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16. Abstract

This study investigates the performance measures for multi-vehicle mobility allowance shuttle transit (MAST) system. Particularly, researchers were primarily concerned with two measures, waiting time and ride time, to evaluate the performance and help design of m-MAST systems. The MAST system is an innovative concept that allows transit vehicles to deviate from a fixed route consisting of a few mandatory checkpoints to serve on-demand customers within a predetermined service area, and thus can be both affordable and convenient enough to attract the general public. For the MAST system, the fixed route can be either a loop or a line between two terminals. The checkpoints are usually located at major transfer stops or high demand zones and are relatively far from each other. Researchers developed analytical results for the waiting time probability distribution and its expected value as well as the expected ride time for different types of customers in terms of the system parameters for both 1-MAST system and multi-vehicle MAST (m-MAST). Researchers also discussed the assumptions behind the estimation. Based on the analytical results, researchers provided the inherent constraints between these parameters and demand.

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Performance Measures for Multi-vehicle Mobility Allowance Shuttle Transit (MAST) System

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EXECUTIVE SUMMARY

Transit service is an inalienable and important part of transportation systems. Nowadays, transit agencies are facing challenges due to urban sprawl. Urban sprawl is a phenomenon that as the population is exploding; the population density of cities is actually dropping. As urban sprawl occurs, cities begin losing their traditional centralized functions, and people spread into the suburbs to seek more space and a lower cost of living. As a result, the urban areas are becoming more and more car-dependent, and it appears that new highway construction may not satisfy the demand. There are a few archetypes of urban sprawl such as the Los Angeles metro area, Houston, and Atlanta, among others. In these areas, traditional transit services are struggling because they are not able to provide a satisfying service in such a low-density context. To face the challenges of urban sprawl, urban planning agencies are proposing policies to regulate unlimited urban sprawl. On the other hand, transit agencies are actively seeking innovative public transit solutions that are attractive enough to serve people in the low-density urban areas.

In terms of their flexibility, public transit services can be divided into two broad categories: fixed-route transit and a more flexible option called the demand-responsive transit. The fixed-route transit systems include the common buses, subway systems, and school shuttles, etc. They are considered to be cost-efficient because of their ride-sharing attribute and sufficiently large loading capacity. Fixed-route transit works well in traditional cities that have a high density, but it is considered to be inconvenient since the fixed stops and schedule cannot meet individual passengers' needs. This lack of flexibility is the most significant constraint of fixed-route transit and prevents it from being effective when used in the urban sprawl context. The demand-responsive transit (DRT) systems are much more flexible due to their door-to-door pickup and drop-off services. DRT has been operated in numerous cities and works as an effective type of flexible transit service especially within low-density residential areas.

Since both fixed-route transit and demand-responsive transit have their advantages and disadvantages, a possible improvement is to be eclectic. The idea is combining the cost-efficient operability of traditional fixed-route transit with the flexibility of demand-responsive systems. This new concept is called the mobility allowance shuttle transit (MAST). It is a hybrid transit system in which vehicles are allowed to deviate from a fixed route to serve a flexible demand.

This study investigates the performance measures for multi-vehicle MAST system, particularly with waiting time and ride time, to evaluate the performance and help with the design of a MAST system. MAST is an innovative concept that allows transit vehicles to deviate from a fixed route consisting of a few mandatory checkpoints to serve on-demand customers within a predetermined service area, and thus can be both affordable and convenient enough to attract the general public. For the MAST system, the fixed route can be either a loop or a line between two terminals. The checkpoints are usually located at major transfer stops or high demand zones and are relatively far from each other. This report develops analytical results for the waiting time probability distribution and its expected value as well as the expected ride time for different types of customers in terms of the system parameters for both 1-MAST system and multi-vehicle MAST (m-MAST). Researchers discuss the assumptions behind the estimation. Based on the analytical results, researchers also provide the inherent constraints between these parameters and demand.

CHAPTER 1 INTRODUCTION

Public transit systems are attracting more attentions of transportation researchers due to urban sprawl and the heavy traffic congestion in urban areas. Generally speaking, transit systems are more cost-efficient than personal vehicles. Thus, with the economic crisis and the increase of fuel prices, transit systems are a better choice for the public. However, the financial support for the whole transportation system has decreased, so it is critical to find a more cost-efficient transit type.

