector defined by Eq. (4) becomes

$$\underline{\varepsilon} = \begin{bmatrix} \frac{\partial h_1}{\partial x} & 0 & \frac{\partial h_2}{\partial x} & 0 & \frac{\partial h_3}{\partial x} & 0 & \frac{\partial h_4}{\partial x} & 0 \\ 0 & \frac{\partial h_1}{\partial y} & 0 & \frac{\partial h_2}{\partial y} & 0 & \frac{\partial h_3}{\partial y} & 0 & \frac{\partial h_4}{\partial y} \\ \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial x} & \frac{\partial h_3}{\partial y} & \frac{\partial h_3}{\partial x} & \frac{\partial h_4}{\partial y} & \frac{\partial h_4}{\partial x} \end{bmatrix} \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{Bmatrix} \quad (12)$$

or in compact form may be expressed as

$$\underline{\varepsilon} = \underline{B}\,\underline{u} \quad (13)$$

The derivatives in the coefficient matrix of Eq. (12) can now be expressed in their final maniputable form

$$\frac{\partial h_1}{\partial x} = \frac{1}{16J}[\,y_2(h_1 + h_2) + y_3(h_4 - h_2) - y_4(h_1 + h_4)]$$
$$\frac{\partial h_2}{\partial x} = \frac{1}{16J}[-y_1(h_1 + h_2) + y_3(h_2 + h_3) + y_4(h_1 - h_3)]$$
$$\frac{\partial h_3}{\partial x} = \frac{1}{16J}[\,y_1(h_2 - h_4) - y_2(h_2 + h_3) + y_4(h_3 + h_4)]$$
$$\frac{\partial h_4}{\partial x} = \frac{1}{16J}[\,y_1(h_1 + h_4) + y_2(h_3 - h_1) - y_3(h_4 + h_3)]$$
$$\frac{\partial h_1}{\partial y} = \frac{1}{16J}[-x_2(h_1 + h_2) + x_3(h_2 - h_4) + x_4(h_1 + h_4)]$$

$$\frac{\partial h_2}{\partial y} = \frac{1}{16J} [x_1(h_1 + h_2) - x_3(h_2 + h_3) + x_4(h_3 - h_1)]$$

$$\frac{\partial h_3}{\partial y} = \frac{1}{16J} [x_1(h_4 - h_2) + x_2(h_2 + h_3) - x_4(h_3 + h_4)]$$

$$\frac{\partial h_4}{\partial y} = \frac{1}{16J} [-x_1(h_1 + h_4) + x_2(h_1 - h_3) + x_3(h_3 + h_4)]$$

(14)

Using standard index notation with i permutating 1 through 4, Eqs. (14) can be written in the two single equations

$$\frac{\partial h_i}{\partial x} = \frac{1}{16J} [y_{i+1}(h_i + h_{i+1}) + y_{i+2}(h_{i+3} - h_{i+1}) - y_{i+3}(h_i + h_{i+3})]$$

$$\frac{\partial h_i}{\partial y} = \frac{1}{16J} [-y_{i+1}(h_i + h_{i+1}) + x_{i+2}(h_{i+1} - h_{i+3}) + x_{i+3}(h_i + h_{i+3})]$$

(15)

3. Element Stiffness and Numerical Integration - Considering a thickness ω (normal to the plane of element shown in Fig. 2), the element stiffness matrix can be expressed in the form

$$\underline{K} = \int_{vol} \underline{B}^T \underline{C}\ \underline{B}\, dv = \int_{area} \underline{B}^T \underline{C}\ \underline{B}\, \omega dA \qquad (16)$$

where $\underline{C}$ is the stress-strain matrix with integration being carried out over the entire area of the element. For purposes of numerical integration, Eq. (16) can be written in terms of the s and t coordinates giving

$$\underline{K} = \int_{-1}^{1} \int_{-1}^{1} \underline{B}^T \underline{C}\ \underline{B}\ (\det J)\ \omega ds\, dt \qquad (17)$$

Upon application of standard one-dimensional numerical integration formulas, Eq. (17) becomes

$$\underline{K} = \sum_{j} \sum_{k} \omega_j \omega_k \quad (\det J)\ \underline{B}^T(s_j, t_k)\ \underline{C}\ \underline{B}\ (s_j, t_k)\omega \qquad (18)$$

in which s_j and t_k are integration points and ω_j and ω_k are the appropriate weighting functions. Using the Gauss-Legendre quadrature formula of degree 2, one obtains

$$\left.\begin{aligned} s_i &= \pm\ 0.577350 \\ t_i &= \pm\ 0.577350 \\ \omega_i &= 1 \end{aligned}\right\} \quad i = 1,2 \qquad (19)$$

For plane stress, matrix $\underline{C}$ has the form

$$\underline{C} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \qquad (20)$$

while for plane strain it becomes

$$\underline{C} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \qquad (21)$$

where E is Young's Modulus and ν is Poisson's ratio.

As shown above, the final form of the element stiffness matrix is a function of geometry, element thickness, and material properties. The overall stiffness matrix of the entire system is assembled by the direct stiffness method [22]. The band width of this matrix depends upon the system of numbering nodal points. Therefore, the nodal points should be numbered in a manner to minimize computer storage requirement and operating time.

B. SOIL BOUNDARY ELEMENT

Boundary elements are used in the mathematical model to account for the elastic action which occurs at the vertical boundaries of the soils being considered. The type of element used for this purpose is a simple elastic spring as shown in Fig. 3. This element has a stiffness matrix $\underline{k}$ in the local coordinate system of the form

$$\underline{k} = E \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{22}$$

where E is the equivalent stiffness of the spring, which can be obtained using standard methods [82,89].

C. PRISMATIC BEAM ELEMENT

Prismatic beam elements are used to model the bridge deck, piers, abutment walls and equivalent columns for soil foundations. The derivation of the stiffness matrix for a beam element considering axial, shear, and bending deformations can be found in many textbooks [86]. Therefore, only the main features of this matrix will be presented here.

The force components acting on the beam element are axial forces s_1 and s_4, shearing forces s_2 and s_5, and bending moments s_3 and s_6 as shown in Fig. 4. The positive directions of these force components (s) and their respective displacement components (u') correspond to those shown in the figure. The stiffness matrix for this element in terms of the local coordinates is given by the standard form

$$\underline{k}_{(6x6)} = \begin{bmatrix} \frac{EA}{\ell} & & & & & \\ 0 & \frac{12EI_z}{\ell^3(1+\phi_y)} & & \text{SYMMETRICAL} & & \\ 0 & \frac{6EI_z}{\ell^2(1+\phi_y)} & \frac{(4+\phi_y)EI_z}{\ell(1+\phi_y)} & & & \\ -\frac{EA}{\ell} & 0 & 0 & \frac{EA}{\ell} & & \\ 0 & \frac{-12EI_z}{\ell^3(1+\phi_y)} & \frac{-6EI_z}{\ell^2(1+\phi_y)} & 0 & \frac{12EI_z}{\ell^3(1+\phi_y)} & \\ 0 & \frac{6EI_z}{\ell^2(1+\phi_y)} & \frac{(2-\phi_y)EI_z}{\ell(1+\phi_y)} & 0 & \frac{-6EI_z}{\ell^2(1+\phi_y)} & \frac{(4+\phi_y)EI_z}{\ell(1+\phi_y)} \end{bmatrix} \quad (23)$$

where

$$\phi_y = \frac{12EI_z}{G\ Asy\ \ell^2} = 24(1+\nu)\ \frac{A}{Asy}\ \left(\frac{\gamma_z}{\ell}\right)^2 \quad (24)$$

represents the shear deformation parameter for reinforced concrete elements and where

ℓ = element length

γ_z = radius of gyration about z axis

I_z = Moment of inertia about z axis

A = cross sectional area

Asy = equivalent shear area in y direction

E = modulus of elasticity

G = modulus of elasticity in shear

ν = Poisson's ratio

The matrix equation relating displacements in the local coordinates u' to displacements in the global coordinates $\underline{u}$ is given by

$$\underline{u}' = \underline{\lambda}\,\underline{u} \tag{25}$$

where λ has the two-dimensional form

$$\underline{\lambda} = \begin{bmatrix} D_x/D_\ell & D_y/D_\ell & 0 & 0 & 0 & 0 \\ -D_y/D_\ell & D_x/D_\ell & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_x/D_\ell & D_y/D_\ell & 0 \\ 0 & 0 & 0 & -D_y/D_\ell & D_y/D_\ell & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

and where

$$\begin{aligned} D_x &= x_j - x_i \\ D_y &= y_j - y_i \\ D_\ell &= \sqrt{D_x^{\,2} + D_y^{\,2}} \end{aligned} \tag{27}$$

Thus, the stiffness matrix $\underline{K}$ in global coordinates becomes

$$\underline{K} = \underline{\lambda}^T\,\underline{k}\,\underline{\lambda} \tag{28}$$

as indicated above, the stiffness matrix is linearly proportional to the areas and moments of inertia. Therefore, a unit width analysis can be carried out, if desired, by dividing all areas and moments of inertia by the width of the bridge deck.

D. FRICTIONAL ELEMENT

A so called frictional element is used to model the frictional action, separation, and impact which take place at the interfaces of soil backfills and abutment walls. This element has the following characteristics (1) the frictional force per unit area is proportional to the normal interface pressure and a coefficient of friction; thus, slippage occurs when the direction angle of the resultant of pressure and friction exceeds the soil angle of friction, (2) impact occurs at the interface upon closure of any gap which may have earlier developed, and (3) no frictional resistance can develop at the interface when wall and soil surfaces have separated. Discontinuous elements similar to this have been developed by Ghaboussi and Wilson, Scholes and Strover, White and Enderly, and Tseng and Penzien [40, 94, 111, 105]; however, the element developed by Goodman and Taylor has been adopted here [41].

This frictional element is a four nodal element as shown in Fig. 5 having length L but with a height equal to zero, i.e. nodal points 1 and 4 coincide as do points 2 and 3. It is shown in a local coordinate system with the origin at the center of the element and the x axis directed along the length of the element.

The relative displacement vector $\underline{u}$ is expressed in terms of the displacement vector $\underline{u}_i$ through the linear interpolation function formula

$$\underline{u} = \begin{Bmatrix} (u_x^{top} - u_x^{bottom}) \\ (u_y^{top} - u_y^{bottom}) \end{Bmatrix}$$

or

$$\underline{u} = \begin{bmatrix} -(1-\frac{2x}{L}) & 0 & -(1+\frac{2x}{L}) & 0 & (1+\frac{2x}{L}) & 0 & (1-\frac{2x}{L}) & 0 \\ 0 & -(1-\frac{2x}{L}) & 0 & -(1+\frac{2x}{L}) & 0 & (1+\frac{2x}{L}) & 0 & (1+\frac{2x}{L}) \end{bmatrix} \begin{Bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{Bmatrix} \quad (29)$$

The material property matrix $\underline{k}$ expressing the stiffness per unit length in the normal and tangential directions is given by

$$\underline{k} = \begin{bmatrix} k_s & 0 \\ 0 & k_n \end{bmatrix} \quad (30)$$

Upon applying the variational principle of solid mechanics, the elastic stiffness matrix in the local coordinate system becomes

$$\underline{K} = \frac{1}{6}\begin{bmatrix} 2k_s & 0 & 1k_s & 0 & -1k_s & 0 & -2k_s & 0 \\ 0 & 2k_n & 0 & 1k_n & 0 & -1k_n & 0 & -2k_n \\ 1k_s & 0 & 2k_s & 0 & -2k_s & 0 & -1k_s & 0 \\ 0 & 1k_n & 0 & 2k_n & 0 & -2k_n & 0 & -1k_n \\ -1k_s & 0 & -2k_s & 0 & 2k_s & 0 & 1k_s & 0 \\ 0 & -1k_n & 0 & -2k_n & 0 & 2k_n & 0 & 1k_n \\ -2k_s & 0 & -1k_s & 0 & 1k_s & 0 & 2k_s & 0 \\ 0 & -2k_n & 0 & -1k_n & 0 & 1k_n & 0 & 2k_n \end{bmatrix} \quad (31)$$

E. EXPANSION JOINT ELEMENT

An expansion joint may be present between the bridge deck and an abutment as shown in Fig. 6. This joint can develop horizontal frictional forces which should be modelled properly. For this purpose, the frictional element previously described can be adopted; however, two additional boundary conditions must be imposed, namely, (1) the relative displacement of the upper and lower part of the joint element, $u_{3x} - u_{2x}$, cannot cause a gap closure (between deck and abutment) greater than the original gap dimension, and (2) frictional forces can be developed only when the relative displacement, $u_{1x} - u_{3x}$, causing a widening of the gap is less than the original "seat" dimension. As soon as the relative displacement $(u_{1x} - u_{3x})$ exceeds the original seat dimension, the bridge deck falls from its support; thus, the computer analysis is stopped at this point.

F. EQUIVALENT COLUMN FOR FOUNDATION

Various mathematical models have been used for structural foundations. As reported in the literature, Parmelee, Whitman and Roesset, Dobry, and others [81, 108] used a simple spring dashpot model; Jennings and Bielak and Richart, Hall, and Woods [58, 89] used the elastic half space; Whitman [109] used an equivalent lumped mass model; and, Dans and Butterfield, Finn, Lysmer and Kuhlemeyer, and Wilson [31, 36, 66, 114] used a finite element mesh. Various methods have been reported for estimating the lateral stiffness of foundations employing piles [84, 88].

The replacement of the foundation flexibility with an equivalent column through use of the elastic half space theory and under the assumption of quasi-static behavior has been reported by Penzien, et. al. [8]. This method has been adopted in the present investigations.

The first step in finding the equivalent column for the foundation is to determine the lateral, vertical, and rotational stiffnesses of the foundation at the footing (or pile cap) level. Once these three stiffnesses have been determined, the foundation is replaced by a column of length L, flexural stiffness EI, and axial stiffness AE which when fixed at its base provides the equivalent lateral, vertical, and rotational stiffnesses to the footing. The stiffness matrix for this column is given in the form of Eq. (23) under the assumption of no shear deformation, i.e. $\phi_y = 0$.

The foundation stiffnesses can be obtained by either the numerical procedure outlined by Penzien or by a closed form approach reported by Gerrand and Harrison [82, 39].

1. <u>Closed Form Solution for Lateral Stiffness of Circular Footing</u> - The closed form solution for lateral stiffness is available for a circular shaped footing as shown in Fig. 7-c. If the footing is rectangular in shape, an equivalent radius r_0 should be calculated for a circular footing having the same area, then

$$k_x = \frac{8\ r_0\ G}{2-\nu} \tag{32}$$

where

k_x = lateral stiffness for a single footing

r_0 = radius of footing

G = shear modulus of soil

ν = Poisson's ratio

For a pier of multiple supports, the interactions between footings are estimated by Eq. (33) which describes the lateral displacement at a distance r from a loaded footing.

$$u = T_h \frac{1}{4\pi r_0 G} [(2-\nu)\cdot\sin^{-1}\frac{1}{r} + \nu \cdot \frac{(r^2-1)^{\frac{1}{2}}}{r^2}] \quad (33)$$

where

u = the displacement

T_h = the total applied force at a single footing

r = the distance in terms of radius r_0

By calculating the average displacement of all footings in a pier and dividing the total force by the average displacement, the equivalent stiffness of a pier with multiple footings is then obtained.

2. Numerical Solution Using the Mindlin Equation to Calculate Lateral Stiffness - The Mindlin equation has the following form. When the x component of displacement on the surface is to be calculated, as produced by a single concentrated force P located at the origin (o, o, o) on the surface of an isotropic half space and acting in the x-direction

$$u_x(x,y,o) = \frac{P(o,o,o)}{16\pi(1-\nu)G} \left\{ \frac{(3-4\nu)}{R} + \frac{1}{R} + \frac{4(1-\nu)(1-2\nu)}{R} + x^2 \left[\frac{1}{R^3} + \frac{3-4\nu}{R^3} - \frac{4(1-\nu)(1-2\nu)}{R^3} \right] \right\} \quad (34)$$

in which

$$R^2 = x^2 + y^2$$

Under the assumption that the lateral force is uniformly distributed over the area of the footing, the procedure to calculate the lateral stiffness of a single footing is (1) replace the uniform pressure by as many concentrated loads P as accuracy requires, (2) calculate the lateral displacement of each load point over the footing area due to each concentrated load, (3) sum the resulting displacements at each load point within the footing as caused by the full set of loads, (4) average the load point displacements, and (5) divide the total resultant force by the average of the load point displacements to obtain the lateral stiffness.

Again, if a pier has many footings, the same averaging procedure is applied by assuming a concentrated force at the center of each footing; thus, the lateral stiffness of each pier foundation can be obtained.

3. <u>Comparison of Two Methods for Lateral Stiffness</u> - To compare the two previously described methods for obtaining foundation lateral stiffness, consider the pier shown in Fig. 8 having four footings. Assuming the footing is 8.5 x 8.5 ft and the distance between footings is 26 ft, the individual footing stiffness k_x and pier stiffness k_p obtained by the two approaches have the following values in k/ft:

$$k_x = \begin{cases} 32.5\ G & \text{Numerical method} \\ 25.5\ G & \text{closed form solution} \end{cases}$$

$$k_p = \begin{cases} 100.6\ G & \text{Numerical method} \\ 80.0\ G & \text{closed form solution} \end{cases}$$

Where the unit of G is in k/ft^2.

In each case the stiffnesses differ by approximately 20%. In the numerical solution the footing is considered to have 36 concentrated forces applied at equal grid intervals over the area. If the resultant force had been discritized at more than 36 points, the differences in stiffness would have decreased.

4. Vertical Stiffness with Friction Piles - Assuming the friction force per unit length of pile varies in a linear manner from a maximum value at the top to zero at the bottom, and assuming zero vertical displacement at the bottom of the pile, the vertical stiffness of the pile k_v is

$$k_v = 3AE/L \tag{35}$$

Where L, A, and E are the length, area, and modulus of elasticity, respectively, of the pile.

5. Vertical Stiffness Without Piles - Assuming uniform vertical displacements as shown in Fig. 7-a, the vertical stiffness of a circular footing without piles is given by

$$k_v = \frac{4\, r_0\, G}{1-\nu} \tag{36}$$

For other footing shapes, similar stiffness relations have been reported by Lysmer and Duncan [67].

6. Rotational Stiffness Without Piles - Assuming rotational displacements as shown in Fig. 7-b, the rotational stiffness of a circular footing without piles is given by

$$k_\theta = \frac{8\, r_0^{\,3}\, G}{3(1-\nu)} \qquad (37)$$

The rotational stiffness of a rigid footing with piles can be calculated directly from the vertical stiffness of each pile.

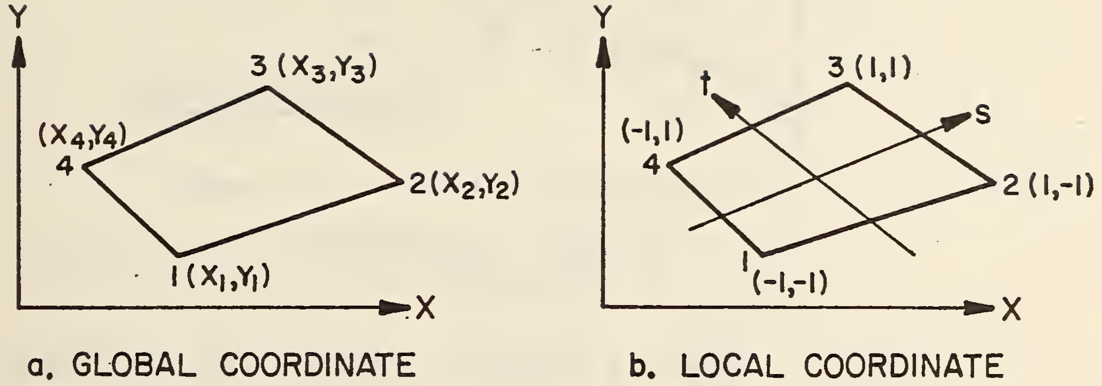


FIG. 2 TWO DIMENSIONAL ISOPARAMETRIC ELEMENT

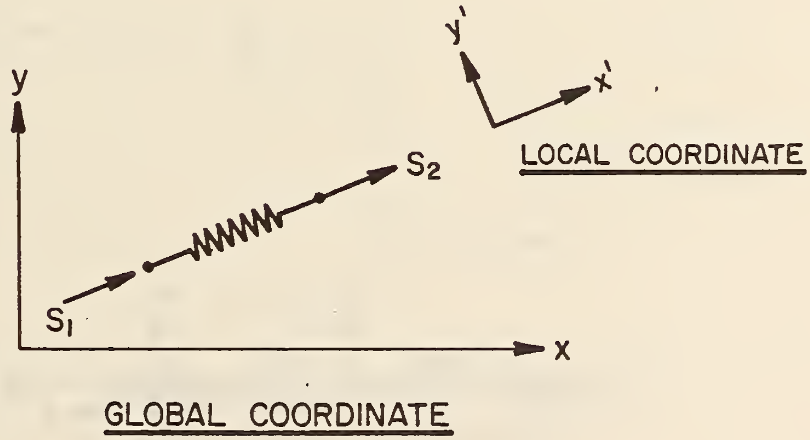


FIG. 3 BOUNDARY ELEMENT

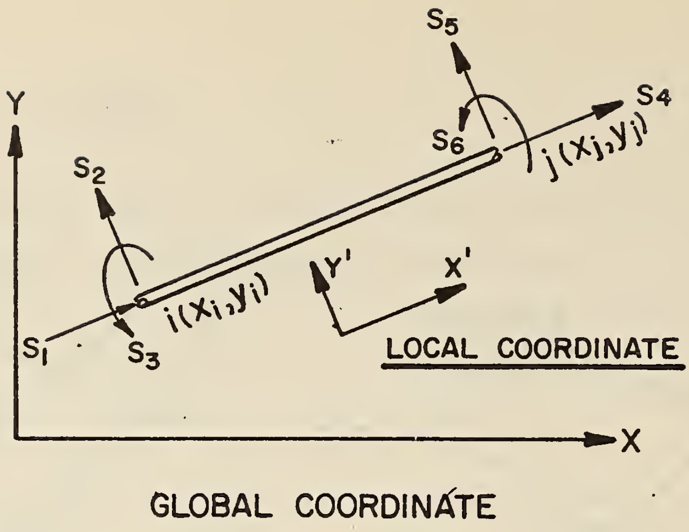


FIG. 4 BEAM ELEMENT COORDINATE SYSTEM

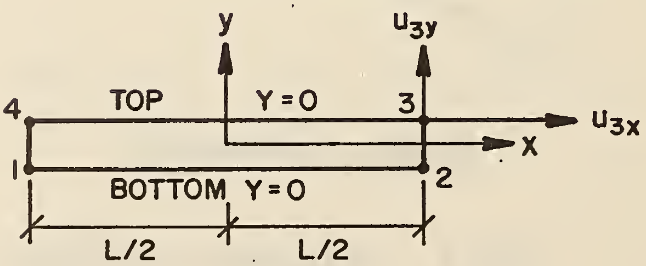
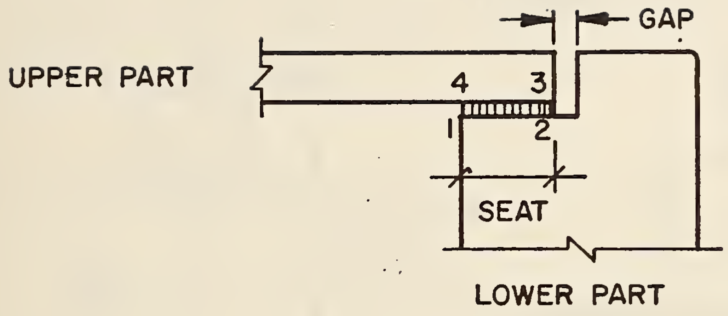
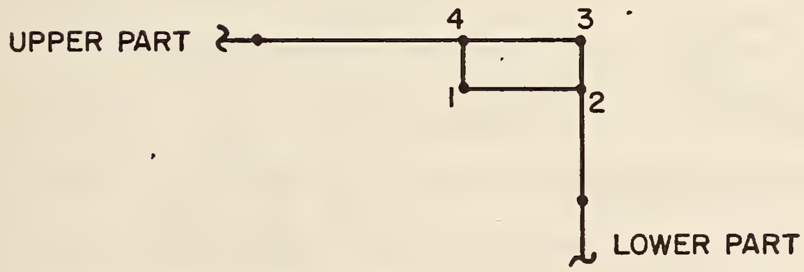


FIG. 5 FRICTIONAL ELEMENT, HEIGHT = 0
LOCAL COORDINATE

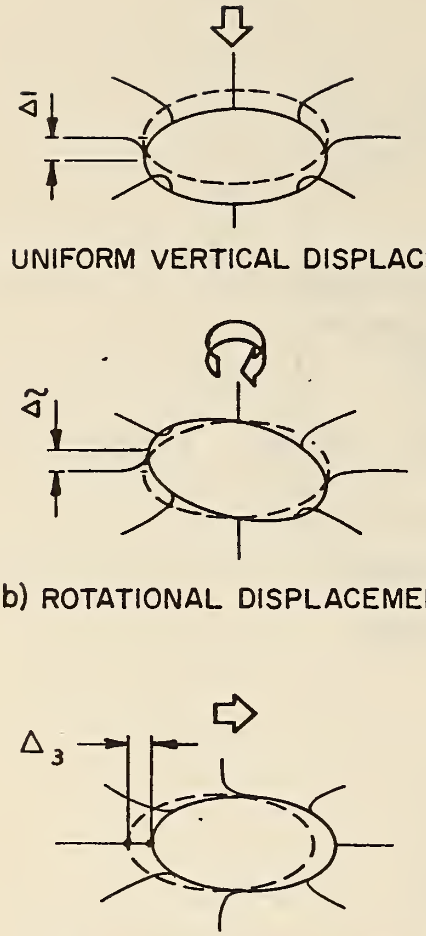


a. PROTOTYPE



b. MODEL

FIG. 6 EXPANSION JOINT ELEMENT



(a) UNIFORM VERTICAL DISPLACEMENT

(b) ROTATIONAL DISPLACEMENT

(c) LATERAL DISPLACEMENT

FIG. 7 DISPLACEMENT TYPES

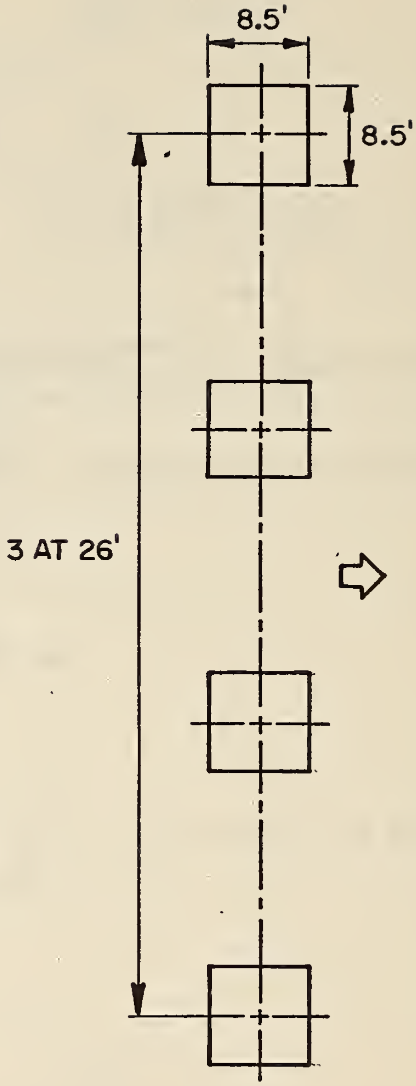


FIG. 8 PIER WITH FOUR COLUMNS

III MATERIAL PROPERTIES OF SOIL

In this chapter, the pertinent soil material properties used to establish a non-linear finite element model are discussed. The non-linear stress-strain relations, Mohr's envelope and p-q diagram, the concepts of active and passive stresses, and the damping characteristics of soil are described.

A. STRESS-STRAIN RELATIONSHIP

Basic to establishing the force-displacement relationships for soil elements as employed in the mathematical model are the stress-strain relationships for the materials involved. These materials include both cohesionless and cohesive soils.

Numerous authors, including Bishop and Henkel, Bishop, Comforth and Seed, have reported procedures for determining the shear modulus of sand [9, 10, 23, 96]. The investigators show that the shear modulus is strongly influenced by confining pressure, strain amplitude, and void ratio. In the present investigation, the equivalent secant shear modulus as determined by extreme points on the hysteresis loop, Fig. 9, is adopted. This modulus can be estimated using the relation proposed by Seed [96], namely,

$$G = 1000\, k_2\, (\sigma'_m)^{\frac{1}{2}} \qquad \text{psf} \qquad (38)$$

where G is the secant modulus of sand, σ'_m is the effective mean stress and k_2 is a parameter which depends upon void ratio e and strain amplitude as shown in Fig. 10. Triaxial tests show that k_2 depends only upon void ratio e at very low strain levels ($\gamma \leq 10^{-3}$ percent). At intermediate strain levels ($10^{-3} \leq \gamma \leq 10^{-1}$ percent), it still depends primarily upon void ratio but is also slightly influenced by state of stress and the friction angle ϕ. At very high strain levels, k_2 is essentially a constant which is almost independent of state of stress, friction angle, and the void ratio. Thus for practical purposes, the values of k_2 can be assumed to vary only with strain amplitude and void ratio as shown in Fig. 10.

For prescribed values of void ratio and confining pressure, the stress-strain relationship is non-linear with the shear modulus changing with shear strain. Upon the initiation of yielding the modulus becomes very small. A tangent modulus curve transformed from the equivalent shear modulus curve, Fig. 10, is shown in Fig. 12. Theoretically, one could use a revised tangent modulus over each time interval of a dynamic analysis; however, it is believed to be more practical to use a more simplified form of stress-strain relationship. In the present investigation a trilinear stress-strain relationship, Fig. 13-a, has been adopted with the shear modulus remaining constant during each of three different loading stages. In the initial stage, the soil element is in its geostatic state, i.e. the vertical stress is equal to the weight of soil (per unit area) above the point of interest as given by

$$\sigma_v = \int_0^y w \, dy \qquad (39)$$

where y is the depth and w is the unit weight of soil. The lateral stress in the principal horizontal direction is

$$\sigma_h = k_0 \sigma_v \tag{40}$$

while in the orthogonal horizontal direction (z) the stress σ_z for plane stress is

$$\sigma_z = 0 \tag{41}$$

and for plane strain is

$$\sigma_z = \mu(\sigma_v + \sigma_h) = \frac{k_0}{1+k_0}(\sigma_v + k_0\sigma_v) = k_0\sigma_v = \sigma_h \tag{42}$$

where μ is Poisson's ratio. The shear modulus in this stage is evaluated by Eq. (38) with k_2 selected in accordance with the very low strain level shown in Fig. 10. In the second stage the soils in the backfill are no longer in the geostatic state due to loadings from the bridge. In this case the shear modulus is calculated using Eq. (38), but k_2 is revised to be consistent with the maximum shear strain in the element. The third stage occurs after the initiation of yielding in which case the stress-strain relation is evaluated in accordance with a theory of plasticity as described in Chapter IV.

In those cases where curves of k_2 vs. γ as shown in Fig. 10 are unavailable, a bilinear stress-strain relationship is adopted with the initial modulus being estimated by an empirical method and the second stage modulus being calculated by the theory of plasticity as shown in Fig. 13-b.

Turning our attention now to certain clay materials, test data show that at very low strain levels, the shear modulus varies almost linearly with shear strength [112]. In a summary report, Seed has presented a curve of secant shear modulus versus shear strain for saturated clays [96]. The shear modulus expressed by this curve, Fig. 14, is in the normalized form G/s_u where s_u is the undrained shear strength of the material. A curve showing the degradation of secant shear modulus with shear strain, as obtained by a number of investigators, is shown in Fig. 11 [96]. These relationships can be used as a guide for estimating the initial modulus of clay when laboratory test data on the specific material are unavailable.

Fig. 15 gives a tangent modulus curve for clay which is consistent with the secant modulus curve of Fig. 11. Test results show that the shear modulus is essentially independent of the confining pressure. A re-evaluation of shear modulus before the initiation of yielding becomes much less important for this material than for sand. For this reason, a bilinear stress-strain relation has been assumed for cohesive soils in the present investigation. This initial modulus is estimated from the data previously described and the tangent stiffness after yielding is obtained using the same basic procedure as used for sands. No hardening effects after the initiation of yielding are considered.

B. MOHR'S ENVELOPE, p-q DIAGRAM

The most widely used yield criterion in soil mechanics is the Mohr-Coulomb criterion which relates the normal stress σ_f and the shear stress τ_f at the failure plane by the equation

$$\tau_f = c + \sigma_f \tan \phi \qquad (43)$$

where c is the cohesion of the soil and ϕ is its friction angle. The soil coefficients are usually obtained by conducting triaxial tests at various confining pressures and by drawing a common tangent line to the resulting Mohr's circles as shown in Fig. 16. It should be noted that since the confining pressure of the triaxial test is uniform, i.e. $\sigma_2 = \sigma_3$, the Mohr's circle can be used in its familiar two-dimensional form. Equation (43) when plotted on the σ-τ plane is called Mohr's yield envelope which implies (1) elastic behavior for a state of stress whose Mohr's circle lies entirely below the envelope, (2) yielding, or impending yielding, for a state of stress whose Mohr's circle has the envelope as its tangent, and (3) that any state of stress whose Mohr's circle crosses the envelope is not permitted.

An alternate way of plotting the results of triaxial tests is to plot the stresses at the plane of maximum shear on the p-q plane, namely,

$$p = \frac{\sigma_1 + \sigma_3}{2} \tag{44}$$

and

$$q = \frac{\sigma_1 - \sigma_3}{2} \tag{45}$$

In Fig. 17, points A, B, and C in the σ-τ plane represent the stresses on planes of maximum shear. A line drawn through these same points in the p-q plane, as shown in Fig. 18, is called the k_f line. The equation of this line is

$$q_f = a + p_f \tan \alpha \tag{46}$$

Strength parameters c and ϕ can be computed from Eq. (46) through the relations

$$\sin \phi = \tan \alpha \tag{47}$$

$$c = a/\cos \phi \quad (48)$$

C. ACTIVE AND PASSIVE STRESS

Using the previously defined yield criterion, active and passive soil stresses can be defined as they relate to the backfill pressures exerted on abutment walls. Starting with soil equilibrium in its geostatic state which has principal axes in the vertical and horizontal directions, decrease the horizontal compressive stress continually until the shear strength of the soil is reached and failure occurs. The horizontal compressive stress at the point of failure is active stress. On the other hand, if the horizontal compressive stress is continually increased, the shear strength of the soil will again be reached at a much higher stress level called the passive stress. The Mohr's circles representing these two failure conditions are shown in Fig. 19 where σ_v and σ_h represent the vertical and horizontal stresses, respectively, in the geostatic state and σ_a and σ_p represent the active and passive stresses, respectively. It should be noted here that since the two horizontal stresses are assumed equal, only two stress components are required to describe the stress conditions.

The stress conditions in the σ-τ diagram of Fig. 19 are again shown in the p-q diagram of Fig. 20, where point A is the geostatic state of stress, point B is the passive state of stress, and point C is the active state of stress. Lines AC and AB are stress paths which depict the successive states of stress which occur in changing from the geostatic condition to the active and passive conditions, respectively.

When applying the concepts of active and passive stresses to more complex stress conditions, certain modifications are needed. For example, the stress conditions of the bridge backfill-soil treated in this investigation is essentially three-dimensional. In this case shear stresses may exist on both vertical and horizontal planes even under gravity loadings. Referring to Figs. 1-A and B for the element coordinates, the Mohr's circle for this condition may be as shown in Fig. 21. Further, it should be noted that if the side slopes of the embankment as seen in Fig. 1-B are not sufficiently flat, the yield plane will show as a line in the y-z plane. In the present study however it is assumed that these slopes are sufficiently flat so that the yield plane shows as a line in the x-y plane. Thus the state of stress at failure as shown on an element of soil in the x-y plane, can be represented by a Mohr's circle tangent to the Mohr's envelope as shown by Circle 1 in Fig. 22. States of stress in the x-z and y-z planes would be represented by Mohr's circles falling entirely within the above defined circle as shown by Circle II and III in Fig. 22. Because of these restrictions, the problem can be treated as a two-dimensional problem in the x-y plane. Finally, in the present investigation, all stresses vary prior to reaching a state of yield. Based on the previous assumptions, active and passive states can be defined in an equivalent manner. Referring to Fig. 23 where the absolute values of the p and q stresses are plotted, p values at static loading and failure conditions can be compared. If $|p|$ is assumed to decrease continually with increasing $|q|$ stress until the Mohr's envelope is reached, active failure occurs. On the other hand, if $|p|$ is assumed to increase continually with increasing $|q|$ stress until the envelope is reached, passive failure occurs. Stresses p_a and p_p represent the active and passive stresses in this case.

D. DAMPING

Soils, like all materials, exhibit energy dissipation when subjected to cyclic loading. Different methods have been used for measuring damping depending upon the strain levels involved [96]. Forced vibration tests have been used for strain levels in the range 10^{-4} to 10^{-2} percent, free vibration tests have been used for strain levels in the range 10^{-3} to 1.0 percent, and static tests have been used to measure the hysteretic energy absorption for strain levels in the range 10^{-2} to 5 percent. The energy dissipation in the latter case has been expressed in terms of equivalent viscous damping ratios [102, 103].

Experimental evidence shows that two major factors influence the amount of damping exhibited by sands, namely, confining pressure and strain amplitude. Damping in this case tends to decrease with an increase in confining pressure and tends to increase with strain amplitude. Clay materials, on the other hand, exhibit damping which is essentially independent of confining pressure but, like sands, the damping increases with strain amplitude.

In modelling the damping as measured in soils, it has generally been carried out by using an equivalent viscous damping system. An important basic study carried out along these lines, using a wide range of yield values, has been reported by Hudson [47]; see Fig. 24. In this case, the equivalent viscous damping ratio corresponding to elasto-plastic hysteretic damping for a single degree of freedom system is given.

In the present investigation, hysteretic damping has not been converted to an equivalent viscous form. Rather, it is treated in its true strain dependent form. Since hysteretic damping treated in this manner accounts for energy dissipation only for strain levels above yield, it is necessary to also include viscous damping to represent the low strain level velocity dependent damping. For this purpose, Rayleigh damping has been used in the present investigation.

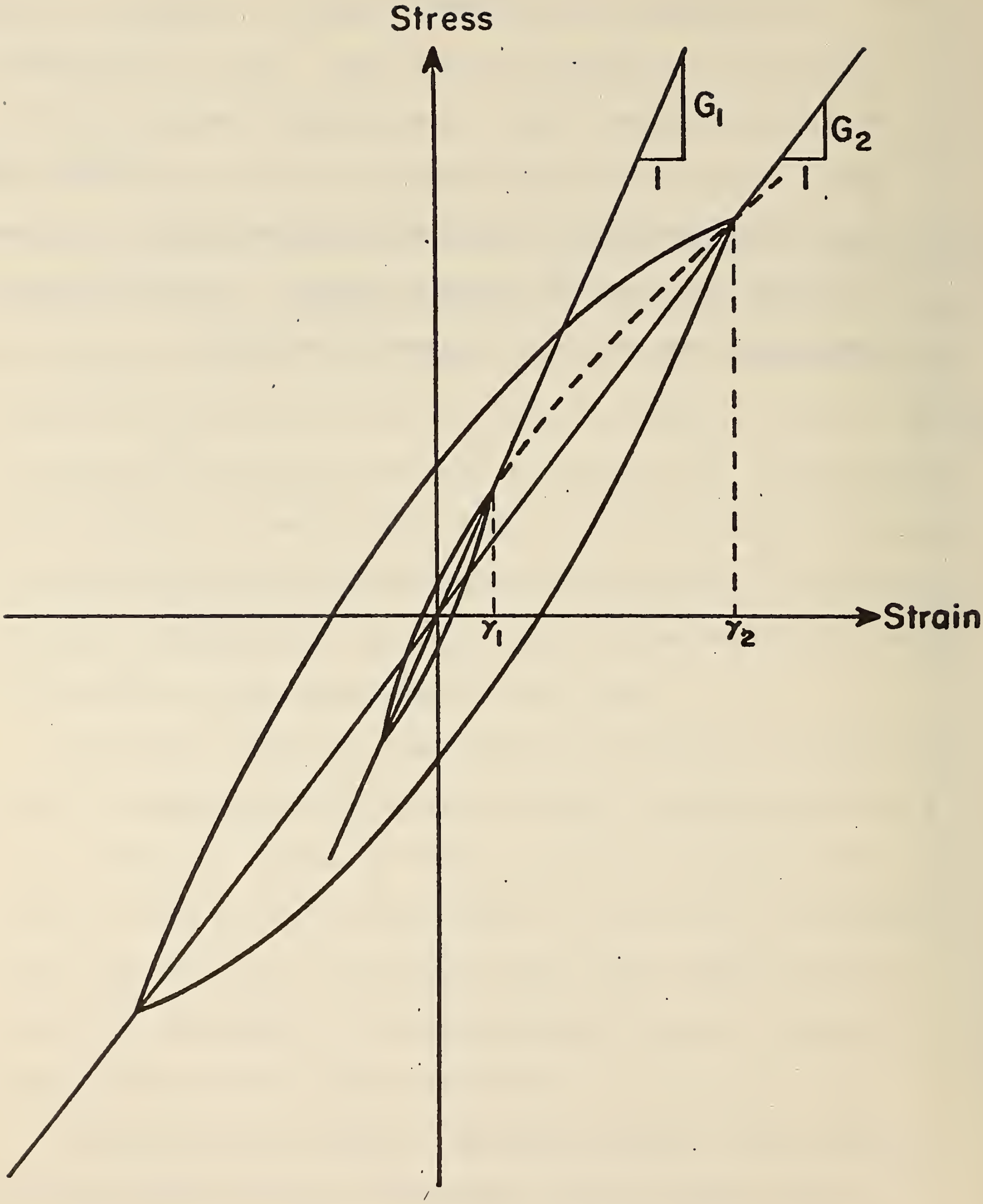


FIG. 9 HYSTERETIC STRESS-STRAIN RELATIONSHIPS AT DIFFERENT STRAIN AMPLITUDES-SECANT MODULUS

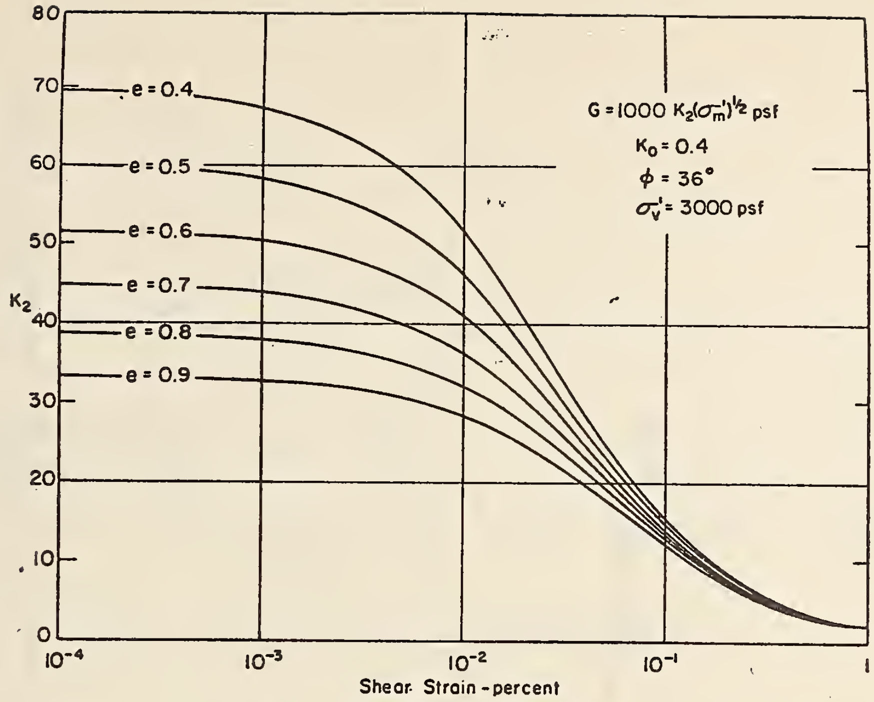


FIG. 10 SECANT SHEAR MODULI OF SANDS AT DIFFERENT VOID RATIOS
(After Seed and Idriss)

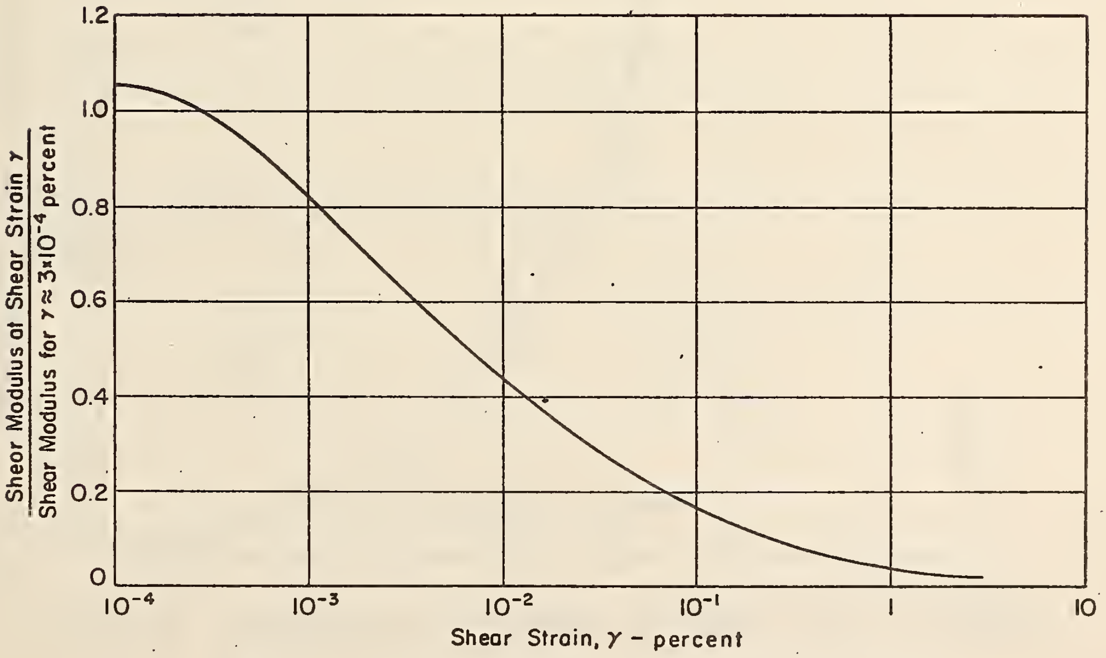


FIG. 11 TYPICAL REDUCTION OF SECANT SHEAR MODULUS WITH SHEAR STRAIN FOR SATURATED CLAYS
(After Seed and Idriss)

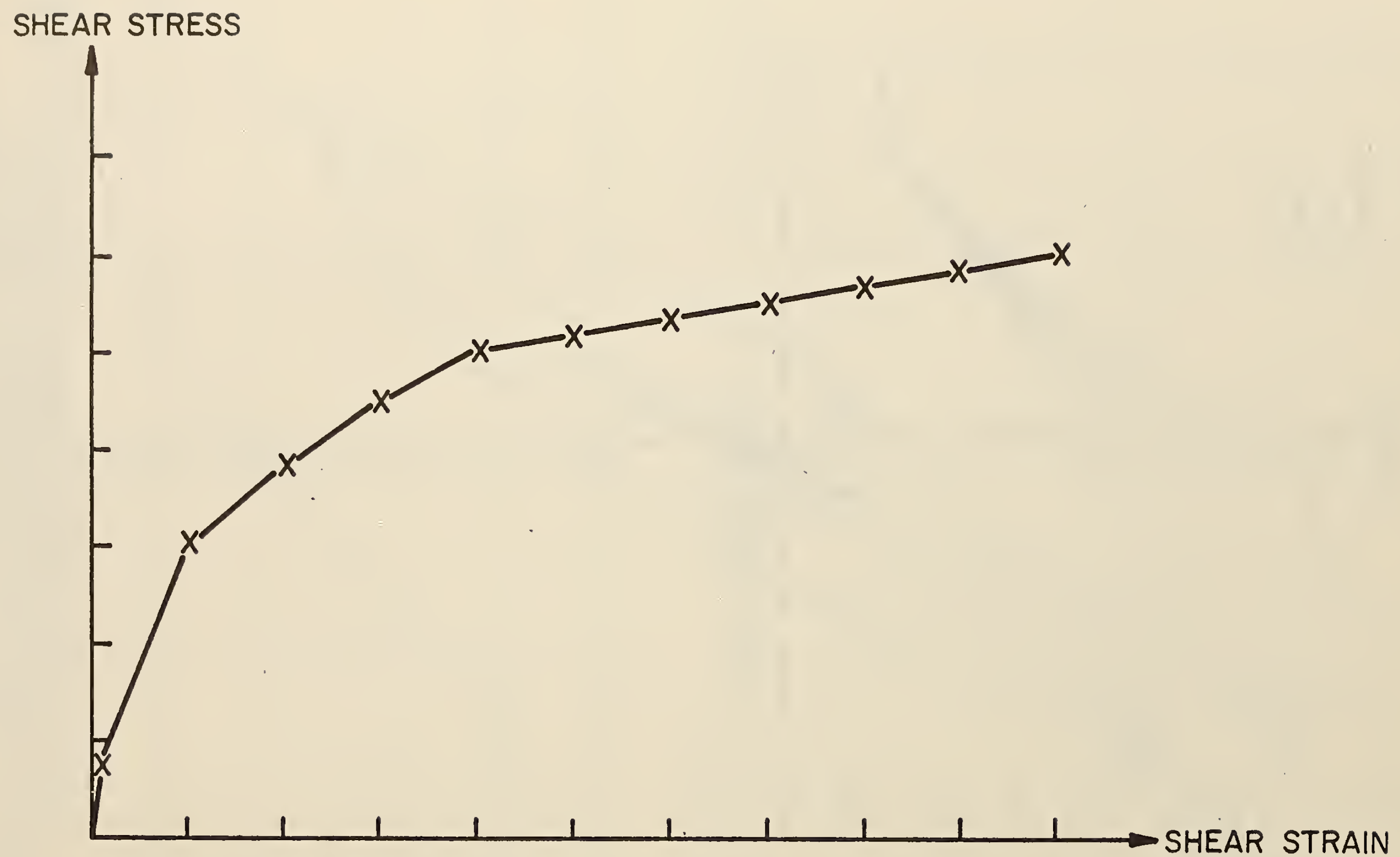
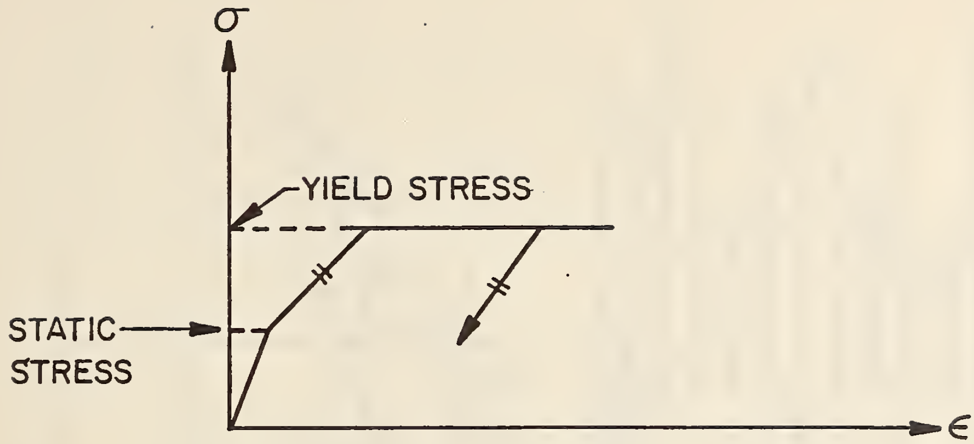
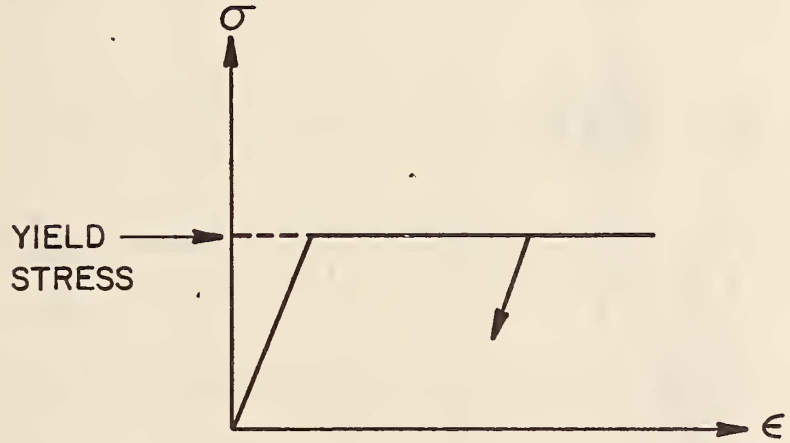


FIG. 12 VARIATION OF TANGENT MODULUS WITH SHEAR STRAIN FOR SAND



(a) TRILINEAR CURVE



(b) BILINEAR CURVE

FIG. 13 IDEALIZED STRESS-STRAIN RELATIONSHIP

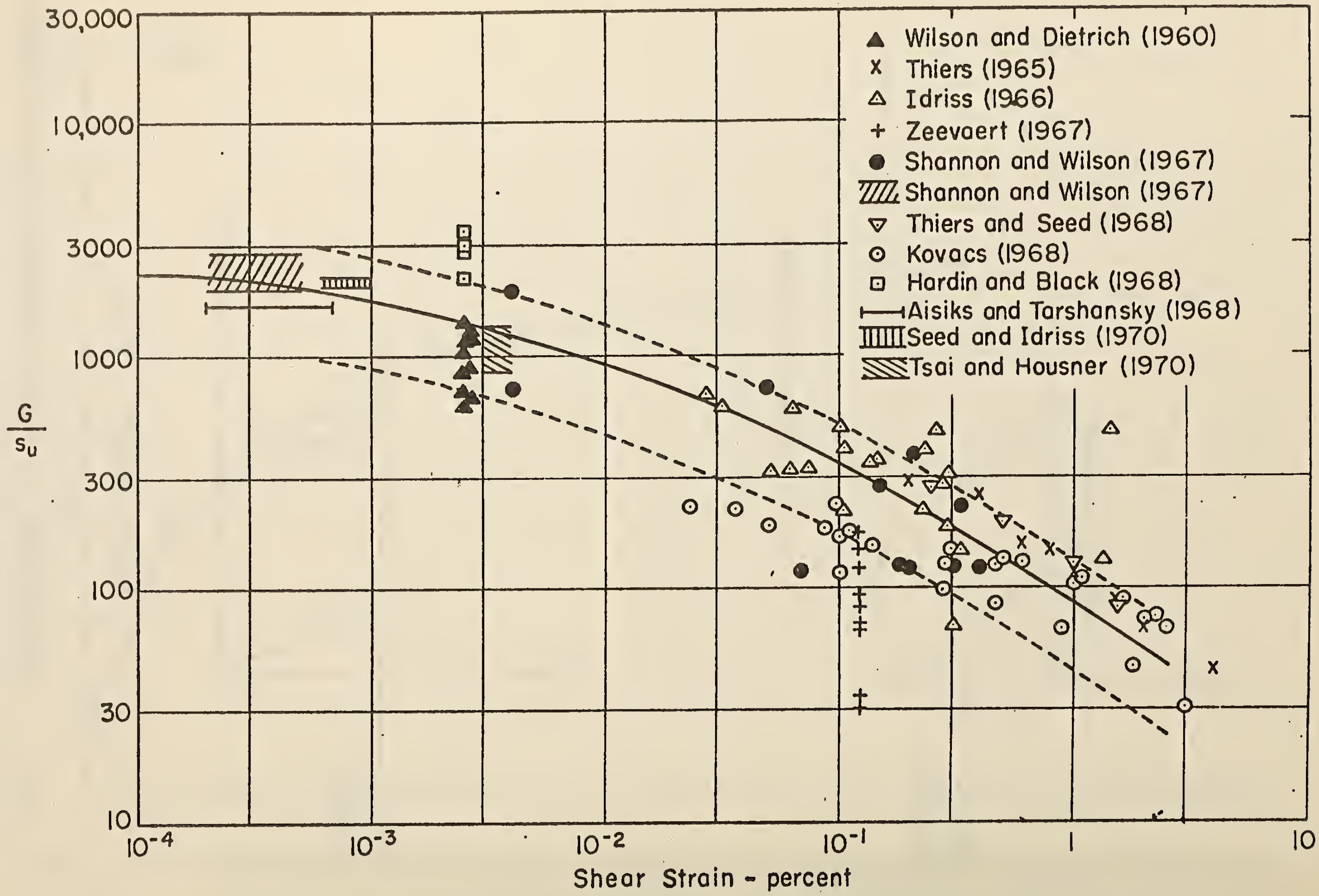


FIG. 14 IN-SITU SECANT SHEAR MODULI FOR SATURATED CLAYS

(After Seed and Idriss)

