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RRELATION OF THEORETICAL AND EXPERIMENTAL TA FOR HIGHWAY BRIDGES

Vol. I. Static Loading

H.A. Miklofsky

May 1974
Final Report

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Washington, D.C. 20590

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16. Abstract A computer programmed analytical routine "SSAP2" is utilized to correlate a theoretical finite element analysis with experimental response of four typical two-lane multi-span girder and slab highway bridges in Tennessee. All bridges give excellent correlation between predicted and experimental responses for deflection and moment distribution to the girders, when the stringers are modelled with beam elements slave connected to plate-shell elements for the deck.

The aspect ratio of element modelling is a critical factor for the finite element analysis. It is concluded that reasonable analytical results can be obtained with the SSAP2 program for non-skew straight composite beam bridges and skew composite beam bridges for low skew angles (less than 30°) provided all element aspect ratios are limited to 1.5 or less. A further restriction in the use of the SSAP2 program is that the bridge load deflection response must be in the linear range.

The appendix of the report contains the theoretical description of the SSAP2 program for the rod, beam, and plate-shell elements. A User's Manual is also included and a computer listing with appropriate control statements for the CDC 6400, IBM 360/370, and Honeywell 6000 machines.

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PREFACE

The authority for conducting this investigation is the Department of Transportation, Contract No. 3-1-1890, Federal Highway Administration, Office of Research, executed with the University of Arizona, Engineering Experiment Station, College of Engineering, Tucson, Arizona 85721.

The principal investigator is Haaren A. Miklofsky, Professor of Civil Engineering, The University of Arizona.

Approximately two-thirds of the CDC computer research was performed on the CDC 6400 computer at the Kitt Peak National Observatory, Tucson, Arizona, while the remaining one-third was performed on the CDC 6400 computer at the University of Arizona. The IBM computer research was performed on the IBM 370 at Hughes Aircraft Company, Tucson, Arizona.

This report was prepared by Haaren A. Miklofsky with the assistance of Shannon Lynch.

ACKNOWLEDGEMENTS

The author is indebted to Dr. Edward L. Wilson, Department of Civil Engineering, the University of California, Berkeley, California, for making available a source deck of computer cards and listing for the SSAP2 program at the start of the research.

Special thanks are extended to Larry Fiock, Director of the Computer Center at the Hughes Aircraft Company, Tucson, Arizona, for permission and assistance in using the IBM 370 computer at Hughes. Thanks are also extended to A. H. Clausen, Jr., computer programmer at Hughes, for assistance in defining the Job Control Statements for the IBM machine at Hughes.

James Smith, Fairbank Highway Research Station, furnished the JCL statements for the IBM 360 computer at FHWA, which are printed herein.

Carl Barber, Kitt Peak National Observatory, was especially helpful as a consultant for the CDC 6400 machine. Thanks are extended to Dr. Stuart Lovell, for permission to use the CDC computer at Kitt Peak.

The author appreciated the opportunity to perform this research under the direction of Robert Varney and Jim Cooper of the Fairbank Highway Research Station, Federal Highway Administration. This research was enjoyable because of their patience and understanding.

Occasional discussion with faculty members, Drs. Hussein Kamel, Ralph Richard, Richmond Neff, Donald Da Deppo, and Hanno Rund produced considerable enlightenment for the author.

Dr. Carlos Felippa, Lockheed Palo Alto Research Laboratory, furnished supplemental documentation for the SLST and SLCCT subroutines.

William Fennell, FHWA, furnished the assembler language subroutine for defining the elapsed CPU time for the IBM log.

Chris Folger, Honeywell Systems Analyst, furnished the control statements and subroutine ETODF for the Honeywell computer.

Finally the author acknowledges with gratitude the able assistance of a dedicated secretary, Shannon Lynch, who programmed, did matrix calculations, and typed the final report.

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CORRELATIONS OF THEORETICAL AND EXPERIMENTAL DATA FOR HIGHWAY BRIDGES

I. Introduction

1. Subject

This report covers the investigation of the use of a computer programmed analytical routine entitled, "SSAP2",[(9)*] to predict displacements, forces, and moments for straight and skew highway bridges. The report also compares selected portions of furnished experimental data for the bridges with results of the theoretical analysis, in which the bridge span is idealized as a grid system. An additional objective is to describe the SSAP2 program in sufficient detail so that a reader, uninitiated in finite element analysis, would be able to understand how the program statements of SSAP2 relate to the finite element analysis theory.

2. Background and Previous Investigation

In 1969 and 1970, four typical two-lane, continuous-span medium-length girder and slab highway bridges in Tennessee were extensively instrumented; and considerable strain and displacement data were obtained at critical points on each structure under a static loading and under the controlled passage of a very heavy vehicle of known weight.[(1)] The four types of bridges tested were:

1. A four-span continuous steel beam structure with composite deck action.
2. A simple span prestressed concrete beam structure.
3. A simple beam reinforced concrete T-beam structure.
4. A three-span continuous steel beam structure with a non-composite deck action.

---

* Superscript numbers in parenthesis ( ) refer to references in Appendix D.

The tests were under the guidance of a Technical Advisory Panel (TAP) consisting of representatives from the Tennessee Department of Highways, the Federal Highway Administration, the Tennessee Valley Authority, and the University of Tennessee. All bridges were located near Winchester, Tennessee. The three Tennessee State highway bridges and one county highway bridge were located in an area to be inundated as a part of the Tennessee Valley Authority Tims Ford Reservoir. Prior to inundation, research personnel from the University of Tennessee instrumented the bridges and tested them first in dynamic loading, and finally static loading to failure. The static loading tests serve as the primary source of background information for the research of this report. The Federal Highway Administration also furnished the author selected sheets of construction drawings of each of the bridges, which are shown in miniature form in Appendix E.

Each of the four bridges was a two-lane deck girder bridge with four longitudinal girders. A description of the bridges is given in Table 1 [(2)] and photographs of the bridges are shown in Figures 1 through 4. Schematic cross-sections of each bridge are shown in Figures 5 and 6. [(3)]

3. Loading

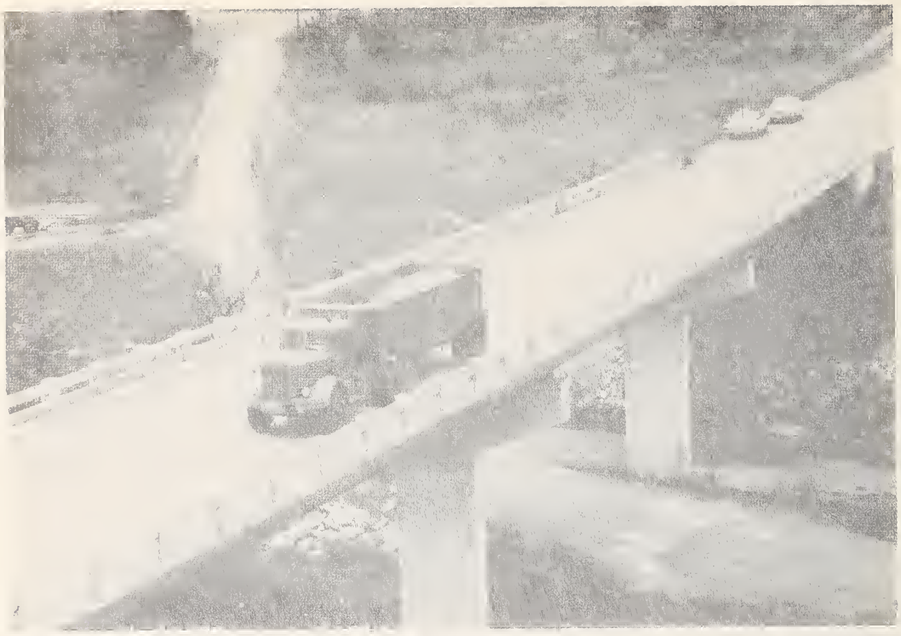
The primary purpose of the static tests was to determine the ultimate capacity and mode of failure of each of the four bridges. The loads were placed on each bridge with the exception of bridge #4 and the test of one span of bridge #2, in such a way as to simulate an HS truck in each lane located in the approximate position to cause maximum positive moment near the center of the span. Some of the loading was done by means of pallets of concrete blocks. Other loadings were done through a series of #18 reinforcing bars connecting a "Rock Anchor System" to a bearing grill on the bridge deck. The reinforcing bars were spliced below the bridge deck with a connection, accommodating a 1 3/8" diameter Stressteel bar for the purpose of measuring load. Full details of the

TABLE 1

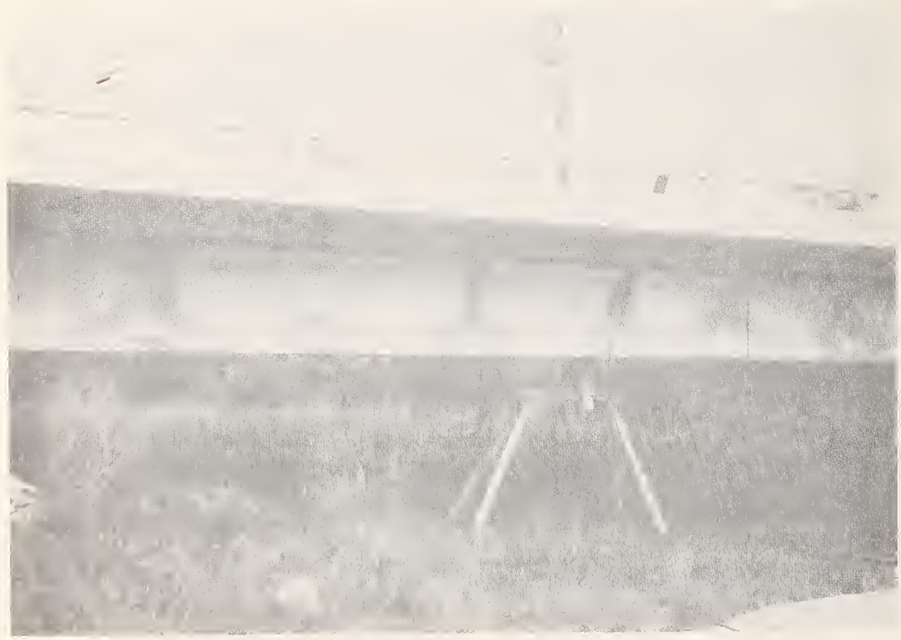
DESCRIPTION OF BRIDGES

Bridge Number	General Description	Span	Girder Spacing	Skew	Location	Design Loading and Date
1	4-Span Continuous, 36" Steel Rolled Beams, Composite in Positive Moment Regions	70'-90'-90'-70'	8' - 4"	90°	Tenn. Rt. 130 over Elk River	HS-20 1963
2	Simple Span Composite with AASHO Type III Precast, Prestressed Concrete Beams	66'	8' - 10"[a]	70°	Tenn. Rt. 130 over Boiling Fork Creek	HS-20 1963
3	Simple Span Reinforced Concrete T-Beams, Monolithic Construction	50'	6' - 10"	60°	US 41A over Elk River	H-15 1938
4	3-Span Continuous, Non-Composite, 27" Steel Rolled Beams	45'-60'-45'	7' - 4"	90°	Mansford Road over Elk River	H-15 1956

[a]Varies due to 4 1/2 degree horizontal curve.



(a) Bridge #1



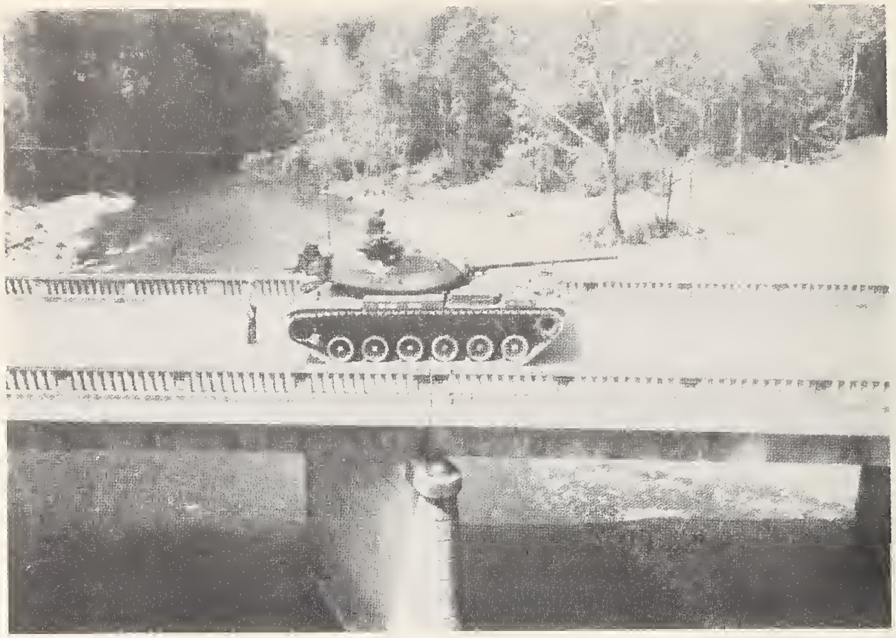
(b) Bridge #2

Fig. 1 - PHOTOGRAPHS OF BRIDGES #1 AND #2

(a) Bridge #2

(b) Bridge #3

Fig. 2 - PHOTOGRAPHS OF BRIDGES #2 AND #3



(a)

(b)

Fig. 3 - PHOTOGRAPHS OF BRIDGE #3

(a)

(b)

Fig. 4 - PHOTOGRAPHS OF BRIDGE #4

loading system and loading procedure can be found in reference (1). Figures 7 through 10 show the location of the loading points for the four bridges,[4] while Table 2 through 5 show the magnitude of loading.[5]

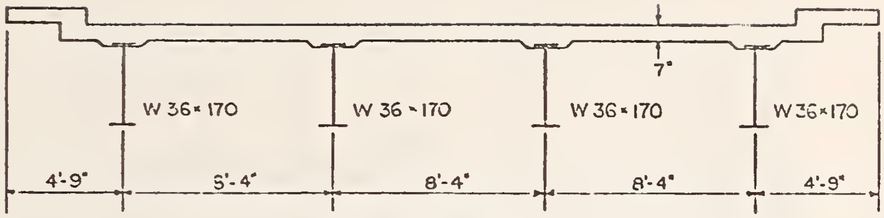
4. Material Property Tests

In order to calculate the theoretical behavior of the four bridges, it was necessary to obtain stress strain relationships for the steel and concrete used in each bridge. During the experimental testing, steel tension coupons were cut from girder webs for bridges #1 and #4; and with these the stress strain diagrams shown in Figure 11[6] were obtained. Figure 12[7] shows a stress strain diagram for one of the samples of girder longitudinal reinforcing bars from bridge #3. Four-inch diameter concrete cores were taken from each bridge deck for determining compressive strengths. The compressive strengths listed in Table 6[8] were corrected to be valid for test cylinders with a diameter of 1/2 the height. Cores were also taken from the precast, prestressed girders of bridge #2. Ten specimens were taken and the test results are listed as follows:

Minimum Compressive Strength	7,600 psi
Maximum Compressive Strength	10,300 psi
Average Compressive Strength	8,700 psi

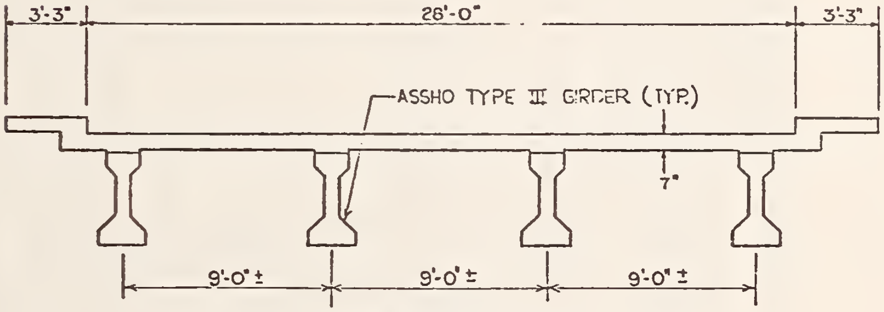
5. Deflection Measurement

Deflectometers were used to measure deflections for both dynamic and static loading of the bridges. Reference (1) contains a detailed discussion of the method of measurement.



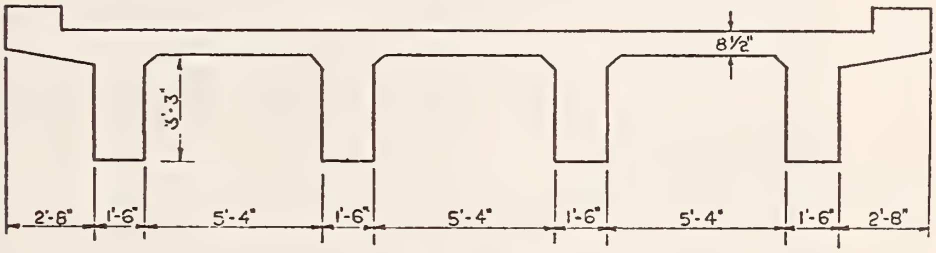
(a) BRIDGE 1, SECTION A-A

(Sections over piers similar except beams are W 36 x 160 with 10 1/2" x 1" cover plates)

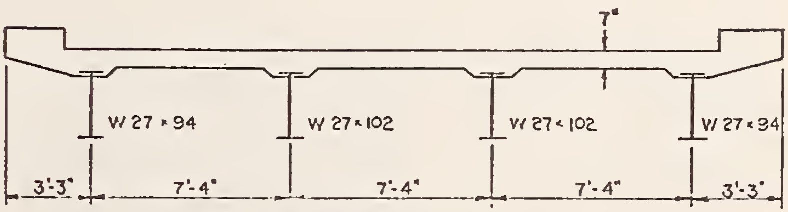


(b) BRIDGE 2, SECTION A-A

FIGURE 5. CROSS-SECTIONS OF BRIDGES 1 AND 2.



(a) BRIDGE 3, SECTION A-A



(b) BRIDGE 4, SECTION A-A

(Sections over interior piers similar except all beams W 27 x 94 with 10" x 3/4" cover plates)

FIGURE 6. CROSS-SECTIONS OF BRIDGES 3 AND 4.

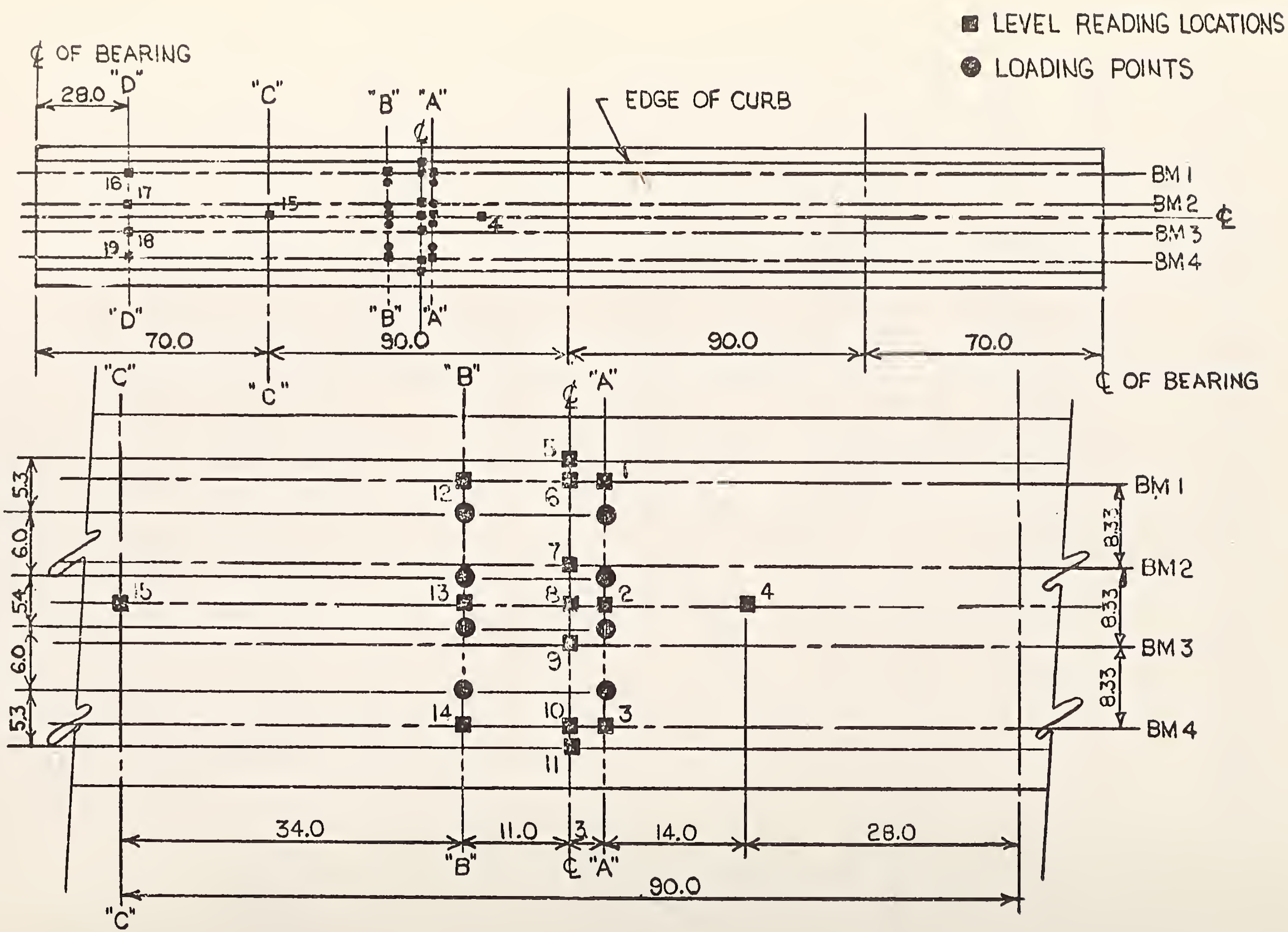


FIGURE 7. LOCATION OF APPLIED LOADS AND POINTS WHERE LEVEL READINGS WERE TAKEN - BRIDGE 1.

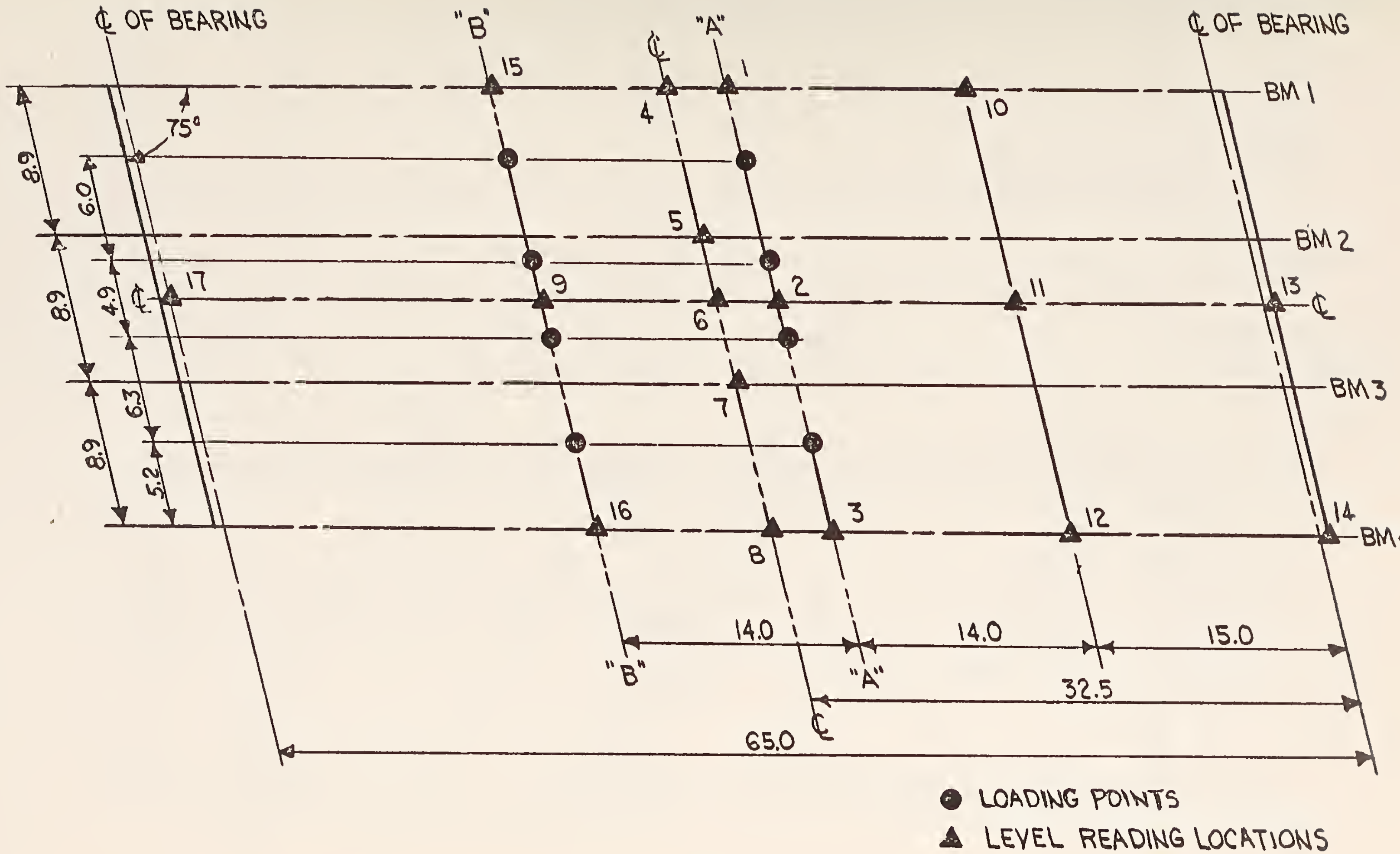


FIGURE 8. LOCATION OF APPLIED LOADS AND POINTS WHERE LEVEL READINGS WERE TAKEN - BRIDGE 2 (Test No. 2500).

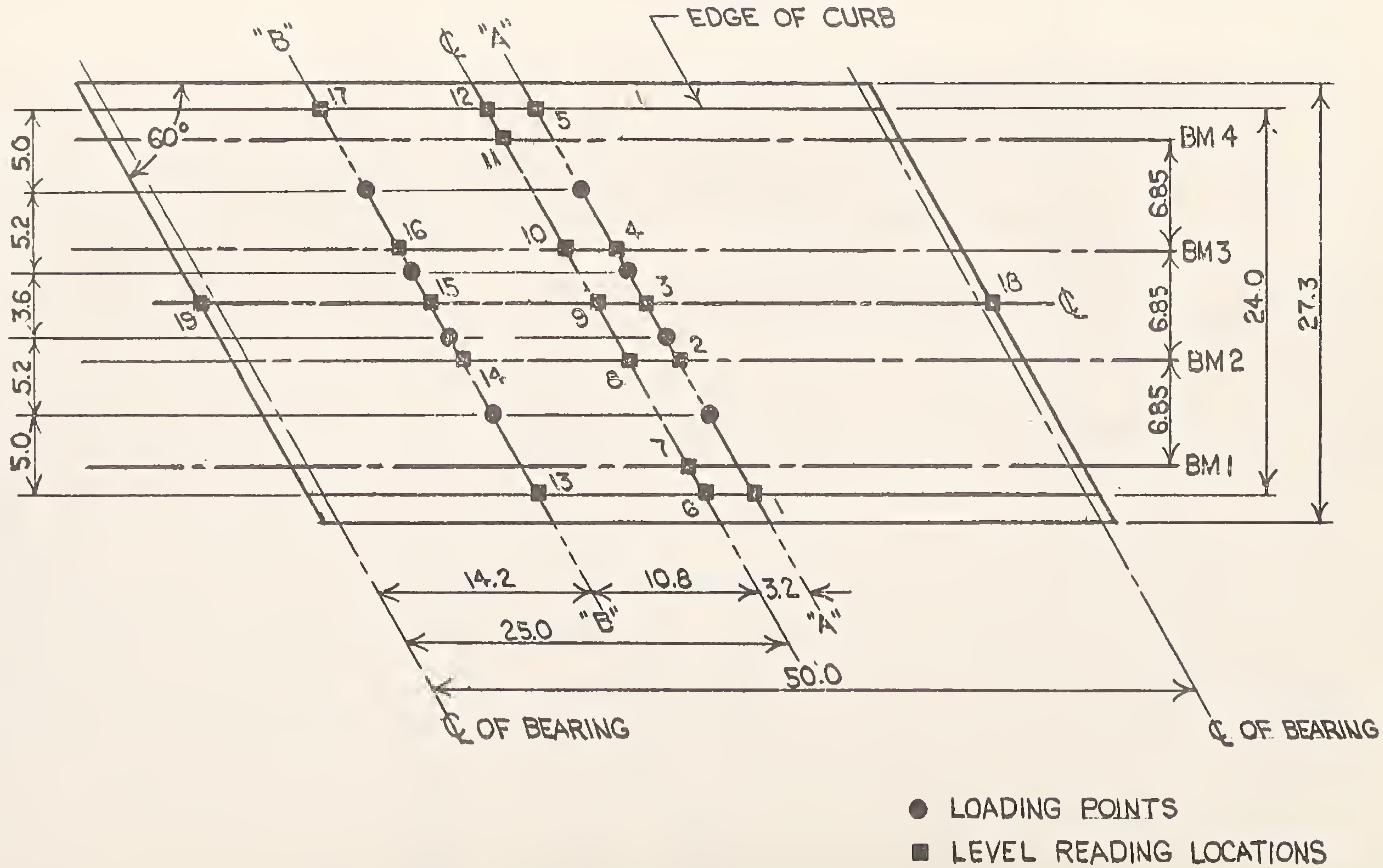


FIGURE 9. LOCATION OF APPLIED LOADS AND POINTS WHERE LEVEL READINGS WERE TAKEN - BRIDGE 3.

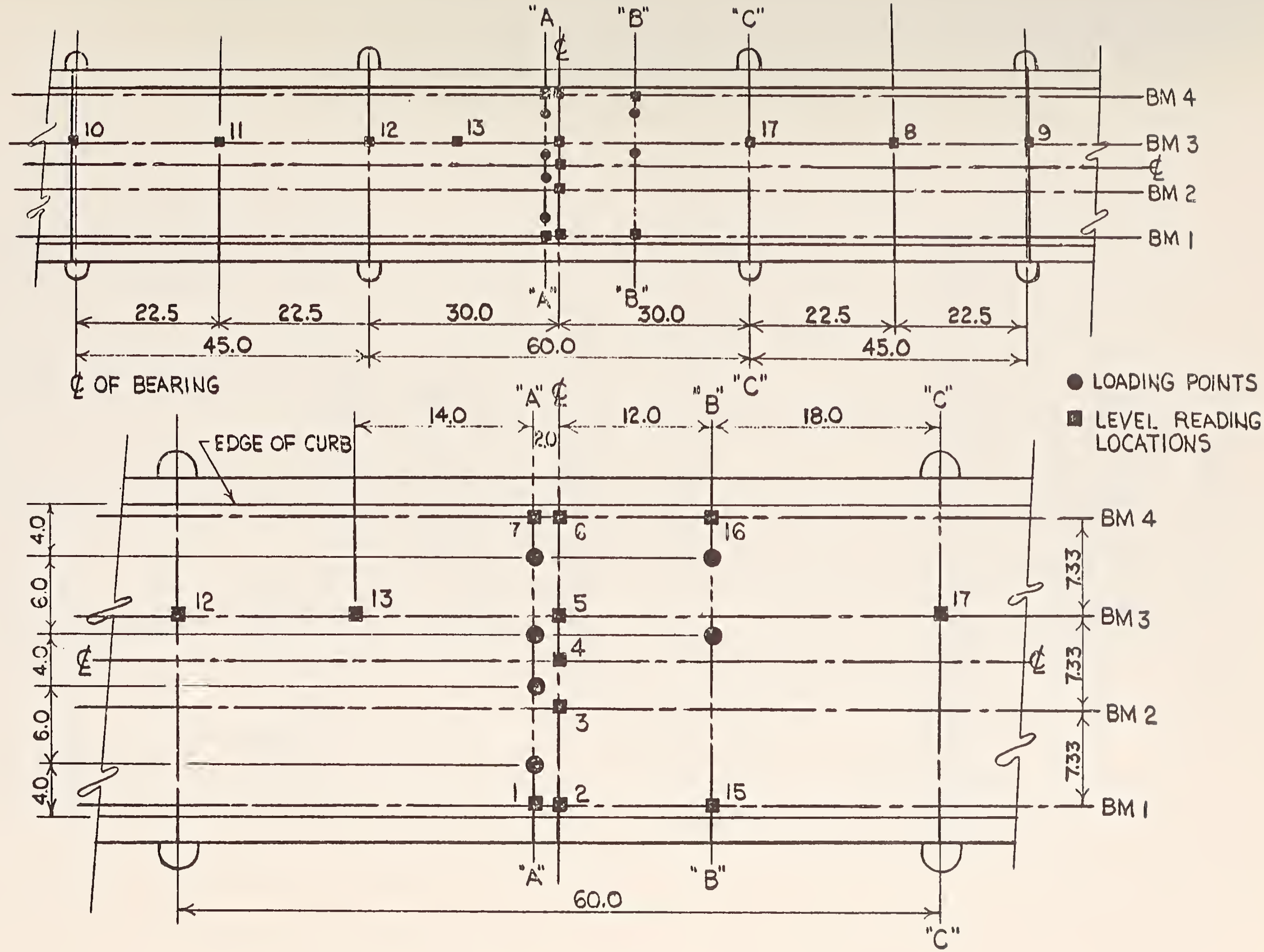
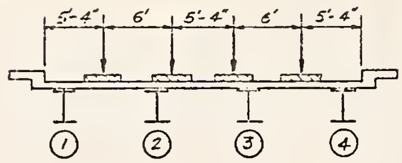


FIGURE 10. LOCATION OF APPLIED LOADS AND POINTS WHERE LEVEL READINGS WERE TAKEN - Bridge 4.

TABLE 2

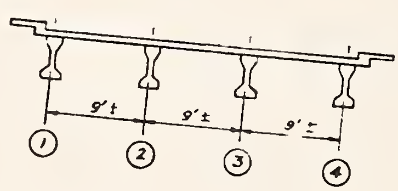
LOAD DISTRIBUTION COEFFICIENTS - BRIDGE 1
STATIC LOAD TESTS



Load At Each Point (kips)	Total Load On Bridge (kips)	Ratio of Bending Moment (%)			
		1	2	3	4
12	96	18	29	30	23
32	256	20	29	29	22
54	432	20	26	29	25
67	536	20	27	29	24
81	648	20	27	30	23
87	696	20	28	30	22
99	792	21	29	30	20

TABLE 3

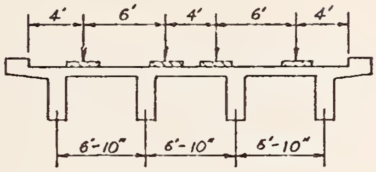
LOAD DISTRIBUTION COEFFICIENTS - BRIDGE 2
STATIC LOAD TESTS



Load At Each Point (kips)	Total Load On Bridge (kips)
15	120
34	370
54	430
75	601
96	765
101	810
117	936
119	950
142	1138

TABLE 4

LOAD DISTRIBUTION COEFFICIENTS - BRIDGE 3
STATIC LOAD TESTS

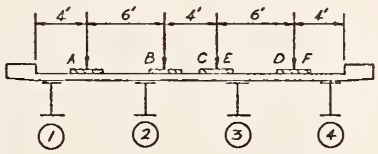


Load At Each Point (kips)	Total Load On Bridge (kips)	Ratio of Bending Moment (%) Interior Girders	Exterior Girders
17.6	141	60	40
48.9	391	57	43
61.5	492	54	46
71.0	567	56	44
80.0	640	56	44
95.3	762	56	44
105.2	845	57	43
118.1	947	58	42

Note: Loads Compiled from Figures 42 through 45.

TABLE 5

LOAD DISTRIBUTION COEFFICIENTS - BRIDGE 4
STATIC LOAD TESTS



Load Location	Load At Each Point (kips)	Total Load On Bridge (kips)	Ratio of Bending Moment (%) 1	2	3	4
A, B,	28	56	38	34	22	6
A, B,	59	118	37	34	22	7
C, D, E, F	28	112	7	20	34	39
C, D, E, F	59	236	7	21	36	36
A, B, C, D	28	112	22	28	29	21
A, B, C, D	59	236	22	28	29	21
All	16	96	20	28	24	28
All	28	168	19	29	26	26
All	38	228	20	28	26	26
All	49	294	20	28	26	26
All	59	354	21	28	25	26

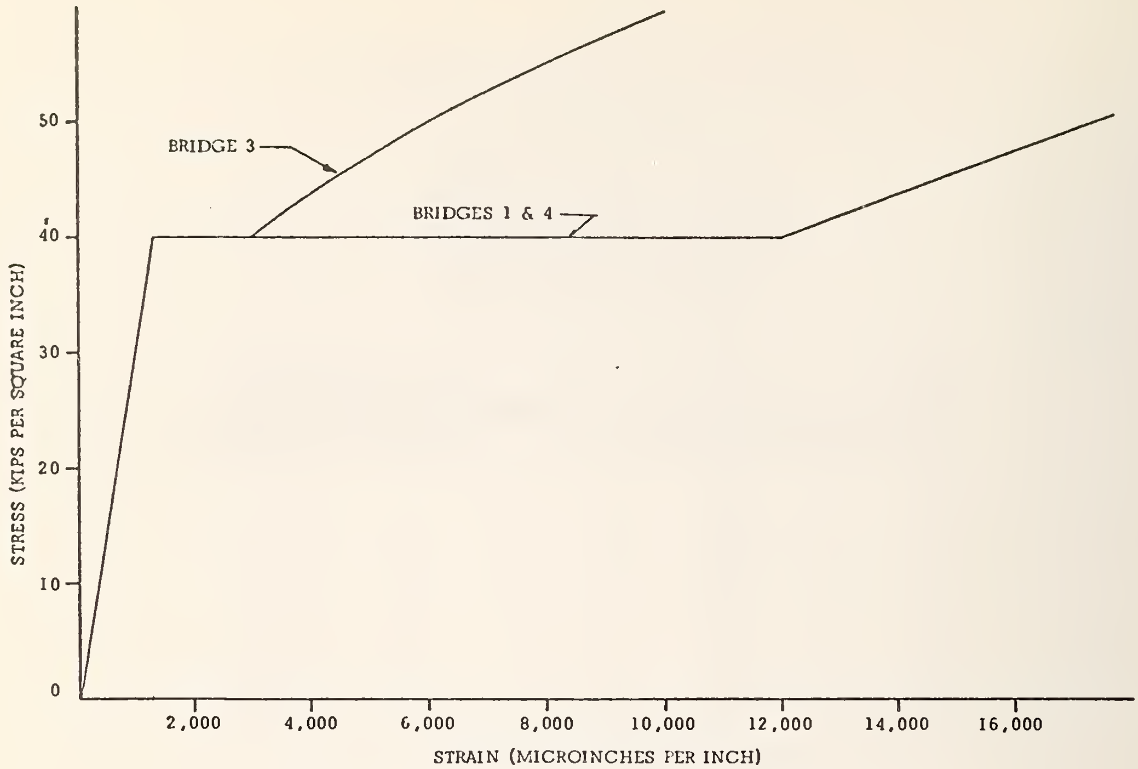


FIGURE 11. STRESS-STRAIN CURVE FOR STEEL IN BRIDGES 1, 3, & 4

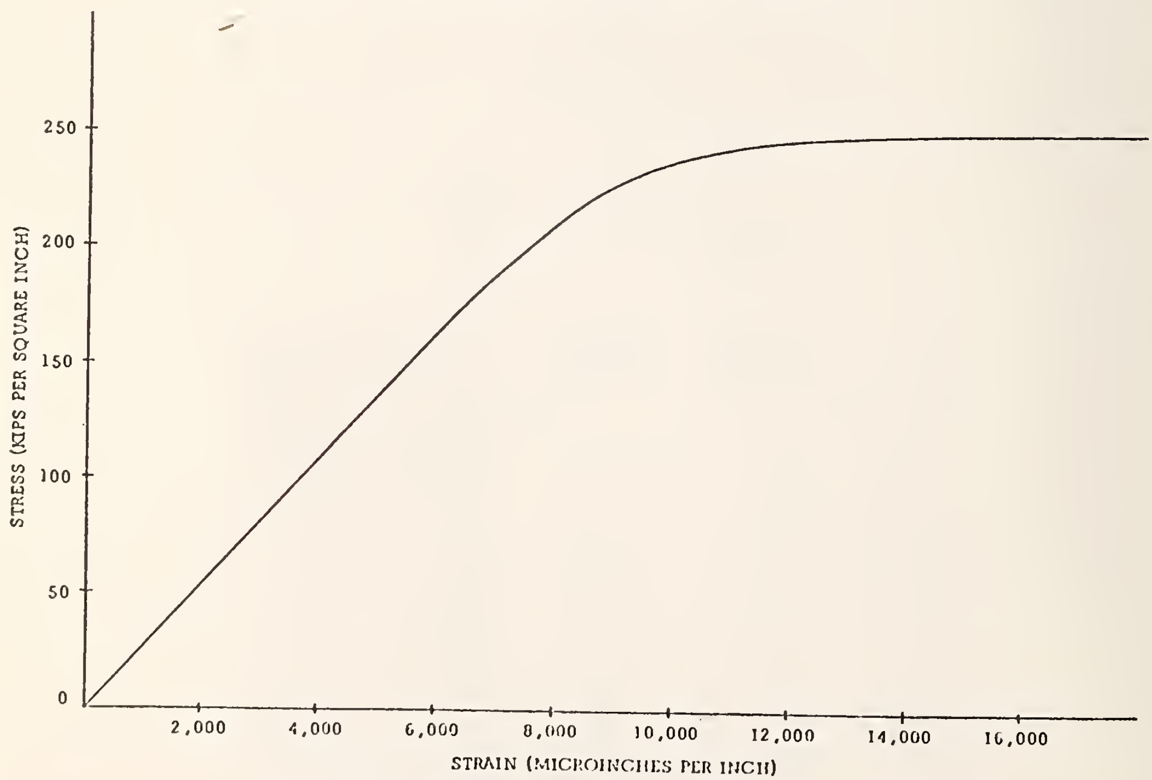


FIGURE 12. STRESS-STRAIN CURVE FOR STEEL IN BRIDGE 2

TABLE 6

TABULATION OF MEASURED COMPRESSIVE STRENGTHS
OF CONCRETE CORES TAKEN FROM BRIDGE DECKS

Bridge Number	Number of Cores Tested	Average Core Height (inches)	Maximum Compressive (psi)	Minimum Compressive (psi)	Average Compressive (psi)
1	42	7.3	8,030	5,170	6,870
2	48	7.0	6,280	4,580	5,500
3	32	8.0	8,200	4,390	6,500
4	32	7.4	7,020	4,090	5,600

II. Computer Analysis of Bridge #1

1. Description

Bridge #1 is a four-span continuous composite steel girder concrete deck bridge built in 1963, on State Route 130 over the Elk River. Figures E-2 through E-5 show details of the bridge reduced from drawings made by the Department of Highways, State of Tennessee. Design specifications were in accordance with the 1961 AASHO* specifications, and the loading was H20-S16-44.

2. Computer Data

Bridge #1 was evaluated by several modeling procedures using the SSAP2 computer program. The most promising results taking into account computer costs, were obtained by using the beam element for the steel stringers, slave connected to quadrilateral plate-shell elements for the concrete slab. The computer input data for this arrangement will be described in detail to familiarize the reader with SSAP2, after which the results of the several modeling procedures will be discussed.

Prior to filling in the input data on standard Fortran form sheets, it was found convenient to prepare a special schematic form sheet for inputing all bridge node numbering and coordinate data. Figure 13 shows this form sheet data arrangement for bridge #1 with respect to node numbering and coordinate location of nodes. A total of 230 nodes were used. Figure 14 shows the beam element numbering for the girders, while Figure 15 shows the plate element numbering for the deck. *Unless the difference in node numbers for each element is kept as small as possible the cost per computer run can be excessive.*

3. Modulus of Elasticity

For the steel girders from Figure 11 the Modulus of Elasticity is:

$$E_s = 31.4 \text{ ksi} \times 10^3$$

ν is assumed as .27

*now AASHTO

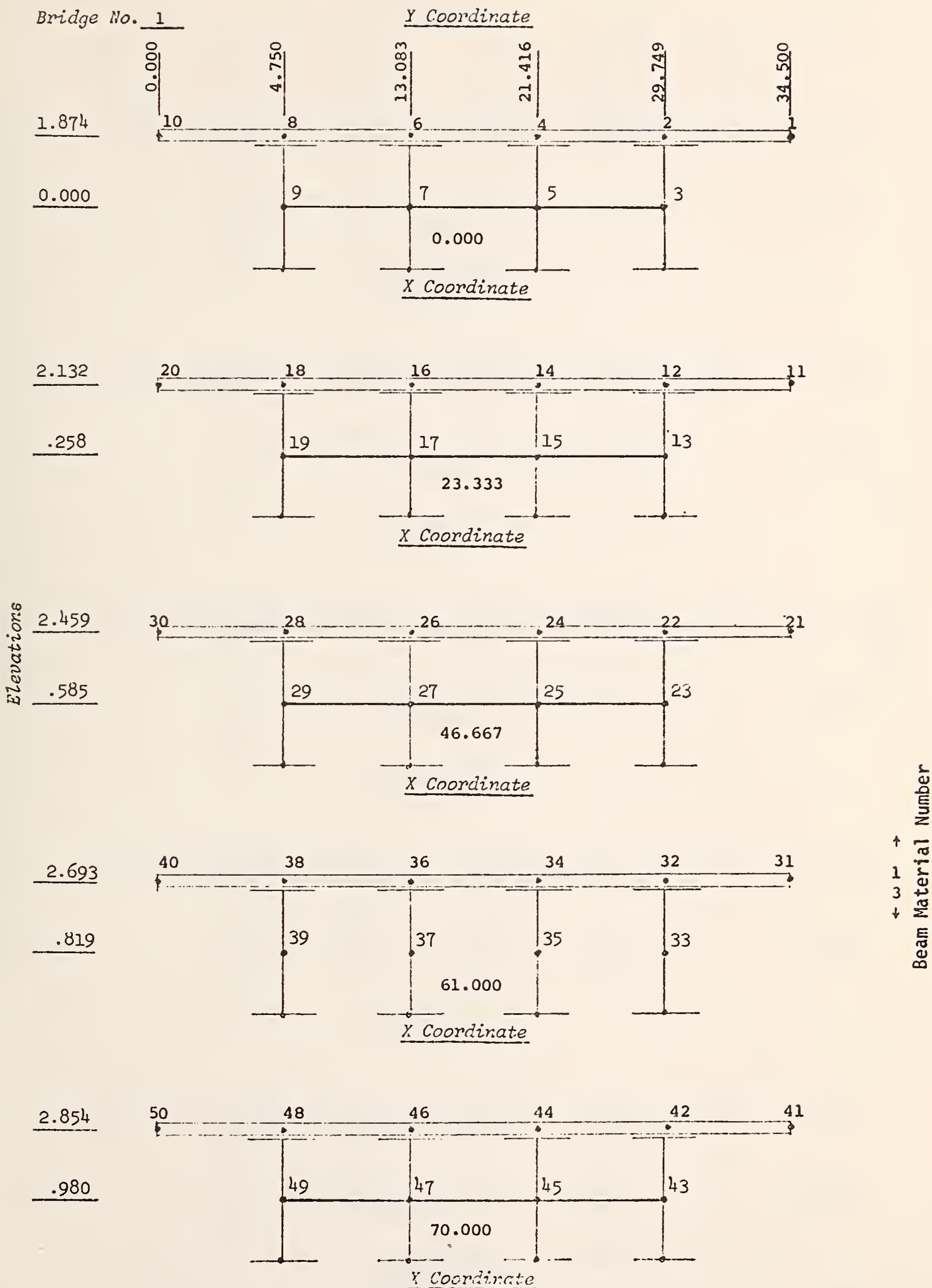


FIG. 13 - NODE COORDINATES FOR BRIDGE #1

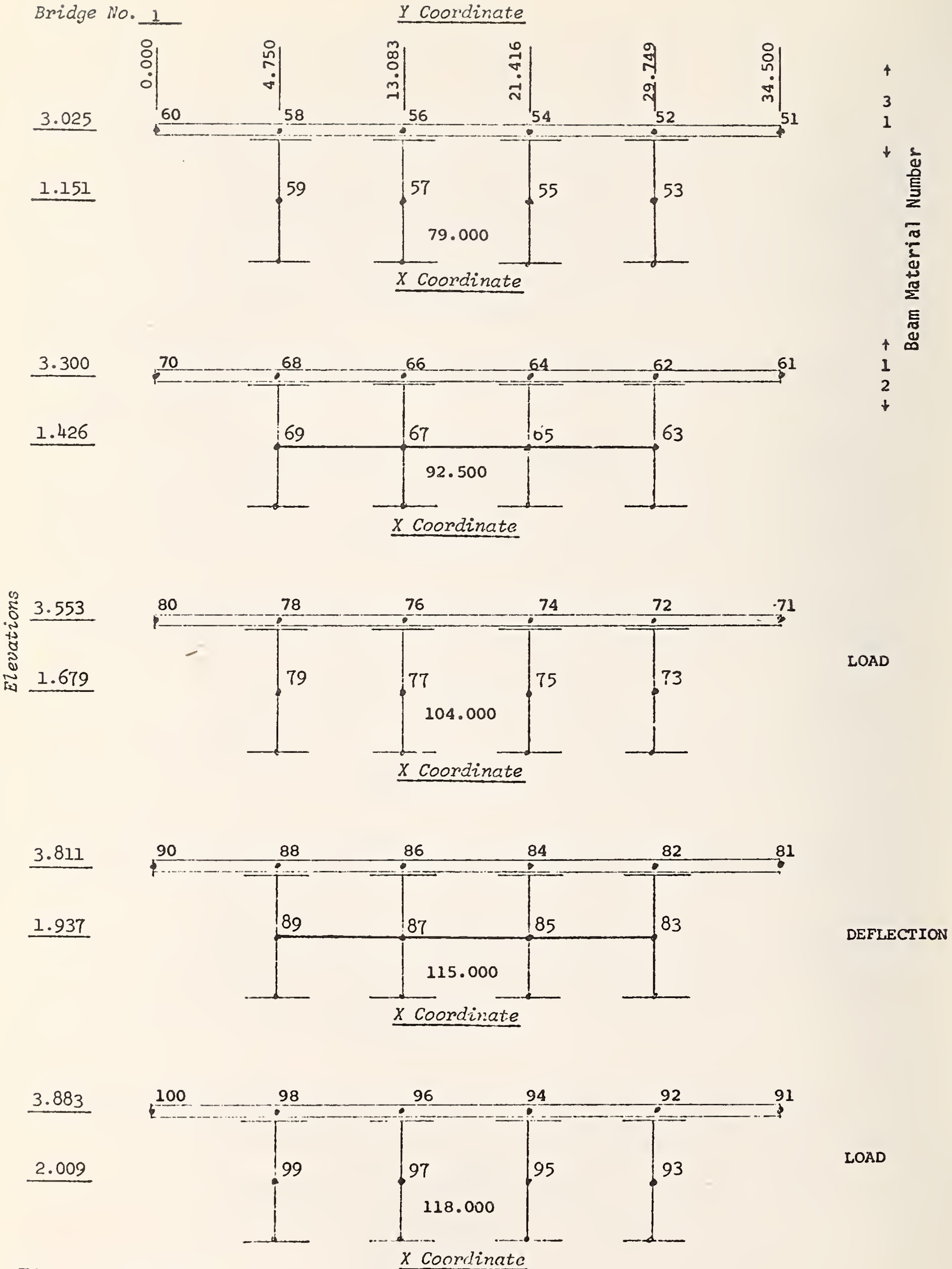


Fig. 13 Continued.

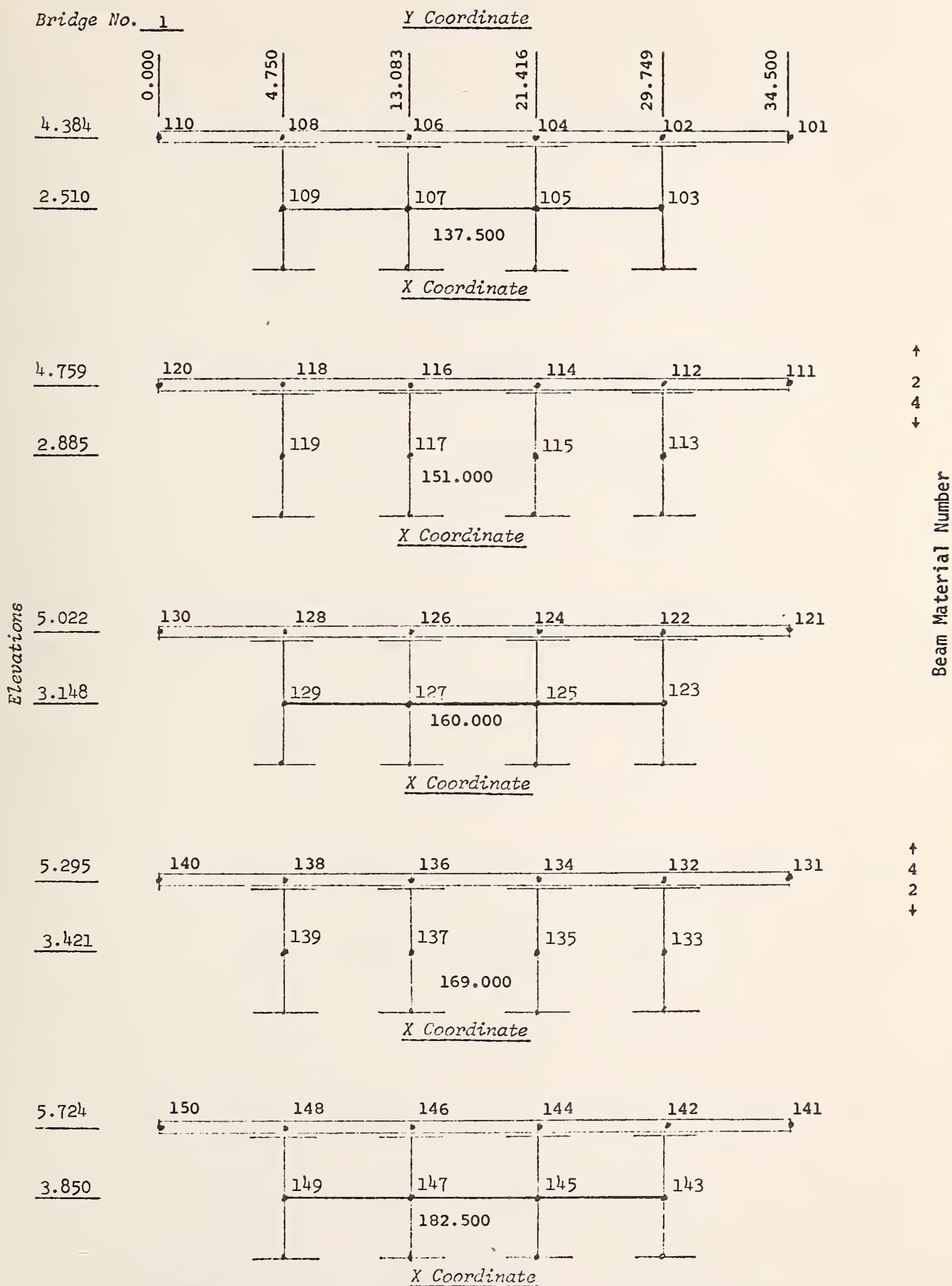


Fig. 13 Continued.

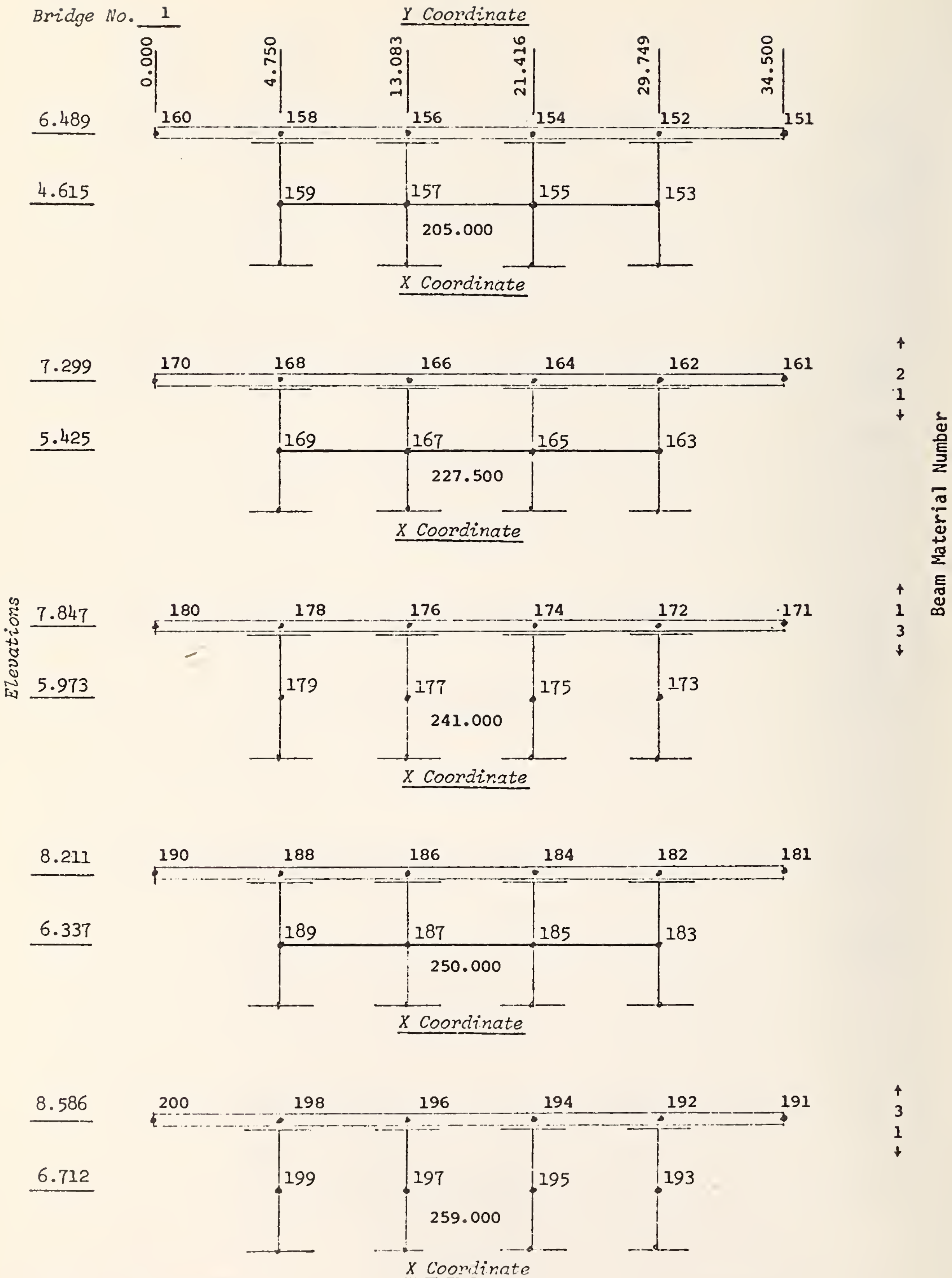


Fig. 13 Continued.

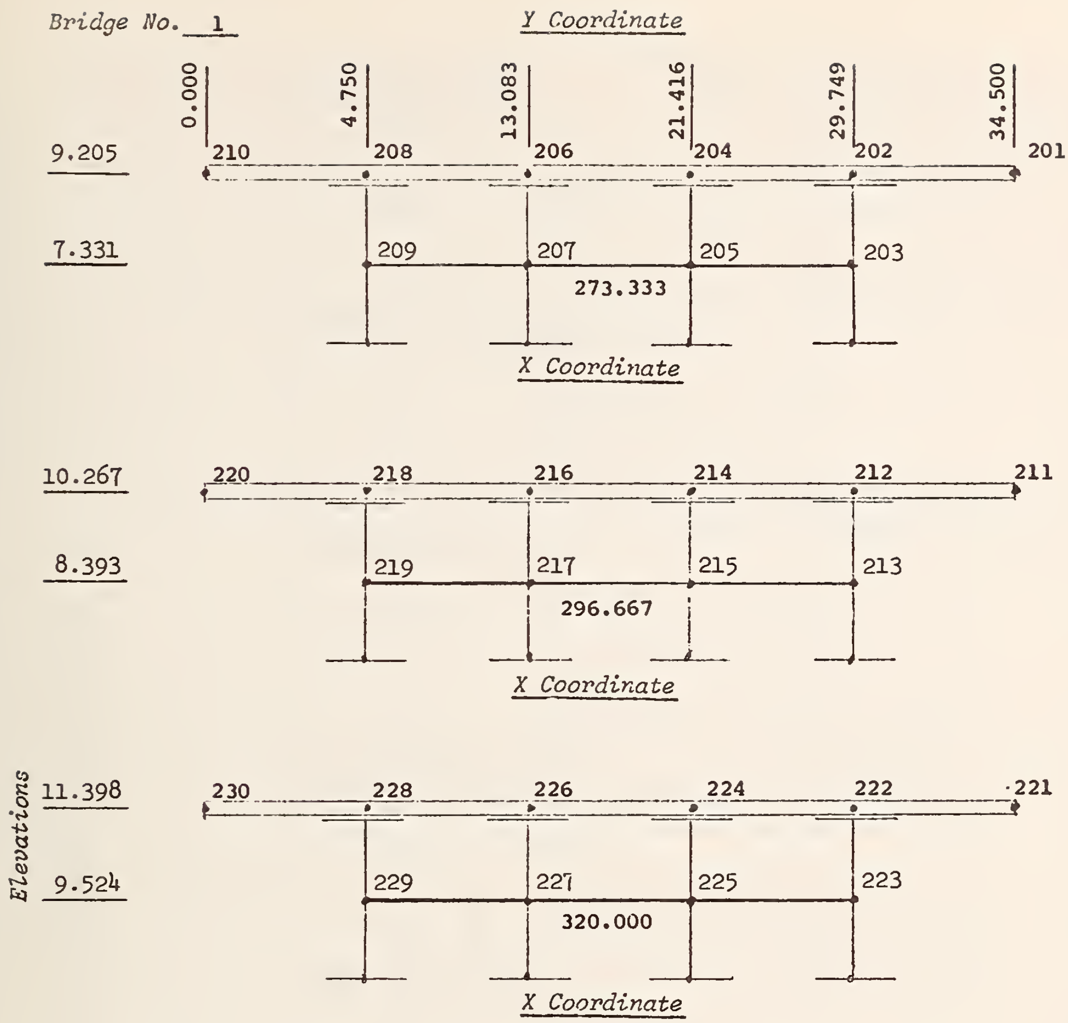


Fig. 13 Continued.

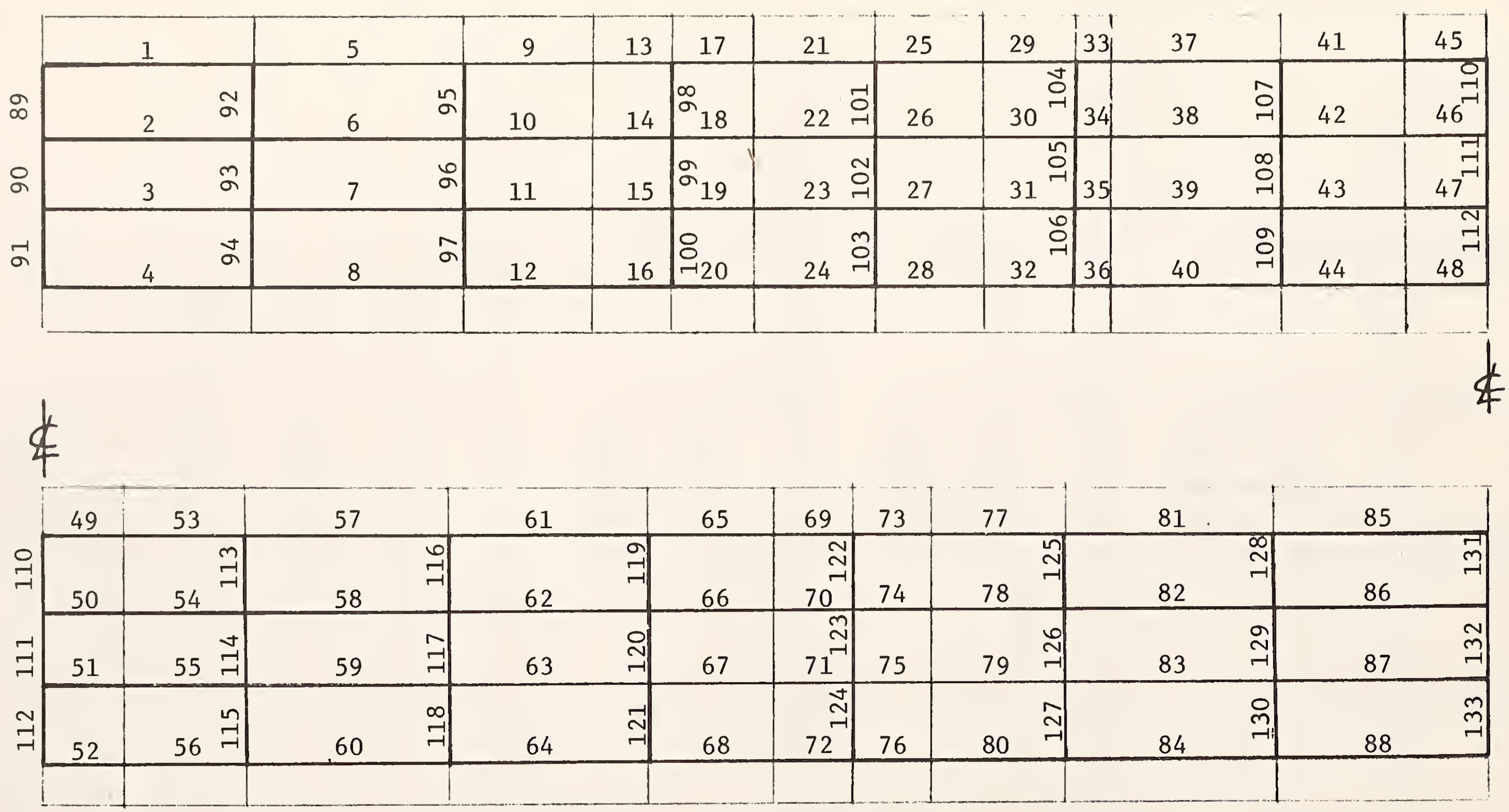


FIG. 14 - BEAM AND DIAPHRAGM NUMBERING FOR BRIDGE #1

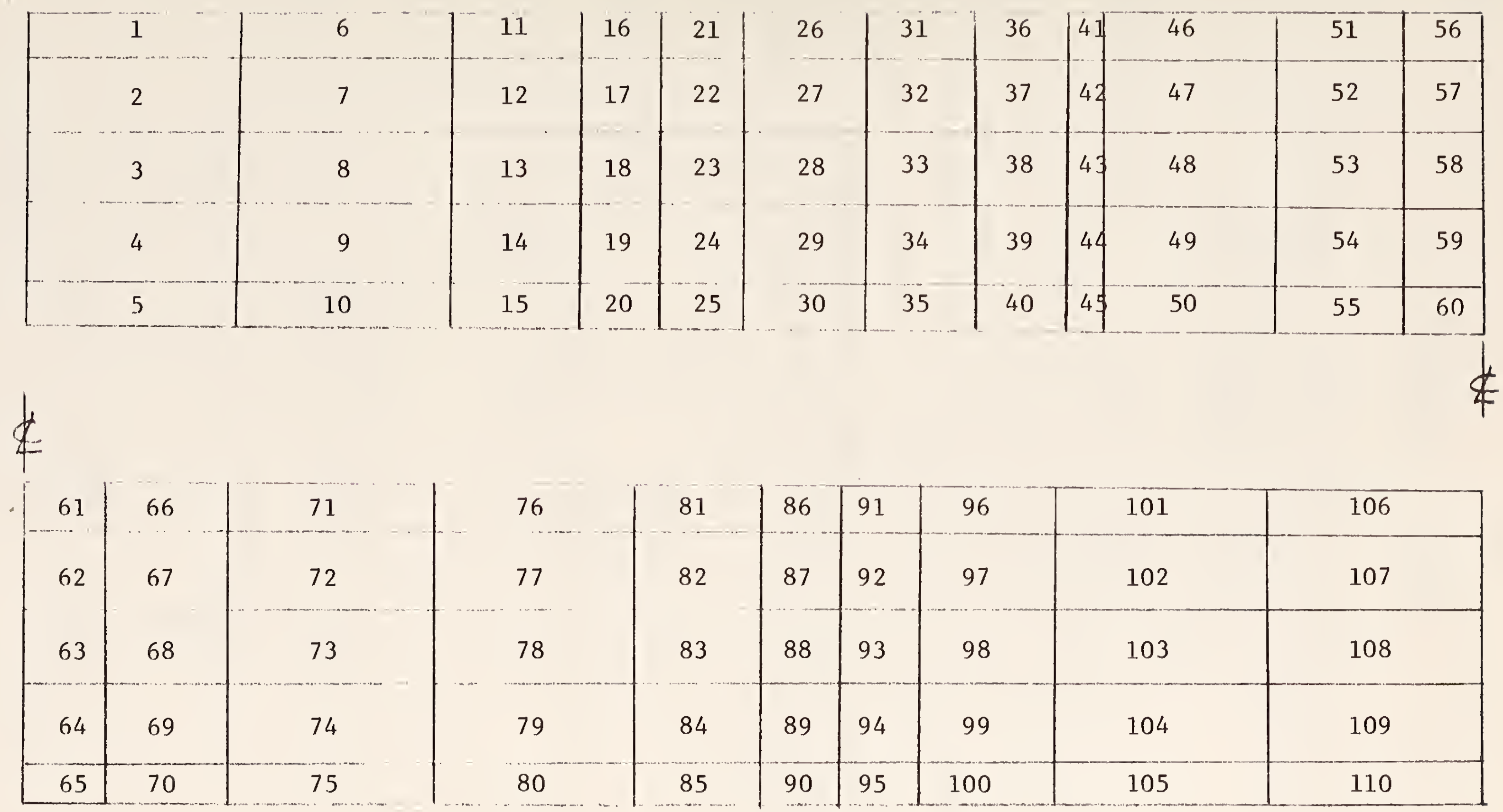


FIG. 15 - PLATE SHELL ELEMENT NUMBERING FOR SLAB OF BRIDGE #1

Since all data for the computer must be in units of kips and feet:

$$E_s = 31,400 \times 144 = 4,521,600 \text{ kips/sq. ft.}$$

From Table 6 the average f'_c for the concrete = 6,870 psi.
Using the ACI formula for Modulus of Elasticity:

$$E_c = 57,500 \times \sqrt{f'_c}$$

$$= \frac{57,500}{1,000} \times \sqrt{6,870}$$

$$= 4766.75 \text{ psi} \times 10^3$$

$$= 4766.75 \times 144$$

$$= 686,412 \text{ ksf}$$

Poisson's ratio ν of 0.15 was assumed for the concrete.

In the shell element the plane stress, stress-strain matrix is used as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Concrete:

$$\frac{E}{1-\nu^2} = \frac{686412}{1-.15^2} = \frac{686412}{.1775} = 702211.76 = C_{xx} = C_{yy}$$

$$\frac{E\nu}{1-\nu^2} = 702211.76(.15) = 105331.76 = C_{xy}$$

$$\frac{E}{2(1+\nu)} = \frac{686412}{2(1.15)} = 298440 = G_{xy}$$

Steel:

$$\frac{E}{1-\nu^2} = \frac{4521600}{1-.27^2} = \frac{4521600}{.9271} = 4877143.7 = C_{xx} = C_{yy}$$

$$\frac{E\nu}{1-\nu^2} = 4877143.7(.27) = 1316828.7 \qquad = C_{xy}$$

$$\frac{E}{2(1+\nu)} = \frac{4521600}{2(1.27)} = 1780157.4 \qquad = G_{xy}$$

4. Elevation Geometry

Finished grade elevations were interpolated from the grade sketch shown on Figure E-1 from Station 7+35 to Station 19+25 using a vertical parabolic curve program routine on a Hewlett-Packard Model 9100B calculator. Using Figure 16 and on the basis that the elevation of node 1 was referenced 0 elevation, the remaining elevations of Figure 13 were so determined. At the time the author prepared the data, the elevations were carried to three significant decimal places. However, upon sober reflection, the author would recommend coordinate data be carried to only one significant figure beyond the decimal point.

5. Moments of Inertia

Using the steel handbook the bending and torsional moments of inertia were determined as shown in Figure 17.

6. Input Listing

Figure 18 shows the input data for bridge #1 in accordance with the User's Manual described in Appendix B. For the convenience of a reader unfamiliar with the SSAP2 program, the top and bottom guides show the column location of numbers on the computer cards.

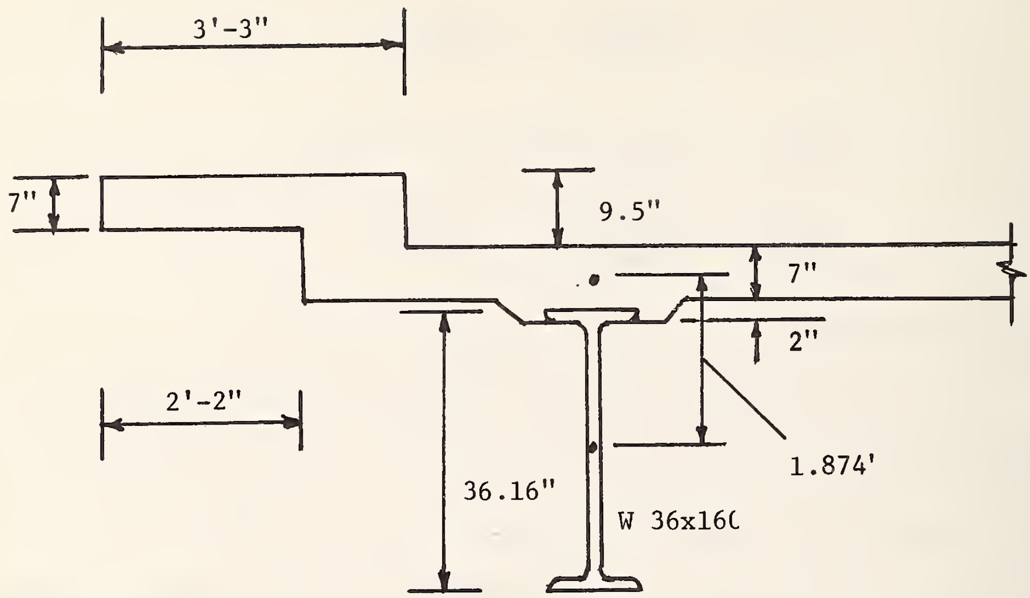


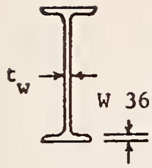
FIG. 16 - BASIC GEOMETRY FOR CROSS SECTION OF BRIDGE #1

Since the boundary condition codes were used in the beam nodes at the bridge supports to slave the beam to the deck with a rigid link, the boundary condition codes for the corresponding deck nodes were used to specify the bearing support conditions for the bridge.

End release codes (pages 34 and B-11) were used for the diaphragms at the exterior girder to reduce the diaphragm end moment to zero, because there is little torsional restraint afforded by the longitudinal girders.

7. Output

Figure 19 is a listing of the output data for bridge #1. Total CPU solution time on a CDC 6400 computer was 126.97 seconds.



Let c = 1728 x 12 d = 144

I_x = 9760/c = .470679 ft^4

I_y = 295/c = .014226

J = 12.4/c = .000597

t_f = 1.02" t_w = .653"

Shear web area = .653 (36) /d = .16325 in^2

Total area = 47.1/d = .327083

I_x = 10500/c = .506365

I_y = 320/c = .015432

J = 15.1/c = .000728

t_f = 1.1 t_w = .68

Shear area = .68 (1.1) /d = .170755

Total area = 50/d = .347222

I_x (covers) = 2 (10.5) (1) $(18.5)^2$ = 7187
 9760
 16947

I_x = 16947/c = .817286

I_y (covers) = 2 (1) $(10.5)^3$/12 = 192.94
 295
 487.94

I_y = 487.94/c = .023531

J (covers @ $at^3/3$ = 2 (10.5) $(1)^3$/3 = 7.000

where $\underline{a}$ is cover area 12.4
 19.4

J = 19.4/d = .000935

By similar reasoning to above:

I_x = .855977 I_y = .024736

J = .001065

Shear area = .1802

Total area = .493055

I_x = .026716

I_y = .000392

J = .000048

Shear area = .05625

Total area = .0875

I_x = .01519

I_y = .000392

J = .000048

Shear area = .041666

Total area = .069166

NOTE: All calculations carried to 6 significant figures on a hand electronic calculator. In retrospect, such accuracy probably unwarranted.

FIG. 17 - MOMENTS OF INERTIA CALCULATIONS FOR STRINGERS AND DIAPHRAGMS, BRIDGE #1

```
**BRIDGE NO 1 - BEAMS AND STRAIGHT SLAB OF PLATES
 230    2    1    0
   1                                1    0.000    34.500    1.874
   2         1    1    1            1    0.000    29.749    1.874
   3    2    2    2    2    2        2    0.000    29.749    0.000
   4         1    1    1            1    0.000    21.416    1.874
   5    4    4    4    4    4        4    0.000    21.416    0.000
   6         1    1    1            1    0.000    13.083    1.874
   7    6    6    6    6    6        6    0.000    13.083    0.000
   8         1    1    1            1    0.000     4.750    1.874
   9    8    8    8    8    8        8    0.000     4.750    0.000
  10                                1    0.000     0.000    1.874
  11                                    23.333    34.500    2.132
  12                                    23.333    29.749    2.132
  13   12   12   12   12   12   12      23.333    29.749    0.258
  14                                    23.333    21.416    2.132
  15   14   14   14   14   14   14      23.333    21.416    0.258
  16                                    23.333    13.083    2.132
  17   16   16   16   16   16   16      23.333    13.083    0.258
  18                                    23.333     4.750    2.132
  19   18   18   18   18   18   18      23.333     4.750    0.258
  20                                    23.333     0.000    2.132
  21                                    46.667    34.500    2.459
  22                                    46.667    29.749    2.459
  23   22   22   22   22   22   22      46.667    29.749    0.585
  24                                    46.667    21.416    2.459
  25   24   24   24   24   24   24      46.667    21.416    0.585
  26                                    46.667    13.083    2.459
  27   26   26   26   26   26   26      46.667    13.083    0.585
  28                                    46.667     4.750    2.459
  29   28   28   28   28   28   28      46.667     4.750    0.585
  30                                    46.667     0.000    2.459
  31                                    61.000    34.500    2.693
  32                                    61.000    29.749    2.693
  33   32   32   32   32   32   32      61.000    29.749    0.819
  34                                    61.000    21.416    2.693
  35   34   34   34   34   34   34      61.000    21.416    0.819
  36                                    61.000    13.083    2.693
  37   36   36   36   36   36   36      61.000    13.083    0.819
  38                                    61.000     4.750    2.693
  39   38   38   38   38   38   38      61.000     4.750    0.819
  40                                    61.000     0.000    2.693
  41                                    70.000    34.500    2.854
  42         1    1    1            1   70.000    29.749    2.854
  43   42   42   42   42   42   42      70.000    29.749    0.980
  44         1    1    1            1   70.000    21.416    2.854
  45   44   44   44   44   44   44      70.000    21.416    0.980
  46         1    1    1            1   70.000    13.083    2.854
  47   46   46   46   46   46   46      70.000    13.083    0.980
  48         1    1    1            1   70.000     4.750    2.854
  49   48   48   48   48   48   48      70.000     4.750    0.980
  50                                    70.000     0.000    2.854
  51                                    79.000    34.500    3.025
  52                                    79.000    29.749    3.025
  53   52   52   52   52   52   52      79.000    29.749    1.151
  54                                    79.000    21.416    3.025
  55   54   54   54   54   54   54      79.000    21.416    1.151
  56                                    79.000    13.083    3.025
  57   56   56   56   56   56   56      79.000    13.083    1.151
  58                                    79.000     4.750    3.025
  59   58   58   58   58   58   58      79.000     4.750    1.151
  60                                    79.000     0.000    3.025
  61                                    92.500    34.500    3.300
  62                                    92.500    29.749    3.300
  63   62   62   62   62   62   62      92.500    29.749    1.426
  64                                    92.500    21.416    3.300
  65   64   64   64   64   64   64      92.500    21.416    1.426
  66                                    92.500    13.083    3.300
  67   66   66   66   66   66   66      92.500    13.083    1.426
  68                                    92.500     4.750    3.300
  69   68   68   68   68   68   68      92.500     4.750    1.426
  70                                    92.500     0.000    3.300
```

FIG. 18 - INPUT LISTING FOR BRIDGE #1

71							104.000	34.500	3.553
72							104.000	29.749	3.553
73	72	72	72	72	72	72	104.000	29.749	1.679
74							104.000	21.416	3.553
75	74	74	74	74	74	74	104.000	21.416	1.679
76							104.000	13.083	3.553
77	76	76	76	76	76	76	104.000	13.083	1.679
78							104.000	4.750	3.553
79	78	78	78	78	78	78	104.000	4.750	1.679
80							104.000	0.000	3.553
81							115.000	34.500	3.811
82							115.000	29.749	3.811
83	82	82	82	82	82	82	115.000	29.749	1.937
84							115.000	21.416	3.811
85	84	84	84	84	84	84	115.000	21.416	1.937
86							115.000	13.083	3.811
87	86	86	86	86	86	86	115.000	13.083	1.937
88							115.000	4.750	3.811
89	88	88	88	88	88	88	115.000	4.750	1.937
90							115.000	0.000	3.811
91							118.000	34.500	3.883
92							118.000	29.749	3.883
93	92	92	92	92	92	92	118.000	29.749	2.009
94							118.000	21.416	3.883
95	94	94	94	94	94	94	118.000	21.416	2.009
96							118.000	13.083	3.883
97	96	96	96	96	96	96	118.000	13.083	2.009
98							118.000	4.750	3.883
99	98	98	98	98	98	98	118.000	4.750	2.009
100							118.000	0.000	3.883
101							137.500	34.500	4.384
102							137.500	29.749	4.384
103	102	102	102	102	102	102	137.500	29.749	2.510
104							137.500	21.416	4.384
105	104	104	104	104	104	104	137.500	21.416	2.510
106							137.500	13.083	4.384
107	106	106	106	106	106	106	137.500	13.083	2.510
108							137.500	4.750	4.384
109	108	108	108	108	108	108	137.500	4.750	2.510
110							137.500	0.000	4.384
111							151.000	34.500	4.759
112							151.000	29.749	4.759
113	112	112	112	112	112	112	151.000	29.749	2.885
114							151.000	21.416	4.759
115	114	114	114	114	114	114	151.000	21.416	2.885
116							151.000	13.083	4.759
117	116	116	116	116	116	116	151.000	13.083	2.885
118							151.000	4.750	4.759
119	118	118	118	118	118	118	151.000	4.750	2.885
120							151.000	0.000	4.759
121							160.000	34.500	5.022
122	1	1	1	1		1	160.000	29.749	5.022
123	122	122	122	122	122	122	160.000	29.749	3.148
124	1	1	1	1		1	160.000	21.416	5.022
125	124	124	124	124	124	124	160.000	21.416	3.148
126	1	1	1	1		1	160.000	13.083	5.022
127	126	126	126	126	126	126	160.000	13.083	3.148
128	1	1	1	1		1	160.000	4.750	5.022
129	128	128	128	128	128	128	160.000	4.750	3.148
130							160.000	0.000	5.022
131							169.000	34.500	5.295
132							169.000	29.749	5.295
133	132	132	132	132	132	132	169.000	29.749	3.421
134							169.000	21.416	5.295
135	134	134	134	134	134	134	169.000	21.416	3.421
136							169.000	13.083	5.295
137	136	136	136	136	136	136	169.000	13.083	3.421
138							169.000	4.750	5.295
139	138	138	138	138	138	138	169.000	4.750	3.421
140							169.000	0.000	5.295
141							182.500	34.500	5.724
142							182.500	29.749	5.724

Fig. 18 Continued.

143	142	142	142	142	142	142	182.500	29.749	3.850
144							182.500	21.416	5.724
145	144	144	144	144	144	144	182.500	21.416	3.850
146							182.500	13.083	5.724
147	146	146	146	146	146	146	182.500	13.083	3.850
148							182.500	4.750	5.724
149	148	148	148	148	148	148	182.500	4.750	3.850
150							182.500	0.000	5.724
151							205.000	34.500	6.489
152							205.000	29.749	6.489
153	152	152	152	152	152	152	205.000	29.749	4.615
154							205.000	21.416	6.489
155	154	154	154	154	154	154	205.000	21.416	4.615
156							205.000	13.083	6.489
157	156	156	156	156	156	156	205.000	13.083	4.615
158							205.000	4.750	6.489
159	158	158	158	158	158	158	205.000	4.750	4.615
160							205.000	0.000	6.489
161							227.500	34.500	7.299
162							227.500	29.749	7.299
163	162	162	162	162	162	162	227.500	29.749	5.425
164							227.500	21.416	7.299
165	164	164	164	164	164	164	227.500	21.416	5.425
166							227.500	13.083	7.299
167	166	166	166	166	166	166	227.500	13.083	5.425
168							227.500	4.750	7.299
169	168	168	168	168	168	168	227.500	4.750	5.425
170							227.500	0.000	7.299
171							241.000	34.500	7.847
172							241.000	29.749	7.847
173	172	172	172	172	172	172	241.000	29.749	5.973
174							241.000	21.416	7.847
175	174	174	174	174	174	174	241.000	21.416	5.973
176							241.000	13.083	7.847
177	176	176	176	176	176	176	241.000	13.083	5.973
178							241.000	4.750	7.847
179	178	178	178	178	178	178	241.000	4.750	5.973
180							241.000	0.000	7.847
181							250.000	34.500	8.211
182		1	1	1		1	250.000	29.749	8.211
183	182	182	182	182	182	182	250.000	29.749	6.337
184		1	1	1		1	250.000	21.416	8.211
185	184	184	184	184	184	184	250.000	21.416	6.337
186		1	1	1		1	250.000	13.083	8.211
187	186	186	186	186	186	186	250.000	13.083	6.337
188		1	1	1		1	250.000	4.750	8.211
189	188	188	188	188	188	188	250.000	4.750	6.337
190							250.000	0.000	8.211
191							259.000	34.500	8.586
192							259.000	29.749	8.586
193	192	192	192	192	192	192	259.000	29.749	6.712
194							259.000	21.416	8.586
195	194	194	194	194	194	194	259.000	21.416	6.712
196							259.000	13.083	8.586
197	196	196	196	196	196	196	259.000	13.083	6.712
198							259.000	4.750	8.586
199	198	198	198	198	198	198	259.000	4.750	6.712
200							259.000	0.000	8.586
201							273.333	34.500	9.205
202							273.333	29.749	9.205
203	202	202	202	202	202	202	273.333	29.749	7.331
204							273.333	21.416	9.205
205	204	204	204	204	204	204	273.333	21.416	7.331
206							273.333	13.083	9.205
207	206	206	206	206	206	206	273.333	13.083	7.331
208							273.333	4.750	9.205
209	208	208	208	208	208	208	273.333	4.750	7.331
210							273.333	0.000	9.205
211							296.667	34.500	10.267
212							296.667	29.749	10.267
213	212	212	212	212	212	212	296.667	29.749	8.393
214							296.667	21.416	10.267

Fig. 18 Continued.

```
215  214  214  214  214  214  214   296.667    21.416     8.393
216                                 296.667    13.083    10.267
217  216  216  216  216  216  216   296.667    13.083     8.393
218                                 296.667     4.750    10.267
219  218  218  218  218  218  218   296.667     4.750     8.393
220                                 296.667     0.000    10.267
221                                1  320.000    34.500    11.398
222          1    1    1           1  320.000    29.749    11.398
223  222  222  222  222  222  222   320.000    29.749     9.524
224          1    1    1           1  320.000    21.416    11.398
225  224  224  224  224  224  224   320.000    21.416     9.524
226          1    1    1           1  320.000    13.083    11.398
227  226  226  226  226  226  226   320.000    13.083     9.524
228          1    1    1           1  320.000     4.750    11.398
229  228  228  228  228  228  228   320.000     4.750     9.524
230                                1  320.000     0.000    11.398
  2  133    6    0    1
  1 4521600.  .27
  1 .3270633  .16325                 .0005979  .0142264  .470679
  2 .3472222  .1707555               .0007282  .015432   .5063657
  3 .4729166  .1723194               .0009355  .023531   .8172863
  4 .4930555  .1802                  .0010557  .0247366  .855977
  5 .0875     .05625                 .0000593  .0006944  .0267168
  6 .0691666  .0416666               .0000487  .000392   .0151909
 0.          0.          0.          0.
 0.          0.          0.          0.
 0.          0.          0.          0.
  1    3   13    2    1    1
  2    5   15    4    1    1
  3    7   17    6    1    1
  4    9   19    8    1    1
  5   13   23   12    1    1
  6   15   25   14    1    1
  7   17   27   16    1    1
  8   19   29   18    1    1
  9   23   33   22    1    1
 10   25   35   24    1    1
 11   27   37   26    1    1
 12   29   39   28    1    1
 13   33   43   32    1    3
 14   35   45   34    1    3
 15   37   47   36    1    3
 16   39   49   38    1    3
 17   43   53   42    1    3
 18   45   55   44    1    3
 19   47   57   46    1    3
 20   49   59   48    1    3
 21   53   63   52    1    1
 22   55   65   54    1    1
 23   57   67   56    1    1
 24   59   69   58    1    1
 25   63   73   62    1    2
 26   65   75   64    1    2
 27   67   77   66    1    2
 28   69   79   68    1    2
 29   73   83   72    1    2
 30   75   85   74    1    2
 31   77   87   76    1    2
 32   79   89   78    1    2
 33   83   93   82    1    2
 34   85   95   84    1    2
 35   87   97   86    1    2
 36   89   99   88    1    2
 37   93  103   92    1    2
 38   95  105   94    1    2
 39   97  107   96    1    2
 40   99  109   98    1    2
 41  103  113  102    1    2
 42  105  115  104    1    2
 43  107  117  106    1    2
 44  109  119  108    1    2
 45  113  123  112    1    4
```

Fig. 18 Continued.

46	115	125	114	1	4		
47	117	127	116	1	4		
48	119	129	118	1	4		
49	123	133	122	1	4		
50	125	135	124	1	4		
51	127	137	126	1	4		
52	129	139	128	1	4		
53	133	143	132	1	2		
54	135	145	134	1	2		
55	137	147	136	1	2		
56	139	149	138	1	2		
57	143	153	142	1	2		
58	145	155	144	1	2		
59	147	157	146	1	2		
60	149	159	148	1	2		
61	153	163	152	1	2		
62	155	165	154	1	2		
63	157	167	156	1	2		
64	159	169	158	1	2		
65	163	173	162	1	1		
66	165	175	164	1	1		
67	167	177	166	1	1		
68	169	179	168	1	1		
69	173	183	172	1	3		
70	175	185	174	1	3		
71	177	187	176	1	3		
72	179	189	178	1	3		
73	183	193	182	1	3		
74	185	195	184	1	3		
75	187	197	186	1	3		
76	189	199	188	1	3		
77	193	203	192	1	1		
78	195	205	194	1	1		
79	197	207	196	1	1		
80	199	209	198	1	1		
81	203	213	202	1	1		
82	205	215	204	1	1		
83	207	217	206	1	1		
84	209	219	208	1	1		
85	213	223	212	1	1		
86	215	225	214	1	1		
87	217	227	216	1	1		
88	219	229	218	1	1		
89	3	5	2	1	5	000001	
90	5	7	4	1	5		
91	7	9	6	1	5		000001
92	13	15	12	1	6	000001	
93	15	17	14	1	6		
94	17	19	16	1	6		000001
95	23	25	22	1	6	000001	
96	25	27	24	1	6		
97	27	29	26	1	6		000001
98	43	45	42	1	6	000001	
99	45	47	44	1	6		
100	47	49	46	1	6		000001
101	63	65	62	1	6	000001	
102	65	67	64	1	6		
103	67	69	66	1	6		000001
104	83	85	82	1	6	000001	
105	85	87	84	1	6		
106	87	89	86	1	6		000001
107	103	105	102	1	6	000001	
108	105	107	104	1	6		
109	107	109	106	1	6		000001
110	123	125	122	1	6	000001	
111	125	127	124	1	6		
112	127	129	126	1	6		000001
113	143	145	142	1	6	000001	
114	145	147	144	1	6		
115	147	149	146	1	6		000001
116	153	155	152	1	6	000001	
117	155	157	154	1	6		

Fig. 18 Continued.

```
 118  157  159  156    1    6                                000001
 119  163  165  162    1    6                     000001
 120  165  167  164    1    6
 121  167  169  166    1    6                                000001
 122  183  185  182    1    6                     000001
 123  185  187  184    1    6
 124  187  189  186    1    6                                000001
 125  203  205  202    1    6                     000001
 126  205  207  204    1    6
 127  207  209  206    1    6                                000001
 128  213  215  212    1    6                     000001
 129  215  217  214    1    6
 130  217  219  216    1    6                                000001
 131  223  225  222    1    5                     000001
 132  225  227  224    1    5
 133  227  229  226    1    5                                000001
   6  110    1
        1
702211.76 105331.76          702211.76            298440.
   0.        0.        0.        0.
   0.        0.        0.        0.
   0.        0.        0.        0.
   0.        0.        0.        0.
   0.        0.        0.        0.
   1    2   12   11    1                   .5833
   2    4   14   12    2                   .5833
   3    6   16   14    4                   .5833
   4    8   18   16    6                   .5833
   5   10   20   18    8                   .5833
   6   12   22   21   11                   .5833
   7   14   24   22   12                   .5833
   8   16   26   24   14                   .5833
   9   18   28   26   16                   .5833
  10   20   30   28   18                   .5833
  11   22   32   31   21                   .5833
  12   24   34   32   22                   .5833
  13   26   36   34   24                   .5833
  14   28   38   36   26                   .5833
  15   30   40   38   28                   .5833
  16   32   42   41   31                   .5833
  17   34   44   42   32                   .5833
  18   36   46   44   34                   .5833
  19   38   48   46   36                   .5833
  20   40   50   48   38                   .5833
  21   42   52   51   41                   .5833
  22   44   54   52   42                   .5833
  23   46   56   54   44                   .5833
  24   48   58   56   46                   .5833
  25   50   60   58   48                   .5833
  26   52   62   61   51                   .5833
  27   54   64   62   52                   .5833
  28   56   66   64   54                   .5833
  29   58   68   66   56                   .5833
  30   60   70   68   58                   .5833
  31   62   72   71   61                   .5833
  32   64   74   72   62                   .5833
  33   66   76   74   64                   .5833
  34   68   78   76   66                   .5833
  35   70   80   78   68                   .5833
  36   72   82   81   71                   .5833
  37   74   84   82   72                   .5833
  38   76   86   84   74                   .5833
  39   78   88   86   76                   .5833
  40   80   90   88   78                   .5833
  41   82   92   91   81                   .5833
  42   84   94   92   82                   .5833
  43   86   96   94   84                   .5833
  44   88   98   96   86                   .5833
  45   90  100   98   88                   .5833
  46   92  102  101   91                   .5833
  47   94  104  102   92                   .5833
  48   96  106  104   94                   .5833
```

Fig. 18 Continued.

49	98	108	106	96		.5833
50	100	110	108	98		.5833
51	102	112	111	101		.5833
52	104	114	112	102		.5833
53	106	116	114	104		.5833
54	108	118	116	106		.5833
55	110	120	118	108		.5833
56	112	122	121	111		.5833
57	114	124	122	112		.5833
58	116	126	124	114		.5833
59	118	128	126	116		.5833
60	120	130	128	118		.5833
61	122	132	131	121		.5833
62	124	134	132	122		.5833
63	126	136	134	124		.5833
64	128	138	136	126		.5833
65	130	140	138	128		.5833
66	132	142	141	131		.5833
67	134	144	142	132		.5833
68	136	146	144	134		.5833
69	138	148	146	136		.5833
70	140	150	148	138		.5833
71	142	152	151	141		.5833
72	144	154	152	142		.5833
73	146	156	154	144		.5833
74	148	158	156	146		.5833
75	150	160	158	148		.5833
76	152	162	161	151		.5833
77	154	164	162	152		.5833
78	156	166	164	154		.5833
79	158	168	166	156		.5833
80	160	170	168	158		.5833
81	162	172	171	161		.5833
82	164	174	172	162		.5833
83	166	176	174	164		.5833
84	168	178	176	166		.5833
85	170	180	178	168		.5833
86	172	182	181	171		.5833
87	174	184	182	172		.5833
88	176	186	184	174		.5833
89	178	188	186	176		.5833
90	180	190	188	178		.5833
91	182	192	191	181		.5833
92	184	194	192	182		.5833
93	186	196	194	184		.5833
94	188	198	196	186		.5833
95	190	200	198	188		.5833
96	192	202	201	191		.5833
97	194	204	202	192		.5833
98	196	206	204	194		.5833
99	198	208	206	196		.5833
100	200	210	208	198		.5833
101	202	212	211	201		.5833
102	204	214	212	202		.5833
103	206	216	214	204		.5833
104	208	218	216	206		.5833
105	210	220	218	208		.5833
106	212	222	221	211		.5833
107	214	224	222	212		.5833
108	216	226	224	214		.5833
109	218	228	226	216		.5833
110	220	230	228	218		.5833
72	1				-20.385	
74	1				-87.615	
76	1				-87.615	
78	1				-20.385	
92	1				-20.385	
94	1				-87.615	
96	1				-87.615	
98	1				-20.385	
0.		0.		0.	0.	

Fig. 18 Continued.

.......NODE DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
230	1	1.835D-03	-7.080D-07	-8.695D-06	1.07D-05	-3.64D-04	0.0
229	1	0.0	0.0	0.0	0.0	0.0	0.0
228	1	1.825D-03	0.0	0.0	0.0	-3.57D-04	0.0
227	1	0.0	0.0	0.0	0.0	0.0	0.0
226	1	1.820D-03	0.0	0.0	0.0	-3.55D-04	0.0
225	1	0.0	0.0	0.0	0.0	0.0	0.0
224	1	1.820D-03	0.0	0.0	0.0	-3.55D-04	0.0
223	1	0.0	0.0	0.0	0.0	0.0	0.0
222	1	1.825D-03	0.0	0.0	0.0	-3.57D-04	0.0
221	1	1.835D-03	7.078D-07	-8.696D-06	-1.07D-05	-3.64D-04	0.0
220	1	2.316D-03	-1.681D-05	-8.399D-03	2.91D-04	-2.31D-04	-5.80D-03
219	1	0.0	0.0	0.0	0.0	0.0	0.0
218	1	2.303D-03	-1.239D-05	-8.333D-03	9.88D-06	-2.24D-04	8.00D-07
217	1	0.0	0.0	0.0	0.0	0.0	0.0
216	1	2.296D-03	-4.162D-06	-8.282D-03	3.30D-06	-2.23D-04	2.62D-08
215	1	0.0	0.0	0.0	0.0	0.0	0.0
214	1	2.296D-03	4.180D-06	-8.282D-03	-3.29D-06	-2.23D-04	-2.69D-08
213	1	0.0	0.0	0.0	0.0	0.0	0.0
212	1	2.303D-03	1.240D-05	-8.333D-03	-9.88D-06	-2.24D-04	-8.01D-07
211	1	2.316D-03	1.683D-05	-8.399D-03	-2.91D-04	-2.31D-04	5.80D-03
210	1	2.636D-03	-3.387D-05	-1.059D-02	3.58D-04	1.67D-04	-7.36D-03
209	1	0.0	0.0	0.0	0.0	0.0	0.0
208	1	2.622D-03	-2.509D-05	-1.048D-02	1.49D-05	1.75D-04	1.49D-06
207	1	0.0	0.0	0.0	0.0	0.0	0.0
206	1	2.617D-03	-8.526D-06	-1.041D-02	4.52D-06	1.73D-04	7.22D-07
205	1	0.0	0.0	0.0	0.0	0.0	0.0
204	1	2.617D-03	8.549D-06	-1.041D-02	-4.52D-06	1.73D-04	-7.22D-07
203	1	0.0	0.0	0.0	0.0	0.0	0.0
202	1	2.622D-03	2.512D-05	-1.048D-02	-1.48D-05	1.75D-04	-1.49D-06
201	1	2.636D-03	3.390D-05	-1.059D-02	-3.58D-04	1.67D-04	7.36D-03
200	1	2.626D-03	-3.590D-05	-5.829D-03	4.84D-04	5.40D-04	-1.07D-02
199	1	0.0	0.0	0.0	0.0	0.0	0.0
198	1	2.625D-03	-2.604D-05	-5.727D-03	1.17D-05	5.51D-04	7.92D-08
197	1	0.0	0.0	0.0	0.0	0.0	0.0
196	1	2.628D-03	-7.894D-06	-5.698D-03	-3.01D-07	5.48D-04	-8.79D-07
195	1	0.0	0.0	0.0	0.0	0.0	0.0
194	1	2.628D-03	7.907D-06	-5.698D-03	3.02D-07	5.48D-04	8.81D-07
193	1	0.0	0.0	0.0	0.0	0.0	0.0
192	1	2.625D-03	2.605D-05	-5.727D-03	-1.17D-05	5.51D-04	-7.72D-08
191	1	2.626D-03	3.591D-05	-5.829D-03	-4.84D-04	5.40D-04	1.07D-02

FIG. 19 - COMPUTER RESULTS FOR BRIDGE #1

190	1	2.516D-03	-1.475D-05	-7.319D-06	-2.860-04	7.700-04	7.120-03
189	1	0.0	0.0	0.0	0.0	0.0	0.0
188	1	2.528D-03	0.0	0.0	0.0	7.50D-04	0.0
187	1	0.0	0.0	0.0	0.0	0.0	0.0
186	1	2.534D-03	0.0	0.0	0.0	7.470-04	0.0
185	1	0.0	0.0	0.0	0.0	0.0	0.0
184	1	2.534D-03	0.0	0.0	0.0	7.47D-04	0.0
183	1	0.0	0.0	0.0	0.0	0.0	0.0
182	1	2.528D-03	0.0	0.0	0.0	7.50D-04	0.0
181	1	2.516D-03	1.475D-05	-7.322D-06	2.86D-04	7.70D-04	-7.12D-03
180	1	2.302D-03	-1.802D-05	7.899D-03	1.160-02	9.570-04	-2.86D-01
179	1	0.0	0.0	0.0	0.0	0.0	0.0
178	1	2.326D-03	-1.304D-05	7.907D-03	-5.40D-06	9.12D-04	2.44D-06
177	1	0.0	0.0	0.0	0.0	0.0	0.0
176	1	2.333D-03	-3.479D-06	7.878D-03	-1.74D-06	9.10D-04	7.85D-07
175	1	0.0	0.0	0.0	0.0	0.0	0.0
174	1	2.333D-03	3.474D-06	7.878D-03	1.74D-06	9.10D-04	-7.84D-07
173	1	0.0	0.0	0.0	0.0	0.0	0.0
172	1	2.326D-03	1.304D-05	7.907D-03	5.40D-06	9.12D-04	-2.44D-06
171	1	2.302D-03	1.801D-05	7.899D-03	-1.16D-02	9.57D-04	2.86D-01
170	1	1.758D-03	1.191D-05	2.217D-02	-4.60D-04	1.02D-03	1.16D-02
169	1	0.0	0.0	0.0	0.0	0.0	0.0
168	1	1.794D-03	8.002D-06	2.213D-02	-9.71D-06	9.93D-04	-2.81D-06
167	1	0.0	0.0	0.0	0.0	0.0	0.0
166	1	1.808D-03	2.380D-06	2.207D-02	-3.38D-06	9.94D-04	-1.18D-06
165	1	0.0	0.0	0.0	0.0	0.0	0.0
164	1	1.808D-03	-2.399D-06	2.207D-02	3.37D-06	9.94D-04	1.18D-06
163	1	0.0	0.0	0.0	0.0	0.0	0.0
162	1	1.794D-03	-8.022D-06	2.213D-02	9.71D-06	9.93D-04	2.81D-06
161	1	1.757D-03	-1.193D-05	2.217D-02	4.60D-04	1.02D-03	-1.16D-02
160	1	6.773D-04	6.405D-05	4.325D-02	-1.12D-03	4.66D-04	3.05D-02
159	1	0.0	0.0	0.0	0.0	0.0	0.0
158	1	7.184D-04	4.619D-05	4.310D-02	-2.02D-05	4.43D-04	-1.57D-06
157	1	0.0	0.0	0.0	0.0	0.0	0.0
156	1	7.352D-04	1.517D-05	4.307D-02	-3.97D-06	4.51D-04	9.17D-07
155	1	0.0	0.0	0.0	0.0	0.0	0.0
154	1	7.352D-04	-1.522D-05	4.307D-02	3.96D-06	4.51D-04	-9.16D-07
153	1	0.0	0.0	0.0	0.0	0.0	0.0
152	1	7.183D-04	-4.624D-05	4.310D-02	2.02D-05	4.43D-04	1.57D-06
151	1	6.773D-04	-6.410D-05	4.325D-02	1.12D-03	4.66D-04	-3.05D-02
150	1	-1.034D-04	1.201D-04	4.228D-02	-8.59D-04	-9.27D-04	2.45D-02

Fig. 19 Continued.

149	1	0.0	0.0	0.0	0.0	0.0	0.0
148	1	-6.500D-05	8.703D-05	4.214D-02	4.86D-06	-9.50D-04	-7.54D-06
147	1	0.0	0.0	0.0	0.0	0.0	0.0
146	1	-5.529D-05	2.820D-05	4.243D-02	1.38D-05	-9.41D-04	-5.04D-06
145	1	0.0	0.0	0.0	0.0	0.0	0.0
144	1	-5.529D-05	-2.826D-05	4.243D-02	-1.38D-05	-9.41D-04	5.04D-06
143	1	0.0	0.0	0.0	0.0	0.0	0.0
142	1	-6.500D-05	-8.708D-05	4.214D-02	-4.87D-06	-9.50D-04	7.54D-06
141	1	-1.034D-04	-1.202D-04	4.228D-02	8.60D-04	-9.27D-04	-2.45D-02
140	1	-1.464D-04	1.324D-04	2.231D-02	-1.10D-03	-2.12D-03	3.36D-02
139	1	0.0	0.0	0.0	0.0	0.0	0.0
138	1	-1.567D-04	9.609D-05	2.222D-02	2.76D-05	-2.17D-03	1.78D-05
137	1	0.0	0.0	0.0	0.0	0.0	0.0
136	1	-1.648D-04	2.878D-05	2.267D-02	4.96D-05	-2.21D-03	1.46D-05
135	1	0.0	0.0	0.0	0.0	0.0	0.0
134	1	-1.648D-04	-2.881D-05	2.267D-02	-4.96D-05	-2.21D-03	-1.46D-05
133	1	0.0	0.0	0.0	0.0	0.0	0.0
132	1	-1.566D-04	-9.613D-05	2.222D-02	-2.76D-05	-2.17D-03	-1.78D-05
131	1	-1.464D-04	-1.324D-04	2.231D-02	1.10D-03	-2.12D-03	-3.36D-02
130	1	5.804D-05	5.380D-05	8.340D-05	1.20D-03	-2.88D-03	-4.12D-02
129	1	0.0	0.0	0.0	0.0	0.0	0.0
128	1	0.0	0.0	0.0	0.0	-2.85D-03	0.0
127	1	0.0	0.0	0.0	0.0	0.0	0.0
126	1	0.0	0.0	0.0	0.0	-2.94D-03	0.0
125	1	0.0	0.0	0.0	0.0	0.0	0.0
124	1	0.0	0.0	0.0	0.0	-2.94D-03	0.0
123	1	0.0	0.0	0.0	0.0	0.0	0.0
122	1	0.0	0.0	0.0	0.0	-2.85D-03	0.0
121	1	5.811D-05	-5.381D-05	8.345D-05	-1.20D-03	-2.88D-03	4.12D-02
120	1	5.808D-04	5.880D-05	-2.883D-02	3.49D-03	-3.41D-03	-1.28D-01
119	1	0.0	0.0	0.0	0.0	0.0	0.0
118	1	4.877D-04	4.804D-05	-2.974D-02	-1.61D-04	-3.34D-03	2.38D-05
117	1	0.0	0.0	0.0	0.0	0.0	0.0
116	1	4.964D-04	1.537D-05	-3.097D-02	-8.43D-05	-3.51D-03	1.29D-05
115	1	0.0	0.0	0.0	0.0	0.0	0.0
114	1	4.964D-04	-1.530D-05	-3.097D-02	8.43D-05	-3.51D-03	-1.29D-05
113	1	0.0	0.0	0.0	0.0	0.0	0.0
112	1	4.878D-04	-4.796D-05	-2.974D-02	1.61D-04	-3.34D-03	-2.38D-05
111	1	5.809D-04	-5.872D-05	-2.883D-02	-3.49D-03	-3.41D-03	1.28D-01
110	1	2.033D-03	-6.856D-05	-7.727D-02	2.85D-03	-3.15D-03	-1.25D-01
109	1	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 19 Continued.

