Appendix F.
Developmental Enforcement Algorithm Definition Document

Predictive Braking Enforcement Algorithm Definition Document

Version 3.3 (DRAFT)

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Revision History

Date	Revision	Description
8/12/10	0.1	Initial Draft of Revised Base Case
8/18/10	0.2	Edits to track curvature, brake force, and air brake model calculations and incorporated comments from internal review
8/19/10	0.3	Minor formatting edits and incorporated additional comments
9/17/10	0.4	Incorporated edits from additional review, modified target offset adjustment, modified resistance calculation, edited nominal brake force calculation, corrected curvature resistance calculation, modified brake state transition logic, corrected recharge rate equations
10/13/10	1.0	Added phase 1 functions, including adaptive functions, handling of distributed power and emergency brake back-up
10/20/10	1.1	Incorporated comments from internal review
11/3/10	1.2	Incorporated edits from additional review, modified filter function
11/10/10	1.3	Added target offset function
11/11/10	1.4	Corrected formulas in target offset function
10/12/11	2.0	Added phase 2 functions, including dynamic braking, locomotive braking and modifications to emergency brake backup
2/27/12	3.0	Added phase 3 functionality for manifest freight and intermodal freight train types, including modifications to nominal brake force calculation, brake pipe propagation time estimation, and target offset calculation. Modified calculation of distributed power brake pipe propagation time calculations and fixed error in dynamic brake estimation.
3/6/12	3.1	Added additional definitions, modified brake force adaptive routine, implemented various other minor edits
4/5/12	3.2	Added relative efficiency of the brake rigging to the calculation of brake shoe force
5/7/12	3.3	Modified nominal brake shoe forces and added unit aluminum coal train type

Table of Contents

F1. Introduction

F1.1 Purpose

The purpose of this document is to fully define and describe the logic flow and mathematical equations for a predictive braking enforcement algorithm intended for implementation in a Positive Train Control (PTC) system.

F1.2 Scope

This document is intended as a comprehensive description of the predictive braking enforcement algorithm defined within. It is not intended as a detailed software requirements specification. It includes a definition of the logic flow and mathematical equations required to develop a functional implementation.

The predictive braking enforcement algorithm described within this document is intended for use in PTC systems for freight trains. Considerations for passenger trains are not included.

The definition of the algorithm contains background on the source of the logic and equations to provide context, but is not intended to provide a complete background on the development of the algorithm.

Information and data pertaining to safety validation of the algorithm is not included in this document, although the intent of the program under which this algorithm was developed is to provide a separate report on the testing and validation of the logic included in this document.

No attempt has been made within this document to consider software implementation techniques, particularly as related to implementation in a safety critical application.

F1.3 Intended Audience

This document is intended for developers of Positive Train Control (PTC) onboard systems and software considering predictive braking and enforcement algorithm options for inclusion in their system.

F1.4 Applicable Documents

The following documents are applicable in that they are either referenced in the algorithm description document or provide useful background information:

- Braking and Prediction Algorithm Definition for the NAJPTC IDOT Project, Rev C
- Hay, William W. (1982). *Railroad Engineering, Second Edition*. John Wiley & Sons, Inc.
- Blaine, David G. (1979). *Modern Freight Car Air Brakes.* Simmons-Boardman Publishing Corporation.

F1.5 Definitions and Acronyms

F1.5.1 Definitions

The following terms are used in the document:

- Auxiliary reservoir – A reservoir located on each car that holds a supply of compressed air that is transferred to the brake cylinder during an air brake application.

- Bail – The act of venting the locomotive brake cylinder pressure generated by the application of the automatic brake to atmosphere.
- Brake cylinder – A reservoir that is supplied with compressed air during an air brake application to control a piston connected to the brake shoes through the brake rigging. The amount of pressure in the brake cylinder determines the amount of force applied by the brake shoes.
- Brake pipe – A pipe that runs the length of the train and is used both to supply compressed air to the brake system on each car and to transmit air brake signals to the control valves on each car via changes to the pressure of the air within the pipe.
- Brake pipe propagation time – The time it takes for an air brake application signal to propagate throughout the length of the train and apply brakes on all cars in the train.
- Brake ratio – The ratio of the sum of the force supplied by all of the brake shoes on a car to the gross rated load of the car, given a 30 psi brake pipe reduction from a brake pipe nominally charged to 90 psi.
- Braking profile – The location/speed curve that describes the response of the train to either a penalty or emergency brake application, given the current conditions.
- Control valve – An air valve on each car that responds to changes in brake pipe pressure by directing air between the brake pipe, auxiliary reservoir, emergency reservoir and brake cylinder.
- Degree of curvature – The central angle turned over a 100 foot chord length, expressed in degrees.
- Distributed power – A descriptive term for a train that includes more than a single group of locomotives distributed throughout the train, separated by groups of non-powered vehicles
- Dynamic Brake – A form of locomotive braking, where the leads of the traction motors are reversed (effectively turning them into generators), providing resistance to the rotating wheels and dissipating the energy generated as heat through a resistor bank.
- Emergency air brake application – A rapid reduction of the brake pipe pressure to atmospheric pressure. An emergency brake application results in higher brake force as a result of the control valve directing air from both the auxiliary and emergency reservoirs to the brake cylinder.
- Emergency reservoir – A reservoir located on each car that is used to supply additional air to the brake cylinder to generate higher brake force during an emergency application.
- Equalization – The point at which the auxiliary reservoir pressure equalizes with the brake cylinder pressure. At this point, no more air can flow from the auxiliary reservoir to the brake cylinder and, therefore, further reduction of the brake pipe pressure at a service rate will not result in any additional brake force.
- Movement authority – authorization given to the train crew by a dispatcher or control operator allowing the train to occupy track limits.

- Onboard computer – The PTC computer onboard the locomotive that is responsible for collecting train status and target information and applying the penalty brake.
- Penalty air brake application – A reduction of the brake pipe pressure at a service rate that results in the control valve directing air from the auxiliary reservoir to the brake cylinder until equalization is reached.
- Percent Grade (% Grade) – The ratio of the change in track elevation over a specified distance, expressed as a percent.
- Positive Train Control (PTC) – A form of train control where train movement authorities and speed limits are transmitted electronically and automatically enforced to prevent violations.
- Predictive braking enforcement algorithm – A computational algorithm that predicts the braking profile of a train and, if necessary, enforces a penalty brake application to prevent a train movement authority or speed limit violation. Also described as "enforcement algorithm" or simply "algorithm".
- Service air brake application – A reduction of the brake pipe pressure at a service rate that results in the control valve directing air from the auxiliary reservoir to the brake cylinder.
- Speed limit – The maximum allowed speed for a train over a particular section of track.
- Target – A location where the train must be at or below a given speed. The target locations are used by the enforcement algorithm to determine if a penalty air brake application is necessary.
- Target offset – A distance that is added to the stopping distance prediction to ensure that the train will stop short of the target with the required confidence, given potential inaccuracies in the prediction calculation.

F1.5.2 Acronyms

The following is a list of acronyms used within this document:

- ARP – Auxiliary reservoir pressure
- BCP – Brake cylinder pressure
- BPP – Brake pipe pressure
- COF – Coefficient of friction
- DP – Distributed power
- EOT – End of train
- IDOT – Illinois Department of Transportation
- NAJPTC – North American Joint Positive Train Control
- OBC – Onboard computer
- PTC – Positive Train Control

F2. Algorithm Overview

The enforcement algorithm described within this document is based on the version developed for the North American Joint Positive Train Control (NAJPTC) program. While other enforcement algorithms have been developed since this algorithm was originally released, the original NAJPTC version serves as a good industry base case and is available in the public domain. The algorithm described within this document seeks to improve on the performance of the NAJPTC algorithm and contains many revisions to the logic, while keeping many of the methods and concepts from the original version.

The primary objective of the predictive braking enforcement algorithm is to enforce PTC train movement authority and speed limits by initiating a penalty air brake application to stop the train from violating any such limit if the train crew fails to take action to prevent the violation, but to be transparent to the train crew when the train is handled properly to prevent the violation. The enforcement algorithm seeks to achieve these objectives by periodically predicting the stopping distance of the train, adding a target offset to the prediction, comparing this result against any authority or speed limits, and initiating a penalty air brake application as necessary.

The stopping distance prediction is performed by employing a simplified longitudinal train energy model to predict the braking profile of the train. The prediction assumes a penalty application is initiated at the time the prediction is made, using a combination of fixed (e.g. consist make-up) and dynamic (e.g. brake pipe pressure) data available to the onboard system. The stopping distance prediction is designed to result in a nominal prediction, which is then adjusted to meet the safety requirements of the system via the calculation of a target offset.

The target offset is a safety buffer added to the stopping location prediction in order to ensure the train will stop short of the target with a certain probability. Figure 1 illustrates this concept by showing a distribution of stopping locations representing the potential variability in stopping location relative to a target for a given scenario. This variability arises from the potential inaccuracies in the prediction attributed to a number of assumptions and unknowns in the prediction calculation. The nominal prediction is located at the mean of this distribution, meaning that, if no target offset were used, the likelihood that the train would overshoot the target would be 50%. As the figure illustrates, the target offset adjusts the target relative to the distribution, so that the likelihood of an overshoot is significantly reduced.

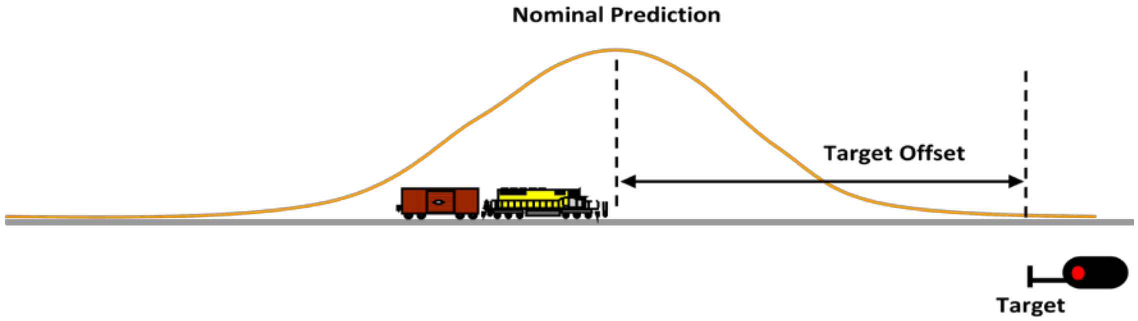

Figure F1 - Illustration of Target Offset

The target offset is based on a regression of the results of a Monte Carlo sensitivity analysis of freight train stopping distance. The target offset function adjusts the stopping location prediction to provide a statistically significant probability of the train stopping short of the target 99.5% of the time.

F3. Detailed Algorithm Definition

This section defines the functions, equations, and logic flow of the predictive braking enforcement algorithm. The intent is that this section will include sufficient detail for developing a working implementation of the algorithm for use in a functional PTC system. The overall architecture of the algorithm is designed to be modular to allow for additional functions to be added or modules to be replaced relatively quickly without affecting other functions or modules within the algorithm. Therefore, the descriptions within this section are organized into a series of functional modules.

F3.1 Initialization

This section describes the functions necessary for initialization of the algorithm. The primary objective of these functions is to set all of the fixed data used by the enforcement algorithm. Although the term initialization is used, these functions are designed to be used to modify these data items at any point, not just when the algorithm is started. For example, if the PTC implementation allows for modification of the consist after the train is en route, the *Update Consist Data* function would be used to update the consist information appropriately.

F3.1.1 Update Consist Data

This function is used to initialize, update, or modify the consist data that is used by the enforcement algorithm. The consist data provided to the enforcement algorithm includes:

- Train type
    - Unit Freight
    - Unit Aluminum Coal
    - Manifest Freight
    - Intermodal Freight
- Number of locomotives
- For each locomotive:
    - Locomotive position in train
    - Locomotive weight in tons
    - Locomotive status, either Run or Isolate
    - Locomotive length in feet
    - Number of axles
    - Locomotive horsepower (not currently used)
- Total trailing weight in tons
- Total number of axles for trailing cars
- Total train length, including locomotives
- Total number of loaded cars
- Total number of empty cars
- Number of cars with inoperative brakes
- Total train brake shoe force (optional input)

From this supplied consist data, the following parameters are derived and stored with the consist data:

- Nominal brake shoe force, $F_{B,NOM}$
- Brake pipe propagation time, T_{APPLY}

Derive Nominal Brake Shoe Force

The nominal brake shoe force is an estimate of the sum of the force applied to the wheels by the brake shoes after the brakes are all fully applied during a penalty air brake application. The total nominal brake shoe force for the train, $F_{B,NOM}$, is the assumed nominal brake shoe force supplied by all of the cars in the train, $F_{B,NOM,CARS}$, plus the assumed nominal brake shoe force supplied by the locomotives, $F_{B,NOM,LOCS}$. The nominal brake shoe force for the cars, $F_{B,NOM,CARS}$, is based on a nominal assumption for the amount of brake shoe force per axle at a brake cylinder pressure of 64psi, for both loaded and empty cars. Table 1 lists the values for the nominal brake shoe force per axle for loaded and empty cars, for each train type.

Table F1 - Nominal Brake Force

Train Type	Nominal Loaded Car Brake Shoe Force per Axle (lbs) at 64 psi BCP, $F_{B,NOM,AXLE,LOADED}$	Nominal Empty Car Brake Shoe Force per Axle (lbs) at 64 psi BCP, $F_{B,NOM,AXLE,EMPTY}$
Unit Freight	$\frac{0.093*W_{CARS}}{N_{AXLES}}$	4962
Unit Aluminum Coal	$\frac{0.11*W_{CARS}}{N_{AXLES}}$	3975
Manifest Freight	5870	5044
Intermodal Freight	6895	3746

W_{CARS} = Trailing Weight of Train, N_{AXLES} = Number of Axles in Train (Locomotives Excluded)

The nominal brake shoe force for the cars, $F_{B,NOM,CARS}$, is equal to:

$$F_{B,NOM,CARS} = N_{AXLES} * \left(F_{B,NOM,AXLE,LOADED} * \frac{N_{LOADED}}{N_{CARS}} + F_{B,NOM,AXLE,EMPTY} * \frac{N_{EMPTY}}{N_{CARS}}\right)$$

Where N_{AXLES} is the total number of trailing axles in the consist, $F_{B,NOM,AXLE,LOADED}$ is the nominal loaded car brake shoe force/axle for the train type from table 1, $F_{B,NOM,AXLE,EMPTY}$ is the nominal empty car brake shoe force/axle for the train type from table 1, N_{LOADED} is the number of loaded cars in the consist, N_{EMPTY} is the number of empty cars in the consist, and N_{CARS} is the total number of cars in the consist, $N_{LOAD} + N_{EMPTY}$. Depending on the implementation, the nominal brake shoe force for the cars, $F_{B,NOM,CARS}$, may also be provided to the enforcement algorithm as a preprocessed value. In this case, the nominal brake shoe force for the cars, $F_{B,NOM,CARS}$, is set to the value provided to the algorithm.

The nominal brake shoe force for the cars, $F_{B,NOM,CARS}$, is then adjusted based on the percent operable brakes. The percent of operable brakes, %OB, is determined as follows:

$$\%OB = 1 - \frac{N_{CARS,IOB}}{N_{CARS}}$$

Where N_{CARS} is the total number of cars and $N_{CARS,IOB}$ is the number of cars with inoperable brakes. The final value of $F_{B,NOM,CARS}$ is therefore determined by:

$$F_{B,NOM,CARS} = F_{B,NOM,CARS} * \%OB$$

The nominal brake shoe force for the locomotives, $F_{B,NOM.LOCS}$, is based on whether or not the locomotive is equipped with bail limiting functionality. The bail limiting functionality prevents a full bail of the locomotive brakes by the train crew during a PTC penalty or emergency application, to provide a more accurate brake profile prediction. If the locomotive is equipped with bail limiting functionality, the nominal brake shoe force for the locomotives, $F_{B,NOM.LOCS}$, is based on the bail limiting brake cylinder pressure threshold (the brake cylinder pressure below which the crew cannot bail to) and the assumption of a 28% net brake ratio at 64 psi BCP for the locomotives:

$$F_{B,NOM,LOCS} = \sum_{1}^{N_{LOCS}} 0.28 * \left(\frac{BCP_{THRESHOLD}}{64}\right) * W_{LOCO}$$

Where N_{LOCS} is the total number of locomotives, W_{LOCO} is the weight of each locomotive, in lbs, and $BCP_{THRESHOLD}$ is the bail limiting brake cylinder pressure threshold.

If the locomotive is not equipped with bail limiting functionality, the nominal brake shoe force for the locomotives, $F_{B,NOM.LOCS}$, is based on the assumption that the locomotive brakes will be bailed off for long trains and applied for short trains. The threshold for the number of cars that determines whether the locomotive brakes are applied or not, N_{SHORT}, should be a configurable parameter, but a value of 8-15 cars is considered to be a reasonable range. If the locomotive brakes are applied, a net brake ratio of 28% at 64 psi BCP for the automatic brake is assumed. The nominal brake shoe force for the locomotives, $F_{B,NOM,LOCS}$, is therefore determined according to the following equation:

$$F_{B,NOM,LOCS} = \begin{cases} \sum_{1}^{N_{LOCS}} 0.28 * W_{LOCO} & if\ N_{CARS} < N_{SHORT} \\ 0 & if\ N_{CARS} \geq N_{SHORT} \end{cases}$$

Where N_{LOCS} is the total number of locomotives in the train, W_{LOCO} is the weight of each locomotive, in lbs, N_{CARS} is the total number of trailing cars in the train and N_{SHORT} is the short train threshold.

The total nominal brake shoe force for the train, $F_{B,NOM}$, is determined by adding the nominal brake shoe force for the cars, $F_{B,NOM,CARS}$, and the nominal brake shoe force for the locomotives, $F_{B,NOM,LOCS}$.