Public transit services are divided into two broad categories: fixed-route transit (FRT) and demand responsive transit (DRT). The FRT systems are thought to be cost-efficient because of their ride-sharing attribute and sufficient loading capacity, but they are considered by the general public to be inconvenient. This inherent lack of flexibility is the most significant constraint of fixed-route transit. The DRT systems are much more flexible since they offer door-to-door pickup and drop-off services. They operate in numerous cities and work as an effective type of flexible transit service, especially within low-density residential areas such as examples in Denver (CO), Raleigh (NC), Akron (OH), Tacoma (WA), Sarasota (FL), Portland (OR), and Winnipeg (Canada) (*1*). However, the associated high cost prevents the DRT from being deployed as a general transit service. As a result they are largely limited to specialized operations such as shuttle service, cab, and Dial-a-Ride services, which are mandated under the Americans with Disabilities Act. Thus, transit agencies are faced with increasing demand for improved and extended DRT service. Thus, a combination of these two types of transit systems is needed to provide a relatively cost-efficient and flexible transit type.

The mobility allowance shuttle transit (MAST) is an innovative concept that combines the cost-efficient operability of traditional FRT with the flexibility of DRT systems. It allows transit vehicles to deviate from a fixed route consisting of a few mandatory checkpoints to serve on-demand customers within a predetermined service area, and thus can be both affordable and convenient enough to attract the general public. For the MAST system, the fixed route can be either a loop or a line between two terminals. The checkpoints are usually located at major transfer stops or high demand zones and are relatively far from each other. A hard constraint of the MAST system is the scheduled departure time from checkpoints. Such a service already exists in Los Angeles County with MTA Line 646 serving as a nighttime bus line transporting mostly night-shift employees of local firms. They developed the insertion heuristic scheduling of a single vehicle MAST system (*2*), but an advanced system can be performed with multiple vehicles, and the scheduling problem became more complicated.

CHAPTER 2 LITERATURE REVIEW

The design and operations of the MAST system have attracted considerable attention in recent years. Quadrifoglio et al. (*3*) evaluated the performance of MAST systems in terms of serving capability and longitudinal velocity. Their results indicate that some basic parameters are helpful in designing the MAST system such as slack time and headway. Quadrifoglio et al. later developed an insertion heuristic scheduling algorithm to address a large amount of demand dynamically (*2*). Quadrifoglio and Dessouky (*4*) carried out a set of simulations to show the sensitivity analysis for the performance of the insertion heuristic algorithm and the capability of the system over different shapes of service area. In 2008, Zhao and Dessouky (*5*) studied the optimal service capacity for the MAST system. Although these studies investigated the design and operations of the MAST system from various aspects, they are all for the single-vehicle MAST system. Not until very recently, research on multiple-vehicle MAST system can be found in the literature. Lu et al. (*6*) developed a mixed-integer program for multiple-vehicle MAST. Insertion-based heuristics were developed to schedule the large-scale multiple-vehicle MAST system efficiently (*7*).

Since the MAST system is a special case of the pickup and delivery problem (PDP) (see [*8*] for complete review), it can be modeled as a mixed integer program (MIP). The PDP has been extensively studied, and many of the exact algorithms are based on integer programming techniques. Sexton and Bodin (*9*) reported a formulation and an exact algorithm using Bender's decomposition. Cordeau introduced an MIP formulation of the multi-vehicle Dial-a-Ride Problem (DARP) (*10*), which is a variant of PDP. He proposed a branch-and-cut algorithm using new valid inequalities for DARP. Cordeau and Laporte gave a comprehensive review on PDP, in which different mathematical formulations and solution approaches were examined and compared (*11*). Lu and Dessouky (*12*) formulated the multi-vehicle PDP as an MIP and developed an exact branch-and-cut algorithm using new valid inequalities to optimally solve multi-vehicle PDP of up to 5 vehicles and 17 customers without clusters and 5 vehicles and 25 customers with clusters within a reasonable time. Cortes et al. proposed an MIP formulation for the PDP with transfers (*13*). Very recently, Ropke and Cordeau (*14*) combined the techniques of row generation and column generation and proposed a branch-cut-and-price algorithm to solve PDP with time windows. In their algorithm, the lower bounds are computed by solving the linear relaxation of a set partitioning problem through column generation, and the pricing subproblems are the shortest path problems. Berbeglia et al. reviewed the most recent literature on dynamic PDPs and provided a general framework for dynamic one-to-one PDPs (*15*). Quadrifoglio et al. proposed an MIP formulation for the static scheduling problem of a single-vehicle MAST system and solved the problem by strengthening the formulation with logic cuts (*16*). Other exact algorithms include dynamic programming. Psaraftis used dynamic programming to solve the single-vehicle DARP (*17*) and its variant with time windows (*18*). Both algorithms have a time complexity of $O(N^2 3^{\mathrm{N}})$ (N for customers) and can solve an instance of N up to 20 in a meaningful time. Very recently, Fortini et al. (*19*) proposed a new heuristic for the traveling salesman problem (TSP) based on computing compatible tours instead of TSP tours. They proved that the best compatible tour has a worst-case cost ratio of 5/3 to that of the optimal TSP tour. A branch-and-cut algorithm was developed to compute the best compatible tour, and Teodorovic and Radivojevic developed a fuzzy logic approach for the DAR problem (*20*).