108	1	1.943D-03	-3.425D-05	-7.976D-02	-4.40D-04	-3.20D-03	1.29D-04
107	1	0.0	0.0	0.0	0.0	0.0	0.0
106	1	1.964D-03	-1.415D-06	-8.459D-02	-2.30D-04	-3.55D-03	6.67D-05
105	1	0.0	0.0	0.0	0.0	0.0	0.0
104	1	1.964D-03	1.654D-06	-8.459D-02	2.30D-04	-3.55D-03	-6.67D-05
103	1	0.0	0.0	0.0	0.0	0.0	0.0
102	1	1.943D-03	3.449D-05	-7.976D-02	4.40D-04	-3.20D-03	-1.29D-04
101	1	2.034D-03	6.881D-05	-7.727D-02	-2.85D-03	-3.15D-03	1.25D-01
100	1	4.654D-03	-2.256D-04	-1.231D-01	-1.75D-03	-6.85D-04	8.00D-03
99	1	0.0	0.0	0.0	0.0	0.0	0.0
98	1	4.748D-03	-1.587D-04	-1.303D-01	-1.81D-03	-6.91D-04	-3.10D-04
97	1	0.0	0.0	0.0	0.0	0.0	0.0
96	1	5.032D-03	-4.601D-05	-1.457D-01	-1.03D-03	-9.11D-04	-2.05D-04
95	1	0.0	0.0	0.0	0.0	0.0	0.0
94	1	5.032D-03	4.644D-05	-1.457D-01	1.03D-03	-9.11D-04	2.05D-04
93	1	0.0	0.0	0.0	0.0	0.0	0.0
92	1	4.748D-03	1.591D-04	-1.303D-01	1.81D-03	-6.91D-04	3.10D-04
91	1	4.654D-03	2.261D-04	-1.231D-01	1.75D-03	-6.85D-04	-8.00D-03
90	1	4.993D-03	-2.422D-04	-1.245D-01	-1.12D-02	-1.41D-04	4.10D-01
89	1	0.0	0.0	0.0	0.0	0.0	0.0
88	1	5.133D-03	-1.422D-04	-1.317D-01	-1.36D-03	-1.22D-04	-2.47D-04
87	1	0.0	0.0	0.0	0.0	0.0	0.0
86	1	5.448D-03	-1.165D-05	-1.473D-01	-7.36D-04	-2.43D-04	-1.61D-04
85	1	0.0	0.0	0.0	0.0	0.0	0.0
84	1	5.448D-03	1.209D-05	-1.473D-01	7.36D-04	-2.43D-04	1.61D-04
83	1	0.0	0.0	0.0	0.0	0.0	0.0
82	1	5.133D-03	1.427D-04	-1.317D-01	1.36D-03	-1.22D-04	2.47D-04
81	1	4.993D-03	2.426D-04	-1.245D-01	1.12D-02	-1.41D-04	-4.10D-01
80	1	5.821D-03	-2.727D-04	-1.128D-01	-3.87D-03	1.87D-03	9.77D-02
79	1	0.0	0.0	0.0	0.0	0.0	0.0
78	1	6.117D-03	-2.086D-04	-1.211D-01	-2.06D-03	1.87D-03	-2.40D-07
77	1	0.0	0.0	0.0	0.0	0.0	0.0
76	1	6.564D-03	-7.703D-05	-1.369D-01	-1.30D-03	2.15D-03	1.33D-05
75	1	0.0	0.0	0.0	0.0	0.0	0.0
74	1	6.564D-03	7.745D-05	-1.369D-01	1.30D-03	2.15D-03	-1.33D-05
73	1	0.0	0.0	0.0	0.0	0.0	0.0
72	1	6.117D-03	2.091D-04	-1.211D-01	2.06D-03	1.87D-03	2.53D-07
71	1	5.820D-03	2.732D-04	-1.128D-01	3.87D-03	1.87D-03	-9.77D-02
70	1	5.994D-03	-6.343D-05	-8.334D-02	-4.16D-03	3.29D-03	1.54D-01
69	1	0.0	0.0	0.0	0.0	0.0	0.0
68	1	6.275D-03	-2.437D-05	-8.746D-02	-7.88D-04	3.33D-03	-2.67D-04
67	1	0.0	0.0	0.0	0.0	0.0	0.0

Fig. 19 Continued.

66	1	6.632D-03	1.029D-05	-9.603D-02	-4.10D-04	3.84D-03	-1.77D-04
65	1	0.0	0.0	0.0	0.0	0.0	0.0
64	1	6.632D-03	-9.977D-06	-9.603D-02	4.10D-04	3.84D-03	1.78D-04
63	1	0.0	0.0	0.0	0.0	0.0	0.0
62	1	6.275D-03	2.469D-05	-8.746D-02	7.88D-04	3.33D-03	2.67D-04
61	1	5.994D-03	6.375D-05	-8.334D-02	4.16D-03	3.29D-03	-1.54D-01
60	1	5.158D-03	6.558D-05	-3.113D-02	-3.77D-03	3.71D-03	1.76D-01
59	1	0.0	0.0	0.0	0.0	0.0	0.0
58	1	5.338D-03	5.728D-05	-3.277D-02	-3.12D-04	3.65D-03	-6.41D-05
57	1	0.0	0.0	0.0	0.0	0.0	0.0
56	1	5.448D-03	1.893D-05	-3.518D-02	-1.67D-04	3.96D-03	-2.91D-05
55	1	0.0	0.0	0.0	0.0	0.0	0.0
54	1	5.448D-03	-1.881D-05	-3.518D-02	1.67D-04	3.96D-03	2.91D-05
53	1	0.0	0.0	0.0	0.0	0.0	0.0
52	1	5.338D-03	-5.716D-05	-3.277D-02	3.12D-04	3.65D-03	6.42D-05
51	1	5.157D-03	-6.546D-05	-3.112D-02	3.78D-03	3.71D-03	-1.76D-01
50	1	4.210D-03	6.289D-05	1.582D-04	-1.06D-03	3.11D-03	5.48D-02
49	1	0.0	0.0	0.0	0.0	0.0	0.0
48	1	4.288D-03	0.0	0.0	0.0	3.09D-03	0.0
47	1	0.0	0.0	0.0	0.0	0.0	0.0
46	1	4.313D-03	0.0	0.0	0.0	3.28D-03	0.0
45	1	0.0	0.0	0.0	0.0	0.0	0.0
44	1	4.313D-03	0.0	0.0	0.0	3.28D-03	0.0
43	1	0.0	0.0	0.0	0.0	0.0	0.0
42	1	4.288D-03	0.0	0.0	0.0	3.09D-03	0.0
41	1	4.210D-03	-6.291D-05	1.583D-04	1.06D-03	3.11D-03	-5.47D-02
40	1	3.250D-03	1.585D-04	2.382D-02	5.48D-04	2.24D-03	-3.37D-02
39	1	0.0	0.0	0.0	0.0	0.0	0.0
38	1	3.266D-03	1.168D-04	2.389D-02	8.15D-05	2.29D-03	-3.11D-05
37	1	0.0	0.0	0.0	0.0	0.0	0.0
36	1	3.243D-03	3.635D-05	2.478D-02	9.49D-05	2.37D-03	-2.33D-05
35	1	0.0	0.0	0.0	0.0	0.0	0.0
34	1	3.243D-03	-3.644D-05	2.478D-02	-9.49D-05	2.37D-03	2.34D-05
33	1	0.0	0.0	0.0	0.0	0.0	0.0
32	1	3.266D-03	-1.169D-04	2.389D-02	-8.15D-05	2.29D-03	3.11D-05
31	1	3.250D-03	-1.586D-04	2.382D-02	-5.49D-04	2.24D-03	3.37D-02
30	1	2.082D-03	1.372D-04	4.422D-02	3.84D-04	7.20D-04	-2.73D-02
29	1	0.0	0.0	0.0	0.0	0.0	0.0
28	1	2.041D-03	9.879D-05	4.421D-02	3.59D-05	7.42D-04	2.26D-05
27	1	0.0	0.0	0.0	0.0	0.0	0.0
26	1	2.002D-03	3.152D-05	4.482D-02	3.15D-05	7.28D-04	2.06D-05

Fig. 19 Continued.

25	1	0.0	0.0	0.0	0.0	0.0	0.0
24	1	2.002D-03	-3.169D-05	4.482D-02	-3.15D-05	7.28D-04	-2.06D-05
23	1	0.0	0.0	0.0	0.0	0.0	0.0
22	1	2.041D-03	-9.895D-05	4.421D-02	-3.60D-05	7.42D-04	-2.26D-05
21	1	2.082D-03	-1.373D-04	4.422D-02	-3.84D-04	7.20D-04	2.73D-02
20	1	1.250D-03	6.801D-05	3.551D-02	2.56D-04	-9.70D-04	-2.42D-02
19	1	0.0	0.0	0.0	0.0	0.0	0.0
18	1	1.209D-03	4.898D-05	3.538D-02	-1.48D-05	-9.48D-04	-1.91D-07
17	1	0.0	0.0	0.0	0.0	0.0	0.0
16	1	1.190D-03	1.584D-05	3.543D-02	-5.27D-07	-9.60D-04	-1.26D-06
15	1	0.0	0.0	0.0	0.0	0.0	0.0
14	1	1.190D-03	-1.597D-05	3.543D-02	4.91D-07	-9.60D-04	1.26D-06
13	1	0.0	0.0	0.0	0.0	0.0	0.0
12	1	1.209D-03	-4.910D-05	3.538D-02	1.47D-05	-9.48D-04	1.82D-07
11	1	1.250D-03	-6.814D-05	3.551D-02	-2.56D-04	-9.70D-04	2.42D-02
10	1	1.323D-03	3.058D-06	4.273D-05	-4.51D-05	-1.54D-03	0.0
9	1	0.0	0.0	0.0	0.0	0.0	0.0
8	1	1.282D-03	0.0	0.0	0.0	-1.52D-03	0.0
7	1	0.0	0.0	0.0	0.0	0.0	0.0
6	1	1.266D-03	0.0	0.0	0.0	-1.52D-03	0.0
5	1	0.0	0.0	0.0	0.0	0.0	0.0
4	1	1.266D-03	0.0	0.0	0.0	-1.52D-03	0.0
3	1	0.0	0.0	0.0	0.0	0.0	0.0
2	1	1.282D-03	0.0	0.0	0.0	-1.52D-03	0.0
1	1	1.323D-03	-3.057D-06	4.271D-05	4.51D-05	-1.54D-03	0.0

.....BEAM FORCES AND MOMENTS

BEAM NO.	LOAD NO.	AXIAL R1	SHEAR R2	SHEAR R3	TORSION M1	BENDING M2	BENDING M3
1	1	4.718D 01	-1.150D 01	-1.321D-03	-6.714D-04	1.536D-02	-8.248D 01
		-4.718D 01	1.150D 01	1.321D-03	6.714D-04	1.547D-02	-1.859D 02
2	1	4.632D 01	-1.129D 01	-1.801D-03	-2.304D-05	1.757D-02	-8.082D 01
		-4.632D 01	1.129D 01	1.801D-03	2.304D-05	2.447D-02	-1.826D 02
3	1	4.632D 01	-1.129D 01	1.795D-03	2.469D-05	-1.747D-02	-8.081D 01
		-4.632D 01	1.129D 01	-1.795D-03	-2.469D-05	-2.441D-02	-1.826D 02
4	1	4.718D 01	-1.150D 01	1.315D-03	6.730D-04	-1.527D-02	-8.248D 01
		-4.718D 01	1.150D 01	-1.315D-03	-6.730D-04	-1.542D-02	-1.859D 02
5	1	1.402D 02	-1.118D 01	6.851D-03	2.327D-03	-1.920D-02	2.368D 01
		-1.402D 02	1.118D 01	-6.851D-03	-2.327D-03	-1.407D-01	-2.847D 02

Fig. 19 Continued.

6	1	1.406D 02	-1.162D 01	8.828D-03	1.475D-03	-4.388D-02	1.828D 01
		-1.406D 02	1.162D 01	-8.828D-03	-1.475D-03	-1.621D-01	-2.896D 02
7	1	1.406D 02	-1.162D 01	-8.826D-03	-1.475D-03	4.384D-02	1.829D 01
		-1.406D 02	1.162D 01	8.826D-03	1.475D-03	1.621D-01	-2.896D 02
8	1	1.402D 02	-1.118D 01	-6.850D-03	-2.327D-03	1.916D-02	2.368D 01
		-1.402D 02	1.118D 01	6.850D-03	2.327D-03	1.407D-01	-2.847D 02
9	1	2.076D 02	-8.881D 00	-4.669D-02	3.317D-03	9.058D-02	1.665D 02
		-2.076D 02	8.881D 00	4.669D-02	-3.317D-03	5.788D-01	-2.938D 02
10	1	2.241D 02	-1.355D 01	-4.134D-02	4.652D-03	9.425D-02	1.474D 02
		-2.241D 02	1.355D 01	4.134D-02	-4.652D-03	4.983D-01	-3.417D 02
11	1	2.241D 02	-1.355D 01	4.132D-02	-4.654D-03	-9.425D-02	1.474D 02
		-2.241D 02	1.355D 01	-4.132D-02	4.654D-03	-4.981D-01	-3.417D 02
12	1	2.076D 02	-8.881D 00	4.669D-02	-3.319D-03	-9.060D-02	1.665D 02
		-2.076D 02	8.881D 00	-4.669D-02	3.319D-03	-5.786D-01	-2.938D 02
13	1	2.129D 02	-7.536D 00	2.157D-01	-1.498D-02	-5.862D-01	2.928D 02
		-2.129D 02	7.536D 00	-2.157D-01	1.498D-02	-1.356D 00	-3.606D 02
14	1	2.528D 02	-1.470D 01	1.779D-01	-1.748D-02	-5.046D-01	3.044D 02
		-2.528D 02	1.470D 01	-1.779D-01	1.748D-02	-1.097D 00	-4.367D 02
15	1	2.528D 02	-1.470D 01	-1.778D-01	1.748D-02	5.044D-01	3.044D 02
		-2.528D 02	1.470D 01	1.778D-01	-1.748D-02	1.096D 00	-4.367D 02
16	1	2.129D 02	-7.536D 00	-2.156D-01	1.498D-02	5.861D-01	2.928D 02
		-2.129D 02	7.536D 00	2.156D-01	-1.498D-02	1.355D 00	-3.606D 02
17	1	1.481D 02	5.320D 01	4.639D-01	-5.790D-02	-2.776D 00	4.697D 02
		-1.481D 02	-5.320D 01	-4.639D-01	5.790D-02	-1.400D 00	9.203D 00
18	1	1.924D 02	5.675D 01	3.119D-01	-3.106D-02	-1.710D 00	5.351D 02
		-1.924D 02	-5.675D 01	-3.119D-01	3.106D-02	-1.097D 00	-2.422D 01
19	1	1.924D 02	5.675D 01	-3.118D-01	3.106D-02	1.709D 00	5.351D 02
		-1.924D 02	-5.675D 01	3.118D-01	-3.106D-02	1.097D 00	-2.422D 01
20	1	1.481D 02	5.320D 01	-4.639D-01	5.789D-02	2.776D 00	4.697D 02
		-1.481D 02	-5.320D 01	4.639D-01	-5.789D-02	1.400D 00	9.199D 00
21	1	-4.510D 01	5.295D 01	-3.482D-01	-3.785D-02	1.431D 00	3.079D 02
		4.510D 01	-5.295D 01	3.482D-01	3.785D-02	3.271D 00	4.071D 02
22	1	-1.812D 01	5.752D 01	-2.670D-01	-1.937D-02	1.119D 00	3.697D 02
		1.812D 01	-5.752D 01	2.670D-01	1.937D-02	2.486D 00	4.069D 02

Fig. 19 Continued.

23	1	-1.812D 01	5.752D 01	2.670D-01	1.936D-02	-1.119D 00	3.697D 02
		1.812D 01	-5.752D 01	-2.670D-01	-1.936D-02	-2.486D 00	4.069D 02
24	1	-4.510D 01	5.295D 01	3.483D-01	3.784D-02	-1.431D 00	3.079D 02
		4.510D 01	-5.295D 01	-3.483D-01	-3.784D-02	-3.272D 00	4.071D 02
25	1	-2.516D 02	4.009D 01	7.639D-01	-1.425D-01	-2.606D 00	-6.064D 01
		2.516D 02	-4.009D 01	-7.639D-01	1.425D-01	-6.181D 00	5.217D 02
26	1	-3.013D 02	7.007D 01	5.634D-01	-9.961D-02	-1.964D 00	6.558D 01
		3.013D 02	-7.007D 01	-5.634D-01	9.961D-02	-4.516D 00	7.404D 02
27	1	-3.013D 02	7.007D 01	-5.633D-01	9.961D-02	1.964D 00	6.558D 01
		3.013D 02	-7.007D 01	5.633D-01	-9.961D-02	4.516D 00	7.404D 02
28	1	-2.516D 02	4.009D 01	-7.640D-01	1.425D-01	2.606D 00	-6.065D 01
		2.516D 02	-4.009D 01	7.640D-01	-1.425D-01	6.182D 00	5.217D 02
29	1	-3.573D 02	1.192D 01	-1.444D 00	8.181D-02	6.278D 00	-3.492D 02
		3.573D 02	-1.192D 01	1.444D 00	-8.181D-02	9.614D 00	4.804D 02
30	1	-4.448D 02	-6.974D-01	-1.050D 00	6.577D-02	4.587D 00	-5.008D 02
		4.448D 02	6.974D-01	1.050D 00	-6.577D-02	6.966D 00	4.931D 02
31	1	-4.448D 02	-6.975D-01	1.050D 00	-6.577D-02	-4.587D 00	-5.008D 02
		4.448D 02	6.975D-01	-1.050D 00	6.577D-02	-6.966D 00	4.931D 02
32	1	-3.573D 02	1.192D 01	1.444D 00	-8.181D-02	-6.278D 00	-3.492D 02
		3.573D 02	-1.192D 01	-1.444D 00	8.181D-02	-9.615D 00	4.804D 02
33	1	-3.734D 02	-1.153D 01	4.457D 00	-1.964D-01	-7.911D 00	-4.517D 02
		3.734D 02	1.153D 01	-4.457D 00	1.964D-01	-5.464D 00	4.171D 02
34	1	-4.569D 02	1.411D 01	2.923D 00	-1.263D-01	-5.245D 00	-4.885D 02
		4.569D 02	-1.411D 01	-2.923D 00	1.263D-01	-3.527D 00	5.309D 02
35	1	-4.569D 02	1.411D 01	-2.923D 00	1.263D-01	5.245D 00	-4.885D 02
		4.569D 02	-1.411D 01	2.923D 00	-1.263D-01	3.527D 00	5.309D 02
36	1	-3.734D 02	-1.153D 01	-4.457D 00	1.964D-01	7.911D 00	-4.517D 02
		3.734D 02	1.153D 01	4.457D 00	-1.964D-01	5.464D 00	4.172D 02
37	1	-2.569D 02	-3.754D 01	-4.398D-01	9.189D-02	5.736D 00	-6.604D 02
		2.569D 02	3.754D 01	4.398D-01	-9.189D-02	2.842D 00	-7.189D 01
38	1	-2.772D 02	-5.240D 01	-2.903D-01	5.342D-02	3.730D 00	-8.207D 02
		2.772D 02	5.240D 01	2.903D-01	-5.342D-02	1.932D 00	-2.015D 02
39	1	-2.772D 02	-5.240D 01	2.903D-01	-5.341D-02	-3.730D 00	-8.207D 02
		2.772D 02	5.240D 01	-2.903D-01	5.341D-02	-1.932D 00	-2.015D 02

Fig. 19 Continued.

40	1	-2.569D 02	-3.754D 01	4.398D-01	-9.189D-02	-5.736D 00	-6.604D 02
		2.5690 02	3.754D 01	-4.398D-01	9.189D-02	-2.842D 00	-7.189D 01
41	1	-2.2860 01	-4.40D0 01	1.840D-01	2.648D-02	-1.827D 00	-3.209D 02
		2.286D 01	4.4DD0 D1	-1.840D-01	-2.648D-02	-6.576D-01	-2.733D 02
42	1	5.0760 00	-4.4350 D1	1.0410-01	1.385D-02	-1.0D2D 00	-2.936D 02
		-5.076D 00	4.4350 01	-1.D41D-D1	-1.385D-D2	-4.0440-01	-3.D53D 02
43	1	5.0760 0D	-4.435D 01	-1.0410-D1	-1.384D-02	1.0020 00	-2.936D 02
		-5.0760 DD	4.435D 01	1.D41D-01	1.384D-02	4.045D-D1	-3.D530 02
44	1	-2.286D 01	-4.400D 01	-1.8400-01	-2.647D-02	1.8270 00	-3.2090 02
		2.2860 01	4.40D0 D1	1.84DD-D1	2.647D-D2	6.577D-01	-2.7330 02
45	1	1.339D 02	-4.398D D1	-2.2960-01	3.371D-02	6.792D-01	1.359D D1
		-1.339D 02	4.398D 01	2.296D-D1	-3.371D-D2	1.3880 DD	-4.096D D2
46	1	1.629D 02	-4.4060 01	-1.351D-01	1.7680-D2	4.1700-01	4.636D 01
		-1.629D 02	4.4D60 01	1.3510-01	-1.768D-D2	7.993D-01	-4.4310 02
47	1	1.629D 02	-4.4060 01	1.3510-01	-1.768D-D2	-4.1710-01	4.6360 01
		-1.6290 D2	4.4D6D 01	-1.351D-01	1.768D-02	-7.989D-D1	-4.4310 02
48	1	1.339D 02	-4.398D 01	2.295D-01	-3.3710-02	-6.792D-01	1.360D 01
		-1.3390 02	4.3980 01	-2.295D-D1	3.371D-D2	-1.3880 0D	-4.0960 02
49	1	1.8650 02	7.225D 00	-1.310D-01	5.916D-D3	8.009D-D1	3.241D 02
		-1.8650 D2	-7.225D 00	1.3100-01	-5.916D-D3	3.788D-01	-2.5910 02
50	1	2.111D 02	1.151D 01	-1.157D-01	1.055D-02	6.833D-D1	3.6740 02
		-2.111D 02	-1.151D 01	1.157D-D1	-1.D55D-D2	3.582D-D1	-2.638D D2
51	1	2.111D 02	1.1510 01	1.157D-D1	-1.D54D-02	-6.831D-01	3.6740 02
		-2.111D 02	-1.1510 01	-1.157D-D1	1.D54D-02	-3.582D-01	-2.638D 02
52	1	1.865D 02	7.225D 0D	1.31DD-D1	-5.916D-03	-8.008D-D1	3.241D 02
		-1.8650 02	-7.225D 00	-1.310D-D1	5.916D-03	-3.788D-01	-2.591D 02
53	1	·1.8090 02	8.07CD 00	3.8770-DZ	-2.254D-03	-3.891D-D1	2.609D 02
		-1.8090 02	-8.070D 00	-3.877D-02	2.254D-03	-1.3450-D1	-1.5190 D2
54	1	1.906D 02	1.091D D1	4.D300-02	-3.497D-03	-3.6770-01	2.888D D2
		-1.9060 02	-1.091D D1	-4.D300-D2	3.497D-D3	-1.7660-D1	-1.415D D2
55	1	1.906D 02	1.091D 01	-4.03CD-02	3.497D-03	3.676D-01	2.8880 02
		-1.906D D2	-1.091D D1	4.D30D-D2	-3.497D-D3	1.766D-D1	-1.415D D2
56	1	1.8D9D 02	8.07CD 00	-3.877D-D2	2.254D-D3	3.891D-01	2.6090 02
		-1.809D 02	-8.D7CO 0D	3.877D-D2	-2.254D-03	1.346D-01	-1.519D 02

Fig. 19 Continued.

57	1	1.251D 02	9.566D 00	-6.510D-04	-1.430D-03	2.845D-02	-2.494D 02
		-1.251D 02	-9.566D 00	6.510D-04	1.430D-03	-1.379D-02	-3.403D 01
58	1	1.252D 02	9.855D 00	-2.802D-04	-1.011D-03	2.348D-02	-2.525D 02
		-1.252D 02	-9.855D 00	2.802D-04	1.011D-03	-1.717D-02	-3.064D 01
59	1	1.252D 02	9.855D 00	2.802D-04	1.011D-03	-2.348D-02	2.525D 02
		-1.252D 02	-9.855D 00	-2.802D-04	-1.011D-03	1.717D-02	-3.064D 01
60	1	1.251D 02	9.566D 00	6.511D-04	1.430D-03	-2.845D-02	2.494D 02
		-1.251D 02	-9.566D 00	-6.511D-04	-1.430D-03	1.380D-02	-3.403D 01
61	1	4.949D 01	9.778D 00	-1.370D-03	6.006D-04	1.042D-02	1.660D 02
		-4.949D 01	-9.778D 00	1.370D-03	-6.006D-04	2.044D-02	5.414D 01
62	1	4.887D 01	9.650D 00	8.557D-04	2.965D-05	-1.620D-02	1.639D 02
		-4.887D 01	-9.650D 00	-8.557D-04	-2.965D-05	-3.061D-03	5.340D 01
63	1	4.887D 01	9.650D 00	-8.553D-04	-2.994D-05	1.620D-02	1.639D 02
		-4.887D 01	-9.650D 00	8.553D-04	2.994D-05	3.058D-03	5.340D 01
64	1	4.949D 01	9.778D 00	1.371D-03	-6.009D-04	-1.042D-02	1.660D 02
		-4.949D 01	-9.778D 00	-1.371D-03	6.009D-04	-2.044D-02	5.414D 01
65	1	-1.172D 01	9.527D 00	4.562D-03	3.559D-04	-6.674D-03	5.153D 01
		1.172D 01	-9.527D 00	-4.562D-03	-3.559D-04	-5.497D-02	7.718D 01
66	1	-1.168D 01	9.433D 00	4.754D-04	1.345D-04	5.838D-03	5.045D 01
		1.168D 01	-9.433D 00	-4.754D-04	-1.345D-04	-1.226D-02	7.700D 01
67	1	-1.168D 01	9.433D 00	-4.756D-04	-1.348D-04	-5.835D-03	5.045D 01
		1.168D 01	-9.433D 00	4.756D-04	1.348D-04	1.226D-02	7.700D 01
68	1	-1.172D 01	9.527D 00	-4.561D-03	-3.563D-04	6.673D-03	5.153D 01
		1.172D 01	-9.527D 00	4.561D-03	3.563D-04	5.496D-02	7.718D 01
69	1	-4.436D 01	9.542D 00	-1.957D-02	9.788D-04	5.680D-02	-2.362D 01
		4.436D 01	-9.542D 00	1.957D-02	-9.788D-04	1.195D-01	1.096D 02
70	1	-4.444D 01	9.416D 00	-5.056D-03	3.161D-04	1.268D-02	-2.433D 01
		4.444D 01	-9.416D 00	5.056D-03	-3.161D-04	3.286D-02	1.092D 02
71	1	-4.444D 01	9.416D 00	5.059D-03	-3.164D-04	-1.268D-02	-2.433D 01
		4.444D 01	-9.416D 00	-5.059D-03	3.164D-04	-3.289D-02	1.092D 02
72	1	-4.436D 01	9.542D 00	1.957D-02	-9.789D-04	-5.680D-02	-2.362D 01
		4.436D 01	-9.542D 00	-1.957D-02	9.789D-04	-1.195D-01	1.096D 02
73	1	-5.477D 01	-2.495D 00	3.807D-03	2.171D-03	-2.201D-02	-9.288D 01
		5.477D 01	2.495D 00	-3.807D-03	-2.171D-03	-1.228D-02	7.041D 01

Fig. 19 Continued.

74	1	-5.455D 01	-2.6200 00	7.976D-03	-6.258D-05	-4.617D-02	-9.342D 01
		5.455D 01	2.6200 00	-7.9760-03	6.2580-05	-2.5670-02	6.982D 01
75	1	-5.455D 01	-2.6200 00	-7.9660-03	6.2370-05	4.610D-02	-9.3420 01
		5.455D 01	2.620D 00	7.9660-03	-6.2370-05	2.566D-02	6.9820 01
76	1	-5.477D 01	-2.4950 00	-3.7990-03	-2.1710-03	2.1950-02	-9.2880 01
		5.4770 01	2.4950 00	3.7990-03	2.1710-03	1.2270-02	7.041D 01
77	1	-5.1040 01	-2.5940 00	-9.7570-04	2.3440-04	1.2740-02	-7.4340 01
		5.1040 01	2.5940 00	9.7570-04	-2.344D-04	1.2620-03	3.7120 01
78	1	-5.026D 01	-2.6130 00	-2.8320-03	3.6220-04	2.656D-02	-7.4360 01
		5.0260 01	2.6130 00	2.8320-03	-3.6220-04	1.4070-02	3.6880 01
79	1	-5.0260 01	-2.6130 00	2.8310-03	-3.6230-04	-2.6540-02	-7.4360 01
		5.0260 01	2.6130 00	-2.8310-03	3.6230-04	-1.4070-02	3.6880 01
80	1	-5.1030 01	-2.5940 00	9.749D-04	-2.3450-04	-1.2720-02	-7.4340 01
		5.103D 01	2.5940 00	-9.7490-04	2.3450-04	-1.2640-03	3.7120 01
81	1	-3.3250 01	-2.6830 00	6.1860-04	-2.2760-04	-8.5040-03	-6.7650 01
		3.3250 01	2.6830 00	-6.1860-04	2.276D-04	-5.9460-03	4.985D 00
82	1	-3.2720 01	-2.6400 00	1.517D-04	-5.702D-05	-3.5290-03	-6.6900 01
		3.272D 01	2.6400 00	-1.5170-04	5.702D-05	-1.3430-05	5.2230 00
83	1	-3.272D 01	-2.6400 00	-1.5150-04	5.704D-05	3.5300-03	-6.6900 01
		3.272D 01	2.6400 00	1.5150-04	-5.7040-05	9.087D-06	5.2230 00
84	1	-3.3250 01	-2.6830 00	-6.1840-04	2.2760-04	8.5050-03	-6.7650 01
		3.3250 01	2.6830 00	6.184D-04	-2.276D-04	5.941D-03	4.985D 00
85	1	-1.113D 01	-2.6770 00	5.9750-04	-4.5140-04	-7.8640-03	-4.3430 01
		1.1130 01	2.6770 00	-5.9750-04	4.5140-04	-6.0940-03	-1.9120 01
86	1	-1.1000 01	-2.649D 00	2.6970-05	-1.500D-04	5.0150-05	-4.3000 01
		1.1000 01	2.6490 00	-2.6970-05	1.5000-04	-6.8030-04	-1.8880 01
87	1	-1.1000 01	-2.6490 00	-2.7670-05	1.501D-04	-4.4260-05	-4.3000 01
		1.100D 01	2.6490 00	2.7670-05	-1.5010-04	6.9060-04	-1.8880 01
88	1	-1.1130 01	-2.6770 00	-5.9810-04	4.515D-04	7.869D-03	-4.3430 01
		1.1130 01	2.6770 00	5.9810-04	-4.5150-04	6.1030-03	-1.9120 01
89	1	0.0	0.0	-6.9090-04	-3.6750-05	2.8790-03	0.0
		0.0	0.0	6.9090-04	3.6750-05	2.8790-03	0.0
90	1	0.0	0.0	-6.0570-07	-9.9580-08	2.5240-06	0.0
		0.0	0.0	6.057D-07	9.958D-08	2.524D-06	0.0

Fig. 19 Continued.

91	1	0.0	0.0	6.891D-04	3.657D-05	-2.871D-03	0.0
		0.0	0.0	-6.891D-04	-3.657D-05	-2.871D-03	0.0
92	1	2.429D-01	-1.829D-02	-1.327D-04	-1.235D-04	3.244D-04	0.0
		-2.429D-01	1.829D-02	1.327D-04	1.235D-04	7.815D-04	-1.524D-01
93	1	1.122D 00	3.997D-06	9.836D-07	-5.837D-08	5.323D-04	-8.379D-03
		-1.122D 00	-3.997D-06	-9.836D-07	5.837D-08	-5.405D-04	8.412D-03
94	1	2.430D-01	1.829D-02	1.345D-04	1.234D-04	-7.887D-04	1.524D-01
		-2.430D-01	-1.829D-02	-1.345D-04	-1.234D-04	-3.318D-04	0.0
95	1	2.837D 00	-1.199D-01	6.189D-03	-1.518D-04	-2.620D-02	0.0
		-2.837D 00	1.199D-01	-6.189D-03	1.518D-04	-2.538D-02	-9.989D-01
96	1	6.806D 00	3.524D-06	1.616D-07	1.984D-08	-8.782D-03	5.197D-01
		-6.806D 00	-3.524D-06	-1.616D-07	-1.984D-08	8.781D-03	-5.197D-01
97	1	2.837D 00	1.199D-01	-6.189D-03	1.519D-04	2.538D-02	9.989D-01
		-2.837D 00	-1.199D-01	6.189D-03	-1.519D-04	2.620D-02	0.0
98	1	0.0	0.0	-1.208D-02	1.966D-03	5.032D-02	0.0
		0.0	0.0	1.208D-02	-1.966D-03	5.032D-02	0.0
99	1	0.0	0.0	6.045D-07	1.562D-07	-2.519D-06	0.0
		0.0	0.0	-6.045D-07	-1.562D-07	-2.519D-06	0.0
100	1	0.0	0.0	1.208D-02	-1.966D-03	-5.033D-02	0.0
		0.0	0.0	-1.208D-02	1.966D-03	-5.033D-02	0.0
101	1	-2.787D 01	1.764D 00	-8.980D-02	5.264D-03	3.932D-01	0.0
		2.787D 01	-1.764D 00	8.980D-02	-5.264D-03	3.551D-01	1.470D 01
102	1	-5.692D 01	1.988D-06	-2.123D-06	1.607D-07	7.552D-02	-6.760D 00
		5.692D 01	-1.988D-06	2.123D-06	-1.607D-07	-7.550D-02	6.760D 00
103	1	-2.787D 01	-1.764D 00	8.980D-02	-5.264D-03	-3.551D-01	-1.470D 01
		2.787D 01	1.764D 00	-8.980D-02	5.264D-03	-3.932D-01	0.0
104	1	-4.867D 01	3.240D 00	-4.258D-02	-1.261D-03	1.957D-01	0.0
		4.867D 01	-3.240D 00	4.258D-02	1.261D-03	1.592D-01	2.700D 01
105	1	-1.044D 02	1.298D-06	1.010D-06	-5.241D-08	6.855D-02	-1.213D 01
		1.044D 02	-1.298D-06	-1.010D-06	5.241D-08	-6.856D-02	1.213D 01
106	1	-4.867D 01	-3.240D 00	4.258D-02	1.2600-03	-1.592D-01	-2.700D 01
		4.867D 01	3.240D 00	-4.258D-02	-1.260D-03	-1.957D-01	0.0
107	1	-1.597D 01	9.958D-01	5.494D-02	-3.647D-03	-2.422D-01	0.0
		1.597D 01	-9.958D-01	-5.494D-02	3.647D-03	-2.156D-01	8.298D 00

Fig. 19 Continued.

108	1	-3.248D 01	-6.071D-06	2.769D-06	-1.715D-07	-2.839D-02	-3.793D 00
		3.248D 01	6.071D-06	-2.769D-06	1.715D-07	2.836D-02	3.793D 00
109	1	-1.597D 01	-9.958D-01	-5.494D-02	3.646D-03	2.156D-01	-8.298D 00
		1.597D 01	9.958D-01	5.494D-02	-3.646D-03	2.422D-01	0.0
110	1	0.0	0.0	6.787D-03	-1.025D-03	-2.828D-02	0.0
		0.0	0.0	-6.787D-03	1.025D-03	-2.828D-02	0.0
111	1	0.0	0.0	2.087D-07	-3.152D-08	-8.695D-07	0.0
		0.0	0.0	-2.087D-07	3.152D-08	-8.695D-07	0.0
112	1	0.0	0.0	-6.787D-03	1.025D-03	2.828D-02	0.0
		0.0	0.0	6.787D-03	-1.025D-03	2.828D-02	0.0
113	1	1.579D 00	-6.042D-02	-2.166D-03	9.012D-05	9.558D-03	0.0
		-1.579D 00	6.042D-02	2.166D-03	-9.012D-05	8.494D-03	-5.035D-01
114	1	4.060D 00	-1.044D-06	2.384D-07	-1.094D-08	2.143D-03	2.275D-01
		-4.060D 00	1.044D-06	-2.384D-07	1.094D-08	-2.145D-03	-2.275D-01
115	1	1.579D 00	6.043D-02	2.167D-03	-9.014D-05	-8.496D-03	5.035D-01
		-1.579D 00	-6.043D-02	-2.167D-03	9.014D-05	-9.560D-03	0.0
116	1	2.303D-02	-1.241D-03	1.599D-05	7.606D-05	4.629D-04	0.0
		-2.303D-02	1.241D-03	-1.599D-05	-7.606D-05	-5.962D-04	-1.034D-02
117	1	5.824D-01	-4.613D-07	7.970D-08	9.770D-09	-3.902D-04	-6.541D-02
		-5.824D-01	4.613D-07	-7.970D-08	-9.770D-09	3.896D-04	6.541D-02
118	1	2.307D-02	1.242D-03	-1.573D-05	-7.604D-05	5.951D-04	1.035D-02
		-2.307D-02	-1.242D-03	1.573D-05	7.604D-05	-4.640D-04	0.0
119	1	-2.345D-01	9.005D-03	-1.389D-04	5.193D-06	9.253D-04	0.0
		2.345D-01	-9.005D-03	1.389D-04	-5.193D-06	2.321D-04	7.504D-02
120	1	-2.953D-01	-3.375D-07	1.340D-08	1.278D-08	5.035D-04	-5.563D-02
		2.953D-01	3.375D-07	-1.340D-08	-1.278D-08	-5.036D-04	5.563D-02
121	1	-2.345D-01	-9.004D-03	1.390D-04	-5.171D-06	-2.326D-04	-7.503D-02
		2.345D-01	9.004D-03	-1.390D-04	5.171D-06	-9.257D-04	0.0
122	1	0.0	0.0	3.902D-04	-2.824D-05	-1.626D-03	0.0
		0.0	0.0	-3.902D-04	2.824D-05	-1.626D-03	0.0
123	1	0.0	0.0	1.761D-07	7.071D-09	-7.338D-07	0.0
		0.0	0.0	-1.761D-07	-7.071D-09	-7.338D-07	0.0
124	1	0.0	0.0	-3.898D-04	2.825D-05	1.624D-03	0.0
		0.0	0.0	3.898D-04	-2.825D-05	1.624D-03	0.0

125	1	1.049D-01	-9.4240-03	3.155D-04	-1.9650-05	-1.4780-03	0.0
		-1.049D-01	9.4240-03	-3.1550-04	1.9650-05	-1.1510-03	-7.853D-02
126	1	-5.460D-03	4.622D-07	1.3550-08	-8.0070-10	-3.0700-04	7.4460-02
		5.460D-03	-4.6220-07	-1.355D-08	8.0070-10	3.0690-04	-7.4460-02
127	1	1.049D-01	9.4240-03	-3.1550-04	1.965D-05	1.1510-03	7.8530-02
		-1.0490-01	-9.4240-03	3.1550-04	-1.9650-05	1.4780-03	0.0
128	1	1.545D-01	-8.1080-03	-1.8090-04	9.8820-06	5.892D-04	0.0
		-1.545D-01	8.108D-03	1.8090-04	-9.8820-06	9.1840-04	-6.756D-02
129	1	1.5050-01	1.2330-07	4.502D-08	-4.2480-09	-1.1480-05	5.433D-02
		-1.505D-01	-1.2330-07	-4.5020-08	4.2480-09	1.1110-05	-5.433D-02
130	1	1.5450-01	8.1080-03	1.8100-04	-9.8880-06	-9.1860-04	6.7560-02
		-1.545D-01	-8.108D-03	-1.810D-04	9.8880-06	-5.895D-04	0.0
131	1	0.0	0.0	-5.1870-04	2.5460-05	2.161D-03	0.0
		0.0	0.0	5.1870-04	-2.5460-05	2.1610-03	0.0
132	1	0.0	0.0	-2.0950-07	-6.2360-09	8.728D-07	0.0
		0.0	0.0	2.0950-07	6.2360-09	8.728D-07	0.0
133	1	0.0	0.0	5.1820-04	-2.546D-05	-2.1590-03	0.0
		0.0	0.0	-5.1820-04	2.546D-05	-2.159D-03	0.0

SHELL ELEMENT STRESSES

ELEMENT NUMBER	LOAD CASE	MEMBRANE STRESS COMPONENTS SXX	SYY	SXY	BENDING MOMENT COMPONENTS MXX	MYY	MXY
1	1	0.54500 01	-0.11350 00	0.11140 01	0.27480 00	-0.82910-02	0.4946D-02
2	1	0.54380 01	0.1972D-01	0.12660 00	0.27860 00	0.3210D-01	0.2247D-02
3	1	0.54140 01	0.4793D-01	-0.66340-04	0.27770 00	0.4094D-01	0.7106D-05
4	1	0.54390 01	0.1972D-01	-0.12660 00	0.27860 00	0.3210D-01	-0.2232D-02
5	1	0.54510 01	-0.11350 00	-0.11140 01	0.27480 00	-0.8297D-02	-0.4926D-02
6	1	0.16380 02	0.36500-01	0.10900 01	0.82210 00	0.1414D-03	0.2890D-01
7	1	0.16300 02	0.3238D-01	0.27320 00	0.83960 00	0.11930 00	0.4059D-01
8	1	0.16220 02	0.1497D 00	-0.6894D-04	0.8465D 00	0.16960 00	0.2223D-05
9	1	0.16300 02	0.32400-01	-0.27330 00	0.83960 00	0.11930 00	-0.4059D-01
10	1	0.16380 02	0.36520-01	-0.10900 01	0.82210 00	0.1539D-03	-0.28890-01
11	1	0.24170 02	0.24840 00	0.20530 00	0.12100 01	-0.3420D-01	0.4449D-01

Fig. 19 Continued.

12	1	0.2527D 02	0.2411D 00	0.2496D 00	0.1294D 01	0.1880D 00	0.4817D-01
13	1	0.2561D 02	0.5741D 00	-0.1823D-04	0.1361D 01	0.3763D 00	-0.6029D-05
14	1	0.2527D 02	0.2412D 00	-0.2496D 00	0.1294D 01	0.1880D 00	-0.4818D-01
15	1	0.2417D 02	0.2484D 00	-0.2052D 00	0.1210D 01	-0.3419D-01	-0.4450D-01
16	1	0.2507D 02	-0.6486D 00	0.3678D 00	0.1033D 01	-0.1041D 00	-0.5280D-01
17	1	0.2751D 02	0.2193D 01	0.1286D 01	0.1094D 01	0.1549D 00	-0.1075D 00
18	1	0.2827D 02	0.2492D 01	0.8648D-04	0.1184D 01	0.3064D 00	-0.1474D-04
19	1	0.2751D 02	0.2193D 01	-0.1286D 01	0.1094D 01	0.1549D 00	0.1074D 00
20	1	0.2507D 02	-0.6484D 00	-0.3679D 00	0.1033D 01	-0.1041D 00	0.5280D-01
21	1	0.1730D 02	-0.4054D 00	-0.4685D 01	0.7263D 00	-0.6195D-01	-0.5279D 00
22	1	0.2019D 02	0.2107D 01	-0.1674D 01	0.7860D 00	0.1910D-01	-0.3549D 00
23	1	0.2107D 02	0.2255D 01	0.1824D-03	0.8438D 00	-0.1021D 00	-0.1990D-04
24	1	0.2019D 02	0.2107D 01	0.1674D 01	0.7860D 00	0.1910D-01	0.3548D 00
25	1	0.1730D 02	-0.4052D 00	0.4685D 01	0.7263D 00	-0.6193D-01	0.5278D 00
26	1	-0.5940D 01	0.4055D 00	-0.4937D 01	-0.3329D 00	-0.1686D 00	-0.5389D 00
27	1	-0.3547D 01	-0.6207D 00	-0.1966D 01	-0.2408D 00	-0.3935D 00	-0.7075D 00
28	1	-0.1895D 01	-0.1678D 01	0.2371D-03	-0.2230D 00	-0.8257D 00	-0.1977D-04
29	1	-0.3547D 01	-0.6207D 00	0.1966D 01	-0.2408D 00	-0.3935D 00	0.7074D 00
30	1	-0.5941D 01	0.4054D 00	0.4936D 01	-0.3329D 00	-0.1687D 00	0.5388D 00
31	1	-0.2992D 02	-0.1413D 00	-0.2566D 01	-0.1390D 01	0.2397D 00	-0.1020D 01
32	1	-0.3261D 02	-0.8972D 00	-0.7394D 00	-0.1713D 01	-0.1034D 01	-0.8057D 00
33	1	-0.3399D 02	-0.1874D 01	0.1175D-03	-0.2069D 01	-0.2642D 01	-0.1399D-04
34	1	-0.3261D 02	-0.8973D 00	0.7393D 00	-0.1713D 01	-0.1034D 01	0.8057D 00
35	1	-0.2993D 02	-0.1415D 00	0.2566D 01	-0.1390D 01	0.2397D 00	0.1020D 01
36	1	-0.4235D 02	0.5602D 00	-0.2085D 01	-0.2057D 01	0.5244D-01	0.2809D 00
37	1	-0.4720D 02	-0.7820D 00	-0.1300D 01	-0.2457D 01	-0.1312D 01	-0.1826D 00
38	1	-0.4986D 02	-0.3198D 01	-0.7256D-04	-0.2947D 01	-0.3217D 01	-0.2067D-06

Fig. 19 Continued.

39	1	-0.47210 02	-0.7823D 00	0.13000 01	-0.24570 01	-0.13120 01	0.18260 00
40	1	-0.42360 02	0.56000 00	0.2085D 01	-0.20570 01	0.5234D-01	-0.28100 00
41	1	-0.43920 02	0.44610 00	0.20460 01	-0.21000 01	0.4532D-01	0.59490 00
42	1	-0.49020 02	-0.1510D 01	0.29750 01	-0.25420 01	-0.13420 01	0.70100 00
43	1	-0.51220 02	-0.4891D 01	-0.2529D-03	-0.29560 01	-0.28560 01	0.87140-05
44	1	-0.49020 02	-0.15100 01	-0.29760 01	-0.25420 01	-0.13420 01	-0.70100 00
45	1	-0.43930 02	0.44580 00	-0.20460 01	-0.21000 01	0.45250-01	-0.59490 00
46	1	-0.30290 02	-0.27510 00	0.32440 01	-0.14170 01	0.19780 00	0.64630 00
47	1	-0.30990 02	-0.11530 01	0.14730 01	-0.16350 01	-0.9228D 00	0.83420 00
48	1	-0.31090 02	-0.23680 01	-0.42500-03	-0.18340 01	-0.19920 01	0.1540D-04
49	1	-0.30990 02	-0.11530 01	-0.14730 01	-0.16350 01	-0.92280 00	-0.83410 00
50	1	-0.30290 02	-0.27520 00	-0.32440 01	-0.14170 01	0.19780 00	-0.64630 00
51	1	-0.24740 01	0.62150 00	0.37360 01	-0.19230 00	-0.15480 00	0.34950 00
52	1	-0.67740 00	-0.9731D-01	0.80570 00	-0.7504D-01	-0.20460 00	0.39730 00
53	1	0.54370 00	-0.58170 00	-0.5479D-03	-0.3543D-01	-0.43050 00	0.1586D-04
54	1	-0.67660 00	-0.97360-01	-0.80620 00	-0.7503D-01	-0.20460 00	-0.3973D 00
55	1	-0.24730 01	0.62150 00	-0.37350 01	-0.19230 00	-0.15480 00	-0.34940 00
56	1	0.1558D 02	-0.38400 00	0.38110 01	0.63970 00	-0.3182D-01	0.32190 00
57	1	0.17840 02	0.18910 01	0.89770 00	0.67660 00	0.4968D-01	0.17660 00
58	1	0.1847D 02	0.20340 01	-0.4945D-03	0.71680 00	-0.6794D-02	0.8378D-05
59	1	0.17840 02	0.18920 01	-0.89840 00	0.67670 00	0.4968D-01	-0.17660 00
60	1	0.15580 02	-0.38380 00	-0.38100 01	0.63970 00	-0.3179D-01	-0.32180 00
61	1	0.2186D 02	-0.51640 00	-0.33930 00	0.89760 00	-0.5440D-01	-0.1114D-01
62	1	0.23400 02	0.18930 01	-0.12640 01	0.90870 00	0.12130 00	0.53850-01
63	1	0.23560 02	0.21500 01	-0.2949D-03	0.95740 00	0.21180 00	0.29490-05
64	1	0.23400 02	0.18930 01	0.12630 01	0.90870 00	0.12130 00	-0.5384D-01
65	1	0.21870 02	-0.51630 00	0.34010 00	0.89760 00	-0.5440D-01	0.11120-01

Fig. 19 Continued.

66	1	0.2080D 02	0.1971D 00	-0.2428D 00	0.1012D 01	-0.1803D-01	-0.3745D-01
67	1	0.2169D 02	0.2232D 00	-0.1989D 00	0.1066D 01	0.1388D 00	-0.2612D-01
68	1	0.2193D 02	0.5497D 00	-0.1779D-03	0.1104D 01	0.2525D 00	0.1375D-05
69	1	0.2169D 02	0.2233D 00	0.1987D 00	0.1066D 01	0.1388D 00	0.2613D-01
70	1	0.2080D 02	0.1972D 00	0.2428D 00	0.1012D 01	-0.1802D-01	0.3745D-01
71	1	0.1449D 02	0.2745D-01	-0.8945D 00	0.7015D 00	-0.5025D-02	-0.1817D-01
72	1	0.1447D 02	0.1245D-01	-0.1623D 00	0.7158D 00	0.9012D-01	-0.2440D-01
73	1	0.1445D 02	0.8086D-01	-0.9020D-04	0.7201D 00	0.1212D 00	-0.1684D-06
74	1	0.1447D 02	0.1247D-01	0.1620D 00	0.7158D 00	0.9013D-01	0.2440D-01
75	1	0.1449D 02	0.2746D-01	0.8944D 00	0.7015D 00	-0.5018D-02	0.1816D-01
76	1	0.5691D 01	-0.6369D-01	-0.9385D 00	0.2793D 00	0.2096D-02	-0.3483D-02
77	1	0.5657D 01	-0.3172D-01	-0.9157D-01	0.2796D 00	0.2648D-01	-0.1504D-02
78	1	0.5627D 01	-0.8918D-03	-0.8424D-04	0.2787D 00	0.3179D-01	-0.9916D-06
79	1	0.5657D 01	-0.3173D-01	0.9135D-01	0.2796D 00	0.2648D-01	0.1502D-02
80	1	0.5691D 01	-0.6370D-01	0.9384D 00	0.2793D 00	0.2096D-02	0.3476D-02
81	1	-0.1212D 01	-0.1369D 00	-0.7558D 00	-0.6237D-01	-0.4017D-02	-0.1855D-01
82	1	-0.1447D 01	-0.1225D 00	-0.9089D-02	-0.7225D-01	-0.1763D-01	0.2894D-02
83	1	-0.1541D 01	-0.1789D 00	-0.8995D-04	-0.7350D-01	-0.1801D-01	-0.9479D-06
84	1	-0.1448D 01	-0.1225D 00	0.8870D-02	-0.7225D-01	-0.1763D-01	-0.2896D-02
85	1	-0.1212D 01	-0.1369D 00	0.7557D 00	-0.6237D-01	-0.4021D-02	0.1853D-01
86	1	-0.4919D 01	0.9335D-01	-0.8296D 00	-0.2196D 00	0.1518D-02	-0.3020D-01
87	1	-0.5296D 01	-0.5646D 00	-0.2787D 00	-0.2099D 00	-0.3392D-01	0.4100D-02
88	1	-0.5314D 01	-0.6300D 00	-0.9802D-04	-0.2101D 00	-0.3385D-01	-0.8808D-06
89	1	-0.5296D 01	-0.5647D 00	0.2785D 00	-0.2099D 00	-0.3392D-01	-0.4102D-02
90	1	-0.4919D 01	0.9329D-01	0.8295D 00	-0.2196D 00	0.1518D-02	0.3019D-01
91	1	-0.6085D 01	0.1242D 00	0.1745D 00	-0.2699D 00	0.4280D-02	0.2788D-01
92	1	-0.6405D 01	-0.5249D 00	0.2214D 00	-0.2553D 00	-0.3006D-01	0.3158D-02

Fig. 19 Continued.

93	1	-0.64320 01	-0.58520 00	0.19090-04	-0.25660 00	-0.3895D-01	-0.6258D-06
94	1	-0.6405D 01	-0.52490 00	-0.22130 00	-0.25530 00	-0.3006D-01	-0.3159D-02
95	1	-0.6085D 01	0.12420 00	-0.17450 00	-0.26990 00	0.4281D-02	-0.27870-01
96	1	-0.56450 01	-0.61380-01	0.96400-01	-0.29660 00	-0.1807D-02	0.98830-02
97	1	-0.5907D 01	-0.52190-01	-0.37040-01	-0.30150 00	-0.2994D-01	0.3148D-02
98	1	-0.60010 01	-0.11040 00	0.10590-04	-0.30260 00	-0.3967D-01	-0.21150-06
99	1	-0.59070 01	-0.5220D-01	0.3707D-01	-0.30150 00	-0.2994D-01	-0.3148D-02
100	1	-0.56460 01	-0.61380-01	-0.96360-01	-0.29660 00	-0.1809D-02	-0.9880D-02
101	1	-0.37830 01	-0.1079D-01	0.24450 00	-0.19340 00	0.8815D-03	-0.10270-03
102	1	-0.38120 01	0.2387D-01	-0.65110-02	-0.19570 00	-0.1780D-01	-0.10280-02
103	1	-0.3836D 01	0.3519D-01	0.49310-05	-0.19510 00	-0.1859D-01	0.13010-07
104	1	-0.38120 01	0.23860-01	0.65400-02	-0.19570 00	-0.1780D-01	0.1028D-02
105	1	-0.3783D 01	-0.1080D-01	-0.2444D 00	-0.19340 00	0.8801D-03	0.10430-03
106	1	-0.12640 01	0.2685D-01	0.25780 00	-0.6425D-01	0.2951D-02	-0.16930-02
107	1	-0.12670 01	0.74620-02	0.2493D-01	-0.65370-01	-0.5300D-02	-0.2921D-02
108	1	-0.12680 01	0.10240-01	0.46350-05	-0.65110-01	-0.5281D-02	0.1594D-06
109	1	-0.12670 01	0.74620-02	-0.24900-01	-0.65370-01	-0.5300D-02	0.2921D-02
110	1	-0.12640 01	0.2686D-01	-0.25760 00	-0.64250-01	0.2951D-02	0.16940-02

Fig. 19 Continued.

OVERALL LOG

NODAL POINT INPUT.............	3.13
FORM ELEMENT STIFFNESSES......	92.01
INPUT NODAL LOADS.............	.18
FORM TOTAL STIFFNESS..........	10.86
EQUATION SOLVING..............	15.72
ELEMENT STRESSES..............	5.07
TOTAL SOLUTION TIME...........	126.97

Fig. 19 Continued.

8. Correlation of Computer and Experimental Data for Bridge #1

Table 2 and Figures 20 through 23 extracted from reference (1) were really the only available static data that could be used to compare the computer results with experimental values. The coefficients listed in Table 2 indicate the lateral distribution of the static loads used to failure. The lateral position of the loads is indicated by a sketch included as a part of Table 2. A more detailed sketch showing the position of the eight load points is given in Figure 7.

In the computer program a total live load of 432 kips was selected. Using a moment distribution procedure the loads were then translated to applied loads directly over the girders at the same cross-section of the bridge. The applied loads are those referenced in the input data.

Figure 20 depicts the experimental data variation in profiles of the bridge cross-section with increasing load at the centerline of an interior span. In Figures 21 through 23 the results of the computer program are overprinted on experimental load deflection curves with the symbol ⊕. Note the excellent agreement between the predicted and measured values for bridge #1. In Figure 23 the measured and computed average load deflection curves are shown. The measured curves were obtained from the average of the deflection readings over the four girders at the centerline of the interior span. The computed load - deflection curve is the result of a computer analysis done by the University of Tennessee using an ICES-STRUDL - II program, wherein the bridge was considered to act as a single beam.* Overprinted with a symbol ⊕ on this graph are the results of SSAP2 in which the bridge was modeled as a grid system. No attempt was made by the author to examine the use of SSAP2 to predict the inelastic deflections of the bridge, although the ICES-STUDL - II computer analysis was carried that far by a stepwise procedure in which the properties of the bridge were changed at each step. It is expected, however, that the SSAP2 program could be used to accomplish the same extension of results.

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* No inference is made that the ICES-STRUDL - II program is any less accurate than SSAP2. The difference in results is based solely on the method of modeling the bridge. Had the author access to ICES-STRUDL - II the results of the grid modelling would probably have been the same as SSAP2.

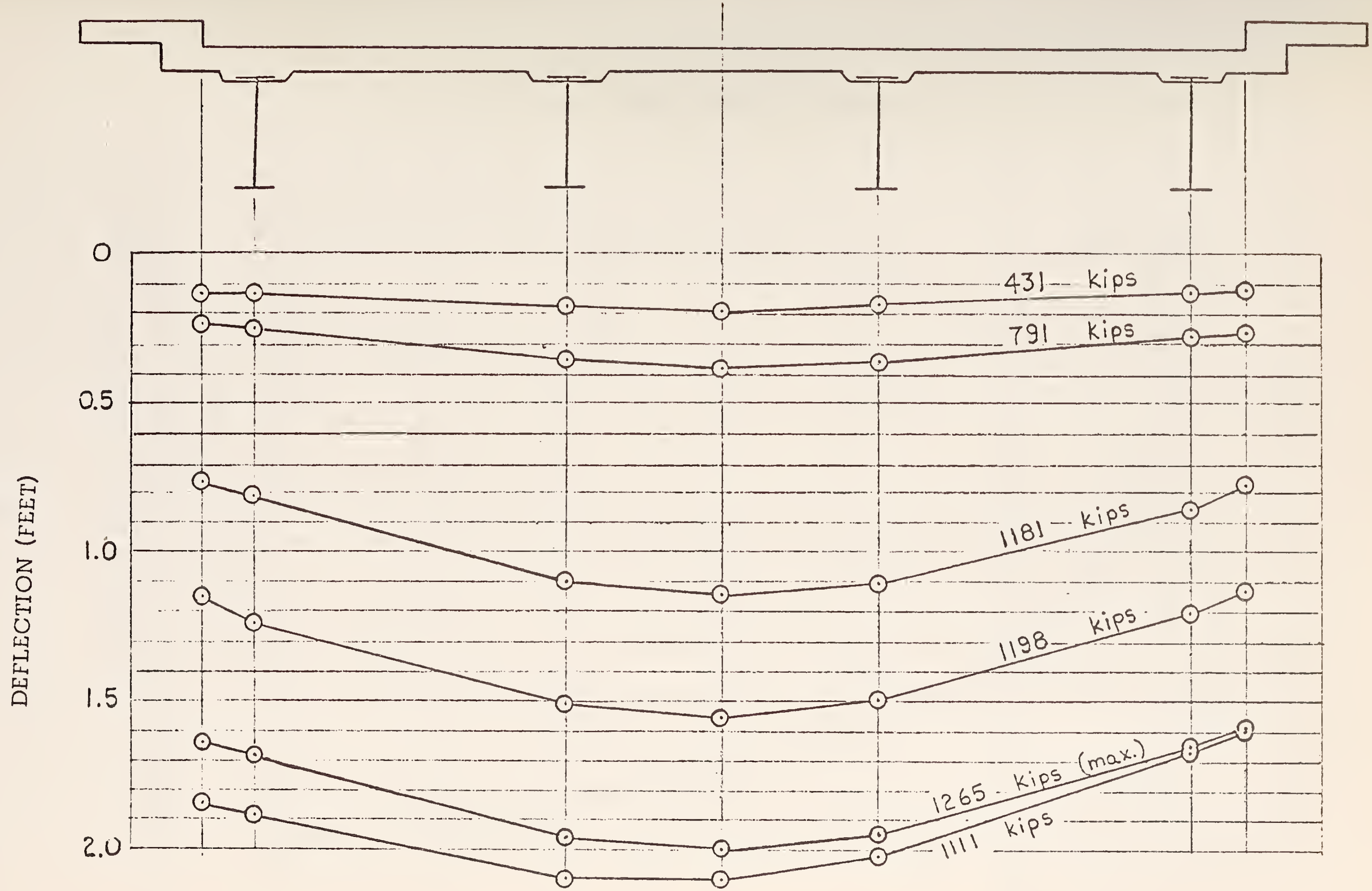


FIGURE 20: CROSS-SECTION PLOTS SHOWING DECK DISTORTION UNDER LOAD - BRIDGE 1

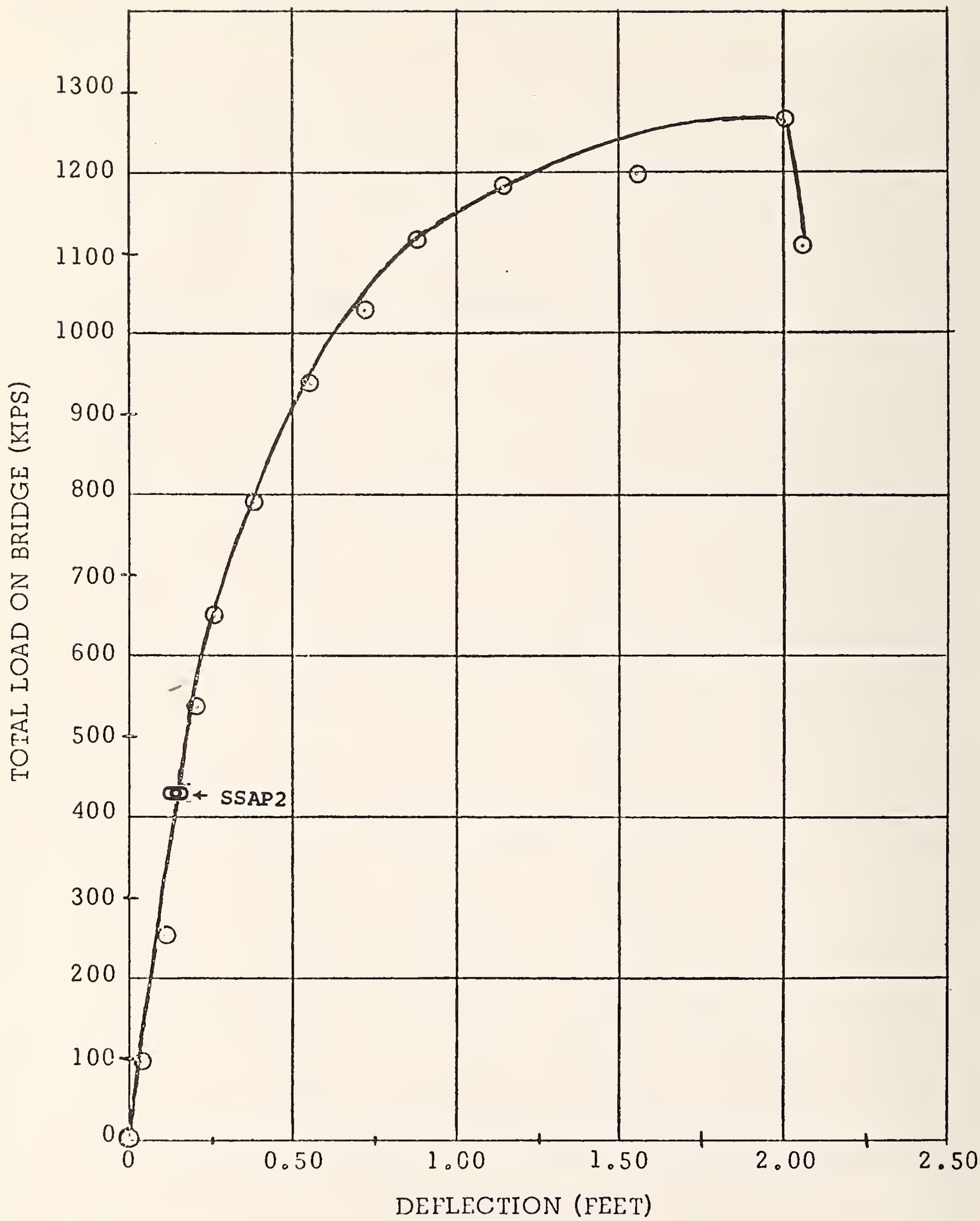


FIGURE 21. LOAD-DEFLECTION CURVE - CENTERLINE OF SPAN AT CENTER OF ROADWAY - BRIDGE 1.

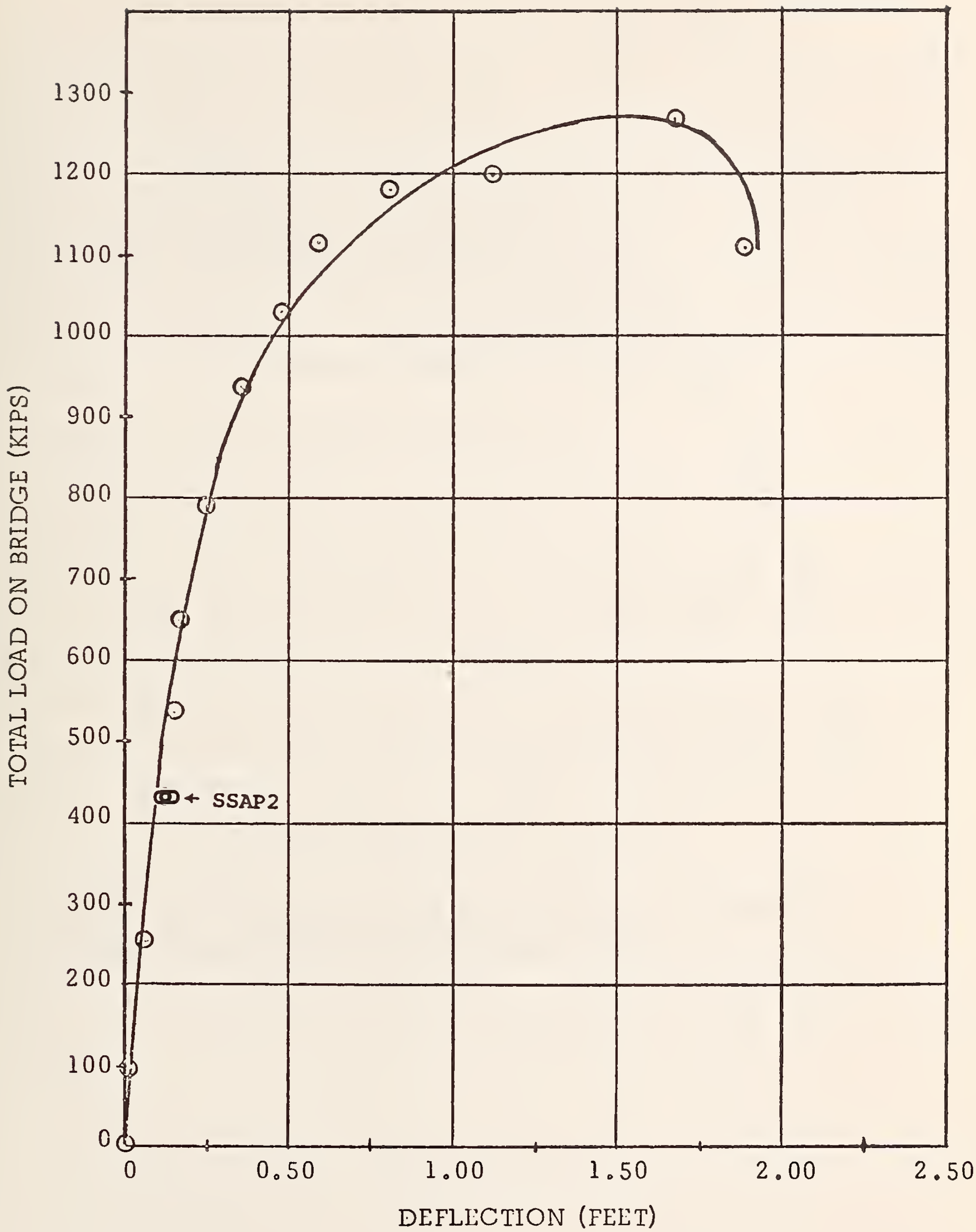


FIGURE 22. LOAD-DEFLECTION CURVE - CENTERLINE OF SPAN OVER EXTERIOR BEAM - BRIDGE 1.

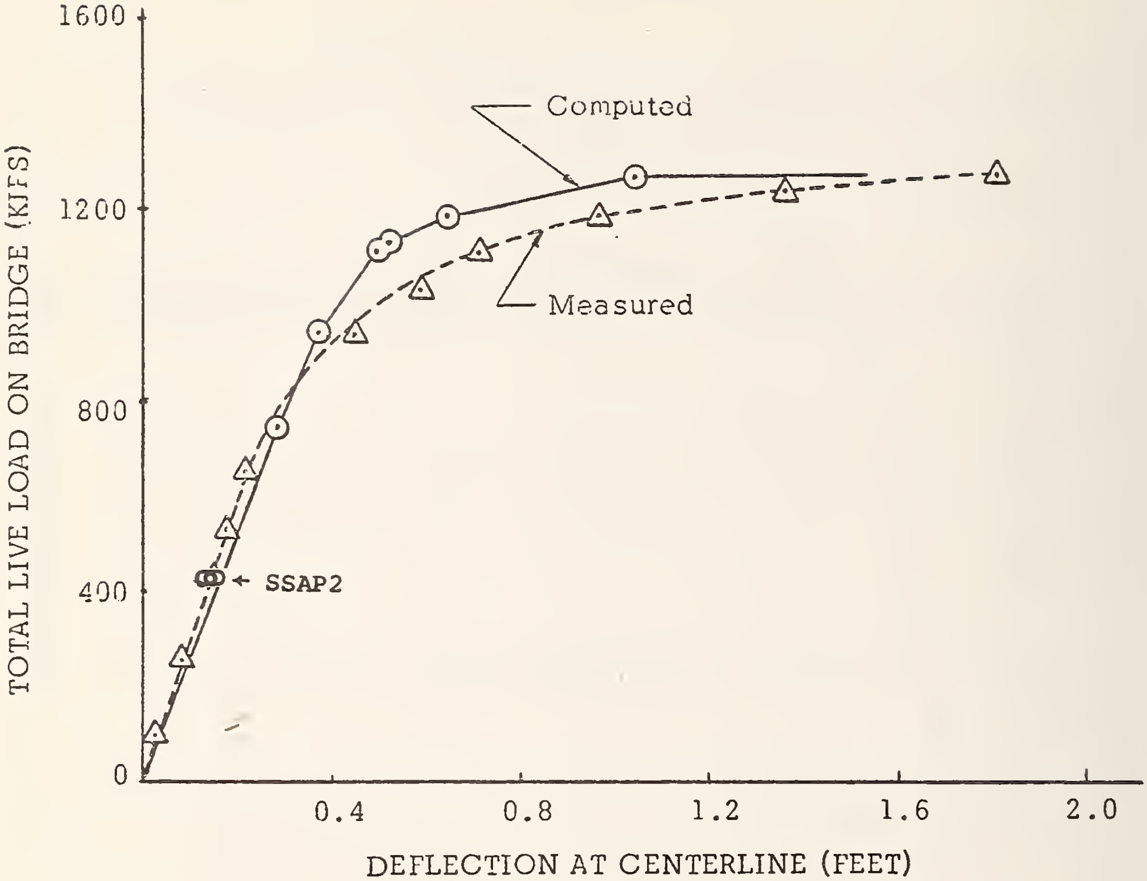


FIGURE **23.** COMPARISON OF MEASURED AND COMPUTED LOAD-DEFLECTION CURVES FOR BRIDGE 1.

Hand calculations were made using the results of the computer analysis to determine the ratio of bridge moment taken by each stringer at the centerline of the interior span. These calculations are reproduced in Figure 24. This time there is only fair agreement between the computer results and the load distribution coefficients of Table 2. It is not known why Table 2 does not show a symmetrical coefficient distribution because of the symmetry of the bridge. These coefficients were calculated from strains measured at the top and bottom of the steel wide flange sections. Note that the computer does indicate a symmetrical distribution of moments to the stringers.

9. Additional Computer Studies for Bridge #1

Other computer models were also used to predict bridge response for bridge #1. Some of these consisted of:

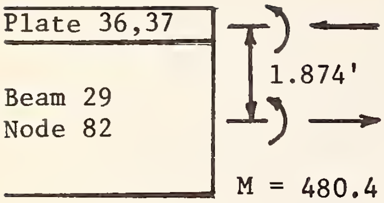
a. The use of plate elements to model the concrete deck and the webs of the steel stringers, while the flanges of the stringers were modeled with rod elements.

b. The use of plate elements for the deck and stringer webs and the use of beam elements for the flanges of the stringers.

c. The use of a thick shell element in place of the thin plate-shell element for the deck.

Table 7 compares the results of items (a) and (b). The thick shell element became impractical because of the number of nodes involved. Additional runs for bridge #1 were made with various aspect ratios resulting in fewer nodes for the bridge. However, the results were not as good. The reason for this will be explained in the next section of this report.

M = 2.057x4.75 = 9.77 ft-kips
2.457x4.17 = 10.24

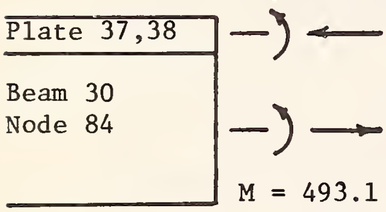


P = 42.35x4.75 = 201.16 kips
47.20x4.17 = 196.68
397.84

P = 357.3 (Slight statics discrepancy, common with finite element analysis unless a fine net spacing is used; however full section statics does check--see below)

397.84x1.874 = 745.55
480.4
9.77
10.24
1245.96 ft-k total

M = 2.704x8.333 = 22.53

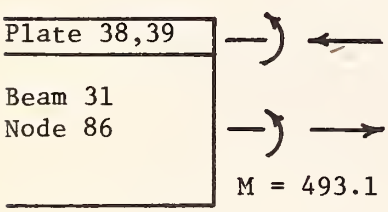


48.53x8.333 = 404.40

444.8

404.40x1.874 = 757.85
493.1
22.53
1273.48

M = 2.702x8.333 = 22.52



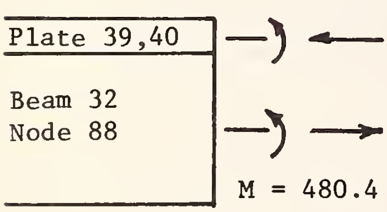
48.53x8.333 = 404.40

444.8

Sum of forces on full section
← 1604
→ 1604

NOTE: When moments are calculated for slab+beam note that moments seem to be approximately 25% for each stringer across the bridge.

2.057x4.75
M = 2.457x4.167



P = 47.21x4.167
42.36x4.75

357.3

		Measured	Computed
Percent of total bending moment to stringer	1	20	24.67
" " "	2	26	25.33
" " "	3	29	25.33
" " "	4	25	24.67

FIG. 24 - CALCULATIONS OF MOMENT TAKEN BY EACH STRINGER FOR BRIDGE #1

TABLE 7 - DEFLECTIONS FROM VARIOUS INPUT MODELS OF BRIDGE #1

		Location Deflection in Feet					
Description of Elements Used	Nodes	Node 82	Node 84	Node 86	Node 88	CPU Sec.	Load Kips
Beam Elements for Stringers Slaved to Plate Elem. for Slab and web of stringer.	230	.132	.147	.147	.132	127	432
Rod Elements for Stringer Upper Flange, Beams for Lower Flange and Plates for Slab and web.	230	.052	.075	.075	.052	261	432
Beam Elements for Both Stringer Flanges, Plates for web and slab.	230	.060	.071	.071	.060	277	432

III. Aspect Ratio and Other Matters

1. Continuous Beam Studies

One of the exceedingly important problems in finite element modeling is to decide on a tolerable aspect ratio for the element size. The more elements used, the more costly becomes each computer run. Yet considerable error can incur if too few nodes are chosen. To examine this aspect of the problem, several computer runs were performed using a single stringer bridge model (primarily to save computer costs). Table 8 shows the results of deflection readings and solution times for four computer runs. All models considered the bridge to consist of a single stringer with a shear connected concrete deck whose width was equal to the center to center distance of the stringers of bridge #1. All models had the same property and load conditions as one stringer of bridge #1.

Example (c) used plate elements for the webs of the stringer, rods for the upper flanges, beams for the lower flanges, and plates for the slab. The aspect ratio for the web-plate elements was approximately 7:1. The resulting center deflection of an interior span was .1036 feet. Example (a) shows the same beam model but using beam elements for the stringers, slave connected to the deck plate elements. Run (d) is the same continuous beam as run (c) with 288 nodes selected in such a way as to keep the aspect ratio below 1.5 for the plane elements in both the web and deck. This time the deflection increased to .1839 feet.

Run (b) used the slave beam with 288 nodes. This time the deflection reduced from .2289 feet for run (a) to .2142 feet. It is evident from these results that the slave connected beams converge faster to the same solution with increasing nodes than do the models using plate elements for the webs.* As the number of nodes are increased, both types of models will tend to converge to the same answer.

However, notice that it takes almost five times as much computer time to model according to run (d) as it does to model according to run (a), and we still do not have the same degree of accuracy with the plate-web

* It is assumed that convergence reaches the true solution.

TABLE 8 - DEFLECTION VALUES AND SOLUTION TIMES FOR FOUR COMPUTER RUNS ON A SINGLE STRINGER OF BRIDGE #1

Run	Deflection in feet	Condition	Nodes	Solution Time Sec.
a	.2289	Slave**	92	44.71
b	.2142	Slave	288	154.33
c	.1036	Web Pl.* Rod Fl. Beam Fl.	92	69.55
d	.1839	"	288	219.61

* Rods were used for the upper flange of the stringer to save on cost. Beams were necessary for the lower flange of the stringer to avoid singularities.

** Beams were used for the stringer slave connected to the plate-shell elements of the slab.

model. Run (a) uses the same spaces between node points as is used for bridge #1; and bridge #1 very closely approximated the experimental results. To the author's way of thinking this is good enough for preliminary design without going to closer node spacing. The web-plate model is prohibitive in cost, especially when we consider that solution time for a bridge input would be approximately four times the above solution time for a single stringer. It should also be noted that runs (a) through (d) were programmed on a CDC 6400 machine. An IBM 360/370 is much slower than a CDC 6400 machine. Consequently, the solution time could be extremely important.

2. Simple Beam Studies

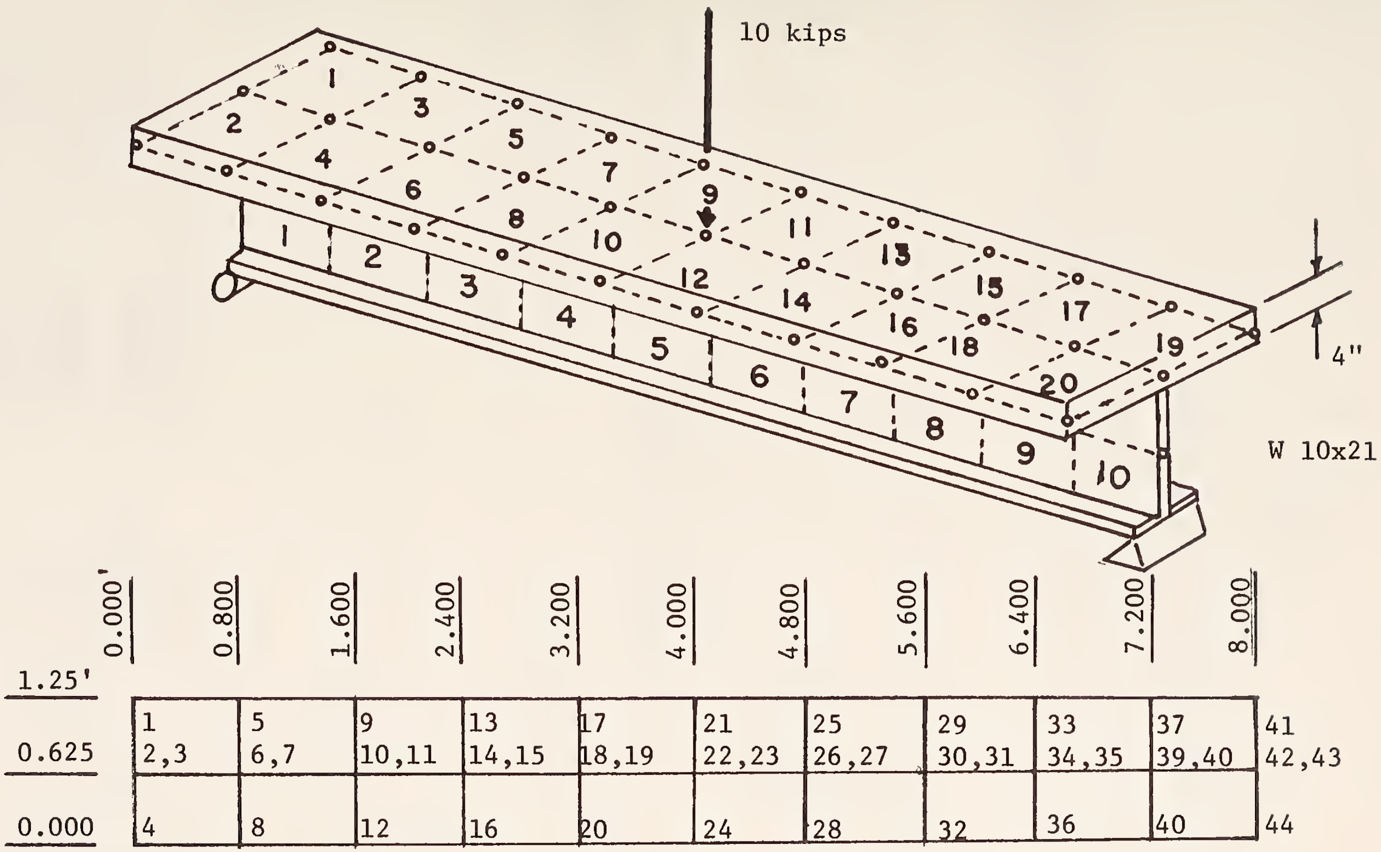
Several simple beam studies were made to assess the accuracy of computer results with theory along several sections of the beam. Figure 25 shows a schematic view of the beam with node, beam element, and slab element numbering. Essentially, a W 10x21 steel beam was chosen for the stringer, shear connected to a 4" thick, 16" wide concrete deck. The length between simple supports was 8 feet.

Figure 26 shows a comparison of moment, shear, and deflection computer values with hand calculations using a composite beam theory. The results are quite favorable when the modeling is done with a beam element for the stringer, slave connected to plate elements for the slab.

Alternately, Figure 27 shows a similar comparison when the beam was modeled with rods for the stringer flanges, plates for the stringer webs and slab. In this run a variation was tried of using a steel plate element to connect the upper beam flange with the center of gravity of the deck slab. Notice that the resulting computer results were not too good.

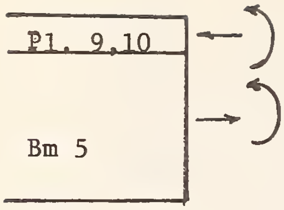
3. Singularities

At this time it might be well to discuss singularities in the program output. When any part of the structure can move without restraint, then singularities in the solution of the stiffness matrix may occur. In



NOTE: Where two numbers exist, the right most number is at the elevation of the centroid of the beam.

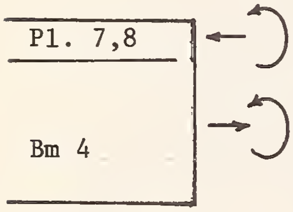
FIG. 25 - SIMPLE BEAM NODE, BEAM, AND SLAB NUMBERING



P = 13.87 (1.25) = 17.34 kips
M = .4863 (1.25) = .6077 ft-kips

P = 17.34
M = 9.212

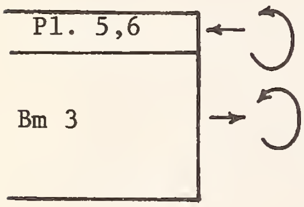
```
                              0.608
                              9.212
17.34 (.579) =               10.039
                             19.859 vs 20
```



P = 10.98 (1.25) = 13.725
M + .3625 (1.25) = .4531

P = 13.72
M = 7.479

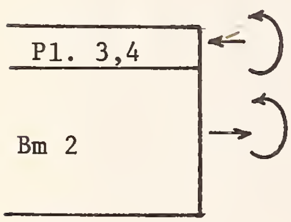
```
                              7.479
                               .4531
13.72 (.579) =                7.9468
                             15.88  vs 16
```



P = 7.844 (1.25) = 9.805
M = .2563 (1.25) = .3204

P = 9.804
M = 5.885

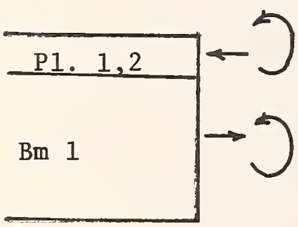
```
                               .3204
                              5.885
9.804 (.579) =                5.6765
                             11.88  vs 12
```



P = 4.608 (1.25) = 5.76
M = .1587 (1.25) = .1984

P = 5.76
M = 4.359

```
                               .1984
                              4.359
5.76  (.579) =                3.3351
                              7.89  vs 8
```



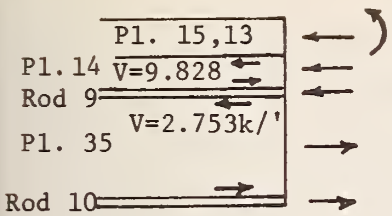
P = .1023 (1.25) = .1279
M = neg.

P = 0
M = 3.744

```
 .1279
3.744
3.87  vs 4
```

x	Shear Computer	Shear Calculated	Deflection Computer	Deflection Calculated
.1L	Practically 4.85 vs 5		.0006139'	.001726'
.2L	all sections.		.001164	.0009802
.3L			.001617	.001367
.4L			.001928	.001629
.5L			.002046	.001726

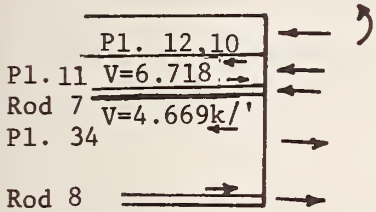
FIG. 26 - COMPARISON OF MOMENT, SHEAR AND DEFLECTION CALCULATIONS FOR SIMPLE BEAM - COMPUTER RESULTS VERSUS THEORY - FOR SLAVE BEAM INPUT.



M = .7249 (1.25) = .906125 ft-k.
P = 11.75 (1.25) = 14.6875 k.
P = 12.15 (.18) = 2.187
P = .055
16.9295

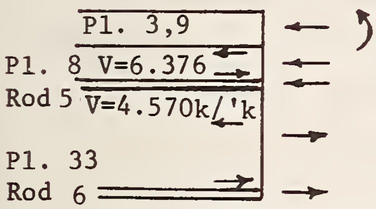
P = 7.711 (.8) = 6.1688
P = 10.652
16.8208

.90612
ΣM = 14.6875 (.98) = 14.39375
2.187 (.89) = 1.94643
.055 (.8) = .044
6.1688 (.4) = 2.46752
(.8) 9.828 (.18) = 1.41523
(.8) 2.753 (.8) = 1.76192
17.99993 vs 20



M = .3784 (1.25) = .46675
P = 9.232 (1.25) = 11.54
P = 9.639 (.18) = 1.73502
P = .301
P = 6.194 (.8) = 4.9552
P = 8.621

ΣM = 11.54 (.98) = 11.3092
1.73502 (.89) = 1.5441678
.301 (.8) = .2408
.46675
4.9552 (.4) = 1.98208
(.8)(6.718) (.18) = .967392
(.8)(4.669) (.8) = 2.698816
15.245045 vs 16



M = .2979 (1.25) = .372375
P = 6.572 (1.25) = 8.215
P = 6.881 (.18) = 1.23858
P = .241
P = 4.464
P = 6.124

.372325
ΣM = 8.215 (.98) = 8.0507
1.23858 (.89) = 1.102336
.241 (.8) = .1928
4.464 (.4) = 1.7856
(.8) 6.376 (.18) = .918144
(.8) 4.570 (.18) = .65808
9.508785 vs 12

x	Shear Computer	Shear Calculated	Deflection Computer	Deflection Calculated
.1L	3.97	5	.0005356	.0005108
.2L	4.94	5	.0009733	.0009802
.3L	4.80	5	.001346	.001367
.4L			.001615	.001629
.5L			.001736	.001726

FIG. 27 - COMPARISON OF MOMENT, SHEAR, AND DEFLECTION FOR SIMPLE BEAM - ROD AND PLATE INPUT.

most cases this will indicate possible instability of the structure. In some instances, these messages serve as a warning that nodal degrees of freedom having an undefined stiffness (such as rotations in an all-truss model or rotations about an axis normal to a shallow shell-plate element) have not been deleted. Table B-1 taken from SAP IV,[9] lists the degrees of freedom that define stiffness for the TRUSS, BEAM and PLATE-SHELL elements. Normally the plate-shell element has all six degrees of freedom defined. However, in the analysis of flat plates the stiffness associated with the rotation normal to the shell surface is not defined; therefore, the rotation normal degrees of freedom must not be included in the analysis.

For this reason, when singularities occurred they were examined to see if they were related to rotations about an axis perpendicular to the plane of the bridge deck. If so, those node points associated with the deck elements were restrained about the axis; i.e., an appropriate number 1 was coded for the boundary condition code in the coordinate input data.* Unfortunately, some of the singularities were attributed to rotation of the web-plate elements of the stringers, where it is impossible to restrain the rotation of these elements about an axis perpendicular to the plane of the web without destroying the true deflection situation. This is another reason for using the plate beam element to model the entire stringer.

In the case where singularities occurred because of rotation of the slab element, similar computer runs were made with and without normal restraint to these elements. There was very little difference in the results for all bridges investigated.

4. Master Slave Relationships

According to SAP IV instructions, a master slave relationship was violated in this program; i.e., master and slave nodes must be beam only nodes; and that no other element type may be connected as a slave. It

* See User's Manual, Appendix B, page B-4.

is supposed that this restriction was placed in the SAP IV User's Manual[10] primarily to insure that beam elements which are slave connected to other elements having undefined degrees of freedom would not in turn have undefined degrees of freedom. In the case of the straight bridges there is really very little tendency for the plate-slab elements to rotate about an axis perpendicular to the plane of the deck. Therefore, the fact that these elements are undefined for this rotation should not cause any really great concern.

In the case of a skewed deck there might be a tendency for significant rotation about a normal to the plate-shell elements. Yet, restraining these elements from that rotation when singularities are listed would make the analysis unreal. The author did not have enough experience with the computer program to assess the error caused by omitting this rotational restraint for the skew deck bridge. In fact, singularities for bridge #3 could not be eliminated. Nevertheless, the good correlation between predicted and measured responses for this bridge reduced the significance of the problem but did not eliminate it.

IV. Bridge #2

1. General

During the remainder of this report, we will present input data, output data, and correlation with experimental results for the beam element girder, shell-plate deck type of modeling, although other modeling schemes were investigated. In all cases it turned out that the use of beam elements for the stringers, slave connected to the plate-shell elements for the deck gave the best correlation, taking into account cost per computer run, aspect ratio, and convergence to the true solution.

2. Description

Bridge #2 shown in Figures 1 and 2 was composed of AASHO Type III precast, prestressed sections and was a widely used type of bridge in Tennessee. It had a 20^o skew, a grade of approximately 4 1/2% and a superelevated roadway due to a 4 1/2 degree horizontal curve.* Bridge #2 was built in 1962 on State Route 130 over the Boiling Fork Creek. Figures E-6 through E-10 show details of the bridge taken from construction drawings. The tests and computer analysis described here is for the end span of the bridge fartherest from Winchester, Tennessee. A choice had to be made between the dimensions of Figure 8 taken from reference (1) and the dimensions shown in Figure E-6 based on construction drawings. Arbitrarily as a first analysis of the bridge, the author decided to use the dimensions shown in Figure 8 as a simplification, and also decided to eliminate the superelevation for the first trial. When the results of the computer analysis showed marginal correlation, the author reprogrammed for further refinement of the geometry of the bridge. Figure 28 describes the node numbering, Figure 29 the beam element numbering, and Figure 30 the plate element numbering for the deck.

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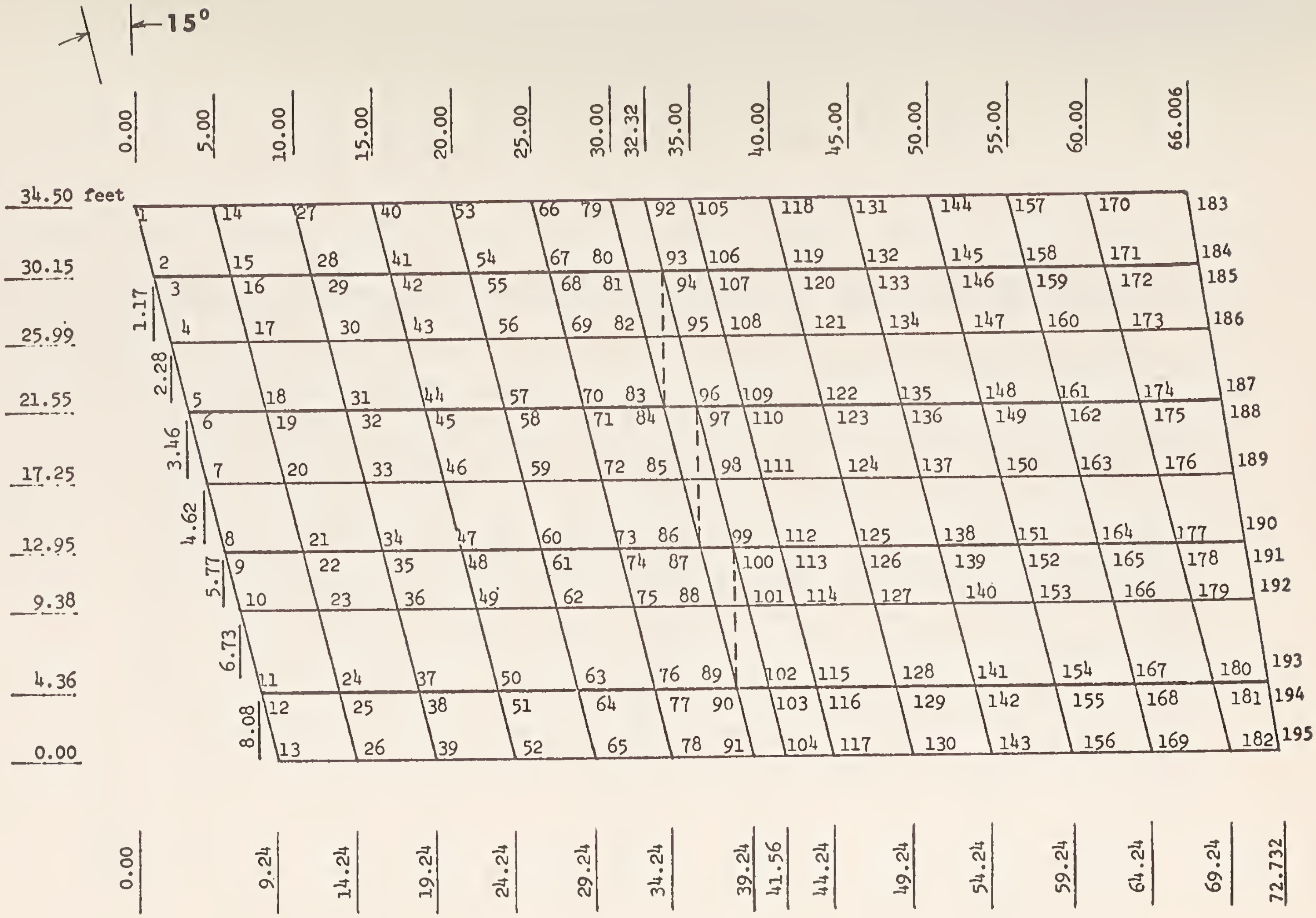
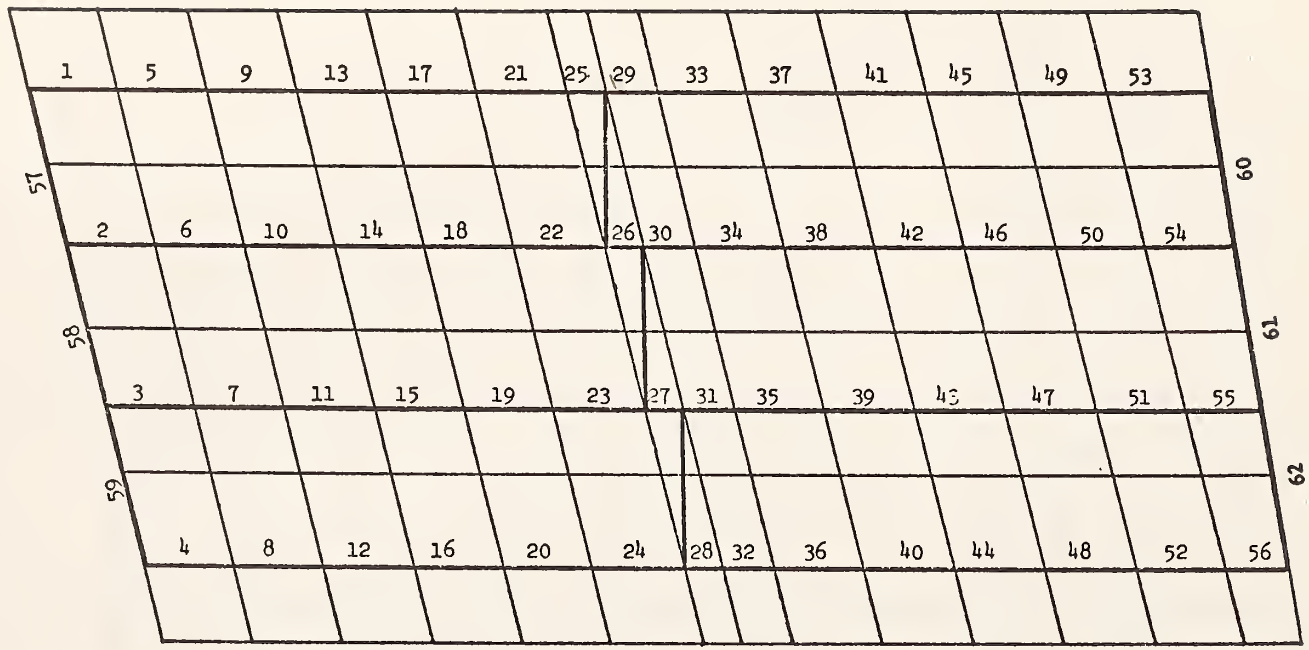
* The end span of the bridge that was modeled in the computer program had a 15^o skew, although the center span had a 20^o skew. See page E-7.



NOTE: Where two numbers exist, the right most number is at the elevation of the centroid of the stringer.

FIG. 28 - LOCATION OF NODES FOR BRIDGE #2



NOTE: Intermediate diaphragms were omitted for this particular input version.

FIG. 29 - BEAM AND END DIAPHRAGM NUMBERING FOR BRIDGE #2

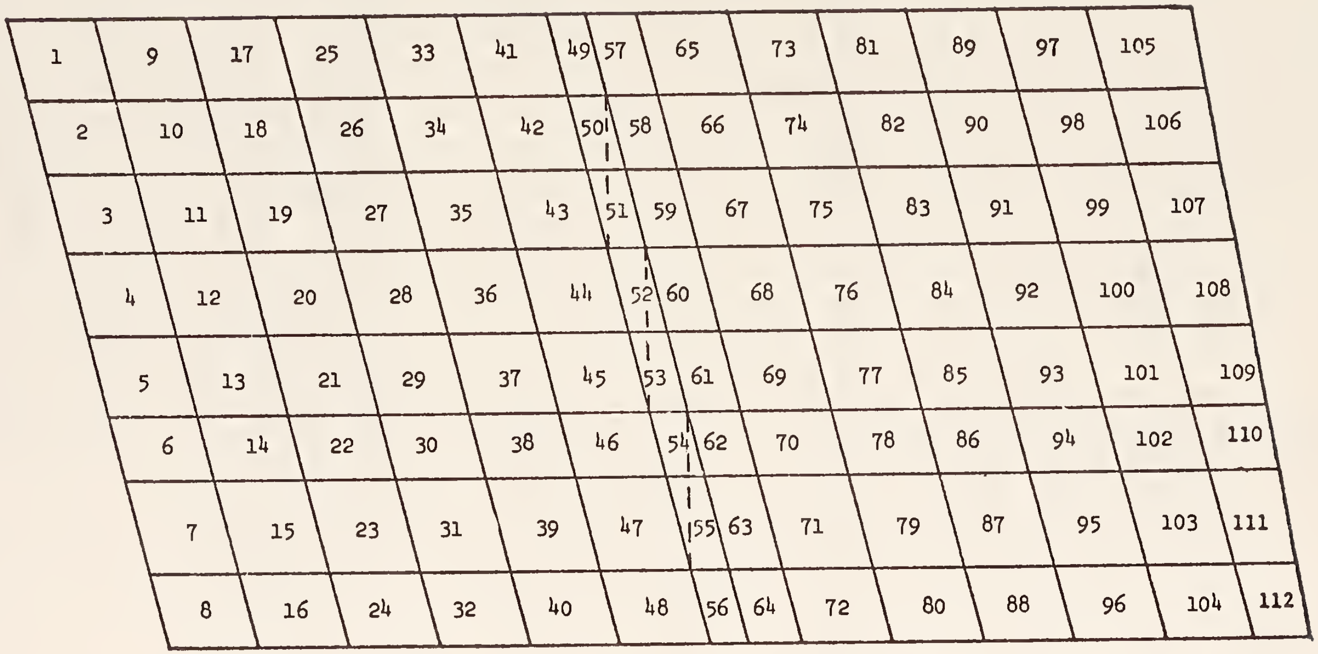


FIG. 30 - PLATE-SHELL ELEMENT NUMBERING FOR SLAB OF BRIDGE #2

3. Modulus of Elasticity

From Table 6:

$$f'_c = 5{,}500$$

$$E_c = 57{,}500 \times \sqrt{f'_c}$$

$$= \frac{57{,}500}{1{,}000} \times \sqrt{5{,}500}$$

$$= 4266.5 \text{ psi} \times 10^3$$

$$= 4266.5 \text{ psi} \times 144$$

$$= 614376 \text{ ksf}$$

The calculations for the plane stress properties are similar to those of bridge #1

$$C_{xx} = C_{yy} = 628517.58$$

$$C_{xy} = 94277.63$$

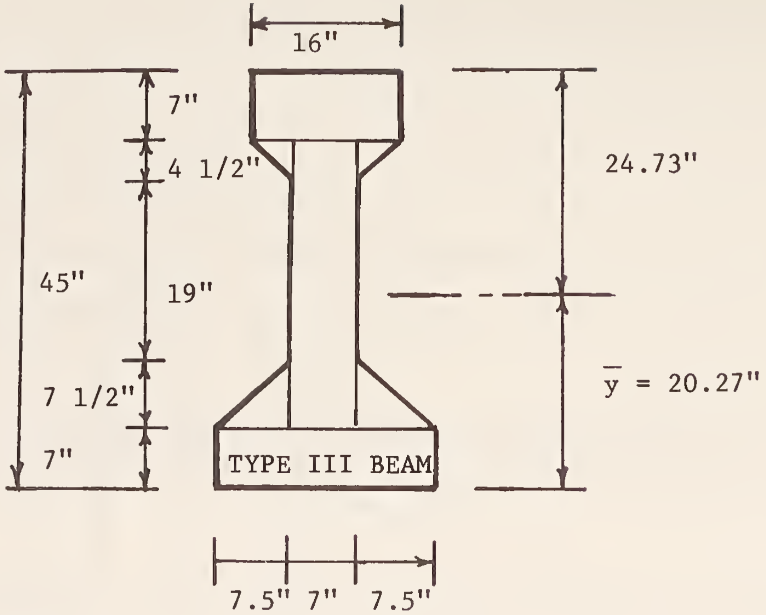
$$G_{xy} = 267119.97$$

4. Moments of Inertia

Bending and torsional moments of inertia were calculated for the Type III concrete beam as shown in Figure 31 considering a homogeneous beam. Figure 32 shows the corresponding calculations for a concrete beam diaphragm at the end bent and the intermediate location as shown in Figure E-8.

5. Input Listing

Figure 33 shows the input data for bridge #2.



