

University Transportation Research Center - Region 2

Final Report



Traffic Volume Estimation using Network Interpolation Techniques

Performing Organization: Rensselaer Polytechnic Institute





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University Transportation Research Center - Region 2

The Region 2 University Transportation Research Center (UTRC) is one of ten original University Transportation Centers established in 1987 by the U.S. Congress. These Centers were established with the recognition that transportation plays a key role in the nation's economy and the quality of life of its citizens. University faculty members provide a critical link in resolving our national and regional transportation problems while training the professionals who address our transportation systems and their customers on a daily basis.

The UTRC was established in order to support research, education and the transfer of technology in the field of transportation. The theme of the Center is "Planning and Managing Regional Transportation Systems in a Changing World." Presently, under the direction of Dr. Camille Kamga, the UTRC represents USDOT Region II, including New York, New Jersey, Puerto Rico and the U.S. Virgin Islands. Functioning as a consortium of twelve major Universities throughout the region, UTRC is located at the CUNY Institute for Transportation Systems at The City College of New York, the lead institution of the consortium. The Center, through its consortium, an Agency-Industry Council and its Director and Staff, supports research, education, and technology transfer under its theme. UTRC's three main goals are:

Research

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The modern professional must combine the technical skills of engineering and planning with knowledge of economics, environmental science, management, finance, and law as well as negotiation skills, psychology and sociology. And, she/he must be computer literate, wired to the web, and knowledgeable about advances in information technology. UTRC's education and training efforts provide a multidisciplinary program of course work and experiential learning to train students and provide advanced training or retraining of practitioners to plan and manage regional transportation systems. UTRC must meet the need to educate the undergraduate and graduate student with a foundation of transportation fundamentals that allows for solving complex problems in a world much more dynamic than even a decade ago. Simultaneously, the demand for continuing education is growing – either because of professional license requirements or because the workplace demands it – and provides the opportunity to combine State of Practice education with tailored ways of delivering content.

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UTRC's Technology Transfer Program goes beyond what might be considered "traditional" technology transfer activities. Its main objectives are (1) to increase the awareness and level of information concerning transportation issues facing Region 2; (2) to improve the knowledge base and approach to problem solving of the region's transportation workforce, from those operating the systems to those at the most senior level of managing the system; and by doing so, to improve the overall professional capability of the transportation workforce; (3) to stimulate discussion and debate concerning the integration of new technologies into our culture, our work and our transportation systems; (4) to provide the more traditional but extremely important job of disseminating research and project reports, studies, analysis and use of tools to the education, research and practicing community both nationally and internationally; and (5) to provide unbiased information and testimony to decision-makers concerning regional transportation issues consistent with the UTRC theme.

UTRC-RF Project No: 49111-23-24

Project Date: December 2013

Project Title: Traffic Volume Estimation using Network Interpolation Techniques

Project's Website:

http://www.utrc2.org/research/projects/improving-seasonal-adjustment-factors

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Sponsor:

University Transportation Research Center - Region 2, A Regional University Transportation Center sponsored by the U.S. Department of Transportation's Research and Innovative Technology Administration

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	1		1	
1. Report No.	2.Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle			5. Report Date	
Traffic Volume Estimation using Networ	k Interpolation Techniqu	es: An Application on	December 31, 2013	
Transit Ridership in NYC Subway System				<u> </u>
			6. Performing Organization	Lode
7. Author(s)			8. Performing Organization	Report No.
Dapang Zhang, Viackup (Caro) Wang			49111-23-24	
Dapeng Zhang, Xiaokun (Cara) Wang			49111-23-24	
9. Performing Organization Name and Address			10. Work Unit No.	
Rensselaer Polytechnic Institute				
110 8th Street, Troy, NY, 12180			11. Contract or Grant No.	
			The Contract of Grant No.	
12. Sponsoring Agency Name and Address			13. Type of Report and Peri	od Covered
University Transportation Research Center	-r		Final Report	
City College of New York-Marshak 910			-	
160 Convent Avenue			14. Sponsoring Agency Cod	le
New York, NY 10031				
15. Supplementary Notes				
16. Abstract				
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the fact that subway stations are only com				-
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17. Key Words		18. Distribution Statement		
Network distance; Kriging; Spatial econo	metrics			
Network distance, Kriging, Spatial econo	meures			
10 Security Classif (of this report)	20 Security Classif (of this read	\ \	21 No of Doggo	22 Price
19. Security Classif (of this report)	20. Security Classif. (of this page)	21. No of Pages	22. Price
Unclassified	Unclassified			
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1. Introduction

