Synthesis of the Advance in and Application of Fractal Characteristics of Traffic Flow

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Lehman Center for Transportation Research
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Synthesis of the Advance in and Application of Fractal Characteristics of Traffic Flow

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METRIC CONVERSION CHART

SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
LENGTH				
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
AREA				
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
SYMBOL	WHEN YOU KNOW	MULTIPLY BY	TO FIND	SYMBOL
VOLUME				
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
NOTE: volumes greater than 1000 L shall be shown in m^3				

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16. Abstract

Fractals are irregular geometric objects that exhibit finite details at all scales, and once magnified, their basic structures remain the same regardless of the scale of magnification. Fractal theory has been successfully applied in different fields of science. This project provides a synthesis of existing applications of fractal theory in various fields, as well as its potential applications in traffic management. The specific information gathered and summarized in this report includes: (1) a synthesis of fractal applications in fields that share similarities with transportation networks, such as electrical networks, and in fields outside of transportation; (2) a synthesis of fractal applications that have been proven effective in traffic flow; (3) additional insights on how fractal theory can be applied in traffic management strategies; and (4) summary of research findings and recommendations for more detailed research.

Two fractal techniques, the fractal dimension and the Hurst exponent, were applied to detect the existence of fractal characteristics in traffic and crash data from Florida. Traffic volume, speed, and occupancy data obtained from 15-min and 1-hr detector data at two locations in Miami-Dade County were found to exhibit fractal characteristics. Furthermore, the speed trend revealed stronger fractal behavior compared to the volume and occupancy trends for the same time period. The existence of fractal characteristics in crash data was detected in both annual and daily frequency trends. However, the daily crash frequency trends exhibited greater extent of fractal behavior compared to the annual trends, mainly due to the existence of more random fluctuations. The fractal investigation of both annual and three-year average crash rates at ten randomly-selected signalized intersections revealed that the annual crash rate trend, in general, exhibited relatively more fractal characteristics than the three-year average trend.

Future research could make use of the insights presented in this study to apply fractal theory in traffic management strategies such as managed lanes, ramp metering, crash analysis, and travel time reliability. Compared to traditional models, fractal theory is anticipated to yield more precise estimates of performance measures. Therefore, a potential future research is to apply fractal theory for predicting short-term traffic flow. Another promising avenue is to apply fractal theory to identify high-crash locations and to predict crash rate at specific locations. These approaches could potentially result in increased efficiency, mobility, and safety of the entire roadway network.

17. Key Word Fractal Theory, Fractal Characteristics, Traffic Flow, Traffic Prediction, Traffic Management, Traffic Safety		18. Distribution Statement	
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EXECUTIVE SUMMARY

The main objective of this project is to provide a synthesis of existing applications of fractal theory in various fields, as well as potential applications of fractal theory in traffic management. The study also identifies those elements of traffic flow which exhibit fractal characteristics. The study further uses crash and traffic flow data from Florida to investigate the existence of fractal characteristics. The specific information gathered and summarized in this report includes:

1. A synthesis of fractal applications in fields that share similarities with transportation networks, such as electrical and water supply networks, and in fields outside of transportation, such as astronomy and ecology;
2. A synthesis of fractal applications that have been proven effective in traffic flow such as in traffic flow prediction;
3. Additional insights on how fractal theory can be applied to traffic management strategies, such as managed lanes, ramp metering, etc.; and
4. Summary of research findings and recommendations for more detailed research and development of future guidance documents.

Fractals are irregular geometric objects that exhibit finite details at all scales, and when magnified, their basic structures remain the same regardless of the scale of magnification. The main characteristics of fractals are self-similarity, iterative process, infinite complexity, and existence of non-integer complex dimension. Common fractal analysis techniques are the Hurst exponent, the fractal dimension, the largest Lyapunov exponent, the power spectrum, and the Kolmogorov entropy. The main indicators of existence of fractal characteristics are as follows: the largest Lyapunov exponent should have a positive sign; the Hurst exponent (H) estimate should lie between 0 and 1; and the Kolmogorov entropy should have a positive sign. Moreover, larger estimates of the Lyapunov exponent, the Hurst exponent, and the fractal dimension indicate more complex fractal nonlinear characteristics.

Fractal theory has been successfully applied in different fields of science, including but not limited to animal behavior studies, human health studies, economics, astronomy, ecology, physics, pavement engineering, environmental engineering, behavior of materials, and electrical networks. Even though application of fractal theory is not uncommon in several areas in transportation, such as traffic flow analysis, urban network analysis, and travel demand modeling, existence of fractal characteristics in safety data is rarely investigated. Moreover, fractal applications in traffic flow prediction showed promising results. Fractal models were found to be appropriate for low traffic volume conditions and for short-term traffic flow predictions and weather forecasts. Additionally, traffic flow and traffic speed were found to be the main elements of traffic data to exhibit fractal characteristics.

In addition to the existing applications, the following traffic management strategies where fractal theory could potentially be applied were discussed: managed lanes, ramp metering, crash analysis, parking management, and travel time reliability. For each application, the data needed to apply fractal theory and the data that could be predicted by applying fractal theory were identified. Fractal behavior was hypothesized to exist in all the five strategies due to random fluctuations in the data.

Two fractal techniques, the fractal dimension and the Hurst exponent, were applied in the **R** Software (*R* Project, 2013) to investigate the existence of fractal characteristics in crash and traffic data from Florida. The fractal dimension estimate was calculated using ten methods: *box count, Hall-Wood, variogram, madogram, rodogram, Incr1, periodogram, DCT-II, genton*, and *wavelet* method. The Hurst exponent was calculated using eight methods: *aggregated variance, differenced aggregated variance, aggregated absolute moment, Higuchi, variance of residuals, R/S, periodogram,* and *modified periodogram*. Fractal behavior exists if the fractal dimension estimate is greater than 1 and whenever the Hurst exponent *(H)* lies between 0 and 1. Further, the higher the value of the estimate, the more predominant the fractal characteristics.

Trends in the statewide annual and daily crash frequency, as well as the annual intersection crash rate were explored. Data from 1990-2011 were used to analyze trends in annual crash frequency, and five-year data from 2007-2011 were used to analyze daily crash frequency trends. The analysis detected the existence of fractal characteristics in both annual and daily crash frequency trends. However, the daily crash trend exhibited greater extent of fractal behavior, mainly due to the existence of more random fluctuations. A common observation was that the fractal dimension estimate from the wavelet method was among the highest estimates. Based on the Hurst exponent estimate, *H*, six of the eight methods yielded estimates between 0 and 1 for annual crash trend, indicating fractal behavior. On the other hand, all the eight Hurst exponent methods for trend in daily crash frequency yielded *H* values between 0 and 1, indicating a fairly consistent fractal behavior.

Additionally, trends in daily crash frequency for different facility types (e.g., freeways, arterials, collectors, etc.) were investigated. The fractal dimension analysis revealed that estimates from all methods were greater than 1 for all facilities. Urban collectors experienced the highest fractal behavior. The Hurst exponent analysis revealed that all *H* estimates were between 0 and 1, which shows the existence of fractal characteristics and long-range dependence. In general, urban facilities resulted in higher *H* estimates than their rural counterparts. More specifically, urban collectors had the highest median *H* estimate.

Since crash rates are used to account for the changes in the annual average daily traffic (AADT) at a location over time, the fractal characteristics of crash rates (i.e., crash frequency per million entering vehicles) were also investigated at ten randomly-selected signalized intersections. To investigate the existence of fractal characteristics in annual crash rates, two types of analysis were conducted: the annual crash rate trend and the three-year moving average crash trend. It was found that the annual crash rate trend, in general, exhibited relatively more fractal characteristics than the three-year average trend in crash rate. This is obvious since most of the *H* estimates for all the ten intersections for the annual trend analysis were between 0 and 1. Furthermore, most of the fractal dimension estimates from the ten methods were above 1 when compared to the three-year average analysis.

Finally, the existence of fractal characteristics in traffic data (i.e., volume, speed, and occupancy) was investigated using 15-min and 1-hr counts at two stations in Miami-Dade County. In general, the volume, speed, and occupancy patterns at both stations exhibited fractal characteristics. The *H* estimates calculated using the eight methods were between 0 and 1 and the fractal dimension estimates from the ten methods were mostly > 1. Furthermore, at both stations, the speed trend revealed stronger fractal behavior compared to the volume and occupancy trends

for the same time period. As previously concluded from the crash analysis, the wavelet method was one of the methods that yielded a relatively high fractal dimension estimate.

Future research could make use of the insights presented in this study to investigate the existence of fractal behavior in traffic management strategies. Compared to traditional statistical models, fractal theory is anticipated to yield more precise estimates of performance measures, such as travel time, traffic flow, traffic speed, rate of arrivals at parking lots, etc. Therefore, a potential future research is to apply fractal theory for prediction purposes. For example, the results of the fractal characteristics investigation for traffic data could be used in short-term traffic predictions (e.g., the next 15 minutes), which could act as a potential proactive traffic management strategy.

Another promising avenue is to use the results of the annual and three-year average analyses in network screening (i.e., identification of high-crash locations) since fractal characteristics are evident in the trends. For example, fractal theory could be applied to crash data to predict whether a high-crash intersection would continue to be listed in the future high-crash location lists if no safety improvements have been made. Similarly, crash rates at specific intersections could be predicted using the fractal extrapolation method. These approaches could potentially result in increased efficiency, mobility, and safety of the entire roadway network.

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LIST OF ACRONYMS/ABBREVIATIONS

AADT	Annual Average Daily Traffic
AID	Automated Incident Detection
ARIMA	Autoregressive Integrated Moving Average
ARRB	Australian Road Research Board
CAR	Crash Analysis Reporting
DCCA	Detrended Cross-Correlation Analysis
DCT	Discrete Cosine Transform
DFA	Detrended Fluctuation Analysis
DOT	Department of Transportation
EEG	Electroencephalogram
FDOT	Florida Department of Transportation
FHWA	Federal Highway Administration
GA	Genetic Algorithm
GIS	Geographic Information Systems
GLM	Generalized Linear Model
GML	Gaussian Maximum Likelihood
HOT	High Occupancy Toll
HOV	High Occupancy Vehicle
IFS	Iterated Function System
IOT	Internet of Things
ITS	Intelligent Transportation Systems
KNN	K Nearest Neighbor
LAN	Local Area Network
MEV	Million Entering Vehicles
MLANN	Multilayer Artificial Neural Network
MLF	Multilayer Feed-Forward
MPEG	Moving Picture Experts Group
O-D	Origin-Destination
OL-SVR	Online Support Vector Machine Regression
PDF	Probability Distribution Function
RCI	Roadway Characteristics Inventory
RCW	Red-Cockaded Woodpeckers
STEWARD	Statewide Transportation Engineering Warehouse for Archived Regional Data
SVM	Support Vector Machine
TDNN	Time Delay Neural Network
TMC	Transportation Management Center
VBR	Variable-Bit-Rate
WAN	Wide Area Network

CHAPTER 1
INTRODUCTION

1.1 Background

Traffic management strategies are essential components of intelligent transportation systems (ITS), and the Florida Department of Transportation (FDOT) is continuously looking for ways to improve traffic safety and mobility through the implementation of these strategies. Examples of such strategies include managed lanes, ramp metering, dynamic message signs, real-time signal timing, etc. FDOT implements these strategies through the transportation management centers (TMCs), which function as nerve centers and provide monitoring and control capabilities for various ITS strategies. Successful implementation of advanced traffic management strategies requires the ability to predict traffic conditions. To date, the process of estimating the impact of various traffic management strategies on traffic flow has been largely based on existing traffic flow models, which view traffic either as a stochastic process or as a kinematic fluid. While these models have the advantages of requiring minimal data and are relatively simple to use, they are based on nonlinear differential equations that have been reported to be unsatisfactory for modeling complex real-world traffic flow conditions (Persaud and Hall, 1989).

Additionally, traditional traffic analysis methodologies which rely on sampling techniques inevitably introduce uncertainties in the analysis results. Hence, the use of sampling techniques may be avoided by using other appropriate models that can analyze large and complex datasets, particularly those generated from vehicle detectors. The wide coverage currently provided by vehicle detectors on both freeway and arterial networks provides unique opportunities to harness the data and offers predictive capabilities for anticipating traffic conditions and enhance system performance based on network-wide historic and real-time data. However, these abundant data have been, at best, underutilized. The SunGuide software currently used by TMCs does not have capabilities to analyze and apply these large datasets to traffic prediction to be used in decision supports. A novel approach to capitalizing on the wealth of traffic data and to explore new traffic flow analysis methods could be beneficial. Such a new approach may emanate from viewing transportation networks from a "fractal" or "chaotic" perspective.

A fractal is "a fragmented geometric shape that can be split into parts, each of which is a reduced-size copy of the whole." The use of fractal dimension analysis is becoming widespread and has been applied in various fields, including medicine, physics, seismology, finance/economics, animal behavior studies, meteorology, and ecology. For example, the finance industry uses the fractal applications to predict stock market movements. Traditional traffic analysis methods may not properly address the nature of traffic flow if the inherent traffic structure was fractal. The main strength of fractal analysis lies in its ability to find patterns in large and complex datasets, for instance, those generated by vehicle detectors. Traffic flow is very dynamic, and several pioneering studies (e.g., Shang et al., 2007 and Li and Shang, 2007) have shown evidence of the presence of a fractal nature in traffic flow and outlined the potential benefits of employing fractal theory in traffic-related applications, such as traffic flow prediction.

Another appeal of the use fractal theory to analyze traffic data is that it does not filter out the roughness nor neglect the fractal interdependencies in the way that basic statistical methods often

do, nor does it round off the roughness in the way that curve fitting does. This is true for, among other things, the understanding of traffic crashes, arrival of vehicles at intersections, the way in which traffic flow/demand varies, and parking management. Several studies have broken ground and begun studying the fractal nature of traffic flow. It is believed that fractal theory has the potential to play an important role in developing predictive models feeding decision support systems that would improve traffic management. However, a unified view of fractal applications in traffic analysis is not well documented. This information is needed before fractal applications can be used in practice, and is documented in this synthesis report.

1.2 Project Objectives

The main objective of this project is to provide a synthesis of existing applications of fractal theory in various fields, as well as potential applications of fractal theory in traffic management. The study further identifies those elements of traffic flow which exhibit fractal characteristics. The specific information gathered and summarized in this synthesis report is:

1. A synthesis of fractal theory applications in fields that share similarities with transportation networks, e.g., electrical and water supply networks.
2. A synthesis of fractal applications that have been proven effective in traffic flow, such as traffic flow prediction.
3. Additional insight on how fractal theory can be applied to traffic management strategies, e.g., managed lanes, ramp metering, traffic safety, parking management, and travel time reliability.
4. Summary of findings and recommendations for more detailed research and development of future guidance documents.

1.3 Report Organization

The rest of the report is organized as follows. Chapter 2 provides an overview of fractal theory and discusses the techniques used to assess the existence of fractal characteristics in the data. Chapter 3 highlights applications of the fractal theory in different fields of science. Chapter 4 provides insights on potential fractal theory applications in traffic management. Chapter 5 investigates the existence of fractal behavior in crash and traffic data. Finally, Chapter 6 provides relevant conclusions and recommendations.

CHAPTER 2
OVERVIEW OF FRACTAL THEORY AND TECHNIQUES

This chapter provides an overview of fractals and multi-fractals, the concept of fractal geometry, and characteristics of fractals in traffic data. It also includes various theoretical techniques used to detect the existence of fractals. It concludes by providing a summary of lessons learned.

2.1 Definition of Fractals

According to Thomas and Dia (2000), the term *fractal* was invented by Mandelbrot (1983) which was extracted from the word *frangere,* meaning *to break*. A fractal is defined as "an irregular geometric object that has finite details at all scales". Whenever a fractal is magnified, its basic structure remains the same regardless of the scale of magnification. This is known as *self-similarity*. Some classic examples of fractals include the Koch Snowflake or Loop (Figure 2-1a) and the Sierpinski Triangle or Gasket (Figure 2-1b). As shown in Figure 2-1(a), the initial pattern of the Koch Snowflake is a triangle. In each iteration, every side of the triangle, called the initiator, is replaced by the generator, and the first three iterations are shown in the figure.

In Figure 2-1(b), the dark Sierpinski Triangle, an equilateral triangle, is shown in the first picture (i.e., prior to Iteration 1). If the middle portion of this triangle, shown in white color, is removed, it results in the second picture. Figure 2-1(b) shows the resulting pictures after each of the four iterations. This iteration process could continue for an infinite number of times.
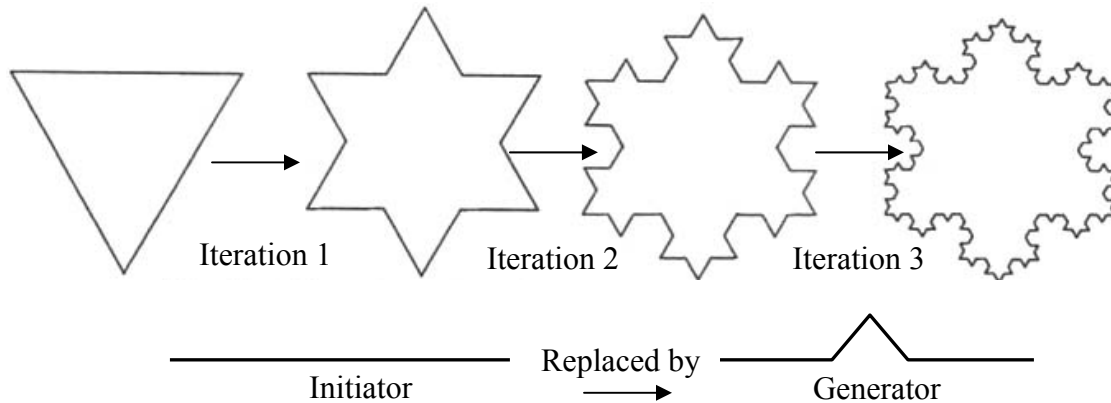
(a) Koch Snowflake/Loop (Source: MIQEL, 2012)
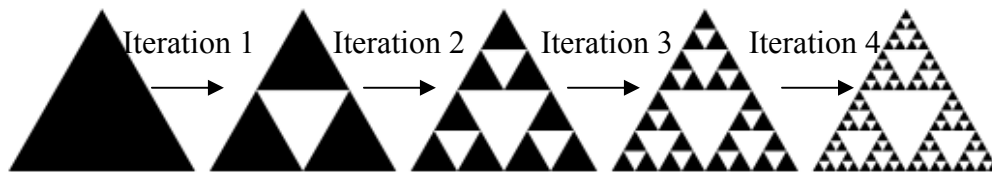
(b) Sierpinski Triangle/Gasket (Source: Wikipedia, 2012a)

Figure 2-1: Examples of Classic Fractals

In nature, examples of fractals include plants (e.g., fern leaf, Figure 2-2a), trees (Figure 2-2b), coastlines (e.g., Britain coastline, Figure 2-2c), clouds (Figure 2-2d), coral reefs (Figure 2-2e),

and mountain landscape (Figure 2-2f). Moreover, blood vessels (shown in Figure 2-3), neurons, membranes, nervous system, etc. of the human body could also represent fractals.
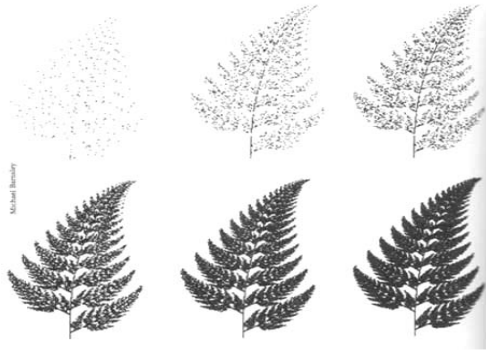
(a) Fern Leaf
(Source: MIQEL, 2012)
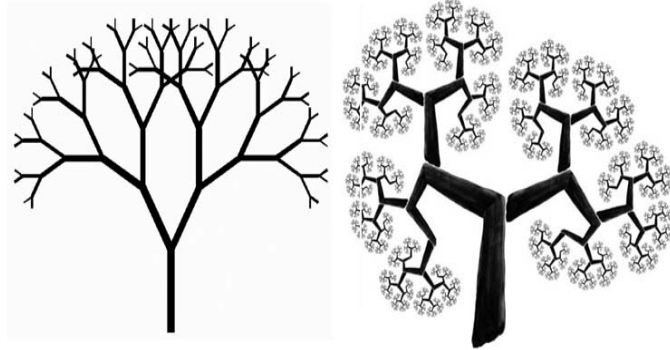
(b) Trees
(Source: MIQEL, 2012)
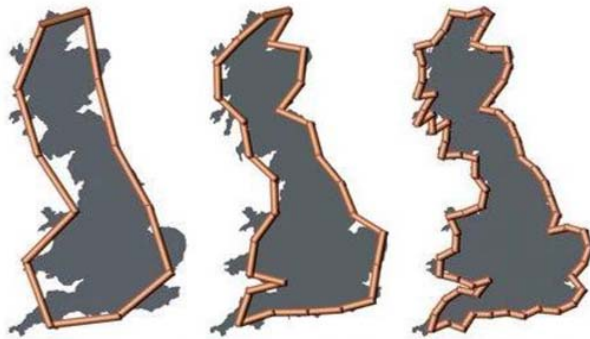
(c) Britain Coastline
(Source: Wikipedia, 2012b)

(d) Clouds
(Source: Worcester Polytechnic Institute, 2012)

(e) Coral Reefs
(Source: deviantART, 2012)

(f) Mountain Landscape
(Source: Vistapro Pictures, 2012)

Figure 2-2: Examples of Fractals in Nature
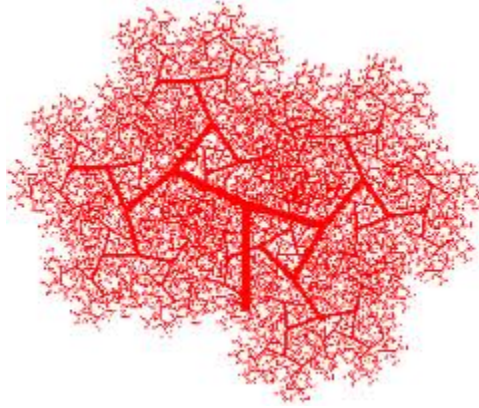
Figure 2-3: Blood Vessels in Human Body That Represent Fractals
(Source: Institute of Cybernetics at Tallinn University of Technology, 2012)

Additionally, as illustrated by Abasolo et al. (2008), the fractal geometry is a new language used to describe, model, and analyze complex forms found in nature. Fractals are different than Euclidean shapes since fractals have no characteristic sizes and are said to be self-similar and independent of scaling, whereas Euclidean shapes possess one or a few length scales and characteristic sizes, and assume regularity of shapes. An important related terminology is *fractal dimension*, which is the non-integer or fractional dimension of any fractal curve, such as coastline dimensions.

As shown in Storkey (1996), fractal geometry rather than Euclidean geometry is the best modeling tool for sets of shapes possessing self-similarity and roughness over a range of scales. This fractal geometry neither neglects the roughness nor the fractal interdependencies in the way that basic statistical and regression methods often do. Moreover, the fractal geometry does not round off the roughness in the way that curve fitting does, and therefore, makes it more superior. The fractal geometry is relevant in situations where the nature of roughness and variations should be accounted for, such as the understanding of crashes, arrival of cars at intersections, and the way traffic demand varies.

2.2 Characteristics of Fractals

The following are the four main characteristics of fractals (Boast 2000):

1. self-similarity,
2. iterative (recursive) process,
3. infinite complexity, and
4. non-integer complex dimension.

2.2.1 Self-Similarity

As previously illustrated, self-similarity is an indication that a smaller object resembles the original bigger object with a different scale. Cai and You (2010) defined the following three main types of self-similarity:

5

1. *Exact self-similarity*: This is the strongest type of self-similarity where the fractal appears identical at different scales. Fractals defined by iterative function systems often display exact self-similarity.

2. *Quasi self-similarity*: This is a loose form of self-similarity where the fractal appears approximately identical at different scales. Quasi self-similar fractals incorporate smaller replica of the entire fractal in a distorted form. Fractals defined by recurrence relations often display quasi self-similarity.

3. *Statistical self-similarity*: This is the weakest type of self-similarity where the fractal has numerical or statistical measures that are preserved across scales. Random fractals are examples of statistically self-similar fractals.

2.2.2 Iterative (Recursive) Process

Fractals are usually characterized by continuity and are subjects of an iterative process. As shown previously, the Koch Snowflake and Sierpinski Triangle are popular examples of the iterative process of fractals.

