University Transportation Research Center - Region 2

Final Report

Finite Element Model Updating and Damage Detection for Bridges Using Vibration Measurements

Performing Organization: Columbia University

December 2013

Sponsor:
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The Region 2 University Transportation Research Center (UTRC) is one of ten original University Transportation Centers established in 1987 by the U.S. Congress. These Centers were established with the recognition that transportation plays a key role in the nation's economy and the quality of life of its citizens. University faculty members provide a critical link in resolving our national and regional transportation problems while training the professionals who address our transportation systems and their customers on a daily basis.

The UTRC was established in order to support research, education and the transfer of technology in the field of transportation. The theme of the Center is "Planning and Managing Regional Transportation Systems in a Changing World." Presently, under the direction of Dr. Camille Kamga, the UTRC represents USDOT Region II, including New York, New Jersey, Puerto Rico and the U.S. Virgin Islands. Functioning as a consortium of twelve major Universities throughout the region, UTRC is located at the CUNY Institute for Transportation Systems at The City College of New York, the lead institution of the consortium. The Center, through its consortium, an Agency-Industry Council and its Director and Staff, supports research, education, and technology transfer under its theme. UTRC's three main goals are:

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16. Abstract

In this report, the results of a study on developing a damage detection methodology based on Statistical Pattern Recognition are presented. This methodology uses a new damage sensitive feature developed in this study that relies entirely on modal characteristics, i.e. natural frequencies and mode shapes, directly identified from measurements of the structural response. A procedure for training the damage detection methodology to account for variability induced by operational conditions, i.e. temperature, traffic, wind, etc., has been proposed to determine boundaries of variability of the cumulative distribution functions of the various features. Two different test setups have been considered: 1) a full set of sensors, and 2) a limited number of sensors. The results show that this methodology has been proven successful in detecting the occurrence of damage and, in the case of full sensor setup, also in accurately locating the element where damage has occurred and the amount of element stiffness reduction. In the case of a limited instrumentation setup, the proposed methodology is successful in identifying the occurrence of damage, and even though it loses accuracy in pinpointing the exact damage location, it still successfully identifies the region containing the damaged element. In conclusion, the results obtained using the proposed SPDSF are promising and allow for damage identification, localization and estimation.

The next step will be to use the same damage detection algorithm but relying only on measurement data of the structural response (output-only). Preliminary results conducted as a part of this study show that using non mass-normalized mode shapes, as usually obtained from output-only identification algorithm, can generate misleading results. The key point will be the determination of the mass normalizing factors of a mode shape matrix identified via output-only system identification algorithms. Also, the development and application of mode shape expansion techniques to obtain the complete mass normalized mode shape matrix in the situation of limited instrumentation will constitute a part of future study.

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Finite Element Model Updating And Damage Detection For Bridges Using Vibration Measurements

1 Introduction

The term Structural Health Monitoring (SHM) is used to indicate the task of analyzing the current condition of a given structural system, e.g. a building, a bridge, an airplane, a rotating machine, etc., in order to establish whether or not damage has occurred. With our nation's infrastructure that is rapidly decaying because of lack of resources and/or proper maintenance, a successful SHM strategy is of obvious interest to engineers and government authorities that are responsible for the up-keeping of complex structural systems such as bridges. Nowadays, bridges have become vital links in the urban landscape (nearly 90 percent of the national food chain moves on state bridges) and need to be kept constantly operational in order to prevent disruption to large sectors of our society. However, according to the latest report of the American Society of Civil Engineers, about 25 percent of the entire national bridge inventory comprises bridges that are either structurally deficient or functionally obsolete. In about 15 years, 50 percent of this nation's bridges will be over 50 years old and this will imply an unprecedented commitment of both financial and human resources. Consequently, there will be the need for more reliable, economic and easy to conduct inspections that can take advantage of the many advances in other areas, like computer technology and material science. It is in this framework that the concept of SHM has to be framed: thanks to the innovations in computer and sensor technology, it is now possible to collect large amounts of data that represent the response of the bridge to the environment excitation and, through the analysis of such data, to provide an instantaneous assessment of the bridge conditions.

For its great potential, SHM has been the object of extensive studies for the past thirty years, starting from the damage assessment in rotating machine to the analysis of the integrity of wind turbines and, now recently, of bridges. Because of such a broad spectrum of applications, various approaches to SHM have been proposed and studied, some of more general breadth and applicability (e.g. vibration based SHM) and some related to well defined applications (e.g. ground penetrating radar to assess delamination in bridge decks). Looking at the more general (global) approaches that look at the overall behavior of the structure, the majority of such SHM approaches focus on vibration based techniques, which employ the measurement of the dynamic response of the structure, measured at some locations on the system of interest, to perform the damage detection assignment. This is becoming the predominant trend in civil engineering applications for bridges, where now monitoring sensor systems are installed permanently, either at the time of initial construction or during the service life, and record, in real time, quantities like displacements and accelerations.

The broad set of vibration based SHM techniques can be divided into two major categories: 1) model based methods and 2) data based methods. As for the first group, model based methods require the selection of a model of the structure that has to be identified from the measured structural response. The basic idea behind such methods is that a model is identified, either directly or iteratively, whose response mimics very closely the measured response from the real structure. Sometimes these models have physical meaning, e.g. a mass-damping-stiffness model, sometimes they do not, e.g. black-box model. Usually, to detect damage, these algorithms often employ the modal characteristics of the model, such as natural frequencies and/or mode shapes, identified from the structural response. As stated by one of the axioms of damage detection [1], the damage detection problem requires the knowledge of the undamaged conditions in order to be solved. In model based techniques exploiting modal analysis, the aforementioned requirement is satisfied by first identifying the modal properties of the structural model that represents the structure in what can be referred to as "undamaged condition" or "healthy state", and then by comparing such properties with those corresponding to a newly identified model obtained from the analysis of the response of the system under unknown conditions. If the novel properties result to be sensibly different from the ones observed on the healthy structure, i.e. different values of corresponding frequencies or different types of modes (translational vs. rotational), then the system can be declared damaged. Of course, how "sensibly different" is something that varies from system to system and it requires some engineering judgment. In addition, a substantial body of literature has been produced on the analysis of the drawbacks of using such approaches in real applications, as modal properties, especially modal frequencies, are highly sensitive to environmental variability due, for instance, to temperature or humidity changes. For example, measurements of the bridge response collected at different times during the year have shown that there is a change in the magnitude of the natural frequencies of about 10 percent between summer and winter, making it difficult for the engineer to separate the damage-induced changes from the seasonal changes. Furthermore, all system identification algorithms are sensitive to measurement noise, making the risk of false alarms error very likely to occur during the damage detection performance. In order to tackle such problems, it has been proposed to cast the structural damage detection problem into a probabilistic framework, which is naturally tailored to cope with variability in the data.

With regard to the second set of methods, data-based methods, as the name suggests, rely entirely on information extracted from the data and do not require the adoption of a physics-based model of the structure. As an example, in this class, we can group all the Autoregressive (AR), moving-average (ARMA), with exogenous terms (ARX) models that are commonly least-square solutions of sets of equations that relate response and input measurements. Because of the nature of such equations that map current outputs to previous outputs and/or input measurements, such data-based models can be successfully used to predict the response of the system (e.g. a bridge) to a future excitation (e.g. predicted ground motion) but cannot be directly applied in detecting structural damage. However, such models become quite useful in damage identification when a statistical pattern recognition approach is followed.

In damage detection, the basic idea behind a statistical pattern recognition approach is the following: first, training data from the undamaged and all possible damaged states is collected and analyzed and a statistical model of the data, for example the probability density function, is created. Once this model is built, then, when the new monitoring data set becomes available, it is possible to see whether its model fits within the statistical model or not. In case it does not, this is an indication that structural damage has occurred and means have to be found to relate such discrepancy to the presence, amount and location of damage.

In the field of SHM, the statistical pattern recognition approach is mainly used to solve "unsupervised verification problems". The term "unsupervised verification problem" is used to represent those problems in which only data from the undamaged structure is available to create the statistical model. The general outline of a unsupervised statistical pattern recognition technique comprises of two phases: 1) the training phase and 2) the testing phase. During the training phase, a large amount of data is collected from the structure in a state that is labeled as the "healthy" condition, then a set of information, called "damage sensitive features", is extracted from such data and the features' probability distribution is determined. This distribution represents the statistical model of the data from the "healthy" structure against which the data from any future monitoring test will be compared. During the testing phase, the damage sensitive features are extracted from the response of the system in an unknown state (either still undamaged or damaged), and the probability of the new features of being realizations of the trained distribution is evaluated. If such probability is lower than a prescribed threshold, the structure is declared damaged. The challenges in this second approach stem mainly from the selection of the most appropriate distribution to model the damage sensitive features' statistical properties, and from the metric to employ to measure the 'distance' of the novel features from the trained distribution. Normal distribution and squared Mahalanobis distance are two popular choices to address the aforementioned issues, as well known results are available for both statistical tools, making the treatment of the overall problem much easier. Nevertheless, the assumption of normally distributed features can be erroneous, especially when the number of available observations is not large. If a set of observations is not normally distributed, mean and covariance matrix are no longer sufficient to completely describe the statistics of the set of observations and consequently, squared Mahalanobis distance does not represent a robust tool to measure the distance of a new observation from the trained distribution. However, pattern recognition remains a viable approach able to account for the physiological changes in structural properties due to external factor effects, even when there are uncertainties, as for the type and distribution of the damage sensitive features. It must be noted, though, that due to the choice of the damage sensitive features, which are often selected as abstract information difficultly relatable to the structural properties, the damage detection algorithm developed within the statistical pattern recognition framework can seldom locate and quantify damage. On the contrary, due to their intuitive relationship with the structure topology and characteristics, modal properties, especially when in the form of mode shapes, can solve the problem of damage location and quantification.

In view of the aforementioned reasons, a 'mixed' approach has been explored recently and presented in this study, where the damage sensitive features are defined using the modal properties identified from the response of a bridge system, while the damage detection, location and quantification is developed according to the statistical pattern recognition paradigm. In this approach, the finite element model updating was not necessary because it was possible to directly obtain the diagonal elements of the mass and stiffness matrices from the identified mode shapes and frequencies. In this way, lengthy and computationally cumbersome calculations for the finite element model updating are avoided. Additionally, while finite element model updating requires the knowledge of the model parameters of an existing finite element model, and the reliability of this knowledge in turn affects the updated model parameters, in the current approach no such *a priori* knowledge of the model parameters are necessary, thereby also providing great advantages in terms of accuracy of the results. The results presented in this report are those related to numerical simulations of the bridge superstructure.

2 Selection of Damage Sensitive Features

One of the most difficult tasks in performing SHM is to select a set of parameters, to be identified from the measurement data, that are sensitive to structural damage and so that can provide early warning. In the past, natural frequencies were used but they were found too insensitive to damage and too much affected by environmental effects. Here, a new set of features, derived from the identified modal characteristics, is proposed.

Consider that the bridge structure can be represented as a discrete mass, damping and stiffness model with N degrees of freedom. The equations describing the motion of such a system can be expressed as:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ represent the $N \times N$ mass, damping and stiffness matrices, respectively, while $\mathbf{x}$, $\dot{\mathbf{x}}$, and $\ddot{\mathbf{x}}$ indicates the $N \times 1$ vectors of the nodal displacements, velocities and accelerations, respectively. The vector $\mathbf{F}$ represents the $N \times 1$ vector containing the nodal input forces representing the external excitation. If the assumption of lumped mass model is used, as it is in this study, the mass matrix can be considered diagonal.

Let $\{\mathbf{M}; \mathbf{K}\}$ and $\{\mathbf{M}^*; \mathbf{K}^*\}$ denote the mass and stiffness matrices of two models of the same structure in two different states, for example the damaged and undamaged states. In damage detection analysis, damage is usually characterized as a change (reduction) of some members' stiffness (e.g. induced by cracking) while the mass is considered unchanged. Hence, it is desirable to derive features that somehow reflect such stiffness reduction. The damage sensitive feature used in the present work is tailored to give a measure of the difference between the diagonal elements of $\mathbf{K}$ and $\mathbf{K}^*$. Equation (2) expresses the relation between the damage sensitive feature evaluated at the jth DOF and the jth diagonal elements of $\mathbf{K}$ and $\mathbf{K}^*$:

$$DSF_j = \frac{K_{j,j} - K^*_{j,j}}{K_{j,j}} = 1 - \frac{K^*_{j,j}}{K_{j,j}}, \text{ for } j = 1, ..., N. \tag{2}$$

Let us denote as $\{\omega, \psi\}$ and $\{\omega^*, \psi^*\}$ the full sets of modal frequencies, ω's, and mode shape vectors, ψ's, identified through any system identification procedure from the states of the system referred to as $\{\mathbf{M}; \mathbf{K}\}$ and $\{\mathbf{M}^*; \mathbf{K}^*\}$, respectively. For convenience in the following derivations, the natural frequencies are stored in an $N \times N$ diagonal matrix $\mathbf{\Lambda}$, whose elements are:

$$\mathbf{\Lambda} = \text{diag}\{\omega_1^2, \quad \omega_2^2, \quad \cdots \quad \cdots \quad \omega_N^2\} \tag{3}$$

while the mode shape vectors are the columns of the eigenvector matrix $\mathbf{\Phi}$,

$$\mathbf{\Phi} = [\boldsymbol{\psi}_1, \quad \boldsymbol{\psi}_2, \quad \cdots \quad \cdots \quad \boldsymbol{\psi}_N] \tag{4}$$

Equivalently, for the model corresponding to the damaged case,

$$\mathbf{\Lambda}^* = \text{diag}\{\omega_1^{*2}, \quad \omega_2^{*2}, \quad \cdots \quad \cdots \quad \omega_N^{*2}\} \tag{5}$$

$$\mathbf{\Phi}^* = [\boldsymbol{\psi}_1^*, \quad \boldsymbol{\psi}_2^*, \quad \cdots \quad \cdots \quad \boldsymbol{\psi}_N^*] \tag{6}$$

If only the response time histories of the system are employed to identify the modal characteristics of the structure (e.g. this could be the case when ambient vibrations of the bridge are monitored), the identified mode shape matrices $\mathbf{\Phi}$ and $\mathbf{\Phi}^*$ are not mass normalized, but rather satisfy the normalizing conditions (7) and (8):

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} = \boldsymbol{\alpha} = diag\{\alpha_1, \quad \alpha_2, \quad \cdots \quad \cdots \quad \alpha_N\} \tag{7}$$

$$\mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} = \mathbf{\Lambda} \boldsymbol{\alpha} \tag{8}$$

where α_i, for $i = 1, ..., N$, are scalars. Similar relationships apply for the state of the system characterized by $\{\mathbf{M}^*; \mathbf{K}^*\}$ and $\{\mathbf{\Lambda}^*, \mathbf{\Phi}^*\}$.

