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Feasibility of a New Indiana Coordinate Reference System (INCRS)

Boudewijn H W van Gelder
Purdue University, vngelder@ecn.purdue.edu

James S. Bethel
Purdue University, bethel@ecn.purdue.edu

Chisaphat Supunyachotsakul
Purdue University, csupunya@purdue.edu

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JOINT TRANSPORTATION RESEARCH PROGRAM

INDIANA DEPARTMENT OF TRANSPORTATION
AND PURDUE UNIVERSITY

FEASIBILITY OF A NEW INDIANA COORDINATE REFERENCE SYSTEM (INCRS)

Boudewijn H. W. van Gelder
Professor Emeritus
School of Civil Engineering
Purdue University
Corresponding Author

James S. Bethel
Associate Professor of Civil Engineering
School of Civil Engineering
Purdue University

Chisaphat Supunyachotsakul
Graduate Research Assistant
School of Civil Engineering
Purdue University

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CORRESPONDING AUTHOR

Professor Boudewijn H. W. van Gelder
School of Civil Engineering
Purdue University
(765) 494-2165
vngelder@ecn.purdue.edu

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16. Abstract

Engineers, Surveyors, and GIS Professionals spend an enormous amount of time correcting field surveys to the classical State Plane Coordinate System (SPCS). The current mapping corrections are in the order of 1:33,000, or 30 parts per million (ppm). Modern surveys (e.g., GPS/InCORS) have an accuracy of a few parts per million. Whenever original surveys made on the surface of the Earth need to be reduced to a mapping reference surface, surveyed distances and angles (azimuths) need to be corrected. Measured distances need to be corrected for two scale factors: 1) due to the mapping scale inherent in conformal mappings, and 2) due to terrain heights. Measured angles (azimuths) need to be corrected for so-called convergence angles. The application of these necessary corrections is time consuming and may add an estimated 15 to 20% to the cost of a survey. The omission of these corrections corrupts the reliability of survey results. A new Indiana Coordinate Reference System (INCRS) allows for so much smaller corrections that when omitted the errors committed are small, and may be even neglected for surveys less accurate than a few ppm. In a few areas of Indiana (e.g. Clark County), terrain heights corrections are still needed because these corrections due to the terrain roughness are at the 14 ppm level. The proposed INCRS not only reduces the scale factor from 30 ppm to a few ppm, but also the convergence angles are reduced by a factor of four (from about 0.5 degree to about 7-8 arcminutes). The new much more accurate mapping system has been developed based on closed formula expressions and simple mathematical coordinate transformations.

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EXECUTIVE SUMMARY

FEASIBILITY OF A NEW INDIANA COORDINATE REFERENCE SYSTEM (INCRS)

Introduction

Engineers, surveyors, and GIS professionals spend an enormous amount of time correcting field surveys to conform to the classical State Plane Coordinate System (SPCS). The current mapping corrections are in the order of 1:30,000, or 33 parts per million (ppm). Modern surveys (e.g., GPS/InCORS) are accurate to a few ppm. Whenever original surveys made on the surface of the Earth need to be reduced to a mapping reference surface, surveyed distances and angles (azimuths) need to be corrected. Measured distances need to be corrected for two scale factors: (1) the mapping scale inherent in conformal mappings, and (2) terrain heights. Measured angles (azimuths) need to be corrected for so-called convergence angles. Applying these necessary corrections is time consuming and adds an estimated 15 to 20% to the cost of a survey. However, omitting these corrections corrupts the reliability of survey results.

A newly proposed Indiana Coordinate Reference System (INCRS) allows for much smaller corrections that, when omitted, result in minor errors that may even be disregarded for surveys that call for less accuracy than a few ppm. (In a few areas of Indiana (e.g., Clark County), terrain height corrections are still needed because these corrections, due to the terrain heights variation, are at the 14 ppm level.) The proposed INCRS not only reduces the scale factor from 33 ppm to a few ppm, but also reduces the convergence angles by a factor of four (from about a maximum of 0.5 degrees to about 7–8 arcminutes). This new, much more accurate mapping system has been developed based on closed formula expressions and simple mathematical coordinate transformations.

Findings

- The current mapping system, Indiana State Plane Coordinate System of 1983 (INSPCS83), based on Transverse Mercator (TM) mapping, causes distortions of survey measurements on the Earth's surface at 33 ppm, or around 0.2 feet (2 inches) per mile (3 cm/km). This level of accuracy is insufficient for modern, highly precise (few ppm) (GPS) surveys (few cm/10 km).
- Two new mapping systems, one based on TM mapping and a second one based on a special case of the Lambert conformal mapping, the Oblique Stereographic (OS), are both capable of reducing mapping errors to the few ppm accuracy level when applied in small geographical areas (counties).
- The TM and the OS systems are equally capable of reducing mapping errors on a county-by-county basis; however, the OS is superior to the TM in equally distributing the small errors in Easting and Northing.
- The mapping related scale factor corrections can be reduced to the less than 2 ppm level in average of all counties in Indiana. The terrain height related errors can also be greatly reduced; however, the terrain heights variation plays a limiting factor. In some areas in Indiana the scale factor error due to the terrain heights variation cannot be reduced to below the 14 ppm level. In this case the classical measurement reductions cannot be omitted and should be applied.
- The proposed INCRS also reduces the convergence angles by a factor of four (INSPCS83 exhibits convergence angles up to the half a degree (30 arcminutes) level).
- An extensive test in Marion County confirmed all the findings stated above.
- The proposed INCRS, based on a spherical approximation that allows closed formula mathematical expressions in conjunction with simple coordinate transformations, models point clouds in the reality (so-called "Real World") with one order of magnitude better (a factor of 10) than the classical INSPCS83 that is based on an ellipsoidal model and extensive series expansions that may have limited accuracy because of truncation errors.

Implementation

Implementation of the INCRS may occur within two to three years after the completion of the feasibility study (time frame: August 2012 to August 2015). During the implementation phase the following tasks need to be completed:

I. Acceptance and approval of the INCRS by the engineering/surveying/GIS communities in Indiana.
II. Delineation of the mapping zones.
III. Official designation of the mapping zones.
IV. Selection of the mapping method (mapping equations) for each mapping zone.
V. Selection of longitude and latitude of the mapping origin (Center of Project (CP)) for each mapping zone.
VI. Selection of the optimum scale factors for each mapping zone.
VII. Selection of False Easting and False Northing for each mapping zone.
VIII. Development of the Indiana Handbook on the New Indiana Coordinate Reference System.
IX. Preparation of legislation (Model Law) that prescribes the use of INCRS and its related mapping parameters for each mapping zone.
X. Adaptation of the Indiana Department of Transportation's Engineering and Survey Design Manuals.
XI. Development of workshops and seminars for the engineering/surveying/GIS communities.

It is foreseen that an implementation SPR is needed to complete tasks I-XI. The request and approval of the implementation SPR should start as soon as possible after August 2012. It is recommended that this charge be led by the ISPLS HARN/INRTN/INCRS/HeightMod Committee.

The implementation of the INCRS should be coordinated with the National Geodetic Survey (NGS) of the National Oceanic and Atmospheric Administration (NOAA), Silver Spring, Maryland, as represented in the State of Indiana by the Office of the Indiana State Geodetic Adviser (OISGA).

ABBREVIATIONS

avg	Average
cont'd	Continued
ppm	Parts per million
Abbrev.	Abbreviation
ACSM	American Congress on Surveying and Mapping
ASCE	American Society of Civil Engineers
Az	Azimuth
B-L&A	Bernardin-Lochmueller and Associates, Inc.
CoM	Center of Mass
CM	Central Meridian
CP	Center of Project
GIS	Geographic Information System
GPS	Global Positioning System
GRS80	Geodetic Reference System 1980
IC	Indiana Code
InCORS	INDOT Continuously Operating Reference Stations
INCRS	Indiana Coordinate Reference System
INDOT	Indiana Department of Transportation
INSPCS83	Indiana State Plane Coordinate System of 1983
ISPLS	Indiana Society of Professional Land Surveyors
INRTN	Indiana Real Time Network
LSQ	Least Squares
Max	Maximum
MED	Median
Min	Minimum
NAD83	North American Datum of 1983
NGS	National Geodetic Survey
OISGA	Office of the Indiana State Geodetic Adviser
OS	Oblique Stereographic
RMS	Root mean squares
STD	Standard Deviation
SPCS	State Plane Coordinate System
TM	Transverse Mercator
USC&GS	United States Coast and Geodetic Survey
USPLSS	United States Public Land Survey Systems
UTM	Universal Transverse Mercator
WISCRS	Wisconsin Coordinate Reference System
WISDOT	Wisconsin Department of Transportation

NOTATION

2D:	Two-dimensional
3D:	Three-dimensional
a:	Semi-major axis of the ellipsoid
b:	Semi-minor axis of the ellipsoid
e:	First eccentricity of the ellipsoid
f:	Ellipsoidal flattening
G:	Gaussian Radius of Curvature
G_A:	Gaussian Radius of Curvature at point A
M:	Radius of Curvature in the Meridian Plane
M_A:	Radius of Curvature in the Meridian Plane at point A
N:	Radius of Curvature in the Prime Vertical Plane
N_A:	Radius of Curvature in the Prime Vertical Plane at point A
R_1:	Rotation matrix about the first axis (by convention the first axis is the X-axis in the Cartesian frame)
R_2:	Rotation matrix about the second axis (by convention the second axis is the Y-axis in the Cartesian frame)
R_3:	Rotation matrix about the third axis (by convention the third axis is the Z-axis in the Cartesian frame)
$R_{G@CP}$:	Gaussian Radius of Curvature at point CP
(X, Y):	X and Y coordinates in the 2D frame
(X′, Y′):	X′ and Y′ coordinates in the 2D Prime frame
(E, N):	Easting and Northing coordinates in the 2D frame
(X, Y, Z):	Cartesian coordinates X, Y, and Z in the 3D frame
(X′, Y′, Z′):	Cartesian coordinates X′, Y′, and Z′ in the 3D Prime frame
(e, n, u):	Cartesian coordinates e (east), n (north), and u (up) in the topocentric frame
(X_M, Y_M, h_v):	3D version of the map coordinates
(λ, Ψ, h_s):	Geodetic coordinates with a sphere as the reference surface; longitude, spherical latitude, and spherical height (height above sphere)
(λ, φ, h_e):	Geodetic coordinates with an ellipsoid as the reference surface; longitude, ellipsoidal latitude, and ellipsoidal height (height above ellipsoid)
$(t_X, t_Y, t_z)^T$:	Translation vector in the 3D Cartesian frame
$(t'_X, t'_Y, t'_z)^T$:	Translation vector in the 3D Cartesian Prime frame

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1. INTRODUCTION

1.1 Background and Problems

Representing the curved surface of the Earth on a flat plane continues to present challenges for map-makers, and subsequently, the entire geospatial community. While the general public may only be concerned with maps that provide the intended accuracy of the typical road atlas, Surveyors, Civil Engineers, GIS and Construction Professionals and the like all demand a much higher standard for their projects.

Selecting the "best" map (or zone) for a certain region or project may or may not be the most appropriate choice for another region or project, given the demands of the particular project. For large scale projects at the state-wide or planning level, selecting the Universal Transverse Mercator, Zone 16 (UTM16), conformal mapping may be the "best" choice, as it covers the State of Indiana in its entirety on one plane. This mapping was developed in the 1940's by the Corps of Engineers, U.S. Army, with an intended maximum scale reduction of 1:2,500. Briefly stated, scale reduction refers to the ratio of error in the lengths of lines as measured on the reference surface when represented in the mapping plane. In other words, the numerically higher the ratio is, the less the error; the numerically lower the ratio is, the more the error.

Another formally recognized conformal mapping system currently in use in Indiana is the Indiana Coordinate System of 1983. Its predecessor, known as the Indiana coordinate system of 1927, was developed by the United States Coast and Geodetic Survey (USC&GS) in the 1930's. It divides Indiana into two regions, the East Zone (designated "1301") and the West Zone ("1302") with intended maximum scale reductions of 1:30,000, and reflected the accuracy of field surveying measurements of that era.

Since the 1930's and 1940's, there has been an indisputable surge of technological advancements in surveying measurement techniques and computer software, enabling users to position themselves on the surface of the earth with greater accuracies than ever before imagined, and, in many cases, instantaneously.

Today, it can be argued that the majority of Surveying and Civil Engineering projects in Indiana are not solely based upon either the State Plane Coordinate System of 1983 or UTM16, but rather modified versions thereof or of local (assumed) coordinate systems with no reference to the North American Datum of 1983 or the GRS80 Ellipsoid.

For the projects that are strictly based upon local or assumed systems, these are mostly extensions of legacy projects that began prior to when Global Positioning Systems became widely used, and are therefore not considered as part of this discussion. For the projects that are based upon modified versions of the State Plane Coordinate System of 1983 or UTM16, they deserve attention as to why they were in fact "modified" from definitive mathematical-based coordinate systems.

The primary reason for such projects to be based upon modified versions of UTM16 and State Plane is to *minimize the difference between field-observed measurements* (referred to herein as "ground distance" or "Real World") *and their corresponding distances as represented in the mapping plane* (referred to herein as "grid distance" or "Mapped World") that calculations will be based upon. After all, it makes good practical sense to base the reports of the distances, acreages, volumes, etc. of activities that have, are or will occur at the local surface of the earth respective to said surface, instead of on a plane that is not localized.

As stated above, UTM16 and the Indiana State Plane Coordinate System of 1983 (INSPCS83) (*1*) were developed with scale reductions of 1:2,500 and 1:30,000, respectively. What this means to Surveying and Civil Engineering projects in Indiana is that, for the UTM16 map projection, there inherently lies approximately 1 foot of discrepancy in field measured distances in 2,500 feet as reduced to the UTM16 mapping plane. The State Plane Coordinate System produces results with approximately one-twelfth of the UTM16 systems, with an average discrepancy across the State of two to three inches in a mile.

While initially this seems to be a much better alternative to the UTM16 systems, it is evident that this still does not meet the "grid versus ground" threshold that Surveyors and Civil Engineers demand for their projects, as so many projects are in fact "scaled to ground."

While this process of "scaling to ground" has been exercised by many practitioners over the years, the drawbacks of not utilizing the parent grid systems have begun taking their toll with mistakes being made across the board from Surveyors, Engineers, Contractors, GIS Professionals, Cartographers, etc. Not scaling correctly, scaling but failing to change the numerical values of the coordinates in order to make them not appear as the original parent grid coordinates and failing to report the process by which the project was scaled are just a few of the problems that are encountered quite frequently.

But even if no mistakes are made and the projects are carried through fruition, the process by which each and every practitioner must endure to ensure he/she has entered the modification parameters correctly and is properly prepared to proceed with his/her duties requires precious time (and consequently, money) that may not have been necessary if the project had not been modified from the parent grid system. This is especially true in the current era with the increasing demand for seamless data sharing amongst professionals. Adding steps to the flow of data (such as "scaling to ground") slows down and complicates the otherwise seamless process, as well as increases the chances for the introduction of errors.

One example in particular is the inclusion of a "scaled to ground" project into a GIS. In order for the GIS practitioner to properly introduce such a project into the GIS requires the modification parameters to be known, and for them to be correct. If the parameters are not known, the GIS practitioner is forced to either best-fit the data into the system respective to other known features or to begin the process of tracking down the party responsible for modifying the specific project. But even if the parameters are known and the GIS practitioner is able to correctly setup his/her coordinate system library with the parameters for each and every new project that comes across their desk, the underlying problems discussed above and further below would still exist.

Another drawback encountered with modified grid coordinate systems is dealing with neighboring systems that also have been "scaled to ground," but with dissimilar parameters. Take for example a long north/south corridor project, "scaled to ground" with its own specific parameters, which gradually rises in elevation from one end to the other. Sometime later, numerous other projects begin that either cross, intersect, border or are in close proximity to the long north/south road project. Because each project's Surveyor would prefer his/her project be locally "scaled to ground," the long/north south project's parameters are ignored and new parameters are calculated for each project.

Now, points in common to the multiple projects have vastly different coordinate values based upon each project's Surveyor's personal preference of a modified system, even though they all may be in as close proximity as less than a mile of one another.

Yet another drawback with modified grid coordinate systems is the loss of the direct relationship of project coordinates with latitude and longitude values from the reference ellipsoid. Maintaining this relationship is the key element to streamlining workflow.

Many enterprise data systems at INDOT (SPMS, DSS, TrnsPort, WMS, EPS, Inspectech, gInt…) use Latitude and Longitude to identify locations of projects, assets, permits, borings and other pertinent information for the agency. Without the direct relationship of local project coordinates to a known reference ellipsoid a large effort of massaging the survey and design data is needed to convert it into a format usable by the end systems.

The last drawback to mention concerning modified grid coordinate systems is simply the seemingly limitless library of coordinate systems that are being generated as time goes on and as new projects begin. There are many, many more coordinate systems that exist in Indiana in addition to UTM16 and State Plane East or West; they are simply the only three that are formally recognized mapping systems. It takes little to no discussion to realize that this constant accumulation of varying coordinates systems adds confusion, increases the possibility of errors and actually hinders the advancement of a seamless work flow environment of the geospatial community.

And so the enigma presents itself at point blank range; ignore the "grid versus ground" separation and use the parent UTM16 or State Plane grid systems, or continue the accumulation of project-specific coordinate systems? The solution is neither, and it was pioneered in the States of Minnesota and Wisconsin several years ago. They remedied the problem by developing multiple grid coordinate systems throughout each state so that the "grid versus ground" separation was reduced so that the desired threshold was achieved. The end results were grid coordinate systems with their artificial boundaries being county lines, with the majority of the Counties having their own individually-assigned systems.

1.2 Multi-Zone Coordinate Reference System (INCRS-BLA)

In recent years, Bernardin-Lochmueller and Associates, Inc., a multi-disciplined Surveying, Planning, Engineering and Environmental firm with its corporate headquarters in Evansville, decided to bring this same concept to Indiana, being that they work in multiple counties across the State.

After developing a dual-zone grid coordinate system for the new-terrain I-69, Evansville to Indianapolis, highway project, B-L&A began developing a new Indiana Coordinate Reference Systems what is now referred to as INCRS-BLA. The end result is a collection of fifty-nine conformal map projections embracing Indiana's ninety-two counties with the claimed of average "grid versus ground" separation of approximately 3/16 of an inch in a mile (+/−1:377,000), the claimed ninety-five percentile separation of 3/4 of an inch in a mile (+/−1:85,000) and a maximum sampled separation of approximately 1-5/16 of an inch in a mile (+/−1:48,000). With the system that BLA developed, Surveying and Civil Engineering projects could utilize these grid coordinate systems in their parent, unmodified form, achieving their "grid versus ground" threshold, seamlessly share their data with other practitioners and, if necessary, properly transform or re-project that data to any other mathematically-based grid coordinate system.

1.3 Research Objectives

Although Bernardin-Lochmueller and Associates, Inc. has completed a sizable portion of the initial development of an alternative grid coordinate system for Indiana, several pieces of the puzzle still need to be researched, tested and developed before Surveying, Engineering and GIS professionals can easily and readily utilize this system. As the research steps proceeded it may turn out that alternatives of B-L&A's multi-zone reference system may have to be developed and test against the one of B-L&A to come up with the system that best represents the Real World and is the most practical for Surveyors Communities.

Based on discussions in the GISLIS/HARN/INCRS/HeightMod Committee of the Indiana Society of Professional Land Surveyors (ISPLS) an alternative mapping system has been further developed for the State of Indiana, based on the theory presented in (*2*). The alternative system is referred to as "INCRS-OISGA" or

"INCRS" for short. OISGA is the acronym for Office of the Indiana State Geodetic Advisor. OISGA developed alternatives, while in a test phase the three mapping systems are compared in order to come up with the "best" suggestions for INDOT, or the Survey and GIS community in general, in the case that any of these systems will be adopted.

2. BASIS OF THE PROPOSED INCRS

2.1 Theory

The existing Indiana Coordinate Reference System consists of two mapping zones. The widely used 2D rectangular reference system has two drawbacks: (1) the scale distortion is not constant in a zone; it varies from location to location, with a maximum of about 1:30,000 from east to west boundary, (2) the ground-to-grid correction factor increases as the separation between the mapping surface and the terrain increases (Randolph County in eastern Indiana). The new InCORS, or better INRTN, makes real-time (GPS) surveying at the 1 ppm accuracy a reality. The measured real world (ground) distances have to be corrected for both effects.

To reduce the corrections between the results of modern 3D (GPS) surveys and 2D conformal mapping systems (State Plane Coordinate Systems, SPCS), the idea is to further limit the size of the zone to be mapped. In this study the extreme case has been considered to limit each zone to the size at the level of a county-by-county area. Details are further explained in Chapter 3, Data Preparation. Also the adoption of a new mapping surface creates the possibility to decrease the corrections considerably. In this study a theory as presented in (*2*) is further developed and tested. The theory is based on simple closed-formula mathematical mapping expressions and coordinate transformations. The corrections are mainly due to two effects: (1) the scale effect from the conformal mapping itself, and (2) the height of the terrain. These two effects will be referred to as "Scale Effect" and "Terrain Effect" respectively. Both effects lower the accuracy of nowadays highly accurate survey data. The details of each effect will be discussed separately in the following sections.

2.1.1 Scale Effect

The Scale Effect lowers the accuracy of the original highly accurate surveys and it is due to the conformal mapping process itself. In this study two different conformal mapping functions have been considered: Transverse Mercator (TM) mapping and Oblique Stereographic (OS) mapping. It should be remarked that the Stereographic Conformal mapping is a special case of the more general Lambert Conformal mapping, see for instance section 52.4, page 1927 in (*3*).

The mathematical details of the mapping functions used in this research study can be found in Appendix A, section A.3. As explained below, the scale variation behavior of a (Transverse) Mercator mapping follows the shape of a cut half-pipe whereas the scale of an (Oblique) Stereographic mapping shows the form or pattern of a bowl.

- Scale Effect of Transverse Mercator (TM) Mapping
  The scale behavior in Normal Mercator Mappings varies originally in north-south (N-S or N for short) or latitudinal direction. The scale behavior in the Transverse aspect of the Mercator mapping varies in an east-west (E-W or E for short) or longitudinal direction. The role of the classical longitude (λ) is played by a new latitude coordinate, the latitude prime (ψ') in the new rotated system, the prime system. That means the further the grid points are removed from the Central Meridian of the transverse mapping the more the scale deviates from 1. The scale behavior for the case of Transverse Mercator mapping can be expressed in the form of equation (2.1).

$$\sigma \propto \frac{1}{\cos\psi'} \quad \sigma = \frac{k}{\cos\psi'} \text{ with } \psi' \approx \frac{\lambda}{\cos\psi} \tag{2.1}$$

  From Eq. 2.1, the scale at each single point varies with the cosine of the original longitude value of that grid point. Initially one sets the k parameter that is considered to be a constant equal to the value of 1. From the nature of cosine values it can be seen that the angle ψ' (which is almost the equivalent of the original longitude) takes on the value of zero (at Central Meridian), making the scale (σ) equal to 1 at the Central Meridian and greater than 1 anywhere else (for k = 1). Figure 2.1 reveals that the overall behavior of the scale (σ) variation of a Transverse Mercator mapping follows the pattern of a cut-half pipe surface.

  If one sets the k value in Eq. 2.1 initially equal to 1, the scale behavior becomes unbalanced as it is greater than 1 everywhere else except at map's Central Meridian. It can be seen from the scale variation curve (as a cross section or profile version of the cut half pipe surface) in Figure 2.2a the variation behavior of the scale, that means the deviation from 1 of the scale value, is only occurring on the so-called "positive side" (greater than 1). In order to balance the scale σ a new appropriate value (1-Δ) is assigned to k. The effect is that the scale variation curve is shifted downwards. The scale behavior has become more balanced as shown in Figure 2.2b. Because of this some points in a mapped zone have scale values greater than 1 (positive side) while other points have scale values less than 1. As an example for the UTM mapping one has adopted k = 1–Δ = 1-1/2,500 = 1-0.0004 = 0.9996. For both SPCS zones in Indiana one has adopted k = 1-Δ = 1-1/30,000 = 1-0.000033 = 0.999967.
- Scale Effect of Oblique Stereographic (OS) Mapping
  The scale behavior of the Normal Stereographic mapping (which a special case of the class of Conformal Lambert mappings) varies with the co-latitude θ, the co-latitude being equal to the 90 degree compliment angle of the latitude (θ = 90°- φ). The scale behavior of the Oblique aspect of the Stereographic mapping varies with the value of the co-latitude prime (θ') which is similarly related to the new (prime) latitude values in the newly rotated prime system. The scale varies when the points radially deviate from the so-called "Computational North Pole" of the mapping, as revealed in Figure 2.3. The overall behavior of the scale variation of the Oblique Stereographic mapping follows the shape of a bowl.

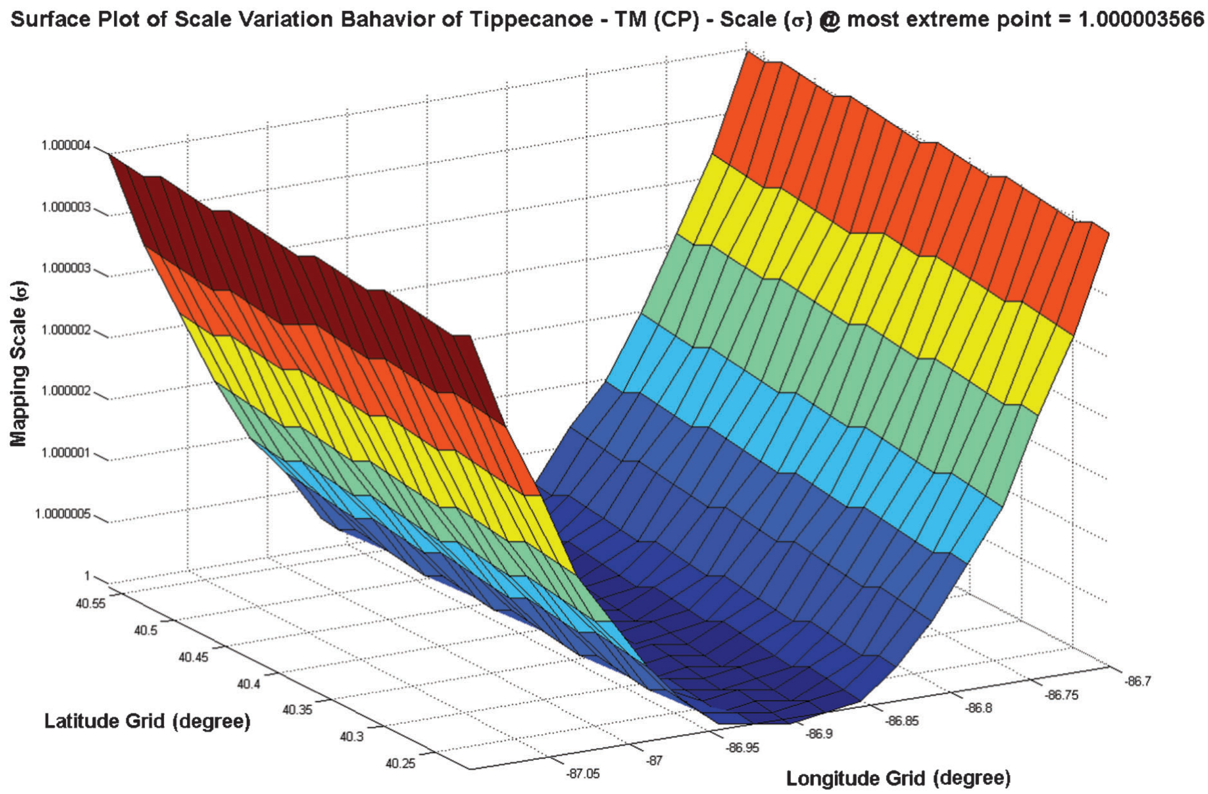


Figure 2.1 Cut half-pipe behavior of the scale (σ) of the Transverse Mercator (TM).

From Eq. 2.2, the scale at each single point varies with the cosine value of co-latitude prime (θ') of that grid point. Initially one sets the constant k equal to the value of 1 in order to have the scale (σ) of value 1 at the Computational North Pole where θ' takes on the value of zero.

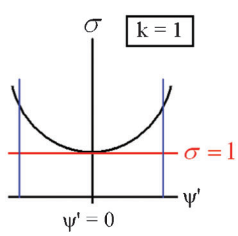


Figure 2.2a Scale $\sigma \geq 1$, or $1 \leq \sigma < 2\Delta$.

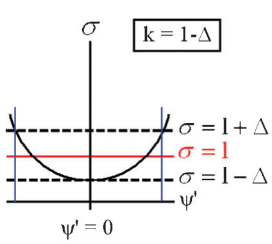


Figure 2.2b Scale σ: $1\text{-}\Delta < \sigma < 1\text{+}\Delta$.

$$\sigma \propto \frac{2}{1+\cos\theta'} \quad \sigma = \frac{2k}{1+\cos\theta'} \quad \sigma = \frac{2}{1+\cos\theta'} \text{ with } k=1 \qquad (2.2)$$

Eq. 2.2 and Figure 2.3 show that because of the k value being equal to 1, the scale behavior seems to be unbalanced as it is greater than 1 everywhere else except at map's Computational North Pole ($\theta'=0$). This issue will be dealt with in the same manner as mentioned in the case of the Transverse Mercator mapping. The scale variation surface is shifted downwards with the appropriately assigned new k value ($k = 1\text{-}\Delta$). Because of this some of the points have scale values greater than 1 while some have scale values smaller than 1.

From the behavior of the scale values discussed above for both mappings (Transverse Mercator and Oblique Stereographic), it is obvious that unavoidably the mapped points have different values of scale, dependent on where the points are located, and how far or how close they are with respect to the map's Central Meridian or Computational North Pole.

For the case of the Transverse Mercator, the scale value of a point that is farther away from the Central Meridian will *deviate much more* from 1 as compared to points that are closer to the Central Meridian. The same idea applies to the Oblique Stereographic mapping, the only difference being that instead of having the Central Meridian, OS deals with a Computational North Pole. In this study this idea has been applied to the investigation of the so-called "worst case scenario" for each mapping. In other words, those points will be addressed where the scale value

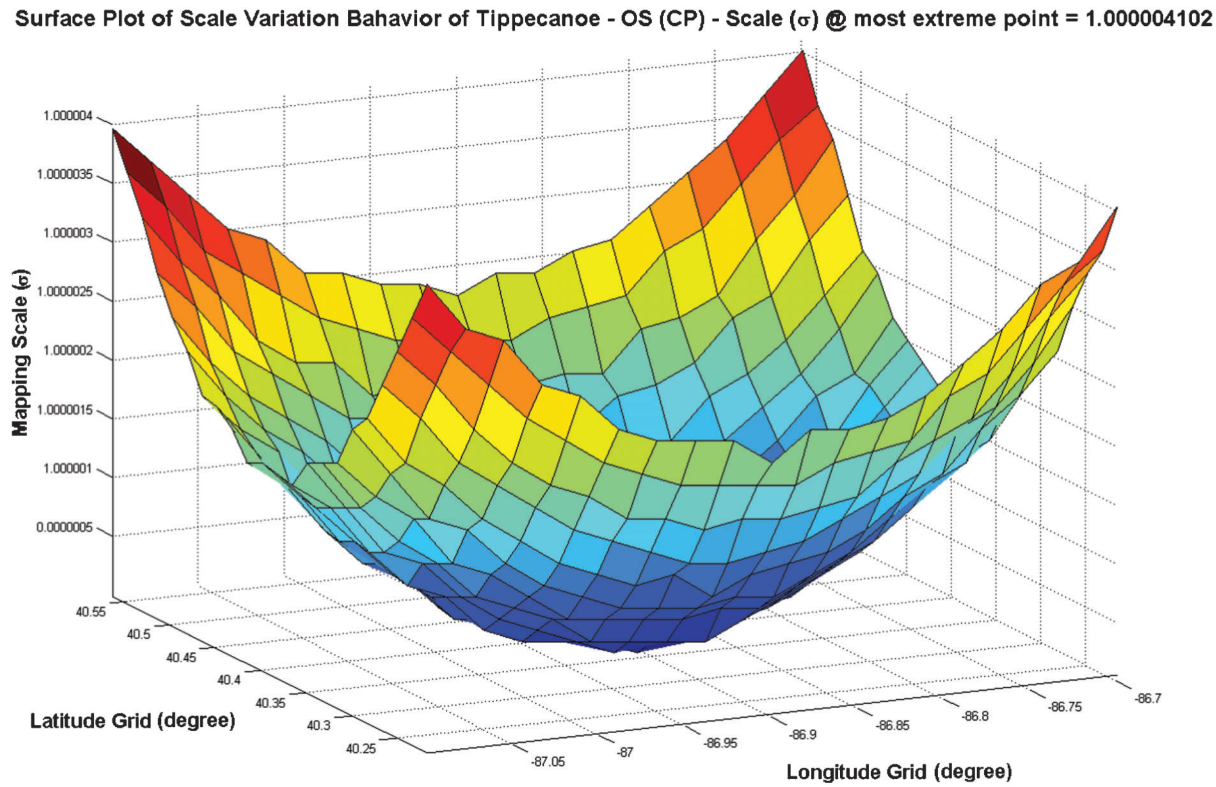


Figure 2.3 Bowl behavior of the scale (σ) of the Oblique Stereographic (OS).

deviates most from 1, as well as the size of that maximum deviation.

The scale behavior in the mapped area (based on county-by-county sized mapping zones, see Chapter 3, sections 3.1) have been investigated for both mappings (TM and OS) with the initial condition of using the k value of 1. Subsequently the scale behavior has been studied in each Test Area (county) where the maximum scale deviation occurs and the corresponding magnitude of that deviation. The complete set of results for all 92 Indiana counties can be found in Table D.1 of Appendix D.

Once the maximum scale deviation values are known (computed from the case that k is equal to 1) from the aforementioned study, the new optimum k value can be assigned in order to balance the scale variation behavior as discussed previously. Due to the fact that the selection of the appropriate optimum value of this new k value depends on a host of decisions that will be made after the conclusion of this study, the selection of k fell automatically beyond the scope of the feasibility study. However, the researchers have recommended (see Chapter 7, section 7.3.4 The Optimization of Scale Corrections of each Mapping) that various options are to be considered on how to arrive at an optimized k. This relatively small investigation should be carried out in a follow-up study once it has been decided to continue with the development of an INCRS.

The idea of balancing the scale variation behavior by assigning new k values ($k = 1-\Delta$) has been applied to all Test Areas (counties) using different optimization methods. The complete results of the mapping correction values for the worst case in all Test Areas have been tabulated in Appendix D.

2.1.2 Terrain Effect

The Terrain Effect lowers the accuracy of the original highly accurate surveys and causes the distances computed on the grid surface (map) not to be equivalent to the actual ground distances (the so-called "Real World" distances). This effect occurs in all mappings (thus also the TM and OS) due to the ground-to-grid (ellipsoid) reductions, the ground-to-grid conversions being dependent on the terrain elevations. This means that higher terrain elevations exhibit more significant differences between ground and grid distances than low terrain elevations. This Terrain Effect behavior is depicted in Figure 2.4. Zone B exhibits larger differences between the ground and grid distances than the ones of Zone A due to the fact that the terrain of Zone B is higher than Zone A. In other words, the overall ellipsoidal heights of the terrain in the area of Zone B are larger than the ones in Zone A.

2.1.3 Convergence Angle or Azimuth Effect

Conformal mappings exhibit not only unavoidable scale distortions, but also angular distortions that are

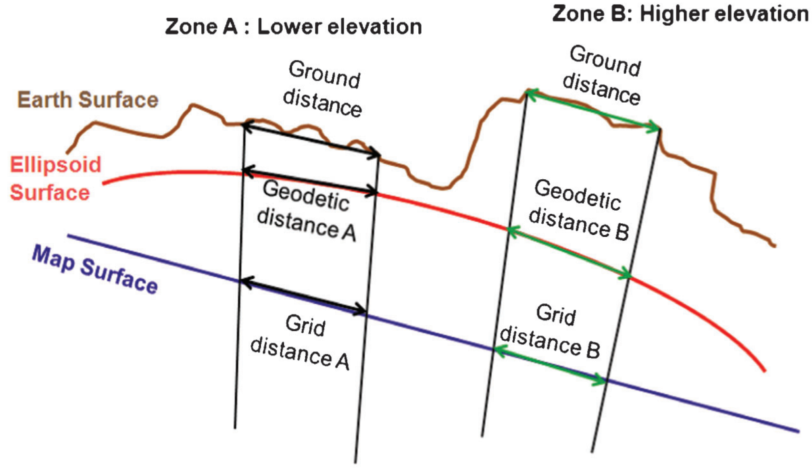


Figure 2.4 The illustration of the Terrain Effect.

better known as the Azimuth Effect or Convergence Angle Effect.

Figure 2.5, as an example, depicts this aforementioned effect in a Transverse Mercator mapping that has the Central Meridian run through the center of Marion County (marked as a pink asterisk) in Indiana. Figure 2.5 clearly shows that the original meridian lines (red-color lines) are mapped in such a way that they do not coincide with the North direction of the map (so-called "Grid North").

This effect causes angular differences between geodetic North which is also known as "True North" and the North direction on the map (Grid North) as shown in Figure 2.6. Corresponding to any particular point on map, the exhibited angular difference is defined as the "Convergence angle," and denoted by the Greek letter γ.

By definition of the Convergence Angle, which states that it is the angle measured from the mapped meridian to Grid North, the sign convention of the convergence angle is positive when the point under consideration is:

1. on the east side of the Central Meridian and above the Equator, e.g., points A and D in Figure 2.6,
2. on the west side of the Central Meridian and below the Equator.

In the other cases the convergence angle has a negative sign, e.g., points B and E in Figure 2.6. It should be noted that Convergence Angle takes on the zero value as well:

1. at the Central Meridian of the map, e.g., points C and F in Figure 2.6,
2. at the (original) equator of the map, e.g., points A, B, and C in Figure 2.6.

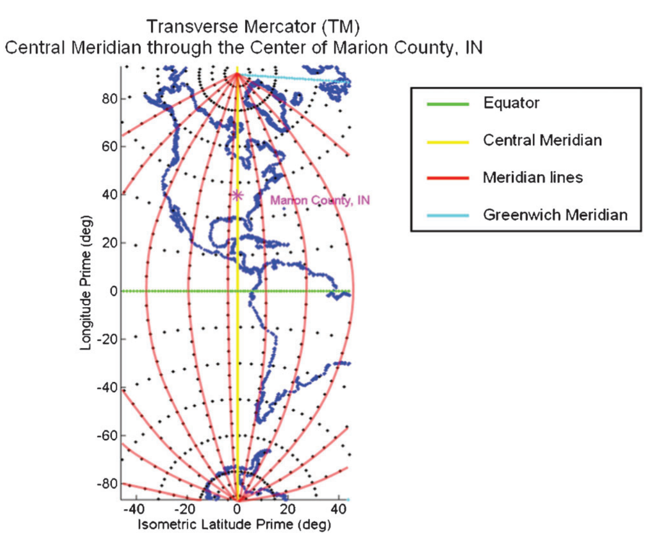


Figure 2.5 Transverse Mercator map, Central Meridian through the center of Marion County, Indiana.

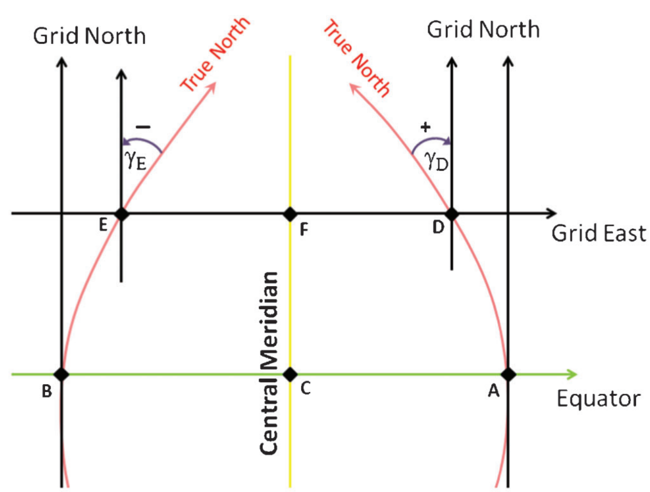


Figure 2.6 Convergence angle and its sign convention.

In addition to the Transverse Mercator map, which is used as an example illustrating Azimuth Effect, or Convergence Angle Effect, a similar explanation can be given for the Oblique Stereographic. This is due to the fact that it exhibits the same behavior of the mapped meridians, i.e., they do not coincide with Gird North (see Figure 2.7).

From the Azimuth Effect (Convergence Angle Effect), the bearing obtained from a conformal (e.g., Transverse Mercator, Oblique Stereographic) map which is known as "Grid Bearing" does not represent the actual geodetic bearing (the so-called True Bearing or Real World Bearing). Therefore, the Convergence Angle (γ) is added to the Grid Bearing in order to arrive at the True Bearing. This process is known as the "Azimuth correction" or "Convergence Angle Correction." As a final conclusion it can be said that the behavior of the Convergence Angles in a mapped area can be judged as a quality measure how well a conformal mapping stays true or close to the Real World. For further discussion, the reader is referred to the Chapter 6 MARION COUNTY TEST, where the Convergence Angle Effect is used to evaluate INSPCS83 vs. INCRS (two alternatives) and INCRS vs. a third submitted mapping solution.

In Figures 2.6 and 2.7 the mappings are shown for a much larger area than e.g., Marion County as the size of the Convergence Angle Effect is small in small areas. It is expected that with an appropriate choice of the location of the Central Meridian (TM) or the Computational North Pole (OS) the convergence angles do not tend to exceed the level of ten arcminutes. For the classical INSPCS83 convergence angles larger than half a degree are not uncommon.

2.2 Basis of INCRS (INCRS-OISGA)

The idea of an alternative mapping system, the so-called "INCRS" that has been developed based on the theory presented in (*2*), will be described in this section. In order to have a mapping system for Indiana that is more commensurate with current high accuracy 3D survey systems an alternative system has been developed based on the following ideas.

- **Small size of zone**
  It is clear that limiting the size of a zone (i.e., the area to be mapped) helps to reduce the Scale Effect when mapping the Real World to the Mapped World. This is of course true for the two mapping functions used in this research (Transverse Mercator and Oblique Stereographic). In principle one may minimize the scale deviations to any acceptable but practical level.
- **Simple closed formula expressions for mappings and transformations**
  The ideas as developed in (*2*) can be efficiently tested in the limited time available of this feasibility study. The ideas are based on the geometric realization that any adopted reference ellipsoid does not deviate much from **an appropriately chosen reference sphere** in a relatively small area. In this study the sphere what is now referred to as the "INCRS Sphere" is designed and chosen to be used as the reference surface for the mapping.

To justify the aforementioned idea that in a relatively small area the deviation of chosen reference sphere (in this case is INCRS Sphere) from the adopted ellipsoid is small, a test has been conducted for each county-by-county area in the State of Indiana. In other words, the size of the differences between the GRS80 ellipsoid surface and the INCRS Sphere surface needed to be investigated. The steps of this test are as follows:

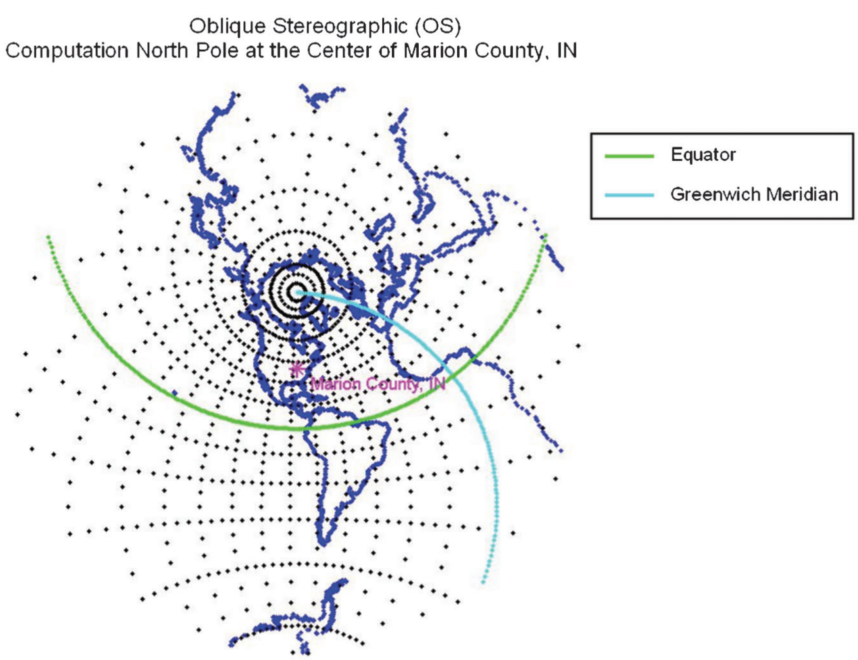


Figure 2.7 Oblique Stereographic map, Computational North Pole at center of Marion County, Indiana.

1.

$$(\lambda, \varphi, h_e) \xrightarrow[\text{Ellipsoidal (GRS80) Model}]{} (X, Y, Z) \quad (2.3)$$

Begin with a set of ellipsoidal coordinates (λ, φ, h_e) of grid points on the surface of the GRS80 ellipsoid. All ellipsoidal heights h_e are set equal to zero in each county. These ellipsoidal coordinates are then converted to Earth-fixed geocentric Cartesian coordinates (X, Y, Z).

2.

$$(X, Y, Z) \xrightarrow[\text{INCRS Sphere Model}]{} (\lambda, \varphi, h_s) \quad (2.4)$$

Convert back Earth-fixed geocentric Cartesian coordinates (X, Y, Z) into the spherical coordinates (λ, ψ, h_s) based on an appropriately chosen INCRS Spherical model.

3. Plot the h_s's (the spherical heights) against the reference (spherical) surface. In other words, the set of h_s's represent the heights of (the grid on) the GRS80 ellipsoid above or below the surface of the reference INCRS sphere. For clarity the sphere itself is shown as a plane.

Figure 2.8 is an example of the surface difference (the deviation) between GRS80 ellipsoid and INCRS Sphere of Tippecanoe County. The differences of these two surfaces are calculated in terms of computed spherical heights (h_s) as in Eq.2.4.

Tippecanoe County exhibits a maximum deviation of 5.8 cm with the average of deviation's size being 1.9 cm when considering the entire county. It should be noted that Tippecanoe County which represents a typical size of a county in Indiana, 24 miles by 24 miles, the deviation reaches hardly the 6 cm level. Relative to the size of the Earth/ellipsoid, the INCRS sphere approximates the ellipsoid to 10 parts per billion (10 ppb, 6 cm/6000 km). In order to be able to visualize these deviations the vertical scale in the plot of Figure 2.8 had to be exaggerated by a factor of approximately 400,000. That means if the deviations were plotted to the same scale in both dimensions (vertical and horizontal), the differences between these two surfaces would not be visible.

Tippecanoe County is just an example that proves how close the INCRS Sphere's surface is to the GRS80 ellipsoid, the differences are considered to be extremely small and insignificant. At this point it should be noted that the ellipsoidal heights of the Real World points (terrain) are NOT sacrificed.

The use of an appropriately chosen reference sphere is solely adopted for (conformal) mapping purposes (the advantage being that use can be made of closed-formula mathematical expressions and simple coordinate transformations).

The investigation of surface difference (the deviation) between GRS80 ellipsoid and INCRS Sphere for every single county-by-county area in the State of Indiana, has been performed, the complete statistics can be found in Table B.3 of Appendix B. In summary, a single averaged value of 6.6 cm represents the average maximum deviation between the GRS80 ellipsoid and

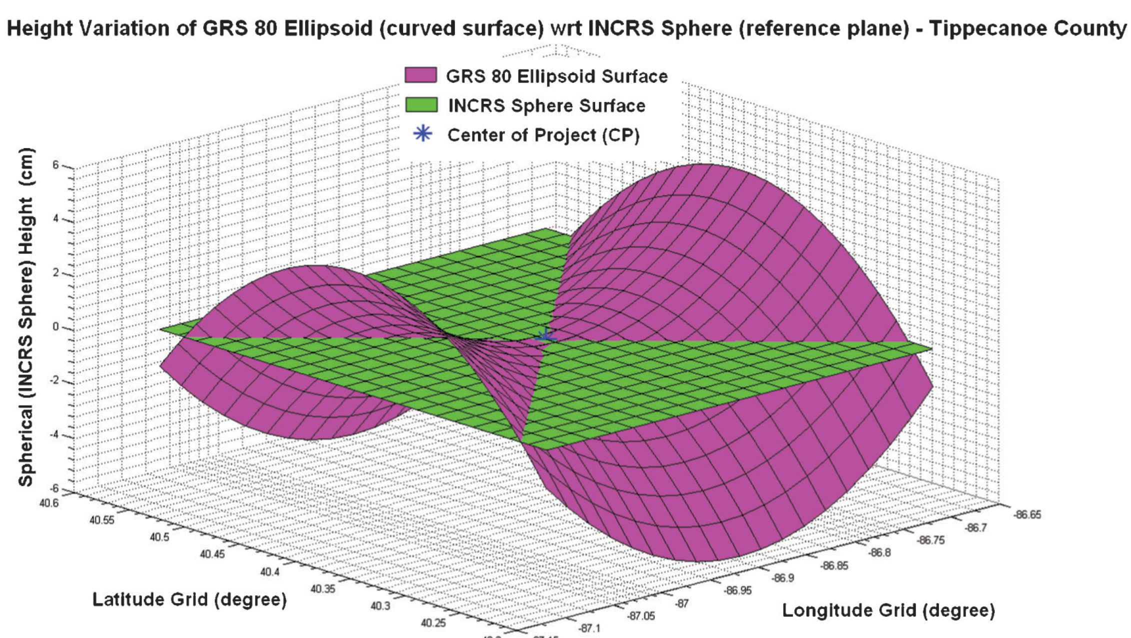


Figure 2.8 The deviation of INCRS Sphere in Tippecanoe County from GRS80 ellipsoid.

local INCRS Spheres of Indiana having computed the maximum surface differences in each county.

The idea of adopting an appropriately sized sphere as the mapping reference surface has been justified by the test procedures as mentioned above. It is clear that the adoption of a reference sphere as the mapping reference surface has many advantages. The largest advantage is that one then needs to use nothing more than simple closed-formula expressions and coordinate transformations. In our case this is true if one defines a reference sphere, the INCRS Sphere, as the reference mapping surface to be used in the Transverse and Oblique aspects of the conformal mappings (in our case, the TM and the OS). The second advantage is that no time needed to be devoted to the investigation of the accuracy of series expansion methods that often accompany the transverse and oblique aspects of the two conformal mappings in conjunction with their reference ellipsoids. The proposed method is completely conformal and transparent to any new developments in the future, e.g., when new versions of reference ellipsoids may be adopted.

- **Choice of an Appropriately Sized Sphere as Mapping Reference Surface (INCRS Sphere)**
  The sphere that has been used as the mapping reference surface is in this case NOT a sphere that has its center coincide with GRS80 ellipsoid's center. Instead the center (origin) of this particular sphere, which has been referred to as "INCRS Sphere," is located along the ellipsoidal normal drawn at the center of the underlying project area (so-called point "CP"). This means that in different counties (different mapping areas) each of them will have its own different reference sphere with its own origin (O_G). Figure 2.9 reveals geometry surrounding the INCRS Sphere.
  R_{normal} of INCRS Sphere is selected in many different ways in this research study (see 4.2.2 Radius of INCRS Sphere) but all selected values of the radii are based on the Gaussian radius of curvature (R_G) of a point. When the Gaussian radius of curvature (R_G) at the center of the project area (point "CP") is used, the radius of the INCRS reference sphere is then called "$R_{G@CP}$." When the averaged value is computed from the Gaussian radii of curvature at all grid points in each area (county) the radius is designated as "$R_{G,\ avg}$." The mathematics of the Gaussian radius of curvature can be found in Appendix A, section A.2.

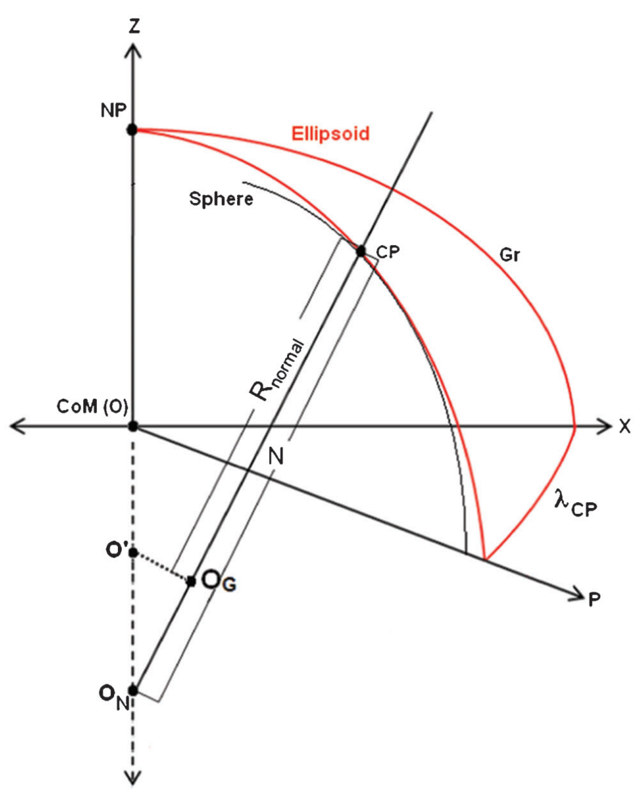


Figure 2.9 The INCRS Sphere.

3. DATA PREPARATION

The whole area of the State of Indiana can be divided in any which way. In contrast to the classical INSPCS83 division into two zones (IN-W and IN-E), another extreme has been considered in this study, a county-by-county division. This division was suggested in meetings with Indiana surveyors, engineers, and other mapping professionals. Since the State of Indiana is divided into 92 counties, 92 different so-called "Test Areas" have been identified. In some experiments all 92 Test Areas are involved, while in other cases only a couple areas (counties) are involved or considered due to the fact that their combined areas are sufficient to represent a whole group of characteristics.

3.1 Selection of the Test Areas

Based on a county-by-county division, 92 different Test Areas were constructed from the geodetic coordinates of the boundaries of each individual county boundary (the West – East longitude and South – North latitude). In this study each Test Area (county) is referred to by its officially adopted county abbreviation and county code, as they are for instance used in the license plate system by Bureau of Motor Vehicles (BMV) in the State of Indiana. The list of all county abbreviations and codes with their boundary coordinates can be found in Table B.1 of Appendix B.

3.2 Point Sampling in a Test Area

The points were sampled over a Test Area in the form of grids. For instance, the mathematical behavior of the Scale Effect has been sampled on an approximately 1 mile by 1 mile grid (with a 1′ by 1′ 20″ angular spacing in the latitude and longitude direction respectively; see Figure 3.1).

It should be noted that some of the boundaries as revealed in Table B.1 of Appendix B are the augmented ones: they have been extended in order that the number of grid points resulting from the angular spacing is an integer number. That means for a county, the easternmost longitude is the start of the angular spacing process going towards the West. The end of the spacing is either right at the westernmost longitude or one step

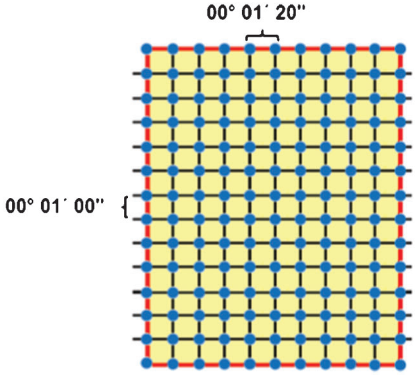


Figure 3.1 Sampled grid points over a Test Area.

beyond in order to ensure that the whole county is covered. Similarly, the county has been sampled from the southernmost boundary going North.

The center of the project (CP) of each Test Area was computed from the corresponding extents of the grid points. The CP's coordinates of all Test Areas (counties) and their corresponding total number of sampled grid points as well as the number of sampled points in both longitudinal and latitudinal directions are presented in Table B.2 of Appendix B.

The erratic behavior of the terrain has been down-sampled from the 1 arc-second resolution Digital Elevation Model (DEM) to meet the same grid points format of the ones designed for studying the Scale Effect. The original Digital Elevation Model (DEM) was retrieved from USGS Seamless Data Warehouse (*4*). It has the following characteristics:

- Original Resolution: 1 arc-second
- Horizontal datum: NAD83
- Vertical datum: NADV88
- Vertical unit: Meters

The ellipsoidal height (h) at each single point was computed by using the following relationship:

$$h = H + N \tag{3.1}$$

Where
h is the ellipsoidal height (height above the reference ellipsoid, unit in meters),
H is the orthometric height (height above the geoid, unit in meters),
N is the geoid undulation below the reference ellipsoid, $N < 0$ (unit in meters), and
the orthometric heights (H) were retrieved from the aforementioned Digital Elevation Model (DEM), and the geoid undulations (N) were computed from the GEOID09 toolkit of NGS (*5*).

Figure 3.2 depicts the above relationship as expressed in Eq. 3.1.

3.3 Groups of Test Areas: Test Areas Scale and Test Areas Terrain

By taking all of the 92 counties (92 Test Areas) in the State of Indiana into account, two groups of Test Areas

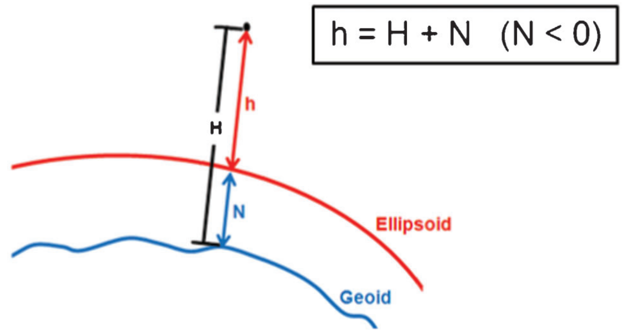


Figure 3.2 Ellipsoidal height (h), orthometric height (H) and geoid undulation (N with $N < 0$).

have been selected: known as "Test Areas Group 1," the so-called "Scale Test Areas" or "Test Areas Scale," and "Test Areas Group 2," the so-called "Terrain Test Areas" or "Test Areas Terrain." These two (subset of) groups of Test Areas represent the extreme scenarios in two different aspects: one to study the varying scale effect, the second to study the varying Terrain Effect.

3.3.1 Test Areas Scale (Test Areas Group 1)

The Test Areas Scale (also known as "Test Areas Group 1") have been selected for the study of the Scale Effect. This group consists of four counties: Tippecanoe, Posey, Madison, and Steuben County. These counties are either far or close to the classical INSPCS83 Central Meridians (CM) as defined in IC 32-19, with two counties for each original INSPCS83 zone. Tippecanoe and Posey County are close and far from the CM of the INSPCS83 West Zone respectively. Madison County is close whereas Steuben is far from the CM of the INSPCS83 East zone. Basically using either the set Tippecanoe and Posey in the West zone or the set Madison and Steuben in the East zone would have been sufficient to check the far vs. close effects from the classical CM of INSPCS83. However, in this study both pairs of counties (Test Areas) were investigated for the purpose of double checking against each other.

3.3.2 Test Areas Terrain (Test Areas Group 2)

The Test Areas Terrain (Test Areas Group 2) represent the extreme cases of terrain heights variations for the study of the Terrain Effect. Initially three different heights: ellipsoidal, orthometric, geoid undulation, have been taken into account.

The complete inventory of the statistical analysis of these heights was conducted in a state-wide, a INSPCS83 zone-wide (East/West) and a county-wide fashion. The complete set of heights statistics can be found in Appendix C.

The initial study revealed that the behavior of the ellipsoidal heights and the orthometric heights in the area of consideration (State of Indiana) agreed with each other. The “agreeable” behavior is noticeable in the way that the ranking of statistical values (max, min, mean, median, etc.) of both heights (ellipsoidal and orthometric) yielded the same identical order (see Table C.4 and Table C.5. of Appendix C). From preliminary results of the initial height analysis it became clear that further study could solely be devoted to the ellipsoidal heights due to the fact they play the main mapping role in the Scale Effect behavior, as well as in the Terrain Effect behavior. A summary of the statistical values of the ellipsoidal heights, evaluated in a state-wide and a INSPCS83 zone-wide (East/West) manner are shown in Table 3.1.

Table 3.1 clearly reveals the fact that the Terrain Effect for the State Plane Coordinate System (SPCS) currently in use in Indiana plays its main role in the East zone itself (see Row group 3 in Table 3.1). The range of the Terrain Effect of the East zone is almost equal to the one for the entire state. Therefore having two separate zones East and West in the INSPCS83 have not been considered any further while studying the reduction of the Terrain Effect.

To come up with the Test Areas Terrain (Test Areas Group 2) that represent the extreme cases of terrain heights variations for the study of the Terrain Effect, two different extreme cases have been focused on: (1) counties that are overall low or overall high, and (2) counties that exhibit large or small height variations within their boundaries.

It could be foreseen that the most problematic terrain type is the one that exhibits large height variations. This is due to the fact that in a fixed mapping area one is able to reduce the Terrain Effect simply by bringing up the reference surface to meet the level of the average terrain height. Areas that exhibit large height variations (range) one is left with some parts of the area being lower or some parts being higher than the reference surface. Therefore the Terrain Effect can be reduced drastically if the area exhibits small height variations while for the case of large variations, not a great deal of improvement is to be expected.

As mentioned above, the ellipsoidal height statistics are the ones of interest. The rankings, that have been performed on all statistical values of the ellipsoidal heights in all Test Areas (counties) in Indiana, yielded a subset of five counties that possess extreme behavior. These five counties are shown in Table 3.2.

Table 3.2 results from a ranking procedure that started from the previously computed ellipsoidal heights statistics (mean, median, standard deviation, range, etc.) of each Test Area (see Table C.1 of Appendix C). Each particular statistical value of the ellipsoidal heights, which in this study are the average (mean), the median (MED), the standard deviation (STD), and the range (Min-Max) of each Test Areas, was used separately in the ranking process. The ranking process began in a descending order from a Test Area (county) that has highest value of average height to the one that possesses the lowest one. The same routine of ranking is applied in similar fashion to the median values, to the standard deviation values and to the range values of the ellipsoidal heights.

The findings are that Randolph is the county that has the highest value of average height, the so-called “overall high,” while Posey is the one that has lowest value of average height, the so-called “overall low.” The ranking of the median of the heights agrees with the one of the average heights therefore the computational consistency is confirmed. It also meant that the median was not further used in this study as a defining characteristic.

The ranking of the standard deviations and the ranges of the heights became the key for the selection of those extreme counties that exhibit large height variations which has been judged earlier to be a problematic terrain type. The results from Table 3.2 show that the ranking of the standard deviations and the ranges of the heights did not agree with each other, they remained subsequently in our studies important parameters describing terrain characteristics. This means that certain terrains that have the same standard deviation or range of heights may have totally different terrain characteristics or terrain patterns as is shown in Figure 3.3. Terrain type 3, 4, and 5 are hardly distinguishable by their values of the height ranges regardless of the fact that they have totally different terrain characteristics.

The reason why different types of terrain characteristics are of interest is because they play a role in defining mapping areas. This is an important issue in

TABLE 3.1
Summary of the ellipsoidal heights statistics of counties in Indiana, in a state-wide and in a zone-wide fashion

		Max	Min	Mean	Median	Range (Min-Max)	Standard deviation
		h_{Max}	h_{Min}	h_{avg}	h_{MED}	h_{Range}	h_{σ} (h_{STD})
Row group		(m)	(m)	(m)	(m)	(m)	(m)
1	Entire State @ County	342.14 Randolph	68.54 Posey	187.04	190.13	273.60	51.63
2	West Zone @ County	271.77 Hendricks	68.54 Posey	156.14	163.31	203.23	43.08
3	East Zone @ County	342.14 Randolph	82.38 Floyd	216.78	219.72	259.76	40.66

TABLE 3.2
Highest and lowest rank of statistical values of ellipsoidal heights in all Test Areas (counties) in Indiana

	Ranking by							
	h_{avg}		h_{MED}		h_{Range}		h_{σ} (h_{STD})	
	Mean of height		Median of height		Range of height (Min-Max)		Standard deviation of height	
	Where	Value (m)	Where	Value (m)	Where	Value (m)	Where	Value (m)
Highest rank	Randolph	297.462	Randolph	298.534	Clark	187.895	Floyd	55.407
Lowest rank	Posey	89.561	Posey	85.192	Pulaski	30.030	Pulaski	5.617

the case when a new coordinate reference system will be adopted for Indiana: the boundaries of each zone are required to be defined beforehand.

As was recognized at the beginning of this study where Test Areas are defined based on their most extreme cases on a county-by-county basis, in general the whole area of the State of Indiana can be divided in any which way in order to achieve (1) an acceptable scale variation and (2) to minimize Terrain Effect as much as possible. Therefore before any new coordinate systems are adopted, defining the mapping areas (zones) can be done logically and even wisely when that the terrain characteristics are known a priori. It means that splitting up or merging zones can be done based on the known terrain characteristics.

If only the variation of heights (height range) are considered to form the extreme cases of terrain that exhibit the largest and smallest height variations, Floyd and Clark would both represent the case of largest height variation regardless of the fact that these two counties may/may not have different terrain characteristics, whereas Pulaski is clearly the one that would represent the county that exhibits the smallest height variation. The members of the Test Areas Terrain (Group 2) were then formed based on five Test Areas (counties) as appeared in Table 3.2. These five counties are referred to as "Test Areas Terrain A (Test Areas Group 2 A)." The members of Test Areas Terrain A are as follows:

- Overall high: Randolph, with a mean of heights (h_{avg}) of 297.462 m.
- Overall low: Posey, with a mean of heights (h_{avg}) of 89.561 m.
- Largest height variation (considering the range of heights): Clark, with a height range (h_{Range}) of 187.895 m.
- Largest height variation (considering the standard deviation of heights): Floyd, with a standard deviation of heights (h_{STD}) of 55.407 m.
- Smallest height variation (considering the range of heights): Pulaski, with a height range (h_{Range}) of 30.030 m.
- Smallest height variation (considering the standard deviation of heights): Pulaski, with a standard deviation of heights (h_{STD}) of 5.617 m.

The reason why the aforementioned five counties are denoted as a sub-group indexed by "A" is because of the fact that it became necessary to denote another set of counties as another sub-group. This sub-group will be referred to as "Test Areas Terrain B (Test Areas Group 2 B)." The selections of the members of "Test Areas Terrain B" are the results from a subsequent study of different terrain characteristics.

As the importance of terrain characteristics was to be foreseen (see the discussion in the two previous paragraphs), the range of heights does not prove to be sufficient to distinguish between different terrain types (see terrain types 4 and 5 in Figure 3.3): **another parameter was needed to improve the differentiation**

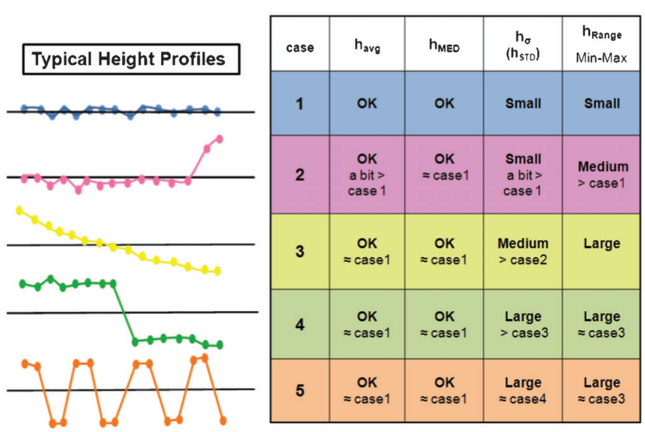


Figure 3.3 Typical terrain height profiles.

between the various types of terrain. This other needed statistical value of the heights is known as the spatial autocorrelation index. This index became subject of further investigation.

Spatial autocorrelation analysis of any observations is a way to investigate the correlation or dependency among observations in the spatial domain. The spatial autocorrelation is a statistic that measures how dependent or in another words how correlated the observations are in the considered spatial domain (or the geographic extent).

In this study of the terrain characteristics the "observations" are the ellipsoidal heights of all grid points in the considered spatial space which is in our case the extent of the Test Area (or county). In practice there are many different ways of computing spatial autocorrelation values. Depending on the different methods used, the results are named differently, for example, Moran's Index (or Moran's I) (*6*), and Geary's C (*7*). In this study, the spatial autocorrelation value, the so-called "Moran's Index" was selected.

- **Moran's Index of Spatial Autocorrelation**. In this study the Moran's Index (or known as Moran's I) is used as the statistical index of spatial autocorrelation due to the fact that Moran's I estimates the overall spatial autocorrelation in a global sense. It is the preferred method for this project where the behavior of the terrain in a Test Area (county) should be investigated as a whole/global spatial/space unit, while some other methods may be more sensitive to a local spatial autocorrelation. The mathematical details of the spatial autocorrelation analysis in term of Moran's Index are available in Appendix A, section A.6.

In order to study whether Moran's Index can be used to discriminate terrain characteristics, a test has been conducted by computing Moran's Index of the ellipsoidal heights of each Test Area (92 counties). The results of all 92 Moran's Index values were ranked in an ascending order. The values of all 92 Moran's Indices of the Indiana counties and their ranking results can be found in Table C.6 and Table C.7 of Appendix C.

Figure 3.4 depicts the terrain in Crawford County exhibiting the smallest Moran's Index value (0.34) among all counties in Indiana while Randolph County possesses the maximum Moran's Index of value (0.95). Switzerland County is in the middle between these two extreme cases: it has a Moran's Index of 0.63.

It is clear from Figure 3.4 that Crawford County has very undulating terrain, whereas Randolph's terrain is quite smooth in the sense that it is hard to find abrupt changes of heights. Switzerland's terrain behavior is neither as rough as Crawford's nor as smooth as Randolph's. Switzerland's terrain seems to be a mixed version of Crawford and Randolph.

Currently, the conclusion drawn above is based on the visualization of the terrain plots and it was explained in the sense that the closer the value of the Moran's Index is to 1 the less undulated the terrain is. When the Moran's Index value approaches zero, the smaller the correlation of heights between neighboring points is, meaning that the rougher the terrain becomes.

