Live Load Effect in Reinforced Concrete Box Culverts Under Soil Fill

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Report

Live Load Effect in Reinforced Concrete Box Culverts Under Soil Fill

Prepared for
Missouri Department of Transportation
Organizational Results

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16. Abstract

Live load effects in box culverts generally diminish with soil fill thickness. In addition, the effect of the live load may be nearly negligible compared to the dead loads when significant fill is placed above the crown of the culvert. The objective of this study is to determine the effects of live load (truck loads) on bridge-size (spans greater than 20 ft) reinforced concrete box culverts under soil fills of different thickness. The study considered the field testing of 10 existing reinforced concrete box culverts with fill depths ranging from 2.5 ft to 13.5 ft. Instrumentation of the culvert consisted of 12 reusable strain transducers and 12 LVDTs. The instrumentation was designed to be applied, used, and removed in one day of testing. Loaded trucks were driven over the culvert to provide live load. The results of the testing show that live load effect does diminish with increasing fill depth. The AASHTO LRFD and LFD Standard Specifications were both overly conservative in predicting strains and displacements compared to the field data for fill depths less than 8 ft. At above 6 ft of fill the measured effect of the live load was less than 10% of the dead load effect. This could be considered as a point at which to ignore the live load effect and therefore not load rate the culvert.

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EXECUTIVE SUMMARY

Live load effects in box culverts generally diminish with soil fill thickness. In addition, the effect of the live load may be nearly negligible compared to the dead loads when significant fill is placed above the crown of the culvert. The objective of this study is to determine the effects of live load (truck loads) on bridge-size (spans greater than 20 ft) reinforced concrete box culverts under soil fills of different thickness. The study considered the field testing of 10 existing reinforced concrete box culverts with fill depths ranging from 2.5 ft to 13.5 ft. Instrumentation of the culvert consisted of 12 reusable strain transducers and 12 LVDTs. The instrumentation was designed to be applied, used, and removed in one day of testing. Loaded trucks were driven over the culvert to provide live load. The results of the testing show that live load effect does diminish with increasing fill depth. The AASHTO LRFD and LFD Standard Specifications were both overly conservative in predicting strains and displacements compared to the field data for fill depths less than 8 ft. At above 6 ft of fill the measured effect of the live load was less than 10% of the dead load effect. This could be considered as a point at which to ignore the live load effect and therefore not load rate the culvert.

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1 INTRODUCTION

1.1 Research Objective

The objective of this study is to determine the effects of live load (truck loads) on bridge-size (spans greater than 20 ft) reinforced concrete box culverts under soil fills of different thickness. Live load effects in box culverts generally diminish with soil fill thickness. In addition, the effect of the live load may be nearly negligible compared to the dead loads when significant fill is placed above the crown of the culvert. Load rating of culverts is currently being mandated by AASHTO for bridge size culverts, defined as culverts with spans greater than 20 feet. However, load rating may not be required if the live load effect can be demonstrated to have a negligible effect on performance. The overall goal of this research is to determine a potential cutoff value for fill depths at which the live load effect becomes negligible. That cutoff value can be used to determine which culverts do not require a load rating. This would save Missouri from the need and expense of load rating a portion of the nearly 3000 culverts.

1.2 Research Plan

This research conducted testing of 10 bridge size culverts to determine the live load effect. The selection of the culverts was conducted in collaboration with MoDOT. The culvert fill depths ranged from about 2.5 ft to 13.5 ft. Testing was conducted by driving a loaded "dump truck" across the culvert. The truck stopped at several locations determined to cause maximum moments and deflections in the culvert. Instrumentation, consisting of strain transducers, LVDT displacement transducers, and an accelerometer, recorded the movements of the culvert. After testing, the data was analyzed to determine the live load effect in the culvert. The data was able to determine how the effect of the live load diminishes with increasing fill depth and distance from the load. This provides a threshold at which the live load effect can be considered negligible. In addition finite element analysis using the computer program Phase II modeled the distribution of live load through the soil.

1.3 Report Organization

This report begins with background information on the culvert design and rating process as well as previous research in the area in Section 2. Section 3 covers the experimental setup used to determine live load effects. Section 3 also gives the results of the testing of the culverts and gives the results of the culvert analyses. Section 4 discusses possible locations for fill depth cutoff values and Section 5 gives the conclusions of the research.

2 BACKGROUND

2.1 Culvert Loads

Figure 2-1 shows the load applied to a reinforced concrete box culvert for design (Modot Engineering Policy Guide section 751.8). On the top slab, EV1 is the vertical earth pressure from the soil, DC1 is the self-weight of the concrete slab, and LL1 is the live load due to truck loading. The distribution of live load on the top slab for fill depths greater than 2 ft is shown in Figure 2-2. This distribution assumes that the wheel load P is distributed over the area E_1 and E_2 as defined in Equation 1.

$$E_1 = 0.83 + \text{LLDF } H \qquad \text{Equation 1}$$
$$E_2 = 1.67 + \text{LLDF } H$$

Where H is the design fill depth and the 0.83 and 1.67 values represent the wheel contact area. The live load distribution factor (LLDF) gives the spread of the load as it moves through the soil as seen in Figure 2-2. Based on basic soil mechanics this load spreads at about a 60 degree angle. In Equation 1 according to the MoDOT EPG the LLDF factor is 1.0. The 2012 AASHTO LRFD bridge design specifications section 3.6.1.2.6 gives a value of 1.15 for the case of select granular backfill and 1.0 for all other cases. The 2002 AASHTO LFD Standard section 6.4.1 gives a value of 1.75 and does not account for the wheel area. The smaller the LLDF factor, the less spread of the load is allowed and the greater the amount of uniform load on the culvert top slab. The slab is then designed as a one way slab, with the live load distributed across the distance E_1.

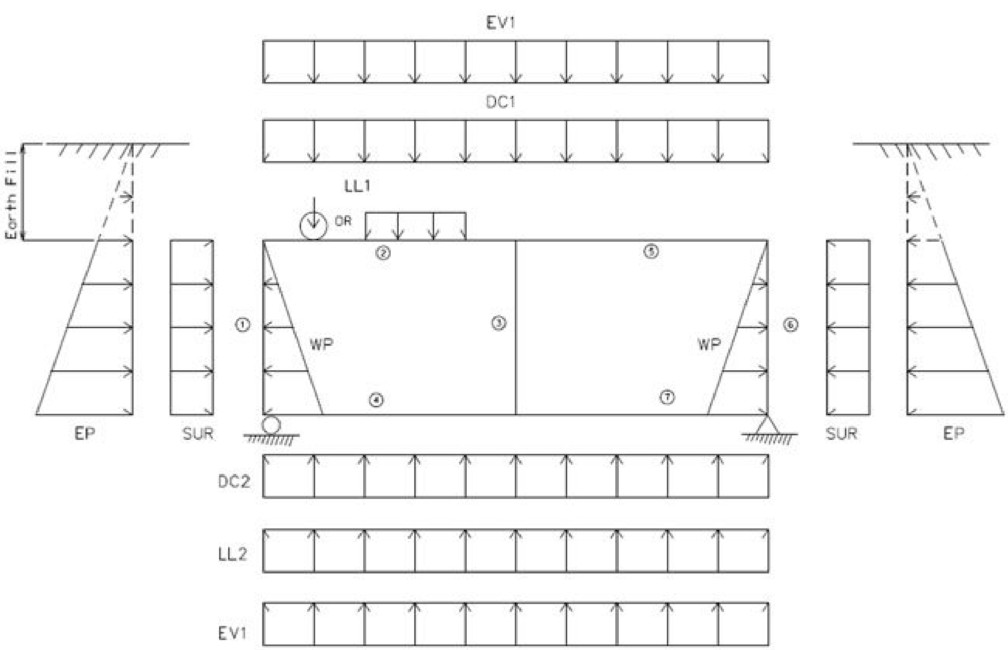

Figure 2-1 Culvert Loads

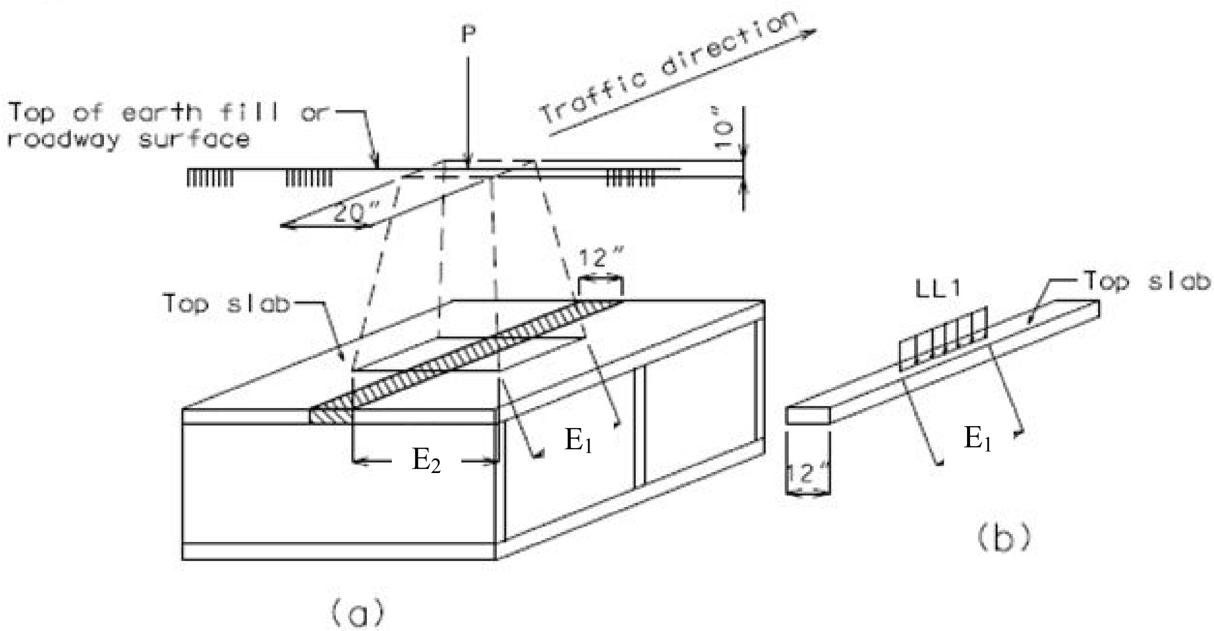

Figure 2-2 – Live load distribution

2.2 Culvert Load Rating

Load rating determines the maximum truck tonnage that can safely cross a bridge or culvert. The load rating is given as an Inventory Rating or an Operating Rating. The Inventory Rating (IR) is the maximum truck load that can safely utilize the culvert for an indefinite period of time (AASHTO, 2003). The Operating Rating (OR) is the absolute maximum permissible truck load that may use the culvert. Load ratings are based on the culvert structural capacity and dead load demand to live load demand as given in Equation 2 (which is based on LFD procedures). The permissible truck load is then the rating factor times the design truck load.

$$RF = \frac{C - A_1 D}{A_2 L(1+I)} \qquad \text{Equation 2}$$

Where: RF = the rating factor
C = the structural capacity of the member
D = the dead load effect on the member
L = the live load effect on the member
I = the impact factor
A_1 = 1.3 = factor for dead loads
A_2 = 2.17 for Inventory Level = factor for live loads
= 1.3 for Operating Level = factor for live loads

The load rating is the lowest rating given for all possible critical sections and demand types. As can be seen in the equation for culverts in which the live load makes up a small part of the total load on the slab, a change in truck weight will not greatly change the rating factor.

2.3 Previous research on Live Load Distribution

A central question in this research is what the actual distribution of truck load is on the culvert. If the distribution is greater than what is used in design equations, then there is less load getting to the culvert and the impact of the live load on the culvert could be smaller. A wider distribution reduces the live load that reaches the culvert and therefore increases the rating factor. At some fill depth the effect of the live load may be small enough that load rating of the culvert may not be required.

The pressure distribution in concrete box culverts has been studied experimentally by Abdel-Karim et al. (1990). The research involved placing pressure sensors on top of a 12 ft by 12 ft reinforced concrete box culvert, covering it with different amounts of soil fill, and then driving a test truck across the culvert. This procedure allowed the researchers to directly determine the vertical stresses just above the culvert due to the soil fill. The research found peak stresses in the soil to be about 300 psf with 2 ft of fill, decreasing to about 100 psf at 8 ft of fill. The researchers further mention the importance of the pavement and two-way action in the slab at helping to distribute the live load, especially at the shallower fill depths.

Abdel-Karim et al. (1990) also commented on the suitability of 1.75 for LLDF in the AASHTO LFD standard. They found that 1.75 is valid and the AASHTO procedure reasonably represents the pressure distribution in the soil for fill depths of 2 ft to 8 ft.

Abdel-Karim et al. (1990) further say that the effect of the live load diminished considerably beyond 8 ft of fill. The point at which to ignore the live load could be when the live load effect is less than 5 percent of the total load effect. On the other hand Gilliland (1986) mentions that this point could be when the live load pressures are less than 10 percent of the pressures due to the dead load only.