By the usual methods of mechanics:

$I_x = 130973 \text{ in}^4 = 6.316206 \text{ ft}^4$

$A = 560 \text{ in}^2 = 3.885416 \text{ ft}^2$

$I_y = 12216 \text{ in}^4 = 0.589147 \text{ ft}^4$

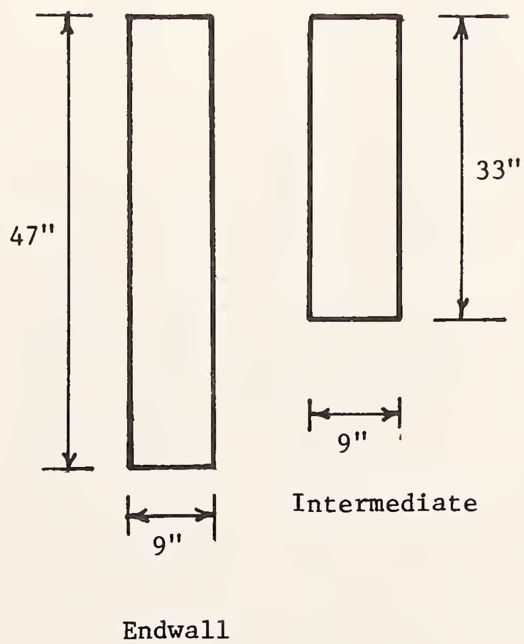
The torsion constant is calculated for concrete by ACI formula

$$C = \Sigma\ (1 - 0.63\ x/y)\frac{x^3y}{3}$$

where x = short rect. length
y = long rect. length

$C = 6817 \text{ in}^4 = 0.328760 \text{ ft}^4$

FIG. 31 - MOMENT OF INERTIA CALCULATIONS FOR GIRDER OF BRIDGE #2



Beams and Diaphragms at Bent and Intermediate locations:

$I_x = 26953 \text{ in}^4 = 1.299805 \text{ ft}^4$

$I_y = 2005 \text{ in}^4 = 0.096679 \text{ ft}^4$

$J = 6641 \text{ in}^4 = 0.320273 \text{ ft}^4$

$A = 297 \text{ in}^2 = 2.0625 \text{ ft}^4$

Endwall Beams:

$I_x = 3.755172 \text{ ft}^4 \quad I_y = 0.137695 \text{ ft}^4$

$J = 0.484336 \text{ ft}^4 \quad A = 2.9375 \text{ ft}^2$

FIG. 32 - MOMENT OF INERTIA CALCULATIONS FOR DIAPHRAGMS OF BRIDGE #2

**BRIDGE NO. 2 SUPERELEVATED-FIRST SPAN (2500) DIMENSIONED AS IN REPORT

NUMBER OF NODAL POINTS = 195

NUMBER OF ELEMENT TYPES = 2

NUMBER OF LOAD CASES = 1

NODAL POINT INPUT DATA

NODE	BOUNDARY CONDITION CODES						NODAL POINT COORDINATES				
NUMBER	X	Y	Z	XX	YY	ZZ	X	Y	Z		T
1	-0	-0	-0	-0	-0	-0	0.000	34.500	3.120	-0	-0.000
2	1	1	1	-0	-0	-0	1.085	30.151	2.730	-0	-0.000
3	2	2	2	2	2	2	1.085	30.151	.252	-0	-0.000
4	-0	-0	-0	-0	-0	-0	2.163	25.850	2.340	-0	-0.000
5	1	1	1	-0	-0	-0	3.240	21.549	1.940	-0	-0.000
6	5	5	5	5	5	5	3.240	21.549	-.538	-0	-0.000
7	-0	-0	-0	-0	-0	-0	4.317	17.250	1.560	-0	-0.000
8	1	1	1	-0	-0	-0	5.394	12.951	1.170	-0	-0.000
9	8	8	8	8	8	8	5.394	12.951	-1.308	-0	-0.000
10	-0	-0	-0	-0	-0	-0	6.473	8.645	.780	-0	-0.000
11	1	1	1	-0	-0	-0	7.551	4.339	.390	-0	-0.000
12	11	11	11	11	11	11	7.551	4.339	-2.088	-0	-0.000
13	-0	-0	-0	-0	-0	-0	8.634	0.000	0.000	-0	-0.000
14	-0	-0	-0	-0	-0	-0	5.000	34.500	3.278	-0	-0.000
15	-0	-0	-0	-0	-0	-0	6.085	30.151	2.888	-0	-0.000
16	15	15	15	15	15	15	6.085	30.151	.410	-0	-0.000
17	-0	-0	-0	-0	-0	-0	7.163	25.850	2.497	-0	-0.000
18	-0	-0	-0	-0	-0	-0	8.240	21.549	2.097	-0	-0.000
19	18	18	18	18	18	18	8.240	21.549	-.381	-0	-0.000
20	-0	-0	-0	-0	-0	-0	9.317	17.250	1.716	-0	-0.000
21	-0	-0	-0	-0	-0	-0	10.394	12.951	1.325	-0	-0.000
22	21	21	21	21	21	21	10.394	12.951	-1.153	-0	-0.000
23	-0	-0	-0	-0	-0	-0	11.473	8.645	.935	-0	-0.000
24	-0	-0	-0	-0	-0	-0	12.551	4.339	.544	-0	-0.000
25	24	24	24	24	24	24	12.551	4.339	-1.934	-0	-0.000
26	-0	-0	-0	-0	-0	-0	13.634	0.000	.154	-0	-0.000
27	-0	-0	-0	-0	-0	-0	10.000	34.500	3.437	-0	-0.000
28	-0	-0	-0	-0	-0	-0	11.085	30.151	3.046	-0	-0.000
29	28	28	28	28	28	28	11.085	30.151	.568	-0	-0.000
30	-0	-0	-0	-0	-0	-0	12.163	25.850	2.654	-0	-0.000
31	-0	-0	-0	-0	-0	-0	13.240	21.549	2.253	-0	-0.000
32	31	31	31	31	31	31	13.240	21.549	-.225	-0	-0.000
33	-0	-0	-0	-0	-0	-0	14.317	17.250	1.872	-0	-0.000
34	-0	-0	-0	-0	-0	-0	15.394	12.951	1.481	-0	-0.000
35	34	34	34	34	34	34	15.394	12.951	-.997	-0	-0.000
36	-0	-0	-0	-0	-0	-0	16.473	8.645	1.090	-0	-0.000
37	-0	-0	-0	-0	-0	-0	17.551	4.339	.699	-0	-0.000
38	37	37	37	37	37	37	17.551	4.339	-1.779	-0	-0.000
39	-0	-0	-0	-0	-0	-0	18.634	0.000	.307	-0	-0.000
40	-0	-0	-0	-0	-0	-0	15.000	34.500	3.595	-0	-0.000
41	-0	-0	-0	-0	-0	-0	16.085	30.151	3.203	-0	-0.000
42	41	41	41	41	41	41	16.085	30.151	.725	-0	-0.000
43	-0	-0	-0	-0	-0	-0	17.163	25.850	2.812	-0	-0.000
44	-0	-0	-0	-0	-0	-0	18.240	21.549	2.410	-0	-0.000
45	44	44	44	44	44	44	18.240	21.549	-.068	-0	-0.000
46	-0	-0	-0	-0	-0	-0	19.317	17.250	2.028	-0	-0.000
47	-0	-0	-0	-0	-0	-0	20.394	12.951	1.636	-0	-0.000
48	47	47	47	47	47	47	20.394	12.951	-.842	-0	-0.000
49	-0	-0	-0	-0	-0	-0	21.473	8.645	1.245	-0	-0.000
50	-0	-0	-0	-0	-0	-0	22.551	4.339	.853	-0	-0.000
51	50	50	50	50	50	50	22.551	4.339	-1.625	-0	-0.000
52	-0	-0	-0	-0	-0	-0	23.634	0.000	4.461	-0	-0.000
53	-0	-0	-0	-0	-0	-0	20.000	34.500	3.754	-0	-0.000
54	-0	-0	-0	-0	-0	-0	21.085	30.151	3.361	-0	-0.000
55	54	54	54	54	54	54	21.085	30.151	.883	-0	-0.000
56	-0	-0	-0	-0	-0	-0	22.163	25.850	2.969	-0	-0.000
57	-0	-0	-0	-0	-0	-0	23.240	21.549	2.566	-0	-0.000
58	57	57	57	57	57	57	23.240	21.549	.088	-0	-0.000
59	-0	-0	-0	-0	-0	-0	24.317	17.250	2.184	-0	-0.000
60	-0	-0	-0	-0	-0	-0	25.394	12.951	1.792	-0	-0.000
61	60	60	60	60	60	60	25.394	12.951	-.686	-0	-0.000
62	-0	-0	-0	-0	-0	-0	26.473	8.645	1.400	-0	-0.000

FIG. 33 - INPUT DATA FOR BRIDGE #2

63	-0	-0	-0	-0	-0	-0	27.551	4.339	1.007	-0	-0.000
64	63	63	63	63	63	63	27.551	4.339	-1.471	-0	-0.000
65	-0	-0	-0	-0	-0	-0	28.634	0.000	.615	-0	-0.000
66	-0	-0	-0	-0	-0	-0	25.000	34.500	3.912	-0	-0.000
67	-0	-0	-0	-0	-0	-0	26.085	30.151	3.519	-0	-0.000
68	67	67	67	67	67	67	26.085	30.151	1.041	-0	-0.000
69	-0	-0	-0	-0	-0	-0	27.163	25.850	3.126	-0	-0.000
70	-0	-0	-0	-0	-0	-0	28.240	21.549	2.723	-0	-0.000
71	70	70	70	70	70	70	28.240	21.549	.245	-0	-0.000
72	-0	-0	-0	-0	-0	-0	29.317	17.250	2.340	-0	-0.000
73	-0	-0	-0	-0	-0	-0	30.394	12.951	1.947	-0	-0.000
74	73	73	73	73	73	73	30.394	12.951	-.531	-0	-0.000
75	-0	-0	-0	-0	-0	-0	31.473	8.645	1.554	-0	-0.000
76	-0	-0	-0	-0	-0	-0	32.551	4.339	1.162	-0	-0.000
77	76	76	76	76	76	76	32.551	4.339	-1.316	-0	-0.000
78	-0	-0	-0	-0	-0	-0	33.634	0.000	.769	-0	-0.000
79	-0	-0	-0	-0	-0	-0	30.000	34.500	4.070	-0	-0.000
80	-0	-0	-0	-0	-0	-0	31.085	30.151	3.677	-0	-0.000
81	80	80	80	80	80	80	31.085	30.151	1.199	-0	-0.000
82	-0	-0	-0	-0	-0	-0	32.163	25.850	3.283	-0	-0.000
83	-0	-0	-0	-0	-0	-0	33.240	21.549	2.880	-0	-0.000
84	83	83	83	83	83	83	33.240	21.549	.402	-0	-0.000
85	-0	-0	-0	-0	-0	-0	34.317	17.250	2.496	-0	-0.000
86	-0	-0	-0	-0	-0	-0	35.394	12.951	2.103	-0	-0.000
87	86	86	86	86	86	86	35.394	12.951	-.375	-0	-0.000
88	-0	-0	-0	-0	-0	-0	36.473	8.645	1.709	-0	-0.000
89	-0	-0	-0	-0	-0	-0	37.551	4.339	1.316	-0	-0.000
90	89	89	89	89	89	89	37.551	4.339	-1.162	-0	-0.000
91	-0	-0	-0	-0	-0	-0	38.634	0.000	.922	-0	-0.000
92	-0	-0	-0	-0	-0	-0	32.320	34.500	4.143	-0	-0.000
93	-0	-0	-0	-0	-0	-0	33.405	30.151	3.750	-0	-0.000
94	93	93	93	93	93	93	33.405	30.151	1.272	-0	-0.000
95	-0	-0	-0	-0	-0	-0	34.483	25.850	3.356	-0	-0.000
96	-0	-0	-0	-0	-0	-0	35.560	21.549	2.952	-0	-0.000
97	96	96	96	96	96	96	35.560	21.549	.474	-0	-0.000
98	-0	-0	-0	-0	-0	-0	36.637	17.250	2.569	-0	-0.000
99	-0	-0	-0	-0	-0	-0	37.714	12.951	2.775	-0	-0.000
100	99	99	99	99	99	99	37.714	12.951	.297	-0	-0.000
101	-0	-0	-0	-0	-0	-0	38.783	8.645	1.781	-0	-0.000
102	-0	-0	-0	-0	-0	-0	39.871	4.339	1.388	-0	-0.000
103	102	102	102	102	102	102	39.871	4.339	-1.090	-0	-0.000
104	-0	-0	-0	-0	-0	-0	40.954	0.000	.994	-0	-0.000
105	-0	-0	-0	-0	-0	-0	35.000	34.500	4.229	-0	-0.000
106	-0	-0	-0	-0	-0	-0	36.085	30.151	3.834	-0	-0.000
107	106	106	106	106	106	106	36.085	30.151	1.356	-0	-0.000
108	-0	-0	-0	-0	-0	-0	37.163	25.850	3.440	-0	-0.000
109	-0	-0	-0	-0	-0	-0	38.240	21.549	3.036	-0	-0.000
110	109	109	109	109	109	109	38.240	21.549	.558	-0	-0.000
111	-0	-0	-0	-0	-0	-0	39.317	17.250	2.652	-0	-0.000
112	-0	-0	-0	-0	-0	-0	40.394	12.951	2.258	-0	-0.000
113	112	112	112	112	112	112	40.394	12.951	-.220	-0	-0.000
114	-0	-0	-0	-0	-0	-0	41.473	8.645	1.864	-0	-0.000
115	-0	-0	-0	-0	-0	-0	42.551	4.339	1.470	-0	-0.000
116	115	115	115	115	115	115	42.551	4.339	-1.008	-0	-0.000
117	-0	-0	-0	-0	-0	-0	43.634	0.000	1.076	-0	-0.000
118	-0	-0	-0	-0	-0	-0	40.000	34.500	4.387	-0	-0.000
119	-0	-0	-0	-0	-0	-0	41.085	30.151	3.992	-0	-0.000
120	119	119	119	119	119	119	41.085	30.151	1.514	-0	-0.000
121	-0	-0	-0	-0	-0	-0	42.163	25.850	3.597	-0	-0.000
122	-0	-0	-0	-0	-0	-0	43.240	21.549	3.193	-0	-0.000
123	122	122	122	122	122	122	43.240	21.549	.715	-0	-0.000
124	-0	-0	-0	-0	-0	-0	44.317	17.250	2.808	-0	-0.000
125	-0	-0	-0	-0	-0	-0	45.394	12.951	2.414	-0	-0.000
126	125	125	125	125	125	125	45.394	12.951	-.064	-0	-0.000
127	-0	-0	-0	-0	-0	-0	46.473	8.645	2.019	-0	-0.000
128	-0	-0	-0	-0	-0	-0	47.551	4.339	1.625	-0	-0.000
129	128	128	128	128	128	128	47.551	4.339	-.853	-0	-0.000
130	-0	-0	-0	-0	-0	-0	48.634	0.000	1.230	-0	-0.000
131	-0	-0	-0	-0	-0	-0	45.000	34.500	4.545	-0	-0.000
132	-0	-0	-0	-0	-0	-0	46.085	30.151	4.150	-0	-0.000
133	132	132	132	132	132	132	46.085	30.151	1.672	-0	-0.000
134	-0	-0	-0	-0	-0	-0	47.163	25.850	3.755	-0	-0.000
135	-0	-0	-0	-0	-0	-0	48.240	21.549	3.349	-0	-0.000
136	135	135	135	135	135	135	48.240	21.549	.871	-0	-0.000
137	-0	-0	-0	-0	-0	-0	49.317	17.250	2.964	-0	-0.000

Fig. 33 Continued.

138	-0	-0	-0	-0	-0	-0	50.394	12.951	2.569	-0	-0.000
139	138	138	138	138	138	138	50.394	12.951	.091	-0	-0.000
140	-0	-0	-0	-0	-0	-0	51.473	8.645	2.174	-0	-0.000
141	-0	-0	-0	-0	-0	-0	52.551	4.339	1.779	-0	-0.000
142	141	141	141	141	141	141	52.551	4.339	-.699	-0	-0.000
143	-0	-0	-0	-0	-0	-0	53.634	0.000	1.384	-0	-0.000
144	-0	-0	-0	-0	-0	-0	50.000	34.500	4.704	-0	-0.000
145	-0	-0	-0	-0	-0	-0	51.085	30.151	4.308	-0	-0.000
146	145	145	145	145	145	145	51.085	30.151	1.830	-0	-0.000
147	-0	-0	-0	-0	-0	-0	52.163	25.850	3.912	-0	-0.000
148	-0	-0	-0	-0	-0	-0	53.240	21.549	3.506	-0	-0.000
149	148	148	148	148	148	148	53.240	21.549	1.028	-0	-0.000
150	-0	-0	-0	-0	-0	-0	54.317	17.250	3.120	-0	-0.000
151	-0	-0	-0	-0	-0	-0	55.394	12.951	2.725	-0	-0.000
152	151	151	151	151	151	151	55.394	12.951	.247	-0	-0.000
153	-0	-0	-0	-0	-0	-0	56.473	8.645	2.329	-0	-0.000
154	-0	-0	-0	-0	-0	-0	57.551	4.339	1.933	-0	-0.000
155	154	154	154	154	154	154	57.551	4.399	-.545	-0	-0.000
156	-0	-0	-0	-0	-0	-0	58.634	0.000	1.537	-0	-0.000
157	-0	-0	-0	-0	-0	-0	55.000	34.500	4.862	-0	-0.000
158	-0	-0	-0	-0	-0	-0	56.085	30.151	4.465	-0	-0.000
159	158	158	158	158	158	158	56.085	30.151	1.987	-0	-0.000
160	-0	-0	-0	-0	-0	-0	57.163	25.850	4.069	-0	-0.000
161	-0	-0	-0	-0	-0	-0	58.240	21.549	3.663	-0	-0.000
162	161	161	161	161	161	161	58.240	21.549	1.185	-0	-0.000
163	-0	-0	-0	-0	-0	-0	59.317	17.250	3.276	-0	-0.000
164	-0	-0	-0	-0	-0	-0	60.394	12.951	2.880	-0	-0.000
165	164	164	164	164	164	164	60.394	12.951	.402	-0	-0.000
166	-0	-0	-0	-0	-0	-0	61.473	8.645	2.484	-0	-0.000
167	-0	-0	-0	-0	-0	-0	62.551	4.339	2.088	-0	-0.000
168	167	167	167	167	167	167	62.551	4.399	-.390	-0	-0.000
169	-0	-0	-0	-0	-0	-0	63.634	0.000	1.691	-0	-0.000
170	-0	-0	-0	-0	-0	-0	60.000	34.500	5.021	-0	-0.000
171	-0	-0	-0	-0	-0	-0	61.085	30.151	4.623	-0	-0.000
172	171	171	171	171	171	171	61.085	30.151	2.145	-0	-0.000
173	-0	-0	-0	-0	-0	-0	62.163	25.850	4.226	-0	-0.000
174	-0	-0	-0	-0	-0	-0	63.240	21.549	3.819	-0	-0.000
175	174	174	174	174	174	174	63.240	21.549	1.431	-0	-0.000
176	-0	-0	-0	-0	-0	-0	64.317	17.250	3.432	-0	-0.000
177	-0	-0	-0	-0	-0	-0	65.394	12.951	3.035	-0	-0.000
178	177	177	177	177	177	177	65.394	12.951	.557	-0	-0.000
179	-0	-0	-0	-0	-0	-0	66.473	8.645	2.639	-0	-0.000
180	-0	-0	-0	-0	-0	-0	67.551	4.339	2.242	-0	-0.000
181	180	180	180	180	180	180	67.551	4.339	-.236	-0	-0.000
182	-0	-0	-0	-0	-0	-0	68.634	0.000	1.845	-0	-0.000
183	-0	-0	-0	-0	-0	-0	66.006	34.500	5.150	-0	-0.000
184	-0	1	1	-0	-0	-0	66.854	30.151	4.760	-0	-0.000
185	184	184	184	184	184	184	66.854	30.151	2.282	-0	-0.000
186	-0	-0	-0	-0	-0	-0	67.693	25.850	4.370	-0	-0.000
187	-0	1	1	-0	-0	-0	68.531	21.549	3.970	-0	-0.000
188	187	187	187	187	187	187	68.531	21.549	1.492	-0	-0.000
189	-0	-0	-0	-0	-0	-0	69.369	17.250	3.590	-0	-0.000
190	-0	1	1	-0	-0	-0	70.207	12.951	3.200	-0	-0.000
191	190	190	190	190	190	190	70.207	12.951	.722	-0	-0.000
192	-0	-0	-0	-0	-0	-0	71.054	8.645	2.810	-0	-0.000
193	-0	1	1	-0	-0	-0	71.886	4.339	2.420	-0	-0.000
194	193	193	193	193	193	193	71.886	4.339	-.058	-0	-0.000
195	-0	-0	-0	-0	-0	-0	72.732	0.000	2.030	-0	-0.000

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z	T
1	-0	-0	-0	-0	-0	-0	0.000	34.500	3.120	-0.000
2	1	1	1	-0	-0	-0	1.085	30.151	2.730	-0.000
3	2	2	2	2	2	2	1.085	30.151	.252	-0.000
4	-0	-0	-0	-0	-0	-0	2.163	25.850	2.340	-0.000
5	1	1	1	-0	-0	-0	3.240	21.549	1.940	-0.000
6	5	5	5	5	5	5	3.240	21.549	-.538	-0.000
7	-0	-0	-0	-0	-0	-0	4.317	17.250	1.560	-0.000
8	1	1	1	-0	-0	-0	5.394	12.951	1.170	-0.000
9	8	8	8	8	8	8	5.394	12.951	-1.308	-0.000
10	-0	-0	-0	-0	-0	-0	6.473	8.645	.780	-0.000
11	1	1	1	-0	-0	-0	7.551	4.339	.390	-0.000
12	11	11	11	11	11	11	7.551	4.339	-2.088	-0.000

Fig. 33 Continued.

13	-0	-0	-0	-0	-0	-0	8.634	0.000	0.000	-0.000
14	-0	-0	-0	-0	-0	-0	5.000	34.500	3.278	-0.000
15	-0	-0	-0	-0	-0	-0	6.085	30.151	2.888	-0.000
16	15	15	15	15	15	15	6.085	30.151	.410	-0.000
17	-0	-0	-0	-0	-0	-0	7.163	25.850	2.497	-0.000
18	-0	-0	-0	-0	-0	-0	8.240	21.549	2.097	-0.000
19	18	18	18	18	18	18	8.240	21.549	-.381	-0.000
20	-0	-0	-0	-0	-0	-0	9.317	17.250	1.716	-0.000
21	-0	-0	-0	-0	-0	-0	10.394	12.951	1.325	-0.000
22	21	21	21	21	21	21	10.394	12.951	-1.153	-0.000
23	-0	-0	-0	-0	-0	-0	11.473	8.645	.935	-0.000
24	-0	-0	-0	-0	-0	-0	12.551	4.339	.544	-0.000
25	24	24	24	24	24	24	12.551	4.339	-1.934	-0.000
26	-0	-0	-0	-0	-0	-0	13.634	0.000	.154	-0.000
27	-0	-0	-0	-0	-0	-0	10.000	34.500	3.437	-0.000
28	-0	-0	-0	-0	-0	-0	11.085	30.151	3.046	-0.000
29	28	28	28	28	28	28	11.085	30.151	.568	-0.000
30	-0	-0	-0	-0	-0	-0	12.163	25.850	2.654	-0.000
31	-0	-0	-0	-0	-0	-0	13.240	21.549	2.253	-0.000
32	31	31	31	31	31	31	13.240	21.549	-.225	-0.000
33	-0	-0	-0	-0	-0	-0	14.317	17.250	1.872	-0.000
34	-0	-0	-0	-0	-0	-0	15.394	12.951	1.481	-0.000
35	34	34	34	34	34	34	15.394	12.951	-.997	-0.000
36	-0	-0	-0	-0	-0	-0	16.473	8.645	1.090	-0.000
37	-0	-0	-0	-0	-0	-0	17.551	4.339	.699	-0.000
38	37	37	37	37	37	37	17.551	4.339	-1.779	-0.000
39	-0	-0	-0	-0	-0	-0	18.634	0.000	.307	-0.000
40	-0	-0	-0	-0	-0	-0	15.000	34.500	3.595	-0.000
41	-0	-0	-0	-0	-0	-0	16.085	30.151	3.203	-0.000
42	41	41	41	41	41	41	16.085	30.151	.725	-0.000
43	-0	-0	-0	-0	-0	-0	17.163	25.850	2.812	-0.000
44	-0	-0	-0	-0	-0	-0	18.240	21.549	2.410	-0.000
45	44	44	44	44	44	44	18.240	21.549	-.068	-0.000
46	-0	-0	-0	-0	-0	-0	19.317	17.250	2.028	-0.000
47	-0	-0	-0	-0	-0	-0	20.394	12.951	1.636	-0.000
48	47	47	47	47	47	47	20.394	12.951	-.842	-0.000
49	-0	-0	-0	-0	-0	-0	21.473	8.645	1.245	-0.000
50	-0	-0	-0	-0	-0	-0	22.551	4.339	.853	-0.000
51	50	50	50	50	50	50	22.551	4.339	-1.625	-0.000
52	-0	-0	-0	-0	-0	-0	23.634	0.000	4.461	-0.000
53	-0	-0	-0	-0	-0	-0	20.000	34.500	3.754	-0.000
54	-0	-0	-0	-0	-0	-0	21.085	30.151	3.361	-0.000
55	54	54	54	54	54	54	21.085	30.151	.883	-0.000
56	-0	-0	-0	-0	-0	-0	22.163	25.850	2.969	-0.000
57	-0	-0	-0	-0	-0	-0	23.240	21.549	2.566	-0.000
58	57	57	57	57	57	57	23.240	21.549	.088	-0.000
59	-0	-0	-0	-0	-0	-0	24.317	17.250	2.184	-0.000
60	-0	-0	-0	-0	-0	-0	25.394	12.951	1.792	-0.000
61	60	60	60	60	60	60	25.394	12.951	-.686	-0.000
62	-0	-0	-0	-0	-0	-0	26.473	8.645	1.400	-0.000
63	-0	-0	-0	-0	-0	-0	27.551	4.339	1.007	-0.000
64	63	63	63	63	63	63	27.551	4.339	-1.471	-0.000
65	-0	-0	-0	-0	-0	-0	28.634	0.000	.615	-0.000
66	-0	-0	-0	-0	-0	-0	25.000	34.500	3.912	-0.000
67	-0	-0	-0	-0	-0	-0	26.085	30.151	3.519	-0.000
68	67	67	67	67	67	67	26.085	30.151	1.041	-0.000
69	-0	-0	-0	-0	-0	-0	27.163	25.850	3.126	-0.000
70	-0	-0	-0	-0	-0	-0	28.240	21.549	2.723	-0.000
71	70	70	70	70	70	70	28.240	21.549	.245	-0.000
72	-0	-0	-0	-0	-0	-0	29.317	17.250	2.340	-0.000
73	-0	-0	-0	-0	-0	-0	30.394	12.951	1.947	-0.000
74	73	73	73	73	73	73	30.394	12.951	-.531	-0.000
75	-0	-0	-0	-0	-0	-0	31.473	8.645	1.554	-0.000
76	-0	-0	-0	-0	-0	-0	32.551	4.339	1.162	-0.000
77	76	76	76	76	76	76	32.551	4.339	-1.316	-0.000
78	-0	-0	-0	-0	-0	-0	33.634	0.000	.769	-0.000
79	-0	-0	-0	-0	-0	-0	30.000	34.500	4.070	-0.000
80	-0	-0	-0	-0	-0	-0	31.085	30.151	3.677	-0.000
81	80	80	80	80	80	80	31.085	30.151	1.199	-0.000
82	-0	-0	-0	-0	-0	-0	32.163	25.850	3.283	-0.000
83	-0	-0	-0	-0	-0	-0	33.240	21.549	2.880	-0.000
84	83	83	83	83	83	83	33.240	21.549	.402	-0.000
85	-0	-0	-0	-0	-0	-0	34.317	17.250	2.496	-0.000
86	-0	-0	-0	-0	-0	-0	35.394	12.951	2.103	-0.000
87	86	86	86	86	86	86	35.394	12.951	-.375	-0.000
88	-0	-0	-0	-0	-0	-0	36.473	8.645	1.709	-0.000
89	-0	-0	-0	-0	-0	-0	37.551	4.339	1.316	-0.000
90	89	89	89	89	89	89	37.551	4.339	-1.162	-0.000

Fig. 33 Continued.

91	-0	-0	-0	-0	-0	-0	38.634	0.000	.922	-0.000
92	-0	-0	-0	-0	-0	-0	32.320	34.500	4.143	-0.000
93	-0	-0	-0	-0	-0	-0	33.405	30.151	3.750	-0.000
94	93	93	93	93	93	93	33.405	30.151	1.272	-0.000
95	-0	-0	-0	-0	-0	-0	34.483	25.850	3.356	-0.000
96	-0	-0	-0	-0	-0	-0	35.560	21.549	2.952	-0.000
97	96	96	96	96	96	96	35.560	21.549	.474	-0.000
98	-0	-0	-0	-0	-0	-0	36.637	17.250	2.569	-0.000
99	-0	-0	-0	-0	-0	-0	37.714	12.951	2.775	-0.000
100	99	99	99	99	99	99	37.714	12.951	.297	-0.000
101	-0	-0	-0	-0	-0	-0	38.783	8.645	1.781	-0.000
102	-0	-0	-0	-0	-0	-0	39.871	4.339	1.388	-0.000
103	102	102	102	102	102	102	39.871	4.339	-1.090	-0.000
104	-0	-0	-0	-0	-0	-0	40.954	0.000	.994	-0.000
105	-0	-0	-0	-0	-0	-0	35.000	34.500	4.229	-0.000
106	-0	-0	-0	-0	-0	-0	36.085	30.151	3.834	-0.000
107	106	106	106	106	106	106	36.085	30.151	1.356	-0.000
108	-0	-0	-0	-0	-0	-0	37.163	25.850	3.440	-0.000
109	-0	-0	-0	-0	-0	-0	38.240	21.549	3.036	-0.000
110	109	109	109	109	109	109	38.240	21.549	.558	-0.000
111	-0	-0	-0	-0	-0	-0	39.317	17.250	2.652	-0.000
112	-0	-0	-0	-0	-0	-0	40.394	12.951	2.258	-0.000
113	112	112	112	112	112	112	40.394	12.951	-.220	-0.000
114	-0	-0	-0	-0	-0	-0	41.473	8.645	1.864	-0.000
115	-0	-0	-0	-0	-0	-0	42.551	4.339	1.470	-0.000
116	115	115	115	115	115	115	42.551	4.339	-1.008	-0.000
117	-0	-0	-0	-0	-0	-0	43.634	0.000	1.076	-0.000
118	-0	-0	-0	-0	-0	-0	40.000	34.500	4.387	-0.000
119	-0	-0	-0	-0	-0	-0	41.085	30.151	3.992	-0.000
120	119	119	119	119	119	119	41.085	30.151	1.514	-0.000
121	-0	-0	-0	-0	-0	-0	42.163	25.850	3.597	-0.000
122	-0	-0	-0	-0	-0	-0	43.240	21.549	3.193	-0.000
123	122	122	122	122	122	122	43.240	21.549	.715	-0.000
124	-0	-0	-0	-0	-0	-0	44.317	17.250	2.808	-0.000
125	-0	-0	-0	-0	-0	-0	45.394	12.951	2.414	-0.000
126	125	125	125	125	125	125	45.394	12.951	-.064	-0.000
127	-0	-0	-0	-0	-0	-0	46.473	8.645	2.019	-0.000
128	-0	-0	-0	-0	-0	-0	47.551	4.339	1.625	-0.000
129	128	128	128	128	128	128	47.551	4.339	-.853	-0.000
130	-0	-0	-0	-0	-0	-0	48.634	0.000	1.230	-0.000
131	-0	-0	-0	-0	-0	-0	45.000	34.500	4.545	-0.000
132	-0	-0	-0	-0	-0	-0	46.085	30.151	4.150	-0.000
133	132	132	132	132	132	132	46.085	30.151	1.672	-0.000
134	-0	-0	-0	-0	-0	-0	47.163	25.850	3.755	-0.000
135	-0	-0	-0	-0	-0	-0	48.240	21.549	3.349	-0.000
136	135	135	135	135	135	135	48.240	21.549	.871	-0.000
137	-0	-0	-0	-0	-0	-0	49.317	17.250	2.964	-0.000
138	-0	-0	-0	-0	-0	-0	50.394	12.951	2.569	-0.000
139	138	138	138	138	138	138	50.394	12.951	.091	-0.000
140	-0	-0	-0	-0	-0	-0	51.473	8.645	2.174	-0.000
141	-0	-0	-0	-0	-0	-0	52.551	4.339	1.779	-0.000
142	141	141	141	141	141	141	52.551	4.339	-.699	-0.000
143	-0	-0	-0	-0	-0	-0	53.634	0.000	1.384	-0.000
144	-0	-0	-0	-0	-0	-0	50.000	34.500	4.704	-0.000
145	-0	-0	-0	-0	-0	-0	51.085	30.151	4.308	-0.000
146	145	145	145	145	145	145	51.085	30.151	1.830	-0.000
147	-0	-0	-0	-0	-0	-0	52.163	25.850	3.912	-0.000
148	-0	-0	-0	-0	-0	-0	53.240	21.549	3.506	-0.000
149	148	148	148	148	148	148	53.240	21.549	1.028	-0.000
150	-0	-0	-0	-0	-0	-0	54.317	17.250	3.120	-0.000
151	-0	-0	-0	-0	-0	-0	55.394	12.951	2.725	-0.000
152	151	151	151	151	151	151	55.394	12.951	.247	-0.000
153	-0	-0	-0	-0	-0	-0	56.473	8.645	2.329	-0.000
154	-0	-0	-0	-0	-0	-0	57.551	4.339	1.933	-0.000
155	154	154	154	154	154	154	57.551	4.399	-.545	-0.000
156	-0	-0	-0	-0	-0	-0	58.634	0.000	1.537	-0.000
157	-0	-0	-0	-0	-0	-0	55.000	34.500	4.862	-0.000
158	-0	-0	-0	-0	-0	-0	56.085	30.151	4.465	-0.000
159	158	158	158	158	158	158	56.085	30.151	1.987	-0.000
160	-0	-0	-0	-0	-0	-0	57.163	25.850	4.069	-0.000
161	-0	-0	-0	-0	-0	-0	58.240	21.549	3.663	-0.000
162	161	161	161	161	161	161	58.240	21.549	1.185	-0.000
163	-0	-0	-0	-0	-0	-0	59.317	17.250	3.276	-0.000
164	-0	-0	-0	-0	-0	-0	60.394	12.951	2.880	-0.000
165	164	164	164	164	164	164	60.394	12.951	.402	-0.000
166	-0	-0	-0	-0	-0	-0	61.473	8.645	2.484	-0.000
167	-0	-0	-0	-0	-0	-0	62.551	4.339	2.088	-0.000

Fig. 33 Continued.

168	167	167	167	167	167	167	62.551	4.399	-.390	-0.000
169	-0	-0	-0	-0	-0	-0	63.634	0.000	1.691	-0.000
170	-0	-0	-0	-0	-0	-0	60.000	34.500	5.021	-0.000
171	-0	-0	-0	-0	-0	-0	61.085	30.151	4.623	-0.000
172	171	171	171	171	171	171	61.085	30.151	2.145	-0.000
173	-0	-0	-0	-0	-0	-0	62.163	25.850	4.226	-0.000
174	-0	-0	-0	-0	-0	-0	63.240	21.549	3.819	-0.000
175	174	174	174	174	174	174	63.240	21.549	1.431	-0.000
176	-0	-0	-0	-0	-0	-0	64.317	17.250	3.432	-0.000
177	-0	-0	-0	-0	-0	-0	65.394	12.951	3.035	-0.000
178	177	177	177	177	177	177	65.394	12.951	.557	-0.000
179	-0	-0	-0	-0	-0	-0	66.473	8.645	2.639	-0.000
180	-0	-0	-0	-0	-0	-0	67.551	4.339	2.242	-0.000
181	180	180	180	180	180	180	67.551	4.339	-.236	-0.000
182	-0	-0	-0	-0	-0	-0	68.634	0.000	1.845	-0.000
183	-0	-0	-0	-0	-0	-0	66.006	34.500	5.150	-0.000
184	-0	1	1	-0	-0	-0	66.854	30.151	4.760	-0.000
185	184	184	184	184	184	184	66.854	30.151	2.282	-0.000
186	-0	-0	-0	-0	-0	-0	67.693	25.850	4.370	-0.000
187	-0	1	1	-0	-0	-0	68.531	21.549	3.970	-0.000
188	187	187	187	187	187	187	68.531	21.549	1.492	-0.000
189	-0	-0	-0	-0	-0	-0	69.369	17.250	3.590	-0.000
190	-0	1	1	-0	-0	-0	70.207	12.951	3.200	-0.000
191	190	190	190	190	190	190	70.207	12.951	.722	-0.000
192	-0	-0	-0	-0	-0	-0	71.054	8.645	2.810	-0.000
193	-0	1	1	-0	-0	-0	71.886	4.339	2.420	-0.000
194	193	193	193	193	193	193	71.886	4.339	-.058	-0.000
195	-0	-0	-0	-0	-0	-0	72.732	0.000	2.030	-0.000

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	1	2	3	4	5	6
2	0	0	0	7	8	9
3	-2	-2	-2	-2	-2	-2
4	10	11	12	13	14	15
5	0	0	0	16	17	18
6	-5	-5	-5	-5	-5	-5
7	19	20	21	22	23	24
8	0	0	0	25	26	27
9	-8	-8	-8	-8	-8	-8
10	28	29	30	31	32	33
11	0	0	0	34	35	36
12	-11	-11	-11	-11	-11	-11
13	37	38	39	40	41	42
14	43	44	45	46	47	48
15	49	50	51	52	53	54
16	-15	-15	-15	-15	-15	-15
17	55	56	57	58	59	60
18	61	62	63	64	65	66
19	-18	-18	-18	-18	-18	-18
20	67	68	69	70	71	72
21	73	74	75	76	77	78
22	-21	-21	-21	-21	-21	-21
23	79	80	81	82	83	84
24	85	86	87	88	89	90
25	-24	-24	-24	-24	-24	-24
26	91	92	93	94	95	96
27	97	98	99	100	101	102
28	103	104	105	106	107	108
29	-28	-28	-28	-28	-28	-28
30	109	110	111	112	113	114
31	115	116	117	118	119	120
32	-31	-31	-31	-31	-31	-31
33	121	122	123	124	125	126
34	127	128	129	130	131	132
35	-34	-34	-34	-34	-34	-34
36	133	134	135	136	137	138
37	139	140	141	142	143	144
38	-37	-37	-37	-37	-37	-37
39	145	146	147	148	149	150
40	151	152	153	154	155	156
41	157	158	159	160	161	162
42	-41	-41	-41	-41	-41	-41
43	163	164	165	166	167	168
44	169	170	171	172	173	174
45	-44	-44	-44	-44	-44	-44
46	175	176	177	178	179	180

Fig. 33 Continued.

47	181	182	183	184	185	186
48	-47	-47	-47	-47	-47	-47
49	187	188	189	190	191	192
50	193	194	195	196	197	198
51	-50	-50	-50	-50	-50	-50
52	199	200	201	202	203	204
53	205	206	207	208	209	210
54	211	212	213	214	215	216
55	-54	-54	-54	-54	-54	-54
56	217	218	219	220	221	222
57	223	224	225	226	227	228
58	-57	-57	-57	-57	-57	-57
59	229	230	231	232	233	234
60	235	236	237	238	239	240
61	-60	-60	-60	-60	-60	-60
62	241	242	243	244	245	246
63	247	248	249	250	251	252
64	-63	-63	-63	-63	-63	-63
65	253	254	255	256	257	258
66	259	260	261	262	263	264
67	265	266	267	268	269	270
68	-67	-67	-67	-67	-67	-67
69	271	272	273	274	275	276
70	277	278	279	280	281	282
71	-70	-70	-70	-70	-70	-70
72	283	284	285	286	287	288
73	289	290	291	292	293	294
74	-73	-73	-73	-73	-73	-73
75	295	296	297	298	299	300
76	301	302	303	304	305	306
77	-76	-76	-76	-76	-76	-76
78	307	308	309	310	311	312
79	313	314	315	316	317	318
80	319	320	321	322	323	324
81	-80	-80	-80	-80	-80	-80
82	325	326	327	328	329	330
83	331	332	333	334	335	336
84	-83	-83	-83	-83	-83	-83
85	337	338	339	340	341	342
86	343	344	345	346	347	348
87	-86	-86	-86	-86	-86	-86
88	349	350	351	352	353	354
89	355	356	357	358	359	360
90	-89	-89	-89	-89	-89	-89
91	361	362	363	364	365	366
92	367	368	369	370	371	372
93	373	374	375	376	377	378
94	-93	-93	-93	-93	-93	-93
95	379	380	381	382	383	384
96	385	386	387	388	389	390
97	-96	-96	-96	-96	-96	-96
98	391	392	393	394	395	396
99	397	398	399	400	401	402
100	-99	-99	-99	-99	-99	-99
101	403	404	405	406	407	408
102	409	410	411	412	413	414
103	-102	-102	-102	-102	-102	-102
104	415	416	417	418	419	420
105	421	422	423	424	425	426
106	427	428	429	430	431	432
107	-106	-106	-106	-106	-106	-106
108	433	434	435	436	437	438
109	439	440	441	442	443	444
110	-109	-109	-109	-109	-109	-109
111	445	446	447	448	449	450
112	451	452	453	454	455	456
113	-112	-112	-112	-112	-112	-112
114	457	458	459	460	461	462
115	463	464	465	466	467	468
116	-115	-115	-115	-115	-115	-115
117	469	470	471	472	473	474
118	475	476	477	478	479	480
119	481	482	483	484	485	486
120	-119	-119	-119	-119	-119	-119
121	487	488	489	490	491	492
122	493	494	495	496	497	498
123	-122	-122	-122	-122	-122	-122
124	499	500	501	502	503	504
125	505	506	507	508	509	510
126	-125	-125	-125	-125	-125	-125
127	511	512	513	514	515	516
128	517	518	519	520	521	522
129	-128	-128	-128	-128	-128	-128
130	523	524	525	526	527	528
131	529	530	531	532	533	534
132	535	536	537	538	539	540
133	-132	-132	-132	-132	-132	-132
134	541	542	543	544	545	546
135	547	548	549	550	551	552
136	-135	-135	-135	-135	-135	-135
137	553	554	555	556	557	558
138	559	560	561	562	563	564
139	-138	-138	-138	-138	-138	-138
140	565	566	567	568	569	570
141	571	572	573	574	575	576
142	-141	-141	-141	-141	-141	-141
143	577	578	579	580	581	582
144	583	584	585	586	587	588
145	589	590	591	592	593	594
146	-145	-145	-145	-145	-145	-145
147	595	596	597	598	599	600
148	601	602	603	604	605	606
149	-148	-148	-148	-148	-148	-148
150	607	608	609	610	611	612
151	613	614	615	616	617	618
152	-151	-151	-151	-151	-151	-151
153	619	620	621	622	623	624
154	625	626	627	628	629	630
155	-154	-154	-154	-154	-154	-154
156	631	632	633	634	635	636
157	637	638	639	640	641	642
158	643	644	645	646	647	648
159	-158	-158	-158	-158	-158	-158
160	649	650	651	652	653	654
161	655	656	657	658	659	660
162	-161	-161	-161	-161	-161	-161
163	661	662	663	664	665	666
164	667	668	669	670	671	672
165	-164	-164	-164	-164	-164	-164
166	673	674	675	676	677	678
167	679	680	681	682	683	684
168	-167	-167	-167	-167	-167	-167
169	685	686	687	688	689	690
170	691	692	693	694	695	696
171	697	698	699	700	701	702
172	-171	-171	-171	-171	-171	-171
173	703	704	705	706	707	708
174	709	710	711	712	713	714
175	-174	-174	-174	-174	-174	-174
176	715	716	717	718	719	720
177	721	722	723	724	725	726
178	-177	-177	-177	-177	-177	-177
179	727	728	729	730	731	732
180	733	734	735	736	737	738
181	-180	-180	-180	-180	-180	-180
182	739	740	741	742	743	744
183	745	746	747	748	749	750
184	751	0	0	752	753	754
185	-184	-184	-184	-184	-184	-184
186	755	756	757	758	759	760
187	761	0	0	762	763	764
188	-187	-187	-187	-187	-187	-187
189	765	766	767	768	769	770
190	771	0	0	772	773	774
191	-190	-190	-190	-190	-190	-190
192	775	776	777	778	779	780
193	781	0	0	782	783	784
194	-193	-193	-193	-193	-193	-193
195	785	786	787	788	789	790

Fig. 33 Continued.

.......THREE DIMENSIONAL BEAM ELEMENTS

NUMBER OF BEAMS = 62
NUMBER OF GEOMETRIC PROPERTY SETS= 3
NUMBER OF FIXED END FORCE SETS = 0
NUMBER OF MATERIALS = 1

MATERIAL	YOUNG S MODULUS	POISSON S RATIO	MASS DENSITY
1	772307.	.15000	-0.00000

BEAM GEOMETRIC PROPERTIES

ELEMENT TYPE	AREA X	AREA Y	AREA Z	INERTIA X	INERTIA Y	INERTIA Z
1	3.885	3.885	-0.000	.329	.589	6.316
2	2.063	2.063	-0.000	.320	.097	1.300
3	2.938	2.938	-0.000	.484	.138	3.755

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	-0.	-0.	-0.	-0.
Y-DIR	-0.	-0.	-0.	-0.
Z-DIR	-0.	-0.	-0.	-0.

BEAM NO	NODES I	J	K	MATL NO	GEOM NO	ELEM LOADS A	B	C	D	END CODES I	J
1	3	16	2	1	1	-0	-0	-0	-0	-0	-0
2	6	19	5	1	1	-0	-0	-0	-0	-0	-0
3	9	22	8	1	1	-0	-0	-0	-0	-0	-0
4	12	25	11	1	1	-0	-0	-0	-0	-0	-0
5	16	29	15	1	1	-0	-0	-0	-0	-0	-0
6	19	32	18	1	1	-0	-0	-0	-0	-0	-0
7	22	35	21	1	1	-0	-0	-0	-0	-0	-0
8	25	38	24	1	1	-0	-0	-0	-0	-0	-0
9	29	42	28	1	1	-0	-0	-0	-0	-0	-0
10	32	45	31	1	1	-0	-0	-0	-0	-0	-0
11	35	48	34	1	1	-0	-0	-0	-0	-0	-0
12	38	51	37	1	1	-0	-0	-0	-0	-0	-0
13	42	55	41	1	1	-0	-0	-0	-0	-0	-0
14	45	58	44	1	1	-0	-0	-0	-0	-0	-0
15	48	61	47	1	1	-0	-0	-0	-0	-0	-0
16	51	64	50	1	1	-0	-0	-0	-0	-0	-0
17	55	68	54	1	1	-0	-0	-0	-0	-0	-0
18	58	71	57	1	1	-0	-0	-0	-0	-0	-0
19	61	74	60	1	1	-0	-0	-0	-0	-0	-0
20	64	77	63	1	1	-0	-0	-0	-0	-0	-0
21	68	81	67	1	1	-0	-0	-0	-0	-0	-0
22	71	84	70	1	1	-0	-0	-0	-0	-0	-0
23	74	87	73	1	1	-0	-0	-0	-0	-0	-0
24	77	90	76	1	1	-0	-0	-0	-0	-0	-0
25	81	94	80	1	1	-0	-0	-0	-0	-0	-0
26	84	97	83	1	1	-0	-0	-0	-0	-0	-0
27	87	100	86	1	1	-0	-0	-0	-0	-0	-0
28	90	103	89	1	1	-0	-0	-0	-0	-0	-0
29	94	107	93	1	1	-0	-0	-0	-0	-0	-0
30	97	110	96	1	1	-0	-0	-0	-0	-0	-0
31	100	113	99	1	1	-0	-0	-0	-0	-0	-0
32	103	116	102	1	1	-0	-0	-0	-0	-0	-0
33	107	120	106	1	1	-0	-0	-0	-0	-0	-0
34	110	123	109	1	1	-0	-0	-0	-0	-0	-0
35	113	126	112	1	1	-0	-0	-0	-0	-0	-0
36	116	129	115	1	1	-0	-0	-0	-0	-0	-0
37	120	133	119	1	1	-0	-0	-0	-0	-0	-0
38	123	136	122	1	1	-0	-0	-0	-0	-0	-0
39	126	139	125	1	1	-0	-0	-0	-0	-0	-0
40	129	142	128	1	1	-0	-0	-0	-0	-0	-0
41	133	146	132	1	1	-0	-0	-0	-0	-0	-0
42	136	149	135	1	1	-0	-0	-0	-0	-0	-0
43	139	152	138	1	1	-0	-0	-0	-0	-0	-0
44	142	155	141	1	1	-0	-0	-0	-0	-0	-0
45	146	159	145	1	1	-0	-0	-0	-0	-0	-0

Fig. 33 Continued.

46	149	162	148	1	1	-0	-0	-0	-0	-0	-0
47	152	165	151	1	1	-0	-0	-0	-0	-0	-0
48	155	168	154	1	1	-0	-0	-0	-0	-0	-0
49	159	172	158	1	1	-0	-0	-0	-0	-0	-0
50	162	175	161	1	1	-0	-0	-0	-0	-0	-0
51	165	178	164	1	1	-0	-0	-0	-0	-0	-0
52	168	181	167	1	1	-0	-0	-0	-0	-0	-0
53	172	185	171	1	1	-0	-0	-0	-0	-0	-0
54	175	188	174	1	1	-0	-0	-0	-0	-0	-0
55	178	191	177	1	1	-0	-0	-0	-0	-0	-0
56	181	194	180	1	1	-0	-0	-0	-0	-0	-0
57	6	3	5	1	3	-0	-0	-0	-0	-0	-0
58	9	6	8	1	3	-0	-0	-0	-0	-0	-0
59	12	9	11	1	3	-0	-0	-0	-0	-0	-0
60	188	185	187	1	2	-0	-0	-0	-0	-0	-0
61	191	188	190	1	2	-0	-0	-0	-0	-0	-0
62	194	191	193	1	2	-0	-0	-0	-0	-0	-0

...THIN ELASTIC SHELL ELEMENTS...

ELEMENT TYPE............ 6

NUMBER OF ELEMENTS...... 112

NUMBER OF MATERIALS..... 1

...MATERIAL PROPERTY INFORMATION...

TYPE	MASS DENSITY	THERMAL EXPANSION COEFFICIENTS AX	AY	AZ
1	-0.	-0.	-0.	-0.

...ELASTIC COEFFICIENTS... CXX	CXY	CXG	CYY	CYG	GXY
.6285E+06	.9428E+05*	.	.6285E+06*	.	.2671E+06

...ELEMENT LOAD MULTIPLIERS...

LOAD CASE	LATERAL LOAD	TEMPERATURE	...GRAVITY ACCELERATION... X-COMPONENT	Y-COMPONENT	Z-COMPONENT
1	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000
2	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000
3	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000
4	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000

ELEMENT DATA

ELEMENT NUMBER	NODAL POINT NUMBERS I	J	K	L	O	MATL TYPE	THICKNESS	PRESSURE	TEMPERATURE	TEMPERATURE GRADIENT
1	2	15	14	1	-0	1	.5833	0.0000	0.0000	0.0000
2	4	17	15	2	-0	1	.5833	0.0000	0.0000	0.0000
3	5	18	17	4	-0	1	.5833	0.0000	0.0000	0.0000
4	7	20	18	5	-0	1	.5833	0.0000	0.0000	0.0000
5	8	21	20	7	-0	1	.5833	0.0000	0.0000	0.0000
6	10	23	21	8	-0	1	.5833	0.0000	0.0000	0.0000
7	11	24	23	10	-0	1	.5833	0.0000	0.0000	0.0000
8	13	26	24	11	-0	1	.5833	0.0000	0.0000	0.0000
9	15	28	27	14	-0	1	.5833	0.0000	0.0000	0.0000
10	17	30	28	15	-0	1	.5833	0.0000	0.0000	0.0000
11	18	31	30	17	-0	1	.5833	0.0000	0.0000	0.0000
12	20	33	31	18	-0	1	.5833	0.0000	0.0000	0.0000
13	21	34	33	20	-0	1	.5833	0.0000	0.0000	0.0000
14	23	36	34	21	-0	1	.5833	0.0000	0.0000	0.0000

Fig. 33 Continued.

15	24	37	36	23	-0	1	.5833	0.0000	0.0000	0.0000
16	26	39	37	24	-0	1	.5833	0.0000	0.0000	0.0000
17	28	41	40	27	-0	1	.5833	0.0000	0.0000	0.0000
18	30	43	41	28	-0	1	.5833	0.0000	0.0000	0.0000
19	31	44	43	30	-0	1	.5833	0.0000	0.0000	0.0000
20	33	46	44	31	-0	1	.5833	0.0000	0.0000	0.0000
21	34	47	46	33	-0	1	.5833	0.0000	0.0000	0.0000
22	36	49	47	34	-0	1	.5833	0.0000	0.0000	0.0000
23	37	50	49	36	-0	1	.5833	0.0000	0.0000	0.0000
24	39	52	50	37	-0	1	.5833	0.0000	0.0000	0.0000
25	41	54	53	40	-0	1	.5833	0.0000	0.0000	0.0000
26	43	56	54	41	-0	1	.5833	0.0000	0.0000	0.0000
27	44	57	56	43	-0	1	.5833	0.0000	0.0000	0.0000
28	46	59	57	44	-0	1	.5833	0.0000	0.0000	0.0000
29	47	60	59	46	-0	1	.5833	0.0000	0.0000	0.0000
30	49	62	60	47	-0	1	.5833	0.0000	0.0000	0.0000
31	50	63	62	49	-0	1	.5833	0.0000	0.0000	0.0000
32	52	65	63	50	-0	1	.5833	0.0000	0.0000	0.0000
33	54	67	66	53	-0	1	.5833	0.0000	0.0000	0.0000
34	56	69	67	54	-0	1	.5833	0.0000	0.0000	0.0000
35	57	70	69	56	-0	1	.5833	0.0000	0.0000	0.0000
36	59	72	70	57	-0	1	.5833	0.0000	0.0000	0.0000
37	60	73	72	59	-0	1	.5833	0.0000	0.0000	0.0000
38	62	75	73	60	-0	1	.5833	0.0000	0.0000	0.0000
39	63	76	75	62	-0	1	.5833	0.0000	0.0000	0.0000
40	65	78	76	63	-0	1	.5833	0.0000	0.0000	0.0000
41	67	80	79	66	-0	1	.5833	0.0000	0.0000	0.0000
42	69	82	80	67	-0	1	.5833	0.0000	0.0000	0.0000
43	70	83	82	69	-0	1	.5833	0.0000	0.0000	0.0000
44	72	85	83	70	-0	1	.5833	0.0000	0.0000	0.0000
45	73	86	85	72	-0	1	.5833	0.0000	0.0000	0.0000
46	75	88	86	73	-0	1	.5833	0.0000	0.0000	0.0000
47	76	89	88	75	-0	1	.5833	0.0000	0.0000	0.0000
48	78	91	89	76	-0	1	.5833	0.0000	0.0000	0.0000
49	80	93	92	79	-0	1	.5833	0.0000	0.0000	0.0000
50	82	95	93	80	-0	1	.5833	0.0000	0.0000	0.0000
51	83	96	95	82	-0	1	.5833	0.0000	0.0000	0.0000
52	85	98	96	83	-0	1	.5833	0.0000	0.0000	0.0000
53	86	99	98	85	-0	1	.5833	0.0000	0.0000	0.0000
54	88	101	99	86	-0	1	.5833	0.0000	0.0000	0.0000
55	89	102	101	88	-0	1	.5833	0.0000	0.0000	0.0000
56	91	104	102	89	-0	1	.5833	0.0000	0.0000	0.0000
57	93	106	105	92	-0	1	.5833	0.0000	0.0000	0.0000
58	95	108	106	93	-0	1	.5833	0.0000	0.0000	0.0000
59	96	109	108	95	-0	1	.5833	0.0000	0.0000	0.0000
60	98	111	109	96	-0	1	.5833	0.0000	0.0000	0.0000
61	99	112	111	98	-0	1	.5833	0.0000	0.0000	0.0000
62	101	114	112	99	-0	1	.5833	0.0000	0.0000	0.0000
63	102	115	114	101	-0	1	.5833	0.0000	0.0000	0.0000
64	104	117	115	102	-0	1	.5833	0.0000	0.0000	0.0000
65	106	119	118	105	-0	1	.5833	0.0000	0.0000	0.0000
66	108	121	119	106	-0	1	.5833	0.0000	0.0000	0.0000
67	109	122	121	108	-0	1	.5833	0.0000	0.0000	0.0000
68	111	124	122	109	-0	1	.5833	0.0000	0.0000	0.0000
69	112	125	124	111	-0	1	.5833	0.0000	0.0000	0.0000
70	114	127	125	112	-0	1	.5833	0.0000	0.0000	0.0000
71	115	128	127	114	-0	1	.5833	0.0000	0.0000	0.0000
72	117	130	128	115	-0	1	.5833	0.0000	0.0000	0.0000
73	119	132	131	118	-0	1	.5833	0.0000	0.0000	0.0000
74	121	134	132	119	-0	1	.5833	0.0000	0.0000	0.0000
75	122	135	134	121	-0	1	.5833	0.0000	0.0000	0.0000
76	124	137	135	122	-0	1	.5833	0.0000	0.0000	0.0000
77	125	138	137	124	-0	1	.5833	0.0000	0.0000	0.0000
78	127	140	138	125	-0	1	.5833	0.0000	0.0000	0.0000
79	128	141	140	127	-0	1	.5833	0.0000	0.0000	0.0000
80	130	143	141	128	-0	1	.5833	0.0000	0.0000	0.0000
81	132	145	144	131	-0	1	.5833	0.0000	0.0000	0.0000
82	134	147	145	132	-0	1	.5833	0.0000	0.0000	0.0000
83	135	148	147	134	-0	1	.5833	0.0000	0.0000	0.0000
84	137	150	148	135	-0	1	.5833	0.0000	0.0000	0.0000
85	138	151	150	137	-0	1	.5833	0.0000	0.0000	0.0000
86	140	153	151	138	-0	1	.5833	0.0000	0.0000	0.0000
87	141	154	153	140	-0	1	.5833	0.0000	0.0000	0.0000
88	143	156	154	141	-0	1	.5833	0.0000	0.0000	0.0000
89	145	158	157	144	-0	1	.5833	0.0000	0.0000	0.0000
90	147	160	158	145	-0	1	.5833	0.0000	0.0000	0.0000
91	148	161	160	147	-0	1	.5833	0.0000	0.0000	0.0000
92	150	163	161	148	-0	1	.5833	0.0000	0.0000	0.0000

Fig. 33 Continued.

93	151	164	163	150	-0	1	.5833	0.0000	0.0000	0.0000
94	153	166	164	151	-0	1	.5833	0.0000	0.0000	0.0000
95	154	167	166	153	-0	1	.5833	0.0000	0.0000	0.0000
96	156	169	167	154	-0	1	.5833	0.0000	0.0000	0.0000
97	158	171	170	157	-0	1	.5833	0.0000	0.0000	0.0000
98	160	173	171	158	-0	1	.5833	0.0000	0.0000	0.0000
99	161	174	173	160	-0	1	.5833	0.0000	0.0000	0.0000
100	163	176	174	161	-0	1	.5833	0.0000	0.0000	0.0000
101	164	177	176	163	-0	1	.5833	0.0000	0.0000	0.0000
102	166	179	177	164	-0	1	.5833	0.0000	0.0000	0.0000
103	167	180	179	166	-0	1	.5833	0.0000	0.0000	0.0000
104	169	182	180	167	-0	1	.5833	0.0000	0.0000	0.0000
105	171	184	183	170	-0	1	.5833	0.0000	0.0000	0.0000
106	173	186	184	171	-0	1	.5833	0.0000	0.0000	0.0000
107	174	187	186	173	-0	1	.5833	0.0000	0.0000	0.0000
108	176	189	187	174	-0	1	.5833	0.0000	0.0000	0.0000
109	177	190	189	176	-0	1	.5833	0.0000	0.0000	0.0000
110	179	192	190	177	-0	1	.5833	0.0000	0.0000	0.0000
111	180	193	192	179	-0	1	.5833	0.0000	0.0000	0.0000
112	182	195	193	180	-0	1	.5833	0.0000	0.0000	0.0000

```
TOTAL NUMBER OF EQUATIONS      =  790
BANDWIOTH                      =   66
NUMBER OF EQUATIONS IN A BLOCK =   51
NUMBER OF BLOCKS               =   16
```

.....NOOAL POINT LOADS

NOOE NO.	LOAD CASE	APPLIED LOADS RX	RY	RZ	MX	MY	MZ
54	1	-0.000	-0.000	-26.334	-0.000	-0.000	-0.000
57	1	-0.000	-0.000	-73.542	-0.000	-0.000	-0.000
60	1	-0.000	-0.000	-80.712	-0.000	-0.000	-0.000
63	1	-0.000	-0.000	-23.298	-0.000	-0.000	-0.000
67	1	-0.000	-0.000	-17.556	-0.000	-0.000	-0.000
70	1	-0.000	-0.000	-49.028	-0.000	-0.000	-0.000
73	1	-0.000	-0.000	-53.808	-0.000	-0.000	-0.000
76	1	-0.000	-0.000	-15.532	-0.000	-0.000	-0.000
106	1	-0.000	-0.000	-35.112	-0.000	-0.000	-0.000
109	1	-0.000	-0.000	-98.056	-0.000	-0.000	-0.000
112	1	-0.000	-0.000	-107.616	-0.000	-0.000	-0.000
115	1	-0.000	-0.000	-31.064	-0.000	-0.000	-0.000
119	1	-0.000	-0.000	-8.778	-0.000	-0.000	-0.000
122	1	-0.000	-0.000	-24.514	-0.000	-0.000	-0.000
125	1	-0.000	-0.000	-26.904	-0.000	-0.000	-0.000
128	1	-0.000	-0.000	-7.766	-0.000	-0.000	-0.000

Fig. 33 Continued.

6. Output

Figure 34 is a listing of the output data for bridge #2. Total solution time on a CDC 6400 was 145.24 seconds.

7. Correlation of Computer and Experimental Data for Bridge #2

Considerable speculation is inferred in the author's comparison of the computer analysis and experimental data for bridge #2, since some of the experimental information for this bridge was not as fully reported in reference (1) as for the other bridges. First of all, it is noticed that the loading coefficients - Table 3 - were reconstructed from the load deflection curve of Figure 36 and the known numbering of load points of Figure 8. Arbitrarily the author selected a total load of 680 kips for input into the computer. As for bridge #1, a moment distribution was used to translate the loading to points on the girders. Secondly, the load deflection curve for the exterior beam was missing. However, Figure 37 does show an average measured deflection curve which was obtained from the average of the deflection readings over the four girders over the centerline of each span. By the use of Figures 36 and 37 we may thus infer an exterior beam deflection for a bridge load of 430 kips to be 0.035 feet. The 430 total kips is considered to be a maximum loading for which a linear relationship of load to deflection is reasonably justified. Figure 35 shows the variation of deck distortion with loads across the bridge.

Table 9 shows the comparison of computer and experimental results for a 430 kip total bridge load. From the examination of these results, it is thus concluded that at least for a linear load deflection relationship the SSAP2 program would predict the responses of this skew bridge rather well. Overprinted on Figures 36 and 37 with a ⊚ are the results of the computer program. Once again note the SSAP2 results modeling the bridge as a grid as compared to a computer analysis modeling the bridge as a single beam.

.......NODE DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
195	1	-7.291E-03	-1.590E-05	1.086E-03	2.19E-04	-1.64E-03	-3.88E-03
194	1	0.	0.	0.	0.	0.	0.
193	1	-7.398E-03	0.	0.	4.29E-04	-2.30E-03	-3.99E-06
192	1	-7.362E-03	8.136E-05	-8.327E-05	2.69E-04	-3.68E-03	7.66E-03
191	1	0.	0.	0.	0.	0.	0.
190	1	-7.593E-03	0.	0.	6.36E-04	-3.38E-03	-2.46E-04
189	1	-7.374E-03	1.883E-05	-1.783E-04	3.71E-05	-5.79E-03	2.20E-02
188	1	0.	0.	0.	0.	0.	0.
187	1	-7.376E-03	0.	0.	6.75E-04	-3.43E-03	-4.00E-04
186	1	-6.953E-03	-6.038E-05	-1.534E-04	2.09E-04	-4.71E-03	1.56E-02
185	1	0.	0.	0.	0.	0.	0.
184	1	-6.753E-03	0.	0.	5.32E-04	-2.38E-03	-5.06E-04
183	1	-6.458E-03	-1.833E-04	1.873E-03	2.40E-05	-6.49E-03	4.64E-02
182	1	-6.890E-03	7.181E-04	-7.105E-03	-1.52E-04	-1.77E-03	-1.44E-03
181	1	0.	0.	0.	0.	0.	0.
180	1	-6.924E-03	1.048E-03	-1.013E-02	-1.22E-04	-2.26E-03	1.40E-04
179	1	-6.810E-03	1.383E-03	-1.350E-02	-1.50E-04	-3.19E-03	1.97E-03
178	1	0.	0.	0.	0.	0.	0.
177	1	-6.967E-03	1.709E-03	-1.665E-02	3.02E-04	-3.33E-03	-1.17E-04
176	1	-6.730E-03	1.883E-03	-1.836E-02	1.50E-03	-1.11E-03	-2.58E-02
175	1	0.	0.	0.	0.	0.	0.
174	1	-6.761E-03	1.915E-03	-1.853E-02	9.77E-04	-3.34E-03	-5.16E-04
173	1	-6.402E-03	1.787E-03	-1.691E-02	1.38E-03	-2.94E-03	-7.15E-04
172	1	0.	0.	0.	0.	0.	0.
171	1	-6.299E-03	1.523E-03	-1.392E-02	1.31E-03	-2.29E-03	-5.89E-04
170	1	-6.078E-03	1.201E-03	-9.928E-03	1.49E-03	-1.97E-03	1.18E-03
169	1	-6.427E-03	1.550E-03	-1.576E-02	2.43E-03	8.54E-03	-1.02E-01
168	1	0.	0.	0.	0.	0.	0.
167	1	-6.367E-03	2.126E-03	-2.133E-02	-9.19E-04	-2.10E-03	1.65E-04

FIG. 34 - COMPUTER RESULTS FOR BRIDGE #2

166	1	-6.159E-03	2.798E-03	-2.777E-02	-4.15E-03	-1.35E-02	1.08E-01
165	1	0.	0.	0.	0.	0.	0.
164	1	-6.233E-03	3.381E-03	-3.331E-02	-1.70E-04	-3.12E-03	-8.84E-05
163	1	-5.947E-03	3.679E-03	-3.587E-02	7.02E-04	-3.26E-03	-1.97E-03
162	1	0.	0.	0.	0.	0.	0.
161	1	-5.978E-03	3.677E-03	-3.518E-02	1.35E-03	-3.10E-03	-4.70E-04
160	1	-5.681E-03	3.331E-03	-3.118E-02	2.21E-03	-1.96E-03	-8.16E-03
159	1	0.	0.	0.	0.	0.	0.
158	1	-5.661E-03	2.814E-03	-2.522E-02	1.99E-03	-2.09E-03	-4.89E-04
157	1	-5.523E-03	2.240E-03	-1.858E-02	3.86E-03	4.13E-03	-5.81E-02
156	1	-5.852E-03	2.215E-03	-2.312E-02	-8.34E-03	-2.37E-02	2.25E-01
155	1	0.	0.	0.	0.	0.	0.
154	1	-5.691E-03	3.082E-03	-3.144E-02	-1.65E-03	-1.83E-03	8.65E-05
153	1	-5.385E-03	4.059E-03	-4.073E-02	7.83E-03	2.77E-02	-3.02E-01
152	1	0.	0.	0.	0.	0.	0.
151	1	-5.374E-03	4.903E-03	-4.857E-02	-6.48E-04	-2.76E-03	-7.35E-05
150	1	-5.033E-03	5.317E-03	-5.189E-02	5.66E-04	-3.08E-03	8.62E-05
149	1	0.	0.	0.	0.	0.	0.
148	1	-5.082E-03	5.259E-03	-5.025E-02	1.67E-03	-2.70E-03	-3.86E-04
147	1	-4.845E-03	4.712E-03	-4.396E-02	2.89E-03	-1.09E-03	-1.30E-02
146	1	0.	0.	0.	0.	0.	0.
145	1	-4.911E-03	3.943E-03	-3.525E-02	2.54E-03	-1.79E-03	-3.66E-04
144	1	-4.858E-03	3.140E-03	-2.613E-02	3.82E-04	-8.01E-03	6.68E-02
143	1	-5.163E-03	2.747E-03	-2.934E-02	8.82E-04	8.21E-03	-9.47E-02
142	1	0.	0.	0.	0.	0.	0.
141	1	-4.915E-03	3.858E-03	-4.000E-02	-2.24E-03	-1.47E-03	8.90E-05
140	1	-4.507E-03	5.108E-03	-5.179E-02	-4.53E-03	-9.36E-03	7.37E-02
139	1	0.	0.	0.	0.	0.	0.
138	1	-4.403E-03	6.188E-03	-6.165E-02	-1.10E-03	-2.24E-03	-5.23E-05
137	1	-4.015E-03	6.700E-03	-6.555E-02	6.57E-04	-1.93E-03	-5.79E-03

Fig. 34 Continued.

136	1	0.	0.	0.	0.	0.	0.
135	1	-4.088E-03	6.588E-03	-6.295E-02	1.92E-03	-2.14E-03	-2.80E-04
134	1	-3.920E-03	5.859E-03	-5.456E-02	3.44E-03	-2.45E-04	-1.65E-02
133	1	0.	0.	0.	0.	0.	0.
132	1	-4.074E-03	4.873E-03	-4.348E-02	2.94E-03	-1.38E-03	-2.27E-04
131	1	-4.103E-03	3.888E-03	-3.239E-02	2.26E-03	-2.91E-03	1.82E-02
130	1	-4.392E-03	3.080E-03	-3.389E-02	-2.42E-03	-2.85E-04	-4.63E-03
129	1	0.	0.	0.	0.	0.	0.
128	1	-4.049E-03	4.397E-03	-4.645E-02	-2.83E-03	-9.94E-04	1.02E-04
127	1	-3.547E-03	5.878E-03	-6.034E-02	4.20E-03	2.12E-02	-2.28E-01
126	1	0.	0.	0.	0.	0.	0.
125	1	-3.323E-03	7.151E-03	-7.174E-02	-1.49E-03	-1.55E-03	-2.18E-05
124	1	-2.914E-03	7.747E-03	-7.598E-02	1.82E-04	-2.34E-03	5.79E-03
123	1	0.	0.	0.	0.	0.	0.
122	1	-3.002E-03	7.586E-03	-7.247E-02	2.07E-03	-1.42E-03	-1.35E-04
121	1	-2.939E-03	6.701E-03	-6.230E-02	3.82E-03	6.48E-04	-1.89E-02
120	1	0.	0.	0.	0.	0.	0.
119	1	-3.180E-03	5.538E-03	-4.942E-02	3.17E-03	-8.81E-04	-6.72E-05
118	1	-3.301E-03	4.424E-03	-3.700E-02	-8.04E-03	-3.53E-02	3.50E-01
117	1	-3.594E-03	3.186E-03	-3.636E-02	-9.71E-03	-2.18E-02	2.19E-01
116	1	0.	0.	0.	0.	0.	0.
115	1	-3.133E-03	4.642E-03	-5.025E-02	-3.28E-03	-4.32E-04	4.50E-05
114	1	-2.544E-03	6.283E-03	-6.568E-02	-3.36E-03	-5.95E-04	-1.58E-04
113	1	0.	0.	0.	0.	0.	0.
112	1	-2.142E-03	7.724E-03	-7.781E-02	-1.82E-03	-6.86E-04	1.25E-05
111	1	-1.794E-03	8.434E-03	-8.232E-02	2.40E-04	-6.17E-04	-6.69E-04
110	1	0.	0.	0.	0.	0.	0.
109	1	-1.861E-03	8.146E-03	-7.783E-02	2.06E-03	-5.34E-04	5.37E-05
108	1	-1.974E-03	7.125E-03	-6.644E-02	2.58E-03	-2.75E-03	2.29E-02
107	1	0.	0.	0.	0.	0.	0.
106	1	-2.265E-03	5.867E-03	-5.260E-02	3.20E-03	-3.03E-04	1.05E-04

Fig. 34 Continued.

105	1	-2.499E-03	4.681E-03	-3.949E-02	2.33E-03	-2.32E-03	2.09E-02
104	1	-3.177E-03	3.173E-03	-3.689E-02	-2.49E-03	1.82E-03	-1.96E-02
103	1	0.	0.	0.	0.	0.	0.
102	1	-2.654E-03	4.654E-03	-5.099E-02	-3.42E-03	-1.11E-04	-2.56E-06
101	1	-2.053E-03	6.256E-03	-6.668E-02	-3.57E-03	-1.19E-04	-7.29E-04
100	1	0.	0.	0.	0.	0.	0.
99	1	-1.665E-03	8.904E-03	-7.896E-02	-1.94E-03	-1.28E-04	4.57E-05
98	1	-1.299E-03	8.546E-03	-8.312E-02	1.51E-04	-5.12E-05	-4.20E-04
97	1	0.	0.	0.	0.	0.	0.
96	1	-1.282E-03	8.192E-03	-7.856E-02	2.00E-03	-3.11E-05	1.58E-04
95	1	-1.487E-03	7.162E-03	-6.701E-02	3.32E-03	3.30E-04	-3.35E-03
94	1	0.	0.	0.	0.	0.	0.
93	1	-1.794E-03	5.884E-03	-5.298E-02	3.14E-03	2.18E-05	1.97E-04
92	1	-2.088E-03	4.699E-03	-3.990E-02	2.38E-03	-1.66E-03	1.67E-02
91	1	-2.830E-03	3.131E-03	-3.692E-02	-9.92E-03	-2.16E-02	2.20E-01
90	1	0.	0.	0.	0.	0.	0.
89	1	-2.265E-03	4.606E-03	-5.093E-02	-3.48E-03	1.66E-04	-4.68E-05
88	1	-1.669E-03	6.284E-03	-6.680E-02	-3.62E-03	2.86E-04	-8.25E-04
87	1	0.	0.	0.	0.	0.	0.
86	1	-1.154E-03	7.728E-03	-7.862E-02	-2.04E-03	3.63E-04	3.78E-05
85	1	-8.600E-04	8.413E-03	-8.288E-02	-1.44E-05	3.39E-04	2.03E-04
84	1	0.	0.	0.	0.	0.	0.
83	1	-8.338E-04	8.120E-03	-7.813E-02	1.89E-03	3.94E-04	2.43E-04
82	1	-1.088E-03	7.094E-03	-6.658E-02	3.95E-03	2.94E-03	-2.56E-02
81	1	0.	0.	0.	0.	0.	0.
80	1	-1.418E-03	5.816E-03	-5.260E-02	3.04E-03	2.99E-04	2.72E-04
79	1	-1.746E-03	4.646E-03	-3.974E-02	6.08E-03	1.04E-02	-1.04E-01
78	1	-2.147E-03	3.009E-03	-3.600E-02	-8.77E-03	-1.88E-02	1.95E-01
77	1	0.	0.	0.	0.	0.	0.
76	1	-1.479E-03	4.374E-03	-4.858E-02	-3.44E-03	7.63E-04	-8.89E-05
75	1	-8.416E-04	6.033E-03	-6.323E-02	3.59E-04	1.37E-02	-1.28E-01

Fig. 34 Continued.

74	1	0.	0.	0.	0.	0.	0.
73	1	-2.487E-04	7.351E-03	-7.460E-02	-2.13E-03	1.26E-03	6.50E-05
72	1	-2.758E-05	7.885E-03	-7.817E-02	-4.86E-04	6.88E-04	7.62E-03
71	1	0.	0.	0.	0.	0.	0.
70	1	1.217E-05	7.623E-03	-7.386E-02	1.55E-03	1.29E-03	3.98E-04
69	1	-3.433E-04	6.635E-03	-6.278E-02	2.68E-03	7.73E-04	3.70E-03
68	1	0.	0.	0.	0.	0.	0.
67	1	-6.986E-04	5.422E-03	-4.960E-02	2.67E-03	8.83E-04	4.22E-04
66	1	-1.078E-03	4.337E-03	-3.777E-02	-2.51E-02	-8.49E-02	8.71E-01
65	1	-1.685E-03	2.747E-03	-3.267E-02	-2.42E-03	1.50E-03	-3.63E-04
64	1	0.	0.	0.	0.	0.	0.
63	1	-7.478E-04	4.018E-03	-4.310E-02	-3.33E-03	1.35E-03	-9.22E-05
62	1	-2.105E-04	5.402E-03	-5.607E-02	-8.20E-03	-1.37E-02	1.57E-01
61	1	0.	0.	0.	0.	0.	0.
60	1	4.671E-04	6.500E-03	-6.588E-02	-2.07E-03	2.12E-03	8.43E-05
59	1	5.521E-04	6.912E-03	-6.873E-02	-6.86E-04	1.76E-03	6.45E-03
58	1	0.	0.	0.	0.	0.	0.
57	1	6.596E-04	6.660E-03	-6.498E-02	1.10E-03	2.13E-03	5.17E-04
56	1	1.934E-04	5.766E-03	-5.515E-02	1.76E-03	3.55E-04	1.55E-02
55	1	0.	0.	0.	0.	0.	0.
54	1	-1.312E-04	4.699E-03	-4.366E-02	2.14E-03	1.42E-03	5.44E-04
53	1	-5.480E-04	3.763E-03	-3.355E-02	1.55E-03	9.68E-06	1.09E-02
52	1	6.126E-03	1.455E-02	-2.264E-02	-3.25E-03	1.52E-03	-9.41E-05
51	1	0.	0.	0.	0.	0.	0.
50	1	-2.980E-04	3.299E-03	-3.474E-02	-3.03E-03	1.80E-03	-1.32E-04
49	1	1.749E-04	4.403E-03	-4.539E-02	5.33E-03	2.86E-02	-2.65E-01
48	1	0.	0.	0.	0.	0.	0.
47	1	8.545E-04	5.216E-03	-5.280E-02	-1.86E-03	2.83E-03	1.03E-04
46	1	8.324E-04	5.507E-03	-5.495E-02	-9.35E-04	2.44E-03	7.62E-03
45	1	0.	0.	0.	0.	0.	0.
44	1	9.806E-04	5.274E-03	-5.191E-02	5.79E-04	2.81E-03	5.97E-04

Fig. 34 Continued.

43	1	4.791E-04	4.549E-03	-4.414E-02	1.13E-03	1.23E-03	1.28E-02
42	1	0.	0.	0.	0.	0.	0.
41	1	2.186E-04	3.698E-03	-3.506E-02	1.47E-03	1.87E-03	6.22E-04
40	1	-1.833E-04	2.975E-03	-2.732E-02	-1.56E-03	-7.29E-03	8.90E-02
39	1	-2.644E-04	1.261E-03	-1.505E-02	-2.79E-03	1.52E-03	2.02E-05
38	1	0.	0.	0.	0.	0.	0.
37	1	-4.719E-05	2.340E-03	-2.468E-02	-2.42E-03	2.15E-03	-2.46E-04
36	1	3.130E-04	3.120E-03	-3.197E-02	-7.02E-03	-1.22E-02	1.54E-01
35	1	0.	0.	0.	0.	0.	0.
34	1	8.819E-04	3.638E-03	-3.675E-02	-1.55E-03	3.32E-03	1.15E-04
33	1	7.971E-04	3.797E-03	-3.805E-02	-8.37E-04	3.72E-03	-3.38E-04
32	1	0.	0.	0.	0.	0.	0.
31	1	9.436E-04	3.614E-03	-3.602E-02	2.14E-05	3.27E-03	6.33E-04
30	1	5.036E-04	3.104E-03	-3.068E-02	3.83E-04	1.69E-03	1.22E-02
29	1	0.	0.	0.	0.	0.	0.
28	1	3.355E-04	2.520E-03	-2.451E-02	7.27E-04	2.20E-03	6.51E-04
27	1	2.618E-06	2.047E-03	-1.946E-02	-6.44E-05	-2.93E-04	2.08E-02
26	1	-3.314E-04	5.590E-04	-7.324E-03	1.75E-03	1.37E-02	-1.22E-01
25	1	0.	0.	0.	0.	0.	0.
24	1	7.380E-05	1.194E-03	-1.287E-02	-1.52E-03	2.42E-03	-2.88E-04
23	1	1.972E-04	1.639E-03	-1.660E-02	5.49E-04	9.97E-03	-6.82E-02
22	1	0.	0.	0.	0.	0.	0.
21	1	5.955E-04	1.866E-03	-1.879E-02	-1.17E-03	3.59E-03	1.50E-04
20	1	4.694E-04	1.913E-03	-1.934E-02	-1.42E-03	2.42E-03	1.53E-02
19	1	0.	0.	0.	0.	0.	0.
18	1	6.191E-04	1.806E-03	-1.837E-02	-5.26E-04	3.52E-03	6.05E-04
17	1	2.860E-04	1.535E-03	-1.563E-02	-1.26E-03	-7.56E-04	3.93E-02
16	1	0.	0.	0.	0.	0.	0.
15	1	2.487E-04	1.253E-03	-1.262E-02	-2.92E-05	2.40E-03	6.13E-04
14	1	-2.699E-05	1.030E-03	-9.999E-03	3.63E-03	1.31E-02	-1.13E-01
13	1	-5.344E-04	-2.024E-04	2.433E-03	-8.55E-04	4.03E-03	-1.96E-02

Fig. 34 Continued.

12	1	0.	0.	0.	0.	0.	0.
11	1	0.	0.	0.	-6.72E-04	2.57E-03	-6.73E-05
10	1	-1.858E-04	4.995E-05	-2.515E-04	9.45E-03	3.61E-02	-3.33E-01
9	1	0.	0.	0.	0.	0.	0.
8	1	0.	0.	0.	-9.19E-04	3.64E-03	3.19E-04
7	1	-3.588E-05	4.169E-05	-2.624E-04	-7.11E-04	4.66E-03	-8.23E-03
6	1	0.	0.	0.	0.	0.	0.
5	1	0.	0.	0.	-8.85E-04	3.56E-03	3.77E-04
4	1	-1.205E-04	-1.932E-05	-1.398E-04	-7.70E-04	3.02E-03	4.77E-04
3	1	0.	0.	0.	0.	0.	0.
2	1	0.	0.	0.	-6.14E-04	2.49E-03	4.04E-04
1	1	-3.201E-04	-2.105E-04	9.102E-04	-6.05E-04	1.18E-03	1.06E-02

.....BEAM FORCES AND MOMENTS

BEAM NO.	LOAD NO.	AXIAL R1	SHEAR R2	SHEAR R3	TORSION M1	BENDING M2	BENDING M3
1	1	-4.203E+01	6.499E+01	4.683E+00	-1.304E+01	-2.907E+01	7.586E+01
		4.203E+01	-6.499E+01	-4.683E+00	1.304E+01	5.644E+00	2.493E+02
2	1	-8.370E+01	1.142E+02	5.598E+00	-8.063E+00	-3.371E+01	2.473E+02
		8.370E+01	-1.142E+02	-5.598E+00	8.063E+00	5.703E+00	3.239E+02
3	1	-8.428E+01	1.196E+02	-4.356E+00	5.738E+00	2.550E+01	2.489E+02
		8.428E+01	-1.196E+02	4.356E+00	-5.738E+00	-3.705E+00	3.492E+02
4	1	-2.733E+01	6.411E+01	-8.097E+00	1.884E+01	3.796E+01	1.546E+01
		2.733E+01	-6.411E+01	8.097E+00	-1.884E+01	2.543E+00	3.053E+02
5	1	-1.304E+02	6.559E+01	1.453E+00	-1.670E+01	-4.919E+00	-3.517E+01
		1.304E+02	-6.559E+01	-1.453E+00	1.670E+01	-2.351E+00	3.633E+02
6	1	-2.382E+02	1.170E+02	1.282E+00	-1.210E+01	-4.223E+00	4.737E+01
		2.382E+02	-1.170E+02	-1.282E+00	1.210E+01	-2.191E+00	5.381E+02
7	1	-2.436E+02	1.183E+02	-1.203E+00	8.258E+00	5.158E+00	2.813E+01
		2.436E+02	-1.183E+02	1.203E+00	-8.258E+00	8.602E-01	5.634E+02
8	1	-1.122E+02	6.255E+01	-1.855E+00	1.973E+01	-1.694E+00	-1.088E+02
		1.122E+02	-6.255E+01	1.855E+00	-1.973E+01	1.097E+01	4.217E+02
9	1	-2.148E+02	6.382E+01	8.208E-01	-1.647E+01	2.721E+00	-1.577E+02
		2.148E+02	-6.382E+01	-8.208E-01	1.647E+01	-6.827E+00	4.770E+02
10	1	-4.022E+02	1.207E+02	5.738E-01	-1.228E+01	3.437E+00	-1.438E+02
		4.022E+02	-1.207E+02	-5.738E-01	1.228E+01	-6.307E+00	7.477E+02
11	1	-4.074E+02	1.192E+02	-2.558E-01	6.922E+00	8.723E-01	-1.760E+02
		4.074E+02	-1.192E+02	2.558E-01	-6.922E+00	4.071E-01	7.721E+02
12	1	-1.807E+02	3.081E+01	-1.346E+00	1.342E+01	-8.771E+00	-2.622E+02
		1.807E+02	-3.081E+01	1.346E+00	-1.342E+01	1.550E+01	4.163E+02
13	1	-2.937E+02	6.105E+01	7.773E-01	-1.460E+01	7.125E+00	-2.846E+02
		2.937E+02	-6.105E+01	-7.773E-01	1.460E+01	-1.101E+01	5.900E+02
14	1	-5.682E+02	1.245E+02	5.270E-01	-1.134E+01	7.482E+00	-3.482E+02
		5.682E+02	-1.245E+02	-5.270E-01	1.134E+01	-1.012E+01	9.712E+02
15	1	-5.707E+02	1.210E+02	-8.920E-02	4.576E+00	1.292E+00	-3.849E+02
		5.707E+02	-1.210E+02	8.920E-02	-4.576E+00	-8.459E-01	9.902E+02

Fig. 34 Continued.

16	1	-2.517E+02	8.299E+01	1.286E+00	6.738E+00	-7.661E+00	-2.360E+02
		2.517E+02	-8.299E+01	-1.286E+00	-6.738E+00	1.225E+00	6.512E+02
17	1	-3.524E+02	3.205E+01	5.752E-01	-1.170E+01	1.116E+01	-4.483E+02
		3.524E+02	-3.205E+01	-5.752E-01	1.170E+01	-1.404E+01	6.086E+02
18	1	-6.937E+02	5.870E+01	4.790E-01	-9.947E+00	1.087E+01	-6.730E+02
		6.937E+02	-5.870E+01	-4.790E-01	9.947E+00	-1.327E+01	9.666E+02
19	1	-6.867E+02	4.750E+01	-1.576E-01	1.405E+00	1.970E+00	-7.199E+02
		6.867E+02	-4.750E+01	1.576E-01	-1.405E+00	-1.182E+00	9.575E+02
20	1	-3.306E+02	3.648E+01	3.380E-01	2.449E+00	-1.464E+00	-4.802E+02
		3.306E+02	-3.648E+01	-3.380E-01	-2.449E+00	-2.270E-01	6.627E+02
21	1	-3.791E+02	1.063E+01	3.038E-01	-7.876E+00	1.390E+01	-5.426E+02
		3.791E+02	-1.063E+01	-3.038E-01	7.876E+00	-1.542E+01	5.958E+02
22	1	-7.476E+02	1.536E+01	6.863E-01	-7.471E+00	1.338E+01	-8.379E+02
		7.476E+02	-1.536E+01	-6.863E-01	7.471E+00	-1.681E+01	9.147E+02
23	1	-7.209E+02	-1.476E+00	4.022E-01	-1.901E+00	1.718E+00	-8.824E+02
		7.209E+02	1.476E+00	-4.022E-01	1.901E+00	-3.730E+00	8.750E+02
24	1	-3.721E+02	7.958E+00	-1.724E+00	6.636E-01	4.000E-01	-5.624E+02
		3.721E+02	-7.958E+00	1.724E+00	-6.636E-01	8.224E+00	6.022E+02
25	1	-3.864E+02	8.814E+00	3.943E-01	-4.722E+00	1.490E+01	-5.726E+02
		3.864E+02	-8.814E+00	-3.943E-01	4.722E+00	-1.582E+01	5.930E+02
26	1	-7.637E+02	1.806E+01	5.604E-01	-4.748E+00	1.658E+01	-8.721E+02
		7.637E+02	-1.806E+01	-5.604E-01	4.748E+00	-1.788E+01	9.140E+02
27	1	-7.245E+02	1.161E+02	-4.024E-01	-4.660E+00	4.505E+00	-8.522E+02
		7.245E+02	-1.161E+02	4.024E-01	4.660E+00	-3.533E+00	1.133E+03
28	1	-3.807E+02	8.151E-01	1.430E-01	-2.812E+00	-8.485E+00	-5.801E+02
		3.807E+02	-8.151E-01	-1.430E-01	2.812E+00	8.153E+00	5.820E+02
29	1	-3.886E+02	5.651E+00	3.055E-01	-2.484E+00	1.543E+01	-5.840E+02
		3.886E+02	-5.651E+00	-3.055E-01	2.484E+00	-1.625E+01	5.992E+02
30	1	-7.718E+02	1.892E+01	5.068E-01	-2.546E+00	1.738E+01	-8.890E+02
		7.718E+02	-1.892E+01	-5.068E-01	2.546E+00	-1.874E+01	9.397E+02
31	1	-7.388E+02	-7.665E+01	2.165E-01	-4.838E+00	1.480E+00	-1.102E+03
		7.388E+02	7.665E+01	-2.165E-01	4.838E+00	-2.071E+00	8.928E+02
32	1	-3.797E+02	-5.307E-01	6.253E-01	-5.806E+00	-8.186E+00	-5.846E+02
		3.797E+02	5.307E-01	-6.253E-01	5.806E+00	6.509E+00	5.832E+02
33	1	-3.700E+02	-3.259E+01	-1.455E-01	8.429E-01	1.597E+01	-6.451E+02
		3.700E+02	3.259E+01	1.455E-01	-8.429E-01	-1.524E+01	4.821E+02
34	1	-7.271E+02	-7.575E+01	-3.273E-01	-8.839E-02	1.798E+01	-1.049E+03
		7.271E+02	7.575E+01	3.273E-01	8.839E-02	-1.635E+01	6.705E+02
35	1	-6.852E+02	-7.599E+01	4.652E-01	-7.268E+00	2.895E+00	-1.030E+03
		6.852E+02	7.599E+01	-4.652E-01	7.268E+00	-5.223E+00	6.499E+02
36	1	-3.568E+02	-3.629E+01	1.182E+00	-9.949E+00	-6.894E+00	-6.391E+02
		3.568E+02	3.629E+01	-1.182E+00	9.949E+00	9.801E-01	4.575E+02
37	1	-3.197E+02	-4.549E+01	-3.292E-01	5.098E+00	1.467E+01	-6.016E+02
		3.197E+02	4.549E+01	3.292E-01	-5.098E+00	-1.302E+01	3.740E+02
38	1	-6.064E+02	-9.861E+01	-8.059E-01	3.384E+00	1.478E+01	-9.552E+02
		6.064E+02	9.861E+01	8.059E-01	-3.384E+00	-1.075E+01	4.618E+02
39	1	-5.689E+02	-9.883E+01	1.456E-01	-8.723E+00	3.526E+00	-9.224E+02
		5.689E+02	9.883E+01	-1.456E-01	8.723E+00	-4.255E+00	4.279E+02
40	1	-3.032E+02	-4.801E+01	1.927E+00	-1.292E+01	-1.973E+00	-5.818E+02
		3.032E+02	4.801E+01	-1.927E+00	1.292E+01	-7.666E+00	3.416E+02
41	1	-2.584E+02	-4.932E+01	-3.546E-01	8.941E+00	1.242E+01	-5.200E+02
		2.584E+02	4.932E+01	3.546E-01	-8.941E+00	-1.064E+01	2.733E+02
42	1	-4.706E+02	-9.628E+01	-1.598E-01	5.595E+00	9.317E+00	-7.842E+02
		4.706E+02	9.628E+01	1.598E-01	-5.595E+00	-8.517E+00	3.026E+02

Fig. 34 Continued.

43	1	-4.350E+02	-9.424E+01	2.245E-01	-9.876E+00	2.637E+00	-7.429E+02
		4.350E+02	9.424E+01	-2.245E-01	9.876E+00	-3.760E+00	2.714E+02
44	1	-2.384E+02	-5.098E+01	-1.872E+00	-1.304E+01	6.569E+00	-4.925E+02
		2.384E+02	5.098E+01	1.872E+00	1.304E+01	2.796E+00	2.375E+02
45	1	-1.936E+02	-5.230E+01	-2.066E-01	1.228E+01	1.012E+01	-4.289E+02
		1.936E+02	5.230E+01	2.066E-01	-1.228E+01	-9.088E+00	1.672E+02
46	1	-3.357E+02	-9.477E+01	-1.964E-01	7.172E+00	7.210E+00	-6.234E+02
		3.357E+02	9.477E+01	1.964E-01	-7.172E+00	-6.228E+00	1.494E+02
47	1	-3.043E+02	-9.104E+01	2.125E-01	-1.054E+01	2.177E+00	-5.791E+02
		3.043E+02	9.104E+01	-2.125E-01	1.054E+01	-3.240E+00	1.237E+02
48	1	-1.733E+02	-5.297E+01	1.401E+00	-1.614E+01	-9.131E+00	-3.888E+02
		1.733E+02	5.297E+01	-1.401E+00	1.614E+01	2.122E+00	1.238E+02
49	1	-1.262E+02	-5.462E+01	-6.184E-01	1.501E+01	8.681E+00	-3.301E+02
		1.262E+02	5.462E+01	6.184E-01	-1.501E+01	-5.587E+00	5.686E+01
50	1	-2.051E+02	-9.009E+01	-9.470E-01	8.261E+00	4.906E+00	-4.643E+02
		2.051E+02	9.009E+01	9.470E-01	-8.261E+00	-1.653E-01	1.329E+01
51	1	-1.766E+02	-8.896E+01	9.219E-01	-1.039E+01	1.644E+00	-4.244E+02
		1.766E+02	8.896E+01	-9.219E-01	1.039E+01	-6.256E+00	-2.064E+01
52	1	-1.061E+02	-5.414E+01	2.787E+00	-1.759E+01	-2.800E+00	-2.799E+02
		1.061E+02	5.414E+01	-2.787E+00	1.759E+01	-1.114E+01	9.070E+00
53	1	-5.121E+01	-5.619E+01	-4.733E+00	1.484E+01	5.618E+00	-2.378E+02
		5.121E+01	5.619E+01	4.733E+00	-1.484E+01	2.169E+01	-8.650E+01
54	1	-6.437E+01	-9.486E+01	-3.737E+00	6.279E+00	-3.589E-01	-3.309E+02
		6.437E+01	9.486E+01	3.737E+00	-6.279E+00	2.013E+01	-1.710E+02
55	1	-5.436E+01	-8.747E+01	3.573E+00	-7.565E+00	4.673E+00	-2.688E+02
		5.436E+01	8.747E+01	-3.573E+00	7.565E+00	-2.188E+01	-1.525E+02
56	1	-3.763E+01	-5.414E+01	3.376E+00	-1.385E+01	1.010E+01	-1.686E+02
		3.763E+01	5.414E+01	-3.376E+00	1.385E+01	-2.475E+01	-6.629E+01
57	1	-2.359E+00	2.595E+00	-6.739E+00	1.997E+01	2.851E+01	1.033E+01
		2.359E+00	-2.595E+00	6.739E+00	-1.997E+01	3.148E+01	1.277E+01
58	1	-7.964E+00	1.685E-01	-8.115E-01	1.540E+00	2.821E+00	-3.374E+00
		7.964E+00	-1.685E-01	8.115E-01	-1.540E+00	4.399E+00	4.873E+00
59	1	-1.298E+01	-4.887E+00	7.400E+00	-2.056E+01	-3.642E+01	-2.850E+01
		1.298E+01	4.887E+00	-7.400E+00	2.056E+01	-2.953E+01	-1.506E+01
60	1	-5.268E+00	4.689E+00	4.767E+00	-1.271E+01	-1.928E+01	1.379E+01
		5.268E+00	-4.689E+00	-4.767E+00	1.271E+01	-2.267E+01	2.747E+01
61	1	-5.774E+00	-9.903E-01	-2.273E-01	7.646E-01	2.266E+00	-7.740E+00
		5.774E+00	9.903E-01	2.273E-01	-7.646E-01	-2.670E-01	-9.689E-01
62	1	-5.191E+00	-2.667E+00	-4.851E+00	1.368E+01	2.258E+01	-1.135E+01
		5.191E+00	2.667E+00	4.851E+00	-1.368E+01	2.015E+01	-1.214E+01

SHELL ELEMENT STRESSES

ELEMENT NUMBER	LOAD CASE	MEMBRANE STRESS COMPONENTS SXX	SYY	SXY	BENDING MOMENT COMPONENTS MXX	MYY	MXY
1	1	-.7672E+01	-.2978E+01	-.6311E+01	-.3366E+00	-.7571E+00	-.6875E+00
2	1	-.8092E+01	.1053E+01	.2930E+01	-.1259E+00	-.5517E+00	-.1180E+01
3	1	-.1622E+01	-.1846E+01	-.7686E+01	.1491E+00	-.8876E+00	-.1346E+01

Fig. 34 Continued.

4	1	-.1841E+01	-.9073E+00	.4944E+01	-.3263E+00	-.8568E+00	-.1464E+00
5	1	-.2751E+01	-.1413E-02	-.3626E+01	.2565E+00	.1061E-01	-.7231E-01
6	1	-.4156E+01	-.1235E+01	.8994E+01	-.5661E+00	-.3739E+00	.1158E+01
7	1	-.1581E+02	.1077E+01	-.6479E+01	-.1948E-01	.6960E+00	.1238E+01
8	1	-.1537E+02	-.2195E+01	.6891E+01	-.8192E+00	-.1292E+01	.1208E+01
9	1	-.1991E+02	.1010E+01	-.5557E+01	-.5352E+00	-.2844E+00	-.1098E+01
10	1	-.1943E+02	.3444E+00	.2081E+01	-.5218E+00	-.4586E+00	-.1400E+01
11	1	-.1731E+02	-.5016E+00	-.1003E+02	-.6357E+00	-.1770E+01	-.1282E+01
12	1	-.1742E+02	-.9891E+00	.8069E+01	-.9127E+00	-.1925E+01	-.4265E+00
13	1	-.1921E+02	-.1583E+01	-.6330E+01	-.4350E+00	-.9759E+00	.3948E-01
14	1	-.2290E+02	-.1313E+01	.1066E+02	-.9847E+00	-.1248E+01	.1083E+01
15	1	-.3024E+02	.7031E+00	-.1454E+01	-.3502E+00	.2292E+00	.1233E+01
16	1	-.2266E+02	.5446E+01	-.6701E+00	-.8903E+00	-.8008E+00	.1733E+01
17	1	-.3156E+02	.2884E+00	-.4538E+01	-.6071E+00	.4361E-01	-.1251E+01
18	1	-.3207E+02	.4393E+00	.1384E+01	-.8255E+00	-.4427E+00	-.1350E+01
19	1	-.3263E+02	.1132E+00	-.1089E+02	-.1080E+01	-.2377E+01	-.1283E+01
20	1	-.3420E+02	-.2673E+00	.7705E+01	-.1612E+01	-.3059E+01	-.5262E+00
21	1	-.3652E+02	-.6914E+00	-.7249E+01	-.1089E+01	-.2204E+01	.4193E-02
22	1	-.3878E+02	-.9436E+00	.1092E+02	-.1519E+01	-.2109E+01	.1036E+01
23	1	-.4020E+02	-.1983E+01	-.3466E+00	-.7274E+00	-.1568E+00	.1185E+01
24	1	-.4702E+02	-.1975E+02	.4269E+01	-.3438E+00	-.1946E+00	.7153E+00
25	1	-.4234E+02	.4371E+00	-.3795E+01	-.7660E+00	.1555E+00	-.1169E+01
26	1	-.4478E+02	.5652E+00	.7062E+00	-.1133E+01	-.3628E+00	-.1230E+01
27	1	-.4855E+02	.1056E+01	-.1016E+02	-.1593E+01	-.2907E+01	-.1222E+01
28	1	-.5135E+02	.1115E+01	.6019E+01	-.2241E+01	-.3984E+01	-.5001E+00
29	1	-.5378E+02	.8186E+00	-.6905E+01	-.1808E+01	-.3401E+01	-.1721E+00
30	1	-.5413E+02	-.5983E-01	.8103E+01	-.2016E+01	-.2948E+01	.8658E+00
31	1	-.5139E+02	-.2827E+01	-.2889E+01	-.9287E+00	-.5585E-01	.6579E+00
32	1	-.4831E+02	-.1347E+02	.1024E+02	-.8520E-01	.8545E+00	-.1063E+01
33	1	-.5071E+02	.5438E+00	-.2571E+01	-.9766E+00	.2115E+00	-.9079E+00
34	1	-.5470E+02	.8393E+00	-.2406E+00	-.1327E+01	-.2143E+00	-.1003E+01

Fig. 34 Continued.

35	1	-.6061E+02	.2256E+01	-.7011E+01	-.2121E+01	-.3340E+01	-.8748E+00
36	1	-.6403E+02	.2845E+01	.2904E+01	-.2582E+01	-.4589E+01	-.5999E+00
37	1	-.6613E+02	.3100E+01	-.4738E+01	-.2419E+01	-.4432E+01	-.1194E+00
38	1	-.6540E+02	.2638E+01	.2751E+01	-.2268E+01	-.3552E+01	.3690E+00
39	1	-.6257E+02	.1195E+01	-.4940E+01	-.1224E+01	.1440E+00	.4022E+00
40	1	-.5165E+02	.5439E+01	.6608E+01	-.1877E+01	.1551E+01	.7325E+00
41	1	-.5520E+02	.5834E+00	-.1201E+01	-.1055E+01	.2554E+00	-.6334E+00
42	1	-.5991E+02	.1235E+01	-.5309E+00	-.1354E+01	-.1166E+00	-.6400E+00
43	1	-.6557E+02	.3058E+01	-.3055E+01	-.2323E+01	-.3455E+01	-.5126E+00
44	1	-.6950E+02	.4359E+01	.1235E+01	-.2652E+01	-.4880E+01	-.7141E+00
45	1	-.7078E+02	.7931E+01	-.2169E+01	-.2867E+01	-.5148E+01	-.1874E-01
46	1	-.6936E+02	.7549E+01	-.2444E+01	-.2245E+01	-.3932E+01	-.3295E+00
47	1	-.6395E+02	.2684E+01	-.3169E+01	-.1545E+01	.1279E+00	.2642E+00
48	1	-.5606E+02	.1037E+01	.1991E+01	-.7421E+00	.1124E+01	.3797E+00
49	1	-.5662E+02	.6675E+00	-.2443E+00	-.1061E+01	.2842E+00	-.4503E+00
50	1	-.6158E+02	.1385E+01	-.2799E+00	-.1412E+01	-.9674E-01	-.3668E+00
51	1	-.6704E+02	.2986E+01	-.6518E+00	-.2447E+01	-.3373E+01	-.3982E+00
52	1	-.6965E+02	.2825E+01	.3294E+01	-.2807E+01	-.5000E+01	-.1370E+00
53	1	-.7810E+02	.4324E+01	.2188E+02	-.2480E+01	-.5176E+01	-.6027E+00
54	1	-.7250E+02	-.1909E+02	-.2151E+02	-.2556E+01	-.4433E+01	.5813E-01
55	1	-.5992E+02	-.2123E+01	-.1537E+01	-.1045E+01	.2155E+00	-.5922E+00
56	1	-.5675E+02	.7504E+00	.8017E+00	-.7242E+00	.6880E+00	-.1151E+00
57	1	-.5708E+02	.6449E+00	.5746E+00	-.1066E+01	.2852E+00	-.2584E+00
58	1	-.6178E+02	.1149E+01	-.4070E+00	-.1349E+01	-.4653E-01	-.1518E+00
59	1	-.6808E+02	.2330E+01	.1415E+01	-.2280E+01	-.3293E+01	-.2517E+00
60	1	-.6866E+02	-.7403E+00	-.3341E+01	-.2433E+01	-.4753E+01	-.4665E+00
61	1	-.7801E+02	-.1918E+02	-.2001E+02	-.3191E+01	-.5614E+01	-.1377E+00
62	1	-.7579E+02	.1555E+01	.1616E+02	-.2081E+01	-.4024E+01	-.4580E+00
63	1	-.6100E+02	.4745E+00	.3070E+01	-.1580E+01	-.1153E+00	-.4258E+00
64	1	-.5705E+02	.9059E+00	-.1507E+00	-.8273E+00	.4781E+00	-.3956E+00
65	1	-.5499E+02	.5087E+00	.1843E+01	-.1105E+01	.2459E+00	.9334E-01

Fig. 34 Continued.

66	1	-.5894E+02	.8657E+00	-.1053E+01	-.1279E+01	.6083E-01	.4671E-01
67	1	-.6441E+02	.2025E+01	.4183E+01	-.2424E+01	-.3227E+01	.2500E+00
68	1	-.6746E+02	.2543E+01	-.7110E+01	-.2642E+01	-.4578E+01	-.1984E-01
69	1	-.6738E+02	.7693E+01	.1081E+01	-.2821E+01	-.5158E+01	-.5460E+00
70	1	-.6438E+02	.7241E+01	-.6406E+01	-.2244E+01	-.3949E+01	-.7024E+00
71	1	-.5820E+02	.2261E+01	.1078E+01	-.1433E+01	-.3068E+00	-.9753E+00
72	1	-.5430E+02	.9686E+00	-.1821E+01	-.8724E+00	.3883E+00	-.8284E+00
73	1	-.4874E+02	.3477E+00	.2982E+01	-.9306E+00	.2376E+00	.3852E+00
74	1	-.5149E+02	.9760E+00	-.1127E+01	-.1095E+01	.4954E-01	.4584E+00
75	1	-.5420E+02	.2588E+01	.7665E+01	-.2032E+01	-.2826E+01	.5602E+00
76	1	-.5539E+02	.4138E+01	-.6964E+01	-.1862E+01	-.3883E+01	-.3637E-01
77	1	-.5456E+02	.4385E+01	.4451E+01	-.2366E+01	-.4611E+01	-.3283E+00
78	1	-.5187E+02	.3152E+01	-.9635E+01	-.1547E+01	-.3344E+01	-.1025E+01
79	1	-.4854E+02	.1695E+01	-.2392E+00	-.1273E+01	-.5119E+00	-.1049E+01
80	1	-.4642E+02	.5656E+00	-.3779E+01	-.7149E+00	.2188E+00	-.1036E+01
81	1	-.4049E+02	.2907E+00	.3808E+01	-.7496E+00	.1986E+00	.6721E+00
82	1	-.4169E+02	.1134E+01	-.2070E+00	-.8611E+00	.3751E-01	.7473E+00
83	1	-.4224E+02	.2078E+01	.9661E+01	-.1609E+01	-.2300E+01	.7581E+00
84	1	-.4216E+02	.2514E+01	-.6503E+01	-.1444E+01	-.3078E+01	.4208E-01
85	1	-.4102E+02	.2296E+01	.4674E+01	-.1885E+01	-.3796E+01	-.3400E+00
86	1	-.3913E+02	.1465E+01	-.1087E+02	-.1227E+01	-.2805E+01	-.1170E+01
87	1	-.3741E+02	.7886E+00	-.8436E+00	-.9923E+00	-.6690E+00	-.1156E+01
88	1	-.3615E+02	.1765E+00	-.4552E+01	-.4802E+00	.1999E+00	-.1198E+01
89	1	-.3081E+02	.2118E+00	.4506E+01	-.5776E+00	.1366E+00	.9124E+00
90	1	-.3100E+02	.7727E+00	.6007E+00	-.6255E+00	.1239E+00	.9560E+00
91	1	-.3053E+02	.9060E+00	.1041E+02	-.1206E+01	-.1728E+01	.8802E+00
92	1	-.2977E+02	.8952E+00	-.6086E+01	-.9524E+00	-.2109E+01	.1229E+00
93	1	-.2853E+02	.4411E+00	.4380E+01	-.1400E+01	-.2827E+01	-.3584E+00
94	1	-.2709E+02	.1380E+00	-.1097E+02	-.8204E+00	-.2138E+01	-.1215E+01
95	1	-.2626E+02	.1525E+00	-.1001E+01	-.8433E+00	-.7905E+00	-.1221E+01
96	1	-.2547E+02	-.5960E-01	-.4705E+01	-.3558E+00	.1668E+00	-.1313E+01

Fig. 34 Continued.

97	1	-.2003E+02	.3197E+00	.4856E+01	-.4798E+00	-.1264E+00	.9910E+00
98	1	-.1993E+02	-.6634E+00	.9146E+00	-.3341E+00	.3427E+00	.1085E+01
99	1	-.1921E+02	-.8766E-01	.1076E+02	-.7873E+00	-.1124E+01	.9696E+00
100	1	-.1811E+02	-.1241E+01	-.6055E+01	-.4564E+00	-.1058E+01	.1682E+00
101	1	-.1648E+02	-.1147E+01	.4523E+01	-.9469E+00	-.1797E+01	-.3096E+00
102	1	-.1537E+02	-.4893E+00	-.1068E+02	-.4581E+00	-.1492E+01	-.1185E+01
103	1	-.1518E+02	-.6793E+00	-.5252E+00	-.6164E+00	-.6741E+00	-.1264E+01
104	1	-.1453E+02	.5808E+00	-.4888E+01	-.3424E+00	-.1971E+00	-.1194E+01
105	1	-.7968E+01	-.1963E+01	.4928E+01	-.3672E+00	-.7302E+00	.8455E+00
106	1	-.7617E+01	.3025E+00	.7552E+00	-.8757E-01	.4957E+00	.1165E+01
107	1	-.7472E+01	-.4684E+01	.1077E+02	-.3760E+00	-.3427E+00	.1078E+01
108	1	-.5979E+01	-.2266E+01	-.5021E+01	-.6708E-01	-.1254E+00	.1262E-01
109	1	-.4930E+01	-.1573E+01	.4614E+01	-.2992E+00	-.7907E+00	-.1450E+00
110	1	-.4378E+01	-.3245E+01	-.9762E+01	.5251E-01	-.7041E+00	-.1281E+01
111	1	-.3822E+01	.2431E+01	.6536E-02	-.2061E+00	-.5944E+00	-.1159E+01
112	1	-.4107E+01	-.2787E+01	-.4998E+01	-.2767E+00	-.7941E+00	-.7233E+00

OVERALL LOG

NODAL POINT INPUT............	2.63
FORM ELEMENT STIFFNESSES......	89.84
INPUT NODAL LOADS............	.25
FORM TOTAL STIFFNESS..........	13.36
EQUATION SOLVING.............	35.18
ELEMENT STRESSES.............	3.99
TOTAL SOLUTION TIME..........	145.24

Fig. 34 Continued.

