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16. Abstract

Commuter rail systems are being introduced into many urban areas as an alternative mode to automobiles for commuting trips. The shift from the auto mode to rail mode is anticipated to greatly help alleviate traffic congestion in urban road networks. However, the right-of-way of many existing commuter rail systems is usually not ideally located. Since the locations of rail systems were typically chosen long ago to serve the needs of freight customers, the majority of current commuter rail passengers have to take a non-walkable connecting trip to reach their final destinations after departing even the most conveniently located rail stations. To make rail a more viable, competitive commuting option, a bus feeder or circulator system is proposed for seamlessly transporting passengers from their departing rail stations to final work destinations. The primary research challenge in modeling such a bus circulator system is to optimally determine a bus route and stop sequence for each circulating tour using the real-time demand information. In this paper, we termed this joint routing and stop optimization problem the *circulator service network design problem*, the objective of which is to minimize the total tour cost incurred by bus passengers and operators while minimizing the walk time of each individual bus passenger. A bi-level nonlinear mixed integer programming model was constructed and a tabu search method with different local search strategies and neighborhood evaluation methods was then developed to tackle the circulator service network design problem.

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REAL-TIME OPTIMIZATION OF PASSENGER COLLECTION FOR COMMUTER RAIL SYSTEMS

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EXECUTIVE SUMMARY

Commuter rail systems are being introduced into many urban areas as an alternative mode to automobiles for commuting trips. The shift from the auto mode to rail mode is anticipated to greatly help alleviate traffic congestion in urban road networks. However, the right-of-way of many existing commuter rail systems is usually not ideally located. Since the locations of rail systems were typically chosen long ago to serve the needs of freight customers, the majority of current commuter rail passengers have to take a non-walkable connecting trip to reach their final destinations after departing even the most conveniently located rail stations. To make rail a more viable, competitive commuting option, a bus feeder or circulator system is proposed for seamlessly transporting passengers from their departing rail stations to final work destinations.

The primary research challenge in modeling such a bus circulator system is to optimally determine a bus route and stop sequence for each circulating tour using the real-time demand information. In this paper, we termed this joint routing and stop optimization problem the *circulator service network design problem*, the objective of which is to minimize the total tour cost incurred by bus passengers and operators while minimizing the walk time of each individual bus passenger.

A bi-level nonlinear mixed integer programming model was constructed and a tabu search method with different local search strategies and neighborhood evaluation methods was then developed to tackle the circulator service network design problem. This exact algorithm was developed because the network size (number of candidate stops) for the feeder system is much smaller than that of an urban transit network. The small size provides some freedom to explore and improve current exact algorithms to solve the problem, as land use information gives us a good idea of where demands will be concentrated. Potential bus stops can be identified geographically before the service is put in place. That is also how the service is going to be operated; the transit operator provides users with all the candidate stops and allows them to choose a stop from the map according to their preference.

Our tentative solution methods take advantage of knowing possible bus stop locations in advance, establishing the route information database through an enumeration process. Each route starts from and ends at the rail station, which is always in the route. We evaluate all possible combinations of candidate stops, and solve it as a TSP to get the minimum travel time. The database will store minimum route costs for all possible stop combinations and our solution method will use them as pre-calculated parameters. Our feeder system is designed so that every

bus leaves the station as the train leaves and comes back to the station before the next train arrives. The minimum travel time for each route should not exceed 30 minutes, which is the headway for the commuter rail. The travel cost converted from the travel time, together with the fixed cost for each route, will determine the route cost. For each route, we have a binary vector a^r with a dimension equal to the total number of candidate stops in route r, a vector r with a dimension of the total number of stops covered by the route and showing the order of visits to stops in route r, and a cost value c^r.

In this research, we only modeled and solved a collective circulator service network design problem, which describes the bus circulator operations in the origin end of workplace-to-home commuting trips. The fleet of buses is used for picking up passengers from selected bus stops close to workplaces and dropping them off at the rail station during the afternoon commuting period. This study treated each bus circulating tour as an independent optimization problem without considering the cooperation and coordination between different buses in the fleet. From the efficiency perspective, it is much desirable to formulate a larger circulator service network design problem, which operates a fleet of buses as a whole instead of operating each individual one separately. Such a fleet-based circulator system can be readily implemented by using the current information and communication technologies.

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1. INTRODUCTION

Peak-hour congestion in urban highway networks is a long-established and hard-to-solve problem. Considering all the practical constraints for highway capacity expansion, many urban areas are implementing commuter rail systems as an alternative mode to alleviate peak-hour congestion in the roadway network. Currently commuter rail development and expansion programs are found at a number of locations in the United States. Most of these programs are using existing tracks or available right-of-way (ROW) in order to eliminate or reduce land takings, relocation effects, and the costs associated with creating an all-new ROW through dense metropolitan areas (Zullig & Phraner, 2000). In fact, the availability of existing rail ROW is a major factor in the evaluation of urban transportation options.

However, as discussed by Grava (2003), one issue related to using existing rail alignments is their location. Since they were built at a different time, often to serve freight traffic in a rather different city layout, they do not necessarily facilitate use for current residential and commercial activities. Distances from potential rail stations to commuter trip destinations often exceed feasible walking distances. If a person wants to take the commuter rail, assume that he drives to the home-end rail station and gets on the rail. When he reaches the work-end station, he is unlikely to have another car ready to use. He may prefer to transfer to another transit mode to get to his final destination, since walking is just not feasible. If he uses the existing urban bus system, he will likely walk to the bus stop, wait for the bus to come, ride the bus to the closest stop to his destination, and then probably walk from the bus stop to his destination. If we consider the total time spent on driving, train riding, transferring, bus riding, and walking to the destination, the commuter rail may lose its attractiveness or even its viability as an alternative mode to driving.

Minimizing access time to rail stations and final destinations thus becomes critical. We propose a bus feeder/circulator system to transport passengers from the rail station to their work destination. This feeder system has two advantages. First, it is embedded with a seamless transfer operation. This seamless transfer concept means that when the train arrives at the station, there is always a bus ready to take rail passengers to their final destinations. By providing a seamless transfer, we are actually eliminating both the waiting time and the possible anxiety experienced by transit users at the transfer point.

Secondly, the recent widespread adoption of smart phones might provide an innovative means of obtaining real-time destination data. The common practice is to design the bus route based upon static demand estimates for a typical day and to operate the feeder buses on a fixed route. In the proposed bus feeder system, the destination information is collected from mobile devices (such as smart phones, tablets, and laptops) on a real-time basis and passengers are allowed to identify their destinations among all demand zones served by the operator. Based on the real-time travel information, the transit operators dynamically identify optimal circulator routes for each and every set of passengers coming into the station. Of course, not every passenger will provide their destination information. Nonetheless, passengers who are not able or willing to provide their destination information can still use the routes that are based on other users' data; as they see the benefits of providing destination information, they might start doing so for their own convenience. Additionally, our work also tested the system performance with only partial traveler information; those results are discussed later in the paper.