Acharya (2012) also investigated the effect of pavements on distributing the load on culverts with low depth soil fills. The research involved testing of two existing culverts with rigid and flexible pavements and fill depths around 2 ft as well as computational modeling. The research found that the current AASHTO live load distribution factors over predicted the vertical live loads especially at low fill depths due to, in part, the additional distribution of load provided by the pavements.

NCHRP report 647 (2010) recommends a slightly different live load distribution based on soil depth than what is presented in the AASHTO code. The main difference is the addition of a term that allows for load spread parallel to the axis of the culvert that is based on the span of the culvert. Longer spans are more flexible and should be able to distribute the loads longitudinally to a greater extent. The report does state that the predicted loads (for the case of a 96 in span culvert) are very conservative, but does not promote a change in the load distribution widths.

A more recent study by TxDOT (Lawson et al. 2010) sought a way to improve the load rating of culverts. Their study involved investigating the accuracy of the current procedures used in load rating culverts. For the research they tested three reinforced concrete box culverts by driving loaded trucks over the culvert and measuring strains and displacements. They found that both the simple and more advanced computational models greatly over predicted the actual moments measured in the culverts. The study resulted in a culvert rating guide (TxDOT 2009) which gives increasingly more complex and sophisticated procedures to determine the load rating of the culvert.

From the previous research it is clear that the effect of the live load diminishes with increasing fill depth. However, the predicted live load per AASHTO design equations may be overly conservative at low fill depths. Also, for fill depths beyond 8 ft the live load effect may be negligible.

3 TECHNICAL APPROACH

3.1 Selection of Representative Culverts

The selection of the culverts was conducted in collaboration with MoDOT. For the first phase, five culverts with depths above and below 8 ft (for example 2 ft, 4ft, 6ft, 8ft, 12ft) were selected and evaluated. The culverts were selected because they represent typical culverts in Missouri. The culverts also have little to no skew because a higher skew would lead to lower measured strain and displacement values. Based on the testing and analysis of Phase 1, five additional culverts were selected for Phase 2 from a range of depths considered critical for the live load effect. The culverts were geographically close to the Columbia/Jefferson City area. Table 3-1 gives a summary of the culverts that were tested.

Table 3-1– Summary of culverts tested.

Bridge #	County	Route	Cell Size	Year Built	Design Load	Design Fill (ft)	Actual Fill (ft)	Clear Height (ft)	Slab Thickness (in)	Wall Thickness (in)	Skew
Phase 1 Culverts											
L0525	Callaway	RT F E	2x12'	1953	H 15	2.58	2.75	9.5	11	11.5	0
N0936	Pettis	RT J S	2x12'	1960	H 15	4	5.0	7.5	10.5	8	0
A2869	Macon	MO 3 S	2x11'	1972	H 15	6	6.48	8	11	9.5	0
R0015	Boone	RT NN	2x13'	1962	H 15	7.8	8.15	9.75	13.5	9.5	15
P0622	Callaway	RT O E	2x12'	1954	H 10	11.51	13.25	10	15.5	10	15
Phase 2 Culverts											
A6330	Audrain	RT AA	3x14’	2001		3	2.547	8	11	8	15
N0793	Montgomery	RT J S	2x14’	1962		1.82	2.555	7.5	10	8.5	0
N0502	Montgomery	MO 94	3x15’	1958		3.82	4.491	6	10.5	8	10
X0749	Boone	RT O S	2x12’	1947	H 10	5.11	6.35	10	12	9	0
N0059	Gasconade	RT Y E	2x10’	1957	H 10	6.1	8.25	8	11	7.5	10

3.2 Culvert Instrumentation and Testing Plan

Instrumentation for the culvert testing consisted of 12 strain transducers and 12 linear variable differential transformers (LVDTs) to measure strain and displacement, respectively. All of the instrumentation was manufactured by Bridge Diagnostics, Inc. A typical strain transducer installation is shown in Figure 3-1. The strain transducers consisted of a full Wheatstone bridge with four active 350-ohm foil gages, contained in a rugged and waterproofed metal casing. The strain transducers were 3 in. long, but were usually installed to the end of an aluminum extension rod that effectively increased the gage length to as great as 24 in. The extended gage length is a mechanical method of amplifying the strain transducer signal, which was particularly important for this research since very small strain measurements were expected. The gage accuracy is about 1×10^{-6} strain which corresponds to about 4 psi in concrete. The gages and extension rods were attached to the culvert via metal tabs that were adhered to the concrete of the culvert with epoxy. The LVDTs had a resolution less than 0.001 in. and were encased in a stainless steel housing. As shown in Figure 3-2, they were mounted to a variable-height tripod, which was extended until the LVDT was in contact with the culvert ceiling with the LVDT midway through its 1-in. range.

Figure 3-1– Two strain transducers, one installed without an extension rod on the culvert ceiling, and one installed with an extension rod on the culvert wall.

Figure 3-2 – LVDT tripods, with a closeup view of the LVDT mount shown in the inset. Data acquisition system can be seen at right.

Locations for the 24 instruments are shown in Figure 3-3. The locations were selected to measure strains and displacements at critical points along the culvert ceiling and walls. One cell, designated the "primary cell", was heavily instrumented, with additional instrumentation in the adjacent ("secondary") cell. This arrangement was chosen to provide a robust data set within reasonable budget constraints. Additional instrument locations were identified under the opposing/unloaded lane of traffic in the primary cell to assess the longitudinal response of each culvert. Figure 3-3 depicts the gage locations for a two-cell culvert; for culverts with 3 cells, the instrument locations were the same as for a two-cell culvert with the center cell as the primary cell. The instrument locations relative to the center of the culvert were generally the same for all tests. In Phase 2, the instrumentation layout was slightly changed from the Phase 1 culverts. Strain gages 2 and 8 were moved to record negative moment in the slab near the center and exterior wall of the culvert.

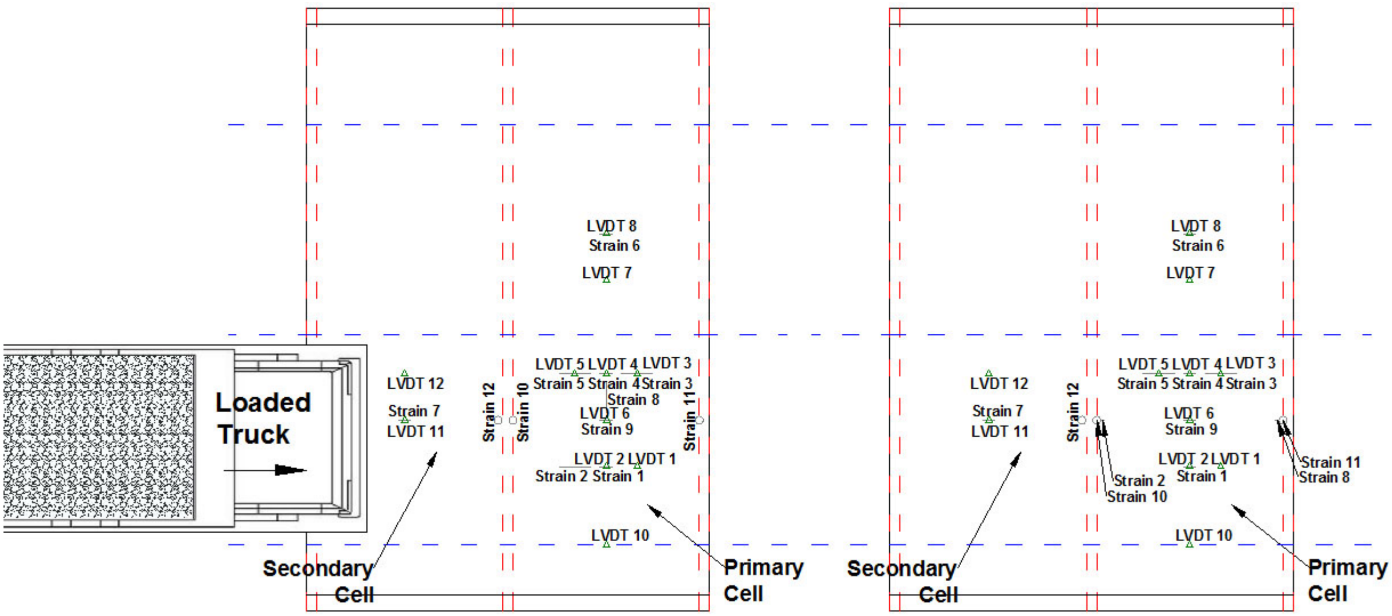

Figure 3-3 – Instrument locations for Phase 1 (left) and Phase 2 (right) culverts.

Typically, the testing routine consisted of installing the instruments according to a location plan similar to that shown in Figure 3-3. Once the gages were installed, a live load was applied to the culvert via a loaded truck. The truck was driven over the culvert and stopped in the 7 static positions shown in Figure 3-4 to gather strain and displacement data. The 7 positions were selected to cause maximum readings in different gage locations. Figure 3-5 shows a loaded truck stopped in Position 2. After data were collected for each of the 7 static positions, the truck was driven over the culvert at several different speeds (1, 5, 10, 20 mph) in an attempt to collect dynamic data. This report focuses on analyzing and interpreting the static data since the speed of the data acquisition system was not sufficient to acquire truly dynamic data.

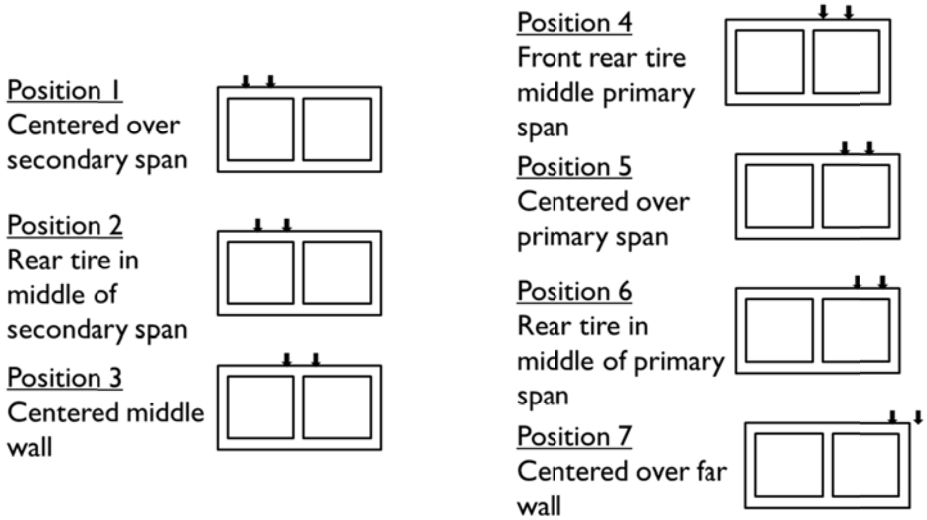

Figure 3-4 – Truck Positions.

Figure 3-5 – Photograph of testing with truck in Position 2.

3.3 Testing Results

A relatively large volume of data was collected for this research. This section presents a summary of the results through a series of tables and plots that show the data as a function of relevant predictor variables (e.g. fill height, position along the ceiling, etc.). A set of tables and plots with more comprehensive results is included in the Appendix.

3.3.1 Strain results

Strain values for each test, truck position, and gage were retrieved from the gages. Several important trends in strain data are summarized in this section. First, the maximum strain values measured on the culvert top slab for each test are presented in Table 3-2 and Figure 3-6. In general the maximum strain occurred in strain gage 9 or 4 located in the middle of the slab beneath the truck when it was at position 5. For some culverts the strain was greater in the secondary (less instrumented) cell when the truck was centered over that cell. There is a clear trend of decreasing strain with increasing fill height, as expected. The rate of strain decrease with fill height appears to drop off notably for fill heights greater than about 5 or 6 ft. Two of the lower fill height tests (A6330 and L0525) had lower maximum strain values than might be predicted based on the other tests. L0525 was the first culvert tested, and the research team encountered some issues developing an adequate bond between the gages and the concrete. It is possible this resulted in recording lower strain values. A6330 is the newest culvert tested and may have less deterioration or different design details that lowered its strain value. The strain for N0793 is much higher than all the other strain values. The high strain could be due to some unique characteristics of the culvert (such as deterioration or cracking) and highlights the variability of the strain values at low fill depths. For greater fill depths, the strain values for culverts of similar depths are closer.

Table 3-2 – Maximum measured strain along top slab.

