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ANALYSIS OF N-LAYERED VISCOELASTIC PAVEMENT SYSTEMS

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FOREWORD

The primary response model programmed in the VESYS IIM pavement design and analysis structural subsystem provides for the solution of a three-layered viscoelastic pavement system only. This report modifies this primary response model by extending the solution for stresses, strains, and deflections, to an N-layered viscoelastic system for any value of Poisson's ratio. An approximate probabilistic solution for the N-layered viscoelastic system is also presented in closed form.

A rut depth prediction model which uses simplified laboratory characterization tests is developed. The resulting pavement design and analysis models are structured into a set of modular subsystems which provide for easy replacement and incorporation of improved components in any one of the subsystems.

An initial concept to optimize the procedure for the design of flexible pavement systems is also presented.

The primary response model for the N-layered pavement system developed in this report has been incorporated into the VESYS IIM computer code. This modified code, known as VESYS G, is presented in FHWA-RD-77-117, "VESYS G - A Computer Program for Analysis of N-Layered Flexible Pavements," Georgia Institute of Technology, James S. Lai, April 30, 1977.

The report is intended primarily for research and development audiences. Copies are being distributed accordingly by transmittal memorandum.

Charles F. Scheffey
Director, Office of Research
Federal Highway Administration

NOTICE

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William L. Hufferd and James S. Lai

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University of Utah
Salt Lake City, Utah 84112

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16. Abstract

The primary response model programmed in the VESYS II-M pavement design and analysis structural subsystem provides for the solution of a three-layered viscoelastic pavement system only. This report modifies this primary response model by extending the solution for stresses, strains, and deflections, to an N-Layered Viscoelastic system for any value of Poisson's ratio. An approximate probabilistic solution for the N-Layered viscoelastic system is also presented in closed form.

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PREFACE

The research reported herein was sponsored by the U.S. Department of Transportation, Federal Highway Administration under Contract DOT-FH-11-8109. Particular gratitude is expressed to Mr. William J. Kenis for his valuable and thoughtful advice and assistance during the course of the program.

Our appreciation is also expressed to former graduate students Mr. Richard Chang for conducting the experimental work and Drs. Thomas Hou and Jim Tseng for programming assistance related to the N-layered probabilistic viscoelastic primary response model. The assistance of colleagues Drs. John Austin and David Mason in exploring and developing appropriate probabilistic models is also appreciated.

Finally, conceptual studies related to design optimization of flexible pavement systems were conducted under subcontract to Austin Research Engineers, Inc., Austin, Texas. The final report, written by Drs. J. B. Rahut and W. R. Hudson, is reproduced in its entirety as Chapter 7 of this report.

Additional program documentation of the primary response model is presented in Appendix A.

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METRIC CONVERSION FACTORS

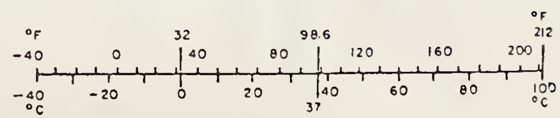
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



CHAPTER 1

PROGRAM SUMMARY

1.1 INTRODUCTION

Current pavement design approaches are mostly empirical; based primarily on field experience and engineering judgement. These approaches involve correlating the required thicknesses of pavement components to certain traffic characteristics, soil strength properties obtained from specified included soil tests, and the properties of the layer materials. These methods have been somewhat successful in the past; however, their applicability to the initial design or rehabilitation of current pavement structures is severely limited. This deficiency is vividly illustrated by the more rapid deterioration of new and rehabilitated pavement structures throughout the country than anticipated by the designers of these structures. That is, the experience gained from one set of materials, traffic loading conditions, environmental conditions, etc. cannot, in general be used to predict pavement performance for a different set of materials, traffic loading, environment, etc. due to the complicated nonlinear relations which exist among these parameters and pavement performance.

The above shortcoming takes on additional importance with the increasing scarcity and cost of petroleum. Accordingly, a need exists for the development of realistic pavement design and analysis procedures which have a firm theoretical foundation. These procedures should be aimed at optimizing pavement performance from a highway user and maintenance point of view. Overall costs should be minimized through trade-offs between initial costs, recurring costs, maintenance strategies and pavement performance. Accomplishment of these objectives requires, in particular:

1. Accurate descriptions of loads and environments throughout the anticipated useful life of the pavement system.

2. Accurate determination of material response and failure properties for each material in the pavement system under laboratory conditions representative of field use conditions.

3. Accurate predictions of the stresses, strains and deflections throughout the pavement structure due to the prescribed loads and environments.

4. Accurate predictions of distress and damage due to the prescribed loads and environments.

5. Accurate relations between pavement system distress and pavement performance.

6. Verification and evaluation of pavement performance predictive models under field use conditions.

7. Modification and re-evaluation of predictive models as required.

In addition, design of overlays requires in site methods for determining reduced or equivalent material response properties (i.e., moduli or compliances) for the layers of the pavement system, and for the evaluation of the load bearing capability of the existing pavement structure at the time the pavement is overlaid.

The Federal Highway Administration (FHWA) has addressed various aspects of the above requirements under Research Project 5C entitled

"New Methodology for Flexible Pavements". Significant and substantial progress has been made in this direction through the development of an overall pavement design framework, known as VESYS II, which is based on mechanistic concepts [1, 2].

The present report summarizes University of Utah work related to the modification and extension of VESYS II. These extensions/ modifications have led to an improved pavement design framework which is referred to as VESYS IIM herein.

The primary improvements to VESYS II consist of:

1. Extension of Three-Layered Viscoelastic solution to an N-layered viscoelastic solution.

2. Development of approximate closed form probabilistic solution for stresses, strains and deflections for and N-layered viscoelastic system.

3. Development of an improved rut depth model which utilizes simplified laboratory characterization tests.

Additional, minor modifications have also been made to VESYS II which are described in subsequent sections of this report.

Initial efforts on the conceptual development of an optimizing and decision making program which interrelates pavement structural make-up, system costs, maintenance strategies and service life were carried out under subcontract by Austin Research Engineers, Inc., Austin, Texas. The results of their study, as submitted in their final report to the University of Utah, are presented in Chapter 7.

Typical input data and examples are included in the discussions of the various system components. Analytical derivations and supporting documentation for certain models are included as appendix material to avoid disruption of the continuity of the main report.

Additional program documentation and a user's manual are contained in Volume II of this final report.

The computer code has been submitted under a separate cover.

1.2 PROJECT OBJECTIVES AND ACCOMPLISHMENTS

The original project work statement consisted of four specific tasks, in addition to a literature review:

1. Develop a VESYS II viscoelastic five layer solution.
2. Develop a closed form probabilistic solution for stresses, strains and deflections due to statistical variations of creep compliance at various times and rates of loading.
3. Develop modular capability of the design framework for substitution of improved primary response or damage models as they are developed. Include simple models for:
    a. thermal fatigue
    b. fatigue fracture mechanics
    c. permanent deformation
    d. power spectral representations of pavement roughness.
4. Develop concepts to interrelate structural make-up, systems costs, maintenance strategies and service life in an optimizing and decision-making program.

All technical objectives have been accomplished or exceeded. In place of a viscoelastic five-layer solution, a general N-layer viscoelastic solution has been developed using the quasi-viscoelastic method of solution. This approach was adopted for a variety of reasons.

The three-layer VESYS II is constrained to consideration of incompressible materials and re-derivation of the solution for five-layers within the standard convolution integral approach for compressible as well as incompressible materials (Poisson's ratio = 1/2) represented a considerable undertaking beyond the scope of the present contract.

Furthermore, introduction of a closed form probabilistic solution for stresses, strains, and deflections within the convolution integrals for the transient viscoelastic solution is complicated. The quasi-viscoelastic method of solution avoids these difficulties as a series of elastic solutions are combined using instantaneous values of relaxation modulus or creep compliance to obtain the viscoelastic solution.

The original intent was to introduce an approximate closed form solution into the CHEVRON 5-layer program [3]. However, the solution method in CHEVRON-5 is specifically applicable to a layered system with no more than five-layers, whereas the approximate closed formed probabilistic solution was obtained by a Taylor's series expansion of the general analytic solution for N-layers about the mean values of the material properties. It appeared that approximately the same effort was required to either program the solution to the probabilistic equations using the method employed in CHEVRON-5 or to completely re-program the solution to the N-layered problem. Since CHEVRON-5 also has other limitations and in the interest of greater program efficiency and flexibility it was decided to develop a new computer code for the deterministic and probabilistic solution for a N-layered system.

An approximate closed form probabilistic solution, as noted above, was obtained using Cornell's first order, second moment theory [4]. A Taylor's series expansion is taken about the mean values of layer creep complicances. The deterministic solution is used to represent the mean solution and the variance of stresses, strains and deflections is expressed in terms of the first partial derivatives.

The various subsystems are assembled in a modular, overlay fashion; however, rather than relay on machine dependent system commands to control execution of the various program elements, a simple command program, written in Fortran, controls execution of the overlays. This procedure, while not as efficient for a specific computer, increases program portability.

The executive program controls execution of all program overlays. The three main elements of the framework consist of the Primary Response Model, the Distress Model and the Serviceability Model, which are not connected by any common statements. Internal logic in the executive program determines which subsystem is to be executed and calls a Read program which reads the necessary input data and creates a data file for access by the subsystems. In this way, the various subsystems can be executed separately and readily replaced as improved mechanistic models are developed.

1.3 DESIGN AND ANALYSIS METHODOLOGY

The recommended methodology for design and analysis of pavement structures consists of the four phases shown schematically in Figure 1-1.

1. Problem Definition
2. Primary Response Analysis
3. Distress Analysis
4. Verification

Additionally, a fifth, surveillance phase, consists of periodically updating analyses and pavement structural integrity assessment during the service life of the pavement system. The approach depicted by the diagram is typical, but may be modified for special types of analyses.

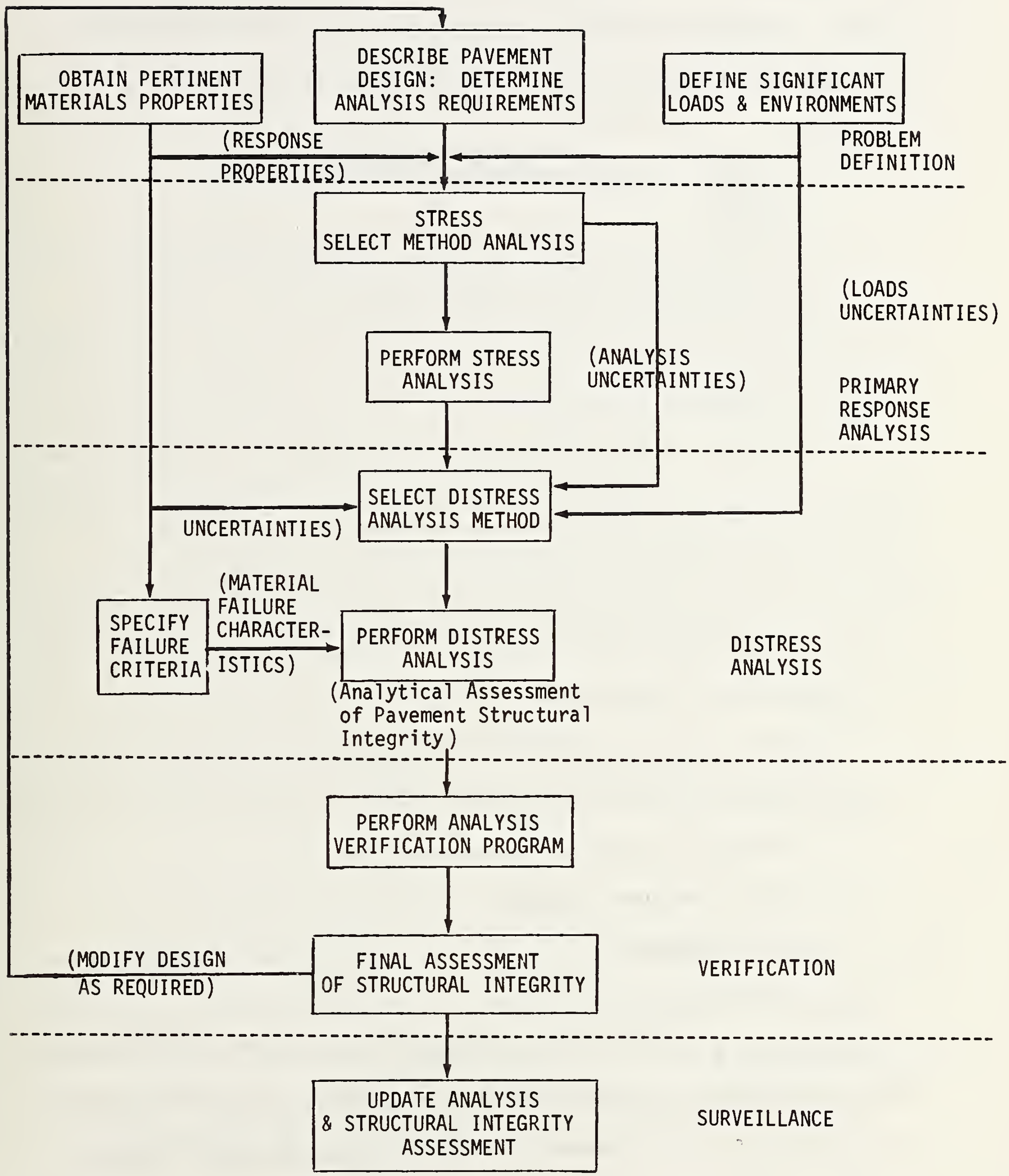


FIGURE 1.1 PAVEMENT DESIGN METHODOLOGY

The scope of analysis for a particular situation must be determined in early phases of the analysis so that data requirements can be clearly defined. Although the nature and precision of the data required will depend upon the type of analysis, minimum requirements generally consist of:

1. Description of the Pavement Structure and System Components.
2. Layer Material Response and Failure Properties.
3. Definition of Loads and Environments.
4. Required Service Life.
5. Maintenance Requirements.

CHAPTER 2

OVERVIEW OF PAVEMENT DESIGN & ANALYSIS METHODS

2.1 INTRODUCTION

Much effort during the past twenty years has gone into the development of pavement design procedures. Some of these methods are widely used and firmly established with some design agencies. These design methods can be classified into three groups: Elasticity methods, Semi-Empirical and Statistical methods, and Empirical methods.

Elacticity methods consider the behavior of pavement under traffic loading assuming the pavement system remains proportional to the applied loads. The principal design criterion consists of limiting stresses or strain as calculated by an elastic-layered theory. The methods belonging to this group include those used by the Kansas Highway Department [5], the U. S. Corps of Engineers (CBR Method) [6], Shell Company [7] and the Texas Highway Department [8].

Semi-Empirical and Statistical methods are based mainly on the correlation of field performance with certain "indirect" strength properties of pavement materials. Methods belonging to this group include those of the state of California [9, 10], the McLeod Method [11], the AASHO Intermin Guidelines for Flexible Pavement design [12], and the Asphalt Institute Method [13].

Distress Mode	Distress Manifestation	Examples of Distress Mechanism
Fracture	Cracking	Excessive loading Repeated loading (i.e., fatigue) Thermal changes Moisture changes Slippage (horizontal forces) Shrinkage
	Spalling	Excessive loading Repeated loading (i.e., fatigue) Thermal changes Moisture changes
Distortion	Permanent deformation	Excessive loading Time-dependent deformation (e.g., creep) Densification (i.e., compaction) Consolidation Swelling
	Faulting	Excessive loading Densification (i.e., compaction) Consolidation Swelling
Disintegration	Stripping	Adhesion (i.e., loss of bond) Chemical reactivity Abrasion by traffic
	Raveling and scaling	Adhesion (i.e., loss of bond) Chemical reactivity Abrasion by traffic Degradation of aggregate Durability of binder

FIGURE 2.1 Partial Listing of Categories of Pavement Distress

Empirical methods relate the pavement thickness to some particular soil conditions. Only the Michigan Highway Department Method [14] and the Canadian Good Road Association Method belong to this group.

Though some of these methods have been used widely and in some sense successfully, none of these methods have the ability to fully take into account all of the pavement distress associated with traffic and non-traffic loading, as illustrated in Figure 2-1. In addition, the rapid changes in traffic volumes, the steadily increasing wheel loads, the use of new and sometimes poorer quality materials, more strength maintenance requirements and longer required service life and the variability of environmental conditions make these methods unsatisfactory. Accordingly, more rational pavement design procedures capable of considering each distress mechanism have been developed or are under development by several agencies. One of the first rational pavement design frameworks was developed by the Massachusetts Institute of Technology under contract from the Federal Highway Administration and is known as the VESYS II viscoelastic pavement design framework [1].

2.2 REVIEW OF VESYS II PAVEMENT DESIGN FRAMEWORK

The VESYS II pavement design system framework developed by MIT and shown schematically in Figure 2-2, consists of five components which, when integrated together, result in a predictive pavement design system. Each component consists of several elements of sub-models which form the rational basis for the design procedure. A brief review of the VESYS-II is given in this chapter. More detailed information is provided in references 1, 2 and 15 to 18.

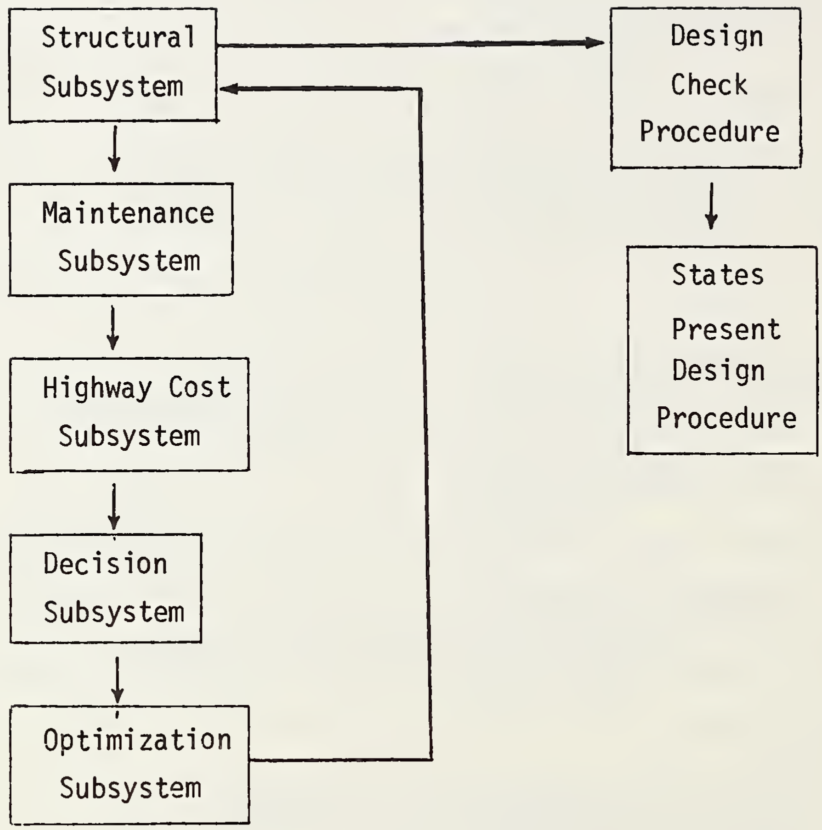


FIGURE 2-2 VESYS II Pavement Design System

In the VESYS II pavement design framework the pavement system is represented by a three-layer model (Figure 2-3); the upper two layers have finite thicknesses and the third layer is infinite. Each layer is infinite in the horizontal directions and may have elastic or viscoelastic behavior.

The pavement system is subjected to axisymmetric loading with varying intensities and durations represented by statistical distributions. Montly mean temperatures are used to represent the prevailing regional climatic conditions. The effects of moisture on the subgrade response is not considered.

Much of the input and output is described in terms of statistical distributions instead of single-valued, deterministic estimates. These probabilistic formulations are of two types: the method of statistical trials (monte carlo simulation) and closed form probabilistic solutions. For the most part, simulation techniques are used in the structural subsystem, although some approximate closed form solutions are employed.

A block diagram of the structural model is presented in Figure 2-4. The structural subsystem consists of a primary response model, a damage model and a serviceability model. In the primary response model, the response of the layered system to a stationary, moving or repeated load applied at the surface is obtained. Using the viscoelastic three-layered system program, materials properties variations are accounted for using Monte Carlo techniques and approximate closed form probabilistic solutions. The statistical estimates of stresses, strains and deflections so determined along with the loading characteristics and temperature history are used as inputs to the Damage Model to obtain the extent of rut depth, roughness or slope variance, and the extent of cracking.

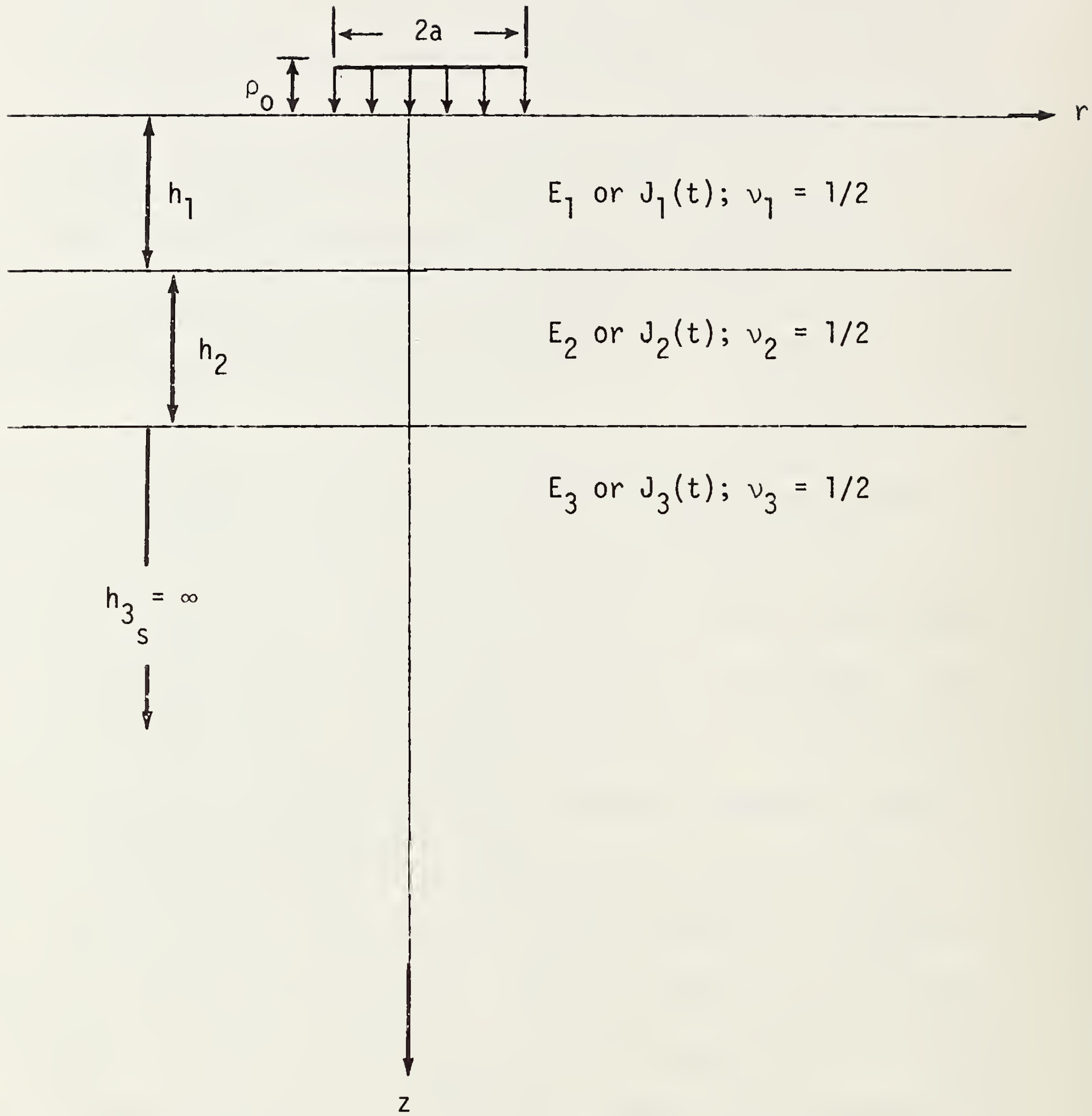


FIGURE 2-3 Idealization of Pavement Structure in VESYS II

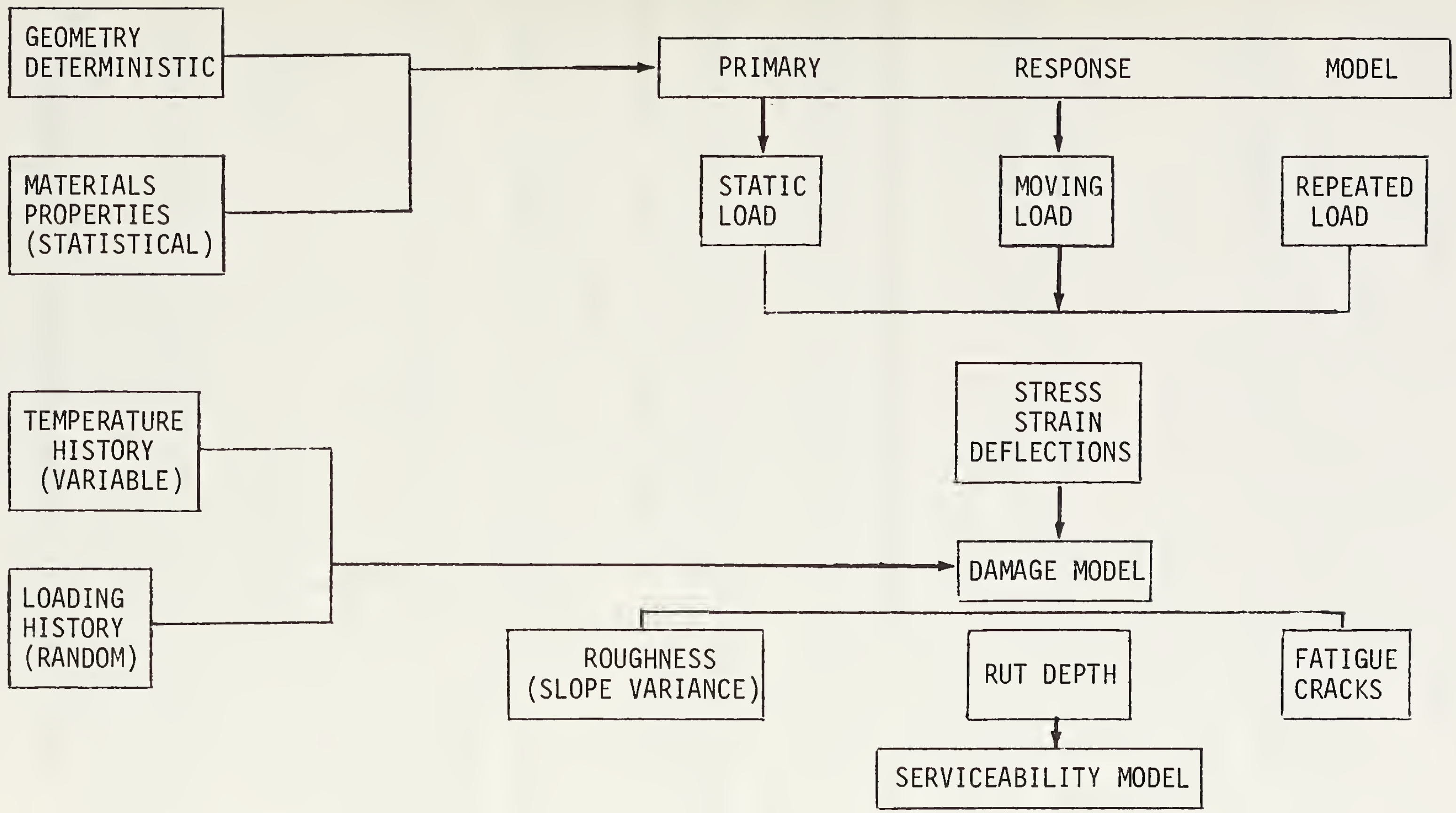


FIGURE 2.4 VESYS II Structural Subsystem (taken from Reference 17)

The distress indicators obtained from the distress model are combined in the form of a equation suggested by AASHO to provide a subjective measure, referred to as the Present Serviceability Index (PSI), of the serviceability level of the system at any time period.

The different elements of the VESYS II design framework are discussed briefly in the following subsections.

2.2.1 INPUT VARIABLES

The inputs to the structural model are divided into four categories: system geometry, materials properties, loads characteristics and temperature history.

The inputs for the SYSTEM GEOMERTRY consist of layer thicknesses for the first two layers. The thicknesses are input as deterministic quantities.

The MATERIALS PROPERTIES input for each layer consist of creep or elastic compliances, time-temperature shift factors, fatigue data in terms of strain level versus number of cycles to failure, and the spatial variation of materials properties. Each layer is assumed to be incompressible (ν = 1/2).

The Creep compliance data are fitted to an exponential (Dirichlet or Prony) series of the form

$$J(t) = \eta \sum_{i=1}^{N} J_i \exp(-t/\tau_i) \qquad (2\text{-}1)$$

using a least squares curve fitting subroutine. In Eq. (1), J(t) represents the creep compliance at time t, and

η = random variable with a mean of unity and a coefficient of variation determined by the scatter of the creep test data.

J_i = coefficients for the mean creep compliance

τ_i = retardation time constants

N = number of terms in the series

When considering the effects of environmental factors such as temperature, Eq. (1) is written:

$$J(t,J) = \alpha_T + \beta_T \eta \sum_{i=1}^{N} J_1 \exp(-t/\tau_k a_T) \qquad (2\text{-}2)$$

where α_T represents a vertical shift on an arithmetic scale, β_T represents a vertical shift on a logarithmic scale and a_T represents a horizontal shift on a logarithmic scale of time. Assuming thermorheologically simple material behavior $\alpha_T = 0$, $\beta_T = \frac{\rho T}{\rho_o T_o} \approx 1$ where ρ denotes the density and the subscript zeros indicate reference values. The term a_T is referred to as the time-temperature shift factor and is a function of temperature (Rankine or Kelvin) only.

Fatigue test data are represented by the equation

$$N = k_1 \Delta\varepsilon^{-k_2} \qquad (2\text{-}3)$$

where N represents a number of cycles to failure at a strain amplitude of $\Delta\varepsilon$. k_1 and k_2 are material property coefficients. k_1 and k_2 are statistically correlated and their temperature dependence determined experimentally. The coefficients k_1 and k_2 are given by their means, variances and coefficients of correlation.

The spatial variation of materials properties is given by a spatial auto-correlation function which measures the statistical correlation of two points a given distance apart. This coefficient denoted by ζ, is represented by the equation

$$\zeta = A\exp(-\Delta x^2/B^2) \qquad (2\text{-}4)$$

where Δx is the distance between the two points, and A and B are constants.

The traffic LOAD HISTORY consists of the duration, intensity and frequency. A circular contact area is assumed for the wheel loads with the contact area assumed to be constant. Hence, the variability of the wheel load is expressed in terms of mean and variance of the tire pressure. The mean and the coefficient of variation of load duration are used to represent the statistical variation of wheel loads duration. Frequency of loading is expressed in terms of a mean number of load applications per month. In addition, a traffic load channelization factor is also input.

The ENVIRONMENTAL VARIABLES are assumed to be temperature and moisture; although moisture effects are yet explicitly included. Temperature is given by its mean and variance for each basic time period, which is usually taken to be a month.

2.2.2 PRIMARY RESPONSE MODEL

The method of solving the three-layer (linear) viscoelastic boundary value problem consists of three steps:

1) the elastic solution due to a stationary load is obtained;
2) the elastic-viscoelastic corresponding principle is applied to the elastic solution to obtain the corresponding viscoelastic solution;
3) the stress-strain and deflection due to a stationary load together with the time functions representing the repeated and moving loads are used to generate the stress, strain and deflection due to repeated and moving loads using linear superposition.

In the formulation of the three-layer elastic solution, the loading and geometry of the system are assumed to be axially symmetric. Therefore, the equilibrium equations and compatibility equations expressed in polar coordinates, and the stress-strain and strain-displacement relations for an incompressible body form a complete set of equations for solution of the stress, strain and deflection in each layer. By using a proper set of exterior boundary conditions (on the free surface), interior boundary conditions (continuity on the interfaces) and the boundary condition at infinity, together with the governing equations in each layer, the stress, strain and deflection anywhere in the three-layer system can be uniquely determined. For example, the deformation at the free surface, w, is expressed in the form:

$$w = 2(1-\mu_1)^2 \frac{J_0(mv)J_1(ma)}{m} \frac{\sum_{i=1}^{18} \phi_i \alpha_i}{\sum_{i=1}^{9} \theta_i \beta_i} \qquad (2\text{-}5)$$

In this expression the ϕ_i's and θ_i's depend on the geometry (thickness of each layer), and the α_i's and β_i's depend on the products of the elastic properties of each layer. For example,

$$\alpha_1 = E_1^2 E_2^2, \ \alpha_4 = E_1^2 E_2 E_3, \text{ and } \alpha_4 = E_1^3 E_2 E_3, \text{ etc.}$$

In the viscoelastic solution of a three-layer system under a stationary load, it is only necessary to replace the modulus for each layer in the previous solution by the corresponding viscoelastic properties (relaxation modulus or creep compliance) using the elastic-viscoelastic correspondence principle. For example, for the deflection at the free surface, Eq. (5) becomes

$$\sum_{i=1}^{9} \theta_i \int_0^t w(t-\tau) \frac{\partial \hat{\beta}_i(\tau)}{\partial \tau} d\tau = R \sum \phi_i \hat{\alpha}_i \qquad (2\text{-}6)$$

where β_i and α_i are integral operators. For example, when $\alpha_4 = E_1^2 E_2 E_3$, then

$$\alpha_4(t) = \int_0^t E_1(t-\tau_1) \frac{\partial}{\partial \tau_1} \int_0^{\tau_1} E_1(\tau_1 - \tau_2) \frac{\partial}{\partial \tau_2} [\int_0^{\tau_2} E_2(\tau_2 - \tau_3) \frac{\partial E_3(\tau_3)}{\partial \tau_3} d\tau_3]$$

and

$$\int_0^t w(t-\tau) \frac{\partial \beta_4(\tau)}{\partial \tau} d\tau$$

$$(2\text{-}7)$$

$$= \int_0^t w(t-\tau) \frac{\partial}{\partial \tau} [\int_0^{\tau_0} E_1(t - \tau_1) \frac{\partial}{\partial \tau_1} \int_0^{\tau_1} E_1(\tau_1 - \tau_2) \frac{\partial}{\partial \tau_2} \int_0^{\tau_2} E_1(\tau_2 - \tau_3) \times \int_0^{\tau_3} E_2(\tau_3 - \tau_4) \frac{\partial E_3(\tau_4)}{\partial \tau_4} d\tau_4]$$

Equation (6) is an integral equation for the deflection w. $E_1(t)$, $E_2(t)$ and $E_3(t)$ are the relaxation moduli for the three layers. Numerical solution of this equation is straightforward although it requires very lengthy calculations, particularly if reasonable accuracy is desired. Once the primary response parameters (stress, strain and deflection) for a stationary load are obtained, then the corresponding parameters for repeated and moving loads are obtained.

The approach mentioned above for the solution of three-layer viscoelastic system is sound. However, it is not without faults. In particular, the computation time required in obtaining the numerical solution of Eq. 7 is substantial. It is well known that the accuracy of the solution depends upon the fineness of the time interval chosen.

Greater accuracy can be achieved using smaller time intervals in the numerical solution. However, the computation time is substantially increased as the number of time intervals increases.

The stationary loading program yields the primary responses of the system which include the deflection at the surface and the radial strain at the first interface beneath the center of the loaded area.

The curve fitting subroutine is used to obtain an exponential series representation of the primary responses. This series along with the traffic and temperature histories and the fatigue properties of the materials, are then used as input to the repeated loading subroutine.

Assuming the loads to be functions of time, the response after repeated loading is determined using linear superposition. This deterministic solution is then converted into an approximate probabilistic closed form solution. This probabilistic solution for the primary responses (i.e., stresses, strains, and deflections) is input to the damage model to predict pavement rutting, cracking and slope variance.

The damage model yields the history of the statistical characteristics of the damage indicators which are input to the serviceability model as described subsequently.

2.2.3 OUTPUT

The outputs of the structural model include the statistical estimates of AASHO's damage indicators (i.e., rut depth, roughness and cracking) determined at different times.

Rut depth is considered to be mainly due to channelization of traffic which causes a differential deformation in the transverse direction, and is given as a differential settlement between the most travelled channel and its surroundings.

Slope variance, or roughness, is an indication of the state of the pavement in the longitudinal direction, and is caused mainly by variation in the properties of the materials. Layers which are assumed to be elastic contribute only to the resilient deformations of the pavement system.

A linear Miner's law is used to account for crack accumulation in the top layer assuming that a fatigue mechanism is responsible for progression of cracking of pavements. The strains at the first interface are used to determine the fatigue life employing Eq. (3).

2.2.4 SERVICEABILITY MODEL

The distress indicators obtained from the damage model (i.e., rut depth, slope variance and cracking) are input to the serviceability model shown in Figure 2-5 to define the life expectancy of the pavement system. Figure 2-5 also includes the maintenance model and highway cost model.

The distress indicators obtained from the damage model are combined in a linear regression form suggested by AASHO to obtain the Present Serviceability Index (PSI).

$$PSI = a_0 + a_1X_1 + a_2X_2 + a_3X_3 \qquad (2\text{-}8)$$

where X_1', X_2 and X_3 are statistical measures of the damage caused by rutting, slope variance and cracking. The probabilities of having any value of PSI at any time are referred to as the state probabilities, and the probability, at any time, of being above some unacceptable value of PSI is defined as the reliability of the system at that time. The unacceptable limit of serviceability is subjectively defined by the user and defines some threshold for failure, from which the life expectancy of the system is determined.

The outputs of the serviceability model consist of the following:

1) Means and variances of AASHO's PSI at different times using the following expression:

$$PSI = a_o + a_1 \log[1 + (SV)] + a_2\sqrt{(GA)} + a_3(RD)^2 \quad (2\text{-}9)$$

where (SV) represents the slope variance, (CA) represents the cracked area and (RD) represents the rut depth. If the damage indicators are statistically independent, the following equations for the expected value of the PSI is used:

$$E[PSI] = a_1 + a_2\{\log[1+(SV)] - \tfrac{1}{2}[1+(SV)^2\ \sigma_{SV}{}^2]\} + a_3\{\overline{\sqrt{(CA)}}^{\,-3/2}\ \sigma^2_{\sqrt{(CA)}}\} + a_4\{\sigma^2_{RD} + (\overline{RD})^2\} \quad (2\text{-}10)$$

The upper bars represent an expected value and the σ^2 represent a variance. The value of the variance of the PSI is similarly derived.

2) Assuming that the serviceability index has a normal distribution and that an unacceptable level of serviceability is defined, the reliability of the system at a given time is computed as the probability of having the system above the failure level.

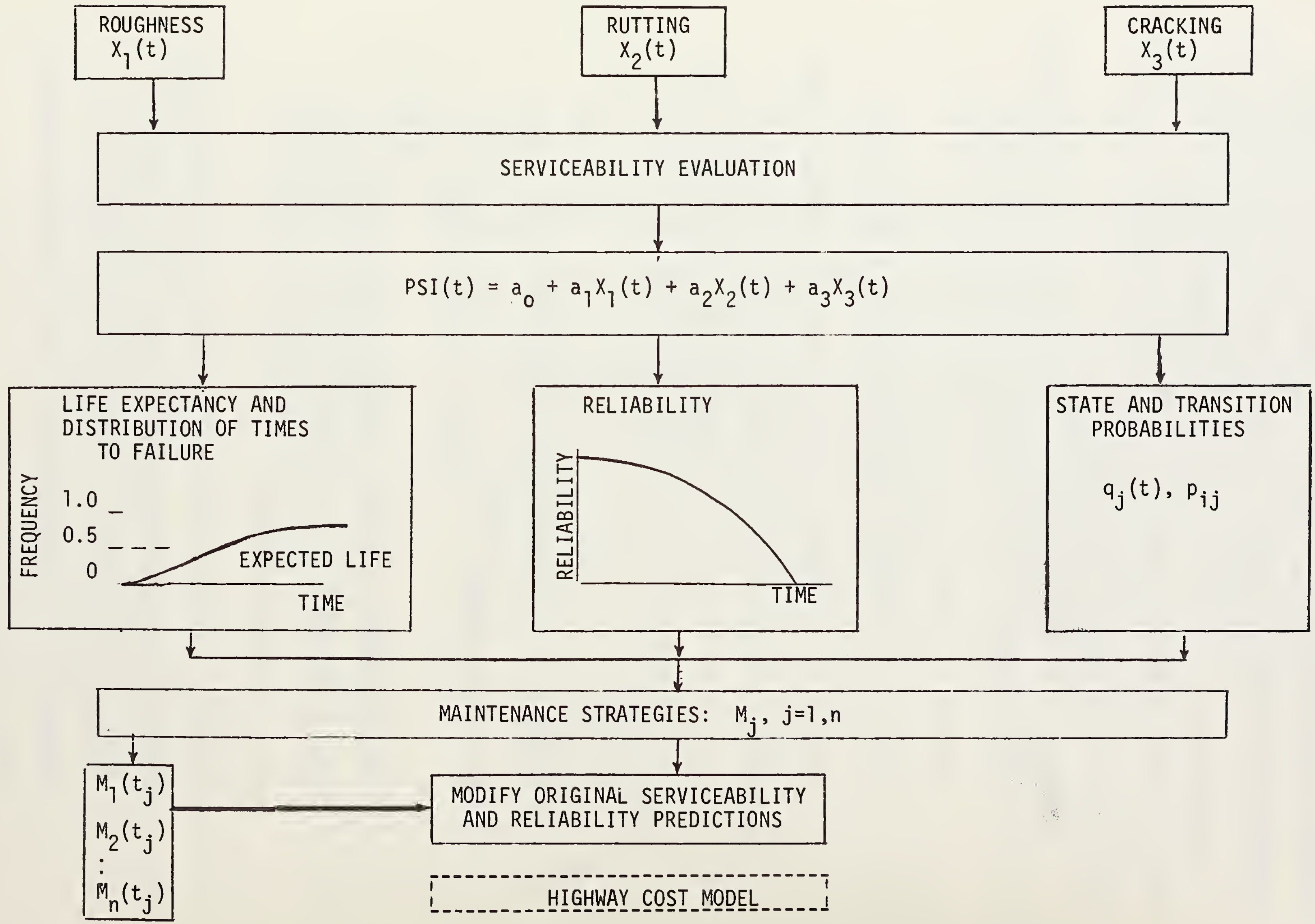


FIGURE 2-5 Serviceability Model

3) The expected life of the system is given as the mean time for the system to reach the failure level in the absence of maintenance activities. Probabilistic distributions of the life are also given.

4) The probability of the system being between specified levels of the serviceability index (i.e., the marginal state probability).

5) The state transition probability, i.e., the probability of going from one level to another at a specified time.

2.2.5 MAINTENANCE SUBMODEL

The maintenance submodel shown in Figure 2.5 generates various maintenance strategies over the operational life of the pavement system and evaluates the consequences of these strategies in terms of serviceability, reliability life and economic attributes. A decision-making algorithm selects those strategies which provide optimal costs for a given design configuration.

2.2.6 COST SUBMODEL

The cost model, also shown in Figure 2-5, addresses itself to the determination of the total costs of construction, operation and maintenance of highway systems, and incorporates three components: construction costs, roadway maintenance costs and vehicle operating costs. Each component in this model provides estimates of resource consumption, and yields the cost estimates of these resources using separately defined unit prices.

Input variables define the project to be analyzed and the structural design-maintenance strategy to be evaluated. Grade, alignment, width and depth of surfacing, maintenance and reconstruction policy are specified along with prevailing unit labor, materials and equipment costs. the submodel estimates the construction, maintenance and user costs for each analysis period which are discounted to find the present value of the structural design - maintenance strategy that is being evaluated. The result is the ability of the highway designer to compare a number of strategies on the basis of their total predicted costs.

These latter two submodels have not been fully implemented with the other models of the structural subsystem.

CHAPTER 3

SYSTEM INPUTS - LOADS AND ENVIRONMENTS

3.1 INTRODUCTION

The system inputs for VESYS-IIM consist of the following six categories:

(1) Geometry of the pavement system

(2) Traffic Loading

(3) Temperature

(4) Moisture

(5) Material Response Properties

(6) Material Damage Properties

All the inputs, except the geometry of the pavement system, are in probablistic format such that the variability of the various parameters can be properly taken into consideration in evaluating the response and performance of the pavement system. Thus, the variability of the pavement performance as well as the mean performance of the pavement system can be estimated in a more realistic manner.

From a practical point of view, in the course of design, the inputs for the system are kept as simple and straight forward as possible. Ultimately, it is hoped that all intermediate data reduction can be handled by a computer. Some of these intermediate calculations require additional knowledge and experience from the one doing the data reduction.

In the chapter the first four categories will be included. The last two categories with regard to the material characterization are discussed in the following chapters.

3.2 GEOMETRY OF THE PAVEMENT SYSTEM

The geometry requirement for the pavement system is the heights of the first n-1 layers with the thickness of the nth layer being infinite. Therefore, for a 5 layer system it is necessary to specify the heights of the first four layers.

3.3 TRAFFIC LOADS

The primary responses (stresses, strains and deflections) depend upon the total weight, the contact area, the contact pressure and wheel configurations. The manifestation of pavement distress in the form of fatigue cracking, rutting and roughness, on the other hand, depends on the stresses, strains and deflections as well as total number of load applications. Hence a more realistic way of handling the traffic loading is needed. Instead of just reading in a certain fixed intensity (such as equivalent 18^k) it is proposed to input (1) the actual wheel load distribution such as the distribution from the standard W-4 form which provides the number of axles per thousand trucks (or vehicles) in each axle load group (10 groups for single axles, and converting the tandem axles into equivalent single axles); (2) Average Daily Traffic, ADT (including mean and variance); (3) monthly variation of traffic; (4) average growth of traffic per year; (5) speed of trucks (mean and variation); (6) average radius of the contact area; (7) channelization distribution factor; (8) percent of truck in traffic stream; (9) percent trucks in the design lane.

Input (1) provides sufficient information with regard to the wheel load distribution. Input (2) and (3) allows generation of the monthly traffic. Input (4) is used to estimate future traffic growth. Inputs (5) and (6) are used to calculate the duration and (7), (8) and (9) are used for calculation of the percent of traffic under the wheel paths in the design lane.

In the present VESYS II primary response model, the contact area (radius = R) is considered to be a fixed value. Hence, the load intensity (or pressure p) is directly proportional to the wheel load P.

$$p = \frac{P}{\pi R^2}$$

Since VESYS II is a linear system, that is the primary responses, strains and deflections at any position in the pavement system are proportional to the load intensity (pressure), the primary responses are thus linearly proportional to the wheel load.* Therefore, in the calculations of the primary responses [$\sigma(1)$, $\varepsilon(1)$ and $\delta(1)$] only the response from a unity pressure need to be calculated. The primary response of $\sigma(1)$, $\varepsilon(1)$ and $\delta(1)$ correspond to a wheel load of $1 \times (\pi R^2)$. The corresponding primary responses for different axle load groups can be obtained straight forwardly by multiplying the factor which equals the wheel load in each axle group divided by $(2\pi R^2)$. Thus a table which consists of frequency versus stresses, strains and deflections per thousand vehicles can be obtained from

*Notice that this argument of proportionality between the primary responses to the wheel load is not valid if the contact area is varied. For example, for a constant pressure, the contact radius is proportional to $\sqrt{P}$. Nonlinearity is introduced.

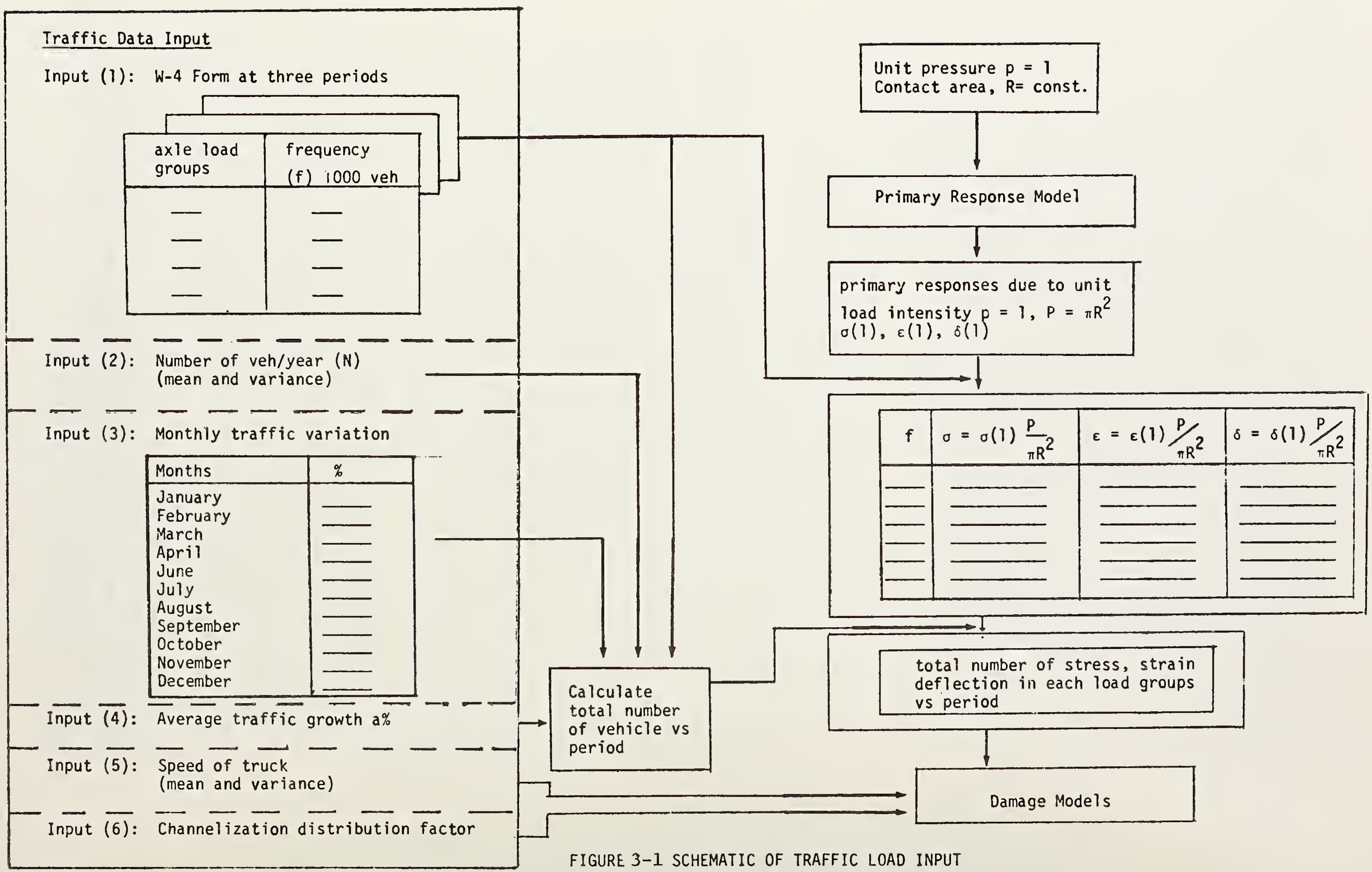


FIGURE 3-1 SCHEMATIC OF TRAFFIC LOAD INPUT

input (1). Input (2), (3) and (4) is then used to calculate the total number of stress, strain and deflection intensities in each category in each month, each year and total accumulated values for any period. This information is then input to the damage models to calculate the total number of stress, strain and deflection intensities in each category in each month, each year and total accumulated values for any period. This inforamtion is then input to the damage models to calculate the distresses. A flow diagram is shown in Figure 3-1.