Since the optimization problem of PDP is known to be strongly NP-hard (*21*) which means an efficient accurate algorithm unlikely exists, researchers have been studying heuristic approaches to solve PDP with large instances in a reasonable (polynomial) time, while not compromising the quality of the solution too much. Along these approaches, insertion heuristics are the most popular because they can quickly provide meaningfully good results and are capable of handling problems with large instances. Another reason that justifies insertion heuristics in practice is that they can be easily implemented in dynamic environments (*22*). Some other efforts in insertion heuristics include research by Lu and Dessouky (*23*). A major disadvantage of the insertion heuristics is that usually it is hard to bound its performance. Another disadvantage is its myopic and greedy approach for current optimum at each time step without having an overview of all the requests. The insertion heuristic controlled by "usable slack time" resolved this issue efficiently (*2*). To evaluate the performance of the proposed heuristics, worst-case analysis can be found for PDP and its fundamental or related problems such as TSP and vehicle routing problem (VRP). Savelsbergh and Sol (*8*) gave a complete review on the pickup and delivery problem and discussed several variants of the problem in terms of different optimization objectives, time-constraints, and fleet sizes. Both exact algorithms based on mathematical modeling and heuristics were reviewed. Christofides (*24*) proposed a heuristic of ratio 3/2 for metric-TSP based on constructing minimum spanning tree and Euler tour. Rosenkrantz et al. (*25*) analyzed the approximation ratio of several heuristics, including the cheapest insertion heuristic for TSP. Archetti et al. (*26*) studied the re-optimization version of TSP, which arises when a new node is added to an optimal solution or when a node is removed. They proved that although the cheapest insertion heuristic has a tight worst-case ratio of 2 (*25*), the ratio decreases to 3/2 when applied to the re-optimization TSP problem. So far the best result on TSP is Arora's polynomial time approximation scheme for Euclidean TSP (*27*).

Categorized by the applicable tools to evaluate the performance of demand-responsive services, the analytic analysis and simulation models are two major methods. Although analytical model that captures all the complexities of DRTs is extremely difficult to develop, analytical models are still found to be superior to simulation models because of their applicability and efficiency in parametric analysis. Usually by applying a reasonable amount of simplicity, analytical models can still maintain a high level of representation.

The approximate analytical model of a demand-responsive transportation system was first developed by Daganzo (*28*). This study focused on the real-time algorithms for dial-a-ride systems. Fu (*29*) provided an analytic model to predict the fleet size and quality-of-service measurements. Diana et al. (*30*) proposed analytic equations to calculate the fleet size for a square service area. Li and Quadrifoglio (*31*) developed an analytic model to determine the optimal service zone for feeder transit service. Lu et al. (*6*) developed an analytical model for fleet-sizing for a two-vehicle MAST system.

CHAPTER 3 SYSTEM DEFINITION AND PROBLEM DESCRIPTION

The MAST system in this study consists of a series of rectangle service segments oriented in a horizontal direction and $c+1$ mandatory checkpoints located at high-density demand zones. All checkpoints are assumed to be evenly spaced and each service segment, called *a basic unit service zone*, is of width w and length $L/c > w$ with two checkpoints $(0, w/2)$ and $(L/c, w/2)$ at the side.[1] Without loss of generality, the checkpoints are numbered starting with 1 from the left to the right. Therefore, the complete MAST system covers a rectangle area of width w and length L with the checkpoint 1 on the left side and checkpoint $c+1$ on the right.