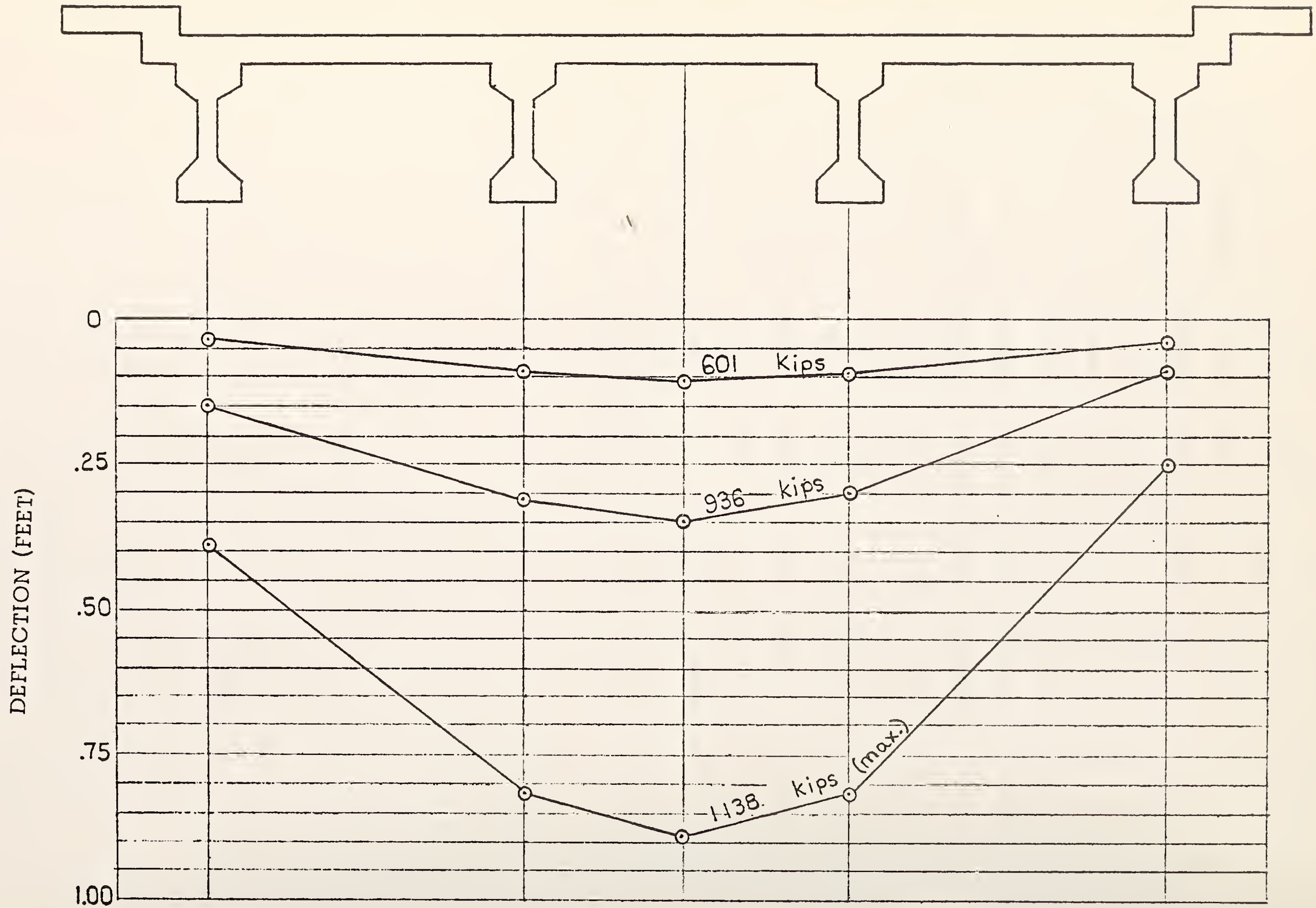


FIGURE 35: CROSS-SECTION PLOTS SHOWING DECK DISTORTION UNDER LOAD - BRIDGE 2 (Test No. 2500)

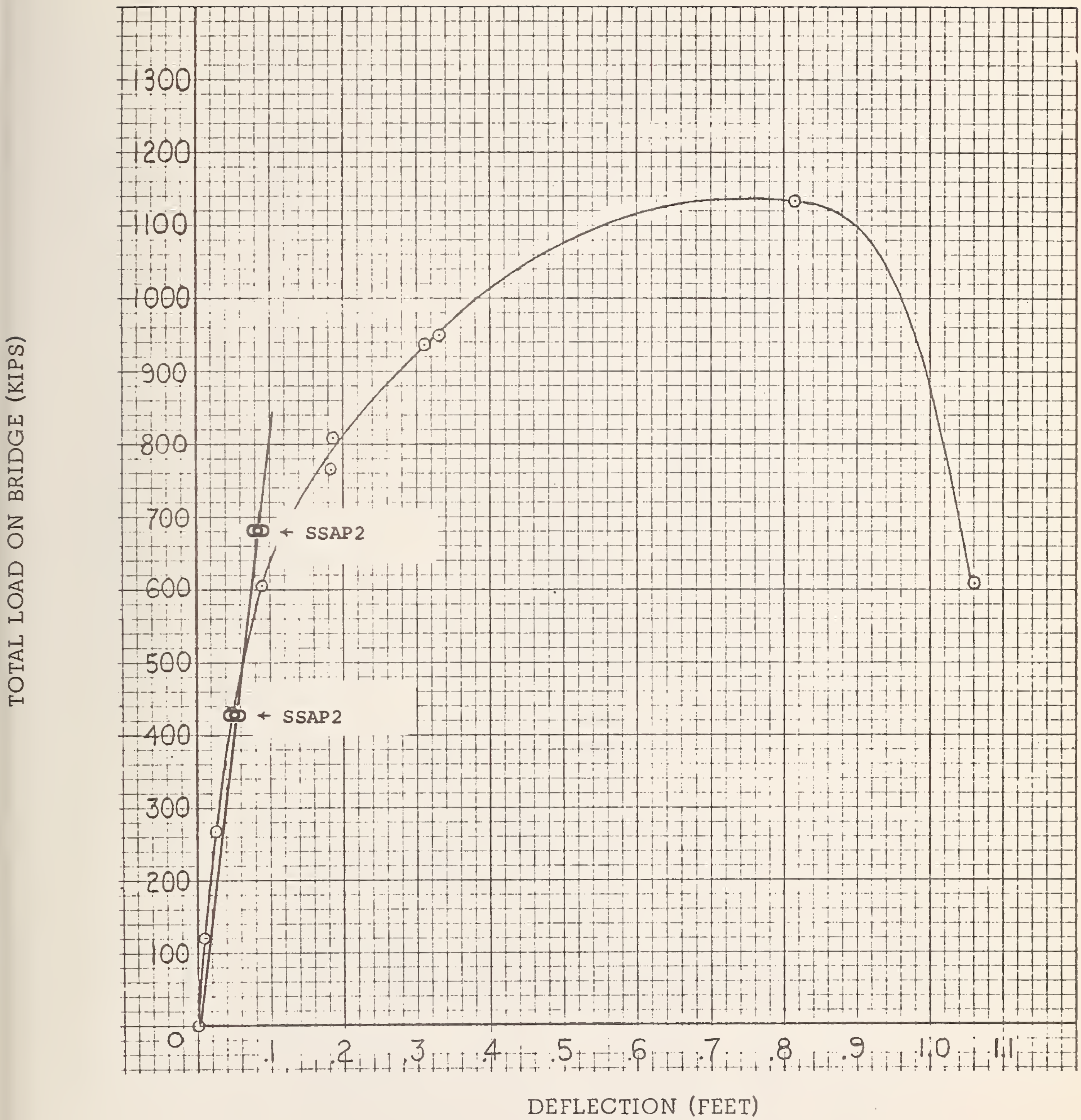


FIGURE 36: LOAD-DEFLECTION CURVE - CENTERLINE OF SPAN OVER INTERIOR BEAM - BRIDGE 2 (Test No. 2500)

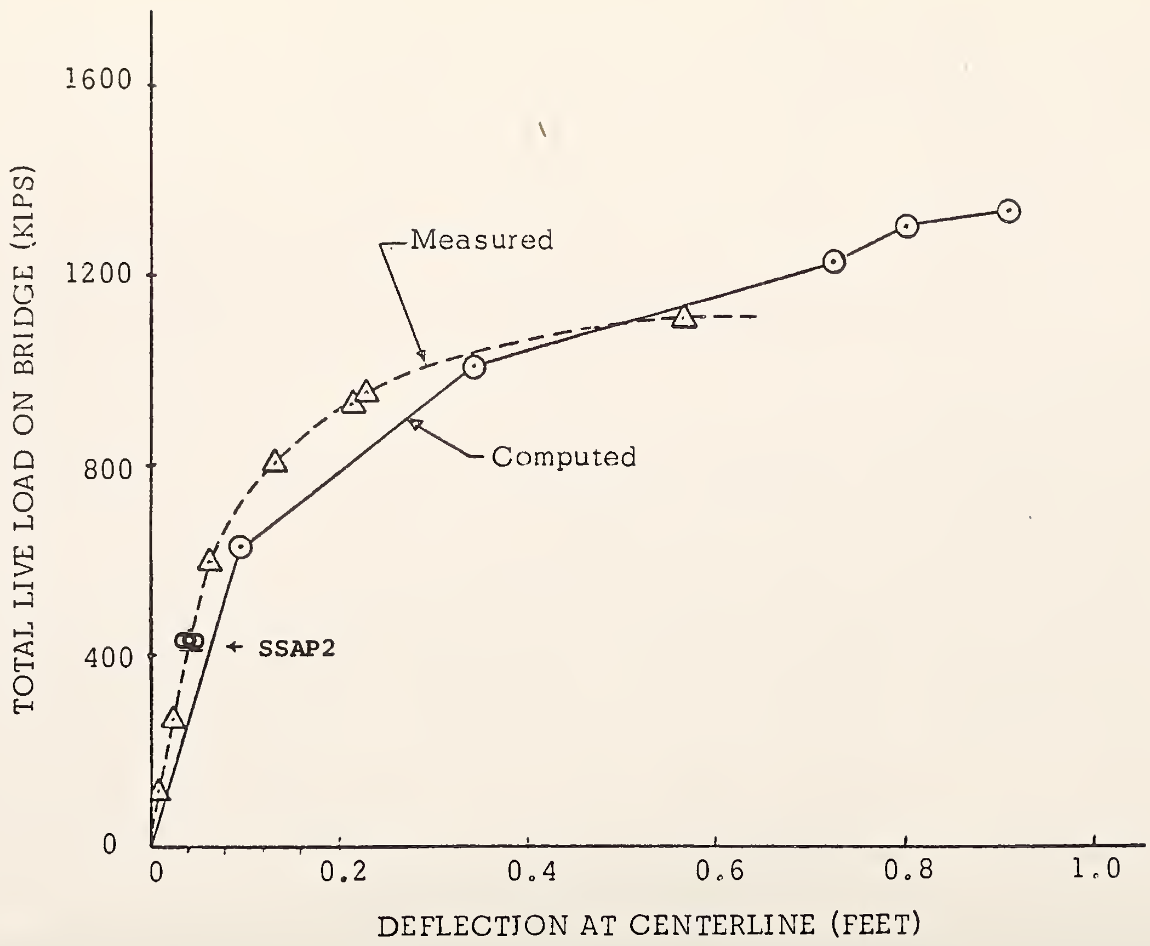


FIGURE **37.** COMPARISON OF MEASURED AND COMPUTED LOAD-DEFLECTION CURVES OF BRIDGE 2 (Test No. 2500).

TABLE 9

COMPARISON OF COMPUTER AND EXPERIMENTAL DEFLECTIONS FOR BRIDGE #2

Description	Nodes	Node Location				Average	Load Kips
		93	96	99	102		
1. Beam elements for girder slave connected to plate elements for the slab -- Center diaphragm omitted. No curvature or super-elevation.	195	.06018	.09267	.09175	.06249		680, located at girder points.
2. Values linearly reduced to 430 kips		.038	.059	.058	.040	.049	430
		Node Location					
		93	96	99	102		
3. Same as No. 1 except that the bridge was curved and superelevated. Entire run shown in report.	195	.05298	.07856	.07896	.05099		680, located at girder points.
4. Values linearly reduced to 430 kips.		.034	.050	.050	.032	.042	430
Figures 36 and 37		.035*	.045	.045	.035*	.040	430
Figure 35		.025	.064	.064	.029		430

* must be .035 for average to be .040.

V. Bridge #3

1. Description

Bridge #3, shown in Figures 2 and 3 is a sloping straight bridge with a 30° skew. Constructed in 1938, it was located on U.S. 41A over the Elk River. It was a simple span of reinforced concrete T-Beam monolithic construction. Figures E-11 through E-13 show details of the bridge taken from construction drawings. Static testing of one end span of bridge #3 was performed in July, 1970. Once again, the dimensions shown on the construction drawings do not match Figure 9. Nevertheless, the author decided to use the dimensions shown in Figure 9 for a first analysis. Figure 38 describes the node numbering, Figure 39 the beam element numbering, and Figure 40 the plate element numbering for the deck.

2. Modulus of Elasticity

From Table 6:

$$f'_c = 6{,}500 \text{ psi}$$

By calculation:

$$E_c = 667782 \text{ ksf}$$

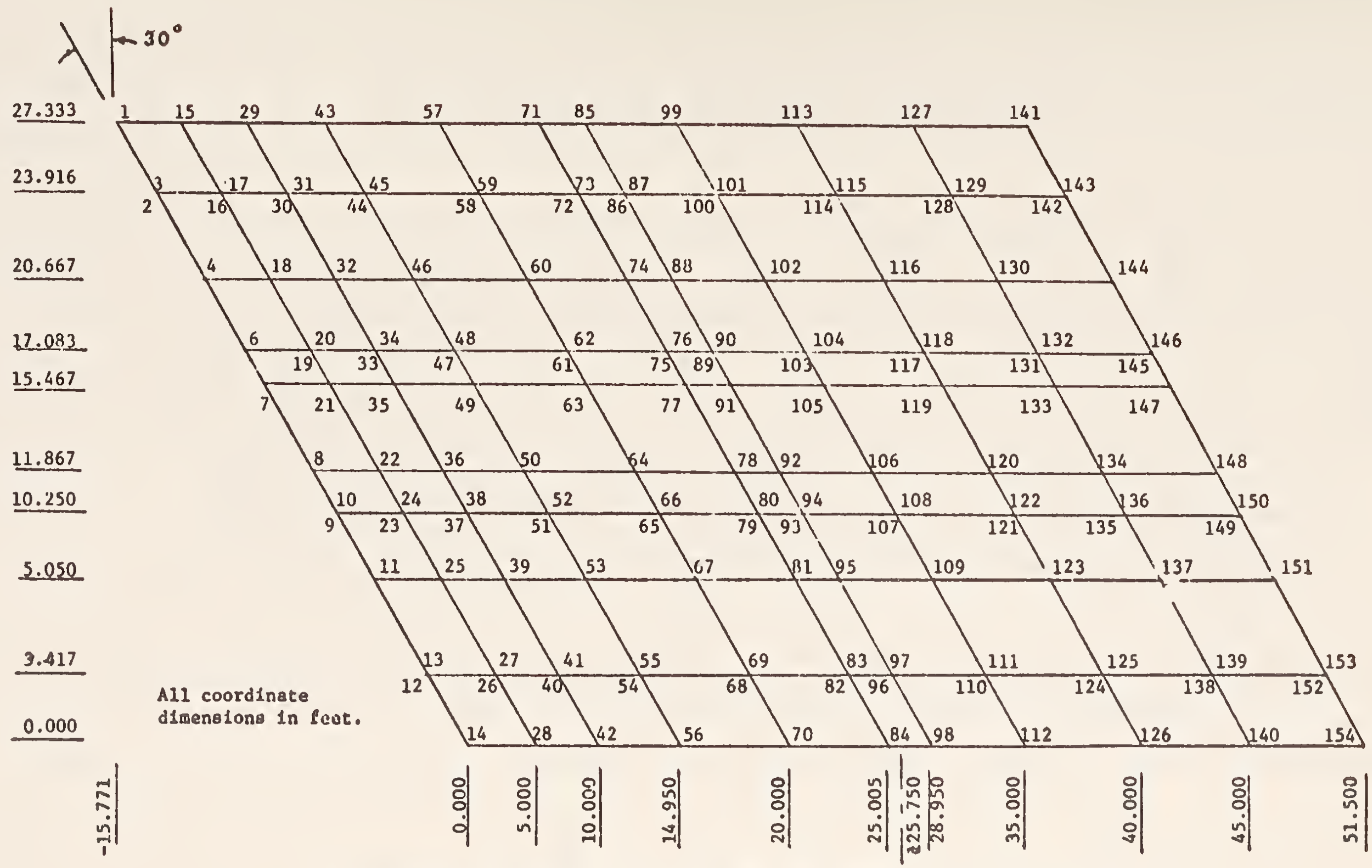
$$C_{xx} = C_{yy} = 683152.85$$

$$C_{xy} = 102472.92$$

$$G_{xy} = 290339.96$$

3. Moments of Inertia

Bending and torsional moments of inertia were calculated as shown in Figure 41 for the concrete stringer and diaphragm, both of rectangular cross-section.



NOTE: Where two numbers exist, the right most number is at the elevation of the centroid of the girder.

FIG. 38 - LOCATION OF NODES FOR BRIDGE #3

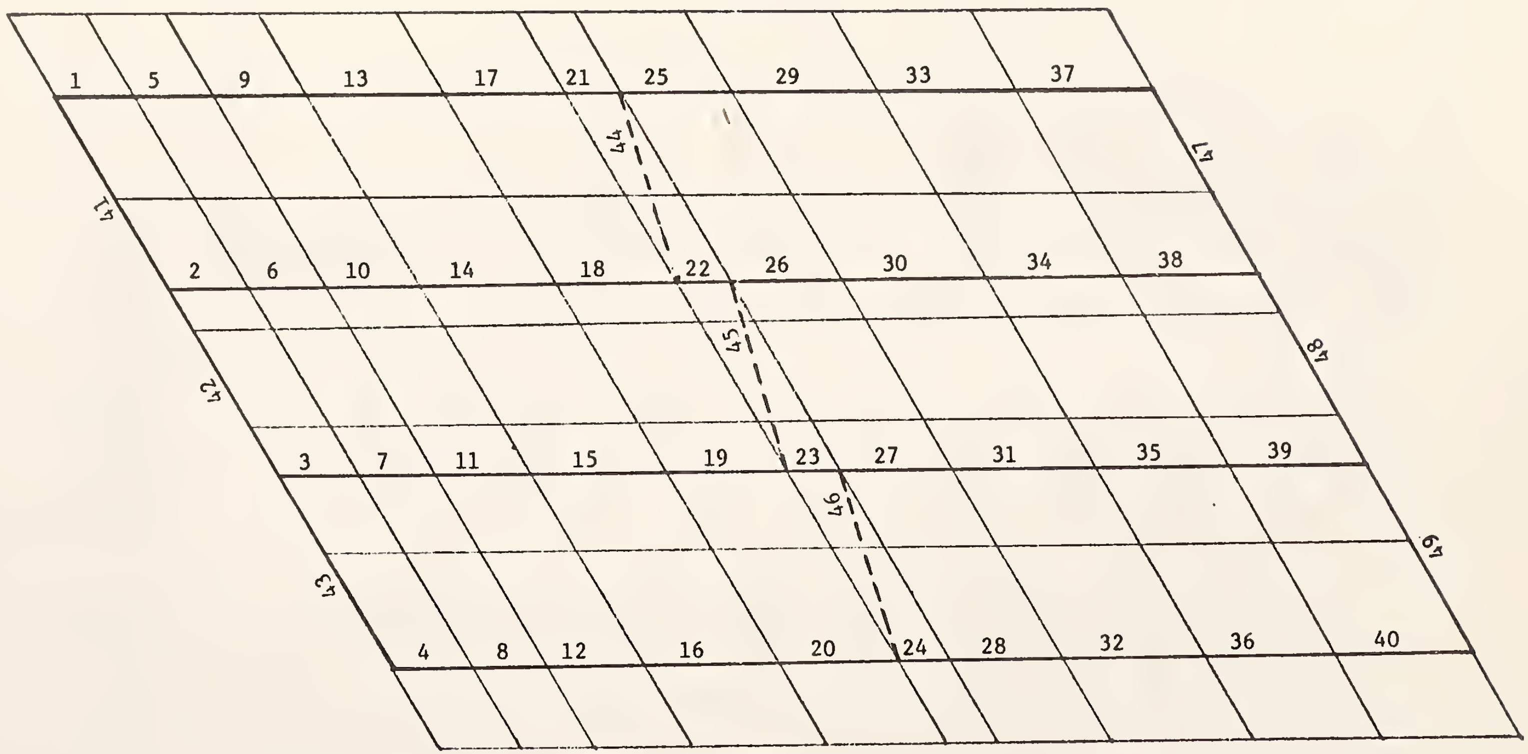


FIG. 39 - BEAM AND DIAPHRAGM NUMBERING FOR BRIDGE #3

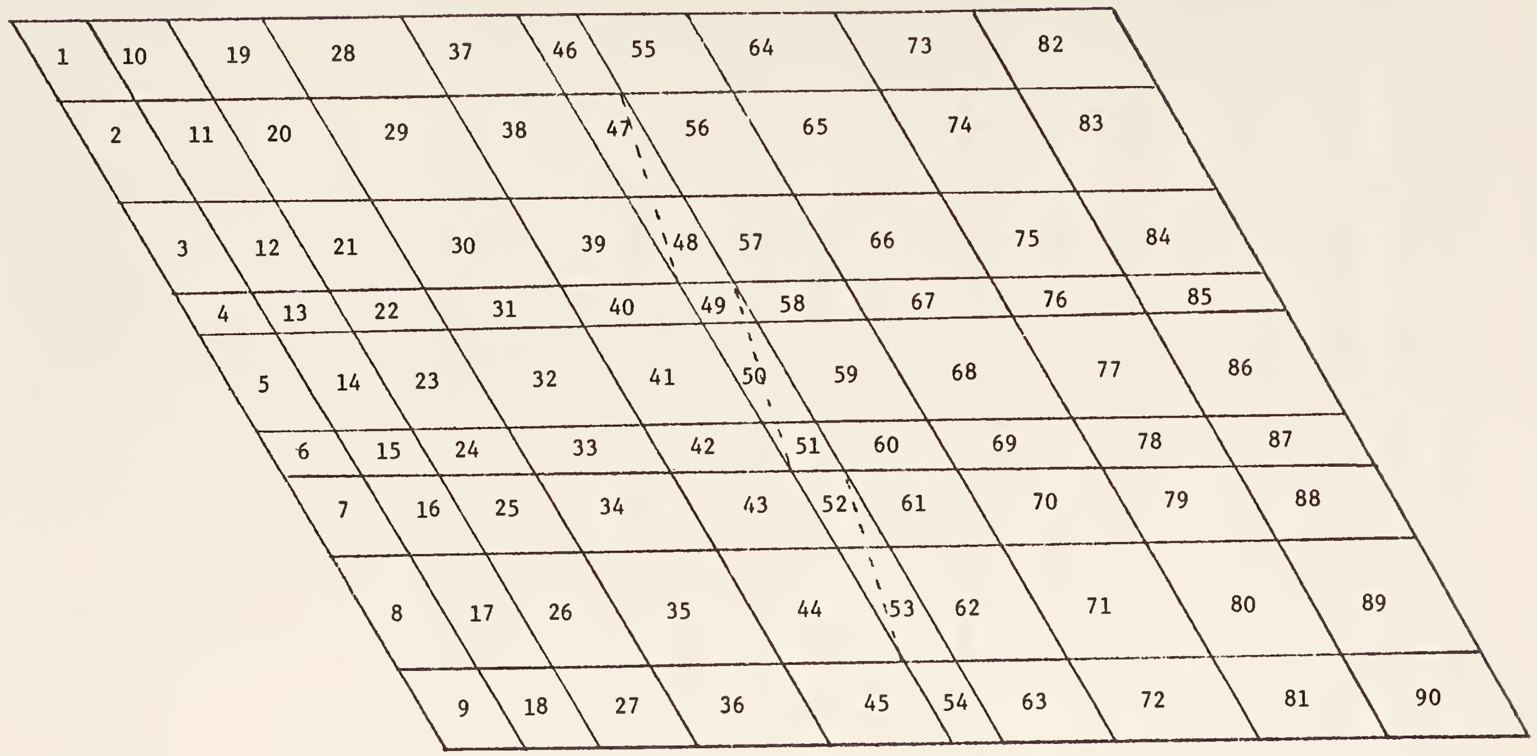
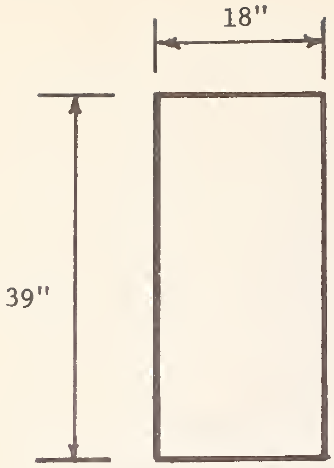


FIG. 40 - PLATE-SHELL ELEMENT NUMBERING FOR SLAB OF BRIDGE #3



By the usual process in mechanics:

I_x = 88978.5 in^4 = 4.291016 ft^4

I_y = 18954.0 in^4 = 0.914063 ft^4

J = (1 - .63x18/39)(39x18^3/3)

= 53771.036 in^4 = 2.593123 ft^4

Area = 702 in^2 = 4.985 ft^2

FIG. 41 - MOMENT OF INERTIA CALCULATIONS FOR GIRDER AND DIAPHRAGMS OF BRIDGE #3

4. Correlation of Computer and Experimental Data for Bridge #3

Figures 42 through 45 show load deflection relationships in a similar way as was described for bridge #2. Overprinted on these figures are the results of the computer program with the symbol ⊙ . As for bridge #2, the SSAP2 results agree fairly well with the experimental data, as long as the load is within the linear portion of the load deflection curve. Table 10 compares these results in a little more detail.

Two runs were performed for bridge #3, one with a center diaphragm and the other without the center diaphragm. The input total load for each run for the bridge was 842 kips, moment distributed to the girders from the loading points shown in Figure 9.[*] When we first take a look at the data for 842 kips, considerable disagreement between computer and experimental results is evident. However, note that in Figure 42, the shape of the deflection's variation across the cross-section changes materially after a load of 492 kips with a rapid dishing out of the center of the roadway of the bridge, indicating some sort of non-linear type of behavior.[**] Therefore, as we reduce the loading to 492 kips and once again make the comparison, we see that there is a fairly good correlation between predicted and measured response. Apparently bridge #3 develops the local non-linearity at a fairly low load. The reason for this phenomenon is not quite understood by the author or described in reference (1). We should also note that modeling with diaphragms does make a significant difference in computer results for this bridge.

---

* 842 kips was estimated as an upper limit for the linear load deflection relationships of Figures 43 and 44.

** The same phenomenon is evidenced for bridge #4 for a large load, but does not appear in the diagrams for bridges #1 or #2.

TABLE 10

COMPARISON OF COMPUTER AND EXPERIMENTAL DEFLECTIONS FOR BRIDGE #3

Description	Nodes	Node Location				Average	Load Kips
		72	75	79	82		
1. Beam Elements for Girder Slave Connected to Plate Elements for Slab--No center Diaphragm	154	.0356	.04837	.0477	.0349		842, at girder points.
2. Values linearly reduced to 492 kips		.029	.039	.039	.028	.034	492
3. Same as No. 1 with center diaphragm	154	.03194	.03716	.03761	.03582		842
4. Values linearly reduced to 492 kips		.026	.030	.030	.029	.029	492
Figures 43-45		.025	.049	.049	.025	.034	492
Figure 42		.025	.039	.039	.029		492

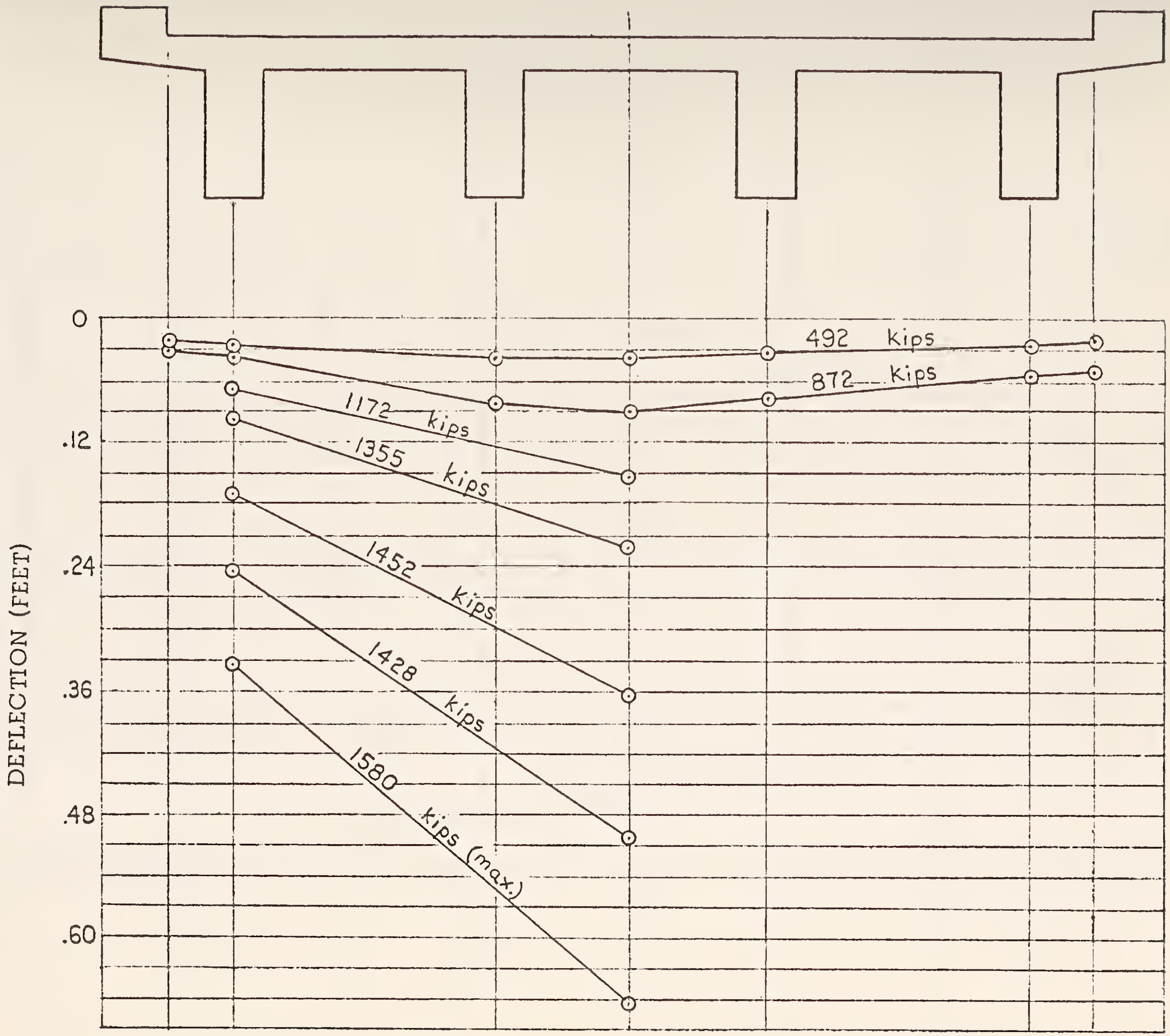


FIGURE **42**: CROSS-SECTION PLOTS SHOWING DECK DISTORTION UNDER LOAD - BRIDGE 3

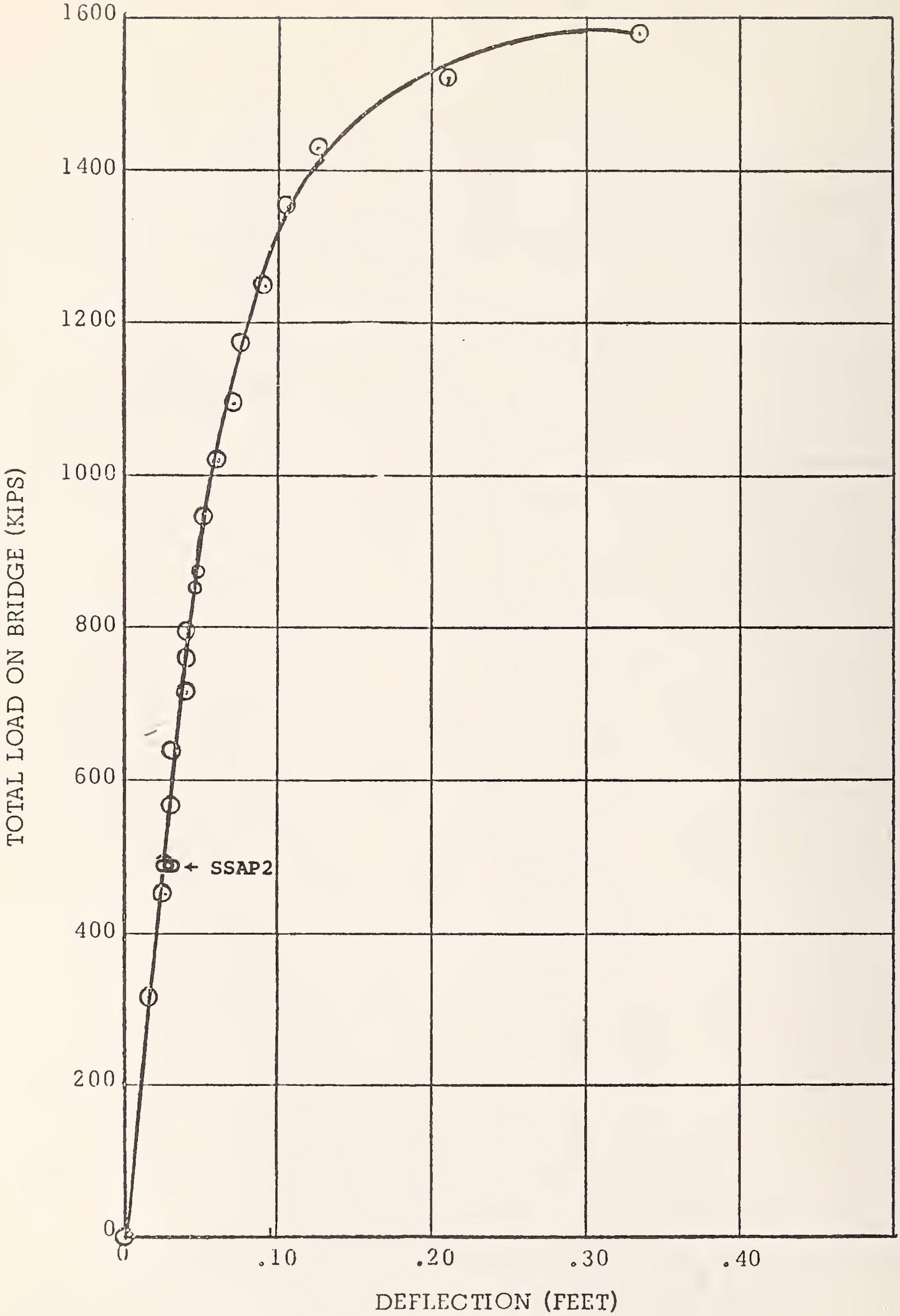


FIGURE 43. LOAD-DEFLECTION CURVE - CENTERLINE OF SPAN OVER EXTERIOR BEAM - BRIDGE 3.

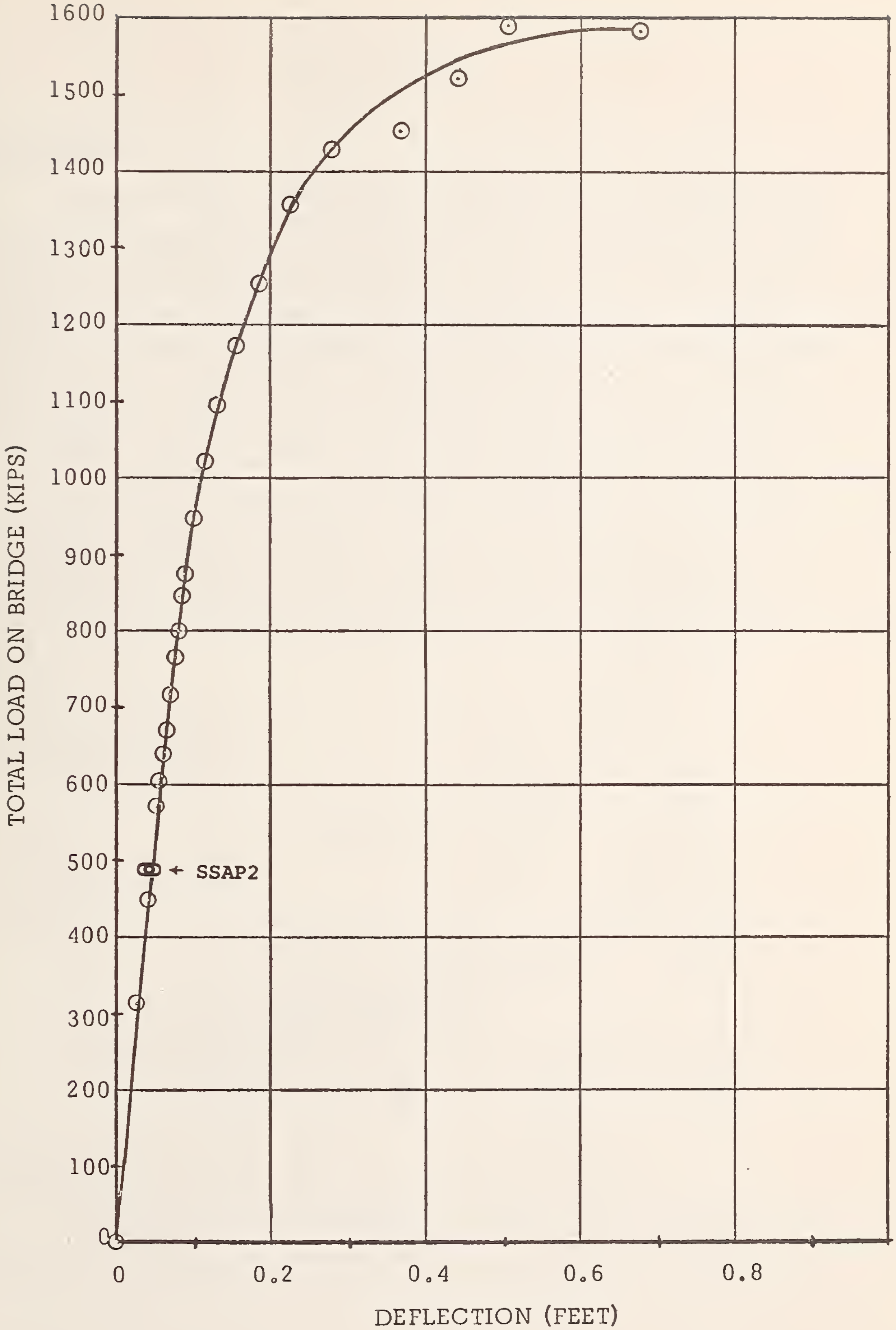


FIGURE 44. LOAD-DEFLECTION CURVE - CENTERLINE OF SPAN AT CENTER OF ROADWAY - BRIDGE 3.

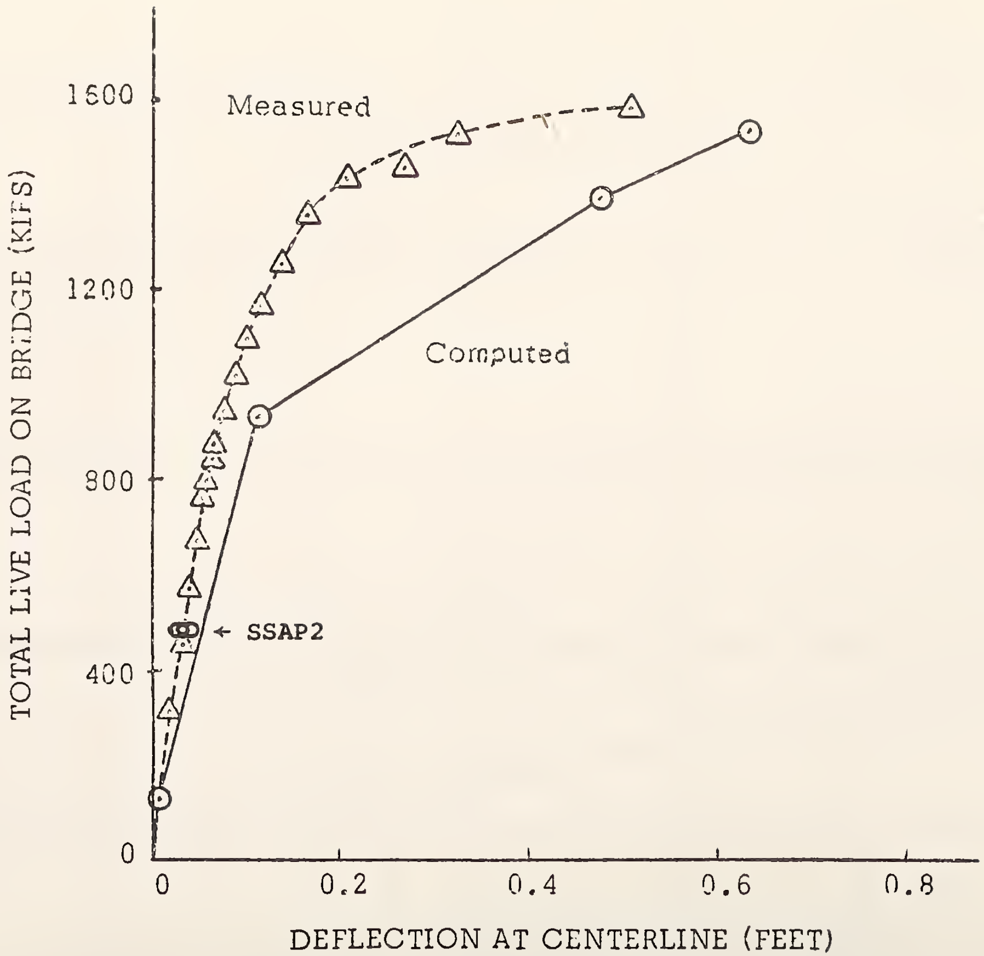


FIGURE **45.** COMPARISON OF MEASURED AND COMPUTED LOAD-DEFLECTION CURVES FOR BRIDGE 3.

VI. Bridge #4

1. Description

Bridge #4 was a three-span, continuous, non-composite design located on Mansford Road over the Elk River. Figure E-14 shows the single construction drawing available to the author. Photographs of the bridge are shown in Figure 4. This steel girder bridge was built in 1956 to support an H-15 load, and it was designed to act non-compositely. However, experimental strain readings indicated that a large degree of composite action did exist up to yielding of the steel at a load of approximately 500 kips.(11) From a pragmatic point of view the computer analysis should have been done without composite action. On the basis of experimental evidence, however, it was decided to input the data for composite action. Figure 46 describes the node numbering, Figure 47 the beam element numbering, and Figure 48 the plate element numbering for the deck.

2. Modulus of Elasticity

From Table 6:

$$f'_c = 5{,}600 \text{ psi}$$

By calculation:

$$E_c = 614369 \text{ ksf}$$

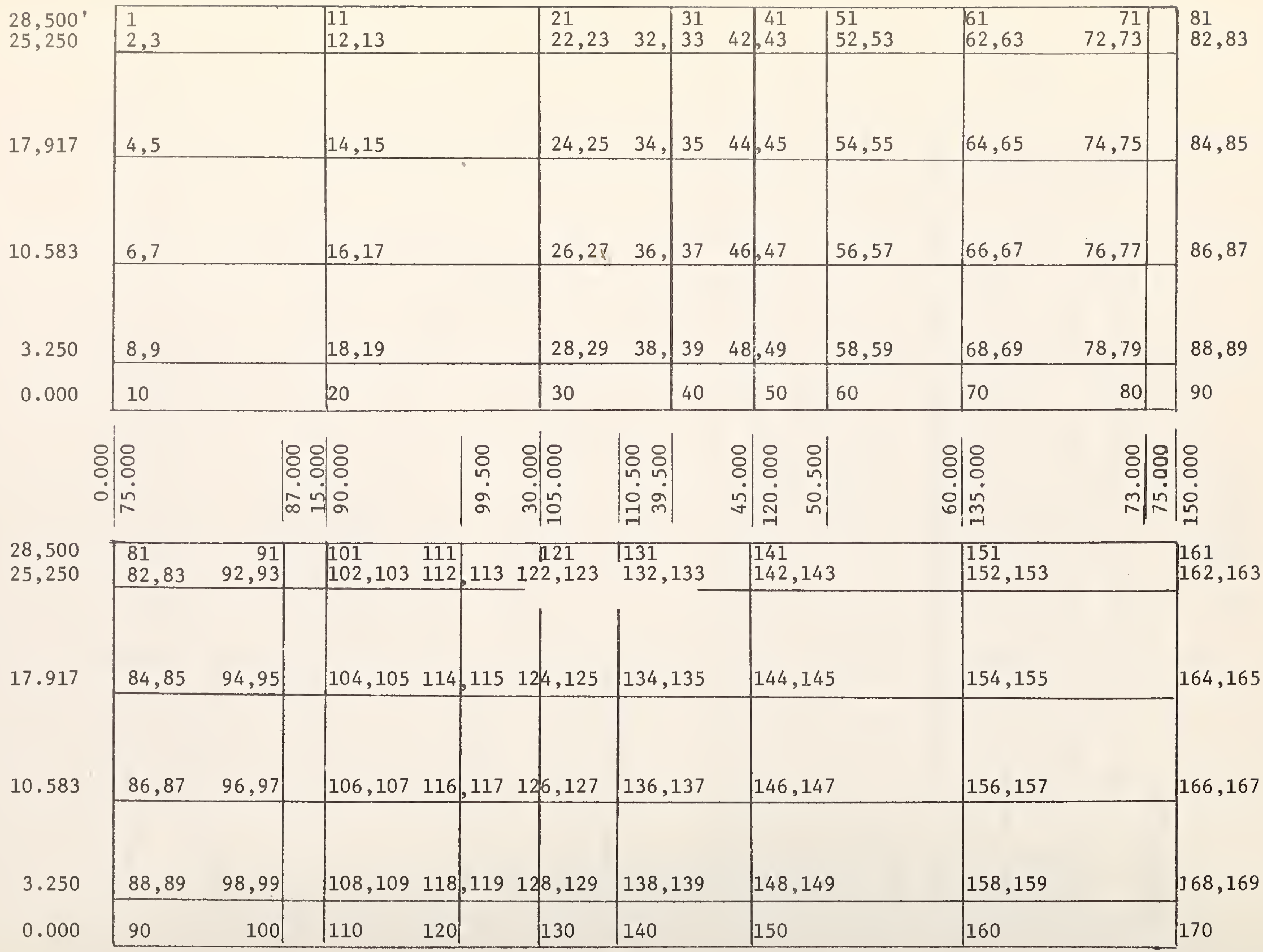
$$C_{xx} = C_{yy} = 628510.28$$

$$C_{xy} = 94276.542$$

$$G_{xy} = 267116.86$$

3. Moments of Inertia

Bending and torsional moments of inertia were calculated for this bridge in a similar way to bridge #1. The results are shown in Figure 49.



NOTE: Where two numbers exist, the rightmost number is at the elevation of the stringer centroid.

FIG. 46 - LOCATION OF NODES FOR BRIDGE #4

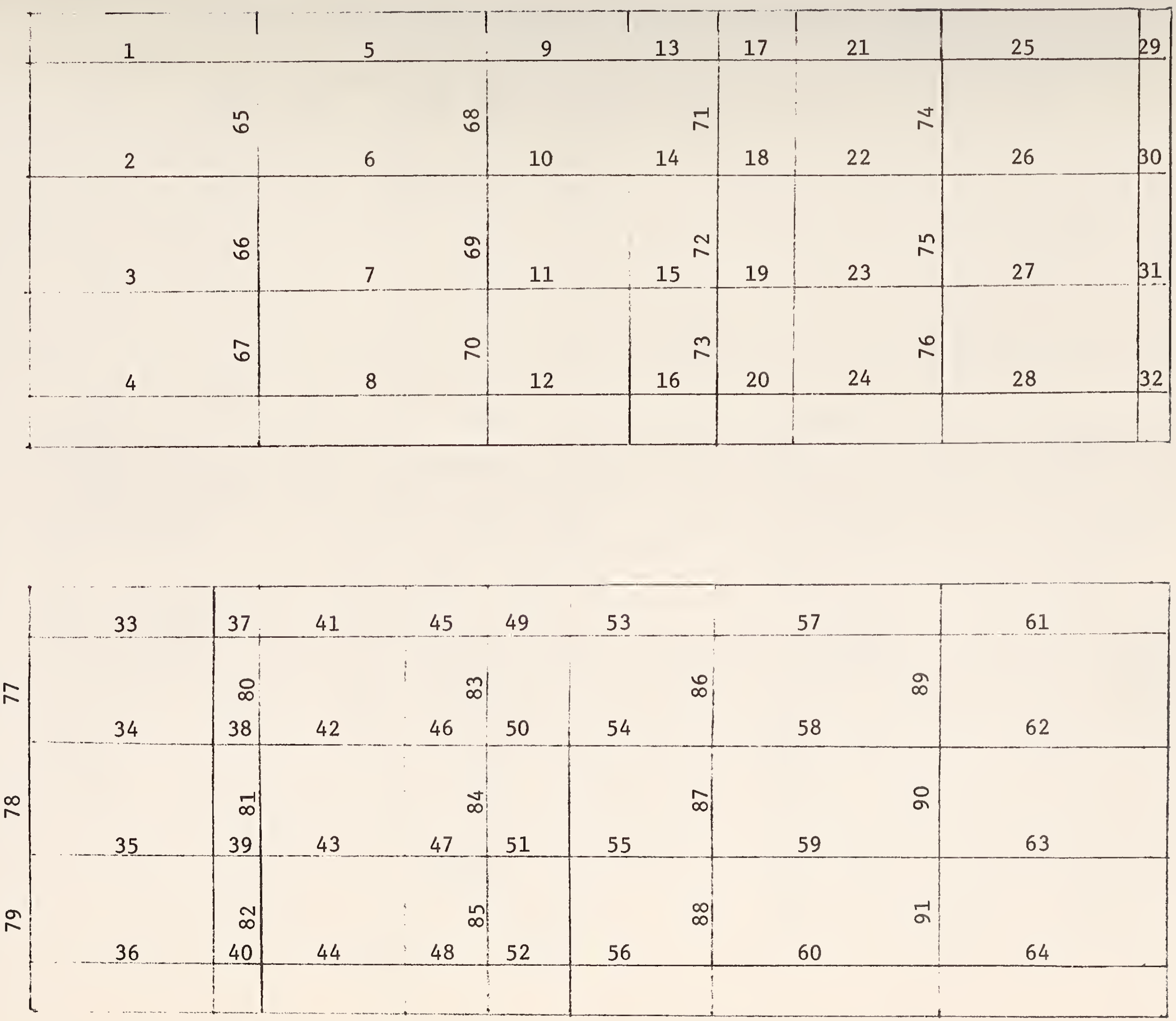


FIG. 47 - STRINGER AND DIAPHRAGM NUMBERING FOR BRIDGE #4

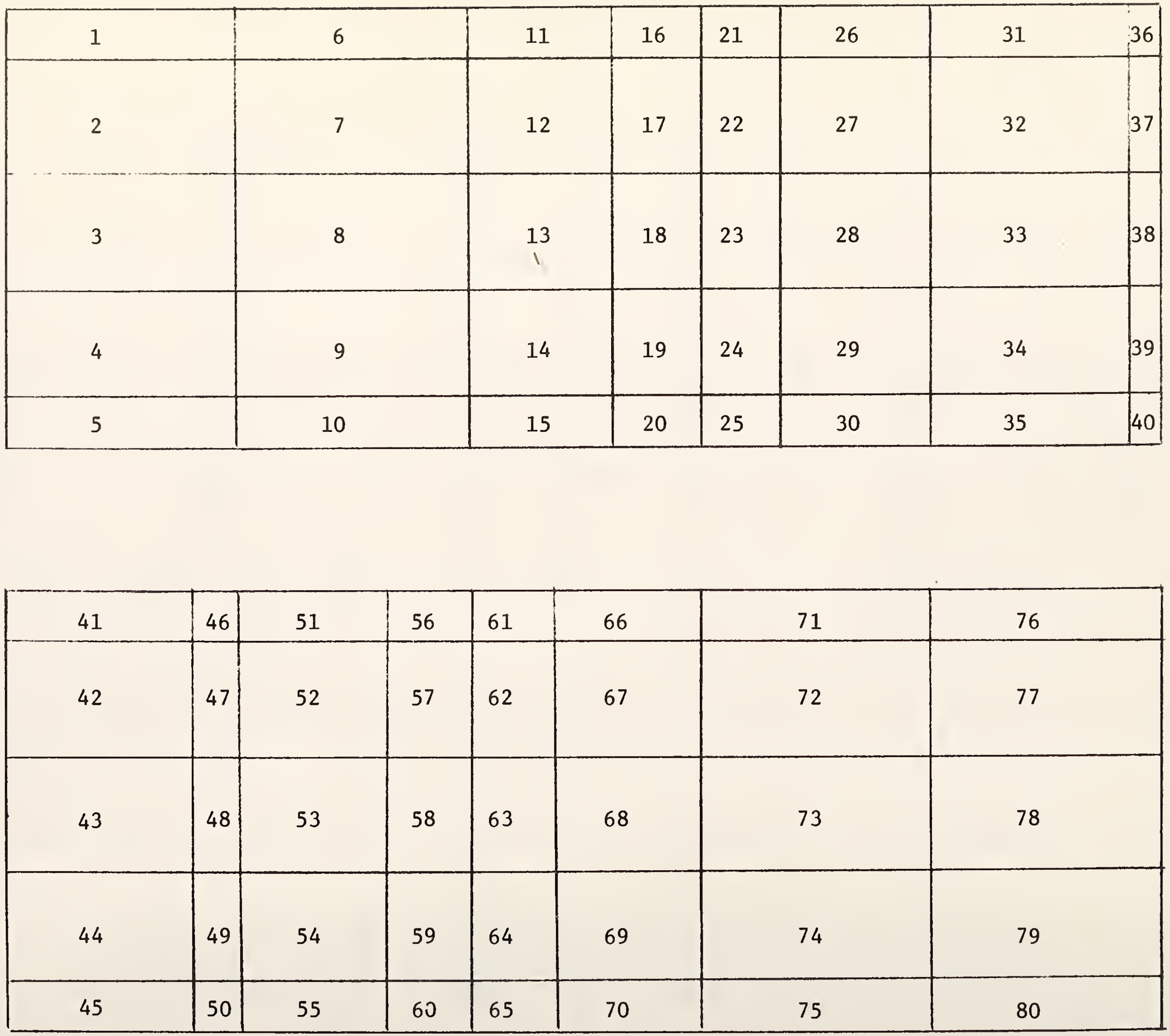
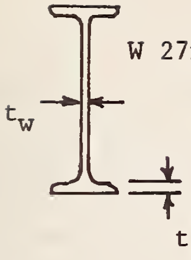


FIG. 48 - PLATE-SHELL ELEMENT NUMBERING FOR SLAB OF BRIDGE #4



Let c = 1728 x 12 d = 144

I_x = 3270/c = .157697 ft^4

I_y = 124/c = .0059799

J = 4.06/c = .0001957

t_f = .747 t_w = .490

Shear web area = 26.91 (.49)/d = .091569 ft^2

Total area = 27.7/d = .192361

I_x = 3610/c = .174093

I_y = 139/c = .006703

J = 5.27/c = .0002541

Shear web area = 27.07 (.518)/d = .097377

Total area = 30.0/d = .208333

I_x (covers) = 2 (10) (.75) $(13.91)^2$ = 2902.3
+ 3610
= 6512.3

I_x = 6512.3/c = .314058

I_y (covers) = 2 (.75) (1000)/12 = 125
+ 139
= 264

I_y = 264/c = .0127314

J (covers) @ $\underline{a}t^3/3$) = 2 (10) $(1)^3/3$) = 6.667
+ 5.27
= 11.937

J = 11.937/c = .000576

I_x = 129/c = .006221

I_y = 3.88/c = .0001871

J = .371/c = .0000178

t_w = .282

Shear Area = 12 (.282)/d = .0235

Total Area = 6.09/d = .042292

FIG. 49 - MOMENTS OF INERTIA CALCULATIONS FOR STRINGERS AND DIAPHRAGMS-BRIDGE #4

4. Correlation of Computer and Experimental Data for Bridge #4

Bridge #4 behaved in practically an identical way to bridge #1. Figures 51 through 53 show experimental load deflection relationships overprinted with the computer results by a symbol ⊕ . This time the loadings were not symmetrically placed as seen in Figure 10. Therefore, for the convenience of the reader the applied loads (after moment distribution to the girders) are given in Table 11. The excellent correlation between predicted and experimental response is self-evident.

Studies were also made for bridge #4 for different modeling techniques. As for bridge #1, the same conclusion applies here with regard to aspect ratio and type of modeling.

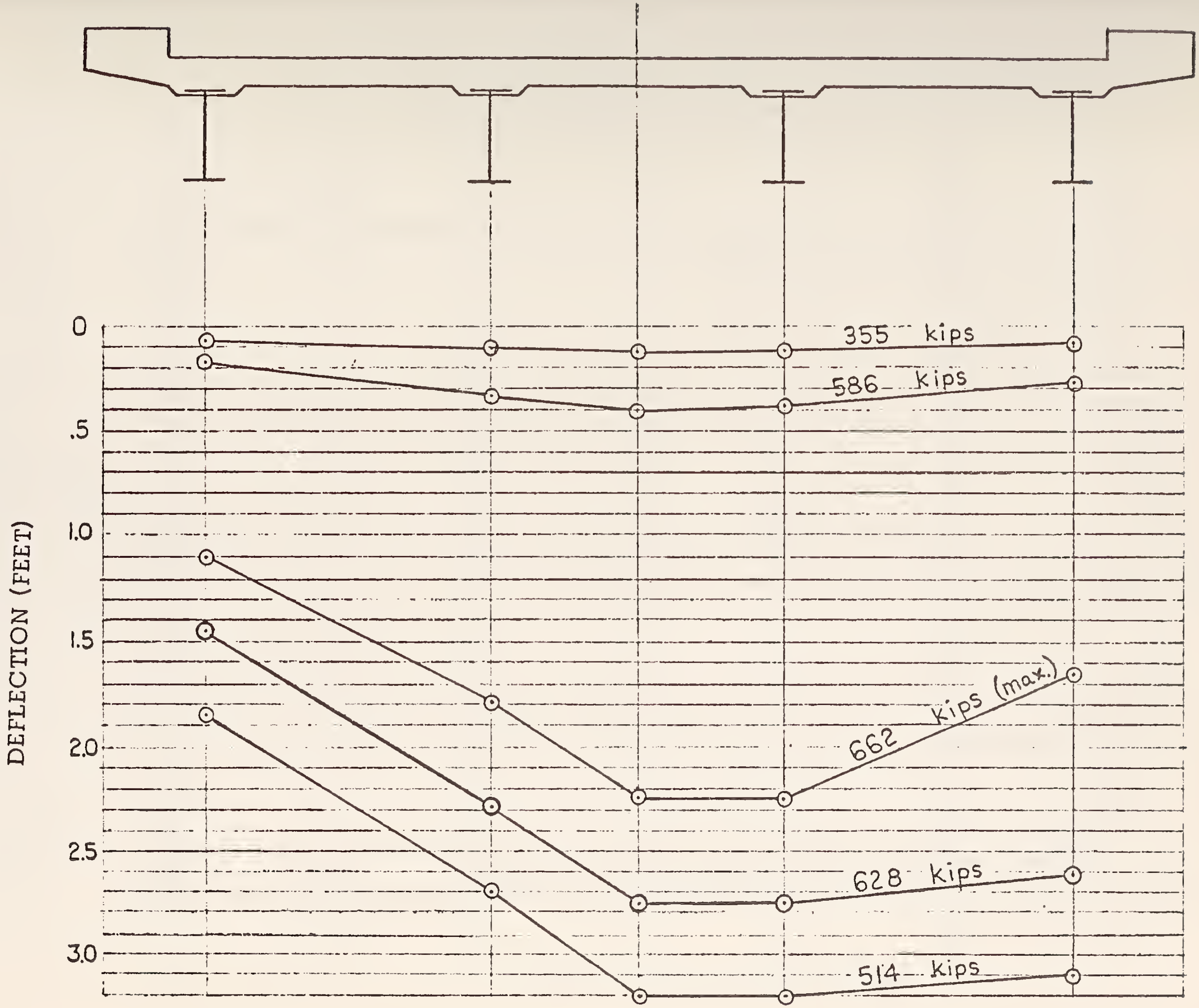


FIGURE 50: CROSS-SECTION PLOTS SHOWING DECK DISTORTION UNDER LOAD - BRIDGE 4

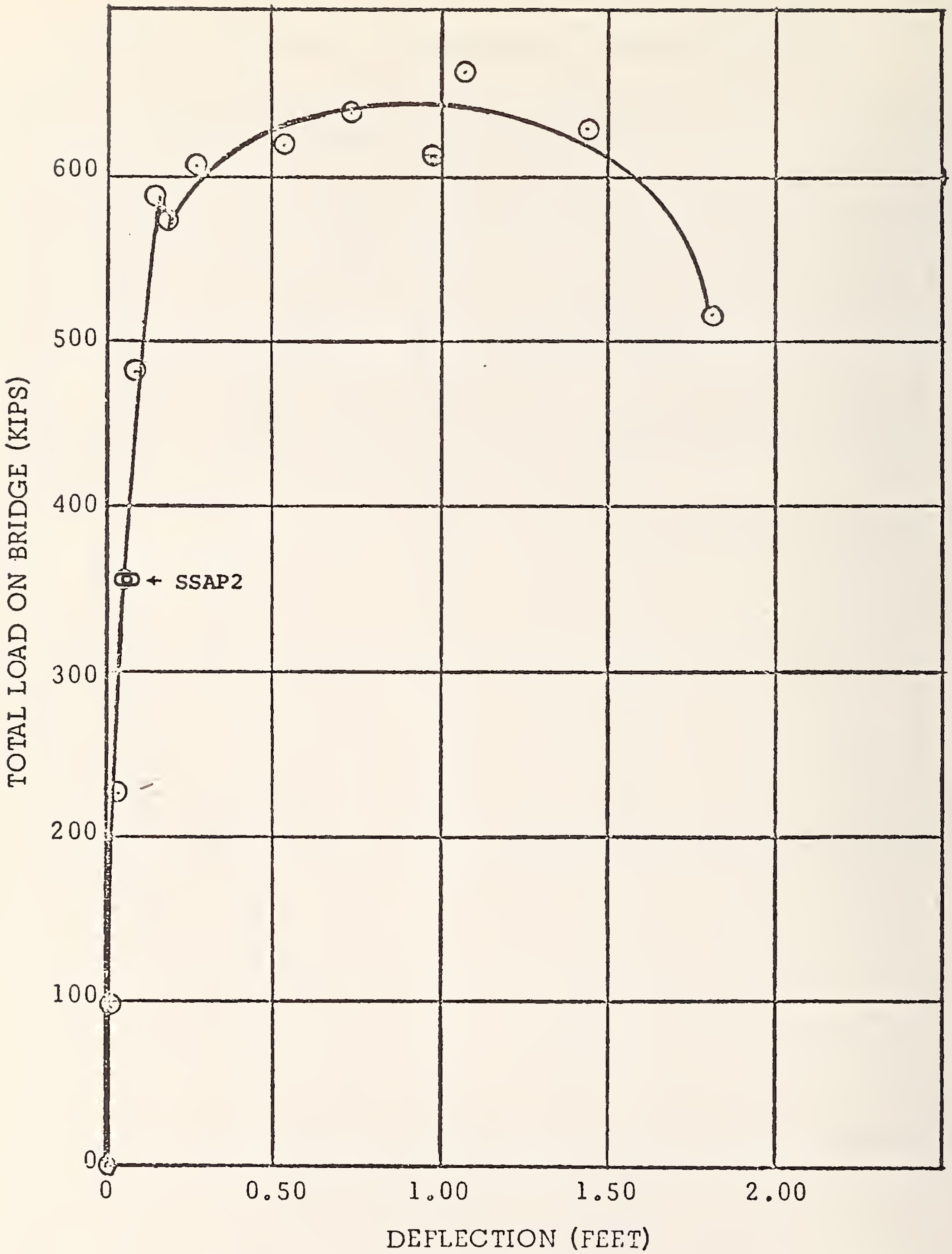


FIGURE 51. LOAD-DEFLECTION CURVE - CENTERLINE OF SPAN OVER EXTERIOR BEAM - BRIDGE 4.

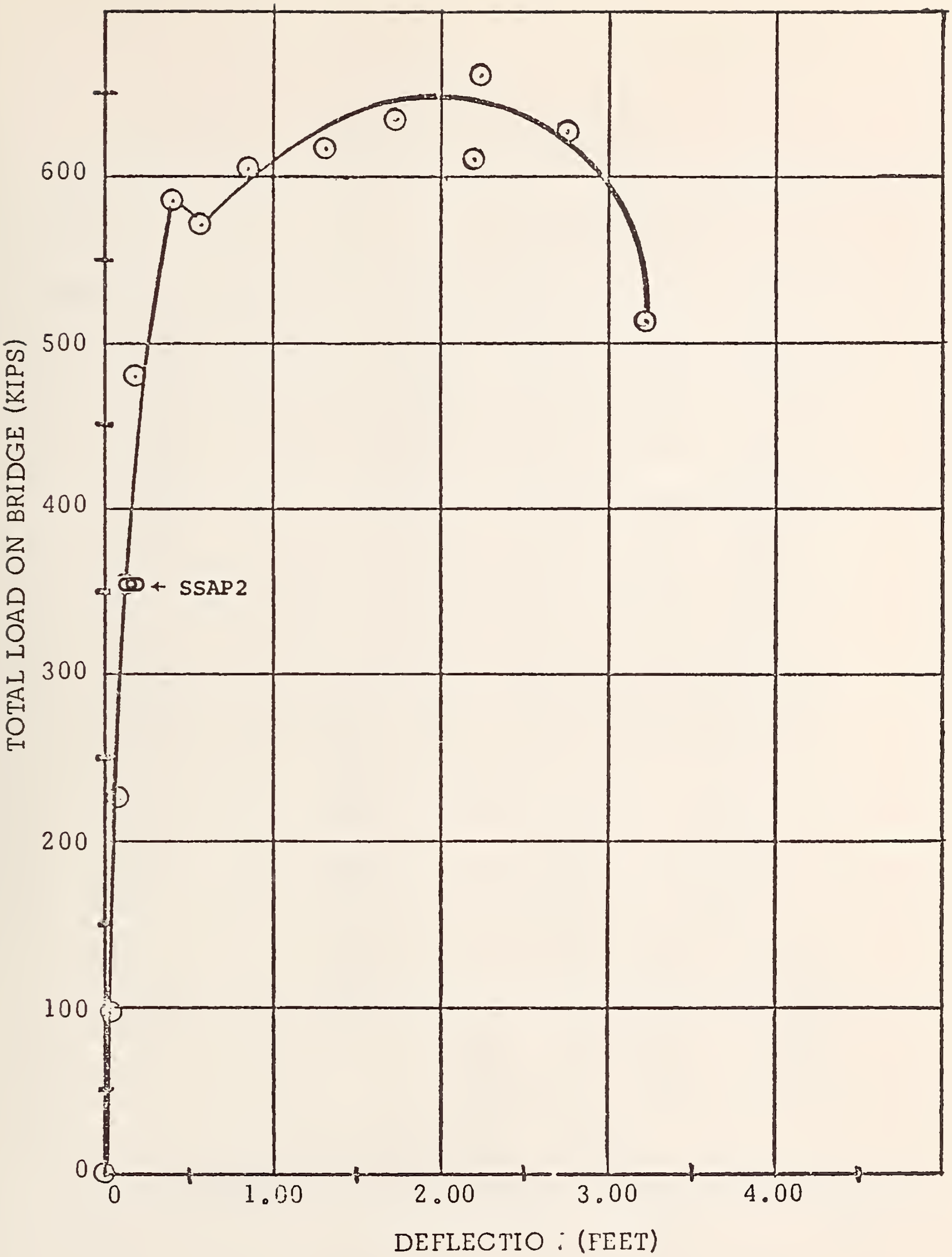


FIGURE 52. LOAD-DEFLECTIO N CURVE - CENTERLINE OF SPAN AT CENTER (F ROADWAY - BRIDGE 4.

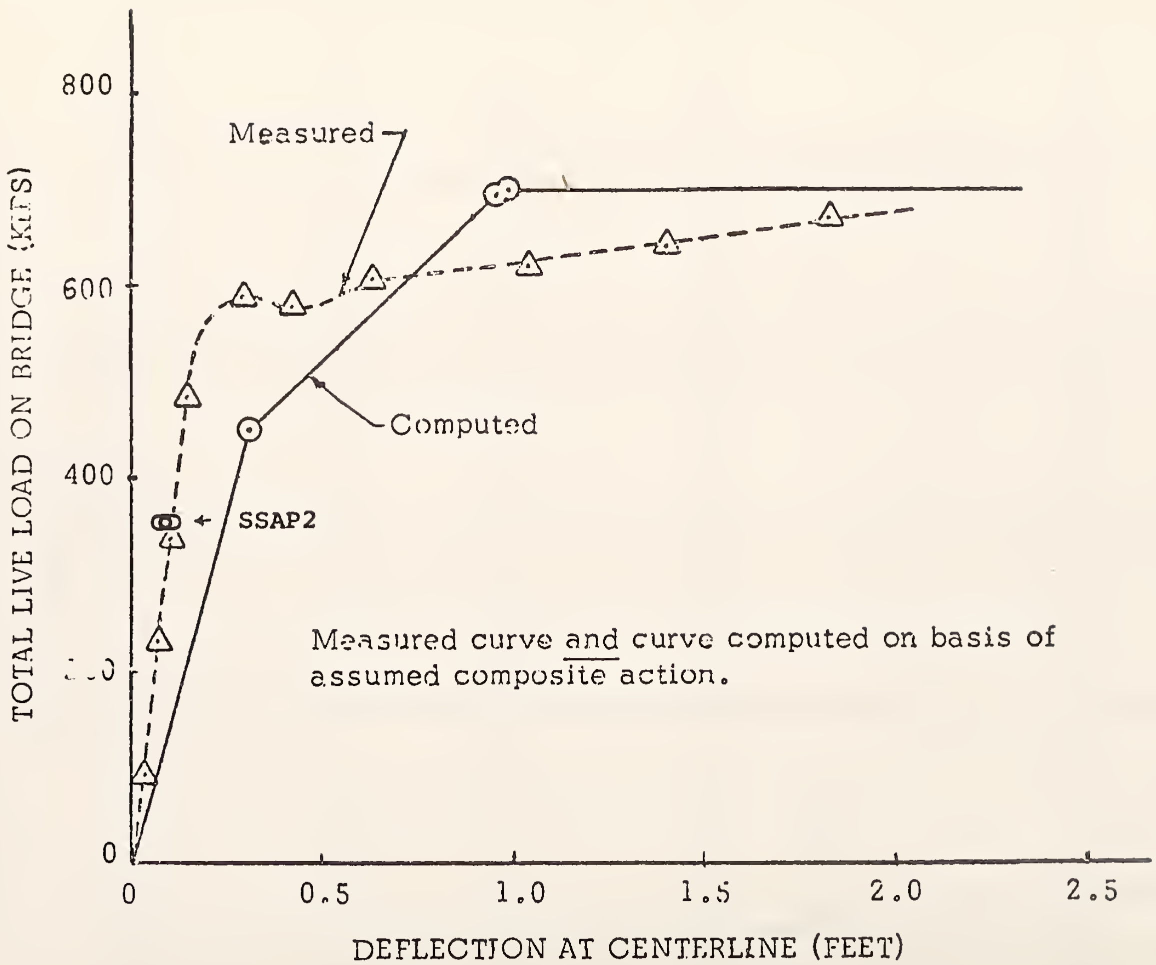


FIGURE **53**. COMPARISON OF MEASURED AND COMPUTED LOAD-DEFLECTION CURVES FOR BRIDGE 4.

TABLE 11 - LOADS APPLIED TO BRIDGE #4

.....NODAL POINT LOADS

NODE NO.	LOAD CASE	RX	RY	RZ	MX	MY	MZ
		APPLIED LOADS					
72	1	-0.000	-0.000	-24.610	-0.000	-0.000	-0.000
74	1	-0.000	-0.000	-93.390	-0.000	-0.000	-0.000
76	1	-0.000	-0.000	-93.390	-0.000	-0.000	-0.000
78	1	-0.000	-0.000	-24.610	-0.000	-0.000	-0.000
92	1	-0.000	-0.000	-25.450	-0.000	-0.000	-0.000
94	1	-0.000	-0.000	-87.730	-0.000	-0.000	-0.000
96	1	-0.000	-0.000	-5.650	-0.000	-0.000	-0.000
98	1	-0.000	-0.000	.830	-0.000	-0.000	-0.000

VII. CONCLUSIONS

1. The SSAP2 program, correctly modeled, gives excellent correlation between predicted and experimental results for straight, continuous, composite steel girder concrete deck bridges and for skew, simple span composite concrete girder and concrete deck bridges when the skew is under 30°. The reason for limiting the angular deviation of the skew line from perpendicularity to 30° is that the computer runs were limited to this maximum value. The author hesitates to extrapolate the information much beyond that without further study. Conversations with Dr. Felippa indicate that the plate-shell element may give poor results (become too stiff) when the deviation angle is more than 30°.

2. This research is limited to a linear analysis program; therefore, the computer program, as it now stands, can only safely be used when the response of the bridge is known to be in a linear range. It should be possible, by a stepwise method, to change the properties of a bridge and thereby extend the range of correlation into the elasto-plastic zone. More research needs to be done in this area to determine the accuracy of this supposition.

3. Modeling the bridge as a beam for computer analysis is not recommended except, perhaps, for very basic preliminary design for estimating purposes.

4. The use of a beam element for the stringers of the bridge, slave connected to a plate-shell element for the deck offers the most economical model, considering such items as cost per computer run, limit on the aspect ratio, and convergence to the true solution.

5. Diaphragms, as they are usually constructed, do not seem to play an important part in changing the vertical deflections of either straight or skew bridges. Their use, however, in bridge design can be mandatory for other purposes.

VIII. RECOMMENDATIONS

1. It is recommended that a curved beam element be investigated for its correlation in the SSAP2 program with experimental data on curved girder bridges.

2. The capabilities of a dynamic version of SSAP2, the SAP-IV program, should be examined with respect to correlation of observed dynamic behavior on some bridges. It is know that a motion picture was made of a pedestrian undercrossing subjected to man-induced vibrations.[12] It is possible to calculate the actual response of the bridge from the motion picture and thus recover the experimental data for verification with the dynamic computer program.

3. The possibility of non-linear programs should be investigated for their correlation with the bridges described in this report on a grid model basis.

IX. APPENDICES

Appendix A contains one version of a theoretical finite element analysis via the statements in SSAP2. The theory is limited to only those elements which were used in this research.

Appendix B consists of excerpts taken directly from the User's Manual for the SSAP2 computer program (13) for the rod, beam, and plate-shell elements. Supplementary information concerning slave-connected nodes and node boundary conditions are excerpted from SAP-IV.(9)

Appendix C contains a listing of the program. Provision is left in the listing to replace additional elements of the SSAP2 program as called for in reference (13). The printout of the listing included in this report will list the additional elements as external references. The IBM 360/370, CDC 6400, and Honeywell 6060 versions of SSAP2 are given in Appendix C.

References are given in Appendix D, and reduced abbridged construction drawings for the four bridges described in this report are given in Appendix E.

Appendix F contains excerpts from Dr. Felippa's paper "A Refined Quadrilateral Element for Analysis of Plate Bending."

Appendix G contains notes received from Dr. Felippa concerning the LST formulation used in Subroutine SLST.

APPENDIX A

Finite Element Background Theory for SSAP2 Computer Program

A. Stiffness of Rod Elements

1.1 Introduction

We have seen in the main body of this report that the bridges were first idealized as an assembly of structural elements and that these elements were attached to each other at node points. The SSAP2 program utilizes the matrix displacement method, in which the stiffness characteristics of elemental structural elements are first determined. Finally, the stiffness matrix for the whole structure is formed by adding the element stiffness matrices; and the governing joint equilibrium equations are written in the following form:

$$\underline{K}\ \underline{u} = \underline{R} \qquad \text{(A.1)}$$

where

$\underline{K}$ = assembled stiffness matrix for entire structure

$\underline{u}$ = coordinate displacement matrix of node points

$\underline{R}$ = coordinate reactions on elements at node points

The derivation of element stiffnesses will now be shown.*

1.2 Stiffness and Flexibility of a Straight One-Dimensional Prismatic Bar

Consider ij as an element in a one-dimensional structure oriented along the X direction as shown in Figure A-1. Node i is held fixed against translation while node j is given a unit force in the X direction. u_j represents the elongation of the bar of length L in the direction of the force.

---

* The derivation can be done in many ways. That shown here is only one viewpoint, that of the author's, and may be considerably different for some of the theory used by Dr. Wilson for SSAP2. Some of the one and two-dimensional presentation for rods and beams follow lecture notes taken by the author while attending Dr. Kamel's lectures at the University of Arizona.

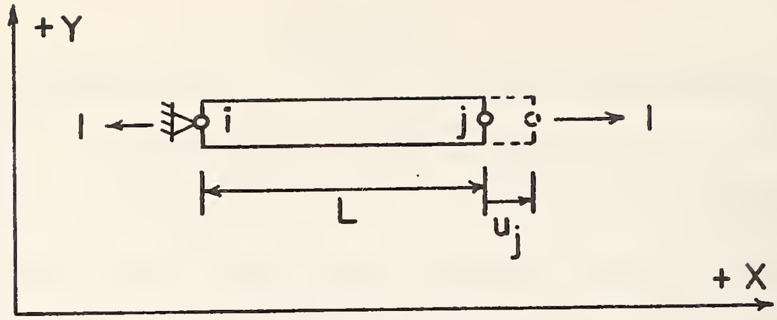


FIG. A-1 ONE DIMENSION ELEMENT WITH UNIT AXIAL LOAD

From Statics: Stress $\sigma_x = \frac{1}{A}$ (A.2)

From Hooke's Law: Strain $\varepsilon_x = \frac{\sigma_x}{E} = \frac{1}{EA}$ (A.3)

From geometry: Displacement $u_j = \varepsilon_x(L) = \frac{L}{EA}$ (A.4)

where

A = member cross sectional area

E = elastic modulus of member material

Since u_j is the displacement due to a unit load at point j, $\frac{L}{EA} = f$ represents the flexibility of the bar at point j when end i is fixed against translation. Now reconsider the same bar, as in Figure A-2, but this time the displacement at point j equals unity.

We wish to ascertain the force k needed for this unit displacement.

From geometry $\varepsilon_x = \frac{1}{L}$ (A.5)

From Hooke's Law: $\sigma_x = E(\varepsilon_x) = \frac{E}{L}$ (A.6)

From statics: $k = \sigma_x(A) = \frac{EA}{L}$ (A.7)

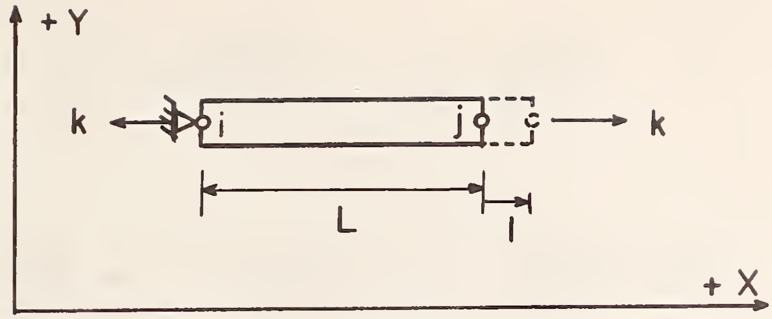


FIG. A-2 ONE DIMENSION ELEMENT WITH UNIT ELONGATION.

Since k is the force at point j for a unit displacement at point j, $k = \frac{EA}{L}$ represents the stiffness of the bar at point j when end i is fixed against translation.

1.3 Stiffness Coefficients for a Single Straight One-Dimensional Prismatic Rod

Consider a one-dimensional structure oriented in the X direction in which bar ij is a single element of the structure, as shown in Figure A-3. We consider three cases:

Case 1: $u_i = 1$ to the right

$u_j = 0$ (fixed)

Figure A-3a shows the equilibrating forces at nodes i and j. Note that the reaction at j is assumed in the positive coordinate direction for use in an equilibrium equation; although it is recognized that the final direction of the force at j would be to the left to produce a compressive force in the bar.

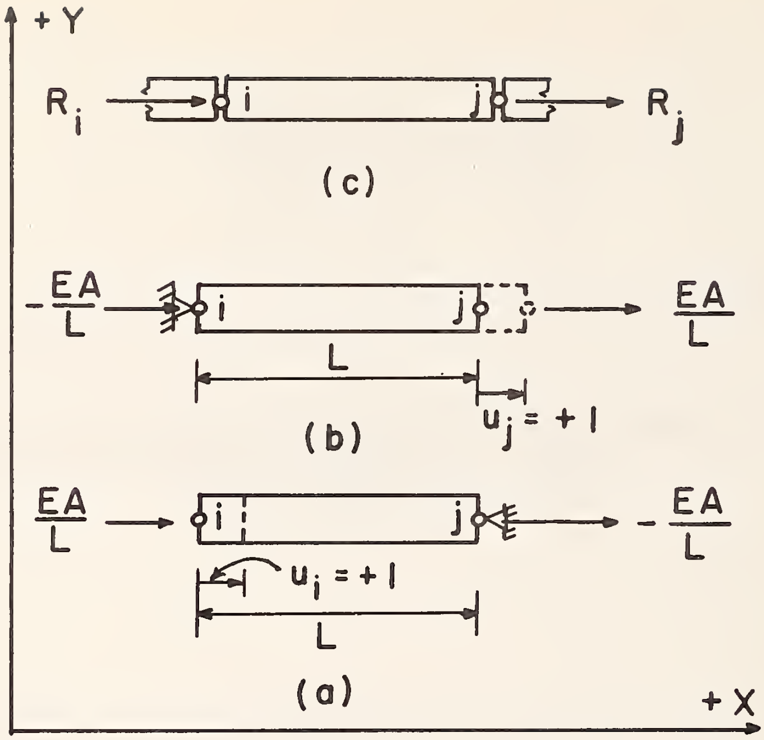


FIG. A-3 SUPERPOSITION OF REACTIONS FOR UNIT ELONGATIONS AT BOTH ENDS OF ONE DIMENSION ELEMENT.

Case 2: $u_i = 0$ (fixed)

$u_j = 1$ to the right

Figure A-3b shows the equilibrating forces for this condition. Once again the forces at all nodes are assumed to be in the positive coordinate direction for insertion in the equilibrium equations.

Case 3:

In the general condition of the structure under load it is probable that neither u_i or u_j will be equal to zero. However, the forces R at nodes i and j can be arrived at by a superposition of Figures A-3a and A-3b as follows:

$$R_i = \frac{EA}{L}(u_i) - \frac{EA}{L}(u_j)$$

$$= \frac{EA}{L}(u_i - u_j)$$

$$= k_{ii}(u_i) + k_{ij}(u_j) \qquad (A.8)$$

$$R_j = -\frac{EA}{L}(u_i) + \frac{EA}{L}(u_j)$$

$$= -\frac{EA}{L}(u_i - u_j)$$

$$= k_{ji}(u_i) + k_{jj}(u_j) \qquad (A.9)$$

where

$$k_{ii} = k_{jj} = +\frac{EA}{L}$$

$$k_{ij} = k_{ji} = -\frac{EA}{L}$$

Equations (A.8) and (A.9) can also be expressed in matrix form:

$$\begin{bmatrix} R_i \\ R_j \end{bmatrix} = \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \end{bmatrix} \qquad (A.10)$$

or

$$\bar{R} = \bar{K}\ \bar{u}$$

$$= \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \qquad (A.11)$$

where

R = node forces of rank [2 x 1]

K = rod stiffness matrix of rank [2 x 2]

u = node deflections of rank [2 x 1]

1.4 Calculation of Strain

The strain of a rod is calculated by the following equation:

$$\varepsilon_x = \frac{u_j - u_i}{L} \tag{A.12}$$

1.5 Stiffness Matrix for a Series of Rods

Consider a structure made up of two prismatic rods connected together at node 2, as shown in Figure A-4.

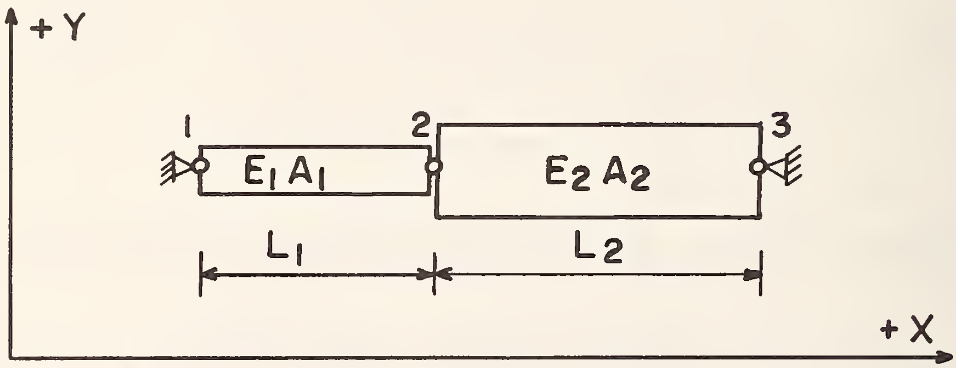


FIG. A-4 TWO PRISMATIC RODS IN ONE DIMENSION STRUCTURE.

Nodes 1 and 3 are supported nodes. We desire to show how each element stiffness matrix is superimposed to form the total structure stiffness matrix.

Let

$$K_1 = \frac{E_1 A_1}{L_1}$$

$$K_2 = \frac{E_2 A_2}{L_2} \tag{A.13}$$

Using equation (A.10)

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and

$$\begin{bmatrix} R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$$

By superposition

$$\begin{bmatrix} R_1 \\ R_2 \\ \\ R_3 \end{bmatrix} = \begin{bmatrix} K_1 & -K_1 & & \\ -K_1 & K_1 & & \\ & & K_2 & -K_2 \\ & & -K_2 & K_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \\ u_3 \end{bmatrix}$$

$$= \begin{bmatrix} K_1 & -K_1 & 0 \\ -K_1 & K_1+K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad \text{(A.14)}$$

or

$$\overline{R} = \overline{K}\,\overline{u} \qquad \text{(A.15)}$$

as previously defined.

1.6 Strain of an Inclined Rod

A prismatic rod element in a one-dimensional structure has two degrees of freedom representing the movement of each point in the one-dimensional direction. For an inclined rod, in a plane, where each node coordinate is represented by two dimensions, there are four degrees of freedom in the global system. A situation of this sort is shown in Figure A-5 where rod ij might be one element of a two dimensional truss.

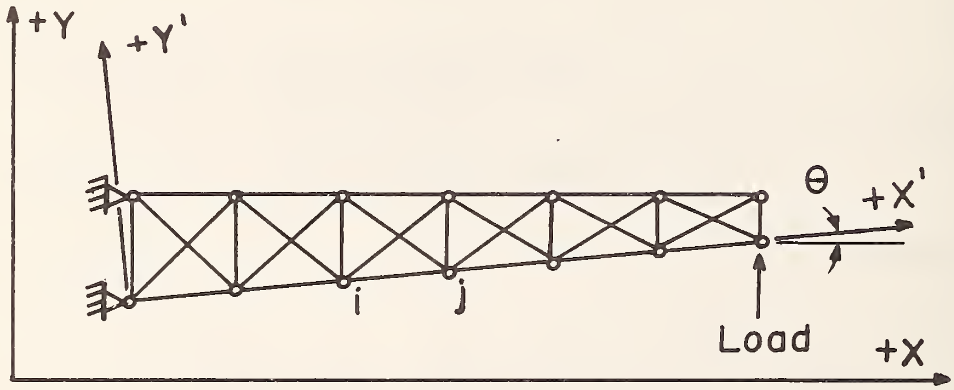


FIG. A-5 PRISMATIC TRUSS ELEMENT.

The truss has global coordinates XY, whereas the element has local coordinates X'Y'. Let us assume that under load, node points i and j move from their original positions. We desire to express the strain of the rod in terms of component movement of its two nodal points.

Let u_i, v_i, u_j, v_j, represent global coordinate movements of node points as shown in Figure A-6. u'_i, v'_i, u'_j, v'_j, represent the components of nodal point movements in terms of the local coordinates of the inclined rod.

Also consider that rod ij makes an angle θ with the X-axis. Define $\ell = \cos\theta$ and $m = \sin\theta$.

From Figure A-7:

$$u'_i = u_i\ell + v_i m$$

$$v'_i = v_i\ell - u_i m \qquad \text{(A.16)}$$

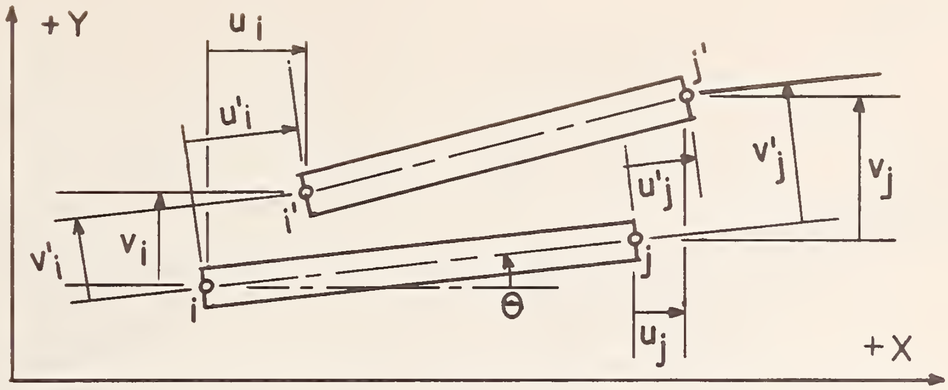


FIG. A-6 GLOBAL AND LOCAL COORDINATE DISPLACEMENTS OF ROD IN TWO DIMENSIONS.

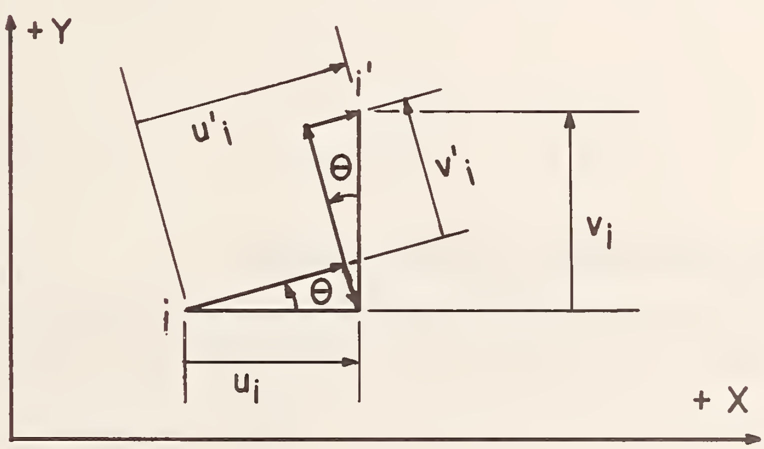


FIG. A-7 GEOMETRY OF COORDINATE TRANSFORMATION IN TWO DIRECTIONS.

Similarly:

$$u'_j = u_j \ell + v_j m$$

$$v'_j = v_j \ell - u_j m \qquad (A.17)$$

The strain becomes:

$$\varepsilon' = \varepsilon'_x = \frac{u'_j - u'_i}{L}$$

$$= \frac{u_j \ell + v_j m - u_i \ell - v_i m}{L}$$

$$= \frac{1}{L} [-\ell \ -m \ +\ell \ +m] \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix} \qquad (A.18)$$

or

$$\underset{1 \times 1}{[\varepsilon']} = \underset{1 \times 4}{[T_{\varepsilon' u}]} \underset{4 \times 1}{[u]} \qquad (A.19)$$

1.7 Stiffness Coefficients for an Inclined Rod

The stiffness matrix for an inclined rod in terms of global displacements can be derived as a superposition of four elementary cases.

Case 1:

$u_i = 1 \rightarrow$ (Figure A-8a)

$u_j = 0$

$v_i = 0$

$v_j = 0$

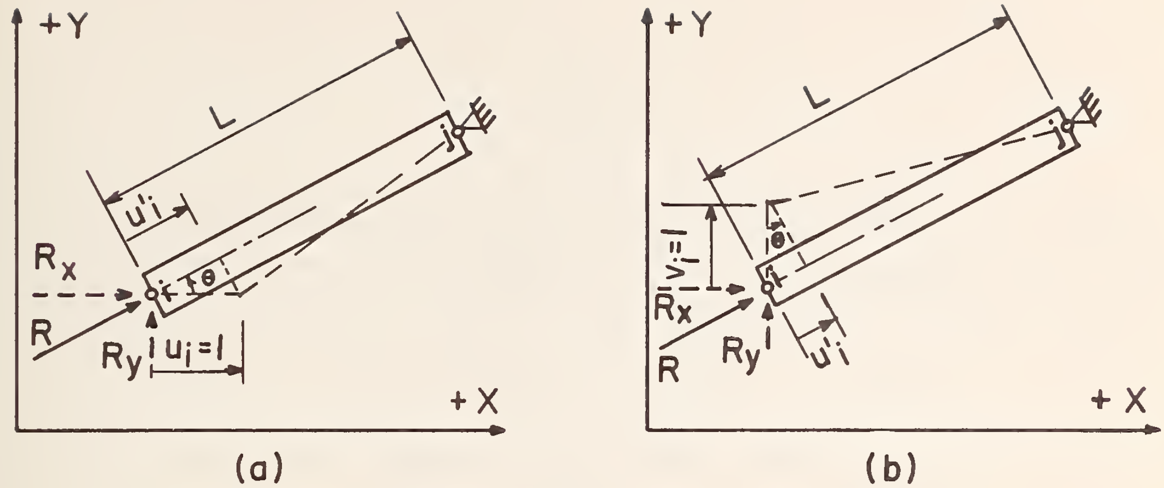


FIG. A-8 END NODAL FORCE COMPONENTS FOR GLOBAL COORDINATE UNIT DISPLACEMENTS.

$$u_i' = \ell$$

Strain: $\varepsilon' = \frac{\ell}{L}$

Stress: $\sigma = E\varepsilon' = \frac{E\ell}{L}$

Force $R = \frac{E\ell A}{L} = \alpha\ell$

where $\alpha = \frac{EA}{L}$

X Comp.: $R_{ix} = R\ell = \alpha\ell^2$

Y Comp.: $R_{iy} = Rm = \alpha\ell m$

<u>Case 2</u>:

$v_i = 1\uparrow$ (Figure A-8b)

$v_j = 0$

$$u_i = 0$$

$$u_j = 0$$

$$u_i' = m$$

Strain: $\varepsilon' = \frac{m}{L}$

Stress: $\sigma = \frac{Em}{L}$

Force: $R = \frac{EmA}{L} = \alpha m$

X Comp.: $R_{ix} = R\ell = \alpha m\ell$

Y Comp.: $R_{iy} = Rm = \alpha m^2$

Case 3:

$$u_j = 1 \rightarrow$$

$$v_j = 0$$

$$u_i = 0$$

$$v_i = 0$$

By similar reasoning as in Case 1:

$$R_{ix} = - \alpha\ell^2$$

$$R_{iy} = - \alpha\ell m$$

Case 4:

$$v_j = 1 \uparrow$$

$$u_j = 0$$

$$v_i = 0$$

$$u_i = 0$$

By similar reasoning as Case 2:

$$R_{ix} = -\alpha m\ell$$

$$R_{iy} = -\alpha m^2$$

In the general condition of the structure under load, it is probable that neither u_i, v_i, u_j, or v_j will be equal to zero. However, the forces at nodes i and j can be arrived at by a superposition of Cases 1 through 4 as follows:

$$R_{ix} = \alpha\ell^2 u_i + \alpha m\ell v_i - \alpha\ell^2 u_j - \alpha m\ell v_j$$

$$R_{jx} = -R_{ix} \text{ by equilibrium}$$

$$R_{iy} = \alpha\ell m u_i + \alpha m^2 v_i - \alpha\ell m u_j - \alpha m^2 v_j$$

$$R_{jy} = -R_{iy} \text{ by equilibrium} \qquad \text{(A.20)}$$

In matrix form the reactions can be written as:

$$\begin{bmatrix} R_{ix} \\ R_{iy} \\ R_{jx} \\ R_{jy} \end{bmatrix} = \begin{bmatrix} \alpha\ell^2 & \alpha\ell m & -\alpha\ell^2 & -\alpha\ell m \\ \alpha\ell m & \alpha m^2 & -\alpha\ell m & -\alpha m^2 \\ -\alpha\ell^2 & -\alpha\ell m & \alpha\ell^2 & \alpha\ell m \\ -\alpha\ell m & -\alpha m^2 & \alpha\ell m & \alpha m^2 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix} \qquad \text{(A.21)}$$

or

$$[R] = [K]\,[u] \qquad \text{(A.22)}$$

where

R = node force components of rank [4 x 1]

K = rod stiffness matrix of rank [4 x 4]

u = node deflections of rank [4 x 1].

Equation A.22 can also be written as

$$\begin{bmatrix} R_{ix} \\ R_{iy} \\ R_{jx} \\ R_{jy} \end{bmatrix} = \alpha \begin{bmatrix} -\ell \\ -m \\ \ell \\ m \end{bmatrix} \begin{bmatrix} -\ell & -m & \ell & m \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix} \qquad (A.23)$$

or

$$[R] = \alpha\ [T^t]\ [T]\ [u] \qquad (A.24)$$

1.8 Global to Local Coordinate Transformation

Using equations (A.16) and (A.17) the local nodal displacements can be written in matrix form as:

$$\begin{bmatrix} u'_i \\ u'_j \end{bmatrix} = \begin{bmatrix} \ell & m & 0 & 0 \\ 0 & 0 & \ell & m \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix} \qquad (A.25)$$

1.9 Three Dimensional Truss Element*

Figure A-9 shows a typical truss element connected to joints i and j. All dimensions are assumed positive; however, the development is correct for elements of different orientation.

* The derivation follows that given in SOLID SAP, except that references to the SSAP2 program have been added.

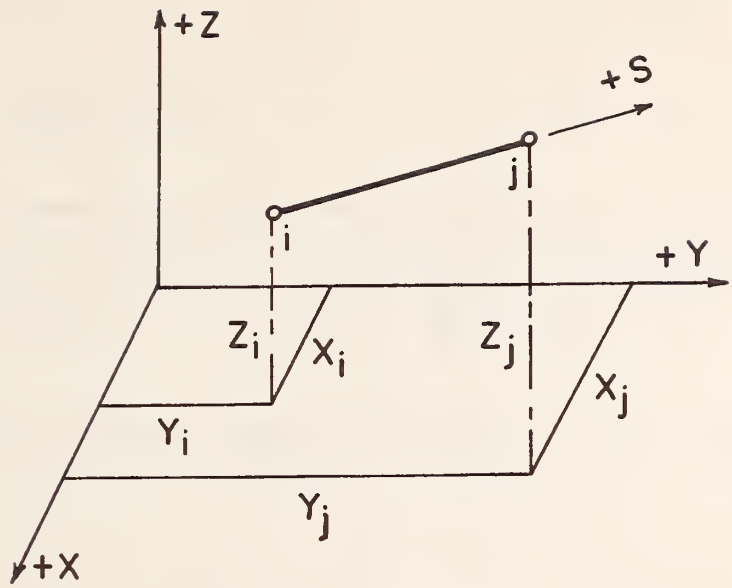


FIG. A-9 TYPICAL TRUSS ELEMENT.*

LET $DX = x_j - x_i$ — *TRUS 74* **, ***

$DY = y_j - y_i$ — *TRUS 75*

$DZ = z_j - z_i$ — *TRUS 76*

$XL = \sqrt{DX^2 + DY^2 + DZ^2}$ — *TRUS 78* (A.26)

The axial strain and displacement of any point between i and j in the s-direction are given by the following equations:

---

* Global axes are oriented as in SOLID SAP. (13)

** Italics type refers to program statement numbers in Appendix C.

*** Note that SSAP2 uses $(x_i - x_j)$ which leads to a negative direction cosine; but the situation is reversed back in *TRUS 80* when computing the strains for equation (A.30).

$$\varepsilon_s = \frac{1}{L}(u_{sj} - u_{si}) \qquad (A.27)$$

$$u_s = u_{si} + S\,\varepsilon_s \qquad (A.28)$$

In terms of global displacements the axial displacement u_s can be written in terms of its global components as:

$$u_s = \frac{DX}{L} u_x + \frac{DY}{L} u_y + \frac{DZ}{L} u_z \qquad (A.29)$$

When equation (A.29) is evaluated at joints i and j and substituted in equation (A.27) the following expression for element strains results:

$$\varepsilon_s = \frac{1}{L^2} [- DX \; - DY \; - DZ \; DX \; DY \; DZ] \begin{bmatrix} u_{xi} \\ u_{yi} \\ u_{zi} \\ u_{xj} \\ u_{yj} \\ u_{zj} \end{bmatrix} \qquad (A.30)$$

```
TRUS 80        ST(1,1) = DX/XL2
TRUS 81        ST(1,2) = DY/XL2
TRUS 82        ST(1,3) = DZ/XL2
TRUS 83        ST(1,4) = - ST(1,1)
TRUS 84        ST(1,5) = - ST(1,2)
TRUS 85        ST(1,6) = - ST(1,3)
```

Therefore, the strain-displacement matrix for a truss element is a 1 x 6 matrix and is given by:

$$\underline{a} = \frac{1}{L^2} [- DX \; - DY \; - DZ \; DX \; DY \; DZ] \qquad (A.31)$$

The axial stress becomes:

$$\sigma_s = E \varepsilon_s \tag{A.32}$$

In SSAP2 the stiffness element matrix is given by a general equation:

$$\underline{K_m} = \int_{vol} \underline{a_m^T} \; \underline{c_m} \; \underline{a_m} \; d V_m \tag{A.33}$$

Since the volume of the prismatic truss element is equal to the cross-sectional area of the element, A, times the length of the element, L, the element stiffness can be calculated from equation (A.33) as:

$$K = \frac{AE}{L^3} \begin{bmatrix} -D_x \\ -D_y \\ -D_z \\ D_x \\ D_y \\ D_z \end{bmatrix} [- D_x - D_y - D_z \; D_x \; D_y \; D_z] \tag{A.34}$$

Note that this stiffness has the same form as Equation (A.23) for the two-dimensional case.

The expansion of Equation (A.34) becomes:

$$K = \frac{AE}{L^3} \left[\begin{array}{ccc|ccc} D_x^2 & D_xD_y & D_xD_z & -D_x^2 & -D_xD_y & -D_xD_z \\ D_yD_x & D_y^2 & D_yD_z & -D_yD_x & -D_y^2 & -D_yD_z \\ D_zD_x & D_zD_y & D_z^2 & -D_zD_x & -D_zD_y & -D_z^2 \\ \hline -D_x^2 & -D_xD_y & -D_xD_z & D_x^2 & D_xD_y & D_xD_z \\ -D_yD_x & -D_y^2 & -D_yD_z & D_yD_z & D_y^2 & D_yD_z \\ -D_zD_x & -D_zD_y & -D_z^2 & D_zD_x & D_zD_y & D_z^2 \end{array} \right] \quad (A.35)$$

In SSAP2:

TRUS 79 *XX = EAL = E(MTYPE)*AREA(MTYPE)*XL*

TRUS 87 *DO 300 L = 1,6*

TRUS 88 *YY = ST(1,L)*XX*

forms $\frac{EAL}{L^2}$ [- DX - DY - DZ DX DY DZ], while

TRUS 89 *DO 250 K = L,6*

TRUS 90 *S(K,L) = ST(1,K)*YY*

fills in the upper triangle of Equation (A.35).

(Examples: $S(1,1) = \frac{-DX}{L^2}\frac{-DX}{L^2} EAL = \frac{DX^2}{L^3} AE$

$S(2,1) = \frac{-DY}{L^2}\frac{-DX}{L^2} EAL = \frac{DXDY}{L^3} AE$)

After the upper triangle is filled in, the lower triangle is filled in by transposing the matrix

TRUS 91 *250 S(L,K) = S(K,L)*

The stress displacement relationship is calculated as:

TRUS 92 $ST(1,L) = E(MTYPE)*ST(1,L)$

$$= E\frac{[D_i]}{L^2} \text{ (see Equations (A.30) and (A.32))}$$

The force displacement relationship becomes:

TRUS 93 $300\ ST(2,L) = AREA(MTYPE)*ST(1,L)$

$$= AE\frac{[D_i]}{L^2}$$

1.10 Thermal Loading on Truss Members

Figure A-10 shows a restrained truss element ij of length L subjected to a temperature change dT.

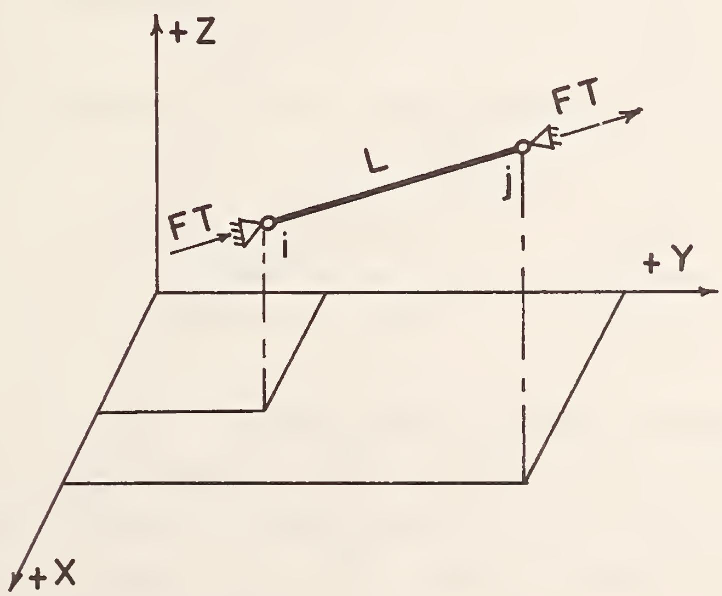


FIG. A-10 THERMAL REACTIONS, TRUSS ELEMENT.

The change of length ΔL of a free bar is:

$$\Delta L = \beta L(dT) \qquad (A.36)$$

where β is the coefficient of expansion or contraction per foot per degree F.

The restraining force to counteract ΔL is:

$$\Delta L = \frac{(FT)\ L}{A\ \ E} \qquad (A.37)$$

where FT is the restraining force due to a temperature change ΔT, E is the modulus of elasticity of the bar, and A its cross-sectional area. Equating Equation (A.36) to (A.37):

$$FT = \beta AE(dT)$$

In SSAP2 the average temperature change in the truss element is used.

```
TRUS  98          TEMP = (T(I) + T(J)) * 0.5 - REFT
```

where REFT is the ambient reference temperature.

```
TRUS  99          FT = TEMP * THERM(MTYPE) * E(MTYPE)
                       * AREA(MTYPE)
```

The end reaction components become (positive coordinate directions assumed):

```
TRUS 100          FX = DX * FT/XL
TRUS 101          FY = DY * FT/XL
TRUS 102          FZ = DZ * FT/XL
```

1.11 Global Coordinate Loading of Truss Members

Figure A-11 shows the directions of element node loading due to gravity forces and effect of temperature. On page B-6 three cards are described specifying the fraction of gravity (element load multipliers) in the three global coordinate directions.* A fourth card specifies the fraction of thermal loading to be added. Four different element load cases may be specified, and are identified as load cases A, B, C, and D

* See note, page B-3.

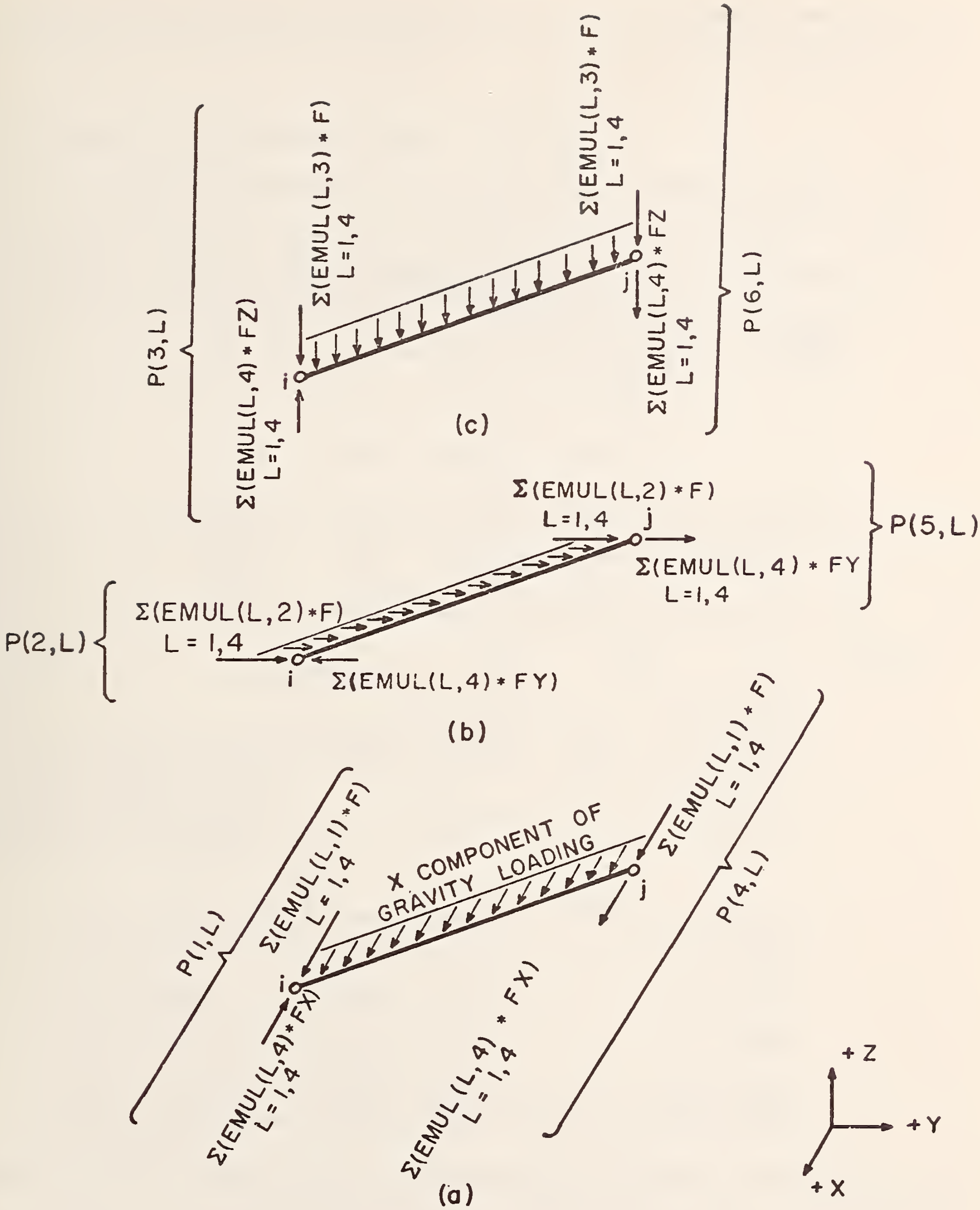


FIG. A-11 COMPONENTS OF GRAVITY LOADING FOR TRUSS MEMBERS.

for the truss elements. Table A-1 protrays the arrangement of information calculated by the following program statements:

TABLE A-1

Gravity and Thermal Loading Cases for SSAP2.

Load Card	L	Element Load Case L = 1 A	L = 2 B	L = 3 C	L = 4 D
1	x	P(1,1)	P(1,2)	P(1,3)	P(1,4)
2	y	P(2,1)	P(2,2)	P(2,3)	P(2,4)
3	z	P(3,1)	P(3,2)	P(3,3)	P(3,4)
4	Temp	EMUL (1,4)	EMUL (2,4)	EMUL (3,4)	EMUL (4,4)

```
TRUS  97          F = WT(MTYPE)*XL/2
TRUS 104          DO 350 L = 1,4
TRUS 105          TT(2,L) = EMUL(L,4)*FT (Fraction of
                                          thermal load)
TRUS 106          TT(1,L) = TT(2,L)/AREA(MTYPE) (Stress
                            due to fraction of thermal
                            load)
TRUS 107          P(1,L) = EMUL(L,1)*F-EMUL(L,4)*FX
TRUS 108          P(2,L) = EMUL(L,2)*F-EMUL(L,4)*FY
TRUS 109          P(3,L) = EMUL(L,3)*F-EMUL(L,4)*FZ
TRUS 110          P(4,L) = EMUL(L,1)*F+EMUL(L,4)*FX
TRUS 111          P(5,L) = EMUL(L,2)*F+EMUL(L,4)*FY
TRUS 112      350 P(6,L) = EMUL(L,3)*F+EMUL(L,4)*FZ
```

Two types of loading are considered (1) dead loading *TRUS 97* through *TRUS 112* or (2) mass loading *TRUS 113* through *TRUS 115*.

```
TRUS 113          F = DEN(MTYPE)*AREA(MTYPE)*XL/2
TRUS 114          DO 375 L = 1,6
TRUS 115      375 XM(L) = F
```

When specifying inertial mass loading the load condition number becomes zero (page B-5).

1.12 Assembly of Stiffness Matrices

We shall show the assembly of stiffness matrices by means of an example. Figure A-12 shows a four-node truss connected by five rod elements. Node 1 is restrained in all coordinate directions, while node 4 is restrained in the Y and Z direction only.*

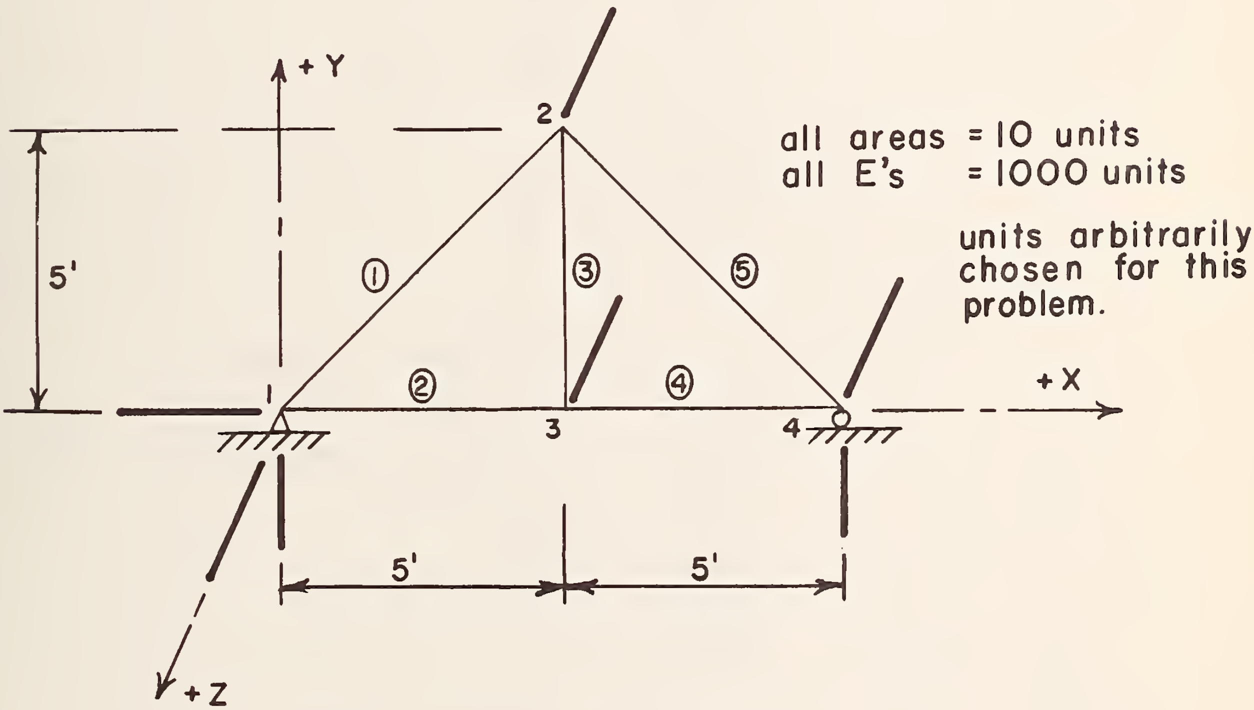


FIG. A-12 TRUSS EXAMPLE

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* Note that all nodes must be restrained in the Z direction for stability, because the example shown is one of a two-dimensional problem used with a three-dimensional computer program.