Derive Nominal Brake Pipe Propagation Time

The nominal service brake pipe propagation time, T_{APPLY}, is the time, in seconds, for the average auxiliary reservoir pressure to reach equalization pressure following a service brake application from a fully charged brake pipe pressure. This value is initialized based on the distance the brake pipe pressure signal has to travel, according to the following equation:

$$T_{APPLY} = \begin{cases} 2.56 * 10^{-7} * L_{TRAVEL}^2 + 1.179 * 10^{-2} * L_{TRAVEL} + 13, & if\ Type = Unit \\ 8.01 * 10^{-7} * L_{TRAVEL}^2 + 8.4792 * 10^{-3} * L_{TRAVEL} + 15.6, & if\ Type = Manifest \\ 1.331 * 10^{-7} * L_{TRAVEL}^2 + 6.871 * 10^{-3} * L_{TRAVEL} + 16.66, & if\ Type = Intermodal \end{cases}$$

Where L_{TRAVEL} is the total distance the brake pipe pressure signal has to travel, in feet.

If the train is operating with all of the locomotives grouped together, the distance the brake pipe pressure signal has to travel, L_{TRAVEL}, is equal to the length of the train, L_{TRAIN}.

If the train is operating with distributed power (DP), meaning there are groups of locomotives (locomotive consists) distributed throughout the train as opposed to all in a single group within the train, the value for L_{TRAVEL} is modified based on the number of locomotive consists within the train:

$$L_{TRAVEL} = \frac{L_{TRAVEL}}{N_{LOCO\_CONSIST}}$$

Where $N_{LOCO\_CONSIST}$ is the number of locomotive consists.

F3.1.2 Update Track Data

This function is used to initialize or update the track data required, which includes:

- Elevation or percent grade and location reference for each grade change
- Track centerline coordinates at frequent intervals for use in determining heading and degree of track curvature

F3.2 Main Process

This section describes the primary high-level functions of the enforcement algorithm that make up the main processing loop. Figure 2 illustrates the flow of the functions within this process. Each of these functions are described generally in this section and described in more detail in subsequent sections, where appropriate.

The main process is to be repeated periodically, as required by the overall PTC system design. Each iteration of the main process will result in a decision on whether or not a penalty or emergency brake application is necessary to prevent a movement authority or speed limit violation. A frequency of 1 Hz is considered typical.

F3.2.1 Update Targets

This function is used to define locations where the train must be at or below a given speed, including movement authorities (zero speed targets) and speed restrictions (non-zero speed targets). The function accepts target data from the onboard system and assigns or removes targets from the target data store, as necessary. Each target contains two data items:

- Target Location – location of the target as referenced to the track database
- Target Speed – speed limit at the target in miles per hour (mph)

When the algorithm completes the brake profile prediction, these targets are used to determine if a penalty brake application is necessary.

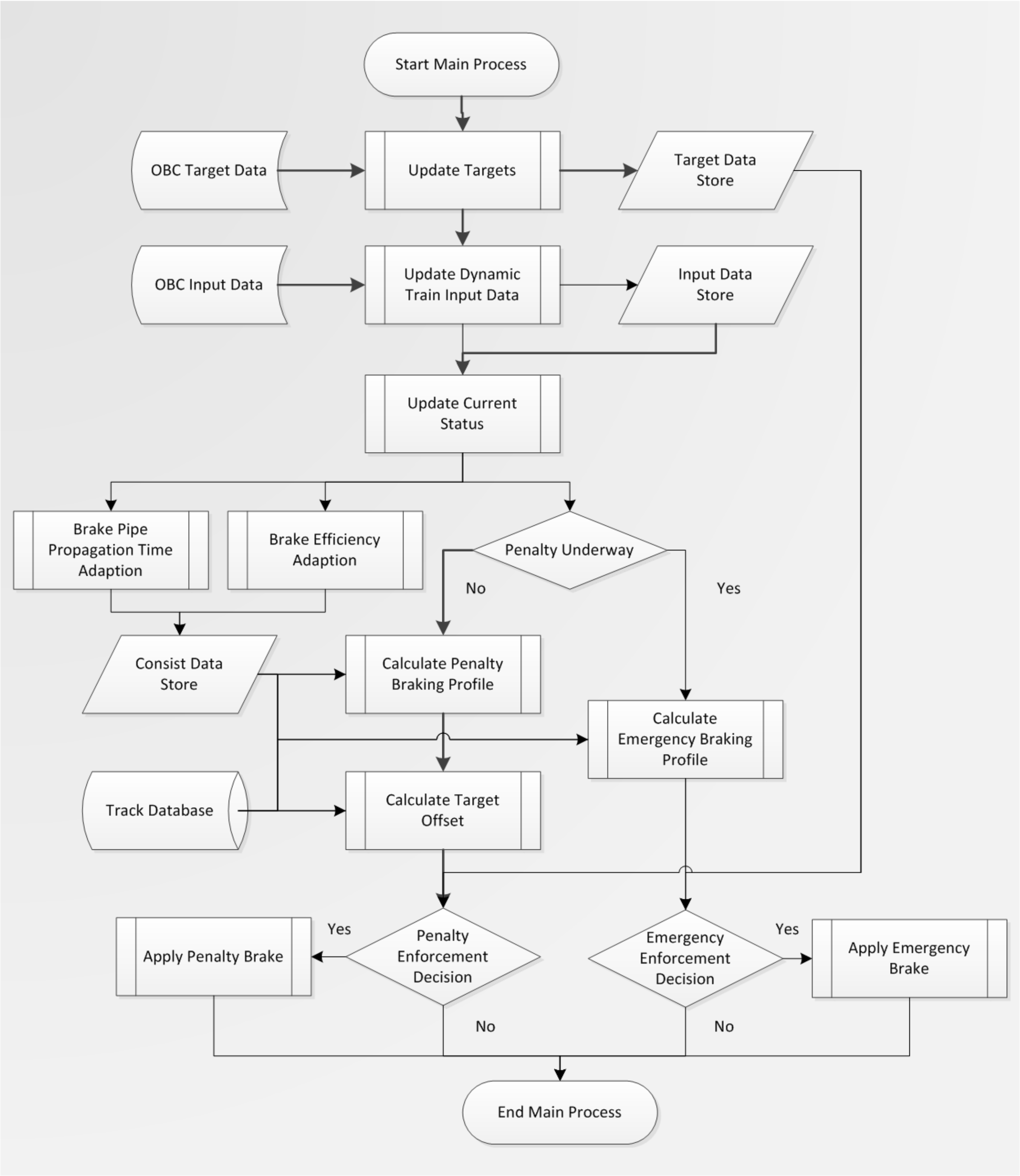

Figure F2 - Main Process Flow Diagram

F3.2.2 Update Dynamic Train Input Data

During each iteration of the main process, this function collects train status information from the onboard system for use elsewhere in the algorithm. The following data items are assigned in this function:

- Location – current location of the lead locomotive as referenced to the track database
- Speed – current speed of the lead locomotive in miles per hour (mph)
- Head-end Brake Pipe Pressure – current brake pipe pressure (BPP) at the lead locomotive in pounds per square inch (psi)
- Tail-end Brake Pipe Pressure – current brake pipe pressure (BPP) at the end of the train in pounds per square inch (psi)
- Direction – current setting of the reverser handle on the lead locomotive. Generally, forward or reverse
- Throttle Notch – current integer notch setting of the throttle handle on the lead locomotive (not currently used)
- Dynamic Brake Setup Status – current setting of the dynamic brake setup status bit (Boolean)
- Dynamic Brake Circuit Active Status – current setting of the dynamic brake circuit active status bit (Boolean)
- Dynamic Brake Voltage – current dynamic brake circuit voltage on the lead locomotive

F3.2.3 Update Current Status

This function updates the algorithm on the current status of the train based on train input data from the onboard system, consist data, and track data from the track database. The current status serves as the initial data point in the braking profile prediction. Specifically, the current state of the air brake system is determined, as described in section 3.3, the average track grade and curvature under the train is determined, as described in section 3.4, forces acting on the train are calculated, as described in section 3.5, and the locomotive dynamic braking force acting on the train, if any, is estimated, as described in section 3.6.

F3.2.4 Brake Pipe Propagation Time Adaption

This function uses input data from the onboard system to adapt the consist data used by the algorithm to more accurately represent the actual brake pipe propagation time for the train. This improves the accuracy of the braking profile prediction and allows for a reduction in the target offset. This function is described in detail in section 3.7.

F3.2.5 Brake Efficiency Adaption

This function uses input data from the onboard system as well as consist and track information to adapt the consist data used by the algorithm to more accurately represent the actual braking efficiency of the train. The output from this function is a modification to the maximum brake force, but this also compensates for inaccuracies in the prediction due to other influences, such as resistive forces, train weight, etc. This function is described in detail in section 3.8.

F3.2.6 Penalty Brake Enforcement Prediction

If the predictive braking enforcement algorithm has not yet enforced a penalty air brake application, the algorithm determines if a penalty air brake application is necessary to avoid violating any of the currently established targets. This comprises three processes; *Calculate Penalty Braking Profile*, *Calculate Target Offset* and the *Penalty Enforcement Decision*.

Calculate Penalty Braking Profile

This function calculates the braking profile of the train by assuming a penalty brake application is made at the time of the calculation, given the current status of the train, the consist data, and the track data from the track database. This calculation represents a nominal prediction of stopping distance, without any conservative assumptions, which are accounted for in the target offset function. The C*alculate Penalty Braking Profile* function is described in detail in section 3.9.

Calculate Target Offset

This function calculates the target offset, based on the consist data, the current status of the train, and the track data over the section of track covered by the braking profile. This function is described in detail in section 3.10.

Penalty Enforcement Decision

This function is used to determine if a penalty brake enforcement is necessary, given the previously calculated braking profile and target offset. All currently active targets are evaluated to determine if a violation is predicted. Multiple targets and combinations of zero speed and non-zero speed targets may need to be evaluated.

For zero speed targets, the predicted zero speed location of the train, according to the braking profile, is added to the calculated target offset and compared against the zero speed target location. If the sum of the predicted zero speed location and the target offset is greater than the target location, a penalty brake application is initiated.

For non zero speed targets, the predicted location of the train at the target speed is added to an adjusted target offset and compared against the target location. The adjusted target offset allows for less conservatism in the algorithm as the train speed approaches the target speed, based on the potential error at various points along the predicted braking curve. The target offset is adjusted based on the current speed and the target speed, according to the following equation:

$$TO_{ADJ} = \begin{cases} TO, & if\ \frac{v_{TGT}}{v_{CUR}} < 0.4 \\ TO * \left(1.4 - \frac{v_{TGT}}{v_{CUR}}\right), & if\ 0.4 < \frac{v_{TGT}}{v_{CUR}} < 0.9 \\ TO * \left(5 - 5 * \frac{v_{TGT}}{v_{CUR}}\right), & if\ 0.9 < \frac{v_{TGT}}{v_{CUR}} < 1 \\ 0, & if\ \frac{v_{TGT}}{v_{CUR}} > 1 \end{cases}$$

Where TO_{ADJ} is the adjusted target offset, TO is the total target offset, v_{TGT} is the target speed and v_{CUR} is the current speed of the train. If the sum of the predicted location at the target speed and the adjusted target offset is greater than the target location, a penalty brake application is initiated.

F3.2.7 Emergency Brake Enforcement Prediction

If a PTC predictive penalty brake enforcement is underway, the algorithm determines if an emergency brake application is necessary to ensure the train will stop short of any target limit. This includes the *Calculate Emergency Braking Profile* and *Emergency Enforcement Decision* processes.

Calculate Emergency Braking Profile

This function calculates the braking profile of the train following the penalty brake application, assuming that an emergency brake application is initiated at the time of the prediction, given the current status of the train, and the consist and track data. The braking profile determined from this function is used to determine if an emergency brake application is necessary to prevent a target violation. This function is described in detail in section 3.11.

Emergency Enforcement Decision

This function uses the calculated emergency braking profile and evaluates it against any currently active zero-speed targets to determine if an emergency brake application is necessary to prevent a target violation. If the predicted stopping position of the train is greater than or equal to the target stopping position, the emergency brake is applied.

F3.3 Update Air Brake System Status

The objective of this function is to determine the current state of the air brake system, including the brake pipe pressure, average auxiliary reservoir pressure, average brake cylinder pressure, and total brake shoe force. This function is used to update the actual air brake system status every iteration through the main processing loop, as well as updating the predicted air brake system status for each time step during the penalty braking profile prediction, as described in section 3.9, and the emergency braking profile prediction, as described in section 3.11.

Ultimately, the total brake shoe force calculated from this process is used by the enforcement algorithm to determine the amount of brake retarding force acting on the train at any given time. However, because of the complexity of the air brake system, there are a number of intermediate values that must be calculated and stored in order to accurately model the brake cylinder pressure response.

The air brake system is controlled by adjusting the amount of pressure in the brake pipe. The control valves, located on each car, respond to changes in brake pipe pressure by allowing air to flow between the various reservoirs on the car. When brake pipe pressure is reduced, the control valve(s) on each car allows air to flow from the auxiliary reservoir(s) to the brake cylinder(s) on that car, which applies the brakes. When the auxiliary reservoir pressure reaches the brake pipe pressure, the system is lapped, and the control valve prevents any more air from flowing between the reservoirs, holding the brake cylinder pressure, and the brake application, constant. When brake pipe pressure is increased, the control valve(s) on each car allows air to flow from the brake pipe to the auxiliary reservoir(s) to recharge the system, and also vents the brake cylinder pressure to atmosphere to release the brakes.

The air brake model employed in the *Update Air Brake System Status* function evaluates the brake pipe pressure to determine the status of the brake system, which is used to estimate the average pressure in the auxiliary reservoirs, which is then used to determine the average pressure in the brake cylinders.

The *Update Air Brake System Status* function flow is illustrated in Figure 3.

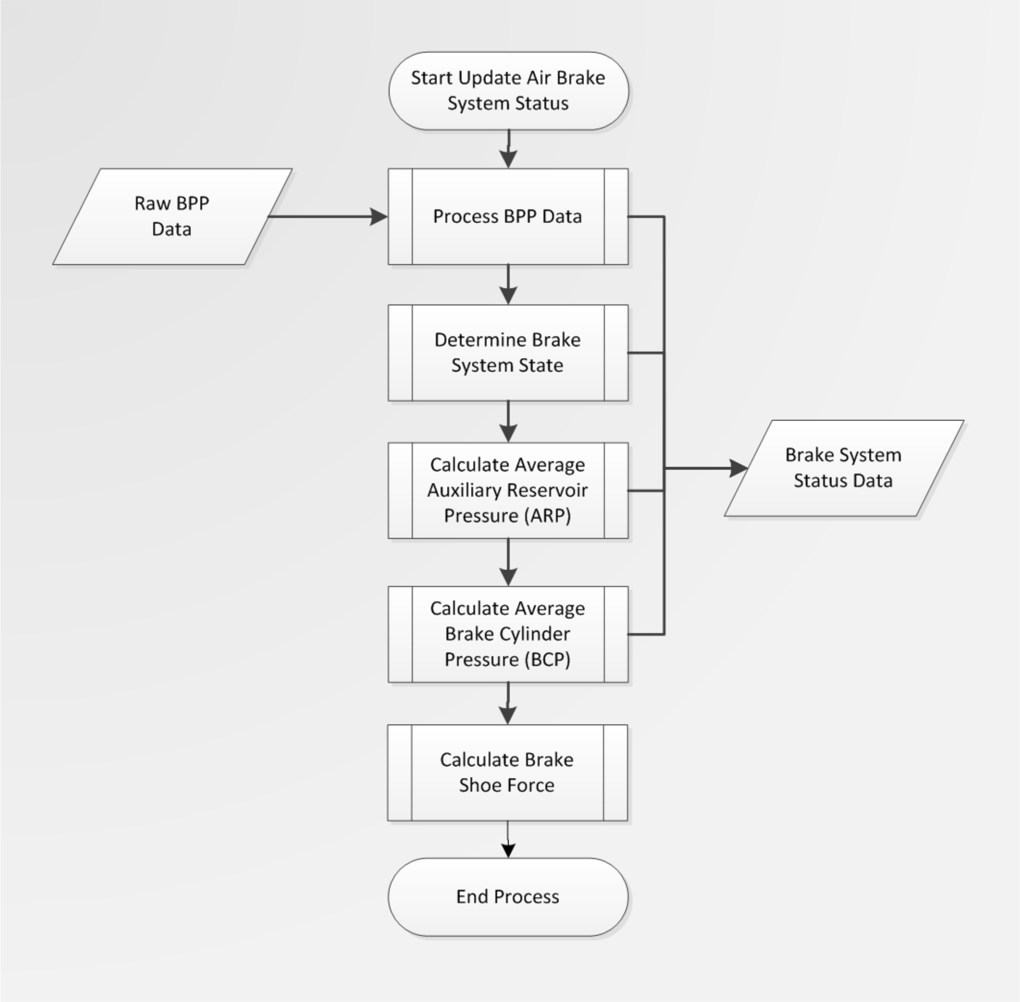

Figure F3 - Update Air Brake System Status Flow Diagram

The function has five primary processes, which are described in detail in the following subsections:

- *Process Brake Pipe Pressure Data* – Filters the raw brake pipe pressure data to determine the brake pipe pressures, brake pipe pressure gradient and brake pipe pressure reduction (if any).
- *Determine Brake System State* – Determines whether the brake system is charged, releasing/charging, applying or lapped, based on the brake pipe data and the brake system data from the previous time step. This information is used to determine the rate of change of the auxiliary reservoir pressure (if any).
- *Calculate Average Auxiliary Reservoir Pressure* – Determines the average auxiliary reservoir pressure based on the rate of change calculated in the determine brake system state process and logical limits.

- *Calculate Average Brake Cylinder Pressure* – Determines the average brake cylinder pressure based on the average auxiliary reservoir pressure, brake system state, and logical limits to be used for estimating brake retarding force.
- *Calculate Total Brake Shoe Force* – Determines the total brake shoe force based on the average brake cylinder pressure.

Because the air brake system status is dependent on previous status data, each of these processes produces data that is saved for the next time step.

The model of the air brake system described in the following subsections includes a number of parameters, defined below. Each of these is initialized at the time the system is started, and the initialization values are defined in the following parameter descriptions.