2.2.3 Infinite Complexity

As shown by Boast (2000), fractals can be magnified infinitely. Once magnified, fractals become complex structures and can be represented by the Mandelbrot and Julia sets shown in Figures 2-4 and 2-5, respectively. The Mandelbrot set is a mathematical set of points whose boundary is an easily recognizable two-dimensional fractal shape. The Julia set is a set in which an arbitrarily minor perturbation can cause drastic behavioral or chaotic changes.
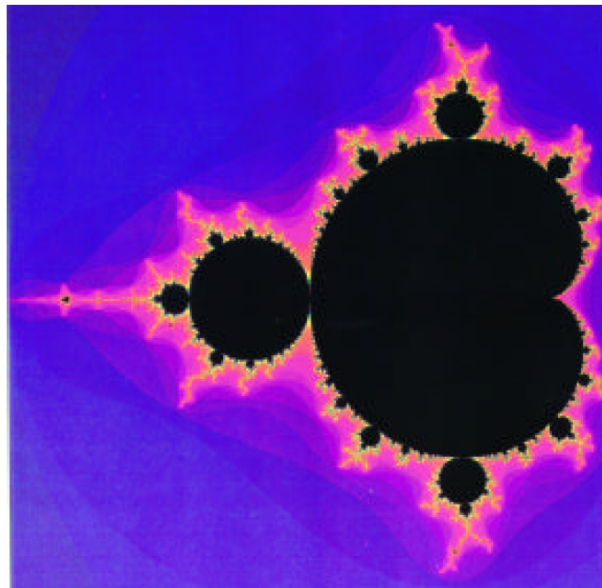
Figure 2-4: Mandelbrot Sets (Source: Boast, 2000)
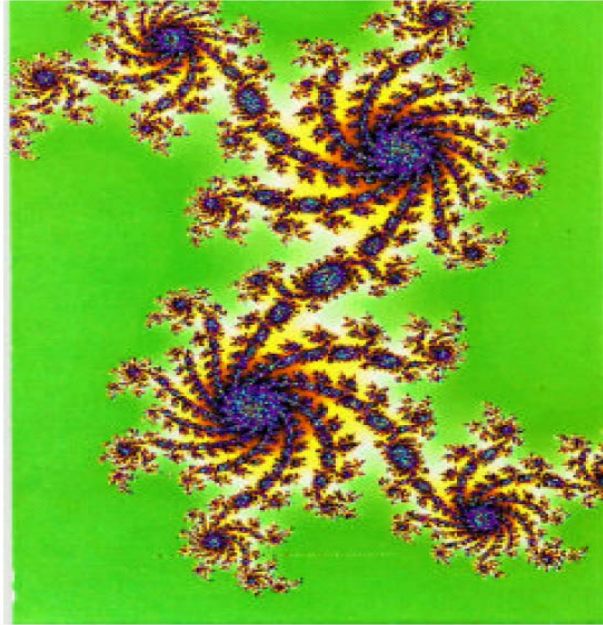
Figure 2-5: Julia Sets (Source: Boast, 2000)

2.2.4 Non-Integer Complex Dimension

Classical geometry deals with objects having integer dimensions, e.g., zero dimensional points, one dimensional lines and curves, two dimensional plane figures (squares and circles), and three dimensional solids (cubes and spheres). However, many natural phenomena are better described using a dimension between two whole numbers, i.e., a fraction. Thus, a fractal curve can have a dimension between one and two in contrast to the straight line that is one dimensional.

The exact fractal dimension depends on how much space the fractal consumes as it curves. The flatter the fractal fills a plane, the closer it approaches to two dimensions. In the same manner, a hilly fractal can have a dimension between two and three. This means that a fractal landscape made up of a large hill covered with tiny mounds would be close to a second-order dimension, while a rough surface composed of many medium-sized hills would be close to a third-order dimension.

2.3 Multi-Fractals

According to Harte (2001), a multi-fractal system is "a generalization of a fractal system in which a single exponent (or the fractal dimension) is insufficient to describe its dynamics, and therefore, a continuous spectrum of exponents (or the so-called singularity spectrum) is needed." This continuous spectrum is known as a multi-fractal system. Multi-fractals are common in nature, especially in geophysics. The multi-fractal analysis has been applied in a variety of practical fields, such as in interpreting medical images (Lopes and Betrouni, 2009), in investigating the human genome behavior (Moreno et al., 2011), and in predicting earthquakes (Enescu et al., 2006).

2.4 Fractals in Highway Traffic Data

Traffic flow models view traffic either as a stochastic process or as a kinematic fluid, and this has been reported to be unsatisfactory for modeling real-world traffic flow conditions (Persaud and Hall, 1989). A new approach could be viewing traffic data from a fractal perspective, mainly characterized by self-similarity and iterative process. Figure 2-6 is extracted from the study by Shang et al. (2007) and indicates signs of fractals in the traffic speed time series data along the Beijing Yuquanying highway. The figure shows a significant variation in speed over time, as well as the distribution similarity of smaller time intervals to larger intervals. By detection of fractal behavior existence in traffic data, fractal analysis could serve as a promising tool for improving traffic flow analysis methods.
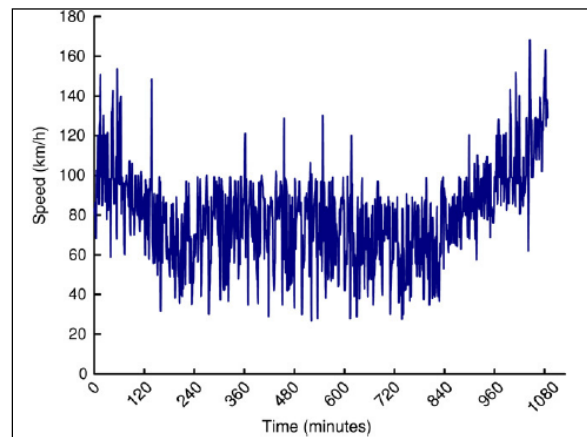
Figure 2-6: Signs of Fractals in Traffic Speed Data (Source: Shang et al., 2007)

2.5 Methods to Detect Fractal Characteristics

A wide variety of techniques have been developed to identify fractal and chaotic behavior, including:

1. power spectrum,
2. empirical probability distribution function (PDF),
3. statistical moment scaling,
4. long-range dependence or Hurst exponent function,
5. fractal dimension,
6. Kolmogorov entropy, and
7. largest Lyapunov exponent.

An overview of the theoretical background of each of the seven methods is provided below.

2.5.1 Power Spectrum

The power spectrum is among the standard and common methods of fractal investigations in nonlinear time series data (Shang et al., 2007). As stated in the power spectrum function $E(f)$:

$$E(f) \propto f^{-\delta} \qquad\qquad (2\text{-}1)$$

where f is the frequency and δ is the spectral exponent.

In the normal process, the power spectrum oscillates randomly about a constant value, indicating that no frequency explains the variance of the sequence. However, the fractal behavior can be identified once noise exists, where the noise adds a continuous elongation to the spectrum. For example, Figure 2-7, extracted from Shang et al. (2007), shows the power spectrum $E(f)$ of the speed time series and the presence of fractal behavior can be observed at the region along the fitted pink regression line. In this region, the fluctuation is minimum. Identification of spectral exponent mainly relies on individual's own judgment and as a result, discrepancies and uncertainties in the identification process are expected to occur.
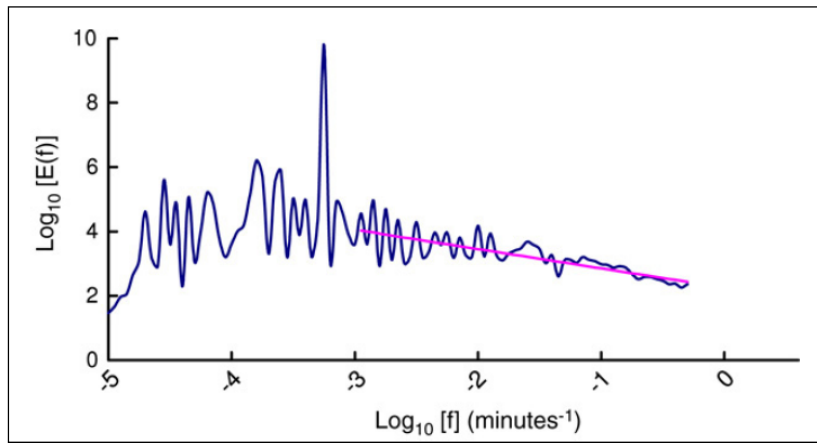
Figure 2-7: Fractal Behavior Indication Using Power Spectrum Method
(Source: Shang et al., 2007)

2.5.2 Empirical Probability Distribution Function

According to Shang et al. (2007), the empirical probability distribution function (PDF) of a time series data can describe the fractal nature of the thresholds of the time series fluctuations at a specific scale. The tail of the probability distribution of the time series X follows a power law of the form shown in Equation 2-2:

$$\Pr(X < x) \propto x^{-D} \qquad\qquad (2\text{-}2)$$

where D is the probability exponent.

In the existence of fractal behavior, the traffic data should exhibit a hyperbolic tail distribution. In Figure 2-8, extracted from Shang et al. (2007), the part of the PDF overlaid by the pink regression line follows a continuous fluctuation and hyperbolic tail distribution and thus exhibits fractal behavior. The value of D determines the type of fractal behavior, i.e., whether mono- or multi-fractal. A value of $D \leq 2$ indicates that a mono-fractal model is sufficient to characterize the traffic data, whereas a multi-fractal model is needed if $D > 2$.
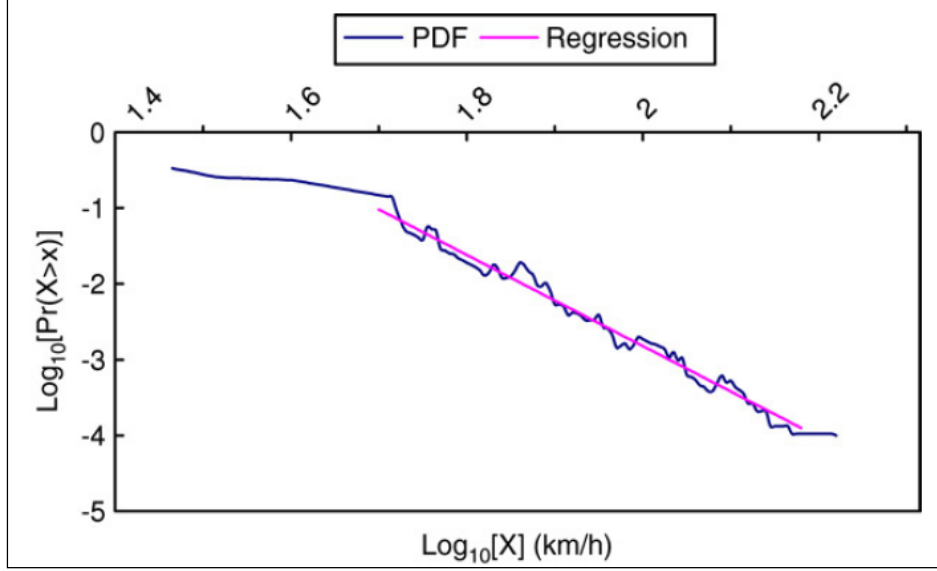
Figure 2-8: Indication of Fractal Behavior Using Empirical Probability Distribution Function (Source: Shang et al., 2007)

2.5.3 Statistical Moment Scaling

To investigate the scaling (or exponential) behavior of time series, it is necessary to examine the variation of the moments with the scale (Shang et al., 2007). The statistical moment scaling method can thus be applied to detect the fractal behavior of the data. In this method, the range of the time series is divided into non-overlapping intervals. The ratio of the maximum scale to the non-overlapping interval is termed as the scale ratio, k. For different scale ratios, the average intensity "$\varepsilon(k, i)$" in each interval i is estimated and raised to the power q. The intensities are then summed up to obtain the statistical moment $M(k, q)$, as follows:

$$M(k,q) = \sum_i \varepsilon(k,i)^q \tag{2-3}$$

The statistical moment $M(k, q)$ relates to the scale ratio k as follows:

$$M(k,q) = k^{\theta(q)} \tag{2-4}$$

where $\theta(q)$ is the function to define fractal behavior.

If the plot $\theta(q)$ versus q is a straight line, the data are said to exhibit mono-fractal behavior. On the other hand, if the plot follows either a convex or a concave pattern, the data are said to exhibit multi-fractal behavior. Figure 2-9 illustrates this point by showing the relationship between q and $\theta(q)$. If the relationship is not straight, i.e., curved, the data are said to exhibit multi-fractal behavior.

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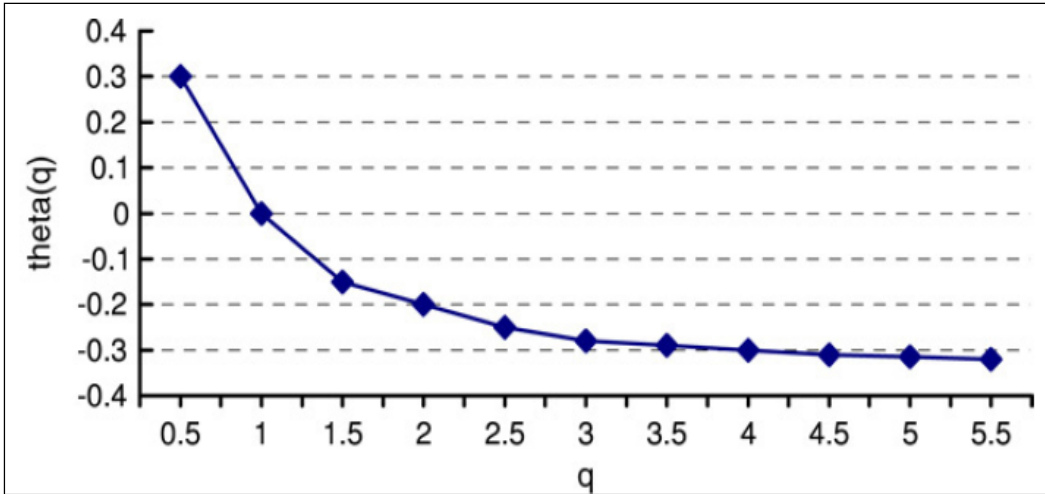
Figure 2-9: Indication of Fractal Behavior Using Statistical Moment Scaling Method
(Source: Shang et al., 2007)

2.5.4 Long-Range Dependence Function or Hurst Exponent Function

As indicated in Shang et al. (2007), the long-range dependence function is used to determine the degree of dependence (or correlation) that exists along the entire range of time series values that are separated by the lag time interval τ. In normal scenarios, the autocorrelation function oscillates around zero, indicating the non-autocorrelation situation. On the other hand, for fractal behavior, the autocorrelation function decays exponentially with increasing lag time interval τ, meaning that a self-similarity characteristic exists in the data. The next sections discuss eight common methods used to estimate the self-similarity behavior and the intensity of long-range dependence in time series data, which is represented by the Hurst exponent (H) estimator. Fractal characteristics are considered to be evident whenever H lies between 0 and 1, and the higher the H value, the more predominant fractal characteristics the data exhibits.

Aggregated Variance Method

The open-access **R** software (*R* Project, 2013) is used for fractal analysis using the Hurst exponent method. As described in the **R** documentation for the *fArma* package (*R* Project, 2013), the function, *aggvarFit*, computes the Hurst exponent (H) from the variance of an aggregated time series process. The original time series data are divided into blocks of size m. The sample variance within each block is then computed. The slope of the least square fit of the logarithm of the sample variances versus the logarithm of the block size is estimated as $2H - 2$. The Hurst exponent (H) can then be calculated from the slope value.

Differenced Aggregated Variance Method

The function, *diffvarFit*, in **R** Software is used to estimate the differences in the sample variances of successive blocks. Again, the slope of the least square fit of the logarithm of the differenced sample variances versus the logarithm of the block size is estimated as $2H - 2$, and H can then be estimated.

Aggregated Absolute Moment Method

The function, *absvalFit*, in **R** Software is used to compute the Hurst exponent from the moments *M* of absolute values of an aggregated time series process. The slope of the regression line of the logarithm of the aggregated absolute versus the logarithm of the block size is estimated as $M \times (H - 1)$. The Hurst exponent (*H*) can then be calculated from the slope value.

Higuchi Method

The Higuchi method is very similar to the aggregated absolute moment method; however, implements a sliding window to compute the aggregated series. In **R** Software, the function, *higuchiFit*, is used to compute the *H* value using the Higuchi method. The slope *D* (or fractal dimension) of the least square fit of the logarithm of the expected path length versus the logarithm of the window sizes equals $2 - H$; *H* can then be computed.

Variance of Residuals Method

In this method, the time series data is divided into blocks of size *m*. Within each block, the cumulative sums are computed up to time *t* and a least-squares line is fitted to the cumulative sums. Next, the sample variance of residuals which is proportional to m^{2H} is computed, and the mean and median values are also computed over the blocks. For this purpose, the function, *pengFit*, is used in **R** Software. The slope of the least square equals $2H$, and *H* can then be computed.

R/S Method

As shown in Shang et al. (2007), let $X = (X_t; t = 1, 2, 3, \ldots)$ denote values of a random variable *X* at different time intervals. The mean μ of *X* equals $E(X_t)$ and the variance σ^2 equals $E[(X_t - \mu)^2]$. Assume that *X* follows an autocorrelation function $R(\tau)$:

$$R(\tau) \approx \tau^{-\beta} L_1(\tau), \quad \tau \to \infty \qquad (2\text{-}5)$$

where $0 < \beta < 1$ and L_1 is a function of the lag time interval that tends to infinity.

Assume that a new time series function $X^{(m)}$ is obtained by averaging the original function *X* over intervals of size *m*. The function *X* is now named "asymptotically second-order self-similar function" with a self-similarity parameter called the Hurst parameter *H* (that determines the fractal behavior), where $H = 1 - \beta/2$. To measure this similarity, there are two methods, the *R/S* plot and the variance-time plot (Shang et al., 2007; Willinger et al., 1995).

In the *R/S* plot method, assume that X_k: $k = 1, 2, 3, \ldots, n$ is a set of *n* observations with mean or expected value of $E[X(n)]$, thus:

$$\frac{R(n)}{S(n)} = \frac{\max(0, W_1, \ldots, W_n) - \min(0, W_1, \ldots, W_n)}{S(n)} \qquad (2\text{-}6)$$

where $S(n)$ is the standard deviation of X_k and W_k is estimated as:

$$W_k = (X_1 + X_2 + ... + X_k) - k E[X(n)], k = 1,2,3,...,n \qquad (2\text{-}7)$$

The *R/S* function can be stated as:

$$R(n)/S(n) \approx c n^H \qquad (2\text{-}8)$$

where *c* is a constant. By taking *log* of both sides in Equation (2-8), it becomes:

$$\log_{10}[R(n)/S(n)] \approx \log_{10} c + H \log_{10}(n) \qquad (2\text{-}9)$$

From Equation (2-9), the Hurst parameter (*H)* is the slope of the plot of $\log_{10}[R(n)/S(n)]$ against $\log_{10}(n)$.

In the variance-time plot method, assume that:

$$Var[X^{(m)}] \approx m^{-\beta} \qquad (2\text{-}10)$$

By taking *log* of both sides in Equation (2-10), it becomes:

$$\log_{10}\{Var[X^{(m)}]\} \approx -\beta \log_{10}(m) \qquad (2\text{-}11)$$

From Equation (2-11), *-ß* is the slope of the plot of $\log_{10}\{Var[X^{(m)}]\}$ against $\log_{10}(m)$. The Hurst parameter (*H*) can then be estimated as *H* = 1 – *ß*/2. As a rule of thumb, the spectral exponent *δ* previously shown in Equation (2-1) can be estimated as *δ* = *2H – 1*.

If $H \leq \frac{1}{2}$, the data are said to exhibit negative long-range autocorrelation or non-independence. On the other hand, if $H > \frac{1}{2}$, the data are known to exhibit a fractal behavior or long-range dependence. Figure 2-10 from Shang et al. (2007) shows the estimation of the Hurst exponent *H* using the *R/S* plot method based on the plot of $\log_{10}[R(n)/S(n)]$ against $\log_{10}(n)$. The fitted line was inclined with a slope of 0.84. Thus, *H* equals 0.84, which is greater than 0.5, and it means that fractal behavior exists in the data. This conclusion is also confirmed by Shang et al. (2007) using the variance-time plot method, as shown in Figure 2-11. In the figure, the slope of the fitted line represents *-ß* and was found to be -0.32. Thus, *H* can be calculated as 1 – *ß*/2 = 0.84, the same value as previously calculated using the *R/S* method.
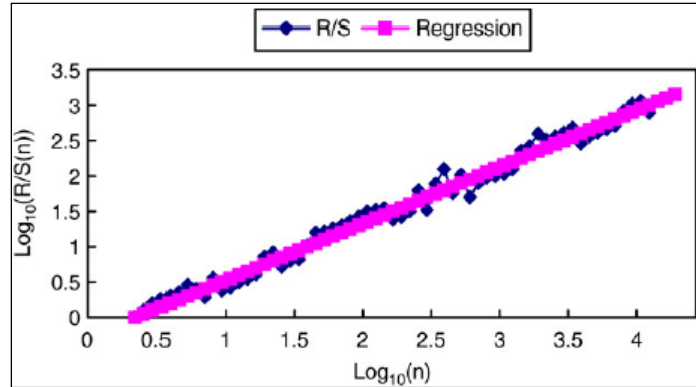
Figure 2-10: Indication of Fractal Behavior Using R/S Autocorrelation Function (Source: Shang et al., 2007)
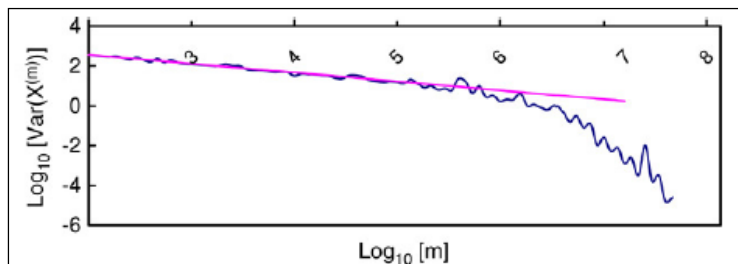
Figure 2-11: Indication of Fractal Behavior Using Variance-Time Autocorrelation Function (Source: Shang et al., 2007)

Periodogram Method

For a finite variance domain, the periodogram is an estimator of the spectral density of time series data. Thus, the log-log plot of the periodogram versus frequency is a straight line and the slope is equivalent to $1 - 2H$. Another approach is the cumulative periodogram, and in this case, the slope of the log-log line fit is $2 - 2H$. In **R** Software, the function, *perFit*, is used to estimate *H*.

Boxed or Modified Periodogram Method

The function, *boxperFit*, in **R** is a modification of the previous periodogram method. In the modified periodogram method, the frequency X-axis is divided into logarithmically equal spaced boxes, and then the average of the periodogram values corresponding to the frequencies inside each box is computed.

2.5.5 Fractal Dimension

As discussed in Hu et al. (2010), fractal dimension describes the degrees of freedom of the dynamic traffic flow system or the system's fractal behavior. The fractal dimension in time series data can be estimated using a correlation dimension statistic. This statistic can be estimated using

the method proposed by Grassberger and Procaccia (1983a) to get the correlation integral $C(r, m)$ between two points i and j in the m-dimensional space:

$$C(r,m)=\frac{1}{N^2}\sum_{i,j=1}^{N}\theta(r-r_{ij})$$ (2-12)

where,

N $=$ number of points in space,
r_{ij} $=$ distance between the two points i and j in space, and
θ $=$ Heaviside function.

The Heaviside function is a discontinuous function whose value is zero for negative arguments and one for positive arguments. In other words,

$$\theta(r-r_{ij})=0 \quad if\ r<r_{ij}$$ (2-13)
$$=1 \quad if\ r\geq r_{ij}$$

The correlation dimension $D(m)$ of the m-dimensional space is then calculated as:

$$D(m)=\lim_{r\to 0}[\ln C(r,m)/\ln r]$$ (2-14)

The larger the fractal or correlation dimension $D(m)$, the more complicated or more fractal the system is. Figure 2-12, from Hu et al. (2010), describes the relationship between the fractal dimension D and the m-dimensional space of traffic flow data collected along both urban and freeway facilities. The figure shows that with the increase of the dimension m, the correlation dimension D becomes constant. When m is greater than 28, D reaches its steady state value of approximately 11 for urban traffic flow data and 7 for freeway facilities. This value is relatively high and indicates that the urban network exhibits a higher fractal behavior compared to the freeway network.
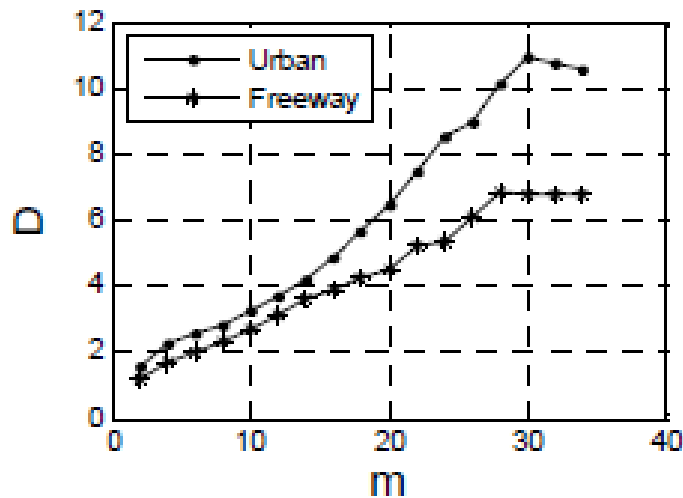
**Figure 2-12: Relationship between Fractal Dimension D and m-Dimensional Space
(Source: Hu et al., 2010)**

There are ten methods to estimate the fractal dimension of the data trend, as follows:

1. Box count method
2. Hall-Wood method
3. Variogram method
4. Variation method (Madogram)
5. Variation method (Rodogram)
6. Increment1 (Incr1) method
7. Periodogram method
8. Discrete cosine transform-II (DCT-II) method
9. Genton method
10. Wavelet method

The methodological approach of each method is summarized in the following sections.

Box Count Method

As documented in Gneiting et al. (2010), assume $n = 2^K$ is a power of 2 and let u denote the range of the data. Also, consider a box scale (size) $\varepsilon_k = 2^{k-K}$, where $k = 0, 1, \ldots, K$. At the largest scale (i.e., at $k = K$), $\varepsilon_K = 1$, which represents a single bounding box of width 1 and height u. At scale ε_k, the bounding box can be represented by 4^{K-k} boxes of width 2^{k-K} and height $u \times 2^{k-K}$ each. The number of such boxes that intersect the linearly interpolated graph are termed $N(\varepsilon)$. The fractal dimension $\hat{D}$ is then estimated as the slope of the ordinary least squares regression fit of the log-log relationship, i.e., $\log[N(\varepsilon)]/\log(\varepsilon)$.