In the case that both input and output histories of the system are measured (e.g. forced vibration tests on the bridge), the mode shapes normalizing α_i factors are all equal to 1, as the following conditions on mass and stiffness apply:

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} = \mathbf{I} \tag{9}$$

$$\mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} = \mathbf{\Lambda} \tag{10}$$

where $\mathbf{I}$ indicates an identity matrix of order N.

In this study, only the time-histories of the structural response will be available for the identification of the modal characteristics of the bridge (output-only identifications), simulating the case that is most commonly occurring in real practice when ambient vibrations of the bridge are recorded. In this case, exploiting conditions (7) and (8) and the mass diagonality assumption, it is possible to express the $\{j, j\}$th elements of the mass matrix, $\mathbf{M}$, and of the stiffness matrix, $\mathbf{K}$, only in terms of the modal properties and normalizing scalars α_i:

$$K_{j,j} = M_{j,j}^2 \sum_{i=1}^{N} \frac{\phi_{j,i}^2 \lambda_i}{\alpha_i} \tag{11}$$

$$M_{j,j} = \left(\sum_{i=1}^{N} \frac{\phi_{j,i}^2}{\alpha_i} \right)^{-1} \tag{12}$$

Substituting Equations (11) and (12) into the initial definition of the proposed damage sensitive feature, Equation (2), it is possible to express the proposed damage sensitive feature only in terms of the identified modal characteristics as:

$$DSF_j = 1 - \frac{\left(\sum_{i=1}^{N} \frac{\phi_{j,i}^{2}}{\alpha_i}\right)^2 \left(\sum_{i=1}^{N} \frac{\phi_{j,i}^{*2}\lambda_i^*}{\alpha_i^*}\right)}{\left(\sum_{i=1}^{N} \frac{\phi_{j,i}^{*2}}{\alpha_i^*}\right)^2 \left(\sum_{i=1}^{N} \frac{\phi_{j,i}^{2}\lambda_i}{\alpha_i}\right)}. \tag{13}$$

This is a quite interesting result because it allows us to look at changes of the stiffness elements directly affected by damage through the identification of modal characteristics as natural frequencies and mode shapes. Because of this link with the stiffness coefficients, we can define this feature as Stiffness Proportional Damage Sensitive Feature (SPDSF).

3 Damage Detection Based on Empirical Cumulative Distribution of Damage Sensitive Features

Inherent in the definition of the Stiffness Proportional Damage Sensitive Feature (SPDSF) presented in the previous section, there is a comparison between two states of the system, so that the feature can actually be thought of as a measure of the variability of the stiffness properties of the system subjected to different conditions. For example, a change in the stiffness value of the element connecting node i and j should be reflected in a change in the values of the ith and jth elements of the main diagonal of the structure's stiffness matrix.

Indeed, it is customary to model damage as a decrease in the stiffness of a part (an element or a joint) of a structure. Nonetheless, as already mentioned, environmental and operational conditions may as well induce changes in the stiffness values of a structure. Therefore, it is extremely important that a damage detection algorithm be able to distinguish between the fluctuations of the damage sensitive features (in this case, the proposed SPDSF) due to the influence of external factors and those due to damage occurrence. Because of this variability that could lead to false alarms or false safety, it is possible, as proposed in [2], to cast the problem in a probabilistic context by introducing the "probability of damage" assigned to each diagonal element of the stiffness matrix. Such a probability can be defined as the probability that the $\{j, j\}$th element, $K^*_{j,j}$, of the stiffness matrix associated to a possibly damaged state be less than a prescribed fraction of the same element, $K_{j,j}$, associated to a known, undamaged, state:

$$P^{damage}(d) = P(K^*_{j,j} < (1-d)K_{j,j}) \text{ for } d \in [0, 1) \tag{14}$$

where d is a parameter indicating the percentage of damage and can range from 0 to 1.

By using the previously derived expressions for the SPDSF (Equation (13)), it is possible to rewrite Equation (14) in terms of the SPDSF defined in Section 2:

$$\begin{aligned} P^{damage}(d) &= P(K^*_{j,j} < (1-d)K_{j,j}) = \\ &= P\left(\frac{K_{j,j} - K^*_{j,j}}{K_{j,j}} > d\right) = \\ &= 1 - CDF\left\{\frac{K_{j,j} - K^*_{j,j}}{K_{j,j}}\right\} \end{aligned} \tag{15}$$

where CDF represents the Cumulative Distribution Function.

Then, the objective of the training phase will be to define boundaries for the fluctuations of the SPDSF that can be considered normal, using data from what can be considered as the undamaged state, in order to set a reference against which new realizations of features extracted from the system under unknown conditions can be compared.

The general procedure of the training phase can be briefly summarized as follows. Let us denote as n_{tr} the number of measurements that have been conducted on the system under various healthy conditions. These measurement sets represent the measurements obtained at different times during the year, with different traffic conditions, etc. They encompass all the different operating conditions of the bridge during which the bridge is considered to be in a healthy state. Let us further denote with m the number of sensors available for each measurement campaign, and with N the number of degrees of freedom of the Finite Element model of the structure in question. From these measurements, a set $Y = \{\boldsymbol{\lambda}^{(i)}, \boldsymbol{\Phi}^{(i)}\},$ for $i = 1, 2, \cdots, n_{tr}$, of modal properties can be identified, where $\boldsymbol{\lambda}^{(i)}$ represents the $N \times 1$ vector containing the squared circular modal frequencies identified from the ith measurement set and $\mathbf{\Phi}^{(i)}$ the corresponding $N \times N$ identified mode shapes. The set Y is then divided into two subsets Y_H and Y_V such that:

$$Y_H \cup Y_V = Y$$

$$Y_H \cap Y_V = \emptyset$$

$$Y_H, Y_V \neq \emptyset$$

$$|Y_H| = n_H$$

$$|Y_V| = n_V$$

The modal properties (natural frequencies and mode shapes) contained in the set Y_H are taken as representative of the reference state and used to evaluate the elements $K_{j,j}^{(i)}$, for $j = 1, 2, \cdots, m$ and $i = 1, 2, \cdots, n_H$, of the stiffness matrix of the entire structure in its reference state. The remaining sets of modal properties contained in Y_V are instead used to estimate the elements $K_{j,j}^{*(i)}$, for $j = 1, 2, \cdots, m$ and $i = 1, 2, \cdots, n_V$. These n_V sets of modal properties might be slightly different from the reference ones because of the possible different operational conditions but are still referring to the healthy state of the structure. By comparing these identified $K_{j,j}^{*(i)}$ with the other identified $K_{j,j}^{(i)}$, it is possible to see a range of variability of such stiffness parameters (and consequently of the proposed DSF) induced by operational conditions. At this point, each $K_{j,j}^{*(i)}$ in Y_V is compared to all the $K_{j,j}^{(i)}$ in Y_H using Equation (13) and, at the end of this procedure, the Empirical Cumulative Distribution Function (ECDF) of each of the n_V sets containing n_H values of

DSF_j can be estimated as:

$$P(d) = \frac{1}{n_H} \sum_{i=1}^{n_H} U(d - DSF_j^{(i)}) \tag{16}$$

where $U(x)$ is the Heaviside function:

$$U(x) = \begin{cases} 0 & x < 0 \\ 0.5 & x = 0 \\ 1 & x > 0. \end{cases} \tag{17}$$

In this way, a set of acceptable values of d and the probability associated to such values to occur are estimated. This set will be indicative of the variability of the operational conditions of the bridge and so a condition that will follow within this range will be considered related to regular operation.

At the time of testing the bridge to assess its condition, a new set of measurement data is collected from the structure in unknown conditions, and a single set of modal properties is identified from such data. The corresponding $K^*_{j,j}$ elements estimated from such modal data are then compared to the $K^{(i)}_{j,j}$ values previously identified, for $i = 1, 2, \cdots, n_H$, obtained from Y_H during the training operation, using again Equation (13). The empirical cumulative distribution function of this new set of data can now be compared to the one obtained in the training phase: a shift towards right of the testing ECDF, outside the boundaries indicative of the operational conditions, indicates the damage occurrence in that specific degree-of-freedom, with a probability $P(d)$ given by Equation (16).

4 Numerical Results

In order to test the validity of the proposed approach, the damage detection algorithm described in the previous section was used to detect and locate damage in a simple bridge model. The bridge was modeled using two interconnected spring-mass chains: each chain consists of alternately placed flexural links and lumped masses, with the corresponding lumped masses of the two chains being additionally connected by flexural links. A total of 8 masses and 14 flexural links were assembled in such a model, as shown in Figure (1). The baseline condition refers to the case where all the 8 masses, m_i^0, for $i = 1, \cdots, 8$, are taken equal to 1.25×10^5 kg and the flexural stiffnesses of all the spring elements, k_i^0, for $i = 1, \cdots, 14$, are set equal to 7.123×10^7 N/m. This simple model can be used to represent simple overpasses or single spans of supported girder bridges, where the spring chains represent the various segments of the girders and the concentrated masses the mass of the deck. Because of the structure of the model, only global bending and global torsional modes have been considered. Such modes, and the corresponding frequencies, are shown in Figure (2) and Figure (3).

Seven different damage scenarios have been considered, as listed in Table (1). The first five scenarios are those that simulate the operational conditions of the bridge: the first case represents the so called “reference” state, in which the initial values of the lumped masses and of the flexural stiffnesses are used, while damage scenarios 2 and 4, with a reduction of the flexural stiffness of 1 percent of the original value for only the elements of one chain, are indicative of a condition where only one part of the bridge is subjected to an increase in temperature with respect to the baseline condition. Analogously, damage scenarios 3

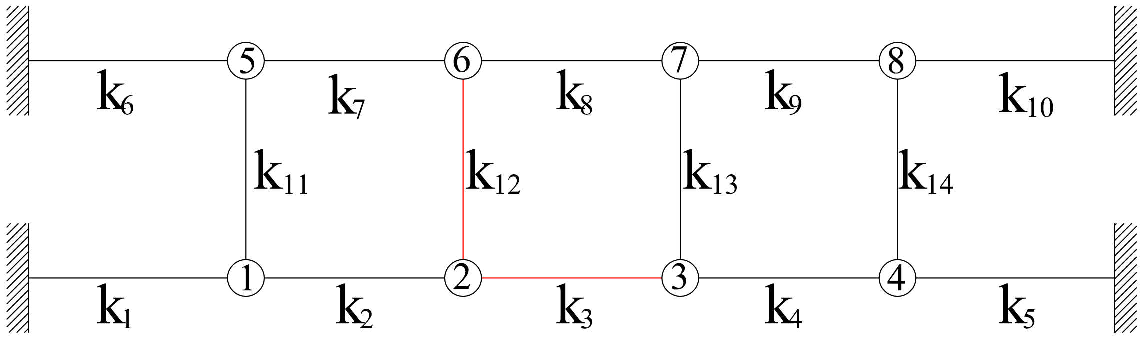

Figure 1: Bridge model considered for numerical case study.

Table 1: Damaged and undamaged states considered.

State	Condition	Description
1	Undamaged (U1)	baseline
2	Undamaged (U2)	$k_i = 0.99k_i^0$, for $i = 1, ..., 5$
3	Undamaged (U3)	$k_i = 1.01k_i^0$, for $i = 1, ..., 5$
4	Undamaged (U4)	$k_i = 0.99k_i^0$, for $i = 6, ..., 10$
5	Undamaged (U5)	$k_i = 1.01k_i^0$, for $i = 6, ..., 10$
6	Damaged (D1)	$k_3 = 0.85k_3^0$
7	Damaged (D2)	$k_{12} = 0.85k_{12}^0$

and 5 represent a condition where only half of the bridge is subjected to a decrease in temperature. The "real" damage scenarios are represented by damage cases 6 and 7. Here, damage is modeled by a 15 percent reduction of stiffness in only one element: even if the reduction might appear substantial, when looking at the overall stiffness at the joint, the reduction is not so evident and so it will represent a good test for the algorithm.