- Service brake application time (sec), T_{APPLY}. The time, in seconds, for the average auxiliary reservoir pressure to reach equalization pressure following a service brake application from a fully charged brake pipe pressure. This value is either estimated or calculated based on the brake propagation time adaption and stored with the consist information.
- Quick service brake application time (sec), $T_{APPLY,QUICKSERV}$. The time, in seconds, for the quick service portion of the brake application to reduce the auxiliary reservoir pressure by the first 10psi of the overall brake application. Initialized to $0.2333 * T_{APPLY} + 4.0314$ sec.
- Emergency brake application time (sec), $T_{APPLY,EMG}$. The time, in seconds for the average auxiliary reservoir pressure to reach equalization pressure following an emergency brake application from a fully charged brake pipe pressure. Initialized to $0.2 * T_{APPLY}$ sec.
- Brake system state – One of six states that identify the behavior of the brake system. Initialized to holding emergency.
- Brake pipe pressure (BPP) parameters
    - Brake pipe pressure set point (psi), BPP_{SET}. The brake pipe pressure at a fully charged (released) state. Initialized to 0 psi.
    - Front brake pipe pressure (psi), BPP_{FRONT}. The brake pipe pressure at the head end of the train, as determined from filtering the data reported to the enforcement algorithm from the onboard computer. Initialized to 0 psi.
    - Rear brake pipe pressure (psi), BPP_{REAR}. The brake pipe pressure at the rear end of the train, as determined from filtering the data measured by an EOT device and reported to the enforcement algorithm from the onboard computer. Initialized to 0 psi.
    - Brake pipe pressure differential (psi), BPP_{DIFF}. The filtered difference in brake pipe pressure between the head and rear end of the train. Initialized to $BPP_{FRONT} – BPP_{REAR}$ psi.

    - Brake pipe pressure reduction (psi), BPP_{RED}. The magnitude of the current brake pipe pressure reduction, if any, determined by the difference between the BPP at a fully charged state and the current BPP. Initialized to $BPP_{SET} - BPP_{FRONT}$ psi.
    - Brake pipe pressure rate of change (psi/sec), BPP_{RATE}. The rate at which the brake pipe pressure is currently changing, determined by the difference in brake pipe pressure reduction. Initialized to 0 psi/sec.
    - Hold brake pipe pressure (psi), BPP_{HOLD}. Reference value for determining brake system state changes. Initialized to 0psi.
- Auxiliary reservoir pressure (ARP) parameters
    - Nominal ARP (psi), ARP_{NOM}. The auxiliary reservoir pressure when the brake system is fully charged. Initialized to $BPP_{SET} - \frac{BPP_{DIFF}}{2}$ psi and recomputed when in the fully charged or charging states.
    - ARP equalization (psi), ARP_{EQU}. The equalization pressure for the current brake application. This value is determined using the volume/pressure relationships between the auxiliary reservoir, emergency reservoir, and brake cylinder. Initialized to $\frac{2.5*ARP_{CUR}+BCP}{3.5}$ psi and changes based on the current ARP and BCP, the nominal ARP, and whether or not it is a service or emergency brake application.
    - Reference ARP (psi), ARP_{REF}. A reference parameter used to compute brake cylinder pressure (BCP). Initialized to ARP_{CUR} psi.
    - Current ARP (psi), ARP_{CUR}. The current average auxiliary reservoir pressure. Initialized to ARP_{NOM} psi.
    - ARP rate of change (psi/sec), ARP_{RATE}. The current auxiliary reservoir pressure rate of change. Initialized to 0 psi/sec.
    - ARP recharge breakpoint (psi), $ARP_{BREAKPOINT}$. The ARP where the ARP rate of change will change from the initial charge rate to the final charge rate for the current brake application. Initialized to 0 psi.
- Brake cylinder pressure (psi), BCP. The current average train brake cylinder pressure. Initialized to 0 psi.

F3.3.1 Process Brake Pipe Pressure Data

This function takes the raw front and rear brake pipe pressures and processes them for use in detecting whether a brake application or release is underway. This function is used both in updating the real time status of the brake system, where the input is provided by the onboard system, and when calculating the brake profile, where the input is calculated and provided as an input to the function. In the latter case the processing of the raw data is not necessary, but does not negatively affect the prediction. Performing the filtering in either case reduces the complexity of the overall process. Figure 4 illustrates the flow of the process.

The first function within this process computes the head end brake pipe pressure, BPP_{FRONT}, by averaging the raw head end BPP data from the onboard system for the most recent sample with the previous two samples.

The next function computes the rear end brake pipe pressure, BPP_{REAR}, by averaging the raw rear end BPP data from the onboard system for the most recent sample with the previous two samples. If any of these raw rear BPP data samples indicates a rear BPP less than zero, the raw rear BPP data sample will be set to $BPP_{FRONT} - BPP_{DIFF}$. The intent is for the onboard system to compensate for bad data by sending the enforcement algorithm a negative value for the rear BPP if there is no EOT device, or the EOT device is reporting bad data.

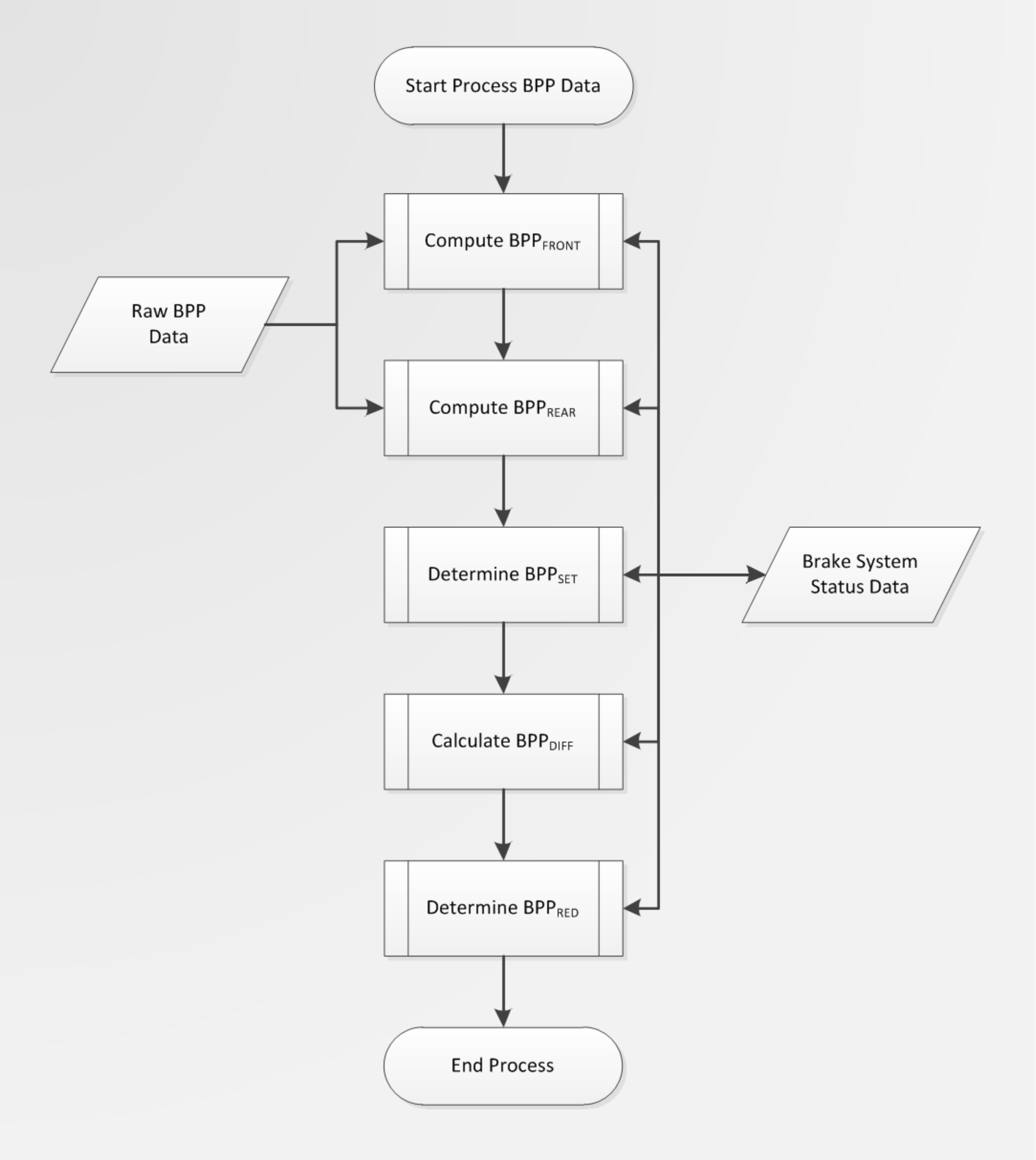

Figure F4 - Process BPP Data Flow Diagram

The Determine BPP_{SET} function is used to determine if the nominal BPP has increased. If $BPP_{FRONT} > BPP_{SET}$, then $BPP_{SET} = BPP_{FRONT}$.

The brake pipe pressure differential is calculated next. The measured brake pipe pressure differential is first determined, according to:

$$BPP_{DIFF,MEAS} = \begin{cases} BPP_{FRONT} - BPP_{REAR}, & if\ BPP_{FRONT} - BPP_{REAR} \leq 40 \\ 5, & if\ BPP_{FRONT} - BPP_{REAR} > 40 \end{cases}$$

The assumption is that if the brake pipe differential is too great, there is likely bad data reported. The brake pipe pressure differential is then filtered using a low pass filter with a 900 second time constant. This compensates for short duration changes in brake pipe pressures to achieve the steady-state brake pipe pressure differential. The formula for this filter is as follows:

$$BPP_{DIFF} = BPP_{DIFF,PREV} + \frac{BPP_{DIFF,MEAS} - BPP_{DIFF,PREV}}{900}$$

The final function of this process determines the depth of the current brake pipe pressure reduction, BPP_{RED} and the current brake pipe pressure rate of change, BPP_{RATE}, if any. These are determined from the following two equations:

$$BPP_{RED} = BPP_{SET} - BPP_{FRONT}$$

$$BPP_{RATE} = BPP_{RED,PREV} - BPP_{RED}$$

These values are used later in the update air brake system status function to identify changes in the brake system state, as described in the next section.

F3.3.2 Determine Brake System State

The D*etermine Brake System State* process uses the current brake pipe pressure, auxiliary reservoir pressure and brake system status to identify changes in the brake system state and compute the auxiliary reservoir pressure rate of change, ARP_{RATE}. This data is used later to determine auxiliary reservoir and brake cylinder pressures and, ultimately, braking force.

The process is a state machine that comprises the following six states:

- Fully charged – The brake pipe and auxiliary reservoir pressures are charged and being held to their set point and the brakes are released.
- Applying service – A service brake pipe pressure reduction is underway, resulting in the control valves directing air from the auxiliary reservoirs to the brake cylinders on each car.
- Applying emergency – The brake pipe pressure is venting at a rapid rate, resulting in the control valves directing air from both the auxiliary reservoirs and emergency reservoirs to the brake cylinders on each car.
- Holding service application – The brake pipe pressure is being held steady at a level below the set point or the auxiliary reservoir and brake cylinder pressures have equalized, resulting in the auxiliary reservoir and brake cylinder pressures holding constant.
- Holding emergency application – The brake pipe pressure is at atmosphere and the auxiliary reservoir, emergency reservoir and brake cylinder pressures have all equalized.
- Charging – The brake pipe pressure is increasing, which results in the control valves directing air from the brake pipe to the auxiliary reservoirs on each car and the brake cylinder pressure venting to atmosphere.

Figure 5 shows a state diagram illustrating the potential state changes between the brake system states listed above. Each state contains its own set of events that will trigger a brake system state change that are reevaluated each time the function is executed. There are also a number of functions that are used in more than one brake system state. The following subsections describe the various brake system states and functions within the determine brake system state process.

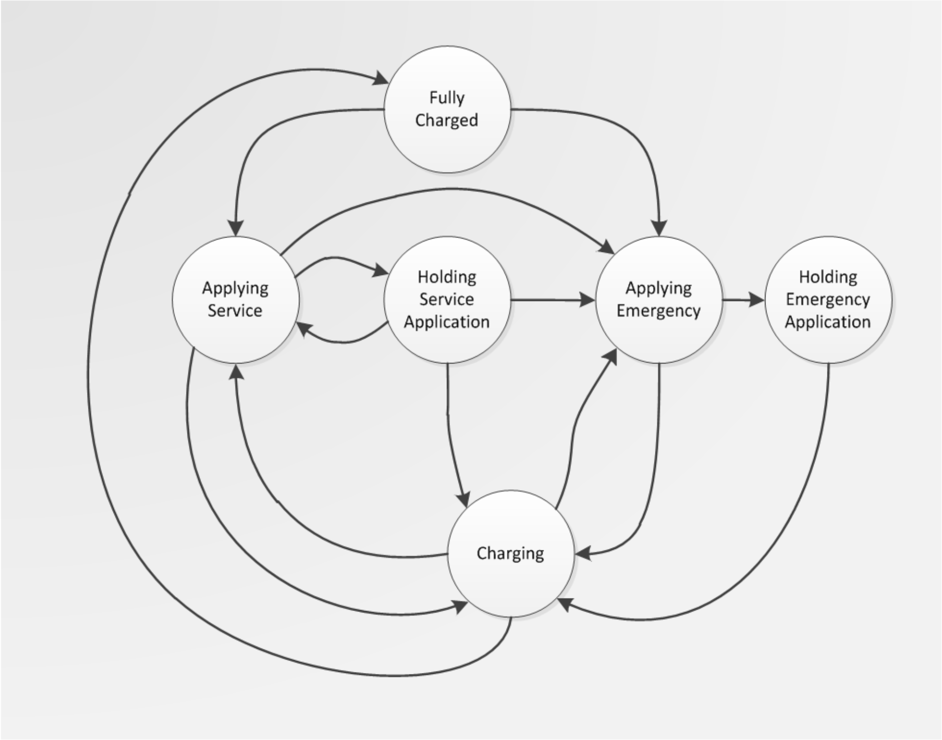

Figure F5 – Brake System State Diagram

Fully Charged Brake System State

When the brake system is fully charged, the brake pipe pressure and auxiliary reservoir pressures are at their set point and there is no pressure in the brake cylinders. From this state, a brake pipe pressure reduction will result in a brake application (service or emergency).

The flow diagram in Figure 6 shows the D*etermine Brake System State* process when the brake system is in the fully charged state. As the diagram shows, when in the fully charged state, the brake system will transition to an applying state (either applying service or applying emergency) if the rate of change of the brake pipe pressure, BPP_{RATE} is less than -1 psi/second or the calculated brake pipe pressure reduction, BPP_{RED}, is greater than 3 psi.

When this occurs, the auxiliary reservoir pressure reference value, ARP_{REF}, is set to the current auxiliary reservoir pressure, ARP_{CUR}, which is used in the brake cylinder pressure calculation, as described in section 3.3.4. The *Initialize Applying State* process, defined in section 3.3.2.7, is then executed, which determines whether a service or emergency brake application is underway and initializes all of the necessary parameters for a brake application.

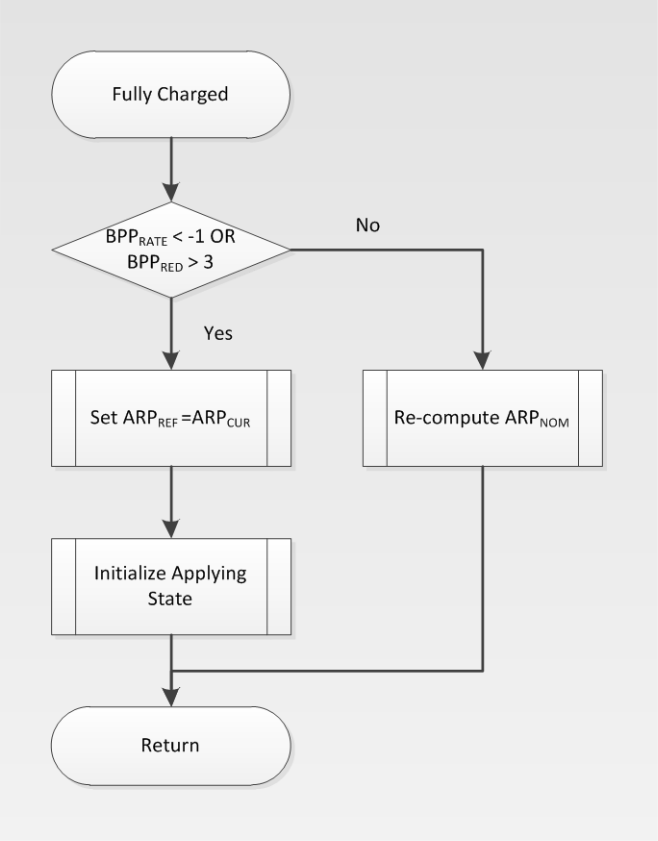

Figure F6 - Fully Charged State Flow diagram

If a brake application is not detected, the brake state remains in the fully charged state and the nominal auxiliary reservoir pressure, ARP_{NOM} is recalculated using the following equation:

$$ARP_{NOM} = BPP_{SET} - \frac{BPP_{DIFF}}{2}$$

Applying Service Brake System State

As the state diagram in Figure 5 shows, the applying service state can transition to the holding service application state, the charging state, or the applying emergency state. The events that trigger these transitions are illustrated in Figure 7, which shows the flow diagram for the applying service state.

As Figure 7 shows, if the head end brake pipe pressure, BPP_{FRONT}, has lowered, the hold pressure, BPP_{HOLD}, is set to this value. This hold pressure is used in other brake states to determine where the brake pipe pressure is relative to the level it was at during the application.