It seems sufficient to list the properties of the Moran's Index value based on the conclusion above, but other possible values of Moran's Indices in some other ranges (such as Moran's Index < 0) had not been explored yet. Therefore the existing conclusion of Moran's Index properties is inconclusive. It was decided at this point in the feasibility study to include a controlled test on the Moran's Index.

Despite computing Moran's Index of all Test Areas, an additional controlled test was introduced by using different simulated types of terrain. Simulated terrains are in the form of black and white images (2-dimensional space) where black pixels represent low-level terrain and the white pixels represent high-level terrain. Subsequently, the Moran's Indices of these simulated terrain images were calculated in order to confirm the perceived behavior of Moran's Index values.

Although different types of simulated terrain were created and run through the test, it can be concluded that complete Moran's Index properties can be summarized based on three distinct terrain types as

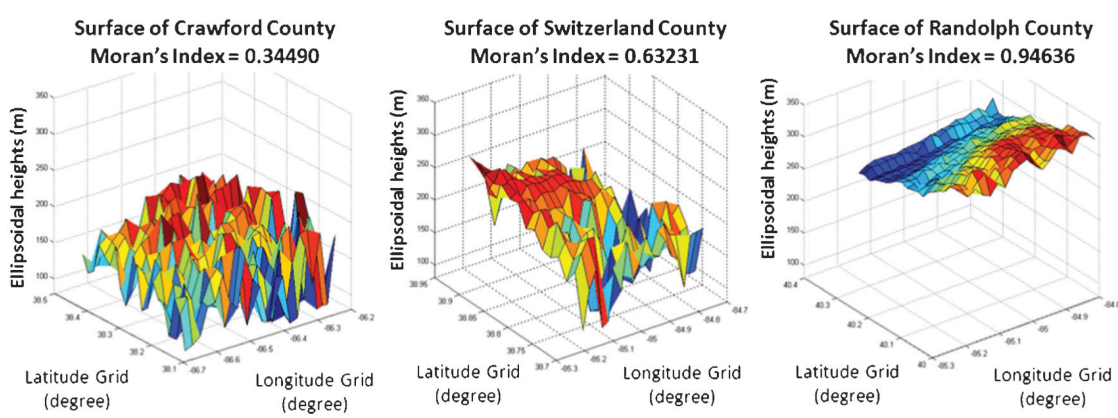


Figure 3.4 Moran's Index values and terrain plots of some Test Areas (counties) at some ranks.

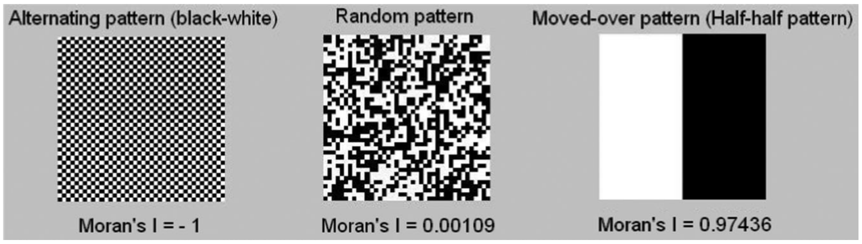


Figure 3.5 Three distinct simulated terrain characteristics and their corresponding Moran's Index values.

shown in Figure 3.5. Our conclusion of the terrain properties are as follows:

1. The Moran's Index can attain any value between +1 and −1. In mathematical terms:

$$-1 \leq \text{Moran's Index} \leq 1 \tag{3.2}$$

2. Moran's I = −1 when the observations are totally dispersed but in a predictable pattern (see Figure 3.5, left image). In this case where heights are the observations, the dispersed pattern means that the terrain behaves totally in this alternating pattern, that is perceived as undulated or rough terrain (up and down terrain) with large height variations in quite a small area.

   That means the terrain exhibits very small height correlation among nearby locations. In the other words, the height values in the area under consideration do not behave in the same way. Instead they behave in an opposite sense (up/down) which makes the Moran's Index value become negative with the smallest value of close to -1 (the behavior is in a very opposite way).

   In our study of the terrain in Indiana, there are no counties that exhibit negative Moran's Index values. The Moran's Index value of 0.34 of Crawford County is the minimum case in our study of all Test Areas (counties).
3. Moran's I ≈ 0 when the observations are in a random pattern (see Figure 3.4, middle image). A random pattern means that the observations appear in the sense that it is hard to be characterized as any specific type but random. For the case of heights as the observations, the random pattern is very hard to find. As said above, the smallest Moran's I was Crawford's: 0.34.
4. Moran's I ≈ 1 when the observations are in moved-over pattern (see Figure 3.4, right image). For the case of heights as the observations, the terrain is smoothly or gradually changes in height. That means that the terrain heights in a certain area exhibit a high height correlation among neighboring points. This is due to the fact that the terrain heights behave in a very similar way which yields a positive Moran's Index of close to 1. Randolph County belongs to this category and it is confirmed by its terrain plot in Figure 3.4 that the terrain is in the form of a non-steep sloping terrain where points of abrupt changes in heights are hard to find. The overall look is smooth, with hardly any or no abrupt height changes.
5. The Moran's Index is positive between 0 and 1, or

$$0 \leq \text{Moran's Index} \leq 1 \tag{3.3}$$

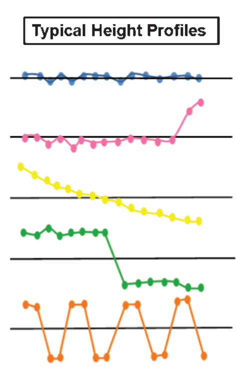
   The observations in an area are neither in a random pattern nor in a moved-over pattern. For the case of heights, the terrain appears to be not so smooth in height variation but also not any close to the random case.

   For example as in Figure 3.4, Switzerland County with a Moran's Index of 0.63 is partly in the form of gradually changing heights but not as smooth as the changes in Randolph. There are some parts of Switzerland's terrain that are quite undulated but not so "up and down" as Crawford. That is why its Moran's Index value is of the medium 0.66 level which falls between of 0.95 (Randolph's) and 0.34 (Crawford's).

In summary the behavior of the Moran's Index value has been investigated. It could be concluded that the Moran's Index can be used to distinguish between different terrain characteristics that would otherwise never be differentiable by the simple statistical values of heights such as the mean, the standard deviation, or the range (see the issues as expressed in Figure 3.3). With Moran's Index computations, terrain types are now differentiable as displayed in Figure 3.6. It should be noted that being able to differentiate between terrain type 4 and 5 is an advantage, due to the fact that dividing original terrain type 4 into two separate zones (i.e., left-right) will dramatically reduce the Terrain Effect, while not a great deal of Terrain Effect reduction is to be expected in doing so (i.e., splitting into two separate zones) for the case of terrain type 5.

* * *

In this study, the Moran's Index was used to classify the roughness of the terrains of all 92 Test Areas (counties) in Indiana. This led to a second group of Test Areas Terrain, the so-called "Test Areas Terrain B" which includes Test Areas (counties) that exhibit extreme roughness (terrain undulation) as expressed in terms of Moran's Index values. The members of the



case	h_{avg}	h_{MED}	h_σ (h_{STD})	h_{Range} Min-Max	Moran's I
1	OK	OK	Small	Small	≈ 1
2	OK a bit > case 1	OK ≈ case1	Small a bit > case 1	Medium > case1	OK
3	OK ≈ case1	OK ≈ case1	Medium > case2	Large	OK
4	OK ≈ case1	OK ≈ case1	Large > case3	Large ≈ case3	≈ 1 >case2
5	OK ≈ case1	OK ≈ case1	Large ≈ case4	Large ≈ case3	Small

Figure 3.6 Different terrain types with their corresponding Moran's Index values' behavior.

TABLE 3.3
Test Areas Terrain A (Test Areas Group 2 A, first sub-group of Test Areas Group 2)

	Test Areas Terrain A (Test Areas Group 2 A)		
Row ID	**Extreme cases**	**County**	**Corresponding ellipsoidal height statistics (m)**
1	Overall high (Maximum h_{avg})	Randolph	h_{avg} = 297.462
2	Overall low (Minimum h_{avg})	Posey	h_{avg} = 89.561
3	Largest height variation (Maximum height range)	Clark	h_{Range} = 187.895
4	Largest height variation (Maximum standard deviation of height)	Floyd	h_{STD} = 55.407
5	Smallest height variation (Minimum height range)	Pulaski	h_{Range} = 30.030
6	Smallest height variation (Minimum standard deviation of height)	Pulaski	h_{STD} = 5.617

TABLE 3.4
Test Areas Terrain B (Test Areas Group 2 B, second sub-group of Test Areas Group 2)

	Test Areas Terrain B (Test Areas Group 2 B)		
Row ID	**Extreme cases**	**County**	**Moran's Index**
1	Smoothest county (Maximum Moran's Index value)	Randolph	0.94636
2	Intermediate county (Moran's Index value falls approximately in the middle between two extremes)	Switzerland	0.63231
3	Roughest county (Minimum Moran's Index value)	Crawford	0.34490

Test Areas Terrain B (Test Areas Group 2 B) are as follows:

- Randolph (smoothest county): Moran's I = 0.94636
- Switzerland (intermediate county): Moran's I = 0.63231
- Crawford (roughest county): Moran's I = 0.34490

Being able to assess the level of terrain undulation by looking at its corresponding Moran's Index value is a very fascinating idea. However, the Moran's Index value alone is not a single final key parameter. The range of height variations (Min-Max) also plays a main role in this study. This serves as the main reason of having two different sub-groups (A and B) in the Test Areas Group 2 (Test Areas Terrain).

Any terrain with small Moran's Index values has a small height correlation. This means that nearby points do not behave in the same way and it results in a pattern of rough or undulated terrain. Rough or undulating terrain may not always be severe as long as its undulation (up-down) is jumping between a small range of height variations. Both Moran's Index value and height variations range can be used for detecting the so-called "worst-case-scenario" among the terrains: the one that is rough (undulated) and possesses a large range of height variations. The "worst-case-scenario" terrain is the one as suggested by its name, that is the worst one because its existing Terrain Effect cannot be reduced by any practical mapping method.

In conclusion, the statistical values of heights and the Moran's Index value computations can be used for distinguishing between terrain characteristics, which is an important factor in designing mapping zones (in a follow-up project?) to minimize the Terrain Effect for any newly adopted coordinate reference system.

Currently we have selected Test Areas (counties) for the study of the Terrain Effect based on a county-by-county selection and the different extreme cases of terrain. The group of Test Areas Terrain (Test Areas Group 2) consists of two sub-groups of Test Areas as shown in Table 3.3 and Table 3.4.

The Test Areas Terrain A (Test Areas Group 2 A) consists of five counties as shown in Table 3.3 whereas three counties form the members of Test Areas Terrain B (Test Areas Group 2 B). Randolph County belongs to both sub-groups of Test Areas Terrain (both A and B). Therefore in conclusion there are altogether seven counties in the Test Areas Terrain (Test Areas Group 2). All these seven Test Areas will be used in the study of the Terrain Effect.

4. RESEARCH APPROACH

In this study 92 Test Areas were created and two different groups of Test Areas were selected: Test Areas Scale (Group 1) and Test Areas Terrain (Group 2), from the data preparation process described in Chapter 3. Each Test Area in both groups is used as the input of the testing process. The testing procedure has been designed in order to assess the performance of the mappings.

The ideas behind the testing schemes (methodology) are summarized in the form of a methodology chart (see Figure 4.1). In summary, the testing scheme is the main idea behind the research approach followed. It describes the structure of how the tests are constructed based on two different sets of tests that focus on the two different effects that affect mapping accuracy. The different mapping configurations consist of three main components: the reference mapping surfaces, the radii of the reference INCRS Spheres and the mapping functions (conformal mapping methods).

It should be noted that the structure of this originally designed testing scheme turned out to be dynamic: it changed from time to time, as preliminary results were obtained. The results of the testing procedures were the cause of continuous changes. Therefore the original designed structure of the testing scheme presented in Figure 4.1 will be referred to as the "Primary Testing Scheme." From Figure 4.1, it is obvious that many procedural steps are involved in the testing as conducted in this research. The main ideas that form the building blocks of the designed testing scheme may be described as follows:

4.1 Division of Study

The tests were divided into two separate sections for different study purposes: (1) Test Section 1 is for studying Scale Effect, hence Test Areas Group 1 (Test Areas Scale) were used as the input dataset and (2) Test Section 2 is for studying Terrain Effect; therefore, Test Areas Group 2 (Test Areas Terrain Group 2 A and Group 2 B) were used as the input dataset of testing process.

4.2 Tailoring the Mapping Configurations

In both Test Sections, different mapping configurations are used in order to study the factors that influence the mapping accuracy. In summary the configurations used constituted of the following:

1. Reference mapping surfaces: two different spheres were used as mapping reference surfaces. One sphere has its origin at the Earth's Center of Mass (CoM) and the INCRS Sphere (not centered at the COM).
2. Radius of INCRS Sphere: four different radii were used for the mapping reference surfaces.
3. Mapping functions: two different mapping functions were used, the Transverse Mercator (TM) and the Oblique Stereographic (OS).

The details of each mapping configuration constituent are discussed one at the time in the following sub-sections:

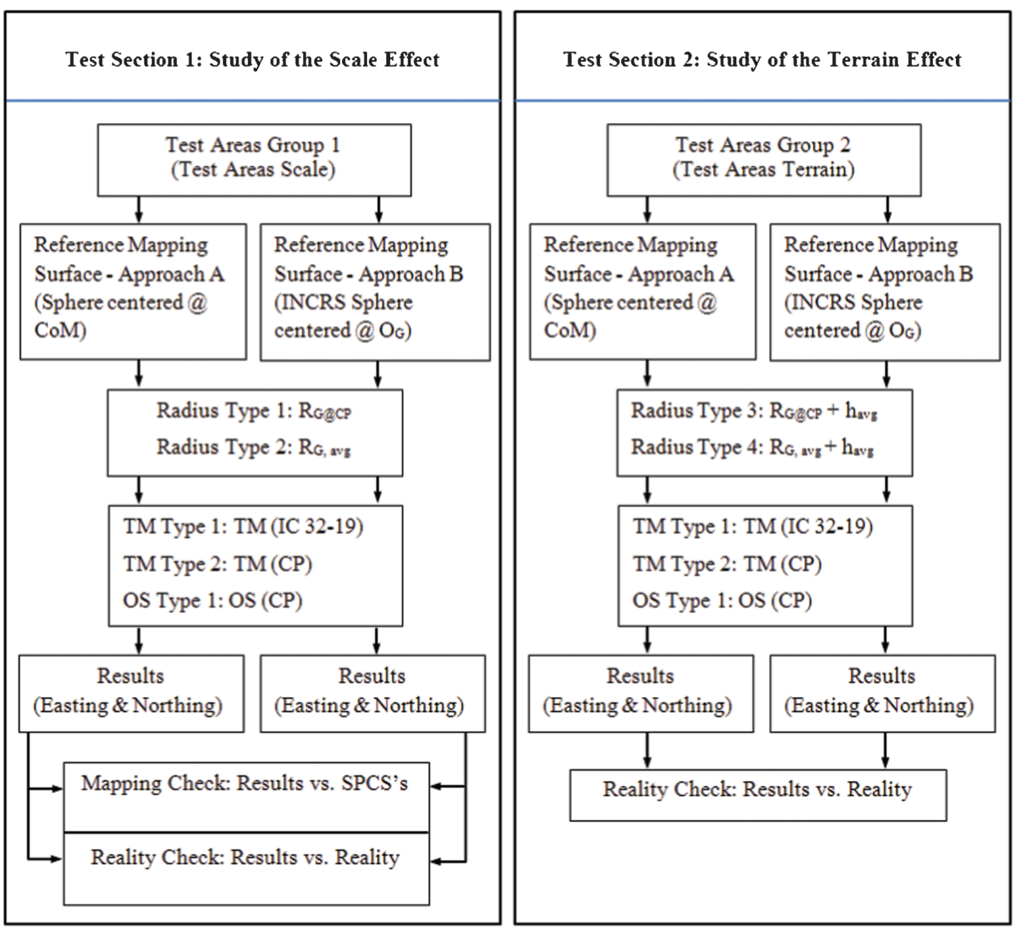


Figure 4.1 Primary Testing Scheme (originally designed testing scheme).

4.2.1 Reference Mapping Surface

All reference surfaces used in both Test Sections (Section 1/Scale and Section 2/Terrain) are pre-designed local spheres, simplifying greatly the mathematics involved without hardly any loss of accuracy. The definitions of the local spheres differed in two approaches.

1. Reference Mapping Surface—Approach A
   In this approach the reference surface is a sphere centered at the Earth's Center of Mass (CoM). It is developed mainly for confirming that the size of the errors committed in the obvious but wrong choice of the center of the reference surface at the COM, are unacceptable. In the course of this feasibility Approach A was abandoned for Approach B.
2. Reference Mapping Surface—Approach B
   The reference surface used in this case is the INCRS Sphere (see Figure 2.9). Its center is located along the ellipsoidal normal drawn at the center (point CP) of underlying project area (county). This means that in different counties, each of them will have its own mapping reference sphere with its own origin (O_G). One county's reference sphere differs from all other counties' reference spheres.

4.2.2 Radius of INCRS Sphere

There are four different values of radii of reference surfaces to be used (Radius Type 1 through Radius Type 4). The definitions of each type are as follows:

- Radius Type 1: $R = R_{G@CP}$
  The Gaussian Radius of Curvature at the project's center ($R_{G@CP}$) is adopted as the radius of the reference sphere.
- Radius Type 2: $R = R_{G,\ avg}$
  The average value of the Gaussian Radius of Curvature is the arithmetic mean of the values of all Gaussian Radii of Curvature at all grid points in a county (Test Area). This average Gaussian Radius of Curvature is adopted as the radius of the reference sphere for that corresponding county.
- Radius Type 3: $R_{G@CP} + h_{avg}$
  This radius makes use of the ellipsoidal heights (h) of all grid points. The average value of the ellipsoidal heights over the considered area is then computed (so-called "h_{avg}"). The sum of $R_{G@CP}$ (as described in Radius Type 1) and h_{avg} is then adopted as the radius of the reference sphere.
- Radius Type 4: $R_{G,\ avg} + h_{avg}$
  The reference sphere's radius is the summation of the averaged Gaussian Radii of Curvature ($R_{G,\ avg}$ as described in Radius Type 2) and the average height h_{avg}.

The radii; Types 3 and 4, were designed to study the influence of the terrain heights (Terrain Effect). Therefore the sampled grid points with their corresponding ellipsoidal heights were used as the input dataset for these cases. In contrast, the radii, Types 1 and 2 were designed to study the Scale Effect when no terrain is involved. For this reason all grid points with zero valued ellipsoidal heights were used as the input dataset. The mapped results from Test Section 1 (Scale), whereby Radius Type 1 or Type 2 were used, revealed that no significant differences in the mapped coordinates could be detected from either using Radius Type 1 or Type 2. Similarly, no significant differences in the mapped coordinates resulting from either using Radius Type 3 or Type 4 in Test Section 2 (Terrain) were detected. Therefore, only Radius Type 1 could have been used during the rest of the Scale Effect studies (Test section 1), and only Radius Type 3 could have been used for the rest of the Terrain Effect tests (Test Section 2).

However, by the time the second semi-annual report was written (the end of December 2011) it was decided that for the rest of the Scale Effect study only Radius Type 2 ($R_{G,\ avg}$) will be pursued. This is due to the fact that the h_{avg} was computed from all sampled grid points of each county. Therefore the thinking was that it would be more logical when "$R_{G,\ avg}$" was used as this value also was computed from all Gaussian Radii of Curvature at all grid points. For the same reason, the preferred use of Radius Type 4 ($R_{G,\ avg} + h_{avg}$) over Type 3 ($R_{G@CP} + h_{avg}$) was used in the Terrain Effect studies.

After further and deep investigation, it turned out that the opposite conclusion should have been drawn from what had been decided before. In computational practice it makes more sense to use Radius Type 1 ($R_{G@CP}$) for the rest of the Scale Effect studies because the Gaussian Radii of Curvature of the grid points are not required to be computed. One reference sphere with the computation of one single Gaussian Radius of Curvature at point CP is needed to model or represent the Test Area (county). It should be noted that, in this research study the center of the project (CP) of each Test Area was located based on its own extent (as previously mentioned in section 3.2). The CP coincided with the middle grid point if one deals with an odd number of grid points north-south, and east-west. The other extreme would be when the CP would fall in the middle of four neighboring grid points in the case when one dealt with a grid consisting of an even number of points north-south and east-west. In case a new INCRS is adopted the extents of each "zone" and its corresponding location of "CP" is one of those issues that needs further consideration (in a potential follow-up study).

Based on the fact that Radius Type 1 ($R_{G@CP}$) is selected to be used for the rest of testing process of the Scale Effect study, it is then obvious for similar reasons that Radius Type 3 ($R_{G@CP} + h_{avg}$) is the preferred choice over Radius Type 4 ($R_{G,\ avg} + h_{avg}$) for the rest of the study of the Terrain Effect.

4.2.3 Mapping Functions

In the testing scheme, two conformal mapping functions have been considered: the Transverse Mercator conformal mapping (TM) and the Oblique Stereographic conformal mapping (OS). They have been applied in each of the two Reference Mapping Surface – Approaches (Approach A, later abandoned, and Approach B). These two mappings have also been used to study the different types of radii. For the Transverse Mercator mapping, the longitude and latitude of the map's origin have been set in two different ways. In summary, three so-called "mapping methods" have been applied in our testing scheme:

1. *Transverse Mercator Type 1: TM(IC 32-19)*. Use of the Transverse Mercator mapping function with the longitude and latitude of the origin as defined in IC 32-19 (8); one Central Meridian (CM) for the IN East zone, and a separate Central Meridian (CM) for the IN West zone.
2. *Transverse Mercator Type 2: TM(CP)*. Use of the Transverse Mercator mapping function with the longitude and latitude of the origin as defined by the geodetic coordinates of the Test Area's project center (CP). In this case each of the areas (counties) will use their own project's center (CP) as the origin of the map. Also the Central Meridian will intersect the CP in a north-south direction.
3. *Oblique Stereographic (only one Type 1): OS(CP)*. Application of the Oblique Stereographic mapping functions uses the project's center (CP) of Test Area (county) under consideration. The CP also referred to as the new defined "Computational North Pole."

TM(IC 32-19) has been designed for two specific purposes: the check on the mathematical consistency, and the detection of any computational errors that may exist in any procedural steps of the INCRS mapping. The mapping accuracy of the mapping method TM(IC 32-19) is anticipated to be in the same ball park as the ones of INSPCS83 (Indiana State Plane Coordinate System of 1983) due to the fact that both mappings have adopted the same Central Meridians as the ones used in the classical INSPCS83. The Test Areas (counties) in the East and West Zones use the same identical Central Meridians as the ones under the INSPCS83. If the mapping accuracy committed from TM(IC 32-19) method is as anticipated, i.e., similar to the INSPCS83, it ensures the correctness of mapping procedure as used in TM(CP) mapping method.

This is because both mapping methods, TM(IC 32-19) and TM(CP), used the same step-by-step mathematical mapping routines with the only difference being the location of Transverse Mercator mapping's Central Meridian. The results obtained from the relevant study are as they were expected to be and hence the mathematical consistency is confirmed. Therefore at a certain step of testing mapping method TM(IC 32-19) has not longer been considered (for other reasons as will be explained later!).

As the preliminary results came in, obtained from tests that have been designed exactly in the way as described in the originally designed testing scheme (Primary Testing Scheme; see Figure 4.1), it became rapidly clear that **some intermediate conclusions during the testing process could be drawn.** This led to the following changes that could be applied to the remaining testing procedures.

1. *Mapping Surface—Approach A*, whereby its reference sphere's center was located at Earth's center of mass (CoM), will no longer be considered. Only the Reference Mapping Surface—Approach B will be used: the origin of the INCRS Sphere is located along the ellipsoidal normal through the CP.
2. *Radius Type 1 ($R_{G@CP}$)* will be solely used in Test Section 1 for the study of the Scale Effect,
3. *Radius Type 3 ($R_{G@CP} + h_{avg}$)* will be solely used in Test Section 2 for the study of the Terrain Effect,
4. *TM(IC 32-19) mapping method* will no longer be considered.

These changes demanded an adapted version of the original designed testing scheme. The adaptation is now referred to as the "Secondary Testing Scheme." The adapted version of testing scheme is depicted in Figure 4.2.

4.3 Results Evaluation Methods

The results of INCRS are the mapped coordinates (Easting and Northing) of the sampled grid points in each Test Area (county). The evaluation procedures have been applied to the results from both mappings of the INCRS (INCRS-OISGA) in order to evaluate the relative quality of both mapping systems.

The evaluation is performed by comparing the new mapping results in two different ways. The first evaluation deals with the ability of INCRS how well it could model the classical Indiana State Plane Coordinate System of 1983 (INSPCS83). This process is the so-called "Mapping Check." The second and most critical evaluation deals with the ability of the two new mappings (INCRS-OISGA/TM and INCRS-OISGA/OS) how well they could model the 3-dimensional undistorted coordinates in the Real World. This process is the so-called "Reality Check." In the Mapping Check, the Easting and Northing (E, N) coordinates of the new mappings are compared against the E, N coordinates from the classical INSPCS83 while in the Reality Check the E, N coordinates of the new mapping are compared against the 3D undistorted original coordinates.

For both comparisons (Mapping Check and Reality Check) an affine fitting model is used with varying numbers of parameters in the fitting procedure. The root mean squares of the fitting residuals are the indicators of how well the new mapping method has modeled the classical INSPCS83 (the Mapping Check). In addition to the use of the affine fitting as one of the evaluation tools, the computed average value of grid distance ratios what is now referred to as "DR" (1 mile and 2 miles) between the new mapping coordinates (E, N) versus the ones of NGS (under INSPCS83) and the ones computed on the Real World grids have also been considered. The results are evaluated and reported in the form of parts per million or ppm. These are the so-called "Average Grid Distance Ratio Computations" which yield averaged grid distance ratio (denoted by "DR_{avg}"). **The DR's may be considered as parts of both the Mapping Checks as well as the Reality Checks in Test Section 1 but not in Test Section 2** because the results of Average Grid Distance Ratio Computations are not meaningful in the case where terrain elevations are involved. This choice is based on the notion how (Indiana) surveyors break down one or two sections of

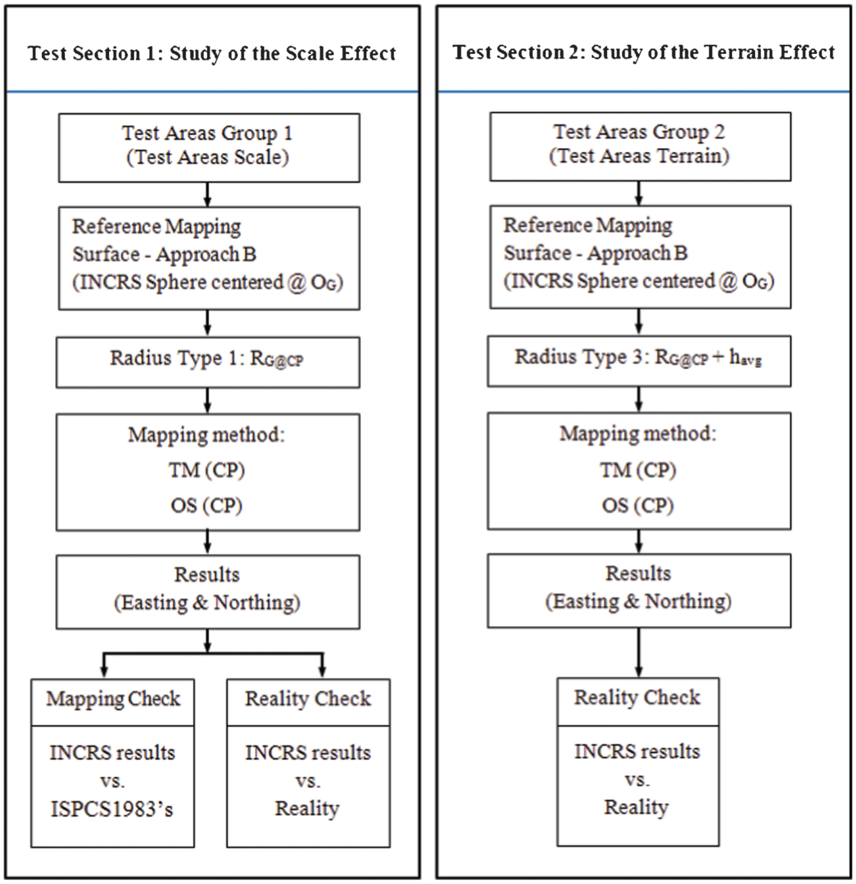


Figure 4.2 Secondary Testing Scheme (adapted version of the original testing scheme).

the United States Public Land Survey Systems (USPLSS) using GPS through INDOTS's INRTN (or InCORS) without reducing the GPS outcomes to the grid.

The Chart in Figure 4.3 shows the summary of ideas of the evaluation tools used in the Mapping Check process. The mathematical details of affine fittings (affine transformations) for different numbers of transformation parameters used in this research study are available in Appendix A, sections A.4 and A.5.

For the case of the Reality Check, two quantities will be mainly monitored: the root mean squares and the average value of (1) the so-called "O-C Differences" (also referred to as the "Differences (D)") *before* a Least Squares (LSQ) fitting is applied, and (2) the fitting residuals "V" *after* a LSQ fitting has been applied. Both variables, "D" and "V," reveal the deviations of the new mapping coordinates (INCRS-OISGA/TM and INCRS-OISGA/OS) with respect to the Real World coordinates.

The after-LSQ residuals V are (1) the indicators of internal consistency if the affine transformation is limited to a seven-parameter similarity transformation, or (2) to detect artifact deformations of the mapped grids, as expressed by suspicious values of the fitting residuals of the seven-parameter transformation, or significant deformation parameters in case of a nine-parameter affine transformation. However, the root mean squares and the average values (in each separate direction (E/N) and in the bidirectional sense (EN), see Chapter 5) of the before-LSQ O-C Differences (D's) in the case of Reality Check are excellent indicators of how well the new mapping has modeled the Real World. The D (= O-C) values are the Observed minus Calculated differences of the seven-parameter LSQ similarity transformation before the first iteration. The O-C Differences are the key for evaluating how well the new mapping coordinates have modeled the Real World. Therefore, in some detail of the O-C Differences will be discussed here. The mathematics of the O-C analysis is explained in Appendix A, section A.7.

In our study the "O" values are the new conformally mapped E and N (and h) coordinates, whereas the "C" values are the Real World undistorted 3D coordinates of the grid or terrain. The O-C Differences result from the subtraction process (differencing) between the "3D" (better 2D+1D) version of new mapping coordinates (E, N, h_v) or "Observed Coordinates (O)" and the 3D undistorted original coordinates in the Real World, the "Calculated Coordinates (C)." In the first iteration in the LSQ process the "Calculated" coordinates are

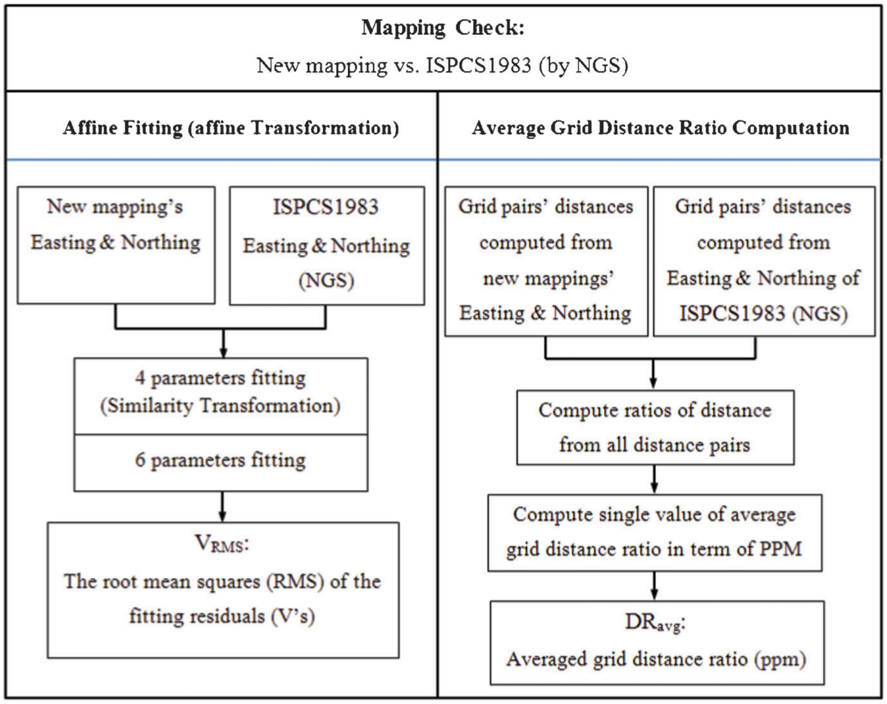


Figure 4.3 Summary of the ideas behind the Mapping Check.

actually nothing else than the coordinates of the original undistorted 3D point cloud, however suitably rotated to a new 3D local (topocentric) coordinate frame.

The Real World Cartesian coordinates (x, y, z) themselves may refer to three different cases of ellipsoidal heights (h_e): (1) the grid in a county on the ellipsoid ($h_e = 0$), or (2) the grid in a county that is situated at all h_e's = h_{avg}, h_{avg} being the average ellipsoidal height of all grid points in that county, or (3) the grid in a county with the actual ellipsoidal terrain heights included (h_e= the actual ellipsoidal heights of points denoted as "h_{Real}").

In order to logically perform the differencing between these two sets of coordinates, the 3D (actually 2D+1D) version of mapped coordinates (E, N, h_v) was introduced. It is clear that the first two elements of (E, N, h_v) are Easting and Northing coordinates respectively, the third element (h_v) of each point represents the height of that point with respect to the new corresponding mapping reference surface, i.e., the relative height of that point when considering the height of the mapping reference surface as the reference (zero reference surface). So h_v denotes the variations of the terrain with respect to the average ellipsoidal height h_{avg} in the Test Area (county). It should be noted that the zero reference surface is not an ellipsoid in the geometrical sense (but very close to it), nor a level or equipotential surface in the physical sense. So, we have for point i in the Test Area:

$$h_v(i) = h_{Real}(i) - h_{avg} \tag{4.1}$$

Although for the O-C Differencing process it seems sufficient to perform a differencing (subtraction) between (E, N, h_v) and the original Real World's Cartesian coordinates (x, y, z) in order to see the performance of any new mapping system, some prior steps are needed before the differencing can be applied in order to obtain the meaningful results. That means the original Real World's Cartesian coordinates (x, y, z) are needed to be in the form of a local still Cartesian coordinate system (as compared to the mapped one which has already been in the local system). In our study the selected form of local coordinate system, that the original Real World's Cartesian coordinates (x, y, z) will be transformed to, is the topocentric system (East, North, Up) or for short (e, n, u).

This local transformation makes the original grid points in the Real World have the "equivalent" physical coordinate components as the ones from new mapping system and makes the results from O-C Differencing process meaningful without altering any properties of the original Real World coordinates. Due to the fact that a few calculation steps were applied in order to transform Real World (x, y, z) coordinates into (e, n, u), these Real World topocentric coordinates are referred to as the "Calculated Coordinates (C)."

The results of O-C Difference process are referred to as "Difference" or "D" and are mainly reported in the form of root mean squares of the Differences (D's) in all components (all directions).

$$D = O - C \tag{4.2}$$

In this study the focus is on the first two components: East (e) and North (n) directions, as they will reflect the

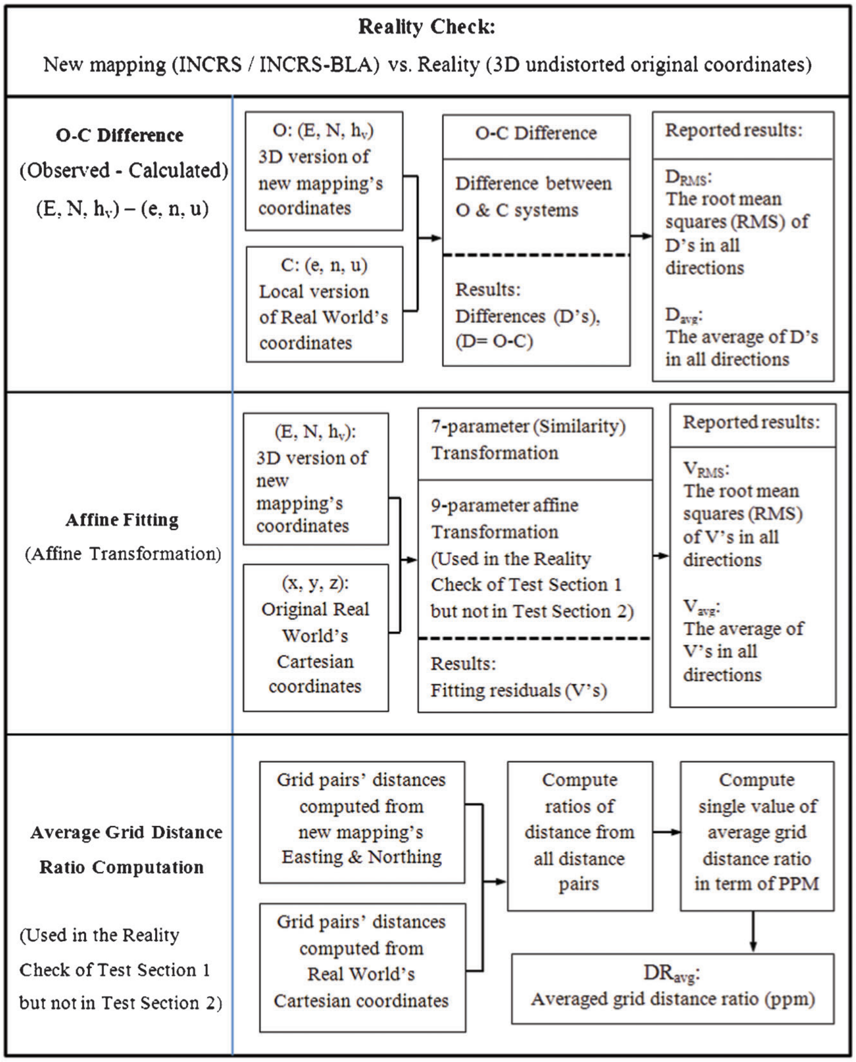


Figure 4.4 Summary of the ideas behind the Reality Check.

performance of the new mapping system on how well they have modeled reality by representing the Real World in the form of the 2D mapped coordinates Easting and Northing. The results from the Reality Check process will be reported in terms of root mean squares and averages (1) of the Differences (D's) and (2) of the fitting residuals (V's).

In case of the Reality Check of Test Section 1, where Test Areas Scale are used as the input dataset and no terrain effects are involved, the O-C Differences, the 7- and 9-parameter affine transformation, as well as the Average Grid Distance Ratio Computations are used as evaluation tools. For the case of Reality Check of Test Section 2, where the Terrain Effect is considered, the evaluation tools used, are not the same as in Test Section 1.

As previously mentioned, in the Reality Check of Test Section 2 the 9-parameter affine fitting models and the Average Grid Distance Ratio Computations have not been used as evaluation tools. In the Reality Check of Test Section 2, the (2D+1D) mapped coordinates (the Mapped World) are compared to the undistorted (3D) positions of the grid or terrain (the Real World) only through a Least Squares (LSQ) 7-parameter affine (similarity) transformation and through the O-C Differences. This is because the results of Average Grid Distance Ratio Computations are not meaningful in the case that terrain elevations are involved.

As the details of the comparison tools used in the Reality Check have already been discussed above, these ideas are still summarized in the form of a chart (see Figure 4.4).

5. RESULTS AND DISCUSSION

The material presented in this Chapter is directly related to the structure of the research approach as described in Chapter 4. The results of each test that has been performed on different groups of Test Areas (see testing scheme in Figure 4.2) are presented in this Chapter, as well as the corresponding discussion. It should be noted that the mapping results which have been subjected to all testing procedures (referred to as the Mapping Check and the Reality Check) are the results from the two INCRS-OISGA mappings only. A third mapping solution was submitted for comparison: the INCRS-S01 mapping solution. However INCRS-S01 is only available for the case of Marion County where the same set of input grid points have been tested through two different OISGA mapping systems (INCRS-OISGA/TM and INCRS-OISGA/OS) and the alternative INCRS-S01 solution. Details of the comparisons of the three INCRS mappings for Marion County dataset can be found in Chapter 6. Chapter 5 deals only with the two INCRS-OISGA mappings.

5.1 Summary Ideas of Research Testing Scheme

In this section a summary of the ideas behind the testing scheme used in this research project will be briefly discussed. It discusses some introductory information before going into detail of the results from each Test Section (Section 1 and 2). Our testing scheme consists of two main Test Sections: Test Section 1 (study of the Scale Effect) and Test Section 2 (study of the Terrain Effect).

- Test Section 1: study of the Scale Effect

- Test Areas used: Test Areas Scale (Test Areas Group 1) consists of 4 counties (see Figure 5.1):

  i. **Tippecanoe** (close to INSPCS83's East Central Meridian),
  ii. **Posey** (far from INSPCS83's East Central Meridian),
  iii. **Madison** (close to INSPCS83's West Central Meridian), and
  iv. **Steuben** (far from INSPCS83's West Central Meridian

- Mapping reference surface: INCRS Sphere (center @O_G), radius = $R_{G@CP}$.
- Mapping Method: TM(IC 32-19), Central Meridian as defined in classical INSPCS83, TM(CP), longitude & latitude of the map origin located at the center of project (CP), Central Meridian coincides with the meridian through the CP, and Oblique Stereographic (OS(CP)), CP as the computational North Pole.

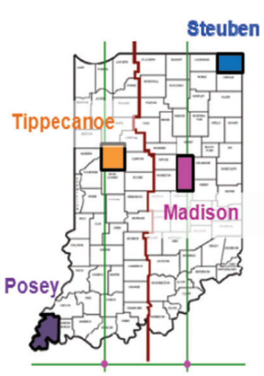


Figure 5.1 Test Areas Scale (4 counties).

- Results evaluation: Mapping Check (INCRS results vs. INSPCS83's), Reality Check (INCRS results vs. Real World).

- Test Section 2: study of the Terrain Effect

- Test Areas used: Test Areas Terrain (Test Areas Group 2) consists of 2 sub-groups: Test Areas Terrain A (Test Areas Group 2 A, see Figure 5.2a) and Test Areas Terrain B (Test Areas Group 2 B, see Figure 5.2b).

Test Areas Terrain A (Test Areas Group 2 A)

  i. **Randolph** (overall high), with a mean of heights (h_{avg}) of 297.462 m.
  ii. **Posey** (overall low), with a mean of heights (h_{avg}) of 89.561 m.
  iii. **Clark** (largest height variation (considering h_{Range})), with a height range (h_{Range}) of 187.895 m.
  iv. **Floyd** (largest height variation (considering h_{STD})), with a standard deviation of heights (h_{STD}) of 55.407 m.
  v. **Pulaski** (smallest height variation (considering h_{Range})), with a height range (h_{Range}) of 30.030 m. Pulaski is also the county that possesses the smallest height variation when considering the standard deviation of heights (h_{STD}), with h_{STD} of 5.617 m.

Test Areas Terrain B (Test Areas Group 2 B)

  i. **Randolph** (smoothest county), with Moran's Index = 0.94636
  ii. **Switzerland** (intermediate county), with Moran's Index = 0.63132
  iii. **Crawford** (roughest county), with Moran's Index = 0.34490

- Mapping reference surface: INCRS Sphere (center @O_G), radius = $R_{G@CP} + h_{avg}$
- Mapping Method: TM(CP), longitude & latitude of map origin located at the center of project (CP), Central Meridian coincides with the meridian through the CP, and Oblique Stereographic (OS(CP)), CP as the computational North Pole.
- Results evaluation: Reality Check (INCRS results vs. Real World).

Since Randolph County is also featured in the Test Areas Group 2 A (as the highest in average county) the final Test Areas Group 2 consists only of seven counties (Randolph, Posey, Clark, Floyd, Pulaski, Switzerland, and Crawford) as shown in Figure 5.3.

Details of the evaluation tools used in the process of the Mapping Check and Reality Check have been described in Chapter 4, section 4.3. For a quick reference, a summary of the ideas behind the Mapping check and the Reality Check have been presented in the form of charts (see Figures 4.3 and 4.4, respectively).

5.2 Results of Test Section 1 (Scale)

In Test Section 1 where the Scale Effect is of interest, the Test Areas Group 1 (Test Areas Scale) has been

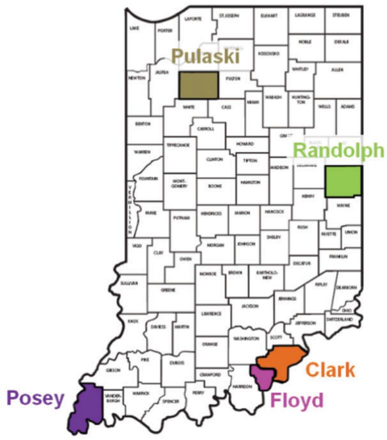


Figure 5.2a Test Areas Terrain A (Test Areas Group 2 A).

Figure 5.2b Test Areas Terrain B (Test Areas Group 2 B).

used throughout all steps of the testing procedures. It should be noted that due to the focus on the Scale Effect and its behavior, elimination of the influence of terrain heights in this particular study is mandatory. Therefore, from the start the sampled grid points used in each Test Area of Group 1 were forced to be on the surface of GRS80 ellipsoid. That means the ellipsoidal heights of all grid points were set to be equal to zero. This means that the test results of this subset may also be directly compared to the Eastings and Northings of the INSPCS83. When one uses the SPCS, the survey observations are ALWAYS reduced to the reference ellipsoid.

A series of procedural steps is needed in order to obtain a set of mapped coordinates as the final results. For a Test Area (a county), it starts with sampling the grid points that cover the county's area by the sampling method as mentioned in Chapter 3, section 3.2. After having sampled the grid points, step-by-step tasks of the INCRS mapping procedure were followed, finally resulting in mapped points expressed in terms of Easting and Northing coordinates. The details of the INCRS mapping's steps have been described in Appendix A, section A.3.

With Easting and Northing coordinates as end results of the INCRS mapping, the quality of the mapped coordinates have been evaluated through two processes, the so-called Mapping Check and the so-called Reality Check. The conclusions about the quality of these mappings were based on these two checks.

5.2.1 Results of the Mapping Check in Test Section 1 (Scale)

In Test Section 1, the Mapping Check process has been used to evaluate the INCRS mapping results against the INSPCS83.

Affine Fitting Transformations, as well as Averaged Grid Distance Ratio Computations were used as tools to evaluate the results. In two dimensions a 4-parameter (similarity transformation) and 6-parameter affine fitting were both considered. The quality of fitting was reported in the terms of root mean squares (RMS) of fitting residuals (V's) denoted by V_{RMS}(E-W) or V_{RMS}(E) for short and V_{RMS}(N-S) or V_{RMS}(N) for short in east-west and north-south direction, respectively. The combined version of the fitting residual V_{RMS} that reports on the overall fitting quality in both directions (not a separate direction) was expressed in term of a single number denoted by V_{RMS}(EN). The latter variable may be referred to as the root mean square of the bidirectional residuals. The fitting quality reflects the performance of INCRS mapping on how well it has modeled the INSPCS83 **(NOT Reality!)**.

The numbers shown in Table 5.1 and Table 5.2 are the results from 4-parameter (similarity transformation) and 6-parameter Affine Fitting transformations between the mapped coordinates under the INCRS mapping and the corresponding ones by NGS mapped under INSPCS83. From now on the results of the INCRS-OISGA mapping (for short INCRS mapping) will be simply referred to as "INCRS coordinates." The corresponding mapped coordinates Easting and Northing by NGS under the INSPCS83 are referred to as the "INSPCS83 Coordinates." The results show the deviations of the INCRS coordinates mapped by different mapping methods (TM(IC 32-19), TM(CP)

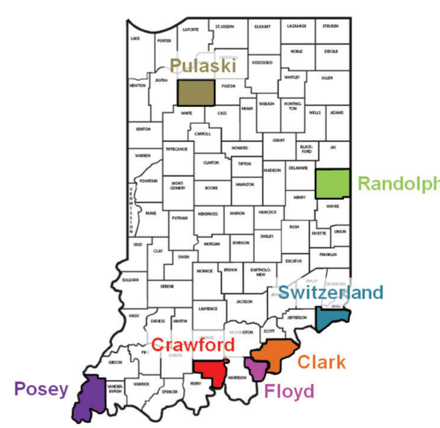


Figure 5.3 Test Areas Terrain (7 counties).

and OS(CP)) from the INSPCS83 Coordinates, after 4- and 6-parameter transformations have been applied.

In this Mapping Check, three different mapping methods were investigated: those are the TM(IC 32-19), TM(CP), and OS(CP), with the details of each mapping method having been described in Chapter 4, section 4.2.3. In summary and for a quick reference, the TM(IC 32-19) is a Transverse Mercator mapping method that uses the INCRS Sphere as mapping reference surface with the radius of $R_{G@CP}$ and the longitude and latitude of the origin as defined in IC 32-19. This means that the INCRS Coordinates are based on the same Central Meridians as the ones that have been defined for the classical INSPCS83. However, the TM(CP) is based on the same concept but longitude and latitude of the origin are defined by the geodetic coordinates of the test area's project center (CP). That means that the (local!) Central Meridian for each Test Area (county) runs north-south through the CP. The OS(CP), an Oblique Stereographic mapping, uses the project's center (CP) as the newly defined "Computational North Pole."

The purpose of this Mapping Check is the evaluation of the performance of the INCRS mapping vs. the classical INSPCS83 mapping (but again, NOT the Real World!!!).

This Mapping Check has been inspired by the question "Does the distance of a grid point being far from or close to the Central Meridians of the two (East and West zones) classical INSPCS83's influence the mapping scale behavior of INCRS mapping?" It seems that the answer to such an easy question may be given by mere visual inspection.

The answer is an obvious "no." Due to the fact that the INCRS mapping, TM(CP), makes use of its own local Central Meridian through its own defined center of the project area (CP), it is not an issue whether the Test Area is close or far to the pre-defined Central Meridian because that is irrelevant. The "no" answer is also held for the case of the OS(CP). In principle, there is also no relationship between the Central Meridian of the classical INSPCS83 and the CP of the OS. If the question could have been answered easily, why bother to conduct the Mapping Check to begin with? The reason is that Mapping Check is not just to reconfirm the aforementioned "no" answer but in fact has many elements of embedded usefulness.

The usefulness of the Mapping Check process is listed below, along with related explanations and discussions of the results.

1. The Mapping Check process may detect the existence of artifact deformations or distortions of the mapped grids.

It is not beyond expectation that the 6-parameter Affine Fitting will produce smaller size of residuals (V's) due to that fact that in general the grid points will be better adjusted through a transformation model that consists of a larger number of parameters.

Although the use of the 6-parameter affine transformation indeed yields smaller size fitting residuals than the ones resulting from a 4-parameter similarity transformation, the difference in residuals size of these two transformations (4- vs. 6-parameter) proved to be insignificant. This became obvious after it was detected that the differences in the size of residuals resulting from these two transformations were extremely small.

It can be seen from a comparison of Tables 5.1 and 5.2 that the mapped grid points have a high internal consistency: no artifact deformation seems to exist after the systematic biases have been removed through the Affine Fitting process.

In contrast, if the grids would contain an artifact deformation that theoretically may be better modeled by a 6-parameter transformation, the 4-parameter

TABLE 5.1
Results of the 4-parameter Affine Fitting (similarity transformation) during the Mapping Check process (INCRS-OISGA vs. INSPCS83) of the Test Areas Scale (Test Area Group 1)

Test Areas Scale	Root mean squares of the fitting residuals (V's)	Results of the 4-parameter Affine Fitting (similarity transformation)					
		TM(IC 32-19) (CM as in classical INSPCS83)		INCRS-OISGA Mapping			
				TM(CP)		OS(CP)	
		(cm)	(ft.)	(cm)	(ft.)	(cm)	(ft.)
Tippecanoe (Close to CM of INSPCS83-W)	V_{RMS} (E)	0.0287	0.0009	*2.976*	*0.098*	*3.103*	*0.102*
	V_{RMS} (N)	0.0285	0.0009	*4.649*	*0.153*	*4.717*	*0.155*
	V_{RMS} (EN)	0.0404	0.0013	*5.520*	*0.181*	*5.646*	*0.185*
Madison (Close to CM of INSPCS83-E)	V_{RMS} (E)	0.0282	0.0009	*1.233*	*0.040*	*1.563*	*0.051*
	V_{RMS} (N)	0.0278	0.0009	*1.406*	*0.046*	*1.569*	*0.051*
	V_{RMS} (EN)	0.0396	0.0013	*1.870*	*0.061*	*2.215*	*0.073*
Posey (Far from CM of INSPCS83-W)	V_{RMS} (E)	0.0303	0.0010	**21.211**	**0.696**	**21.275**	**0.698**
	V_{RMS} (N)	0.0292	0.0010	**30.558**	**1.003**	**30.586**	**1.003**
	V_{RMS} (EN)	0.0421	0.0014	**37.198**	**1.220**	**37.258**	**1.222**
Steuben (Far from CM of INSPCS83-E)	V_{RMS} (E)	0.0280	0.0009	**7.367**	**0.242**	**7.379**	**0.242**
	V_{RMS} (N)	0.0294	0.0010	**11.078**	**0.363**	**11.090**	**0.364**
	V_{RMS} (EN)	0.0406	0.0013	**13.304**	**0.436**	**13.320**	**0.437**

TABLE 5.2
Results of the 6-parameter Affine Fitting during the Mapping Check process (INCRS-OISGA vs. INSPCS83) of the Test Areas Scale (Test Areas Group 1)

Test Areas Scale	Root mean squares of the fitting residuals (V's)	Results of the 6-parameter Affine Fitting					
		TM(IC 32-19) (CM as in classical INSPCS83)		INCRS-OISGA Mapping			
				TM(CP)		OS(CP)	
		(cm)	(ft.)	(cm)	(ft.)	(cm)	(ft.)
Tippecanoe (Close to CM of INSPCS83-W)	V_{RMS} (E)	0.0286	0.0009	*2.976*	*0.098*	*3.083*	*0.101*
	V_{RMS} (N)	0.0283	0.0009	*4.649*	*0.153*	*4.707*	*0.154*
	V_{RMS} (EN)	0.0403	0.0013	*5.520*	*0.181*	*5.626*	*0.185*
Madison (Close to CM of INSPCS83-E)	V_{RMS} (E)	0.0281	0.0009	*1.233*	*0.040*	*1.512*	*0.050*
	V_{RMS} (N)	0.0277	0.0009	*1.406*	*0.046*	*1.555*	*0.051*
	V_{RMS} (EN)	0.0395	0.0013	*1.870*	*0.061*	*2.169*	*0.071*
Posey (Far from CM of INSPCS83-W)	V_{RMS} (E)	0.0302	0.0010	**21.211**	**0.696**	**21.264**	**0.698**
	V_{RMS} (N)	0.0291	0.0010	**30.557**	**1.003**	**30.582**	**1.003**
	V_{RMS} (EN)	0.0420	0.0014	**37.197**	**1.220**	**37.248**	**1.222**
Steuben (Far from CM of INSPCS83-E)	V_{RMS} (E)	0.0280	0.0009	**7.367**	**0.242**	**7.377**	**0.242**
	V_{RMS} (N)	0.0294	0.0010	**11.078**	**0.363**	**11.088**	**0.364**
	V_{RMS} (EN)	0.0406	0.0013	**13.304**	**0.436**	**13.318**	**0.437**

transformation would exhibit large differences in residual size in comparison to the residuals of the 6-parameter transformation. If large (affine) deformations or distortions would be present in the mapped grid, the 6-parameter transformation would exhibit much smaller residuals than the ones of the 4-parameter transformation.

2. The Mapping Check process may disprove the (illogical) idea that the classical INSPCS83 coordinates are used as the reference coordinates against which new mapping systems are compared.

As a matter of fact, it is not a logical conclusion that new mapping coordinates (INCRS coordinates) are compared against the ones of the INSPCS83. This is due to the fact that the quality of any (new) mapping system should be tested against the undistorted point cloud coordinates as they exist in reality (the so-called "Real World"), not against any existing mapped coordinates such as the INSPCS83 coordinates. Therefore the logical comparison that would assess the quality or the performance of any new mapping system is the comparison of its mapped coordinates against the Real World (The Reality Check).

Considering the three different mapping methods as shown in Tables 5.1 and 5.2, it seems tempting to conclude that the OS(CP) mapping is not as good as the TM(CP) mapping, because it has produced larger fitting residuals. Also the TM(IC 32-19) is the best as it possesses the smallest size residuals among all of three mapping methods in all Test Areas under consideration. The aforementioned conclusion is correct only in the case of answering the question "What is the best mapping method **in modeling the classical INSPCS83** among these three mapping systems? As a matter of fact, the TM(IC 32-19) is capable of closer mimicking the INSPCS83 than either TM(CP) or OS(CP) because it did make use of the same Central Meridians as the ones being used by the INSPCS83.

Considering the effect of being either far from or close to the Central Meridians of classical INSPCS83 of the INCRS mappings (TM(CP) and OS(CP)), the results in Table 5.1 and Table 5.2 show that Posey and Steuben exhibit larger fitting residuals (identified in **boldface** at the lower part of the Tables) than the ones of Tippecanoe and Madison (identified in *italics* at the upper part of the Tables). It may be implied that being either far or close from the Central Meridians of INSPCS83 is the key as Tippecanoe is close to the West Central Meridian of the classical INSPCS83 whereas Posey is far removed from it. The same argument is valid for Madison and Steuben in the East zone of the INSPCS83. It should be noted again that **the above conclusion is only true for the case that high quality modeling of the INSPCS83 coordinates is the goal**. Since being far or close to the Central Meridians does matter to the original INSPCS83 coordinates, any new mapping systems that try to mimic the classical INSPCS83, this effect will be embedded also in those new mapping coordinates. The quality of the mapped coordinates will be accordingly. That is why the ones identified in **boldface** of Tables 5.1 and 5.2 has larger residuals than the ones identified in *italics*.

It can also be explained in the way that the results from the TM(CP) and the OS(CP) of Posey and Steuben are totally different from the ones that were mapped under the classical INSPCS83, because under the INSPCS83 Posey and Steuben are so far away from Central Meridians. This is not the case for the TM(CP) and the OS(CP) because the Central Meridians run through the centers of the counties. Due to that fact that the INSPCS83 results (regardless of their quality) are used *as a reference in this comparison*, the large difference between the mapped results from the reference (INSPCS83) and the ones from TM(CP) and OS(CP) are reflected in terms of large fitting residuals identified in **boldface** in both Tables 5.1 and 5.2.

It has become clear now that the results shown in Table 5.1 and Table 5.2 should only be used to draw conclusions in case of modeling the classical INSPCS83 is the goal. In other words, the large residuals exhibited in Posey and Steuben and the smaller residuals of Tippecanoe and Madison while being mapped under the INCRS (TM(CP) and OS(CP)) did reflect the specific ability of INCRS mapping in modeling INSPCS83 in the case that the Test Areas are either far from or close to the Central Meridians of the classical INSPCS83 respectively.

The same results do not reflect the general quality of the INCRS mapping. As a matter of fact, it does not reflect the mapping accuracy of INCRS at all.

* * *

In addition to the use of Affine Fitting process during the Mapping Check of this Test Section 1, Average Grid Distance Ratio Computations (see Chapter 4, section 4.3) have been considered as well. The results of the Average Grid Distance Ratio Computations are shown in Table 5.3.

The distances between each grid pair in each direction (N-S and E-W) have been computed from the INCRS coordinates. The ratios between them and the corresponding ones computed from INSPCS83 coordinates were then calculated resulting in a grid distance ratio for each grid pair what has been referred to as "DR."

The average value of those ratios (DR's) was then computed. In the actual process, grids distances were computed by two different methods: distance computed by adjacent point pair (approximately 1 mile) and by every other points (approximately 2 miles) resulting in two separate results.

It was discovered as presented in the second semi-annual report that without Terrain Effect or no terrain elevations involved both methods of computing DR_{avg} yielded insignificantly different results. Therefore the results presented in Table 5.3 are the averaged grid distance ratios DR_{avg}'s as computed from only the adjacent point pairs 1 mile apart.

The results of Table 5.3 show the agreement with the Affine Fitting results shown in Tables 5.1 and 5.2. The TM(IC 32-19) that closely mimics INSPCS83 (because it adopts the same Central Meridians as the ones defined by INSPCS83) yields the same results (33 ppm) regardless of the position of the Test Areas. It can be concluded that the TM(IC 32-19) system shows the scale offset as the INSPCS83 at the same level of 33 ppm. That means that TM(IC 32-19) has modeled the INSPCS83 with an accuracy of 33 ppm. The results of TM(IC 32-19) shown in Table 5.3 agree with the results shown in Tables 5.1 and 5.2 because the fitting residuals in the case of TM(IC 32-19) of all Test Areas (regardless of being far or close to the Central Meridians of the classical INSPCS83) have stayed in the same ball park: 0.0009 – 0.0010 ft. when considering residuals in separate directions E/N, and 0.0013 – 0.0014 ft. when considering bidirectional residuals.

In contrast, the Averaged Grid Distance Ratio (DR_{avg}) of the TM(CP) and OS(CP) shown in Table 5.3 do not clearly reflect the ability of INCRS in modeling the INSPCS83. This is due to the fact that the TM(CP) of each Test Area made use of its own latitude and longitude of origin (local Central Meridian). The OS(CP) is a somewhat different mapping. For these reasons each **Test Area behaved in its own individual manner** (unlike the TM(IC 32-19) mapping that has similar results as INSPCS83). Therefore the averaged grid distance ratio DR_{avg} computed for the cases of TM(CP) and OS(CP) shows embedded systematic biases (as understood in terms of the individual behavior of the grids). Hence the DR_{avg} is not a good indicator of the ability of the INCRS mappings in modeling the classical INSPCS83. Consequently, the Affine Fitting process is introduced as an indicator of the capability of the INCRS mapping in modeling INSPCS83. The Least Squares fitting process will be able to remove the systematic biases when comparing the three mapping methods under consideration.

The usefulness of the Mapping Check process that has been discussed in this section has provided insights and understanding in the INCRS mapping behavior. It is clear at this point that the results shown in the Tables show the ability of INCRS mapping in modeling INSPCS83 but not the Real World (reality).

TABLE 5.3
Results of the Average Grid Distance Ratio Computations during the Mapping Check process (INCRS-OISGA vs. INSPCS83) of the Test Areas Scale (Test Areas Group 1)

	Average Grid Distance Ratios (Grid pairs distances of INCRS vs. Grid pairs distances of INSPCS83 (NGS)) (ppm)					
	TM(IC 32-19) (CM as in classical INSPCS83)		INCRS-OISGA Mapping			
			TM(CP)		OS(CP)	
Test Areas Scale	N-S	E-W	N-S	E-W	N-S	E-W
Tippecanoe	33	33	30	30	30	31
Madison	33	33	33	33	34	34
Posey	33	33	29	29	29	28
Steuben	33	33	5	5	5	5

Comparison of the proposed mapping methods against the Real World is the most important task remaining. As a matter of fact modeling the Real World (Reality) is the goal of any mapping system. This consideration leads to the introduction of the Reality Check process whereby the ability of any mapping in modeling the Real World is thoroughly investigated.

5.2.2 Results of the Reality Check in Test Section 1 (Scale)

Previously during the Mapping Checks (5.2.1) the INSPCS83 was used as the reference mapping system. The ability of the INCRS mapping to model the classical INSPCS83 has been discussed. The results of the Mapping Check have pointed out the differences between the Scale Effect behavior of the INCRS mapping and the classical INSPCS83. There are some remaining questions of which the answers have not been addressed yet. For example, "How well can the INCRS model the Real World (and not model any existing system such as INSPCS83)?" or "How does the Scale Effect (which comes from conformal mapping process itself) behave in INCRS mappings?" The results from the Reality Check process discussed in this section will lead to the answers and explanations of the aforementioned questions.

During this Reality Check of Test Areas Scale, the Scale Effect behavior is focused on while the ability of the INCRS mapping in modeling the Real World when no Terrain Effect is involved is of interest. The results of the Reality Check process in this study will reflect ability of INCRS mapping in modeling reality (the Real World) the best when the Scale Effect is the only existing factor that affects the quality of the mapping). That quality is understood in terms of the "mapping accuracy." Without the existence of the Terrain Effect, the Affine Fitting results will indeed be a good indicator of the relative mapping accuracy when comparisons are made among different Test Areas and different mapping methods (TM(IC-32-19), TM(CP), and OS(CP)). The term "relative mapping accuracy" is used due to the fact that the only relative quality among candidates are addressed in this section of study.

That means that the Affine Fitting results can be used to point out the better mapping method; better in the sense which mapping handles the behavior of the scale well, and that it possesses a better mapping accuracy in comparison to other candidate mappings. Therefore in this section analysis of the Affine Fitting results themselves are sufficient to be used as the indicators in comparing the relative quality among three different mapping methods.

The Affine Least Square Fittings used in this Reality Check study are the 7-parameter (3D similarity) and 9-parameter (3D affine) transformation because the point clouds in 3-dimensional space (reality) are used as the reference. The results of the 7- and 9-parameter Affine Fittings are shown in Table 5.4 and 5.5 respectively. The affine fittings were performed as part of the Reality Check whereby the mapped coordinates were fitted to the real 3D undistorted original point clouds for the three mapping methods (TM(IC 32-19), TM(CP) and OS(CP)).

It is expected that for all three mapping cases (TM(IC 32-19), TM(CP) and OS(CP)), the 9-parameter Affine Fitting produces smaller size of fitting residuals (V's) than the ones from the 7-parameter similarity fitting. The same explanation applies to this case as for the cases of the 4- and 6-parameter fitting, i.e., suspicious fitting residuals in both the 7- and 9-parameter Affine fitting are non-existent. This guarantees that the mapped grids have internal consistency and are free from artifact deformations.

Considering three different mapping methods (TM(IC 32-19), TM(CP), and OS(CP)) in Table 5.4 and Table 5.5, the differences in the size of the fitting residuals of each mapping method reflect the relative abilities in modeling the Real World. In this case, the two INCRS-OISGA mappings (TM(CP) and OS(CP)) possess smaller size fitting residuals. The conclusion may be drawn that the two INCRS-OISGA mappings are considered to be of higher quality than TM(IC 32-19) in modeling the Real World point clouds. It should be noted that the results indicate that the Real World has been better modeled by TM(CP) and OS(CP) than TM(IC 32-19) in a relative sense. The absolute quality of INCRS-OISGA mappings themselves in modeling reality is yet unknown.

Comparing the mapped coordinates under the TM(IC 32-19), closely mimicking the classical INSPCS83 against the Real World coordinates, large fitting residuals can be seen when the mapped areas (Test Areas) are far removed from the adopted Central Meridians of the mapping system. It is also noticeable from the results of Steuben and Posey County, that are far from the adopted East and West Central Meridians respectively, that they exhibit larger fitting residuals (as identified in **boldface** in both Tables 5.4 and 5.5) than the ones of Tippecanoe and Madison.

It is obvious that the level of being far away from or close to the INSPCS83's Central Meridians is not of relevance for the cases of the TM(CP) and the OS(CP) at all. The TM(CP) has adopted its own locally defined mapping origin (local Central Meridian). Therefore, as shown in Tables 5.4 and 5.5, the size of the fitting residuals for all Test Areas in case of the TM(CP) are in the same ball park regardless of the location of the Test Area. OS(CP) makes use of its own local computational North Pole and hence behaves independently regardless of the Test Area being far away from or close to the INSPCS83's Central Meridians. In summary, the mapping scale (Scale Effect) behavior of TM(CP) for a Test Area can be thought of as a small individual version of INSPCS83 where a mapped zone instead of composed of many counties (as with the case of the INSPCS83) consists only of a single area with the size not larger than that of a county.

Considering the fitting residuals of case TM(CP) (every row in columns 5 and 6 of both Table 5.4 and Table 5.5) for all Test Areas Scale (4 counties), the differences in size of these fitting residuals are not significant. The size of the residuals may only depend on the difference in size of Test Areas. For a Test Area

TABLE 5.4
Results of the 7-parameter Affine Fitting (similarity transformation) during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Scale (Test Areas Group 1)

Test Areas Scale	Root mean square of the fitting residuals (V's)	Results of the 7-parameter Affine Fitting (similarity transformation)					
		TM(IC 32-19) (CM as in classical INSPCS83)		INCRS-OISGA Mapping			
				TM(CP)		OS(CP)	
		(cm)	(ft.)	(cm)	(ft.)	(cm)	(ft.)
Tippecanoe (Close to CM of INSPCS83-W)	V_{RMS} (E)	3.063	0.100	0.722	0.024	1.085	0.036
	V_{RMS} (N)	4.997	0.164	1.836	0.060	1.305	0.043
	V_{RMS} (EN)	5.861	0.192	1.973	0.065	1.697	0.056
Madison (Close to CM of INSPCS83-E)	V_{RMS} (E)	1.276	0.042	0.329	0.011	0.924	0.030
	V_{RMS} (N)	2.565	0.084	2.143	0.070	2.112	0.069
	V_{RMS} (EN)	2.865	0.094	2.168	0.071	2.305	0.076
Posey (Far from CM of INSPCS83-W)	V_{RMS} (E)	**21.236**	**0.697**	0.904	0.030	1.776	0.058
	V_{RMS} (N)	**30.742**	**1.009**	3.373	0.111	2.772	0.091
	V_{RMS} (EN)	**37.363**	**1.226**	3.492	0.115	3.292	0.108
Steuben (Far from CM of INSPCS83-E)	V_{RMS} (E)	**7.400**	**0.243**	0.745	0.024	0.792	0.026
	V_{RMS} (N)	**11.124**	**0.365**	1.020	0.033	0.567	0.019
	V_{RMS} (EN)	**13.361**	**0.438**	1.263	0.041	0.974	0.032

it is obvious that the different magnitudes of the fitting residuals in each direction (E/N) are dependent on the shape of that Test Area. This explains the scale behavior of the TM(CP).