From the perspective of commuter rail operators, the bus distributor system is a means to connect rail stations to commuter trip destinations to produce competitive overall trip times and potentially increase ridership. From the perspective of commuter rail users, the demand-responsive circulator system allows them to avoid highway system congestion without losing access to their final destinations. Additionally, they would benefit more from this system than from a regular fixed-route bus system since the route is designed specifically for the particular set of passengers on the train.

When solving transit network design problems, two branches of algorithms are primarily used: analytical optimization models and heuristic algorithms. Classical analytical optimization models were used in the early stages of the research on the transit route network design problem. These models focus on developing a continuous convex objective function under assumptions that simplify and idealize the transit network. By solving first-order equations of the objective functions in these models, optimal solutions for stop spacing, headway, frequency, or other route characteristics can be efficiently generated. As noted by Ceder and Wilson (1986), analytic methods are suited to early stage screening in the planning process or conceptual policy decisions where approximate design parameters are adequate but have little practical benefit in solving real-world problems. Heuristic solution frameworks are more often employed in solving real-size network problems. Chakroborty (2003) provided a detailed discussion regarding why

the urban transit network design problem (UTNDP) cannot be solved with exact algorithms such as Branch-and-Bound or Branch-and-Cut. Problems arise with inclusion of discrete decision variables in the UTNDP, the property of the nonlinearity of the UTNDP, and definition of logical conditions to better describe a realistic transit network in the mathematical program, as also mentioned by Baaj and Mahmassani (1990). All these facts lead to a common problem: a significant computational burden.

Given the limitation of exact algorithms in solving realistic transit network design problems, approximation techniques—heuristics and meta-heuristics—are usually preferred in many practical situations. They enable one to solve the real-size network design problem in a reasonable time frame, compared to exact algorithms.

Kuah and Perl (1988) present a heuristic method to solve the feeder bus network design problem for an optimal route set and operating frequencies to provide feeder bus service to access an existing rail system. Extended from the heuristics proposed by Kuah and Perl (1988), Martins and Pato (1998) employed a tabu search algorithm. Tabu restrictions prevent the replacement of a stop in its previous position for a chosen number of iterations. The intensification strategy of the tabu search method accentuates the search in a region of good solutions by decreasing the tenure of moves marked tabu. Lownes and Machemehl (2010) propose both exact and meta-heuristic methods to specifically address the circulator service network design project (CSNDP) problem by formulating a mixed integer programming approach with the objective accounting for transit user travel cost, transit agency operation cost, and social cost related to unserved demand. The exact method utilizes lower bound and additional stopping criteria to reduce computational effort, yet it is still suitable for small to medium-sized networks while the tabu search method is used to solve large networks. However, the solution framework based on the tabu search does not solve the CSNDP within a strict time limit. In this paper, the computational time and solution quality of this algorithm is compared against our proposed algorithms.

2.   BACKGROUND

In performing a literature review, the research team found that commuter rail is increasingly being used for a variety of non-commute purposes during the off-peak periods and on weekends. The majority of commuter rail systems connect to other transit routes and only a few seem to utilize circulator, or shuttle, systems to provide passengers access to their final destinations (these include Austin, Albuquerque, Nashville, and New Haven). Interestingly, the feasibility studies do not seem to account for the impact of a well-designed collector-distributor system on quality of service and ridership numbers 1, 2, 3, 4, 5, and 6. While only a limited number of customer satisfaction surveys were found, in all these surveys were indications of the significant impact of a feeder bus system on commuter rail ridership e.g., 5 and 6. For example, for the Tri-Rail commuter rail service in southeast Florida, the limitations of existing feeder bus services are believed to be a constraint in the attainment of the system's ridership potential. The transit systems that currently provide feeder bus services are not designed to serve the commuter rail corridor, but are oriented to serving urban areas along the coast, which is east of the commuter rail line (Shaw, 1989).

In general, the conclusion of this literature review is that commuter rail service is transitioning from its original role of serving work commute trips and peak hours to serving other trip purposes and operating during non-peak hours as well as weekends. Thus, circulators are becoming even more popular and significant in encouraging rail ridership. These findings emphasize the importance of (and increased market for) demand-responsive circulator systems, as demand for commuter rail can vary significantly throughout the day and thus a demand-responsive system can significantly minimize the cost of operation as compared with a fixed-route circulator system. Additionally, this literature review determined the scale at which the problem should be addressed. The feeder bus system, depending on the range of commuter rail service, has the potential to accommodate the demand spatially. These findings are important when developing formulations and heuristic methods to solve the optimization problem.

Previous Network Design Solutions

Previous work on the bus transit route network design problem involved finding a bus transit route network configuration and other associated operational decisions that achieve a

desired objective with a variety of constraints. Depending on the problem characteristic and modeler's perspective, objective and decision variables can be defined in various ways. Kuah and Perl (1988) considered both user travel cost and transit agency operation cost while formulating a feeder bus to rail access system. Dubois, Bel, and Llibre (1979) argue that minimizing the total travel time is an appropriate objective for modifying a transportation network to better serve its existing demand. Instead of including operation cost in the objective function, they include a budget cost constraint. Ceder and Israeli (1998) include empty-space hours of vehicles in their model objective to represent unproductivity from the transit operator side. Lee and Vuchic (2005) contend that minimization of user travel cost is a proper objective for public transit agencies; however, for private transit agencies, profit maximization would be more appropriate. The combination of the two objectives, which represents social benefit maximization or social cost minimization, tends to be favored by transit planners.

Jaw et al. (1984) presented a heuristic algorithm for a time-constrained version of the multi-vehicle, many-to-many "Dial-A-Ride" system problem (this system is discussed in Section 3). The time constraints used in this study consist of (1) the time difference between the actual pickup or delivery of a customer and the desired pickup or delivery time and (2) the maximum customer riding time. They defined the objective function such that it balances the cost of providing service with the customers' preferences for pickup and delivery times close to those requested and for short ride times.

In a study done in 2000 in the San Francisco Bay Area about traveler response to a personalized demand-responsive transit system, Khattak and Yim found that in general, people take time to carefully plan their commute, determining their best option by weighing several different factors such as travel, walking, and waiting times; cost; safety; and accessibility (Khattak and Yim, 2004).