In this study, only acceleration time-histories will be used in the identification of the modal parameters. Accelerations are the most common type of data obtained during the moni-

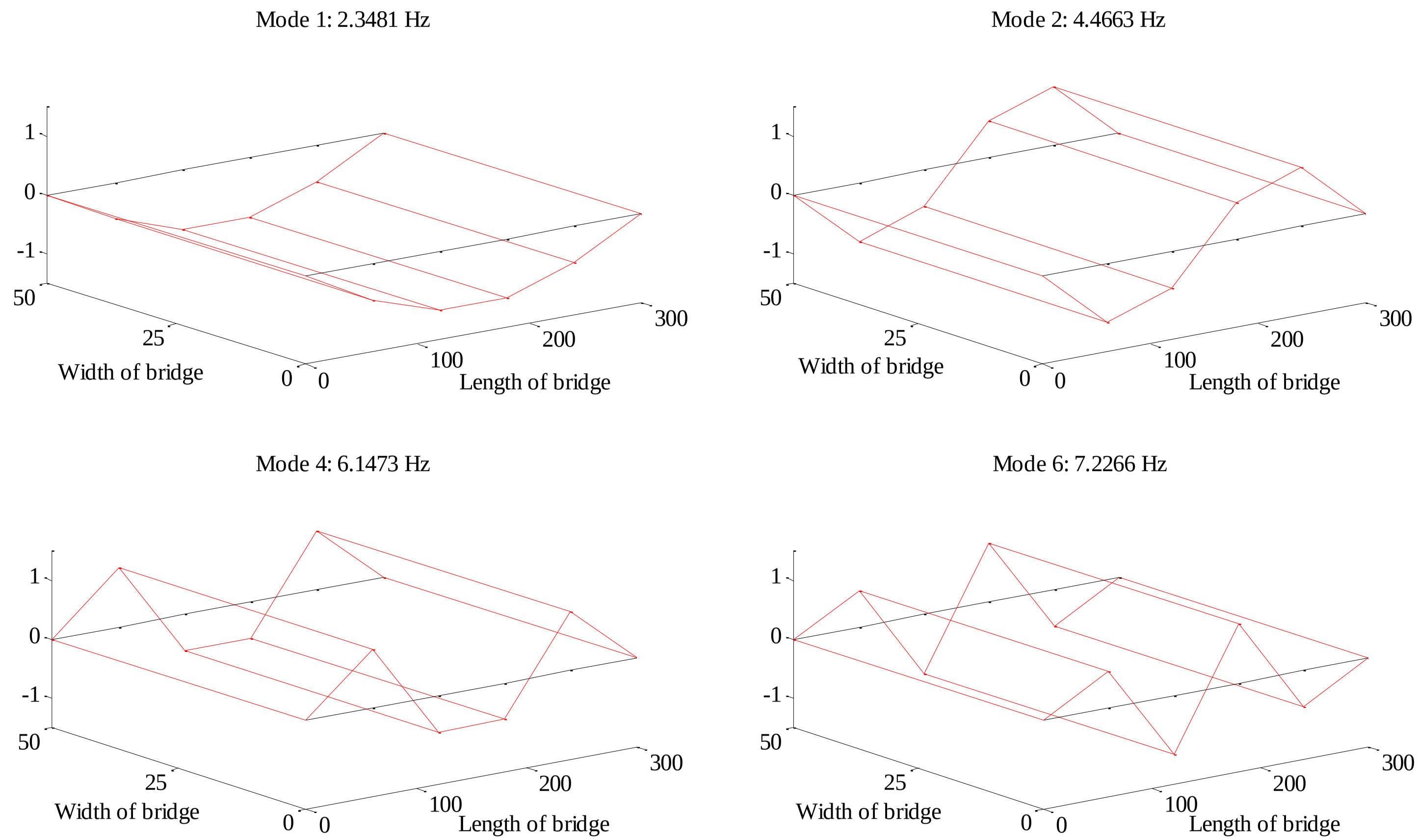

Figure 2: Bending modes of baseline model.

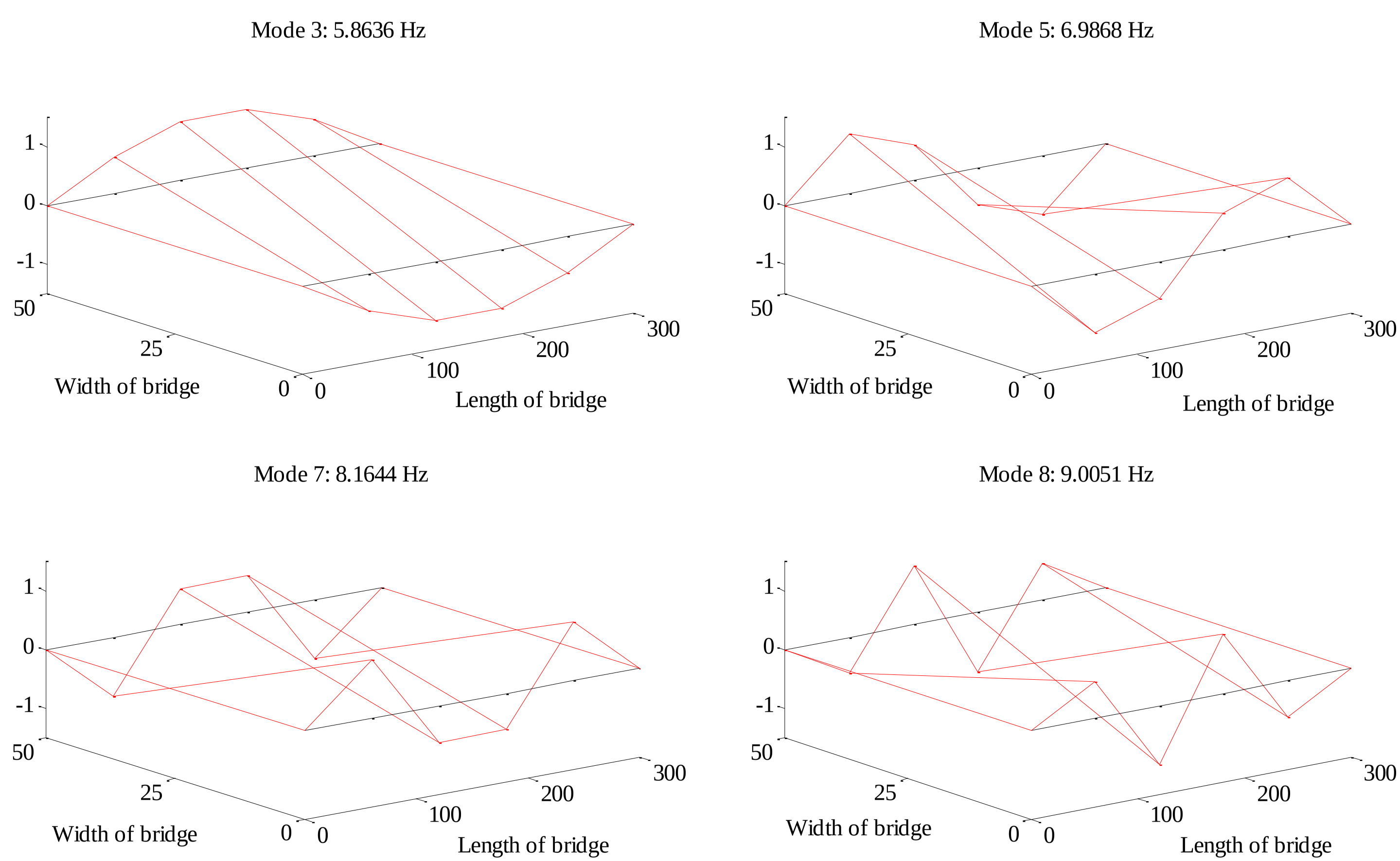

Figure 3: Torsional modes of baseline model.

toring of a bridge and so the algorithm will be tested using only acceleration data. In this version of the algorithm, input-output data will be considered in the identification. However, in the future extension, cases when only time-histories of the structural response are available will be considered. The general formulation of the damage detection algorithm will not change. For the simulations analyzed in this study, the input is represented by a white Gaussian noise applied at some nodal points.

In this study, the modal parameters of the various models are identified using the input-output system identification algorithm OKID/ERA (Observer Kalman filter IDentification/Eigensystem Realization Algorithm) [3]. This algorithm provides a first-order realization of the system using any arbitrary number of time-histories of the structural response and of the input, from which it is possible to extract modal frequencies, modal damping ratios and mode shapes. However, the mode shapes can only be determined at the locations where the time-histories of the structural response were measured (e.g. sensor locations) and this introduces a differentiation in the analysis between 1) the case of a full set of sensors (e.g. sensors at every degree of freedom), and 2) the case of a limited set of sensors (e.g. sensors at only few degrees of freedom). This distinction will be highlighted in the analysis of the results that follows.

4.1 Full Set of Sensors

The training data set is obtained by simulating the response of the system under states from 1 to 5 at each degree of freedom, using white Gaussian noise as input. For each undamaged state, 20 different sets of input have been used, so that n_{tr} is equal to 100 in this example. In each simulation, the mass and stiffness values given in Table (1) are perturbed by ± 1 percent, so as to simulate operational condition variability. Following the procedure detailed in the previous section, one hundred sets of modal properties have been then identified: these sets have been distributed to create the two sets Y_H and Y_V according to the criterion that the even realizations have been taken to construct the set Y_H, i.e. $Y_H = \{\boldsymbol{\lambda}^{(i)}, \boldsymbol{\Phi}^{(i)}\}$ for $i = 2, 4, \cdots, 50$, while the odd realizations have been used to form the set Y_V, $Y_V = \{\boldsymbol{\lambda}^{(i)}, \boldsymbol{\Phi}^{(i)}\}$ for $i = 1, 3, \cdots, 49$. With this choice of sets Y_H and Y_V, each set contains 10 realizations of each of the 5 undamaged scenarios. For testing, only a single set of input has been used to simulate the response of the system in all the 7 states of Table (1): the idea of using also the response of the system in operational conditions was dictated by the need to test the ability of the algorithm not only to identify one of the damage states but also to identify as undamaged a healthy state.

Figures (4) to (7) shows the results for a test performed using data simulated from state 1, not used to construct the training ECDF. The red curves represent the ECDFs of the SPDSF values considered to be acceptable: if the new identified ECDF obtained from the testing set of data is within these two lines, the system can be considered undamaged. While the mean of the lowest values assumed by the SPDSFs in healthy conditions are close to 0, the mean of the maximum values assumed by the features can also exceed 2 percent. This can be explained considering the probabilistic framework in which this analysis is conducted. In fact, within a deterministic framework, a value of the SPDSF higher than 0 would have to be considered indicative of damage, as only one reference structure would be considered, i.e. only one value for each $K_{j,j}$ would be estimated, and all instances whose SPDSF would depart from 0 would raise a damage alarm. On the contrary, the initial training phase performed in the currently proposed approach enables us to set a reasonable range of values within which the departure of $K^*_{j,j}$ from $K_{j,j}$ can be considered as due to the influence of external factors, e.g. temperature, traffic, wind, etc.

Figures (8) to (11) shows the ECDF results for a test performed using data simulated from damage state 6, in which the flexural stiffness of the third element had been reduced by 15 percent. This change should have affected directly the stiffness components $K_{2,2}$ and $K_{3,3}$. In this case, it is interesting to see that the ECDF estimates for $K_{2,2}$, $K_{3,3}$, $K_{4,4}$, $K_{6,6}$, $K_{7,7}$ and $K_{8,8}$ fall outside the boundaries defining the admissible range of the SPDSF values. Nonetheless, the mean of DSF_2 and DSF_3 are about 7 percent, while those of the remaining features are around 1 percent. This clearly indicates that the damage is between degrees of freedom 2 and 3, i.e. in the element k_3. Moreover, by exploiting the ECDFs of DSF_2 and DSF_3, it is possible to state that the variation of $K_{2,2}$ and $K_{3,3}$ from the healthy states exceeds 6 percent with a probability of 0.97. It can be concluded that the proposed approach is then able to locate damage and also to give an estimate of the damage severity. For example, for the current damage scenario, damage is located between degrees of freedom 2 and 3. Degree of freedom 2 is shared between elements 2, 3 and 12; therefore, assuming that all the elements have the same stiffness, k_0, and that only one element stiffness has been decreased to an amount equal to x, the following equation can be solved to estimate which fraction of the original stiffness characterize the stiffness

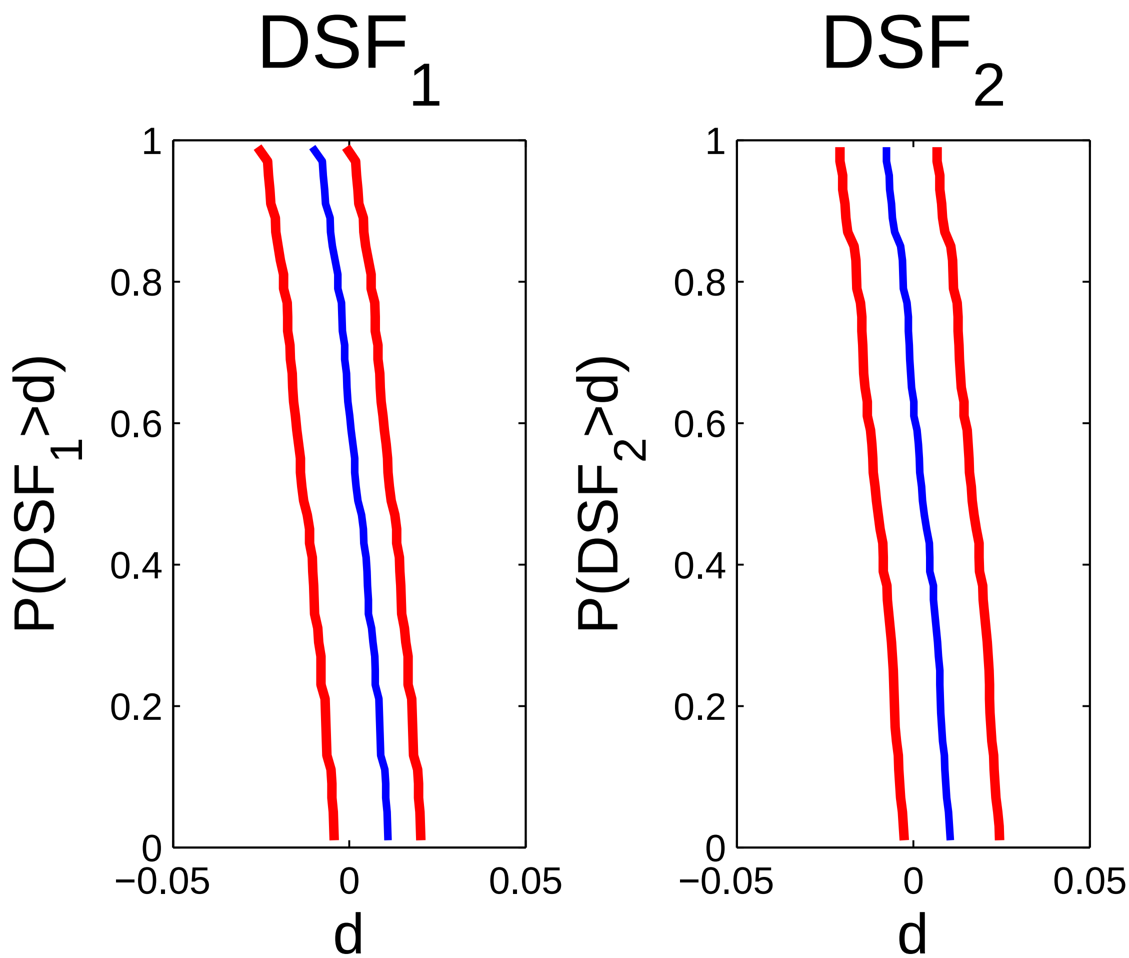

Figure 4: Test using data simulated from state 1: ECDFs for DSF_1 and DSF_2.

