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A FINITE ELEMENT FORMULATION WITH COMBINED LOADINGS FOR SHEAR DOMINANT RC STRUCTURES

by

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16. Abstract Inelastic failure of reinforced concrete (RC) structures under seismic loadings can be due either to loss of flexural, shear or bond capacity. Specifically, the effect of combined loadings can lead to a complex failure mechanism that plays a vital role in concrete mechanics. This report describes the formulation of an inelastic nonlinear beam element with axial, bending, and shear force interaction. The element considers shear deformation and is based on the section discretization into fibers with hysteretic material models for the constituent materials. The steel material constitutive law follows the Menegotto-Pinto model. The concrete material model uses an orthotropic constitutive relation in which the directions of orthotropy are the principal directions of total strain. These directions will change during the loading history, in accordance with the well-known rotating crack model. The concrete model accounts for the biaxial state of stress in the directions of orthotropy, in addition to degradation under reversed cyclic loading. Shear deformations are coupled with bending effects. Transverse strains are internal variables determined by imposing equilibrium at each fiber between the concrete and the vertical steel stirrups. Element forces are obtained by performing equilibrium based numerical integration on section axial, flexural, and shear behavior along the length of the element. In order to establish the validity of the proposed model correlation studies were conducted between analytical results and experimental tests of columns tested under cyclic loading. A structural analysis of a shear sensitive bridge pier subjected to ground input motion is also presented.			
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ABSTRACT:

Inelastic failure of reinforced concrete (RC) structures under seismic loadings can be due either to loss of flexural, shear or bond capacity. Specifically, the effect of combined loadings can lead to a complex failure mechanism that plays a vital role in concrete mechanics. This report describes the formulation of an inelastic nonlinear beam element with axial, bending, and shear force interaction. The element considers shear deformation and is based on the section discretization into fibers with hysteretic material models for the constituent materials. The steel material constitutive law follows the Menegotto-Pinto model. The concrete material model uses an orthotropic constitutive relation in which the directions of orthotropy are the principal directions of total strain. These directions will change during the loading history, in accordance with the well-known rotating crack model. The concrete model accounts for the biaxial state of stress in the directions of orthotropy, in addition to degradation under reversed cyclic loading. Shear deformations are coupled with bending effects. Transverse strains are internal variables determined by imposing equilibrium at each fiber between the concrete and the vertical steel stirrups. Element forces are obtained by performing equilibrium based numerical integration on section axial, flexural, and shear behavior along the length of the element. In order to establish the validity of the proposed model correlation studies were conducted between analytical results and experimental tests of columns tested under cyclic loading. A structural analysis of a shear sensitive bridge pier subjected to ground input motion is also presented.

INTRODUCTION

The analytical modeling of the behavior of structures under combined loadings has received considerable attention in recent years. One of the main applications is the modeling of bridge columns, deep beams and several other highway structures and offshore RC structures, which are prone to fail in a brittle manner under bending, shear and axial force interaction. Concrete constitutive relations are simplified with orthotropic models in which the axes of orthotropy coincide with the principal longitudinal and transverse strain of concrete. Vecchio and Collins (1982), and Vecchio (1990,1992) have developed a concrete model which addresses compressive strength increase due to biaxial compression and compressive strength degradation as a function of tensile strain after cracking.

RESEARCH SIGNIFICANCE

This report presents a model with an orthotropic material property and coupled shear and bending interaction. During an earthquake, RC structures undergo large cyclic deformations with the concrete undergoing crack opening and closing. The problem has been further complicated by considering softening, confinement, tension stiffening and dilatancy, which accompanies large shear strains induced in the stirrups. This study tackles the extension of the fiber method formulation (e.g. Spacone et al. 1996) for application in axial bending and shear interaction. Timoshenko beam theory with quadratic shape functions for deformation and linear shape functions for rotation has been used in the model. After a brief description of the model, numerical analyses were performed using the developed model to investigate the effect of the structural variables. This model was developed using the finite element program FEAP developed by Taylor (2005).