As a matter of fact, it is the well-known behavior of the mapping scale of the Transverse Mercator mapping. The same can be said about the difference in size of the fitting residuals for the different Test Areas in case of the OS(CP).

It is noticeable from Tables 5.4 and 5.5 that for a Test Area the difference between fitting residuals in east-west (V_{RMS}(E)) and north-south (V_{RMS}(N)) direction for the TM(CP) is larger than the ones of OS(CP). It can be concluded that in a Test Area, the OS(CP) distributes the errors better and more equally in east-west and north-south direction than the TM(CP). This superior property of the OS(CP) will be more obvious if the considered Test Area is "squarish."

The question "How does the Scale Effect (which comes from conformal mapping process itself) behave in INCRS mappings?" has been answered from the discussions of the Affine Fitting (7- and 9-parameter) results as discussed in the two paragraphs above.

Yet another question to be answered is "How well does an INCRS mapping model the Real World?" Even though the results from Affine Fitting have revealed the fact that the Real World can be better modeled by TM(CP) and OS(CP) than by TM(IC 32-19), the ability of the TM(CP) and the OS(CP) in modeling the reality itself in an absolute sense has not been revealed yet.

TABLE 5.5
Results of the 9-parameter Affine Fitting during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Scale (Test Areas Group 1)

Test Areas Scale	Root mean square of the fitting residuals (V's)	Results of the 9-parameters Affine Fitting					
		TM(IC 32-19) (CM as in classical INSPCS83)		INCRS-OISGA Mapping			
				TM(CP)		OS(CP)	
		(cm)	(ft.)	(cm)	(ft.)	(cm)	(ft.)
Tippecanoe (Close to CM of INSPCS83-W)	V_{RMS} (E)	2.964	0.097	0.698	0.023	0.943	0.031
	V_{RMS} (N)	4.873	0.160	1.466	0.048	1.035	0.034
	V_{RMS} (EN)	5.704	0.187	1.624	0.053	1.400	0.046
Madison (Close to CM of INSPCS83-E)	V_{RMS} (E)	0.695	0.023	0.277	0.009	0.899	0.029
	V_{RMS} (N)	1.937	0.064	1.329	0.044	1.513	0.050
	V_{RMS} (EN)	2.057	0.068	1.357	0.045	1.760	0.058
Posey (Far from CM of INSPCS83-W)	V_{RMS} (E)	**17.520**	**0.575**	0.904	0.030	1.635	0.054
	V_{RMS} (N)	**30.654**	**1.006**	2.456	0.081	2.085	0.068
	V_{RMS} (EN)	**35.308**	**1.158**	2.617	0.086	2.649	0.087
Steuben (Far from CM of INSPCS83-E)	V_{RMS} (E)	**6.636**	**0.218**	0.659	0.022	0.609	0.020
	V_{RMS} (N)	**11.115**	**0.365**	0.914	0.030	0.510	0.017
	V_{RMS} (EN)	**12.946**	**0.425**	1.127	0.037	0.794	0.026

The Averaged Grid Distance Ratio Computation in this Reality Check process can be the key to the above unanswered question.

The results of the Averaged Grid Distance Ratio Computation are shown in Table 5.6. The results agree with the ones from the Affine Fitting (7- and 9-parameter transformations), and hence superiority of both the TM(CP) and the OS(CP) over the TM(IC 32-19) is confirmed. It seems that the TM(IC 32-19) has produced good results at an acceptable level for the case of Madison and Tippecanoe.

This is due to the fact that under the TM(IC 32-19) mapping that uses the same Central Meridians as the classical INSPCS83's, the latter two Test Areas are close to their Central Meridians. The opposite quality is revealed in the case of Posey and Steuben as these counties are so far away from their Central Meridians (especially Posey). Being sensitive to the level of being far away from or close to the adopted Central Meridian, consequently the TM(IC 32-19) mapping method will no longer been investigated in Test Section 2. It has already been proved that even with the absence of the Terrain Effect its ability in modeling the Real World is not as good as the ones of the two INCRS-OISGA mappings TM(CP) and OS(CP).

The last four columns of Table 5.6 reveal the excellent ability of INCRS-OISGA mappings in modeling the Real World when only the Scale Effect is considered (no terrain heights are involved). This quality is reflected by the term of what has been known as the "mapping correction." It can be concluded that for Test Areas Group 1 the INCRS-OISGA mappings may bring the INSPCS83 mapping corrections from the typical 33 ppm (in some instances even more than 33 ppm for some areas where one is very far removed from the INSPCS83's Central Meridians) down to the level of 2.2 ppm of mapping corrections (see the statistical summary of mapping corrections of counties in Indiana in Chapter 7, Table 7.2). The mapping correction values of INCRS-OISGA may vary depending on the size and shape of the mapped area (Test Area). For all Test Areas it can be concluded that the mapping corrections are in the same ball park as may be seen from the last four columns of Table 5.6. This is because Posey County itself is quite large in size. For other similar large size counties, the mapping corrections may only increase from 2.2 ppm by a slight amount. The worst case and the average case scenario of these INCRS mapping correction values vary over Indiana counties can be found in Table 7.2 of Chapter 7.

From Table 5.6, one other interesting aspect can be seen (which also has been revealed and confirmed by the results of the Affine Fitting as shown in Tables 5.4 and 5.5): the "beauty" of the Oblique Stereographic mapping (OS(CP)). Unlike the Transverse Mercator (TM(CP)), the Oblique Stereographic (OS(CP)) has distributed the errors equally in both north-south (N-S) and east-west (E-W) directions. This effect can be clearly seen in the last two columns of Table 5.6.

5.2.3 Conclusions of Test Section 1 (Scale)

From the results of the Mapping Check and the Reality Check performed in the Test Section 1, the following conclusions can be summarized:

1. The absence of artifact deformations in the mapped grids of all Test Areas (Test Areas Scale: all of 4 counties) that were subjected to a variety of tests, because the 4- and 6-parameter Affine Fittings produced insignificantly different results during the Mapping Check, and equally so by the 7- and 9-parameter Affine Fittings during the Reality Check process.
2. Comparing the mapped coordinates under any new mapping system against the ones of the classical INSPCS83 is not logical and may be even deceiving. The quality of any new mapping system depends how well this new mapping models reality (the Real World) not the existing INSPCS83. That means that any new mapping system should only be evaluated against their capability in modeling the Real World, not how they model existing mapping systems such as the INSPCS83.
3. TM(CP) and OS(CP) of INCRS-OISGA mapping (INCRS mapping) have modeled the Real World better than TM(IC 32-19) has done. Therefore, TM(IC 32-19) has no longer been considered in Test Section 2.
4. The scale behavior of TM(CP) of the INCRS mapping is independent from the level how close to or how far away the Test Area is from the Central Meridian of the classical INSPCS83 because each zone (Test Area) of the TM(CP) in INCRS mapping has adopted its own local Central Meridian. This independency is also valid for the OS(CP) that has its own "Computational North Pole" adopted for each zone (Test Area).

TABLE 5.6
Results of the Average Grid Distance Ratio Computation during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Scale (Test Areas Group 1)

	Average Grid Distance Ratios (Grid pairs distances of INCRS vs. Grid pairs distances of Real World) (ppm)					
	TM(IC 32-19)(CM as in classical INSPCS83)		INCRS-OISGA Mapping			
			TM(CP)		OS(CP)	
Test Areas Scale	N-S	E-W	N-S	E-W	N-S	E-W
Tippecanoe	4.3	4.2	1.3	1.2	1.4	1.4
Madison	1.1	1.0	0.7	0.6	1.6	1.6
Posey	64.0	63.9	1.6	1.4	2.2	2.2
Steuben	39.2	39.1	1.3	1.1	1.0	1.0

5. Ignoring the influence of the terrain elevations the OS(CP) of INCRS mapping has distributed its errors equally in both north-south (N-S) and east-west (E-W) directions whereas this balanced property is not present with the TM(CP) of INCRS mapping.

5.3 Results of Test Section 2 (Terrain)

In section 5.3 the results of Test Section 2 (Terrain) are presented along with the related discussions. In Test Section 2 where Terrain Effect is of interest, the Test Areas Group 2 (Test Areas Terrain) has been used throughout all steps of the testing procedures. It should be noted that this study has focused on the Terrain Effect which are directly related to the actual ellipsoidal heights of the sampled grid points. For that reason the real ellipsoidal heights of the grid points were used in this study. The ellipsoidal heights have been computed from the down-sampled Digital Elevation Model (DEM) and the geoid undulation model with related computational steps and added information as explained in Chapter 3, section 3.2. With the focus on the Terrain Effect consequently only the Reality Check process was used in the evaluation of the results.

From the results of Test Section 1, it was concluded that the TM(IC 32-19) did not need to be considered any longer. Therefore only two mapping methods TM(CP) and OS(CP) were investigated in the Test Section 2.

5.3.1 Results of Reality Check in Test Section 2 (Terrain)

The ability of the INCRS mappings in modeling the Real World when the Terrain Effect is involved is of prime interest in this study. Due to the fact that the terrain heights directly affect the quality of mapped grids, the Reality Check have been used to study this Terrain Effect, and how it influences the underlying mapping system (INCRS). The size of the deviations of the mapped grids from the Real World coordinates (in terms of the so-called O-C Differences) reflects the ability of the mapping, i.e., the INCRS mapping, in modeling reality. The seven-parameter (similarity transformation) Affine Fitting and the O-C Differencing were used as the evaluating tools in the Reality Check process of this Test Section 2. As the Terrain Effect is focused on both sub-groups of the Test Areas Group 2 (Test Areas Group 2 A and Group 2 B) resulting from the selection method as explained in Chapter 3 (section 3.3) were used as representative Test Areas (counties). These two groups represent some extreme cases of the Terrain Effect. The height statistics of all seven counties belonging to Test Areas Group 2 (Test Areas Terrain) are summarized in Table 5.7.

The Affine fitting is performed between the INCRS coordinates and the 3D undistorted grid points in reality. It should be noted that unlike the Reality Check in Test Section 1, the Reality Check in this Test Section (section 2) has used all grid points with their real ellipsoidal heights. Similar to the Reality Check process in Test Section 1, the qualities of Affine Fitting in this Test Section 2 are reported in terms of the root mean squares in each east-west (E-W) and north-south (N-S) directions as well as the bidirectional direction (EN). That is in terms of $V_{RMS}(E)$, $V_{RMS}(N)$, and $V_{RMS}(EN)$, respectively.

Similar to the way of expressing the Affine Fitting results, the results of the O-C Differencing process (also referred to as the Differences D) are also presented in similar fashion. That is in terms of $D_{RMS}(E)$, $D_{RMS}(N)$, and $D_{RMS}(EN)$.

Additionally to the root mean squares values, the fitting residuals (V's) and Differences (D's) are also reported in terms of the average values in the same set of directions (E, N, and EN). That is $V_{avg}(E)$, $V_{avg}(N)$, and $V_{avg}(EN)$ when considering fitting residuals, and $D_{avg}(E)$, $D_{avg}(N)$, and $V_{avg}(EN)$ when the O-C Differences (D's) are considered. These averaged values give an overall insight in how the mapped coordinates deviate from reality (Real World).

It should be noted that for the quality comparisons between all mapped results of the different Test Areas (counties), the quality is reported in terms of single number bidirectional statistical values of the O-C Differences (D's) and the Affine Fitting residuals (V's). $D_{RMS}(EN)$, $D_{avg}(EN)$, $V_{RMS}(EN)$, and $V_{avg}(EN)$ are used as the preferred quality indicators. The quality indicators of the separate direction (E-W or E for short) or (N-S of N for short) statistics are less convenient to handle and to compare. Only when the quality in a specific direction (E/N) is needed, the statistical values of those directions will be reported.

5.3.1a Results of the Reality Check of Test Areas Terrain A (Test Areas Group 2 A)

The results of the Reality Check from the O-C Differencing process of Test Areas Terrain A (Test Areas Group 2 A) that consists of five counties (Randolph, Posey, Clark, Floyd, and Pulaski) are shown in Tables 5.8 and 5.9. For discussion purposes, the O-C Differences of Randolph County with a maximum average value of heights (overall high), and Posey County with a minimum average value of heights (overall low) from all 92 counties are presented in Table 5.8. The O-C Differences of Clark County (largest height variation when considering the range of heights (h_{Range}) as the key), Floyd County (largest height variation when considering the standard deviation of heights (h_{STD}) as the key), and Pulaski County (smallest height variation when considering either h_{Range} or h_{STD}) are presented in Table 5.9.

1. Results of the O-C Differences: a Test Area being overall high or overall low (Test Area possessing the highest or lowest value of h_{avg}). The O-C Differences in Table 5.8 for Randolph County and Posey County are in the same ball park (see values identified in **boldface** in Table 5.8). This is also true when either the root mean squares or the average values are considered.

One may draw the incorrect conclusion from the results of Posey and Randolph as shown in Table 5.8

TABLE 5.7
Summary of the ellipsoidal height statistics of the Test Areas Terrain (Test Areas Group 2)

County	h_{Max} (m)	h_{Min} (m)	h_{Range} (m)	h_{avg} (m)	h_{STD} (m)	Moran's I
Randolph: Overall high and maximum value of Moran's Index	342.139	252.026	90.114	297.462	22.974	0.94636
Posey: Overall low	135.174	68.544	66.630	89.561	13.298	0.69487
Clark: Largest height range	270.372	82.477	187.895	169.377	43.297	0.77396
Floyd: Largest h_{STD}	266.777	82.384	184.393	175.612	55.407	0.82959
Pulaski: Smallest height range and smallest h_{STD}	198.924	168.894	30.030	180.402	5.617	0.80360
Switzerland: Intermediate value of Moran's Index	264.068	94.177	169.891	187.122	43.441	0.63132
Crawford: Minimum value of Moran's Index	230.581	82.631	147.950	163.181	33.533	0.34490

that the county that is in overall (in average) lower will get mapped better under the INCRS mappings, both TM(CP) and OS(CP), than the county that is in overall higher (has a larger value of h_{avg}). Posey possesses smaller RMS's and smaller average values of D's than Randolph. *It is in fact insufficient to draw any conclusion whether the level of being in overall high or low of a Test Area is the factor that mainly causes the O-C Differences to differ per county, i.e., the difference of the quality of mapped coordinates, solely based on the results as shown in Table 5.8*. This issue (high/low in overall) will no further be a questionable one when the results of the other Test Areas as shown in Table 5.9 are addressed.

If the potentially incorrect implication in the paragraph above is true, i.e., the overall lower (low h_{avg} value) the county is, the better it is mapped (it has smaller RMS and smaller average values for the O-C Differences (D's)), Randolph should exhibit the worst results (the largest RMS and average values of D's) while Posey should display the best results (the smallest RMS and average value of D's). The quality (RMS and average value of D's) of the mapped coordinates of other counties with h_{avg} values that are in between these two extremes should fall in between the best and the worst in an orderly fashion.

The results of Table 5.9 prove that the above claim is not true, in fact Clark (h_{avg} = 169.377 m) and Floyd County (h_{avg} = 175.612 m), both with h_{avg} values that are even lower than the one of Randolph (h_{avg} = 297.462 m), instead exhibit worse results (i.e., they possess a larger RMS and average value of D's; see values identified in **boldface** in Table 5.9) than Randolph. Another result that supports that the claim is not true, is that Pulaski County (h_{avg} = 180.402 m) with a h_{avg} value that is even higher than the one of Posey (it has the smallest h_{avg} of 89.561 m of all counties in Indiana), instead gets mapped better in comparison to Posey because Pulaski exhibits smaller (and even the smallest!) size of RMS and average value of the D's (see values identified in *italics* in Table 5.9).

The results from Tables 5.8 and 5.9 have shown that the level of being an overall high or low county:

1. does not cause any problems in the INCRS mapping; as a matter of fact the high Randolph County does not exhibit large RMS and average values of the Differences,
2. is not the factor that mainly influences the quality of the INCRS mappings. This has been proven by the fact expressed by (1), as well as the fact that the results of Randolph and Posey are not so significantly different: the quality of the mapping is in the same ball park.

The conclusions stated in (1) and (2) are justified by the results shown in Tables 5.8 and 5.9 that have been analyzed and discussed throughout the paragraphs

TABLE 5.8
Results of the O-C Differences during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Terrain A (overall high and overall low counties)

Test Areas Terrain A (overall high/low counties)	Statistical values of the O-C Difference results (D's)		INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
Randolph: Overall high h_{avg} (297.462 m), h_{Range} (90.114 m), h_{STD} (22.974m)	Root mean squares (RMS) of the Differences (D's)	D_{RMS} (E)	3.835	0.126	3.851	0.126
		D_{RMS} (N)	4.791	0.157	4.648	0.152
		D_{RMS} (EN)	**6.137**	**0.201**	**6.035**	**0.198**
	Average values of the Differences (D's)	D_{avg} (E)	−1.342	−0.044	−1.342	−0.044
		D_{avg} (N)	3.346	0.110	3.347	0.110
		D_{avg} (EN)	**4.895**	**0.161**	**4.813**	**0.158**
Posey: Overall low h_{avg} (89.561 m), h_{Range} (66.630 m), h_{STD} (13.298)	Root mean squares (RMS) of the Differences (D's)	D_{RMS} (E)	2.595	0.085	3.015	0.099
		D_{RMS} (N)	4.286	0.141	3.662	0.120
		D_{RMS} (EN)	**5.010**	**0.164**	**4.743**	**0.156**
	Average values of the Differences (D's)	D_{avg} (E)	−1.044	−0.034	−1.044	−0.034
		D_{avg} (N)	−1.546	−0.051	−1.545	−0.051
		D_{avg} (EN)	**3.909**	**0.128**	**3.534**	**0.116**

TABLE 5.9
Results of the O-C Differences during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Terrain A (counties with largest and smallest height variation)

Test Areas Terrain A (largest/smallest height variation)	Statistical values of the O-C Difference results (D's)		Results of the O-C Differences			
			INCRS TM(CP)		INCRS OS(CP)	
			(cm)	(ft.)	(cm)	(ft.)
Clark: Largest height range (187.895 m), h_{avg} (169.377 m), h_{STD} (43.297 m)	Root mean squares (RMS) of Differences (D's)	D_{RMS} (E)	10.868	0.357	10.071	0.330
		D_{RMS} (N)	6.543	0.215	6.949	0.228
		D_{RMS} (EN)	**12.686**	**0.416**	**12.235**	**0.401**
	Average values of Differences (D's)	D_{avg} (E)	3.218	0.106	3.218	0.106
		D_{avg} (N)	−1.093	−0.036	−1.090	−0.036
		D_{avg} (EN)	**9.632**	**0.316**	**9.201**	**0.302**
Floyd: Largest h_{STD} (55.407 m), h_{avg} (175.612 m), h_{Range} (184.393 m)	Root mean squares (RMS) of Differences (D's)	D_{RMS} (E)	6.344	0.208	6.328	0.208
		D_{RMS} (N)	7.221	0.237	7.164	0.235
		D_{RMS} (EN)	**9.612**	**0.315**	**9.558**	**0.314**
	Average values of Differences (D's)	D_{avg} (E)	4.184	0.137	4.184	0.137
		D_{avg} (N)	−2.557	−0.084	−2.556	−0.084
		D_{avg} (EN)	**8.187**	**0.269**	**8.162**	**0.268**
Pulaski: Smallest h_{Range} (30.03 m) & smallest h_{STD} (5.617 m), h_{avg} (180.402m)	Root mean squares (RMS) of Differences (D's)	D_{RMS} (E)	1.581	0.052	1.422	0.047
		D_{RMS} (N)	1.301	0.043	0.879	0.029
		D_{RMS} (EN)	*2.047*	*0.067*	*1.672*	*0.055*
	Average values of Differences (D's)	D_{avg} (E)	−0.758	−0.025	−0.758	−0.025
		D_{avg} (N)	−0.198	−0.007	−0.197	−0.006
		D_{avg} (EN)	*1.817*	*0.060*	*1.318*	*0.043*

above. It should be noted that the stated conclusions can be explained and visually confirmed by the fact that the mapping reference surface of the INCRS for each county locally approaches the actual terrain. The radius of the INCRS reference sphere has been increased in such a fashion that the reference surface meets the average level of the grid points of each county (Test Area) to a high degree. That leads to the "inflated" version of the reference sphere with extended radius $R_{G@CP} + h_{avg}$. Therefore, being overall high or low makes no difference and has no opposing influences on the quality of the INCRS mapping.

2. Results of the O-C Differences: a Test Area having large or small height variations. Table 5.9 presents the influence of the height variation in three extreme Test Areas: Clark County (largest h_{Range}), Floyd County (largest h_{STD}), and Pulaski County (smallest h_{Range} and smallest h_{STD}) (see also the corresponding statistics of the heights of each county in Table 5.7).

As previously mentioned in Chapter 3 (section 3.3) that based on the geometry of the INCRS mapping reference surface, it is likely that the county that exhibits a large height range will also potentially be the problematic one. This idea has been proven by the results shown in Table 5.9. Clark and Floyd exhibit larger RMS's and average values of the Differences as compared to the considerably smaller ones of Pulaski.

This is due to the reason that Pulaski County has a small height range of only 30.030 m whereas Clark and Floyd possess ranges of height of 187.895 m and 184.393 m respectively. It should be noted that the height range (h_{Range}) of Clark and Floyd are about a factor of six times larger than the height range of Pulaski.

Based on the known structure of the INCRS mappings and the results of the three extreme counties shown in Table 5.9, it seems tempting to initially recognize a pattern or trend in the mapping quality: counties that exhibits smaller height variations (having smaller values of h_{Range} or h_{STD}) tend to be mapped with a better quality than the ones with larger height variations, i.e., the former produce smaller RMS's and average values of the O-C Differences D. However, it seems insufficient to confirm the above claimed pattern by solely relying on the results of Table 5.9 because it is not always true that the county with smaller height variations will always produce better results. Therefore this claimed pattern of quality needs further investigation and justification. This will be discussed later when other related information has already been introduced.

It should be noted that the term large/small height variation that has been used so far are connected to two different issues when a Test Area is labeled as having large/small height variation: (1) the county exhibits a large/small range of heights (h_{Range}), or the other issue (2) the county exhibits a large/small standard deviation of heights (h_{STD}). Currently both issues have been considered and have contributed to different Test Areas in the Test Areas Terrain A. It is noticeable that both Clark (the largest h_{Range}) and Floyd (the largest h_{STD}) are considered and both are representatives of counties with the largest height variations, whereas Pulaski is only the representative of a county with the smallest height variation because it exhibits both the smallest range of heights (h_{Range}) and the smallest standard deviation of heights (h_{STD}).

With the results from Table 5.8 and Table 5.9, Clark County (with the largest h_{Range}) has produced the worst quality among those Test Areas in Test Areas Terrain A. It may be concluded that **using range of heights (h_{Range}) as the indicator of height variations is a better idea than using the standard deviation of heights (h_{STD}).**

This is due to the fact that Clark actually exhibits the largest h_{Range} but not the largest h_{STD}. And Clark proves also to be the one that produces the worst mapping results in comparison to the rest of the considered Test Areas of this Test Areas Group 2 A, while Floyd is the one that actually exhibits the largest h_{STD} but its mapping results have been proven to be better than Clark's.

Even though the above idea seems to be a correct conclusion for the time being based on the results of Clark and Floyd (as shown in Tables 5.8 and 5.9), but (again) it is not sufficient to draw the concrete conclusion whether from now on only the range of heights (h_{Range}) should be the sole indicator for a county with large/small height variations or whether and the large/small standard deviations of the heights (h_{STD}) should be left unused or not. This topic will be returned to after some other related information and results have been discussed.

3. Affine Fitting results of the Test Areas Terrain A (all five counties). Additional to the use of O-D Differences in the Reality Check process, the 7-parameter (similarity transformation) Affine Fitting has also been used as an evaluation tool. The results of the Affine Fitting applied to all Test Areas Terrain A (5 counties: Randolph, Posey, Clark, Floyd, and Pulaski) are shown in Table 5.10.

The results shown in Table 5.10 agree with the O-C Differences in Tables 5.8 and 5.9. Even with the influence of the Terrain Effect, it is noticeable that in some cases, e.g., the Test Area that exhibits small height variations, the INCRS mapping can handle the (negative) influence of the terrain very well and actually produces remarkable results. Pulaski is such an example. Pulaski's deviations in terms of D's (O-C Differences) and V's (Affine Fitting residuals) are not significantly different (see section of Pulaski in Table 5.9 against the corresponding one in Table 5.10). This proves that even before an Affine Fitting (which is a LSQ process) is applied, the mapping coordinates have modeled the Real World already with considerably small deviations. This means that the mapping coordinates of Pulaski are of an extreme high quality.

Furthermore, the important findings from Tables 5.8 and 5.9 are that for every Test Area in the Test Areas Terrain A (altogether five counties), in overall the OS(CP) has distorted the Real World less than the TM(CP) has done, i.e., the deviations from the Real World of the TM(CP) are larger than the ones of the OS(CP) (for each Test Area, compare the (EN) sections of the OS(CP) against the corresponding ones of the TM(CP) in both Table 5.8 and 5.9).

5.3.1b The Reality Check of the Test Areas Terrain B (Test Areas Group 2 B)

This section presents the results of the Reality Check process performed with Test Areas Terrain B (Test Areas Group 2 B). This group consists of three Test Areas (counties) that possess the extreme values of the Moran's Index (i.e., the spatial autocorrelation of heights). In summary, the members of Test Areas Terrain B are Randolph (smoothest), Switzerland (intermediate smooth), and Crawford (most undulated). Test Areas Terrain B is designed for the study of the Terrain Effect in the sense of the roughness of the terrain heights. The purpose is to study whether or not the roughness or the undulation of terrain heights plays the main role or being the main factor in the mapping quality.

The results of the O-C Differences for the Test Areas Terrain B are shown in Table 5.11 whereas the corresponding Affine Fitting results are shown in Table 5.12.

1. The results of the O-C Differences: a Test Area being rough or smooth. From the O-C results shown in Table 5.11 of the Test Areas Terrain B, it is noticeable that the mapping quality in Switzerland County is the worst one whereas the mapping quality in Randolph County is the best one among these three considered counties. These results prove that it is not necessary that a Test Area (county) with rougher terrain heights (the smaller Moran's Index value) will produce worse result: it is clear that Crawford with the roughest terrain is even better than Switzerland that has a smoother terrain (larger value of Moran's Index).

The results in Table 5.11 show that Randolph County possesses the best results among these three counties. It should be noted that this is **not** because Randolph County is the smoothest county (otherwise Switzerland should have performed better than Crawford).

The conclusion is that the roughness or the undulation of the terrain heights do not contribute much to the role of the Terrain Effect, i.e., roughness or undulation of terrain has no influence on the mapping quality. This claim has been substantiated by the results shown in Table 5.11.

2. Affine Fitting results of the Test Areas Terrain B. The 7-parameter (similarity transformation) Affine Fitting results of Test Areas Terrain B shown in Table 5.12 agree with the corresponding O-C Differences shown in Table 5.11. As a matter of fact among these three counties Randolph possesses the smallest fitting residuals whereas the largest fitting residuals belong to Switzerland County, its fitting residuals only being slightly larger than Crawford County's.

It should be noted that if among of these three counties (Randolph, Switzerland, and Crawford) the quality of the mapped results are to be ranked from best to worst, the ranking order will be in the form of: first place: Randolph, second place: Crawford and last place:

TABLE 5.10
Results of the 7-parameter Affine Fitting (similarity transformation) during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Terrain A (Test Areas Group 2 A)

Test Areas Terrain A	Statistical values the Affine Fitting residuals (V's)		INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
Randolph: Overall high h_{avg} (297.462 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	3.620	0.119	3.669	0.120
		V_{RMS} (N)	3.507	0.115	3.276	0.107
		V_{RMS} (EN)	5.040	0.165	4.918	0.161
	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	−0.000	−0.000
		V_{avg} (N)	0.000	0.000	−0.000	−0.000
		V_{avg} (EN)	4.418	0.145	4.300	0.141
Posey: Overall low h_{avg} (89.561 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	2.341	0.077	2.840	0.093
		V_{RMS} (N)	4.210	0.138	3.838	0.126
		V_{RMS} (EN)	4.817	0.158	4.774	0.157
	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	0.000	0.000
		V_{avg} (N)	0.000	0.000	0.000	0.000
		V_{avg} (EN)	3.921	0.129	3.815	0.125
Clark: Largest height range (187.895 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	8.523	0.280	8.395	0.275
		V_{RMS} (N)	6.784	0.223	6.996	0.230
		V_{RMS} (EN)	10.893	0.357	10.928	0.359
	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	−0.000	−0.000
		V_{avg} (N)	0.000	0.000	−0.000	−0.000
		V_{avg} (EN)	8.701	0.285	8.714	0.286
Floyd: Largest h_{STD} (55.407 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	4.732	0.155	4.721	0.155
		V_{RMS} (N)	6.758	0.222	6.697	0.220
		V_{RMS} (EN)	8.250	0.271	8.194	0.269
	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	0.000	0.000
		V_{avg} (N)	0.000	0.000	0.000	0.000
		V_{avg} (EN)	7.277	0.239	7.250	0.238
Pulaski: Smallest h_{Range} (30.03 m) & smallest h_{STD} (5.617 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	1.263	0.041	1.341	0.044
		V_{RMS} (N)	1.453	0.048	0.935	0.031
		V_{RMS} (EN)	1.925	0.063	1.635	0.054
	Average values of the fitting residuals (V's)	V_{avg} (E)	−0.000	−0.000	−0.000	−0.000
		V_{avg} (N)	−0.000	−0.000	−0.000	−0.000
		V_{avg} (EN)	1.444	0.047	1.255	0.041

Results of the 7-parameter Affine Fitting (similarity transformation)

Switzerland. This ranking will be denoted as "Randolph – Crawford – Switzerland." With this ranking order it is noticeable that the orders coincide with the ranking order of the range of height values (h_{Range}) when they are ranked from small to large values. We get the same ranking: "Randolph – Crawford – Switzerland." These matching orders may be judged as one of the supporting reasons of the previously stated idea (as mentioned on page 32) that the counties that exhibit smaller height variations tends to be mapped better than the ones with larger height variations.

Additionally, the important findings from the results of the O-C Differences of the Test Areas Terrain B as shown in Table 5.11 are that for every considered Test Area the OS(CP) distorts the Real World generally less than TM(CP) does. It means that the deviations from the Real World of the TM(CP) are larger than the ones of the OS(CP) (see values identified in **boldface** in Table 5.11).

Even though at this point some previously stated ideas or some preliminary conclusions have been re-enforced or confirmed by many supporting results, more analysis is needed in order to draw concrete conclusions about the study of the Terrain Effect. Therefore some further analysis will be made in the next section (5.3.2 Extended Version of the Mapping Quality Analysis) of this Chapter. An earlier temporarily closed discussion will be reopened, see the closing remarks at the bottom of page 82.

5.3.2 Extended Version of the Mapping Quality Analysis

The performance and the ability of INCRS mapping in presence of the Terrain Effect have been studied through the Reality Check process of this Test Section 2. Remarkable results of the Reality Check through the O-C Differences and the 7-parameter (similarity transformation) Affine Fitting have been presented in Tables 5.8 through 5.12 in a separate manner based on each sub-group of Test Areas Terrain. The level of being overall high/low and having large/small height variations of the Test Areas have been studied based on

TABLE 5.11
Results of the O-C Differences during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Terrain B (counties with extreme Moran's Index values)

Test Areas Terrain B (extreme Moran's Index values)	Statistical values of the O-C Difference results (D's)		Results of the O-C Differences INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
Randolph: Maximum Moran's I (0.94636), h_{Range} (90.114 m), h_{STD} (22.974 m)	Root mean squares (RMS) of the Differences (D's)	D_{RMS} (E)	3.835	0.126	3.851	0.126
		D_{RMS} (N)	4.791	0.157	4.648	0.152
		D_{RMS} (EN)	**6.137**	**0.201**	**6.035**	**0.198**
	Average values of the Differences (D's)	D_{avg} (E)	−1.342	−0.044	−1.342	−0.044
		D_{avg} (N)	3.346	0.110	3.347	0.110
		D_{avg} (EN)	**4.895**	**0.161**	**4.813**	**0.158**
Switzerland: Medium Moran's I (0.63132), h_{Range} (169.891 m), h_{STD} (43.441 m)	Root mean squares (RMS) of the Differences (D's)	D_{RMS} (E)	8.178	0.268	8.200	0.269
		D_{RMS} (N)	5.491	0.180	5.394	0.177
		D_{RMS} (EN)	**9.851**	**0.323**	**9.815**	**0.322**
	Average values of the Differences (D's)	D_{avg} (E)	3.380	0.111	3.380	0.111
		D_{avg} (N)	−1.348	−0.044	−1.347	−0.044
		D_{avg} (EN)	**7.608**	**0.250**	**7.429**	**0.244**
Crawford: Minimum Moran's I (0.34490), h_{Range} (147.950 m), h_{STD} (33.533 m)	Root mean squares (RMS) of the Differences (D's)	D_{RMS} (E)	6.371	0.209	6.481	0.213
		D_{RMS} (N)	6.536	0.214	6.263	0.205
		D_{RMS} (EN)	**9.127**	**0.299**	**9.012**	**0.296**
	Average values of the Differences (D's)	D_{avg} (E)	−0.350	−0.011	−0.350	−0.011
		D_{avg} (N)	−2.093	−0.069	−2.091	−0.069
		D_{avg} (EN)	**7.068**	**0.232**	**6.886**	**0.226**

the Test Areas Terrain A whereas the level of having smooth/rough terrain heights have been studied based on the Test Areas Terrain B. However, it turns out that the results from the different sub-groups A and B of the Test Areas Terrain have supported each other in many aspects. This leads to the idea of inspecting all the results for each member of the Test Areas Terrain (Test Areas Group 2) and to the idea of investigating the behavior of the Terrain Effect based on these all Test Areas by taking into account all aspects of the terrain heights (aspects of being overall high/low, having large/small height variation, and smooth/rough terrain) in order to draw firm conclusions about possible trends in the Terrain Effect behavior.

TABLE 5.12
Results of the 7-parameter Affine Fitting (similarity transformation) during the Reality Check process (INCRS-OISGA vs. Reality) of the Test Areas Terrain B (Test Areas Group 2 B)

Test Areas Terrain B (extreme Moran' Index values)	Statistical values of the Affine Fitting residuals (V's)		Results of the 7-parameter Affine Fitting (similarity transformation) INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
Randolph: Maximum Moran's I (0.94636), h_{Range} (90.144 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	3.620	0.119	3.669	0.120
		V_{RMS} (N)	3.507	0.115	3.276	0.107
		V_{RMS} (EN)	5.040	0.165	4.918	0.161
	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	−0.000	−0.000
		V_{avg} (N)	0.000	0.000	−0.000	−0.000
		V_{avg} (EN)	4.418	0.145	4.300	0.141
Switzerland: Medium Moran's I (0.63132), h_{Range} (169.891 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	7.465	0.245	7.498	0.246
		V_{RMS} (N)	5.353	0.176	5.233	0.172
		V_{RMS} (EN)	9.186	0.301	9.143	0.300
	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	−0.000	−0.000
		V_{avg} (N)	0.000	0.000	0.000	0.000
		V_{avg} (EN)	7.340	0.241	7.311	0.240
Crawford: Minimum Moran's I (0.34490), h_{Range} (147.950 m)	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	6.407	0.210	6.522	0.214
		V_{RMS} (N)	6.261	0.205	5.938	0.195
		V_{RMS} (EN)	8.958	0.294	8.820	0.289
	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	0.000	0.000
		V_{avg} (N)	0.000	0.000	−0.000	−0.000
		V_{avg} (EN)	7.101	0.233	6.942	0.228

TABLE 5.13
Quality ranking of the mapped grids under the INCRS mapping of the Test Areas Terrain (Test Areas Group 2)

	Quality ranking of mapped grids of the Test Areas Terrain (Consists of 2 sub-groups: A and B) (Ranking by the average values of the bidirectional Differences (D_{avg} (EN)) from the O-C Differences)					
			INCRS TM(CP)		INCRS OS(CP)	
Rank	County (h_{Range} (m) / h_{STD} (m) / Moran's I)	Average values of (D's)	(cm)	(ft.)	(cm)	(ft.)
1	Pulaski (30.030 / 5.617 / 0.80360)	D_{avg} (EN)	1.817	0.060	1.318	0.043
2	Posey (66.630 / 13.298 / 0.69487)	D_{avg} (EN)	3.909	0.128	3.534	0.116
3	Randolph (90.144 / 22.974 / 0.94636)	D_{avg} (EN)	4.895	0.161	4.813	0.158
4	Crawford (147.950 / 33.533 / 0.34490)	D_{avg} (EN)	7.068	0.232	6.886	0.226
5	Switzerland (169.891 / 43.441 / 0.63132)	D_{avg} (EN)	7.608	0.250	7.429	0.244
6	Floyd (184.393 / 55.407 / 0.82959)	D_{avg} (EN)	8.187	0.269	8.162	0.268
7	Clark (187.895 / 43.297 / 0.77396)	D_{avg} (EN)	9.632	0.316	9.201	0.302

One can tell something about the quality of the mapped grids or the ability/performance of a mapping system by inspecting its corresponding deviations from the Real World that are monitored in terms of the O-C Differences (D). In this case the overall deviation expressed in terms of the average value of bidirectional Differences D_{avg}(EN) are used as the key parameters. The quality of the mapped grids of all the Terrain Test Areas has been ranked from best to worst based on the D_{avg}(EN) values. The quality ranking results are shown in Table 5.13.

Table 5.13 shows the best result for Pulaski County whereas the worst one belong to Clark County. That means when terrain heights get involved Pulaski gets mapped under the INCRS with the smallest deviations from the Real World with the level of the average bidirectional deviations at 1.817 cm (0.060 ft.) for the TM(CP) and at 1.318 cm (0.043 ft.) for the OS(CP).

Clark represents the worst case with the largest deviations from the Real World with the level of the average bidirectional deviation at 9.632 cm (0.316 ft.) for the TM(CP) and at 9.201 cm (0.302 ft.) for the OS(CP).

Furthermore, the quality ranking of the mapped grids of Test Areas Terrain as shown in Table 5.13 are also tabulated in Table 5.14 next to the ranking of the statistical values of heights in the Terrain Test Areas. In column 3 of Table 5.14, the range of heights (h_{Range}) of all considered Test Areas have been ranked in ascending order from the Test Area that possesses the minimum value of h_{Range} to the one that exhibits the largest value of h_{Range}. In a similar fashion the ranking results of the standard deviation of heights (h_{STD}) are tabulated in the column 4 of Table 5.14 whereas column 5 holds the ranking results of Moran's Index values in a descending order that is equivalent to the ranking of the terrain roughness from the smoothest to most undulated.

It is noticeable that the ranking order of the ranges of heights (h_{Range}) presented in column 3 of Table 5.14 matches with the quality ranking order of the mapped grids (column 2 of Table 5.14) whereas the ranking order of the standard deviations of the heights (h_{STD}) and the Moran's Index values do not agree with the quality ranking.

It can be concluded that range of heights (h_{Range}) plays the main role and is the main factor of the Terrain Effect that describes the quality of mapped results whereas the standard deviations of heights (h_{STD}) and the roughness of terrain that is expressed in terms of the Moran's Indices are not the key parameters that designate the quality of mapped grids. This means that the Terrain Effect's extremes for the quality of the INCRS mappings can be based on the ranges of heights (h_{Range}).

TABLE 5.14
Quality rankings of the mapped grids (under the INCRS mapping) and the ranking of the statistical values of the ellipsoidal heights (h_{Range} (m) / h_{STD} (m) / Moran's Index) of the Test Areas Terrain (Test Areas Group 2)

	Quality Ranking of mapped grids (ranking by D_{avg} (EN))	Ranking of h_{Range} (m)	Ranking of h_{STD} (m)	Ranking of Moran's I
Rank	Min → Max	Min → Max	Min → Max	Smoothest (Max) → Roughest (Min)
1	Pulaski (30.030 / 5.617 / 0.80360)	Pulaski (30.030)	Pulaski (5.617)	Randolph (0.94636)
2	Posey (66.630 / 13.298 / 0.69487)	Posey (66.630)	Posey (13.298)	Floyd (0.82959)
3	Randolph (90.114 / 22.974 / 0.94636)	Randolph (90.114)	Randolph (22.974)	Pulaski (0.80360)
4	Crawford (147.950 / 33.533 / 0.34490)	Crawford (147.950)	Crawford (33.533)	Clark (0.77396)
5	Switzerland (169.891 / 43.441 / 0.63132)	Switzerland (169.891)	Clark (43.297)	Posey (0.69487)
6	Floyd (184.393 / 55.407 / 0.82959)	Floyd (184.393)	Switzerland (43.441)	Switzerland (0.63132)
7	Clark (187.895 / 43.297 / 0.77396)	Clark (187.895)	Floyd (55.407)	Crawford (0.34490)

TABLE 5.15
Worst/best case under the INCRS mapping of the Test Areas Terrain (Test Areas Group 2)

Test Areas Terrain (worst/best case)	Statistical values of the O-C Difference results (D's)		Results of the O-C Differences: INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
Clark	Root mean squares (RMS) of the Differences (D's)	D_{RMS} (E)	10.868	0.357	10.071	0.330
Largest height range (187.895 m)		D_{RMS} (N)	6.543	0.215	6.949	0.228
Worst case		D_{RMS} (EN)	12.686	0.416	12.235	0.401
	Average values of the Differences (D's)	D_{avg} (E)	3.218	0.106	3.218	0.106
		D_{avg} (N)	−1.093	−0.036	−1.090	−0.036
		D_{avg} (EN)	9.632	0.316	9.201	0.302
Pulaski	Root mean squares (RMS) of the Differences (D's)	D_{RMS} (E)	1.581	0.052	1.422	0.047
Smallest h_{Range} (30.030 m)		D_{RMS} (N)	1.301	0.043	0.879	0.029
Best case		D_{RMS} (EN)	2.047	0.067	1.672	0.055
	Average values of the Differences (D's)	D_{avg} (E)	−0.758	−0.025	−0.758	−0.025
		D_{avg} (N)	−0.198	−0.007	−0.197	−0.006
		D_{avg} (EN)	1.817	0.060	1.318	0.043

5.3.3 Conclusions of Test Section 2 (Terrain)

From the results of the Reality Check performed in the Test Section 2 in which the Terrain Effect is considered, the following conclusions can be drawn in summary:

1. No artifact deformations are present in the mapped grids of all Test Areas (Test Areas Terrain: all of 7 counties), because no suspicious patterns in the fitting residuals can be recognized in the process of the 7-parameter (similarity transformation) Affine Fitting. The values of the fitting residuals logically agree with the corresponding O-C Differences (the ones before the Affine Fitting is applied).
2. With the existence of the Terrain Effect when real terrain heights get involved, the beauty of the OS(CP) in distributing errors equally in both north-south (N-S) and east-west (E-W) directions are obscured.
3. Even though the standard deviations of heights (h_{STD}) tend to behave in a same pattern as the ranges of heights (h_{Range}), they do not necessarily behave in the same way per county. It means when a county has a larger range of heights (h_{Range}) does not always mean that it will have also a larger value for the standard deviation of the heights (h_{STD}) (Clark and Floyd are the prime examples).
4. Roughness or undulation of the terrain does not have a recognizable relationship with the height range (h_{Range}) or the standard deviation of heights (h_{STD}), i.e., being rough or smooth has nothing to do with having large/small ranges of heights or large/small standard deviations of heights.
5. When considering the Terrain Effect, the range of heights (h_{Range}) is the key parameter that describes the quality of the mapped grids of the INCRS mappings (see columns 2 and 3 of Table 5.14). A county that exhibits a smaller range of heights (h_{Range}) will likely be mapped better under the INCRS mapping as compared to a county that possesses a larger range of heights (h_{Range}). That means when one considers the magnitude of the height variations, the range of heights (h_{Range}) should be used as an indicator on how severe the heights values vary.
6. Directly related to (5), having a small standard deviation of heights (h_{STD}) or being quite a smooth terrain (a large value of the Moran's Index) does not warrant a high quality (less deviation from the Real World) of the mapped grids. As a matter of fact the quality of the mapped grids does not depend on whether or not the terrain is smooth/rough or having large/small standard deviation of heights (h_{STD}) (instead it mainly depends on the range of heights (h_{Range}), see (5)).
7. Based on the structure of the INCRS mapping and the already proven results, being an overall high/low (highest/lowest value of h_{avg}) county has no relationship to the mapped grids' quality.
8. Based on the average values and the root mean squares of the bidirectional deviations, D_{avg}(EN) and D_{RMS}(EN), respectively, the OS(CP) deviates less from the Real World as compared to the TM(CP). It means that the OS(CP) produces smaller values of D_{avg}(EN) and D_{RMS}(EN) as compared to the corresponding ones of the TM(CP).
9. When the Terrain Effect is considered, Clark County is the representative of the worst case (the largest h_{Range}) of the INCRS mapping: it exhibits the largest values for the deviations between the Real World and the Mapped World. In contrast, Pulaski with the smallest height variation (smallest h_{Range}) has produced the smallest values of these deviations. The statistics of the deviations D of these two extreme cases present the ball park figures of the best and the worst case when considering the effect of the terrain in the INCRS mappings. These are summarized in Table 5.15.

6. MARION COUNTY TEST

On February 6, 2012, in a meeting with members of the Study Advisory Committee of this JTRP project, surveyors, engineers, and other mapping professionals it was proposed that a test data set be made available to the community (1) to test the proposed mapping algorithm, (2) to test whether existing mapping algorithms could handle the proposed method. At the same time the researchers decided to use the dataset to iron out any mathematical and numerical differences that may exist by developing a second completely independent algorithm (code).

Despite the multi-county results presented in Chapter 5, it was proposed that this separate investigation was going to be devoted solely to Marion County. After duly preparation this dataset (Marion County Test Area) was distributed among all volunteers who had indicated interest in the test. In the end the JTRP research team and one other surveyor analyzed the Marion County Test Dataset. Three mapping solutions were submitted: two versions of the JTRP team (INCRS-OISGA), and one by a surveyor (INCRS-S01). The dataset has gone through all the same evaluation processes as described in Chapter 4: the Reality Check.

The main purpose of this Chapter is the comparison between the mapping results of INCRS -OISGA and INCRS-S01 in order to get a better understanding of the behavior of the mapping algorithms and the resulting mapping coordinates. As a matter of fact, during the same February meeting the desire was expressed that the Indiana surveyor in general, and the Indiana Department of Transportation (INDOT) in particular, would like to see the question answered as the result of this research project: "What is the preferred mapping method, and why?"

6.1 Marion County Metadata

Before the mapping results of Marion County from both INCRS-OISGA and INCRS-S01 are discussed, details of the Marion County Test Dataset and its related information are first introduced in the form of Metadata Sheet (Figure 6.1) which describes settings and parameters of Marion County as well as every other value needed beforehand in order to be able to calculate the final INCRS mapping results: the Easting and Northing coordinates.

The Metadata Sheet, is presented in the form of a framed text as shown in the following pages. The Metadata Sheet starts with the reference code of Marion County which is 49 under the reference system of IN.gov, or its other equivalent 097 under the county reference system of NGS (NGS FIPS Code). In this study, geodetic coordinate system (longitude, latitude) referring to the NAD83 datum is used. NAD83 uses the GRS80 ellipsoid. Its ellipsoidal parameters are presented on the Metadata Sheet. Furthermore, the Marion County boundaries are described in terms of their geodetic extents in both longitude and latitude directions while the sampled grid points overlaying the county are displayed in Figure A (see Figure 6.1).

Figure A depicts the sampled grid points of Marion County with their reference ID in a chess-board type naming system: alphabetical characters run from A to S in longitude direction, and numbers run from 1 to 19 in latitude direction. Hence the points in Marion County have their ID in the form of A01 to S19. Marion County has 19 points in both directions (longitude and latitude) which constitute a square grid. It should be noted that having an equal number of points in both directions are not mandatory, because counties in Indiana do not necessarily have a square form. The Test Areas, as featured in Chapter 5, the number of (grid) points in both directions vary constituting rectangular grids. For Marion's case (19 x 19 points grid), the center of the project (CP) coincides with grid point J10; hence the CP and J10 have the same geodetic coordinates. In some other Test Areas, the point CP may fall between adjacent grid points. The geodetic coordinates of the selected CP are defined in the NAD83 frame and presented in the Metadata Sheet.

In this study, the INCRS Sphere serves as the basis of the mapping reference surfaces. Its radius is equal to the Gaussian Radius of Curvature at the center of the project (CP), The so-called $R_{G@CP}$ is listed on the Metadata Sheet. In the case of Marion County the INCRS Sphere's radius = $R_{G@CP}$ = 6374224.337 m. With the pre-defined geodetic coordinates of point CP $(\lambda, \varphi, h)_{CP}$ and its computed corresponding value of Gaussian Radius of Curvature ($R_{G@CP}$) the origin of INCRS Sphere (XO_G, YO_G, ZO_G) is then located by simply moving downwards along the ellipsoidal normal at the CP by a distance equal to the Gaussian Radius of Curvature. These values are displayed in the Metadata Sheet as well.

The actual ellipsoidal heights of all the grid points have been analyzed. The statistical summaries of Marion County's ellipsoidal heights are presented in Table A of the Metadata Sheet (see Figure 6.1). The average value of ellipsoidal heights (h_{avg}) is used in the case of the "inflated" INCRS Sphere minimizing the Terrain Effect.

The INCRS mapping in the "overall" study was performed for two different purposes: (1) the study of Scale Effect and (2) the study of Terrain Effect. In both cases the INCRS Sphere with its fixed origin (XO_G, YO_G, ZO_G) serves as the basis of the reference mapping surfaces as previously mentioned. In the first case where solely the scale effect is studied, no terrain heights are involved, all original sampled grid points reside on the GRS80 ellipsoidal surface (all ellipsoidal heights are equal to zero). These points were then mapped onto (reduced to) the mapping reference surface which in this case is the "original" INCRS Sphere of Marion County, whose radius equals to $R_{G@CP}$ (the word "original" is used to represent INCRS Sphere with the unaltered value of the radius).

For the second case where the Terrain Effect was the focus of the investigation, all grid points with their real corresponding ellipsoidal heights were mapped onto the mapping reference surface which is the "inflated" INCRS Sphere with the increased radius being equal to $R_{G@CP} + h_{avg}$. These two different cases of the INCRS mapping explained above are reflected in term of using two different radii of reference: $R_{G@CP}$ for the first (Scale Effect) case, and $R_{G@CP} + h_{avg}$ for the second (Terrain Effect) case.

In summary with the computed average value of Marion County's ellipsoidal height (h_{avg}) and the pre-defined

Marion County Dataset Metadata Sheet

County Abbreviation: Ma

County Code: 49 (reference in IN.gov system)

County FIPS code: 097 (NGS FIPs code system)

Datum: NAD83 with GRS80 ellipsoid

GRS80 ellipsoidal parameters are:

- Semi-major axis (a) = 6378137.0 m
- Inverse flattening (1/f) = 298.257222101

County Boundary:

- West – East: From longitude of 86° 21' 00" W to longitude of 85° 57' 00" W
- South – North: From latitude of 39° 38' 00" N to latitude of 39° 56' 00" N

Sampled Grid Points:

To simulate a regular grid point separation in the real world with one mile spacing in east-west and north-south direction, the points based on geodetic coordinates (longitude, latitude) were sampled with the following intervals:

- Points' spacing of 00° 01' 00" in latitude direction
- Points' spacing of 00° 01' 20" in longitude direction

The allocation of grid points and their chess-board naming system are also demonstrated in Figure A.

Figure 6.1 Marion County dataset metadata sheet.

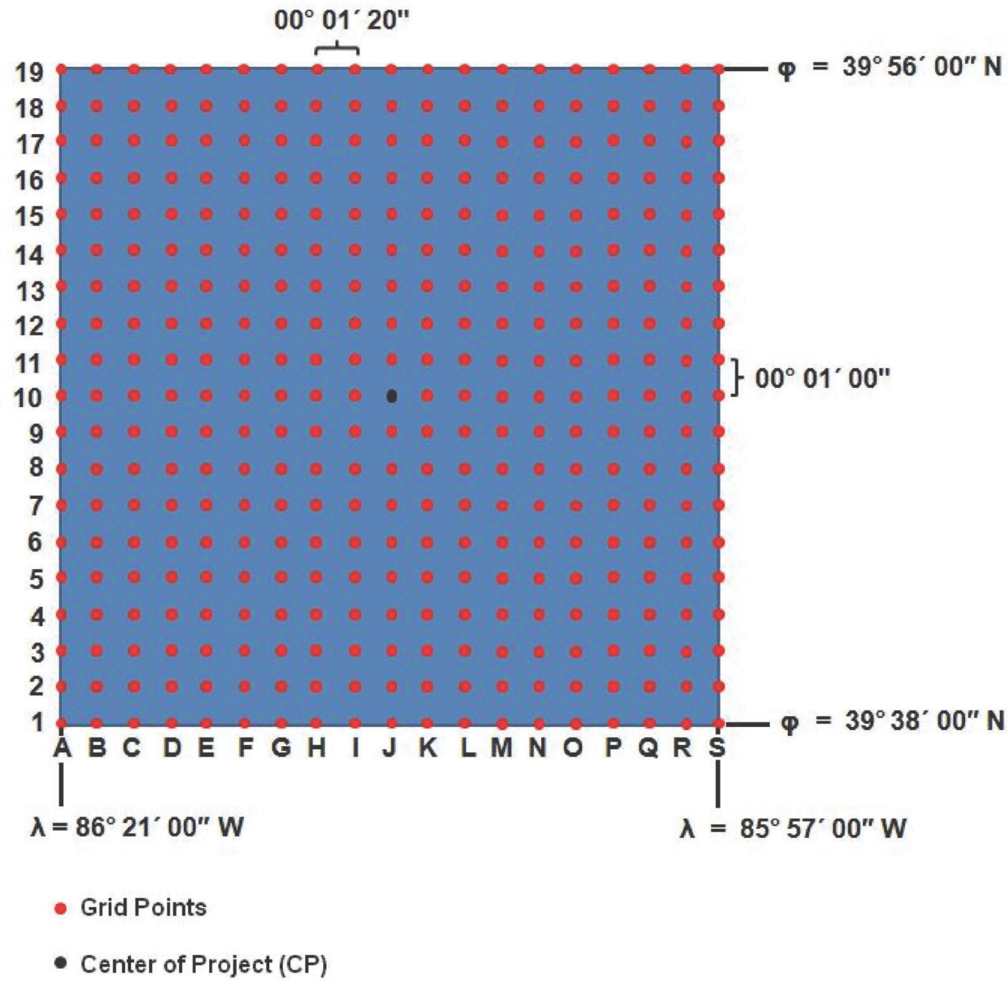


Figure A. Grid points allocation and chess-board naming system of Marion County.

<u>**Center of Project (CP)**</u>**:**

The geodetic coordinates of point CP as referred to the NAD83 datum are as follows:

$(\lambda, \varphi, h)_{CP}$ = (86° 09' 00" W, 39° 47' 00" N, 0)

It should be noted that the ellipsoidal height of point CP is equal to zero. This means that the CP lays on the ellipsoidal reference surface.

Figure 6.1 Continued.

Radius of the Reference Sphere (INCRS Sphere):

The Gaussian Radius of Curvature at the center of the project (CP) ($R_{G@CP}$) is adopted as the radius of INCRS Sphere.

For Marion County: INCRS Sphere's radius = $R_{G@CP}$ = 6374224.337 m

Origin of the INCRS Sphere:

INCRS sphere's origin (O_G) does not coincide with the geocentric origin of ellipsoid "GRS80." The ellipsoidal coordinates (λ, φ, h) and Cartesian coordinates (X, Y, Z) of INCRS sphere's origin are as follows:

- $(\lambda, \varphi, h)_G$ = (86° 09' 00" W, 39° 47' 00" N, -$R_{G@CP}$),
  = (86° 09' 00" W, 39° 47' 00" N, -6374224.337)
- (XO_G, YO_G, ZO_G) : (XO_G, YO_G, ZO_G) = (653.860 m, -9716.109 m, -19250.504 m)

Statistics of the Ellipsoidal Heights (h):

Table A. Summary of the Ellipsoidal Heights Statistics of Marion County

Statistical Values of Ellipsoidal Heights in Marion County					
Max (h_{Max})	Min (h_{Min})	Range (Min – Max) (h_{Range})	Mean (h_{avg})	Median (h_{MED})	STDEV (h_{STD})
(m)	(m)	(m)	(m)	(m)	(m)
248.056	119.812	128.245	207.022	208.631	18.425

Figure 6.1 Continued.

The Radii of the Mapping Reference Surface

INCRS Sphere serves as the basis of mapping reference surfaces. The two different cases of mappings were applied for two different study purposes: the Scale Effect and Terrain Effect. In each case, a different value of the radius R_{INCRS} of the mapping reference surface is adopted:

- $R_{INCRS} = R_{G@CP} + 000.000$ m = 6374224.337 + 000.000 = 6374224.337 m
- $R_{INCRS} = R_{G@CP} + h_{avg}$ = 6374224.337 + 207.022 = 6374431.359 m

False Easting and False Northing:

For both the Transverse Mercator (TM) as well as the Oblique Stereographic (OS) the same False Easting and the same False Northing have been used for the CP. The values of the INCRS coordinates are identical to the classical INSPCS83 coordinates for point CP (center of project).

This means for point CP (for both h = 000.000 m and h = 207.022 m) its Easting and Northing coordinates are as follows:

- ETM49 = EOS49 = ESPCS49 (East Zone) = 58597.440 m
- NTM49 = NOS49 = NSPCS49 (East Zone) = 503573.131 m

Scale Factors k:

To minimize the scale factor effect (due to the mapping and the terrain height) different scale factors have been adopted for the Transverse Mercator mapping (TM) and the Oblique Stereographic mapping (OS):

- Transverse Mercator:
  kTM = 0.999998186, for h = 000.000 m AND h = actual ellipsoidal height
- Oblique Stereographic:
  kOS = 0.999998242, for h = 000.000 m AND h = actual ellipsoidal height

Figure 6.1 Continued.

Spatial Autocorrelation of the Ellipsoidal Heights in Marion County:

Moran's Index = 0.82434

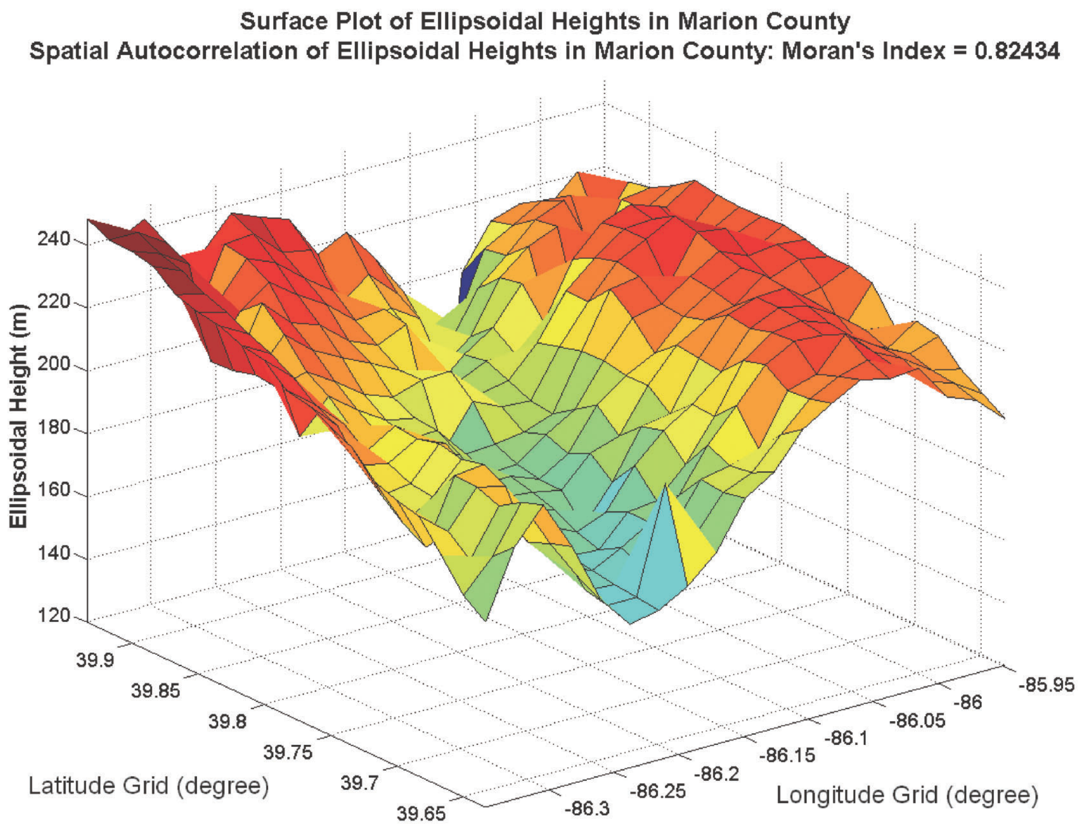


Figure B. Surface plot of ellipsoidal heights in Marion County.

Figure 6.1 Continued.

geodetic position of point CP $(\lambda, \varphi, h)_{CP}$ the remaining values of INCRS mapping's parameters are simply "derived" values as follows:

- Gaussian Radius of Curvature at the center of project (CP) ($R_{G@CP}$),
- Origin of INCRS Sphere (XO_G, YO_G, ZO_G)
- The two different radii of the reference sphere ($R_{G@CP}$ and $R_{G@CP} + h_{avg}$)

In addition to the discussed INCRS mapping parameters presented in the Metadata Sheet of Marion County which are either pre-defined ones or derived ones, there are other values used in this Marion County Test project that have also been described on the Metadata Sheet. These are the "False Easting" and the "False Northing," the "Scale Factors k" and the "Spatial Autocorrelation of the Ellipsoidal Heights" expressed in term of Moran's Index value.

In this study, the False Easting and the False Northing were adopted and applied to the mapped coordinates in order to arrive at the so-called INCRS coordinates which constitute then the final Easting and Northing coordinates belonging to this INCRS mapping. The False Easting and False Northing values on the Metadata Sheet are adopted in such a fashion that the INCRS coordinates of point CP were forced to be identical to the Easting and Northing coordinates of the same point as mapped by NGS under the classical INSPCS83.

The scale factor values (k) used in either the Transverse Mercator mapping (TM) or the Oblique Stereographic mapping (OS) have been obtained from the analysis of the scale behavior (see Chapter 2, section 2.1.1) in the whole county. This makes it possible 1) to quantify the maximum scale deviation, and 2) to deduce and adopt optimum scale factor values (kTM and kOS as shown in the Metadata Sheet) that show a balanced scale behavior.

Additionally, the spatial autocorrelation behavior of the ellipsoidal heights in Marion County was expressed in terms of the Moran Index value (see Chapter 3, section 3.3). The Moran's Index value for Marion County is 0.82434. The surface plot of the ellipsoidal terrain heights in Marion County is displayed in Figure B of the Metadata Sheet (see Figure 6.1). It can be concluded from the value of Moran Index which is close to 1 that the terrain of Marion County is not rough (very undulated).

Overall the terrain gradually changes in height which makes that the terrain heights have a high height correlation among neighboring points. The terrain height behavior is also confirmed by the surface plot in Figure B.

6.2 Marion County Mapping Results

After all related (input) information presented in Marion County Dataset Metadata Sheet for INCRS mapping of Marion County has been thoroughly discussed in the previous section, in this section the actual mapping results will be introduced.

As already mentioned in the Metadata Sheet that the sampled grid points in Marion County is in the form of 19 x 19 square grid which totals 361 grid points, some extra known coordinate points in Marion County have been included in this test for double checking purposes. Those are the HARN station points: ZID A, ZID B, F 350, and IMAGIS 47. The list of geodetic coordinates of all points used (361 grid points + 4 HARN station points) can be found in Table E.1 of Appendix E, as well as the corresponding mapped coordinates Easting and Northing by NGS under the classical INSPCS83.

The final INCRS mapping results (Easting and Northing coordinates) have been obtained by using two different ways of mapping which are now referred to as two different cases.

1. The case whereby all points' ellipsoidal heights are set to be equal to zero before starting any mapping procedures. This case in general is referred to as "Case 1" or "Case h_0."
2. The case whereby the real ellipsoidal heights of points were used before starting any mapping procedures. This case in general is referred to as "Case 2" or "Case h_{Real}."

The final INCRS results what generally have been referred to as "INCRS coordinates" which have been mapped in the following two different manners are displayed in the Table E.2 of Appendix E, respectively.

- "Case 1" or "Case h_0" indicates that all grid points have ellipsoidal heights equal to zero. The grid points were then mapped using the INCRS Sphere ($R_{G@CP}$) as the mapping reference surface. This INCRS mapping is called "INCRS Case h_0" with its final mapped coordinates that will be referred to as "INCRS coordinates Case h_0"
- "Case 2" or "Case h_{Real}" indicates that the real ellipsoidal heights of the grid points have been used. The grid points were then mapped by using an "inflated" version of INCRS Sphere ($R_{G@CP} + h_{avg}$) as the mapping reference surface. This INCRS mapping is called "INCRS Case h_{Real}" with its final mapped coordinates that will be referred to as "INCRS coordinates Case h_{Real}"

For each method of INCRS-OISGA mapping (INCRS Case h_0 or INCRS Case h_{Real}) both the Transverse Mercator mapping TM(CP) and the Oblique Stereographic mapping OS(CP) were investigated. The investigation can be sub-divided into the following four sub-cases:

1. INCRS TM(CP) Case h_0, with corresponding "INCRS coordinates TM(CP) Case h_0."
2. INCRS OS(CP) Case h_0, with corresponding "INCRS coordinates OS(CP) Case h_0."
3. INCRS TM(CP) Case h_{Real}, with corresponding "INCRS coordinates TM(CP) Case h_{Real}."
4. INCRS OS(CP) Case h_{Real}, with corresponding "INCRS coordinates OS(CP) Case h_{Real}."

For both the TM(CP) and OS(CP) of any INCRS mapping the selected scale factors k (kTM and kOS) have already been applied and resulted in the INCRS coordinates displayed in the Tables E.2 and E.3 of Appendix E.

The final mapping results (Easting and Northing coordinates) of INCRS-S01 that will be referred to as "INCRS-S01 coordinates" are displayed in Appendix E, Table E.3. These INCRS-S01 coordinates form the only available dataset from the INCRS-S01 mapping.

Based on the assumption that the real ellipsoidal heights of all grid points were requested by the surveyor who submitted the INCRS-S01 mapping solution, the existing version of the INCRS-S01 coordinates are (may be optimistically) assumed to be the one of the previously called "Case 2 (Case h_{Real})" where the real terrain heights are involved.

This INCRS-S01 specific manner of mapping is now referred to as "INCRS-S01 Case h_{Real}" and the corresponding mapping results are now specifically referred to as "INCRS-S01 coordinates Case h_{Real}." It should be noted that the INCRS-S01 coordinates displayed in Appendix E, Table E.4 are supposedly a "**modified version**" of the raw results (parent results) from the original INCRS-S01 mapping system. The research team received word that the mapped grid has been rotated in such a way to force the bearing of the grids' center line (line J10-J19) to have the same bearing as the one of the INSPCS83. Despite requests a Metadata Sheet for the INCRS-S01 solution was never received.

In summary the results available for comparison are the following:

1. INCRS-OISGA coordinates Case h_0
   a. No Terrain Effect was involved.
   b. All points have zero ellipsoidal heights.
   c. Mapping reference surface is the original INCRS Sphere (with radius of $R_{G@CP}$).
   d. Two different mapping functions were investigated: the TM(CP) and the OS(CP) which resulted in the different mapping results. They have been referred to as "INCRS coordinates TM(CP) Case h_0" and "INCRS coordinates OS(CP) Case h_0."
2. INCRS-OISGA coordinates Case h_{Real}
   a. Terrain Effect was involved.
   b. All points have their real terrain ellipsoidal heights.
   c. Mapping reference surface is the "inflated" INCRS Sphere (with radius of $R_{G@CP} + h_{avg}$).
   d. Two different mapping functions were investigated: the TM(CP) and the OS(CP) which resulted in the different mapping results. They will be referred to as "INCRS coordinates TM(CP) Case h_{Real}" and "INCRS coordinates OS(CP) Case h_{Real}."
3. INCRS-S01 coordinates Case h_{Real}. Results from INCRS-S01 mapping, with the assumptions as follows:
   a. Terrain Effect was involved.
   b. All points have their real ellipsoidal heights.

"INCRS-S01 coordinates Case h_{Real}" are the results from the INCRS-S01 mapping. This mapping made use of only the Transverse Mercator mapping functions (as said, no Metadata Sheet was provided, so it is unknown what mapping parameters were used to create the mapping coordinates). In this case the results are referred to as "INCRS-S01 coordinates TM(??) Case h_{Real}." The addition "TM(??)" denotes the fact that the adopted central meridian is unknown. It should be noted that also the INCRS-S01 TM(??) coordinates Case h_0 were not available.

All available results of Marion County mapped under different mapping systems (INCRS-OISGA, sometimes shortened to INCRS, and INCRS-S01) and their reference names are summarized in Table 6.1.

6.3 Evaluation of the Results

This section will describe the results of each different mapping system: INCRS-OISGA (INCRS) and INCRS-S01. Unlike the INCRS-OISGA mappings of which the results from both INCRS Case h_0 and INCRS Case h_{Real} will be discussed, the discussion of the results of INCRS-S01 system has been solely devoted to INCRS-S01 Case h_{Real}, the only results available.

The results from each mapping system have first been evaluated by the evaluation tools as discussed in Chapter 4, section 4.3. The process named "Reality Check" was used (the idea behind the Reality Check has been summarized in Figure 4.4) to evaluate the virtually 3D version of mapped results known in terms of (E, N, h_v) from each mapping method (INCRS and INCRS-S01) in the sense how well they have modeled the Real World.