Figure A-13 lists the input data.

In subroutine *RUSS* the following numbers are generated for element 1:*

DX	= -5
DY	= -5
DZ	= 0
XL2	= 50
XL	= 7.07
XX	= 1000(10)7.07 = 70700
ST(1,1)	$= \frac{-5}{7.07} = -.707$
ST(1,2)	$= \frac{-5}{7.07} = -.707$
ST(1,3)	= 0
ST(1,4)	= .707
ST(1,5)	= .707
ST(1.6)	= 0
YY	= -.707(70700) = 50000
S(1,1)	= ST(1,1)*YY = -.707(50000) = - 35350
S(2,1)	= ST(1,2)*YY = -.707(50000) = - 35350
S(3,1)	= ST(1,3)*YY = 0
S(4,1)	= 35350
S(5,1)	= 35350
S(6,1)	= 0
ST(1,1)	= E*(-.707) = - 707
ST(1,2)	= - 707
ST(1,3)	= 0
ST(1,4)	= 707
ETC.	

---

* Program calculates i - j first.

IBM

FORTRAN Coding Form

PROGRAM		PUNCHING INSTRUCTIONS	GRAPHIC	PAGE
PROGRAMMER	DATE		PUNCH	CARD ELECTRO

FORTRAN STATEMENT

```
**SPECIAL - TRUSS EXAMPLE
    4    1    1    0
    1    1    1    1   -1   -1   -1     0.000     0.000     0.000
    2                                   5.000     5.000     0.000
    3                                   5.000     0.000     0.000
    4         1                        10.000     0.000     0.000
    1    5    1
    1 1000.                            10.
    0.        0.        0.        0.
    0.        0.        0.        0.
    0.        0.        0.        0.
    0.        0.        0.        0.
    1    1    2    1
    2    1    3    1
    3    2    3    1
    4    3    4    1
    5    2    4    1
    3    1            -10.000
BLANK CARD
    0.        0.        0.        0.
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms

G

FIG. A-13 INPUT DATA FOR TRUSS EXAMPLE PROBLEM.

At the end of subroutine *RUSS* the following are entered on File 1 in sequence

```
   6                      ND
   2                      NS
 (0,0,0,1,2,0)            LM(I),L+1,ND
  707  ──────┐
  707        │
    0        │
 -707        │            ST(L,K)
 -707        │
    0        │
  707        │
  707        │
    0        │
 -707        │
 -707        │
    0        │
  etc.       │
```

RUSS calls subroutine *CALBAN* which in turn rewrites the information on File 2 as follows (after calculating the bandwidth):

```
   73                     LRD
    6                     ND
 (0,0,0,1,2,0)            LM(I)
 -35350 ─────┐
 -35350      │
     0       │
  35350      │            S(I,J)
  35350      │
     0       │
   etc.      │
```

Eventually subroutine *ADDSTF* picks this information off of File 2 to use it in the following way:

For member 1 connecting node 1 to node 2

$$K_1 = \frac{AE}{L^3}\begin{bmatrix}-5\\-5\\0\\5\\5\\0\end{bmatrix}[-5\ -5\ 0\ 5\ 5\ 0]$$

$$
K_1 = \frac{10^4}{353.4}
\begin{array}{c|cccccc|}
 & U_{x1} & U_{y1} & U_{z1} & U_{x2} & U_{y2} & U_{z2} \\
\hline
U_{x1} & 25 & 25 & 0 & -25 & -25 & 0 \\
U_{y1} & 25 & 25 & 0 & -25 & -25 & 0 \\
U_{z1} & 0 & 0 & 0 & 0 & 0 & 0 \\
U_{x2} & -25 & -25 & 0 & 25 & 25 & 0 \\
U_{y2} & -25 & -25 & 0 & 25 & 25 & 0 \\
U_{z2} & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

where

$$\frac{10(1000)(25)}{353.4} = 707$$

In similar fashion for member 2 connecting node 1 and node 3.

$$
K_2 = \frac{10^4}{125}
\begin{array}{c|cccccc|}
 & U_{x1} & U_{y1} & U_{z1} & U_{x3} & U_{y3} & U_{z3} \\
\hline
U_{x1} & 25 & 0 & 0 & -25 & 0 & 0 \\
U_{y1} & 0 & 0 & 0 & 0 & 0 & 0 \\
U_{z1} & 0 & 0 & 0 & 0 & 0 & 0 \\
U_{x3} & -25 & 0 & 0 & 25 & 0 & 0 \\
U_{y3} & 0 & 0 & 0 & 0 & 0 & 0 \\
U_{z3} & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

where

$$\frac{10000(25)}{125} = 2000$$

For member 3:

$$K_3 = \frac{10^4}{125} \begin{array}{c} \\ U_{x2} \\ U_{y2} \\ U_{z2} \\ U_{x3} \\ U_{y3} \\ U_{z3} \end{array} \begin{array}{c} \begin{array}{cccccc} U_{x2} & U_{y2} & U_{z2} & U_{x3} & U_{y3} & U_{z3} \end{array} \\ \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 & -25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -25 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

For member 4:

$$K_4 = \frac{10^4}{125} \begin{array}{c} \\ U_{x3} \\ U_{y3} \\ U_{z3} \\ U_{x4} \\ U_{y4} \\ U_{z4} \end{array} \begin{array}{c} \begin{array}{cccccc} U_{x3} & U_{y3} & U_{z3} & U_{x4} & U_{y4} & U_{z4} \end{array} \\ \left[\begin{array}{cccccc} 25 & 0 & 0 & -25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -25 & 0 & 0 & 25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

For member 5:

$$K_5 = \frac{10^4}{353.4} \begin{array}{c} \\ U_{x2} \\ U_{y2} \\ U_{z2} \\ U_{x4} \\ U_{y4} \\ U_{z4} \end{array} \begin{array}{c} \begin{array}{cccccc} U_{x2} & U_{y2} & U_{z2} & U_{x4} & U_{y4} & U_{z4} \end{array} \\ \left[\begin{array}{cccccc} 25 & -25 & 0 & -25 & 25 & 0 \\ -25 & 25 & 0 & 25 & -25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -25 & 25 & 0 & 25 & -25 & 0 \\ 25 & -25 & 0 & -25 & 25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

By superposition the stiffness K for the entire structure is generated and inserts into Equation (A.1).

$$
\begin{bmatrix} R_{x1} \\ R_{y1} \\ R_{z1} \\ R_{x2} \\ R_{y2} \\ R_{z2} \\ R_{x3} \\ R_{y3} \\ R_{z3} \\ R_{x4} \\ R_{y4} \\ R_{z4} \end{bmatrix}
= Ku =
\begin{bmatrix}
2707 & 707 & 0 & -707 & -707 & 0 & -2000 & 0 & 0 & 0 & 0 & 0 \\
707 & 707 & 0 & -707 & -707 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-707 & -707 & 0 & 1414 & 0 & 0 & 0 & 0 & 0 & -707 & 707 & 0 \\
-707 & -707 & 0 & 0 & 3414 & 0 & 0 & -2000 & 0 & 707 & -707 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2000 & 0 & 0 & 0 & 0 & 0 & 4000 & 0 & 0 & -2000 & 0 & 0 \\
0 & 0 & 0 & 0 & -2000 & 0 & 0 & 2000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -707 & 707 & 0 & -2000 & 0 & 0 & 2707 & -707 & 0 \\
0 & 0 & 0 & 707 & -707 & 0 & 0 & 0 & 0 & -707 & 707 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \cancel{U}_{x1} \\ \cancel{U}_{y1} \\ \cancel{U}_{z1} \\ U_{x2} \\ U_{y2} \\ \cancel{U}_{z2} \\ U_{x3} \\ U_{y3} \\ \cancel{U}_{z3} \\ U_{x4} \\ \cancel{U}_{y4} \\ \cancel{U}_{z4} \end{bmatrix}
$$

But from Figure A-12 we note that many nodal component deflections will be zero, and the corresponding matrix multiplications drop out.

Therefore, subroutine *GAUSS* has only five equations left to solve for the following nodal displacements:

U_{x2} = .0025 feet

U_{y2} = -.0096 "

U_{x3} = .0025 "

U_{y3} = -.0146 "

U_{x4} = .0050 "

Using Equations (A.30) and (A.32) the element forces are generated as follows:

Force in member 1	=	-7.07 kips
Force in member 2	=	5
Force in member 3	=	10
Force in member 4	=	5
Force in member 5	=	-7.07

Although the example shown was for a statically determinate structure, the solution of a statically indeterminate structure would be prepared the same way. The stiffness method thus eliminates the traditional subdivisions in many undergraduate curricula between determinate and indeterminate structures.

Note: For a pin connected truss rod elements would be used. For a welded truss beam elements are recommended, especially if the secondary stresses might be of importance.

B. Stiffness of a Prismatic Beam Element

2.1 Bending and Shear versus Displacement or Rotation

In Figure A-14 beam ij is given a unit y displacement at i. The resulting end moments and shear reactions are desired. Since the tangents at ends i and j equal zero:

$$-M_i = M_j$$

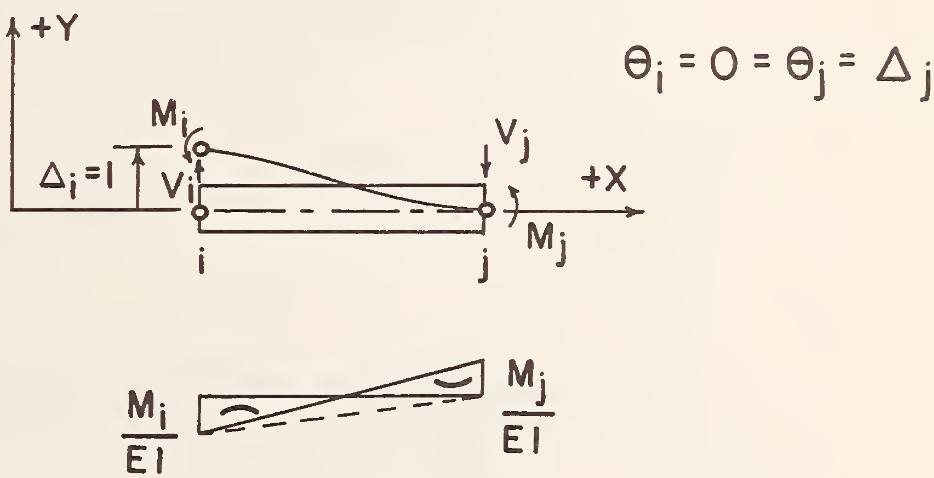


FIG. A-14 BEAM VERTICAL DISPLACEMENT AT NODE i.

Use of moment-area concepts give the following relationships:
The deflection at i from a tangent at j is:

$$\Delta_i = -\frac{M_i L}{2EI}\frac{L}{3} + \frac{M_j L}{2EI}\frac{2}{3}L = \frac{ML^2}{6EI}^{*,**} \qquad \text{(A.38)}$$

Therefore, the bending stiffness about the z axis becomes:

$$M = -M_i = M_j = \frac{6EI\Delta}{L^2} = \frac{6EI(1)}{L^2} = \frac{6EI}{L^2} \qquad \text{(A.39)}$$

---

* Sign Convention: +M is considered to act in a clockwise direction at end i and in a counterclockwise direction at end j.

** $I = I_z$ for the two-dimensional derivation.

The shears become

$$V_i = -V_j = \frac{2M}{L} = \frac{12EI}{L^3} \qquad (A.40)$$

In Figure A-15 beam ij is given a unit y displacement at end j. By analogy to Equations (A.38) and (A.39) the following equations describe the moment and shear reactions:

$$M_i = -M_j = \frac{6EI}{L^2} \qquad (A.41)$$

$$-V_i = V_j = \frac{12EI}{L^3} \qquad (A.42)$$

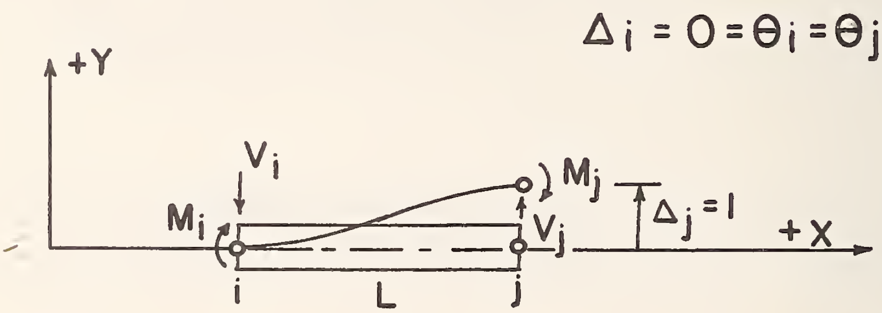


FIG. A-15 BEAM VERTICAL DEFLECTION AT NODE j.

In Figure A-16 beam ij is given a unit counterclockwise rotation at end i.* The deflection at i from a tangent at j is zero

$$\Delta_i = 0 = -\frac{M_i L}{2EI}\frac{L}{3} + \frac{M_j L}{2EI}\frac{2}{3}L \qquad (A.43)$$

Therefore

$$M_j = \frac{M_i}{2} \qquad (A.44)$$

* A counterclockwise rotation is considered a positive rotation for this situation only.

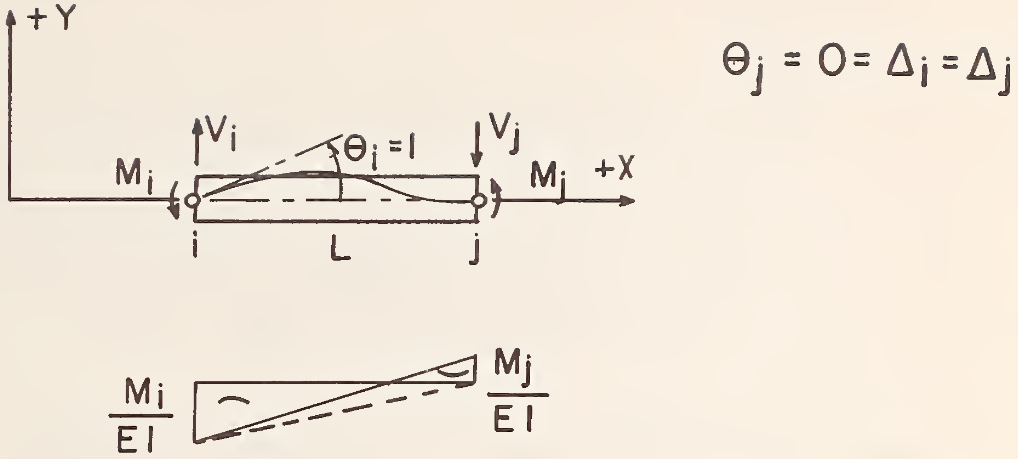


FIG. A-16 BEAM ROTATION AT END i

The rotation at i equals the area under the M/EI diagram.

$$\theta_i = -\frac{M_i L}{2EI} + \frac{M_i}{2EI}\frac{L}{2} = -\frac{M_i L}{4EI} \quad \text{(A.45)}$$

Thus:

$$M_i = -\frac{4EI\theta_i}{L} = -\frac{4EI(1)}{L} = -\frac{4EI}{L} \quad \text{(A.46)}$$

represents the moment stiffness at end i due to a unit rotation at i.

Also:

$$M_j = \frac{-M_i}{2} = \frac{2EI}{L} \quad \text{(A.47)}$$

The shears become:

$$V_i = -V_j = \frac{-M_i - \frac{M_i}{2}}{L} = \frac{-3M_i}{2L} = \frac{6EI}{L^2} \quad \text{(A.48)}$$

In Figure A-17 beam ij is given a unit clockwise rotation at end j:

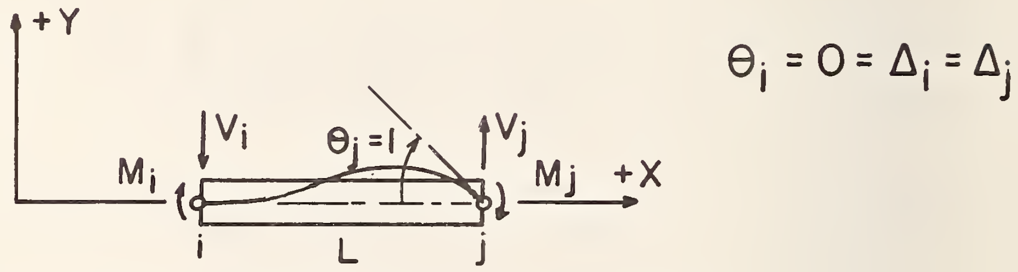


FIG. A-17 BEAM ROTATION AT END j.

By analogy to Equations (A.46) through (A.48)

$$M_i = \frac{-2EI}{L}$$

$$M_j = \frac{4EI}{L}$$

$$-V_i = V_j = \frac{6EI}{L^2} \qquad (A.49)$$

2.2 Translation of Prismatic Beam Ends without Displacement or Rotation

See Section 1.3 for the stiffness relationships developed for truss members.

2.3 Torsional Moments Versus End Rotations of a Prismatic Beam

Figure A-18 shows beam ij subjected to a twisting moment at end i while restrained at end j.

From mechanics of materials the angle of twist θ can be derived as:

$$\theta_i = \frac{T_i L}{JG} \qquad (A.50)$$

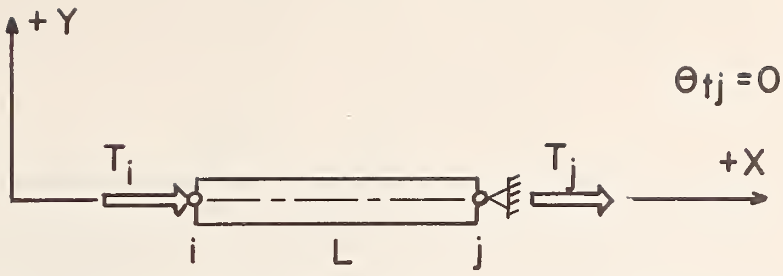


FIG. A-18 BEAM TORSION AT END i.

where G is the shearing modulus of elasticity, and J is the torsional constant expressed as a function of the member's cross-section. For a circle J is the polar moment of inertia. For a rectangular cross-section, where warping is considered, the torsional constant can be approximated by the expression:*

$$\underset{h > t}{J_1} = \frac{ht^3}{3} \left[1 - 0.63 \frac{t}{h} + 0.052 \left(\frac{t}{h}\right)^5\right]$$

as defined in Figure A-19b

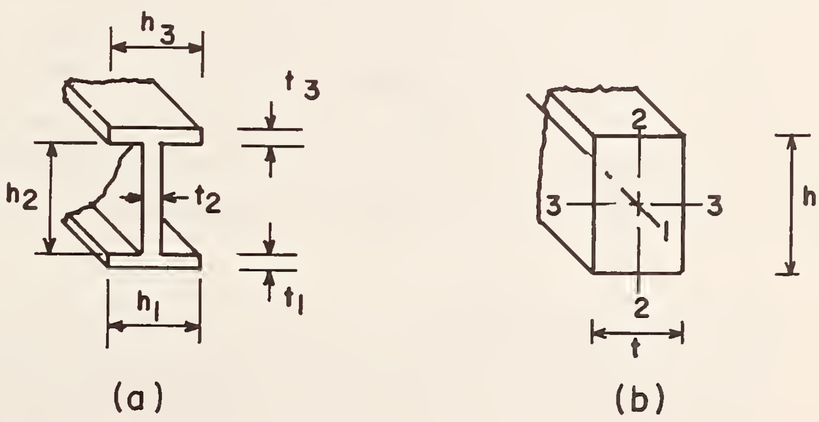


FIG. A-19 DIMENSIONS FOR EQUATION A-50.

* Reference 14, pg. 227.

For a prismatic member compounded of slender rectangular areas, such as an I beam, the torsional constant can be expressed as:

$$J_1 = \sum \frac{1}{3} ht^3$$

where h is the long dimension of each rectangular area considered.

For a unit rotation the torsional stiffness can be expressed from Equation (A.50) as:

$$K_{ti} = \frac{GJ}{L} \qquad (A.51)$$

where J is the torsion constant about the beam local 1 axis as defined in Figure (A-19).

When end i is restrained and end j is given a unit twist rotation, the stiffness becomes:

$$K_{tj} = -\frac{GJ}{L} \qquad (A.52)$$

2.4 Assembly of Beam Element Stiffness in Local Coordinates

Figure A-20 is taken from SOLID SAP and defines the reference coordinate system used in SSAP2 for the three dimensional beam element. The place which locates the principal axis of the beam is defined by node k in the local coordinate system $\hat{S}_1$, $\hat{S}_2$, $\hat{S}_3$. Table 2 shows the assembly of the previous subsections into a matrix relating the beam element end forces and its corresponding end displacements and rotations in terms of local rectangular coordinates, when the 1 axis is along the beam centroidal axis, as shown in Figure A-20.*

---

* For the computer program the plus sign convention corresponds to plus vectors in the plus coordinate direction. For the end i of the beam in Fig. A-14 along the z axis this means reversing the sign convention for M_i used in the derivation of the theory. It is unfortunate that the deflected structure makes a unified sign convention between moment area theory and computer analysis impractical.

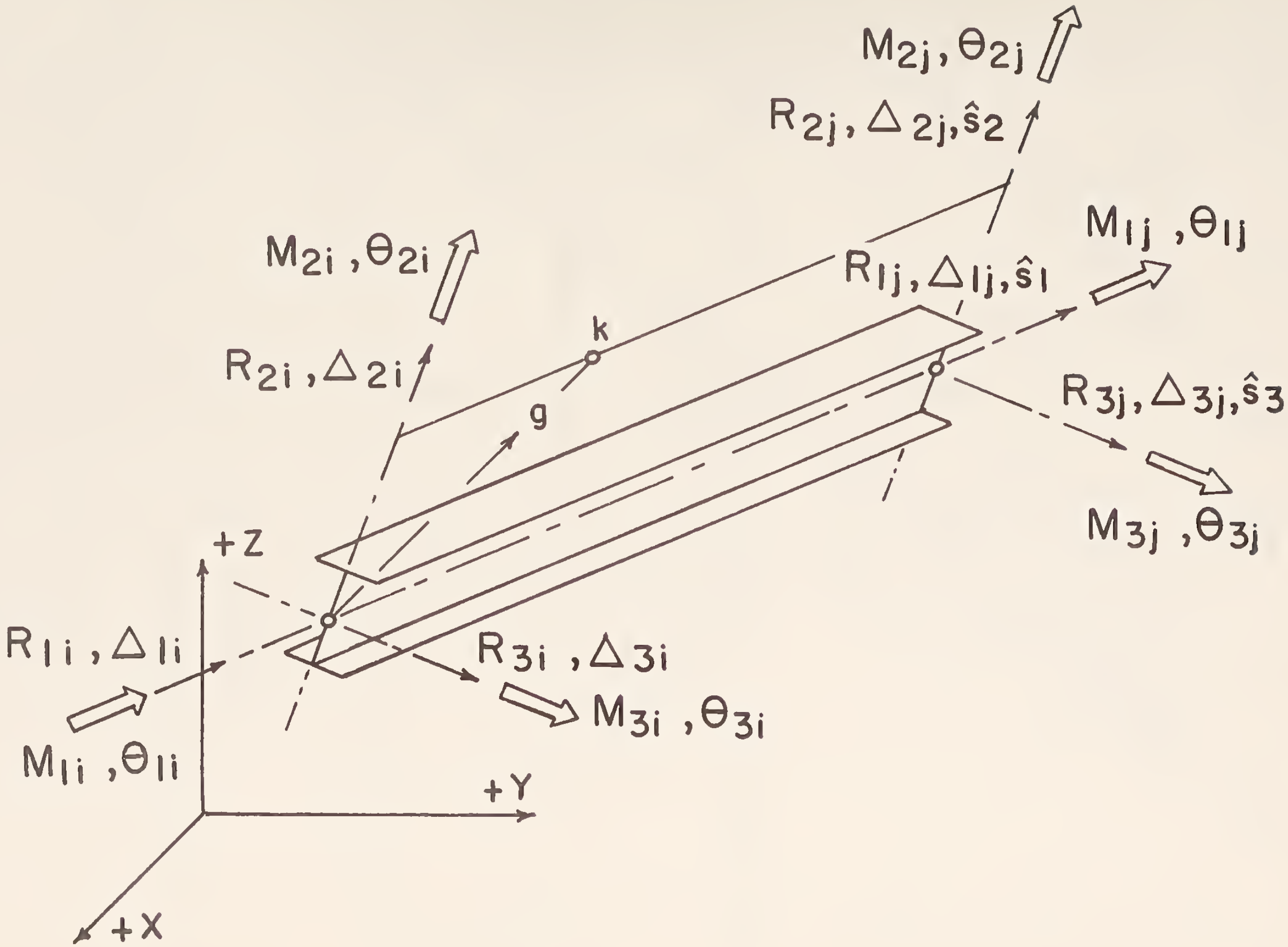


FIG. A-20 THREE DIMENSIONAL BEAM ELEMENT.

$$
\begin{bmatrix} R_{i1} \\ R_{i2} \\ R_{i3} \\ M_{i1} \\ M_{i2} \\ M_{i3} \\ R_{j1} \\ R_{j2} \\ R_{j3} \\ M_{j1} \\ M_{j2} \\ M_{j3} \end{bmatrix}
=
\left[\begin{array}{cccccc|cccccc}
\frac{EA_1}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA_1}{L} & 0 & 0 & 0 & 0 & 0 \\
 & \frac{12EI_3}{L^3} & 0 & 0 & 0 & \frac{6EI_3}{L^2} & 0 & -\frac{12EI_3}{L^3} & 0 & 0 & 0 & \frac{6EI_3}{L^2} \\
 & & \frac{12EI_2}{L^3} & 0 & -\frac{6EI_2}{L^2} & 0 & 0 & 0 & -\frac{12EI_2}{L^3} & 0 & -\frac{6EI_2}{L^2} & 0 \\
 & & & \frac{GI_1}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_1}{L} & 0 & 0 \\
 & & & & \frac{4EI_2}{L} & 0 & 0 & 0 & \frac{6EI_2}{L^2} & 0 & \frac{2EI_2}{L} & 0 \\
 & & & & & \frac{4EI_3}{L} & 0 & -\frac{6EI_3}{L^2} & 0 & 0 & 0 & \frac{2EI_3}{L} \\
\hline
 & & & & & & \frac{EA_1}{L} & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \frac{12EI_3}{L^3} & 0 & 0 & 0 & -\frac{6EI_3}{L^2} \\
 & & & & & & & & \frac{12EI_2}{L^3} & 0 & \frac{6EI_2}{L^2} & 0 \\
 & & & \text{SYMMETRIC} & & & & & & \frac{GI_1}{L} & 0 & 0 \\
 & & & & & & & & & & \frac{4EI_2}{L} & 0 \\
 & & & & & & & & & & & \frac{4EI_3}{L}
\end{array}\right]
\begin{bmatrix} u_{i1} \\ u_{i2} \\ u_{i3} \\ \theta_{i1} \\ \theta_{i2} \\ \theta_{i3} \\ u_{j1} \\ u_{j2} \\ u_{j3} \\ \theta_{j1} \\ \theta_{j2} \\ \theta_{j3} \end{bmatrix}
$$

2.5 Inclusion of Shear Deflection in Beam Analysis

Figure A-21 shows a portion of a beam, dx in length, subjected to a shear stress τ which is assumed uniformly distributed across the section of the beam.

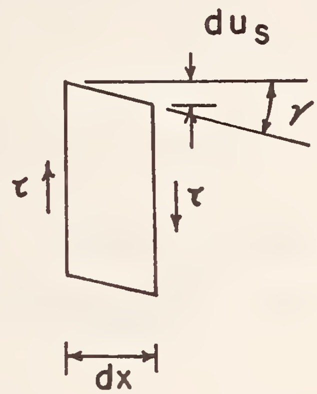


FIG. A-21 LOCAL SHEAR DISTORTION OF BEAM ELEMENT.

Under these circumstances the shearing strain γ is given by the relationship:

$$\gamma = \frac{du_s}{dx} = \tan \gamma \tag{A.54}$$

for small deformation, where du_s is the shearing deflection in the dx distance. Also:

$$G = -\frac{\tau}{\gamma} = -\frac{V}{A\gamma} = -\frac{Vdx}{Adu_s} \tag{A.55}$$

In Figure A-22, a beam ij is distorted by shearing forces under the same assumptions of Figure A-14.

From Figure A-21, and Equation (A.55):

$$du_s = -\frac{Vdx}{AG} = -\frac{V_i dx}{AG} \tag{A.56}$$

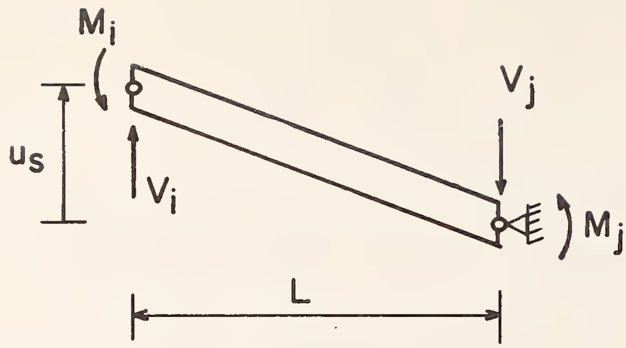


FIG. A-22 REACTIONS ON BEAM SUBJECTED TO SHEARING FORCES.

or

$$\frac{EIdu_s}{dx} = -\frac{EIV_i}{AG} \qquad (A.57)$$

Integrating

$$EIu_s = -\frac{EIV_iX}{AG} + C \qquad (A.58)$$

When $X = L, \quad u_s = 0$

$$C = \frac{EIV_iL}{AG} \qquad (A.59)$$

Therefore

$$EIu_s = -\frac{EIV_iX}{AG} + \frac{EIV_iL}{AG} \qquad (A.60)$$

If we let $\phi = \frac{12EI}{AGL^2}$, Equation (A.60) becomes:

$$EIu_s = -\frac{\phi V_i XL^2}{12} + \frac{\phi V_i L^3}{12} \qquad (A.61)$$

Now the total deflection u is the superposition of the bending deflection u_b of Figure A-14 and the shearing deflection u_s of Figure A-22, or

$$u = u_b + u_s \qquad (A.62)$$

When X = 0

$$u = u_i = 1 = u_b + \frac{\phi V_i L^3}{12EI} \qquad (A.63)$$

From Equations (A.39) and (A.40)

$$\frac{V_i}{u_b} = \frac{12EI}{L^3} \qquad (A.64)$$

Therefore

$$u_b = \frac{V_i L^3}{12EI} \qquad (A.65)$$

and

$$u_i = \frac{V_i L^3}{12EI} + \phi \frac{V_i L^3}{12EI} = (1 + \phi) \frac{V_i L^3}{12EI} \qquad (A.66)$$

The bending stiffness coefficient of Equations (A.39) and (A.40) is thus modified to include the shearing deflections, and this becomes:

$$K_{iv} = \frac{V_i}{u_i} = \frac{12EI}{(1 + \phi)L^3} = -\frac{V_j}{u_i} \qquad (A.67)$$

But

$$M_i = V_i \frac{L}{2} \tag{A.68}$$

So

$$K_{im} = \frac{M_i}{u_i} = \frac{6EI}{(1 + \phi)L^2} = - K_i M_j \tag{A.69}$$

Similar modification can be made for the stiffness coefficients associated with the beam end rotations, and are derived in reference (15). When these modifications are included in the beam element matrix we obtain:

$$
\begin{bmatrix} R_{i1} \\ R_{i2} \\ R_{i3} \\ M_{i1} \\ M_{i2} \\ M_{i3} \\ R_{j1} \\ R_{j2} \\ R_{j3} \\ M_{j1} \\ M_{j2} \\ M_{j3} \end{bmatrix}
=
\left[\begin{array}{cccccc|cccccc}
\frac{EA_1}{L} & & & & & & -\frac{EA_1}{L} & & & & & \\
 & \frac{12EI_3}{L^3(1+\phi_2)} & & & & \frac{6EI_3}{L^2(1+\phi_2)} & & \frac{-12EI_3}{L^3(1+\phi_2)} & & & & \frac{6EI_3}{L^2(1+\phi_2)} \\
 & & \frac{12EI_2}{L^3(1+\phi_3)} & & \frac{-6EI_2}{L^2(1+\phi_3)} & & & & \frac{-12EI_2}{L^3(1+\phi_3)} & & \frac{-6EI_2}{L^2(1+\phi_3)} & \\
 & & & \frac{GI_1}{L} & & & & & & \frac{-GI_1}{L} & & \\
 & & & & \frac{(4+\phi_3)EI_2}{L(1+\phi_3)} & & & & \frac{6EI_2}{L^2(1+\phi_3)} & & \frac{(2+\phi_3)EI_2}{1(1+\phi_3)} & \\
 & & & & & \frac{(4+\phi_2)EI_3}{L(1+\phi_3)} & & \frac{-6EI_3}{L^2(1+\phi_2)} & & & & \frac{(2+\phi_2)EI_3}{L(1+\phi_2)} \\
\hline
 & & & & & & \frac{EA_1}{L} & & & & & \\
 & & & & & & & \frac{12EI_3}{L^3(1+\phi_3)} & & & & \frac{-6EI_3}{L^2(1+\phi_2)} \\
 & \text{SYMMETRIC} & & & & & & & \frac{12EI_2}{L^3(1+\phi_3)} & & \frac{6EI_2}{L^2(1+\phi_3)} & \\
 & & & & & & & & & \frac{GI_1}{L} & & \\
 & & & & & & & & & & \frac{(4+\phi_3)EI_2}{L(1+\phi_3)} & \\
 & & & & & & & & & & & \frac{(4+\phi_2)EI_3}{L(1+\phi_2)}
\end{array} \right]
\begin{bmatrix} u_{i1} \\ u_{i2} \\ u_{i3} \\ \theta_{i1} \\ \theta_{i2} \\ \theta_{i3} \\ u_{j1} \\ u_{j2} \\ u_{j3} \\ \theta_{j1} \\ \theta_{j2} \\ \theta_{j3} \end{bmatrix}
\qquad (A.70)
$$

In the assembly of the complete 12 x 12 beam element stiffness matrix different values of I and A about the two bending axes are accounted for, where:

$$\phi_2 = \frac{12EI_3}{GA_{s2}L^2} = 24(1 + \nu) \frac{A}{A_{s2}} \left(\frac{r_3}{L}\right)^2 \qquad \text{(A.71)}$$

and

$$\phi_3 = \frac{12EI_2}{GA_{s3}L^2} = 24(1 + \nu) \frac{A}{A_{s3}} \left(\frac{r_2}{L}\right)^2 \qquad \text{(A.72)}$$

When the radii of gyration r_3 and r_2 are small in comparison to member length, the shear deflections can be ignored.

Equation (A.70) is assembled in the *NEWBM* subroutine, *BEAM 288 - BEAM 304* and *BEAM 320 - BEAM 345*.

COPROP = Common Properties

MELTYPE = Material Element Type or Geometry Number

SHFY = $L^2\phi_2/2$

SHFZ = $L^2\phi_3/2$

2.6 Transformation Equations for the Beam Element*

The geometric location of a typical beam element is defined by joint number i and j. The place which locates the principal axis of the beam is defined by a third joint number k as shown in Figure (A-20). One way of developing the transformation equations from local to global coordinates is conveniently developed by the use of vector notation. The unit vector in the $\hat{S}_1$ and $\hat{g}$ directions is given by:

$$\hat{S}_1 = S_{1x}\hat{X} + S_{1y}\hat{Y} + S_{1z}\hat{Z} \qquad \text{(A.73)}$$

$$\hat{g} = g_x\hat{X} + g_y\hat{Y} + g_z\hat{Z} \qquad \text{(A.74)}$$

* Taken directly from SOLID SAP.

where $\hat{X}$, $\hat{Y}$, and $\hat{Z}$ are unit vectors in the X, Y, and Z directions. The direction-cosines are

$$S_{1x} = \frac{DX}{L} \qquad S_{1y} = \frac{DY}{L} \qquad S_{1z} = \frac{DZ}{L} \tag{A.75}$$

$$g_x = \frac{GX}{G} \qquad g_y = \frac{GY}{G} \qquad g_z = \frac{GZ}{G} \tag{A.76}$$

where

$$\begin{aligned} DX &= x_j - x_i & \qquad GX &= x_k - x_i \\ DY &= y_j - y_i & \qquad GY &= y_k - y_i \\ DZ &= z_j - z_i & \qquad GZ &= z_k - z_i \end{aligned} \tag{A.77}$$

$$L = \sqrt{DX^2 + DY^2 + DZ^2} \qquad G = \sqrt{GX^2 + GY^2 + GZ^2} \tag{A.78}$$

The unit vector in the $\hat{S}_3$ direction is given by the vector product of $\hat{S}_1$ and $\hat{g}$ divided by the length of vector in that direction.

$$\hat{S}_3 = \frac{\hat{S}_1 \times \hat{g}}{|\hat{S}_1 \times \hat{g}|} = S_{3x}\hat{X} + S_{3y}\hat{Y} + S_{3z}\hat{Z} \tag{A.79}$$

where the vector product is by definition the evaluation of the determinent

$$\underline{a} \times \underline{b} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \tag{A.80}$$

The unit vector in the $\hat{S}_2$ direction is given by the vector product

$$\hat{S}_2 = \hat{S}_3 \times \hat{S}_1 = S_{2x}\hat{X} + S_{2y}\hat{Y} = S_{2z}\hat{Z} \tag{A.81}$$

The three unit vectors may be summarized by the following matrix equations:

$$\begin{bmatrix} \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \end{bmatrix} = \begin{bmatrix} S_{ix} & S_{1y} & S_{1z} \\ S_{2x} & S_{2y} & S_{2z} \\ S_{3x} & S_{3y} & S_{3z} \end{bmatrix} \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix} \qquad \text{(A.82)}$$

Within the computer program, local displacements, forces, and moments are transformed to the global system by the 3 x 3 matrix of Equation (A.82).

In SSAP2, Equation (A.82) is evaluated by statements *BEAM 142* through *BEAM 163* in subroutine *TEAM*. Since the calculation process is rather tricky, these calculations will be explained further.

Let us examine Equation (A.79) in more detail.

$$\hat{S}_1 \times \hat{g} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ S_{1x} & S_{1y} & S_{1z} \\ g_x & g_y & g_z \end{vmatrix} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{DX}{L} & \frac{DY}{L} & \frac{DZ}{L} \\ \frac{GX}{G} & \frac{GY}{G} & \frac{GZ}{G} \end{vmatrix} \qquad \text{(A.83)}$$

$$= x\left(\frac{DY}{L}\frac{GZ}{G} - \frac{DZ}{L}\frac{GY}{G}\right)$$

$$+ \; y\left(\frac{DZ}{L}\frac{GX}{G} - \frac{DX}{L}\frac{GZ}{G}\right)$$

$$+ \; z\left(\frac{DX}{L}\frac{GY}{G} - \frac{DY}{L}\frac{GX}{G}\right)$$

The length of vector $|\hat{S}_1 \times \hat{g}|$ is obtained as the square root of the sum of components squared.

$$\left[\frac{1}{LG}(DYGZ - DZGY)\right]^2 = \frac{1}{L^2G^2}(DY^2GZ^2 - 2DYDZGZGY + DZ^2GY^2)$$

$$\left[\frac{1}{LG}(DZGX - DXGZ)\right]^2 = \frac{1}{L^2G^2}(DZ^2GX^2 - 2DXDZGXGZ + DX^2GZ^2)$$

$$\left[\frac{1}{LG}(DXGY - DYGX)\right]^2 = \frac{1}{L^2G^2}(DX^2GY^2 - 2DXDYGXGY + DY^2GX^2) \qquad \text{(A.84)}$$

$$= |\hat{S}_1 \times \hat{g}|^2 \tag{A.85}$$

Therefore:

$$S_{3x} = \frac{DYGZ - DZGY}{LG\, |\hat{S}_1 \times \hat{g}|} \tag{A.86}$$

$$S_{3y} = \frac{DZGX - DXGZ}{LG\, |\hat{S}_1 \times \hat{g}|} \tag{A.87}$$

$$S_{3z} = \frac{DXGY - DYGX}{LG\, |\hat{S}_1 \times \hat{g}|} \tag{A.88}$$

Using Equations (A.75) and (A.81)

$$\hat{S}_2 = \hat{S}_3 \times \hat{S}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ S_{3x} & S_{3y} & S_{3z} \\ S_{1x} & S_{1y} & S_{1z} \end{vmatrix} \tag{A.89}$$

$$= \frac{1}{L^2G|\hat{S}_1 \times \hat{g}|} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ DYGZ-DZGY & DZGX-DXGZ & DXGY-DYGX \\ DX & DY & DZ \end{vmatrix}$$

Therefore:

$$S_{2x} = \frac{DZ^2GX-DXDZGZ - DYDXGY+DY^2GX}{L^2G|\hat{S}_1 \times \hat{g}|}$$

$$= \frac{(DZ^2 + DY^2)\ GX-DXDZGZ-DYDXGY}{L^2G|\hat{S}_1 \times \hat{g}|} \tag{A.90}$$

$$S_{2y} = \frac{DX^2GY-DXDYGX - DYDZGZ+DZ^2GY}{L^2G|\hat{S}_1 \times \hat{g}|}$$

$$= \frac{(DX^2 + DZ^2)GY - DXDYGX - DYDZGZ}{L^2G|\hat{S}_1 \times \hat{g}|} \quad (A.91)$$

$$S_{2z} = \frac{DY^2GZ-DYDZGY - DXDZGX+DX^2GZ}{L^2G|\hat{S}_1 \times \hat{g}|}$$

$$= \frac{(DY^2 + DX^2)GZ - DYDZGY - DXDZGX}{L^2G|\hat{S}_1 \times \hat{g}|} \quad (A.92)$$

Following the statements in SSAP2

BEAM 142	$A1 = DX$
BEAM 143	$A2 = DY$
BEAM 144	$A3 = DZ$
BEAM 145	$B1 = GX$
BEAM 146	$B2 = GY$
BEAM 147	$B3 = GZ$
BEAM 148	$AA = L^2$
BEAM 149	$AB = DXGX + DYGY + DZGZ$
BEAM 150	$U1 = L^2GX - \overline{AB}DX$
BEAM 151	$U2 = L^2GY - \overline{AB}DY$
BEAM 152	$U3 = L^2GZ - \overline{AB}DZ$
BEAM 153	$UU = U1^2 + U2^2 + U3^2$

or

$$\begin{aligned} UU = {} & L^4GX^2 - 2L^2GX\overline{AB}DX + \overline{AB}^2DX^2 \\ & + L^4GY^2 - 2L^2GY\overline{AB}DY + \overline{AB}^2DY^2 \\ & + L^4GZ^2 - 2L^2GZ\overline{AB}DZ + \overline{AB}^2DZ^2 \\ = {} & L^4G^2 - 2L^2\overline{AB}\ \overline{AB} + \overline{AB}^2L^2 \end{aligned}$$

$$= L^4G^2 - L^2\overline{AB}^2 = L^2[L^2G^2 - \overline{AB}^2] \qquad (A.93)$$

But:

$$L^2G^2 - \overline{AB}^2 = (DX^2 + DY^2 + DZ^2)(GX^2 + GY^2 + GZ^2) - (DXGX + DYGY + DZGZ)(DXGX + DYGY + DZGZ)$$

$$= \left\{ \begin{array}{l} DX^2GX^2 + DY^2GX^2 + DZ^2GX^2 + \\ DX^2GY^2 + DY^2GY^2 + DZ^2GY^2 + \\ DX^2GZ^2 + DY^2GZ^2 + DZ^2GZ^2 \end{array} \right\}$$

$$- \left\{ \begin{array}{l} DXGXDXGX + DYGYDXGX + DZGZDXGX + \\ DXGXDYGY + DYGYDYGY + DZGZDYGY + \\ DXGXDZGZ + DYGYDZGZ + DZGZDZGZ \end{array} \right\}$$

$$= \left\{ \begin{array}{l} DY^2GZ^2 - 2DYDZGZGY + DZ^2GY^2 + \\ DZ^2GZ^2 - 2DXDZGXGZ + DX^2GZ^2 + \\ DX^2GY^2 - 2DXDYGXGY + DY^2GX^2 \end{array} \right\} \qquad (A.94)$$

By comparison with Equation (A.85):

$$UU = L^4G^2|\hat{S}_1 \times \hat{g}|^2 \qquad (A.95)$$

BEAM 154 $\quad UU = L^2G|\hat{S}_1 \times \hat{g}|$

BEAM 158 $\quad T(2,1) = \dfrac{U1}{UU} = \dfrac{L^2GX - \overline{AB}DX}{L^2G|\hat{S}_1 \times \hat{g}|}$

or

$$\frac{DX^2GX + DY^2GX + DZ^2GX - DX^2GX - DXDYGY - DXDZGZ}{L^2G|\hat{S}_1 \times \hat{g}|}$$

$$= \frac{(DY^2 + DZ^2)GX - DXDZGZ - DXDYGY}{L^2G|\hat{S}_1 \times \hat{g}|}$$

which is identical to Equation (A.90).

Similarily:

$$\text{BEAM 159} \qquad T(2,2) = \frac{U2}{UU} = \frac{L^2GY - \overline{ABDY}}{L^2G|\hat{S}_1 \times \hat{G}|}$$

is identical to Equation (A.91) and:

$$\text{BEAM 160} \qquad T(2,3) = \frac{U3}{UU} = \frac{L^2GZ - \overline{ABGZ}}{L^2G|\hat{S}_1 \times \hat{G}|}$$

is identical to Equation (A.92).

Equation (A.81) is then used to evaluate $\hat{S}_3$, which can be rewritten as:

$$\hat{S}_3 = \hat{S}_1 \times \hat{S}_2$$

But

$$\text{BEAM 136} \qquad T(1,1) = S_{1x}$$

$$\text{BEAM 137} \qquad T(1,2) = S_{1y}$$

$$\text{BEAM 138} \qquad T(1,3) = S_{1z}$$

Therefore:

$$\hat{S}_3 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ T(1,1) & T(1,2) & T(1,3) \\ T(2,1) & T(2,2) & T(2,3) \end{vmatrix} \qquad (A.96)$$

$$\text{BEAM 161} \qquad T(3,1) = T(1,2)\ T(2,3) - T(1,3)\ T(2,2)$$

$$\text{BF4M 162} \qquad T(3,2) = T(1,3)\ T(2,1) - T(1,1)\ T(2,3)$$

$$\text{BEAM 163} \qquad T(3,3) = T(1,1)\ T(2,2) - T(1,2)\ T(2,1)$$

Equation (A.82) becomes:

$$\begin{bmatrix} \hat{S}_1 \\ \hat{S}_2 \\ \hat{S}_3 \end{bmatrix} = \begin{bmatrix} T(1,1) & T(1,2) & T(1,3) \\ T(2,1) & T(2,2) & T(2,3) \\ T(3,1) & T(3,2) & T(3,3) \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} \qquad (A.97)$$

2.7 Global to Local Coordinate Transformation in Three Dimensions

The matrix equation relating local displacements u' to global displacement u can be derived by resolving element displacement vectors from the global set of coordinates into the local coordinate system , as was shown for the two dimentional case. For more details, see reference (15), page 81.

C. Stiffness of Plate and Shell Elements (Quadrilateral)

3.1 Introduction

The SSAP2 computer program utilizes three different triangular elements for constructing plate and shell quadrilaterals. Two of these elements (Constant Strain Triangle - CST) and (Linear Strain Triangle - LST) are used to describe the inplane or membrane stresses, while a third element (Linearly Varying Curvature Triangle - LCCT) is used to describe the flexural stresses. The total stress in the quadrilateral is considered to be a superposition of the membrane and flexural stresses without interaction. In this report the stiffness matrix is described for each of the above triangles; however, the correlation between theory and program statements will be limited to the flat plate element of constant thickness, which was used in the bridge research. These derivations follow that of Fellipa's dissertation[16] - further extensions of the theory for variable thickness plates is contained in his dissertation.

3.2 The Quadrilateral Element

The thin plate and shell element included in this report is a quadrilateral of arbitrary geometry formed from four compatible triangles (Figure A-23). These triangles are known as refined triangles since additional nodes are included at the midpoints of their sides. One external midside node has been eliminated from each of the triangles for programming purposes to be described later. We first concern ourselves with the development of the stiffness matrix for the individual triangular elements in the quadrilateral.

3.3 Area Coordinate System

The geometry and evaluation of stiffness matrices of arbitrary triangular elements can be expressed by rectangular cartesian coordinates. However, a more elegant formulation of the stiffness and geometry by means of natural or area coordinates is often used in the literature;[17] [18] [19]

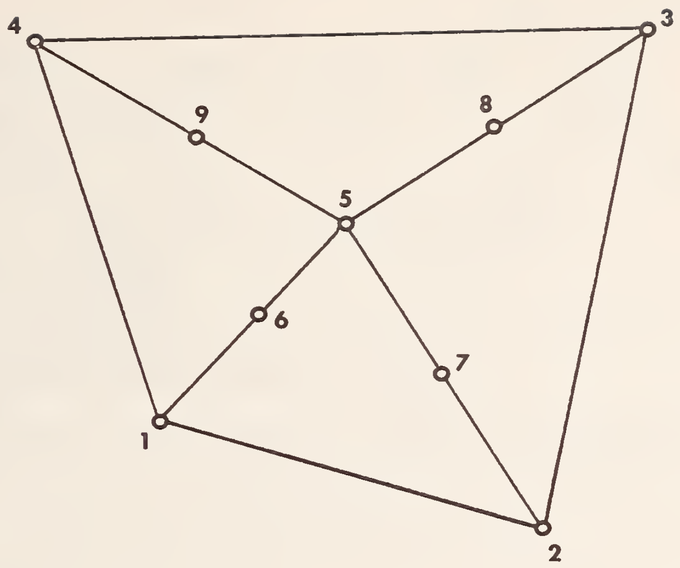


FIG. A-23 QUADRILATERAL COMPOSED FROM FOUR COMPATIBLE REFINED TRIANGLES.

with one advantage in that the natural coordinates are invariant with respect to the shape and orientation of the triangle, and that all integration formulas become remarkably simple. The natural coordinate system is used in SSAP2. In the natural coordinate system the specification of a point within the triangular element is done with a set of dimensionless numbers whose magnitude never exceeds unity.

3.4 Geometry of Triangular Element

Consider the triangular element of Figure A-24a. Let j and k denote the first and second cyclic permutations of i = 1,2,3 (i.e., j = 2,3,1 and k = 3,1,2); then the projected dimensions on the xy plane may be expressed as:

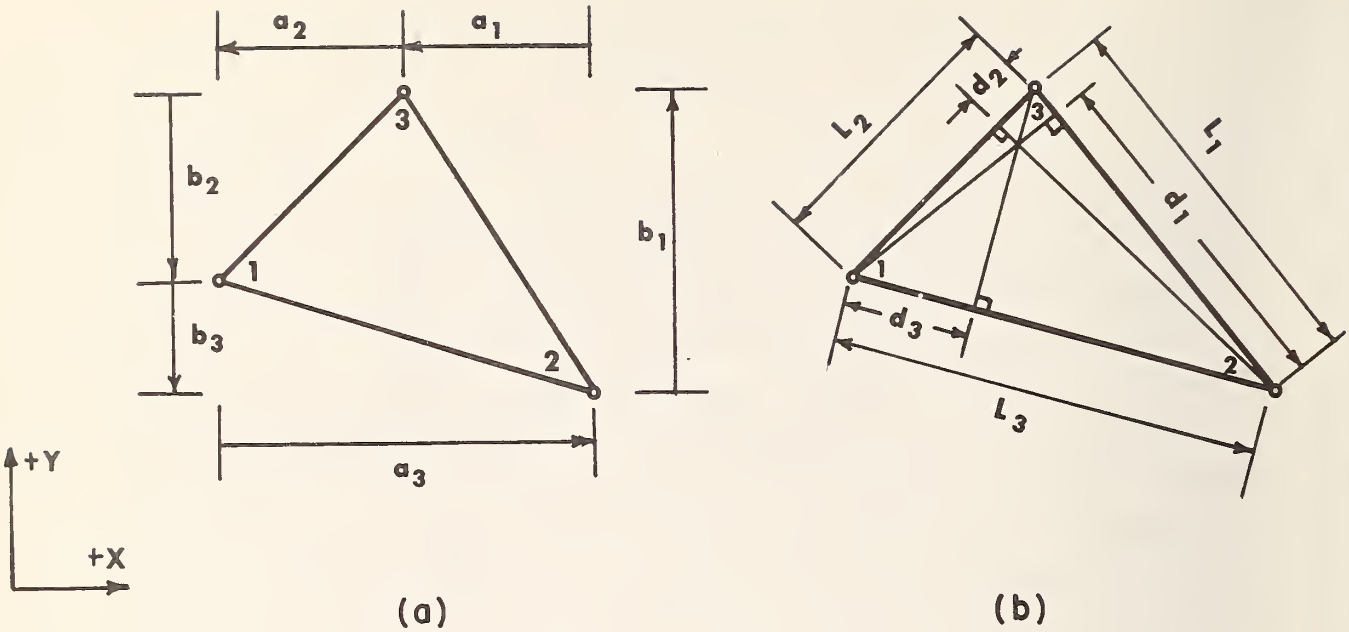


FIG. A-24 PROJECTED AND INTRINSIC DIMENSIONS OF ARBITRARY TRIANGLE.

$$a_i = x_k - x_j \qquad b_i = y_j - y_k$$

or:

$$\begin{aligned} a_1 &= x_3 - x_2 & b_1 &= y_2 - y_3 \\ a_2 &= x_1 - x_3 & b_2 &= y_3 - y_1 \\ a_3 &= x_2 - x_1 & b_3 &= y_1 - y_2 \end{aligned} \qquad (A.98)$$

Also some intrinsic dimensions as defined in Figure A-24b may be expressed in terms of these projected dimensions.

$$l_i^2 = a_i^2 + b_i^2$$

$$H_i = \frac{2A}{l_i}$$

where A is the area of the triangle,

$$d_i = -\frac{1}{l_i}\,(a_i a_k + b_i b_k)$$

$$\lambda_i = \frac{d_i}{l_i} = -\,\frac{a_i a_k + b_i b_k}{a_i^{\,2} + b_i^{\,2}}$$

$$\mu_i = 1 - \lambda_i = -\,\frac{a_i a_j + b_i b_j}{a_i^{\,2} + b_i^{\,2}} \qquad \text{(A.99)}$$

3.5 Natural Triangular Coordinates

The triangular coordinates ξ_1, ξ_2, ξ_3 of any point P in the triangle may be defined either as the ratios of the areas A_i of the subtriangles subtended by that point to the total area of the triangle, or as the ratios of the normal distances n_i to the heights H_i, i.e.,

$$\xi_i = \frac{A_i}{A} = \frac{n_i}{H_i} \qquad \text{(A.100)}$$

as shown in Figure A-25. At any point $\xi_1 + \xi_2 + \xi_3 = 1$.

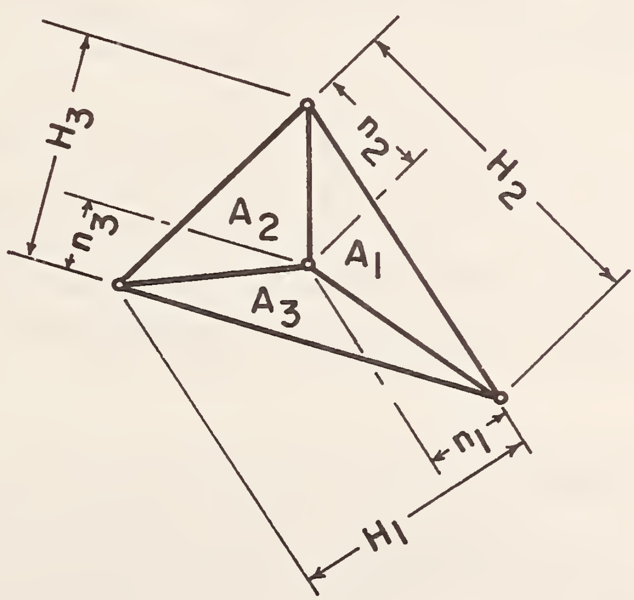


FIG. A-25 HEIGHTS OF MAIN AND SUBAREAS OF TRIANGLE.

The relationships between cartesian and triangular coordinates may be expressed as:

$$\begin{bmatrix} 1 \\ x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \qquad (A.101)$$

or by inversion:

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} 2A_{23} & b_1 & a_1 \\ 2A_{31} & b_2 & a_2 \\ 2A_{12} & b_3 & a_3 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \qquad (A.102)$$

in which $2A_{ij} = x_i y_j - x_j y_i$

3.6 Variation of Natural Triangular Coordinates

Figure A-26 shows the variation of ξ_1, ξ_2, and ξ_3 over the area of the triangle. At any point P the value of ξ_1, ξ_2, and ξ_3 is merely the height of the corresponding sloping plane above the triangular base. Within a triangular element the displacements may be considered to vary in a similar way to the ξ's for a unit coordinate displacement at one node with the other two nodes restrained. For example, Figure A-26d shows a displacement pattern for $u_x = 1$, which is similar to the variation of ξ_1.

3.7 Triangular Subelements

For compatibility requirements it is necessary to further subdivide the triangle into three subelements as shown in Figure A-27, in which the internal point "0" is the centroid of the element area, and the subtriangles are numbered to correspond with the opposite corner number.

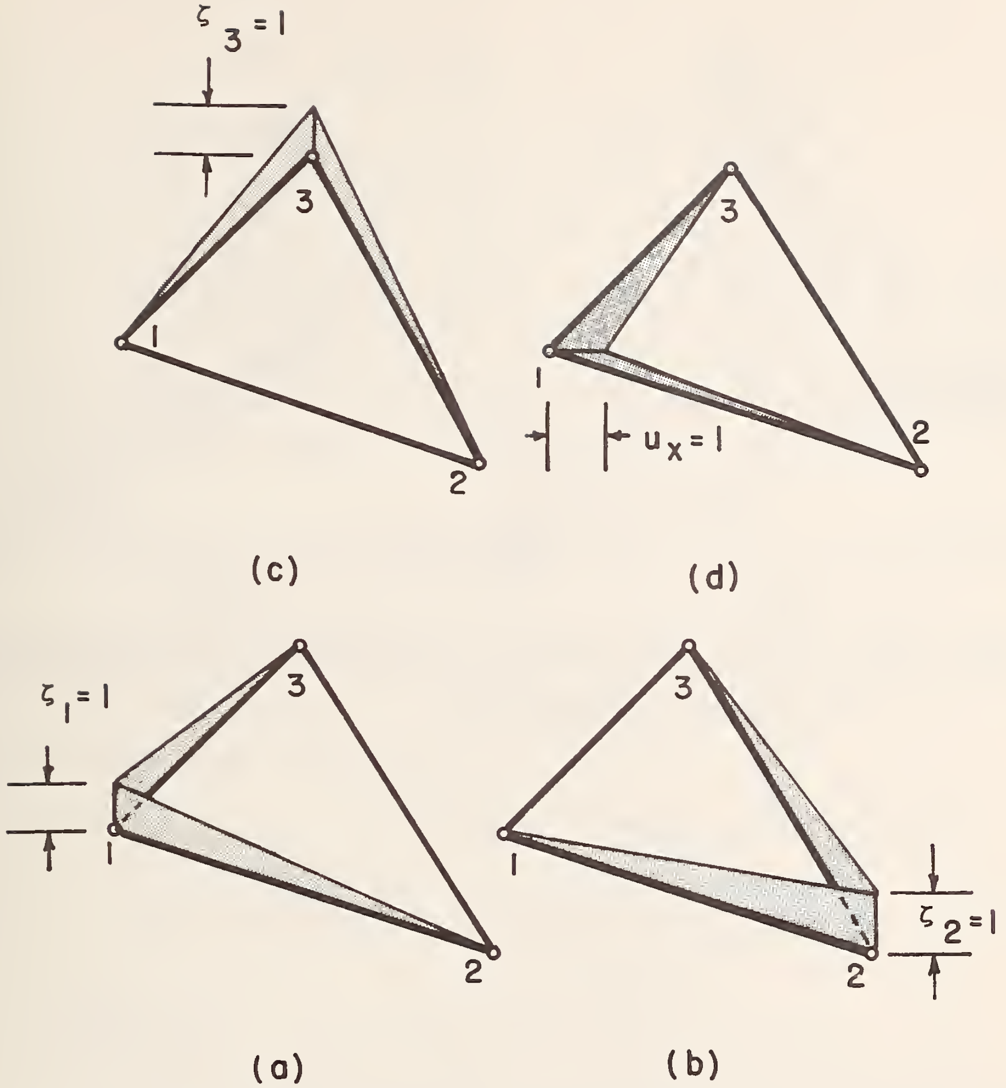


FIG. A-26 LINEAR VARIATIONS OF ζ_1, ζ_2, AND ζ_3 OVER TRIANGLE AREA.

The geometric relationships discussed above apply to each of the subtriangles if its three corners are renumbered 1 - 2 - 3, with 3 being taken as the internal point. The subtriangle is then identified in the forthcoming expressions by means of a superscript. The renumbering scheme is shown in Figure A-27b for subelement (1). For example, $\theta_{x2}^{(1)}$ would be the rotation about the x axis for node 2 of subelement (1).

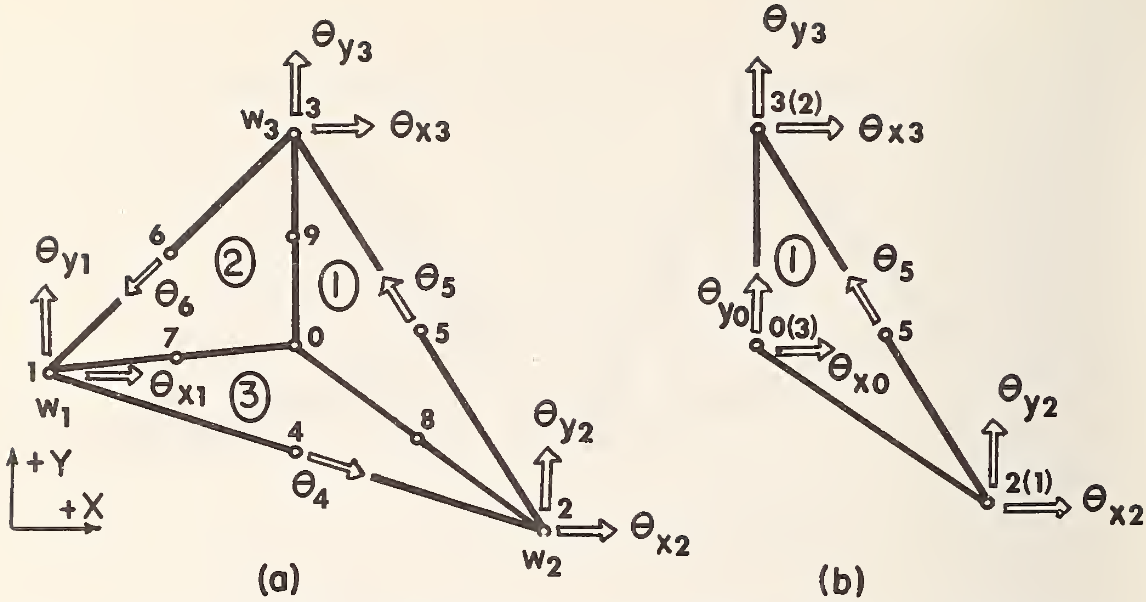


FIG. A-27 RENUMBERING SCHEME FOR TRIANGLE SUBELEMENTS.

The area of the subelement is given by:

$$A_k = \left(\frac{y + y_j}{2}\right)(x_j - x) + \left(\frac{y + y_i}{2}\right)(x - x_i) - \left(\frac{y_j + y_i}{2}\right)(x_j - x_i) \qquad (A.103)$$

where the coordinates of point "0" (Figure A-27(a)) located in any position within the triangle 1-2-3 are given by x and y, or in dimensionless form:

$$\xi_k = \frac{1}{2A}\left[(y + y_j)(x_j - x) + (y + y_i)(x - x_i) - (y_j + y_i)(x_j - x_i)\right] \qquad (A.104)$$

The derivatives with respect to the cartesian coordinates are in general:

$$\frac{\partial \xi_k}{\partial x} = \frac{y_i - y_j}{2A} \quad \text{and} \quad \frac{\partial \xi_k}{\partial y} = \frac{x_j - x_i}{2A} \qquad (A.105)$$

These evaluated for the three subelement corner coordinates are:

$$\frac{\partial \xi_1}{\partial x} = \frac{b_1}{2A} \qquad \frac{\partial \xi_1}{\partial y} = \frac{a_1}{2A}$$

$$\frac{\partial \xi_2}{\partial x} = \frac{b_2}{2A} \qquad \frac{\partial \xi_2}{\partial y} = \frac{a_2}{2A}$$

$$\frac{\partial \xi_3}{\partial x} = \frac{b_3}{2A} \qquad \frac{\partial \xi_3}{\partial y} = \frac{a_3}{2A} \qquad \text{(A.106)}$$

where the a's and b's are the corresponding projected dimensions of the subelement.

3.8 Linear Displacement or Constant Strain Field for Triangular Element

Any linear displacement function u(x,y) over the triangular element with final nodal displacements u_1, u_2, u_3 is a linear combination of $f_1(\xi_1, \xi_2, \xi_3)$, $f_2(\xi_1, \xi_2, \xi_3)$, and $f_3(\xi_1, \xi_2, \xi_3)$ with scalar coefficients u_1, u_2, u_3, or

$$u(\xi_1, \xi_2, \xi_3) = f_1(\xi_1, \xi_2, \xi_3)u_1 + f_2(\xi_1, \xi_2, \xi_3)u_2 + f_3(\xi_1, \xi_2, \xi_3)u_3 \qquad \text{(A.107)}$$

Similarly:

$$v(\xi_1, \xi_2, \xi_3) = f_1(\xi_1, \xi_2, \xi_3)v_1 + f_2(\xi_1, \xi_2, \xi_3)v_2 + f_3(\xi_1, \xi_2, \xi_3)v_3 \qquad \text{(A.108)}$$

For linear interpolation these functions are identical to the natural functions for the element; that is:

$$\theta_i = f_i(\xi_1, \xi_2, \xi_3) = \xi_i \quad i = 1,2,3 \qquad \text{(A.109)}$$

The constant strain triangle has 6° of freedom for membrane action. In a linear displacement field we have constant strain; and the strains are given by:

$$\varepsilon_x(\xi_i) = \frac{\partial u(\xi_i)}{\partial x} = \overline{\phi_x^T}\,\overline{u}$$

$$\varepsilon_y(\xi_i) = \frac{\partial v(\xi_i)}{\partial y} = \overline{\phi_y^T}\,\overline{v}$$

$$\gamma_{xy}(\xi_i) = \frac{\partial v(\xi_i)}{\partial x} + \frac{\partial u(\xi_i)}{\partial y} = \overline{\phi_y^T}\,\overline{u} + \overline{\phi_x^T}\,\overline{v} \qquad \text{(A-110)}$$

where:

$$\overline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad \overline{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

By use of the chain rule we may write:

$$\frac{\partial u(\xi_i)}{\partial x} = \frac{\partial u}{\partial \xi_1}\frac{\partial \xi_1}{\partial x} + \frac{\partial u}{\partial \xi_2}\frac{\partial \xi_2}{\partial x} + \frac{\partial u}{\partial \xi_3}\frac{\partial \xi_3}{\partial x}$$

$$= \frac{b_1}{2A}\frac{\partial u}{\partial \xi_1} + \frac{b_2}{2A}\frac{\partial u}{\partial \xi_2} + \frac{b_3}{2A}\frac{\partial u}{\partial \xi_3}$$

$$\frac{\partial v(\xi_i)}{\partial y} = \frac{a_1}{2A}\frac{\partial v}{\partial \xi_1} + \frac{a_2}{2A}\frac{\partial v}{\partial \xi_2} + \frac{a_3}{2A}\frac{\partial v}{\partial \xi_3}$$

$$\frac{\partial u(\xi_i)}{\partial y} = \frac{a_1}{2A}\frac{\partial u}{\partial \xi_1} + \frac{a_2}{2A}\frac{\partial u}{\partial \xi_2} + \frac{a_3}{2A}\frac{\partial u}{\partial \xi_3}$$

$$\frac{\partial v(\xi_i)}{\partial x} = \frac{b_1}{2A}\frac{\partial v}{\partial \xi_1} + \frac{b_2}{2A}\frac{\partial v}{\partial \xi_2} + \frac{b_3}{2A}\frac{\partial v}{\partial \xi_3} \qquad \text{(A-111)}$$

Since the strains are constant, we may select any convenient point to evaluate the derivatives $\frac{\partial u}{\partial \xi}$, $\frac{\partial v}{\partial \xi}$. Choosing the nodal points of the triangle and noting that the variation ϕ is identical to the variation ξ, and that $\frac{\partial \phi_i}{\partial \xi_i} = 1$ in Equation (A.109) , the strains can be written from Equation (A.110) as:

$$\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} b_1 & b_2 & b_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 & b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \bar{B}\bar{r} \qquad \text{(A-112)}$$

where $\bar{B}$ is known as the strain-nodal displacement matrix.

3.9 Stress Strain Relations

For a linear elastic material the two-dimensional stress strain relations may be written as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad \text{(A-113)}$$

or:

$$\bar{\sigma}(\xi_i) = \bar{C}(\xi_i)\, \bar{\varepsilon}(\xi_i) \qquad \text{(A-114)}$$

where $\bar{C}$ is symmetric, and contains the elements of the plane stress material matrix read into the program from data input. For isotropic materials

$$\bar{C} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & \cdot \\ \nu & 1 & \cdot \\ \cdot & \cdot & \lambda \end{bmatrix} \qquad \lambda = \frac{1-\nu}{2} \qquad \text{(A-115)}$$

3.10 Stiffness Matrix - Constant Strain Triangle

The element stiffness represents the nodal forces in the element resulting from unit nodal displacements. It can be arrived at from strain energy considerations.

The total strain energy stored in an element of constant thickness h is:

$$\Phi = \frac{1}{2} \int_V \overline{\varepsilon}^T \overline{\sigma} \, dv$$

$$= \frac{\overline{r}^T}{2} \int_V \overline{B}^T \overline{C} \, \overline{B} \, dv \, \overline{r} \qquad \text{(A-116)}$$

where the stiffness matrix by definition is the quantity under the integral sign. Since matrices B and C are not functions of the (x,y) coordinates, the matrix product $\overline{B}^T \overline{C} \, \overline{B}$ may be taken outside the integral sign and we have:

$$K = A \, h \, \overline{B}^T \overline{C} \, \overline{B} \qquad \text{(A-117)}$$

where h is the element thickness. Substituting Equations (A-112) and (A-113) we have:

$$K = \frac{h}{4A} \begin{bmatrix} b_1 & \cdot & a_1 \\ b_2 & \cdot & a_2 \\ b_3 & \cdot & a_3 \\ \cdot & a_1 & b_1 \\ \cdot & a_2 & b_2 \\ \cdot & a_3 & b_3 \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ \text{symm.} & & C_{33} \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 & b_1 & b_2 & b_3 \end{bmatrix} \qquad \text{(A.118)}$$

After carrying out the indicated matrix manipulation we arrive at a stiffness matrix which can be identified in terms of the SSAP2 program statements; i.e.:

$$K = \frac{h}{4A} \begin{bmatrix} \begin{array}{c} C_{11}b_1b_1 + C_{13}a_1b_1 \\ +C_{13}b_1a_1 + C_{33}a_1a_1 \end{array} & \begin{array}{c} C_{11}b_2b_1 + C_{13}a_2b_1 \\ +C_{13}b_2a_1 + C_{33}a_2a_1 \end{array} & \begin{array}{c} C_{11}b_3b_1 + C_{13}a_3b_1 \\ +C_{13}b_3a_1 + C_{33}a_3a_1 \end{array} & \begin{array}{c} C_{12}a_1b_1 + C_{13}b_1b_1 \\ +C_{23}a_1a_1 + C_{33}b_1a_1 \end{array} & \begin{array}{c} C_{12}a_2b_1 + C_{13}b_2b_1 \\ +C_{23}a_2a_1 + C_{33}b_2a_1 \end{array} & \begin{array}{c} C_{12}a_3b_1 + C_{13}b_3b_1 \\ +C_{23}a_3a_1 + C_{33}b_3a_1 \end{array} \\ & \begin{array}{c} C_{11}b_2b_2 + C_{13}a_2b_2 \\ +C_{13}b_2a_2 + C_{33}a_2a_2 \end{array} & \begin{array}{c} C_{11}b_3b_2 + C_{13}a_3b_2 \\ +C_{13}b_3a_2 + C_{33}a_3a_2 \end{array} & \begin{array}{c} C_{12}a_1b_2 + C_{13}b_1b_2 \\ +C_{23}a_1a_2 + C_{33}b_1a_2 \end{array} & \begin{array}{c} C_{12}a_2b_2 + C_{13}b_2b_2 \\ +C_{23}a_2a_2 + C_{33}b_2a_2 \end{array} & \begin{array}{c} C_{12}a_3b_2 + C_{13}b_3b_2 \\ +C_{23}a_3a_2 + C_{33}b_3a_2 \end{array} \\ & & \begin{array}{c} C_{11}b_3b_3 + C_{13}a_3b_3 \\ +C_{13}b_2a_2 + C_{33}a_3a_3 \end{array} & \begin{array}{c} C_{12}a_1b_3 + C_{13}b_1b_3 \\ +C_{23}a_1a_3 + C_{33}b_1a_3 \end{array} & \begin{array}{c} C_{12}a_2b_3 + C_{23}b_2b_3 \\ +C_{13}a_2a_3 + C_{33}b_2a_3 \end{array} & \begin{array}{c} C_{12}a_3b_3 + C_{13}b_3b_3 \\ +C_{23}a_3a_3 + C_{33}b_3a_3 \end{array} \\ & & & \begin{array}{c} C_{22}a_1a_1 + C_{23}b_1a_1 \\ +C_{23}a_1b_1 + C_{33}b_1b_1 \end{array} & \begin{array}{c} C_{22}a_2a_1 + C_{23}b_2a_1 \\ +C_{23}a_2b_1 + C_{33}b_2b_1 \end{array} & \begin{array}{c} C_{22}a_3a_1 + C_{23}b_3a_1 \\ +C_{23}a_3b_1 + C_{33}b_3b_1 \end{array} \\ & \text{SYMMETRICAL} & & & \begin{array}{c} C_{22}a_2a_2 + C_{23}b_2a_2 \\ +C_{23}a_2b_2 + C_{33}b_2b_2 \end{array} & \begin{array}{c} C_{22}a_3a_2 + C_{23}b_3a_2 \\ +C_{23}a_3b_2 + C_{33}b_3b_2 \end{array} \\ & & & & & \begin{array}{c} C_{22}a_3a_3 + C_{23}b_3a_3 \\ +C_{23}a_3b_3 + C_{33}b_3b_3 \end{array} \end{bmatrix} \qquad \text{(A.119)}$$

3.11 Computer Generation of Constant Strain Triangle Stiffness

The SSAP2 program generates the CST stiffness in three steps:

a. A call to subroutine *QDCOS* where the local cartesian coordinate system (x,y,z) of a quadrilateral or triangle is defined with respect to the global cartesian coordinate system ($\bar{X}$, $\bar{Y}$, $\bar{Z}$).

b. A call to subroutine *TDCOS* where the projected dimensions of a triangle subelement are computed with respect to the local coordinate system x - y - z in the plane of the subelement.

c. A call to *SLST* which forms the plane stress stiffness matrix for the CST triangle.

3.12 Subroutine QDCOS

A triangle plate-shell element of arbitrary slope and orientation in space is shown in Figure A-28a. The element location is specified by global coordinates

$$\bar{X}_i = \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{bmatrix} \quad i = 1,2,3 \qquad \text{(A.120)}$$

of the three corners 1, 2, and 3.

In addition to the global coordinate system, a local rectangular coordinate system (x,y,z) is specified with the x and y axes located in the plane of the element,* while the z axis is perpendicular to the plane of the element. Positive direction of the z axis is taken to coincide with the positive direction of the rotating vector corresponding to the sequence 1, 2, 3.

Within subroutine *QDCOS* the z-axis is determined by cross multiplying vectors 1-3 and 3-2. The x-axis is determined for the triangle by completing a parallelogram using the addition of vectors 1-3 and 1-2, after which the vector 1-g is determined as its diagonal.

* If quadrilaterial element is not plane, an "average" element plane defined by the four external midside points is assumed.

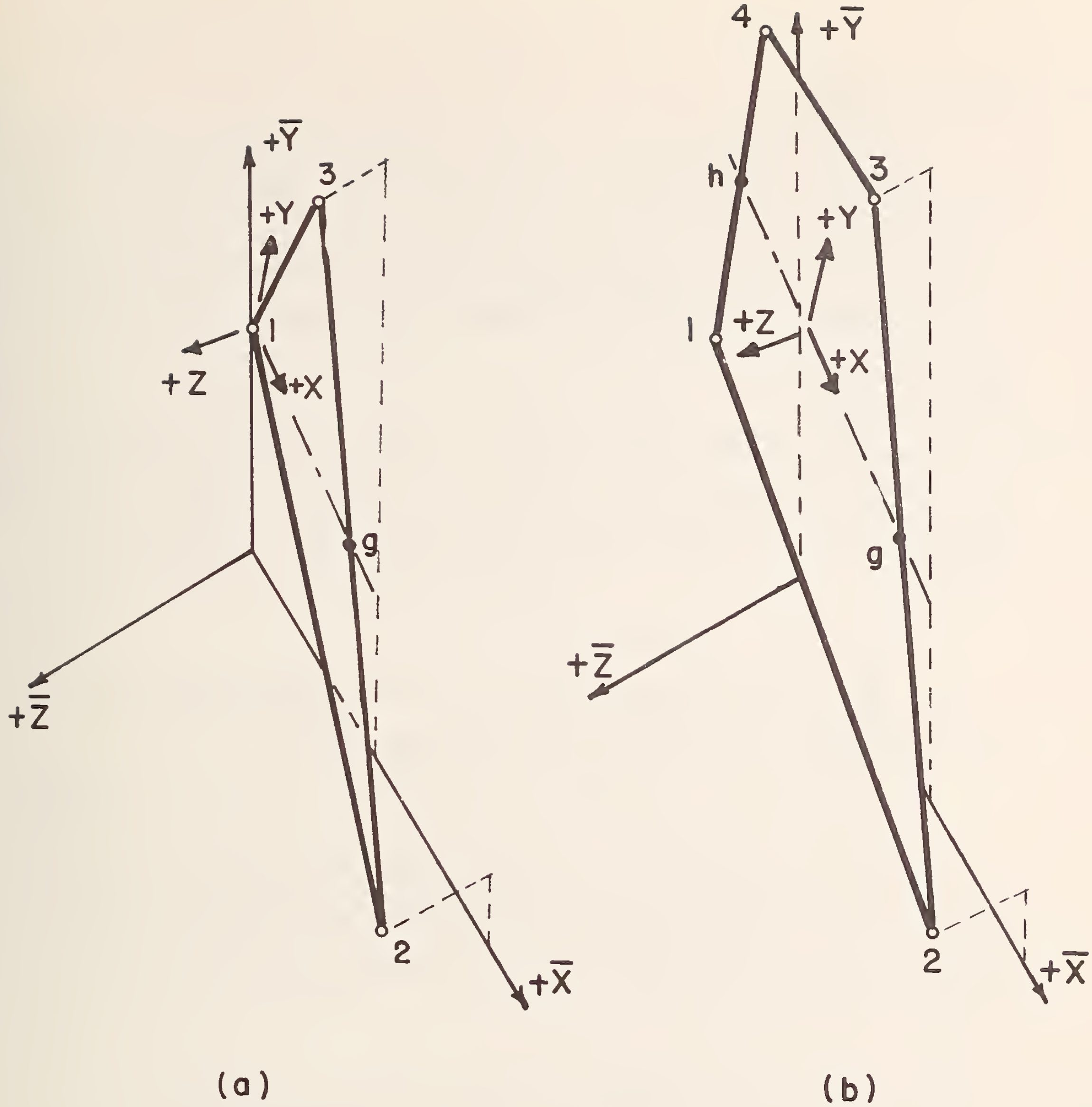


FIG. A-28 GLOBAL AND LOCAL AXES FOR TRIANGLE AND QUADRILATERAL.

"g" therefore becomes located at the midpoint of the line 2-3. Now the y axis can be established by cross-multiplying axes x and z.

For the quadrilateral, Figure A-28b, a similar computation locates x as along the line joining the midpoints of sides 1-4 and 3-2. Points h and g are on this line as shown. Once again the z axis is taken as perpendicular to the plane of the plate; and the y axis is then located by cross multiplying x and z. Finally subroutine *QDCOS* calculates the direction cosines of x, y, and z with respect to the $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes.

3.13 Subroutine TDCOS

In subroutine *TDCOS* the projected dimensions of the triangular element as defined by Figure A-24 are established in the plane of the element with the direction of x and y as previously established in *QDCOS*. Thus the projected lengths are in the local planar axes of the element. These projected lengths become the a's and b's of Equation (A.112).

3.14 Subroutine SLST

Subroutine *SLST* actually does the filling in of the appropriate terms of Equation (A.119) for the Constant Strain Triangle. Note that Equation (A.119) as given defines the nodal displacement vectors in the order:

$$r^T = [\ u_1,\ u_2,\ u_3,\ v_1,\ v_2,\ v_3\ ] \qquad \text{(A-121)}$$

For the sake of printing out all component information of each node in sequence, the order of r^T is changed to:

$$r^T = [\ u_1,\ v_1,\ u_2,\ v_2,\ u_3,\ v_3\ ] \qquad \text{(A-122)}$$

This is accomplished by an interchange of columns and rows in Equation (A.119). For example, if we label the locations in the matrix K of Equation (A.119) by its a_{ij} value, we would have the following sequence:

$$K = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{bmatrix} \end{array} \qquad \text{(A-123)}$$

If the columns and rows are interchanged we get the following order of terms:

$$K = \begin{array}{c} \\ 1 \\ 3 \\ 5 \\ 2 \\ 4 \\ 6 \end{array} \begin{array}{c} \begin{matrix} 1 & 3 & 5 & 2 & 4 & 6 \end{matrix} \\ \begin{bmatrix} 1,1 & 1,3 & 1,5 & 1,2 & 1,4 & 1,6 \\ 3,1 & 3,3 & 3,5 & 3,2 & 3,4 & 3,6 \\ 5,1 & 5,3 & 5,5 & 5,2 & 5,4 & 5,6 \\ 2,1 & 2,3 & 2,5 & 2,2 & 2,4 & 2,6 \\ 4,1 & 4,3 & 4,5 & 4,2 & 4,4 & 4,6 \\ 6,1 & 6,3 & 6,5 & 6,2 & 6,4 & 6,6 \end{bmatrix} \end{array} \qquad \text{(A-124)}$$

In *SLST* the stiffness matrix is packed in the upper triangular form by the sequence A,B,C,D,E,F as shown:

$$\begin{bmatrix} A & A & B & B & D & D \\ A & A & B & B & D & D \\ & & C & C & E & E \\ & & C & C & E & E \\ & \text{Symmetric} & & & F & F \\ & & & & F & F \end{bmatrix}$$

As calculated by the program statements *SHEL 1134* thru *SHEL 1137* the *SLST* routine automatically transfers the equations to the correct matrix location as established by Equation (A.124). To check this out the reader should calculate the terms:

ST(K-1,L-1) ST(K ,L) ST(K-1,L) ST(K ,L-1)

For example for J=1, L=2, I=1, K=2 one would get:

$$ST(1,1) = C_{11}b_1b_1 + C_{33}a_1a_1 + C_{13}(a_1b_1 + b_1a_1)$$

which fits term 1,1 of Equation (A.119).

$$ST(2,2) = C_{22}a_1a_1 + C_{33}b_1b_1 + C_{23}(a_1b_1 + b_1a_1)$$

which fits term 4,4 of Equation (A.119).

$$ST(1,2) = C_{12}b_1a_1 + C_{33}a_1b_1 + C_{13}b_1b_1 + C_{23}a_1a_1$$

which fits term 1,4 of Equation (A.119).

$$ST(2,1) = C_{12}a_1b_1 + C_{33}b_1a_1 + C_{13}b_1b_1 + C_{23}a_1a_1$$

which fits term 4,1 of Equation (A.119).

The transformation of terms thus can be stated:

1. Calculate ST(number, number)
2. Read the appropriate row and column of Equation (A.124) represented by the numbers. The corresponding location in Equation (A.124) represents the matrix location for Equation (A.119) which will match the values calculated by ST.

3.15 Stiffness Matrix - Linear Strain Triangle.

The original formulation cited in reference (16), based on Dr. Felippa's dissertation, does not quite match the *SLST* program statements of SSAP2, but probably does match another program written by Dr. Felippa in the Appendix to reference (16). The *SLST* statements follow another version of the LST Triangle prepared by Dr. Felippa two years later. The reformulation, as received by the author, is given in Appendix G. Essentially for the LST Triangle the stiffness can be written in the following form:

$$
K = \frac{h}{48A}
\begin{bmatrix}
b_1 & b_1 & b_1 & \cdot & \cdot & \cdot & a_1 & a_1 & a_1 \\
b_2 & b_2 & b_2 & \cdot & \cdot & \cdot & a_2 & a_2 & a_2 \\
b_3 & b_3 & b_3 & \cdot & \cdot & \cdot & a_3 & a_3 & a_3 \\
\cdot & 4b_3 & 4b_2 & \cdot & \cdot & \cdot & \cdot & 4a_3 & 4a_2 \\
4b_3 & \cdot & 4b_1 & \cdot & \cdot & \cdot & 4a_3 & \cdot & 4a_1 \\
4b_2 & 4b_1 & \cdot & \cdot & \cdot & \cdot & 4a_2 & 4a_1 & \cdot \\
\cdot & \cdot & \cdot & a_1 & a_1 & a_1 & b_1 & b_1 & b_1 \\
\cdot & \cdot & \cdot & a_2 & a_2 & a_2 & b_2 & b_2 & b_2 \\
\cdot & \cdot & \cdot & a_3 & a_3 & a_3 & b_3 & b_3 & b_3 \\
\cdot & \cdot & \cdot & \cdot & 4a_3 & 4a_2 & \cdot & 4b_3 & 4b_2 \\
\cdot & \cdot & \cdot & 4a_3 & \cdot & 4a_1 & 4b_3 & \cdot & 4b_1 \\
\cdot & \cdot & \cdot & 4a_2 & 4a_1 & \cdot & 4b_2 & 4b_1 & \cdot
\end{bmatrix}
\begin{bmatrix}
2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2
\end{bmatrix}
\begin{bmatrix}
b_1 & b_2 & b_3 & \cdot & 4b_3 & 4b_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
b_1 & b_2 & b_3 & 4b_3 & \cdot & 4b_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
b_1 & b_2 & b_3 & 4b_2 & 4b_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_1 & a_2 & a_3 & \cdot & 4a_3 & 4a_2 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_1 & a_2 & a_3 & 4a_3 & \cdot & 4a_1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_1 & a_2 & a_3 & 4a_2 & 4a_1 & \cdot \\
a_1 & a_2 & a_3 & \cdot & 4a_3 & 4a_2 & b_1 & b_2 & b_3 & \cdot & 4b_3 & 4b_2 \\
a_1 & a_2 & a_3 & 4a_3 & \cdot & 4a_1 & b_1 & b_2 & b_3 & 4b_3 & \cdot & 4b_1 \\
a_1 & a_2 & a_3 & 4a_2 & 4a_1 & \cdot & b_1 & b_2 & b_3 & 4b_2 & 4b_1 & \cdot
\end{bmatrix}
\qquad (A.125)
$$

Equation (A.126) is the result of carrying out the indicated matrix manipulations specified by Equation (A.125).

Equation (A.127) is analogous to Equation (A.123).

Equation (A.128) is analogous to Equation (A.124).

The matching of SSAP2 statements *SHEL 1183* thru *SHEL 1186* to theory is accomplished by the same procedure as outlined for the CST triangle.

For example for J=3, JM=6, I=1, K=2 one would get:

$$\begin{aligned} ST(1,7) = 16[&c_{11}(b_1b_2 + b_1b_3) = c_{33}(a_1a_2 + a_1a_3) \\ &+ c_{13}(a_1b_2 + a_1b_3 + b_1a_2 + b_1a_3)] \end{aligned}$$

which fits term 1,4 of Equation (A.126).

$$\begin{aligned} ST(2,8) = 16[&c_{22}(a_1a_2 + a_1a_3) + c_{33}(b_1b_2 + b_1b_3) \\ &+ c_{23}(a_1b_2 + a_1b_3 + b_1a_2 + b_1a_3)] \end{aligned}$$

which fits term 7,10 of Equation (A.126)

$$\begin{aligned} ST(1,8) = 16[&c_{12}(b_1a_2 + b_1a_3) + c_{33}(a_1b_2 + a_1b_3) \\ &+ c_{13}(b_1b_2 + b_1b_3) + c_{23}(a_1a_2 + a_1a_3)] \end{aligned}$$

which fits term 1,10 of Equation (A.126).

$$\begin{aligned} ST(2,7) = 16[&c_{12}(a_1b_2 + a_1b_3) + c_{33}(b_1a_2 + b_1a_3) \\ &+ c_{13}(b_1b_2 + b_1b_3) + c_{23}(a_1a_2 + a_1a_3)] \end{aligned}$$

which fits term 7,4 of Equation (A.126).

$$
\mathbf{K} = \frac{h}{48A}
\left[\begin{array}{cccccc}
\begin{array}{l}12[b_1(c_{11}b_1 + c_{13}a_1)\\ \quad + a_1(c_{13}b_1 + c_{33}a_1)]\end{array} &
\begin{array}{l}12[b_1(c_{11}b_2 + c_{13}a_2)\\ \quad + a_1(c_{13}b_2 + c_{33}a_2)]\end{array} &
\begin{array}{l}12[b_1(c_{11}b_3 + c_{13}a_3)\\ \quad + a_1(c_{13}b_3 + c_{33}a_3)]\end{array} &
\begin{array}{l}16\{b_1[c_{11}(b_3+b_2) + c_{13}(a_3+a_2)]\\ \quad + a_1[c_{13}(b_3+b_2) + c_{33}(a_3+a_2)]\}\end{array} &
\begin{array}{l}16\{b_1[c_{11}(b_3+b_1) + c_{13}(a_3+a_1)]\\ \quad + a_1[c_{13}(b_3+b_1) + c_{33}(a_3+a_1)]\}\end{array} &
\begin{array}{l}16\{b_1[c_{11}(b_2+b_1) + c_{13}(a_2+a_1)]\\ \quad + a_1[c_{13}(b_2+b_1) + c_{33}(a_2+a_1)]\}\end{array} \\
 &
\begin{array}{l}12[b_2(c_{11}b_2 + c_{13}a_2)\\ \quad + a_2(c_{13}b_2 + c_{33}a_2)]\end{array} &
\begin{array}{l}12[b_2(c_{11}b_3 + c_{13}a_3)\\ \quad + a_2(c_{13}b_3 + c_{33}a_3)]\end{array} &
\begin{array}{l}16\{b_2[c_{11}(b_3+b_2) + c_{13}(a_3+a_2)]\\ \quad + a_2[c_{13}(b_3+b_2) + c_{33}(a_3+a_2)]\}\end{array} &
\begin{array}{l}16\{b_2[c_{11}(b_3+b_1) + c_{13}(a_3+a_1)]\\ \quad + a_2[c_{13}(b_3+b_1) + c_{33}(a_3+a_1)]\}\end{array} &
\begin{array}{l}16\{b_2[c_{11}(b_2+b_1) + c_{13}(a_2+a_1)]\\ \quad + a_2[c_{13}(b_2+b_1) + c_{33}(a_2+a_1)]\}\end{array} \\
 & &
\begin{array}{l}12[b_3(c_{11}b_3 + c_{13}a_3)\\ \quad + a_3(c_{13}b_3 + c_{33}a_3)]\end{array} &
\begin{array}{l}16\{b_3[c_{11}(b_3+b_2) + c_{13}(a_3+a_2)]\\ \quad + a_3[c_{13}(b_3+b_2) + c_{33}(a_3+a_2)]\}\end{array} &
\begin{array}{l}16\{b_3[c_{11}(b_3+b_1) + c_{13}(a_3+a_1)]\\ \quad + a_3[c_{13}(b_3+b_1) + c_{33}(a_3+a_1)]\}\end{array} &
\begin{array}{l}16\{b_3[c_{11}(b_2+b_1) + c_{13}(a_2+a_1)]\\ \quad + a_3[c_{13}(b_2+b_1) + c_{33}(a_2+a_1)]\}\end{array} \\
 & & &
\begin{array}{l}16\{b_3[c_{11}(2b_3+b_2) + c_{13}(2a_3+a_2)]\\ \quad + b_2[c_{11}(b_3+2b_2) + c_{13}(a_3+2a_2)]\\ \quad + a_3[c_{13}(2b_3+b_2) + c_{33}(2a_3+a_2)]\\ \quad + a_2[c_{13}(b_3+2b_2) + c_{33}(a_3+2a_2)]\}\end{array} &
\begin{array}{l}16\{b_3[c_{11}(b_3+b_1) + c_{13}(a_3+a_1)]\\ \quad + b_2[c_{11}(b_3+2b_1) + c_{13}(a_3+2a_1)]\\ \quad + a_3[c_{13}(b_3+b_1) + c_{33}(a_3+a_1)]\\ \quad + a_2[c_{13}(b_3+2b_1) + c_{33}(a_3+2a_1)]\}\end{array} &
\begin{array}{l}16\{b_3[c_{11}(b_2+2b_1) + c_{13}(a_2+2a_1)]\\ \quad + b_2[c_{11}(b_2+b_1) + c_{13}(a_2+a_1)]\\ \quad + a_3[c_{13}(b_2+2b_1) + c_{33}(a_2+2a_1)]\\ \quad + a_2[c_{13}(b_2+b_1) + c_{33}(a_2+a_1)]\}\end{array} \\
 & & \text{SYMMETRICAL} & &
\begin{array}{l}16\{b_3[c_{11}(2b_3+b_1) + c_{13}(2a_3+a_1)]\\ \quad + b_1[c_{11}(b_3+2b_1) + c_{13}(a_3+2a_1)]\\ \quad + a_3[c_{13}(2b_3+b_1) + c_{33}(2a_3+a_1)]\\ \quad + a_1[c_{13}(b_3+2b_1) + c_{33}(a_3+2a_1)]\}\end{array} &
\begin{array}{l}16\{b_3[c_{11}(2b_2+b_1) + c_{13}(2a_2+a_1)]\\ \quad + b_1[c_{11}(b_2+b_1) + c_{13}(a_2+a_1)]\\ \quad + a_3[c_{13}(2b_2+b_1) + c_{33}(2a_2+a_1)]\\ \quad + a_1[c_{13}(b_2+b_1) + c_{33}(a_2+a_1)]\}\end{array} \\
 & & & & &
\begin{array}{l}16\{b_2[c_{11}(2b_2+b_1) + c_{13}(2a_2+a_1)]\\ \quad + b_1[c_{11}(b_2+2b_1) + c_{13}(a_2+2a_1)]\\ \quad + a_2[c_{13}(2b_2+b_1) + c_{33}(2a_2+a_1)]\\ \quad + a_1[c_{13}(b_2+2b_1) + c_{33}(a_2+2a_1)]\}\end{array}
\end{array}\right]
$$

$12[b_1(c_{12}a_1 + c_{13}b_1) + a_1(c_{23}a_1 + c_{33}b_1)]$	$12[b_1(c_{12}a_2 + c_{13}b_2) + a_1(c_{23}a_2 + c_{33}b_2)]$	$12[b_1(c_{12}a_3 + c_{13}b_3) + a_1(c_{23}a_3 + c_{33}b_3)]$	$16\{b_1[c_{12}(a_3+a_2) + c_{13}(b_3+b_2)] + a_1[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)]\}$	$16\{b_1[c_{12}(a_3+a_1) + c_{13}(b_3+b_1)] + a_1[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)]\}$	$16\{b_1[c_{12}(a_2+a_1) + c_{13}(b_2+b_1)] + a_1[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
$12[b_2(c_{12}a_1 + c_{13}b_1) + a_2(c_{23}a_1 + c_{33}b_1)]$	$12[b_2(c_{12}a_2 + c_{13}b_2) + a_2(c_{23}a_2 + c_{33}b_2)]$	$12[b_2(c_{12}a_3 + c_{13}b_3) + a_2(c_{23}a_3 + c_{33}b_3)]$	$16\{b_2[c_{12}(a_3+a_2) + c_{13}(b_3+b_2)] + a_2[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)]\}$	$16\{b_2[c_{12}(a_3+a_1) + c_{13}(b_3+b_1)] + a_2[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)]\}$	$16\{b_2[c_{12}(a_2+a_1) + c_{13}(b_2+b_1)] + a_2[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
$12[b_3(c_{12}a_1 + c_{13}b_1) + a_3(c_{23}a_1 + c_{33}b_1)]$	$12[b_3(c_{12}a_2 + c_{13}b_2) + a_3(c_{23}a_2 + c_{33}b_2)]$	$12[b_3(c_{12}a_3 + c_{13}b_3) + a_3(c_{23}a_3 + c_{33}b_3)]$	$16\{b_3[c_{12}(a_3+a_2) + c_{13}(b_3+b_2) + a_3[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)]\}$	$16\{b_3[c_{12}(a_3+a_1) + c_{13}(b_3+b_1)] + a_3[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)]\}$	$16\{b_3[c_{12}(a_2+a_1) + c_{13}(b_2+b_1)] + a_3[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
$16[(b_3+b_2)(c_{12}a_1+c_{13}b_1) + (a_3+a_2)(c_{23}a +c_{33}b_1)]$	$16[(b_3+b_2)(c_{12}a_2 + c_{13}b_2) + (a_3+a_2)(c_{23}a_2 + c_{33}b_2)]$	$16[(b_3+b_2)(c_{12}a_3 + c_{13}b_3) + (a_3+a_2)(c_{23}a_3 + c_{33}b_3)]$	$16\{b_3[c_{12}(2a_3+a_2) + c_{13}(2b_3+b_2)] + b_2[c_{12}(a_3+2a_2) + c_{13}(b_3+2b_2)] + a_3[c_{23}(2a_3+a_2) + c_{33}(2b_3+b_2)] + a_2[c_{23}(a_3+2a_2) + c_{33}(b_3+2b_2)]\}$	$16\{b_3[c_{12}(a_3+a_1) + c_{13}(b_3+b_1)] + b_2[c_{12}(a_3+2a_1) + c_{13}(b_3+2b_1)] + a_3[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)] + a_2[c_{23}(a_3+2a_1) + c_{33}(b_3+2b_1)]\}$	$16\{b_3[c_{12}(a_2+2a_1) + c_{13}(b_2+2b_1)] + b_2[c_{12}(a_2+2a_1) + c_{13}(b_2+b_1)] + a_3[c_{23}(a_2+2a_1) + c_{13}(b_2+2b_1)] + a_2[c_{23}(a_2+a_1) + c_{13}(b_2+b_1)]\}$
$16[(b_3+b_1)(c_{12}a_1+c_{13}b_1) + a_3+a_1)(c_{23}a_1+c_{33}b_1)]$	$16[(b_3+b_1)(c_{12}a_2 + c_{13}b_2) + (a_3+a_1)(c_{23}a_2 + c_{33}b_2)]$	$16[(b_3+b_1)(c_{12}a_3 + c_{13}b_3) + (a_3+a_1)(c_{23}a_3 + c_{33}b_3)]$	$16\{b_3[c_{12}(a_3+a_2) + c_{13}(b_3+b_2)]\ b_1[c_{12}(a_3+2a_2)+ c_{13}(b_3+2b_2)] + a_3[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)] + a_1[c_{23}(a_3+2a_2) + c_{33}(b_3+2b_2)]\}$	$16\{b_3[c_{12}(2a_3+a_1) + c_{13}(2b_3+b_1)] + b_1[c_{12}(a_2+2a_1) + c_{13}(b_3+2b_1)] + a_3[c_{23}(2a_3+a_1) + c_{33}(2b_3+b_1)] + a_1[c_{23}(a_3+2a_1) + c_{33}(b_3+2b_1)]\}$	$16\{b_3[c_{12}(2a_3+a_1) + c_{13}(2b_2+b_1)] + b_1[c_{12}(a_2+a_1) + c_{13}(b_2+b_1)] + b_3[c_{23}(2a_2+a_1) + c_{33}(2b_2+b_1)] + b_1[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
$16[(b_2+b_1)(c_{12}a_1+c_{13}b_1) + (a_2+a_1)(c_{23}a_1+c_{33}b_1)]$	$16[(b_2+b_1)(c_{12}a_2 + c_{13}b_2) + (a_2+a_1)(c_{23}a_2 + c_{33}b_2)]$	$16[(b_2+b_1)(c_{12}a_3 + c_{13}b_3) + (a_2+a_1)(c_{23}a_3 + c_{33}b_3)]$	$16\{b_2[c_{12}(a_3+a_2) + c_{13}(b_3+b_2)] + b_1[c_{12}(2a_3+a_2) + c_{13}(2b_3+b_2)] + a_2[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)] + a_1[c_{23}(2a_3+a_2) + c_{33}(2b_3+b_2)]\}$	$16\{b_2[c_{12}(2a_3+a_1) + c_{13}(2b_3+b_1)] + b_1[c_{12}(a_3+a_1) + c_{13}(b_3+b_1)] + a_2[c_{23}(2a_3+a_1) + c_{33}(2b_3+b_1)] + a_1[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)]\}$	$16\{b_2[c_{12}(2a_2+a_1) + c_{13}(2b_2+b_1)] + b_1[c_{12}(a_2+2a_1) + c_{13}(b_2+2b_1)] + a_2[c_{23}(2a_2+a_1) + c_{33}(2b_2+b_1)] + a_1[c_{23}(a_2+2a_1) + c_{33}(b_2+2b_1)]\}$
$12[a_1(c_{22}a_1+c_{23}b_1) + b_1(c_{23}a_1+c_{33}b_1)]$	$12[a_1(c_{22}a_2 + c_{23}b_2) + b_1(c_{23}a_2 + c_{33}b_2)]$	$12[a_1(c_{22}a_3 + c_{23}b_3) + b_1(c_{23}a_3 + c_{33}b_3)]$	$16\{a_1[c_{22}(a_3+a_2) + c_{23}(b_3+b_2)] + b_1[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)]\}$	$16\{a_1[c_{22}(a_3+a_1) + c_{23}(b_3+b_1)] + b_1[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)]\}$	$16\{a_1[c_{22}(a_2+a_1) + c_{23}(b_2+b_1)] + b_1[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
	$12[a_2(c_{22}a_2 + c_{23}b_2) + b_2(c_{23}a_2 + c_{33}b_2)]$	$12[a_2(c_{22}a_3 + c_{23}b_3) + b_2(c_{23}a_3 + c_{33}b_3)]$	$16\{a_2[c_{22}(a_3+a_2) + c_{23}(b_3+b_2)] + b_2[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)]\}$	$16\{a_2[c_{22}(a_3+a_1) + c_{23}(b_3+b_1)] + b_2[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)]\}$	$16\{a_2[c_{22}(a_2+a_1) + c_{23}(b_2+b_1)] + b_2[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
		$12[a_3(c_{22}a_3 + c_{23}b_3) + b_3(c_{23}a_3 + c_{33}b_3)$	$16\{a_3[c_{22}(a_3+a_2) + c_{23}(b_3+b_2)] + b_3[c_{23}(a_3+a_2) + c_{33}(b_3+b_2)]\}$	$16\{a_3[c_{22}(a_3+a_1) + c_{23}(b_3+b_1)] + b_3[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)]\}$	$16\{a_3[c_{22}(a_2+a_1) + c_{23}(b_2+b_1)] + b_3[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
			$16\{a_3[c_{22}(2a_3+a_2) + c_{23}(2b_3+b_2)] + a_2[c_{22}(a_3+2a_2) + c_{23}(b_3+2b_2)] + b_3[c_{23}(2a_3+a_2) + c_{33}(2b_3+b_2)] + b_2[c_{23}(a_3+2a_2) + c_{33}(b_3+2b_2)]\}$	$16\{a_3[c_{22}(a_3+a_1) + c_{23}(b_3+b_1)] + a_2[c_{22}(a_3+2a_1) + c_{23}(b_3+2b_1)] + b_3[c_{23}(a_3+a_1) + c_{33}(b_3+b_1)] + b_2[c_{22}(a_3+2a_1) + c_{33}(b_3+2b_1)]\}$	$16\{a_3[c_{22}(a_2+a_1) + c_{23}(b_2+b_1)] + a_2[c_{22}(a_2+2a_1) + c_{23}(b_2+2b_1)] + b_3[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)] + b_2[c_{23}(a_2+2a_1) + c_{33}(b_2+2b_1)]\}$
				$16\{a_3[c_{22}(2a_3+a_1) + c_{23}(2b_3+b_1)] + a_1[c_{22}(a_3+2a_1) + c_{23}(b_3+2b_1)] + b_3[c_{23}(2a_3+a_1) + c_{33}(2b_3+b_1)] + b_1[c_{23}(a_3+2a_1) + c_{33}(b_3+2b_1)]\}$	$16\{a_3[c_{22}(2a_2+a_1) + c_{23}(2b_2+b_1)] + a_1[c_{22}(a_2+a_1) + c_{23}(b_2+b_1)] + b_3[c_{23}(2a_2+a_1) + c_{33}(b_2+b_1)\} + b_1[c_{23}(a_2+a_1) + c_{33}(b_2+b_1)]\}$
					$16\{a_2[c_{22}(2a_2+a_1) + c_{23}(2b_2+b_1)] + a_1[c_{22}(a_2+2a_1) + c_{23}(b_2+2b_1)] + b_2[c_{23}(2a_2+a_1) + c_{33}(2b_2+b_1)] + b_1[c_{23}(a_2+2a_1) + c_{33}(b_2+2b_1)]\}$

(A-126)

	1	2	3	4	5	6	7	8	9	10	11	12
1	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,10	1,11	1,12
2	2,1	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	2,10	2,11	2,12
3	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3.8	3,9	3,10	3,11	3,12
4	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	4,10	4,11	4,12
5	5,1	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9	5,10	5,11	5,12
6	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9	6,10	6,11	6,12
K = 7	7,1	7,2	7,3	7,4	7,5	7,6	7,7	7,8	7,9	7,10	7,11	7,12
8	8,1	8,2	8,3	8,4	8,5	8,6	8,7	8,8	8,9	8,10	8,11	8,12
9	9,1	9,2	9,3	9,4	9,5	9.6	9,7	9,8	9,9	9.10	9.11	9.12
10	10,1	10,2	10,3	10,4	10,5	10,6	10,7	10,8	10,9	10,10	10,11	10,12
11	11,1	11,2	11,3	11,4	11,5	11.6	11,7	11,8	11,9	11,10	11,11	11,12
12	12,1	12,2	12,3	12,4	12,5	12,6	12,7	12,8	12,9	12,10	12,11	12,12

(A.127)

	1	3	5	7	9	11	2	4	6	8	10	12
1	1,1	1,3	1,5	1,7	1,9	1,11	1,2	1,4	1,6	1,8	1,10	1,12
3	3,1	3,3	3,5	3,7	3,9	3,11	3,2	2,4	3,6	3,8	3,10	3,12
5	5,1	5,3	5,5	5,7	5,9	5,11	5,2	5,4	5,6	5,8	5,10	5,12
7	7,1	7,3	7,5	7,7	7,9	7,11	7,2	7,4	7,6	7,8	7,10	7,12
9	9.1	9,3	9,5	9,7	9,9	9,11	9,2	9,4	9,6	9,8	9,10	9,12
K = 11	11,1	11,3	11,5	11,7	11,9	11,11	11,2	11,4	11,6	11,8	11,10	11,12
2	2,1	2,3	2,5	2,7	2,9	2,11	2,2	2,4	2,6	2,8	2,10	2,12
4	4,1	4,3	4,5	4,7	4,9	4,11	4,2	4,4	4,6	4,8	4,10	4,12
6	6,1	6,3	6,5	6,7	6,9	6,11	6,2	6,4	6,6	6,8	6,10	6,12
8	8,1	8,3	8,5	8,7	8,9	8,11	8,2	8,4	8,6	8,8	8,10	8,12
10	10,1	10,3	10,5	10,7	10,9	10,11	10,2	10,4	10,6	10,8	10,10	10,12
12	12,1	12,3	12,5	12,7	12,9	12,11	12,2	12,4	12,6	12,8	12,10	12,12

(A.128)

3.16 Stiffness Matrix - LCCT Triangle.

Appendix F contains the derivation of the stiffness matrix for the general quadrilateral plate bending element composed of four partially constrained linear curvature compatible triangles. Also described is a shear distortion mechanism which may be incorporated into the element without changing its basic structure.

Equation (F-22) contains the bending stiffness without amplification for shear distortion. The terms T_n are obtained by double differentiating Equation (F-13), and calculating the values at the nodes of the subelement triangles. These values then match SSAP2 statements *SHEL* 1364 thru *SHEL* 1391. The identification of these statements will be described in some detail.

As for Equation (A.111) the chain rule is used to express the first derivatives of a ϕ function, $\phi = F(\zeta_1,\zeta_2,\zeta_3)$, as follows:

$$\frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial\zeta_1}\frac{\partial\zeta_1}{\partial x} + \frac{\partial\phi}{\partial\zeta_2}\frac{\partial\zeta_2}{\partial x} + \frac{\partial\phi}{\partial\zeta_3}\frac{\partial\zeta_3}{\partial x}$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial\phi}{\partial\zeta_1}\frac{\partial\zeta_1}{\partial y} + \frac{\partial\phi}{\partial\zeta_2}\frac{\partial\zeta_2}{\partial y} + \frac{\partial\phi}{\partial\zeta_3}\frac{\partial\zeta_3}{\partial y} \qquad \text{(A.129)}$$

Again differentiating each of the above products gives the second derivatives, such as

$$\frac{\partial^2\phi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial x}\right) = \frac{\partial\zeta_1}{\partial x}\left[\frac{\partial^2\phi}{\partial\zeta_1\partial\zeta_1}\frac{\partial\zeta_1}{\partial x} + \frac{\partial^2\phi}{\partial\zeta_1\partial\zeta_2}\frac{\partial\zeta_2}{\partial x} + \frac{\partial^2\phi}{\partial\zeta_1\partial\zeta_3}\frac{\partial\zeta_3}{\partial x}\right] + \frac{\partial\phi}{\partial\zeta_1}\frac{\partial^2\zeta_1}{\partial x^2}$$

$$+ \frac{\partial\zeta_2}{\partial x}\left[\frac{\partial^2\phi}{\partial\zeta_2\partial\zeta_1}\frac{\partial\zeta_1}{\partial x} + \frac{\partial^2\phi}{\partial\zeta_2\partial\zeta_2}\frac{\partial\zeta_2}{\partial x} + \frac{\partial^2\phi}{\partial\zeta_2\partial\zeta_3}\frac{\partial\zeta_3}{\partial x}\right] + \frac{\partial\phi}{\partial\zeta_2}\frac{\partial^2\zeta_2}{\partial x^2}$$

$$+ \frac{\partial\zeta_3}{\partial x}\left[\frac{\partial^2\phi}{\partial\zeta_3\partial\zeta_1}\frac{\partial\zeta_1}{\partial x} + \frac{\partial^2\phi}{\partial\zeta_3\partial\zeta_2}\frac{\partial\zeta_2}{\partial x} + \frac{\partial^2\phi}{\partial\zeta_3\partial\zeta_3}\frac{\partial\zeta_3}{\partial x}\right] + \frac{\partial\phi}{\partial\zeta_3}\frac{\partial^2\zeta_3}{\partial x^2}$$

(A.130)

where the last term of each line becomes zero since the displacement function is linear.

Substituting Equation (A.106) into Equation (A.130), we obtain:

$$\frac{\partial^2\phi}{\partial x^2} = \frac{b_1}{4A^2}\left[\frac{\partial^2\phi}{\partial\zeta_1\partial\zeta_1}b_1 + \frac{\partial^2\phi}{\partial\zeta_1\partial\zeta_2}b_2 + \frac{\partial^2\phi}{\partial\zeta_1\partial\zeta_3}b_3\right]$$
$$+ \frac{b_2}{4A^2}\left[\frac{\partial^2\phi}{\partial\zeta_2\partial\zeta_1}b_1 + \frac{\partial^2\phi}{\partial\zeta_2\partial\zeta_2}b_2 + \frac{\partial^2\phi}{\partial\zeta_2\partial\zeta_3}b_3\right]$$
$$+ \frac{b_3}{4A^2}\left[\frac{\partial^2\phi}{\partial\zeta_3\partial\zeta_1}b_1 + \frac{\partial^2\phi}{\partial\zeta_3\partial\zeta_2}b_2 + \frac{\partial^2\phi}{\partial\zeta_3\partial\zeta_3}b_3\right]$$

(A.131)

To obtain $\frac{\partial^2\phi}{\partial y^2}$ all b's in Equation (A.131) are replaced by a's. To obtain the mixed derivative $\frac{\partial^2\phi}{\partial x\partial y}$ all b's within the brackets of Equation (A.131) only are replaced by a's.

To match the computer statements each of the compatible displacement functions (page F-15) are differentiated twice, and substituted in Equation (A.131) in accordance with the schedule given in Table A-2.

When I=1 (*SHEL* 1294), the calculations are for subelement 3.

When I=2 (*SHEL* 1294), the calculations are for subelement 1.

When I=3 (*SHEL* 1294), the calculations are for subelement 2.

TABLE A-2 IDENTIFICATION OF COMPATIBLE DISPLACEMENT FUNCTION SECOND DERIVATIVES FOR SUBELEMENT 3.

Statement Number		N = 1	N = 2	N = 3
SHEL 1364	$\zeta_1=1 \quad \zeta_2=0 \quad \zeta_3=0$	$P(1,1) = \frac{\partial^2 \Phi_{\omega 1}^{(3)}}{\partial x^2}$	$P(8,1) = \frac{\partial^2 \Phi_{\omega 1}^{(3)}}{\partial y^2}$	$P(13,1) = \frac{\partial^2 \Phi_{\omega 1}^{(3)}}{\partial x \partial y}$
SHEL 1365		$P(1,2) = \frac{\partial^2 \Phi_{\theta x 1}^{(3)}}{\partial x^2}$	$P(8,2) = \frac{\partial^2 \Phi_{\theta x 1}^{(3)}}{\partial y^2}$	$P(13,2) = \frac{\partial^2 \Phi_{\theta x 1}^{(3)}}{\partial x \partial y}$
SHEL 1366		$P(1,3) = \frac{\partial^2 \Phi_{\theta y 1}^{(3)}}{\partial x^2}$	$P(8,3) = \frac{\partial^2 \Phi_{\theta y 1}^{(3)}}{\partial y^2}$	$P(13,3) = \frac{\partial^2 \Phi_{\theta y 1}^{(3)}}{\partial x \partial y}$
SHEL 1367		$P(1,4) = \frac{\partial^2 \Phi_{\omega 2}^{(3)}}{\partial x^2}$	$P(8,4) = \frac{\partial^2 \Phi_{\omega 2}^{(3)}}{\partial y^2}$	$P(13,4) = \frac{\partial^2 \Phi_{\omega 2}^{(3)}}{\partial x \partial y}$
SHEL 1368		$P(1,5) = \frac{\partial^2 \Phi_{\theta x 2}^{(3)}}{\partial x^2}$	$P(8,5) = \frac{\partial^2 \Phi_{\theta x 2}^{(3)}}{\partial y^2}$	$P(13,5) = \frac{\partial^2 \Phi_{\theta x 2}^{(3)}}{\partial x \partial y}$
SHEL 1369		$P(1,6) = \frac{\partial^2 \Phi_{\theta y 2}^{(3)}}{\partial x^2}$	$P(8,6) = \frac{\partial^2 \Phi_{\theta y 2}^{(3)}}{\partial y^2}$	$P(13,6) = \frac{\partial^2 \Phi_{\theta y 2}^{(3)}}{\partial x \partial y}$
SHEL 1370		$P(1,7) = \frac{\partial^2 \Phi_{\omega 3}^{(3)}}{\partial x^2}$	$P(8,7) = \frac{\partial^2 \Phi_{\omega 3}^{(3)}}{\partial y^2}$	$P(13,7) = \frac{\partial^2 \Phi_{\omega 3}^{(3)}}{\partial x \partial y}$
SHEL 1371		$P(1,8) = \frac{\partial^2 \Phi_{\theta x 3}^{(3)}}{\partial x^2}$	$P(8,8) = \frac{\partial^2 \Phi_{\theta x 3}^{(3)}}{\partial y^2}$	$P(13,8) = \frac{\partial^2 \Phi_{\theta x 3}^{(3)}}{\partial x \partial y}$
SHEL 1372		$P(1,9) = \frac{\partial^2 \Phi_{\theta y 3}^{(3)}}{\partial x^2}$	$P(8,9) = \frac{\partial^2 \Phi_{\theta y 3}^{(3)}}{\partial y^2}$	$P(13,9) = \frac{\partial^2 \Phi_{\theta y 3}^{(3)}}{\partial x \partial y}$
SHEL 1373		$P(1,10) = \frac{\partial^2 \Phi_{\theta 5}^{(3)}}{\partial x^2}$	$P(8,10) = \frac{\partial^2 \Phi_{\theta 5}^{(3)}}{\partial y^2}$	$P(13,10) = \frac{\partial^2 \Phi_{\theta 5}^{(3)}}{\partial x \partial y}$
SHEL 1374		$P(1,11) = \frac{\partial^2 \Phi_{\theta 6}^{(3)}}{\partial x^2}$	$P(8,11) = \frac{\partial^2 \Phi_{\theta 6}^{(3)}}{\partial y^2}$	$P(13,11) = \frac{\partial^2 \Phi_{\theta 6}^{(3)}}{\partial x \partial y}$
SHEL 1375		$P(1,12) = \frac{\partial^2 \Phi_{\theta 4}^{(3)}}{\partial x^2}$	$P(8,12) = \frac{\partial^2 \Phi_{\theta 4}^{(3)}}{\partial y^2}$	$P(13,12) = \frac{\partial^2 \Phi_{\theta 4}^{(3)}}{\partial x \partial y}$

Statement Number		N = 1	N = 2	N = 3
SHEL 1376		$P(4,1) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 1}}{\partial x^2}$	$P(11,1) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 1}}{\partial y^2}$	$P(16,1) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 1}}{\partial x \partial y}$
SHEL 1377		$P(4,2) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x1}}{\partial x^2}$	$P(11,2) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x1}}{\partial y^2}$	$P(16,2) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x1}}{\partial x \partial y}$
SHEL 1378		$P(4,3) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y1}}{\partial x^2}$	$P(11,3) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y1}}{\partial y^2}$	$P(16,3) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y1}}{\partial x \partial y}$
SHEL 1379		$P(4,4) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 2}}{\partial x^2}$	$P(11,4) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 2}}{\partial y^2}$	$P(16,4) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 2}}{\partial x \partial y}$
SHEL 1380		$P(4,5) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x2}}{\partial x^2}$	$P(11,5) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x2}}{\partial y^2}$	$P(16,5) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x2}}{\partial x \partial y}$
SHEL 1381	$\zeta_1=0 \quad \zeta_2=1 \quad \zeta_3=0$	$P(4,6) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y2}}{\partial x^2}$	$P(11,6) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y2}}{\partial y^2}$	$P(16,6) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y2}}{\partial x \partial y}$
SHEL 1382		$P(4,7) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 3}}{\partial x^2}$	$P(11,7) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 3}}{\partial y^2}$	$P(16,7) = \frac{\partial^2 \hat{\phi}^{(3)}_{\omega 3}}{\partial x \partial y}$
SHEL 1383		$P(4,8) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x3}}{\partial x^2}$	$P(11,8) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x3}}{\partial y^2}$	$P(16,8) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta x3}}{\partial x \partial y}$
SHEL 1384		$P(4,9) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y3}}{\partial x^2}$	$P(11,9) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y3}}{\partial y^2}$	$P(16,9) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta y3}}{\partial x \partial y}$
SHEL 1385		$P(4,10) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta 5}}{\partial x^2}$	$P(11,10) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta 5}}{\partial y^2}$	$P(16,10) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta 5}}{\partial x \partial y}$
SHEL 1386		$P(4,11) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta 6}}{\partial x^2}$	$P(11,11) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta 6}}{\partial y^2}$	$P(16,11) = \frac{\partial^2 \hat{\phi}^{(3)}_{\theta 6}}{\partial x \partial y}$
SHEL 1387		$P(4,12) = \frac{\partial^2 \hat{\phi}^{3}_{\theta 4}}{\partial x^2}$	$P(11,12) = \frac{\partial^2 \hat{\phi}^{3}_{\theta 4}}{\partial y^2}$	$P(16,12) = \frac{\partial^2 \hat{\phi}^{3}_{\theta 4}}{\partial x \partial y}$

Table A-12 Continued.

Statement Number		N = 1	N = 2	N = 3
SHEL 1388	$\zeta_1 = \frac{1}{3}$ $\zeta_2 = \frac{1}{3}$ $\zeta_3 = \frac{1}{3}$	$P(19,1) = \frac{\partial^2 \Phi^{(3)}_{\omega 1}}{\partial x^2}$	$P(20,1) = \frac{\partial^2 \Phi^{(3)}_{\omega 1}}{\partial y^2}$	$P(21,1) = \frac{\partial^2 \Phi^{(3)}_{\omega 1}}{\partial x \partial y}$
SHEL 1389		$P(19,8) = \frac{\partial^2 \Phi^{(3)}_{\theta x3}}{\partial x^2}$	$P(20,8) = \frac{\partial^2 \Phi^{(3)}_{\theta x3}}{\partial y^2}$	$P(21,8) = \frac{\partial^2 \Phi^{(3)}_{\theta x3}}{\partial x \partial y}$
SHEL 1390		$P(19,9) = \frac{\partial^2 \Phi^{(3)}_{\theta y3}}{\partial x^2}$	$P(20,9) = \frac{\partial^2 \Phi^{(3)}_{\theta y3}}{\partial y^2}$	$P(21,9) = \frac{\partial^2 \Phi^{(3)}_{\theta y3}}{\partial x \partial y}$
SHEL 1391		$P(19,12) = \frac{\partial^2 \Phi^{(3)}_{\theta 4}}{\partial x^2}$	$P(20,12) = \frac{\partial^2 \Phi^{(3)}_{\theta 4}}{\partial y^2}$	$P(21,12) = \frac{\partial^2 \Phi^{(3)}_{\theta 4}}{\partial x \partial y}$

Table A-2 Concluded.

Notes: $\Phi^{(3)}_{\theta x3}$, page F-15, is modified prior to differentiation to

$$\Phi^{(3)}_{\theta x3} = \frac{1}{6} \zeta_3{}^2 [3(2b_1 - b_3 + b_1\lambda_1]\zeta_2 + (b_2\mu_2 - b_1\lambda_1)\zeta_3 - 3(-b_3 + 2b_2 + b_2\mu_2)\zeta_1]$$

by subtracting $[b_1+b_2+b_3=0]$ from $(3b_1+b_2+b_1\lambda_1)$ and $(b_1+3b_2+b_2\mu_2)$ respectively.

$\Phi^{(3)}_{\theta 4}$ is also modified to

$$\Phi^{(3)}_{\theta 4} = \frac{4A}{3L_3} [6\zeta_1\zeta_2\zeta_3 + \zeta_3{}^2(-3\zeta_1 - 3\zeta_2 + 2\zeta_3)]$$

prior to differentiation by noting that

$$-3(\zeta_1+\zeta_2+\zeta_3=1) + 3\zeta_3 + 2\zeta_3 = 5\zeta_3 - 3$$

APPENDIX B

User's Manual for Truss, Beam, and Plate-Shell Elements

B.1 Introduction

Appendix B contains excerpts from the SAP IV Manual (Reference 9) and the SSAP2 Manual (Reference 13, known as SOLID SAP). Only those sections of the manuals which pertain to the Truss, Beam and Plate-Shell elements have been excerpted. For the interested reader the aforementioned publications describe the Membrane, 2D Quadrilateral, 3D Brick, Boundary, Thick Shell, and 3D Pipe elements.

B.2 Boundary Condition Codes (Excerpted from Reference 9)

Boundary condition codes can only be assigned the following values (M = 1,2,...,6):

IX(N,M) = 0;	unspecified (free) displacement (or rotation) component
IX(N,M) = 1;	deleted (fixed) displacement (or rotation) component
IX(N,M) = K:	node number "K" (1< K > NUMNP and K ≠ N) is the "master" node to which the Mth degree of freedom at node "N" is a "slave"

An unspecified (IX(N,M) = 0) degree of freedom is free to translate or rotate as the solution dictates. Concentrated forces (or moments) may be applied in this degree of freedom. One (1) system equilibrium equation is required for each unspecified degree of freedom in the model. The maximum number of equilibrium equations is always less than six (6) times the total number of nodes in the model.

Deleted (IX(N,M) = 1) degrees of freedom are removed from the final set of equilibrium equations. Deleted degrees of freedom are fixed (points of reaction), and any loads applied in these degrees of freedom are ignored by the program. Nodes that are used for geometric reference only (i.e., nodes not assigned to any element) must have all six (6) degrees of freedom deleted. Nodal degrees of freedom having undefined stiffness (such as rotations in an all TRUSS model, out-of-plane components in a two-dimensional planar model, etc.) should be deleted. Deletions have the beneficial effect of reducing the size of the set of equations that must be solved. The table below lists the types of degrees of freedom that are defined by each different element type. The table was prepared assuming that the element has general orientation in (X,Y,Z) space.

TABLE B-1 DEGREES OF FREEDOM WITH DEFINED STIFFNESS

Element Type	δX	δY	δZ	$\delta\theta_X$	$\delta\theta_Y$	$\delta\theta_Z$
Truss	x	x	x			
Beam	x	x	x	x	x	x
Plate-Shell	x	x	x	x	x	x'

Note: x' is not defined for the flat plate.

B.3 Slave Elements

A "master/slave" option is allowed to model rigid links in the system. For this case, IX(N,M) = K means that the Mth degree of freedom at node "N" is "slave" to (dependent on) the same (Mth) degree of freedom at node "K"; node "K" is said to be the master node to which node N is slave. Note that no actual beam need to run from node K to node N, however the following restrictions hold:

(a) Node one (1) cannot be a master node; i.e., $K \neq 1$.
(b) Nodes "n" and "K" must be beam-only nodes; i.e., no other element type may be connected to either node N or K.
(c) A node "N" can be slave to only one master node, "K"; multiple nodes, however, can be slave to the same master.
(d) If the beam from "N" to "K" is to be a rigid link arbitrarily oriented in the X,Y,Z space, then all six (6) degrees of freedom at node "N" must be made slaves to node "K".

Displacement/rotation components for slave degrees of freedom at node "N" are not recovered for printing: i.e., zeroes appear as output for slave degrees of freedom.

The remainder of Appendix B is excerpted directly from SSAP2 without modification, except for some page numbering.

NOTE: Pages B-6 and B-10 specify multipliers of the gravity load along the global coordinate axes. This may be considered as a function of the slope of the vertical direction with respect to the global coordinate axes. On page B-5 element load multipliers are specified. These may be considered as multipliers for the Dead Loads (Dead Load Factors) of the element in question. The program gives four possible choices of a dead load which may be added to the live load condition per program run.

B-4 Input Data

The geometry of the joints, the boundary conditions, and the joint concentrated loads are numerically defined by a sequence of punched cards. The properties of the different structural elements are described separately.

I. Heading Card (12A6)

Columns	1 - 72	Contain information to be printed with output

II. Control Card (4I5)

Columns	1 - 5	Number of joints in system
	6 - 10	Number of element groups
	11 - 15	Number of load conditions
	16 - 20	Number of frequencies (= 0 for static analysis)

III. Joint Data (7I5,3F10.0,I5,F10.0)

The following information must be given for each joint in the system:

Columns	1 - 5	Joint Number	
	6 - 10	X-direction	Boundary Condition Codes: Zero or blank indicates that the joint is free to move in that direction and loads may be applied. 1 indicates that the joint is fixed in that direction.
	11 - 15	Y-direction	
	16 - 20	Z-direction	
	21 - 25	Rotation about X-axis	
	26 - 30	Rotation about Y-axis	
	31 - 35	Rotation about Z-axis	
	36 - 45	X-ordinate	For a slave node specify the number of the master node. Number 1 may not be used.
	46 - 55	Y-ordinate	
	56 - 65	Z-ordinate	
	66 - 70	KN	
	71 - 80	Joint Temperature	

Joint cards need not be in joint-order sequence. If cards are omitted, the joint data for a series of joints is generated. KN is a mesh generation parameter on the last card of a mesh generation sequence. KN is the increment to be added to the previous nodal point number. The intermediate joints are located at equal intervals along the straight line. The boundary condition codes for the generated joint data are set equal to the boundary condition codes on the first joint card in the series.

If a particular degree of freedom is fixed for a series of cards, this may be indicated by a boundary condition code of -1 on the first joint card in the series and +1 on the last joint card in the series. See element descriptions for determination of temperature dependent material properties and thermal loads from joint temperatures.

IV. Element Data

A sequence of cards is required for each type of element in the structure. The form of this data for each type is described in section B-5.

V. Concentrated Load Data (2I5,6F10.0)

One card per load case for each joint which has nonzero concentrated loads or moments applied. The cards must be in joint-number sequence.

Columns	1 - 5	Joint number
	6 - 10	Load condition number
	11 - 20	Load X-direction
	21 - 30	Load Y-direction
	31 - 40	Load Z-direction
	41 - 50	Moment X-axis
	51 - 60	Moment Y-axis
	61 - 70	Moment Z-axis

This sequence of cards (if any) must be terminated with ONE BLANK CARD.

VI. Element Load Multipliers (4F10.0)

Four different types of loads associated with the element are possible. These element loads are referred to as load cases A, B, C and D. By the use of "Element Load Multipliers," it is possible to add fractions of the basic element loads to any of the concentrated load conditions.

One card must be supplied for each load condition which contains the following information:

Columns	1 - 10	Multiplier for element load A
	11 - 20	Multiplier for element load B
	21 - 30	Multiplier for element load C
	31 - 40	Multiplier for element load D

These cards must be in load-order sequence. The definitions of the element loads associated with a particular element type are discussed in detail under the section "Element Data".

B-5 Element Data

Type 1 - Three-Dimensional Truss Members

Truss elements are identified by the number 1. Axial forces and stresses are calculated for each member. A uniform temperature change and inertia loads in three directions can be considered as the basic member load conditions. The truss members are described by the following sequence of cards:

A. Control Card (3I5)

Columns	1 - 5	The number 1
	6 - 10	Number of truss members
	11 - 15	Number of members with different properties

B. Member Property Cards (I5,5F10.0)

One card is required for each member which has a different cross-section or different material properties.

Columns	1 - 5	Material identification number
	6 - 15	Modulus of elasticity
	16 - 25	Coefficient of thermal expansion
	26 - 35	Blank
	36 - 45	Cross-sectional area
	46 - 55	Weight per unit length (used to calculate gravity loads)

C. Element Load Factors (4F10.0) Four cards

Three cards specifying the fraction of gravity (in each of the three global coordinate directions) to be added to each element load case.

Card 1: Multiplier of gravity load in the +X direction

Columns	1 - 10	Element load case A
	11 - 20	Element load case B
	21 - 30	Element load case C
	31 - 40	Element load case D

Card 2: As above for gravity in the +Y direction

Card 3: As above for gravity in the +Z direction

Card 4: This indicates the fraction of the thermal load to be added to each of the element load cases.

D. <u>Member Data Cards</u> (4I5,F10,0,I5)

One card per member in increasing numerical order starting with one.

Columns	1 - 5	Member number (n)
	6 - 10	Joint number I
	11 - 15	Joint number J
	16 - 20	Member material identification number
	21 - 30	Reference temperature for zero stress
	31 - 35	Optional parameter K causing automatic generation of number data.

If a series of elements exist such that the member number, N_i, is one greater than the previous member number (i.e. $N_i = N_{i-1} + 1$) and the joint number can be given by

$$I_i = I_{i-1} + K$$

$$J_i = J_{i-1} + K$$

Then only the first element in the series need be provided. The member identification number and the temperature for the generated elements are set equal to the values on the first card. If K is input as zero it is set to 1 by the program.

The member temperature increase ΔT used to calculate thermal loads is given by

$$\Delta T = (T_i + T_j)/2.0 - T_r$$

where $(T_i + T_j)/2.0$ is the average of the nodal temperatures specified on the joint data cards for nodes i and j; and T_r is the zero stress reference temperature specified on the element card. For truss elements it is generally more convenient to set $T_i = T_j = 0.0$ such that $\Delta T = - T_r$ (note the minus sign). Other types of member loadings can be specified using an equivalent ΔT. If a truss member has an initial lack of fit by an amount d (positive if too long) then $\Delta T = d/(\alpha L)$ If an initial prestress force P (positive if tensile) is applied to the member ends that is released after the member is connected to the rest of the structure then $\Delta T = - P/(\alpha A E)$. In the above formulas A = cross section area, L = member length and α = coefficient of thermal expansion.

Type 2 - Three-Dimensional Beam Elements

Beam elements are identified by the number 2. Forces (axial and shear) and moments (bending and torsion) are calculated (in the beam local coordinate system) for each beam. Gravity loadings in each coordinate direction and specified fixed end forces form the basic member load conditions.

The beam members are described by the following sequence of cards:

A. Control Card (5I5)

Columns	1 - 5	The number 2
	6 - 10	Number of beam elements
	11 - 15	Number of geometric property cards
	16 - 20	Number of fixed end force sets
	21 - 25	Number of different materials

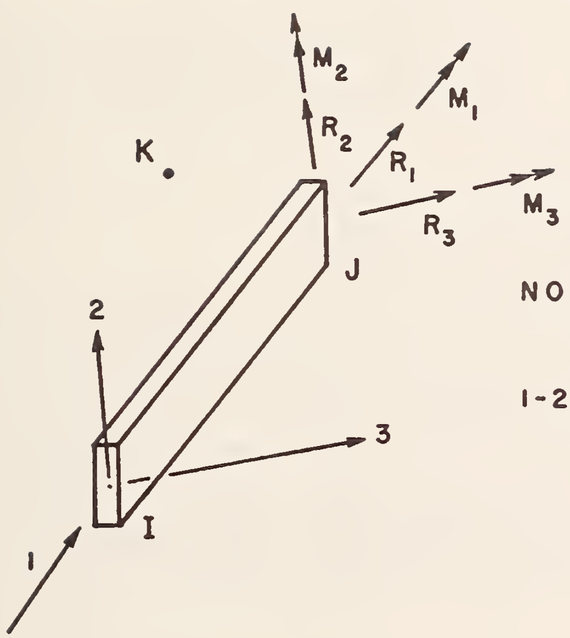
B. Material Property Cards (I5,3F10.0)

Columns	1 - 5	Material identification number
	6 - 15	Young's modulus
	16 - 25	Poisson's ratio

C. Geometric Property Cards (I5,6F10.0)

Columns	1 - 5	Geometric property number
	6 - 15	Axial area
	16 - 25	Shear area associated with shear forces in local 2-direction
	26 - 35	Shear area associated with shear forces in local 3-direction
	36 - 45	Torsional inertia
	46 - 55	Flexural inertia about local 2-axis
	56 - 65	Flexural inertia about local 3-axis

One card is required for each unique set of properties. Shear area is included only if shear deformations are to be included in the analysis.



NOTE:
K IS ANY NODAL POINT WHICH LIES IN THE LOCAL 1-2 PLANE (NOT ON THE 1-AXIS)

LOCAL COORDINATE SYSTEM FOR BEAM ELEMENT

D. Element Load Factors (4F10.0)

Three cards specifying the fraction of gravity (in each of the three global coordinate directions) to be added to each element load case.

Card 1: Multiplier of gravity load in the +X direction

Columns	1 - 10	Element load case A
	11 - 20	Element load case B
	21 - 30	Element load case C
	31 - 40	Element load case D

Card 2: As above for gravity in the +Y direction

Card 3: As above for gravity in the +Z direction

Fixed-End Forces are not computed within the program for gravity loads

E. Fixed-End Forces (I5,6F10.0/I5,6F10.0)

Two cards are required for each unique set of fixed-end forces occurring in the analysis. Distributed loads and thermal loads are input using fixed end forces.

Card 1:

Columns	1 - 5	Fixed-end force number
	6 - 15	Fixed-end force in local 1-direction at Node I
	16 - 25	Fixed-end force in local 2-direction at Node I
	26 - 35	Fixed-end force in local 3-direction at Node I
	36 - 45	Fixed-end moment about local 1-direction at Node I
	46 - 55	Fixed-end moment about local 2-direction at Node I
	56 - 65	Fixed-end moment about local 3-direction at Node I

Card 2:

Columns	1 - 5	Blank
	6 - 15	Fixed-end force in local 1-direction at Node J
	16 - 25	Fixed-end force in local 2-direction at Node J
	26 - 35	Fixed-end force in local 3-direction at Node J
	36 - 45	Fixed-end moment about local 1-direction at Node J
	46 - 55	Fixed-end moment about local 2-direction at Node J
	56 - 65	Fixed-end moment about local 3-direction at Node J

Note that values input are literally fixed-end values. Corrections due to hinges and rollers are performed within the program. Directions 1, 2 and 3 indicate principal directions in the local beam coordinates

F. Beam Data Cards (10I5,2I6,I8)

Columns		
	1 - 5	Identification - beam number
	6 - 10	Node I number
	11 - 15	Node J number
	16 - 20	Node K number - see Figure 4-1.
	21 - 25	Material number
	26 - 30	Geometric property number
	31 - 35	A — Fixed-end force identification for element load cases A, B, C, and D respectively
	36 - 40	B
	41 - 45	C
	46 - 50	D
	51 - 56	End release code - Node I
	57 - 62	End release code - Node J
	63 - 70	Optional parameter k used for automatic generation of element data. This option is described below under a separate heading. If the option is not used, the field is left blank.

The end release code at each node is a six digit number of ones and/or zeros. The 1st, 2nd, ... 6th digits respectively correspond to the force components R1, R2, R3, M1, M2, M3 at each node.

If any one of the above member end forces is known to be zero (hinge or roller), the digit corresponding to that component is a one.

Automatic Element Data Generation

If a series of elements occurs in which each element number NE_i is one greater than the previous number NE_{i-1}

i.e., $$NE_i = NE_{i-1} + 1$$

only the element data card for the first element in the series need be given as input.

IF The end nodal point numbers $NI_i = NI_{i-1} + k$

$$NJ_i = NJ_{i-1} + k$$

AND THE

a) material identification number

b) geometric property identification number

c) fixed-end force identification numbers for each element load case

d) element code

e) orientation of local 2-axis

are the same for each element in the series.

The generator option is the value of k and if left blank is taken to be one. The element data card for the last element in the structure must always be given.

*Where successive beam elements have the same stiffness, orientation and element loading, the program automatically skips recomputation of the stiffness. Note this when numbering the beams to obtain maximum efficiency.

Type 6 - Plate and Shell Elements (Quadrilateral)

A. Control Card (3I5)

Columns	1 - 5	The number 6
	6 - 10	Number of shell elements
	11 - 15	Number of different materials

B. Material Property Information

Anisotropic material properties are possible. For each different material, two cards must be supplied.

Card 1: (I10,20X,4F10.0)

Columns	1 - 10	Material identification number
	31 - 40	Mass density
	41 - 50	Thermal expansion coefficient α_x
	51 - 60	Thermal expansion coefficient α_y
	61 - 70	Thermal expansion coefficient α_z

Card 2: (6F10.0)

Columns	1 - 10	Elasticity element C_{xx}
	11 - 20	Elasticity element C_{xy}
	21 - 30	Elasticity element C_{xs}
	31 - 40	Elasticity element C_{yy}
	41 - 50	Elasticity element C_{ys}
	51 - 60	Elasticity element G_{xy}

Elements in plane stress material matrix [C]

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{xx} & C_{xy} & C_{xs} \\ C_{xy} & C_{yy} & C_{ys} \\ C_{xs} & C_{ys} & G_{xy} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

C. Element Load Multiplers (5 cards)

Card 1: (4F10.0)

Columns	1 - 10	Distributed lateral load multiplier for load case A
	11 - 20	Distributed lateral load multiplier for load case B
	21 - 30	Distributed lateral load multiplier for load case C
	31 - 40	Distributed lateral load multiplier for load case D

Card 2: (4F10.0)

Columns	1 - 10	Temperature multiplier for load case A
	11 - 20	Temperature multiplier for load case B
	21 - 30	Temperature multiplier for load case C
	31 - 40	Temperature multiplier for load case D

Card 3: (4F10.0)

Columns	1 - 10	X-direction acceleration for load case A
	11 - 20	X-direction acceleration for load case B
	21 - 30	X-direction acceleration for load case C
	31 - 40	X-direction acceleration for load case D

Card 4: (4F10.0) Same as Card 3 for Y-direction

Card 5: (4F10.0) Same as Card 3 for Z-direction

D. Element Cards (8I5,F10.0)

One card for each element

Columns	1 - 5	Element number
	6 - 10	Node I
	11 - 15	Node J
	16 - 20	Node K
	21 - 25	Node L
	26 - 30	Node 0*
	31 - 35	Material identification (If left blank, taken as one)
	36 - 40	Element data generator K_n**
	41 - 50	Element thickness
	51 - 60	Distributed lateral load (pressure)
	61 - 70	Mean temperature variation T from the reference level in undeformed position
	71 - 80	Mean temperature gradient $\partial T/\partial z$ across the shell thickness (a positive temperature gradient produces a negative curvature).

* When columns 26 - 30 are left blank, mid-node properties are computed by averaging the four nodes.

** Element cards must be in element number sequence. If element cards are omitted, the program automatically generates the omitted information as follows:

The increment for element number is one

$$\text{i.e.} \quad NE_{i+1} = NE_i + 1$$

The corresponding increment for nodal number is K_n

$$\text{i.e.} \quad NI_{i+1} = NI_i + K_n$$

$$NJ_{i+1} = NJ_i + K_n$$

$$NK_{i+1} = NK_i + K_n$$

$$NL_{i+1} = NL_i + K_n$$

Material identification, element thickness, distributed lateral load, temperature and temperature gradient for generated elements are the same as the first element in the series. The last element card is always needed.

NOTE

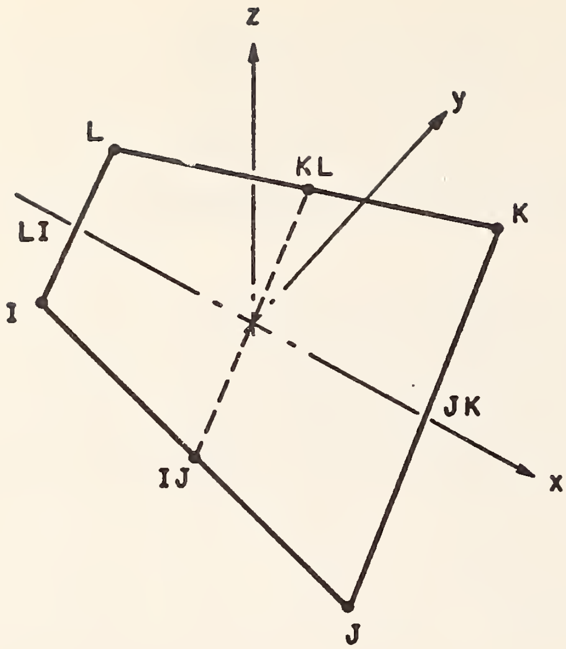
The nodal point numbers I, J, K and L are in sequence in a counter-clockwise direction around the element. The local element coordinate system (x, y, z) is defined as follows:

x Specified by LI - JK, where LI and JK are midpoints of sides L-I and J-K.

z Normal to x and to the line joining midpoints IJ and KL.

y Normal to x and z to complete the right-handed system.

This system is used to express all physical and kinematic shell properties (stresses, strains, material law, etc.), except that the body force density is referred to the global coordinate system (X, Y, Z).



For the analyses of smooth shells, rotational constraints normal to the surface may be imposed by the addition of Boundary elements at the nodes (element type #7).

APPENDIX C - PROGRAM CAPACITY

C-1 High Speed Storage Requirements

The high speed storage requirements of the program can be changed depending on the size of the problem to be solved. This is done by changing the two Fortran statements at the start of SAP2, i.e.