Hall–Wood Method

Hall and Wood (1993) introduced a newer version of the box count estimator that is appropriate for the smallest observed box scales. Let $A(\varepsilon)$ denote the total area of the boxes with scale (width) ε that intersect with the linearly interpolated data. As previously discussed, the number of boxes is $N(\varepsilon)$. The Hall–Wood fractal dimension estimator is based on ordinary least squares regression fit of $\log \hat{A} (l/n)$ on $\log(l/n)$, i.e.,:

$$\hat{D}_{HW} = 2 - \left\{ \sum_{l=1}^{L} (s_l - \bar{s}) \log \hat{A}(l/n) \right\} \left\{ \sum_{l=1}^{L} (s_l - \bar{s})^2 \right\}^{-1} \tag{2-15}$$

where $l = 1, 2, \ldots, L \geq 2$, $s_l = \log(l/n)$, and $\bar{s} = \dfrac{1}{L} \sum_{l=1}^{L} s_l$.

Variogram Method

The method of moments estimator for the variogram or structure function at time lag $t = l/n$ from time series data is:

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$$\hat{V}_2(l/n) = \frac{1}{2(n-l)} \sum_{i=l}^{n} (X_{i/n} - X_{(i-l)/n})^2 \qquad (2\text{-}16)$$

where $X_t: t \in \mathbb{R}$ is a stochastic process.

The variogram fractal dimension estimator is based on ordinary least squares regression fit of log $\hat{V}_2(l/n)$ on log t, i.e.,:

$$\hat{D}_{V_2} = 2 - \frac{1}{2} \left\{ \sum_{l=1}^{L} (s_l - \bar{s}) \log \hat{V}_2(l/n) \right\} \left\{ \sum_{l=1}^{L} (s_l - \bar{s})^2 \right\}^{-1} \qquad (2\text{-}17)$$

where $l = 1, 2, \ldots, L \geq 2$, $s_l = \log(l/n)$, and $\bar{s}$ is the mean of $s_1, s_2, s_3, \ldots, s_L$.

Variation Method

The variation method is a generalization of the variogram estimator that is based on the first order differences and the variogram of order p of a stochastic process with stationary increments. The general variation fractal dimension estimator is based on ordinary least squares regression fit of log $\hat{V}_p(l/n)$ on log (l/n), i.e.,:

$$\hat{D}_{V_p} = 2 - \frac{1}{p} \left\{ \sum_{l=1}^{L} (s_l - \bar{s}) \log \hat{V}_p(l/n) \right\} \left\{ \sum_{l=1}^{L} (s_l - \bar{s})^2 \right\}^{-1} \qquad (2\text{-}18)$$

where $l = 1, 2, \ldots, L \geq 2$, $s_l = \log(l/n)$, and $\bar{s}$ is the mean of $s_1, s_2, s_3, \ldots, s_L$. The variation method converges to the variogram method when $p = 2$, converges to the madogram method when $p = 1$, and converges to the rodogram method when $p = 0.5$ (Gneiting et al., 2010).

Increment1 (Incr1) Method

The increment1 (incr1) method is very similar to the variation method; however, unlike the variation method, it is based on the second order differences.

Periodogram Method

As described in Gneiting et al. (2010), the periodogram method assumes the number of observations $n = 2m + 1$ and the random variable X_t at time $t = i/(2m)$, where $i = 0, 1, \ldots, 2m$. Based on these assumptions, Chan et al. (1995) defined the Brownian or random function $\hat{B}(\omega)$ as:

$$\hat{B}_\omega = \frac{1}{m} \left[\frac{X_0 + X_1}{2} + \sum_{i=1}^{2m-1} X_{i/(2m)} \cos\left(\omega \frac{i-m}{m} \right) \right] \qquad (2\text{-}19)$$

Chan et al. (1995) argued that the cosine part of the periodogram is more appropriate than the full periodogram for the fractal dimension estimation; thus, the cosine part that is shown in Equation (2-19) reveals a semi-periodogram function. The semi-periodogram estimator of the fractal dimension is then estimated as follows:

$$\hat{D}_{SP} = \frac{5}{2} + \frac{1}{2} \left\{ \sum_{l=1}^{L} (s_l - \bar{s}) \log \hat{J}(\omega_l) \right\} \left\{ \sum_{l=1}^{L} (s_l - \bar{s})^2 \right\}^{-1} \tag{2-20}$$

where $\hat{J}(\omega_l) = \hat{B}(\omega_l)^2$ is the semi-periodogram function, $\omega_l = 2\Pi l$, $l = 1, 2, \ldots, L \geq 2$, $s_l = \log(\omega_l)$, and $\bar{s}$ is the mean of $s_1, s_2, s_3, \ldots, s_L$.

The periodogram (or more precisely, the semi-periodogram) estimator of the fractal dimension $\hat{D}_{SP}$ is then derived from the ordinary least squares regression fit of $\log(\omega_l)$ against $\log(\hat{J}(\omega_l))$.

Discrete Cosine Transform-II (DCT-II) Method

As described in Gneiting et al. (2010) and following the same assumptions as those for the periodogram method, the Brownian $\hat{B}(\omega)$ function for the DCT-II method is:

$$\hat{B}_\omega = \left(\frac{2}{2m+1} \right)^{0.5} \left[\sum_{i=0}^{2m} X_{i/(2m)} \cos\left(\omega \frac{2i+1}{4m} \right) \right] \tag{2-21}$$

where $\hat{J}(\omega) = \hat{B}(\omega)^2$ is the DCT-II function, $\omega_l = 2\Pi l/(2m+1)$, $l = 1, 2, \ldots$, $s_l = \log(\omega_l)$, and $\bar{s}$ is the mean of $s_1, s_2, s_3, \ldots, s_L$, and $L = \min(2m, 4n^{2/3})$.

The DCT-II fractal dimension estimator is derived from the ordinary least squares regression fit of $\log(\omega_l)$ against $\log(\hat{J}(\omega_l))$. This estimator uses roughly four times more points in the log-log regression compared to the semi-periodogram estimator (Gneiting et al., 2010).

Genton Method

A highly robust variogram fractal dimension estimator was proposed by Genton (1998). Assume $U_i(d) = X_{i/n} - X_{(i-d)/n}$, then:

$$V(d) = [2.2191\{|U_i(d) - U_j(d)|; \ i < j\}_{(k)}]^2 \tag{2-22}$$

where $k = \begin{pmatrix} \frac{n-d}{2}+1 \\ 2 \end{pmatrix}$

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The genton fractal dimension estimator $\hat{D}_k$ is then derived from the ordinary least squares regression fit of log(d) against log($V(d)$).

Wavelet Method

As described in Gneiting et al. (2010), assume J_0 vectors of wavelet coefficients W_j, $j = 1, 2, \ldots, J_0$, each of which contains $2n$ coefficients (where n is the number of observations). The coefficients in the jth vector are associated with the scale parameter $\tau_j = 2^{j-1}$. The wavelet variance is then estimated as the average of these coefficients squared, i.e., $\|W_j\|^2 / 2n$. The ordinary least squares estimator of the slope of the regression fit of the variance on log(τ_j) can then be replaced with the weighted least squares estimator, $\hat{\alpha}_{WL}$ (Gneiting et al., 2010). Based on this, the wavelet estimator of the fractal dimension $\hat{D}_{WL}$ can be estimated as follows:

$$\hat{D}_{WL} = 2 - 0.5 \times \hat{\alpha}_{WL} \tag{2-23}$$

2.5.6 Kolmogorov Entropy

The Kolmogorov entropy explains the extent of the chaotic and fractal behavior of the system (Hu et al., 2010). The Kolmogorov entropy is usually denoted as K and the average prediction scale of the system is denoted as $1/K$. Using the method proposed in Grassberger and Procaccia (1983b), Kolmogorov entropy can be estimated as:

$$K(r,m) = \frac{1}{\tau} \ln[C(r,m)/C(r,m+1)] \tag{2-24}$$

where τ is the delay time. As shown in Figure 2-13, also extracted from Hu et al. (2010), when $m > 30$, the value of the Kolmogorov entropy K for both urban and freeway networks is steady at around 0.3.
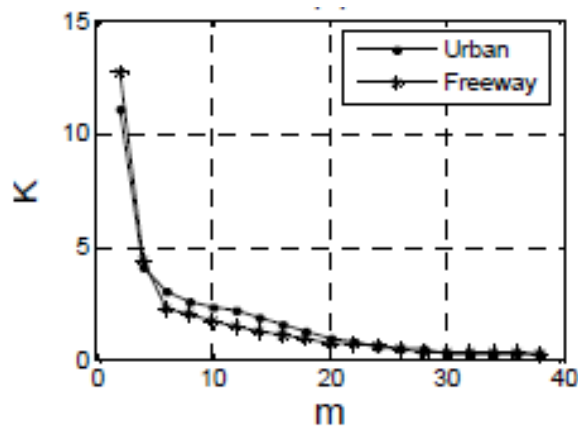
Figure 2-13: Relationship between Kolmogorov Entropy K and m-Dimensional Space (Source: Hu et al., 2010)

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2.5.7 Largest Lyapunov Exponent

As discussed in McCue and Troesch (2004), the Lyapunov exponent is used to measure the convergence or divergence rates of nearby trajectories (Wolf, 1986; Ziehmann et al., 1999). The sign of the exponent can determine if a fractal behavior is existent. A negative Lyapunov exponent denotes convergence, i.e., non-existence of fractal behavior, while a positive Lyapunov exponent indicates divergence, i.e., existence of fractal behavior. Two main methods for calculating the largest Lyapunov exponent are the direct method and the tangent space method.

As shown by Rosenstein et al. (1993) and Kantz (1994), in the direct method, the largest Lyapunov exponent is calculated by searching for all neighbors within the vicinity of the reference trajectory and computing the average distance between the neighbors and the reference trajectory as a function of time. To calculate the largest Lyapunov exponent λ, the stretching function $S(\tau)$ is first estimated as follows:

$$S(\tau) = \frac{1}{T} \sum_{t=1}^{T} \ln \left(\frac{1}{|U_t|} \sum_{i \in U_t} |x_{t+\tau} - x_{i+\tau}| \right) \tag{2-25}$$

where,

x_t = arbitrary point in time series,
U_t = point in the vicinity of x_t,
x_i = neighbor of x_t,
τ = relative time scaled by sampling rate, and
T = total length of the analyzed time series.

The largest Lyapunov exponent λ can then be computed as follows:

$$e^{\lambda \tau} \propto e^{S(\tau)} \tag{2-26}$$

The second method, the tangent space method, calculates the full spectrum of Lyapunov exponents through local predictions of the Jacobian matrix J along the time series trajectory. J is the partial derivative of the function $f(x)$ with respect to x, where x is a trajectory function (Sano and Sawada, 1985; McCue and Troesch, 2004).

$$J = \frac{\partial f(x)}{\partial x} \tag{2-27}$$

The largest Lyapunov exponent λ can then be computed as follows:

$$\lambda_i = \lim_{n \to \infty} \frac{1}{n\tau} \sum_{j=1}^{n} \ln \left\| A_j \, e_i^j \right\| \tag{2-28}$$

where,

τ = time increment,
n = number of observations or data points,
A_j = ratio of covariance matrices, and

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e_i^j =   set of basis vectors of the tangent space at x_j ($i = 1, 2, 3, \dots$).

As discussed in Kantz (1994) and McCue and Troesch (2004), compared to the direct method, the tangent space method is more sensitive to the selection of the dimensional space m. A very small value of m can yield an erroneously large Lyapunov exponent value, while a very large value of m can yield a spurious Lyapunov exponent value. Therefore, selection of an appropriate value of the dimensional space m while using the tangent space method requires greater attention. However, when the value of m is correctly specified, the tangent space method is found to be more reliable than the direct method.

Figure 2-14, extracted from Hu et al. (2010), explains how to compute the largest Lyapunov exponent λ. The figure represents the relationship between the stretching function ($y(i)$) and the time increment (i) for both urban and freeway traffic flow networks. The largest Lyapunov exponent (λ) can be calculated as the slope of the straight line in each case. It can be noticed from the figure that both lines are approximately parallel, resulting in very close estimates of λ. As found in Hu et al. (2010), the value of λ for urban traffic flow is 0.011, while that for traffic flow along the freeway section is 0.015. The values of both exponents are positive, indicating that fractal behavior exists in both urban and freeway flow networks.
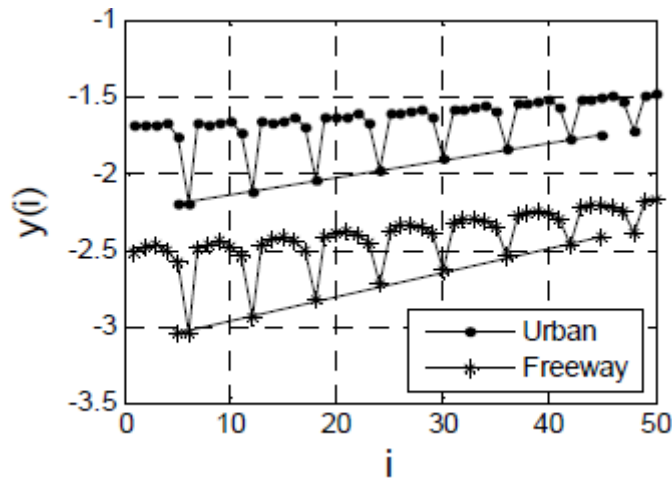
Figure 2-14: Identifying the Largest Lyapunov Exponent (Source: Hu et al., 2010)

2.6 Summary

An overview of the fractal theory and the various techniques used to detect fractal existence in the data were discussed in detail. Fractals are mainly irregular geometric objects that exhibit finite details at all scales, and when magnified, their basic structures remain the same regardless of the scale of magnification. The main characteristics of fractals are self-similarity, iterative process, infinite complexity, and existence of non-integer complex dimension. Traffic flow data are hypothesized to exhibit fractal characteristics due to their random fluctuations during a selected time period, for example, 24 hours.

Common fractal analysis techniques are the Hurst exponent, the fractal dimension, the Lyapunov exponent, the power spectrum, and the Kolmogorov entropy. The fractal dimension technique includes ten different methods, the box count, Hall-Wood, variogram, madogram, rodogram, Incr1, periodogram, DCT-II, genton, and wavelet methods. The Hurst exponent technique includes eight methods, aggregated variance, differenced aggregated variance, aggregated absolute moment, Higuchi, variance of residuals, R/S, periodogram, and modified periodogram. In summary, common signs to identify the existence of fractal/chaotic characteristics are:

- The Hurst exponent (H) estimate should lie in the range of 0 and 1.
- The fractal dimension estimate should be greater than 1.
- The largest Lyapunov exponent should have a positive sign.
- The Kolmogorov entropy should have a positive sign.
- Larger estimates of the fractal dimension, the Hurst exponent, the Kolmogorov entropy, and the largest Lyapunov exponent values indicate more complex fractal nonlinear characteristics.

CHAPTER 3
EXISTING APPLICATIONS OF FRACTAL THEORY

This chapter reviews the existing real-world applications of fractal theory in different fields of science, both within and outside transportation. The fields outside transportation include those that share similarities to transportation networks, such as electrical and water supply networks, as well as other diverse fields, such as animal behavior, human health, economics, astronomy, ecology, computer science, seismic analysis, etc. The chapter also presents fractal applications in transportation research, such as applications in traffic flow and traffic speed prediction, pavement evaluation, urban planning, and vehicle demand modeling. Applications of multi-fractal theory in traffic flow prediction are also discussed. Finally, the chapter concludes with a summary of the main findings, lessons learned, and gaps in the literature.

3.1 Fields outside Transportation

3.1.1 Electrical Networks

Boyle et al. (2007) studied self-similar energy forms on a group of fractal N-gaskets. The authors constructed electric circuit (resistor) networks such that every resistor was a scalar multiple of the corresponding resistor in the next-order network. They determined all the resistances and scaling constant for every N-gasket, and the fractal existence was revealed. The authors further studied the asymptotic behavior of these scaling constants as the number of sides "N" tended to infinity.

3.1.2 Water Supply

Chai and Li (2007) discussed hierarchical structures of fractal growth of water supply networks and their fractal dimensions. They applied a statistical mechanics method for exploring interactions of the water consumption elements. The authors observed a self-organized emergence of evolving water supply networks, which was an indication of fractal existence.

3.1.3 Control of Network Traffic/Internet Simulation

The use of fractals in controlling and simulating network traffic has been widely applied by researchers, as seen in Lam and Wornell (1998), Ryu and Lowen (2000), Liu (2006), and Yu and Fei (2008). Lam and Wornell (1998) applied multi-scale techniques to explore fractal point processes in various networking scenarios with self-similar traffic. These multi-scale techniques were also developed for analyzing fractal queuing scenarios. Examples included the steady-state customer distribution for a queue servicing self-similar arrivals and for Poisson customers serviced with self-similar holding times. The authors showed that multi-scale methods could be used in conjunction with dynamic programming techniques to develop efficient control policies for the fractal queuing scenarios. The implemented multi-scale techniques performed much better than the traditionally used queuing strategies.

In the similar manner, Ryu and Lowen (2000) hypothesized that both the internet Local Area Network (LAN) and Wide Area Network (WAN) exhibit fractal behavior. They could

successfully demonstrate that LAN and WAN exhibit fractal behavior using quantitative fractal point process models. The authors further explained how the internet flow-level fractal dynamics (such as user activity, session/flow arrivals, and duration) affected the packet-level fractal dynamics.

Liu (2006) applied the multi-fractal theory to analyze internet traffic data. The objective was to convey a better understanding of the fractal nature of network traffic to lead to more efficiency and better internet quality of service. Liu (2006) applied two techniques in the analysis: (1) the second-order self-similarity method, and (2) multi-fractal method. Multi-fractal analysis is a broader view of fractals as opposed to the second-order self-similarity method, and is based on the concept of a local signal singularity characterized by the Holder exponent. Both techniques showed that network traffic exhibited fractal behavior. The study also proposed a measurement of the "efficient bandwidth" for fractal traffic control to improve network efficiency.

Yu and Fei (2008) analyzed traffic collected from wireless mesh network and hypothesized that traffic in wireless networks exhibited fractal characteristics or long-range dependence (i.e., existence of correlations over a broad range of time series). The authors applied the Hurst index, which is a key characteristic of the bursty traffic, and observed the existence of long-range dependence or fractals in the wireless network time series data.

3.1.4 Video Traffic Flow Predictions

The prediction of video traffic flow has been the focus of several researchers for many years, e.g., Willinger et al. (1995), Fei and Zhimei (2003), and Shenghui and Zhengding (2006). Willinger et al. (1995) provided a physical explanation of the existence of self-similarity (i.e., fractal behavior indication) in high-speed internet traffic. The authors attributed this to the high variability characteristic or infinite variance of the LAN internet process. The authors further discussed that the superposition of ON/OFF sources (or packet trains) where ON-periods and OFF-periods exhibit a high variability could produce aggregate network traffic that was characterized by self-similarity and long-range dependence. Its empirical application was the development of parsimonious models to predict the complex nature of internet traffic data.

Fei and Zhimei (2003) explored the multi-fractal characteristics of moving picture experts group-4 (MPEG-4) encoded video traffic in wire line and wireless networks. The authors could detect multi-fractal characteristics by using multi-fractal and spectra analysis. Furthermore, a multi-fractal wavelet model was used to analyze video traffic data. This model captured the characteristics of video traffic data, including the temporal and spatial localities. The results showed that the wavelet model was flexible and accurate in the MPEG-4 video modeling.

Shenghui and Zhengding (2006) applied the multi-fractal method to analyze and predict variable-bit-rate (VBR) video traffic in real-time. According to the authors, since traffic is non-stationary, traditional linear methods which use short-range dependence feature were not suitable for real-time video traffic prediction at large time scales. The authors therefore used the multi-fractal autoregressive integrated moving average (ARIMA) method. The results confirmed the authors' hypothesis, where the multi-fractal method yielded higher prediction accuracy than the traditional linear models.

3.1.5 Animal Behavior

One of the studies that explored animal behavior using the fractal theory is Seuront and Cribb (2011). They examined stress in dolphins as a result of boat presence and type of boat in South Australia. The stress level was inferred from the distribution patterns of their dive durations. Dive durations were recorded from land-based stations in the absence of boat traffic, as well as in the presence of kayaks, inflatable motor boats, powerboats, and fishing boats. For this study, the authors recorded close to 6,000 observations. The authors used the fractal method in terms of the scaling exponents of the cumulative probability distribution of dive durations in the absence of boat traffic and under different conditions of boat interferences. The presence of boats was found to affect the complexity of dive duration patterns. Further, a significant increase in behavioral stress was induced by the presence of fishing boats, motorized inflatable boats, and powerboats. According to the authors, the use of fractals provided a non-invasive, quantitative framework used to assess the changes in stress response of animals.

3.1.6 Human Health

Two of the studies that investigated human health are Goldberger et al. (2002) and Abasolo et al. (2008). Goldberger et al. (2002) applied fractal theory to assess cardiac risk and forecast sudden cardiac death in humans. The authors presumed that the normal human heartbeat fluctuates in a complex manner, even under resting conditions, which made the fractal method more appealing. The scaling techniques adapted from statistical physics revealed the presence of long-range, power-law correlations as part of multi-fractals operation over a wide range of time series.

Abasolo et al. (2008) argued that nonlinearity was found in human brain at cellular level. They applied the fractal dimension method to characterize the electroencephalogram (EEG) background complexity in Alzheimer's disease patients, where EEG is the tool used for diagnosing mental diseases. They further emphasized that fractal dimension method was characterized by self-similarity and independence of scaling, making it reliable to measure signal complexity of Alzheimer's disease patients.

3.1.7 Landform Features and Landscapes

Cai and You (2010) compared the simple fractal dimension and multi-fractal spectra to explore an area before and after mountain landslide. The authors used the simple fractal dimension to describe the overall landform terrain. They found that the simple dimension of the landform became bigger after the mountain landslide. As expected, the geological environment became complicated after the landslide; therefore, the use of multi-fractals was considered to be more appropriate.

Al-Hamdan et al. (2012) analyzed forest structure characteristics using a fractal dimension analysis and a spatial autocorrelation technique called Moran's I. The analyzed forest was the Oakmulgee National Forest located in Alabama. They found that the fractal dimension value was close to 2.7, a fairly high value which confirms the data's complex fractal behavior. The objective of the study was to differentiate between hardwood and softwood species to facilitate forest landscape mapping for performing environmental impact analysis.

3.1.8 Surface Roughness

Ganti and Bhushan (1995) simulated a number of engineering surface profiles using the generalized fractal analysis. The authors simulated different types of surfaces including magnetic tapes, thin-films, rigid disks, steel disks, plastic disks, and diamond films. Based on the measured surfaces, all isotropic engineering surfaces and texture surfaces were represented as non-stationary processes with stationary increments. Moreover, for magnetic tapes, steel disks, and diamond films, the isotropic nature in the surface was found to produce similar structure functions in the two perpendicular directions. On the other hand, for magnetic tapes and plastic disks, different structure functions were found, hence yielding different fractal parameter values.

3.1.9 Astronomy

Mouradian and Soru-Escaut (1991) analyzed the solar activity cycle from the aspect of large scale magnetic fields, i.e., unipolar and bipolar magnetic regions. The authors used fractal dimension analysis and found that bipolar magnetic regions became turbulent as sunspot activity grew.

3.1.10 Environmental Engineering

Svensson et al. (1996) investigated the characteristics of daily rainfall in two different climates: one with extreme rainfall variability in China and another with moderate rainfall variability in Sweden. The authors were able to detect multi-fractal properties in both the climates. The rainfall mechanisms were grouped into different categories and were analyzed separately, and the authors concluded that the multi-fractal framework could be used for analyzing rainfall in different regional climates.

3.1.11 Economics

Richards (2000) indicated that recent studies have found that the financial market time series patterns could be seen as fractals. This methodology was derived from Econophysics, a specialty field in Physics. According to the author, the conducted empirical tests could demonstrate that exchange and interest rates exhibited fractal characteristics since they showed non-integer dimensionality. However, the degree of fractality could diminish with the increase in time scale.

3.1.12 Weather Forecast/Meteorology

Suresh et al. (1999) developed models to forecast surface temperature and pressure using both fractal dimensions and auto regressive processes to help aircrafts plan ahead more efficiently. These models were reliable for short-range forecast, preferably one-to three-day forecasts.

3.1.13 Mechanical Behavior of Materials

Arasan et al. (2010) investigated the impact of the shape of aggregate particles on their mechanical behavior. According to the authors, the shape of aggregate particles affects durability, workability, shear resistance, tensile strength, stiffness, and fatigue. The authors used fractal dimension analysis because of the irregularity of particles. It was found that the flow of

asphalt concrete decreased and the stability increased when the fractal dimension of aggregate was increased.

3.1.14 Seismic Analysis

Tosi et al. (1999) used a fractal dimension method to improve seismic signal recognition. This method was based on the difference between seismic signals and background random noise. The method was compared to traditional methods for seismic analysis. It was found that the fractal dimension method could recognize seismic phases and had greater sensitivity to smaller signals.

3.1.15 Ecology

Wigley et al. (1999) monitored the reproductive performance of red-cockaded woodpeckers (RCW) on private lands in Louisiana from 1993 to 1995. The authors measured the area of foraging habitat, amount of foraging substrate, and landscape characteristics. It was found that variables representing landscape metrics such as fractal dimension were related to the RCW measures of reproduction.

3.2 Traffic Flow Theory

Traffic prediction is an important application of the traffic flow theory. The ability to predict traffic variables (such as speed, travel time, and flow) using real-time and historical data collected by loop detectors plays a vital role in improving travelers' experience, especially in congested conditions. For this, short-term traffic flow predictions to estimate traffic flow in the next 10-30 minutes time interval have significant applications in the field of ITS. On the other hand, long-term traffic prediction is not of importance since travelers are more interested in knowing traffic conditions in the next 10-15 minute time intervals rather than in the next few hours or in the following day. The following section discusses short-term traffic prediction studies.