Kriging method is a frequently used interpolation methodology in geography, which enables estimations of unknown values at certain places with the considerations of distances among locations. When it is used in transportation field, network distance is a better measurement of distance as traffic follows the network. The original work proposed was to use network kriging to estimate seasonal adjustment factors for the annual average daily traffic (AADT). The work later on changed the effort slightly. The key focus is still the development of the network kirging model, but the application is transit ridership in the NYC subway system. Revision on the scope of work is due to the following reasons: (1) the estimation of subway ridership is crucial to transportation planners, investors, and public agencies, especially for the region; (2) Kriging in subway ridership estimation is a reasonable and valuable research attempt, thus contributes to the literature; (3) After careful assessment, the PI has found that the available data at count stations are insufficient to support reliable estimation of AADT seasonal adjustment factor as the count station distribution is too sparse. The work thus develops the network Kriging method and demonstrates its application using the transit ridership in NYC subway system.

Reliable transit ridership estimation is important for passengers, transit companies, and public agencies. With reliable estimation, passengers can make confident decisions on their travel paths, modes, and departure time. Transit companies can assign proper capacity, make reasonable service schedules, and operate economically. Public agencies can propose cost-effective transportation investments, manage financial and labor force, and enhance sustainable city developments.

Most existing studies estimate transit ridership as part of travel demand modeling using four-step methods (Horowitz, 1984) or activity-based models (Hildebrand, 2003). The four step method predicts traffic patterns at an aggregate level while activity-based models predict individual travel behavior at disaggregate level. Both approaches are within the general framework of travel demand modeling, where transit ridership is treated as demand of a specific transportation mode. Although behavior consistent, estimating transit ridership as part of the overall travel demand has high data requirement. An alternative approach is to use regression models to build direct connections between ridership and a set of factors whose information can be easily obtained. These factors often include local demographic features, economic indexes, and geographic information, etc. Regression models present a quick and convenient alternative for ridership estimation. Current regression models tend to assume that the ridership estimates are independent across stations. However, a lot of the uncontrolled factors, such as crime rates in the neighborhood and conditions of sidewalk may

influence ridership, causing strong correlation between ridership estimates of nearby stations. The correlation weakens as the distance between stations get longer. There should be a way to fully recognize and utilize such spatial interdependency pattern instead of simply treating the unobservables as white noise.

In light of the deficiency of current regression models, this study uses a Network Kriging model to estimate transit ridership. Kriging models are often used to estimate unknown variables using known variables at nearby locations. A standard Kriging estimator is a weighted average of known variables where weights are determined by distance. A universal Kriging method can further consider the "shift" caused by local conditions. For ridership estimation, this means ridership at new stations can be estimated based on local factors and current ridership at existing stations. Furthermore, network distance, instead of Euclidean distance, is used to determine weights as it reflects the fact that subway stations are connected via subway tunnels. This study will estimate the subway ridership of a new subway line, the Second Avenue Subway, in New York City using Network Kriging model. Features of stations' geographical locations and network connectivity are captured in the model, resulting in more reliable ridership estimates.

The next section reviews current literature on transit ridership estimation and Kriging models. Data is then described and the model specification is discussed. Finally results are presented and analyzed, followed by conclusions.

2. Literature Review

2.1 Kriging Models

Kriging in geo-statistics is synonymous with "optimally predicting" in space, using observations with known values at nearby locations (Cressie, 1990). This method has been applied in a variety of research fields and derived to many sophisticated formats. For instance, Odeh et al. (1995) used a heterotopic cokriging and a regression-Kriging to predict soil properties. Goovaerts (2000) presented Kriging models for the spatial prediction of rainfall. Lefohn et al. (1987) used Kriging technique to predict seasonal mean ozone concentrations for estimating crop losses. There are also literatures incorporating Kriging in the transportation field. Briggs et al. (2000) modeled spatial patterns of traffic related air pollution, using Kriging method to generate accurate, high-resolution air pollution maps. Braxmeier et al. (2009) used Kriging methods to forecast the traffic on a road network in Berlin, Germany. Wang and Kockelman (2009) applied a Kriging model to predict annual average daily traffic using Texas highway count data. Given the strong predictive power of Kriging method for forecasting spatially distributed data, this study also adopts a Kriging model to forecast transit ridership.

