

Real-time Rideshare Matching Problem

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Real-time Rideshare Matching Problem

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16. Abstract According to the Commuting in America report, more than 88% of American workers commute to work in private vehicles, which accounts for a daily sum of 166 million miles. The report also indicates that more than 76% of the commuters drive alone, resulting in inefficient use of the transportation infrastructure. Development of programs that encourage ridesharing can alleviate this problem; however, past efforts to promote ridesharing have not achieved full potential due to rigid spatial and temporal requirements of the travel schedules of participating parties. A dynamic rideshare system that takes advantage of real-time passenger demand, vehicle supply, and travel time information can overcome these issues. Real-time rideshare matching differs from the classical rideshare matching in two ways. First, traditional systems assume that the travelers have a fixed schedule and a fixed set of origins and destinations. Real-time systems must take into account each trip individually and be able to match the rides to arbitrary origins and destinations based on the passengers' and drivers' preferences. The second major difference is that real time rideshare systems must be able to respond to instant requests in a very short period of time. Numerous papers exist that deal with various aspects of ridesharing; however, few studies have considered the rideshare problem as an optimization problem. Recent technological advances in information technology, communication, and the improvements in the ITS infrastructure (i.e., availability of real time travel time information and live accident and congestion reports) have added a new dimension to the ridesharing problem. Motivated by the use of technology to improve mobility through efficient use of existing transportation capacity, this project proposes an optimization framework for Automated Real-Time Rideshare Network.			
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Introduction

Why ridesharing?

The price of oil again began rising in February 2009 and it reached the two-year record level of USD 91.15 a barrel in January 2011 (Figure 1). The analysts predict that oil prices will continue to rise and consumers' budgets will be more under pressure. The world economy is running fast out of the cheap oil that has powered the economy development since the 1950s [Londarev and Baláž, 2005]. The problem related to traffic congestion and environmental pollutions in big cities are increasing [Slack et al., 2006].



Figure 1: Weekly all countries oil price FOB weighted

Source: U.S. Energy Information Administration

In the face of increasing the price of transportation fuel cost and worsening the effects of traffic congestion and environmental pollutions, wise usage of personal automobiles are gaining more attraction. Rideshare is a solution for car travel reduction aiming to bring together travelers with similar itineraries and time schedules. Ridesharing has generated much interest in recent years with media coverage (the Wall Street Journal [Saranow, 2006], Time [Sayre, 2006] Newsweek [Levy, 2007], Business Week [Walters, 2007], ABC News [Bell, 2007], The NY Times [Wiedenkeller, 2008], USA Today's [Jesdanun, 2008], and NBC4 News [McPeck, 2011], among many others.)

Mean occupancy rates of personal vehicle trips (the average number of travelers per vehicle trip) in the United States is 1.6 persons per vehicle mile (Table 1) ranging from 1.14 for work-related

trips to 2.05 for social or recreational trips and weekday trips have a weighted (by miles travelled in trip) occupancy of 1.5 compared to 2 people per vehicle mile on weekend trips [BTS, 2001].

Table 1: Vehicle Occupancy per Vehicle Mile by Daily Trip Purpose

	Mean	SE
All personal vehicle trips	1.63	0.012
Work	1.14	0.007
Work-related	1.22	0.020
Family/personal	1.81	0.016
Church/school	1.76	0.084
Social/recreational	2.05	0.028
Other	2.02	0.130

NOTE: SE = standard error.

SOURCE: The 2001 National Household Travel Survey, daily trip file, U.S. Department of Transportation.

The large travel demand for personal car transportation together with low occupancies leads to traffic congestion that is an increasingly important issue in many urban areas with rapid population and economic growth. Congestion has gotten worse in regions of all sizes in the United States. In 2007, congestion caused urban Americans to travel 4.2 billion hours more and to purchase an extra 2.8 billion gallons of fuel for a congestion cost of \$87.2 billion which is an increase of more than 50% over the previous decade (Table 2). This was a decrease of 40 million hours and a decrease of 40 million gallons, but an increase of over \$100 million from 2006 due to an increase in the cost of fuel and truck delay [UMR 2009]. An effective ridesharing system that encourages the travelers to share their personal car could be an effective countermeasure against traffic congestion with reducing personal car travel demand.

In the United States more than 87% of commuters travel in private vehicles which accounts for a daily sum of 166 million miles and single occupancy vehicles make up a big portion of 77% of the travels (Table 3), resulting in inefficient use of the transportation infrastructure [CIAIII, 2006] and giving a big opportunity for developing a rideshare system.

Table 2: Major Findings for 2009 – The Important Numbers for the 439 U.S. Urban Areas

Measures of...	1982	1997	2006	2007
... Individual Traveler Congestion				
Annual delay per peak traveler (hours)	14	32	37	36
Travel Time Index	1.09	1.20	1.25	1.25
"Wasted" fuel per peak traveler (gallons)	9	21	25	24
Congestion Cost (constant 2007 dollars)	\$290	\$621	\$758	\$757
Urban areas with 40+ hours of delay per peak traveler	1	10	27	23
... The Nation's Congestion Problem				
Travel delay (billion hours)	0.79	2.72	4.20	4.16
"Wasted" fuel (billion gallons)	0.50	1.82	2.85	2.81
Congestion cost (billions of 2007 dollars)	\$16.7	\$53.6	\$87.1	\$87.2
... Travel Needs Served				
Daily travel on major roads (billion vehicle-miles)	1.68	2.93	3.79	3.82
Annual public transportation travel (billion person-miles)	38.8	42.6	53.4	55.8
... Expansion Needed to Keep Today's Congestion Level				
Lane-miles of freeways and major streets added every year	15,500	16,532	15,032	12,676
Public transportation riders added every year (million)	3,456	3,876	3,779	3,129
... The Effect of Some Solutions				
Travel delay saved by				
Operational treatments (million hours)	7	116	307	308
Public transportation (million hours)	290	455	622	646
Congestion costs saved by				
Operational treatments (billions of 2007 dollars)	\$.02	\$2.3	\$6.4	\$6.5
Public transportation (billions of 2007 dollars)	\$6.3	\$9.3	\$13.1	\$13.7

Travel Time Index (TTI) – The ratio of travel time in the peak period to travel time at free-flow conditions. A Travel

Time Index of 1.35 indicates a 20-minute free-flow trip takes 27 minutes in the peak.

Delay per Peak Traveler – The extra time spent traveling at congested speeds rather than free-flow speeds divided by the number of persons making a trip during the peak period.

Wasted Fuel – Extra fuel consumed during congested travel.

Vehicle-miles – Total of all vehicle travel (10 vehicles traveling 9 miles is 90 vehicle-miles).

Expansion Needed – Either lane-miles or annual riders to keep pace with travel growth (and maintain congestion).

Source: 2009 Urban Mobility Report, Texas Transportation Institute, the Texas A&M University System, July 2009

Single occupancy vehicles commute a daily sum of 127 million miles [CIAIII, 2006]. Composite national average driving cost per mile is 54.1 cents including average fuel, routine maintenance, tires, insurance, license and registration, loan finance charges and depreciation costs [AAA 2008] (Table 4 represents more detailed breakdown by miles driven and vehicle type.). Therefore, a successful ridesharing program that increases the occupancy of vehicles may make a significant saving on driving costs of the roadway system.

Table 3: Mode share Trends, 2000-2004

Mode	Census 2000	2000 ACS	2001 ACS	2002 ACS	2003 ACS	2004 ACS
	128,279,228*	127,731,766*	128,244,898*	128,617,952*	129,141,982*	130,832,187*
	Percent					
Private vehicle	87.88	87.51	87.58	87.81	88.20	87.76
Drive alone	75.70	76.29	76.84	77.42	77.76	77.68
Carpool	12.19	11.22	10.74	10.39	10.44	10.08
Transit	4.57	5.19	5.07	4.96	4.82	4.57
Bus	2.50	2.81	2.79	2.71	2.63	2.48
Streetcar	0.06	0.07	0.06	0.06	0.06	0.07
Subway	1.47	1.57	1.51	1.45	1.44	1.47
Railroad	0.51	0.55	0.54	0.56	0.53	0.53
Ferry	0.03	0.04	0.04	0.04	0.04	0.03
Taxi	0.16	0.16	0.13	0.14	0.12	0.12
Motorcycle	0.11	0.12	0.12	0.11	0.11	0.15
Bike	0.38	0.44	0.42	0.36	0.37	0.37
Walk	2.93	2.68	2.55	2.48	2.27	2.38
Other	0.70	0.85	0.87	0.82	0.72	0.81
Work at home	3.26	3.21	3.38	3.46	3.50	3.84
All	100.00	100.00	100.00	100.00	100.00	100.00

*Total workers
 Note: ACS excludes group quarters population.

Source: Commuting in America III: The Third National Report on Commuting Patterns and Trends, 2006, Transportation Research Board, 2006

Table 4: Driving cost by miles driven and vehicle type

miles per year	10,000	15,000	20,000
small sedan	55.1 cents	42.1 cents	35.7 cents
medium sedan	71.9 cents	55.2 cents	46.9 cents
large sedan	85.8 cents	65.1 cents	54.8 cents
composite average *	71.0 cents	54.1 cents	45.8 cents

Source: 2008 Your Driving Costs, American Automobile Association

Private automobile is also the most pollutant transportation mode [Hensher, 2008]. Transportation is also a significant source of greenhouse gas (GHG) emissions. In 2003, the transportation sector accounted for about 27 percent of total U.S. GHG emissions and it was predicted to continue increasing rapidly, reflecting the anticipated impact of factors such as economic growth, increased movement of freight by trucks and aircraft, and continued growth in personal travel. About 81 percent of transportation GHG emissions in the United States came

from “on-road” vehicles. Personal transport accounted for 62 percent of total transportation emissions (35 percent for passenger cars and 27 percent for light-duty trucks including SUVs, minivans and pickup trucks and less than 1 percent for motorcycles) and heavy-duty vehicles including trucks and buses, were responsible for 19 percent of total transportation emissions. (Figure 2) [Transportation GHG Emissions Report, 2006].