```
COMMON A(n)
MTOT = n
```

The minimum value of n needed is computed as follows:

n = 10* (number of joints) + M

where

M = the maximum value of each of the following:

(1) Truss elements

M = 5* NMAT   NMAT = number of material types

(2) Beam elements

M = 3*NMAT+12*NFIX+6*NPROP

NFIX = number of fixed end force sets

NPROP = number of different beam properties

(3) Plane stress and plane strain elements.

M = 4*NMAT+11*NMAT*NTC

NTC = number of material temperatures

(4) Axisymmetric quadrilateral

M = 4*NMAT

(5) Three-dimensional solid elements

M = 4*NMAT+4*NLD+2475   NLD = number of element load sets

(6) Plate and shell elements

M = 12*NMAT

(7) Boundary elements

M = 0

(8) Solid thick shell element

M = 4*NMAT+4NLD+6615

Note: (1) A convenient general rule for computing a minimum value of n (except for solid elements) is:

n = 11* (number of joints)

(2) For optimum efficiency, however, a value of n, considerably greater than the minimum, should be used.

(3) If the value of n is set too small an error message is printed and program execution is terminated.

C-2 Low Speed Storage Requirements

For very large problems, the amount of low speed backup storage on the computer will govern the maximum size of structure which can be solved. Six temporary storage files are used with the following maximum storage requirements:

Tape 1

For element stress-displacement transformation matrices

Number of locations on Tape 1 =

$$\sum_{i=1}^{Ne} [(N_s + 2) * N_d + N_s * 4 + 2]_i$$

Where N_e = total number of elements

N_s = number of output stresses associated with an element

N_d = number of displacement degrees of freedom associated with an element

Tape 2

For element stiffness matrices

Number of locations on Tape 2 =

$$\sum_{i=1}^{Ne} [N_d * (N_d + 6) + 1]_i$$

or for temporary storage during solution of equations

Number of locations on Tape 2 =

$$\left| \frac{N_{band}}{N_{eb}} \right| (N_{band} * N_{eb})$$

Where N_{band} = half band width of equations

N_{eb} = number of equations in a block

$$= \frac{(MTOT - 4 * N_\ell)}{(N_{band} + N_\ell + 1) * 2}$$

N_ℓ = number of load conditions

or, for storage of displacements

Number of locations on Tape 2 = $N_{eq} * N_\ell$

Where N_{eq} = total number of equations

Tape 3

For temporary storage during solution of equations

Number of locations on Tape 3 =

$$N_{eq} * (N_{band} + N_{\ell})$$

Tape 4

For storage of total stiffness and load matrices.

Number of locations on Tape 4 =

$$N_{eq} * (N_{band} + N_{\ell})$$

Tape 7

For temporary storage in the formation of the total stiffness matrix

Number of locations on Tape 2 =

$$\sqrt{N_e} * (N_d * (N_d + 6) + 1)$$

or, for temporary storage during solution of equations; same as second option for Tape 2.

Tape 8

For storage of boundary condition array and load multipliers

Number of locations on Tape 8 =

$$6 * N_j + 4 * N_{\ell}$$

where N_j = total number of joints in the system

C-3 Solid-SAP Common Storage Allocation

COMMON A (MTOT)

COMMON/ELPAR/NPAR (14), NUMNP, MBAND, N1, N2, N3, N4, N5, NEQ

COMMON/EM/2594 locations max.

COMMON/JUNK/222 locations max.

APPENDIX C

Computer Program Listing

Appendix C contains a listing of the SSAP2 version used by the author to process the CDC 6400, IBM 360/370, and the Honeywell 6060 runs. In the CDC version a single statement *CALL SECOND (T(i))* is sufficient to call the computer clock for calculating elapsed CPU time through the various phases of the program *(OVERALL LOG)*. For the IBM machine two assembler subroutines must be called; *ETODF* for starting the clock and *ETODF1* for providing the elapsed CPU time from start time. *ETODF* in turn calls a macro instruction *STIMER* (Start time) to set the clock. *ETODF1* calls a macro instruction *TTIMER* (Total time) to stop the clock. Elapsed CPU time is thus contained in *TTIMER*. For calling the Honeywell computer clock the program statements are similar to that of the CDC machine, except that a short subroutine is used to return clock time in the same units (seconds x 10^{-2} as an integer number) to the main program.

In the IBM and Honeywell runs underflows occur at statements *SSAP 584* or *SHEL 442* when the shell element becomes nearly singular. The underflows are considered in the nature of a warning signal that the results of a computation are smaller than the recording capacity of the computer. The standard fixup for the IBM machine is to set the quantity equal to zero. The underflows have not yet interfered with the program to cause any errors; therefore the author has suppressed the underflow warning messages by inclusion of a single card *CALL ERRSET (208,256,-1,1)* in the main program. Removal of this card will once again provide the underflow messages.

```
C  ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** **  *
C     SAP2 A STATIC ANALYSIS PROGRAM FOR THREE-DIMENSIONAL STRUCTURES
C                       REVISED MARCH 1972
C  ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** **  *
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
      COMMON /JUNK / HED(12),JUK(210)
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
      COMMON / EM / QQQ(2596)
      DIMENSION T(7)
C
C     PROGRAM CAPACITY CONTROLLED BY THE FOLLOWING TWO STATEMENTS ...
      COMMON A(11000)
      MTOT=11000
C
      CALL ERRSET(208,256,-1,1)
C
C     PROGRAM CONTROL DATA
C
    5 CALL ETODF(ITOD)
      READ(ISYSCR,100) HED,NUMNP,NELTYP,LL,NF,NOYN
      IF (NUMNP.EQ.0) STOP
      WRITE(ISYSLP,200) HED,NUMNP,NELTYP,LL
C
C     INPUT JOINT DATA--ID ARRAY ON TAPE 8
C
      N1=1
      N2=N1+6*NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+NUMNP
      N6=N5+NUMNP
      IF(N6.GT.MTOT) CALL ERROR(N6-MTOT)
      CALL INPUTJ(A(N1),A(N2),A(N3),A(N4),A(N5),NUMNP,NEQ)
C
C     FORM ELEMENT STIFFNESSES--STIFF. ON TAPE 2 -STRESS MATRIX ON TAPE1
C
      CALL ETODF1(ITOD)
      TOD=ITOD
      T(1)=TOD*0.01
      CALL ETODF(ITOD)
      MBAND=0
      NUMEL=0
      REWIND ISCRF1
      REWIND ISCRF2
      DO 900 M=1,NELTYP
      READ(ISYSCR,1001) NPAR
      WRITE(ISCRF1) NPAR
      NUMEL=NUMEL+NPAR(2)
      MTYPE=NPAR(1)
      CALL ELTYPE(MTYPE)
  900 CONTINUE
      CALL ETODF1(ITOD)
      TOD=ITOD
      T(2)=TOD*0.01
      CALL ETODF(ITOD)
C
C     INPUT NODAL LOADS AND JOINT MASSES --- WRITE ON TAPE 3
C
      NEQB=(MTOT-4*LL)/(MBAND+LL+1)/2
      NBLOCK=(NEQ-1)/NEQB +1
      IF (NEQB.GT.NEQ) NEQB=NEQ
      N3=N2+NEQB*LL
      N4=N3+6*LL
      WRITE (ISYSLP,201) NEQ,MBAND,NEQB,NBLOCK
      CALL INL(A(N1),A(N2),A(N3),A(N4),NUMNP,NEQB,LL)
      CALL ETODF1(ITOD)
      TOD=ITOD
      T(3)=TOD*0.01
      CALL ETODF(ITOD)
C
C     FORM TOTAL STIFFNESS MATRIX --ON TAPE 4
C
      NE2B=2*NEQB
      N2=N1+NEQB*MBAND
      N3=N2+NEQB*LL
      N4=N3+4*LL
      NN2=N1+NE2B*MBAND
      NN3=NN2+NE2B*LL
      NN4=NN3+4*LL
      CALL ADDSTF(A(N1),A(NN2),A(NN3),A(NN4),NUMEL,NBLOCK,NE2B,LL,MBAND)
```

```
      CALL ETODF1(ITOD)
      TOD=ITCO
      T(4)=TOO*0.01
      CALL ETODF(ITOD)
C                                                                       SSAP  72
C     SOLVE FOR DISPLACEMENT UNKNOWNS                                   SSAP  73
C                                                                       SSAP  74
      NSB=(MBAND+LL)*NEQB                                               SSAP  75
      NSBB=NEQB*LL*(2+(MBAND-1)/NEQB)                                   SSAP  76
      IF(NSBB.LT.NSB) NSBB=NSB                                          SSAP  77
      N4=N3+NSBB                                                        SSAP  78
      CALL USOL(A(N1),A(N3),A(N4),NEQB,MBAND,LL,NBLOCK,NSB,4,3,7,2,2)   SSAP  79
      CALL ETODF1(ITOD)
      TOD=ITCO
      T(5)=TOD*0.01
      CALL ETODF(ITOD)
C                                                                       SSAP  81
C     PRINT DISPLACEMENT                                                SSAP  82
C                                                                       SSAP  83
      N2=N1+NUMNP*6                                                     SSAP  84
      N3=N2+6*LL                                                        SSAP  85
      CALL PRINTD(A(N1),A(N2),A(N3),NEQB,NUMNP,LL,NBLOCK,NEQ,2)         SSAP  86
C                                                                       SSAP  87
C     COMPUTE STRESSES                                                  SSAP  88
C                                                                       SSAP  89
   30 N2=N1+4*LL                                                        SSAP  90
      N3=N2+NEQB*LL                                                     SSAP  91
      LB=(MTOT-N3)/(NEQ +12)                                            SSAP  92
      NDYN=0                                                            SSAP  93
      CALL STRESS(A(N1),A(N2),A(N3),NEQB,LB,LL,NEQ,NBLOCK)              SSAP  94
C                                                                       SSAP  95
      CALL ETODF1(ITOD)
      TOD=ITOO
      T(6)=TOO*0.01
      T(7)=T(1)+T(2)+T(3)+T(4)+T(5)+T(6)                                SSAP  99
      WRITE (ISYSLP,203) (T(I),I=1,7)
C
C                                                                       SSAP 101
      GO TO 5                                                           SSAP 102
C                                                                       SSAP 103
  100 FORMAT(12A6/6I5)                                                  SSAP 104
  200 FORMAT(1H1,12A6///                                                SSAP 105
     .  28H  NUMBER OF NODAL POINTS   =  ,I5//                          SSAP 106
     .  28H  NUMBER OF ELEMENT TYPES  =  ,I5//                          SSAP 107
     .  28H  NUMBER OF LOAD CASES     = ,I5)                            SSAP 108
  201 FORMAT(34H2 TOTAL NUMBER OF EQUATIONS       =,I5,                 SSAP 109
     1      /34H  BANDWIDTH                       =,I5,                 SSAP 110
     2      /34H  NUMBER OF EQUATIONS IN A BLOCK =,I5,                  SSAP 111
     3      /34H  NUMBER OF BLOCKS                =,I5)                 SSAP 112
  203 FORMAT ( 12H2OVERALL LOG //                                       SSAP 113
     .          33H  NODAL POINT INPUT.............. ,F8.2//            SSAP 114
     .          33H  FORM ELEMENT STIFFNESSES...... ,F8.2//             SSAP 115
     .          33H  INPUT NODAL LOADS............. ,F8.2//             SSAP 116
     .          33H  FORM TOTAL STIFFNESS.......... ,F8.2//             SSAP 117
     .          33H  EQUATION SOLVING.............. ,F8.2//             SSAP 118
     .          33H  ELEMENT STRESSES.............. ,F8.2///            SSAP 119
     .          33H  TOTAL SOLUTION TIME........... ,F8.2)              SSAP 120
C                                                                       SSAP 121
 1001 FORMAT (14I5)                                                     SSAP 122
      END                                                               SSAP 123

      BLOCK DATA
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
      DATA ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9/1,3,11,12,13,14,17,18,19/
      END


      SUBROUTINE INPUTJ(ID,X,Y,Z,T,NUMNP,NEQ)                           SSAP 124
C-----E. WILSON JULY 1971                                               SSAP 125
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
      DIMENSION X(1),Y(1),Z(1),ID(NUMNP,6),T(1)                         SSAP 126
C                                                                       SSAP 127
C---- READ OR  GENERATE NODAL POINT DATA--------------------------------SSAP 128
```

```
      WRITE (ISYSLP,2000)
      WRITE (ISYSLP,2001)
      NOLD=0
   10 READ(ISYSCR,1000) N,(ID(N,I),I=1,6),X(N),Y(N),Z(N),KN,T(N)
      WRITE (ISYSLP,2002) N,(ID(N,I),I=1,6),X(N),Y(N),Z(N),KN,T(N)
      IF(NOLD.EQ.0) GO TO 50
C-----CHECK IF GENERATION IS REQUIRED-----------------------------------
      DO 20 I=1,6
      IF(ID(N,I).EQ.0.AND.ID(NOLD,I).LT.0) ID(N,I)=ID(NOLD,I)
   20 CONTINUE
      IF(KN.EQ.0) GO TO 50
      NUM=(N-NOLD)/KN
      NUMN=NUM-1
      IF(NUMN.LT.1) GO TO 50
      XNUM=NUM
      DX=(X(N)-X(NOLD))/XNUM
      DY=(Y(N)-Y(NOLD))/XNUM
      DZ=(Z(N)-Z(NOLD))/XNUM
      DT=(T(N)-T(NOLD))/XNUM
      K=NOLD
      DO 30 J=1,NUMN
      KK=K
      K=K+KN
      X(K)=X(KK)+DX
      Y(K)=Y(KK)+DY
      Z(K)=Z(KK)+DZ
      T(K)=T(KK)+DT
      DO 30 I=1,6
      ID(K,I)=ID(KK,I)
      IF (ID(K,I).GT.1) ID(K,I)=ID(KK,I)+KN
   30 CONTINUE
C
   50 NOLD=N
      IF(N.NE.NUMNP) GO TO 10
C
C---- PRINT ALL NODAL POINT DATA----------------------------------------
C
      WRITE(ISYSLP,2003)
      WRITE (ISYSLP,2001)
      WRITE (ISYSLP,2005) (N,(ID(N,I),I=1,6),X(N),Y(N),Z(N),T(N),
     1N=1,NUMNP)
C
C-----NUMBER UNKNOWNS AND SET MASTER NODES NEGATIVE---------------------
C
      NEQ=0
      DO 60 N=1,NUMNP
      DO 60 I=1,6
      ID(N,I)=IABS(ID(N,I))
      IF(ID(N,I)-1) 57,58,59
   57 NEQ=NEQ+1
      ID(N,I)=NEQ
      GO TO 60
   58 ID(N,I)=0
      GO TO 60
   59 ID(N,I)=-ID(N,I)
   60 CONTINUE
      WRITE (ISYSLP,2004) (N,(ID(N,I),I=1,6),N=1,NUMNP)
      REWIND ISCRF8
      WRITE (ISCRF8) ID
C
      RETURN
 1000 FORMAT (7I5,3F10.0,I5,F10.0)
 2000 FORMAT (//23H NODAL POINT INPUT DATA )
 2001 FORMAT (5HONODE 3X 24HBOUNDARY CONDITION CODES 11X
     .. 23HNODAL POINT COORDINATES / 7H NUMBER 2X 1HX 4X 1HY 4X 1HZ 3X
     . 2HXX 3X 2HYY 3X 2HZZ12X 1HX 12X 1HY 12X 1HZ 12X 1HT )
 2002 FORMAT (I5,6I5,3F13.3,I5,F13.3)
 2003 FORMAT (//21H GENERATED NODAL DATA)
 2004 FORMAT (//17H EQUATION NUMBERS/
     1 35H     N     X     Y     Z    XX    YY    ZZ /(7I5))
 2005 FORMAT (I5,6I5,4F13.3)
      END


      SUBROUTINE ELTYPE(MTYPE)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
```

```
      GO TO (1,2,3,4,5,6,7,8),MTYPE                                     SSAP 201
C                                                                       SSAP 202
C     THREE OIMENSIONAL TRUSS ELEMENTS                                  SSAP 203
C                                                                       SSAP 204
    1 CALL TRUSS                                                        SSAP 205
      GO TO 900                                                         SSAP 206
C                                                                       SSAP 207
C     THREE DIMENSIONAL BEAM ELEMENTS                                   SSAP 208
C                                                                       SSAP 209
    2 CALL BEAM                                                         SSAP 210
      GO TO 900                                                         SSAP 211
C                                                                       SSAP 212
C     PLANE STRESS ELEMENTS                                             SSAP 213
C                                                                       SSAP 214
    3 CALL PLANE                                                        SSAP 215
      GO TO 900                                                         SSAP 216
C                                                                       SSAP 217
C     AXISYMMETRIC SOLID ELEMENTS                                       SSAP 218
C                                                                       SSAP 219
    4 CALL PLANE                                                        SSAP 220
      GO TO 900                                                         SSAP 221
C                                                                       SSAP 222
C     THREE DIMENSIONAL SOLIO ELEMENTS                                  SSAP 223
C                                                                       SSAP 224
    5 CALL THREED                                                       SSAP 225
      GO TO 900                                                         SSAP 226
C                                                                       SSAP 227
C     PLATE BENDING ELEMENTS                                            SSAP 228
C                                                                       SSAP 229
    6 CALL SHELL                                                        SSAP 230
      GO TO 900                                                         SSAP 231
C                                                                       SSAP 232
C                                                                       SSAP 233
    7 CALL BOUNO                                                        SSAP 234
      GO TO 900                                                         SSAP 235
C                                                                       SSAP 236
C     THICK SHELL ELEMENTS                                              SSAP 237
C                                                                       SSAP 238
    8 CALL THKSHL                                                       SSAP 239
C                                                                       SSAP 240
  900 RETURN                                                            SSAP 241
      ENO                                                               SSAP 242


      SUBROUTINE INL(IO,B,TR,TMASS,NUMNP,NEQB,LL)                       SSAP 243
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C                                                                       SSAP 244
C     INPUT NODAL LOAOS ANO MASSES                                      SSAP 245
C                                                                       SSAP 246
      DIMENSION ID(NUMNP,6),B(NEQB,LL),TR(6,LL),TMASS(NEQB)             SSAP 247
      COMMON / JUNK / R(6),TXM(6)                                       SSAP 248
C                                                                       SSAP 249
      NT=3                                                              SSAP 250
      REWIND ISCRF3
      KSHF=0                                                            SSAP 252
      WRITE (ISYSLP,2002)
      DO 750 I=1,NEQB                                                   SSAP 254
      TMASS(I)=0.                                                       SSAP 255
      DO 750 K=1,LL                                                     SSAP 256
  750 B(I;K)=0.0                                                        SSAP 257
C                                                                       SSAP 258
      DO 900 NN=1,NUMNP                                                 SSAP 259
C                                                                       SSAP 260
      OO 100 I=1,6                                                      SSAP 261
      TXM(I)=0.                                                         SSAP 262
      OO 100 J=1,LL                                                     SSAP 263
  100 TR(I,J)=0.0                                                       SSAP 264
C                                                                       SSAP 265
      IF(NN.EQ.1) GO TO 300                                             SSAP 266
  150 IF(N.NE.NN) GO TO 400                                             SSAP 267
      OO 200 I=1,6                                                      SSAP 268
      IF (L) 180,180,190                                                SSAP 269
  180 TXM(I)=R(I)                                                       SSAP 270
      GO TO 200                                                         SSAP 271
  190 TR(I,L)=R(I)                                                      SSAP 272
  200 CONTINUE                                                          SSAP 273
```

```
  300 READ (ISYSCR,1001) N,L,R
      IF (N.EQ.0) GO TO 150
      WRITE (ISYSLP,2001) N,L,R
      GO TO 150
C
  400 DO 800 J=1,6
      II=ID(NN,J)-KSHF
      IF (II) 800,800,500
  500 DO 600  K=1,LL
  600 B(II,K)=TR(J,K)
      TMASS(II)=TXM(J)
  610 IF(II.NE.NEQB) GO TO 800
      WRITE(ISCRF3) B,TMASS
      KSHF=KSHF+NEQB
      DO 700 I=1,NEQB
      TMASS(I)=0.
      DO 700 K=1,LL
  700 B(I,K)=0.0
  800 CONTINUE
  900 CONTINUE
      WRITE(ISCRF3) B,TMASS
C
      RETURN
 1001 FORMAT (2I5,7F10.0)
 2001 FORMAT (2I5,7F10.3)
 2002 FORMAT (23H2.....NODAL POINT LOAOS // 10H NODE LOAD 23X
     . 14HAPPLIEO LOADS              / 10H  NO. CASE   6X   2HRX   8X
     .  2HRY 8X 2HRZ BX 2HMX BX 2HMY BX 2HMZ )
      END


      SUBROUTINE ERROR(N)
      WRITE (ISYSLP,2000) N
 2000 FORMAT (// 20H STORAGE EXCEEOED BY  I6)
      STOP
      END


      SUBROUTINE AOOSTF(A,B,STR,TMASS,NUMEL,NBLOCK,NE2B,LL,MBAND)
      IMPLICIT REAL*B (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRFB,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRFB,ISCRF9
C     FORMS GLOBAL EQUILIBRIUM EQUATIONS IN BLOCKS
      DIMENSION A(NE2B,MBANO),B(NE2B,LL),STR(4,LL),TMASS(NE2B),SS(1)
      COMMON /EM/ LRD,NO,LM(2592)
      EQUIVALENCE (SS(1),LM(1))
      NEQB=NE2B/2
      K=NEQB+1
      X=NBLOCK
      MB=DSQRT(X)
      MB=MB/2+1
      NEBB=MB*NE2B
      MM=1
C
      NSHIFT=0
      REWIND ISCRF3
      REWIND ISCRF4
C
C     READ ELEMENT LOAD MULTIPLIERS
C
      WRITE (ISYSLP,2000)
      DO 50 L=1,LL
      READ(ISYSCR,1002) (STR(I,L),I=1,4)
   50 WRITE (ISYSLP,2002) L,(STR(I,L),I=1,4)
      WRITE (ISCRFB) STR
C
C     FORM EQUATIONS IN BLOCKS     ( 2 BLOCKS AT A TIME)
C
      DO 1000 M=1,NBLOCK ,2
      DO 100 I=1,NE2B
      DO 100 J=1,MBANO
  100 A(I,J)=0.
      REAO (ISCRF3) ((B(I,L),I=1,NEQB),L=1,LL),(TMASS(I),I=1,NEQB)
      IF (M.EQ.NBLOCK) GO TO 200
      REAO (ISCRF3) ((B(I,L),I=K,NE2B),L=1,LL),(TMASS(I),I=K,NE2B)
  200 CONTINUE
C
      REWINO ISCRF7
```

```
      REWIND ISCRF2
      NA=ISCRF7
      NUME=NUM7                                                         SSAP 347
      IF (MM.NE.1) GO TO 75                                             SSAP 348
      NA=ISCRF2
      NUME=NUMEL                                                        SSAP 350
      NUM7 =0                                                           SSAP 351
C                                                                       SSAP 352
   75 DO 700 N=1,NUME                                                   SSAP 353
      READ (NA) LRD,ND,(SS(I),I=1,LRD)
      DO 600 I=1,ND                                                     SSAP 355
      LMN=1-LM(I)                                                       SSAP 356
      II=LM(I)-NSHIFT                                                   SSAP 357
      IF (II.LE.0.OR.II.GT.NE2B) GO TO 600                              SSAP 358
      DO 300 L=1,LL                                                     SSAP 359
      DO 300 J=1,4                                                      SSAP 360
      KK=ND*(ND+J)+1                                                    SSAP 361
  300 B(II,L)=B(II,L)+SS(I+KK)*STR(J,L)                                 SSAP 362
      DO 500 J=1,ND                                                     SSAP 363
      JJ=LM(J)+LMN                                                      SSAP 364
      IF(JJ) 500,500,390                                                SSAP 365
  390 KK=ND*J+1                                                         SSAP 366
  400 A(II,JJ)=A(II,JJ)+SS(I+KK)                                        SSAP 367
  500 CONTINUE                                                          SSAP 368
  600 CONTINUE                                                          SSAP 369
C                                                                       SSAP 370
C     DETERMINE IF STIFFNESS IS TO BE PLACED ON TAPE 7
C                                                                       SSAP 372
      IF (MM.GT.1) GO TO 700                                            SSAP 373
      DO 650 I=1,ND                                                     SSAP 374
      II=LM(I) -NSHIFT                                                  SSAP 375
      IF(II.GT.NE2B.AND.II.LE.NE8B) GO TO 660                           SSAP 376
  650 CONTINUE                                                          SSAP 377
      GO TO 700                                                         SSAP 378
  660 WRITE (ISCRF7) LRD,ND,(SS(I),I=1,LRD)
      NUM7=NUM7+1                                                       SSAP 380
C                                                                       SSAP 381
  700 CONTINUE                                                          SSAP 382
      WRITE (ISCRF4) ((A(I,J),I=1,NEQB),J=1,MBAND),((B(I,L),I=1,NEQB),
     1L=1,LL)
      IF(M.EQ.NBLOCK) GO TO 1000                                        SSAP 384
      WRITE (ISCRF4) ((A(I,J),I=K,NE2B),J=1,MBAND),((B(I,L),I=K,NE2B),
     1L=1,LL)
      IF (MM.EQ.MB) MM=0                                                SSAP 386
      MM=MM+1                                                           SSAP 387
 1000 NSHIFT=NSHIFT+NE2B                                                SSAP 388
C                                                                       SSAP 389
      RETURN                                                            SSAP 390
 1002 FORMAT (4F10.0)                                                   SSAP 391
 2000 FORMAT (10H2STRUCTURE 12X 25HELEMENT LOAD MULTIPLIERS /           SSAP 392
     . 10H LOAD CASE 9X 1HA 9X 1HB 9X 1HC 9X 1HD/)                      SSAP 393
 2002 FORMAT (I6,7X,4F10.3)                                             SSAP 394
      END                                                               SSAP 395


      SUBROUTINE PRINTD(ID,D,B,NEQB,NUMNP,LL,NBLOCK,NEQ,NT)             SSAP 396
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C                                                                       SSAP 397
      DIMENSION ID(NUMNP,6),B(NEQB,LL),D(6,LL)                          SSAP 398
C                                                                       SSAP 399
      NT=ISCRF2
      REWIND NT                                                         SSAP 400
      REWIND ISCRF8
      READ (ISCRF8) ID
      M=NEQ                                                             SSAP 403
      NN=NEQB*NBLOCK                                                    SSAP 404
      WRITE (ISYSLP,2003)
      N=NUMNP                                                           SSAP 406
C                                                                       SSAP 407
      DO 500 KK=1,NUMNP                                                 SSAP 408
C                                                                       SSAP 409
      I=6                                                               SSAP 410
      DO 250 II=1,6                                                     SSAP 411
      DO 100 L=1,LL                                                     SSAP 412
  100 D(I,L)=0.                                                         SSAP 413
      IF(M.GT.NN) GO TO 150                                             SSAP 414
```

```
      IF (M.EQ.0) GO TO 150
      READ (NT) B
      NN=NN-NEQB
  150 IF(ID(N,I).LT.1) GO TO 250
      K=M-NN
      M=M-1
C
      DO 200 L=1,LL
  200 D(I,L)=B(K,L)
  250 I=I-1
C
      WRITE (ISYSLP,2004) N,(L,(D(I,L),I=1,6),L=1,LL)
C
  500 N=N-1
C
      RETURN
C
 2003 FORMAT (40H1.......NOOE DISPLACEMENTS ANO ROTATIONS//
     . 5H NOOE  5H LOAO  11X  1HX  11X  1HY  11X  1HZ  9X  2HXX
     .  9X  2HYY  9X  2HZZ)
 2004 FORMAT (1H0,I4,I5,1P3E12.3,3E11.2/(I10,3E12.3,3E11.2))
      END



      SUBROUTINE USOL (A,B,MAXB,NEQB,MB,LL,NBLOCK,NSB,NORG,NBKS,NT1,
     .                                                   NT2,NRST)
      IMPLICIT REAL*B (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRFB,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRFB,ISCRF9
      DIMENSION A(NSB),B(NSB),MAXB(NEQB)
C-----------------------------------------------------------------------
      NC=MB+LL
      NBR=(MB-1)/NEQB+1
      INC=NEQB-1
      NMB=NEQB*MB
      N2=ISCRF2
      N1=ISCRF7
      NORG=ISCRF4
      NBKS=ISCRF3
      NRST=ISCRF2
      REWIND NORG
      REWIND NBKS
C
C---- REDUCE EQUATIONS  BLOCK-BY-BLOCK ------------------------------
C
      DO 900 N=1,NBLOCK
      IF (N.GT.1.ANO.NBR.EQ.1) GO TO 110
      IF (NBR.EQ.1) GO TO 105
      REWIND N1
      REWINO N2
  105 NI=N1
      IF(N.EQ.1) NI=NORG
      REAO (NI) A
  110 00 300 I=1,NEQB
      0=A(I)
      IF(0) 115,300,120
  115 M=NEQB*(N-1)+I
      WRITE (ISYSLP,116) M,0
  116 FORMAT (33HOSET OF EQUATIONS MAY BE SINGULAR    /
     . 26H OIAGONAL TERM OF EQUATION   I8, BH  EQUALS   1PE12.4)
C
  120 II=I
      DO 125 J=2,NC
      II=II+NEQB
  125 A(II)=A(II)/0
C
      DO 130 J=I,NMB,NEQB
      IF (A(J).NE.0.) MAXB(I)=J
  130 CONTINUE
C
      JL=I+1
      IF (JL.GT.NEQB) GO TO 300
      II=I
      DO 200 J=JL,NEQB
      II=II+NEQB
      IF(II.GT.NMB) GO TO 200
      C=A(II)
      IF (C.EQ.0.0) GO TO 200
```

```
      C=C*A(I)
C
      KK=J-II
      MAX=MAXB(I)
      DO 150 JJ=II,MAX,NEQB
  150 A(JJ+KK)=A(JJ+KK)-C*A(JJ)
C
      KK=J +NMB
      JJ=I+NMB
      DO 175 L=1,LL
      A(KK)=A(KK)-C*A(JJ)
      KK=KK+NEQB
  175 JJ=JJ+NEQB
  200 CONTINUE
  300 CONTINUE
      WRITE (NBKS) A,MAXB
C
C---- SUBSTITUTE INTO REMAINING EQUATIONS ---------------------------
C
      DO 800 NN=1,NBR
      IF(N+NN.GT.NBLOCK) GO TO 800
      NI=N1
      IF(N.EQ.1) NI=NORG
      IF(NN.EQ.NBR) NI=NORG
      READ (NI) B
      IL=1+NN*NEQB*NEQB
      DO 700 I=1,NEQB
      II=IL
      DO 690 K=1,NEQB
      IF (II.GT.NMB) GO TO 690
      C=A(II)
      IF (C.EQ.0.0) GO TO 690
      C=C*A(K)
      MAX=MAXB(K)
C
      KK=I-II
      DO 640 JJ=II,MAX,NEQB
  640 B(JJ+KK)=B(JJ+KK)-C*A(JJ)
C
      KK=I+NMB
      JJ=K+NMB
      DO 650 L=1,LL
      B(KK)=B(KK)-C*A(JJ)
      KK=KK+NEQB
  650 JJ=JJ+NEQB
C
  690 II=II-INC
  700 IL=IL+NEQB
C
      IF(NBR.NE.1) GO TO 750
      DO 740 I=1,NSB
  740 A(I)=B(I)
      GO TO 800
  750 WRITE (N2) B
  800 CONTINUE
C
      M=N1
      N1=N2
  900 N2=M
C
C---- BACKSUBSTITUTION - RESULTS ON TAPE NRST ----------------------
C
      LS=LL*NEQB
      NEB=NEQB*(NBR+1)
      NUM=NBR*NEQB
      MAX=NEB*LL
      DO 905 I=1,MAX
  905 B(I)=0.
      REWIND NRST
C--------------------------------------------------------------------
      DO 1000 N=1,NBLOCK
      BACKSPACE NBKS
      READ (NBKS) A,MAXB
      BACKSPACE NBKS
      DO 910 L=1,LL
      K=L*NEB
      DO 910 J=1,NUM
      I=K-NEQB
      B(K)=B(I)
  910 K=K-1
C
```

```
      I=NMB
      DO 920 L=1,LL
      K=(L-1)*NEB
      DO 920 J=1,NEQB
      I=I+1
      K=K+1
  920 B(K)=A(I)
C
      DO 955 I=1,NEQB
      J=NEQB+1-I
      MAX=MAXB(J)
      IF (A(J).EQ.0.) GO TO 955
      DO 950 L=1,LL
      KK=J+(L-1)*NEB
      JJ=KK+1
      IL=J+NEQB
      C=B(KK)
      DO 940 II=IL,MAX,NEQB
      C=C-A(II)*B(JJ)
  940 JJ=JJ+1
  950 B(KK)=C
  955 CONTINUE
C
      I=0
      DO 960 L=1,LL
      K=(L-1)*NEB
      DO 960 J=1,NEQB
      K=K+1
      I=I+1
  960 A(I)=B(K)
C
      WRITE (NRST) (A(I),I=1,LS)
 1000 CONTINUE
C-----------------------------------------------------------------------
      RETURN
      END


      SUBROUTINE STRESS(STR,B,D,NEQB,LB,LL,NEQ,NBLOCK)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
      DIMENSION D(NEQ,LB),B(NEQB,LL),STR(4,LL)
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,MEQ
      COMMON /JUNK/ LT,LH
C
      READ (ISCRF8) STR
      NT=(LL-1)/LB +1
      LH=0
      REWIND ISCRF3
C
      DO 1000 II=1,NT
C
      LT =LH+1
      LLT=1-LT
      LH=LT+LB-1
      IF(LH.GT.LL) LH=LL
C
C     MOVE DISPLACEMENTS INTO CORE FOR LB LOAD CONDITIONS
C
      REWIND ISCRF2
      IF (NDYN.EQ.3) READ (ISCRF2)
      NQ=NEQB*NBLOCK
      DO 200 NN=1,NBLOCK
      READ (ISCRF2) B
      N=NEQB
      IF (NN.EQ.1) N=NEQ-NQ+NEQB
      NQ=NQ-NEQB
      DO 200 J=1,N
      I=NQ+J
      DO 200 L=LT,LH
      K=L+LLT
  200 D(I,K)=B(J,L)
      LK=LH-LT+1
C
C     CALCULATE STRESSES FOR ALL ELEMENTS FOR LB LOAD CONDITIONS
C
      REWIND ISCRF1
```

```
      DO 1000 M=1,NELTYP                                                SSAP 640
      READ (ISCRF1) NPAR
      MTYPE=NPAR(1)                                                     SSAP 642
      NPAR(1)=0                                                         SSAP 643
      CALL ELTYPE(MTYPE)                                                SSAP 644
 1000 CONTINUE                                                          SSAP 645
C                                                                       SSAP 646
      RETURN                                                            SSAP 647
      END                                                               SSAP 648


      SUBROUTINE CALBAN(MBAND,NDIF,LM,XM,S,P,ND,NDM)                    SSAP 649
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C-----CALCULATES BAND WIDTH AND WRITES STIFFNESS MATRIX ON TAPE 2       SSAP 650
      DIMENSION LM(1),XM(1),S(NDM,NDM),P(NDM,4)                         SSAP 651
      MIN=100000                                                        SSAP 652
      MAX=0                                                             SSAP 653
      DO 800 L=1,ND                                                     SSAP 654
      IF (LM(L).EQ.0) GO TO 800                                         SSAP 655
      IF (LM(L).GT.MAX) MAX=LM(L)                                       SSAP 656
      IF (LM(L).LT.MIN) MIN=LM(L)                                       SSAP 657
  800 CONTINUE                                                          SSAP 658
      NDIF=MAX-MIN+1                                                    SSAP 659
      IF (NDIF.GT.MBAND) MBAND=NDIF                                     SSAP 660
      LRD=1+ND*(ND+6)                                                   SSAP 661
      WRITE (ISCRF2) LRD,ND,(LM(I),I=1,ND),LRD,ND,(LM(I),I=1,ND),((S(I,J
     1),I=1,ND),J=1,ND),((P(I,J),I=1,ND),J=1,4),(XM(I),I=1,ND)
      RETURN                                                            SSAP 664
      END                                                               SSAP 665



      SUBROUTINE STRSC(STR,D,NEQ,NTAG)                                  SSAP 666
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
      DIMENSION STR(4,1),D(NEQ,1)                                       SSAP 667
      COMMON /JUNK/ LT,LH,L,LZ,SG(20),SIG(7),EXTRA(150)
      COMMON /EM/ NS,ND,LM(48),B(48,48),TI(48,4)                        SSAP 669
C                                                                       SSAP 670
      IF (NTAG.EQ.0) GO TO 800                                          SSAP 671
      LL=L-LT+1                                                         SSAP 672
      DO 300 I=1,NS                                                     SSAP 673
      SG(I)=0.0                                                         SSAP 674
      DO 300 J=1,4                                                      SSAP 675
  300 SG(I)=SG(I)+TI(I,J)*STR(J,L)                                      SSAP 676
      DO 500 J=1,ND                                                     SSAP 677
      JJ=LM(J)                                                          SSAP 678
      IF(JJ.EQ.0) GO TO 500                                             SSAP 679
      DO 400 I=1,NS                                                     SSAP 680
  400 SG(I)=SG(I)+B(I,J)*D(JJ,LL)                                       SSAP 681
C                                                                       SSAP 682
  500 CONTINUE                                                          SSAP 683
      GO TO 900                                                         SSAP 684
  800 READ (ISCRF1) ND,NS,(LM(I),I=1,ND),((B(I,J),I=1,NS),J=1,ND),
     1((TI(I,J),I=1,NS),J=1,4)
  900 RETURN                                                            SSAP 687
      END                                                               SSAP 688


      SUBROUTINE TRUSS                                                  TRUS   1
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C                                                                       TRUS   2
      COMMON A(1)                                                       TRUS   3
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQTRUS   4
      COMMON / JUNK / LT,LH,L,LZ,SIG(20)
C                                                                       TRUS   6
      IF(NPAR(1).EQ.0) GO TO 500                                        TRUS   7
      N6=N5+NUMNP                                                       TRUS   8
      N7 =N6+NPAR(3)                                                    TRUS   9
```

```
      N8 =N7+NPAR(3)
      N9 =N8+NPAR(3)
      N10=N9+NPAR(3)
      MM=N10+NPAR(3)-MTOT
      IF(MM.GT.0) CALL ERROR(MM)
      CALL RUSS(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
     .          A(N10),NUMNP)
      RETURN
C
  500 WRITE (ISYSLP,2002)
      NUME=NPAR(2)
      DO 800 MM=1,NUME
      CALL STRSC (A(N1),A(N3),NEQ,0)
      WRITE (ISYSLP,2001)
      DO 800 L=LT,LH
      CALL STRSC (A(N1),A(N3),NEQ,1)
      WRITE (ISYSLP,3002) MM,L,SIG(1),SIG(2)
  800 CONTINUE
      RETURN
C
 2001 FORMAT (/)
 2002 FORMAT (//23H   TRUSS MEMBER ACTIONS //
     .          46H0 MEMBER    LOAD          STRESS            FORCE  )
 3002 FORMAT (2I8,F15.5,F15.3)
      END


      SUBROUTINE RUSS (ID,X,Y,Z,T,E,THERM,DEN,AREA,WT,NUMNP)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
      DIMENSION X(1),Y(1),Z(1),ID(NUMNP,1),E(1),THERM(1),DEN(1),AREA(1)
     . ,T(1),WT(1)
      COMMON /ELPAR/ NPAR(14),NNNNN,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
      COMMON /EM/LM(24),ND,NS,S(24,24),P(24,4),XM(24),ST(12,24),TT(12,4)
      COMMON /JUNK/ EMUL(4,4),I,J,K,L,M,N,II,JJ,KK,MTYPE,TEMP,DX,DY,DZ,
     1 XL2,XL,XX,YY,F,FT,FX,FY,FZ,MIN,MAX,NDIF,KKK,TEM,MTYP
C
C     CONTROL INFORMATION AND MEMBER PROPERTIES
C
      NUME=NPAR(2)
      NUMMAT=NPAR(3)
      WRITE (ISYSLP,2000) NUME,NUMMAT
      WRITE (ISYSLP, 2001)
      DO 10 I=1,NUMMAT
      READ (ISYSCR,1001) N,E(N),THERM(N),DEN(N),AREA(N),WT(N)
   10 WRITE (ISYSLP,2002) N,E(N),THERM(N),DEN(N),AREA(N),WT(N)
C
C     ELEMENT LOAD MULTIPLIERS
C
      READ (ISYSCR,1003) EMUL
      WRITE (ISYSLP,2003) EMUL
C
C     ELEMENT INFORMATION
      WRITE (ISYSLP, 2005)
C
      N=1
  100 READ (ISYSCR,1004) M,II,JJ,MTYP,TEM,KK
      IF(KK.EQ.0) KK=1
  120 IF(M.NE.N) GO TO 200
      I=II
      J=JJ
      MTYPE=MTYP
      REFT=TEM
      KKK=KK
C
C     1. FORM ELEMENT STIFFNESS AND STRESS MATRICES
C
  200 DX=X(I)-X(J)
      DY=Y(I)-Y(J)
      DZ=Z(I)-Z(J)
      XL2=DX*DX+DY*DY+DZ*DZ
      XL=DSQRT(XL2)
      XX=E(MTYPE)*AREA(MTYPE)*XL
      ST(1,1)=DX/XL2
      ST(1,2)=DY/XL2
      ST(1,3)=DZ/XL2
```

```
      ST(1,4)=-ST(1,1)
      ST(1,5)=-ST(1,2)
      ST(1,6)=-ST(1,3)
C
      DO 300 L=1,6
      YY=ST(1,L)*XX
      DO 250 K=L,6
      S(K,L)=ST(1,K)*YY
  250 S(L,K)=S(K,L)
      ST(1,L)=E(MTYPE)*ST(1,L)
  300 ST(2,L)=AREA(MTYPE)*ST(1,L)
C
C     2. INERTIA AND THERMAL LOADS
C
      F=WT(MTYPE)*XL/2.
      TEMP=(T(I)+T(J))*0.5 - REFT
      FT=-TEMP*THERM(MTYPE)*E(MTYPE)*AREA(MTYPE)
      FX=DX*FT/XL
      FY=DY*FT/XL
      FZ=DZ*FT/XL
C
      DO 350 L=1,4
      TT(2,L)=EMUL(L,4)*FT
      TT(1,L)=TT(2,L)/AREA(MTYPE)
      P(1,L)=EMUL(L,1)*F-EMUL(L,4)*FX
      P(2,L)=EMUL(L,2)*F-EMUL(L,4)*FY
      P(3,L)=EMUL(L,3)*F-EMUL(L,4)*FZ
      P(4,L)=EMUL(L,1)*F+EMUL(L,4)*FX
      P(5,L)=EMUL(L,2)*F+EMUL(L,4)*FY
  350 P(6,L)=EMUL(L,3)*F+EMUL(L,4)*FZ
      F=DEN(MTYPE)*AREA(MTYPE)*XL/2.
      DO 375 L=1,6
  375 XM(L)=F
C
C     3. FORM LOCATION MATRIX AND COMPUTE BAND WIDTH
C
      DO 400 L=1,3
      LM(L)=ID(I,L)
  400 LM(L+3)=ID(J,L)
C
      ND=6
      NS=2
      NDM=24
      CALL CALBAN (MBAND,NDIF,LM,XM,S,P,ND,NDM)
      WRITE (ISCRF1) ND,NS,(LM(L),L=1,ND),((ST(L,K),L=1,NS),K=1,ND),
     1((TT(L,K),L=1,NS),K=1,4)
C
C     4. CHECK FOR MORE ELEMENTS
C
      WRITE (ISYSLP,2004) N,I,J,MTYPE,REFT,NDIF
      IF (N.EQ.NUME) RETURN
      N=N+1
      I=I+KKK
      J=J+KKK
      IF(N.GT.M) GO TO 100
      GO TO 120
C
 1001 FORMAT (I5,5F10.0)
 1003 FORMAT (4F10.0)
 1004 FORMAT (4I5,1F10.0,I5)
 2000 FORMAT (///25H1NUMBER OF TRUSS MEMBERS= I5/
     1 25H NUMBER OF DIFF. MEMBERS= I5)
 2001 FORMAT (///1X,4HTYPE,14X,1HE,10X,5HALPHA,12X,3HDEN,11X,4HAREA
     1 11X,4HWT/L )
 2002 FORMAT (I5,5E15.7)
 2003 FORMAT(///25H ELEMENT LOAD MULTIPLIERS / 20X,1HA,14X,1HB,14X,1HC,
     1 14X,1HD,/6H X-DIR4E15.6/ 6H Y-DIR4E15.6/ 6H Z-DIR4E15.6/
     2 6H  TEMP4E15.6)
 2004 FORMAT (4I6,F10.2,I7)
 2005 FORMAT (///42H1     N       I       J  TYPE       TEMP    BAND   )
      END


      SUBROUTINE BEAM
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTDT,NEQ
```

```
      COMMUN / JUNK / LT,LH,L,LZ,SIG(20)
      CUMMUN A(1)
C
      IF(NPAR(1).EQ.0) GU TO 500
      N6=N5+NPAR(5) + NUMNP
      N7=N6+NPAR(5)
      N8=N7+NPAR(5)
      N9=N8+12*NPAR(4)
      N10=N9+6*NPAR(3)
      IF(N10.GT.MTOT) CALL ERRUR(N10-MTOT)
      CALL TEAM(NPAR(2),NPAR(3),NPAR(4),NPAR(5),A(N1),A(N2),A(N3),
     1  A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),NUMNP,MBAND)
      RETURN
C
  500 WRITE (ISYSLP,2002)
      NUME=NPAR(2)
      DO 800 MM=1,NUME
      CALL STRSC (A(N1),A(N3),NEQ,0)
      WRITE (ISYSLP,2001)
      DO 800 L=LT,LH
      CALL STRSC (A(N1),A(N3),NEQ,1)
      WRITE (ISYSLP,3002) MM,L,(SIG(I),I=1,12)
  800 CONTINUE
      RETURN
 2001 FORMAT (/)
 2002 FORMAT(/29H0.....BEAM FORCES AND MOMENTS//
     . 10HOBEAM LOAD 5X 5HAXIAL  2(7X,5HSHEAR),5X 7HTORSION
     .  2(5X,7HBENDING)/ 10H  NO.  NO.  8X  2HR1  10X  2HR2  10X
     .  2HR3 10X 2HM1 10X 2HM2 10X 2HM3)
 3002 FORMAT (I5,I4,1PE11.3,5E12.3/8X,6E12.3/)
      END


      SUBROUTINE TEAM(NBEAM,NUMETP,NUMFIX,NUMMAT,ID,X,Y,Z,E,G,RO,
     .SFT, COPROP,NUMNP,MBAND)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C     FORMS 3-D BEAM STIFFNESS AND STRESS ARRAYS
C
      COMMON/EM/LM(24),ND,NS,ASA(24,24),RF(24,4),XM(24),SA(12,24),
     .    SF(12,4)
      DIMENSIUN X(1),Y(1),Z(1),ID(NUMNP,1),E(1),G(1),SFT(NUMFIX,1)
     .  ,COPROP(NUMETP,1),RU(1),EMUL(3,4)
      COMMON /NEWB/ LC(4),T(3,3),JK(6),MELTYP,IDUM,DL,MATTYP
      DIMENSION ILC(4),TI(3,3),TJ(3,3),STIF(722),TS(2,2),LS(4)
      EQUIVALENCE (STIF,LM)
C
C     INITIALIZATION
C
      WRITE (ISYSLP,2005) NBEAM,NUMETP,NUMFIX,NUMMAT
      N=0
      DO 5 I=1,1058
    5 STIF(I)=0.
C
C     READ AND PRINT MATERIAL PRUPERTY DATA
C
      WRITE (ISYSLP,2001)
      DO 10 I=1,NUMMAT
      READ (ISYSCR,1001) N,E(N),G(N),RO(N)
      WRITE (ISYSLP,2002) N,E(N),G(N),RO(N)
   10 G(N)=0.5*E(N)/(1.+G(N))
C
C     READ AND PRINT GEOMETRIC PROPERTIES OF COMMON ELEMENTS.
C
      WRITE (ISYSLP,2003)
      DO 30 I=1,NUMETP
      READ (ISYSCR,1002) N,(COPROP(N,J),J=1,6)
      IF((CUPROP(N,1).NE.0.0).AND.(COPROP(N,4).NE.0.0).AND.
     1   (COPROP(N,5).NE.0.0).AND.(CUPRUP(N,6).NE.0.0))  GO TO 20
      WRITE (ISYSLP,2013)
      CALL EXIT
   20 WRITE (ISYSLP,2004) N,(CUPRUP(N,J),J=1,6)
   30 CUNTINUE
C
C     ELEMENT LOAD MULTIPLIERS
C
      READ (ISYSCR,1006) ((EMUL(I,J),J=1,4),I=1,3)
```

```
      WRITE (ISYSLP,2006) ((EMUL(I,J),J=1,4),I=1,3)
C
C     READ AND PRINT FIXED END FORCES IN LOCAL COORDINATES
C
      IF(NUMFIX .EQ. 0) GO TO 56
      WRITE (ISYSLP,2010)
      DO 55 I=1,NUMFIX
      READ (ISYSCR,1005) N,(SFT(N,J),J=1,12)
   55 WRITE (ISYSLP,2011) N,(SFT(N,J),J=1,12)
   56 CONTINUE
C
C     READ AND PRINT ELEMENT DATA. GENERATE MISSING INPUT.
C
      WRITE (ISYSLP,4000)
      L=0
   60 KKK=0
      READ (ISYSCR,3000)INEL,INI,INJ,INK,IMAT,IMEL,ILC,INELKI,INELKJ,INC
      IF (INEL.NE.1) GO TO 15
      NI=INI
      NJ=INJ
      NK=INK
   15 IF (INC.EQ.0) INC=1
   65 L=L+1
      KKK=KKK+1
      ML=INEL-L
      IF (ML)  66,67,68
   66 WRITE (ISYSLP,4003) INEL
      CALL EXIT
   67 NEL=INEL
       NI     =INI
       NJ     =INJ
      NK=INK
       MATTYP=IMAT
       MELTYP=IMEL
      DO 90 I=1,4
   90 LC(I)=ILC(I)
       NLOAD=LC(1)+LC(2)+LC(3)+LC(4)
       NEKODI=INELKI
       NEKODJ=INELKJ
      DO 91 I=1,3
   91 T(2,I)=TI(2,I)
      GO TO 69
   68 NEL=INEL-ML
       NI     =IN+KKK*INCR
       NJ     =JN+KKK*INCR
   69 CONTINUE
      WRITE (ISYSLP,4001) NEL,NI,NJ,NK,MATTYP,MELTYP,LC,NEKODI,NEKODJ
C
   74 DX=X(NJ)-X(NI)
      DY=Y(NJ)-Y(NI)
      DZ=Z(NJ)-Z(NI)
      DL=DSQRT(DX*DX+DY*DY+DZ*DZ)
      IF(DL) 75,75,76
   75 WRITE (ISYSLP,4005) NEL
      CALL EXIT
C
C     FORM GLOBAL TO LOCAL COORDINATE TRANSFORMATION.
C
   76 T(1,1)=DX/DL
      T(1,2)=DY/DL
      T(1,3)=DZ/DL
C
C     COMPUTE DIRECTION COSINES OF LOCAL Y-AXIS
C
      A1=X(NJ)-X(NI)
      A2=Y(NJ)-Y(NI)
      A3=Z(NJ)-Z(NI)
      B1=X(NK)-X(NI)
      B2=Y(NK)-Y(NI)
      B3=Z(NK)-Z(NI)
      AA=A1*A1+A2*A2+A3*A3
      AB=A1*B1+A2*B2+A3*B3
      U1=AA*B1-AB*A1
      U2=AA*B2-AB*A2
      U3=AA*B3-AB*A3
      UU=U1*U1+U2*U2+U3*U3
      UU=DSQRT(UU)
      IF (UU.GT.0.) GO TO 40
      WRITE (ISYSLP,4002) INEL
      STOP
   40 T(2,1)=U1/UU
```

```
      T(2,2)=U2/UU
      T(2,3)=U3/UU
      T(3,1)=T(1,2)*T(2,3)-T(1,3)*T(2,2)
      T(3,2)=T(1,3)*T(2,1)-T(1,1)*T(2,3)
      T(3,3)=T(1,1)*T(2,2)-T(1,2)*T(2,1)
C
C     CHECK IF NEW STIFFNESS NEEDED
C
      IF (NEL.GE.1) GO TO 80
      IF (DABS(DS-DL).GT.DL/100.) GO TO 80
      IF ((MT.NE.MATTYP).OR.(ME.NE.MELTYP)) GO TO 80
      IF ((JK(1).NE.NEKODI).OR.(JK(2).NE.NEKODJ)) GO TO 80
      DO 81 I=1,4
      IF (LS(I).NE.LC(I)) GO TO 80
   81 CONTINUE
      DO 82 I=1,2
      DO 82 J=1,2
      IF (DABS(TS(I,J)-T(I,J)).GT.DABS(T(I,J)/100.)) GO TO 80
   82 CONTINUE
      GO TO 185
C
   80 DS=DL
      MT=MATTYP
      ME=MELTYP
      DO 77 I=1,2
      DO 77 J=1,2
   77 TS(I,J)=T(I,J)
      DO 78 I=1,4
   78 LS(I)=LC(I)
      JK(1)=NEKODI
      JK(2)=NEKODJ
C
C     FORM NEW STIFFNESS
C
      CALL NEWBM (E,G,RO,COPROP,SFT,NUMFIX,NUMETP)
C
C     ADD GRAVITY LOADING ... POINT LOADS ONLY COMPUTED
C
      DO 180 I=1,3
      DO 180 J=1,4
      RF(I,J)=RF(I,J)+EMUL(I,J)*XM(I)
  180 RF(I+6,J)=RF(I+6,J)+EMUL(I,J)*XM(I+6)
C
C     FORM ELEMENT LOCATION MATRIX
C
  185 CONTINUE
      DO 170 M=1,6
      LM(M)=ID(NI,M)
      LM(M+12)=0
      LM(M+18)=0
  170 LM(M+6)=ID(NJ,M)
C
      NS=12
      ND=12
C
C     TRANSFORM TO MASTER DEGREES OF FREEDOM
C
      CALL SLAVE (X,Y,Z,ID,NUMNP,NI,NJ)
C
C     WRITE ELEMENT INFORMATION ON TAPE
C
      NDM=24
      CALL CALBAN (MBAND,NDIF,LM,XM,ASA,RF,ND,NDM)
      WRITE (ISCRF1) ND,NS,(LM(I),I=1,ND),((SA(I,J),I=1,NS),J=1,ND),
     1((SF(I,J),I=1,NS),J=1,4)
C
C         CHECK FOR LAST ELEMENT
C
        IF (NBEAM-NEL)  66,500,260
  260 CONTINUE
      IF (ML.GT.0)  GO TO 65
       IN      =INI
       JN      =INJ
      INCR=INC
      GO TO 60
  500 RETURN
C
 1001 FORMAT(I5,3F10.0)
 1002 FORMAT(I5,6F10.0)
 1005 FORMAT(I5,6F10.0/F15.0,5F10.0)
 1006 FORMAT (4F10.0)
```

```
2001 FORMAT(55H1MATERIAL    YOUNG S     POISSON S           MASS        BEAM 240
   1       /55H             MODULUS        RATIO            DENSITY    )BEAM 241
2002 FORMAT(1H ,I5,3X,F12.0,F14.5,F14.5)                                 BEAM 242
2003 FORMAT(1H1/                                                         BEAM 243
    .  26H BEAM GEOMETRIC PROPERTIES//                                   BEAM 244
    1   48H ELEMENT       AREA         AREA         AREA                 BEAM 245
    2  60H INERTIA      INERTIA      INERTIA                             BEAM 246
    3 / 48H  TYPE           X            Y            Z                  BEAM 247
    4                30H    X            Y            Z                 )BEAM 248
2004 FORMAT(1H ,I5,2X,6F12.3)                                            BEAM 249
2005 FORMAT(39H1.......THREE DIMENSIONAL BEAM ELEMENTS//                 BEAM 250
    .  36H  NUMBER OF BEAMS                   =,I5/                      BEAM 251
    .  36H  NUMBER OF GEOMETRIC PROPERTY SETS=,I5/                       BEAM 252
    .  36H  NUMBER OF FIXEO END FORCE SETS    =,I5/                      BEAM 253
    .  36H  NUMBER OF MATERIALS               =,I5)                      BEAM 254
2006 FORMAT(///25H ELEMENT LOAO MULTIPLIERS / 20X,1HA,14X,1HB,14X,1HC, BEAM 255
    1 14X,1HO,/6H X-OIR4E15.6/ 6H Y-DIR4E15.6/ 6H Z-OIR4E15.6/ )         BEAM 256
2010 FORMAT(1H1,1H ,                                                     BEAM 257
    1  30X40H FIXED ENO FORCES IN LOCAL COOROINATES                      BEAM 258
    2//53H TYPE   NODE        FORCE X      FORCE Y      FORCE Z          BEAM 259
    3                    35H MOMENT X     MOMENT Y     MOMENT Z         )BEAM 260
2011 FORMAT(1H ,I3,6X,1HI,3X,6F12.3/1H ,9X,1HJ,3X,6F12.3/)               BEAM 261
2013 FORMAT(1HO/                                                         BEAM 262
    1  60H SECTION PROPERTIES OTHER THAN SHEAR AREAS MAY NOT BE SPECIF BEAM 263
    2  34HIED AS ZERO. EXECUTION TERMINATEO.)                            BEAM 264
3000 FORMAT (10I5,2I6,I8)                                                BEAM 265
4000 FORMAT(1H1/                                                         BEAM 266
    . 5HOBEAM 5X 5HNODES 5X 5H MATL 5H GEOM 5X 10HELEM LOADS 4X   10X  BEAM 267
    . 12H  END COOES / 5H  NO      4X 1HI 4X 1HJ 4X 1HK 5H  NO  5H  NO  BEAM 268
    .   4X 1HA 4X 1HB 4X 1HC 4X 1HO 9X 1HI 9X 1HJ)                       BEAM 269
4001 FORMAT (10I5,2I10)                                                  BEAM 270
4002 FORMAT (9HOBEAM NO ,I5, 26H     K NUOE ON BEAM X-AXIS               BEAM 271
    .  26H......EXECUTION TERMINATED )                                   BEAM 272
4003 FORMAT( 36HOELEMENT CARD ERROR, ELEMENT NUMBER= I6)                 BEAM 273
4004 FORMAT(1H ,31HNOOAL POINT NUMBERS FOR ELEMENT,I5,36HARE IOENTICAL.BEAM 274
   1 EXECUTION TERMINATEO.)                                              BEAM 275
4005 FORMAT(8HOELEMENT,I5,39H HAS ZERO LENGTH. EXECUTION TERMINATED.)  BEAM 276
     END                                                                 BEAM 277


     SUBROUTINE NEWBM(E,G,RO,COPROP,SFT,NUMFIX,NUMETP)                   BEAM 278
     IMPLICIT REAL*8 (A-H,O-Z)
     INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
    1ISCRF9
     COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
    1ISCRF8,ISCRF9
C                                                                        BEAM 279
C     FORM NEW BEAM STIFFNESS                                            BEAM 280
C                                                                        BEAM 281
     DIMENSION E(1),G(1),RO(1),COPROP(NUMETP,1),SFT(NUMFIX,1)            BEAM 282
     COMMON/EM/LM(24),ND,NS,ASA(24,24),RF(24,4),XM(24),SA(12,24),        BEAM 283
    .     SF(12,4)                                                       BEAM 284
     COMMON /NEWB/ LC(4),T(3,3),JK(6),MELTYP,IOUM,DL,MATTYP
     DIMENSION R(12),S(12,12),C(12)                                      BEAM 286
C                                                                        BEAM 287
     DO 5 I=1,12
     DO 5 J=1,12
   5 S(I,J)=0.
     AX=COPROP(MELTYP,1)                                                 BEAM 290
      AY=COPROP(MELTYP,2)                                                BEAM 291
      AZ=COPROP(MELTYP,3)                                                BEAM 292
     \AX=COPROP(MELTYP,4)                                                BEAM 293
     AAY=COPROP(MELTYP,5)                                                BEAM 294
     AAZ=COPROP(MELTYP,6)                                                BEAM 295
     SHFY=0.0                                                            BEAM 296
     SHFZ=0.0                                                            BEAM 297
     ZY=E(MATTYP)/(OL*DL)                                                BEAM 298
     EIY=ZY*AAY                                                          BEAM 299
     EIZ=ZY*AAZ                                                          BEAM 300
     IF(AY.NE.0.0) SHFY=6.*EIZ/(G(MATTYP)*AY)                            BEAM 301
     IF(AZ.NE.0.0) SHFZ=6.*EIY/(G(MATTYP)*AZ)                            BEAM 302
     COMMY=EIY/(1.+2.*SHFZ)                                              BEAM 303
     COMMZ=EIZ/(1.+2.*SHFY)                                              BEAM 304
C                                                                        BEAM 305
C        FIXED END FORCES IN LOCAL COOROS                                BEAM 306
C                                                                        BEAM 307
     DO 73 N=1,4                                                         BEAM 308
     M=LC(N)                                                             BEAM 309
     IF (M.GT.0)  GO TO 71                                               BEAM 310
     DO 70 I=1,12                                                        BEAM 311
  70 SF(I,N)=0.                                                          BEAM 312
```

```
      GO TO 73                                                          BEAM 313
   71 DO 72 I=1,12                                                      BEAM 314
   72 SF(I,N)=SFT(M,I)                                                  BEAM 315
   73 CONTINUE                                                          BEAM 316
C                                                                       BEAM 317
C     FORM ELEMENT STIFFNESS IN LOCAL COORDINATES                       BEAM 318
C                                                                       BEAM 319
      S(1,1)= F(MATTYP)* AX/DL                                          BEAM 320
      S(4,4)= G(MATTYP)*AAX/DL                                          BEAM 321
      S(2,2)= COMMZ*12./DL                                              BEAM 322
      S(3,3)= COMMY*12./DL                                              BEAM 323
      S(5,5)= COMMY* 4.*DL*(1.+0.5*SHFZ)                                BEAM 324
      S(6,6)= COMMZ* 4.*DL*(1.+0.5*SHFY)                                BEAM 325
      S(2,6)= COMMZ* 6.                                                 BEAM 326
      S(3,5)=-COMMY* 6.                                                 BEAM 327
      DO 102 I=1,6                                                      BEAM 328
      J=I+6                                                             BEAM 329
  102 S(J,J)=S(I,I)                                                     BEAM 330
      DO 104 I=1,4                                                      BEAM 331
      J=I+6                                                             BEAM 332
  104 S(I,J)=-S(I,I)                                                    BEAM 333
      S(6,12)= S(6,6)*(1.-SHFY)/(2.+SHFY)                               BEAM 334
      S(5,11)= S(5,5)*(1.-SHFZ)/(2.+SHFZ)                               BEAM 335
      S(2,12)= S(2,6)                                                   BEAM 336
      S(6, 8)=-S(2,6)                                                   BEAM 337
      S(8,12)=-S(2,6)                                                   BEAM 338
      S(3,11)= S(3,5)                                                   BEAM 339
      S(5, 9)=-S(3,5)                                                   BEAM 340
      S(9,11)=-S(3,5)                                                   BEAM 341
      DO 106 I=2,12                                                     BEAM 342
      K=I-1                                                             BEAM 343
      DO 106 J=1,K                                                      BEAM 344
  106 S(I,J)=S(J,I)                                                     BEAM 345
C                                                                       BEAM 346
C     MODIFY ELEMENT STIFFNESS AND ELEMENT FIXED END FORCES FOR KNOWN   BEAM 347
C     ZERO MEMBER END FORCES.                                           BEAM 348
C                                                                       BEAM 349
      IF ((JK(1)+JK(2)).EQ.0) GO TO 145                                 BEAM 350
      DO 140 K=1,2                                                      BEAM 351
      KK=JK(K)                                                          BEAM 352
      KD=100000                                                         BEAM 353
      I1=6*(K-1)+1                                                      BEAM 354
      I2=I1+5                                                           BEAM 355
      DO 140 I=I1,I2                                                    BEAM 356
      IF (KK.LT.KD)  GO TO 140                                          BEAM 357
      SII=S(I,I)                                                        BEAM 358
      DO 125 N=1,12                                                     BEAM 359
  125 R(N)=S(I,N)                                                       BEAM 360
      DO 130 M=1,12                                                     BEAM 361
      C(M)=S(M,I)/SII                                                   BEAM 362
      DO 130 N=1,12                                                     BEAM 363
  130 S(M,N)=S(M,N)-C(M)*R(N)                                           BEAM 364
      DO 135 N=1,4                                                      BEAM 365
      SFI=SF(I,N)                                                       BEAM 366
      DO 135 M=1,12                                                     BEAM 367
  135 SF(M,N)=SF(M,N)-C(M)*SFI                                          BEAM 368
  136 KK=KK-KD                                                          BEAM 369
  140 KD=KD/10                                                          BEAM 370
  145 CONTINUE                                                          BEAM 371
C                                                                       BEAM 372
C     OBTAIN SA(12,12) RELATING ELEMENT END FORCES (LOCAL) AND          BEAM 373
C     JOINT DISPLACEMENTS (GLOBAL).                                     BEAM 374
C                                                                       BEAM 375
      DO 31 I=1,12
      DO 31 J=1,24
   31 SA(I,J)=0.
      DO 150 LA=1,10,3                                                  BEAM 378
      LB=LA+2                                                           BEAM 379
      DO 150 MA=1,10,3                                                  BEAM 380
      MB=MA-1                                                           BEAM 381
      DO 150 I=LA,LB                                                    BEAM 382
      DO 150 JM=1,3                                                     BEAM 383
      J=JM+MB                                                           BEAM 384
      XX=0.                                                             BEAM 385
      DO 151 K=1,3                                                      BEAM 386
  151 XX=XX+S(I,K+MB)*T(K,JM)                                           BEAM 387
  150 SA(I,J)=XX                                                        BEAM 388
C                                                                       BEAM 389
C     ELEM STIFF ASA(12,12) AND FIXED END FORCES RF(12) IN GLOBAL COORDSBEAM 390
```

```
C
      DO 32 I=1,24
      DO 32 J=1,24
   32 ASA(I,J)=0.
      DO 160 LA=1,10,3
      LB=LA-1
      DO 160 MA=1,10,3
      MB=MA+2
      DO 160 IL=1,3
      I=IL+LB
      DO 160 J=MA,MB
      XX=0.
      DO 161 K=1,3
  161 XX=XX+T(K,IL)*SA(K+LB,J)
  160 ASA(I,J)=XX
C
      DO 165 LA=1,10,3
      LB=LA-1
      DO 165 IL=1,3
      I=IL+LB
      DO 165 N=1,4
      XX=0.
      DO 162 K=1,3
  162 XX=XX-T(K,IL)*SF(K+LB,N)
  165 RF(I,N)=XX
C
C     FORM MASS MATRIX
C
      XXM=RO(MATTYP)*AX*DL/2.
      DO 180 M=1,3
      XM(M)=XXM
      XM(M+3)=0.
      XM(M+9)=0.
  180 XM(M+6)=XXM
      RETURN
      END



      SUBROUTINE SLAVE (X,Y,Z,IO,NUMNP,NI,NJ)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C     PERFORMS SLAVE...MASTER DISPLACEMENT TRANSFORMATION
C       ( FOR NODES CONNECTED TO BEAM ELEMENTS ONLY)
C
      DIMENSION X(1),Y(1),Z(1),IO(NUMNP,1)
      COMMON /EM/ LM(24),NO,NS,S(24,24),R(96),XM(24),SA(12,24) ,TT(12,4)
C
C     DETERMINE REQUIRED TRANSLATION OEGREES OF FREEDOM
C
      DO 54 NF=1,12,6
      NOD=NI
      IF (NF.EQ.7) NOD=NJ
      DO 30 K=1,3
      I=K+NF-1
      IF (LM(I).GE.0) GO TO 30
      M=-LM(I)
      LM(I)=ID(M,K)
      IF(K-2) 35,45,55
   35 D1=-(Y(NOO)-Y(M))
      D2=   Z(NOO)-Z(M)
      LM(NO+1)=IO(M,6)
      LM(NO+2)=IO(M,5)
      GO TO 50
   45 D1=-(Z(NOO)-Z(M))
      D2=   X(NOD)-X(M)
      LM(NO+1)=IO(M,4)
      LM(ND+2)=IO(M,6)
      GO TO 50
   55 01=-( X(NOO)-X(M))
      D2=   Y(NOO)-Y(M)
      LM(ND+1)=IO(M,5)
      LM(NO+2)=ID(M,4)
   50 CONTINUE
C
```

```
C      TRANSFORMATION...ARRAYS INCREASE IN SIZE                          BEAM 461
C                                                                        BEAM 462.
       DO 60 II=1,ND                                                     BEAM 463
       S(ND+1,II)=S(I,II)*D1                                             BEAM 464
       S(ND+2,II)=S(I,II)*D2                                             BEAM 465
       XM(ND+1)=XM(I)*D1*D1                                              BEAM 466
       XM(ND+2)=XM(I)*D2*D2                                              BEAM 467
       S(II,ND+1)=S(II,I)*D1                                             BEAM 468
       S(II,ND+2)=S(II,I)*D2                                             BEAM 469
   60 CONTINUE                                                           BEAM 470
C                                                                        BEAM 471
       DO 70 II=1,NS                                                     BEAM 472
       SA(II,ND+1)=SA(II,I)*D1                                           BEAM 473
   70 SA(II,ND+2)=SA(II,I)*D2                                            BEAM 474
C                                                                        BEAM 475
       S(ND+1,ND+1)=S(I,I)*D1**2                                         BEAM 476
       S(ND+2,ND+2)=S(I,I)*D2**2                                         BEAM 477
       S(ND+1,ND+2)=S(I,I)*D1*D2                                         BEAM 478
       S(ND+2,ND+1)=S(ND+1,ND+2)                                         BEAM 479
       ND=ND+2                                                           BEAM 480
   30 CONTINUE                                                           BEAM 481
C                                                                        BEAM 482
C      SET ROTATIONS                                                     BEAM 483
C                                                                        BEAM 484
       DO 54 J=1,3                                                       BEAM 485
       K=NF+J+2                                                          BEAM 486
       IF(LM(K).GE.0)  GO TO 54                                          BEAM 487
       M=-LM(K)                                                          BEAM 488
       LM(K)=ID(M,J+3)                                                   BEAM 489
   54 CONTINUE                                                           BEAM 490
C                                                                        BEAM 491
       RETURN                                                            BEAM 492
       END                                                               BEAM 493


       SUBROUTINE SHELL                                                  SHEL   1
       IMPLICIT REAL*8 (A-H,O-Z)
       INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
      1ISCRF9
       COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
      1ISCRF8,ISCRF9
       COMMON A(1)                                                       SHEL   2
       COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQSHEL   3
       COMMON / JUNK / LT,LH,L,LZ,SIG(20)
C                                                                        SHEL   5
       IF (NPAR(1).EQ.0) GO TO 500                                       SHEL   6
C      PROTECT NODAL TEMPERATURES                                        SHEL   7
       N6=N5+NUMNP+12*NPAR(3)                                            SHEL   8
       IF (N6.GT.MTOT) CALL ERROR (N6-MTOT)                              SHEL   9
       CALL TPLATE(NPAR(2),NPAR(3),A(N1),A(N2),A(N3),A(N4),A(N5),NUMNP,  SHEL  10
      1  MBAND)                                                          SHEL  11
       RETURN                                                            SHEL  12
C                                                                        SHEL  13
  500 WRITE (ISYSLP,2002)
       NUME=NPAR(2)                                                      SHEL  15
       DO 800 MM=1,NUME                                                  SHEL  16
       CALL STRSC (A(N1),A(N3),NEQ,0)                                    SHEL  17
       WRITE (ISYSLP,2001)
       DO 800 L=LT,LH                                                    SHEL  19
       CALL STRSC (A(N1),A(N3),NEQ,1)                                    SHEL  20
       WRITE ( ISYSLP,3002) MM,L,(SIG(I),I=1,6)
  800 CONTINUE                                                           SHEL  22
C                                                                        SHEL  23
       RETURN                                                            SHEL  24
C                                                                        SHEL  25
 2001 FORMAT (/)                                                         SHEL  26
 2002 FORMAT (24H1 SHELL ELEMENT STRESSES//                              SHEL  27
     1 102H              ELEMENT       LOAD     MEMBRANE STRESS COMPONENTSSHEL  28
     2          BENDING MOMENT COMPONENTS        /                       SHEL  29
     3 102H              NUMBER       CASE       SXX           SYY      SSHEL  30
     4XY          MXX          MYY          MXY      //)                 SHEL  31
 3002 FORMAT (10X,2I10,6E12.4)                                           SHEL  32
      END                                                                SHEL  33


       SUBROUTINE TPLATE(NUMEL,NUMMAT,ID,X,Y,Z,C,NUMNP,MBAND)            SHEL  34
       IMPLICIT REAL*8 (A-H,O-Z)
       INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
      1ISCRF9
```

```
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
      COMMON/OTSARG/
     1XX(5),YY(5),ZZ(5),HM(5),HP(5),CM(3,3),ALFA(3),HW(5),RHO(5,3),P(5)
     2,T(5),DT(5),SM(5,3),BM(5,3),TDIS(3,3,4),TROT(3,3,4),S(42,42),R(42)
      COMMON/EM/LM(24),ND,NS,ASA(24,24),RF(24,4),XM(24),SA(12,24)
     1,SF(12,4)
      DIMENSION X(1),Y(1),Z(1),ID(NUMNP,1),C(12,1),IX(7),IY(7),EL(4)
     1,TO(3,3),TLO(5,4)
C
      LL=4
C
      DO 4 I=1,12
      DO 4 J=1,4
    4 SF(I,J)=0.
      DO 5 I=1,24
      DO 5 J=1,4
    5 RF(I,J)=0.0
      NPF=6
      NS=6
      MID=4
      MID=0
      IDIS=0
      IROT=0
      ISTOP=0
      MTYPE=6
      WRITE (ISYSLP,2000) MTYPE,NUMEL,NUMMAT
C
C *** READ AND PRINT OF MATERIAL PROPERTIES
C
      WRITE (ISYSLP,2001)
      DO 10 J=1,NUMMAT
      READ (ISYSCR,1000) N,(C(I,N),I=1,12)
   10 WRITE (ISYSLP,2002) N,(C(I,N),I=3,12)
C
C *** READ AND PRINT OF ELEMENT LOAD MULTIPLIERS
C
      READ (ISYSCR,1002) ((TLO(I,J),J=1,4),I=1,5)
      WRITE (ISYSLP,2006)
      WRITE (ISYSLP,2007) (J,(TLO(I,J),I=1,5),J=1,4)
C
C *** READ AND PRINT OF ELEMENT DATA
C
      WRITE (ISYSLP,2003)
      NN=0
  100 READ (ISYSCR,1001) MM,IY,EL
  110 NN=NN+1
      IF (MM-NN) 440,50,60
   50 DO 45 I=1,7
   45 IX(I)=IY(I)
      INCL=IY(7)
      IF (IY(6).EQ.0) IY(6)=1
      IM=IY(6)
      IF(INCL.EQ.0) INCL=1
      NNS=5
      IF (IY(5).EQ.0) NNS=4
      IF (IY(4).EQ.0) NNS=3
      RHOM=C(3,IM)
      ALFA(1)=C(4,IM)
      ALFA(2)=C(5,IM)
      ALFA(3)=C(6,IM)
      CM(1,1)=C(7,IM)
      CM(1,2)=C(8,IM)
      CM(1,3)=C(9,IM)
      CM(2,2)=C(10,IM)
      CM(2,3)=C(11,IM)
      CM(3,3)=C(12,IM)
      CM(2,1)=CM(1,2)
      CM(3,1)=CM(1,3)
      CM(3,2)=CM(2,3)
C
      DO 30 I=1,NNS
      HM(I)=EL(1)
      HP(I)=EL(1)
      HW(I)=EL(1)
      SM(I,1)=0.0
      SM(I,2)=0.0
      SM(I,3)=0.0
      BM(I,1)=0.0
```