Bridge #	Design Fill (ft)	Actual Fill (ft)	Cell Size	Slab Thickness (in)	Year Built	Skew	Truck Weight (lb)	Max top slab Strain	Gage	Wheel Position
A6330	3	2.547	3x14’	11	2001	15	59,660	1.97E-05	9	5
N0793	1.82	2.555	2x14’	10	1962	0	50,800	4.26E-05	4	6
L0525	2.58	2.75	2x12'	11	1953	0	50,660	1.19E-05	7	1
N0502	3.82	4.491	3x15’	10.5	1958	10	59,660	2.17E-05	7	1
N0936	4	5.0	2x12'	10.5	1960	0	46,350	1.83E-05	9	5
X0749	5.11	6.35	2x12’	12	1947	0	53,690	1.35E-05	9	4
A2869	6	6.48	2x11'	11	1972	0	56,600	7.31E-06	9	5
R0015	7.8	8.15	2x13'	13.5	1962	15	45,880	9.22E-06	9	5
N0059	6.1	8.25	2x10’	11	1957	10	50,900	1.19E-05	4	5
P0622	11.51	13.25	2x12'	15.5	1954	15	54,560	5.51E-06	4	6

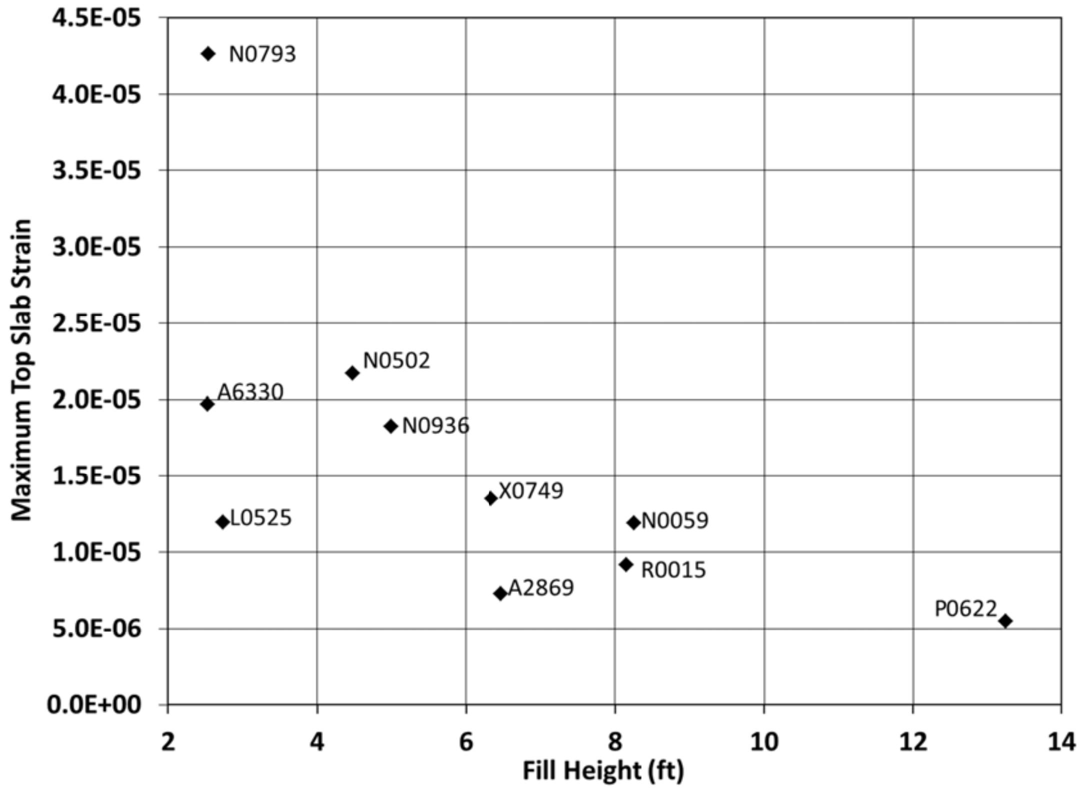

Figure 3-6– Maximum strain measured along top slab versus fill height.

Table 3-3 and Figure 3-7 show similar data but for strain measured along the culvert walls rather than the top slab. The magnitudes of the maximum strain values in the wall are about half of those measured in the top slab. The trend of decreasing strain with increasing fill height is similar to that observed for the ceiling data, and a similar reduction in the rate of strain decrease is observed around 5 ft of fill height.

Table 3-3 – Maximum measured strain along wall.

Bridge #	Design Fill (ft)	Actual Fill (ft)	Cell Size	Slab Thickness (in)	Year Built	Skew	Truck Weight (lb)	Max Wall Strain	Gage	Wheel Position
A6330	3	2.547	3x14’	11	2001	15	59,660	1.22E-05	10	4
N0793	1.82	2.555	2x14’	10	1962	0	50,800	1.83E-05	11	2
L0525	2.58	2.75	2x12'	11	1953	0	50,660	1.78E-05	12	1
N0502	3.82	4.491	3x15’	10.5	1958	10	59,660	2.17E-05	10	5
N0936	4	5.0	2x12'	10.5	1960	0	46,350	5.52E-06	12	1
X0749	5.11	6.35	2x12’	12	1947	0	53,690	6.12E-06	12	4
A2869	6	6.48	2x11'	11	1972	0	56,600	5.70E-06	10	4
R0015	7.8	8.15	2x13'	13.5	1962	15	45,880	3.98E-06	10	4
N0059	6.1	8.25	2x10’	11	1957	10	50,900	6.09E-06	11	7
P0622	11.51	13.25	2x12'	15.5	1954	15	54,560	3.64E-06	12	6

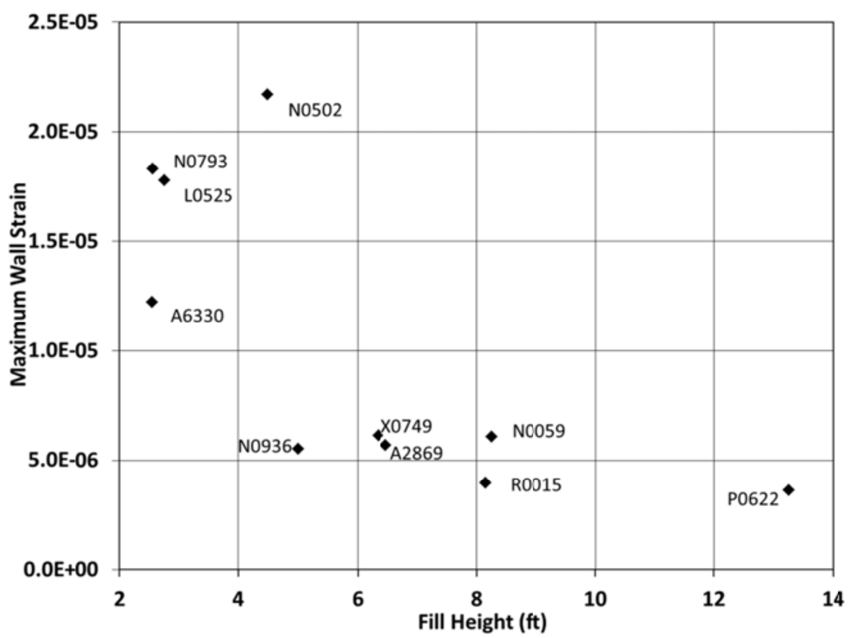

Figure 3-7. Maximum wall strain versus fill height.

Figure 3-8 shows the strain either side of the center wall measured in strain gages 10 and 12 when the truck is in position 5. The difference in the strain from one side of the wall to the other indicates bending in the wall. Culverts A6330 and N0502, which were designed as fixed, have the steepest gradients, indicating more bending as expected with a fixed connection. However data for the other culverts also indicate some bending in the center wall even though they are designed as pinned connections. The stain data indicate that there is some fixity in the connection.

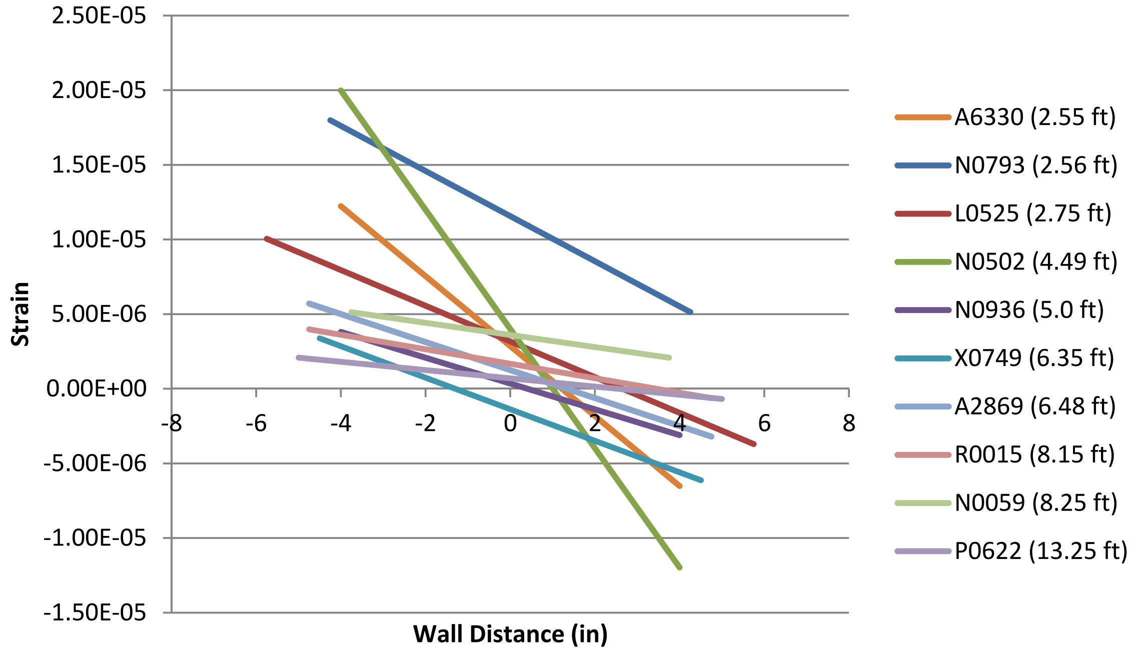

Figure 3-8 – Strain in center wall

Figure 3-9 shows profiles of strain across the cross section of the culvert (i.e. looking through culvert if standing in the stream) for each test with the truck in Position 5. The shapes of the profiles are all similar and are all consistent with predictions based on basic structural theory. The magnitude of the strain values for each test generally decreases with increasing fill height, consistent with results discussed above. Culvert N0793 had significantly greater positive moment strains than the other culverts.

The profiles for the Phase 2 tests include strain values along the top slab right next to the walls (Gages 2 and 8) to demonstrate the negative moment at the connection between the wall and the top slab. The Phase 1 culverts did not measure the negative moment so those values are not plotted. The field strain data show both fixed and pinned like behavior in the culverts. A culvert with pinned connection would see no moment at the exterior wall. However the strain profiles as in Figure 3-9 do not indicate that kind of pure behavior. Culvert N0793 does show a negative moment at the exterior wall and a high negative moment at the center wall. However the other culverts for which negative moment data are available (N0059 and X0749) show little to no negative moment at the exterior wall and their strain distributions are consistent with a culvert with pinned supports. The culverts that were designed as fixed (A6330 and N0502) do show fixed strain behavior in the field data. Unfortunately, for the other culverts the negative moments were not measured. It is likely that even in culverts that were designed as pinned, their true behavior falls between the fixed and pinned regions.

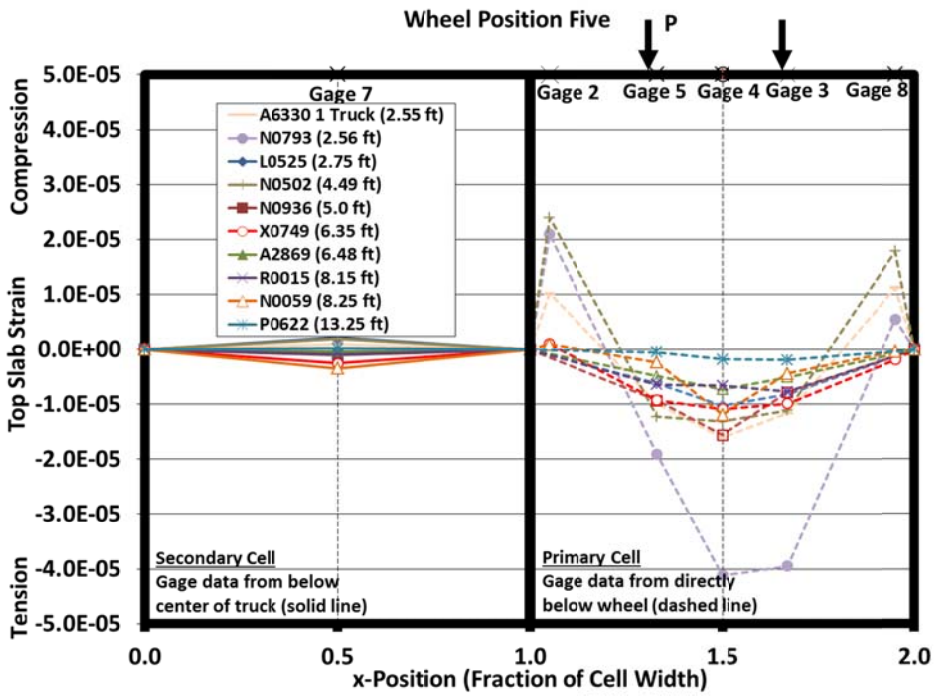

Figure 3-9. Strain profiles across culvert cross-section for truck in position 5 for all 10 tests.