At the present time the effects of wheel load configurations, and shear force on the stresses induced in the pavement system are not included in the primary response model of the VESYS IIM system. This is mainly due to the assumption of axial symmetry for loading as well as geometry on the solution of layered system which excluded the multiple wheel loads and shear forces automatically. Although it should not be too difficult a incorporate non-symmetric loading into the layered system. The computer program developed recently by Harrison, Wardle, and Gerrard [19] in Australia can handle the horizontal shear force.

3.4 TEMPERATURE

The need for temperature is two fold: one is the calculation of time-temperature superposition used to calculate the effect of temperature on the creep compliances of the materials and the other is to calculate thermal fracture. The temperature is given by the mean and variance of air temperature for each basic period. The basic period is usually taken to be a month. This data may be obtained from the weather bureau in the region.

The temperature in the pavement can be obtained from a heat transfer analysis.

3.5 MOISTURE

Subgrade moisture is read in to calculate the moisture-shift factor to be used to relate the effect of moisture on the creep compliance of subgrade materials. This information is also needed in order to calculate the freezing and thawing effect of the subgrade in each basic period.

In the VESYS IIM it is assumed that surface and base materials are relatively insensitive to moisture change. Hence moisture effect is considered only in the subgrade materials.

CHAPTER 4

SYSTEM INPUTS - MATERIALS PROPERTIES

4.1 INTRODUCTION

The properties of the materials in each layer in pavement system are dependent upon the constitutents of the materials and the manner in which they are prepared and constructed. Since these factors are statistical quantities, the material properties in each layer are expected to be statistical quantities. The material properties which are pertinent to pavement performance are divided into the following two categories:

(1) Material Response Properties: Creep compliances (for viscoelastic materials)or elastic modulus (for elastic response) and Poisson's ratio. These properties are needed in order to calculate the stresses, strains and deflection responses in a pavement system under the application of external loadings.

(2) Material Damage Properties: Fatigue properties and permanent deformation. These properties together with the stresses, strains and deflection responses in the pavement system are used to estimate the pavement damage of cracking, rutting and roughness under various stages of its life.

4.2 CREEP COMPLIANCES AND ELASTIC MODULUS

In the VESYS IIM framework, the pavement system is considered to be an n layer linear viscoelastic system. For example, in a five-layered system, the first and second layers represent the bituminous surface course and base course respectively, the third layer represents the compacted s bgrade, the fourth and fifth layers can be considered to be natural ground. The materials in each layer are considered to be linear viscoelastic or linear elastic if the time-dependency of the material response under a constant stress can be neglected.

4.3 LINEAR VISCOELASTIC CONSTITUTIVE EQUATION

For a linear viscoelastic material, the creep strains ε_{ij} (t) under a given stress history σ_{ij} (t) can be represented by the following constitutive equations

$$\varepsilon_{ij}(t) = \delta_{ij} \int_0^t \phi_1 (t-\xi) \frac{\partial \sigma_{kk}(\xi)}{\partial \xi} d\xi + \int_0^t \phi_2 (t-\xi) \frac{\partial \sigma_{ij}(\xi)}{\partial \xi} d\xi \qquad (4\text{-}1)$$

In this equation, ε_{ij}, σ_{ij} represent the strain and stress tensors respectively and each consists of six independent components. Three normal strains and stresses and three shear strains and stresses: σ_{kk} represents the dilational (confining) stress; and δ_{ij} is a kronecker tensor, which equals 1 when i=j, and o when i≠j. Thus, under a uniaxial stress (unconfined)state, $\sigma_{ij} = \sigma_{11} = \sigma$ and $\sigma_{kk} = \sigma/3$, the axial creep strain ε_{11} (t) becomes

$$\varepsilon_{11}(t) = \frac{1}{3} \int_0^t [\phi_1 (t-\xi) + 3\phi_2 (t-\xi)] \frac{\partial \sigma(\xi)}{\sigma \xi} d\xi \qquad (4\text{-}2)$$

or

$$\varepsilon_{11}(t) = \int_0^t J_1\ (t-\xi)\ \frac{\partial\sigma(\xi)}{\partial\xi}\, d\xi$$

$$J_1(t) = \frac{1}{3}\ [\phi_1\ (t) + 3\ \phi_2(t)] \qquad (4\text{-}3)$$

The radial creep strain (transverse strain) $\varepsilon_{22} = \varepsilon_{33}$ (t) becomes

$$\varepsilon_{22}(t) = \frac{1}{3} \int_0^t \phi_1(t-\xi)\ \frac{\partial\sigma(\xi)}{\partial\xi}\, d\xi \qquad (4\text{-}4)$$

Under a confining test in which the radial stress equals P and axial stress $\sigma_{11} = \sigma' + P$, the creep strain components become

$$\varepsilon_{11}(t) = \frac{1}{3} \int_0^t J_1(t-\xi)\ \frac{\partial\sigma\ (\xi)}{\partial\xi}\, d\xi + \int_0^t \phi_1(t-\xi)\ \frac{\partial p(\xi)}{\partial\xi}\, d\xi \qquad (4\text{-}5)$$

$$\varepsilon_{22}(t) = \varepsilon_{33}(t) = \frac{1}{3} \int_0^t \phi_1(t-\xi)\ \frac{\alpha[\sigma(\xi)+3p(\xi)]}{\partial\xi}\, d\xi \qquad (4\text{-}6)$$

In both cases, $\phi_1(t)$ and $\phi_2(t)$ or $\phi_1(t)$ and $J_1(t)$ can be determined.

When the axial stress is a constant $\sigma(t) = \sigma_o$, (4-3) and (4-4) become

$$\varepsilon_{11}(t) = J_1(t)\sigma_o \qquad (4\text{-}7a)$$

$$\varepsilon_{22} = \varepsilon_{33}(t) = \phi_1(t)\sigma_o \qquad (4\text{-}7b)$$

Thus the axial creep compliance $J_i(t)$ can be determined by dividing the axial creep strain by the applied stress.

$$J_1(t) = \frac{\varepsilon_{11}(t)}{\sigma_o} \qquad (4\text{-}8)$$

Using the definitions of Poisson's ratio, the Poisson ratio ν of a linear viscoelastic material can be determined as follows

$$\nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = -\frac{\phi_1(t)\sigma_0}{J_1(t)\sigma_0} = -\frac{\phi_1(t)}{J_1(t)} \tag{4-9}$$

Equation (4-9) implies that the Poisson's ratio for a linear viscoelastic material, in general, is time dependent. However, in the VESYS IIM program, ν is considered to be time-independent, though it is not necessarily equal to one half.*

When the material response is insensitive to time, the creep strain $\varepsilon_{11}(t)$ under a constant stress is independent of time or nearly so, then the creep compliance in (4-8) becomes a constant, and the reciprocal is the elastic modulus. Further, if only the resilient strain instead of total strain is used, then the modulus is called resilient modulus.

4.4 CREEP COMPLIANCE OF ASPHALT CONCRETE

The mix design, gradation of aggregates, density and void ratio of the asphalt concrete samples made in the laboratory should simulate the in-place asphalt concrete material. The test samples are usually cylindrical in shape with the height of the specimen about 1-1/2 to 2 times the diameter. In creep tests, a constant stress is applied and strain is measured as a function of time. Normally this test is conducted at constant temperature. The creep compliance can be determined from (4.8). A typical result of creep compliance versus time is shown in Figure 4-1.

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*The original VESYS II set the Poisson's ratio equal to one-half.

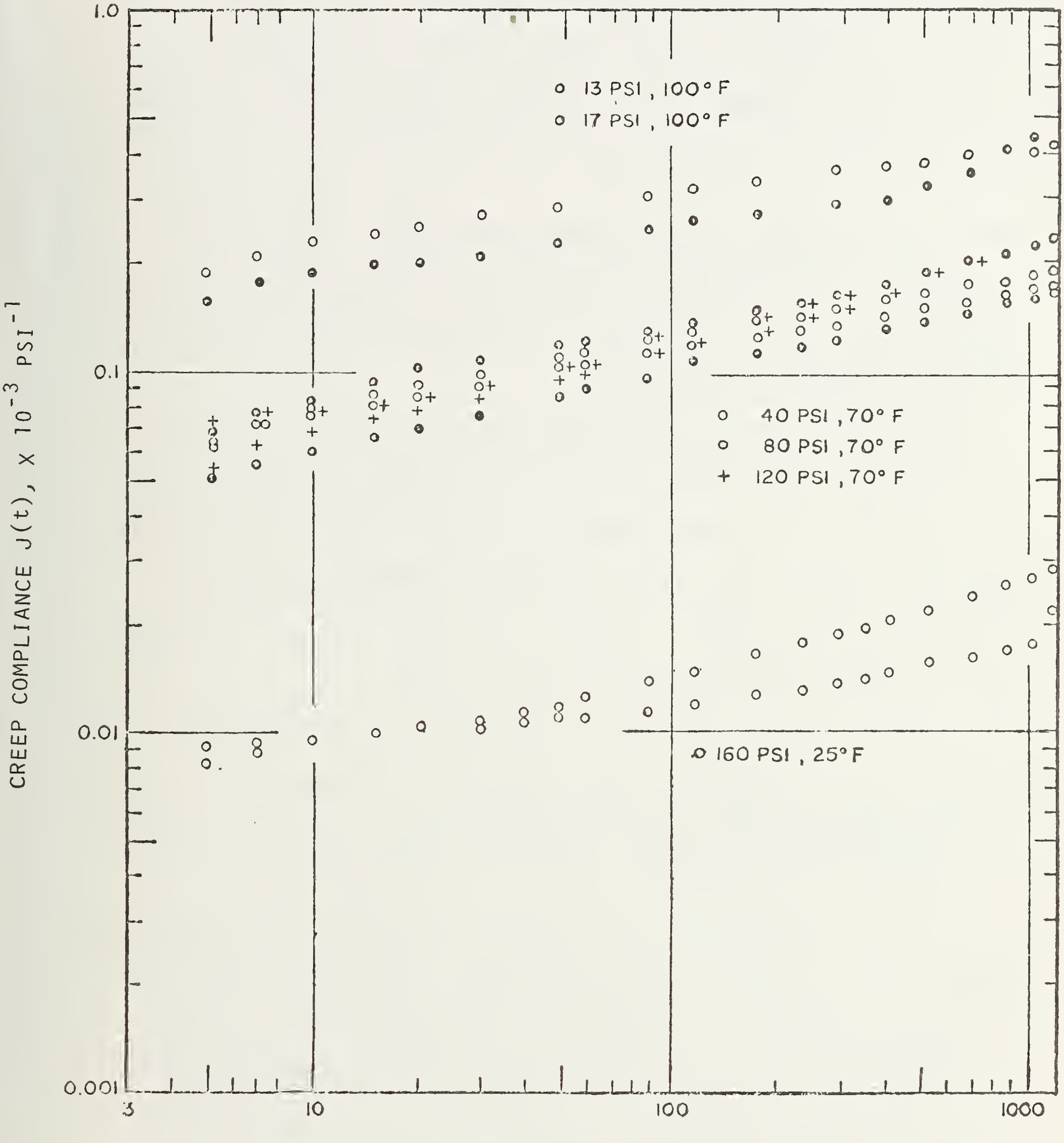

FIGURE 4-1 CONSTANT STRESS CREEP TESTS
(4.9% AC, HIGH VIS., 3% RUBBER)

Laboratory test results of creep compliances versus time are input to the VESYS IIM program. The program curve fits the creep compliances to an exponential (Dirichlet) series of the form:

$$J\ (t)\quad \sum_{i=1}^{N} G_i \exp\ (-t\delta_i) \tag{4-10}$$

where G_i are the coefficients of the Dirichlet series for the creep compliance; δ_i are the exponents of the series, they are generally chosen to be one order of magnitude apart from each other; and N is the number of terms in the series.

In the present VESYS IIM program, the coefficient of variation of J(t) is used to take into consideration the variability of the creep compliance. It can be estimated partially from a set of laboratory creep tests results and partially from the in-place quality control and the construction method. The above discussion is directly applicable to any pavement material exhibiting time dependency. This includes the bituminous treated base, clay soils. Creep of soil and the effect of moisture are discussed in a later section of this chapter.

4.5 TEMPERATURE EFFECT ON CREEP COMPLIANCE OF ASPHALT CONCRETE AND TIME-TEMPERATURE SUPERPOSITION

At different temperatures, the creep compliance and relaxation modulus of asphalt concrete will be different from those determined at a reference temperature. This dependency on temperature as well as on time of creep compliance can be expressed by the following equation:

$$J = J\ (T,t) \tag{4-11}$$

where T is the temperature and t is the time. This creep compliance function can be determined in principle, from a proper set of tests which require several creep tests under different isothermal temperature levels. This poses a problem if the number of tests required are very large. Furthermore, even if this approach is workable the analysis of pavement layer systems involving transient temperatures becomes very involved.

Fortunately, theoretical and experimental results for many viscoelastic materials [21, 22] and experimental results on asphalt concrete [20, 24, 25] indicate the effects due to time and temperature can be combined into a single parameter though the concept of the "time-temperature superposition principle" which implies that the following relations exist

$$J\ (T,t) = J\ (T_0,\zeta) \tag{4-12}$$

and

$$\zeta = t/a_T(T) \tag{4-12a}$$

where t is the actual time of observation measured from first application of load, T is the temperature and ζ is the "reduced time", and T_0 is the reference temperature. The reduced time is related to the real time t by the temperature shift factor $A_T(t)$, which is function of temperature. The time-temperature superposition principle cited above states that the effect of temperature on the time dependent mechanical behavior is equivalent to a stretching (or shrinking) of the real time for temperature above (or below) the reference. In other words, the behavior of material at high temperature and high strain rate is similar to that at low temperature and low strain rate. Materials exhibiting this property are called "Thermoheologically simple". Thus determination of the temperature shift factor $A_T(t)$ as a function of temperature will provide the necessary information for determination of the reduced time ζ. Figure 4-2 and 4-3 are the master creep curve and shift factor versus temperature from the creep compliances shown in Figure 4-1.

Therefore (4.10) becomes

$$J9t,(T) = \sum_{i=1}^{N} G_i \exp(-\zeta\delta_i) \tag{4.13}$$

for creep compliance at temperatures other than reference temperature.

4.6 RESILIENT MODULUS OF AGGREGATE BASE

Aggregate base materials are found to exhibit no creep under sustained load and therefore the response of aggregates under loading can best be represented by the resilient modulus. The resilient modulus of aggregates is usually obtained through a triaxial-compression repeated-load tests. Under the application of repeated stresses confining pressure plus axial deviatoric stress, the axial strain response is measured. The resilient modulus is defined as the ratio of a repeatedly applied deviatoric stress to the recoverable axial strain. The resilient modulus E_R of aggregate base material is found to be nonlinear function of mean normal stress,

$$E_R = K(\sigma)^n \tag{4-14}$$

Figure 4-4 shows the effect of confining stress on the resilient

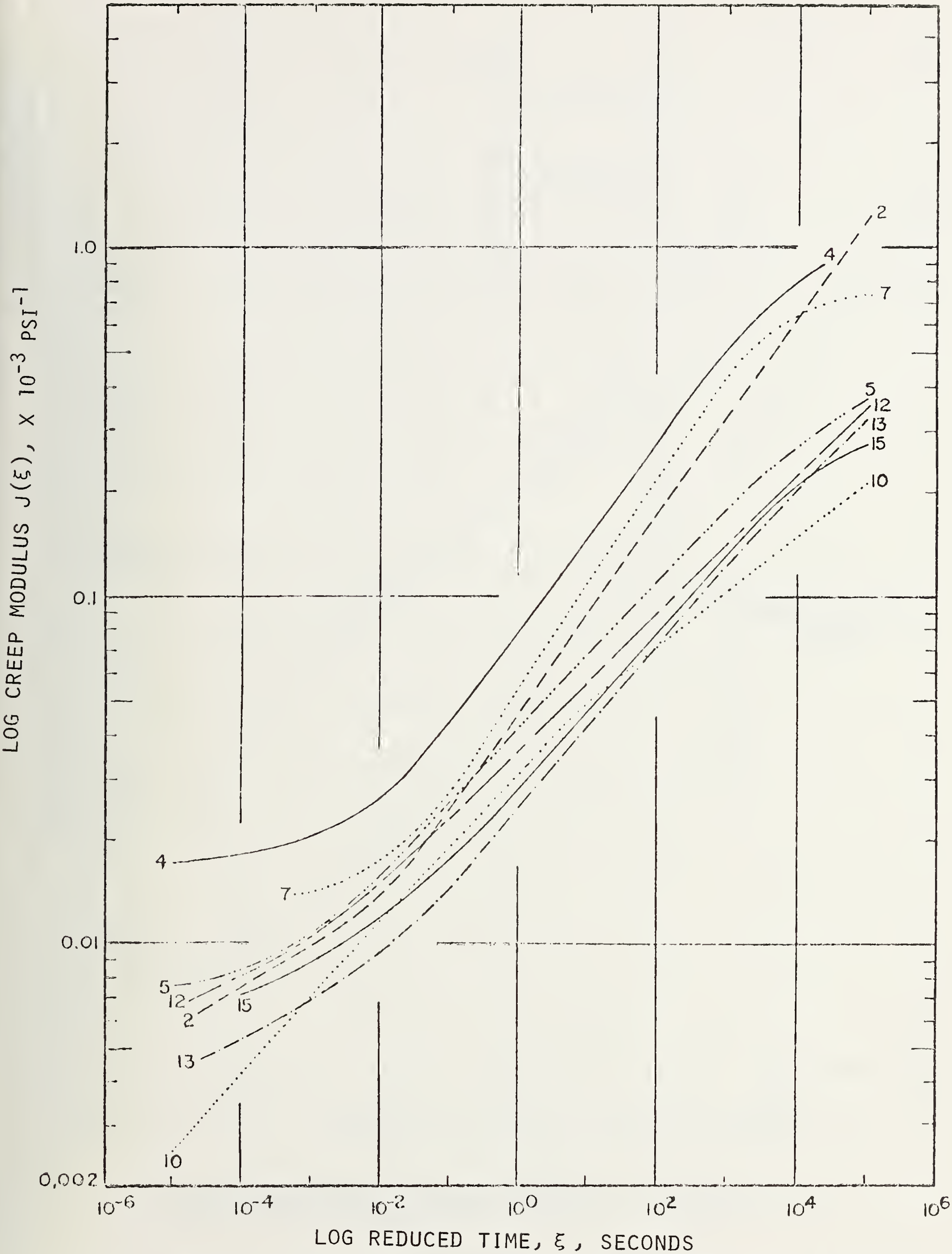

FIGURE 4-2 MASTER CURVES FOR UNCONFINED COMPRESSIVE CREEP MODULUS AT 70°F

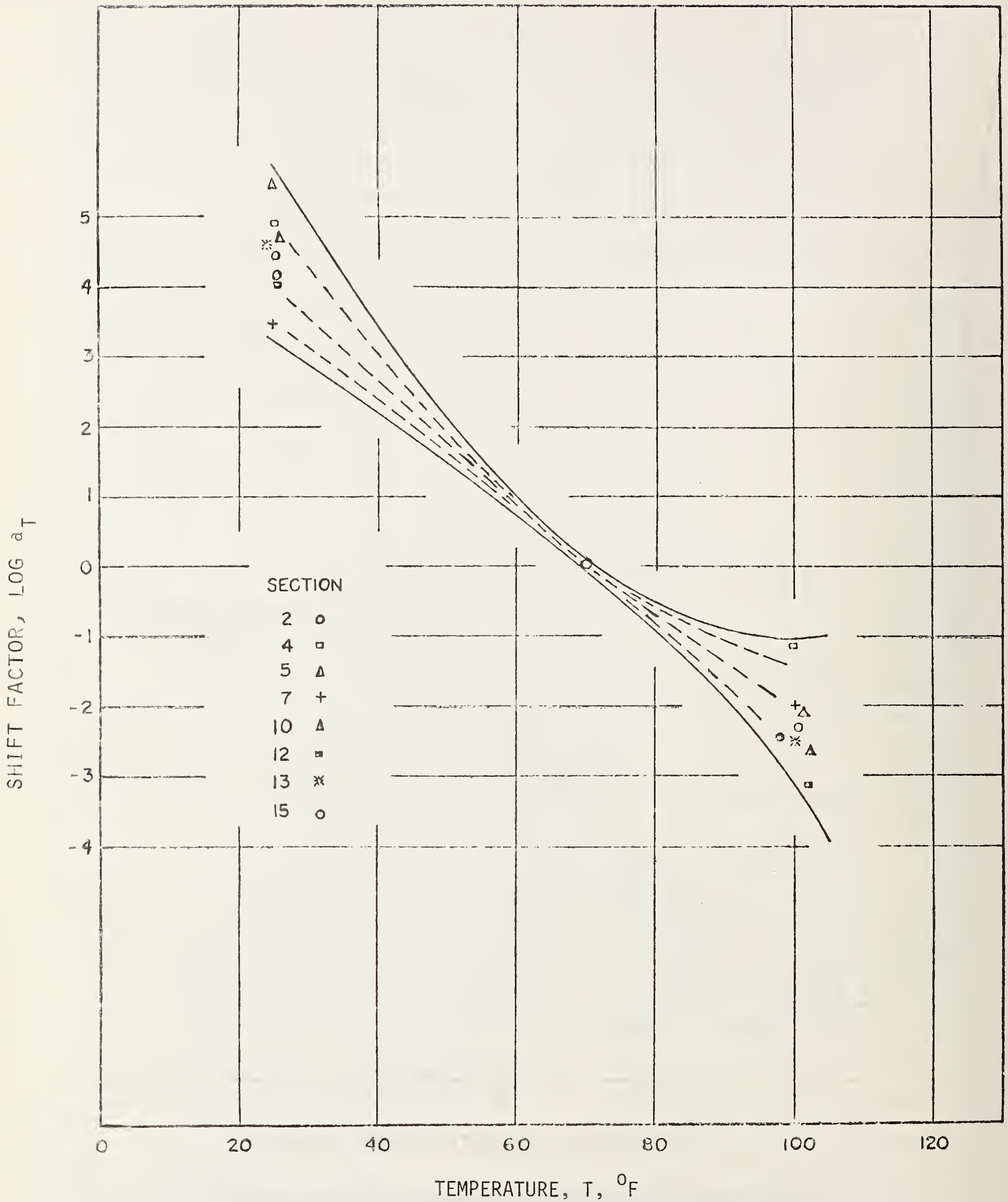

Figure 4-3 SHIFT FACTOR VERSUS TEMPERATURE

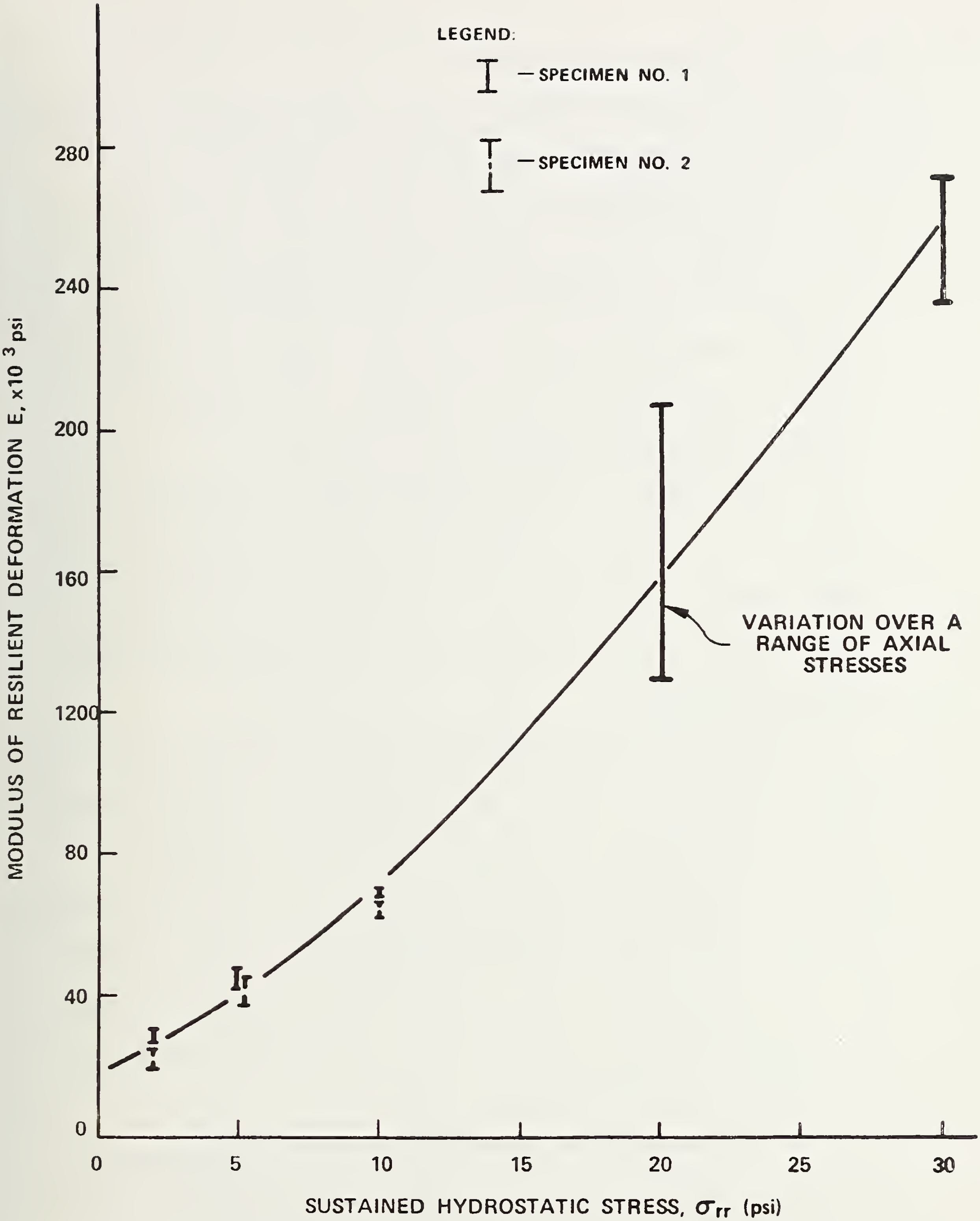

Figure 4-4 RESILIENT MODULUS FUNCTION FOR GRANULAR BASE VS. CONFINING STRESS

modulus of a granular base material.

In order to select a resilient modulus for an aggregate base material for use in the layerd system, the approximate mean normal stress induced in the aggregate base of the layered system when subjected to the anticipated external load has to be known as priori. This approximate mean normal stress can be obtained, through estimation or through iteration between the estimation of mean normal stress, the resilient modulus and the actual response of the system.

4.7 CREEP COMPLIANCE AND MOISTURE EFFECT OF SUBGRADE

The creep compliance of subgrade materials can be determined in a manner similar to the determination of the creep compliance of asphalt concrete. The only difference is that confining compression creep tests are usually used for subgrade materials instead of unconfined compression creep tests.

The significance of time-dependence of creep compliance of subgrade materials depends on the type of subgrade material and moisture content. Usually, temperature has little effect on the creep of subgrade materials. For silts and clays of high liquid limit and high plasticity, the materials exhibit more time-dependency than those with low liquid limit, and coarse grained soils. Moisture content in soils has very significant effect on the creep behavior of the material. This has been demonstrated in [26] and Figure 4-5.

From a phenomenological standpoint, the effect of moisture and time on the creep of subgrade materials can be combined into a single parameter using the concept of "time-moisture superposition principle" similar to the "time-temperature superposition principle" combining the

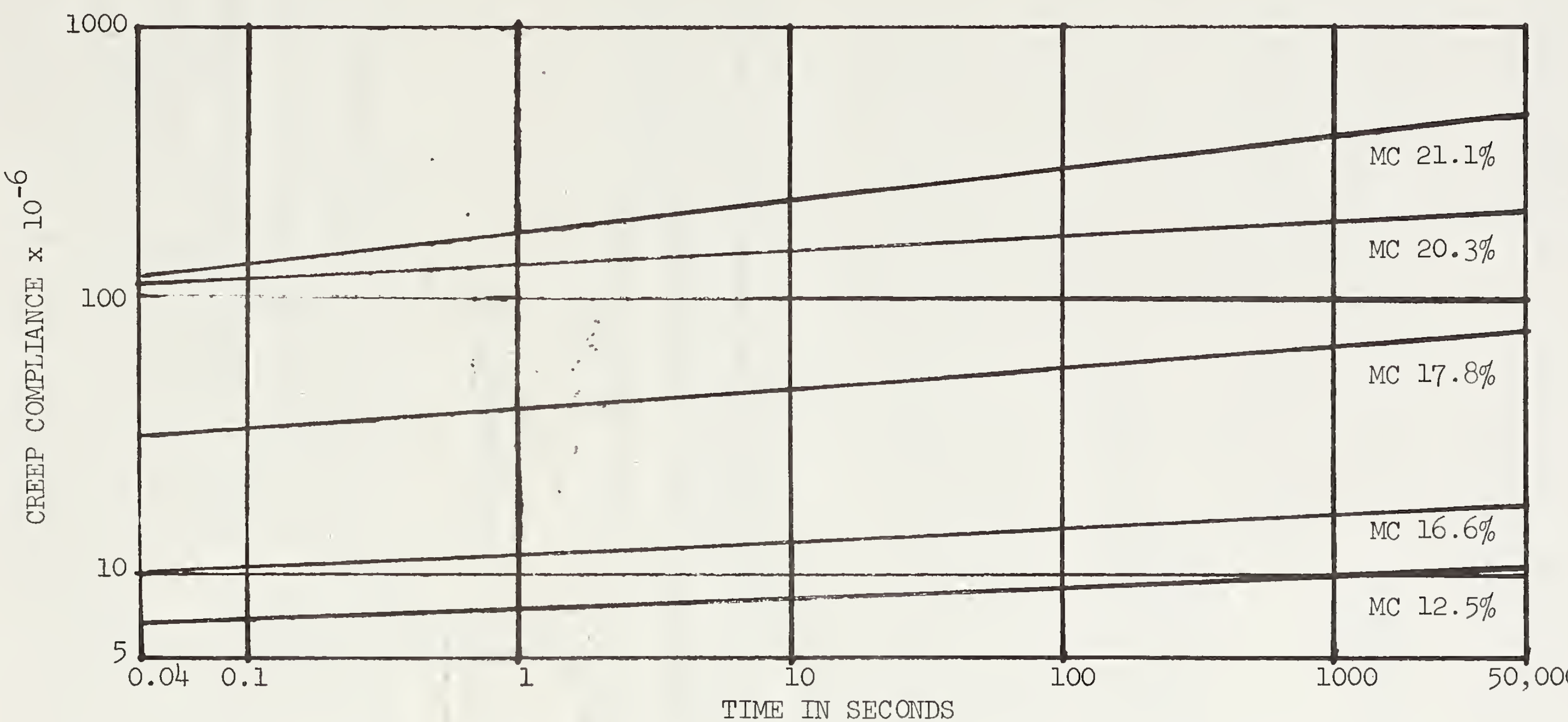

FIGURE 4-5 CREEP COMPLIANCE VS TIME FOR VARIOUS MOISTURE CONTENTS IN PERCENT OF THE PENN STATE TEST TRACK CLAY

effect of time and moisture into a single parameter called reduced time. Hence the creep compliance obtained at different moisture levels can be reduced to the following form

$$J(M,t) = J(M_o,\xi) \tag{4-15}$$

where

M = moisture content

M_o = reference moisture content

t = real time

ξ = moisture reduced time

and the real time and the reduced time are related by

$$\xi = t/A_m(M) \tag{4-16}$$

where $A_m(M)$ is called moisture shift factor which is a function of moisture content. Inserting (4.15) (4.16) into (4.10) yields the following expression for creep compliance at different moisture levels

$$(M,t) = \sum_{i=1}^{N} G_i \exp(-\xi\delta_i) \tag{4-17}$$

As mentioned above, the creep behavior of subgrade material is not dependent on temperature so long as the temperature is above freezing temperature. However, if the temperature does become a significant factor; that is, the creep compliance depends on both temperature and moisture, then

$$J(T, M, t) = J(T_o, M_o, \zeta) \tag{4-18}$$

where the temperature moisture reduced time ζ equals

$$\zeta = \frac{t}{A_T(t)\ A_m(M)} \tag{4-19}$$

and the moisture shift factor $A_m(M)$ and temperature shift factor $A_T(t)$ can be determined from (4.10) and (4.12a).

4.8 FATIGUE PROPERTIES OF ASPHALT CONCRETE

In the present VESYS IIM system conventional fatigue properties of asphalt concrete are used to predict the fatigue cracking of pavement surface; though eventually a fracture mechanics approach to the determination of pavement cracking should be used. Results of the fatigue properties of asphalt concrete have been collected by many investigators in the past ten years, see for example [24]. Typically fatigue test results follow a power-law function of the tensile strain amplitude. Hence, in the present VESYS IIM the following relation is used to represent the fatigue life (N_f) under a constant strain amplitude ($\Delta\varepsilon$)

$$N_f = K_1\ (\Delta\varepsilon)^{K_2} \tag{4-20}$$

K_1 and K_2 are coefficients related to the materials characteristics which can be determined from a set of fatigue tests. In general, these coefficients are generally sensitive to temperature and are also statistical in nature. The dependency of K_1 and K_2 on temperature is to be determined from further experimental data.

The coefficients K_1 and K_2 may be statistically correlated. The coefficient of correlation could vary between -1 and +1. This coefficient of correlation is an important effect on the analysis. If this coefficient is (-1) it means that an increase of K_1 corresponds necessarily to a decrease of K_2 while if the coefficient is zero it means that K_1 and K_2 are statistically independent of each other. These coefficients are given by their means, variances and coefficient of correlation.

The cumulative damage law based on Miner's rule as shown in the following:

$$\text{Damage} = \sum_{i=1}^{m} \frac{n}{N_f}\;_i \tag{4-21}$$

where

n = number of repetions of the strain designated by the range i

and

N_f = number of repetions in the range i required to produce failure.

CHAPTER 5

PRIMARY RESPONSE MODEL

5.1 INTRODUCTION

The VESYS II pavement design framework, as noted previously, solves for the stresses, strains and deflections in a three-layered, incompressible, viscoelastic system using a convolution integral formulation. The solution is only valid for three layers which are all incompressible. In the course of extending the solution to five layers and incorporating an approximate closed form probabilistic solution, it was decided that greater efficiencies could be obtained by discarding this primary response model and developing a general layered solution which is valid for an arbitrary number of layers (the limiting factor being computer storage capabilities) and for any value of Poisson's ratio. In addition, due to the time scale of loading for pavement systems the quasi-elastic approximations to the viscoelastic response is employed to further reduce computation time. It is demonstrated that acceptable solutions are obtained with this approximation.

An approximate closed-form probabilistic solution is obtained using the first order, second moment theory.

5.2 QUASI-ELASTIC ANALYSIS

The quasi-elastic method of analysis is based upon the work of Schapery [27] and is applicable to viscoelastic problems in which the derivative of the time-dependent solution with respect to logarithmic time, log t, is a slowly varying function of log t. The method involves replacing elastic moduli (or compliances) by instantaneous values of the relaxation moduli, and, as necessary, making a finite difference approximation to the Boltzmann Superposition integral of the loading function.

As an illustration of the procedure of performing a quasi-elastic analysis, the steps followed in obtaining a solution to a problem for a single layer are outlined below. The same procedure is followed for any number of layers.

1) Determine a value for the layer temperature

2) Subdivide the loading curve into a sequence of time intervals in such a manner that the curve is reasonably well approximated by a series of step functions

$$
\begin{aligned}
P(t) &= 0 && , \; t = 0 \\
P(t) &= \Delta P_1 && , \; \tau_1 < t < \tau_2 \\
P(t) &= \Delta P_1 + \Delta P_2 && , \; \tau_2 < t < \tau_3 \\
&\;\;\vdots \\
P(t) &= \Delta P_1 + \ldots + \Delta P_k && , \; \tau_K < t < \tau_{k+1}
\end{aligned}
$$

3) Perform a stress analysis for a unit load ($P(t) = 1$ psi)

for each of the time increments, τ_k, by some appropriate means using as an elastic compliance the value of the creep compliance corresponding to the reduced times τ_k/a_T.

4) Denoting the components of the displacment vector at the point x_k (k = 1, 2, 3) and time obtained from the analysis carried out in Step 3 above by w_i, the components W_i of displacement corresponding to the step-wise loading may be determined from the following set of equations*

$$w_i(x_k,t) = \Delta P_1 W_i(s_k,t-\tau_1) \; , \quad \tau_1 < t < \tau_2$$

$$w_i(x_k,t) = \Delta P_1 W_i(x_k,t-\tau_2) + P_i W_i(x_k,t-\tau_2) \; , \; \tau_2 < t < \tau_3$$

$$\vdots \qquad\qquad \vdots$$

$$w_i(x_k,t) = \sum_{n=1}^{N+1} \Delta P_n W_i(x_k,t-\tau_n)$$

The values, W_i, can be found from computer solutions at a few log t points plus linear interpolations in log t.

Although the above illustration was for a single layer, and statistical variations of the loading history and materials properties were excluded, inclusion of these effects is straight-forward, and material property variability is included in the computer code developed.

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* If the limits $N \to \infty$ and $\Delta P_k \to 0$ $\tau_n < t < t_{n+1}$ are taken (corresponding to a continuously varying pressure P(t)), this set of equations become the Boltzman Superposition Integeral

$$w_i(x_k,t) = \int_0^t W_i(x_k,t-\tau) \frac{dP(\tau)}{\partial \tau} d\tau$$

Figures 5-1 and 5-2 on Pages 53 and 54, respectively compare MIT published data for a three-layered viscoelastic system [15] with the solution for stress and deflection obtained using discrete moduli values and the N-layer program. The excellent agreement obtained indicates the applicability of the quasi-elastic approach.

5.3 ELASTIC N-LAYER SOLUTION

An idealized layered system can be represented by multilayers of different thicknesses and elastic material characteristics as shown in Figure 5-3, in which a normal load of intensity q is distributed over a circular area of radius a.

The layer materials are assumed to be linearly elastic, isotropic, and weightless. The layers are assumed to be infinite in horizontal extent and the lower layer (n-th layer) is considered to be semi-infinite in the vertical direction.

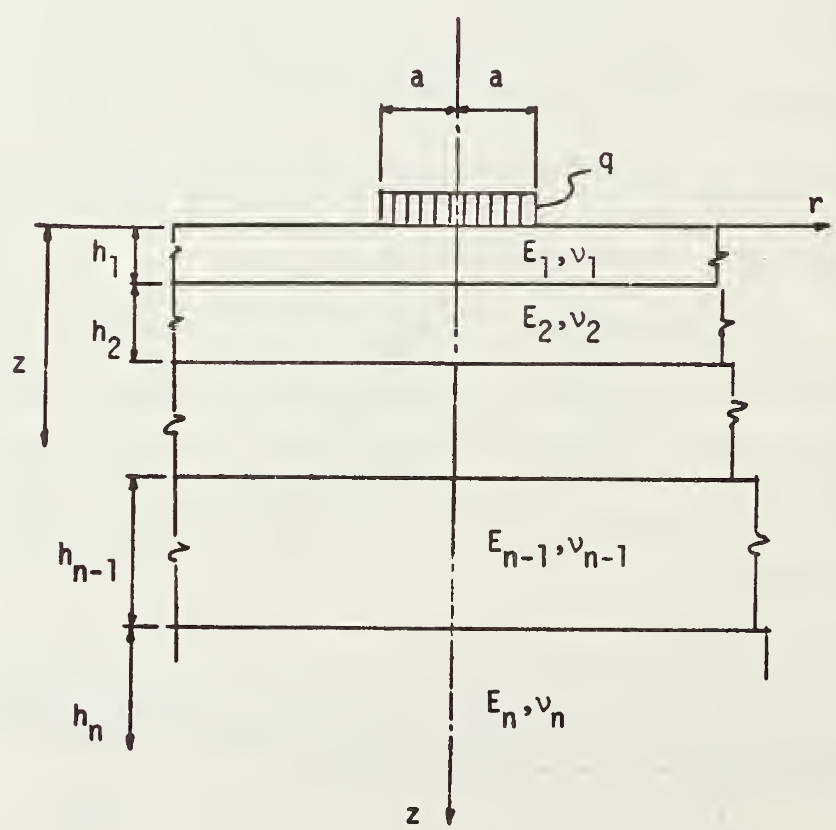

Figure 5-3 **An n-Layer Pavement System**

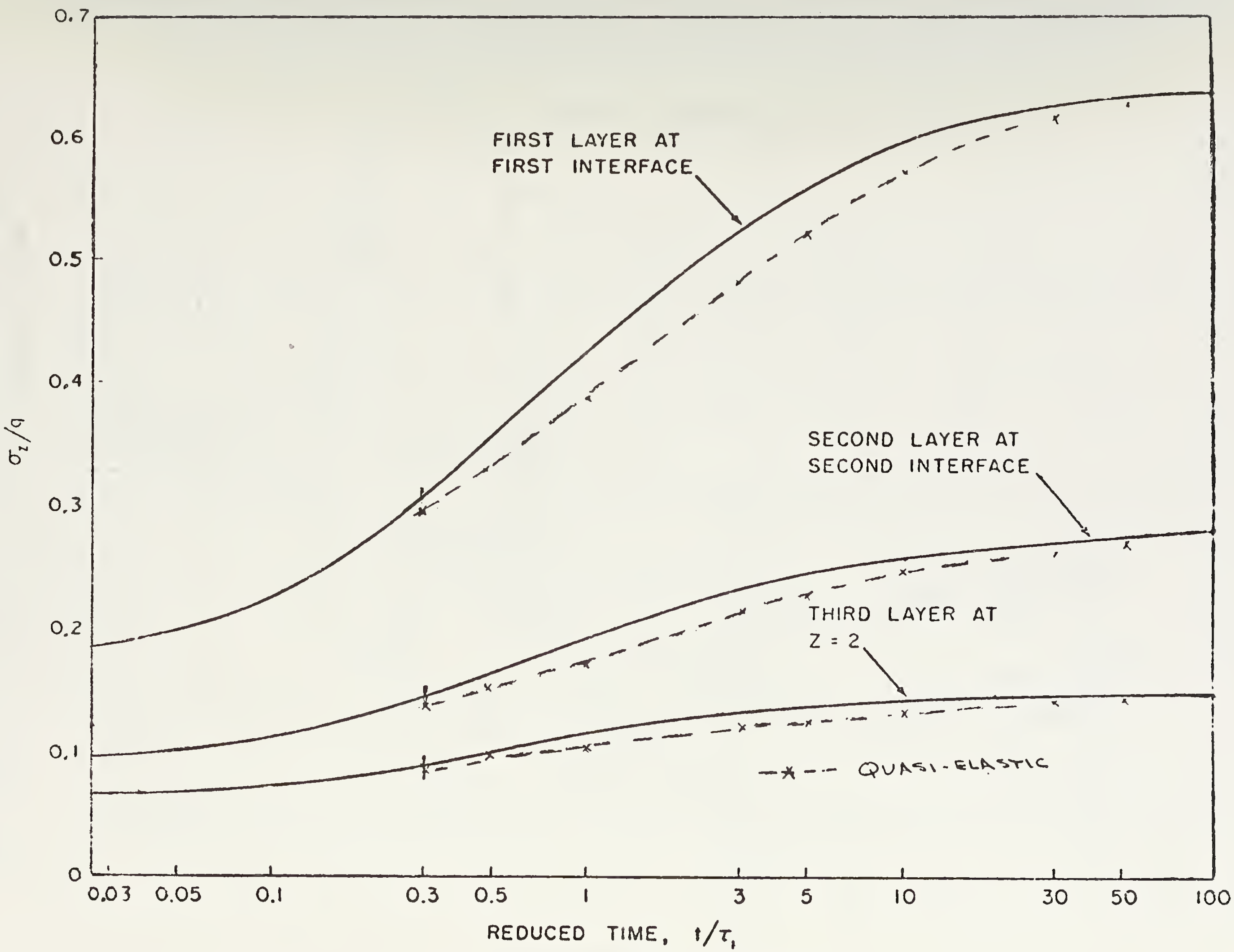

Figure 5-1 NORMAL STRESS VS REDUCED TIME

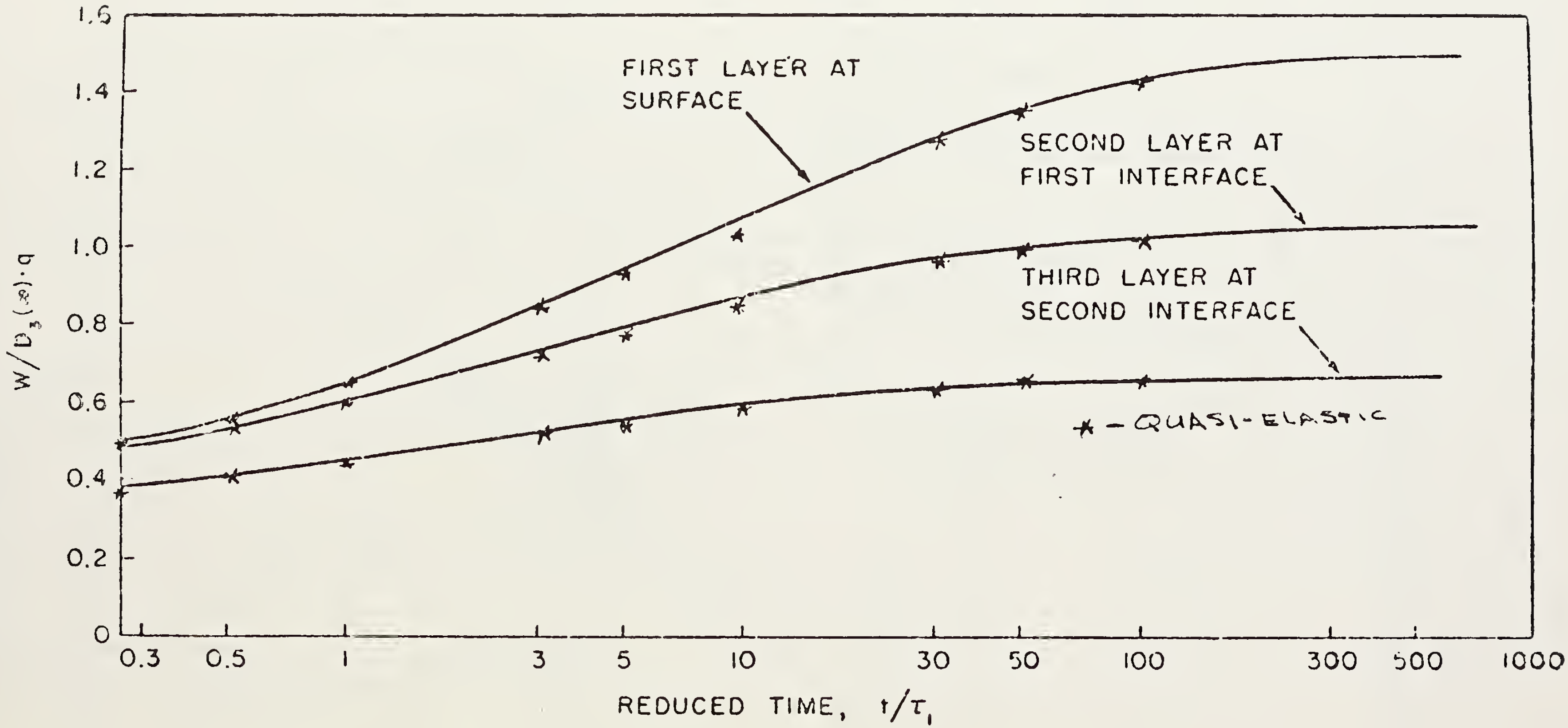

Figure 5-2 VERTICAL DEFLECTION VS REDUCED TIME

In order to solve for the components of stresses and strains at any point of the structure, linear small deformation elasticity theory is employed. A stress function $\phi(r,z)$ which satisfies the governing differential equation is determined for each layer.

$$\nabla^4\phi = 0 \tag{5-1}$$

In cylindrical coordinates, (Figure 5-4) the equations of equilibrium have the form

$$\frac{\partial\sigma_r}{\partial r} + \frac{\partial\tau_{rz}}{\partial z} + \frac{\sigma_r-\sigma_\theta}{r} = 0 \tag{5-2}$$

$$\frac{\partial\sigma_z}{\partial z} + \frac{\partial\tau_{zr}}{\partial r} + \frac{\tau_{zr}}{r} = 0 \tag{5-3}$$

where σ_r, σ_z, τ_{rz}, τ_{zr}, $\tau_{\theta r}$, and $\tau_{r\theta}$ are components of the stress tensor with

$$\tau_{r\theta} = \tau_{\theta r} = 0$$

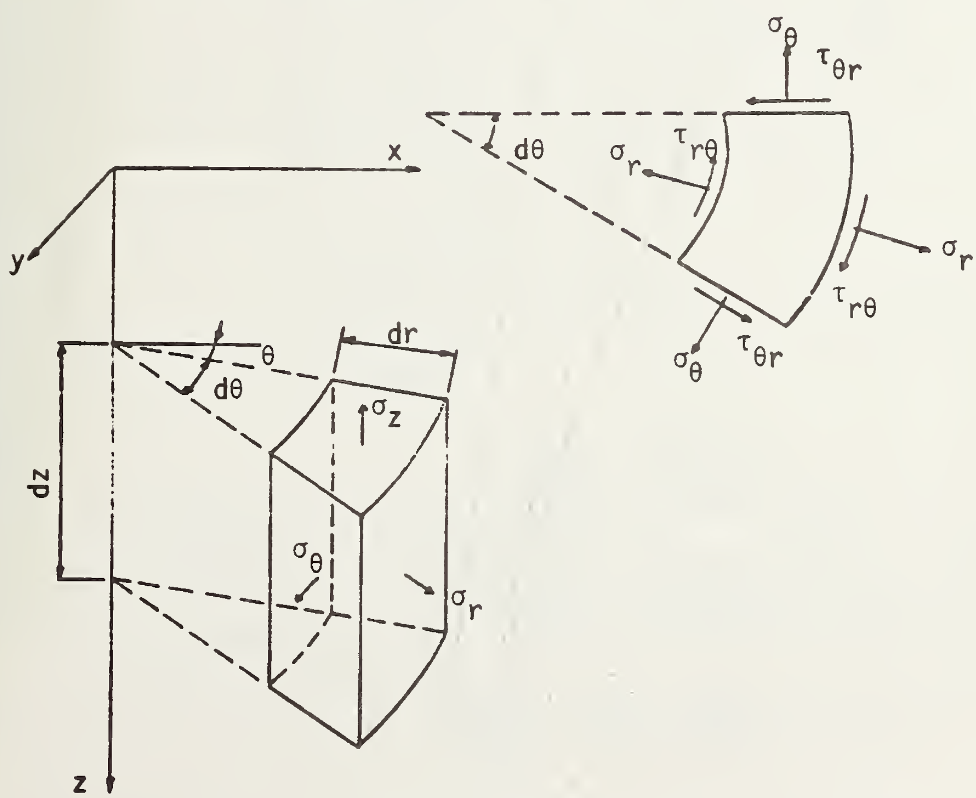

Figure 5-4 Stress State of an Element in Pavement System

The equations of compatibility in terms of stresses are given by

$$\nabla^2\sigma_r - \frac{2}{r^2}(\sigma_r - \sigma_\theta) + \frac{1}{1+\nu}\frac{\partial^2\Theta}{\partial r^2} = 0 \tag{5-4}$$

$$\nabla^2\sigma_\theta + \frac{2}{r^2}(\sigma_r - \sigma_\theta) + \frac{1}{1+\nu}\frac{\partial\Theta}{r\partial r} = 0 \tag{5-5}$$

$$\nabla^2\sigma_z + \frac{1}{1+\nu}\frac{\partial^2\Theta}{\partial z^2} = 0 \qquad \text{(5-6}$$

$$\nabla^2\tau_{rz} - \frac{1}{r^2}\tau_{rz} + \frac{1}{1+\nu}\frac{\partial^2\Theta}{\partial r\partial z} = 0 \tag{5-7}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{5-8}$$

$$\nabla^4 = \nabla^2\nabla^2$$

$$\Theta = \sigma_r + \sigma_\theta + \sigma_z$$

The stresses and displacements are given by

$$\sigma_r = \frac{\partial}{\partial z}\left[\nu\nabla^2\phi - \frac{\partial^2\phi}{\partial r^2}\right] \tag{5-9}$$

$$\sigma_\theta = \frac{\partial}{\partial z}\left[\nu\nabla^2\phi - \frac{1}{r}\frac{\partial\phi}{\partial r}\right] \tag{5-10}$$

$$\sigma_z = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{5-11}$$

$$\tau_{rz} = \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{5-12}$$

$$u = -\frac{1+\nu}{E}\,\frac{\partial^2\phi}{\partial r\partial z} \tag{5-13}$$

$$w = \frac{1+\nu}{E}\left[2(1-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{5-14}$$

$$v = 0 \tag{5-15}$$

where

u is the radial displacement

v is the tangential displacement, and

w is the vertical displacement

The continuity conditions at the i-th interface are given by

$$\sigma_z^i = \sigma_z^{i+1} \tag{5-16}$$

$$\tau_{rz}^i = \tau_{rz}^{i+1} \tag{5-17}$$

$$u^i = u^{i+1} \tag{5-18}$$

$$w^i = w^{i+1} \tag{5-19}$$

Equations (5-16 to 5-19) state that each layer at the i-th interface is considered to be "rough", that is the components of stress σ_z and τ_{rz} as well as the components of displacement u and w are continuous at $z = h_i$ (i - 1, 2, ..., n-1).

