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Analysis of Lane Change Crash Avoidance

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ABSTRACT

For designing crash avoidance systems it is necessary to know the dynamic conditions that characterize the accidents. This paper examines the dynamic conditions that set apart safe from unsafe lane changes. They are determined on the basis of the relative distances and velocities between the vehicles at the time the lane change is initiated, and upon consideration of the geometry of the lane change path. The analysis provides means to quantify the significance of the errors of the measurements and estimations that the countermeasure system must carry out to achieve its goal. The discussion includes consideration of system's latency and reaction times of driver and vehicle, to compare them with the time between lane change initiation and the moment when the crash would occur. The paper considers possible different levels of capability for lane change countermeasure systems, and how they could be verified by tests without crash risk.

INTRODUCTION

Among the multiple matters relevant to the establishment of the so-called "Intelligent Vehicle-Highway Systems," recently renamed "Intelligent Transportation Systems,'* the introduction of automotive crash avoidance systems is particularly important. Consequently, there is currently a great interest in those countermeasure systems. But to design them properly it is necessary to know the dynamic conditions that characterize the accidents, and how they could be changed to avoid or mitigate their occurrence. These dynamic conditions differ for the various types of automotive accidents.

This paper is concerned with the analysis of the dynamic conditions that set apart safe from unsafe automotive lane changes, and with the discussion of related matters. Using the pertinent variables, the paper first determines the conditions that identify safe and unsafe lane changes. The analysis continues with the determination of means to quantify the 'importance of the different variables, as well as the significance of the errors of the measurements and estimations that the countermeasure system must carry out to achieve its function. Then, the paper discusses the impact of the reaction times of the countermeasure system, driver, and vehicle. Comparing these times with-the time between the initiation of the lane change and the moment when the potential crash would occur is important to judge the effectiveness of the countermeasure system for unsafe lane changes. Finally, the paper deliberates on possible capability levels of the lane change countermeasure systems and how they could be verified by tests without collision risk.

BOUNDARIES FOR SAFE LANE CHANGE

To determine the dynamic conditions that set apart safe from unsafe lane changes, it is convenient to subdivide the analysis according to pertinent considerations and values of certain variables. Naming 1 and 2 the two vehicles involved, we designate 1 as the vehicle that makes the lane change. The analysis is carried out for lane change to the right; the analysis for lane change-to the left is just symmetric.

The analysis subdivisions consider which vehicle is faster, i.e., whether $V_1 = V_2$. In other words, the subdivisions

consider whether the closing velocity, $V_c=V_2-V_1$, is positive or negative. For $V_1 < V_2$, only the cases in which the front bumper of vehicle 1 is ahead the front bumper of vehicle 2 at the time the lane change is initiated need to be considered. Similarly, for $V_1 > V_2$ the reverse is true. The subdivisions further consider whether vehicle 1 completes the lane change in front or behind vehicle 2.

The results for each subdivision are presented in the following corresponding subsections. The details for obtaining the results are presented in the appendixes.

CASE $V_1 < V_2$, WITH 1 COMPLETING THE LANE CHANGE BEHIND 2 - This case is depicted in Figure 1.

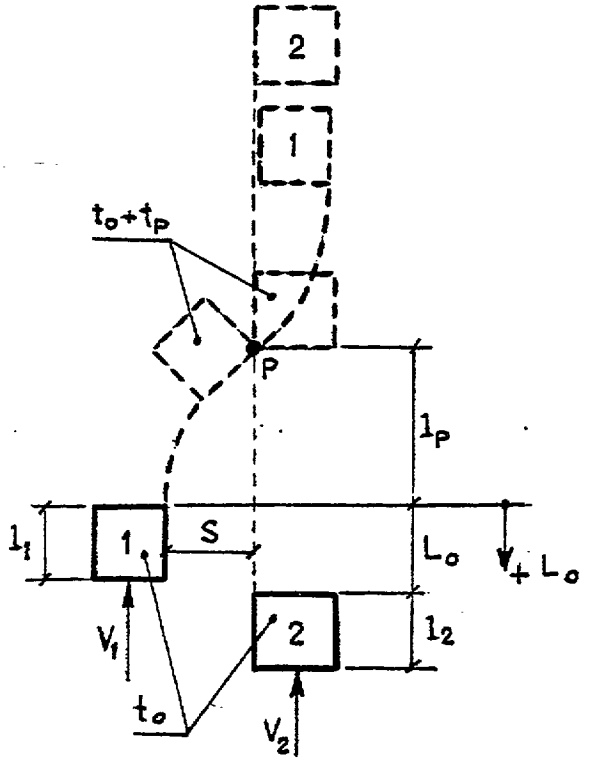

Figure 1. $V_1 < V_2$, with 1 completing the lane change behind 2

As it is shown in Appendix 1, the lane change will be safe if, at the initiation of the lane change, the longitudinal distance between the front bumpers of the vehicles is

$$L_o < V_c \cdot t_p - l_2 , \qquad (1)$$

where l_2 is the length of vehicle 2, and t_p is the time between the initiation of the lane change and the moment when the front of vehicle 1 would reach the interception point P. Appendix 2 shows that t_p can be determined as a function of the initial lateral separation, S, between the vehicles, the shape and dimensions of the lane change path, and the velocity of the lane changing vehicle.

Thus, the line given by $L_o = V_c \cdot t_p - l_2$ separates the safe and unsafe regions. This is shown in Figure 2, for lines of different values of t_p.