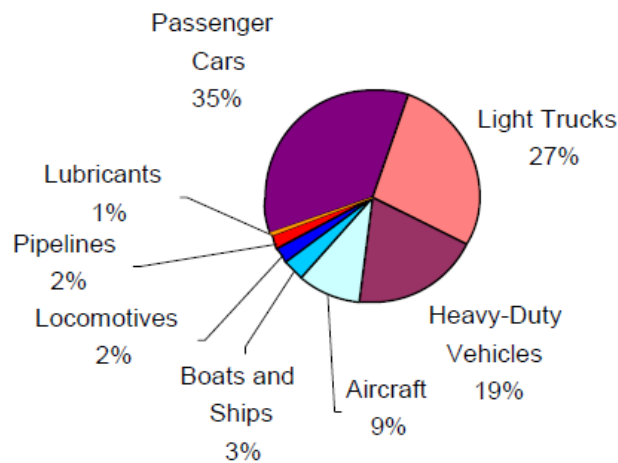


Figure 2. 2003 Transportation Greenhouse Gas Emissions, by Source

Source: U.S. Environmental Protection Agency, 2005. Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990–2003. Washington, DC, Table 2-9.

Ridesharing with increasing the rate of occupancy per vehicle represents an opportunity to decrease the cost and undesirable impacts of traffic congestion, fuel consumption, and pollution.

Why Real-Time ridesharing?

Although several organized ridesharing projects have been attempted but successful ridesharing systems are still in short supply. Certainly, in order to be widely adopted, ride-sharing must be easy, safe, flexible, efficient and economical and must be able to compete with one of the greatest advantages of private car usage, i.e., immediate access to door-to-door transportation.

[Agatz et al., 2010].

Dynamic ridesharing (also called real-time ridesharing) is a form of carpooling system that provides rides for single, one-way trips. Dynamic ridesharing differs from regular carpooling and vanpooling in that ridesharing is arranged on a per trip basis rather than for trips made on a regular basis [Casey et al., 2000].

Traditional carpooling, however, is too limiting to accommodate the unconventional schedules of today's rideshare demand, where many commuters will only respond to flexible commuting options [Levofsky et al., 2001].

Some of the transportation agencies have been working on innovative technology to provide this flexibility. The focus has been on the concept of "smart travelers" riding in "smart vehicles". "Smart", in the most advanced sense, means that both the people and vehicles are continuously connected via wireless communications and the "smart traveler" is a person who has access to real-time and reliable information in order to make travel decisions [Schweiger et al., 1994].

In dynamic ridesharing system, individuals submit requests for a ride to an operations center or central database, either by telephone, e-mail, or direct input to a system residing on the Internet. The database of trips that have been offered by registered drivers is searched by the ride matching software to see if any match the approximate time and destination of the trip request. A request may be made for any destination or time of day, but matches are more likely to be found for travel in peak periods and in principal commute directions. Requests for ride matches can be made well in advance or close to the time when the ride is desired. A return trip would be a separate trip request and could be matched with a different driver. The ITS element in dynamic ridesharing is the automation of the trip request matching and arrangement process, which allows trips to be arranged on short notice. This can be done by either the traveler using the Internet or by a customer service representative at a transit agency call center. The technology involved is rideshare software and possibly the Internet [Casey et al., 2000].

Dynamic ridesharing benefits both drivers and passengers. Passengers benefit by having an alternative when their usual mode is unavailable, and by possibly eliminating the need for an additional car for occasional use. Dynamic ridesharing is particularly valuable when public transportation is not an option. Drivers benefit by having someone to share the cost of the trip

(although this may not always happen) or to gain enough passengers to qualify for high occupancy vehicle (HOV) lanes and reduce the travel time of their trip [Casey et al., 2000].

Dynamic ridesharing could combat the increase in the numbers of vehicle trips, levels of Vehicle Miles Traveled, and amounts of congestion on the road. According to the United States Department of Transportation, 17% of the growth of VMT in the United States between 1983 and 1990 was caused by a decrease in vehicle occupancy – accounting for far more than the 13% increase due to population growth [Surface Transportation Policy Project. 1999]. But addressing this growth through traditional means is difficult because only 11% of the United States urban population lives within one-quarter mile of a transit stop with non-rush hour frequency of 15 minutes or less [National Science and Technology Council, 1999]. Dynamic ridesharing, in contrast, has the potential to reduce each of these factors; 35% of participants in a Bellevue Smart Traveler project focus group [Haselkorn et al. 2005] and 50% of respondents to a Hawaii Department of Transportation study [Flannelly and McLeod, 2000] expressed a willingness to use such a service if it were available. The failure of the experiment in the large (open to the general public) dial-a-ride, door-to-door transit service in San Jose, CA, showed the great potential that door to door services have in attracting users. The transit system abolished less than six months after it opened because it was more successful in luring riders than its originators expected it to be [Lindsey 1975]. An expensive U.S. average \$13 per-ride cost, however, prohibits conventional dial-a-ride service from becoming a viable option for a large number of trips [John A. Volpe National Transportation Systems Center (U.S.), 2000].

Recent technological advances in internet based communication devices such as PDAs, smart phones, and wireless laptops could be key enablers to increase popularity of Dynamic ridesharing. According to comScore report, 234 million Americans subscribed to mobile phone plans in January 2010. Of these, 42.7 million owned Internet-accessible smart phones, which represented an 18 percent increase over the three months ended in October.

Definition and Features of Real-Time Ridesharing

Dynamic ridesharing also known as dynamic carpooling, real-time ridesharing, ad-hoc ridesharing, and instant ridesharing has been defined differently by different scholars. An early effort to increase the industry's knowledge and adoption of successful applications of advanced

technologies defined dynamic car-pooling as “a mode of transportation that is ready when you are. They are multipurpose and can be arranged either in real-time or close to it (near term). Participants pre-qualify and are put into a database. Upon receipt of a trip enquiry, the database is searched for others who are traveling in the same direction at the same time. Participants can not only use this database to arrange for carpools to and from work, but also to a shopping center, medical facility or any other trip generator” [Schweiger et al., 19994] . One of the other first definitions proposed was developed in preparation for a field operational test in Sacramento, CA in 1994 that defined dynamic ridesharing as “a one-time rideshare match obtained for a one-way trip either the same day or the evening before” [Kowshik et al., 1996]. Another trial in 1997 which was aimed to test the concept of dynamic rideshare matching services using Internet and e-mail at the University of Washington in Seattle defined dynamic ridesharing as “two or more people sharing a single trip, without regard to previous arrangements or history among the individuals involved. In comparison to traditional ridematching services, which focus on commuters traveling to and from the same origins and destinations on fixed schedules, a dynamic ridesharing system must be able to match random trip requests at any time. Thus, the system must be able to match potential carpoolers quickly to respond to same-day trip requests, as well as the more traditional commute trips” [Dailey et al., 1997].

‘dynamicridesharing.org’ defines dynamic ridesharing as “A system that facilitates the ability of drivers and passengers to make one-time ride matches close to their departure time, with sufficient convenience and flexibility to be used on a daily basis” [Kirshner, 2008].

A recent definition proposed for dynamic ridesharing described it as “an automated system that facilitates drivers and riders to share one-time trips close to their desired departure times” and characterized it by the following features: Dynamic, independent private entities, cost sharing, non-recurring trips, prearranged, and automated matching [Agatz, et al., 2010].

Another recent work suggests real-time ridesharing as “A single or recurring rideshare trip with no fixed schedule, organized on a one-time basis, with matching of participants occurring as little as a few minutes before departure or as far in advance as the evening before a trip is scheduled to take place” [Amey, 2010].

All the definitions emphasize that dynamic ridesharing is occasional in their nature and has no fixed amount of advanced notice required for establishing the shared trip. For the purposes of the study presented in this dissertation proposal, real-time ridesharing is defined as:

“A non-recurring multipurpose rideshare trip which is prearranged on a per trip basis on a short-notice to establish shared trips close to the desired departure times and locations of the participants to gain HOV lanes privileges or share the cost of the trip. ”

Optimization Models and Dynamic Ride-Sharing

A viability analysis for dynamic rideshare system that examined both theoretical concepts and actual implementation of a dynamic rideshare system in Los Angeles [[Hall and Qureshi, 1997](#)] concluded that in theory dynamic ridesharing is a viable concept and a user should be successful to find a ride-match but in practice the story is different and one at best one can might expect a one in five chance of someone offering a ride. In another study, A GIS approach analysis to identify common clusters of commuters in University of Toronto [[Sarraino et al., 2008](#)] found that during morning commute hours (7:00–10:30am), 1,461 of 3,030 drive trips (48%) were suitable for ridesharing based on residential proximity and similar residential departure times. A similar study in Massachusetts Institute of Technology suggested that between 50% and 77% of the commuting population could rideshare on a maximum-effort day that is significantly higher than the 8% of the MIT community that currently choose to rideshare [[Amey, 2011](#)].

A simulation study in Metro Atlanta showed that the use of sophisticated optimization methods substantially increases the likelihood to find the ride-matches and also that dynamic ridesharing has potential for success in large U.S. metropolitan areas [[Agatz et al., 2010](#)].

While technological advances have greatly eased the communication and reputation systems and social network tools have tackled the fear of sharing a ride with strangers, the development of optimization algorithms for matching the participant in real-time and ultimately increasing the rate of participation in the ridesharing system has largely ignored by transportation research community. This research is the first of its kind to develop an optimization algorithm for real-time rideshare matching problem.

State-of-the-Art

The aim of the following is to review state of the art of dynamic ridesharing projects and researches.

Bellevue Smart Traveler

The goal of the Bellevue Smart Traveler (BST) project was to design and test a traveler information center (TIC) prototype in downtown Bellevue, Washington, east of Seattle that is an area with concentrated employment facilities and a high percentage of single occupancy vehicle (SOV) commuters. The idea was to provide the participant with convenient off-site access to the TIC's information including up-to-the-minute traffic congestion information, transit information, and carpool/vanpool ride-matches using a telephone, and/or a hand-held alpha-numeric pager.