In addition to the fixed checkpoints where the customers can get on or off the vehicles, the customer can request the place to within the basic unit service zone to be picked up or dropped off. The customer demand is measured in terms of a series of pick-up and drop-off stops, each uniformly scattered in the restricted study area with the density 2ρ per unit square area. Therefore, all the stop points together follow the spatial Poisson distribution (*32*, *33*). Also, think of the points of stops within the study area as a subset of the spatial Poisson points distributed in the whole space. At any non-checkpoint stop point, assume that there is a constant service time of s_0. Moreover, assume that vehicles have infinite loading capacity and travel at the constant speed v (*2*). When vehicles approach to any stop within the service zone, they follow rectilinear paths. That is to say that the L_1 distance is used to measure the distance between any two stops.

The customers are generally classified as three types: regular (denoted by PD), hybrid (denoted by NPD or PND), and random (denoted by $NPND$). By regular it means that the customers with pick-up and drop-off at checkpoints. The random customers are those whose pick-up and drop-off points within basic unit service zones, while the hybrid customers have both checkpoint stops and random stops. The portions of $PD, NPD, PND, and\ NPND$ requests are $\alpha, \beta, \gamma,$ and η, respectively, with the constraint $\alpha + \beta + \gamma + \eta = 1$.

Similar to the assumptions in (*3*), vehicles are assumed to travel either from the left checkpoint to the right checkpoint following the left-right direction, or the vice versa. The left-right (right-left) vehicles are responsible for the customers whose drop-off stops locate to their right (left). Although such assumption services the purpose to simplify the following analysis, it is indeed reasonable in the practice.

Suppose that there are m (an even number) times per vehicle during the day operating on the left-right and right-left MAST routes, respectively. Although a vehicle can operate in multiple times, simply assume $m/2$ vehicles on each direction, i.e., each vehicle completes a full service cycle (two times full MAST route). The density 2ρ represents the total demand for each vehicle

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[1] The point (0,0) refers to the left-bottom point of a basic unit service zone.

in this service cycle. Therefore, there are ρ demand stops per unit square area for each left-right or right-left vehicle. The total number of stops for a vehicle in one direction in the whole MAST system is:

$$n = \rho \cdot L w. \tag{3.1}$$

Two vehicles travel simultaneously in the left-right and right-left directions, i.e., two vehicles at the same time starting from the 1st to the right and from the $(c+1)$th to the left, respectively. After h unit time (defined as headway), the other two vehicles operate in the same way.

Vehicles at each checkpoint have a fixed departure time. The time between fixed scheduled departures at two consecutive checkpoints is denoted as t_f and assumed the same for both left-right and right-left directions. It implies that each vehicle must complete the travel in a basic unit service zone within t_f unit time. Accordingly, the travel of vehicles from the 1st checkpoint to the $(c+1)$th will take up $c \cdot t_f$ unit time.

The above requirement for the fixed departure time implies that the vehicles should satisfy all demands in a basic unit service zone within time t. Therefore, there is a constraint between the random demand density and time t_f if it is required that the vehicle should satisfy all demand in each run of operation. This constraint is discussed later.

There are many variants of policy for the vehicle to make up their schedule. Without explicit statement, researchers were primarily concerned with the non-backtracking policy, which allows the vehicle to move in a straight forward progression through the service zone. Based on the previous assumption that all customer origins should be to the right (left) of the destinations for a left-right (right-left) vehicle, such policy permits all customers to be served. Obviously, this policy is not optimal. However, under this policy the system performance is not trivial to analyze. Researchers adopted this policy as a reasonable starting point to analyze the MAST system.

In the following, researchers investigated the performance measures for both 1-MAST system and multi-vehicle MAST (m-MAST) system. Researchers were primarily concerned with two performance measures, waiting time and ride time, as well as the associated assumptions.

3.1 Notations

The system parameters and notations are as follows.

- s_0 Service time at a non-checkpoint stop;

- L Length of MAST line, i.e., distance from the first to the last checkpoint (miles);
- w Allowed lateral deviation (miles);
- v Average vehicle travel speed (miles/unit time);
- l Time of traveling L_1 distance from (x, y) to $(0, w/2)$, equal to $\frac{x + |y - \frac{w}{2}|}{v}$;
- t_f Time interval between fixed scheduled departure times of two consecutive checkpoints (unit time);
- h Time headway between two consecutive buses at the starting checkpoint (hour);
- ρ Demand density per unit area for left-right or right-left direction;
- PD (Regular) pick-up and drop-off at checkpoints;
- PND (Hybrid) pick-up at a checkpoint and drop-off at a random point;
- NPD (Hybrid) pick-up at a random point and drop-off at a checkpoint;
- NPND (Random) pick-up and drop-off at random points;
- $\alpha, \beta, \gamma, \eta$ Portions of PD, PND, NPD, NPND requests, respectively, and $\alpha + \beta + \gamma + \eta = 1$;
- T_{rd}^{PD} Expected ride time of a PD passenger (unit time);
- W^{PD} Expected waiting time of a PD passenger (unit time);