Lam and Xie (2002) conducted a study in Singapore that examined passengers' path choices in a complex transit network. A survey asked passengers to choose between three transit path options between the same origin and destination. Each path varied in walking distance, wait time, fare, and number of stops and transfers. The passengers were not informed of the total travel time of the three paths. Without that information, the responses were nearly evenly split between the three path choices. Next, the travel times of each path were revealed and the passengers were again asked to choose. Just over half of those surveyed chose the path with the

shortest travel time. This result shows that passengers prefer a shorter travel time, even if it means a longer walk or more transfers. However, it also reinforces the fact that, although travel time is a very important factor in passengers' decision-making, mode choice is based on a trade-off of many factors, not just travel time. Nearly half of the passengers chose the two longer paths of travel, presumably because they found some advantage in the other aspects of the trip, such as the shorter walk or the fewer stops.

A major component of transit accessibility is walking distance. Convenient walk access on both trip ends is very important to make a transit option viable. Roughly a 5-minute or quarter-mile (about 400 meters) walk is generally accepted as reasonable transit walk access. Crowley et al. (2009) examined travel data from the travel-habits database of Toronto, Canada, to analyze how walking distance to transit affects mode choice. Part of the study compared walking distance from a subway station to the mode share of passengers accessing the subway by walking. For those within 200 meters of the station, the mode share was at 36%. The mode share decreased slightly to 32% as the walking distance increased to between 200 and 400 meters, then decreased significantly to 17% as the walking distance increased to between 400 and 800 meters. Some studies report that the perceived "cost" of time spent walking is two to three times more than the "cost" associated with in-vehicle time.

Beimborn et al. (2003) looked at accessibility and connectivity and the relationship they have on mode captivity and mode choice. A study was done in the Portland, Oregon, area using data from various sources. The results of the analysis showed that people who have the option of using either transit or automobile are influenced little by the overall travel times of the modes. The access to and time spent waiting for the transit system are much more important issues when a passenger is deciding whether or not to use a transit system.

In the simplest case, a simple network with a set of nodes to be visited by a single vehicle was considered. In this case, the nodes may be visited in any order, there are no precedence relationships, the travel costs between two nodes are the same regardless of the direction traveled, and there are no delivery-time restrictions. In addition, vehicle capacity is not considered. The output for the single-vehicle problem is a route or a tour where each node is visited only once and the route begins and ends at the depot node. The tour is formed with the goal of minimizing the total tour cost. This simplest case is referred to as a *traveling salesman problem* (TSP), which was used for the preliminary analysis in this study. The formulation of

this problem is based on the formulation developed by Lownes (2007). However, instead of having a set of centroids, this set now represents a set of demand calls received. This analysis used an adaptive tabu search method that consisted of an initial solution by nearest neighbor and swap-and-insert method for neighborhood search strategy.

The project team reviewed the literature on the application of a heuristic tabu search to TSPs and closely related problems, such as vehicle routing problems (VRP). Heuristic optimization methods trade optimality of the solutions that they output with execution times. Many details go into the structure of a mathematical programming problem and a heuristic method (a tabu search in this context), which are not within the scope of this report. Papers on the application of a tabu search to these problems were classified based on problem size; generation of initial solutions; selection of moves; the choice of short-, medium-, and long-term memory structures; and aspiration criteria.

It is interesting to note that even though heuristics are meant to handle large problems, most current research deals only with TSPs with up to 100 nodes. A tabu search is an improvement heuristic. It needs to start with a feasible tour in the graph describing the TSP. Overall, eight methods were identified to generate initial solutions. Some of these methods include nearest neighbor, sweep, and Solomon. A tabu search, as an improvement heuristic, moves from one solution to the next in search of an optimal solution. The method of moving from one solution to another is described by a set of rules and called a *move*. The set of all solutions that can be reached from a given solution using a pre-specified move is called the *neighborhood* of the solution. A variety of move types have been used in the literature in the context of TSPs and related problems, including the following: 2-opt move, insertion, vertex insertion, and generalized insertion. (For more information about TSPs, please consult Gendreau et al., 1999; Johnson et al., 2002; and Reinelt, 1991).

This problem class is appropriate for the preliminary analysis of this problem. In general, this problem class is appropriate for a single VRP or when the service area is divided up into subareas of service and a single TSP optimization is conducted within each region. However, consideration of service time windows for this problem is essential as passenger pickup time determined by the optimization algorithm cannot be violated outside a predetermined time frame (or will be costly to violate). The provision of a time window into the formulation of this problem is part of the ongoing work. The simple TSP formulation uses some assumptions that

may be unrealistic. As the project progresses, these assumptions will be relaxed and other variations of the TSP will be considered, as discussed next.

An extension of the TSP, referred to as the *multiple traveling salesman problem* (MTSP), occurs when a fleet of vehicles must be routed from a single depot. The goal is to generate a set of routes, one for each vehicle in the fleet. The characteristics of this problem are that a node may be assigned to only one vehicle, but a vehicle will have more than one node assigned to it. No restrictions are placed on the size of the load or number of passengers a vehicle may carry. The solution to this problem will give the order in which each vehicle is to visit its assigned nodes. As in the single-vehicle case, the objective is to develop the set of minimum-cost routes, where "cost" may be represented by a dollar amount, distance, or travel time. This class seems appropriate if the number of vehicles is determined prior to the optimization and we are assured that demand can be accommodated through this predetermined number of vehicles. The MTSP naturally seems to be an appropriate next step for this study.

If the capacity of the multiple vehicles is restricted and coupled with the possibility of varying demands at each node, the problem is classified as a VRP. The VRP is more versatile than any kind of TSP, but is more difficult to solve optimally. The VRP and application of tabu search algorithms to this type of problem have been more studied recently. While tabu search algorithms developed for TSPs and VRPs are partially similar, structurally they can be significantly different. Based on the findings of a comprehensive study of tabu search approaches for the VRP, algorithms by Gehring and Homberger (2001) and Cordeau et al. (2001) show the best performance.

In this paper, we propose a mathematical model to solve the circulator service network design problem (CSNDP) and develop methods to solve for the optimal routes in a demand-responsive manner. To have the solution methods running for an optimal route for each set of passengers coming into the station, the computational time for each run of the algorithm should be limited to no more than 5 minutes, which is the time for all passengers in the train to get off and walk to the bus stop. This constraint poses a challenge to the algorithm development. This research fills the gap in the literature, since it formulates the model for and rapidly optimizes a seamless transfer feeder bus system. The rest of this paper is organized as follows. Section 3 gives the bi-level model formulation of the CSNDP problem, where the upper level aims to

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minimize the total costs incurred by both transit operators and transit users, and the lower level accounts for passenger travel behavior to minimize their own walk trips. Section 4 explains the adaptive tabu search algorithm designed for solving this CSNDP problem. Four local search strategies are proposed. In Section 5, computational performance of the four adaptive tabu search algorithms is evaluated and compared against the enumeration method and the tabu search algorithm proposed by Lownes (2010). Conclusions and further research are given in the final section.