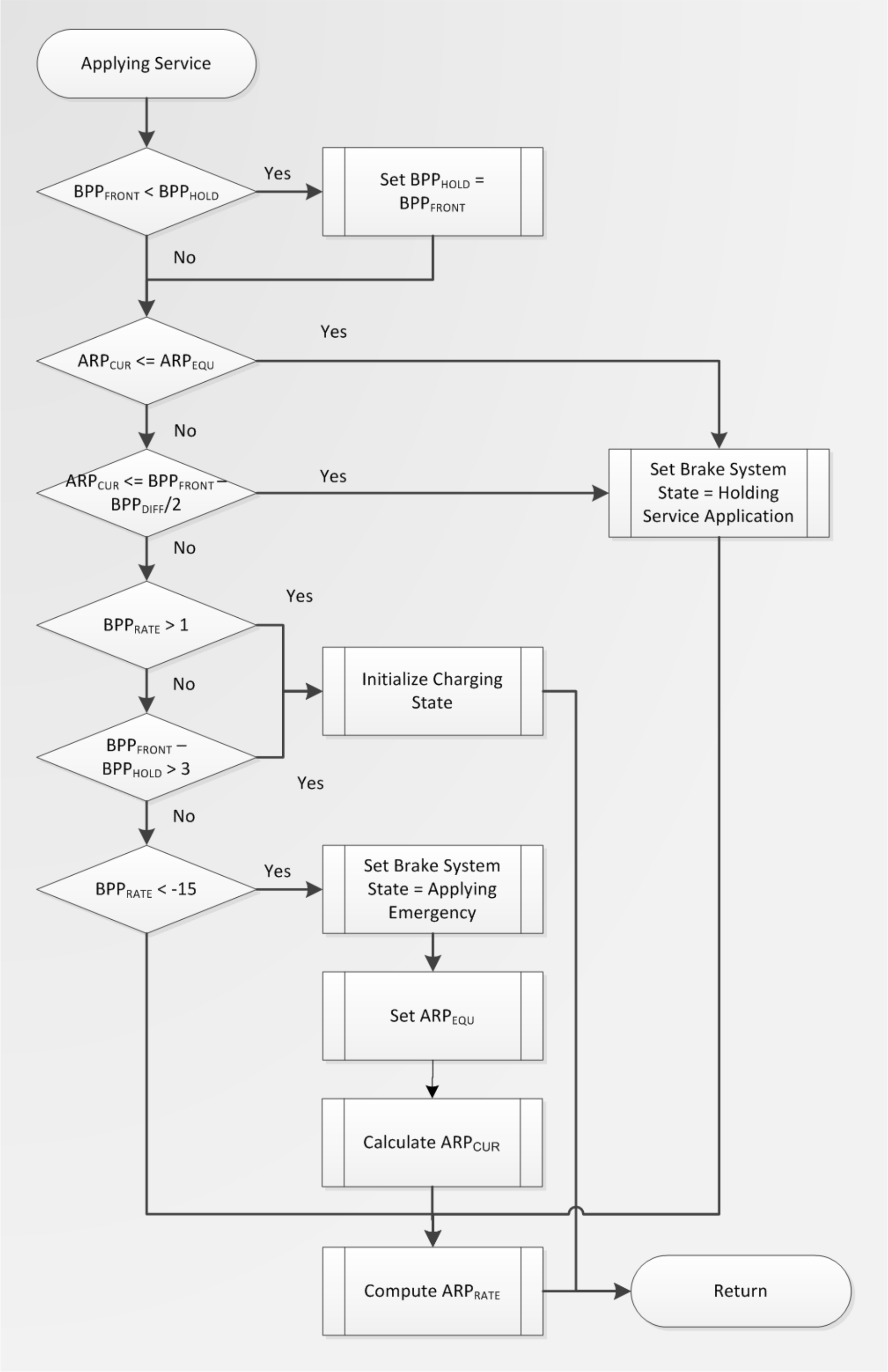

Figure F7 - Applying Service State Flow Diagram

There are two potential conditions that, if met, will cause the brake system to transition to the holding service application state:

1. $ARP_{CUR} \leq ARP_{EQU}$
2. $ARP_{CUR} \leq BPP_{FRONT} - \frac{BPP_{DIFF}}{2}$

In the first condition, the brake system has reached equalization and the auxiliary reservoir pressure cannot drop any further. In the second, the brake pipe pressure has steadied, meaning that the auxiliary reservoir pressure will not drop any further until a deeper brake pipe pressure reduction has been made. When either of these conditions is met, the brake system state is set to the holding service application state and the C*ompute Auxiliary Reservoir Pressure Rate of Change* function, defined in section 3.3.2.9, is executed.

A transition to the charging state will be triggered by either of the following:

1. $BPP_{RATE} > 1$
2. $BPP_{FRONT} - BPP_{HOLD} > 3$

In the first condition, the brake pipe pressure has started to rise fast enough to indicate that a brake release is in progress, and in the second, the brake pipe pressure has risen far enough to indicate a brake release has occurred. In either case, the I*nitialize Charging State* function, defined in section 3.3.2.8, is executed.

If the rate of change of the brake pipe pressure, $\mathrm{BPP}_{\mathrm{RATE}}$, is less than -15 psi/second, the brake pipe pressure is venting rapidly enough for the control valves to trigger an emergency application, and the brake system state will transition to the applying emergency state. At this point, the control valves will direct air from both the auxiliary and emergency reservoirs to the brake cylinder until all three reservoirs equalize. Therefore, the equalization pressure, $\mathrm{ARP}_{\mathrm{EQU}}$, must be re-calculated. The new equalization pressure is determined using Boyle's law, as described in section 3.3.2.7:

$$ARP_{EQU} = \frac{2.5 * ARP_{CUR} + BCP + 3.5 * ARP_{NOM}}{7}$$

The auxiliary reservoir pressure, $\mathrm{ARP}_{\mathrm{CUR}}$, is then recalculated to model the combining of the emergency reservoir and the auxiliary reservoir, as follows:

$$ARP_{CUR} = \frac{2.5 * ARP_{CUR} + 3.5 * ARP_{NOM}}{6}$$

The rate of change of the auxiliary reservoir pressure, $\mathrm{ARP}_{\mathrm{RATE}}$, is then recalculated, as described in section 3.3.2.9.

If none of the above conditions are met, the brake system will remain in the applying service state. The rate of change of the auxiliary reservoir pressure is then recalculated by calling the *Compute Auxiliary Reservoir Pressure Rate of Change* function, which is defined in section 3.3.2.9.

Applying Emergency Brake System State

The process flow for the applying emergency brake state is very similar to that of the applying service brake state. Figure 8 shows the flow diagram for the applying emergency brake state.

Similar to the applying service brake state function, this function begins by setting the hold pressure, BPP_{HOLD}, to the head end brake pipe pressure, BPP_{FRONT}, when the head end brake pipe pressure has lowered.

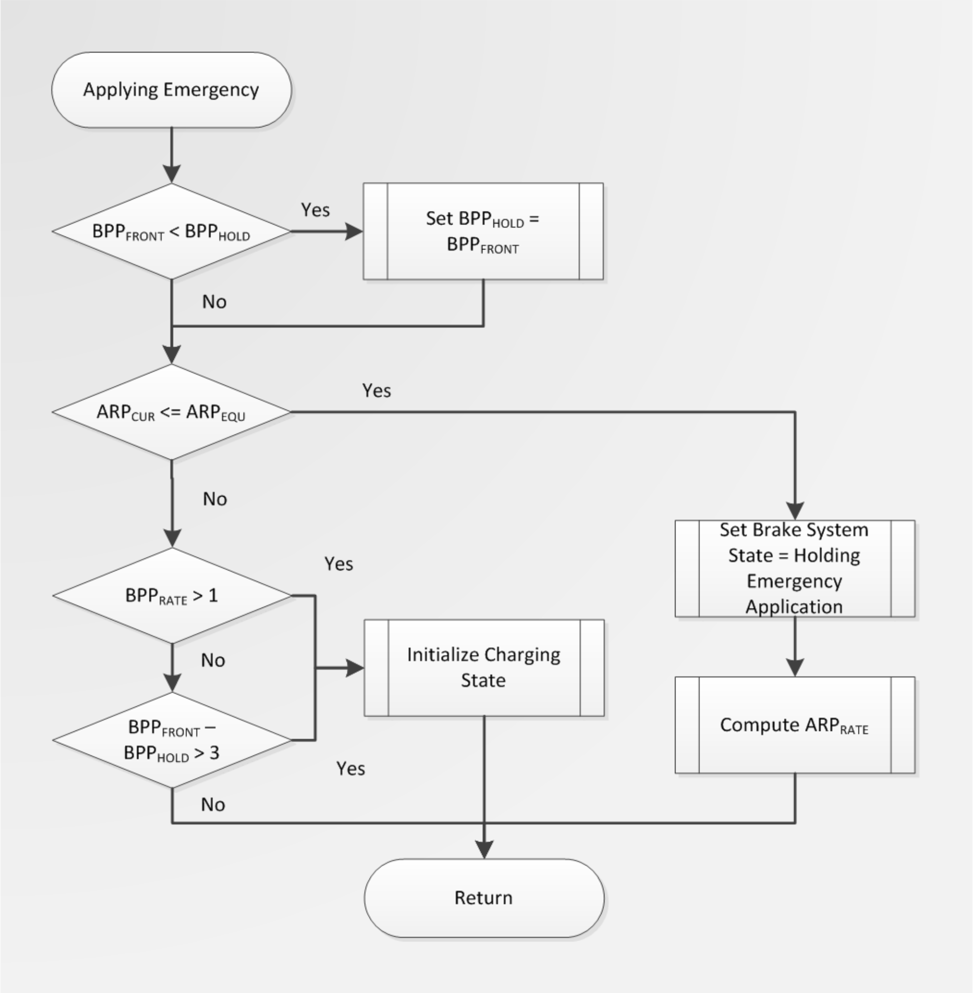

Figure F8 - Applying Emergency State Flow Diagram

In the applying emergency state, the brake system will transition to the holding emergency application state, when the auxiliary reservoir pressure has reached equalization:

$$ARP_{CUR} \leq ARP_{EQU}$$

When this occurs, the brake system state is set to holding emergency application and the auxiliary reservoir pressure rate of change, ARP_{RATE}, is recomputed, as described in section 3.3.2.9.

The brake system will transition to the charging state upon satisfying either of the following conditions:

1. $BPP_{RATE} > 1$
2. $BPP_{FRONT} - BPP_{HOLD} > 3$

Similar to the applying service brake state, the first condition indicates the brake pipe pressure has started to rise fast enough to determine that a brake release is in progress, and in the second condition, the brake pipe pressure has risen far enough to determine that a brake release has occurred. When this happens, the *Initialize Charging State* function, which is described in section 3.3.2.8, is executed.

If none of the conditions described above are satisfied, the brake state will remain in the applying emergency state until the next time step.

Holding Service Application Brake System State

If the brake system state is set to holding service application, the process flow depicted in Figure 9 is followed. As the diagram shows, a brake release is detected when either of the following is true, as discussed in the applying service and applying emergency brake system states:

- $BPP_{RATE} > 1$
- $BPP_{FRONT} - BPP_{HOLD} > 3$

When a brake release is detected, the *Initialize Charging State* function, defined in section 3.3.2.8, is called.

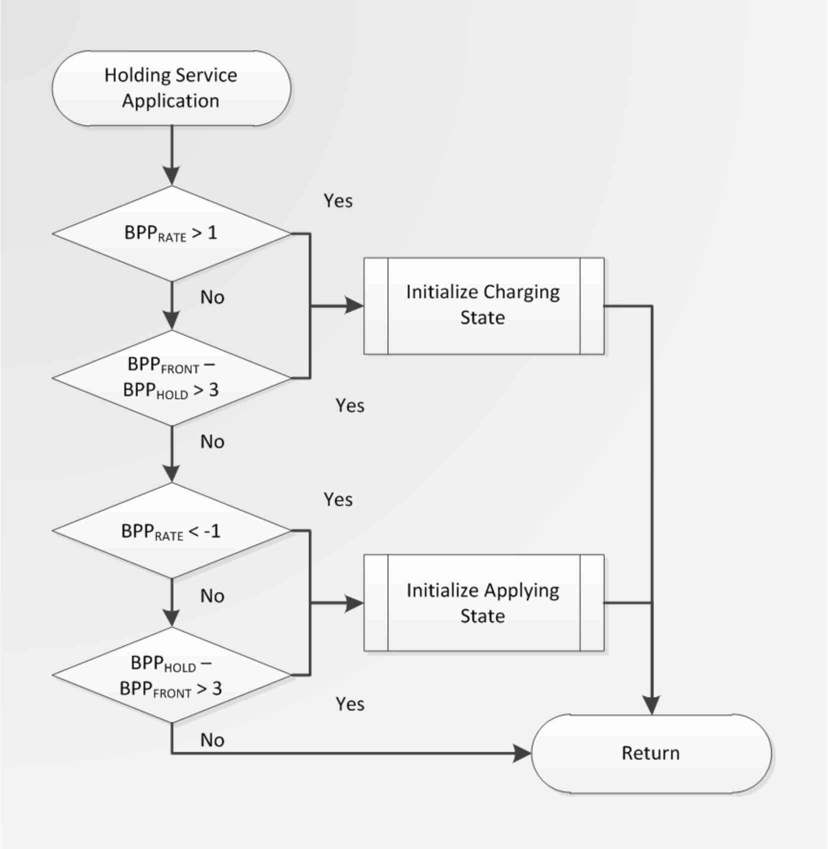

Figure F9 - Holding Service Application State Flow Diagram

If, instead, the brake pipe pressure begins to drop, as indicated by the following conditions, a deeper brake application is detected:

- $BPP_{RATE} < -1$
- $BPP_{HOLD} - BPP_{FRONT} > 3$

When this occurs, the *Initialize Applying State* function, defined in section 3.3.2.7, is called, which transitions the brake system back to one of the applying states (applying service or applying emergency).

If neither a brake set nor a brake release is detected, the brake system will remain in the holding service application state.

Holding Emergency Application Brake System State

When the brake system is in the holding emergency application state, the only transition that can occur is to the charging state when a brake release is detected. No further application can occur, because the brake pipe pressure has been depleted.

This flow of this process is shown in Figure 10. A brake release is detected when the brake pipe pressure rate of change, BPP_{RATE}, is greater than 1 psi/second. If this occurs, the *Initialize Charging State* function, defined in section 3.3.2.8 is called. If a brake release is not detected, the brake system remains in the holding emergency application state.

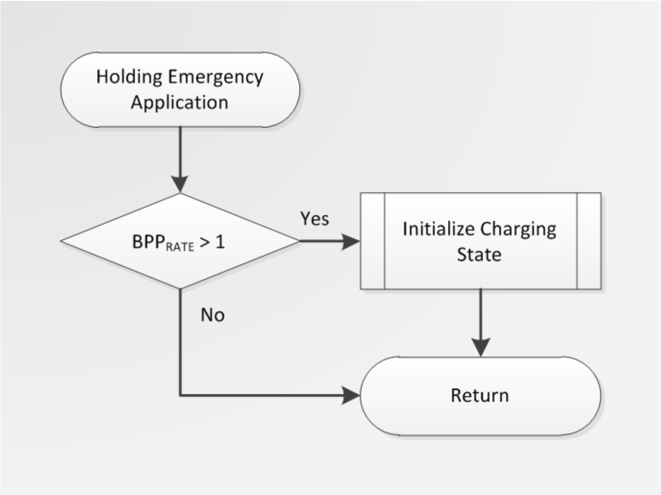

Figure F10 - Holding Emergency Application State Flow Diagram

Charging Brake System State

The process flow for the charging brake system state is illustrated in Figure 11. From this state, a brake application is detected when the brake pipe pressure is reduced, as indicated by either of the following two conditions:

- $BPP_{RATE} < -1$
- $BPP_{HOLD} - BPP_{FRONT} > 3$

When this occurs, the reference pressure, ARP_{REF}, is reset to the current auxiliary reservoir pressure, ARP_{CUR}. This value is used in the *Compute Average Brake Cylinder Pressure* function, defined in section 3.3.4, and effectively limits the amount of brake cylinder pressure that can build, due to the fact that there is less than full auxiliary reservoir pressure when the brake application is initiated. The *Initialize Applying State* function, defined in section 3.3.2.7, is then called to transition the brake state to the appropriate applying state.

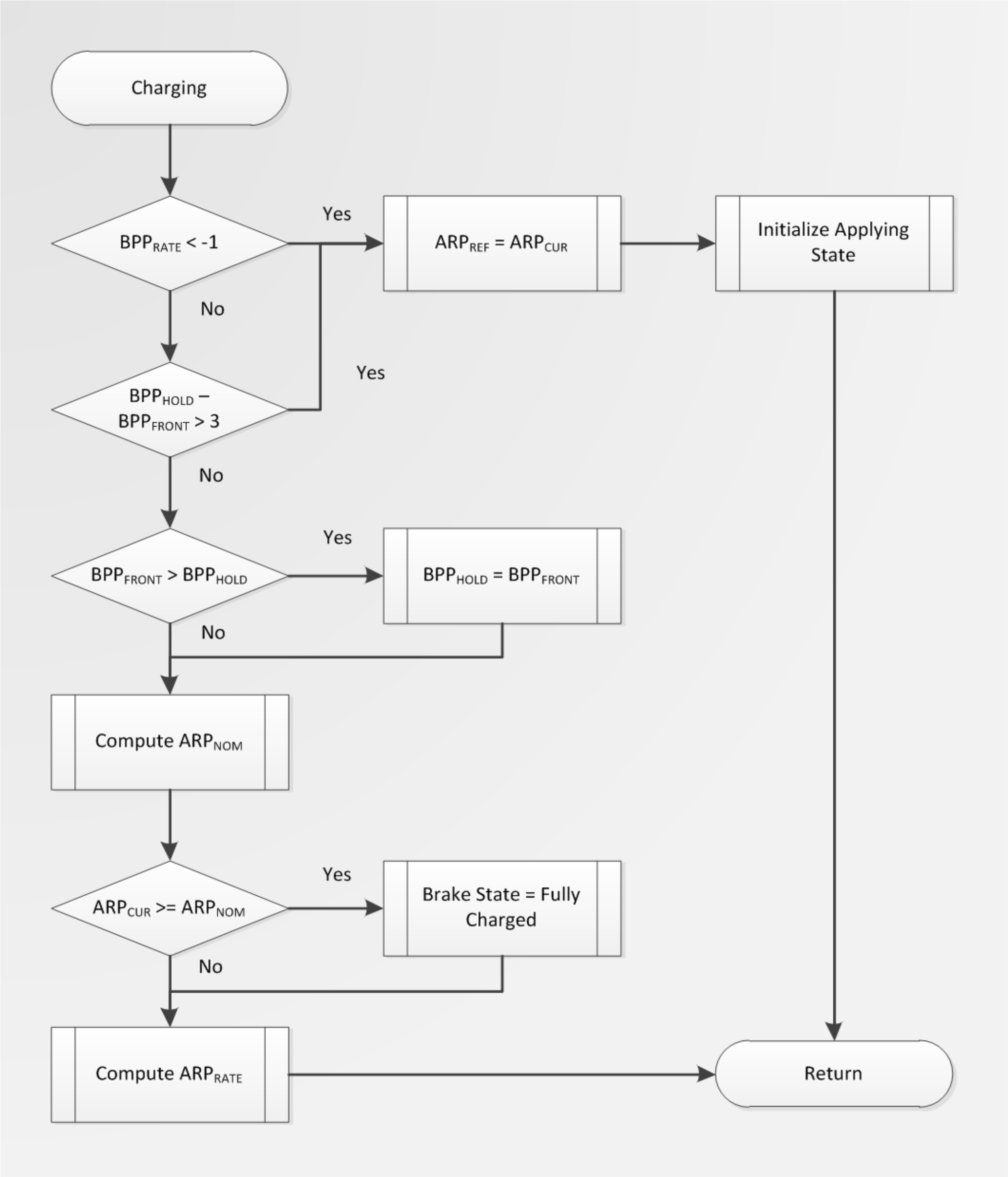

Figure F11 - Charging State Flow Diagram

If a brake application is not detected, the brake pipe hold pressure, BPP_{HOLD}, is increased to the current head end brake pipe pressure, BPP_{FRONT}. The nominal auxiliary reservoir pressure, ARP_{NOM}, is recomputed, according to the following equation:

$$ARP_{NOM} = BPP_{SET} - \frac{BPP_{DIFF}}{2}$$

A check is then made to determine if the brake system has completed the recharge process. If the current auxiliary reservoir pressure, ARP_{CUR}, is greater than or equal to the nominal auxiliary reservoir pressure, ARP_{NOM}, the brake state is set to fully charged. Otherwise, the brake system remains in the charging state.