For the Reality Check, two evaluation tools have been used: the so-called O-C Differences, and the Affine Fitting. It should be understood that the general 9-parameter Affine Fitting includes also the 7-parameter similarity transformation. It was mentioned in Chapter 4 that the O-C Differencing process is the calculation of the difference between the mapped coordinates, referred to as "O" and the real 3D grid (the Real World) referred to as "C." The difference (the subtraction "O" minus "C") was referred to as "Difference(s)" or "D('s)" for short.

The results from the O-C Difference process (D's) will be reported in the same fashion as it was presented in Chapter 5. That is in terms of "D_{RMS}(E)" and "D_{RMS}(N)," the root mean squares of the Differences (D's) in Easting and Northing direction respectively, and in terms of "D_{RMS}(EN)" which is the root mean square of the bidirectional "Differences (D's) that reflects the overall behavior of Differences (D's) in a single number.

Additional to the root mean squares of the Differences (D's), the average value of them are also presented. These are expressed in terms of "D_{avg}(E)," "D_{avg}(N)," and "D_{avg}(EN)." Similarly to the presentation of the results as used for the case of O-C Differencing, the same approach has been applied to the results of Affine Fitting process, the fitting residuals (V's).

These averaged values give an overall insight how the mapped coordinates deviate from reality (Real World) in both the "before" (for the case of D_{avg}) and "after" the Affine Fitting is applied (for the case of V_{avg}) in two particular directions (E and N) or in the sense of bidirectional deviations (EN).

It should be noted that the Affine Fitting procedure which is a Least Squares (LSQ) Adjustment process is a computational procedure that will not be applied in real mapping practice. As a matter of fact, surveyors perform mapping procedures to obtain mapped coordinates, the "O" values, without having to adjust them through for instance Least Squares methods. That means a surveyor would not apply a LSQ fit after having mapped his/her survey to any new adopted INCRS. The purpose of employing the Affine Fitting as an evaluation tool is purely to check the internal consistency of the mapped grid. It brings to light possible artifact deformations of the mapped grid, as expressed by suspicious values of the fitting residuals.

6.3.1 Results of the INCRS-OISGA Case h_0

The INCRS results from Case h_0 whereby all grid points were assumed to be on the ellipsoid surface (it means before the starting of any mapping procedures ellipsoidal heights were set to be equal to zero) will be discussed in this section.

When solely the INCRS mapping is considered, not only the performance of INCRS mapping itself gets evaluated through the Reality Check process (O-C Difference and Affine Fitting) but the evaluation can also be based on **the comparison between the TM(CP) mapping vs. the OS(CP)** mapping. It should be noted that some properties of a mapping system can be easier evaluated when the results get mapped without the inference of terrain elevations, i.e., the quality of mapping does not get influenced by the Terrain Effect. Hence the so-called "Case h_0" mapping comparison was conducted. INCRS-OISGA mapping in terms of both TM(CP) and OS(CP) were implemented in the sense of "Case h_0" mapping. The mapped results have been investigated through the Reality Check process (O-C Difference and Affine Fitting) on how well the Real World can be modeled.

It should be noted that as the two different mapping methods TM(CP) and OS(CP) under INCRS are to be compared becomes the main focus of this test, at this point it is sufficient to employ only the 7-parameter Affine Fitting (similarity transformation).

It is not necessary to perform both the 7- and 9-parameter Affine Fitting least squares analyses because possible artifact deformations of the mapped grid (which can be detected by performing both 7- and 9-parameter affine transformations) are not of the interest. It was revealed from the test of the INCRS Case h_{Real} (see section 6.3.2), which was actually been conducted prior to the INCRS Case h_0, that actually no artifact grid deformations could be detected.

The results of O-C (=D) Differencing and 7-parameter Affine Fitting (similarity transformation) process will be discussed in following separate sections:

1. Results of the O-C Differences of the INCRS-OISGA Case h_0. It should be noted that in this study more than the necessary significant digits were displayed in the tables that report on the results of the O-C Differencing and Affine Fitting (up to the 10^{-5} m level or 10 micron level). This has been done intentionally for interpretation purposes.

The results of the O-C Differencing for Marion County as the Test Area (see Table 6.2) will reflect the quality of the performance of the INCRS-OISGA mapping of modeling the Real World. With initially no terrain involved the INCRS-OISGA mapping is able to model reality (Real World) in the east direction at the level of 0.752 cm (0.025 ft.) for the case of INCRS TM(CP) and of 0.801 cm (0.026 ft.) for the case of INCRS OS(CP). Similar is the ability to model reality in the north direction (see Rows 1 and 2 of Table 6.2).

It is very noticeable that without any modifications, i.e., no Affine Fitting has been yet applied, the mapped coordinates belonging to the INCRS-OISGA are already very close to reality: the magnitude of the deviations in each direction (E and N) from the Real World do not exceed approximately 1.3 cm or 0.04 ft. (see Rows 1 and 2 of Table 6.2).

When considering the deviations in the sense of bidirectional deviations, the TM(CP) produces a root mean squares of the bidirectional deviations of 1.487 cm (0.049 ft.) which is larger than 1.119 cm (0.037 ft.) of the OS(CP) (see Row 3 of Table 6.2). Similarly, the results of Marion County mapped under INCRS Case h_0 has also shown that the averaged bidirectional deviations from the Real World (D_{avg}(EN)) are at the level of 1.275 cm (0.042 ft.) for the TM(CP) which is larger than 0.899 cm (0.029 ft.) of the OS(CP) (see Row 6 of Table 6.2).

TABLE 6.1
Reference names of mapped results of Marion County

	INCRS-OISGA coordinates	
	INCRS coordinates Case h_0	**INCRS coordinates Case h_{Real}**
TM	INCRS coordinates TM(CP) Case h_0	INCRS coordinates TM(CP) Case h_{Real}
OS	INCRS coordinates OS(CP) Case h_0	INCRS coordinates OS(CP) Case h_{Real}
	INCRS-S01 coordinates	
	INCRS-S01 coordinates Case h_0	**INCRS-S01 coordinates Case h_{Real}**
TM	—	INCRS-S01 TM(??) coordinates Case h_{Real}
OS	—	—

TABLE 6.2
Results of the O-C Differences during the Reality Check process of the INCRS-OISGA Case h_0 of Marion County

Row ID	Statistical values of the O-C Difference results (D's)		Results of the O-C Differences of the INCRS-OISGA Case h_0 of Marion County: INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
1	Root mean squares (RMS)	D_{RMS} (E)	0.752	0.025	0.801	0.026
2	of the O-C Differences (D's)	D_{RMS} (N)	1.282	0.042	0.782	0.026
3		D_{RMS} (EN)	1.487	0.049	1.119	0.037
4	Average values of	D_{avg} (E)	0.000	0.000	−0.000	−0.000
5	the O-C Differences (D's)	D_{avg} (N)	−0.002	−0.000	−0.001	−0.000
6		D_{avg} (EN)	1.275	0.042	0.899	0.029

Furthermore, the averaged results of Differences (D_{avg}) presented in Table 6.2 reveal that the average deviations of the Easting and Northing coordinates from the Real World as mapped by INCRS stay approximately within the level of 0.002 cm (0.000 ft.) for both the TM(CP) and the OS(CP) (see Rows 4 and 5 of Table 6.2).

The results of the O-C Differencing as shown in Table 6.2 also reveal that without the influence of the Terrain Effect, the INCRS OS(CP) has distributed the errors more equally or more symmetrically (in the same ball park) in both directions (east-west (E) and north-south (N)) as compared to INCRS TM(CP). It should be noted that stating that the errors were distributed equally means that the errors for both directions (E and N) are in the same ball park magnitude-wise. The meaning of "equal" in this sense does not imply exact equality in a numerical sense.

The vertical axis in Figure 6.2 represents the results of the O-C Differences of the INCRS TM(CP) in east-west direction (D(E)) plotted at each corresponding grid point. The O-C differences in north-south (N) direction (D(N)) are presented in Figure 6.3. In a similar fashion the O-C Differences are displayed for the INCRS OS(CP) in Figures 6.4 and 6.5. The more symmetric behavior of the error distribution of the INCRS OS(CP) can be easily detected in Figure 6.4 (E-W or E direction for short) and Figure 6.5 (N direction). When compared to the O-C plots of the INCRS TM(CP), the error distribution of the latter mapping is not in a symmetric manner, i.e., the errors were not distributed equally in both directions E and N, as shown in Figures 6.2 and 6.3.

Considering the deviations from the Real World (Differences (D's)) of each grid point in a bidirectional manner (denoted by "EN") one realizes first that at each single grid point there consists a deviation in both east-west (E) directions and north-south (N) directions. These have been referred to as D(E) and D(N) respectively. These deviations may be either positive or negative. The size of the summation vector of vector D(E) and D(N) at each point were then computed [in math: $D(EN) = sqrt[(D(E)^2 + D(N)^2)]$ resulting in a single semi-positive number that will be referred to as "D(EN)." The bidirectional deviation "D(EN)" represents the size of the total deviation as contributed by the separate deviations in both directions (E and N) of the grid point under consideration. The vertical axis in Figure 6.6 represents bidirectional deviations of INCRS TM(CP) plotted at each corresponding grid point, whereas the ones of INCRS OS(CP) are presented in Figure 6.7.

It should be noted that the bidirectional deviations (D(EN)'s) as plotted in Figures 6.6 and 6.7 that belong to the TM(CP) and the OS(CP) mappings respectively, show a remarkable consistency between both mapped

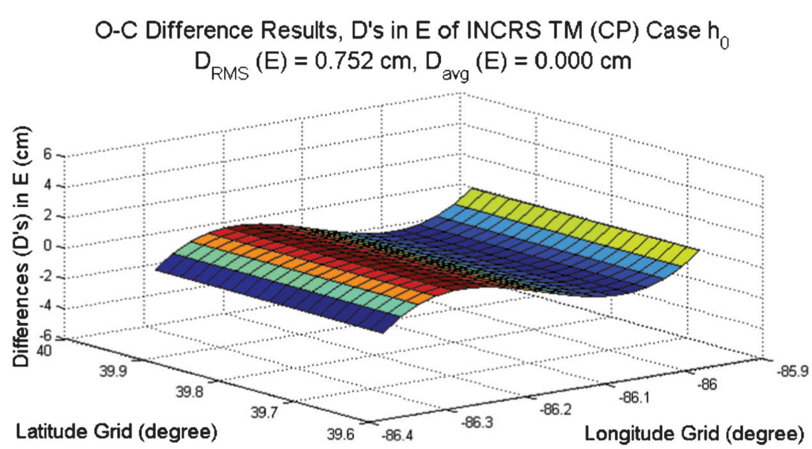


Figure 6.2 Plot of the Differences (D's) in E direction of INCRS TM(CP) Case h_0.

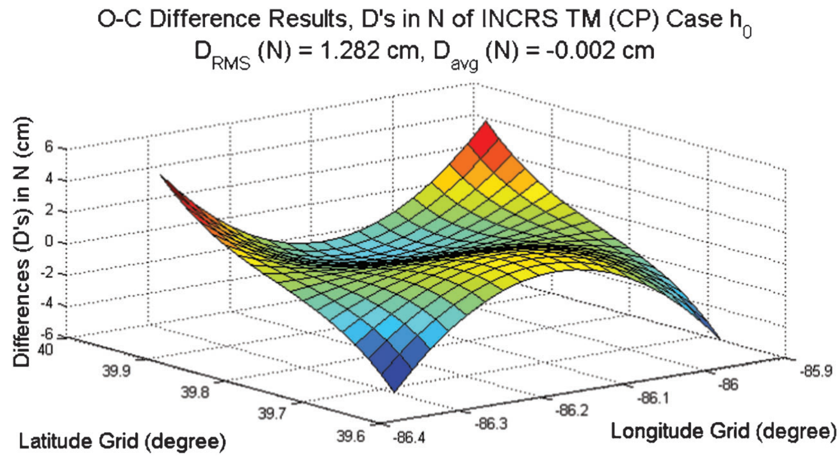


Figure 6.3 Plot of the Differences (D's) in N direction of INCRS TM(CP) Case h_0.

grids. It also reconfirms that both grids do not contain any artifact deformation.

Also Figures 6.6 and 6.7 confirm the somewhat higher quality of the Oblique Stereographic mapping OS(CP) over the Transverse Mercator mapping TM(CP). The asymmetric behavior of the O-C values in the Easting for the TM(CP) (Figure 6.2) apparently causes the somewhat lower quality of the TM(CP). The average of the O-C Differences for the bidirectional deviations for the INCRS-OISGA TM(CP) Case h_0 is 1.28 cm with an RMS of 1.49 cm. The same values for the INCRS-OISGA OS(CP) Case h_0 are 0.90 cm and 1.12 cm, respectively.

2. Results of the 7-parameter Affine Fitting (similarity transformation) of the INCRS Case h_0. The results shown in Table 6.3 are from the 7-parameter Affine Fitting (similarity transformation) between INCRS coordinates and the corresponding topocentric local coordinates of grid points in the Real World frame. The results show the deviations of the INCRS with respect to the Real World without any Terrain Effect involved after a 7-parameter (three shifts, three rotations, and one scale) 3D similarity transformation was applied through a Least Square fitting.

The results shown in Table 6.3 agree with the corresponding O-C Differencing results as shown in Table 6.2. For Marion County without terrain elevations the INCRS TM(CP) exhibits somewhat larger deviations from the Real World than the INCRS OS(CP). In Table 6.3 the deviations are expressed in terms of fitting residuals.

6.3.2 Results of the INCRS-OISGA Case h_{Real}

In this section the INCRS mapping coordinates Case h_{Real} have been evaluated through the Reality Check's evaluation tools (O-C Differencing and Affine Fitting). It should be noted that in the "Case h_{Real}" the actual ellipsoidal heights of all grid points were used.

It should be noted that in the case of Case h_0 (section 6.3.1) the mappings of INCRS TM(CP) and INCRS OS(CP) can be compared and the performance of TM(CP) and OS(CP) can be cleanly evaluated based on the fact that no Terrain Effect was involved. Therefore in the Affine Fitting process only a 7-parameter Affine

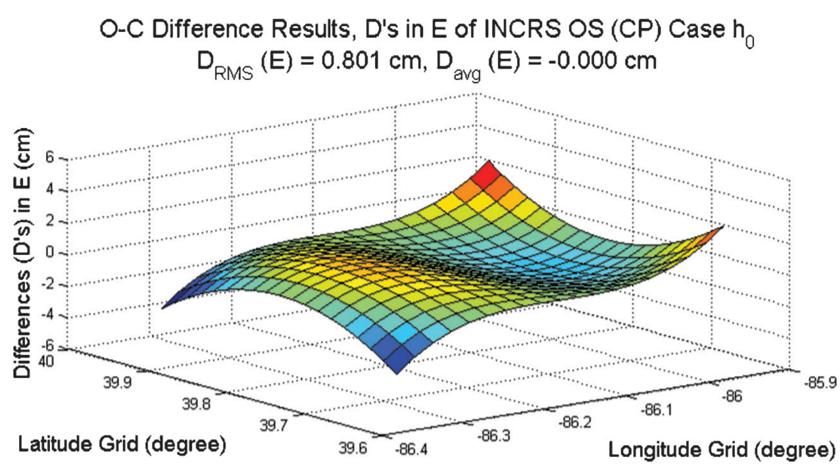


Figure 6.4 Plot of the Differences (D's) in E direction of INCRS OS(CP) Case h_0.

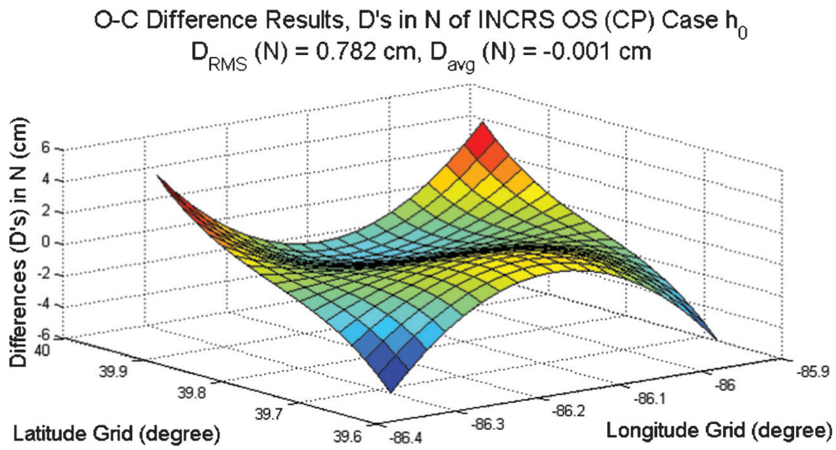


Figure 6.5 Plot of the Differences (D's) in N direction of INCRS OS(CP) Case h_0.

Fitting (similarity transformation) is sufficient (see explanation in the 3rd and 4th paragraphs of section 6.3.1). Here in this section the purpose of evaluations still remains the same, i.e., to evaluate the performance of INCRS mapping in modeling the Real World. The only difference this time is that the Terrain Effect is not neglected because the actual ellipsoidal heights are used. In this section, the comparison has not been discussed in the same way as in section 6.3.1. It instead focusing on the aspect of INCRS TM(CP) vs. INCRS OS(CP), the absolute sense of the quality of the INCRS mappings will be discussed. The absolute quality which will be expressed in terms of the numerical values of the deviations is more of interest.

In this section both the 7- and 9-parameter Affine Fittings are also considered. It is to confirm and ensure that there does not exist any artifact deformations in the mapped grids. It should be noted that the INCRS Case h_0 made use of the results from the INCRS Case h_{Real} to claim the absence of artifact deformations (see section 6.3.1) which led to the idea of using only the 7-parameter (not the 9-) Affine Fitting in any tests where INCRS Case h_0 was involved.

1. Results of the O-C Differences of the INCRS Case h_{Real}. Considering the quality of INCRS that is reflected by the O-C (= D) Differencing results as shown in Table 6.4, the root mean squares of the Differences D reflect the deviation of the INCRS mapped results from the Real World. The (impressive!) results show that the INCRS mapping is able to model reality (Real World) in the east direction at the level of 2.909 cm (0.095 ft.) for the case of TM(CP) and at 2.888 cm (0.095 ft.) for the case of OS(CP). Similar is the ability to model reality in the north direction (see Rows 1 and 2 of Table 6.4).

It should be noted that the O-C Differences are the result from the comparison of the raw mapped coordinates against the reality without any modifications applied to the original mapped coordinates. The only exception is a local shift which does not alter the relative location of the original grid points. It is very noticeable that without any modifications, the mapped coordinates belonging to the INCRS are already very close to reality: the magnitude of the deviations in each direction (E and N) from the Real World do not exceed approximately 3.1 cm or 0.10 ft. (see Rows 1 and 2 of Table 6.4)

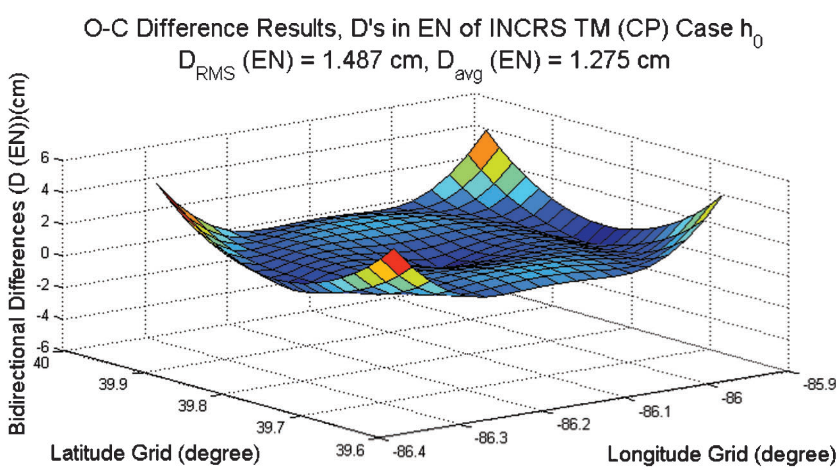


Figure 6.6 Plot of the bidirectional Differences (D(EN)'s) of INCRS TM(CP) Case h_0.

TABLE 6.3
Results of the 7-parameter Affine Fitting (similarity transformation) during the Reality Check process of the INCRS-OISGA Case h_0 of Marion County

Row ID	Statistical values of the Affine Fitting residuals (V's)		Results of the 7-parameter Affine Fitting (similarity transformation) of the INCRS Case h_0 of Marion County			
			INCRS TM(CP)		INCRS OS(CP)	
			(cm)	(ft.)	(cm)	(ft.)
1	Root mean squares (RMS) of the fitting residuals (V's)	V_{RMS} (E)	0.772	0.025	0.969	0.032
2		V_{RMS} (N)	1.475	0.048	0.933	0.031
3		V_{RMS} (EN)	1.665	0.055	1.345	0.044
4	Average values of the fitting residuals (V's)	V_{avg} (E)	0.000	0.000	0.000	0.000
5		V_{avg} (N)	0.000	0.000	0.000	0.000
6		V_{avg} (EN)	1.221	0.040	0.932	0.031

When considering the deviations in the sense of bidirectional deviations, the TM(CP) has produced a root mean square of the bidirectional deviations of 4.224 cm (0.139 ft.) whereas the RMS for the OS(CP) is 4.241 cm (0.139 ft.) (see Row 3 of Table 6.4). It should be noted that the deviations from the Real World for both the TM(CP) and the OS(CP) of the INCRS-OISGA mapping in all considered directions (E, N and EN) are all in the same ball park with insignificant differences.

The averaged results (D_{avg}) presented in Table 6.4 reveal that the average deviations of the Easting and Northing coordinates from the Real World as mapped by INCRS-OISGA stay approximately within 1.1 cm (0.03 ft.) for both the TM(CP) and the OS(CP) (see Rows 4 and 5 of Table 6.4).

Additionally, in Table 6.4 the averaged bidirectional deviations from the Real World D_{avg}(EN), at the level of 3.204 cm (0.105 ft.) for the TM(CP), which is slightly larger than the 3.149 cm (0.103 ft.) for the OS(CP) (see Row 6 of Table 6.4). It should be noted that the average values of the deviations from the Real World for both the TM(CP) and the OS(CP) in all considered directions (E, N and EN) are insignificantly different from each other (see Rows 4, 5 and 6 of Table 6.4)

It is noticeable that the Terrain Effect influences somewhat negatively the quality of the mapped results. It also obscures the superiority of the INCRS OS(CP) in distributing the errors equally in both E and N directions. These somewhat negative and obscuring features are also visible in the plots of the O-C Differences in Figure 6.8 through Figure 6.11.

Figures 6.10 and 6.11 illustrate the results of the O-C Differencing for the INCRS OS(CP) in east-west (E) and north-south (N) directions, respectively. In these two Figures, it is obvious that the symmetric behavior of the error distribution for the OS(CP) is obscured. This has been confirmed by the corresponding numerical results of the O-C Differences as shown in Table 6.4: the "equality" of the deviations in both directions (E and N) is not anymore obvious for the case of OS(CP) when the Terrain Effect gets involved. It should be noted that this superiority in the distribution of errors (deviations) equally was not obtained for the INCRS TM(CP) in neither cases, whether the Terrain Effect was incorporated or not.

2. Results of the 7-parameter Affine Fitting (similarity transformation) of the INCRS-OISGA Case h_{Real}. The results in Table 6.5 show the deviations of the INCRS Case h_{Real} with respect to the Real World after a 7-parameter 3D similarity transformation was applied by a Least Squares fitting.

After the completion of a 7-parameter Affine Fitting process (this means the adjustment has been applied to all mapped coordinates), the results show that the mapped coordinates are improved as the root mean

TABLE 6.4
Results of the O-C Differences during the Reality Check process of the INCRS-OISGA Case h_{Real} of Marion County

Row ID	Statistical values of the O-C Difference results (D's)		Results of the O-C Differences of the INCRS Case h_{Real} of Marion County			
			INCRS TM(CP)		INCRS OS(CP)	
			(cm)	(ft.)	(cm)	(ft.)
1	Root mean squares (RMS)	D_{RMS} (E)	2.909	0.095	2.888	0.095
2	of the O-C Differences (D's)	D_{RMS} (N)	3.063	0.100	3.107	0.102
3		D_{RMS} (EN)	4.224	0.139	4.241	0.139
4	Average values of the	D_{avg} (E)	−0.615	−0.020	−0.615	−0.020
5	O-C Differences (D's)	D_{avg} (N)	−1.030	−0.034	−1.029	−0.034
6		D_{avg} (EN)	3.204	0.105	3.149	0.103

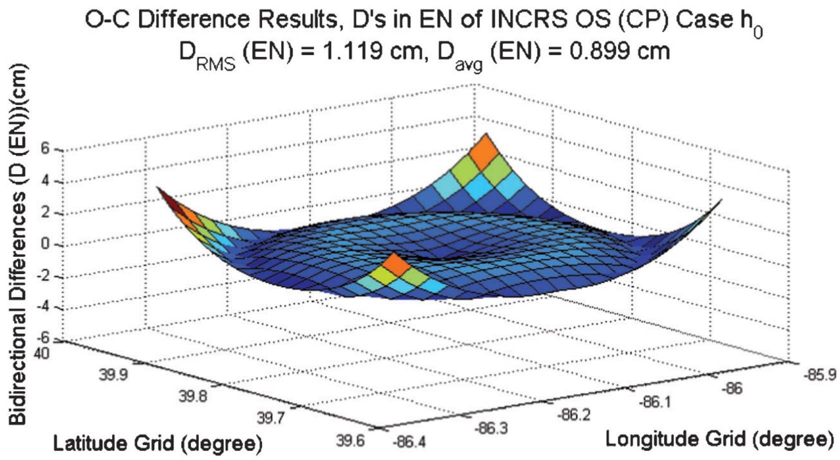


Figure 6.7 Plot of the bidirectional Differences (D(EN)'s) of INCRS OS(CP) Case h_0.

squares of the bidirectional fitting residual "V_{RMS}(EN)" are smaller than the one of the O-C Differences "D_{RMS}(EN)" (compare Table 6.5 against Table 6.4).

It is noticeable that the improvement after the adjustment is not so significant. That means that the results as shown in Table 6.4 (the O-C Differences) were already good enough since there was not much improvement to be obtained by the adjustment procedure (Affine Fitting). Therefore, the results from Table 6.5 have proved and re-confirmed the impressive performance of INCRS-OISGA (as it has already been perceived by the results shown in Table 6.4) even when the Terrain Effect is not neglected (Case h_{Real}).

3. Results of the 9-parameter Affine Fitting of the INCRS-OISGA Case h_{Real}. The results shown in Table 6.6 are from the 9-parameter Affine Fitting between INCRS coordinates and the corresponding topocentric local coordinates of the grid points in the Real World frame. Table 6.6 shows that the 9-parameter Affine Fitting yields smaller residuals than the ones from 7-parameter (similarity transformation) fitting (compare Table 6.6 against Table 6.5).

This is not beyond expectation: when in general the mathematical model consists of more parameters the more likely it will be that smaller size residuals are produced. This is due to the fact that the extended model (with more parameters) is generally better suited to accommodate the observations involved.

An important conclusion can be drawn from the 7- and 9-parameter fittings: even though the residuals from the 7-parameter fitting are not as small as the ones from the 9-parameter fitting (due to the reasons that have been clarified above), yet in this particular case the differences between the residual size from these two fittings (7- and 9-parameters) are not significant. This means that the internal consistency of the mapped grid points as compared to the unmapped Real World points is ensured. In other words, no artifact deformations exist. In the case of significant deformations the residuals from the 9-parameter fitting should be significantly better (and different) from the ones of the 7-parameter fitting, because it is expected that deformations or distortions can be modeled better by a 9-parameter affine transformation than a 7-parameter similarity transformation.

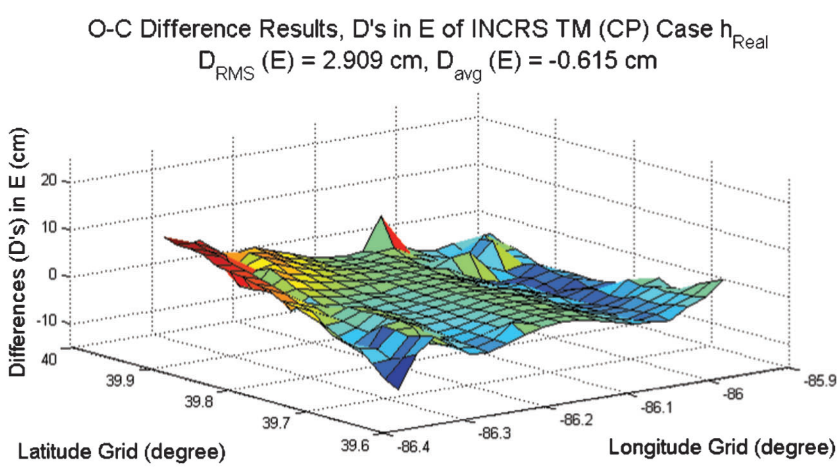


Figure 6.8 Plot of the Differences (D's) in E direction of INCRS TM(CP) Case h_{Real}.

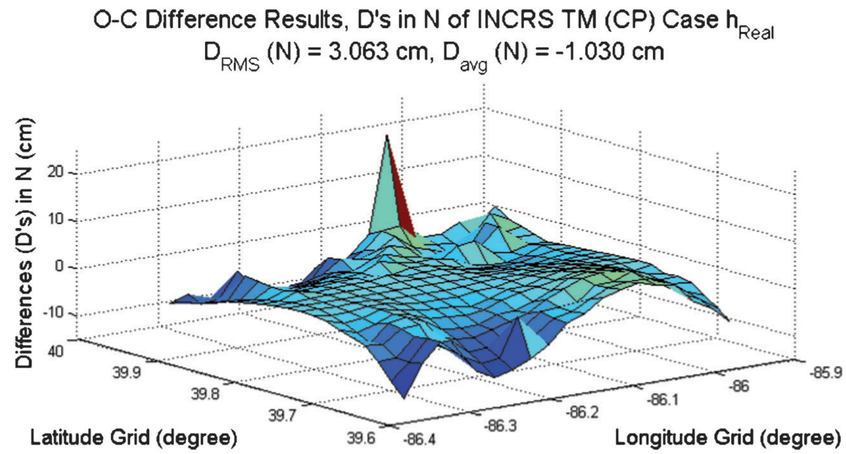


Figure 6.9 Plot of the Differences (D's) in N direction of INCRS TM(CP) Case h_{Real}.

Therefore it can be concluded from Tables 6.5 and 6.6 that the grid points do not contain any significant deformations or distortions.

6.3.3 Results of the INCRS-S01 Case h_{Real}

This section evaluates the results of the INCRS-S01 mapping. Due to the fact that the real ellipsoidal heights had been requested by a surveyor, it is assumed that the INCRS-S01 coordinates have been produced using the real ellipsoidal heights (h_{Real}). Similar to section 6.3.1 and section 6.3.2, the INCRS-S01 have been evaluated through the so-called Reality Check process by using O-C Differencing and Affine Fitting as evaluation tools.

It is foreseen that the results of INCRS-S01 Case h_{Real} will be considered in the comparison between two different mapping systems INCRS-OISGA and INCRS-S01 under the same Case h_{Real}. Therefore, the Reality Check process applied to INCRS-S01 is simply the same as what has been applied to the case of INCRS Case h_{Real}. These are: the O-C Differencing and the 7- (similarity transformation) and the 9-parameter Affine Fitting. The results from the O-C Differencing will be discussed in a separate section whereas another section will combine the discussion of both the 7- and 9-parameter Affine Fitting.

1. Results of the O-C Difference of the INCRS-S01 Case h_{Real}. The O-C Differences shown in Table 6.7 reflect the performance of the INCRS-S01 mapping in modeling reality (Real World). The INCRS-S01 mapping shows an average deviation (D_{avg}) from the Real World in both E and N directions of approximately not exceeding 3.3 cm (0.11 ft.) (see Rows 4 and 5 of Table 6.7). However the values of the root mean squares of the O-C Differences in both E and N directions, that reflect the size of deviation of mapped results from reality, are at an alarming level.

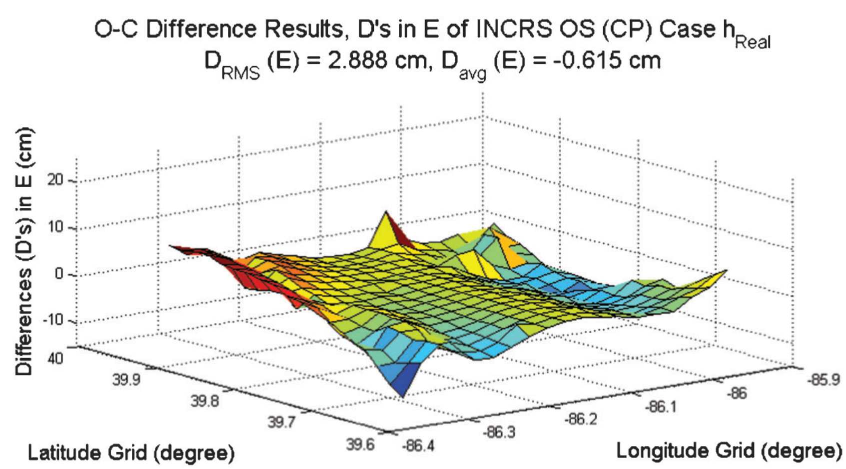


Figure 6.10 Plot of the Differences (D's) in E direction for INCRS OS(CP) Case h_{Real}.

TABLE 6.5
Results of the 7-parameter Affine Fitting (similarity transformation) during the Reality Check process of the INCRS-OISGA Case h_{Real} of Marion County

Row ID	Statistical values of the Affine Fitting residuals (V's)		Results of the 7-parameter Affine Fitting (similarity transformation) of the INCRS-OISGA Case h_{Real} of Marion County INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
1	Root mean squares (RMS)	V_{RMS} (E)	2.498	0.082	2.580	0.085
2	of the fitting residuals (V's)	V_{RMS} (N)	3.065	0.101	3.036	0.100
3		V_{RMS} (EN)	3.954	0.130	3.984	0.131
4	Average values of the	V_{avg} (E)	0.000	0.000	−0.000	−0.000
5	fitting residuals (V's)	V_{avg} (N)	−0.000	−0.000	0.000	0.000
6		V_{avg} (EN)	3.005	0.099	3.013	0.099

They show that the INCRS-S01 mapping is able to model reality (Real World) in the East direction merely at the level of 54.807 m (179.81 ft.) and of 56.357 m (184.90 ft.) for the North direction. The (maximum) magnitudes of these deviations are as large as (more than!) 55 m (180 ft.).

When considering the deviations in the sense of bidirectional deviation, the INCRS-S01 has produced the root mean squares of bidirectional deviations D_{RMS}(EN) of 78.613 m (257.92 ft.) and the averaged bidirectional deviation D_{avg}(EN) is at the level of 73.689 m (241.76 ft.) (see Rows 3 and 6 of Table 6.7).

The deviations of more than 55 m (180 ft.) are caused by the fact that results of INCRS-S01 mapping have been rotated ("modified") in such a way that the azimuth of the J10-J19 line perfectly agrees with the azimuth of J10-J19 under the classical INSPCS83. This modification may have altered the behavior of the original (parent) mapped grid.

The results of the O-C Differencing for the INCRS-S01 TM Case h_{Real} in east-west (E) and north-south (S) direction are illustrated in Figures 6.12 and 6.13 respectively. The huge deviations in both directions are obviously depicted by these two plots.

2. Results of 7-parameter (similarity transformation) and 9-parameter Affine Fitting of the INCRS-S01 coordinates resulted from the INCRS-S01 TM(??) Case h_{Real}. The results shown in Table 6.8 are from the 7-parameter (similarity transformation) and 9-parameter Affine Fitting between INCRS-S01 coordinates and the corresponding topocentric local coordinates of the grid points in the Real World frame.

Table 6.8 shows the deviations of the INCRS-S01 with respect to the Real World after a 7- and 9-parameter 3D affine transformation was applied by a Least Square fitting. It shows that both Affine Fittings brought down the large deviations considerably (as compared to those shown in Table 6.7). The deviations are down to the level of approximately 3.2 cm (0.1 ft.) in both E and N directions. The sizes of residuals resulting from these two fittings (7- and 9-parameter) are not significantly different which reflects the internal consistency of the grids with no artifact deformations embedded.

It is important to be aware that surveyors would not apply a Least Squares fitting to the final mapped coordinates. The Affine Fitting was only used as an evaluation tool but will never be applied to the mapped coordinates in practice. Therefore the critical evaluation of the quality of the proposed new mapping systems should be based on the O-C Differences as they indeed reflect the ability of any mapping system to model the Real World.

6.4 Comparisons of the Results

Even though an evaluation of the results from both systems (INCRS-OISGA and INCRS-S01) have been presented and thoroughly discussed in section 6.3, some additional discussion is warranted whereby the results

TABLE 6.6
Results of the 9-parameter Affine Fitting during the Reality Check process of the INCRS-OISGA Case h_{Real} of Marion County

Row ID	Statistical values of the Affine Fitting residuals (V's)		Results of the 9-parameter Affine Fitting of the INCRS-OISGA Case h_{Real} of Marion County INCRS TM(CP) (cm)	INCRS TM(CP) (ft.)	INCRS OS(CP) (cm)	INCRS OS(CP) (ft.)
1	**Root mean squares (RMS)** of	V_{RMS} (E)	2.445	0.080	2.580	0.085
2	fitting the residuals (V's)	V_{RMS} (N)	2.919	0.096	2.969	0.097
3		V_{RMS} (EN)	3.808	0.125	3.934	0.129
4	**Average values** of the fitting	V_{avg} (E)	0.000	0.000	−0.000	−0.000
5	residuals (V's)	V_{avg} (N)	0.000	0.000	−0.000	−0.000
6		V_{avg} (EN)	2.865	0.094	3.010	0.099

TABLE 6.7
Results of the O-C Differences during the Reality Check process of the INCRS-S01 TM(??) Case h_{Real} of Marion County

Row ID	Statistical values of the O-C Difference results (D's)		Results of the O-C Differences of the INCRS-S01 Case h_{Real} of Marion County INCRS-S01 TM(??) (cm)	(ft.)
1	**Root mean squares (RMS)** of the O-C Differences (D's)	D_{RMS} (E)	5480.704	179.813
2		D_{RMS} (N)	5635.727	184.899
3		D_{RMS} (EN)	7861.268	257.916
4	**Average values** of the O-C Differences (D's)	D_{avg} (E)	3.245	0.106
5		D_{avg} (N)	−1.040	−0.034
6		D_{avg} (EN)	7368.785	241.758

of INCRS-OISGA and INCRS-S01 mappings will be compared against each other side-by-side in order to point out some important aspects of these two different mapping systems.

6.4.1 Results of the O-C Differences of INCRS-OISGA and INCRS-S01

This section will describe the comparisons between the available mappings by INCRS-OISGA and INCRS-S01 (see the available dataset in Table 6.1). Since the INCRS-S01 mapping provided only the mapped results (1) for the case h_{Real}, i.e., the ellipsoidal coordinates of the Marion County grid whereby the ellipsoidal height reflects the "Real" terrain height, and (2) for the Transverse Mercator case, the comparison between the "INCRS coordinates TM(CP) Case h_{Real}" and the "INCRS-S01 TM(??) coordinates Case h_{Real}" are of main interest. The INCRS-S01 mapping did not consider the "Case h_0." In other words the undulating terrain of Marion County was not reduced to the ellipsoid, as it would have happened under the classical INSPCS83. Therefore the comparisons between INCRS and INCRS-S01 will be solely devoted to the case whereby the real ellipsoidal heights are used. The absence of a INCRS-S01 TM(??) Case h_0 dataset (although requested) prevented an in-depth analysis of the pure conformal mapping process behind the INCRS-S01 mapping (because the Case h_{Real} unfortunately masks the properties of the conformal mapping mathematics itself).

The results from the O-C Differences are of interest due to the fact that they do reflect the deviations of a mapping system with respect to the Real World; consequently, it represents the performance and quality of a mapping system on how well it can model, or how close it stays to the Real World. The ultimate goal of the surveyor/engineer is that the 2D (+1D) mapped positions represent as truthfully the 3D Real World positions of the points that have been surveyed. Table 6.9 shows that for INCRS-OISGA mappings (either TM(CP) or OS(CP)) the deviations from reality (Real World) in both E and N directions stay within approximately 3.1 cm (0.10 ft.). In contrast to the INCRS-OISGA, the deviations of the INCRS-S01 mapping are as large as (or even more than) 55 meters (180 ft.) (see Rows 1 and 2 of Table 6.9).

Table 6.9 that represents the averaged values of the O-C Differences (D's) from both INCRS-OISGA

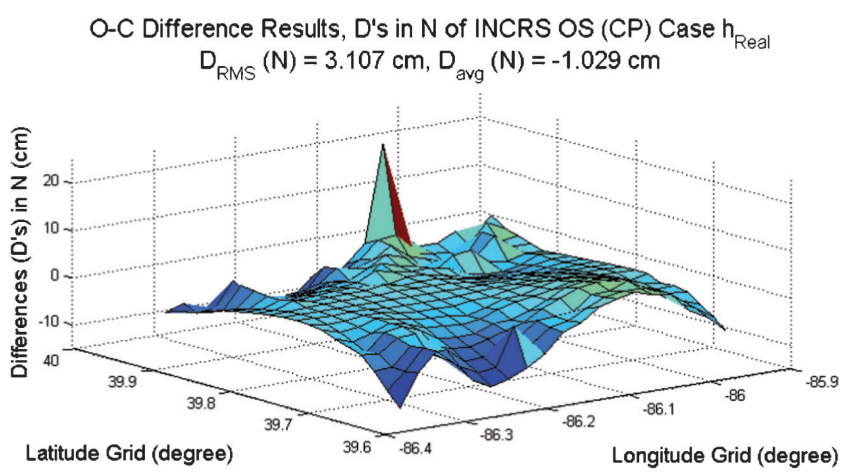


Figure 6.11 Plot of the Differences (D's) in N direction for INCRS OS(CP) Case h_{Real}.

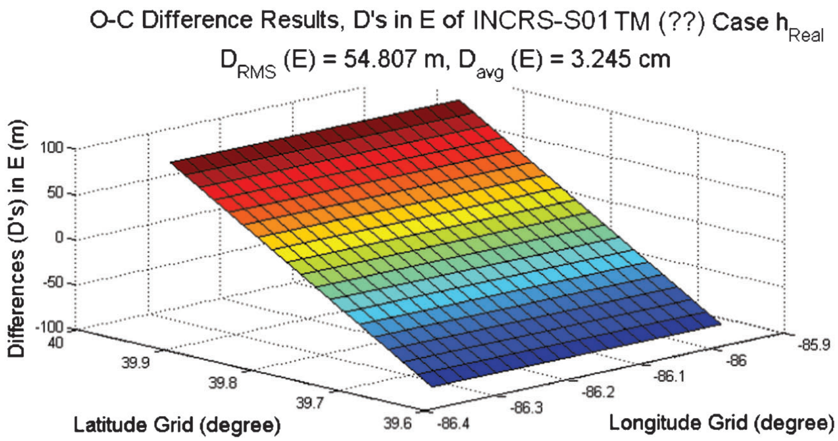


Figure 6.12 Plot of the Differences (D's) in E direction of INCRS-S01 TM(??) Case h_{Real}.

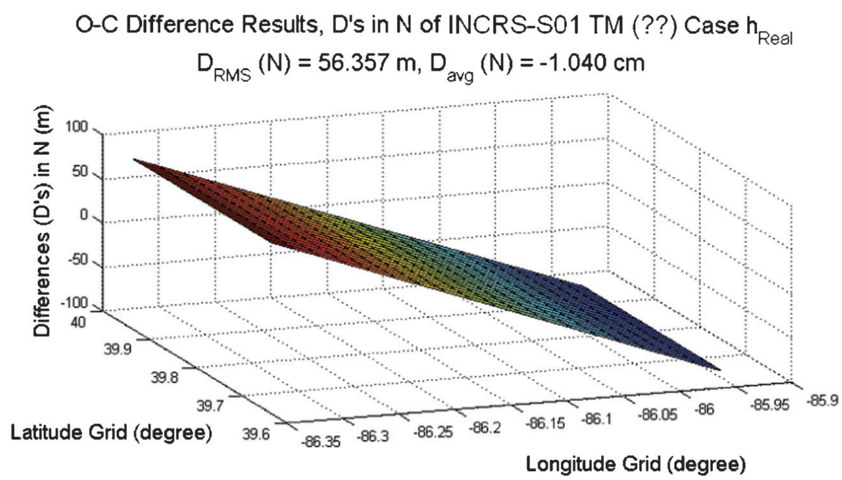


Figure 6.13 Plot of the Differences (D's) in N direction of INCRS-S01 TM(??) Case h_{Real}.

TABLE 6.8
Results of the 7-parameter (similarity transformation) and 9-parameter Affine Fitting during the Reality Check process of the INCRS-S01 TM(??) Case h_{Real} of Marion County

			INCRS-S01 TM(??) Case h_{Real} of Marion County			
			7-parameter Affine Fitting (similarity transformation)		9-parameter Affine Fitting	
Row ID	Statistical values of the Affine Fitting residuals (V's)		(cm)	(ft.)	(cm)	(ft.)
1	**Root mean squares (RMS)** of fitting residuals (V's)	V_{RMS} (E)	2.540	0.083	2.503	0.082
2		V_{RMS} (N)	3.126	0.103	3.027	0.099
3		V_{RMS} (EN)	4.028	0.132	3.927	0.129
4	**Average values** of fitting residuals (V's)	V_{avg} (E)	0.000	0.000	−0.000	−0.000
5		V_{avg} (N)	0.000	0.000	0.000	0.000
6		V_{avg} (EN)	3.087	0.101	3.033	0.100

TABLE 6.9
Results of the O-C Differences during the Reality Check process of the INCRS-OISGA TM(CP), INCRS-OISGA OS(CP), and the INCRS-S01 TM(??) (all under Case h_{Real})

			Results of the O-C Differences of INCRS-OISGA Case h_{Real} and the INCRS-S01Case h_{Real} of Marion County					
			INCRS TM(CP)		INCRS OS(CP)		INCRS-S01 TM(??)	
Row ID	Statistical values of the O-C Difference results (D's)		(cm)	(ft.)	(cm)	(ft.)	(cm)	(ft.)
1	**Root mean squares (RMS)** of	D_{RMS} (E)	2.909	0.095	2.888	0.095	5480.704	179.813
2	the O-C Differences (D's)	D_{RMS} (N)	3.063	0.100	3.107	0.102	5635.727	184.899
3		D_{RMS} (EN)	4.224	0.139	4.241	0.139	5558.756	182.374
4	**Average values** of the O-C	D_{avg} (E)	**−0.615**	**−0.020**	**−0.615**	**−0.020**	*3.245*	*0.106*
5	Differences (D's)	D_{avg} (N)	**−1.030**	**−0.034**	**−1.029**	**−0.034**	*−1.040*	*−0.034*
6		D_{avg} (EN)	**3.204**	**0.105**	**3.149**	**0.103**	*7368.785*	*241.758*

mappings, TM(CP) and OS(CP), is summarized in the the values identified in **boldface**: its averaged mapping deviations from the Real Word stay within approximately 1.1 cm or 0.03 ft. in both E and N directions. The comparable deviations of the INCRS-S01 TM mapping are of the order of approximately 3.3 cm (0.11 ft.), as shown with the values identified in *italics*.

Although an average deviation of around 3.3 cm of the INCRS-S01 mapping may be considered quite a small number, the average deviation of both INCRS-OISGA mappings virtually never exceed the 1.1 cm level. Since the average deviation of INCRS-S01 mapping is 3 times larger than the INCRS-OISGA mappings it may be concluded that for this test dataset in Marion County both INCRS-OISGA mappings are superior to the INCRS-S01 mapping by a factor of three.

The root mean squares analysis of both mapping systems shows that the quality of the INCRS-S01 is inferior, see the last two columns in Rows 1, 2, 3, and 6 of Table 6.9. The low quality is most likely introduced by a deliberate "correction" to the orientation of the parent grid.

6.4.2 Results of Convergence Angle Analysis

In this section an additional issue, the "Convergence Angle Issue" will be discussed. It is directly related to the "Convergence Effect" described in Chapter 2, section 2.1.3. It also follows from the large O-C deviations observed in the INCRS-S01 mapping. Therefore a closer look at the behavior of convergence angle from the three different mappings, INCRS-OISGA TM(CP), INCRS-OISGA OS(CP), and INCRS-S01 TM(??), is warranted.

For this side-issue the convergence angle (γ) at the extreme NW corner (point A18, see Figure 6.1) of Marion County has been analyzed for the different mapping systems. To a large degree the convergence angle at point A18 can be approximated by the azimuth A18 - A19 ($\gamma_{@A18} \approx$ Az A18-A19). The results of these computations for the different mapping systems are shown in Table 6.10. The convergence angle at A18 in the case of the classical INSPCS83 is also presented in Table 6.10.

Table 6.10 shows that the INCRS-OISGA TM(CP) and INCRS-OISGA OS(CP) mappings also reduce the convergence angles approximately by a factor of four: the INSPCS83 convergence angle of around 28′ is reduced to the level of around 7′. The convergence angle of INCRS-S0 TM(??) of around 26′ stayed in the same ball park as the one of the classical INSPCS83.

Furthermore a Least Squares similarity transformation analysis shows that the average convergence angle behavior over all Marion County is about 18′ for the INCRS-S01 TM(??) mapping while the average convergence angles belonging to both the INCRS-OISGA TM(CP) and INCRS-OISGA OS(CP) mappings are exactly equal to 0′ (as expected).

In summary, the study of the Marion County dataset has clarified the three following issues:

1. The error committed in the case that a surveyor omits or neglects to apply the ground-to-grid (or grid-to-ground) scale correction, the so-called "mapping scale" and "terrain height correction" is minimized by the INCRS-OISGA TM(CP) and INCRS-OISGA OS(CP) mappings to the 4.2 cm (0.14 ft) level.
2. The errors in Easting and Northing are more balanced for the INCRS-OISGA OS(CP) mapping than for the INCRS-OISGA TM(CP) mapping. This effect is clearly visible in the tests of Case h_0, i.e., when all grid points are reduced to the ellipsoidal surface, or equivalently in the case of the INCRS-OISGA mapping that adopts the "inflated" version of the INCRS Sphere (the "inflated" INCRS Sphere adds to the Gaussian Radius of Curvature at the center of the

TABLE 6.10
The convergence angle at extreme NW corner (A18) of Marion County computed from different mapping systems

	Convergence angle @ A18		
Mapping systems	degree	minute	second
Classical INSPCS83 (NGS)	00	28	14.07
INCRS TM(CP) (INCRS-OISGA TM(CP))	00	07	42.08
INCRS OS(CP) (INCRS-OISGA OS(CP))	00	07	41.40
INCRS-S01 TM(??)	00	26	17.25

project ($R_{G@CP}$) the average value of ellipsoidal heights (h_{avg}) as computed from the area to be mapped) and the grid points are reduced to this new reference surface level. When the actual terrain heights are used, the superiority of the OS mapping over the TM mapping is masked.

3. The error committed in the case a surveyor omits or neglects to apply the ground-to-grid (or grid-to-ground) azimuth corrections, the so-called "convergence angle" is also minimized in the INCRS-OISGA TM(CP) as well as the INCRS-OISGA OS(CP) mappings. The INCRS-S01 TM(??) mapping exhibits large convergence angles, mainly due to the fact that the (original or parent?) INCRS-S01 TM(??) grid has been rotated in such a way that the central meridian between the points J10 and J19 (see Figure 6.1) had the same azimuth correction as for the INSPCS83. This rotation seems unnecessary. Moreover, the rotation causes deviations of 90 meters (300 feet) around the perimeter of Marion County if one compares the Real World coordinates to the INCRS-S01 mapping coordinates.

7. SUMMARY, CONCLUSIONS, IMPLEMENTATION, AND RECOMMENDATIONS

Chapter 5 describes in detail the results of the many testing procedures that have been employed during this study. Similarly, in Chapter 6 the results are discussed of a separate test solely devoted to Marion County. In section 7.1 the core results of these findings of this research study will be summarized. In section 7.2 the ideas of INCRS implementation are discussed. This leads to the question that if INCRS will be adopted what other issues or topics needed to be addressed, re-visited, and/or further investigated in a possible follow-up project? The answers of this question are discussed in sections 7.3 (Recommendations) and 7.4 (Implementation Recommendation).

7.1 Summary and Conclusions

The behavior of the INCRS-OISGA system and its preference over the INCRS-S01 solution submitted by a surveyor will be summarized. The overall mapping improvements obtained from the proposed INCRS as compared to the current INSPCS83 will also be discussed.

7.1.1 Preference of INCRS over INCRS-S01

In Chapter 6 the mapping results of a pilot Test Area "Marion County" under INCRS and INCRS-S01 have been rigorously compared and evaluated. In this section the conclusions from a comparison between these two systems will be re-drawn in a very concise manner, namely in the form of Table 7.1. Mapping issues will be tabulated, as well with corresponding remarks for both the INCRS and the INCRS-S01.

The feasibility study deals with the development on a new mapping system (INCRS-OISGA) and with the comparison between this county-based INCRS (-OISGA) versus the multi-county coordinate reference system originally proposed by Bernardin-Lochmueller and Associates, Inc. (INCRS-BLA). It may be concluded that the new system INCRS-OISGA (or for short INCRS) which has been developed based on the theory as developed in (*2*) is to be preferred over INCRS-S01 because of (1) reduced errors if corrections due to scale, terrain height, and convergence angle are omitted, (2) despite the similar relative accuracies of both INCRS's the absolute accuracy of INCRS-OISGA is superior, and (3) mapping corrections show the most balanced behavior in Easting and Northing for the INCRS-OISGA OS(CP).

7.1.2 INCRS-OISGA (INCRS) in a Nutshell

- **Mapping Corrections of INCRS.** Due to the access to the defining parameters of INCRS (and the lack of access to metadata of INCRS-S01), this section will be solely devoted to the former alternative system. Table 7.2 presents a summary of the mapping corrections of the INCRS mapping. Based on the county-by-county zoning as a pilot idea used in the studies of INCRS, the study of the mapping scale behavior is addressed in the form of mapping corrections for all 92 Test Areas (Counties). From the results of all 92 Test Areas (see Appendix D) the averaged values of the mapping corrections (units in parts per million (ppm)) have been computed for both mappings TM(CP) and OS(CP) as analyzed for the INCRS. In overall, TM(CP) of INCRS exhibits mapping corrections of 1.10 ppm. This is the average value computed from the mapping corrections of all 92 Test Areas. The mapping corrections have a standard deviation of 0.50 ppm. For the OS(CP), the average mapping correction is in the order of 1.39 ppm with a standard deviation of 0.44 ppm. The average values of the mapping corrections for these two different mapping methods (TM(CP) and OS(CP)) are in the same ball park and insignificantly different based on their standard deviations.

Based on a county-by-county zoning system, Gibson County exhibits the worst case of the mapping corrections of TM(CP) at a level of 2.96 ppm while Knox County forms the worst case for the mapping corrections of OS(CP) at the level of 3.34 ppm. The best case scenarios that exhibit the smallest mapping corrections in the case of TM(CP) is Vermillion County at a level of only 0.19 ppm while the best one for OS(CP) is Ohio County with a minimum mapping correction of 0.45 ppm.

The superior scale behavior of the Transverse Mercator mapping (TM(CP)) can be explained by the fact that Vermillion county possesses an extremely small scale variation because its narrow longitudinal shape. The scale variation in E-W direction is minimal. This direction coincides with the direction in which scale variation is most prominent for the Transverse Mercator mapping. In contrast, Gibson County with its large E-W extension, results in the maximum mapping corrections. Knox County possesses the largest distance as measured from center of project (point CP)

TABLE 7.1
Summary of the properties of the INCRS vs. INCRS-S01 based on a pilot Test Area (Marion County)

	Mapping issues	Remarks	INCRS-OISGA (INCRS)	INCRS-S01	Preference
1	Combined Scale/Terrain factors	Size and variation of the effect of ignored scale/terrain height corrections	- Size and variation of ignored scale/terrain corrections only slightly better than INCRS-S01	- Size and variation of ignored scale/terrain corrections only slightly worse than INCRS.	INCRS slightly better than INCRS-S01
			- Corrections in E and N are balanced	- Corrections in E and N are not balanced	INCRS OS(CP) better than INCRS TM(CP) and INCRS-S01 TM
2	(Relative) Precision of the mapped coordinates	Relative precision between the mapped coordinates	Precise (no discernible artifact deformations present)	Precise (no discernible artifact deformations present)	Equal
3	(Absolute) Accuracy of the mapped coordinates	Deviations from reality (Real World) (Results of O-C Difference)	Mapping deviates hardly from reality, much less than INCRS-S01 does = High accuracy	Mapping shows large deviations from reality, much more than INCRS does = Very low accuracy	INCRS
4	Convergence angle	Size variation of Convergence angles	Approximately a factor of 4 times smaller than the ones of INSPCS83	Approximately the same size as INSPCS83	INCRS
		Convergence angle correction	Omission of convergence angle corrections results in smaller errors	Omission of convergence angle corrections leads to same size errors as in INSPCS83.	
5	Implementation	System development	Very simple Sphere: - simple geometry - closed formulae mapping expressions - no series expansions	More complicated Customized ellipsoid: - complicated mapping routines - accuracy depends on series - expansions - series expansions need to be re-evaluated	INCRS
		Future amendments (e.g., if a new improved datum/ellipsoid is to be adopted)	Requires no changes because of mapping parameters are all derived parameters (from new ellipsoidal parameters)	Requires adaptation of regionally defined ellipsoids	INCRS

to its furthest point; this causes the maximum mapping correction value for the case of the OS(CP). As a matter of fact, the bowl-shaped scale behavior of Stereographic mapping is most extreme in the furthest point away from the Computational North Pole. Consequently, when one considers the shape and relatively small size of Ohio County, the scale variation is minimized. It is then also reasonable to assume that Ohio County exhibits the smallest mapping corrections for the OS(CP).

From the numbers shown in Table 7.2, it seems tempting to select TM(CP) over OS(CP) for the INCRS mapping. However, there are other factors related to this issue that need to be taken into consideration. This makes the selection of a single method over another one (and the use of that selected method for all mapping zones) not obvious. For an explanation of this statement and the related discussion the reader is referred to section 7.3 (Recommendations) further in this Chapter.

- **Terrain Corrections of INCRS.** Inevitably the Terrain Effect hampers the mapping accuracy. A study of the terrain height behavior has been conducted, and the results have been presented in terms of a set of statistical values of the terrain heights variations (see Chapter 3, section 3.3). The results from an initial assessment of the terrain height behavior have led to the idea of a whole series of tests. These tests evaluate INCRS's mapped coordinates for the TM(CP) as well as the OS(CP). These coordinates have embedded the

TABLE 7.2
Summary of the Mapping Scale corrections of the INCRS-OISGA

Maximum errors committed ignoring Mapping Scale corrections; all 92 Indiana counties	INCRS-OISGA Mapping			
	TM(CP)		OS(CP)	
	(ppm)	County	(ppm)	County
Max (worst)	2.96	Gibson	3.34	Knox
Min (best)	0.19	Vermillion	0.45	Ohio
Average (92 counties)	1.10		1.39	
St-Dev (92 counties)	0.50		0.44	

TABLE 7.3
Summary of the Terrain Height corrections of the INSPCS83 and the INCRS-OISGA

Mapping	Errors committed ignoring Terrain Height corrections; all 92 Indiana counties			
	Minimum		Maximum	
	(ppm)	County	(ppm)	County
INSPCS83 (Indiana State Plane Coordinate System of 1983)	14.1	The lowest area in the state of Indiana: Posey County: Average ellipsoidal height (h_{avg}) of 89.561 m	46.7	The highest area in the state of Indiana: Randolph County: Average ellipsoidal height (h_{avg}) of 297.46 m
INCRS-OISGA (INCRS)	2.4	The county with the smallest height range: Pulaski County: Height range ($h_{Min-Max}$) of 30.030 m	14.7	The county with the largest height range: Clark County: Height range ($h_{Min-Max}$) of 187.89 m

Note:
2.4 ppm = 2.4 cm/10 km = 0.08 ft./6 miles = 0.01 ft./mile = 0.12 in./mile.
14.1 ppm = 14.1 cm/10 km = 0.45 ft./6 miles = 0.08 ft./mile = 0.96 in./mile.
14.7 ppm = 14.7 cm/10 km = 0.47 ft./6 miles = 0.08 ft./mile = 0.96 in./mile.
46.7 ppm = 46.7 cm/10 km = 1.48 ft./6 miles = 0.25 ft./mile = 3.00 in./mile.

effect of the terrain height. The Terrain Effect is based on the idea that the State of Indiana is divided up in a county-by-county zoning system. Table 7.3 shows the Terrain Effect in ppm as compared to the Terrain Effect inherent in the classical Indiana State Plane Coordinate System of 1983 (INSPCS83).

Looking at the minimum terrain corrections the best county under INSPCS83 was the lowest county in southwest Indiana, the confluence area of the Wabash and Ohio river, Posey County. The omission of terrain corrections would yield 14.1 ppm errors. With the adoption of the INCRS the best county becomes Pulaski County, because of its small height variations the omission of terrain corrections yields only errors of 2.4 ppm.

In contrast looking at the maximum terrain corrections the worst county under INSPCS83 was the highest county Randolph: 46.7 ppm errors when terrain corrections were not applied. With the introduction of INCRS the worst county is Clark County. Because of the more erratic nature of the terrain the best one can do by bringing the reference mapping surface to the average height of this county is still at the level of 14.7 ppm errors (when intentionally omitting terrain height corrections).

In summary one may say, that with the introduction of an INCRS the Indiana Survey community replaces its best case under the INSPCS83 (14.1 ppm in Posey County) by its worst (same level) case under the INCRS (14.7 ppm in Clark County). Especially, for boundary surveys in conjunction with the use of GPS in the INRTN (InCORS) the introduction of an INCRS will be an improvement. In some areas in Indiana, among them Clark County is the worst, it remains to be seen whether the introduction of an INCRS will be an improvement as far as engineering surveys are concerned.

It should also be noted that the maximum and minimum terrain corrections as shown in Table 7.3 are based on the county-by-county zoning system. The results may vary with different zoning definitions (larger or smaller size of zones, by combining counties or subdividing counties respectively).

7.2 INCRS Implementation

Contacts have been made with the NGS Wisconsin State Geodetic Adviser and the Wisconsin Department of Transportation (WISDOT) to discuss implementation issues of a similar alternative (area-by-area) conformal mapping system in use in this state. It could be concluded that the developed coordinate system called WISCRS (Wisconsin Coordinate Reference System (*9*)) has been accepted by surveyors and is being widely used. However, the WISCRS has not been incorporated in the Wisconsin Code. Currently, WISCRS is compatible with commercial software and equipment. As an example, ArcGIS software has made WISCRS available for the users with predefined projection parameters and coordinates in the so-called subsection "County systems." It is estimated that the omission of the classical survey reductions to the ellipsoid may lead to a cost savings of anywhere between 15% and 20%. This may be true for only those surveys that do not require 1 ppm accuracies (as boundary surveys). Engineering surveys that require higher accuracies than the INCRS can guarantee, the proper reductions should be carried out, better yet, the engineering surveys should be pursued and kept in 3D, retaining their high 3D accuracies. This will become feasible in the future when engineering design software becomes capable of accepting 3D point clouds, without the (current) reduction to split into 2D + 1D models. However, the acceptance of 3D after-design operations may be even farther away in the future.

It is expected that the implementation of the INCRS can be done in a similar fashion as Wisconsin did with the WISCRS. It is foreseen that the Surveying, Engineering, and GIS professional communities have to be made aware of the new INCRS. This crucial task should be undertaken by the Professional Societies in Surveying (ISPLS), Engineering (ASCE/IN), and GIS (IGIC). In parallel, popular mapping software companies (e.g., ESRI, Trimble, Intergraph, etc.) are to be requested and stimulated to integrate INCRS into their system.

If INCRS is to be adopted as the new Indiana Coordinate Reference System, it is clear at this point that in order to successfully implement an INCRS, the very first step that **must** be taken right after the completion of this feasibility study is the preparation of documentation or a **Handbook and User Guide of INCRS.** This guide will serve as the reference for educating users about INCRS. Without the availability of Handbook and User Guide of INCRS, the implementation of INCRS is impossible.

It is expected that the Indiana Society of Professional Land Surveyors (ISPLS) and the Indiana Department of Transportation (INDOT) will have to play a leading role in formulating this handbook, or none of any implementation steps can be started. In parallel, INDOT will have to rewrite or augment their Survey and Design Manuals.

As an example may serve the 90 page manual developed by and for the State of Oregon (*10*). It is estimated that the tasks of the development (writing) of (ISPLS) Handbook and User Guide and Survey/Design Manuals may take two to two-and-a-half years of a few dedicated individuals. For one thing, the feasibility study did not address the definition of all parameters that are necessary to completely define a full INCRS, see below and section 7.3.

It is necessary to point out that in order to come up with a complete version of a Handbook and User Guide of INCRS, there are other related issues that needed to be fully investigated and finalized. As a matter of fact, the construction of a complete set of INCRS defining parameters was beyond the scope of this study. The choices of some parameters during the INCRS feasibility study were set to arbitrary or temporary values. In the future Indiana Code defining decisions need to be made concerning the size of mapping zones, the values of adopted mapping corrections, the resolution of the mapping grid, etc.

The temporarily adopted values were set for the purpose of studying the feasibility of the system itself but not for finalizing the final "look" or "face" of INCRS. Hence there still exist many related issues or factors that needed to be considered and decided upon. Therefore in the next section (7.3 Recommendations) of this document other issues that needed to be investigated or even re-visited for further analysis will be discussed.

7.3 Recommendations

In order to formulate and finalize the complete "look" of INCRS, the following issues as listed below need further investigation and defining conclusions.

- **Technical and Numerical Issues To Be Decided (see also sections 7.3.1 through 7.3.6):**

1. The definition of (the extent of) the mapping zones,
2. The definition of longitude and latitude of the mapping origin in a zone,
3. The INCRS mapping method, the TM(CP) or OS(CP), for each zone,
4. The optimization of scale corrections of each mapping zone,
5. The reference codes and/or abbreviations of each mapping zone,
6. The False Easting and False Northing coordinates.

- **Professional and Political Issues To Be Decided/ Completed (see also section 7.3.7):**

1. Development (and writing) of the (ISPLS) INCRS Handbook and User Guide,
2. Augmenting (and rewriting) of (parts of) the (INDOT) Survey and Engineering Design Manuals,
3. Assessing whether Model Law/Code for the INCRS should be developed,
4. Sponsoring of the Model Law/Code,
5. Dissemination of the new INCRS through special dedicated (ACSM/ISPLS and ASCE/IN Chapter) workshops, presentations at (ISPLS) Annual meetings, Road School, County Surveyor meetings, GIS meetings etc.

7.3.1 The Definition of (the extent of) Mapping Zones

In this study, each mapping zone of the INCRS (they have been referred to as "Test Area") is based on a per-county size. This led to 92 Test Areas (zones/counties). This county-size zoning system of INCRS in this feasibility study has been designed based on the fact that researchers concluded that in the case of INCRS the smallest zone size needs to be adopted for optimal minimization of the mapping errors. Such a system may reasonably coincide with the political division of the State in counties. A finer zoning than at the county-size level will not be practical and is difficult to administrate. It should also be noted that a county-based system in Indiana is also suitable for the fact that the areas covered by each county are reasonably equal. For this feasibility study of INCRS in terms of its technical aspects of assessing the size of the committed errors could serve as the foundation for further study. The about equal-area zones are suited to finalize the zone defining parameters. It may well be that certain mapping parameters may be adopted as group values for a group of zones (counties).

Basically, the INCRS zoning can be done in such a way that it minimizes the mapping scale variation and Terrain Effect for a group of zones (counties). The size of each zone may vary (intra-county or extra-county): some areas may exhibit similar terrain behavior, combining counties into one zone, or subdividing zones (counties) under the condition that combining or subdividing those areas do not make the mapping correction exceed predefined limits. No specific rules for the size of mapping zones have been defined (yet) as long as they yield accuracies at predefined satisfactory levels. Beside the physical properties of the areas that play a key role in defining mapping zones, other factors related to political aspects may also influence the final definition of zones.

The scope of this feasibility study did not contain the final definition of INCRS mapping zones. Hence this issue of INCRS mapping zones definition is discussed in the recommendation section.

7.3.2 The Definition of Longitude and Latitude of Mapping Origin in a Zone

It is highly recommended that the position of the origin of the mapping should be finalized after the delineations of the zones are defined. That means that the mapping's origin or Central Meridian positions should be tailored to the defined boundaries of the zones, but not the other way around. In principle any arbitrary position can be adopted as the mapping's origin such as the selection of the position of the Town Hall, the Court House, a historical landmark, etc. The designation of these types of landmarks as the CP is not a practical idea and is not recommended.

The design of the final definition of INCRS mapping zones and its parameters were not within the scope of this study. Therefore no attempt has been made by its researchers to draw conclusions about the final position of the mapping's origin of each zone. In this feasibility study of INCRS, the physical center of the area to be mapped (center of project area) that has been referred to as point CP, was used as the mapping's origin (or as the longitude of the Central Meridian/CM). The idea is practical in the sense that the boundaries of zone were pre-defined, and that it also creates the symmetrical effect in the mapping when considering the whole zone as the project area. It is recommended that if INCRS is to be adopted and the definitions of zones are set, the physical center of zone is likely to be the best position of the mapping's origin. Considering all pros and cons, rounded value (to the nearest arcminute, or the nearest five arcminutes?) for the longitude and latitude of the center of the zone, or Center of Project (CP)/Central Meridian (CM), is probably most preferred. One should be reminded that under the current INSPCS83 the location (longitude) of the two Central Meridians in Indiana is rounded to the nearest five arcminutes.