The user population was employees of downtown Bellevue companies taking part in the BST demonstration project. Registered users had access to pagers in addition to the phone-based system and would have been tracked to determine how they used the system and whether or not the system were effective in encouraging their use of HOV transportation options. The registration application acquired information such as: full name, gender, employer, Washington state driver's license number, work address, home address, work phone number, home phone number (public or private), work days, work hours, preferred arrival time to work, preferred departure time from work, schedule flexibility (in terms of time), preferred pickup points (three of them, selected from a list, in ranked order), smoking preference, gender preference (exclusive and nonexclusive), willingness to be a driver (how often, how many seats available), willingness to be a rider (how often). For ridesharing purposes, registered users were divided into "ride groups". All registered users were working in a four square block area of downtown Bellevue but lived throughout the Puget Sound area. Hence, ride groups were based on where users lived so that each ride group was consisting of users that commute to and from the same general areas to increase the potential for successful dynamic ridesharing; each ride group had enough users so that a reasonable number of ride-matches were possible. However, each ride group was not so large to prevent overflow of information for riders looking for rides. Ride groups covered a small enough geographical area so that drivers and riders could meet and be dropped off at convenient locations. The formation of ride groups was based on zip codes and preferred pick-up/drop-off

points (as specified on the application). The TIC was tested and demonstrated over a five-month period (from late November 1993 to late April 1994). During that time, 53 users were registered. Of the registered users, 48 formed three ride groups: 23 from areas south of Bellevue, 10 from areas east of Bellevue, and 15 from areas north of Bellevue. Members from the ride groups offered 509 rides and only six ride-matches were logged. Results from the usage patterns and various surveys that were conducted suggested that participants liked the idea of dynamic ridesharing, the presentation of the information and the technology. However, for various reasons they were either unable or unwilling to form ride matches. Some of the reasons were: insufficient rideshare choices due to the limited size of rideshare groups, being uncomfortable getting into someone else's car, limited time saving incentives due to lack of HOV lanes in the Bellevue area, and technology limitations that reduced the effectiveness of pager delivery. Another possible reason for failure of the project may have been the inconvenience of the rideshare service. The system did not actually match the riders. When users received potential matches from their ride groups, they were left to coordinate the trip.

The BST project conclusions suggested that rideshare group is a new social entity and more work was needed to determine (1) how to encourage ride acceptance and (2) the dynamics of a viable ride group. Incentives such as management support and encouragement could have played a stronger role. Placing the BST TIC on the Internet would help people more easily obtain and respond to rideshare information [[Haselkorn et al., 1995](#)].

Since participants were placed in location-based ride groups, trips were limited to work and home, with time of the trip as the sole variable. For maximum benefits, dynamic ride matching systems need to allow both location and time to vary to enable matching for work and non-work trips [[Dailey et al., 1997](#)].

Los Angeles Smart Traveler Field Operational Test

The Los Angeles Smart Traveler Field Operational Test (FOT) was one of the largest and most comprehensive Automated Rideshare Matching System (ARMS) experiments to date. The purpose of the study was to evaluate the performance and effectiveness of the Advanced Traveler Information System (ATIS). This project was implemented in Los Angeles as part of the new technology demonstrations being carried out by the California Advanced Public Transportation

Systems Group (CAPTS) at Caltrans District 7. It was designed as a field operational test of three different media approaches for providing traveler information: fully automated telephone systems; automated multi-media touch screen kiosks; and PC via modem. The information included: transit routes, fares and services; traffic conditions on the freeways; and ride-matching information for ridesharing on both frequent and one time occasions. Survey results indicated a high degree of user satisfaction for the kiosks that provided a new medium for obtaining pre-trip traveler information, yet the overall usage rate was low (an average of 25 transactions per day), relative to the cost of providing the kiosk service. Low usage combined with high capital and operating costs yielded a total cost per use of approximately \$2.00 (over a five-year lifetime of the kiosk). Kiosks placed in office locations had the lowest usage while kiosks placed in Union Station in downtown Los Angeles and kiosks placed in shopping malls had the highest usage. This finding suggests that the kiosks may be used more for non-work related trip information when users have more time, such as for shopping trips or by tourists. Smart Traveler Automated Ride-matching Service (ARMS) allowed users to use their touch tone phone to find rideshare partners. It was designed to provide individuals with lists of potential compatible rideshare partners for either regular carpooling or an occasional emergency ride home. As with the kiosks, the service was available in both English and Spanish. For the purposes of finding either regular rideshare partners or a once only ride, those using the system used the touch tone phone to enter changes in preferred travel times. They received a computer generated list of people to contact who live and work near them with similar schedules. The user could then choose to call some or all of the people on the list, or record a message that Smart Traveler automatically delivers to potential carpool partners, allowing them to call the individual back if they are interested in sharing a ride. The ARMS was found to have very little usage (34 persons per week). From a small telephone survey of ARMS users it was concluded that most users used the service to seek regular ridesharing opportunities and not the featured one-time ride service. The researchers concluded that there is not enough interest in ARMS to justify its cost of operation. The modem service was found to have significant usage. In a period of 35 weeks a total of 83,155 uses were recorded (on an average weekday there were circa 400 uses per day). These levels of use indicated that there was indeed a demand for the service. This component of the ATIS system did not have the multi-modal component at the time of evaluation and instead only reported

Caltrans congestion information. Usage was found to be higher in the mornings and evenings, consistent with commuter trip planning [Giulian et al., 1995].

Los Angeles Smart Traveler program operated only from July 1994 to September 1994 and it was therefore limited to the approximately 68,000 people. The research conducted from October 1994 to March 1995 showed that an average of 34 people per week used the system. Users calling a toll free number could select dynamic ridesharing from an options menu. An AutoText interface allowed users to input and change their travel times and to search for new matches based on the new times. Ride match lists were provided over the phone to the users who then had the option of calling the potential matches or having a computer send a message. In order to use ARMS, individuals had to be registered with Commuter Transportation Services. There is no way to know how many matches were actually made because users were not required to report them. The evaluation concluded that the market for “one-day-only” rides was very limited because of participants’ concerns over safety [Golob and Giuliano, 1996].

Sacramento Rideshare Matching Field Operational Test

A real-time rideshare matching field operational test evaluation was conducted in Sacramento, California which began in late 1994 and terminated in 1995 with the participation of the Federal Transit Administration, Caltrans, PATH, Sacramento Rideshare, and U.C. Davis Institute of Transportation Studies. The service was not automated, but operator-based. Users answered questions over the telephone about origin and destination locations, purpose of trip, etc. Trip matches were made by sorting from database orientation and destination zip codes, and then prioritizing by the closeness of desired trip times. Three hundred and sixty people (from a database of 5,000 who expressed interest in carpooling) registered as drivers willing to offer on-demand rides. The rate of match was very low and from the ten requests made for dynamic ridesharing, only one potential match was made, and it is not known if the match was secured.

The final report concluded there were several reasons for the poor performance of the program: Poor marketing of the service and personal security concerns. As part of the system design, user needs were assessed through a review of literature and focus group discussions. Six user needs were identified: background screening; information security; matching and system reliability; system access; flexibility; and, a compensation scheme. The user needed flexible ridesharing

arrangements that would allow users with non-identical origins and destinations to be matched as well as a reliable system that would be able to generate a large number of potential matches for any given trip [[Kowshik et al., 1996](#)].

Coachella Valley TransAction Network

Commuter Transportation Services, Inc. (CTS) developed the Coachella Valley TransAction Network (TAN) in 1994 as a pilot test for providing information on transit and ridesharing.

The project was similar to the Los Angeles Smart Traveler project, in that real-time traffic and transit information and rideshare information were provided on over 700,000 registrants throughout the Riverside area via four stand-alone commuter information kiosks. During the seven-month test period, more than 21,510 people accessed the kiosk system. Approximately one-third of them accessed information on ridesharing and only 256 printouts were rideshare match lists. The project was expensive to implement and usage was low. CTS concluded that kiosks were probably not the best medium for obtaining real-time rideshare information and recommended it not be included in future models [[Haselkorn et al., 1995](#)].

Seattle Smart Traveler

Seattle Smart Traveler (SST) project was part of a larger Intelligent Transportation System Field Operational Test conducted by the Washington State Department of Transportation, the University of Washington, King County Metro, and five private sector partners from 1995 to 1997 that was designed to test the concept of automated dynamic rideshare matching using the Internet and electronic mail at the University of Washington in Seattle [[Dailey et al., 1999](#)].

The SST project defined dynamic ridesharing as “two or more people sharing a single trip, without regard to previous arrangements or history among the individuals involved” and addressed the differences between dynamic ridesharing with traditional ride-matching services, which focus on commuters traveling to and from the same origins and destinations on fixed schedules, as “a dynamic ridesharing system must be able to match random trip requests at any time” [[Federal Transit Administration, 1996](#)].

User group was limited to faculty, students, and staff from the University of Washington. The SST was designed to respond to the request of three types of matches: regular commute trips, additional regular trips, and occasional trips. A user entered the origin, destination, day of week,

departure time, and arrival time for each trip type. The system then identified potential matches using a search structure a search tree containing four levels of detail. To provide flexibility in the matching of trips, a time range or window was used for both the requested departure and arrival times. The SST automatically generated and sent an e-mail message with this information if the user desired or the participant could call the potential matches [[Federal Transit Administration, 1996](#)].

The evaluation report found that faculty and staff made up 68% of users, with students comprising the remaining 32%. Approximately 700 ride-matches were requested during the 15-month test period, of those 150 potential matches generated, and At least 41 matches actually made. It was possible that more ride matches were made, as since there was no requirement that actual trips be reported [[Casey et al., 2000](#)].

SST suggested that the relationship between the number of users and the number of carpools formed was quadratic, i.e., rate of carpooling would increase with the number of users. It also suggested that carpooling has the potential to have a larger effect on traffic demand management (TDM) if large groups of people participate. Further, SST suggested that a web-based ridematch system can be as effective as traditional ridematching. SST suggested the following quantitative relationships between numbers of users, matches, and carpools: SST estimated the number matches expected (T_m) given U users is:

$$T_m(U) = \frac{\alpha^2 U^2 - U}{2} P_m \quad (1)$$

And, the actual number of carpools (C_p) is:

$$C_p = \beta T_m(U) \quad (2)$$

Where α and β are constant coefficients, and P_m is the probability for a pair of trips matching constant across the population of rideshare trips assuming that: (1) the probability of trips matching is approximately constant across the population of trips, (2) the relationship between

the number of users and the number of trips is linear, and (3) the relationship between matches and actual carpools is linear [Dailey et al., 1999].