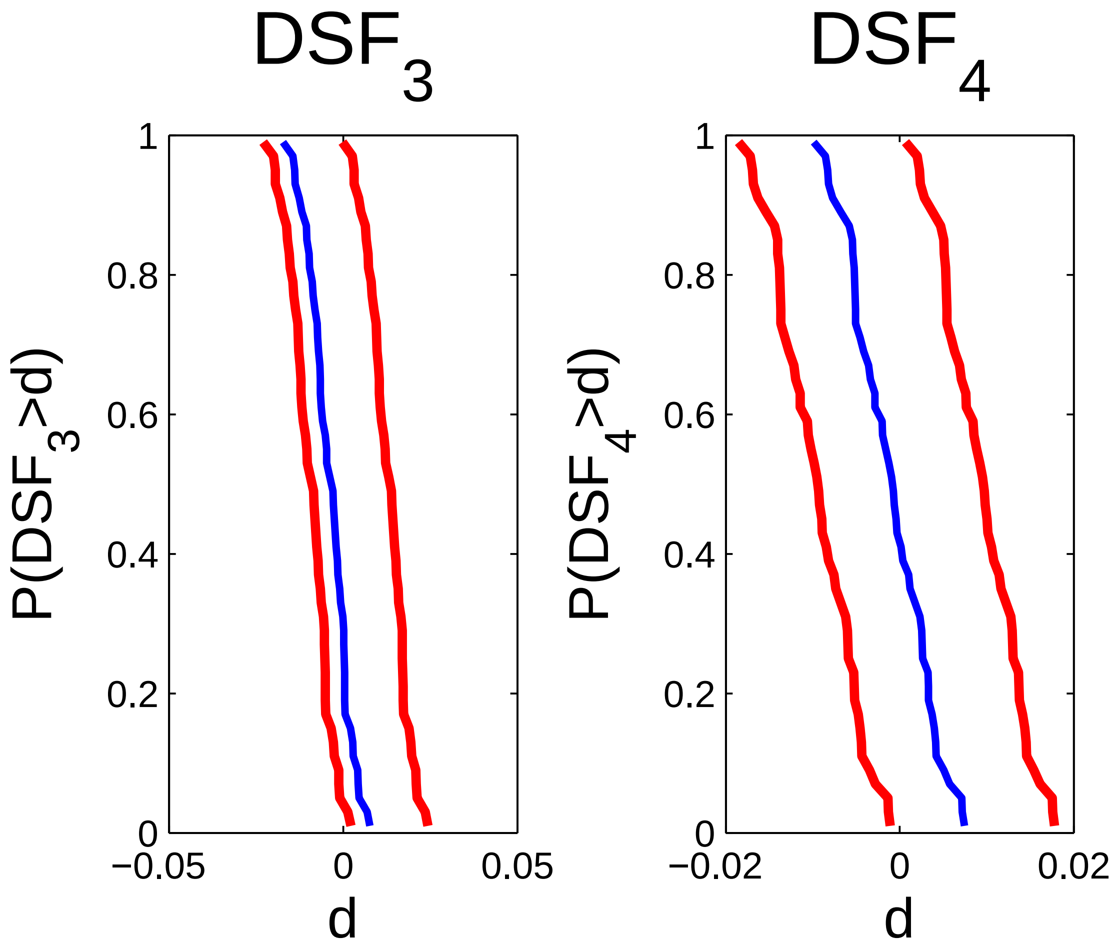

Figure 5: Test using data simulated from state 1: ECDFs for DSF_3 and DSF_4.

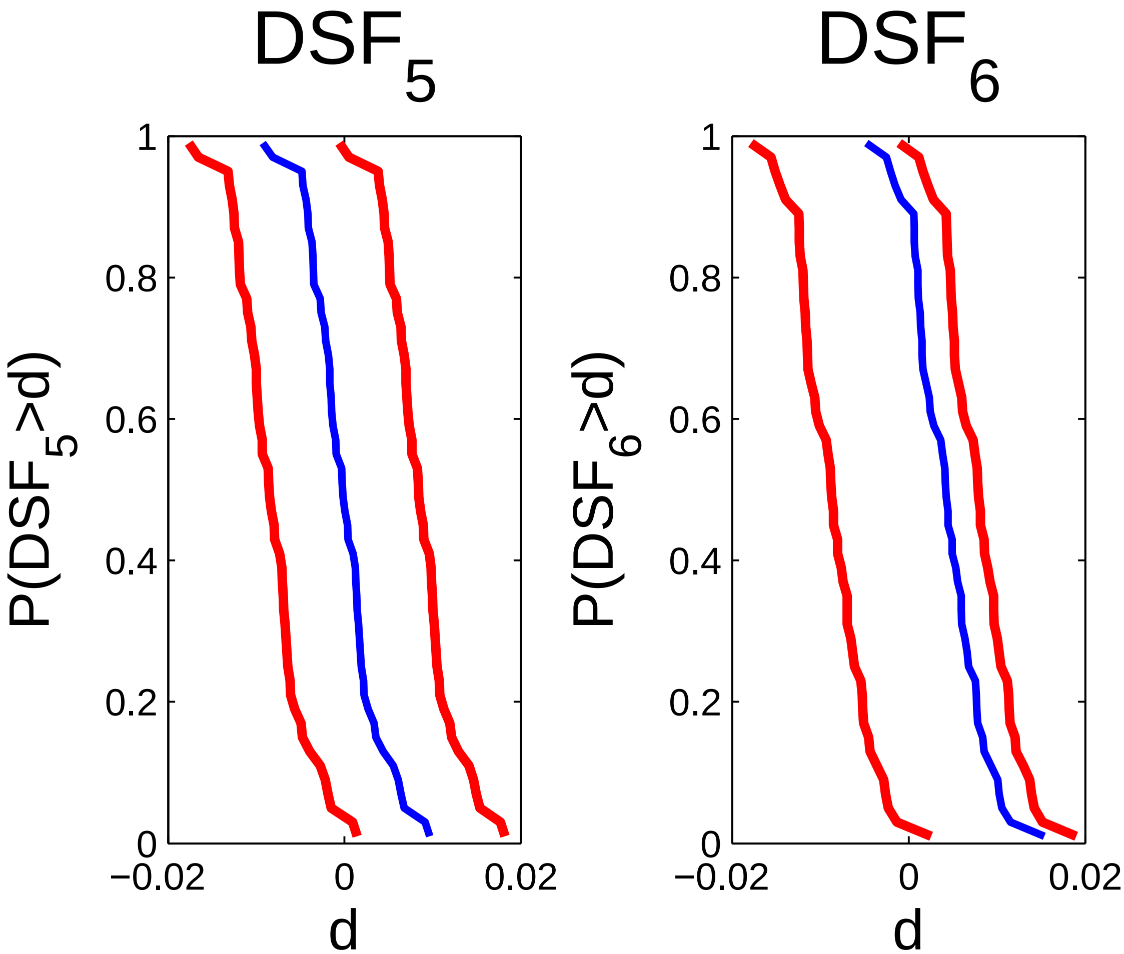

Figure 6: Test using data simulated from state 1: ECDFs for DSF_5 and DSF_6.

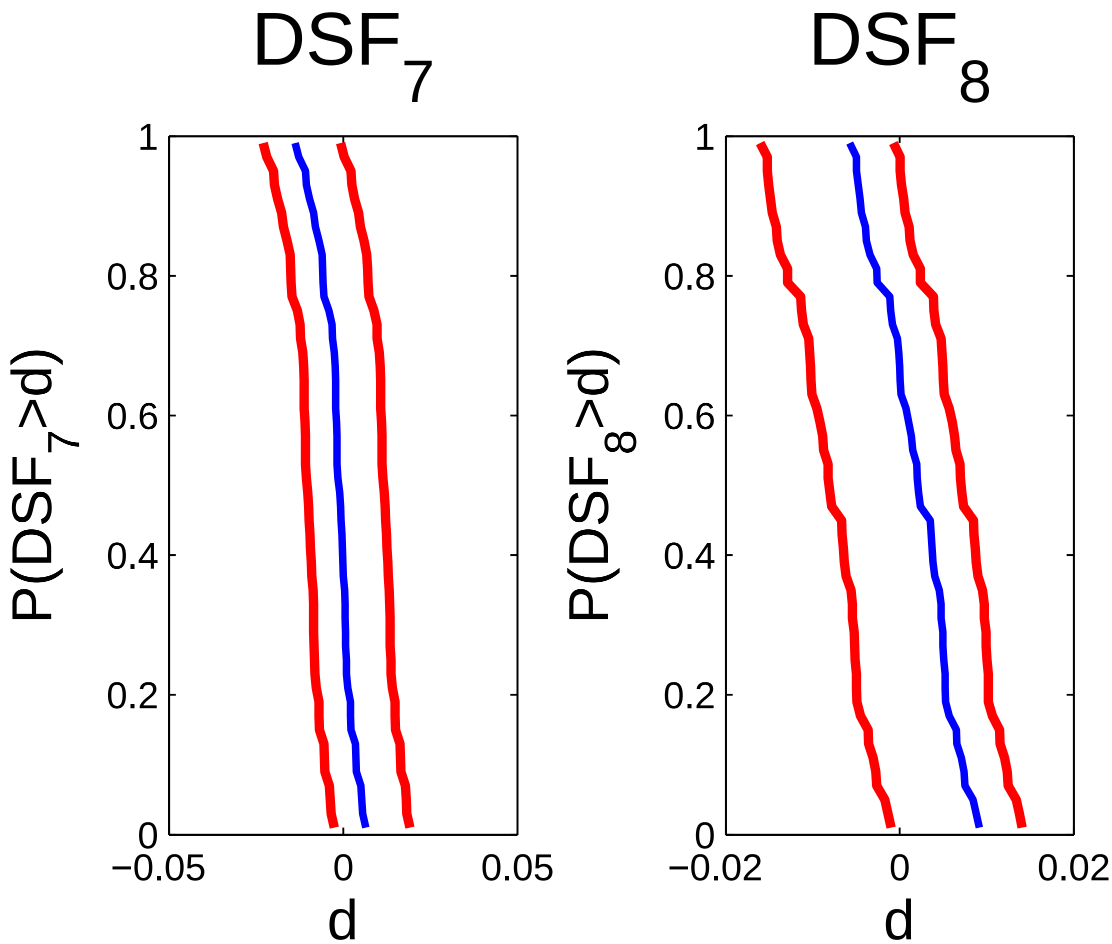

Figure 7: Test using data simulated from state 1: ECDFs for DSF_7 and DSF_8.

value of the damaged element:

$$\frac{3k_0 - (2+x)k_0}{3k_0} = 0.06 \Rightarrow x = 0.82 \tag{18}$$

Therefore, from the observation of Figures (8) and (9), it is possible to conclude that the stiffness of the flexural element 3 has decreased to 18 percent, very close to the actual value of 15 percent reduction.

Finally, Figures (12) to (15) show the results for a test performed using data simulated from damage state 7. Following the considerations made for the previous damage scenario, in this case it can be concluded that damage has occurred between degrees of freedom 2 and 6, and that the stiffness value of the flexural element 12 has decreased at least to 85 percent its original value. This represents the actual reduction of stiffness introduced in the model, confirming the great potential of the proposed approach in locating and quantifying structural damage.