Standard Kriging inference for spatially distributed variables is based on the relationship between distance and variability, which is called a semivariogram in Kriging profession. Various theoretical semivariogram have been proposed, and Dale Zimmerman and Bridget Zimmerman (1991) did a comprehensive comparison of those functions. Laslett (1994) also compared the choice of theoretical semivariogram functions and concluded that the precision of prediction is based on the real data. Typical application of Kriging relies on Euclidean distance, assuming spatial dependency over the continuous space. However, Euclidean distance may not be appropriate for certain occasions. For example, Hoef et al. (2006) developed a Kriging model that uses flow volume and stream distance in a research for predicting stream flows at unmeasured locations. Wang and Kockelman (2009) estimated annual average daily traffic using highway network distance measurement. The reason for using non-Euclidean distance is that locations are not related in Euclidean space, but through a certain intermedia. For example, the transport of smog is blocked by hills and mountains. Animals migrate around lakes, mountains, and settlements. The dependence of seasonal adjustments occurs over road networks. All these spatial autocorrelation occur over specific areas. which can be guite different from the continuous space. In the case of road network, the Euclidean distance between sites on two parallel roads could be short, but if the roads are not connected, the spatial dependence of these two sites could be minimal. In such a case, the "distance" in standard Kriging method needs to be updated with something that can indicate the special feature of network structure. As Wang and Kockelman

(2009) also indicated, "spatial autocorrelation functions based on network (rather than Euclidean) distances" would make the Kriging method more promising. Unfortunately, the conversion from Euclidean distance to other distance-related indicators is far from trivial: The computational burden can increase dramatically if non-Euclidean distances are used and sample size is large. The more challenging issue caused by the use of non-Euclidean distance is that the covariance matrix is no longer warranted to be positive semi-definite (PSD), which compromises the model validity.

2.2 Transit Ridership Estimation

The most commonly used methodologies of transit ridership estimation in practice are professional judgments, contracts with similar routes, service elasticities analysis (Litman, 2004), four step travel demand models, and econometric and regression models (Boyle, 2006). Among them, the first three methods are primarily used for route or service changes, while four step travel demand model is commonly used for new services. Although four step model has been popular for decades in ridership estimation, sometimes it is not a handy method in practice and the resolutions are too gross to capture refined built-in environment characteristics near transit stops (Cervero, 2006a). Econometric and regression models, on the other hand, are less costly to use (Marshall and Grady, 2006) and they can establish the relationship between a variety of influential factors Many current literatures have exploited regression methods. Standard multiple regression models appeared in a large amount of literatures to test various influential factors of transit ridership. (Gomez-Ibanez, 1996; Hendrickson, 1986; Kain, 1997; Kitamura, 1989; Kuby et al., 2004; Taylor et al., 2003; Wang and Skinner, 1984). Apart from the operation of transit systems, influential factors include regional geography, metropolitan economy, population characteristics, and auto/highway system characteristics, etc. (Taylor et al., 2009) Their magnitudes of effect are found different in case studies and should be analyzed on a case by case basis. Moreover, more advanced models are used to accommodate the nature of the ridership data generating process. For example, discrete outcome models are better techniques when the dependent variables are not continuous. Whether or not people choose transit is often treated as explained variables, and binary logit models are suitable in such cases (Baum-Snow and Kahn, 2000; Syed and Khan, 2000). Koppelman (1983) developed a simplified form of multinomial logit model on an application of travel mode shares for a set of transit services. Abdel-Aty (2001) used an ordered probit model to explain the likelihood of using transit based on a stated preference survey. Cervero (2006b) established nested logit models for explaining rail location and commute choices in order to promote transit oriented development in California. In addition, Simultaneous equations models are able to take fully use of available information and avoid potential the exogeneity problem. Transit ridership is always simultaneous correlated with its

demand/supply. Studies found such exogeneity problem did exist and simultaneous equations models can offer a better model fitting (Peng et al., 1997; Taylor et al., 2009). Another frequently used model is the time series model, which can extract ridership information from previous data and formulate the trends (Kain and Liu, 1999; Kyte et al., 1988).

All the aforementioned studies focus on the effects of controlled factors and time trend. while few attempt to reduce estimation errors by considering the spatial dependency of transit ridership, which is an important feature underlying the transit system. In the general field of transit research, a few studies have considered spatial dependency using spatial econometric methods. For example, Goetzke (2008) applied a spatial autoregressive logit model to formulate transit use in New York City. Kim and Zhang (2005) investigated the interaction between land price and transit use in Seoul using spatial autoregressive model, spatial error model, and spatial autocorrelation model. These models presume that dependent variables or error terms are correlated among observations. By assuming the spatial patterns, models' explanatory power normally increases. Another method with spatial consideration is geographically weighted regression (GWR). Chow et al. (2006) developed a geographically weighted regression (GWR) to improve the accuracy of ridership estimation using data of Broward County, Florida. Cardozo et al. (2012) also used a GWR model in a Madrid metro ridership analysis and concluded that GWR model had a better fit. GWR models allow for specifications of local spatial effects and capture the geographic heterogeneity of influential factors' effects. However, above mentioned models with spatial considerations lack of abilities to forecast values at locations that are new in the system. In light of the remarkable forecast ability of Kriging models, this study uses Kriging methods to investigate transit ridership, which has not yet studied by considering spatial dependency.