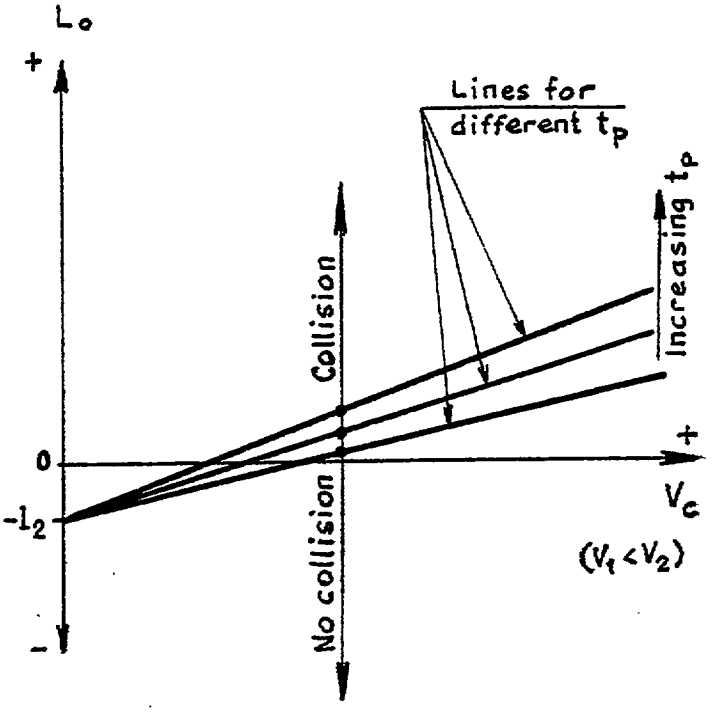

Figure 2. No collision for $L_o < V_c \cdot t_p - l_2$

EQ(1) applies specifically for constant velocities V_1 and V_2. If these velocities do not remain constant during the lane change, the equation would have to also include particular functions (which can be innumerably different) for the corresponding longitudinal accelerations or decelerations. It can be noticed, however, that EQ(1) provides automatically an additional margin of safety if vehicle 1 decelerates or vehicle 2 accelerates during the lane change. On the other hand, for 1 accelerating or 2 decelerating, a margin of safety would have to be estimated and added to the equation.

CASE $V_1 < V_2$, WITH 1 COMPLETING THE LANE CHANGE IN FRONT OF 2 - This is represented in Figure 3. Of course, if vehicle 2 does not decelerate there will always be a collision. Assuming that it decelerates, it is considered here a possible worst situation, i.e., the case in which driver 2 does not start decelerating (because he/she does not want, or because he/she is inattentive) until vehicle 1 completes the lane change.

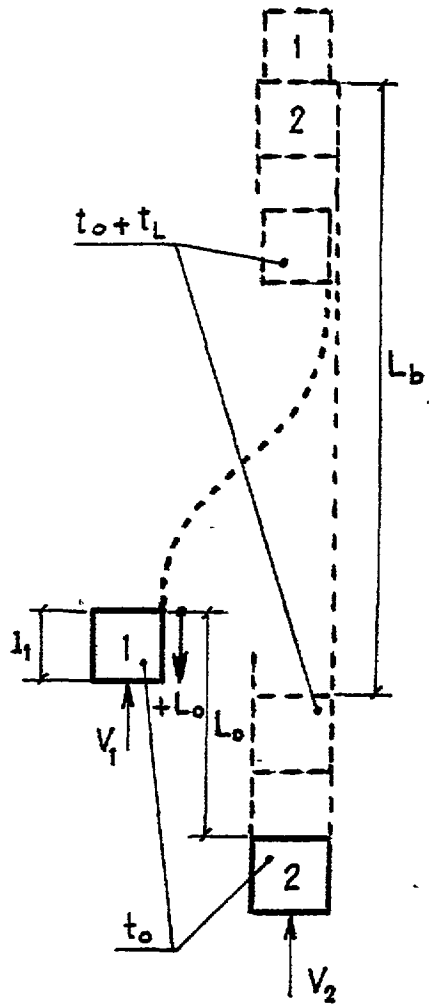

Figure 3. $V_1 < V_2$, with 1 completing the lane change in front of 2

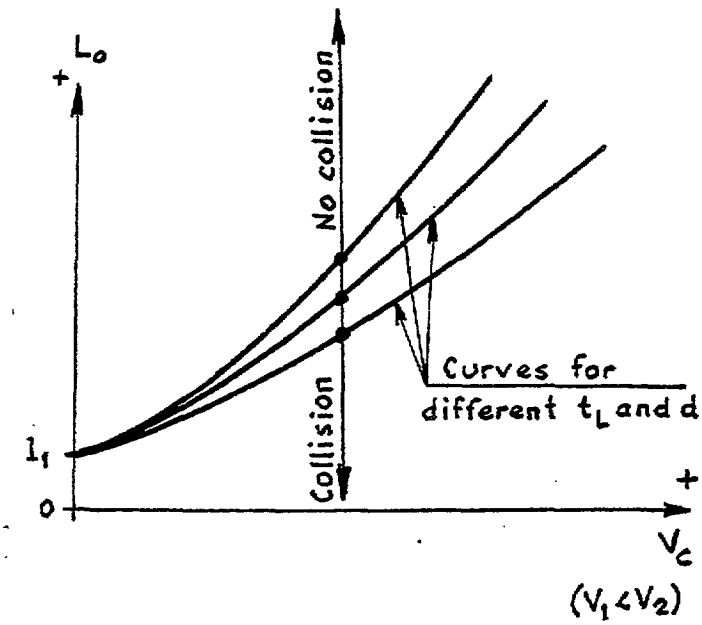

Figure 4. No collision for
$L_o > l_1 + V_c \cdot t_L + V_c^2/2d$

Thus, it is assumed that 2 starts braking at time $t_o + t_L$, and that in the braking distance L_b the velocity of vehicle 2 decreases from V_2 to V_1 under constant deceleration d. Then, as shown in Appendix 1, the lane change will be safe if at the time of initiating the lane change the distance