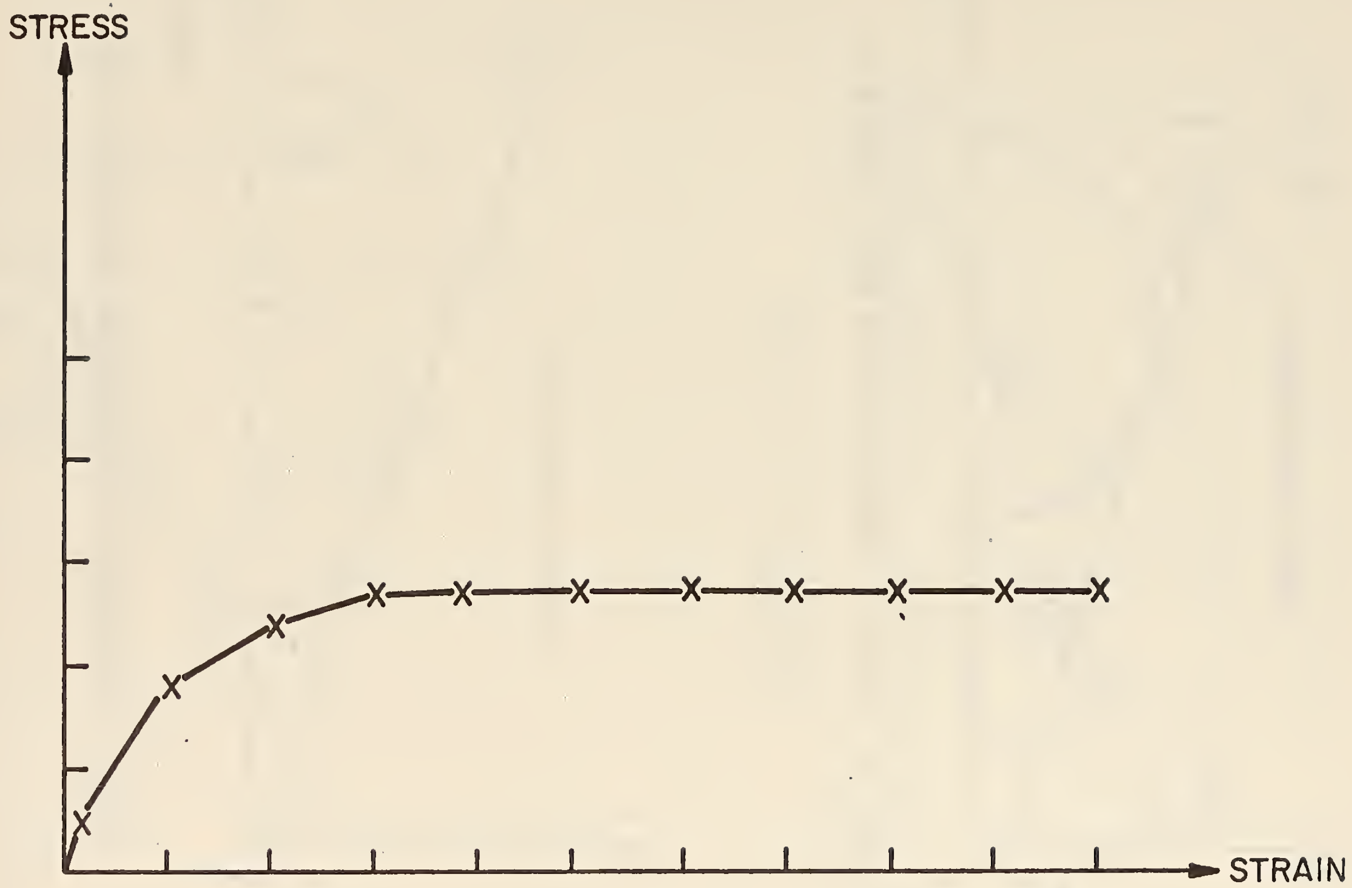


FIG. 15 VARIATION OF TANGENT MODULUS WITH SHEAR STRAIN FOR CLAYS

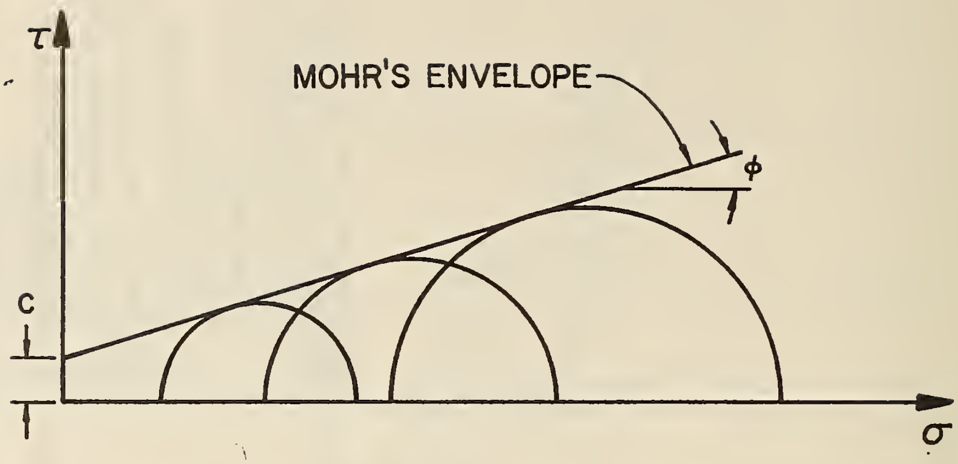


FIG. 16 MOHR'S ENVELOP FOR SOIL SAMPLE

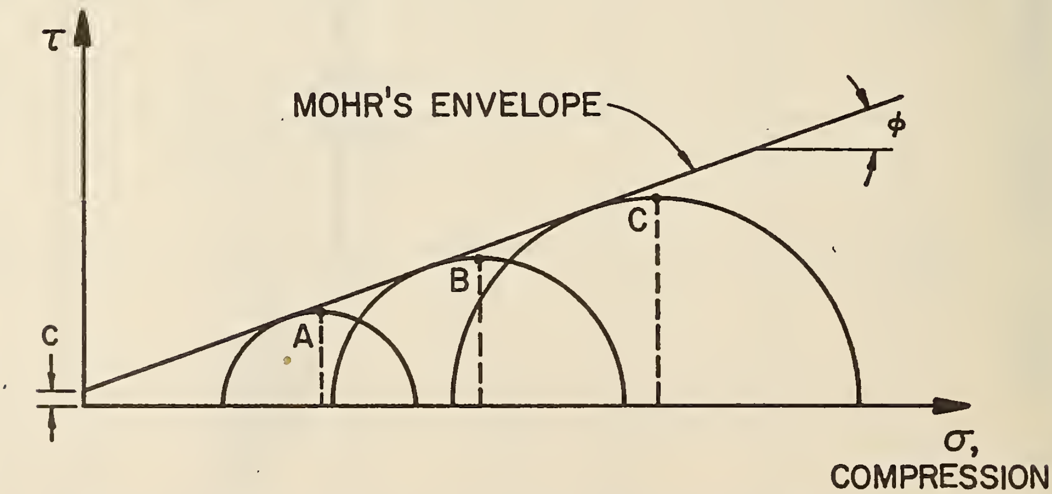


FIG. 17 STRESSES AT MAXIMUM SHEAR PLANE

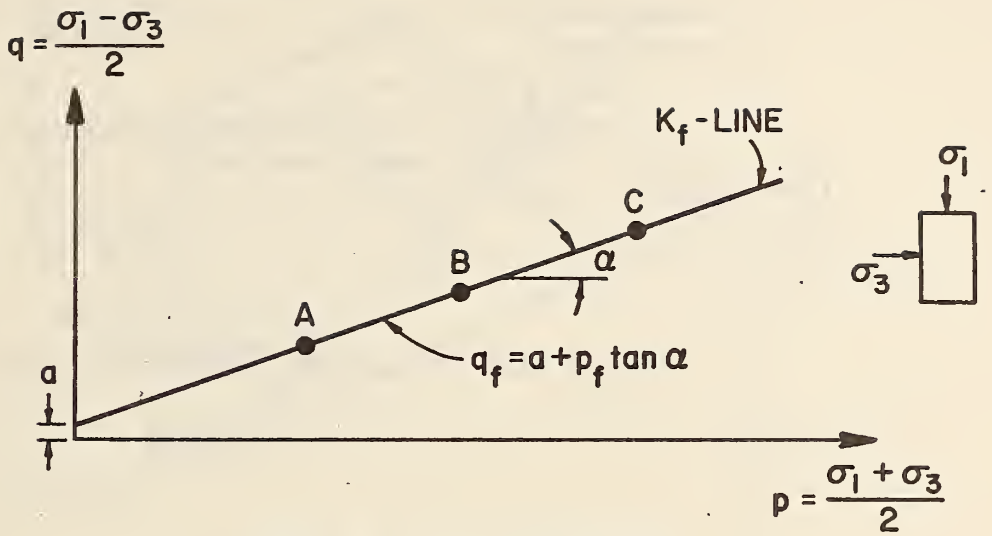


FIG. 18 p-q DIAGRAM

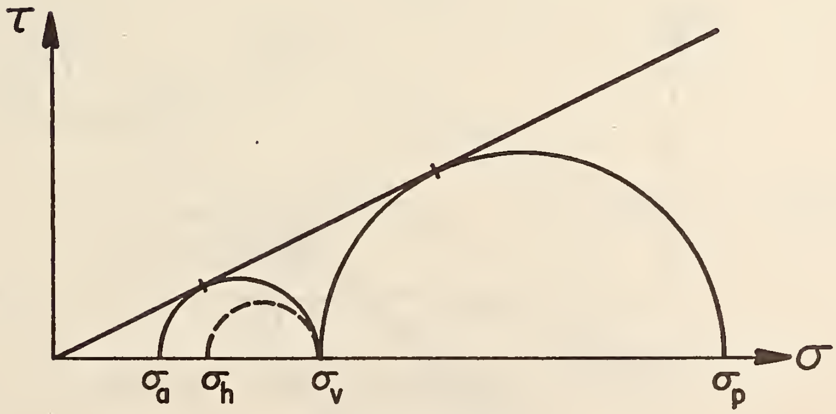


FIG. 19 MOHR CIRCLE

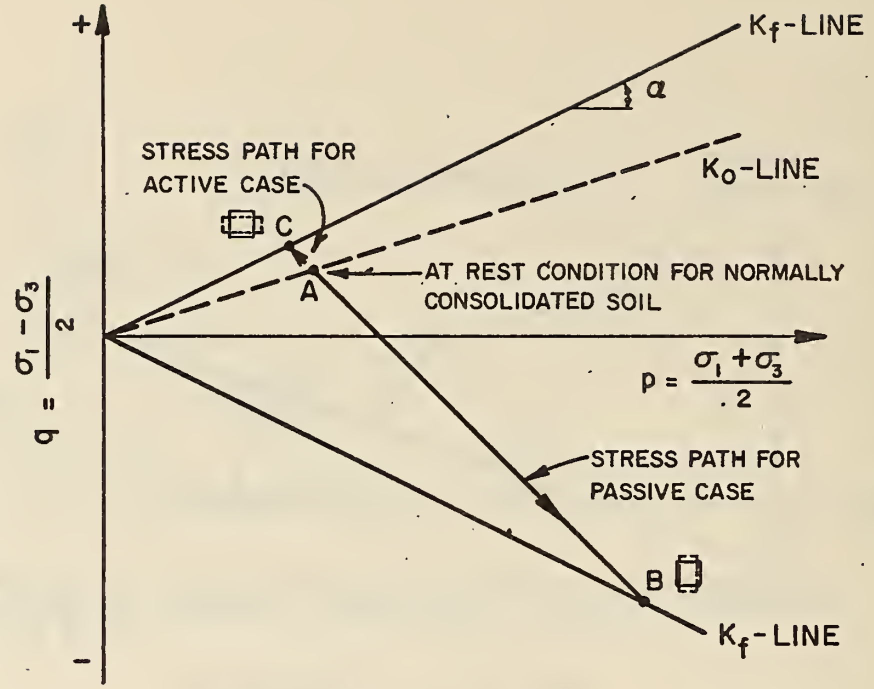


FIG. 20 STRESS PATHS FOR RANKINE ACTIVE AND PASSIVE CONDITIONS

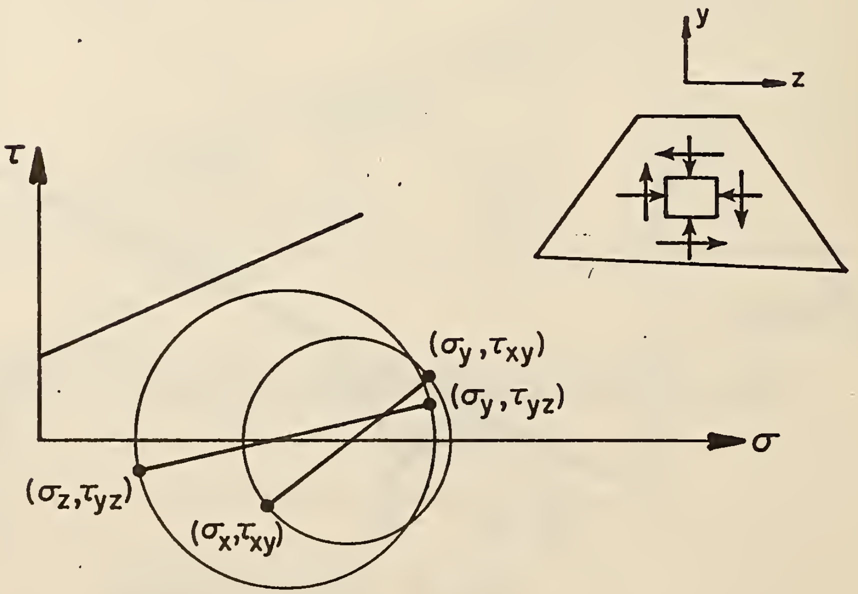


FIG. 21 STRESS CONDITIONS AT FIELD

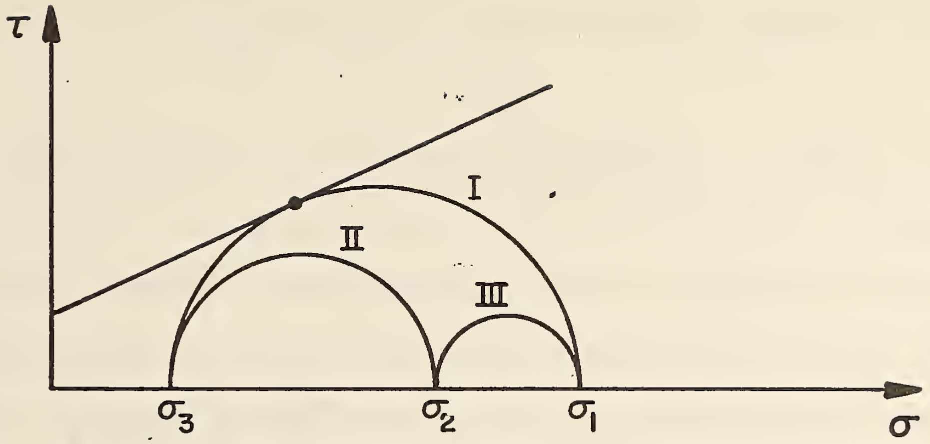


FIG. 22 MOHR CIRCLE IN 3 PLANES

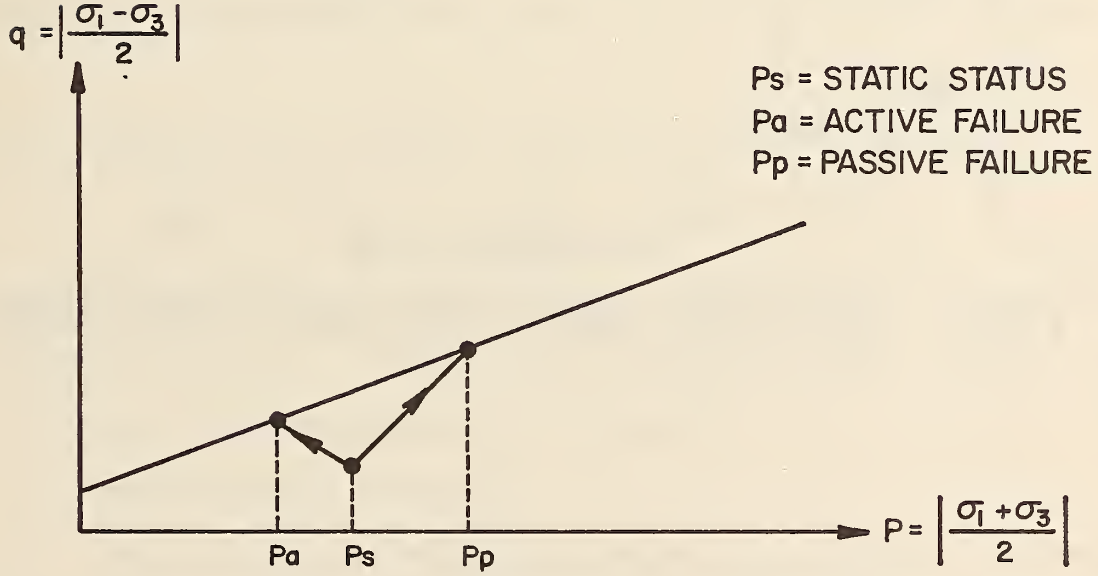


FIG. 23 ACTIVE AND PASSIVE FAILURE

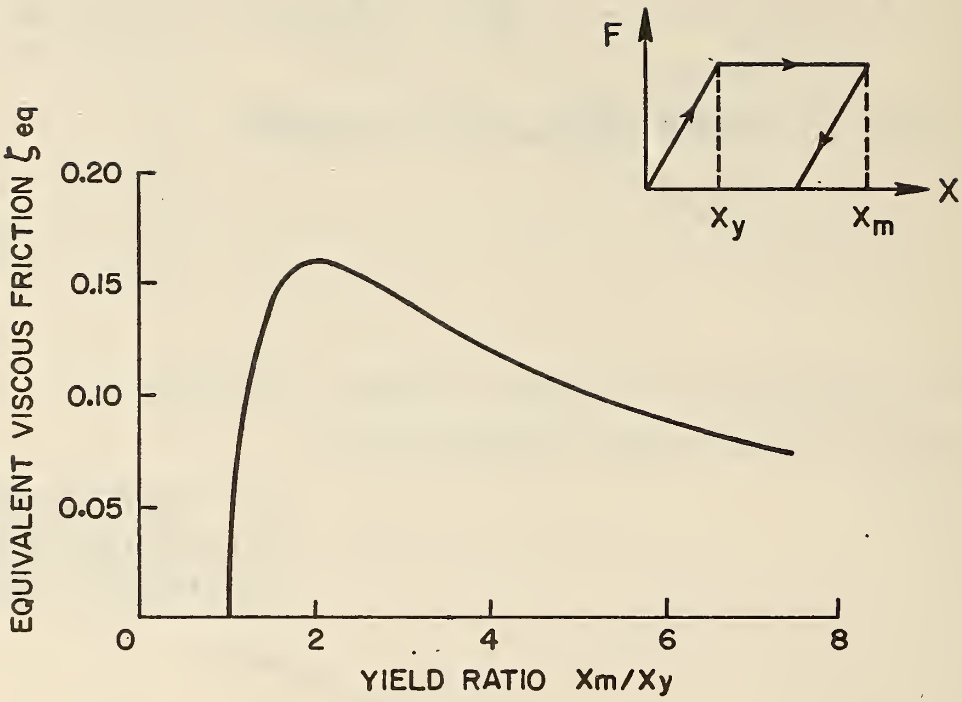


FIG.24 EQUIVALENT VISCOUS FRICTION VS YIELD RATIO

IV NON-LINEAR STIFFNESSES OF MODEL ELEMENTS

During periods of low amplitude oscillation, a bridge-soil system can be modelled using the linear elements as described in Chapter II. It may be necessary in this case to treat the friction and expansion joint elements in a piece-wise linear fashion. For a severe earthquake however inelastic deformations may occur in the concrete columns and/or backfill soils, separations and impacts may develop between the abutments and backfills, slippages may take place in the expansion joints, and yielding may take place at the soil boundaries or in the foundation to complicate the behavior.

It is the purpose of this chapter to describe the non-linear behaviors of all elements and to derive their non-linear stiffnesses.

A. GENERAL ELASTIC-PERFECTLY PLASTIC STRESS-STRAIN RELATIONS

Presently, extensive literature exists on the theory of plasticity and its application to different types of materials and structural elements [30, 46, 85]. Recently, its application has been extended to soil structures and frame structures as reported by Dibaj and Penzien, and by Porter and Powell [28, 83].

The first step in deriving the stress-strain relations for an elastic-perfectly plastic material is to assume a yield function expressed in terms of the stress space. This stress function can be expressed as

$$f(\tau_{ij}) = 0 \qquad (49)$$

By application of the flow rule, the plastic strain increment tensor is derivable from this function using the relation

$$\delta \varepsilon^{P}_{ij} = \lambda \frac{\partial f}{\partial \tau_{ij}} \tag{50}$$

where λ is a non-negative scale factor. The total strain increment tensor is thus decomposed into its elastic and plastic components as expressed by

$$\delta \varepsilon_{ij} = \delta \varepsilon^{E}_{ij} + \delta \varepsilon^{P}_{ij} \tag{51}$$

The generalized Hooke's law, relating the increment of stress tensor to the increment of elastic strain tensor, can be written in the form

$$\delta \tau_{ij} = C^{E}_{ijk\ell} \; \delta \varepsilon^{E}_{ij} \tag{52}$$

Using Eq. (51), Eq. (52) can be rewritten as

$$\delta \tau_{ij} = C^{E}_{ijk\ell} \; (\delta \varepsilon_{k\ell} - \delta \varepsilon^{P}_{k\ell}) \tag{53}$$

For an elastic-perfectly plastic material

$$\delta f = \frac{\partial f}{\partial \tau_{ij}} \delta \tau_{ij} = 0 \tag{54}$$

Substituting Eqs. (53) and (50) into Eq. (54), one obtains

$$\frac{\partial f}{\partial \tau_{ij}} C^{E}_{ijk\ell} \; (\delta \varepsilon_{k\ell} - \delta \varepsilon^{P}_{k\ell}) = 0 \tag{55}$$

or

$$\frac{\partial f}{\partial \tau_{ij}} C^{E}_{ijk\ell} \; \delta \varepsilon_{k\ell} - \frac{\partial f}{\partial \tau_{ij}} C^{E}_{ijk\ell} \; \frac{\partial f}{\partial \tau_{k\ell}} \lambda = 0 \tag{56}$$

Solving for λ gives

$$\lambda = h\, C^E_{ijk\ell} \frac{\partial f}{\partial \tau_{ij}} \delta\, \varepsilon_{k\ell} \tag{57}$$

where

$$\frac{1}{h} = C^E_{ijk\ell} \frac{\partial f}{\partial \tau_{ij}} \frac{\partial f}{\partial \tau_{k\ell}} \tag{58}$$

Substituting Eq. (57) into Eq. (50) results in the relation

$$\delta\, \varepsilon^P_{ij} = h\, C^E_{mnk\ell} \frac{\partial f}{\partial \tau_{mn}} \delta\, \varepsilon_{k\ell} \frac{\partial f}{\partial \tau_{ij}} \tag{59}$$

A further substitution of this relation into Eq. (53) gives

$$\delta\, \tau_{ij} = C^E_{ijk\ell} \left(\delta\, \varepsilon_{k\ell} - h\, C^E_{ijmn} \frac{\partial f}{\partial \tau_{ij}} \frac{\partial f}{\partial \tau_{k\ell}} \delta\, \varepsilon_{mn}\right) \tag{60}$$

or

$$\delta\, \tau_{ij} = C^E_{ijk\ell} \left(\delta\, \varepsilon_{k\ell} - A_{k\ell mn}\, \delta\, \varepsilon_{mn}\right) \tag{61}$$

where

$$A_{k\ell mn} = h\, C^E_{ijk\ell} \frac{\partial f}{\partial \tau_{ij}} \frac{\partial f}{\partial \tau_{mn}} \tag{62}$$

The final form of the stress-strain relation for elasto-perfectly plastic material can now be written in the form

$$\delta\, \tau_{ij} = C_{ijk\ell}\, \delta\, \varepsilon_{k\ell} \tag{63}$$

where

$$C_{ijk\ell} = C^E_{ijk\ell} - C^E_{ijmn}\, A_{k\ell mn} \tag{64}$$

and where

$$C^{P}_{ijk\ell} = C^{E}_{ijmn} A_{k\ell mn} \qquad (65)$$

B. SOIL FINITE ELEMENT

The Mohr-Coulomb criterion as stated in Eq. (43) of Chapter III may be expressed in terms of the principal stresses, namely

$$(\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3) \sin \phi = 2c \cos \phi \qquad (66)$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ with tension as positive. In the case of two dimensional stress in the x,y plane, this relationship takes the form

$$(\sigma_x + \sigma_y) \sin \phi + 2R - 2c \cos \phi = 0 \qquad (67)$$

where

$$R = [(\frac{\sigma_x - \sigma_y}{2})^2 + (\tau_{xy})^2]^{\frac{1}{2}} \qquad (68)$$

is the radius of the failure stress circle as shown in Fig. 25.

1. Tangent Stiffness In Plain Strain - The derivative of tangent stiffness given by Eqs. (49) through (65) will now be presented in matrix form for the case of plain strain. Using the Mohr-Coulomb criterion as the yield function $f(\tau_{ij})$, and the index notation for stresses such that $\tau_{11} = \sigma_x$, $\tau_{22} = \sigma_y$, and $\tau_{12} = \tau_{xy}$, one obtains

$$f = (\tau_{11} + \tau_{22}) \sin \phi + 2 [(\frac{\tau_{11} - \tau_{22}}{2})^2 + \tau_{12}^2]^{\frac{1}{2}} - 2c \cos \phi = 0 \qquad (69)$$

and

$$q_{ij} = \frac{\partial f}{\partial \tau_{ij}} \tag{70}$$

Equation (50) becomes

$$\{\delta\varepsilon^P\} = \lambda\{q\} \tag{71}$$

where

$$\{q\}^T = < q > = < q_{11} \; q_{22} \; q_{12} >$$

$$= < [\frac{1}{2} \sin \phi + (\frac{\tau_{11} - \tau_{12}}{R})], [\frac{1}{2} \sin \phi - (\frac{\tau_{11} - \tau_{22}}{R})], [2 \frac{\tau_{12}}{R}] > \tag{72}$$

Equation (51) in matrix form can be written as

$$\{\delta \varepsilon\} = \{\delta \varepsilon^E + \delta \varepsilon^P\} \tag{73}$$

where

$$\{\delta \varepsilon\}^T = <\delta \varepsilon> = < \delta \varepsilon_{11} \; \delta \varepsilon_{22} \; \delta \varepsilon_{12} >$$

$$\{\delta \varepsilon^P\}^T = < \delta \varepsilon^P > = < \delta \varepsilon^P_{11} \; \delta \varepsilon^P_{22} \; \delta \varepsilon^P_{12} >$$

$$\{\delta \varepsilon^E\}^T = < \delta \varepsilon^E > = < \delta \varepsilon^E_{11} \; \delta \varepsilon^E_{22} \; \delta \varepsilon^E_{12} >$$

$$\{\delta \tau\} = [C^E] \{\delta \varepsilon^E\}$$

and where

$$\{\delta \tau\}^T = < \delta \tau> = < \delta \tau_{11} \; \delta \tau_{22} \; \delta \tau_{12} > \tag{74}$$

$$[C^E] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \tag{75}$$

$$\lambda = h\{q\}^T [C^E] \{\delta \varepsilon\} \tag{76}$$

$$\frac{1}{h} = \{q^T\} [C^E] \{q\}$$

$$\doteq \frac{E}{1+\nu} \left[\frac{\sin^2 \phi}{2(1-2\nu)} + \frac{1}{2} + \frac{3}{2} \frac{\tau_{12}^2}{R^2} \right] \tag{77}$$

$$\frac{1}{h} = \frac{E\ B}{1+\nu} \tag{78}$$

where

$$B = \frac{\sin^2 \phi}{2(1-2\nu)} + \frac{1}{2} + \frac{3}{2} \frac{\tau_{12}^{\ 2}}{R^2} \tag{79}$$

Equations (62) through (65) in matrix form become

$$[A] = h \{q\} \{q\}^T [C^E] \tag{80}$$

$$\{\delta \tau\} = [C] \{\delta \varepsilon\} \tag{81}$$

$$[C] = [C^E] - [C^E][A]$$

$$= [C^E] - h[C^E] \{q\} \{q^T\} [C^E]$$

$$= [C^E] - [C^P] \tag{82}$$

Letting

$$\{Q\}^T = \{q\}^T [C^E]$$

$$= \frac{E}{1+\nu} < Q_1\ Q_2\ Q_3 >$$

$$= \frac{E}{1+\nu} < \frac{\sin \phi}{2(1-2\nu)} + \frac{\tau_{11} - \tau_{12}}{4R}, \frac{\sin \phi}{2(1-2\nu)} - \frac{\tau_{11} - \tau_{22}}{4R}, \frac{\tau_{12}}{R} > \tag{83}$$

, the matrix $[C^P]$ can be expressed in the form

$$[C^P] = h[C^E]\{q\}\{q\}^T[C^E]$$

$$= h\{Q\}\{Q\}^T$$

$$= \frac{E}{B(1+\nu)}\begin{bmatrix} Q_1^{\;2} & Q_1Q_2 & Q_1Q_3 \\ Q_2Q_1 & Q_2^{\;2} & Q_2Q_3 \\ Q_3Q_1 & Q_3Q_2 & Q_3^{\;2} \end{bmatrix} \tag{84}$$

Finally, the tangent stiffness in explicit matrix notation becomes

$$[C] = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} - \frac{E}{B(1+\nu)}\begin{bmatrix} Q_1^{\;2} & Q_1Q_2 & Q_1Q_3 \\ Q_2Q_1 & Q_2^{\;2} & Q_2Q_3 \\ Q_3Q_1 & Q_3Q_2 & Q_3^{\;2} \end{bmatrix} \tag{85}$$

2. Postulate for Application of Tangent Stiffness to the Case of Plane Stress - In order to use the previously derived tangent stiffness for the case of plane stress, one must make two assumptions as follows: (1) no yielding occurs in the third direction and (2) the results of triaxial tests are directly applicable to the case of plain stress. Under these assumptions, the above derivations for plane strain also apply to the case of plane stress except that the matrix $[C^E]$ is changed to the form

$$[C^E] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{86}$$

This results in the following changes

$$\{q\}^T\ [C^E]\ \{q\} = \frac{2E}{1-\nu^2}\ [(1+\nu)\ \sin^2\phi + (1-\nu)\ (\frac{\tau_{11} - \tau_{22}}{4R})^2 + (1-\nu)\ (\frac{\tau_{12}}{R})^2\ ] \tag{87}$$

$$\frac{1}{h} = \frac{2EB}{1-\nu^2} \tag{88}$$

$$B = [(1+\nu)\ \sin^2\phi + (1-\nu)\ (\frac{\tau_{11} - \tau_{22}}{4R})^2 + (1-\nu)\ (\frac{\tau_{12}}{R})^2\ ] \tag{89}$$

$$\begin{aligned} \{Q\}^T &= \{q\}^T\ [C^E] \\ &= \frac{E}{1-\nu^2} < Q_1\ Q_2\ Q_3 > \\ &= \frac{E}{1-\nu^2} < \frac{1}{2}\ (1+\nu)\ \sin\phi + (1+\nu)\ \frac{\tau_{11} - \tau_{22}}{4R}, \\ &\quad \frac{1}{2}\ (1+\nu)\ \sin\phi - (1+\nu)\ \frac{\tau_{11} - \tau_{22}}{4R},\ (1-\nu)\ \frac{\tau_{12}}{R} > \end{aligned} \tag{90}$$

$$\begin{aligned} [C^P] &= h[C^E]\ \{q\}\ \{q^T\}\ [C^E] \\ &= h\{Q\}\ \{Q\}^T \\ &= \frac{E}{2B(1-\nu^2)} \begin{bmatrix} Q_1{}^2 & Q_1Q_2 & Q_1Q_3 \\ Q_2Q_1 & Q_2{}^2 & Q_2Q_3 \\ Q_3Q_1 & Q_3Q_2 & Q_3{}^2 \end{bmatrix} \end{aligned} \tag{91}$$

Finally, one obtains

$$\begin{aligned} [C] &= [C^E] - [C^P] \\ &= \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} - \frac{E}{2B(1+\nu)} \begin{bmatrix} Q_1{}^2 & Q_1Q_2 & Q_3 \\ Q_2Q_1 & Q_2{}^2 & Q_2Q_3 \\ Q_3Q_1 & Q_3Q_2 & Q_3{}^2 \end{bmatrix} \end{aligned} \tag{92}$$

C. COLUMN ELEMENT

1. Trilinear Yield Surface of the Moment-Axial Force Interaction Diagram - Moment-axial force interaction curves for the most commonly used sections of reinforced concrete columns are available in handbooks published by the American Concrete Institute [2]. These sections include the spirally reinforced, circular and square columns, and the symmetrically reinforced, rectangular tied columns. Three typical sections are shown in Fig. 27. For other types of sections, a computer program has been developed to obtain the interaction curves by a direct analysis method [105].

There are three controlling points on the interaction curve which can be used to approximate its form using a trilinear relationship. These points are the minimum eccentricity point B, the balanced point C, and the pure moment point D, as shown in Fig. 26. Segment AB having a horizontal slope defines the ultimate axial load capacity as that axial load given by the ACI code for the minimum eccentricity condition [35]. Segment BC defining the compression failure zone connects point B with the balanced point C which corresponding to a concrete strain of 0.003 and a steel strain equal to the yield strain. Finally, segment CD connects point C with point D which corresponds to the yield moment in the presence of no axial load. This latter segment defines the tension failure zone. Since tension seldom occurs in the columns of short bridges, it is not necessary to define the interaction diagram in the negative (tension) region of P. Line segment OD therefore is considered the boundary line for the yield surface which signifies zero tension capacity.

Using the approximate trilinear form of the interaction curve, the normalized yield stress function can be written as

$$f_i (S_1, S_2) = a_i S_1 + b_i S_2 + c_i = 0 \tag{93}$$

where

$$S_1 = \frac{P}{P^2} \qquad ; \qquad S_2 = \frac{M}{M^2} \tag{94}$$

For line segments AB, BC, and CD, coefficients a_i, b_i, and c_i are, respectively,

$$\begin{aligned} a_1 &= 1 & a_2 &= \frac{M_1}{M_2} - 1 & a_3 &= \frac{M_3}{M_2} - 1 \\ b_1 &= 0 \quad ; & b_2 &= 1 - \frac{P_1}{P_2} & b_3 &= 1 - \frac{P_3}{P_2} \\ c_1 &= - \frac{P_1}{P_2} & c_2 &= \frac{P_1}{P_2} - \frac{M_1}{M_2} & c_3 &= \frac{P_3}{P_2} - \frac{M_3}{M_2} . \end{aligned} \tag{95}$$

2. Tangent Stiffness - Using the trilinear interaction relationship, the derivation of the elastic-perfectly plastic tangent stiffness of a column follows the same procedure used previously for soils except one must consider that (1) plastic deformations are concentrated at the ends of the element with the deformations taking place independently over zero lengths at each end of the element, (2) plastic deformations are independent of the shear forces present, and (3) the stiffnesses are expressed in terms of element end forces and displacements rather than in terms of stress and strain as in the case of soil elements. In this case, one obtains

$$\{du^P\} = \begin{Bmatrix} du_I^P \\ du_J^P \end{Bmatrix} = \begin{bmatrix} \{q\}_I & \{0\} \\ \{0\} & \{q\}_J \end{bmatrix} \begin{Bmatrix} \lambda_I \\ \lambda_J \end{Bmatrix} = [q]\ \{\lambda\} \tag{96}$$

where du_I^P and du_J^P are the plastic deformation increments at ends I and J of the element, respectively, and λ_I and λ_J are the associated proportionality factors. It follows therefore that

$$\{du^P\}^T = < du_1^P \quad du_2^P \quad du_3^P \quad du_4^P \quad du_5^P \quad du_6^P > \tag{97}$$

$$\{q\}_m^T = \begin{Bmatrix} \dfrac{\partial f_i}{\partial s_j} \end{Bmatrix}_m^T = < a_i \ 0 \ b_i >_m, \quad m = I \text{ or } J \tag{98}$$

$$\{du\} = \begin{Bmatrix} du_I \\ du_J \end{Bmatrix} = \begin{Bmatrix} du_I^E \\ du_J^E \end{Bmatrix} + \begin{Bmatrix} du_J^P \\ du_J^P \end{Bmatrix} \tag{99}$$

$$\{ds\} = \begin{Bmatrix} ds_I \\ ds_J \end{Bmatrix} = [K^E] \begin{Bmatrix} du_I^E \\ du_J^E \end{Bmatrix} \tag{100}$$

where $[K^E]$ is the elastic stiffness matrix appearing in Eq. 23. Further, one obtains

$$\{\lambda\} = \begin{Bmatrix} \lambda_I \\ \lambda_J \end{Bmatrix} = [h]\ [q]^T\ [K^E]\ \{du\} \tag{101}$$

$$[h]^{-1} = [q]^T\ [K^E]\ [q] \tag{102}$$

$$[A] = [q]\ [h]\ [q]^T\ [K^E] \tag{103}$$

$$\{ds\} = [K]\ [du] \tag{104}$$

$$
\begin{aligned}
[K] &= [K^E] - [K^E]\,[A] \\
&= [K^E] - [K^E]\,[q]\,[h]\,[q]^T\,[K^E] \\
&= [K^E] - [K^P] \qquad (105)
\end{aligned}
$$

3. <u>An Approximation Used in Numerical Iteration</u> - Due to the occurence of impact upon the frictional element at the interface of soil and abutment wall elements, the time interval used in a dynamic analysis to obtain a stable numerical solution must be quite small. It must be sufficiently small so that "overshooting" of the interaction diagram during a single interval as shown in Fig. 28 is minimized. Even though this interval is kept small, the error introduced into a solution by "overshooting' has been corrected [16, 60, 105].

In the present investigation, a simple procedure has been adopted for the overshooting. If, as shown in Fig. 28, the elastic stress state assumed during an interval moves the applied forces from point A to point B then a transition from an elastic to a yield state is indicated. While the new force vector $S_{t+\Delta t}$ as represented by point B is adopted without correction, point C which is the intersection of the new stress vector $S_{t+\Delta t}$ and the yield segment EF is used to calculate the slope of the plastic deformation vector $\dot{du}^P$. This same procedure is used when overshooting occurs at a discontinuity point on the interaction curve as shown in Fig. 29. In this case, the elastic stress state assumed during an interval moves the applied forces from point A' to point B'. The new force vector $S_{t+\Delta t}$ intersects line EF; thus, the slope of EF is used to calculate the plastic deformation increment. Other investigators have used somewhat different procedures for this correction [73, 77, 83, 107].

D. FRICTIONAL ELEMENT

The non-linear behavior of the frictional element is described in terms of normal and shear stiffnesses k_n and k_s during three distinct stages, namely, (1) when reparation occurs, in which case $k_n = k_s = 0$, (2) when compression occurs at the interface but the shear strength of the element is not exceeded in which case k_n and k_s are assigned high values, and (3) when compression occurs but the shear strength of the element is exceeded in which case $k_s = 0$ and k_n retains a high value. The shear yield strength of the element can be defined by the Mohr-Coulomb yield criterion, i.e.,

$$\tau = \sigma \tan \phi_w \tag{106}$$

where ϕ_w is the angle of wall friction between the soil and the abutment wall.

Before discussing the value of ϕ_w to be used, two terms must be defined [63]. Firstly, the constant volume frictional angle is defined by $\phi_{cv} = \sin^{-1} \left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \right)_{cv}$, where σ_1 and σ_3 are the axial stress and confining pressure, respectively, in a triaxial test at that stage when the sand strains without further volume change. Secondly, peak friction angle ϕ is defined as the slope of Mohr envelope which is a function of the stresses indicated at the peak of stress-strain curve of a triaxial test.

For backfill against a concrete wall, Lamb suggests that the angle of wall friction ϕ_w is about equal to ϕ_{cv} and that it typically has a numerical value of about 30° [63]. Seed and Whitman in a discussion of dynamically active pressure against walls suggest that $\phi_w = \phi/2$ is satisfactory for most practical purposes [97].

E. EXPANSION JOINT ELEMENT

The stiffnesses of the expansion joint element can be defined in terms of normal and shear stiffnesses k_n and k_s during three stages, (1) when the frictional resistance between the deck and abutment is not exceeded in which case k_n and k_s are assigned high values, (2) when the frictional resistance is exceeded, but the gap between deck and abutment as shown in Fig. 6a is not closed, in which case $k_s = 0$ and k_n retains a high value, (3) when the frictional resistance is exceeded and the gap is closed in which case, if the relative displacement indicated is consistent with gap closure, high values k_n and k_s are retained.

F. SOIL BOUNDARY ELEMENT AND EQUIVALENT COLUMN OF FOUNDATION

The non-linear behavior of the soil boundary element and the equivalent column foundation element can be approximated by defining yield levels using standard methods and adopting elasto-plastic hysteretic models. In the present investigation, only elastic behavior has been considered.

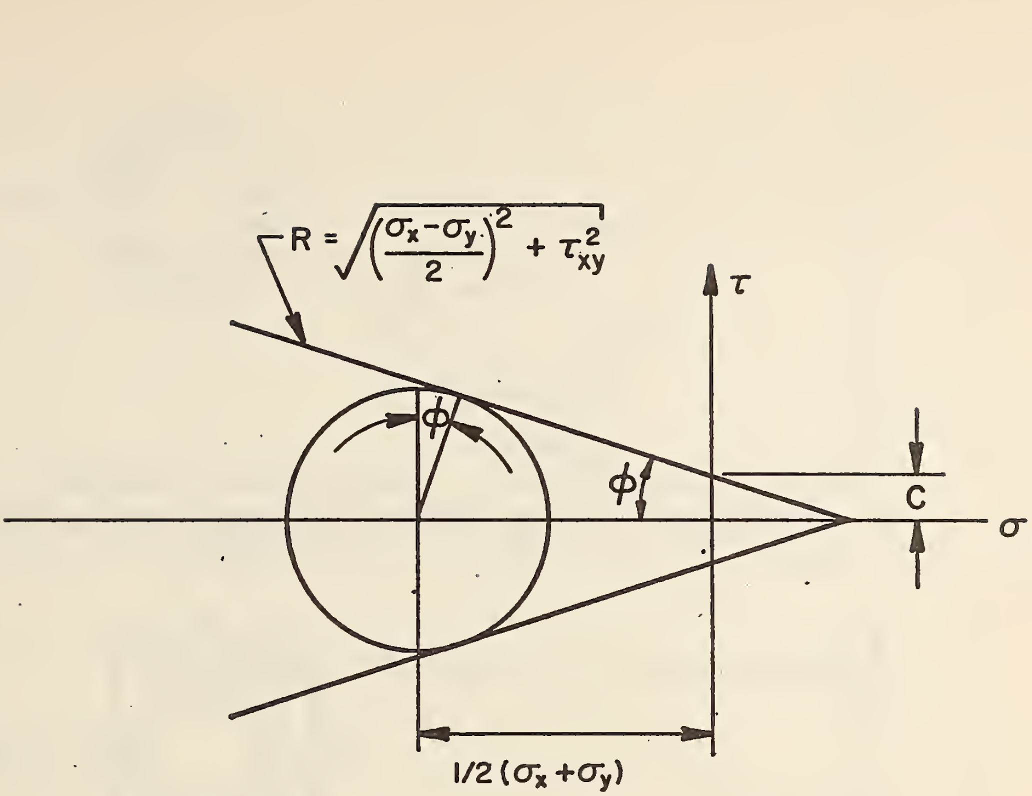


FIG. 25 MOHR-COULOMB YIELD FUNCTION

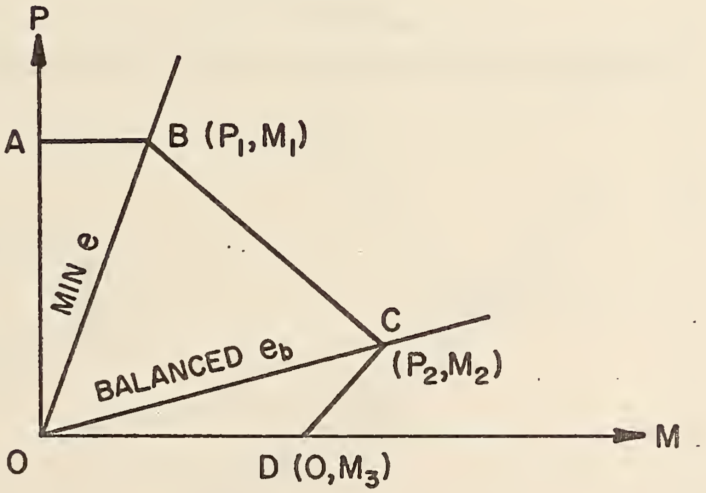
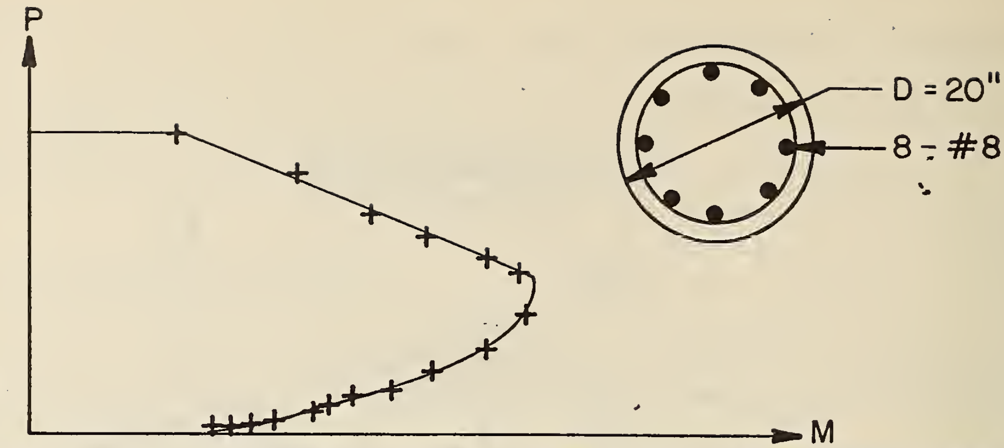
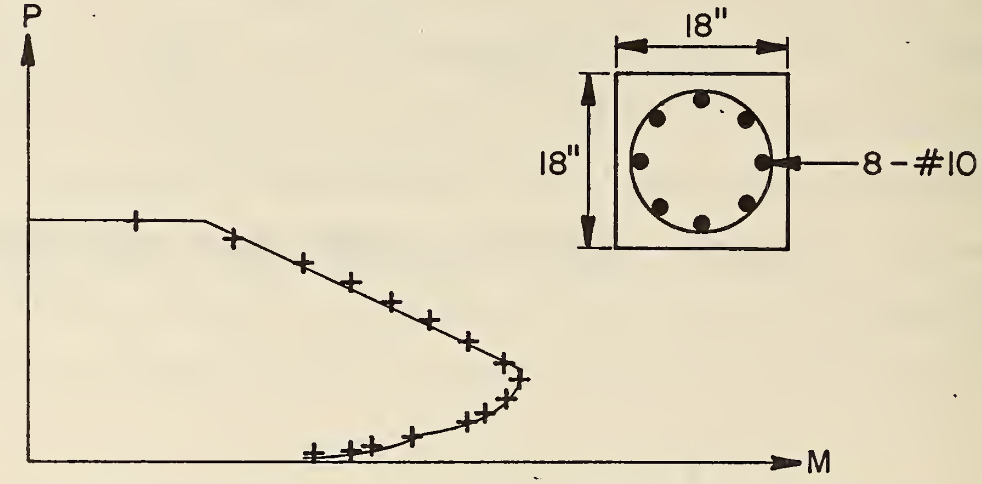


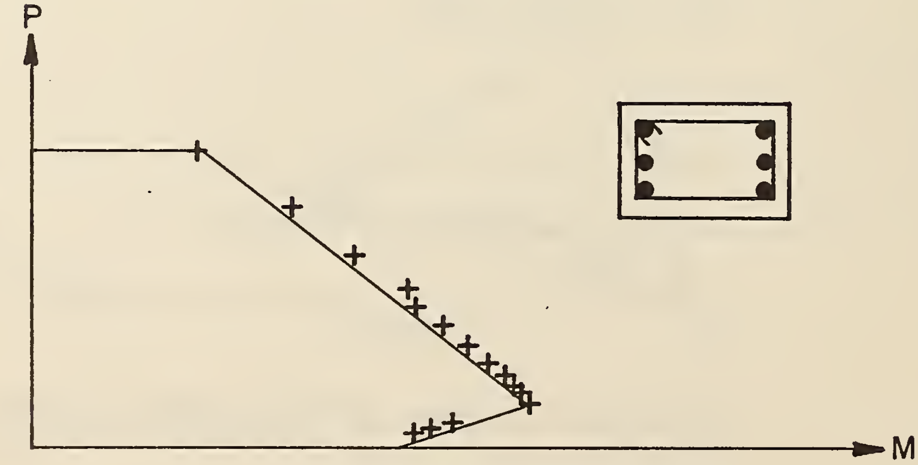
FIG. 26 COLUMN INTERACTION CURVE



(a) CIRCULAR SECTION WITH BARS CIRCULARLY ARRANGED



(b) SQUARE SECTIONS WITH BAR CIRCULARLY ARRANGED



(c) RECTANGULAR SECTION

FIG. 27 INTERACTION DIAGRAM OF CONCRETE COLUMN

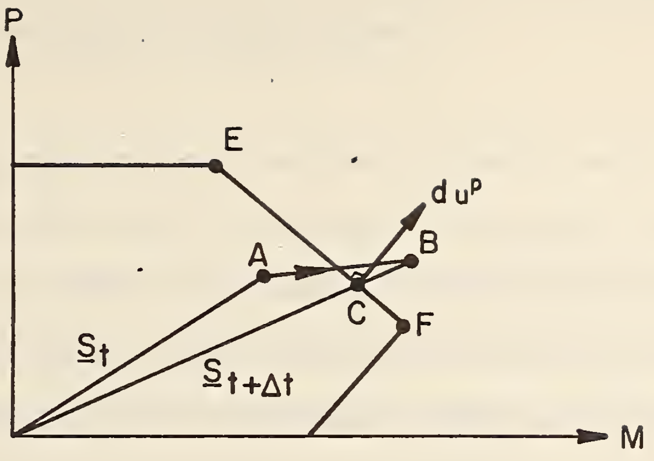


FIG. 28 SITUATION AT OVERSHOOTING

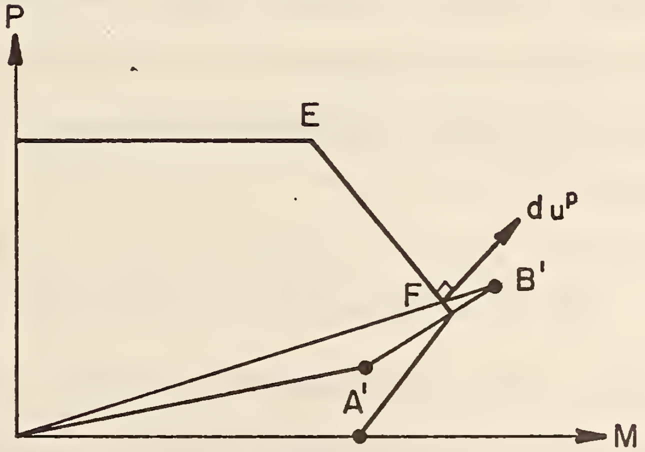


FIG. 29 OVERSHOOTING AT DISCONTINUITY

V DYNAMIC ANALYSIS PROCEDURES

In the subsequent sections, the nonlinear coupled equations of motion for the discrete parameter soil-structure system are formulated and the method used to establish their associated stiffness, mass, and damping matrices are described. Also presented are the step-by-step integration techniques employed in their solution.

A. EQUATIONS OF MOTION

Although some previous investigations have considered spacial variations in the earthquake ground motions [27, 32, 43], the present investigation assumes identical motions at all base points of the soil-structural system. This assumption is considered reasonable due to the relatively short lengths of bridges being considered.

The coupled equations of motion at time t for an N degree of freedom system subjected to rigid base excitation can be expressed in the matrix form

$$[M]\ \{\ddot{u}\}_t + [C]_t\ \{\dot{u}\} + [K]_t\ \{u\}_t = \{R\}_t \tag{107}$$

where [M] is the constant mass matrix, $[C]_t$, and $[K]_t$ are the time dependent damping and stiffness matrices, respectively, and where $\{u\}_t$, $\{\dot{u}\}_t$, and $\{\ddot{u}\}_t$ are the nodal point displacement, velocity, and acceleration vectors, respectively. The excitation force vector $\{R\}_t$ due to rigid base motions is given by the relation

$$\{R\}_t = -\ [M]\ \{I\}\ (\ \ddot{u}^x_{gt} + \ddot{u}^y_{gt}\ ) \tag{108}$$

where {I} is the unit vector and $\ddot{u}^x_{gt}$ and $\ddot{u}^y_{gt}$ are the horizontal and vertical components of ground acceleration.

B. STIFFNESS MATRIX

The complete stiffness matrix $[K]_t$ is assembled from the individual element stiffness matrices using the direct stiffness method [22]. The individual element stiffnesses during the elastic and inelastic ranges in each time interval are obtained by the procedures described in Chapters II and IV. The complete stiffness matrix takes on a symmetric banded form; thus, only the diagonal and the off-diagonal terms on one side need be stored in the computer.

C. MASS MATRIX

The mathematical model used assumes all mass as concentrated at the nodal points. The diagonal mass matrix which results represents a significant saving in computer storage and computational time when compared with similar requirements for the consistent mass matrix [21]. One-third of the mass of each triangular element, one-fourth of the mass of each quadrilateral element, and one-half of the mass of each beam element are lumped at their respective nodal points. No rotational moments of inertia are assigned to these masses. The resulting mass matrix for the complete soil-structural system takes the form

$$[M] = \text{diag} < M_1 \; M_2 \; \text{---} \; M_n > \qquad (109)$$

where M_i is the mass associated with the ith degree of freedom and n is the total number of degrees of freedom present in the system. The static condensation procedure is used to eliminate the degree of freedom of zero rotational masses in the solution of eigenvalues.

D. DAMPING MATRIX

Various methods have been used by investigators to determine the viscous damping matrix corresponding to matrix $[C]_t$ in Eq. (107) [44]. Wilson and Penzien have described two methods for evaluting orthogonal damping matrices [115]. The first method relates the modal damping ratios to the coefficients in the Caughey series form [15]. The second method is a direct approach which expresses the damping matrix as a sum of a series of matrices each of which produces damping in only one particular mode. The second approach has the advantage that prescribed damping ratios in all modes are easily controlled.

The Rayleigh damping matrix which constructs a damping matrix from a scaled linear combination of the mass and stiffness matrices is used in the present investigation. This type of damping matrix has the advantage that it can be calculated directly using the relation

$$[C]_t = \alpha[M] + \beta[K]_t \tag{110}$$

where α and β are scalar quantities to be prescribed. By properly selecting these scalar values, the damping ratios can be controlled in two normal modes. It can be shown that these quantities are related to the damping ratios (ξ) and circular frequencies (ω) of the ith and jth normal modes through the equations

$$\alpha = \frac{2\,\omega_i \omega_j\,(\xi_j \omega_i - \xi_i \omega_j)}{(\omega_i^2 - \omega_j^2)} \tag{111}$$

$$\beta = \frac{2\,(\xi_i \omega_i - \xi_j \omega_j)}{(\omega_i^2 - \omega_j^2)} \tag{112}$$

Further, it can be shown that if α and β satisfy Eqs. (111) and (112), the damping ratio in nth normal has the value given by

$$\xi_n = \frac{\alpha + \beta\,\omega_n^2}{2\,\omega_n} \tag{113}$$

In the present investigation, the numerical values of α and β are determined by using the initial elastic soil-structural system and by prescribing the damping ratios of any two modes of the system. These quantities are then held constant at these values throughout the time history of response including those periods of time when the system responds inelastically.

As shown by Eq. (107), the stiffness matrix varies with time due to nonlinear effects; therefore, the damping matrix also varies with time. Because the stiffnesses in the system decrease considerably during periods of element yielding, the viscous damping present during these periods also decreases. It should be kept in mind, however, that the major sources of energy dissipation during these periods are the hysteresis loops in the force-deformation relations as described in Chapter III.

E. STEP-BY-STEP INTEGRATION TECHNIQUES

Having the solution of the coupled equations of motion at time t, the step-by-step integration procedure allows one to obtain their solution at a later time $t+\Delta t$. To develop this procedure, the matrix equation of motion is transformed to its incremental form by subtracting Eq. (107) for time t from the corresponding equation for time $t+\Delta t$ as given by

$$[M]\ (\{\ddot{u}\}_t + \{\Delta\ddot{u}\}_t) + [C]_t\ (\{\dot{u}\}_t + \{\Delta\dot{u}\}_t) + [K]_t\ (\{u\}_t + \{\Delta u\}_t)$$
$$= \{R\}_t + \{\Delta R\}_t \tag{114}$$

In this equation, the incremental quantities represent those changes taking place during the interval Δt following time t. Thus, one obtains the incremental form

$$[M]\ \{\Delta\ddot{u}\}_t + [C]_t\ \{\Delta\dot{u}\}_t + [K]_t\ \{\Delta u\}_t = \{\Delta R\}_t \tag{115}$$

To find the incremental changes, various procedures can be employed [7, 76]. The differences in these procedures relate to the analytical form of the variation in response over the time interval Δt. In the present investigation, two different analytical forms have been programmed for computer solution, namely, constant acceleration and linear acceleration. These forms lead to the following equations for velocity and displacement at time $t+\Delta t$ expressed in terms of the state vectors at time t and the acceleration vector at time $t+\Delta t$:

Constant Acceleration

$$\{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + \frac{1}{2}\Delta t\ \{\ddot{u}\}_t + \frac{1}{2}\Delta t\ \{\ddot{u}\}_{t+\Delta t} \tag{116}$$

$$\{u\}_{t+\Delta t} = \{u\}_t + \Delta t\ \{\dot{u}\}_t + \frac{1}{4}\,\Delta t^2\{\ddot{u}\}_t + \frac{1}{4}\,\Delta t^2\{\ddot{u}\}_{t+\Delta t} \tag{117}$$

<u>Linear Acceleration</u>

$$\{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + \frac{1}{2}\,\Delta t\ \{\ddot{u}\}_t + \frac{1}{2}\,\Delta t\ \{\ddot{u}\}_{t+\Delta t} \tag{118}$$

$$\{u\}_{t+\Delta t} = \{u\}_t + \Delta t\ \{\dot{u}\}_t + \frac{1}{3}\,\Delta t^2\ \{\ddot{u}\}_t + \frac{1}{6}\,\Delta t^2\ \{\ddot{u}\}_{t+\Delta t} \tag{119}$$

Using the following definitions for the incremental vectors

$$\{\Delta\ddot{u}\}_t = \{\ddot{u}\}_{t+\Delta t} - \{\ddot{u}\}_t \tag{120}$$

$$\{\Delta\dot{u}\}_t = \{\dot{u}\}_{t+\Delta t} - \{\dot{u}\}_t \tag{121}$$

$$\{\Delta u\}_t = \{u\}_{t+\Delta t} - \{u\}_t \tag{122}$$

the incremental velocity and acceleration vectors can be expressed in the form

<u>Constant Acceleration</u>

$$\{\Delta\ddot{u}\}_t = \frac{4}{\Delta t^2}\,\{\Delta u\}_t - \{A\}_t \tag{123}$$

$$\{\Delta\dot{u}\}_t = \frac{2}{\Delta t}\ \{\Delta u\}_t - \{B\}_t \tag{124}$$

where

$$\{A\}_t = \frac{4}{\Delta t}\,\{\dot{u}\}_t + 2\ \{\ddot{u}\}_t \tag{125}$$

$$\{B\}_t = 2\ \{\dot{u}\}_t \tag{126}$$

Linear Acceleration

$$\{\Delta\ddot{u}\}_t = \frac{6}{\Delta t^2}\{\Delta u\}_t - \{A\}_t \tag{127}$$

$$\{\Delta\dot{u}\}_t = \frac{3}{\Delta t}\{\Delta u\}_t - \{B\}_t \tag{128}$$

where

$$\{A\}_t = \frac{6}{\Delta t}\{\dot{u}\}_t + 3\{\ddot{u}\}_t \tag{129}$$

$$\{B\}_t = 3\{\dot{u}\}_t + \frac{1}{2}\Delta t\{\ddot{u}\}_t \tag{130}$$

Using these relations, the incremental equation of motion, Eq. (115), can be written in the form

$$[\bar{K}]_t\{\Delta\bar{u}\}_t = \{\Delta\bar{R}\}_t \tag{131}$$

where

$$\{\Delta\bar{R}\}_t = \{\Delta R\}_t + [M]\{A\}_t + C_2[M]\{B\}_t \tag{132}$$

$$[\bar{K}]_t = [K]_t + C_1[M] \tag{133}$$

and the actual incremental displacement vector can be expressed as

$$\{\Delta u\}_t = C_3(\{\Delta\bar{u}\}_t + \beta\{B\}_t) \tag{134}$$

Constants C_1, C_2, and C_3 are given by the relations

Constant Acceleration

$$C_1 = \frac{2\alpha\Delta t + 4}{\Delta t^2 + 2\beta\Delta t} \tag{135}$$

$$C_2 = \alpha - C_1\beta \tag{136}$$

$$C_3 = \frac{\Delta t}{\Delta t + 2\beta} \tag{137}$$

<u>Linear Acceleration</u>

$$C_1 = \frac{3\,\alpha\,\Delta t + 6}{\Delta t^2 + 3\,\beta\,\Delta t} \tag{138}$$

$$C_2 = \alpha - C_1\,\beta \tag{139}$$

$$C_3 = \frac{\Delta t}{\Delta t + 3\beta} \tag{140}$$

After computing the incremental displacement vector using Eq. (134), the corresponding incremental acceleration and velocity vectors are determined using Eqs. (123) and (124), or Eqs. (127) and (128), respectively. The displacement, velocity and acceleration vectors at time $t+\Delta t$ are then evaluated using Eqs. (120), (121) and (122), respectively. The tangent stiffness, strain, and stress for each element can now be calculated for time $t+\Delta t$.