CHAPTER 4 PERFORMANCE MEASURES FOR 1-MAST SYSTEMS

This section serves to characterize the performance of the MAST systems in terms of waiting time and ride time. Because of the random nature of demand scattered within the service zone, the requested customer could not know in advance the exact number of stops that the vehicle has before picking up him/her. Similarly, when a customer gets on the vehicle, he/she is uncertain about the exact ride time to the next checkpoint due to the random stops in between. In order to tackle these problems, researchers have to answer the question: what is the best information to provide about the waiting time and ride time for an arbitrary customer? At this point, it is natural to present the waiting and ride time in terms of their expected value and/or distribution. Therefore, researchers have to describe not only how stop points scatter in the service zones, but the time when each demand stop appears. Researchers employ some probability techniques to investigate these problems for 1-MAST system in this section. Then researchers generalize the results to the m-MAST system in the next section.

4.1 Waiting Time at a Given Pick-up Location

Focus on a basic unit service zone and consider the left-right vehicle in this section.
Assume first that the demand stops within a basic unit service zone are known right before the vehicle leaves the left-side checkpoint. By the non-backtracking policy, the vehicle serves the demand sequentially based on their longitudinal positions. Now suppose that a pick-up request is given at the location (x, y) within the service zone.[2] To estimate its waiting time, denoted by $W(x, y)$, one can calculate the length of the path that the bus reaches this stop and then divide it by *v*. Since such path depends on other demand stops, researchers need to know the other stops distribution. Although the total demand distribution is given in the assumption, the situation here is different because the distribution of other stops can be thought of as a conditional distribution conditioning on the event that there is point at (x, y). Such probability is known as Palm probability (*33*). For a general distribution, the Palm probability does not necessarily have the same form. But for the spatial Poisson point process, the Palm probability distribution is still the Poisson with the intensity ρ as if the given point located at the origin. Hence, consider the other stops as Poisson distributed points even given a point at (x, y). This result considerably reduces the complexity of the problem.

As other stops form a Poisson point process, the probability that no point appears in the rectangle area with length Δx and width y is $e^{-\rho \cdot w\Delta x}$. Hence, the longitudinal positions of these stop points are Poisson process with rate ρw (*3*). It is more convenient to track the path back from the stop (x, y) to the left-side checkpoint to estimate its length. One can seek the last random stop point position before the vehicle reaches (x, y). There are two cases that would occur: no such stop exits or such stop lies at the (s, t) with $0 \leq s \leq x$. For the first case, the length of the path is

[2] Researchers also assume the left-side check point position is (0, w/2).

simply the L_1 distance between (x, y) and the checkpoint $(0, w/2)$. For the second case, the length can be decomposed into the length from (x, y) to (s, t) and that from (s, t) to the checkpoint. Because of the homogeneity of random stops, the latter would be viewed as the case given a point at (s, t) and calculated in a similar way to the path length estimation given the point (x, y).

Let the probability of waiting time $W(x, y)$ larger than u be $P(W(x, y) > u)$, and let $l = \frac{x + |y - \frac{w}{2}|}{v}$. $P(W(x, y) > u) = 1$ if $u < l$, because l is the least length that the vehicle travels. Based on the previous analysis, $P(W(x, y) = l) = 1 - e^{-\rho \cdot xw}$. Note that although there could be the cases that some random stop points lie on the path with length l, such event's probability is zero. Hence, the only case with a positive probability for the event $W(x, y) = l$ is the first case discussed in the previous paragraph. Based on the previous analysis, the following equation for $u > l$:

$$P(W(x, y) > u) = \int_0^u \int_0^x \left(W(x - s, t) > u - s_0 - \frac{x - s + |y - t|}{v} \right) \cdot \rho\, e^{-\rho\, w(x - s)} ds dt. \quad (4.1)$$

Equation (4.1) is actually analogue to the Kolmogorov backward equation (*34*). There is also an analogue to the forward equation but not elaborated here.