$$L_o > l_1 + V_c \cdot t_L + \frac{V_c^2}{2d} \cdot \qquad (2)$$

For d = o, this equation confirms that L_o should be theoretically infinite for no collision. t_L has a range of values which depend on the length of the lane change path and V_1. The deceleration d also has a range of values, depending on tire-road friction, and driver and vehicle attributes.

Figure 4 shows the boundaries for safe and unsafe lane changes, for different values of t_L and d.

EQ(2) applies specifically for constant V_1 during the lane change. The equation automatically provides an additional margin of safety if vehicle 1 accelerates longitudinally during the lane change. On the other hand, if vehicle 1 decelerates, a margin of safety would have to be estimated and added to the equation.

CASE $V_1 > V_2$, WITH 1 COMPLETING THE LANE CHANGE IN FRONT OF 2 - This case is depicted in Figure 5.

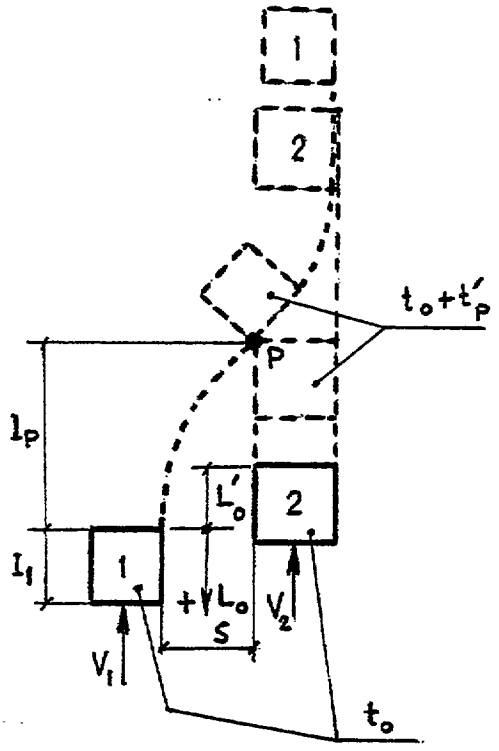

Figure 5. $V_1 > V_2$, with 1 completing the lane change in front of 2

Appendix 1 shows that in this case the lane change will be safe when

$$L_o > V_c \cdot t'_p + l_1 \quad , \qquad (3)$$

where l_1 is the length of vehicle 1, and t'_p is the time between the initiation of the lane change and the moment when the rear of vehicle 1 would reach the interception point P. (Thus $t'_p = t_p + l_1/V_1$, and t_p can be determined as indicated previously for the case of Figure 1).

Figure 6 indicates the boundary between collision and noncollision regions, for different values of t'_p.

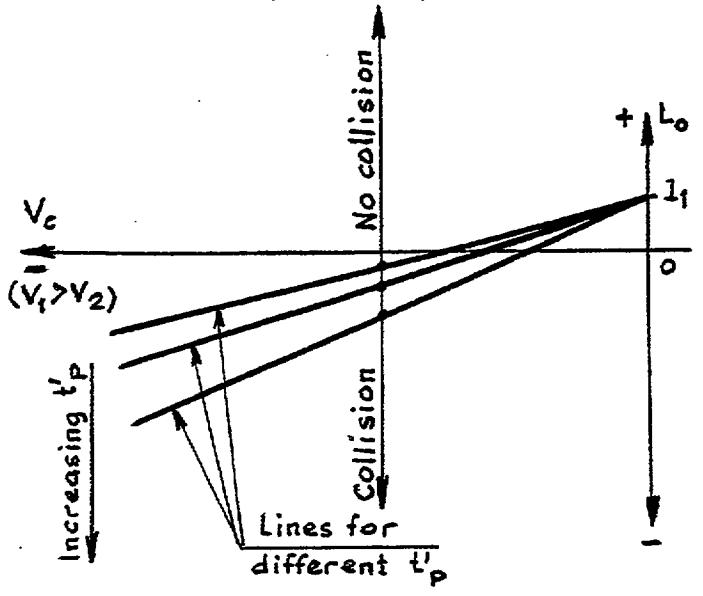

Figure 6. No collision for $L_o > V_c \cdot t'_p + l_1$

If V_1 and V_2 are not constant, comments analogous to those presented before for the case of Figure 1 apply here. However, the automatic additional margin of safety provided by EQ(3) only occurs if vehicle 1 accelerates or vehicle 2 decelerates during the lane change.

CASE $V_1 > V_2$, WITH 1 COMPLETING THE LANE CHANGE BEHIND 2 - This situation is described in Figure 7. Needless to say, if vehicle 1 does not decelerate there will always be a collision. It will be assumed here a possible worst situation, i.e., that 1 starts decelerating rather late, specifically when it completes the lane change.