BASIC ASSUMPTIONS

At first glance, Concrete is considered as an orthotropic material. The 3D problem is converted to a 2D problem with plane stress orthotropic approximation. All the stresses and strains are assumed to be averaged across the thickness.

ORTHOGONAL STRESS STATE DETERMINATION

COMPRESSION-COMPRESSION

Current research uses the Vecchio's (1992) simplified version of Kupfer et al. (1969) biaxial compression strength equation. The concrete compressive strength increase in lateral direction depends on the confining stress on the orthogonal direction. Strength enhancement and increase in ductility are dependent on the biaxial compressive stresses. Concrete in compression exhibits lateral expansion and increase in Poisson ratio. An upper limit of 0.5 has been considered for Poisson ratio.

TENSION-COMPRESSION REGION

Vecchio (1990) derived a softening equation in the tension-compression region, which is implemented in the current model (Figure. 1). Based on panel testing as proposed by Hsu et al. (1995), Belarbi and Hsu (1995) also developed a similar expression based on the softened concrete constitutive laws. The lateral tensile stress reduces the compressive strength in the orthogonal direction. The equation for compressive strength reduction factor proposed by Vecchio (1990) is:

$$\beta = \frac{1}{0.8 + \frac{0.34|\epsilon_1|}{|\epsilon_0|}} \leq 1 \quad (1)$$

The ultimate stresses in the orthogonal directions is

$$\sigma_{2p} = \beta f'_c \quad (2)$$

Where β is softening coefficient, ϵ_1 is the lateral tensile strain and ϵ_0 is the concrete strain at peak compressive strength.

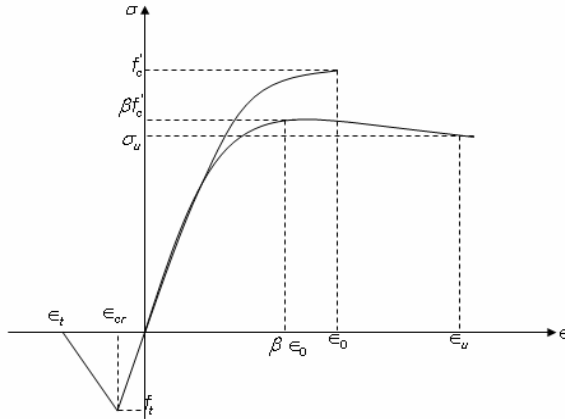


Figure. 1. Stress-Strain curve with softening (1990)

TENSION-TENSION

In the current model, tensile strength is not affected by its orthogonal compressive or tensile stresses. Always the tensile strength is equal to the uniaxial tensile strength at cracking and tension stiffening is considered only after cracking.

SHEAR ELEMENT FORMULATION

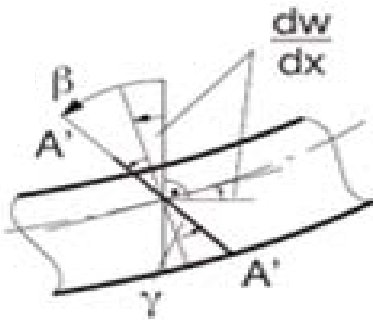


Figure. 2 Section Deformations

A section model was developed based on the Timoshenko type displacement formulation. The deformation w along a semi decimal distance dx has been described with three kinematic variables ϵ_x for axial strain, γ for shear strain and β for section rotation (Figure. 2)

GLOBAL MATERIAL MATRIX

The concrete model is simplified with plane stress orthotropic assumption. The model has perpendicular planes of elastic symmetry 1 and 2 with the local principal axes that are normal to these planes of symmetry

The stresses in the orthogonal directions are

$$\sigma = D_{LO} \epsilon \quad (3)$$

Where σ is the local stress vector, ϵ is the local strain vector and D_{LO} is the local material stiffness matrix in the principal direction (Chen 1976).

RC structures are considered orthotropic materials with material axes oriented at an angle θ between the principal and x-axes.