3. MODEL DEVELOPMENT

Designing a Collector Bus System

A comprehensive investigation was conducted to classify the design of a collector bus system under a well-defined class of mathematical programming problems depending on certain characteristics of the service delivery system, such as size of demand and spatial coverage, consideration of capacities of the vehicles, and routing and scheduling (time windows) objectives.

A limited number of studies examine the use of a demand-responsive transportation system concept in designing collector-distributor systems. The most famous of these, conducted in the early 1970s, is what is called a "Dial-A-Ride" Transportation (DART) system, which mostly addresses many-origins-to-many-destination problems. Wilson et al. (1971, 1976) developed a real-time algorithm for a DART system in Haddonfield, NJ, and Rochester, NY. They defined their objective function as the weighted sum of current passengers' wait times, ride times, given arrival times, and additional distance travelled. Wilson et al. (1980) found that DART system demand is quite sensitive to the level of service provided; thus, collection and delivery times become a crucial component of DART problem structure. This study also noted the importance of developing measures of reliability as well as models to predict both levels of reliability and the impacts of reliability on demand. Perhaps, to some extent, dissatisfactory DART operation can be attributed to lack of constraints on passengers' wait and riding times. Although most studies aimed to minimize a combined cost to both operators and users, the resulting quality of service was not satisfactory to consumers, which resulted in low demand.

4.   PROBLEM DESCRIPTION

The real-time CSNDP problem aims to provide an optimal route to collect each set of passengers from their work locations and deliver them to the commuter rail station on a real-time basis. The solution methods determine both the bus stop locations and the route connecting them. In this operation, all the candidate stops in the service area (the small circle nodes in Figure 1) are shown to the rail users.
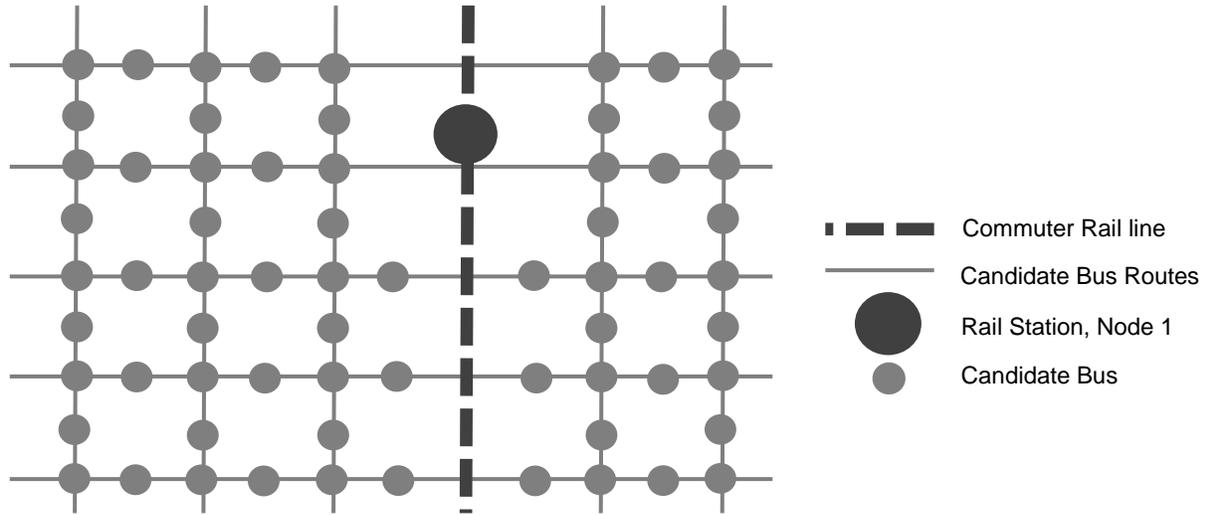

Figure 1. Network Representation.

The candidate stops are included in set I; i, j, k represent individual candidate stops, where $i, j, k \in I$. Rail passengers will contact the operating center to request pickup at their indicated bus stops. We assume that the headway of the commuter rail system is 30 minutes and the first PM peak hour train arrives at the station at 5:30 p.m. and every 30 minutes is counted as a time period. Starting from 5:00 p.m. as time point 0, 5:00–5:30 p.m. is regarded as period 1, 5:30–6:00 p.m. as period 2, and so on. The set of time periods is represented as T. In this problem, $t = 0, 1, 2, 3$ or 4. In this circulator, we propose to have both fixed stops and responsive stops. Hence, in the model formulation, we have I_f^t and I_r^t as subset of set I, where I_f^t is the set of fixed stops in period t and I_r^t is the set of responsive stops in period t. The detailed information about fixed stops and responsive stops will be discussed later. For each time period t, the fixed stops set and responsive stops set compose the set I.

In this problem, the number of vehicles V_n dedicated to this service is defined as a constraint in the model formulation rather than a decision variable. The system provides maximum service while minimizing the total cost. Related costs include two parts: route costs and penalty cost. Route costs account for drivers' wages and benefits, equipment investment, and maintenance and fuel/gas costs, and is calculated as the sum of fixed costs and operational costs. For every possible combination of stops, a small TSP will be solved for the minimum travel time along the stops and this minimum travel time is converted into operation cost. In this model, the route cost c^r is calculated as the sum of the fixed cost and operation cost. The set of all candidate routes is represented as R and individual routes r belong to R. We have a_i^r as a node-route incident matrix and this matrix can be considered as a set of column vectors a^r. Each vector a^r is associated with route r and the binary element a_i^r in the vector indicates whether route r covers stop i. To make sure that the maximum number of passengers is served by the routes, a penalty cost is applied to missed fixed stops and missed responsive stops. Since missing two consecutive trains is considered unacceptable for this feeder service, we enforce the system to serve the stops with demand leftover from the previous time period and these stops are regarded as fixed stops. The stops with demand coming in during current time period are regarded as responsive stops. The penalty cost for fixed stops is defined as P_F and penalty cost for responsive stops as P_R. The penalty costs associated with not providing service to fixed points are much higher than those applied to the responsive points. If the available vehicle is able to fulfill the task, the problem is an optimal route combination choice that achieves the least total cost. Otherwise, demands from the last period have priority over the new arrivals. The high penalty cost drives the solution to meet the demand for the fixed stops first.