Figure 3-10 shows profiles of strain along the length of the culvert (i.e. looking at the culvert from the backfill) for each test with the truck in Position 5. The shapes of the profiles show a more pronounced gradient of strain versus location for the culverts with the lower fill depths. This supports the idea that the lower fill depth is not as capable of distributing the strain across a large area. For the high fill depths (i.e. P0622) the strain seems almost uniform across the culvert, supporting the idea that stresses from a truck at centered at wheel position 5 spread out significantly due to the high fill depth. The magnitude of the strain values for each test generally decreases with increasing fill height, consistent with results discussed above. However culvert N0793 still has significantly greater strains.

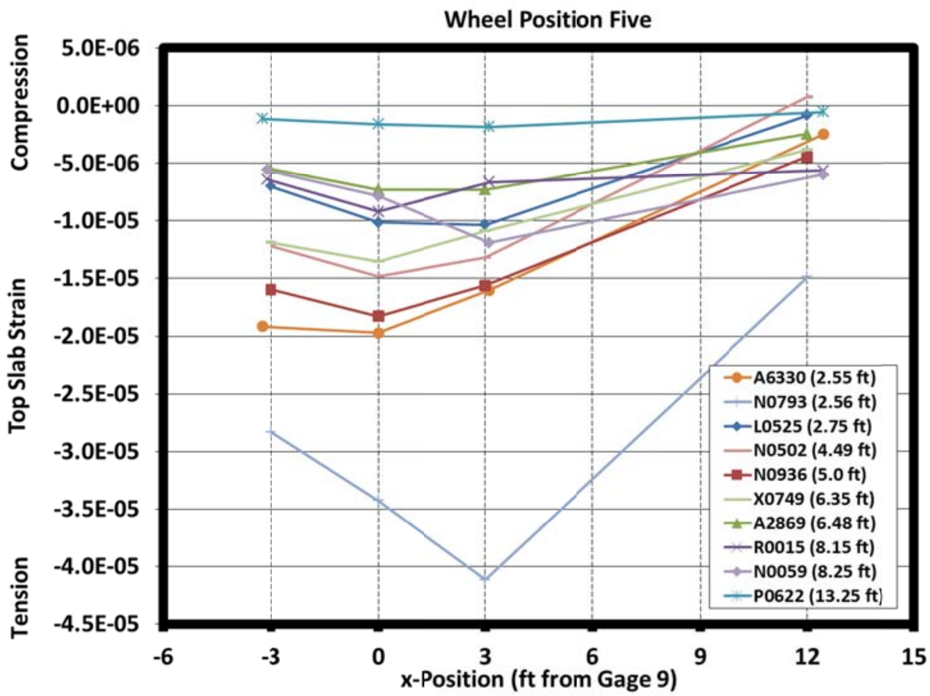

Figure 3-10. Strain profiles along length of culvert for truck in position 5 for all 10 tests. Gage 9 (the reference point for x values) was always centered below the truck (i.e. in the center of the loaded traffic lane).

For the Phase 1 tests, Gage 8 was oriented along the length of the culvert to measure longitudinal strain (whereas all the other gages were oriented perpendicular to the length of the culvert to measure strain along the cross section). The longitudinal strain gives an indication of two-way action in the culvert top slab. If the slab were truly a one-way slab, as it is designed, the longitudinal strain would be zero. Maximum strain values for this gage are presented in Table 3-4. The values are always notably less than the maximum values recorded for the transverse ceiling gages: for the three tests with shallower fill, the longitudinal value was about 20% of the maximum ceiling strain (for the same test); for the three tests with deeper fill, the longitudinal strain was closer to 50% of the maximum ceiling strain. This could indicate the proportion of longitudinal strain increases with fill height therefore there is more two-way action in the slab with greater fill depth, but the increase in percentage values could also be a result of less precise measurements for the deeper (smaller strain) tests.

Table 3-4 – Maximum measured strain in gage oriented along length of culvert (Gage 8 for Phase 1 only).

Bridge #	Design Fill (ft)	Actual Fill (ft)	Cell Size	Slab Thickness (in)	Year Built	Skew	Truck Weight (lb)	Max Long Strain	% of Max Ceiling Strain	Wheel Position
L0525	2.58	2.75	2x12'	11	1953	0	50,660	2.37E-06	20	5
N0936	4	5.0	2x12'	10.5	1960	0	46,350	3.77E-06	21	4
A2869	6	6.48	2x11'	11	1972	0	56,600	1.64E-06	22	5
R0015	7.8	8.15	2x13'	13.5	1962	15	45,880	3.83E-06	42	5
P0622	11.51	13.25	2x12'	15.5	1954	15	54,560	3.10E-06	56	6

Figure 3-11 shows two profiles of strain across the cross section of the culvert (i.e. looking through culvert if standing in the stream) for the test of A6330: one with one truck, and one with two trucks side-by-side. A6330 was the only culvert tested with two trucks. The results indicate an increase in strain, as expected. At the center of the primary cell, the increase is about 50 percent, relative to the value for one truck ($1.6x10^{-6}$ versus $2.5x10^{-6}$).

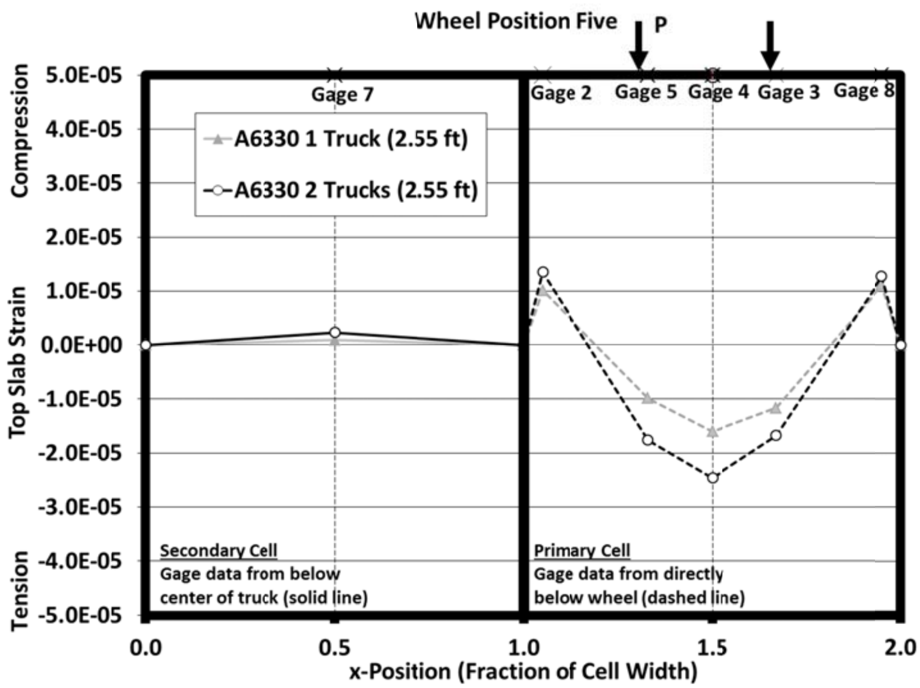

Figure 3-11. Strain profiles across culvert cross-section for Culvert A6330 with one truck in Position 5 and two trucks in Position 5.

3.3.2 Displacement results

Displacement values for each test, truck position, and gage were calculated from the LVDTs using calibration factors provided by the manufacturer. Several important trends in displacement data are summarized in this section. First, the maximum displacement values measured along the culvert ceiling for each test are presented in Table 3-5 and Figure 3-12. There is a clear trend of decreasing displacement with increasing fill height, as expected and as noted above for the strain data. The rate of displacement decrease with fill height appears to drop off notably for fill heights greater than about 5 ft, similar to the trend noted above for strain data. Again the displacements for culvert N0793 are the highest and there is the greatest variability in the displacements at the lower fill depths. In all cases the displacement is less than 0.02 in.

Table 3-5 – Maximum measured displacement along ceiling.

Bridge #	Design Fill (ft)	Actual Fill (ft)	Cell Size	Slab Thickness (in)	Year Built	Skew	Truck Weight (lb)	Max Displacement (in)	Gage	Wheel Position
A6330	3	2.547	3x14'	11	2001	15	59,660	0.0091	6	5
N0793	1.82	2.555	2x14'	10	1962	0	50,800	0.0195	12	1
L0525	2.58	2.75	2x12'	11	1953	0	50,660	0.0069	10	4
N0502	3.82	4.491	3x15'	10.5	1958	10	59,660	0.0130	12	1
N0936	4	5.0	2x12'	10.5	1960	0	46,350	0.0042	12	1
X0749	5.11	6.35	2x12'	12	1947	0	53,690	0.0042	11	1
A2869	6	6.48	2x11'	11	1972	0	56,600	0.0021	4	5
R0015	7.8	8.15	2x13'	13.5	1962	15	45,880	0.0029	7	5
N0059	6.1	8.25	2x10'	11	1957	10	50,900	0.0024	12	1
P0622	11.51	13.25	2x12'	15.5	1954	15	54,560	0.0020	5	6

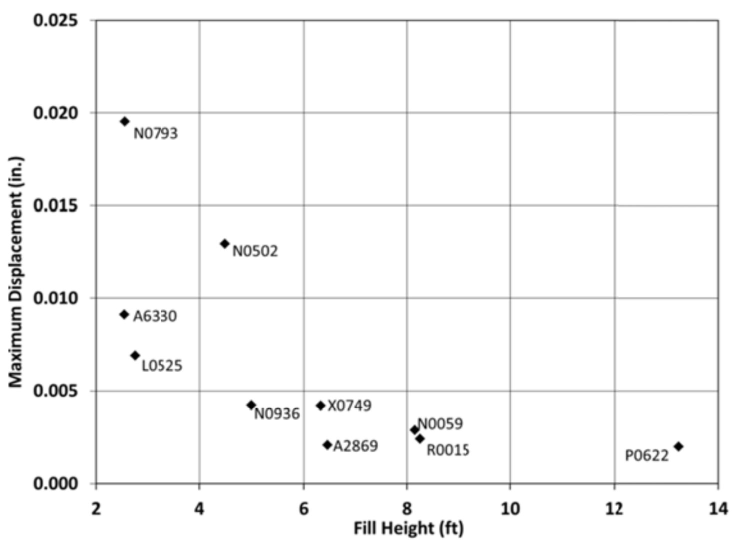

Figure 3-12 – Maximum measured displacement versus fill height.

Figure 3-13 shows profiles of displacement across the cross section of the culvert (i.e. looking through culvert if standing in the stream) for each test with the truck in Position 5. The shapes of the profiles are all similar and are all consistent with predictions based on basic structural theory. The displacement profile shapes are also consistent with the strain profile shapes shown in Figure 3-9. The magnitude of the displacement values for each test generally decreases with increasing fill height, consistent with results discussed above.

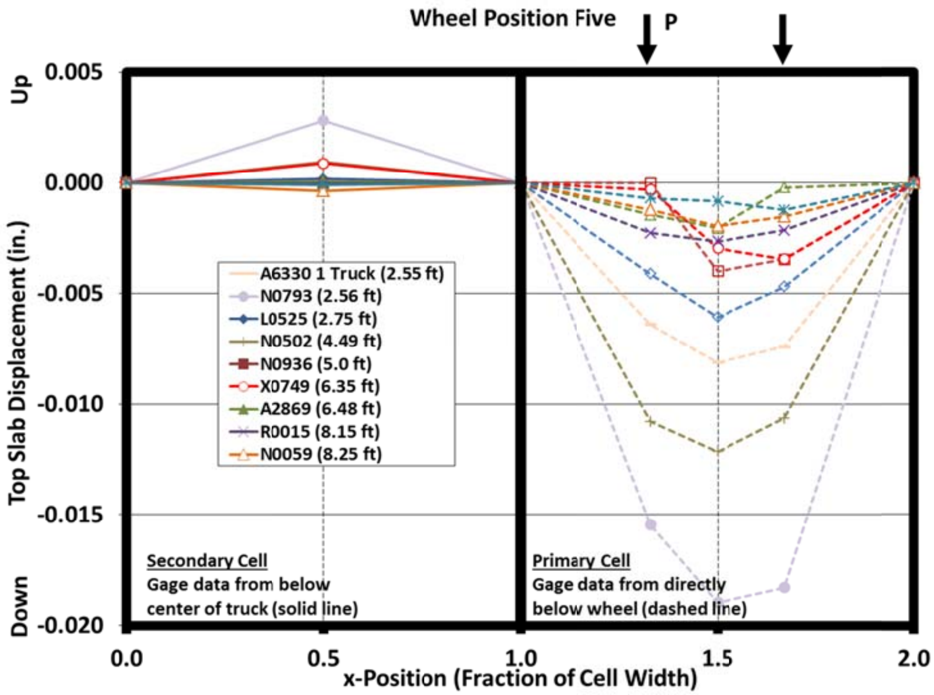

Figure 3-13– Displacement profiles across culvert cross-section for truck in Position 5 for all 10 tests.

Figure 3-14 shows profiles of displacement along the length of the culvert (i.e. looking at the culvert from the backfill) for each test with the truck in Position 5. The shapes of the profiles are all similar, and they all indicate less displacement in the side of the culvert that was not loaded (i.e. under the lane of traffic opposite the truck), especially for the tests with shallower fill heights. The magnitude of the displacement values for each test generally decreases with increasing fill height, consistent with results discussed above. Furthermore, the gradient of the displacement values versus distance from the truck location decreases with increasing fill depth indicating that the greater fill depths distribute the load to a wider area.