3.2.1 Short-Term Traffic Prediction

There has been a lot of research in predicting short-term traffic conditions. Abdulhai et al. (1998) applied a time delay neural network (TDNN) model for short-term traffic flow and occupancy prediction. The model was further optimized using genetic algorithm (GA) and the prediction performance was tested using both simulated and actual traffic flow data along the I-5 freeway in Orange County, California on April 2, 1997 from 4 to 6 pm. They found that an increase in data aggregation resulted in an increase in prediction accuracy. The model performed acceptably using both simulated and actual data. Using the same investigated data, the TDNN model showed superior prediction accuracy compared to the multilayer feed-forward (MLF) neural network model.

Similarly, Chen and Grant-Muller (2001) applied sequential or dynamic learning neural network models to improve traffic flow prediction accuracy. The authors used aggregated 15-min traffic count data from 6 am to 9 pm from the M25 London orbital motorway in England, UK. The models were named sequential since data points were presented to the freeway network in

sequence and then discarded once used. It was found that the adopted models yielded relatively high prediction accuracy of traffic flows, especially for a medium-sized freeway network.

The application of time series techniques in traffic prediction is extensive in the literature. For example, Al-Deek et al. (2001) applied a nonlinear time series model for short-term traffic prediction along an 18-km stretch of the I-4 freeway in Central Florida. Speed, volume, and occupancy data were collected from 25 double inductance loop detectors spaced every 0.5 miles. The data was collected during the 4-hr morning peak period from 6 am until 10 am for 15 months. The developed model performed very well except during heavy congestion times that have unstable traffic conditions . Ishak and Al-Deek (2002) used the same corridor and the same time period to assess the accuracy of traffic speed and travel time predictions based on the proposed nonlinear time series model. The authors used the relative travel time prediction errors as a performance measure and applied multivariate generalized linear model (GLM) to identify significant factors affecting the model's performance. The analysis revealed that congestion index, which reflects the traffic state at the time of prediction, was the most significant factor in determining travel time prediction accuracy. Interestingly, the error in predicting travel times was found to increase drastically in congested conditions, mainly due to the high fluctuations in speed observations. Furthermore, short prediction horizons (e.g., 5-min speed prediction ahead of present time) were more favorable in congested conditions. However, in light or moderate traffic conditions, longer prediction horizons could be more useful.

The application of ARIMA models for short-term traffic prediction is widespread. Cetin and Comert (2006) proposed an ARIMA model that explicitly accounted for occasional regime changes using statistical detection algorithms and the model was compared to the traditional ARIMA model. The authors used this model type to improve short-term traffic speed prediction using traffic data collected from loop detectors along the I-880 freeway for four days. The speed data were then aggregated into 1-min intervals and fed into the ARIMA models. It was found that the proposed model resulted in a significant improvement in prediction accuracy compared to the traditional ARIMA model. In addition, Zeng et al. (2008) applied a combined ARIMA-multilayer artificial neural network (ARIMA-MLANN) model for 8-min traffic flow prediction. Daily 8-min traffic flow data from 7 am to 7 pm along a 45-km section of the Guangyuan Highway in Guangzhou, China were used in the analysis. The authors found that the combined ARIMA-MLANN model yielded higher prediction accuracy compared to the application of each model separately.

Since nonparametric techniques do not require underlying relationship between the dependent variable and independent variables, they were extensively applied in traffic predictions. For example, Clark (2003) adopted the K nearest neighbor (KNN) technique to predict traffic speed, occupancy, and flow along the M25 London orbital motorway in England, UK. Training data included traffic speed, occupancy, and flow that were collected for three weeks in 10-min intervals. On the other hand, test data were collected for one-week period in 10-min intervals. The KNN technique could be described as a pattern-matching method, where recent observations were matched with historical observations. The author found that short-term predictions of the three traffic parameters were accurate, especially for traffic flow and occupancy. This was mainly since traffic speed data exhibited the least pattern variation for most periods of the day. Castro-Neto et al. (2009) proposed a supervised machine learning technique, the online support

vector machine regression (OL-SVR) for predicting short-term traffic flow in typical and atypical conditions. Typical conditions included traffic on weekdays with no incidents and accidents. On the other hand, atypical conditions include occurrence of crashes, presence of inclement weather or work zones, and holidays. The authors used 5-min aggregated real-time data collected from more than 8100 freeway sections in California. The OL-SVR model was compared with three predictive models, the Gaussian maximum likelihood (GML), Holt exponential smoothing, and artificial neural network models. Under typical conditions, GML provided higher traffic flow prediction accuracy compared to other models, since GML models rely on recurring characteristics of daily traffic flow. During non-recurring atypical conditions, OL-SVR provided the best prediction accuracy.

Using the same concept of support vector machine (SVM) techniques, Duan et al. (2011) improved the traditional modeling structure of traffic prediction models by adopting a rough-set structure that allowed for data reduction pretreatment prior to the model fit. The authors then applied an SVM model with a rough-set structure and compared the prediction performance with neural network and traditional SVM models. One week of historical traffic data was used in the analysis. It was found that the rough-set SVM provided better prediction accuracy compared to the neural network and traditional SVM models.

It can be concluded from the aforementioned studies that there has been no lack of effort in short-term traffic predictions mainly used to predict flow, speed, and occupancy. Short-term traffic prediction could be viewed as the main application of the fractal theory in the field of ITS, e.g., forecasting traffic flow in the next 15-min period based on both the previous real-time data on the same day, as well as historical data for three-week period for example. This information would assist travelers in avoiding congested freeway sections and therefore, improve the freeway network performance. The following sections focus on the application of fractals in traffic flow theory. The review includes research in analyzing roadway traffic data (e.g., traffic volume, speed, and occupancy), traffic incidents, and congestion levels.

3.2.2 Roadway Traffic Data

Storkey (1996) discussed several common characteristics of fractals that exist in traffic data. Storkey indicated that the main characteristics of fractals include non-integer fractal dimensions, multi-fractal spectra, self-affinity, and long-run dependence. Fractal roughness characterized by non-integer fractal dimensions implies that viewing traffic as a process involving smooth densities is inaccurate. Self-affinity in traffic data and long-run dependence can be demonstrated through periods of high or low traffic "bunch" over all time periods. Therefore, characterizing data using mean and variance is considered quite poor representation. In addition, fractal characteristics could describe traffic patterns better than viewing traffic as a Poisson process.
Zhang and Jarrett (1998) investigated the dynamic behavior of roadway traffic flows in a region represented by an origin-destination (O-D) network. The authors modified the conventional gravity model proposed by Dendrinos and Sonis (1990) to be dynamic to describe the variations of O-D traffic flows over discrete-time periods, e.g., by day, by week, etc. When the system is one or two dimensional, the O-D flow pattern could reach equilibrium. On the other hand, when the system dimension is higher, the behavior found in the model included equilibrium,

oscillations, periodic doubling, and chaos/fractals. These chaotic attractors or fractals were characterized by positive Lyapunov exponents.

Liu and Tang (2007) validated the complexity characteristics of traffic flow by introducing characteristic quantities of complexity, such as Lempel-Ziv complexity, statistical complexity, approximate entropy, and combining the criterion methods of chaos and fractal. Additionally, using traffic flow data that varies with time, Zheng (2006) applied fractal geometry theory in terms of the fractal interpolation method to simulate the entire complex functional graph of traffic flow data. Afterwards, forward fractal prediction method was introduced. The results showed that traffic flow statistically possessed a self-similar characteristic to effectively predict future traffic data.

Hu et al. (2010) investigated if urban traffic flow data collected from Beijing, China exhibited nonlinear fractal behavior. The authors used fractal dimension, Kolmogorov entropy, and largest Lyapunov exponent methods to detect the existence of fractal. They found that fractal behavior existed in the data since the Kolmogorov entropy and largest Lyapunov exponent values were positive. The existence of fractals could be useful in forecasting the changing trends in traffic flow as well as unusual traffic incidents and traffic congestion.

Real-time traffic flow data along highways was analyzed by Frazier and Kockelman (2004), Shang et al. (2005 & 2007), Lin and Lan (2005), and Xu et al. (2010). Frazier and Kockelman (2004) explored a series of chaotic techniques to analyze traffic flow data, including Fourier power series, correlation dimension, and the largest Lyapunov exponent. The authors also compared the chaos theory to the traditional nonlinear least-squares method. The traffic flow data were extracted from inductive-loop detectors embedded on I-80 near Sacramento, California collected over a one-month period in 2003, and the 30-second vehicle counts were aggregated to five-minute counts. The traffic flow data were found to be chaotic. Further, when used with a low-pass filter, predictions based on chaos theory had greater predictive power than the nonlinear least-squares method.

Shang et al. (2005) applied nonlinear time series techniques to analyze traffic speed data collected from the Beijing Xizhimen highway over six months in peak periods. Speed, volume, and occupancy data were collected every 20 seconds using dual loop detectors placed every half a mile for each lane of the freeway locations. Data were then aggregated into two-minute intervals and the largest Lyapunov exponent was used to detect the existence of fractal or chaotic behavior. It was found that chaotic characteristics obviously exist in the traffic data. The authors recommended that short-term traffic predictions (8-min) should be used instead of long-term predictions. Confirming this finding, Li and Huang (2011) compared the observed and predicted traffic flow data collected in Shanghai for a relatively short time period. They concluded that both data had very close estimates.

Using the same concept on another Chinese highway, Shang et al. (2007) demonstrated the existence of fractal behavior in traffic speed data observed at the Beijing Yuquanying highway. The observed traffic data included speed, volume, and occupancy data over a period of 40 months. The data were collected for each lane every 20 seconds using dual loop detectors spaced at half-mile intervals. The methods used to detect the fractal behavior were the power spectrum,

empirical probability distribution function, statistical moment scaling function, and autocorrelation function. However, the explored methods had some limitations. For example, there were insufficient experiments, and the presence of noise might have influenced the outcomes of the fractal identification methods.

Using traffic flow data collected along I-135 in Minneapolis, Minnesota, Lin and Lan (2005) examined whether nonlinear dynamics (or fractals) of traffic time series data existed for different time scales, including 1-min, 5-min, and 10-min counts. The authors implemented a three-step parsimony procedure that included three indices, the largest Lyapunov exponent, the power spectrum, and the iterated function system (IFS) map. It was found that if the traffic flow data are measured in 1-min counts, the traffic dynamics in the morning hours exhibit chaotic/fractal behavior. However, if the same flow data are measured in 5-min and 10-min counts, these chaotic structures may disappear and exhibit a quasi-periodic nature. The practical implication of the study was that in predicting traffic flow, if flow is measured with a longer time interval, such as 5-10 minute counts, it is better to make use of the quasi-periodic techniques by using seasonal ARIMA, wavelet analysis, or Fourier analysis techniques. However, if traffic flow is measured with a shorter time interval such as 1-min counts, the aforementioned techniques may not accurately predict traffic flow.

Additionally, by collecting traffic flow time series data on the Lenovo-Bridge highway in the Beijing Third Ring Road, Xu et al. (2010) explored the autocorrelations and cross-correlations that exist in the observed traffic flow time series. The authors implemented a powerful fractal analysis technique known as detrended fluctuation analysis (DFA) and detrended cross-correlation analysis (DCCA), a generalization of the DFA method. The DCCA method provides a systematic means to identify cross-correlations between datasets obtained from a traffic system. Both methods were implemented to avoid false detection of autocorrelations and cross-correlation due to noise superimposed on the collected data. It was found that the traffic flow fluctuation time series exhibited autocorrelations; however, the positive and negative traffic fluctuation signals exhibited anti-correlated behavior. It was also observed that the traffic speed fluctuation time series derived from adjacent sections exhibited stronger cross-correlations than the traffic speed fluctuation derived from adjacent lanes.

3.2.3 Traffic Incident Data

The following studies focus on the application of fractal theory in analyzing roadway traffic incidents. Thomas and Dia (2000) developed a fractal threshold model and tested it on a dataset containing 100 incidents (60 for training and 40 for validation) representing speed, occupancy, and flow data. These models were compared with two other models, artificial neural network and a calibrated Australian Road Research Board (ARRB) model. The fractal threshold model was found to perform well in detecting lane-blocking incidents, and worked better in low traffic flow or off-peak periods.

A later study by Thomas and Dia (2006) reviewed the most widely used freeway Automated Incident Detection (AID) models, including the comparative California models, time series models, McMaster models, neural network models, fuzzy logic models, and macroscopic models. These models could analyze real-time data collected from traffic detection equipment to

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determine whether an incident has occurred. The authors also discussed the refinement of an AID model based on fractal dimension analysis of speed and occupancy data. This idea was proposed since traffic parameters upstream of incidents and bottlenecks show substantial irregular behavior when compared to downstream conditions. The aforementioned models were implemented using the same dataset of 100 field incidents. It was observed that fractal analysis and neural network models outperformed the comparative California models. These results were also found in their earlier study (Thomas and Dia, 2004).

A similar study that examined the existence of traffic incidents was done by Lan et al. (2003). The authors used several chaotic techniques, such as the largest Lyapunov exponent, the capacity dimension, the correlation dimension, the Kolmogorov entropy, delay time, and the Hurst exponent. The results showed that the largest Lyapunov exponent had the most significant impact both before and after the traffic incident. In fact, the average prediction rate of incidents based on the Lyapunov exponent chaotic parameter reached 93.75%. This percentage was considered better than conventional incident detection algorithms based on microscopic or macroscopic traffic parameters which have an average incident prediction rate of 90%.

3.2.4 Traffic Congestion Data

A unique study that described traffic congestion by applying the fractal geometry was performed by Torok and Kertesz (1996). They applied computer simulation using green wave model and a multi-spin coding technique to investigate the infinite system size which indicates a non-zero density transition from the free flow to the congested state. It was found that the geometry of the traffic jam was characterized by a non-trivial fractal dimension of 1.58.

3.3 Other Transportation Areas

This section reviews fractal theory applications in other transportation areas, which include urban roadway network analysis, travel demand modeling, crash analysis, pavement texture evaluation, railway ballast evaluation, and logistics.

3.3.1 Urban Roadway Network Exploration/Urban Analysis

Fractal theory has been extensively applied in urban planning and analysis. Tang (2003) investigated the relationship between urban roads and population density in San Antonio, Texas by applying fractal geometry analysis. The approach was based on the box dimension of fractal theory for calculating the road dimension. The dimension of roads was found to be more volatile than that of population density. Furthermore, it was found that a close relation existed between roadway patterns and population density.

Sun et al. (2007) introduced a distributive continuous fractal analysis method to evaluate the roadway network in the City of Dalian in China. The authors treated the city as distributive continuous space and applied fractal analysis with the aid of a geographic information systems (GIS) tool to different parts of the space. This approach was different from the traditional approach of treating the city as a whole. It was found that the entire transportation network in city as well as parts of the network in the city exhibited the property of self-similarity or fractal.

This microscopic analysis could help policy makers and planners see details of a city more clearly and make accurate predictions.

McAdams (2009) discussed the applicability of fractal theory for urban analysis. The author found that the fractal dimensions of urban network were not infinite, but rather incorporated a defined fuzzy range of values based on the characteristics of urban network. Further, the author recommended combining fractal analysis and chaotic techniques (e.g., neural networks) with spatial analytical techniques (e.g., GIS) to more adequately represent urban characteristics.

Following McAdams' (2009) recommendation by adopting the fractal theory along with GIS, Bai et al. (2010) separately analyzed the spatial patterns of roadway networks in the northern and southern Jiangsu Province in China in 1999 and 2009. The fractal dimension value and covering depth of the road network were calculated in the two regions in 1999 and 2009. Covering depth was estimated by covering the study roadway into square grids and was calculated as the function of number of grids and the square's length of the side. It was shown that within the same year, no significant differences existed in the fractal dimension and covering depth between the two regions, whereas for the same region, there were differences between the years 1999 and 2009. This indicated a rapid construction and development of the roadway network in Jiangsu province over the last decade, and a well-maintained regional balance at the same time.

3.3.2 Travel Demand Modeling

Two applications of fractals in demand modeling can be found in Burnett and Pongou (2006) and Moret et al. (2012). Burnett and Pongou (2006) investigated the relationship between travel behavioral models and the geometry of the metropolitan transport network in the Boston area in Massachusetts. The fractal dimension analysis was used to analyze eight classes of roadways: all roadways, major highways, other highway types, major roads, local roads, minor roads, ramps, and private roads. The applied fractal dimension method had its basis from the Hausdorff dimension method. The analysis revealed that different trip purposes and scales of analysis existed between highway systems and major and local road systems.

Moret et al. (2012) applied fractal theory to analyze vehicle demand on the ferry-boat system between Salvador City and Itaparica Island in Bahia, Brazil. The authors used the classical ARIMA model and fractal-based models. It was found that all used methods indicated a decreasing vehicle demand between the two regions over time.

3.3.3 Crash Analysis

Peng et al. (2008) applied fractal theory to crash analysis. The authors used the power spectrum method and found that the time series annual distribution of crash data exhibited self-affinity, confirming the existence of fractals. Furthermore, the fractal extrapolation method was implemented and yielded reasonable crash predictions.

3.3.4 Logistics

Chen et al. (2011) proposed a model of pallet dispatch optimization using fractal theory. The idea evolved with the development of logistics information such as the application of Internet of

Things (IOT); therefore, the pallet dispatch could be optimized through a co-shared information resource fractal unit. As part of the study, the authors also discussed some issues of the fractal model for different business partners.

3.3.5 Railway Engineering/Ballast Evaluation

Ho et al. (2013) used the fractal dimension analysis to classify the grain size distribution of railway ballast. The authors used a sample three-year data from a railway section in Nebraska. They found that the fractal dimension was increasing over time for the entire grain size distribution. However, the fractal dimension was found to be decreasing for the portion representing fouled material, which is small-sized particles or dirt that is filled in voids within the ballast layer. This fouled material is the main reason for degrading the ballast performance. They further concluded that the initial grain size distribution of the ballast might not exhibit fractal behavior, but it could exhibit fractal behavior over time due to the increase in fouling forces.

3.3.6 Pavement Evaluation

Fractal theory has been extensively applied in pavement and material evaluations. For example, Kokkalis et al. (2002) applied fractal theory to analyze pavement texture since irregularities in pavement surface were continuous from pavement macro-texture to pavement micro-texture. These irregularities were characterized by repetitive pattern, and exhibited self-similarities. The authors could successfully evaluate pavement surface wear with respect to its impact on skid resistance using fractal dimension modeling. Further, they found that variations in fractal dimension values yielded consistent results with the variations in skid resistance values.

Yuan et al. (2009) measured the asphalt pavement surface texture using the fractal theory. The pavement surface texture images were acquired using ordinary cameras, and both field and laboratory tests were conducted. Afterwards, fractal dimension analysis on contour of gray level was performed. It was found that fractal dimension could successfully detect the roughness of pavement surface texture.

Additionally, using fractal dimension theory, Vallejo (2011a) determined the optimum size of shale particles to resist slaking and point loads under drying and wetting conditions. The fractal dimension found was 1.425 and the optimum shale size was ≤ 0.06 mm (1.8×10^{-4} ft). In another study, Vallejo (2011b) used fractal dimension theory to identify three levels of pavement cracking (i.e., low, medium, and high) to develop various crack distress indices. The author observed that fractal dimension increased with increase in cracking level. It was found that pavements with low cracking level had a fractal dimension of 1.05. For pavements with moderate level of cracking, the fractal dimension was 1.17. Finally, pavements with high cracking level had a fractal dimension that varied between 1.50 and 1.71. Figures 3-1(a), (b), and (c) show fractal dimension estimations for low, moderate, and high levels of cracking, respectively. In the figure, fractal dimension is estimated as the slope of the relationship between the number of cells "N" and the size of the cell "r". N is determined using the box method where a sequence of square grids with a dimension r is drawn on a white paper. Afterwards, each square grid is placed over the area containing the cracks and N is counted as the number of cells

intersected by the crack traces. From the figure, it can be inferred that fractal dimension increases with level of cracking.

(a) Low Cracking Level   **(b) Moderate Cracking Level**   **(c) High Cracking Level**
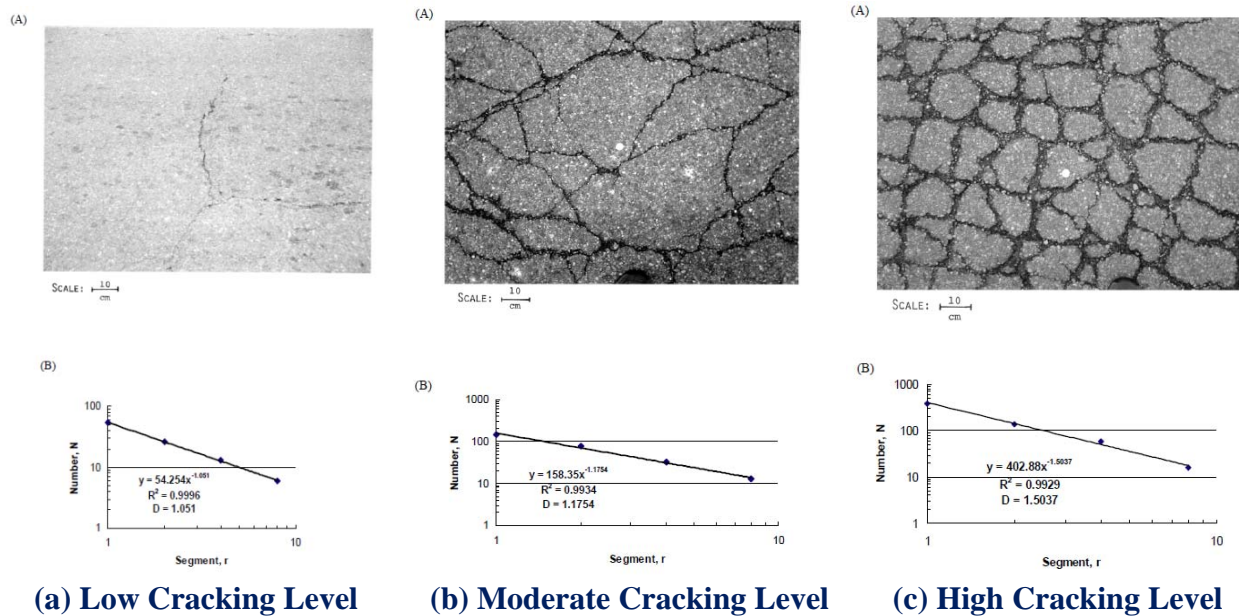
Figure 3-1: Fractal Dimension Estimation for Different Cracking Levels
(Source: Vallejo, 2011b)

3.4 Multi-Fractals in Traffic Flow Theory

This section highlights the applications of multi-fractals in the traffic flow analysis and car-following models. It is observed that several multi-fractal studies were found in analyzing traffic flows. One of the earliest studies in applying multi-fractals theory in traffic flow analysis is Vojak et al. (1995). The authors explored the presence of multi-fractals in traffic flow data collected every 40 seconds over a 19-day period on the Boulevard Peripherique in Paris, France. Flow data were aggregated to 10-min intervals, and the presence of multi-fractals was demonstrated through the irregularities of the time series plot of traffic flow. In addition, the existence of multi-fractals was found to be promising for short-term traffic flow predictions.

Furthermore, Shang et al. (2006), Shang and Shen (2007), and Li and Shang (2007) applied multi-fractals in traffic flow analysis using data collected from Beijing Yuquanying highway. All the three studies agreed that multi-fractals existed in traffic flow data. Shang et al. (2006) applied multi-fractal modeling techniques using the Holder exponent. The results indicated that multi-fractal characteristics existed in the traffic system, where the degree of fractality of traffic data tended to increase with congestion level. The authors further concluded that simple proactive traffic congestion avoidance can be developed based on the fractal theory of traffic flow data.

Shang and Shen (2007) investigated the presence of multi-fractal behavior in traffic speed, volume, and occupancy time series by applying both statistical and geometrical approaches. The raw data for speed, volume, and occupancy were collected every 20 seconds for each lane on the highway; however, these data were aggregated into one-min intervals. The authors applied the

Holder exponent and statistical moment scaling methods to detect multi-fractality. It was found that all the three traffic parameters exhibited multi-fractal behavior. Li and Shang (2007) explored the presence of multi-fractality in traffic flow data observed over a period of about 40 months (from January 16, 2001 to June 17, 2004). Using multi-fractal spectrum analysis in terms of the Hurst exponent and spectrum width spread, the traffic flow was shown to detect multi-fractality.

Along the same facility (i.e., the Beijing Yuquanying highway), Yue et al. (2010) proposed a pioneering method for capturing the degree of variability of traffic flow at each time t, called the time-dependent Hurst exponent $H(t)$. This exponent was calculated for traffic flow time series observed on the highway. The authors detected obvious time series variability of explored data, indicating a non-monotonic multi-fractal behavior in Beijing's traffic flow data. Moreover, it was found that these multi-fractal distributions were suitable for modeling highly complicated traffic processes. The authors highlighted the advantage of using multi-fractal Hurst exponent $H(t)$ over mono-fractal Hurst exponent, i.e., the multi-fractal model can accurately measure the exact degree of variability of traffic flow at each time t. Additionally, the proposed multi-fractal model does not require prior assumptions on the stochastic process and on the probability distribution function of random variables.

Zhiheng et al. (2008) proposed a new mixed-flow model to capture the main features of urban traffic volume flows over Beijing transportation network. The model consisted of the following three distributions: 1) the major relatively-stable distribution that follows an M-shape curve; 2) the random fluctuation distribution that follows time variant super-Gaussian distribution; and 3) the peak distribution that follows the negative exponential distribution. The traffic flow data were collected via detectors buried along the second and third ring roads of Beijing in 2006 and 2007. Traffic flow volume was found to follow a multi-fractal time series distribution. The results highlighted the characteristics of urban traffic flow that are useful for traffic flow prediction.