Rotational matrix R is introduced as

$$R = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -\cos\theta\sin\theta \\ -2\cos\theta\sin\theta & 2\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \quad (4)$$

The orthotropic stiffness matrix in global direction D_{gl} is

$$D_{gl} = R^T D_{LO} R \quad (5)$$

STIRRUP CONSIDERATION

Transverse strains are internal variables determined by imposing equilibrium at each fiber between concrete and vertical steel stirrups. Transverse strains are not known in advance and due to the non-linear behavior of the concrete and steel, an iterative procedure is needed to satisfy the equilibrium in the transverse direction following the procedure proposed by Petrangeli et al. (1999).

$$\sigma_{y,c}^i A_{y,c}^i + \sigma_{y,s}^i A_{y,s}^i = 0 \quad (6)$$

Where $\sigma_{y,c}^i$, $\sigma_{y,s}^i$ are concrete and steel stresses in the transverse direction at fiber i, and $A_{y,c}^i$, $A_{y,s}^i$ are area of concrete and steel in the transverse direction at fiber i.

STIFFNESS AND RESISTANCE CALCULATION

The total stiffness of the section is the sum of concrete and steel stiffness, and is evaluated as follows.

$$K_{Section} = \sum_1^{nC} (K_{Section})_C + \sum_1^{nS} (K_{Section})_S \quad (7)$$

Where the subscripts C, S denotes the parameters related to concrete and steel respectively, nC and nS are the number of concrete and steel fibers in the section and $K_{Section}$ is the section stiffness.

The total force of the section is the sum of concrete and steel forces in the respective direction, and is evaluated as follows.

$$F_{Section} = \sum_1^{nC} (F_{Section})_C + \sum_1^{nS} (F_{Section})_S \quad (8)$$

Where $F_{Section}$ is the section force.

NUMERICAL VERIFICATIONS

FLEXURAL COLUMN

For further validation, the model was used to simulate the behavior of a reinforced concrete bridge column (Column M07) tested by Belarbi et al. (2007) at the University of Missouri, Rolla (Figure. 3).

GEOMETRY AND MATERIAL PROPERTIES

The actual test specimen had a 24 in diameter and was 144 in long from the top of the footing to the centerline of the applied load. A 2.5 in hole in the center of the column cross section was used to run seven high strength steel strands that were stressed to apply the axial load. The load was applied at the top of the column using two hydraulic actuators in a displacement-control mode. The reinforcement consisted of twelve No. 8 longitudinal bars, and No. 3 spiral transverse reinforcement spaced at 2.75 in.

A concrete compressive strength of $f'_c=4.8$ ksi (33.1 MPa) and tensile strength of $f_t=0.5$ ksi (3.5 MPa) are used to analyze the reinforced concrete column. A Yield stress of longitudinal as well as transverse reinforcement of $f_y=67.0$ ksi (462 MPa) is used in the analysis. The Young's modulus of concrete and steel are $E_c = 4000$ ksi (27600 MPa) and $E_s = 29000$ ksi (200,000 MPa) respectively.