Estimate the expectation of waiting time $W(x, y)$, denoted by $\bar{w}(x, y)$, as an example of the application of the Equation (4.1). One can integrate u and notice that $\bar{w}(x, y) = \int \mathrm{P(W(x,y)} > u\,)\mathrm{du}$, then:

$$\bar{w}(x, y) = \int_0^w \int_0^x \left(\bar{w}(x - s, t) + s_0 + \frac{x - s + |y - t|}{v} \right) \cdot \rho\, e^{-\rho\, w(x - s)} ds dt + l \cdot (1 - e^{-\rho\, wx}). \quad (4.2)$$

The explicit expression of $\bar{w}(x, y)$ will be tedious. Nevertheless, the following approximate formula when x is relatively large is:

$$\bar{w}(x, y) = \frac{w(\rho\, w\, x - 1)}{3} + x + \frac{3w}{4} - \frac{1}{w}(yw - y^2). \quad (4.3)$$

It is important to obtain the probability $P(W(x, y) > u)$. Since the fixed scheduled departure time at checkpoints requires each vehicle to travel the entire unit service zone within t_f , this probability can be used to evaluate some risk measure in a particular area that the vehicle would violate such requirement.

4.2 Waiting Time for the 1-MAST System

When one seeks to measure the average waiting time for the whole 1-MAST system, i.e., a series of basic unit service zones served by one vehicle, the problem involves some arbitrary solutions. This is because the waiting time for the 1-MAST system depends strongly on the assumption of the arrivals of the customers. Researchers investigate a few reasonable assumptions about the arrivals of demand and obtain associated results.

One of the assumptions is to assume all the demand stops are now right before the vehicle leaves and all pick-up customers wait at the corresponding stops. The customers within the area between checkpoints i and $i+1$ will wait for a time period during which the vehicle leaves the checkpoint i and then travel to his place. The vehicle reaches point i will take $(i-1)t_f$ unit time. The average time for the vehicle from checkpoint i to the customer stop, $\bar{t}$, can be estimated as a half of the time of vehicle traveling within the basic unit zone. By the results in (*3*), the average waiting time for the 1-MAST system is:

$$\bar{W}_1 = \bar{t} + \frac{(c-1)\cdot t_f}{\rho w L} = \frac{1}{2}\left(\frac{L}{v} + \frac{w}{v}\left[\frac{1}{2} + \frac{\rho w L - 1}{3}\right] + \rho w L s_0\right) + \frac{(c-1)\cdot t_f}{\rho w L}. \tag{4.4}$$

The above assumption may not be exactly accurate in the real world as the customer far away from the checkpoint 1 may not show up that early. A more realistic assumption about arrivals is that once the vehicle stops at a picks-up point, the next pick-up demand appears and the vehicle travels to that stop. However, there might be some drop-off points between these two pick-up stops. If the event of a stop being a drop-off point has probability p, then the average number of stops between two pick-up points is $p(1-p)^{-1}$. Hence, the average customer waiting time under this assumption is approximately:

$$\bar{W}_2 = \frac{p}{1-p}s_0 + \frac{1}{(1-p)\rho w} + \frac{w}{3(1-p)v}. \tag{4.5}$$

The general assumption about arrival time may be viewed as a time stamp associated with each stop point. Therefore, it would be convenient to think of the demand stops as a marked Poisson point process. The detailed analysis would be more complicated and not discussed here.

4.3 Ride Time

The ride time refers to the time that a customer gets on the vehicle until he/she arrives at the destination. If one seeks the solution of ride time within a basic unit service zone, the approach will be similar to the problem of waiting time. If one examines the whole service area consisting of several basic unit service zones, the ride for a customer may cover consecutive zones. Focus on the type of PD customers and let $E(T_{rd}^{PD})$ denote the average ride time of all the possible pairs of pick-up and drop-off checkpoints.

Assuming the vehicle is progressing from left to right as depicted in Figure 1, the customer who got the ride at the checkpoint i should have $c+1-i$ choices of destination checkpoints. When this customer's destination is $i+1$ (i.e., within a unit zone), the ride time (denoted by t_0^{PD}) is the length that vehicle travels divided by v plus some service time.