4.2 Reduced Number of Sensors

This case is considered to represent situations that usually occur in real life, when the number of available sensors, e.g. accelerometers, is less than the number of degrees-of-freedom used in the structure model. Here, the same 7 states considered for the previous case have been considered, but, in this second case study, only the response of the system at degrees of freedom 1, 3, 6 and 8 are used to identify the modal properties. The

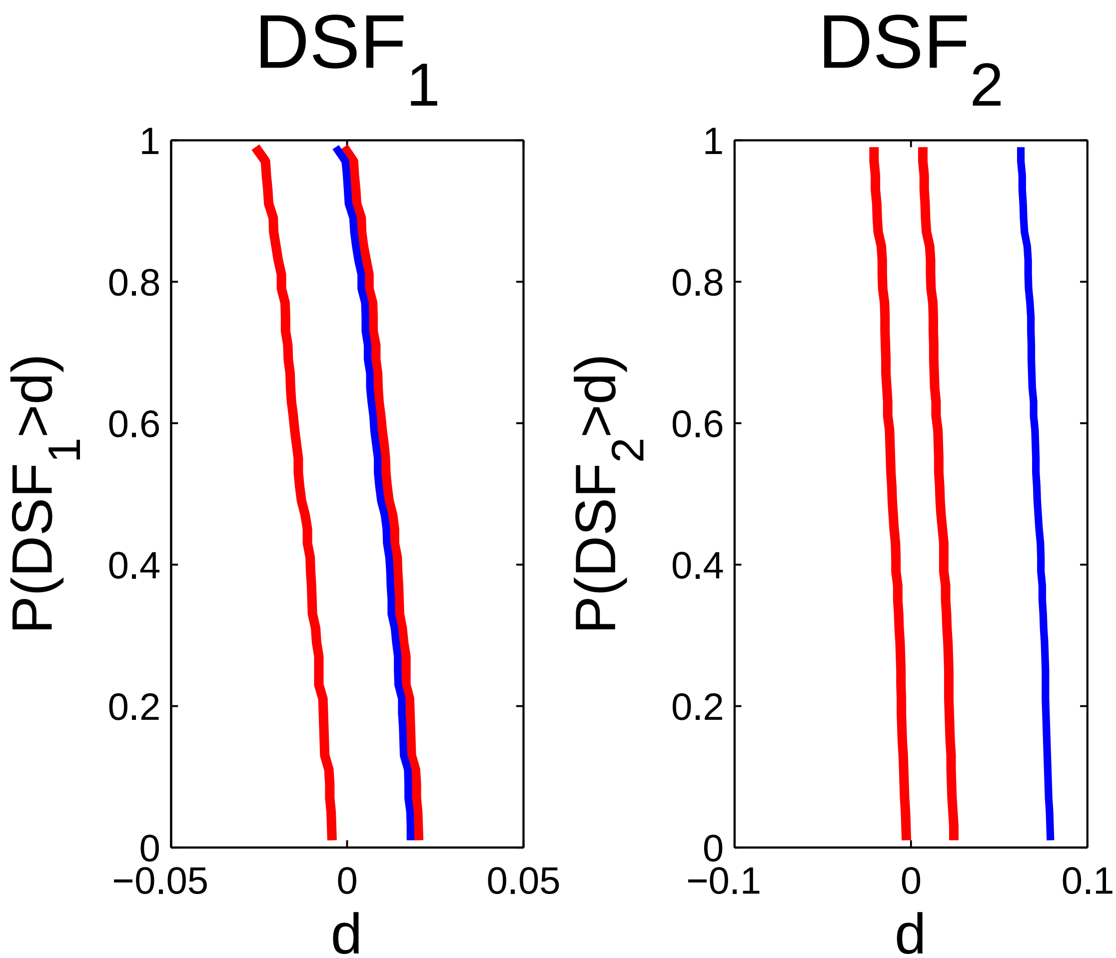

Figure 8: Test using data simulated from state 6: ECDFs for DSF_1 and DSF_2.

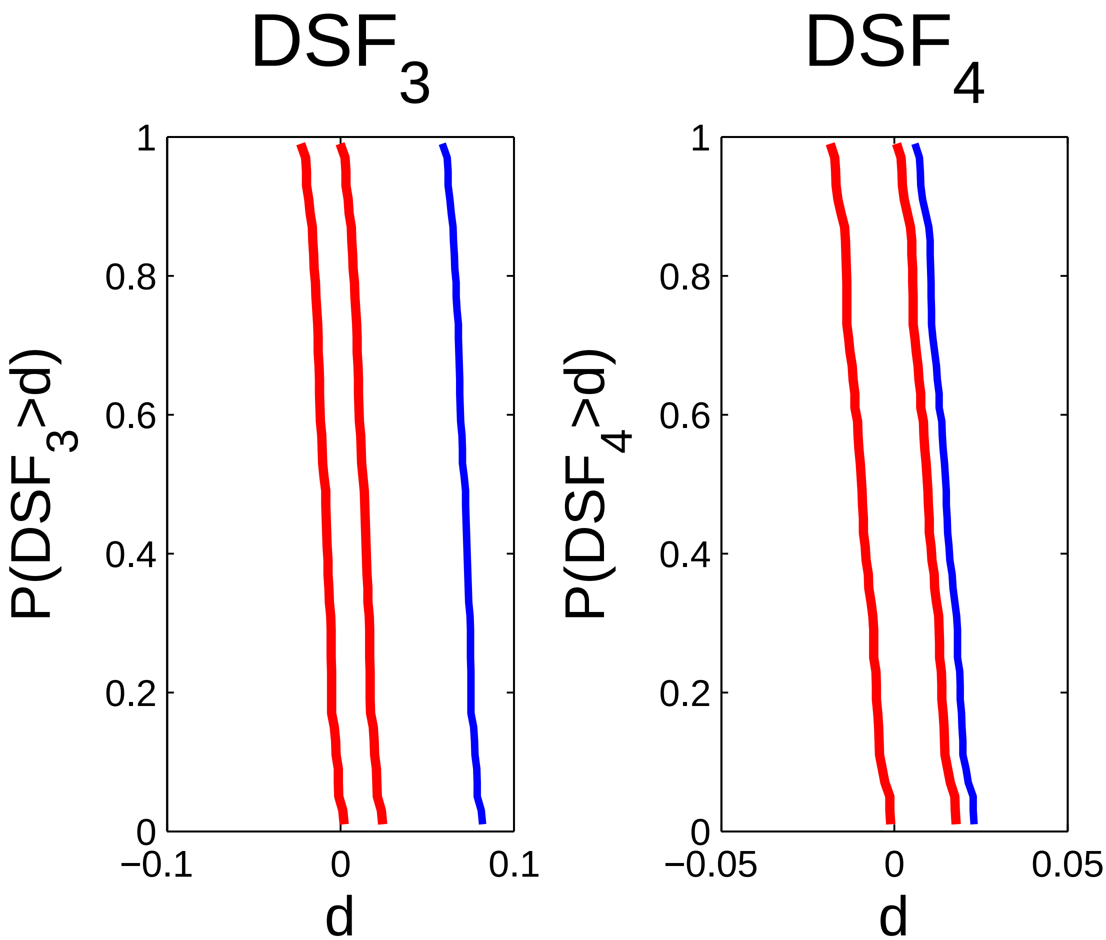

Figure 9: Test using data simulated from state 6: ECDFs for DSF_3 and DSF_4.

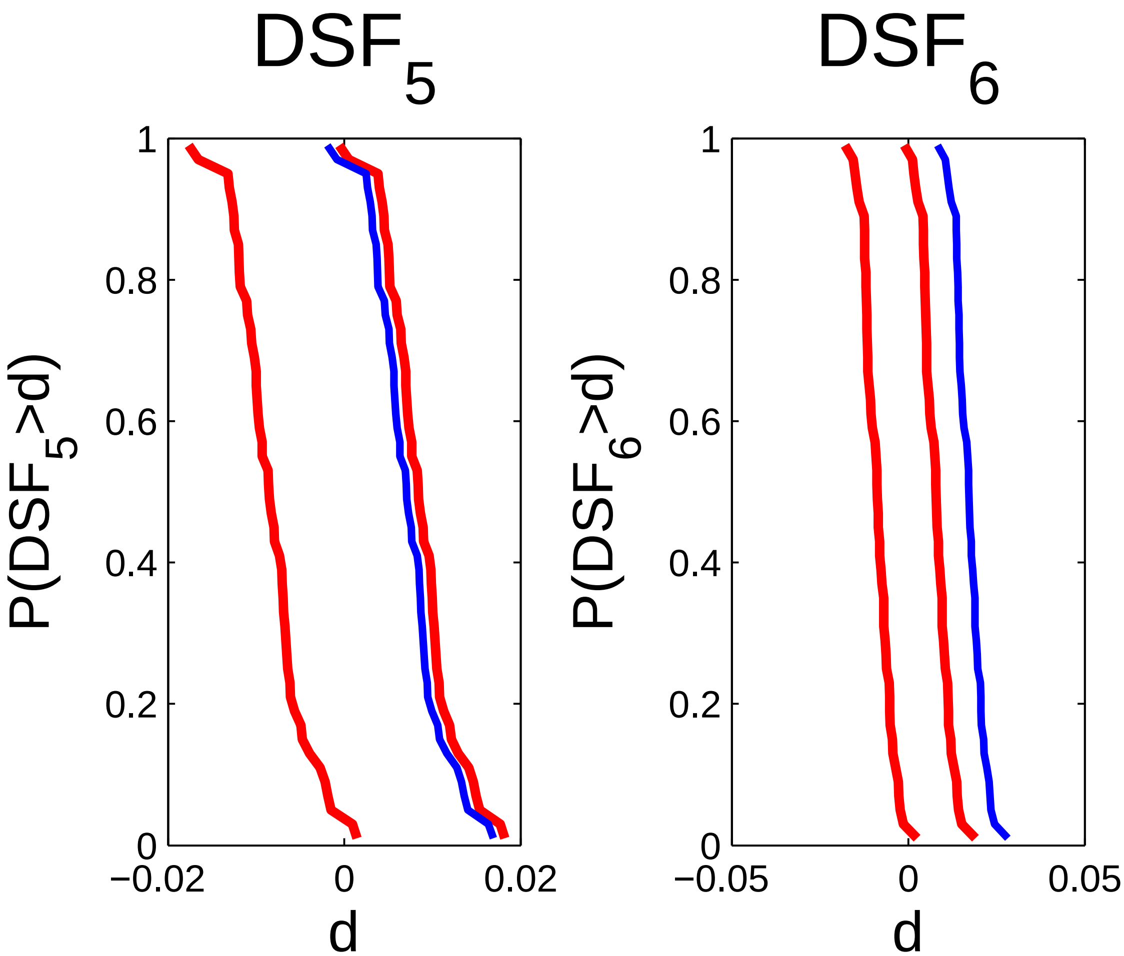

Figure 10: Test using data simulated from state 6: ECDFs for DSF_5 and DSF_6.

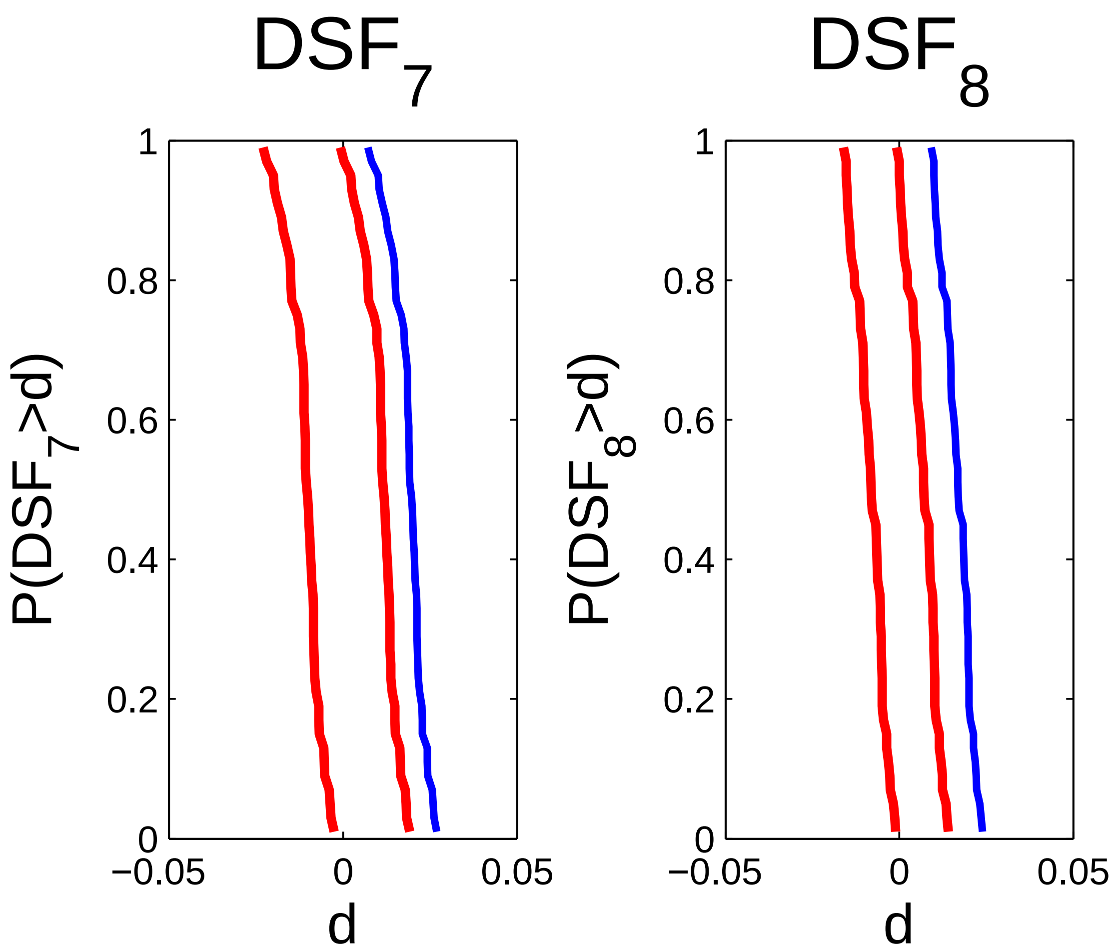

Figure 11: Test using data simulated from state 6: ECDFs for DSF_7 and DSF_8.