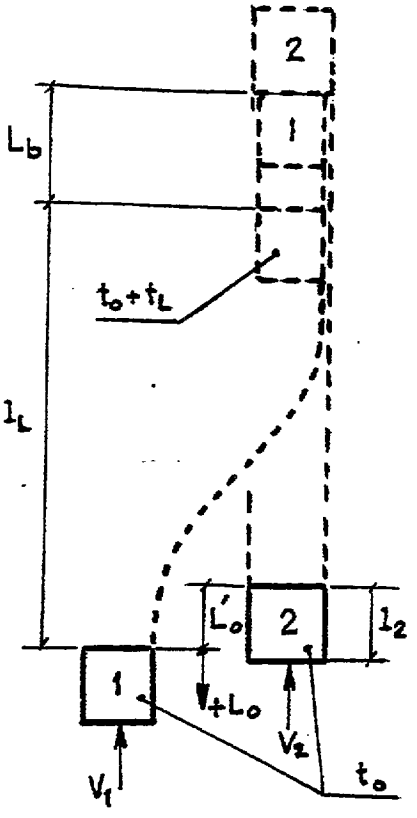

Figure 7. $V_1 > V_2$, with 1 completing the lane change behind 2

Appendix 1 shows that in this situation, for a constant deceleration d, the lane change will be safe if at the time of its initiation the distance

$$L_o < - l_2 + V_c \cdot t_L - \frac{V_c^2}{2d} \quad . \qquad (4)$$

For d=o, this equation verifies that

L_0 should be theoretically infinite to avoid the collision.

Figure 8 shows the boundaries for safe and unsafe lane changes, for different values of t_L and d.

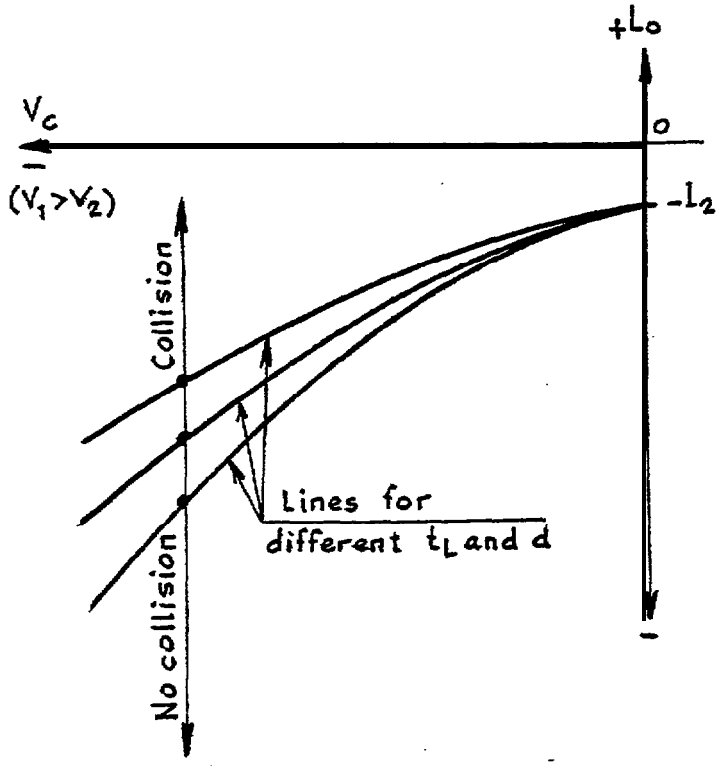

Figure 8. No collision for $L_0 < -1_2 + v_c \cdot t_L - v_c^2 / 2d$

EQ(4) applies specifically for constant V1 and V2 during the lane change. The equation automatically provides *an* additional safety margin if during the lane change vehicle 1 decelerates or vehicle 2 accelerates. On the other hand, for 1 accelerating or 2 decelerating during the lane change, a margin of safety would have to be estimated and added to the equation.

COMPILATION OF SAFE AND UNSAFE REGIONS FOR LANE CHANGE

The results for the various cases considered above are compiled in Figure 9. The hatched areas cover the conditions for which collisions would always occur. The other areas are safe as long as the lane changes **are** conducted according to the asumptions indicated in the derivation of the equations that determine the safe areas.