An alternative solution of the incremental equilibrium equation, Eq. (115) can be obtained by separating the tangent stiffness matrix $[K]_t$ into its elastic and plastic parts, $[K^E]$ and $[K^P]_t$, as described in Chapter IV. That term associated with plastic deformation is then transferred to the right hand side of the equation of motion and is treated as an equivalent load vector [21].

In the above described step-by-step integration procedures, the initial displacements and velocities at time $t = 0$ are assumed equal to zero. The very first incremental acceleration vector is then computed directly from Eq. (115), i.e.,

$$[M]\,\{\Delta\ddot{u}\}_0 + [C]_0\{0\}_0 + [K]_0\,\{0\}_0 = \{\Delta R\}_0$$

or

$$\{\Delta\ddot{u}\}_0 = [M]^{-1}\{\Delta R\}_0 = [M]^{-1}\{R\}_0 = -\{\ddot{u}_g\}_0$$

The initial forces existing in the elements cannot be assumed zero but must be taken equal to the static gravitational forces since tangent stiffness is dependent upon total force (gravity plus seismic).

While step-by-step integration procedures similar to those described above must be used for nonlinear analyses, they may or may not be used for linear analyses since the mode superposition method is an alternate method which can be used for linear analyses [116]. In the present investigation, it was found computationally convenient to use the step-by-step method for both linear and nonlinear analyses.

F. TIME INTERVAL Δt

The step-by-step integration method is accurate only if the time interval Δt is small compared with the shortest period T of the soil-structural system and is also small compared with the predominant periods in the excitation. Assuming the latter condition is satisfied, the ratio $\Delta t/T$ must be selected less than a certain critical value to insure a convergent and stable solution in the case of the linear acceleration method; however, it can be shown that the constant acceleration method is always stable for a linear system [76]. In the present study, the presence of the nonlinear friction elements tend to encourage an unstable response, if the $\Delta t/T$ ratio is taken too large. Therefore, extreme care must be taken in selecting the numerical value of this ratio. The effects of this particular parameter on dynamic response are discussed subsequently in Chapter VI.

G. EARTHQUAKE INPUT

In the present investigation, the horizontal ground motion was prescribed in accordance with the acceleration time-history shown in Fig. 30. This artificial accelerogram was generated by A. K. Chopra to simulate the ground motions produced by the San Fernando earthquake at the site of the Olive View Hospital located about 6 miles southwest of the epicenter [18]. It has a peak acceleration of 0.5 g and a uniform phase of high intensity shaking for 8 seconds.

The vertical ground motions were assumed zero for the present study, but the computer program has the option to permit input of vertical ground motions.

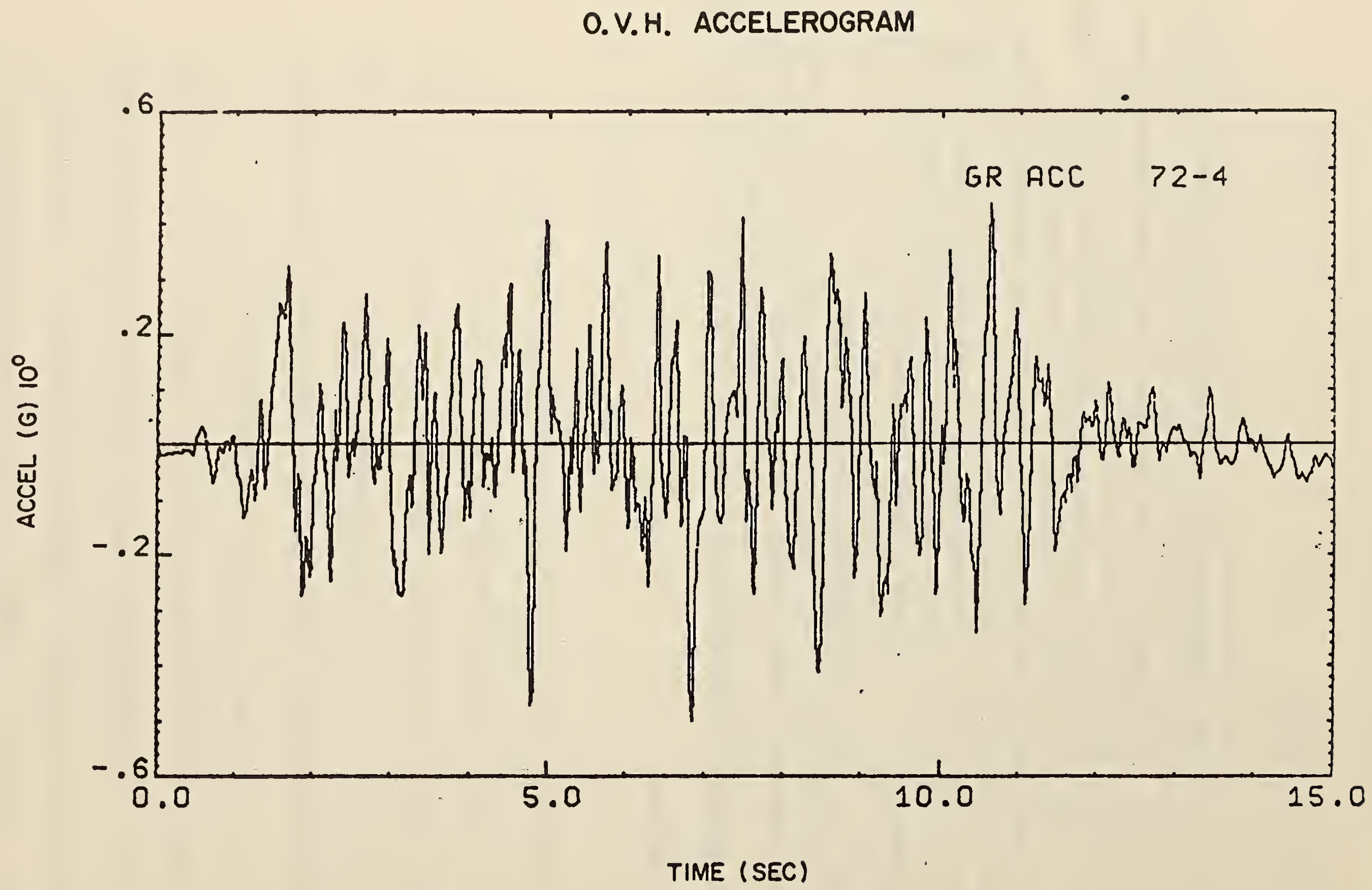


FIG.30 SIMULATED GROUND ACCELERATION RECORD OF THE SAN FERNANDO EARTHQUAKE AT THE OLIVE VIEW HOSPITAL SITE

VI PARAMETER STUDIES

The previously defined mathematical modelling and dynamic analysis procedures have been applied to a straight version of an existing slightly-curved skewed bridge, namely, the North Connector Undercrossing located approximately 800 ft. northerly of Route 5 - San Fernando Road Interchange in the city and county of Los Angeles. Plan and elvation views of this bridge are shown in Fig. 31 along with cross-sectional views of the decks and the centrally located supporting columns. Figure 32 shows a sectional view of the left abutment with a portion of the backfill of extent L and height H.

It is the purpose of this chapter to present the results of dynamic analyses for the North Connector Undercrossing when subjected to one component of earthquake excitation in its longitudinal direction. The ground motions used in this study were the Olive View Hospital accelerations shown in Fig. 30 during the time interval 2-4 seconds. This relatively short duration was chosen to minimize computer time and yet to provide sufficient time for a representative dynamic response to occur. The peak acceleration in this excitation is approximately 0.3 g.

In order to establish an appropriate mathematical model of the entire bridge-soil system, parameter studies were first carried out using a rigid wall with uniform elastic backfill. The complete bridge-soil system was then analyzed with certain parameter variations. The results of these studies along with the results of an integration-time-interval sensitivity analysis are presented in the subsequent sections of this chapter.

A. RIGID WALL-BACKFILL SYSTEM

1. Lateral Extent of Backfills - To study the longitudinal dimension or extent of backfill required in the mathematical modelling of bridge-soil systems, analyses were conducted using a single rigid wall and a uniform linear-elastic backfill having the properties $\gamma = 110$ pcf, $\nu = 0.35$, and G = 10 ksi. Five cases with L/H ratios equal to 4.8, 6.0, 10.0, 14.0, and 22.0, as shown in Fig. 33, were used in this investigation. The finite element idealization consists of 5 elements in the vertical direction and a variable number in the horizontal direction. The element length to height ratio R = a/b was maintained at a constant value equal to 2 throughout the extent of the backfill in Cases 1 and 2 but was maintained only over a distance 2H from the wall in Cases 3, 4, and 5. This ratio was set equal to 4 for all elements beyond the distance 2H in the latter cases. No friction elements were placed between the rigid wall and the backfill in these particular studies and the left vertical boundary of the backfill was assumed free in each case. The fundamental period of the soil system under these conditions is about 0.08 seconds in each case with slight increases occurring with increasing values of L/H as shown in Table 1.

Assuming rigid body earthquake excitations to occur along the entire base of the backfill and at the rigid-wall vertical-boundary and assuming 5 percent of critical damping in the first two modes of vibration, time histories of response were obtained for all 5 cases. Response quantities of greatest interest were the maximum or peak values of (1) total lateral dynamic force exerted on the rigid wall, and (2) horizontal displacements and accelerations at various location in the backfill.

Figure 34 shows the maximum total lateral dynamic force F_d exerted on the wall for each of the 5 cases studied. This force is reasonably constant at a value of approximately 14.5 kips over the range studied; i.e. $4.8 < L/H < 22$, and is about 3.9 times greater than the average static value which is approximately 3.7 kips. The mean location of the resultant force F_d was found to be at a distance 0.52 H above the base which is considerably higher than the position of the resultant static force F_s located at 0.34 H.

The maximum horizontal displacements relative to the moving base at 6 different locations in the backfill are shown in Fig. 35 for all 5 cases. While these displacements are reasonably constant over the range of L/H studied, large differences are noted from one point to another. The displacements for points 1 and 2 are very small due to the fact they are located only one element away from the rigid wall. Points 3 and 4 which are 5 elements away from the wall experienced much larger displacements than points 1 and 2 and points 5 and 6 at the left boundary experienced even larger displacements. These relative displacements reflect the manner in which the rigid wall boundary effects decay with increasing distance from the wall. The displacements for points 3 and 5 are considerably larger than the corresponding displacements for points 4 and 6, respectively; thus, demonstrating the increase in displacements with vertical distance above the rigid base.

Figure 36 shows the maximum total horizontal acceleration at locations 1-6 for all 5 cases studied. The relative variations of this response quantity with L/H, horizontal distance from the wall, and vertical distance above the base are similar to those previously described for horizontal displacement. It should be noted that the average acceleration with L/H ranges from about 0.5 g for points 1 and 2 to about 1.1 g for point 5. These values represent a rather small amplification of acceleration at points 1 and 2 and a fairly large amplification of the acceleration at point 5 over the peak excitation acceleration of about 0.3 g.

Based on the numerical results shown in Figs. 34-36 and summarized in Table 2, it appears that an L/H ratio equal to 14 is sufficient for use in any bridge-soil analysis. This ratio may even be reduced to a value of 6 with little loss in accuracy in determining the maximum values of abutment backfill forces. However, this reduction could introduce significant changes in the predicted maximum horizontal accelerations.

2. <u>Length to Height Ratio of Finite Elements</u> - Length to height ratios R = a/b for the finite elements of the backfill model were rather arbitrarily assigned values as shown in Table 1 for the 5 cases previously defined. To investigate the influence of changing these ratios, 4 new cases as shown in Fig. 37, each having an L/H ratio equal to 10, are defined. The length to height ratio equals 2 throughout the model for Case 1 and over a distance 2H from the wall for Cases 2, 3, and 4. Beyond 2H, the length to height ratio equals 4, 6, and 10 for Cases 2, 3, and 4, respectively. Five elements are again used in the vertical direction of the model and fixed and free boundaries are prescribed at the right and left ends, respectively. Linear elastic soil properties are again assumed equal to the values previously assigned.

As before, rigid body earthquake excitations are assumed to occur along the entire base of the backfill and 5 percent of critical damping is assigned to the first two modes of vibration. Time histories of response are obtained for all 4 cases, including (1) total dynamic force exerted on the rigid wall, and (2) horizontal displacements and accelerations at 6 locations in the backfill.

Figure 38 shows total static force F_s and the maximum total dynamic lateral force F_d exerted on the wall for each of the 4 cases defined above. This force is fairly constant at a value of approximately 14.5 kips over the range of R assigned beyond x = 2H and is about 3.9 times greater than the average static value. The mean location of the resultant force F_d is at a distance about 0.52H above the base. The average position of the static resultant is 0.34H.

The maximum horizontal displacements relative to the moving base and the maximum total horizontal accelerations at the 6 different locations are shown in Figs. 39 and 40, respectively, for all 4 cases. The results in these figures are quite similar to the results shown the corresponding Figs. 35 and 36. Thus, it is apparent that the changes introduced in R for x > 2H have introduced relatively small changes in overall response and that Case 4, Fig. 37, can be considered a reasonable model of the backfill.

Table 3 presents a summary of maximum response for the above described 4 cases.

3. Number of Finite Elements in the Vertical Direction - All previous cases analyzed have used 5 finite elements in the vertical direction of the backfill. To check the adequacy of this number the distributions and resultant magnitudes of the static and dynamic backfill pressures on the rigid retaining wall are compared using 5 and 10 elements

in this direction.

Two cases as defined in Fig. 41 are used for this comparison. Both cases use an L/H ratio equal to 10 and R ratios equal to 2 and 10 for $x < 2H$ and $x > 2H$, respectively. Case 1 uses 5 elements in the vertical direction while Case 2 uses 10 elements.

It is quite clear from the results in Fig. 41 that the distributions and maximum resultant magnitudes of the backfill forces are very similar for Cases 1 and 2 and that the positions of the corresponding resultants almost coincide. Considering this fact and the fact that their time histories (see Fig. 42) are very similar, one may conclude that 5 finite elements in the vertical direction of the backfill is sufficient for engineering purposes.

4. Soil Stiffness - To study the influence of soil stiffness on the dynamic response of the backfill soil system, the finite element model identified as Case 4 in Fig. 37 was analyzed for three soil conditions, namely, a uniform soil modulus equal 10 ksi, a uniform soil modulus equal to 2.5 ksi, and a variable soil modulus in accordance with Eq. (38) for $K_2 = 50$. Poisson's ratio ν was assumed equal to 0.35 and the unit weight was again assigned the value 110 pcf. These new soil conditions are identified as Cases 1 - 3 in Fig. 43.

While the shapes of the distributions of maximum total wall pressures and their resultant force positions are quite similar for all three cases, the magnitudes of the resultant dynamic wall pressures vary considerably with the soil stiffness condition. This variation ranges from F_d = (100)(13.06) for Case 3 to F_d = (154.0)(13.06) for Case 2. Obviously, for the particular excitation used, the less stiff backfill soils produce higher backfill forces on the rigid wall. If an excitation having a different amplitude distribution with frequency had been used, this observation could be changed considerably. Therefore, one must use caution in interpreting the results of this particular study. It is important to recognize however that soil stiffness can be an important parameter which should be studied using realistic earthquake excitations.

5. Frictional Element Stiffnesses - As pointed out previously, the shear and normal stiffnesses of the frictional elements located between backfill soil and abutment walls are arbitrarily assigned finite but very high values rather than infinite values to avoid discontinuities in their force displacement relations. To study the influences of these stiffnesses on dynamic response, the rigid wall-soil system shown in Fig. 44 was analyzed assuming linear soil behavior. In these studies the shear and normal stiffnesses (K_S and K_N) for each frictional element were assigned equal values ranging from 1 to 10^9 ksi.

The total static and total maximum dynamic lateral wall forces (F_s and F_d) obtained in these studies are plotted in Fig. 45 for stiffnesses $K_S = K_N$ equal to 1, 10^3, 10^6, and 10^9 ksi. As one would expect, these forces are reasonably constant over a wide range of stiffnesses. However, as the stiffnesses approach zero, F_s and F_d also approach zero which represent unrealistic values. Theoretically both F_s and F_d should approach constant values asymptotically with increasing stiffnesses, however the value of F_d for $K_S = K_N = 10^9$ ksi is much larger than for the smaller values of stiffness. This large increase is due to a numerical instability which developed in the analysis procedures and therefore should be ignored. It appears therefore that realistic wall forces can be obtained by selecting stiffnesses in the range $10^3 < K_S = K_N < 10^6$ ksi.

Although the asymptotic static value of wall force in Fig. 45 is consistent with values previously presented for cases having no friction elements, the maximum dynamic force of about 40.0 kips is considerably larger than the average value (14.5) previously presented. This increase in dynamic wall force is due to the separations and associated impacts which occur between the backfill soil and the upper part of the rigid wall. Thus it appears that for high intensity excitations, the friction element is essential to realistic modelling.

B. INTEGRATION-TIME-INTERVAL SENSITIVITY ANALYSIS

Throughout the rigid wall-backfill parameter studies previously described, the numerical integration time step was assigned a value equal to 0.01 seconds and the constant acceleration method was used which enables response to be stable, but not necessarily convergent, for all modes of vibration. To study the adequacy of using 0.01 seconds for Δt, the rigid wall-soil system defined by Case 4, Fig. 37, was re-analyzed using $\Delta t = 0.001$ seconds. The total number of degrees of freedom for this system is 90 with the fundamental period being 0.084 seconds and the highest period estimated at 0.0065 seconds. The convergent limit of the ratio of time step duration to period, i.e. $\Delta t/T$, is 0.39 for the constant acceleration method. Thus, for $\Delta t = 0.01$ seconds, the convergent period T is 0.026 seconds. Since this period corresponds to the period of the 22nd mode of vibration, only the lowest 22 modes are convergent in the constant acceleration method of analysis when $\Delta t = 0.01$ seconds. If on the other hand, $\Delta t = 0.001$ seconds, all modes are convergent.

To check the accuracy of response obtained for the above case using $\Delta t = 0.01$ seconds, the time histories of lateral dynamic wall force and horizontal acceleration at point No. 1 (see Fig. 40) are obtained for $\Delta t = 0.001$ and are plotted in Fig. 46 where they can be compared with corresponding results for $\Delta t = 0.01$ seconds. The vertical acceleration time histories for point No. 1 are plotted in Fig. 47 for $\Delta t = 0.01$ and 0.001 seconds. A summary of the maximum values of response for this case is presented in Table 4.

Obviously, the results of Fig. 46 indicate that a time step interval of 0.01 seconds is quite adequate in predicting total lateral wall force and horizontal acceleration time histories. However, the results of Fig. 47 indicate the very low level vertical acceleration time histories caused primarily by very high frequency modal responses cannot be predicted accurately by $\Delta t = 0.01$ seconds. Since this high frequency response is relatively unimportant from an engineering point of view, it is concluded that $\Delta t = 0.01$ seconds is adequate for the previously described rigid wall-soil parameter studies and also for the bridge-soil system studies to be described subsequently.

C. BRIDGE-SOIL SYSTEM

To study the dynamic response of the combined bridge-soil system, 3 mathematical models of the North Connector Undercrossing as shown in Fig. 48 were defined. Model A has fixed boundary conditions at depth H of the backfills, at the base of abutments, and at the base of all columns. Model B has fixed boundary conditions at depths 2.2H and 2.5H of the backfills, leveling with bases of pier columns, which allows the base of abutments to translate and rotate with the soil system, and fixed boundary conditions are also provided at the base of all columns. Model C has fixed boundary conditions only along the base of the backfills as in Model B. The bases of abutments and columns of this model are attached to equivalent columns representing the foundation flexibility. These equivalent columns of course, have fixed boundary conditions at their bases.

The soil elements in all three models can be assumed linear or nonlinear as desired and friction elements can be included in Models A and C, but not in Model B. The backfills extend a distance 6H in all models as shown. The bridge deck is linear in each model; however, either linear or nonlinear columns can be used. Backfill and foundation soils were assumed to have the properties $G = 10.0$ ksi, $\nu = 0.35$, $\gamma = 110$ lb/ft^3, $c = 0$, and $\phi = 30°$.

1. <u>Soil Pressures on Abutments</u> - The static and maximum dynamic pressure distributions on one abutment wall for two cases are shown in Fig. 49. The model used for Case 1 is Model A with no friction elements and with all other elements assumed linear. The model used for Case 2 is Model B in its complete linear form. Due to the characteristic response of the bridge-soil system, the static and dynamic pressure distributions are quite different in form in each case. The resultant lateral static force F_s for Case 2 is about 17% less than for Case 1, due to the change in abutment flexibility and the maximum resultant

dynamic force F_d for Case 2 exceeds the value for Case 1 by 170%. This latter difference is undoubtedly due to a closer matching of the lower mode periods of vibration for Case 2 with the predominant periods in the excitation. The location of the resultant static force is at about H/3 for Case 1 and at about H/2 for Case 2 while the location of the maximum dynamic resultant force is at about 0.53H and 0.6H, respectively.

Another check on the influence of bridge structure flexibility on the resultant abutment soil forces can be made by comparing the results for Cases 1 and 2 in Fig. 50. In this figure, Case 1 is identical to Case 1 in Fig. 49 but Case 2 is different. Here, Case 2 is actually the same as Case 1 except that the abutment wall and column stiffnesses have been reduced by a factor of 10. It is seen that Case 2 shows a 75% increase in the maximum dynamic resultant force over Case 1 due to the decrease in bridge structure flexibility. This increase is consistent with the similar increase previously noted for Case 2 in Fig. 49. The location of the resultant dynamic lateral force is again at about mid-height. The increase in the resultant static force for Case 2 over the value for Case 1 is due to the increase in rotation (due to deck dead loads) at the top of the abutment caused by the reduced abutment flexibility.

Further results of analysis are shown in Fig. 51 identified as Cases 1, 2, and 3. In this figure, Case 1 is again the complete linear version of Model A. Cases 2 and 3 are also based on Model A but Case 2 has introduced one nonlinearity, namely the friction elements, and Case 3 has employed two nonlinearities - friction elements and nonlinear soil elements. The very large increase in maximum dynamic force F_d for

Cases 2 and 3 over Case 1 is due primarily to the impact wall forces following separations between wall and backfill.

Finally the results of two additional analyses are shown in Figure 52. Cases 1 and 2 in this figure are based on Model C using friction elements and linear soil elements; however, Case 2 uses the equivalent foundation columns while Case 1 does not. The most significant result to note in Fig. 52 is that force F_d is more than twice as great for Case 2 over the value shown for Case 1. Again this increase is due to the fact that the more flexible bridge system has a closer matching of frequencies with the predominant frequencies in the excitation.

2. Total Seismic Force Carried by Columns and Abutments - To investigate the maximum total base shear carried by columns and abutments, five cases were analyzed as indicated in Table 5. Case 1 represents the bridge alone with no soil-structure interaction, i.e. Model A, Fig. 48, but with no backfill; Case 2 is Model A with linear backfill and no friction elements; Case 3 is Model B, Fig. 48, with linear backfill; Case 4 is Model A with linear backfill and friction elements; and Case 5 is Model A with non-linear backfill and friction elements. Clearly, the presence of backfill contributes significantly to the maximum total base shear, also it appears from Case 3 that the total base shear increases with overall flexibility which is again evidence of a better matching of the lower natural frequencies with the predominant frequencies in the excitation. The relatively large displacement shown for Case 3 is due to the large flexibility of the system for this case in comparison with the other cases.

D. COMPARISON OF RESULTANT ABUTMENT BACKFILL FORCE OBTAINED BY ANALYSIS AND THE MONONOBE-OKABE METHOD

One commonly used formula in calculating the resultant dynamic lateral force on the abutment wall is the Mononobe-Okabe formula [53, 55, 97]. This formula has the following form

$$P_p = (1-K_v) \cdot r \cdot x \cdot K_{Ep} \tag{141}$$

where

P_p = Passive earthpressure at depth x

K_v = vertical seismic coefficient

γ = Unit weight of soil

x = arbitrary depth

K_{EP} = Passive earthpressure coefficient during earthquake

The coefficient K_{Ep} is given by

$$K_{Ep} = \frac{\cos^2 (\phi-\theta_0+\theta)}{\cos\theta_0 \cdot \cos\theta^2 \cdot \cos(\theta-\theta_0)\left[1 - \sqrt{\dfrac{\sin\phi \cdot \sin(\phi+\alpha-\theta_0)}{\cos(\theta-\theta_0) \cdot \cos(\theta-\alpha)}}\right]^2} \tag{142}$$

where

ϕ = Angle of friction of soil

$\theta_0 = \tan^{-1} \dfrac{K_h}{1-K_v}$

K_h = Horizontal seismic coefficient

θ = Angle between the backline of the wall and the vertical line

α = Angle between ground surface and the horizontal line

Using $\phi = 30°$; $\theta = 0°$; $\theta_0 = \tan^{-1} \frac{0.1}{1-0.0} = 6°$; $\alpha = 0°$; $\gamma = 110$ lb/ft^3

$$K_{Ep} = 2.86$$

The earthpressure at bottom of wall is

$$p_p = (1-0.0) * 110 * 13.5 * 2.86 = 4.25 \text{ K/ft}^2$$

and the total lateral force on the wall is

$$F_d = \frac{1}{2} * 13.5 * 4.25 = 28.6 \text{ K/ft}$$

Using the non-linear form of Model A, Fig. 48, or Case 3 of Fig. 51, analysis gives

$$F_d = 35.0 \text{ K/ft}$$

This analytical result is a higher value than that formula given by the Monobe-Okabe. The position of the resultant force is assumed a distance H/3 above the base when using the formula; however, the analysis shows it to be 0.44H above the base. This higher position given by an analysis is consistent with other investigations [97, 117].

Table 1 Details of Rigid Wall Systems for Study of Lateral Extent

Case	L/H	Subdivision of System	Fundamental Period (Sec.)	Notes
1	4.8	R=2 Uniformly	0.0825	1. Time step for all cases is 0.01 sec. 2. Damping ratio is 0.05 for first two modes 3. G = 10.0 ksi
2	6.0	R=2 Uniformly	0.0838	
3	10.0	R=2 for x < 2H R=4 for x > 2H	0.0846	
4	14.0	R=2 for x < 2H R=4 for x > 2H	0.0848	
5	22.0	R=2 for x < 2H R=4 for x > 2H	0.0849	

Table 2 Comparison of Responses of Rigid Wall System at Different Lateral Extent, Using Case 5 as a Basis for Calculating the Percentage

Case	F_s	F_d	A_1	A_2	A_3	A_4	A_5	A_6	U_1	U_2	U_3	U_4	U_5	U_6
	%													
1	99.1	105.8	100.3	98.9	124.4	116.8	97.4	96.2	115.2	104.1	115.9	115.1	98.4	97.6
2	99.1	103.3	100.2	100.1	115.8	110.2	106.8	103.1	108.9	102.5	110.5	109.7	107.3	104.7
3	100	99.4	98.9	99.5	102.3	99.5	117.3	113.2	99.1	99.2	94.8	96.7	113.5	110.9
4	100	97.9	97.9	98.9	104.5	99.0	98.8	100.3	98.2	97.5	99.4	99.3	100.3	100.2
5	100	100	100	100	100	100	100	100	100	100	100	100	100	100
Case 5 in Absolute Value	Kips		$in/sec^2 \times 10^2$						$in \times 10^{-2}$					
	3.73	14.54	1.90	1.88	2.21	1.96	4.23	2.87	1.12	1.21	4.65	2.99	8.75	5.31

1. A_i and U_i are the Maximum Acceleration and Displacement at point i respectively

2. F_s and F_d are the Static and Maximum Dynamical Lateral Forces.

Table 3 Comparison of Responses of Rigid Wall System With Different Length to Height Ratio R = a/b, Using Case 1 as Basis for Calculating the Percentage

Case	F_s	F_d	A_1	A_2	A_3	A_4	A_5	A_6	U_1	U_2	U_3	U_4	U_5	U_6
	%													
1	100	100	100	100	100	100	100	100	100	100	100	100	100	100
2	100	98.8	98.9	99.5	99.6	97.0	97.3	100	97.4	99.2	97.6	97.6	95.8	97.4
3	100	99.0	98.4	98.9	97.4	93.0	92.2	97.2	94.7	97.5	95.1	94.3	89.7	93.1
4	100	98.6	98.4	99.5	99.1	96.0	75.9	87.4	97.4	99.2	100.4	99.0	75.0	80.5
Case 1 in Absolute Value	Kips		$in/sec^2 \times 10^2$						$INCH \times 10^{-2}$					
	3.73	14.63	1.90	1.88	2.27	2.01	5.10	3.25	1.14	1.21	4.52	2.96	10.36	6.05

1. A_i and U_i are the maximum acceleration and displacement at point i respectively

2. F_s and F_d are the static and maximum dynamical lateral force

3. Refer to Fig. 37 for details of subdivision

Table 4 Comparison of Responses of Rigid Wall System With R = 2 for x < 2H and R = 10 for x > 2H at Different Time Step. Using Case 2 as basis

Case	Time Step	F_s	F_d	A_{1h}	A_{1v}	A_{2h}	A_{2v}	A_{3h}	A_{3v}
	Sec	%							
1	0.01	100	99.0	96.4	233.3	96.0	109.0	95.3	82.1
2	0.001	100	100	100	100	100	100	100	100
Case 2 in Absolute Value	Sec	Kips		in/sec^2 x 10^2					
	0.001	3.725	14.58	1.94	0.056	2.01	0.041	2.98	0.51

1. F_s, F_d are the static and maximum dynamical lateral forces

2. A_{ih}, A_{iv} are the maximum horizontal and vertical acceleration at point i.

Table 5 Comparison of Maximum Total Seismic Force on the Bridge With or Without Soil Interaction

Case	Bridge Model	A_x (%)	U_x (%)	F_{max} (%)	Remarks
1	Bridge alone	100	100	100	1. F_{max} is the maximum total seismic force on the bridge
2	linear bridge-soil system, abutment base fixed	73	140	168	2. A_x, U_x is the horizontal acceleration and displacement at the center of deck at the time of F_{max}
3	linear bridge-soil system, abutment on soil	237	1120	205	
4	Bridge-soil system, linear soil, non-linear friction	138	186	276	3. Case 2 - Fig. 48-A, no friction element
5	Bridge-soil system, non-linear soil and friction	163	151	225	Case 3 - Fig. 48-B Case 4, 5 - Fig. 48-A, with friction element
Case 1	Absolute Value	g's	INCH 10^{-2}	KIPS	
		0.50	3.63	408.0	