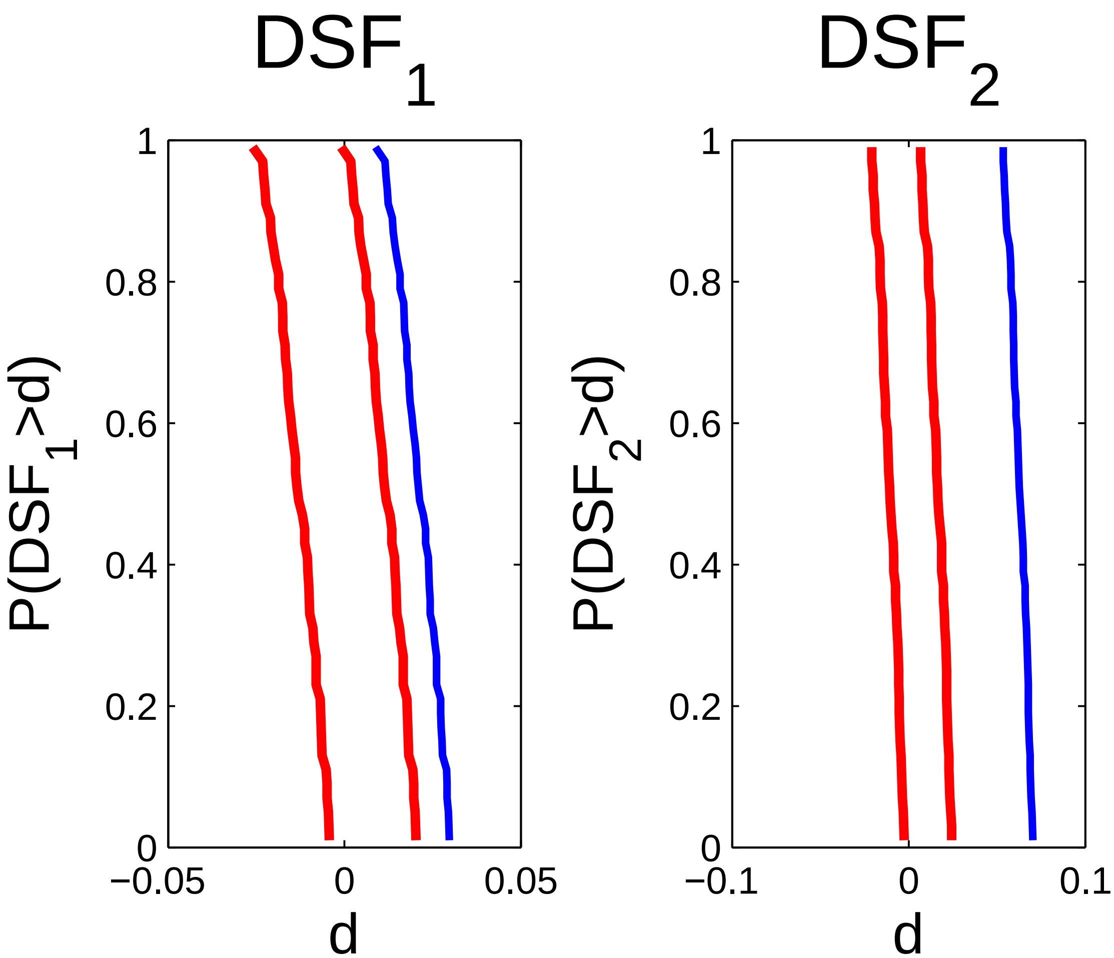

Figure 12: Test using data simulated from state 7: ECDFs for DSF_1 and DSF_2.

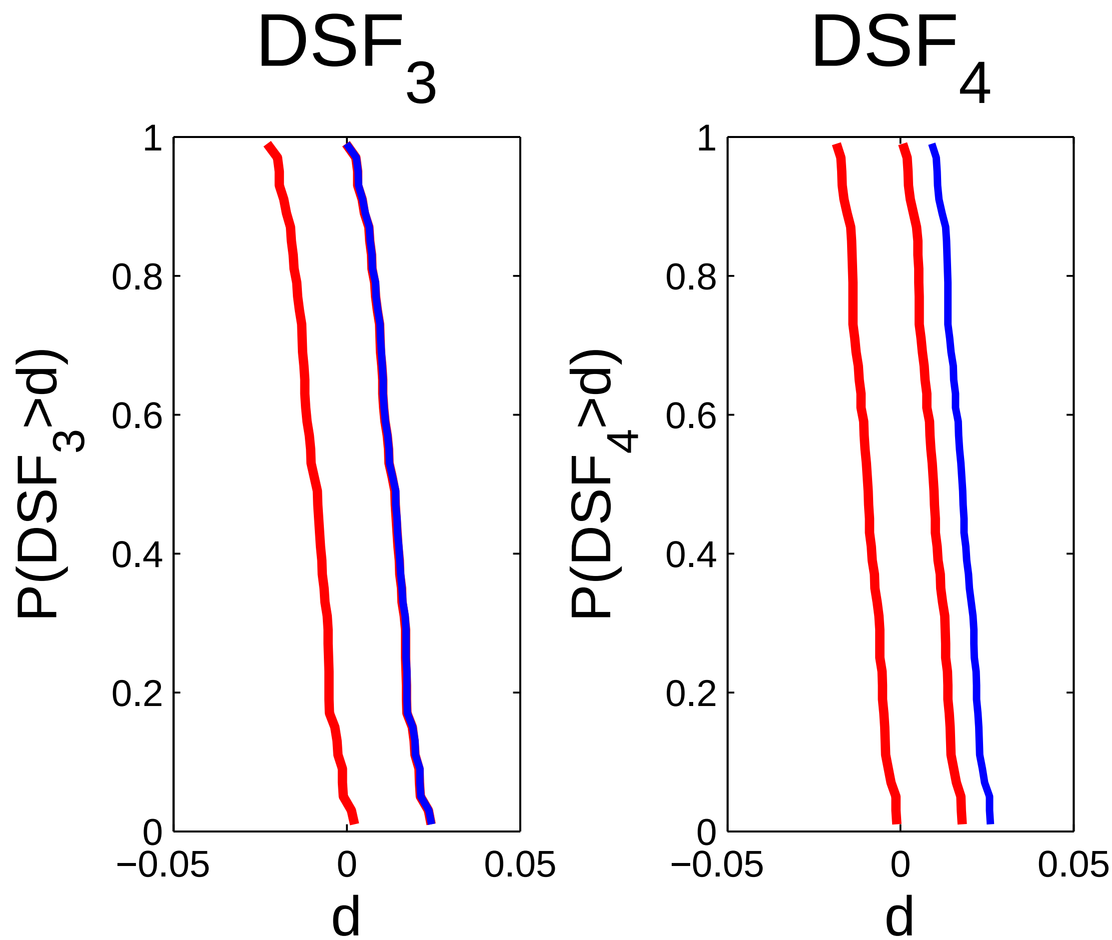

Figure 13: Test using data simulated from state 7: ECDFs for DSF_3 and DSF_4.

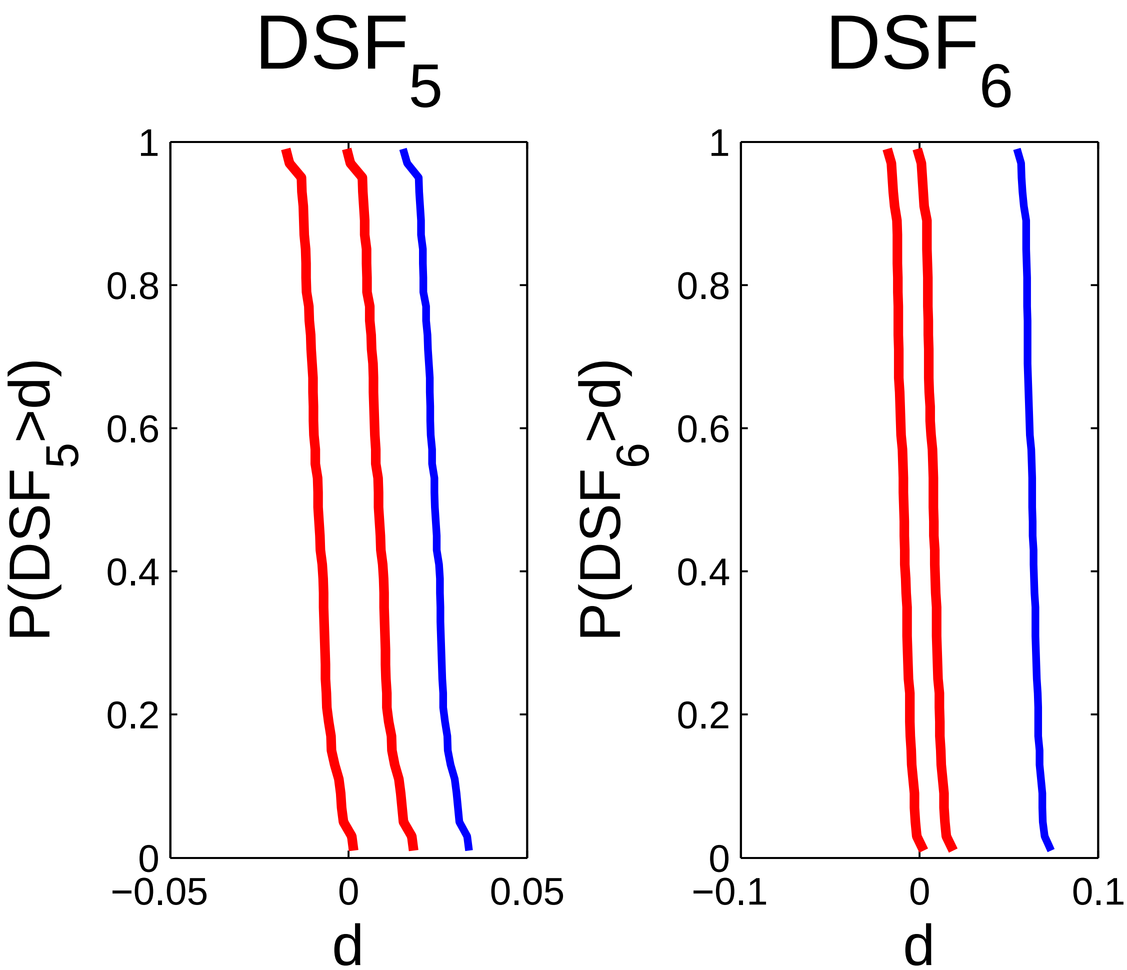

Figure 14: Test using data simulated from state 7: ECDFs for DSF_5 and DSF_6.

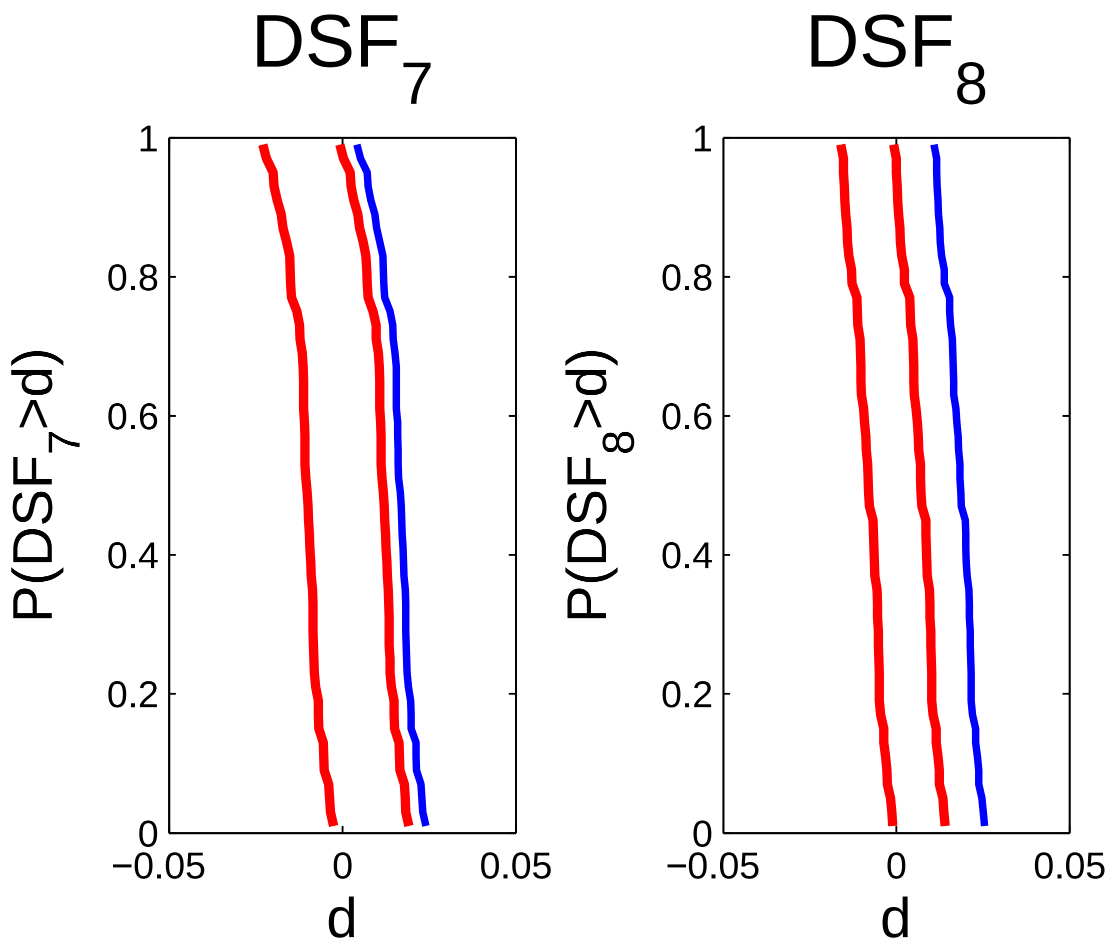

Figure 15: Test using data simulated from state 7: ECDFs for DSF_7 and DSF_8.