Finally, the auxiliary reservoir pressure rate of change, ARP_{RATE}, is recomputed, as described in section 3.3.2.9.

Initialize Applying State Process

The I*nitialize Applying State Process* is called whenever a brake application is detected during the *Determine Brake State* process. The process determines what type of brake application (service or emergency) has been initiated and initializes the appropriate parameters for monitoring the brake system during the application. The process flow is illustrated in Figure 12.

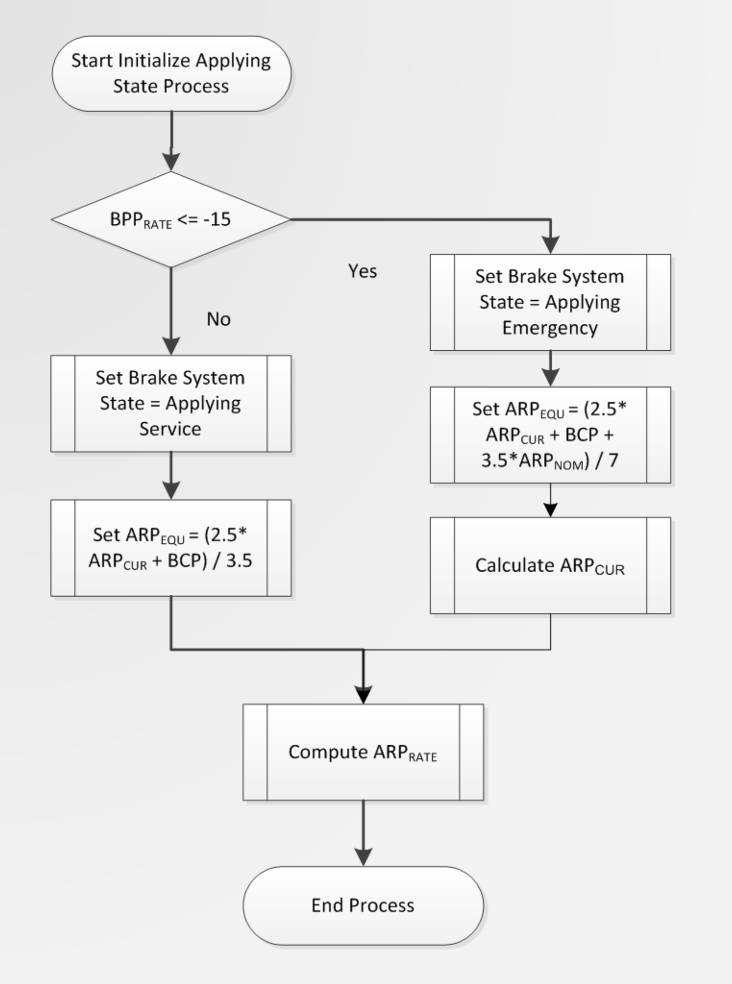

Figure F12 - Initialize Applying State Flow Diagram

The type of brake application that has been initiated is determined by the brake pipe pressure rate of change, BPP_{RATE}. An emergency brake application is indicated by the brake pipe pressure venting at a high rate. If the following condition is met, the air brake model assumes an emergency brake application is underway, and the brake state is set to applying emergency:

$$BPP_{RATE} \leq -15$$

If the above condition is not met, it is assumed that a service brake application is underway, and the brake state is set to applying service.

The equalization pressure, ARP_{EQU}, is then calculated for the application, using Boyle's law, which states that the product of the volume and pressure in a system must remain constant. If an emergency application is underway, the control valves will direct air from both the auxiliary and emergency reservoirs to the brake cylinder until all three reservoirs equalize. The volume pressure relationship can be described as follows:

$$ARP_{EQU} * (V_{AR} + V_{BC} + V_{ER}) = V_{AR} * ARP_{CUR} + V_{BC} * BCP + V_{ER} * ERP_{CUR}$$

Where V_{AR} is the volume of the auxiliary reservoir, V_{BC} is the volume of the brake cylinder, V_{ER} is the volume of the emergency reservoir, and ERP_{CUR} is the current emergency reservoir pressure. Since the emergency reservoir is fully charged, the nominal auxiliary reservoir pressure, ARP_{NOM}, can be used in place of the emergency reservoir pressure. The ratio of the volumes of the emergency reservoir, auxiliary reservoir, and brake cylinder is known to be 3.5:2.5:1. Using these assumptions, the new equalization pressure, ARP_{EQU}, can be calculated as follows:

$$ARP_{EQU} = \frac{2.5 * ARP_{CUR} + BCP + 3.5 * ARP_{NOM}}{7}$$

The auxiliary reservoir pressure, ARP_{CUR}, is then recalculated to model the combining of the emergency reservoir and the auxiliary reservoir, as follows:

$$ARP_{CUR} = \frac{2.5 * ARP_{CUR} + 3.5 * ARP_{NOM}}{6}$$

If a service application is determined to be in progress, the control valves will only direct air from the auxiliary reservoirs to the brake cylinders, and therefore the equalization pressure, ARP_{EQU}, is set according to the following equation:

$$ARP_{EQU} = \frac{2.5 * ARP_{CUR} + BCP}{3.5}$$

Once the equalization pressure is set, the auxiliary reservoir pressure rate of change, ARP_{RATE} is computed, according to the function defined in section 3.3.2.9.

Initialize Charging State Process

The *Initialize Charging State* process initializes the brake system parameters necessary to monitor the brake system during a brake release. The following three tasks are performed as part of this process:

- The brake system state is set to charging
- The ARP recharge breakpoint, $ARP_{BREAKPOINT}$, is set
- The auxiliary reservoir pressure rate of change is set

The ARP recharge breakpoint, $ARP_{BREAKPOINT}$, is used in the calculation of the auxiliary reservoir pressure rate of change, to determine which charging rate should be used. The first half of the auxiliary reservoir pressure is charged at the initial rate, and the remaining is charged at the secondary charging rate. The ARP recharge breakpoint, $ARP_{BREAKPOINT}$, is therefore set according to the following equation:

$$ARP_{BREAKPOINT} = \frac{ARP_{NOM} + ARP_{CUR}}{2}$$

The auxiliary reservoir pressure rate of change is calculated according to the process described in section 3.3.2.9.

Compute Auxiliary Reservoir Pressure Rate of Change Process

The Compute Auxiliary Reservoir Pressure Rate of Change process is used by many of the brake states to determine the rate at which the auxiliary reservoir pressure will change over the next time step. Figure 13 shows a flow diagram for the process.

The calculation of the auxiliary reservoir pressure rate of change, ARP_{RATE}, depends primarily on the current state of the brake system. If the brake system is in the applying emergency state, the auxiliary reservoir pressure rate of change, ARP_{RATE}, is calculated based on the emergency application time, $T_{APPLY,EMG}$, according to the following equation:

$$ARP_{RATE} = \frac{ARP_{EQU} - ARP_{CUR}}{T_{APPLY,EMG}}$$

If the brake system is in the applying service state, a check is made to determine if the quick service rate should be used. The quick service portion of the brake application is a result of a function on modern control valves where the application rate of the brakes is increased at the beginning of the application by local venting of a small amount of brake pipe pressure. If the auxiliary reservoir pressure, ARP_{CUR} is within 10 psi of the brake pipe pressure set point, BPP_{SET}, the quick service rate is used and ARP_{RATE} is set according to the following equation:

$$ARP_{RATE} = \frac{-10}{T_{APPLY,QUICKSERV}}$$

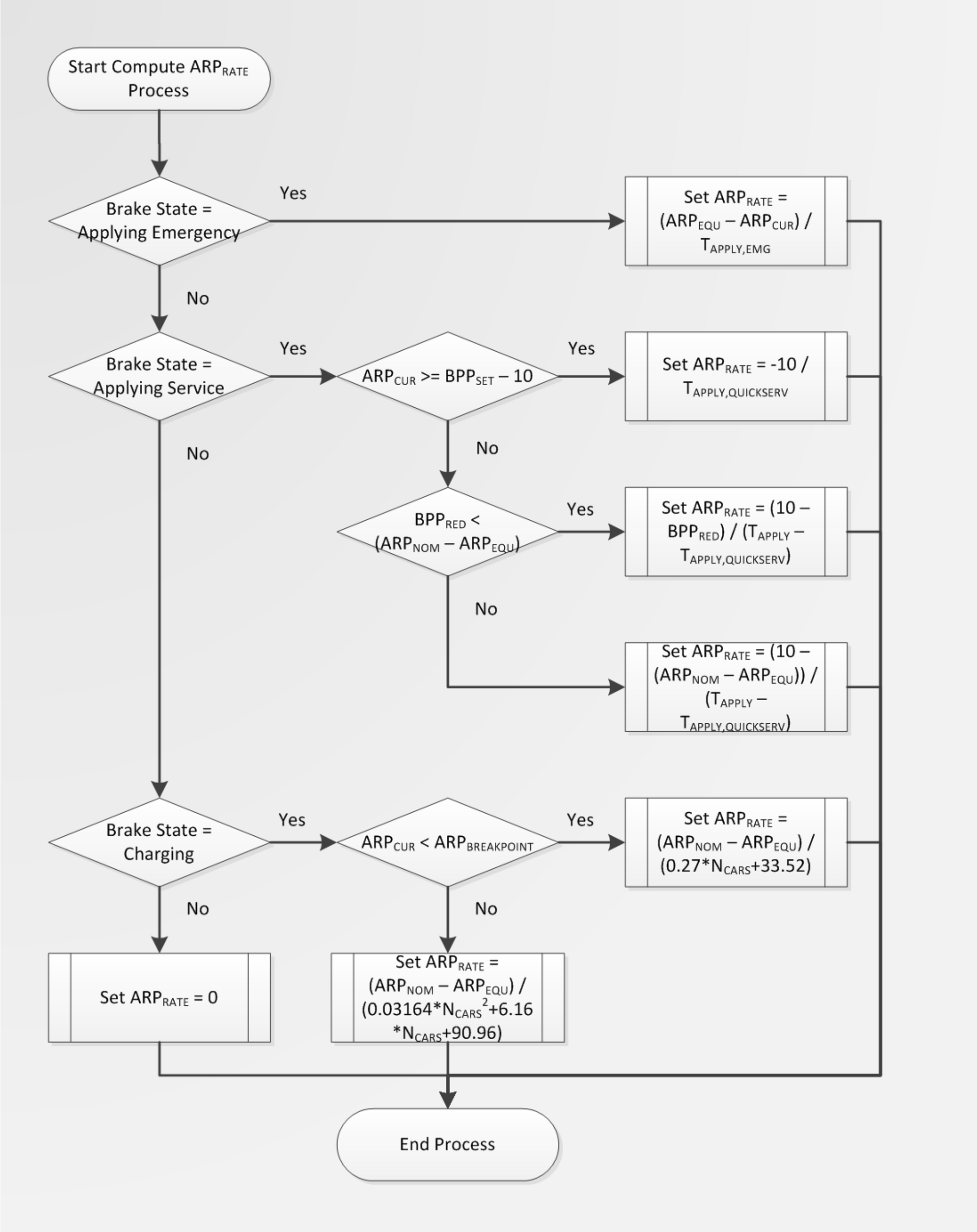

Figure F13 - Compute ARP$_{\mathrm{RATE}}$ Process Flow Diagram

If the auxiliary reservoir pressure, $\mathrm{ARP}_{\mathrm{CUR}}$, is more than 10 psi below the brake pipe pressure set point, $\mathrm{BPP}_{\mathrm{SET}}$, the normal service rate is used, which is dependent on the magnitude of the brake pipe pressure reduction, $\mathrm{BPP}_{\mathrm{RED}}$, up to the equalization point. If the brake pipe pressure reduction, $\mathrm{BPP}_{\mathrm{RED}}$, is less than the difference between the nominal auxiliary reservoir pressure, $\mathrm{ARP}_{\mathrm{NOM}}$, and the equalization pressure, $\mathrm{ARP}_{\mathrm{EQU}}$, the rate is calculated according to:

$$ARP_{RATE} = \frac{10 - BPP_{RED}}{T_{APPLY} - T_{APPLY,QUICKSERV}}$$

If the brake pipe pressure reduction, BPP_{RED}, is greater than or equal to the difference between the nominal auxiliary reservoir pressure, ARP_{NOM}, and the equalization pressure, ARP_{EQU}, the rate is calculated according to:

$$ARP_{RATE} = \frac{10 - \left(ARP_{NOM} - ARP_{EQU}\right)}{T_{APPLY} - T_{APPLY,QUICKSERV}}$$

If the brake system is in the charging state, the air brake model predicts that the first half of the recharge process will be at a faster rate than the second half. The two recharge rates are related to the number of cars in the train. A check is made to determine which charge rate should be used. If the auxiliary reservoir pressure, ARP_{CUR}, is less than the recharge breakpoint, $ARP_{BREAKPOINT}$, the initial charge rate is used and ARP_{RATE} is set according to:

$$ARP_{RATE} = \frac{ARP_{NOM} - ARP_{EQU}}{0.27 * N_{CARS} + 33.52}$$

If the auxiliary reservoir pressure, ARP_{CUR}, is greater than or equal to the recharge breakpoint, $ARP_{BREAKPOINT}$, the final charge rate is used and ARP_{RATE} is set according to:

$$ARP_{RATE} = \frac{ARP_{NOM} - ARP_{EQU}}{0.03164 * N_{CARS}^2 + 6.16 * N_{CARS} + 90.96}$$

If the brake system is in any state other than those discussed above (i.e. fully charged, holding service application or holding emergency application), the auxiliary reservoir pressure is not changing and ARP_{RATE} is set to 0 psi/second.

F3.3.3 Calculate Average Auxiliary Reservoir Pressure

The *Calculate Average Auxiliary Reservoir Pressure* process uses the auxiliary reservoir pressure rate of change, ARP_{RATE}, calculated during the determine brake system state process to update the current auxiliary reservoir pressure, ARP_{CUR}, according to the following equation:

$$ARP_{CUR} = ARP_{PREV} + ARP_{RATE}$$

The auxiliary reservoir pressure is limited to the nominal auxiliary reservoir pressure, ARP_{NOM}, and the equalization pressure, ARP_{EQU}, by the following relationship:

$$ARP_{EQU} \leq ARP_{CUR} \leq ARP_{NOM}$$

F3.3.4 Calculate Average Brake Cylinder Pressure

The C*alculate Average Brake Cylinder Pressure* process determines the current brake cylinder pressure, BCP, based on the brake system state, the auxiliary reservoir pressure, ARP_{CUR}, and the auxiliary reservoir reference pressure, ARP_{REF}. The average brake cylinder pressure is used later in the algorithm to determine the magnitude of the retarding force on the train due to the air brakes.

If the brake state is either charging or fully charged, the brake cylinder pressure, BCP, is set to 0 psi. This assumes that the brake cylinder pressure is vented to atmosphere instantaneously upon detecting a brake release. Although this is not actually true, it represents a conservative assumption and eliminates the complexity of modeling the venting of the brake cylinder.

If the brake state is either applying emergency or holding emergency application, the brake cylinder pressure is set according to the following equation:

$$BCP = 6 * (ARP_{REF} - ARP_{CUR})$$

If the brake state is either applying service or holding service application, the brake cylinder pressure is set to the greater of the result of the following two equations:

1. $BCP = \left(3.127 * (ARP_{REF} - ARP_{CUR})\right) - 9.9$
2. $BCP = 1.579 * (ARP_{REF} - ARP_{CUR})$

F3.3.5 Calculate Brake Shoe Force

The brake shoe force, $F_{B,SHOE}$, is the sum of the force applied by the brake shoes to the wheels. It is calculated based on the nominal brake force for the train, $F_{B,NOM}$, the current average brake cylinder pressure, BCP, and the efficiency of the brake rigging at the current brake cylinder pressure.

The total current brake shoe force, $F_{B,SHOE}$, is calculated by multiplying the nominal brake force, $F_{B,NOM}$, the ratio of the current brake cylinder pressure, BCP, to the control pressure of 64 psi, and the relative efficiency of the brake rigging, Eff:

$$F_{B,SHOE} = F_{B,NOM} * \frac{BCP}{64} * Eff$$

As brake cylinder pressure increases, the spring in the brake cylinder is overcome and the slack in the brake rigging is taken up, increasing the efficiency of the brake rigging. The relative efficiency of the brake rigging is the efficiency at the current brake cylinder pressure relative to the efficiency at the control pressure of 64 psi. The relative efficiency of the brake rigging at the current brake cylinder pressure is determined by:

$$Eff = \begin{cases} 0 & for\ BCP < 8psi \\ 1.14 - \dfrac{8.97}{BCP} & for\ BCP \geq 8psi \end{cases}$$

F3.4 Update Track Grade and Curvature

The purpose of the update track grade and curvature process is to determine the average grade and total curvature under the train to be used later in calculating the forces acting on the train. This function is used both to monitor the real time track grade and curvature under the train and to provide track grade and curvature data for the braking profile prediction.