Since the region of interest is infinite in the r direction, the biharmonic equation is readily solved using Hankel Transforms [28]

$$\hat{\phi}(m,z) = \int_0^\infty r\phi(r,z)\, J_0(mr)dr \tag{5-20}$$

The inverse function is

$$\phi(r,z) = \int_0^\infty m\hat{\phi}(m,z)\, J_0(mr)dm \tag{5-21}$$

For partial differential $\nabla^2\phi$, it follows that

$$\nabla^2\phi = \int_0^\infty m\left[\frac{d^2\hat{\phi}}{dz^2} - m^2\hat{\phi}\right] J_0(mr)dm \tag{5-22}$$

and

$$\int_0^\infty r\nabla^2\phi\, J_0(mr)dr = \left(\frac{d^2}{dz^2} - m^2\right) \int_0^\infty r\phi\, J_0(mr)dr$$

$$\int_0^\infty r\nabla^4\phi\, J_0(mr)dr = \left(\frac{d^2}{dz^2} - m^2\right)^2 \int_0^\infty r\phi\, J_0(mr)dr \tag{5-23}$$

Thus instead of solving biharmonic equation (5-1), one is led to the solution of the ordinary differential equation

$$\left(\frac{d^2}{dz^2} - m^2\right)^2 \hat{\phi}(m,z) = 0 \tag{5-24}$$

where $\hat{\phi}$ is defined in Equation (5-20). The solution of Equation (5-24) leads to the expression

$$\hat{\phi}(m,z) = (A+Bz)e^{mz} + (C+Dz)e^{-mz} \tag{5-25}$$

For the i-th layer of the system, Equation (5-25) can be written

$$\hat{\phi}_i(m,z) = (A_i+B_iz)e^{mz} + (C_i+D_iz)e^{-mz} \tag{5-26}$$

where A_i, B_i, C_i and D_i, are constants of integration, to be determined in such a way that the boundary conditions are satisfied.

Now, multiplying $\hat{\phi}_i$ by $J_o(mr)$ in Equation (5-26), and defining $\tilde{\phi}_i(m,r,z)$ as the product of $\hat{\phi}(m,z)\ J_o(mr)$

$$\tilde{\phi}_i(m,r,z) = J_o(mr)\left[(A_i+B_iz)e^{mz} + (C_i+D_iz)e^{-mz}\right] \tag{5-27}$$

it can be verified that Equation (5-1) is satisfied by the function $\tilde{\phi}_i(m,r,z)$.

Making use of Equation (5-27), Equations (5-9) through (5-14) become

$$\tilde{\sigma}_z^i = m^2J_o(mr)\{(1-2\nu_i)[B_ie^{mz} + D_ie^{-mz}] - m[(A_i+B_iz)e^{mz} - (C_i+D_iz)e^{-mz}]\} \tag{5-28}$$

$$\tilde{\tau}_{rz}^i = m^2J_1(mr)\{2\nu_i[B_ie^{mz} - D_ie^{-mz}] + m[(A_i+B_iz)e^{mz} + (C_i+D_iz)e^{-mz}]\} \tag{5-29}$$

$$\tilde{\sigma}_r^i = m^2J_o(mr)\{(1+2\nu_i)[B_ie^{mz}+D_ie^{-mz}] + m[(A_i+B_iz)e^{mz} - (C_i+D_iz)e^{-mz}]\}$$

$$-m^2\frac{J_1(mr)}{mr}\{B_ie^{mz}+D_ie^{-mz} + m[(A_i+B_iz)e^{mz} - (C_i+D_iz)e^{-mz}]\} \tag{5-30}$$

$$\tilde{\sigma}_\theta^i = 2\nu_im^2J_o(mr)[B_ie^{mz}+D_ie^{-mz}] + m^2\frac{J(mr)}{mr}\{B_ie^{mz}+D_ie^{-mz}+m[(A_i+B_iz)e^{mz}$$

$$-(C_i+D_iz)e^{-mz}]\} \tag{5-31}$$

$$\tilde{u}^i = \frac{1+\nu_i}{E_i}mJ_1(mr)\{B_ie^{mz}+D_ie^{-mz}+m[(A_i+B_iz)e^{mz} - (C_i+D_iz)e^{-mz}]\} \tag{5-32}$$

$$\tilde{w}^i = \frac{1+\nu_i}{E_i} J_o(mr)m\{2(1-2\nu_i)[B_i e^{mz}-D_i e^{-mz}] -m[(A_i+B_i z)e^{mz}+(C_i+D_i z)e^{-mz}]\} \quad (5\text{-}33)$$

Substituting Equations (5-28) to (5-31) into Equations (5-16) and (5-19) and rewriting,

$$S_i(z) = \begin{bmatrix} \tilde{\sigma}_z^i \\ \tilde{\tau}_{rz}^i \\ \tilde{u}^i \\ \tilde{w}^i \end{bmatrix} = K(\nu_i,E_i)\ M(z,\nu_i)\ D(z) \begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix} \quad (5\text{-}34)$$

where

$$K(\nu_i,E_i) = \begin{bmatrix} -m^2 J_o(mr) & & & \\ & m^2 J_1(mr) & & \\ & & \frac{1+\nu_i}{E_i} mJ_1(mr) & \\ & & & \frac{1+\nu_i}{E_i} mJ_o(mr) \end{bmatrix} \quad (5\text{-}35)$$

$$M(z,\nu_i) = \begin{bmatrix} 1 & mz+2\nu_i-1 & -1 & -mz+2\nu_i-1 \\ 1 & mz+2\nu_i & 1 & mz-2\nu_i \\ 1 & mz+1 & -1 & -mz+1 \\ 1 & mz+4\nu_i-2 & 1 & mz-4\nu_i+2 \end{bmatrix} \quad (5\text{-}36)$$

and

$$D(z) = \begin{bmatrix} me^{mz} & & & \\ & e^{mz} & & \\ & & me^{-mz} & \\ & & & e^{-mz} \end{bmatrix} \quad (5\text{-}37)$$

Continuity conditions at the i-th interface can be expressed by

$$S_i(h_i-0) = S_{i+1}(h_i) \tag{5-38}$$

or

$$\begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix} = \frac{D^{-1}(h_i)X_iD(h_i)}{4(\nu_i-1)} \begin{bmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{bmatrix} \tag{5-39}$$

and

$$\begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix} = \prod_{j=i}^{n-1} \frac{D^{-1}(h_j)X_jD(h_j)}{4(\nu_j-1)} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix} \tag{5-40}$$

$$x_i = \begin{bmatrix} (4\nu_i-3)-L_i & \begin{matrix}-q(-h_i,\nu_i,\nu_{i+1}) \\ +L_iq(-h_i,\nu_{i+1},\nu_i)\end{matrix} & \begin{matrix}(2mh_i+4\nu_i-1) \\ (1-L_i)\end{matrix} & \begin{matrix}p(h_i,\nu_i,\nu_{i+1}) \\ -L_ip(h_i,\nu_{i+1},\nu_i)\end{matrix} \\ 0 & L_i(4\nu_{i+1}-3)-1 & 2(L_i-1) & \begin{matrix}(2mh_i-4\nu_{i+1}+1) \\ (L_i-1)\end{matrix} \\ \begin{matrix}(2mh_i-4\nu_i+1) \\ (L_i-1)\end{matrix} & \begin{matrix}-p(-h_i,\nu_i,\nu_{i+1}) \\ +L_ip(h_i,\nu_{i+1},\nu_i)\end{matrix} & (4\nu_i-3)-L_i & \begin{matrix}q(h_i,\nu_i,\nu_{i+1}) \\ -L_iq(h_i,\nu_{i+1},\nu_i)\end{matrix} \\ 2(1-L_i) & \begin{matrix}(2mh_i+4\nu_{i+1}-1) \\ (1-L_i)\end{matrix} & 0 & L_i(4\nu_{i+1}-3)-1 \end{bmatrix} \tag{5-41}$$

where $p(h,\nu,\mu) = 2(mh)^2 + 4mh(\nu-\mu) + 1 - 8\nu\mu + 2\mu$

$q(h,\nu,\mu) = 2mh(2\nu-1) - (1 + 8\mu\nu - 6\mu)$

$L_i = \frac{E_i}{E_{i+1}} \frac{1+\nu_{i+1}}{1+\nu_i}$

So far the boundary conditions at the interface have been considered. At the loading surface a vertical uniform loading may be assumed in the form

$$p(m)J_0(mr) \tag{5-42}$$

By imposing $\sigma_z^1\big|_{z=o} = -p(m)J_0(mr)$, Equation (5-28) reduces to

$$mA_1 + (2\nu_1-1)B_1 - m\,C_1 + (2\nu_1-1)\,D_1 = \frac{p(m)}{m^2} = K(m) \tag{5-43}$$

By imposing $\tau_{rz}^1\big|_{z=o} = 0$, Equation (5-29) reduces to

$$mA_1 + 2\nu_1B_1 + mC_1 - 2\nu_1D_1 = 0 \tag{5-44}$$

Equations (5-43) and (5-44) can be written as

$$\begin{bmatrix} 0 \\ K(m) \end{bmatrix} = \begin{bmatrix} m & 2\nu_1 & m & -2\nu_1 \\ m & 2\nu_1-1 & -m & 2\nu_1-1 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} \tag{5-45}$$

or

$$\begin{bmatrix} 0 \\ K(m) \end{bmatrix} = \begin{bmatrix} Q \end{bmatrix} \begin{bmatrix} \prod_{j=1}^{n-1} \dfrac{D(h_j)^{-1}X_jD(h_j)}{4(\nu_j-1)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix} \tag{5-46}$$

where

$$Q = \begin{bmatrix} m & 2\nu_1 & m & -2\nu_1 \\ m & 2\nu_1-1 & -m & 2\nu_1-1 \end{bmatrix} \tag{5-47}$$

The constants C_n and D_n are therefore determined. Knowing C_n and D_n, the values of A_i, B_i, C_i and D_i are obtainable from Equation (5-40) since Equations (5-9) and (5-14) are linear functions in ϕ, the stress function

$$\phi(r,z) = \int_0^\infty \tilde{\phi}(m,r,z)dm \tag{5-48}$$

will solve Equations (5-9) and (5-14) when subjected to a load

$$\int_0^a p(m)J_0(mr)dm \tag{5-49}$$

The inverse formula for the Hankel Transform is

$$\frac{1}{2}[f(r-0) + f(r+0)] = \int_0^\infty m[r\ f(r)J_0(mr)dr]\ J_0(mr)dm \tag{5-50}$$

Therefore p(m) can be expressed as

$$\begin{aligned} p(m) &= m\int_0^\infty r\ f(r)J_0(mr)dr \\ &= a\ J_1(ma) \end{aligned} \tag{5-51}$$

where

$$f(r) = \begin{cases} 1 & r^2 < a^2 \\ \frac{1}{2} & r^2 = a^2 \\ 0 & r^2 > a^2 \end{cases}$$

The stresses and displacements at any depth in the i-th layer are then given by

$$\bar{\sigma}_z^i = a\int_0^\infty J_1(ma)\tilde{\sigma}_z^i dm \tag{5-52}$$

$$\bar{\tau}_{rz}^i = a\int_0^\infty J_1(ma)\tilde{\tau}_{rz}^i dm \tag{5-53}$$

$$\bar{\sigma}_r^i = a\int_0^\infty J_1(ma)\tilde{\sigma}_r^i dm \tag{5-54}$$

$$\bar{\sigma}_\theta^i = a\int_0^\infty J_1(ma)\tilde{\sigma}_\theta^i dm \tag{5-55}$$

$$\bar{u}^i = a\int_0^\infty J_1(ma)\tilde{u}^i dm \tag{5-56}$$

$$\bar{w}^i = a\int_0^\infty J_1(ma)\tilde{w}^i dm \tag{5-57}$$

5.4 PROBABILISTIC N-LAYER SOLUTION

Since the external loading, environmental conditions and internal system parameters of a pavement structure are generally random quantities, the underlying reliability theories must employ probablistic concepts. However, most structural analyses are deterministic with uncertainties reflected in design safety factors. These safety factors are not normally based on mathematical models describing the interplay between the various uncertainties.

Generally speaking, it is impossible to determine the characteristics of an uncertain quantity given as a nonlinear function of other uncertain quantities soley by the characteristics of the set of these uncertain quantities. Such a determination is possible only if the distribution function of the set is known and satisfies certain integrability requirements [29].

Solving stochastic differential equations is one currently prevalent method for structural problems with simple geometries [30]. In applying this method, however, not only must the problems be limited to simple geometries and material properties, but simplifying assumptions must also be made for the random variables, such as assuming normal distributions, because of the complexity of the differential operators involved. Moreover, the mathematics involved is often so complicated that the problem becomes intractable [31]. An appropriate probabilistic model was developed in this program since the application of full probability theory has several practical limitations [4]:

1) The mathematics becomes complicated.
2) Available data and information about uncertainties are often too scarce for practical implementation.

Additional practical considerations which support development of approximate probabilistic models include [4].

1) Near the mean, probabilities are insensitive to the distribution type.
2) At the tails, data and models are too weak to predict reliable absolute probabilities; but many distributions give similar relative changes in probability per unit increase in the standardization abscissa.
3) When expected cost optimization is used to determine engineering design, first and second moments are sufficient if the cost function is quadratic.
4) Engineering design and cost are very insensitive to small changes in probabilities (the relationship usually being approximately logarithmic).

The approach followed in the program is based on Cornell's first-order, second moment theory [4]. The term "first-order" indicates the application of linearization to nonlinear equations and the term "second-moment" refers to the fact that only the first two moments of the random variables involved are needed.

This approach is believed to be superior to other approaches, such as the use of maximum likelihood methods or moment generating functions, since these approaches require knowledge of the distributions of the random variables [32] whereas the first-order, second-moment theory does not require knowledge of the distributions of the random variables.

5.4.1 First Order, Second Moment Theory

In the application of a first-order, second-moment approximation, the term "first-order: indicates linearization of nonlinear equations and the term "second-moment" refers to the fact that only the first two moments of the random variables are needed. For example, some random variables X can be characterized by their mean, $\overline{X}$, and variance, Var $[X] = \sigma_X^2$ (or equavalently their standard deviation, σ_X, or coefficient of variation $V_X = \sigma_X/\overline{X}$). The stochastic dependence between a random variable X and a random variable Y is measured by their covariance, Cov[X,Y], or equivalently by their correlation coefficient

$$\rho_{XY} = \frac{\text{Cov }[X,Y]}{\sigma_X \, \sigma_Y} \tag{5-58}$$

Similarly, a random process X(t) is defined by its mean function $\overline{X}(t)$ and is covariance function Cov[X(t), X(s)] (possibly cross-covariance function).

Linearization of a nonlinear equation is illustrated in the following example. Let Y be a continuous function of X,

$$Y = g(X) \tag{5-59}$$

where X and Y are random variables. The mean of X is denoted by $\overline{X}$. Expanding g(X) about the mean $\overline{X}$ in a Taylor series expansion, then

$$g(X) = g(\overline{X}) + (X-\overline{X}) \frac{dg(X)}{dX} \bigg|_{X=\overline{X}} + (X-\overline{X})^2 \frac{d^2g(X)}{dX^2} \bigg|_{X=\overline{X}} + \ldots \tag{5-60}$$

Retaining only the first two terms of the expansion and taking the expectation of both sides then yields the approximation

$$E[Y] = E[g(X)] \doteq g(E(X)) \tag{5-61}$$

for E[g(x)] since $E(X-\overline{X}) = 0$. Similarly, retaining the same terms, the approximation for the variance is given by

$$\mathrm{Var}[Y] = \mathrm{Var}[g(X)] \doteq \mathrm{Var}[X] \left(\frac{dg(X)}{dX} \bigg|_{X-\overline{X}} \right)^2 \tag{5-62}$$

Since $g(\overline{X})$ is a constant,

$$\mathrm{Var}[g(\overline{X})] = 0 \tag{5-63}$$

and

$$\mathrm{Var}\left[\left(\frac{dg(X)}{dX} \bigg|_{X=\overline{X}} \right) (X-\overline{X}) \right] = \mathrm{Var}[X] \left(\frac{dg(X)}{dX} \bigg|_{X=\overline{X}} \right)^2 \tag{5-64}$$

Using this same principle for a variable, say Z, which is a continuous function of several variables, $X_1, X_2, \ldots, X_n$, - whose means, variances, and covariances are known - the first order approximation of the mean and variance of Z can be obtained by expanding the function $g(X_1, X_2, \ldots, X_n)$ in a multiple-variable Taylor series, about the mean values of the X's. Keeping only the linear terms in the deviation from the means and applying the same linear transformation rules for the mean and variance yields the approximations

$$E[Z] \doteq \overline{Z} \equiv g\,(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n) \tag{5-65}$$

and

$$\mathrm{Var}[Z] \doteq \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i} \Big|_{X=\overline{X}} \right)^2 \mathrm{Var}[X_i]$$

$$+ \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(\frac{\partial g}{\partial X_i} \Big|_{X=\overline{X}} \right) \left(\frac{\quad}{\partial X_j} \Big|_{X=\overline{X}} \right) \mathrm{Cov}[X_i, X_j] \qquad (5\text{-}66)$$

in which the partial derivatives are evaluated at the mean values of X_i and X_j, i.e., $\overline{X}$; and $\overline{X}_j$.

If the random variables X_i and X_j are independent, then Equation (5-66) simplifies to

$$\mathrm{Var}[Z] \doteq \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i} \Big|_{X=\overline{X}} \right)^2 \mathrm{Var}\,[X_i] \qquad (5\text{-}67)$$

Since $\mathrm{Cov}[X_i, X_j] = 0$ for X_i and X_j uncorrelated.

Many details described by full probability theory are not described by the first order second-moment theory; however, the main idea is to extend a deterministic description to obtain the essential behavior of the uncertainty interplay with as few new concepts as possible. The results obtained are independent of any assumptoins about the form of the underlying probability distributions. If appropriate assumptions can be adopted, the description of the moments of the random variables may be extended to some probability laws.

5.4.2 Closed Form Probabilistic Solution

The development of an approximate closed form probabilistic solution for the stresses and deflections given in Section 5.3 proceeds in the following manner. Let

$$S_i = g_i(E_1, E_2, \ldots, E_n) \qquad (5\text{-}68)$$

where S_i represents either the desired stress or displacement at i-th layer; $E_1, E_2, \ldots, E_n$ are the elastic moduli, which are assumed to be the only random variables in the system. The E_i are further assumed to be independent. Then, from the previous section the expected value of S_i may be approximated by the expression

$$E[S_i] = g_i(\bar{E}_1, \bar{E}_2, \ldots, \bar{E}_n) + \frac{1}{2}\sum_{j=1}^{n} \left.\frac{\partial^2 g_i}{\partial E_j^2}\right|_{\bar{E}_j} \sigma_{E_j}^2 \qquad (5\text{-}69)$$

The variance of S_i is given by

$$\mathrm{Var}[S_i] = \sum_{j=1}^{n} \left(\left.\frac{\partial g_i}{\partial E_j}\right|_{\bar{E}_j} \right)^2 \sigma_{E_j}^2 \qquad (5\text{-}70)$$

The term $g_i(\bar{E}_1, \bar{E}_2, \ldots, \bar{E}_n)$ represents the mean value of S_i , i.e., the value of S_i obtained using mean moduli values $\bar{E}_i$. The terms

$$\left.\frac{\partial^2 g_i}{\partial E_j^2}\right|_{\bar{E}_j} \sigma_{F_j}^2 \qquad \text{and} \qquad \left(\left.\frac{\partial g_i}{\partial E_j}\right|_{\bar{E}_j} \right)^2 \sigma_{E_j}^2$$

are the 2nd and 1st partial derivatives of S_i with respect to E_j evaluated at $\bar{E}_j$ and multiplied by $\sigma_{E_j}^2$ respectively. The $\bar{E}_j$ and $\sigma_{E_j}^2$ are assumed to have been determined from an experimental characterization program.

From Equation (5-34)

$$\frac{\partial g_i}{\partial E_j} = \frac{\partial S_i}{\partial E_j} = \frac{\partial K(\nu_i,\, E_i)}{\partial E_j} M(z_i,\, \nu_i)\; D(z) \begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix}$$

$$+ K(\nu_i,\, E_i)\; M(z_i,\, \nu_i)\; D(z) \begin{bmatrix} \frac{\partial A_i}{\partial E_j} \\ \frac{\partial B_i}{\partial E_j} \\ \frac{\partial C_i}{\partial E_j} \\ \frac{\partial D_i}{\partial E_j} \end{bmatrix} \tag{5-71}$$

$$\frac{\partial^2 g_i}{\partial E_j^2} = \frac{\partial^2 S_i}{\partial E_j^2} = \frac{\partial^2 K(\nu_i,\, E_i)}{\partial E_j^2}\; M(z_i,\, \nu_i)\; D(z) \begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix}$$

$$+ 2\, K(\nu_i,\, E_i)\; M(z_i,\, \nu_i)\; D(z) \begin{bmatrix} \frac{\partial A_i}{\partial E_j} \\ \frac{\partial B_i}{\partial E_j} \\ \frac{\partial C_i}{\partial E_j} \\ \frac{\partial D_i}{\partial E_j} \end{bmatrix}$$

$$+ K(\nu_i,\, E_i)\; M(z_i,\, \nu_i)\; D(z) \begin{bmatrix} \frac{\partial^2 A_i}{\partial E_j^2} \\ \frac{\partial^2 B_i}{\partial E_j^2} \\ \frac{\partial^2 C_i}{\partial E_j^2} \\ \frac{\partial^2 D_i}{\partial E_j^2} \end{bmatrix} \tag{5-72}$$

$$\frac{\partial K(\nu_i, E_i)}{\partial E_j} = \begin{bmatrix} 0 & -- & -- & -- \\ -- & 0 & -- & -- \\ -- & -- & \frac{-(1+\nu)}{E_j^2}\,\delta_{ij}\, m\, J_1(mr) & -- \\ -- & -- & -- & \frac{+(1+\nu)}{E_j^2}\,\delta_{ij}\, m\, J_o\,(mr) \end{bmatrix} \tag{5-73}$$

$$\frac{\partial^2 K(\nu_i, E_i)}{\partial E_j^2} = \begin{bmatrix} 0 & -- & -- & -- \\ -- & 0 & -- & -- \\ -- & -- & \frac{+2(1+\nu)}{E_j^3}\,\delta_{ij}\, m\, J_1(mr) & -- \\ -- & -- & -- & \frac{-2(1+\nu)}{E_j^3}\,\delta_{ij}\, m\, J_o(mr) \end{bmatrix} \tag{5-74}$$

and

$$\delta_{ij} = \begin{cases} 1 & \text{when } i=j \\ 0 & \text{when } i\neq j \end{cases}$$

Differentiating Equation (5-46) and rearranging the terms

$$Q \prod_{j=1}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ \frac{\partial C_n}{\partial E_j} \\ \frac{\partial D_n}{\partial E_j} \end{bmatrix} =$$

$$- \sum_{\ell=1}^{n-1} \frac{D^{-1}(h_\ell)\ \frac{\partial X_\ell}{\partial E_j}\ D(h_\ell)}{4(\nu_\ell - 1)} \prod_{\substack{j=1 \\ j\neq\ell}}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix} \tag{5-75}$$

where $\frac{\partial X_i}{\partial E_j}$ is defined in Equation (5-76)

$$\frac{\partial X_i}{\partial E_j} = \begin{bmatrix} -\frac{\partial L_i}{\partial E_j} & \frac{\partial L_i}{\partial E_j} \cdot q(-h_i, \nu_{i+1}, \nu_i) & -(2mh_i + 4\nu_i - 1)\frac{\partial L_i}{\partial E_j} & \frac{-\partial L_i}{\partial E_j}(-h_i, \nu_{i+1}, \nu_i) \\ 0 & \frac{\partial L_i}{\partial E_j}(4\nu_{i+1} - 3) & 2\frac{\partial L_i}{\partial E_j} & (2mh_i - 4\nu_{i+1} + 1)\frac{\partial L_i}{\partial E_j} \\ (2mh_i - 4\nu_i + 1)\frac{\partial L_i}{\partial E_j} & \frac{\partial L_i}{\partial E_j} p(h_i, \nu_{i+1}, \nu_i) & -\frac{\partial L_i}{\partial E_j} & -\frac{\partial L_i}{\partial E_j} q(h_i, \nu_{i+1}, \nu_i) \\ -2\frac{\partial L_i}{\partial E_j} & -(2mh_i + 4\nu_{i+1} - 1)\frac{\partial L_i}{\partial E_j} & 0 & (4\nu_{i+1} - 3)\frac{\partial L_i}{\partial E_j} \end{bmatrix} \tag{5-76}$$

where

$$\frac{\partial L_i}{\partial E_j} = \frac{\partial \left[\frac{E_i}{E_{i+1}} \frac{1 + \nu_{i+1}}{1 + \nu_i}\right]}{\partial E_j} = \left(\frac{1 + \nu_{i+1}}{1 + \nu_i}\right) \left[\frac{1}{E_{i+1}} \delta_{ij} - \frac{E_i}{E_{i+1}^2} \delta_{\alpha j}\right] \quad ; \alpha = i+1$$

$$p(h,\nu,\mu) = m^2(2h^2) + 4mh(\nu-\mu) + (1-8\nu\mu + 2\mu)$$

$$q(h,\nu,\mu) = 2mh(2\nu-1) - (1 + 8\nu\mu-6\mu)$$

From Equation (5-40) we obtain

$$\begin{bmatrix} \frac{\partial A_i}{\partial E_j} \\ \frac{\partial B_i}{\partial E_j} \\ \frac{\partial C_i}{\partial E_j} \\ \frac{\partial D_i}{\partial E_j} \end{bmatrix} = \prod_{j=1}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ \frac{\partial C_n}{\partial E_j} \\ \frac{\partial D_n}{\partial E_j} \end{bmatrix}$$

$$+ \sum_{\ell=i}^{n-1} \frac{D^{-1}(h_\ell)\ \frac{\partial X_\ell}{\partial E_j}\ D(h_\ell)}{4(\nu_\ell - 1)} \prod_{\substack{j=i \\ j\neq\ell}}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix} \qquad (5\text{-}77)$$

Since C_n, D_n, A_i, B_i, C_i, D_i are determined using the mean values, $\overline{E}_i$, $\frac{\partial C_n}{\partial E_j}$, $\frac{\partial D_n}{\partial E_j}$, $\frac{\partial A_i}{\partial E_j}$, $\frac{\partial B_i}{\partial E_j}$, $\frac{\partial C_i}{\partial E_j}$, $\frac{\partial D_i}{\partial E_j}$ $(i=1, \ldots, n-1)$ may be determined using Equations (5-75) and (5-77). The second partial derivatives of S_i with respect to E_j are obtained from Equations (5-75) and (5-77):

$$Q \prod_{j=1}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ \frac{\partial^2 C_n}{\partial E_j^2} \\ \frac{\partial^2 D_n}{\partial E_j^2} \end{bmatrix}$$

$$
= -2 \sum_{\ell=1}^{n-1} \frac{D^{-1}(h_\ell) \frac{\partial X_\ell}{\partial E_j} D(h_\ell)}{4(\nu_\ell - 1)} \prod_{\substack{j=1 \\ j\neq\ell}}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ \frac{\partial C_n}{\partial E_j} \\ \frac{D_n}{E_j} \end{bmatrix}
$$

$$
- \sum_{\ell=1}^{n-1} \frac{D^{-1}(h_\ell) \frac{\partial^2 X}{\partial E_j^2} D(h_\ell)}{4(\nu_\ell - 1)} \prod_{\substack{j=1 \\ j\neq}}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix}
$$

$$
- \sum_{\ell=1}^{n-1} \frac{D^{-1}(h_\ell) \frac{\partial X_\ell}{\partial E_j} D(h_\ell)}{4(\nu_\ell - 1)} \sum_{\substack{k=1 \\ k\neq\ell}}^{n-1} \frac{D^{-1}(h_j) \frac{\partial X_k}{\partial E_j} D(h_k)}{4(\nu_k - 1)}
$$

$$
\prod_{\substack{j=1 \\ j\neq k \\ i\neq\ell}}^{n-1} \frac{D^{-1}(h_j)\ X_j\ D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix} \qquad (5\text{-}78)
$$

and

$$\begin{bmatrix} \dfrac{\partial^2 A_i}{\partial E_j^{\,2}} \\ \dfrac{\partial^2 B_i}{\partial E_j^{\,2}} \\ \dfrac{\partial^2 C_i}{\partial E_j^{\,2}} \\ \dfrac{\partial^2 D_i}{\partial E_j^{\,2}} \end{bmatrix} = \prod_{j=1}^{n-1} \frac{D^{-1}(h_j)\; X_j\; D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial^2 C_n}{\partial E_j^{\,2}} \\ \dfrac{\partial^2 D_n}{\partial E_j^{\,2}} \end{bmatrix}$$

$$+ 2 \sum_{\ell=i}^{n-1} \frac{D(h_\ell)\, \dfrac{\partial X_\ell}{\partial E_j}\, D(h_\ell)}{4(\nu_\ell - 1)} \prod_{\substack{j=i \\ j\neq\ell}}^{n-1} \frac{D^{-1}(h_j)\; X_j \quad (h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ \dfrac{\partial C_n}{\partial E_j} \\ \dfrac{\partial D_n}{\partial E_j} \end{bmatrix}$$

$$+ \sum_{\ell=i}^{n-1} \frac{D^{-1}(h_\ell)\, \dfrac{\partial^2 X_\ell}{\partial E_j^{\,2}}\, D(h_\ell)}{4(\nu_\ell - 1)} \prod_{\substack{j=i \\ j\neq\ell}}^{n-1} \frac{D^{-1}(h_j)\; X_j\; D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix}$$

$$+ \sum_{=i}^{n-1} \frac{D^{-1}(h_\ell)\, \dfrac{\partial X_\ell}{\partial E_j}\, D(h\ )}{4(\nu_\ell - 1)} \sum_{\substack{k=i \\ k\neq\ell}}^{n-1} \frac{D^{-1}(h_k)\, \dfrac{\partial X_k}{\partial E_j}\, D(h_k)}{4(\nu_k - 1)}$$

$$\prod_{\substack{j=i \\ j\neq k \\ j\neq\ell}}^{n-1} \frac{D^{-1}(h_j)\; X_j\; D(h_j)}{4(\nu_j - 1)} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix} \qquad (5\text{-}79)$$

where

$$\frac{\partial^2 X_i}{\partial E_j^{\,2}} = \begin{bmatrix} -\frac{\partial^2 L_i}{\partial E_j} & \frac{\partial^2 L_i}{\partial E_j^{\,2}}\, q(-h_i \;, \nu_{i+1}, \nu_i) & -(2mh_i + 4\nu_i - 1)\frac{\partial^2 L_i}{\partial E_j^{\,2}} & -\frac{\partial^2 L_i}{\partial E_j^{\,2}}\,(-h_i, \nu_{i+1}, \nu_i) \\ 0 & \frac{\partial^2 L_i}{\partial E_j^{\,2}}\,(4\nu_{i+1} - 3) & 2\,\frac{\partial^2 L_i}{\partial E_j^{\,2}} & (2mh_i - 4\nu_{i+1} + 1)\,\frac{\partial^2 L_i}{\partial E_j^{\,2}} \\ (2mh_i - 4\nu_i + 1)\,\frac{\partial^2 L_i}{\partial E_j^{\,2}} & \frac{\partial^2 L_i}{\partial E_j^{\,2}}\, p(h_i, \nu_{i+1}, \nu_i) & -\;\frac{\partial^2 L_i}{\partial E_j^{\,2}} & -\frac{\partial^2 L_i}{\partial E_j^{\,2}}\, q(h_i, \nu_{i+1}, \nu_i) \\ -2\,\frac{\partial^2 L_i}{\partial E_j^{\,2}} & -(2mh_i + 4\nu_{i+1} - 1)\,\frac{\partial^2 L_i}{\partial E_j^{\,2}} & 0 & \frac{\partial^2 L_i}{\partial E_j^{\,2}} \end{bmatrix} \tag{5-80}$$

and

$$\frac{\partial^2 L_i}{\partial E_j^{\,2}} = 2\left(\frac{1 + \nu_{i+1}}{1 + \nu_i}\right)\left[\frac{-1}{E_{i+1}^{\,2}}\,\delta_{\alpha j} \qquad \delta_{ij} + \frac{E_i}{E_{i+1}^{\,3}}\,\delta_{\alpha j}\right] \;;\quad \alpha = i+1$$

Now that $\frac{\partial A_i}{\partial E_j}$, $\frac{\partial B_i}{\partial E_j}$, $\frac{\partial^2 C_i}{\partial E_j^2}$, $\frac{\partial^2 D_i}{\partial E_j^2}$ are known, these may be substituted into Equations (5-71) and (5-72) and then (5-69) and (5-70) to obtain $E[S_i]$ and $Var[S_i]$.

In evaluating the computer code for the means, pavement response it was found that the terms involving the second deviations were not significant and hence they were deleted from the code.

To obtain the solution for a circular load, the stresses or displacement must be integrated from zero to infinity with respect to m and multiplied by a. Hence

$$\begin{aligned} E[\tilde{S}_i] &= a\, E\left[\int_0^\infty J_1(ma)\, S_i\, dm\right] \\ &\simeq a\, E\left[\sum_{j=1}^{n} J_1(ma)_j\, S_{ij}\, \Delta\,(m_j)\right] \\ &\simeq a\, \left[\sum_{j=1}^{m} J_1(ma)_j\, E[S_i]_j\, \Delta\,(m_j)\right] \\ &= a \int_0^\infty J_1(ma)\, E[S_i]\, dm \end{aligned} \tag{5-81}$$

where $J_1(ma)_j$, S_{ij} denote $J_1(ma)$, S_i evaluated at $m = m_j$.

Similarly,

$$\begin{aligned} Var[\tilde{S}_i[&= a^2\, Var\left[\int_0^\infty J_1(ma)\, S_i\, dm\right] \\ &= a^2 \sum_{j=i}^{n} Var[E_j] \left[\int_0^\infty J_1(ma) \left.\frac{\partial S_i}{\partial E_j}\right|_{E_j=\bar{E}_j} dm\right]^2 \\ &\simeq a^2 \sum_{j=i}^{n} Var[E_j] \left[\sum_{\ell=1}^{M} J_1(ma)_\ell \left(\left.\frac{\partial S_i}{\partial E_j}\right|_{E_j=\bar{E}_j}\right)_\ell \Delta m\ell\right]^2 \end{aligned} \tag{5-82}$$

5.5 EXAMPLE SOLUTIONS

The probabilistic responses for surface deflection, strain or the bottom of the surface layer and stresses in the surface layer of a three-layer pavement system subjected to a circular wheel load are illustrated in Figures 5-5, 5-6 and 5-7. The molulus of the first layer is assumed to have a coefficient of variation of 0.25 while the properties of the other two layers are deterministic. It may be observed that the variability of the responses are quite different.

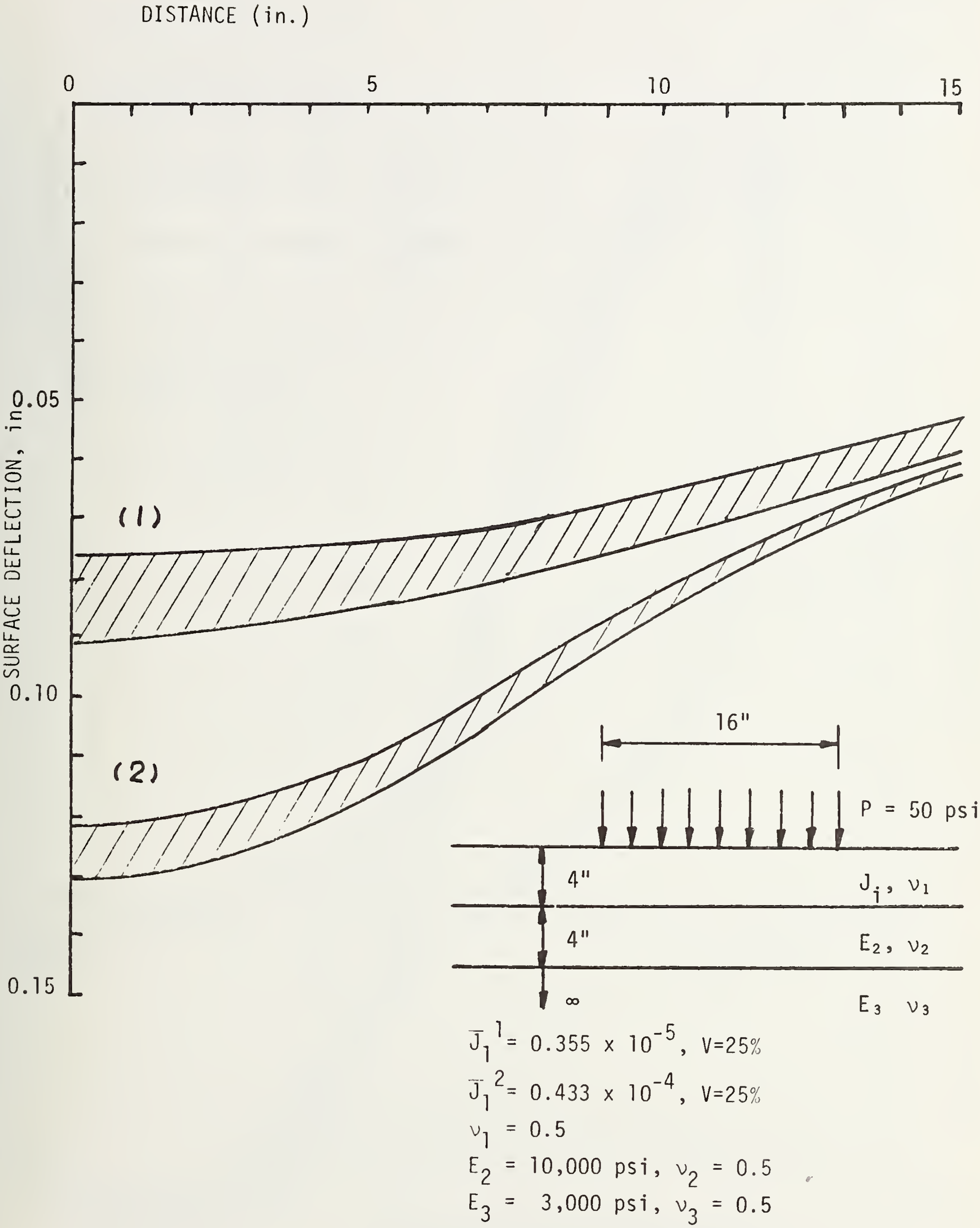

Figure 5-5 SURFACE DEFLECTION ($\delta \pm 3\sigma$)

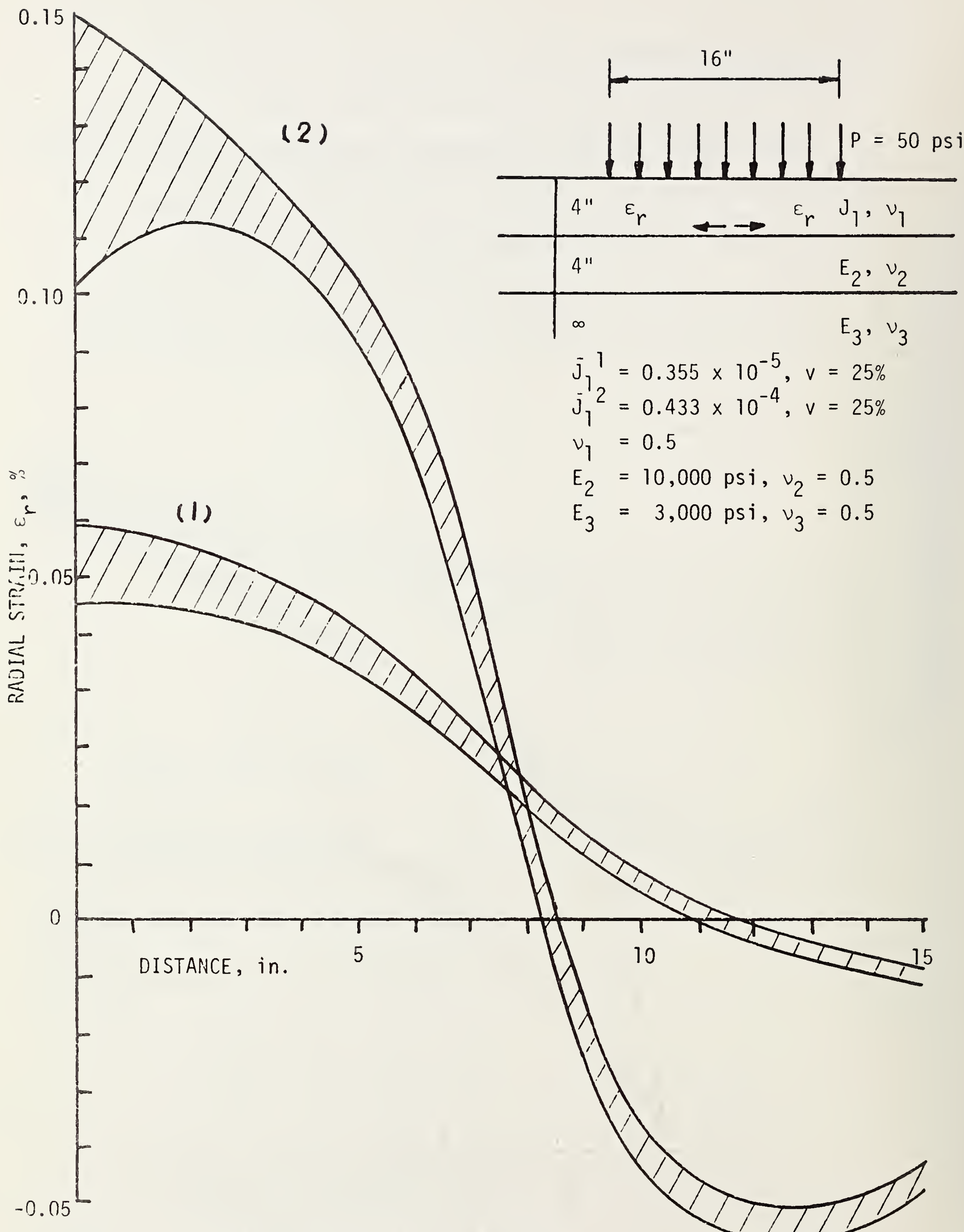

Figure 5-6 RADIAL STRAIN AT FIRST INTERFACE
($\varepsilon_r \pm \sigma_\varepsilon$)

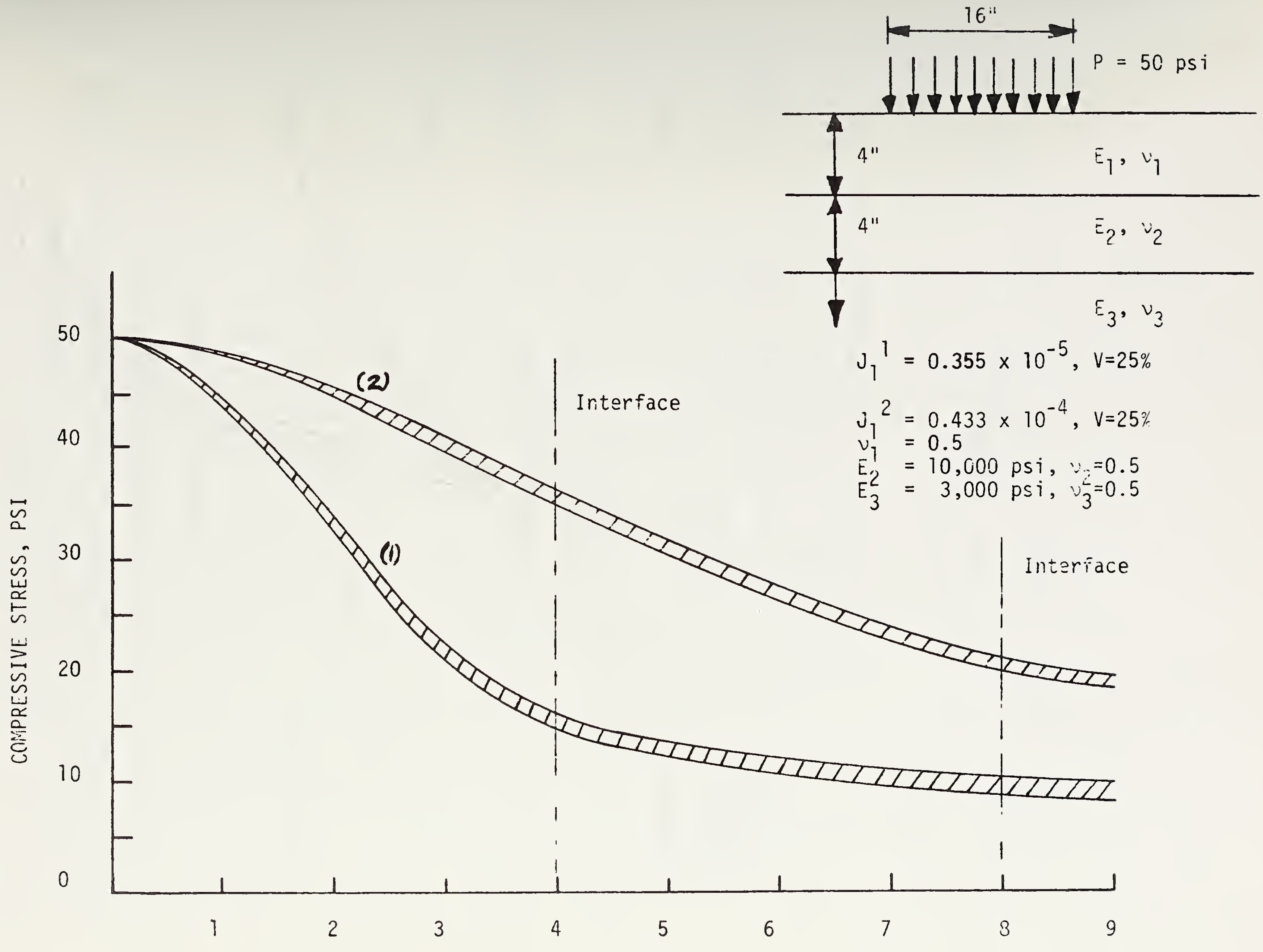

Figure 5-7 VERTICAL STRESS AT CENTER LOADING ($\sigma_z \pm \sigma_\sigma$)

CHAPTER 6

RUT DEPTH PREDICITON MODEL

6.1 INTRODUCTION

The design framework proposed by Moavenzadeh [1] and continued in VESYS IIM is shown schematically in Figure 6-1. It consists of five components which, when integrated together, result in a predictive pavement design system. Each component consists of several elements or subsystems which form the rational basis for the design pro edure. Among the five subsystems shown in Figure 6-1, the structural subsystem is perhaps the most important one. As shown in Figure 6-2, the structural subsystem consists of three models - the primary response model, the damage model and the serviciability model.