3. Data Description

The case study of this research is the proposed Second Avenue Subway line in the New York City (NYC) subway system.

The NYC subway is one of the oldest, most extensive, and busiest rapid rail systems in the world with an annual ridership of 1.665 billion in 2012 (MTA, 2012a). Stations are located throughout the boroughs of Manhattan, Brooklyn, Queens, and Bronx, and are mostly open 24 hours a day (MTA, 2012b). Manhattan, the study area, has a dense subway network with 20 lines. As shown in the left map on Figure 1, the service on Upper East Side of Manhattan is sparse, which is served only by Lexington avenue lines (MTA, 2003). Lexington avenue lines also work as the major transportation between Manhattan and Bronx. Moreover, Upper East Side has a high transportation demand because it is one of the most affluent neighborhoods in the City with many diplomatic missions, museums, hotels, and shopping centers. Currently, people have to suffer from the crowed service at stations along the Upper East Side. To relieve the traffic burden, the Metropolitan Transportation Authority (MTA) revealed a set of construction projects, including the one for the Second Avenue Subway from 125th street to Hanover Square (Bennett, 2009).

As proposed in 2006, the Second Avenue Subway construction consists of four phases (MTA, 2013b): the first phase is scheduled to complete and open to public at the end of 2016 (Donohue, 2013), aiming to split the flows on the most crowded segments of Lexington Avenue lines (96th street to 63rd street). Upon the accomplishment of the first phase, the current Q trains will be rerouted to serve Upper East Side and be connected with the current Broadway lines via existing 63rd Street line. A new designation of T train will serve the entire length of second avenue line, sharing tracks and stations with Q trains at the Upper East Side (Reeves, 2006). The plan of routes and stations are shown in the right side map on figure 1 and the list of new stations is shown in Table 1.



Figure 1 Current Manhattan Subway Map and Potential Second Avenue Subway Map (MTA, 2013a)

Sq

Station	Location	Transfers
125th Street	125th St and Lexington Ave	4 5 6 trains
		M60 bus to LaGuardia Airport Metro-North Railroad at Harlem - 125th Street
116th Street	116th St and 2 Ave	
106th Street	106th St and 2 Ave	
96th Street	96th St and 2 Ave	
86th Street	86th St and 2 Ave	
72nd Street	72nd St and 2 Ave	
55th Street	55th St and 2 Ave	6 E M trains
42nd Street	42nd St and 2 Ave	4 5 6 7 S trains Metro-North Railroad & Long Island Rail Road
34th Street	34th St and 2 Ave	
23rd Street	23rd St and 2 Ave	
14th Street	14th St and 2 Ave	L train
Houston Street	Houston St and 2 Ave	F train
Grand Street	Grand St and 2 Ave	B D trains
Chatham Square	Worth St and Chatham Square	
Seaport	Fulton St and Water St	
Hanover Square	Water St and Hanover Square	Pier 11

Table 1 List of New Stations (MTA, 2013b)

The goal of this study is to estimate ridership at these proposed stations using easily obtainable data and considering the spatial dependency. More specifically, the focus will be the forecast of average weekday ridership as stations in Manhattan are much busier in weekdays than weekends and holidays. Supporting information mainly comes from four sources: ridership at existing stations provided by MTA (2013c), demographic information in the surrounding neighborhoods derived from the American Community Survey (ACS) (U.S. Census Bureau, 2010a), employment information provided by County Business Patterns (CBP) (U.S. Census Bureau, 2010b), and information regarding built environment derived from the Primary Land Use Tax Lot Output (PLUTO) (NYC Department of Planning, 2010).

3.1 Ridership Information

Ridership information on existing lines is collected by MTA via the MetroCard ticket system. The average ridership data is provided on MTA's website (MTA, 2013c). The ridership consists of all passengers who enter the subway system, including those

transferring from buses, but not those transferring from other subway lines. It should be noted that ridership at some stations are counted together as they are internally connected to facilitate transfer. In the NYC subway system, there are two pairs of such stations. One is the 14th Street - 6 Avenue station and the 14th Street - 7 Avenue station; the other is the Time Square-42nd Street station and the Port Authority-42nd Street station. In these cases, ridership of individual station is allocated based on the number of tracks in each station. Such processing leads to ridership data for 117 individual stations in Manhattan.