```
      BM(I,2)=0.0
   30 BM(I,3)=0.0
      GO TO 70
C
   60 DO 65 I=1,NNS
   65 IX(I)=IX(I)+INCL
C
   70 DO 40 I=1,NNS
      J=IX(I)
      XX(I)=X(J)
      YY(I)=Y(J)
   40 ZZ(I)=Z(J)
C
      DO 550 IL=1,LL
      DO 520 I=1,NNS
      RHO(I,1)=TLO(3,IL)*RHOM
      RHO(I,2)=TLO(4,IL)*RHOM
      RHO(I,3)=TLO(5,IL)*RHOM
      P(I)=TLO(1,IL)*EL(2)
      T(I)=TLO(2,IL)*EL(3)
  520 OT(I)=TLO(2,IL)*EL(4)
      IF (IL.GE.2) GO TO 530
C
      CALL QTSHEL  (0,NNS,NPF,MID,IOIS,IROT,ND ,NTF)
C
      DO 510 I=1,ND
      XM(I)=0.
      DO 510 J=1,ND
  510 ASA(I,J)=S(I,J)
      GO TO 550
C
  530 CALL QTSHEL  (1,NNS,NPF,MID,IOIS,IROT,ND ,NTF)
C
      DO 540 I=1,ND
  540 RF(I,IL)=R(I)
  550 CONTINUE .
      WRITE (ISYSLP,2004) NN,(IX(I),I=1,6),HM(1),P(1),T(1),OT(1)
C
C *** FORM LM ARRAY AND COMPUTE BANDWIDTH
C
      DO 410 I=1,NNS
      J=NPF*I-NPF
      N=IX(I)
      DO 410 K=1,NPF
  410 LM(J+K)=ID(N,K)
C
      CALL QOCOS (4,XX,YY,ZZ,TO)
      H=HM(1)
C
      CALL STRETR (TO,XX,YY,ZZ,CM,SA,H,RHOM,XM)
C
      NDM=24
      CALL CALBAN (MBAND,NDIF,LM,XM,ASA,RF,ND,NDM)
      WRITE (ISCRF1) ND,NS,(LM(I),I=1,ND),((SA(I,J),I=1,NS),J=1,ND),
     1((SF(I,J),I=1,NS),J=1,4)
C
      GO TO 500
  440 WRITE (ISYSLP,2005) MM
      ISTOP=1
  500 IF (NN.LT.MM) GO TO 110
      IF (NN.EQ.NUMEL) RETURN
      IF (ISTOP.EQ.1) STOP
      GO TO 100
C
 2000 FORMAT (34H1...THIN ELASTIC SHELL ELEMENTS...//
     1 25H ELEMENT TYPE............I5//
     2 25H NUMBER OF ELEMENTS......I5//
     3 25H NUMBER OF MATERIALS.....I5)
 1000 FORMAT (I10,6F10.0/6F10.0)
 2001 FORMAT (36H1...MATERIAL PROPERTY INFORMATION...//
     1 131H                                  MASS   THERMAL EXPANSION COEF
     2FICIENTS      ...................ELASTIC COEFFICIENTS............
     3..../
     4 131H TYPE                           DENSITY          AX         AY
     5       AZ         CXX         CXY         CXG         CYY         CYG
     6GXY //)
 2002 FORMAT (1H I4,22X,4E11.4,6E10.4)
 1002 FORMAT (4F10.0)
 2006 FORMAT (31H1...ELEMENT LOAD MULTIPLIERS...//
     1 90H                                                ..........GRAVI
```

```
     2TY ACCELFRATION........... /
     3 90H   LOAD CASE     LATERAL LOAD     TEMPERATURE     X-COMPONENT
     4Y-COMPONENT     Z-COMPONENT  /)
 2007 FORMAT (1X,I10,5X,5F15.5)
 2003 FORMAT (15H1*ELEMENT DATA*//
     1 96H ELEMENT            NODAL POINT NUMBERS        MATL
     2                         TEMPERATURE/
     3 96H  NUMBER         I     J     K     L     O       TYPE    THICKNESS
     4  PRESSURE TEMPERATURE   GRADIENT /)
 1001 FORMAT (8I5,4F10.0)
 2004 FORMAT (3X,I5,5X,5I5,5X,I5,4F12.4)
 2005 FORMAT (23H0 ELEMENT CARD IN ERROR,I5)
C
      END


      SUBROUTINE STRETR (T,X,Y,Z,CM,SA,H,RHOM,XM)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C *** FORM DISPLACEMENT-STRESS TRANSFORMATION MATRIX
C
      DIMENSION T(3,3),X(1),Y(1),Z(1),CM(3,3),SA(12,24),AA(12,12),MM(12)
     1,A(12,12),AM(4),S1(3,9),S2(3,9),S3(3,12),V(3,3),XY(3,2),XM(1)
C
      DO 5 I=1,12
      DO 5 J=1,24
    5 SA(I,J)=0.
      DO 10 I=1,3
      V(I,1)=X(I+1)-X(1)
      V(I,2)=Y(I+1)-Y(1)
   10 V(I,3)=Z(I+1)-Z(1)
      DO 20 I=1,3
      DO 20 J=1,2
      XY(I,J)=0.
      DO 20 K=1,3
   20 XY(I,J)=XY(I,J)+V(I,K)*T(J,K)
      XO=(XY(1,1)+XY(2,1)+XY(3,1))/4.0
      YO=(XY(1,2)+XY(2,2)+XY(3,2))/4.0
C
C *** FORM LUMP MASS MATRIX
C
      AM1=XY(1,1)*YO-XO*XY(1,2)
      AM2=(XY(1,1)-XO)*(XY(2,2)-YO)-(XY(2,1)-XO)*(XY(1,2)-YO)
      AM3=(XY(2,1)-XO)*(XY(3,2)-YO)-(XY(3,1)-XO)*(XY(2,2)-YO)
      AM4=XO*XY(3,2)-XY(3,1)*YO
      AC=0.25*H*RHOM
      AM(1)=AC*(AM1+AM4)
      AM(2)=AC*(AM1+AM2)
      AM(3)=AC*(AM2+AM3)
      AM(4)=AC*(AM3+AM4)
C
      DO 25 L=1,4
      XX=AM(L)
      DO 25 J=1,3
      JS=6*(L-1)+J
      XM(JS)=XX
   25 XM(JS+3)=0.
C
C *** FORM STRESS TRANSFORMATION FOR IN PLANE STRESS
C
      S=0.5
      R=0.5
      DO 100 I=1,3
      DO 100 J=1,8
  100 S1(I,J)=0.0
C
      XYJ=XY(1,1)*XY(3,2)-XY(3,1)*XY(1,2)+(XY(1,1)*(XY(2,2)-XY(3,2))
     1 -(XY(2,2)-XY(3,1))*XY(1,2))*S-((XY(1,1)-XY(2,1))*XY(3,2)
     2 -XY(3,1)*(XY(1,2)-XY(2,2)))*R
      X1=XY(1,2)-XY(3,2)-(XY(2,2)-XY(3,2))*S-(XY(1,2)-XY(2,2))*R
      X2=XY(3,2)+(XY(2,2)-XY(3,2))*S-XY(3,2)*R
      X3=-XY(1,2)*S+XY(3,2)*R
      X4=-XY(1,2)+XY(1,2)*S+(XY(1,2)-XY(2,2))*R
      Y1=-XY(1,1)+XY(3,1)+(XY(2,1)-XY(3,1))*S+(XY(1,1)-XY(2,1))*R
      Y2=-XY(3,1)-(XY(2,1)-XY(3,1))*S+XY(3,1)*R
      Y3=XY(1,1)*S-XY(3,1)*R
```

```
      Y4=XY(1,1)-XY(1,1)*S-(XY(1,1)-XY(2,1))*R
C
      S1(1,1)=X1/XYJ
      S1(1,3)=X2/XYJ
      S1(1,5)=X3/XYJ
      S1(1,7)=X4/XYJ
      S1(2,2)=Y1/XYJ
      S1(2,4)=Y2/XYJ
      S1(2,6)=Y3/XYJ
      S1(2,8)=Y4/XYJ
      S1(3,1)=S1(2,2)
      S1(3,2)=S1(1,1)
      S1(3,3)=S1(2,4)
      S1(3,4)=S1(1,3)
      S1(3,5)=S1(2,6)
      S1(3,6)=S1(1,5)
      S1(3,7)=S1(2,8)
      S1(3,8)=S1(1,7)
      DO 105 I=1,3
      DO 105 J=1,8
      S2(I,J)=0.
      DO 104 K=1,3
  104 S2(I,J)=S2(I,J)+CM(I,K)*S1(K,J)
  105 S2(I,J)=S2(I,J)*H
C
      H=H**3/12.
      DO 110 L=1,4
      DO 110 I=1,3
      DO 110 J=1,3
      JS=6*(L-1)+J
      KA=2*(L-1)+1
  110 SA(I,JS)=S2(I,KA)*T(1,J)+S2(I,KA+1)*T(2,J)
C
C *** FORM STRESS TRANSFORMATION FOR MOMENT
C
      DO 30 I=1,12
      DO 30 J=1,12
      AA(I,J)=0.
   30 A(I,J)=0.
C
      DO 40 I=1,3
      J=3*I+1
      A(1,J)=1.
      A(2,J)=XY(I,1)
      A(3,J)=XY(I,2)
      A(4,J)=XY(I,1)*XY(I,1)
      A(5,J)=XY(I,1)*XY(I,2)
      A(6,J)=XY(I,2)*XY(I,2)
      A(7,J)=A(4,J)*XY(I,1)
      A(8,J)=A(5,J)*XY(I,1)
      A(9,J)=A(5,J)*XY(I,2)
      A(10,J)=A(6,J)*XY(I,2)
      A(11,J)=XY(I,1)*XY(I,1)*XY(I,1)*XY(I,2)
      A(12,J)=XY(I,1)*XY(I,2)*XY(I,2)*XY(I,2)
      A(3,J+1)=1.0
      A(5,J+1)=XY(I,1)
      A(6,J+1)=2.0*XY(I,2)
      A(8,J+1)=XY(I,1)*XY(I,1)
      A(9,J+1)=2.0*XY(I,1)*XY(I,2)
      A(10,J+1)=3.0*XY(I,2)*XY(I,2)
      A(11,J+1)=XY(I,1)*XY(I,1)*XY(I,1)
      A(12,J+1)=3.0*XY(I,1)*XY(I,2)*XY(I,2)
      A(2,J+2)=-1.0
      A(4,J+2)=-2.0*XY(I,1)
      A(5,J+2)=-XY(I,2)
      A(7,J+2)=-3.0*XY(I,1)*XY(I,1)
      A(8,J+2)=-2.0*XY(I,1)*XY(I,2)
      A(9,J+2)=-XY(I,2)*XY(I,2)
      A(11,J+2)=-3.0*XY(I,1)*XY(I,1)*XY(I,2)
      A(12,J+2)=-XY(I,2)*XY(I,2)*XY(I,2)
   40 CONTINUE
      A(1,1)=1.0
      A(3,2)=1.0
      A(2,3)=-1.0
C
      CALL INVERT (A,12,12,MM,AA)
C
      DO 210 I=1,3
      DO 210 J=1,9
  210 S1(I,J)=0.
```

```
C
      S1(1,1)=2.0
      S1(1,4)=6.0*XO
      S1(1,5)=2.0*YO
      S1(1,8)=6.0*XO*YO
      S1(2,3)=2.0
      S1(2,6)=2.0*XO
      S1(2,7)=6.0*YO
      S1(2,9)=6.0*XO*YO
      S1(3,2)=2.0
      S1(3,5)=4.0*XO
      S1(3,6)=4.0*YO
      S1(3,8)=6.0*XO*XO
      S1(3,9)=6.0*YO*YO
C
      DO 220 I=1,3
      DO 220 J=1,9
      S2(I,J)=0.
      DO 215 K=1,3
  215 S2(I,J)=S2(I,J)+CM(I,K)*S1(K,J)
  220 S2(I,J)=-S2(I,J)*H
C
      DO 230 I=1,3
      DO 230 J=1,12
      S3(I,J)=0.
      DO 230 K=1,9
  230 S3(I,J)=S3(I,J)+S2(I,K)*A(J,3+K)
C
      DO 240 L=1,4
      DO 240 I=1,3
      DO 240 J=1,3
      JS=6*(L-1)+J
      KA=3*(L-1)+1
      SA(I+3,JS)=S3(I,KA)*T(3,J)
      JS=JS+3
      SA(I+3,JS)=S3(I,KA+1)*T(1,J)+S3(I,KA+2)*T(2,J)
  240 CONTINUE
      RETURN
      END


      SUBROUTINE INVERT(A,NN,N,M,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C          GENERAL MATRIX INVERSION SUBROUTINE
C
      DIMENSION A(1),M(1),C(1)
C
      DO 90 I=1,NN
   90 M(I)=-I
C
      DO 140 I=1,NN
C
C     LOCATE LARGEST ELEMENT
C
      D=0.0
      DO 112 L=1,NN
      IF (M(L)) 100,100,112
  100 J=L
      DO 110 K=1,NN
      IF (M(K)) 103,103,108
  103 IF (DABS(D)-DABS(A(J))) 105,105,108
  105 LO=L
      KO=K
      D=A(J)
  108 J=J+N
  110 CONTINUE
  112 CONTINUE
C
C     INTERCHANGE ROWS
C
      TEMP=-M(LO)
      M(LO)=M(KO)
      M(KO)=TEMP
      L=LO
```

```
      K=KD
      DO 114 J=1,NN
      C(J)=A(L)
      A(L)=A(K)
      A(K)=C(J)
      L=L+N
  114 K=K+N
C
C     DIVIDE COLUMN BY LARGEST ELEMENT
C
      NR=(KD-1)*N+1
      NH=NR+N-1
      DO 115 K=NR,NH
  115 A(K)=A(K)/D
C
C     REDUCE REMAINING ROWS AND COLUMNS
C
      L=1
      DO 135 J=1,NN
      IF (J-KD) 130,125,130
  125 L=L+N
      GO TO 135
  130 DO 134 K=NR,NH
      A(L)=A(L)-C(J)*A(K)
  134 L=L+1
  135 CONTINUE
C
C     REDUCE ROW
C
      C(KD)=-1.D
      J=KD
      DO 140 K=1,NN
      A(J)=-C(K)/D
  140 J=J+N
C
C     INTERCHANGE COLUMNS
C
      DO 200 I=1,NN
      L=0
  150 L=L+1
      IF(M(L)-I) 150,160,150
  160 K=(L-1)*N+1
      J=(I-1)*N+1
      M(L)=M(I)
      M(I)=I
      DO 200 L=1,NN
      TEMP=A(K)
      A(K)=A(J)
      A(J)=TEMP
      J=J+1
  200 K=K+1
C
      RETURN
C
      END


      SUBROUTINE QTSHEL (KKK,NNS,NPF,MID,IDIS,IROT,NEF,NTF)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C     THIS SUBROUTINE CAN EVALUATE
C             ... ELEMENT STIFFNESS MATRIX       ...
C             ... CONSISTENT NODAL FORCE VECTOR ...
C             ... INTERNAL STRESSES AND MOMENTS ...
C     OF A SHALLOW QUADRILATERAL SHELL ELEMENT ASSEMBLED WITH 4 FLAT
C     TRIANGLES, OR OF A SINGLE TRIANGULAR SHELL ELEMENT.
C
C
C * * * * * * * * * * * * CALLING ARGUMENTS * * * * * * * * * * * * * *
C
C     INPUTS
C
C      KKK        INTEGER FLAG SPECIFYING OPERATION TO BE PERFORMED
C                  IF KKK =-1, FORM STIFFNESS MATRIX ONLY.
C                  IF KKK = 0, FORM STIFFNESS MATRIX AND LOAD VECTOR.
```

```
C                   IF KKK = 1, FORM LOAD VECTOR ONLY.
C                   IF KKK = 2, EVALUATE STRESSES AND MOMENTS.
C
C     NNS         NUMBER OF SUPPLIED NODAL POINTS
C                  IF NNS = 5, QTSHEL FORMS A QUADRILATERAL, AND THE
C                  PROPERTIES AT THE INTERNAL NODE 5 MUST BE INPUT.
C                  IF NNS = 4, QTSHEL FORMS A QUADRILATERAL, AND THE
C                  PROPERTIES AT THE INTERNAL NODE 5 ARE SET BY QTSHEL
C                  TO BE THEIR CORNER AVERAGE.
C                  IF NNS = 3, QDSTIF FORMS A SINGLE TRIANGLE.
C
C     NPF         NUMBER OF GLOBAL DEGREES OF FREEDOM AT EACH
C                 EXTERNAL NODE (3, 5 OR 6)
C                  IF NPF = 6, THE 3 DISPLACEMENTS U, V AND W AND THE
C                  3 ROTATIONS RX, RY AND RZ ARE INCLUDED AS D.O.F.
C                  IF NPF = 5, THE ROTATION RZ IS IGNORED.
C                  IF NPF = 3, ONLY U, V AND W ARE CONSIDERED AND
C                  THE BENDING STIFFNESS IS NOT INCLUDED (MEMBRANE
C                  SHELL ELEMENT)
C
C     MID         NUMBER OF INTERNAL MIDPOINTS IN QUADRILATERAL (0 OR 4)
C                  IF MID = 0, THE MEMBRANE ELEMENTS ARE   CST
C                         AND THE BENDING  ELEMENTS ARE   LCCT-9
C                  IF MID = 4, THE MEMBRANE ELEMENTS ARE   LST-10
C                         AND THE BENDING  ELEMENTS ARE   LCCT-11
C                 IF NNS = 3 (SINGLE TRIANGLE) MID IS ASSUMED TO BE 0
C
C     IDIS        INTEGER FLAG FOR THE NODAL DISPLACEMENTS U,V,W
C                  IF IDIS = 0, U,V,W ARE SPECIFIED IN THE GLOBAL SYSTEM
C                  IF IDIS = 1, U,V,W ARE SPECIFIED IN THE NODAL DISPL
C                  SYSTEMS DEFINED BY THE DIRECTION COSINE ARRAY TDIS.
C
C     IROT        INTEGER FLAG FOR THE NODAL ROTATIONS RX,RY,RZ.
C                  IF IROT = 0, RX,RY,RZ ARE SPECIFIED IN GLOBAL SYSTEM
C                  IF IROT = 1, RX,RY,RZ ARE SPECIFIED IN THE NODAL ROT
C                  SYSTEMS DEFINED BY THE DIRECTION COSINE ARRAY TROT.
C
C    OUTPUTS
C
C     NEF         NUMBER OF EXTERNAL DEGREES OF FREEDOM (NEF = NPF*NEN,
C                 WHERE NEN=4 FOR QUADRILATERAL, =3 FOR SINGLE TRIANGLE)
C
C     NTF         TOTAL NUMBER OF DEGREES OF FREEDOM (EXTERNAL+INTERNAL)
C
C
C * * * * * * * * * * * ARRAYS IN  COMMON /QTSARG/  * * * * * * * * * * *
C
C X(I),Y(I),Z(I)  I=1...NNS   GLOBAL NODAL COORDINATES
C
C    CM(I,J)      I=1...3, J=1...3   PLANE STRESS MATERIAL MATRIX
C                 RELATING STRESSES TO STRAINS IN THE LOCAL SYSTEM
C
C    ALFA(I)      I=1...3   DILATATION COEFFICIENTS RELATING IN-PLANE
C                 THERMAL STRAINS IN THE LOCAL SYSTEM TO TEMPERATURES
C
C     HM(I)       I=1...NNS   THICKNESS RESISTING MEMBRANE STRESSES
C
C     HP(I)       I=1...NNS   THICKNESS RESISTING BENDING MOMENTS
C
C    RHO(I,J)     I=1...NNS, J=1...3  GLOBAL COMPONENTS RHOX (J=1),
C                 RHOY (J=2) AND RHOZ (J=3) OF BODY FORCES PER UNIT
C                 OF VOLUME
C
C     HW(I)       I=1...NNS   THICKNESS FOR COMPUTING BODY FORCES
C                 RHO*HW PER UNIT OF ELEMENT AREA
C
C     P(I)        I=1...NNS   LATERAL PRESSURE (NORMAL TO THE FACES OF
C                 THE COMPONENT TRIANGLES)
C
C     T(I)        I=1...NNS   MEAN TEMPERATURE VARIATIONS
C
C     DT(I)       I=1...NNS   MEAN TEMPERATURE THICKNESS GRADIENTS
C
C    SM(I,J)      I=1...NNS, J=1...3   ARRAY OF MEMBRANE STRESS
C                 COMPONENTS IN THE LOCAL SYSTEM  SIG-XX (J=1), SIG-YY
C                 (J=2) AND SIG-XY (J=3). SM CONTAINS
C                  MEMBRANE STRESSES IN THE INITIAL POSITION AS INPUT
C                  WHEN KKK=0,1,2 (EXCLUDING THERMAL ACTIONS)
C                  MEMBRANE STRESSES IN THE DEFORMED POSITION AS OUTPUT
C                  WHEN KKK=2 (INCLUDING THERMAL ACTIONS)
```

```
C
C    BM(I,J)        I=1...NNS, J=1...3   ARRAY OF BENDING MOMENT
C                   COMPONENTS IN THE LOCAL SYSTEM  MOM-XX (J=1), MOM-YY
C                   (J=2) AND MOM-XY (J=3). BM CONTAINS
C                    BENDING MOMENTS IN THE INITIAL POSITION AS INPUTS
C                    WHEN KKK=0,1,2 (EXCLUDING THERMAL ACTIONS)
C                    BENDING MOMENTS IN THE DEFORMED POSITION AS OUTPUT
C                    WHEN KKK=2 (INCLUDING THERMAL ACTION)
C
C   TDIS(I,J,K)     I=1...3, J=1...3, K=1...NEN    NOT REQUIRED IF
C                   IDIS=0. IF IDIS=1, TDIS(1..3,1..3,K) MUST CONTAIN
C                   THE (3,3) DIRECTION COSINE MATRIX OF THE NODAL
C                   DISPLACEMENT SYSTEM AT THE K-TH ELEMENT NODE WITH
C                   RESPECT TO THE GLOBAL SYSTEM
C
C   TROT(I,J,K)     I=1...3, J=1...3, K=1...NEN    NOT REQUIRED IF
C                   IROT=0. IF IROT=1, TROT(1..3,1..3,K) MUST CONTAIN
C                   THE (3,3) DIRECTION COSINE MATRIX OF THE NODAL
C                   ROTATION SYSTEM AT THE K-TH ELEMENT NODE WITH
C                   RESPECT TO THE GLOBAL SYSTEM
C
C     S(I,J)        I=1...NEF, J=1...NEF  EXTERNAL STIFFNESS MATRIX
C                   (OUTPUT IF KKK=-1,0)
C
C     S(I,J)        I=1...NTF, J=NEF+1...NTF   REDUCED INTERNAL STIFFNESS
C                   OF QUADRILATERAL ELEMENT. OUTPUT IF KKK=-1,0.
C                   REQUIRED INPUT IF KKK=1,2. NOT USED FOR SINGLE
C                   TRIANGLE.
C
C      R(I)         I=1...NEF    OUTPUT EXTERNAL NODAL FORCES IF KKK=0,1.
C                   INPUT EXTERNAL NODAL DISPLACEMENTS IF KKK=2.
C
C      R(I)         I=NEF+1...NTF   REDUCED INTERNAL NODAL FORCE VECTOR
C                   OF QUADRILATERAL ELEMENT. OUTPUT IF KKK=0,1.
C                   REQUIRED INPUT IF KKK=2 (RETURNS INTERNAL NODAL
C                   DISPLACEMENTS). NOT USED FOR SINGLE TRIANGLE.
C
C
C * * * * * * * * * * ROLE OF ARRAYS IN COMMON /QTSARG/ * * * * * * * *
C
C       ARRAYS       OPERATION  KKK =-1    KKK = 0    KKK = 1    KKK = 2
C                                 Q   T      Q   T      Q   T      Q   T
C
C  X,Y,Z,CM,ALFA,HM,HP (*)        I   I      I   I      I   I      I   I
C      RHO,HW,P                   -   -      I   I      I   I      -   -
C        T,DT  (*)                -   -      I   I      I   I      I   I
C        SM,BM (*)                -   -      I   I      I   I     I/O I/O
C      TDIS,TROT (**)             I   I      I   I      I   I      I   I
C   S(1..NEF,1..NEF)              O   O      O   O      -   -      -   -
C S(1..NTF,NEF+1..NTF)            O   -      O   -      I   -      I   -
C      R(1..NEF)                  -   -      O   O      O   O      I   I
C    R(NEF+1..NTF)                -   -      O   -      O   -     I/O  -
C
C        WHERE   Q=QUADRILATERAL (NNS=4,5), T=SINGLE TRIANGLE (NNS=3)
C                I=INPUT, O=OUTPUT, I/O=INPUT/OUTPUT, -=NOT USED.
C
C    NOTES    (*)  HP,DT AND BM ARE NOT USED IF NPF=3.
C            (**)  TDIS IS NOT USED IF IDIS=0, AND TROT IS NOT USED
C                  IF IROT=0.
C
C
      COMMON /QTSARG/ X(5),Y(5),Z(5), HM(5),HP(5), CM(3,3),ALFA(3),
     1 HW(5),RHO(5,3),P(5), T(5),DT(5), SM(5,3),BM(5,3), TDIS(3,3,4),
     2 TROT(3,3,4), S(42,42), R(42)
      COMMON /TRIARG/ A(3),B(3), HMT(3),HPT(3), C(3,3), SMT(3,3),
     1 BMT(3,3), FT(12),    P1(3),P2(3),P3(3),RM(3), ST(12,12)
      COMMON /TRANSF/  T1(3),T2(3),T3(3), T0(3,3)
      DIMENSION F(1), IPERMQ(4), MFR(5), LOC(5), NC(3), CA(3), WGT(3),
     1 TD1(13),TD2(13),TD3(9), TR1(9),TR2(9),TR3(9), U(1),V(1),W(1),
     2 RX(1),RY(1)
      EQUIVALENCE (T11,T1),(T12,T1(2)),(T13,T1(3)),(T21,T2),(T22,T2(2)),
     1 (T23,T2(3)),(T31,T3),(T32,T3(2)),(T33,T3(3)), (R,F),
     2  (U,FT), (V,FT(7)), (W,P1), (RX,P2), (RY,P3)
      LOGICAL QUAD, TRIG, NOMP, NOST, SIST, NOLO, SILO, NOSM, SISM, SKMP
      DATA  IPERMQ /2,3,4,1/, MFR /3,3,3,2,2/, WGT /.50,.50,.25/
C
C     INITIALIZE
C
      SIST = KKK.LE.0
      NOST = .NOT.SIST
```

```
      SILO = KKK.EQ.0.OR.KKK.EQ.1
      NOLO = .NOT.SILO
      SISM = KKK.GE.2
      NOSM = .NOT.SISM
      IF ((NNS.NE.3).AND.(NNS.NE.5))  NNS = 4
      IF ((NPF.NE.3).AND.(NPF.NE.6))  NPF = 5
      NEN = MIN0 (NNS,4)
      QUAD = NEN.EQ.4
      TRIG = NEN.EQ.3
      WG = 1.
      N3 = 2*NEN - 3
      NTRI = 3*NEN - 8
      NEF = NEN*NPF
      NSF = NEF + (NEN-3)*NPF
      IF (MID.NE.4)  MID = 0
      MIOP = MID
      IF (TRIG)  MIOP = 0
      NFM = 3
      IF (NPF.EQ.3)  NFM = 2
      NTF = NSF + NFM*MIOP
      NOMP = MIOP.LE.0
      SKMP = NOMP.OR.NOST
      IF (NNS.NE.4)  GO TO 130
      X(5) = 0.25*(X(1)+X(2)+X(3)+X(4))
      Y(5) = 0.25*(Y(1)+Y(2)+Y(3)+Y(4))
      Z(5) = 0.25*(Z(1)+Z(2)+Z(3)+Z(4))
      HM(5) = 0.25*(HM(1)+HM(2)+HM(3)+HM(4))
      HP(5) = 0.25*(HP(1)+HP(2)+HP(3)+HP(4))
      IF (KKK.LT.0)  GO TO 130
      T(5) = 0.25*(T(1)+T(2)+T(3)+T(4))
      DT(5) = 0.25*(DT(1)+DT(2)+DT(3)+DT(4))
      DO 110  J = 1,3
      SM(5,J) = 0.25*(SM(1,J)+SM(2,J)+SM(3,J)+SM(4,J))
  110 BM(5,J) = 0.25*(BM(1,J)+BM(2,J)+BM(3,J)+BM(4,J))
      IF (NOLO)  GO TO 130
      P(5) = 0.25*(P(1)+P(2)+P(3)+P(4))
      HW(5) = 0.25*(HW(1)+HW(2)+HW(3)+HW(4))
      DO 120  J = 1,3
  120 RHO(5,J) = 0.25*(RHO(1,J)+RHO(2,J)+RHO(3,J)+RHO(4,J))
  130 IF (NOST)  GO TO 150
      DO 140  I = 1,NTF
      DO 140  J = 1,NTF
  140 S(I,J) =  0.
  150 IF (SISM)  GO TO 170
      DO 160  I = 1,NTF
  160 F(I) = 0.
  170 IF (NOSM.OR.TRIG)  GO TO 200
      NEF1 = NEF + 1
      DO 180  L = NEF1,NTF
      M = L - 1
      DO 180  I = 1,M
  180 R(L) = R(L) - S(I,L)*R(I)
  200 DO 210  I = 1,63
  210 A(I) = 0.
      DO 220  I = 1,3
      CA(I) = CM(1,I)*ALFA(1)+CM(2,I)*ALFA(2)+CM(3,I)*ALFA(3)
      DO 220  J = 1,3
  220 C(I,J) = CM(I,J)
C
C     COMPUTE DIRECTION COSINE MATRIX TD OF LOCAL ELEMENT SYSTEM
C
      CALL QDCOS (NTRI,X,Y,Z,TD)
C
C     LOOP OVER THE NTRI TRIANGLE COMPONENTS
C
      DO 700  NT = 1,NTRI
      N1 = NT
      N2 = IPERMQ(N1)
      NC(1) = N1
      NC(2) = N2
      NC(3) = N3
      MT = MIOP/2
      NDD = 3 + MT
C
C     COMPUTE DIRECTION COSINES OF LOCAL TRIANGLE SYSTEM
C     AND THE TRIANGLE PROJECTIONS A,B ONTO IT
C
      CALL TDCOS (N1,N2,N3,X,Y,Z,A,B)
C
```

```
C     SET UP INPUTS FOR TRIANGLE SUBROUTINES
C
      DO 240  I = 1,3
      L = NC(I)
      LOC(I) = NPF*(L-1)
      HMT(I) = HM(L)
      HPT(I) = HP(L)
      IF (NOLD)  GO TO 240
      ROX = RHO(L,1)
      ROY = RHO(L,2)
      ROZ = RHO(L,3)
      RO1 = T11*ROX+T12*ROY+T13*ROZ
      RO2 = T21*ROX+T22*ROY+T23*ROZ
      RO3 = T31*ROX+T32*ROY+T33*ROZ
      H1 = HW(L)
      P1(I) = RO1*H1
      P2(I) = RO2*H1
      P3(I) = RO3*H1 + P(L)
      TEMP = T(L)
      TMOM = DT(L)*HP(L)**3/12.
      DO 230  J = 1,3
      SMT(I,J) = SM(L,J) - CA(J)*TEMP
  230 BMT(I,J) = BM(L,J) - CA(J)*TMOM
  240 CONTINUE
C
C     FORM TRANSFORMATIONS BETWEEN ELEMENT AND NODAL SYSTEMS
C
      L1 = 9*N1 - 8
      L2 = 9*N2 - 8
      CALL TRFPRD (IDIS,NEN,TDIS(1,1,N1),TDIS(1,1,N2),TDIS(1,1,3),TD1,
     1TD2,TD3)
      IF (NPF.NE.3)
     1CALL TRFPRD (IROT,NEN,TROT(1,1,N1),TROT(1,1,N2),TROT(1,1,3),TR1,
     2TR2,TR3)
      DO 250  I = 7,8
      TD1(I+3) = TD1(I)
      TD1(I+5) = TD1(I)
      TD2(I+3) = TD2(I)
  250 TD2(I+5) = TD2(I)
      LOC(4) = NSF + NFM*(N2-1)
      LOC(5) = NSF + NFM*(N1-1)
      N4 = LOC(4) + 3
      N5 = LOC(5) + 3
C
C     MEMBRANE CONTRIBUTION
C
  260 IF (SISM)  GO TO 320
C     MEMBRANE STIFFNESS AND/OR LOAD VECTOR
      CALL SLST (MT,KKK)
      LT = 0
      DO 300  JJ = 1,NOD
      J = JJ + JJ
      M = LOC(JJ)
      LL = MFR(JJ)
      DO 300  L = 1,LL
      M = M + 1
      LT = LT + 1
      C1 = TD1(LT)
      C2 = TD2(LT)
      IF (SILD)  F(M) = F(M) + FT(J-1)*C1 + FT(J)*C2
      IF (NOST)  GO TO 300
      KT = 0
      DO 290  II = 1,JJ
      I = II + II
      KK = MFR(II)
      IF (II.EQ.JJ)  KK = L
      H1 = ST(I-1,J-1)*C1 + ST(I-1,J)*C2
      H2 = ST(I  ,J-1)*C1 + ST(I  ,J)*C2
      N = LOC(II)
      N = LOC(II)
      DO 290  K = 1,KK
      N = N + 1
      KT = KT + 1
      SQ = S(N,M) + TD1(KT)*H1 + TD2(KT)*H2
      S(N,M) = SQ
  290 S(M,N) = SQ
  300 CONTINUE
  320 CONTINUE
  400 IF (NPF.EQ.3)  GO TO 560
C
```

```
C     PLATE BENDING CONTRIBUTION
      IF (SISM)  GO TO 600
C
C     BENDING STIFFNESS AND/OR LOAD VECTOR
      CALL SLCCT (MT,KKK)
      DO 500  JJ = 1,3
      JT = 3*JJ-3
      J = JT + 1
      DO 450  L = 1,NPF
      M = LOC(JJ) + L
      L3 = L - 3
      IF (L3.GT.0)  GO TO 420
      C3 = TD3(JT+L)
      IF (SILD)  F(M) = F(M) + FT(J)*C3
      IF (SKMP)  GO TO 450
      S4 = S(M,N4) + ST(J,10)*C3
      S5 = S(M,N5) - ST(J,11)*C3
      GO TO 430
  420 C1 = TR1(JT+L3)
      C2 = TR2(JT+L3)
      IF (SILD)  F(M) = F(M) + FT(J+1)*C1 + FT(J+2)*C2
      IF (SKMP)  GO TO 450
      S4 = S(M,N4) + ST(J+1,10)*C1 + ST(J+2,10)*C2
      S5 = S(M,N5) - ST(J+1,11)*C1 - ST(J+2,11)*C2
  430 S(M,N4) = S4
      S(N4,M) = S4
      S(M,N5) = S5
      S(N5,M) = S5
  450 CONTINUE
      IF (NOST)  GO TO 500
      DO 480  II = 1,JJ
      IT = 3*II-3
      I = IT + 1
      KK = NPF
      DO 480  L = 1,NPF
      IF (II.EQ.JJ)  KK = L
      M = LOC(JJ) + L
      L3 = L - 3
      IF (L3.GT.0)  GO TO 460
      C3 = TD3(JT+L)
      H1 = ST(I   ,J)*C3
      H2 = ST(I+1,J)*C3
      H3 = ST(I+2,J)*C3
      GO TO 470
  460 C1 = TR1(JT+L3)
      C2 = TR2(JT+L3)
      H1 = ST(I   ,J+1)*C1 + ST(I   ,J+2)*C2
      H2 = ST(I+1,J+1)*C1 + ST(I+1,J+2)*C2
      H3 = ST(I+2,J+1)*C1 + ST(I+2,J+2)*C2
  470 N = LOC(II)
      DO 480  K = 1,KK
      N = N + 1
      K3 = K - 3
      K1 = IT + K
      K2 = IT + K3
      IF (K3.LE.0)  SQ = S(N,M) + TD3(K1)*H1
      IF (K3.GT.0)  SQ = S(N,M) + TR1(K2)*H2 + TR2(K2)*H3
      S(N,M) = SQ
  480 S(M,N) = SQ
  500 CONTINUE
      IF (NOMP)  GO TO 700
      IF (NOLD)  GO TO 540
      F(N4) = F(N4) + FT(10)
      F(N5) = F(N5) - FT(11)
  540 IF (NOST)  GO TO 700
      S(N4,N4) = S(N4,N4) + ST(10,10)
      S(N5,N5) = S(N5,N5) + ST(11,11)
      S(N4,N5) = S(N4,N5) - ST(10,11)
      S(N5,N4) = S(N4,N5)
      GO TO 700
  560 IF (NOLD)  GO TO 700
      FL = (P3(1)+P3(2)+P3(3))*(A(3)*B(2)-A(2)*B(3))/12.
      JT = 0
      DO 580  JJ = 1,2
      DO 580 L=1,3
      JT=JT+1
      M = LOC(JJ) + L
  580 F(M) = F(M) + FL*TD3(JT)
  600 CONTINUE
  700 CONTINUE
```

```
      IF (SISM.OR.TRIG)  GO TO 900
C
C    CHECK FOR POSSIBLE INTERNAL STIFFNESS SINGULARITY (FLAT
C     OR NEARLY FLAT QUADRILATERAL WHEN NPF = 3 OR 6)
      IF ((NPF.EQ.5).OR.NOST)  GO TO 730
      DO 720  N = 3,6,3
      IF (NPF.NE.N)  GO TO 720
      M = 5*N
      M1 = M - 1
      M2 = M - 2
      IF (S(M,M).GT.(S(M1,M1)+S(M2,M2))*1.0E-08)  GO TO 720
      DO 710  I = 1,NTF
      S(I,M) = 0.
  710 S(M,I) = 0.
  720 CONTINUE
C
C     CONDENSATION OF INTERNAL DEGREES OF FREEDOM
C
  730 NIF = NTF - NEF
      DO 800  N = 1,NIF
      K = NTF - N
      L = K + 1
      PIVOT = S(L,L)
      FL = F(L)
      IF (PIVOT.LE.0.)  GO TO 800
      F(L) = F(L)/PIVOT
      DO 780  I = 1,K
      G = S(I,L)
      IF (G)  740,780,740
  740 IF (NOST)  GO TO 770
      G = G/PIVOT
      S(I,L) = G
      DO 760  J = I,K
      SQ =  S(I,J) - G*S(L,J)
      S(I,J) = SQ
  760 S(J,I) = SQ
  770 F(I) = F(I) - G*FL
  780 CONTINUE
  800 CONTINUE
  900 RETURN
      END


      SUBROUTINE QDCOS (N,X,Y,Z,T)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C     THIS SUBROUTINE COMPUTES THE DIRECTION COSINES OF THE LOCAL
C     ELEMENT SYSTEM OF A QUADRILATERAL (N=4) OR SINGLE TRIANGLE (N=1)
C
      DIMENSION X(1), Y(1), Z(1), T(1)
      X1 = X(2)+X(3)-X(N)-X(1)
      Y1 = Y(2)+Y(3)-Y(N)-Y(1)
      Z1 = Z(2)+Z(3)-Z(N)-Z(1)
      X2 = X(3)+X(N)-X(1)-X(2)
      Y2 = Y(3)+Y(N)-Y(1)-Y(2)
      Z2 = Z(3)+Z(N)-Z(1)-Z(2)
      S1 = X1**2+Y1**2+Z1**2
      C  = (X1*X2+Y1*Y2+Z1*Z2)/S1
      X2 = X2 - C*X1
      Y2 = Y2 - C*Y1
      Z2 = Z2 - C*Z1
      S1=DSQRT(S1)
      S2=DSQRT(X2**2+Y2**2+Z2**2)
      X1 = X1/S1
      Y1 = Y1/S1
      Z1 = Z1/S1
      X2 = X2/S2
      Y2 = Y2/S2
      Z2 = Z2/S2
      T(1) = X1
      T(2) = X2
      T(3) = Y1*Z2-Y2*Z1
      T(4) = Y1
      T(5) = Y2
      T(6) = Z1*X2-Z2*X1
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      T(7) = Z1                                                         SHEL 959
      T(8) = Z2                                                         SHEL 960
      T(9) = X1*Y2-X2*Y1                                                SHEL 961
      RETURN                                                            SHEL 962
      ENO                                                               SHEL 963



      SUBROUTINE TDCOS (N1,N2,N3,X,Y,Z,A,B)                             SHEL 964
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C                                                                       SHEL 965
C     THIS SUBROUTINE COMPUTES THE OIRECTION COSINES OF THE LOCAL       SHEL 966
C     SYSTEM AND THE PROJECTEO OIMENSIONS OF A TRIANGLE COMPONENT       SHEL 967
C                                                                       SHEL 968
      COMMON /TRANSF/  T1(3),T2(3),T3(3), T(9)                          SHEL 969
      EQUIVALENCE (T11,T1),(T12,T1(2)),(T13,T1(3)),(T21,T2),(T22,T2(2)),SHEL 970
     1 (T23,T2(3)),(T31,T3),(T32,T3(2)),(T33,T3(3))                     SHEL 971
      DIMENSION  X(1), Y(1), Z(1), A(1), B(1)                           SHEL 972
      A1 = X(N1)-X(N3)                                                  SHEL 973
      B1 = Y(N1)-Y(N3)                                                  SHEL 974
      C1 = Z(N1)-Z(N3)                                                  SHEL 975
      A2 = X(N2)-X(N3)                                                  SHEL 976
      B2 = Y(N2)-Y(N3)                                                  SHEL 977
      C2 = Z(N2)-Z(N3)                                                  SHEL 978
      T31 = B1*C2-B2*C1                                                 SHEL 979
      T32 = C1*A2-C2*A1                                                 SHEL 980
      T33 = A1*B2-A2*B1                                                 SHEL 981
      S=OSQRT(T31**2+T32**2+T33**2)
      T31 = T31/S                                                       SHEL 983
      T32 = T32/S                                                       SHEL 984
      T33 = T33/S                                                       SHEL 985
      T11 = T33*T(5)-T32*T(8)                                           SHEL 986
      T13 = T32*T(2)-T31*T(5)                                           SHEL 988
      T12 = T31*T(8)-T33*T(2)                                           SHEL 987
      S=DSQRT(T11**2+T12**2+T13**2)
      T11 = T11/S                                                       SHEL 990
      T12 = T12/S                                                       SHEL 991
      T13 = T13/S                                                       SHEL 992
      T21 = T13*T32-T12*T33                                             SHEL 993
      T23 = T12*T31-T11*T32                                             SHEL 995
      T22 = T11*T33-T13*T31                                             SHEL 994
      A(1) = -T11*A2-T12*B2-T13*C2                                      SHEL 996
      A(2) =  T11*A1+T12*B1+T13*C1                                      SHEL 997
      B(1) =  T21*A2+T22*B2+T23*C2                                      SHEL 998
      B(2) = -T21*A1-T22*B1-T23*C1                                      SHEL 999
      A(3) = -A(1)-A(2)                                                 SHEL1000
      B(3) = -B(1)-B(2)                                                 SHEL1001
      RETURN                                                            SHEL1002
      ENO                                                               SHEL1003


      SUBROUTINE TRFPRO (M,NEN,Q1,Q2,Q3,P1,P2,P3)                       SHEL1004
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C       THIS SUBROUTINE GENERATES THE TRANSFORMATIONS RELATING A LOCAL
      COMMON /TRANSF/  T1(3),T2(3),T3(3), T(9)                          SHEL1009
C                                                                       SHEL1008
C     SUBTRIANGLE SYSTEM TO THE NODAL DIS/ROT SYSTEMS AT ITS 3 CORNERS  SHEL1007
        OIMENSION P1(1),P2(1),P3(1),Q1(1),Q2(1),Q3(1)
      EQUIVALENCE (T11,T1),(T12,T1(2)),(T13,T1(3)),(T21,T2),(T22,T2(2)),SHEL1011
     1 (T23,T2(3)),(T31,T3),(T32,T3(2)),(T33,T3(3))                     SHEL1012
      DO 300  I = 1,3                                                   SHEL1013
      J = I + 3                                                         SHEL1014
      K = I + 6                                                         SHEL1015
      P1(I) = T1(I)                                                     SHEL1016
      P1(J) = T1(I)                                                     SHEL1017
      P2(1) = T2(1)                                                     SHEL1018
      P2(J) = T2(I)                                                     SHEL1019
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      P3(I) = T3(I)                                                     SHEL1020
      P3(J) = T3(I)                                                     SHEL1021
      IF (NEN.NE.4)  GO TO 150                                          SHEL1022
      CI = T(I)                                                         SHEL1023
      CJ = T(J)                                                         SHEL1024
      CK = T(K)                                                         SHEL1025
      IF (M)  260,260,240                                               SHEL1026
  150 IF (M)  180,180,200                                               SHEL1027
  180 P1(K) = T1(I)                                                     SHEL1028
      P2(K) = T2(I)                                                     SHEL1029
      P3(K) = T3(I)                                                     SHEL1030
      GO TO 300                                                         SHEL1031
  200 CI = Q3(I)                                                        SHEL1032
      CJ = Q3(J)                                                        SHEL1033
      CK = Q3(K)                                                        SHEL1034
  240 P1(I) = T11*Q1(I) + T12*Q1(J) + T13*Q1(K)                         SHEL1035
      P1(J) = T11*Q2(I) + T12*Q2(J) + T13*Q2(K)                         SHEL1036
      P2(I) = T21*Q1(I) + T22*Q1(J) + T23*Q1(K)                         SHEL1037
      P2(J) = T21*Q2(I) + T22*Q2(J) + T23*Q2(K)                         SHEL1038
      P3(I) = T31*Q1(I) + T32*Q1(J) + T33*Q1(K)                         SHEL1039
      P3(J) = T31*Q2(I) + T32*Q2(J) + T33*Q2(K)                         SHEL1040
  260 P1(K) = T11*CI    + T12*CJ    + T13*CK                            SHEL1041
      P2(K) = T21*CI    + T22*CJ    + T23*CK                            SHEL1042
      P3(K) = T31*CI    + T32*CJ    + T33*CK                            SHEL1043
  300 CONTINUE                                                          SHEL1044
      RETURN                                                            SHEL1045
      END                                                               SHEL1046


      SUBROUTINE SLST (M,KKK)                                           SHEL1047
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C                                                                       SHEL1048
C     THIS SUBROUTINE FORMS THE PLANE STRESS STIFFNESS MATRIX AND/OR    SHEL1049
C     THE CONSISTENT LOAD VECTOR OF A LINEAR STRAIN TRIANGLE (LST) WITH SHEL1050
C     6, 5 OR 4 NODAL POINTS, OR OF A CONSTANT STRAIN TRIANGLE (CST).   SHEL1051
C     LINEAR ELASTIC ANISOTROPIC MATERIAL                               SHEL1052
C                                                                       SHEL1053
C * * * * * * * * * * * * * * * * INPUTS * * * * * * * * * * * * * * * SHEL1054
C                                                                       SHEL1055
C      M        NUMBER OF MIDPOINTS INCLUDED AS NODAL POINTS (M=3,2,1   SHEL1056
C               FOR LST, M=0 FOR CST). NOTE.. MIDPOINTS 4-5-6 ARE       SHEL1057
C               LOCATED ON THE SIDES 2-3, 3-1 AND 1-2, RESPECTIVELY.    SHEL1058
C                                                                       SHEL1059
C     KKK       OPERATION FLAG                                          SHEL1060
C                 KKK LE 0 = FORM STIFFNESS MATRIX AND LOAD VECTOR.     SHEL1061
C                 KKK GT 0 = FORM LOAD VECTOR ONLY.                     SHEL1062
C                                                                       SHEL1063
C  A(I),B(I)    I=1...3   PROJECTIONS OF SIDES 2-3, 3-1 AND 1-2 ONTO    SHEL1064
C               X AND -Y, RESPECTIVELY.                                 SHEL1065
C                                                                       SHEL1066
C   C(I,J)      I=1...3, J=1...3   PLANE STRESS MATERIAL MATRIX.        SHEL1067
C                                                                       SHEL1068
C    H(I)       I=1...3   CORNER THICKNESSES (LINEAR VARIATION ASSUMED). SHEL1069
C                                                                       SHEL1070
C PX(I),PY(I)   I=1...3   CORNER VALUES OF X-Y COMPONENTS OF BODY FORCES SHEL1071
C               PER UNIT OF ELEMENT AREA (LINEAR VARIATION ASSUMED).    SHEL1072
C                                                                       SHEL1073
C   SMT(I,J)    I=1...3, J=1...3   INITIAL MEMBRANE STRESS COMPONENTS   SHEL1074
C               SIG-XX (J=1), SIG-YY (J=2) AND SIG-XY (J=3 AT THE       SHEL1075
C               CORNERS I=1,2,3  (LINEAR VARIATION ASSUMED).            SHEL1076
C                                                                       SHEL1077
C                                                                       SHEL1078
C * * * * * * * * * * * * * * * * OUTPUTS * * * * * * * * * * * * * * * SHEL1079
C                                                                       SHEL1080
C   ST(I,J)     I=1..NOF, J=1..NOF  WITH NOF (NUMBER OF DOF) = 6+2*M, IS SHEL1081
C               THE ELEMENT STIFFNESS MATRIX ASSOCIATED WITH THE NODAL  SHEL1082
C               DISPLACEMENT ORDERING                                   SHEL1083
C                   U(1),V(1),U(2),V(2),U(3), ..... V(3+M)              SHEL1084
C               WHERE U(4), ... V(3+M), IF M GT 0, ARE DEVIATIONS       SHEL1085
C               FROM LINEARITY AT THE MIDPOINTS 1...M.                  SHEL1086
C                                                                       SHEL1087
C    FT(I)      I=1..NDF   CONSISTENT NODAL FORCE VECTOR ASSOCIATED     SHEL1088
C               WITH THE NODAL DISPLACEMENT ORDERING DESCRIBED ABOVE.   SHEL1089
C                                                                       SHEL1090
C                                                                       SHEL1091
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      COMMON /TRIARG/ A(3),B(3), H(3),  HPT(3), C(3,3), SMT(3,3),
     1 BMT(3,3), FT(12),     PX(3),PY(3),PT(3),RM(3), ST(12,12)
      DIMENSION Q(3,3), QA(3), QB(3), A4(3), B4(3), IPERM(3),
     1 SXX(3),SYY(3), SXY(3)
      EQUIVALENCE (SXX,SMT),(SYY,SMT(4)),(SXY,SMT(7))
      LOGICAL NOS
      DATA  IPERM /2,3,1/
      NOS = KKK.GT.0
      NDF = 6 + 2*M
      AREA = A(3)*B(2)-A(2)*B(3)
      SUMH = H(1)+H(2)+H(3)
      HO = SUMH/3.
      IF (HO) 500,500,140
  140 PXS = PX(1)+PX(2)+PX(3)
      PYS = PY(1)+PY(2)+PY(3)
      SXXH = 0.
      SYYH = 0.
      SXYH = 0.
      DO 150  I = 1,3
      CH = (SUMH+H(I))/24.
      SXXH = SXXH + CH*SXX(I)
      SYYH = SYYH + CH*SYY(I)
  150 SXYH = SXYH + CH*SXY(I)
      FAC = HO/(2.*AREA)
      C11 = C(1,1)*FAC
      C22 = C(2,2)*FAC
      C33 = C(3,3)*FAC
      C12 = C(1,2)*FAC
      C13 = C(1,3)*FAC
      C23 = C(2,3)*FAC
      DO 200  J = 1,3
      L = J + J
      FT(L-1) = (PXS+PX(J))*AREA/24. - (B(J)*SXXH+A(J)*SXYH)
      FT(L)   = (PYS+PY(J))*AREA/24. - (A(J)*SYYH+B(J)*SXYH)
      IF (NOS)  GO TO 200
  180 DO 190  I = 1,J
      K = I + I
      AA = A(I)*A(J)
      BB = B(I)*B(J)
      AB = A(I)*B(J)
      BA = B(I)*A(J)
      ABA = AB+BA
      ST(K-1,L-1) = C11*BB + C33*AA + C13*ABA
      ST(K  ,L  ) = C22*AA + C33*BB + C23*ABA
      ST(K-1,L  ) = C12*BA + C33*AB + C13*BB + C23*AA
  190 ST(K  ,L-1) = C12*AB + C33*BA + C13*BB + C23*AA
  200 CONTINUE
      IF (M) 350,350,220
  220 DO 240  I = 1,3
      A4(I) = 4.*A(I)
      B4(I) = 4.*B(I)
      J = IPERM(I)
      K = IPERM(J)
      R = H(I)/HO
      Q(I,I) = 0.1+R/15.
      Q(J,K) = 0.1-R/60.
  240 Q(K,J) = Q(J,K)
      DO 300  J = 1,M
      J1 = IPERM(J)
      J2 = IPERM(J1)
      L = J + J + 6
      FX = 0.
      FY = 0.
      DO 250  N = 1,3
      Q1 = Q(N,J1)
      Q2 = Q(N,J2)
      QA(N) = Q2*A4(J1)+Q1*A4(J2)
      QB(N) = Q2*B4(J1)+Q1*B4(J2)
      FX = FX - QB(N)*SXX(N)-QA(N)*SXY(N)
  250 FY = FY - QA(N)*SYY(N)-QB(N)*SXY(N)
      FT(L-1) = (PXS-PX(J))*AREA/12. + FX*HO/2.
      FT(L)   = (PYS-PY(J))*AREA/12. + FY*HO/2.
      IF (NOS)  GO TO 300
      SUMQA = QA(1)+QA(2)+QA(3)
      SUMQB = QB(1)+QB(2)+QB(3)
      JM = J + 3
      DO 290  I = 1,JM
      K = I + I
      IF (I.GT.3) GO TO 260
      AA = A(I)*SUMQA
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      AB = A(I)*SUMQB
      BA = B(I)*SUMQA
      BB = B(I)*SUMQB
      GO TO 280
  260 I1 = IPERM(I-3)
      I2 = IPERM(I1)
      AA = A4(I2)*QA(I1)+A4(I1)*QA(I2)
      AB = A4(I2)*QB(I1)+A4(I1)*QB(I2)
      BA = B4(I2)*QA(I1)+B4(I1)*QA(I2)
      BB = B4(I2)*QB(I1)+B4(I1)*QB(I2)
  280 ABA = AB+BA
      ST(K-1,L-1) = C11*BB + C33*AA + C13*ABA
      ST(K  ,L  ) = C22*AA + C33*BB + C23*ABA
      ST(K-1,L  ) = C12*BA + C33*AB + C13*BB + C23*AA
  290 ST(K  ,L-1) = C12*AB + C33*BA + C13*BB + C23*AA
  300 CONTINUE
  350 DO 400  I = 2,NDF
      OO 400  J = 1,I
  400 ST(I,J) = ST(J,I)
  500 RETURN
      END


      SUBROUTINE SLCCT (M,KKK)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
C
C     THIS SUBROUTINE FORMS THE PLATE BENDING STIFFNESS ANO/OR THE
C     CONSISTENT LOAO VECTOR OF A LINEAR CURVATURE COMPATIBLE TRIANGLE
C     (LCCT) WITH 6, 5, 4 OR 3 NOOAL POINTS
C
C
C * * * * * * * * * * * * * * * * * INPUTS  * * * * * * * * * * * * * * * * *
C
C      M         NUMBER OF MIOPOINT OEGREES OF FREEDOM (M =3,2,1,0).
C                NOTE.. MIDPOINTS 4-5-6 (IF INCLUDEO) ARE LOCATEO ON
C                SIDES 2-3, 3-1 ANO 1-2, RESPECTIVELY.
C
C     KKK        OPERATION FLAG
C                  KKK LE 0 = FORM STIFFNESS MATRIX ANO LOAD VECTOR.
C                  KKK GT 0 = FORM LOAO VECTOR ONLY.
C
C  A(I),B(I)     I=1...3   PROJECTIONS OF SIOES 2-3, 3-1 ANO 1-2 ONTO
C                X AND -Y, RESPECTIVELY.
C
C   C(I,J)       I=1...3, J=1...3    PLANE STRESS MATERIAL MATRIX.
C
C    H(I)        I=1...3   CORNER THICKNESSES (LINEAR VARIATION ASSUMEO).
C
C    PT(I)       I=1...3   CORNER VALUES OF LATERAL OISTRIBUTEO LOAO
C                (LINEAR VARIATION ASSUMEO).
C
C   BMT(I,J)     I=1...3, J=1...3    INITIAL BENOING MOMENT COMPONENTS
C                MOM-XX (J=1), MOM-YY (J=2) ANO MOM-XY (J=3) AT THE
C                CORNERS I=1...3  (LINEAR VARIATION ASSUMEO).
C
C
C * * * * * * * * * * * * * * * * OUTPUTS * * * * * * * * * * * * * * * * *
C
C   ST(I,J)      I=1..NOF, J=1..NOF  WITH NOF (NUMBER OF DOF) = 9+M, IS
C                THE ELEMENT STIFFNESS MATRIX ASSOCIATEO WITH THE NODAL
C                DISPLACEMENT ORDERING
C                  W(1),RX(1),RY(1),W(2), .... RY(3),RM(1), ... RM(M)
C                WHERE RM(1), ... RM(M), IF M GT 0, ARE MIOPOINT
C                DEVIATIONS FROM NORMAL SLOPE LINEARITY
C
C     FT(I)      I=1..NOF   CONSISTENT NOOAL FORCE VECTOR ASSOCIATED
C                WITH THE NODAL OISPLACEMENT ORDERING DESCRIBEO ABOVE.
C
C
      COMMON /TRIARG/ A(3),B(3), HMT(3), H(3),  C(3,3), SMT(3,3),
     1 BMT(3,3), FT(12),    PX(3),PY(3),PT(3),RM(3), ST(12,12)
      OIMENSION  P(21,12), G(21), Q(3,6), QB(3,6), T(3), U(3), HT(3),
     1 TX(3), TY(3), IPERM(3), XM(3,3), XMO(3)
      EQUIVALENCE (CM11,C(1)),(CM12,C(2)),(CM13,C(3)),(CM22,C(5)),
     1 (CM23,C(6)),(CM33,C(9))
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      LOGICAL NOS, FLAT                                                 SHFL1244
      DATA IPERM/2,3,1/                                                 SHFL1245
      HO = (H(1)+H(2)+H(3))/3.                                          SHFL1246
      IF (HO.LE.0.)  GO TO 1000                                         SHFL1247
      NDF = 9 + M                                                       SHFL1248
      NUS = KKK.GT.0                                                    SHEL1249
      FLAT = (H(1).EQ.H(2)).AND.(H(2).EQ.H(3))                          SHFL1250
      AREA = A(3)*B(2)-A(2)*B(3)                                        SHEL1251
      FAC = HO**3*AREA/864.                                             SHFL1252
      PTF = AREA/6480.                                                  SHEL1253
      T(3) = 1.                                                         SHEL1254
      DO 150  I = 1,3                                                   SHEL1255
      J = IPERM(I)                                                      SHEL1256
      K = IPERM(J)                                                      SHEL1257
      X = A(I)**2+B(I)**2                                               SHEL1258
      U(I) = -(A(I)*A(J)+B(I)*B(J))/X                                   SHEL1259
      X=DSQRT(X)
      Y = 2.*AREA/X                                                     SHEL1261
      HT(I) =  2.*Y                                                     SHEL1262
      TX(I) =  Y*A(I)/X                                                 SHEL1263
      TY(I) = -Y*B(I)/X                                                 SHEL1264
      A1 = A(I)/AREA                                                    SHEL1265
      A2 = A(J)/AREA                                                    SHEL1266
      B1 = B(I)/AREA                                                    SHEL1267
      B2 = B(J)/AREA                                                    SHEL1268
      Q(1,I)   = B1*B1                                                  SHEL1269
      Q(2,I)   = A1*A1                                                  SHEL1270
      Q(3,I)   = 2.*A1*B1                                               SHEL1271
      Q(1,I+3) = 2.*B1*B2                                               SHEL1272
      Q(2,I+3) = 2.*A1*A2                                               SHEL1273
      Q(3,I+3) = 2.*(A1*B2+A2*B1)                                       SHEL1274
      DO 120  N = 1,3                                                   SHEL1275
  120 XM(N,I) = BMT(N,I)*AREA/72.                                       SHEL1276
      IF (FLAT)  GO TO 150                                              SHEL1277
      DO 140  N = 1,3                                                   SHEL1278
      L = IPERM(N)                                                      SHEL1279
      T(1) = H(N)/HO                                                    SHEL1280
      T(2) = H(L)/HO                                                    SHEL1281
      IF (T(1).GT.0.)  XM(N,I) = XM(N,I)/T(1)**3                        SHEL1282
      C1 = T(I)                                                         SHEL1283
      C2 = T(J)                                                         SHEL1284
      C3 = T(K)                                                         SHEL1285
      C4 = C2+C3                                                        SHEL1286
      C11 = C1*C1                                                       SHEL1287
      C23 = C2*C3                                                       SHEL1288
      C5 = C4*(3.*C1+C4) + 6.*C11 - 2.*C23                              SHEL1289
      C6 = C5 + 3.*C4*C4 - 4.*(C11+C23)                                 SHEL1290
      QB(N,I  ) = (C1*(10.*C11-3.*C23)+C4*C5)/17.5 - 2.0                SHEL1291
  140 QB(N,I+3) = (C1*(C11-2.*C23)+C4*C6)/35.0 - 1.0                    SHEL1292
  150 CONTINUE                                                          SHEL1293
      DO 200  I = 1,3                                                   SHEL1294
      J = IPERM(I)                                                      SHEL1295
      K = IPERM(J)                                                      SHEL1296
      II = 3*I                                                          SHEL1297
      JJ = 3*J                                                          SHEL1298
      KK = 3*K                                                          SHEL1299
      A1 = A(I)                                                         SHEL1300
      A2 = A(J)                                                         SHEL1301
      A3 = A(K)                                                         SHEL1302
      B1 = B(I)                                                         SHEL1303
      B2 = B(J)                                                         SHEL1304
      B3 = B(K)                                                         SHEL1305
      U1 = U(I)                                                         SHEL1306
      U2 = U(J)                                                         SHEL1307
      U3 = U(K)                                                         SHEL1308
      W1 = 1.-U1                                                        SHEL1309
      W2 = 1.-U2                                                        SHEL1310
      W3 = 1.-U3                                                        SHEL1311
      B1D = B1 + B1                                                     SHEL1312
      B2D = B2 + B2                                                     SHEL1313
      B3D = B3 + B3                                                     SHEL1314
      A1D = A1 + A1                                                     SHEL1315
      A2D = A2 + A2                                                     SHEL1316
      A3D = A3 + A3                                                     SHEL1317
      C21 = B1-B3*U3           + TX(K)                                  SHEL1318
      C22 = -B1D+B2*W2+B3*U3 + TX(J)-TX(K)                              SHEL1319
      C31 = A1-A3*U3           + TY(K)                                  SHEL1320
      C22 = -B1D+B2*W2+B3*U3 + TX(J)-TX(K)                              SHFL1321
      C31 = A1-A3*U3           + TY(K)                                  SHFL1322
      C32 = -A1D+A2*W2+A3*U3 + TY(J)-TY(K)                              SHEL1323
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      C51 = B3*W3-B2          + TX(K)
      C52 = B2D-B3*W3-B1*U1   + TX(I)-TX(K)
      C61 = A3*W3-A2          + TY(K)
      C62 = A2D-A3*W3-A1*U1   + TY(I)-TY(K)
      C81 = B3-B2D-B2*U2      + TX(J)
      C82 = B1D-B3+B1*W1      + TX(I)
      C91 = A3-A2D-A2*U2      + TY(J)
      C92 = A1D-A3+A1*W1      + TY(I)
      P1 = PT(I)*PTF
      P2 = PT(J)*PTF
      P3 = PT(K)*PTF
      U37 = 7.*U3
      W27 = 7.*W2
      W24 = 4.*W2
      U34 = 4.*U3
      C1 = 54.+W27
      C2 = 54.+U37
      C3 = 15.+W24
      C4 = 39.+U37
      C5 = 39.+W27
      C6 = 15.+U34
      TXS = TX(J)+TX(K)
      TYS = TY(J)+TY(K)
      FT(II-2) = 6.*((90.+U37+W27)*P1+(36.+U37+W24)*P2+(36.+U34+W27)*P3)
      FT(II-1) = (C1*B2-C2*B3+7.*TXS)*P1 + (C3*B2-C4*B3+4.*TXS+
     1 3.*TX(K))*P2 + (C5*B2-C6*B3+4.*TXS+3.*TX(J))*P3
      FT(II)   = (C1*A2-C2*A3+7.*TYS)*P1 + (C3*A2-C4*A3+4.*TYS+
     1 3.*TY(K))*P2 + (C5*A2-C6*A3+4.*TYS+3.*TY(J))*P3
      FT(K+9)  = (7.*(P1+P2)+4.*P3)*HT(K)
      XMO(I) = (XM(1,I)+XM(2,I)+XM(3,I))/3.
      DO 200  N = 1,3
      L = 6*(I-1) + N
      Q11 = Q(N,I)
      Q22 = Q(N,J)
      Q33 = Q(N,K)
      Q12 = Q(N,I+3)
      Q23 = Q(N,J+3)
      Q31 = Q(N,K+3)
      Q2333 = Q23-Q33
      Q3133 = Q31-Q33
      P(L   ,II-2) = 6.*(-Q11+W2*Q33+U3*Q2333)
      P(L   ,II-1) = C21*Q23+C22*Q33-B3D*Q12+B2D*Q31
      P(L   ,II  ) = C31*Q23+C32*Q33-A3D*Q12+A2D*Q31
      P(L   ,JJ-2) = 6.*(Q22+W3*Q2333)
      P(L   ,JJ-1) = C51*Q2333+B3D*Q22
      P(L   ,JJ  ) = C61*Q2333+A3D*Q22
      P(L   ,KK-2) = 6.*(1.+U2)*Q33
      P(L   ,KK-1) = C81*Q33
      P(L   ,KK  ) = C91*Q33
      P(L   ,I+9 ) = 0.
      P(L   ,J+9 ) = HT(J)*Q33
      P(L   ,K+9 ) = HT(K)*Q2333
      P(L+3 ,II-2) = 6.*(Q11+U3*Q3133)
      P(L+3 ,II-1) = C21*Q3133-B3D*Q11
      P(L+3 ,II  ) = C31*Q3133-A3D*Q11
      P(L+3 ,JJ-2) = 6.*(-Q22+U1*Q33+W3*Q3133)
      P(L+3 ,JJ-1) = C51*Q31+C52*Q33+B3D*Q12-B1D*Q23
      P(L+3 ,JJ  ) = C61*Q31+C62*Q33+A3D*Q12-A1D*Q23
      P(L+3 ,KK-2) = 6.*(1.+W1)*Q33
      P(L+3 ,KK-1) = C82*Q33
      P(L+3 ,KK  ) = C92*Q33
      P(L+3 ,I+9 ) = HT(I)*Q33
      P(L+3 ,J+9 ) = 0.
      P(L+3 ,K+9 ) = HT(K)*Q3133
      P(N+18,II-2) = 2.*(Q11+U3*Q12+W2*Q31)
      P(N+18,KK-1) = ((B1D-B2D)*Q33+C82*Q23+C81*Q31)/3.
      P(N+18,KK  ) = ((A1D-A2D)*Q33+C92*Q23+C91*Q31)/3.
  200 P(N+18,K+9 ) = HT(K)*Q12/3.
  300 DO 400  J = 1,NDF
      DO 340  L = 1,3
      II = L
      KK = L + 18
      P3 = P(KK,J)
      G(KK) = 0.
      DO 340  N = 1,3
      I = IPERM(N)
      JJ = II + 3
      P1 = P(II,J)
      P2 = P(JJ,J)
      SUM = P1 + P2 + P3
```

```
      G1 = SUM + P1
      G2 = SUM + P2
      G3 = SUM + P3
      IF (FLAT)  GO TO 320
      G1 = G1 + QB(N,1)*P1 + QB(N,6)*P2 + QB(N,5)*P3
      G2 = G2 + QB(N,6)*P1 + QB(N,2)*P2 + QB(N,4)*P3
      G3 = G3 + QB(N,5)*P1 + QB(N,4)*P2 + QB(N,3)*P3
  320 G(II) = G1
      G(JJ) = G2
      G(KK) = G3 + G(KK)
      II = II + 6
  340 FT(J) = FT(J) - XM(N,L)*G1 - XM(I,L)*G2 - XMO(L)*G3
      IF (NOS)  GO TO 400
      DO 360  N = 1,19,3
      G1 = G(N)
      G2 = G(N+1)
      G3 = G(N+2)
      G(N)   = CM11*G1 + CM12*G2 + CM13*G3
      G(N+1) = CM12*G1 + CM22*G2 + CM23*G3
  360 G(N+2) = CM13*G1 + CM23*G2 + CM33*G3
      DO 390  I = 1,J
      X = 0.
      DO 380  N = 1,21
  380 X = X + G(N)*P(N,I)
      X = X*FAC
      ST(I,J) = X
  390 ST(J,I) = X
  400 CONTINUE
 1000 RETURN
      END
```

IBM 360/370 CONTROL STATEMENTS FOR EXECUTING PROGRAM AT FHWA

```
//R4111BR1 JOB (011B,FHRS),'MIKLOF',CLASS=F,TIME=10
//SSAP EXEC PGM=SSAP,TIME=7
//SSAP EXEC PGM=TEMPNAME,TIME=60
//STEPLIB DD DSN=RS41.SSAP2,UNIT=2314,VOL=SER=FHRS01,DISP=SHR
//FT03F001 DD SYSOUT=A,DCB=(RECFM=UA,BLKSIZE=133)
//FT11F001 DD DSN=&&FT11,UNIT=SYSDA,DISP=(NEW,DELETE,DELETE),
//   DCB=(RECFM=VBS,LRECL=2044,BLKSIZE=2048),SPACE=(CYL,(35,5))
//FT12F001 DD DSN=&&FT12,UNIT=SYSDA,DISP=(NEW,DELETE,DELETE),
//   DCB=(RECFM=VBS,LRECL=4092,BLKSIZE=4096),SPACE=(CYL,(35,5))
//FT13F001 DD DSN=&&FT13,UNIT=SYSDA,DISP=(NEW,DELETE,DELETE),
//   DCB=(RECFM=VBS,LRECL=2044,BLKSIZE=2048),SPACE=(CYL,(35,5))
//FT14F001 DD DSN=&&FT14,UNIT=SYSDA,DISP=(NEW,DELETE,DELETE),
//   DCB=(RECFM=VBS,LRECL=2044,BLKSIZE=2048),SPACE=(CYL,(35,5))
//FT17F001 DD DSN=&&FT17,UNIT=SYSDA,DISP=(NEW,DELETE,DELETE),
//   DCB=(RECFM=VBS,LRECL=4092,BLKSIZE=4096),SPACE=(CYL,(35,5))
//FT18F001 DD DSN=&&FT18,UNIT=SYSDA,DISP=(NEW,DELETE,DELETE),
//   DCB=(RECFM=VBS,LRECL=84,BLKSIZE=4241,SPACE=(TRK,(50,20))
//FT19F001 DD DSN=&&FT19,UNIT=SYSDA,DISP=(NEW,DELETE,DELETE),
//   DCB=(RECFM=VBS,LRECL=84,BLKSIZE=424),SPACE=(TRK,(50,20))
//FT01F001 DD *
```

DATA DECK

```
/*
```

IBM 360/370 CONTROL STATEMENTS FOR CREATING THE LOAD MODULE AT FHWA

```
//R4111BR1 JOB (0118,FHRS),'MIKLOF',CLASS=F,TIME=2
//ASM1 EXEC ASMFC,PARM.ASM=(LOAD)
//ASM.SYSGD DD DSN=&&LOADSET,UNIT=SYSDA,
// SPACE=(TRK,(100,10)),DISP=(MOD,PASS)
//ASM.SYSIN DD *
```

ETODF ASSEMBLER LISTING

```
R0       EQU    0
R1       EQU    1
R2       EQU    2
RBASE    EQU    12
R13      EQU    13
R15      EQU    15
ETOD     CSECT
         SAVE   (14,12),,*
         LR     RBASE,R15
         USING  ETODF,RBASE
         LA     R2,SAVEAREA
         ST     R13,4(R2)
         ST     R2,8(R13)
         LR     R13,R2
         L      R2,0(R1)
         STIMER TASK,DINTVL=SIXTY
         ST     R0,0(R2)
         L      R13,4(R13)
         RETURN (14,12)
SAVEAREA DS     18F
SIXTY    DC     C'01000000'
         END
```

```
/*
//ASM2 EXEC ASMFC,PARM.ASM=(LOAD)
//ASM.SYSGO DD DSN=&&LOADSET,UNIT=SYSDA,
// SPACE=(TRK,(100,10)),DISP=(MOD,PASS)
//ASM.SYSIN DD *
```

ETODF1 ASSEMBLER LISTING

```
R0       EQU    0
R1       EQU    1
R2       EQU    2
RBASE    EQU    12
R13      EQU    13
R15      EQU    15
ETODF1   CSECT
         SAVE   (14,12),,*
         LR     RBASE,R15                ESTABLISH ADDRESSABILITY
         USING  ETODF1,RBASE             AND TELL TO THE ASSEMBLER
         LA     R2,SAVEAREA
         ST     R13,4(R2)                CHAIN CALLER"S SAVE AREA TO MINE
         ST     R2,8(R13)                CHAIN MY SAVE AREA TO CALLER"S
         LR     R13,R2                   ESTABLISH NEW SAVE AREA
         L      R2,0(R1)                 PICK UP ADDRESS OF PARAMETER AREA
         TTIMER CANCEL
         SR 6,6                          R6=0
         CR 0,6                          TIME ALL ELAPSED?
         BNE CONT                        NO,CONTINUE TO COMPUTE
         B FINISH                        YES,GO TO RETURN
CONT     LA 6,26                         R6=#MICRO SEC. PER TIM. UNIT
         LR     1,0                      LOAD R1 FOR MULTIPLICATION
         MR 0,6                          MICRO SEC=TIM IN UNITS X 26
         L      6,CONST1
         DR 0,6                          CONVERT MICRO TO HUNDREDS
         L      6,CONST2
         SR 6,1                          SUBTRACT TO FIND ELAPSED TIME
FINISH   ST 6,0(R2)                      STORE RESULT IN PARM AREA
         L      R13,4(R13)               RESTORE CALLER"S SAVE AREA
         RETURN (14,12)                  AND RETURN TO CALLER
SAVEAREA DS     18F
CONST1   DC     F'10000'                 R6=10 TO THE FOURTH
CONST2   DC     F'360000'                #HUNDREDS SECS IN AN HOUR
         END
```

```
/*
//CLG EXEC FORTGCL,PARM.FORT=(SOURCE,NOMAP,NOLIST),
//  PARM.LKED=(MAP,LET)
//FORT.SYSIN DD *
```

IBM SOURCE DECK

```
/*
//LKED.SYSLMOD DD DSN=RS41.SSAP2,
//  UNIT=2314,VOL=SER=FHRS01,
//  DCB=BLKSIZE=1024,
//  SPACE=(TRK,(300,100,1),RLSE),
//  DISP=(NEW,KEEP)
//LKED.SYSIN DD *
   ENTRY MAIN
   NAME SSAP
```

To compile the main program for the CDC 6400 machine, the unlabelled statements (those not containing an SSAP number should be removed and the following statements inserted in their sequence locations.

```
      PROGRAM   SAP2 (INPUT,OUTPUT,TAPE5=INPUT, TAPE6=OUTPUT,TAPE1,         SSAP   1
     1TAPE2,TAPE3,TAPE4,TAPE7,TAPE8)                                        SSAP   2

      COMMON / EM / QQQ(2594)                                               SSAP   9

    5 CALL SECOND (T(1))                                                    SSAP  18
      READ (5,100)   HED,NUMNP,NELTYP,LL,NF,NDYN                            SSAP  19

      WRITE(6,200)   HED,NUMNP,NELTYP,LL                                    SSAP  21


      CALL SECOND (T(2))                                                    SSAP  36


      REWIND 1                                                              SSAP  39
      REWIND 2                                                              SSAP  40

      READ (5,1001) NPAR                                                    SSAP  42
      WRITE (1) NPAR                                                        SSAP  43

      WRITE (6,201) NEQ,MBAND,NEQB,NBLOCK                                   SSAP  56
      CALL SECOND(T(3))                                                     SSAP  57

      CALL SECOND(T(4))                                                     SSAP  59


      CALL SECOND(T(5))                                                     SSAP  71


      CALL SECOND(T(6))                                                     SSAP  80


      CALL SECOND(T(7))                                                     SSAP  96
      DO 40 I=1,6                                                           SSAP  97
   40 T(I)=T(I+1)-T(I)                                                      SSAP  98

      WRITE (6,203) T                                                       SSAP 100

      WRITE (6,200)                                                         SSAP 129
      WRITE (6,2001)                                                        SSAP 130

   10 READ  (5,1000) N,(ID(N,I),I=1,6),X(N),Y(N),Z(N),KN,T(N)               SSAP 132
      WRITE (6,2002) N,(ID(N,I),I=1,6),X(N),Y(N),Z(N),KN,T(N)               SSAP 133
```

```
      WRITE (6,2003)                                                          SSAP 166
      WRITE (6,2001)                                                          SSAP 167
      WRITE(6,2005) (N,(ID(N,I),I=1,6),X(N),Y(N),Z(N),T(N),N=1,NUMNP)         SSAP 168


      WRITE(6,2004) (N,(ID(N,I),I=1,6),N=1,NUMNP                              SSAP 184
      REWIND 8                                                                SSAP 185
      WRITE (8) ID                                                            SSAP 186


      REWIND NT                                                               SSAP 251

      WRITE (6,2002)                                                          SSAP 253

  300 READ(5,1001) N,L,R                                                      SSAP 274

      WRITE (6,2001) N,L,R                                                    SSAP 276


      WRITE (NT) B,TMASS                                                      SSAP 286


      WRITE (NT) B,TMASS                                                      SSAP 294


      WRITE (6,2000) N                                                        SSAP 304


      COMMON /EM/ LRD,ND,LM(2592)                                             SSAP 311
      EQUIVALENCE (SS,ND)                                                     SSAP 312


      MB=SQRT(X)                                                              SSAP 316


      REWIND 3                                                                SSAP 322
      REWIND 4                                                                SSAP 323


      WRITE (6,2000)                                                          SSAP 327


      READ (5,1002)    (STR(I,L),I=1,4)                                       SSAP 329
   50 WRITE (6,2002) L,(STR(I,L),I=1,4)                                       SSAP 330
      WRITE (8) STR                                                           SSAP 331


      READ (3) ((B(I,L),I=1,NEQB),L=1,LL),(TMASS(I),I=1,NEQB)                 SSAP 339

      READ (3) ((B(I,L),I=K,NE2B),L=1,LL),(TMASS(I),I=K,NE2B)                 SSAP 341
```

```
      REWIND 7                                                          SSAP 344
      REWIND 2                                                          SSAP 345
      NA=7                                                              SSAP 346

      NA=2                                                              SSAP 349

      READ (NA) LRD,(SS(I),I=1,LRD)                                     SSAP 354

C     DETERMINE IF STIFFNESS IS TO BE PLACED ON TAPE 7                  SSAP 371

  660 WRITE (7) LRD,(SS(I),I=1,LRD)                                     SSAP 379

      WRITE (4) ((A(I,J),I=1,NEQB),J=1,MBAND),((B(I,L),I=K,NE2B),L=1,LL)SSAP 383

      WRITE (4) ((A(I,J),I=K,NE2B),J=1,MBAND),((B(I,L),I=K,NE2B),L=1,LL)SSAP 385

      REWIND 8                                                          SSAP 401
      READ (8) ID                                                       SSAP 402

      WRITE (6,2003)                                                    SSAP 405

      WRITE (6,2004) N,(L,(D(I,L),I=1,6),L=1,LL)                        SSAP 426

      N2=NT2                                                            SSAP 445
      N1=NT1                                                            SSAP 446

      WRITE (6,116) M,D                                                 SSAP 464

      READ (8) STR                                                      SSAP 608

      REWIND 3                                                          SSAP 611

      REWIND 2                                                          SSAP 622
      IF(NDYN.EQ.3) READ (2)                                            SSAP 623

      READ (2) B                                                        SSAP 626

      REWIND 1                                                          SSAP 639

      READ (1) NPAR                                                     SSAP 641
```

```
      WRITE (2) LRD,ND,(LM(I),I=1,ND),((S(I,J),I=1,ND),J=1,ND),              SSAP 662
     1 ((P(I,J),I=1,ND),J=1,4),(XM(I),I=1,ND)                               SSAP 663

      COMMON /JUNK/LT,LH,L,SG(20), SIG(7),EXTRA(150)                        SSAP 668

  800 READ (1) ND,NS,(LM(I),I=1,ND),((B(I,J),I=1,NS),J=1,ND),               SSAP 685
     1 ((T1(I,J),I=1,NS),J=1,4)                                             SSAP 686

      COMMON / JUNK / LT,LH,L,SIG(20)                                       TRUS   5

  500 WRITE (6,2002)                                                        TRUS  19

      WRITE (6,2001)                                                        TRUS  23

      WRITE (6,3002) MM,L,SIG(1),SIG(2)                                     TRUS  26

      WRITE (6,2000) NUME,NUMMAT                                            TRUS  48
      WRITE (6,2001)                                                        TRUS  49

      READ (5,1001) N,E(N),THERM(N),DEN(N),AREA(N),WT(N)                    TRUS  51
   10 WRITE (6,2002) N,E(N),THERM(N),DEN(N),AREA(N),WT(N)                   TRUS  52

      READ (5,1003) EMUL                                                    TRUS  56
      WRITE (6,2003) EMUL                                                   TRUS  57

      WRITE (6,2005)                                                        TRUS  60

  100 READ (5,1004) M,II,JJ,MTYP,TEM,KK                                     TRUS  63

      XL=SQRT(XL2)                                                          TRUS  78

      WRITE (1) ND,NS,(LM(L),L=1,ND),((ST(L,K),L=1,NS),K=1,ND),             TRUS 127
     1 ((TT(L,K),L=1,NS),K=1,4)                                             TRUS 128

      WRITE (6,2004) N,I,J,MTYPE,REFT,NDIF                                  TRUS 132

      COMMON / JUNK / LT,LH,L,SIG(20)                                       BEAM   4

  500 WRITE (6,2002)                                                        BEAM  18

      WRITE (6,2001)                                                        BEAM  22

      WRITE (6,3002) MM,L,(SIG(I),I=1,12)                                   BEAM  25

      COMMON /NEWB/ LC(4),T(3,3),JK(6),MELTYP,DL,MATTYP                     BEAM  44
```

```
      WRITE (6,2005) NBEAM,NUMETP,NUMFIX,NUMMAT                           BEAM  50

      WRITE (6,2001)                                                      BEAM  57

      READ (5,1001) N,E(N),G(N),RO(N)                                     BEAM  59
      WRITE (6,2002) N,E(N),G(N),RO(N)                                    BEAM  60

      WRITE (6,2003)                                                      BEAM  65

      READ (5,1002) N,(COPROP(N,J), J=1,6)                                BEAM  67

      WRITE (6,2013)                                                      BEAM  70

   20 WRITE (6,2004) N,(COPROP(N,J),J=1,6)                                BEAM  72

      READ (5,1006) ((EMUL(I,J),J=1,4),I=1,3)                             BEAM  77
      WRITE (6,2006) ((EMUL(I,J),J=1,4),I=1,3)                            BEAM  78

      WRITE (6,2010)                                                      BEAM  83

      READ (5,1005) N,(SFT(N,J),J=1,12)                                   BEAM  85
   55 WRITE (6,2011) N,(SFT(N,J),J=1,12)                                  BEAM  86

      WRITE (6,4000)                                                      BEAM  91

      READ (5,3000) INEL,INI,INJ,INK,IMAT,IMEL,ILC,INELKI,INELKJ,INC      BEAM  94

   66 WRITE (6,4003) INEL                                                 BEAM 104

      WRITE (6,4001) NEL,NI,NJ,NK,MATTYP,MELTYP,LC,NEKODI,NEKODJ          BEAM 124

      DL=SQRT(DX*DX+DY*DY+DZ*DZ)                                          BEAM 129

   75 WRITE (6,4005) NEL                                                  BEAM 131

      UU=SQRT(UU)                                                         BEAM 154

      WRITE (6,4002) INEL                                                 BEAM 156

      IF (ABS(DS-DL).GT.DL/100.) GO TO 80                                 BEAM 168

      IF (ABS(TS(I,J)-T(I,J)).GT.ABS(T(I,J/100.) GO TO 80                 BEAM 176

      WRITE (1) ND,NS,(LM(I),I=1,ND),((SA(I,J),I=1,NS),J=1,ND),           BEAM 222
     1 ((SF(I,J),I=1,NS),J=1,4)                                           BEAM 223

      COMMON/NEWB/ LC(4),T(3,3),JK(6),MELTYP,DL,MATTYP                    BEAM 285

      DO 5 I=1,144                                                        BEAM 288
    5 S(I)=0                                                              BEAM 289
```

```
      DO 31 I=1,288                                                          BEAM 376
   31 SA(I)=0.                                                               BEAM 377

      DO 32 I=1,576                                                          BEAM 392
   32 ASA(I)=0.                                                              BEAM 393


      COMMON / JUNK / LT,LH,L,SIG(20)                                        SHEL   4

  500 WRITE (6,2002)                                                         SHEL  14

      WRITE (6,2001)                                                         SHEL  18

      WRITE (6,3002) MM,L,(SIG(I),I=1,6)                                     SHEL  21

      WRITE (6,2000) MTYPE,NUMEL,NUMMAT                                      SHEL  60

      WRITE (6,2001)                                                         SHEL  64

      READ (5,1000) N,(C(I,N)I=1,12)                                         SHEL  66

   10 WRITE (6,2002) N,(C(I,N)I=3,12)                                        SHEL  67

      READ (5,1002) ((TLO(I,J),J=1,4),I=1,5)                                 SHEL  71
      WRITE (6,2006)                                                         SHEL  72
      WRITE (6,2007) (J,(TLO(I,J),I=1,5),J=1,4)                              SHEL  73

      WRITE(6,2003)                                                          SHEL  77

  100 READ (5,1001)MM,IY,EL                                                  SHEL  79

      WRITE (6,2004) NN,(IX(I),I=1,6),NM(1),P(1),T(1),DT(1)                  SHEL 150

      WRITE (1) ND,NS,(LM(I),I=1,ND),((SA(I,J),I=1,NS),J=1,ND),              SHEL 167
     1 ((SF(I,J)I=1,NS),J=1,4)                                               SHEL 168

  440 WRITE (6,2005) MM                                                      SHEL 171

  103 IF (ABS(D)-ABS(A(J))) 105,105,108                                      SHEL 405

      CALL TRFPRD (IDIS,NEN,TDIS(L1),TDIS(L2),TDIS(19),TD1,TD2,TD3)          SHEL 759

     1CALL TRFPRD (IROT,NEN,TROT(L1),TROT(L2),TROT(19),TR1,TR2,TR3)          SHEL 761

      S1 = SQRT (S1)                                                         SHEL 945
      S2 = SQRT (X2**2+Y2**2+Z2**2)                                          SHEL 946

      S = SQRT (T31**2+T32**2+T33**2)                                        SHEL 982

      T13 = T32*T(2)-T31*T(5)                                                SHEL 988
      S = SQRT (T11**2+T12**2+T13**2)                                        SHEL 989
```

```
C                                                                      SHEL1005
C   THIS SUBROUTINE GENERATES THE TRANSFORMATIONS RELATING A LOCAL    SHEL1006

    DIMENSION  P1(1), P2(1), P3(1), Q1(1), Q2(1), Q3(1)               SHEL1010

    X = SQRT(X)                                                       SHEL1260
```

CONTROL STATEMENTS FOR COMPILING CDC 6400

```
JOBCARD-NAME,BN____________,CM77000,T200.
FTN(B=PUNCHB,OPT=2)
789 (ALL IN COLUMN ONE)

                OBJECT DECK

6789(ALL IN COLUMN ONE)
```

CONTROL STATEMENTS FOR DATA INPUT CDC 6400

```
JOBCARD-NAME,BN____________,CM77000,T200.
LIBRARY (SCOPE33)
INPUT.
789 (ALL IN COLUMN ONE)

            BINARY DECK

789 (ALL IN COLUMN ONE)

            DATA DECK

6789 (ALL IN COLUMN ONE)
```

CONTROL STATEMENTS FOR THE HONEYWELL 6060 COMPUTER

```
$       SNUMB     82050
$       IDENT     8515001,CHRIS
$       USERID    SYCSA$WASP/DNM
$       OPTION    FORTRAN
$       FORTY     NLSTIN
$       LIMITS    ,30K
```

*IBM 360/370 SOURCE LISTING**

```
$       GMAP      NDECK,NLSTOU
        TTL       ELAPSED PROCESSOR TIME ROUTINE
*                 THIS SUBROUTINE DETERMINES ELAPSED PROCESSOR TIME SINCE
*                 START OF PROGRAM EXECUTION; TIME IS RETURNED AS NO. OF
*                 1/100 SECONDS IN INTEGER FORMAT.
QPLACE  BSS       1
CONST1  DEC       640
ETODF   SAVE      0,2,3,4,5,6,7
        LDXO      2,1
        STXO      QPLACE     SAVE ADDR OF CALL PARAM TIME
        MME       GELAPS     ELAPSED TIME IN Q
        DIV       CONST1     # OF 1/100 SECS IN Q
        STQ       QPLACE,*
        RETURN    ETODF
        END
$       EXECUTE
$       LIMITS    999,60K,,20K
$       FILE      11,X1R,10L
$       FILE      12,X2R,10L
$       FILE      13,X3R,10L
$       FILE      14,X4R,10L
$       FILE      17,X5R,10L
$       FILE      18,X6R,10L
$       FILE      19,X7R,10L
$       SYSOUT    03
$       DATA      01
```

DATA DECK

```
$       ENDJOB
***EOF
```

* *Substitute the main program listing on Pages C-50 -C-51 in place of the IBM listing on Pages C-2 and C-3.*

```
C  ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** *
C     SSAP2 A STATIC ANALYSIS PROGRAM FOR THREE-DIMENSIONAL STRUCTURES *
C                          REVISED MARCH 1972
C  ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** *
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,ISCRF8,
     1ISCRF9
      COMMON/INOUT/ISYSCR,ISYSLP,ISCRF1,ISCRF2,ISCRF3,ISCRF4,ISCRF7,
     1ISCRF8,ISCRF9
      COMMON /JUNK / HED(12),JUK(210)
      COMMON /ELPAR/ NPAR(14),NUMNP,MBAND,NELTYP,N1,N2,N3,N4,N5,MTOT,NEQ
      COMMON / EM / QQQ(2596)
      DIMENSION T(7)
C
C     PROGRAM CAPACITY CONTROLLED BY THE FOLLOWING TWO STATEMENTS ...
      COMMON A(11000)
      MTOT=11000
C
C
C     PROGRAM CONTROL DATA
C
    5 CALL ETODF(ITOD)
      TOD=ITOD
      T(1)=TOD*0.01
      READ(ISYSCR,100) HED,NUMNP,NELTYP,LL,NF,NDYN
      IF (NUMNP.EQ.0) STOP
      WRITE(ISYSLP,200) HED,NUMNP,NELTYP,LL
C
C     INPUT JOINT DATA--ID ARRAY ON TAPE 8
C
      N1=1
      N2=N1+6*NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+NUMNP
      N6=N5+NUMNP
      IF(N6.GT.MTOT) CALL ERROR(N6-MTOT)
      CALL INPUTJ(A(N1),A(N2),A(N3),A(N4),A(N5),NUMNP,NEQ)
C
C     FORM ELEMENT STIFFNESSES--STIFF. ON TAPE 2-STRESS MATRIX ON TAPE1
C
      CALL ETODF(ITOD)
      TOD=ITOD
      T(2)=TOD*0.01
      MBAND=0
      NUMEL=0
      REWIND ISCRF1
      REWIND ISCRF2
      DO 900 M=1,NELTYP
      READ(ISYSCR,1001) NPAR
      WRITE(ISCRF1) NPAR
      NUMEL=NUMEL+NPAR(2)
      MTYPE=NPAR(1)
      CALL ELTYPE(MTYPE)
      CALL ETODF(ITOD)
      TOD=ITOD
      T(3)=TOD*0.01
  900 CONTINUE
C
C     INPUT NODAL LOADS AND JOINT MASSES --- WRITE ON TAPE 3
C
      NEQB=(MTOT-4*LL)/(MBAND+LL+1)/2
      NBLOCK=(NEQ-1)/NEQB+1
      IF (NEQB.GT.NEQ) NEQB=NEQ
      N3=N2+NEQB*LL
      N4=N3+6*LL
      WRITE (ISYSLP,201) NEQ,MBAND,NEQB,NBLOCK
      CALL INL(A(N1),A(N2),A(N3),A(N4),NUMNP,NEQB,LL)
      CALL ETODF(ITOD)
      TOD=ITOD
      T(4)=TOD*0.01
```

```
      CALL INL(A(N1),A(N2),A(N3),A(N4),NUMNP,NEQB,LL)
      CALL ETODF(ITOD)
      TOD=ITOD
      T(4)=TOD*0.01
C
C     FORM TOTAL STIFFNESS MATRIX -- ON TAPE 4
C
      NE2B=2*NEQB
      N2=N1+NEQB*MBAND
      N3=N2+NEQB*LL
      N4=N3+4*LL
      NN2=N1+NE2B*MBAND
      NN3=NN2+NE2B*LL
      NN4=NN3+4*LL
      CALL ADDSTF(A(N1),A(NN2),A(NN3),A(NN4),NUMEL,NBLOCK,NE2B,LL,MBAND)
      CALL ETODF(ITOD)
      TOD=ITOD
      T(5)=TOD*0.01
C
C     SOLVE FOR DISPLACEMENT UNKNOWNS
C
      NSB=(MBAND+LL)*NEQB
      NSBB=NEQB*LL*(2+(MBAND-1)/NEQB)
      IF(NSBB.LT.NSB) NSBB=NSB
      N4=N3+NSBB