The primary response model accepts the basic material properties, (i.e., creep compliance, Poisson's ratio), pavement system geometry (i.e., layer thickness), traffic loading (e.g., wheel load, tire pressure, frequency or distribution of load applications, etc.) and environmental variables (e.g., temperature, moisture level, etc.), and computes the stresses, strains and deflections at every point in the pavement system. The primary response model in VESYS IIM accepts stochastic moduli or compliances in order to account for the inherent variability of the input variables and the pavement system.

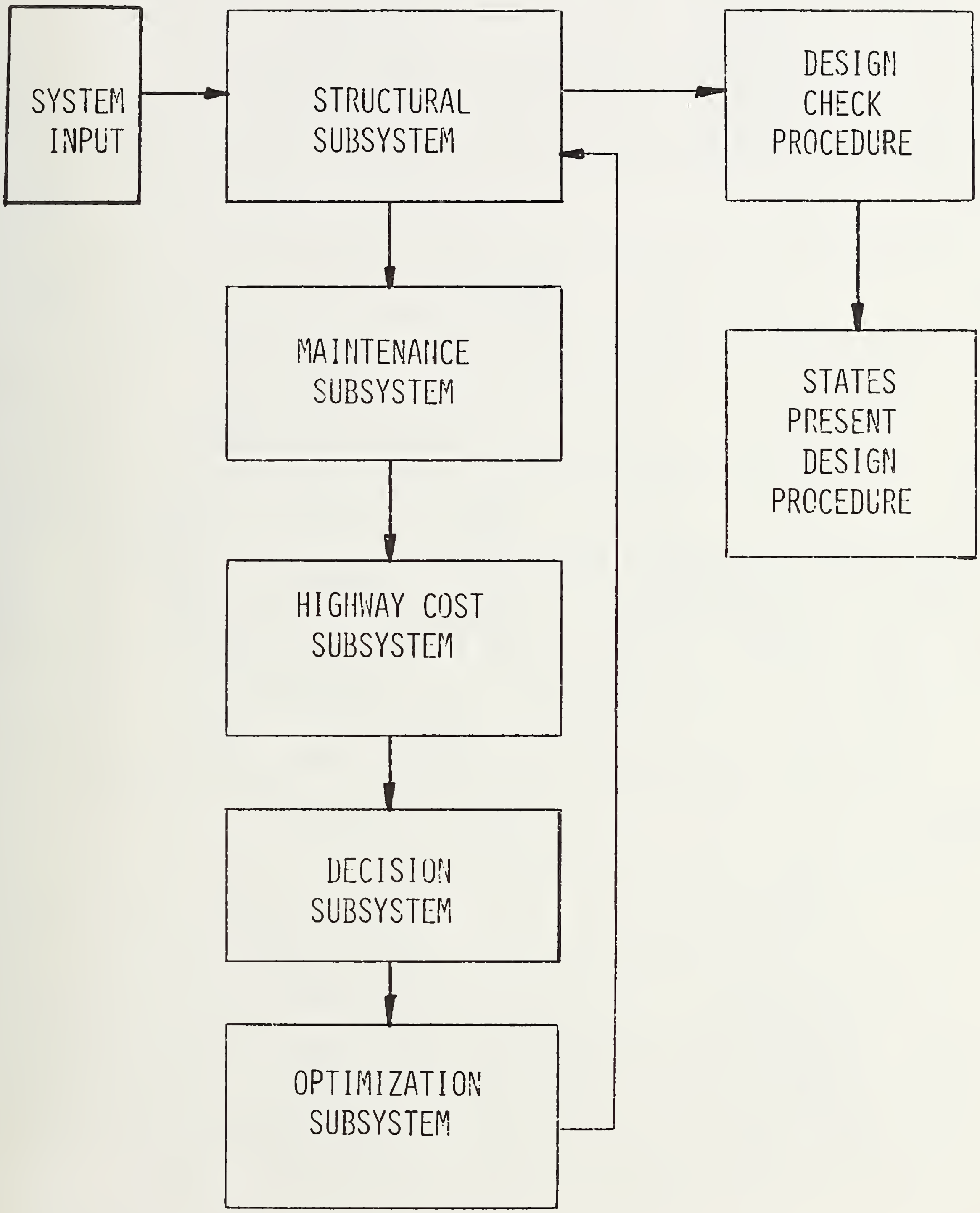

FIGURE 6-1 PAVEMENT DESIGN SYSTEM

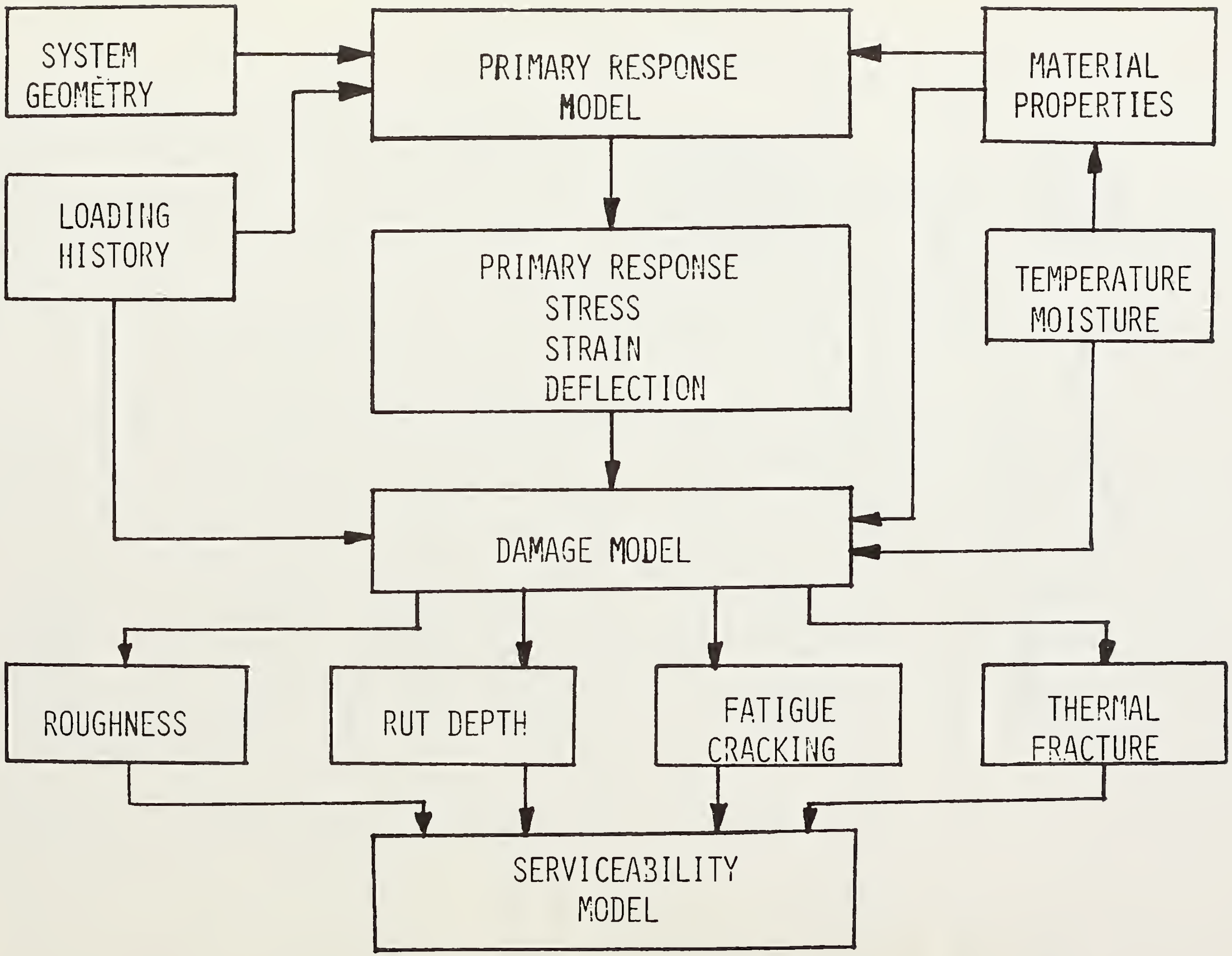

FIGURE 6-2
STRUCTURAL SUBSYSTEM

The probabilistic solution for the primary responses i.e., the stresses, strains and deflections, obtained from the primary response model and the material failure properties are inputs to the damage model, which predicts pavement distress. To date, fatigue cracking, rutting (permanent deformation), and roughness have been considered to be the msot significant modes of pavement distress. The pavement distress modes are calculated separately in three submodels even though there is some interaction between them.

The calculated pavement distresses are input to the serviceability model to define the life expectancy of the pavement system.

Pavement rutting* is the result of a channelized traffic causing differential surface deformation under the areas of intensive load applications in the wheel paths. The amount of pavement rutting depends upon the following parameters:

1) The distribution of traffic loads, particularly the transverse distribution of traffic loads, i.e., channelized traffic;
2) The stresses induced in the pavement system, which depend upon the response characteristics of the layer materials;
3) The permanent strain induced as the result of the stresses developed in (2). This permanent strain depend on the permanent deformation characteristics of the layer materials.

Although satisfactory means for rut depth prediction have not yet been developed, the mechanism of rutting under repeated traffic loading is relatively well know from various field and laboratory investigations. In the AASHO Road Test [33] an extensive study was made of the rutting phenomenon. Two conclusions were drawn from this

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* "rutting" as used in this report refers to the depressions under a wheel path to repeated wheel loads which cause consolidation and/or lateral movement in the pavement layers. With this definition, other mechanisms which may cause rutting, such as abrasion, and war due to studded tires etc., are excluded.

study which revealed two very improtant aspects of the mechanism of rutting:

1. Rutting of the asphalt pavement surface can be attributed to the change of thickness of all layers of the pavement system (see Figure 6-3).
2. Rutting is due primarily to lateral displacement of the pavement materials, rather than from densification.

These findings have been confirmed by other field test results [34].

Realizing that many varieties of materials are used for paving materials, and that the characteristics of the materials' response properties and permanent deforamtion characteristics are not well known, a pavement system rut depth model must be flexible so that different pabing materials can be in the model with minimum inconvenience to the user. Thus, the rut depth model is developed in modular form so that refinements and new developments related to the permanent defomration characteristics of the various paving materials, as well as refinements of the primary response model or other subsystems can be readily adapted without haveing to re-develop the pavement design framework.

Basically, a model for predicting pavement rutting consists of two parts. The first part is the characterization of permanent deformation of pavement materials including materials to be used for surface, base, subbase, and subgrade, under repeated loading. The second part is the analytical pavement model, or primary response model, in which the stress distributions in the pavement system under a vehicle wheel load can be calculated. These stresses together with permanent deformation characteristics of the material are used to estimate the permanent

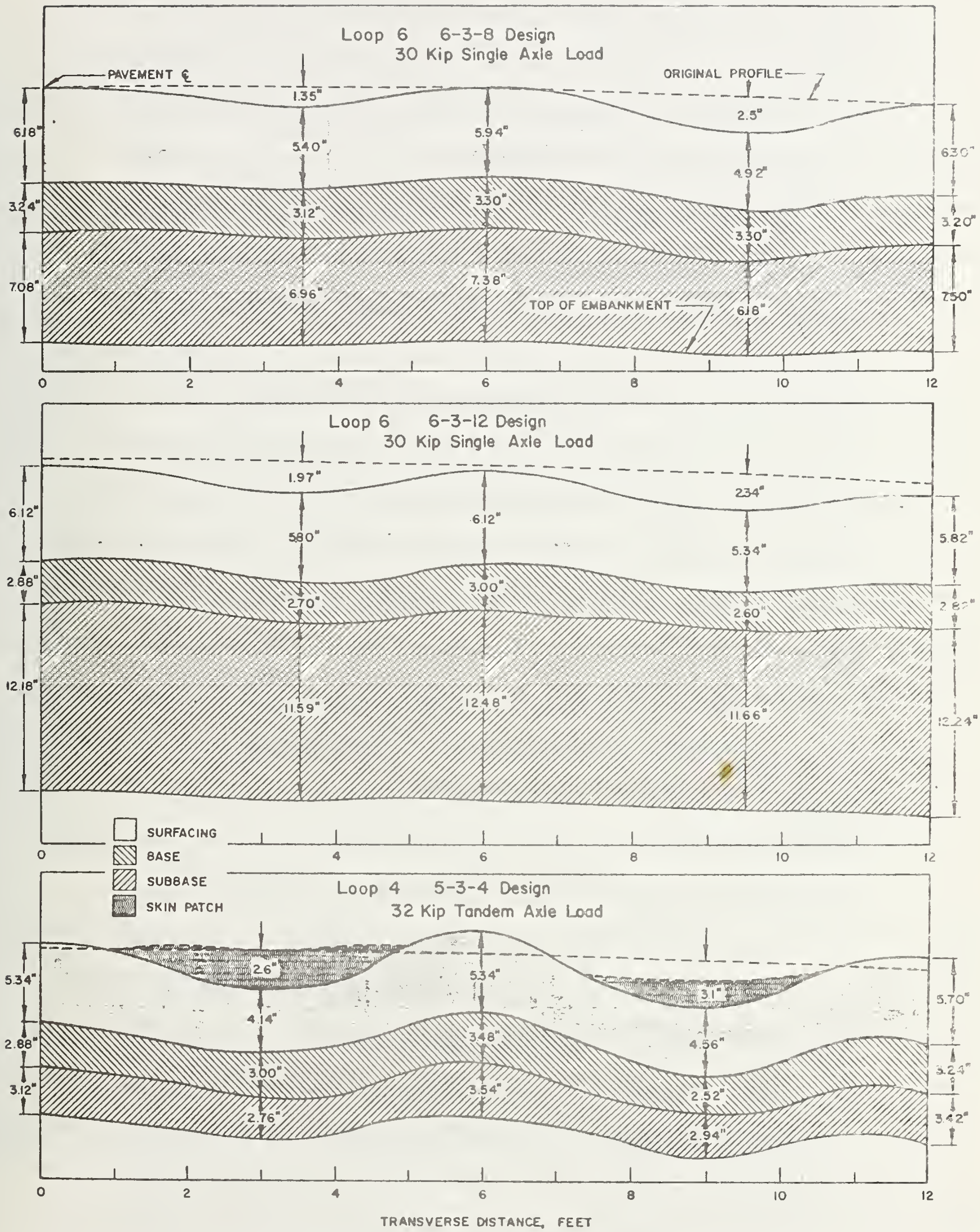

Figure 6-3 TRANSVERSE PROFILES AASHO TEST ROAD
TRENCH STUDY (FROM REFERENCE 33)

strains. Summation of the permanent strain with depth under the wheel paths yields the permanent deformation of the surface. The total rut depth under a sequence of traffic loading (varying in magnitude as well as in duration) is calculated by properly summing the rut-depth obtained from each single load application.

The general approach to the characterization of the permanent deformation characteristics of pavement materials under repeated loading has been to test the material under conditions as nearly representative of field conditions as possible. The test procedures include the use of the stresses (stress levels for axial and confining stresses) simulating those induced by the wheel loads (intensity, and duration), temperature, and moisture in the field. Using a well planned statistical factorial design, the effects of each parameter on the permanent strains can be obtained. Inasmuch as the relationships are derived for the permanent strains of the paving materials under simulated field conditions, the predictions using the equations so derived should be very reliable. A main disadvantage of this approach, however, is that when the number of parameters involved and the levels for each parameter are too large, the experimental program becomes much too cumbersone and too expensive to be of practical use.

Thus, an alternate approach for predicting the permanent deformation characteristics of asphalt concrete, in particular, under repetitive loading has been developed involving a simplified testing procedure in which only a relativly small number of creep tests are needed for the characterization of the constitutive equation for predicting permanent strains.

6.2 GENERAL FRAMEWORK OF THE RUT DEPTH MODEL

The rut depth model developed herein is compatible with the structural subsystem in VESYS II or VESYS IIM. As noted previously, the rut depth model is modular to provide for maximum flexibility for substituting various primary response models and permanent deformation realtionships of the various paving materials. The analysis provedure for calcuating rut depth is illustrated schematically in Figure 6-4, and a simplified flow diagram for the rut depth model is shown in Figure 6-5.

The geometry of the pavement system (e.g., layer thickness), the loading (considering a unit step laod intensity) and material response properties (moduli, or compliances) are input to the primary response model which calculates stress, strain and deflection throughout the pavement structure for a step load intensity. The stresses under the wheel path thus calculated are the only input from the primary respons model to the rut depth model. These may be input directly from the primary response model or read from a file storage unit if the two subsystems are ran at different times. As shown in Figure 6-5, the vertical stress σ_z and radial stress (confining stress), σ_r, at several depths directly under the wheel load are calculated. The stresses for different load graoups (see Chapter 4) are also calculated.

The rut depth model consists of several submodels for calculating the permanent deforamtion of each layer of the pavement system. The rut depth of the pavement system under the given one step loading (for a given load intensity and given load duration) is the sum of the permanent deformation accumulated in each layer. The total rut depth under a sequence of traffic loading (varying in magnitude as well as in

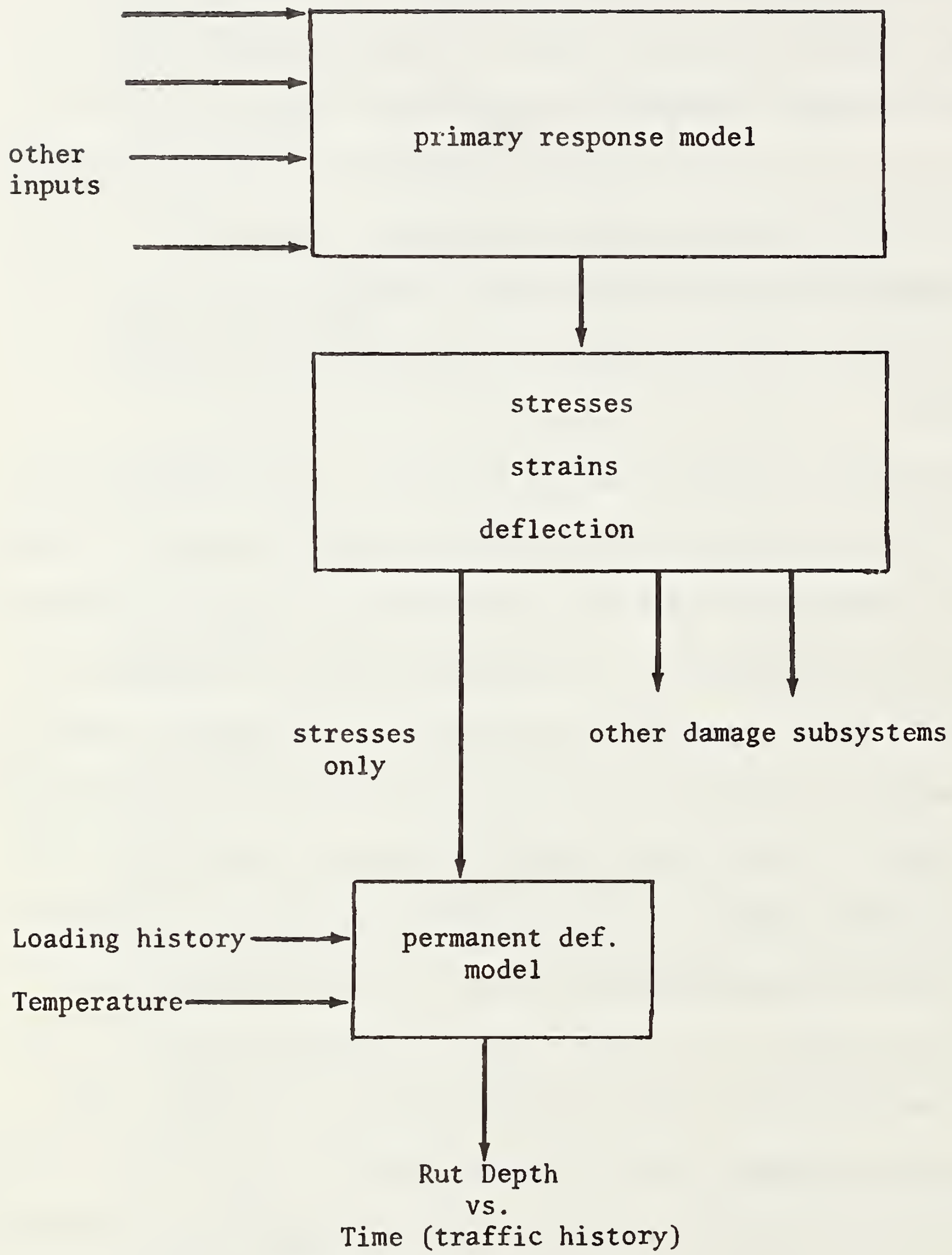

FIGURE 6-4 ANALYSIS PROCEDURE FOR CALCULATING RUT DEPTH

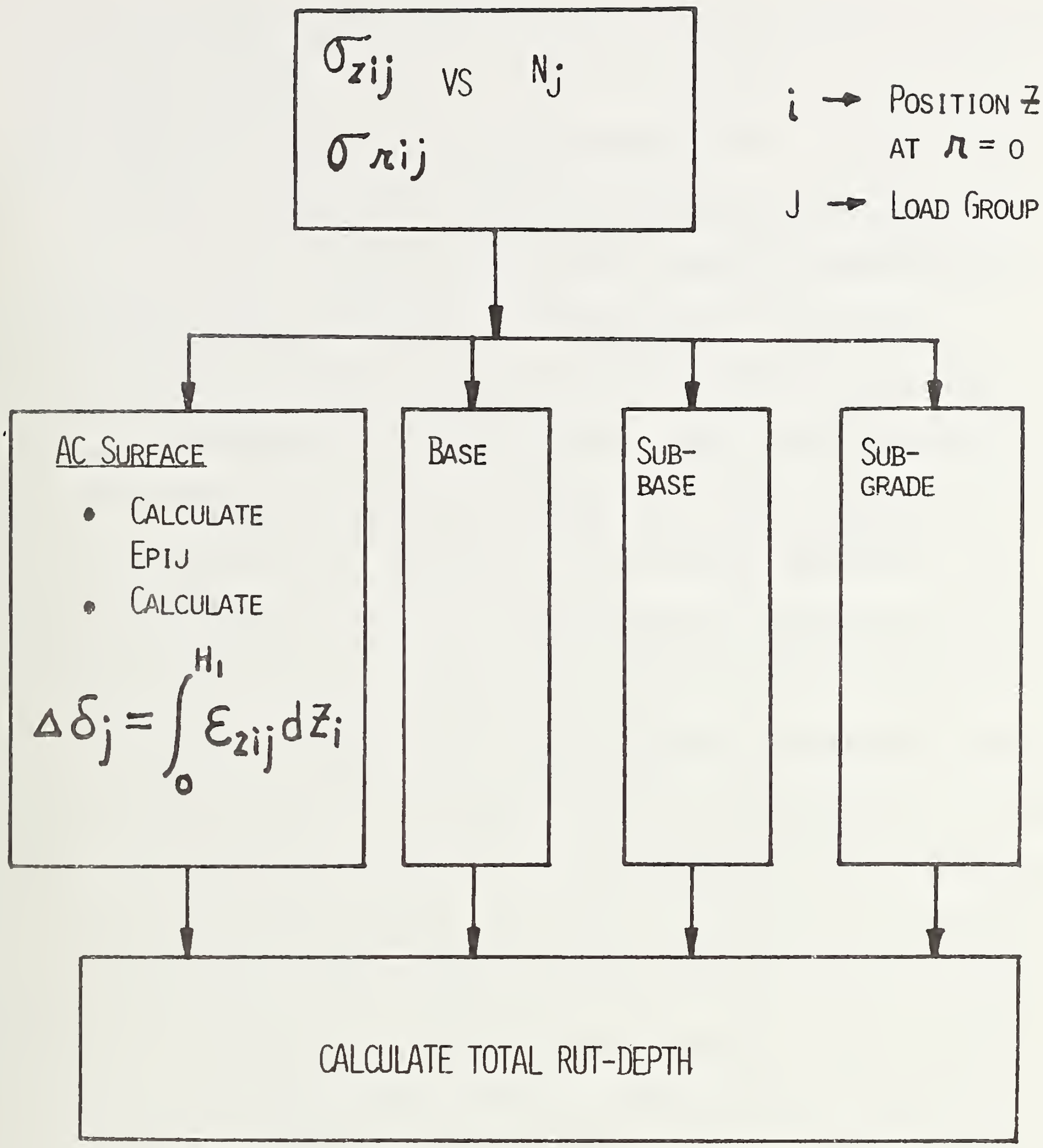

FIGURE 6.5 RUT-DEPTH MODEL

duration) is calculated by properly summing the rut depth contribution from each single load application. The calculation may prove to be quite lengthy. A similar approach has also been used by Romain [35], McLean and Monismith [36], Morris [37], and Barksdale [38] for estimating the rut depth of pabement systems due to repeated loading.

Although the logic of rut-depth prediction, as mentioned herein, is rather straightforward, the detailed calculations are not quite so straight-forward. First, the relationship of the permanent strains to the applied stresses, temperature, moisture levels, etc., for most paving materials are still relatively unknown. Second, the problem is complicated by the complex nature of traffic load characteristics. The stresses induced in the pavement system due to traffic depend on the total weight, contact area, contact pressure and wheel configuration. The transverse distribution of vehicles along the traffic lane will affect the actual stresse induced and hence the permanent deformation under the "wheel path"*. Load duration also affects the accumulation of permanent deformation. Although under normal conditions only the stresses induced from normal wheel loads are important to the permanent deformation, the shear force associated with acceleration, braking and cornering (e.g., highway curves) could conceivably contribute a significant amount to the rutting. Third, the environmental effects of temperature and moisture must be considered. Temperature and moisture not only affect creep compliances which in turn affect the stress/strain response in the primary response model, temperature and moisture also effect the pavement deforamtion characteristics of the layer materials. Pavement

---

* The "wheel path" as used in this report, is defined loosely as the positions in the transverse direction of the traffic lane in which the highest distribution of wheel loads pass.

rutting due to uneven subgrade heaving as a result of freeze thaw action is not considered.

6.3 TRAFFIC LOADING AND ENVIRONMENTAL INPUTS

The inputs for traffic loading and environment (temperature, moisture) for the rut depth medel are the same as for the primary response model and are described in Chapter 4.

To briefly review, instead of using a certain fixed intensity (such as the equivalent 18 kip axle load), the traffic load input consists of: (1) the actual wheel load distribution, such as the distribution from the standard W-4 form which provides the number of axles per thousand trucks (or vehicles) in each axle load group (10 groups for single axles, converting tandem axles into equivalent single axles*); (2) average daily traffic, ADT (including mean and variance); (3) monthly variation of traffic; (4) average growth of traffic per year; (5) speed of trucks (mean and variation); (6) average radius of the contact area; (7) channelization distribution factor; (8) percentage of trucks in traffic stream; and, (9) percentage of trucks in the design lane. A table which consists of frequency versus stresses, strains and deflections per thousand vehicles can be obtained from input (1). Input (20), (3) and (4) are then used to calculate the total stress, strain and deflection in each axle load group in each month, each year and the total accumulated values for any period. This information is then input to the damage models to calculate the pavement distresses.

---

* At the present time, the tandem-wheel is considered as two single wheels; that is, the tandem axle load is divided by two and each is considered as a single axle wheel load.

The effects of wheel load congifurations, and shear force on the stresses induced in the pavement system are not included in the primary response model of the VESYS II system. This is due to the assumption of axial symmetry for loading as well as geometry of the layered system and automoatically excludes multiple wheel loads and shear forces.

The temperature is given by the mean and variance of air temperature for each basic period, and moisture level is also given by mean and variance in the subgrade for the same basic period. The basic period is usually taken to be a month.

A heat transfer analysis may be used to obtain more accurate estimates of the temperature in the pavement structure (e.g., Dempsey and Thompson [39]); however, at the present time this type of analysis is not included for the following reasons: (1) inputing only mean and variance of monthly temperature and moisture is not accurate enough to warrant the use of more accurate heat transfer analyses; (2) only the asphalt concrete surface and asphalt treated base course are strongly dependent on temperature. The subgrade material is not too sensitive to temperature level so long as the temperature is above freezing; hence, only a relatively rough estimation of subgrade temperature with air temperature variation is required. Since temperature variations in the surface and base courses are relatively close to the surface temperature variations, temperatures in these layers can be estimated rather accurately using as approximate method.

6.4 MATERIAL CHARACTERIZATION

The material properties which are pertinent to the analysis of rutting can be catagorized into two groups: the material response properties and permanent deformation properties.

Material response properties consist of creep compliances (for viscoelastic materials) or elastic moduli (for elastic materials) and Poisson's ratio. These properties of each layer material are needed in the primary response modul for calculating the stresses, strain and deflections in a pavement system under the application of external loadings. Characterization of material response properties of pavement materials has been the subject of extensive investigation in the past ten years, (see for example, [24, 40]), and therefore is not discussed herein. This section concentrates on characterization of permanent deformation characteristics.

The permanent strain induced in various paving materials under an applied stress depends, in a complex manner, upon the following parameters:

stress state, (axial and confining pressure)

stress level

stress duration

number of load repetitions

environmental factors (temperature, moisture, etc.)

Therefore, a constitutive equation, relating the permanent strains (ε_p) to the stress state and stress levels σ_{ij}, load duration Δt, number of load repetitions N, temperature T, and moisture content M, is required for each kind of paving material to facilitate rut-depth prediction. The general form of this relationship is

$$\varepsilon_p = f[\sigma_{ij}, \Delta t, N, T, M] \tag{6-1}$$

Not all materials necessarily depend upon the totality of these parameters, however, Asphalt concrete is strongly dependent upon temperature variation, but may not be too sensitive to moisture level; whereas, the effect of these factors upon clay soil could be exactly opposite. In the same vein, if the permanent strains induced from repeated loading are due solely to "viscous flow" such as may occur in a well compacted asphalt concrete with relatively high asphalt content, the effect of duration (Δt) under each cycle and the total number of load application (N) can be combined into a single parameter (Δt N). For A material such as agregates, which shows an exactly opposite behavior, the permanent strain due to repeated loading arises entirely from "time independent plastic deformation or friction", and the permanent strain depends only on the total number of load applications (N) and is independent of the duration (Δt).

It is also important to realize that, strictly speaking, an equation determined for a given paving material under a given set of conditions cannot be used to describe the behavior of the same material under different conditions. For example, changing the asphalt content, compaction, or amount of air voids in asphalt concrete results in a different response.

A limited number of research efforts toward development of constitutive equations for permanent strain for paving materials have been initiated recently. At the present time, the development of the constitutive equation for permanent strains can be categorized by two approaches:

(1) Simulative statistical approach,

(2) Viscoelastic - plastic approach.

In the first approach, laboratory samples of the material are tested under conditions as nearly representative of field service conditions as possible. The test procedures include use of the actual stresses (actual stress levels for axial stress, confining stress) induced by the wheel loads, duration of loading, temperatures, and moisture levels, etc. Based on this preliminary information, a statistically designed test program is undertaken to determine the effects of the main design parameters and their interactions on the permanent strains using factorial analysis [41]. Using multiple regression analysis (linear or nonlinear), an equation relating the permanent strains to the design factors can be obtained.

Inasmuch as the relationships are derived for the permanent strains of the paving materials as a function of the number of load applications, stress state and environmental conditions under simulated service conditions, the predictions of pavement rutting using these equations should be very reliable; depending only upon the reproducibility of field service conditions in the laboratory. In principle, by using the factorial experiment design, the effect of any parameter upon permanent strain can be obtained as accurately as desired by using small increments or selecting many levels for each factor in the experimental design.

The effort necessary for acceptable reliability unfortunately turns out to be the main disadvantage of this approach. If the number of factors involved and the levels for each factor are too large the experimental program quickly becomes much too cumbersome and expensive to be of practical use. For example, for a three factor design, with each at three levels, a factor design would require $3^3 = 27$ tests. Allowing for replications to determine inherent material variabilities, the total number of tests could easily exceed 100. In order to investigate permanent strain as a function of the number of load applications, a minimum of 10,000 load repetitions per test is required. Thus, the test program can easily exceed 200-300 hours. In addition, the apparatus and instrumentation needed to obtain reliable results are rather extensive and expensive. Thus, the most urgent research need in this area is the development of simplified tests which decrease the total number of tests, shorten the amount of time required for each test, and simplify the test mehtods and instrumentation requirements.

The second approach for development of constitutive equations for permanent strain is based upon viscoelastic-plastic theories. There is no universal theory that can be used indiscriminantly for all materials, rather, a specific mechanics theory can only be used for certain paving materials, which meet certain conditions. Therefore, it is anticipated that it may be necessary to treat each material differently with a different theory. This fact makes this approach less attractive and not as easily understood as the first approach; however, once a particular theory is shown to be applicable for a particular material, the functional relationship between permanent strain and the influencing factors can be explicitly developed from the theory. In this approach the size of the test program is reduced by an order of magnitude over the laboratory based methods, and only a relatively small number of tests are needed to determine the coefficients in the equation describing permanemt deformation. This result makes this approach very appealing from a practical point of view. More importantly, the characteristics of permanent strains obtained in this approach are, in many cases, explicitly obtained in terms of certain fundamental mechanical properties. The effects of physical characteristics and environmental factors on those properties are relatively well known. In contrast, for example, a change in mix design of asphalt concrete requires a whole new test program to characterize permanent strains in the first approach.

Despite its obvious advantages, the second approach is not without shortcomings. For example, the applicability and the accuracy of the theory for predicting the permanent strain of a given material is undoubtedly the central question to be answered. As will be seen, the present state of knowledge in some cases, is still not quite sufficient to answer this question convincingly. These obstacles will certainly be overcome in the future as more effort is expended on this approach.

In the following subsections the constitutive equations for permanent strain developed by other researchers is briefly reviewed before presenting the specific viscoelastic-plastic model recommended for asphalt concrete.

6.4.1 Asphalt Concrete

Morris [37] used the axial stress σ_1, confining pressure σ_3, and temperature T as the three main design factors. Each has two levels in a 2^3 factorial design experiment for investigating the relationship between the permanent strain of asphalt concrete and the number of load repetitions. For each given set of loading and temperature conditions, an ashphalt concrete specimen (4 inches in diameter and 8 inches long) was subjected to a minimum of 10^6 load repetitions and the permanent strains vs. number of repetitions were measured.

For the compression model he obtained

$$\log\left(\frac{\varepsilon_p}{\log N}\right) = \theta_o + \theta_1 \sigma_1 + \theta_2 \sigma_3 + \theta_3 T + \theta_3 \sigma_3 T \qquad (6\text{-}2)$$

For the tension model,

$$\log\left(\frac{\varepsilon_p}{10^5} N\right) = \beta_o + \beta_1 \sigma_1 + \beta_2 \sigma_3 + \beta_3 T$$

where ε_p is the permanent strain, N is the number of load repetitions, and $\theta_o \ . \ . \ . \ \theta_4$; $\beta_o \ . \ . \ . \ \beta_3$ are constants.

McLean and Monismith [36] used a similar approach in investigating the permanent strain of asphalt concrete under repeated loading, with the following results:

$$\log \varepsilon_p = K[(\sigma_1 - \sigma_3)\varepsilon_e]^{\alpha} + C_1 \log N + C_2 (\log N)^2 + C_3 (\log N)^3 \qquad (6\text{-}3)$$

where ε_p is the axial permanent strain, σ_1 and σ_3 are the axial and lateral stresses, ε_e is the elastic strain and K and α are temperature dependent coefficients.

Soussou, Moavenzadeh and Findakly [1] have suggested the following equation to account for the permanent strain of asphalt concrete:

$$\frac{\varepsilon_p}{\sigma_o}(t) = [J(t) - J(t-\Delta t)] + (1-\mu)J(t-\Delta t)e^{-\beta\varepsilon} \qquad (6\text{-}4)$$

In this equation ε_p represents the permanent deformation under a given loading with a load duration equal to Δt, $J(t)$ is the creep compliance of the material, μ is the ratio of the creep compliance under unloading and the creep compliance under loading,and $e^{-\beta\varepsilon}$ is introduced to account for the decrease in the accumulation rate of permanent strain as the degree of compaction increases. When $\mu=1$, i.e., when the loading and unloading behavior are the same (6-4) becomes

$$\varepsilon_p (t) = [J(t) - J(t-\Delta t)]\sigma_o. \tag{6-5}$$

Under this situation the permanent strain is accounted for only through the rate sensitivity of the linear viscoelastic material. Although linear viscoelasticity theory has been used by many researchers to characterize the time-dependent behavior of asphalt concrete, asphalt concretes are not, strictly speaking, linear viscoelastic materials. In particular, the unloading and loading behavior of these materials are quite different as demonstrated by Lai [42,43]. Because of this, the use of (6-5) to describe the permanent strain in the original version of the VESYS II was not very successful as pointed out in [2,36,37]. Equation (6-4) should improve the predictive capability. In (6-4) when the stress pulse shape (constant shape throughout or varying shape) is given, the accumulated permanent strain for any period (i.e., at any number of load repetitions) can be calculated so long as $J(t)$, μ, and β for the material are known. In principle, $J(t)$ is the creep compliance determined from the loading portion of the creep test, and μ is determined from both the loading and unloading portion of the creep curve, the constant β must be determined from a repetitive test. It is important to recognize the fact that μ, the ratio of creep compliances under unloading $J_u(t - \Delta t)$, loading $J(t - \Delta t)$, as illustrated in Figure 6-6,

$$\mu = \frac{J_u(t-\Delta t)}{J(t-\Delta t)} \tag{6-6}$$

depends upon the duration of loading time (Δt), and even at a given duration, the ratio of these two time dependent curves depends on time.

6.4.2 Granular Base Material

The available results on the permanent strain characteristics of granular materials are relatively meager. Barksdale[38] conducted rather extensive repeated load laboratory triaxial tests on untreated base course materials. The results indicated that the effect of axial and confining pressure on the permanent strain can be fitted by the following equation, which was originally developed for the static triaxial test for sand and clay [44].

$$\varepsilon_p = \frac{(\sigma_1-\sigma_3)\,/\,(K\sigma_3^{\,n})}{1-\left\{\dfrac{\dfrac{(\sigma_1-\sigma_3)R_f}{2(c\cos\phi+\sigma_3\sin\phi)}}{(1-\sin\phi)}\right\}} \tag{6-7}$$

where ε_p = axial plastic strain at 100,000 repetitions

c = cohesion

ϕ = angle of internal friction

k, n, R_f are constants

σ_1, σ_3 = axial and confining pressure respectively.

It was also indicated by Barksdale that the total accumulated plastic strain is related linearly to the logarithm of the number of load applications. Hence, the right side of (6-7) should be multiplied by $\frac{1}{5}\log_{10}N$. Thus,

$$\varepsilon_p = F\,\frac{1}{5}\log_{10}N \tag{6-8}$$

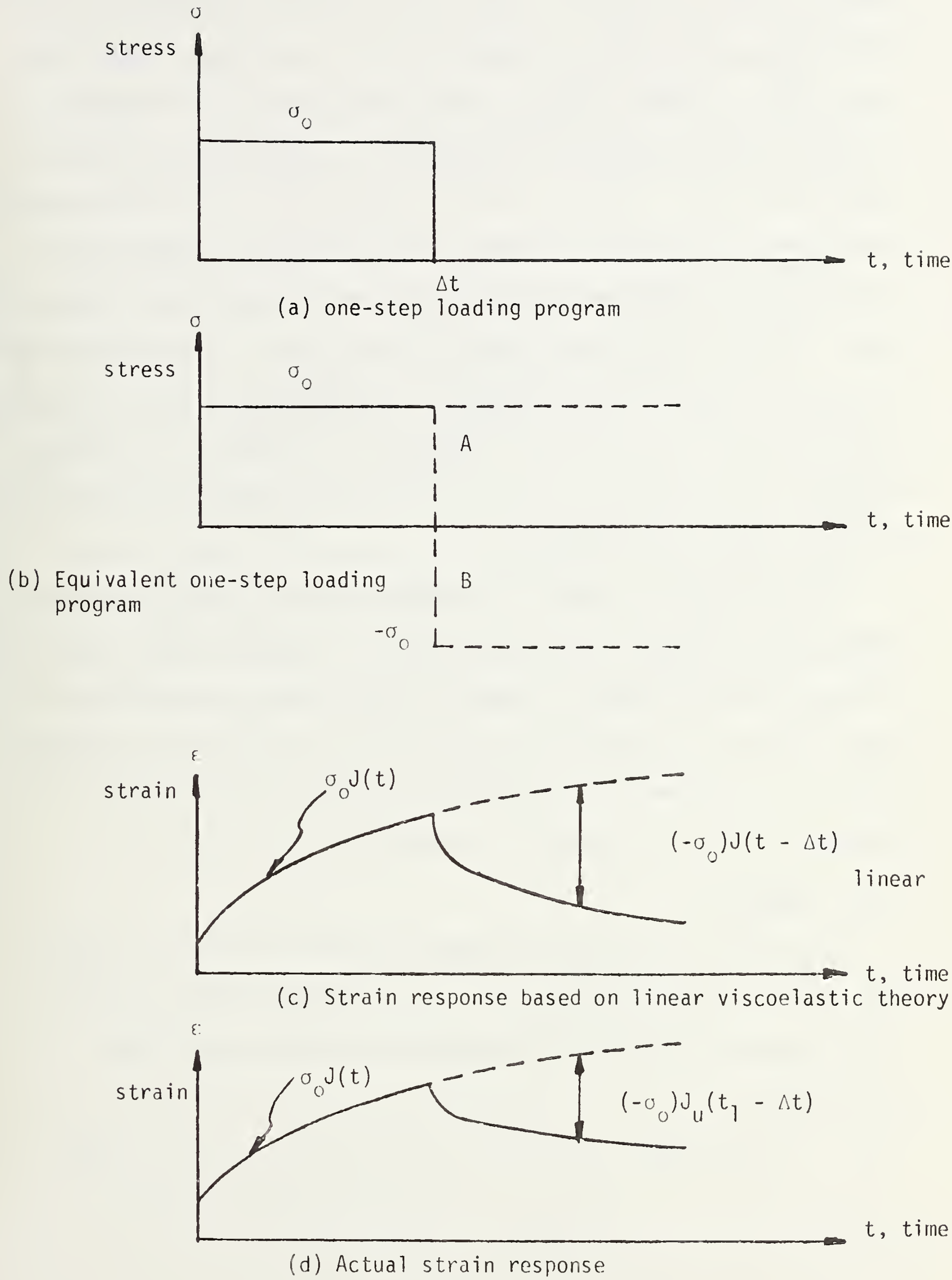

Figure 6-6 CREEP AND RECOVERY BEHAVIOR OF ASPHALT CONCRETE

where F is given in (6-7) and is the actual accumulated permanent strain at 100,000 load applications. Although the factor F can be estimated from static tests, as noted in [44], a more practical way of using (6-8) to estimate the permanent strain is to determine F directly by performing repetitive triaxial tests (up to 100,000 cycles per test) for the proposed base course materials with the applied axial and lateral stresses chosen to simulate the anticipated actual stresses occurring in the pavement structure. With F thus determined, (6-8) can then be used to estimate the permanent strain for different load applications. If, however, the results of triaxial confining repetitive tests are not available , due to lack of a testing apparatus or other reasons, a static triaxial test may be used to estimate the F factor for a given material.

So far, there is no theoretical method available which is capable of predicting the accumulative permanent strain of aggregates under repetitive loading. The work by Mandl and Lague [45], in which plasticity theory is used to describe the deformation characteristics of aggregates, may offer promise for development of a theoretical framework for prediction of the permanent strain due to repetitive loadings.

6.4.3 Clay Soil

The results of triaxial load tests on clay soil [46, 47] and some test results on Penn State Test Track Clay conducted by FHWA, indicate a linear permanent strain vs. the logarithum of the number of load applications

$$\varepsilon_p = A \log N \tag{6-9}$$

The coefficient A depends on axial and lateral stresses, moisture, compaction, and temperature for a given type of clay. At the present time, it seems that the most practical way of estimating the permanent strain under repeated loading is to use (6-9) with the coefficient A determined in the laboratory under tesing conditions, such as axial and lateral stresses, moisture, simulating the conditions anticipated in the field.

It has been pointed out [48, 49] that clay soils exhibit creep phenomenon when subjected to a constant stress and exhibit partial recovery and partial permanent strain upon stress removal. This phenomenon seems to indicate that the theoretical approaches discussed subsequently can be applied to develop a simplified method for estimating the accumulated permanent strain of clay soils from creep tests. Figure 6-7 shows the results of permanent strain induced from different load durations versus the laoding duration of a series of creep tests of Penn State Test Track Clay. A power law relation between the permanent strains and the duration of loading time for this material is clearly indicated.

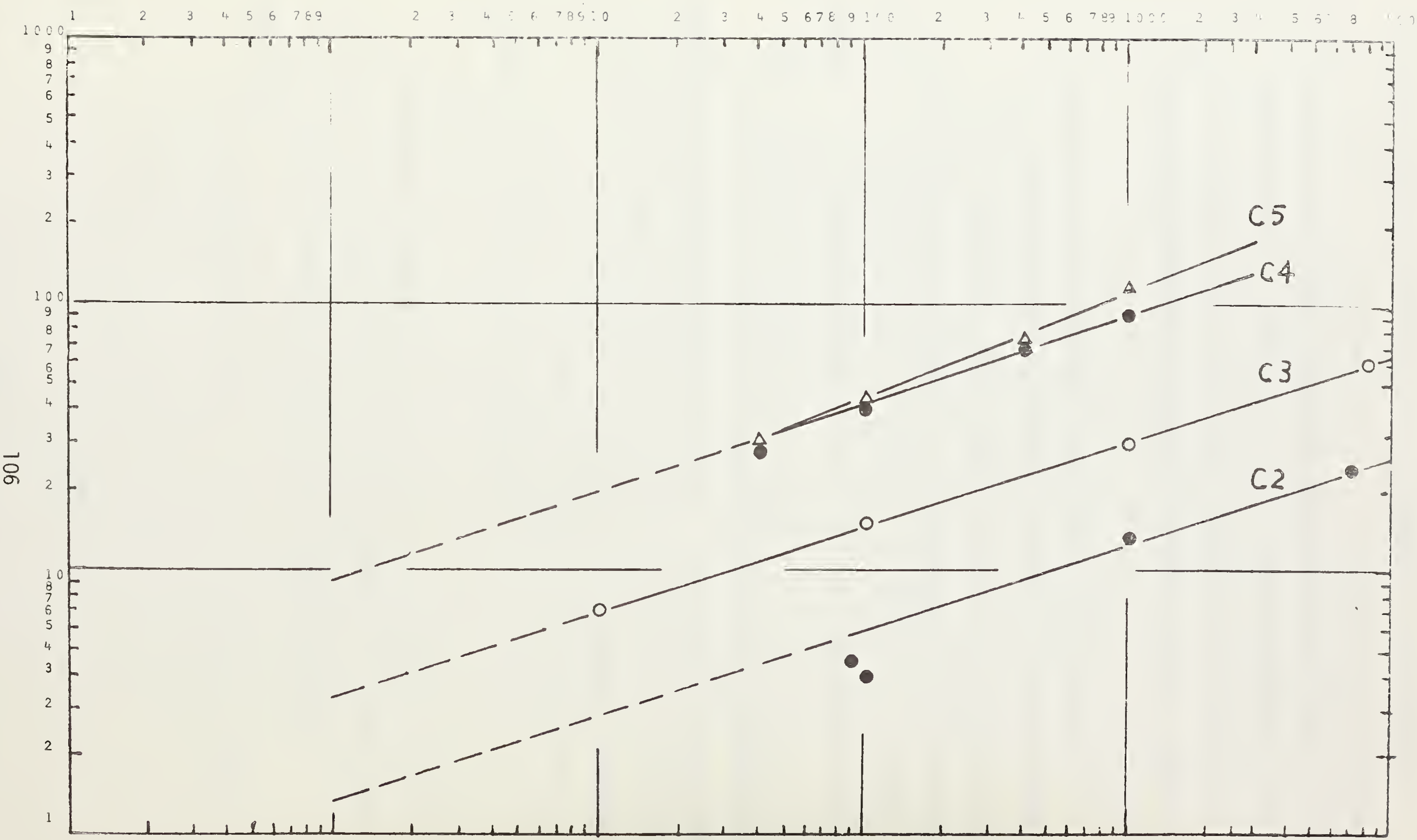

Figure 6-7 PERMANENT STRAIN VERSUS LOADING TIME OF A CLAY SOIL
(Courtesy of W.J. Kenis, FHWA)

For cement-treated base, it is assumed in VESYS IIM that this material does not exhibit permanent deformation.