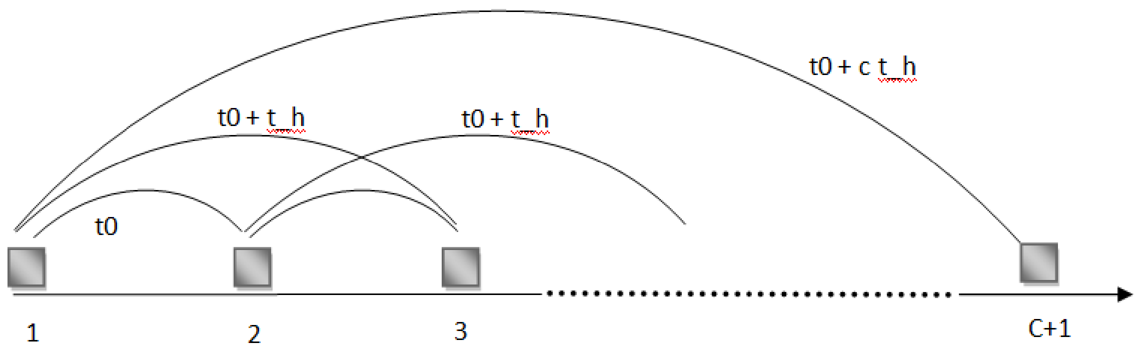

Figure 1: Illustration for derivation of $E(T_{rd}^{PD})$

Due to the fixed departure schedule of the vehicle at checkpoints, the ride time from i to $i+j$ is equal to $t_0^{PD}+(j-1)t_h$ where $j>1$.

This argument is similar to that in (*3*). If one assumes the customer's destination choices are of equal chance, then the following expression for the average ride time is:

$$E(T_{rd}^{PD})=\frac{\sum_{j=1}^{c}(c+1-j)[t_0^{PD}+(j-1)t_h]}{\sum_{j=1}^{c}(c+1-j)}=t_0^{PD}+\frac{c-1}{3}t=\left(\frac{L}{v}+\frac{w}{v}\left[\frac{1}{2}+\frac{\rho wL-1}{3}\right]+\rho wLs_0\right)+\frac{c-1}{3}t\,. \quad (4.6)$$

CHAPTER 5 ANALYTICAL MODEL FOR M-MAST PROBLEM

The section covers the performance measures for the m-MAST system. As described previously, there are *m* times per vehicle operating on each direction and researchers focus on the left-right direction operation. Although many performance can be deduced by using the previous results for the 1-MAST system, there are some unique features and system constraints that distinguish the m-MAST from the 1-MAST system.

There is a system constraint on the number of vehicles and the headway in an m-MAST system. When the first left-right vehicle reaches the $c+1$ checkpoint, it should not have more than $m/2$ vehicles operating in the left-right direction as there are $m/2$ vehicles at the 1st checkpoint and each vehicle only operates in a full service cycle. Since the headway of two vehicles is h :

$$h \cdot m/2 \geq c \cdot t_h. \tag{5.1}$$

If $2c/m$ is about 1, one can see from the equation (5.1) that h can be the same with t_h . In this case, it implies that there is at least one vehicle serving any basic unit zone at any time on average. The following will discuss performance measures and other system constraints.

5.1 Waiting Time

In the m-MAST system, a customer at random stops close to a left-side checkpoint in a basic unit service zone would wait for no time as a vehicle just passed his/her place or for about a headway time h to be picked up by the coming next vehicle. However, when the customer is away from the checkpoint, the waiting time might not be exactly h . Such description rules out some assumptions for the 1-MAST system as in the m-MAST system; the reasonable customer arrivals pattern might be random in time, independent from the vehicle schedule.

Suppose the customer arrives at the place (x, y) within the basic unit zone. If a vehicle just passes around the place $(x, w/2)$, it will not come back to pick the customer due to the non-backtracking policy. Although the travel path for the next vehicle before passing the horizontal position x is uncertain, the vehicle passes the vertical line with x at the horizontal position in about h unit time on average. It can be easily seen from the Figure 2. Since the service zone is symmetric about the line from $(0, w/2)$ to $(L/(2c), w/2)$, every random path to the line with horizontal position x has its own symmetric path about this line. Considering the arrivals will be uniformly distributed on the segment from $(x, 0)$ to (x, w), the next arrival vehicle should be about *h* unit time after the first vehicle just passes a demand pick-up stop. Although such argument is heuristic, the result can be checked in a rigor approach.