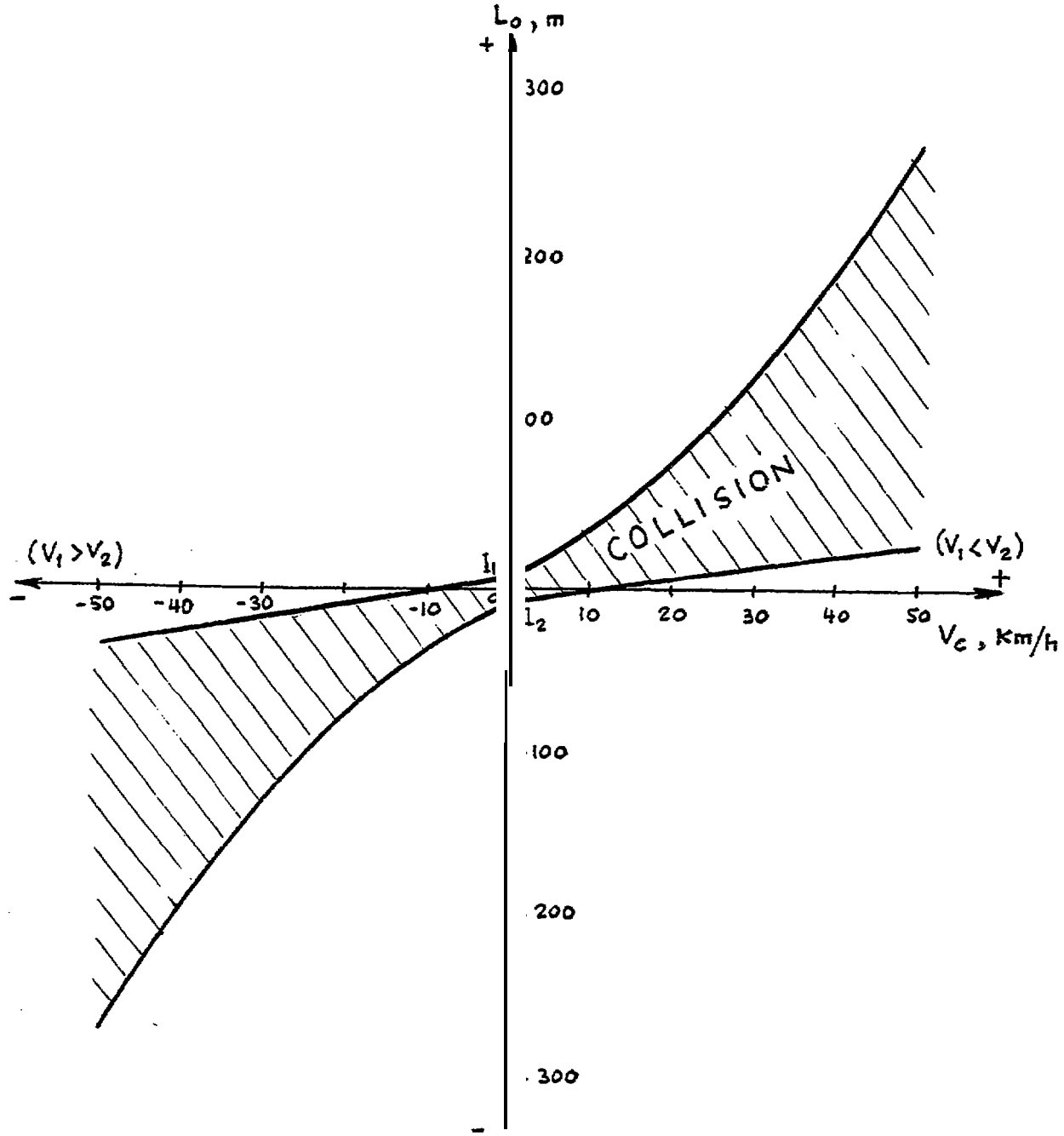

Figure 9. Collision region for lane change.
(The plot is an example for
t_L=5s , t_p = l.5s, d=2m/s^2)

QUANTIFYING. THE IMPACT OF THE VARIOUS VARIABLES, AND OF THE ERRORS OF MEASUREMENT AND ESTIMATION

Partial differentiation of the equations EQ(1) through EQ(4) provides quantification of the impact that each of the independent variables, zf has individually on the determination of the boundaries for safe lane change. Thus,

$$dL_o\Big|_{z_i} = \frac{\partial L_o}{\partial z_i}\, dz_i, \qquad (5)$$

and the percentage variation of L_o caused by the percentage variation of z_i is

$$\%L_o\Big|_{z_i} = \frac{z_i}{L_o}\cdot\frac{\partial L_o}{\partial z_i}\,\%z_i \,. \qquad (6)$$

The warnings or controlling outputs of the countermeasure system may depend on measurements (for L_o, V_c, S, for example) and estimates (for l_p, l_L, for example) that the system must carry out during its operation. These measurements and estimates will intrinsically contain some degree of error, which will result in some total error for the system's output. Applying error theory to the pertinent equation EQ(1) through EQ(4) above, the total error, EL_o, of the quantity L_o is given by

$$EL_o = \sum^{n} \frac{\partial L_o}{\partial z_i}\, Ez_i \qquad (7)$$

where Ez_i is the error of the corresponding independent variable z_i (i=1,.....n).

CONSIDERATION OF THE TIME TO CRASH, SYSTEM'S LATENCY, AND REACTION TIME

The following times must also be taken into account for lane change crash avoidance:

- The time to crash, t_c, i.e., the time between the initiation of the lane change and the moment of the potential crash.

- The latency time of the system, t_s, i.e., the time taken by the countermeasure system to provide the warning.

- The reaction time, t_r, i.e., the time between the warning signal and the moment the vehicle's countermeasure maneuver is started. This t_r includes the driver's reaction time (which generally will be the largest component) and the vehicle's reaction time.

The significance of these times depends on the inequality $t_s + t_r \lessgtr t_c$.

Thus, if

$$t_s + t_r < t_c \qquad (8)$$

there is a recovery time, $t_c - t_s - t_r$, for conducting a maneuver that could avoid or mitigate the crash. On the other hand, if

$$t_s + t_r > t_c \qquad (9)$$

the crash cannot be avoided because of a lack of recovery time.

During the time $t_s + t_r$ that passes without correcting the course, vehicle 1 has a lateral displacement h. Since S was the initial lateral separation between the two vehicles, to avoid the crash vehicle 1 has to recover a lateral distance equal to h-S in the recovery time $t_c-t_s-t_r$. Accordingly, the magnitude of the average lateral acceleration, a, which vehicle 1 must exercise, to move laterally away from vehicle 2 and avoid the crash, can be estimated using the simple equation "distance = (acceleration/2) x time^2 ". That is,

$$a = 2(h-S)/(t_c-t_s-t_r)^2 . \qquad (10)$$

Several observations regarding the variables related to the potential crash will be now indicated.