7.3.3 The INCRS Mapping Method, the TM(CP) or the OS(CP), for each Zone

The selection of the mapping method used in each zone is directly related to the definition of the sizes and shapes of the zones. The findings of this study proved that for a zone (the considered Test Area) either the TM(CP) or the OS(CP) yielded better mapping accuracies. In some cases neither the TM(CP) nor OS(CP) produced any significantly better results than the other. Therefore selecting an INCRS mapping method of the TM(CP) over the OS(CP) depended on the physical properties (shape and size) of the zone under consideration. For example, a zone such as Vermillion County that exhibits a much longer extent in N-S (latitudinal) direction than E-W (longitudinal) direction will be better mapped under TM(CP) than under the OS(CP). This means that the specific shape and size of an area or zone, based on the commonality of characteristics between those neighboring zones, directly drives the choice of the preferred mapping method. Therefore it is highly recommended that the selection of the mapping method may be taken into account parallel to the definition of the delineation of the mapping zones.

Referring back to the results as shown in Table 7.2, one may draw the incorrect conclusion that the selection of the TM(CP) over the OS(CP) for all mapping zones will be the best solution for Indiana warranting the best results, i.e., the smallest mapping corrections. The explanation as given in the previous paragraph proves that not necessarily one single mapping method should be selected and applied to all zones. The adoption of an optimal mapping method that yields better results for each zone under consideration may be a better approach. One should be reminded that if the behavior of the mapping corrections (or committed errors by omission of mapping corrections) should be balanced in Easting and Northing, the OS(CP) is preferred "hands-down" over the TM(CP).

7.3.4 The Optimization of Scale Corrections of each Mapping Zone

After the process of defining of the boundaries of the INCRS mapping zones and their corresponding mapping origins is completed as well as the corresponding mapping methods for each zone have been selected, finding the appropriate mapping correcting scale values (what has been referred to as the scale factor 1-Δ) is the next issue to be considered. [NB: for the current INSPCS83 the scale factor is equal to 1 – 1/30,000, with Δ being 1/30,000]. In this research the mapping correction (Δ) of each Test Area for the case of TM(CP) and OS(CP) mapping methods were computed based on the idea of balancing the scale variation behavior (see the explanations presented in Chapter 2, section 2.1.1). The idea of finding the optimum mapping correction value itself has embedded two different aspects to be considered as follows:

1. Single or multiple value(s) of mapping corrections Δ (global vs. local Δ's)? This is related to the aspect of whether or not a single optimized mapping correction value ("global" Δ) should be adopted for all mapping zones regardless the different mapping methods used in different zones. The other extreme is that each mapping zone gets assigned its corresponding optimum mapping correction value ("local" Δ). Subsequent considerations of the mapping/scale correction values could also be dependent on the mapping method being used. An outcome of further study could be that each mapping method, the TM(CP) and OS(CP), have their own optimal "global" mapping/scale correction values.

2. Choice of optimization method used to arrive at the mapping correction value. The selection of the preferable computation method used for optimizing mapping correction value has not been considered so far. In the other words, the method of how to balance the scale variation behavior in each area under consideration has to be selected.

In the Marion County Test study, the mapping correction value (Δ) has been computed based on the method of using the extreme scale values on both ends of the scale values profiles (σ_{Min} and σ_{Max}) to balance the overall scale variation behavior. At that time of the study, the mentioned optimization method was used in order to be able to proceed to the next step in the analysis. It was not yet possible to draw a conclusion about what the best way of optimizing mapping correction values is.

Another method of balancing the scale variation behavior which has also been investigated, is the use of the scale value at the 50th percentile level (σ_{50}, $\sigma_{50} = 1 + \Delta_{50}$) as the key to redistribute the scale values over all points in the area. It means that the newly adopted mapping correction (k) will be equal to 1 - Δ_{50}.

Instead of using the scale value at the 50th percentile (σ_{50}) to balance scale variation behavior, another value such as the average scale (σ_{avg}, $\sigma_{avg} = 1 + \Delta_{avg}$) computed from the scale values at all grid points was also investigated. The mapping correction (k) is then equal to 1 - Δ_{avg}. The mapping correction (k) resulted from using all these different methods of balancing the scale variation behavior can be found in Appendix D.

Despite the fact that some methods of balancing the scale variation have been investigated, the conclusion about the best way of optimizing mapping correction value is yet unclear. There exist also many other different computational methods to optimize the mapping correction value, each of which are subjected to different mathematical theories, and hence will yield different solutions.

Exercising methods that are linear/profile-based or area-based, constitute different methods of optimizing the mapping correction values. This is another research topic in itself and was beyond the scope of this research. It is highly recommended that in order to efficiently determine the appropriate mapping correction values, (although not a major issue) further investigation of different array of methods of optimizing mapping corrections as outlined above is pursued for a limited time.

7.3.5 The Reference Codes and/or Abbreviations of each Mapping Zone

After the definition of the INCRS zones is completely finalized, reference codes should be assigned to each zone. In this study, each zone has its own reference code and abbreviation which complied with the ones that have officially been adopted in the license plate system by the Bureau of Motor Vehicles (BMV) in the State of Indiana. However, the final reference code and/or the abbreviations of each zone depends on the how the zones boundaries are defined, and on the total number of zones in the final design of INCRS system. The final reference code and/or abbreviation of the zones may be changed from the one that was used in this study. In the case a county-by-county zoning system is used, it is highly recommended that the same referencing method as used in this study is to be adopted. The reference code for each county that has been administrated by the National Geodetic Survey (NGS) may also be a good alternative. The NGS Code is referred to as the NGS FIPS Code.

7.3.6 The False Easting and False Northing Coordinates

After the definition of zones, the reference zone codes and/or zone abbreviations, map origins, mapping methods, and mapping correction values have been finalized, an almost complete "look" or "face" of INCRS starts to appear. What is left to be considered is the form the mapped coordinates are going to take. Assuming that a similar but renewed type of Easting and Northing system is going to replace or augment the current INSPCS83, values for the False Easting and Northing coordinates (of the CP) need to be adopted. A solution may be that each zone possesses its own False Easting and False Northing coordinates that are related to some specific property of each zone such as reference code number. This will aid map users in recognizing zone location from their distinct coordinates.

In contrast, if the case of having distinct coordinates number for each zone is not possible, other approaches of setting False Easting and False Northing coordinates may be considered. In conclusion, some thoughts need to be given to the name-giving of these coordinates so that they can be differentiated from the classical Easting and Northing coordinates belonging to the INSPCS83 (E_{INCRS} vs. $E_{INSPCS83}$, and N_{INCRS} vs. $N_{INSPCS83}$, etc.).

7.3.7 Professional and Political Issues To Be Decided/Completed

The five issues (Tasks) mentioned at the beginning of section 7.3 (Recommendations) are crucial to the success of a new INCRS, once the outcome of this feasibility is judged to be positive.

These five issues are:

1. Development (and writing) of the (ISPLS) INCRS Handbook and User Guide,
2. Augmenting (and rewriting) of (parts of) the (INDOT) Survey and Engineering Design Manuals,
3. Assessing whether Model Law/Code for the INCRS should be developed,
4. Sponsoring of the Model Law/Code,
5. Dissemination of the new INCRS through special dedicated (ACSM/ISPLS and ASCE/IN Chapter) workshops, presentations at (ISPLS) Annual meetings, Road School, County Surveyor meetings, GIS meetings etc.

Based on the example of the development of the Oregon Handbook and the time to complete that task with success it is estimated that a period of two to two-and-a-half years is needed by a set of dedicated professionals to complete Tasks 1 and 2. At the Board level of ISPLS, INDOT, IGIC, and last but not least OISGA decisions need to be made to pursue (and if positive) develop, promote, sponsor, and adopt Model Law that prescribes the use of a new INCRS (Tasks 3 and 4). It may be that the Indiana professionals expect that Task 5 is "outsourced" to OISGA.

7.4 Implementation Recommendation

To realize the Technical and Numerical issues (sections 7.3.1 through 7.3.6) and the Professional and Political issues (section 7.3.7) it is recommended that an "Implementation SPR" stretching over a period of two to two-and-a-half years be formulated, proposed, and approved.

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APPENDIX A. MATHEMATICAL EXPRESSIONS

A.1 INTRODUCTION

Appendix A explains all the mathematics that has been used during the development of the INCRS-OISGA (known as INCRS for short).

All coordinate frames are defined in a right-handed sense. This holds for 2D frames as well as 3D frames. So, we have:

2D: (X, Y), (X′, Y′), (E, N), (E_{Final}, N_{Final}), etc.
3D: (x, y, z), (X, Y, Z), (X′, Y′, Z′), (X_M, Y_M, h_v), (λ, ψ, h_s), (λ, φ, h_e), (e, n, u), etc.

All rotations are defined in a right-handed sense, meaning that the argument (angle) of rotation has been defined as positive when the sense of rotation is counterclockwise as viewed from the positive end of the rotation axis looking towards the origin of the frame.

The original (relative) geometries of the 3D point clouds, whether they are expressed in geocentric (x, y, z) or in topocentric (e, n, u) frames, or in rotated/translated "primed" (X′, Y′, Z′) frames, whether they are expressed using ellipsoidal or spherical coordinates, are all identical in each representation. It is investigated how the 3D-to-2D mapping process distorts the original (relative) geometry of these 3D point clouds.

A.2 GAUSSIAN RADIUS OF CURVATURE

The Gaussian Radius of Curvature (G) is a quantity that tries to approximate the local curvature of an ellipsoid in all directions in an average sense. It is the geometric mean of the two extreme radii if one integrates and averages all azimuth-dependent radii over the interval [$0^\circ \leq R_{Az} < 360^\circ$]. This means that G is a position dependent value. The Gaussian Radius of Curvature at any point A (G_A) can be computed from the (maximum) Radius of Curvature in the Prime Vertical Plane at point A (N_A) and the (minimum) Radius of Curvature in the Meridian Plane at point A (M_A) with the relationship expressed in the form of Eq. A.2.1.

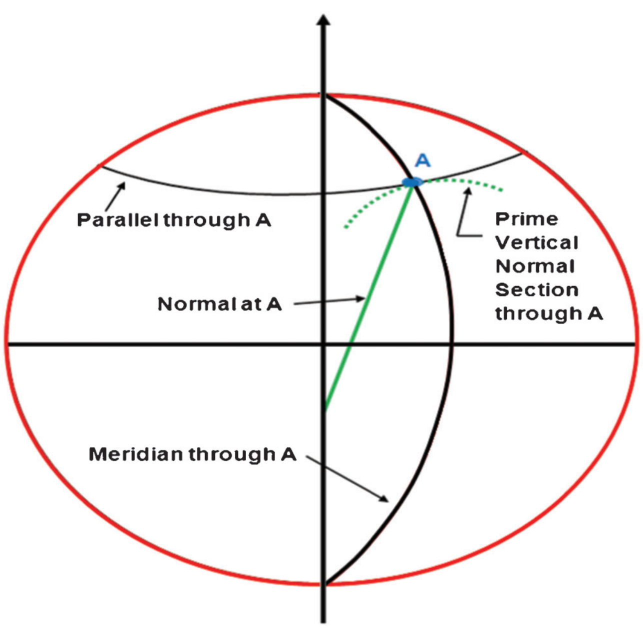

Figure A.2.1 Prime vertical normal section through point A and meridian plane through point A.

$$G_A = \sqrt{M_A N_A} \tag{A.2.1}$$

Where

N_A is the Radius of Curvature in the Prime Vertical Plane (E-W) at point A, and
M_A is the Radius of Curvature in the Meridian Plane (N-S) at point A.

- Radius of Curvature in the Prime Vertical (N)

The Radius of Curvature in the Prime Vertical Plane (N) (see Figures A.2.1 and A.2.2) is a point-dependent value. N can be viewed as the radius of the best fitting circle to the intersecting curve between the ellipsoidal surface and the E-W plane through the (ellipsoidal) normal through point A. At any point A with ellipsoidal latitude of φ_A, N_A can be computed from the expression written in the form of Eq. A.2.2.

$$N_A = \frac{a}{\sqrt{1-(e^2 \sin^2 \varphi_A)}} \tag{A.2.2}$$

Where
a is the semi-major axis of the ellipsoid, and
e is the (first) eccentricity of the ellipsoid which can be computed from Eq. A.2.3.

$$e^2 = 2f - f^2, \ e = \sqrt{2f - f^2} \tag{A.2.3}$$

Where
f is the ellipsoid flattening.

$$f = \frac{a-b}{a} \tag{A.2.4}$$

Where
a is the semi-major axis of the ellipsoid, and
b is the semi-minor axis of the ellipsoid.

NOTE: For NAD83 datum with GRS80 ellipsoid, the parameters of the GRS80 ellipsoid (a, f) are as follows:
a_{GRS80} = 6378137.0 m
f_{GRS80} = 1/298.257222101

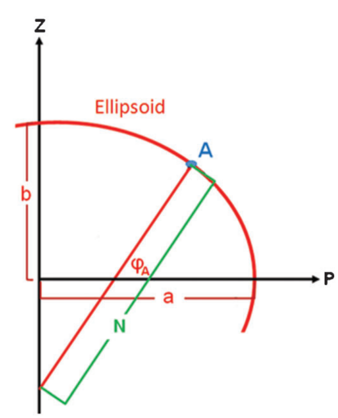

Figure A.2.2 Meridian plane illustrating the Radius of Curvature in the Prime Vertical at point A.

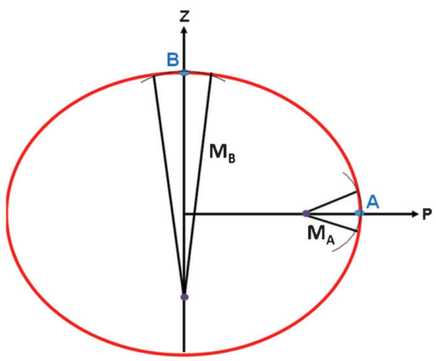

Figure A.2.3 Geometry of the Radii of Curvature in the Meridian Plane.

• Radius of Curvature in the Meridian Plane (M)

The Radius of Curvature in the Meridian Plane (M) is also a point-dependent value. M can be viewed as the radius of the locally best fitting circle to the ellipse. The radius M is constantly changing along the meridian. Therefore the value of M is a function of the ellipsoidal latitude. Figure A.2.3 illustrates the geometry of the Radii of Curvature in the Meridian Plane (M) at two different points; point A and point B, as examples.

Figure A.2.4 illustrates the ellipsoidal latitude dependency of the Radius of Curvature in the Meridian Plane (M): the value of M continuously changes with the value of ellipsoidal latitude (φ).

At any point A with ellipsoidal latitude of φ_A, the Radius of Curvature in the Meridian Plane at point A (M_A) can be computed from the formula as written in the form of Eq. A.2.5.

$$M_A = \frac{a\left(1 - e^2\right)}{\sqrt{\left(1 - e^2 \sin^2 \varphi_A\right)^3}} \qquad \text{(A.2.5)}$$

Where
a is the semi-major axis of the ellipsoid, and
e is the (first) eccentricity of the ellipsoid (see Eq. A.2.3)

It should be noted that the values of the Gaussian Radius of Curvature (G), the Radius of Curvature in the Prime Vertical (N), and the Radius of Curvature in the Meridian Plane (M) have the following properties:

1. $N > 0$, $M > 0$, and $G > 0$
2. $N \geq M$ for all φ

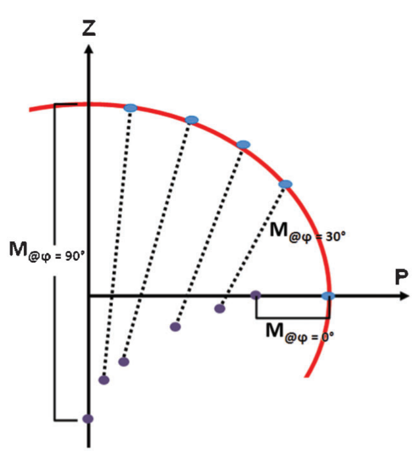

Figure A.2.4 The Radius of Curvature in the Meridian Plane (M) as a function of the ellipsoidal latitude.

3. At $\varphi = \pm 90°$, $N = M$, and take on their maximum values
4. At $\varphi = 0°$, N and M take on their own corresponding minimum values
5. $M \leq G \leq N$ for all φ

A.3 INCRS MAPPING PROCEDURES

The INCRS mapping makes use of the pre-defined "INCRS Sphere" as the mapping reference surface. The center (origin: O_G) of INCRS Sphere is located along the ellipsoidal normal drawn at the center of the underlying project area (so-called point "CP"). The geometry of INCRS has already been revealed in Figure 2.9, for convenience Figure A.3.1 repeats the illustration of Figure 2.9.

Steps in the INCRS Mapping Procedure

The INCRS mapping procedure is composed of four main steps as follows:

Step 0 Define the INCRS Sphere,
Step 1 Dissimilar Coordinate Transformation,
Step 2 Similar Coordinate Transformation,
Step 3 Mapping Procedures.

The details of each step are discussed in separate sections. Each step may consist of many different sub-steps.

Step 0 Define the INCRS Sphere The so-called Step 0 is the first fundamental step for all other steps. In this step the INCRS Sphere is to be defined and it will serve as the mapping reference surface. The defining parameters of the INCRS Sphere are the center of the project known as point CP and the radius of the sphere known under the generic name as "R_{normal}." The value of R_{normal} is equal to the Gaussian Radius of Curvature at point CP; hence it is specifically called "$R_{G@CP}$." In order to locate the INCRS Sphere, the geodetic coordinates (λ, φ, h_e) of point CP for the NAD83 datum (with the GRS80 ellipsoid) are needed. With the help of these ellipsoidal coordinates the value of $R_{G@CP}$ that serves as the radius of INCRS Sphere, can then be calculated. Also the origin (O_G) of INCRS Sphere can be located (see Figure A.3.1).

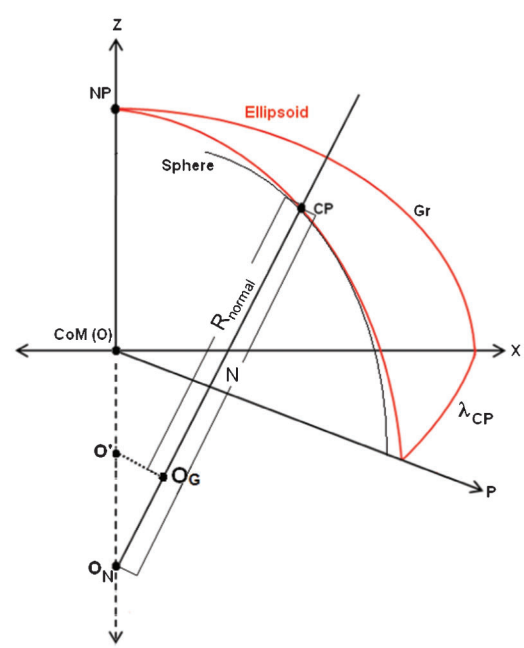

Figure A.3.1 INCRS Sphere (same as Figure 2.9).

Step 0.1 Define point CP (center of the project). The geodetic coordinates of point CP are defined on the ellipsoid. This means that the ellipsoidal height of point CP is forced to be zero. In this research report the point CP is the center of the project area and was computed from the coordinates that reflect the extent of the area (county). The geodetic coordinates of point CP are denoted as $(\lambda_{CP}, \varphi_{CP}, 0)_{NAD83}$.

Step 0.2 Computation of Gaussian Radius of Curvature at point CP (G_{CP}). After point CP is defined, the Gaussian Radius of Curvature at point CP (G_{CP}) can be computed based on the mathematical expressions as described in section A.2. The value of the computed Gaussian Radius of Curvature at point CP (G_{CP}) is then defined as the radius of the fundamental INCRS Sphere, and is indicated by "$R_{G@CP}$."

Step 0.3 Locate the origin (O_G) of INCRS Sphere. The coordinates of the INCRS Sphere's origin (O_G) are derived values from the known position of point CP and the radius of the INCRS Sphere ($R_{G@CP}$). The Cartesian coordinates of the origin (O_G) are defined in a geocentric Earth-fixed frame, the CoM frame. This frame is a right handed Cartesian coordinate frame that has its origin located at the Center of Mass (CoM) of the Earth. It is that frame that is realized by the Geodetic Reference Frame ellipsoid (GRS80) and the North American Datum 1983 (NAD83). The Cartesian coordinates of the origin of the INCRS Reference Sphere are expressed in terms of $(XO_G, YO_G, ZO_G)_{CoM}$ and can be calculated from its derived geodetic coordinates that are $(\lambda_{CP}, \varphi_{CP}, h_{CP} = -R_{G@CP})_{NAD83}$ through a so-called "Dissimilar Coordinate Transformation" that converts geodetic coordinates into Cartesian coordinates in the CoM frame (see Step 1 for more details on the Dissimilar Coordinate Transformation).

The new Cartesian coordinate frame that has the origin at the O_G is referred to as the "G" frame. The G frame (or G coordinate system) is simply the new Cartesian coordinate frame that is exactly parallel to the CoM frame: the orientations of the axes of both frames are identical. That means that the G frame is simply a translated version of the CoM frame. The well-known relationship is expressed in Eq. A.3.1.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{CoM} - \begin{bmatrix} XO_G \\ YO_G \\ ZO_G \end{bmatrix}_{CoM} \tag{A.3.1}$$

The geocentric coordinates of the points $(X, Y, Z)_{CoM}$ in the CoM frame can be transformed into Cartesian coordinates in the G frame $(X, Y, Z)_G$, by the relationship as expressed in Eq. A.3.1.

Step 1 Dissimilar coordinate transformation (Ellipsoidal coordinates → Cartesian coordinates) Ellipsoidal coordinates in the NAD83 datum → Cartesian coordinates in CoM frame

$$(\lambda, \varphi, h_e)_{NAD83(GRS80)} \longrightarrow (X, Y, Z)_{CoM}$$

Transform all points (points to be mapped) which were originally defined in the geodetic coordinate system under NAD83 datum into the Cartesian coordinate frame (CoM) by the so-called Dissimilar Coordinate Transformation as expressed in Eq. A.3.2. **It should be noted that for the INCRS mapping, any mentioning of geodetic coordinates and its related parameters (a, f) are for the NAD83 datum (GRS80 ellipsoid).**

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{CoM} = \begin{bmatrix} (N+h_e)\cos\varphi\cos\lambda \\ (N+h_e)\cos\varphi\sin\lambda \\ (N+h_e)\sin\varphi - Ne^2\sin\varphi \end{bmatrix} \tag{A.3.2}$$

Where
λ is the ellipsoidal longitude,
φ is the ellipsoidal latitude
h_e is the ellipsoidal height (height above ellipsoid, or hae), and
N is the Radius of Curvature in the Prime Vertical Plane (see section A.2).

Step 2 Similar Coordinate Transformation (Cartesian coordinates (CoM) → Cartesian coordinates (G)) Cartesian coordinates in CoM frame → Cartesian coordinates in G frame

$$(X, Y, Z)_{CoM} \longrightarrow (X, Y, Z)_G$$

Transform the Cartesian coordinates as defined in the CoM frame into Cartesian coordinates in the G frame by using the relationship as expressed in Eq. A.3.1.

Up to this Step 2, all points to be mapped are now in the form of Cartesian coordinates defined in the G frame. They are now ready to be mapped by any selected set of mapping functions (mapping methods). In this research study, different mapping methods were investigated (as discussed in Chapter 4, section 4.2.3). Those are as follows:

- *Transverse Mercator Type 1: TM(IC 32-19)*. Use of the Transverse Mercator mapping function with the longitude and latitude of the origin as defined in IC 32-19 (one Central Meridian (CM) for the IN East zone, and a separate Central Meridian (CM) for the IN West zone).
- *Transverse Mercator Type 2: TM(CP)*. Use of the Transverse Mercator mapping function with the longitude and latitude of the origin as defined by the geodetic coordinates of the Test Area's project center (CP). In this case each of the areas (counties) will use their own project centers (CP) as the origin of the map. Also the Central Meridian will intersect the CP in a north-south direction.
- *Oblique Stereographic (only one Type 1): OS(CP)*. Application of the Oblique Stereographic mapping functions uses the project's center (CP) of Test Area (county) under consideration. The CP also referred to as the new defined "Computational North Pole."

Step 3 Mapping Procedures The tasks in Step 3 are dependent on the selected method of mapping. In this study different mapping methods were exercised. Therefore Step 3 will be subdivided into 3 different sub-sections as follows:

Step 3A: Mapping Procedures for the TM(IC 32-19) mapping,
Step 3B: Mapping Procedures for the TM(CP) mapping, and
Step 3C: Mapping Procedures for the OS(CP) mapping.

The details of each sub-section will be described in separate sections. It should be noted that Step 3A and Step 3B are almost the same with the difference being the definition of the Central Meridian.

Choice 1: Step 3A Mapping Procedures for the TM(IC 32-19) Mapping In this case, the TM(IC 32-19) mapping is used. Right from Step 2 that all the points are ready to be mapped, the next steps are as follows:

Step 3A.1 Apply Transverse Aspect.

$$(X, Y, Z)_G \xrightarrow[\text{Apply Transverse Aspect}]{} (X', Y', Z')$$

Applying the Transverse Aspect is accomplished by transforming all the points from the G frame into the new so-called "Prime" system. This new Prime system has adopted a new map's origin and the Central Meridian as defined under IC 32-19. The transformation is dependent on the Test Area under consideration: is it located in the original East or West zone of the INSPCS83? The East and West zone have each their own new "Equator," which is the fundamental idea behind the Transverse Mercator mapping. The Transverse Aspect is obtained by the following coordinate transformation, see Eq. A.3.3.

$$\vec{X}' = R_1(-90^\circ) R_3(\lambda_{CM})_G \vec{X}_G \quad (A.3.3)$$

Where
$(\lambda_{CM})_G$ is the longitude in the G system of the original INSPCS83's Central Meridian, and the remaining terms are described in Eqs. A.3.4 through A.3.7.

$$\vec{X}' = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \quad (A.3.4)$$

$$\vec{X}_G = \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} \quad (A.3.5)$$

$$R_3(\lambda_{CM})_G = \begin{bmatrix} \cos(\lambda_{CM})_G & \sin(\lambda_{CM})_G & 0 \\ -\sin(\lambda_{CM})_G & \cos(\lambda_{CM})_G & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A.3.6)$$

$$R_1(-90^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90^\circ) & \sin(-90^\circ) \\ 0 & -\sin(-90^\circ) & \cos(-90^\circ) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & +1 & 0 \end{bmatrix} \quad (A.3.7)$$

Step 3A.2 Dissimilar Coordinate Transformation (Cartesian coordinates → Spherical coordinates). Cartesian coordinates in Prime system → Sphericl coordinates

$$(X', Y', Z') \xrightarrow[\text{Spherical model (RadiusType i)}]{} (\lambda', \psi', h_s')$$

Transform all Cartesian coordinates in the Prime system into spherical coordinates in the same frame by the relationship as expressed by Eqs. A.3.8 through A.3.10. The equations are based on the value of the INCRS Sphere's radius. In this study four different radii of the mapping reference sphere have been used (see Chapter 4, section 4.2.2) for various purposes as explained in Chapter 4.

In this case the subscript "i" has been used to explain the fact that the Radius Type 1/2/3/4 may be selected to be used which is dependent on the purpose of the study. In summary, for the Study of the Scale Effect, Radius Type 1 and Type 2 will be used whereas for the case of Terrain Effect study the Radius Type 3 and Type 4 are in charge.

$$\lambda' = \arctan\left(\frac{Y'}{X'}\right) \quad (A.3.8)$$

$$\psi' = \arctan\left(\frac{Z'}{\sqrt{X'^2 + Y'^2}}\right) \quad (A.3.9)$$

$$h_s' = \sqrt{X'^2 + Y'^2 + Z'^2} - R_i \quad (A.3.10)$$

Where
λ' is the spherical longitude in the Prime system,
ψ' is the spherical latitude in the Prime system,
h_s' is the spherical height (height above INCRS Sphere) in the Prime system, and
R_i is the radius of mapping reference sphere (INCRS Sphere) with $i = 1, \ldots, 4$. Radius Types 1 to 4 depend on the purpose of study.

Step 3A.3 Apply Mapping Function (Spherical coordinates → coordinates in mapped frame).

$$(\lambda', \psi', h_s') \xrightarrow[\text{Transverse Mercator mapping function}]{} (X_M, Y_M)$$

Transform all spherical coordinates (λ', ψ', h_s') as defined in the Prime system into 2-dimensional (X_M, Y_M) mapping coordinates (the mapped frame) by applying the Transverse Mercator mapping functions as expressed in Eqs. A.3.11 and A.3.12.

$$X_M := R_i(\lambda') \quad (A.3.11)$$

$$Y_M := R_i\left\{\ln\left[\tan\left(\frac{\psi'}{2} + \frac{\pi}{4}\right)\right]\right\} \quad (A.3.12)$$

Step 3A.4 Transformation to Easting and Northing frame.

$$(X_M, Y_M) \longrightarrow (E, N)$$

The (X_M, Y_M) coordinates in the mapped frame are then transformed into coordinates in the conventional Easting and Northing (E, N) system by the transformation expressed in Eq. A.3.13

$$\begin{bmatrix} E \\ N \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} X_M \\ Y_M \end{bmatrix} + \begin{bmatrix} T_E \\ T_N \end{bmatrix} \quad (A.3.13)$$

Where
T_E and T_N are the computed translations in Easting and Northing directions respectively. They are defined in the Easting and Northing frame in such a fashion that any desired values of the Eastings and Northings may be achieved by applying an additional translation terms "False Easting (FE)" and "False Northing (FN)" (see Eq. A.3.14) to the coordinates.

$$\begin{bmatrix} E_{Final} \\ N_{Final} \end{bmatrix} = \begin{bmatrix} E \\ N \end{bmatrix} + \begin{bmatrix} FE \\ FN \end{bmatrix} \quad (A.3.14)$$

Choice 2: Step 3B Mapping Procedures for the TM(CP) mapping In this choice, the TM(CP) mapping is used. The mapping steps of TM(CP) are almost the same as the case of TM(IC 32-19) with the only difference being the definition of the Central Meridian. Instead of using the pre-defined Central Meridians of INSPCS83, TM(CP) makes use of the longitude value at its point CP as the local Central Meridian.

The steps of TM(CP) mapping are the same as TM(IC 32-19) (Step 3A.1 through Step 3A.4) with only one difference of the value used in the step of applying Transverse Aspect (Step 3A.1, Eq. A.3.3). In this case the Transverse Aspect is applied as in Eq. A.3.15.

$$\vec{X}' = R_1(-90^\circ) R_3(\lambda_{CP})_G \vec{X}_G \quad (A.3.15)$$

Where
$(\lambda_{CP})_G$ is the spherical longitude of point CP in the G frame. The remaining terms except the rotation about the third axis $(R_3(\lambda_{CP})_G)$ were already described in Eqs. A.3.4, A.3.5 and A.3.7. In this case, the rotation about the third axis $(R_3(\lambda_{CP})_G)$ is expressed in the form as shown in Eq. A.3.16.

$$R_3(\lambda_{CP})_G = \begin{bmatrix} \cos(\lambda_{CP})_G & \sin(\lambda_{CP})_G & 0 \\ -\sin(\lambda_{CP})_G & \cos(\lambda_{CP})_G & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A.3.16)$$

Choice 3: Step 3C Mapping Procedures with OS(CP) mapping

In this choice, the Oblique Stereographic mapping is used. Right from Step 2 all the points are ready to be mapped. The next steps are as follows:

Step 3C.1 Similar Coordinate Transformation (Cartesian coordinates (G) → Cartesian coordinates (Prime)).

$$(X, Y, Z)_G \xrightarrow[\text{Computational North Pole as the new North Pole in Prime system}]{} (X', Y', Z')$$

Transform the Cartesian coordinates in G frame into the Cartesian coordinates in the Prime system of which the point CP is the Computational North Pole. The CP will serve as the new North Pole of the Prime coordinate frame. The transformation is expressed as in Eq. A.3.17. **It should be noted that the so-called Prime frame in the case of the Oblique Stereographic is not the same as the Prime coordinate frame in the case of Transverse Mercator mapping. The "Prime Frame" is used as the generic term to represent the transformed (rotated and/or translated) frame for which the transformation parameters may vary. The "Prime Frame" has a different "look" (orientation) in the Transverse Mercator mapping as in the Oblique Stereographic mapping.**

For the Oblique Stereographic the Prime frame is obtained from

$$\vec{X'} = R_2(\theta_{CP})_G R_3(\lambda_{CP})_G \overrightarrow{X_G} \quad (A.3.17)$$

Where

$(\lambda_{CP})_G$ is the spherical longitude of point CP in the G frame,

$(\theta_{CP})_G$ is the spherical co-latitude of point CP in the G frame. The remaining terms are described in Eqs. A.3.18 through A.3.21.

$$\vec{X'} = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \quad (A.3.18)$$

$$\vec{X'}_G = \begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} \quad (A.3.19)$$

$$R_3(\lambda_{CP})_G = \begin{bmatrix} \cos(\lambda_{CP})_G & \sin(\lambda_{CP})_G & 0 \\ -\sin(\lambda_{CP})_G & \cos(\lambda_{CP})_G & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A.3.20)$$

$$R_2(\theta_{CP})_G = \begin{bmatrix} \cos(\theta_{CP})_G & 0 & -\sin(\theta_{CP})_G \\ 0 & 1 & 0 \\ \sin(\theta_{CP})_G & 0 & \cos(\theta_{CP})_G \end{bmatrix} \quad (A.3.21)$$

Step 3C.2 Dissimilar Coordinate Transformation (Cartesian coordinates → Spherical coordinates). Cartesian coordinates in Prime system → Spherical coordinates

$$(X', Y', Z') \xrightarrow[\text{Spherical model(Radius Type i)}]{} \left(\lambda', \psi', h_s{}'\right)$$

This step is the same as Step 3A.2: the Cartesian coordinates in the Prime frame are transformed into the corresponding spherical coordinates in the same frame through the use of the relationship as expressed in Eqs. A.3.8 through A.3.10 based on the use of INCRS Sphere as the mapping reference sphere.

Where

λ' is the spherical longitude in the Prime system,

ψ' is the spherical latitude in the Prime system,

h'_s is the spherical height (height above INCRS Sphere) in the Prime system, and

R_i is the radius of mapping reference sphere (INCRS Sphere) which can be varied from Radius Type 1 – Radius Type 4 depends on the purpose of study.

Step 3C.3 Apply Mapping Function (Spherical coordinates → coordinates in mapped frame).

$$\left(\lambda', \psi', h_s{}'\right) \xrightarrow[\text{Stereographic mapping function}]{} (X_M, Y_M)$$

Transform all spherical coordinates (λ', ψ', h_s') in the Prime system into 2-dimenstional (X_M, Y_M) coordinates in the mapped frame by applying Stereographic mapping function as expressed in Eqs. A.3.22 and A.3.23.

$$X_M := R_i\left[2\tan\left(\frac{\theta'}{2}\right)\cos(\lambda')\right] \quad (A.3.22)$$

$$Y_M := R_i\left[2\tan\left(\frac{\theta'}{2}\right)\sin(\lambda')\right] \quad (A.3.23)$$

Where

λ' is the spherical longitude in the Prime system,

θ' is the co-latitude in the Prime system ($\theta' = 90\text{-}\ \psi'$; ψ' is the spherical latitude in the Prime system),

h_s' is the spherical height (height above INCRS Sphere) in the Prime system, and

R_i is the radius of mapping reference sphere (INCRS Sphere) with i = 1,…, 4. The Radius Types 1 to 4 depends on the purpose of its use.

Step 3C.4 Transformation to the Easting and Northing frame.

$$(X_M, Y_M) \longrightarrow (E, N)$$

The (X_M, Y_M) coordinates in the mapped frame are then transformed into coordinates in the conventional Easting and Northing (E, N) frame by the transformation expressed in Eq. A.3.24. The final Easting and Northing coordinates (E_{Final}, N_{Final}) may be achieved by applying an additional translation terms "False Easting (FE)" and "False Northing (FN)" to the coordinates in the Easting and Northing (E, N) frame (see Eq. A.3.14). The translation terms FE and FN have been set up in such a way that the map's origin will take on the value of this False Easting (FE) and False Northing (FN) coordinates.

$$\begin{bmatrix} E \\ N \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} X_M \\ Y_M \end{bmatrix} \quad (A.3.24)$$

A.4 TWO-DIMENSIONAL LINEAR TRANSFORMATION

A two-dimensional linear transformation is a 2D Affine transformation. Figure A.4.1 illustrates the idea of a transformation that connects two systems together under the relationship as expressed in terms of the transformation parameters.

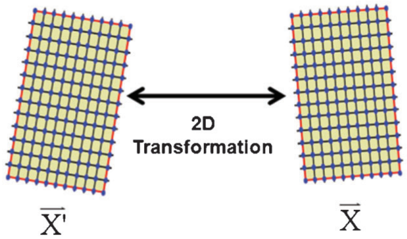

Figure A.4.1 The idea of the 2-dimensional transformation

One of many conventional ways of naming two different systems is in the form of 2-dimensional coordinates vectors in a so-called "Normal" or "Original" system (as in Eq. A.4.1) and the "Prime" system (as in Eq. A.4.2)

$$\vec{X} = \begin{bmatrix} X \\ Y \end{bmatrix} \quad \text{(A.4.1)}$$

$$\vec{X}' = \begin{bmatrix} X' \\ Y' \end{bmatrix} \quad \text{(A.4.2)}$$

The linear transformation (either 2-dimensional or 3-dimensional) expresses the relationship between two sets of coordinates (point clouds) in the form as shown in Eq. A.4.3

$$\vec{X}' = M\vec{X} + \vec{T}' \quad \text{(A.4.3)}$$

Where matrix M is called the "Transformation Matrix" which is the end result of the matrix multiplications that consist of **fundamental element(s) in a linear transformation**, and $\vec{T}'$ is a vector so-called "Translation Vector" expressed under the Prime system.

The fundamental elements of a 2D linear transformation are as follows:

1. T: Translation (Shift). It can be written in the vector form as in Eq. A.4.4

$$\vec{T}' = \begin{bmatrix} t'_X \\ t'_Y \end{bmatrix} \text{ or } \vec{T} = \begin{bmatrix} t_X \\ t_Y \end{bmatrix} \quad \text{(A.4.4)}$$

2. U: Uniform Scale factor. With the same scale factor value equals to "u" in all directions. It can be written in the matrix form as in Eq. A.4.5.

$$U = \begin{bmatrix} u & 0 \\ 0 & u \end{bmatrix} \quad \text{(A.4.5)}$$

3. S: Stretch (Non-uniform scale factor). With the different scale factor values in each individual directions. It can be written in the matrix form as in Eq. A.4.6.

$$S = \begin{bmatrix} u_X & 0 \\ 0 & u_Y \end{bmatrix} \quad \text{(A.4.6)}$$

4. R: Rotation. Rotates from axis X to Y in the counter-clockwise direction with angle θ. It can be written in the matrix form as in Eq. A.4.7.

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \text{(A.4.7)}$$

5. K: Skew (Shear). The skewness of the axes or the non-orthogonality of the axes. The skewness that may be expressed in terms of a small angle ω, may be written in the form of Eq. A.4.8.

$$K = \begin{bmatrix} 1 & \sin\omega \\ 0 & \cos\omega \end{bmatrix} \quad \text{(A.4.8)}$$

6. F: Reflection. For 2D linear transformation, the reflection is either about the X-axis or Y-axis. These two cases of reflection can be expressed in term of matrix as written in Eqs. A.4.9 and A.4.10 respectively.

$$F = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{(A.4.9)}$$

$$F = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{(A.4.10)}$$

In a transformation not all fundamental elements may be used. Different combinations of elements with their different values of parameters resulting in indefinite numbers of the "look" of the Transformation Matrix M.

Four-parameter Affine Transformation (2D Similarity Transformation)

It is commonly known as 2D similarity transformation. It is a linear transformation consisting of these following four transformation parameters.

1. Uniform Scale (scale factor u for both X and Y directions),
2. Rotation (rotation angle θ),
3. Translation terms (t'_X and t'_Y expressed in the Prime system under the Translation Vector $\vec{T}'$)

The four-parameter (u, θ, t'_X, and t'_Y) 2D similarity transformation can be written in the form of Eq.A.4.11.

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} u & 0 \\ 0 & u \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t'_X \\ t'_Y \end{bmatrix} \quad \text{(A.4.11)}$$

After the manipulation of the terms in Eq. A.4.11, the results can be re-written in the form of Eqs. A.4.12 and A.4.13.

$$X' = u\cos\theta(X) + u\sin\theta(Y) + t'_X \quad \text{(A.4.12)}$$

$$Y' = -u\sin\theta(X) + u\cos\theta(Y) + t'_Y \quad \text{(A.4.13)}$$

The expressions in Eqs. A.4.12 and A.4.13 can be re-parameterized with four dummy parameters (a, b, c, and d) in the form as shown in Eq. A.4.14.

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} \quad \text{(A.4.14)}$$

Where
a = $u\cos\theta$,
b = $u\sin\theta$,
c = t'_X, and
d = t'_Y.

Four dummy parameters: a, b, c, and d, as written in the form of Eq. A.4.14 may then be solved for. The relationships between the geometrically better interpretable parameters and the dummy parameters as shown in Eqs. A.4.15 through A.4.18 will be solved in order to arrive at the geometrical parameters of the transformation.

$$u = \sqrt{(a^2 + b^2)} \quad \text{(A.4.15)}$$

$$\theta = \arctan\left(\frac{b}{a}\right) \quad \text{(A.4.16)}$$

$$t'_X = c \quad \text{(A.4.17)}$$

$$t'_Y = d \quad \text{(A.4.18)}$$

Six-parameter Affine Transformation

The six-parameter linear transformation consists of the following six transformation parameters.

1. 1 Stretch which in 2D transformation is equivalent to 2 non-uniform scale factors (scale factor u_X and u_Y for separate X and Y directions),
2. 1 Rotation (rotation angle θ),
3. 1 Skew (skew angle ω),
4. 2 Translation terms (t'_X and t'_Y expressed in the Prime system under the Translation Vector T').

The six-parameter (u_X, u_Y, θ, α, t'_X, and t'_Y) 2D transformation can be written in the form of Eq. A.4.19.

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & \sin\alpha \\ 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} u_X & 0 \\ 0 & u_Y \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t'_X \\ t'_Y \end{bmatrix} \quad \text{(A.4.19)}$$

After the manipulation of terms in Eq. A.4.19, the results can be re-written in the form of Eq. A.4.20.

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} u_x\cos\theta & u_y\sin\omega\cos\theta + u_Y\cos\omega\sin\theta \\ -u_x\sin\theta & -u_y\sin\omega\sin\theta + u_Y\cos\omega\cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t'_X \\ t'_Y \end{bmatrix} \quad \text{(A.4.20)}$$

The expressions in Eq. A.4.20 can be re-parameterized with six dummy parameters (a, b, c, d, e and f) in the form as shown in Eq. A.4.21.

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \quad \text{(A.4.21)}$$

Where
$a = u_X \cos\theta$,
$b = -u_X \sin\theta$,
$c = (u_Y \sin\omega \cos\theta) + (u_Y \cos\omega \sin\theta)$,
$d = (-u_Y \sin\omega \sin\theta) + (u_Y \cos\omega \cos\theta)$,
$e = t'_X$, and
$f = t'_Y$.

Six dummy parameters: a, b, c, d, e, as written in the form of Eq. A.4.21 can then be solved for. The relationships between the geometrical parameters and the dummy parameters as shown in in Eqs. A.4.22 through A.4.27 will be solved in order to arrive at the geometrical parameters of the transformation.

$$u_X = \sqrt{(a^2 + b^2)} \quad \text{(A.4.22)}$$

$$u_Y = \sqrt{(c^2 + d^2)} \quad \text{(A.4.23)}$$

$$\theta = \arctan\left(\frac{-b}{a}\right) \quad \text{(A.4.24)}$$

$$\omega = \arctan\left(\frac{ac + bd}{cb + ad}\right) \quad \text{(A.4.25)}$$

$$t'_X = e \quad \text{(A.4.26)}$$

$$t'_Y = f \quad \text{(A.4.27)}$$

NOTE: It should be noted that both the four- and six-parameter linear transformations presented in the section above of which the transformation is in the form of Eq. A.4.3 may now referred to as "Model A":

Model A:

$$\vec{X'} = M + \vec{T'} \quad \text{(A.4.3)}$$

The transformations can also be written in the form as presented in Eq. A.4.28 what may now referred to as the "Model B" both for the four- (similarity transformation) and six-parameter Affine transformation.

Model B:

$$\vec{X'} = M\left(\vec{X} - \vec{T}\right) \quad \text{(A.4.28)}$$

Where matrix M is the Transformation Matrix, and $\vec{T}$ is a Translation Vector expressed under the Normal or Original Frame ($\vec{X}$).

In this report the four- (similarity) and six-parameter Affine transformations were used during the Mapping Check in order to evaluate how well the INSPCS83 agree with the INCRS mappings. The two systems are the INCRS coordinates (mapped coordinates under INCRS mapping) and the mapped coordinates under INSPCS83 by NGS. In the Mapping Check of this report the Affine transformations written only in the form of "Model A" wereinvestigated.

In contrast to the Mapping Check, the Reality Check process, that has been conducted in this report in order to evaluate how well the new mappings have modeled reality (the Real World), use has been made of a 3D Affine transformation (7- and 9-parameters) written in both models (Model A and Model B). The purpose of using these two different Affine transformation models is for double-checking purposes as the Affine Fitting's residuals from both models have to be identical with only differences in the final values of adjusted parameters (such as scale, shifts, and rotations).

The Affine Fitting results (statistical values of residuals) from both models (Model A and Model B) have been confirmed to be identical which warrants the internal mathematical consistency; hence only one set of results (either from Model A or Model B) was displayed in this Final Report due to the fact the statistical values of fitting residuals from both Model A and Model B are just simply identical for both sets of data.

Nevertheless, both the 7- (similarity transformation) and the 9-parameter Affine transformation applied to both models (Model A and Model B) are also described in the next section of this Appendix (A.5).

A.5 THREE-DIMENSIONAL AFFINE TRANSFORMATION

The three-dimensional Affine transformation is applied to 3D grid points' coordinates in two different frames. In general these two different systems are denoted in the form as written in Eqs. A.5.1 andA.5.2.

$$\vec{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{A.5.1}$$

$$\vec{X}' = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \tag{A.5.2}$$

Seven-parameter Affine Transformation (3D Similarity Transformation): Model A

For Model A, the seven-parameter similarity transformation is written in the form of Eq.A.4.3 where the Translation Vector is expressed in the Prime frame. The 7-parameter similarity transformation that is written in the form of Model A consists of these following 7 transformation parameters.

1. 1 Uniform Scale (scale factor u for all X, Y, and Z directions),
2. 3 Rotations (α, β, and γ) about each axis:
   a. rotation angle α about X-axis (first axis),
   b. rotation angle β about Y-axis (second axis), and
   c. rotation angle γ about Z-axis (third axis),
3. 3 Translation terms (t'_X, t'_Y, and t'_Z expressed in the Prime frame under the Translation Vector $\vec{T}'$).

The 7-parameter (u, α, β, γ, t'_X, t'_Y, and t'_Z) similarity transformation that is written in the form of Model A can be expressed in the form as shown in Eq.A.5.3.

$$\vec{X}' = URX + \vec{T}' \tag{A.5.3}$$

Where

U is the Scale Matrix. It can be expressed in the form as shown in Eq. A.5.4,

R is the Rotation Matrix which has been contributed from all 3 rotations of different directions. It may be expressed in the form, e.g., as shown in Eq. A.5.5, and $\vec{T}'$ is the Translation Vector in the Prime frame. It can be expressed in the form as shown in Eq.A.5.6.

$$U = \begin{bmatrix} u_X & 0 & 0 \\ 0 & u_Y & 0 \\ 0 & 0 & u_Z \end{bmatrix} \tag{A.5.4}$$

$$R = R_3(\gamma)R_2(\beta)R_1(\alpha) \tag{A.5.5}$$

$$\vec{T}' = \begin{bmatrix} t'_X \\ t'_Y \\ t'_Z \end{bmatrix} \tag{A.5.6}$$

Where

$R_1(\alpha)$ is the rotation with angle α about X-axis (first axis),

$R_2(\beta)$ is the rotation with angle β about Y-axis (second axis), and

$R_3(\gamma)$ is the rotation with angle γ about Z-axis (third axis).

These three rotations can be expressed in the matrix forms as shown in Eqs. A.5.7 through A.5.9.

$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \tag{A.5.7}$$

$$R_2(\beta) = \begin{bmatrix} \cos(\beta) & 0 & -\sin(\beta) \\ 0 & 1 & 0 \\ \sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \tag{A.5.8}$$

$$R_3(\gamma) = \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.5.9}$$

From Eqs. A.5.4 through A.5.9, the 7-parameter similarity transformation as written in Eq. A.4.3 in the form of Model A can be re-written in the form as shown in Eq. A.5.10.

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} u_X & 0 & 0 \\ 0 & u_Y & 0 \\ 0 & 0 & u_Z \end{bmatrix} R_3(\gamma)R_2(\beta)R_1(\alpha) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t'_X \\ t'_Y \\ t'_Z \end{bmatrix} \tag{A.5.10}$$

Seven-parameter Affine Transformation (3D Similarity Transformation): Model B

For Model B, the seven-parameter similarity transformation is written in the form as shown in Eq. A.4.28 where the Translation Vector is expressed in the Normal or Original frame. The 7-parameter similarity transformation in the form of Model B consists of the following 7 transformation parameters.

1. 1 Uniform Scale (scale factor u for all X and Y directions),
2. 3 Rotations (α, β, and γ) about each axis:
   a. rotation angle α about X-axis (first axis),
   b. rotation angle β about Y-axis, (second axis),and
   c. rotation angle γ about Z-axis (third axis),
3. 3 Translation terms (t_X, t_Y, and t_Z expressed in the original frame under the Translation Vector $\vec{T}$).

The 7-parameter (u, α, β, γ, t_X, t_Y, and t_Z) similarity transformation that is written in the form of Model B can be expressed in a similar way as what has been applied in the case of Model A. The clear difference is that in Model B the Translation Vector is expressed in the original frame and not in the Prime frame. The Translation Vector of the 7-parameter similarity transformation in Model B can be written in the form as shown in Eq.A.5.11.

$$\vec{T} = \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix} \tag{A.5.11}$$

The remaining transformation's parameters (rotations and scales) are expressed in the same forms as they were written in Eqs. A.5.4, A.5.5, and Eqs. A.5.7 through A.5.9 that contribute to the "look" of 7-parameter similarity transformation which has been written in the form of Model B which can be re-written in the form of Eq. A.5.12.

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} u_X & 0 & 0 \\ 0 & u_Y & 0 \\ 0 & 0 & u_Z \end{bmatrix} R_3(\gamma)R_2(\beta)R_1(\alpha)\left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix}\right) \quad (A.5.12)$$

Nine-parameter Affine Transformation: Model A

For Model A, the nine-parameter Affine transformation is written in the form of Eq. A.4.3 where the Translation Vector is expressed in the Prime frame. Nine-parameter Affine transformation in the form of Model A consists of the following 9 transformation parameters.

1. 1 Stretch which in 3D transformation is equivalent to 3 non-uniform scale factors (scale factor u_X, u_Y, and u_Z for separate X, Y, and Z directions),
2. 3 Rotations (α, β, and γ) about each axis:

   a. rotation angle α about X-axis (first axis),
   b. rotation angle β about Y-axis (second axis), and
   c. rotation angle γ about Z-axis (third axis),

3. 3 Translation terms (t'_X, t'_Y, and t'_Z expressed in the Prime system).

The 9-parameter (u_X, u_Y, u_Z, α, β, γ, t'_X, t'_Y, and t'_Z) Affine transformation in the form of Model A can be expressed in the form as shown in Eq. A.5.13.

$$\vec{X}' = SRX + \vec{T}' \quad (A.5.13)$$

Where
S is the Stretch Matrix (non-uniform Scale Matrix). It can be expressed in the form as shown in Eq.A.5.14,
R is the Rotation Matrix which has been contributed from all three rotations of different directions.
The matrix form of R is the same as it was written in Eq. A.5.5 as well as its separate rotation elements as were written in Eqs. A.5.7 through A.5.9, and $\vec{T}'$ is the Translation Vector in the Prime frame which has already been shown in Eq. A.5.6.

$$S = \begin{bmatrix} u_X & 0 & 0 \\ 0 & u_Y & 0 \\ 0 & 0 & u_Z \end{bmatrix} \quad (A.5.14)$$

The "look" of 9-parameter Affine transformation which has been written in the form of Model A as shown in Eq. A.4.13 can then be re-written in the form as shown in Eq. A.5.15.

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} u_X & 0 & 0 \\ 0 & u_Y & 0 \\ 0 & 0 & u_Z \end{bmatrix} R_3(\gamma)R_2(\beta)R_1(\alpha) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t'_X \\ t'_Y \\ t'_Z \end{bmatrix} \quad (A.5.15)$$

Nine-parameter Affine Transformation: Model B

The nine-parameter Affine transformation in the form of Model B consists of the following 9 transformation parameters.

1. 1 Stretch which in 3D transformation is equivalent to 3 non-uniform scale factors (scale factor u_X, u_Y, and u_Z for separate X, Y, and Z directions),
2. 3 Rotations (α, β, and γ) about each axis:

   a. rotation angle α about X-axis (first axis),
   b. rotation angle β about Y-axis (second axis), and
   c. rotation angle γ about Z-axis (third axis).

3. 3 Translation terms (t_X, t_Y, and t_Z expressed in the Normal system under the Translation Vector $\vec{T}$).

For Model B, the 9-parameter Affine transformation is written in the form as shown in Eq. A.5.16.

$$\vec{X}' = SR(\vec{X} - \vec{T}) \quad (A.5.16)$$

Where
T is the Translation Vector in the Normal or Original frame which has already been shown in Eq. A.5.11.
The remaining parameters in Eq. A.5.16 are expressed in the same as in the case of Model A, that is the S Matrix as it was expressed in Eq. A.5.14 and the rotation matrix R as it was expressed by the Eq. A.5.5, with has been expanded to Eqs. A.5.7 through A.5.9.
The "look" of 9-parameter Affine transformation which has been written in the form of Model B as shown in Eq. A.5.16 can then be re-written in the form as shown in Eq. A.5.17.

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} u_X & 0 & 0 \\ 0 & u_Y & 0 \\ 0 & 0 & u_Z \end{bmatrix} R_3(\gamma)R_2(\beta)R_1(\alpha)\left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix}\right) \quad (A.5.17)$$

A.6 MORAN'S INDEX OF SPATIAL AUTOCORRELATION

In this research report the spatial autocorrelation of the ellipsoidal heights, as considered in small geographic areas, has been investigated. The details of this analysis have already extensively been discussed in Chapter 3, section 3.3.2. The Moran's Index developed by Patrick A.P. Moran (*6*) is used as the index of the spatial autocorrelation of the heights in the Test Areas investigated in this research study. In this section, the mathematical details are given of how the spatial autocorrelation of the observations in term of Moran's Index is computed. In our case the Moran's Index of the ellipsoidal heights of the terrain in the Test Areas (counties) are discussed.

The explanations will be given in a form of an example of the Moran's Index computation of an area with 3 points by 4 points grid size.

In the example it is assumed that the heights of 12 points (in the form of 3 by 4 points grid, see Figure A.6.1) are given. The so-called "h-grid" is put in the form of a 3 by 4 matrix that stores the values of heights at each grid point. It should be noted that the geographic relationship between the grid points is already embedded in the matrix. As an example the point that is displayed as position (1,2) in the grid is actually located (in the Real World) on the east side of point with position (1,1) and right to the north of grid point with position (2,2).

The Moran's Index (I) of heights in Marion County, or for that matter in any county, can be computed from the formula as given in Eq. A.6.1.

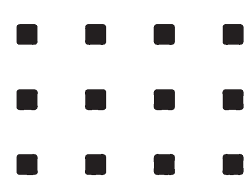

Figure A.6.1 Example of a 3 by 4 point grid.

$$I=\left(\frac{N}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}}\right)\frac{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}\left(h_i-\bar{h}\right)\left(h_j-\bar{h}\right)}{\sum_{i=1}^{N}\left(h_i-\bar{h}\right)^2} \qquad (A.6.1)$$

Where

N is the total number of points (for the grid example above, N = 12),

W is the Weight Matrix with dimension of N by N (for this example, [W] = 12 x 12), and

w_{ij} is the element of Weight Matrix at the i^{th} row and the j^{th} column.

The w_{ij} is the weight value that defines the relationship between point i^{th} and point j^{th}; hence the Weight Matrix has the dimension of N by N as at each single point the relationship between that point and the rest of the points (including itself) are to be tabulated in Weight matrix form.

The steps of defining Weight Matrix are as follows:

1. Assignment of points' indices is running from 1 to N (in this case is running from 1 to 12). The points can be indexed in any which way with no specific pattern as long as the position of them in the spatial domain (geographic extent) can be tracked. In case of this example, the points have been indexed in the way as shown in Figure A.6.2 where "P1" is the point with ID number = 1, "P2" is the point with ID number = 2, and so on.
2. The rule of defining the weight value (w_{ij}) is that w_{ij} is assigned to be equal to 1 if "Pj" (point with ID = j) is adjacent to "Pi" (point with ID = i); otherwise w_{ij} is assigned to be equal to 0. In this case a point is judged to be adjacent only if it is next to the considered point in one of these four different manners: left/right or east/west side, and over/beneath or north/south side. Points that are separated in a diagonal direction have also been given a zero weight. With this rule the Weight Matrix of this grid example gets the form as shown in Eq. A.6.2.

$$W=\begin{bmatrix} 0&1&0&0&1&0&0&0&0&0&0&0\\ 1&0&1&0&0&1&0&0&0&0&0&0\\ 0&1&0&1&0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&1&0&0&0&0\\ 1&0&0&0&0&1&0&0&1&0&0&0\\ 0&1&0&0&1&0&1&0&0&1&0&0\\ 0&0&1&0&0&1&0&1&0&0&1&0\\ 0&0&0&1&0&0&1&0&0&0&0&1\\ 0&0&0&0&1&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0&1&0&1&0\\ 0&0&0&0&0&0&1&0&0&1&0&1\\ 0&0&0&0&0&0&0&1&0&0&1&0 \end{bmatrix} \qquad (A.6.2)$$

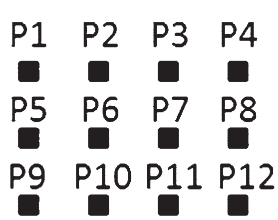

Figure A.6.2 Example of grid points with their corresponding assigned ID's.

With the assigned w_{ij} values at every element of the Weight Matrix the Moran's Index of spatial autocorrelation of heights can be computed from the formula as written in Eq. A.6.1. In the case that the heights at each grid point are considered, the end result is one single value of the Moran's Index that characterizes the spatial autocorrelation of the heights in terms of the terrain undulations.

A.7 LEAST SQUARES ADJUSTMENT

In this study during the evaluation of the mapped results in either Mapping Check process or Reality Check process, the Affine Fitting is used with varying numbers of parameters (4- and 6-parameter in the Mapping Check process and 7- and 9-parameter in the Reality Check process). The Affine transformations between two systems which in this study are (1) the mapped results in terms of Easting and Northing coordinates and (2) the reference coordinates which varies in different tests, in the Mapping Check the reference coordinates are the mapped coordinates under INSPCS83 by NGS whereas for the Reality Check the reference coordinates are the 3D undistorted points in the Real World. It should be noted that for the case of the Mapping Check the Affine transformation of use is the 2D Affine transformation whereas for the Reality Check which the mapped coordinates are compared against the 3D reality, the 3D Affine transformation is of use instead.

Here in this section, the Affine Fitting through the Least Square (LSQ) Adjustment model will be discussed. It should be noted that the discussion of LSQ adjustment here will be in the scope of only how the final fitting residuals are achieved based on the presented formulae. The complete descriptions of Least Squares Adjustments with the mathematical proofs are not within the scope of this section.

In this study the Least Squares Adjustment model in the form of the so-called "Observation Equation Model" is exercised. The model is written in the vector form as shown in Eq. A.7.1.

$$L_a=F(X_a) \qquad (A.7.1)$$

Where

L_a is the adjusted observations written in the vector form, and

X_a is the adjusted parameters.

Eq. A.7.1 is understood in terms of the expression as shown in Eq. A.7.2.

$$L_b+V=F(X_0+X) \qquad (A.7.2)$$

Where

L_b is the (original/raw) observations written in vector form,

V is the residuals of the LSQ process,

X_0 is the approximated values of the parameters (initially guessed parameter values), and

X is the corrections to parameters, see Eq. A.7.3.

$$X=X_a-X_0 \qquad (A.7.3)$$

It should be noted that in this study the observations are the mapped coordinates under new mapping system and the parameters are the transformation parameters that link the observations to the reference coordinates through the transformation (4- or 6-parameter transformation for the Mapping Check and 7- or 9-parameter transformation for the Reality Check). Through the LSQ process the parameters values are solved for as well as the residuals. The size of the residuals indicates how close the observations (mapped results) are to the reference coordinates (NGS's INSPCS83 coordinates/3D reality).

With the observations and the reference coordinates written in the form of Affine transformation (see Eqs. A.4.3 and A.4.28) the LSQ process can be starts in steps as presented in the order of following equations starting with linearized version (see Eq. A.7.4) of the (non-linear) original form in Eq. A.7.1.

$$L_b + V = F(X_0) + \left(\frac{\partial F}{\partial X}\right) X \tag{A.7.4}$$

Eq. A.7.4 is equivalent to Eqs. A.7.5 and A.7.6.

$$L_b + V = L_0 + AX \tag{A.7.5}$$

$$L + V = AX \tag{A.7.6}$$

Where

$$L_0 = F(X_0) \tag{A.7.7}$$

$$A = \left(\frac{\partial F}{\partial A}\right) \tag{A.7.8}$$

$$L = L_b - L_0 \tag{A.7.9}$$

Where

L is the residual or linearized observations,

L_0 is the approximated observations (computed from X_0), and

A is the design or Jacobian matrix (partial derivative matrix ∂observations/∂parameters).

The LSQ solutions are written in the formulae as shown in the following equations which finally lead to the solved values of parameters and residuals in the iteration manner till difference between the correction values to the parameters (X) between iterations do not exceed a predefined threshold value.

$$X = (A^t PA)^{-1} (A^t PL) \tag{A.7.10}$$

Where

P is the weight matrix of observations.

X_a gets adjusted through iterations by the newly computed corrections (X, see Eq. A.7.10) from the latest iteration as demonstrated in the form as shown in Eq. A.7.11.

$$X_{a,it+1} = X_{0,it+1} + X_{it+1} \text{ with } X_{0,it+1} = X_{a,it} \tag{A.7.11}$$

Where

"it" denoted the iteration (1, 2, 3,....).

At the last iteration (when the criterion to stop iteration is satisfied) the latest updated values of X_a are the final adjusted parameters. The residuals (V) can be solved from the final adjusted parameters (X_a) by the relationship as written in Eq. A.7.12.

$$V = L_a - L_b = F(X_a) - L_b \tag{A.7.12}$$

It should be noted that in this study, the Affine Fitting residuals (V) computed from the steps as mentioned above are used as the indicators on how close the observations are to the reference values (reference coordinates). In this case the observations are the mapped coordinates under the new mapping systems. The reference values are the mapped coordinates under the INSPCS83 by NGS during the Mapping Check process and the 3D points in the Real World in case of the Reality Check process.

A.8 EVALUATION OF THE O-C DIFFERENCES DURING THE REALITY CHECK

The research deals with one main question: is there room for improvement if one conformally maps 3D reality into a 2D+1D mapped world, or symbolically: $(x, y, z) \rightarrow (X_M, Y_M, h_v)$?

During this research the undistorted 3D (real) (x, y, z) point cloud is compared to the 3D (actually 2D+1D) version of mapped coordinates (X_M, Y_M, h_v) point cloud in general or more precisely (E, N, h_v). In our case the generic mapped coordinates (X_M, Y_M) are in the form of Easting and Northing coordinates (E, N). The third element (h_v) of each point represents the height of that point with respect to the newly adopted mapping reference surface which is understood in terms of the height variations with respect to the mapping reference surface level (h_{avg}) (see Eq. 4.1 in section 4.3 of Chapter 4).

Comparing the mapped coordinates against the 3D reality, it is obvious that the mapping distorts the geometry of the 3D (x, y, z) point cloud somewhat. In a least squares sense, one compares the original point cloud $\vec{X}$: (x, y, z) to the primed (mapped) point cloud $\vec{X}'$: (X_M, Y_M, H_v). This immediately evokes the same 6- or 7 similarity transformation model, e.g., Model B for evaluation, see Eq. A.8.1.

$$\vec{X}' = \sigma R(\vec{X} - \vec{T}) \tag{A.8.1}$$

In Eq. A.8.1 $\vec{X}$: (x, y, z) plays the role of the original or the reference values (not to be altered!) 3D point cloud in the Real World, and $\vec{X}'$: (X_M, Y_M, h_v) represents the (somewhat) distorted mapped point cloud. The original vector $\vec{X}$ consists of the independent undistorted variables (i.e., the point cloud (x, y, z).The $\vec{X}'$ vector represents the stochastic (somewhat distorted) variables, the mapped point cloud (X_M, Y_M, h_v). The $\vec{X}'$ are subject to improvement. In a least squares sense they are the observations vector L_b (more details of LSQ process can be found in section A.7) that need improvement by the residuals V (in order to make the raw observations fit the model).

The Least Squares Model of this problem can be written in the form of Eq. A.8.2.

$$L_a = L_b + V = F(X_a) = \vec{X}' + V = \sigma R(\vec{X} - \vec{T}) \tag{A.8.2}$$

With X_a in this case is the vector that contains the six or seven (similarity transformation) or nine (affine transformation) parameters.

In the linearization process of the non-linear model as written in Eq. A.8.1 one arrives at the linearized form as shown in Eq. A.7.4 which equivalent to the ones of Eqs. A.7.5 and A.7.6.

The relationship of the approximated observations (L_0) and the original observation (L_b) as was written in the Eq. A.7.9 ($L = L_b - L_0$) is directly related to the O-C Difference process during the Reality Check of the mapped results.

The relationship of the O-C Differencing and the LSQ model is shown in Eq. A.8.3.

$$L = L_b - L_0 = O - C \tag{A.8.3}$$

According to Eq. A.8.3, in the O-C Differencing process the original/raw observations (L_b) are denoted as "Observed (O)" and the approximate observations (L_0) that have been computed from the initial approximate parameters (($L_0 = F(X_0)$) is denoted as "Calculated (C)."

The mathematical idea behind the O-C Difference as presented in Eq. A.8.3 form the basis of the discussion that follows in the following paragraphs.

The matrix R as shown in Eq. A.8.2 is in principle an arbitrary rotation matrix that consists of three rotations. This triplet of rotations may have a variety of forms, e.g., $R = R_1 * R_2 * R_3$, or $R = R_3 * R_1 * R_3$, etc.

If one makes smart choices for the rotation matrices R, i.e., select the initial rotation parameters in terms of X_0 in the LSQ process, one realizes that the L_0 (= $F(X_0)$) vector in the LSQ process attains a very special meaning during the first iteration: the L_0 vector represents nothing else than the Cartesian Topocentric coordinates (e, n, u) of the original 3D point cloud. This means that the L vector contains the differences between the

"Observations O or L_b," i.e., the mapped point cloud (X_M, Y_M, h_v), and the original undistorted point cloud L_0, however expressed in the local topocentric frame (e, n, u) centered in point CP (center of project). This means that the L vector is simply a direct evaluation of the (somewhat) distorting mapping process during the first iteration in the Least Squares estimation process.

In our study the choice for the rotation matrices (forming in terms of X_0 in the LSQ process) are in the form as written in Eq. A.8.4.

$$R = R_3(f(Az)) * R_1(f(\varphi)) * R_3(f(\lambda)) \tag{A.8.4}$$

Where

$R_3(f(Az))$ is an azimuth rotation around the normal to the mapping reference surface (see Eq. A.8.5).

$$R_3(f(Az)) = R_3(0^\circ + \delta Az) \tag{A.8.5}$$

$R_1(f(\varphi))$ is a latitude related rotation. The actual argument of rotation is equal to $90^\circ - \varphi + \delta\varphi$ (see Eq. A.8.6).

$$R_1(f(\varphi)) = R_1(90^\circ - \varphi + \delta\varphi) \tag{A.8.6}$$

$R_3(f(\lambda))$ is a longitude related rotation around the original third axis, the z-axis. The argument of rotation is equal to $90^\circ + \lambda + \delta\lambda$ (see Eq. A.8.7).

$$R_3(f(\lambda)) = R_3(90^\circ + \lambda + \delta\lambda) \tag{A.8.7}$$

During the Marion County test (Chapter 6) the L_b vector contains the relative coordinates of the mapped point cloud which known in generic terms as (X_M-X_{CP}, Y_M-Y_{CP}, h_v) or in this case are (E-E_{CP}, N-N_{CP}, h_v). The fact that the Lvector reflects directly the mapped distortions is revealed in Eqs. A.8.8 through A.8.10.

$$L_E = L_{b,E} - L_{0,e} = (E - E_{CP}) - e \tag{A.8.8}$$

$$L_N = L_{b,N} - L_{0,n} = (N - N_{CP}) - n \tag{A.8.9}$$

$$L_{h_v} = L_{b,h_v} - L_{0,u} = h_v - u \tag{A.8.10}$$

As expected the least squares analysis of the 19x19 grid in Marion County finds that the rotational similarity transformation parameters δAz and $\delta\lambda$ are both equal to zero, even after several iterations during the Reality Check. The third angle $\delta\varphi$ is a very small angle indeed (not equal to zero), reflecting the fact that the ellipsoidal normal in the central point CP does not coincide with the normal to the (E, N) mapping plane due to the slight (non-symmetrical!) N-S changes in the radius of curvature M.

APPENDIX B. BASIC INFORMATION FOR ALL 92 TEST AREAS IN INDIANA

This section presents the basic information of all 92 Test Areas (counties) that have been used in the different tests in this feasibility study. The basic information are the names, the abbreviations, the reference codes, the geodetic boundary extents of the Test Areas (counties) as well as the computed geometric centers (the center project points, or CP's) of the Test Areas. It also includes the information about the number of points in the grid of each Test Area with its corresponding number of points in the two directions of longitude and latitude. Table B.1 presents the reference information of the Test Areas (names, abbreviations, and codes) and the geodetic coordinates (with respect to the NAD83 datum) of the boundary extents of each of the Test Areas. In Table B.2, along with reference information of each Test Area, the geodetic coordinates (with respect to the NAD83 datum) of point CP of each Test Area are tabulated. Table B.2 also presents the numbers of points in total and in both directions (longitude and latitude) of the Test Areas. The deviations of the INCRS Sphere from the GRS80 ellipsoid of the NAD83 datum for each Test Area (county) are tabulated in Table B.3.