The process described here assumes that the weight of the train is uniformly distributed throughout the length of the train. A method for determining track grade and track curvature forces for a train with non-uniform distribution of weight along the train may be provided in later versions.

F3.4.1 Update Track Grade

Track grade information is obtained using the location of the head end of the train, the length of the train, and the track grade in the track database. The average grade under the train is used to determine the grade force acting on the train.

The average percent grade under the train is calculated using the following equation:

$$\%Grd_{AVG} = \frac{Elev_{HE} - Elev_{TE}}{L_{TRAIN}} \times 100$$

Where $\%Grd_{AVG}$ is the average percent grade under the train, $Elev_{HE}$ is the track elevation at the location of the head end of the train, $Elev_{TE}$ is the track elevation at the location of the tail end of the train, and L_{TRAIN} is the total length of the train.

F3.4.2 Update Track Curvature

The degree of track curvature is traditionally defined as the central angle turned over a 100 foot section of track. This definition is useful for determining train resistance due to track curvature. To calculate the resistance over the entire length of the train, the average degree of curvature over the entire length of the train is used. The average degree of track curvature over the length of the train is defined as the equivalent constant degree of curvature over the length of the train expressed in degrees per 100 foot track section. For example, if one half of a train of uniformly distributed mass was on a 2 degree curve, while the other half was on a tangent section of track, the average track curvature over the length of the train would be 1 degree.

To determine the average degree curvature, the track centerline coordinates along the length of the train are used. Figure 14 shows three consecutive track centerline coordinates from the track database, points A, B and C. The lines connecting each consecutive set of track centerline coordinates define two sections of track, AB and BC, between the sets of coordinates. The heading over each track section is defined by the angle created between the track section and the vertical line. The change in heading over section AB is the difference between these angles, as shown on the figure.

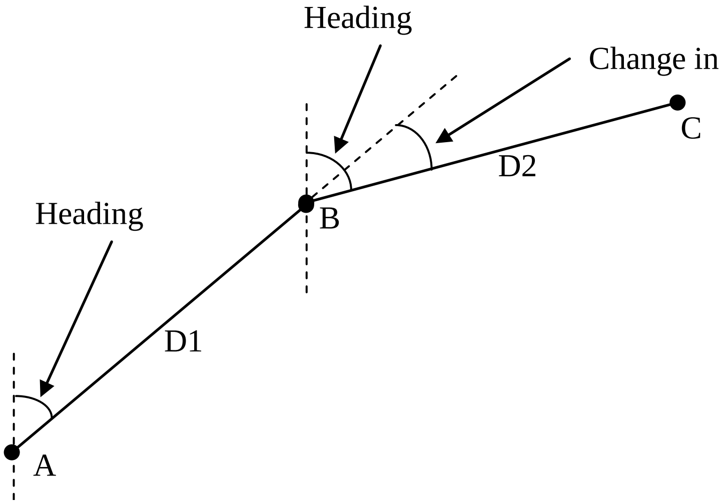

Figure F14 – Track Curvature Determination

The total change in heading over the length of the train, ΔHdg_{TOT}, can then be determined by summing the absolute value of the change in heading of each individual track section, ΔHdg_{SEC}, between the head and rear of the train:

$$Hdg_{TOT} = \sum |Hdg_{SEC}|$$

The absolute value in this equation ensures that the total change in heading will be additive regardless of any change in curve direction over the length of the train.

The average degree of curvature, Crv_{AVG}, can then be determined by dividing the total change in heading, Hdg_{TOT}, by the train length, L_{TRAIN}, and multiplying by 100:

$$Crv_{AVG} = \frac{Hdg_{TOT}}{L_{TRAIN}} * 100$$

F3.5 Calculate Train Forces

The *Calculate Train Forces* process performs calculations to determine the net force acting on the train (without dynamic brake force, which is determined later), both in real time and during the braking profile prediction. The net force acting on the train at any given time can be modeled as the sum of the various independent forces acting on the train, as follows:

$$F_{NET} = \sum F = F_{LOC} + F_{GRD} + F_{CRV} + F_{RES} + F_{BRK}$$

Where F_{NET} is the net force acting on the train, F_{LOC} is the tractive force generated by the locomotives, F_{GRD} is the grade force, F_{CRV} is the curving resistance, F_{RES} is the net resistive forces acting on the train due to aerodynamic, wheel/rail, and bearing resistance, and F_{BRK} is the retarding force from the air brake system.

F3.5.1 Calculate Locomotive Force

During a brake application, the tractive effort produced by the locomotives, F_{LOC}, is assumed to be zero.

F3.5.2 Calculate Grade and Curving Forces

The grade force, F_{GRD}, is computed using the following equation:

$$F_{GRD} = -20 * W_{TRAIN} * \%Grd_{AVG}$$

Where W_{TRAIN} is the weight of the train in tons and $\%Grd_{AVG}$ is the average percent grade under the train, as described in section 3.4.1. The negative sign in the above equation serves to produce a positive force for a negative (downhill) grade, tending to accelerate the train, and a negative force for a positive (uphill) grade, tending to decelerate the train.

The curving force, F_{CRV}, is determined by from the following equation:

$$F_{CRV} = -0.8 * W_{TRAIN} * Crv_{AVG}$$

Where W_{TRAIN} is the weight of the train in tons and Crv_{AVG} is the average degree of curvature over the length of the train, as described in section 3.4.2. The negative sign in this equation serves to produce a result that is always negative, tending to decelerate the train, regardless of the direction of the curve.

F3.5.3 Calculate Resistive Force

The total train resistive force, F_{RES}, is the sum of the resistive forces acting on the locomotives and the resistive forces acting on the trailing cars. The resistive forces are calculated using a form of the Modified Davis Equation, which is used to calculate the resistance of a given rail vehicle:

$$R_u = 0.6 + \frac{20}{w} + 0.01V + \frac{KV^2}{wn}$$

Where R_u is the vehicle resistance in lbs/ton, w is the weight per axle in tons, n is the number of axles on the vehicle, V is the vehicle speed in mph, and K is the aerodynamic drag coefficient for the vehicle. Multiplying this equation by the weight of the vehicle in tons, W_{VEH}, gives the resistance in lbs/vehicle, R_{VEH}:

$$R_{VEH} = 0.6W_{VEH} + 20n + 0.01W_{VEH}V + KV^2$$

Multiplying this equation by the number of cars, N_{CARS}, and locomotives, N_{LOCS}, gives the resistance in lbs for the train:

$$R_{TRAIN} = 0.6W_{TRAIN} + 20n_{TOTAL} + 0.01W_{TRAIN}V + (K_{LOCOS}N_{LOCS} + K_{CARS}N_{CARS})V^2$$

Where W_{TRAIN} is the total weight of the train in tons, n_{TOTAL} is the total number of axles in the train, K_{LOCS} is the aerodynamic coefficient for locomotives and K_{CARS} is the aerodynamic coefficient for trailing cars. The following aerodynamic coefficients for locomotives and trailing cars are assumed:

$$K = \begin{cases} 0.294\ for\ locomotives \\ 0.07\ for\ trailing\ cars \end{cases}$$

Substituting in the aerodynamic coefficients and introducing a negative sign to produce a negative result, tending to decelerate the train, results in the following equation for the resistive forces acting on the train:

$$F_{RES} = -(0.6W_{TRAIN} + 20n_{TOTAL} + 0.01W_{TRAIN}V + (0.294N_{LOCS} + 0.07N_{CARS})V^2)$$

F3.5.4 Calculate Brake Force

The brake force, F_{BRK}, is the product of the brake shoe force, $F_{B,SHOE}$, and the coefficient of friction between the brake shoe and the wheel, $COF_{SHOE,WHEEL}$. The coefficient of friction between the brake shoe and the wheel is given by the following equation:

$$COF_{SHOE,WHEEL} = 0.255 + 0.11e^{-0.07v}$$

Where v is the current velocity in mph. The total brake force, F_{BRK}, is then given by:

$$F_{BRK} = -F_{B,SHOE} * COF_{SHOE,WHEEL}$$

Where the negative sign results in a negative value for the brake force, tending to decelerate the train.

F3.6 Estimate Dynamic Brake Force

The *Estimate Dynamic Brake Force* process uses acceleration and force data acting on the train to estimate the level of dynamic braking, if any, to be used in the braking profile prediction processes. The algorithm assumes that the current dynamic brake setting will remain following a PTC penalty or emergency brake application. The *Estimate Dynamic Brake Force* process flow is shown in figure 15.

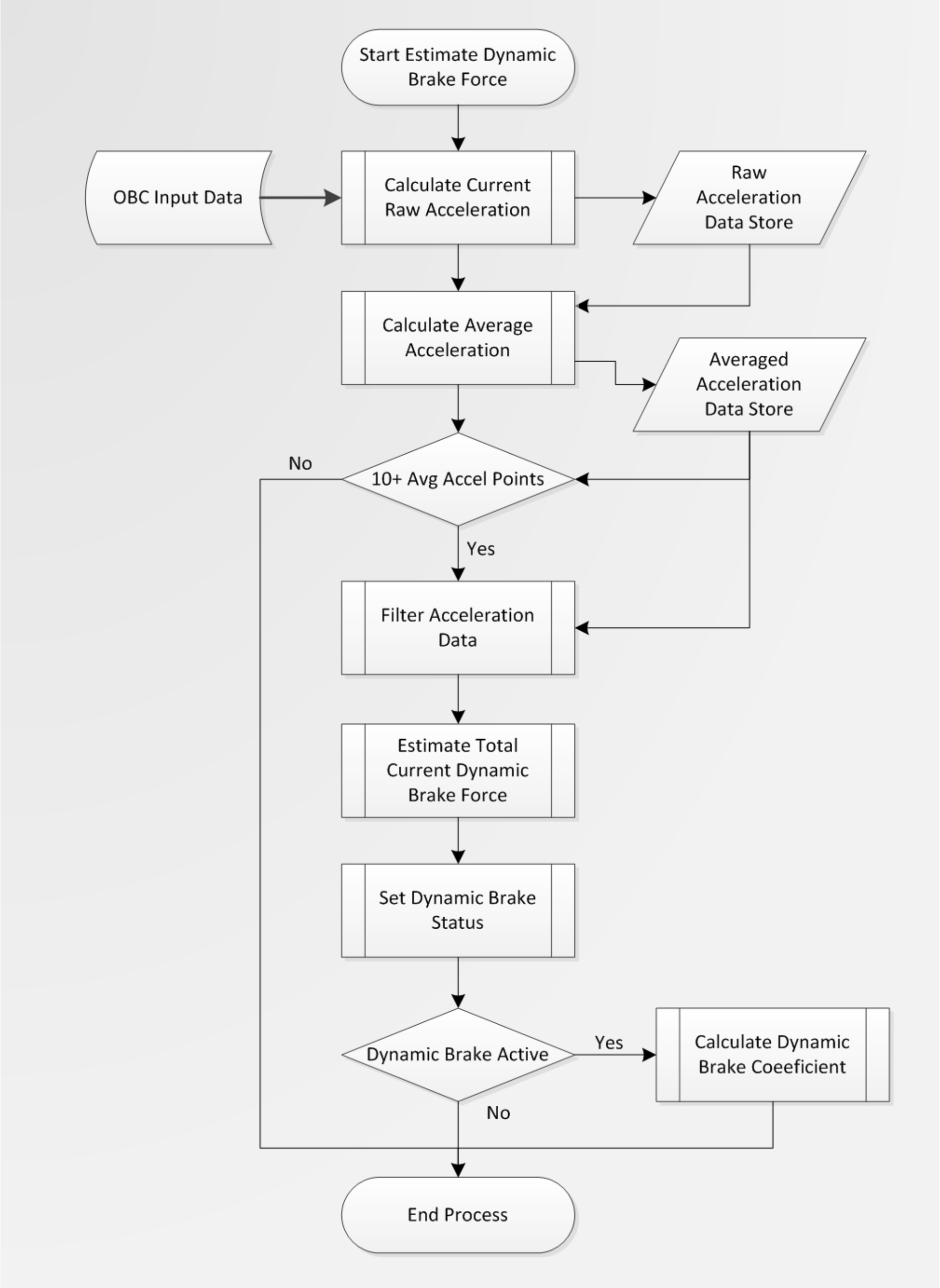

Figure F15 – Estimate Dynamic Brake Force Flow Diagram

The process begins by estimating the current raw acceleration of the train. The current instantaneous acceleration is measured by dividing the difference between the current velocity and the velocity from the previous iteration of the main process by the time interval between main process iterations:

$$a_{CUR,RAW} = \frac{v_{CUR} - v_{PREV}}{\Delta t}$$

The raw acceleration, $a_{CUR,RAW}$ is stored. The raw acceleration is then averaged over the previous three time steps to compensate for short duration spikes in the speed data. For the first two iterations, this step is skipped, as there is not enough stored acceleration data.

$$a_{CUR,AVG} = \begin{cases} a_{CUR,RAW}, & if\ t < 3s \\ \dfrac{a_{CUR,RAW} + a_{(t-1),RAW} + a_{(t-2),RAW}}{3}, & if\ t \geq 3s \end{cases}$$

This averaged acceleration, $a_{CUR,AVG}$ is then stored. Once there are at least 10 averaged acceleration data points, the most recent 10 averaged acceleration data points are filtered, using a least squares regression line method, as described in section 3.12. The filtered acceleration is then determined using the slope and intercept of the least squares regression line, as follows:

$$a_{FILT} = b + 4m$$

The filtered acceleration is then used to estimate the dynamic brake force, if any. The dynamic brake force, F_{DB}, is calculated using Newton's Second Law of Motion:

$$F_{DB} = ma - F_{GRD} - F_{CRV} - F_{RES} - F_{BRK}$$

Where m is the train mass in slugs (equal to the total weight of the train in lbs divided by the acceleration due to gravity ~ 32.2 ft/s^2), a is the filtered acceleration data, F_{GRD} is the force due to track grade, F_{CRV} is the force due to track curvature, and F_{RES} is the sum of the resistive forces acting on the train. If F_{DB} is greater than or equal to zero, the dynamic brake force, F_{DB} is set to zero.

The dynamic brake status is then determined, based on the result of the dynamic brake estimation and feedback from the onboard system. If F_{DB} is less than zero, the dynamic brake setup bit from the onboard system is set, the dynamic brake active bit from the onboard system is set, and the dynamic brake voltage from the onboard system is greater than zero, the dynamic brake status is set to active. If any of these conditions are not met, the dynamic brake status is set to inactive, and the process ends.

If the dynamic brake status is set to active, a dynamic brake coefficient is calculated for determining dynamic brake force in the braking profile prediction. If the dynamic brake status is not set to active, the dynamic brake coefficient is set to zero, so that no dynamic brake is assumed in the penalty and emergency brake profile predictions.

The dynamic brake coefficient is calculated according to:

$$DB_{COEFF} = \frac{F_{DB}/N_{LOCOS}}{v^{-1.01145}}$$

Where N_{LOCOS} is the number of locomotives in the consist and v is the current speed of the train in mph.

F3.7 Brake Pipe Propagation Time Adaption

The *Brake Pipe Propagation Time Adaption* function uses brake pipe pressure data from the onboard system during a service air brake application to estimate the actual brake pipe propagation time of the train. If a brake application is underway, the *Brake Pipe Propagation Time Adaption* function records the time and pressure each time the rear end brake pipe pressure drops and evaluates whether or not the brake application is complete, and if so, uses the time and pressure data to adjust the brake pipe propagation time parameter, T_{APPLY}, accordingly. To allow the brake pipe propagation time adaption to run in parallel with the normal prediction loop, the time and pressure data is stored and an index is used to track the progress of the adaption process. Figure 16 illustrates the flow of the function.

The function begins with the index initialized to 0 and waits for a brake pipe pressure reduction. During each iteration of the main process, the front brake pipe pressure, BPP_{FRONT}, is compared against the brake pipe pressure set point, BPP_{SET}, to determine if a brake pipe pressure reduction has occurred. If the front brake pipe pressure, BPP_{FRONT}, is at least three psi less than the brake pipe pressure set point, BPP_{SET}, the rear brake pipe pressure, BPP_{REAR}, is recorded as p_0, and a timer is started. The index is then incremented for the next iteration.

At the next iteration of the main process, the rear brake pipe pressure, BPP_{REAR}, is compared against the initial rear brake pipe pressure, p_0, to determine if the brake pipe pressure reduction has propagated to the rear of the train. If the rear brake pipe pressure, BPP_{REAR}, is at least three psi less than the initial rear brake pipe pressure, p_0, the time on the timer is recorded as t_{INIT}, and the timer is reset to zero. The rear brake pipe pressure at this point, BPP_{REAR}, is recorded as p_1, and the time, t_1, is set to zero. The index is again incremented for the next iteration.

Starting with the next iteration of the main process, each time the function is called, the rear brake pipe pressure, BPP_{REAR}, is compared against the rear brake pipe pressure from the previous index, p_{N-1}, to determine if the rear brake pipe pressure has further dropped. If it has, the rear brake pipe pressure is recorded as p_N the time on the timer is recorded as t_N, and the index is incremented.

After each time the rear brake pipe pressure drops, starting after t_3 and p_3 are recorded, the function calculates the slope and the intercept of the pressure drop between t_1 and t_N, $Slope_{TOT,N}$ and I_N, using a least-squares line method, as described in section 3.12.