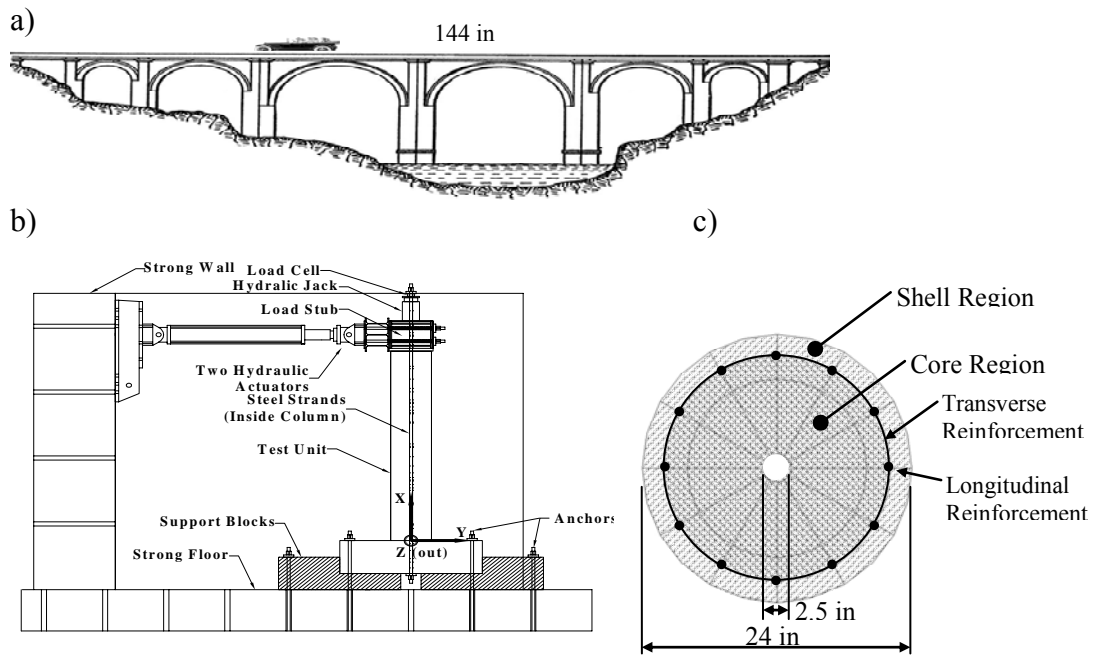


Figure 3. (a) Bridge schematic view (b) Bridge column test setup (c) Bridge column section