Safonov et al. (2002a & b) explored the existence of chaos and fractals in a car-following traffic model. Safonov et al. (2002a) applied a system of delay-differential equations and found that for low and high values of car density, the system had a stable steady-state solution. Moreover, it was observed that above a certain time delay and for intermediate density values, the system switched to the chaos situation with multi-fractal nature. In addition, Safonov et al. (2002b) hypothesized that the presence of car delays could be the reason for chaotic and multi-fractal behavior. They applied the same system of delay-differential equations on single-lane roadway traffic and their hypothesis was found to be true.

3.5 Summary

Existing studies have demonstrated that fractal theory has been successfully applied in many different fields of science, including but not limited to animal behavior studies, human health studies, economics, astronomy, ecology, physics, pavement engineering, environmental engineering, behavior of materials, electrical networks, traffic flow analysis, urban analysis, crash analysis, and travel demand modeling. However, fractal applications in highway traffic flow analysis are still emerging and showed promising results in traffic flow prediction.

Common chaotic techniques used by researchers for detecting fractal characteristics are the largest Lyapunov exponent, the power spectrum, the fractal dimension, the Kolmogorov entropy, and the Hurst exponent.

The majority of studies that compared multiple models agreed that fractal techniques outperformed conventional methods, such as the linear models, nonlinear least-squares method, and artificial neural networks. The review indicated that traffic flow and traffic speed were the main elements of traffic data to exhibit fractal characteristics. Furthermore, fractal models were found to be more appropriate for short-term traffic flow prediction and weather forecast, as well as for low traffic volume conditions. This is because, when traffic flow data are collected in short time intervals (e.g., 1-min counts), chaotic and fractal characteristics are quite clear. On the other hand, if the same flow data are collected in relatively long time intervals (e.g., 5-10 min counts), the chaotic structures disappear (Lin and Lan 2005). Furthermore, the literature review has identified the following gaps:

1. The need of more comprehensive ITS data having traffic flow and traffic speed estimates collected from more than a single roadway over multiple years. These data are expected to validate the existence of fractal behavior in traffic data and could improve long-term traffic predictions.
2. Limited applications of fractal theory in traffic safety analysis were noticed from the literature (Peng et al., 2008).

CHAPTER 4
POTENTIAL APPLICATIONS OF FRACTAL THEORY IN TRANSPORTATION

This chapter discusses potential applications of fractal theory in transportation. Fractal theory could potentially be applied to the following traffic management strategies:

1. managed lanes,
2. ramp metering,
3. crash analysis,
4. parking management, and
5. travel time reliability studies.

In the following sections, each of the five strategies is first introduced. Then, the data needed for performing fractal analysis are discussed. The data elements that could be predicted are also identified. Finally, the chapter concludes with the summary of each strategy.

4.1 Managed Lanes

"Managed lanes" is a term that refers to any lane or corridor that controls its usage by vehicle type, tolling, or access management. These lanes are typically separated from the general purpose lanes. The following are the four common types of managed lanes:

- High occupancy vehicle (HOV) lanes.
- Congestion priced lanes (High occupancy toll or HOT lanes).
- General express toll lanes.
- Exclusive lanes (e.g., bus or truck lanes).

HOV lanes allow those vehicles with two or more occupants (e.g., buses and carpools) to use free-tolled lanes that are separated from the main traffic or general purpose lanes during times of high congestion. HOV lanes provide travel time and fuel savings, travel time reliability, and an incentive for ridesharing or carpooling. They also reduce riders' stress.

HOT lanes allow single-occupant vehicles access HOV lanes by enforcing toll payment via toll booths, automatic license plate recognition, or electronic toll collection systems. The toll amount varies based on freeway congestion conditions, which increases with the increase in traffic volume and density. This process is known as "congestion pricing" or "dynamic tolling" and mainly aims at minimizing traffic congestion in general purpose lanes.

Unlike HOV and HOT lanes, the general use express toll lanes charge all vehicles, regardless of occupancy. Exclusive lanes restrict lane access to only buses, trucks, or other slower moving vehicles and allow higher speeds in adjacent lanes for passenger cars. For this, exclusive bus lanes provide an added travel time benefit for using transit, reduce the effects of congestion felt by transit passengers, and decrease waiting time for passengers. This helps increase both mobility and safety.

Since congestion pricing is a dynamic strategy that adjusts toll amount with the variation in traffic flow and/or density, the "managed lanes" strategy is a subject of a random process as a

result of freeway traffic flow and density fluctuations during hours of the day, days of the month, and on different seasons. Furthermore, the rate of emission of vehicles near loop detectors can play a role in fractal analysis since higher emissions could be a result of lower average speed, and could be an estimator of traffic congestion. The following data elements are recommended to be collected for performing the fractal analysis, as well as for prediction purposes. Note that prediction is intended to predict the same data elements for different time periods. Depending on data availability, fractal analysis can be performed by time of the day, day of the month, or month of the year.

1. Traffic flow, density, speed, occupancy, and travel time just upstream and downstream of the loop detector of interest.
2. Traffic flow, density, and speed up to 3 miles upstream and downstream of the detector of interest.
3. Emission rate upstream and downstream of the detector of interest.

4.2 Ramp Metering

Ramp metering is a common ITS strategy to reduce traffic congestion on freeways. Ramp meters are traffic signals placed on on-ramps to control when and how many vehicles can enter the freeway (Gayah, 2006 and Haleem, 2007). As documented in Gayah (2006) and Haleem (2007), the earliest use of ramp metering was on the Eisenhower Expressway (I-290) in Chicago, Illinois in 1963, where a police officer directed traffic onto the freeway and one vehicle was only allowed to enter the mainline freeway at a time. The use of ramp metering strategy has progressed a lot in the last decade throughout the United States and worldwide. For example, in the United States, ramp metering was implemented in Minnesota, California, New York, Washington, and Florida. Outside the United States, ramp metering is implemented in the Netherlands, France, Germany, and Scotland.

Originally, ramp metering signals were pre-timed and they allowed vehicles onto the freeway at a constant green time (Gayah, 2006). However, nowadays, actuated signals are used that take into account traffic flow conditions on the mainline freeway to determine the green time needed for the ramp meter. Actuated signals include two main algorithms, local and coordinated. Local or uncoordinated ramp metering takes into account traffic conditions near the ramp that is being metered and the metering time of a specific ramp is independent of the time at another ramp. On the other hand, coordinated ramp metering requires the metering time of a particular ramp to be based on traffic data from both nearby and farther locations within the corridor. In other words, the green time for every ramp in the corridor is dependent on the adjacent network.

One of the most common uncoordinated ramp metering algorithms is ALINEA. The ALINEA algorithm calculates the metering rate for a particular on-ramp on the freeway by using only occupancy measurements taken from the nearest loop detector downstream of the on-ramp of interest. If the occupancy is higher than a pre-determined value, the metering time rate along the on-ramp is reduced to provide additional time for congestion to dissipate. On the other hand, a popular coordinated ramp metering algorithm is the Zone algorithm, which aims to balance the traffic volume entering the zone with the traffic leaving the zone. In this algorithm, the freeway is broken up into small sections, such as three-mile long sections. These sections are termed as

zones, and all on-ramps within each zone are coordinated together. The upstream area of each zone is typically a free flow area, whereas the downstream area is a bottleneck area.

Both ALINEA and Zone ramp metering algorithms might be promising for analysis using fractal theory since these two strategies use actuated signals. Moreover, the two algorithms are examples of a stochastic random process as a result of freeway traffic flow, density, and speed fluctuation as a result of temporal variations. For integrating ramp metering algorithms in fractal analysis for traffic management, the following sets of data are recommended for investigation and prediction purposes.

1. Traffic flow, speed, density, and occupancy just downstream of the ramp of interest.
2. Traffic flow, speed, density, and occupancy up to 3 miles upstream and downstream of the ramp of interest (for the Zone algorithm).
3. Traffic flow and metering time/rate at the on-ramp of interest.

4.3 Crash Analysis

Crash data required for performing safety analysis are characterized by the random fluctuation of crash frequency over multiple years. In addition, during the same day, crash frequency varies by time of the day, for example, morning and afternoon peak periods are quite different from non-peak periods. Moreover, crash frequency through days of the month varies randomly. All these raise concerns about whether crash data exhibit fractal characteristics.

Multiple years of crash data and roadway characteristics data are required to perform fractal analysis. More specifically, daily and annual trends in crash frequencies and rates are recommended for investigation. It is believed that the existence of fractal characteristics in crash data could be beneficial in predicting crashes at specific locations, i.e., intersections and segments. It would provide preliminary perspective about locations which might continue to show an increase in crashes or might experience reduction in crashes in the future. This information would be useful for better allocation of safety resources.

4.4 Parking Management

According to the Parking Management Strategies Toolbox (2013) of the City of Laguna Beach, California, parking management refers to strategies that result in a more efficient use of parking resources. The main step toward effective parking management is identifying the cause of parking concerns affecting residents, business owners, and visitors. Several steps must then be undertaken to ensure that parking management strategies are supported by the stakeholders. Essential parking management strategies include, but are not limited to:

- *Shared parking*: parking serves multiple users during various peak parking demands.
- *Accurate and flexible standards*: parking requirements are adjusted based on factors, such as geographic location, land use, demographics (e.g., driver's age), and demand hour.
- *Parking maximums*: an upper limit for the period of parking is enforced.
- *Remote parking and shuttle service*: parking is provided away from the destination, and shuttle buses are used to move to the required destination, e.g., in airports.

- *Walking and cycling improvements*: walking and cycling are facilitated to and from parking facilities.
- *Increasing capacity of parking facilities*: parking spaces can be increased with major land construction.
- *Parking pricing*: money charge for parking should be used efficiently to increase revenue at an acceptable level.
- *Financial incentives*: travelers are offered financial benefits for reducing their automobile trips, e.g., providing parking cash-outs or choosing cash money in the case of subsidized parking.
- *Unbundle parking*: parking spaces are sold or rented separately from the building space.
- *Bicycle parking facilities*: sufficient and secure bicycle parking spaces should be provided.
- *Improve enforcement and control*: parking spaces should be frequently monitored and trespassing and loitering should be prevented.
- *Overflow parking plans*: efficient parking procedures in special events should be provided, such as commencements and official celebrations.
- *Addressing spillover problems*: undesirable parking at neighboring businesses' parking lots should be efficiently monitored in situations of increased demand.
- *Improving dissemination of parking information*: accurate information on parking availability and pricing should be provided while using maps, signs, brochures, and internet.
- *Smart parking technology*: real-time guidance to available spaces and use of smart payment technology.

Due to the random arrivals and stochastic demand patterns at parking facilities throughout a typical day and from day to day, fractal behavior might exist in the parking management strategy. The recommended data for collection for performing fractal analysis and for prediction purposes are:

1. Number of occupied parking spots on weekdays and weekends.
2. Rate of arrivals per hour on weekdays and weekends.
3. Average parking time per day and average percentage of capacity utilization per day for the entire year.

4.5 Travel Time Reliability

Travel time reliability is an emerging topic in the context of traffic operations and safety. As indicated by the Federal Highway Administration (FHWA) 2006 report, "*Travel Time Reliability: Making It There on Time, All the Time*", travel time reliability is defined as "the consistency or dependability in travel times, as measured from day to day or across different times of day." Reliability can be considered in a real-time sense, where a trip being taken is compared to a preset standard time (Lyman and Bertini, 2008). Reliable travel times are related to improved safety, enhanced efficiencies for freight transport, and improved quality of life for road users in terms of less delay and frustration. Unreliable travel times are mainly caused due to either recurring congestion (e.g., bottlenecks and poor traffic signal timing) or nonrecurring congestion (e.g., traffic incidents, weather condition, work zones, and special events).

According to Lomax et al. (2003), travel time reliability is a measure of the amount of congestion experienced by roadway users. Measures of travel time reliability attempt to quantify the variability in travel time across different days and months and the variability across different times of day. A network that provides a high level of service exhibits a high level of travel time reliability. Furthermore, at the same congestion level, a facility with a higher measure of reliability is the one least influenced by events. There are several measures of travel time reliability; among them are five main measures as follows (FHWA, 2006; Lyman and Bertini, 2008; Wakabayashi, 2010):

1. *90th or 95th percentile travel time*: 90th and 95th travel time values based on travel time distribution during the most congested days.

2. *Travel time index*: estimated as mean travel time divided by free-flow travel time, it is the mean time it takes to travel during peak hours compared to free-flow conditions.

3. *Buffer index*: it is the difference between 95th percentile travel time and mean travel time, divided by mean travel time. For example, a buffer index of 30% means that, for a 20-min average travel time, a traveler requires an additional 6 minutes (0.3×20 minutes) to reach destination.

4. *Planning time index*: it is estimated as the 95th percentile travel time divided by free-flow travel time. For example, a planning time index of 1.5 means that, for a 20-min trip in free-flow traffic, the total time that should be planned for the trip with a planning time index of 1.5 is 30 minutes (1.5×20 minutes).

5. *Percent or degree of congestion*: it is the percent of days the mean speed falls below a predetermined speed.

In addition to the aforementioned performance measures, several other measures are available to quantify travel time reliability, such as standard deviation and coefficient of variation. However, these measures are discouraged, as they are not easily related to everyday commuting experiences nor readily understood by non-technical audiences (FHWA, 2006; Lyman and Bertini, 2008).

Due to the stochastic characteristics of travel time reliability measures on different facilities, e.g., freeways and major arterials, it is hypothesized that travel time reliability measures exhibit fractal behavior. Again, fractal analysis can be performed for the same day or for different days of the month or year. The recommended data for collection for performing fractal analysis are:

1. Free-flow travel time, mean travel time, 90th and 95th percentile travel time, and mean travel speed upstream of the station of interest.
2. Free-flow travel time, mean travel time, 90th and 95th percentile travel time, and mean travel speed downstream of the station of interest.

Relevant measures for prediction purposes from the fractal analysis include the aforementioned elements of data on different time periods in addition to travel time reliability measures, which include:

1. Travel time index, buffer index, planning time index, and degree of congestion upstream of the station of interest.
2. Travel time index, buffer index, planning time index, and degree of congestion downstream of the station of interest.

4.6 Summary

This chapter discussed potential applications of fractal theory in the following traffic management strategies: managed lanes, ramp metering, crash analysis, parking management, and travel time reliability analysis. For each application, data required for collection, as well as data to be predicted were identified. Due to the random fluctuations in crash frequency, traffic flow, and speed data, and pattern of arrivals at parking lots, fractal behavior was hypothesized to exist in all the five aforementioned traffic management strategies.

Since ramp metering and managed lane strategies are commonly implemented along freeways, most of the recommended data for collection and prediction included traffic flow, speed, density, and occupancy upstream and downstream the ramp/station of interest. For crash analysis, fractal characteristics could be investigated in temporal trends in crash frequency and crash rate. For parking management strategy, data recommended for collection and prediction included number of occupied parking spots through the day, number of arrivals through the day, and average parking time per day. For analyzing travel time reliability, data recommended for collection included mean travel time, free-flow travel time, 90th and 95th percentile travel time, and mean travel speed. Predicted measures from the reliability analysis could include common performance measures such as travel time index, buffer index, and degree of congestion.

CHAPTER 5
ASSESSMENT OF FRACTAL CHARACTERISTICS IN CRASH AND TRAFFIC DATA

This chapter uses crash and traffic data from Florida to detect the existence of fractal characteristics. Daily and annual trends in crash frequency are explored. Trends in daily crash frequency for different facility types (e.g., freeways, arterials, collectors, etc.) are also investigated. Additionally, fractal characteristics in annual crash rate trend at ten randomly-selected signalized intersections are investigated. Further, volume, speed, and occupancy data are used to detect the existence of fractals. Finally, the chapter concludes with the summary of findings.

5.1 Fractal Characteristics in Crash Data

This section focuses on the application of the fractal dimension and the Hurst exponent techniques, as introduced previously in Chapter 2, to detect the existence of fractal characteristics in Florida's crash data for the following:

1. annual crash frequency,
2. daily crash frequency,
3. daily crash frequency for different facility types, and
4. annual crash rate for intersections.

Statewide crash data from 1990 to 2011 were used to analyze trends in annual crash frequency, and five-year data from 2007-2011 were used to analyze the daily crash frequency trend for the entire state road network and for different facility types. The analysis was performed by extracting crash data from the Crash Analysis Reporting (CAR) system which were then merged with the most recent Roadway Characteristics Inventory (RCI) data (i.e., from 2011). To account for variations in the annual average daily traffic (AADT) between locations, fractal characteristics for the 1991-2011 annual crash rates (i.e., crash frequency normalized by entering traffic volumes) were investigated for ten randomly-selected signalized intersections.

5.1.1 Annual Crash Frequency

Figure 5-1 shows the annual crash frequency in Florida from 1990 to 2011. From the figure, it can be observed that there exists a random fluctuation of crash frequency over the years; hence, the data is considered to exhibit fractal behavior. Using the *fractaldim* package in **R** Software (*R* Project, 2013), fractal dimension was estimated using each of the 10 methods discussed in Chapter 2. The fractal analysis is based on relatively fewer data points (22 in total). Since the existence of fractal behavior in crash data needs validation using more data points, these results have to be used with caution.
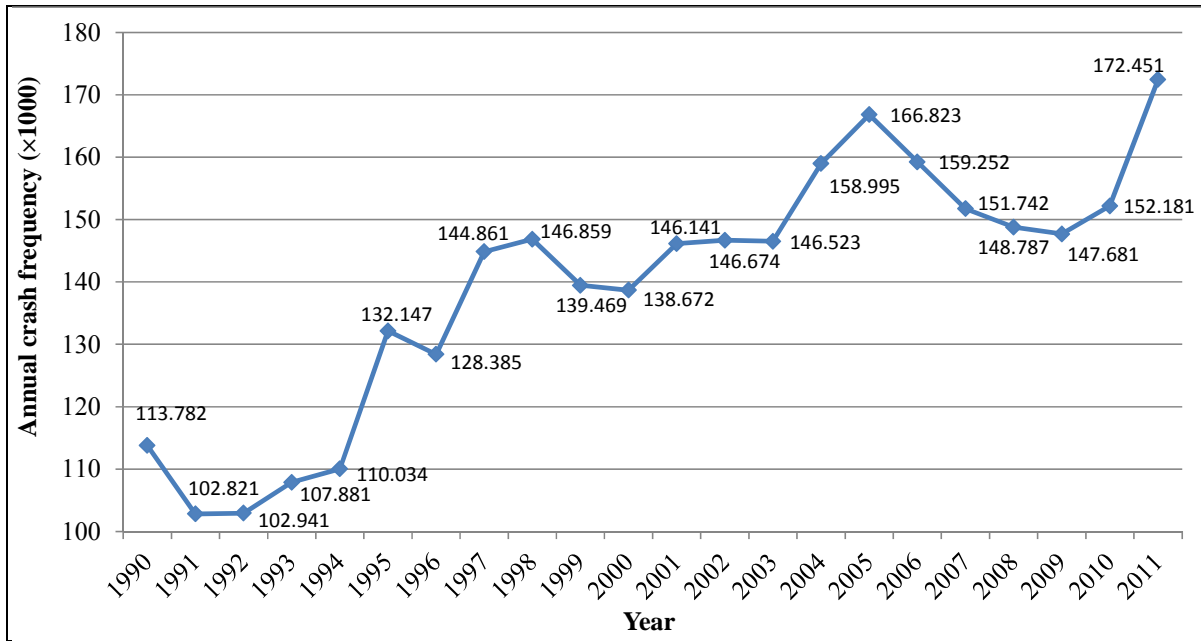
Figure 5-1: Annual Crash Frequency in Florida

Figure 5-2 shows the estimation of fractal dimension using each of the 10 fractal dimension methods. As shown in the plots, almost all the fractal dimension estimates (except from periodogram method) are ≥ 1, which reveals the existence of fractal behavior. The fractal dimension estimate using the DCT-II method was the highest at 2.01. Four methods, Hall-Wood, variogram, genton, and wavelet, yielded relatively high fractal dimension estimates that are close to 1.50. The incr1 and madogram methods also yielded a relatively high fractal dimension estimate of 1.44 and 1.33, respectively. Of all the methods, periodogram method yielded the lowest fractal dimension estimate of 0.95.

The Hurst exponent, H, was used to detect the existence of fractal characteristics. Fractal behavior exists when $0 < H \leq 1$ and the higher the H value, the higher is the fractal behavior (Shang et al., 2007). Table 5-1 gives the estimates of H using the various methods in the *fArma* package in *R* Software for two temporal trends: crash frequency by day of month and year. The table reveals that the temporal variations in crash frequency exhibit fractal characteristics using the majority of methodological approaches. The periodogram and modified periodogram methods yielded H estimates greater than 1, implying that the fractal behavior does not exist in the data. The differenced aggregated variance method yielded an H estimate of 1. However, all the other methods yielded H estimates ≤ 1; of these, the variance of residuals method yielded the highest estimate.
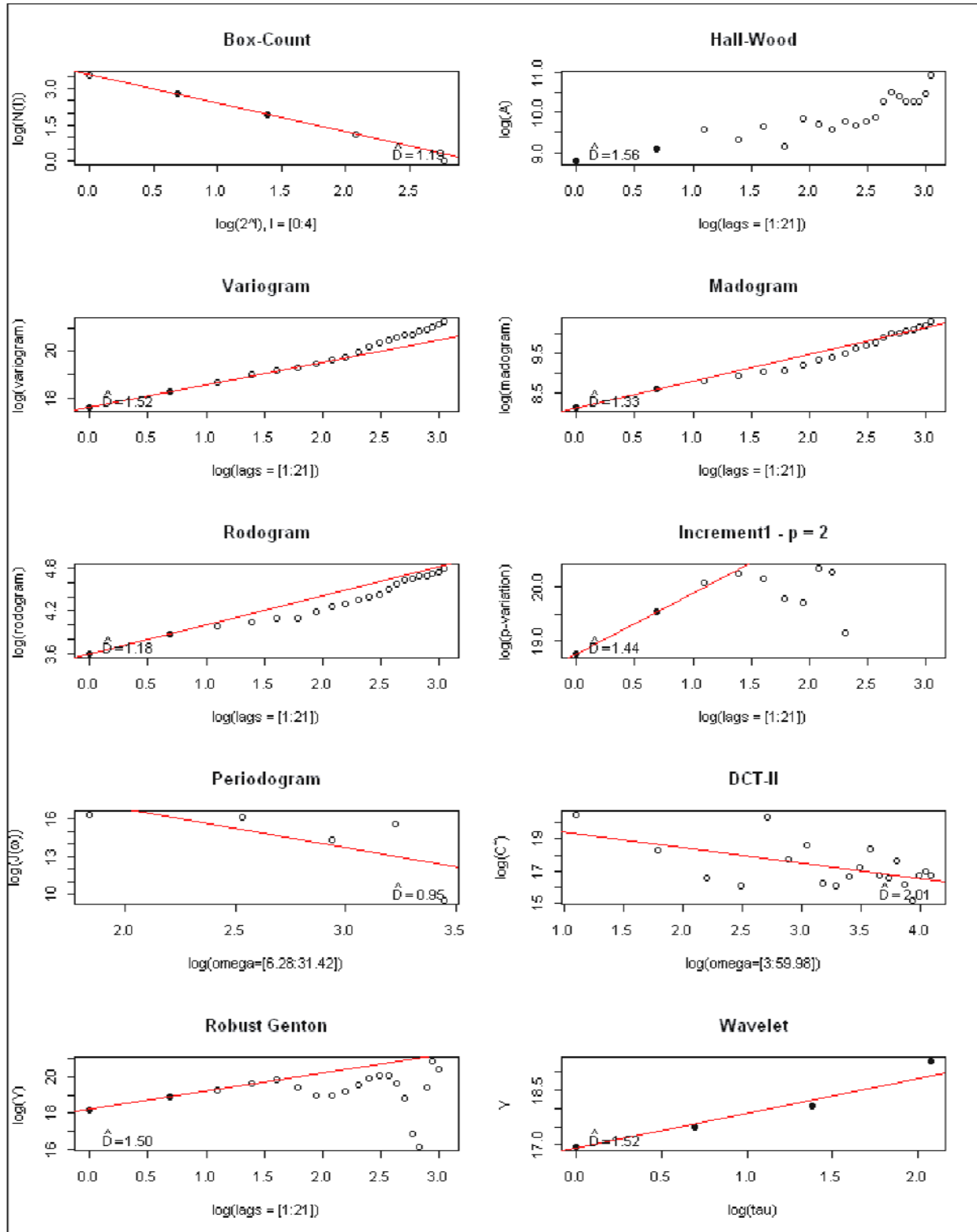
Figure 5-2: Fractal Dimension Estimation for Annual Trend in Crash Frequency

Table 5-1: Hurst Exponent Estimation Method by Type of Analysis

Hurst Exponent Method	Crash Analysis by Year	Crash Analysis by Day of Month
Higuchi	0.876	0.963
R/S	---[a]	0.867
Aggregated variance	0.747	0.837
Differenced aggregated variance	1.000	0.990
Aggregated absolute moment	0.883	0.849
Variance of residuals	0.901	0.689
Periodogram	1.575[b]	0.785
Modified periodogram	1.773 [b]	0.663

[a] Could not be estimated due to matrix singularity.
[b] H estimate is > 1, implying that fractal behavior does not exist.

5.1.2 Daily Crash Frequency

Figure 5-3 shows the trend in daily crash frequency from 2007 to 2011. The figure also displays the overall trend line. The data might exhibit fractal behavior due to its random fluctuations. Figure 5-4 shows the estimations of the fractal dimension using each of the 10 fractal dimension methods. As shown in the plots, all the fractal dimension values are > 1, which confirms the existence of fractal behavior. Furthermore, fractal dimension estimates of periodogram, wavelet, and DCT-II methods were > 2. This shows that the daily crash frequency demonstrates high fractal and self-similarity characteristics. The highest estimate was provided by the periodogram method, while the lowest was provided by the incr1 method. It can be also seen that the variogram, madogram, Hall-Wood, box count, and genton yielded close fractal dimension estimates between 1.70 and 1.80. Again, the wavelet method was one of the methods that yielded a relatively high fractal dimension estimate (the second highest).