TABLE B.1
Geodetic coordinates (NAD83) of the boundaries of all 92 Test Areas (counties) in Indiana

			West - East Boundary						South - North Boundary					
		IN	From Longitude (West)			To Longitude (West)			From Latitude (North)			To Latitude (North)		
County Name	County Abbrev.	County Code	deg.	min.	sec.	deg.	min.	sec.	deg.	min.	sec.	deg.	min.	sec.
Adams	A	01	85	04	00.00000	84	48	00.00000	40	34	00.00000	40	55	00.00000
Allen	Al	02	85	20	00.00000	84	48	00.00000	40	55	00.00000	41	16	00.00000
Bartholomew	B	03	86	05	00.00000	85	41	00.00000	39	02	00.00000	39	21	00.00000
Benton	Bn	04	87	32	40.00000	87	06	00.00000	40	28	00.00000	40	44	00.00000
Blackford	Bl	05	85	28	00.00000	85	12	00.00000	40	23	00.00000	40	34	00.00000
Boone	Bo	06	86	43	00.00000	86	15	00.00000	39	56	00.00000	40	11	00.00000
Brown	Br	07	86	23	40.00000	86	05	00.00000	39	02	00.00000	39	21	00.00000
Carroll	C	08	86	46	00.00000	86	22	00.00000	40	26	00.00000	40	44	00.00000
Cass	Ca	09	86	35	20.00000	86	10	00.00000	40	34	00.00000	40	55	00.00000
Clark	Cl	10	86	01	00.00000	85	25	00.00000	38	16	00.00000	38	36	00.00000
Clay	Cy	11	87	14	20.00000	86	57	00.00000	39	10	00.00000	39	37	00.00000
Clinton	Cn	12	86	43	00.00000	86	15	00.00000	40	11	00.00000	40	26	00.00000
Crawford	Cr	13	86	41	40.00000	86	15	00.00000	38	06	00.00000	38	25	00.00000
Daviess	Da	14	87	18	00.00000	86	54	00.00000	38	30	00.00000	38	55	00.00000
Dearborn	D	15	85	09	00.00000	84	49	00.00000	38	56	00.00000	39	19	00.00000
Decatur	De	16	85	42	00.00000	85	18	00.00000	39	08	00.00000	39	27	00.00000
DeKalb	Dk	17	85	12	00.00000	84	48	00.00000	41	16	00.00000	41	32	00.00000
Delaware	Dl	18	85	35	40.00000	85	13	00.00000	40	05	00.00000	40	23	00.00000
Dubois	Du	19	87	05	00.00000	86	41	00.00000	38	12	00.00000	38	32	00.00000
Elkhart	E	20	86	03	00.00000	85	39	00.00000	41	26	00.00000	41	46	00.00000
Fayette	F	21	85	18	00.00000	85	02	00.00000	39	31	00.00000	39	47	00.00000
Floyd	Fl	22	86	02	00.00000	85	46	00.00000	38	11	00.00000	38	25	00.00000
Fountain	Fo	23	87	26	00.00000	87	06	00.00000	39	57	00.00000	40	22	00.00000
Franklin	Fr	24	85	18	20.00000	84	49	00.00000	39	16	00.00000	39	32	00.00000
Fulton	Fu	25	86	29	00.00000	85	57	00.00000	40	55	00.00000	41	10	00.00000
Gibson	Gi	26	87	59	00.00000	87	19	00.00000	38	10	00.00000	38	32	00.00000
Grant	G	27	85	52	20.00000	85	27	00.00000	40	23	00.00000	40	39	00.00000
Greene	Gr	28	87	14	20.00000	86	41	00.00000	38	54	00.00000	39	10	00.00000
Hamilton	H	29	86	15	00.00000	85	51	00.00000	39	56	00.00000	40	13	00.00000
Hancock	Ha	30	85	57	20.00000	85	36	00.00000	39	42	00.00000	39	57	00.00000
Harrison	Hr	31	86	20	40.00000	85	54	00.00000	37	57	00.00000	38	25	00.00000
Hendricks	He	32	86	42	40.00000	86	20	00.00000	39	36	00.00000	39	56	00.00000
Henry	Hn	33	85	36	00.00000	85	12	00.00000	39	47	00.00000	40	05	00.00000
Howard	Ho	34	86	22	40.00000	85	52	00.00000	40	22	00.00000	40	34	00.00000
Huntington	Hu	35	85	40	00.00000	85	20	00.00000	40	39	00.00000	41	00	00.00000
Jackson	J	36	86	17	20.00000	85	48	00.00000	38	44	00.00000	39	03	00.00000
Jasper	Js	37	87	17	20.00000	86	56	00.00000	40	44	00.00000	41	17	00.00000
Jay	Ja	38	85	13	20.00000	84	48	00.00000	40	19	00.00000	40	34	00.00000
Jefferson	Je	39	85	41	20.00000	85	12	00.00000	38	35	00.00000	38	55	00.00000
Jennings	Jn	40	85	48	20.00000	85	27	00.00000	38	49	00.00000	39	12	00.00000
Johnson	Jo	41	86	15	40.00000	85	57	00.00000	39	21	00.00000	39	38	00.00000
Knox	K	42	87	46	00.00000	87	06	00.00000	38	25	00.00000	38	55	00.00000
Kosciusko	Ko	43	86	05	40.00000	85	39	00.00000	41	03	00.00000	41	26	00.00000
Lagrange	L	44	85	40	00.00000	85	12	00.00000	41	32	00.00000	41	46	00.00000
Lake	La	45	87	33	00.00000	87	13	00.00000	41	10	00.00000	41	42	00.00000
LaPorte	Le	46	86	57	00.00000	86	29	00.00000	41	14	00.00000	41	46	00.00000

TABLE B.1
(Continued)

County Name	County Abbrev.	IN County Code	West - East Boundary						South - North Boundary					
			From Longitude (West)			To Longitude (West)			From Latitude (North)			To Latitude (North)		
			deg.	min.	sec.	deg.	min.	sec.	deg.	min.	sec.	deg.	min.	sec.
Lawrence	Lr	47	86	41	00.00000	86	17	00.00000	38	41	00.00000	39	00	00.00000
Madison	M	48	85	52	20.00000	85	35	00.00000	39	57	00.00000	40	23	00.00000
Marion	Ma	49	86	21	00.00000	85	57	00.00000	39	38	00.00000	39	56	00.00000
Marshall	Mr	50	86	28	20.00000	86	03	00.00000	41	10	00.00000	41	29	00.00000
Martin	Mn	51	86	55	40.00000	86	41	00.00000	38	30	00.00000	38	54	00.00000
Miami	Mi	52	86	10	40.00000	85	52	00.00000	40	34	00.00000	41	00	00.00000
Monroe	Mo	53	86	41	40.00000	86	19	00.00000	39	00	00.00000	39	21	00.00000
Montgomery	My	54	87	06	00.00000	86	42	00.00000	39	52	00.00000	40	13	00.00000
Morgan	Mg	55	86	41	40.00000	86	15	00.00000	39	21	00.00000	39	38	00.00000
Newton	N	56	87	32	00.00000	87	16	00.00000	40	44	00.00000	41	14	00.00000
Noble	No	57	85	40	00.00000	85	12	00.00000	41	16	00.00000	41	32	00.00000
Ohio	O	58	85	08	40.00000	84	50	00.00000	38	54	00.00000	39	02	00.00000
Orange	Or	59	86	42	00.00000	86	18	00.00000	38	24	00.00000	38	41	00.00000
Owen	Ow	60	87	03	20.00000	86	38	00.00000	39	10	00.00000	39	28	00.00000
Parke	P	61	87	26	20.00000	87	01	00.00000	39	37	00.00000	39	57	00.00000
Perry	Pe	62	86	50	00.00000	86	26	00.00000	37	50	00.00000	38	16	00.00000
Pike	Pi	63	87	28	00.00000	87	04	00.00000	38	14	00.00000	38	33	00.00000
Porter	Pr	64	87	13	20.00000	86	56	00.00000	41	14	00.00000	41	43	00.00000
Posey	Po	65	88	06	20.00000	87	41	00.00000	37	46	00.00000	38	14	00.00000
Pulaski	Pl	66	86	56	00.00000	86	28	00.00000	40	55	00.00000	41	10	00.00000
Putnam	Pm	67	87	02	00.00000	86	38	00.00000	39	28	00.00000	39	52	00.00000
Randolph	R	68	85	13	00.00000	84	49	00.00000	40	00	00.00000	40	19	00.00000
Ripley	Ri	69	85	28	00.00000	85	04	00.00000	38	55	00.00000	39	19	00.00000
Rush	Ru	70	85	38	00.00000	85	18	00.00000	39	27	00.00000	39	47	00.00000
St. Joseph	Sj	71	86	31	00.00000	86	03	00.00000	41	26	00.00000	41	46	00.00000
Scott	S	72	85	54	00.00000	85	34	00.00000	38	34	00.00000	38	50	00.00000
Shelby	Sh	73	85	58	00.00000	85	38	00.00000	39	21	00.00000	39	42	00.00000
Spencer	Sp	74	87	16	40.00000	86	46	00.00000	37	47	00.00000	38	12	00.00000
Starke	St	75	86	56	00.00000	86	28	00.00000	41	10	00.00000	41	26	00.00000
Steuben	Sn	76	85	12	00.00000	84	48	00.00000	41	32	00.00000	41	46	00.00000
Sullivan	Su	77	87	40	40.00000	87	14	00.00000	38	54	00.00000	39	16	00.00000
Switzerland	Sw	78	85	12	20.00000	84	47	00.00000	38	42	00.00000	38	56	00.00000
Tippecanoe	T	79	87	06	00.00000	86	42	00.00000	40	13	00.00000	40	34	00.00000
Tipton	Ti	80	86	16	00.00000	85	52	00.00000	40	13	00.00000	40	25	00.00000
Union	U	81	85	02	20.00000	84	49	00.00000	39	31	00.00000	39	44	00.00000
Vanderburgh	Vg	82	87	41	40.00000	87	27	00.00000	37	50	00.00000	38	10	00.00000
Vermillion	Ve	83	87	32	20.00000	87	23	00.00000	39	37	00.00000	40	09	00.00000
Vigo	Vi	84	87	38	20.00000	87	13	00.00000	39	16	00.00000	39	37	00.00000
Wabash	Wb	85	85	57	40.00000	85	39	00.00000	40	39	00.00000	41	03	00.00000
Warren	Wa	86	87	32	40.00000	87	06	00.00000	40	09	00.00000	40	28	00.00000
Warrick	W	87	87	33	00.00000	87	01	00.00000	37	52	00.00000	38	14	00.00000
Washington	Ws	88	86	19	00.00000	85	51	00.00000	38	25	00.00000	38	47	00.00000
Wayne	Wy	89	85	13	00.00000	84	49	00.00000	39	43	00.00000	40	00	00.00000
Wells	We	90	85	28	00.00000	85	04	00.00000	40	34	00.00000	40	55	00.00000
White	Wh	91	87	07	00.00000	86	35	00.00000	40	34	00.00000	40	55	00.00000
Whitley	Wi	92	85	41	40.00000	85	19	00.00000	41	00	00.00000	41	18	00.00000

TABLE B.2
Geodetic coordinates (NAD83) of the centers of project (point CP's) of all 92 Test Areas (counties) in Indiana

			Geodetic coordinates of the centers of project (point CP)						Number of points		
			Longitude (West)			Latitude (North)			Total	In the λ	In the φ
County Name	County Abbrev.	IN County Code	deg.	min.	sec.	deg.	min.	sec.		direction	direction
Adams	A	01	84	56	00.00000	40	44	30.00000	286	13	22
Allen	Al	02	85	04	00.00000	41	05	30.00000	550	25	22
Bartholomew	B	03	85	53	00.00000	39	11	30.00000	380	19	20
Benton	Bn	04	87	19	20.00000	40	36	00.00000	357	21	17
Blackford	Bl	05	85	20	00.00000	40	28	30.00000	156	13	12
Boone	Bo	06	86	29	00.00000	40	03	30.00000	352	22	16
Brown	Br	07	86	14	20.00000	39	11	30.00000	300	15	20
Carroll	C	08	86	34	00.00000	40	35	00.00000	361	19	19
Cass	Ca	09	86	22	40.00000	40	44	30.00000	440	20	22
Clark	Cl	10	85	43	00.00000	38	26	00.00000	588	28	21
Clay	Cy	11	87	05	40.00000	39	23	30.00000	392	14	28
Clinton	Cn	12	86	29	00.00000	40	18	30.00000	352	22	16
Crawford	Cr	13	86	28	20.00000	38	15	30.00000	420	21	20
Daviess	Da	14	87	06	00.00000	38	42	30.00000	494	19	26
Dearborn	D	15	84	59	00.00000	39	07	30.00000	384	16	24
Decatur	De	16	85	30	00.00000	39	17	30.00000	380	19	20
DeKalb	Dk	17	85	00	00.00000	41	24	00.00000	323	19	17
Delaware	Dl	18	85	24	20.00000	40	14	00.00000	342	18	19
Dubois	Du	19	86	53	00.00000	38	22	00.00000	399	19	21
Elkhart	E	20	85	51	00.00000	41	36	00.00000	399	19	21
Fayette	F	21	85	10	00.00000	39	39	00.00000	221	13	17
Floyd	Fl	22	85	54	00.00000	38	18	00.00000	195	13	15
Fountain	Fo	23	87	16	00.00000	40	09	30.00000	416	16	26
Franklin	Fr	24	85	03	40.00000	39	24	00.00000	391	23	17
Fulton	Fu	25	86	13	00.00000	41	02	30.00000	400	25	16
Gibson	Gi	26	87	39	00.00000	38	21	00.00000	713	31	23
Grant	G	27	85	39	40.00000	40	31	00.00000	340	20	17
Greene	Gr	28	86	57	40.00000	39	02	00.00000	442	26	17
Hamilton	H	29	86	03	00.00000	40	04	30.00000	342	19	18
Hancock	Ha	30	85	46	40.00000	39	49	30.00000	272	17	16
Harrison	Hr	31	86	07	20.00000	38	11	00.00000	609	21	29
Hendricks	He	32	86	31	20.00000	39	46	00.00000	378	18	21
Henry	Hn	33	85	24	00.00000	39	56	00.00000	361	19	19
Howard	Ho	34	86	07	20.00000	40	28	00.00000	312	24	13
Huntington	Hu	35	85	30	00.00000	40	49	30.00000	352	16	22
Jackson	J	36	86	02	40.00000	38	53	30.00000	460	23	20
Jasper	Js	37	87	06	40.00000	41	00	30.00000	578	17	34
Jay	Ja	38	85	00	40.00000	40	26	30.00000	320	20	16
Jefferson	Je	39	85	26	40.00000	38	45	00.00000	483	23	21
Jennings	Jn	40	85	37	40.00000	39	00	30.00000	408	17	24
Johnson	Jo	41	86	06	20.00000	39	29	30.00000	270	15	18
Knox	K	42	87	26	00.00000	38	40	00.00000	961	31	31
Kosciusko	Ko	43	85	52	20.00000	41	14	30.00000	504	21	24
Lagrange	L	44	85	26	00.00000	41	39	00.00000	330	22	15
Lake	La	45	87	23	00.00000	41	26	00.00000	528	16	33
LaPorte	Le	46	86	43	00.00000	41	30	00.00000	726	22	33
Lawrence	Lr	47	86	29	00.00000	38	50	30.00000	380	19	20
Madison	M	48	85	43	40.00000	40	10	00.00000	378	14	27
Marion	Ma	49	86	09	00.00000	39	47	00.00000	361	19	19
Marshall	Mr	50	86	15	40.00000	41	19	30.00000	400	20	20
Martin	Mn	51	86	48	20.00000	38	42	00.00000	300	12	25
Miami	Mi	52	86	01	20.00000	40	47	00.00000	405	15	27
Monroe	Mo	53	86	30	20.00000	39	10	30.00000	396	18	22
Montgomery	My	54	86	54	00.00000	40	02	30.00000	418	19	22
Morgan	Mg	55	86	28	20.00000	39	29	30.00000	378	21	18
Newton	N	56	87	24	00.00000	40	59	00.00000	403	13	31
Noble	No	57	85	26	00.00000	41	24	00.00000	374	22	17
Ohio	O	58	84	59	20.00000	38	58	00.00000	135	15	9
Orange	Or	59	86	30	00.00000	38	32	30.00000	342	19	18

TABLE B.2
(Continued)

County Name	County Abbrev.	IN County Code	Geodetic coordinates of the centers of project (point CP)						Number of points		
			Longitude (West)			Latitude (North)			Total	In the λ	In the φ
			deg.	min.	sec.	deg.	min.	sec.		direction	direction
Owen	Ow	60	86	50	40.00000	39	19	00.00000	380	20	19
Parke	P	61	87	13	40.00000	39	47	00.00000	420	20	21
Perry	Pe	62	86	38	00.00000	38	03	00.00000	513	19	27
Pike	Pi	63	87	16	00.00000	38	23	30.00000	380	19	20
Porter	Pr	64	87	04	40.00000	41	28	30.00000	420	14	30
Posey	Po	65	87	53	40.00000	38	00	00.00000	580	20	29
Pulaski	Pl	66	86	42	00.00000	41	02	30.00000	352	22	16
Putnam	Pm	67	86	50	00.00000	39	40	00.00000	475	19	25
Randolph	R	68	85	01	00.00000	40	09	30.00000	380	19	20
Ripley	Ri	69	85	16	00.00000	39	07	00.00000	475	19	25
Rush	Ru	70	85	28	00.00000	39	37	00.00000	336	16	21
St. Joseph	Sj	71	86	17	00.00000	41	36	00.00000	462	22	21
Scott	S	72	85	44	00.00000	38	42	00.00000	272	16	17
Shelby	Sh	73	85	48	00.00000	39	31	30.00000	352	16	22
Spencer	Sp	74	87	01	20.00000	37	59	30.00000	624	24	26
Starke	St	75	86	42	00.00000	41	18	00.00000	374	22	17
Steuben	Sn	76	85	00	00.00000	41	39	00.00000	285	19	15
Sullivan	Su	77	87	27	20.00000	39	05	00.00000	483	21	23
Switzerland	Sw	78	84	59	40.00000	38	49	00.00000	300	20	15
Tippecanoe	T	79	86	54	00.00000	40	23	30.00000	418	19	22
Tipton	Ti	80	86	04	00.00000	40	19	00.00000	247	19	13
Union	U	81	84	55	40.00000	39	37	30.00000	154	11	14
Vanderburgh	Vg	82	87	34	20.00000	38	00	00.00000	252	12	21
Vermillion	Ve	83	87	27	40.00000	39	53	00.00000	264	8	33
Vigo	Vi	84	87	25	40.00000	39	26	30.00000	440	20	22
Wabash	Wb	85	85	48	20.00000	40	51	00.00000	375	15	25
Warren	Wa	86	87	19	20.00000	40	18	30.00000	420	21	20
Warrick	W	87	87	17	00.00000	38	03	00.00000	575	25	23
Washington	Ws	88	86	05	00.00000	38	36	00.00000	506	22	23
Wayne	Wy	89	85	01	00.00000	39	51	30.00000	342	19	18
Wells	We	90	85	16	00.00000	40	44	30.00000	418	19	22
White	Wh	91	86	51	00.00000	40	44	30.00000	550	25	22
Whitley	Wi	92	85	30	20.00000	41	09	00.00000	342	18	19

TABLE B.3
Maximum surface differences between the INCRS Sphere and the GRS80 ellipsoid of all 92 Test Areas (counties) in Indiana

			Size of deviations of INCRS Sphere from GRS80 ellipsoid (GRS80 being the reference surface)			
County Name	County Abbrev.	IN County Code	Maximum deviation in higher-zone (cm)	Maximum deviation in lower-zone (cm)	Maximum deviation overall (cm)	Average deviation (cm)
Adams	A	01	5.8	1.9	5.8	1.8
Allen	Al	02	5.7	7.5	7.5	2.5
Bartholomew	B	03	4.9	4.7	4.9	1.8
Benton	Bn	04	3.4	5.4	5.4	1.7
Blackford	Bl	05	1.6	1.9	1.9	0.7
Boone	Bo	06	3.0	6.1	6.1	1.9
Brown	Br	07	4.9	2.8	4.9	1.6
Carroll	C	08	4.2	4.4	4.4	1.6
Cass	Ca	09	5.7	4.8	5.7	1.9
Clark	Cl	10	5.6	11.1	11.1	3.4
Clay	Cy	11	9.9	2.4	9.9	3.1
Clinton	Cn	12	3.0	6.0	6.0	1.9
Crawford	Cr	13	5.1	6.1	6.1	2.1
Daviess	Da	14	8.7	4.8	8.7	2.7
Dearborn	D	15	7.2	3.3	7.2	2.2
Decatur	De	16	4.9	4.7	4.9	1.8
DeKalb	Dk	17	3.3	4.1	4.1	1.4
Delaware	Dl	18	4.3	4.0	4.3	1.5
Dubois	Du	19	5.6	5.0	5.6	1.9
Elkhart	E	20	5.1	4.1	5.1	1.7
Fayette	F	21	3.4	2.0	3.4	1.1
Floyd	Fl	22	2.7	2.2	2.7	1.0
Fountain	Fo	23	8.3	3.1	8.3	2.5
Franklin	Fr	24	3.5	7.0	7.0	2.2
Fulton	Fu	25	2.9	7.5	7.5	2.3
Gibson	Gi	26	6.8	13.8	13.8	4.2
Grant	G	27	3.3	4.9	4.9	1.6
Greene	Gr	28	3.5	9.2	9.2	2.8
Hamilton	H	29	3.8	4.5	4.5	1.6
Hancock	Ha	30	3.0	3.6	3.6	1.3
Harrison	Hr	31	11.0	6.2	11.0	3.4
Hendricks	He	32	5.4	4.1	5.4	1.8
Henry	Hn	33	4.3	4.5	4.5	1.6
Howard	Ho	34	1.9	7.2	7.2	2.3
Huntington	Hu	35	5.7	3.0	5.7	1.8
Jackson	J	36	5.0	7.2	7.2	2.3
Jasper	Js	37	14.1	3.3	14.1	4.3
Jay	Ja	38	2.9	4.9	4.9	1.6
Jefferson	Je	39	5.5	7.2	7.2	2.4
Jennings	Jn	40	7.3	3.8	7.3	2.2
Johnson	Jo	41	3.9	2.8	3.9	1.3
Knox	K	42	12.5	13.5	13.5	4.6
Kosciusko	Ko	43	6.8	5.2	6.8	2.2
Lagrange	L	44	2.5	5.6	5.6	1.7
Lake	La	45	13.1	2.9	13.1	4.1
LaPorte	Le	46	13.1	5.6	13.1	3.9
Lawrence	Lr	47	5.0	4.8	5.0	1.8
Madison	M	48	9.0	2.3	9.0	2.8
Marion	Ma	49	4.3	4.6	4.6	1.6
Marshall	Mr	50	4.6	4.6	4.6	1.7
Martin	Mn	51	8.0	1.8	8.0	2.5
Miami	Mi	52	8.8	2.6	8.8	2.7
Monroe	Mo	53	6.0	4.2	6.0	1.9
Montgomery	My	54	5.9	4.5	5.9	1.9
Morgan	Mg	55	3.9	5.7	5.7	1.9
Newton	N	56	11.7	1.9	11.7	3.7
Noble	No	57	3.3	5.6	5.6	1.8
Ohio	O	58	0.9	2.9	2.9	0.9

TABLE B.3
(Continued)

County Name	County Abbrev.	IN County Code	Size of deviations of INCRS Sphere from GRS80 ellipsoid (GRS80 being the reference surface): Maximum deviation in higher-zone (cm)	Maximum deviation in lower-zone (cm)	Maximum deviation overall (cm)	Average deviation (cm)
Orange	Or	59	4.0	4.9	4.9	1.7
Owen	Ow	60	4.4	5.2	5.2	1.8
Parke	P	61	5.4	5.1	5.4	1.9
Perry	Pe	62	9.5	5.0	9.5	2.9
Pike	Pi	63	5.0	4.9	5.0	1.8
Porter	Pr	64	10.7	2.1	10.7	3.4
Posey	Po	65	11.1	5.6	11.1	3.4
Pulaski	Pl	66	2.9	5.8	5.8	1.8
Putnam	Pm	67	7.8	4.6	7.8	2.4
Randolph	R	68	4.8	4.5	4.8	1.7
Ripley	Ri	69	7.9	4.7	7.9	2.5
Rush	Ru	70	5.4	3.2	5.4	1.7
St. Joseph	Sj	71	5.1	5.6	5.6	2.0
Scott	S	72	3.5	3.4	3.5	1.3
Shelby	Sh	73	5.9	3.2	5.9	1.9
Spencer	Sp	74	8.8	8.2	8.8	3.1
Starke	St	75	3.3	5.7	5.7	1.8
Steuben	Sn	76	2.5	4.1	4.1	1.3
Sullivan	Su	77	6.6	5.9	6.6	2.3
Switzerland	Sw	78	2.7	5.4	5.4	1.7
Tippecanoe	T	79	5.8	4.4	5.8	1.9
Tipton	Ti	80	1.9	4.4	4.4	1.4
Union	U	81	2.3	1.4	2.3	0.8
Vanderburgh	Vg	82	5.6	1.9	5.6	1.8
Vermillion	Ve	83	13.7	0.7	13.7	4.6
Vigo	Vi	84	6.0	5.2	6.0	2.0
Wabash	Wb	85	7.5	2.6	7.5	2.3
Warren	Wa	86	4.8	5.5	5.5	1.9
Warrick	W	87	6.8	9.0	9.0	2.9
Washington	Ws	88	6.7	6.7	6.7	2.4
Wayne	Wy	89	3.9	4.5	4.5	1.6
Wells	We	90	5.8	4.3	5.8	1.9
White	Wh	91	5.8	7.7	7.7	2.5
Whitley	Wi	92	4.2	3.8	4.2	1.5

APPENDIX C. RESULTS OF THE ANALYSES OF HEIGHTS

This section presents the results of the analyses of heights (ellipsoidal height, orthometric heights, and geoid undulations) of all 92 Test Areas (counties) that have been used in the different tests in this feasibility study. The statistical values of ellipsoidal heights (h) of all 92 Test areas in terms of their extreme height values (maximum and minimum), average (mean/median) of heights, range of heights, and the standard deviation of heights are presented in Table C.1. In the same fashion of ellipsoidal heights, the statistical values of orthometric heights (H) are presented in Table C.2 whereas the ones of the geoid undulations are shown in Table C.3.

In order to arrive at the Test Areas that represent the extreme cases of the terrain, the complete ranking of the statistical values of the ellipsoidal heights and orthometric heights of all 92 Test Areas (counties) in Indiana is performed. The complete ranking results are shown in Table C.4 and Table C.5, respectively. Table C.6 shows the value of spatial autocorrelation of ellipsoidal heights (h) (Moran's Index of h) of each Test Area (county) for all 92 Test Areas (counties) in Indiana, whereas the results of descending ranking of the Moran's Index values of ellipsoidal heights of all Test Areas (counties) are shown in Table C.7.

TABLE C.1
Statistical values of the ellipsoidal heights (h's) of all 92 Test Areas (counties) in Indiana

County Name	County Abbrev.	IN County Code	Maximum (h_{Max}) (m)	Minimum (h_{Min}) (m)	Range (Min-Max) (h_{Range}) (m)	Mean (h_{avg}) (m)	Median (h_{MED}) (m)	St-Dev (h_{STD}) (m)
Adams	A	01	235.276	201.747	33.529	219.233	219.160	6.394
Allen	Al	02	243.148	182.268	60.880	211.201	210.614	11.488
Bartholomew	B	03	252.137	140.712	111.425	173.418	169.127	20.620
Benton	Bn	04	230.295	169.551	60.744	196.529	196.599	10.886
Blackford	Bl	05	257.857	227.767	30.090	239.214	237.991	6.378
Boone	Bo	06	261.189	210.530	50.659	244.285	248.481	12.414
Brown	Br	07	288.269	130.652	157.617	196.123	196.460	30.594
Carroll	C	08	217.976	126.615	91.361	174.667	175.659	16.404
Cass	Ca	09	215.634	134.109	81.525	186.733	186.616	16.038
Clark	Cl	10	270.372	82.477	187.895	169.377	171.261	43.297
Clay	Cy	11	216.508	119.511	96.997	154.313	152.049	18.189
Clinton	Cn	12	251.740	165.527	86.213	223.844	228.021	19.207
Crawford	Cr	13	230.581	82.631	147.950	163.181	165.813	33.533
Daviess	Da	14	168.573	91.281	77.292	118.090	115.444	14.478
Dearborn	D	15	276.568	102.980	173.588	209.843	218.373	49.804
Decatur	De	16	293.556	183.108	110.448	238.614	238.472	25.712
DeKalb	Dk	17	286.078	197.635	88.443	237.612	233.885	16.539
Delaware	Dl	18	296.149	223.742	72.407	252.199	250.656	13.850
Dubois	Du	19	212.094	93.370	118.723	130.290	124.372	23.218
Elkhart	E	20	262.079	186.033	76.047	219.648	220.503	14.412
Fayette	F	21	315.091	197.832	117.259	260.928	264.119	24.641
Floyd	Fl	22	266.777	82.384	184.393	175.612	195.141	55.407
Fountain	Fo	23	200.901	112.895	88.006	165.964	169.380	21.246
Franklin	Fr	24	288.856	129.981	158.875	244.605	255.213	31.598
Fulton	Fu	25	241.812	184.023	57.789	210.506	208.807	13.880
Gibson	Gi	26	148.879	75.325	73.554	100.899	99.660	12.525
Grant	G	27	248.452	200.484	47.968	228.140	228.684	7.986
Greene	Gr	28	229.927	108.379	121.548	147.428	139.596	29.243
Hamilton	H	29	259.551	186.732	72.819	226.083	224.525	15.676
Hancock	Ha	30	274.029	204.235	69.794	235.883	231.660	14.761
Harrison	Hr	31	249.502	82.699	166.803	174.451	178.038	36.531
Hendricks	He	32	271.773	166.726	105.047	231.484	236.532	21.801
Henry	Hn	33	324.336	240.566	83.770	284.726	285.886	16.716
Howard	Ho	34	240.009	181.984	58.025	220.412	221.045	9.825
Huntington	Hu	35	243.450	180.727	62.723	215.552	216.353	11.345
Jackson	J	36	247.855	118.490	129.365	155.454	142.800	30.290
Jasper	Js	37	202.311	160.684	41.627	174.384	174.357	7.256
Jay	Ja	38	303.756	219.628	84.129	248.364	246.567	16.064
Jefferson	Je	39	260.639	94.155	166.484	190.174	194.096	35.394
Jennings	Jn	40	238.765	130.615	108.150	184.042	186.091	26.872
Johnson	Jo	41	249.032	160.814	88.219	199.034	199.102	17.023
Knox	K	42	150.363	81.154	69.209	108.625	107.638	12.916
Kosciusko	Ko	43	261.402	200.723	60.680	227.599	226.598	11.437
Lagrange	L	44	280.273	213.573	66.700	244.422	243.809	14.450
Lake	La	45	202.332	141.611	60.721	164.833	161.647	15.820

TABLE C.1
(Continued)

County Name	County Abbrev.	IN County Code	Maximum (h_{Max}) (m)	Minimum (h_{Min}) (m)	Range (Min-Max) (h_{Range}) (m)	Mean (h_{avg}) (m)	Median (h_{MED}) (m)	St-Dev (h_{STD}) (m)
LaPorte	Le	46	234.775	141.373	93.402	185.014	181.776	18.023
Lawrence	Lr	47	243.814	110.066	133.748	169.468	171.580	27.823
Madison	M	48	273.097	208.241	64.856	233.386	232.691	9.020
Marion	Ma	49	248.056	119.812	128.245	207.022	208.631	18.425
Marshall	Mr	50	238.943	182.003	56.941	210.587	212.247	10.840
Martin	Mn	51	221.149	97.698	123.452	146.374	144.224	27.154
Miami	Mi	52	232.788	159.767	73.021	203.620	206.560	15.420
Monroe	Mo	53	260.563	119.217	141.346	188.014	190.445	29.418
Montgomery	My	54	249.841	146.333	103.508	209.888	209.296	15.196
Morgan	Mg	55	246.271	136.488	109.783	189.838	194.781	23.769
Newton	N	56	202.311	157.605	44.705	170.784	170.208	7.765
Noble	No	57	286.078	224.567	61.511	250.620	249.095	12.585
Ohio	O	58	258.495	102.509	155.986	188.843	201.799	43.135
Orange	Or	59	247.952	110.064	137.889	175.756	175.965	27.490
Owen	Ow	60	249.144	121.778	127.366	172.163	171.345	25.781
Parke	P	61	214.531	108.914	105.617	162.535	164.252	26.444
Perry	Pe	62	217.905	75.363	142.543	132.852	132.293	32.774
Pike	Pi	63	153.886	89.749	64.137	112.583	111.368	13.296
Porter	Pr	64	223.538	141.407	82.131	173.143	170.319	19.971
Posey	Po	65	135.174	68.544	66.630	89.561	85.192	13.298
Pulaski	Pl	66	198.924	168.894	30.030	180.402	179.784	5.617
Putnam	Pm	67	265.876	143.772	122.104	209.507	209.824	23.488
Randolph	R	68	342.139	252.026	90.114	297.462	298.534	22.974
Ripley	Ri	69	277.790	124.038	153.752	247.825	253.749	20.848
Rush	Ru	70	300.760	225.086	75.674	263.465	264.789	16.916
St. Joseph	Sj	71	238.810	165.466	73.345	203.519	203.216	16.531
Scott	S	72	261.341	124.586	136.755	158.549	154.482	25.687
Shelby	Sh	73	246.938	167.506	79.432	205.846	206.367	17.094
Spencer	Sp	74	155.156	75.363	79.794	100.562	97.520	14.821
Starke	St	75	201.388	168.074	33.314	179.039	178.995	6.933
Steuben	Sn	76	310.379	238.936	71.443	271.281	270.793	13.568
Sullivan	Su	77	159.484	94.399	65.085	120.257	120.575	14.008
Switzerland	Sw	78	264.068	94.177	169.891	187.122	196.781	43.441
Tippecanoe	T	79	219.408	120.637	98.771	174.958	176.014	21.053
Tipton	Ti	80	249.274	215.886	33.388	233.769	231.564	6.762
Union	U	81	313.021	192.853	120.169	266.129	271.412	27.143
Vanderburgh	Vg	82	139.361	73.278	66.083	96.620	95.365	15.427
Vermillion	Ve	83	172.465	109.392	63.073	145.571	153.010	18.008
Vigo	Vi	84	183.430	101.567	81.863	135.880	137.659	17.333
Wabash	Wb	85	245.802	162.546	83.256	208.911	208.052	14.663
Warren	Wa	86	215.430	118.116	97.314	173.598	178.297	18.067
Warrick	W	87	155.277	73.253	82.025	97.848	95.060	13.911
Washington	Ws	88	273.014	116.087	156.927	189.681	195.399	36.728
Wayne	Wy	89	341.776	225.011	116.765	287.240	288.698	24.053
Wells	We	90	246.674	191.488	55.186	220.248	221.084	8.390
White	Wh	91	215.254	126.665	88.589	175.050	174.988	11.693
Whitley	Wi	92	265.484	195.081	70.403	232.773	229.932	13.221

TABLE C.2
Statistical values of the orthometric heights (H's) of all 92 Test Areas (counties) in Indiana

County Name	County Abbrev.	IN County Code	Maximum (H_{Max}) (m)	Minimum (H_{Min}) (m)	Range (Min-Max) (H_{Range}) (m)	Mean (H_{avg}) (m)	Median (H_{MED}) (m)	St-Dev (H_{STD}) (m)
Adams	A	01	268.881	235.181	33.700	252.735	252.715	6.444
Allen	Al	02	275.976	215.660	60.316	244.351	243.784	11.367
Bartholomew	B	03	285.741	174.558	111.183	207.223	202.889	20.596
Benton	Bn	04	263.546	202.961	60.585	229.846	229.999	10.868
Blackford	Bl	05	291.702	261.672	30.030	273.238	272.098	6.349
Boone	Bo	06	294.486	244.209	50.277	277.987	282.139	12.438
Brown	Br	07	321.626	163.876	157.750	229.408	229.759	30.605
Carroll	C	08	252.218	160.567	91.651	208.730	209.727	16.476
Cass	Ca	09	249.475	168.121	81.354	220.795	220.601	16.010
Clark	Cl	10	303.913	115.824	188.089	202.837	204.750	43.278
Clay	Cy	11	249.552	152.234	97.318	187.136	184.936	18.290
Clinton	Cn	12	285.850	199.454	86.396	257.840	262.070	19.278
Crawford	Cr	13	263.481	115.811	147.670	196.242	198.921	33.595
Daviess	Da	14	201.072	123.804	77.268	150.505	147.838	14.598
Dearborn	D	15	310.602	137.135	173.467	243.896	252.308	49.779
Decatur	De	16	327.637	217.228	110.409	272.714	272.569	25.721
DeKalb	Dk	17	319.195	231.115	88.080	270.868	267.177	16.478
Delaware	Dl	18	329.884	257.782	72.102	286.119	284.594	13.771
Dubois	Du	19	244.940	125.180	119.759	162.723	156.560	23.366
Elkhart	E	20	295.408	219.679	75.730	253.153	254.013	14.393
Fayette	F	21	349.094	231.895	117.199	294.905	298.116	24.632
Floyd	Fl	22	300.199	115.675	184.524	208.903	228.419	55.399
Fountain	Fo	23	234.182	145.741	88.441	199.115	202.509	21.315
Franklin	Fr	24	322.904	163.803	159.101	278.500	288.826	31.579
Fulton	Fu	25	275.422	217.640	57.782	244.203	242.649	13.934
Gibson	Gi	26	180.389	106.187	74.202	132.255	131.054	12.635
Grant	G	27	282.590	234.821	47.769	262.403	263.006	7.933
Greene	Gr	28	262.905	140.866	122.039	180.149	172.379	29.369
Hamilton	H	29	293.644	220.683	72.961	260.195	258.707	15.683
Hancock	Ha	30	307.972	238.267	69.705	269.833	265.636	14.781
Harrison	Hr	31	282.916	115.824	167.092	207.585	211.134	36.604
Hendricks	He	32	304.789	199.588	105.201	264.584	269.789	21.910
Henry	Hn	33	358.055	274.471	83.584	318.567	319.736	16.667
Howard	Ho	34	274.380	216.237	58.143	254.763	255.442	9.839
Huntington	Hu	35	277.741	214.735	63.006	249.537	250.218	11.383
Jackson	J	36	281.220	151.757	129.463	189.032	176.493	30.152
Jasper	Js	37	235.681	194.323	41.358	207.971	207.957	7.218
Jay	Ja	38	337.128	253.231	83.898	281.997	280.245	16.021
Jefferson	Je	39	294.624	128.016	166.608	224.011	227.922	35.415
Jennings	Jn	40	272.923	164.472	108.451	218.026	220.097	26.934
Johnson	Jo	41	282.230	193.744	88.487	232.227	232.188	16.975
Knox	K	42	182.632	112.663	69.969	140.524	139.823	13.028
Kosciusko	Ko	43	294.779	234.387	60.393	261.172	260.157	11.393
Lagrange	L	44	313.463	246.807	66.655	277.673	277.080	14.425
Lake	La	45	236.024	175.186	60.838	198.376	195.233	15.823
LaPorte	Le	46	268.609	175.192	93.417	218.858	215.575	18.024
Lawrence	Lr	47	276.826	143.180	133.646	202.529	204.702	27.832
Madison	M	48	307.030	242.438	64.592	267.479	266.801	8.969
Marion	Ma	49	281.670	153.754	127.917	240.559	242.187	18.541
Marshall	Mr	50	272.549	215.826	56.724	244.330	245.992	10.839
Martin	Mn	51	253.996	130.246	123.751	179.166	177.031	27.200
Miami	Mi	52	266.709	193.958	72.751	237.773	240.891	15.395
Monroe	Mo	53	293.573	152.281	141.292	221.096	223.547	29.404
Montgomery	My	54	283.038	179.600	103.438	243.259	242.682	15.190
Morgan	Mg	55	279.102	169.477	109.625	222.757	227.679	23.745
Newton	N	56	235.681	190.924	44.756	204.117	203.578	7.732
Noble	No	57	319.195	257.917	61.278	283.798	282.302	12.539
Ohio	O	58	292.621	136.524	156.097	222.933	235.957	43.123

TABLE C.2
(Continued)

County Name	County Abbrev.	IN County Code	Maximum (H_{Max}) (m)	Minimum (H_{Min}) (m)	Range (Min-Max) (H_{Range}) (m)	Mean (H_{avg}) (m)	Median (H_{MED}) (m)	St-Dev (H_{STD}) (m)
Orange	Or	59	281.385	143.083	138.303	208.949	208.996	27.573
Owen	Ow	60	282.160	154.507	127.653	205.045	204.319	25.815
Parke	P	61	247.700	141.696	106.004	195.611	197.206	26.544
Perry	Pe	62	250.841	107.290	143.552	165.248	164.515	32.955
Pike	Pi	63	185.739	121.559	64.180	144.456	143.200	13.323
Porter	Pr	64	257.282	175.192	82.090	206.869	204.060	20.000
Posey	Po	65	166.172	98.755	67.417	120.196	115.898	13.471
Pulaski	Pl	66	232.509	202.563	29.946	214.090	213.473	5.608
Putnam	Pm	67	298.887	176.717	122.170	242.546	242.950	23.510
Randolph	R	68	375.756	285.848	89.909	331.060	332.063	22.937
Ripley	Ri	69	311.828	158.118	153.710	281.918	287.822	20.852
Rush	Ru	70	334.777	259.039	75.738	297.454	298.763	16.921
St. Joseph	Sj	71	272.607	199.114	73.494	237.307	237.036	16.540
Scott	S	72	294.928	158.374	136.554	192.316	188.253	25.650
Shelby	Sh	73	280.880	201.081	79.799	239.648	240.117	17.194
Spencer	Sp	74	187.340	106.680	80.660	132.025	128.844	14.998
Starke	St	75	235.110	201.934	33.176	212.854	212.846	6.897
Steuben	Sn	76	343.706	272.317	71.389	304.640	304.125	13.602
Sullivan	Su	77	192.030	126.371	65.659	152.451	152.695	14.106
Switzerland	Sw	78	298.026	128.083	169.943	221.055	230.667	43.445
Tippecanoe	T	79	253.062	154.088	98.974	208.591	209.699	20.992
Tipton	Ti	80	283.527	250.139	33.388	268.107	265.958	6.747
Union	U	81	346.715	226.636	120.080	299.808	304.814	27.092
Vanderburgh	Vg	82	170.394	104.242	66.152	127.619	126.311	15.498
Vermillion	Ve	83	205.013	142.213	62.800	178.303	185.611	17.950
Vigo	Vi	84	215.749	133.989	81.760	168.448	170.183	17.336
Wabash	Wb	85	279.561	196.783	82.778	243.015	242.252	14.618
Warren	Wa	86	248.586	151.063	97.523	206.761	211.573	18.074
Warrick	W	87	187.048	104.242	82.807	129.141	126.193	14.051
Washington	Ws	88	306.632	149.560	157.072	223.154	228.835	36.683
Wayne	Wy	89	375.386	258.824	116.562	321.022	322.530	23.979
Wells	We	90	280.591	225.407	55.184	254.110	254.952	8.444
White	Wh	91	248.790	160.658	88.132	208.775	208.721	11.603
Whitley	Wi	92	298.428	228.413	70.015	265.955	263.404	13.156

TABLE C.3
Statistical values of the geoid undulations (N's) from NGS's Geoid09 model of all 92 Test Areas (counties) in Indiana

County Name	County Abbrev.	IN County Code	Maximum (N_{Max}) (m)	Minimum (N_{Min}) (m)	Range (Min-Max) (N_{Range}) (m)	Mean (N_{avg}) (m)	Median (N_{MED}) (m)	St-Dev (N_{STD}) (m)
Adams	A	01	-33.306	-33.728	0.422	-33.503	-33.499	0.103
Allen	Al	02	-32.737	-33.675	0.938	-33.150	-33.186	0.214
Bartholomew	B	03	-33.325	-34.054	0.729	-33.805	-33.838	0.177
Benton	Bn	04	-33.040	-33.557	0.517	-33.317	-33.335	0.146
Blackford	Bl	05	-33.845	-34.199	0.354	-34.024	-34.025	0.090
Boone	Bo	06	-33.261	-34.164	0.903	-33.702	-33.698	0.202
Brown	Br	07	-33.047	-33.523	0.476	-33.285	-33.273	0.116
Carroll	C	08	-33.796	-34.269	0.473	-34.062	-34.063	0.119
Cass	Ca	09	-33.722	-34.340	0.618	-34.062	-34.088	0.174
Clark	Cl	10	-33.201	-33.724	0.523	-33.460	-33.442	0.130
Clay	Cy	11	-32.454	-33.081	0.627	-32.823	-32.825	0.154
Clinton	Cn	12	-33.601	-34.345	0.744	-33.997	-34.001	0.190
Crawford	Cr	13	-32.422	-33.460	1.038	-33.061	-33.086	0.232
Daviess	Da	14	-31.959	-32.786	0.827	-32.415	-32.421	0.194
Dearborn	D	15	-33.687	-34.164	0.477	-34.053	-34.081	0.096
Decatur	De	16	-33.962	-34.159	0.197	-34.100	-34.103	0.043
DeKalb	Dk	17	-32.895	-33.597	0.702	-33.256	-33.235	0.150
Delaware	Dl	18	-33.695	-34.106	0.411	-33.919	-33.922	0.087
Dubois	Du	19	-31.771	-32.923	1.152	-32.433	-32.448	0.261
Elkhart	E	20	-33.237	-33.797	0.560	-33.504	-33.485	0.138
Fayette	F	21	-33.862	-34.083	0.221	-33.978	-33.976	0.054
Floyd	Fl	22	-33.110	-33.476	0.366	-33.292	-33.299	0.085
Fountain	Fo	23	-32.844	-33.414	0.570	-33.151	-33.172	0.130
Franklin	Fr	24	-33.372	-34.131	0.759	-33.896	-33.956	0.194
Fulton	Fu	25	-33.533	-34.045	0.512	-33.697	-33.679	0.114
Gibson	Gi	26	-30.782	-31.966	1.184	-31.356	-31.339	0.259
Grant	G	27	-34.043	-34.385	0.342	-34.262	-34.273	0.084
Greene	Gr	28	-32.354	-32.992	0.638	-32.721	-32.742	0.178
Hamilton	H	29	-33.753	-34.318	0.565	-34.112	-34.117	0.116
Hancock	Ha	30	-33.696	-34.033	0.337	-33.950	-33.962	0.060
Harrison	Hr	31	-32.574	-33.461	0.887	-33.134	-33.149	0.201
Hendricks	He	32	-32.840	-33.636	0.796	-33.099	-33.054	0.184
Henry	Hn	33	-33.657	-33.951	0.294	-33.841	-33.856	0.074
Howard	Ho	34	-34.186	-34.406	0.220	-34.351	-34.368	0.047
Huntington	Hu	35	-33.352	-34.300	0.948	-33.985	-34.026	0.201
Jackson	J	36	-33.126	-33.905	0.779	-33.577	-33.618	0.228
Jasper	Js	37	-33.316	-33.837	0.521	-33.587	-33.592	0.127
Jay	Ja	38	-33.364	-33.946	0.582	-33.634	-33.626	0.159
Jefferson	Je	39	-33.692	-33.999	0.307	-33.836	-33.840	0.069
Jennings	Jn	40	-33.849	-34.158	0.309	-33.984	-33.968	0.094
Johnson	Jo	41	-32.896	-33.640	0.744	-33.193	-33.169	0.186
Knox	K	42	-31.251	-32.561	1.310	-31.899	-31.893	0.309
Kosciusko	Ko	43	-33.286	-33.780	0.494	-33.573	-33.596	0.113
Lagrange	L	44	-33.139	-33.387	0.248	-33.251	-33.241	0.050
Lake	La	45	-33.299	-33.711	0.412	-33.543	-33.551	0.086
LaPorte	Le	46	-33.706	-33.929	0.223	-33.844	-33.847	0.040
Lawrence	Lr	47	-32.918	-33.303	0.385	-33.061	-33.049	0.085
Madison	M	48	-33.933	-34.318	0.385	-34.094	-34.079	0.095
Marion	Ma	49	-32.920	-34.013	1.093	-33.537	-33.558	0.266
Marshall	Mr	50	-33.582	-33.858	0.276	-33.744	-33.762	0.069
Martin	Mn	51	-32.515	-32.942	0.427	-32.792	-32.804	0.099
Miami	Mi	52	-33.754	-34.384	0.630	-34.153	-34.188	0.167
Monroe	Mo	53	-32.961	-33.262	0.301	-33.082	-33.070	0.074
Montgomery	My	54	-33.143	-33.628	0.485	-33.372	-33.355	0.105
Morgan	Mg	55	-32.824	-33.108	0.284	-32.918	-32.903	0.062
Newton	N	56	-33.069	-33.656	0.587	-33.332	-33.318	0.133
Noble	No	57	-32.823	-33.425	0.602	-33.178	-33.180	0.154
Ohio	O	58	-33.987	-34.161	0.174	-34.090	-34.101	0.049

TABLE C.3
(Continued)

County Name	County Abbrev.	IN County Code	Maximum (N_{Max}) (m)	Minimum (N_{Min}) (m)	Range (Min-Max) (N_{Range}) (m)	Mean (N_{avg}) (m)	Median (N_{MED}) (m)	St-Dev (N_{STD}) (m)
Orange	Or	59	-32.817	-33.459	0.642	-33.193	-33.215	0.176
Owen	Ow	60	-32.655	-33.019	0.364	-32.883	-32.891	0.079
Parke	P	61	-32.679	-33.300	0.621	-33.076	-33.108	0.150
Perry	Pe	62	-31.396	-33.188	1.792	-32.396	-32.401	0.399
Pike	Pi	63	-31.453	-32.346	0.893	-31.873	-31.855	0.198
Porter	Pr	64	-33.578	-33.846	0.268	-33.726	-33.730	0.068
Posey	Po	65	-30.002	-31.229	1.227	-30.634	-30.632	0.266
Pulaski	Pl	66	-33.567	-33.807	0.240	-33.688	-33.680	0.047
Putnam	Pm	67	-32.869	-33.256	0.387	-33.039	-33.032	0.101
Randolph	R	68	-33.382	-33.836	0.454	-33.598	-33.596	0.098
Ripley	Ri	69	-33.951	-34.159	0.208	-34.093	-34.108	0.051
Rush	Ru	70	-33.895	-34.067	0.172	-33.989	-33.986	0.046
St. Joseph	Sj	71	-33.513	-33.877	0.364	-33.788	-33.820	0.076
Scott	S	72	-33.534	-33.864	0.330	-33.768	-33.790	0.075
Shelby	Sh	73	-33.410	-34.082	0.672	-33.802	-33.838	0.161
Spencer	Sp	74	-30.758	-32.441	1.683	-31.463	-31.428	0.368
Starke	St	75	-33.606	-33.882	0.276	-33.815	-33.829	0.053
Steuben	Sn	76	-33.138	-33.705	0.567	-33.359	-33.309	0.158
Sullivan	Su	77	-31.754	-32.566	0.812	-32.193	-32.211	0.175
Switzerland	Sw	78	-33.854	-34.073	0.219	-33.933	-33.915	0.058
Tippecanoe	T	79	-33.309	-33.977	0.668	-33.633	-33.620	0.163
Tipton	Ti	80	-34.187	-34.404	0.217	-34.338	-34.344	0.042
Union	U	81	-33.372	-33.894	0.522	-33.679	-33.695	0.146
Vanderburgh	Vg	82	-30.577	-31.370	0.793	-30.999	-31.014	0.187
Vermillion	Ve	83	-32.411	-32.994	0.583	-32.733	-32.741	0.143
Vigo	Vi	84	-32.168	-33.044	0.876	-32.568	-32.555	0.208
Wabash	Wb	85	-33.661	-34.349	0.688	-34.104	-34.146	0.179
Warren	Wa	86	-32.674	-33.519	0.845	-33.163	-33.186	0.180
Warrick	W	87	-30.772	-31.968	1.196	-31.292	-31.283	0.234
Washington	Ws	88	-33.191	-33.823	0.632	-33.474	-33.444	0.121
Wayne	Wy	89	-33.605	-33.976	0.371	-33.782	-33.772	0.094
Wells	We	90	-33.342	-34.205	0.863	-33.861	-33.868	0.191
White	Wh	91	-33.490	-34.088	0.598	-33.725	-33.707	0.145
Whitley	Wi	92	-32.741	-33.839	1.098	-33.182	-33.165	0.278

TABLE C.4
Descending orders of all 92 Test Areas (counties) in Indiana, ranked by different statistical values of the ellipsoidal heights

	By h_{Max}		By h_{Min}		By h_{Range}		By h_{avg}		By h_{MED}		By h_{STD}	
Rank	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)
1	R	342.139	R	252.026	Cl	187.895	R	297.462	R	298.534	Fl	55.407
2	Wy	341.776	Hn	240.566	Fl	184.393	Wy	287.240	Wy	288.698	D	49.804
3	Hn	324.336	Sn	238.936	D	173.588	Hn	284.726	Hn	285.886	Sw	43.441
4	F	315.091	Bl	227.767	Sw	169.891	Sn	271.281	U	271.412	Cl	43.297
5	U	313.021	Ru	225.086	Hr	166.803	U	266.129	Sn	270.793	O	43.135
6	Sn	310.379	Wy	225.011	Je	166.484	Ru	263.465	Ru	264.789	Ws	36.728
7	Ja	303.756	No	224.567	Fr	158.875	F	260.928	F	264.119	Hr	36.531
8	Ru	300.760	Dl	223.742	Br	157.617	Dl	252.199	Fr	255.213	Je	35.394
9	Dl	296.149	Ja	219.628	Ws	156.927	No	250.620	Ri	253.749	Cr	33.533
10	De	293.556	Ti	215.886	O	155.986	Ja	248.364	Dl	250.656	Pe	32.774
11	Fr	288.856	L	213.573	Ri	153.752	Ri	247.825	No	249.095	Fr	31.598
12	Br	288.269	Bo	210.530	Cr	147.950	Fr	244.605	Bo	248.481	Br	30.594
13	Dk	286.078	M	208.241	Pe	142.543	L	244.422	Ja	246.567	J	30.290
14	No	286.078	Ha	204.235	Mo	141.346	Bo	244.285	L	243.809	Mo	29.418
15	L	280.273	A	201.747	Or	137.889	Bl	239.214	De	238.472	Gr	29.243
16	Ri	277.790	Ko	200.723	S	136.755	De	238.614	Bl	237.991	Lr	27.823
17	D	276.568	G	200.484	Lr	133.748	Dk	237.612	He	236.532	Or	27.490
18	Ha	274.029	F	197.832	J	129.365	Ha	235.883	Dk	233.885	Mn	27.154
19	M	273.097	Dk	197.635	Ma	128.245	Ti	233.769	M	232.691	U	27.143
20	Ws	273.014	Wi	195.081	Ow	127.366	M	233.386	Ha	231.660	Jn	26.872
21	He	271.773	U	192.853	Mn	123.452	Wi	232.773	Ti	231.564	P	26.444
22	Cl	270.372	We	191.488	Pm	122.104	He	231.484	Wi	229.932	Ow	25.781
23	Fl	266.777	H	186.732	Gr	121.548	G	228.140	G	228.684	De	25.712
24	Pm	265.876	E	186.033	U	120.169	Ko	227.599	Cn	228.021	S	25.687
25	Wi	265.484	Fu	184.023	Du	118.723	H	226.083	Ko	226.598	F	24.641
26	Sw	264.068	De	183.108	F	117.259	Cn	223.844	H	224.525	Wy	24.053
27	E	262.079	Al	182.268	Wy	116.765	Ho	220.412	We	221.084	Mg	23.769
28	Ko	261.402	Mr	182.003	B	111.425	We	220.248	Ho	221.045	Pm	23.488
29	S	261.341	Ho	181.984	De	110.448	E	219.648	E	220.503	Du	23.218
30	Bo	261.189	Hu	180.727	Mg	109.783	A	219.233	A	219.160	R	22.974
31	Je	260.639	Bn	169.551	Jn	108.150	Hu	215.552	D	218.373	He	21.801
32	Mo	260.563	Pl	168.894	P	105.617	Al	211.201	Hu	216.353	Fo	21.246
33	H	259.551	St	168.074	He	105.047	Mr	210.587	Mr	212.247	T	21.053
34	O	258.495	Sh	167.506	My	103.508	Fu	210.506	Al	210.614	Ri	20.848
35	Bl	257.857	He	166.726	T	98.771	My	209.888	Pm	209.824	B	20.620
36	B	252.137	Cn	165.527	Wa	97.314	D	209.843	My	209.296	Pr	19.971
37	Cn	251.740	Sj	165.466	Cy	96.997	Pm	209.507	Fu	208.807	Cn	19.207
38	My	249.841	Wb	162.546	Le	93.402	Wb	208.911	Ma	208.631	Ma	18.425
39	Hr	249.502	Jo	160.814	C	91.361	Ma	207.022	Wb	208.052	Cy	18.189
40	Ti	249.274	Js	160.684	R	90.114	Sh	205.846	Mi	206.560	Wa	18.067
41	Ow	249.144	Mi	159.767	Wh	88.589	Mi	203.620	Sh	206.367	Le	18.023
42	Jo	249.032	N	157.605	Dk	88.443	Sj	203.519	Sj	203.216	Ve	18.008
43	G	248.452	My	146.333	Jo	88.219	Jo	199.034	O	201.799	Vi	17.333
44	Ma	248.056	Pm	143.772	Fo	88.006	Bn	196.529	Jo	199.102	Sh	17.094
45	Or	247.952	La	141.611	Cn	86.213	Br	196.123	Sw	196.781	Jo	17.023
46	J	247.855	Pr	141.407	Ja	84.129	Je	190.174	Bn	196.599	Ru	16.916
47	Sh	246.938	Le	141.373	Hn	83.770	Mg	189.838	Br	196.460	Hn	16.716
48	We	246.674	B	140.712	Wb	83.256	Ws	189.681	Ws	195.399	Dk	16.539
49	Mg	246.271	Mg	136.488	Pr	82.131	O	188.843	Fl	195.141	Sj	16.531
50	Wb	245.802	Ca	134.109	W	82.025	Mo	188.014	Mg	194.781	C	16.404
51	Lr	243.814	Br	130.652	Vi	81.863	Sw	187.122	Je	194.096	Ja	16.064
52	Hu	243.450	Jn	130.615	Ca	81.525	Ca	186.733	Mo	190.445	Ca	16.038
53	Al	243.148	Fr	129.981	Sp	79.794	Le	185.014	Ca	186.616	La	15.820
54	Fu	241.812	Wh	126.665	Sh	79.432	Jn	184.042	Jn	186.091	H	15.676
55	Ho	240.009	C	126.615	Da	77.292	Pl	180.402	Le	181.776	Vg	15.427
56	Mr	238.943	S	124.586	E	76.047	St	179.039	Pl	179.784	Mi	15.420
57	Sj	238.810	Ri	124.038	Ru	75.674	Or	175.756	St	178.995	My	15.196
58	Jn	238.765	Ow	121.778	Gi	73.554	Fl	175.612	Wa	178.297	Sp	14.821
59	A	235.276	T	120.637	Sj	73.345	Wh	175.050	Hr	178.038	Ha	14.761

TABLE C.4
(Continued)

Rank	By h_{Max} County Abbrev.	By h_{Max} Value (m)	By h_{Min} County Abbrev.	By h_{Min} Value (m)	By h_{Range} County Abbrev.	By h_{Range} Value (m)	By h_{avg} County Abbrev.	By h_{avg} Value (m)	By h_{MED} County Abbrev.	By h_{MED} Value (m)	By h_{STD} County Abbrev.	By h_{STD} Value (m)
60	Le	234.775	Ma	119.812	Mi	73.021	T	174.958	T	176.014	Wb	14.663
61	Mi	232.788	Cy	119.511	H	72.819	C	174.667	Or	175.965	Da	14.478
62	Cr	230.581	Mo	119.217	Dl	72.407	Hr	174.451	C	175.659	L	14.450
63	Bn	230.295	J	118.490	Sn	71.443	Js	174.384	Wh	174.988	E	14.412
64	Gr	229.927	Wa	118.116	Wi	70.403	Wa	173.598	Js	174.357	Su	14.008
65	Pr	223.538	Ws	116.087	Ha	69.794	B	173.418	Lr	171.580	W	13.911
66	Mn	221.149	Fo	112.895	K	69.209	Pr	173.143	Ow	171.345	Fu	13.880
67	T	219.408	Lr	110.066	L	66.700	Ow	172.163	Cl	171.261	Dl	13.850
68	C	217.976	Or	110.064	Po	66.630	N	170.784	Pr	170.319	Sn	13.568
69	Pe	217.905	Ve	109.392	Vg	66.083	Lr	169.468	N	170.208	Po	13.298
70	Cy	216.508	P	108.914	Su	65.085	Cl	169.377	Fo	169.380	Pi	13.296
71	Ca	215.634	Gr	108.379	M	64.856	Fo	165.964	B	169.127	Wi	13.221
72	Wa	215.430	D	102.980	Pi	64.137	La	164.833	Cr	165.813	K	12.916
73	Wh	215.254	O	102.509	Ve	63.073	Cr	163.181	P	164.252	No	12.585
74	P	214.531	Vi	101.567	Hu	62.723	P	162.535	La	161.647	Gi	12.525
75	Du	212.094	Mn	97.698	No	61.511	S	158.549	S	154.482	Bo	12.414
76	La	202.332	Su	94.399	Al	60.880	J	155.454	Ve	153.010	Wh	11.693
77	Js	202.311	Sw	94.177	Bn	60.744	Cy	154.313	Cy	152.049	Al	11.488
78	N	202.311	Je	94.155	La	60.721	Gr	147.428	Mn	144.224	Ko	11.437
79	St	201.388	Du	93.370	Ko	60.680	Mn	146.374	J	142.800	Hu	11.345
80	Fo	200.901	Da	91.281	Ho	58.025	Ve	145.571	Gr	139.596	Bn	10.886
81	Pl	198.924	Pi	89.749	Fu	57.789	Vi	135.880	Vi	137.659	Mr	10.840
82	Vi	183.430	Hr	82.699	Mr	56.941	Pe	132.852	Pe	132.293	Ho	9.825
83	Ve	172.465	Cr	82.631	We	55.186	Du	130.290	Du	124.372	M	9.020
84	Da	168.573	Cl	82.477	Bo	50.659	Su	120.257	Su	120.575	We	8.390
85	Su	159.484	Fl	82.384	G	47.968	Da	118.090	Da	115.444	G	7.986
86	W	155.277	K	81.154	N	44.705	Pi	112.583	Pi	111.368	N	7.765
87	Sp	155.156	Pe	75.363	Js	41.627	K	108.625	K	107.638	Js	7.256
88	Pi	153.886	Sp	75.363	A	33.529	Gi	100.899	Gi	99.660	St	6.933
89	K	150.363	Gi	75.325	Ti	33.388	Sp	100.562	Sp	97.520	Ti	6.762
90	Gi	148.879	Vg	73.278	St	33.314	W	97.848	Vg	95.365	A	6.394
91	Vg	139.361	W	73.253	Bl	30.090	Vg	96.620	W	95.060	Bl	6.378
92	Po	135.174	Po	68.544	Pl	30.030	Po	89.561	Po	85.192	Pl	5.617

TABLE C.5
Descending orders of all 92 Test Areas (counties) in Indiana, ranked by different statistical values of the orthometric heights

	By h_{Max}		By h_{Min}		By h_{Range}		By h_{avg}		By h_{MED}		By h_{STD}	
Rank	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)	County Abbrev.	Value (m)
1	R	375.756	R	285.848	Cl	188.089	R	331.060	R	332.063	Fl	55.399
2	Wy	375.386	Hn	274.471	Fl	184.524	Wy	321.022	Wy	322.530	D	49.779
3	Hn	358.055	Sn	272.317	D	173.467	Hn	318.567	Hn	319.736	Sw	43.445
4	F	349.094	Bl	261.672	Sw	169.943	Sn	304.640	U	304.814	Cl	43.278
5	U	346.715	Ru	259.039	Hr	167.092	U	299.808	Sn	304.125	O	43.123
6	Sn	343.706	Wy	258.824	Je	166.608	Ru	297.454	Ru	298.763	Ws	36.683
7	Ja	337.128	No	257.917	Fr	159.101	F	294.905	F	298.116	Hr	36.604
8	Ru	334.777	Dl	257.782	Br	157.750	Dl	286.119	Fr	288.826	Je	35.415
9	Dl	329.884	Ja	253.231	Ws	157.072	No	283.798	Ri	287.822	Cr	33.595
10	De	327.637	Ti	250.139	O	156.097	Ja	281.997	Dl	284.594	Pe	32.955
11	Fr	322.904	L	246.807	Ri	153.710	Ri	281.918	No	282.302	Fr	31.579
12	Br	321.626	Bo	244.209	Cr	147.670	Fr	278.500	Bo	282.139	Br	30.605
13	Dk	319.195	M	242.438	Pe	143.552	Bo	277.987	Ja	280.245	J	30.152
14	No	319.195	Ha	238.267	Mo	141.292	L	277.673	L	277.080	Mo	29.404
15	L	313.463	A	235.181	Or	138.303	Bl	273.238	De	272.569	Gr	29.369
16	Ri	311.828	G	234.821	S	136.554	De	272.714	Bl	272.098	Lr	27.832
17	D	310.602	Ko	234.387	Lr	133.646	Dk	270.868	He	269.789	Or	27.573
18	Ha	307.972	F	231.895	J	129.463	Ha	269.833	Dk	267.177	Mn	27.200
19	M	307.030	Dk	231.115	Ma	127.917	Ti	268.107	M	266.801	U	27.092
20	Ws	306.632	Wi	228.413	Ow	127.653	M	267.479	Ti	265.958	Jn	26.934
21	He	304.789	U	226.636	Mn	123.751	Wi	265.955	Ha	265.636	P	26.544
22	Cl	303.913	We	225.407	Pm	122.170	He	264.584	Wi	263.404	Ow	25.815
23	Fl	300.199	H	220.683	Gr	122.039	G	262.403	G	263.006	De	25.721
24	Pm	298.887	E	219.679	U	120.080	Ko	261.172	Cn	262.070	S	25.650
25	Wi	298.428	Fu	217.640	Du	119.759	H	260.195	Ko	260.157	F	24.632
26	Sw	298.026	De	217.228	F	117.199	Cn	257.840	H	258.707	Wy	23.979
27	E	295.408	Ho	216.237	Wy	116.562	Ho	254.763	Ho	255.442	Mg	23.745
28	S	294.928	Mr	215.826	B	111.183	We	254.110	We	254.952	Pm	23.510
29	Ko	294.779	Al	215.660	De	110.409	E	253.153	E	254.013	Du	23.366
30	Je	294.624	Hu	214.735	Mg	109.625	A	252.735	A	252.715	R	22.937
31	Bo	294.486	Bn	202.961	Jn	108.451	Hu	249.537	D	252.308	He	21.910
32	H	293.644	Pl	202.563	P	106.004	Al	244.351	Hu	250.218	Fo	21.315
33	Mo	293.573	St	201.934	He	105.201	Mr	244.330	Mr	245.992	T	20.992
34	O	292.621	Sh	201.081	My	103.438	Fu	244.203	Al	243.784	Ri	20.852
35	Bl	291.702	He	199.588	T	98.974	D	243.896	Pm	242.950	B	20.596
36	Cn	285.850	Cn	199.454	Wa	97.523	My	243.259	My	242.682	Pr	20.000
37	B	285.741	Sj	199.114	Cy	97.318	Wb	243.015	Fu	242.649	Cn	19.278
38	Ti	283.527	Wb	196.783	Le	93.417	Pm	242.546	Wb	242.252	Ma	18.541
39	My	283.038	Js	194.323	C	91.651	Ma	240.559	Ma	242.187	Cy	18.290
40	Hr	282.916	Mi	193.958	R	89.909	Sh	239.648	Mi	240.891	Wa	18.074
41	G	282.590	Jo	193.744	Jo	88.487	Mi	237.773	Sh	240.117	Le	18.024
42	Jo	282.230	N	190.924	Fo	88.441	Sj	237.307	Sj	237.036	Ve	17.950
43	Ow	282.160	My	179.600	Wh	88.132	Jo	232.227	O	235.957	Vi	17.336
44	Ma	281.670	Pm	176.717	Dk	88.080	Bn	229.846	Jo	232.188	Sh	17.194
45	Or	281.385	Le	175.192	Cn	86.396	Br	229.408	Sw	230.667	Jo	16.975
46	J	281.220	Pr	175.192	Ja	83.898	Je	224.011	Bn	229.999	Ru	16.921
47	Sh	280.880	La	175.186	Hn	83.584	Ws	223.154	Br	229.759	Hn	16.667
48	We	280.591	B	174.558	W	82.807	O	222.933	Ws	228.835	Sj	16.540
49	Wb	279.561	Mg	169.477	Wb	82.778	Mg	222.757	Fl	228.419	Dk	16.478
50	Mg	279.102	Ca	168.121	Pr	82.090	Mo	221.096	Je	227.922	C	16.476
51	Hu	277.741	Jn	164.472	Vi	81.760	Sw	221.055	Mg	227.679	Ja	16.021
52	Lr	276.826	Br	163.876	Ca	81.354	Ca	220.795	Mo	223.547	Ca	16.010
53	Al	275.976	Fr	163.803	Sp	80.660	Le	218.858	Ca	220.601	La	15.823
54	Fu	275.422	Wh	160.658	Sh	79.799	Jn	218.026	Jn	220.097	H	15.683
55	Ho	274.380	C	160.567	Da	77.268	Pl	214.090	Le	215.575	Vg	15.498
56	Jn	272.923	S	158.374	Ru	75.738	St	212.854	Pl	213.473	Mi	15.395
57	Sj	272.607	Ri	158.118	E	75.730	Or	208.949	St	212.846	My	15.190
58	Mr	272.549	Ow	154.507	Gi	74.202	Fl	208.903	Wa	211.573	Sp	14.998
59	A	268.881	T	154.088	Sj	73.494	Wh	208.775	Hr	211.134	Ha	14.781