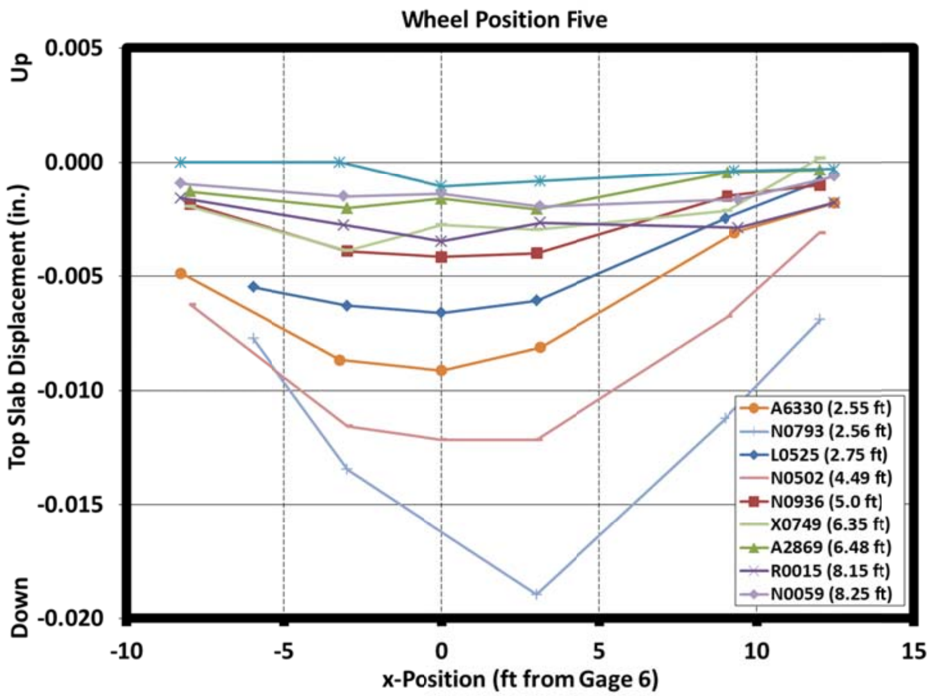

Figure 3-14 – Displacement profiles along length of culvert for truck in Position 5 for all 10 tests. Gage 6 (the reference point for x values) was always centered below the truck (i.e. in the center of the loaded traffic lane).

Figure 3-15 shows two profiles of displacement across the cross section of the culvert (i.e. looking through culvert if standing in the stream) for the test of A6630: one with one truck, and one with two trucks side-by-side. A6330 was the only culvert tested with two trucks. The results indicate an increase in displacement with the addition of the second truck, as expected. At the center of the primary cell, the increase is about 40 percent, relative to the value for one truck (0.008 in. versus 0.011 in.).

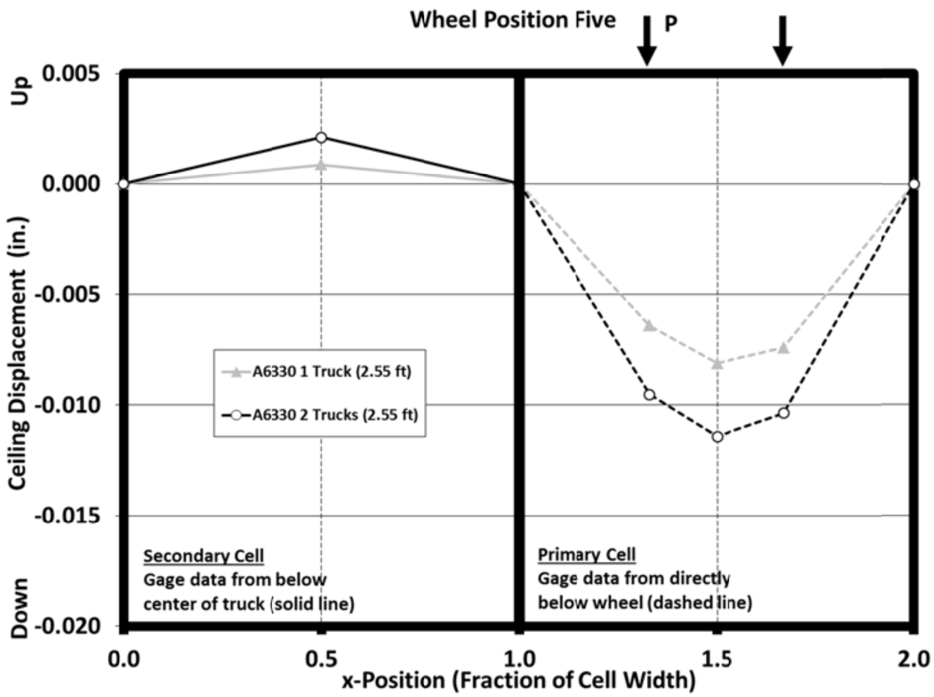

Figure 3-15 – Displacement profiles across culvert cross-section for Culvert A6330 with one truck in Position 5 and two trucks in Position 5.

3.4 Analysis of Culverts

Analysis of the culverts was undertaken to determine the distribution of live load through the fill depth. This analysis will help establish a depth at which the live load could be considered negligible. The finite element analysis was conducted using the program Phase II. The analysis was then compared to current LFD and LRFD design procedures.

3.4.1 Phase II analysis

Phase II is a 2-dimensional elasto-plastic finite element program for calculating stresses and displacements around underground openings. Each concrete box culvert was modeled and analyzed with the Phase II program. Assumptions were made about the soil and structural properties to complete each model and set a basis for a parametric study of soil properties effects on live load distribution factors. Borings were completed at each site to determine soil classification and basic properties. For modeling purposes, these properties were similar at each site and assumed to be the same. Manipulation of assumed soil properties was found to have insignificant effects on the program's desired intention, which was to observe load transfer distribution through the soil above the excavation.

Structural assumptions were made based on an elastic, un-cracked concrete slab. The concrete in the culvert was modeled using elements with the modulus of elastic for concrete. An assumed modulus of elasticity was used to model the overlying roadway surface.
A complete list of assumptions is as follows:

- Modulus of Elasticity for Concrete is 3,600 ksi.
- Modulus of Elasticity for the Roadway is 2,900 ksi.

- Concrete Unit Weight is 150 pcf.
- Roadway Unit Weight is 140 pcf.
- Soil Unit Weight is 120 pcf.
- All soils considered to be plastic.
- All "structural soils" considered to be elastic.
- Peak and Residual friction angles for plastic soils is 25 degrees.
- Peak and Residual friction angles for elastic soils is 35 degrees.
- Cohesion for all soils is 220 psf.
- Poison's Ratio is 0.3 for all soils.
- No vertical deflection is allowed in the foundation beneath the culvert.
- Field Stress Type is Gravity.
- Total Stress Ratio (horiz/vert for in/out of plane) is equal to 0.5.
- Only 3 inches of roadway pavement thickness is used.
- Two concentrated loads spaced 4 ft on center.

The truck load used in the Phase II program consisted of two 9 kip loads spaced at 6 ft which represented one wheel line of the dump truck used to test the culverts. Considering Phase II is only a 2-dimensional modeling program, the live load in the longitudinal direction must be accounted for. Therefore, the vertical stresses found in the program were divided by the distribution width according to the AASHTO LRFD design equation. The purpose of the Phase 2 models was to focus on the live load stress distribution, so the dead load was subtracted out in all analyses.

Figure 3-16 shows the stress distribution in the soil for culverts A6330 and P0622. As can be seen in the figure for A6330, which has a low fill depth, the stresses in the soil are directly distributed on the top slab of the culvert with the highest stresses being between the two wheel loads. For P0622, which has a high fill depth, the stresses are spread out in the soil to cause a more evenly distributed stress on the top slab. Figure 3-17 shows the distribution of vertical stresses just above the top slab coming from the live load for all culverts. As can be seen in the figure the highest stresses are for the culverts with the lowest fill depths. In addition, the gradient of the stress versus distance is much more pronounced at the lower fill depths. For the greater fill depths the pressure is more evenly distributed across the culvert.

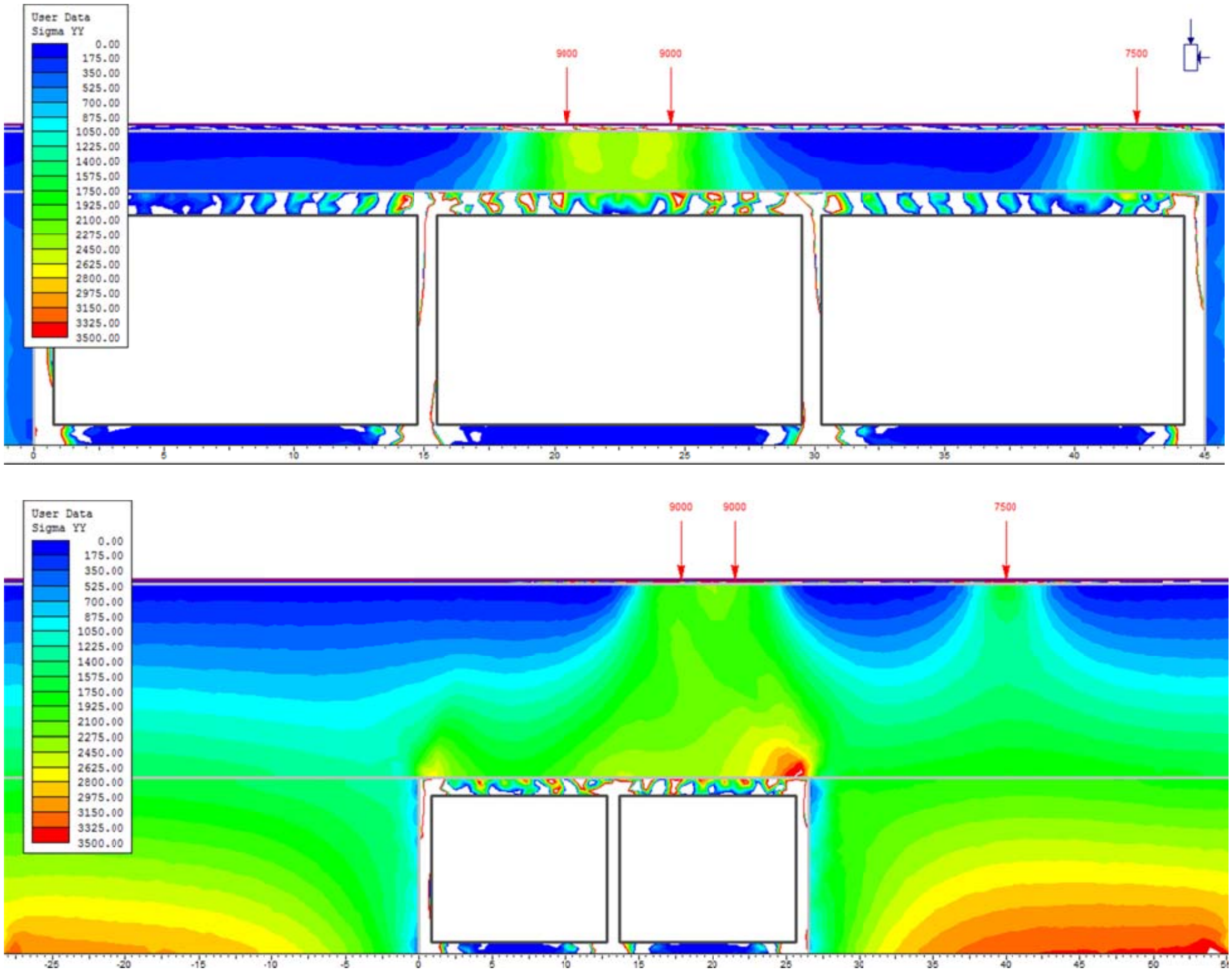

Figure 3-16 – Phase II vertical stress distribution in soil for culvert A6330 and P0622

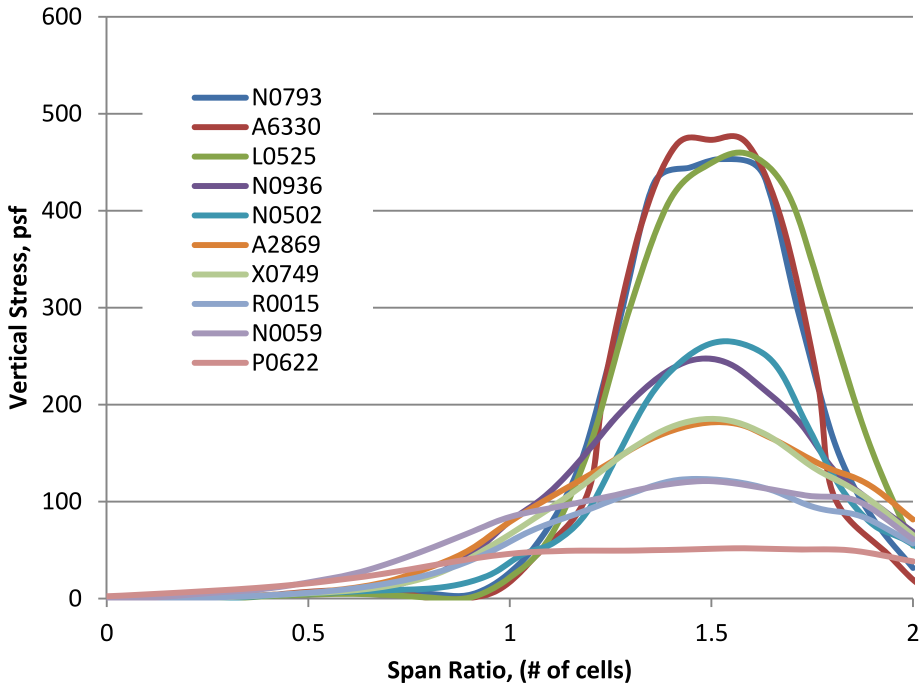

Figure 3-17 – Vertical stresses just above top slab due to live load from Phase II analysis

3.4.2 Design loads

Design loads were calculated based on AASHTO procedures so that the loads could be compared to the field data and Phase II analysis. Although dead loads were not measured in the field data, information about the percentage of dead to live load may help determine a cutoff depth for load rating. Dead loads overlying the box culvert are taken as the unit weight of the soil, assumed to be 120 pcf, multiplied by the fill depth of the soil. The weight of the soil is increased due to the soil-structure interaction by a factor of F_e.

$$F_e = 1 + 0.20\frac{H}{B_c} \leq 1.15 \quad \text{Equation 3}$$

Where H is the fill depth and B_c is the total width of the culvert normal to the centerline. For the culverts in this study the vertical stress in the soil due to dead load is given in

Table 3-6.