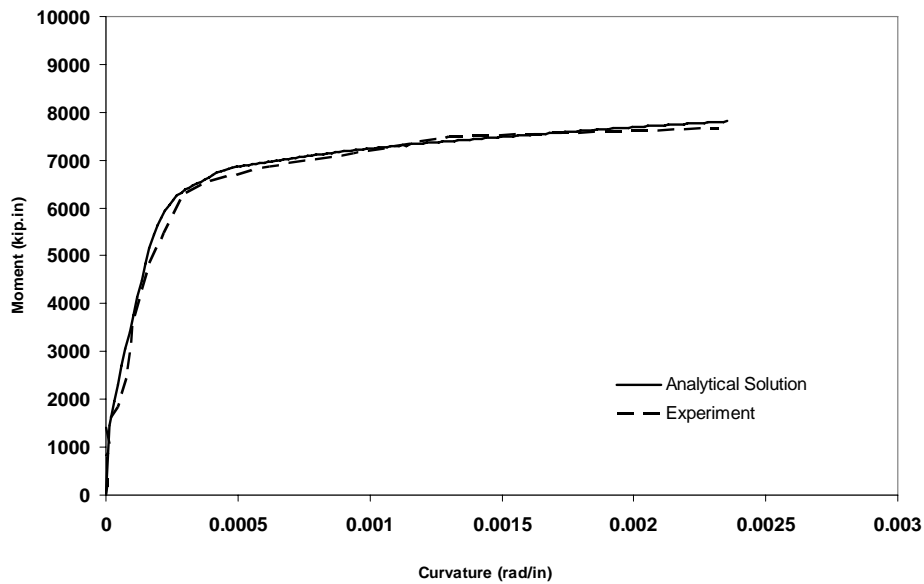


Figure 4 Monotonic Moment versus Curvature

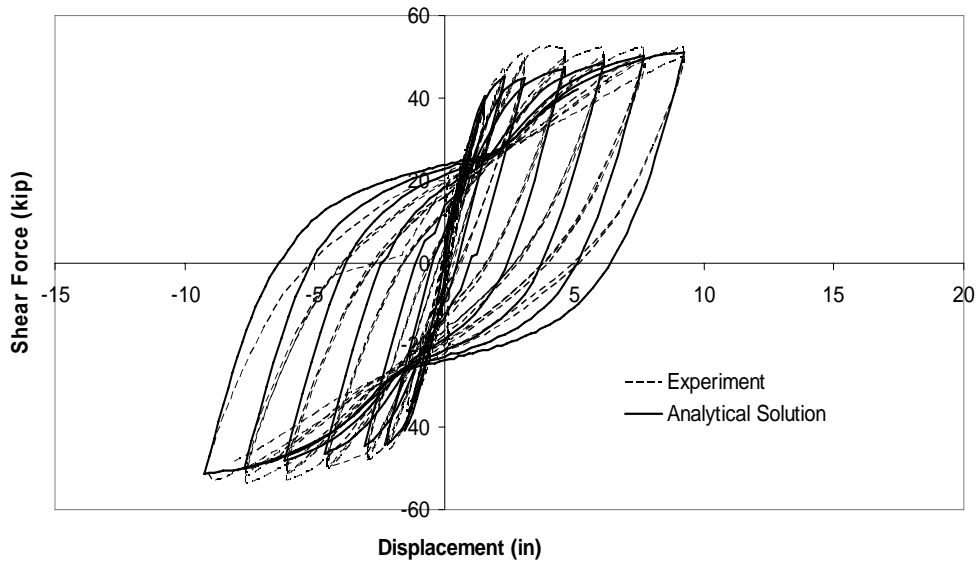


Figure. 5 Cyclic Shear force Displacement hysteresis comparison with Experiment

Column M07 is the control specimen and is tested under cyclic uni-axial bending. The experimental moment-rotation behavior of the specimen along with the analytical results is shown in Figure. 4 and its cyclic force-displacement result are shown in Figure.5. The model failed in a flexure mode followed by core degradation. The model curvature - moment capacity and force – displacements matched well with the experiment. The flexural resistance is stable with a capacity of 52 kip (231.3 kN). The model showed a ductile flexural failure mode and good correlation is observed between the analytical and experimental results.

SHEAR COLUMN

The proposed element was validated by modeling a high strength reinforced concrete squat column (Column HC4-8L 16-T6-0.1P) tested by Xiao and Martirosyan (1998) at the University of Southern California, Los Angeles (Figure. 6a). An axial load of 534 kN was applied constantly to the column. Rotations were fixed at the column bottom and top so that the column deforms anti-symmetrically with respect to the mid height under combined axial and lateral loading (Figure. 6b). This column failed with a shear cracks following the degradation of the transverse shear reinforcement. In the numerical analysis, the column was divided into several finite elements until converged was achieved.

The width of the column section is 254 mm and its depth is also 254 mm. The longitudinal reinforcement consists of 8 No.16 (15.9 mm diameter) bars, uniformly spaced along the perimeter with a clear cover of 13mm. No. 6 (6.4 mm diameter) steel stirrups are provided with a spacing of 51mm.

An average concrete compressive strength $f'_c=86$ MPa is used to analyze the reinforced concrete column. A yield stress $f_y=510$ MPa and 449 MPa are being used for the longitudinal and transverse reinforcements respectively. Young's modulus of concrete and steel are $E_c=45500$ MPa and $E_s=200,000$ MPa respectively.