3.2 Neighborhood Information

Ridership at a station can also be influenced by the characteristics of the neighborhood surrounding it. Most existing studies define "neighborhood" as the zone within certain walkable distance from the station; and 0.25 miles is often used as the industry standard (Sallis, 2008). As a result, neighborhoods are delineated by circles around stations. However, the Manhattan subway stations are so densely located that these circles overlap substantially. In such cases, Thiessen Polygons can be created to represent neighborhoods. Thiessen polygons are generated by (1) drawing inerratic circles around each station; (2) creating bisector lines by connecting the points where circles intersect; and (3) connecting the bisector lines. Essentially, Thiessen Polygon avoids neighborhood overlapping by allocating a location to its nearest station. In a study of Madrid Metro network, Gutiérrez, Cardozo (2011) created Thiessen Polygons to represent neighborhoods around metro stations, and results are satisfactory. This study generates Thiessen Polygons based on locations of subway stations and the boundary of Manhattan, implying that the entire island is assumed to be served by the subway system. Such assumption is justifiable as most of these polygons are within the circles of 0.25 mile radius. Some locations (e.g., along the edge) have longer distance from the stations, but not exceeding 1 mile. Taking into account the condition of walking facilities in Manhattan, such distance is still considered walkable. Figure 2 shows the neighborhoods generated with Thiessen Polygon.

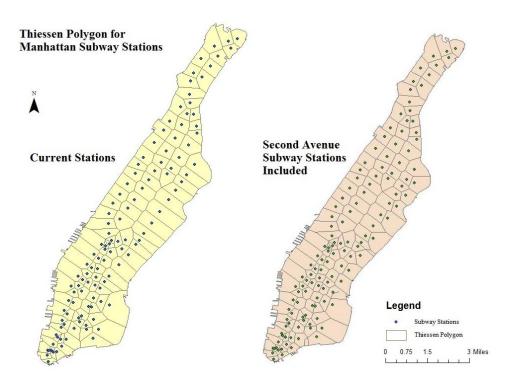


Figure 2 Thiessen Polygon for Manhattan Subway Stations

The Thiessen Polygons are then spatially aligned with the ACS data (in census tract level), CBP data (at ZIP code level), and the Primary Land Use Tax Lot Output (PLUTO) data, generating variables indicating neighborhood characteristics. Table 2 lists their descriptions and summary statistics. Variables *In_pop, gender*, and *In_inc* are derived from the ACS. *In_emp* is derived from the CBP. *retail* and *storage* originate from PLUTO. Variables *In_ridership*, *In_pop*, *In_emp*, *In_retail* and *In_storage* are in the form of natural logarithm as they have large values and their distributions are right-skewed. Besides, a log transformed model offers convenient interpretation as the estimated coefficients can be directly interpreted as elasticity. The variable *attract* is a binary variable, indicating whether the station serves at least one of the most popular attractions(Timeout.com, 2013). The variable *line* indicates the number of subway lines serving a station. The variable *othermode* is an indicator variable, showing the connectivity to other modes of transportation.

Variables	Definitions	Mean	Standard Deviation	Min	Max
Dependent V	ariable				
In_ridership	logarithm of average weekday ridership in persons	9.70	0.89	7.58	11.88
Independent	Variables				
ln_pop	logarithm of population in persons	8.46	1.16	4.75	10.60
gender	ratio of male to female	0.94	0.19	0.48	2.15
In_income	logarithm of residents' income in dollar	11.10	0.57	9.14	12.23
In_emp	logarithm of employment in persons	9.01	1.16	6.27	11.53
In_retail	logarithm of retail area in square miles	13.43	0.92	11.44	15.23
In_storage	logarithm of storage area in square miles	11.41	2.36	6.87	15.84
attract	indicator variable: 1, if there are top 20 NYC attractions; 0, if not	0.16	0.37	0	1
line	number of subway lines	2.19	1.61	1	8
othermode	indicator variable: 1, if available to transfer to trunk bus, intercity bus, train, path, and ferry; 0, if not	0.17	0.38	0	1

Table 2 Summary Statistics of Variables

The above statistics are derived from variable values for current neighborhoods. When interpolating the ridership on the Second Avenue Subway stations, the neighborhoods will be re-delineated and the above independent variables will be recalculated.