Starting after t_4 and p_4 are recorded, the function also calculates the slope of the pressure drop between t_{N-1} and t_N, $Slope_{CUR,N}$, as follows:

$$Slope_{CUR,N} = \frac{p_N - p_{N-1}}{t_N - t_{N-1}}$$

The ratio between the current slope, $Slope_{CUR,N}$, and the total slope, $Slope_{TOT,N}$, is then calculated to determine if the brake pipe pressure is beginning to level off:

$$R = \frac{Slope_{CUR,N}}{Slope_{TOT,N}}$$

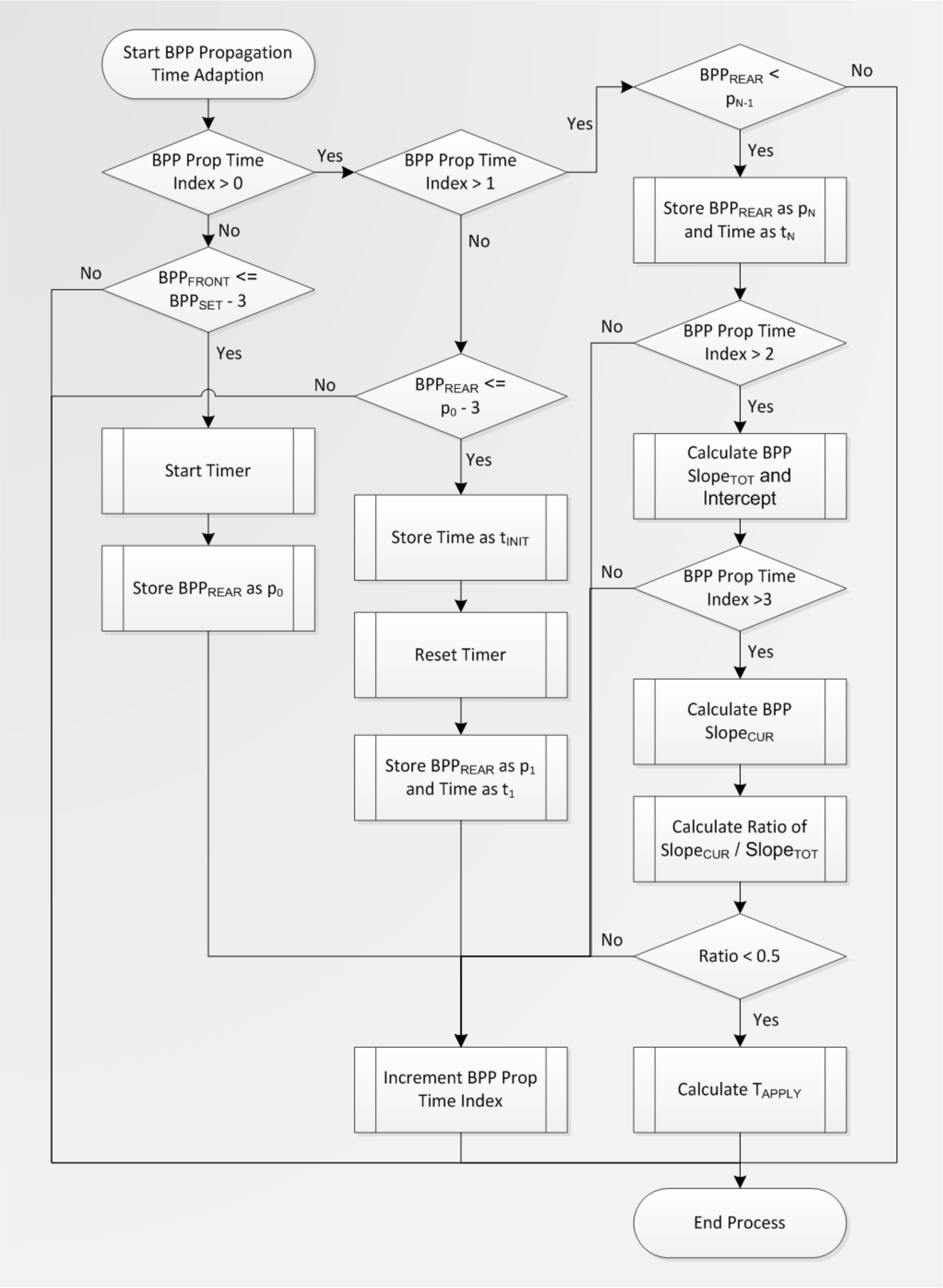

Figure F16 - Brake Pipe Propagation Time Adaption Flow Diagram

If the ratio, R, is greater than 0.5, the function waits for the next rear brake pipe pressure drop and re-evaluates. If the slope is less than 0.5, the brake pipe propagation is determined to be complete, and the brake pipe propagation time, T_{APPLY}, can be calculated. The slope used in the calculation of the brake pipe propagation time, T_{APPLY}, is based on the total slope from the previous time step, $Slope_{TOT,N-1}$, and is dependent on whether or not the brake pipe pressure reduction was a full service reduction or less than a full service reduction:

$$Slope_{FINAL} = \begin{cases} 1.1 * Slope_{TOT,N-1} - 0.0933 & if\ p_N \geq ARP_{EQU} \\ Slope_{TOT,N-1} & if\ p_N < ARP_{EQU} \end{cases}$$

The propagation time, T_{APPLY}, for a penalty brake application is then calculated as follows:

$$T_{APPLY} = \frac{ARP_{EQU} - I_{N-1}}{Slope_{FINAL}} + t_{INIT} + 1$$

If the train is operating with distributed power (DP), the brake pipe propagation time adaption cannot be run with the DP active, meaning that this function should be skipped if the train has DP active. However, brake pipe propagation time adaption can still be performed on trains equipped with DP by running with the DP cut out, presumably prior to train departure. In this case, the brake pipe propagation time adaption is performed as described above, and when the DP is cut in, an adjustment is made to the brake pipe propagation time, T_{APPLY}. The adjustment to the brake pipe propagation time is based on the distance that the brake pipe pressure signal has to travel with DP cut in, L_{TRAVEL}, calculated during consist initialization.

The estimated brake pipe propagation time with DP cut out, $T_{APPLY,EST}$ is determined using the full train length, L_{TRAIN}:

$$T_{APPLY,EST} = \begin{cases} 2.56 * 10^{-7} * L_{TRAIN}^2 + 1.179 * 10^{-2} * L_{TRAIN} + 13, & if\ Type = Unit \\ 8.01 * 10^{-7} * L_{TRAIN}^2 + 8.4792 * 10^{-3} * L_{TRAIN} + 15.6, & if\ Type = Manifest \\ 1.331 * 10^{-7} * L_{TRAIN}^2 + 6.871 * 10^{-3} * L_{TRAIN} + 16.66, & if\ Type = Intermodal \end{cases}$$

The estimated brake pipe propagation time with DP cut in, $T_{APPLy,DP,EST}$ is determined using the total distance the brake pipe pressure signal has to travel with DP, L_{TRAVEL}, determined during consist initialization:

$$T_{APPLY,DP,EST} = \begin{cases} 2.56 * 10^{-7} * L_{TRAVEL}^2 + 1.179 * 10^{-2} * L_{TRAVEL} + 13, & if\ Type = Unit \\ 8.01 * 10^{-7} * L_{TRAVEL}^2 + 8.4792 * 10^{-3} * L_{TRAVEL} + 15.6, & if\ Type = Manifest \\ 1.331 * 10^{-7} * L_{TRAVEL}^2 + 6.871 * 10^{-3} * L_{TRAVEL} + 16.66, & if\ Type = Intermodal \end{cases}$$

The ratio of the measured adapted brake pipe propagation time with DP cut out to the estimated brake pipe propagation time with DP cut out is calculated:

$$R = \frac{T_{APPLY,MEAS}}{T_{APPLY,EST}}$$

Where $T_{APPLY,MEAS}$ is the brake pipe propagation time with DP cut out, measured by the brake pipe propagation time adaption function. The brake pipe propagation time with DP cut in, $T_{APPLY,DP}$, is then set according to the following equation:

$$T_{APPLY,DP} = T_{APPLY,,DP,EST} * R$$

The new value calculated for the brake pipe propagation time, T_{APPLY}, replaces the nominal value and is used in the brake profile prediction functions going forward. At this point, the index is reset to zero so that the next brake application can be used to recalculate the brake pipe propagation time.

As a practical implementation issue for the brake pipe propagation time adaption function, checks may be required to ensure the function does not calculate erroneous results when unexpected air brake system operation is encountered. For example, the rear brake pipe pressure may not drop for enough time to compute the propagation time if the brake application is not deep enough. Likewise, if the brake application is released shortly after being initiated, the rear brake pipe pressure may not drop enough to compute the propagation time. In either of these cases, the function will continue to wait for another rear brake pipe pressure drop. A timeout function or a check to ensure the brake pipe pressure is still dropping may alleviate this potential issue.

F3.8 Brake Efficiency Adaption

The *Brake Efficiency Adaption* function is designed to improve the stopping distance prediction performed by the enforcement algorithm by adapting to the actual braking efficiency of the train. The function estimates a value for the total nominal train brake shoe force, $F_{B,NOM}$, based on the braking performance of the train during a normal service brake application. Figure 17 illustrates the flow of the *Brake Efficiency Adaption* function.

The *Brake Efficiency Adaption* function is run with each iteration of the main process. The function starts by evaluating conditions to determine if a deep enough air brake application has been made to perform the adaption. The following criteria must be met, in order to perform the adaption:

- Brake system state is Holding Service Application
- Brake pipe pressure reduction, BPP_{RED}, is greater than or equal to 10psi
- Train speed is greater than or equal to 10mph
- Dynamic brake is set to inactive

If all of the above criteria are met, the function records the following current train status data:

- Train speed, v
- Brake cylinder pressure, BCP
- Auxiliary reservoir equalization pressure, ARP_{EQU}
- Force due to track grade, F_{GRD}
- Force due to track curvature, F_{CRV}
- Force due to train resistance, F_{RES}
- Coefficient of friction between the brake shoes and wheels, $COF_{SHOE,WHEEL}$

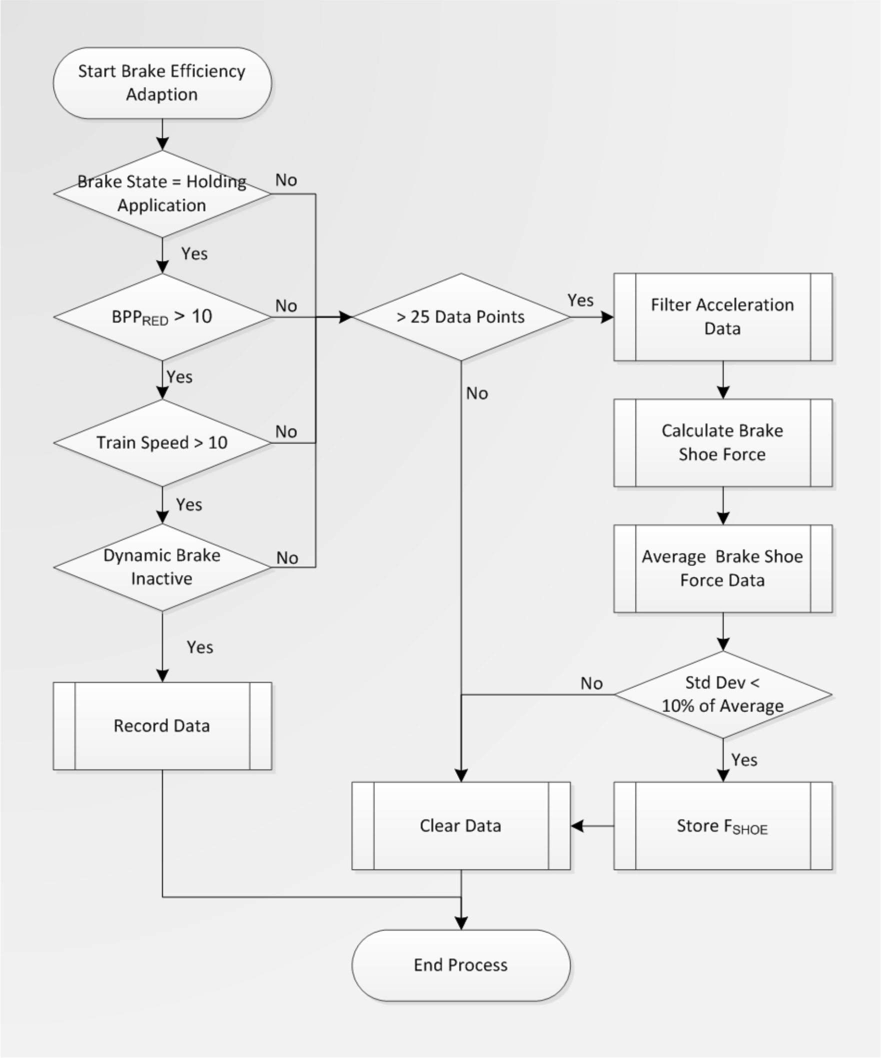

Figure F17 - Brake Efficiency Adaption Flow Diagram

The acceleration is also calculated, using the following equation:

$$a = \frac{v_{CUR} - v_{PREV}}{\Delta t}$$

Where v_{CUR} is the current train speed in mph, v_{PREV} is the train speed at the previous iteration of the main process in mph, and Δt is the integration time step of the main process. The raw acceleration is then averaged over the previous three time steps to compensate for short duration spikes in the speed data. For the first two iterations, this step is skipped, as there is not enough stored acceleration data since the brake application was initiated.

$$a_{CUR,AVG} = \begin{cases} a_{CUR,RAW}, & if\ t < 3s \\ \dfrac{a_{CUR,RAW} + a_{(t-1),RAW} + a_{(t-2),RAW}}{3}, & if\ t \geq 3s \end{cases}$$

Where t is the current time step. The averaged acceleration is stored and the process ends for this iteration.

If any of the criteria above are not met, the function uses the data that has been collected to perform the brake efficiency adaption. In order to filter the acceleration data, a minimum of 10 data points are required. If fewer than 10 data points have been collected, the data is cleared and the function ends without performing the adaption. If there are 10 or more data points, the entire averaged acceleration data set is filtered using a least squares regression line method. The filter function is described in detail in section 3.12.

The filtered accelerations are determined using the slope and intercept of the least squares regression line, as follows:

$$a_{FILT,i=1\ldots N} = b + (i-1) * m$$

For each data point in the data set, the brake force, F_{BRK}, is determined as per Newton's Second Law of Motion:

$$F_{BRK} = ma - F_{GRD} - F_{CRV} - F_{RES}$$

Where m is the train mass in slugs (equal to the total weight of the train in lbs divided by the acceleration due to gravity ~ 32.2 ft/s^2), a is the filtered acceleration data, F_{GRD} is the force due to track grade, F_{CRV} is the force due to track curvature, and F_{RES} is the sum of the resistive forces acting on the train. The brake force is equal to the product of the brake shoe force, $F_{B,SHOE}$, and the coefficient of friction between the brake shoes and the wheels, $COF_{SHOE,WHEEL}$:

$$F_{BRK} = -F_{B,SHOE} * COF_{SHOE,WHEEL}$$

Where the negative sign serves to produce a negative brake force, tending to decelerate the train. Substituting this for the brake force, F_{BRK}, and solving for the brake shoe force, $F_{B,SHOE}$, results in the following relationship:

$$F_{B,SHOE} = -\left(\frac{ma - F_{GRD} - F_{CRV} - F_{RES}}{COF_{SHOE,WHEEL}}\right)$$

The total nominal brake shoe force, $F_{B,NOM}$, at each data point is then calculated based on the brake shoe force, the ratio of the control pressure of 64 psi to the current brake cylinder pressure, BCP, and the relative efficiency of the brake rigging at the current brake cylinder pressure, Eff:

$$F_{B,NOM} = F_{B,SHOE} * \frac{64}{BCP} * \frac{1}{Eff}$$

The relative efficiency of the brake rigging is determined according to the following equation:

$$Eff = \begin{cases} 0 & for\ BCP < 8psi \\ 1.14 - \dfrac{8.97}{BCP} & for\ BCP \geq 8psi \end{cases}$$

After the total nominal brake shoe force, $F_{B,NOM}$, is calculated for all of the data points, they are averaged and the standard deviation is calculated for the data set to determine if the measurements show good correlation. If the standard deviation of the values is less than 10% of the average value, the average is stored in the consist data store, replacing the nominal value and used in the prediction calculation going forward. If the standard deviation is greater than or equal to the average value, the data is cleared and the *Brake Efficiency Adaption* process ends without applying the adaption.

F3.9 Calculate Penalty Braking Profile

The *Calculate Penalty Braking Profile* process is responsible for computing the braking profile for the train, prior to any PTC air brake enforcement, by assuming a penalty brake application is initiated at the time of the calculation. The process is run once each time through the main process, as shown in Figure 2, and the result is used, along with the target offset, to determine if a penalty air brake enforcement is necessary. The *Calculate Penalty Braking Profile* process flow is shown in Figure 18.

The prediction of the brake profile is performed by employing a numerical integration process whereby the acceleration is determined based on the forces acting on the train and then integrated with respect to time to determine the velocity, which is again integrated with respect to time to determine the position at each time step. The value of the integration time step used in this process is considered an implementation issue, influenced by the required accuracy of the prediction and the processing capabilities of the system. However, the following should be taken into consideration when selecting an appropriate value:

- A sufficient number of time steps should be allowed between air brake state transitions to ensure an accurate prediction of auxiliary reservoir and brake cylinder pressures
- The distance traveled in one time step should not include a large change in track grade
- The change in both acceleration and velocity over a single time step should be kept to a minimal

A value of one second is considered typical for the integration time step.

The process begins by calculating the current acceleration of the train, given the current force status, previously determined. The acceleration is calculated according to Newton's Second Law of Motion:

$$\sum F = ma$$

Where $\sum F$ is the sum of the forces acting on the train in lbs, m is the total mass of the train in slugs (equal to the total weight of the train in lbs divided by the acceleration due to gravity ~ 32.2 ft/s^2), and a is the instantaneous acceleration of the train in ft/s^2.

If the dynamic brake status is set to inactive, the sum of the forces acting on the train, $\sum F$, is equal to the net force acting on the train, F_{NET}, determined from the *Calculate Train Forces* function. If the dynamic brake status is set to active, the sum of the forces acting on the train, $\sum F$, is equal to the net force acting on the train, F_{NET}, determined from the *Calculate Train Forces* function, plus the dynamic brake force, F_{DB}, determined according to:

$$F_{DB} = \begin{cases} DB_{COEFF} * v^{-1.01145} * N_{LOCOS}, & if\ v > 20\ mph \\ DB_{COEFF} * 20^{-1.01145} * N_{LOCOS}, & if\ 20\ mph \geq v > 5\ mph \\ 0, & if\ v \leq 5\ mph \end{cases}$$

where DB_{COEFF} is the dynamic brake coefficient determined from the *Estimate Dynamic Brake Force* function, v is the speed of the train in mph and N_{LOCOS} is the number of locomotives in the consist.