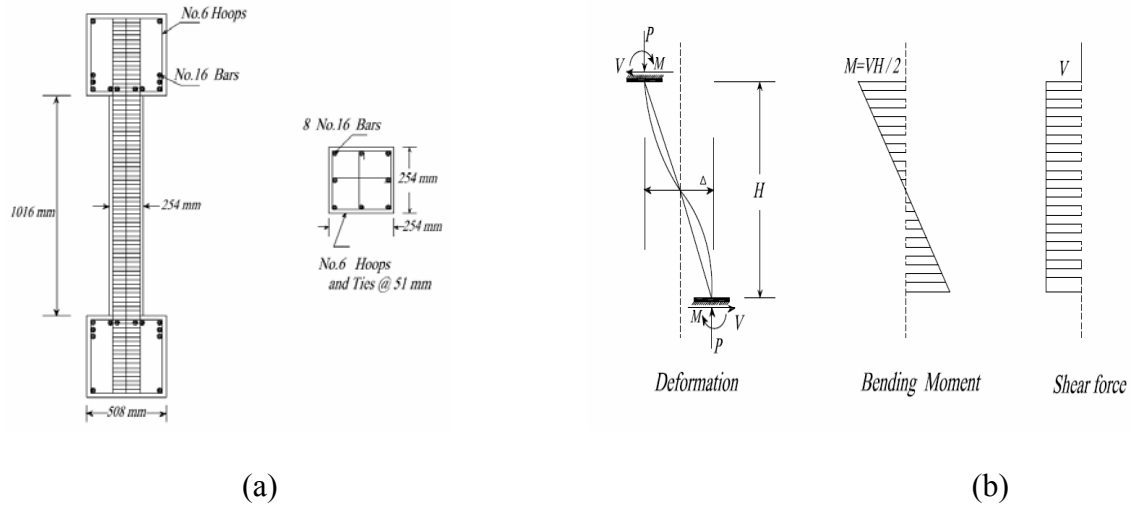


Figure 6 (a) Column details and (b) Loading pattern.

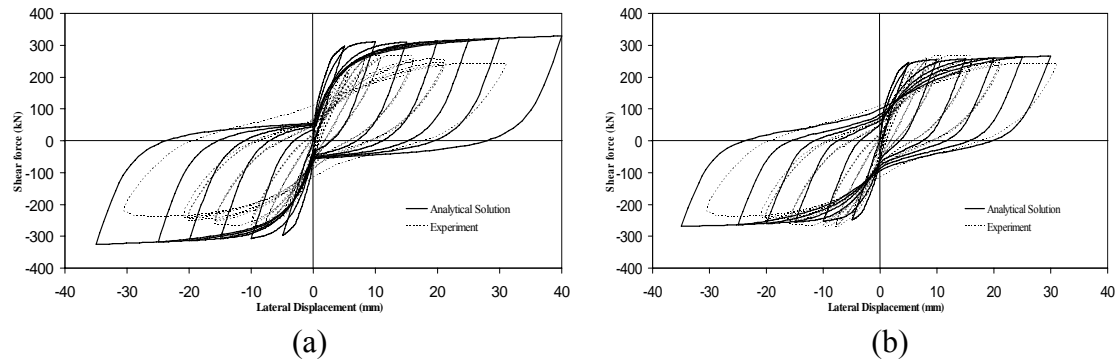


Figure 7 Cyclic shear force displacement hysteresis comparison with experiment (a) flexural element (b) shear element

Column HC4-8L 16-T6-0.1P has low shear reinforcement ($\rho_s=1.63\%$) and is tested under cyclic uni-axial bending. The experimental and analytical cyclic force-displacement results are shown in Figure 7. Figure 7(a) refers to the Bernoulli type flexural element with infinite shear strength which tends to overestimate the strength and ductility, while Figure 7(b) refers to the shear element which shows a very good correlation with the experimental results. The shear model showed a flexural shear failure mode, where shear failure was observed at the column ends with the increase of transverse strains, followed by core degradation and stirrup yielding. The new element

predicted accurately the amount of energy dissipation and displaced shape. Because of the formation of a plastic hinge after yielding of the reinforcement, the resisting load is stable with a capacity of 260 kN until a ductility level of 6. Figure 8 shows the analytical stress-strain behavior of the transverse reinforcement using the shear element. Analytical model was well agreed with the experimental transverse strain.