4. Methodology

This study develops a Network Kriging approach to estimate ridership on newly constructed subway stations. At each station, the ridership can be expressed as

$$Z_s = X_s \beta + \varepsilon_s \tag{1}$$

where Z_s is the ridership at station *s*. X_s is the vector of *K* influential factors at station *s*. β is the corresponding estimable vector of parameters. ε_s is the error term and can be further formulated by a semivariogram function $\gamma(h)$, where *h* is the network distance between any two stations *s* and *s'*.

The commonly used semivariogram functions $\gamma(h)$ include exponential function, spherical function, and Gaussian function. All functions are monotonically increasing and have maximal values. Such a trend is consistent with the geo-statistic assumption that closer objects tend to have similar performance. Statisticians find that these functions provide similar results in practice (Zimmerman et al., 1998). This study chooses the exponential function so that

$$\gamma(h) = \begin{cases} c_0 + c_1 \left[1 - \exp\left(-\frac{h}{a}\right) \right] & \text{if } h > 0 \\ 0 & \text{otherwise} \end{cases}$$
(2)

where c_0 , c_1 , and a are parameters to be estimated. c_0 is called the "nugget effect" (Geoff, 2005), which reflects discontinuity at the semivariogram's origin, as caused by factors such as sampling error at subway stations. $c_0 + c_1$ is the maximum of γ , called the "sill", indicating the maximum variance in the error terms between a pair of subway stations. Thus, c_1 refers to the "partial sill". a is called the "range", determining the threshold distance between two stations where the variance in the error term stabilizes.

In order to estimate the Kriging model, Z_s is first regressed on X_s to obtain the empirical value of the error. The empirical semivariogram can be then created and fitted to the exponential function in Equation (2), obtaining c_0 , c_1 , and a. Once the theoretical semivariogram with the estimated c_0 , c_1 , and a is derived, it can be used to construct a variance-covariance matrix of the error term V in the form of

$$V_{ss'} = c_0 + c_1 - \gamma_{ss'}(h)$$
(3)

With the updated error term, the β is re-estimated using a feasible generalized least square (GLS) method. Such a process is iterated until converge.

The ridership on the Second Avenue Subway stations can be derived by

$$Z_{new} = X_{new} \widehat{\beta} + V_{new,old}^T V_{old,old}^{-1} \varepsilon_{old}$$
(4)

where subscripts *new* and *old* indicate the newly-constructed stations and existing ones respectively. Essentially, ridership at a new station is predicted as the summation of local neighborhood influence (captured by the variables characterizing the neighborhood) and contribution of unobserved factors (captured by the interpolated error term based on estimation residuals at existing stations). The second component in Equation (4) can be considered as the main contribution that distinguishes this study from previous ones: instead of assuming unobserved factors as white noise and treating them as nuisance, they are fully utilized to improve the prediction by considering the spatial dependency. The estimation and forecasting processes are coded in MATLAB.

5. Model Validation

This section validates the reliability Network Kriging models on ridership estimation. Current ridership data on 15 randomly selected stations in current subway network are set to be unmeasured. The unmeasured stations are estimated from other ridership data by three models, linear regression, Kriging with Euclidean distance, and Network Kriging. Then, the estimation accuracy can be indicated by comparing the actual data and estimated values.

Statistics Mean Squared Error (MSE) and Percent Squared Error (%SE) measure the difference between the estimated ridership (Z_{est}) and the observed ridership Z_{obs} . *n* is the number of randomly selected stations where ridership counts need to be estimated.

$$MSE = \frac{\sum_{n} (Z_{est} - Z_{obs})^{2}}{n}$$
$$\% SE = \frac{MSE}{\sum Z_{obs}}$$

The validation results are shown in Table 3. Both measures indicate that Network Kriging has better estimation on the unmeasured ridership and linear regression model has the largest bias. That is, the Network Kriging model has a little improvement on estimation accuracy and should be appropriate for the Second Avenue Subway ridership estimation.

	Linear Regression	Kriging with Euclidean Dist	Network Kriging
Mean Squared Error	12.03	11.99	11.97
% Square Error	19.94%	19.88%	19.84%

Table 3 Validation Results of Three Models

6. Results Analysis

The Network Kriging method is then applied to the ridership data over the entire Manhattan subway network. As shown in Table 4, the values of coefficient estimates are close in all models. This is expected as Ordinary Least Square (OLS) and Generalized Least Square (GLS) both produce unbiased estimators. Unbiased estimators for the same parameter set should be very close to each other. However, the t-statistics in both Kriging models are much larger than those in the linear regression. In other words, the efficiency of estimation is much better with the Kriging models. The Network Kriging method produces estimates with narrower confidence intervals thus more reliable forecasting results for the Second Avenue Subway ridership.