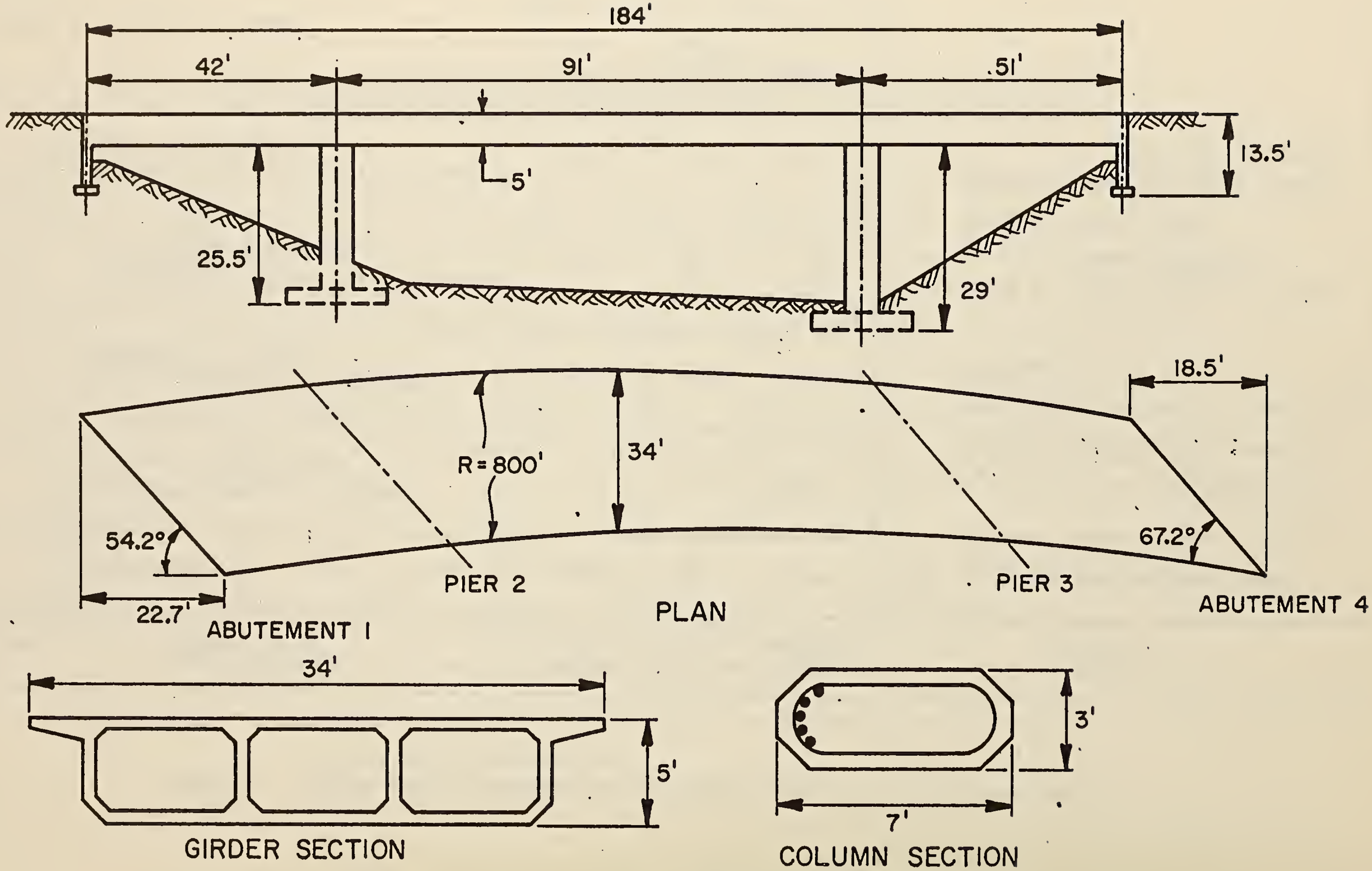


FIG. 31 GENERAL PLAN OF NORTH CONNECTOR UNDERCROSSING

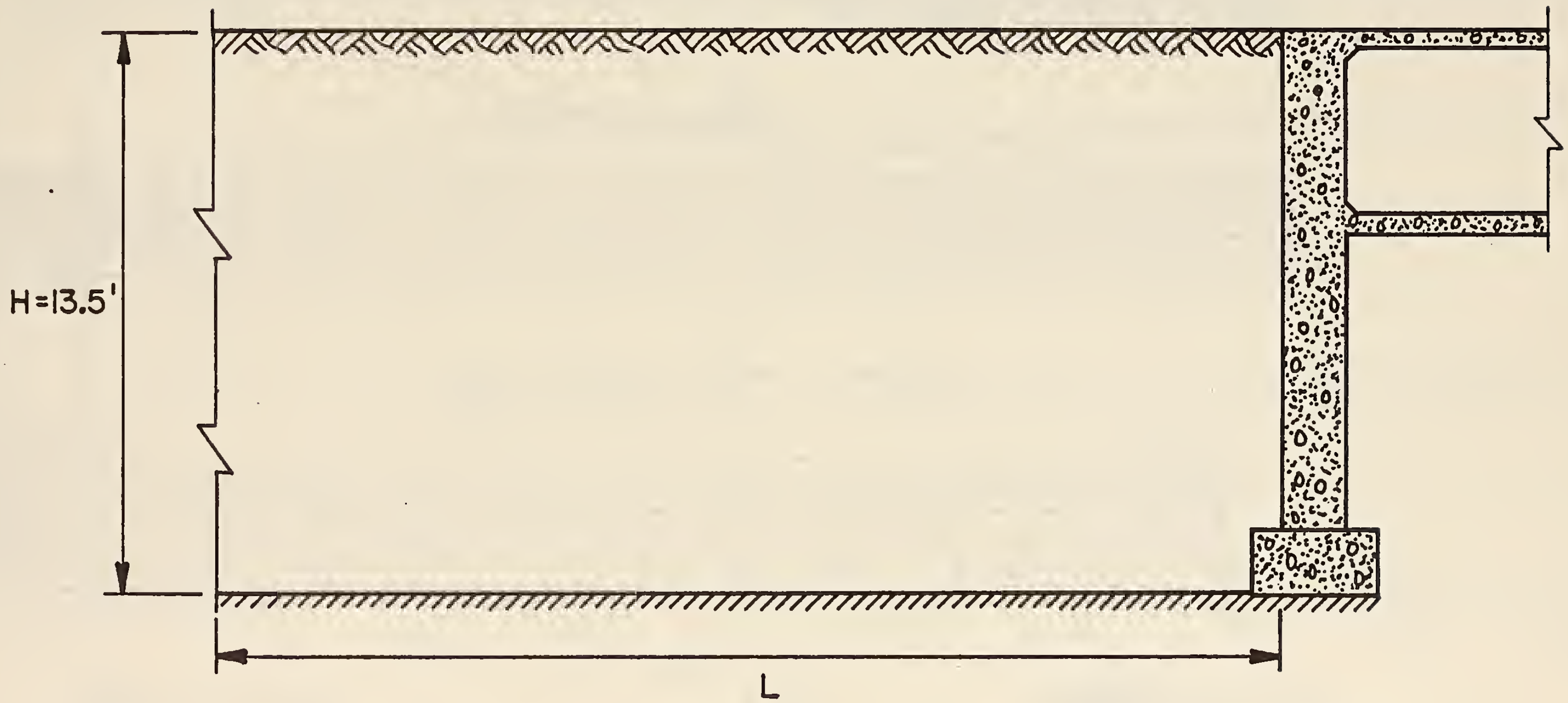


FIG. 32 ABUTMENT AND BACKFILLS

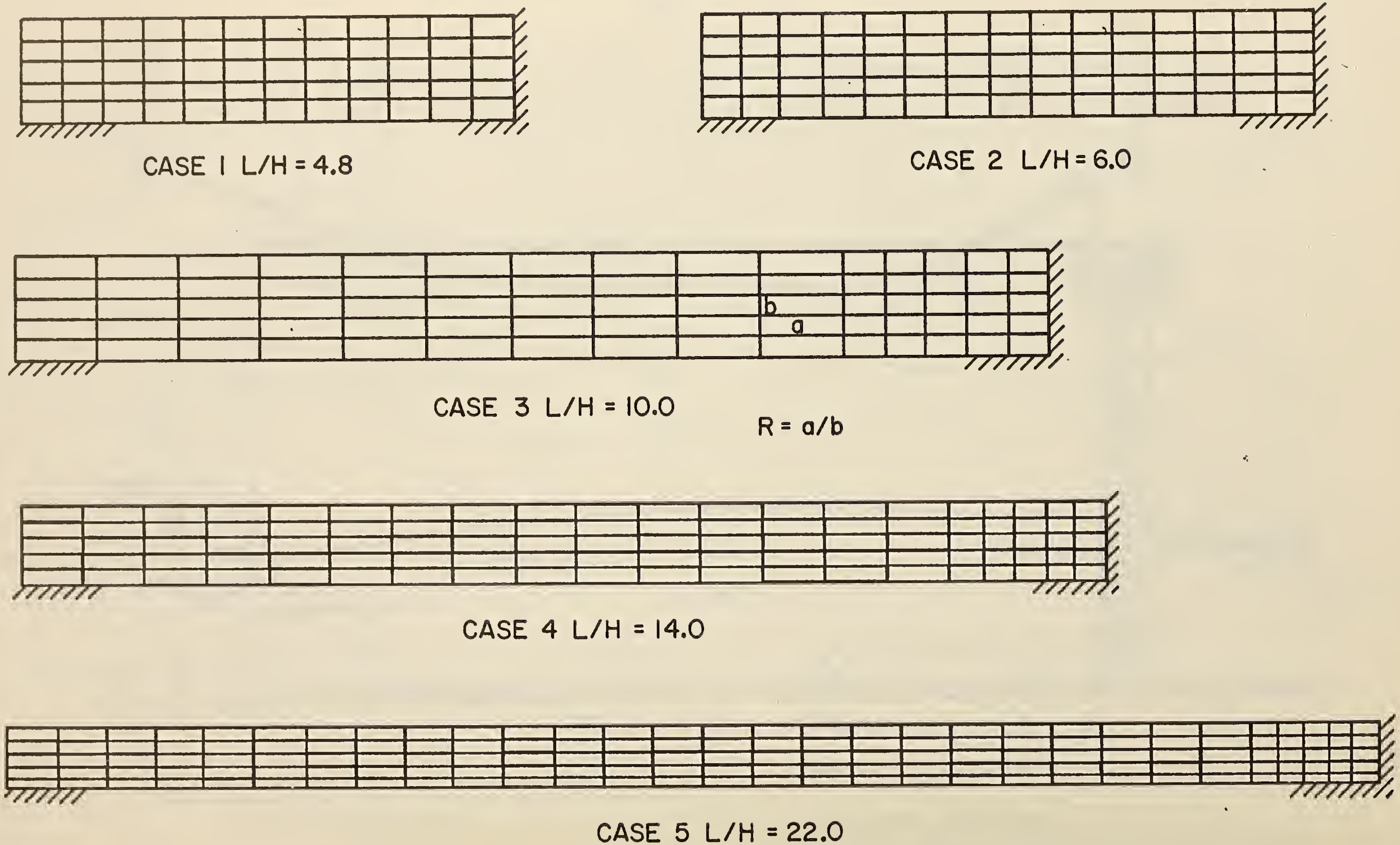


FIG. 33 RIGID WALL SYSTEM FOR STUDYING THE EFFECTS OF LATERAL EXTENT OF BACKFILLS

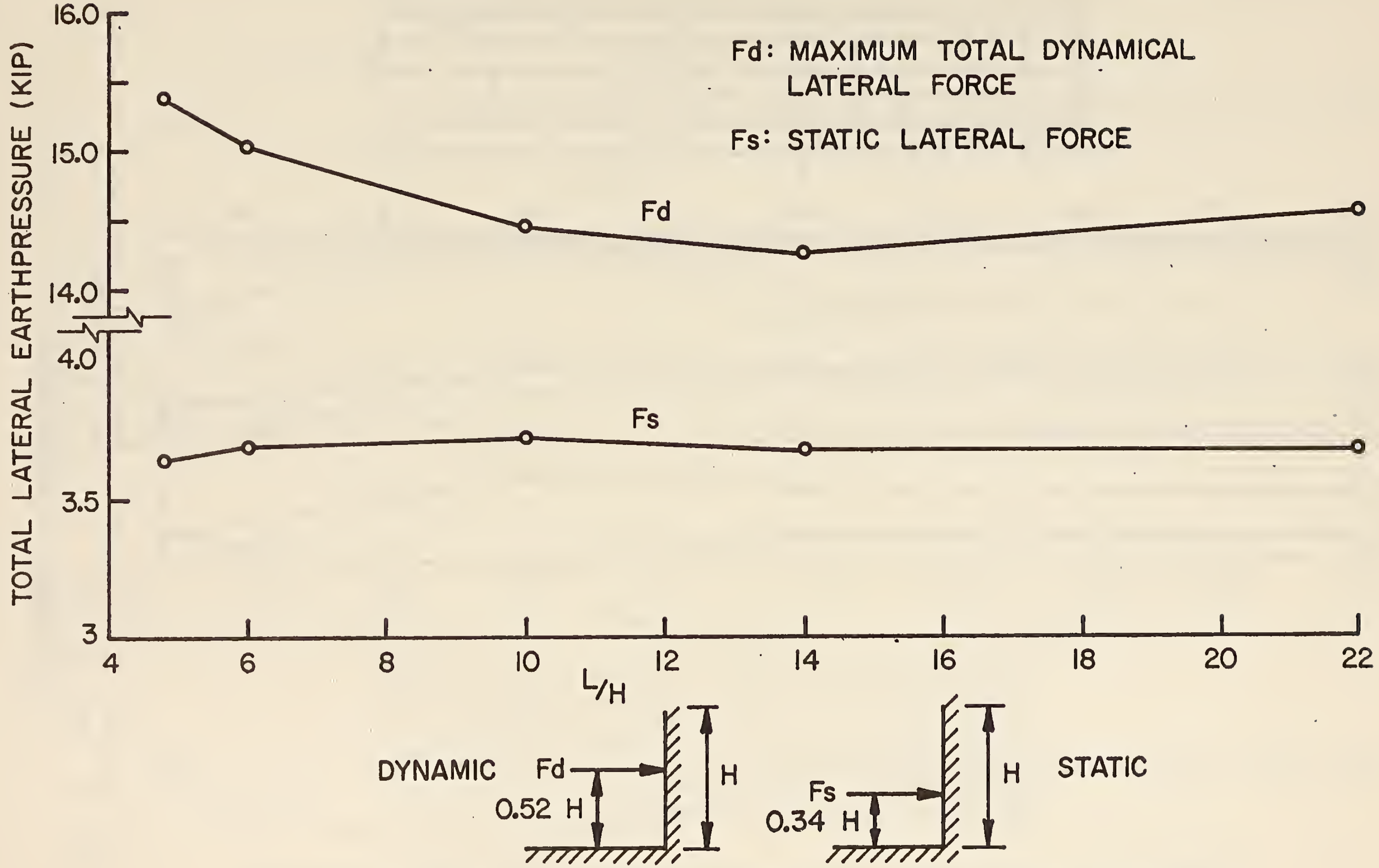


FIG. 34 STATIC AND MAXIMUM DYNAMICAL TOTAL LATERAL EARTHPRESSURE FOR DIFFERENT LATERAL EXTENT L/H

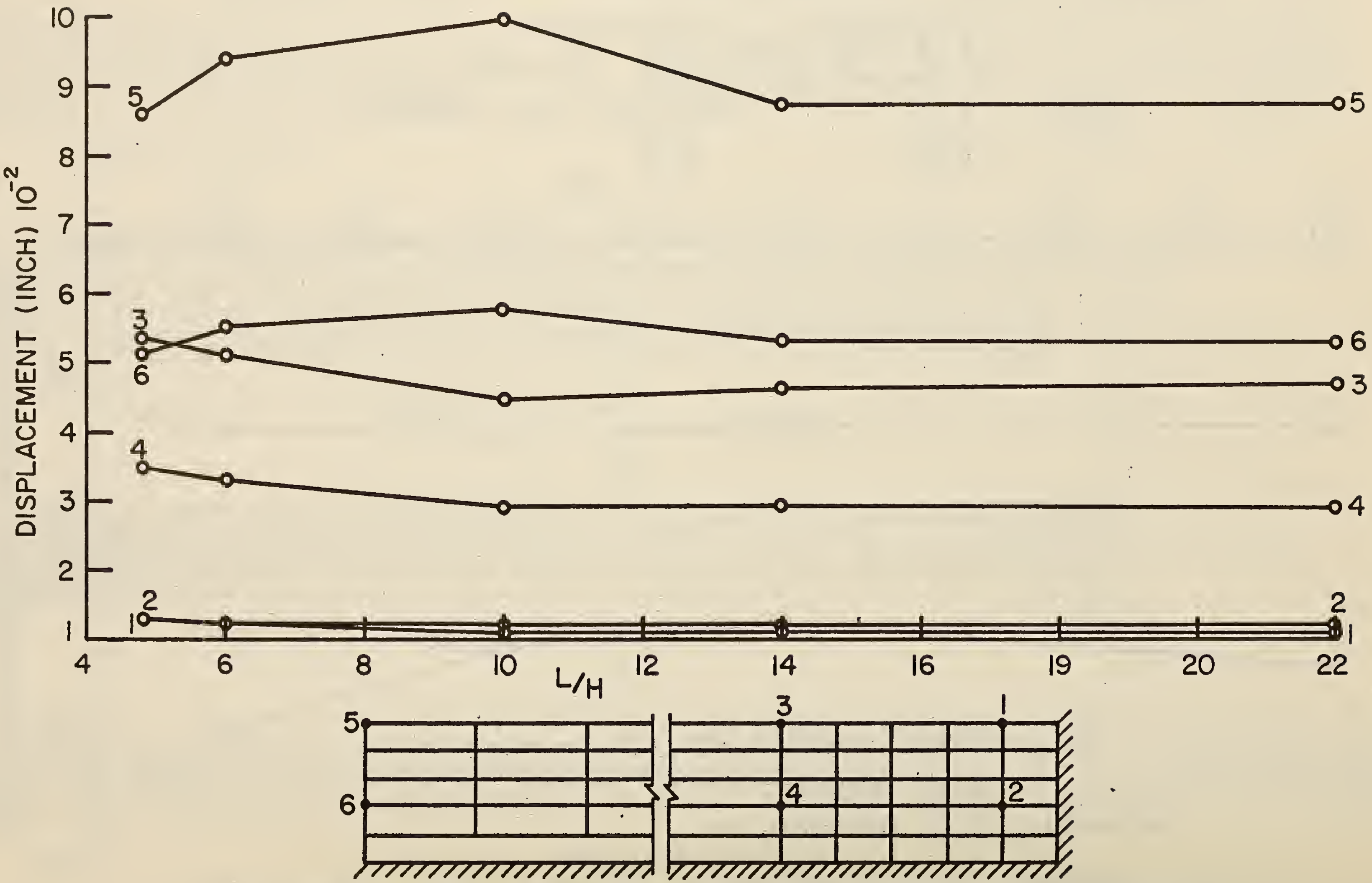


FIG. 35 MAXIMUM HORIZONTAL DISPLACEMENT AT POINTS 1 TO 6 FOR DIFFERENT LATERAL EXTENT L/H

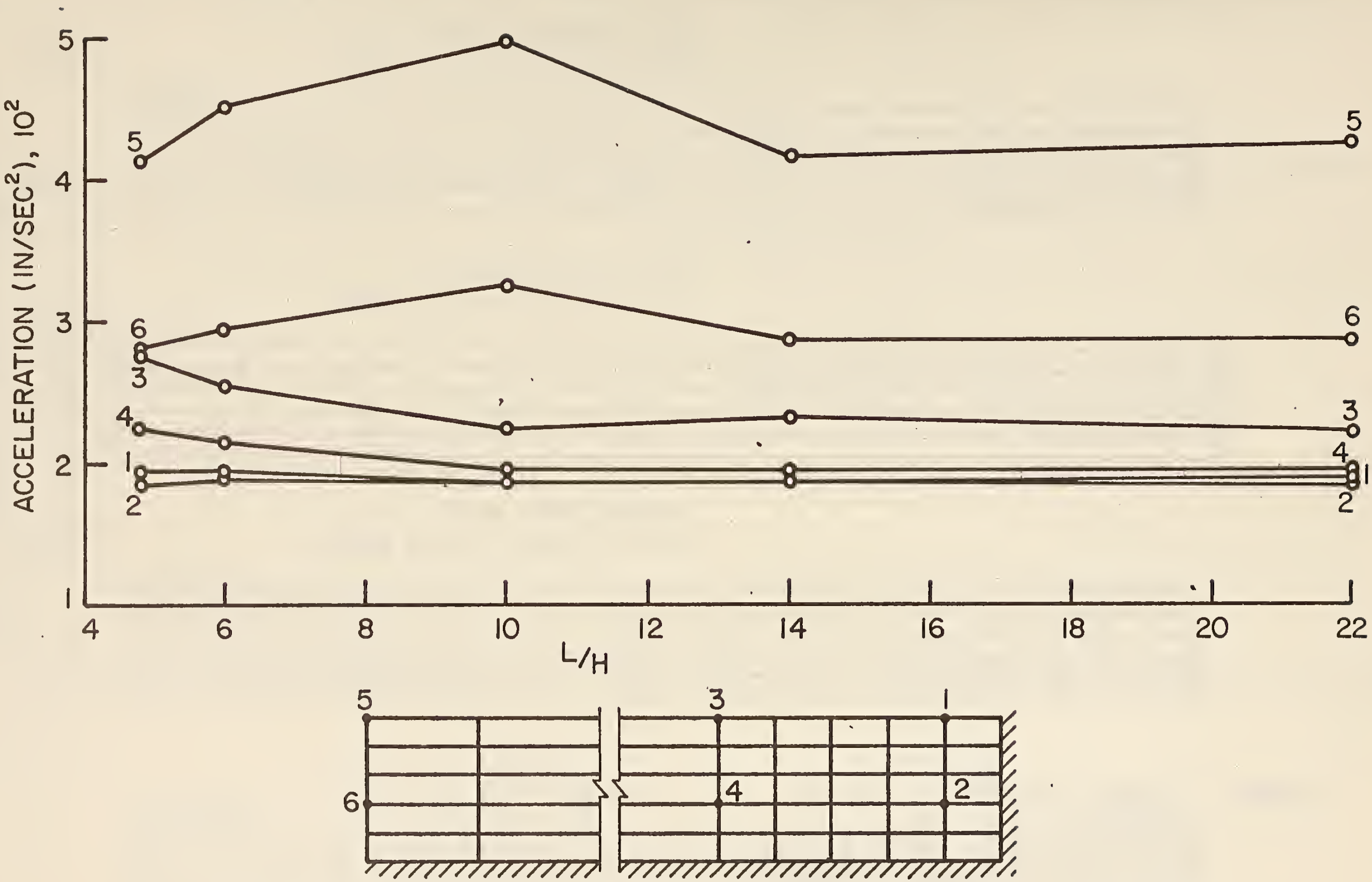


FIG. 36 MAXIMUM HORIZONTAL ACCELERATIONS AT POINTS 1 TO 6 FOR DIFFERENT LATERAL EXTENT L/H

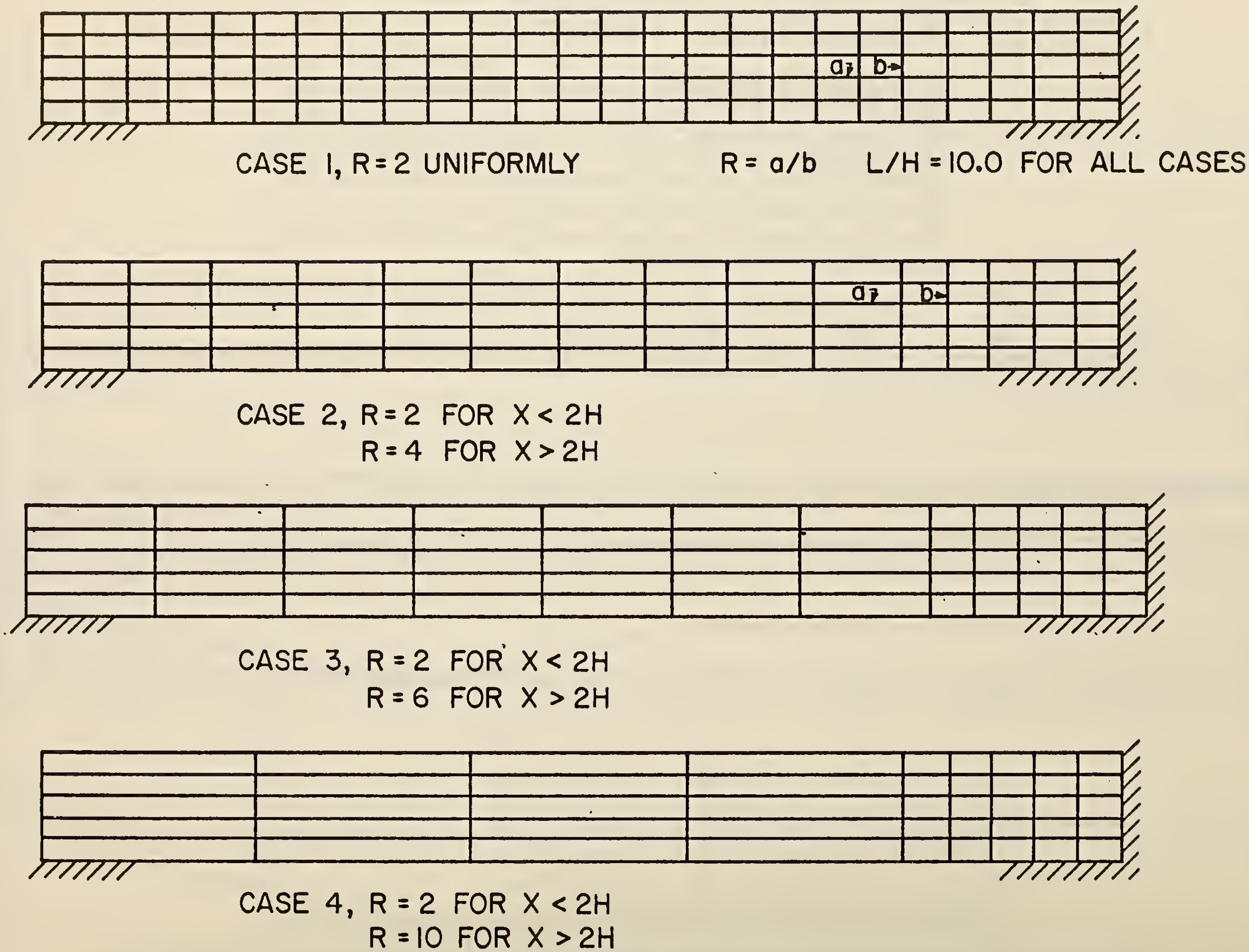


FIG. 37 FINITE ELEMENT MODEL FOR STUDYING THE LENGTH TO HEIGHT RATIO R

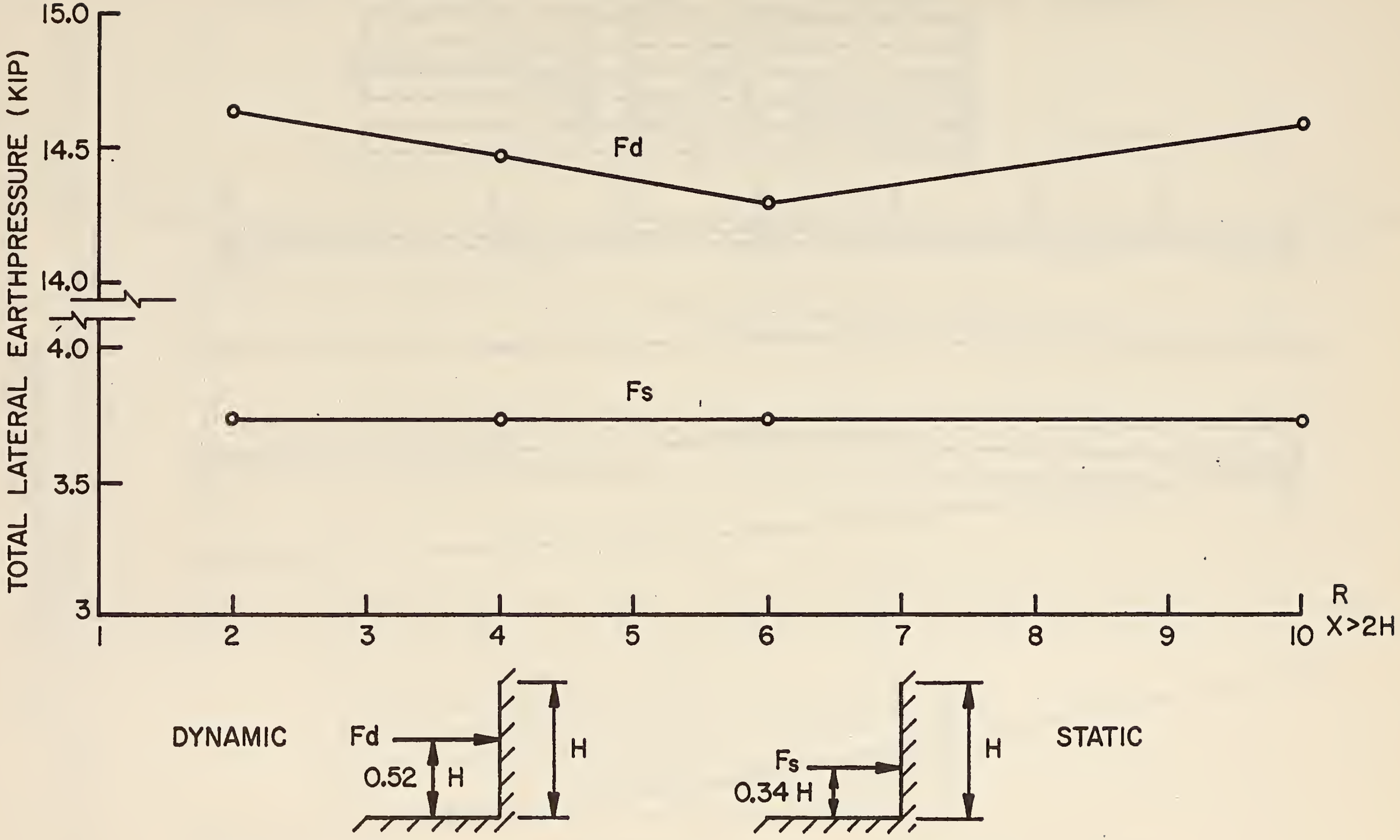


FIG.38 STATIC AND MAXIMUM DYNAMICAL TOTAL LATERAL PRESSURE FOR DIFFERENT ELEMENT SIZE R= a/b

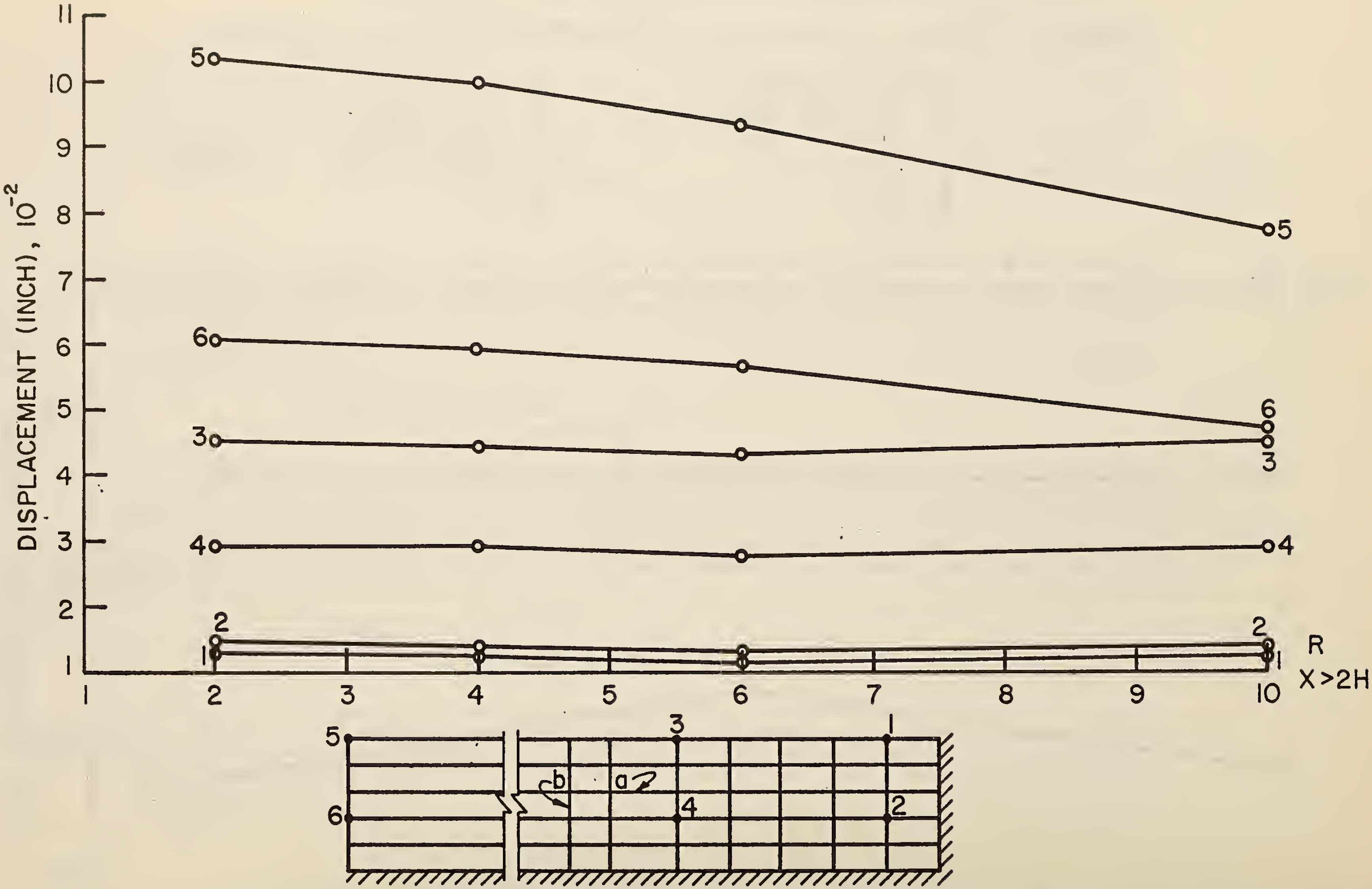


FIG.39 MAXIMUM HORIZONTAL DISPLACEMENT AT POINTS 1 TO 6 FOR DIFFERENT ELEMENT SIZE R = a/b

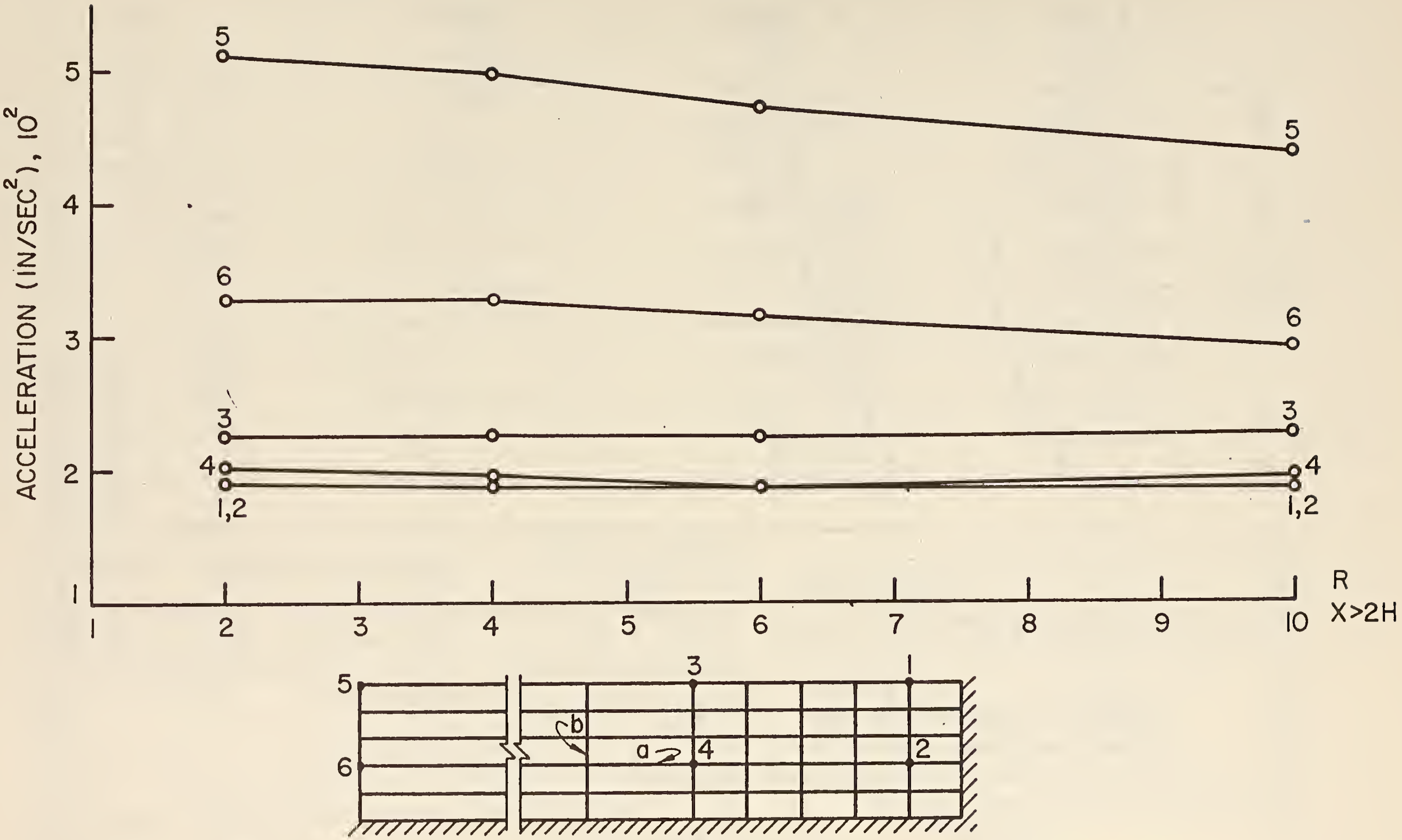


FIG. 40 MAXIMUM HORIZONTAL ACCELERATION AT POINTS
1 TO 6 FOR DIFFERENT ELEMENT SIZE $R = a/b$

CASE	TOTAL LAYERS	SUBDIVISION	Fs (%)	Fd (%)	REMARKS
1	5	R = 2 FOR X<2H R = 10 FOR X>2H	95.2	94.2	1. Fs, Fd ARE THE STATIC AND MAXIMUM DYNAMIC LATERAL FORCES 2. CASE 2 AS BASIS FOR CALCULATING PERCENTAGES
2	10	L/H = 10	100	100	
CASE 2, ABSOLUTE VALUES			KIPS 3.91	KIPS 15.32	

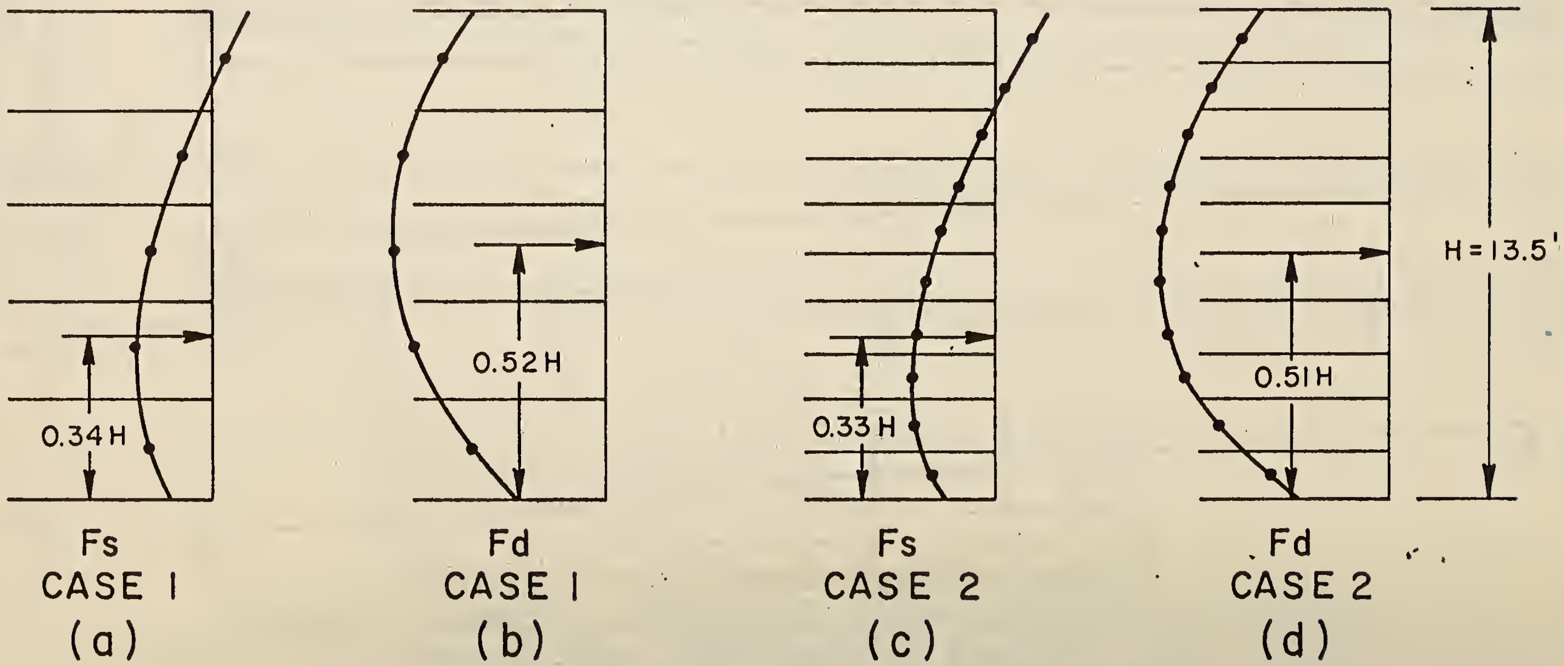
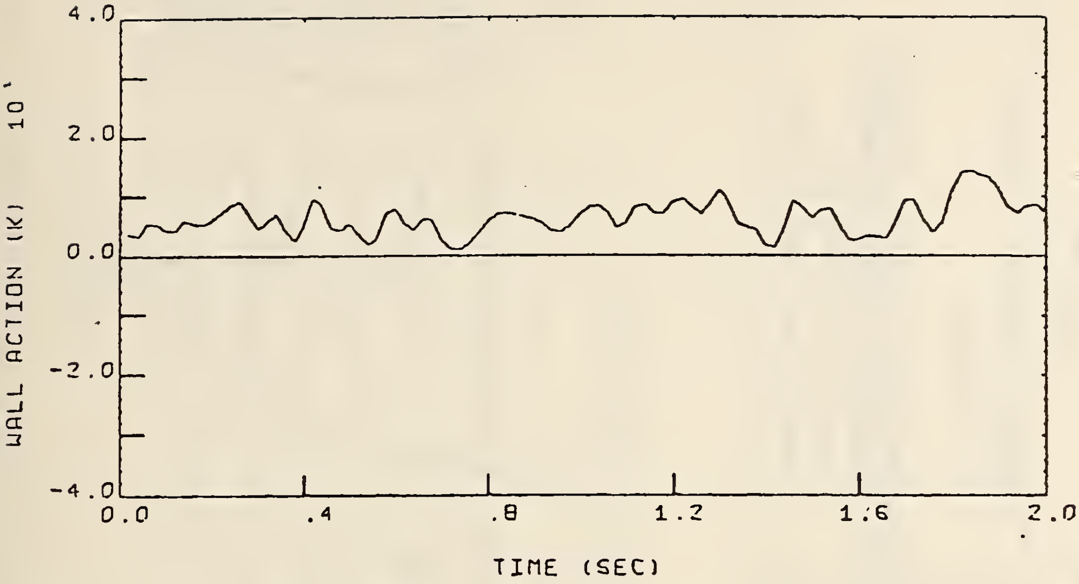
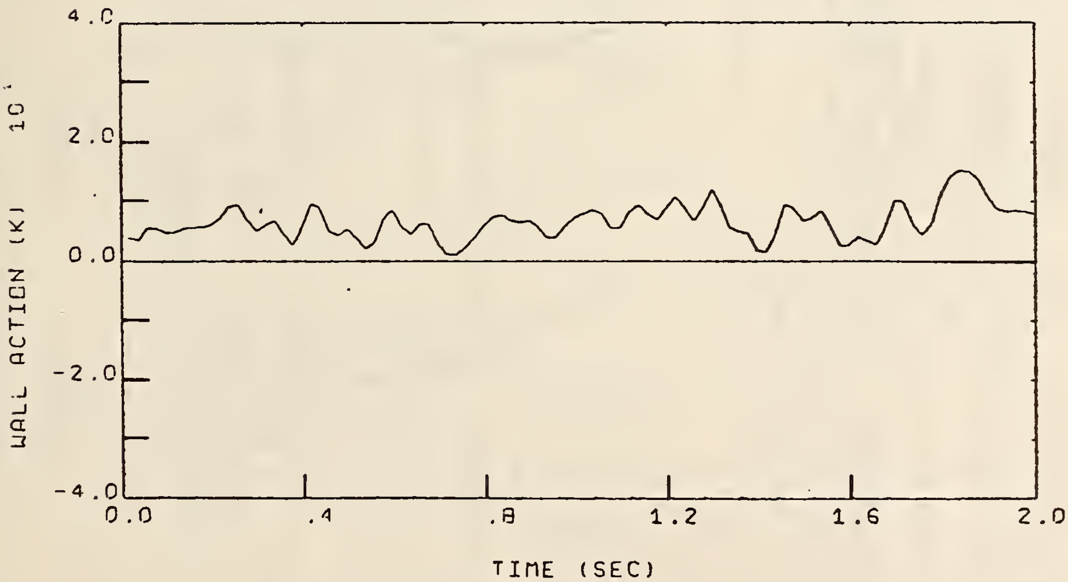


FIG. 41 STATIC AND MAXIMUM DYNAMICAL PRESSURE DISTRIBUTION WITH DIFFERENT NUMBER OF LAYERS - RIGID WALL SYSTEM



(a) 5 LAYERS



(b) 10 LAYERS

FIG. 42 TOTAL LATERAL EARTHPRESSURE – RIGID WALL SYSTEM

CASE	SUBDIVISION	SOIL PROPERTY	Fs	Fd	REMARKS
			%		
1	R = 2 FOR X < 2H	G = 10.0 UNIFORM	101.3	110.5	1. REFER TO FIG. 37 , CASE 4 FOR MODEL 2. CASE 3, THE NON-UNIFORM MODULUS CASE AS BASES FOR CALCULATING THE PERCENTAGE FOR $K_2 = 50$
2	R = 10 FOR X > 2H	G = 2.5 UNIFORM	101.3	154.0	
3	L/H = 10.0	$G = 1000.K_2(\sigma m)^{1/2}$	100	100	
CASE 3, ABSOLUTE VALUE			KIPS		
			3.67	13.06	

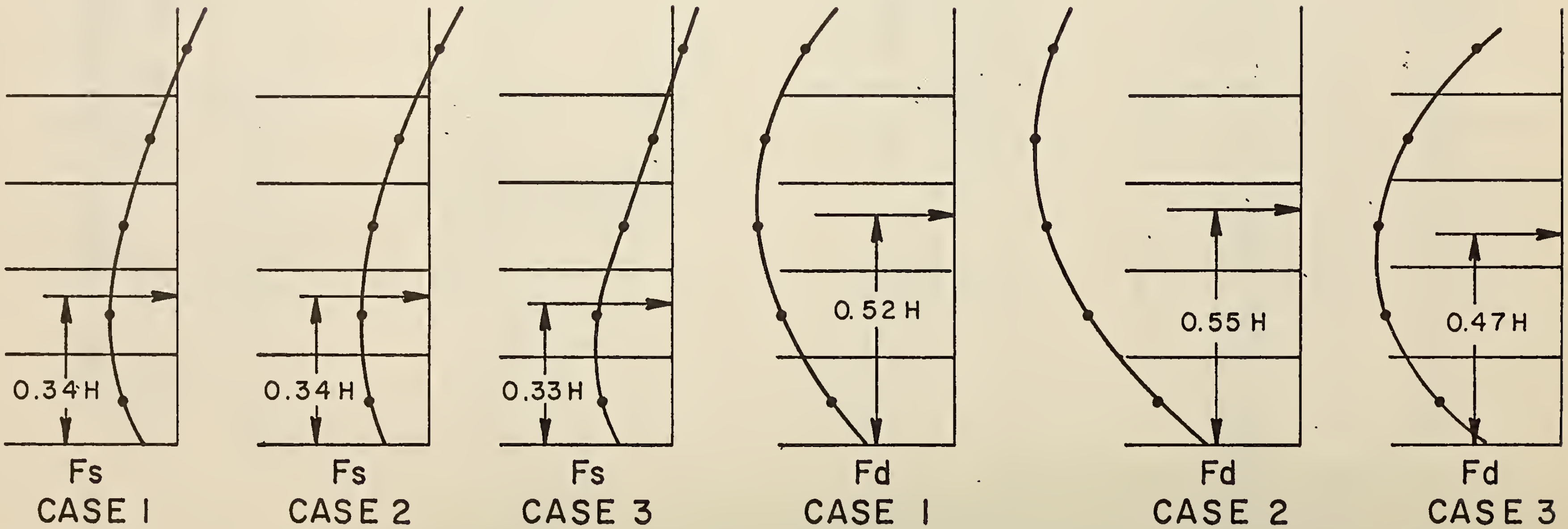
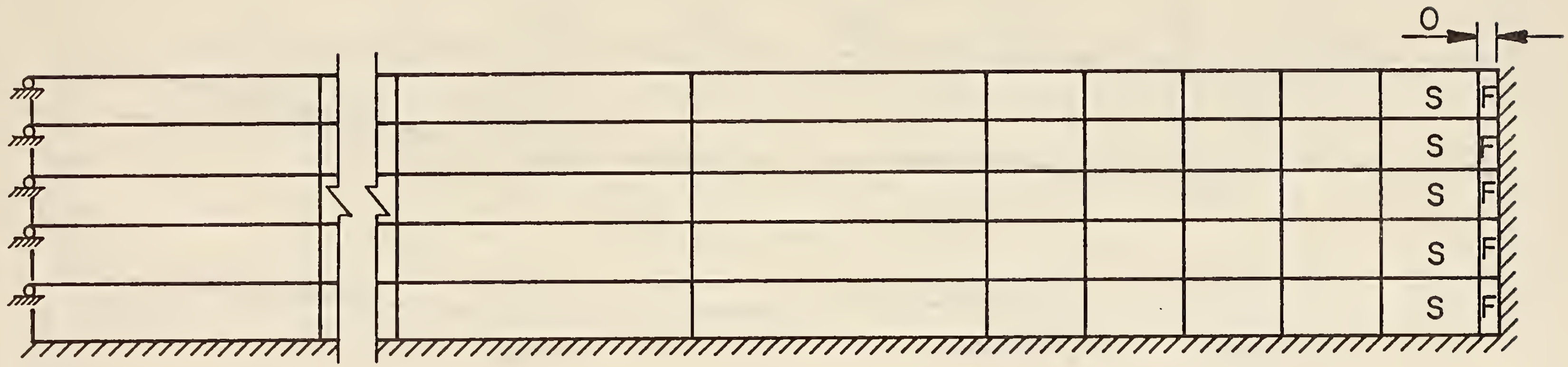


FIG. 43 STATIC AND MAXIMUM DYNAMICAL PRESSURE DISTRIBUTION WITH DIFFERENT SOIL PROPERTIES - RIGID WALL SYSTEM



$R \approx 2$ FOR $X < 2H$
$R \approx 6$ FOR $X > 2H$
$L/H = 115/13.5 = 8.5$
$G = 10.0$ KSI

$K_S = K_N$ RANGING $1, 10^3, 10^6, 10^9$ KSI
F: THE FRICTIONAL ELEMENTS NEAR WALL
S: SOIL ELEMENT

FIG. 44 MODEL FOR STUDYING FRICTIONAL ELEMENT, RIGID WALL SYSTEM

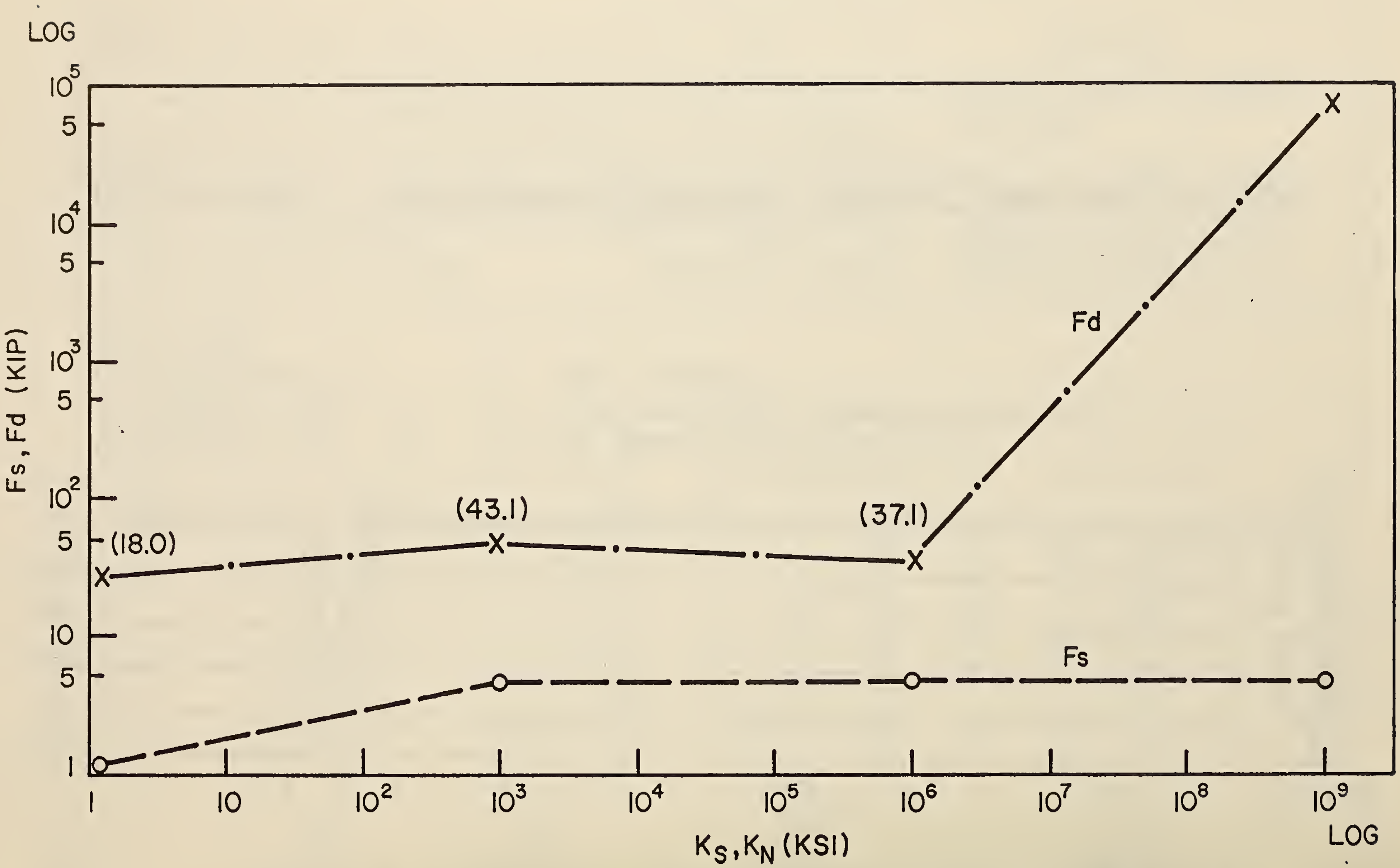


FIG. 45 EFFECT OF STIFFNESS OF FRICTIONAL ELEMENT ON TOTAL LATERAL FORCE - RIGID WALL SYSTEM

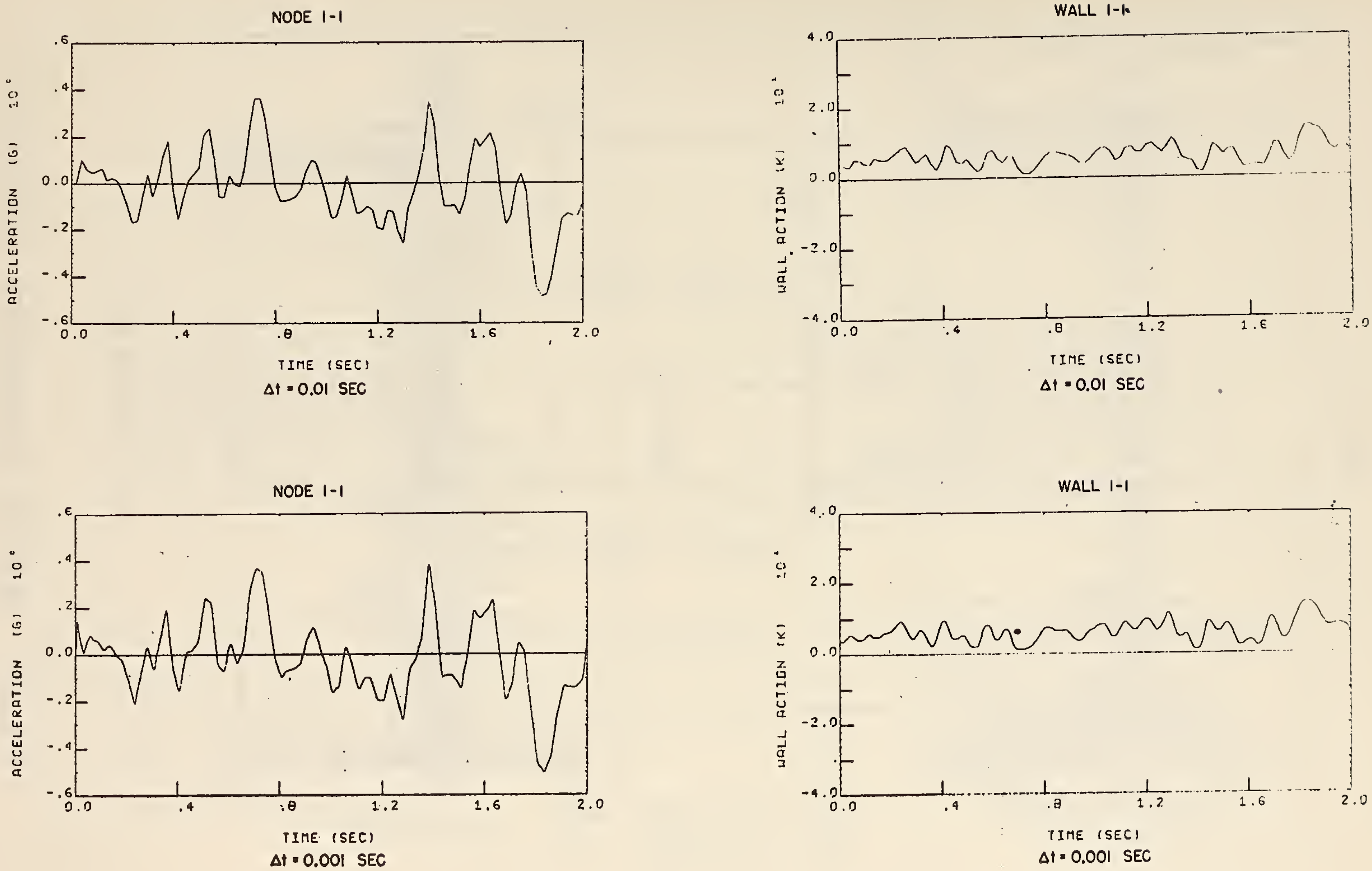
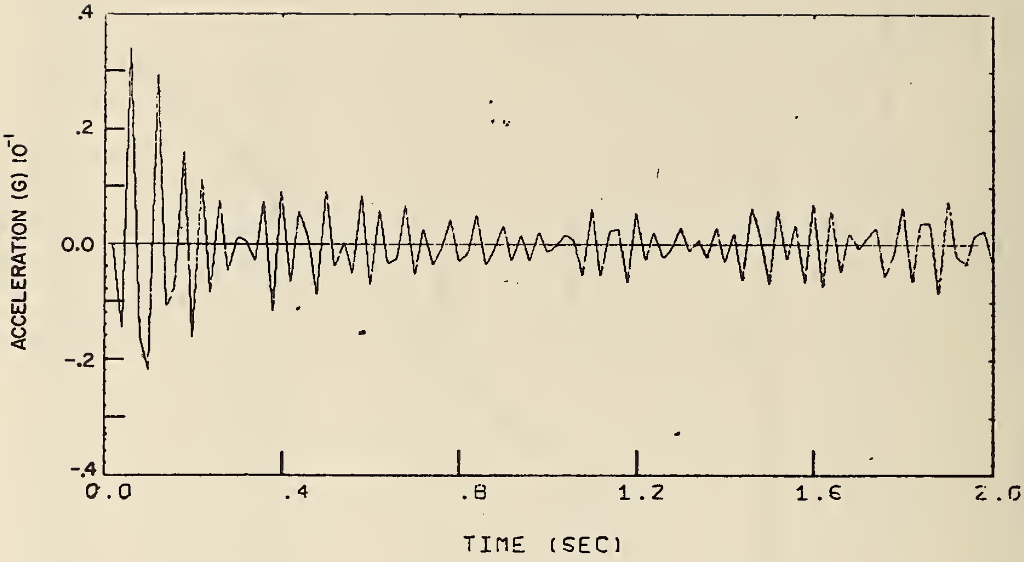


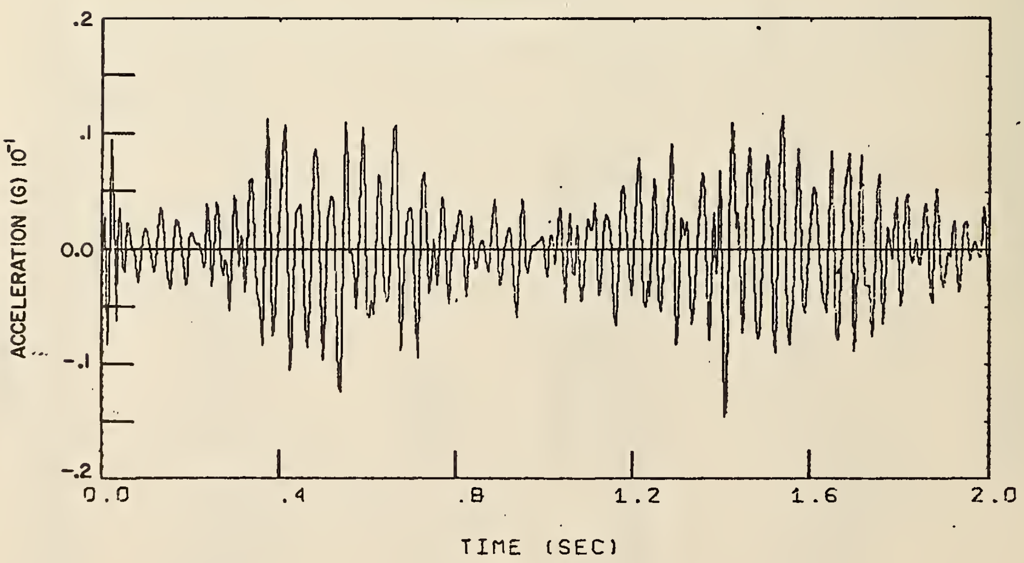
FIG. 46 LATERAL FORCE AND HORIZONTAL ACCELERATION AT POINT NO. I - RIGID WALL SYSTEM

NODE I-2



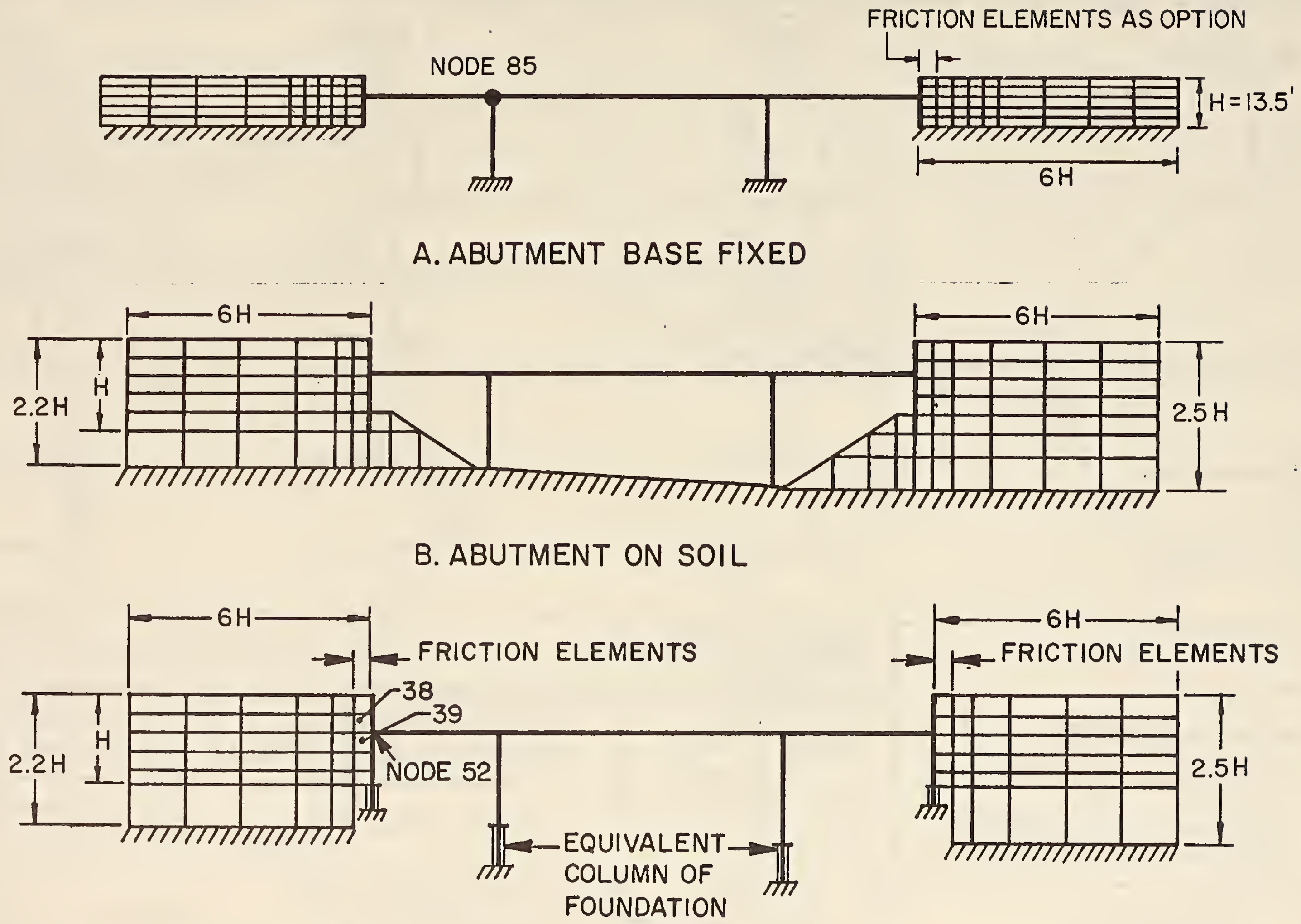
(a) $\Delta t = 0.01$ SEC

NODE I-2



(b) $\Delta t = 0.001$ SEC

FIG. 47 VERTICAL ACCELERATION AT POINT NO. I



A. ABUTMENT BASE FIXED

B. ABUTMENT ON SOIL

C. SUBSTRUCTURES ON EQUIVALENT COLUMNS

FIG. 48 NORTH CONNECTOR UNDERCROSSING, BRIDGE-SOIL SYSTEMS

CASE	SYSTEM	MOVEMENT AT ABUTMENT BASE	F_s	F_d	.REMARKS
			%		
1	BRIDGE-SOIL SYSTEM FIXED BASE	NO MOVEMENT	100	100	1. ALL SOILS ARE ASSUMED UNIFORM, WITH G=10 KSI 2. CASE 1, REFER TO FIG. 48-A CASE 2, REFER TO FIG. 48-B
2	BRIDGE-SOIL SYSTEM COLUMN ON SOIL	ROTATION AND TRANSLATION	83.0	270.0	
CASE 1, ABSOLUTE VALUE			KIPS		
			3.42	6.32	

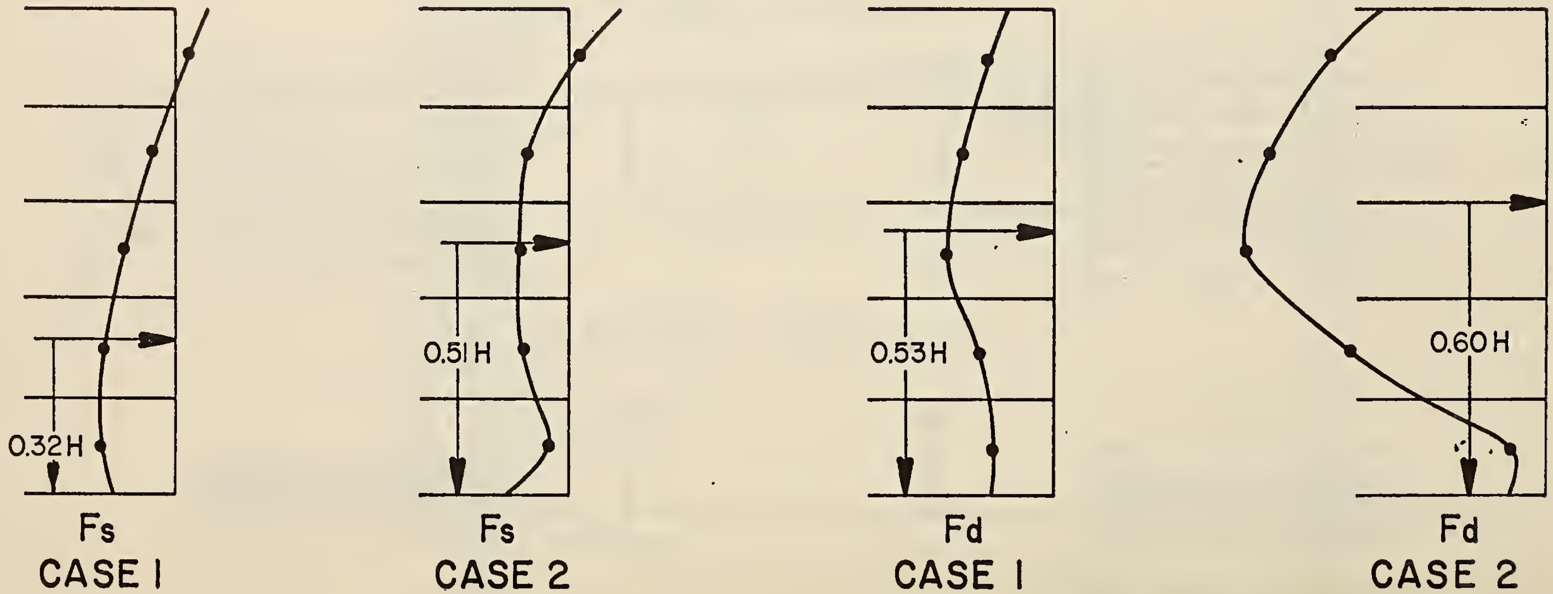


FIG. 49 STATIC AND MAXIMUM DYNAMICAL PRESSURE DISTRIBUTION WITH DIFFERENT BOUNDARY CONDITIONS AT THE BASE OF ABUTMENT

CASE	SYSTEM	PERIOD, SEC.		Fs	Fd	REMARKS
		1 st.	10 th.	%		
1	BRIDGE-SOIL SYSTEM FIXED BASE	0.195	0.063	100	100	1. ALL SOILS ARE ASSUMED UNIFORM, WITH G = 10 KSI 2. REFER TO FIG. 48-A FOR THE MODEL
2	SAME AS 1, EXCEPT WITH SOFTER SUB-STRUCTURE	0.232	0.076	120.0	175.0	
CASE 1, ABSOLUTE VALUE				KIPS		
				3.42	6.32	

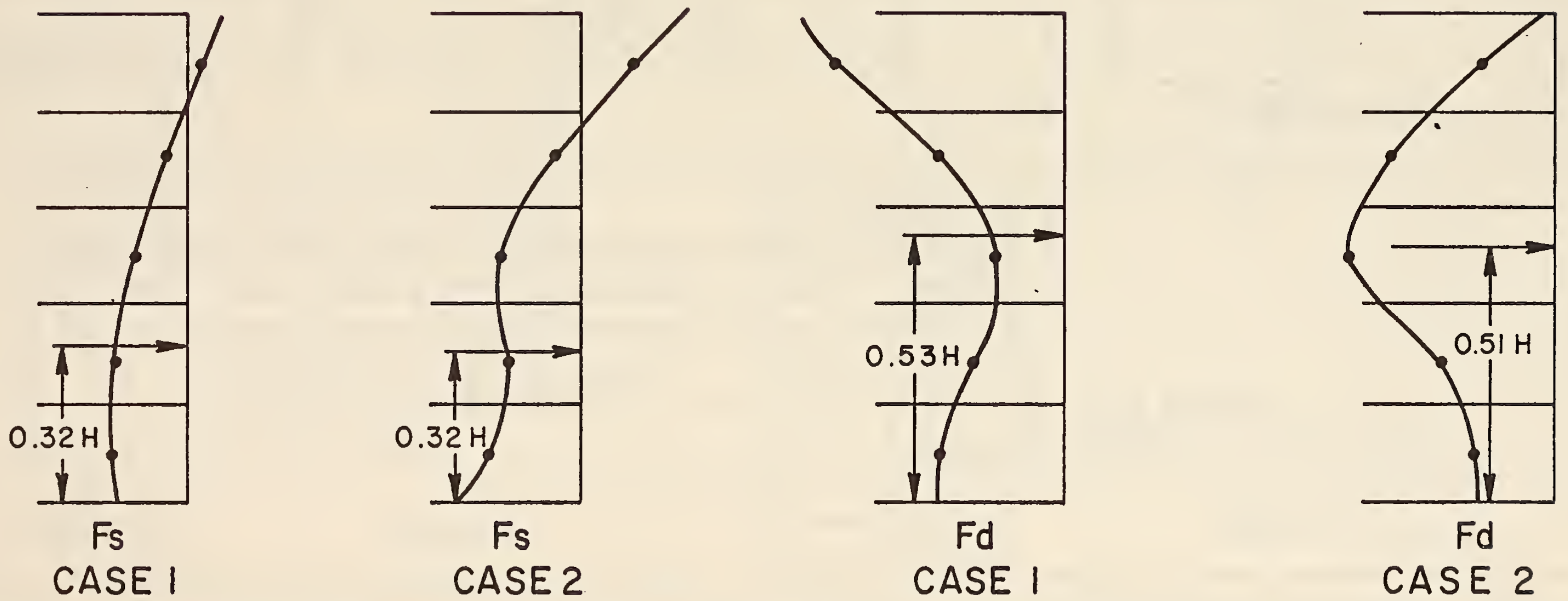


FIG. 50 STATIC AND MAXIMUM DYNAMICAL PRESSURE DISTRIBUTION OF BRIDGE-SOIL SYSTEM WITH DIFFERENT SUBSTRUCTURES

CASE	SYSTEM	Fs (%)	Fd (%)	REMARKS
1	LINEAR MODEL	100	100	1. REFER TO FIG. 48-A FOR MODEL
2	LINEAR SOIL, NON-LINEAR FRICTION	88.0	612.0	
3	NON-LINEAR SOIL, FRICTION	80.5	520.0	
CASE 1, ABSOLUTE VALUES		KIPS 3.42	KIPS 6.75	

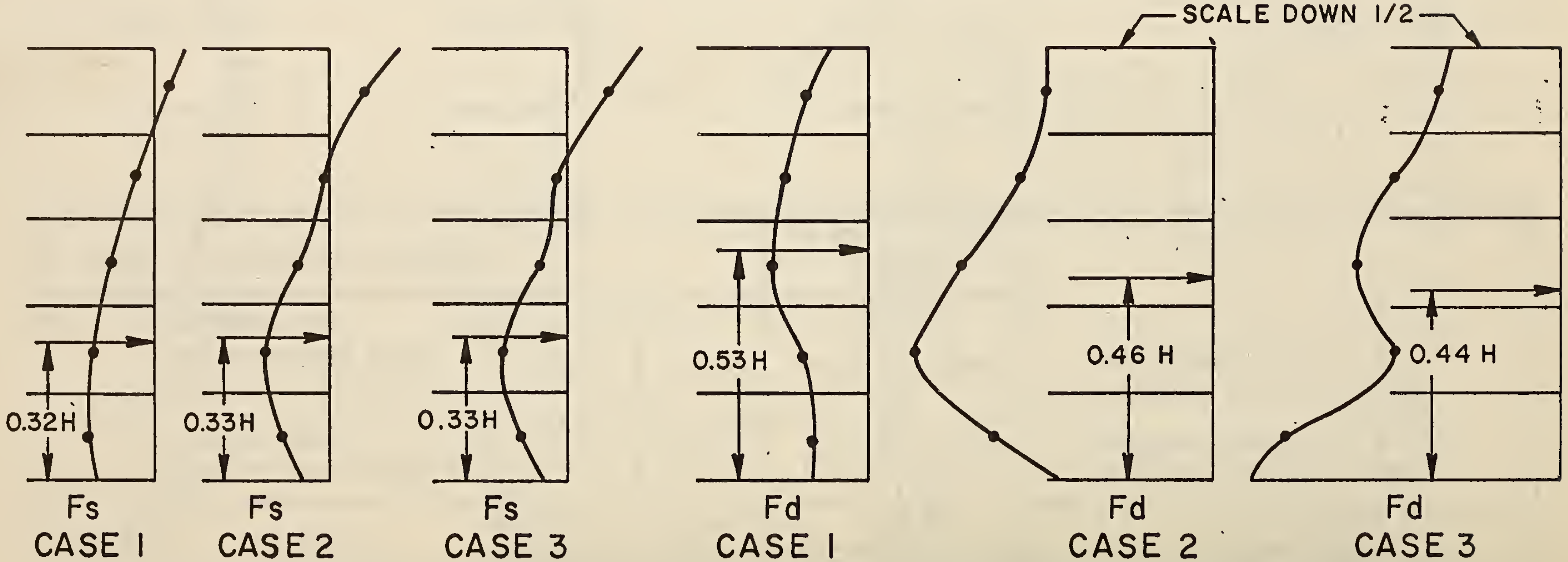


FIG. 51 STATIC AND DYNAMIC PRESSURE DISTRIBUTION OF BRIDGE-SOIL SYSTEM WITH DIFFERENT DEGREES OF NON-LINEARITY

CASE	SYSTEM	FOUNDATION	Fs	Fd	REMARKS
			%		
1	BRIDGE - SOIL SYSTEM WITH NON-LINEAR FRICTION ELEMENT, THE REST IS LINEAR	FIXED AT TOP OF EQ.COL.	100	100	1. REFER TO FIG. 48-C FOR MODEL
2		WITH EQUIVALENT COLUMN	129.0	225.0	
CASE 1, ABSOLUTE VALUES			KIPS		
			3.30	15.3	

SCALE DOWN 1/2 FOR Fd

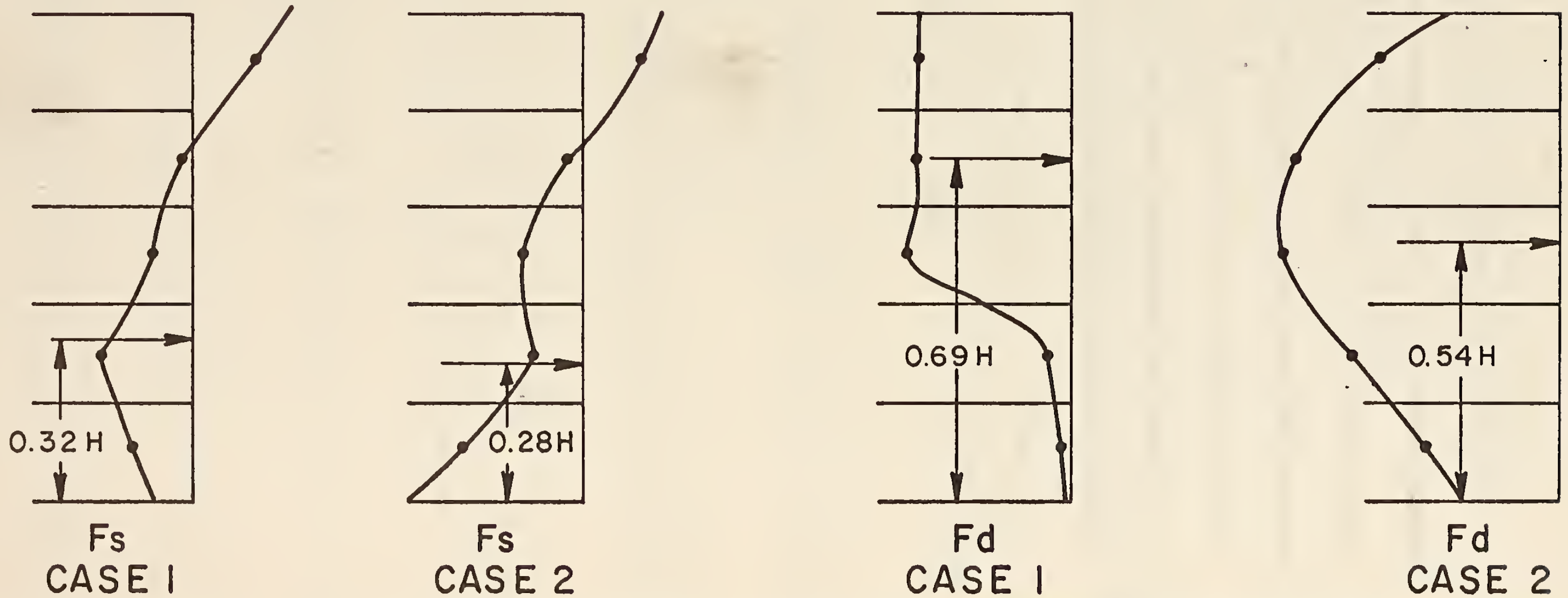


FIG. 52 STATIC AND MAXIMUM DYNAMICAL PRESSURE DISTRIBUTION OF BRIDGE - SOIL SYSTEM WITH DIFFERENT FOUNDATION

VII GENERAL CONCLUSION

Based on the results of this investigation, it is concluded that soil-structure interaction effects must be considered when analyzing the dynamic response of short, stiff, single or multiple span bridges. The mathematical modelling and computer programs presented herein provide an effective means of conducting such analyses.

Since the numerical results obtained in this investigation are very limited, caution should be exercised when interpreting them in a quantitative sense. Further analyses are recommended to complete the parameter studies.