The distance h can be estimated. For a sinusoidal lane change path, EQ(A17) of Appendix 2 yields

$$h = \frac{H}{2}\left[1-\sin\left(\frac{\pi}{l_L}x+\frac{\pi}{2}\right)\right] .$$

But $x = V_1(t_s + t_r)$; therefore,

$$h = \frac{H}{2}\left\{1-\sin\left[\frac{\pi}{l_L}V_1\left(t_s+t_r\right)+\frac{\pi}{2}\right]\right\}. \qquad (11)$$

The latency t_s is a characteristic of the individual countermeasure system. The reaction time t_r depends on the characteristics of the driver and the vehicle. However, the time to crash, t_c, is a function of the particular dynamic scenario. Therefore, some remarks on the determination of t_c are presented in the following subsection. These remarks apply, as an example, to the scenario depicted in Figure 1; different scenarios can yield equations different from EQ(12), EQ(13), EQ(14), and EQ(15) given below.

ESTIMATION OF THE TIME TO CRASH FOR THE SCENARIO OF FIGURE 1- Since l_p/V_1 is the time needed by vehicle 1 to reach the path of vehicle 2, the eventual crash can only occur at

$$t_c \geqslant l_p/V_1, \qquad (12)$$

and this equation indicates that the minimum time to crash would be

$$t_{cmin} = l_p/V_1 \quad (13)$$

For a sinusoidal lane change, the value of l_p is given by EQ(A19) of Appendix 2. Accordingly, for such a path

$$t_{cmin} = \frac{\left[\arcsin(1-2S/H) - \pi/2\right]\cdot l_L}{\pi V_1}. \quad (14)$$

The upper time to crash would be just when vehicle 1 completes the lane change, i.e.,

$$t_{cmax} = l_L/V_1. \quad (15)$$

This is because if the accident has not occurred at $t_c = l_L/V_1$, it cannot occur given that $V_2 > V_1$.

LEVELS OF COUNTERMEASURE SYSTEM CAPABILITY

Crash avoidance systems can be designed for different levels of capability, which of course will have different levels of costs and benefits. Basically, for lane change countermeasure systems the following levels can be envisioned:

a) Systems which simply inform about the existance of targets in the "blind spots," without indicating whether or not a safe lane change could be initiated. The information could be presented whenever there is a target in the "blind spots," or only when requested by the driver (by activating the turning signal, for example). Providing the information whenever there is a target could be helpful for inattentive or uncareful drivers, but on the other hand it could be distractive or annoying in many cases.

b) Systems which provide a warning when an unsafe lane change is initiated. If the system's latency, and the reaction times of the driver and the vehicle are too high to allow for a countermeasure maneuver, collisions could still happen. The warning would also be provided if, upon activation of the turning signal by the driver, the system determines that it would then be unsafe to change lanes. The combination of this type of system and a conscious driver (who would request the information by activating the turning signal) would insure safe lane changes.

c) Systems such as systems b) but which also introduce an automatic control of the vehicle to avoid the collision if the driver's reaction time exceeds a permissible value.

VERIFICATION OF THE COUNTERMEASURE SYSTEMS' CAPABILITIES

This refers only to the verification of the fundamental capabilities of the countermeasure systems, including those that will allow classifying the systems within the levels a), b) or c) mentioned in the previous section. (The verification of more particular matters, such as the appropriateness of the interface between the system and the driver, for example, are not dealt with here.)

To verify the fundamental capabilities of a countermeasure system it is necessary to record its outputs during track tests. The tests must cover all pertinent conditions under which lane changes are conducted. Thus, the test will include different target vehicles, usual ranges of absolute and relative vehicles' velocities, usual ranges of distances between vehicles, different weather, day and night light, pertinent degree of clutter, etc. Then, the initiation and end of the outputs of the countermeasure system, for each test run, can be located on theoretical plots of the type shown in Figure 9, to verify that the outputs are provided when needed according to the claims made for the system.

The verification tests should be carried out without crash risk. For systems of the categories a) and b) described in the previous section, the verification data can be recorded without actually deviating the vehicles from their straight paths. In fact, simple "blind spot" systems can only claim the detection of targets in the blind areas, they cannot claim whether or not the lane change would be safe. For testing the b) systems, it will be necessary to incorporate some device capable of simulating the steering of the vehicle and providing an input to the countermeasure system without actually steering the vehicle. And in this case the test data must include for each run the recording of the instant when the initiation of the lane change would be simulated. As indicated previously, the recording of the lane change initiation must be done for different runs with pertinent different distances between the vehicles, while the other variables are kept constant. Also, the tests should verify the outputs of the countermeasure system with and without having on the turning signals of the vehicle. Obviously, a countermeasure system that only provides output while the turning signals are on will not yield benefits when the drivers do

not use the turning signals.

The countermeasure systems described under c) in the previous section would eliminate the risk of unsafe lane changes if their capability for automatic corrective maneuvers works properly. But until their automatic capability is confirmed, preliminary tests without actual lane changes should be conducted, in the same manner that it has just been indicated for the b) countermeasure systems.

SUMMARY

The paper describes a methodology for determining the dynamic conditions that differentiate safe from unsafe automotive lane changes.