	Linear Regression		UK with Euclidean Dist		UK with Network Dist	
Variables	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
In_pop	0.103	2.22	0.104	7.65	0.103	10.09
gender	0.263	0.97	0.280	3.51	0.270	4.44
In_income	0.252	2.82	0.253	9.53	0.252	12.75
In_emp	0.168	2.98	0.172	10.35	0.168	13.47
In_retail	0.299	3.66	0.297	12.35	0.299	16.58
In_storage	-0.054	-2.24	-0.055	-7.74	-0.054	-10.17
attract	0.315	2.41	0.301	7.71	0.314	10.87
line	0.240	7.95	0.240	26.78	0.239	35.98
othermode	0.248	2.12	0.247	7.18	0.249	9.64
constant	0.258	0.23	0.224	0.68	0.255	1.04
c0			0.083	5.04	0.047	4.11
c1			0.003	0.78	0.025	2.31
а			0.901	42.21	0.899	63.55

Table 4 Results of the Three Models

These estimates also provide interesting insights into the subway ridership problem.

The coefficient of population is 0.103, indicating that population has a positive effect on subway ridership. A 1% increase of population in the surrounding neighborhood is associated with 0.103% increase of the ridership at the subway station. Male to female ratio is estimated to be positively related to ridership: an additional 0.01 of the gender ratio is associated with 0.270% more ridership. This estimate implies that men tend to generate more subway trips than women do. Wealthier neighborhoods tend to generate more subway ridership, as indicated by the positive coefficient of *In_income*. Every 1% increase in the neighborhood's average household income is associated with 0.252% increase in the ridership. This finding is consistent with most previous studies where income is found to be positively related to the travel frequency.

Many people working in Manhattan rely on subway to commute from Queens, Brooklyn and Bronx to their workplaces. As a result, employment contributes significantly to subway ridership. Stations serving areas with high employment densities are often crowded. It is estimated that a station's ridership will increase by 0.168% if the number of employments in the surrounding neighborhood increases by 1%. Manhattan is also well known for its commerce and shops generate large travel demand, as captured by the variable *In_retail*. It is estimated that ridership at a subway station will be 0.299% higher with 1% additional retail area. In contrast, storage areas do not attract passengers. Ridership decreases by 0.054% if the storage area increases 1%.

Besides local residents and workers, a large portion of NYC subway riders are tourists. Tourists visiting New York City often find subway as the most convenient transportation mode. Millions of tourists come to the City every year, contributing to the subway ridership. Therefore, stations serving scenic spots tend to have higher ridership. The coefficient of *attract* is 0.314, indicating the ridership at these stations are 31.4% higher than those not serving any tourism attractions.

The subway ridership is not only influenced by surrounding neighborhood's travel demand, but also quality of supplies. Some stations more attractive to riders because they are served by both local and express subway lines, or both east-west and south-north lines. In order to reflect the impact of "supply," the effect of number of lines is evaluated. Results suggest that ridership is 23.9% higher when there is one additional subway line serving a certain station.

The NYC subway is also the main connector of NYC's various transportation terminals. In New York City and surroundings, there are three international airports, one national train hub, several regional train stations, and multiple inter-city bus terminals. Passengers may use the subway system to transfer from one to another, contributing to the subway ridership. The estimate of *othermode* shows that ridership at stations serving major transportation terminals are 24.9% higher.

The parameters in the semivariogram are estimated simultaneously with the coefficients of independent variables. The "nugget effect" c_0 is 0.047, which is caused by the measurement error or the short scale variability. The "partial sill" c_1 is 0.025, indicating the variance does not improve much when the network distance increases. The "sill" $c_0 + c_1$ is thus 0.072, indicating the maximum variability of regression error terms is low. The reason may be that the exogenous independent variables have captured most variance. When checking the R-square statistics of the Network Kriging model, more than 99% of the variability of ridership is explained by the independent variables. The "range" parameter is 0.899, indicating that the variability stabilizes when the network distance between two stations is longer than 0.899 miles. In other words, 0.899 miles is the threshold to see whether there is an increasing variability.

The ridership at the Second Avenue Subway stations can thus be interpolated using the estimated coefficients. The independent variables of the new stations are calculated by new Thiessen Polygons which are delineated based on the planned station locations. The before and after ridership are shown in Table 5. Four of the Second Avenue Subway stations use the existing stations and the others are new stations. The ridership changes at the four stations mainly result from the re-delineation of the covered area. Comparison of the results from Network Kriging and those from linear regression shows

some differences. For example, the ridership at the 125th station is 39,900 by linear regression, but 32,219 by Network Kriging. The lower ridership estimated by Network Kriging may be due to the consideration of nearby stations (116th station, 110th station, and etc.) which have low ridership as Kriging assumes that the error variability is low for close objects.