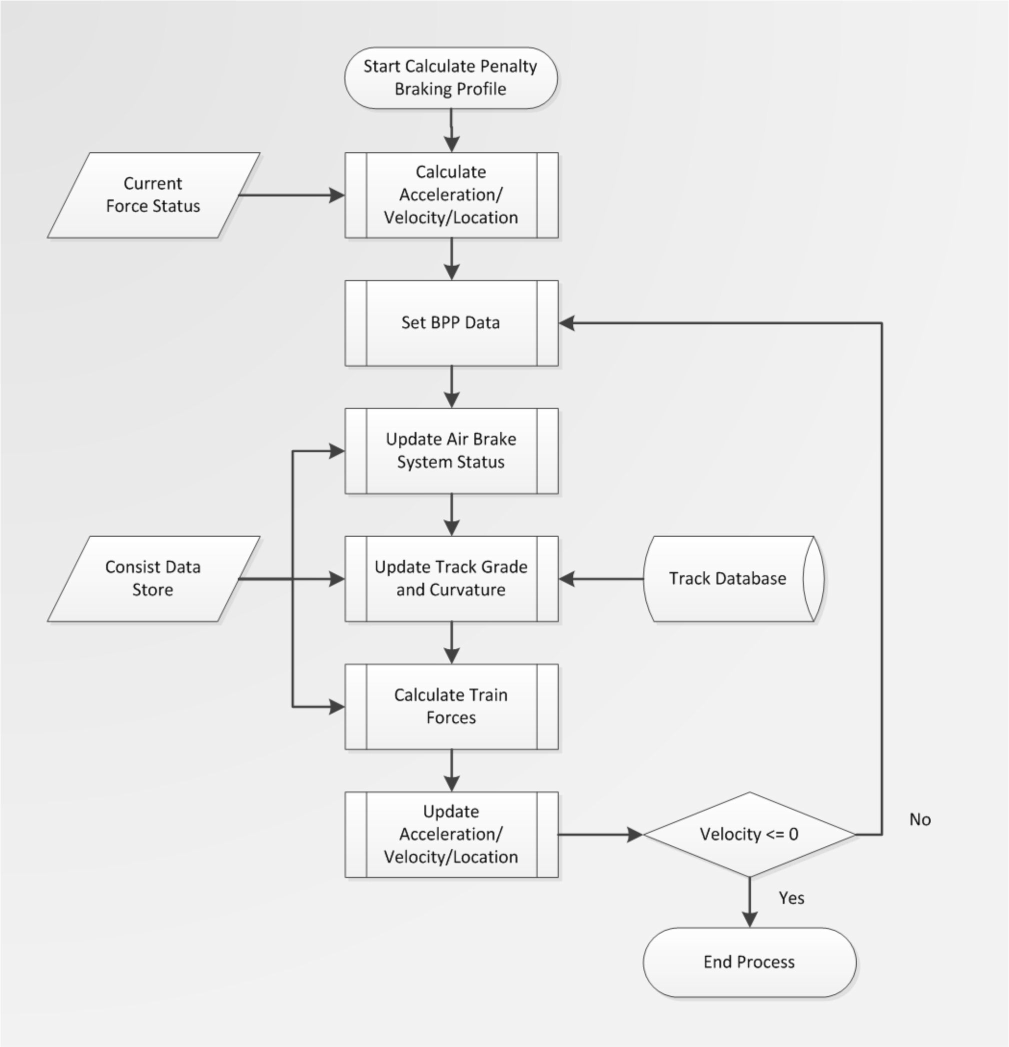

Figure F18 - Calculate Penalty Braking Profile Flow Diagram

The predicted velocity in ft/s, v, over the integration time step, Δt, can then be determined, using the current velocity according to:

$$v = v_{PREV} + a\Delta t$$

The predicted location, x, can then be determined, using the current location according to:

$$x = x_{PREV} + v_{PREV}\Delta t + \frac{a}{2}\Delta t^2$$

Next, the predicted brake pipe pressure is set, based on the assumption that the penalty brake has been applied. The service rate of reduction of the brake pipe pressure is assumed to be 4 psi/second, meaning that the brake pipe pressure at the head end at any given time into the brake profile calculation can be determined according to:

$$BPP_{FRONT,t} = BPP_{FRONT,CUR} - 4.0t$$

Where $BPP_{FRONT,t}$ is the predicted head end brake pipe pressure at the given number of seconds into the brake profile prediction, $BPP_{FRONT,CUR}$ is the actual current head end brake pipe pressure, and t is the number of seconds into the brake profile prediction.

This new predicted brake pipe pressure status is used in the air brake model to update all of the brake system parameters for the next predicted time step using the *Update Air Brake System Status* process defined in section 3.3.

The grade and curvature data is then updated for the next predicted time step, based on the predicted location from the previous time step, using the *Update Track Grade and Curvature* process defined in section 3.4.

The forces acting on the train at the next predicted time step are then calculated, based on the predicted values using the *Calculate Train Forces* process defined in section 3.5. If the dynamic brake status is set to active, the dynamic brake force is added to the net force calculated from the *Calculate Train Forces* function, as described earlier in this section.

The forces acting on the train are used to recalculate the acceleration and this numerical integration process is repeated until the predicted velocity is less than or equal to zero.

F3.10 Calculate Target Offset

The *Calculate Target Offset* function generates the buffer distance to offset the predicted stopping distance necessary to provide a high level of statistical confidence that the enforcement will result in the train stopping short of the target 99.5% of the time. The function is the result of a regression analysis on a large number of stopping distance simulations with Monte Carlo variation of the parameters that affect stopping distance for a variety of operating scenarios.

The target offset function calculates the target offset based on the following:

- train type
- train loading condition
- power configuration (head end or distributed power)
- current train speed, v, in mph
- equivalent constant grade over the predicted stopping distance, g
- trailing weight, W_{CARS}, in tons
- total length, L_{TRAIN}, in feet
- total number of axles on the train, n_{TOTAL}
- number of empty, N_{EMPTY}, and loaded, N_{LOAD}, cars.

The equivalent constant percent grade over the predicted stopping distance, g, is determined by summing the average percent grade under the train, $\%Grd_{AVG}$, at each step during the stopping distance prediction, and dividing by the total number of steps in the prediction:

$$g = \frac{\sum_{1}^{N_{STEP}} \%Grd_{AVG}}{N_{STEP}}$$

F3.10.1 Unit Freight/Unit Aluminum Coal Train Target Offset

For empty unit trains, the target offset, TO, is equal to:

$$TO = \ (0.326v - 1.358g + 0.000573W_{CARS} + 0.000103L_{TRAIN} - 0.891)^2$$

For loaded unit trains, the function used to determine the target offset is dependent on the power configuration. For loaded unit trains with distributed power, the target offset, TO, is equal to:

$$TO = (0.465v - 2.071g + 0.0000287W_{CARS} + 0.000145L_{TRAIN} + 0.709)^2$$

For loaded unit trains with head end power, the target offset, TO, is computed from one of three equations, depending on the equivalent constant percent grade over the predicted stopping distance, g:

$$TO = \begin{cases} TO_{DEC}, & if\ g \leq 0 \\ \left(1 - \frac{g}{0.5}\right) * TO_{DEC} + \frac{g}{0.5} * TO_{INC}, & if 0 < g < 0.5 \\ TO_{INC}, & if\ g \geq 0.5 \end{cases}$$

Where the values for incline and decline are determined from the following two equations:

$$TO_{DEC} = (0.536v - 2.97g - 0.000147W_{CARS} + 0.0013L_{TRAIN} - 3.698)^2$$

$$TO_{INC} = e^{0.0774v - 0.911g + 0.0000104W_{CARS} + 0.0000566L_{TRAIN} + 2.933}$$

F3.10.2 Manifest Freight Train Target Offset

For manifest freight trains with distributed power, the target offset, TO, is equal to:

$$TO = e^{0.0457v - 0.36g + 0.0000278W_{CARS} - 0.00795n_{TOTAL} + 0.03364N_{LOAD} + 0.03223N_{EMPTY} + 3.568}$$

For manifest freight trains with head end power, the target offset, TO, is computed from one of three equations, depending on the equivalent constant percent grade over the predicted stopping distance, g:

$$TO = \begin{cases} TO_{DEC}, & if\ g \leq -0.5 \\ \left(1 + \frac{g}{0.5}\right) * TO_{INC} - \frac{g}{0.5} * TO_{DEC}, & if -0.5 < g < 0 \\ TO_{INC}, & if\ g \geq 0 \end{cases}$$

Where the values for incline and decline are determined from the following two equations:

$$TO_{DEC} = (0.475v - 1.03g + 0.0004W_{CARS} + 0.0031n_{TOTAL})^2$$

$$TO_{INC} = (0.436v - 2.12g + 0.00011W_{CARS} + 0.0037n_{TOTAL})^2$$

F3.10.3 Intermodal Freight Train Target Offset

For intermodal freight trains with distributed power, the target offset, TO, is equal to:

$$TO = (0.338v - 2.031g + 0.0000475W_{CARS} + 0.00605n_{TOTAL})^2$$

For intermodal freight trains with head end power, the target offset, TO, is equal to:

$$TO = (0.335v - 2.412g + 0.0000415W_{CARS} + 0.00445n_{TOTAL} + 2.024)^2$$

F3.11 Calculate Emergency Braking Profile

The *Calculate Emergency Braking Profile* process is similar to the *Calculate Penalty Braking Profile* process. It is responsible for computing the braking profile for the train, following a PTC penalty air brake enforcement, to ensure that the train will stop short of the target with the specified level of safety, in the event the penalty air brake enforcement alone is insufficient. As in the *Calculate Penalty Braking Profile* process, the emergency braking profile is determined by performing a numerical integration on the equation of motion, but with the assumption that an emergency brake application is initiated at the time the function is called. The process is run once each time through the main process, following a PTC penalty air brake enforcement, as shown in Figure 2. The *Calculate Emergency Braking Profile* process flow is shown in Figure 19.

As in the *Calculate Penalty Braking Profile* process, the value of the integration time step used in this process is considered an implementation issue, but it is assumed that the same integration time step is used.

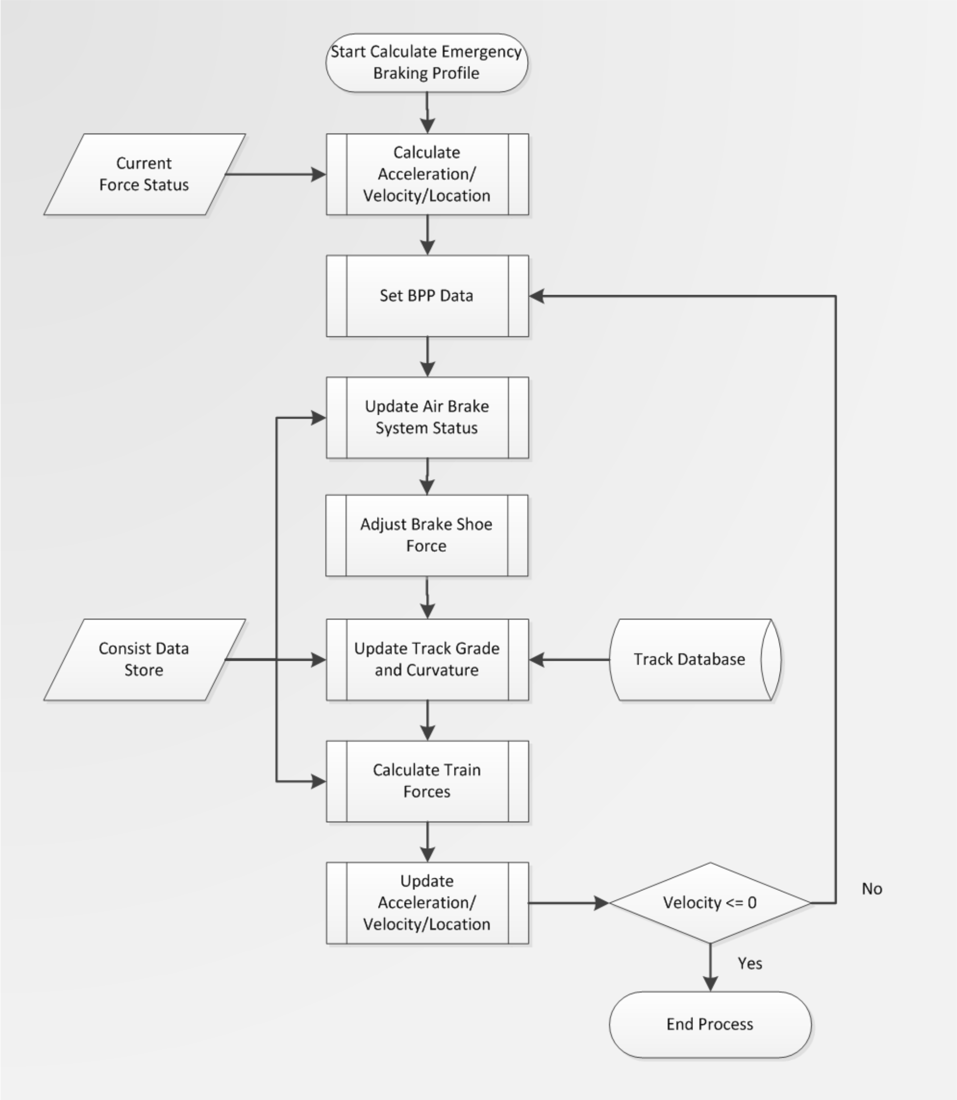

Figure F19 - Calculate Emergency Braking Profile Flow Diagram

The process begins by calculating the current acceleration of the train, given the current force status, previously determined. The acceleration is calculated according to Newton's Second Law of Motion:

$$\sum F = ma$$

Where $\sum F$ is the sum of the forces acting on the train in lbs, m is the total mass of the train in slugs (equal to the total weight of the train in lbs divided by the acceleration due to gravity ~ 32.2 ft/s^2), and a is the instantaneous acceleration of the train in ft/s^2.

If the dynamic brake status is set to inactive, the sum of the forces acting on the train, ∑F, is equal to the net force acting on the train, F_{NET}, determined from the *Calculate Train Forces* function. If the dynamic brake status is set to active, the sum of the forces acting on the train, ∑F, is equal to the net force acting on the train, F_{NET}, determined from the *Calculate Train Forces* function, plus the dynamic brake force, F_{DB}, determined according to:

$$F_{DB} = \begin{cases} DB_{COEFF} * v^{-1.01145} * N_{LOCOS}, & if\ v > 20\ mph \\ DB_{COEFF} * \ 20^{-1.01145} * N_{LOCOS}, & if\ 20\ mph\ \geq\ v > 5\ mph \\ 0, & if\ v\ \leq 5\ mph \end{cases}$$

where DB_{COEFF} is the dynamic brake coefficient determined from the Estimate Dynamic Brake Force function, v is the speed of the train in mph and N_{LOCOS} is the number of locomotives in the consist.

The predicted velocity in ft/s, v, over the integration time step, Δt, can then be determined, using the current velocity according to:

$$v = v_{PREV} + a\Delta t$$

The predicted location, x, can then be determined, using the current location according to:

$$x = x_{PREV} + v_{PREV}\Delta t + \frac{a}{2}\Delta t^2$$

Next, the predicted brake pipe pressure is set, based on the assumption that the emergency brake has been applied. The emergency rate of reduction of the brake pipe pressure is assumed to be 40 psi/second, meaning that the brake pipe pressure at the head end at any given time into the brake profile calculation can be determined according to:

$$BPP_{FRONT,t} = BPP_{FRONT,CUR} - 40.0t$$

Where $BPP_{FRONT,t}$ is the predicted head end brake pipe pressure at the given number of seconds into the brake profile prediction, $BPP_{FRONT,CUR}$ is the actual current head end brake pipe pressure, and t is the number of seconds into the brake profile prediction.

This new predicted brake pipe pressure status is used in the air brake model to update all of the brake system parameters for the next predicted time step using the *Update Air Brake System Status* process defined in section 3.3.

In order to provide a reasonably conservative estimate of the level of braking achievable with the emergency brake application, the total current brake shoe force, $F_{B,SHOE}$, is multiplied by a safety factor:

$$F_{B,SHOE} = F_{B,SHOE} * 0.9$$

The grade and curvature data is then updated for the next predicted time step, based on the predicted location from the previous time step, using the *Update Track Grade and Curvature* process defined in section 3.4.

The forces acting on the train at the next predicted time step are then calculated, based on the predicted values using the *Calculate Train Forces* process defined in section 3.5. If the dynamic brake status is set to active, the dynamic brake force is added to the net force calculated from the *Calculate Train Forces* function, as described earlier in this section.

The forces acting on the train are used to recalculate the acceleration and this numerical integration process is repeated until the predicted velocity is less than or equal to zero.

F3.12 Filter Function

The filter function is used by the *Estimate Dynamic Brake Force, Brake Pipe Propagation Time Adaption*, and *Brake Efficiency Adaption* functions to filter data using a least squares regression line method. The filter function is designed to filter any number of data points. The following parameters are set to perform the least squares regression line method:

- $X_1 = N$
- $X_2 = t_1 + t_2 + \cdots + t_N$
- $X_3 = X_2$
- $X_4 = {t_1}^2 + {t_2}^2 + \cdots + {t_N}^2$
- $Y_1 = s_1 + s_2 + \cdots + s_N$
- $Y_2 = t_1 * s_1 + t_2 * s_2 + \cdots + t_N * s_N$
- $Z = X_1 * X_4 - X_2 * X_3$

Where N is the number of raw data samples, $s_{i=1\ldots N}$ are the raw data samples, and $t_{i=1\ldots N}$ are the time steps associated with the data samples.

The slope, m, and intercept, b, of the least squares regression line are then calculated as follows:

$$m = \frac{X_1 * Y_2 - X_2 * Y_1}{Z}$$

$$b = \frac{X_4 * Y_1 - X_3 * Y_2}{Z}$$