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APPENDIX

Computer Program Listings

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      PROGRAM BRISOT(INPUT,OUTPUT,TAPE6=OUTPUT,TAPE1,TAPE3,TAPE4,TAPE10,BRISOT.1
     1               TAPE11,TAPE12,TAPE13,TAPE14,TAPE21,TAPE22,TAPE23,  BRISOT.2
     2              TAPE7,TAPE30,TAPE20,TAPE40,TAPE41)                  BRISOT.3
C                                                                       BRISOT.4
C     ******************************************************************BRISOT.5
C     TWO DIMENSIONAL NONLINEAR DYNAMIC ANALYSIS OF SOIL-BRIDGE INTERACTBRISOT.6
C     JULY,1973-NA-CHI CHEN                                             BRISOT.7
C     ******************************************************************BRISOT.8
      COMMON/TITLE/HEAD(12),EQNAME(12)                                  BRISOT.9
      COMMON/MATER/NUMATS,NUMATC,NUMATF,NUMATE,NUMATB,NUNGE,NINTCV,THICKBRISOT.10
     1,NPLAN                                                            BRISOT.11
      COMMON/ELPAR/NUMNP,NUMEL,NEQ,MBAND,KLIN                           BRISOT.12
      COMMON/TIME/JUMP,T,DT,NPRTN,NTAPE,KPRINT                          BRISOT.13
      COMMON/ABS/UGXT,UGYT,VGXT,VGYT,OACCX,OACCY                        BRISOT.14
      COMMON/SPECIL/ISTOP                                               BRISOT.15
      COMMON/OMEGA/OMEGA1,OMEGA2,NF1,NF2                                BRISOT.16
      COMMON/TAPES/NSTIF,NMASS                                          BRISOT.17
      DIMENSION S(13),NELW(10)                                          BRISOT.18
      COMMON A(35500)                                                   BRISOT.19
C                                                                       BRISOT.20
C     PROGRAM CONTROL DATA                                              BRISOT.21
C                                                                       BRISOT.22
 1000 CALL SECOND(S(1))                                                 BRISOT.23
      ISTOP=0                                                           BRISOT.24
      JUMP=0                                                            BRISOT.25
      READ 1,HEAD,NUMNP,NUMEL,NETYPE,KLIN,NUMATS,NUMATC,NUMATF,NUMATE,  BRISOT.26
     1       NUMATB,NUNGE,NINTCV,NPLAN,THICK                            BRISOT.27
      IF(NUMNP .EQ. 0) STOP                                             BRISOT.28
      WRITE(6,101)  HEAD,NUMNP,NUMEL,NETYPE,KLIN,NUMATS,NUMATC,NUMATF,  BRISOT.29
     1              NUMATE,NUMATB,NUNGE,NINTCV,NPLAN,THICK              BRISOT.30
      READ 6,NF,NF1,NF2                                                 BRISOT.31
      WRITE(6,106) NF,NF1,NF2                                           BRISOT.32
C                                                                       BRISOT.33
C     INPUT JOINT DATA--ID(3,NUMNP)                                     BRISOT.34
C                                                                       BRISOT.35
      N1=1                                                              BRISOT.36
      N2=N1+3*NUMNP                                                     BRISOT.37
      N3=N2+NUMNP                                                       BRISOT.38
      N4=N3+NUMNP                                                       BRISOT.39
      CALL INPUTJ(A(N1),A(N2),A(N3),NUMNP,NEQ)                          BRISOT.40
      CALL SECOND(S(2))                                                 BRISOT.41
C                                                                       BRISOT.42
C     READ IN MATERIAL PROPERTIES DATA,AND BEAM,COLUMN GEOMETRIC DATA   BRISOT.43
C                                                                       BRISOT.44
      N5=N4+8*NUMATS                                                    BRISOT.45
      N6=N5+5*NUMATC                                                    BRISOT.46
      N7=N6+4*NUMATF                                                    BRISOT.47
      N8=N7+4*NUMATE                                                    BRISOT.48
      N9=N8+2*NUMATB                                                    BRISOT.49
      N10=N9+3*NUNGE                                                    BRISOT.50
      N11=N10+20*NUMATS                                                 BRISOT.51
      N12=N11+9*NINTCV                                                  BRISOT.52
      DO 301 I=1,NETYPE                                                 BRISOT.53
      READ 2,NTYPE                                                      BRISOT.54
      WRITE(6,102) NTYPE                                                BRISOT.55
      CALL MATPRO(NTYPE,A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),A(N10),     BRISOT.56
     1A(N11))                                                           BRISOT.57
  301 CONTINUE                                                          BRISOT.58
      CALL SECOND(S(3))                                                 BRISOT.59
C                                                                       BRISOT.60
C     READ IN ELEMENT DATA                                              BRISOT.61
C                                                                       BRISOT.62
      N13=N12+6*NUMEL                                                   BRISOT.63
      N14=N13+4*NUMEL                                                   BRISOT.64
      CALL ELDATA(A(N1),A(N12),A(N13),NBAND,NUMEL)                      BRISOT.65
      CALL SECOND(S(4))                                                 BRISOT.66
C                                                                       BRISOT.67
C     INPUT AND CALCULATE NODAL MASS,LOAD,AND FIXED END MOMENT OF BEAM  BRISOT.68
C                                                                       BRISOT.69
      N15=N14+NEQ                                                       BRISOT.70
      N16=N15+NEQ                                                       BRISOT.71
      N17=N16+NEQ                                                       BRISOT.72
      NT18=N17+NEQ                                                      BRISOT.73
      NT19=NT18+NEQ                                                     BRISOT.74
      CALL LOAD(A(N1),A(N2),A(N3),A(N4),A(N5),A(N9),A(N12),A(N13),      BRISOT.75
     1A(N14),A(N15),A(N16),A(N17),A(NT18),NUMEL,NEQ)                    BRISOT.76
      CALL SECOND(S(5))                                                 BRISOT.77
C                                                                       BRISOT.78
C                                                                       BRISOT.79
C     FORM ELASTIC STRESS-STRAIN RELATIONSHIP                           BRISOT.80
      NT18=N17+5*NUMEL                                                  BRISOT.81
      NT19=NT18+NUNGE                                                   BRISOT.82
      NT20=NT19+5*NUMATS                                                BRISOT.83
      NT21=NT20+5*NUMATC*NUNGE                                          BRISOT.84
      NT22=NT21+5*NUMATF                                                BRISOT.85
      NT23=NT22+5*NUMATE                                                BRISOT.86
      NT24=NT23+5*NUMATB                                                BRISOT.87
      CALL ESGEPS(A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),      BRISOT.88
     1A(N12),A(N13),A(N17),A(NT18),A(NT19),A(NT20),A(NT21),A(NT22),     BRISOT.89
     2A(NT23),NUMATS,NUMATC,NUMATF,NUMATE,NUMATB,NUNGE,NPLAN)           BRISOT.90
      CALL SECOND(S(6))                                                 BRISOT.91
C                                                                       BRISOT.92
C                                                                       BRISOT.93
C     FORM TOTAL ELASTIC STIFFNESS FOR STATIC ANALYSIS                  BRISOT.94
C                                                                       BRISOT.95
      N18=N17+5*NUMEL                                                   BRISOT.96
      N19=N18+NEQ*NBAND                                                 BRISOT.97
      NT20=N19+NUMEL                                                    BRISOT.98
      CALL ETSTIF(A(N1),A(N2),A(N3),A(N5),A(N9),A(N12),A(N13),A(N14),   BRISOT.99
     1            A(N17),A(N18),A(N19),NUMEL,NEQ,NBAND)                 BRISOT.100
      CALL SECOND(S(7))                                                 BRISOT.101
C                                                                       BRISOT.102
C     CALCULATE LOWEST ?NF? FREQUENCIES                                 BRISOT.103
C                                                                       BRISOT.104
      CALL MODES(NEQ,NBAND,1,NF,NTOT,N18)                               BRISOT.105
C                                                                       BRISOT.106
C     SOLVE FOR DISPLACEMENT-STATIC CASE                                BRISOT.107
C                                                                       BRISOT.108
      JUMP=0                                                            BRISOT.109
      N20=N19+NEQ                                                       BRISOT.110
      N21=N20+NEQ                                                       BRISOT.111
      N22=N21+NEQ                                                       BRISOT.112
      N23=N22+NEQ                                                       BRISOT.113
      N24=N23+NEQ                                                       BRISOT.114
      N25=N24+NEQ                                                       BRISOT.115
      CALL STATIC(A(N14),A(N18),A(N19),A(N20),A(N21),A(N22),A(N23),     BRISOT.116
     1A(N24),NEQ,NBAND)                                                 BRISOT.117
      CALL SECOND(S(8))                                                 BRISOT.118
C                                                                       BRISOT.119
C     COMPUTE STRAIN                                                    BRISOT.120
C                                                                       BRISOT.121
      N26=N25+3*NUMEL                                                   BRISOT.122
      N27=N26+3*NUMEL                                                   BRISOT.123
      N28=N27+3*NUMEL                                                   BRISOT.124
      CALL STRAIN(A(N1),A(N2),A(N3),A(N12),A(N22),A(N25),A(N26),A(N27)) BRISOT.125
      CALL SECOND(S(9))                                                 BRISOT.126
C                                                                       BRISOT.127
C     COMPUTE STRESS                                                    BRISOT.128
C                                                                       BRISOT.129
      N29=N28+8*NUMEL                                                   BRISOT.130
      N30=N29+8*NUMEL                                                   BRISOT.131
      N31=N30+4*NUMEL                                                   BRISOT.132
      CALL STRESS(A(N12),A(N22),A(N26),A(N28),A(N29),A(N30))            BRISOT.133
      CALL SECOND(S(10))                                                BRISOT.134
C                                                                       BRISOT.135
C     INPUT ABUTMENT WALL INFORMATION,CALCULATE EARTHPRESS ON WALL      BRISOT.136
C                                                                       BRISOT.137
      READ 3,NTW                                                        BRISOT.138
      IF(NTW .EQ. 0) GO TO 402                                          BRISOT.139
      READ 4,(NELW(I),I=1,NTW)                                          BRISOT.140
      WRITE(6,103) NTW,(NELW(I),I=1,NTW)                                BRISOT.141
      ML=0                                                              BRISOT.142
      DO 302 I=1,NTW                                                    BRISOT.143
      LENGTH=3*NELW(I)                                                  BRISOT.144
      IF(LENGTH .GT. ML) ML=LENGTH                                      BRISOT.145
  302 CONTINUE                                                          BRISOT.146
      GO TO 403                                                         BRISOT.147
  402 CONTINUE                                                          BRISOT.148
      N32=N31                                                           BRISOT.149
      N33=N32                                                           BRISOT.150
      N34=N33                                                           BRISOT.151
      N35=N34                                                           BRISOT.152
      IF(NTW .EQ. 0) GO TO 401                                          BRISOT.153
  403 CONTINUE                                                          BRISOT.154
      LENGTH=NTW*ML                                                     BRISOT.155
      N32=N31+LENGTH                                                    BRISOT.156
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      CALL WADATA(NELW,A(N31),NTW)                                      BRISOT.157
      N33=N32+NTW                                                       BRISOT.158
      N34=N33+NTW                                                       BRISOT.159
      N35=N34+NTW                                                       BRISOT.160
      CALL WFORCE(A(N3),A(N12),A(N29),NELW,A(N31),A(N32),A(N33),A(N34), BRISOT.161
     1            NTW)                                                  BRISOT.162
  401 CONTINUE                                                          BRISOT.163
      CALL SECOND(S(11))                                                BRISOT.164
C                                                                       BRISOT.165
C     PRINT RESULTS,AND STORE ON TAPE2 FOR PLOTTING                     BRISOT.166
C                                                                       BRISOT.167
      NUMELE=NUMEL                                                      BRISOT.168
      N36=N35+NUMEL                                                     BRISOT.169
      N37=N36+NUMEL                                                     BRISOT.170
      NT38=N37+9*NUMEL                                                  BRISOT.171
      CALL PRINTR(A(N1),A(N12),A(N19),A(N20),A(N21),A(N25),A(N27),A(N29)BRISOT.172
     1,A(N30),A(N32),A(N33),A(N34),A(N35),A(N36),A(N37),NTW,NUMELE)     BRISOT.173
      CALL SECOND(S(12))                                                BRISOT.174
C                                                                       BRISOT.175
C     START DYNAMIC ANALYSIS-STEP-BY-STEP                               BRISOT.176
C     FORM NONLINEAR TOTAL STIFFNESS                                    BRISOT.177
C                                                                       BRISOT.178
      N38=NT38+36                                                       BRISOT.179
      CALL NTSTIF(A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),      BRISOT.180
     1A(N10),A(N11),A(N12),A(N13),A(N14),A(N17),A(N18),A(N19),A(N20),   BRISOT.181
     2A(N22),A(N25),A(N26),A(N27),A(N28),A(N29),A(N30),A(N35),A(N36),   BRISOT.182
     3A(NT38),GS,NEQ,MBAND,NUMEL,KLIN)                                  BRISOT.183
      N39=N38+NEQ                                                       BRISOT.184
      N40=N39+NEQ                                                       BRISOT.185
      N41=N40+NEQ                                                       BRISOT.186
      N42=N41+NEQ                                                       BRISOT.187
      N43=N42+NEQ                                                       BRISOT.188
      N44=N43+NEQ                                                       BRISOT.189
      CALL STEP(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),  BRISOT.190
     1A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),   BRISOT.191
     2A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),A(N25),A(N26),A(N27),   BRISOT.192
     3A(N28),A(N29),A(N30),A(N31),A(N32),A(N33),A(N34),A(N35),A(N36),   BRISOT.193
     4A(N37),A(NT38),A(N38),A(N39),A(N40),A(N41),A(N42),A(N43),NELW,NTW,BRISOT.194
     5GS,NUMNP,NEQ,MBAND,RLIN,NUMEL)                                    BRISOT.195
      CALL OUTPUT(A(N1),NUMNP,NUMEL,NEQ,NTW)                            BRISOT.196
      CALL SECOND(S(13))                                                BRISOT.197
      DO 303 I=1,12                                                     BRISOT.198
  303 S(I)=S(I+1)-S(I)                                                  BRISOT.199
      TT=0.0                                                            BRISOT.200
      DO 304 I=1,12                                                     BRISOT.201
  304 TT=TT+S(I)                                                        BRISOT.202
      S(13)=TT                                                          BRISOT.203
      WRITE(6,104) S                                                    BRISOT.204
      WRITE(6,105) N44                                                  BRISOT.205
C     ****************                                                  BRISOT.206
C     FORMAT STATEMENT                                                  BRISOT.207
C     ****************                                                  BRISOT.208
    1 FORMAT(12A6/(12I5,F10.4))                                         BRISOT.209
    2 FORMAT(I5)                                                        BRISOT.210
    3 FORMAT(I5)                                                        BRISOT.211
    4 FORMAT(16I5)                                                      BRISOT.212
    6 FORMAT(3I5)                                                       BRISOT.213
  101 FORMAT(1H1,12A6//                                                 BRISOT.214
     1       * NO. OF NODAL POINTS                          *,I5/       BRISOT.215
     2       * NO. OF ELEMENTS                              *,I5/       BRISOT.216
     3       * NO. OF ELEMENT TYPES                         *,I5/       BRISOT.217
     4       * NONLINEAR OR LINEAR ANALYSIS,0-LINEAR,1-NONLINEAR*,I5/   BRISOT.218
     5       * NO. OF SOIL MATERIAL TYPES                   *,I5/       BRISOT.219
     6       * NO. OF CONCRETE MATERIAL TYPES               *,I5/       BRISOT.220
     7       * NO. OF FRICTION ELEMENT TYPES                *,I5/       BRISOT.221
     8       * NO. OF EXPANSION JOINT TYPES                 *,I5/       BRISOT.222
     9       * NO. OF BOUNDARY ELEMENT TYPES                *,I5/       BRISOT.223
     1       * NO. OF CONCRETE GEOMETRIC PROPERTIES         *,I5/       BRISOT.224
     2       * NO. OF P-M INTERACTION CURVES OF CONCRETE    *,I5/       BRISOT.225
     3       * PLANE STRAIN-0,PLANE STRESS-1                *,I5/       BRISOT.226
     4       * ELEMENT THICKNESS                            *,F10.3)BRISOT.227
  102 FORMAT(*1MATERIAL TYPE NTYPE=*,I5//)                              BRISOT.228
  103 FORMAT(* INFORMATIONS OF WALLS*/                                  BRISOT.229
     1       * TOTAL NO. OF WALL=*, I5/                                 BRISOT.230
     2       * NO. OF ELEMENT ON WALL 1,2,...=*,10I5)                   BRISOT.231
  104 FORMAT(*1TIME LOG*//                                              BRISOT.232
     1       * NODAL POINT INPUT                          *,F8.2//      BRISOT.233
     2       * INPUT MATERIAL PROPERTIES                  *,F8.2//      BRISOT.234
     3       * INPUT ELEMENT DATA                         *,F8.2//      BRISOT.235
     4       * INPUT AND CALCULATE NODAL MASS,LOAD        *,F8.2//      BRISOT.236
     5       * FORM ELASTIC STRESS-STRAIN RELATIONSHIP *,F8.2//         BRISOT.237
     6       * FORM ELASTIC TOTAL STIFFNESS               *,F8.2//      BRISOT.238
     7       * STATIC SOLUTION OF DISPLACEMENT            *,F8.2//      BRISOT.239
     8       * COMPUTE STRAIN OF SOIL ELEMENT             *,F8.2//      BRISOT.240
     9       * COMPUTE STRESS OF ALL ELEMENT              *,F8.2//      BRISOT.241
     .       * EARTHPRESSURE ON WALL                      *,F8.2//      BRISOT.242
     .       * PRINT RESULTS                              *,F8.2//      BRISOT.243
     .       * DYNAMIC SOLUTION                           *,F8.2//      BRISOT.244
     .       * TOTAL SOLUTION TIME                        *,F8.2)       BRISOT.245
  105 FORMAT(* LENGTH OF BLANK COMMON A*,I8)                            BRISOT.246
  106 FORMAT(//* NO. OF FREQUENCIES TO BE CALCULATED*,I5/               BRISOT.247
     1       * THE MODES TO CONTROL THE RAYLEIGH DAMPING*,2I5)          BRISOT.248
      END                                                               BRISOT.249

      SUBROUTINE INPUTJ(ID,X,Y,NUMNP,NEQ)                               INPUTJ.2
C                                                                       INPUTJ.3
C     *****************************************************************INPUTJ.4
C     READ OR GENERATE NODAL POINT DATA                                 INPUTJ.5
C     *****************************************************************INPUTJ.6
C                                                                       INPUTJ.7
      DIMENSION X(1),Y(1),ID(3,NUMNP)                                   INPUTJ.8
      WRITE(6,501)                                                      INPUTJ.9
      WRITE(6,502)                                                      INPUTJ.10
      NOLD=0                                                            INPUTJ.11
 1000 CONTINUE                                                          INPUTJ.12
      READ 1,N,(ID(I,N),I=1,3),X(N),Y(N),KN                             INPUTJ.13
      WRITE(6,101) N,(ID(I,N),I=1,3),X(N),Y(N),KN                       INPUTJ.14
      IF(NOLD .EQ. 0) GO TO 401                                         INPUTJ.15
C                                                                       INPUTJ.16
C     CHECK IF GENERATION IS REQUIRED                                   INPUTJ.17
      IF(KN .EQ. 0) GO  TO 401                                          INPUTJ.18
C                                                                       INPUTJ.19
C     NUM=NO. OF INTERVALS BETWEEN INPUT NODE                           INPUTJ.20
C     NUMN=NO. OF NODES TO BE GENERATED                                 INPUTJ.21
C     DX,DY, THE LENGTH OF INTERVAL                                     INPUTJ.22
      NUM=N-NOLD                                                        INPUTJ.23
      NUMN=NUM-1                                                        INPUTJ.24
      IF(NUMN .LT. 1) GO TO 401                                         INPUTJ.25
      XNUM=NUM                                                          INPUTJ.26
      DX=(X(N)-X(NOLD))/XNUM                                            INPUTJ.27
      DY=(Y(N)-Y(NOLD))/XNUM                                            INPUTJ.28
C                                                                       INPUTJ.29
C     GENERATE NEW NODAL POINT                                          INPUTJ.30
      K=NOLD                                                            INPUTJ.31
      DO 301 J=1,NUMN                                                   INPUTJ.32
      KK=K                                                              INPUTJ.33
      K=K+1                                                             INPUTJ.34
      X(K)=X(KK)+DX                                                     INPUTJ.35
      Y(K)=Y(KK)+DY                                                     INPUTJ.36
      DO 301 I=1,3                                                      INPUTJ.37
      ID(I,K)=ID(I,KK)                                                  INPUTJ.38
  301 CONTINUE                                                          INPUTJ.39
  401 CONTINUE                                                          INPUTJ.40
      NOLD=N                                                            INPUTJ.41
      IF(N .NE. NUMNP) GO TO 1000                                       INPUTJ.42
C                                                                       INPUTJ.43
C     PRINT ALL NODAL POINT DATA                                        INPUTJ.44
      WRITE(6,503)                                                      INPUTJ.45
      WRITE(6,502)                                                      INPUTJ.46
      WRITE(6,201)(N,(ID(I,N),I=1,3),X(N),Y(N),N=1,NUMNP)               INPUTJ.47
C                                                                       INPUTJ.48
C     CALCULATE THE EQUATION NUMBERS FOR EVERY DEGREE OF FREEDOM        INPUTJ.49
      NEQ=0                                                             INPUTJ.50
      DO 302 N=1,NUMNP                                                  INPUTJ.51
      DO 302 I=1,3                                                      INPUTJ.52
      IF(ID(I,N)-1) 402,403,403                                         INPUTJ.53
  402 CONTINUE                                                          INPUTJ.54
      NEQ=NEQ+1                                                         INPUTJ.55
      ID(I,N)=NEQ                                                       INPUTJ.56
      GO TO 302                                                         INPUTJ.57
```

```
  403 IO(I,M)=0
  307 CONTINUE
C
C     WRITE EQUATION NUMBERS
      WRITE(6,202) (N,(IO(I,M),I=1,3),N=1,NUMNP)
      RETURN
    1 FORMAT(4I5,2F10.2,I5)
  101 FORMAT(4I5,14X,2F10.2,I7)
  201 FORMAT(4I5,14X,2F10.2)
  501 FORMAT(23H NODAL POINT INPUT DATA)
  202 FORMAT(//17H EQUATION NIMBERS/
     1       20H    N     X     Y   ZZ/(4I5))
  502 FORMAT(5HONODE,3X,24HBOUNDARY CONDITION CODES,11X,
     1       23HNODAL POINT COORDINATES/
     2       7H NUNBER,2X,1HX,4X,1HY,4X,2HZZ,
     3       22X,1HX, 9X,1HY,4X,2HKN//)
  503 FORMAT(* GENERATED NODAL DATA*)
      END


      SUBROUTINE MATPRO(NTYPE,PARASO,PARACO,PARAFR,PARAEX,PARABO,
     1COPROP,K2S,CNP)
C
C*************************************************************************
C     INPUT MATERIAL PROPERTIES,BEAM,COLUMN GEOMETRIC PROPERTY
C*************************************************************************
C
      COMMON/MATER/NUMATS,NUMATC,NUNATF,NUMATE,NUMATB,NUMGE,NINTCV,THICK
     1,NPLAN
      DIMENSION PARASO(8,1),PARACO(5,1),PARAFR(4,1),PARAEX(4,1),
     1          PARABO(2,1),COPROP(3,1),K2S(20,1),CNP(9,1)
      COMMON/ELPAR/NUMNP,NUMEL,NEO,MBAND,KLIN
      REAL K2S
      WRITE(6,501)
      GO TO (401,402,403,404,405),NTYPE
  401 CONTINUE
C
C     READ IN SOIL PROPERTY DATA
C
      DO 306 I=1,NUMATS
      READ 2, N, (PARASO(J,N),J=1,8)
  306 CONTINUE
      WRITE(6,502)
      WRITE(6,102) IN,(PARASO(J,N),J=1,8),N=1,NUMATS)
      IF(KLIN .EQ. 0) GO TO 312
      WRITE(6,503)
      DO 301 I=1,NUMATS
      READ 2, N,(K2S(J,M),J=1,20)
      WRITE(6,103) N,(K2S(J,M),J=1,20)
  301 CONTINUE
      GO TO 312
C
C     READ IN CONCRETE PROPERTY DATA
C
  402 CONTINUE
      DO 307 I=1,NUMATC
      READ 2,N,(PARACO(J,N),J=1,5)
  307 CONTINUE
C
C     READ IN BEAM,COLUMN GEOMETRIC DATA AND INTERACTION CURVE
C
      WRITE(6,505)
      WRITE(6,105) (N,(PARACO(J,N),J=1,5),N=1,NUMATC)
      WRITE(6,507)
      DO 308 I=1,NUMGE
      READ 2,N,(COPROP(J,N),J=1,3)
      WRITE(6,107) N,(COPROP(J,N),J=1,3)
  308 CONTINUE
      IF(KLIN .EQ. 0) GO TO 312
      IF(NINTCV .EQ. 0) GO TO 312
      WRITE(6,508)
      DO 302 I=1,NINTCV
      READ 3,N,(CMP(J,N),J=1,9)
      WRITE(6,108) N,(CMP(J,N),J=1,9)
  302 CONTINUE
      GO TO 312
C
C     READ IN FRICTIONAL ELEMENT PROPERTY
C
  403 CONTINUE
      DO 309 I=1,NUMATF
      READ 1,M,(PARAFR(J,M),J=1,4)
  309 CONTINUE
      WRITE(6,510)
      WRITE(6,110)(N,(PARAFR(J,N),J=1,4),N=1,NUMATF)
      GO TO 312
C
C     READ EXPANSION JOINT DATA
C
  404 CONTINUE
      DO 310 I=1,NUMATE
      READ 1,N,(PARAEX(J,N),J=1,4)
  310 CONTINUE
      WRITE(6,512)
      WRITE(6,112) (N,(PARAEX(J,N),J=1,4),N=1,NUMATE)
      GO TO 312
C
C     READ IN BOUNDARY ELEMENT DATA
C
  405 CONTINUE
      DO 311 I=1,NUMATB
      READ 2,N,(PARABO(J,N),J=1,2)
  311 CONTINUE
      WRITE(6,514)
      WRITE(6,114)(N,(PARABO(J,N),J=1,2),N=1,NUMATB)
  312 CONTINUE
      RETURN
C
C     FORMAT STATEMENT
C
    1 FORMAT(I10/(4E10.2))
    2 FORMAT(I10/(8F10.3))
    3 FORMAT(I10/(9F8.3))
  102 FORMAT(I2,8F10.3)
  103 FORMAT(I2,2F10.3/(2X,2F10.3))
  105 FORMAT(I2,5F10.3)
  107 FORMAT(1X,I2,3F10.1)
  108 FORMAT(I2,10X,F10.3,10X,F10.3,10X,F10.3/
     1I12X,F10.3,10X,F10.3,10X,F10.3))
  110 FORMAT(I2,4E10.2)
  112 FORMAT(I2,4E10.2)
  114 FORMAT(I2,2F10.3)
  501 FORMAT(* MATERIAL PROPERTY,BEAM AND COLUMN GEOMETRICAL PROPERTY DA
     *TA*///)
  502 FORMAT(* SOIL PROPERTY DATA*//
     1        * ELASTIC SHEAR MODULUS=G1    KSI*/
     2        * PLASTIC SHEAR MODULUS=G2    KSI*/
     3        * POISSON,S RATIO       =NU       */
     4        * COHESION OF CLAY       =C  LB/INCH*/
     5        * FRICTIONAL ANGLE       =PHI*/
     6        * YIELD SHEAR STRAIN    =GAMMAY*/
     7       * MASS DENSITY      =RHO LB-SEC.SQ/FT/CU.FT*/
     8        * UNIT WEIGHT OF SOIL  =WT LB/CU.FT*/
     92H N,8X,2HG1,8X,2HG2,8X,2HNU,9X,1HC,7X,3HPHI,5X,
     .5HGAMMA,7X,3HRHO,8X,2HWT//)
  503 FORMAT(* SHEAR MODULUS CURVE DATA *//
     1        * MATERIAL NO.=N*/
     2        * COEFFICIENT OF TANGENT SHEAR MODULUS=K2S*/
     3        * SHEAR STRAIN=GAMMAY */
     42H N,7X,3HK2S,4X,6HGAMMAY//)
  505 FORMAT(* CONCRETE PROPERTY DATA *//
     1        * ELASTIC MODULUS=E1  KSI*/
     2        * PLASTIC MODULUS=E2    KSI*/
     3        * POISSION,S RATIO=NU*/
     4       * MASS DENSITY=RHO LB-SEC.SQ/FT/CU.FT*/
     5        * UNIT WEIGHT=WT  LB/CU.FT*/
     62H N,8X,2HE1,8X,2HE2,8X,2HNU,7X,3HRHO,8X,2HWT//)
  507 FORMAT(* GEOMETRIC PROPERTY OF BEAN,COLUMN *//
     1        * GEOMETRIC PROPERTY NUMBER=N */
     2        * AXIAL AREA=A  SQ.INCHES*/
     3        * SHEAR AREA=AV  SQ. INCH*/
     4        * MOMENT OF INERTIA=I INCH.4*//
```

```
     52H N,9X,1HA,8X,2HAV,9X,1HI//)
 508 FORMAT(* AXIAL LOAD,MOMENT INTERACTION CURVE*//
     12H N,3X,12HECCENTRICITY,4X,16HAXIAL LOAD  KIPS,TX,
     213HMOMENT   K-IN//)
 510 FORMAT(* FRICTION ELEMENT DATA*//
     1         * SHEAR STIFF=KS  KSI*/
     2         * NORMAL STIFF =KN  KSI*/
     3         * FRICTION COEFFICIENT=U*/
     4         * ANGLE OF WALL TO VERTICAL AXIS=THETA*//
     52H N,8X,2HKS,8X,2HKN,9X,1HU,5X,5HTHETA//)
 512 FORMAT(* EXPANSION JOINT DATA *//
     1         * GAP AT JOINT=GAP  INCH*/
     2         * SEAT FOR ALLOWING SLIDING=SEAT  INCH*/
     3         * FRICTIONAL COEFFICIENT=U*//
     4         * LEFT OR RIGHT END INDICATOR=IND*/
     52H N,TX,3HGAP,6X,4HSEAT,9X,1HU,TX,3HIND//)
 514 FORMAT(* BOUNDARY ELEMENT DATA*//
     1         * ELASTIC MODULUS=EI  KSI*/
     2         * YIELD STRESS=FY  KSI*/
     32H N,8X,2HEI,8X,2HFY//)
      END
```

```
      SUBROUTINE ELDATA(ID,IX,IBC,MBAND,NUMEL)
C
C     ************************************************************
C     IDENTIFY THE ELEMENT NODAL POINT,ELEMENT TYPE,MATERIAL TYPE,
C     BEAM GEOMETRIC PROPERTY NO.,END RELEASE CODE
C     ************************************************************
C
      DIMENSION ID(3,1),IX(6,NUMEL),IBC(4,NUMEL),LM(12)
      DIMENSION IDE(10)
      WRITE(6,501)
      N=0
      MBAND=0
      DO 301 I=1,10
      IDE(I)=0
 301  CONTINUE
      DO 351 I=1,4
      DO 352 J=1,NUMEL
 352  IBC(I,J)=0
 351  CONTINUE
C
C     READ IN ELEMENT DATA
 1000 READ 1,M,(IDE(I),I=1,10)
 2000 N=N+1
      IF(M .EQ. N) GO TO 401
C
C     ELEMENT DATA GENERATEION NEEDED
      DO 303 I=1,4
      IX(I,N)=IX(I,N-1)+1
      NODE=IX(I,N)
      JJJ=3*I
      JJ=JJJ-1
      J=JJ-1
      LM(JJJ)=ID(3,NODE)
      LM(JJ)=ID(2,NODE)
      LM(J)=ID(1,NODE)
 303  CONTINUE
      MAX=0
      MIN=10000
      DO 355 L=1,12
      IF(LM(L) .EQ. 0) GO TO 355
      IF(LM(L) .GT. MAX) MAX=LM(L)
      IF(LM(L) .LT. MIN) MIN=LM(L)
 355  CONTINUE
      IF(MAX .EQ. 0) MIN=1
      MB=MAX-MIN+1
      IF(MB .GT. MBAND) MBAND=MB
      IX(5,N)=IX(5,N-1)
      IX(6,N)=IX(6,N-1)
      IF(IDE(T) .NE. 0) GO TO 406
      GO TO 402
 406  CONTINUE
      DO 306 I=1,4
      IBC(I,N)=IBC(I,N-1)
 306  CONTINUE
      GO TO 403
 401  DO 304 I=1,6
      IX(I,N)=IDE(I)
 304  CONTINUE
      DO 354 I=1,4
      NODE=IX(I,N)
      JJJ=3*I
      JJ=JJJ-1
      J=JJ-1
      LM(JJJ)=ID(3,NODE)
      LM(JJ)=ID(2,NODE)
      LM(J)=ID(1,NODE)
 354  CONTINUE
      MIN=10000
      MAX=0
      DO 353 L=1,12
      IF(LM(L) .EQ. 0) GO TO 353
      IF(LM(L) .GT. MAX) MAX=LM(L)
      IF(LM(L) .LT. MIN) MIN=LM(L)
 353  CONTINUE
      IF(MAX .EQ. 0) MIN=1
      MB=MAX-MIN+1
      IF(MB .GT. MBAND) MBAND=MB
C
C     NOT BEAM OR COLUMN
      IF(IDE(T) .EQ. 0) GO TO 402
C
C     BEAM OR COLUMN ELEMENT
      DO 305 I=1,4
      IBC(I,N)=IDE(I+6)
 305  CONTINUE
      GO TO 403
 402  CONTINUE
      WRITE(6,101) N,(IX(I,N),I=1,6),MB
      GO TO 404
 403  CONTINUE
      WRITE(6,102) N,(IX(I,N),I=1,6),(IBC(J,N),J=1,4),MB
 404  IF(N .EQ. NUMEL) GO TO 405
      IF(N .EQ. M) GO TO 1000
      GO TO 2000
 405  CONTINUE
      RETURN
    1 FORMAT(11I5)
  101 FORMAT(9X,I3,4I4,9X,I4,9X,I4,59X,I4)
  102 FORMAT(9X,I3,4I4,9X,I4,9X,I4,17X,I4,12X,2I4,TX,I2,9X,I4)
  501 FORMAT(54H1ELEMENT NO.   I   J   K   L ELEMENT TYPE MATERIAL NO.,3
     1X,60HGEOMETRIC PROPERTY   END RELEASE I   J   MINTCV   BAND WIDTH)
      END
```

```
      SUBROUTINE LOAD(ID,X,Y,PARASO,PARACO,COPROP,IX,IBC,
     1SLV,DLX,DLY,TMASS,TLOAD,NUMEL,NEQ)
C
C     ******************************************************************
C     CALCULATE STATIC LOAD VECTOR-SLV,
C     DYNAMIC LOAD IN X DIRECTION-DLX
C     DYNAMIC LOAD VECTOR IN Y DIRECTION-DLY
C     ******************************************************************
C
      DIMENSION ID(3,1),X(1),Y(1),PARASO(8,1),PARACO(5,1),COPROP(3,1),
     1IX(6,1),IBC(4,1)
      DIMENSION XX(4),YY(4),LN(8)
      DIMENSION SLV(NEQ),DLX(NEQ),DLY(NEQ),TMASS(NEQ),TLOAD(NEQ)
C
C     INITIZATION
      DO 301 I=1,NEQ
      SLV(I)=0.0
      DLX(I)=0.0
      DLY(I)=0.0
  301 CONTINUE
C
C     CALCULATE LOAD ELEMENT BY ELEMENT
      DO 302 N=1,NUMEL
C
C     DETERMINE WHICH ELEMENT TYPE
      NTYPE=IX(5,N)
      GO TO (401,402,403,403,403),NTYPE
C
C     TYPE 1,SOIL ELEMENT
  401 CONTINUE
      DO 303 J=1,4
      NODE=IX(J,N)
      JJ=J+J
      LN(JJ)=ID(2,NODE)
      LN(JJ-1)=ID(1,NODE)
      XX(J)=X(NODE)
      YY(J)=Y(NODE)
  303 CONTINUE
      MATYPE=IX(6,N)
      CALL FSOIL(XX,YY,AMASS,WT,PARASO(1,MATYPE),IX(1,N))
      DO 304 J=1,4
      JJ=J+J
      IT=LN(JJ)
      ITT=LN(JJ-1)
      IF(IT .EQ. 0) GO TO 404
      SLV(IT)=-WT+SLV(IT)
      DLX(IT)=0.0+DLX(IT)
      DLY(IT)=AMASS+DLY(IT)
  404 CONTINUE
      IF(ITT .EQ. 0) GO TO 304
      SLV(ITT)=0.0+SLV(ITT)
      DLX(ITT)=AMASS+DLX(ITT)
      DLY(ITT)=0.0+DLY(ITT)
  304 CONTINUE
      GO TO 403
C
C     TYPE 2,CONCRETE ELEMENT
  402 CONTINUE
      DO 305 J=1,2
      JJ=3*J
      NODE=IX(J,N)
      LN(JJ)=ID(3,NODE)
      LN(JJ-1)=ID(2,NODE)
      LN(JJ-2)=ID(1,NODE)
      XX(J)=X(NODE)
      YY(J)=Y(NODE)
  305 CONTINUE
      MATYPE=IX(6,N)
      NG=IBC(1,N)
      AXA=COPROP(1,NG)
      WT=AXA*PARACO(5,MATYPE)/144.0
      AMASS=AXA*PARACO(4,MATYPE)/144.0
      CALL FBECOL(XX,YY,AMASS,WT,FEN)
      DO 306 J=1,2
      JJ=3*J
      IT=LM(JJ)
      ITT=LM(JJ-1)
      ITTT=LM(JJ-2)
      IF(IT .EQ. 0) GO TO 405
      SLV(IT)=(-1)**(J)*FEN+SLV(IT)
      DLY(IT)=0.0+DLY(IT)
      DLX(IT)=0.0+DLX(IT)
  405 CONTINUE
      IF(ITT .EQ. 0) GO TO 406
      SLV(ITT)=-WT+SLV(ITT)
      DLX(ITT)=0.0+DLX(ITT)
      DLY(ITT)=AMASS+DLY(ITT)
  406 CONTINUE
      IF(ITTT .EQ. 0) GO TO 306
      SLV(ITTT)=0.0+SLV(ITTT)
      DLX(ITTT)=AMASS+DLX(ITTT)
      DLY(ITTT)=0.0+DLY(ITTT)
  306 CONTINUE
  403 CONTINUE
  302 CONTINUE
C
C     ADD UP CONCENTRATED MASS,OR/AND, LOAD
      G=386.07
      CALL INCML(ID,TMASS,TLOAD)
      DO 307 I=1,NEQ
      SLV(I)=SLV(I)+TMASS(I)*G+TLOAD(I)
      DLX(I)=DLX(I)+TMASS(I)
      DLY(I)=DLY(I)+TMASS(I)
  307 CONTINUE
      WRITE(6,501)
      WRITE(6,101)(SLV(I),DLX(I),DLY(I),I=1,NEQ)
C
C     PUT MASS MATRIX INTO TAPE4,KIP,INCH,SEC
      DO 308 I=1,NEQ
  308 TMASS(I)=DLX(I)+DLY(I)
      WRITE(4) TMASS

      RETURN
  101 FORMAT(F10.3,2F10.6)
  501 FORMAT(* LOAD VECTOR *//
     1*        SLV*,*        DLX*,*        DLY*)
      END
```

```
      SUBROUTINE FSOIL(XX,YY,AMASS,WT,FARASO,IX)
C
C     ******************************************************************
C     CALCULATE LUMF MASS OF SOIL ELEMENT
C     ******************************************************************
C
      COMMON/MATER/NUMATS,NUMATC,NUMATF,NUMATE,NUMATB,NUNGE,NINTCV,THICK
     1,NPLAN
      DIMENSION PARASO(8),XX(4),YY(4),IX(6)
      Y24=YY(2)-YY(4)
      Y31=YY(3)-YY(1)
      Y42=-Y24
      Y13=-Y31
      AREA=XX(1)*Y24+XX(2)*Y31+XX(3)*Y42+XX(4)*Y13
      AREA=AREA/2.0
C
C     TOTAL MASS,AND WEIGHT
      RHO=FARASO(7)
      W=PARASO(8)
      TMASS=RHO*AREA*THICK
      WT=W*AREA*THICK
C
C     LUMPED MASS,WEIGHT AT NODE,FOR QUADLATERAL ELEMENT
      IF(IX(3) .EQ. IX(4)) GO TO 401
      AMASS=TMASS/48000.0
      WT=WT/4000.0
      GO TO 402
  401 CONTINUE
C
C     TRIANGULAR ELEMENT
      AMASS=TMASS/36000.0
      WT=WT/3000.0
  402 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FBECOL(XX,YY,AMASS,WT,FEN)
C
C     ******************************************************************
C     CALCULATE THE DEAD LOAD AND F.E.N. OF CONCRETE
C     ******************************************************************
C
      DIMENSION XX(2),YY(2)
C
C     LENGTH=BL
      HL=XX(2)-XX(1)
      VL=YY(2)-YY(1)
      BL=SQRT(HL**2+VL**2)
      BL=ABS(BL)
       H=ABS(HL)
      FEN=HL**2*WT/12.0
      WT=BL*WT/2.0
      AMASS=BL*AMASS/2.0
      FEN=FEN*12.0/1000.0
      WT=WT/1000.0
      AMASS=AMASS/12000.0
      RETURN
      END
```

```
      SUBROUTINE INCNL(IO,TMASS,TLOAD)
C
C     ******************************************************************
C     INPUT CONCENTRATED MASS OR LOAD
C     ******************************************************************
C
      DIMENSION IO(3,1),TMASS(1),TLOAD(1),RN(3),RL(3)
      COMMON/ELPAR/NUMNP,NUMEL,NEQ,NBAND,KLIN
      WRITE(6,501)
      DO 301 I=1,NEQ
      TMASS(I)=0.0
      TLOAD(I)=0.0
  301 CONTINUE
C
C     INPUT JOINT MASS AND LOAD
      DO 302 NN=1,NUMNP
      READ 1,N,(RN(J),J=1,3),(RL(J),J=1,3)
      IF(N .EQ. 0) GO TO 304
      WRITE(6,101) N,RN,RL
      IF( N .GT. NUMNF) GO TO 304
      DO 303 J=1,3
      II=IO(J,N)
      IF(II .EQ. 0) GO TO 303
      TMASS(II)=RN(J)
      TLOAD(II)=RL(J)
      TMASS(II)=TMASS(II)/12000.0
      TLOAD(II)=TLOAD(II)/1000.0
  303 CONTINUE
  302 CONTINUE
  304 CONTINUE
      RETURN
    1 FORMAT(I10,6F10.3)
  101 FORMAT(I5,6F10.3)
  501 FORMAT(* INPUT DATA OF CONCENTRATED MASS AND LOAD IN LB,FT,S*//
     15H NODE,4X,6HMASS-X,4X,6HMASS-Y,3X,7HMASS-ZZ,
     26HLOAD-X,4X,6HLOAD-Y,3X,THLOAD-ZZ//)
      END
```

```
      SUBROUTINE ESGEPS(X,Y,PARASO,PARACO,PARAFR,PARAEX,PARABO,COPROP,
     1                  IX,IBC,C,BCL,D1,D2,D3,D4,D5,NUMATS,NUMATC,
     2                  NUMATF,NUMATE,NUMATB,NUNGE,NPLAN)
C
C     ******************************************************************
C     CALCULATE ELASTIC STRESS-STRAIN,OR ELEMENT FORCE-DEFORMATION RELATE
C     ******************************************************************
C
      DIMENSION PARASO(8,1),PARACO(5,1),PARAFR(4,1),PARAEX(4,1),
     1          PARABO(2,1),COPROP(3,1),IX(6,1),IBC(4,1),C(5,1),
     2          D1(5,1),D2(5,NUMATC,1),D3(5,1),D4(5,1),D5(5,1),
     3          X(1),Y(1),BCL(1),XX(2),YY(2)
      COMMON/ELPAR/NUMNP,NUMEL,NEQ,NBAND,KLIN
C
C     INITILIZATION
      DO 301 I=1,5
      DO 302 J=1,NUMEL
      C(I,J)=0.0
  302 CONTINUE
      DO 303 J=1,NUMATS
      D1(I,J)=0.0
  303 CONTINUE
      DO 304 J=1,NUMATC
      DO 305 K=1,NUNGE
      D2(I,J,K)=0.0
  305 CONTINUE
  304 CONTINUE
      DO 306 J=1,NUMATF
      D3(I,J)=0.0
  306 CONTINUF
      DO 307 J=1,NUMATE
```

```
      D4(I,J)=0.0                                                       ESGEPS.33
  307 CONTINUE                                                          ESGEPS.34
      DO 308 J=1,NUMATB                                                 ESGEPS.35
      D5(I,J)=0.0                                                       ESGEPS.36
  308 CONTINUE                                                          ESGEPS.37
  301 CONTINUE                                                          ESGEPS.38
C                                                                       ESGEPS.39
C     FORM BASIC MATERIAL CONSTANTS                                     ESGEPS.40
C     CHECK PLANE STRESS OR STRAIN                                      ESGEPS.41
C     SOIL STRESS-STRAIN RELATIONSHIP                                   ESGEPS.42
      IF(NUMATS .EQ. 0) GO TO 407                                       ESGEPS.43
      IF(NPLAN .EQ. 1) GO TO 406                                        ESGEPS.44
C                                                                       ESGEPS.45
C     PLANE STRAIN                                                      ESGEPS.46
      DO 309 I=1,NUMATS                                                 ESGEPS.47
      G=PARASO(1,I)                                                     ESGEPS.48
      ANU=PARASO(3,I)                                                   ESGEPS.49
      B1=2.0*(1.0-ANU)                                                  ESGEPS.50
      B2=1.0-2.0*ANU                                                    ESGEPS.51
      D1(1,I)=G*B1/B2                                                   ESGEPS.52
      D1(2,I)=G*2.0*ANU/B2                                              ESGEPS.53
      D1(3,I)=G                                                         ESGEPS.54
  309 CONTINUE                                                          ESGEPS.55
      GO TO 407                                                         ESGEPS.56
  406 CONTINUE                                                          ESGEPS.57
C                                                                       ESGEPS.58
C     PLANE STRESS                                                      ESGEPS.59
      IF(NUMATS .EQ. 0) GO TO 407                                       ESGEPS.60
      DO 310 I=1,NUMATS                                                 ESGEPS.61
      G=PARASO(1,I)                                                     ESGEPS.62
      ANU=PARASO(3,I)                                                   ESGEPS.63
      B1=1.0-ANU                                                        ESGEPS.64
      D1(1,I)=G*2.0/B1                                                  ESGEPS.65
      D1(2,I)=G*2.0*ANU/B1                                              ESGEPS.66
      D1(3,I)=G                                                         ESGEPS.67
  310 CONTINUE                                                          ESGEPS.68
  407 CONTINUE                                                          ESGEPS.69
C                                                                       ESGEPS.70
C     CONCRETE END FORCES-DEFORMATION RELATIONSHIP                      ESGEPS.71
      IF(NUMATC .EQ. 0) GO TO 409                                       ESGEPS.72
C     CALCULATE BEAM AND COLUMN LENGTH ACCORDING TO GEOMETRIC NO.       ESGEPS.73
      DO 317 I=1,NUMGE                                                  ESGEPS.74
      BCL(I)=0.0                                                        ESGEPS.75
  317 CONTINUE                                                          ESGEPS.76
      DO 318 N=1,NUMEL                                                  ESGEPS.77
      NG=IBC(1,N)                                                       ESGEPS.78
      IF(NG .EQ. 0) GO TO 318                                           ESGEPS.79
      DO 319 I=1,2                                                      ESGEPS.80
      NODE=IX(I,N)                                                      ESGEPS.81
      XX(I)=X(NODE)                                                     ESGEPS.82
      YY(I)=Y(NODE)                                                     ESGEPS.83
  319 CONTINUE                                                          ESGEPS.84
      HL=XX(2)-XX(1)                                                    ESGEPS.85
      VL=YY(2)-YY(1)                                                    ESGEPS.86
      BL=SQRT(HL**2+VL**2)                                              ESGEPS.87
      BCL(NG)=BL*12.0                                                   ESGEPS.88
  318 CONTINUE                                                          ESGEPS.89
      DO 311 I=1,NUMATC                                                 ESGEPS.90
      E=PARACO(1,I)                                                     ESGEPS.91
      ANU=PARACO(3,I)                                                   ESGEPS.92
      G=E/(2.0*(1.0+ANU))                                               ESGEPS.93
      DO 312 J=1,NUMGE                                                  ESGEPS.94
      AA=COPROP(1,J)                                                    ESGEPS.95
      AV=COPROP(2,J)                                                    ESGEPS.96
      AI=COPROP(3,J)                                                    ESGEPS.97
      BE=12.0*E*AI                                                      ESGEPS.98
      BET=BE/(AV*G)                                                     ESGEPS.99
      BETA=BET/BCL(J)**2.0                                              ESGEPS.100
      D2(1,I,J)=E*AA/BCL(J)                                             ESGEPS.101
      D2(2,I,J)=BE/(BCL(J)**3.0*(1.0+BETA))                             ESGEPS.102
      D2(3,I,J)=(4.0+BETA)/(BCL(J)*(1.0+BETA))                          ESGEPS.103
      D2(3,I,J)=D2(3,I,J)*E*AI                                          ESGEPS.104
      D2(4,I,J)=BE/(2.0*BCL(J)**2.0*(1.0+BETA))                         ESGEPS.105
      D2(5,I,J)=(2.0-BETA)*E*AI/(BCL(J)*(1.0+BETA))                     ESGEPS.106
  312 CONTINUE                                                          ESGEPS.107
  311 CONTINUE                                                          ESGEPS.108
C                                                                       ESGEPS.109
C     FRICTIONAL ELEMENT NODAL FORCE-DISPLACEMENT RELATION IN LOCAL COORESGEPS.110
  409 CONTINUE                                                          ESGEPS.111
      IF(NUMATF .EQ. 0) GO TO 410                                       ESGEPS.112
      DO 313 I=1,NUMATF                                                 ESGEPS.113
      AKS=PARAFR(1,I)                                                   ESGEPS.114
      AKN=PARAFR(2,I)                                                   ESGEPS.115
      D3(1,I)=AKS/3.0                                                   ESGEPS.116
      D3(2,I)=AKN/3.0                                                   ESGEPS.117
      D3(3,I)=AKS/6.0                                                   ESGEPS.118
      D3(4,I)=AKN/6.0                                                   ESGEPS.119
  313 CONTINUE                                                          ESGEPS.120
  410 CONTINUE                                                          ESGEPS.121
C                                                                       ESGEPS.122
C     EXPANSION JOINT NODAL FORCE-DISPLACEMENT RELATIONSHIP IN LOCAL COOESGEPS.123
      IF(NUMATE .EQ. 0) GO TO 411                                       ESGEPS.124
      AKS=3000000.0                                                     ESGEPS.125
      AKN=3000000.0                                                     ESGEPS.126
      DO 314 I=1,NUMATE                                                 ESGEPS.127
      D4(1,I)=AKS/3.0                                                   ESGEPS.128
      D4(2,I)=AKN/3.0                                                   ESGEPS.129
      D4(3,I)=AKS/6.0                                                   ESGEPS.130
      D4(4,I)=AKN/6.0                                                   ESGEPS.131
  314 CONTINUE                                                          ESGEPS.132
  411 CONTINUE                                                          ESGEPS.133
C                                                                       ESGEPS.134
C     BOUNDARY ELEMENT FORCE-DISPLACEMENT RELATIONSHIP                  ESGEPS.135
      IF(NUMATB .EQ. 0) GO TO 408                                       ESGEPS.136
      DO 315 I=1,NUMATB                                                 ESGEPS.137
      E=PARABO(1,I)                                                     ESGEPS.138
      D5(1,I)=E                                                         ESGEPS.139
  315 CONTINUE                                                          ESGEPS.140
C                                                                       ESGEPS.141
C     FORM STRESS-STRAIN MATRIX--C(5,N),ELEMENT BY ELEMENT              ESGEPS.142
  408 CONTINUE                                                          ESGEPS.143
      DO 316 N=1,NUMEL                                                  ESGEPS.144
C                                                                       ESGEPS.145
C     DETERMINE WHICH ELEMENT TYPE                                      ESGEPS.146
      NTYPE=IX(5,N)                                                     ESGEPS.147
      GO TO (401,402,403,404,405),NTYPE                                 ESGEPS.148
  401 CONTINUE                                                          ESGEPS.149
C                                                                       ESGEPS.150
C     DETERMINE MATERIAL NUMBER                                         ESGEPS.151
      MAT=IX(6,N)                                                       ESGEPS.152
      C(1,N)=D1(1,MAT)                                                  ESGEPS.153
      C(2,N)=D1(2,MAT)                                                  ESGEPS.154
      C(3,N)=D1(3,MAT)                                                  ESGEPS.155
      GO TO 316                                                         ESGEPS.156
C                                                                       ESGEPS.157
C     CONCRETE ELEMENT                                                  ESGEPS.158
C     DETERMINE MATERIAL NO. AND GEOMETRIC NO.                          ESGEPS.159
  402 CONTINUE                                                          ESGEPS.160
      MAT=IX(6,N)                                                       ESGEPS.161
      NG=IBC(1,N)                                                       ESGEPS.162
      C(1,N)=D2(1,MAT,NG)                                               ESGEPS.163
      C(2,N)=D2(2,MAT,NG)                                               ESGEPS.164
      C(3,N)=D2(3,MAT,NG)                                               ESGEPS.165
      C(4,N)=D2(4,MAT,NG)                                               ESGEPS.166
      C(5,N)=D2(5,MAT,NG)                                               ESGEPS.167
      GO TO 316                                                         ESGEPS.168
C                                                                       ESGEPS.169
C     FRICTIONAL ELEMENT                                                ESGEPS.170
  403 CONTINUE                                                          ESGEPS.171
      MAT=IX(6,N)                                                       ESGEPS.172
      C(1,N)=D3(1,MAT)                                                  ESGEPS.173
      C(2,N)=D3(2,MAT)                                                  ESGEPS.174
      C(3,N)=D3(3,MAT)                                                  ESGEPS.175
      C(4,N)=D3(4,MAT)                                                  ESGEPS.176
      GO TO 316                                                         ESGEPS.177
C     EXPANSION JOINT ELENANT                                           ESGEPS.178
  404 CONTINUE                                                          ESGEPS.179
      MAT=IX(6,N)                                                       ESGEPS.180
      C(1,N)=D4(1,MAT)                                                  ESGEPS.181
      C(2,N)=D4(2,MAT)                                                  ESGEPS.182
      C(3,N)=D4(3,MAT)                                                  ESGEPS.183
      C(4,N)=D4(4,MAT)                                                  ESGEPS.184
      GO TO 316                                                         ESGEPS.185
C     BOUNDARY SPRING                                                   ESGEPS.186
                                                                        ESGEPS.187
  405 CONTINUE                                                          ESGEPS.188
      MAT=IX(6,N)                                                       ESGEPS.189
      C(1,N)=D5(1,MAT)                                                  ESGEPS.190
  316 CONTINUE                                                          ESGEPS.191
      RETURN                                                            ESGEPS.192
      END                                                               ESGEPS.193
```

```
      SUBROUTINE ETSTIF(ID,X,Y,PARACO,COPROP,IX,IBC,SLV,C,AE,NIND,NUMEL,ETSTIF.2
     1                  NEQ,NBAND)                                      ETSTIF.3
C                                                                       ETSTIF.4
C     ******************************************************************ETSTIF.5
C     FORM ELASTIC TOTAL STIFFNESS MATRIX AE(NEQ,NBAND)                 ETSTIF.6
C     ******************************************************************ETSTIF.7
C                                                                       ETSTIF.8
      DIMENSION ID(3,1),X(1),Y(1),PARACO(5,1),COPROP(3,1),IX(6,1),      ETSTIF.9
     1          IBC(4,1),SLV(1),C(5,1),AE(NEQ,1),XX(4),YY(4),NIND(1),   ETSTIF.10
     2          SY(6,6)                                                 ETSTIF.11
      COMMON/EM/LM(8),ASA(8,8)                                          ETSTIF.12
      COMMON/NONL/NREAD                                                 ETSTIF.13
C                                                                       ETSTIF.14
C     FORM ELEMENT STIFF MATRIX,ONE BY ONE,AND STORE ON TAPE 1          ETSTIF.15
C                                                                       ETSTIF.16
C     INITILIZATION                                                     ETSTIF.17
      NREAD=0                                                           ETSTIF.18
      DO 351 I=1,NEQ                                                    ETSTIF.19
      DO 351 J=1,NBAND                                                  ETSTIF.20
  351 AE(I,J)=0.0                                                       ETSTIF.21
      DO 352 I=1,6                                                      ETSTIF.22
      DO 352 J=1,6                                                      ETSTIF.23
  352 SY(I,J)=0.0                                                       ETSTIF.24
      DO 353 I=1,NUMEL                                                  ETSTIF.25
  353 NIND(I)=0                                                         ETSTIF.26
      DO 301 N=1,NUMEL                                                  ETSTIF.27
      NTYPE=IX(5,N)                                                     ETSTIF.28
C                                                                       ETSTIF.29
C     DETERMINE THE ELEMENT TYPE                                        ETSTIF.30
      GO TO (401,402,403,403,405),NTYPE                                 ETSTIF.31
C                                                                       ETSTIF.32
C     TYPE 1,SOIL ELEMENT                                               ETSTIF.33
  401 CONTINUE                                                          ETSTIF.34
C                                                                       ETSTIF.35
C     LOCATION OF MASS                                                  ETSTIF.36
      CALL LAM(ID,X,Y,IX,XX,YY,N)                                       ETSTIF.37
      CALL QUADS(XX,YY,C(1,N),SY,NIND(N))                               ETSTIF.38
C                                                                       ETSTIF.39
C     STORE ON TOTAL STIFFNESS MATRIX AE                                ETSTIF.40
      CALL STORE(AE,NEQ)                                                ETSTIF.41
      GO TO 406                                                         ETSTIF.42
C                                                                       ETSTIF.43
C     TYPE 2,BEAM,COLUMN ELEMENT                                        ETSTIF.44
  402 CONTINUE                                                          ETSTIF.45
C                                                                       ETSTIF.46
C     LOCATION OF MASS                                                  ETSTIF.47
      CALL LAM(ID,X,Y,IX,XX,YY,N)                                       ETSTIF.48
      CALL CONCTS(XX,YY,PARACO,COPROP,IX(1,N),IBC(1,N),C(1,N),SLV,SY,   ETSTIF.49
     1            NIND(N))                                              ETSTIF.50
C     STORE ON TOTAL STIFFNESS MATRIX AE                                ETSTIF.51
      CALL STORE(AE,NEQ)                                                ETSTIF.52
      GO TO 406                                                         ETSTIF.53
C                                                                       ETSTIF.54
C     TYPE 3,FRICTIONAL ELEMENT STIFFNESS                               ETSTIF.55
C     TYPE 4,EXPANSION JOINT                                            ETSTIF.56
  403 CONTINUE                                                          ETSTIF.57
C                                                                       ETSTIF.58
C     LOCATION OF MASS                                                  ETSTIF.59
      CALL LAM(ID,X,Y,IX,XX,YY,N)                                       ETSTIF.60
C                                                                       ETSTIF.61
      CALL FRICTS(XX,YY,C(1,N))                                         ETSTIF.62
C     STORE ON TOTAL STIFFNESS MATRIX AE                                ETSTIF.63
      CALL STORE(AE,NEQ)                                                ETSTIF.64
      GO TO 406                                                         ETSTIF.65
C                                                                       ETSTIF.66
C     TYPE 5,BOUNDARY ELEMENT                                           ETSTIF.67
  405 CONTINUE                                                          ETSTIF.68
      DO 314 I=1,2                                                      ETSTIF.69
      NODE=IX(I,N)                                                      ETSTIF.70
      II=ID(1,NODE)                                                     ETSTIF.71
      LM(I)=II                                                          ETSTIF.72
  314 CONTINUE                                                          ETSTIF.73
      CALL BOUNDS(C(1,N))                                               ETSTIF.74
C     STORE ON TOTAL STIFFNESS MATRIX AE                                ETSTIF.75
      DO 315 I=1,2                                                      ETSTIF.76
      II=LM(I)                                                          ETSTIF.77
      IF(II .EQ. 0) GO TO 315                                           ETSTIF.78
      DO 316 J=1,2                                                      ETSTIF.79
      JJ=LM(J)-II+1                                                     ETSTIF.80
      IF(JJ .LT. 1) GO TO 316                                           ETSTIF.81
      AE(II,JJ)=AE(II,JJ)+ASA(I,J)                                      ETSTIF.82
  316 CONTINUE                                                          ETSTIF.83
  315 CONTINUE                                                          ETSTIF.84
  406 CONTINUE                                                          ETSTIF.85
  301 CONTINUE                                                          ETSTIF.86
C     FORM ELASTIC TOTAL STIFFNESS AE(NEQ,NBAND),AND STORE ON TAPE 3    ETSTIF.87
      WRITE(3) ((AE(I,J),I=1,NEQ),J=1,NBAND)                            ETSTIF.88
      RETURN                                                            ETSTIF.89
      END                                                               ETSTIF.90
```

```
      SUBROUTINE LAM(ID,X,Y,IX,XX,YY,N)                                 LAM.2
C     ******************************************************************LAM.3
C     CALCULATE LOCATION OF MASS--LM                                    LAM.4
C     ******************************************************************LAM.5
      COMMON/EM/LM(8),ASA(8,8)                                          LAM.6
      DIMENSION ID(3,1),X(1),Y(1),IX(6,1),XX(4),YY(4)                   LAM.7
      I2=IX(2,N)                                                        LAM.8
      I3=IX(3,N)                                                        LAM.9
      IF(I2 .EQ. I3) GO TO 401                                          LAM.10
C                                                                       LAM.11
C     QUADRATERAL ELEMENT--SOIL,FRICTON,EXPANSION JOINT ELEMENT         LAM.12
      DO 301 I=1,4                                                      LAM.13
      NODE=IX(I,N)                                                      LAM.14
      JJ=I+I                                                            LAM.15
      LM(JJ)=ID(2,NODE)                                                 LAM.16
      LM(JJ-1)=ID(1,NODE)                                               LAM.17
      XX(I)=X(NODE)                                                     LAM.18
      YY(I)=Y(NODE)                                                     LAM.19
  301 CONTINUE                                                          LAM.20
      GO TO 402                                                         LAM.21
  401 CONTINUE                                                          LAM.22
C     ONE DIMENSIONAL ELEMENT --BEAM,COLUMN                             LAM.23
C                                                                       LAM.24
      LM(7)=0                                                           LAM.25
      LM(8)=0                                                           LAM.26
      DO 302 I=1,2                                                      LAM.27
      NODE=IX(I,N)                                                      LAM.28
      JJ=3*I                                                            LAM.29
      LM(JJ)=ID(3,NODE)                                                 LAM.30
      LM(JJ-1)=ID(2,NODE)                                               LAM.31
      LM(JJ-2)=ID(1,NODE)                                               LAM.32
      XX(I)=X(NODE)                                                     LAM.33
      YY(I)=Y(NODE)                                                     LAM.34
  302 CONTINUE                                                          LAM.35
  402 CONTINUE                                                          LAM.36
      RETURN                                                            LAM.37
      END                                                               LAM.38
```