During each time period, we have real-time passenger data d_i^t as the new demand requesting service. The decision variables x_r^t indicated whether route r is selected in time period t. At the end of each period we have the state variable D_i^t showing the amount of demand left out of service at each stop. The relationships between decision variables and state variables are shown in Equations (1) and (2). Depending on whether there is demand carried over from the last period $(t-1)$ for stop i, the model will identify whether stop i is fixed or responsive during this period t, as shown in Equations (3) and (4).

$$D_i^t = 0 \quad \forall t = 0 \tag{1}$$

$$D_i^t = \left(1 - \sum_r a_{ir}x_{t-1}^r\right)(D_i^{t-1} + d_i^t) \quad \forall i \in I, t \in \{1, 2, ..., T\} \tag{2}$$

$$I_F^t = \{i | D_i^{t-1} > 0\} \tag{3}$$

$$I_R^t = \{i | D_i^{t-1} = 0\} \tag{4}$$

At time point 0, we assume that service is starting and there is no demand left over from previous periods, as shown in Equation (1). At the end of each period, the demand left out of service is calculated as shown in Equation (2). The term $\sum_r a_{ir}x_r^{t-1}$ shows whether stop i is covered by any selected route. If it is covered, the leftover demand and newly arriving demand at this stop will be become unserved demand at the end of current period. This unserved demand is a state variable updated at the end of every period for all the stops and the mathematical model will apply a penalty to these stops. The operating center will determine on a real-time basis the best route combination for each time period to reduce the overall cost to the system along the peak-hour time horizon.

The feeder service has a goal to keep rail users from missing two trains in a row. With the total demand staying within the capacity of the available vehicles, the proper parameter setting—the penalty cost P_F and P_R—will force the service to achieve this goal. We have the following model:

For each time period t,

$$\min z = \sum_{r \in R} c^r x_t^r + \sum_{i \in I_F^t} P_F \left(1 - \sum_r a_i^r x_t^r\right) + \sum_{i \in I_R^t} P_R \left(1 - \sum_r a_i^r x_t^r\right) \tag{5}$$

$$\sum_r x_t^r \leq V_n \tag{6}$$

$$\sum_i a_i^r x_t^r \leq 1 \quad \forall \ i \in I \tag{7}$$

$$x_t^r \in \{0,1\} \quad \forall \ r \in R \tag{8}$$

where Equation (3) and (4) hold for I_F^t and I_R^t.

The objective function in (5) collects the total route cost and penalty costs along the planning time horizon. The first term is all the route costs across all the time periods. The second term is the penalty costs applied to the fixed stops where passengers were not served for more

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than one time period. The last term is the penalty cost applied to responsive stops with new demand coming in during the current period but left out of service for this time period. Constraint (6) sets a limit on the total number of routes at each time period, which should be more than the number of available vehicles. Constraint (7) gives the property of set partitioning to our route selection problem. Each stop will be visited no more than once in each time period. However, the penalty costs applied to the stops left out of service will be the driving force to serve as many nodes as possible. Constraint (8) restricts the route selection variables to a binary format.

5. RESULTS

The Tentative Exact Algorithm

This section presents an exact algorithm based on a column selection method to solve the route optimization for each time period, although most work reported in the literature solved the transit route optimization problem with heuristic methods. This exact algorithm was developed because the network size (number of candidate stops) for the feeder system is much smaller than that of an urban transit network. The small size provides some freedom to explore and improve current exact algorithms to solve the problem, as land use information gives us a good idea of where demands will be concentrated. Potential bus stops can be identified geographically before the service is put in place. That is also how the service is going to be operated; the transit operator provides users with all the candidate stops and allows them to choose a stop from the map according to their preference.

Our tentative solution methods take advantage of knowing possible bus stop locations in advance, establishing the route information database through an enumeration process. Each route starts from and ends at the rail station, which is always in the route. We evaluate all possible combinations of candidate stops, and solve it as a TSP to get the minimum travel time. The database will store minimum route costs for all possible stop combinations and our solution method will use them as pre-calculated parameters. Our feeder system is designed so that every bus leaves the station as the train leaves and comes back to the station before the next train arrives. The minimum travel time for each route should not exceed 30 minutes, which is the headway for the commuter rail. The travel cost converted from the travel time, together with the fixed cost for each route, will determine the route cost. For each route, we have a binary vector a^r with a dimension equal to the total number of candidate stops in route r, a vector r with a dimension of the total number of stops covered by the route and showing the order of visits to stops in route r, and a cost value c^r.

We consider the LP-relaxation obtained from the binary IP (5)–(8) by replacing (8) with $x_t^r \geq 0.$ The column selection method applied to the primal LP-relaxation, or the LP-master (LPM) consists of two steps: initialization and pricing.

Initialization. Construct a restricted LPM by restricting the LPM to the columns that are vectors with one element equal to one, which means each route covers only one stop. Solve the

restricted LPM to get the primal optimal solution x as well as the dual optimal solution (π, μ), where π and $\mu = \mu_i$ are the dual variables corresponding to constraint (6) and (7), respectively.

Pricing. Select a column whose reduced cost is negative. If there is no such column, the current solution x is an optimal solution of the LPM. Otherwise, enter such a column to get a new pair of solutions x' and (π', μ') and repeat this pricing step.

The pricing step amounts to finding a variable x_t^r of the rth route whose reduced cost is negative. In terms of the dual variables (π, μ), the reduced cost z_r is given by $c^r - \sum_{i \in I}(\mu_i + P_F \delta_i(I_F^t) + P_R \delta_i(I_R^t))a_i^p - \pi$, where $\delta_i(A) = \begin{cases} 1 \ if \ i \in A \\ 0 \ if \ i \notin A \end{cases}$. If $z_r \geq 0$ for all routes, the current solution is optimal. Otherwise, we enter the corresponding column to the LPM to improve the solution. In our algorithm, the pricing step simply evaluates all possible routes for their reduced cost and then selects the most negative column to enter the model to get a new pair of primary and dual solution values.

The Improved Exact Algorithm

As the tentative solution methods take advantage of knowing possible bus stop locations in advance and establish the route information database by an enumeration process before computing the route configuration, the improved algorithm allows us to generate the new column to enter the problem without intensive pre-calculation based on stop location prediction for all the possible route information.

The improved method relaxes the integer programming model into a linear one by dropping the integrity constraints in the original model and then solves the linear programming model based on the Dantizig-Wolfe decomposition method. The Dantizig-Wolfe method starts from an initial solution and generates the columns for entering the problem based on their reduced cost until no more candidate columns have negative reduced costs. In the column generation step, we always choose the columns with the most negative cost. This sub-problem transforms into a constrained shortest path problem.