The SST project identified some issues that may have limited the use of the system. First, the project was implemented before the real boom in Internet use. Second, the developing technology for the dynamic ride-matching capabilities was somewhat cumbersome. Third, the SST had been viewed by some targeted users as a temporary endeavor. Fourth, there were no sufficient incentives to encourage greater ridesharing. Finally, there were safety concerns regarding sharing rides with strangers. Although the test ended in June 1997, the SST continues to operate for a few years later even though no staff was assigned to the project. Without staff support, the database was not updated or purged of former users [Turnbull, 1999].

The SST system is no longer operational; however, an offline demonstration of the project can be viewed by following the SST link: <http://sst.its.washington.edu/sst/>

Missoula Ravalli Transportation Management Association

The Missoula Ravalli Transportation Management Association (MRTMA) operated a ridesharing program in Missoula., Montana using GeoMatch information system. for matching new applicants with existing carpools [Casey, 2000].

GeoMatch is a geographic based system that matches people with carpools, vanpools, and provides transit information. The program runs on personal computers using the Microsoft Access database software.

Rideshare requests were provided by telephone and generating a matchlist usually took about four minutes. The rideshare program was in operation since 1997 and had over 300 names in the carpool database by September 2000. During that time period, It forms 30 regular carpools and four vanpools and received three to five rideshare request calls per week, one to two of those were one-time rides [Casey, 2000].

King County Metro's Regional Ridematch System

King County is located in Washington State, comprises 2,134 mi² with more than 1.8 million people. Major cities include Seattle and Bellevue, with numerous smaller suburban cities throughout the county. Washington State's Commute Trip Reduction (CTR) Act that was passed in 1991 and reauthorized in 2006, as a part of the Washington Clean Air Act, required major employers to reduce drive-alone commuting by their employees and provides a regulatory framework for measuring employer success. Since passage of the CTR Act, King County Metro Transit has worked closely with major employers to design products and programs to help them meet the CTR goals. Almost all these efforts focus on working with employers to reach employees and providing tools and incentives to employees to use alternatives like busing, carpooling, biking, telecommuting, and compressed work schedules [[Travel Behavior, Environmental, and Health Impacts of Community Design and Transportation Investment, 2005](#)].

King County Metro with about 1,300 transit coaches and more than 700 vans in its vanpool fleet and a well-integrated bicycle support program, has incorporated special event ride matching into its regional rideshare program, *rideshareonline*, that is a self-serve, public, internet-based rideshare matching service in association with regional carpool/vanpool providers [[Cooper, 2007](#)].

RideshareOnline.com instantly matches registered commuters with carpool or vanpool partners with a similar daily commute in the area. Users enter their commuting times and locations and can instantly see a list of ridematches to whom they may e-mail a rideshare request anytime for everything from carpools, vanpools, SchoolPools and biking to work, to one-time special events like ballgames and concerts and conferences [[King County Metro Transit, 2010](#)].

Redmond Transportation Management Association's Ridematch system

Redmond is the seventh most populous city in King County and the fifteenth most populous city in the State of Washington, with a residential population of over 46,000. It encompasses an area of over 16.6 square miles. The city is well known as a center of technology and the location for a

number of known high-tech and biomedical companies such as Microsoft, Nintendo, AT&T Wireless, and Medtronic Physio-Control.

The Greater Redmond Transportation Management Association (GRTMA) has established an automated ridematching system for carpools and vanpools on the Internet. *RideQuest*, is an employer and geographic information system (GIS) based system with the database accessible by SQL Server. Registered users enter a street address or a nearby intersection, and the software produces a map showing that location for verification by the registrant. Then the request is entered into the database along with information on the users travel needs and preferences such as whether they wish to drive or ride, ride with smokers or non-smokers, or ride with employees of specific companies. People are matched based on their origin address and final destination with a numbering system of the best match to less potential match [[Knapp, 2005](#)].

The system can send automatic emails to other registered commuters who may be able to rideshare. A map showing the requestor's location and the location of potential matches are displayed on the screen together with their names and methods of contacting them. Individuals can change their information at any time or remove themselves from the system if they have found satisfactory ridesharing arrangements, moved, changed jobs, etc. Every three months, e-mails are automatically sent to all registrants asking for their continued interest in participation. Non-respondents are automatically removed along with those responding in the negative. An early version of the system was tested in April 1999 with 1,200 registrants. There are no statistics available on carpool formation [[Casey, 2000](#)].

GRTMA promotes the program using posters, post cards, email and the web site and has a variety of promotions throughout the year to encourage people to register in the rideshare system including a trip to Hawaii, and a 12-oz Starbucks Coffee beverage free. Vanpool drivers don't have to pay the monthly vanpool fare and they also receive up to 40 personal use miles on the van [[Knapp, 2005](#)].

Minerva Dynamic Ridesharing System

Aegis Transportation Systems developed a system called MINERVA in Oregon that takes advantage of ATHENA smart traveler system. ATHENA developed in City of Ontario,

California, with funding from the FTA in 1994, but the project was abandoned in 1996 due to a turnover of the city council. The ATHENA project differed from other dynamic ridesharing programs in that trip requestors would not have received a list of potential drivers, and would not have had to contact trip providers to arrange travel. Instead, a central computer would have arranged the match and advised the rider and driver of pickup points, times, and fares. The ATHENA project incorporated a central database that interfaced with personal digital assistants (PDA's), hand held devices that have messaging and GIS capabilities. Interested parties would have pre-registered with ATHENA. Once registered, all ATHENA drivers would have received a PDA for their car, and all potential passengers would also used telephone-based information systems and other computer and communications technologies to integrate these new personalized transportation services with conventional transit (e.g. bus, rail, ferry), paratransit (e.g. taxi, shuttle, dial-a-ride), and ridesharing (e.g. carpool, vanpool, buspool) modes to develop more cost-effective public transportation systems. Market research studies indicate that this approach would reduce vehicle trips and vehicle miles traveled (VMT) per capita significantly, at a low cost to taxpayers. MINERVA used “smart” technology including cellular phones, palmtop computers, and wireless data communications to provide low-cost alternatives to transportation in low-density areas and low travel corridors. MINERVA took the ATHENA concept one step further. MINERVA integrated the smart traveler system with other online information services—home shopping, telebanking, e-mail, and interactive games—in an attempt to reduce the need for some trips altogether [[Levofsky et al., 2001](#)].

The Oregon State legislature committed \$1.5 million to the project, with additional commitments of \$3 million in matching funds from local pilot sites, and \$1 million in in-kind support from private management consulting outfits. A dozen Oregon cities expressed their interest in piloting MINERVA [[Victoria Transport Policy Institute, 2010](#)].

Both ATHENA and MINERVA did not progress beyond the developmental stage and were never implemented. However, their Internet and GIS components formed the basis of many ridesharing programs in use today [[Chan and Shaheen, 2011](#)].

Online Ridematching and Traveler Information Services

With respect to the fact that most of the dynamic ridematching applications and pilot tests of the 1980s and 90s failed to provide enough users to consistently create a successful instant ridesharing match, next generation of the most dynamic ridesharing focused on more reliable strategies to encourage ridesharing including online ridematching and traveler information services. Before 1999, the websites for ridematching applications were either simple pages listing agency contact information, online forms for users to email the agency to receive a matchlist, or online notice boards for users to manually post or search carpool listings [[Bower, 2004](#)]. Between 1999 and 2004, private software companies began developing ridematching platforms. Although it became much easier to find ridesharing in a larger online database, the carpools still suffered from the same inflexibility drawback as traditional carpools. Online ridematching programs tended to be more static and inflexible and best suited for commutes with regular prearranged schedules and were not competitive enough to compete with the flexibility that private auto travel offered [[Chan and Shaheen, 2011](#)].

In another attempt, on July 2000, the Federal Communications Commission designated a uniform “511” as the traveler information telephone number to make real-time traveler information more widely available for local, regional, and state agencies across the U.S. including carpool and/or vanpool information services [[Profiles of 511 Traveler Information Systems Update, 2009](#)].

Dynamic Ridesharing in the era of Internet Enabling Technologies

From 2004 to the present, dynamic ridesharing takes advantage of the incentive strategies that encourage ridesharing such as HOV lanes, and park-and-ride efforts and it integrated with Internet enabling technologies such as World Wide Web, Smart phones, Global Positioning System (GPS), Data Repository, Automated Financial Transactions, and social networking.

Based on the research of the author, there are approximately 33 notable applications and software platforms that offer ridesharing services. However, the systems typically serve as platforms that bring users together, rather than as active mechanisms that generate rideshare

plans and provide fair payments [Kamar and Horvitz, 2009]. Following is a brief description for those applications and software platforms:

Aktalita is an under development application that combines the Web, a geospatially enabled database, and a Java enabled cellphone to provide real-time carpooling between drivers and passengers. When a driver is about to travel or a passenger needs a ride, they enter an offer or request to the system via the web or Java enabled cellphone. The system then queries its geospatial database to attempt to match passenger and driver, and notifies them for further negotiation [<http://www.aktalita.com/>].

AlterNetRides.com works nationwide but also can be tailored for a community. It is completely automated, a person can become a member, set up a ride and be viewing others wanting to rideshare in just minutes [<http://alternetrides.com/>].

Avego is a proprietary application for Apple iPhone. It uses GPS technologies and presents an intuitive user interface. The application relies on a proprietary service called Futurefleet, on which no implementation details are given [<http://www.avego.com/>].

Carpoolconnect.com matches up carpooling commuters based on similar commutes defined by home and work zip codes [<http://carpoolconnect.com/>].