```
      SUBROUTINE STORE(AE,NEQ)
C     ******************************************************************
C     STORE ELEMENT STIFFNESS INTO TOTAL STIFFNESS MATRIX AE(NEQ,MBAND)
C     ******************************************************************
      COMMON/EM/LM(8),ASA(8,8)
      DIMENSION AE(NEQ,1)
      DO 301 I=1,8
      II=LM(I)
      IF(II .EQ. 0) GO TO 301
      DO 302 J=1,8
      JJ=LM(J)-II+1
      IF(JJ .LT. 1) GO TO 302
      AE(II,JJ)=AE(II,JJ)+ASA(I,J)
  302 CONTINUE
  301 CONTINUE
      RETURN
      END



      SUBROUTINE QUADS(XX,YY,E,SY,IND)
C
C     ******************************************************************
C     FORM SOIL ELEMENT STIFFNESS MATRIX,AND STORE ON TAPE 1
C     ******************************************************************
C
      DIMENSION XX(4),YY(4),E(5),SS(4),TT(4),H(4),HS(4),HT(4),
     1          HX(4),HY(4)
      DIMENSION B(3,8),S(6,6),SA(3,8),SY(6,6)
      COMMON/EM/LM(8),ASA(8,8)
      COMMON/MONL/MREAD
      COMMON/MATER/NUMATS,NUMATC,NUMATF,NUMATE,NUMATB,MUMGE,NIMTCV,THICK
     1,NPLAN
      COMMON/ELASTC/SE(8,8)
        COMMON/TIME/JUMP,T,DT,MPRTM,MTAPE,KPRINT
      DATA SS/-1.0,1.0,1.0,-1.0/, TT/-1.0,-1.0,1.0,1.0/
C
C     INITIALIZATION
      DO 301 I=1,8
      DO 302 J=1,8
  302 ASA(I,J)=0.0
  301 CONTINUE
      DO 303 I=1,3
      DO 304 J=1,8
  304 B(I,J)=0.0
  303 CONTINUE
C
C
      DO 305 II=1,4
      SC=SS(II)*0.57735026918963
      TC=TT(II)*0.57735026918963
      SM=1.0-SC
      SP=1.0+SC
      TM=1.0-TC
      TP=1.0+TC
C
C     FORM INTERPOLATION FCTNS
      H(1)=SM*TM/4.0
      H(2)=SP*TM/4.0
      H(3)=SP*TP/4.0
      H(4)=SM*TP/4.0
C
C     DERIVATIVE OF H W.R.T. S,D(H(I))/DS=HS(I)
      HS(1)=-TM/4.0
      HS(2)=-HS(1)
      HS(3)=TP/4.0
      HS(4)=-HS(3)
C
C     DERIVATIVE W.R.T. T,D(H(I))/DT=HT(I)
      HT(1)=-SM/4.0
      HT(2)=-SP/4.0
      HT(3)=-HT(2)
      HT(4)=-HT(1)
C
C     FORM PARTIAL DERIVATIVE OF X,Y W.R.T. S,T
C     PYT=DY/DT,PYS=DY/DS,PXS=DX/DS,PXT=DX/DT
C     XJ=JACOBIAN=(DX/DS)*(DY/DT)-(DX/DT)*(DY/DS)
      PYT=HT(1)*YY(1)+HT(2)*YY(2)+HT(3)*YY(3)+HT(4)*YY(4)
      PYS=HS(1)*YY(1)+HS(2)*YY(2)+HS(3)*YY(3)+HS(4)*YY(4)
      PXT=HT(1)*XX(1)+HT(2)*XX(2)+HT(3)*XX(3)+HT(4)*XX(4)
      PXS=HS(1)*XX(1)+HS(2)*XX(2)+HS(3)*XX(3)+HS(4)*XX(4)
      XJ=PXS*PYT-PXT*PYS
C
C     FORM D(H(I))/DX=HX(I) AND D(H(I)/DY)=HY(I) IN STRAIN-DISPLACEMENT
      PSX=PYT/XJ
      PTX=-PYS/XJ
      PSY=-PXT/XJ
      PTY=PXS/XJ
      DO 306 I=1,4
      HX(I)=PSX*HS(I)+PTX*HT(I)
      HY(I)=PSY*HS(I)+PTY*HT(I)
      HX(I)=HX(I)/12.0
      HY(I)=HY(I)/12.0
  306 CONTINUE
C
C     FORM STRAIN-DISPLACEMENT MATRIX B(3,8)
      B(1,1)=HX(1)
      B(1,3)=HX(2)
      B(1,5)=HX(3)
      B(1,7)=HX(4)
      B(2,2)=HY(1)
      B(2,4)=HY(2)
      B(2,6)=HY(3)
      B(2,8)=HY(4)
C
      B(3,1)=B(2,2)
      B(3,2)=B(1,1)
      B(3,3)=B(2,4)
      B(3,4)=B(1,3)
      B(3,5)=B(2,6)
      B(3,6)=B(1,5)
      B(3,7)=B(2,8)
      B(3,8)=B(1,7)
C
      IF(IND .EQ. 1) GO TO 401
C     ELASTIC CASE
C     STRESS-STRAIN MATRIX S(3,3)
      S(1,1)=E(1)
      S(1,2)=E(2)
      S(1,3)=0.0
      S(2,1)=E(2)
      S(2,2)=E(1)
      S(2,3)=0.0
      S(3,1)=0.0
      S(3,2)=0.0
      S(3,3)=E(3)
  403 CONTINUE
      GO TO 402
  401 CONTINUE
C
C     PLASTIC STRESS-STRAIN MATRIX S(3,3)
      DO 309 I=1,3
      DO 309 J=1,3
  309 S(I,J)=SY(I,J)
  402 CONTINUE
C
C     FORM ELEMENT STIFFNESS MATRIX ASA(I,J)
      FAC=THICK*XJ*1728.0
      DO 307 J=1,8
      D1=FAC*(S(1,1)*B(1,J)+S(1,2)*B(2,J)+S(1,3)*B(3,J))
      D2=FAC*(S(2,1)*B(1,J)+S(2,2)*B(2,J)+S(2,3)*B(3,J))
      D3=FAC*(S(3,1)*B(1,J)+S(3,2)*B(2,J)+S(3,3)*B(3,J))
      DO 308 I=J,8
      ASA(I,J)=ASA(I,J)+B(1,I)*D1+B(2,I)*D2+B(3,I)*D3
      ASA(J,I)=ASA(I,J)
  308 CONTINUE
  307 CONTINUE
  305 CONTINUE
      IF(NREAD .EQ. 1) GO TO 410
C
C     STORE ELEMENT STIFFNESS INFORMATION ON TAPE 1
      ND=8
      NS=3
```

```
      WRITE(1) ND,NS,(LM(I),I=1,ND),((S(I,J),I=1,NS),J=1,NS),          QUADS.135
     1  ((B(I,J),I=1,NS),J=1,ND)                                       QUADS.136
      GO TO 411                                                        QUADS.137
  410 CONTINUE                                                         QUADS.138
C     WRITE NON-LINEAR INFORMATION ON TAPE 41                          QUADS.139
      DO 310 I=1,3                                                     QUADS.140
      DO 310 J=1,3                                                     QUADS.141
      S(I,J)=SE(I,J)-S(I,J)                                            QUADS.142
  310 CONTINUE                                                         QUADS.143
      WRITE(41)ND,NS,(LM(I),I=1,ND),((S(I,J),I=1,NS),J=1,NS),          QUADS.144
     1          ((B(I,J),I=1,NS),J=1,ND)                               QUADS.145
  411 CONTINUE                                                         QUADS.146
      RETURN                                                           QUADS.147
      END                                                              QUADS.148
```

```
      SUBROUTINE CONCTS(XX,YY,PARACO,COPROP,IX,IBC,C,SLV,SY,NIND)      CONCTS.2
C                                                                      CONCTS.3
C     *****************************************************************CONCTS.4
C     FORM ELEMENT STIFFNESS OF BEAM,COLUMN IN GLOBAL COORD --ASA      CONCTS.5
C     *****************************************************************CONCTS.6
C                                                                      CONCTS.7
      DIMENSION XX(4),YY(4),PARACO(5,1),COPROP(3,1),IX(6),C(5),IBC(4), CONCTS.8
     1          SLV(1),SY(6,6)                                         CONCTS.9
      DIMENSION T(3,3),JK(2),S(8,8),R(6),CC(6),SF(6),SA(8,8)           CONCTS.10
      COMMON/EM/LM(8),ASA(8,8)                                         CONCTS.11
      DIMENSION TI(3,3),SFL(6)                                         CONCTS.12
      COMMON/NONL/NREAD                                                CONCTS.13
      DIMENSION SL(6,6)                                                CONCTS.14
      COMMON/ELASTC/SAL(8,8)                                           CONCTS.15
      DIMENSION SFGU(6),RF(6)                                          CONCTS.16
      COMMON/TIME/JUMP,B,DT,NPRTM,NTAPE,KPRINT                         CONCTS.17
C                                                                      CONCTS.18
C     FORM GLOBAL TO LOCAL COORDS TRANFORMATION MATRIX--T(2,2)         CONCTS.19
      DX=XX(2)-XX(1)                                                   CONCTS.20
      DY=YY(2)-YY(1)                                                   CONCTS.21
      DL=SQRT(DX**2+DY**2)                                             CONCTS.22
      T(1,1)=DX/DL                                                     CONCTS.23
      T(1,2)=DY/DL                                                     CONCTS.24
      T(1,3)=0.0                                                       CONCTS.25
      T(2,1)=-T(1,2)                                                   CONCTS.26
      T(2,2)=T(1,1)                                                    CONCTS.27
      T(2,3)=0.0                                                       CONCTS.28
      T(3,1)=0.0                                                       CONCTS.29
      T(3,2)=0.0                                                       CONCTS.30
      T(3,3)=1.0                                                       CONCTS.31
C                                                                      CONCTS.32
C     BEAM END RELEASE CODE                                            CONCTS.33
      JK(1)=IBC(2)                                                     CONCTS.34
      JK(2)=IBC(3)                                                     CONCTS.35
C                                                                      CONCTS.36
C     FORM ELEMENT STIFFNESS MATRIX IN LOCAL COORD--S(8,8)             CONCTS.37
      DO 302 I=1,8                                                     CONCTS.38
      DO 302 J=1,8                                                     CONCTS.39
  302 S(I,J)=0.0                                                       CONCTS.40
      IF(NIND .NE. 0) GO TO 401                                        CONCTS.41
      S(1,1)=C(1)                                                      CONCTS.42
      S(2,2)=C(2)                                                      CONCTS.43
      S(3,3)=C(3)                                                      CONCTS.44
      S(3,2)=C(4)                                                      CONCTS.45
      S(4,1)=-C(1)                                                     CONCTS.46
      S(4,4)=S(1,1)                                                    CONCTS.47
      S(5,2)=-S(2,2)                                                   CONCTS.48
      S(5,3)=-S(3,2)                                                   CONCTS.49
      S(5,5)=S(2,2)                                                    CONCTS.50
      S(6,2)=S(3,2)                                                    CONCTS.51
      S(6,3)=C(5)                                                      CONCTS.52
      S(6,5)=-S(6,2)                                                   CONCTS.53
      S(6,6)=S(3,3)                                                    CONCTS.54
      DO 303 I=1,6                                                     CONCTS.55
      DO 304 J=I,6                                                     CONCTS.56
  304 S(I,J)=S(J,I)                                                    CONCTS.57
  303 CONTINUE                                                         CONCTS.58
      DO 322 I=1,6                                                     CONCTS.59
      DO 322 J=1,6                                                     CONCTS.60
      SL(I,J)=S(I,J)                                                   CONCTS.61
  322 CONTINUE                                                         CONCTS.62
      GO TO 402                                                        CONCTS.63
  401 CONTINUE                                                         CONCTS.64
      DO 353 I=1,6                                                     CONCTS.65
      DO 353 J=1,6                                                     CONCTS.66
  353 S(I,J)=SY(I,J)                                                   CONCTS.67
  402 CONTINUE                                                         CONCTS.68
      IF(NREAD .EQ. 1) GO TO 411                                       CONCTS.69
C                                                                      CONCTS.70
C     FIXED END FORCES--SF(6)                                          CONCTS.71
C     FORM FIXED END FORCES                                            CONCTS.72
      MATYPE=IX(6)                                                     CONCTS.73
      NG=IBC(1)                                                        CONCTS.74
      AXA=COPROP(1,NG)                                                 CONCTS.75
      WT=AXA*PARACO(5,MATYPE)/144.0                                    CONCTS.76
      AMASS=AXA*PARACO(4,MATYPE)/144.0                                 CONCTS.77
      CALL FBECOL(XX,YY,AMASS,WT,FEM)                                  CONCTS.78
      DO 352 I=1,6                                                     CONCTS.79
      SF(I)=0.0                                                        CONCTS.80
  352 CONTINUE                                                         CONCTS.81
C     GLOBAL FORCES                                                    CONCTS.82
      SF(1)=0.0                                                        CONCTS.83
      SF(2)=WT                                                         CONCTS.84
      SF(3)=FEM                                                        CONCTS.85
      SF(4)=0.0                                                        CONCTS.86
      SF(5)=WT                                                         CONCTS.87
      SF(6)=-FEM                                                       CONCTS.88
      DO 334 I=1,6                                                     CONCTS.89
      SFGU(I)=SF(I)                                                    CONCTS.90
  334 CONTINUE                                                         CONCTS.91
C     TRANSFORM TO LOCAL COORDINATES                                   CONCTS.92
      TI(1,1)=T(1,1)                                                   CONCTS.93
      TI(1,2)=T(1,2)                                                   CONCTS.94
      TI(1,3)=0.0                                                      CONCTS.95
      TI(2,1)=T(2,1)                                                   CONCTS.96
      TI(2,2)=T(2,2)                                                   CONCTS.97
      TI(2,3)=0.0                                                      CONCTS.98
      TI(3,1)=0.0                                                      CONCTS.99
      TI(3,2)=0.0                                                      CONCTS.100
      TI(3,3)=1.0                                                      CONCTS.101
      SFL(1)=TI(1,1)*SF(1)+TI(1,2)*SF(2)                               CONCTS.102
      SFL(2)=TI(2,1)*SF(1)+TI(2,2)*SF(2)                               CONCTS.103
      SFL(3)=SF(3)                                                     CONCTS.104
      SFL(4)=TI(1,1)*SF(4)+TI(1,2)*SF(5)                               CONCTS.105
      SFL(5)=TI(2,1)*SF(4)+TI(2,2)*SF(5)                               CONCTS.106
      SFL(6)=SF(6)                                                     CONCTS.107
      DO 321 I=1,6                                                     CONCTS.108
      SF(I)=SFL(I)                                                     CONCTS.109
  321 CONTINUE                                                         CONCTS.110
  411 CONTINUE                                                         CONCTS.111
C                                                                      CONCTS.112
C     MODIFY ELEMENT STIFFNESS AND FIXED END FORCES FOR KNOWN ZERO MEMBECONCTS.113
      IF((JK(1)+JK(2)) .EQ. 0) GO TO 413                               CONCTS.114
      DO 306 K=1,2                                                     CONCTS.115
      KK=JK(K)                                                         CONCTS.116
      KD=100                                                           CONCTS.117
      I1=3*(K-1)+1                                                     CONCTS.118
      I2=I1+2                                                          CONCTS.119
      DO 306 I=I1,I2                                                   CONCTS.120
      IF(KK .LT. KD) GO TO 306                                         CONCTS.121
      SII=S(I,I)                                                       CONCTS.122
      DO 307 N=1,6                                                     CONCTS.123
  307 R(N)=S(I,N)                                                      CONCTS.124
      DO 308 M=1,6                                                     CONCTS.125
      CC(M)=S(M,I)/SII                                                 CONCTS.126
      DO 308 N=1,6                                                     CONCTS.127
  308 S(M,N)=S(M,N)-CC(M)*R(N)                                         CONCTS.128
      IF(NREAD .EQ. 1) GO TO 412                                       CONCTS.129
      SFI=SF(I)                                                        CONCTS.130
      DO 309 M=1,6                                                     CONCTS.131
  309 SF(M)=SF(M)-CC(M)*SFI                                            CONCTS.132
  412 CONTINUE                                                         CONCTS.133
      KK=KK-KD                                                         CONCTS.134
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  306 KD=KD/10                                                      CONCTS.135
      IF(NREAD .EQ. 1) GO TO 413                                    CONCTS.136
C                                                                   CONCTS.137
C     FIXED END FORCE RF(6) IN GLOBAL COORDINATE                    CONCTS.138
      DO 331 LA=1,4,3                                               CONCTS.139
      LB=LA-1                                                       CONCTS.140
      DO 331 IL=1,3                                                 CONCTS.141
      I=IL+LB                                                       CONCTS.142
      XA=0.0                                                        CONCTS.143
      DO 332 K=1,3                                                  CONCTS.144
  332 XA=XA-T(K,IL)*SF(K+LB)                                        CONCTS.145
  331 RF(I)=XA                                                      CONCTS.146
  413 CONTINUE                                                      CONCTS.147
C                                                                   CONCTS.148
C     OBTAIN SA(6,6) RELATING ELEMENT END FORCES(LOCAL) TO          CONCTS.149
C     JOINT DISPLACEMENT(GLOBAL)                                    CONCTS.150
      DO 311 I=1,64                                                 CONCTS.151
      SA(I)=0.0                                                     CONCTS.152
  311 CONTINUE                                                      CONCTS.153
      DO 312 LA=1,4,3                                               CONCTS.154
      LB=LA+2                                                       CONCTS.155
      DO 312 MA=1,4,3                                               CONCTS.156
      MB=MA-1                                                       CONCTS.157
      DO 312 I=LA,LB                                                CONCTS.158
      DO 312 JM=1,3                                                 CONCTS.159
      J=JM+MB                                                       CONCTS.160
      XA=0.0                                                        CONCTS.161
      DO 313 K=1,3                                                  CONCTS.162
  313 XA=XA+S(I,K+MB)*T(K,JM)                                       CONCTS.163
  312 SA(I,J)=XA                                                    CONCTS.164
C                                                                   CONCTS.165
C     ELEMENT STIFFNESS MATRIX ASA(6,6) IN GLOBAL COORDS            CONCTS.166
      DO 314 I=1,64                                                 CONCTS.167
  314 ASA(I)=0.0                                                    CONCTS.168
      DO 315 LA=1,4,3                                               CONCTS.169
      LB=LA-1                                                       CONCTS.170
      DO 315 MA=1,4,3                                               CONCTS.171
      MB=MA+2                                                       CONCTS.172
      DO 315 IL=1,3                                                 CONCTS.173
      I=IL+LB                                                       CONCTS.174
      DO 315 J=MA,MB                                                CONCTS.175
      XA=0.0                                                        CONCTS.176
      DO 316 K=1,3                                                  CONCTS.177
  316 XA=XA+T(K,IL)*SA(K+LB,J)                                      CONCTS.178
      ASA(I,J)=XA                                                   CONCTS.179
  315 CONTINUE                                                      CONCTS.180
      IF(NREAD .EQ. 1) GO TO 410                                    CONCTS.181
      NS=6                                                          CONCTS.182
      ND=6                                                          CONCTS.183
      WRITE(1) ND,NS,(LM(I),I=1,ND),((SL(I,J),I=1,NS),J=1,ND),      CONCTS.184
     1         ((SA(I,J),I=1,NS),J=1,ND),                           CONCTS.185
     2         (SF(I),I=1,ND)                                       CONCTS.186
      GO TO 421                                                     CONCTS.187
  410 CONTINUE                                                      CONCTS.188
C     WRITE NON-LINEAR INFORMATION ON TAPE 41                       CONCTS.189
      DO 318 I=1,6                                                  CONCTS.190
      DO 318 J=1,6                                                  CONCTS.191
      SA(I,J)=SAL(I,J)-SA(I,J)                                      CONCTS.192
  318 CONTINUE                                                      CONCTS.193
      WRITE(41)ND,NS,(LM(I),I=1,ND),((SL(I,J),I=1,NS),J=1,ND),      CONCTS.194
     1         ((SA(I,J),I=1,NS),J=1,ND),                           CONCTS.195
     2         (SF(I),I=1,ND)                                       CONCTS.196
  421 CONTINUE                                                      CONCTS.197
      RETURN                                                        CONCTS.198
      END                                                           CONCTS.199
```

```
      SUBROUTINE FRICTS(XX,YY,C)                                    FRICTS.2
C                                                                   FRICTS.3
C     ****************************************************************FRICTS.4
C     FORM FRICTIONAL ELEMENT STIFFNESS MATRIX AND STORE ON TAPE 1  FRICTS.5
C     ****************************************************************FRICTS.6
C                                                                   FRICTS.7
      DIMENSION XX(4),YY(4),C(5),S(8,8),SA(8,8),T(2,2)              FRICTS.8
      COMMON/EM/LM(8),ASA(8,8)                                      FRICTS.9
      COMMON/NONL/NREAD                                             FRICTS.10
      COMMON/TIME/JUMP,B,DT,NPRTM,NTAPE,KPRINT                      FRICTS.11
      COMMON/ELASTC/SEA(8,8)                                        FRICTS.12
C                                                                   FRICTS.13
C     INITILIZATION                                                 FRICTS.14
      DO 301 I=1,64                                                 FRICTS.15
      S(I)=0.0                                                      FRICTS.16
  301 CONTINUE                                                      FRICTS.17
C                                                                   FRICTS.18
C     FORM ELEMENT STIFFNESS IN LOCAL COORDS--S(8,8)                FRICTS.19
      S(1,1)=C(1)                                                   FRICTS.20
      S(1,3)=C(3)                                                   FRICTS.21
      S(2,2)=C(2)                                                   FRICTS.22
      S(2,4)=C(4)                                                   FRICTS.23
      S(3,3)=S(1,1)                                                 FRICTS.24
      S(4,4)=S(2,2)                                                 FRICTS.25
      S(1,5)=-S(1,3)                                                FRICTS.26
      S(1,7)=-S(1,1)                                                FRICTS.27
      S(2,6)=-S(2,4)                                                FRICTS.28
      S(2,8)=-S(2,2)                                                FRICTS.29
      S(3,5)=-S(1,1)                                                FRICTS.30
      S(3,7)=-S(1,3)                                                FRICTS.31
      S(4,6)=-S(2,2)                                                FRICTS.32
      S(4,8)=-S(2,4)                                                FRICTS.33
      DO 303 I=1,4                                                  FRICTS.34
      DO 303 J=1,4                                                  FRICTS.35
  303 S(I+4,J+4)=S(I,J)                                             FRICTS.36
      DO 304 I=2,8                                                  FRICTS.37
      K=I-1                                                         FRICTS.38
      DO 304 J=1,K                                                  FRICTS.39
      S(I,J)=S(J,I)                                                 FRICTS.40
  304 CONTINUE                                                      FRICTS.41
C                                                                   FRICTS.42
C     FORM TRANSFORMATION MATRIX T(2,2)                             FRICTS.43
      DX=XX(2)-XX(1)                                                FRICTS.44
      DY=YY(2)-YY(1)                                                FRICTS.45
C     CASE OF HORIZONTAL FRICTION ELEMENT                           FRICTS.46
      IF(DX .NE. 0.0 .OR. DY .NE. 0.0) GO TO 421                    FRICTS.47
      DX=XX(4)-XX(1)                                                FRICTS.48
      DY=YY(4)-YY(1)                                                FRICTS.49
  421 CONTINUE                                                      FRICTS.50
      DL=SQRT(DX**2+DY**2)                                          FRICTS.51
C     STIFFNESS FOR TOTAL LENGTH L,K/INCH                           FRICTS.52
      DO 351 I=1,8                                                  FRICTS.53
      DO 351 J=1,8                                                  FRICTS.54
      S(I,J)=S(I,J)*DL*12.0                                         FRICTS.55
  351 CONTINUE                                                      FRICTS.56
      T(1,1)=DX/DL                                                  FRICTS.57
      T(1,2)=DY/DL                                                  FRICTS.58
      T(2,2)=T(1,1)                                                 FRICTS.59
      T(2,1)=-T(1,2)                                                FRICTS.60
C     FORM SA(8,8),RELATING ELEMENT END FORCES(LOCAL) TO            FRICTS.61
C     JOINT DISPLACEMENT(GLOBAL)                                    FRICTS.62
      DO 305 I=1,64                                                 FRICTS.63
  305 SA(I)=0.0                                                     FRICTS.64
      DO 306 LA=1,7,2                                               FRICTS.65
      LB=LA+1                                                       FRICTS.66
      DO 306 MA=1,7,2                                               FRICTS.67
      MB=MA-1                                                       FRICTS.68
      DO 306 I=LA,LB                                                FRICTS.69
      DO 306 JM=1,2                                                 FRICTS.70
      J=JM+MB                                                       FRICTS.71
      XA=0.0                                                        FRICTS.72
      DO 307 K=1,2                                                  FRICTS.73
  307 XA=XA+S(I,K+MB)*T(K,JM)                                       FRICTS.74
      SA(I,J)=XA                                                    FRICTS.75
  306 CONTINUE                                                      FRICTS.76
```

```
C
C     ELEMENT STIFFNESS ASA(8,8) IN GLOBAL COORDS
      DO 308 I=1,64
  308 ASA(I)=0.0
      DO 309 LA=1,7,2
      LB=LA-1
      DO 309 MA=1,7,2
      MB=MA+1
      DO 309 IL=1,2
      I=IL+LB
      DO 309 J=MA,MB
      XA=0.0
      DO 310 K=1,2
  310 XA=XA+T(K,IL)*SA(K+LB,J)
      ASA(I,J)=XA
  309 CONTINUE
      IF(NREAD .EQ. 1) GO TO 410
C
C     WRITE ELEMENT STIFFNESS INFORMATION ON TAPE 1
      ND=8
      NS=8
      WRITE(1) ND,NS,(LN(I),I=1,ND),((S(I,J),I=I,NS),J=1,ND),
     1         ((SA(I,J),I=1,NS),J=1,ND)
      GO TO 411
  410 CONTINUE
C     WRITE NON-LINEAR INFORMATION ON TAPE 41
      DO 312 I=1,8
      DO 312 J=1,8
      SA(I,J)=SEA(I,J)-SA(I,J)
  312 CONTINUE
      WRITE(41)ND,NS,(LN(I),I=1,ND),((S(I,J),I=1,NS),J=1,ND),
     1         ((SA(I,J),I=1,NS),J=1,ND)
  411 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BOUNDS(C)
C
C     ******************************************************************
C     FORM BOUNDARY ELEMENT STIFFNESS MATRIX ANA STORE ON TAPE 1
C     ******************************************************************
C
      COMMON/EM/LN(8),ASA(8,8)
      DIMENSION C(5),S(8,8),SA(8,8)
      COMMON/NONL/NREAD
      COMMON/ELASTC/SE(8,8)
      DO 301 I=1,64
      S(I)=0.0
  301 CONTINUE
      S(1,1)=C(1)
      S(1,2)=-S(1,1)
      S(2,1)=S(1,2)
      S(2,2)=S(1,1)
      DO 302 I=1,8
      DO 302 J=1,8
      SA(I,J)=S(I,J)
      ASA(I,J)=S(I,J)
  302 CONTINUE
      IF(NREAD .EQ. 1) GO TO 410
      ND=2
      NS=2
      WRITE(1) ND,NS,(LN(I),I=1,ND),((S(I,J),I=1,NS),J=1,ND)
      GO TO 411
  410 CONTINUE
C     WRITE NON-LINEAR INFORMATION ON TAPE 41
      DO 304 I=1,2
      DO 304 J=1,2
      S(I,J)=SE(I,J)-S(I,J)
  304 CONTINUE
      WRITE(41)ND,NS,(LN(I),I=1,ND),((S(I,J),I=1,NS),J=1,ND)
  411 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STATIC(SLV,AE,U,V,ACC,DU,OV,DA,NEQ,MBAND)
C
C     ******************************************************************
C     SOLUTION OF STATIC CASE--DISPLACEMENT AT NODES
C     ******************************************************************
C
      DIMENSION SLV(1),U(1),V(1),ACC(1),DU(1),OV(1),DA(1),AE(NEQ,1)
      COMMON/TIME/JUMP,T,DT,NPRTN,NTAPE,KPRINT
C
C      INITILIZATION
      DO 351 I=1,NEQ
      V(I)=0.0
      ACC(I)=0.0
      DU(I)=0.0
      DV(I)=0.0
      DA(I)=0.0
  351 CONTINUE
      JUMP=0
      NPRTN=0
      KPRINT=0
      T=0.0
      REWIND 3
      READ(3) ((AE(I,J),I=1,NEQ),J=1,NBAND)
      DO 301 I=1,NEQ
      IF(AE(I,1) .NE. 0) U(I)=SLV(I)
  301 CONTINUE
      CALL TRIA(AE,NEQ,NBAND)
      CALL BACKS(AE,U,NEQ,MBAND)
      DO 302 I=1,NEQ
      DU(I)=U(I)
  302 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRIA(A,NEQ,MBAND)
C
C     ******************************************************************
C     TRIANGULIZE STIFFNESS MATRIX A OF AX=B
C     ******************************************************************
C
      DIMENSION A(1)
      NE=NEQ-1
      NN=MBAND-1
      NM=NN*NEQ
      MK=NEQ-MN
      DO 301 N=1,NE
      NT=N-MK
      IF(NT .GT. 0) NM=NM-NEQ
      IF(A(N) .EQ. 0.0) GO TO 301
      L=N
      IL=N+NEQ
      IH=N+NM
      DO 302 I=IL,IH,NEQ
      L=L+1
      J=L
      C=A(I)/A(N)
      DO 303 K=I,IH,NEQ
      A(J)=A(J)-C*A(K)
  303 J=J+NEQ
      A(I)=C
  302 CONTINUE
  301 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BACKS(A,B,NEQ,MBAND(
C
C     ****************************************************************
C      SOLVE EQUATIONS BY GAUSSIAN ELIMINATION
C     ****************************************************************
C
      DIMENSION A(1),B(1)
      MMM=MBAND-1
      N=0
 1000 N=N+1
      C=B(N)
      IF(A(N( .NE. 0.0( B(N(=B(N(/A(N(
      IF(N .EQ. NEQ) GO TO 401
      IL=N+1
      IH=MIN0(NEQ,N+MMM)
      M=N
      DO 301 I=IL,IH
      M=M+NEQ
  301 B(I)=B(I)-A(M(*C
      GO TO 1000
C
  401 IL=N
      N=N-1
      IF(N .EQ. 0( RETURN
      IH=MIN0(NEQ,N+MMM)
      M=N
      DO 302 I=IL,IH
      M=M+NEQ
  302 B(N)=B(N(-A(M(*B(I(
      GO TO 401
      END


      SUBROUTINE STRAIN(ID,X,Y,IX,DU,TEPS,DEPS,PEPS(
C
C     ****************************************************************
C      CALCULATE THE INCREMENTAL STRAIN,AND THEN TOTAL STRAIN OF SOIL EL
C     IN BOTH X,Y COORDS AND PRINCIPAL DIRECTION,AT CENTER OF ELEMENT
C     ****************************************************************
C
      COMMON/ELPAR/NUMNP,NUMEL,NEQ,MBAND,KLIN
      COMMON/TIME/JUMP,T,DT,NPRTM,NTAPE,KPRINT
      DIMENSION HX(4(,HY(4),B(3,8),LM(8(
      DIMENSION ID(3,1(,X(1),Y(1),IX(6,1),DU(1(,TEPS(3,1),DEPS(3,1),
     1           PEPS(3,1(
      IF(JUMP .NE. 0) GO TO 401
C
C     INITIALIZATION
      DO 301 I=1,3
      DO 301 J=1,NUMEL
      TEPS(I,J)=0.0
      DEPS(I,J(=0.0
      PEPS(I,J)=0.0
  301 CONTINUE
      DO 302 I=1,3
      DO 302 J=1,8
      B(I,J)=0.0
  302 CONTINUE
  401 CONTINUE
      DO 303 N=1,NUMEL
C
C     MATERIAL TYPE=NTYPE
      NTYPE=IX(5,N)
      IF(NTYPE .NE. 1( GO TO 303
      DO 304 I=1,4
      JJ=I+I
      NODE=IX(I,N)
      LM(JJ)=ID(2,NODE)
      LM(JJ-1)=ID(1,NODE)
  304 CONTINUE
      I=IX(1,N)
      J=IX(2,N)
      K=IX(3,N)
      L=IX(4,N(
C
C     DISPLACEMENT STRAIN TRANFORMATION MATRIX--B(3,8)
C     AT POINT S=0,T=0 IN STRAIN -DISPLACEMENT MATRIX B(3,8)
      X13=X(I)-X(K)
      X24=X(J)-X(L(
      Y13=Y(I(-Y(K)
      Y24=Y(J(-Y(L(
      XJ=X13*Y24-X24*Y13
C
C     FORM D(H(I)/DX=HX(I),AND D(H(I(/DY=HY(I)
      HX(1)=Y24/XJ
      HX(2(=-Y13/XJ
      HX(3)=-HX(1(
      ....

      HY(1(=-X24/XJ
      HY(2(=X13/XJ
      HY(3(=-HY(1(
      HY(4)=-HY(2(
      DO 305 I=1,4
      II=I+I
      JJ=I(-1
      B(1,JJ(=HX(I)/12.0
      B(2,II(=HY(I)/12.0
      B(3,I((=HX(I(/12.0
      B(3,JJ)=HY(I)/12.0
  305 CONTINUE
C
C     EVALUATION OF INCREMENTAL STRAIN-DEPS(3,N)
      DO 306 I=1,3
      DO 307 J=1,8
      JJ=LM(J(
      IF(JJ .EQ. 0( GO TO 307
      DEPS(I,N(=DEPS(I,N)+B(I,J)*DU(JJ)
  307 CONTINUE
  306 CONTINUE
C
C     ADD UP TOTAL STRAIN--TEPS(3,N)
      DO 308 I=1,3
      TEPS(I,N(=TEPS(I,N(+DEPS(I,N)
  308 CONTINUE
C
C     EVALUATE PRINCIPAL STRAIN-PEPS(I,N),AND
C     MAX SHEAR STRAIN PEPS(3,N)
      DE=TEPS(1,N(-TEPS(2,N(
      IF(DE .EQ. 0( GO TO 402
      TAN=TEPS(3,N(/DE
      THETA1=ATAN(TAN)/2.0
      THETA2=THETA1+1.570796326B
      TEMP1=TEPS(1,N(*COS(THETA1)*COS(THETA1)
      TEMP1=TEMP1+TEPS(2,N)*SIN(THETA1(*SIN(THETA1(
      TEMP1=TEMP1+TEPS(3,N)*SIN(THETA1)*COS(THETA1)
      TEMP2=TEPS(1,N(*COS(THETA2)*COS(THETA2)
      TEMP2=TEMP2+TEPS(2,N(*SIN(THETA2(*SIN(THETA2)
      TEMP2=TEMP2+TEPS(3,N)*SIN(THETA2)*COS(THETA2)
      PEPS(1,N(=AMAX1(TEMP1,TEMP2(
      PEPS(2,N)=AMIN1(TEMP1,TEMP2)
      PEPS(3,N(=PEPS(1,N(-PEPS(2,N(
      GO TO 303
  402 CONTINUE
      PEPS(1,N)=TEPS(1,N(
      PEPS(2,N)=TEPS(2,N(
      PEPS(3,N)=TEPS(3,N(
  303 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STRESS1(X,DU,DEPS,DSIG,SIG,PSIG)                       STRESS.2
C                                                                       STRESS.3
C     ******************************************************************STRESS.4
C     CALCULATE INCREMENTAL STRESS,AND TOTAL STRESS OF ALL ELEMENTS     STRESS.5
C     ******************************************************************STRESS.6
C                                                                       STRESS.7
      COMMON/ELPAR/NUMNP,NUMEL,NEQ,MBAND,KLIN                           STRESS.8
      DIMENSION IX(6,1),DU(1),DEPS(3,1),DSIG(8,1),SIG(8,1),PSIG(4,1)    STRESS.9
      COMMON/TIME/JUMP,T,DT,MPRTN,NTAPE,KPRINT                          STRESS.10
      DIMENSION LN(8),SS(138),S1(3,3),SA2(6,6),SA34(8,8),SA5(2,2)       STRESS.11
      DIMENSION ISS(138)                                                STRESS.12
      EQUIVALENCE(SS,ISS)                                               STRESS.13
      COMMON/NONL/NREAD                                                 STRESS.14
      IF(JUMP .NE. 0) GO TO 406                                         STRESS.15
C                                                                       STRESS.16
C     INITIALIZATION                                                    STRESS.17
      DO 301 I=1,8                                                      STRESS.18
      DO 301 J=1,NUMEL                                                  STRESS.19
      SIG(I,J)=0.0                                                      STRESS.20
      DSIG(I,J)=0.0                                                     STRESS.21
  301 CONTINUE                                                          STRESS.22
  406 CONTINUE                                                          STRESS.23
      REWIND 41                                                         STRESS.24
      REWIND 1                                                          STRESS.25
      DO 302 N=1,NUMEL                                                  STRESS.26
C                                                                       STRESS.27
C     DETERMINE ELEMENT TYPE                                            STRESS.28
      NTYPE=IX(5,N)                                                     STRESS.29
      GO TO (401,402,403,403,405),NTYPE                                 STRESS.30
C                                                                       STRESS.31
C     TYPE 1,SOIL ELEMENT                                               STRESS.32
  401 CONTINUE                                                          STRESS.33
      IF(KLIN .EQ. 0) GO TO 411                                         STRESS.34
      IF(NREAD .EQ. 0) GO TO 411                                        STRESS.35
C     NON-LINEAR CASE                                                   STRESS.36
      READ(41) (SS(I),I=1,19)                                           STRESS.37
      GO TO 412                                                         STRESS.38
  411 CONTINUE                                                          STRESS.39
      READ(1) (SS(I),I=1,19)                                            STRESS.40
  412 CONTINUE                                                          STRESS.41
      KK=7                                                              STRESS.42
C     SET UP STRESS-STRAIN MATRIX-S1(3,3)                               STRESS.43
C                                                                       STRESS.44
      DO 352 I=1,3                                                      STRESS.45
      KI=KK+3*I                                                         STRESS.46
      DO 303 J=1,3                                                      STRESS.47
      JI=KI+J                                                           STRESS.48
  303 S1(J,I)=SS(JI)                                                    STRESS.49
  352 CONTINUE                                                          STRESS.50
C                                                                       STRESS.51
C     CALCULATE INCREMENTAL STRESS-DSIG(3,N),AND ADD UP TOTAL STRESS SIGSTRESS.52
      DO 304 I=1,3                                                      STRESS.53
      DO 305 J=1,3                                                      STRESS.54
  305 DSIG(I,N)=DSIG(I,N)+S1(I,J)*DEPS(J,N)                             STRESS.55
      SIG(I,N)=SIG(I,N)+DSIG(I,N)                                       STRESS.56
  304 CONTINUE                                                          STRESS.57
C     CALCULATE PRINCIPAL STRESSES,DIRECTION,AND MAX. SHEAR             STRESS.58
      D=(SIG(1,N)-SIG(2,N))/2.0                                         STRESS.59
      S=(SIG(1,N)+SIG(2,N))/2.0                                         STRESS.60
      TAU=SIG(3,N)                                                      STRESS.61
      R=SQRT(D*D+TAU*TAU)                                               STRESS.62
      PSIG(1,N)=S+R                                                     STRESS.63
      PSIG(2,N)=S-R                                                     STRESS.64
      PSIG(3,N)=(PSIG(1,N)-PSIG(2,N))/2.0                               STRESS.65
      IF(D .NE. 0.0) GO TO 407                                          STRESS.66
      PSIG(4,N)=45.0                                                    STRESS.67
      GO TO 302                                                         STRESS.68
  407 CONTINUE                                                          STRESS.69
      PHI2=2.0*SIG(3,N)/(SIG(1,N)-SIG(2,N))                             STRESS.70
      PHI2=ABS(PHI2)                                                    STRESS.71
      PSIG(4,N)=0.5*ATAN(PHI2)                                          STRESS.72
      PSIG(4,N)=PSIG(4,N)*57.29577                                      STRESS.73
      GO TO 302                                                         STRESS.74
C                                                                       STRESS.75
C     TYPE 2,BEAM,COLUMN ELEMENT                                        STRESS.76
  402 CONTINUE                                                          STRESS.77
      IF(KLIN .EQ. 0) GO TO 421                                         STRESS.78
      IF(NREAD .EQ. 0) GO TO 421                                        STRESS.79
C     NON-LINEAR CASE                                                   STRESS.80
      READ(41) (SS(I),I=1,80)                                           STRESS.81
      GO TO 422                                                         STRESS.82
  421 CONTINUE                                                          STRESS.83
      READ(1) (SS(I),I=1,86)                                            STRESS.84
  422 CONTINUE                                                          STRESS.85
      KK=38                                                             STRESS.86
C                                                                       STRESS.87
C     SET UP STRESS-DISPLACEMENT MATRIX-SA2(6,6)                        STRESS.88
      DO 306 I=1,6                                                      STRESS.89
      KI=KK+6*I                                                         STRESS.90
      DO 307 J=1,6                                                      STRESS.91
      JI=KI+J                                                           STRESS.92
  307 SA2(J,I)=SS(JI)                                                   STRESS.93
  306 CONTINUE                                                          STRESS.94
C                                                                       STRESS.95
C     SET UP LOCATION OF MASS                                           STRESS.96
      DO 308 I=1,6                                                      STRESS.97
      LN(I)=ISS(2+I)                                                    STRESS.98
  308 CONTINUE                                                          STRESS.99
C                                                                       STRESS.100
C     ADD UP FIXED END FORCES                                           STRESS.101
      IF(JUMP .NE. 0) GO TO 451                                         STRESS.102
      DO 351 I=1,6                                                      STRESS.103
      SIG(I,N)=SS(80+I)                                                 STRESS.104
  351 CONTINUE                                                          STRESS.105
  451 CONTINUE                                                          STRESS.106
C                                                                       STRESS.107
C     CALCULATE INCREMENTAL STRESS-DSIG(6,N),AND TOTAL STRESS SIG(6,N)  STRESS.108
      DO 309 I=1,6                                                      STRESS.109
      DO 310 J=1,6                                                      STRESS.110
      JJ=LN(J)                                                          STRESS.111
      IF(JJ .EQ. 0) GO TO 310                                           STRESS.112
      DSIG(I,N)=DSIG(I,N)+SA2(I,J)*DU(JJ)                               STRESS.113
  310 CONTINUE                                                          STRESS.114
      SIG(I,N)=SIG(I,N)+DSIG(I,N)                                       STRESS.115
  309 CONTINUE                                                          STRESS.116
      GO TO 302                                                         STRESS.117
C                                                                       STRESS.118
C     TYPE 3,FRICTIONAL ELEMENT                                         STRESS.119
C     TYPE 4,EXPANSION JOINT ELEMENT                                    STRESS.120
  403 CONTINUE                                                          STRESS.121
      IF(KLIN .EQ. 0) GO TO 431                                         STRESS.122
      IF(NREAD .EQ. 0) GO TO 431                                        STRESS.123
      READ(41) (SS(I),I=1,138)                                          STRESS.124
      GO TO 432                                                         STRESS.125
  431 CONTINUE                                                          STRESS.126
      READ(1) (SS(I),I=1,138)                                           STRESS.127
  432 CONTINUE                                                          STRESS.128
      KK=66                                                             STRESS.129
C                                                                       STRESS.130
C     SET UP STRESS-DISPLACEMENT MATRIX SA34(8,8)                       STRESS.131
      DO 311 I=1,8                                                      STRESS.132
      KI=KK+8*I                                                         STRESS.133
      DO 312 J=1,8                                                      STRESS.134
      JI=KI+J                                                           STRESS.135
  312 SA34(J,I)=SS(JI)                                                  STRESS.136
  311 CONTINUE                                                          STRESS.137
C                                                                       STRESS.138
C     SET UP LOCATION OF MASS                                           STRESS.139
      DO 313 I=1,8                                                      STRESS.140
      LN(I)=ISS(2+I)                                                    STRESS.141
  313 CONTINUE                                                          STRESS.142
C                                                                       STRESS.143
C     CALCULATE INCREMENTAL STRESS DSIG(8,N),AND TOTAL STRESS SIG(8,N)  STRESS.144
      DO 314 I=1,8                                                      STRESS.145
      DO 315 J=1,8                                                      STRESS.146
      JJ=LN(J)                                                          STRESS.147
      IF(JJ .EQ. 0) GO TO 315                                           STRESS.148
      DSIG(I,N)=DSIG(I,N)+SA34(I,J)*DU(JJ)                              STRESS.149
  315 CONTINUE                                                          STRESS.150
      SIG(I,N)=SIG(I,N)+DSIG(I,N)                                       STRESS.151
  314 CONTINUE                                                          STRESS.152
      GO TO 302                                                         STRESS.153
C                                                                       STRESS.154
C     TYPE 5, BOUNDARY ELEMENT                                          STRESS.155
  405 CONTINUE                                                          STRESS.156
```

```
      IF(KLIN .EQ. 0) GO TO 441
      IF(NREAD .EQ. 0) GO TO 441
      READ(41) (SS(I),I=1,8)
      GO TO 442
  441 CONTINUE
      READ(1) (SS(I),I=1,8)
  442 CONTINUE
      KK=?
C
C     SET UP STRESS--DISPLACEMENT MATRIX SA5(2,2)
      DO 316 I=1,2
      KI=KK+2*"
      DO 317 J=1,2
      J(=KI+J
  317 SA5(J,((=SS(JI)
  316 CONTINUE
C
C     SET UP LOCATION OF MASS
      DO 318 (=1,2
      LN(I)=ISS(2+I(
  318 CONTINUE
C
C     CALCULATE INCREMSNTAL STRESS OSIGI2,N),AND TOTAL STRESS SIG(2,N)
      DO 319 I=1,2
      DO 320 J=1,2
      JJ=LN(J)
      IF(JJ .EO. D) GO TO 320
      OSIG(I,N(=DSIG(I,N(+SA5II,J)*DUIJJ)
  320 CONTINUE
      SIG(I,N(=SIG(I,N(+OSIG(I,N(
  319 CONTINUE
  302 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WADATA(NELW,INFW,NTW)
C
C     ****************************************************************
C     INPUT OATA OF ELEMENTS AGAINST WALL FOR CALCULATE THE TOTAL FORCES
C     ****************************************************************
C
      COMMON/MATER/NUMATS,NUMATC,NUMATF,NUMATE,NUMATB,NUMGE,NINTCV,THICK
     1,NPLAN
      COMMON/ELPAR/NUMNP,NUREL,NEQ,MBAND,KLIN
      DIMENSION NELW(I(,INFW(NTW,1(
C
C     READ CONTROL INFORMATION
      WRITE(6,5D2(
C
C     READ IN ELEMENT OATA ON WALL, FROM TOP TO BOTTOM
      DO 301 (=1,NTW
      NN=3*NELW(I(
      READ 2,NW,(INFW(NW,II),II=1,NN)
      WRITE(6,102( NW
      WRITE(6,103) ((NFW(NW,II(,II=1,NN)
  301 CONTINUE
    2 FORMAT(I5/I3I5))
  102 FORMAT(I9)
  103 FORMAT(18X,(5,19X,(5,19X,I5)
  502 FORMAT(* WALL NO.*,3X,*ELEMENT NO.*,3X,*UPPER NODAL POINT NO.*,
     I       3X,*LOWER NODAL POINT NO.*(
      RETURN
      END
```

```
      SUBROUTINE WFORCE(Y,IX,SIG,NELW,INFW,WFX,YBAR,WFY,NTW(
C
C     ****************************************************************
C     CALCULATE THE TOTAL FORCES FX,FY,(LOCAL( ACTING ON THE WALL,
C     AND ITS LINE OF ACTION FROM THE BASE
C     ****************************************************************
C
      COMMON/MATER/NUNATS,NUMATC,NUMATF,NUMATE,NUMATB,NUMGE,NINTCY,THICK
     I,NPLAN
      DIMENSION Y(1),(X(6,1(,SIG(8,1(,NELW(1(,INFW(NTW,1(
      DINENSION WFX(I(,YBAR(I),WFY(1),FNOD(8(
      DO 301 I=I,NTW
C
C     IN(TILIZATION
      WNT=D.0
      WFX(I(=D.D
      WFY(I)=0.0
      N=NELW(I(
      NP=3*N
      NL=NP-2
C     LOWEST ELEMENT NO.
      LNEL=INFW(I,NL)
C     LOWEST NODAL POINT NO.
      LNOD=INFW(I,NP(
      DO 302 J=1,N
      JJ=3*IJ-1)+1
      NEL=INFW(I,JJ(
      NOD1=INFW(I,JJ+1(
      NOD2=INFW(I,JJ+2)
      YL=ABS(YINOD1)-YINOD2))
C
C     CHECK THE SEQUENCE OF NODE ON ELEMENT
      DO 303 K=1,4
      NOD=IX(K,NEL(
      IFINOD1 .NE. NOO) GO TO 401
      K2=K+K
      K1=K2-1
C
C     CHANGE THE CENTER STRESS TO NODAL FORCES IN CASE OF SOIL ELEMENT
      NTYPE=IX(5,NEL(
      IF(NTYPE .EQ. 3( GO TO 402
      IF(SIG(1,NEL) .GE. 0.0( GO TO 403
      FNOD(K1)=0.5*THICK*YL*SIG(1,NEL)
      IF(K .EQ. 1 .OR. K .EQ. 2( GO TO 404
      FNOD(K2(=-D.5*THICK*YL*SIG(3,NEL)
      GO TO 406
  404 FNOD(K2(=D.5*THICK*YL*SIG(3,NEL(
      GO TO 406
  403 FNOD(K1)=0.D
      FNOD(K2)=D.0
      GO TO 406
  402 CONTINUE
C
C     FRICTION ELEMENT AT WALL
      SIGN12=D.5*(SIG(2,NEL)+SIG(4,NEL))
      SIGN34=D.5*(SIG(6,NEL(+SIG(8,NEL))

      IF(S(GNI2 .GT. D.D .AND. SIGN34 .LT. 0.0) GO TO 405
C
C     TENSION
      FNOD(K1)=D.D
      FNOD(K2(=D.0
      GO TO 406
  405 CONTINUE
      FNOD(K1)=ABS(S(GNI2(
      FNOO(K2)=-SIG(K1,NEL)
  406 CONT(NUE
      WFX(((=WFX(I(+ABS(FNOD(KI()
      WNT=WNT+ABS(FNOD(K1)(*ABS(Y(NOO)-Y(LNOD((
      WFY(I(=WFY(I)+FNDO(K2(
      GO TO 303
  401 CONT(NUE
      IF(NOD2 .NE. NOD) GO TO 303
      K2=K+K
      K1=K2-1
      NTYPE=(X(5,NEL)
      (F(NTYPE .EQ. 3) GO TD 407
      IF(SIG(1,NEL) .GE. D.0) GO TD 408
      FNOO(K1)=0.5*TH(CK*YL*S(G(1,NEL)
```