Dropping the integrity constraints from Model (5)–(8), the model (5)–(7) becomes a linear program. Assuming π_i and π_0 are the dual values corresponding to Eqs. (6) and (7), the reduced cost for a new column r is

$$\bar{c}^r = c^r + \sum_{i \in I_F} (\pi_i + P_F)(1 - a_i^r) + \sum_{i \in I_R} (\pi_i + P_R)(1 - a_i^r) - \pi_0 \quad (9)$$

Since the column with the most negative reduced cost enters the problem, the pricing step becomes a constrained shortest path problem with respect to the following arc cost:

$$rc(i,j) = c_{ij} - \pi_j - Pd_j \quad \forall\ i,j \in \{0, 1, ..., I\} \quad (10)$$

where $P = \begin{cases} P_F \ if\ j \in I_F \\ P_R \ if\ j \in I_r \end{cases}$

Dynamic programming is widely used to solve the elementary shortest path problem with resource constraint (ESPPRC). We illustrate the exact algorithm for solving the ESPPRC developed by Feillet (2004) in the following sections.

Shortest Path Problem with Resource Constraint (SPPRC)

Before we introduce algorithms for the ESPPRCs, we give a brief illustration of the problem context and solution method of SPPRC. SPPRC is a relaxed version of ESPPRC.

Let $G = (N, A)$ be a network where A is the set of arcs and N is the set of nodes including an origin p and a destination q. A cost c_{ij} is associated with each arc $(i, j) \in A$. The cost of a path is defined as the sum of cost of the arcs of the path. Let L be the number of resources and $d_{ij}^l \geq 0$ be the consumption of l resource along arc (i, j). For each resource l, each node $i \in N$ has the feasibility window $[a_i^l, b_i^l]$ such that the consumption of resource l along the path from p to i is constrained to belong to the interval $[a_i^l, b_i^l]$. If the consumption resource l is lower than a_i^l when the path reaches i, it is set to a_i^l. Note that this notation is natural for the time resource, but also allows us to represent capacity constraints by defining intervals $[0, Q]$ on nodes, where Q is the capacity limit. For the SPPRC a node can be used more than once in a path (i_j can be equal to $i_k, j \neq k$). The objective is to generate a minimum cost path from p to d that satisfies all the resource constraints.

Desrochers' algorithm is a label-correcting reaching algorithm. It is an extension of the Ford-Bellman algorithm, taking resource constraint into account. Desrochers defines a state for each partial path from the origin node p to a node j as $(T_j^1, T_j^2, ..., T_j^L)$, where T_j^l is the amount of

the lth resource required to reach node j using a path from p to j, and a cost $C(T_j^1, T_j^2, ..., T_j^L)$. For a given node j, the feasible states respect the resource constraints at this node, and they form a set $\{(T_j^1, T_j^2, ..., T_j^L) | a_i^l \leq T_j^l \leq T_i^l \; for \; l = 1, ..., L\}$. Each path has a defined label as $(T_j^1, T_j^2, ..., T_j^L, C_i)$. These labels represent both the state value and the cost of a path X_{pi}. For simplicity, $T_i^\blacksquare$ is later on used to represent the state values. Each node receives labels throughout the computation. For two distinct paths X_{pi}' and X_{pi}^- from p to j associated with labels $(T_j^{\blacksquare'}, C_j')$ and $(T_j^{\blacksquare'}, C_j')$, they define that X_{pi}' dominates X_{pi}^- or $(T_j^{\blacksquare'}, C_j') < (T_j^{\blacksquare'}, C_j')$ if and only if $C_j' < C_j', T_j^{l'} < T_i^{l'}$ for $l = 1, ..., L$ and $(T_j^{\blacksquare'}, C_j') \neq (T_j^{\blacksquare'}, C_j')$. Non-dominated labels are treated until no new labels can be created. When a label is treated, all its new labels are extended from its associated node toward every possible successor node. The dominance rules are for saving the computational effort.

Elementary Shortest Path Problem with Resource Constraints (ESPPRC)

The pricing step as a sub-problem to our column generation method is actually an ESPPRC. In ESPPRC, paths must be elementary, so that no node can be visited more than once in a feasible path. As pointed out by Feillet, adjusting Desrochers' label-correcting algorithm to solve the ESPPRC instead of the SPPRC is not trivial: "One can neither find an optimal elementary path by just solving an SPPRC and selecting an elementary path among the set of optimal paths nor by solving the SPPRC and by enforcing that only elementary paths be generated at each step of process (Feillet et al., 2006). The major difficulty in modifying the labeling algorithm for ESPPRC comes from the fact, as discussed by Chabrier (2006), that a label cannot be directly fathomed using the dominance rule defined for SPPRC. If we denote $N(p)$ as the set of nodes in the partial path p, we cannot prolong p to j if we already have $j \in N(p)$ in ESPPRC. In this case, even if some partial path p_1 dominates some other partial path p_2, a continuation of p_2 with some node $i \in N(p_1)$ might be useful later

Beasley and Christofides proposed to include in the path label an extra binary resource for each node $n_k \in N$. This resource will take value 0 initially and will be set to 1 when the node is visited in the label. Feillet adapted this idea to Desrocher's label-correcting algorithm. They add another resource s_i, which counts the number of nodes visited by a path X_{pi} to compare the

label more efficiently. So in the label definition, each path $X_{p,i}$ from the origin to node j is associated with a state $R_j = (T_j^1, T_j^2, ..., T_j^L, s_j, N_j^{\div}, ..., N_j^{TZ})$, corresponding to each resource consumption, the number of visited nodes, and the visitation vector ($N_i^{\vec{k}} = 1$ if the path visits node n_k, 0 otherwise).

The new dominance rule is the following:

Let $X_{p,i}^1$ and $X_{p,i}^-$ be two distinct paths from p to j with associated labels (R_j^1, C_j^1) and $(R_j^{'}, C_j^{'})$. Then $X_{p,i}^1$ dominates $X_{p,i}^-$ or $(R_j^1, C_j^1) < (R_j^{'}, C_j^{'})$ if and only if $C_j^1 < C_j^{'}, T_j^{l'} < T_j^{l'}$ for $l = 1, ..., L$, $V_j^{k'} \le V_j^{k'}$ for $k = 1, ..., n$, and $(R_j^1, C_j^1) \ne (R_j^{'}, C_j^{'})$.

Feillet proved the claim that during the execution of the modified algorithm, we need only to consider non-dominated paths. A complete description of the algorithm is given below.

Description of the Algorithm

Denote the index set of labels on node i by I_i. For each $k \in I_i$, there is a corresponding path p_i^k from the origin p to i having state R_i^k and cost C_i^k. We refer to (R_i^k, C_i^k) as the label. We denote $\textbf{Extend}(\lambda_i, j)$ as a function that returns the label resulting from the extension of label λ_i towards node j when the extension is feasible; otherwise nothing is returned. For the ESPPRC, feasible extensions return new labels satisfying all the resource constraints, the physical constraint defined in $T_i^{\blacksquare}$, and the resource constraints for node visitation defined in $(s_i, N_j^{\div}, ..., N_i^{TZ})$. We define the procedure $EFF(\Lambda)$ to keep only non-dominated labels in the list of labels Λ.