6.4.4 Spatial Variations of Materials Properties

This coefficient is a measure of the statistical correlation of two points at a given distance of each other. This coefficient ρ may be represented by the following equation

$$\rho = A + B \exp\left(\frac{-x^2}{c^2}\right)$$

$$A + B = 1$$

where x represents the distance between two points to be correlated. This equation implies that two neighboring locations (x smaller) are moer likely to have similar properties (larger ρ) than two locations which are farther apart.

This coefficient of spartial variations is used to facilitate the roughness prediction.

6.5 DEVELOPMENT OF A MODEL FOR PREDICTING PERMANENT DEFORMATION OF ASPHALT CONCRETE

6.5.1 Model Development

Asphalt concrete, as noted previously and shown schematically in Figure 6-6, and also in Figure 6-8, exhibits creep behavior under a sustained load. When the applied stress is small (about 30 psi unconfined, at room temperature), the creep strain is approximately proportional to the applied stres. At higher stress, the creep strain become nonlinearly dependent on the applies stress [42].

Thus, the creep compliance $J(t,\sigma)$ of that material, which is defined as the time dependent creep strain divided by the applied constant stress, can be characterized from Figure 6-8.

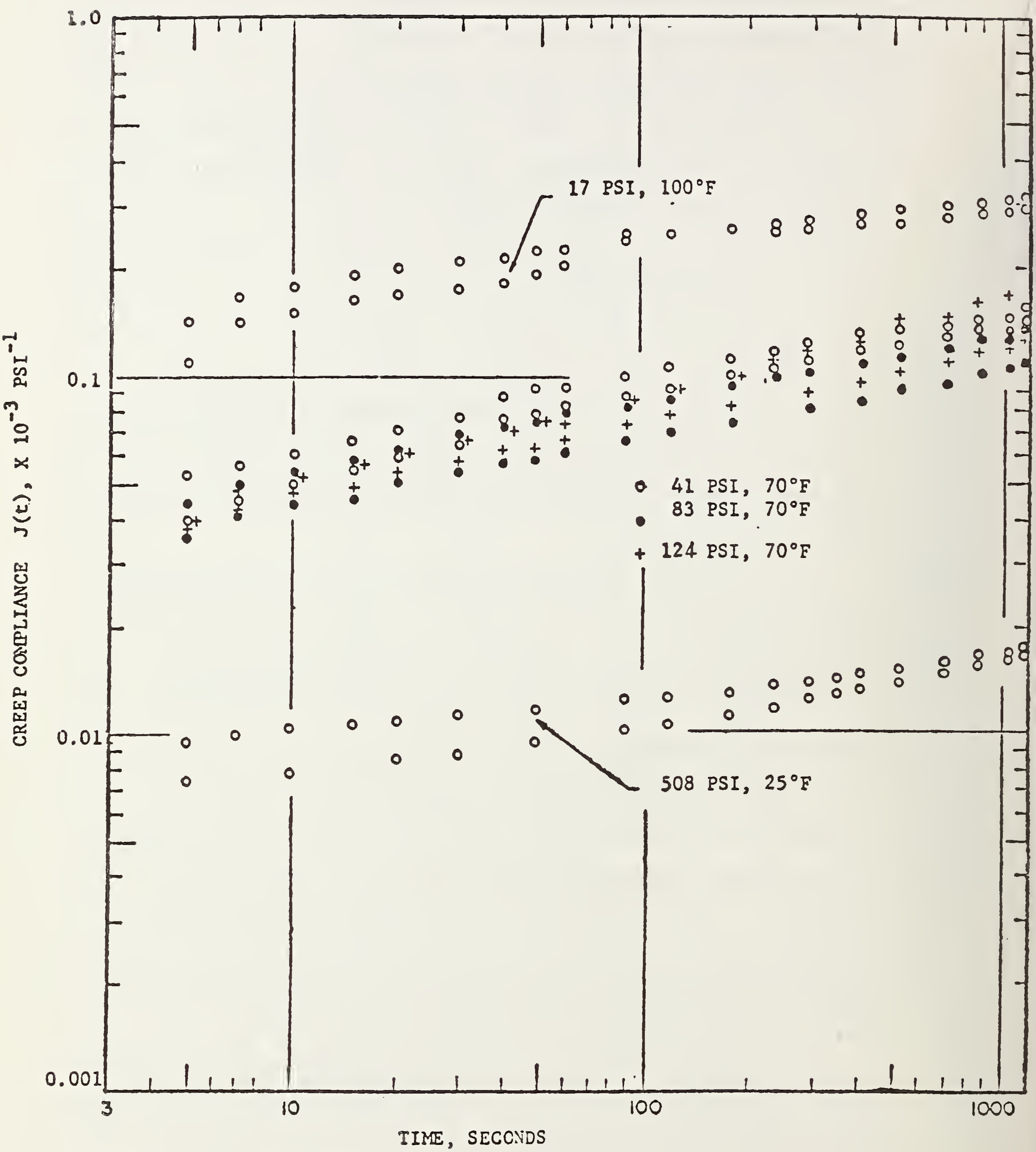

FIGURE 6-8 CONSTANT STRESS CREEP TESTS
(5.0% AC, LOW VIS., 3% RUBBER)

$$J(t,\sigma) = \frac{\varepsilon(t)}{\sigma} \tag{6-10}$$

Creep compliance could be stress dependent if the response is non-linear. For example, from Figure 6-9, the following results can be obtained

$$J(t,\sigma) = A(\sigma)\ t^n \tag{6-11}$$

$$A(\sigma) = A_0 + A_1\sigma + A_2\sigma^2 \tag{6-12}$$

The creep behavior of asphalt concrete becomes more complicated when the material is subjected to a more complex loading history. For example, the recovery behavior of the material as shown in Figure 6-9 indicates that the material exhibits permanent strain which depends upon the duration as well as the intensity of the loading. The recovery curves as shown in Figure 6-9 cannot be predicted with reasonable accuracy by using existing viscoelastic theory. For example, using the superposition principle, or modified superposition principle [50] the recovery strains, $\varepsilon_r(t)$, can be predicted from creep compliance as follows

$$\varepsilon_r(t) = \sigma_0[J(\sigma_0,t) - J(\sigma_0,\ t-t_1)] \tag{6-13}$$

$$t > t_1$$

where σ_0 is the applied constant stress and t_1 is the duration of the loading. Results of the predictions using (6-13) are shown in Figure 6-9. Similarly, using the modified superposition principle which has been used quite successfully on many other materials [51, 52] yeilds unaccpetable prediction of the creep behavior of asphalt concrete under multiple steps loading as shown in Figure 6-10.

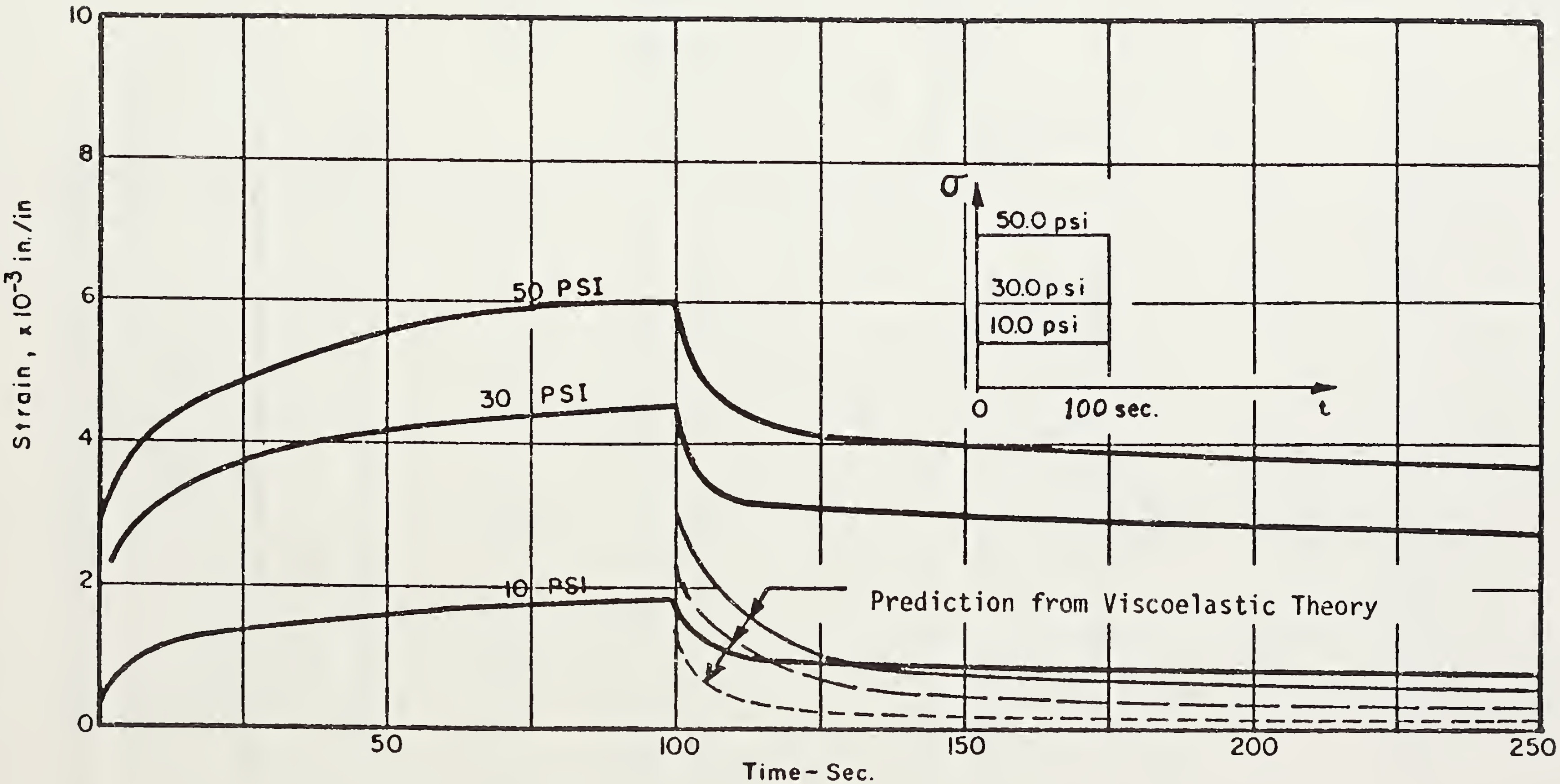

Figure 6-9 **Creep and Recovery of Constant Stress Creep Tests, $t_1 = 100$ sec.**

From [42]

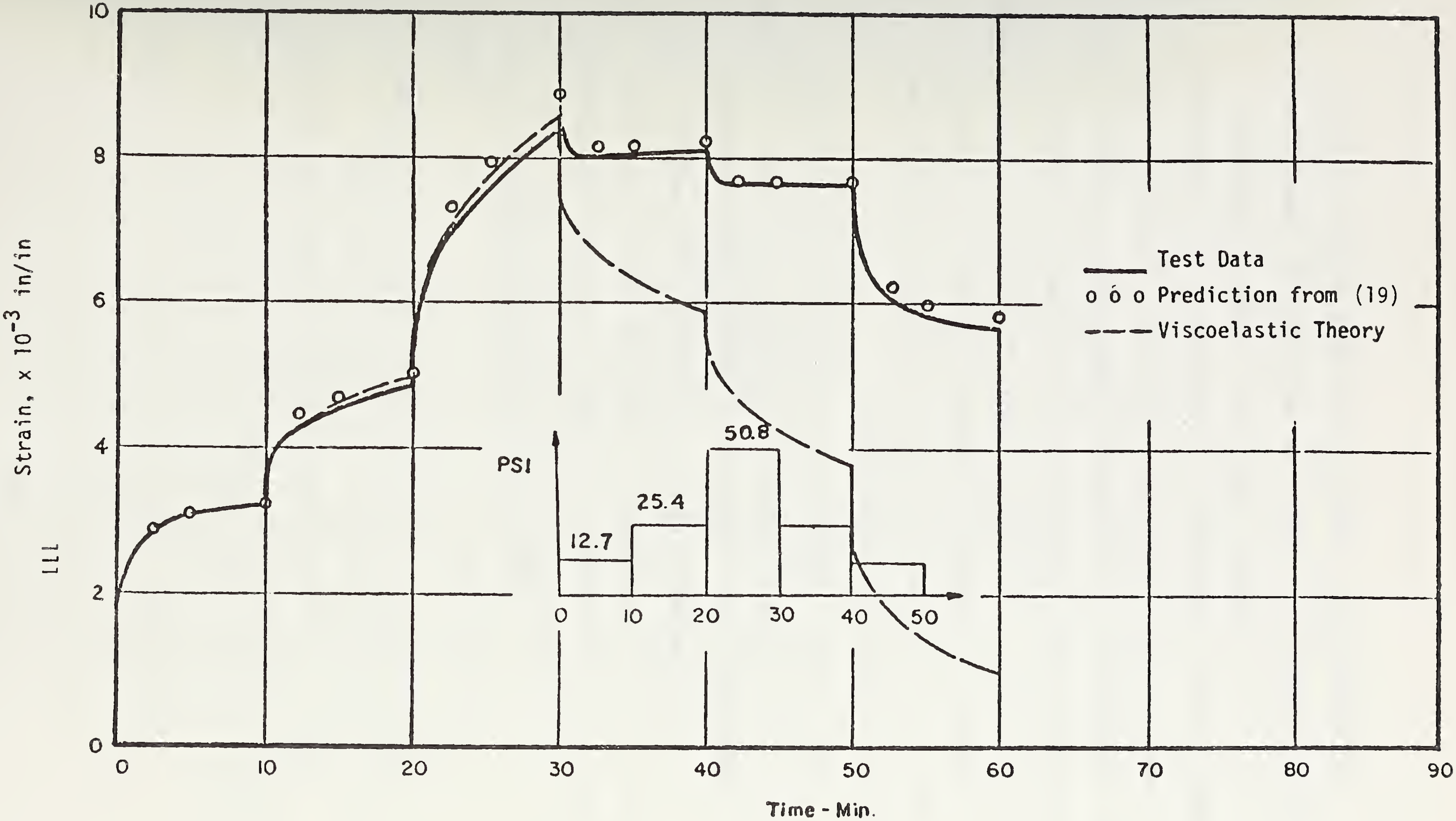

Figure 6-10 Results and Predictions of Creep Behavior under Step Loading

The large portion of unrecoverable strain of each creep test as shown in Fiture 6-9 and the observation that the amount of unrecoverable strain is dependent on the length of the loading period, suggests that the creep behavior may be represented by a generalized nonlinear Kelvin model as shown in Figure 6-11. The nonlinear dashpot contributes to the irrecoverable strain, while the series of Kelvin models (Kelvin chain) contribute to the power-law creep behavior as discussed in [51]. Therefore, the total creep strains (ε_T) are separated into two parts, the irrecoverable strains (ε_p) due to the nonlinear dashpot and the completely recoverable strains (ε_v) due to the nonlinear Kelvin chain (see Figure 6-12).

These two strain components can be characterized from a series of creep tests. Each creep test has different stress level and duration of loading. During each test, both the loading creep curve and recovery curve including the permanent strain have to be measured. The permanent strain from each test is plotted against the duration of loaidng time can be obtained. For example, (6-14) was obtained from the test results discussed in a later part of this section.

$$\varepsilon_p(t) = A(\sigma)\ t^{\alpha} \tag{6-14}$$

This equation can also be interpreted as the amount of irrecoverable strain induced at any instant of time t during a creep test.

Once this is done, the recoverable strain in each creep test can be estimated by subtracting the ε_p calculated from (6-14) from the total measured creep strain $\varepsilon_T(t)$. Thus

$$\varepsilon_v(t) = \varepsilon_T(t) - \varepsilon_p(t) \tag{6-15}$$

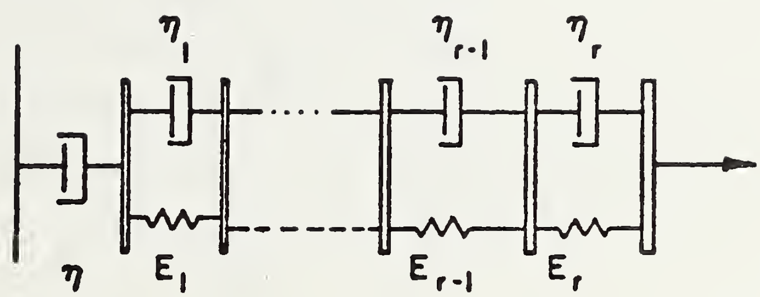

Figure 6-11 Nonlinear Generalized Kelvin Model

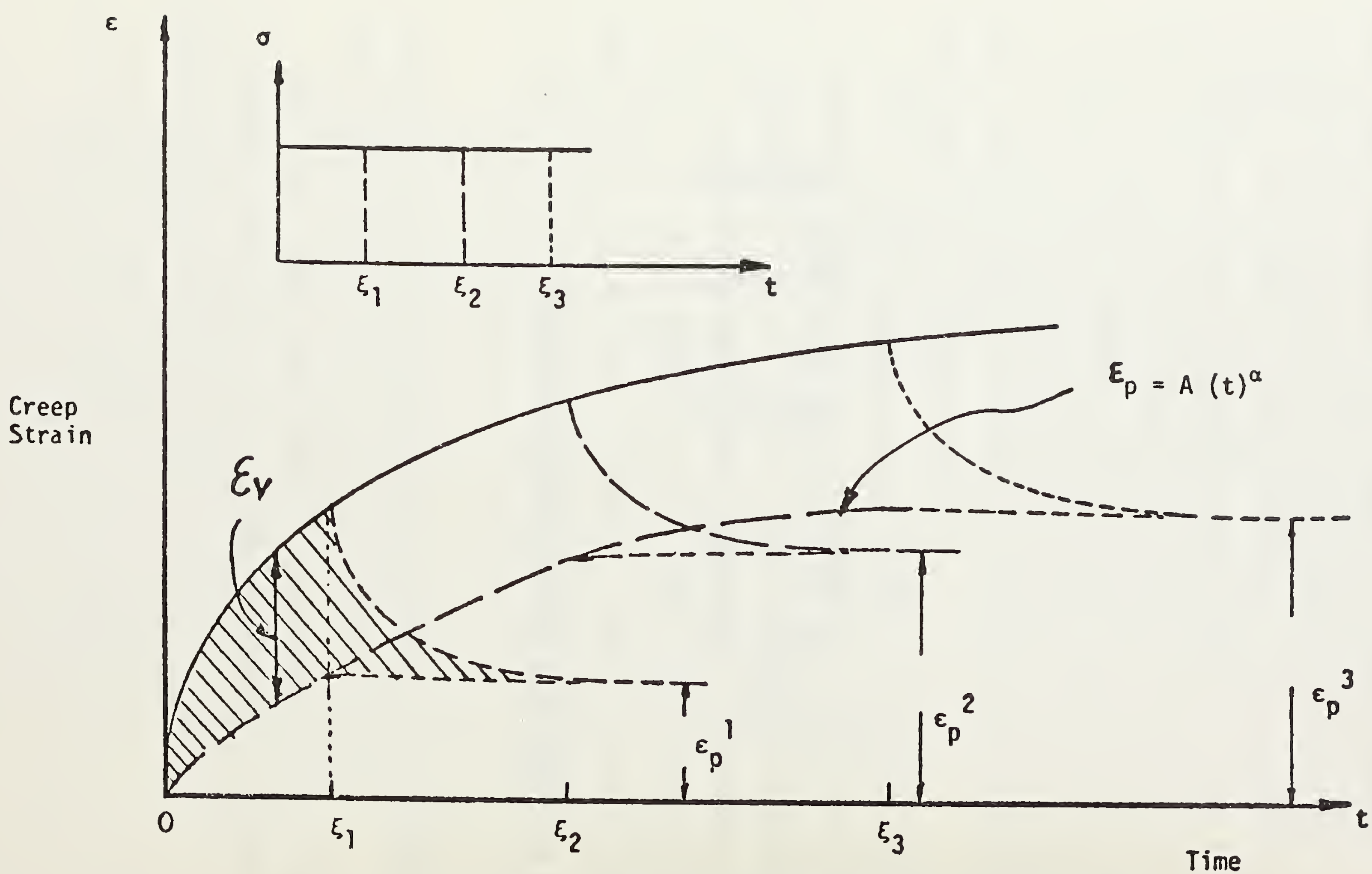

Figure 6-12 **Creep and Recovery Curves Under Different Loading Period (ξ_i)**

Again, using the same technique of cross-plotting $\varepsilon_v(t)$ versus t, an analytical form for $\varepsilon_v(t)$ can be obtained. Equation (6-16) was thus obtained

$$\varepsilon_v(t) = B(\sigma)\ t^{\beta} \tag{6-16}$$

Thus under a constant stress creep and recovery test, the creep strain ε_T and recovery strain ε_r can be predicted as follows:

$$\varepsilon_T(t) = B(\sigma)\ t^{\beta} + A(\sigma)\ t^{\alpha}\ ,\ t < t \tag{6-17}$$

$$\varepsilon_r(t) = B(\sigma)\ [t^{\beta} - (t - t_1)^{\beta}] + A(\sigma)\ t_1{}^{\alpha}\ ,\ t > t \tag{6-18}$$

Although the coefficients $A(\sigma)$, $B(\sigma)$, α, and β in (6-14) and (6-16), which may depend upon test temperature, were determined separately as mentioned above, it seems that the nonlinear optimization techniques as proposed by Distefano [53, 54] and Hufferd [55] can be used to evaluate the coefficients from the complete creep and recovery test results.

It has been shown in [42] that under time-dependent stress input, the recoverable strain can be predicted by using the superposition principle (for linear behavior) or the modified superpostion principle (for nonlinear behavior) and the irrecoverable strains can be predicted more accurately by using a strain-hardening theory. Thus,

$$\varepsilon_T = \int_0^t (t - \xi)^{\beta}\ \frac{\partial B(\sigma)}{\partial \sigma}\ \dot{\sigma}\ (\xi)d\xi + \int_0^t \dot{\varepsilon}_p(\xi)d\xi \tag{6-19}$$

where the premanent strain rate can be determined from (6-14)

$$\dot{\varepsilon}_p = \alpha\ A\ t^{\alpha-1}$$

By eliminating t from the previous equation, $\dot{\varepsilon}_p$ becomes

$$\dot{\varepsilon}_p = \alpha(\varepsilon_p)^{\frac{\alpha-1}{\alpha}} (A)^{\frac{1}{\alpha}} \tag{6-20}$$

Equation (6-19) and (6-20) were used in [42] to predict the creep of asphalt concrete under the multi-step laoding as shown here in Figure 6-10. The results compare much more closer with the test results than the predictions using strictly viscoelasticity theory.

6.5.2 Behavior of Asphalt Concrete Under Repeated Loading

Equation (6-19) and (6-20) can be used to predict the creep behavior of asphalt concrete under repeated loading. In general, the multiple steps stress inputs as illustrated in Figure 6-13A can be expressed in a single algebraic equation as follows:

$$\sigma(t) = \sum_{i=0}^{N} (\sigma_i - \sigma_{i-1}) H(t - t_i) \tag{6-21}$$

where $H(t - t_i)$ is the Heaviside's unit function which has the value 1 when $t > t_i$, o when $t < t_o$, σ-1 = 0 and $t_o = 0$. For repeated loading with variable stress amplitude but constant duration of loading (aT) and constant recovery period (1-a)T in each load application as illustrated in Figure 6-13B (6-21) becomes

$$\sigma(t) = \sigma_{2N} H (t-NT) + \sum_{i=0}^{N-1} \sigma_{2i} \left[H(t-iT) - H(t-it-aT) \right] \tag{6-22}$$

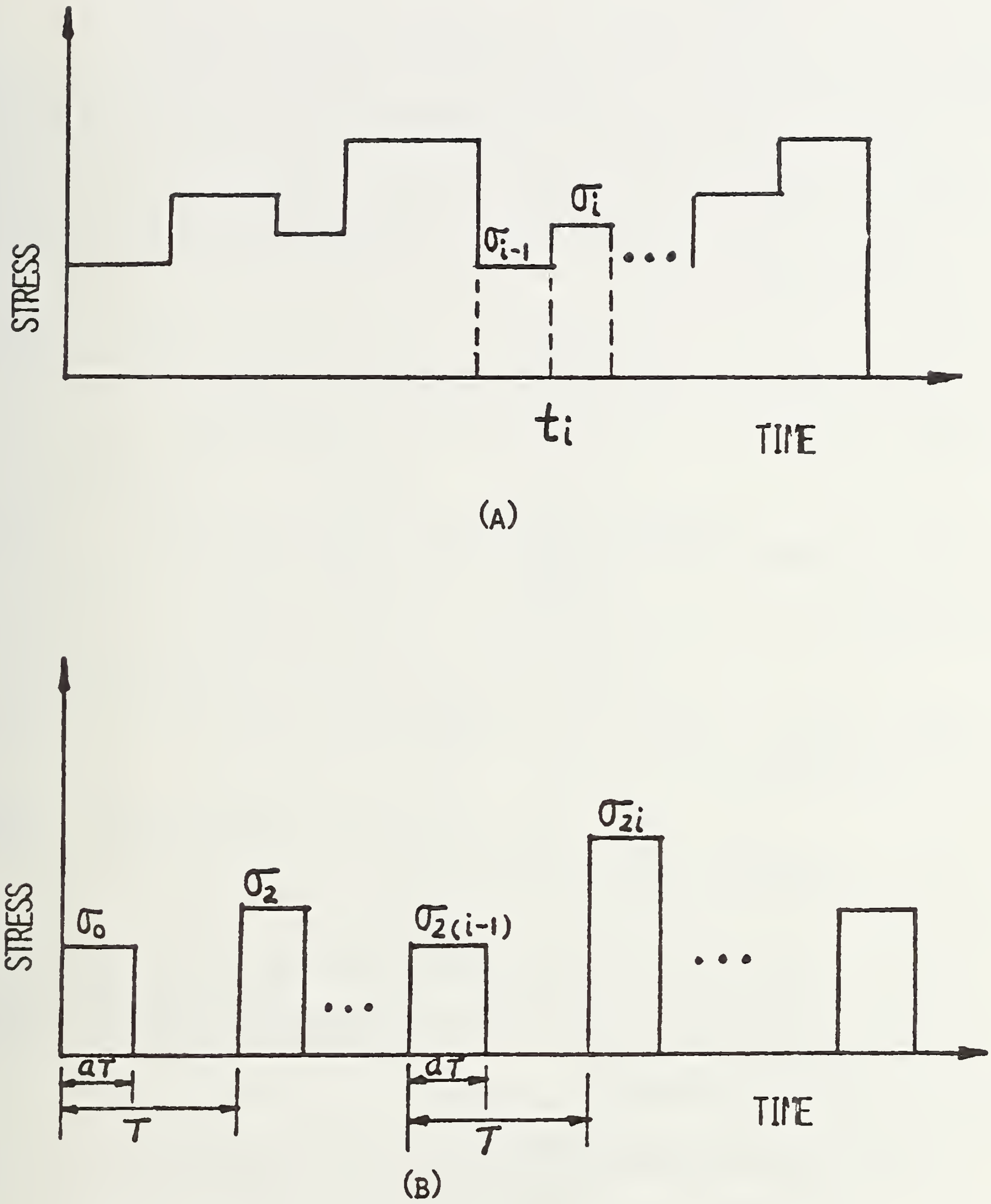

FIGURE 6-13 MULTIPLE STEP STRESS INPUTS

for $NT < t < NT + aT$ (loading portion at n-th cycle), and

$$\sigma(t) = \sum_{i=0}^{N} \sigma_{2i} \left[H(t-iT) - H(t-iT-aT)\right] \tag{6-23}$$

for $NT+aT < t < (N+1)T$ (recovery portion at n-th cycle).

Inserting (6-22) and (6-23) into (6-19) and (6-20), the recoverable and irrecoverable strains at any instance of time can be calculated. For example, the strains at the end of the loading portion in the n-th cycle ($t=NT+aT$) are

$$\varepsilon_v = B(\sigma_{2N})(aT)^{\beta} + (T)^{\beta} \sum_{i=0}^{N-1} B(\sigma_{2i}) \left[(N+a-i)^{\beta} - (N-i)^{\beta}\right], \tag{6-24}$$

$$\varepsilon_p = (aT)^{\alpha} \left[\sum_{i=0}^{N} A(\sigma_{2i})^{1/\alpha}\right]^{\alpha}. \tag{6-25}$$

The strains at the end of the recovery period in the n-th cycle, $t = (n+1)T$, are

$$\varepsilon_v = (T)^{\beta} \sum_{i=0}^{N} B(\sigma_{2i}) \left[(N+1-i)^{\beta} - (N+1-i-a)^{\beta}\right] \tag{6-26}$$

and ε_p si the same as given in (6-25). Some useful results can be obtained from (6-24), (6-25) and (6-26) which are discussed briefly in the following sections.

6.5.3 Effect of Load Duration

From (6-25), it can be seen that the permanent strain, ε_p, under repeated loading is dependent on the duration of loading time (aT). If, the stress amplitude is the same throughout the test peiod, (6-25) reduces to

$$\varepsilon_p = A(\sigma)(aTN)^{\alpha} = A(\sigma)(aT)^{\alpha}\,(N)^{\alpha} \tag{6-27}$$

This power law relationship has been found to be suitable in representing the accumulative permanent strain under repeated stress cycles from the test results shown in the next section, and has also been reported by Brown and Snaith [56]. In [56] it was also confirmed that the total permanent strain was dependent on the total effective duration of loading (aTN) instead of the total number of repetitions (N). It can also be shown that for α less than one (true in all cases), the actual permanent strain during each loading period decreases. That is, the accumulation rate decreases as the number of load applications increases.

The cumulative strain contributed from the recovery strains ε_v, as given in (6-24) and (6-25) are relatively small. For example, under constant stress amplitude, (6-26) becomes

$$\varepsilon_v = B(\sigma)(T)^{\beta} \sum_{i=0}^{N} (N+1-i)^{\beta} - (N+1-i-a)^{\beta} \tag{6-28}$$

For $0 < \beta \leq 0.2$ which is valid for most asphalt materials and for $0 < \alpha < < 0.5$, the contribution of (6-28) compared with (6-27) for a large number of load repetitions becomes very small (about 10%). The calculations of (6-28) for large N (such as 10^6) becomes very involved. However, by assuming that α and β in (6-27) and (6-28) are the same, the ratio of ε_v and ε_p is approximately equal to

$$\frac{\varepsilon_v}{\varepsilon p} \simeq \frac{B(\sigma)}{A(\sigma)}\,(\alpha \cdot \beta) \tag{6-29}$$

6.5.4 Effect of Rest Period

Equations (6-27) or (6-25) indicate that ε_p is independent on the rest period, while ε_v as given in (6-26) or (6-28) shows that, for a constant T, the longer the rest period (smaller a), the smaller the recoverable strain. For smaller a, or equivalently, longer resting period, (1-a), the contribution of ε_v to the total cumulative strain is small, particularly when β is small. The fact that the rest period has little effect on the accumulation of the permanent strain under cyclic loading has also been reported in [56].

6.5.5 Effect of Mixed Stress Amplitude

Since the contribution of ε_v to the total permanent strain is small, the discussion on the effect of mixed stress amplitude is concentrated on the ε_p part. Equation (6-25) indicates that under constant duration of loading in each cycle, the total accumulated strain is independent of the dequence in which the different stress amplitudes are applied, so long as the combination of the various stress amplitudes in the entire period is the same.

Equation (6-25) and (6-27) can also be used to determine an equivalent stress amplitude which will yield an equivalent total permanent strain at the same total number of repetitive loadings. These equations can also be used to determine different combinations of stress levels versus number of repetitions which will all yield the same total permanent strain under a sequence of mixed stress levels. This can be done so long as the constant α and the coefficient A as a function of stress have been determined from a series of creep tests.

6.6 COMPARISON OF PERMANENT DEFORMATION MODEL WITH EXPERIMENTAL RESULTS

In order to test the accuracy of (6-24), (6-25) and (6-26) in predicting the accumulative permanent strain under repetitive loading, a series of creep tests and repetitive loading tests were conducted.

6.6.1 Materials and Specimens*

A single gradation of aggregates as shown in Table 6-1 were used.

Table 6-1 Gradation of Aggregates

Sieve Size	3/8"	#4	#8	#16	#50	#200
% Passing	100	95.0	60.0	33.0	15.0	6.0

The asphalt cement used in preparing test specimens was from Phillips Petroleum Company with 85-100 penetration grade. The asphalt content was 8% by weight.

The test specimens used in this study were compressed asphalt concrete cylinders 2" in diameter and 3" in length. The asphalt mixture was pressed in a 2" diameter mold at 300 psi for 5 minutes at a temperature of 200°F. The specimens were cured in an oven at 120°F for 72 hours prior to the testing. Also, in order to minimize the randomness of the test results, each specimen was subjected to a 50 psi prestress for 12 minutes.

6.6.2 Creep Tests

A series of creep tests were conducted with 30 psi unconfined compressive stress. Each specimen was tested at different duration of loading time (2, 10, 100, 1000 seconds). At the end of the loading

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* The materials and specimens used are the same as that used in [42] except that 9% asphalt content was used in [42].

period, the specimens were allowed to recover until the recovery curve leveled off. Figure 6-14 shows the results of permanent strain induced for different loading durations versus the loading duration. These data were fitted by a power law of the form for both the irrecoverable and recoverable strain components

$$\varepsilon_p = A_p(t_o)^{\alpha} \;, \quad = 0.4$$

$$\varepsilon_v = B(t)^{\beta} \;, \quad = 0.115$$

6.6.3 Repetitive Tests

A series of repetitive tests were conducted at frequencies between 0.2 and $5H_z$. The first set of the repetitive tests had equal loading and unloading time, though each test had different frequency. The accumulative permanent strain versus number of load repetitions is shown in Figure 6-15. It is shown clearly in this figure that the accumulative permanent strain depends on the frequency. However, replotting the results of accumulative permanent strains versus effective loading time as shown in Figure 6-16 indicates that the effective load duration time is perhaps a better measure of the accumulative permanent strains in repetitive loading tests. Superimposing the equation determined from creep tests as shown in Figure 6-14 on Figure 6-16 indicates that (6-27) which can be determined from a series of creep and recovery tests, can also be used to predict the accumulative permanent strain under repetitive loading with resonable accuracy.

The second series of repetitive tests were conducted in which the loading times were kept constant and the recovery time were varied.

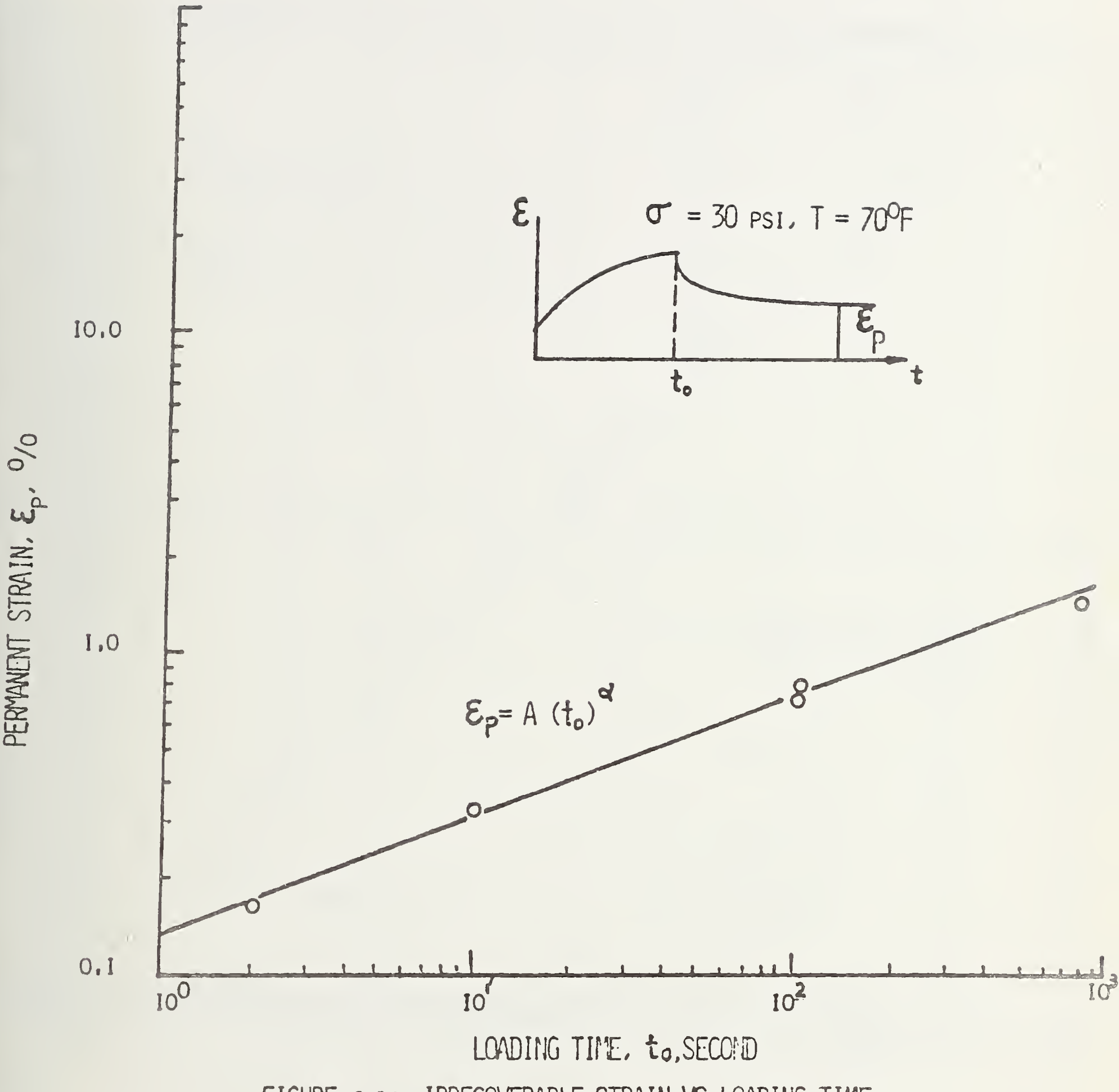

FIGURE 6-14 IRRECOVERABLE STRAIN VS LOADING TIME

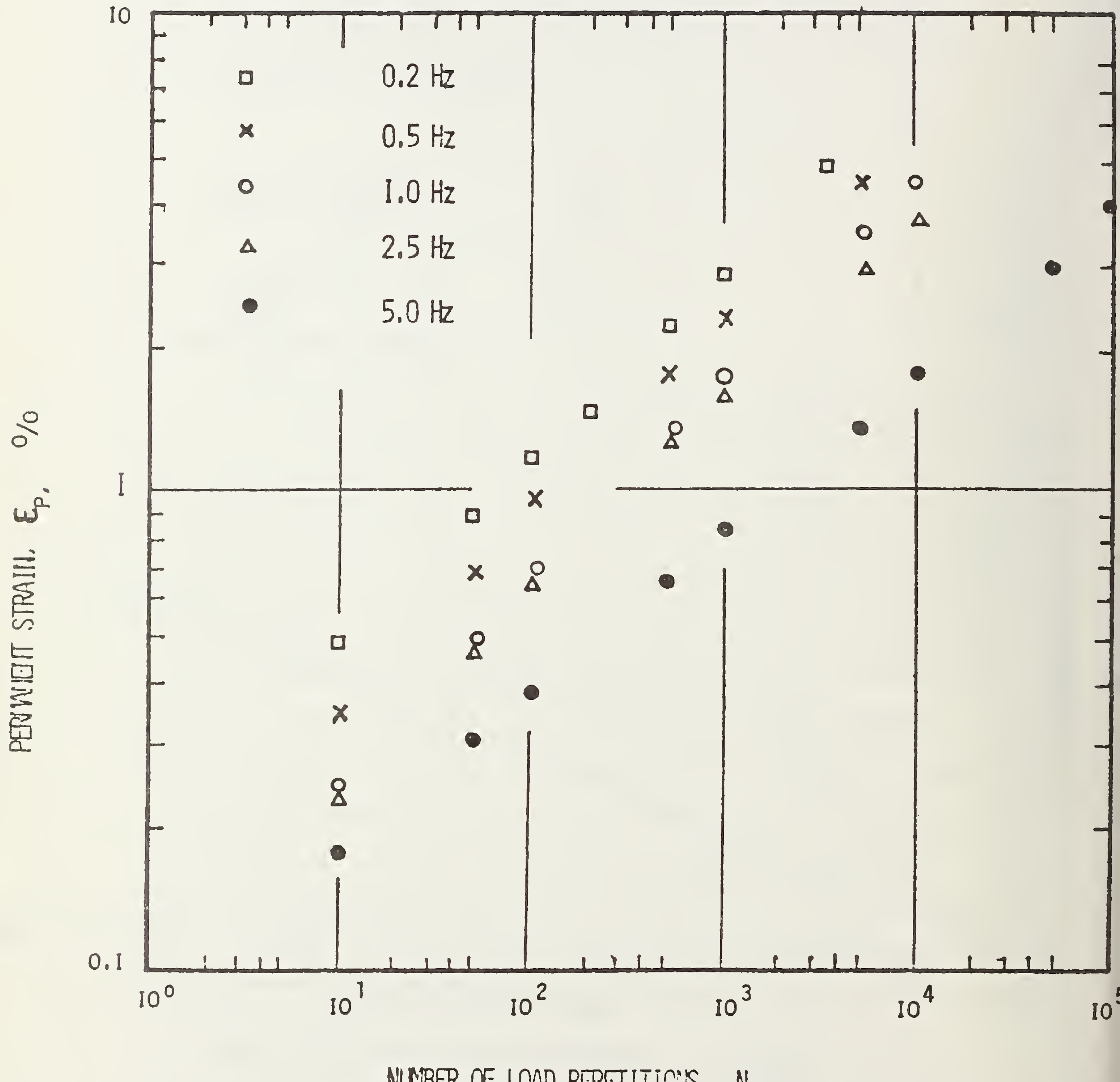

FIGURE 6-15 ACCUMULATIVE PERMANENT STRAINS VERSUS N

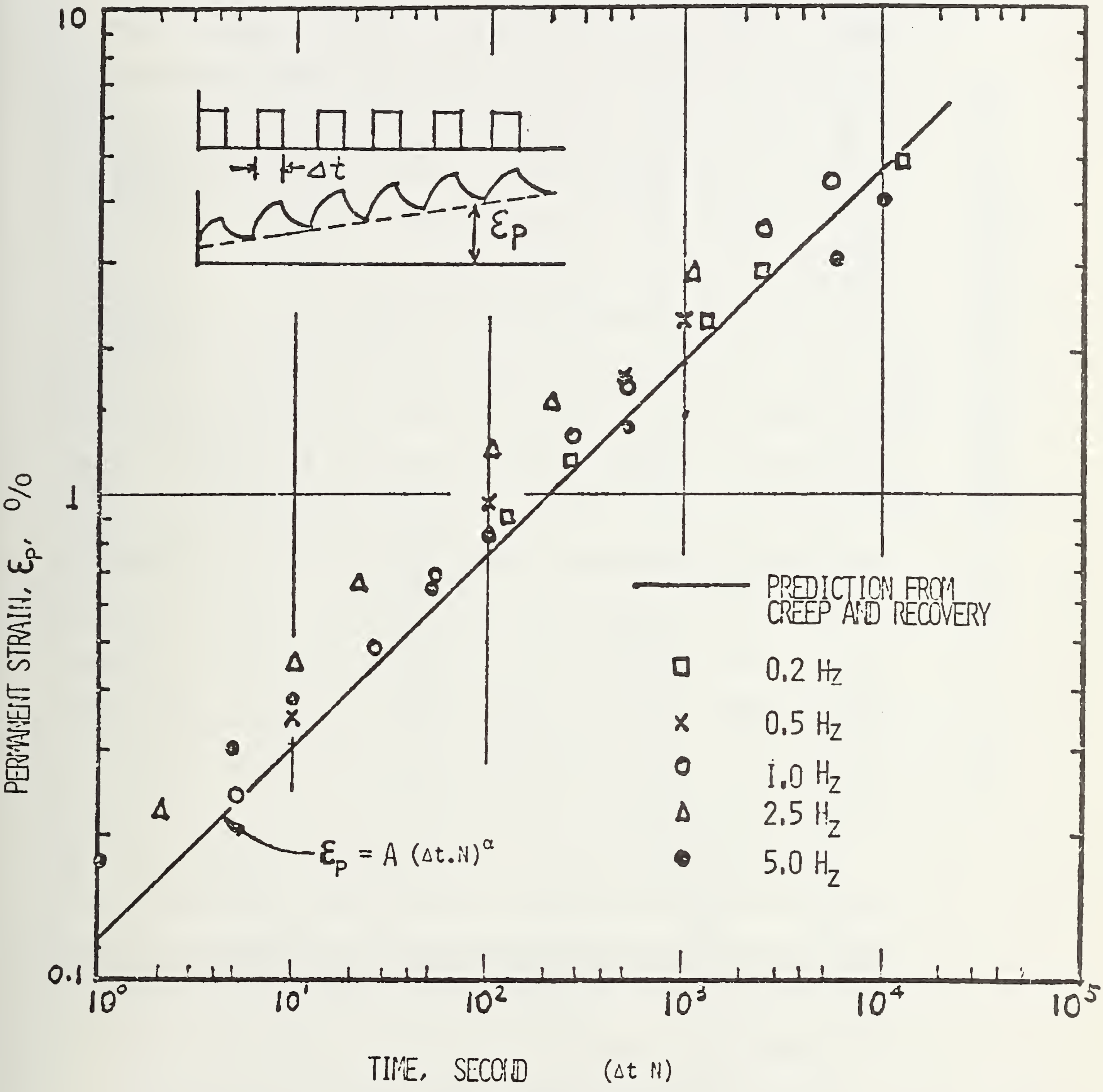

FIGURE 6-16 ACCUMULATIVE PERMANENT STRAINS VERSUS EFFECTIVE LOADING TIME

The results shown in Figure 6-17 indicate that the longer the resting time, the smaller the accumulative permanent strain.

In the third series of repetitive tests, each repetitive test consisted of mixing stress amplitude. The results shown in Figure 6-18, in which three stress levels of 20 psi, 30 psi and 40 psi were applied in different sequence, indicate that at the end of 54 cycles the total acumulation of permanent strain is the same. Additional tests at higher frequencies and for a larger number of cycles are needed, however, to further verify this result.

6.7 PROBABILISTIC PERMANENT DEFORAMTION MODEL

One of the primary objectives in the development of the rationally based pavement design framework is to take into account the variability of the material properties, traffic loading and environmental factors. In the primary response model, as mentioned before, the input material properties (creep compliance, resilience modulus, etc.) are considered probabilistic in nature and the output stresses, strains, and deflections are also probability functions, even when the input traffic loading adn environmental factors are deterministic. The variability of the input from the primary response model is further compounded by the variability of temperature and moisture.

Thus, the output from the rut-depth model is expected to be stchastic inasmuch as the output from the primary response model is input to the rut-depth model plus the fact that the permanent deformation characteristics of paving materials under repetitive loadings are expected to be variable.