Carpoolworld.com uses the commuter's precise latitude and longitude coordinates to find the best matches for their trip among the other commuters in the database, based on exactly how close together they live and exactly how close together they work [<http://www.carpoolworld.com/>].

Carpool.ca is available via the internet and uses home and destination locations, driving route and other personal information to help commuters identify potential carpool partners. This self-serve system has various levels of security, limiting individual access to personal rideshare information while providing rideshare program administrators broader access. The program includes a built in CO2 savings calculator [<http://www.carpool.ca/>].

Carriva is a proprietary solution using phone calls as communication system and a fixed price of 0,10€ / km. Currently it has got 1118 active users [<https://www.carriva.org/MFC/app>].

Carticipate is a proprietary iPhone application that integrates with Facebook. It has a interface looking like Google Maps mobile. It is an experiment in social transportation. According to the website, it is available on 59 countries [<http://www.carticipate.com>].

Commuter Register is a multimedia publication that provides listings of car and vanpools, transit routes and schedules, Park and Ride lots, and articles and helpful travel tips focusing on employee commute matching [<http://www.2plus.com/>].

Divide The Ride is a static, web-based solution organized around children and family activities. Families invite other trusted families to join their group. Groups get notifications when a ride is needed [<http://www.dividetheride.com/>].

Ecolane DRT and *Ecolane Dynamic Carpool* are two ridesharing softwares offered by Ecolane Company integrated with Nokia touchscreen device. Among the features, they declare that the device is capable of real-time data communication, reports of arrivals and departures with time information, device locking mechanisms, GPS location and direction, mileage tracking, detailed trip information. It is a completely web-based, turn-key scheduling and dispatching solution with user interfaces that are accessed securely using a standard web-browser using seamless integration with multiple Mobile Data Terminal (MDT) and Automatic Vehicle Location (AVL) platforms. It enables commuters to overcome the biggest obstacles of traditional carpooling today - irregular working schedules and finding a carpool partner. Commuters are able to select if they want to rideshare in as little as 15 minutes and create an instant carpool with the mobile phone or web-based applications. The Ecolane Dynamic Carpool software communicates the needs of both drivers and passengers, and automatically matches potential carpoolers based on digital maps, individual profiles, user groups, and user ratings [<http://www.ecolane.com/>].

eCommuter is an internet-based technology application specializing in Real-Time Internet traveler solutions. It is the first-to-market in the category of Internet ride-matching that gives commuters the power to find their own partners for sharing a carpool or vanpool to work [<http://www.ecommuter.com>].

eRideShare.com is a free service for connecting travelers going the same way. According to Yahoo and Google it is the leading carpool/ridesharing website and has been recognized as "Best

of the Net" by About.com. The site has over 17,000 commuter and traveler throughout the US and Canada [<http://www.erideshare.com/>].

Flinc comes from Germany and is a dynamic carpooling application system that can be used on smart phones or online. This application utilizes mobile phones' location based capabilities and navigational software to connect passengers and drivers, offering a customer to customer (C2C) interaction both for ride coordination and financial interaction. The system analyzes real-time traffic and brings riders and drivers together, eliminating the need for coordination methods such as phone calls, emails, or text messaging. Passengers can identify available seats in cars belonging to drivers in their network and send a request to be picked up at their location. The driver, after confirming the pickup, receives instructions via the navigation software and arrives to pick up the passenger [<http://www.flinc.org/world/>].

GoLoco is a proprietary web application that also relies on Facebook. It uses a private payment system and coordinates carpool and vanpools for work, campus, religious and group events [<http://goloco.org/>].

Goose Networks is a web-based ridematching services that allows commuters to connect with each other for flexible, one-way carpools or for regular, recurring trips. Users simply input their commute schedule online; existing matches are immediately shown, and built-in email and SMS text message notifications help keep users informed of new options as they become available [<http://www.goosenetworks.com/>].

GreenRide Connect Metro has two employer and campus editions that combines a user-friendly interface with rich content management with multi-tiered administration, social network integration, content management, GIS capabilities, employer management, single-trip matching, raffle management, cluster mapping, savings tracking (energy, economic, environmental), comprehensive and exportable reports, vanpool management are all available through the suite of GreenRide solutions [<http://www.greenride.com/>].

Hover (High Occupancy Vehicles in Express Routes) is a casual carpooling system that was inspired in Auckland, New Zealand when a city manager observed that "if everyone shared a ride one day a week there would be 20% less traffic". Hover creates a community of rideshare commuters who share benefit of the savings through the own credit system. Each time a driver

provides a ride, he receives one credit from each passenger and each time a rider takes a ride, he uses one credit. The members are approved after security check and with two personal references. Hover uses RFID technology to identify members and cars and operates to agreed destinations. In morning, each member drives or walks to a Hover park that is a secure and safe place to leave the cars. Each Hover park, along the route to destination, has parking areas set up for agreed destinations. In general they require about 100 participants from one Hover park to a given destination area to keep the waiting times to a workable level and form a trip with at least 3 people. In the evening, participants make their way either on foot or by car to a Hover Port. Riders from the morning might wait at a Hover Point, like a bus stop. Drivers going by these Hover Point will stop and pick up riders and take them to the Hover Port. At the Hover Port passengers will get out of the car they came in and change to a car that is going back to their Hover park. On exiting the Hover Park, the system recognizes driver and passengers and distributes credit points. It also offers a guaranteed back-to-home system, by using taxis [<http://www.hoverport.org/>].

iCarpool is a static, online and custom branded hosted solution for employers and regional public agencies with interactive maps, privacy protection, high precision trip matching and support for all trip types such as daily commute, one time trips or real time (dynamic carpool) trips. The application also supports - multiple modes such as carpool, vanpool, bike, walk and transit, integrated GIS data such as park-and-ride lots, bike routes, multi modal trip calendar and integrated incentives provided by employers or regional public agencies. Matching criteria includes social relationships, but no details are given [<http://www.icarpool.com/>].

KOMOTOR TDM management system offered by Base Technologies is a comprehensive, web-based total TDM service that combines ride matching, management, measurement, and reporting tools in one product site [<http://www.basetech.com/>].

MyCasualCarpool.com helps users find others with similar daily commuting patterns and create rideshare lots using only resources available in virtually every residential neighborhood [<http://www.MyCasualCarpool.com/>].

NuRide Network is the incentive-based ride network that rewards people every time they share a ride. Through the NuRide Network®, individuals can easily arrange individual ridesharing trips for work or pleasure and earn rewards for every confirmed trip they take. Unlike a traditional

carpool, NuRide is flexible and casual with users being able to share a single ride without any ongoing commitments. The miles-based reward points can be redeemed for gift cards, gift certificates and other rewards from their corporate sponsors [<http://www.nuride.com>].

Pathway EnRoute is a turnkey solution to enable and track carpooling and vanpooling both public, and across organizations in Metropolitan Toronto and Ontario [<http://www.carpoolzone.ca>] and British Columbia [<http://www.online.ride-share.com>] in Canada. Pathway EnRoute claims that it has the most sophisticated route-based ride-matching available today as it is not only finds passengers whose endpoints the driver pass by, but also the Pathway EnRoute Search Engine finds driver routes that pass by the driver endpoints and claims that this second category typically accounts for increase in up to 50% of matches, and other systems are blind to these matches. Additional EnRoute Search Engine features include: Instantly adding and dragging waypoints by clicking on maps for editing a route, viewing multiple routes together with capability of switching between routes without page reloads, and continually searches for matches and automatically sending notifications of suitable matches even after users log off. Pathway Rewards include: a calendar-based incentive tracking system and an online emergency ride home service [<http://www.pathwayintelligence.com/>].

Piggyback is an Android application using a step-by-step approach (maximum one user input at each application screen) and makes wide use of graphical representations instead of text. When a driver and passengers are matched their compatibility is showed, represented with stars (0 to 5) and categorized as friendliness, reliability, driving skills and car. After the ride, the feedback system lets the user set the points for the aspects listed above. The application lets also plan rides using a static carpooling approach [<http://www.piggybackmobile.com/>].

Ridegrid is another under-development proprietary application that uses mobile Internet and location technology to enable individuals to obtain rides to and from any location, spontaneously. RideGrid works by dynamically combining routes and evaluates the change required in a driver's route such that it passes through the desired source and destination of a compatible rider, and broker the agreement [<http://www.highregardsoftware.com/ridegrid-dynamic-ridesharing.html>].

RideNow is a Web and cell-phone ("Interactive Voice Response") interface system with parking space incentives for instant ridesharing where each ride match request is the basis for potentially new carpool arrangements. The system can give users a ride match within 10 minutes [<http://www.ridenow.org/>].

RidePro is an integrated desktop and web based rideshare information solution. It is a client-server, menu-driven, Windows®-based application with integrated GIS mapping that sends rideshare match report directly to an e-mail message. The web interface allows the public to create their own registrations and run their own match reports in a secure, confidential environment. The web component uses the same database as the local area network interface. Both interfaces support matching to carpools, vanpools, park-and-ride lots, public transit, telecommute centers, day care centers, bike partners, and bike routes [<http://www.ridepro.net/>].

RideshareOnline.com is a Seattle-based online ridematching system. Registered users enter their work location and the starting point of their commute that is either a home address or a nearby intersection and they enter their weekly work schedule and any daily variations. They can instantly see a list of rideshare matches to whom they may email a rideshare request [<http://www.rideshareonline.com/>].

RideShark is an online map-based rideshare solution that enables registrants to find rideshare partners based on customized search criteria that includes ridematching based on a regional, TMA or secure cluster or private organization. *RideShark* utilizes Geographic Information System (GIS) technology from Microsoft MapPoint [<http://www.RideShark.com>].

RM 21 is the proprietary route-based carpool /vanpool ridematching software system to power the Chicago Area Transportation Study in northeastern Illinois. This system departs from typical "mile-radius" searching by allowing users to chart their travel path. This path is then used to find matches of varying quality as determined by sameness of route, closeness of schedule, and matching of individual preferences [<http://www.ShareTheDrive.org/>].