Table 3-6 – Design dead loads in culvert

Culvert	Fill Depth (ft)	Design Dead Load (plf)
A6330	2.55	450
N0793	2.56	438
L0525	2.75	474
N0502	4.49	681
N0936	5	743
X0749	6.35	927
A2869	6.48	931
R0015	8.15	1166
N0059	8.25	1147
P0622	13.25	1816

The live load projected onto the culvert is distributed across lengths in the longitudinal and transversal directions. These dimensions are represented by E_1 and E_2, respectively (Figure 2-2), and account for the wheel contact area and distribution of load through the overlying soil depth. Distribution through the fill depth (H) is given as an effective distribution length (E) in Equation 4. E varies according to AASHTO LFD or LRFD designs. The 2012 AASHTO LRFD Standard Section 3.6.1.2.6 gives an LLDF value of 1.15 for the case of select granular backfill and 1.0 for all other cases. AASHTO specifications (3.6.1.2.6) for the tire contact area (L_T) are 20 in. (1.67 ft) in the transversal direction and 10 in. (0.83 ft) in the longitudinal direction. The 2002 AASHTO LFD Standard Section 6.4.1 gives a value of 1.75 and does not account for the wheel area. Smaller LLDF factors result in narrower load distributions and greater uniform loads on the culvert top slab.

$$E = L_T + LLDF * H \qquad \text{Equation 4}$$

The distribution area is calculated as E_1*E_2 and the corresponding stress on the culvert is calculated as $LL=P/(E_1*E_2)$, where P is the wheel load. The wheel load P is initially taken as a single wheel load applied to the contact area mentioned above. As the fill depth increases, the load distributions overlap and the new distribution lengths should be taken as the total length between the outer distribution slopes. In the longitudinal direction, the rear axles are spaced 4 ft on center. In the transversal direction, the wheel lines are spaced 6 ft on center. The distribution lengths will vary according to the LLDF used. When the load distribution of the axles overlap, the distribution length is increased by the axle spacing and the load is doubled. The resulting stress distributed across the adjusted area will be found using the concentrated load of 2*P. When the load distribution of the wheel lines overlap, the transversal distribution length is increased by the wheel line spacing and the load is increased to 4*P. The fill depths at which the load and load distribution lengths increase can be seen in Table 3-7.

Table 3-7 - Fill Depths at which the distribution lengths and load must be increased to account for multiple wheel/axle loads.

E_1	Fill Depth >	3.17	ft	Factor:	1.00	LRFD
	Fill Depth >	2.75	ft		1.15	LRFD
	Fill Depth >	2.29	ft		1.75	LFD
E_2	Fill Depth >	4.33	ft	Factor:	1.00	LRFD
	Fill Depth >	3.77	ft		1.15	LRFD
	Fill Depth >	3.43	ft		1.75	LFD

Other considerations for design live loads include stresses on the bottom slab and outer walls of the culverts. The uplift pressures on the bottom slab of the culvert are considered to be the total live load on the top slab divided by the entire length of the culvert. This results in an equal uplift force that is distributed across the full length of the bottom slab in counterpart to the more concentrated top slab live load.

Table 3-8 gives the design live loads on the top slab based on a 9 kip wheel load. According current design procedures the design wheel load is actually 16 kips (half a 32 kip axel on a HS 20 truck). However, in order to make the design loads comparable to the field data (which used a loaded dump truck), a 9 kip wheel load was used. The difference in moment between the 4 rear 9 kip wheels used in the field and subsequent analysis and the 2 rear 16 kip wheels used in design is less than 8%.

3.4.3 Comparison of Phase II and design values

The Phase II analyses give the live load vertical stress distribution across the top slab. In order to make comparisons of these values to the design values an approximate uniform distribution was found. This distribution is based on a load value that is 90% of the peak value and a distribution width set so that the area under the Phase II live load curve is the same as the area under the uniform distribution. An example of the two distributions can be seen in Figure 3-18. These values are compared to what would be computed as the design loads per the AASHTO LFD (1.75 LLDF) and LRFD (1.15 LLDF) procedures. The design values are calculated without any load factors and are based on a 9 kip wheel load so that they are comparable to the field and Phase II data. The widths for the lower fill depths (where the loading does not overlap) are doubled to account for both of the rear axles. As seen in Table 3-8 and Figure 3-19 the loads and the widths from the LRFD (1.15 LLDF) are close to those predicted by the Phase II model. This indicates that the simplified distribution used in the design equations is close to the computer model.

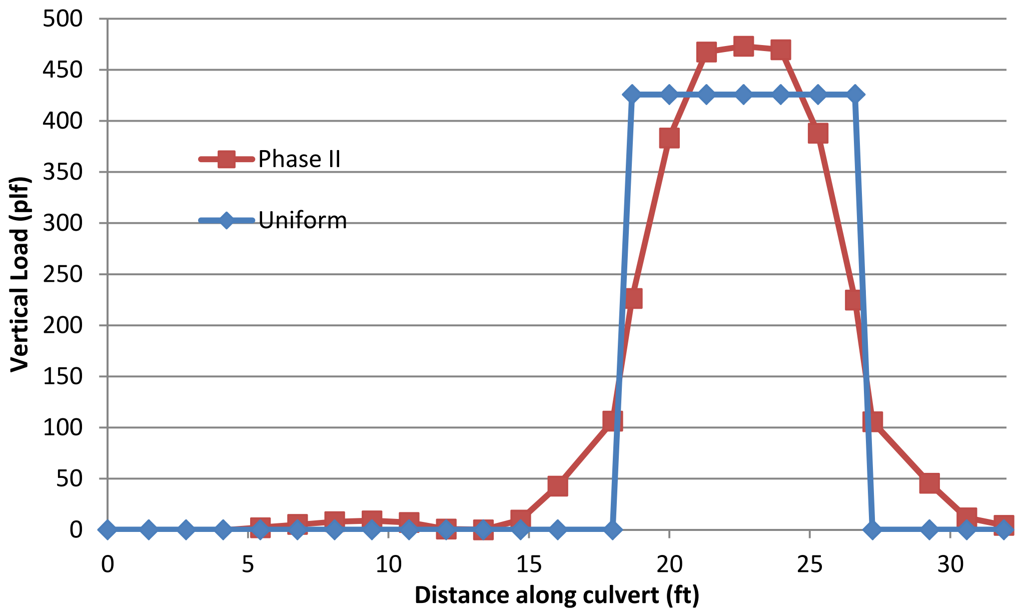

Figure 3-18 – Phase II and equivalent uniform distribution for culvert A6330

Table 3-8 – Phase II and design loads and distribution widths

Bridge #	Fill Depth (ft)	Peak vertical load in Phase II (plf)	Equivalent uniform Phase II load (plf)	Distribution width for equivalent load (ft)	Uniform load per AASHTO LFD (plf)	Width per AASHTO LFD (ft)	Uniform load per AASHTO LRFD (plf)	Width per AASHTO LRFD (ft)
A6330	2.55	355	426	9.3	452	8.9	520	7.5
N0793	2.56	340	408	9.7	448	9.0	517	7.5
L0525	2.75	342	412	9.1	389	9.6	466	8.0
N0502	4.49	189	237	9.6	183	12.7	281	10.0
N0936	5	176	222	10.9	161	13.6	254	10.6
X0749	6.35	130	166	12.0	120	15.9	198	12.1
A2869	6.48	127	163	11.9	117	16.2	194	12.3
R0015	8.15	85	110	14.7	86	19.1	149	14.2
N0059	8.25	84	109	13.4	85	19.3	147	14.3
P0622	13.25	35	47	19.7	42	28.0	78	20.1

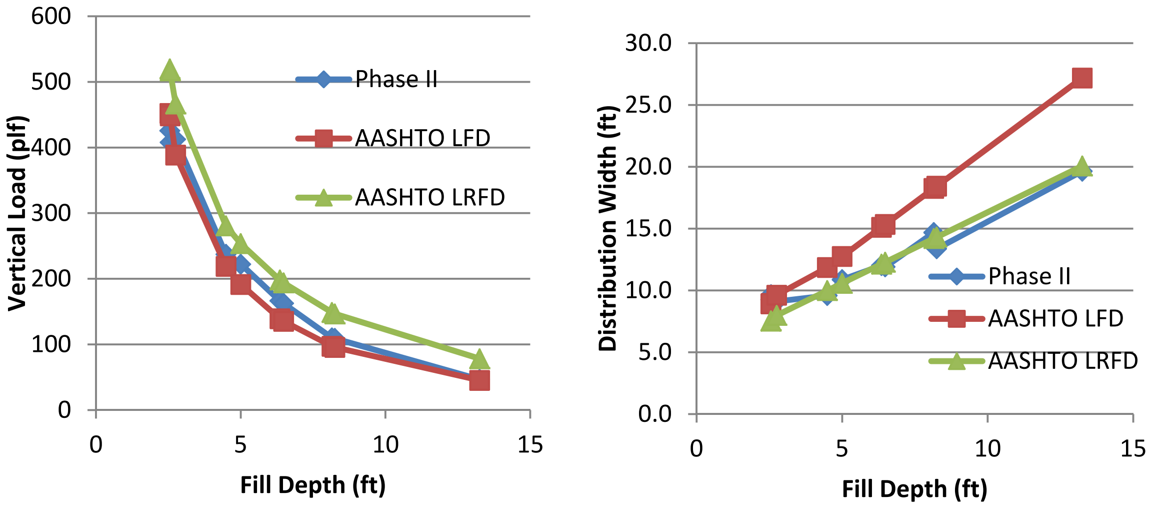

Figure 3-19 – Comparison of Phase II and design loads and distribution widths

3.5 Predicted moments and vertical loads from field measurements

In order to compare the vertical loads from the Phase II and design analysis, the strains and displacements from the field data were converted into moments and loads. In order to convert the strain measurement into a moment Equation 5 was used.

$$\varepsilon E = \sigma = \frac{Mc}{I} \qquad \text{Equation 5}$$

Where ε is the field strain, E is the Young's modulus of the concrete (assumed to be 3600 ksi), σ is the stress in the slab, M is the moment, c the neutral axis depth (assumed to be ½ the slab thickness), and I the uncracked moment of inertia. Although reinforced concrete is generally considered to be cracked in service with a moment of inertia value equal to 0.3 times the gross moment of inertia, the uncracked moment of inertia was used because it represented the most conservative estimate of the moment. Therefore the moments and distributed loads calculated from the field data could be as little as 30% of those reported in this report.

Once the moment from the field strain was determined, a computer model of the culvert was constructed in SAP 2000 and a uniform load over a distribution width equal to the AASHTO LRFD values was found so that the maximum positive moment in the model equaled the field based moment. The same procedure was followed with the displacements. Figure 3-20 shows the SAP model and uniform load for culvert A6330. Table 3-1 gives the calculated moments and loads based on the field data.