In (6-2), (6-3), (6-8) and (6-9), the variability of permanent

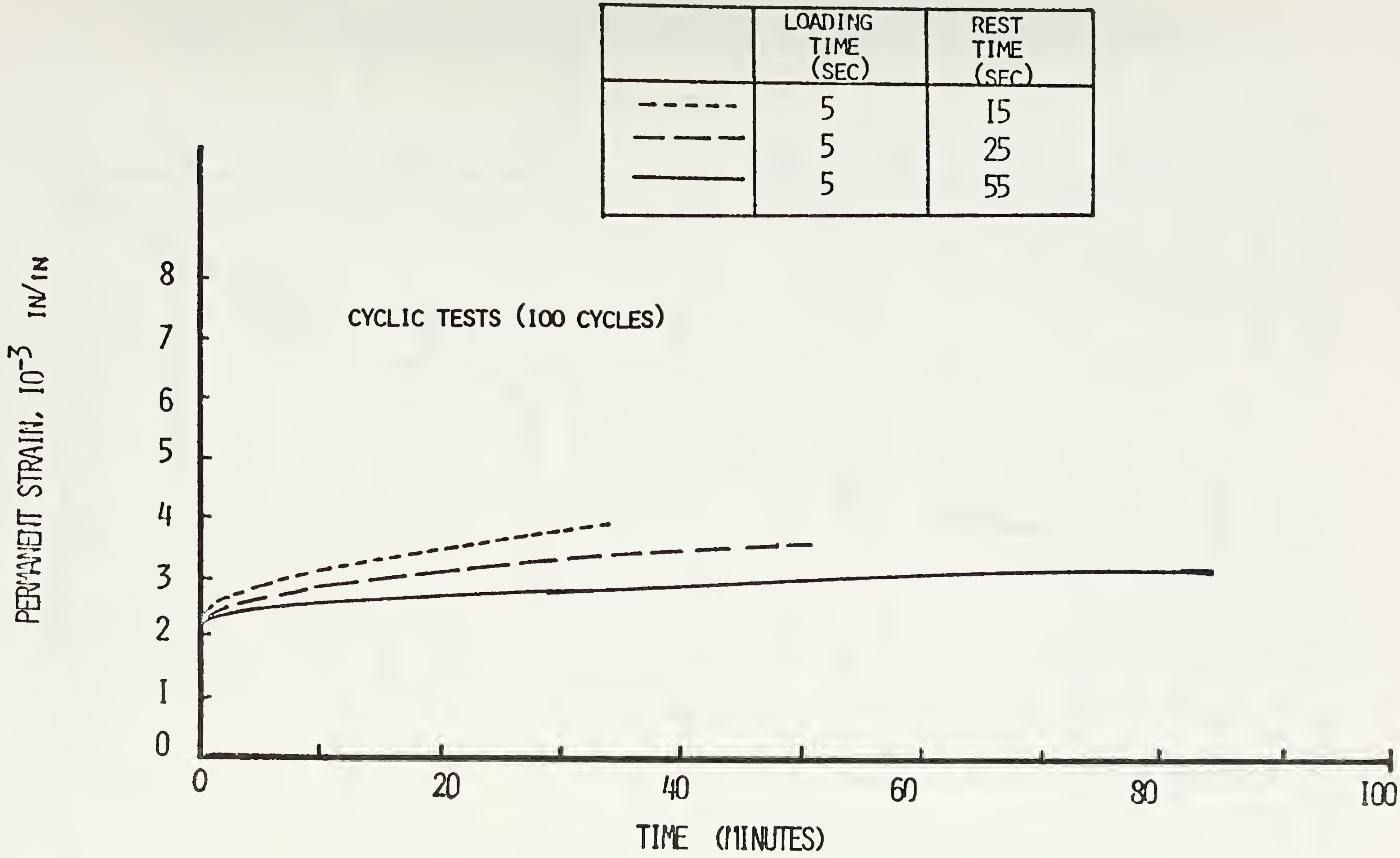

FIGURE 6-17 ACCUMULATIVE PERMANENT STRAIN VERSUS TOTAL TIME FOR DIFFERENT RESTING PERIOD

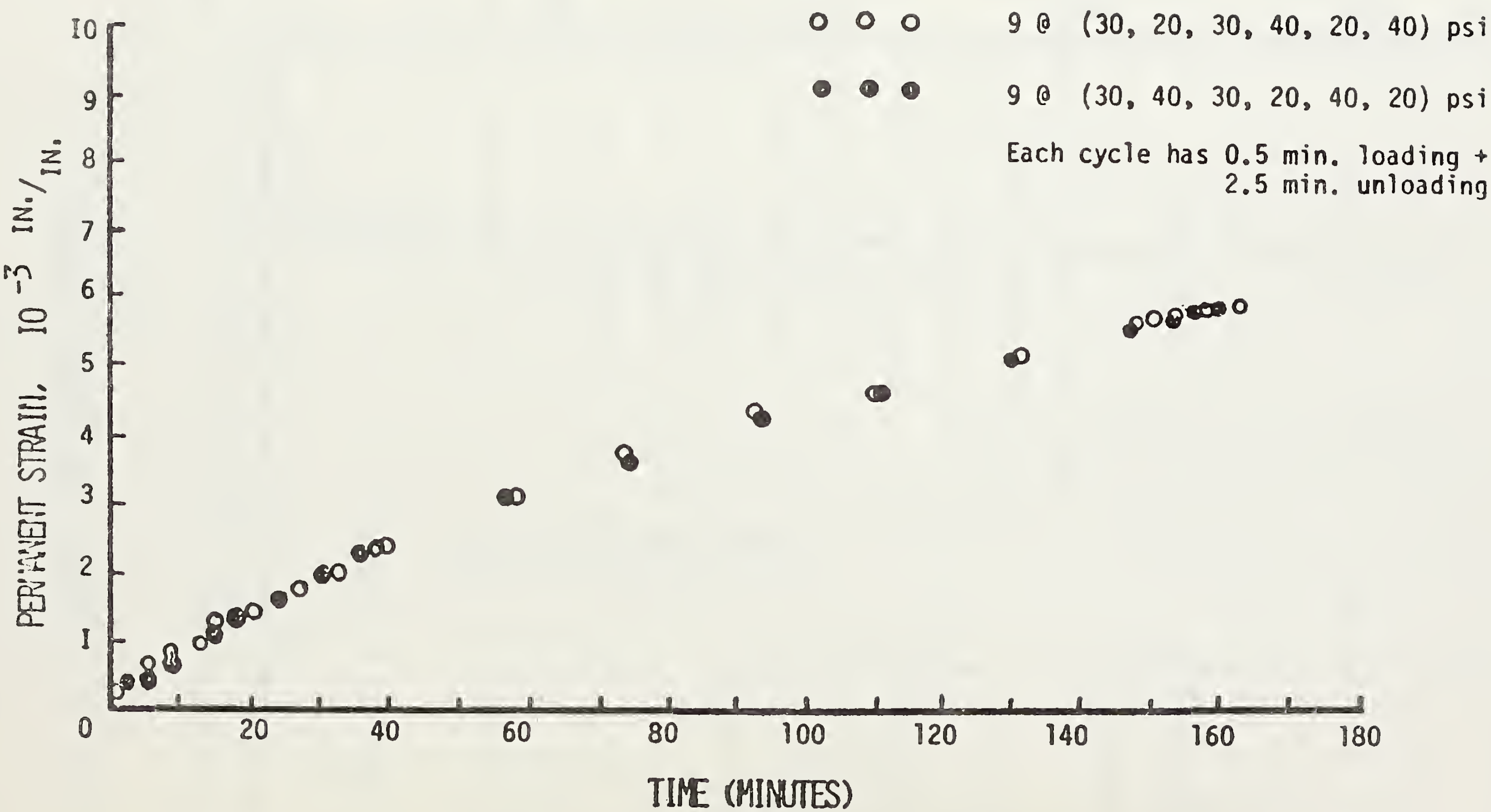

FIGURE 6-18 THE EFFECT OF LOADING SEQUENCE ON THE ACCUMULATIVE PERMANENT STRAINS

strain under a given set of conditions (given stresses, temperature, etc.) can be obtained directly from a number of repeated tests under the given conditions. This is shown schematically in Figure 6-19. In principle, if a sufficient number of repetitive tests can be conducted, a constitutive equation for permanent strains whcih take into account the variability as a function of stresses, etc., can be developed, for example

$$\varepsilon_p = A(\sigma_1, \sigma_3, \ldots) \log N + D_A (\sigma_1, \sigma_3, \ldots) \log N \tag{6-30}$$

In (6-30) D_A is the standard deviation of A. Both A and D_A are functions of the testing conditions (σ_1, σ_3, etc.). Since (6-30) is determined in the laboratory, σ_1, σ_3, etc., in (6-30) are treated as deterministic variables.

One of the disadvantages of (6-30) is that even though the probabilistic nature of the stresses obtianed from the primary response model are directly influenced by material variability, it is not possible to separate the influence of material variability upon permanent strain from that of the stresses.

In the primary response model in VESYS IIM the variability of stress is expressed in terms of its mean and deviation ($\bar{\sigma}$, D_σ). The variability of permanent strain can be estimated in terms of mean stress and deviation as follows:

$$\bar{\varepsilon}_p = \bar{A}(\bar{\sigma}_1, \bar{\sigma}_3, \ldots) \log N + D_A(\bar{\sigma}_1, \bar{\sigma}_3, \ldots) \log N \tag{6-31}$$

$$\left(D_{\varepsilon_p}\right)^2 = \left\{\left(\frac{\partial A}{\partial \sigma_1} + \frac{\partial D_A}{\partial \sigma_1}\right)^2 \left(D_{\sigma_1}\right)^2 + \left(\frac{\partial A}{D\sigma_3} + \frac{\partial D_A}{\partial \sigma_3}\right)^2 \left(D_{\sigma_3}\right)^2\right\} (\log N)^2 \tag{6-32}$$

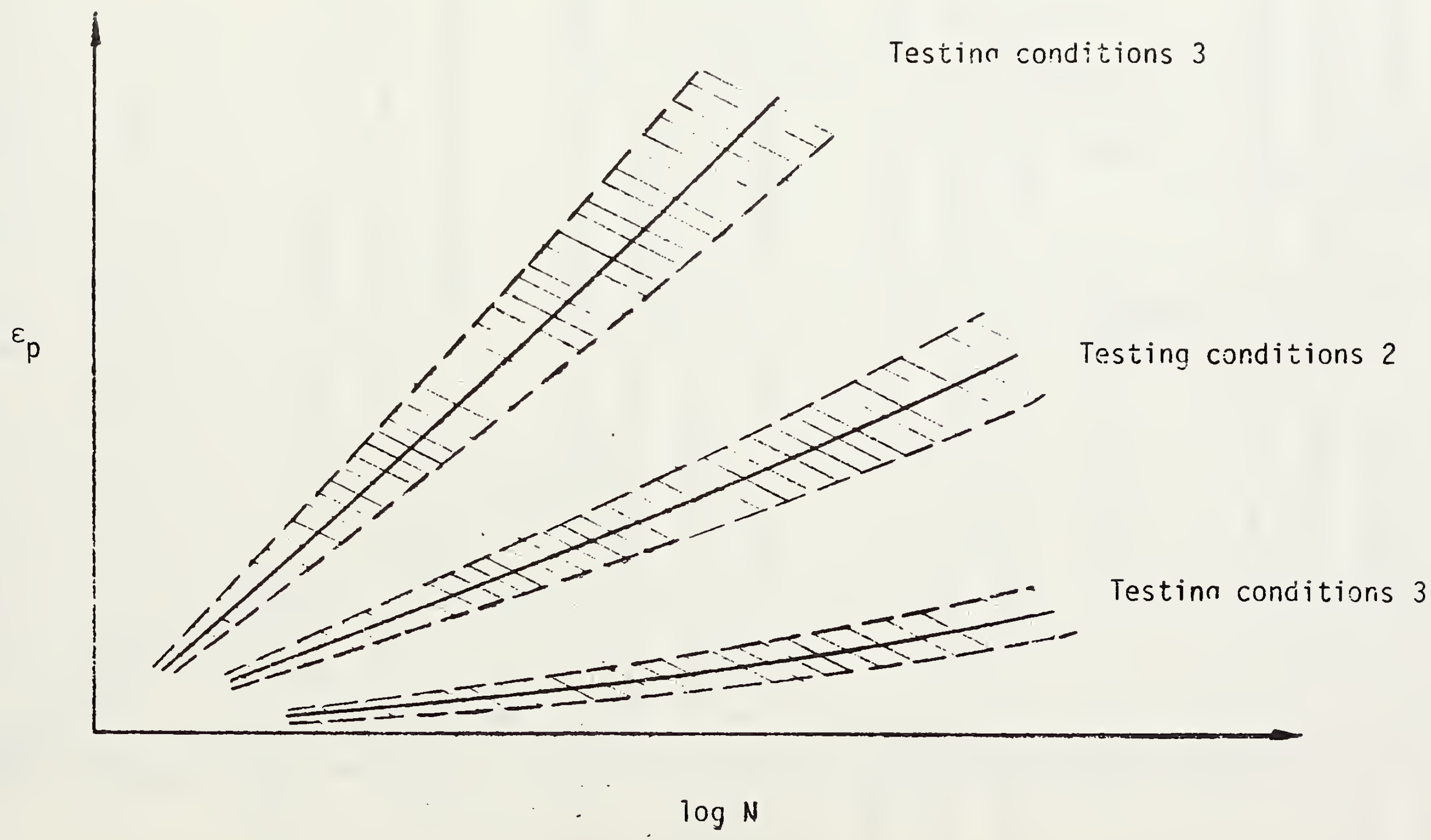

Figure 6-19 Variability of Permanent Strains in Repetititive test under different testing conditions.

In (6-4) and (6-27) however, the variability of permanent strain can be obtained indirectly from creep tests. The permanent strain including the variability of (6-4) can be rewritten as follows:

$$\varepsilon_p = F(t)\ \sigma + D_F\ \sigma \tag{6-33}$$

where

$$F(t) = [J(t) - J(t-\Delta t] + (1-\mu)\ J(t-\Delta t)\ e^{-\beta\varepsilon} \tag{6-34}$$

$$D_F(t) = (1-\mu)\ D_J(t-\Delta t)\ e^{-\beta\varepsilon} \tag{6-35}$$

In (6-33) and (6-35), D_F is the deviation of the permanent strain, and D_J in (6-34) is the deviation of the creep compliance. The mean and deviation of permanent strain for a variable stress is given by

$$\bar{\varepsilon}_p = F(t)\ \bar{\sigma} + D_F(t)\ \bar{\sigma} \tag{6-36}$$

and

$$\left(D_{\varepsilon_p}\right)^2 = [F(t) = D_F(t)]^2\ (D_\sigma)^2 \tag{6-37}$$

In (6-27) the variability of the permanent strain is obtained from the variability of the irrecoverable portion of the creep tests.

$$\varepsilon_p = A_p(\sigma)\ [N(aT)]^\alpha + D_A\ (\sigma)\ [N(aT)]^\alpha \tag{6-38}$$

In (6-38) $D_A(\sigma)$ is the standard deviation of the irrecoverable part of the creep tests. For example, the mean values and the standard deviations of the irrecoverable part of the creep tests of the asphalt concrete cores taken from the Green River Test Road [57] indicate that both of these properties depend on the mix design and the age of the materials.

Higher coefficient of variabions were observed for low asphalt content mixtures and for older asphalt concrete cores. When the stress varies, the permanent strains of (6-38) become

$$\bar{\varepsilon}_p = A_p(\bar{\sigma})\ [N(aT)]^{\alpha} + D_Z(\bar{\sigma})\ [N(aT)]^{\alpha}$$

$$\left(D_{\varepsilon_p}\right)^2 = \left[\left(\frac{\partial A_p}{\partial \sigma}\right)^2 + \left(\frac{\partial D_A}{\partial \sigma}\right)^2\right] (D_\sigma)^2 \quad [N(aT)]^{\alpha} \qquad (6\text{-}39)$$

The variability of the loading duration time aT under each application can also be incorporated in (6-39).

It seems that (6-38) and (6-30) offer a more practical applicatio for those materials which exhibit flow (plastic deformation) under constant loading. Not only are no additional tests needed, aside from those required for the primary response model, but more importantly, the variability of the permanent deformation is tied in directly with the variability of the materils properties used in the primary response model in which the stresses and strains in the pavement system are to be evaluated.

At the present time, the major obstacle in using these approaches is the lack of sufficient experimental results to verify these theories. For that matter, more experimental results are needed ont only to verify (6-38) and (6-39), but also to simplify them and to simplify (6-31) and (6-32) as well. Eventually, when sufficient experimental data has been obtained, it may be possible to decide on which approach should be taken and develop a concise material characterization program. At the present time, it is best to allow users the flexibility of selecting the type of theory to be used for different materials. This selection be based upon user's perference and the availability of resources in conducting the necessary testing.

6.8 MODULAR FRAMEWORK FOR RUT-DEPTH PREDICTIONS

As mentioned earlier, the VESYS II Computer program and design framework have not yet reached the final stage of development. There is still room for modifications and improvements to the program. In the primary response model, different versions of layer systems and finite element methods can be used to improve the predictive capability of the model. On the other had, in the rut-depth submodel, there is still need for a rather large effort directed at development of constitutive equations for permanent strain for various materials. Similarly, other damage submodels require further modifications and improvement as well. Because of this, the VESYS IIM is developed in modular form so that any improvement or modifications on any part of the system can be readily incorporated into the system without having to change the rest of the system.

The modular concept for the rut-depth submodel is shown schematically in Fiture 6-20. In this submodel, the input data is the stresses (means and deviations) at various prsitions in the pavement sustem. The stresses are the output from the VESYS IIM primary response model or any other source. The calculations of the total permanent deformation within each layer is also a modular form; that is, the modules for each can be readily replaced or omitted. For example, for a full depth asphalt pavement, the modules for calculating the permanent deforamtion, other than the ones for asphalt concrete and subgrade, can be omitted. Within each module for calculation of the permanent deforamtion of each layer, the constitutive equation for permanent strain is also modular in that

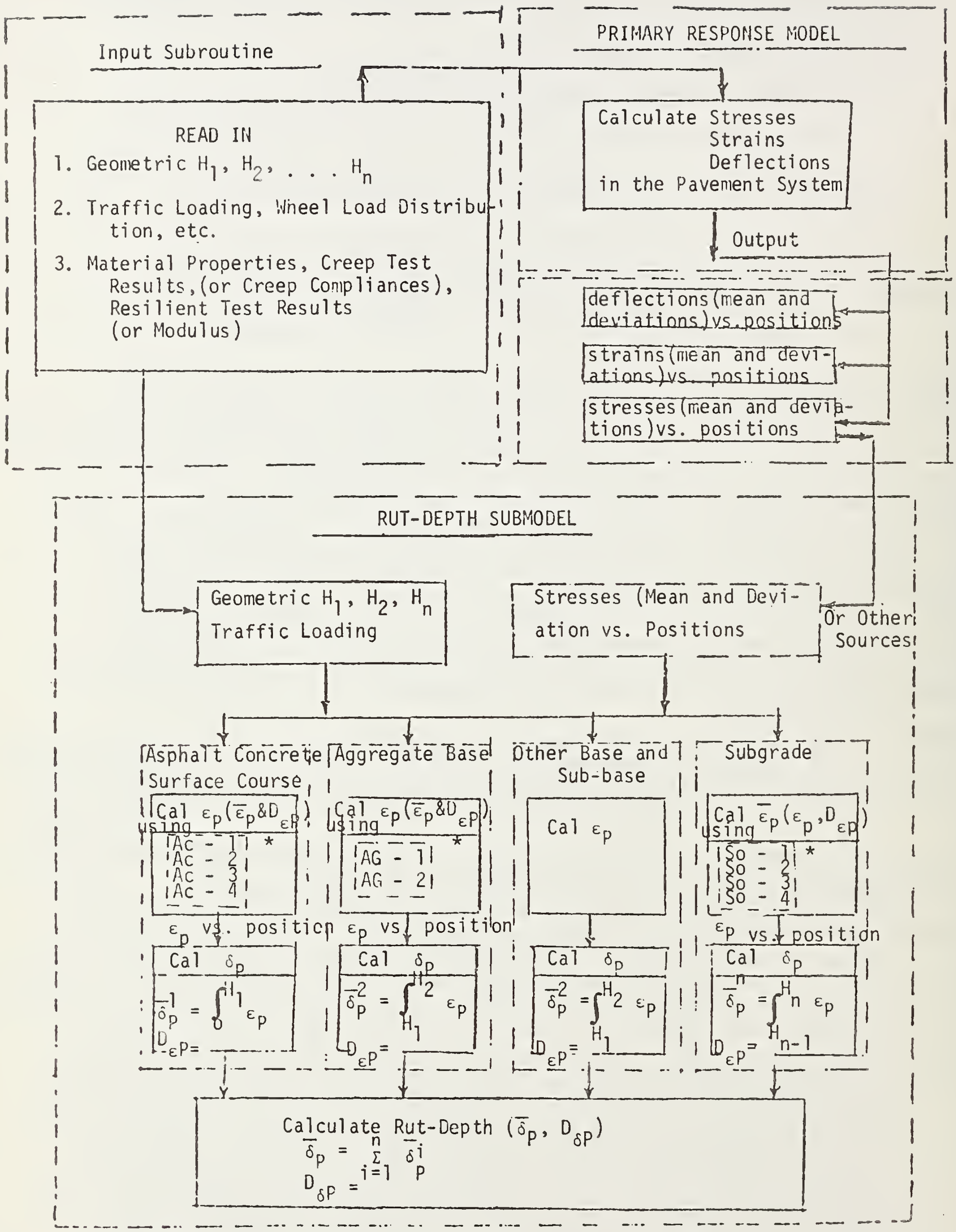

Figure 6-20 Modular Form of the Rut-Depth Submodel

(Note: Subroutines enclosed by dashed lines are independent modules which can be replaced or in some cases omitted without affecting the overall calculations.)

*See Table 6-2

different constitutive equations can be substituted depending upon the user's preference with minimal incovenience to the user. For example, in the module for calculatio of permanent deformation of an asphalt concrete surface course, different constitutive equations such as AC-1, AC-2, AC-3, or AC-4 (see Table 6-2) may be used.

6.9 CONCLUDING REMARKS

The philosophy and methodology for the development of a realistic model for rut-depth predictions has been presented.

The primary motivation for the procedure adopted is the need to provide for maximum flexibility in the particular permanent deformation characteristics observed of different paving materials and the different layers of the pavement system, and the establishment and verification of simplified experimental methods for rut-depth predictions which offer substantial appeal from a practical highway designer's point of view.

A simplified method for predicting permanent deformation of asphalt concrete under repeated loading using the results of creep and recovery tests is proposed. The prediction of permanent deformation using this method compared with the experimental results of asphalt concrete under repeated loading is excellent. Further verification and implementation of this simplified method and the overall rut=depth model proposed herein is required, however.

The approach is based on the premise that the permanent strain under repeated loading is due solely to "viscous flow," which for a well compacted rich mix asphalt concrete should be quite accurate. The permanent strain of asphalt concrete inducted during creep tests at different loading periods is used to predict the permanent strains induced under repetitive loadings. This is accomplished by conducting several creep tests at several stress levels

TABLE 6-2

CONSTITUTIVE EQUATIONS FOR PERMANENT STRAINS AND TESTING PROGRAM FOR FLEXIBLE PAVEMENT SYSTEM

Materials	Constitutive Equations can be used		Test Required
	Designation	Equation No. in the text	
Asphalt Concrete (AC)	AC-1	(6-31),(6-32),(6-2),(6-3)	Repetitive loading tests, varying stress state
	AC-2	(6-36),(6-37),(6-4)	A few repetitive loading tests, creep tests
	AC-3	(6-39),(6-27)	Creep tests
	AC-4	Other types	——
Aggregate (AG)	AG-1	(6-31),(6-32),(6-7),(6-8)	Repetitive loading tests
	AG-2	Other types	——
Clay-soil (SO)	SO-1	(6-31),(6-32)	Repetitive loading tests
	SO-2	(6-36),(6-37),(6-4)	Repetitive loading tests, creep tests
	SO-3	(6-39),(6-27)	Creep tests
	SO-4	Other types	——
Other Materials			

for different loading durations. During each test, both the loading and recovery portions of the creep test are recorded. A creep compliance is determined from the loaidng portion of the test. From the unloading portion of the results for different durations of loading time an equation relating the amount of total permanent strain to the loading time duration can be established. The effect of load duration per cycle and the number of load applications on the permanent deformation can be combined into a single "effective" load duration parameter. The approach discussed is more applicable to a rich mix asphalt concrete, for the reasons meantioned before. For asphalt concrete with low asphalt content, the total permanent strain due to repeated loading can no longer be considered as solely due to "viscous flow" and hence (6-24) - (6-26) alone are no longer adequate to describe the permanent deformation. It is more likely that the following equation should yield a better prediction

$$\varepsilon_p = A(\Delta tN)^{\alpha} + (N)^{\xi} \tag{6-40}$$

The first term in this equation is the same at (6-27) and the second term is considered due to "time indipendent plastic deformation" or "friction" as against "viscous flow". In principle, from the creep and recovery test results, and plotting irrecoverable strain versus the load duration, similar to Figure 6-14, the following representation can be obtained

$$\varepsilon_p = A(t_o)^{\alpha} + \gamma \tag{6-41}$$

The exponent for the second term of (6-41) can be determined from a single additional repeated loading test.

CHAPTER 7

CONCEPTS FOR DESIGN OPTIMIZAITON OF FLEXIBLE PAVEMENT SYSTEMS*

7.1 INTRODUCTION

The primary objective of this report is the development of concepts for optimization of design decisions for flexible pavement systems based on viscoelastic structural models. The research approach adopted was to review previous work; study conceptual interrelations of system components, specific decision criteria and optimizing techniques; define and recommend phasing for the future developments needed and to propose a conceptual program for the first phase. The results of these studies are discussed in detail in subsequent sections.

There has been a very considerable amount of work expended toward development of a rational flexible pavement design and management system that would allow optimal design decisions based on the predicted lifetime performance and economics of flexible pavement systems. Such systems exist and are in use [58 - 60]. However, knowledgeable persons understand that this is a beginning and that years of continued use, feedback, and refinement are required to complete this effort. The potential interim and long-term payback on this investment is quite large when one considers the billions of dollars spent for pavement maintenance and the huge savings in time and money that may be realized through very small improvements in our ability to optimize design decisions.

The successful development of the VESYS System into a complete Pavement Analysis and Design System (PADS) is considered to be heavily dependent on expediting realization of its capability as a complete system. Recommendations have been made in a subsequent section for accomplishing this as the next phase of research and other recommendations are included for further development in subsequent phases.

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* This chapter was prepared under subcontract to Austin Research Engineers (ARE) by J.B. Ranhut and W.R. Hudson. The ideas presented represent those of ARE. Important contributions to the ideas presented were made by Drs. B.F. McCullough and M. Darter and Mr. Harvey J. Treybig, to whom the authors express their appreciation.

7.2 REVIEW AND SYNTHESIS OF PREVIOUS WORK

Conceptual work on the development of a systematic approach to pavement design was carried out by Hudson, Finn and McCullough in 1967 to 1970 [60] with the support of the National Cooperative Highway Research Program (NCHRP), Highway Research Board. This work modified and extended previous work by Scrivner, McFarland and Carey [61] that had in 1968 culminated in a computerized flexible pavement design method. An updated and improved system called Systems Analysis Method for Pavements (SAMP) with a computer program designated in [60] as SAMP5 was one product of the NCHRP study by Hudson, Finn and McCullough.

Project 123 was initiated in 1969 as a joint effort by the Center for Highway Research, University of Texas at Austin and the Texas Transportation Institute (TTI) under the support and leadership of the Texas Highway Department. This project has provided continued development and extension of the flexible pavement design and SAMP concepts and their implementation for the design of flexible pavements in Texas. This has resulted in a series of Flexible Pavement System (FPS) Computer programs currently through FPS 13 with increasingly sophisticated submodels that have advanced a long way toward rational idealization of the total flexible pavement design problem. A Rigid Pavement System (RPS) was also developed at the University of Texas [59] and its approach to optimization approximated that in FPS.

The NCHRP continued its work during the period 1972 to 1974 through a contract with the Texas Transportation Institute to conduct pilot implementation tests in Kansas, Lousiana and Florida with SAMP5 and to improve SAMP5 based on the experience gained in implementation. Considerable experience had already been gained during implementation in Texas, but it was desired to test implementation in other states. The principal results from this effort as reported by Lytton and McFarland in [62] was a somewhat improved computer program called SAMP 6, and the conclusion that this type system was readily implementable in other states, particularly those using the AASHO Interim Design guides.

The Federal Highway Administration has also carried out a research program to develop a mechanistic structural submodel and concepts for implementation within an overall management system. This research was conducted largely at MIT [1, 18, 63], and produced a computer program called VESYS II that considers pavement materials as linearly visco-elastic and predicts their initial life on a probabilistic basis.

Additional work has also been carried out at the University of Waterloo, Canada [64], but will not be discussed here.

The FPS and SAMP pavement systems are basically the same since the overall pavement system development in Texas has evolved through cooperative efforts and important improvements in one system are generally applied to the other. The choice of a specific computer program for use between FPS 14 and SAMP6 for example would be primarily a choice between specific features of the particular programs. From an optimization viewpoint there are no important differences at all. It will be convenient for the rest of this report to consider FPS and SAMP as similar systems in terms of optimization with the understanding that specific computer programs contain some variations in the capabilities of their submodels. This group of concepts and computer programs will subsequently be called the Texas FPS or FPS.

The goals of each of the existing pavement systems are the same, although the approach toward achievement of these goals varies considerably. Each of these systems is aimed at a rational design that will provide the riding public a pavement that meets their requirements for comfort and safety. Fig. 7-1 describes the design process simply in systems analysis terms. Fig. 7-2 shows the conceptual basis for VESYS II as reported by Soussou, Moavenzadeh and Findakly in [1]. Only Phase 1 of this effort has currently been developed.

The most basic difference between the Texas and the FHWA systems is that VESYS II seeks to characterize material properties as visco-elastic and to predict the stress damage and life expectancy on that basis, while the Texas FPS characterizes materials on their resilient properties after an appropriate number of cycles of loading. Both use modification of the equation resulting from the AASHO Road Test to predict the present serviceability index with time. Both consider some variability of design parameters, and FPS calculates rehabilitation strategies required to restore the serviceability of the pavement to a higher level when necessary and optimizes the design choices on the basis of costs. Each method looks forward to a development of sufficient

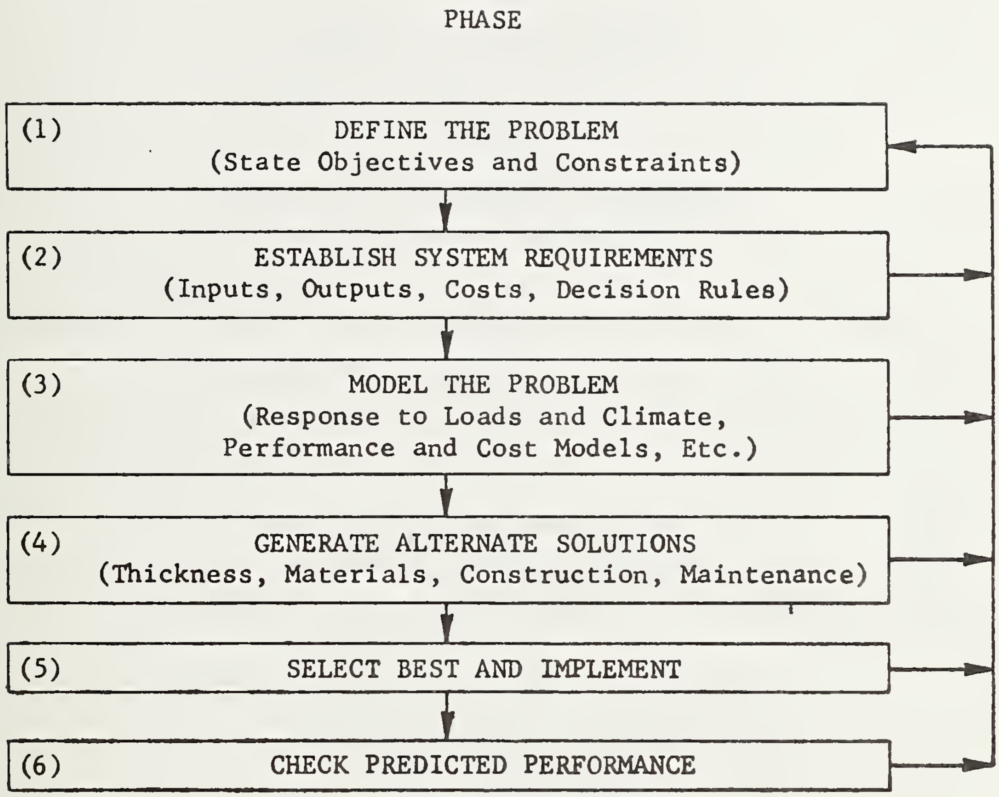


Figure 7-1 MAJOR PHASES OF THE SYSTEMS ANALYSIS METHOD [1]

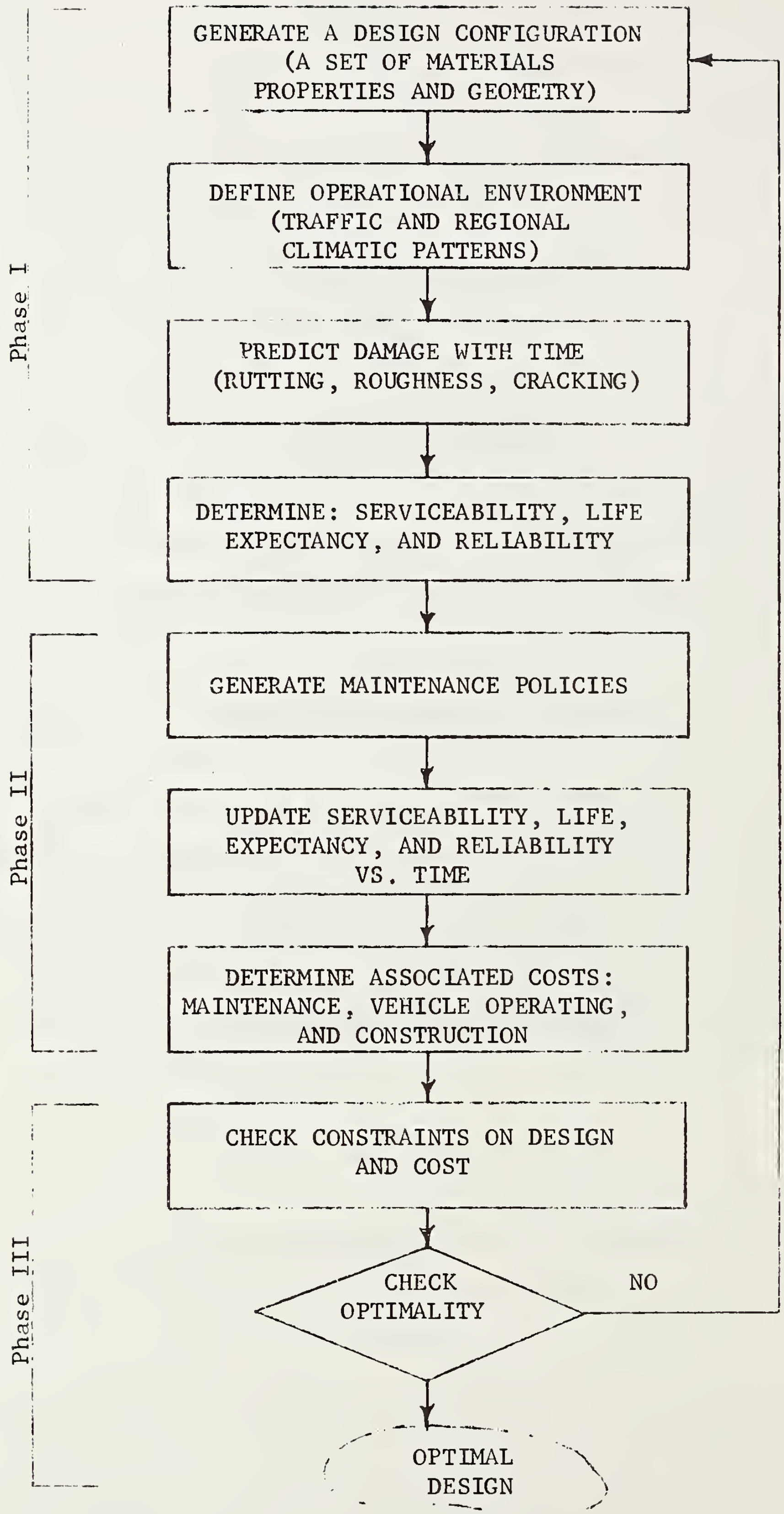


Figure 7-2 GRAPHICAL REPRESENTATION OF A PAVEMENT DESIGN PROCESS (After [1].

data on social and user benefits to broaden the optimization model past the cost basis currently possible.

Another very important difference between the two systems is the lack of any consideration of the effects of swelling clays on pavement serviceability by VESYS II. It is to be expected and has been clearly proven that loss of serviceability for pavement structures founded on clays subject to seasonal or other volume change is a strong function of random volume change as well as traffic effects. A means of including these effects in serviceability predictions exists in the Texas FPS and means of quantifying input values from laboratory tests has been developed [65]. These effects cannot be related to an assumed elasticity or viscoelasticity in the subgrade as the mechanism involved is not primarily dependent on loading and response to loading, so inclusion of a subsystem such as the one in FPS will ultimately be necessary if meaningful serviceability predictions are to accrue from VESYS where clay subgrades are involved.

Development has been much greater on the Texas FPS than on VESYS II. The Texas FPS in earlier versions had used AASHO equations altogether (no elastic layer theory) and has been implemented in selected districts of the Texas Highway Department for some time. A more sophisticated model with elastic layer theory and probabilistic considerations has been developed but not yet implemented [66]. VESYS II as currently configured follows the life cycle through a probabilistic prediction of the life of only one design strategy for the initially constructed pavement only. While conceptually stated in [1], the models for generating maintenance policy, updating serviceability and life expectance, the cost model, and the optimization model do not yet exist for VESYS II. The FPS model on the other hand includes these capabilities except that cost is the only decision criteria utilized directly for optimization at the present time. This does, however, include user costs for user delays occurring during overlay periods.

7.2.1 VESYS Conceptual Decision Structure for Pavement Maintenance Developed at MIT

The VESYS concepts for arriving at maintenance decisions as developed at MIT are described in Appendix 9 (Part III) of [1]. This subsection is intended to provide a condensed version of the most pertinent concepts described therein.

The basis for the conceptual analysis is the Bayes approach to the theory of decision-making, utilizing a multi-attribute utility analysis. The Markovian property of probabilities of future occurrences being dependent on current state and independent of past events is assumed to apply. Strategies yielding maximum utilities or minimum cost/benefit ratios constitute the optimal strategies to be selected for maintenance of the particular design configurations under study.

The factors to be considered are "cost, safety, comfort, performance, life, etc." In recognition of the subjectivity of some of the factors, expected utilities (expressions of judgemental preferences of individuals for certain commodity) is the proposed basis for analysis. Tradeoff is then to be conducted between the different attributes for the selection of the optimal set of strategies.

The AASHO serviceability index defines performance at any particular time and the attributes safety, comfort, and cost are defined as decision criteria in addition to serviceability.

Serviceability may be increased to a higher level at any time with application of maintenance, the level of serviceability being dependent on the level of maintenance selected. The three levels of maintenance considered are high, medium and low.

Safety and comfort may also be increased due to upgrading of the system such as an increase in skid resistance, overlay, etc.

Maintenance level is taken to define the effort (in terms of labor and materials) which is expended to upgrade the state of the system from its present level. A future cost model is assumed to assign costs for any maintenance effort discounted to present worth, providing a criterion for alternative evaluation.

Fig. 7-3 shows the decision flow diagram from one state with an associated probability to the next. "M" means a decision to maintain and "N/M" a decision not to. If maintaining, the different levels of maintenance each have an associated cost and the resulting state has a probability of occurring.

The utilities for cost, performance, safety, etc. will be evaluated for each time period and these utilities accumulated to ammount to an established utility for each strategy. These "multi-attribute utilities" are ultimately scaled down to "monetary equivalent" for optimization.

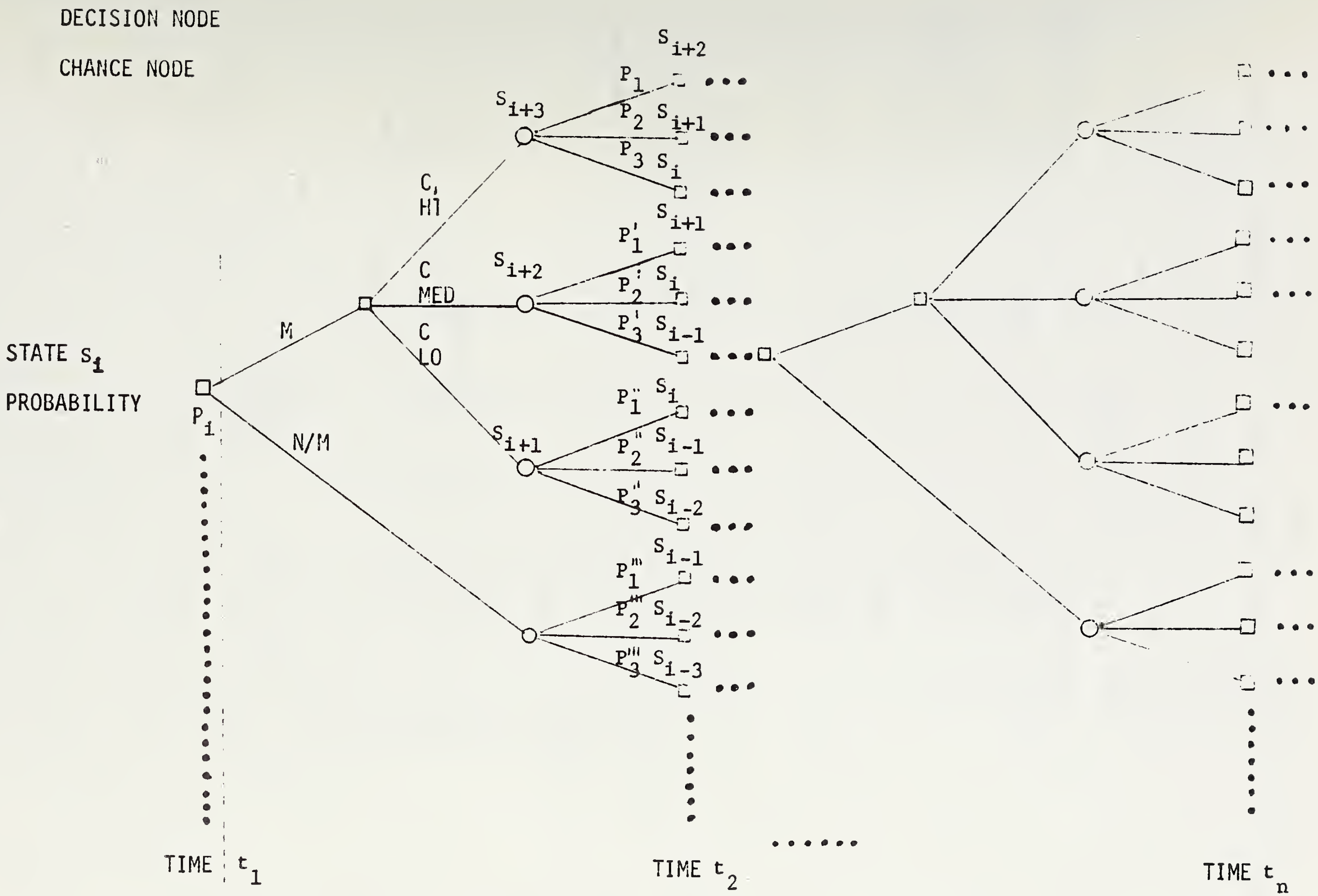


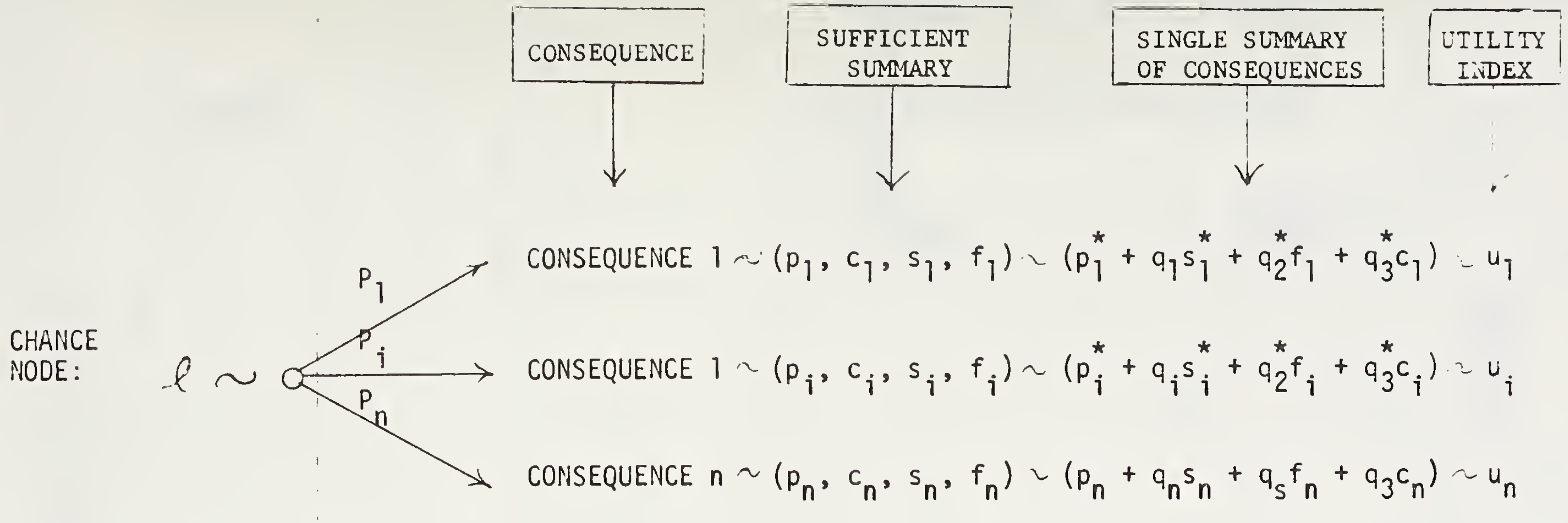
FIGURE 7-3 DECISION FLOW DIAGRAM FOR MAINTENANCE (After Ref [1]).

A method of approach is described in Appendix 9 of [1] using the "mid-value splitting technique" to scale the individual's preference or utility for each attribute. This depends on personal interviews between engineers and prospective users on the assumptions that (1) the description of the state of the highway is articulate and (2) "the user is educated and is fully aware of the consequences of his needs and responses." Using a rating system from 0 to 10, the engineer asks questions and raises and lowers his values until the user becomes indifferent to the differences. The end result from this would be a rather complex multi-dimensional "joint preference space". The evaluation of the strategies is illustrated in Fig. 7-4 which relates to the "chance nodes" of Fig. 7-3 Each chance node is represented by a lottery with probabilities and consequences shown at the top of each branch. All multi-dimensional utilities are reduced to scalar utilities expressed in terms of performance indices.

After some decision in [1] that in effect recognizes that the assumptions may not be sufficiently valid and that most utilities may not as yet be numerically compared, the authors decided to scale utilities into monetary values and deal with costs as the only attribute directly compared in decision-making (as in the Texas FPS system) pending more research on multi-attribute utility analysis.

7.2.2 Texas FPS System for Decisions and Optimization

The method for benefit analysis applied in the Flexible Pavement System currently in use in Texas is the "cost-effectiveness method", utilizing the "equal-effectiveness criterion". That is, a range of design strategies, each providing a serviceability level across the design life equal or greater than some minimum PSI, are compared on a cost basis and ranked in order of ascending costs. The costs considered by the economics subsystem for each strategy include those for initial construction, routine maintenance, overlays, user delays during overlays, and salvage costs. All of these costs are reduced to present worth and combined into a total cost for the design strategy for use in ranking. SAMP6 also includes subroutines to include costs for initial construction of shoulders and shoulder overlays concurrent with pavement overlays and for overall cross-section effort on a volumetric rather than area of pavement surface basis.



$x \sim y$: INDICATES THE PREFERENTIAL INDIFFERENCE BETWEEN ATTRIBUTES x AND y

$p^* = p + q_3(c - c^*) + q_1(s - s^*) + q_2(f - f^*)$.

p^*, c^*, s^*, f^* ARE MINIMUM VALUES FOR PERFORMANCE, COST, SAFETY, AND COMFORT REQUIRED FOR FOR THE SYSTEM TO FUNCTION.

Figure 7-4 Utility Evaluation for the Multiattribute Problem (After Ref [1]).

The actual design decision or final selection of strategy is accomplished external to the computer program by an engineer. The basis for selection is engineering judgement in consideration of the cost comparisons printed out, his working experience with the various materials and their combinations and a variety of other personal (or possibly political) decision criteria. (See Fig. 7-5 for a partial listing.)

FPS13, a version developed but not yet implemented, includes a capability for considering the stochastic nature of the design variables leading to design strategies at differing levels of confidence or reliability. Standard deviations or variations are input for:

1. Average daily traffic
2. Percent trucks
3. Axles per truck
4. Temperature constant
5. PSI of initial structure
6. Axles load/equivalency parameter
7. Lack-of-fit for:
    a. Performance model
    b. Deflection model for new construction
    c. Deflection model for overlay
8. Swell of clay subgrades
9. Stiffness coefficient of the subgrade
10. Stiffness of pavement base and subbase.

SAMP6 also includes stochastics on a limited basis by establishing a reliability confidence level requirement and a single overall coefficient of variation on traffic variables.

A number of decision criteria have been defined in conceptual systems from various FPS reference sources including primarily:

1. Costs
2. Safety
3. Maintainability
4. Reliability
5. Function
6. Riding quality
7. Availability of funds
8. Time
9. Existing practice
10. Administrative requirements
11. Public view

The decision criteria are conceived as having weighting functions assigned by the design engineer (Fig. 7-5) so that the computer may make

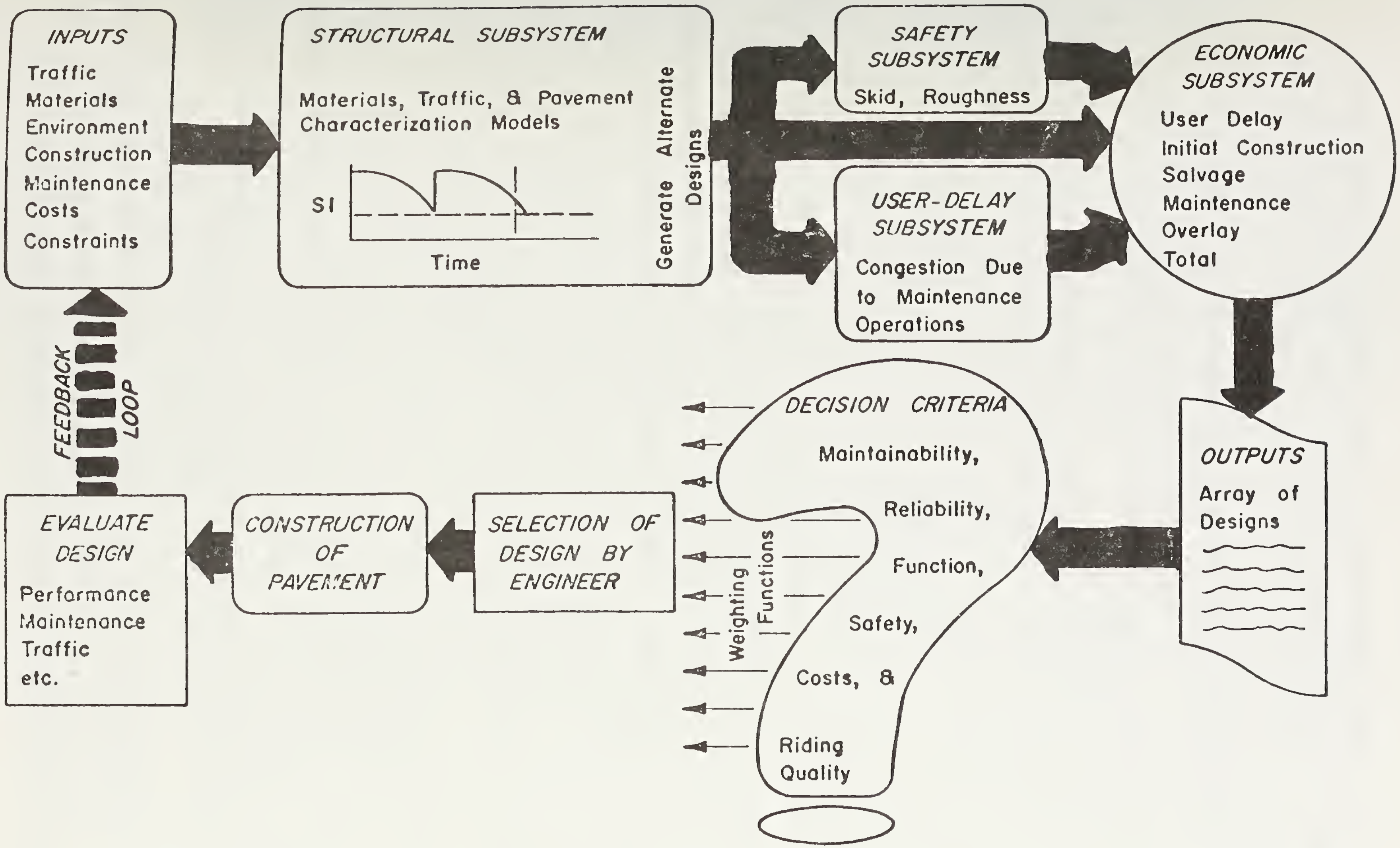


Fig. 7-5 SIMPLIFIED CONCEPTUAL PAVEMENT DESIGN SYSTEM SHOWING BASIC SUBSYSTEMS AND OTHER CONSIDERATIONS (From [66]).

selections based on assessments of these attributes accruing to the separate design strategies. Costs are specifically considered in the Economic Subsystem and reliability may be established as a design parameter.