Visual BACSCAP 2007 is a user-friendly transportation program designed by the Marketing Institute at Florida State University College of Business for use by commuter assistance programs. The primary function of the program is to provide commuters with information

regarding pools. An online demo of the program, EzRide, can be viewed at <http://nctr.cob.fsu.edu/ezridedemo> [<http://www.tmi.cob.fsu.edu/vbacscap/download07.htm>].

VivaCommute is a web-based commuter rideshare services for all geographical locations in Canada and the United States. This web-based application matches people who travel the same route and share the same driving schedule. The system uses nearest neighbor logic [<http://www.vivacommute.com/>].

Zimride.com combines Google Maps and optional social network integration and a proprietary route-matching algorithm. Zimride has partnered with 50 U.S. colleges, universities, and companies that each has their own network of members. In addition to each network's website, Zimride also uses the Facebook platform to attract public users [<http://www.zimride.com/>].

Summary of Reviews

Before 2004, almost all the pilot test projects shared a number of common characteristics: all but the Seattle project abandoned for low usage. They all suffered from a small number of requests for rides and a smaller number of matches made. This failure could be attributed to how each was designed. Commuter behavior is important to understanding what happened.

While there were many potential reasons for low dynamic ridesharing before 2004 including lack of awareness of the ridesharing programs, a deficiency in the number of riders and drivers participation, insufficient incentives to encourage people to rideshare, safety concerns about sharing rides with strangers, inflexibility of the existing rideshare programs, lack of funding for the systems' operation, lack of institutional support and incentives, time consuming process to receive a match list, and then burdensome attempting to make contact with possible drivers with no guarantee that a match would be made, after 2004 technological and computing advances help to overcome many of those potential obstacles. Internet-enabled technologies such as World Wide Web, Smart phones, Global Positioning System (GPS), Data Repository, Automated Financial Transactions, social networks, and automated ridematching softwares are enabling technologies for ridesharing to organize rides in real time either a few minutes before the trip takes place or while the trip is occurring with passengers picked up and dropped off along the way.

Moreover, there has been significant growth and overall success with the strategy of partnerships between ridematching software companies and the large-scale clients. This partnership strategy has gained more users and is most suited for commuters with regular schedules. Many public agencies and companies have started promoting ridesharing by providing incentives. The rise of social networks has enabled ridesharing companies to better address the security concerns of sharing a ride between potential riders and drivers and their friends [Chan and Shaheen, 2011].

There are many applications and software platforms that offer dynamic ridesharing services. However, the systems typically serve only as platforms that bring users together, rather than as active mechanisms that generate rideshare plans. All of the underlying systems use some form of algorithm to match riders and passengers. Some of the algorithms do so based only on origin and destination, while some of the newer algorithms match drivers and passengers based on the commonality of their travel route. The review of the literature revealed that the development of optimization algorithms for real-time matching of the participant has largely ignored by transportation research community. This research is the first of its kind to develop an optimization algorithm for real-time rideshare matching problem.

Problem Definition

This research considers a Dynamic Rideshare Matching Problem which is responsible to spontaneously identify suitable matches between passengers requesting rideshare services with relevant drivers available to carpool for credits and HOV lane privileges. DRMP receives passengers and drivers information and preferences continuously over time.

At any time moment t the set of passengers in the system P_t is partitioned as $P_t = O_t \cup W_t$ where O_t is the set of onboard passengers already started their service and have not left the system at t and W_t is the set of passengers in the system waiting for a ride. The set of drivers D_t is further partitioned as $D_t = D_t^{s_0} \cup D_t^{s_1} \cup D_t^{s_2} \cup \dots$ where $D_t^{s_j}$ is the set of drivers in the system with j seats available for passengers to be assigned. For those drivers belonging to the subset $D_t^{s_0}$, it means that there is no more seat available for passengers to be assigned. As time Δ elapses, new request for rideshare from passengers and drivers arrive and also some passengers and drivers depart the system. This addition and deletion needs to be incorporated into the existing carpool paths or new carpool created to handle them. Thus, at any time moment $t + \Delta$,

$P_{t+\Delta} = P_t + P_{t,t+\Delta} - P'_{t,t+\Delta}$ where $P_{t,t+\Delta}$ is the set of newly arrived passengers and $P'_{t,t+\Delta}$ are the passengers left or gave up the system in the time slot $(t, t+\Delta)$. $P'_{t,t+\Delta}$ is further partitioned as $P'_{t,t+\Delta} = P_{t,t+\Delta}^{\text{given up}} \cup P_{t,t+\Delta}^{\text{left}}$. Likewise, $D_{t+\Delta} = D_t + D_{t,t+\Delta} - D'_{t,t+\Delta}$, where $D_{t,t+\Delta}$ and $D'_{t,t+\Delta}$ are the set of newly added drivers and the drivers who left the system in the time slot $(t, t+\Delta)$.

For each passenger $i \in P_t$ there are origin and destination points v_o^i and v_d^i . At first, DRMP with respect to the availability of a driver matches pickup and drop off points of passenger $i \in P_t$ at his/her origin and destination points, and in the case of unavailability of a driver, with respect to the preferences defined by the passenger, the system assigns different points to pickup and drop off which are denoted by v_p^i and v_d^i . Associated with each passenger $i \in P_t$ are requested time to start the service at the origin point denoted by t_o^i . The system may assign a different time to pickup that is denoted by t_p^i .

Likewise, for each driver $j \in D_t$ there is an origin-destination pair point denoted by (v_o^j, v_d^j) . Associated with each driver $j \in D_t$ is departing time t_o^j from origin, the number Q^j of places available on the vehicle, the origin to destination route in the form of successive nodes as well as personal information and ridesharing preferences. In additions, for each pick up and drop off route per passenger $i \in P_t$ there would be a credit b_{ij} assigned to the driver $j \in D_t$.

Along the aforementioned information, DRMP takes in other personal information and ridesharing preferences for each passenger $i \in P_t$ and driver $j \in D_t$. Table 4 shows most relevant information and preferences.

DRMP asks to assign passengers to drivers and to identify the feasible routes to be driven by the drivers in order to:

- The total number of matching in a given planning horizon is maximized

Such that:

- Seat capacity is satisfied.
- The number of connection for each passenger is less than a predetermined parameter.
- Waiting time to pickup for each passenger is less than a predetermined parameter.
- Detour distance for each driver is less than a predetermined parameter.
- Relocating distance to pickup for each passenger is less than a predetermined parameter.

- Ridesharing preferences is secured.
 - Age matching preferences are satisfied.
 - Gender matching preferences are met.
 - Smoking matching preferences are secured.
 - Pet restrictions are met.
 - Preferences on maximum number of people sharing a ride are met.

Table 4: Personal information and ridesharing preferences

Information/preferences	Passenger p_i	Driver d_k	Type of input
Gender	✓	✓	Male or Female
Age	✓	✓	Young, Middle age, Old
Smoker	✓	✓	Yes or No
Number of passengers	✓	-	1,2,...
Number of seats	-	✓	1,2,..., Q_k
Pet friendly	✓		Yes or No
Pet restriction		✓	Yes or No
Smoke restriction		✓	Yes or No
Flexible with relocating to a nearby walking distance point	✓		Yes or No
Flexible with detour		✓	Yes or No
Flexible with reconnection	✓		Yes or No

Problem Formulation

Proximity in time and space

For each driver j , a route consisting of successive points, $v_1^j, v_2^j, v_3^j, \dots, v_n^j$ are specified at the time that he/she signs into the system. Also, there are points of origin and destination for each passenger i in the system denoted by v_O^i and v_D^i .

When the point of origin for passenger i is one of the points visiting by the driver j or it is in an acceptable walking distance, φ , from a point visiting by the driver j , or the point of origin for the passenger is within the acceptable detour distance, β , from the current location of driver j . A driver could be a promising driver to pick up the passenger for his first lag of journey with respect to proximity in space assuming all other conditions satisfied.

$$PS_{ijtv} = \begin{cases} 0 \text{ or } 1 & ; D_{v_o^i, v_m^j} \leq \varphi \text{ or } D_{v_m^j, v_o^i} + D_{v_o^i, v_{m+1}^j} \leq D_{v_m^j, v_{m+1}^j} + \beta; v_m^j, v_{m+1}^j \in V^j \\ 0 & ; \text{ otherwise} \end{cases} \quad (3)$$

Where v_o^i and $D_{v_o^i, v_m^j}$ are the original point of origin and the relocating distance for passenger i , respectively. $D_{v_m^j, v_{m+1}^j}$ is the distance of travel independent of time between two successive

A driver could be a promising driver to pick up the passenger for his first lag of journey with respect to proximity in time assuming all other conditions satisfied.

$$PT_{ijtv} = \begin{cases} 0 \text{ or } 1 & ; t_o^i + t_{v_o^i, v_m^j}^i \leq t_{v_m^j}^j \leq \gamma_i + t_o^i + t_{v_o^i, v_m^j}^i \text{ or } t_o^i \leq t_{v_m^j}^j + t_{v_m^j, v_o^i}^j \leq \gamma_i + t_o^i; v_m^j, v_{m+1}^j \in V^j \\ 0 & ; \text{ otherwise} \end{cases} \quad (4)$$

When the point of destination for passenger i is one of the points visiting by the driver j or it is in an acceptable walking distance, φ , from a point visiting by the driver j , or the point of destination for the passenger is within an acceptable detour distance, β , from the location points of driver j . That driver could be a promising driver to drop off the passenger for his last lag of journey with respect to proximity in time assuming all other conditions satisfied.

$$DS_{ijtv} = \begin{cases} 0 \text{ or } 1 & ; D_{v_D^i, v_m^j} \leq \varphi \text{ or } D_{v_m^j, v_D^i} + D_{v_D^i, v_{m+1}^j} \leq D_{v_m^j, v_{m+1}^j} + \beta; v_m^j, v_{m+1}^j \in V^j \\ 0 & ; \text{ otherwise} \end{cases} \quad (5)$$

Where v_D^i and $D_{v_D^i, v_m^j}$ are the original point of destination and the relocating distance for passenger i , respectively. $D_{v_m^j, v_{m+1}^j}$ is the distance of travel independent of time between two successive points v_m^j, v_{m+1}^j .