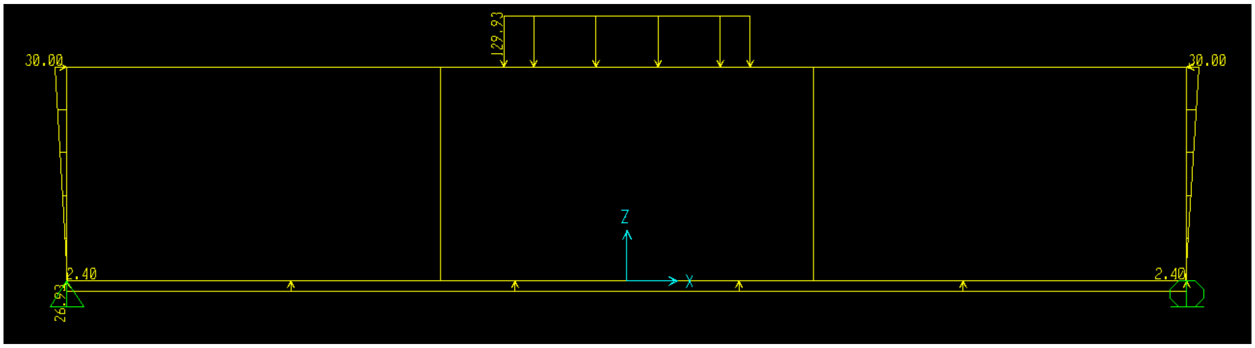

Figure 3-17.1.

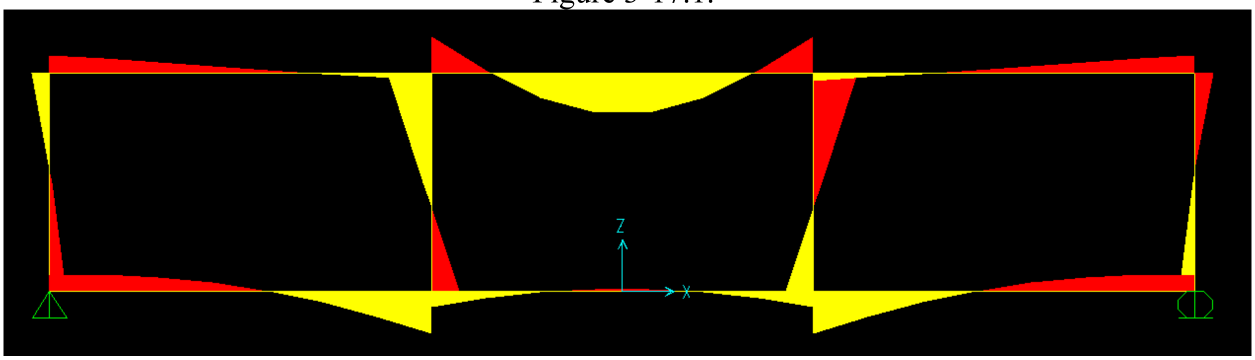

Figure 3-20 – Design Live Loads and moment diagram on culvert A6330

Table 3-9 – Moments and loads calculated from field data

Culvert	Fill Depth	Maximum field strains	Moment from strain (kip-in)	Distributed load from strain (plf)	Maximum field displacements (in)	Moment from displacement (kip-in)	Distributed load from displacement (plf)
A6330	2.55	1.97E-05	17.2	140	0.0091	20.0	163
N0793	2.56	4.26E-05	30.7	221	0.0195	28.7	207
L0525	2.75	1.19E-05	10.4	141	0.0069	19.7	269
N0502	4.49	2.17E-05	17.2	118	0.0130	21.8	149
N0936	5	1.83E-05	14.5	130	0.0042	9.5	85
X0749	6.35	1.35E-05	14.0	111	0.0042	13.4	106
A2869	6.48	7.31E-06	6.4	63	0.0021	6.2	61
R0015	8.15	9.22E-06	12.1	75	0.0029	11.0	68
N0059	8.25	1.19E-05	10.4	84	0.0024	6.2	51
P0622	13.25	5.51E-06	9.5	66	0.0020	12.8	89

3.5.1 Fixed versus Pinned Supports

The preceding analysis was based on the assumption that all connections in the culverts are fixed, however only culverts A6330 and N0502 were designed as fixed connections. Based on structural analysis, a culvert with fixed connections would see higher negative moments near the culvert wall and lower positive moments at the center span. Table 3-10 gives the predicted moments for the design live load based on both fixed and pinned connections and shows approximately a 40% increase in positive moment if the connections are pinned.

Table 3-10 Moments based on fixed or pinned connections

Culvert	Fill Depth (ft)	Positive moment based on fixed ends (kip-in)	Negative moment fixed ends (kip-in)	Positive moment with pinned ends (kip-in)
A6330	2.55	63.7	64	81
N0793	2.56	71.6	60.2	107
L0525	2.75	34.27	33.5	50.53
N0502	4.49	41	48	54
N0936	5	28.38	36.15	44.75
X0749	6.35	25.1	27.3	37.02
A2869	6.48	19.5	24.2	31.2
R0015	8.15	23.89	23.44	26.5
N0059	8.25	18	20.7	27
P0622	13.25	11.27	9	16.2

The field strain data showed both fixed and pinned like behavior in the culverts, as discussed earlier. A comparison of the calculated distributed loads (based on the measured field strains) based on the assumption of fixed or pinned connections are given in Figure 3-21. The assumption of a pinned connection causes the back-calculated distributed load to be about 70% of that based on the assumption of a fixed connection. Because the true behavior of the culverts lies between the fixed and pinned connections, the following calculations and conclusions are based conservatively on the fixed connection assumption.

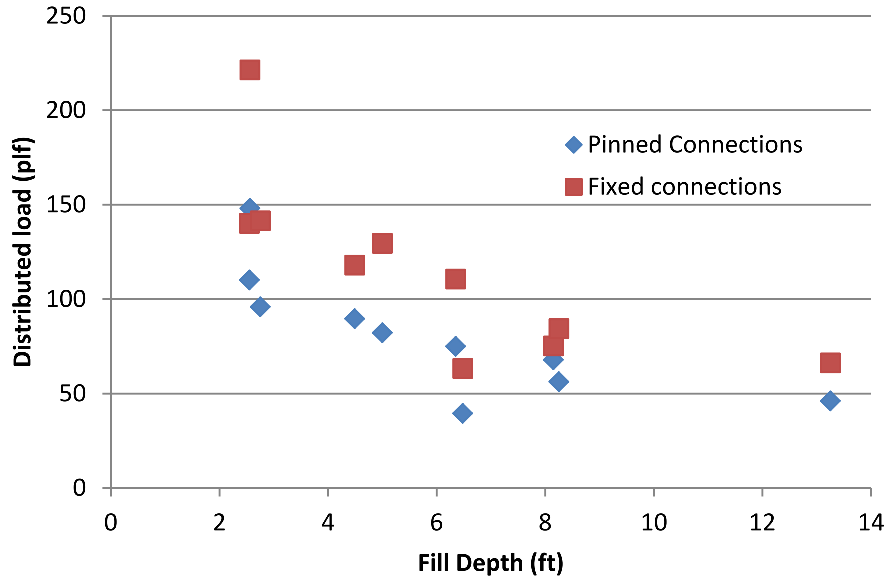

Figure 3-21 Comparison of distributed load based on fixed or pinned connection

3.6 Load Ratings

Another way to interpret the results is to consider how much the truck weight would have to increase to reach the ultimate moment capacity of the reinforced concrete section. This is essentially the LFD based load rating given in Equation 2. To determine the rating factor the positive moment capacity of the top slab was found and compared to the dead plus the live load. Figure 3-22 shows the rating factor plotted against the fill depth. As can be seen in the figure all of the culverts are over-designed. There is less correlation with fill depth but there does seem to be a slight increase in rating factor as the fill depth increases. The data is distorted by the fact that the culverts were designed at different times for different design vehicles. Overall, the design of the culverts is very conservative.

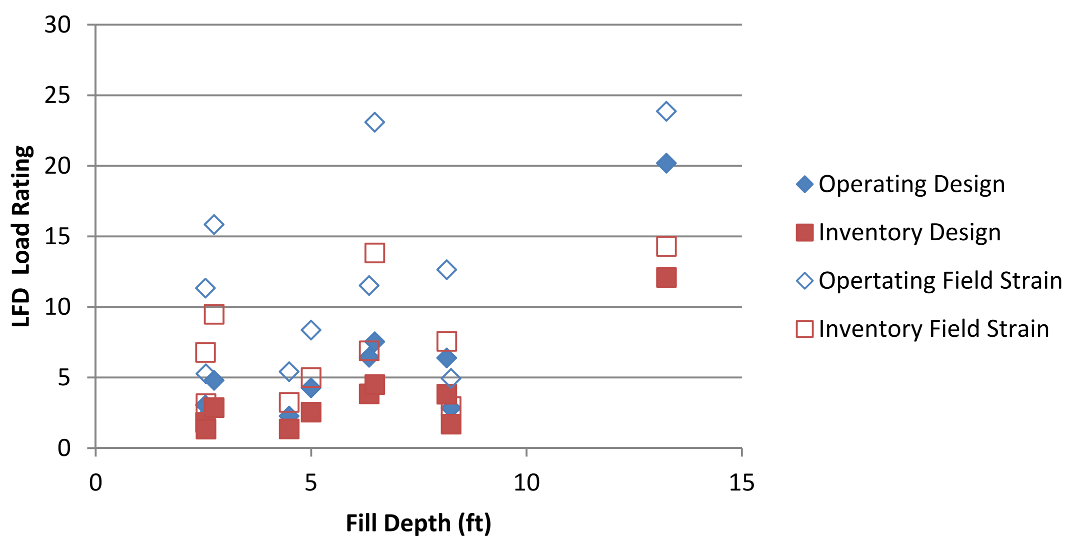

Figure 3-22 – Rating factors for culverts

3.7 Comparison of Phase II, design, and field loads

Figure 3-23 and Figure 3-24 show the comparison of the vertical loads on the top slab and positive moment, respectively, versus fill depth based on the Phase II, design, and field data. The Phase II results are very close to the design results. However, the field data shows significantly smaller values than both the Phase II and design, especially at the lower fill depths. Phase II can only model the soil in 2D; it cannot account for additional distribution longitudinally in the culvert, which may be important at low fill depths. In addition the two-way action of the reinforced concrete slab plays a role. To investigate the effect of two-way action, a shell element model of the top slab of culvert L0525 was made in SAP 2000. When the entire top slab was modeled and pressures placed on the slab according to the AASHTO LRFD design, a stress of 172 psi was generated at the bottom of the slab. When only a one foot wide slice of the slab was modeled, representing the purely one-way action assumed in design, the stress increased to 327 psi. The difference in the stresses is due to the two-way action in the slab. In the field the two-way action was measured via strains longitudinally in the culvert as given in Table 3-4. The two-way action would be less prevalent in slabs with a higher fill depth in which the load is more uniformly distributed. The difference between field and design values may also be due to the effect of the pavement helping to distribute the load being more pronounced at lower fill depths as found in Acharya (2012).

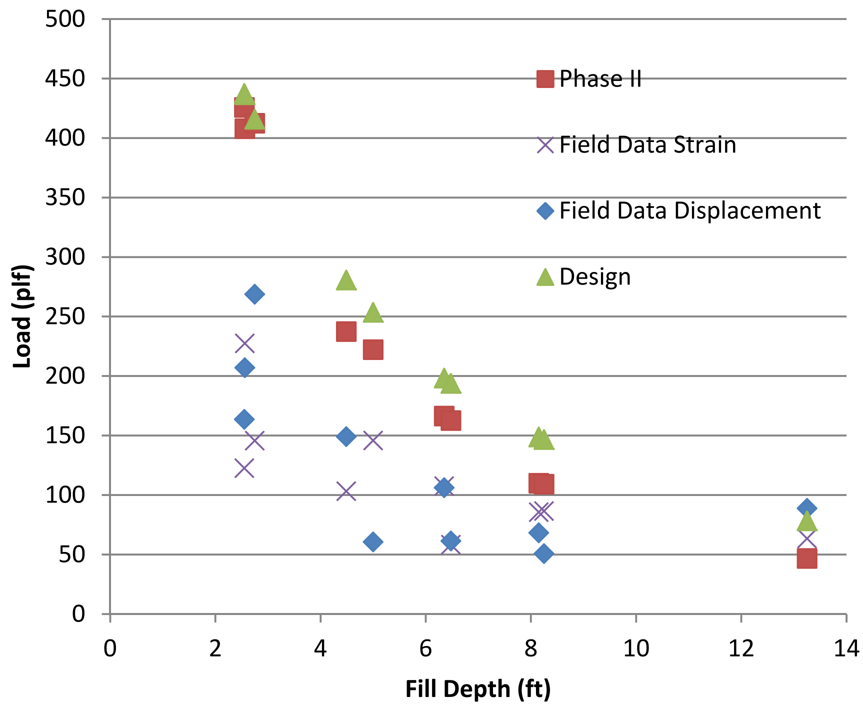

Figure 3-23 – Comparison of vertical load based on Phase II, design, and field data