Riding quality is implicitly established through a minimum allowable PSI, but there can be a very considerable variation overall in levels of riding quality between design strategies, as PSI is not solely a function of riding quality.

Maintainability is implicitly satisfied when a design strategy is predicted to produce a satisfactory level of serviceability over the design life at an acceptable cost, but other maintenance characteristics such as ease or speed are not considered. Availability of funds is satisfied since "funds available" is input as a design constraint. "Function" is satisfied by the input description of the traffic necessary to support the function and by the Terminal PSI level selected.

While a safety subsystem has not yet been fully implemented into the FPS, work is underway to relate skid requirements in terms of skid numbers to laboratory tests for rate of polish such that skid numbers may be predicted in terms of traffic applications to predict decay to a minimum acceptable skid number. Loss of skid number to the minimum would trigger seal coats with specific materials on the basis of their rates of polish. The design strategy would then consider combinations of overlays (can improve skid resistance as well as ride quality) and seal coats to maintain minimum levels of PSI and skid number. While safety is also dependent on roughness, the minimum allowable PSI will generally exceed the minimum requirement for safety. Safety implications of pavement geometrics would necessarily be considered external to the computer program.

Time as a decision criterion is not presently planned for consideration internal to the computer program, except to the extent that overlay periods increase user costs in the economic model. Such considerations as early completion of initial construction could be programmed, but are currently left as a design engineer selection criterion.

7.2.3 Comprison of VESYS and Texas FPS Concepts for Decision and Optimization

The comparison of the two concepts is more specifically the comparison of the conceptual development for VESYS as described and of an existing operable optimization system planned for expansion in the future to consider attributes in addition to those now considered.

While the FPS as implemented includes some decision criteria implicitly as previously described and optimizes on costs (including user costs during overlays), VESYS cannot presently generate design strategies and the optimization features exist only conceptually.

The two approaches to decision making and optimization appear to vary greatly in theory. The VESYS approach would generate a "tree" with many branches with nodes occurring at succeeding increments in time, while the FPS system in effect only creates nodes in the tree when serviceability reaches the minimum allowed. Both would evaluate the resulting attributes for separate design strategies at the "tips of the decision tree" or at the end of the design life period.

The approach to considering stochastic variations is also somewhat different in the two methods and could result in different selections. The probability of attaining the condition state under consideration is used as a multiplier at each node of the VESYS decision tree to reduce the anticipated benefit in relation to its uncertainty of attainment. The Texas FPS, on the other hand, establishes a required reliability level as a constraint and eliminates design strategies of lesser reliability from consideration. The reliability for a particular design strategy is a function of coefficients of variation or standard deviation input for various design parameters as derived from field data, testing analysis, and planned level of construction control. This procedure is discussed in [66] by Darter and Hudson.

Although a direct comparison would be difficult without considerable development work on the VESYS concepts, it seems possible that similar design strategies might result from the two if initial condition states were the same, decisions to maintain for VESYS were limited to nodes when serviceability approached the same minimum, condition states after overlay were the same and if the probabilities assigned to VESYS approximated the confidence levels and variations input to FPS.

Both systems consider the same type of decision criteria (sometimes called benefits, utilities, attributes, etc., in the literature) and neither has advanced toward a serious total analysis of all decision criteria, primarily because meaningful bases for quantifiying them for comparison do not yet exist.

7.3 CONCEPTUAL INTERRELATIONSHIP OF SYSTEM COMPONENTS

The development of a pavement design system involves as one task the organization of the system components or subsystems in a rational and efficient manner. Each must not only satisfy its specified functions rationally and efficiently, but it must relate to and interact with other subsystems so that the overall system objectives are achieved.

As each subsystem has numerous components and is complex within itself, the level of complexity and abstraction as the entire system is assembled exceeds the comprehension of the human brain unless some tool is used to allow it to focus on a comprehensive amount of operations at one time without loss of the rest of the detail. Interaction and flow diagrams are used to graphically show the structure of a system and to retain it generally intact while a subsystem is studied and possibly revised. These diagrams may be simple with entire subsystems shown in one block or in great detail, depending on their intended use. Such diagrams to describe conceptual interrelationships of the system components for a pavement analysis and design system require more detail than in Fig. 7-1 and 7-2 but not so much that they cannot be easily comprehended.

A simplified diagram of a pavement design system appears in Fig. 7-5. The process begins with inputs and continues along the paths of the arrows. In this very general diagram, the structural subsystem generates alternate design strategies including overlay strategies and predicts serviceability levels with time. The various design strategies are checked for safety in terms of skid resistance and roughness, user delays during overlay operations are evaluated and costed, total costs are calculated for the various design strategies and the results are arrayed by total cost and printed out. The decision criteria block is shown as a question mark due to current inability to apply all the major criteria effectively. A design in this case is selected by the engineer after reviewing the cost optimization by the system and applying the remaining decision criteria himself. At this point, the design is complete, but it is most important that the process be closed by a "data feedback loop" to allow future comparisons of actual responses to predicted responses so that the subsystems may be continually upgraded to improve their predictive ability.

McFarland [67] has developed a very approximate means of evaluating user costs during overlays and reviews the pros and cons of five methods of benefit analysis. The method used in the Texas FPS seems to be as good as or better than any other available until a capability for meaningful quantification of subjective benefits is developed.

The method of approach proposed in [1] for establishing utility values is theoretically interesting, but is not believed to be practically implementable as described. It suffers from lack of the relationship to actual physical roadways that the usual "rating sessions" have enjoyed. Also, "utilities" for ride quality have already been established at great expense and efforts are well along to characterize safety in terms of skid number and roughness.

While both pavement systems in concept approach optimization on the basis of selecting the best overall design strategy from an array of design strategies that have been developed, the practicality of this approach is strongly related to the computer time required in the particular system for developing a strategy. As VESYS II utilizes a series of solutions at points in time to arrive at its life predictions, the computer time required for generating an array of design strategies is expected to be many times greater than for FPS. While FPS as implemented may consider several hundred or even thousands of design strategies to reach a selection of the ten or twenty best for consideration, only a small fraction of that many would be practical for VESYS II and very possibly for more sophisticated versions of FPS as well. It appears then that VESYS II will require optimization within the submodels to reduce considerably the array of design strategies produced without loss of generality.

It must be remembered that the models leading to serviceability predictions include a mix of theory and empirical developments that range from fairly gross to reasonably accurate approximations. Research in progress will result in improvements that must be considered. This expectation of change emphasizes the necessity for a formalized data feedback system as discussed in detail in [68].

It should be noted that the decision criteria may be applied as input constraints or on a comparative basis after all design strategies have been processed or as a combination of the two.

Having studied the simplified conceptual system of Fig. 7-5 additional detail may now be considered as in Fig. 7-6. Note that some decision criteria will be applied as constraints and that stochastic concepts are now included to account for variability in the input parameters. Probability distributions are output for all the predicted responses. This concept does not establish the engineer's specific involvement with optimization. He may apply the decision criteria himself or share the application with the system.

The basic building block to be developed and expanded is the VESYS IIM system, which at present has only been developed through Phase I as depicted in Fig. 7-2. This capability is represented by part of the inputs and the structural subsystem in Fig. 7-5, except that VESYS II at present does not generate alternate designs and considers only the initial constructed pavement (no overlays or restorative maintenance). The relationships betweem the components of VESYS II are portrayed in Fig. 7-7. Material properties fitted to a mathematical relation, the geometry (primarily layer thickness, keyed to material properties) and the radius of the applied load are used in a static elastic layer analysis to arrive at the primary responses of radial strains and vertical displacements directly beneath a unit load. These responses are curve fit with relation to means and variances and become input to a repetitive load subprogram.

Using 1) inputs from the static unit load analysis, 2) temperature data including a system shift function related to the material, 3) the initial serviceability and 4) the minimum allowable serviceability, the repetitive load subsystem applies fatigue, rut depth and roughness models to arrive at predictions of roughness, rut depth and cracking. This data is then used in a serviceability prediction subprogram to produce predictions of service life and reliability.

As there is nothing to optimize until multiple design strategies are generated, optimization of VESYS IIM implies its expansion to essentially embody those components or subsystems and capabilities shown in Fig. 7-5 and 7-6.

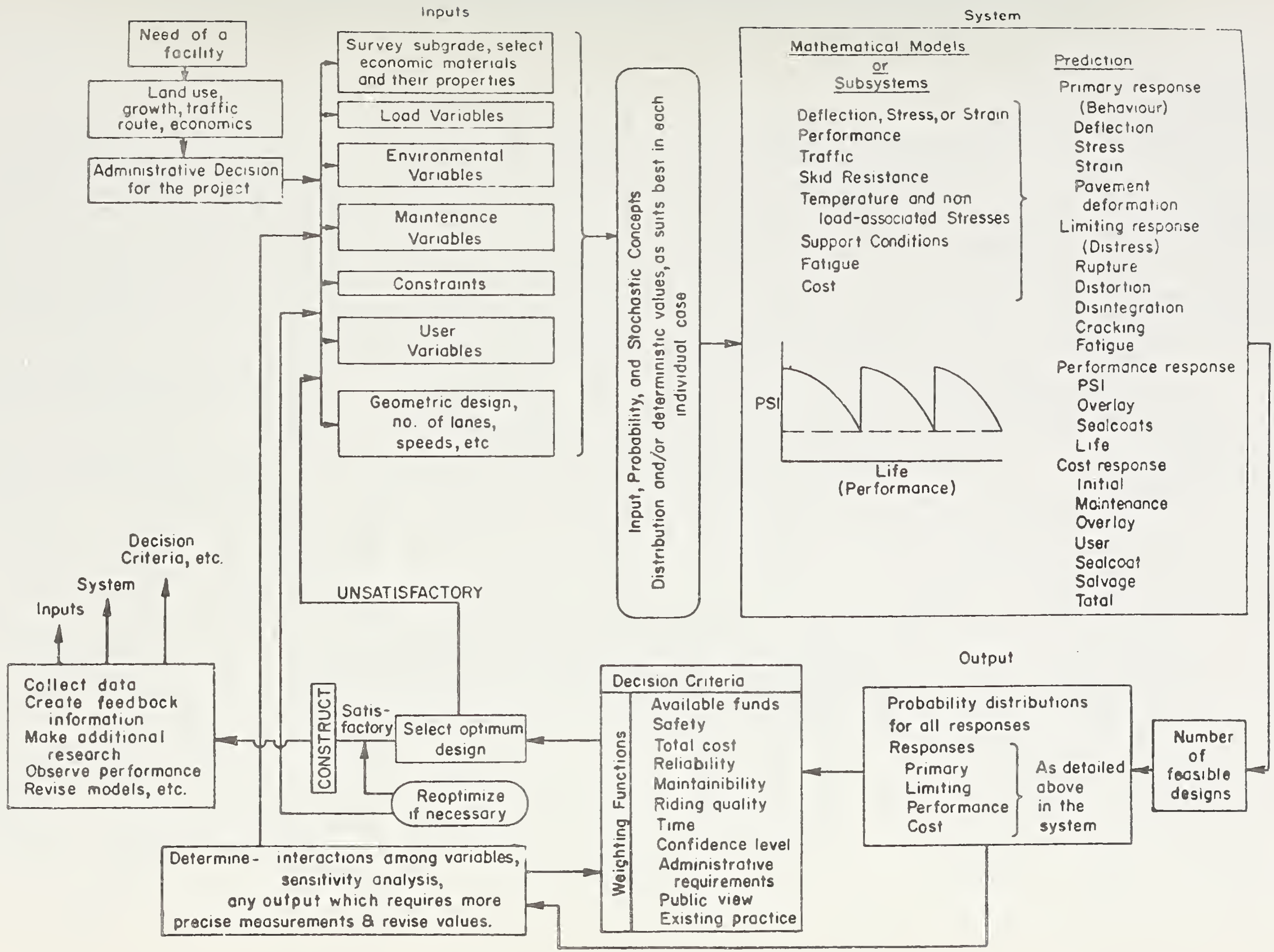


Fig. 7-6 CONCEPTUAL PAVEMENT DESIGN SYSTEM (After [69]).

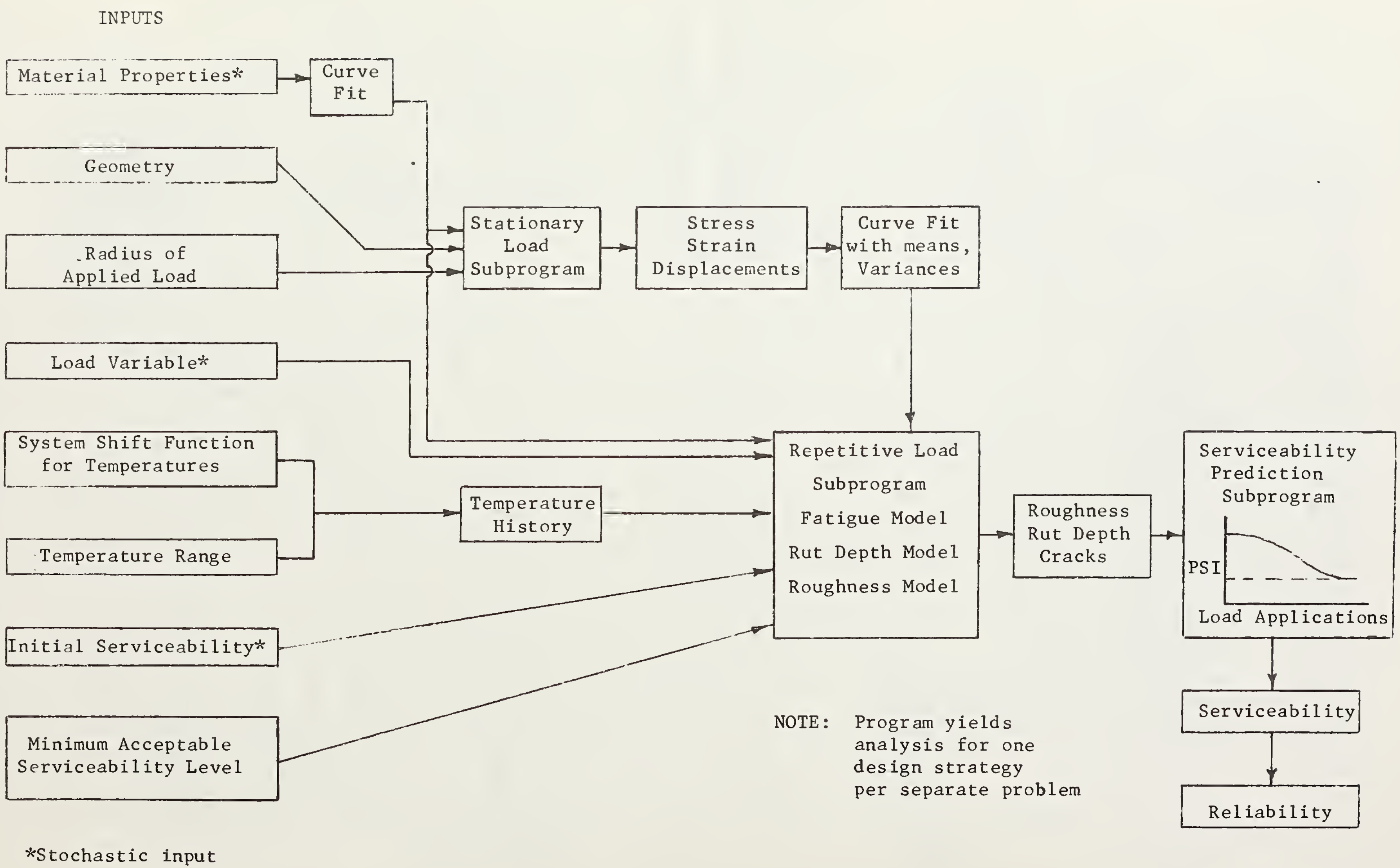


Figure 7-7 VESYS II PAVEMENT ANALYSIS AND DESIGN SYSTEM.

7.4 SPECIFIC DECISION CRITERIA

Decision criteria for a pavement system should include all those characteristics or attributes that have impact on the safety, cost of or benefits available from the pavement system and that are subject to control. Some decisions may be reached internal to the design system through establishment of constraints (usually as input data) such as maximum allowable cost and minimum time between overlays. Others must be made by comparison of the "outcomes" or the characteristics of the design strategies predicted by the pavement analysis and design system.

The following are decision criteria identified by study of the literature with some discussion of each:

1. Costs - in a very general sense, costs could include vehicular wearout and damage to sensitive freight as a function of lateral and longitudinal roughness, tire wear as a function of surface texture and many other such parameters as well as those costs previously identified as being considered in the FPS. The reason such parameters are omitted is that insufficient data is available upon which to make useful comparisons. Even the user cost data developed by McFarland [67] for the Texas FPS is based on limited data although part of it is in use. The cost parameters that may reasonably be compared with present data are initial construction cost, routine maintenance cost, cost for overlays and seal coats, user costs due to delays during overlay construction, and the negative cost of salvage value after the design period.

   McFarland [67] developed costs for vehicle operation, rural accidents and passenger discomfort as functions of pavement serviceability level, but considered them too limited for use pending further research. One limitation was that the user costs were related only to PSI rather than to other parameters also affecting them such as type of road and skid number. The other important limitation is the lack of sufficient reliable data upon which to base the benefit model.

2. Safety - Ignoring the safety aspects of geometrics that are not considered in the pavement analysis and design, safety may be

considered in terms of the effects of the pavement surface on ability of a vehicle operator to control his vehicle in dry or wet weather. Specifically, skid resistance may be related to a skid number, which is primarily related to the initial skid resistance of the surface materials and the rate of polish and consequent decay of skid resistance with traffic applications. Rutting and longitudinal roughness affect the driver's control, but only seriously when rutting and roughness are severe.

Substantial research is underway to learn about skid resistance and to relate it to safety and types of surfaces. Other research is underway to relate decay of skid number to traffic applications and to laboratory testing to establish rates of polish of aggregates. These studies may be expected to allow development of an ability to predict decay of skid number with time for a specific surface and traffic estimate much as serviceability is now predicted using the AASHO equation. The implementation of this capability into the Pavement Analysis and Design System (PADS) will require optimization of combinations of overlays that may restore serviceability and skid resistance and intermediate seal coats to restore skid resistance only. The requirement will then be to maintain minimum established levels of both serviceability and skid resistance with trade-offs between performance, safety and other benefits.

It is believed that the safety aspects of rutting and longitudinal roughness will be implicitly satisfied by meeting ride quality standards.

3. Maintainability - is defined in [70, 71] as a measure of the degree to which effort may be required during the service life to keep serviceability at a satisfactory level. This includes normal or routine maintenance such as repairing random potholes and sealing random cracks and repair maintenance required to restore adequate serviceability when a serious loss has occurred or seems imminent.

Alexander [70] proposes an "index of maintainability", which amounts to the arbitrary number 1000 divided by predicted maintenance cost to avoid fractions. A high index number in this case implies low maintenance costs. These indices could be used as numerical measures of maintainability for comparison as decision criteria.

Alexander also proposes a probabilistic expected cost analysis basis as follows:

$$\bar{EC} = \sum_{i=1}^{i=n} (p\ (X_i) \times V_i)$$

where:

$\bar{EC}$ = the expected cost of a situation that may have any one of n outcomes

$p(X_i)$ = probability that the outcome will be X_i

V_i = cost of outcome X_i

While the definitions above imply a comparison of the maintenance required for various design strategies, a design strategy must practically meet some established rules to compete as to maintainability and these rules will vary in different environments and traffic situations. For instance, the freeze-thaw cycles in the Northeast cause a generally higher level of acceptable maintenance than in the Southwest, but an expansive clay subgrade in the Southwest would also markedly increase the maintenance frequency expectation. Frequent maintenance on a major urban freeway would be less pallatable than the same frequency of maintenance on a rural highway.

Some of the rules related to maintenance used in FPS as input constraints follow:

a. Minimum period before the first overlay is required. Design strategies that are predicted to deteriorate to the minimum established serviceability in a lesser time than that allowed are eliminated from further consideration.
b. Minimum period between overlays.
c. Time to first seal coat.
d. Time between seal coats.
e. Allowable thickness each layer.
f. Minimum overlay thickness.
g. Maximum total thickness of all overlays.
h. Maximum funds available for initial construction.

Another rule suggested by Alexander [70] is that required maintenance should not exceed that amount that can practically be made available.

With the growing shortages of some aggregates and other materials, constraints on excessive uses of materials in short supply for maintenance might be appropriate in some locales.

It appears that maintainability may best be insured for the present through input constraints that insure that candidate design strategies satisfy the rules. There is a possibility that a pavement requiring frequent maintenance might have other attributes warranting its consideration, but the probability is so low that the inefficiency of processing these marginal strategies through the system does not appear warranted. In general, excessive maintenance will generate so much construction and user costs that the strategy in question cannot be a strong contender. However, if desirable, maintainability could be developed along with other decision criteria for utility optimization in the future.

4. Reliability is defined in [71] as a measure of the probability that serviceability will be at an adequate level throughout the design service life. Darter and Hudson [66] extended this definition to make it more complete as follows:

"Reliability is the probability that the pavement system will perform its intended function over its design life (time) and under the conditions (or environment) encountered during operation."

Reliability may also be viewed as the confidence level that the pavement system will perform in accordance with its stipulated design functions.

Virtually every aspect of pavement design, construction, operation and maintenance is subject to uncertainty. Traffic estimates, environmental predictions, anticipated material characteristics, the design models themselves, costs for construction and maintenance and many more variables are approximations at best and educated guesses in some cases. While many are subject to some degree of control, even they are sufficiently variable that probabilities could appropriately be assigned to all the design parameters. The limitation is lack of data upon which to assign reasonable values of means and variances for use with the usually assumed normal probability distribution.

Increases in reliability may generally be obtained at additional cost by use of better quality materials, closer control of material variations, providing more maintenance and increasing pavement layer thicknesses in general [66]. Fig. 7-8 illustrates conceptually the relationship between reliability, performance and costs. As can be seen, practical limits exist past which little additional serviceability or reliability may be obtained regardless of additional expense.

There are two approaches that may be taken in the application of reliability as a decision criterion:

a. It may be established at an appropriate level for the system considered and input as a constraint, thus eliminating design strategies of lesser reliability (this capability already exists in FPS) or

b. It may be defined in time and considered on a comparative basis along with other decision criteria.

Lemer and Moavenzadeh [71] offer a very general matheamtical expression for failure in terms of reliability and N possible failure modes:

$$\begin{aligned} &D_1 \, (e_1, e_2, \ldots, e_L) \le R_1(c_1, c_2, \ldots, c_M) \\ &D_2 \, (e_1, \ldots, e_L) \le R_2 \, (c_1, \ldots, c_M) \\ &\quad\vdots \qquad\qquad\quad \vdots \\ &D_N \, (e_1, \ldots, e_L) \le R_N \, (c_1, \ldots, c_M) \end{aligned}$$

where:

D_i = environmental loads placed on the pavement as determined by a set of environmental qualities (e_1, e_2, ..., e_L) such as wheel loads, temperature and vehicle speeds

R_i = capacity of the pavement to resist environmental load D_i.

c_i = system characteristics that determine pavement response.

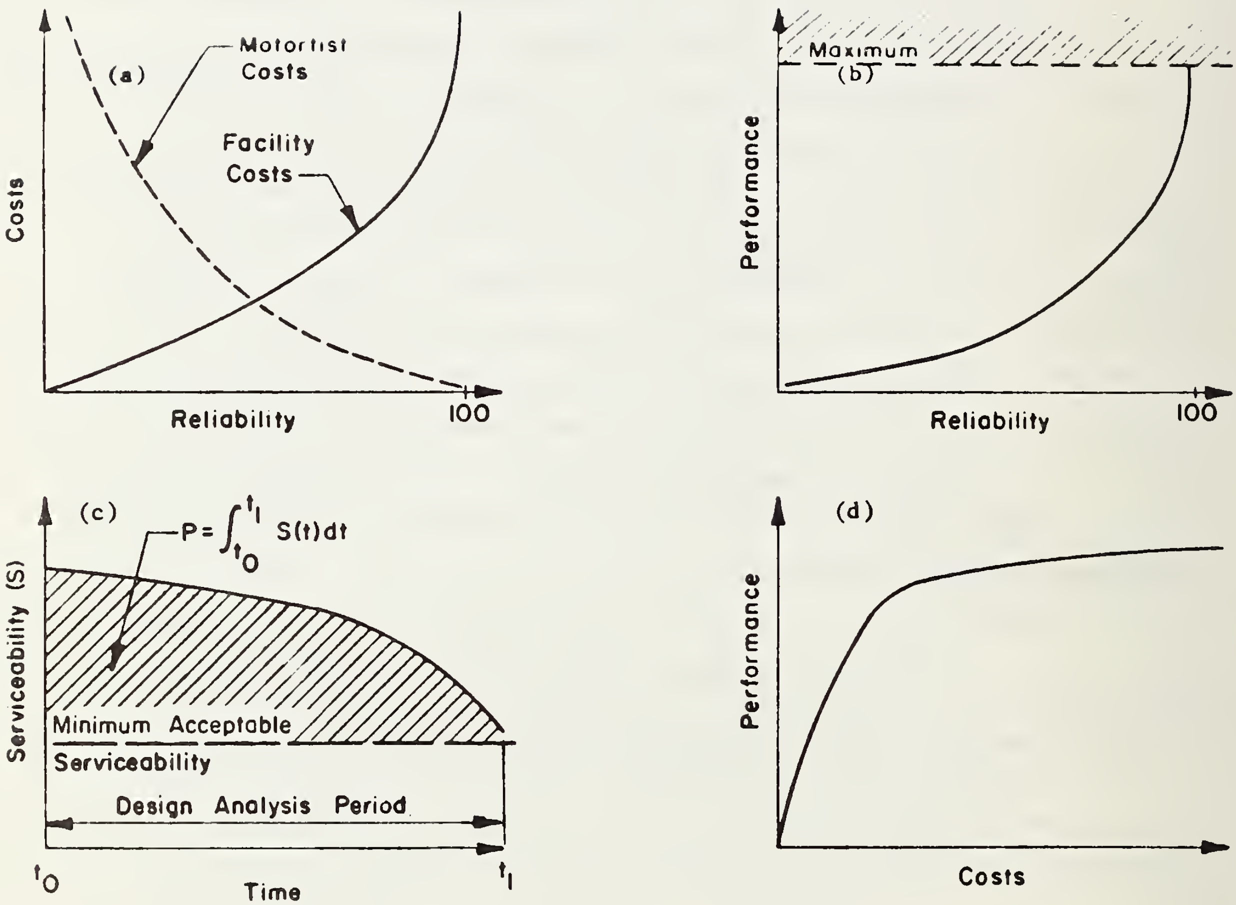


Figure 7-8 ILLUSTRATION OF RELATIONSHIPS BETWEEN RELIABILITY (R), PERFORMANCE (P), AND COST (C) (After [66]).

Failure is the condition in which one or more of the inequalities $D_i \leq R_i$ is not satisfied. The reliability is then the probability that all the N inequalties are true and may be expressed as:

$$R = p(\text{no failure}) = p(D_i \leq R_i)$$

$$\text{for } i = 1, \ldots, N$$

In FPS, a much more specific mathematical definition is used:

$$R = p(N > n)$$

where:

N = number of load applications that a section of pavement can withstand before minimum allowable serviceability is reached within a limited maintenance input.

n = number of load applications which are applied to the pavement section.

This definition of reliability is developed from the basic concept that a no-failure probability exists when the number of load repetitions to minimum acceptable serviceability is not exceeded by the number of load applications applied n.

In FPS, R is taken to represent the number of 0.2 mile sections that will survive the design period and it is assumed that an overlay will be applied when 1-R sections have failed. After an overlay, each section is assumed to have an equal R chance of success during the next performance period and the same for still other performance periods.

A choice of six reliability levels from 50 to 99.99 percent are available in FPS. Selection of a reliability level is based on the functional classification of the highway traffic volume and 18 kip equivalent single axle load applications. Table 7-1 gives recommendations by Darter and Hudson [66], where C, D, and E represent confidence or reliability levels of 95, 99 and 99.9 percent, respectively.

TABLE 7-1 RECOMMENDED DESIGN RELIABILITY LEVELS FOR FPS PROGRAM (After [66]).

Traffic Handling situation	Functional classification	Equivalent 18-kip single axle loads: <500,000	500,000 to 2,000,000	>2,000,000
Satisfactory	Collector	C	C or D*	D
	Arterial	C	C or D	D or E
Some Problems	Collector	C	C or D	D
	Arterial	C or D	D	D or E
Considerable Problems	Collector	C or D	D or E	E
	Arterial	D	D or E	E

*Note: If pavement is located in urban area, use higher reliability level wherever range is given.

Referring again to the possible approaches of implementing reliability as a decision criterion, it appears much more practical to utilize it as a constraint for the present until a broader capability exists for assessing and applying utilities from a full range of benefits. As the state-of-the-art advances, a basis for applying reliability as a decision criterion should be developed along with other weighted decision criteria.

Reliability will increase in complexity as other decision criteria are applied. For instance, FPS considers in effect one failure mode (or N = 1 in the Lemer and Moavenzadeh formulation). As it becomes possible to qualify and define safety requirements (in terms of material characteristics, traffic applications and other parameters), a second failure mode or inequality must be considered. If other rules for failure are applied, other inequalities must be considered and probabilities defined for each.

Reliability is only meaningful to the extent that the probability estimates themselves are reliable. Considerable work has been done [66] to learn the sensitivity of design life predictions to variations in design parameters from actual data collected in Texas and elsewhere. Much more research will be required to develop more data.

5. Function of a pavement system involves the purpose it is expected to serve. For instance, it may be a high speed expressway, an access road, a farm-to-market road, an interstate highway, a collector street in a city street system or others.

The varying functional requirements of the users for different types of highway or street systems implies variations in performance requirements and benefits. For instance, the amount of tolerable variations in vertical profile (or roughness) for a high-speed expressway would be much less than that for a neighborhood street. Maintenance frequency may be greater on a rural road with little traffic than an interstate highway with thousands of vehicles per day to be delayed.

It appears that the important effects of pavement system functions are "imbedded" in other decision criteria and that it will serve as an implicit decision criterion through its effect on them. The other decision criteria for which function is a consideration are:

a. Safety - minimum skid number and allowable variations in lateral and longitudinal profile will be a function of traffic velocities.

b. Maintainability is dependent on the level of serviceability required, which is a variable with function.

c. Reliability levels selected should reflect function in terms of the impact that failures and consequent maintenance would have on satisfactory support of functional requirements.

d. Riding quality in terms of allowable minimum serviceability levels is directly dependent on the function the pavement is to serve.

e. Availability of funds is directly dependent also on the function the pavement is to serve. Much higher funding is required and justifiable for an expressway to serve masses of vehicles than for a small collector street.

6. Riding quality primarily expresses the degree of comfort or lack of discomfort experienced by a highway user when riding in an average automobile. It is represented in the PADS by the by the Present Serviceability Index (PSI) calculated at any time by modified AASHO equations in terms of rut depth, cracking and slope variance (and also characteristics of a clay subgrade in FPS). While this is a reasonable parameter for interim use, it should be understood that the data from the AASHO Road Test are basically 50 m.p.h. data and that discomfort is dependent on a amplitudes of roughness and frequency of occurrence, which depends on vehicle velocity. That is, the same profile on a 30 m.p.h. city street and a 55 m.p.h. expressway would not be expected to yeild the same PSI rating by the same raters in the same automobiles traveling at the different velocities.

Riding quality may be implicitly enforced by input of a minimum allowable PSI as in FPS, but should also be considered in terms of performance across the design period. This would allow selection of the design strategy with higher overall performance if costs and other benefits were approximately equivalent. Performance may be measured as the area under the Pavement Serviceability Index vs. Time Curve:

$$\text{Performance} = \int_{0}^{t_{final}} PSI(t)dt$$

Potential performance past the end of the design period should also be considered as a part of the salvage value or in some other way to take this benefit into account during optimization. The degree of importance to be attached to this secondary benefit could be controlled by an input weighting function. This benefit could be considered separately from ride quality if desired.

7. Availability of funds for initial construction is presently imposed in FPS as a constraint on the allowable initial construction cost of the pavement system. Design strategies are eliminated that require more funding to construct than that available. While this appears generally reasonable, there is a good probability that the total costs over the design period might be considerably less for some design strategies eliminated because of initial construction costs than for the optimum strategy selected. Knowledge of such an advantage might warrant some realignment of funding to attain it.

   Study might be warranted to determine whether high initial cost strategies should be considered or eliminated as in FPS.

   The availability of funds for overlay and seal coat might also warrant consideration in selection between design strategies having fewer increments of costlier major repair or those having more increments of cheaper major repair.

8. Time is designated in the literature as a decision criterion. It is indeed a very important parameter and appears as input constraints (time between overlays, etc.), as a parameter for calculating other design criteria (such as user delay costs)

and as a calculated output (such as time for PSI to deteriorate to minimum allowable).

Although not currently planned, it could be considered as an independent design criterion for possible future design criteria such as initial construction time and maintenance periods on a utility basis rather than user costs.

9. Existing Practice is the empirical result of years of good and bad experience with various materials and designs over a range of conditions. It exists in codes, design manuals, research reports, textbooks, in the minds if practicing engineers and in other locations.

   It would be very difficult to devise a means of applying existing practice as a decision criterion internal to the PADS, but it may be applied externally by the design engineer as he reviews the design strategies and makes his selection. Kenis and McMahon [18] propose initial use of the VESYS II as a design check procedure for the various states' present design procedures. Such an interaction between practice and the new PADS may result in changes in the PADS that would amount to an imbedded consideration of the useful knowledge reflected in the existing practice.

10. Administrative requirements - Within any public agency there has built up a set of administrative rules and procedures which govern one or more phases of the operation of the agency. The rules may be derived from state or federal laws as part of the system of rules associated with a particular funding source such as the governing Board of the Agency; i.e., the State Highway Engineers or the Highway Commission.

    These rules may deal with political issues or issues of special local concern such as "buy American" policies, which dictate the use of local products in lieu of imported substitutes. Energy sensitive issues may also be treated in this light until they can become quantifiable in direct "cost" terms in the program. In any event, these administrative rules must be considered implicitly or explicitly in the decision process of pavement design.

11. Public view - the condition of a pavement system sometimes becomes a political issue with strong expression of public dissatisfaction. Such a situation might warrant replacement of a particular pavement at a high level of initial serviceability and perhaps maintenance of a relatively high level. Other kinds of public pressure might also be brought to bear.

The implementation of this decision criterion would be most complex in any manner other than as consideration with other decision criteria. Therefore, it is proposed that the public view be taken into account implicitly rather than as a separate decision criterion.

Another contemporary criterion is availabilty of materials. As many aggregates and asphaltic materials are in short supply around the country, it is appropriate to establish a vehicle for utilizing the materials in short supply only when necessary. As an example, let us say that two alternatives for base material are to be considered and the total cost and other benefits are about equal. It is obviously advantageous to consume from the largest supply rather than the limited one. The relative importance of such conservation of resources in short supply could be established by an input weight for use with this design criterion.

7.5 OPTIMIZING TECHNIQUES

A complete Pavement Analysis and Design System (PADS) implies a "black box" such as that illustrated in Fig. 7-9 that can:

1. Accept real life inputs on a) available materials characteristics and costs, b) traffic and load predictions, c) performance requirements, d) environment characteristics, e) construction quality control expectations and f) means and variances for appropriate input parameters.
2. Analyze efficiently a range of material combinations and thicknesses on a probabilistic basis to arrive at reasonable predictions of pavement responses (roughness, rutting, cracking and loss of skid resistance) with time and traffic applications.
3. Translate pavement response predictions into sufficiently realistic predictions of performance and reliability for the various design strategies in terms of both ride quality and safety.
4. Generate appropriate maintenance policies (generally comprising overlays and seal coats of various thicknesses and using various materials) for consideration with the various initial designs.
5. Analyze and predict responses and performance on a probabilistic basis for the upgraded pavement system.
6. Repeat Item 5 as required to provide the required performance over the design period.
7. Calculate the costs reduced to present worth for initial construction, routine maintenance, overlays and seal coats, user costs during maintenance, and the negative salvage value of the materials at the end of the design period.
8. Assess on a numerical basis the utility of each overall design strategy for each established design criterion.
9. Make comparisons between the design strategies on the basis of combinations of numerically defined and weighted utilities for the established design criteria.
10. Display (printout) the results of the analysis and evaluation of the various design strategies so that a final selection and design implementation may be made by the design engineer or administrator.

VESYS II is capable of the first three items, except that it only handles one design strategy at a time on an input basis and does not consider

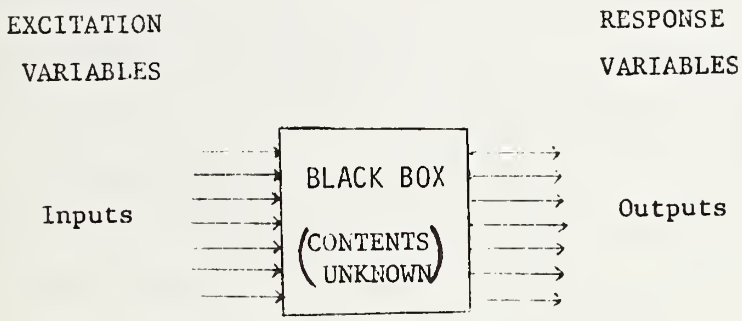


Figure 7-9 BLACK BOX SYSTEM (After [72]).

effects of subgrade volume change or skid resistance at all. A later version of FPS developed but not yet implemented is capable of the first eight items, except it does not yet consider skid resistance and optimization is only on costs. The accuracy of predictions for either depends on the validity of its materials characterizations, other input parameters, and its models, and these have yet to be fully tested.

The task to be accomplished is not simply one of optimization, but includes extension of the analysis model to produce a general set of design strategies for initial construction, overlays and seal coats instead of one very specific analysis and set of probabilistic predictions.

The general process of system analysis and optimization as defined in [73] and illustrated in Fig. 7-10 includes essentially the following steps:

1. The outcomes for each possible action or feasible solution are predicted on the basis of analysis.
2. These outcomes are then evaluated according to some scale of value or desirability.
3. A criterion of decision, based on the objectives of the system, is used to determine the most desirable action or optimal solution.

Viewing the effort required to produce a complete PADS from this angle indicates that we are starting with the capability for Block 1 and the basis for Block 2, but must develop the capability for generating multiple feasible solutions, for scaling values or desirability, for implementation of decision criteria, and for selecting the optimum solution.

The objectives of this section are to discuss optimization briefly and some working techniques available for use, recommend the next phase of effort to be undertaken to develop an optimization capacity for VESYS II and to discuss some possibilities of subsystem optimization to increase the efficiency of the PADS as it develops.

7.5.1 Mechanics of Optimization

Decision criteria may be applied in the form of input constraints that limit the generation of design strategies of no interest or in an optimization subroutine after the strategies are generated. The best approach will undoubtedly be application of some factors as constraints and others in an optimization subroutine.

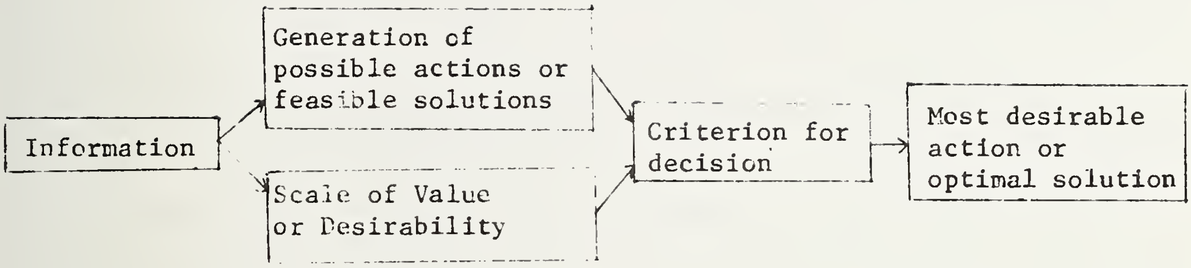


Figure 7-10 SYSTEM ANALYSIS AND OPTIMIZATION (After [73]).

Fig. 7-11 is an FPS flow diagram that illustrates how the constraints are applied. This system both limits the range of parameters analyzed and the number of solutions processed further in the system. Such constraints are required to limit consideration to suitably practical design strategies. While the number of strategies generated is not directly limited to some practical number, limitations may be applied by selection of magnitude of incremental layer thickness variations or as proposed previously for subsystem optimization. Other constraints not known in Fig. 7-11 such as specified reliability and maximum average annual maintenance costs can be applied in the same manner.

The bottom block of Fig. 7-11 represents the optimization on total cost of the whole range of feasible designs generated within the framework of the constraints.

Fig. 7-12 portrays an optimization system used in a Rigid Pavement System (RPS), which could also be applied to a flexible pavement system. This system accumulates only a specified input number of the least expensive design strategies. Design strategies are generated up to the number specified without any optimization. The most expensive is then identified and subsequent design strategies compared to it. More expensive design strategies are not stored, but less expensive strategies replace the current most expensive in storage. New comparisons are made as each design strategy is processed to determine which strategy is the most costly.

While both Fig. 7-11 and 7-12 represent systems to be optimized on the basis of total costs, there is no reason that the same general procedures could not be applied even if the basis for optimization was a number representing a combination of weighted utilities.

7.5.2 Subsystem Optimization to Improve Efficiency and Reduce Computation Time Requirements

While not directly applicable to the primary objective of optimizing the selection of design strategies, the relative optimization of some of the subsystems in the PADS would allow more flexibility in design strategy generation and consideration within the constraints of practical computer time requirements.

Versions of the Texas FPS implemented were deterministic rather than stochastic and are not as complex and detailed as those subsequently developed and the VESYS program. There was no constraint to solving

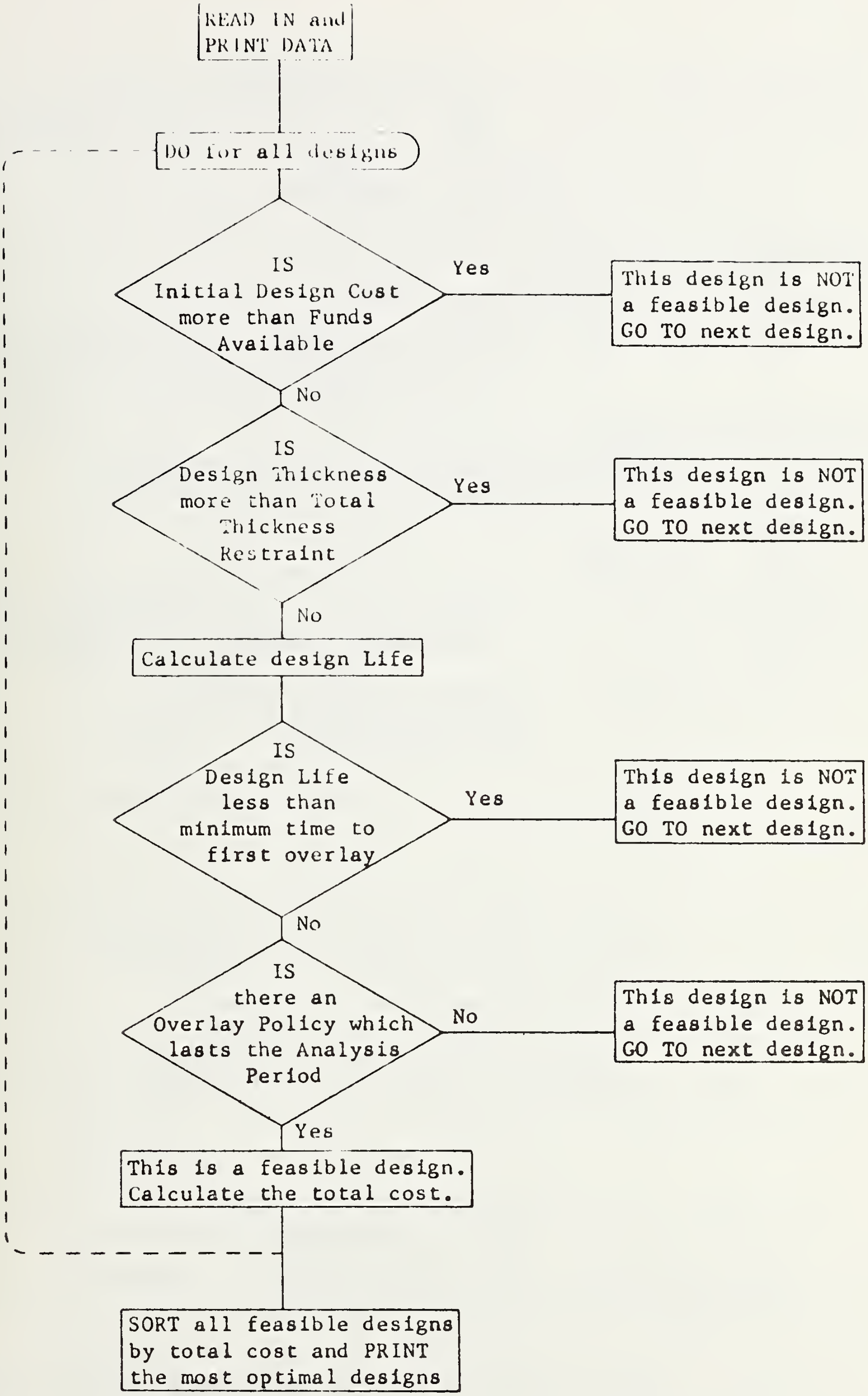
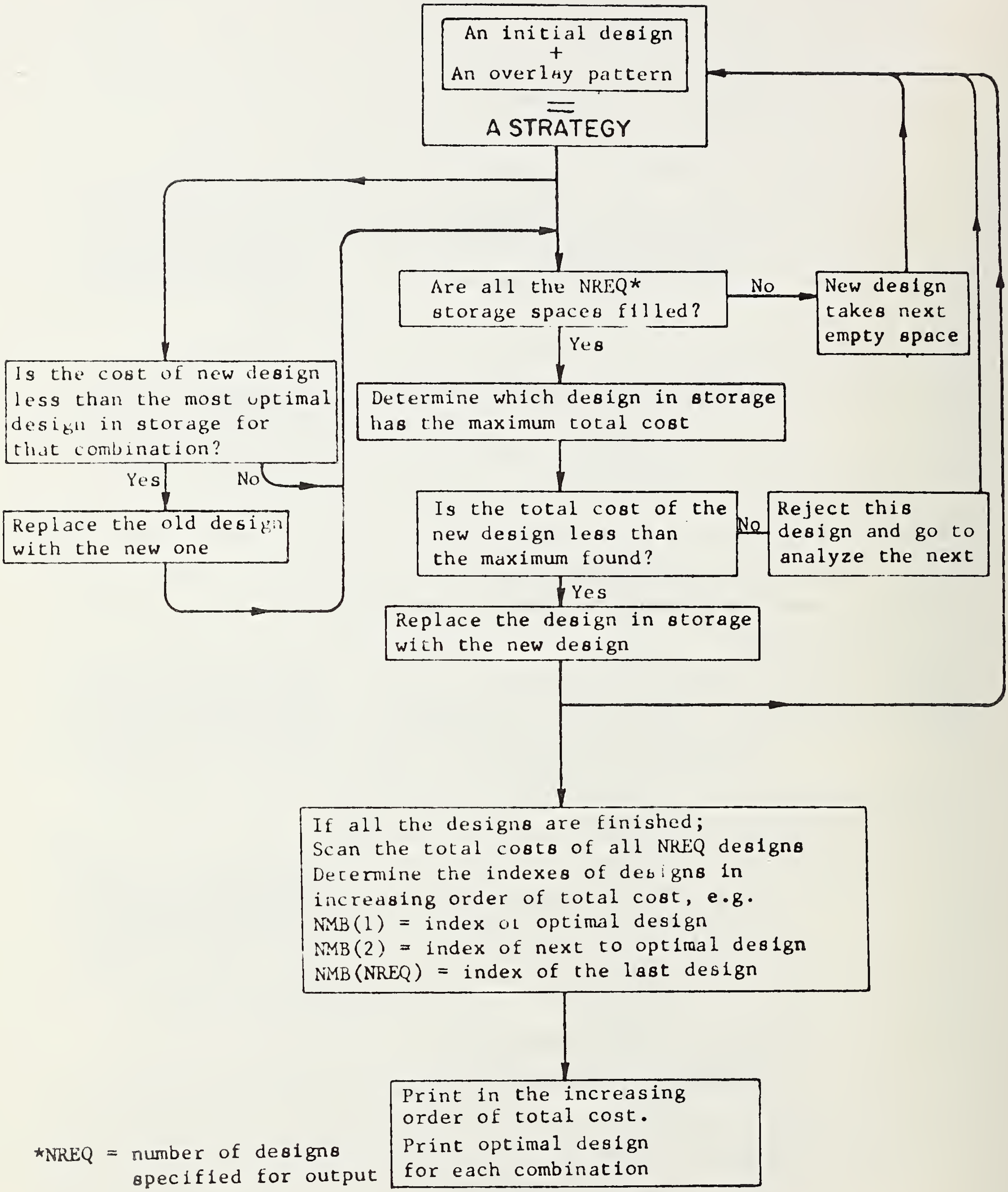


Figure 7-11 SUMMARY FLOW CHART ILLUSTRATING MECHANICS OF THE FPS PROGRAM (After [58]).