Table 5 also lists the before-after ridership on parallel Lexington Avenue stations. Without considering induced demand, ridership of most stations decreases. The main reason is that the Second Avenue Subway covers part of area that is currently covered by Lexington Avenue lines. In other words, the Lexington lines ridership is split to the Second Avenue Subway. This is one of the most important targets of constructing the Second Avenue Subway. When the new lines are open to public, Lexington Avenue lines will not be crowded as it is today and mainly serve as the connection between Manhattan and Bronx.

Station on Second Avenue Subway	Current Ridership	Estimated Ridership by Linear Regression	Estimated Ridership by Kriging with Network Dist	Parallel Lexington Avenue Stations	Current Ridership	Estimated Ridership by Linear Regression	Estimated Ridership by Kriging with Network Dist
125th Street	27,990	39,900	32,219	125th Street	27,990	39,900	32,219
116th Street	New	13,017	18,283	116th Street	15,655	7,556	8,336
106th Street	New	15,192	13,017	110th Street	11,630	8,954	10,568
Tooth Street	INEW	15,192	13,017	103th Street	15,210	9,176	10,165
96th Street	New	18,941	16,209	96th Street	24,870	18,904	19,411
86th Street	New	44,338	37,907	86th Street	60,965	60,272	55,880
				77th Street	35,579	29,498	28,752
72nd Street	New	54,776	47,028	68th Street / Hunter College	34,984	21,244	17,996
EEth Street	Now	45 749	12 5 4 0	59th Street	63,138	79,255	58,201
55th Street	New	45,748	42,540	51th Street	62,774	40,862	34,431
42nd Street	New	37,813	37,172	42nd Street / Grand Central	144,350	133,415	93,920
34th Street	New	23,723	21,722	33rd Street	30,497	23,667	31,357
sath Street	INEW	23,723	21,722	28th Street	22,274	12,993	17,327
23rd Street	New	22,241	17,840	23rd Street	30,929	11,227	12,346
14th Street	6,123	28,441	32,778	14th Street / Union Square	106,380	120,183	55,407
				Astor Plaza	17,630	15,212	15,020
Houston Street	17,090	24,328	17,815	Bleecker Street	34,191	57,492	44,084
	17,090	24,320	17,010	Spring Street	11,132	11,117	8,522
Grand Street	23,304	15,994	12,690	Canal Street	46,435	98,357	46,770
Chatham Square	New	12,890	13,845	Brooklyn Bridge	36,939	23,423	16,095
Seaport	New	14,294	13,384	Fulton Street	64,287	62,977	33,157
Hanover Square	New	10,600	9,866	Wall Street	22,551	8,063	9,705

Table 5 Ridership on Second Avenue Subway stations and Parallel Lexington stations

7. Conclusion

This study uses a Network Kriging method to estimate ridership at subway stations, using the Second Avenue Subway in Manhattan as an example.

With this Network Kriging model, independent variables capture the deterministic part of ridership. The stochastic part is fitted by a semivariogram which is an exponential function of distance between two stations. The distance is calculated by the subway network distance instead of the Euclidean distance in standard Kriging because network distance is a more pattern-consistent index in measuring adjacency in a subway network. The fitted semivariogram shows that close stations have less variability and distant stations generally have large variability, but the variability keeps constant beyond 0.899 miles. The reliability of Network Kriging model is first validated by estimating ridership at 15 randomly-selected stations in the current network. Results show that Network Kriging improves the estimation accuracy compared to a standard linear regression model and a Kriging model with Euclidean distance. The ridership along the new Second Avenue Subway and the parallel Lexington Avenue Subway is then estimated. Results show that Second Avenue Subway will serve a considerable number of passengers and the congestion on Lexington lines will be relieved. However, the total fare revenue would not increase much right after the operation of new services. Transit companies may need to dispatch fewer trains on the new lines to operate economically. Public agencies can give residents and businesses incentives to move along the east coast of Manhattan to take fully use the Second Avenue Subway.

The Network Kriging model developed and applied in this study improves Kriging model by using network distance instead of Euclidean distance. The methodology is also applicable in other transportation issues that involve measurements of adjacency. Besides, the study of spatial dependency on transit ridership highly contributes to public transportation research, serving as an important reference for future works on transitoriented cities.

8. Acknowledgment

Funding for this research was provided by the Region II University Transportation Research Center and by Rensselaer Polytechnic Institute.

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