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Final Report

Project Title:

Stochastic Fundamental Diagram for Probabilistic Traffic Flow Modeling

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Flowing water in river, transported gas or oil in pipe, electric current in wire, moving goods on conveyor, molecular motors in living cell, and driving vehicles on a highway are various kinds of flow from physical or non-physical systems, yet each exhibits distinct characteristics. One of the attributes that distinguishes vehicular traffic flow from other flows as a special kind of 'fluid' is the so-called fundamental diagram - the relationships among traffic flow characteristics (e.g. flow, speed, and density) which are typically represented graphically, see an illustration in Figure 1. The fundamental diagram plays an essential role in transportation analysis and operations. For example, the study on traffic flow dynamics relies on input from the speed-density relationship to understand how a perturbation propagates among vehicles; a highway capacity analysis utilizes the speed-flow relationship to determine the level of service that the highway provides. Hence, sound mathematical models that better represent these relationships build a solid foundation for traffic flow analysis and efficient traffic control. Such an observation has motivated a variety of speed-density models since the path-breaking attempt by Greenshields in 1935. Departing from the Greenshields model, a number of models are proposed with varying degrees of success. Note that these models are in a single-equation form (single-regime model). Further improvements are made by decomposing the speed-density relationship into multiple pieces for better fitting. However, no matter single- or multi-regime, these models are deterministic in nature which essentially describes average system behaviors from a statistical perspective.
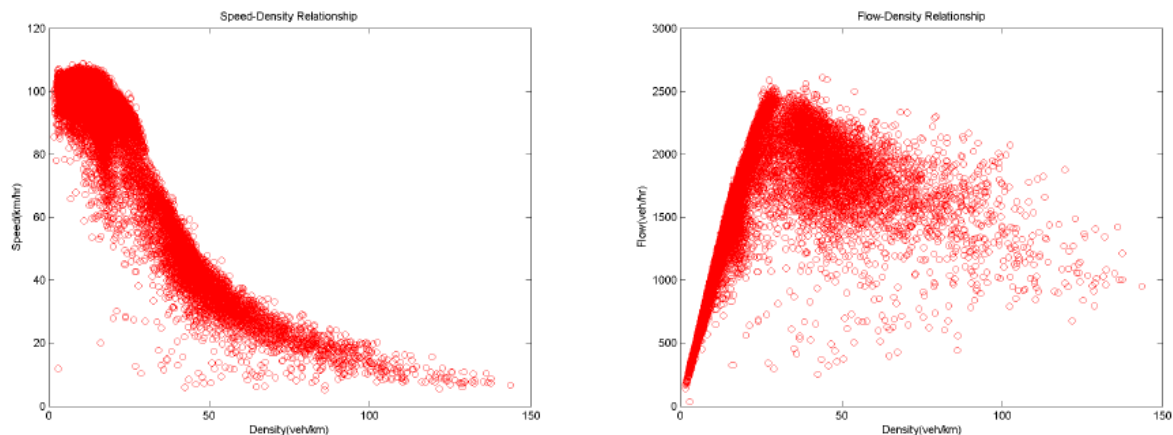

Figure 1 A fundamental diagram observed on a freeway

The scattering effect can be readily seen in the speed-density and flow-density plot as shown in Figure 1. The underlying mechanism behind the observed scattering phenomenon is frequently the effect of a large number of factors (e.g., driver behaviors, highway geometric, vehicles and the environmental conditions etc) that are not modeled explicitly. It is such an intrinsic property of transportation system that deterministic models should be considered as insufficient.

Currently, it is still controversial regarding whether the scattering effect is due to the measurement errors, the inherent nature of traffic flow, or a combination of both. Essentially, there are two main sources of randomness. The first type of randomness comes intrinsically from the data collection system and the computational process that follows. This type of randomness is well-understood and can be statistically controlled. The second type of randomness is inherently due to traffic dynamics, and it relates to a general lack of knowledge about the finer

details of traffic system. For example, drivers' behaviors vary on an individual basis; the collective behaviors of driver groups would be better described in a distributional law rather than in deterministic terms. This type of randomness is assumed to underlie the proposed stochastic speed-density model, refer to Figure 2 for an illustrative stochastic model.
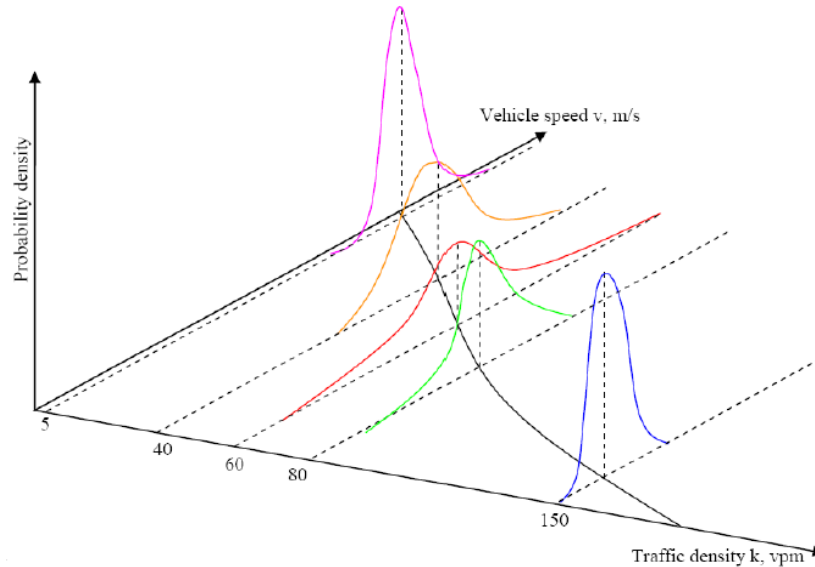

Figure 2 An illustrative Stochastic Speed-Density Relationship Model

In all deterministic models, either single-regime or multi-regime, the speed-density model has a functional relation; that is, given a density there exists a fixed speed computed from a deterministic formula, or vice versa. By investigating empirical observations from field observations, it is clear that multiple traffic speeds can correspond to one traffic density, or vice versa. This observation motivates our work.

The aim of this research, rather than to resolve the current controversies regarding sources of randomness, is to advance the modeling effort of the speed-density relationship from the deterministic domain to stochastic while still achieving mathematical elegance and empirical accuracy. To fulfill this end, a conceptual model was formulated assuming that speed is a random process of density and a random variable. Then the random process is discretized in a probabilistic framework using the spectral stochastic finite element method. It is recognized that the stochastic behavior of the real-world traffic system is often difficult to describe or predict exactly when the influence of unknown randomness is sizable. Nevertheless, it is quite possible to capture the chance that a particular value will be observed during a certain interval in a probabilistic sense. The analyses above readily lend themselves to the stochastic representation of the fundamental diagram of traffic flow, an approach that is empirically more appealing.

The stochastic model is derived through discretizing the stochastic speed process into deterministic and stochastic components with prescribed correlation using the spectral stochastic finite element method (SSFEM). By incorporating the first and second moments (i.e., mean, variance and correlation) of the stochastic speed process, a stochastic fundamental diagram is formulated which can describe the observed phenomena as well as match the empirical data. On the application side, the proposed model can potentially be used for real-time prediction and to explain phenomena to finer details.