OKID/ERA algorithm is again used for the identification: the algorithm is able to identify all 8 modes, but only the rows corresponding to the instrumented degrees of freedom of the mass normalized mode shape matrix may be estimated. This identification can be carried out using input-output balance equations in the time domain [4].

The results for the case of a limited number of sensors are presented in Figure (16) to Figure (21). Also for this case, the damage identification is successfully completed, declaring the structure healthy for the first test, and damaged in the last two tests. Nonetheless, using this limited sensor setup, the probable area of damage is now less confined than in the first case: for the first damage case (state 6 in Table (1)) where the structure is declared damaged, it is possible to state that damage has occurred around degree of freedom 3, i.e. either element 3, 4 or 13 could be damaged. However, in this case, the extent of such damage is uncertain owing to the absence of measurements at degrees of freedom 2, 4 and 7: it could range from mild and extended, in the case where all three elements are damaged (in such scenario, each element would have decreased its stiffness to 96 percent of its original value), to substantial but localized, in the case where only one element has its stiffness decreased to 82 percent of its original value. Similar considerations apply for the case where the response of the system under damage state 7 is considered: damage may be present in elements 7, 8 and 12, and, in such case, either each element would have been subjected to a decrease to 96 percent of its original value, or only one of such element could have been damaged leading to a reduction in stiffness of at least 15 percent of its original value. Nevertheless, even though the results in this second case are less accurate than the case when the full set of sensors is available, the proposed approach guarantees a reasonable indication of damage presence and location.

The damage detection in the situation of limited instrumentation may be further improved by applying mode shape expansion to expand the mass normalized mode shapes, identified at the instrumented degrees of freedom using OKID/ERA and input-output balance, from these instrumented to the non-instrumented degrees of freedom. In this way, the complete mass normalized mode shape matrix, needed for the calculation of all the DSF_j's, can be estimated, and will hence improve the accuracy of damage location and severity estimation. To this end, the mode shape expansion equations of [4], based on the eigenvalue equations and on equations from the structural topology, may be considered in conjunction with the damage detection method proposed here. This approach has the advantage of obtaining reliable estimates of the model's complete mass normalized mode shapes without any *a priori* information on the structural properties, of being robust in the presence of significant measurement noise and of being flexible with respect to the type of measurement data necessary, e.g. accelerations and/or velocities and/or displacements.

5 Conclusions and Future Studies

In this report, the results of a study on developing a damage detection methodology based on Statistical Pattern Recognition are presented. This methodology uses a new damage sensitive feature developed in this study that relies entirely on modal characteristics, i.e. natural frequencies and mode shapes, directly identified from measurements of the struc-

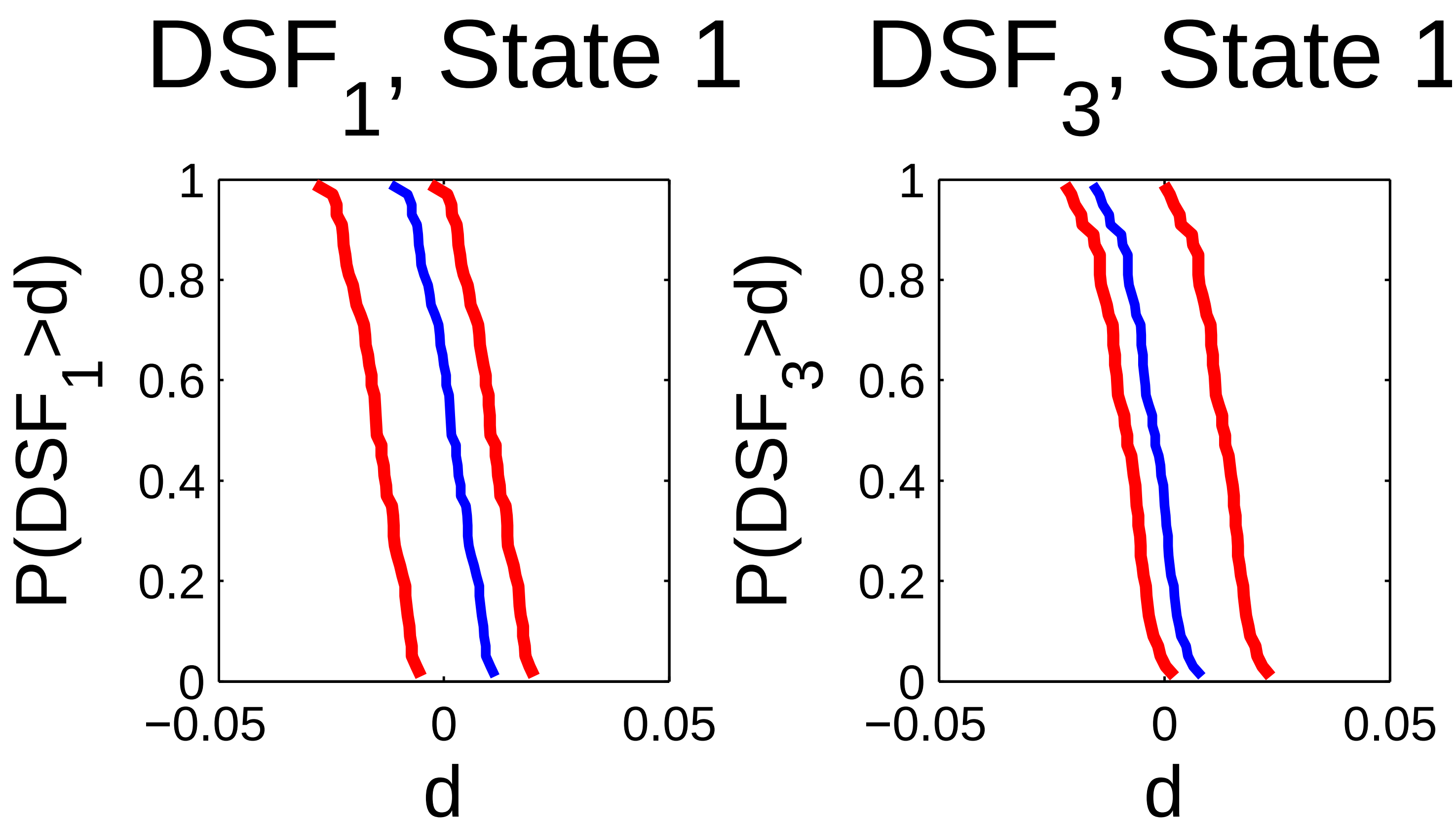

Figure 16: Test using data simulated from state 1 with reduced number of sensors: ECDFs for DSF_1 and DSF_3.

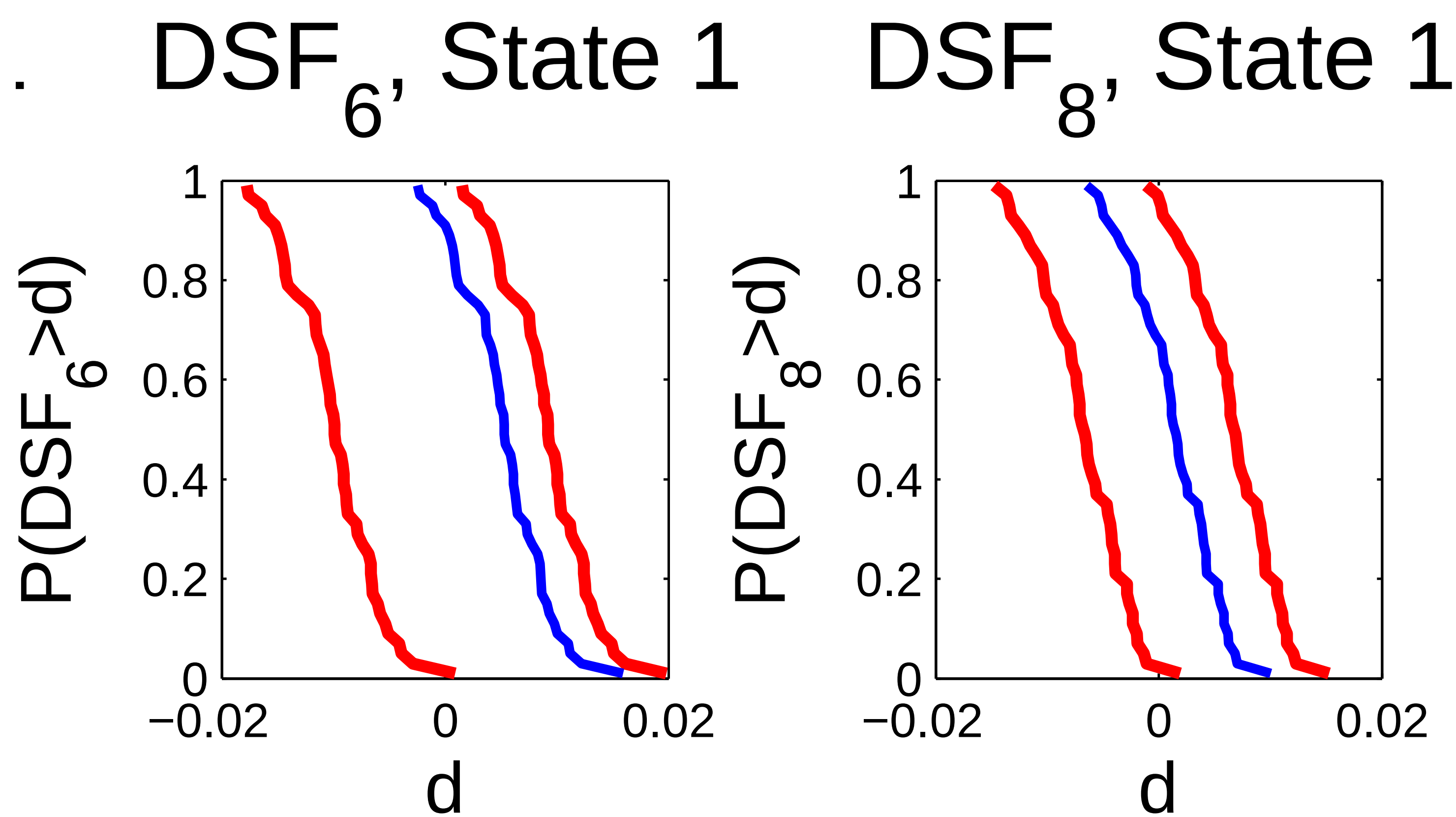

Figure 17: Test using data simulated from state 1 with reduced number of sensors: ECDFs for DSF_6 and DSF_8.

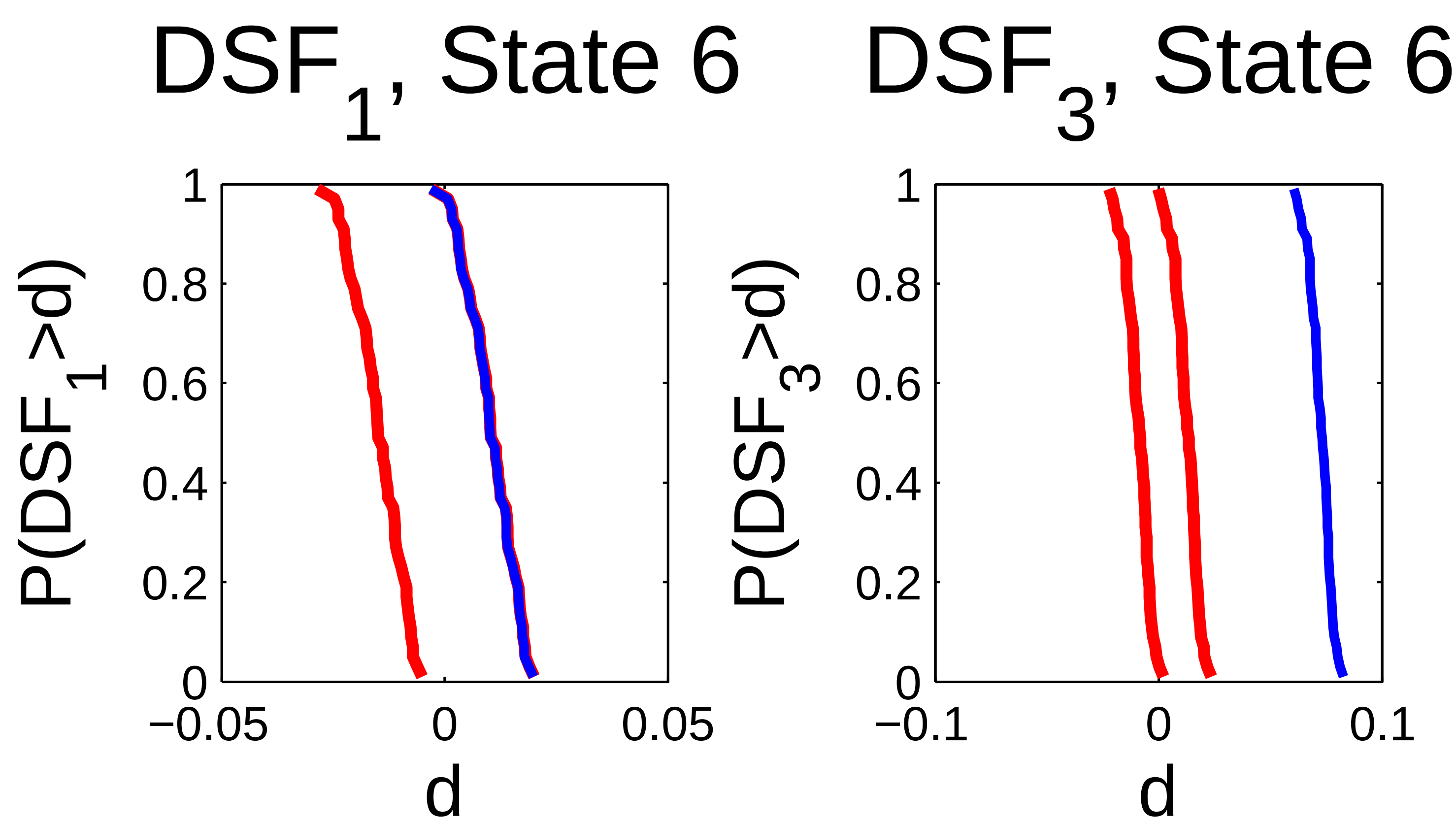

Figure 18: Test using data simulated from state 6 with reduced number of sensors: ECDFs for DSF_1 and DSF_3.