Again, as given in Table 5-1, the Hurst exponent, H, was used to detect the existence of fractal characteristics. From the table, it is observed that all methods yielded an H estimate in the range of 0 and 1, which confirms the existence of fractal behavior. The two closest H estimates to 1 were generated from the differenced aggregated variance (0.990) and Higuchi (0.963) methods. Thus, the two methods were associated with the highest self-similarity characteristics.
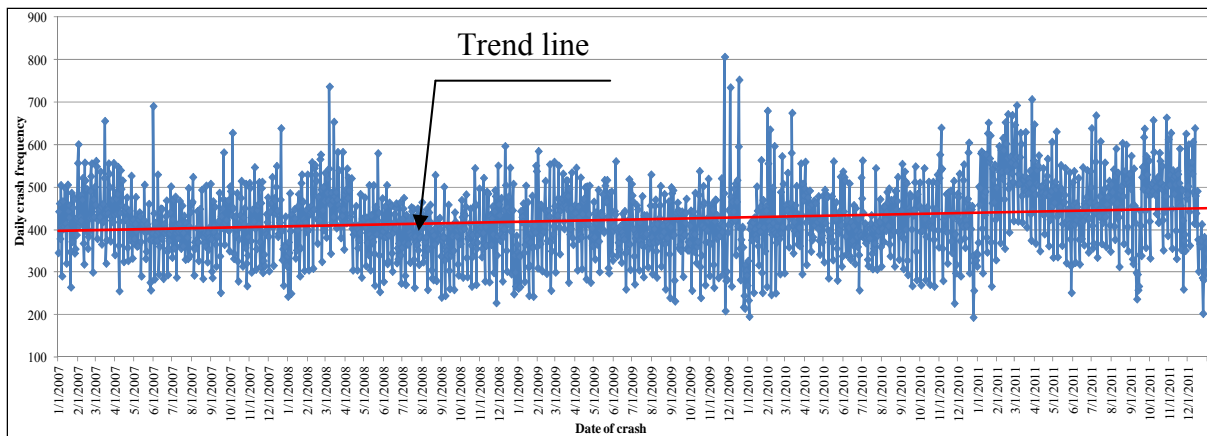
Figure 5-3: Daily Crash Frequency Trend
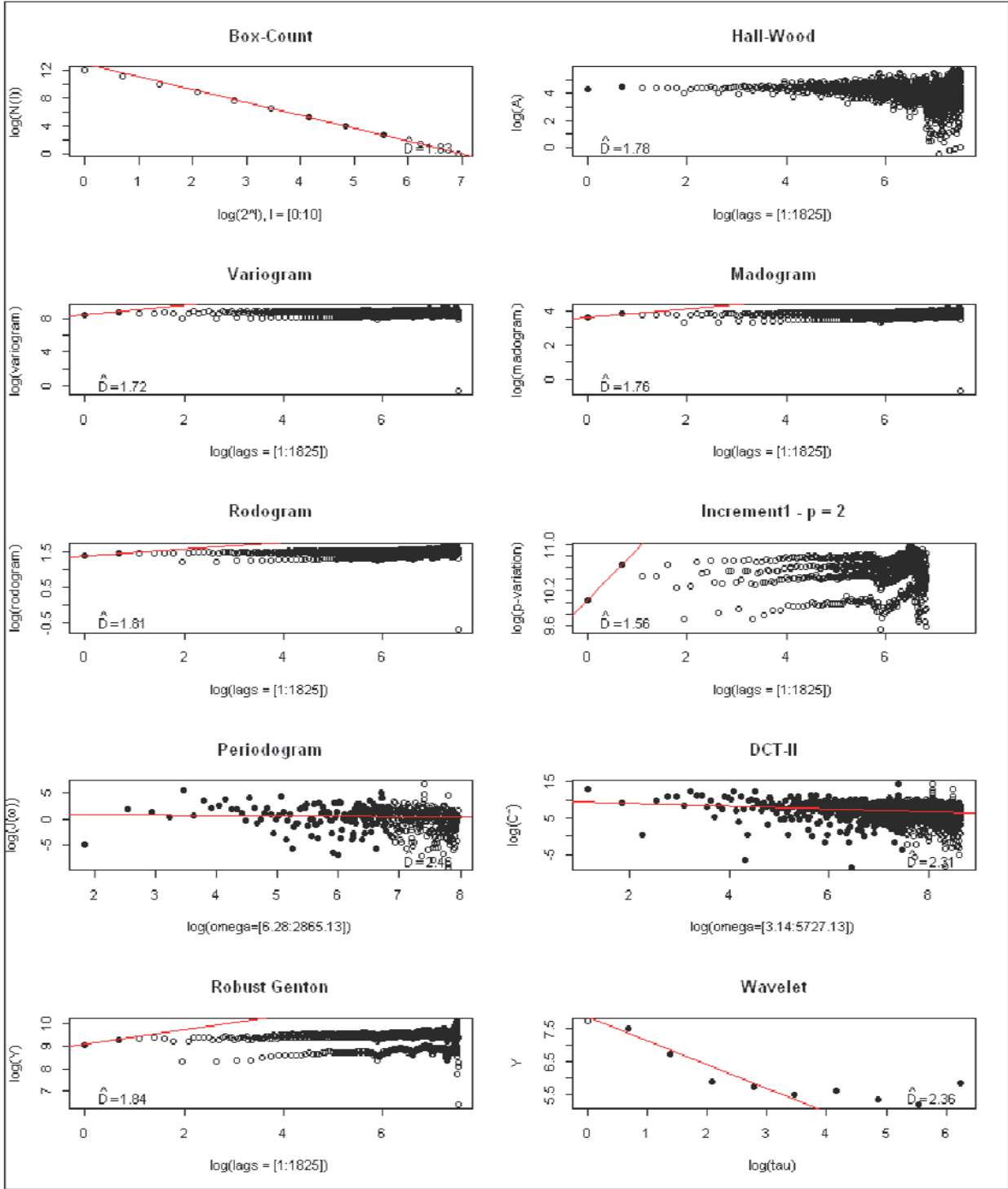
Figure 5-4: Fractal Dimension Estimations for Daily Trend in Crash Frequency

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5.1.3 Daily Crash Frequency for Different Facility Types

This section focuses on investigating the existence of fractal characteristics in daily crash data at different facilities such as freeways and arterials. The main objective of this analysis is to explore the extent of fractal behavior exhibited by each facility type. Table 5-2 shows the different facility types along with the number of crashes that occurred in the five-year period from 2007 to 2011.

Table 5-2: Five-Year Crash Frequency by Facility Type

Facility Type	Five-Year Crash Frequency
Rural Freeways and Expressways	17,856
Rural Major Principal Arterials	28,829
Rural Minor Arterials	11,677
Rural Collector and Local Roadways	1,239[+]
Urban Freeways and Expressways	128,480
Urban Major Principal Arterials	380,109
Urban Minor Arterials	142,575
Urban Collector and Local Roadways	6,054
Unknown	56,023
Total	**772,842**

[+] Daily crash frequency ranged between one and two crashes; hence, rural collector and local roadways were excluded from analysis.

As seen in Table 5-2, urban facilities experienced more crashes than rural facilities. Furthermore, urban major arterials experienced the highest crash frequency, followed by urban minor arterials and urban freeways. This could be because major and minor arterials have comparatively more miles of roadways than freeways, which increases the likelihood of a crash. The facility type of 56,023 crashes could not be identified and was excluded from further analysis. Rural collectors and local roads were also excluded since they experienced very few crashes. Finally, 715,580 crashes were included in the analysis.

Fractal Dimension Analysis

Table 5-3 gives fractal dimension estimates for each of the seven facility types. It is observed that all estimates are greater than 1, revealing fractal behavior. Particularly, periodogram, DCT-II, and wavelet methods yielded the highest fractal dimension estimates, which are greater than 2. Box plots are used to visually compare the fractal dimension estimates of different facility types. Figure 5-5 gives the general description of a box plot.

As shown in Figure 5-5, a box plot includes five critical levels. The lowest level is section 1-1, which represents the minimum value. Section 2-2 represents the value at the 25th percentile. Section 3-3 is the median value or the 50th percentile value, which is then followed by the 75th percentile value at section 4-4. Finally, section 5-5 represents the maximum possible value. Data range is represented by the distance between the maximum and minimum values (i.e., between sections 1-1 and 5-5).

Table 5-3: Fractal Dimension Estimate by Facility Type

Fractal Dimension Method	Rural Freeways	Rural Major Arterials	Rural Minor Arterials	Urban Freeways	Urban Major Arterials	Urban Minor Arterials	Urban Collectors
Box count	1.65	1.77	1.86	1.81	1.84	1.84	1.81
Hall-Wood	1.89	1.89	1.93	1.79	1.77	1.81	1.91
Variogram	1.87	1.94	1.94	1.77	1.74	1.79	1.96
Madogram	1.85	1.94	1.92	1.74	1.79	1.82	1.95
Rodogram	1.82	1.92	1.91	1.73	1.82	1.84	1.93
Incr1	1.82	1.91	1.93	1.68	1.58	1.67	1.95
Periodogram	2.48	2.49	2.46	2.22	2.29	2.41	2.38
DCT-II	2.42	2.42	2.41	2.30	2.27	2.38	2.44
Genton	1.42	2.00	2.00	1.71	1.89	1.86	2.00
Wavelet	2.36	2.39	2.41	2.34	2.43	2.44	2.41
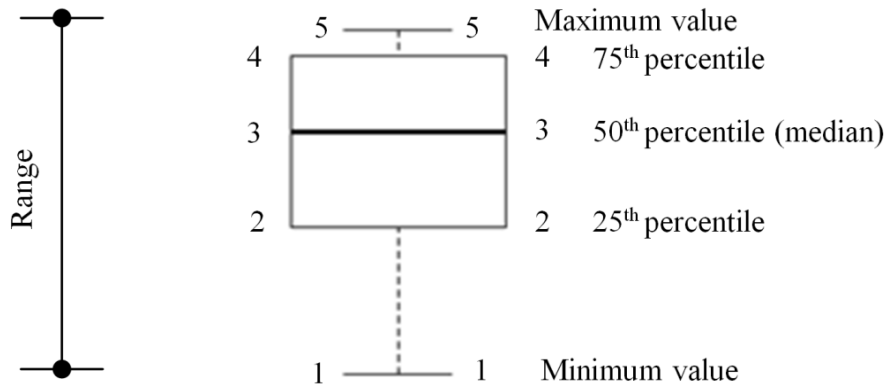
Figure 5-5: Box Plot Description

Figure 5-6 gives box plots of fractal dimension estimates using the ten fractal dimension methods. From the figure, it is observed that the periodogram, wavelet, and DCT-II yielded the highest fractal dimension estimates. Furthermore, the median fractal dimension value of the other seven methods is between 1.8 and 2, with a higher propensity to 1.8, which shows that these methods yielded consistent estimates.

Figure 5-7 compares fractal behavior of the daily crash frequency pattern along the seven facility types. Urban collectors and local roadways experienced the highest fractal behavior; however, their estimates were relatively close to that of both rural major and minor arterials. Rural facilities experienced higher fractal dimension estimates compared to their urban counterparts. Among the rural facilities, rural major and minor arterials experienced higher fractal behavior than rural freeways. Similarly, urban major and minor arterials experienced higher fractal behavior than urban freeways, which is consistent with the findings of Hu et al. (2010).
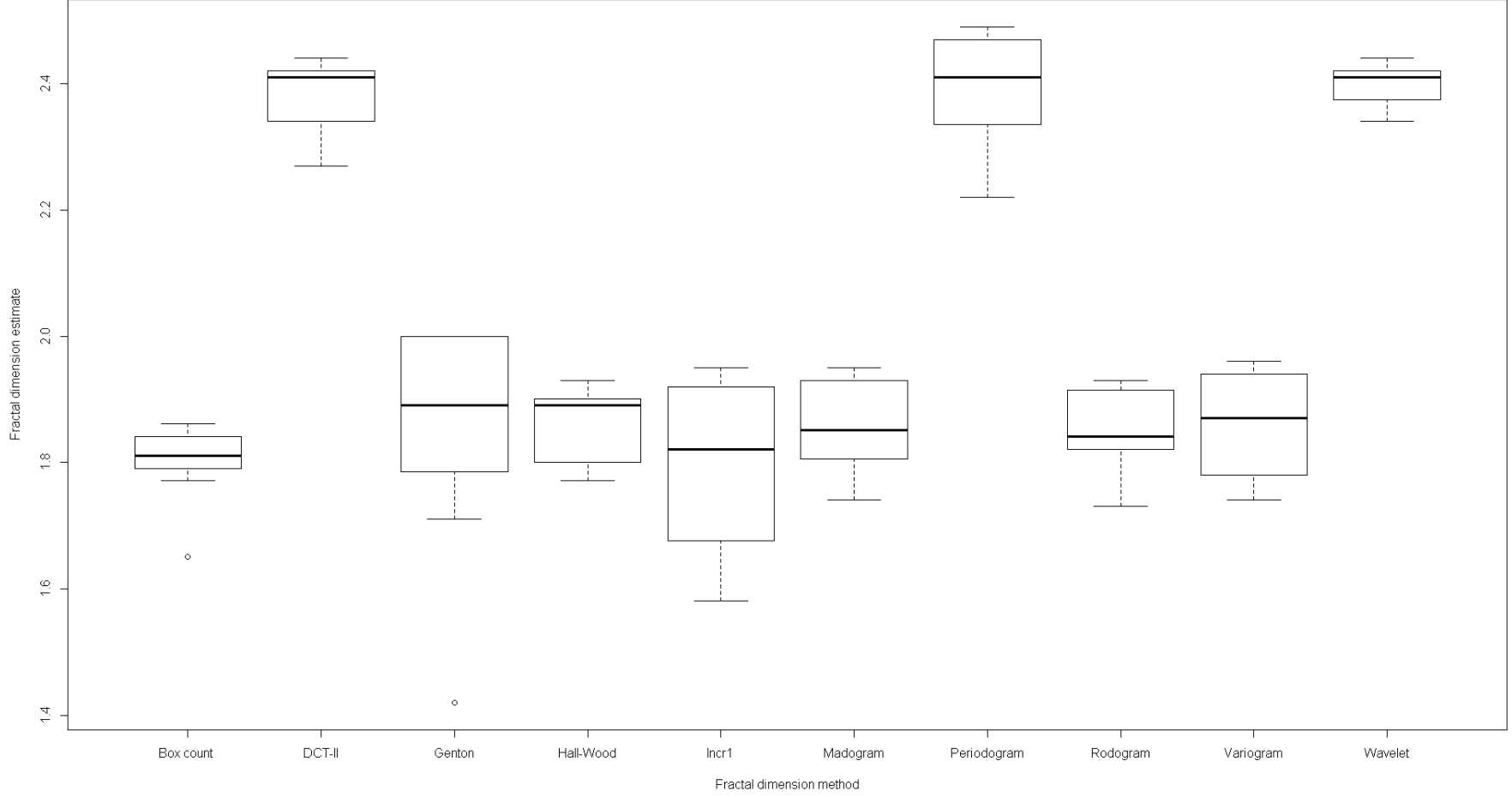
Figure 5-6: Box Plot of Fractal Dimension Estimate by Estimation Method
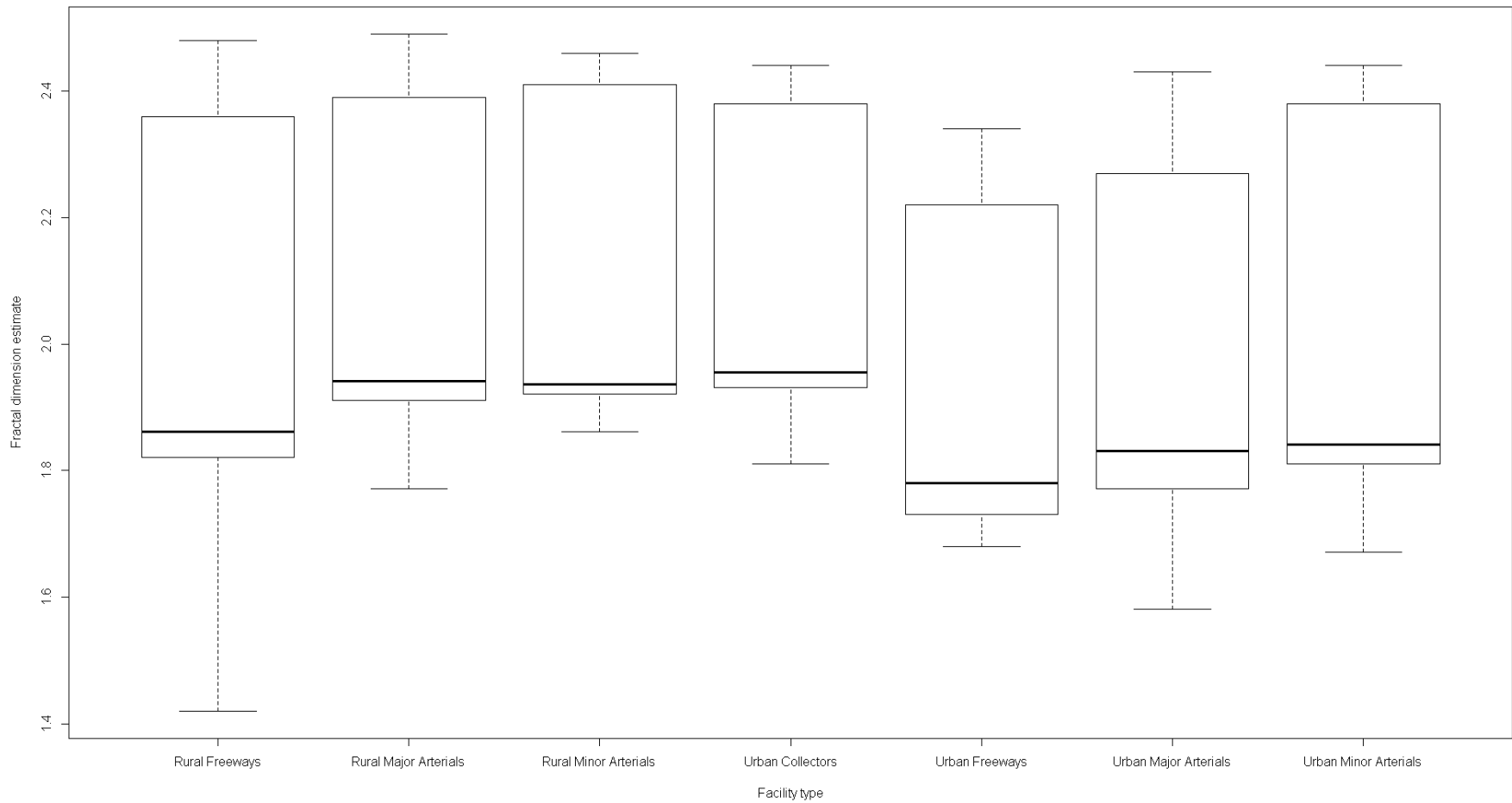
Figure 5-7: Box Plot of Fractal Dimension Estimate by Facility Type

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Hurst Exponent Analysis

Table 5-4 gives the Hurst exponent estimate, *H*, for each of the seven facility types using different estimation methods. It is observed that all estimates are between 0 and 1, which reveals fractal behavior. For each of the seven facility types, the Higuchi method yielded the highest *H* estimate, which was close to 1. This shows that the Higuchi method was associated with the highest self-similarity characteristics. Moreover, the periodogram, modified periodogram, and variance of residuals methods roughly yielded the lowest *H* estimates.

Table 5-4: Hurst Exponent Estimate by Facility Type

Hurst Exponent Method	Rural Freeways	Rural Major Arterials	Rural Minor Arterials	Urban Freeways	Urban Major Arterials	Urban Minor Arterials	Urban Collectors
Higuchi	0.971	0.964	0.967	0.967	0.967	0.967	0.956
R/S	0.694	0.652	0.543	0.817	0.767	0.850	0.735
Aggregated variance	0.666	0.713	0.553	0.635	0.750	0.708	0.790
Differenced aggregated variance	0.879	0.949	0.802	0.945	0.926	0.889	0.817
Aggregated absolute moment	0.696	0.754	0.577	0.687	0.800	0.744	0.836
Variance of residuals	0.644	0.640	0.599	0.660	0.673	0.645	0.605
Periodogram	0.647	0.588	0.582	0.747	0.700	0.608	0.573
Modified periodogram	0.616	0.597	0.578	0.733	0.686	0.642	0.519

Figure 5-8 gives box plots of *H* estimates using the eight *H* methods. It is observed that the *H* estimates from the Higuchi and differenced aggregated variance methods were highest, revealing that these methods incorporated the highest self-similarity characteristics. Furthermore, the median *H* value of the aggregated absolute moment, *R/S*, and aggregated variance were between 0.7 and 0.8, with a higher propensity to 0.7. Moreover, unlike for the other methods, the majority of estimates for the Higuchi method were in the vicinity of the median estimate. Figure 5-9 gives box plots by facility type. In general, urban facilities experienced higher *H* estimates than their rural counterparts. More specifically, urban collectors had the highest median *H* estimate.
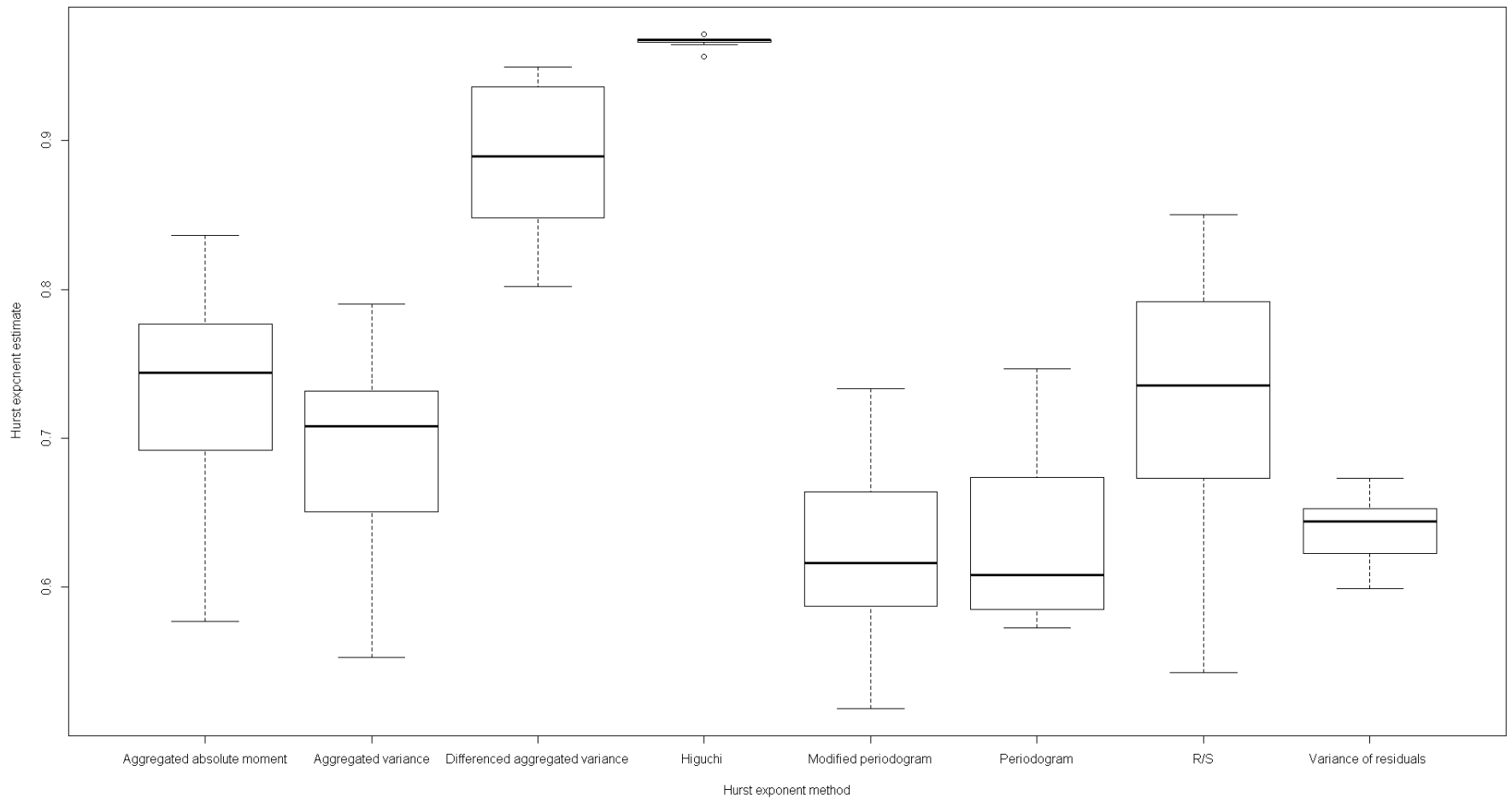
Figure 5-8: Box Plot of Hurst Exponent Estimate by Estimation Method
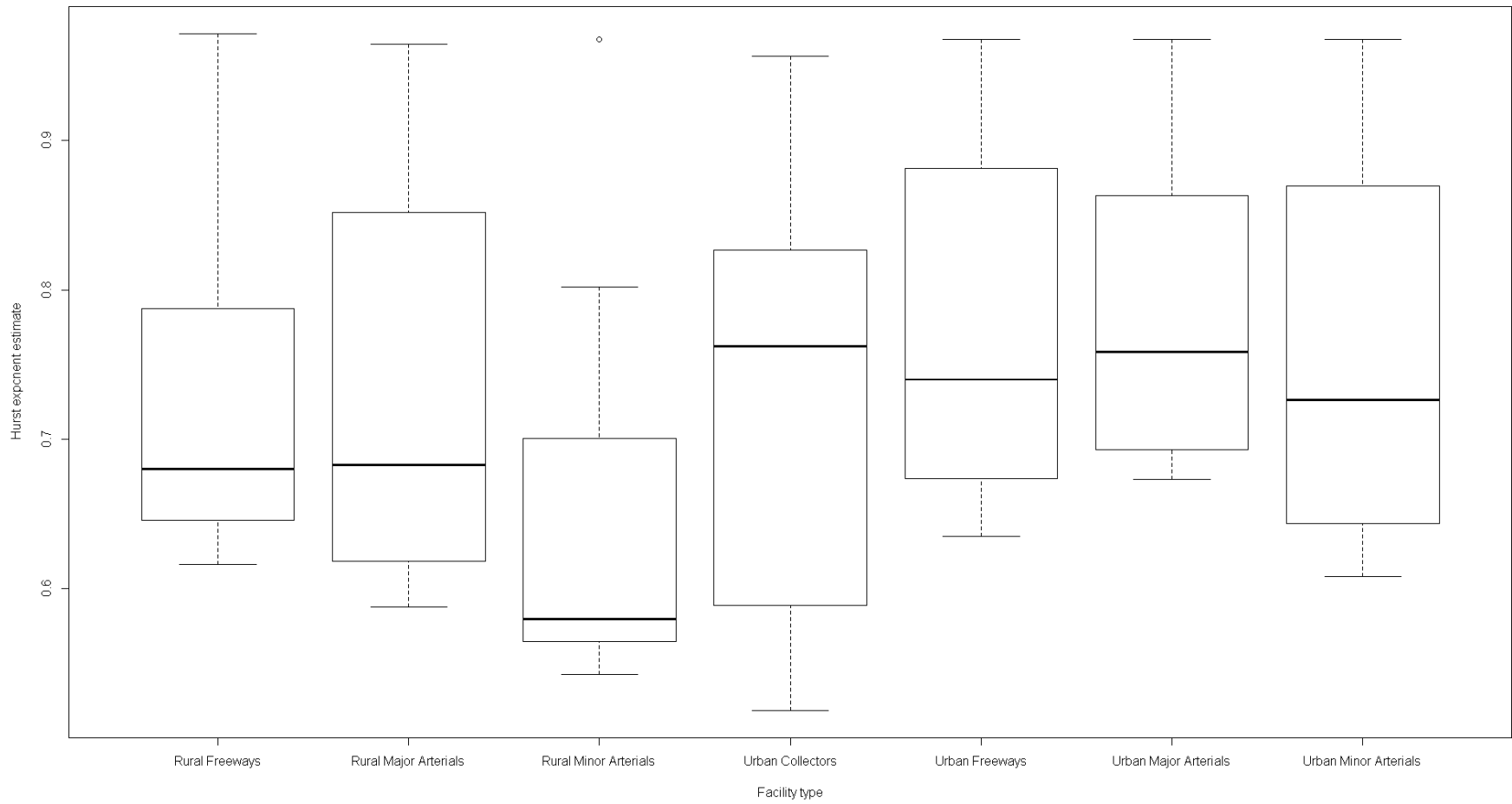
Figure 5-9: Box Plot of Hurst Exponent Estimate by Facility Type

This section investigates the existence of fractal characteristics in crash rates at signalized intersections. Crash rates are used to account for the changes in AADT at a location over time. As shown in Table 5-5, ten signalized intersections on the state network were randomly selected for this analysis. At each intersection, crash rates were calculated by normalizing the number of crashes by the million entering vehicles (MEV). A 250-ft buffer radius was used to identify intersection-related crashes. ArcGIS (ESRI, 2011) to extract these crashes at each intersection for the 21-year analysis period (1991 through 2011).