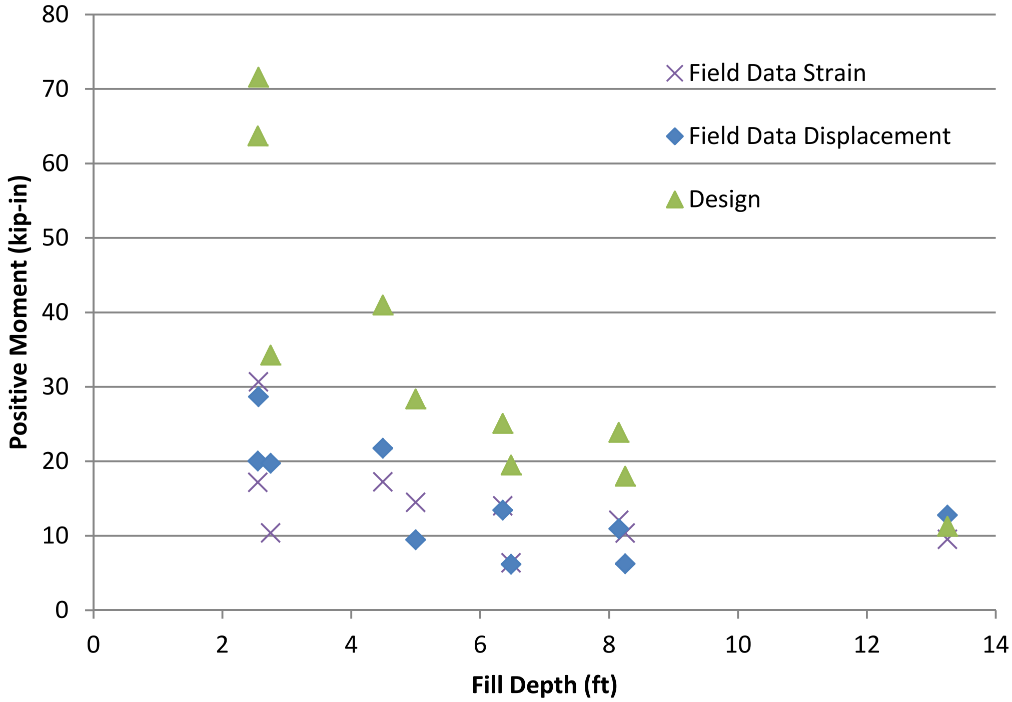

Figure 3-24 - Comparison of positive moment based on design and field data

4 RESULTS AND DISCUSSION

4.1 Proposed fill depth for load rating cutoff

The testing results show clearly that as the fill depth is increased the effect of the live load on the culvert is diminished. The question that remains is at what depth can the effect of the live load be ignored. Abdel-Karim et al. (1990) say the point at which to ignore the live load could be when the live load effect is less than 5 percent of the total load effect. On the other hand Gilliland (1986) mentions that this point could be when the live load pressures are less than 10 percent of the pressures due to the soil load only. Figure 4-1 gives the ratio of the live load to the total load for both the field data and the unfactored AASHTO LRFD design loads. According to the field data the point at which the live load is less than 5% of the total load occurs around 10 ft of fill. Figure 4-2 gives the ratio of the positive moment due to live load and positive moment due to total dead plus live load. Again the point at which live moment is less than 5% of the total moment is around 10 ft. Figure 4-3 and Figure 4-4 gives the ratio of the live to dead load and moment, respectively. According to the field data the point at which the live load is less than 10% of the dead load occurs around 6 ft of fill.

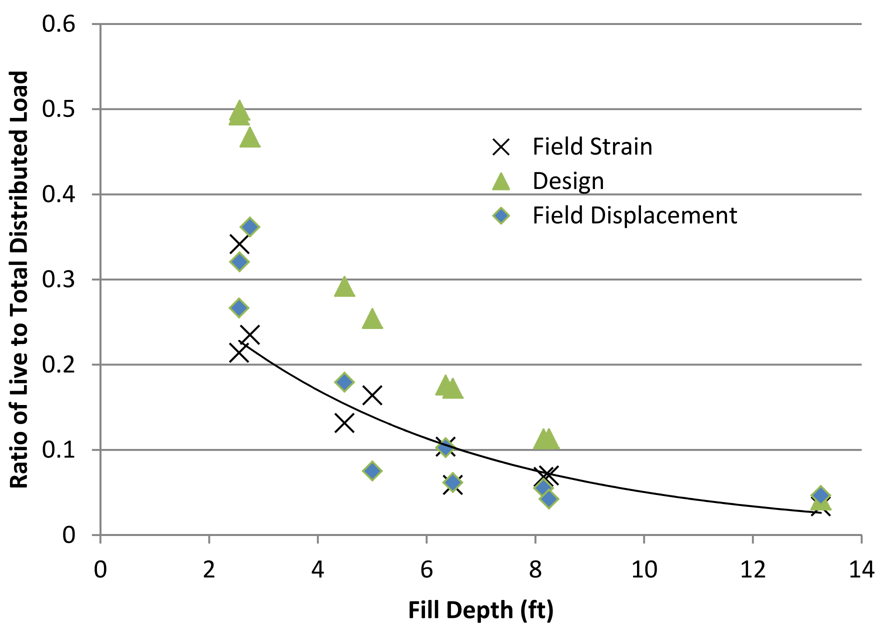

Figure 4-1- Ratio of live load to total load

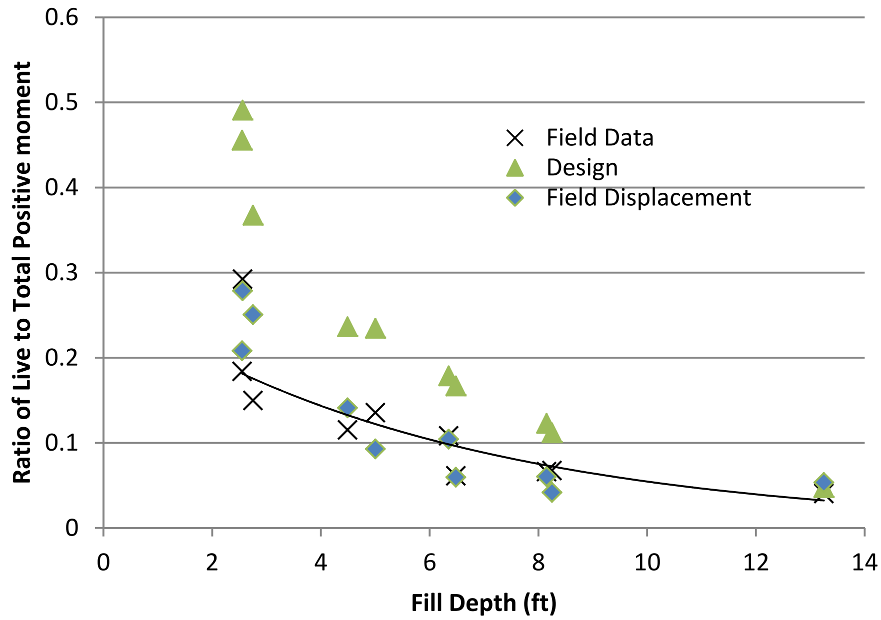

Figure 4-2 - Ratio of live load to total moment

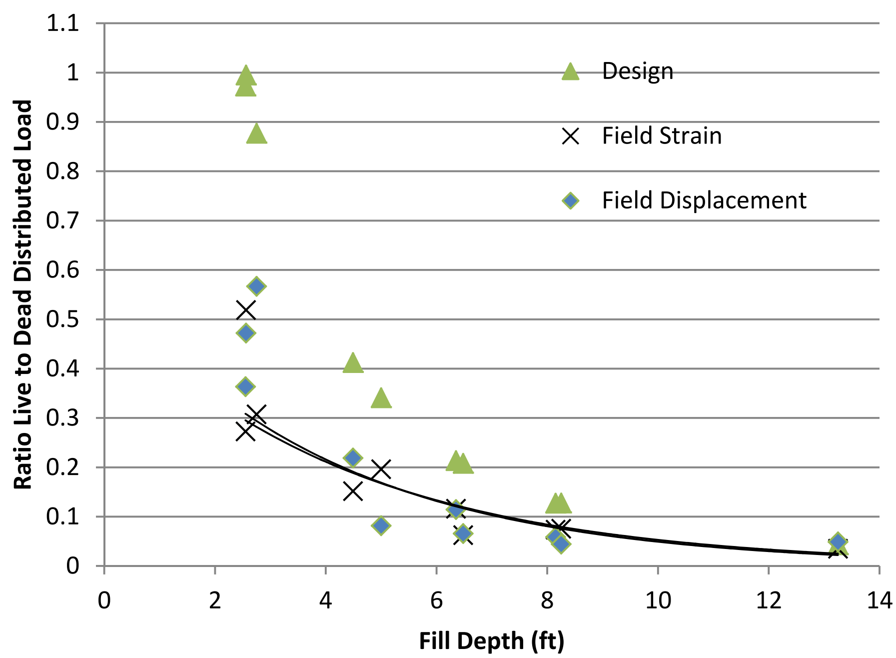

Figure 4-3 – Ratio of live load to dead load

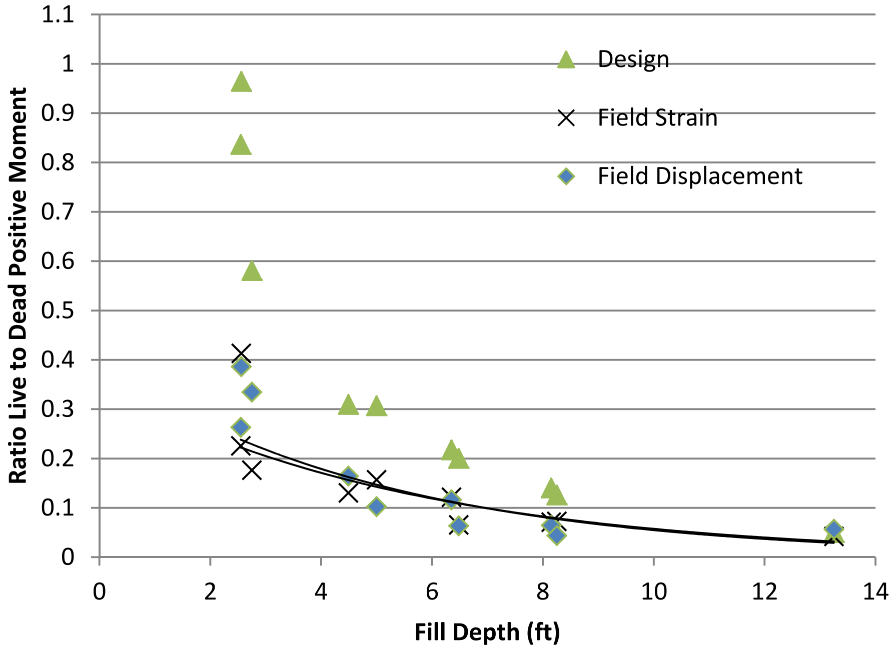

Figure 4-4 - Ratio of live load to dead moment

It is important to note that in the above figures the design live load is based on a 9 kip wheel load so that it is comparable to the field data which used dump trucks with approximately 9 kip wheel loads. In the AASHTO LRFD design the wheel load would correspond to a HS 20 truck that has a 32 kip axle and a 16 kip wheel load. When the HS20 truck loading was used in culverts N0793 and N0936 the difference in the positive moment was less than 8%.

In addition there were many assumptions in the preceding calculations to determine the distributed load and moment from the field data. These assumptions led to a conservative estimate of the distributed loads.

- Multiple trucks – the calculations assume only one tuck on the culvert. For culvert A6330 (2.55 ft of fill) a field test with two trucks resulted in a 50% increase in the strains measured in the culvert.
- The analysis of the culverts was made on the assumption that the connections of the top slab to the wall behaved as fixed, even though most were designed as pinned. The strain data indicated that there was some fixity in the behavior of all culverts. The assumption of fixed connections reduced the calculated distributed loads by about 70%.
- The analysis of the culverts was based on the assumption that the concrete section was not cracked. If a cracked section were used the moments and distributed loads from the field data could be reduced to as little as 30% of the reported values.
- Two-way action was found to be significant in helping to distribute the live load at shallow fill depths. Two-way action can reduce the stress in the top slab by as much as 50% at a fill depth of 2.75ft. The effect of two-way action diminishes as the fill depth increases and the load distribution becomes more uniform.

5 CONCLUSIONS

Field testing measured the strains and displacements of 10 culverts with varying fill depths. The field data showed that the culverts with the lowest fill depths had the highest strains and displacements. The variance of the data was greater at lower fill depths than at the higher depths indicating that the design and condition of the culvert is likely more influential at the lower fill depths. The distribution of the strains and displacements were consistent with what would be expected from structural analysis, and their values were comparable to those from previous research. The strain and displacement data were analyzed to compute representative moments and distributed live loads on the culvert top slab. These loads were compared to what would be predicted using AASHTO design equations, and a finite element analysis (Phase II). The field data compared well with the design and Phase II analysis at fill depths greater than about 8 ft. At smaller fill depths the field data were significantly less than the design values likely due to additional distribution of the loads via two-way action in the slab.

The point at which to ignore live load effects could be taken as where the live load effect is less than 5% of the total load. This point would correspond to about 10 ft of fill based on the field data. Another possible cutoff is where the live load is less than 10% of the dead load which would correspond to about 6 ft of fill. In all cases the design of the culverts is very conservative with the ability to carry 10 times or more of the dump truck weight that was applied to the culvert.

6 RECOMMENDATIONS

The field testing of the culverts showed that as fill depth increases the effect of the live load does diminish. At approximately 6 ft of fill the live load on the culvert (based on the field data) is approximately 10% of the dead load. This is a possible point at which to no longer load rate the culverts as the increase in live load has little effect on the culvert.

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