The differences between safe and unsafe lane changes are quantified in terms of the pertinent variables. These variables include the longitudinal and lateral distances between the vehicles at the initiation of the lane change, their absolute and relative velocities, and the shape and dimensions of the lane change path.

The analysis is made for changing lanes on straight roads. It could be extended in an analogous fashion for anomalous lane changes, such as for lane changes on curves.

The analysis includes the determination of the significance of the errors of the measurements and estimations. that the countermeasure systems must carry out during its operation to achieve its goal.

A discussion is included regarding the impact that the countermeasure system's latency, and the reaction times of the driver and the vehicle have on crash avoidance effectiveness.

. A discussion is also included on, possible levels -of capability of the countermeasure systems and how the capabilities can be verified by tests without crash risk.

The scope of the paper is limited to the outcome for potential individual lane change accidents. To determine the outcome of introducing lane change crash avoidance systems in a population of vehicles, the analysis should be complemented with pertinent statistical procedures which will take into account the variabilities within the given population. This, of course, will require knowledge of (or estimates of) the distributions of the variable quantities, i.e., the distributions for: absolute and relative velocities of the vehicles, longitudinal and lateral distances between the vehicles, drivers' reaction times and other pertinent human factors, systems' latencies, lateral accelerations for recovery, etc.

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(These references are for general information related to the subject of the paper; they are not referred to in the text of the paper.)

The contents of this paper express the views of the author on the subject, and do not necessarily- represent the views or positions of the National Highway Traffic Safety Administration.

APPENDIXES

NOMENCLATURE

Vehicle 1 is the one that makes the lane change.
d=longitudinal deceleration.
H=total lateral displacement of vehicle 1 for the complete lane change.
h=lateral displacement of vehicle 1, from t_o until it starts a corrective maneuver.
L_b=longitudinal distance traveled by a vehicle while braking with deceleration d.
L_o=longitudinal distance between front bumpers of vehicles 1 and 2 at time t_o.
l_L=total longitudinal distance traveled by vehicle 1 during its lane change.
l_p=longitudinal distance traveled by vehicle 1 from t_o until it reaches the interception point P (Fig 1).
l_1=length of vehicle 1.
l_2=length of vehicle 2.
S=lateral distance between vehicles 1 and 2 at time t_o.
t=time
t_b=time braking with constant deceleration d.
t_L=time taken by vehicle 1 to execute the lane change.
t_p=time taken by vehicle 1 to travel the distance l_p.
t_p'=time taken by vehicle 1 to travel the distance $l_p + l_1$.
t_o=time at which vehicle 1 initiates the lane change.
V= velocity.
$V_c = V_2 - V_1$, the longitudinal closing velocity.
V_1=longitudinal (i.e., along x) velocity of vehicle 1.
V_2=longitudinal velocity of vehicle 2.
x=longitudinal coordinate of a point on the lane change curve.
y=lateral coordinate of a point on the lane change curve.

APPENDIX 1 - DETAILS ON THE DETERMINATION OF THE BOUNDARIES FOR SAFE LANE CHANGE

The details follow, for each specific case of the corresponding figure presented in the main text.

Case of Figure 1, for $V_1 < V_2$, with 1 completing the lane change behind 2.

There will not be a collision if

$$L_o + l_p < V_2 \cdot t_p - l_2. \qquad (A1)$$

But we are considering $l_p = V_1 \cdot t_p$; therefore, substituting in EQ(A1):

$$L_o + V_1 \cdot t_p < V_2 \cdot t_p - l_2, \text{ or}$$

$$L_o < (V_2 - V_1) \cdot t_p - l_2.$$

Consequently, there will not be a collision if

$$L_o < V_c \cdot t_p - l_2 \qquad (A2),$$

which is EQ(1) of the main text.

Case of Figure 3, for $V_1 < V_2$, with 1 completing the lane change in front of 2.

As it was indicated in the main text, it is assumed that vehicle 2 starts braking at time $t_o + t_L$ and continues with constant deceleration d until V_2 becomes V_1. Then, there will not be a collision if

$$L_o + V_1 \cdot t_L + V_1 \cdot t_b - l_1 > V_2 \cdot t_L + L_b. \qquad (A3)$$

But $t_b = (V_2 - V_1)/d$, (A4)

and $L_b = \int_{t_L}^{t_L+t_b} V.dt = \int_o^{t_b} V.dt = \int_o^{t_b} (V_2 - t.d)\,dt =$

$$= \int_o^{t_b} V_2.dt - \int_o^{t_b} t.d.dt = V_2.t_b - \frac{d}{2}t_b^2. \qquad (A5)$$

Therefore,, using EQ(A4) and EQ(A5), EQ(A3) can be written

$$L_o + V_1 \cdot t_L + V_1 \frac{V_2 - V_1}{d} - l_1 > V_2 \cdot t_L + V_2 \frac{V_2 - V_1}{d} - \frac{d}{2}\left(\frac{V_2 - V_1}{d}\right)^2,$$

$$\text{or } L_o > (V_2 - V_1)t_L + (V_2 - V_1)\frac{V_2 - V_1}{d} - \frac{(V_2 - V_1)^2}{2d} + l_1. \qquad (A6)$$

Consequently, there will not be a collision if

$$L_o > l_1 + V_c \cdot t_L + \frac{V_c^2}{2d}, \qquad (A7)$$

which is EQ(2) of the main text.

Case of Figure 5, for $V_1 > V_2$, with 1 completing the lane change in front of 2.