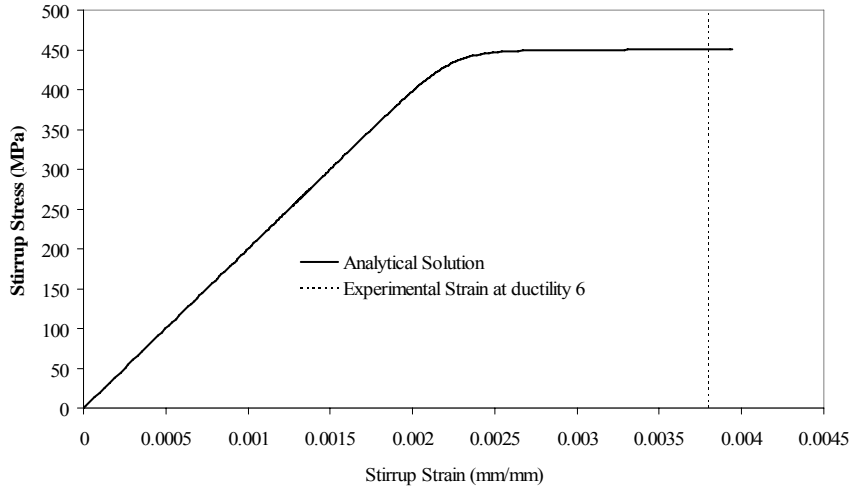


Figure 8 Transverse reinforcement stress-strain behavior

EARTHQUAKE ANALYSIS

BOUSIAS SPECIMEN

An earthquake analysis was conducted with the RC column tested by Bousias et al. (1995) using the Erzincan (1992) record recorded in Turkey. The column is 1490 mm long with a rectangular cross section of 250x250 mm. A cover of 15 mm in all directions is provided. The longitudinal reinforcement consists of 3 bars 16mm in diameter placed at the top and bottom, and 2 bars 16 mm in diameter placed at the center. The concrete compressive strength f'_c is equal 30.75 MPa, the yield strength of the steel rebars f_y is equal 460 MPa, and the steel elastic modulus E_s is equal 210,000 MPa. The column is subject to a constant axial force $N= 300$ kN

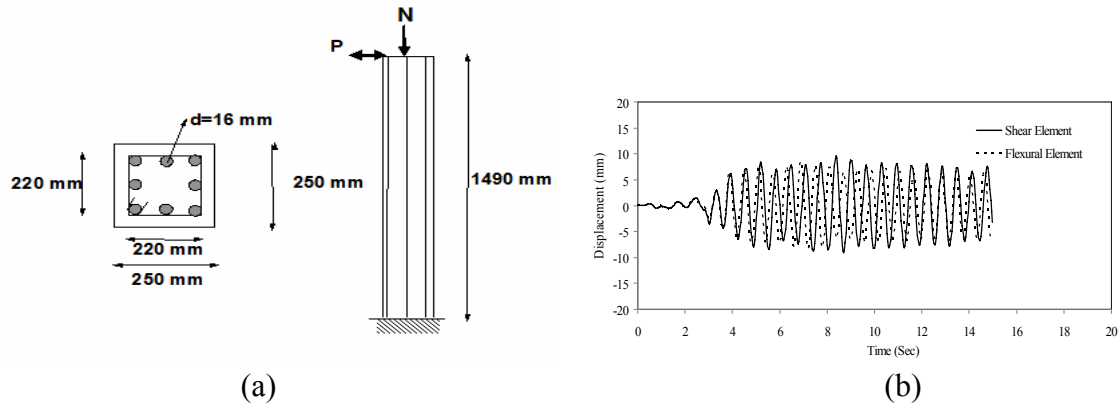


Figure 9 (a) Column details (b) Earthquake response of Bousias column with Erzincan earthquake record

Figure 9(b) shows the column displacement time history. The shear effect is small in the elastic region, but becomes predominant after yielding of the reinforcement.

CONCLUSIONS

This work represents a new approach to the numerical, static and dynamic analysis of structures made up of non-linear materials. A Fiber beam element was developed to analyze the reinforced concrete structure with the incorporation of mechanisms of shear deformation and strength. The emphasis of the work is on the evaluation of the effect of orthotropic model parameters to account for shear sensitive structures and the action of transverse reinforcement. The section deformations are determined from the strain state using the classical plane section hypothesis. The proposed model is able to capture the interaction between axial, flexural, and shear responses.

The results of a numerical correlation with the experimental results of a bridge pier section show that the model is able to capture the shear failure and provides reasonable results.

Future studies will concentrate on effect of combined loadings on shear sensitive sections and that of bond slip at the interface between the reinforcing bars and concrete following the model proposed by Ayoub (2006) and Ayoub and Filippou (2003).

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