*NREQ = number of designs specified for output

Figure 7-12 OPTIMIZATION PROCEDURE RPS1 (After [59]).

hundreds of design strategies that would maintain a minimum serviceability level and optimizing on the basis of costs. As the subsystems increase in complexity, there is more need for means of eliminating strategies of little promise without processing through the entire system.

There are a number of promising possibilities that may be considered and implemented in later research efforts to optimize strategy generation such that only strategies with a reasonable probability of successful competition with other strategies would be generated in detail and analyzed. Some possibilities follow:

1. It appears possible to run a large factorial of representative and selected solutions to develop a regression equation capable of a reasonably accurate prediction of serviceability, thus temporarily bypassing most of the time-consuming portions of the structure model. The predicted serviceability in combination with other parameters could be used as a basis for elimination of the majority of possible strategies prior to use of the viscoelastic structural model. A similar analysis could be used to develop regression curves for overlay strategies to reduce those considered for each of these surviving initial construction strategies.
2. A similar but somewhat different approach might be used to develop probabilities of successful competition for different strategies after several viscoelastic performance predictions had been completed. For instance, after three solutions at varying thickness of a layer are completed, considerable insight may be gained as to the feasibility of using other thicknesses. This approach would basically amount to creating trends with a few solutions and extrapolating this data to eliminate other variations of a particular parameter. This might be accomplished for several parameters simultaneously or separately.
3. Use the AASHO equations for predicting primary responses as in FPS9 to establish bounds on the array before proceeding with viscoelastic solutions.
4. Use a few viscoelastic solutions at large increments of thickness to "home-in" on the layer thicknesses for initial construction, AASHO equations over the limited array including overlays and then visco-elastic solutions again over the final small array of strategies for selection.

5. Use some version of the Texas FPS system to optimize to a small array of design strategies and then apply VESYS to that small array of strategies to optimize on the viscoelastic basis.

Another very attractive option would be to maintain both the FPS and VESYS structural models in one computer program and to develop the flexibility to utilize either or both, preferably with the capability of using one to reduce the array of possible design strategies and the other or both to generate the array. Such a computer program would be a powerful tool in the continuing refinement of the models toward increasingly accurate predictions of real-life responses and performance of pavement systems. A similar multiplicity could be added for candidate optimizational techniques to allow their comparison for the same set of design strategies.

7.6 CONCEPTUAL PROGRAM

It is quite clear that development of full optimization of the VESYS methods must necessarily be phased, both due to the complexity involved and because the means of defining all the decision criteria identified do not yet exist in such manner that they may be compared and utilized directly. This does not mean, however, that useful and implementable systems may not be produced at each phase beginning now.

The purpose of this section is to explore the additional work which appears to be appropriate, to develop reasonable recommendations for phasing this work and to define in some detail the proposed first phase for continued development.

7.6.1 Definition of Effort Required to Achieve a Rational Flexible Pavement Analysis and Design System

The required capabilities of the PADS are illustrated broadly in Fig. 7-2. It was pointed out previously that VESYS IIM essentially represents the completion of Phase I and that no optimization could occur until Phases II and III had been completed. Also, VESYS IIM has no submodel for considering the effects of differential volume change in expansive clay subgrades.

There appears to be no reason not to take advantage of the state-of-the-art efforts already expended for the Texas FPS to expand VESYS so that the VESYS "black box" may with minimal effort encompass the Phase II and III capabilities shown in Fig. 7-2. Other than the disparity in state of development, the principal difference between VESYS and FPS is in the nature of materials characterizations, the method of considering traffic and the model for developing the primary responses of rutting, cracking and roughness. There should be no major constraints to replacing the FPS structural model with the five-layer version of VESYS developed under this project as the first phase of continued development effort. The resulting version of VESYS would have the capabilities presently available in FPS encompassing the visco-elastic structural model and would provide a "platform" for the next phase of optimization. Care should be taken to maintain "modularity" in the ensuing computer programs so that subsequent refinements may be made as easily as possible.

The required effort for continued development of a PADS is discussed above with the exception of implementation. Implementation itself is a fairly major undertaking involving 1) careful education of those involved, 2) salesmanship, 3) interfacing with existing data collection and design systems and 4) establishment of a data feedback loop and means of monitoring it and refining the models. Also included with the major tasks will be a number of related tasks such as these:

1. Further development of information on important design parameters. The relative importance of the parameters will have for the most part been established through sensitivity studies such as those reported in [66] and those for VESYS IIM in progress by Austin Research Engineers, Inc under FHWA Contract DOT-FH-11-8258. Those with the greatest impact on design selections should be emphasized in research.
2. Optimization of the subsystem to improve efficiency and reduce computational time requirements as previously discussed.
3. Comparative studies of the results obtained from VESYS and FPS prediction models for rut depth, fatigue and roughness to identify causes of any variations. This should lead to possibilities for refinement of the models.
4. Continued development of stochastic data on design parameters for use in design on a probablistic basis. Primary emphasis should be given those parameters that have been found to have strong importance to design results.

7.6.2 Phasing of Continued PADS Development

The first phase recommended is the "marriage" of the five-layer VESYS system with the most advanced version of the Texas FPS, Computer Program FPS-13. Appropriate features from SAMP6 such as the cost subroutines for consideration of shoulders and cross-section volumes may also be included. The resulting computer program would be a quite advanced system capable of developing full-life design strategies to particular levels of desired reliability. Such a program could serve both as an implementable system and a platform for continued research.

Phase II should include the following:

1. Improvement of the computer program developed in Phase I to increase its efficiency through application of some or all of

the concepts described previously as subsystem optimization.

2. Comparative studies of results obtained from the VESYS and FPS prediction models for rut depth, fatigue and roughness. Refinement of models as indicated by this or other studies.
3. Studies aimed at rationally including seal coat or other surface repair strategies for maintaining acceptable levels of skid resistance.
4. Other studies or developments identified as requirements during Phase I.

Phase III could involve several separate efforts to include:

1. Implementation in cooperating state highway departments.
2. Development of a basis for consideration of other decision criteria and for assigning relative weights to reflect their importance. The expanded optimization capability should also be added to the latest version of the computer program and experiments run to study its effects.
3. Study of other distress mechanisms and limiting responses to continue improvement of the predictive model. Refine the predictive model to reflect the results of these studies.
4. Further study of the stochastic characteristics of the important design parameters.

Future work past Phase III toward realization of an accurate and reliable PADS will generally be defined as work progresses, but it should include improvement of the implemented versions as new refinements warrant.

7.6.3 Concept for Phase I PADS

A summary flow diagram is shown in Fig. 7-13 for the proposed combination of computer program FPS-13 with the five-layer VESYS computer program developed under this project. While this flow diagram indicates fairly clearly the nature of the procedure, it may be useful to follow through the various steps and discuss them separately.

As indicated in the upper right-hand corner of Fig. 7-13, a series of initial designs may be considered including variations in thicknesses and properties of the asphalt concrete, base and subbase (if any) and in properties for the subgrade (i.e., treated and untreated). The specified combinations of layers may be considered iteratively.

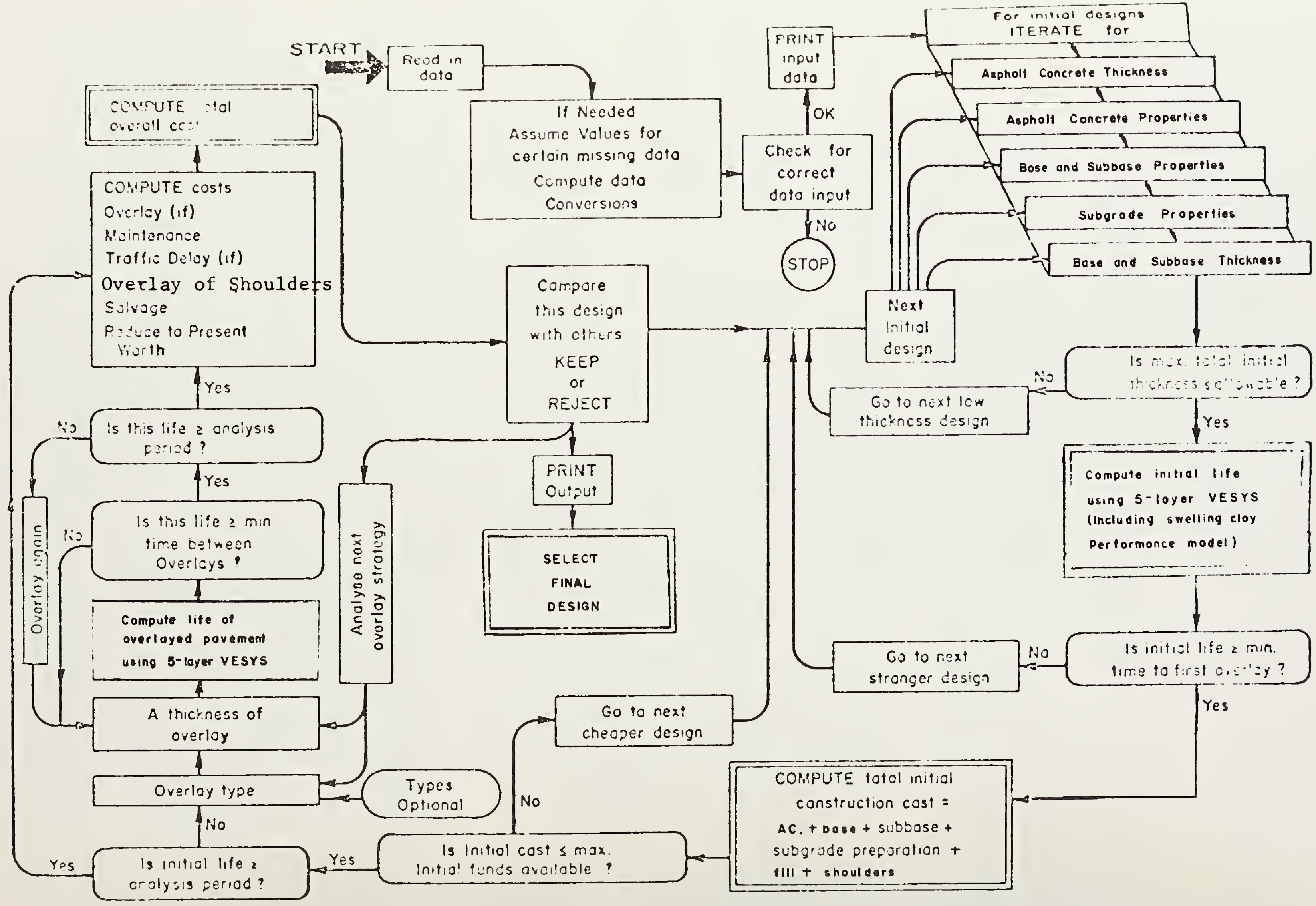


Figure 7-13 SUMMARY FLOW DIAGRAM, PHASE I PADS WITH FIVE-LAYER VESYS STRUCTURAL MODEL.

The model for volume change and resulting roughness due to clay subgrades could also be included in VESYS at the same time to refine the performance predictions for this fairly strong parameter.

The objective of this recommendation is to produce a PADS as quickly as possible so that the business of refinement and development may proceed. The cost base procedure in use in the Texas FPS and proposed above for VESYS provides a reasonable selection procedure pending development of more sophisticated benefit or attribute analysis and the result of this "marriage" should represent an implementable PADS.

The structural model itself needs to be subjected to continued development and scrutiny to include other distress mechanisms and refinement of those presently considered as new knowledge is developed from other studies. One facet of this effort should be continuing long-term comparisons between predicted and actual performance within a "data feedback" system such as that described in [68]. Study of causes of disparities could lead to identification of deficiencies in the models that could then be corrected. These efforts may probably best be made cooperatively with states where the PADS is implemented.

The optimization for the Texas FPS system is currently limited to indirect imposition of several of the desired decision criteria as constraints and arraying the best design strategies on the basis of costs so that a designer may apply this and other decision criteria of his choice in making a selection. As indicated previously, it is conceptually intended that the rational PADS will include a broader optimization involving weighted decision criteria other than the costs now considered. Such optimization would involve some means of assigning weights to the various social, political and cost criteria in terms of their relative benefits or utilities.

The constraint on implementation of such an expanded optimization system is not in the mechanics, but in the identification of all the criteria that should be included and the relative importance to attach to each. The identification and quantification of these criteria represents a research phase of its own, which could be carried on separately and concurrently with other development of the PADS. The approach here should be one of continued refinement of a cost optimization system that may be reasonably and effectively used in the interim as it represents a much-improved extension of the cost-based design decision well-established in the profession.

For a particular initial design i, the total initial thickness is compared to a specified maximum value. If too thick, this design is discarded and the next lower thickness tried. If not too thick, design i enters the five-layer VESYS structural and serviceability prediction model.

The five-layer VESYS, which is illustrated in more detail in Fig 7 and previously described, will predict periods of satisfactory serviceability at different levels of reliability, but it is necessary to select one period for use. The most straightforward method of selection is to establish a required level of reliability and interpolate for a period of time to reach the minimum serviceability level between pairs of time to minimum serviceability level and reliability values calculated by the five-layer VESYS subsystem.

The initial life (period for serviceability to decay to the minimum allowable level) would then be compared to a specified minimum time to first overlay. If the initial life is less than the minimum, design i is discarded and the next stronger design is considered. If equal or greater, the initial construction cost is calculated and compared to the specified maximum initial funds available.

If the estimated initial cost exceeds the maximum initial funds available, design i is discarded and a new initial design is considered. If the estimated initial cost is within the initial funds available, a check is made to see if the initial life predicted equals or exceeds the analysis period. If so, no overlay is required and the initial construction cost plus annual maintenance costs reduced to present worth represent the total overall cost for the analysis period.

If the initial life is less than the analysis period, various overlay strategies must be considered, also on an iterative basis for increasing overlay thicknesses and material properties. The present serviceability index after an overlay must be estimated and specified as for the initial pavement.

The "life" or period of time before the serviceability of the overlayed pavement structure deteriorates to the established minimum level may be predicted using the five-layer VESYS subsystem as for the initial design, but it will be necessary to decide how to model the various layers. For instance, an initial design having a subgrade, a treated subgrade layer, a subbase layer, a base layer and the asphaltic concrete layer does not leave another available layer in the "model" for an overlay. Also, a second or third overlay may be required. It is possible to consider both the original asphaltic concrete and subsequent overlays as one thicker layer if their material properties are similar. Similarly, the subbase and base may be combined if appropriate. Another decision is required whether the initial material properties will be considered to apply or should they be revised to reflect the anticipated effects from repetitive loading and environment prior to overlay. These problems are subject to solution with a reasonable amount of effort, but must be studies during the Phase I development.

After the life for the overlay is computed, it would be compared to the specified minimum period between overlays. If less than the minimum, the overlay strategy is abandoned and another thickness or material tried. If greater than the minimum, the combined lives for the initial construction and the overlayed pavement system are compared to the analysis period. If the combined total life is greater than the analysis period, design i and the one overlay enter the subsystem for calculation of total overall costs. If lesser, another overlay is required and the procedure is repeated.

When sufficient overlays have been included to extend the life through the analysis period, the total overall cost for this design strategy is computed. The total overall cost will include the initial construction cost, overlay costs, annual maintenance costs and traffic delays less the salvage value. All future costs are reduced to present worth for comparison.

The total overall cost is compared to those for other design strategies as illustrated in Fig. 7-12. The new strategy will be retained in storage if fewer than the number of design strategies specified for printing out are stored or if it is cheaper than some strategy in storage that it may replace.

After the comparison with other design strategies, a new overlay iteration begins with a new overlay thickness or material. After all specified overlay strategies have been considered and compared, a new iteration of the entire procedure begins with a new initial design. After the array of initial designs and overlay strategies for each have been processed, the surviving overall design strategies are printed out and the designer may study them and make the final design selection.

It would be helpful for the reader to read over the data input guide for FPS-13 [66] or SAMP6 [62] to become familar with the input data required and its organization. The majority of the data could be input without revision, but some revisions will be required including the following:

1. FPS-13 assumes an analysis period of twenty years and flexibilty for consideration of other specified periods if desired, would require revision.
2. FPS-13 operates with 18-kip axle equivalencies, while VESYS would require specific tire pressures and radii of the loaded areas.
3. VESYS required stochastic creep compliance data while FPS-13 uses strength coefficients for characterization of the layer materials.
4. VESYS requires a vector of temperature values that are used to describe an annual cycle, a reference temperature for the master creep compliance curves, a time-temperature shift factor and other inputs for considering the effects of temperature on material characteristics, whereas FPS-13 uses a stochastic district or reginal temperature constant with internal subroutines.
5. Traffic data is required by VESYS in vehicles per day (usually only trucks considered) and a stochastic repeated load intensity and duration while FPS-13 uses average daily traffic and percent trucks on a stochastic basis.
6. Other VESYS input requirements would have to be provided for.

The expansion to include the shoulder and cross-section volume costs in optimization will involve integration of these cost subroutines from the SAMP6 computer program into the cost subroutines for FPS-13. This should not be difficult due to the similarity of the two computer programs. Detail on these SAMP6 subroutines and input requirements appears in Chapter 9 of Appendix C, of [62].

It would be desirable to develop a research version of the combined Phase I computer program as discussed previously to include the structural and predictive models for both the VESYS and FPS systems with as few variations in input as necessary. This would allow relatively easy comparative studies that might lead to refinements of the prediction models for rut depth, fatigue and roughness or other improvements. The effort involved in development of such a research program concurrently with the development of the the implementable VESYS/FPS-13 combination should be rather nominal. If this research version of the combined Phase I program was not produced, comparative studies recommended in Phase II could still be conducted using the separate computer programs.

7.7 SUMMARY

The very considerable body of previous work toward development of rational analysis and design systems for flexible pavements has been discussed and synthesized. The structures of the systems and interrelationships of the various system components were displayed in diagram form and the flow of system operations discussed.

Specific decision criteria for use in optimization of design were defined and discussed in detail. While it is possible to define the majority of the decision criteria, no feasible basis for assigning utilities and comparison has been developed for most of them. This has led to the use of total cost as a basis for optimization. Total cost as currently used includes initial construction costs, routine maintenance costs, costs for overlays and seal costs, user costs due to delays during overlay construction and the negative cost of salvage value after the design period all considered at their "present worth".

Techniques for design optimization were discussed, but there can be no optimization for the VESYS system until capabilities for considering multiple initial design strategies and generating maintenance policies has been incorporated. A "marriage" of the new five-layer VESYS computer program is proposed as the next phase of research to provide a working VESYS Pavement Analysis and Design System. Other research and development needs are defined and their phasing proposed.

There appears to be no serious constraint on the development of an implementable PADS during the next research phase proposed.

CHAPTER 8

MODULAR FRAMEWORK OF VESYS IIM

Modifications made to VEYSY II to develop VSYSY IIM have been made in such a fashion as to facilitate future changes and in corporation of new subsystems. The VESYS IIM computer routines have not been completely coupled to VESYS II, however. Also, a significant protion of VESYS II is not modular. Coupling our portions of VESYS IIM with the remaining VESYS II distress, serviceability and maintainability subsystems in a modular fashion is not difficult, however,

The various computer programs in VESYS IIM have been assembled in a modular fashion into three program elements:

READ PROGRAM
PRIMARY RESPONSE
DISTRESS ANALYSIS

Rather than rely on machine dependent system commands to control execution of the various program elements, a simple executive program written if Fortran languate is used to execute the various subsystems. This procedure, while not as efficient for a specific computer, greatly simplifies the task of transferring the program from one machine to another. Figure 8-1 presents a shcematic of the organization of VESYS IIM.

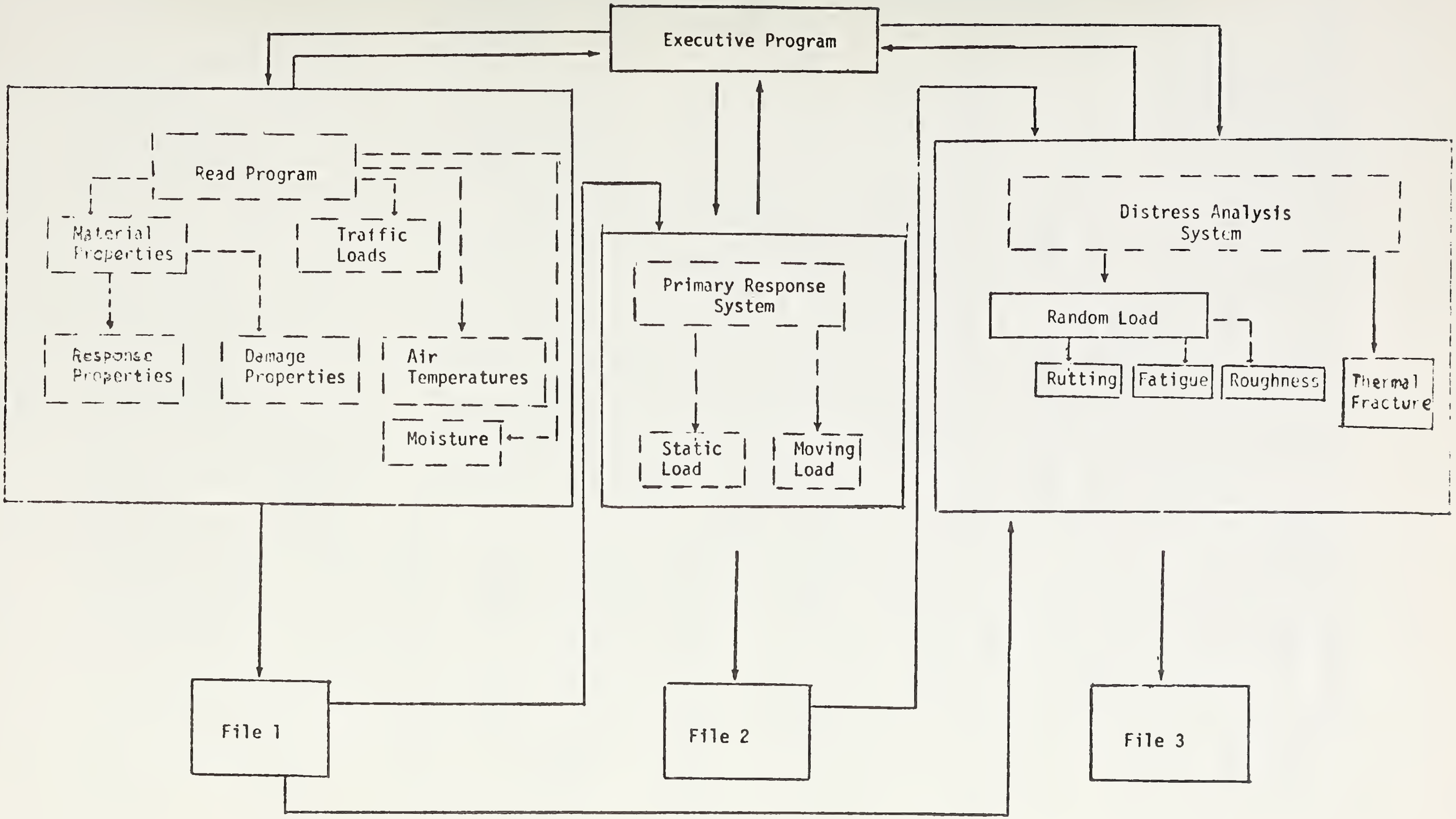


Figure 8-1 SCHEMATIC OF VESYS II-M FRAMEWORK

The executive program controls executions of all program elements. The READ, PRIMARY RESPONSE, AND DISTRESS ANALYSIS programs are sotred on separately accessible files. Each subsystem of these programs represents a subroutine which may contain further subelements depending upon the calculations being made.

The EXECUTIVE program calls the READ program and tells it what input data to read. This data is stored on a file or on a tape and/or printed. Control is returned to the EXECUTIVE program which may call for an exit or execution of any of the subsystems of the PRIMARY RESPONSE OR DISTRESS ANALYSIS systems. If the PRIMARY RESPONSE system is called, the required input data is read from FILE 1. The output of PRIMARY RESPONSE is stored in FILE 2 and/or printed, and control is returned to EXECUTIVE. If the DISTRESS ANALYSIS system is called, the input data required is read from FILE 1 and FILE 2 and the output is stored in FILE 3 or printed.

With the organization outlined above, it is possible to execute certain portions of the program separately as well as make modifications to any system in a straight forward manner as long as consistent input-output formats are maintained, without having to change significant numbers of common blocks.

CHAPTER 9

SUMMARY AND RECOMMENDATIONS

The significant accomplishments during this contract consist of:

1. Extension three-layered viscoelastic solution to an N-layered viscoelastic solution.
2. Development of an approximate flosed form probabilistic solution for stresses, strains and deflections for an N-layered viscoelastic system.
3. Development of an improved rut depth model which uses simplified laboratory characterization tests.

Additional work is required, however, to complete development of a rational pavement design framework. First, VESYS II and VESYS IIM should be merged completely and a series of sensitivity analyses conducted on the resulting design system.

An approximate probabilistic model developed for the primary response nodel should be validated and the limits of applicability established. If the range of applicability is not as large as desired, the first four moments, rather than the first two used in VESYS IIM, may be necessary to determine probability distritution functions.

The primary response model should also be extended to include multiple wheel loads with variable contact areas and variable spacing.

The extension is straight forward assuming linear behavior; however, it may be worth while to investigate a finite element formulation to allow even greater flexibility in the loading and material behavior characteristics. Particularly, if the non-linear behavior of the base and subgrade materials is to be adequately accounted for in evaluating pavement performance.

The rut depth model developed under this contract should be validated with field studies. Additional validation of other distress systems in VESYS II is also required, including incorporation of viscoelastic fracture mechanics concepts rather that the elastic approaches currently used.

Also, due to the importance of pavement roughness in evaluating pavement performance, a mechanistic roughness model is needed. A simple approach is this problem is to model the interaction between vehicle and pavement as a simple two degree-of-freedom, spring-mass system moving on a Maxwell model viscoelastic pavement. Roughness may then be characterized by the root mean square value of the pavement profile assuming that the pavement-wheel interaction is the main contributor to the change in pavement roughness. The change in the random pavement profile can be investigated with spectral and correlation analyses.

Finally, simplified material characterization procedures and automated data reduction routines are needed to increase the appeal and usefulness of a rational pavement design framework to highway designers. A simplified testing procedure is incorporated in the permanent defermation model developed; however, data reduction may be more complicated requiring more training of laboratory technicians. Accordingly, it is recommended that a material characterization subsystem based on numerical optimization techniques be incorporated into VESYS IIM to perform the necessary data reduction for materials' response and damage behavior.

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APPENDIX A

PRIMARY RESPONSE MODEL
PROGRAM DOCUMENTATION

I. SUBROUTINES AND VARIABLES

MAIN

MAIN is the executinve control routine. All subroutines are called from this program and all input data and output are printed.

Several variables pertaining to control of data executions are defined in this program:

ITN = number of terms used in numerical integration (default value = 46)

ITN4 = ITN x 4 (internally calculated)

REED

All data are read-in in this subprogram. The input is then passed to MAIN.

TITLE (I) = job title

NS = number of layers

N = NS-1 (calculated internally)

IR = number of radial points where output is desired

IT = number of axial points where output is desired

WGT = total wight of load

AR = load radius

PSI = tire pressure

V(I) = Poisson's ratio for i-th layer elastic

E(I) = modulus or compliance for i-th layer

PR(I) = radial coordinate

AA(I) = axial coordinate

COVE(I) = coefficient of variation of elastic modulus or compliance of i-th layer

NT(I) = number of time for which lab creep data is curve fit for i-th layer

TYME(I,J) = vector of times for lab creep data

VALVE(J,I) = creep, compliance at time J for layer I

COV(J,I) = coefficient of variation of creep compliance or modulus at time J for layer I

NTO = number of times for which output is desired

T1(I) = vector of times for which output is desired

COF(K) = coefficient in power law curve-fit of compliance

DN(K) = exponent in power law curve-fit of compliance

A flag is read to determine if an elastic or viscoelastic analysis is to be performed and if laboratory data are to be curve-fit. Either modulus or compliance may be input. The program compares the magnitude with unity to determine which has been input. If the coefficients of variation input are zero for all layers then a deterministic solution is generated.

If compliances are input, then the mean and variance of moduli used internally in the program are calculated from*

$$E[E_i] = \frac{1}{\overline{E}_i} \left[1 + \left(\frac{\sigma_{E_i}}{\overline{E}_i} \right) \right]^2$$

* Our thanks are due Dr. R.L. Lytton of Texas A & M University for these expressions.

$$Var[E_i] = \frac{\sigma_{E_i}^{\;2}}{\overline{E}_i^{\,4}} \left[1 - \left(\frac{\sigma_{E_i}}{\overline{E}_i}\right)\right]^2$$

PART

This subroutine performs the numberical partition of $\{X_i\}$ and the four points in each interval (X_{i-1}, X_i) at which the integration of $\int_0^\infty T(m)dm$ is evaluated. The set of X_i's are so chosen from the zeros of T(m) such that in the interior of any interval (X_{i-1}, X_i) there is at most, one zero of T(m).

The variables in this subroutine are:

BZ(I) = the roots of the Bessel functions J_0 and J_1

G1,G2 = the roots of the Legendre polynomial

AZ(K) = the parameter m required in the evaluation of

$\int_0^\infty T(m)dm$

The value of ITN, which represents the number of terms used in the series approximation to the integral is set at 46. This value runs on the UNIVAC 1108 Computer, however, the size of a double precision word is less on IBM. Therefore, the maximum number of terms may have to be decreased.

COEF

The constants A_i, B_i, C_i and D_i, and $\frac{\partial A_i}{\partial E_j}$, $\frac{\partial B_i}{\partial E_j}$, $\frac{\partial C_i}{\partial E_j}$ and $\frac{\partial D_i}{\partial E_j}$ are computed in this subroutine. Most of CPU time spent is due to the evaluation of these constants:

The variables defined here include:

$$PI = \prod_{j=1}^{n-1} \frac{D^{-1}(h_j) X_j D(h_j)}{4(\nu_j - 1)}$$

DPI $= \prod_{j=i}^{n-1} \dfrac{D^{-1}(h_j) \frac{\partial X_j}{\partial E_k} D(h_j)}{4(\nu_j-1)}$

DEN = the determinant of the 2 x 2 matrix for solving for C_n and D_n

A(M), B(M), C(M) and D(M)

= constants A_i, B_i, C_i and D_i, respectively, where M=1,2,·····,ITN4

CN $= C_n$

DN $= D_n$

DPX, DPXX, DPG

= intermediate values used in computing DPI

DGA(M,LL),DGB(M,LL), DGC(M,LL) and DGD(M,LL)

$= \frac{\partial A_i}{\partial E_j}, \frac{\partial B_i}{\partial E_j}, \frac{\partial C_i}{\partial E_j}$ and $\frac{\partial D_i}{\partial E_j}$, respectively, where i=M, M=1,2,·····ITN4, j=LL, LL= layer index, representing the layer to be differentiated

<u>TERMS</u>

Before the constants A_i,B_i,···, etc., can be evaluated, certain elements used in obtaining these constants have to be defined.

This subroutine defines the following variables:

F1(G,Y1,Y2) $= p(h,\nu,u) = 2(mh)^2+4mh(\nu-\mu)+1-8\nu\mu+2\mu$

F2(G,Y1,Y2) $= q(h,\nu,u) = 2mh(2\nu-1)-(1+8\mu\nu-6\mu)$

X1(I,J,K) = part of X(I,J,K)

X2(I,J,K) = part of X(I,J,K)

X(I,J,K) = X1(I,J,K) + EV X2(I,J,K)

EV $= \dfrac{E_i}{E_{i+1}} \dfrac{1+\nu_{i+1}}{1+\nu_i}$

DMULT

This subroutine performs matrix multiplications of $D^{-1}(h_j)\ X_j\ D(h_j)$ or $D^{-1}(h_j)\ \frac{\partial X_j}{\partial E_k}\ D(h_j)$ which is required for the evaluation of constants A(M), B(M), C(M), D(M), DGA(M,LL), DGB(M,LL), DGC(M,LL) and DGD(M,LL).

BESSEL

This subprogram computes either $J_o(m)$ or $J_1(m)$ to within an absolute tolerance of less than 1.1×10^{-7}. The computation was the following approximations

(1) Function $J_o(x)$

For $X \geq 3$, Let $Y = 3/X$

$$F(x) = .79788456 - .00000077Y - .0055274Y^2 - .00009512Y^3 + .00137237Y^4 - .00072805Y^5 + .00014476Y^6$$

$$T(x) = X-.78539816-.04166397Y-.00003954Y^2+.00262573Y^3 - .00054125Y^4-.0029333Y^5+.00013558Y^6$$

$$J_o(x) = F(x)\ T(x)/\sqrt{X}$$

For $X < 3$, Let $Y = X/3$

$$J_o(x) = 1.-2.2499997Y^2+1.2656208Y^4-.3163866Y^6+.0444479Y^8 - .0039444Y^{10} + .00021Y^{12}$$

(2) Function $J_1(x)$

For $X \leq 3$, Let $Y = 3/X$

$$F(x) = .79788456+.00000156Y+.01659667Y^2+.00017105Y^3 - .00249511Y^4 + .00113653Y^5 - .00020033Y^6$$

$$T(x) = X - 2.35619449 + .12499612Y + .0000565Y^2 - .00637879Y^3$$
$$+ .00074348Y^4 + .00079824Y^5 - .00029166Y^6$$

$$J_1(x) = F(x)\ T(x)/\sqrt{X}$$

<u>For $X < 3$</u>, Let $Y = X/3$

$$J_1(x) = .5 - .56249985Y^2 + .21093573Y^4 - .03954289Y^6$$
$$+ .00443319Y^8 - .00031761Y^{10} + .00001109Y^{12}$$

<u>CALCIN</u>

This subroutine performs all deterministic calculations.

<u>CALCN2</u>

This subroutine performs probabilistic calculations.

<u>PROBL</u>

This subroutine is called by CALCN2 and evaluates the sums

$$\sum_{j=1}^{n} \left(\frac{\partial^2 g_i}{\partial E_j^{\,2}}\right) \sigma_{E_j}^{\,2}$$

and

$$\sum_{J=1}^{n} \left(\frac{\partial g_i}{\partial E_j}\right)^2 \sigma_{E_j}^{\,2}$$

<u>CURVIT</u>

A least squares curve-fit of laboratory creep compliance or relaxation modulus data is obtained for the power law representation

$$J(t) = A\ t^n$$

where

$$A = \exp\left[\frac{\begin{vmatrix} \sum_{i=1}^{n} \log J_i & \sum_{i=1}^{n} \log t_i \\ \sum_{i=1}^{n} \log J_i \log t_i & \sum_{i=1}^{n} (\log t_i)^2 \end{vmatrix}}{DET}\right]$$

$$n = \frac{\begin{vmatrix} N & \sum_{i=1}^{N} \log J_i \\ \sum_{i=1}^{N} \log t_i & \sum_{i=1}^{N} \log J_i \log t_i \end{vmatrix}}{DET}$$

and

$$DET = \begin{vmatrix} N & \sum_{i=1}^{N} \log t_i \\ \sum_{i=1}^{N} \log t_i & \sum_{i=1}^{N} (\log t_i)^2 \end{vmatrix}$$

II. EVALUATION OF EQUATIONS

Evaluations of terms in the probabilistic solution is described to provide familiarity with the structure of the computer program.

In solving $\frac{\partial A_i}{\partial E_j}$, $\frac{\partial B_i}{\partial E_j}$, $\frac{\partial C_i}{\partial E_j}$ and $\frac{\partial D_i}{\partial E_j}$

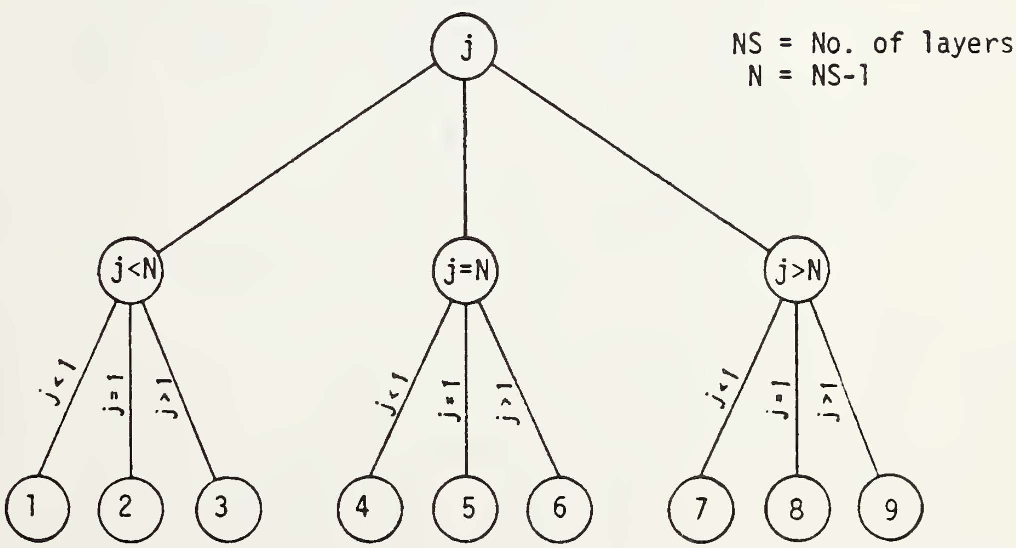
$$\begin{bmatrix} \frac{\partial A_1}{\partial E_j} \\ \frac{\partial B_1}{\partial E_j} \\ \frac{\partial C_1}{\partial E_j} \\ \frac{\partial D_1}{\partial E_j} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial E_j} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ C_n \\ D_n \end{bmatrix} + \begin{bmatrix} P_1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{\partial C_n}{\partial E_j} \\ \frac{D_n}{E_j} \end{bmatrix} \tag{1}$$

where

$$P_1 = \prod_{\ell=1}^{n-1} \frac{D_\ell^{-1} \; X_\ell D_\ell}{4(\nu_\ell - 1)} \tag{2}$$

$$\frac{\partial P_1}{\partial E_j} = \frac{\partial}{\partial E_j} \left[\prod_{\ell=1}^{n-1} \frac{D_\ell^{-1} \; X_\ell D_\ell}{4(\nu_L - 1)} \right] \tag{3}$$

In the evaluation of Equation (3) nine cases must be considered. The cases form a tree structure:



Case 1 $\quad 0 = j > N$

This case does not exist.

Case 2 $\quad 1 = j < N$

$$\frac{\partial P_1}{\partial E_j} = \frac{\partial P_1}{\partial E_1} = \left[\frac{D_1^{-1}\frac{\partial X_1}{\partial E_1}D_1}{4(\nu_1-1)}\right]\left[\prod_{\ell=2}^{N}\frac{D_\ell^{-1}X_\ell D_\ell}{4(\nu_\ell-1)}\right]$$

$$= \frac{L_1}{E_1}\left[\frac{D_1^{-1}\frac{\partial X_1}{\partial L_1}D_1}{4(\nu_1-1)}\right]\left[\prod_{\ell=2}^{N}\frac{D_\ell^{-1}X_\ell D_\ell}{4(\nu_\ell-1)}\right] \qquad (4)$$

where

$$L_k = \frac{E_k}{E_{k+1}}\,\frac{1+\nu_{k+1}}{1+\nu_k} \qquad k = 1,2,3,\cdots\cdots,n-1$$

Case 3 $\quad 2 \leq j < N$

In this case, the possible conditions are:

$j = 2$	$N = 3$
$j = 2,3$	$N = 4$
$j = 2,3,4$	$N = 5$
.	.
.	.
.	.
.	.
.	.
$j = 2,3,4,\cdots,n-2$	$N = n-1$

$$\frac{\partial P_1}{\partial E_j} = \left[\prod_{\ell=1}^{j-2} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)}\right] \left\{ \left[\frac{D_{j-1}^{-1} \frac{\partial X_{j-1}}{\partial E_j} D_{j-1}}{4(\nu_{j-1}-1)}\right]\left[\frac{D_j^{-1} X_j D_j}{4(\nu_j - 1)}\right] + \left[\frac{D_{j-1}^{-1} X_{j-1} D_{j-1}}{4(\nu_{j-1}-1)}\right]\right.$$

$$\left.\left[\frac{D_j^{-1} \frac{\partial X_j}{\partial E_j} D_j}{4(\nu_j - 1)}\right]\right\}\left[\prod_{\ell=j+1}^{n-1} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)}\right]$$

$$= \left[\prod_{\ell=1}^{n-2} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)}\right] \left\{ \frac{-L_{j-1}}{E_j}\left[\frac{D_{j-1}^{-1} \frac{\partial X_{j-1}}{\partial L_{j-1}} D_{j-1}}{4(\nu_{j-1}-1)}\right]\left[\frac{D_j^{-1} X_j D_j}{4(\nu_j - 1)}\right] + \frac{L_j}{E_j}\left[\frac{D_{j-1}^{-1} X_{j-1} D_{j-1}}{4(\nu_{j-1}-1)}\right]\right.$$

$$\left.\left[\frac{D_j^{-1} \frac{\partial X_j}{\partial L_j} D_j}{4(\nu_j - 1)}\right]\right\}\left[\prod_{\ell=j+1}^{n=1} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)}\right] \qquad (5)$$

where

$$\frac{\partial X_i}{\partial E_k} = \frac{\partial X_i}{\partial L_i}\frac{\partial L_i}{\partial E_k} = \frac{\partial X_i}{\partial L_i}\left[\frac{L_i}{E_i}\delta_{ik} - \frac{L_i}{E_{i+1}}\delta_{\alpha k}\right]_{\alpha=i+1} \qquad (6)$$

<u>Case 4</u> $j < 1$

This case does not exist.

<u>Case 5</u> $j = N = 1$

$$\frac{\partial P_1}{\partial E_1} = \frac{L_1}{E_1}\left[\frac{D_1^{-1} \frac{\partial X_1}{\partial L_1} D_1}{4(\nu_1 - 1)}\right] \qquad (7)$$

Case 6 $\quad 1 < j = N$

$$\frac{\partial P_1}{\partial E_j} = \left[\prod_{\ell=1}^{j=2} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)}\right]\left\{\left[\frac{D_{j-1}^{-1} \frac{\partial X_{j-1}}{\partial E_j} D_{j-1}}{4(\nu_{j-1}-1)}\right]\left[\frac{D_j^{-1} X_j D_j}{4(\nu_j - 1)}\right] + \left[\frac{D_{j-1}^{-1} X_{j-1} D_{j-1}}{4(\nu_{j-1}-1)}\right]\right.$$

$$\left.\left[\frac{D_j^{-1} \frac{\partial X_j}{\partial E_j} D_j}{4(\nu_j - 1)}\right]\right\}$$

$$= -\frac{1}{E_j} \prod_{\ell=1}^{n=2} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)} \left\{ L_{j-1} \left[\frac{D_{j-1}^{-1} \frac{\partial X_{j-1}}{\partial L_{j-1}} D_{j-1}}{4(\nu_{j-1}-1)}\right]\left[\frac{D_j^{-1} X_j D_j}{4(\nu_j - 1)}\right]\right.$$

$$\left. - L_j \left[\frac{D_{j-1}^{-1} X_{j-1} D_{j-1}}{4(\nu_{j-1}-1)}\right]\left[\frac{D_j^{-1} \frac{\partial X_j}{\partial L_j} D_j}{4(\nu_j - 1)}\right]\right\} \qquad (8)$$

Case 7 $\quad j < 1$

This case does not exist.

Case 8 $\quad j = NS = 1$

This case exists for $N = 0$ (semi-infinite space) only.

Case 9 $\quad j = NS > 1$

$$\frac{\partial P_1}{\partial E_j} = \left[\prod_{\ell=1}^{j-2} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)}\right]\left[\frac{D_{j-1}^{-1} \frac{\partial X_{j-1}}{\partial E_j} D_{j-1}}{4(\nu_{j-1}-1)}\right]$$

$$= -\frac{L_N}{E_{NS}} \left[\prod_{\ell=1}^{j-2} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)}\right]\left[\frac{D_N^{-1} \frac{\partial X_N}{\partial E_N} D_N}{4(\nu_N - 1)}\right] \qquad (9)$$

Note: $\displaystyle\prod_{\ell=1}^{j-2} \frac{D_\ell^{-1} X_\ell D_\ell}{4(\nu_\ell - 1)} = 1$ for $NS = 2$

III. USER'S MANUAL

The input requirements for the primary response model are defined in Table I below:

TABLE I

INPUT REQUIREMENTS
PRIMARY RESPONSE MODEL

CARD		PARAMETER
1	ITITLE (I) (20A4) -	Title and Job information; columns 5-80 are printed out with job results.
2	IFLG, NS, IR, IZ, WGT, AR, PSI (4I5, 3E10.5)	
	IFLG = 0 →	Elastic Analysis.
	IFLG = 1 →	Viscoelastic Analysis with lab creep data to be curve-fit to power law $A\ t^n$.
	IFLG ≠ or 1 →	Viscoelastic Analysis; but coefficient "A" and exponent "n" of power law At^n are to be input.
	NS =	Number of Layers.
	IR =	Number of radial points where output is desired.
	IZ =	Number of axial points where output is desired.
	WGT =	Axle Load
	AR =	Load Radius
	PSI =	Tire Pressure
		(WGT, AR, PSI:) Only two of these three may be input; however, any two will do.
3	HH (I) (I=1,NS-1)(8E10.5) -	Layer thicknesses for first n-1 layers.
4	RR (I) (I=1,IR) (8E10.5) -	Radial points where output is desired.
5	ZZ (I) (I=1,IZ) (8E10.5) -	Axial points where output is desired.
IFLG = 0:		
6	E(I),C E(I), V(I), I=1, NS (8E10.5)	
	E(I) =	Elastic Modulus (or compliance of the Ith layer.

COVE(I) = Coefficient of variation of elastic modulus (or compliance of Ith layer.

V(I) = Poisson's ratio of Ith layer.

Note: Either modulus or compliance may be input. If COVE(I) = 0 for all layers; then a deterministic solution is obtained.

IFLG = 1:

6 NT(J) (I5) = Number of times for which laboratory creep data is input for J-layer.

7 TYME(I,J) (I=1,NT)(8E10.5) = Vector of times for laboratory creep data.

8 VALUE (I,J), COV(I,J) (I=1,NT) (8E10.5)

VALUE (I,J) = Creep compliance at time I for layer J.

COV(I,J) = Coefficient of variation of creep compliance at time I for layer J

Note: Cards 6, 7, & e are repeated for as many layers as there are. Also, if anyone layer has creep compliance data to be input, then they all must have complainces input (not moluli); however, they need not be viscoelastic (i.e., all values can be the same for elastic layer).

9 V(J) (J=1,NS) (8E10.5) = Poisson's ratio.

10 NTO (I5) = Number of times for which output is desired.

11 = Vector of times for which output is desired.

IFLG ≠ 0 or 1:

6 COF(K), DN(K), COVE(K) (K-1,NS) (8E10.5)

COF(K) = Coeffocient in power law At^n.

DN(K) = Exponent in power law At^n.

COVE(K) = Coefficeint of variation of creep compliance (constant with time for each layer).

Note: If any layer is elastic input compliance = COF(K) and ND(K) = zero. Any COVE(K) ≠ 0 again signals probabilistic solution.

7	V(J) (J=1,NS) (8E10.5)	= Poisson's ratio.
8	NTO (I5)	= Number of times for which output is desired.
9	T1(I) (I=1,NTO) (8E10.5)	= Vector of times for which output is desired.

Note:

LAST END (A4) : The above sequences of cards may be repeated any number of times for different problems; however, the very Last card of job must have END in the first three columns.

* U.S. GOVERNMENT PRINTING OFFICE :

FEDERALLY COORDINATED PROGRAM OF HIGHWAY RESEARCH AND DEVELOPMENT (FCP)

The Offices of Research and Development of the Federal Highway Administration are responsible for a broad program of research with resources including its own staff, contract programs, and a Federal-Aid program which is conducted by or through the State highway departments and which also finances the National Cooperative Highway Research Program managed by the Transportation Research Board. The Federally Coordinated Program of Highway Research and Development (FCP) is a carefully selected group of projects aimed at urgent, national problems, which concentrates these resources on these problems to obtain timely solutions. Virtually all of the available funds and staff resources are a part of the FCP, together with as much of the Federal-aid research funds of the States and the NCHRP resources as the States agree to devote to these projects.*

FCP Category Descriptions

1. Improved Highway Design and Operation for Safety

Safety R&D addresses problems connected with the responsibilities of the Federal Highway Administration under the Highway Safety Act and includes investigation of appropriate design standards, roadside hardware, signing, and physical and scientific data for the formulation of improved safety regulations.

2. Reduction of Traffic Congestion and Improved Operational Efficiency

Traffic R&D is concerned with increasing the operational efficiency of existing highways by advancing technology, by improving designs for existing as well as new facilities, and by keeping the demand-capacity relationship in better balance through traffic management techniques such as bus and carpool preferential treatment, motorist information, and rerouting of traffic.

3. Environmental Considerations in Highway Design, Location, Construction, and Operation

Environmental R&D is directed toward identifying and evaluating highway elements which affect the quality of the human environment. The ultimate goals are reduction of adverse highway and traffic impacts, and protection and enhancement of the environment.

4. Improved Materials Utilization and Durability

Materials R&D is concerned with expanding the knowledge of materials properties and technology to fully utilize available naturally occurring materials, to develop extender or substitute materials for materials in short supply, and to devise procedures for converting industrial and other wastes into useful highway products. These activities are all directed toward the common goals of lowering the cost of highway construction and extending the period of maintenance-free operation.

5. Improved Design to Reduce Costs, Extend Life Expectancy, and Insure Structural Safety

Structural R&D is concerned with furthering the latest technological advances in structural designs, fabrication processes, and construction techniques, to provide safe, efficient highways at reasonable cost.

6. Prototype Development and Implementation of Research

This category is concerned with developing and transferring research and technology into practice, or, as it has been commonly identified, "technology transfer."

7. Improved Technology for Highway Maintenance

Maintenance R&D objectives include the development and application of new technology to improve management, to augment the utilization of resources, and to increase operational efficiency and safety in the maintenance of highway facilities.

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* The complete 7-volume official statement of the FCP is available from the National Technical Information Service (NTIS), Springfield, Virginia 22161 (Order No. PB 242057, price $45 postpaid). Single copies of the introductory volume are obtainable without charge from Program Analysis (HRD-2), Offices of Research and Development, Federal Highway Administration, Washington, D.C. 20590.

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