Step 0: Initialization

Set the label associated with the partial path starting from the origin p as $L_p = (0, ..., 0)$ and the labels associated with the partial paths from the origin p to a node i as $L_i = \emptyset$ for all $i \in N \backslash \{p\}$.

Set $T_i = \emptyset$ for each $i \in N$.

Step 1: Selection of the label to be treated

If $\bigcup_{i \in N} (L_i \backslash T_i) = \emptyset$ then STOP; all efficient labels have been generated.

Else choose $i \in N$ and $k \in L_i \backslash T_i$ and select the label

$\lambda_i^k = (T_i^{1k}, T_i^{2k}, ..., T_i^{Lk}, s_i^k, N_i^{1k}, ..., N_i^{nk}, C_i^k)$ /*so that (T_i^k, C_i^k) is lexicographically minimal*/.

Step 2: Treatment of label λ_i^k

    for all $(i, j) \in A$

      if $N_i^{jk} = 0$ then

$$L_j := EFF(L_j \cup \{\textbf{Extend}(\lambda_i^k, N^j)\})$$

    enddo

    set $T_i := T_i \cup \{k\}$.

    goto Step 1

In the pricing step sub-problem, we define rail station represented by 0 as the origin p, and a copy of the rail station represented by $I+1$ as the destination q. Note I is the candidate bus stop set. Node $I+1$ is just a virtual copy of the rail station. It has exactly the same travel time and cost information to all the other nodes in set I, and its travel time and cost to the origin q are zeros. Besides, the virtual node $I+1$ has a virtual dual variable π_{I+1} with value zero. The origin, the destination, and all the other candidate bus stops form the node set N. We assume that the network is well connected, and the arc set A includes all the links from every node to every other node in set N. With the previously described solution method, we are able to find the candidate route with negative reduced cost as the pricing condition.

Computational Results

The network, which contains 10 candidate bus stops, was developed from the Martin Luther King, Jr. Station network for testing the exact solution method. Figure 2 shows the 12-stop network, in which the locations of candidate bus stops and shortest driving routes

connecting these stops over the urban street map are highlighted. In this network, demands might be waiting at or coming into each stop.
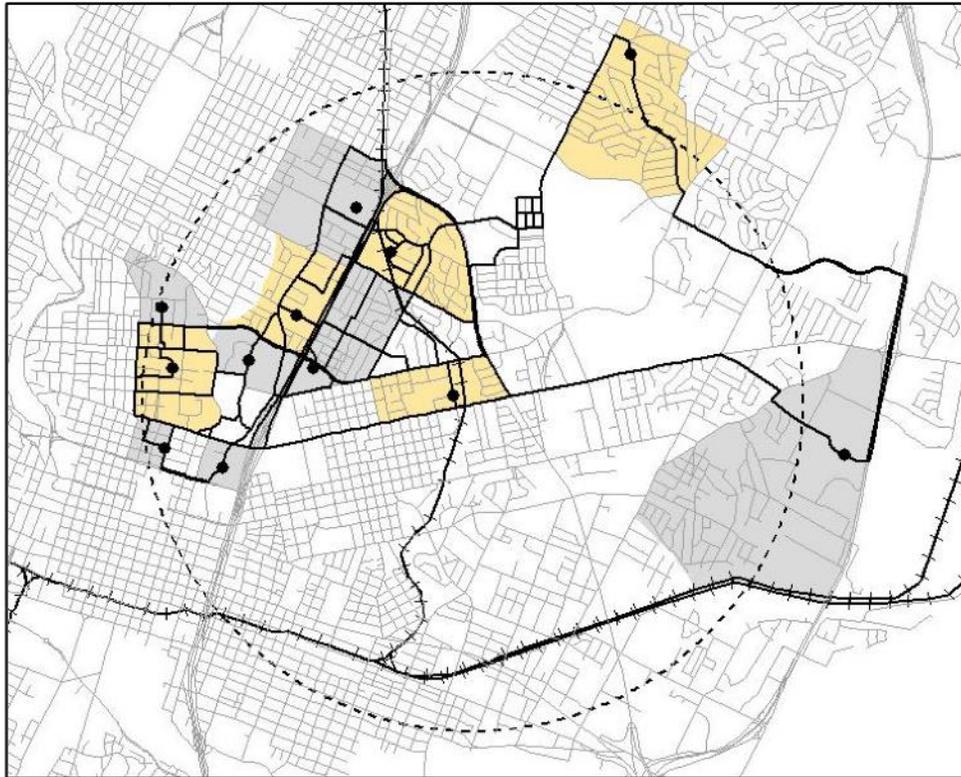

Figure 2. A Bus Circulator Network Located in Austin, Texas.

If we consider the problem at the first period, all stops will be responsive—no demand is left out of service from the last time period so no stop is labeled as a fixed stop. For this small test, we know that to cover all the stops with minimum total route costs, we need at least three vehicles. The test results start with three available vehicles and no fixed stops in the route. In Figure 3, the objective function value or total costs are shown with the penalty cost and the increment of penalty cost applied to each unserved responsive stop. We can see that as the penalty cost per stop increases, the objective value, which is the sum of route cost and penalty cost, also increases. If the operator wants to minimize unserved or delayed users, they can simply increase the penalty cost for unserved stops to drive the model to cover more stops to avoid high penalty costs. In this model, route costs are the only cost to transit operators; thus, as there is more emphasis on meeting all the demand, the route cost increases. The trend of penalty costs is different from both total costs and route costs. The curve of penalty costs can be regarded as

concave. When the increment of unit penalty cost is not significant enough to affect the optimal solution, the total penalty cost increases linearly with the unit penalty cost.
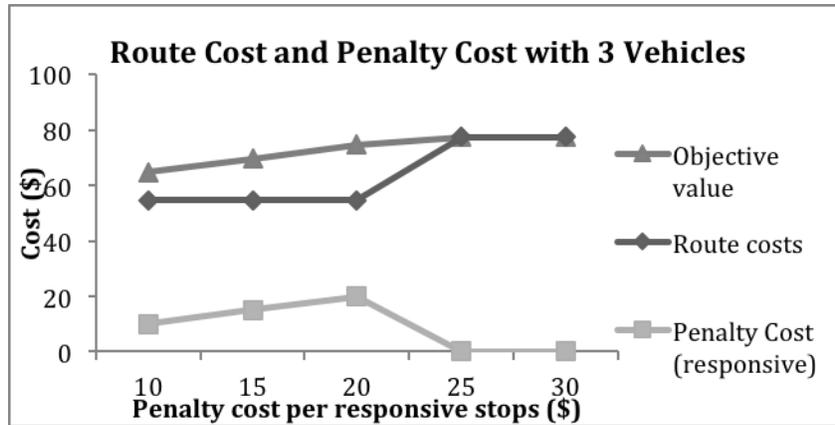

Figure 3. Route Cost and Penalty Cost with Three Vehicles.