There will not be a collision if

$$L_o' + V_2 \cdot t_p' < l_p. \qquad (A8)$$

We are considering $l_1 + l_p = V_1 \cdot t_p'$; therefore, substituting in EQ(A8):

$$L_o' + V_2 \cdot t_p' < V_1 \cdot t_p' - l_1, \text{ or}$$

$$L_o' < (V_1 - V_2) \cdot t_p' - l_1, \text{ , or}$$

$$L_o' < -V_c \cdot t_p' - l_1 \qquad \text{(A9)}$$

But letting $L_o' = -L_o$, EQ(A9) can be written

$$-L_o < -V_c \cdot t_p' - l_1 \cdot$$

Consequently, there will not be a collision if

$$L_o > V_c \cdot t_p' + l_1 \qquad \text{(A10)}$$, which is

EQ(3) of the main text.

Case of Figure 7, for $V_1 \geq V_2$, with 1 completing the lane change behind 2.

As it was indicated in the main text, it is assumed that vehicle 1 starts braking at time $t_o + t_L$ and continues with constant deceleration d until V_1 becomes V_2. Then, there will not be a collision if

$$l_L + L_b < L_o' + V_2 (t_L + t_b) - l_2 \qquad \text{(A11)}$$

But we have

$$l_L = V_1 \cdot t_L, \qquad \text{(A12)}$$

$$t_b = (V_1 - V_2)/d, \qquad \text{(A13)}$$

and $$L_b = \int_{t_L}^{t_L+t_b} V \cdot dt = \int_{o}^{t_b} (V_1 - t \cdot d)\,dt =$$

$$= \int_{o}^{t_b} V_1 \cdot dt - \int_{o}^{t_b} t \cdot d \cdot dt = V_1 \cdot t_b - \frac{d \cdot t_b^2}{2}. \qquad \text{(A14)}$$

Therefore, using EQ(A12), EQ(A13), and EQ(A14), EQ(A11) can be written

$$V_1 \cdot t_L + V_1 \frac{V_1 - V_2}{d} - \frac{(V_1 - V_2)^2}{2d} < L_o' + V_2 \left(t_L + \frac{V_1 - V_2}{d}\right) - l_2 \cdot$$

$$\text{or } L_o' > (V_1 - V_2) \cdot t_L + \frac{(V_1 - V_2)^2}{2d} + l_2 \cdot \qquad \text{(A15)}$$

Then, letting $L_o' = -L_o$, EQ(A15) can be written

$$-L_o > (V_1 - V_2) \cdot t_L + \frac{(V_1 - V_2)^2}{2d} + l_2 .$$

Consequently, there will not be a collision if

$$L_o < -l_2 + V_c \cdot t_L - \frac{V_c^2}{2d} \cdot \qquad \text{(A16)}$$

which is EQ(4) of the main text.

APPENDIX 2 - RELATIONSHIPS FOR POSITIONS ALONG THE LANE CHANGE PATH

The relationships given here are for a sinusoidal lane change path as depicted in Figure A1. For other paths the corresponding formulas would be more or less analogous, unless the path would differ significantly from a sinusoidal one.

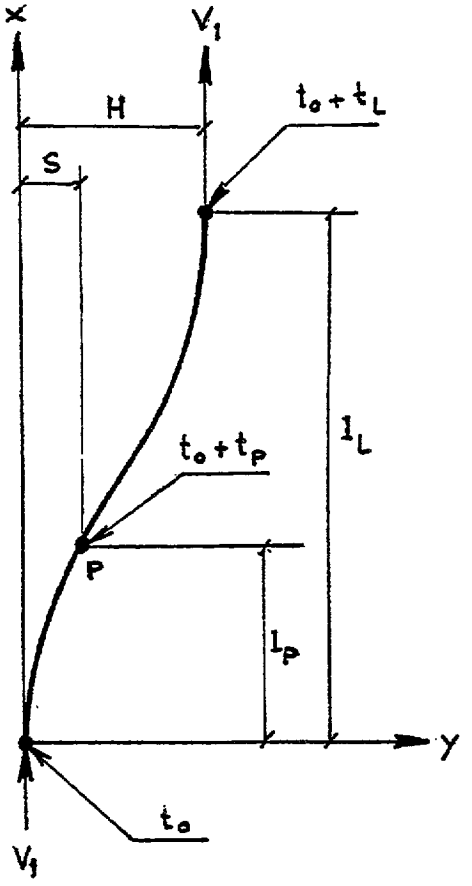

Figure A1. Lane change path

The longitudinal and lateral coordinates of the sinusoidal path are related by the corresponding equation

$$y = \frac{H}{2}\left[1 - \sin\left(\frac{\pi}{l_L} x + \frac{\pi}{2}\right)\right], \qquad \text{(A17)}$$

$$\text{or} \quad x = \left[\arcsin\left(1 - \frac{2y}{H}\right) - \frac{\pi}{2}\right] \cdot \frac{l_L}{\pi} \cdot \qquad \text{(A18)}$$

Therefore, for any point P:

$$l_p = \left[\arcsin\left(1 - \frac{2S}{H}\right) - \frac{\pi}{2}\right] \cdot \frac{l_L}{\pi} \cdot \qquad \text{(A19)}$$

Also, for constant longitudinal velocity V_1:

$$t_p = l_p / V_1 , \qquad \text{(A20)}$$

$$\text{and} \quad t_L = l_L / V_1 . \qquad \text{(A21)}$$