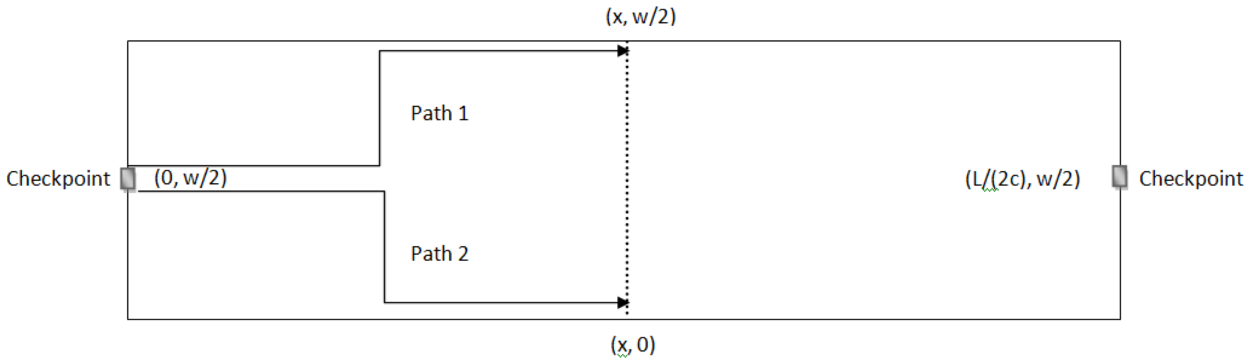

Figure 2: Path 1 and Path 2 are symmetric about the line $(0, w/2)$ to $(L/(2c), w/2)$.

Note that a customer also uniformly arrives at any time during h. Therefore, the expected waiting time for a customer is:

$$\mathrm{E}(W^{PD}) = \mathrm{E}(W^{NPD}) = \mathrm{E}(W^{PND}) = \mathrm{E}(W^{NPND}) = h/2. \tag{5.2}$$

The total average waiting time for both directions and all customers is:

$$\mathrm{E}(W_{total}) = \rho \cdot cLw \cdot h. \tag{5.3}$$

The above analysis rules out the possibility that a later starting vehicle takes over the earlier one due to the random paths. If this case occurs, the analysis approach will be more complicated than the above. The future study will discuss this case.

5.2 Ride Time

This section only covers the total average ride time for each customer type. If E_T^{PD} is the expected ride time of a PD customer within a basic unit service zone, the total ride distance is the sum of L and the lateral deviation. Any NPND type customer within a basic unit service zone has an expected lateral distance $w/3$ between pick-up and drop-off points, while any NP or ND type customer has an expected lateral distance $w/4$. Hence, the expectation of ride time of a PD customer is the sum of travel time and the total service time:

$$E_T^{PD} = \frac{L}{v} + \frac{1}{v}\left(2 \cdot \frac{w}{4} + \frac{w}{3}(\rho L w(1-\alpha) - 1)\right) + s_0 \cdot \rho L w(1-\alpha). \tag{5.4}$$

Hence, the total ride time of PD customers for m vehicles is:

$$\mathrm{E}(T_{rt}^{PD}) = m \cdot \rho L w \alpha \cdot (E_T^{PD} + (c-1)t/3). \tag{5.5}$$

Similarly:

$$\mathrm{E}(T_{rt}^{PND}) = m \cdot \rho L w \beta \cdot \left(\frac{E_T^{PD}}{2} + \frac{(c-1)}{3} t \right), \tag{5.6}$$

and:

$$\mathrm{E}(T_{rt}^{NPND}) = m \cdot \rho L w \eta \cdot \left(\frac{E_T^{PD}}{3c} + \frac{(c+1)(c-1)}{3c} t \right). \tag{5.7}$$

5.3 Relationship between t_h and Ride Time

As described earlier, there should be some regularity constraints for the system parameters. The inequality (5.1) reflects a basic relationship between h and t_h. There is a more important relationship about the fixed vehicle schedule and the demand density. Intuitively, the increasing the demand density should lead to more frequent vehicle dispatches. Such relationship may be revealed by examining the ride time and system parameter t_h. If it is required that the vehicle should satisfy all demand in each run of operation in the basic service unit zone, the requirement on t_h will be met for any vehicle schedule. Therefore, the expected ride time E_T^{PD} and time t must satisfy:

$$E_T^{PD} \le t_h. \tag{5.8}$$

CHAPTER 6 CONCLUSION

In this study, researchers were primarily concerned with two measures, waiting time and ride time, to evaluate the performance and help design of MAST system. Researchers developed analytical results for the waiting time probability distribution and its expected value as well as the expected ride time for different types of customers in terms of the system parameters for both 1-MAST system and m-MAST. Researchers also discussed the assumptions behind the estimation. Based on the analytical results, researchers provided the inherent constraints between these parameters and demand. More realistic assumptions and associated results will be studied in the future.

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