```
      IF(K .EQ. 1 .OR. K .EQ. 2) GO TO 409                              WFORCE.80
      FNOD(K2)=-0.5*THICK*YL*SIG(3,NEL)                                 WFORCE.81
      GO TO 411                                                         WFORCE.82
  409 FNOD(K2)=0.5*THICK*YL*SIG(3,NEL)                                  WFORCE.83
      GO TO 411                                                         WFORCE.84
  408 CONTINUE                                                          WFORCE.85
      FNOD(K1)=0.0                                                      WFORCE.86
      FNOD(K2)=0.0                                                      WFORCE.87
      GO TO 411                                                         WFORCE.88
  407 CONTINUE                                                          WFORCE.89
C                                                                       WFORCE.90
C     FRICTION ELEMENT ON THE WALL                                      WFORCE.91
      SIGN12=0.5*(SIG(2,NEL)+SIG(4,NEL))                                WFORCE.92
      SIGN34=0.5*(SIG(6,NEL)+SIG(8,NEL))                                WFORCE.93
      IF(SIGN12 .GT. 0.0 .AND. SIGN34 .LT. 0.0) GO TO 410               WFORCE.94
C     TENSION                                                           WFORCE.95
      FNOD(K1)=0.0                                                      WFORCE.96
      FNOD(K2)=0.0                                                      WFORCE.97
      GO TO 411                                                         WFORCE.98
  410 CONTINUE                                                          WFORCE.99
      FNOD(K1)=ABS(SIGN12)                                              WFORCE.100
      FNOD(K2)=-SIG(K1,NEL)                                             WFORCE.101
  411 CONTINUE                                                          WFORCE.102
      WFX(I)=WFX(I)+ABS(FNOD(K1))                                       WFORCE.103
      WMT=WMT+ABS(FNOD(K1))*ABS(Y(NOD)-Y(LNOD))                         WFORCE.104
      WFY(I)=WFY(I)+FNOD(K2)                                            WFORCE.105
  303 CONTINUE                                                          WFORCE.106
  302 CONTINUE                                                          WFORCE.107
      IF(NTYPE .EQ. 3) GO TO 421                                        WFORCE.108
      IF(WFX(I) .EQ. 0.0) GO TO 412                                     WFORCE.109
      YBAR(I)=WMT/WFX(I)                                                WFORCE.110
      WFX(I)=WFX(I)*144.0                                               WFORCE.111
      WFY(I)=WFY(I)*144.0                                               WFORCE.112
      GO TO 301                                                         WFORCE.113
  412 CONTINUE                                                          WFORCE.114
      YBAR(I)=0.0                                                       WFORCE.115
      GO TO 301                                                         WFORCE.116
C     CASE OF FRICTION ELEMENT                                          WFORCE.117
  421 CONTINUE                                                          WFORCE.118
      IF(WFX(I) .EQ. 0) GO TO 422                                       WFORCE.119
      YBAR(I)=WMT/WFX(I)                                                WFORCE.120
      GO TO 301                                                         WFORCE.121
  422 CONTINUE                                                          WFORCE.122
      YBAR(I)=0.0                                                       WFORCE.123
  301 CONTINUE                                                          WFORCE.124
      RETURN                                                            WFORCE.125
      END                                                               WFORCE.126
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      SUBROUTINE PRINTR(ID,IX,U,V,ACC,TEPS,PEPS,SIG,PSIG,WFX,YBAR,WFY,  PRINTR.2
     1              NPASS,NIND,HSIG,NTW,NUMELE)                         PRINTR.3
C                                                                       PRINTR.4
C     ******************************************************************PRINTR.5
C     PRINT THE CALCULATED RESULTS AND STORE ON TAPE 2 FOR PLOTTING     PRINTR.6
C     ******************************************************************PRINTR.7
C                                                                       PRINTR.8
      COMMON/NATER/NUNATS,NUMATC,NUMATF,NUMATE,NUNATB,NUMGE,NINTCV,THICKPRINTR.9
     1,NPLAN                                                            PRINTR.10
      COMMON/ELPAR/NUMNP,NUMEL,NEQ,NBAND,KLIN                           PRINTR.11
      COMMON/TIME/JUMP,T,DT,NPRTM,NTAPE,KPRINT                          PRINTR.12
      COMMON/ABS/UGXT,UGYT,VGXT,VGYT,OACCX,OACCY                        PRINTR.13
      DIMENSION ID(3,1),IX(6,1),U(1),V(1),ACC(1),TEPS(3,1),PEPS(3,1),   PRINTR.14
     1          SIG(8,1),PSIG(4,1),NPASS(1),NIND(1),WFX(1),YBAR(1),     PRINTR.15
     2          WFY(1),D(9)                                             PRINTR.16
      COMMON/SPECIL/ISTOP                                               PRINTR.17
      DIMENSION HSIG(9,NUMELE)                                          PRINTR.18
C                                                                       PRINTR.19
C                                                                       PRINTR.20
C     DETERMINE TO BE PRINT OR NOT                                      PRINTR.21
      IF(JUMP .EQ. 1) KPRINT=NPRTM                                      PRINTR.22
      IF(JUMP .EQ. 1) GO TO 401                                         PRINTR.23
      IF(ISTOP .EQ. 1) GO TO 401                                        PRINTR.24
      IF(JUMP .NE. KPRINT) GO TO 411                                    PRINTR.25
      ISTOP=2                                                           PRINTR.26
      KPRINT=KPRINT+NPRTN                                               PRINTR.27
C     STATIC RESULT                                                     PRINTR.28
      IF(JUMP .NE. 0) GO TO 401                                         PRINTR.29
      DO 351 I=1,NUMEL                                                  PRINTR.30
      NPASS(I)=0                                                        PRINTR.31
  351 NIND(I)=0                                                         PRINTR.32
      DO 352 N=1,NUMEL                                                  PRINTR.33
      DO 352 I=1,9                                                      PRINTR.34
      HSIG(I,N)=0.0                                                     PRINTR.35
  352 CONTINUE                                                          PRINTR.36
      WRITE(6,501)                                                      PRINTR.37
      GO TO 402                                                         PRINTR.38
  401 CONTINUE                                                          PRINTR.39
      WRITE(6,502)                                                      PRINTR.40
  402 CONTINUE                                                          PRINTR.41
C                                                                       PRINTR.42
C     NODAL POINT DISPLACEMENT,VELOCITY,ACCELERATION                    PRINTR.43
      WRITE(6,503)                                                      PRINTR.44
      WRITE(6,101) JUMP,T                                               PRINTR.45
C                                                                       PRINTR.46
C     GROUND DISPLACEMENT,VELOCITY,ACCELERATION                         PRINTR.47
      IF(JUMP .EQ. 0) GO TO 405                                         PRINTR.48
      WRITE(6,104) UGXT,UGYT,VGXT,VGYT,OACCX,DACCY                      PRINTR.49
  405 CONTINUE                                                          PRINTR.50
      DO 301 N=1,NUMNP                                                  PRINTR.51
      DO 302 I=1,9                                                      PRINTR.52
  302 D(I)=0.0                                                          PRINTR.53
      DO 303 I=1,3                                                      PRINTR.54
      M=ID(I,N)                                                         PRINTR.55
      IF(M .EQ. 0) GO TO 303                                            PRINTR.56
      D(I)=U(M)                                                         PRINTR.57
      D(I+3)=V(M)                                                       PRINTR.58
      D(I+6)=ACC(M)                                                     PRINTR.59
  303 CONTINUE                                                          PRINTR.60
      WRITE(6,102) N,(D(I),I=1,9)                                       PRINTR.61
  301 CONTINUE                                                          PRINTR.62
C                                                                       PRINTR.63
C     STRESS,STRAIN AT CENTER OF SOIL ELEMENT                           PRINTR.64
      WRITE(6,504)                                                      PRINTR.65
  411 CONTINUE                                                          PRINTR.66
      DO 304 N=1,NUMEL                                                  PRINTR.67
      NTYPE=IX(5,N)                                                     PRINTR.68
      IF(NTYPE .NE. 1) GO TO 304                                        PRINTR.69
      DO 305 I=1,9                                                      PRINTR.70
  305 D(I)=0.0                                                          PRINTR.71
      DO 306 I=1,3                                                      PRINTR.72
      HSIG(I,N)=SIG(I,N)                                                PRINTR.73
      D(I)=SIG(I,N)                                                     PRINTR.74
      D(I+3)=TEPS(I,N)                                                  PRINTR.75
      D(I+6)=PEPS(I,N)                                                  PRINTR.76
  306 CONTINUE                                                          PRINTR.77
      HSIG(9,N)=FLOAT(NPASS(N))                                         PRINTR.78
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 416                       PRINTR.79
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      IF(ISTOP .NE. 2) GO TO 304                                        PRINTR.80
  416 CONTINUE                                                          PRINTR.81
      WRITE(6,102) N,(D(I),I=1,9)                                       PRINTR.82
  304 CONTINUE                                                          PRINTR.83
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 417                       PRINTR.84
      IF(ISTOP .NE. 2) GO TO 412                                        PRINTR.85
  417 CONTINUE                                                          PRINTR.86
      WRITE(6,509)                                                      PRINTR.87
      DO 321 N=1,NUMEL                                                  PRINTR.88
      NTYPE=IX(5,N)                                                     PRINTR.89
      IF(NTYPE .NE. 1) GO TO 321                                        PRINTR.90
      WRITE(6,105) N,(PSIG(I,N),I=1,4),NPASS(N)                         PRINTR.91
  321 CONTINUE                                                          PRINTR.92
  412 CONTINUE                                                          PRINTR.93
      IF(NUMATC .EQ. 0) GO TO 406                                       PRINTR.94
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 418                       PRINTR.95
      IF(ISTOP .NE. 2) GO TO 413                                        PRINTR.96
  418 CONTINUE                                                          PRINTR.97
C                                                                       PRINTR.98
C     END FORCES OF BEAM,COLUMN                                         PRINTR.99
      WRITE(6,505)                                                      PRINTR.100
  413 CONTINUE                                                          PRINTR.101
      DO 307 N=1,NUMEL                                                  PRINTR.102
      NTYPE=IX(5,N)                                                     PRINTR.103
      IF(NTYPE .NE. 2) GO TO 307                                        PRINTR.104
      DO 353 I=1,6                                                      PRINTR.105
      HSIG(I,N)=SIG(I,N)                                                PRINTR.106
  353 CONTINUE                                                          PRINTR.107
      HSIG(9,N)=FLOAT(NIND(N))                                          PRINTR.108
      DO 308 I=1,4                                                      PRINTR.109
  308 D(I)=0.0                                                          PRINTR.110
      D(1)=SIG(4,N)                                                     PRINTR.111
      D(2)=SIG(2,N)                                                     PRINTR.112
      D(3)=SIG(3,N)                                                     PRINTR.113
      D(4)=SIG(6,N)                                                     PRINTR.114
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 419                       PRINTR.115
      IF(ISTOP .NE. 2) GO TO 307                                        PRINTR.116
  419 CONTINUE                                                          PRINTR.117
      WRITE(6,102) N,(D(I),I=1,4),HSIG(9,N)                             PRINTR.118
  307 CONTINUE                                                          PRINTR.119
  406 CONTINUE                                                          PRINTR.120
      IF(NUMATF .EQ. 0 .AND. NUMATE .EQ. 0) GO TO 407                   PRINTR.121
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 420                       PRINTR.122
      IF(ISTOP .NE. 2) GO TO 414                                        PRINTR.123
  420 CONTINUE                                                          PRINTR.124
C                                                                       PRINTR.125
C     NODAL FORCE AT FRICTIONAL,AND EXPANSION JOINT ELEMENT             PRINTR.126
      WRITE(6,506)                                                      PRINTR.127
  414 CONTINUE                                                          PRINTR.128
      DO 358 N=1,NUMEL                                                  PRINTR.129
      NTYPE=IX(5,N)                                                     PRINTR.130
      IF(NTYPE .NE. 3 .AND. NTYPE .NE. 4) GO TO 358                     PRINTR.131
      N1=IX(1,N)                                                        PRINTR.132
      N2=IX(2,N)                                                        PRINTR.133
      N3=IX(3,N)                                                        PRINTR.134
      N4=IX(4,N)                                                        PRINTR.135
      DO 309 I=1,8                                                      PRINTR.136
  309 D(I)=0.0                                                          PRINTR.137
      DO 310 I=1,8                                                      PRINTR.138
      HSIG(I,N)=SIG(I,N)                                                PRINTR.139
  310 D(I)=SIG(I,N)                                                     PRINTR.140
      HSIG(9,N)=FLOAT(NIND(N))                                          PRINTR.141
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 421                       PRINTR.142
      IF(ISTOP .NE. 2) GO TO 358                                        PRINTR.143
  421 CONTINUE                                                          PRINTR.144
      WRITE(6,103) N,N1,N2,N3,N4,(D(I),I=1,8)                           PRINTR.145
  358 CONTINUE                                                          PRINTR.146
  407 CONTINUE                                                          PRINTR.147
      IF(NUMATB .EQ. 0) GO TO 408                                       PRINTR.148
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 422                       PRINTR.149
      IF(ISTOP .NE. 2) GO TO 415                                        PRINTR.150
  422 CONTINUE                                                          PRINTR.151
C                                                                       PRINTR.152
C     AXIAL FORCE AT BOUNDARY SPRING                                    PRINTR.153
      WRITE(6,507)                                                      PRINTR.154
  415 CONTINUE                                                          PRINTR.155
      DO 311 N=1,NUMEL                                                  PRINTR.156
      NTYPE=IX(5,N)                                                     PRINTR.157
      IF(NTYPE .NE. 5) GO TO 311                                        PRINTR.158
      D(1)=SIG(2,N)                                                     PRINTR.159
      HSIG(1,N)=SIG(2,N)                                                PRINTR.160
      HSIG(9,N)=FLOAT(NIND(N))                                          PRINTR.161
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 423                       PRINTR.162
      IF(ISTOP .NE. 2) GO TO 311                                        PRINTR.163
  423 CONTINUE                                                          PRINTR.164
      WRITE(6,102) N,D(1)                                               PRINTR.165
  311 CONTINUE                                                          PRINTR.166
  408 CONTINUE                                                          PRINTR.167
C                                                                       PRINTR.168
C     FORCES AGAINST THE WALL                                           PRINTR.169
      IF(NTW .EQ. 0) GO TO 410                                          PRINTR.170
      IF(ISTOP .EQ. 1 .OR. JUMP .EQ. 1) GO TO 424                       PRINTR.171
      IF(ISTOP .NE. 2) GO TO 410                                        PRINTR.172
  424 CONTINUE                                                          PRINTR.173
      WRITE(6,508)                                                      PRINTR.174
      DO 312 N=1,NTW                                                    PRINTR.175
      WRITE(6,102) N,WFX(N),YBAR(N),WFY(N)                              PRINTR.176
  312 CONTINUE                                                          PRINTR.177
C                                                                       PRINTR.178
C     WRITE INFORMATION ON TAPE 2 FOR PLOTTING                          PRINTR.179
  410 CONTINUE                                                          PRINTR.180
      IF(JUMP .EQ. 0) GO TO 403                                         PRINTR.181
      KJJ=(JUMP/NTAPE)*NTAPE                                            PRINTR.182
      IF(KJJ .NE. JUMP) GO TO 404                                       PRINTR.183
  403 CONTINUE                                                          PRINTR.184
      WRITE(21) (U(I),I=1,NEQ),(ACC(I),I=1,NEQ)                         PRINTR.185
      WRITE(22) ((HSIG(I,N),I=1,9),N=1,NUMEL)                           PRINTR.186
      IF(NTW .EQ. 0) GO TO 404                                          PRINTR.187
      WRITE(23) (WFX(I),YBAR(I),WFY(I),I=1,NTW)                         PRINTR.188
  404 CONTINUE                                                          PRINTR.189
      ISTOP=0                                                           PRINTR.190
       RETURN                                                           PRINTR.191
  101 FORMAT(I5,F10.4)                                                  PRINTR.192
  102 FORMAT(15X,I5,9E12.4)                                             PRINTR.193
  103 FORMAT(15X,I8,4I3,8E11.3)                                         PRINTR.194
  104 FORMAT(' GROUND MOTION',6X,2E12.4,12X,2E12.4,12X,2E12.4)          PRINTR.195
  105 FORMAT(15X,I5,4E12.4,I10)                                         PRINTR.196
  501 FORMAT('1STATIC ANALYSIS RESULT'//)                               PRINTR.197
  502 FORMAT('1DYNAMIC ANALYSIS RESULT'//)                              PRINTR.198
  503 FORMAT(' RELATIVE DISP-U,VELOC-V,TOTAL ACCEL-ACC'//               PRINTR.199
     1       ' TIME        TIME NODE',                                  PRINTR.200
     2       4X,'      U-X',4X,'      U-Y',4X,'     U-ZZ',              PRINTR.201
     3       4X,'      V-X',4X,'      V-Y',4X,'     V-ZZ',              PRINTR.202
     4       4X,'    ACC-X',4X,'    ACC-Y',4X,'   ACC-ZZ'/              PRINTR.203
     5       ' STEP        SEC  NO.',                                   PRINTR.204
     6       4X,'     INCH',4X,'     INCH',4X,'      RAD.',             PRINTR.205
     7       4X,'INCH/SEC',4X,'INCH/SEC',4X,'RAD./SEC',                 PRINTR.206
     8       4X,' IN/S..2',4X,' IN/S..2',4X,'RAD/S..2'//)               PRINTR.207
  504 FORMAT(15X,' STRESS,STRAIN AT THE CENTER OF SOIL ELEMENT'/        PRINTR.208
     1   15X,' SIGX ,SIGY , SIGXY-THE STRESSES'/                        PRINTR.209
     2   15X,' TEPSX,TEPSY,TEPSXY-THE STRAINS IN X,Y COORDS'/           PRINTR.210
     3   15X,' PEPS1,PEPS2,PEPS12-THE PRINCIPAL STRAINS,MAX. SHEAR'//   PRINTR.211
     4    '                ELEM',                                       PRINTR.212
     5    6X,' SIGX',6X,'  SIGY',6X,' SIGXY',                           PRINTR.213
     6    6X,' TEPSX',6X,' TEPSY',6X,'TEPSXY',                          PRINTR.214
     7    6X,' PEPS1',6X,' PEPS2',6X,'PEPS12'/                          PRINTR.215
     8    '                 NO.',                                       PRINTR.216
     9    6X,'   KSI',6X,'   KSI',6X,'   KSI'//)                        PRINTR.217
  505 FORMAT(15X,' BEAM,COLUMN FORCES,MOMENT'//                         PRINTR.218
     1    '                ELEM',                                       PRINTR.219
     2    5X,'  AXIAL',5X,'  SHEAR',5X,'BENDING',5X,'BENDING',5X,'YIELD'PRINTR.220
     3    /'                NO.',                                       PRINTR.221
     4    5X,'    KIP',5X,'    KIP',5X,'M3,K-IN',5X,'M6,K-IN'//)        PRINTR.222
  506 FORMAT(15X,' NODAL FORCES AT FRICTION AND EXPANSION ELEMENT'/     PRINTR.223
     1    '                ELEMENT NODAL ORDER',                        PRINTR.224
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2     8X,* H1*,8X,* V1*,8X,* H2*,8X,* V2*,                           PRINTR.225
3     8X,* H3*,8X,* Y3*,8X,* H4*,8X,* V4* /                          PRINTR.226
4        *                      NO. N1 N2 N3 N4*,8X,*KIP*//)          PRINTR.227
507 FORMAT(15X,* AXIAL FORCES AT BOUNDARY SPRING*//                   PRINTR.228
1       15X,* ELEMENT*,6X,* AXIAL*/                                   PRINTR.229
2       15X,*      NO.*,6X,*   KIP*/)                                 PRINTR.230
508 FORMAT(15X,* TOTAL FORCES ACTING ON THE WALL*/                    PRINTR.231
1    15X,* FORCE PERPEDICULAR TO WALL..X-FORCE*/                      PRINTR.232
2    15X,* FORCE PARALLEL TO THE WALL...Y-FORCE*/                     PRINTR.233
3    15X,* LINE OF ACTION FROM THE BASE..YBAR*//                      PRINTR.234
4    15X,* WALL*,4X,* X-FORCE*,TX,* YBAR*,4X,* Y-FORCE*/              PRINTR.235
5    15X,* NO.*,4X,*      KIP*,TX,*   FT*,4X,*      KIP*//)           PRINTR.236
509 FORMAT(15X,* PSIG1,PSIG2,PSIG3,PSIG4,-PRINCIPAL STRESSES,MAX.SHEARPRINTR.237
  1PRINCIPAL DIRECTION*/                                              PRINTR.238
  2   15X,* NPASS=0 ELASTIC, 1 ACTIVE FAILURE, 2 PASSIVE FAILURE*//   PRINTR.239
  3      *                      ELEMENT*,                             PRINTR.240
  4     6X,* PSIG1*,6X,* PSIG2*,6X,* PSIG3*,6X,* PSIG4*,4X,* NPASS*//)PRINTR.241
   END                                                                PRINTR.242


      SUBROUTINE NTSTIF(X,Y,PARASO,PARACO,PARAFR,PARAEX,PARABO,COPROP, NTSTIF.2
     1K2S,CMP,IX,IBC,SLV,C,AEN,U,V,DU,TEPS,DEPS,PEPS,OSIG,SIG,PSIG,     NTSTIF.3
     2NPASS,NINO,SY,GS,NEQ,MBAND,NUMEL,KLIN)                           NTSTIF.4
C     ******************************************************************NTSTIF.5
C     FORM NON-LINEAR TOTAL TANGENT STIFFNESS MATRIX AEN(NEQ,MBAND)     NTSTIF.6
C     ******************************************************************NTSTIF.7
      COMMON/EM/LN(8),ASA(8,8)                                         NTSTIF.8
      COMMON/NELEM/N                                                    NTSTIF.9
      DIMENSION X(1),Y(1),PARASO(8,1),PARACO(5,1),PARAFR(4,1),         NTSTIF.10
     1PARAEX(4,1),PARABO(2,1),K2S(20,1),CMP(9,1),IX(6,1),IBC(4,1),U(1), NTSTIF.11
     2V(1),DU(1),TEPS(3,1),OEPS(3,1),PEPS(3,1),DSIG(8,1),SIG(8,1),      NTSTIF.12
     3PSIG(4,1),SY(6,6),C(5,1),NPASS(1),NIND(1),AEN(NEQ,1)              NTSTIF.13
      DIMENSION SS(138),SA2(6,6),SA4(8,8),UE(4),YE(8),DTU(8),F(8)       NTSTIF.14
      DIMENSION XX(4),YY(4),AC(3),BC(3),CC(3)                           NTSTIF.15
      COMMON/TIME/JUMP,T,DT,MPRTM,MTAPE,KPRINT                          NTSTIF.16
      DIMENSION COPROP(3,1),SLV(1)                                      NTSTIF.17
      COMMON/NONL/NREAD                                                 NTSTIF.18
      COMMON/MATER/NUMATS,NUMATC,NUMATF,NUMATE,NUMATB,NUMGE,NINTCV,THICKNTSTIF.19
     1,NPLAN                                                            NTSTIF.20
      EQUIVALENCE(SS,ISS)                                               NTSTIF.21
      DIMENSION ISS(138)                                                NTSTIF.22
      DIMENSION SL(6,6)                                                 NTSTIF.23
      COMMON/ELASTC/SE(8,8)                                             NTSTIF.24
      REAL K2S                                                          NTSTIF.25
C                                                                       NTSTIF.26
C     IF LINEAR ANALYSIS,GO BACK                                        NTSTIF.27
      NREAD=1                                                           NTSTIF.28
      IF(KLIN .EQ. 0)RETURN                                             NTSTIF.29
C     INITILIZATION                                                     NTSTIF.30
      DO 351 I=1,5                                                      NTSTIF.31
      DO 351 J=1,NUMEL                                                  NTSTIF.32
  351 C(I,J)=0.0                                                        NTSTIF.33
      DO 347 I=1,8                                                      NTSTIF.34
      DO 347 J=1,8                                                      NTSTIF.35
      SE(I,J)=0.0                                                       NTSTIF.36
  347 CONTINUE                                                          NTSTIF.37
      NYIELD=0                                                          NTSTIF.38
C                                                                       NTSTIF.39
C     READ TOTAL ELASTIC STIFFNESS FROM TAPE 3,AND PUT INTO--AEN        NTSTIF.40
      REWIND 3                                                          NTSTIF.41
      READ(3) ((AEN(I,J),I=1,NEQ),J=1,MBAND)                            NTSTIF.42
      REWIND 41                                                         NTSTIF.43
      REWIND 1                                                          NTSTIF.44
C                                                                       NTSTIF.45
C     CHECK YIELD CONDITION OF ELEMENTS,ONE BY ONE                      NTSTIF.46
      DO 302 N=1,NUMEL                                                  NTSTIF.47
      NTYPE=IX(5,N)                                                     NTSTIF.48
      MATYPE=IX(6,N)                                                    NTSTIF.49
      GO TO (401,402,403,403,405),NTYPE                                 NTSTIF.50
  401 CONTINUE                                                          NTSTIF.51
C                                                                       NTSTIF.52
C     TYPE 1,SOIL ELEMENT                                               NTSTIF.53
      CALL SOILNE(PARASO(1,MATYPE),K2S(1,MATYPE),TEPS(1,N),DEPS(1,N),   NTSTIF.54
     1PEPS(1,N),OSIG(1,N),SIG(1,N),PSIG(1,N),SY,GS,SIGMST,NINO(N),      NTSTIF.55
     2NPASS(N))                                                         NTSTIF.56
      READ(1) (SS(I),I=1,19)                                            NTSTIF.57
      KK=7                                                              NTSTIF.58
      DO 341 I=1,3                                                      NTSTIF.59
      KI=KK+3*I                                                         NTSTIF.60
      DO 342 J=1,3                                                      NTSTIF.61
      JI=KI+J                                                           NTSTIF.62
  342 SE(J,I)=SS(JI)                                                    NTSTIF.63
  341 CONTINUE                                                          NTSTIF.64
C                                                                       NTSTIF.65
C     CHECK IF IT IS YIELD                                              NTSTIF.66
      IF(NINO(N) .NE. 0) GO TO 321                                      NTSTIF.67
      WRITE(41) (SS(I),I=1,19)                                          NTSTIF.68
      GO TO 302                                                         NTSTIF.69
  321 CONTINUE                                                          NTSTIF.70
C                                                                       NTSTIF.71
C     LOCATION OF MASS                                                  NTSTIF.72
      DO 303 I=1,8                                                      NTSTIF.73
  303 LM(I)=ISS(2+I)                                                    NTSTIF.74
      DO 304 I=1,4                                                      NTSTIF.75
      NODE=IX(I,N)                                                      NTSTIF.76
      XX(I)=X(NODE)                                                     NTSTIF.77
      YY(I)=Y(NODE)                                                     NTSTIF.78
  304 CONTINUE                                                          NTSTIF.79
      IF(NINO(N) .EQ. 2) GO TO 451                                      NTSTIF.80
C                                                                       NTSTIF.81
C     FORM TANGENT STIFFNESS,BY SUBTRACTING ASA FROM AEN                NTSTIF.82
      CALL QUADS(XX,YY,C(1,N),SY,NINO(N))                               NTSTIF.83
      CALL SUBTRC(AEN,NEQ)                                              NTSTIF.84
      GO TO 302                                                         NTSTIF.85
  451 CONTINUE                                                          NTSTIF.86
C                                                                       NTSTIF.87
C     CASE OF CHANGE STIFFNESS BUT NOT YIELD AFTER STATIC NANLYSIS      NTSTIF.88
      ANU=PARASO(3,MATYPE)                                              NTSTIF.89
      IF(NPLAN .EQ. 1) GO TO 452                                        NTSTIF.90
C                                                                       NTSTIF.91
C     PLANE STRAIN                                                      NTSTIF.92
      B1=2.0*(1.0-ANU)                                                  NTSTIF.93
      B2=1.0-2.0*ANU                                                    NTSTIF.94
      C(1,N)=GS*B1/B2                                                   NTSTIF.95
      C(2,N)=GS*2.0*ANU/B2                                              NTSTIF.96
      C(3,N)=GS                                                         NTSTIF.97
      GO TO 453                                                         NTSTIF.98
C                                                                       NTSTIF.99
C     PLANE STRESS                                                      NTSTIF.100
  452 CONTINUE                                                          NTSTIF.101
      B1=1.0-ANU                                                        NTSTIF.102
      C(1,N)=GS*2.0/B1                                                  NTSTIF.103
      C(2,N)=GS*2.0*ANU/B1                                              NTSTIF.104
      C(3,N)=GS                                                         NTSTIF.105
  453 CONTINUE                                                          NTSTIF.106
      CALL QUADS(XX,YY,C(1,N),SY,NINO(N))                               NTSTIF.107
      CALL SUBTRC(AEN,NEQ)                                              NTSTIF.108
      NYIELD=2                                                          NTSTIF.109
      GO TO 302                                                         NTSTIF.110
C                                                                       NTSTIF.111
C     TYPE 2,BEAM,COLUMN ELEMENT                                        NTSTIF.112
  402 CONTINUE                                                          NTSTIF.113
C                                                                       NTSTIF.114
C     READ ELASTIC INFORMATION FROM TAPES 1                             NTSTIF.115
      READ(1) (SS(I),I=1,80)                                            NTSTIF.116
C                                                                       NTSTIF.117
C     IF ELASTIC BEAM,GO BACK                                           NTSTIF.118
      IF(PARACO(1,MATYPE) .NE. PARACO(2,MATYPE)) GO TO 322              NTSTIF.119
      WRITE(41) (SS(I),I=1,80)                                          NTSTIF.120
      GO TO 302                                                         NTSTIF.121
  322 CONTINUE                                                          NTSTIF.122
C                                                                       NTSTIF.123
C     LOCATION OF MASS                                                  NTSTIF.124
C     FORM ELASTIC DISPLACEMENT(GLOBAL)-STRESS(LOCAL) MATRIX-SA2(6,6)   NTSTIF.125
      KK=38                                                             NTSTIF.126
      DO 354 I=1,6                                                      NTSTIF.127
      KI=KK+6*I                                                         NTSTIF.128
```

```
      DO 305 J=1,6
      JI=KI+J
  305 SA2(J,I)=SS(JI)
  354 CONTINUE
      DO 346 I=1,6
      DO 346 J=1,6
      SE(I,J)=SA2(I,J)
  346 CONTINUE
C     FORM DISPLACEMENT STRESS MATRIX,LOCAL--SL(6,6)
      KK=2
      DO 355 I=1,6
      KI=KK+6*I
      DO 356 J=1,6
      JI=KI+J
  356 SL(J,I)=SS(JI)
  355 CONTINUE
      DO 307 I=1,6
  307 LM(I)=ISS(2+I)
      LM(7)=0
      LM(8)=0
C
C     FIND INCREMENTAL NODAL DISPLACEMENT
      DO 308 I=1,6
      JJ=LM(I)
      IF(JJ .NE. 0) GO TO 406
      DTU(I)=0.0
      GO TO 407
  406 DTU(I)=DU(JJ)
  407 CONTINUE
      F(I)=SIG(I,N)-DSIG(I,N)
  308 CONTINUE
C
C     CALCULATE COEFFICIENT OF YIELD FCTN
      NINT=IBC(4,N)
      AC(1)=1.0
      BC(1)=0.0
      CC(1)=-CMP(2,NINT)/(CMP(5,NINT))
      AC(2)=1.0-CMP(3,NINT)/CMP(6,NINT)
      BC(2)=CMP(2,NINT)/CMP(5,NINT)-1.0
      CC(2)=-BC(2)-AC(2)
      AC(3)=1.0-CMP(9,NINT)/CMP(6,NINT)
      BC(3)=CMP(8,NINT)/CMP(5,NINT)-1.0
      CC(3)=-AC(3)-BC(3)
      CALL CONCNE(CMP(1,NINT),F,DTU,SA2,AC,BC,CC,SY,NINDIN),SL)
C
      IF(JUMP .EQ. 0) NIND(N)=0
C     CHECK IF IT IS YIELD
      IF(NIND(N) .NE. 0) GO TO 323
      WRITE(41) (SS(I),I=1,80)
      GO TO 302
  323 CONTINUE
C
C     FORM YIELD STIFFNESS ASA-GLOBAL
      DO 309 I=1,2
      NODE=IX(I,N)
      XX(I)=X(NODE)
      YY(I)=Y(NODE)
  309 CONTINUE
       CALL CONCTS(XX,YY,PARACO,COPROF,IX(1,N),IBC(1,N),C(1,N),SLV,SY,
     1             NIND(N))
C
C     FORM TANGENT STIFFNESS BY SUBTRACTING ASA FROM AEN
      CALL SUBTRC(AEN,NEQ)
      GO TO 302
  403 CONTINUE
C
C     TYPE 3,FRICTION ELEMENT
C     TYPE 4,EXPANSION JOINT
C
C     READ ELEMENT INFORMATION FROM TAPE 1
      READ(1) (SS(I),I=1,138)
C     SET UP DISPLACEMENT(GLOBAL)-STRESS(LOCAL) MATRIX-SA4(8,8)
      KK=66
      DO 310 I=1,8
      KI=KK+8*I
      DO 311 J=1,8
      JI=KI+J
  311 SA4(J,I)=SS(JI)
  310 CONTINUE
      DO 343 I=1,8
      DO 343 J=1,8
      SE(I,J)=SA4(I,J)
  343 CONTINUE
C
C     LOCATION OF MASS
      DO 312 I=1,8
  312 LM(I)=ISS(2+I)
C
C     FIND INCREMENTAL NODAL DISPLACEMENT,AND NODAL FORCES
      DO 313 I=1,8
      II=(I+1)/2
      JJ=LM(I)
      IF(JJ .NE. 0) GO TO 408
      UE(I)=0.0
      DTU(I)=0.0
      GO TO 409
  408 CONTINUE
      DTU(I)=DU(JJ)
      IF(MOD(I,2) .EQ. 0) GO TO 409
      UE(II)=U(JJ)
  409 F(I)=SIG(I,N)
      VE(I)=V(JJ)
  313 CONTINUE
      IF(NTYPE .EQ. 4) GO TO 410
C     FRICTION ELEMENT
C
      CALL PFRICT(PARAFR(1,MATYPE),SA4,DTU,F,C(1,N),NINDIN))
      GO TO 411
  410 CONTINUE
      CALL PEXPAN(PARAFR(1,MATYPE),PARAEX(1,MATYPE),SA4,UE,DTU,VE,F,
     1            C(1,N),NINDIN))
  411 CONTINUE
C
C     CHECK IF IT IS YIELD
      IF(NINDIN) .NE. 0) GO TO 324
      WRITE(41) (SS(I),I=1,138)
      GO TO 302
  324 CONTINUE
      DO 314 I=1,4
      NODE=IX(I,N)
      XX(I)=X(NODE)
      YY(I)=Y(NODE)
  314 CONTINUE
C
C     FORM TANGENT STIFFNESS-ASA(GLOBAL)
      CALL FRICTS(XX,YY,C(1,N))
      CALL SUBTRC(AEN,NEQ)
      GO TO 302
C
  405 CONTINUE
C     TYPE 5,BOUNDARY ELEMENT
      READ(1) (SS(I),I=1,8)
C
C     LOCATION OF MASS
      DO 315 I=1,2
  315 LM(I)=ISS(2+I)
      DO 360 I=3,8
      LM(I)=0
  360 CONTINUE
      KK=2
      DO 344 I=1,2
      KI=KK+2*I
      DO 345 J=1,2
      JI=KI+J
  345 SE(J,I)=SS(JI)
  344 CONTINUE
      DO 316 I=1,8
      JJ=LM(I)
      IF(JJ .NE. 0) GO TO 412
      DTU(I)=0.0
      GO TO 413
  412 CONTINUE
      DTU(I)=DU(JJ)
```

```
413 F(I)=SIG(I,N)-DSIG(I,N)
316 CONTINUE
    CALL PBOUND(PARABO(1,MATYPE),DTU,F,C(1,N),NIND(N))
    IF(NIND(N) .NE. 0) GO TO 325
    WRITE(41) (SS(I),I=1,8)
    GO TO 302
325 CONTINUE
    CALL BOUNDS(C(1,N))
    CALL SUBTRC(AEN,NEQ)
302 CONTINUE
C   SET NEW ELASTIC STIFFNESS AFTER STATIC ANALYSIS
    IF(JUMP .NE. 0) GO TO 414
    IF(NYIELD .NE. 2) GO TO 414
    WRITE(3) ((AEN(I,J),I=1,NEQ),J=1,MBAND)
414 CONTINUE
    RETURN
    END
```

```
    SUBROUTINE SUBTRC(AEN,NEQ)
C
C   ****************************************************************
C   FORM TANGENT STIFFNESS BY SUBTRACTING THE YIELD STIFFNESS ASA FROM
C   TOTAL ELASTIC STIFFNESS-AEN
C   ****************************************************************
    DIMENSION AEN(NEQ,1)
    COMMON/EN/LM(8),ASA(8,8)
    DO 301 I=1,8
    II=LM(I)
    IF(II .EQ. 0) GO TO 301
    DO 302 J=1,8
    JJ=LM(J)-II+1
    IF(JJ .LT. 1) GO TO 301
302 AEN(II,JJ)=AEN(II,JJ)-ASA(I,J)
301 CONTINUE
    RETURN
    END
```

```
    SUBROUTINE SOILNE(PARASO,K2S,TEPS,DEPS,PEPS,DSIG,SIG,PSIG,
   1                  SY,GS,SIGNST,NIND,NPASS)
C
C   ****************************************************************
C   CHECK THE SOIL IS YIELD,AND FORM YIELD STIFFNESS MATRIX--
C   SY(LOCAL)--BY CALING SUBROUTINE PSOIL
C   ****************************************************************
C
    DIMENSION PARASO(8),K2S(20),TEPS(3),DEPS(3),PEPS(3),DSIG(8),
   1          SIG(8),PSIG(4),SY(6,6),PSSIG(3)
    COMMON/TIME/JUMP,T,DT,NPRTN,MTAPE,KPRINT
    COMMON/MATER/NUMATS,NUMATC,NUMATF,NUMATE,NUMATB,NUMGE,NINTCV,THICK
   1,NPLAN
    DIMENSION SIGNSN(350)
    COMMON/NELEN/N
    REAL K2S
C
C   SET UP LINEAR INDICATOR,AND BASIC PARAMETERS OF SOIL
    NIND=0
    NPASS=0
    G1=PARASO(1)
    G2=PARASO(2)
    ANU=PARASO(3)
    C=PARASO(4)
    PHI=PARASO(5)
    PHI=PHI/57.2978
    GAMMAY=PARASO(6)
C
C   DETERMINE WHICH STRESS-STRAIN LAW IS USED
C   LINEAR CASE
    IF(G1 .EQ. G2) RETURN
C
C   BILINEAR CASE WITH SHEAR STRAIN CONTROL--FOR ONE DIMENSIONAL SHEAR
C   BEAN CASE ONLY
    IF(GAMMAY .NE. 0.0) GO TO 408
    IF(JUMP .NE. 0) GO TO 403
C
C   SAVE STATIC MEAN STRESS FOR DETECTING ACTIVE OR PASSIVE FAILURE,
C   CALCULATE NEW STIFFNESS AT THE END OF STATIC ANALYSIS
    IF(NPLAN .EQ. 0) GO TO 402
C
C   PLANE STRESS,MEAN SQ. STRESS
    SIGMST=(SIG(1)+SIG(2))/2.0
    SIGMSN(N)=SIGMST
    SIGMST=ABS(SIGMST)
    SIGME=SQRT(SIGMST)
    GO TO 403
C
C   PLAIN STRAIN
402 CONTINUE
    SIGME=(SIG(1)+SIG(2))*(1.0+ANU)/3.0
    SIGMST=(SIG(1)+SIG(2))/2.0
    SIGMSN(N)=SIGMST
    SIGME=ABS(SIGME)
    SIGME=SQRT(SIGME)
403 CONTINUE
C
C   BILINEAR CASE WITH COLOMB-MOHR,WITHOUT STRAIN-HARDENING
    IF(K2S(1) .EQ. 0.0) GO TO 407
    IF(JUMP .NE. 0) GO TO 451
C   ********************************************************
C   TRILINEAR CASE WITH CURVE FROM EERC-70-10,NO HARDENING
C   ********************************************************
    GAMMAX=PEPS(3)
C
C   CHECK WITH SHEAR MODULUS CURVE K2S FOR COFF. K2
    IF(GAMMAX .LT. K2S(2)) RETURN
    DO 301 I=1,10
    IF(GAMMAX .GE. K2S(2*I) .AND. GAMMAX .LT. K2S(2*I+2))  GO TO 405
301 CONTINUE
405 CONTINUE
C
C   CHECK IF IT IS YIELD
    GS=G1
    CALL PSOIL(DSIG,SIG,PSIG,SY,GS,ANU,C,PHI,NIND)
    IF(NIND .EQ. 1) GO TO 406
    IF(GAMMAX .NE. K2S(2*I)) GO TO 409
```

```
      AK2=K2S(2*I-1)
      GO TO 410
  409 AK2=(K2S(2*I-1)+K2S(2*I+1))/2.0
  410 CONTINUE
C
C     MODIFIED ELASTIC STIFFNESS IN KSI
      GS=AK2*2.638*SIGNE
      GS=G1-GS
      NIND=2
      RETURN
  451 CONTINUE
      CALL PSOIL(DSIG,SIG,PSIG,SY,GS,ANU,C,PHI,NIND)
      IF(NIND .EQ. 1) GO TO 406
      RETURN
  406 CONTINUE
C
C     CHECK IF IT IS ACTIVE OR PASSIVE FAILURE,NPASS=1 ACTIVE FAILURE,
C     NPASS=2 PASSIVE FAILURE
C     NPASS=0 EALSTIC
      IF(NIND .EQ. 0) RETURN
      SIGM=(SIG(1)+SIG(2))/2.0
      IF(SIGM .GE. SIGMSN(N)) NPASS=1
      IF(SIGM .LT. SIGMSN(N)) NPASS=2
      RETURN
C     ******************************************
C     BILINEAR WITH COULOMB-MOHR NO HARDENING
C     ******************************************
  407 CONTINUE
      GS=G1
      CALL PSOIL(DSIG,SIG,PSIG,SY,GS,ANU,C,PHI,NIND)
      IF( NIND .EQ. 1) GO TO 411
      RETURN
  411 CONTINUE
C     CHECK IF IT IS ACTIVE OR PASSIVE YIELD
C
      SIGM=(SIG(1)+SIG(2))/2.0
      IF(SIGM .GE. SIGMSN(N)) NPASS=1
      IF(SIGM .LT. SIGMSN(N)) NPASS=2
      RETURN
C     ******************************************
C     SHEAR STRAIN CONTROL-SHEAR BEAM ONLY
C     ******************************************
  408 CONTINUE
      GAMMAX=TEPS(3)
C
C     CALCULATE UPPER LIMIT AND LOWER LIMIT LINES
      TAU=GAMMAX*(G1-G2)+G2*GAMMAX
      TAL=GAMMAX*(G2-G1)+G2*GAMMAX
C     ELASTIC CASE
      IF(SIG(3) .LT. TAU .AND. SIG(3) .GT. TAL) RETURN
C
C     CHECK LOADING OR UNLOADING WHEN YIELDING
      PSSIG(3)=SIG(3)+DEPS(3)*G1
C     UNLOADING FROM YIELDING,ELASTIC CASE
      IF(PSSIG(3) .LT. TAU .AND. PSSIG(3) .GT. TAL) RETURN
C
C     PLANE STRAIN FOR SHEAR BEAM ANALYSIS
      IF(G2 .EQ. 0.0) G2=G1/100000000.0
      GP=G1-G2
      DO 452 I=1,6
      DO 452 J=1,6
  452 SY(I,J)=0.0
      SY(1,1)=2.0*(1.0-ANU)*GP/(1.0-2.0*ANU)
      SY(1,2)=2.0*ANU*GP/(1.0-2.0*ANU)
      SY(3,3)=GP
      SY(2,2)=SY(1,1)
      SY(2,1)=SY(1,2)
      NIND=1
      RETURN
      END
```

```
      SUBROUTINE PSOIL(DSIG,SIG,PSIG,SY,GS,ANU,C,PHI,NIND)
C
C     ****************************************************************
C     FORM PLASTIC STRESS-STRAIN RELATIONSHIP-SY(LOCAL)
C     ****************************************************************
C
      DIMENSION PARASO(8),K2S(20),TEPS(3),DEPS(3),PEPS(3)
      DIMENSION DSIG(8),SIG(8),PSIG(4),SY(6,6),OF(3),QP(3)
C
C     CHECK IF IT IS  YIELD FOR BILINEAR CASE
      T3=2.0*C*COS(PHI)/1000.0
      T1=(1.0+SIN(PHI))*PSIG(1)
      T2=(1.0-SIN(PHI))*PSIG(2)
      F=T1-T2-T3
      IF(F .LT. 0.0) RETURN
C
C     CHECK IF IT IS LOADING OR UNLOADING
      T=(SIG(1)+SIG(2))/2.0
      T2=T**2
      R=T2+SIG(3)**2
      R=SQRT(R)
      OF(1)=T/R/2.0+0.5*SIN(PHI)
      OF(2)=OF(1)
      OF(3)=SIG(3)/R
      DL=OF(1)*DSIG(1)+OF(2)*DSIG(2)+OF(3)*DSIG(3)
      IF(DL .LT. 1.0E-10) RETURN
      NIND=1
      B=0.5*SIN(PHI)
      B1=2.0*B*B/(1.0-2.0*ANU)
      B2=SIG(3)**2/R**2
      AC=B1+0.5+1.5*B2
C
C     FORM COMPONENTS OF PLASTIC STRESS-STRAIN LAW-QP(3)
      TE=(SIG(1)-SIG(2))/4.0/R
      QP(1)=B/(1.0-2.0*ANU)+TE
      QP(2)=B/(1.0-2.0*ANU)-TE
      QP(3)=SIG(3)/R
      EA=GS*2.0/AC
C
C     FORM PLASTIC MODULUS-SY(3,3)
      DO 302 I=1,3
      DO 302 J=1,3
      SY(I,J)=QP(I)*QP(J)
      SY(I,J)=EA*SY(I,J)
  302 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CONCNE(CMP,F,DTU,S,A,B,C,SY,NIND,SL)
C
C     ******************************************************************
C     CHECK IF THE ELEMENT IS YIELD,AND FORM YIELD STIFFNESS SY(LOCAL)
C     BY CALLING PCOL
C     ******************************************************************
C
      DIMENSION F(6),DF(6),DTU(6),S(6,6),FN(2),FP(2),FV(2),
     1          FPN(2),FMN(2),A(3),B(3),C(3),CMP(9),PHI(2)
      DIMENSION SY(6,6),SL(6,6)
      COMMON/TIME/JUMP,T,DT,NPRTN,MTAPE,KPRINT
C
C     SET UP LINEAR INDICATOR NIND=0 ELASTIC,NIND=1 PLASTIC
      NIND=0
C
C     CALCULATE APPARENT ELASTIC INCREMENTAL END FORCES-DF(6)
      DO 301 I=1,6
      XA=0.0
      DO 302 J=1,6
      XA=XA+S(I,J)*DTU(J)
  302 CONTINUE
      DF(I)=XA
  301 CONTINUE
C     CHECK YIELDING CONDITION
C
      FP(1)=F(1)+DF(1)
      FV(1)=F(2)+DF(2)
      FN(1)=F(3)+DF(3)
      FP(2)=F(4)+DF(4)
      FV(2)=F(5)+DF(5)
      FN(2)=F(6)+DF(6)
C
C     TENSIONAL YIELD AT BOTH ENDS
      IF(FP(1) .LT. 0.0 .AND. FP(2) .GT. 0.0) GO TO 405
C
C     NORMALIZE THE END FORCES
      FPN(1)=FP(1)/CMP(5)
      FMN(1)=FN(1)/CMP(6)
      FPN(2)=FP(2)/CMP(5)
      FMN(2)=FN(2)/CMP(6)
      FMN(1)=ABS(FMN(1))
      FMN(2)=ABS(FMN(2))
      FPN(2)=ABS(FPN(2))
C
C     DETERMINE THE ECCENTRICITY IN INCHES
      IF(FP(1) .EQ. 0.0) GO TO 411
      EC1=FN(1)/FP(1)
      EC1=ABS(EC1)
      GO TO 413
  411 EC1=100.0
  413 CONTINUE
      IF(FP(2) .EQ. 0.0) GO TO 412
      EC2=FN(2)/FP(2)
      EC2=ABS(EC2)
      GO TO 414
  412 EC2=100.0
  414 CONTINUE
C
C     FORM YIELD FCTN AT I END
      JJ=0
      DO 303 I=1,3
      II=I-1
      JJ=JJ+1
      IF(EC1 .LE. CMP(3*II+1)) GO TO 401
  303 CONTINUE
  401 CONTINUE
      AM1=A(JJ)
      BM1=B(JJ)
      CM1=C(JJ)
      PHI(1)=1.0+CM1/(AM1*FPN(1)+BM1*FMN(1))
      SIGN=FP(1)*FP(2)
C     CASE OF ONE END TENSION,ONE END COMPRESSION TREAT AS ELASTIC
      IF(SIGN .GT. 0.0) PHI(1)=-0.1
C
C     FORM YIELD FCTN AT J END
      JJ=0
      DO 304 I=1,3
      II=I-1
      JJ=JJ+1
      IF(EC2 .LE. CMP(3*II+1)) GO TO 402
  304 CONTINUE
  402 CONTINUE
      AM2=A(JJ)
      BM2=B(JJ)
      CM2=C(JJ)
      PHI(2)=1.0+CM2/(AM2*FPN(2)+BM2*FMN(2))
C     CASE OF ONE END TENSION,ONE END COMPRESSION TREAT AS ELASTIC
      IF(SIGN .GT. 0.0) PHI(2)=-0.1
      AM1=-AM1/CM1
      BM1=-BM1/CM1
      AM2=-AM2/CM2
      BM2=-BM2/CM2
C
C     BOTH ENDS ELASTIC
      IF(PHI(1) .LT. 0.0 .AND. PHI(2) .LT. 0.0) GO TO 407
C
C     I END YIELD
      IF(PHI(1) .GE. 0.0 .AND. PHI(2) .LT. 0.0) GO TO 403
C
C     J END YIELD
      IF(PHI(1) .LT. 0.0 .AND. PHI(2) .GE. 0.0) GO TO 404
C
C     BOTH ENDS YIELD
      IF(PHI(1) .GE. 0.0 .AND. PHI(2) .GE. 0.0) GO TO 406
C
C     ELASTIC-PLASTIC  BEAM WITH I END YIELD
  403 CONTINUE
      NIND=1
      CALL PCOL(1,AM1,BM1,AM2,BM2,S,SY,SL)
      GO TO 407
C
C     ELASTIC-PLASTIC BEAM WITH J END YIELD
  404 CONTINUE
      NIND=1
      CALL PCOL(2,AM1,BM1,AM2,BM2,S,SY,SL)
      GO TO 407
  405 CONTINUE
C     BOTH ENDS YIELD IN TENSION
      NIND=1
      AM1=-CMP(5)
      AM2=-CMP(5)
      BM1=0.0
      BM2=0.0
  406 CONTINUE
C     ELASTIC-PLASTIC BEAM WITH BOTH ENDS YIELD
      CALL PCOL(3,AM1,BM1,AM2,BM2,S,SY,SL)
  407 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PCOL(NODE,AM1,BM1,AM2,BM2,S,SY,SL(
C
C     ****************************************************************
C     CALCULATE THE STIFFNESS-SY(LOCAL(
C     ****************************************************************
C
      OIMENSION S(6,6(,SY(6,6(,YSI6,2(,BB(2,2(,AAI2,2(,BAI6,2(,BSI2,6(
      OIMENSION SL(6,6(
      COMMON/TIME/JUMP,T,OT,MPRTN,NTAPE,KPRINT
C
C     INITILIZATION
      00 301 I=1,6
      DO 301 J=1,2
      YS((,J)=0.0
  301 CONTINUE
      DO 311 (=1,2
      00 311 J=1,2
      AA((,J(=0.0
  311 CONTINUE
C     SET UP LOCAL STRESS-STRAIN RELATIONSHIP
      00 312 I=1,6
      00 312 J=1,6
      S(I,J(=SL(I,J(
  312 CONTINUE
C
C     NODE=1 I END YIELD,NC=1
C     NODE=2,J END YIELD,NC=1
C     NODE=3,BOTH END YIELO,NC=2
      NC=1
      IFINODE .EQ. 3( GO TO 402
      IF(NODE .EQ. 2( GO TO 401
C
C     ( ENO YIELD
      YS(1,1)=AM1
      YS(3,1(=BM1
      GO TO 403
  401 CONTINUE
C
C     J ENO YIELO
      YS(4,1(=AM2
      YSI6,1(=BM2
      GO TO 403
  402 CONTINUE
C
C     BOTH END YIELD
      NC=2
      YSI1,1(=AM1
      YSI3,1(=BM1
      YS(4,2(=AM2
      YS(6,2(=BM2
  403 CONTINUE
      00 303 K=1,NC
      00 303 I=1,6
      XA=D.0
      00 304 J=1,6
      [illegible]
  304 CONTINUE
      BS(K,I(=XA
  303 CONTINUE
      DO 305 I=1,NC
      00 305 J=1,NC
      XA=D.0
      00 306 K=1,6
      XA=XA+BS(J,K(*YS(K,I(
  306 CONTINUE
      BB(J,((=XA
  305 CONTINUE
      OET=BB(1,1(
      IF(NODE .NE. 3( GO TO 404
      OET=BB(1,1(*BB(2,2(-BB(1,2(*BBI2,1(
  404 CONTINUE
      (F(OET .LT. 1.0E-30( GO TO 405
      AA(1,1)=1.0/DET
      IF(NOOE .NE. 3( GO TO 405
      AA(1,1)=BBI2,2(/OET
      AAI1,2(=-BB(1,2(/DET
      AA(2,1(=AA(1,2(
      AA(2,2(=BB(1,1(/OET
  405 CONTINUE
      00 307 I=1,6
      00 307 J=1,NC
      XA=D.0
      DO 308 K=1,NC
      XA=XA+BS(K,I(*AA(K,J(
  308 CONTINUE
      BA((,J)=XA
  307 CONTINUE
      DO 309 I=1,6
      DO 309 J=1,6
      XA=0.0
      00 310 K=1,NC
      XA=XA+BA((,K(*BS(K,J(
  310 CONTINUE
      SY(I,J(=XA
  309 CONTINUE
      00 313 I=1,6
      IF(SY(I,I( .LE. D.0( SYII,I(=0.9*SLI1,I(
      00 313 J=1,6
      IF(SY(I,J( .GT. 0.9*SLII,J(( SYII,JI=D.9*SL(I,J(
  313 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PFRICT(PARAFR,SA,DTU,P,C,NINO)  .
C
C     ****************************************************************
C     CHECK IF THE FRICTIONAL ELEMENT IS YIELO,AND THEN FORN THE YIELD
C     STRESS-STRAIN CONSTANTS--CYI4(=CI4(
C     ****************************************************************
C
      OIMENSION PARAFRI4(,SAI8,8(,DTUI8(,FI8(,CI4I,DFI8I
      COMMON/TIME/JUMP,T,DT,MPRTN,NTAPE,KPRINT
C
C     SET LINEAR INDICATOR--NIND
      NIND=0
      AKS=PARAFRI1(
      AKN=PARAFRI2I
C
C     CALCULATE APPARENT ELATIIC INCREMENTAL END FORCES--DFI8(
      DO 301 I=1,8
      XA=D.0
      DO 302 J=1,8
      XA=XA+SAII,J(*DTUIJ(
  302 CONTINUE
      DFII(=XA
  301 CONTINUE
C
C     AVERAGE APPARENT STRESS AT CENTER OF ELEMENT
      SIGN12=D.5*IF(2(+DFI2(+F14(+OFI4((
      SIGN34=0.5*(FI6(+DF16(+F(8(+DFI8(I
      SIGS12=0.5*(FI1(+DF(1(+FI3(+DFI3((
      SIGS34=D.5*(F(5(+DF(5(+FI7(+DFI7()
      IFISIGN12 .GT. 0.0 .AND. SIGN34 .LT. 0.D( GO TO 401
C
C     TENSION CASE
      CI1(=AKS/3.0-AKS/10000000.D
      C12(=AKN/3.0-AKN/10000000.D
      C(3(=AKS/6.0-AKS/10000000.0
      CI4(=AKN/6.0-AKN/10000000.D
      NINO=2
      GO TO 402
  401 CONTINUE
C
C     COMPRESSION,ELASTIC CASE
      FU=PARAFR(3(
      SHEAR=0.5*(ABS(SIGS12)+ABS(SIGS34)(
      STREG=FU*0.5*(ABS(SIGN12(+ABS(SIGN34((
      (F(SHEAR .LT. STREG) RETURN
```

```
C
C     COMPRESSION,SHEAR FAILURE
      NIND=1
      C(1(=AKS/3.0-AKS/10000000.0
      C(2(=0.0
      C(3)=AKS/6.0-AKS/10000000.0
      C(4(=0.0
  402 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PEXPAN(PARAFR,PARAEX,SA,UE,DTU,V,F,C,NIND(
C
C     ******************************************************************
C     CHECK IF EXPANSION JOINT IS YIELD,OR IF THE BRIDGE FALLS OFF
C     SUPPORT,AND FORM TANGENT MODULUS CONSTANTA-C(4)
C     ******************************************************************
C
      DIMENSION PARAEX(4),SA(8,8(,UE(4),DTU(8(,F(8),C(4(,V(8(,DF(8)
      DIMENSION PARAFR(4(
C     CHECK IF THE SUPERSTRUCTURE FALLS OFF
      SEAT=PARAEX(2(
      AKS=PARAFR(1)
      AKN=PARAFR(2)
      IF(PARAEX(4) .EQ. 0.0( GO TO 401
C
C     LEFT LOWER
      SLIP=UE(4)-UE(1)
      IF(SLIP .GT. SEAT( GO TO 402
      GO TO 403
  401 CONTINUE
C
C     RIGHT HAND LOWER
      SLIP=UE(2(-UE(3)
      IF(SLIP .GT. SEAT) GO TO 402
      GO TO 403
  402 CONTINUE
      PRINT 501
      STOP
  403 CONTINUE
C
C     SET LINEAR INDICATOR
      NIND=0
C     CALCULATE APPARENT ELASTIC INCREMENTAL END FORCES FORCES OF(8)
      DO 301 I=1,8
      XA=0.0
      DO 302 J=1,8
      XA=XA+SA(I,J(*DTU(J(
  302 CONTINUE
      DF(I)=XA
  301 CONTINUE
C
C     AVERAGE APPARENT FORCES AT CENTER OF ELEMENT
      SIGN12=0.5*(F(2)+DF(2)+F(4(+DF(4()
      SIGN34=0.5*(F(6(+DF(6(+F(8)+DF(8)(
      SIGS12=0.5*(F(1(+DF(1)+F(3(+DF(3((
      SIGS34=0.5*(F(5(+DF(5(+F(7(+DF(7((
      IF(SIGN12 .GT. 0.0 .AND. SIGN34 .LT. 0.0( GO TO 404
C
C     TENSION CASE
      C(1(=AKS/3.0-AKS/10000000.0
      C(2(=AKN/3.0-AKN/10000000.0
      C(3(=AKS/6.0-AKS/10000000.0
      C(4)=AKN/6.0-AKN/10000000.0
      NIND=2
      GO TO 411
  404 CONTINUE
C
C     COMPRESSION,ELASTIC CASE
      FU=PARAEX(3(
      STREG=FU*0.5*(ABS(SIGN12(+ABS(SIGN34((
      SHEAR=0.5*(ABS(SIGS12(+ABS(SIGS34((
      IF(SHEAR .LT. STREG( RETURN
C
C     COMPRESSION,SHEAR FAILURE,LEFT LOWER
      GAP=PARAEX(1(
      IF(PARAEX(4( .NE. 0.0) GO TO 405
      SHORT=UE(4(-UE(1(
      IF(SHORT .GE. GAP( GO TO 407
      GO TO 408
  407 CONTINUE
      RV=V(1(-V(4(
      IF(RV .GT. 0.0( RETURN
  408 CONTINUE
C
C     SLIPPAGE OCCUR
      C(1(=AKS/3.0-AKS/10000000.0
      C(2(=0.0
      C(3)=AKS/6.0-AKS/10000000.0
      C(4(=0.0
      NIND=1
      GO TO 411
C
C     RIGHT HAND LOWER CASE
  405 CONTINUE
      SHORT=UE(3(-UE(2)
      IF(SHORT .GE. GAP( GO TO 409
      GO TO 410
  409 CONTINUE
      RV=V(3(-V(2(
      IF(RV .GT. 0.0( RETURN
  410 CONTINUE
      C(1)=AKS/3.0-AKS/10000000.0
      C(2(=0.0
      C(3(=AKS/6.0-AKS/10000000.0
      C(4)=0.0
      NIND=1
  411 CONTINUE
  501 FORMAT(* BRIDGE FALLS OFF*)
      RETURN
      END
```

```
      SUBROUTINE PBOUND(PARABD,DTU,F,C,NIND(
C     ******************************************************************
C     CHCEK IF BOUNDARY JOINT IS YIELD,AND THEN FORM YIELD STIFFNESS-C
C     ******************************************************************
      DIMENSION PARABD(2(,DTU(8),F(8),C(1),PSSIG(1)
      NIND=0
C
C     CALCULATE UPPER LIMIT AND LOWER LIMIT LINES
      E1=PARABD(1(
      FY=PARABD(2)
      FU=FY
      FL=-FY
C     APPARENT END FORCES DUE TO ELASTIC INCREMENT
      PSSIG(1(=F(1(+DTU(1)*E1
C     CHECK YIELD CONDITION
      IF(PSSIG(1( .LT. FU .AND. PSSIG(1( .GT. FU( RETURN
C
C     YIELD
      C(1)=0.9*E1
      NIND=1
      RETURN
      END
```

```
      SUBROUTINE STEP(ID,X,Y,PARASO,PARACO,PARAFR,PARAEX,PARABO,COPROP,
     1K2S,CNP,IX,IBC,SLY,DLX,DLY,C,AEN,U,V,ACC,DU,DV,DA,TEPS,DEPS,
     2PEPS,DSIG,SIG,PSIG,INFW,WFX,YBAR,WFY,NPASS,NINO,HSIG,SY,AT,BT,
     3TRU,TRY,TRA,US,NELW,NTW,GS,NUMNP,NEQ,MBAND,KLIN,NUMEL)
C
C     ******************************************************************
C     STEP-BY-STEP INTEGRATION- -NEWMARK LINEAR OR CONSTANT
C     ACCELERATION METHOD
C     ******************************************************************
C
      COMMON/SPECIL/ISTOP
      COMMON/TITLE/HEAD(12),EQNAME(12)
      COMMON/MATER/NUMATS,NUMATC,NUMATF,NUNATE,NUMATB,NUNGE,NINTCV,THICK
     1,NPLAN
      COMMON/TIME/JUMP,T,DT,MPRTM,NTAPE,KPRINT
      COMMON/ABS/UGXT,UGYT,VGXT,YGYT,DACCX,DACCY
      COMMON/OMEGA/OMEGA1,OMEGA2,NF1,NF2
      DIMENSION ID(3,1),X(1),Y(1),PARASO(8,1),PARACO(5,1),PARAFR(4,1),
     1PARAEX(4,1),PARABO(2,1),K2S(20,1),CNP(9,1),IX(6,1),IBC(4,1),U(1),
     2V(1),ACC(1),DU(1),DV(1),DA(1),TEPS(3,1),DEPS(3,1),PEPS(3,1),
     3DSIG(8,1),SIG(8,1),PSIG(4,1),WFX(1),YBAR(1),WFY(1),NPASS(1),
     4NINO(1),AEN(NEQ,1),SY(6,6),AT(1),BT(1),TRU(1),TRY(1),TRA(1),US(1)
      DIMENSION U2G(9,2),DLX(1),DLY(1),C(5,1),NELW(10),INFW(1)
      COMMON/EQ/DURANA,DDT
      DIMENSION HSIG(9,NUMEL)
      DIMENSION COPROP(3,1),SLY(1)
      REAL K2S
      READ 1,EQNAME
      PRINT 101,EQNAME
      READ 2,EQMUL,DTEQ,DT,DUREQ,DURANA,VERTAC,MPRTM,NTAPE,KPRINT
      PRINT 102,EQMUL,DTEQ,DT,DUREQ,DURANA,VERTAC,MPRTM,NTAPE,KPRINT
      DDT=DT*NTAPE
      READ 3,X1,X2,NSLOPE
      WRITE(6,103) X1,X2,NSLOPE
      WRITE(6,105) OMEGA1,OMEGA2
      T1=X2*OMEGA1-X1*OMEGA2
      T2=2.0*OMEGA1*OMEGA2
      TC=OMEGA1**2-OMEGA2**2
      ALFA=T1*T2/TC
      T3=2.0*(X1*OMEGA1-X2*OMEGA2)
      BETA=T3/TC
C
C     FORM CONSTANTS FOR INTEGRATION METHOD
      IF(NSLOPE .EQ. 0) GO TO 402
C
C     LINEAR ACCELERATION METHOD
      TEMP1=3.0*ALFA*DT+6.0
      TEMP2=DT*DT+3.0*BETA*DT
      C1=TEMP1/TEMP2
      C2=ALFA-BETA*C1
      C3=DT/(DT+3.0*BETA)
      AT1=6.0/DT
      AT2=3.0
      BT1=3.0
      BT2=DT/2.0
      COM=6.0/DT/DT