TABLE C.5
(Continued)

Rank	By h_{Max} County Abbrev.	By h_{Max} Value (m)	By h_{Min} County Abbrev.	By h_{Min} Value (m)	By h_{Range} County Abbrev.	By h_{Range} Value (m)	By h_{avg} County Abbrev.	By h_{avg} Value (m)	By h_{MED} County Abbrev.	By h_{MED} Value (m)	By h_{STD} County Abbrev.	By h_{STD} Value (m)
60	Le	268.609	Ma	153.754	H	72.961	C	208.730	C	209.727	Wb	14.618
61	Mi	266.709	Mo	152.281	Mi	72.751	T	208.591	T	209.699	Da	14.598
62	Bn	263.546	Cy	152.234	Dl	72.102	Js	207.971	Or	208.996	L	14.425
63	Cr	263.481	J	151.757	Sn	71.389	Hr	207.585	Wh	208.721	E	14.393
64	Gr	262.905	Wa	151.063	Wi	70.015	B	207.223	Js	207.957	Su	14.106
65	Pr	257.282	Ws	149.560	K	69.969	Pr	206.869	Cl	204.750	W	14.051
66	Mn	253.996	Fo	145.741	Ha	69.705	Wa	206.761	Lr	204.702	Fu	13.934
67	T	253.062	Lr	143.180	Po	67.417	Ow	205.045	Ow	204.319	Dl	13.771
68	C	252.218	Or	143.083	L	66.655	N	204.117	Pr	204.060	Sn	13.602
69	Pe	250.841	Ve	142.213	Vg	66.152	Cl	202.837	N	203.578	Po	13.471
70	Cy	249.552	P	141.696	Su	65.659	Lr	202.529	B	202.889	Pi	13.323
71	Ca	249.475	Gr	140.866	M	64.592	Fo	199.115	Fo	202.509	Wi	13.156
72	Wh	248.790	D	137.135	Pi	64.180	La	198.376	Cr	198.921	K	13.028
73	Wa	248.586	O	136.524	Hu	63.006	Cr	196.242	P	197.206	Gi	12.635
74	P	247.700	Vi	133.989	Ve	62.800	P	195.611	La	195.233	No	12.539
75	Du	244.940	Mn	130.246	No	61.278	S	192.316	S	188.253	Bo	12.438
76	La	236.024	Sw	128.083	La	60.838	J	189.032	Ve	185.611	Wh	11.603
77	Js	235.681	Je	128.016	Bn	60.585	Cy	187.136	Cy	184.936	Ko	11.393
78	N	235.681	Su	126.371	Ko	60.393	Gr	180.149	Mn	177.031	Hu	11.383
79	St	235.110	Du	125.180	Al	60.316	Mn	179.166	J	176.493	Al	11.367
80	Fo	234.182	Da	123.804	Ho	58.143	Ve	178.303	Gr	172.379	Bn	10.868
81	Pl	232.509	Pi	121.559	Fu	57.782	Vi	168.448	Vi	170.183	Mr	10.839
82	Vi	215.749	Cl	115.824	Mr	56.724	Pe	165.248	Pe	164.515	Ho	9.839
83	Ve	205.013	Hr	115.824	We	55.184	Du	162.723	Du	156.560	M	8.969
84	Da	201.072	Cr	115.811	Bo	50.277	Su	152.451	Su	152.695	We	8.444
85	Su	192.030	Fl	115.675	G	47.769	Da	150.505	Da	147.838	G	7.933
86	Sp	187.340	K	112.663	N	44.756	Pi	144.456	Pi	143.200	N	7.732
87	W	187.048	Pe	107.290	Js	41.358	K	140.524	K	139.823	Js	7.218
88	Pi	185.739	Sp	106.680	A	33.700	Gi	132.255	Gi	131.054	St	6.897
89	K	182.632	Gi	106.187	Ti	33.388	Sp	132.025	Sp	128.844	Ti	6.747
90	Gi	180.389	Vg	104.242	St	33.176	W	129.141	Vg	126.311	A	6.444
91	Vg	170.394	W	104.242	Bl	30.030	Vg	127.619	W	126.193	Bl	6.349
92	Po	166.172	Po	98.755	Pl	29.946	Po	120.196	Po	115.898	Pl	5.608

TABLE C.6
Spatial autocorrelation (Moran's Index) values of the ellipsoidal heights of all 92 Test Areas (counties) in Indiana

County Name	County Abbrev.	IN County Code	Moran's Index of ellipsoidal heights	County Name	County Abbrev.	IN County Code	Moran's Index of ellipsoidal heights
Adams	A	01	0.79642	Lawrence	Lr	47	0.46477
Allen	Al	02	0.85535	Madison	M	48	0.77537
Bartholomew	B	03	0.75286	Marion	Ma	49	0.82434
Benton	Bn	04	0.82805	Marshall	Mr	50	0.78169
Blackford	Bl	05	0.67331	Martin	Mn	51	0.44520
Boone	Bo	06	0.91104	Miami	Mi	52	0.81131
Brown	Br	07	0.45186	Monroe	Mo	53	0.53502
Carroll	C	08	0.75460	Montgomery	My	54	0.76821
Cass	Ca	09	0.81109	Morgan	Mg	55	0.64544
Clark	Cl	10	0.77396	Newton	N	56	0.82398
Clay	Cy	11	0.71447	Noble	No	57	0.73632
Clinton	Cn	12	0.92364	Ohio	O	58	0.57858
Crawford	Cr	13	0.34490	Orange	Or	59	0.48796
Daviess	Da	14	0.71288	Owen	Ow	60	0.63869
Dearborn	D	15	0.74265	Parke	P	61	0.78469
Decatur	De	16	0.91476	Perry	Pe	62	0.48847
DeKalb	Dk	17	0.89169	Pike	Pi	63	0.55650
Delaware	Dl	18	0.87429	Porter	Pr	64	0.91844
Dubois	Du	19	0.63811	Posey	Po	65	0.69487
Elkhart	E	20	0.84167	Pulaski	Pl	66	0.80360
Fayette	F	21	0.71643	Putnam	Pm	67	0.78720
Floyd	Fl	22	0.82959	Randolph	R	68	0.94636
Fountain	Fo	23	0.74507	Ripley	Ri	69	0.67153
Franklin	Fr	24	0.53150	Rush	Ru	70	0.86423
Fulton	Fu	25	0.87979	St. Joseph	Sj	71	0.87810
Gibson	Gi	26	0.62209	Scott	S	72	0.78780
Grant	G	27	0.76185	Shelby	Sh	73	0.86731
Greene	Gr	28	0.76997	Spencer	Sp	74	0.58497
Hamilton	H	29	0.93281	Starke	St	75	0.89227
Hancock	Ha	30	0.86412	Steuben	Sn	76	0.67700
Harrison	Hr	31	0.57622	Sullivan	Su	77	0.73426
Hendricks	He	32	0.90940	Switzerland	Sw	78	0.63231
Henry	Hn	33	0.81716	Tippecanoe	T	79	0.82395
Howard	Ho	34	0.85663	Tipton	Ti	80	0.90195
Huntington	Hu	35	0.75170	Union	U	81	0.72528
Jackson	J	36	0.72705	Vanderburgh	Vg	82	0.70603
Jasper	Js	37	0.85309	Vermillion	Ve	83	0.61376
Jay	Ja	38	0.88586	Vigo	Vi	84	0.79344
Jefferson	Je	39	0.54778	Wabash	Wb	85	0.75170
Jennings	Jn	40	0.87982	Warren	Wa	86	0.74681
Johnson	Jo	41	0.80065	Warrick	W	87	0.67679
Knox	K	42	0.68899	Washington	Ws	88	0.75949
Kosciusko	Ko	43	0.81682	Wayne	Wy	89	0.80916
Lagrange	L	44	0.85600	Wells	We	90	0.81282
Lake	La	45	0.92670	White	Wh	91	0.78490
LaPorte	Le	46	0.91946	Whitley	Wi	92	0.82444

TABLE C.7
Descending orders of all 92 Test Areas (counties) in Indiana, ranked by spatial autocorrelation (Moran's Index) values of the ellipsoidal heights

Rank	County Name	County Abbrev.	IN County Code	Moran's Index of ellipsoidal heights	Rank	County Name	County Abbrev.	IN County Code	Moran's Index of ellipsoidal heights
1	Randolph	R	68	0.94636	47	Madison	M	48	0.77537
2	Hamilton	H	29	0.93281	48	Clark	Cl	10	0.77396
3	Lake	La	45	0.92670	49	Greene	Gr	28	0.76997
4	Clinton	Cn	12	0.92364	50	Montgomery	My	54	0.76821
5	LaPorte	Le	46	0.91946	51	Grant	G	27	0.76185
6	Porter	Pr	64	0.91844	52	Washington	Ws	88	0.75949
7	Decatur	De	16	0.91476	53	Carroll	C	8	0.75460
8	Boone	Bo	6	0.91104	54	Bartholomew	B	3	0.75286
9	Hendricks	He	32	0.90940	55	Huntington	Hu	35	0.75170
10	Tipton	Ti	80	0.90195	56	Wabash	Wb	85	0.75170
11	Starke	St	75	0.89227	57	Warren	Wa	86	0.74681
12	DeKalb	Dk	17	0.89169	58	Fountain	Fo	23	0.74507
13	Jay	Ja	38	0.88586	59	Dearborn	D	15	0.74265
14	Jennings	Jn	40	0.87982	60	Noble	No	57	0.73632
15	Fulton	Fu	25	0.87979	61	Sullivan	Su	77	0.73426
16	St. Joseph	Sj	71	0.87810	62	Jackson	J	36	0.72705
17	Delaware	Dl	18	0.87429	63	Union	U	81	0.72528
18	Shelby	Sh	73	0.86731	64	Fayette	F	21	0.71643
19	Rush	Ru	70	0.86423	65	Clay	Cy	11	0.71447
20	Hancock	Ha	30	0.86412	66	Daviess	Da	14	0.71288
21	Howard	Ho	34	0.85663	67	Vanderburgh	Vg	82	0.70603
22	Lagrange	L	44	0.85600	68	Posey	Po	65	0.69487
23	Allen	Al	2	0.85535	69	Knox	K	42	0.68899
24	Jasper	Js	37	0.85309	70	Steuben	Sn	76	0.67700
25	Elkhart	E	20	0.84167	71	Warrick	W	87	0.67679
26	Floyd	Fl	22	0.82959	72	Blackford	Bl	5	0.67331
27	Benton	Bn	4	0.82805	73	Ripley	Ri	69	0.67153
28	Whitley	Wi	92	0.82444	74	Morgan	Mg	55	0.64544
29	Marion	Ma	49	0.82434	75	Owen	Ow	60	0.63869
30	Newton	N	56	0.82398	76	Dubois	Du	19	0.63811
31	Tippecanoe	T	79	0.82395	77	Switzerland	Sw	78	0.63231
32	Henry	Hn	33	0.81716	78	Gibson	Gi	26	0.62209
33	Kosciusko	Ko	43	0.81682	79	Vermillion	Ve	83	0.61376
34	Wells	We	90	0.81282	80	Spencer	Sp	74	0.58497
35	Miami	Mi	52	0.81131	81	Ohio	O	58	0.57858
36	Cass	Ca	9	0.81109	82	Harrison	Hr	31	0.57622
37	Wayne	Wy	89	0.80916	83	Pike	Pi	63	0.55650
38	Pulaski	Pl	66	0.80360	84	Jefferson	Je	39	0.54778
39	Johnson	Jo	41	0.80065	85	Monroe	Mo	53	0.53502
40	Adams	A	1	0.79642	86	Franklin	Fr	24	0.53150
41	Vigo	Vi	84	0.79344	87	Perry	Pe	62	0.48847
42	Scott	S	72	0.78780	88	Orange	Or	59	0.48796
43	Putnam	Pm	67	0.78720	89	Lawrence	Lr	47	0.46477
44	White	Wh	91	0.78490	90	Brown	Br	7	0.45186
45	Parke	P	61	0.78469	91	Martin	Mn	51	0.44520
46	Marshall	Mr	50	0.78169	92	Crawford	Cr	13	0.34490

APPENDIX D. RESULTS OF THE SCALE VARIATION ANALYSES

This section presents the results of the analyses of the scale variations (see Chapter 2, section 2.1.1) of each Test Areas (counties). Table D.1 presents the values of the maximum scale deviations (from 1) of each of all 92 Test Areas (counties) that are mapped under INCRS mapping with two selected mapping functions. The two mapping functions are the Transverse Mercator (TM(CP)) and the Oblique Stereographic (OS(CP)), without adopting any newly optimized scale factor values, i.e., the scale factor "k" equals to 1.

Table D.2 presents the scale variations results when mapped under INCRS by adopting the new optimized scale factor "k" (k = 1-Δ) computed by the method which is denoted as "Extreme Values Shifting" that arrives at the value of Δ by using the extreme scale values on both ends of the scale values profiles (σ_{Min} and σ_{Max}) to balance the overall scale variation behavior.

Instead of using extreme scale values on both ends of the scale values profiles to balance scale variation behavior, another value such as the average scale (σ_{avg}, $\sigma_{avg} = 1 + \Delta_{avg}$) computed from the scale values at all grid points was also investigated. The mapping correction k is then equal to 1 - Δ_{avg}. The results of the scale variation analysis based on this method can be found in Table D.3.

Another method of balancing the scale variation behavior which has also been investigated, is the use of the scale value at the 50th percentile level (σ_{50}, $\sigma_{50} = 1 + \Delta_{50}$) as the key to redistribute the scale values over all points in the area. It means that the newly adopted mapping correction k will be equal to 1 - Δ_{50}. The scale variation results are presented in Table D.4.

TABLE D.1
Scale values with maximum deviation from 1 (k = 1) of all 92 Test Areas (counties) in Indiana

			Maximum scale value deviation from 1 (when k = 1)			
			INCRS TM(CP)		INCRS OS(CP)	
County Name	County Abbrev.	IN County Code	Scale value	Maximum deviation (ppm)	Scale value	Maximum deviation (ppm)
Adams	A	01	1.000001569	1.57	1.000003105	3.11
Allen	Al	02	1.000006208	6.21	1.000005419	5.42
Bartholomew	B	03	1.000003691	3.69	1.000003743	3.74
Benton	Bn	04	1.000004370	4.37	1.000003529	3.53
Blackford	Bl	05	1.000001577	1.58	1.000001425	1.43
Boone	Bo	06	1.000004895	4.90	1.000003628	3.63
Brown	Br	07	1.000002233	2.23	1.000003015	3.02
Carroll	C	08	1.000003543	3.54	1.000003475	3.48
Cass	Ca	09	1.000003932	3.93	1.000004284	4.28
Clark	Cl	10	1.000008485	8.49	1.000006339	6.34
Clay	Cy	11	1.000001918	1.92	1.000004796	4.80
Clinton	Cn	12	1.000004859	4.86	1.000003610	3.61
Crawford	Cr	13	1.000004677	4.68	1.000004235	4.24
Daviess	Da	14	1.000003747	3.75	1.000005160	5.16
Dearborn	D	15	1.000002570	2.57	1.000004068	4.07
Decatur	De	16	1.000003680	3.68	1.000003737	3.74
DeKalb	Dk	17	1.000003455	3.46	1.000003073	3.07
Delaware	Dl	18	1.000003194	3.19	1.000003300	3.30
Dubois	Du	19	1.000003778	3.78	1.000003991	3.99
Elkhart	E	20	1.000003437	3.44	1.000003822	3.82
Fayette	F	21	1.000001618	1.62	1.000002156	2.16
Floyd	Fl	22	1.000001680	1.68	1.000001871	1.87
Fountain	Fo	23	1.000002496	2.50	1.000004537	4.54
Franklin	Fr	24	1.000005477	5.48	1.000004082	4.08
Fulton	Fu	25	1.000006208	6.21	1.000004284	4.28
Gibson	Gi	26	1.000010504	10.50	1.000007788	7.79
Grant	G	27	1.000003954	3.95	1.000003322	3.32
Greene	Gr	28	1.000007147	7.15	1.000004915	4.92
Hamilton	H	29	1.000003596	3.60	1.000003317	3.32
Hancock	Ha	30	1.000002861	2.86	1.000002613	2.61
Harrison	Hr	31	1.000004696	4.70	1.000006470	6.47
Hendricks	He	32	1.000003239	3.24	1.000003723	3.72
Henry	Hn	33	1.000003612	3.61	1.000003509	3.51
Howard	Ho	34	1.000005797	5.80	1.000003653	3.65
Huntington	Hu	35	1.000002445	2.45	1.000003542	3.54
Jackson	J	36	1.000005560	5.56	1.000004675	4.68
Jasper	Js	37	1.000002775	2.78	1.000007118	7.12
Jay	Ja	38	1.000003962	3.96	1.000003162	3.16
Jefferson	Je	39	1.000005584	5.58	1.000004892	4.89
Jennings	Jn	40	1.000002934	2.93	1.000004249	4.25
Johnson	Jo	41	1.000002213	2.21	1.000002626	2.63

TABLE D.1
(Continued)

County Name	County Abbrev.	IN County Code	Maximum scale value deviation from 1 (when k = 1)			
			INCRS TM(CP)		INCRS OS(CP)	
			Scale value	Maximum deviation (ppm)	Scale value	Maximum deviation (ppm)
Knox	K	42	1.000010431	10.43	1.000009938	9.94
Kosciusko	Ko	43	1.000004294	4.29	1.000004928	4.93
Lagrange	L	44	1.000004664	4.66	1.000003361	3.36
Lake	La	45	1.000002407	2.41	1.000006593	6.59
LaPorte	Le	46	1.000004707	4.71	1.000007739	7.74
Lawrence	Lr	47	1.000003728	3.73	1.000003761	3.76
Madison	M	48	1.000001875	1.88	1.000004495	4.50
Marion	Ma	49	1.000003628	3.63	1.000003517	3.52
Marshall	Mr	50	1.000003861	3.86	1.000003828	3.83
Martin	Mn	51	1.000001399	1.40	1.000003731	3.73
Miami	Mi	52	1.000002135	2.14	1.000004625	4.63
Monroe	Mo	53	1.000003295	3.30	1.000003966	3.97
Montgomery	My	54	1.000003603	3.60	1.000004120	4.12
Morgan	Mg	55	1.000004516	4.52	1.000003775	3.78
Newton	N	56	1.000001561	1.56	1.000005519	5.52
Noble	No	57	1.000004703	4.70	1.000003695	3.70
Ohio	O	58	1.000002241	2.24	1.000001457	1.46
Orange	Or	59	1.000003757	3.76	1.000003397	3.40
Owen	Ow	60	1.000004097	4.10	1.000003751	3.75
Parke	P	61	1.000004044	4.04	1.000004124	4.12
Perry	Pe	62	1.000003816	3.82	1.000005462	5.46
Pike	Pi	63	1.000003775	3.78	1.000003784	3.78
Porter	Pr	64	1.000001804	1.80	1.000005329	5.33
Posey	Po	65	1.000004260	4.26	1.000006252	6.25
Pulaski	Pl	66	1.000004753	4.75	1.000003557	3.56
Putnam	Pm	67	1.000003645	3.65	1.000004851	4.85
Randolph	R	68	1.000003589	3.59	1.000003692	3.69
Ripley	Ri	69	1.000003703	3.70	1.000004880	4.88
Rush	Ru	70	1.000002533	2.53	1.000003370	3.37
St. Joseph	Sj	71	1.000004679	4.68	1.000004441	4.44
Scott	S	72	1.000002597	2.60	1.000002644	2.64
Shelby	Sh	73	1.000002540	2.54	1.000003590	3.59
Spencer	Sp	74	1.000006239	6.24	1.000006402	6.40
Starke	St	75	1.000004717	4.72	1.000003702	3.70
Steuben	Sn	76	1.000003427	3.43	1.000002743	2.74
Sullivan	Su	77	1.000004574	4.57	1.000004830	4.83
Switzerland	Sw	78	1.000004151	4.15	1.000003105	3.11
Tippecanoe	T	79	1.000003566	3.57	1.000004102	4.10
Tipton	Ti	80	1.000003566	3.57	1.000002539	2.54
Union	U	81	1.000001123	1.12	1.000001451	1.45
Vanderburgh	Vg	82	1.000001425	1.43	1.000002818	2.82
Vermillion	Ve	83	1.000000549	0.55	1.000005667	5.67
Vigo	Vi	84	1.000004085	4.09	1.000004360	4.36
Wabash	Wb	85	1.000002130	2.13	1.000004096	4.10
Warren	Wa	86	1.000004411	4.41	1.000004102	4.10
Warrick	W	87	1.000006778	6.78	1.000005930	5.93
Washington	Ws	88	1.000005111	5.11	1.000005098	5.10
Wayne	Wy	89	1.000003619	3.62	1.000003328	3.33
Wells	We	90	1.000003529	3.53	1.000004083	4.08
White	Wh	91	1.000006274	6.27	1.000005452	5.45
Whitley	Wi	92	1.000003107	3.11	1.000003257	3.26

TABLE D.2
Maximum scale value deviations from 1 when adopting k = 1 - Δ of all 92 Test Areas (counties) in Indiana

		Δ = Maximum scale value deviation from 1 (when k = 1 - Δ)					
		INCRS TM(CP)			INCRS OS(CP)		
County Name	IN County Code	Original Max. deviation (ppm)	Δ (ppm)	k = 1 - Δ	Original Max. deviation (ppm)	Δ (ppm)	k = 1 - Δ
Adams	01	1.57	0.78	0.999999216	3.11	1.55	0.999998447
Allen	02	6.21	3.10	0.999996896	5.42	2.71	0.999997290
Bartholomew	03	3.69	1.85	0.999998155	3.74	1.87	0.999998129
Benton	04	4.37	2.19	0.999997815	3.53	1.77	0.999998235
Blackford	05	1.58	0.79	0.999999211	1.43	0.71	0.999999288
Boone	06	4.90	2.45	0.999997553	3.63	1.81	0.999998186
Brown	07	2.23	1.12	0.999998884	3.02	1.51	0.999998492
Carroll	08	3.54	1.77	0.999998228	3.48	1.74	0.999998263
Cass	09	3.93	1.97	0.999998034	4.28	2.14	0.999997858
Clark	10	8.49	4.24	0.999995758	6.34	3.17	0.999996830
Clay	11	1.92	0.96	0.999999041	4.80	2.40	0.999997602
Clinton	12	4.86	2.43	0.999997571	3.61	1.81	0.999998195
Crawford	13	4.68	2.34	0.999997661	4.24	2.12	0.999997883
Daviess	14	3.75	1.87	0.999998127	5.16	2.58	0.999997420
Dearborn	15	2.57	1.29	0.999998715	4.07	2.03	0.999997966
Decatur	16	3.68	1.84	0.999998160	3.74	1.87	0.999998131
DeKalb	17	3.46	1.73	0.999998272	3.07	1.54	0.999998464
Delaware	18	3.19	1.60	0.999998403	3.30	1.65	0.999998350
Dubois	19	3.78	1.89	0.999998111	3.99	2.00	0.999998004
Elkhart	20	3.44	1.72	0.999998281	3.82	1.91	0.999998089
Fayette	21	1.62	0.81	0.999999191	2.16	1.08	0.999998922
Floyd	22	1.68	0.84	0.999999160	1.87	0.94	0.999999065
Fountain	23	2.50	1.25	0.999998752	4.54	2.27	0.999997732
Franklin	24	5.48	2.74	0.999997262	4.08	2.04	0.999997959
Fulton	25	6.21	3.10	0.999996896	4.28	2.14	0.999997858
Gibson	26	10.50	5.25	0.999994748	7.79	3.89	0.999996106
Grant	27	3.95	1.98	0.999998023	3.32	1.66	0.999998339
Greene	28	7.15	3.57	0.999996427	4.92	2.46	0.999997542
Hamilton	29	3.60	1.80	0.999998202	3.32	1.66	0.999998342
Hancock	30	2.86	1.43	0.999998570	2.61	1.31	0.999998693
Harrison	31	4.70	2.35	0.999997652	6.47	3.24	0.999996765
Hendricks	32	3.24	1.62	0.999998380	3.72	1.86	0.999998139
Henry	33	3.61	1.81	0.999998194	3.51	1.75	0.999998246
Howard	34	5.80	2.90	0.999997102	3.65	1.83	0.999998174
Huntington	35	2.45	1.22	0.999998778	3.54	1.77	0.999998229
Jackson	36	5.56	2.78	0.999997220	4.68	2.34	0.999997662
Jasper	37	2.78	1.39	0.999998613	7.12	3.56	0.999996441
Jay	38	3.96	1.98	0.999998019	3.16	1.58	0.999998419
Jefferson	39	5.58	2.79	0.999997208	4.89	2.45	0.999997554
Jennings	40	2.93	1.47	0.999998533	4.25	2.13	0.999997875
Johnson	41	2.21	1.11	0.999998894	2.63	1.31	0.999998687
Knox	42	10.43	5.22	0.999994784	9.94	4.97	0.999995031
Kosciusko	43	4.29	2.15	0.999997853	4.93	2.46	0.999997536
Lagrange	44	4.66	2.33	0.999997668	3.36	1.68	0.999998320
Lake	45	2.41	1.20	0.999998797	6.59	3.30	0.999996703
LaPorte	46	4.71	2.35	0.999997646	7.74	3.87	0.999996131
Lawrence	47	3.73	1.86	0.999998136	3.76	1.88	0.999998120
Madison	48	1.88	0.94	0.999999063	4.50	2.25	0.999997752
Marion	49	3.63	1.81	0.999998186	3.52	1.76	0.999998242
Marshall	50	3.86	1.93	0.999998069	3.83	1.91	0.999998086
Martin	51	1.40	0.70	0.999999300	3.73	1.87	0.999998134
Miami	52	2.14	1.07	0.999998933	4.63	2.31	0.999997687
Monroe	53	3.30	1.65	0.999998352	3.97	1.98	0.999998017
Montgomery	54	3.60	1.80	0.999998198	4.12	2.06	0.999997940
Morgan	55	4.52	2.26	0.999997742	3.78	1.89	0.999998112
Newton	56	1.56	0.78	0.999999220	5.52	2.76	0.999997241
Noble	57	4.70	2.35	0.999997649	3.70	1.85	0.999998152
Ohio	58	2.24	1.12	0.999998879	1.46	0.73	0.999999272

TABLE D.2
(Continued)

County Name	IN County Code	Δ = Maximum scale value deviation from 1 (when k = 1 - Δ)					
		INCRS TM(CP)			INCRS OS(CP)		
		Original Max. deviation (ppm)	Δ (ppm)	k = 1 - Δ	Original Max. deviation (ppm)	Δ (ppm)	k = 1 - Δ
Orange	59	3.76	1.88	0.999998121	3.40	1.70	0.999998302
Owen	60	4.10	2.05	0.999997952	3.75	1.88	0.999998125
Parke	61	4.04	2.02	0.999997978	4.12	2.06	0.999997938
Perry	62	3.82	1.91	0.999998092	5.46	2.73	0.999997269
Pike	63	3.78	1.89	0.999998113	3.78	1.89	0.999998108
Porter	64	1.80	0.90	0.999999098	5.33	2.67	0.999997335
Posey	65	4.26	2.13	0.999997870	6.25	3.13	0.999996874
Pulaski	66	4.75	2.38	0.999997623	3.56	1.78	0.999998221
Putnam	67	3.65	1.82	0.999998177	4.85	2.43	0.999997574
Randolph	68	3.59	1.80	0.999998205	3.69	1.85	0.999998154
Ripley	69	3.70	1.85	0.999998148	4.88	2.44	0.999997560
Rush	70	2.53	1.27	0.999998734	3.37	1.69	0.999998315
St. Joseph	71	4.68	2.34	0.999997661	4.44	2.22	0.999997780
Scott	72	2.60	1.30	0.999998701	2.64	1.32	0.999998678
Shelby	73	2.54	1.27	0.999998730	3.59	1.80	0.999998205
Spencer	74	6.24	3.12	0.999996880	6.40	3.20	0.999996799
Starke	75	4.72	2.36	0.999997641	3.70	1.85	0.999998149
Steuben	76	3.43	1.71	0.999998287	2.74	1.37	0.999998629
Sullivan	77	4.57	2.29	0.999997713	4.83	2.42	0.999997585
Switzerland	78	4.15	2.08	0.999997924	3.11	1.55	0.999998448
Tippecanoe	79	3.57	1.78	0.999998217	4.10	2.05	0.999997949
Tipton	80	3.57	1.78	0.999998217	2.54	1.27	0.999998730
Union	81	1.12	0.56	0.999999438	1.45	0.73	0.999999274
Vanderburgh	82	1.43	0.71	0.999999287	2.82	1.41	0.999998591
Vermillion	83	0.55	0.27	0.999999726	5.67	2.83	0.999997166
Vigo	84	4.09	2.04	0.999997957	4.36	2.18	0.999997820
Wabash	85	2.13	1.07	0.999998935	4.10	2.05	0.999997952
Warren	86	4.41	2.21	0.999997794	4.10	2.05	0.999997949
Warrick	87	6.78	3.39	0.999996611	5.93	2.97	0.999997035
Washington	88	5.11	2.56	0.999997444	5.10	2.55	0.999997451
Wayne	89	3.62	1.81	0.999998190	3.33	1.66	0.999998336
Wells	90	3.53	1.77	0.999998235	4.08	2.04	0.999997958
White	91	6.27	3.14	0.999996863	5.45	2.73	0.999997274
Whitley	92	3.11	1.55	0.999998446	3.26	1.63	0.999998372

TABLE D.3
Maximum scale value deviations from 1 when adopting k = 1 - Δ_{avg} of all 92 Test Areas (counties) in Indiana

		Δ_{avg} = Maximum scale value deviation from 1 when adopting k = 1 - Δ_{avg}					
		INCRS TM(CP)			INCRS OS(CP)		
County Name	IN County Code	Original Max. deviation (ppm)	Δ_{avg} (ppm)	k = 1 - Δ_{avg}	Original Max. deviation (ppm)	Δ_{avg} (ppm)	k = 1 - Δ_{avg}
Adams	01	1.57	0.61	0.999999393	3.11	1.15	0.999998848
Allen	02	6.21	2.23	0.999997770	5.42	1.96	0.999998037
Bartholomew	03	3.69	1.36	0.999998639	3.74	1.38	0.999998619
Benton	04	4.37	1.60	0.999998404	3.53	1.30	0.999998696
Blackford	05	1.58	0.61	0.999999388	1.42	0.56	0.999999443
Boone	06	4.90	1.78	0.999998220	3.63	1.34	0.999998662
Brown	07	2.23	0.85	0.999999153	3.02	1.12	0.999998876
Carroll	08	3.54	1.31	0.999998693	3.47	1.29	0.999998715
Cass	09	3.93	1.44	0.999998559	4.28	1.57	0.999998431
Clark	10	8.49	3.02	0.999996976	6.34	2.28	0.999997716
Clay	11	1.92	0.73	0.999999267	4.80	1.74	0.999998259
Clinton	12	4.86	1.77	0.999998233	3.61	1.33	0.999998669
Crawford	13	4.68	1.71	0.999998292	4.23	1.55	0.999998446
Daviess	14	3.75	1.38	0.999998620	5.16	1.87	0.999998125
Dearborn	15	2.57	0.97	0.999999034	4.07	1.49	0.999998508
Decatur	16	3.68	1.36	0.999998643	3.74	1.38	0.999998621
DeKalb	17	3.46	1.27	0.999998726	3.07	1.14	0.999998857
Delaware	18	3.19	1.18	0.999998815	3.30	1.22	0.999998776
Dubois	19	3.78	1.39	0.999998607	3.99	1.47	0.999998531
Elkhart	20	3.44	1.27	0.999998733	3.82	1.41	0.999998594
Fayette	21	1.62	0.63	0.999999373	2.16	0.82	0.999999181
Floyd	22	1.68	0.65	0.999999349	1.87	0.72	0.999999281
Fountain	23	2.50	0.94	0.999999063	4.54	1.65	0.999998346
Franklin	24	5.48	1.98	0.999998016	4.08	1.50	0.999998502
Fulton	25	6.21	2.23	0.999997767	4.28	1.56	0.999998435
Gibson	26	10.50	3.72	0.999996284	7.79	2.78	0.999997215
Grant	27	3.95	1.45	0.999998549	3.32	1.23	0.999998769
Greene	28	7.15	2.56	0.999997437	4.92	1.79	0.999998213
Hamilton	29	3.60	1.33	0.999998674	3.32	1.23	0.999998770
Hancock	30	2.86	1.07	0.999998931	2.61	0.98	0.999999018
Harrison	31	4.70	1.71	0.999998289	6.47	2.33	0.999997670
Hendricks	32	3.24	1.20	0.999998799	3.72	1.37	0.999998627
Henry	33	3.61	1.33	0.999998668	3.51	1.30	0.999998702
Howard	34	5.80	2.09	0.999997906	3.65	1.34	0.999998658
Huntington	35	2.45	0.92	0.999999081	3.54	1.31	0.999998692
Jackson	36	5.56	2.01	0.999997987	4.68	1.71	0.999998293
Jasper	37	2.78	1.03	0.999998968	7.12	2.54	0.999997456
Jay	38	3.96	1.45	0.999998546	3.16	1.17	0.999998825
Jefferson	39	5.58	2.02	0.999997979	4.89	1.78	0.999998217
Jennings	40	2.93	1.09	0.999998906	4.25	1.56	0.999998443
Johnson	41	2.21	0.84	0.999999160	2.63	0.99	0.999999013
Knox	42	10.43	3.68	0.999996317	9.94	3.53	0.999996473
Kosciusko	43	4.29	1.57	0.999998435	4.93	1.79	0.999998208
Lagrange	44	4.66	1.70	0.999998303	3.36	1.24	0.999998758
Lake	45	2.41	0.90	0.999999098	6.59	2.36	0.999997638
LaPorte	46	4.71	1.70	0.999998295	7.74	2.76	0.999997237
Lawrence	47	3.73	1.37	0.999998626	3.76	1.39	0.999998612
Madison	48	1.88	0.72	0.999999283	4.50	1.64	0.999998363
Marion	49	3.63	1.34	0.999998662	3.52	1.30	0.999998699
Marshall	50	3.86	1.42	0.999998584	3.83	1.41	0.999998591
Martin	51	1.40	0.55	0.999999452	3.73	1.37	0.999998630
Miami	52	2.14	0.81	0.999999192	4.63	1.68	0.999998318
Monroe	53	3.30	1.22	0.999998778	3.97	1.46	0.999998541
Montgomery	54	3.60	1.33	0.999998672	4.12	1.51	0.999998488
Morgan	55	4.52	1.65	0.999998351	3.78	1.39	0.999998608
Newton	56	1.56	0.60	0.999999398	5.52	1.99	0.999998013
Noble	57	4.70	1.71	0.999998290	3.70	1.36	0.999998639
Ohio	58	2.24	0.85	0.999999148	1.46	0.57	0.999999433

TABLE D.3
(Continued)

County Name	IN County Code	Δ_{avg} = Maximum scale value deviation from 1 when adopting k = 1 - Δ_{avg}					
		INCRS TM(CP)			INCRS OS(CP)		
		Original Max. deviation (ppm)	Δ_{avg} (ppm)	k = 1 - Δ_{avg}	Original Max. deviation (ppm)	Δ_{avg} (ppm)	k = 1 - Δ_{avg}
Orange	59	3.76	1.39	0.999998614	3.40	1.26	0.999998740
Owen	60	4.10	1.50	0.999998497	3.75	1.38	0.999998616
Parke	61	4.04	1.48	0.999998517	4.12	1.51	0.999998486
Perry	62	3.82	1.41	0.999998595	5.46	1.98	0.999998019
Pike	63	3.78	1.39	0.999998608	3.78	1.40	0.999998604
Porter	64	1.80	0.69	0.999999311	5.33	1.92	0.999998077
Posey	65	4.26	1.56	0.999998441	6.25	2.25	0.999997746
Pulaski	66	4.75	1.73	0.999998271	3.56	1.31	0.999998688
Putnam	67	3.65	1.34	0.999998658	4.85	1.77	0.999998233
Randolph	68	3.59	1.32	0.999998677	3.69	1.36	0.999998638
Ripley	69	3.70	1.36	0.999998636	4.88	1.78	0.999998223
Rush	70	2.53	0.95	0.999999048	3.37	1.25	0.999998751
St. Joseph	71	4.68	1.70	0.999998301	4.44	1.62	0.999998378
Scott	72	2.60	0.98	0.999999023	2.64	0.99	0.999999006
Shelby	73	2.54	0.95	0.999999045	3.59	1.33	0.999998675
Spencer	74	6.24	2.25	0.999997752	6.40	2.31	0.999997691
Starke	75	4.72	1.72	0.999998285	3.70	1.36	0.999998637
Steuben	76	3.43	1.26	0.999998735	2.74	1.03	0.999998974
Sullivan	77	4.57	1.67	0.999998332	4.83	1.76	0.999998239
Switzerland	78	4.15	1.52	0.999998476	3.10	1.16	0.999998845
Tippecanoe	79	3.57	1.31	0.999998686	4.10	1.51	0.999998495
Tipton	80	3.57	1.32	0.999998683	2.54	0.95	0.999999047
Union	81	1.12	0.45	0.999999552	1.45	0.57	0.999999434
Vanderburgh	82	1.43	0.56	0.999999441	2.82	1.05	0.999998948
Vermillion	83	0.55	0.23	0.999999767	5.67	2.03	0.999997973
Vigo	84	4.09	1.50	0.999998502	4.36	1.60	0.999998403
Wabash	85	2.13	0.81	0.999999194	4.10	1.50	0.999998501
Warren	86	4.41	1.61	0.999998390	4.10	1.51	0.999998494
Warrick	87	6.78	2.44	0.999997565	5.93	2.14	0.999997855
Washington	88	5.11	1.86	0.999998143	5.10	1.86	0.999998145
Wayne	89	3.62	1.33	0.999998665	3.33	1.23	0.999998765
Wells	90	3.53	1.30	0.999998700	4.08	1.50	0.999998502
White	91	6.27	2.25	0.999997746	5.45	1.98	0.999998025
Whitley	92	3.11	1.15	0.999998848	3.26	1.21	0.999998792

TABLE D.4
Maximum scale value deviations from 1 when adopting $k = 1 - \Delta_{50}$ of all 92 Test Areas (counties) in Indiana

		Δ_{50} = Maximum scale value deviation from 1 when adopting $k = 1 - \Delta_{50}$					
		INCRS TM(CP)			INCRS OS(CP)		
County Name	IN County Code	Original Max. deviation (ppm)	Δ_{50} (ppm)	$k = 1 - \Delta_{50}$	Original Max. deviation (ppm)	Δ_{50} (ppm)	$k = 1 - \Delta_{50}$
Adams	01	1.57	0.39	0.999999609	3.11	0.98	0.999999023
Allen	02	6.21	1.55	0.999998452	5.42	1.87	0.999998133
Bartholomew	03	3.69	1.13	0.999998869	3.74	1.28	0.999998724
Benton	04	4.37	1.09	0.999998910	3.53	1.23	0.999998772
Blackford	05	1.58	0.39	0.999999606	1.42	0.53	0.999999467
Boone	06	4.90	1.34	0.999998662	3.63	1.21	0.999998786
Brown	07	2.23	0.72	0.999999276	3.02	1.04	0.999998963
Carroll	08	3.54	1.09	0.999998914	3.47	1.23	0.999998771
Cass	09	3.93	1.09	0.999998907	4.28	1.54	0.999998464
Clark	10	8.49	2.28	0.999997719	6.34	2.08	0.999997923
Clay	11	1.92	0.55	0.999999447	4.80	1.42	0.999998579
Clinton	12	4.86	1.33	0.999998672	3.61	1.21	0.999998787
Crawford	13	4.68	1.17	0.999998833	4.23	1.50	0.999998505
Daviess	14	3.75	1.15	0.999998854	5.16	1.73	0.999998271
Dearborn	15	2.57	0.74	0.999999262	4.07	1.33	0.999998673
Decatur	16	3.68	1.13	0.999998872	3.74	1.28	0.999998724
DeKalb	17	3.46	1.06	0.999998940	3.07	1.10	0.999998902
Delaware	18	3.19	0.89	0.999999109	3.30	1.19	0.999998808
Dubois	19	3.78	1.16	0.999998842	3.99	1.40	0.999998597
Elkhart	20	3.44	1.05	0.999998947	3.82	1.37	0.999998630
Fayette	21	1.62	0.40	0.999999596	2.16	0.76	0.999999241
Floyd	22	1.68	0.42	0.999999581	1.87	0.71	0.999999291
Fountain	23	2.50	0.72	0.999999284	4.54	1.41	0.999998586
Franklin	24	5.48	1.62	0.999998380	4.08	1.37	0.999998629
Fulton	25	6.21	1.55	0.999998451	4.28	1.38	0.999998621
Gibson	26	10.50	2.96	0.999997035	7.79	2.51	0.999997486
Grant	27	3.95	1.10	0.999998899	3.32	1.18	0.999998822
Greene	28	7.15	1.93	0.999998075	4.92	1.49	0.999998507
Hamilton	29	3.60	1.10	0.999998897	3.32	1.19	0.999998809
Hancock	30	2.86	0.71	0.999999286	2.61	0.93	0.999999067
Harrison	31	4.70	1.17	0.999998830	6.47	2.17	0.999997826
Hendricks	32	3.24	0.90	0.999999097	3.72	1.31	0.999998695
Henry	33	3.61	1.11	0.999998893	3.51	1.23	0.999998767
Howard	34	5.80	1.58	0.999998417	3.65	1.11	0.999998887
Huntington	35	2.45	0.70	0.999999298	3.54	1.21	0.999998794
Jackson	36	5.56	1.64	0.999998357	4.68	1.57	0.999998427
Jasper	37	2.78	0.69	0.999999309	7.12	2.06	0.999997941
Jay	38	3.96	1.10	0.999998896	3.16	1.08	0.999998916
Jefferson	39	5.58	1.65	0.999998350	4.89	1.71	0.999998294
Jennings	40	2.93	0.73	0.999999268	4.25	1.46	0.999998541
Johnson	41	2.21	0.72	0.999999282	2.63	0.94	0.999999059
Knox	42	10.43	2.94	0.999997064	9.94	3.34	0.999996662
Kosciusko	43	4.29	1.07	0.999998930	4.93	1.71	0.999998285
Lagrange	44	4.66	1.28	0.999998725	3.36	1.08	0.999998919
Lake	45	2.41	0.69	0.999999312	6.59	1.94	0.999998059
LaPorte	46	4.71	1.28	0.999998719	7.74	2.44	0.999997565
Lawrence	47	3.73	1.14	0.999998857	3.76	1.28	0.999998723
Madison	48	1.88	0.54	0.999999460	4.50	1.35	0.999998646
Marion	49	3.63	1.11	0.999998888	3.52	1.23	0.999998767
Marshall	50	3.86	1.07	0.999998926	3.83	1.32	0.999998676
Martin	51	1.40	0.42	0.999999575	3.73	1.08	0.999998916
Miami	52	2.14	0.69	0.999999310	4.63	1.38	0.999998621
Monroe	53	3.30	0.92	0.999999081	3.97	1.35	0.999998650
Montgomery	54	3.60	1.10	0.999998897	4.12	1.44	0.999998557
Morgan	55	4.52	1.13	0.999998873	3.78	1.27	0.999998725
Newton	56	1.56	0.39	0.999999611	5.52	1.57	0.999998431
Noble	57	4.70	1.29	0.999998715	3.70	1.24	0.999998764

TABLE D.4
(Continued)

County Name	IN County Code	Δ_{50} = Maximum scale value deviation from 1 when adopting $k = 1 - \Delta_{50}$ INCRS TM(CP) Original Max. deviation (ppm)	INCRS TM(CP) Δ_{50} (ppm)	INCRS TM(CP) $k = 1 - \Delta_{50}$	INCRS OS(CP) Original Max. deviation (ppm)	INCRS OS(CP) Δ_{50} (ppm)	INCRS OS(CP) $k = 1 - \Delta_{50}$
Ohio	58	2.24	0.73	0.999999270	1.46	0.45	0.999999550
Orange	59	3.76	1.15	0.999998847	3.40	1.21	0.999998792
Owen	60	4.10	1.14	0.999998860	3.75	1.31	0.999998691
Parke	61	4.04	1.12	0.999998875	4.12	1.44	0.999998556
Perry	62	3.82	1.17	0.999998833	5.46	1.83	0.999998166
Pike	63	3.78	1.16	0.999998843	3.78	1.28	0.999998722
Porter	64	1.80	0.52	0.999999481	5.33	1.53	0.999998467
Posey	65	4.26	1.18	0.999998817	6.25	2.07	0.999997933
Pulaski	66	4.75	1.30	0.999998701	3.56	1.20	0.999998798
Putnam	67	3.65	1.12	0.999998885	4.85	1.62	0.999998377
Randolph	68	3.59	1.10	0.999998900	3.69	1.27	0.999998727
Ripley	69	3.70	1.13	0.999998867	4.88	1.64	0.999998357
Rush	70	2.53	0.73	0.999999273	3.37	1.17	0.999998828
St. Joseph	71	4.68	1.28	0.999998723	4.44	1.55	0.999998454
Scott	72	2.60	0.75	0.999999253	2.64	0.99	0.999999008
Shelby	73	2.54	0.73	0.999999271	3.59	1.22	0.999998778
Spencer	74	6.24	1.70	0.999998301	6.40	2.20	0.999997802
Starke	75	4.72	1.29	0.999998711	3.70	1.24	0.999998762
Steuben	76	3.43	1.05	0.999998948	2.74	0.95	0.999999051
Sullivan	77	4.57	1.14	0.999998859	4.83	1.71	0.999998294
Switzerland	78	4.15	1.16	0.999998843	3.10	1.04	0.999998962
Tippecanoe	79	3.57	1.09	0.999998908	4.10	1.44	0.999998563
Tipton	80	3.57	1.10	0.999998904	2.54	0.85	0.999999154
Union	81	1.12	0.40	0.999999597	1.45	0.52	0.999999484
Vanderburgh	82	1.43	0.43	0.999999567	2.82	0.90	0.999999101
Vermillion	83	0.55	0.19	0.999999812	5.67	1.55	0.999998446
Vigo	84	4.09	1.14	0.999998864	4.36	1.55	0.999998451
Wabash	85	2.13	0.69	0.999999311	4.10	1.30	0.999998701
Warren	86	4.41	1.10	0.999998900	4.10	1.44	0.999998561
Warrick	87	6.78	1.69	0.999998309	5.93	2.03	0.999997973
Washington	88	5.11	1.40	0.999998605	5.10	1.75	0.999998249
Wayne	89	3.62	1.11	0.999998890	3.33	1.19	0.999998806
Wells	90	3.53	1.08	0.999998919	4.08	1.43	0.999998569
White	91	6.27	1.56	0.999998435	5.45	1.89	0.999998114
Whitley	92	3.11	0.87	0.999999133	3.26	1.17	0.999998826

APPENDIX E. RESULTS OF THE MARION COUNTY TEST

This section presents the results of the Marion County Test. Table E.1 presents the geodetic coordinates of the sampled grid points in Marion County as well as the HARN station points: ZID A, ZID B, F 350, and IMAGIS 47. The map coordinates of the points in the form of Easting and Northing coordinates under Indiana State Plane Coordinate System of 1983 (INSPCS83) by NGS are also listed in this table.

Table E.2 presents the map coordinates (Easting and Northing) of points the proposed INCRS mapping by two different mapping functions: Transverse Mercator (TM(CP)) and Oblique Stereographic (OS(CP)). Table E.2 includes the results of both methods (INCRS TM(CP) and INCRS OS(CP)) mapped under two different cases. These are (1) the case whereby no terrain elevations are involved (Case h_0). It uses the INCRS Sphere with the radius of $R_{G@CP}$ as the reference surface and (2) the case whereby terrain elevations are involved (Case h_{Real}). It uses the "inflated" INCRS Sphere with the radius of $R_{G@CP} + h_{avg}$ as the reference surface.

The INCRS-S01 coordinates of points in Marion County Test are available in Table E.3 under the assumption that the real ellipsoidal heights are used. The coordinates are also in the form of Easting and Northing coordinates.

TABLE E.1
Geodetic coordinates of points in Marion County Test and the corresponding map coordinates under the INSPCS83 by NGS

		Geodetic coordinates of points in Marion County						Height above GRS80 ellipsoid	INSPCS83 (NGS)	
		Longitude (West)			Latitude (North)				Easting	Northing
Row ID	Point ID	deg.	min.	sec.	deg.	min.	sec.	(m)	(m)	(m)
01	A01	86	21	00.00000	39	38	00.00000	178.600	41338.317	487030.727
02	A02	86	21	00.00000	39	39	00.00000	183.808	41352.397	488881.153
03	A03	86	21	00.00000	39	40	00.00000	193.289	41366.482	490731.584
04	A04	86	21	00.00000	39	41	00.00000	202.392	41380.573	492582.020
05	A05	86	21	00.00000	39	42	00.00000	204.182	41394.668	494432.461
06	A06	86	21	00.00000	39	43	00.00000	206.779	41408.768	496282.908
07	A07	86	21	00.00000	39	44	00.00000	214.173	41422.873	498133.360
08	A08	86	21	00.00000	39	45	00.00000	217.668	41436.983	499983.817
09	A09	86	21	00.00000	39	46	00.00000	221.400	41451.099	501834.280
10	A10	86	21	00.00000	39	47	00.00000	222.852	41465.219	503684.747
11	A11	86	21	00.00000	39	48	00.00000	229.161	41479.344	505535.220
12	A12	86	21	00.00000	39	49	00.00000	230.494	41493.474	507385.698
13	A13	86	21	00.00000	39	50	00.00000	233.460	41507.609	509236.182
14	A14	86	21	00.00000	39	51	00.00000	234.654	41521.749	511086.671
15	A15	86	21	00.00000	39	52	00.00000	236.882	41535.894	512937.164
16	A16	86	21	00.00000	39	53	00.00000	242.573	41550.044	514787.664
17	A17	86	21	00.00000	39	54	00.00000	244.219	41564.199	516638.168
18	A18	86	21	00.00000	39	55	00.00000	244.459	41578.359	518488.678
19	A19	86	21	00.00000	39	56	00.00000	248.056	41592.524	520339.193
20	B01	86	19	40.00000	39	38	00.00000	199.902	43246.030	487016.449
21	B02	86	19	40.00000	39	39	00.00000	193.607	43259.652	488866.874
22	B03	86	19	40.00000	39	40	00.00000	189.319	43273.279	490717.303
23	B04	86	19	40.00000	39	41	00.00000	195.182	43286.911	492567.737
24	B05	86	19	40.00000	39	42	00.00000	186.681	43300.548	494418.177
25	B06	86	19	40.00000	39	43	00.00000	190.884	43314.190	496268.622
26	B07	86	19	40.00000	39	44	00.00000	196.753	43327.836	498119.073
27	B08	86	19	40.00000	39	45	00.00000	202.960	43341.488	499969.528
28	B09	86	19	40.00000	39	46	00.00000	212.363	43355.144	501819.989
29	B10	86	19	40.00000	39	47	00.00000	215.763	43368.805	503670.455
30	B11	86	19	40.00000	39	48	00.00000	221.271	43382.470	505520.927
31	B12	86	19	40.00000	39	49	00.00000	225.459	43396.141	507371.404
32	B13	86	19	40.00000	39	50	00.00000	228.466	43409.816	509221.885
33	B14	86	19	40.00000	39	51	00.00000	232.152	43423.496	511072.373
34	B15	86	19	40.00000	39	52	00.00000	234.698	43437.181	512922.865
35	B16	86	19	40.00000	39	53	00.00000	238.653	43450.871	514773.363
36	B17	86	19	40.00000	39	54	00.00000	240.853	43464.566	516623.866
37	B18	86	19	40.00000	39	55	00.00000	244.425	43478.265	518474.374
38	B19	86	19	40.00000	39	56	00.00000	238.267	43491.969	520324.887
39	C01	86	18	20.00000	39	38	00.00000	210.785	45153.742	487002.644
40	C02	86	18	20.00000	39	39	00.00000	208.330	45166.906	488853.066
41	C03	86	18	20.00000	39	40	00.00000	203.921	45180.075	490703.494
42	C04	86	18	20.00000	39	41	00.00000	206.705	45193.249	492553.927
43	C05	86	18	20.00000	39	42	00.00000	207.053	45206.427	494404.366

TABLE E.1
(Continued)

		Geodetic coordinates of points in Marion County						Height above GRS80 ellipsoid	INSPCS83 (NGS)	
		Longitude (West)			Latitude (North)				Easting	Northing
Row ID	Point ID	deg.	min.	sec.	deg.	min.	sec.	(m)	(m)	(m)
44	C06	86	18	20.00000	39	43	00.00000	197.071	45219.610	496254.809
45	C07	86	18	20.00000	39	44	00.00000	197.984	45232.798	498105.258
46	C08	86	18	20.00000	39	45	00.00000	201.160	45245.990	499955.712
47	C09	86	18	20.00000	39	46	00.00000	201.619	45259.187	501806.172
48	C10	86	18	20.00000	39	47	00.00000	207.113	45272.389	503656.636
49	C11	86	18	20.00000	39	48	00.00000	208.982	45285.595	505507.106
50	C12	86	18	20.00000	39	49	00.00000	196.573	45298.806	507357.581
51	C13	86	18	20.00000	39	50	00.00000	208.800	45312.022	509208.062
52	C14	86	18	20.00000	39	51	00.00000	209.645	45325.242	511058.547
53	C15	86	18	20.00000	39	52	00.00000	207.733	45338.467	512909.038
54	C16	86	18	20.00000	39	53	00.00000	207.326	45351.696	514759.535
55	C17	86	18	20.00000	39	54	00.00000	235.046	45364.931	516610.036
56	C18	86	18	20.00000	39	55	00.00000	238.526	45378.170	518460.543
57	C19	86	18	20.00000	39	56	00.00000	242.852	45391.413	520311.055
58	D01	86	17	00.00000	39	38	00.00000	195.031	47061.452	486989.310
59	D02	86	17	00.00000	39	39	00.00000	192.793	47074.158	488839.731
60	D03	86	17	00.00000	39	40	00.00000	189.789	47086.869	490690.158
61	D04	86	17	00.00000	39	41	00.00000	204.304	47099.585	492540.589
62	D05	86	17	00.00000	39	42	00.00000	208.084	47112.305	494391.026
63	D06	86	17	00.00000	39	43	00.00000	206.471	47125.029	496241.468
64	D07	86	17	00.00000	39	44	00.00000	208.167	47137.758	498091.916
65	D08	86	17	00.00000	39	45	00.00000	206.464	47150.491	499942.368
66	D09	86	17	00.00000	39	46	00.00000	207.273	47163.229	501792.826
67	D10	86	17	00.00000	39	47	00.00000	199.911	47175.972	503643.289
68	D11	86	17	00.00000	39	48	00.00000	200.340	47188.719	505493.758
69	D12	86	17	00.00000	39	49	00.00000	206.953	47201.470	507344.232
70	D13	86	17	00.00000	39	50	00.00000	211.437	47214.226	509194.711
71	D14	86	17	00.00000	39	51	00.00000	219.949	47226.986	511045.195
72	D15	86	17	00.00000	39	52	00.00000	226.728	47239.751	512895.685
73	D16	86	17	00.00000	39	53	00.00000	220.830	47252.520	514746.179
74	D17	86	17	00.00000	39	54	00.00000	220.414	47265.294	516596.679
75	D18	86	17	00.00000	39	55	00.00000	212.099	47278.073	518447.185
76	D19	86	17	00.00000	39	56	00.00000	228.284	47290.855	520297.695
77	E01	86	15	40.00000	39	38	00.00000	175.597	48969.160	486976.449
78	E02	86	15	40.00000	39	39	00.00000	179.285	48981.409	488826.868
79	E03	86	15	40.00000	39	40	00.00000	187.181	48993.662	490677.293
80	E04	86	15	40.00000	39	41	00.00000	182.677	49005.919	492527.723
81	E05	86	15	40.00000	39	42	00.00000	194.567	49018.180	494378.159
82	E06	86	15	40.00000	39	43	00.00000	196.171	49030.446	496228.600
83	E07	86	15	40.00000	39	44	00.00000	197.564	49042.716	498079.046
84	E08	86	15	40.00000	39	45	00.00000	202.362	49054.991	499929.497
85	E09	86	15	40.00000	39	46	00.00000	201.153	49067.270	501779.953
86	E10	86	15	40.00000	39	47	00.00000	186.956	49079.553	503630.415
87	E11	86	15	40.00000	39	48	00.00000	195.159	49091.841	505480.882
88	E12	86	15	40.00000	39	49	00.00000	200.051	49104.132	507331.355
89	E13	86	15	40.00000	39	50	00.00000	205.748	49116.429	509181.832
90	E14	86	15	40.00000	39	51	00.00000	212.438	49128.729	511032.315
91	E15	86	15	40.00000	39	52	00.00000	217.970	49141.034	512882.803
92	E16	86	15	40.00000	39	53	00.00000	227.623	49153.343	514733.297
93	E17	86	15	40.00000	39	54	00.00000	231.842	49165.657	516583.796
94	E18	86	15	40.00000	39	55	00.00000	233.086	49177.974	518434.300
95	E19	86	15	40.00000	39	56	00.00000	227.136	49190.296	520284.809
96	F01	86	14	20.00000	39	38	00.00000	165.396	50876.867	486964.059
97	F02	86	14	20.00000	39	39	00.00000	168.123	50888.658	488814.477
98	F03	86	14	20.00000	39	40	00.00000	169.594	50900.453	490664.901
99	F04	86	14	20.00000	39	41	00.00000	173.779	50912.252	492515.330
100	F05	86	14	20.00000	39	42	00.00000	175.591	50924.055	494365.764
101	F06	86	14	20.00000	39	43	00.00000	181.692	50935.862	496216.203

TABLE E.1
(Continued)

		Geodetic coordinates of points in Marion County						Height above GRS80 ellipsoid	INSPCS83 (NGS)	
		Longitude (West)			Latitude (North)				Easting	Northing
Row ID	Point ID	deg.	min.	sec.	deg.	min.	sec.	(m)	(m)	(m)
102	F07	86	14	20.00000	39	44	00.00000	185.566	50947.674	498066.648
103	F08	86	14	20.00000	39	45	00.00000	188.186	50959.489	499917.098
104	F09	86	14	20.00000	39	46	00.00000	186.157	50971.309	501767.553
105	F10	86	14	20.00000	39	47	00.00000	187.053	50983.133	503618.013
106	F11	86	14	20.00000	39	48	00.00000	191.571	50994.961	505468.479
107	F12	86	14	20.00000	39	49	00.00000	194.261	51006.793	507318.950
108	F13	86	14	20.00000	39	50	00.00000	199.246	51018.630	509169.427
109	F14	86	14	20.00000	39	51	00.00000	207.643	51030.471	511019.908
110	F15	86	14	20.00000	39	52	00.00000	214.740	51042.315	512870.395
111	F16	86	14	20.00000	39	53	00.00000	221.011	51054.164	514720.887
112	F17	86	14	20.00000	39	54	00.00000	228.039	51066.017	516571.385
113	F18	86	14	20.00000	39	55	00.00000	233.923	51077.875	518421.887
114	F19	86	14	20.00000	39	56	00.00000	237.240	51089.736	520272.395
115	G01	86	13	00.00000	39	38	00.00000	167.294	52784.573	486952.142
116	G02	86	13	00.00000	39	39	00.00000	168.235	52795.906	488802.559
117	G03	86	13	00.00000	39	40	00.00000	168.918	52807.242	490652.981
118	G04	86	13	00.00000	39	41	00.00000	170.179	52818.583	492503.408
119	G05	86	13	00.00000	39	42	00.00000	171.545	52829.928	494353.841
120	G06	86	13	00.00000	39	43	00.00000	177.675	52841.277	496204.279
121	G07	86	13	00.00000	39	44	00.00000	179.864	52852.629	498054.722
122	G08	86	13	00.00000	39	45	00.00000	180.505	52863.986	499905.171
123	G09	86	13	00.00000	39	46	00.00000	181.032	52875.347	501755.625
124	G10	86	13	00.00000	39	47	00.00000	187.539	52886.712	503606.084
125	G11	86	13	00.00000	39	48	00.00000	191.748	52898.080	505456.549
126	G12	86	13	00.00000	39	49	00.00000	192.446	52909.453	507307.018
127	G13	86	13	00.00000	39	50	00.00000	195.842	52920.830	509157.493
128	G14	86	13	00.00000	39	51	00.00000	203.341	52932.211	511007.974
129	G15	86	13	00.00000	39	52	00.00000	207.228	52943.595	512858.459
130	G16	86	13	00.00000	39	53	00.00000	215.516	52954.984	514708.950
131	G17	86	13	00.00000	39	54	00.00000	220.589	52966.377	516559.446
132	G18	86	13	00.00000	39	55	00.00000	225.510	52977.774	518409.948
133	G19	86	13	00.00000	39	56	00.00000	233.435	52989.174	520260.455
134	H01	86	11	40.00000	39	38	00.00000	172.391	54692.278	486940.696
135	H02	86	11	40.00000	39	39	00.00000	201.773	54703.152	488791.112
136	H03	86	11	40.00000	39	40	00.00000	171.819	54714.031	490641.533
137	H04	86	11	40.00000	39	41	00.00000	174.882	54724.913	492491.959
138	H05	86	11	40.00000	39	42	00.00000	170.996	54735.800	494342.391
139	H06	86	11	40.00000	39	43	00.00000	173.368	54746.690	496192.827
140	H07	86	11	40.00000	39	44	00.00000	177.158	54757.584	498043.269
141	H08	86	11	40.00000	39	45	00.00000	179.458	54768.482	499893.717
142	H09	86	11	40.00000	39	46	00.00000	181.300	54779.383	501744.169
143	H10	86	11	40.00000	39	47	00.00000	178.766	54790.289	503594.627
144	H11	86	11	40.00000	39	48	00.00000	181.785	54801.198	505445.091
145	H12	86	11	40.00000	39	49	00.00000	179.645	54812.111	507295.559
146	H13	86	11	40.00000	39	50	00.00000	198.783	54823.029	509146.033
147	H14	86	11	40.00000	39	51	00.00000	198.638	54833.949	510996.512
148	H15	86	11	40.00000	39	52	00.00000	203.299	54844.874	512846.996
149	H16	86	11	40.00000	39	53	00.00000	207.925	54855.803	514697.486
150	H17	86	11	40.00000	39	54	00.00000	214.031	54866.735	516547.981
151	H18	86	11	40.00000	39	55	00.00000	223.736	54877.671	518398.481
152	H19	86	11	40.00000	39	56	00.00000	230.231	54888.611	520248.987
153	I01	86	10	20.00000	39	38	00.00000	180.651	56599.981	486929.723
154	I02	86	10	20.00000	39	39	00.00000	182.313	56610.397	488780.137
155	I03	86	10	20.00000	39	40	00.00000	181.513	56620.818	490630.557
156	I04	86	10	20.00000	39	41	00.00000	180.392	56631.242	492480.982
157	I05	86	10	20.00000	39	42	00.00000	180.471	56641.670	494331.412
158	I06	86	10	20.00000	39	43	00.00000	177.086	56652.102	496181.848
159	I07	86	10	20.00000	39	44	00.00000	169.365	56662.537	498032.289

TABLE E.1
(Continued)

		Geodetic coordinates of points in Marion County						Height above GRS80 ellipsoid	INSPCS83 (NGS)	
		Longitude (West)			Latitude (North)				Easting	Northing
Row ID	Point ID	deg.	min.	sec.	deg.	min.	sec.	(m)	(m)	(m)
160	I08	86	10	20.00000	39	45	00.00000	177.584	56672.976	499882.735
161	I09	86	10	20.00000	39	46	00.00000	177.846	56683.419	501733.186
162	I10	86	10	20.00000	39	47	00.00000	179.363	56693.865	503583.643
163	I11	86	10	20.00000	39	48	00.00000	182.447	56704.315	505434.105
164	I12	86	10	20.00000	39	49	00.00000	192.735	56714.769	507284.572
165	I13	86	10	20.00000	39	50	00.00000	199.140	56725.226	509135.045
166	I14	86	10	20.00000	39	51	00.00000	181.538	56735.687	510985.523
167	I15	86	10	20.00000	39	52	00.00000	207.434	56746.152	512836.006
168	I16	86	10	20.00000	39	53	00.00000	209.032	56756.620	514686.495
169	I17	86	10	20.00000	39	54	00.00000	200.138	56767.092	516536.988
170	I18	86	10	20.00000	39	55	00.00000	204.128	56777.568	518387.488
171	I19	86	10	20.00000	39	56	00.00000	216.613	56788.047	520237.992
172	J01	86	09	00.00000	39	38	00.00000	195.392	58507.683	486919.221
173	J02	86	09	00.00000	39	39	00.00000	193.941	58517.641	488769.635
174	J03	86	09	00.00000	39	40	00.00000	194.913	58527.604	490620.053
175	J04	86	09	00.00000	39	41	00.00000	186.482	58537.570	492470.477
176	J05	86	09	00.00000	39	42	00.00000	181.557	58547.539	494320.906
177	J06	86	09	00.00000	39	43	00.00000	189.447	58557.512	496171.341
178	J07	86	09	00.00000	39	44	00.00000	180.960	58567.489	498021.780
179	J08	86	09	00.00000	39	45	00.00000	186.855	58577.469	499872.225
180	J09	86	09	00.00000	39	46	00.00000	183.439	58587.453	501722.676
181	J10	86	09	00.00000	39	47	00.00000	186.235	58597.440	503573.131
182	J11	86	09	00.00000	39	48	00.00000	183.745	58607.430	505423.592
183	J12	86	09	00.00000	39	49	00.00000	188.542	58617.425	507274.058
184	J13	86	09	00.00000	39	50	00.00000	189.321	58627.422	509124.530
185	J14	86	09	00.00000	39	51	00.00000	189.838	58637.423	510975.007
186	J15	86	09	00.00000	39	52	00.00000	184.373	58647.428	512825.489
187	J16	86	09	00.00000	39	53	00.00000	198.353	58657.436	514675.976
188	J17	86	09	00.00000	39	54	00.00000	203.621	58667.448	516526.469
189	J18	86	09	00.00000	39	55	00.00000	210.982	58677.463	518376.967
190	J19	86	09	00.00000	39	56	00.00000	221.185	58687.482	520227.470
191	K01	86	07	40.00000	39	38	00.00000	201.189	60415.383	486909.192
192	K02	86	07	40.00000	39	39	00.00000	197.745	60424.884	488759.604
193	K03	86	07	40.00000	39	40	00.00000	196.689	60434.389	490610.021
194	K04	86	07	40.00000	39	41	00.00000	199.996	60443.896	492460.444
195	K05	86	07	40.00000	39	42	00.00000	190.570	60453.408	494310.872
196	K06	86	07	40.00000	39	43	00.00000	198.052	60462.922	496161.306
197	K07	86	07	40.00000	39	44	00.00000	196.061	60472.440	498011.744
198	K08	86	07	40.00000	39	45	00.00000	189.824	60481.961	499862.188
199	K09	86	07	40.00000	39	46	00.00000	196.374	60491.486	501712.637
200	K10	86	07	40.00000	39	47	00.00000	191.695	60501.013	503563.092
201	K11	86	07	40.00000	39	48	00.00000	187.045	60510.545	505413.552
202	K12	86	07	40.00000	39	49	00.00000	187.943	60520.079	507264.017
203	K13	86	07	40.00000	39	50	00.00000	191.125	60529.617	509114.487
204	K14	86	07	40.00000	39	51	00.00000	194.048	60539.159	510964.963
205	K15	86	07	40.00000	39	52	00.00000	192.831	60548.703	512815.444
206	K16	86	07	40.00000	39	53	00.00000	185.237	60558.251	514665.930
207	K17	86	07	40.00000	39	54	00.00000	197.707	60567.803	516516.422
208	K18	86	07	40.00000	39	55	00.00000	202.340	60577.358	518366.919
209	K19	86	07	40.00000	39	56	00.00000	210.137	60586.916	520217.421
210	L01	86	06	20.00000	39	38	00.00000	212.290	62323.083	486899.634
211	L02	86	06	20.00000	39	39	00.00000	207.262	62332.126	488750.045
212	L03	86	06	20.00000	39	40	00.00000	199.382	62341.172	490600.462
213	L04	86	06	20.00000	39	41	00.00000	210.025	62350.222	492450.884
214	L05	86	06	20.00000	39	42	00.00000	203.332	62359.275	494301.311
215	L06	86	06	20.00000	39	43	00.00000	203.874	62368.330	496151.743
216	L07	86	06	20.00000	39	44	00.00000	197.559	62377.390	498002.180
217	L08	86	06	20.00000	39	45	00.00000	203.355	62386.452	499852.623

TABLE E.1
(Continued)