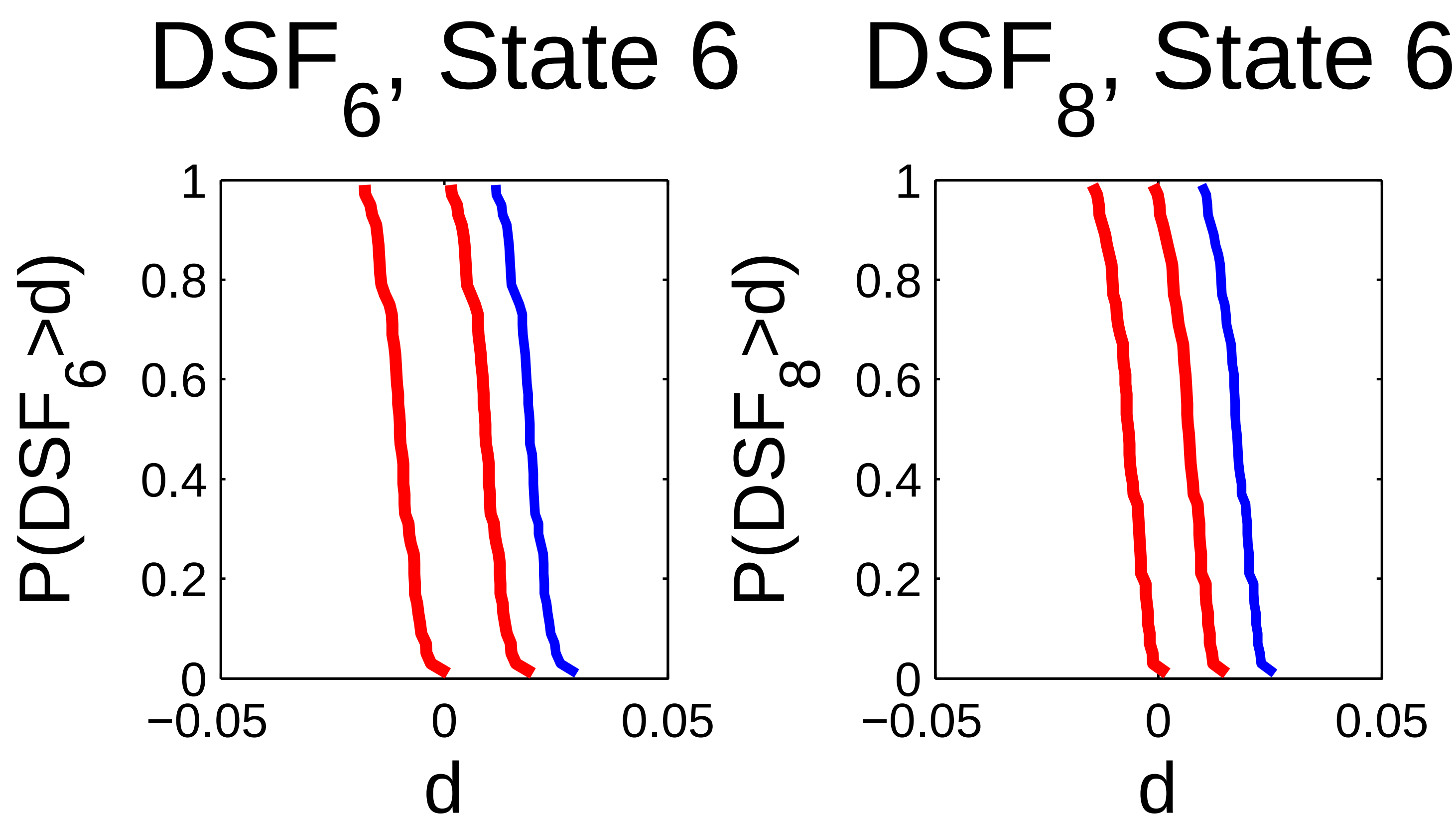

Figure 19: Test using data simulated from state 6 with reduced number of sensors: ECDFs for DSF_6 and DSF_8.

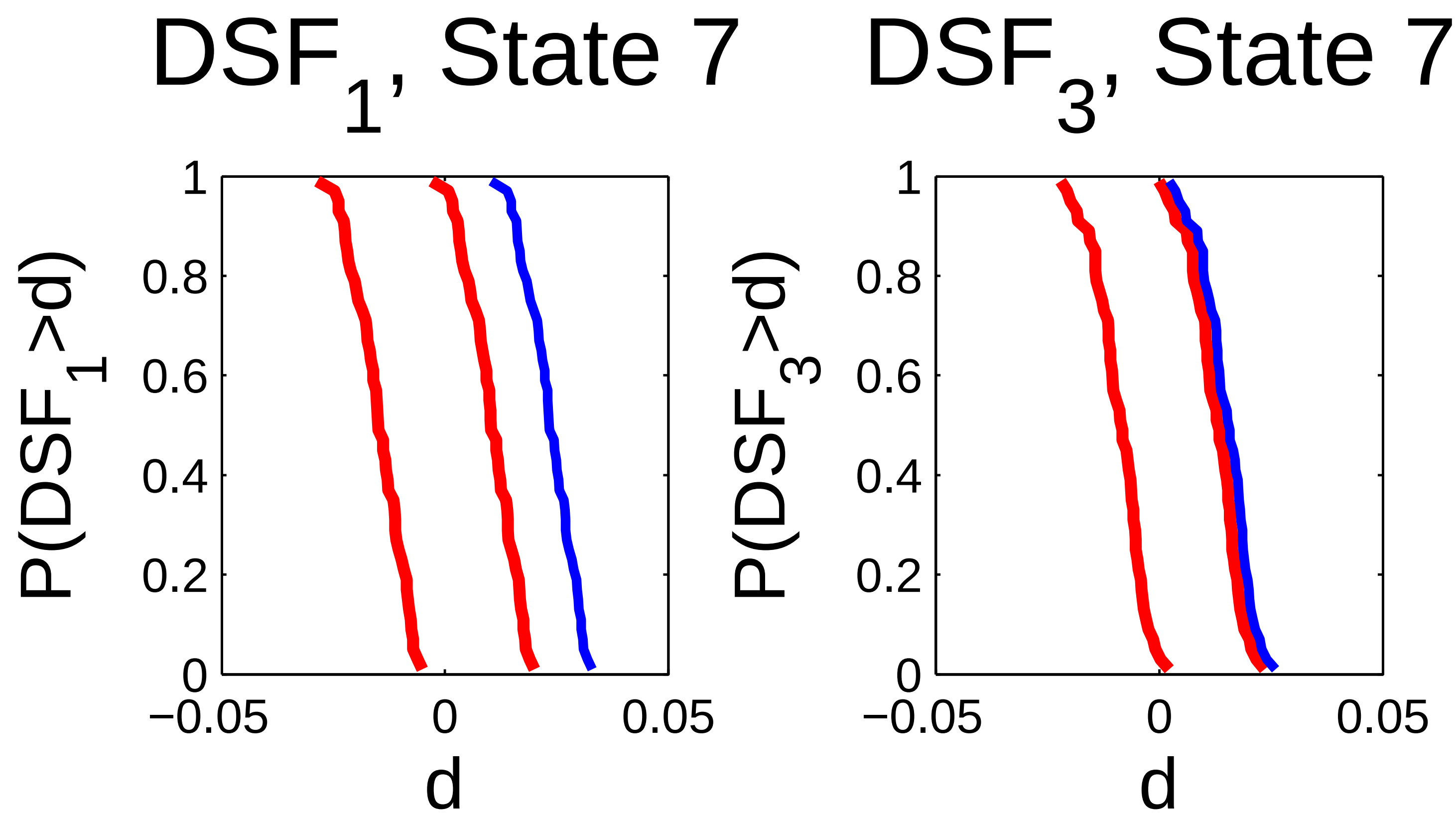

Figure 20: Test using data simulated from state 7 with reduced number of sensors: ECDFs for DSF_1 and DSF_3.

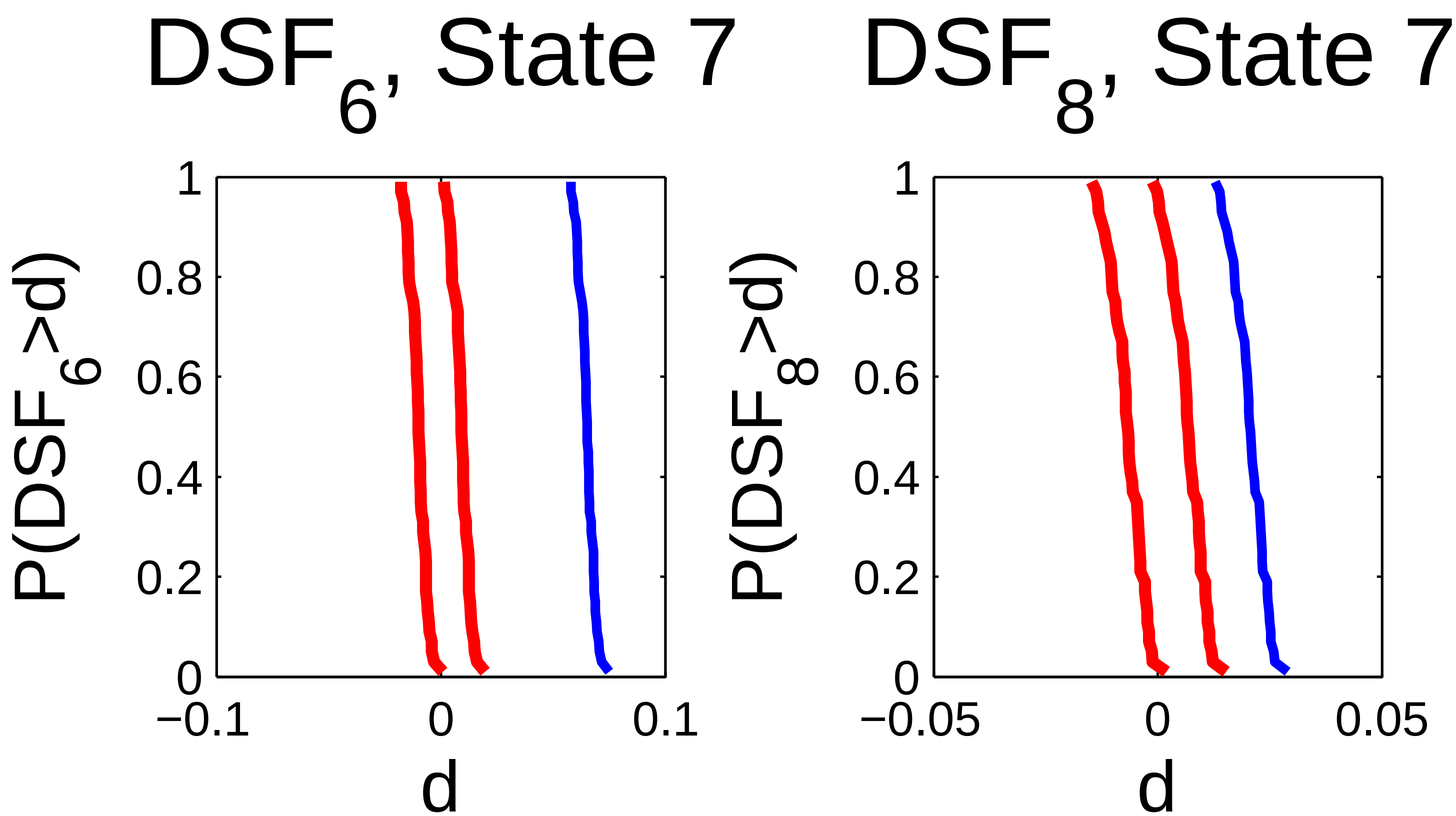

Figure 21: Test using data simulated from state 7 with reduced number of sensors: ECDFs for DSF_6 and DSF_8.

tural response. A procedure for training the damage detection methodology to account for variability induced by operational conditions, i.e. temperature, traffic, wind, etc., has been proposed to determine boundaries of variability of the cumulative distribution functions of the various features. Two different test setups have been considered: 1) a full set of sensors, and 2) a limited number of sensors. The results show that this methodology has been proven successful in detecting the occurrence of damage and, in the case of full sensor setup, also in accurately locating the element where damage has occurred and the amount of element stiffness reduction. In the case of a limited instrumentation setup, the proposed methodology is successful in identifying the occurrence of damage, and even though it loses accuracy in pinpointing the exact damage location, it still successfully identifies the region containing the damaged element. In conclusion, the results obtained using the proposed SPDSF are promising and allow for damage identification, localization and estimation.

The next step will be to use the same damage detection algorithm but relying only on measurement data of the structural response (output-only). Preliminary results conducted as a part of this study show that using non mass-normalized mode shapes, as usually obtained from output-only identification algorithm, can generate misleading results. The key point will be the determination of the mass normalizing factors of a mode shape matrix identified via output-only system identification algorithms. Also, the development and application of mode shape expansion techniques to obtain the complete mass normalized mode shape matrix in the situation of limited instrumentation will constitute a part of future study.

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