Table 5-5: Sample Intersections for Fractal Investigation of Crash Rate Trend

Int. ID	County	Milepost	Roadway ID1	Roadway ID2	1991-2011 Average Crash Rate (Crashes/MEV)
1	Alachua	13.649	26090000	26250000	1.17
2	Alachua	3.225	26250000	26070068	1.39
3	Bay	2.212	46060000	46001000	1.21
4	Brevard	20.999	70020000	70100000	0.77
5	Brevard	0.000	70012000	70180000	0.37
6	Broward	8.782	86020000	86130000	0.89
7	Hillsborough	5.685	10010000	10120000	1.79
8	Miami-Dade	5.773	87140000	87080001	0.76
9	Miami-Dade	1.477	87170000	87034000	1.39
10	Miami-Dade	0.638	87220000	87027000	0.75

Existence of fractal characteristics in annual crash rates at signalized intersections was investigated using the following two types of analysis: trend in annual crash rate and trend in three-year moving average crash rate. The random fluctuations in annual crash frequency were addressed (i.e., smoothened) using the moving average method. For each intersection, the three-year moving average crash rate was calculated by taking the average value of crash rate for three years. For example, the three-year moving average crash rate for the year 1993 was the average value of the crash rates for the years 1991, 1992, and 1993. A sample plot of the annual and three-year average trend for Intersection ID 10 in Miami-Dade County is shown in Figures 5-10(a) and (b), respectively. It is observed that both plots show a long-term decline in crash rate trend.

Table 5-6 gives the results of the fractal characteristics investigation at each of the ten intersections for both annual and three-year moving average crash rate trends. Similar to earlier sections, the fractal dimension (using ten methods) and the Hurst exponent (using seven methods) techniques were used in to investigate the existence of fractal characteristics. However, results from the modified periodogram method, one of the methods used to estimate Hurst exponent, are not included in the table since the *H* estimate could not be estimated for a few intersections due to matrix singularity. From the table, it is observed that the annual crash rate trend, in general, exhibited relatively more fractal characteristics than the three-year average trend. For all the ten intersections, most of the *H* estimates for the annual trend analysis were

between 0 and 1. Again, most of the fractal dimension estimates were above 1 when compared to the three-year average analysis.

It is also observed that the H7 method, the periodogram method, mostly yielded H estimates more than 1 for the majority of intersections for both the analyses (i.e., annual crash trend and moving average crash trend). In the three-year moving average analysis, the three intersection IDs, 4, 6, and 9 did not exhibit strong fractal behavior using three of the Hurst exponent methods, aggregated absolute moment, variance of residuals, and periodogram. However, they exhibited strong fractal behavior using the ten fractal dimension methods. This could reveal that these intersections generally experienced fractal characteristics while considering the majority of methods in both techniques combined.

(a) Annual Trend (1991-2011)
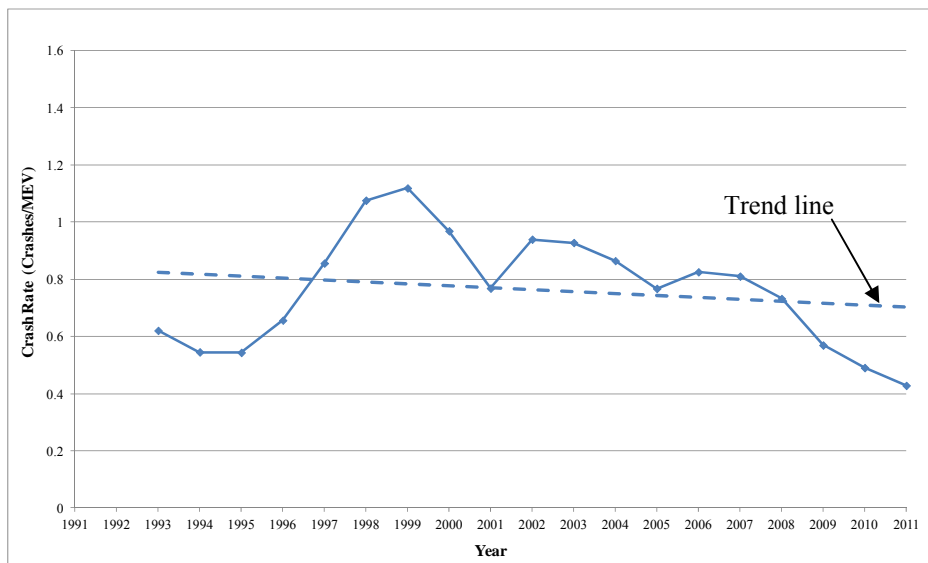
(b) Three-Year Average Trend (1993-2011)

Figure 5-10: Crash Rate Trend at Intersection 10 in Miami-Dade County

Table 5-6: Fractal Characteristics Investigation of Crash Rates at Intersections

Trends in Annual Crash Rate

ID	County	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	Fractal?
1	Alachua	1.49	1.96	1.95	2.01	2.04	2.00	2.70	2.85	2.18	1.94	0.71	0.56	0.86	0.49	0.94	0.49	1.48	Yes
2	Alachua	1.22	2.14	1.62	1.56	1.51	1.64	3.28	1.76	2.11	1.66	0.67	0.65	0.76	3.28	0.96	1.36	1.86	Yes
3	Bay	1.40	1.43	1.42	1.45	1.51	1.39	2.88	1.35	1.72	1.58	0.59	0.62	0.71	2.00	0.98	1.78	1.11	Yes
4	Brevard	1.15	1.44	1.93	1.72	1.56	2.17	2.74	2.57	1.70	2.00	0.73	0.70	0.80	0.71	1.05	0.31	2.33	Yes
5	Brevard	1.58	1.73	1.99	2.03	2.03	1.87	3.48	2.45	2.12	2.48	0.77	0.67	0.50	0.74	0.79	0.30	0.95	Yes
6	Broward	1.29	1.68	1.87	1.75	1.69	1.86	2.71	2.75	1.91	2.09	0.74	0.72	0.84	0.60	1.07	1.45	1.27	Yes
7	Hillsborough	1.54	1.76	2.12	2.05	2.00	2.33	1.57	2.51	2.03	2.25	0.77	0.88	0.82	0.47	1.11	1.07	0.72	Yes
8	Miami-Dade	1.35	1.97	2.07	1.82	1.65	2.15	0.14	2.52	1.91	2.26	0.82	0.88	0.79	1.21	0.91	0.37	1.54	Yes
9	Miami-Dade	1.49	1.54	2.02	2.02	1.97	2.22	1.55	1.97	2.09	1.94	0.79	0.60	0.89	1.69	0.96	0.18	2.17	Yes
10	Miami-Dade	1.69	1.67	1.69	1.53	1.35	1.54	2.75	2.23	1.52	2.07	0.85	0.69	0.52	0.82	0.82	0.45	0.94	Yes

Trends in Three-Year Moving Average Crash Rate

ID	County	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	Fractal?
1	Alachua	1.51	1.24	1.35	1.35	1.26	1.20	1.11	2.34	1.54	1.41	0.68	0.85	0.85	1.00	0.75	0.00	1.61	Yes
2	Alachua	1.00	1.04	1.15	1.12	1.11	0.73	1.64	1.14	1.29	1.25	0.68	0.81	0.26	1.85	0.57	1.63	2.30	Yes
3	Bay	0.94	1.17	1.09	1.11	1.14	0.39	0.01	0.39	0.86	1.34	0.60	0.78	0.61	0.54	0.66	1.19	1.35	Yes
4	Brevard	1.09	1.06	1.26	1.11	0.99	1.23	1.09	1.70	1.08	1.40	0.93	0.99	0.85	1.00	1.18	2.61	2.93	Yes
5	Brevard	1.58	1.83	1.59	1.53	1.50	1.50	1.20	1.71	1.45	1.81	0.70	0.92	0.72	0.26	0.99	0.55	0.92	Yes
6	Broward	1.28	1.21	1.44	1.33	1.21	1.56	2.87	2.03	1.94	1.39	0.93	0.39	0.95	---[a]	1.10	2.24	1.79	Yes
7	Hillsborough	1.22	1.38	1.43	1.37	1.33	1.48	1.54	1.67	1.13	1.61	0.79	0.95	0.61	0.03	1.00	1.37	0.27	Yes
8	Miami-Dade	1.29	1.62	1.57	1.75	1.90	1.73	2.81	2.04	1.90	1.58	0.74	0.69	0.84	---[a]	1.00	0.00	4.15	Yes
9	Miami-Dade	1.07	1.20	1.32	1.42	1.53	1.50	1.25	1.33	1.39	1.30	0.80	0.78	0.93	2.85	1.07	1.57	3.43	Yes
10	Miami-Dade	1.43	1.04	1.24	1.25	1.20	0.83	0.99	1.28	1.22	1.43	0.88	0.90	-0.02	0.37	0.28	2.00	1.79	Yes

FD1 = Box Count   FD2 = Hall-Wood   FD3 = Variogram   FD4 = Madogram   FD5 = Rodogram   FD6 = Incr1
FD7 = Periodogram   FD8 = DCT-II   FD9 = Genton   FD10 = Wavelet   H1 = Higuchi   H2 = *R/S*
H3 = Aggregated Variance   H4 = Differenced Aggregated Variance   H5 = Aggregated Absolute Moment
H6 = Variance of Residuals   H7 = Periodogram   H8 = Modified Periodogram

[a] Could not be estimated due to matrix singularity.

5.2 Fractal Characteristics in Traffic Data

This section investigates the existence of fractal characteristics in traffic volume, speed, and occupancy data. The raw data were extracted from the STEWARD (Statewide Transportation Engineering Warehouse for Archived Regional Data) website http://cce-trc-cdwserv.ce.ufl.edu/steward/index.html. The website includes 5-min, 15-min, and 1-hr traffic counts at different stations in the seven districts and the Turnpike in Florida since January 2007. The readings were recorded using loop detectors embedded in the roadway pavement. For investigating the existence of fractal characteristics, two stations were used along the US 1 route in Miami-Dade County, District 6. The first station ID is 630082 and is located along US 1 between SW 152 Street and SW 160 Street. The second station, with ID 630022, is located along US 1 between SW 27 Avenue and Bird Road (SW 40 Street).

Traffic volume (i.e., number of vehicles crossing the loop detector at a unit time), speed (i.e., actual speed of vehicles crossing the detector), and occupancy (i.e., percent of time the detection zone of the detector is occupied by vehicles) data for 15-min and 1-hr intervals were extracted from the website for May 2010 at the two stations of interest. Since the traffic behavior usually differs on weekends compared to weekdays, weekend days in the month of May 2010 were excluded from further analysis. Peak and off-peak periods in each of the 15-min and 1-hr intervals at both stations were analyzed. The 15-min analysis at each station included weekday periods from 3:30 pm to 7:45 pm, while the 1-hr analysis at each station included weekday periods from 4:00 pm to 8:00 pm.

Figure 5-11 shows the volume (in vehicles per 15-min), speed (in mph), and occupancy (%) patterns at both stations for a 15-min period from 4:30 pm to 4:45 pm in May 2010. From Figure 5-11(a), it is observed that the volume trends at both stations are very similar with a higher volume counts at Station 630082. The speed trends at both stations, as shown in Figure 5-11(b), are relatively different, with greater fluctuations at Station 630022. Furthermore, the speed trend at Station 630082 is relatively smooth and fluctuates from 35 mph to 45 mph. Similarly, the occupancy trends at both stations as shown in Figure 5-11(c) are different, with higher fluctuations at Station 630022. On the other hand, the occupancy trend at Station 630082 is smooth and ranges from 5% to 10%.
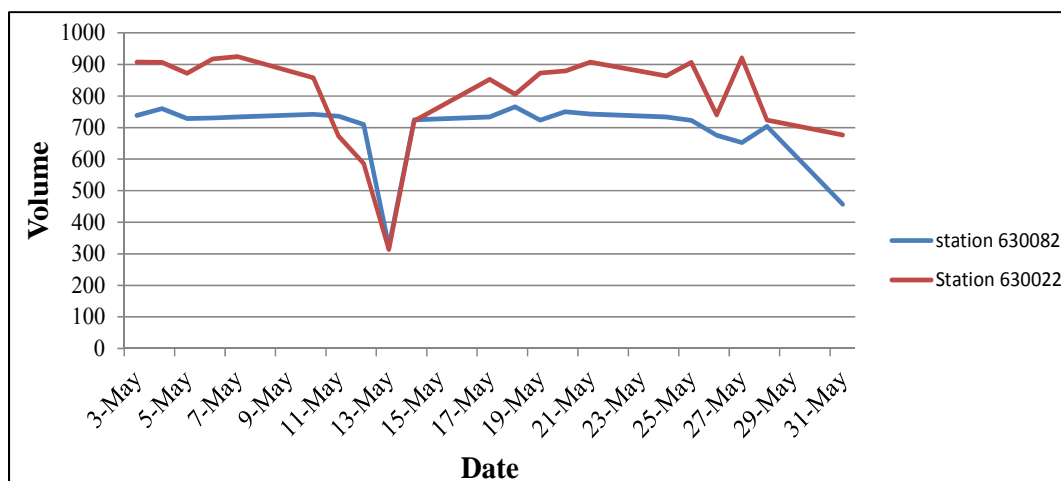
(a) Volume Trend
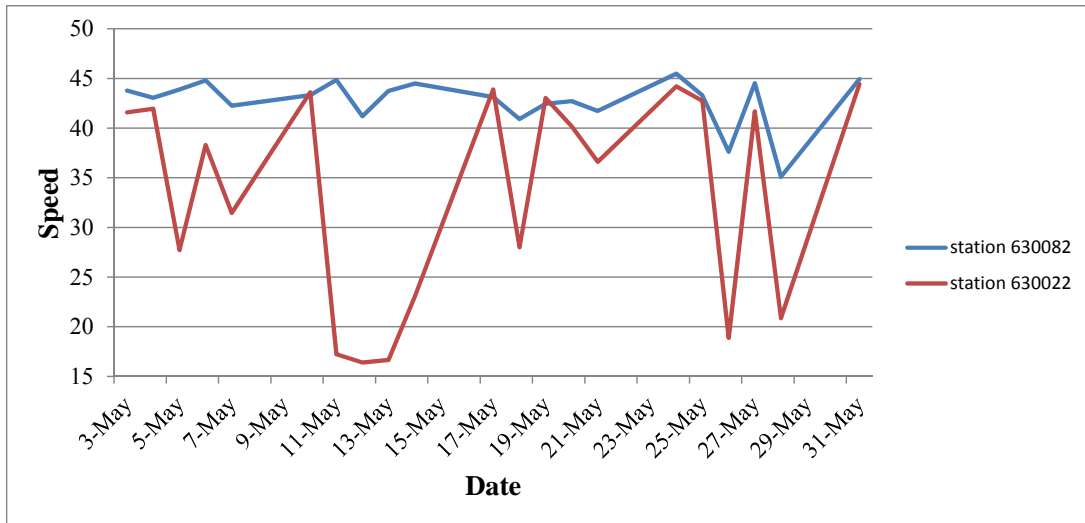
(b) Speed Trend
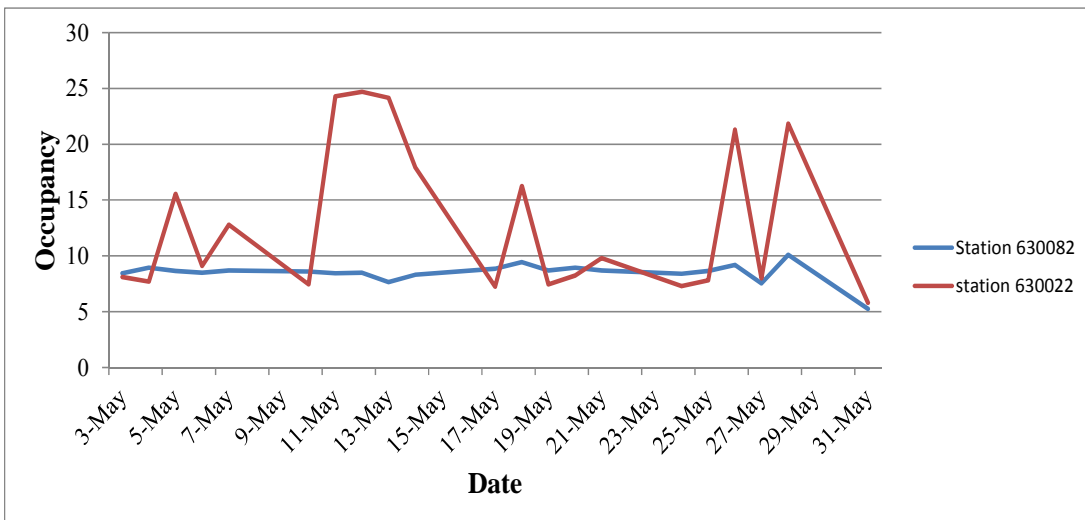
(c) Occupancy Trend

**Figure 5-11: 15-Min Traffic Trends at Two Stations in Miami-Dade County
(4:30 pm-4:45 pm)**

The results of the fractal characteristics investigation of volume, speed, and occupancy trends at each period of the 15-min and 1-hr intervals at Station 630082 are shown in Tables 5-7 through 5-9, respectively. Similarly, Tables 5-10 through 5-12, respectively, provide the results of the fractal characteristics investigation of volume, speed, and occupancy trends at each period of the 15-min and 1-hr intervals at Station 630022.

It is observed from the tables that, in general, the volume, speed, and occupancy patterns at both stations exhibited fractal characteristics since the *H* estimates were between 0 and 1 and the fractal dimension estimates were above 1. Furthermore, at both stations, the speed trend revealed stronger fractal behavior compared to the volume and occupancy trends for the same time period.

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At Station 630082, the fractal dimension estimates for speed data at 15-min and 1-hr intervals were higher than the corresponding estimates for volume and occupancy data. The H estimates for speed data were also higher than the corresponding estimates for each of the volume and occupancy fractal analysis and were consistently in the range of 0 and 1. The occupancy trend for the time period from 7 pm to 8 pm did not exhibit strong fractal behavior using six Hurst exponent methods, as the H estimates calculated using these methods were negative. However, the trend exhibited strong fractal behavior using the ten fractal dimension methods, where the estimates were greater than 1. Therefore, when a majority of methods in both techniques were considered, the occupancy trend was found to experience fractal behavior.

At Station 630022, the volume and occupancy trends from 7:30 pm to 7:45 pm did not exhibit strong fractal behavior from the Hurst exponent analysis, where most of the H estimates were negative. However, the trend exhibited strong fractal behavior using the ten fractal dimension methods, where the estimates were greater than 1. Therefore, when a majority of methods in both techniques were considered, the volume and occupancy trends were found to experience fractal behavior.