Figure 4 shows the cost performance with only two vehicles available for the feeder service. Similar to the results in Figure 3, there are no assumed fixed stops in this test, which means that no passengers from the last period missed their train, and all travelers waiting for service are new arrivals during the current period. We can see from the results that no matter how much we increment the unit penalty cost applied to unserved demand, route costs stay constant. This is due to the limitation of the vehicle availability. Since the number of available vehicles becomes a tight constraint in this case, increments of unit penalty cost will only increase penalty cost and then total costs, but do nothing to change the route configuration.
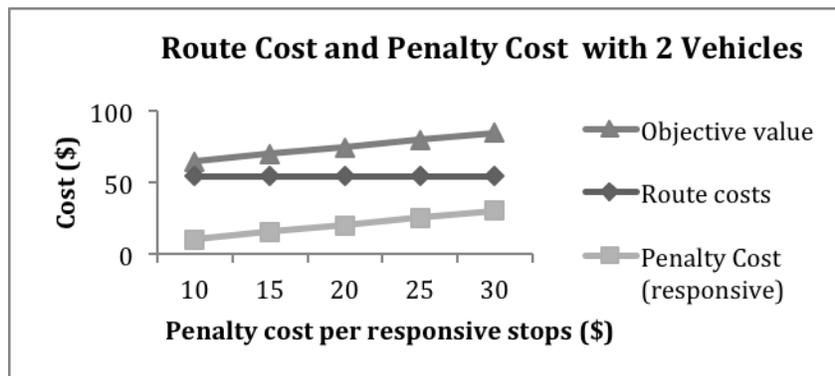

Figure 4. Route Cost and Penalty Cost with Two Vehicles.

The above two cases both assume that there are no fixed stops; however, the period after the period shown in Figure 4 would be a different case. There is demand unserved from the last period, which means passengers already missed one train. One might logically assume that the transit operator would want to help passengers avoid missing a second train. To simulate this desire, we set a much higher penalty value. Assume that this is the intermediate time period and demand at some stops missed the train from the last time period. The cost performance with two available vehicles is shown in Figure 5.
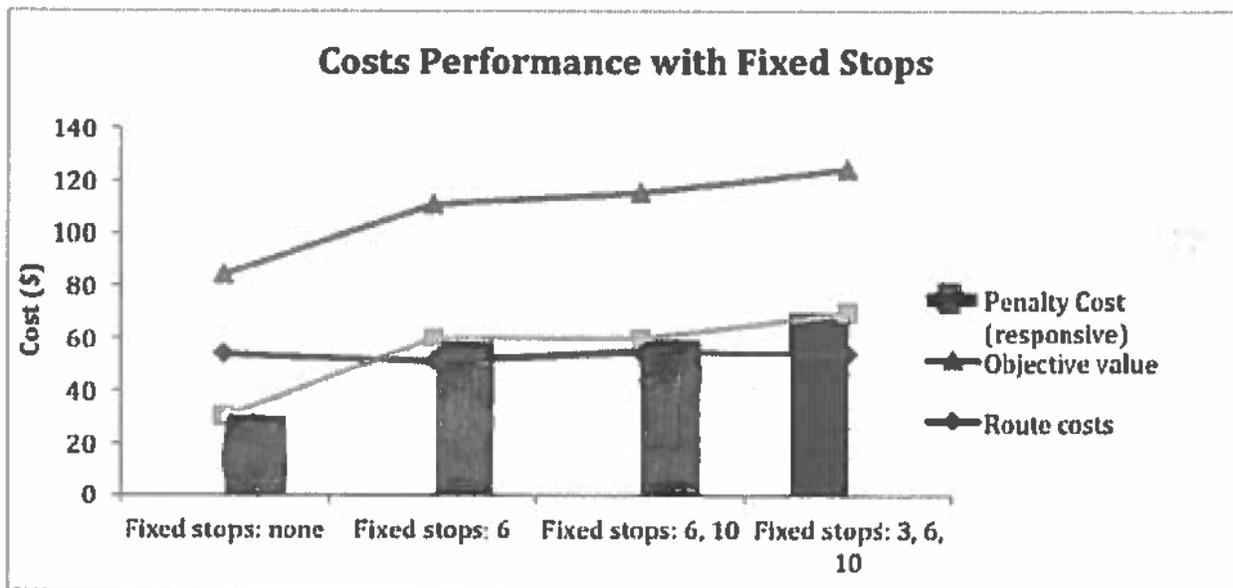

Figure 5. Cost Performance with Fixed Stops and Two Vehicles.

The total costs increase when more stops are considered fixed stops. From the modeling perspective, more fixed stops mean more constraints in the model and a smaller feasible region. In a minimization problem, the objective value increases as the feasible region shrinks. From the practical point of view, to be able to serve a larger demand, transit operators have to pay more, either as route costs or penalty costs. As we can see, when more stops are considered fixed stops, the total penalty costs increase. Breaking these penalty costs down, the model first avoids penalties applied to fixed stops shown as solid black bards in Figure 5. The penalty cost is only applied to responsive stops shown as grey bars when there are zero to two fixed stops. When more fixed stops come into the model, the resource limit becomes the constraint and it is not possible to serve all the fixed stops.

6. CONCLUSIONS

This report formulates the circulator service network design problem into a nonlinear mixed integer programming problem. The model formulation is rooted from minimization objectives for both transit operators' operation costs and transit users' travel costs. As a special case of VRP, this integer programming problem presents very challenging computational complexity.

To find an efficient procedure that could be implemented for real-time operations, we proposed and tested a column-generation-based solution method. The computational experiments suggest that both the solution quality and computational efficiency would meet the requirement for a real-time operation.

In this research, we only modeled and solved a collective circulator service network design problem, which describes the bus circulator operations in the origin end of workplace-to-home commuting trips. The fleet of buses is used for picking up passengers from selected bus stops close to workplaces and dropping them off at the rail station during the afternoon commuting period. This study treated each bus circulating tour as an independent optimization problem without considering the cooperation and coordination between different buses in the fleet. From the efficiency perspective, it is much desirable to formulate a larger circulator service network design problem, which operates a fleet of buses as a whole instead of operating each individual one separately. Such a fleet-based circulator system can be readily implemented by using the current information and communication technologies.

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