      CALL USOL(A(N1),A(N3),A(N4),NEQB,MBAND,LL,NBLOCK,NSB,4,3,7,2,2)
      CALL ETODF(ITOD)
      TOD=ITOD
      T(6)=TOD*0.01
C
C     PRINT DISPLACEMENT
C
      N2=N1+NUMNP*6
      N3=N2+6*LL
      CALL PRINTD(A(N1),A(N2),A(N3),NEQB,NUMNP,LL,NBLOCK,NEQ,2)
C
C     COMPUTE STRESSES
C
   30 N2=N1+4*LL
      N3=N2+NEQB*LL
      LB=(MTOT-N3)/(NEQ+12)
      NDYN=0
      CALL STRESS(A(N1),A(N2),A(N3),NEQB,LB,LL,NEQ,NBLOCK
C
      CALL ETODF(ITOD)
      TOD=ITOD
      T(7)=TOD*0.01
      DO 40 I=1,6
   40 T(I)=T(I+1)-T(I)
      T(7)=T(1)+T(2)+T(3)+T(4)+T(5)+T(6)
      WRITE (ISYSLP,203) T
C
      GO TO 5
C
  100 FORMAT(12A6/6I5)
  200 FORMAT(1H1,12A6///
     .  28H  NUMBER OF NODAL POINTS    =  ,I5//
     .  28H  NUMBER OF ELEMENT TYPES   =  ,I5//
     .  28H  NUMBER OF LOAD CASES      =  ,I5)
  201 FORMAT(34H2 TOTAL NUMBER OF EQUATIONS        =,I5,
     1       /34H  BANDWIDTH                        =,I5,
     2       /34H  NUMBER OF EQUATIONS IN A BLOCK =,I5,
     3       /34H  NUMBER OF BLOCKS                 =,I5)
  203 FORMAT ( 12H2OVERALL LOG //
     .            33H  NODAL POINT INPUT.............. ,F8.2//
     .            33H  FORM ELEMENT STIFFNESSES...... ,F8.2//
     .            33H  INPUT NODAL LOADS.............. ,F8.2//
     .            33H  FORM TOTAL STIFFNESS........... ,F8.2//
     .            33H  EQUATION SOLVING............... ,F8.2//
     .            33H  ELEMENT STRESSES............... ,F8.2///
     .            33H  TOTAL SOLUTION TIME............ ,F8.2)
C
 1001 FORMAT (14I5)
      END
```

APPENDIX D

REFERENCES

1. Burdette, E.G. and Goodpasture, D.W., *"FINAL REPORT ON FULL SCALE BRIDGE TESTING, AN EVALUATION OF BRIDGE DESIGN CRITERIA"*, a draft report submitted to the Office of Research and Planning, Tennessee Highway Department and The Department of Transportation, Federal Highway Administration, December 31, 1971.

2. Reference 1, pg. 60.

3. Reference 1, pg. 83-84.

4. Reference 1, pg. 86,89,90,91.

5. Reference 1, pg. 67,71,73.

6. Reference 1, pg. 94.

7. Reference 1, pg. 95.

8. Reference 1, pg. 63.

9. Bathe, Klaus-Jurgen, Wilson, Edward L., Peterson, Fred E., *"SAP IV - A STRUCTURAL ANALYSIS PROGRAM FOR STATIC AND DYNAMIC RESPONSE OF LINEAR SYSTEMS"*, a report to the National Science Foundation, Grant GI-36387 or Report No. EERC 73-11, College of Engineering, University of California, Berkeley, June 1973.

10. Reference 9, pg. III.3.

11. Reference 1, pg. 43.

12. N.E. 80th Pedestrian Undercrossing, King County, State of Washington, a drawing of the Washington State Highway Commission, Department of Highways; and a letter report entitled "History of Vibration Problems" available to the author from FHWA.

13. Wilson, Edward L., *"SOLID SAP - A STATIC ANALYSIS PROGRAM FOR THREE DIMENSIONAL STRUCTURES"*, Report No. UC SESM 71-19, Structural Engineering Laboratory, University of California, Berkeley, March 1972.

14. Beaufait, Fred W., Rowan, William H. Jr., Hoadley, Peter G. and Hackett, Robert M., *"COMPUTER METHODS OF STRUCTURAL ANALYSIS"*, Prentice Hall Inc., Englewood Cliffs, New Jersey, 1970, pg. 227.

15. Przemieniecki, J. S., *"THEORY OF MATRIC STRUCTURAL ANALYSIS"*, McGraw Hill, New York, 1968, pg. 74-80.

16. Felippa, Carlos A. *"REFINED FINITE ELEMENT ANALYSIS OF LINEAR AND NONLINEAR TWO-DIMENSIONAL STRUCTURES"*, PB 178 418, National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia, October, 1966. The appendices to this work are accession number PB 178 419.

17. Zienkiewicz, O. C., *"THE FINITE ELEMENT METHOD IN ENGINEERING SCIENCE"*, McGraw Hill, London, 1971, pg. 116.

18. Holand, Ivar, and Bell, Kolbein, *"FINITE ELEMENT METHODS IN STRESS ANALYSIS"*, TAPIR,Trondheim, Norway, 1970, pg. 170.

19. Desai, Chandrakant S., and Abel, John S., *"A NUMERICAL METHOD FOR ENGINEERING ANALYSIS"*, Van Nostrand Reinhold Co., New York, 1972, pg. 88.

20. Clough, Ray W., and Felippa, Carlos A, A REFINED QUADRILATERAL ELEMENT FOR ANALYSIS OF PLATE BENDING, *"PROC. 2ND CONFERENCE ON MATRIX METHODS IN STRUCTURAL MECHANICS"*, Wright-Patterson AFB, Ohio, AFFDL-TR-68-150, October 15-17, 1968.

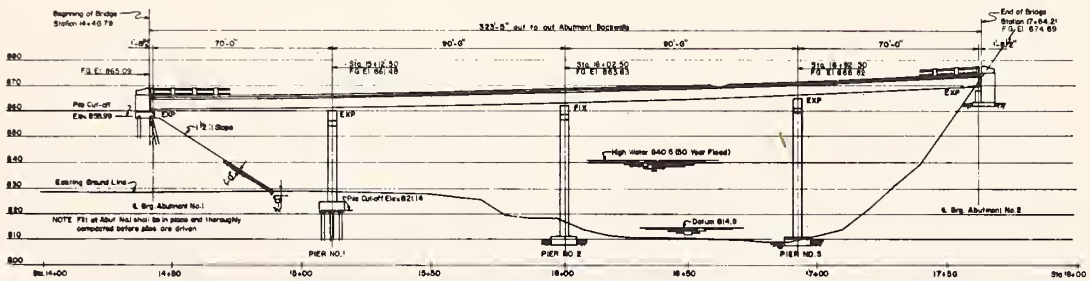
21. Martin, Harold C, and Carey, Graham F.,*"INTRODUCTION TO FINITE ELEMENT ANALYSIS--THEORY AND APPLICATIONS"*, McGraw Hill, New York, 1973, pg. 300.

APPENDIX E

Selected Construction Drawings of Bridges 1 thru 4

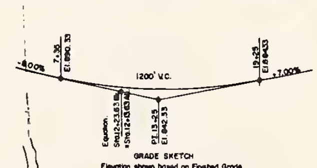
Henry W. Derthick, Engineer of Structures, Department of Highways, State of Tennessee, furnished reproducible copies of the bridge drawings which are presented in miniaturized form in this Appendix

FED. ROAD DIST. NO.	STATE	FED. AID PROJ. NO.	FISCAL YEAR	SHEET NO.	TOTAL SHEETS
3	TENN.		19		



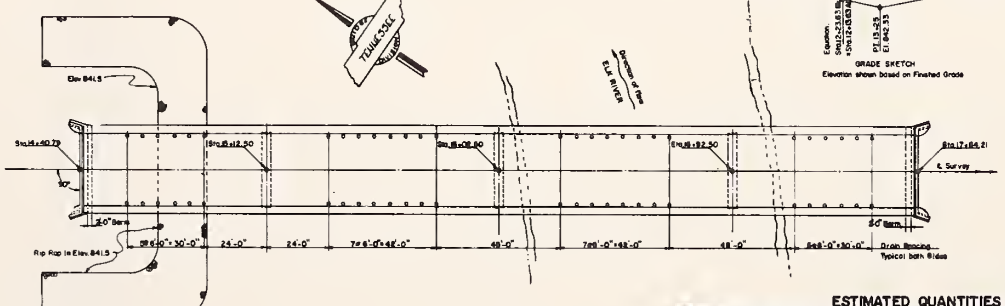
ELEVATION

FG = Finished Grade

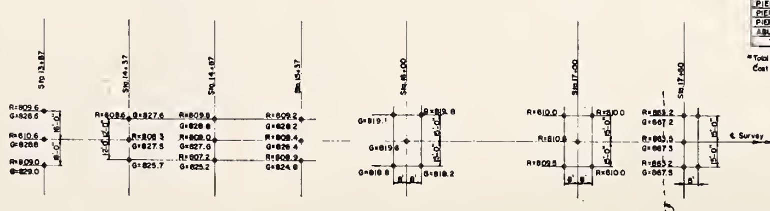


GRADE SKETCH

Elevation shown based on Finished Grade



PLAN



SOUNDING SKETCH

R = ROCK ELEVATION
G = GROUND ELEVATION

DESIGN DATA

Drainage Area = 349 Sq. Mi.
Discharge (50 Year Flood) = 31,100 CFS
Area Provided = 5290 Sq. Ft.
Mean Velocity Through Constriction = 5.9 FPS

GENERAL NOTES

DESIGN SPECIFICATIONS: 1961 Edition of AASHO Standard Specifications for Highway Bridges
SPECIFICATIONS: Standard Road & Bridge Specifications of the Tennessee Department of Highways
LOADING: H20-S16-44
CONCRETE: To be Class "A"
REINFORCING STEEL: To be intermediate or hard grade. See Specifications. Standard hook details as recommended by C.R.S.I. shall apply.
STRUCTURAL STEEL: See notes on Dwg. No. K-27-98.
PILES: To be 10" 42# Piles shall be driven to rock or a minimum bearing of 36 Tons.
HIGH TENSILE BOLTS: See AASHO Specifications Article 2.10.20.
STUD WELDING: See Special Provision.
PAINT: See Specifications.

FOUNDATION NOTE

When rock foundations at Piers 2 & 3 have been uncovered holes 6' deep shall be drilled into rock at points designated by the Engineer. Footing for Pier No. 1 shall be excavated to the pile cut-off elevation shown and rod soundings made as directed by the Engineer to determine if piles are to be driven or the footing excavated to rock. No column steel shall be ordered until final footing elevations have been determined. The cost of rod soundings shall be included in the cost of bid items.

HANDRAIL NOTE

Build Handrail according to Dwg. No. H-5-110 except use endpost shown on Dwg. No. K-27-100
Use Q=320'-11" with 38 spaces @ L=6'-11⅛"

LIST OF DRAWINGS

	NO.
LAYOUT	K-27-97
SUPERSTRUCTURE	K-27-98 & K-27-99
ABUTMENT NO. 1	K-27-100
PIERS NO. 1, 2 & 3	K-27-101
ABUTMENT NO. 2	K-27-102
PILE SPLICE & ROADWAY EXPANSION DEVICE	H-S-146
HANDRAIL	H-5-110

Required 56-3"ϕx12" Transite Drains (or equal)

ESTIMATED QUANTITIES

ITEM	EXCAVATION			CONCRETE CLASS A	STEEL - POUNDS		CONCRETE HANDRAIL	ROCK DRILLING	STEEL PILES 10 BP 42	RIP-RAP
	DRY	WET	ROCK	CU. YDS.	REINFORCING	STRUCTURAL	LIN. FT.	LIN. FT.	LIN. FT.	CU. YDS.
SUPERSTRUCTURE				267.3	68,950					
ABUTMENT NO. 1				20.7	2,871					
PIER NO. 1	148	62		71.7	9,065					
PIER NO. 2	34	68	14	85.7	11,267					
PIER NO. 3		14	14	88.9	11,705					
ABUTMENT NO. 2	166		16	40.6	3,039					
TOTALS	346	144	44	574.9	104,597	Lump Sum*	642	18	550	186

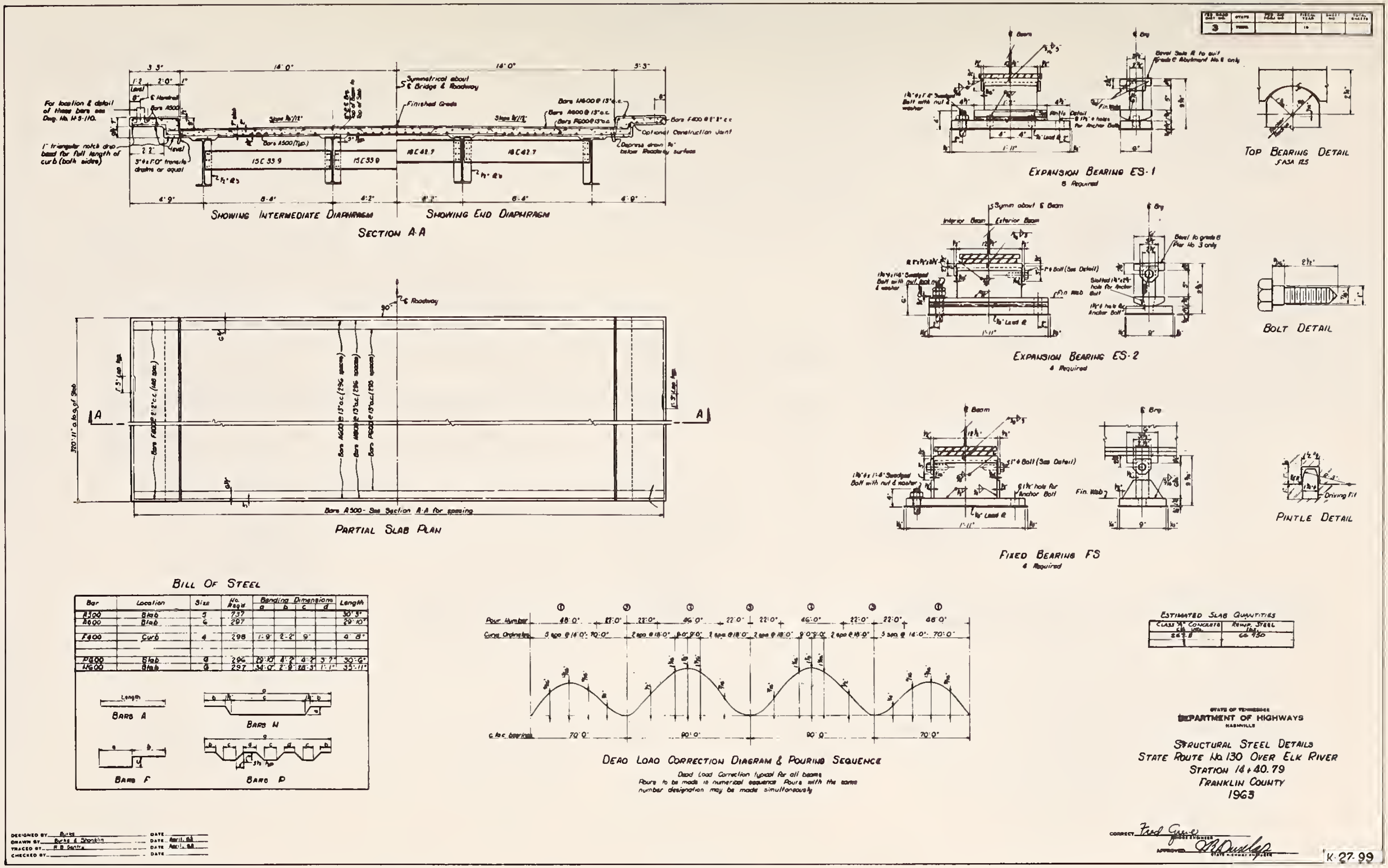
* Total estimated weight Structural Steel 266,500 lbs. including Bearing Devices, Roadway Expansion Devices & Shear connectors
Cost of 3"ϕ Transite Drains (or equal) shall be included in the Unit Price Bid for Class "A" Concrete

28'-0" ROADWAY WITH SAFETY CURBS
STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS
NASHVILLE
LAYOUT
STATE ROUTE 130 OVER ELK RIVER
STATION 14+40.79
FRANKLIN COUNTY
1963

DESIGNED BY Billy Burke & Al Bates DATE
DRAWN BY GRB & ARB DATE
TRACED BY JACK CROCKETT DATE 4/29/63
CHECKED BY DATE

CORRECT Fred Greene
APPROVED W.R. Dunlap

K-27-97

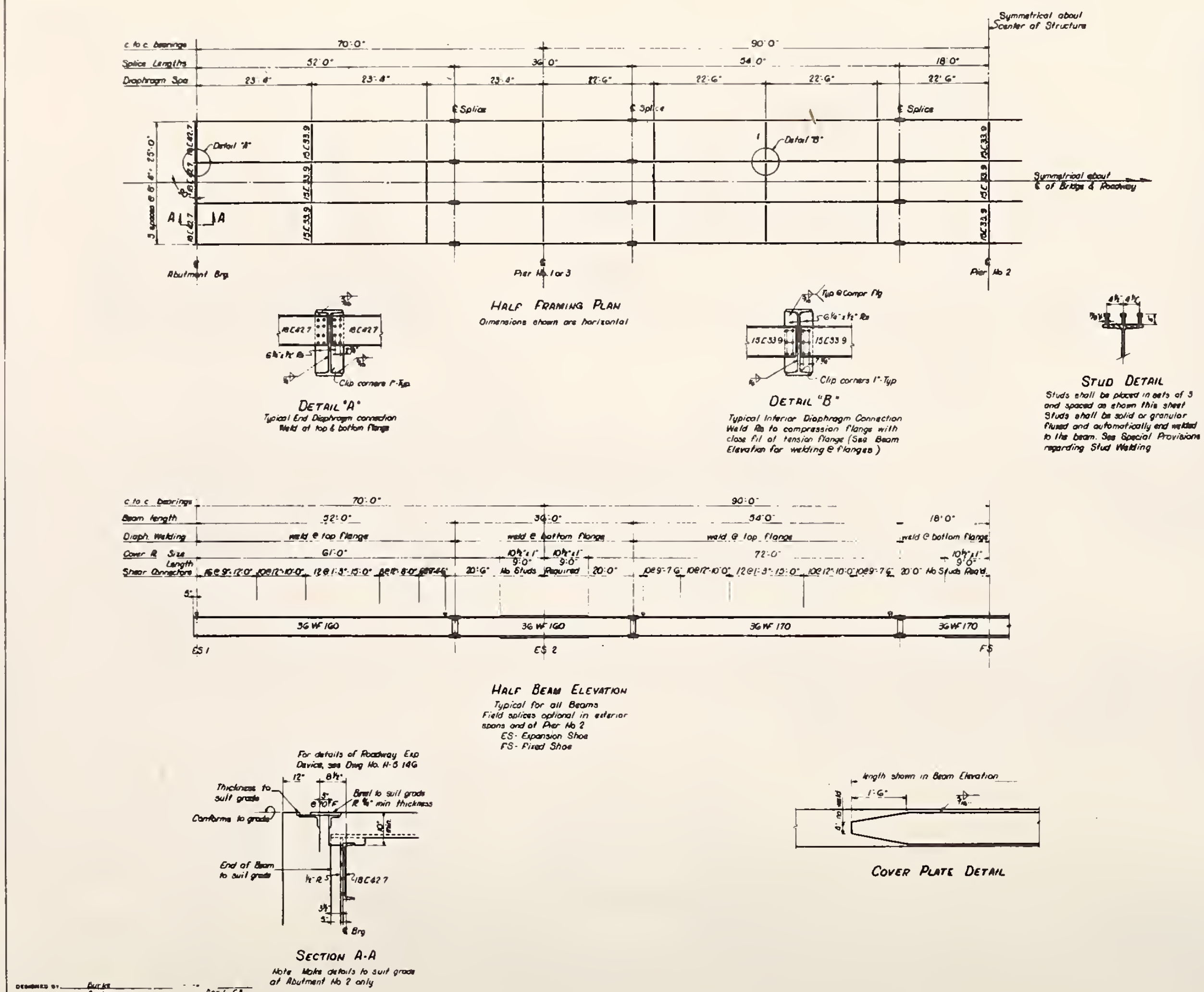


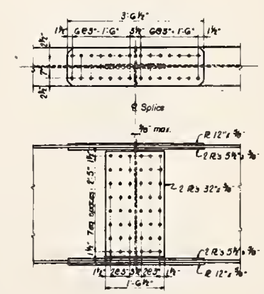
Bill Of Steel

Bar	Location	Size	No. Req'd	Bending Dimensions a	b	c	d	Length
A500	Slab	5	737					30'-5"
A600	Slab	6	297					29'-10"
F400	Curb	4	298	1'-9"	2'-2"	9"		4'-8"
P600	Slab	6	296	29'-10"	4'-2"	4'-2"	3'-7"	30'-6"
H600	Slab	6	297	34'-0"	2'-9"	28'-5"	1'-1"	35'-11"

Estimated Slab Quantities

Class "A" Concrete cu. yds.	Reinf. Steel lbs.
267.8	66,430





NOTES

Structural Steel shall conform to ASTM A36-62T.

Field Connections to be made with ⅞" high strength bolts. See AASHO Specifications, Article 2.10.20. All high strength bolted connections are the friction type.

Paint: See Specifications. No paint shall be applied to the top surfaces of the top flanges or at any point of field weld or bolt connection. Splices and other field connections shall be cleaned and primed before forming slab.

Welding to be in conformance with the current edition of the American Welding Society Specifications for Welded Highway & Railway Bridges.

Bearings: See details on Dwg. No. K-27-99.

Roadway Expansion Devices: See Dwg. No. H-5-146 and this sheet.

Camber: All beams shall be cambered to compensate for dead load deflection and vertical curve. See Dwg No K-27-99 for Dead Load Correction Curve.

Concrete Deck: The concrete deck shall not be poured until all structural steel is erected and all bolting is complete.

For General Notes, see Dwg. No. K 27-97

For other Structural Steel Details, see Dwg. No. K 27-99.

SPLICE DETAIL

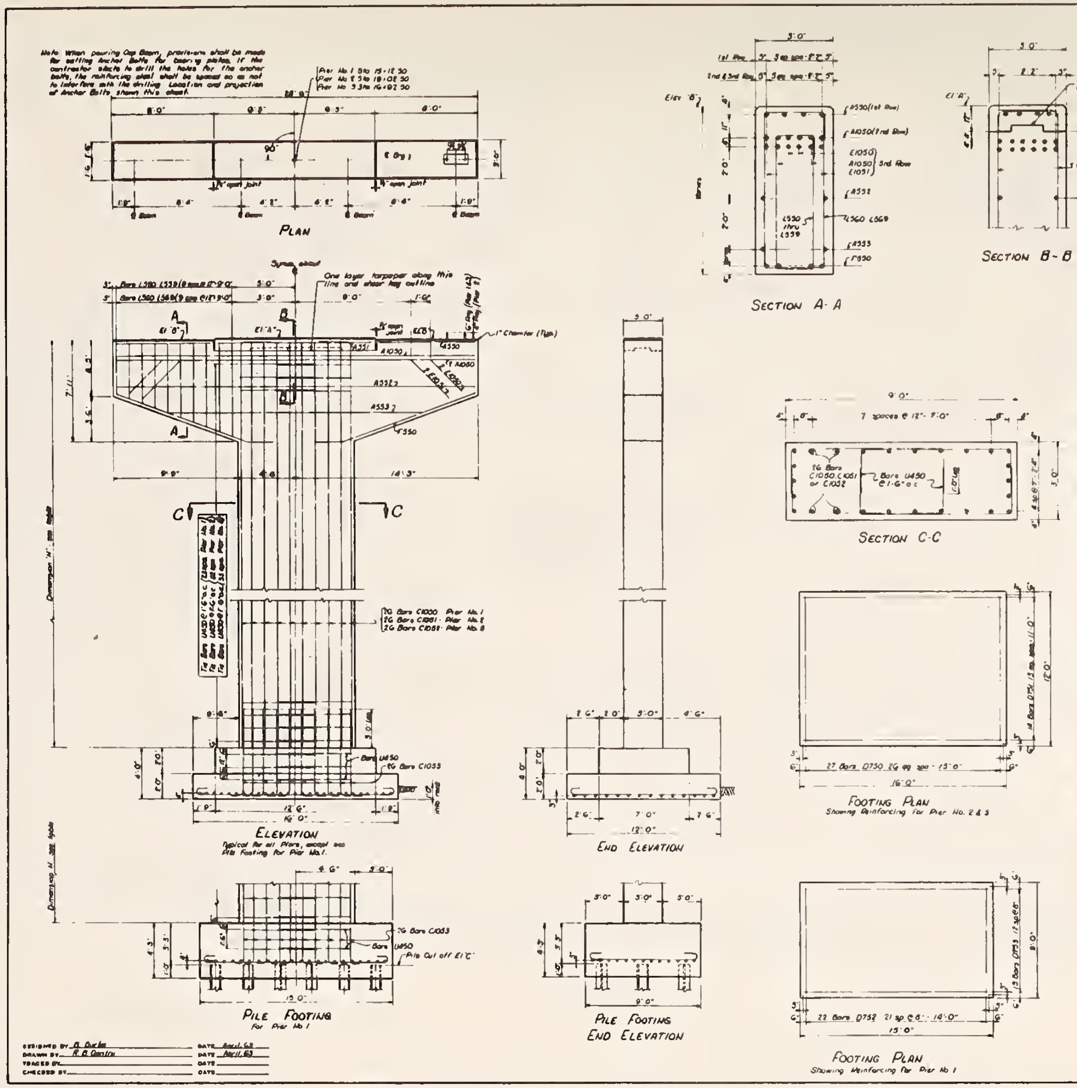
Splices to be sub-punched or sub-drilled ⅛" smaller and reamed to size and match marked while assembled in the shop
Use fill ℞s as required

STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS
NASHVILLE

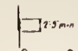
STRUCTURAL STEEL DETAILS
STATE ROUTE No. 130 OVER ELK RIVER
STATION 14+40.79
FRANKLIN COUNTY
1963

K-27-98

Note: When pouring Cap Beam, provisions shall be made for setting Anchor Bolts for bearing plates. If the contractor elects to drill the holes for the anchor bolts, the reinforcing steel shall be spaced so as not to interfere with the drilling. Location and projection of Anchor Bolts shown this sheet.



Optional Splice Detail



Note: Not more than 50% of the column steel in any one face shall be spliced at the same location. A bar shall have only one splice. The cost of additional steel required for the optional splices shall be at the contractors expense.

Bill Of Steel

Bar	Location	Size	No. Required Pier 1	No. Required Pier 2	No. Required Pier 3	Bending Dimensions a	b	c	d	Length
A550	Cap Beam	5	8	8	8					7'-6"
A551	Cap Beam	5	4	4	4					11'-0"
A552	Cap Beam	5	2	2	2					28'-0"
A553	Cap Beam	5	2	2	2					30'-0"
A1050	Cap Beam	10	8	8	8					28'-0"
C1050	Column	10	26			36'-9"	2'-3"			38'-10"
C1051	Column	10		26		31'-5"	2'-3"			33'-4"
C1052	Column	10			26	34'-8"	2'-5"			36'-7"
C1053	Col. & Footing	10	26	26	26	6'-8"	11"			7'-4"
D750	Footing	7		27	27	11'-6"				13'-2"
D751	Footing	7		14	14	15'-6"				17'-2"
D752	Footing	7	22			8'-6"				10'-2"
D753	Footing	7	13			14'-6"				16'-2"
E1050	Cap Beam	10	2	2	2	21'-0"	3'-6"			28'-0"
E1051	Cap Beam	10	2	2	2	18'-0"	4'-5"			26'-0"
F550	Cap Beam	5	8	8	8	2'-0"	11'-0"	5'-9"		13'-0"
L550	Cap Beam	5	1	1	1	1'-6"	2'-11"	12"		9'-10"
L551		5	1	1	1	1'-6"	3'-3"	12"		10'-9"
L552		5	1	1	1	1'-9"	3'-7"	12"		11'-2"
L553		5	1	1	1	1'-6"	4'-0"	12"		12'-0"
L554		5	1	1	1	1'-6"	4'-4"	12"		12'-8"
L555		5	1	1	1	1'-9"	4'-9"	12"		13'-6"
L556		5	1	1	1	1'-9"	5'-1"	12"		14'-2"
L557		5	1	1	1	1'-9"	5'-5"	12"		14'-10"
L558		5	1	1	1	1'-9"	5'-9"	12"		15'-6"
L559	Cap Beam	5	1	1	1	1'-6"	6'-2"	12"		16'-4"
L560	Cap Beam	5	1	1	1	2'-4"	3'-10"	12"		13'-4"
L561		5	1	1	1	2'-4"	4'-2"	12"		14'-0"
L562		5	1	1	1	2'-4"	4'-7"	12"		14'-10"
L563		5	1	1	1	2'-4"	4'-11"	12"		15'-6"
L564		5	1	1	1	2'-4"	5'-3"	12"		16'-2"
L565		5	1	1	1	2'-4"	5'-7"	12"		16'-10"
L566		5	1	1	1	2'-4"	6'-0"	12"		17'-8"
L567		5	1	1	1	2'-4"	6'-4"	12"		18'-4"
L568		5	1	1	1	2'-4"	6'-8"	12"		19'-0"
L569	Cap Beam	5	1	1	1	2'-4"	7'-1"	12"		19'-10"
U450	Column	4	108	144	156	5'-10"	1'-9"			9'-4"

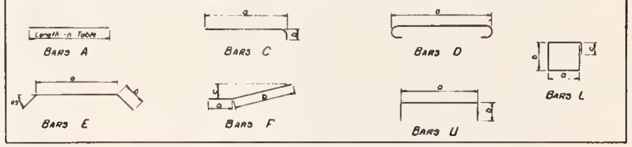
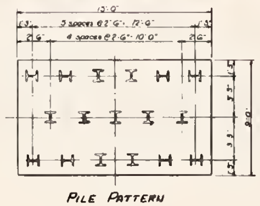


Table Of Dimensions & Elevations

Location	Dimension "H"	Elevation "A"	Elevation "B"	Elevation "C"
Pier No. 1	37'-0"	861.48	861.39	821.14
Pier No. 2	31'-6"	863.63	863.54	—
Pier No. 3	34'-8"	866.82	866.73	—

Estimated Quantities

Item	Concrete Cl. "A" Cu. Yds.	Reinf. Steel Lbs.
Pier No. 1	71.7	9,065
Pier No. 2	85.7	11,267
Pier No. 3	88.9	11,705



Pile Pattern
Typical @ Pier No. 1

State of Tennessee
DEPARTMENT OF HIGHWAYS
Nashville

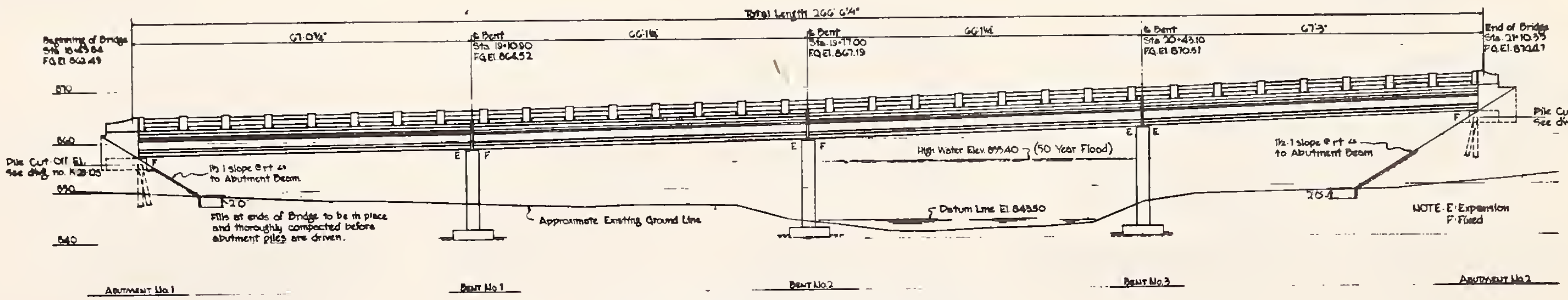
Piers No. 1, 2 & 3
State Route No. 130 Over Elk River
Station 14+40.79
Franklin County
1963

Correct: Fred Green
Approved: [signature]

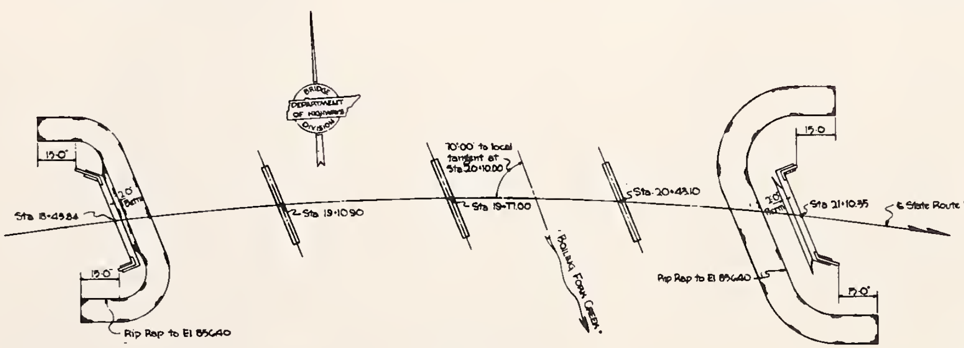
K-27-101

Designed by: A. Burke — Date: April 63
Drawn by: R. B. Gentry — Date: April 63
Traced by: — Date:
Checked by: — Date:

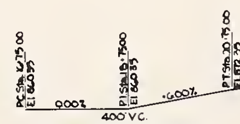
E — 5



· ELEVATION AT RIGHT ANGLES TO STATE ROUTE 130 ·



· PLAN ·



SKETCH OF GRADES
STATE ROUTE 130
NOTE: Elevations shown are based on finished grade.

· GENERAL NOTES ·

SPECIFICATION: Standard Road & Bridge Specification of the Tennessee Department of Highways.
LOADING: H20-S16-44.
CONCRETE: (Cast In Place) To be Class 'A'.
REINFORCING STEEL: To be Intermediate or Hard Grade. Standard Hook Details as recomended by CRSI shall apply.
DESIGN SPECIFICATIONS: AASHO 1961
NEOPRENE BEARING PAD: See Special Provisions.
JOINT SEALER: To be Class 'A' or 'B'. See Special Provisions.

· DESIGN DATA (USGS REPORT) ·

Drainage Area 77.1 sq. mi.
Q: 12,000 cfs Area Provided 1140 sq. ft.
V: 7.3 fps

· FOUNDATION NOTE ·

When foundations for Bents have been uncovered, holes 6 ft. deep shall be drilled into the rock at points designated by the Engineer. No reinforcing steel for columns shall be ordered until final footing elevations are established.

· PILE NOTE ·

All piles to be driven to refusal on rock or a minimum bearing of 36 tons.

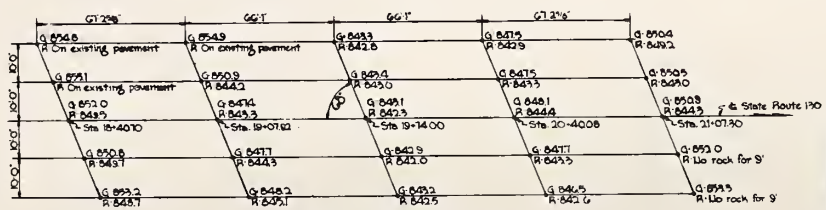
· HANDRAIL NOTE ·

Build Handrail according to dwg. no. K-15-58 except use Endpost as shown on dwg. no. K-28-125 & K-28-127
"X"-1' ℄ Lt. 263' 9⅜" "Lt" 6'-1⅞"(+) (31 Spaces)
℄ Rt. 264' 7" "Lt" 7'-0⅛"(+) (31 Spaces)

· ESTIMATED QUANTITIES ·

Item	Excavation Cu. Yds. Rock	Wet	Dry	Concrete Class 'A' Cu. Yds.	Reinf. Steel lbs.	Prestress. Beams* AASHO Type III	Concrete Handrail Lin. Ft.	Rock Drilling Lin. Ft.	Steel Piles 10" BP 42 Lin. Ft.	Rip Rap Cu. Yds.
Superstructure				244.0	55,665	16	528			
Abutment No. 1				16.9	1,969				80	56
Abutment No. 2				17.9	2,010				180	94
Bent No. 1	4	5	35	31.5	7,770					
Bent No. 2	4	18	1	35.6	8,552					
Bent No. 3	4	10	37	38.1	9,207					
Total	12	33	73	384.0	85,173	16	528	36	260	150

* Average length 66'0"
NOTE: Cost of 88-5"⌀-9" Transite Drains (or equal) to be included in the unit price bid for Class 'A' Concrete.
NOTE: Lump Sum for removal of existing bridge.



· SOUNDING SKETCH ·
NOTE: G: Ground; R: Rock
NOTE: Stations for soundings are on ℄ of original survey approximately 20'0" upstream from Bridge.

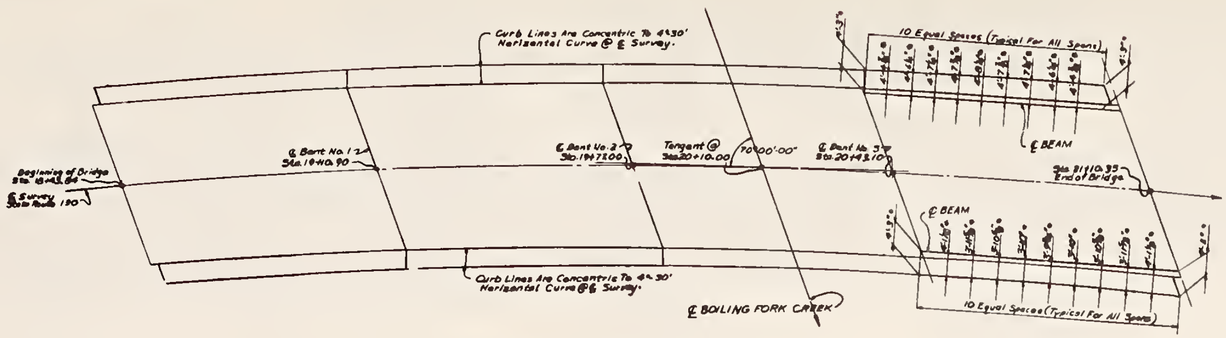
LIST OF DRAWINGS — DWG. NO.

28'0" ROADWAY WITH 2'0" SAFETY CURBS
STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS
NASHVILLE

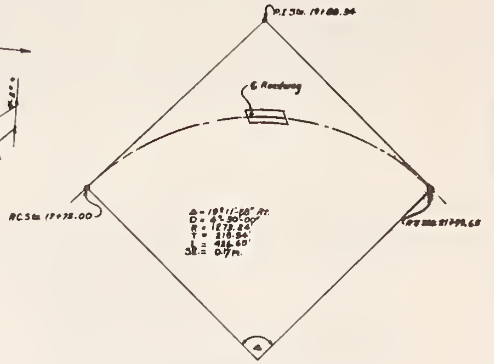
LAYOUT OF BRIDGE
STATE ROUTE 130 OVER BOILING FORK CREEK
STATION 18+43.84
FRANKLIN COUNTY
1962

CORRECT ________
APPROVED ________

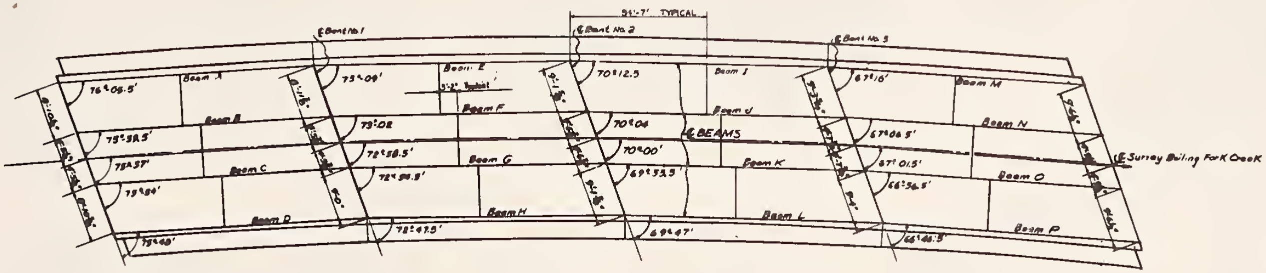
K 28-121



GEOMETRICAL PLAN
Note: Angle Shown Above is with Local Tangent
⊕ Radial Distance
Note: All Substructure is Parallel to ℄ of Boiling Fork Creek as Shown on this Sheet.



HORIZONTAL CONTROL DATA



GEOMETRICAL LAYOUT OF BEAMS
Note: Angles Shown on Layout are to Chords

TABLE OF BEAM LENGTHS

Beam	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Length	65'-11 1/8"	65'-11 3/4"	66'-0 5/8"	66'-0 1/2"	65'-11 1/4"	65'-11 7/8"	66'-0 1/8"	66'-0 3/4"	65'-10 7/8"	65'-11 5/8"	66'-0 3/8"	66'-1 1/8"	65'-10 1/2"	65'-11 1/2"	66'-0 1/2"	66'-1 1/2"

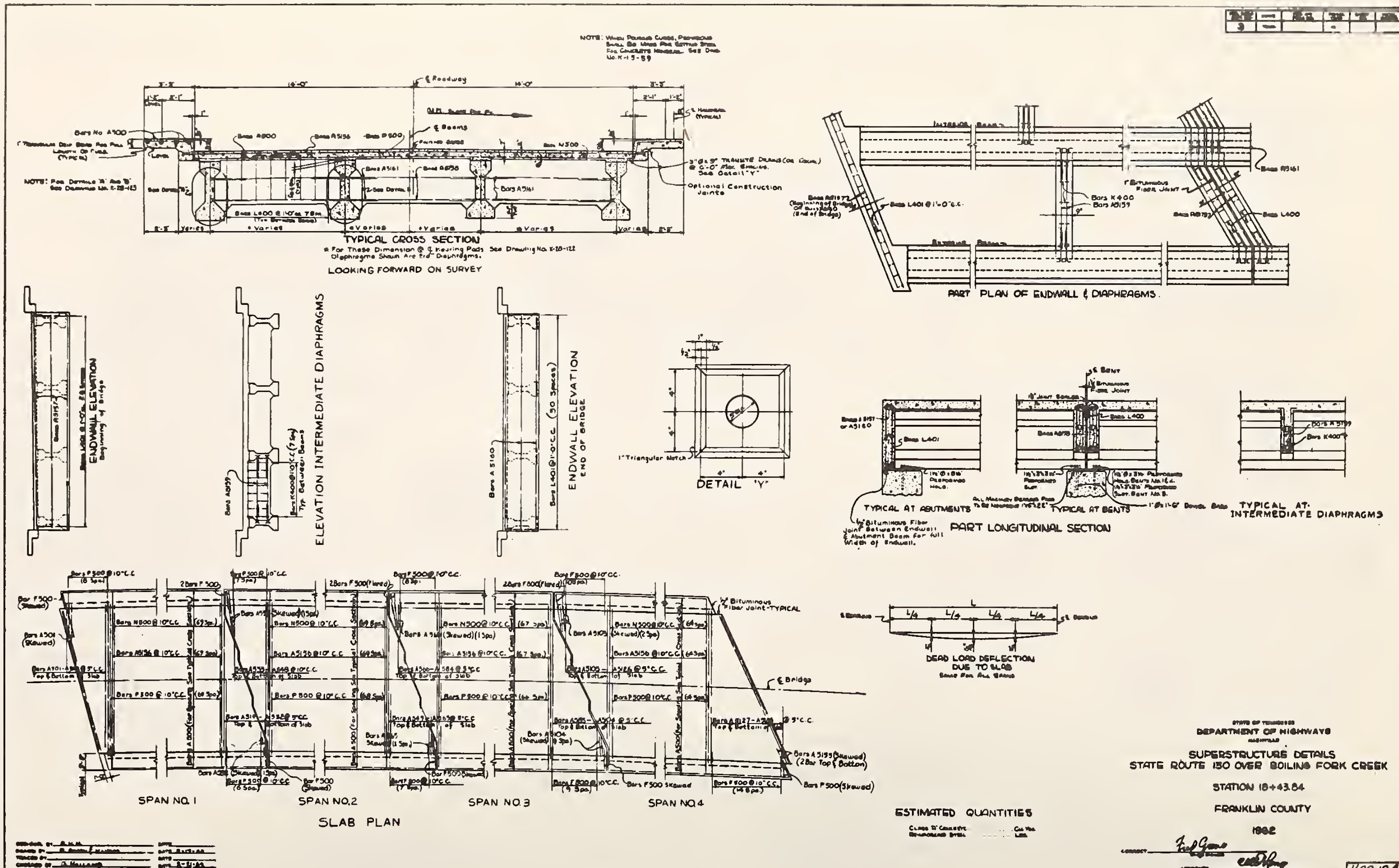
NOTES:

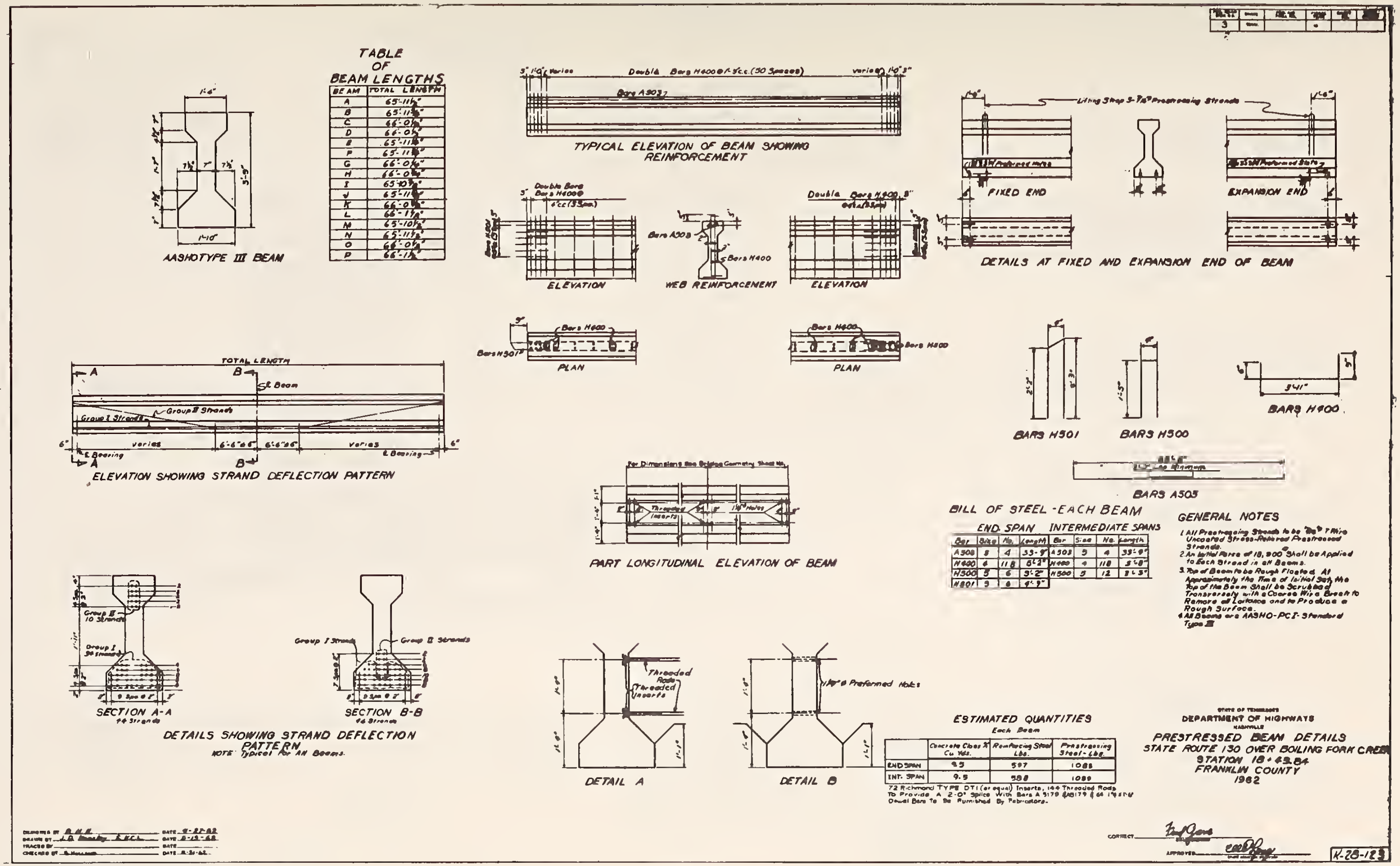
1. The Entire Bridge is on A Horizontal Circular Curve. The Curb Lines and ℄ of Roadway Are Concentric To A 4°30'-00" Horizontal Circular Curve Along ℄ Roadway.
2. The Beams are on Chords of Concentric Arcs Between Supports.
3. The Stations Shown on the Geometrical Plan are Based on the ℄ Survey of State Route 150.

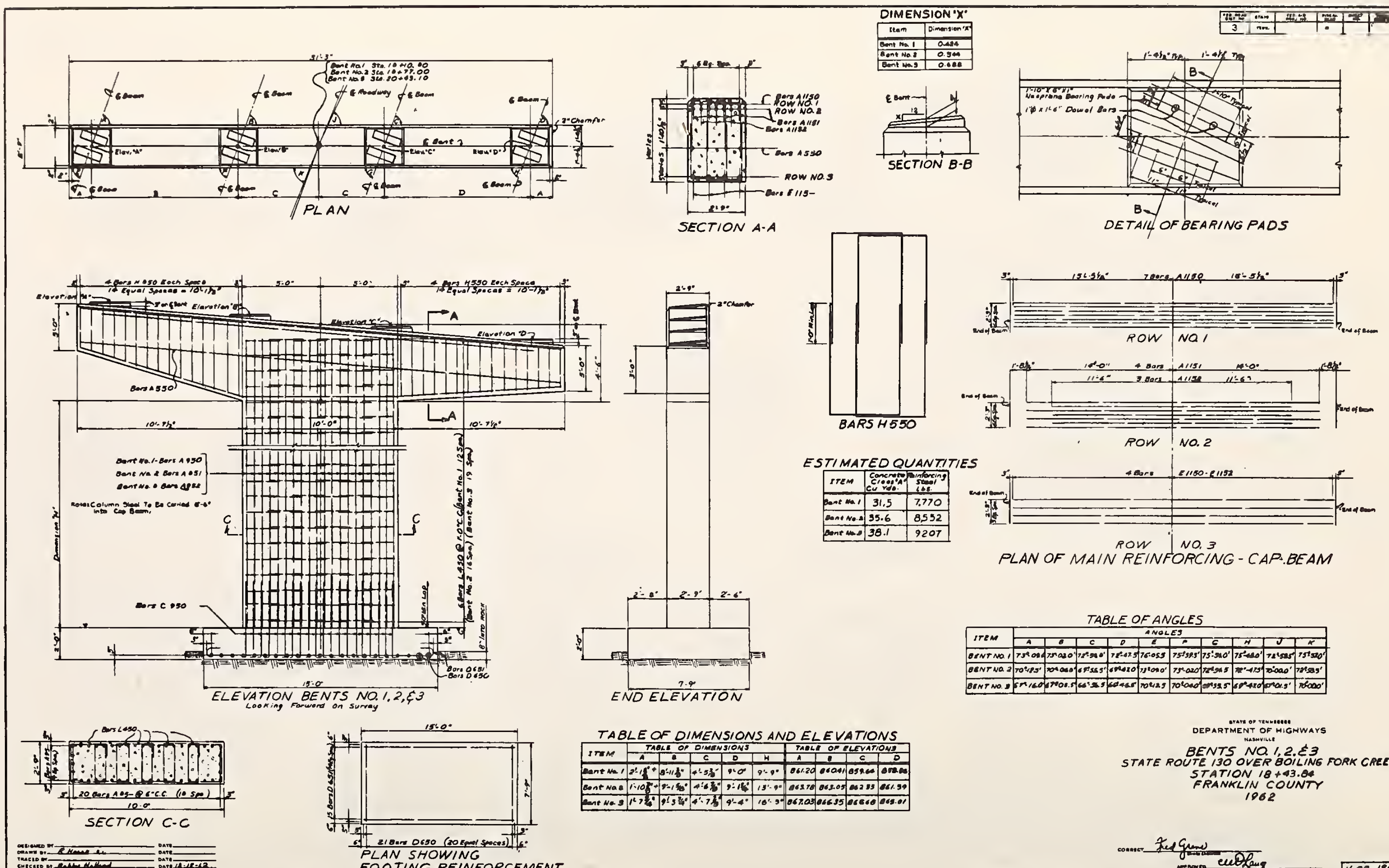
STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS
NASHVILLE

BRIDGE GEOMETRY
STATE ROUTE 130 OVER BOILING FORK CREEK
STATION 18+43.84
FRANKLIN COUNTY
1962

K-23-122


TYPICAL CROSS SECTION
LOOKING FORWARD ON SURVEY
PART PLAN OF ENDWALL & DIAPHRAGMS
ENDWALL ELEVATION
Beginning of Bridge
ELEVATION INTERMEDIATE DIAPHRAGMS
ENDWALL ELEVATION
END OF BRIDGE
DETAIL 'Y'
TYPICAL AT ABUTMENTS
TYPICAL AT BENTS
TYPICAL AT INTERMEDIATE DIAPHRAGMS
PART LONGITUDINAL SECTION
DEAD LOAD DEFLECTION DUE TO SLAB
SPAN NO. 1
SPAN NO. 2
SPAN NO. 3
SPAN NO. 4
SLAB PLAN
ESTIMATED QUANTITIES
DEPARTMENT OF HIGHWAYS
SUPERSTRUCTURE DETAILS
STATE ROUTE 130 OVER BOILING FORK CREEK
STATION 18+43.84
FRANKLIN COUNTY
1962
K-28-124


AASHOTYPE III BEAM
TABLE OF BEAM LENGTHS
BEAM | TOTAL LENGTH
A | 65'-11
B | 65'-11
C | 66'-0
D | 66'-0
E | 65'-11
F | 65'-11
G | 66'-0
H | 66'-0
I | 65'-10
J | 65'-11
K | 66'-0
L | 66'-1
M | 65'-10
N | 65'-11
O | 66'-0
P | 66'-1
TYPICAL ELEVATION OF BEAM SHOWING REINFORCEMENT
Double Bars H400 @ 1'-5" c.c. (50 Spaces)
Bars A503
ELEVATION
WEB REINFORCEMENT
Bars A503
Bars H400
PLAN
Bars H301
Lifting Strap 3-7/16" Prestressing Strands
FIXED END
EXPANSION END
DETAILS AT FIXED AND EXPANSION END OF BEAM
TOTAL LENGTH
Group II Strands
Group I Strands
varies
ELEVATION SHOWING STRAND DEFLECTION PATTERN
BARS H501
BARS H500
BARS H400
BARS A505
PART LONGITUDINAL ELEVATION OF BEAM
Threaded Inserts
BILL OF STEEL - EACH BEAM
END SPAN
Bar | Size | No. | Length
A503 | 5 | 4 | 33'-9"
H400 | 4 | 118 | 6'-2"
H500 | 5 | 6 | 3'-2"
H501 | 5 | 6 | 4'-9"
INTERMEDIATE SPANS
Bar | Size | No. | Length
A503 | 5 | 4 | 33'-9"
H400 | 4 | 118 | 5'-8"
H500 | 5 | 12 | 3'-3"
GENERAL NOTES
1. All Prestressing Strands to be 7 Wire Uncoated Stress-Relieved Prestressed Strands.
2. An Initial Force of 18,900# Shall be Applied to Each Strand in all Beams.
3. Top of Beam to be Rough Floated. At Approximately the Time of Initial Set, the Top of the Beam Shall be Scrubbed Transversely with a Coarse Wire Brush to Remove all Laitance and to Produce a Rough Surface.
4. All Beams are AASHO-PCI-Standard Type III
SECTION A-A
94 Strands
Group II 10 Strands
Group I
SECTION B-B
94 Strands
DETAILS SHOWING STRAND DEFLECTION PATTERN
NOTE: Typical for All Beams.
DETAIL A
Threaded Rods
Threaded Inserts
DETAIL B
Preformed Holes
ESTIMATED QUANTITIES
Each Beam
 | Concrete Class X Cu. Yds. | Reinforcing Steel Lbs. | Prestressing Steel - Lbs.
END SPAN | 9.5 | 597 | 1088
INT. SPAN | 9.5 | 588 | 1089
72 Richmond TYPE DTI (or equal) Inserts, 144 Threaded Rods To Provide A 2-0" Splice With Bars A 5179 Dowel Bars To Be Furnished By Fabricators.
STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS
NASHVILLE
PRESTRESSED BEAM DETAILS
STATE ROUTE 130 OVER BOILING FORK CREEK
STATION 18+43.84
FRANKLIN COUNTY
1962
CORRECT
APPROVED
K-28-123



DIMENSION 'X'

Item	Dimension 'X'
Bent No. 1	0.424
Bent No. 2	0.544
Bent No. 3	0.688

ESTIMATED QUANTITIES

ITEM	Concrete Class 'A' Cu. Yds.	Reinforcing Steel Lbs.
Bent No. 1	31.5	7,770
Bent No. 2	35.6	8,552
Bent No. 3	38.1	9207

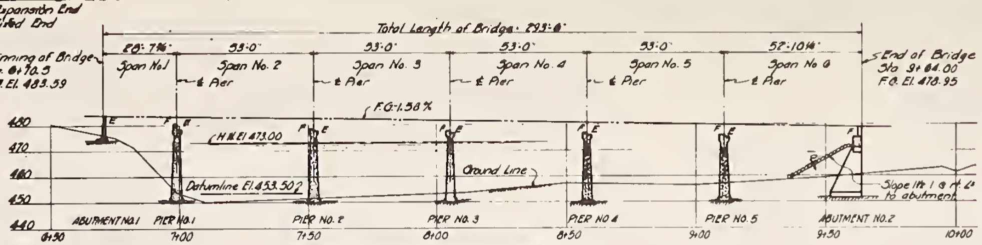
TABLE OF ANGLES

ITEM	ANGLES A	B	C	D	E	F	G	H	J	K
BENT NO. 1	73°09.6'	75°02.0'	72°58.0'	72°47.5'	76°05.5'	75°39.5'	75°51.0'	75°48.0'	72°58.5'	75°52.0'
BENT NO. 2	70°48.5'	70°06.0'	69°55.5'	69°41.0'	73°09.0'	73°02.0'	72°54.5	72°47.5'	70°00.0'	72°58.5'
BENT NO. 3	67°16.0'	67°08.5'	[illegible]	[illegible]	70°42.5	70°04.0'	[illegible]	69°48.0'	67°01.5'	70°00.0'

TABLE OF DIMENSIONS AND ELEVATIONS

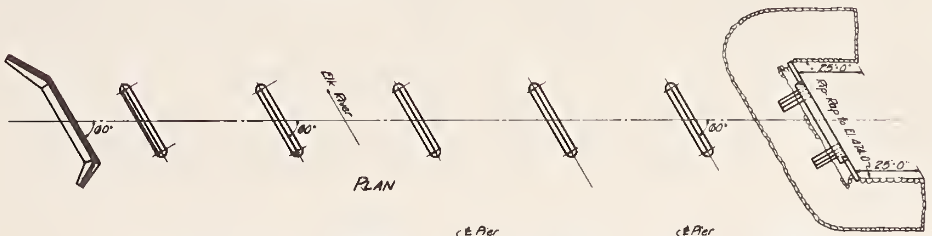
ITEM	TABLE OF DIMENSIONS A	B	C	D	H	TABLE OF ELEVATIONS A	B	C	D
Bent No. 1	2'-1[illegible]"	8'-11[illegible]"	4'-5[illegible]"	9'-0"	9'-9"	861.20	860.41	859.66	[illegible]
Bent No. 2	1'-10[illegible]"	9'-1[illegible]"	4'-6[illegible]"	9'-1[illegible]"	13'-9"	863.76	863.05	862.35	861.59
Bent No. 3	1'-7[illegible]"	9'-5[illegible]"	4'-7[illegible]"	9'-4"	16'-9"	867.03	866.35	866.68	865.01

E - Expansion End
F - Fixed End



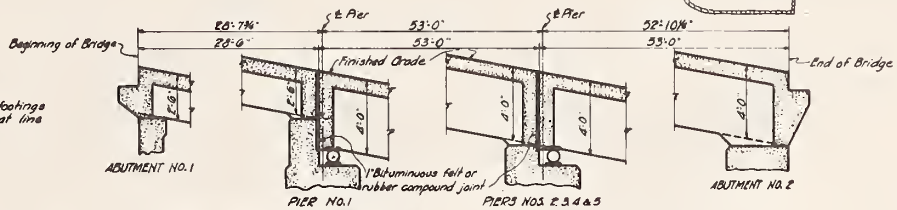
SECTION ALONG ℄ OF BRIDGE & ROADWAY
Scale 1" = 20'-0"

NOTE: All Pier footings shall be carried 1'-0" into rock.

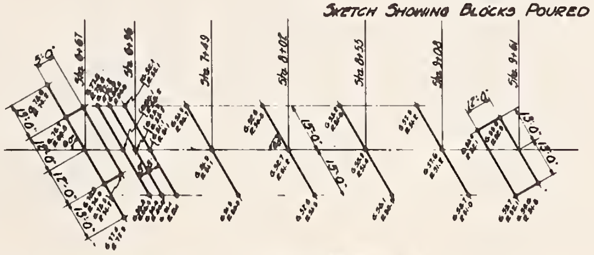


PLAN

SPECIAL NOTE:
Rock Excavation for all footings shall be made to the neat line of the footings.



SKETCH SHOWING BLOCKS POURED WITH BEAM STEMS TO OBTAIN LEVEL BEARINGS



SKETCH OF ROCK SOUNDINGS
G - Ground.
R - Solid Rock.
After the foundation has been uncovered, holes shall be drilled into the rock 6'-0" deep at points designated by the Engineer.

DESIGN DATA

Drainage Area - 108320 Ac.
Using a factor of C = 0.5 in Talbot's Formula gives a req'd area of 4155 Sq. Ft.
Area provided 4200 Sq. Ft.

GENERAL NOTES

Specifications: Standard Road and Bridge Specifications of the Tennessee Department of Highways and Public Works.
Concrete in Handrail shall be Class "S"; in Piers 1, 2 and 3 to be Class "B"; Concrete in remainder of Structure shall be Class "A".
Reinforcing Steel: See Specifications.
Forms and Finish:
Rock Foundation:
Rip-Rap: See Specifications.
Phosphor Bronze A.S.T.M. B 100-35T Class B Material rolled and annealed.

LIST OF DRAWINGS

For Details of Handrail see dwg. — B-2-10
For Details of Span No. 1 see dwg. — B-6-70
For Skewed Slab Plan see dwg. — B-2-41
For Details of Spans No. 2 - No. 6 see dwg. — A-14-60
For Skewed Slab Plan see dwg. — B-10-27
For Details of Abutment No. 1 see dwg. — B-14-19
For Details of Piers No. 1-5 see dwg. — B-14-20
For Details of Abutment No. 2 see dwg. — B-14-21

24'-0" ROADWAY

ESTIMATED QUANTITIES

Item	Excavation Cu. Yds.			Concrete Cu. Yds.			Steel Lbs.			Phosphor Bronze Plates Lbs.	Rip Rap Cu. Yds.	Rock Drilling Lin. Ft.
	Dry	Wet	Rock	Class "S"	Class "A"	Class "B"	Reinforc.	Structural	Forgings			
Superstructure	—	—	—	40.0	—	—	4407	—	—	—		
Deck Girders	—	—	—	—	471.3	—	111532	3405	5150	170		
Abutment No. 1	75	—	10	—	23.7	—	1092	—	—	—		
Pier No. 1	20	0	35	—	—	114.2	668	—	—	—		
Pier No. 2	0	15	12	—	—	109.3	323	—	—	—		
Pier No. 3	0	15	15	—	—	104.0	323	—	—	—		
Pier No. 4	25	20	15	—	80.4	—	2072	—	—	—		
Pier No. 5	45	30	12	—	59.8	—	2583	—	—	—		
Abutment No. 2	80	25	10	—	48.2	—	4993	—	—	—	190	
Totals	215	105	109	40.0	683.4	327.5	129193	3405	5150	170	190	120

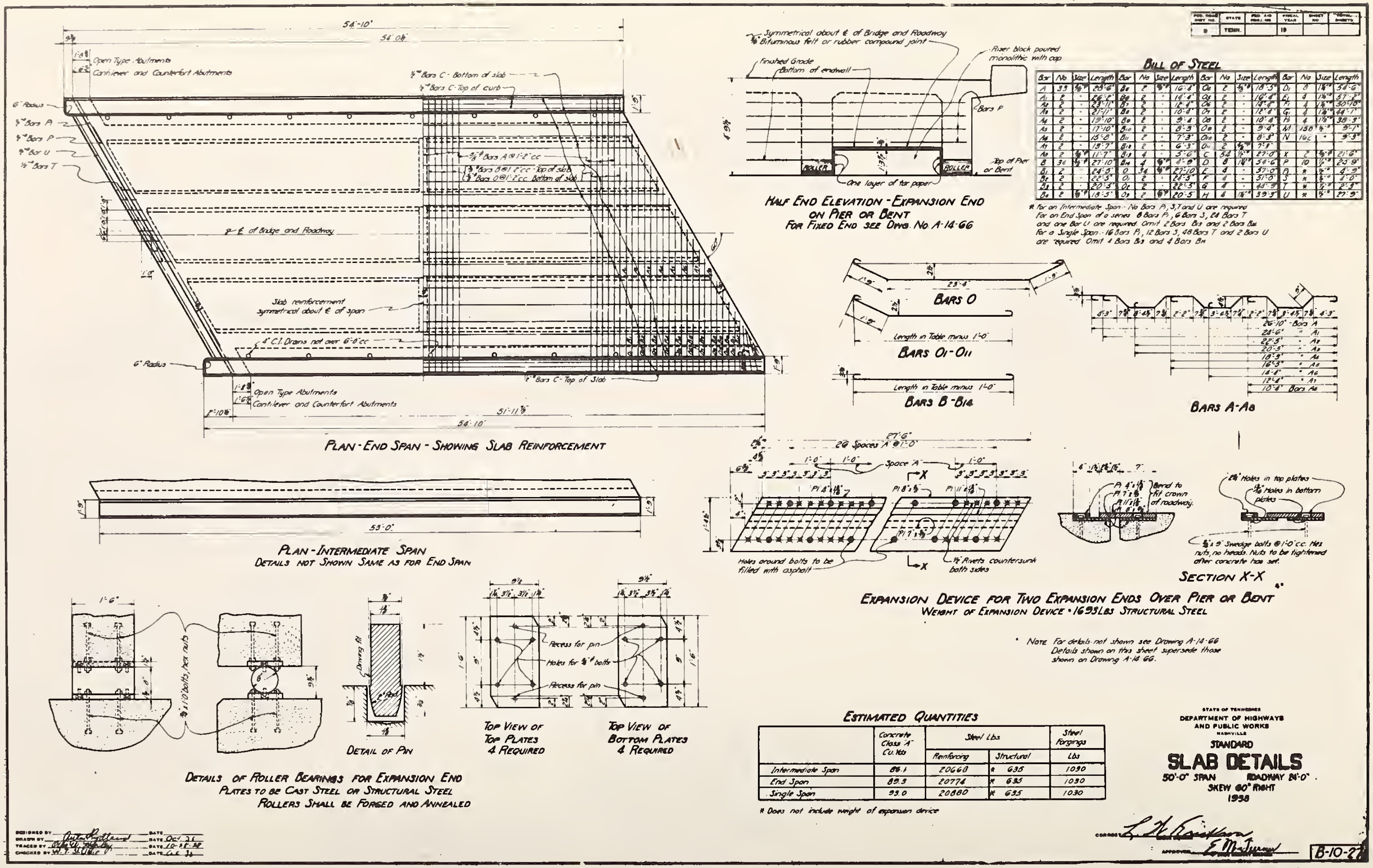
Fed. Road Dist. No.	State	Fed. Aid Proj. No.	Fiscal Year	Sheet No.	Total Sheets
8	TENN.	[illegible]	[illegible]	8	30

STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS AND PUBLIC WORKS
NASHVILLE

LAYOUT OF BRIDGE

ELK RIVER
NEAR ESTILL SPRINGS
FRANKLIN CO.
1936

Designed by — Date
Drawn by — Date
Traced by — Date Sept. 37
Checked by — Date



Bill of Steel

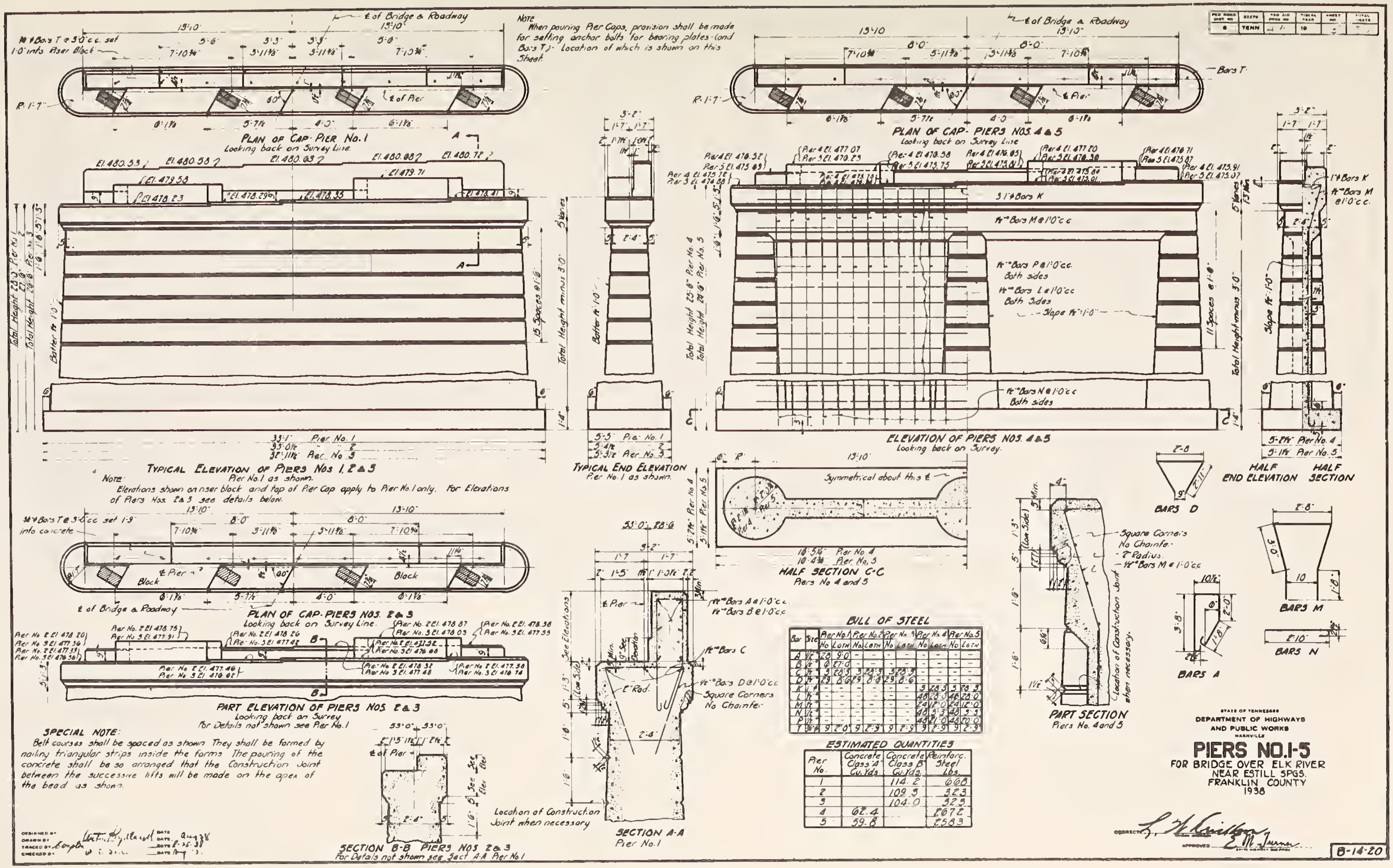
Bar	No	Size	Length	Bar	No	Size	Length	Bar	No	Size	Length	Bar	No	Size	Length
A	33	⅝"	28'-6"	B5	2	⅝"	16'-4"	O4	2	⅝"	18'-3"	D1	8	1¼"	54'-6"
A1	2	"	26'-2"	B6	2	"	14'-4"	O5	2	"	16'-4"	E1	4	1¼"	57'-2"
A2	2	"	23'-11"	B7	2	"	12'-4"	O6	2	"	14'-4"	F1	4	1¼"	50'-10"
A3	2	"	21'-11"	B8	2	"	10'-6"	O7	2	"	12'-6"	G1	4	1¼"	44'-1"
A4	2	"	19'-10"	B9	2	"	9'-4"	O8	2	"	10'-4"	H1	4	1¼"	38'-9"
A5	2	"	17'-10"	B10	2	"	8'-5"	O9	2	"	9'-4"	M	150	½"	5'-1"
A6	2	"	15'-8"	B11	2	"	7'-3"	O10	2	"	8'-3"	N	162	"	9'-3"
A7	2	"	13'-7"	B12	2	"	6'-3"	O11	2	⅝"	7'-3"				
A8	2	⅝"	11'-7"	B13	4	"	3'-6"	C	54	½"	27'-0"	X	2	⅝"	21'-6"
B	34	⅝"	27'-10"	B14	4	⅝"	4'-9"	D	8	1¼"	54'-6"	P	10	½"	23'-9"
B1	2	"	24'-5"	O	34	⅝"	27'-10"	E	4	"	57'-0"	P1	*	½"	4'-9"
B2	2	"	22'-5"	O1	2	"	24'-5"	F	4	"	51'-0"	S	*	½"	4'-0"
B3	2	"	20'-5"	O2	2	"	22'-5"	G	4	"	45'-9"	T	*	½"	2'-9"
B4	2	⅝"	18'-5"	O3	2	⅝"	20'-5"	H	4	1¼"	39'-5"	U	*	½"	27'-9"

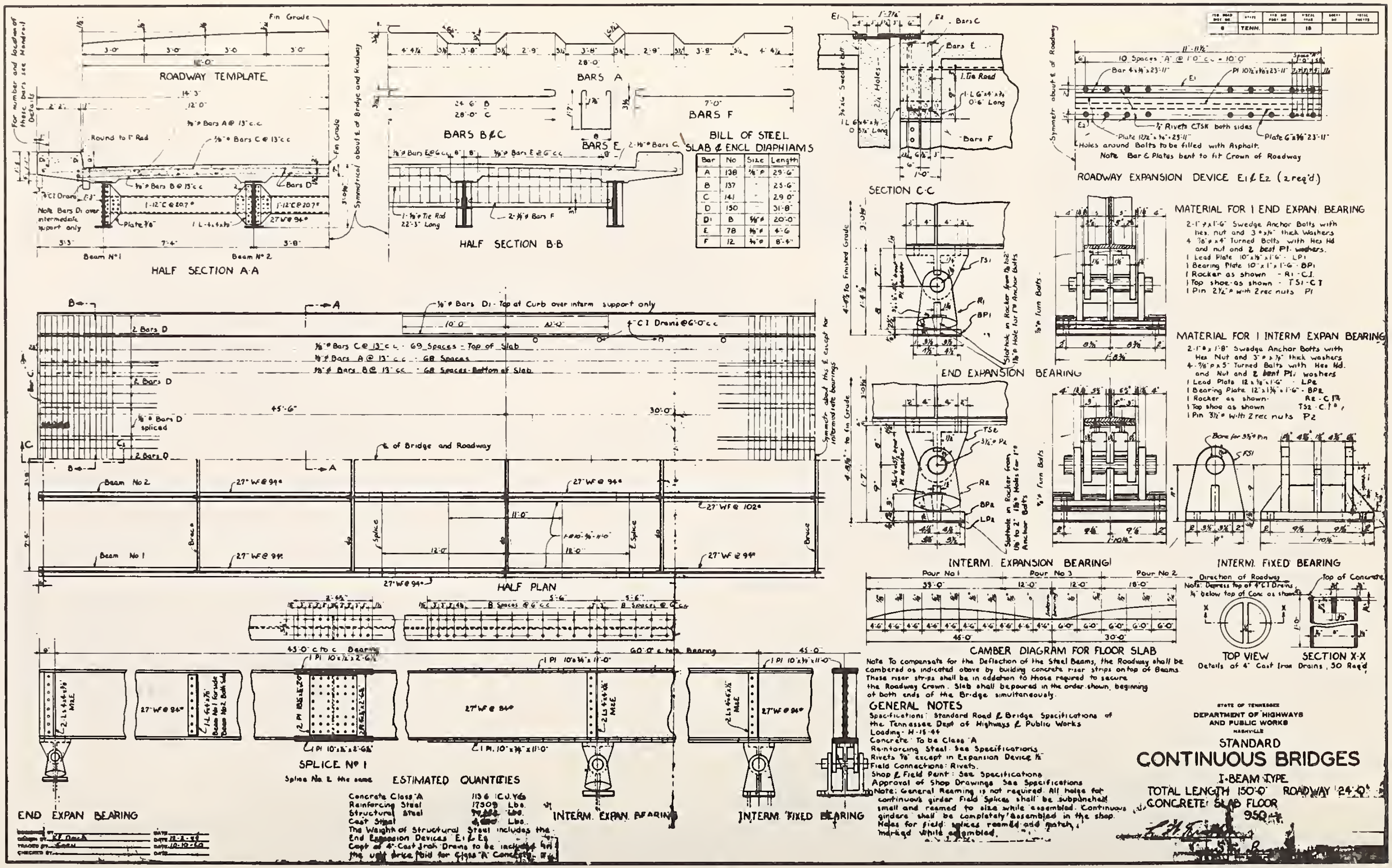
* For an Intermediate Span:- No Bars P1, S, T and U are required
For an End Span of a series 6 Bars P1, 6 Bars S, 24 Bars T and one Bar U are required. Omit 2 Bars B13 and 2 Bars B14
For a Single Span:- 16 Bars P1, 12 Bars S, 48 Bars T and 2 Bars U are required. Omit 4 Bars B13 and 4 Bars B14

Estimated Quantities

	Concrete Class "A" Cu. Yds	Steel Lbs		Steel Forgings
		Reinforcing	Structural	Lbs
Intermediate Span	86.1	20668	* 635	1030
End Span	89.3	20774	* 635	1030
Single Span	93.0	20880	* 635	1030

* Does not include weight of expansion device


STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS AND PUBLIC WORKS
NASHVILLE
PIERS NO.1-5
FOR BRIDGE OVER ELK RIVER
NEAR ESTILL SPGS.
FRANKLIN COUNTY
1938
B-14-20
PLAN OF CAP-PIER No.1
Looking back on Survey Line
TYPICAL ELEVATION OF PIERS Nos 1, 2 & 3
Pier No.1 as shown.
Note: Elevations shown on riser block and top of Pier Cap apply to Pier No.1 only. For Elevations of Piers Nos. 2 & 3 see details below.
TYPICAL END ELEVATION
Pier No.1 as shown.
Note: When pouring Pier Caps, provision shall be made for setting anchor bolts for bearing plates (and Bars T). Location of which is shown on this Sheet.
PLAN OF CAP-PIERS NOS. 4 & 5
Looking back on Survey Line
ELEVATION OF PIERS NOS. 4 & 5
Looking back on Survey.
HALF END ELEVATION
HALF SECTION
PLAN OF CAP-PIERS NOS. 2 & 3
Looking back on Survey Line.
PART ELEVATION OF PIERS NOS 2 & 3
Looking back on Survey
For Details not shown see Pier No.1
SPECIAL NOTE:
Belt courses shall be spaced as shown. They shall be formed by nailing triangular strips inside the forms. The pouring of the concrete shall be so arranged that the Construction Joint between the successive lifts will be made on the apex of the bead as shown.
SECTION B-B PIERS NOS 2 & 3
For Details not shown see Sect A-A Pier No.1
SECTION A-A
Pier No.1
Location of Construction Joint when necessary
HALF SECTION C-C
Piers No 4 and 5
Symmetrical about this ℄
BILL OF STEEL
ESTIMATED QUANTITIES
Pier No. | Concrete Class "A" Cu. Yds. | Concrete Class "B" Cu. Yds. | Reinforc. Steel Lbs.
1 | | 114.2 | 668
2 | | 109.3 | 323
3 | | 104.0 | 323
4 | 62.4 | | 2672
5 | 59.8 | | 2583
PART SECTION
Piers No. 4 and 5
BARS D
BARS M
BARS N
BARS A


ROADWAY TEMPLATE
HALF SECTION A-A
BARS A
BARS B & C
BARS E
BARS F
HALF SECTION B-B
BILL OF STEEL
SLAB & ENCL DIAPHIAMS
SECTION C-C
ROADWAY EXPANSION DEVICE E1 & E2 (2 req'd)
MATERIAL FOR 1 END EXPAN BEARING
MATERIAL FOR 1 INTERM EXPAN BEARING
END EXPANSION BEARING
INTERM. EXPANSION BEARING
INTERM. FIXED BEARING
HALF PLAN
CAMBER DIAGRAM FOR FLOOR SLAB
TOP VIEW
SECTION X-X
Details of 4" Cast Iron Drains, 30 Req'd
SPLICE No 1
ESTIMATED QUANTITIES
END EXPAN BEARING
INTERM. EXPAN. BEARING
INTERM. FIXED BEARING
GENERAL NOTES
STATE OF TENNESSEE
DEPARTMENT OF HIGHWAYS AND PUBLIC WORKS
NASHVILLE
STANDARD
CONTINUOUS BRIDGES
I-BEAM TYPE
TOTAL LENGTH 150'-0" ROADWAY 24'-0"
CONCRETE SLAB FLOOR

E-14

APPENDIX F

A Refined Quadrilateral Element for Analysis of Plate Bending

1. INTRODUCTION

Several texts have referred to and restated Dr. Felippa's paper concerning the stiffness of triangular elements in bending (18,19,21). The author prefers to excerpt the original version verbatim in this appendix for reference to a discussion concerning the tie-in of program statements to the theory, since it fits so well with the previous discussion on the SLST subroutine. Only the formula numbers have been changed in keeping with the format of this report. The discussion is in Appendix A.

2. EXCERPT

Displacement Interpolation Functions

The nodal displacement degrees of freedom which are to be considered in the stiffness matrix of the complete element also are shown in Fig. A-27a . These include the transverse displacements of each corner, w_i, the rotations at each corner about the x and y axes, θ_{xi} and θ_{yi}, as well as the rotation at the three mid-side nodes about an axis parallel to the side, θ_4, θ_5 and θ_6. In order to develop displacement patterns for the complete triangle, displacement interpolation functions were assumed independently for each subtriangle.

The displacement interpolation functions for each subelement express the relationship between the displacements $w^{(i)}$ within the element and the ten displacement components of its nodal points $r^{(i)}$, as follows:

$$w^{(i)} = \phi^{(i)} r^{(i)} \qquad \text{(F-1)}$$

As may be noted in Fig. A-27b.. The nodal displacement vector for subelement 1 is:

$$r^{(1)^T} = \left[w_2 \; \theta_{x2} \; \theta_{y2} \; w_3 \; \theta_{x3} \; \theta_{y3} \; w_0 \; \theta_{x0} \; \theta_{y0} \; \theta_5 \right] \qquad \text{(F-2)}$$

The set of 10 cubic interpolation functions for Subelement 1 may be expressed in triangular coordinates as follows:

$$
\phi^{(1)^T} = \begin{bmatrix}
\zeta_1^2(3-2\zeta_1) + 6\mu_3^{(1)}\zeta_1\zeta_2\zeta_3 \\
\zeta_1^2(b_3^{(1)}\zeta_2 - b_2^{(1)}\zeta_3) + (b_3^{(1)}\mu_3^{(1)} - b_1^{(1)})\zeta_1\zeta_2\zeta_3 \\
\zeta_1^2(a_3^{(1)}\zeta_2 - a_2^{(1)}\zeta_3) + (a_3^{(1)}\mu_3^{(1)} - a_1^{(1)})\zeta_1\zeta_2\zeta_3 \\
\zeta_2^2(3-2\zeta_2) + 6\lambda_3^{(1)}\zeta_1\zeta_2\zeta_3 \\
\zeta_2^2(b_1^{(1)}\zeta_3 - b_3^{(1)}\zeta_1) + (b_2^{(1)} - b_3^{(1)}\lambda_3^{(1)})\zeta_1\zeta_2\zeta_3 \\
\zeta_2^2(a_1^{(1)}\zeta_3 - a_3^{(1)}\zeta_1) + (a_2^{(1)} - a_3^{(1)}\lambda_3^{(1)})\zeta_1\zeta_2\zeta_3 \\
\zeta_3^2(3-2\zeta_3) \\
\zeta_3^2(b_2^{(1)}\zeta_1 - b_1^{(1)}\zeta_2) \\
\zeta_3^2(a_2^{(1)}\zeta_1 - a_1^{(1)}\zeta_2) \\
4H_3^{(1)}\zeta_1\zeta_2\zeta_3
\end{bmatrix} \tag{F-3}
$$

where the subscripts correspond to the renumbered nodes of the subelement. With this convention, the interpolation functions for subelements 2 and 3 are the same as Eq. F-3, except with appropriately changed superscripts. It should be noted, however, that the nodal displacements in Eq. F-2 are identified by node numbers defined for the complete element assembly.

Now if the vector $\bar{r}$ of all nodal displacements of the complete element assembly are written in the sequence

$$
\begin{aligned}
\bar{r}^T &= \left[\; w_1\ \theta_{x1}\ \theta_{y1}\ w_2\ \theta_{x2}\ \theta_{y2}\ w_3\ \theta_{x3}\ \theta_{y3}\ \theta_4\ \theta_5\ \theta_6 \;\vdots\; w_0\ \theta_{x0}\ \theta_{y0} \;\right] \\
&= \left[\, r^T \;\vdots\; r_0^T \,\right]
\end{aligned} \tag{F-4}
$$

the displacements in subelement 1 can be expressed

$$
w^{(1)} = \bar{\phi}^{(1)}\bar{r} = \left[\, \phi_e^{(1)} \;\vdots\; \phi_0^{(1)} \,\right] \begin{bmatrix} r \\ \hline r_0 \end{bmatrix} \tag{F-5}
$$

where $\bar{\phi}^{(1)}$ is similar to Eq. F-3, but expanded with 5 zeros to account for the nodal displacements not associated with Element 1, and with appropriate rearrangement of terms. The submatrices $\phi_e^{(1)}$ and $\phi_0^{(1)}$ represent the interpolation functions for the external and internal nodal displacements respectively.

Expressing the displacements in the other subelements similarly, the complete system of displacements can be written:

$$\begin{bmatrix} w^{(1)} \\ w^{(2)} \\ w^{(3)} \end{bmatrix} = \left[\begin{array}{c|c} \phi_e^{(1)} & \phi_0^{(1)} \\ \phi_e^{(2)} & \phi_0^{(2)} \\ \phi_e^{(3)} & \phi_0^{(3)} \end{array}\right] \begin{bmatrix} r \\ \hline r_0 \end{bmatrix} \qquad \text{(F-6)}$$

Establishing Internal Compatibility

Eq. F-6 is an expression of the cubic displacement patterns developed in the three subelements. Because of the common displacements imposed at the nodes, the transverse displacements of two adjacent elements are identical along their juncture line. However, their normal slopes differ between the nodes; hence, Eq. F-6 does not represent an internally compatible displacement field. To establish slope compatibility along the internal edges of the subelements, additional nodes 7, 8, and 9 were located at the mid-points of these edges, as shown in Fig. A-27a. The normal slope was computed at each of these nodes in each subelement, for example

$$\left(\frac{\partial w^{(1)}}{\partial n}\right)_7 \equiv \theta_7^{(1)} = \left[\begin{array}{c|c} b_7^{(1)} & b_{70}^{(1)} \end{array}\right] \begin{bmatrix} r \\ \hline r_0 \end{bmatrix} \qquad \text{(F-7)}$$

where $b_7^{(1)}$ is $\dfrac{\partial \phi_e}{\partial n}$ and $b_{70}^{(1)}$ is $\dfrac{\partial \phi_0}{\partial n}$ both evaluated at node 7. Similar expressions can be developed for each nodal slope in each subelement. To maintain internal slope compatibility, it is necessary to match the nodal slopes in adjacent subelements, i.e., $\theta_7^{(1)} = -\theta_7^{(3)}$, etc. (where the negative sign results from the convention that the positive normal is directed outward), or stating all three compatibility requirements together:

$$\begin{bmatrix} \theta_7^{(1)} \\ \theta_8^{(2)} \\ \theta_9^{(3)} \end{bmatrix} + \begin{bmatrix} \theta_7^{(3)} \\ \theta_8^{(1)} \\ \theta_9^{(2)} \end{bmatrix} = \left[\begin{array}{c|c} (b_7^{(1)} + b_7^{(3)}) & (b_{70}^{(1)} + b_{70}^{(3)}) \\ (b_8^{(2)} + b_8^{(1)}) & (b_{80}^{(2)} + b_{80}^{(1)}) \\ (b_9^{(3)} + b_9^{(2)}) & (b_{90}^{(3)} + b_{90}^{(2)}) \end{array}\right] \begin{bmatrix} r \\ \hline r_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(F-8)}$$

Eq. F-8 may be expressed symbolically as follows:

$$\left[B \; \vdots \; B_o \right] \left[\frac{r}{r_o} \right] = 0 \qquad \text{(F-9)}$$

Now the values of r_o which will satisfy these compatibility conditions may be computed from Eq. F-9; i.e.

$$r_o = - B_o^{-1} B \, r \equiv L \, r \qquad \text{(F-10)}$$

Finally, introducing the compatibility constraint of Eq. F-10 into Eq. F-6, the fully compatible displacement field in the three subelements becomes:

$$\begin{bmatrix} w^{(1)} \\ w^{(2)} \\ w^{(3)} \end{bmatrix} = \left(\begin{bmatrix} \phi_e^{(1)} \\ \phi_e^{(2)} \\ \phi_e^{(3)} \end{bmatrix} + \begin{bmatrix} \phi_o^{(1)} \\ \phi_o^{(2)} \\ \phi_o^{(3)} \end{bmatrix} L \right) r \equiv \begin{bmatrix} \hat{\phi}^{(1)} \\ \hat{\phi}^{(2)} \\ \hat{\phi}^{(3)} \end{bmatrix} r \qquad \text{(F-11)}$$

Although it is straightforward in concept, the derivation of the compatible interpolation functions of Eq. F-11 involves long algebraic manipulations and is a tedious operation. Explicit expressions for these functions are presented on Page F-15 for the convenience of the reader.

Internal Curvature Field

In order to define the stiffness matrix corresponding with the derived displacement field, it is necessary to establish the curvatures within the subelements. The most convenient means of defining the curvature distribution is by means of appropriate interpolation functions; in the present case where the assumed displacement functions are cubic, it is important to note that the curvature must vary linearly within each subelement. Thus, the curvature field can be expressed as the product of linear interpolation functions multiplied by curvature values defined at the corner nodes.

Within any subelement "i," the curvature $\chi^{(i)}$ can be obtained by differentiation of the displacement field, thus

$$\chi^{(i)} \equiv \begin{bmatrix} w_{xx}^{(i)} \\ w_{yy}^{(i)} \\ 2 \cdot w_{xy}^{(i)} \end{bmatrix} = T^{(i)} r \qquad \text{(F-12)}$$

where

$$T^{(i)} \equiv \begin{bmatrix} \dfrac{\partial^2 \hat{\phi}^{(i)}}{\partial x^2} \\ \dfrac{\partial^2 \hat{\phi}^{(i)}}{\partial y^2} \\ 2\dfrac{\partial^2 \hat{\phi}^{(i)}}{\partial x \, \partial y} \end{bmatrix} \tag{F-13}$$

Now the nodal values of the curvatures in subelement "i" may be determined by evaluating $T^{(i)}$ at corner points; thus, if the nodal curvatures are designated $\chi_n^{(i)}$ and the nodal values of $T^{(i)}$ are $T_n^{(i)}$ the relationship may be expressed

$$\chi_n^{(i)} = T_n^{(i)} r \tag{F-14}$$

in which, for example,

$$\chi_n^{(i)T} = \left[w_{xx2}^{(i)} \; w_{xx3}^{(i)} \; w_{xxo}^{(i)} \; w_{yy2}^{(i)} \; w_{yy3}^{(i)} \; w_{yyo}^{(i)} \; 2w_{xy2}^{(i)} \; 2w_{xy3}^{(i)} \; 2w_{xyo}^{(i)} \right] \tag{F-15}$$

The linear curvature variation within the subelement can now be expressed by means of the linear interpolation functions ϕ_x as follows:

$$\chi^{(i)} = \begin{bmatrix} \phi_x & 0 & 0 \\ 0 & \phi_x & 0 \\ 0 & 0 & \phi_x \end{bmatrix} \chi_n^{(i)} \equiv \bar{\phi}_x \, \chi_n^{(i)} \tag{F-16}$$

where the linear interpolation functions are merely the three triangular coordinates:

$$\phi_x = \left[\zeta_1 \; \zeta_2 \; \zeta_3 \right] \tag{F-17}$$

It is of interest to note that the curvatures in the three subelements are identical at the nodal point "O"; thus, these quantities need be evaluated only once.

Element Stiffness

The strain energy due to bending of the subelement may be obtained by integrating the product of the moments and the curvatures over the area of the element, i.e.

$$U^{(i)} = \frac{1}{2} \int_A m^{(i)^T} \chi^{(i)} dA \qquad \text{(F-18)}$$

where the internal moments are given by

$$m^{(i)} \equiv \begin{bmatrix} m_{xx}^{(i)} \\ m_{yy}^{(i)} \\ m_{xy}^{(i)} \end{bmatrix} = D\chi^{(i)} \qquad \text{(F-19)}$$

The matrix D in Eq. F-19 represents the constitutive relationship for the element material. For a plate with constant material properties through the thickness "h," this relationship may be expressed

$$D = \frac{h^3}{12} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ \text{(Symm.)} & & C_{33} \end{bmatrix} \qquad \text{(F-20)}$$

where the coefficients C_{ij} represent the elastic properties. Substituting Eq. F-14, F-16 and F-19 into Eq. F-18 leads to

$$U^{(i)} = \frac{1}{2} r^T T_n^{(i)^T} \int_A \bar{\phi}_x^T D \bar{\phi}_x \, dA \; T_n^{(i)} r$$

or

$$U^{(i)} = \frac{1}{2} r^T K^{(i)} r \qquad \text{(F-21)}$$

where

$$K^{(i)} = T_n^{(i)T} G^{(i)} T_n^{(i)} \qquad \text{(F-22)}$$

is the stiffness matrix contribution of Subelement "i" expressed in terms of the nodal Displacements r , and

$$G^{(i)} = \int_A \bar{\phi}_x^T D^{(i)} \bar{\phi}_x \, dA \equiv \begin{bmatrix} G_{11}^{(i)} & G_{12}^{(i)} & G_{13}^{(i)} \\ & G_{22}^{(i)} & G_{23}^{(i)} \\ (\text{Symm.}) & & G_{33}^{(i)} \end{bmatrix} \quad \text{(F-23)}$$

If the material properties and thickness are uniform over the area of the subelement, the individual terms of the Matrix $G^{(i)}$ become

$$G_{ij}^{(i)} = \frac{h^3}{12} C_{ij} \; R^{(i)}$$

where $R^{(i)}$ is given by

$$R^{(i)} = \int_A \bar{\phi}_x^T \; \bar{\phi}_x \; dA = \frac{A^{(i)}}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad \text{(F-24)}$$

If material properties or thickness of the element vary over its area, it is recommended to evaluate the intergral of Equation 29 numerically.

It will be noted that the stiffness matrix of the complete triangle is obtained by merely adding the contributions of the three subelements because they are all expressed in terms of the same set of nodal coordinates. Thus

$$K = K^{(1)} + K^{(2)} + K^{(3)} \quad \text{(F-25)}$$

ELEMENT STIFFNESS INCLUDING SHEAR DISTORTION

Assumed Displacements

The basic assumption of the element stiffness analysis presented in the preceding paragraphs is the Kirchhoff hypothesis, which may be expressed mathematically as follows:

$$\left. \begin{aligned} w(x,y,z) &= w(x,y) \\ u &= -z \frac{\partial w}{\partial x} = z\,\theta_y \\ v &= -z \frac{\partial w}{\partial y} = -z\,\theta_x \end{aligned} \right\} \quad \text{(F-26)}$$

This assumption imposes the condition that normals to the undeformed plate mid-surface remain undistorted and normal to the deformed mid-surface. The fact that they remain normal during deformation effectively eliminates shear distortion from the assumed behavior, and thus leads to results which are applicable only to relatively thin plates. The theory can be extended to account for shear distortion in an approximate way, however, by adding a simple shear distortion mechanism to the Kirchhoff deformation hypothesis.

The shear distortion mechanism assumed in this study can be explained conveniently by reference to Fig. F-1a, which represents a cross-section view of a plate element. The rotation of the cross-section is shown to depend on the rotation of the mid-surface $\frac{\partial w}{\partial y}$ plus an additional shear distortion β_y which is assumed to be a simple straight line rotation (uniform shear strain through the thickness). The total rotation ϕ_x thus is given by

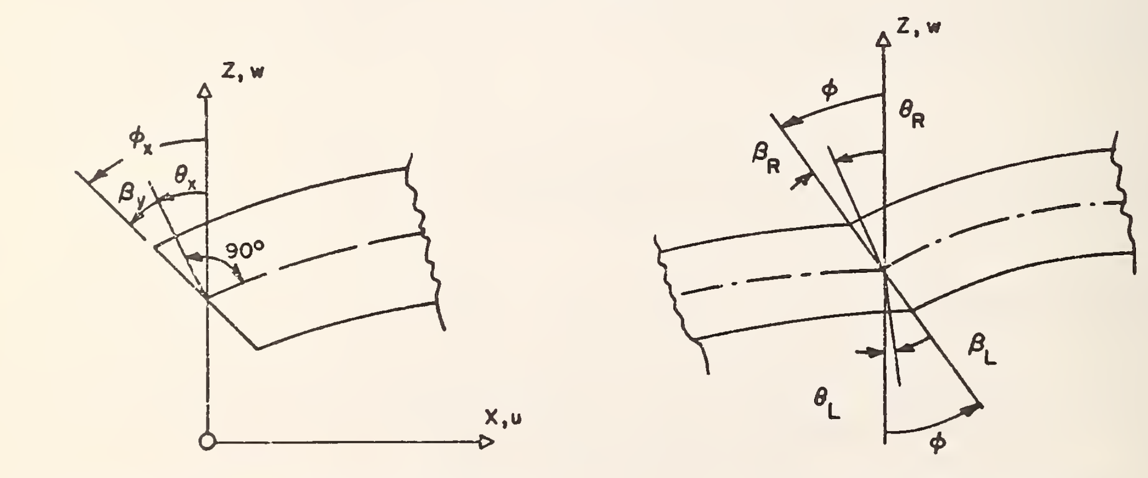
$$\phi_x = \frac{\partial w}{\partial y} + \beta_y = \theta_x + \beta_y \tag{F-27}$$

A similar assumption is made along the other axis, thus

$$\phi_y = -\frac{\partial w}{\partial x} - \beta_x = \theta_y - \beta_x \tag{F-28}$$

To define this distribution of cross-section rotations through the element field, it is now assumed that the transverse displacements w(x,y) are given by the compatible interpolation functions defined by Eq. F-11 , for each subelement:

$$w^{(i)} = \hat{\phi}^{(i)} r \tag{F-29}$$



(a) Definition of Mean Shear Distortion (b) Continuity Requirement

Fig. F-1 Shear Distortion Geometry

In addition, it is assumed that the shear distortions vary linearly over the entire triangle:

$$\begin{bmatrix} \beta_x \\ \beta_y \end{bmatrix} = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta_1 & \zeta_2 & \zeta_3 \end{bmatrix} \begin{bmatrix} \beta_{x1} \\ \beta_{x2} \\ \beta_{x3} \\ \beta_{y1} \\ \beta_{y2} \\ \beta_{y3} \end{bmatrix} \qquad \text{(F-30)}$$

or symbolically

$$\beta = \phi_\beta \, r_s \qquad \text{(F-31)}$$

Now it is convenient to substitute total rotation of the section ϕ for the slope θ in the nodal degrees of freedom of Eq. F-29 . Thus, using relationships derived from Eq. F-27 and F-28 , the following expressions may be defined for each corner nodal point:

$$\begin{bmatrix} \theta_{xi} \\ \theta_{yi} \end{bmatrix} = \begin{bmatrix} \phi_{xi} \\ \phi_{yi} \end{bmatrix} + \begin{bmatrix} -\beta_{yi} \\ +\beta_{xi} \end{bmatrix} \qquad \text{(F-32)}$$

while for a typical mid-side node, such as node 4:

$$\theta_4 = \phi_4 - (\beta_{x1} + \beta_{x2})\frac{c_3}{2} - (\beta_{y1} + \beta_{y2})\frac{s_3}{2} \qquad \text{(F-33)}$$

where $c_3 = a_3/L_3$ and $s_3 = -b_3/L_3$. Similar expressions may be obtained for nodes 5 and 6 by cyclic permutation of indices.

Using these expressions, the interpolation functions for displacement in each subelement Equation 35 may be rewritten as follows:

$$w^{(i)} = \left[\hat{\phi}^{(i)} \;\vdots\; \hat{\phi}_s^{(i)} \right] \begin{bmatrix} r_B \\ \hdashline r_s \end{bmatrix} \qquad \text{(F-34)}$$

where

$$r_B^T = \left[w_1 \; \phi_{x1} \; \phi_{y1} \; w_2 \; \phi_{x2} \; \phi_{y2} \; w_3 \; \phi_{x3} \; \phi_{y3} \; \phi_4 \; \phi_5 \; \phi_6 \right]$$

r_s = displacement vector in Eq. F-30

$\hat{\phi}^{(i)}$ = interpolation functions of Eq. F-11 and

$$
\hat{\phi}_s^{(i)^T} = \begin{bmatrix}
-\phi_{\theta y1} & -(\phi_{\theta 4}\, c_3 + \phi_{\theta 6}\, c_2)/2 \\
-\phi_{\theta y2} & -(\phi_{\theta 5}\, c_1 + \phi_{\theta 4}\, c_3)/2 \\
-\phi_{\theta y3} & -(\phi_{\theta 6}\, c_2 + \phi_{\theta 5}\, c_1)/2 \\
-\phi_{\theta x1} & -(\phi_{\theta 4}\, s_3 + \phi_{\theta 6}\, s_2)/2 \\
-\phi_{\theta x2} & -(\phi_{\theta 5}\, s_1 + \phi_{\theta 4}\, s_3)/2 \\
-\phi_{\theta x3} & -(\phi_{\theta 6}\, s_2 + \phi_{\theta 5}\, s_1)/2
\end{bmatrix} \tag{F-35}
$$

in which the subscripts of the interpolation functions ϕ identify their nodal displacement components. Eqs. F-30, F-31 and F-34 define completely the deformation of the element, expressing the transverse displacements Eq. F-34 in terms of 18 nodal displacements, and the shearing distortions Eq. F-30 in terms only of the 6 nodal shear distortions.

Internal Curvature Field

Where shearing distortions are included in the displacement field, the total curvatures χ^* which define the normal strain distribution depend on both the transverse displacements and the shear distortions as follows:

$$
\chi^* = \begin{bmatrix} \dfrac{\partial \phi_y}{\partial x} \\[2ex] \dfrac{\partial \phi_x}{\partial y} \\[2ex] \dfrac{\partial \phi_y}{\partial y} + \dfrac{\partial \phi_x}{\partial x} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\[2ex] \dfrac{\partial^2 w}{\partial y^2} \\[2ex] 2\dfrac{\partial^2 w}{\partial x\, \partial y} \end{bmatrix} + \begin{bmatrix} \dfrac{\partial \beta_x}{\partial x} \\[2ex] \dfrac{\partial \beta_y}{\partial y} \\[2ex] \dfrac{\partial \beta_x}{\partial y} + \dfrac{\partial \beta_y}{\partial x} \end{bmatrix} = \chi + \chi_\beta \tag{F-36}
$$

The first curvature term must be defined separately for each subelement, and from Eq. F-34 is given by:

$$
\chi^{(i)} = \left[\, T^{(i)} \;\vdots\; T_s'^{(i)} \,\right] \begin{bmatrix} r_B \\ \hline r_s \end{bmatrix} \tag{F-37}
$$

where $T^{(i)}$ is given by Eq. F-12 and $T_s'^{(i)}$ has an equivalent definition in terms of $\hat{\Phi}_s^{(i)}$. However, the second curvature term in Eq. F-36 may be expressed for the entire element, i.e.:

$$\chi_\beta = T_s'' \, r_s \tag{F-38}$$

where

$$T_s'' = \frac{1}{2A}\begin{bmatrix} b_1 & b_2 & b_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 & b_1 & b_2 & b_3 \end{bmatrix} \tag{F-39}$$

applying the derivative definitions of Eq. A-105 to Eq. F-30. Thus substituting Eq. F-37 and F-38 into Eq. F-36 leads to

$$\chi^{*(i)} = \left[\, T^{(i)} \,\vdots\, T_s^{(i)} \right]\begin{bmatrix} r_B \\ \hdashline r_s \end{bmatrix} \tag{F-40}$$

where

$$T_s^{(i)} = T_s'^{(i)} + T_s'' \tag{F-41}$$

Now because these combined curvatures still vary linearly within the subelements, their distribution can be expressed in terms of nodal curvature values as before; i.e. by analogy with Eq. F-14 and F-16:

$$\chi_n''^{(i)} \quad \left[\, T_n^{(i)} \,\vdots\, T_{sn}^{(i)} \right]\begin{bmatrix} r_B \\ \hdashline r_s \end{bmatrix} \tag{F-42}$$

and

$$\chi^{*(i)} = \bar{\Phi}_x \, \chi_n^{*(i)} \tag{F-43}$$

Element Stiffness

The strain energy due to bending deformations may be expressed in terms of the bending stiffness matrix by analogy with Eq. F-25:

$$K_B^{(i)} = \begin{bmatrix} T_n^{(i)T} \\ \hdashline T_{sn}^{(i)T} \end{bmatrix}\left[G^{(i)}\right]\left[\, T_n^{(i)} \,\vdots\, T_{sn}^{(i)} \right] = \begin{bmatrix} K_{BB} & K_{BS} \\ \hdashline K_{SB} & K'_{SS} \end{bmatrix} \tag{F-44}$$

in which $G^{(i)}$ is exactly as in Eq. F-24. An additional strain energy contribution U_s results from the shear distortion, however, which is given as follows:

$$U_s = \frac{1}{2} \int_A Q^T \beta \, dA \tag{F-45}$$

in which the shearing forces Q are

$$Q \equiv \begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = D_s \beta \tag{F-46}$$

In this expression, D_s represents the shear distortion constitutive relationship. For a plate with uniform properties through the thickness, and assuming that z is a principal elastic direction, it may be expressed:

$$D_s = h \begin{bmatrix} C_{44} & C_{45} \\ C_{45} & C_{55} \end{bmatrix} \tag{F-47}$$

in which C_{ij} are material constants.

Introducing Eqs. F-47, F-46 and F-31 into Eq. F-45 leads to

$$U_s = \frac{1}{2} r_s^T \int_A \phi_\beta^T D_s \phi_\beta \, dA \, r_s$$

or

$$U_s = \frac{1}{2} r_s^T K''_{ss} r_s \tag{F-48}$$

where K''_{ss} is the shear stiffness matrix. In the case where material properties and thickness are uniform over the element, the shear stiffness becomes

$$K''_{ss} = h \begin{bmatrix} C_{44} R & C_{45} R \\ C_{45} R & C_{55} R \end{bmatrix} \tag{F-49}$$

in which R is given by Eq. F-24. For nonuniform thickness or material properties, the integral should be evaluated numerically.

The total stiffness is obtained finally by superposition of the bending and shear contributions,

$$K = \left[\begin{array}{c|c} K_{BB} & K_{BS} \\ \hline K_{SB} & (K'_{SS} + K''_{SS}) \end{array} \right] \tag{F-50}$$

in which it should be noted that the bending terms must be computed separately for each subelement and added, whereas the shear stiffness term K''_{SS} may be computed for the entire element.

Condensation of Shear Distortion Degrees of Freedom

The element stiffness matrix of Eq. F-50 represents the element force-displacement relationship, which may be expressed as follows:

$$\begin{bmatrix} P_B \\ P_S \end{bmatrix} = \begin{bmatrix} K_{BB} & K_{BS} \\ K_{SB} & K_{SS} \end{bmatrix} \begin{bmatrix} r_S \\ r_S \end{bmatrix} \qquad \text{(F-51)}$$

in which $K_{SS} = K'_{SS} + K''_{SS}$. A total of 18 degrees of freedom are included in this relationship, 6 shear distortion components r_S in addition to the 12 basic translation and rotation displacements of the nodes. At this point, however, it should be noted that the shear distortion angle β need not be continuous between adjacent elements. As shown in Fig. F-1b, the total rotation ϕ must be the same for two adjacent elements in order to satisfy compatibility conditions, but β and θ need not be the same. Consequently, the shear distortion degrees of freedom are not needed to achieve compatibility and they may be eliminated from the element by static condensation.

As a result of the condensation process, the modified element stiffness relationship becomes:

$$\overline{P} = \overline{K}\, r \qquad \text{(F-52)}$$

in which

$$\begin{aligned} \overline{K} &= K_{BB} - K_{BS} K_{SS}^{-1} K_{SB} \\ \overline{F} &= P_B - K_{BS} K_{SS}^{-1} P_S \end{aligned} \qquad \text{(F-53)}$$

It is of interest to note that the degrees of freedom of this reduced element stiffness matrix are exactly equivalent to those of the original LCCT-12 element; thus, an element including this shear distortion capability can easily be incorporated into existing plate bending analysis programs based on the original element.

GENERAL QUADRILATERAL ELEMENT Q-19

Although the LCCT-12 element employs an optimum compatible cubic displacement field and, therefore, will yield the best possible results for a given triangular element mesh involving compatible cubic displacements, its mid-point nodes are a somewhat undesirable feature. They tend to complicate mesh generation procedures, increase the band width of the assembled equation systems, and require special identification in the development of computer programs. To overcome these disadvantages, while retaining most of the flexibility of the LCCT-12 element, it is convenient to develop a special version of the element by constraining the normal slope to vary linearly along one side.

Consider, for example, Subelement 3 of the element shown in Fig. A-27a. The mid-side node 4 of this subelement can be eliminated by introducing the condition that its value is the average of the corresponding slopes at nodes 1 and 2,

$$\theta_4 = \theta_{x4} \, C_3 - \theta_{y4} \, S_3 = (\theta_{x1} + \theta_{x2}) \frac{C_3}{2} - (\theta_{y1} + \theta_{y2}) \frac{S_3}{2} \qquad \text{(F-54)}$$

where c_3 and s_3 are the same as in Eq. F-33. Using this condition, the displacement interpolation expressions in Eq. F-11 can be reduced to only 11 components, and the stiffness matrix thus reduced accordingly.

The resulting partially constrained element is designated LCCT-11. (Similar constraints must be applied to the other sides to develop the LCCT-10 and LCCT-9 elements.) Four LCCT-11 elements may then be assembled into the Q-19 quandrilateral having no mid-side nodes on the exterior edges as shown in Fig. A-23. Although this element has 19 degrees of freedom, the 7 internal degrees of freedom of the assemblage are eliminated by a static condensation process equivalent to Eq. F-50 before the quadrilateral is assembled into the complete structure. Thus, the final quadrilateral has only 12 degrees of freedom, corresponding to a translation and two rotations at each node. It is a fully compatible element, having linear variations of normal slopes along all exterior edges.

Also it should be noted that triangular elements including shear distortion effects can be assembled to form a quadrilateral element in a similar fashion. In this case, however, it has been found most convenient to condense the shear distortion degrees of freedom at the quadrilateral assemblage level rather than in the individual triangles.

COMPATIBLE DISPLACEMENT FUNCTIONS

For subelement 3, Eq. F-11 gives $w^{(3)} = \hat{\phi}^{(3)}\, r$ where the displacement functions may be expressed:

$$\hat{\phi}^{(3)} = \hat{\phi}^{(3)}_{w1}\ \hat{\phi}^{(3)}_{\theta x1}\ \hat{\phi}^{(3)}_{\theta y1}\ \hat{\phi}^{(3)}_{w2}\ \hat{\phi}^{(3)}_{\theta x2}\ \hat{\phi}^{(3)}_{\theta y2}\ \hat{\phi}^{(3)}_{w3}\ \hat{\phi}^{(3)}_{\theta x3}\ \hat{\phi}^{(3)}_{\theta y3}\ \hat{\phi}^{(3)}_{\theta 4}\ \hat{\phi}^{(3)}_{\theta 5}\ \hat{\phi}^{(3)}_{\theta 6}$$

in which (in terms of the dimensions and coordinates of the <u>complete</u> element):

$$\hat{\phi}^{(3)}_{w1} = \zeta_1^2(3-2\zeta_1) + 6\mu_3\zeta_1\zeta_2\zeta_3 + \zeta_3^2\left[3(\lambda_2-\mu_3)\zeta_1 + (2\mu_3-\lambda_2)\zeta_3 - 3\mu_3\zeta_2\right]$$

$$\hat{\phi}^{(3)}_{\theta x1} = \zeta_1^2(b_2\zeta_3 - b_3\zeta_2) + (b_1 - b_3\mu_3)\zeta_1\zeta_2\zeta_3 + \frac{1}{6}\zeta_3^2\left[3(b_2\lambda_2 + b_3\mu_3 - 2b_1)\zeta_1 + 3(b_3\mu_3 - b_1)\zeta_2 + (3b_1 - b_2\lambda_2 - 2b_3\mu_3)\zeta_3\right]$$

$$\hat{\phi}^{(3)}_{w2} = \zeta_2^2(3-2\zeta_2) + 6\lambda_3\zeta_1\zeta_2\zeta_3 + \zeta_3^2\left[3(\mu_1-\lambda_3)\zeta_2 + (2\lambda_3-\mu_1)\zeta_3 - 3\lambda_3\zeta_1\right]$$

$$\hat{\phi}^{(3)}_{\theta x2} = \zeta_2^2(b_3\zeta_1 - b_1\zeta_3) + (b_3\lambda_3 - b_2)\zeta_1\zeta_2\zeta_3 + \frac{1}{6}\zeta_3^2\left[3(2b_2 - b_3\lambda_3 - b_1\mu_1)\zeta_2 + 3(b_2 - b_3\lambda_3)\zeta_1 + (-3b_2 - b_1\mu_1 + 2b_3\lambda_3)\zeta_3\right]$$

$$\hat{\phi}^{(3)}_{w3} = \zeta_3^2\left[3(1+\mu_2)\zeta_1 + 3(1+\lambda_1)\zeta_2 + (1-\mu_2-\lambda_1)\zeta_3\right]$$

$$\hat{\phi}^{(3)}_{\theta x3} = \frac{1}{6}\zeta_3^2\left[3(3b_1 + b_2 + b_1\lambda_1)\zeta_2 + (b_2\mu_2 - b_1\lambda_1)\zeta_3 - 3(b_1 + 3b_2 + b_2\mu_2)\zeta_1\right]$$

$$\hat{\phi}^{(3)}_{\theta 4} = \frac{4A}{3L_3}\left[6\zeta_1\zeta_2\zeta_3 + \zeta_3^2(5\zeta_3 - 3)\right]$$

$$\hat{\phi}^{(3)}_{\theta 5} = \frac{4A}{3L_1}\left[\zeta_3^2(3\zeta_2 - \zeta_3)\right]$$

$$\hat{\phi}^{(3)}_{\theta 6} = \frac{4A}{3L_2}\left[\zeta_3^2(3\zeta_1 - \zeta_3)\right]$$

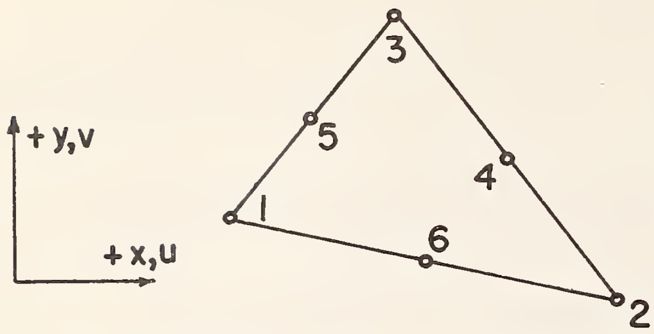
For $\hat{\phi}^{(3)}_{\theta yi}$ (i = 1,2,3) change all b's in $\hat{\phi}^{(3)}_{\theta xi}$ to a's.

Above expressions apply to subelement 3, where $\zeta_1 \geq \zeta_3,\ \zeta_2 \geq \zeta_3$

In subelements 1 (where $\zeta_2 \geq \zeta_1, \zeta_3 \geq \zeta_1$) and in subelement 2 (where $\zeta_3 \geq \zeta_2,\ \zeta_1 \geq \zeta_2$) permute cyclically all subscripts and superscripts (1-2-3 permutes to 2-3-1 and 4-5-6 to 5-6-4); for example: $\hat{\phi}^{(1)}_{wi} = \zeta_1^2\left[3(1+\mu_3)\zeta_2 + 3(1+\lambda_2)\zeta_3 + (1-\mu_3-\lambda_2)\zeta_1\right]$

APPENDIX G

LST Formulation Used in Subroutine SLST



Note: Node Numbering differs from thesis.

Nodal Freedoms

$$\underset{\sim}{q}_u^T = [\, u_1,\ u_2,\ u_3,\ \bar{u}_4,\ \bar{u}_5,\ \bar{u}_6 \,]$$

$$\underset{\sim}{q}_v^T = [\, v_1,\ v_2,\ v_3,\ \bar{v}_4,\ \bar{v}_5,\ \bar{v}_6 \,]$$

in which

$$\bar{u}_{i+3} = u_{i+3} - .5(\, u_j + u_k)$$

$$\bar{v}_{i+3} = v_{i+3} - .5(\, v_j + v_k)$$

with j = 2,3,1 and k = 3,1,2 for i = 1,2,3 respectively.

Displacement Field Assumption

$$\underset{2x1}{\begin{bmatrix} u \\ v \end{bmatrix}} = \underset{2x6}{\begin{bmatrix} \underset{\sim}{q}_u^T \\ \underset{\sim}{q}_v^T \end{bmatrix}} \underset{6x1}{\begin{bmatrix} \phi \end{bmatrix}}$$

where the shape function vector is (ξ_i = triangular coordinates)

$$\underset{6x1}{\phi} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ 4\xi_1\xi_2 \\ 4\xi_2\xi_3 \\ 4\xi_3\xi_1 \end{bmatrix} \begin{matrix} \\ \text{CST} \\ \\ \\ \text{LST (deviation from CST)} \\ \\ \end{matrix}$$

To get 5-node element set $\bar{u}_6 = \bar{v}_6 = 0$ (delete last column of $\underline{q}_u^T$, $\underline{q}_v^T$ and last element of ϕ). To get 4-node element set $\bar{u}_6 = \bar{u}_5 = \bar{v}_6 = \bar{v}_5 = 0$, etc.

Strain-Displacement Relations

$$[\varepsilon] = \underline{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \underset{3\times 12}{\begin{bmatrix} \underline{\phi}_2^T & \cdot \\ \cdot & \underline{\phi}_1^T \\ \underline{\phi}_1^T & \underline{\phi}_2^T \end{bmatrix}} \underset{12\times 1}{\begin{bmatrix} \underline{q}_u \\ \underline{q}_v \end{bmatrix}}$$

where

$$\phi_1 = \frac{1}{2A} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 4\xi_3 a_2 + 4\xi_2 a_3 \\ 4\xi_1 a_3 + 4\xi_3 a_1 \\ 4\xi_2 a_1 + 4\xi_1 a_2 \end{bmatrix} \qquad \phi_2 = \frac{1}{2A} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 4\xi_3 b_2 + 4\xi_2 b_3 \\ 4\xi_1 b_3 + 4\xi_3 b_1 \\ 4\xi_2 b_1 + 4\xi_1 b_2 \end{bmatrix}$$

In matrix form

$$\underline{\varepsilon} = \underline{D}\,\underline{q}$$

Stiffness Matrix

$$\underline{K} = \int_A \underline{D}^T \underline{C}\,\underline{D}\, h\, dA \qquad (C = \text{stress-strain matrix})$$

$$= \int_A \begin{bmatrix} \underline{\phi}_2 & \cdot & \underline{\phi}_1 \\ \cdot & \underline{\phi}_1 & \underline{\phi}_2 \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ & C_{22} & C_{23} \\ & & C_{33} \end{bmatrix} \begin{bmatrix} \underline{\phi}_2^T & \cdot \\ \cdot & \underline{\phi}_1^T \\ \underline{\phi}_2^T & \underline{\phi}_1^T \end{bmatrix} h\, dA$$

Consider a typical term viz (each C_{ij} is assumed constant over Δ.)

$$C_{ij} \int_A \phi_m \phi_n^T \, h \, dA \qquad \begin{array}{l} i,j = 1,2,3 \\ m,n = 1,2 \end{array}$$

Assuming linear thickness variation over Δ:

$$\begin{aligned} h &= h_1\xi_1 + h_2\xi_2 + h_3\xi_3 \\ &= h_o\,(r_1\xi_1 + r_2\xi_2 + r_3\xi_3) \end{aligned}$$

where

$$r_i = h_i/h_o \qquad (i = 1,2,3)$$

and

$$h_o = \frac{1}{3}\,(h_1 + h_2 + h_3) = \text{mean thickness}$$

Note that $r_1 + r_2 + r_3 = 3$.

The required integration formulas over the triangle are:

$$\int_A \xi_i \, dA = A/3$$

$$\int_A \xi_i\xi_j \, dA = A(1 + \delta_{ij})/12, \qquad \delta_{ij} = \text{Kronecker } \delta$$

$$\int_A \xi_i\xi_j\xi_k \, dA = A\, e_{ijk}/60 \qquad \begin{aligned} e_{ijk} &= 6 \text{ if } i = j = k \\ &= 3 \text{ if } i = j \neq k \text{ or} \\ &\quad\; i \neq j = k \text{ or} \\ &\quad\; i = k \neq j \\ &= 1 \text{ if } i \neq j \neq k \end{aligned}$$

It is trivial now to show that with linearly-varying h:

$$\int_A \phi_m \phi_n^T \, h \, dA = \frac{1}{4A^2}\, P_m^T\, Q\, P_n\, h_o\, A \qquad (m,n = 1,2)$$

where

$$\underline{P}_1 = \begin{bmatrix} a_1 & a_2 & a_3 & \cdot & 4a_3 & 4a_2 \\ a_1 & a_2 & a_3 & 4a_3 & \cdot & 4a_1 \\ a_1 & a_2 & a_3 & 4a_2 & 4a_1 & \cdot \end{bmatrix}$$

$\underset{\sim}{P}_2$ = similar to $\underset{\sim}{P}_1$, but with b's instead of a's

and

$$\underset{\sim}{Q} = \frac{1}{60}\begin{bmatrix} 6+4r_1 & 6-r_2 & 6-r_3 \\ & 6+4r_2 & 6-r_1 \\ \text{Symmetric} & & 6+4r_3 \end{bmatrix}$$

If the triangle has constant thickness ($r_1 = r_2 = r_3 = 1$) $\underset{\sim}{Q}$ reduces to the well known integration weight matrix:

$$\underset{\sim}{Q} = \frac{1}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Notes:

1. The node freedom vector $\underset{\sim}{q}$ is actually assembled as:

$$[u_1, v_1, u_2, v_2, \cdot \cdot \cdot]$$

i.e. with staggered components of $\underset{\sim}{q}_u$ and $\underset{\sim}{q}_v$

2. The entries of the CST stiffness (upper 6x6 of $\underset{\sim}{K}$) turn out to be independent of r_1, r_2, r_3 and are only affected by the mean thickness h_o. For instance, the upper 3x3 of $\int_A \phi_1 \phi_1^T h \, dA$ is

$$\frac{h_o}{240A}\begin{bmatrix} a_1 & a_1 & a_1 \\ a_2 & a_2 & a_2 \\ a_3 & a_3 & a_3 \end{bmatrix}\begin{bmatrix} 6+4r_1 & 6-r_2 & 6-r_3 \\ & 6+4r_2 & 6-r_1 \\ \text{Symmetric} & & 6+4r_3 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{bmatrix}$$

$$= \frac{h_o}{8A}\begin{bmatrix} a_1a_1 & a_1a_2 & a_1a_3 \\ & a_2a_2 & a_2a_3 \\ \text{Symmetric} & & a_3a_3 \end{bmatrix}$$

in which the fact that $r_1 + r_2 + r_3 = 3$ has been used to reduce out r_1, r_2, r_3. So if the element is CST (M = 0) there is no need for forming $\underset{\sim}{Q}$. (Cards SHEL 1140 to 1148)

FHWA
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