Table 5-7: Fractal Characteristics Investigation of Traffic Volume at Station 630082

Interval	PM Period	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	H8	Fractal?
15-min	3:30-3:45	1.25	1.96	2.03	1.99	2.06	2.15	1.09	2.06	1.78	2.17	0.81	0.88	0.02	-0.80	0.18	1.62	0.36	0.41	Yes
	4:30-4:45	1.19	3.46	1.94	1.90	1.80	1.88	3.48	2.42	1.69	2.19	0.74	0.77	0.49	0.84	0.77	0.67	0.27	0.24	Yes
	5:30-5:45	1.11	2.70	1.71	1.83	1.89	1.57	2.10	1.68	1.75	1.82	0.87	0.29	0.38	0.50	0.73	1.94	0.72	0.88	Yes
	6:30-6:45	1.49	3.29	2.28	2.30	2.35	2.52	2.06	2.14	2.40	2.06	0.87	0.91	0.48	0.05	0.82	1.29	0.40	0.64	Yes
	7:30-7:45	1.62	2.76	2.17	2.41	2.64	2.44	2.47	2.34	2.41	2.30	0.90	1.05	-0.10	0.46	0.19	084	1.02	0.72	Yes
1-hr	4:00-5:00	1.28	3.14	1.82	1.77	1.77	1.73	2.88	2.30	1.87	2.09	0.76	0.15	0.45	0.45	0.79	0.94	0.42	0.39	Yes
	5:00-6:00	1.30	2.97	1.77	1.98	2.17	1.62	2.65	1.60	2.54	1.85	0.86	1.25	0.32	0.39	0.73	1.68	0.61	0.74	Yes
	6:00-7:00	1.43	3.40	2.01	2.10	2.11	2.13	2.38	1.97	2.26	2.04	0.87	0.89	0.24	0.19	0.60	2.29	0.24	0.25	Yes
	7:00-8:00	1.50	2.04	1.93	1.65	1.38	1.77	1.81	2.29	1.13	2.25	0.86	0.70	0.23	-1.10	0.34	1.53	0.06	-0.10	Yes

Table 5-8: Fractal Characteristics Investigation of Traffic Speed at Station 630082

Interval	PM Period	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	H8	Fractal?
15-min	3:30-3:45	1.35	1.70	2.02	1.86	1.75	2.00	4.34	2.54	1.63	2.42	0.82	0.43	0.53	0.25	0.85	0.30	0.39	0.67	Yes
	4:30-4:45	1.63	2.09	2.63	2.54	2.45	2.78	3.35	3.01	2.57	2.63	0.91	0.14	0.59	0.50	0.67	0.51	0.20	0.69	Yes
	5:30-5:45	1.37	2.04	2.29	2.23	2.14	2.44	2.82	2.89	2.05	2.58	0.90	0.69	0.34	0.49	0.59	0.99	0.44	0.34	Yes
	6:30-6:45	1.35	3.22	2.60	2.67	2.76	2.92	2.41	2.90	2.61	2.64	0.90	0.72	0.45	0.89	0.64	0.98	0.19	0.35	Yes
	7:30-7:45	1.63	2.43	2.45	2.46	2.57	2.51	2.62	2.73	2.29	2.43	0.85	1.72	0.72	0.60	1.09	0.89	0.43	0.79	Yes
1-hr	4:00-5:00	1.54	2.01	2.75	2.42	2.14	3.08	2.46	2.80	2.28	2.53	0.90	0.36	0.64	0.59	0.83	1.33	0.39	0.84	Yes
	5:00-6:00	1.30	2.40	2.44	2.31	2.13	2.56	2.90	2.89	2.11	2.57	0.90	0.28	0.17	0.32	0.40	1.01	0.58	0.58	Yes
	6:00-7:00	1.27	2.70	2.52	2.34	2.18	2.75	3.42	2.97	2.17	2.66	0.89	1.00	0.24	0.52	0.34	0.82	0.36	0.29	Yes
	7:00-8:00	1.43	2.49	2.26	2.15	2.11	2.31	2.63	2.54	1.93	2.42	0.86	0.59	0.40	-1.10	0.70	1.58	0.61	0.55	Yes

Table 5-9: Fractal Characteristics Investigation of Traffic Occupancy at Station 630082

Interval	PM Period	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	H8	Fractal?
15-min	3:30-3:45	1.47	1.63	1.83	1.77	1.78	1.76	2.34	1.85	1.32	2.09	0.89	-0.30	0.26	-0.10	0.63	0.52	0.80	1.12	Yes
	4:30-4:45	1.38	2.66	2.74	2.42	2.35	3.13	3.53	2.58	2.17	2.35	0.83	1.04	-0.10	0.78	0.34	1.13	-0.10	0.44	Yes
	5:30-5:45	1.21	2.59	2.13	2.20	2.62	2.14	3.21	2.16	2.66	2.18	0.87	0.21	0.50	0.55	0.81	1.49	0.22	0.56	Yes
	6:30-6:45	1.27	3.57	2.44	2.45	2.51	2.65	2.42	2.42	2.50	2.27	0.84	0.93	0.50	-0.20	0.78	1.29	0.22	0.41	Yes
	7:30-7:45	1.52	2.37	2.19	2.35	2.43	2.42	2.86	2.46	2.66	2.27	0.93	1.09	-0.60	0.81	-0.30	0.95	0.72	0.65	Yes
1-hr	4:00-5:00	1.43	2.33	2.25	1.96	1.69	2.36	2.89	2.06	1.44	2.12	0.85	0.76	-0.90	0.43	-0.40	2.18	0.33	0.55	Yes
	5:00-6:00	1.50	2.32	2.09	1.91	1.68	2.02	2.98	2.10	2.17	2.15	0.86	1.15	0.18	0.30	0.64	1.41	0.32	0.45	Yes
	6:00-7:00	1.25	3.49	2.13	2.21	2.62	2.22	2.87	2.13	2.17	2.20	0.86	0.82	0.03	0.01	0.33	1.87	0.11	0.04	Yes
	7:00-8:00	1.55	1.95	1.98	1.81	1.72	1.84	3.02	2.15	1.37	2.30	0.87	0.40	-0.30	-0.10	-0.20	1.39	-0.20	-0.40	Yes

FD1 = Box Count   FD2 = Hall-Wood   FD3 = Variogram   FD4 = Madogram   FD5 = Rodogram   FD6 = Incr1
FD7 = Periodogram   FD8 = DCT-II   FD9 = Genton   FD10 = Wavelet   H1 = Higuchi   H2 = *R/S*
H3 = Aggregated Variance   H4 = Differenced Aggregated Variance   H5 = Aggregated Absolute Moment
H6 = Variance of Residuals   H7 = Periodogram   H8 = Modified Periodogram

Table 5-10: Fractal Characteristics Investigation of Traffic Volume at Station 630022

Interval	PM Period	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	H8	Fractal?
15-min	3:30-3:45	1.63	1.82	2.20	2.34	2.46	2.45	2.88	2.28	2.41	2.29	0.90	0.52	0.23	1.32	0.14	0.45	0.84	0.58	Yes
	4:30-4:45	1.08	2.39	1.62	1.76	1.94	1.62	3.18	2.12	2.26	1.84	0.65	0.91	0.66	0.84	0.84	0.86	1.26	1.74	Yes
	5:30-5:45	1.63	1.27	1.43	1.37	1.35	1.04	4.04	2.28	1.09	1.94	0.80	0.83	0.09	0.79	0.05	1.23	0.02	1.72	Yes
	6:30-6:45	1.42	1.99	1.69	1.96	2.16	1.65	2.35	2.57	2.10	2.02	0.91	0.67	0.94	-0.10	0.96	1.03	0.15	0.69	Yes
	7:30-7:45	1.62	1.61	1.63	1.57	1.46	1.39	2.72	2.32	1.58	2.45	0.87	0.66	-0.10	-0.10	-0.30	0.82	-0.40	0.55	Yes
1-hr	4:00-5:00	1.18	2.32	1.54	1.43	1.35	1.44	2.93	1.97	1.16	1.95	0.83	0.64	0.26	-0.10	0.01	0.89	-0.10	-0.20	Yes
	5:00-6:00	1.40	2.67	2.13	1.85	1.62	2.26	2.28	2.08	1.65	2.02	0.88	0.99	0.83	0.56	0.61	2.50	0.68	0.47	Yes
	6:00-7:00	1.51	3.09	1.91	1.89	1.82	1.86	2.45	1.81	1.88	1.88	0.90	0.57	1.69	0.31	0.48	2.43	0.65	0.59	Yes
	7:00-8:00	1.71	3.46	1.91	1.89	1.80	1.88	2.44	1.41	2.07	1.89	0.87	0.54	2.75	0.22	0.17	2.37	0.55	0.54	Yes

Table 5-11: Fractal Characteristics Investigation of Traffic Speed at Station 630022

Interval	PM Period	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	H8	Fractal?
15-min	3:30-3:45	1.70	2.29	2.22	2.38	2.58	2.27	2.62	2.21	2.21	2.41	0.75	0.54	0.11	1.66	0.34	0.51	1.13	0.59	Yes
	4:30-4:45	1.55	2.37	2.30	2.53	2.64	2.43	3.24	2.38	3.07	2.26	0.61	0.93	0.40	1.21	0.99	0.49	1.31	0.91	Yes
	5:30-5:45	1.62	1.91	1.77	1.77	1.72	1.58	0.56	2.37	1.80	2.25	0.67	0.73	0.31	1.04	0.59	0.69	0.51	0.90	Yes
	6:30-6:45	1.25	1.51	1.83	1.78	1.79	1.72	3.42	2.55	1.92	2.37	0.95	0.77	-0.60	0.66	0.32	1.07	0.16	0.01	Yes
	7:30-7:45	1.54	1.92	1.78	1.79	1.88	1.69	2.02	2.62	1.82	2.32	0.85	0.75	0.64	-0.60	0.32	1.56	0.71	0.74	Yes
1-hr	4:00-5:00	1.19	1.86	1.71	1.58	1.56	1.52	2.38	1.81	1.36	2.07	0.88	0.64	0.57	4.45	0.54	1.35	0.51	0.47	Yes
	5:00-6:00	1.21	2.32	2.01	1.87	1.73	2.07	0.21	2.11	1.82	2.08	0.88	0.60	1.16	-0.30	0.21	1.27	0.14	1.14	Yes
	6:00-7:00	1.51	2.37	1.87	1.97	2.09	1.89	1.11	1.63	2.03	1.86	0.88	0.37	1.00	0.47	0.12	1.64	1.25	0.80	Yes
	7:00-8:00	1.51	3.80	2.01	2.05	2.14	2.45	2.15	1.92	2.13	2.02	0.88	0.65	0.85	-0.30	2.70	1.00	0.90	1.00	Yes

Table 5-12: Fractal Characteristics Investigation of Traffic Occupancy at Station 630022

Interval	PM Period	FD1	FD2	FD3	FD4	FD5	FD6	FD7	FD8	FD9	FD10	H1	H2	H3	H4	H5	H6	H7	H8	Fractal?
15-min	3:30-3:45	1.66	2.20	2.20	2.36	2.50	2.24	2.62	2.36	2.53	2.41	0.97	0.57	0.31	1.70	0.39	0.60	1.42	0.64	Yes
	4:30-4:45	1.55	2.40	2.25	2.53	2.76	2.37	2.80	2.11	3.46	2.19	1.21	0.95	0.48	1.82	0.95	0.50	1.22	1.15	Yes
	5:30-5:45	1.58	1.88	1.77	1.79	1.73	1.64	2.20	2.33	1.65	2.26	1.07	0.68	0.26	1.04	0.70	0.64	0.50	1.10	Yes
	6:30-6:45	1.27	1.56	1.92	1.87	1.93	1.86	3.38	2.27	1.57	2.39	0.68	0.75	-0.10	0.59	0.06	1.04	0.18	-0.10	Yes
	7:30-7:45	1.68	1.47	1.63	1.58	1.49	1.38	3.65	2.97	1.40	2.52	0.91	0.68	-0.10	-0.90	-0.10	0.71	-1.20	-0.40	Yes
1-hr	4:00-5:00	1.19	1.86	1.71	1.58	1.56	1.52	2.38	1.81	1.36	2.07	1.02	0.71	-1.60	0.04	1.32	0.79	1.00	1.20	Yes
	5:00-6:00	1.21	2.32	2.01	1.87	1.73	2.07	0.21	2.11	1.82	2.08	0.92	0.94	0.85	0.60	0.34	2.21	0.46	0.57	Yes
	6:00-7:00	1.51	2.37	1.87	1.97	2.09	1.89	1.11	1.63	2.03	1.86	0.91	0.48	1.44	0.30	0.43	2.32	0.66	0.68	Yes
	7:00-8:00	1.51	3.80	2.01	2.05	2.14	2.45	2.15	1.92	2.13	2.02	0.87	0.61	0.69	0.57	0.49	3.78	0.97	0.84	Yes

FD1 = Box Count FD2 = Hall-Wood FD3 = Variogram FD4 = Madogram FD5 = Rodogram FD6 = Incr1
FD7 = Periodogram FD8 = DCT-II FD9 = Genton FD10 = Wavelet H1 = Higuchi H2 = *R/S*
H3 = Aggregated Variance H4 = Differenced Aggregated Variance H5 = Aggregated Absolute Moment
H6 = Variance of Residuals H7 = Periodogram H8 = Modified Periodogram

As an example, Figure 5-12 shows the estimations of fractal dimension using each of the 10 fractal dimension methods at Station 630082 for traffic volume data. As shown in the plots, all the fractal dimension estimates are ≥ 1, which reveals the existence of fractal behavior. The highest four values were estimated using the peridogram (3.48), Hall-Wood (3.46), DCT-II (2.42), and wavelet (2.19), respectively. Thus, as previously concluded, the wavelet method was one of the methods that yielded a relatively high fractal dimension estimate. One the other hand, the box count method yielded the lowest fractal dimension estimate of 1.19.
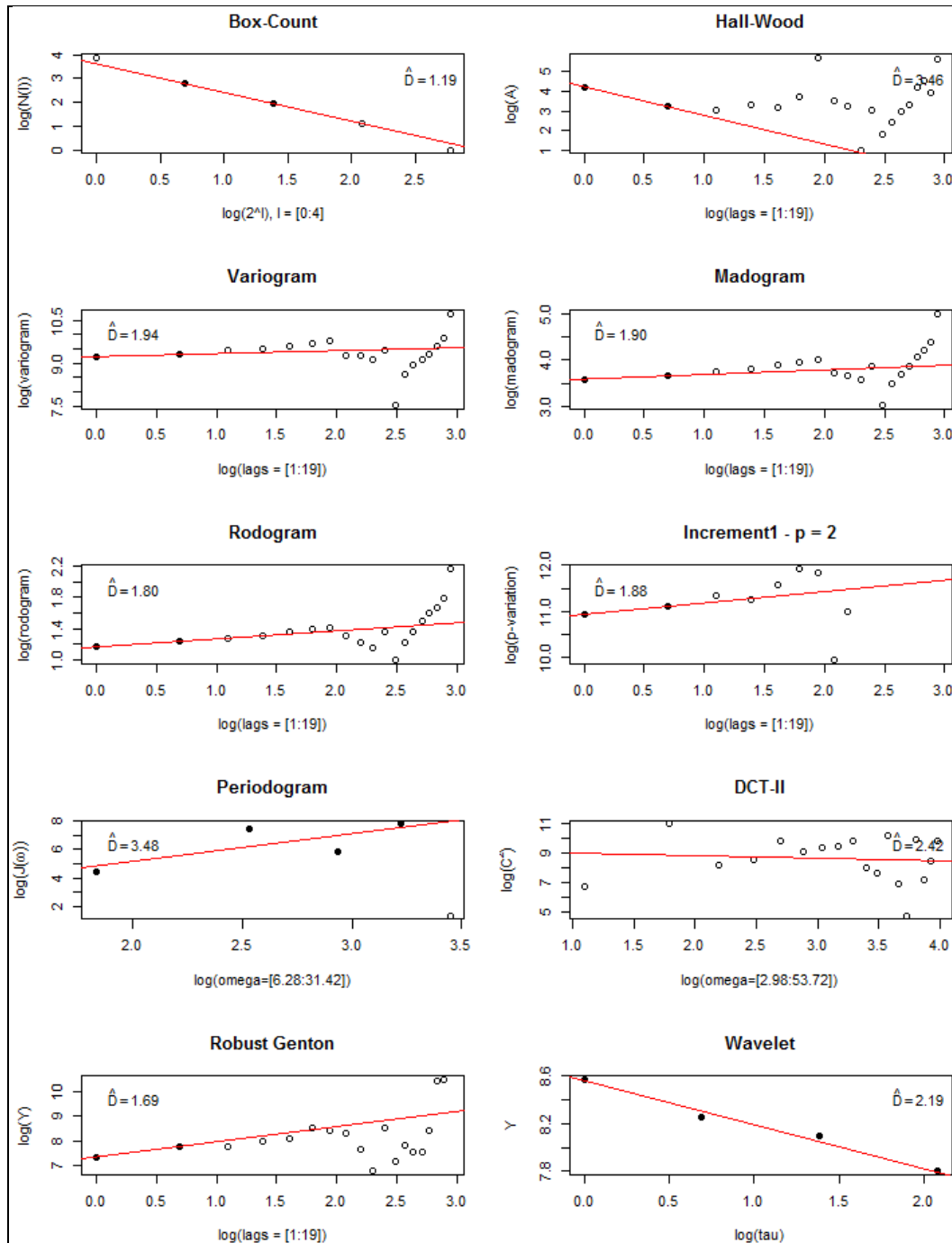

Figure 5-12: Fractal Dimension Estimations for Volume at Stn 630082 (4:30 pm-4:45 pm)

5.3 Summary

Two fractal techniques, the fractal dimension and the Hurst exponent, were applied using the *fractaldim* and *fArma* packages in **R** Software to detect the existence of fractal characteristics in Florida's crash and traffic data. Temporal trends in crash data, specifically; trends in the statewide annual and daily crash frequency, and the annual intersection crash rate were explored. Data from 1990-2011 were used to analyze trends in annual crash frequency, and five-year data from 2007-2011 were used to analyze daily crash frequency trends. For analyzing annual crash rate, ten signalized intersections on state roads were randomly selected and investigated using crash data from 1991 to 2011. For the traffic data, volume, speed, and occupancy trends were investigated at two stations in Miami-Dade County for the entire month of May in 2010.

Fractal dimension was estimated using the following ten different methods: box count, Hall-Wood, variogram, madogram, rodogram, Incr1, periodogram, DCT-II, genton, and wavelet methods. The Hurst exponent, H, was estimated using the following eight different methods: Higuchi, R/S, aggregated variance, differenced aggregated variance, aggregated absolute moment, variance of residuals, periodogram, and modified periodogram methods. Fractal behavior was considered to be evident if fractal dimension estimate is >1 and/or if H lies between 0 and 1. In the two techniques, the higher the estimate value, the more predominant the fractal characteristics.

The analysis detected the existence of fractal characteristics in both daily and annual crash frequency trends. However, the daily crash trend exhibited greater extent of fractal behavior, mainly due to the existence of more random fluctuations. For the annual crash trends, nine of the ten fractal dimension methods yielded estimates greater than 1. On the other hand, all the ten methods yielded estimates greater than 1 for the daily crash trends. A common observation was that the fractal dimension estimate from the wavelet method was among the highest estimates. Based on the Hurst exponent estimate, H, six of the eight methods yielded estimates between 0 and 1 for the annual crash trend, indicating fractal existence. On the other hand, all the eight Hurst exponent methods for trend in daily crash frequency yielded H values between 0 and 1, indicating a fairly consistent fractal behavior.

The trends in daily crash frequency for different facility types (e.g., freeways, arterials, collectors, etc.) were also investigated. The fractal dimension analysis revealed that estimates from all methods were greater than 1 for all the facilities. Urban collectors experienced the highest fractal behavior; however, its estimate was relatively close to that of rural major and rural minor arterials. The Hurst exponent analysis revealed that all H estimates were between 0 and 1, which shows the existence of fractal characteristics and long-range dependence. In general, urban facilities resulted in higher H estimates than their rural counterparts. More specifically, urban collectors had the highest median H estimate.

Since crash rates are used to account for the changes in AADT at a location over time, the fractal characteristics of crash rates were also investigated at ten randomly-selected signalized intersections. To investigate the existence of fractal characteristics in annual crash rates, two types of analysis were conducted: the annual crash rate trend and the three-year moving average crash trend. It was found that the annual crash rate trend, in general, exhibited relatively more

fractal characteristics than the three-year average trend in crash rate. This is obvious since most of the *H* estimates for all the ten intersections for the annual trend analysis were between 0 and 1. Furthermore, most of the fractal dimension estimates from the ten methods were above 1 when compared to the three-year average analysis.

Finally, the existence of fractal characteristics in traffic data (i.e., volume, speed, and occupancy) was investigated using 15-min and 1-hr counts at two stations. In general, the volume, speed, and occupancy patterns at both stations exhibited fractal characteristics. The *H* estimates calculated using the eight methods were between 0 and 1 and the fractal dimension estimates from the ten methods were mostly > 1. Furthermore, at both stations, the speed trend revealed stronger fractal behavior compared to the volume and occupancy trends for the same time period. It was also observed that the occupancy and volume trends at one of the stations at a specific time period (4:30 pm to 4:45 pm) exhibited relatively little fractal behavior compared to other periods. As previously concluded from the crash analysis, the wavelet method was one of the methods that yielded a relatively high fractal dimension estimate.

CHAPTER 6
CONCLUSIONS AND RECOMMENDATIONS

Fractals are irregular geometric objects that exhibit finite details at all scales, and when magnified, their basic structures remain the same regardless of the scale of magnification. The main characteristics of fractals are self-similarity, iterative process, infinite complexity, and existence of non-integer complex dimension. Common fractal analysis techniques are the Hurst exponent, the fractal dimension, the largest Lyapunov exponent, the power spectrum, and the Kolmogorov entropy. The main indicators of existence of fractal characteristics are as follows: the largest Lyapunov exponent should have a positive sign; the Hurst exponent (H) estimate should lie between 0 and 1; and the Kolmogorov entropy should have a positive sign. Moreover, larger estimates of the Lyapunov exponent, the Hurst exponent, and the fractal dimension indicate more complex fractal nonlinear characteristics.

Fractal theory has been successfully applied in different fields of science, including but not limited to animal behavior studies, human health studies, economics, astronomy, ecology, physics, pavement engineering, environmental engineering, behavior of materials, and electrical networks. Even though application of fractal theory is not uncommon in several areas in transportation such as traffic flow analysis, urban network analysis, and travel demand modeling, existence of fractal characteristics in safety data is rarely investigated. Moreover, fractal applications in traffic flow prediction showed promising results. Fractal models were found to be more appropriate for short-term traffic flow predictions and weather forecasts, and for low traffic volume conditions. Additionally, traffic flow and traffic speed were found to be the main elements of traffic data to exhibit fractal characteristics.

In addition to the existing applications, the following traffic management strategies where fractal theory could potentially be applied were discussed: managed lanes, ramp metering, crash analysis, parking management, and travel time reliability. For each application, the data needed to apply fractal theory and the data that could be predicted by applying fractal theory were identified. Fractal behavior was hypothesized to exist in these strategies due to random fluctuations in the data.

Ramp metering and managed lane strategies are implemented along freeways, and therefore, most of the recommended data included traffic flow, speed, and occupancy upstream and downstream of the ramp/station of interest. For crash analysis, existence of fractal characteristics could be investigated based on the distribution of crash frequency and crash rate. For parking management, data on turnover rate, occupancy of parking spots, etc. were recommended. Finally, for travel time reliability, data on mean travel time, free-flow travel time, 90th and 95th percentile travel time, and mean travel speed were recommended. For travel time reliability, common performance measures such as travel time index, buffer index, and degree of congestion could be predicted.

Two fractal techniques, the fractal dimension and the Hurst exponent, were applied using the *fractaldim* and *fArma* packages in **R** Software to investigate the existence of fractal characteristics in crash and traffic data from Florida. The fractal dimension technique includes ten different methods, box count, Hall-Wood, variogram, madogram, rodogram, Incr1,

periodogram, DCT-II, genton, and wavelet methods. The Hurst exponent technique includes the following eight methods: aggregated variance, differenced aggregated variance, aggregated absolute moment, Higuchi, variance of residuals, *R/S*, periodogram, and modified periodogram methods. More fractal behavior exists for higher fractal dimension estimate calculated using the aforementioned methods, specifically if the estimate is greater than 1. Again, fractal behavior is evident if *H* lies between 0 and 1. Further, the higher the estimate value, the more predominant the fractal characteristics.

Temporal trends in crash frequency data, specifically; trends in the statewide annual and daily crash frequency, and the annual intersection crash rate were explored. Data from 1990-2011 were used to analyze trends in annual crash frequency, and five-year data from 2007-2011 were used to analyze daily crash frequency trends. The analysis detected the existence of fractal characteristics in both annual and daily crash frequency trends. However, the daily crash trend exhibited greater extent of fractal behavior, mainly due to the existence of more random fluctuations. Nine of the ten fractal dimension methods yielded estimates greater than 1 for annual crash trends, while all the ten methods yielded estimates greater than 1 for daily crash trends. A common observation was that the fractal dimension estimate from the wavelet method was among the highest estimates. Based on the Hurst exponent estimate, *H*, six of the eight methods yielded estimates between 0 and 1 for annual crash trend, indicating fractal behavior. On the other hand, all the eight Hurst exponent methods for trend in daily crash frequency yielded *H* values between 0 and 1, indicating a fairly consistent fractal behavior.

Additionally, trends in daily crash frequency for different facility types (e.g., freeways, arterials, collectors, etc.) were investigated. The fractal dimension analysis revealed that estimates from all methods were greater than 1 for all facilities. Urban collectors experienced the highest fractal behavior. The Hurst exponent analysis revealed that all *H* estimates were between 0 and 1, which shows the existence of fractal characteristics and long-range dependence. In general, urban facilities resulted in higher *H* estimates than their rural counterparts. More specifically, urban collectors had the highest median *H* estimate.

Since crash rates are used to account for changes in AADT at a location over time, the fractal characteristics of crash rates (i.e., crash frequency per million entering vehicles) were also investigated at ten randomly-selected signalized intersections. To investigate the existence of fractal characteristics in annual crash rates, two types of analysis were conducted: the annual crash rate trend and the three-year moving average crash trend. It was found that the annual crash rate trend, in general, exhibited relatively more fractal characteristics than the three-year average trend in crash rate. This is obvious since most of the *H* estimates for all the ten intersections for the annual trend analysis were between 0 and 1. Furthermore, most of the fractal dimension estimates from the ten methods were above 1 when compared to the three-year average analysis.

Finally, the existence of fractal characteristics in traffic data (i.e., volume, speed, and occupancy) was investigated using 15-min and 1-hr counts at two stations in Miami-Dade County. In general, the volume, speed, and occupancy patterns at both stations exhibited fractal characteristics. The *H* estimates calculated using the eight methods were between 0 and 1 and the fractal dimension estimates from the ten methods were mostly > 1. Furthermore, at both stations, the speed trend revealed stronger fractal behavior compared to the volume and occupancy trends

for the same time period. It was also observed that the occupancy and volume trends at one of the stations at a specific time period (4:30 pm to 4:45 pm) exhibited relatively small fractal behavior compared to other periods. As previously concluded from the crash analysis, the wavelet method was one of the methods that yielded a relatively high fractal dimension estimate.

Future research could make use of the insights presented in this study to investigate the existence of fractal behavior in traffic management strategies. A good starting point will be to use the recommended data elements to detect fractal behavior in the following traffic management strategies: managed lanes, ramp metering, crash analysis, parking management, and travel time reliability analysis. For these strategies, the recommended list of data variables could be refined and expanded. Compared to traditional statistical regression models, fractal theory is anticipated to yield more precise estimates of performance measures, such as travel time, traffic flow, traffic speed, rate of arrivals at parking lots, etc. Therefore, a potential future research is to apply fractal theory for prediction purposes, e.g., to estimate short-term traffic flow on general purpose and managed lanes, to predict number of arrivals to a parking lot and average parking time per day, etc. This approach could potentially result in increased efficiency, mobility, and safety of the entire roadway network. The results of the fractal characteristics investigation for traffic data could be used in short-term traffic predictions (e.g., the next 15 minutes), which could act as a potential proactive traffic management strategy. Further, fractal behavior can also be investigated in complex scenarios e.g., implementation of both ramp metering and managed lanes along freeway sections.

Another promising avenue is to use the results of the annual and three-year average analyses in network screening (i.e., identification of high-crash locations) since fractal characteristics are evident in the trends. For example, a potential application of fractal theory in the three-year moving average trend analysis could be to predict whether a high-crash intersection would continue to be listed in the future high-crash location lists if no safety improvements have been made. This is important since FDOT identifies high-crash locations using the most recent three-year data, e.g., high-crash intersections in 2009 were identified using data from 2007-2009. Since the Hurst exponent (H) method has been successfully applied in forecasting future trends in various applications, the same concept can be applied for predicting the trends in crash rate at intersections listed as high-crash locations. The Hurst exponent can be estimated using the current trend data. If the H value is greater than 0.5 for an intersection with an increasing trend that is already in the high-crash list, then, in the future, the intersection is expected to continue to have an increasing trend. In such a case, there is a high probability that this intersection will continue to be in the high-crash location list. On the other hand, if H is less than 0.5, the increasing trend at the intersection is anticipated to reverse in the future (i.e., have a decreasing trend), and there is a high probability that this intersection would not be included in the high-crash location list. Similarly, crash rates at specific intersections could be predicted using the fractal extrapolation method.

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