Origin-Destination route related constraints

When the first and last lag of journey for passenger i are shared with driver j , it means that driver j could be assigned to give a ride to passenger i for all the trip from the origin to destination without a need to reconnection. i.e.,

$$R_{ijtv} = \begin{cases} 0 \text{ or } 1 & ; P_{ijt} = DS_{ijt} \text{ AND } P_{ijt} \neq 0 \\ 0 & ; \text{ otherwise} \end{cases} \quad (6)$$

When the first and last lag of journey for passenger i are not shared with driver j , it means that we need a feasible reconnection to connect the origin and destination of the passenger i . Let suppose that the first lag of journey for the passenger is shared with driver j and the last lag is shared with driver j' . At time moment t , the successive visiting points for driver j after he picks up passenger i are: $R_t^j = \{v_m^j, \dots, v_{m+n}^j\}$ and the remaining successive points for driver j' before he drops off passenger i are $R_t^{j'} = \{v_k^{j'}, \dots, v_{k+l}^{j'}\}$.

From the viewpoint of proximity in space, passenger i would like to change the ride from driver j to driver j' when there is a node en route for driver j within φ miles walking distance of a node visiting by driver j' , or there is an acceptable distance β_j from a visiting point of driver j to a visiting point of driver j' in order to make a detour to drop off the passenger to be picked up by driver j' , or there is an acceptable distance $\beta_{j'}$ from a visiting point of driver j' to a visiting point of driver j in order to make a detour to pick up the passenger who is already dropped off by driver j , i.e., $R_t^{jj'} = R_t^j \cap R_t^{j'} \neq \emptyset$.

Continuity constraints

When a passenger matches with a driver and the passengers on board with respect to the matching preferences, the original route of driver and arrival times may change. There are a few possibilities:

1) when there is a detour to pick up and/or drop off the passenger without connection, i.e.,

$$P_{ijv_m^j}^2 = 1 \text{ and/or } DS_{ijtv_m^j}^2 = 1 \text{ and } C_{ijv_m^j v_k^{j'} t} \neq 1 \text{ for } j \in D_t; v_m^j \in V^j; v_k^{j'} \in V^{j'}$$

2) when there is a connection, i.e., $C_{ijv_m^j v_k^{j'} t} = 1$ for $j \in D_t; v_m^j \in V^j; v_k^{j'} \in V^{j'}$, then

3) when none of the above situations happens,

$$t_{v_{m+1}}^j = t_{v_m}^j + t_{v_m, v_{m+1}}^j \quad \text{for } j \in D_t; v_m^j \in V^j \quad (7)$$

Ridesharing preferences constraints

The rideshare system considers matching between drivers and passengers when their age, gender, smoking, and pet preferences match.

Age preferences matching formulation

The rideshare system considers matching between drivers and passengers when their age preferences matches. That is,

$$\rho_{i'jt}^A = \begin{cases} 0 \text{ or } 1; \text{ Age pref. of driver } j \in D_t \text{ and pass. on} \\ \text{board } i' \in O_t^j \text{ match with the pass. } i \in W_t \\ 0 ; \text{ otherwise} \end{cases} \quad (8)$$

To formulate the constraints, it is supposed that age attitude of all individuals denoted by ξ is decomposed into 3 classes: young, middle age and Old. Numerical values 1, 2, 3 are defined for the classes as 1 for Young, 2 for Middle age, or 3 for Old. Each person defines the age preferences for the individuals whom he/she will share the trip. The preferences which are denoted by η would be specified by selecting one of the 7 possible combinations.

Table 5: Age attitude and preferences classification

Individual A (passenger /passenger on board/driver)	
Age Attitude	Age Preference
1, 2, or 3	{1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

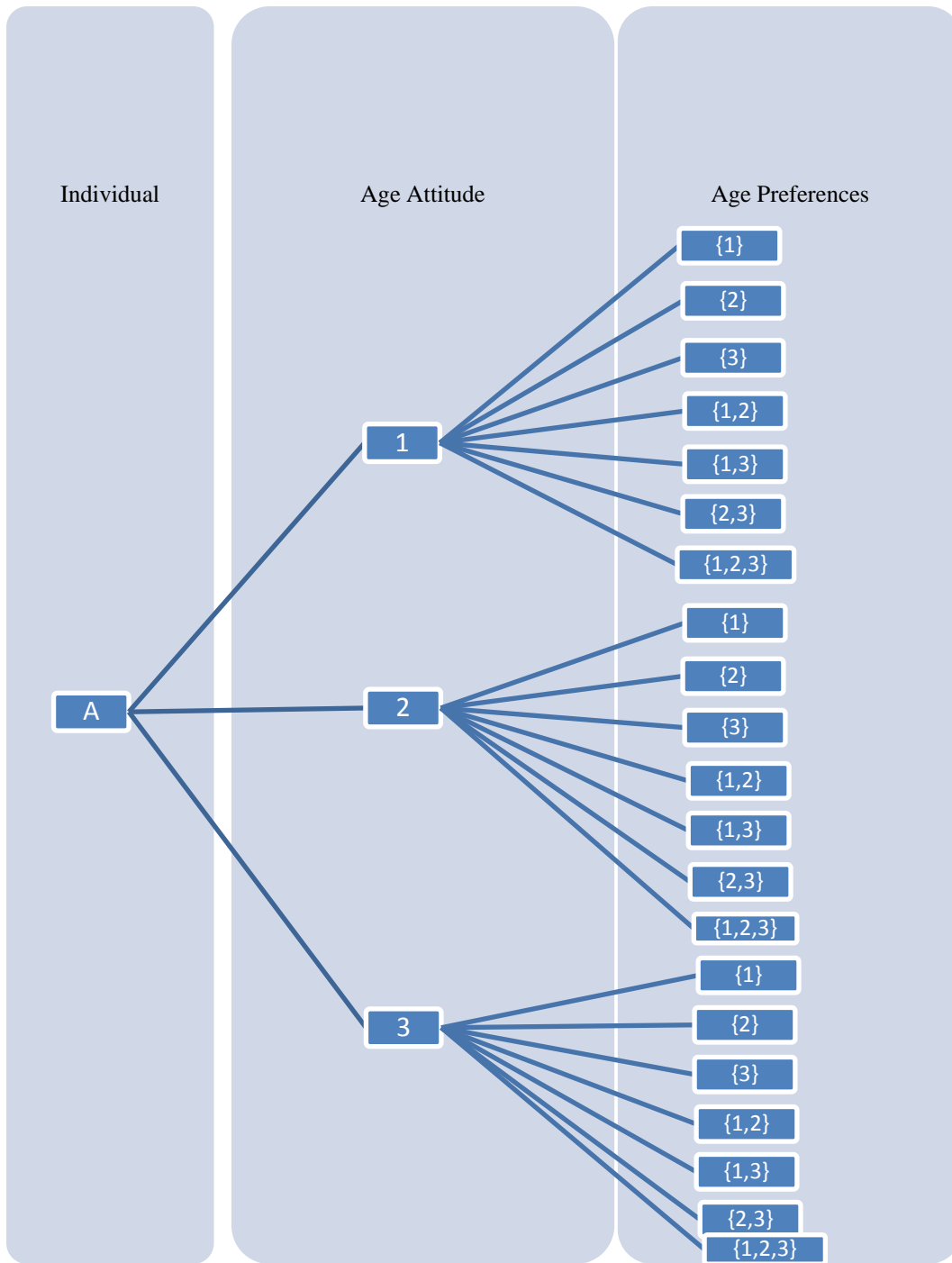


Figure 3: Age attitude and preferences classification

Case 1: For example, if there is an individual i whose age attitude falls in Young category ($a^i = 1$) willing to share the ride only with Young individuals (i.e., $\eta^i = \{1\}$) and driver j is a young person ($a^j = 1$) willing to give a ride to young and middle age individuals (i.e., $\eta^j =$

{1,2}), and there are already 1 Young passenger i' on board ($a^{i'} = 1$) whose age preference is young ($\eta^{i'} = \{1\}$), and another Young passenger i'' on board ($a^{i''} = 1$) who has no age preferences (i.e., $\eta^{i''} = \{1,2,3\}$), then individual i can share the ride with passenger i' , passenger i'' and driver j .

Case 2: For example, if there is a passenger i whose age attitude falls in Young category ($a^i = 1$) willing to share the ride only with Young individuals (i.e., $\eta^i = \{1\}$) and driver j is a young person ($a^j = 1$) willing to give a ride to young and middle age individuals (i.e., $\eta^j = \{1,2\}$), and there are already 1 middle age passenger i' on board ($a^{i'} = 2$) who has no age preference (i.e., $\eta^{i'} = \{1,2,3\}$), then the passenger i is not willing to share the ride with passenger i' and driver j .

Case 3: For example, if there is a passenger i whose age attitude falls in Young category ($a^i = 1$) willing to share the ride only with Young individuals (i.e., $\eta^i = \{1\}$) and driver j is a middle age person ($a^j = 2$) willing to give a ride to young and middle age individuals (i.e., $\eta^j = \{1,2\}$), and there are already 1 middle age passenger i' on board ($a^{i'} = 2$) who prefers sharing the ride with middle and old ages individual, i.e., $\eta^{i'} = \{2,3\}$), then the passenger i will not share the ride with passenger i' and driver j .