		Geodetic coordinates of points in Marion County						Height above GRS80 ellipsoid	INSPCS83 (NGS)	
		Longitude (West)			Latitude (North)				Easting	Northing
Row ID	Point ID	deg.	min.	sec.	deg.	min.	sec.	(m)	(m)	(m)
218	L09	86	06	20.00000	39	46	00.00000	203.236	62395.517	501703.072
219	L10	86	06	20.00000	39	47	00.00000	204.681	62404.586	503553.525
220	L11	86	06	20.00000	39	48	00.00000	204.446	62413.658	505403.984
221	L12	86	06	20.00000	39	49	00.00000	202.543	62422.733	507254.448
222	L13	86	06	20.00000	39	50	00.00000	200.615	62431.811	509104.917
223	L14	86	06	20.00000	39	51	00.00000	193.839	62440.893	510955.392
224	L15	86	06	20.00000	39	52	00.00000	198.937	62449.978	512805.872
225	L16	86	06	20.00000	39	53	00.00000	192.827	62459.065	514656.357
226	L17	86	06	20.00000	39	54	00.00000	186.532	62468.156	516506.848
227	L18	86	06	20.00000	39	55	00.00000	188.017	62477.251	518357.344
228	L19	86	06	20.00000	39	56	00.00000	196.829	62486.348	520207.845
229	M01	86	05	00.00000	39	38	00.00000	217.989	64230.781	486890.549
230	M02	86	05	00.00000	39	39	00.00000	216.158	64239.367	488740.959
231	M03	86	05	00.00000	39	40	00.00000	215.525	64247.955	490591.374
232	M04	86	05	00.00000	39	41	00.00000	220.477	64256.546	492441.795
233	M05	86	05	00.00000	39	42	00.00000	218.555	64265.140	494292.221
234	M06	86	05	00.00000	39	43	00.00000	211.766	64273.738	496142.652
235	M07	86	05	00.00000	39	44	00.00000	212.756	64282.338	497993.089
236	M08	86	05	00.00000	39	45	00.00000	213.671	64290.942	499843.531
237	M09	86	05	00.00000	39	46	00.00000	207.853	64299.548	501693.978
238	M10	86	05	00.00000	39	47	00.00000	215.751	64308.158	503544.431
239	M11	86	05	00.00000	39	48	00.00000	221.691	64316.770	505394.888
240	M12	86	05	00.00000	39	49	00.00000	214.426	64325.386	507245.351
241	M13	86	05	00.00000	39	50	00.00000	214.561	64334.004	509095.820
242	M14	86	05	00.00000	39	51	00.00000	201.327	64342.626	510946.294
243	M15	86	05	00.00000	39	52	00.00000	210.403	64351.251	512796.773
244	M16	86	05	00.00000	39	53	00.00000	208.934	64359.878	514647.257
245	M17	86	05	00.00000	39	54	00.00000	202.233	64368.509	516497.747
246	M18	86	05	00.00000	39	55	00.00000	187.347	64377.143	518348.242
247	M19	86	05	00.00000	39	56	00.00000	119.812	64385.780	520198.742
248	N01	86	03	40.00000	39	38	00.00000	222.991	66138.479	486881.935
249	N02	86	03	40.00000	39	39	00.00000	220.745	66146.606	488732.345
250	N03	86	03	40.00000	39	40	00.00000	218.238	66154.736	490582.759
251	N04	86	03	40.00000	39	41	00.00000	225.432	66162.869	492433.179
252	N05	86	03	40.00000	39	42	00.00000	224.971	66171.005	494283.604
253	N06	86	03	40.00000	39	43	00.00000	222.965	66179.144	496134.034
254	N07	86	03	40.00000	39	44	00.00000	224.062	66187.286	497984.470
255	N08	86	03	40.00000	39	45	00.00000	219.034	66195.431	499834.911
256	N09	86	03	40.00000	39	46	00.00000	216.987	66203.578	501685.357
257	N10	86	03	40.00000	39	47	00.00000	217.752	66211.728	503535.809
258	N11	86	03	40.00000	39	48	00.00000	223.149	66219.881	505386.265
259	N12	86	03	40.00000	39	49	00.00000	224.654	66228.038	507236.728
260	N13	86	03	40.00000	39	50	00.00000	222.618	66236.196	509087.195
261	N14	86	03	40.00000	39	51	00.00000	218.936	66244.358	510937.668
262	N15	86	03	40.00000	39	52	00.00000	194.361	66252.523	512788.146
263	N16	86	03	40.00000	39	53	00.00000	216.454	66260.690	514638.629
264	N17	86	03	40.00000	39	54	00.00000	210.286	66268.861	516489.118
265	N18	86	03	40.00000	39	55	00.00000	209.052	66277.034	518339.612
266	N19	86	03	40.00000	39	56	00.00000	198.334	66285.210	520190.112
267	O01	86	02	20.00000	39	38	00.00000	220.793	68046.175	486873.794
268	O02	86	02	20.00000	39	39	00.00000	223.441	68053.845	488724.202
269	O03	86	02	20.00000	39	40	00.00000	228.468	68061.517	490574.616
270	O04	86	02	20.00000	39	41	00.00000	224.882	68069.192	492425.035
271	O05	86	02	20.00000	39	42	00.00000	226.260	68076.869	494275.459
272	O06	86	02	20.00000	39	43	00.00000	227.067	68084.550	496125.888
273	O07	86	02	20.00000	39	44	00.00000	227.167	68092.233	497976.323
274	O08	86	02	20.00000	39	45	00.00000	221.138	68099.918	499826.763
275	O09	86	02	20.00000	39	46	00.00000	222.911	68107.607	501677.208

TABLE E.1
(Continued)

		Geodetic coordinates of points in Marion County						Height above GRS80 ellipsoid	INSPCS83 (NGS)	
		Longitude (West)			Latitude (North)				Easting	Northing
Row ID	Point ID	deg.	min.	sec.	deg.	min.	sec.	(m)	(m)	(m)
276	O10	86	02	20.00000	39	47	00.00000	223.928	68115.298	503527.659
277	O11	86	02	20.00000	39	48	00.00000	217.772	68122.992	505378.115
278	O12	86	02	20.00000	39	49	00.00000	229.252	68130.688	507228.576
279	O13	86	02	20.00000	39	50	00.00000	229.216	68138.388	509079.043
280	O14	86	02	20.00000	39	51	00.00000	227.947	68146.090	510929.515
281	O15	86	02	20.00000	39	52	00.00000	193.390	68153.794	512779.992
282	O16	86	02	20.00000	39	53	00.00000	220.247	68161.502	514630.475
283	O17	86	02	20.00000	39	54	00.00000	219.139	68169.212	516480.963
284	O18	86	02	20.00000	39	55	00.00000	214.124	68176.924	518331.456
285	O19	86	02	20.00000	39	56	00.00000	212.236	68184.640	520181.954
286	P01	86	01	00.00000	39	38	00.00000	221.607	69953.871	486866.124
287	P02	86	01	00.00000	39	39	00.00000	218.522	69961.083	488716.532
288	P03	86	01	00.00000	39	40	00.00000	220.225	69968.297	490566.944
289	P04	86	01	00.00000	39	41	00.00000	221.873	69975.513	492417.362
290	P05	86	01	00.00000	39	42	00.00000	222.875	69982.733	494267.786
291	P06	86	01	00.00000	39	43	00.00000	225.170	69989.954	496118.214
292	P07	86	01	00.00000	39	44	00.00000	227.619	69997.179	497968.648
293	P08	86	01	00.00000	39	45	00.00000	228.458	70004.405	499819.088
294	P09	86	01	00.00000	39	46	00.00000	227.462	70011.635	501669.532
295	P10	86	01	00.00000	39	47	00.00000	229.460	70018.867	503519.982
296	P11	86	01	00.00000	39	48	00.00000	232.257	70026.101	505370.437
297	P12	86	01	00.00000	39	49	00.00000	234.157	70033.338	507220.898
298	P13	86	01	00.00000	39	50	00.00000	229.885	70040.578	509071.363
299	P14	86	01	00.00000	39	51	00.00000	231.252	70047.820	510921.835
300	P15	86	01	00.00000	39	52	00.00000	217.920	70055.065	512772.311
301	P16	86	01	00.00000	39	53	00.00000	214.018	70062.312	514622.793
302	P17	86	01	00.00000	39	54	00.00000	222.768	70069.562	516473.280
303	P18	86	01	00.00000	39	55	00.00000	219.441	70076.814	518323.772
304	P19	86	01	00.00000	39	56	00.00000	215.149	70084.069	520174.270
305	Q01	85	59	40.00000	39	38	00.00000	210.058	71861.566	486858.927
306	Q02	85	59	40.00000	39	39	00.00000	211.736	71868.319	488709.333
307	Q03	85	59	40.00000	39	40	00.00000	215.661	71875.076	490559.745
308	Q04	85	59	40.00000	39	41	00.00000	216.009	71881.834	492410.163
309	Q05	85	59	40.00000	39	42	00.00000	214.514	71888.595	494260.585
310	Q06	85	59	40.00000	39	43	00.00000	219.746	71895.358	496111.013
311	Q07	85	59	40.00000	39	44	00.00000	220.559	71902.124	497961.446
312	Q08	85	59	40.00000	39	45	00.00000	218.005	71908.892	499811.885
313	Q09	85	59	40.00000	39	46	00.00000	221.645	71915.662	501662.328
314	Q10	85	59	40.00000	39	47	00.00000	225.466	71922.435	503512.777
315	Q11	85	59	40.00000	39	48	00.00000	230.885	71929.210	505363.232
316	Q12	85	59	40.00000	39	49	00.00000	226.996	71935.987	507213.691
317	Q13	85	59	40.00000	39	50	00.00000	224.921	71942.767	509064.156
318	Q14	85	59	40.00000	39	51	00.00000	228.116	71949.550	510914.627
319	Q15	85	59	40.00000	39	52	00.00000	225.196	71956.334	512765.102
320	Q16	85	59	40.00000	39	53	00.00000	197.612	71963.121	514615.583
321	Q17	85	59	40.00000	39	54	00.00000	200.350	71969.911	516466.070
322	Q18	85	59	40.00000	39	55	00.00000	218.541	71976.703	518316.561
323	Q19	85	59	40.00000	39	56	00.00000	211.833	71983.497	520167.058
324	R01	85	58	20.00000	39	38	00.00000	206.450	73769.260	486852.201
325	R02	85	58	20.00000	39	39	00.00000	208.631	73775.556	488702.607
326	R03	85	58	20.00000	39	40	00.00000	209.448	73781.854	490553.018
327	R04	85	58	20.00000	39	41	00.00000	211.537	73788.154	492403.435
328	R05	85	58	20.00000	39	42	00.00000	209.885	73794.456	494253.857
329	R06	85	58	20.00000	39	43	00.00000	212.462	73800.761	496104.284
330	R07	85	58	20.00000	39	44	00.00000	209.535	73807.068	497954.716
331	R08	85	58	20.00000	39	45	00.00000	219.456	73813.377	499805.154
332	R09	85	58	20.00000	39	46	00.00000	225.113	73819.688	501655.597
333	R10	85	58	20.00000	39	47	00.00000	226.596	73826.002	503506.045

TABLE E.1
(Continued)

		Geodetic coordinates of points in Marion County						Height above GRS80 ellipsoid	INSPCS83 (NGS)	
		Longitude (West)			Latitude (North)				Easting	Northing
Row ID	Point ID	deg.	min.	sec.	deg.	min.	sec.	(m)	(m)	(m)
334	R11	85	58	20.00000	39	48	00.00000	221.969	73832.318	505356.499
335	R12	85	58	20.00000	39	49	00.00000	222.735	73838.636	507206.958
336	R13	85	58	20.00000	39	50	00.00000	221.210	73844.956	509057.422
337	R14	85	58	20.00000	39	51	00.00000	218.340	73851.278	510907.892
338	R15	85	58	20.00000	39	52	00.00000	222.127	73857.603	512758.367
339	R16	85	58	20.00000	39	53	00.00000	217.974	73863.930	514608.847
340	R17	85	58	20.00000	39	54	00.00000	218.052	73870.259	516459.333
341	R18	85	58	20.00000	39	55	00.00000	205.094	73876.591	518309.823
342	R19	85	58	20.00000	39	56	00.00000	220.359	73882.924	520160.320
343	S01	85	57	00.00000	39	38	00.00000	198.059	75676.953	486845.948
344	S02	85	57	00.00000	39	39	00.00000	202.725	75682.791	488696.353
345	S03	85	57	00.00000	39	40	00.00000	207.389	75688.631	490546.763
346	S04	85	57	00.00000	39	41	00.00000	214.505	75694.473	492397.179
347	S05	85	57	00.00000	39	42	00.00000	216.083	75700.317	494247.600
348	S06	85	57	00.00000	39	43	00.00000	211.264	75706.163	496098.027
349	S07	85	57	00.00000	39	44	00.00000	221.638	75712.011	497948.458
350	S08	85	57	00.00000	39	45	00.00000	223.650	75717.862	499798.895
351	S09	85	57	00.00000	39	46	00.00000	223.568	75723.714	501649.338
352	S10	85	57	00.00000	39	47	00.00000	225.646	75729.568	503499.785
353	S11	85	57	00.00000	39	48	00.00000	227.009	75735.425	505350.238
354	S12	85	57	00.00000	39	49	00.00000	226.471	75741.283	507200.697
355	S13	85	57	00.00000	39	50	00.00000	226.255	75747.144	509051.160
356	S14	85	57	00.00000	39	51	00.00000	226.798	75753.007	510901.629
357	S15	85	57	00.00000	39	52	00.00000	228.148	75758.871	512752.104
358	S16	85	57	00.00000	39	53	00.00000	223.153	75764.738	514602.583
359	S17	85	57	00.00000	39	54	00.00000	219.277	75770.607	516453.068
360	S18	85	57	00.00000	39	55	00.00000	217.810	75776.478	518303.558
361	S19	85	57	00.00000	39	56	00.00000	212.575	75782.351	520154.054
362	ZID A	86	16	47.56322	39	44	22.56171	208.707	47438.663	498785.706
363	ZID B	86	17	16.84533	39	44	18.12656	207.505	46740.513	498653.724
364	F 350	86	18	09.38083	39	45	51.75507	204.961	45510.124	501550.091
365	IMAGIS 47	86	01	06.38127	39	40	53.27057	224.190	69822.640	492210.419

TABLE E.2
Map coordinates of points in Marion County Test under the INCRS mapping

		Case h_0: Radius of INCRS Sphere = $R_{G@CP}$				Case h_{Real}: Radius of INCRS Sphere = $R_{G@CP} + h_{avg}$			
		INCRS TM(CP)		INCRS OS(CP)		INCRS TM(CP)		INCRS OS(CP)	
Row ID	**Point**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**
01	A01	41427.696	486937.921	41427.676	486937.941	41427.139	486937.380	41427.119	486937.400
02	A02	41431.817	488788.383	41431.803	488788.402	41431.260	488787.902	41431.247	488787.921
03	A03	41435.939	490638.849	41435.931	490638.868	41435.383	490638.429	41435.375	490638.447
04	A04	41440.063	492489.321	41440.060	492489.338	41439.507	492488.961	41439.503	492488.977
05	A05	41444.188	494339.799	41444.189	494339.813	41443.632	494339.498	41443.633	494339.513
06	A06	41448.315	496190.281	41448.319	496190.293	41447.759	496190.041	41447.763	496190.053
07	A07	41452.444	498040.769	41452.450	498040.779	41451.888	498040.589	41451.894	498040.599
08	A08	41456.573	499891.262	41456.581	499891.269	41456.018	499891.143	41456.026	499891.149
09	A09	41460.704	501741.761	41460.713	501741.764	41460.149	501741.701	41460.158	501741.705
10	A10	41464.837	503592.265	41464.846	503592.265	41464.282	503592.266	41464.291	503592.266
11	A11	41468.971	505442.774	41468.980	505442.771	41468.416	505442.835	41468.425	505442.832
12	A12	41473.107	507293.289	41473.115	507293.282	41472.552	507293.410	41472.560	507293.403
13	A13	41477.244	509143.809	41477.250	509143.799	41476.689	509143.990	41476.695	509143.981
14	A14	41481.382	510994.334	41481.386	510994.322	41480.827	510994.575	41480.831	510994.563
15	A15	41485.522	512844.864	41485.522	512844.850	41484.967	512845.166	41484.968	512845.152
16	A16	41489.663	514695.400	41489.660	514695.384	41489.109	514695.762	41489.105	514695.746
17	A17	41493.806	516545.941	41493.798	516545.923	41493.252	516546.364	41493.243	516546.346
18	A18	41497.950	518396.488	41497.937	518396.469	41497.396	518396.970	41497.382	518396.951
19	A19	41502.096	520247.040	41502.076	520247.020	41501.542	520247.583	41501.522	520247.563
20	B01	43335.446	486933.910	43335.427	486933.923	43334.952	486933.368	43334.932	486933.382
21	B02	43339.109	488784.370	43339.095	488784.384	43338.615	488783.889	43338.601	488783.903
22	B03	43342.774	490634.837	43342.765	490634.850	43342.279	490634.416	43342.270	490634.429
23	B04	43346.440	492485.308	43346.434	492485.321	43345.945	492484.947	43345.940	492484.960
24	B05	43350.106	494335.785	43350.105	494335.796	43349.612	494335.485	43349.611	494335.496
25	B06	43353.775	496186.267	43353.776	496186.277	43353.281	496186.027	43353.282	496186.036
26	B07	43357.444	498036.755	43357.448	498036.762	43356.950	498036.575	43356.954	498036.582
27	B08	43361.115	499887.248	43361.120	499887.253	43360.621	499887.128	43360.626	499887.133
28	B09	43364.787	501737.746	43364.793	501737.748	43364.294	501737.686	43364.300	501737.688
29	B10	43368.461	503588.249	43368.467	503588.249	43367.967	503588.250	43367.974	503588.250
30	B11	43372.136	505438.758	43372.142	505438.755	43371.642	505438.819	43371.648	505438.816
31	B12	43375.812	507289.272	43375.817	507289.267	43375.318	507289.393	43375.323	507289.388
32	B13	43379.489	509139.792	43379.492	509139.784	43378.996	509139.973	43378.999	509139.965
33	B14	43383.167	510990.316	43383.169	510990.307	43382.674	510990.558	43382.676	510990.548
34	B15	43386.847	512840.846	43386.846	512840.835	43386.354	512841.148	43386.353	512841.137
35	B16	43390.529	514691.382	43390.523	514691.369	43390.036	514691.744	43390.031	514691.731
36	B17	43394.211	516541.923	43394.202	516541.909	43393.718	516542.345	43393.709	516542.332
37	B18	43397.895	518392.469	43397.881	518392.455	43397.402	518392.951	43397.388	518392.938
38	B19	43401.580	520243.020	43401.560	520243.007	43401.087	520243.563	43401.068	520243.550
39	C01	45243.196	486930.370	45243.178	486930.378	45242.764	486929.828	45242.745	486929.836
40	C02	45246.402	488780.830	45246.388	488780.839	45245.969	488780.349	45245.955	488780.358
41	C03	45249.608	490631.296	45249.598	490631.305	45249.175	490630.875	45249.166	490630.884
42	C04	45252.816	492481.767	45252.810	492481.776	45252.383	492481.406	45252.377	492481.415
43	C05	45256.024	494332.244	45256.021	494332.252	45255.592	494331.943	45255.589	494331.951
44	C06	45259.234	496182.725	45259.233	496182.732	45258.801	496182.485	45258.801	496182.492
45	C07	45262.445	498033.213	45262.446	498033.218	45262.012	498033.032	45262.014	498033.038
46	C08	45265.657	499883.705	45265.660	499883.709	45265.225	499883.585	45265.228	499883.589
47	C09	45268.870	501734.203	45268.874	501734.205	45268.438	501734.143	45268.442	501734.145
48	C10	45272.084	503584.706	45272.088	503584.706	45271.652	503584.706	45271.656	503584.706
49	C11	45275.299	505435.214	45275.303	505435.212	45274.868	505435.275	45274.871	505435.273
50	C12	45278.516	507285.728	45278.519	507285.724	45278.084	507285.849	45278.087	507285.845
51	C13	45281.734	509136.247	45281.735	509136.242	45281.302	509136.428	45281.304	509136.423
52	C14	45284.952	510986.771	45284.952	510986.765	45284.521	510987.013	45284.521	510987.006
53	C15	45288.172	512837.301	45288.169	512837.293	45287.741	512837.603	45287.738	512837.595
54	C16	45291.393	514687.836	45291.387	514687.828	45290.962	514688.198	45290.956	514688.189
55	C17	45294.616	516538.377	45294.606	516538.368	45294.184	516538.799	45294.175	516538.790
56	C18	45297.839	518388.922	45297.825	518388.914	45297.408	518389.405	45297.394	518389.396
57	C19	45301.063	520239.473	45301.045	520239.466	45300.632	520240.016	45300.614	520240.008
58	D01	47150.946	486927.302	47150.929	486927.305	47150.575	486926.760	47150.558	486926.763

TABLE E.2
(Continued)

		Case h_0: Radius of INCRS Sphere = $R_{G@CP}$				Case h_{Real}: Radius of INCRS Sphere = $R_{G@CP} + h_{avg}$			
		INCRS TM(CP)		INCRS OS(CP)		INCRS TM(CP)		INCRS OS(CP)	
Row ID	**Point**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**
59	D02	47153.694	488777.762	47153.681	488777.766	47153.323	488777.281	47153.310	488777.285
60	D03	47156.442	490628.228	47156.432	490628.233	47156.071	490627.806	47156.062	490627.812
61	D04	47159.191	492478.698	47159.185	492478.704	47158.820	492478.337	47158.814	492478.343
62	D05	47161.941	494329.174	47161.938	494329.180	47161.571	494328.874	47161.567	494328.879
63	D06	47164.693	496179.656	47164.691	496179.661	47164.322	496179.415	47164.321	496179.420
64	D07	47167.445	498030.143	47167.445	498030.146	47167.074	498029.962	47167.074	498029.966
65	D08	47170.198	499880.635	47170.199	499880.637	47169.827	499880.515	47169.829	499880.517
66	D09	47172.952	501731.132	47172.954	501731.134	47172.582	501731.072	47172.584	501731.074
67	D10	47175.707	503581.635	47175.709	503581.635	47175.337	503581.635	47175.339	503581.635
68	D11	47178.463	505432.143	47178.465	505432.142	47178.093	505432.204	47178.095	505432.202
69	D12	47181.220	507282.656	47181.222	507282.654	47180.850	507282.777	47180.852	507282.775
70	D13	47183.978	509133.175	47183.978	509133.171	47183.608	509133.356	47183.608	509133.352
71	D14	47186.737	510983.699	47186.736	510983.695	47186.367	510983.940	47186.366	510983.936
72	D15	47189.497	512834.229	47189.493	512834.223	47189.127	512834.530	47189.124	512834.525
73	D16	47192.258	514684.763	47192.252	514684.758	47191.888	514685.125	47191.882	514685.120
74	D17	47195.020	516535.303	47195.010	516535.298	47194.650	516535.725	47194.641	516535.720
75	D18	47197.783	518385.849	47197.770	518385.845	47197.413	518386.331	47197.400	518386.327
76	D19	47200.546	520236.400	47200.529	520236.397	47200.177	520236.942	47200.160	520236.939
77	E01	49058.696	486924.706	49058.681	486924.705	49058.386	486924.164	49058.371	486924.163
78	E02	49060.985	488775.166	49060.974	488775.167	49060.676	488774.684	49060.664	488774.685
79	E03	49063.275	490625.631	49063.267	490625.633	49062.966	490625.210	49062.958	490625.212
80	E04	49065.566	492476.102	49065.560	492476.104	49065.257	492475.741	49065.251	492475.743
81	E05	49067.858	494326.577	49067.855	494326.580	49067.549	494326.277	49067.546	494326.280
82	E06	49070.151	496177.059	49070.149	496177.061	49069.842	496176.818	49069.840	496176.821
83	E07	49072.444	498027.545	49072.444	498027.548	49072.136	498027.365	49072.135	498027.367
84	E08	49074.739	499878.037	49074.739	499878.039	49074.430	499877.917	49074.430	499877.918
85	E09	49077.034	501728.534	49077.035	501728.535	49076.725	501728.474	49076.726	501728.475
86	E10	49079.330	503579.037	49079.331	503579.037	49079.021	503579.037	49079.022	503579.037
87	E11	49081.626	505429.544	49081.627	505429.543	49081.318	505429.605	49081.319	505429.604
88	E12	49083.924	507280.057	49083.924	507280.056	49083.616	507280.178	49083.616	507280.176
89	E13	49086.222	509130.576	49086.222	509130.574	49085.914	509130.757	49085.913	509130.754
90	E14	49088.521	510981.100	49088.519	510981.097	49088.213	510981.341	49088.211	510981.338
91	E15	49090.821	512831.629	49090.817	512831.626	49090.513	512831.930	49090.509	512831.927
92	E16	49093.122	514682.163	49093.116	514682.161	49092.814	514682.525	49092.808	514682.522
93	E17	49095.424	516532.703	49095.415	516532.701	49095.116	516533.125	49095.107	516533.123
94	E18	49097.726	518383.248	49097.714	518383.248	49097.418	518383.730	49097.406	518383.730
95	E19	49100.029	520233.799	49100.014	520233.800	49099.721	520234.341	49099.706	520234.342
96	F01	50966.445	486922.582	50966.432	486922.578	50966.197	486922.040	50966.185	486922.036
97	F02	50968.276	488773.042	50968.267	488773.040	50968.029	488772.560	50968.019	488772.558
98	F03	50970.109	490623.507	50970.101	490623.506	50969.861	490623.085	50969.854	490623.085
99	F04	50971.941	492473.977	50971.936	492473.978	50971.694	492473.616	50971.689	492473.616
100	F05	50973.775	494324.453	50973.771	494324.454	50973.528	494324.152	50973.524	494324.153
101	F06	50975.609	496174.934	50975.607	496174.935	50975.362	496174.693	50975.360	496174.694
102	F07	50977.444	498025.420	50977.443	498025.421	50977.197	498025.239	50977.196	498025.241
103	F08	50979.279	499875.911	50979.279	499875.912	50979.032	499875.791	50979.032	499875.792
104	F09	50981.115	501726.408	50981.116	501726.409	50980.868	501726.348	50980.869	501726.349
105	F10	50982.952	503576.911	50982.953	503576.911	50982.705	503576.911	50982.706	503576.911
106	F11	50984.789	505427.418	50984.790	505427.418	50984.543	505427.478	50984.543	505427.478
107	F12	50986.627	507277.931	50986.627	507277.930	50986.381	507278.052	50986.381	507278.051
108	F13	50988.466	509128.449	50988.465	509128.448	50988.219	509128.630	50988.218	509128.629
109	F14	50990.305	510978.973	50990.303	510978.971	50990.059	510979.214	50990.057	510979.212
110	F15	50992.145	512829.502	50992.142	512829.501	50991.899	512829.803	50991.895	512829.802
111	F16	50993.986	514680.036	50993.981	514680.035	50993.739	514680.397	50993.734	514680.397
112	F17	50995.827	516530.576	50995.820	516530.576	50995.581	516530.997	50995.573	516530.998
113	F18	50997.669	518381.120	50997.659	518381.123	50997.423	518381.602	50997.413	518381.605
114	F19	50999.512	520231.671	50999.499	520231.675	50999.265	520232.213	50999.253	520232.217
115	G01	52874.194	486920.930	52874.184	486920.923	52874.008	486920.388	52873.999	486920.381
116	G02	52875.568	488771.390	52875.560	488771.385	52875.382	488770.908	52875.374	488770.904
117	G03	52876.942	490621.854	52876.936	490621.852	52876.756	490621.433	52876.750	490621.430

TABLE E.2
(Continued)

		Case h_0: Radius of INCRS Sphere = $R_{G@CP}$				Case h_{Real}: Radius of INCRS Sphere = $R_{G@CP} + h_{avg}$			
		INCRS TM(CP)		INCRS OS(CP)		INCRS TM(CP)		INCRS OS(CP)	
Row ID	Point	Easting (m)	Northing (m)	Easting (m)	Northing (m)	Easting (m)	Northing (m)	Easting (m)	Northing (m)
118	G04	52878.316	492472.324	52878.312	492472.323	52878.131	492471.963	52878.127	492471.962
119	G05	52879.691	494322.800	52879.688	494322.800	52879.506	494322.499	52879.503	494322.499
120	G06	52881.067	496173.281	52881.065	496173.281	52880.882	496173.040	52880.880	496173.040
121	G07	52882.443	498023.767	52882.442	498023.767	52882.258	498023.586	52882.257	498023.587
122	G08	52883.820	499874.258	52883.819	499874.259	52883.634	499874.138	52883.634	499874.138
123	G09	52885.197	501724.755	52885.197	501724.755	52885.011	501724.695	52885.011	501724.695
124	G10	52886.574	503575.257	52886.574	503575.257	52886.389	503575.257	52886.389	503575.257
125	G11	52887.952	505425.764	52887.952	505425.764	52887.767	505425.825	52887.767	505425.824
126	G12	52889.331	507276.277	52889.330	507276.277	52889.146	507276.398	52889.145	507276.397
127	G13	52890.710	509126.795	52890.709	509126.795	52890.525	509126.976	52890.524	509126.975
128	G14	52892.089	510977.319	52892.087	510977.318	52891.904	510977.560	52891.902	510977.559
129	G15	52893.469	512827.847	52893.466	512827.848	52893.284	512828.148	52893.281	512828.149
130	G16	52894.850	514678.381	52894.845	514678.383	52894.665	514678.743	52894.661	514678.744
131	G17	52896.231	516528.921	52896.225	516528.923	52896.046	516529.342	52896.040	516529.345
132	G18	52897.612	518379.465	52897.604	518379.470	52897.427	518379.947	52897.420	518379.952
133	G19	52898.994	520230.016	52898.984	520230.023	52898.809	520230.558	52898.800	520230.565
134	H01	54781.943	486919.750	54781.936	486919.741	54781.819	486919.208	54781.812	486919.199
135	H02	54782.858	488770.210	54782.853	488770.203	54782.735	488769.728	54782.730	488769.722
136	H03	54783.775	490620.674	54783.770	490620.670	54783.651	490620.253	54783.647	490620.249
137	H04	54784.691	492471.144	54784.688	492471.142	54784.567	492470.783	54784.564	492470.781
138	H05	54785.608	494321.619	54785.606	494321.618	54785.484	494321.318	54785.482	494321.317
139	H06	54786.525	496172.100	54786.523	496172.100	54786.401	496171.859	54786.400	496171.859
140	H07	54787.442	498022.586	54787.441	498022.586	54787.319	498022.405	54787.318	498022.405
141	H08	54788.360	499873.077	54788.359	499873.077	54788.236	499872.957	54788.236	499872.957
142	H09	54789.278	501723.574	54789.278	501723.574	54789.154	501723.514	54789.154	501723.514
143	H10	54790.196	503574.076	54790.196	503574.076	54790.073	503574.076	54790.073	503574.076
144	H11	54791.115	505424.583	54791.115	505424.583	54790.992	505424.643	54790.991	505424.643
145	H12	54792.034	507275.096	54792.033	507275.096	54791.911	507275.216	54791.910	507275.216
146	H13	54792.953	509125.614	54792.952	509125.614	54792.830	509125.794	54792.829	509125.795
147	H14	54793.873	510976.137	54793.872	510976.138	54793.750	510976.378	54793.748	510976.378
148	H15	54794.793	512826.666	54794.791	512826.667	54794.670	512826.967	54794.667	512826.968
149	H16	54795.713	514677.200	54795.710	514677.202	54795.590	514677.561	54795.587	514677.563
150	H17	54796.634	516527.739	54796.630	516527.743	54796.511	516528.160	54796.507	516528.164
151	H18	54797.555	518378.283	54797.549	518378.290	54797.432	518378.765	54797.426	518378.771
152	H19	54798.476	520228.833	54798.469	520228.842	54798.353	520229.375	54798.346	520229.384
153	I01	56689.691	486919.042	56689.688	486919.032	56689.630	486918.500	56689.626	486918.490
154	I02	56690.149	488769.501	56690.147	488769.494	56690.087	488769.020	56690.085	488769.013
155	I03	56690.607	490619.966	56690.605	490619.961	56690.545	490619.545	56690.543	490619.540
156	I04	56691.065	492470.436	56691.064	492470.433	56691.004	492470.075	56691.002	492470.072
157	I05	56691.524	494320.911	56691.523	494320.909	56691.462	494320.610	56691.461	494320.608
158	I06	56691.982	496171.392	56691.982	496171.391	56691.921	496171.151	56691.920	496171.150
159	I07	56692.441	498021.878	56692.441	498021.877	56692.379	498021.697	56692.379	498021.696
160	I08	56692.900	499872.369	56692.900	499872.369	56692.838	499872.248	56692.838	499872.248
161	I09	56693.359	501722.865	56693.359	501722.865	56693.297	501722.805	56693.297	501722.805
162	I10	56693.818	503573.367	56693.818	503573.367	56693.756	503573.367	56693.756	503573.367
163	I11	56694.277	505423.874	56694.277	505423.874	56694.216	505423.935	56694.216	505423.935
164	I12	56694.737	507274.387	56694.737	507274.387	56694.675	507274.507	56694.675	507274.508
165	I13	56695.197	509124.905	56695.196	509124.905	56695.135	509125.085	56695.135	509125.086
166	I14	56695.656	510975.428	56695.656	510975.429	56695.595	510975.669	56695.594	510975.670
167	I15	56696.116	512825.957	56696.115	512825.958	56696.055	512826.258	56696.054	512826.260
168	I16	56696.577	514676.490	56696.575	514676.494	56696.515	514676.852	56696.513	514676.855
169	I17	56697.037	516527.030	56697.035	516527.034	56696.975	516527.451	56696.973	516527.456
170	I18	56697.497	518377.574	56697.495	518377.581	56697.436	518378.056	56697.433	518378.063
171	I19	56697.958	520228.124	56697.955	520228.134	56697.896	520228.666	56697.893	520228.676
172	J01	58597.440	486918.806	58597.440	486918.796	58597.440	486918.264	58597.440	486918.254
173	J02	58597.440	488769.265	58597.440	488769.258	58597.440	488768.784	58597.440	488768.776
174	J03	58597.440	490619.730	58597.440	490619.725	58597.440	490619.309	58597.440	490619.303
175	J04	58597.440	492470.200	58597.440	492470.196	58597.440	492469.839	58597.440	492469.835
176	J05	58597.440	494320.675	58597.440	494320.673	58597.440	494320.374	58597.440	494320.372

TABLE E.2
(Continued)

		Case h_0: Radius of INCRS Sphere = $R_{G@CP}$				Case h_{Real}: Radius of INCRS Sphere = $R_{G@CP} + h_{avg}$			
		INCRS TM(CP)		INCRS OS(CP)		INCRS TM(CP)		INCRS OS(CP)	
Row ID	Point	Easting (m)	Northing (m)	Easting (m)	Northing (m)	Easting (m)	Northing (m)	Easting (m)	Northing (m)
177	J06	58597.440	496171.156	58597.440	496171.154	58597.440	496170.915	58597.440	496170.914
178	J07	58597.440	498021.641	58597.440	498021.641	58597.440	498021.461	58597.440	498021.460
179	J08	58597.440	499872.133	58597.440	499872.132	58597.440	499872.012	58597.440	499872.012
180	J09	58597.440	501722.629	58597.440	501722.629	58597.440	501722.569	58597.440	501722.569
181	J10	58597.440	503573.131	58597.440	503573.131	58597.440	503573.131	58597.440	503573.131
182	J11	58597.440	505423.638	58597.440	505423.638	58597.440	505423.698	58597.440	505423.698
183	J12	58597.440	507274.151	58597.440	507274.151	58597.440	507274.271	58597.440	507274.271
184	J13	58597.440	509124.669	58597.440	509124.669	58597.440	509124.849	58597.440	509124.850
185	J14	58597.440	510975.192	58597.440	510975.193	58597.440	510975.433	58597.440	510975.434
186	J15	58597.440	512825.720	58597.440	512825.722	58597.440	512826.021	58597.440	512826.023
187	J16	58597.440	514676.254	58597.440	514676.257	58597.440	514676.615	58597.440	514676.619
188	J17	58597.440	516526.793	58597.440	516526.798	58597.440	516527.215	58597.440	516527.220
189	J18	58597.440	518377.338	58597.440	518377.345	58597.440	518377.819	58597.440	518377.827
190	J19	58597.440	520227.888	58597.440	520227.898	58597.440	520228.430	58597.440	520228.440
191	K01	60505.189	486919.042	60505.192	486919.032	60505.250	486918.500	60505.254	486918.490
192	K02	60504.731	488769.501	60504.733	488769.494	60504.793	488769.020	60504.795	488769.013
193	K03	60504.273	490619.966	60504.275	490619.961	60504.335	490619.545	60504.337	490619.540
194	K04	60503.815	492470.436	60503.816	492470.433	60503.876	492470.075	60503.878	492470.071
195	K05	60503.356	494320.911	60503.357	494320.909	60503.418	494320.610	60503.419	494320.608
196	K06	60502.898	496171.392	60502.898	496171.391	60502.959	496171.151	60502.960	496171.150
197	K07	60502.439	498021.878	60502.439	498021.877	60502.501	498021.697	60502.501	498021.696
198	K08	60501.980	499872.369	60501.980	499872.369	60502.042	499872.248	60502.042	499872.248
199	K09	60501.521	501722.865	60501.521	501722.865	60501.583	501722.805	60501.583	501722.805
200	K10	60501.062	503573.367	60501.062	503573.367	60501.124	503573.367	60501.124	503573.367
201	K11	60500.603	505423.874	60500.603	505423.874	60500.664	505423.935	60500.664	505423.935
202	K12	60500.143	507274.387	60500.143	507274.387	60500.205	507274.507	60500.205	507274.508
203	K13	60499.683	509124.905	60499.684	509124.905	60499.745	509125.085	60499.745	509125.086
204	K14	60499.224	510975.428	60499.224	510975.429	60499.285	510975.669	60499.286	510975.670
205	K15	60498.764	512825.957	60498.765	512825.958	60498.825	512826.258	60498.826	512826.260
206	K16	60498.303	514676.490	60498.305	514676.494	60498.365	514676.852	60498.367	514676.855
207	K17	60497.843	516527.030	60497.845	516527.034	60497.905	516527.451	60497.907	516527.456
208	K18	60497.383	518377.574	60497.385	518377.581	60497.444	518378.056	60497.447	518378.063
209	K19	60496.922	520228.124	60496.925	520228.134	60496.984	520228.666	60496.987	520228.676
210	L01	62412.937	486919.750	62412.944	486919.741	62413.061	486919.208	62413.068	486919.199
211	L02	62412.022	488770.210	62412.027	488770.203	62412.145	488769.728	62412.150	488769.722
212	L03	62411.105	490620.674	62411.110	490620.670	62411.229	490620.253	62411.233	490620.249
213	L04	62410.189	492471.144	62410.192	492471.142	62410.313	492470.783	62410.316	492470.780
214	L05	62409.272	494321.619	62409.274	494321.618	62409.396	494321.318	62409.398	494321.317
215	L06	62408.355	496172.100	62408.357	496172.100	62408.479	496171.859	62408.480	496171.859
216	L07	62407.438	498022.586	62407.439	498022.586	62407.561	498022.405	62407.562	498022.405
217	L08	62406.520	499873.077	62406.521	499873.077	62406.644	499872.957	62406.644	499872.957
218	L09	62405.602	501723.574	62405.602	501723.574	62405.726	501723.514	62405.726	501723.514
219	L10	62404.684	503574.076	62404.684	503574.076	62404.807	503574.076	62404.807	503574.076
220	L11	62403.765	505424.583	62403.765	505424.583	62403.888	505424.643	62403.889	505424.643
221	L12	62402.846	507275.096	62402.847	507275.096	62402.969	507275.216	62402.970	507275.216
222	L13	62401.927	509125.614	62401.928	509125.614	62402.050	509125.794	62402.051	509125.795
223	L14	62401.007	510976.137	62401.008	510976.138	62401.130	510976.378	62401.132	510976.378
224	L15	62400.087	512826.666	62400.089	512826.667	62400.210	512826.967	62400.213	512826.968
225	L16	62399.167	514677.200	62399.170	514677.202	62399.290	514677.561	62399.293	514677.563
226	L17	62398.246	516527.739	62398.250	516527.743	62398.369	516528.160	62398.373	516528.164
227	L18	62397.325	518378.283	62397.331	518378.290	62397.448	518378.765	62397.454	518378.771
228	L19	62396.404	520228.833	62396.411	520228.842	62396.527	520229.375	62396.534	520229.384
229	M01	64320.686	486920.930	64320.696	486920.923	64320.872	486920.388	64320.881	486920.381
230	M02	64319.312	488771.390	64319.320	488771.385	64319.498	488770.908	64319.506	488770.903
231	M03	64317.938	490621.854	64317.944	490621.852	64318.124	490621.433	64318.130	490621.430
232	M04	64316.564	492472.324	64316.568	492472.323	64316.749	492471.963	64316.753	492471.962
233	M05	64315.189	494322.800	64315.192	494322.800	64315.374	494322.499	64315.377	494322.499
234	M06	64313.813	496173.281	64313.815	496173.281	64313.998	496173.040	64314.000	496173.040
235	M07	64312.437	498023.767	64312.438	498023.767	64312.622	498023.586	64312.623	498023.587

TABLE E.2
(Continued)

		Case h_0: Radius of INCRS Sphere = $R_{G@CP}$				Case h_{Real}: Radius of INCRS Sphere = $R_{G@CP} + h_{avg}$			
		INCRS TM(CP)		INCRS OS(CP)		INCRS TM(CP)		INCRS OS(CP)	
Row ID	**Point**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**
236	M08	64311.060	499874.258	64311.061	499874.259	64311.246	499874.138	64311.246	499874.138
237	M09	64309.683	501724.755	64309.683	501724.755	64309.868	501724.695	64309.869	501724.695
238	M10	64308.306	503575.257	64308.306	503575.257	64308.491	503575.257	64308.491	503575.257
239	M11	64306.928	505425.764	64306.928	505425.764	64307.113	505425.825	64307.113	505425.824
240	M12	64305.549	507276.277	64305.550	507276.277	64305.734	507276.398	64305.735	507276.397
241	M13	64304.170	509126.795	64304.171	509126.795	64304.355	509126.976	64304.356	509126.975
242	M14	64302.791	510977.319	64302.793	510977.318	64302.976	510977.560	64302.978	510977.559
243	M15	64301.411	512827.847	64301.414	512827.848	64301.596	512828.148	64301.599	512828.149
244	M16	64300.030	514678.381	64300.035	514678.383	64300.215	514678.743	64300.219	514678.744
245	M17	64298.649	516528.921	64298.655	516528.923	64298.834	516529.342	64298.840	516529.345
246	M18	64297.268	518379.465	64297.276	518379.470	64297.453	518379.947	64297.460	518379.952
247	M19	64295.886	520230.016	64295.896	520230.023	64296.071	520230.557	64296.081	520230.564
248	N01	66228.435	486922.582	66228.448	486922.578	66228.682	486922.040	66228.695	486922.036
249	N02	66226.604	488773.042	66226.613	488773.040	66226.851	488772.560	66226.861	488772.558
250	N03	66224.771	490623.507	66224.779	490623.506	66225.019	490623.085	66225.026	490623.085
251	N04	66222.939	492473.977	66222.944	492473.978	66223.186	492473.616	66223.191	492473.616
252	N05	66221.105	494324.453	66221.109	494324.454	66221.352	494324.152	66221.356	494324.153
253	N06	66219.271	496174.934	66219.273	496174.935	66219.518	496174.693	66219.520	496174.694
254	N07	66217.436	498025.420	66217.437	498025.421	66217.683	498025.239	66217.684	498025.241
255	N08	66215.601	499875.911	66215.601	499875.912	66215.848	499875.791	66215.848	499875.792
256	N09	66213.765	501726.408	66213.764	501726.409	66214.012	501726.348	66214.011	501726.349
257	N10	66211.928	503576.911	66211.927	503576.911	66212.175	503576.911	66212.174	503576.911
258	N11	66210.091	505427.418	66210.090	505427.418	66210.337	505427.478	66210.337	505427.478
259	N12	66208.253	507277.931	66208.253	507277.930	66208.499	507278.052	66208.499	507278.051
260	N13	66206.414	509128.449	66206.415	509128.448	66206.661	509128.630	66206.661	509128.629
261	N14	66204.575	510978.973	66204.577	510978.971	66204.821	510979.214	66204.823	510979.212
262	N15	66202.735	512829.502	66202.738	512829.501	66202.981	512829.803	66202.985	512829.802
263	N16	66200.894	514680.036	66200.899	514680.035	66201.141	514680.397	66201.146	514680.397
264	N17	66199.053	516530.576	66199.060	516530.576	66199.299	516530.997	66199.307	516530.998
265	N18	66197.211	518381.120	66197.221	518381.123	66197.457	518381.602	66197.467	518381.605
266	N19	66195.368	520231.671	66195.381	520231.675	66195.615	520232.213	66195.627	520232.217
267	O01	68136.184	486924.706	68136.199	486924.705	68136.493	486924.164	68136.508	486924.163
268	O02	68133.895	488775.166	68133.906	488775.167	68134.204	488774.684	68134.216	488774.685
269	O03	68131.605	490625.631	68131.613	490625.633	68131.914	490625.210	68131.922	490625.212
270	O04	68129.314	492476.102	68129.320	492476.104	68129.622	492475.740	68129.628	492475.743
271	O05	68127.022	494326.577	68127.025	494326.580	68127.331	494326.276	68127.334	494326.279
272	O06	68124.729	496177.059	68124.731	496177.061	68125.038	496176.818	68125.040	496176.821
273	O07	68122.436	498027.545	68122.436	498027.548	68122.744	498027.365	68122.745	498027.367
274	O08	68120.141	499878.037	68120.141	499878.039	68120.450	499877.917	68120.449	499877.918
275	O09	68117.846	501728.534	68117.845	501728.535	68118.155	501728.474	68118.154	501728.475
276	O10	68115.550	503579.037	68115.549	503579.037	68115.859	503579.037	68115.858	503579.037
277	O11	68113.254	505429.544	68113.253	505429.543	68113.562	505429.605	68113.561	505429.604
278	O12	68110.956	507280.057	68110.956	507280.056	68111.264	507280.178	68111.264	507280.176
279	O13	68108.658	509130.576	68108.658	509130.574	68108.966	509130.757	68108.967	509130.754
280	O14	68106.359	510981.100	68106.361	510981.097	68106.667	510981.341	68106.669	510981.338
281	O15	68104.059	512831.629	68104.063	512831.626	68104.367	512831.930	68104.371	512831.927
282	O16	68101.758	514682.163	68101.764	514682.161	68102.066	514682.525	68102.072	514682.522
283	O17	68099.456	516532.703	68099.465	516532.701	68099.764	516533.125	68099.773	516533.123
284	O18	68097.154	518383.248	68097.166	518383.248	68097.462	518383.730	68097.474	518383.729
285	O19	68094.851	520233.799	68094.866	520233.800	68095.159	520234.341	68095.174	520234.342
286	P01	70043.934	486927.302	70043.951	486927.305	70044.305	486926.760	70044.322	486926.763
287	P02	70041.186	488777.762	70041.199	488777.766	70041.557	488777.280	70041.570	488777.285
288	P03	70038.438	490628.228	70038.448	490628.233	70038.809	490627.806	70038.818	490627.811
289	P04	70035.689	492478.698	70035.695	492478.704	70036.060	492478.337	70036.066	492478.343
290	P05	70032.939	494329.174	70032.942	494329.180	70033.309	494328.874	70033.313	494328.879
291	P06	70030.187	496179.656	70030.189	496179.661	70030.558	496179.415	70030.559	496179.420
292	P07	70027.435	498030.143	70027.435	498030.146	70027.806	498029.962	70027.805	498029.966
293	P08	70024.682	499880.635	70024.681	499880.637	70025.052	499880.515	70025.051	499880.517
294	P09	70021.928	501731.132	70021.926	501731.134	70022.298	501731.072	70022.296	501731.074

TABLE E.2
(Continued)

		Case h_0: Radius of INCRS Sphere = $R_{G@CP}$				Case h_{Real}: Radius of INCRS Sphere = $R_{G@CP} + h_{avg}$			
		INCRS TM(CP)		INCRS OS(CP)		INCRS TM(CP)		INCRS OS(CP)	
Row ID	Point	Easting (m)	Northing (m)	Easting (m)	Northing (m)	Easting (m)	Northing (m)	Easting (m)	Northing (m)
295	P10	70019.173	503581.635	70019.171	503581.635	70019.543	503581.635	70019.541	503581.635
296	P11	70016.417	505432.143	70016.415	505432.142	70016.787	505432.204	70016.785	505432.202
297	P12	70013.660	507282.656	70013.658	507282.654	70014.030	507282.777	70014.028	507282.775
298	P13	70010.902	509133.175	70010.902	509133.171	70011.272	509133.356	70011.272	509133.352
299	P14	70008.143	510983.699	70008.144	510983.695	70008.513	510983.940	70008.514	510983.936
300	P15	70005.383	512834.229	70005.387	512834.223	70005.753	512834.530	70005.756	512834.525
301	P16	70002.622	514684.763	70002.628	514684.758	70002.992	514685.125	70002.998	514685.120
302	P17	69999.860	516535.303	69999.870	516535.298	70000.230	516535.725	70000.239	516535.720
303	P18	69997.097	518385.849	69997.110	518385.845	69997.467	518386.331	69997.480	518386.327
304	P19	69994.334	520236.400	69994.351	520236.397	69994.703	520236.942	69994.720	520236.939
305	Q01	71951.684	486930.370	71951.702	486930.378	71952.116	486929.828	71952.135	486929.836
306	Q02	71948.478	488780.830	71948.492	488780.839	71948.911	488780.349	71948.925	488780.358
307	Q03	71945.272	490631.296	71945.282	490631.305	71945.705	490630.875	71945.714	490630.884
308	Q04	71942.064	492481.767	71942.070	492481.776	71942.497	492481.406	71942.503	492481.415
309	Q05	71938.856	494332.244	71938.859	494332.252	71939.288	494331.943	71939.291	494331.951
310	Q06	71935.646	496182.725	71935.647	496182.732	71936.078	496182.485	71936.079	496182.492
311	Q07	71932.435	498033.213	71932.434	498033.218	71932.867	498033.032	71932.866	498033.038
312	Q08	71929.223	499883.705	71929.220	499883.709	71929.655	499883.585	71929.652	499883.589
313	Q09	71926.010	501734.203	71926.006	501734.205	71926.442	501734.143	71926.438	501734.145
314	Q10	71922.796	503584.706	71922.792	503584.706	71923.228	503584.706	71923.224	503584.706
315	Q11	71919.581	505435.214	71919.577	505435.212	71920.012	505435.275	71920.008	505435.273
316	Q12	71916.364	507285.728	71916.361	507285.724	71916.796	507285.849	71916.793	507285.845
317	Q13	71913.146	509136.247	71913.145	509136.242	71913.578	509136.428	71913.576	509136.423
318	Q14	71909.928	510986.771	71909.928	510986.765	71910.359	510987.013	71910.359	510987.006
319	Q15	71906.708	512837.301	71906.711	512837.293	71907.139	512837.603	71907.142	512837.595
320	Q16	71903.487	514687.836	71903.493	514687.828	71903.918	514688.198	71903.924	514688.189
321	Q17	71900.264	516538.377	71900.274	516538.368	71900.696	516538.799	71900.705	516538.790
322	Q18	71897.041	518388.922	71897.055	518388.914	71897.472	518389.405	71897.486	518389.396
323	Q19	71893.817	520239.473	71893.835	520239.466	71894.248	520240.016	71894.266	520240.008
324	R01	73859.434	486933.910	73859.453	486933.923	73859.928	486933.368	73859.948	486933.382
325	R02	73855.771	488784.370	73855.785	488784.384	73856.265	488783.889	73856.279	488783.903
326	R03	73852.106	490634.837	73852.115	490634.850	73852.601	490634.416	73852.610	490634.429
327	R04	73848.440	492485.308	73848.446	492485.321	73848.935	492484.947	73848.940	492484.960
328	R05	73844.774	494335.785	73844.775	494335.796	73845.268	494335.484	73845.269	494335.496
329	R06	73841.105	496186.267	73841.104	496186.277	73841.599	496186.027	73841.598	496186.036
330	R07	73837.436	498036.755	73837.432	498036.762	73837.930	498036.575	73837.926	498036.582
331	R08	73833.765	499887.248	73833.760	499887.253	73834.259	499887.128	73834.253	499887.133
332	R09	73830.093	501737.746	73830.087	501737.748	73830.586	501737.686	73830.580	501737.688
333	R10	73826.419	503588.249	73826.413	503588.249	73826.913	503588.250	73826.906	503588.250
334	R11	73822.744	505438.758	73822.738	505438.755	73823.238	505438.819	73823.232	505438.816
335	R12	73819.068	507289.272	73819.063	507289.267	73819.562	507289.393	73819.557	507289.388
336	R13	73815.391	509139.792	73815.388	509139.784	73815.884	509139.973	73815.881	509139.965
337	R14	73811.713	510990.316	73811.711	510990.307	73812.206	510990.558	73812.204	510990.548
338	R15	73808.033	512840.846	73808.034	512840.835	73808.526	512841.148	73808.527	512841.137
339	R16	73804.351	514691.382	73804.357	514691.369	73804.844	514691.744	73804.850	514691.731
340	R17	73800.669	516541.923	73800.678	516541.909	73801.162	516542.345	73801.171	516542.331
341	R18	73796.985	518392.469	73796.999	518392.455	73797.478	518392.951	73797.492	518392.937
342	R19	73793.300	520243.020	73793.320	520243.007	73793.793	520243.563	73793.812	520243.549
343	S01	75767.184	486937.921	75767.204	486937.941	75767.741	486937.380	75767.761	486937.400
344	S02	75763.063	488788.383	75763.077	488788.402	75763.620	488787.902	75763.633	488787.921
345	S03	75758.941	490638.849	75758.949	490638.868	75759.497	490638.428	75759.505	490638.447
346	S04	75754.817	492489.321	75754.820	492489.338	75755.373	492488.961	75755.377	492488.977
347	S05	75750.692	494339.799	75750.691	494339.813	75751.248	494339.498	75751.247	494339.513
348	S06	75746.565	496190.281	75746.561	496190.293	75747.121	496190.041	75747.117	496190.053
349	S07	75742.436	498040.769	75742.430	498040.779	75742.992	498040.589	75742.986	498040.598
350	S08	75738.307	499891.262	75738.299	499891.269	75738.862	499891.143	75738.854	499891.149
351	S09	75734.176	501741.761	75734.167	501741.764	75734.731	501741.701	75734.722	501741.705
352	S10	75730.043	503592.265	75730.034	503592.265	75730.598	503592.266	75730.589	503592.266
353	S11	75725.909	505442.774	75725.900	505442.771	75726.464	505442.835	75726.455	505442.832

TABLE E.2
(Continued)

		Case h_0: Radius of INCRS Sphere = $R_{G@CP}$				Case h_{Real}: Radius of INCRS Sphere = $R_{G@CP} + h_{avg}$			
		INCRS TM(CP)		INCRS OS(CP)		INCRS TM(CP)		INCRS OS(CP)	
Row ID	**Point**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**	**Easting (m)**	**Northing (m)**
354	S12	75721.773	507293.289	75721.765	507293.282	75722.328	507293.410	75722.320	507293.403
355	S13	75717.636	509143.809	75717.630	509143.799	75718.191	509143.990	75718.185	509143.981
356	S14	75713.498	510994.334	75713.494	510994.322	75714.053	510994.575	75714.049	510994.563
357	S15	75709.358	512844.864	75709.358	512844.850	75709.913	512845.166	75709.912	512845.152
358	S16	75705.217	514695.400	75705.220	514695.384	75705.771	514695.762	75705.775	514695.746
359	S17	75701.074	516545.941	75701.082	516545.923	75701.628	516546.364	75701.637	516546.345
360	S18	75696.930	518396.488	75696.943	518396.469	75697.484	518396.970	75697.498	518396.951
361	S19	75692.784	520247.040	75692.804	520247.020	75693.338	520247.582	75693.358	520247.563
362	ZID A	47464.604	498725.545	47464.604	498725.548	47464.243	498725.387	47464.244	498725.390
363	ZID B	46767.176	498589.800	46767.177	498589.804	46766.793	498589.638	46766.794	498589.642
364	F 350	45521.184	501479.481	45521.187	501479.483	45520.760	501479.412	45520.763	501479.415
365	IMAGIS 47	69883.929	492270.930	69883.936	492270.935	69884.295	492270.562	69884.302	492270.567

TABLE E.3
Map coordinates of points in Marion County Test under the INCRS-S01 mapping

		INCRS-S01 coordinates				INCRS-S01 coordinates	
		Easting	Northing			Easting	Northing
Row ID	Point ID	(m)	(m)	Row ID	Point ID	(m)	(m)
01	A01	41338.317	487030.727	48	C10	45272.389	503656.636
02	A02	41352.397	488881.153	49	C11	45285.595	505507.106
03	A03	41366.482	490731.584	50	C12	45298.806	507357.581
04	A04	41380.573	492582.020	51	C13	45312.022	509208.062
05	A05	41394.668	494432.461	52	C14	45325.242	511058.547
06	A06	41408.768	496282.908	53	C15	45338.467	512909.038
07	A07	41422.873	498133.360	54	C16	45351.696	514759.535
08	A08	41436.983	499983.817	55	C17	45364.931	516610.036
09	A09	41451.099	501834.280	56	C18	45378.170	518460.543
10	A10	41465.219	503684.747	57	C19	45391.413	520311.055
11	A11	41479.344	505535.220	58	D01	47061.452	486989.310
12	A12	41493.474	507385.698	59	D02	47074.158	488839.731
13	A13	41507.609	509236.182	60	D03	47086.869	490690.158
14	A14	41521.749	511086.671	61	D04	47099.585	492540.589
15	A15	41535.894	512937.164	62	D05	47112.305	494391.026
16	A16	41550.044	514787.664	63	D06	47125.029	496241.468
17	A17	41564.199	516638.168	64	D07	47137.758	498091.916
18	A18	41578.359	518488.678	65	D08	47150.491	499942.368
19	A19	41592.524	520339.193	66	D09	47163.229	501792.826
20	B01	43246.030	487016.449	67	D10	47175.972	503643.289
21	B02	43259.652	488866.874	68	D11	47188.719	505493.758
22	B03	43273.279	490717.303	69	D12	47201.470	507344.232
23	B04	43286.911	492567.737	70	D13	47214.226	509194.711
24	B05	43300.548	494418.177	71	D14	47226.986	511045.195
25	B06	43314.190	496268.622	72	D15	47239.751	512895.685
26	B07	43327.836	498119.073	73	D16	47252.520	514746.179
27	B08	43341.488	499969.528	74	D17	47265.294	516596.679
28	B09	43355.144	501819.989	75	D18	47278.073	518447.185
29	B10	43368.805	503670.455	76	D19	47290.855	520297.695
30	B11	43382.470	505520.927	77	E01	48969.160	486976.449
31	B12	43396.141	507371.404	78	E02	48981.409	488826.868
32	B13	43409.816	509221.885	79	E03	48993.662	490677.293
33	B14	43423.496	511072.373	80	E04	49005.919	492527.723
34	B15	43437.181	512922.865	81	E05	49018.180	494378.159
35	B16	43450.871	514773.363	82	E06	49030.446	496228.600
36	B17	43464.566	516623.866	83	E07	49042.716	498079.046
37	B18	43478.265	518474.374	84	E08	49054.991	499929.497
38	B19	43491.969	520324.887	85	E09	49067.270	501779.953
39	C01	45153.742	487002.644	86	E10	49079.553	503630.415
40	C02	45166.906	488853.066	87	E11	49091.841	505480.882
41	C03	45180.075	490703.494	88	E12	49104.132	507331.355
42	C04	45193.249	492553.927	89	E13	49116.429	509181.832
43	C05	45206.427	494404.366	90	E14	49128.729	511032.315
44	C06	45219.610	496254.809	91	E15	49141.034	512882.803
45	C07	45232.798	498105.258	92	E16	49153.343	514733.297
46	C08	45245.990	499955.712	93	E17	49165.657	516583.796
47	C09	45259.187	501806.172	94	E18	49177.974	518434.300

TABLE E.3
(Continued)

		INCRS-S01 coordinates				INCRS-S01 coordinates	
		Easting	Northing			Easting	Northing
Row ID	Point ID	(m)	(m)	Row ID	Point ID	(m)	(m)
95	E19	49190.296	520284.809	142	H09	54779.383	501744.169
96	F01	50876.867	486964.059	143	H10	54790.289	503594.627
97	F02	50888.658	488814.477	144	H11	54801.198	505445.091
98	F03	50900.453	490664.901	145	H12	54812.111	507295.559
99	F04	50912.252	492515.330	146	H13	54823.029	509146.033
100	F05	50924.055	494365.764	147	H14	54833.949	510996.512
101	F06	50935.862	496216.203	148	H15	54844.874	512846.996
102	F07	50947.674	498066.648	149	H16	54855.803	514697.486
103	F08	50959.489	499917.098	150	H17	54866.735	516547.981
104	F09	50971.309	501767.553	151	H18	54877.671	518398.481
105	F10	50983.133	503618.013	152	H19	54888.611	520248.987
106	F11	50994.961	505468.479	153	I01	56599.981	486929.723
107	F12	51006.793	507318.950	154	I02	56610.397	488780.137
108	F13	51018.630	509169.427	155	I03	56620.818	490630.557
109	F14	51030.471	511019.908	156	I04	56631.242	492480.982
110	F15	51042.315	512870.395	157	I05	56641.670	494331.412
111	F16	51054.164	514720.887	158	I06	56652.102	496181.848
112	F17	51066.017	516571.385	159	I07	56662.537	498032.289
113	F18	51077.875	518421.887	160	I08	56672.976	499882.735
114	F19	51089.736	520272.395	161	I09	56683.419	501733.186
115	G01	52784.573	486952.142	162	I10	56693.865	503583.643
116	G02	52795.906	488802.559	163	I11	56704.315	505434.105
117	G03	52807.242	490652.981	164	I12	56714.769	507284.572
118	G04	52818.583	492503.408	165	I13	56725.226	509135.045
119	G05	52829.928	494353.841	166	I14	56735.687	510985.523
120	G06	52841.277	496204.279	167	I15	56746.152	512836.006
121	G07	52852.629	498054.722	168	I16	56756.620	514686.495
122	G08	52863.986	499905.171	169	I17	56767.092	516536.988
123	G09	52875.347	501755.625	170	I18	56777.568	518387.488
124	G10	52886.712	503606.084	171	I19	56788.047	520237.992
125	G11	52898.080	505456.549	172	J01	58507.683	486919.221
126	G12	52909.453	507307.018	173	J02	58517.641	488769.635
127	G13	52920.830	509157.493	174	J03	58527.604	490620.053
128	G14	52932.211	511007.974	175	J04	58537.570	492470.477
129	G15	52943.595	512858.459	176	J05	58547.539	494320.906
130	G16	52954.984	514708.950	177	J06	58557.512	496171.341
131	G17	52966.377	516559.446	178	J07	58567.489	498021.780
132	G18	52977.774	518409.948	179	J08	58577.469	499872.225
133	G19	52989.174	520260.455	180	J09	58587.453	501722.676
134	H01	54692.278	486940.696	181	J10	58597.440	503573.131
135	H02	54703.152	488791.112	182	J11	58607.430	505423.592
136	H03	54714.031	490641.533	183	J12	58617.425	507274.058
137	H04	54724.913	492491.959	184	J13	58627.422	509124.530
138	H05	54735.800	494342.391	185	J14	58637.423	510975.007
139	H06	54746.690	496192.827	186	J15	58647.428	512825.489
140	H07	54757.584	498043.269	187	J16	58657.436	514675.976
141	H08	54768.482	499893.717	188	J17	58667.448	516526.469

TABLE E.3
(Continued)

		INCRS-S01 coordinates				INCRS-S01 coordinates	
		Easting	Northing			Easting	Northing
Row ID	Point ID	(m)	(m)	Row ID	Point ID	(m)	(m)
189	J18	58677.463	518376.967	236	M08	64290.942	499843.531
190	J19	58687.482	520227.470	237	M09	64299.548	501693.978
191	K01	60415.383	486909.192	238	M10	64308.158	503544.431
192	K02	60424.884	488759.604	239	M11	64316.770	505394.888
193	K03	60434.389	490610.021	240	M12	64325.386	507245.351
194	K04	60443.896	492460.444	241	M13	64334.004	509095.820
195	K05	60453.408	494310.872	242	M14	64342.626	510946.294
196	K06	60462.922	496161.306	243	M15	64351.251	512796.773
197	K07	60472.440	498011.744	244	M16	64359.878	514647.257
198	K08	60481.961	499862.188	245	M17	64368.509	516497.747
199	K09	60491.486	501712.637	246	M18	64377.143	518348.242
200	K10	60501.013	503563.092	247	M19	64385.780	520198.742
201	K11	60510.545	505413.552	248	N01	66138.479	486881.935
202	K12	60520.079	507264.017	249	N02	66146.606	488732.345
203	K13	60529.617	509114.487	250	N03	66154.736	490582.759
204	K14	60539.159	510964.963	251	N04	66162.869	492433.179
205	K15	60548.703	512815.444	252	N05	66171.005	494283.604
206	K16	60558.251	514665.930	253	N06	66179.144	496134.034
207	K17	60567.803	516516.422	254	N07	66187.286	497984.470
208	K18	60577.358	518366.919	255	N08	66195.431	499834.911
209	K19	60586.916	520217.421	256	N09	66203.578	501685.357
210	L01	62323.083	486899.634	257	N10	66211.728	503535.809
211	L02	62332.126	488750.045	258	N11	66219.881	505386.265
212	L03	62341.172	490600.462	259	N12	66228.038	507236.728
213	L04	62350.222	492450.884	260	N13	66236.196	509087.195
214	L05	62359.275	494301.311	261	N14	66244.358	510937.668
215	L06	62368.330	496151.743	262	N15	66252.523	512788.146
216	L07	62377.390	498002.180	263	N16	66260.690	514638.629
217	L08	62386.452	499852.623	264	N17	66268.861	516489.118
218	L09	62395.517	501703.072	265	N18	66277.034	518339.612
219	L10	62404.586	503553.525	266	N19	66285.210	520190.112
220	L11	62413.658	505403.984	267	O01	68046.175	486873.794
221	L12	62422.733	507254.448	268	O02	68053.845	488724.202
222	L13	62431.811	509104.917	269	O03	68061.517	490574.616
223	L14	62440.893	510955.392	270	O04	68069.192	492425.035
224	L15	62449.978	512805.872	271	O05	68076.869	494275.459
225	L16	62459.065	514656.357	272	O06	68084.550	496125.888
226	L17	62468.156	516506.848	273	O07	68092.233	497976.323
227	L18	62477.251	518357.344	274	O08	68099.918	499826.763
228	L19	62486.348	520207.845	275	O09	68107.607	501677.208
229	M01	64230.781	486890.549	276	O10	68115.298	503527.659
230	M02	64239.367	488740.959	277	O11	68122.992	505378.115
231	M03	64247.955	490591.374	278	O12	68130.688	507228.576
232	M04	64256.546	492441.795	279	O13	68138.388	509079.043
233	M05	64265.140	494292.221	280	O14	68146.090	510929.515
234	M06	64273.738	496142.652	281	O15	68153.794	512779.992
235	M07	64282.338	497993.089	282	O16	68161.502	514630.475

TABLE E.3
(Continued)

Row ID	Point ID	INCRS-S01 coordinates Easting (m)	INCRS-S01 coordinates Northing (m)	Row ID	Point ID	INCRS-S01 coordinates Easting (m)	INCRS-S01 coordinates Northing (m)
283	O17	68169.212	516480.963	330	R07	73807.068	497954.716
284	O18	68176.924	518331.456	331	R08	73813.377	499805.154
285	O19	68184.640	520181.954	332	R09	73819.688	501655.597
286	P01	69953.871	486866.124	333	R10	73826.002	503506.045
287	P02	69961.083	488716.532	334	R11	73832.318	505356.499
288	P03	69968.297	490566.944	335	R12	73838.636	507206.958
289	P04	69975.513	492417.362	336	R13	73844.956	509057.422
290	P05	69982.733	494267.786	337	R14	73851.278	510907.892
291	P06	69989.954	496118.214	338	R15	73857.603	512758.367
292	P07	69997.179	497968.648	339	R16	73863.930	514608.847
293	P08	70004.405	499819.088	340	R17	73870.259	516459.333
294	P09	70011.635	501669.532	341	R18	73876.591	518309.823
295	P10	70018.867	503519.982	342	R19	73882.924	520160.320
296	P11	70026.101	505370.437	343	S01	75676.953	486845.948
297	P12	70033.338	507220.898	344	S02	75682.791	488696.353
298	P13	70040.578	509071.363	345	S03	75688.631	490546.763
299	P14	70047.820	510921.835	346	S04	75694.473	492397.179
300	P15	70055.065	512772.311	347	S05	75700.317	494247.600
301	P16	70062.312	514622.793	348	S06	75706.163	496098.027
302	P17	70069.562	516473.280	349	S07	75712.011	497948.458
303	P18	70076.814	518323.772	350	S08	75717.862	499798.895
304	P19	70084.069	520174.270	351	S09	75723.714	501649.338
305	Q01	71861.566	486858.927	352	S10	75729.568	503499.785
306	Q02	71868.319	488709.333	353	S11	75735.425	505350.238
307	Q03	71875.076	490559.745	354	S12	75741.283	507200.697
308	Q04	71881.834	492410.163	355	S13	75747.144	509051.160
309	Q05	71888.595	494260.585	356	S14	75753.007	510901.629
310	Q06	71895.358	496111.013	357	S15	75758.871	512752.104
311	Q07	71902.124	497961.446	358	S16	75764.738	514602.583
312	Q08	71908.892	499811.885	359	S17	75770.607	516453.068
313	Q09	71915.662	501662.328	360	S18	75776.478	518303.558
314	Q10	71922.435	503512.777	361	S19	75782.351	520154.054
315	Q11	71929.210	505363.232	362	ZID A	47438.663	498785.706
316	Q12	71935.987	507213.691	363	ZID B	46740.513	498653.724
317	Q13	71942.767	509064.156	364	F 350	45510.124	501550.091
318	Q14	71949.550	510914.627	365	IMAGIS 47	69822.640	492210.419
319	Q15	71956.334	512765.102				
320	Q16	71963.121	514615.583				
321	Q17	71969.911	516466.070				
322	Q18	71976.703	518316.561				
323	Q19	71983.497	520167.058				
324	R01	73769.260	486852.201				
325	R02	73775.556	488702.607				
326	R03	73781.854	490553.018				
327	R04	73788.154	492403.435				
328	R05	73794.456	494253.857				
329	R06	73800.761	496104.284				