      COM1=DT*DT/3.0
      COM2=DT*DT/6.0
      D3=3.0/DT
      GO TO 401
  402 CONTINUE
C
C     CONSTANT ACCELERATION METHOD
      TEMP1=2.0*ALFA*DT+4.0
      TEMP2=DT*DT+2.0*BETA*DT
      C1=TEMP1/TEMP2
      C2=ALFA-BETA*C1
      C3=DT/(DT+2.0*BETA)
      AT1=4.0/DT
      AT2=2.0
      BT1=2.0
      BT2=0.0
      COM=4.0/DT/DT
      COM1=DT*DT/4.0
      COM2=DT*DT/4.0
      D3=2.0/DT
  401 CONTINUE
C
      ACCY=0.0
      G=386.0T
      DO 301 I=1,9
      DO 301 J=1,2
      U2G(I,J)=0.0
  301 CONTINUE
C
      READ 4,(U2G(I,1),I=2,9)
      PRINT 104,(U2G(I,1),I=1,9)
      ACCX=U2G(2,1)*EQMUL*G
      IF(VERTAC .EQ. 0.0) GO TO 403
      READ 4,(U2G(I,2),I=2,9)
      PRINT 104,(U2G(I,2),I=1,9)
      ACCY=U2G(2,2)*EQMUL*G
  403 CONTINUE
      DUR1=T.0*DTEQ
      DUR=8.0*DTEQ
      DO 302 I=1,NEQ
      AT(I)=0.0
      BT(I)=0.0
  302 CONTINUE
C     CALCULATE AT,BT FOR 1ST STEP
      DO 303 I=1,NEQ
      IF(DLX(I) .EQ. 0.0) GO TO 303
      AT(I)=-AT2*ACCX
      BT(I)=-BT2*ACCX
  303 CONTINUE
      IF(VERTAC .EQ. 0.0) GO TO 404
      DO 304 I=1,NEQ
      IF(DLY(I) .EQ. 0.0) GO TO 304
      AT(I)=-AT2*ACCY
      BT(I)=-BT2*ACCY
  304 CONTINUE
  404 CONTINUE
C
C     INITILIZATION
      DO 305 I=1,NEQ
      TRU(I)=0.0
      TRY(I)=0.0
      TRA(I)=0.0
  305 CONTINUE
      UGXT=0.0
      UGYT=0.0
      VGXT=0.0
      YGYT=0.0
      DACCX=ACCX
      DACCY=ACCY
C     ******************************
C     START OF STEP-BY-STEP CYCLE
C     ******************************
 1000 T=T+DT
      JUMP=JUMP+1
      IF(T .GT. DURANA) GO TO 414
      IF(T .GT. DUREQ) GO TO 406
      IF(T .LE. DUR1) GO TO 405
      U2G(1,1)=U2G(9,1)
      U2G(1,2)=U2G(9,2)
      READ 4,(U2G(I,1),I=2,9)
      PRINT 104,(U2G(I,1),I=1,9)
      IF(VERTAC .NE. 0.0) GO TO 453
      GO TO 454
  453 READ 4,(U2G(I,2),I=2,9)
      PRINT 104,(U2G(I,2),I=1,9)
  454 CONTINUE
      DUR1=DUR1+DUR
  405 CONTINUE
C
C     INTERPOLATION OF EARTHQUAKE INPUT IF DT NOT EQUAL TO DTEQ
      TEMP=(DUR1-T)/DTEQ
      INO=TEMP
      FRACT=TEMP-FLOAT(INO)
      INO=9-INO
      ACCX=(U2G(INO,1)+FRACT*(U2G(INO-1,1)-U2G(INO,1)))*EQMUL*G
      ACCY=(U2G(INO,2)+FRACT*(U2G(INO-1,2)-U2G(INO,2)))*EQMUL*G
      GO TO 407
  406 CONTINUE
      ACCX=0.0
```

```
      ACCY=0.0
  407 CONTINUE
      DTACCX=OACCX-ACCX
      DTACCY=OACCY-ACCY
      DO 351 I=1,NEQ
  351 DU(I)=0.0
C
C     INCREMENTAL FORCE VECTOR-DU(I)
      DO 306 I=1,NEQ
      DU(I)=DLX(I)*(DTACCX+AT(I)+C2*BT(I))
      DU(I)=DU(I)+DLY(I)*(DTACCY+AT(I)+C2*BT(I))
  306 CONTINUE
C
C     INTEGRATION OF EQ ACCELERATION RECORD INTO VELO. AND DISP. OF
C     GROUND MOTION
      UGXT=UGXT+DT*VGXT+COM1*OACCX+COM2*ACCX
      UGYT=UGYT+DT*VGYT+COM1*OACCY+COM2*ACCY
      VGXT=VGXT+0.5*DT*(OACCX+ACCX)
      VGYT=VGYT+0.5*DT*(OACCY+ACCY)
      OACCX=ACCX
      OACCY=ACCY
      IF(KLIN .EQ. 1) GO TO 430
      IF(JUMP .NE. 1) GO TO 451
      REWIND 3
      READ(3) ((AEN(I,J),I=1,NEQ),J=1,MBAND)
  430 CONTINUE
C     FORM EFFECTIVE MATRIX KBAR=K+C1*M
      DO 311 I=1,NEQ
      TEMP=C1*(DLX(I)+DLY(I))
      AEN(I,1)=AEN(I,1)+TEMP
  311 CONTINUE
  451 CONTINUE
      CALL TRIA(AEN,NEQ,MBAND)
      CALL BACKS(AEN,DU,NEQ,MBAND)
C     ****************************************************************
C     INCREMENTAL DISPLACEMENT,VELOCITY,AND ACCELERATION IS CALCULATED
C     ****************************************************************
      DO 307 I=1,NEQ
      DISP=C3*DU(I)+C3*BETA*BT(I)
      DU(I)=DISP
      VEL=DISP*D3-BT(I)
      DV(I)=VEL
      ACCE=DISP*CON-AT(I)
      DA(I)=ACCE
  307 CONTINUE
C
C     THE TOTAL RELATIVE RESPONSE
      DO 308 I=1,NEQ
      TRU(I)=TRU(I)+DU(I)
      TRV(I)=TRV(I)+DV(I)
      TRA(I)=TRA(I)+DA(I)
      IF(TRU(I) .LE. 100.0) GO TO 308
      PRINT 500,JUMP,I,TRU(I)
  500 FORMAT(*1 STRUCTURE YIELD,ANALYSIS QUIT,AT JUMP,I,TRU=*,2I5,E12.4)
      DURANA=T-DT*FLOAT(NTAPE)
      ISTOP=1
      CALL PRINTR(ID,IX,TRU,TRV,ACC,TEPS,PEPS,SIG,PSIG,WFX,YBAR,WFY,
     1            NPASS,NIND,MSIG,NTW,NUMELE)
      RETURN
  308 CONTINUE
      DO 320 J=1,NUMEL
      DO 321 I=1,3
  321 DEPS(I,J)=0.0
      DO 322 I=1,8
  322 DSIG(I,J)=0.0
  320 CONTINUE
C
C     CALCULATE STRAIN
      CALL STRAIN(ID,X,Y,IX,DU,TEPS,DEPS,PEPS)

C
C     CALCULATE STRESS
      CALL STRESS(IX,DU,DEPS,DSIG,SIG,PSIG)
C     NO WALL
      IF(NTW .EQ. 0) GO TO 420
C
C     CALCULATE WALL FORCES
      CALL WFORCE(Y,IX,SIG,NELW,INFW,WFX,YBAR,WFY,NTW)
C
C     TOTAL RESPONSE OF ALL NODAL POINT FROM STATIC
  420 CONTINUE
      IF(JUMP .NE. 1) GO TO 452
      DO 352 I=1,NEQ
  352 US(I)=U(I)
      GO TO 415
  452 CONTINUE
      DO 353 I=1,NEQ
  353 US(I)=0.0
  415 CONTINUE
      DO 309 I=1,NUMNP
      NX=ID(1,I)
      IF(NX .EQ. 0) GO TO 408
      IF(NX .GT. NEQ) STOP
      TRU(NX)=TRU(NX)+US(NX)
      U(NX)=TRU(NX)+UGXT
      V(NX)=TRV(NX)+VGXT
      ACC(NX)=TRA(NX)+ACCX
  408 CONTINUE
      NY=ID(2,I)
      IF(NY .EQ. 0) GO TO 409
      TRU(NY)=TRU(NY)+US(NY)
      U(NY)=TRU(NY)+UGYT
      V(NY)=TRV(NY)+VGYT
      ACC(NY)=TRA(NY)+ACCY
  409 CONTINUE
      NT=ID(3,I)
      IF(NT .EQ. 0) GO TO 309
      TRU(NT)=TRU(NT)+US(NT)
      U(NT)=TRU(NT)
      V(NT)=TRV(NT)
      ACC(NT)=TRA(NT)
  309 CONTINUE
C
C     PRINT OUT RESULTS
      NUMELE=NUMEL
      CALL PRINTR(ID,IX,TRU,TRV,ACC,TEPS,PEPS,SIG,PSIG,WFX,YBAR,WFY,
     1            NPASS,NIND,MSIG,NTW,NUMELE)
C
C     CALCULATE AT(I),BT(I),FOR USE IN NEXT STEP
      DO 310 I=1,NEQ
      AT(I)=AT1*TRV(I)+AT2*TRA(I)
  310 CONTINUE
      DO 313 I=1,NEQ
      BT(I)=BT1*TRV(I)+BT2*TRA(I)
  313 CONTINUE
C
C     CHECK IF IT IS A LINEAR OR NONLINEAR ANALYSIS
      IF(KLIN .NE. 0) GO TO 412
C
C     LINEAR ANALYSIS
      REWIND 3
      READ(3) ((AEN(I,J),I=1,NEQ),J=1,MBAND)
C     FORM EFFECTIVE MATRIX
      DO 312  I=1,NEQ
      TEMP=C1*(DLX(I)+DLY(I))
      AEN(I,1)=AEN(I,1)+TEMP
  312 CONTINUE
      GO TO 413
  412 CONTINUE
      CALL NTSTIF(X,Y,PARASO,PARACO,PARAFR,PARAEX,PARABO,COPROP,K2S,
     1CMP,IX,IBC,SLV,C,AEN,U,V,DU,TEPS,DEPS,PEPS,DSIG,SIG,PSIG,NPASS,
     2NIND,SY,GS,NEQ,MBAND,NUMEL,KLIN)
  413 GO TO 1000
  414 CONTINUE
      RETURN
    1 FORMAT(12A6)
    2 FORMAT(6F10.5,3I5)
    3 FORMAT(2F10.6,I5)
    4 FORMAT(8F7.4)
  101 FORMAT(1H1,12A6)
  102 FORMAT(* EARTHQUAKE MULTIPLICATION FACTOR   *F10.5/
     1       * TIME INTERVAL ON DATA CARDS        *,F10.5/
     2       * TIME INCREMENT OF THE ANALYSIS     *,F10.5/
     3       * DURATION OF EARTHQUAKE             *,F10.5/
     4       * DURATION OF ANALYSIS               *,F10.5/
     5       * WITH OR WITHOUT VERTICAL EQ        *,F10.5/
     6       * TIME INCREMENT OF PRINT OUT/DT     *,I10/
```

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  7       * TIME INTERVAL FOR TAPE WRITING/DT *,I10/
  8       * TIME OF FIRST PRINT OUT/DT        *,I10//)
103 FORMAT(* DAMPING RATIO 1ST MODE         *,F10.6/
  1       * DAMPING RATIO 2ND MODE         *,F10.6/
  2       * CONSTANT OR LINEAR ACCELERATION*,I10//)
104 FORMAT(* INPUT EARTHQUAKE*/(9F7.4))
105 FORMAT(* TWO CONTROL FREQ FOR DAMP,OMEGA1,OMEGA2*/(2E12.4))
    END
```

```
      SUBROUTINE MODES(NEQ,MBAND,NBLOCK,NF,MTOT,N18)
C     **********************************************************************
C     PROGRAM TO COMPUTE 1ST TWO LOWEST FREQUENCIES FOR CALCULATE
C     DAMPING COEFFICIENTS,ALFA,BETA
C     **********************************************************************
      COMMON A(35500)
      COMMON/OMEGA/OMEGA1,OMEGA2,NF1,NF2
      COMMON/TAPES/NSTIF,NMASS
      MTOT=35500
      NSTIF=3
      NMASS=4
      WRITE(6,101) NEQ,MBAND,NF
      NIM=3
      NC=NF+NIM
      NWA=NEQ*MBAND
      NM2=N18+NWA
      NM3=NM2+NEQ
      NM4=NM3+NEQ
      NM5=NM4+NEQ
      NM6=NM5+NEQ
      NM7=NM6+NEQ*NC
      NM8=NM7+NEQ*NC
      NM9=NM8+NC
      NM10=NM9+NC
      NM11=NM10+NC
      NM12=NM11+NC
      NM13=NM12+NC
      IF(MTOT-NM13) 401,402,402
  401 WRITE(6,102) NM13
      STOP
  402 CONTINUE
      CALL SECANTD(A(N18),A(NM2),A(NM3),A(NM4),A(NM5),A(NM6),A(NM7),
     1A(NM8),A(NM9),A(NM10),A(NM11),A(NM12),NEQ,MBAND,NWA,NF,NC)
      OMEGA1=A(NM8+NF1-1)
      OMEGA2=A(NM8+NF2-1)
      OMEGA1=6.28318/OMEGA1
      OMEGA2=6.28318/OMEGA2
       RETURN
  101 FORMAT(*1PROBLEM INFORMATION      *,//
    1       * NO. OF EQUATIONS         *,I4/
    2       * NO. OF BANDWIDTH OF A    *,I4/
    3       * NO. OF FREQUENCIESREQ    *,I4)
  102 FORMAT(* FOR EXECUTION NEED TO INCREASE MTOT TO*,I6)
       END
```

```
      SUBROUTINE SECANTD (A,B,V,MAXA,W,VV,WW,ROOT,TIM,ERRVL,ERRVR,
     *                  NITE,N,MA,NWA,NROOT,NC)
C
     COMMON/TAPES/NSTIF,NMASS
      DIMENSION A(NWA),B(N),V(N),VS(1),W(M),VV(N,NC),WW(N,NC),ROOT(NC),
     *          TIM(NC),ERRVL(NC),ERRVR(NC)
      INTEGER NITE(NC),MAXA(NC)
C
C     FOLLOWING TOLERANCES ARE SET FOR CDC 6400.....
C
      ACTOL=1.0E-04
      RCBTOL=1.0E-06
      RTOL=1.0E-10
      RQTOL=1.0E-12
      SCALE=2.0**900
C
      NTF=5
      IITEM=10
      NITEM=40
C
      REWIND NMASS
      READ (NMASS) B
C
      ETA=2.0
      NOV=0
      JR=1
      MSK=0
      NWA=M*MA
      ISC=1000
C
      CALL SECOND (TIM1)
      RA=0.0
      RR=0.0
      CALL BANDET (A,B,V,MAXA,N,NWA,RA,MSCH,DETA,ISC,1)
      FA=DETA
      FR=FA
      DETR=DETA
C
C     FIND LOWER BOUND ON SMALLEST EIGENVALUE
C
      WRITE (6,1010)
      DO 100 I=1,M
  100 W(I)=B(I)
      RT=0.0
      IITE=0
     KK=2
  110 IITE=IITE+1
      DO 120 I=1,M
  120 V(I)=W(I)
      CALL BANDET (A,B,V,MAXA,M,NWA,RA,NSCH,DETA,ISC,KK)
      KK=2
      RQT=0.0
      DO 130 I=1,M
  130 RQT=RQT+W(I)*V(I)
      DO 180 I=1,N
  180 W(I)=B(I)*V(I)

      RQB=0.0
      DO 140 I=1,N
  140 RQB=RQB+W(I)*V(I)
      RQ=RQT/RQB
      WRITE (6,1004) RQ
      BS=SQRT(RQB)
      TOL=ABS(RQ-RT)/RQ
      IF (TOL.LT.RCBTOL) GO TO 150
      DO 160 I=1,N
  160 W(I)=W(I)/BS
      RT=RQ
      IF (IITE.LT.IITEM) GO TO 110
C
  150 DO 170 I=1,N
  170 V(I)=V(I)/BS
      RB=RQ*(1.0-AMIN1(0.1,100*TOL))
      IS=0
  230 CALL BANDET (A,B,V,MAXA,N,NWA,RB,NSCH,DETB,ISC,1)
      WRITE (6,1020) RB,NSCH
      FB=DETB
      IF (NSCH.EQ.0) GO TO 300
```

```
      )S=IS+1
      IF ))S.LE.MTF) GO TO 240
      WR)TE (6,1030)
      STOP
240   RB=RB/(NSCH+I)
      GO TO 230
C
C     )TERAT(ON FOR IMO)V)DUAL ROOT
C
300   WRITE (6,1040)
      M)TE(JR)=1
      WR)TE (6,1050) JR,NITE(JR),RA,OETA,FA,ETA,ISC
      N)TE(JR)=2
      WRITE (6,1050) JR,N)TE(JR),RB,OETB,FB,ETA,ISC
C
C     STOP WHEN REQUIREO NO. OF ROOTS SMALLER THAM RC ANO NOV=0 FOUNO
C
310   )F (NSCH.GE.MROOT) GO TO 900
C
      O)F=FB-FA
      IF )OIF.NE.O.O) GO TO 320
      WR)TE )6,1060)
      GO TO 900
320   OEL=FB*)RB-RA)/O)F
      RC=RB-ETA*DEL
      TCL=RCBTOL*RC
      IF (ABS)RC-RB).GT.TOL) GO TO 330
      WRITE (6,1070)
      ROOT(JR)=RB
      GO TO 400
C
330   CALL BANOET )A,B,V,MAXA,M,MWA,RC,MSCH,OETC,ISC,1)
      FC=OETC
      NITE(JR)=MITE)JR)+1
      IF (JR.EQ.1) GO TO 340
      JJ=JR-1
      OO 350 K=1,JJ
350   FC=FC/(RC-ROOT(K))
340   WR)TE )6,1050) JR,N)TE(JR),RC,OETC,FC,ETA,ISC
C
C     START )MVERSE )TERATIONS
C
      MES=0
      IF )JR.EQ.1) GO TO 380
      DO 360 )=1,JJ
360   IF )ROOT)I).LT.RC) MES=MES+1
380   NOV=MSCH-MES
      IF )NOV.EQ.O) GO TO 370
      WRITE )6,1080) NOV
      ROOT(JR)=RC
      )F )NOV.GT.1) MSK=1
C
      GO TO 400
370   RR=RA
      FR=FA
      OETR=OETA
      RA=RB
      FA=FB
      DETA=OETB
      RB=RC
      FB=FC
      OETB=OETC
C
C     RESET ETA IF MECESSARY
C
      TOL=RB*ACTOL
      IF (ABS(RA-RB).LT.TOL) ETA=ETA*2
      IF (NITE(JR).LE.MITEM) GO TO 310
      WRITE (6,1015) NITE)JR),JR
      GO TO 900
C
C     CHECK FOR STORAGE
C
400   IF )JR.LE.NC) GO TO 405
      WR)TE (6,1090)
      GO TO 900
C
405   NOR=JR-1
      CALL SECONO (T)M3)
      WRITE (6,1100) NOR
      (F (JR.EQ.1) GO TO 410
      OO 420 I=1,N
420   V)))=1.0
      KK=2
410   OO 430 )=1,M
430   W())=B)I)*V)I)
      )S=0
      RTA=0.0
      GO TO 510
C
C     INVERSE )TERATION
C
440   N)TE(JR)=MITE)JR)+1
      OO 450 I=1,M
450   V())=W(I)
      CALL BANDET )A,B,V,MAXA,M,MWA,RC,MSCH,OETC,ISC,KK)
      )F ))S.EQ.1) GO TO 460
      KK=2
      RQT=0.0
      DO 470 I=1,M
470   RQT=RQT+W)I)*V)I)
      OO 475 I=1,M
475   W)))=B)I)*V)I)
      RQB=0.0
      DO 480 I=1,N
480   RQB=RQB+W(I)*V)I)
      RQ=RQT/RQB
      RT=ROOT)JR)+RQ
      WRITE )6,1110) JR,NITE(JR),RT,RQ
      TOL=RT*RQTOL
      IF (ABS)RT-RTA).GT.TOL) GO TO 510
      IS=1
      GO TO 440
C
510   RTA=RT
      BS=SQRT)RQB)
      OO 490 )=1,M
490   W)I)=W)I)/BS
      IF (NOR.EQ.O) GO TO 550
      DO 520 K=1,NOR
      AL=O.0
      OO 530 I=1,M
530   AL=AL+VV)I,K)*W(I)
      DO 540 I=1,M
540   W)I)=W())-AL*WW(I,K)
520   CONT)NUE
C
550   )F )MITE)JR).LE.MITEM) GO TO 440
      WRITE )6,1015) NITE(JR),JR
      GO TO 900
C
460   RQT=0.0
      ERRT=RQB
      OO 570 I=1,M
570   RQT=RQT+V(I)*W(I)
      DO 560 I=1,M
560   W(I)=B(I)*V(I)
      RQB=0.0
      OO 580 I=1,M
580   RQB=RQB+V(I)*W(()
C
C     OBTAIM A RATHER LARGE ERROR BOUMO
C
      RQ=RQT/RQB
      ROOT)JR)=ROOT)JR)+RQ
      ERR=SQRT)ERRT/RQB)
      ERRVL)JR)=ROOT)JR)-ERR
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      ERRVR(JR)=ROOT(JR)+ERR
C
      BS=SQRT(RDB)
      DO 590 I=1,N
      W(I)=W(I)/BS
  590 V(I)=V(I)/BS
      DO 600 I=1,N
      WW(I,JR)=W(I)
  600 VV(I,JR)=V(I)
C
      CALL SECOND (TIM2)
      TIM3=TIM2-TIM3
      WRITE (6,1120) TIM3
      TIM(JR)=TIM2-TIM1
      TIM1=TIM2
C
C     DECIDE STRATEGY FOR ITERATION TOWARD NEXT ROOT
C
      TOL=RTOL*ROOT(JR)
      IF (NOV.GT.0) GO TO 700
      IF (ABS(ROOT(JR)-RB).GT.TOL) GO TO 710
      IF (RA.GT.0.0) GO TO 720
      RA=RB/2.
      CALL BANDET (A,B,V,MAXA,N,NWA,RA,NSCH,DETA,ISC,1)
      FA=DETA
  720 RB=RA
      FB=FA
      DETB=DETA
      RA=RR
      FA=FR
      DETA=DETR
      GO TO 710
C
  700 IF (ROOT(JR).GT.RC) NSK=1
      IF (NSK.EQ.1) GO TO 730
      IF (ABS(RC-ROOT(JR)).LT.TOL) GO TO 740
      IF (ABS(ROOT(JR)-RB).LT.TOL) GO TO 750
      RA=RB
      FA=FB
      DETA=DETB
  750 RB=RC
      FB=FC
      DETB=DETC
      GO TO 710
  740 IF (ABS(ROOT(JR)-RB).GT.TOL) GO TO 710
      IF (RA.GT.0.0) GO TO 760
      RA=RB/2.
      CALL BANDET (A,B,V,MAXA,N,NWA,RA,NSCH,DETA,ISC,1)
      FA=DETA
  760 RB=RA
      FB=FA
      DETB=DETA
      RA=RR
      FA=FR
      DETA=DETR
  710 FA=FA/(RA-ROOT(JR))
      FB=FB/(RB-ROOT(JR))
      JR=JR+1
      ETA=2.0
      GO TO 300
C
  730 IF (RA.GT.0.0) GO TO 780
      RA=RB/2.
      CALL BANDET (A,B,V,MAXA,N,NWA,RA,NSCH,DETA,ISC,1)
      FA=DETA
  780 IF (ABS(ROOT(JR)-RB).GT.TOL) GO TO 770
      RB=RA
      FB=FA
      DETB=DETA
      RA=RR
      FA=FR
      DETA=DETR
  770 FA=FA/(RA-ROOT(JR))
      FB=FB/(RB-ROOT(JR))
      FR=FR/(RR-ROOT(JR))
      IF (ROOT(JR).LE.RC) NOV=NOV-1
      JR=JR+1
      NITE(JR)=0
      ROOT(JR)=RC
      IF (NOV.GT.0) GO TO 400
      NSK=0
      ETA=2.0
      GO TO 300
C
  900 NROOT=JR-1
      IF (NROOT.EQ.0) RETURN
      WRITE (6,1130)
      WRITE (6,1004) (ROOT(J),J=1,NROOT)
      WRITE (6,1140)
      WRITE (6,1006) (NITE(J),J=1,NROOT)
      WRITE (6,1150)
      WRITE (6,1008) (TIM(J),J=1,NROOT)
      WRITE (6,1160)
      WRITE (6,1004) (ERRVL(J),J=1,NROOT)
      WRITE (6,1004) (ERRVR(J),J=1,NROOT)
C
C     ARRANGE EIGENVALUES AND VECTORS IN ASCENDING ORDER
C
      IF (JR.EQ.2) GO TO 950
      JR=JR-2
  910 IS=0
      DO 920 I=1,JR
      IF (ROOT(I+1).GE.ROOT(I)) GO TO 920
      IS=IS+1
      RT=ROOT(I+1)
      ROOT(I+1)=ROOT(I)
      ROOT(I)=RT
      DO 930 K=1,N
      RT=VV(K,I+1)
      VV(K,I+1)=VV(K,I)
  930 VV(K,I)=RT
  920 CONTINUE
      IF (IS.GT.0) GO TO 910
C
  950 WRITE (6,1170)
      NROOT=NSCH
      DO 960 I=1,NROOT
      IF (ROOT(I).LE.0.0) GO TO 960
      ROOT(I)=SQRT(ROOT(I))
  960 CONTINUE
      WRITE (6,3000) NT
      IF (NT.NE.7) NT=7
      REWIND NT
      WRITE (NT) (ROOT(I),I=1,NROOT)
C
C     PRINT FREQUENCIES AND MODE SHAPES
C
      WRITE (6,2000)
      DO 990 I=1,NROOT
      PERIOD=6.2831853/ROOT(I)
      WRITE (6,2001) I,ROOT(I),PERIOD
  990 ROOT(I)=PERIOD
C
      WRITE (NT) ((VV(I,J),I=1,N),J=1,NROOT)
      RETURN
C
 1002 FORMAT (1H ,12E11.4)
 1004 FORMAT (1H0,6E20.12)
 1006 FORMAT (1H0,6I20)
 1008 FORMAT (1H0,6F20.2)
 1010 FORMAT (1H1,63HINVERSE ITERN GIVES FOLLOWING APPROXIMATN TO LOWES
     1T EIGENVALUE  )
 1015 FORMAT (41HWE ABANDON ITERN BECAUSE NO OF ITERN IS I3,9H FOR ROO
     1T I3 )
 1020 FORMAT (5HORB = E20.12,7H NSCH = I4)
 1030 FORMAT (30HOWE BETTER CHECK THE MATRICES  )
 1040 FORMAT (1H1,4X,4HROOT,4X,4HNITE,18X,2HRC,15X,12HDET (A-RC*B),15X,
     /2HFC,13X,3HETA,4X,3HISC)
 1050 FORMAT (1H0,4X,I4,4X,I4,8X,3E22.14,F7.2,I6)
 1060 FORMAT (42HOTHE DEFLATED POLYNOMIAL HAS NO MORE ROOTS  )
 1070 FORMAT (29HO(RC-RB) IS SMALLER THAN TOL )
 1080 FORMAT (16HOWE JUMPED OVER (I4,16H UNKNOWN ROOT(S)  )
 1090 FORMAT (1H1,36HNO MORE STORAGE FOR VECTORS  WE QUIT  )
 1100 FORMAT (1H0,34X,4HROOT,18X,2HRO,18X,4HNOR=,I2)
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1110  FORMAT (1HO,4X,I4,4X,I4,8X,2E22.14)                              MODES.424
1120  FORMAT (20HOTIME FOR INV ITERN F5.2)                              MODES.425
1130  FORMAT (20HOTHE EIGENVALUES ARE  /)                               MODES.426
1140  FORMAT (42HONO OF ITERATIONS FOR EACH EIGENVALUE ARE    /)        MODES.427
1150  FORMAT (30HOTIME USEO FOR EACH EIGENVALUE /)                      MODES.428
1160  FORMAT (43HOFOLLOWING ARE ERROR BOUNDS ON EIGENVALUES )           MODES.429
1170  FORMAT (1H1,62HWE ACCEPT FOLLOWING FREQUENCIES AND MODES ARRANGEOMODES.430
    . IN ORDER    )                                                     MODES.431
2000  FORMAT (39H1......PRINT OF FREQUENCIES AND PERIOOS//              MODES.432
    .  6X,4HMODE,9X,11HFREQUENCIES,13X,7HPERIODS /                      MODES.433
    .  6X,4H NO ,9X,11H (RAO/SEC) ,13X,7H (SEC)  /)                     MODES.434
2001  FORMAT (I10,2F20.4)                                               MODES.435
3000  FORMAT (////46H EIGENVALUE AND EIGENVECTOR ARE STOREO IN TAPE,I3)MODES.436
C                                                                       MODES.437
      ENO                                                               MODES.438


      SUBROUTINE BANOET (A,B,V,MAXA,NN,NWA,RA,NSCH,OET,ISCALE,KK)       MODES.440
C                                                                       MODES.441
      OIMENSION A(NWA),B(1),V(1),MAXA(1)                                MODES.442
     COMMON/TAPES/NSTIF,NMASS                                           MODES.443
C                                                                       MODES.444
C     TRIANGULARIZE BANOEO STIFFNESS MATRIX                             MODES.445
C                                                                       MODES.446
      NR=NN-1                                                           MODES.447
      IF (KK-2) 100,700,800                                             MODES.448
C                                                                       MODES.449
100   TOL=1.0E+07                                                       MODES.450
      RTOL=1.0E-10                                                      MODES.451
      SCALE=2.0**900                                                    MODES.452
      NTF=3                                                             MODES.453
      IS=1                                                              MODES.454
120   REWINO NSTIF                                                      MODES.455
      REAO (NSTIF) A                                                    MODES.456
      00 140 I=1,NN                                                     MODES.457
140   A(I)=A(I)-RA*B(I)                                                 MODES.458
160   IF (NWA.EQ.NN) GO TO 230                                          MODES.459
      00 200 N=1,NR                                                     MODES.460
      IH=N+NWA-NN                                                       MODES.461
210   IF (A(IH)) 220,215,220                                            MODES.462
215   IH=IH-NN                                                          MODES.463
      GO TO 210                                                         MODES.464
220   MAXA(N)=IH                                                        MODES.465
      PIV=A(N)                                                          MODES.466
      IF (PIV) 221,226,221                                              MODES.467
221   IL=N+NN                                                           MODES.468
      L=N                                                               MODES.469
      00 240 I=IL,IH,NN                                                 MODES.470
      L=L+1                                                             MODES.471
      C=A(I)                                                            MODES.472
      IF (C) 225,240,225                                                MODES.473
225   C=C/PIV                                                           MODES.474
      IF (ABS(C).LT.TOL) GO TO 235                                      MODES.475
226   IS=IS+1                                                           MODES.476
      IF (IS.LE.NTF) GO TO 245                                          MODES.477
      WRITE (6,1000) NTF                                                MODES.478
      STOP                                                              MODES.479
245   RA=RA*(1.0-RTOL)                                                  MODES.480
      GO TO 120                                                         MODES.481
235   J=L-I                                                             MODES.482
      00 260 K=I,IH,NN                                                  MODES.483
260   A(K+J)=A(K+J)-C*A(K)                                              MODES.484
      A(I)=C                                                            MODES.485
240   CONTINUE                                                          MODES.486
200   CONTINUE                                                          MODES.487
230   IF (A(NN).NE.0.0) GO TO 280                                       MODES.488
      AA=ABS(A(1))                                                      MODES.489
      OO 290 I=2,NR                                                     MODES.490
290   AA=AA+ABS(A(I))                                                   MODES.491
      A(NN)=-(AA/NR)*1.0E-16                                            MODES.492
C                                                                       MODES.493
280   NSCH=0                                                            MODES.494
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      ISC=0                                                             MODES.495
      OET=1.0                                                           MODES.496
      00 300 I=1,NN                                                     MODES.497
      IF (ABS(OET).LT.SCALE) GO TO 320                                  MODES.498
      OET=OET/SCALE                                                     MODES.499
      ISC=ISC+1                                                         MODES.500
320   OET=OET*A(I)                                                      MODES.501
300   IF (A(I).LT.0.) NSCH=NSCH+1                                       MODES.502
C                                                                       MODES.503
      IF (ISCALE.LT.1000) GO TO 340                                     MODES.504
      ISCALE=ISC                                                        MODES.505
      GO TO 900                                                         MODES.506
340   IF (ISC-ISCALE) 350,900,370                                       MODES.507
350   OET=OET/SCALE                                                     MODES.508
      GO TO 900                                                         MODES.509
370   DET=OET*SCALE                                                     MODES.510
      GO TO 900                                                         MODES.511
C                                                                       MODES.512
700   IL=NN                                                             MODES.513
      00 400 N=1,NR                                                     MODES.514
      C=V(N)                                                            MODES.515
      V(N)=C/A(N)                                                       MODES.516
      IF (NWA-NN) 410,400,410                                           MODES.517
410   IL=IL+1                                                           MODES.518
      IH=MAXA(N)                                                        MODES.519
      K=N                                                               MODES.520
      00 420 I=IL,IH,NN                                                 MODES.521
      K=K+1                                                             MODES.522
420   V(K)=V(K)-C*A(I)                                                  MODES.523
400   CONTINUE                                                          MODES.524
      V(NN)=V(NN)/A(NN)                                                 MODES.525
C                                                                       MODES.526
800   IF (NWA-NN) 430,900,430                                           MODES.527
430   N=NN                                                              MODES.528
      00 440 L=2,NN                                                     MODES.529
      N=N-1                                                             MODES.530
      IL=N+NN                                                           MODES.531
      IH=MAXA(N)                                                        MODES.532
      K=N                                                               MODES.533
      00 460 I=IL,IH,NN                                                 MODES.534
      K=K+1                                                             MODES.535
460   V(N)=V(N)-A(I)*V(K)                                               MODES.536
440   CONTINUE                                                          MODES.537
900   RETURN                                                            MODES.538
1000  FORMAT (1H1,20H TRIANG FACTORIZATN I3,32H TIMES ABONDONED,CHECK   MODES.539
     1MATRICES  )                                                       MODES.540
      END                                                               MODES.541
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```
      SUBROUTINE OUTPUT(IO,NUMNP,NUMEL,NEQ,NTW)                          OUTPUT.2
C                                                                        OUTPUT.3
C     ********************************************************************OUTPUT.4
C     OUTPUT THE SELECTED RESULTS IN TIME HISTORY                        OUTPUT.5
C     ********************************************************************OUTPUT.6
C                                                                        OUTPUT.7
      COMMON A(1)                                                        OUTPUT.8
      COMMON/TIME/JUMP,T,DT,MPRTM,MTAPE,KPRINT                           OUTPUT.9
      DIMENSION ID(3,NUMNP)                                              OUTPUT.10
      COMMON/BLOCK/NSD,NSS,NSW,KK1,KK2,KK3,MDIS,MSTR,MWAL,NBD,NBS,NBW    OUTPUT.11
      COMMON/EQ/DURANA,DDT                                               OUTPUT.12
C                                                                        OUTPUT.13
C      INPUT SPECIFICATION FOR OUTPUT OF RESPONSE TIME HISTORY           OUTPUT.14
      NH1=1                                                              OUTPUT.15
      NH2=NH1+3*NUMNP                                                    OUTPUT.16
      NH3=NH2+NEQ                                                        OUTPUT.17
      NH4=NH3+9*NUMEL                                                    OUTPUT.18
      NH5=NH4+3*NTW                                                      OUTPUT.19
      CALL INOUT(A(NH1),A(NH2),A(NH3),A(NH4),NUMNP,NUMEL,NEQ,NTW)        OUTPUT.20
C                                                                        OUTPUT.21
C     PACK TIME HISTORY IN BLOCK                                         OUTPUT.22
      NNTOT=20000                                                        OUTPUT.23
      DDT=DT*MTAPE                                                       OUTPUT.24
      NDS=DURANA/DDT+1                                                   OUTPUT.25
      NDISB=(NNTOT-NH5-NEQ*2)/(MDIS*2)                                   OUTPUT.26
      NDISB=(NNTOT-NH5-9*NDS)/(MDIS*2)                                   OUTPUT.27
      IF(NDISB .GT. NDISB) NDISB=NDISB                                   OUTPUT.28
      IF(NDISB .GT. NDS) NDISB=NDS                                       OUTPUT.29
      NBD=(NDS-1)/NDISB+1                                                OUTPUT.30
C                                                                        OUTPUT.31
      NSTRB=(NNTOT-NH5-9*NUMEL)/(MSTR*2)                                 OUTPUT.32
      MSTRB=(NNTOT-NH5-9*NDS)/(MSTR*2)                                   OUTPUT.33
      IF(NSTRB .GT. NSTRB) NSTRB=MSTRB                                   OUTPUT.34
      IF(NSTRB .GT. NDS) NSTRB=NDS                                       OUTPUT.35
      NBS=(NDS-1)/NSTRB+1                                                OUTPUT.36
C                                                                        OUTPUT.37
      IF(NWAL .EQ. 0) GO TO 402                                          OUTPUT.38
      NWFB=(NNTOT-NH5-3*NTW)/NWAL                                        OUTPUT.39
      NWFB=(NNTOT-NH5-9*NDS)/NWAL                                        OUTPUT.40
      IF(NWFB .GT. MWFB) NWFB=MWFB                                       OUTPUT.41
      IF(NWFB .GT. NDS) NWFB=NDS                                         OUTPUT.42
      NBW=(NDS-1)/NWFB+1                                                 OUTPUT.43
C                                                                        OUTPUT.44
      GO TO 403                                                          OUTPUT.45
  402 CONTINUE                                                           OUTPUT.46
      NWFB=0                                                             OUTPUT.47
      NBW=0                                                              OUTPUT.48
  403 CONTINUE                                                           OUTPUT.49
      NH6=NH5+NEQ                                                        OUTPUT.50
      NH7=NH6+NEQ                                                        OUTPUT.51
      NH8=NH7+MDIS*NDISB                                                 OUTPUT.52
      NH10=NH5+9*NUMEL                                                   OUTPUT.53
      NH12=NH5+3*NTW                                                     OUTPUT.54
      NH13=NH12+MWAL*NWFB                                                OUTPUT.55
      CALL REPACK(A(NH1),A(NH2),A(NH3),A(NH4),A(NH5),A(NH6),A(NH7),      OUTPUT.56
     1            A(NH8),A(NH5),A(NH10),A(NH5),A(NH12),NDS,NEQ,MDIS,     OUTPUT.57
     2            NDISB,NUMEL,MSTR,NSTRB,NTW,MWAL,NWFB,NBD,NBS,NBW,      OUTPUT.58
     3            NUMNP)                                                 OUTPUT.59
C                                                                        OUTPUT.60
      MT=0                                                               OUTPUT.61
      NFILE=0                                                            OUTPUT.62
      IF(KK1 .EQ. 2) NFILE=NFILE+NSD*2                                   OUTPUT.63
      IF(KK2 .EQ. 2) NFILE=NFILE+NSS                                     OUTPUT.64
      IF(KK3 .EQ. 2) NFILE=NFILE+NSW                                     OUTPUT.65
      IF(NFILE .EQ. 0) GO TO 401                                         OUTPUT.66
      MT=30                                                              OUTPUT.67
      REWIND MT                                                          OUTPUT.68
      WRITE(MT) NFILE,NDS,DDT                                            OUTPUT.69
  401 CONTINUE                                                           OUTPUT.70
      NH6=NH5+NDS                                                        OUTPUT.71
      NH7=NH6+8*NDS                                                      OUTPUT.72
      NH8=NH7+MDIS*NDISB                                                 OUTPUT.73
      NH9=NH7+NSTR*NSTRB                                                 OUTPUT.74
      NH10=NH7+NWAL*NWFB                                                 OUTPUT.75
      IFF=0                                                              OUTPUT.76
C                                                                        OUTPUT.77
C      OUTPUT SELECTED ABSOLUTE DISPLACEMENT TIME HISTORY                OUTPUT.78
      REWIND 10                                                          OUTPUT.79
      CALL OUTHIS(A(NH1),A(NH2),A(NH3),A(NH4),A(NH5),A(NH6),A(NH7),      OUTPUT.80
     1            NUMNP,NUMEL,NTW,NDS,MDIS,NDISB,NSD,NBD,1,KK1,10,11,MT, OUTPUT.81
     2            IFF,NEQ)                                               OUTPUT.82
C                                                                        OUTPUT.83
C      OUTPUT SELECTED ABSOLUTE ACCELERATION TIME HISTORY                OUTPUT.84
      REWIND 10                                                          OUTPUT.85
      CALL OUTHIS(A(NH1),A(NH2),A(NH3),A(NH4),A(NH5),A(NH6),A(NH7),      OUTPUT.86
     1            NUMNP,NUMEL,NTW,NDS,MDIS,NDISB,NSD,NBD,2,KK1,10,12,MT, OUTPUT.87
     2            IFF,NEQ)                                               OUTPUT.88
C                                                                        OUTPUT.89
C     OUTPUT SELECTED STRESS TIME HISTORY                                OUTPUT.90
      REWIND 20                                                          OUTPUT.91
      CALL OUTHIS(A(NH1),A(NH2),A(NH3),A(NH4),A(NH5),A(NH6),A(NH7),      OUTPUT.92
     1            NUMNP,NUMEL,NTW,NDS,NSTR,NSTRB,NSS,NBS,3,KK2,20,13,MT, OUTPUT.93
     2            IFF,NEQ)                                               OUTPUT.94
C     CASE OF NO WALL                                                    OUTPUT.95
      IF(NTW .EQ. 0) GO TO 404                                           OUTPUT.96
C                                                                        OUTPUT.97
C     OUTPUT SELECTED WALL FORCES TIME HISTORY                           OUTPUT.98
      REWIND 40                                                          OUTPUT.99
      CALL OUTHIS(A(NH1),A(NH2),A(NH3),A(NH4),A(NH5),A(NH6),A(NH7),      OUTPUT.100
     1            NUMNP,NUMEL,NTW,NDS,MWAL,NWFB,NSW,NBW,4,KK3,40,14,MT,  OUTPUT.101
     2            IFF,NEQ)                                               OUTPUT.102
  404 CONTINUE                                                           OUTPUT.103
      RETURN                                                             OUTPUT.104
      END                                                                OUTPUT.105


      SUBROUTINE INOUT(ID,IDIS,ISTR,IWALL,NUMNP,NUMEL,NEQ,NTW)           OUTPUT.106
C                                                                        OUTPUT.107
C     ********************************************************************OUTPUT.108
C     INPUT SPECIFICATIONS FOR OUTPUT RESPONSE TIME HISTORY              OUTPUT.109
C     ********************************************************************OUTPUT.110
C                                                                        OUTPUT.111
      DIMENSION IDIS(NEQ),ISTR(9,NUMEL),IWALL(3,NTW),IC(3),IS(9),IW(3)   OUTPUT.112
      DIMENSION ID(3,NUMNP),KD(4,8)                                      OUTPUT.113
      COMMON/BLOCK/NSD,NSS,NSW,KK1,KK2,KK3,MDIS,MSTR,MWAL,NBD,NBS,NBW    OUTPUT.114
C                                                                        OUTPUT.115
C     INITILIZATION                                                      OUTPUT.116
      DO 301 I=1,NEQ                                                     OUTPUT.117
      IDIS(I)=0                                                          OUTPUT.118
  301 CONTINUE                                                           OUTPUT.119
      DO 302 N=1,NUMEL                                                   OUTPUT.120
      DO 302 I=1,9                                                       OUTPUT.121
      ISTR(I,N)=0                                                        OUTPUT.122
  302 CONTINUE                                                           OUTPUT.123
      DO 303 N=1,NTW                                                     OUTPUT.124
      DO 303 I=1,3                                                       OUTPUT.125
      IWALL(I,N)=0                                                       OUTPUT.126
  303 CONTINUE                                                           OUTPUT.127
C                                                                        OUTPUT.128
C      READ AND PRINT DISPLACEMENT,ACCELERATION SPECIFICATION            OUTPUT.129
C     L=COLUMN NO.  KK1=1 PRINT  =2 PRINT + PLOT                         OUTPUT.130
      L=0                                                                OUTPUT.131
      K=0                                                                OUTPUT.132
      READ 1,KK1                                                         OUTPUT.133
 1000 CONTINUE                                                           OUTPUT.134
C     A BLOCK CARD FOR ENDING                                            OUTPUT.135
      READ 1,NP,IC                                                       OUTPUT.136
      IF(NP .GT. 0) GO TO 401                                            OUTPUT.137
      IF(L .EQ. 0) GO TO 402                                             OUTPUT.138
      WRITE(10) KD,L                                                     OUTPUT.139
      K=K+1                                                              OUTPUT.140
      GO TO 402                                                          OUTPUT.141
  401 CONTINUE                                                           OUTPUT.142
      IF(K .EQ. 0 .AND. L.EQ. 0) WRITE(6,501)                            OUTPUT.143
      WRITE(6,101) NP,IC                                                 OUTPUT.144
      DO 304 I=1,3                                                       OUTPUT.145
      II=IC(I)                                                           OUTPUT.146
      IF(II .EQ. 0) GO TO 304                                            OUTPUT.147
      JJ=ID(I,NP)                                                        OUTPUT.148
```

```
      IF(JJ .LE. NEQ) GO TO 403
  404 CONTINUE
      WRITE(6,502) NP,II
      GO TO 304
  403 CONTINUE
      IF(JJ .LE. 0) GO TO 404
      L=L+1
      KO(1,L)=NP
      KO(2,L)=II
      KO(3,L)=JJ
      KO(4,L)=0
      IDIS(JJ)=1
      IF(L .LT. 8) GO TO 304
      WRITE(10) KO,L
      K=K+1
      L=0
  304 CONTINUE
      GO TO 1000
C
C     READ AND INPUT STRESS OUTPUT SPECIFICATION
  402 CONTINUE
      NSD=K
      WRITE(6,503) KK1
      L=0
      K=0
      READ 1,KK2
 2000 CONTINUE
      READ 1,NTYPE,NEL,IS
C     A BLANK CARD FOR ENDING
      IF(NEL .GT. 0) GO TO 405
      IF(L .EQ. 0) GO TO 406
      WRITE(20) KO,L
      K=K+1
      GO TO 406
  405 CONTINUE
      IF(K .EQ. 0 .AND. L .EQ. 0) WRITE(6,504)
      WRITE(6,103) NTYPE,NEL,IS
      DO 305 I=1,9
      II=IS(I)
      IF(II .LE. 0) GO TO 305
      JJ=9*(NEL-1)+II
      L=L+1
      KO(1,L)=NTYPE
      KO(2,L)=NEL
      KO(3,L)=II
      KO(4,L)=JJ
      ISTR(II,NEL)=1
      IF(L .LT. 8) GO TO 305
      WRITE(20) KO,L
      K=K+1
      L=0
  305 CONTINUE
      GO TO 2000
  406 CONTINUE
      NSS=K
C
C     READ AND PRINT WALL FORCE OUTPUT SPECIFICATION
      L=0
      K=0
      READ 1,KK3
 3000 CONTINUE
      READ 1,NW,IW
      IF(NW .GT. 0) GO TO 407
      IF(L .EQ. 0) GO TO 408
      WRITE(40) KO,L
      K=K+1
      GO TO 408
  407 CONTINUE
      IF(K .EQ. 0 .AND. L.EQ. 0) WRITE(6,505)
      WRITE(6,104) NW,IW
      DO 306 I=1,3
      II=IW(I)
      IF(II .EQ. 0) GO TO 306
      JJ=3*(NW-1)+I
      IWALL(I,NW)=1
      L=L+1
      KO(1,L)=NW
      KO(2,L)=II
      KO(3,L)=JJ
      KO(4,L)=0
      IF(L .LT. 6) GO TO 306
      WRITE(40) KO,L
      K=K+1
      L=0
  306 CONTINUE
      GO TO 3000
  408 CONTINUE
      NSW=K
C     CALCULATE LOCATION IN BUFFER FOR DISPLACEMENT(ACCELERATION),
C     STRESS,WALL FORCE
      MDIS=0
      MSTR=0
      MWAL=0
      DO 307 I=1,NEQ
      IF(IDIS(I) .EQ. 0) GO TO 307
      MDIS=MDIS+1
      IDIS(I)=MDIS
  307 CONTINUE
C
      DO 308 N=1,NUMEL
      DO 308 I=1,9
      IF(ISTR(I,N) .EQ. 0) GO TO 308
      MSTR=MSTR+1
      ISTR(I,N)=MSTR
  308 CONTINUE
      DO 309 N=1,NTW
      DO 309 I=1,3
      IF(IWALL(I,N) .EQ. 0) GO TO 309
      MWAL=MWAL+1
      IWALL(I,N)=MWAL
  309 CONTINUE
      RETURN
    1 FORMAT(14I5)
  101 FORMAT(I5,4X,3I4)
  103 FORMAT(2I4,5X,12I3)
  104 FORMAT(4X,I4,6X,3I4)
  501 FORMAT(*1DISPLSEMENT AND ACCELERATION COMPOENTS FOR*/
     1       * WHICH OUTPUT TIME HISTORY IS REQUIRED     *//
     2       * NODE    DISPLACEMENT(ACCELERATION) COMPOENTS*/)
  502 FORMAT(I5,4X,I4,4X,*FIXED DOF...NO OUTPUT*)
  503 FORMAT(* OUTPUT TYPE=1 PRINT, =2 PRINT + PLOT*,*...........*,I2/)
  504 FORMAT(*1ELEMENT STRESS COMPOENTS FOR WHICH*/
     1       * OUTPUT TIME HISTORY IS REQUIRED    *//
     2       * ELEMENT         STRESS COMPOENTS(1,8),YIELD(9)*/
     3       * TYPE  NO.*/)
  505 FORMAT(*1WALL FORCES FOR WHICH OUTPUT*,
     1       * TIME HISTORY IS REQUIRED    */
     2       * WALL NO.        COMPOENTS(1-FX,2-YBAR,3-FY)*/)
      END
```

```
      SUBROUTINE REPACK(ID,IDIS,ISTR,IWALL,X,X2,XH,X2H,HSIG,STH,HWALL,
     1                  WFH,NOS,NEQ,MDIS,NDISB,NUMEL,MSTR,NSTRB,NTW,
     2                  MWAL,NWFB,NBD,NBS,NBW,NUMNP)
C
C     ************************************************************
C     PACK RESPONSE TIME HISTORY IN OPTIMAL AVAILABLE STORAGE BLOCKS
C     ************************************************************
C
      DIMENSION X(NEQ),X2(NEQ),XH(MDIS,NDISB),K2H(MDIS,NDISB),
     1          HSIG(9,NUMEL),ISTR(9,NUMEL),STH(MSTR,NSTRB),HWALL(3,NTW)
     2          ,IWALL(3,NTW),WFH(MWAL,NWFB)
      DIMENSION ID(3,NUMNP),IDIS(NEQ)
      REWIND 21
      REWIND 22
      REWIND 23
      L=0
      K=0
      DO 301 M=1,NDS
C     PACK DISPLACEMENT ARRAY-KH,ACCELERATION ARRAY-K2H FROM TAPE 21
      READ(21) X,X2
      L=L+1
      DO 302 I=1,NEO
      II=IDIS(I)
      IF(II) 302,302,402
  402 CONTINUE
      XH(I,L)=X(I)
      X2H(I,L)=K2(I)
  302 CONTINUE
      IF(L .LT. NDISB) GO TO 301
      WRITE(11) L,XH
      WRITE(12) L,X2H
      K=K+1
      L=0
  301 CONTINUE
      IF(L) 403,403,404
  404 CONTINUE
      WRITE(11) L,XH
      WRITE(12) L,X2H
      K=K+1
  403 CONTINUE
      IF(NBD .NE. K) NBD=K
C
C      PACK STRESS-STH FROM TAPE 22
      L=0
      K=0
      DO 306 M=1,NDS
      READ(22) HSIG
      L=L+1
      DO 303 N=1,NUMEL
      DO 303 I=1,9
      II=ISTR(I,N)
      IF(II) 303,303,406
  406 CONTINUE
      STH(I,L)=HSIG(I,N)
  303 CONTINUE
      IF(L .LT. NSTRB) GO TO 306
      WRITE(13) L,STH
      K=K+1
      L=0
  306 CONTINUE
      IF(L) 407,407,408
  408 CONTINUE
      WRITE(13) L,STH
      K=K+1
  407 IF(NBS .NE. K) NBS=K
C     CASE OF NO WALL
      IF(NTW .EQ. 0) GO TO 412
C
C     PACK WALL FORCE ARRAY-WFH FROM TAPE 23
      L=0
      K=0
      DO 304 M=1,NOS
      READ(23) HWALL
      L=L+1
      DO 305 N=1,NTW
      DO 305 I=1,3
      II=IWALL(I,N)
      IF(II) 305,305,409
  409 CONTINUE
      WFH(I,L)=HWALL(I,N)
  305 CONTINUE
      IF(L.LT. NWFB) GO TO 304
      WRITE(14) L,WFH
      K=K+1
      L=0
  304 CONTINUE
      IF(L) 410,410,411
  411 WRITE(14) L,WFH
      K=K+1
  410 IF(NBW .NE. K) NBW=K
  412 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUTHIS(ID,IDIS,ISTR,IWALL,TA,K,UH,NUMNP,NUMEL,NTW,
     1                  NDS,NDI,NDJ,NOB,NHB,KKK,KKI,IT,JT,MT,IFF,NEQ)
C
C     ************************************************************
C     OUTPUT RESPONSE TIME HISTORY ON SPECIFIED DISPLAY MEDIUM
C     ************************************************************
C
      DIMENSION TM(8),XM(8),ID(3,NUMNP),ISTR(9,NUMEL),IWALL(3,NTW),TA(1
     1          ,X(8,NDS),UH(NO),NDJ),IDIS(NEQ)
      DIMENSION KD(4,8)
      COMMON/TIME/JUMP,T,DT,NPRTH,NTAPE,KPRINT
      COMMON/EQ/DURANA,DDT
C
C     TAPE IT INPUT TAPE STORE KD(4,8),L
C     TAPE JT INPUT TAPE STORE XH(MDIS,NDISB),K2H(MDIS,NDISB)
C                              STH(MSTR,NSTRB),WFH(MWAL,NWFB)
C
      IF(NOB .EQ. 0) RETURN
      DO 301 M=1,NOB
      IFF=IFF+1
      REWIND JT
      READ(IT) KD,L
      DO 302 I=1,8
      TM(I)=0.0
      XM(I)=0.0
  302 CONTINUE
C
C     PRINT APPROPRIATE TITLE
C
      GO TO (401,402,403,404) KKK
  401 CONTINUE
      WRITE(6,501) M,IFF
      WRITE(6,101) (KD(1,I),KD(2,I),I=1,L)
      GO TO 405
  402 CONTINUE
      WRITE(6,502) M,IFF
      WRITE(6,101) (KD(1,I),KD(2,I),I=1,L)
      GO TO 405
  403 CONTINUE
      WRITE(6,503) M,IFF
      WRITE(6,102) (KD(1,I),KD(2,I),KD(3,I),I=1,L)
      GO TO 405
  404 CONTINUE
      WRITE(6,504) M,IFF
      WRITE(6,103) (KD(1,I),KD(2,I),I=1,L)
C
C     ARRANGE TIME HISTORY IN OUTPUT FORM
  405 CONTINUE
      TT=-DDT
      N=0
      DO 303 NB=1,NHB
      READ(JT) K,UH
      DO 304 J=1,K
      N=N+1
      TT=TT+DDT
      DO 305 I=1,L
      GO TO (411,412,413,414) KKK
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411 CONTINUE                                                      OUTPUT.427
    JJ=KO(3,I)                                                    OUTPUT.428
    II=IDIS(JJ)                                                   OUTPUT.429
    XX=UH(II,J)                                                   OUTPUT.430
    GO TO 415                                                     OUTPUT.431
412 CONTINUE                                                      OUTPUT.432
    JJ=KO(3,I)                                                    OUTPUT.433
    II=IDIS(JJ)                                                   OUTPUT.434
    XX=UH(II,J)                                                   OUTPUT.435
    GO TO 415                                                     OUTPUT.436
413 CONTINUE                                                      OUTPUT.437
    LL=KO(2,I)                                                    OUTPUT.438
    II=KO(3,I)                                                    OUTPUT.439
    II=ISTR(II,LL)                                                OUTPUT.440
    XX=UH(II,J)                                                   OUTPUT.441
    GO TO 415                                                     OUTPUT.442
414 CONTINUE                                                      OUTPUT.443
    LL=KO(1,I)                                                    OUTPUT.444
    II=KO(2,I)                                                    OUTPUT.445
    II=IWALL(II,LL)                                               OUTPUT.446
    XX=UH(II,J)                                                   OUTPUT.447
C   ABSOLUTE MAX                                                  OUTPUT.448
415 CONTINUE                                                      OUTPUT.449
    AX=ABS(XX)                                                    OUTPUT.450
    IF(AX-XM(I)) 416,416,417                                      OUTPUT.451
417 CONTINUE                                                      OUTPUT.452
    XM(I)=AX                                                      OUTPUT.453
    TM(I)=TT                                                      OUTPUT.454
416 CONTINUE                                                      OUTPUT.455
    X(I,M)=XX                                                     OUTPUT.456
    TA(M)=TT                                                      OUTPUT.457
305 CONTINUE                                                      OUTPUT.458
304 CONTINUE                                                      OUTPUT.459
303 CONTINUE                                                      OUTPUT.460
C                                                                 OUTPUT.461
    GO TO (418,419) KKI                                           OUTPUT.462
418 CONTINUE                                                      OUTPUT.463
    DO 306 N=1,MOS                                                OUTPUT.464
    WRITE(6,104) TA(N),(X(I,N),I=1,L)                             OUTPUT.465
306 CONTINUE                                                      OUTPUT.466
    WRITE(6,105) (XM(I),I=1,L)                                    OUTPUT.467
    WRITE(6,106) (TM(I),I=1,L)                                    OUTPUT.468
    GO TO 420                                                     OUTPUT.469
419 CONTINUE                                                      OUTPUT.470
    IF(KKI .EQ. 2) WRITE(NT) IFF,KKK,L,KD,XM,X                    OUTPUT.471
    GO TO 418                                                     OUTPUT.472
420 CONTINUE                                                      OUTPUT.473
301 CONTINUE                                                      OUTPUT.474
    RETURN                                                        OUTPUT.475
101 FORMAT(8H    TIME,2X,8(I8,1H-,I2,X))                          OUTPUT.476
102 FORMAT(8H    TIME,2X,8(I4,2H--,I3,1H-,I2))                    OUTPUT.477
103 FORMAT(8H    TIME,2X,8(I8,1H-,I2,X))                          OUTPUT.478
104 FORMAT(F8.3,2X,8E12.4)                                        OUTPUT.479
105 FORMAT(* MAXIMUM ABSOLUTE VALUES*/,* MAXIMUM  *,8E12.4)       OUTPUT.480
106 FORMAT(* MAXMUM VALUE TIME*,F10.5)                            OUTPUT.481
501 FORMAT(*1TIME HISTORY FOR SELECTED DISPLACEMENT COMPOENTS*,   OUTPUT.482
   1       5H.....I3,3TX,*FILE NO.*,I3//                          OUTPUT.483
   2       20X,*NODE NUMBERS AND DISPLACEMENT COMPOENTS*)         OUTPUT.484
502 FORMAT(*1TIME HISTORY FOR SELECTED ACCELERATION COMPOENTS*,   OUTPUT.485
   1       5H.....,I3,3TX,*FILE NO.*,I3//                         OUTPUT.486
   2       20X,*NODE NUMBERS AND ACCELERATION COMPOENTS*)         OUTPUT.487
503 FORMAT(*1TIME HISTORY FOR SELECTED STRESS COMPONENTS*,        OUTPUT.488
   1       5H.....,I3,41X,*FILE NO.*,I3//                         OUTPUT.489
   2       20X,*ELEMENT TYPE-ELEMENT NO.-STRESS(1,8),YIELD(9)*/)  OUTPUT.490
504 FORMAT(*1TIME HISTORY FOR SELECTED WALL FORCE*,               OUTPUT.491
   1       5H.....,I3,4TX,*FILE NO.*,I3//                         OUTPUT.492
   2       20X,*WALL NO.-COMPOENTS,1=X,2=YBAR,3=Y*/)              OUTPUT.493
    END                                                           OUTPUT.494
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