For each passenger i pending to be assigned to driver j , there are four decision checks:

- 1) **Passenger - Driver age matching check:** does the age preference of the passenger $i \in W_t$ match with the age of the driver $j \in D_t$. That is,

$$A_{tv}^{ij} = \begin{cases} 1 & ; \text{ Age pref. of pass. } i \in W_t \text{ matches with the age of driver } j \in D_t \\ 0 & ; \text{ otherwise} \end{cases} \quad (9)$$

- 2) **Driver - Passenger age matching check:** does the age preferences of the driver match with the age of the passenger. That is,

$$A_{tv}^{ji} = \begin{cases} 1 & ; \text{ age pref. of driver } j \in D_t \text{ matches with the age of pass. } i \in W_t \\ 0 & ; \text{ otherwise} \end{cases} \quad (10)$$

- 3) **Passenger - Passenger onboard age matching check:** does the age preference of the passenger $i \in W_t$ match with the age of the passengers on board $i' \in O_t^j$.

$$A_{tv}^{i'}$$

$$= \begin{cases} 1 & ; \text{ age pref. of pass. } i \in W_t \text{ matches with age of pass. on board } i' \in O_t^j \\ 0 & ; \text{ otherwise} \end{cases}$$

(11)

- 4) **Passenger onboard - Passenger age matching check:** do the age preferences of the passengers on board $i' \in O_t^j$ match with the age of the passenger $i \in W_t$.

$$A_{tv}^{i'}$$

$$= \begin{cases} 0 \text{ or } 1 & ; \text{ age pref. of pass. on board } i' \in O_t^j \text{ matches with age of pass. } i \in W_t \\ 0 & ; \text{ otherwise} \end{cases}$$

(12)

If all the four questions above mentioned are positive, then the passenger $i \in W_t$ is considered to be assigned to the driver $j \in D_t$ with respect to the age and age preferences criterion.

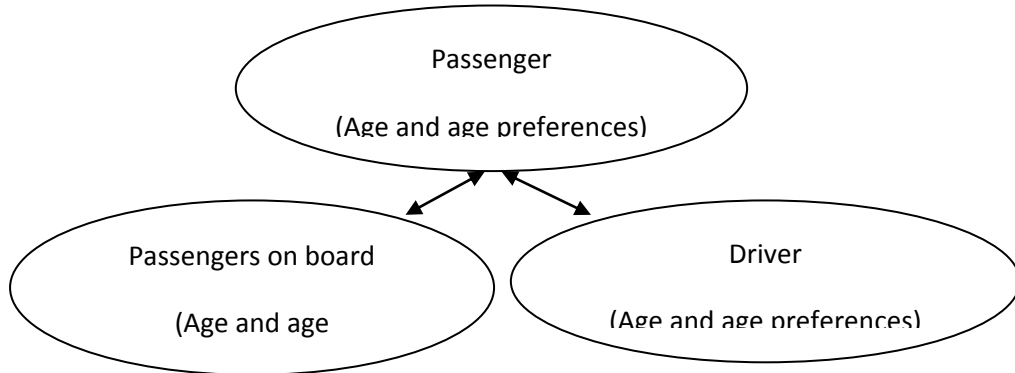


Figure 4: Age matching relations

In other words,

$$\rho_{ijtv}^A \leq A_{tv}^{ij} \cdot A_{tv}^{ji} \cdot A_{tv}^{i'j} \cdot A_{tv}^{j'i}$$

(13)

Gender preferences matching formulation

The rideshare system considers matching between drivers and passengers when their gender preferences matches. That is,

$$\rho_{ijt}^G = \begin{cases} 0 \text{ or } 1; & \text{Gender pref. of driver } j \in D_t \text{ and pass. on board } i' \in O_t^j \\ & \text{match with the gender of pass } i \in W_t \\ 0 & ; \text{ otherwise} \end{cases} \quad (14)$$

To formulate the constraints, it is supposed that gender denoted by g is decomposed into 2 classes: Male and female. Numerical values 1 and 2 are assigned to each class as 1 for Male and 2 for female. Each person defines the gender preferences for the individuals whom he/she will share the trip. The preferences which are denoted by ∂ would be specified by selecting one of the 3 possible combinations.

Table 6: Gender attitude and preferences classification

Individual A (passenger /passenger on board/driver)	
Gender	Gender Preference
1 or 2	{1}, {2}, {1,2}

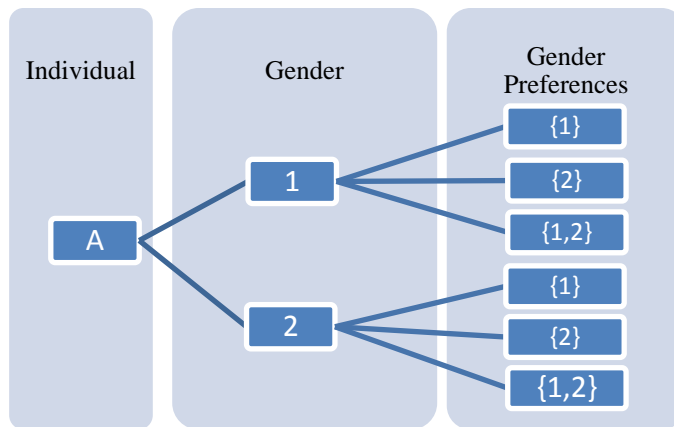


Figure 5: Gender attitude and preferences classification

Case 1: For example, if there is a male individual i ($g^i = 1$) willing to share the ride only with male individuals (i.e., $\partial^i = \{1\}$) and driver j is a male person ($g^j = 1$) who has no gender preferences (i.e., $\partial^j = \{1,2\}$), and there are already 2 male passengers i' and i'' on board ($g^{i'} = g^{i''} = 1$) who have no gender preference $\partial^{i'} = \partial^{i''} = \{1,2\}$), then the individual i can share the ride with passengers i' and i'' and driver j .

Case 2: For example, if there is a female individual i ($g^i = 2$) willing to share the ride only with female individuals (i.e., $\partial^i = \{2\}$) and driver j is a female person ($g^j = 2$) who has no gender preferences (i.e., $\partial^j = \{1,2\}$), and there are already 1 male passengers i' on board ($g^{i'} = 1$) who has no gender preference $\partial^{i'} = \{1,2\}$), then the individual i is not willing to share the ride with passenger i' and driver j .

For each passenger $i \in W_t$ pending to be assigned to driver $j \in D_t$, there are four decision checks:

- 1) **Passenger - Driver gender matching check:** does the gender preference of the passenger $i \in W_t$ match with the gender of the driver $j \in D_t$. That is,

$$G_{tv}^{ij} = \begin{cases} 1 & ; \text{ gender pref. for passenger } i \in W_t \text{ matches with driver } j \in D_t \\ 0 & ; \text{ otherwise} \end{cases} \quad (15)$$

- 2) **Driver - Passenger gender matching check:** does the gender preferences of the driver $j \in D_t$ match with the age of the passenger $i \in W_t$. That is,

$$G_{tv}^{ji} = \begin{cases} 1 & ; \text{ gender pref. of driver } j \in D_t \text{ matches with the pass. } i \in W_t \\ 0 & ; \text{ otherwise} \end{cases} \quad (16)$$

- 3) **Passenger - Passenger onboard gender matching check:** does the gender preference of the passenger $i \in W_t$ match with the gender of the passengers on board $i' \in O_t^j$.

$$G_{tv}^{ii'} = \begin{cases} 1 & ; \text{ gender pref. for pass. } i \in W_t \text{ matches with pass. on board } i' \in O_t^j \\ 0 & ; \text{ otherwise} \end{cases} \quad (17)$$

- 4) **Passenger onboard - Passenger gender matching check:** do the gender preferences of the passengers on board $i' \in O_t^j$ match with the gender of the passenger $i \in W_t$.

$$G_{tv}^{i'i} = \begin{cases} 0 \text{ or } 1 & ; \text{ gender pref. of pass. on board } i' \in O_t^j \text{ matches with pass. } i \in W_t \\ 0 & ; \text{ otherwise} \end{cases} \quad (18)$$

If answers to all the four above mentioned questions are positive, then the passenger is considered to be assigned to the driver $j \in D_t$ with respect to the gender and gender preferences criterion.

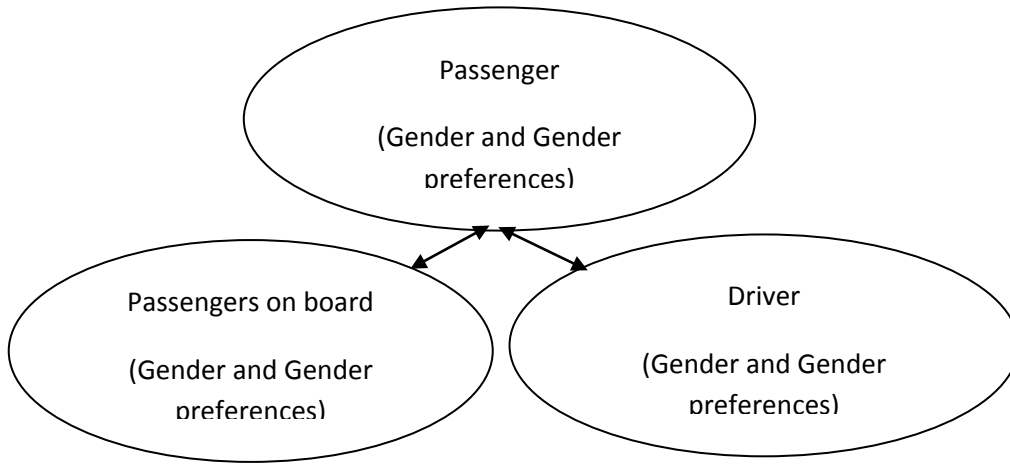


Figure 6: Gender matching relations

In other words,

$$\rho_{ijtv}^G \leq G_{tv}^{ij} \cdot G_{tv}^{ji} \cdot G_{tv}^{ii'} \cdot G_{tv}^{i'i} \quad (19)$$

Smoking preferences matching formulation

The rideshare system considers matching between drivers and passengers when their smoking preferences match. That is,

$$\rho_{ijtv}^S = \begin{cases} 0 \text{ or } 1 & ; \text{ Smoking pref. of driver and pass. on board match with the pass.} \\ 0 & ; \text{ otherwise} \end{cases} \quad (20)$$

To formulate the constraints, it is supposed that smoking tendency denoted by s is decomposed into 2 classes: Smoker and Nonsmoker. Numerical values 1 and 2 are assigned to each class as 1 for Smoker and 2 for Nonsmoker. Each person defines the smoking preferences for the individuals whom he/she will share the trip. The preferences which are denoted by Ω would be specified by selecting one of the 3 possible combinations.

Table 7: Smoking attitude and preferences classification

Individual A (passenger /passenger on board/driver)	
Smoker	Smoking Preference
1 or 2	{1}, {2}, {1,2}

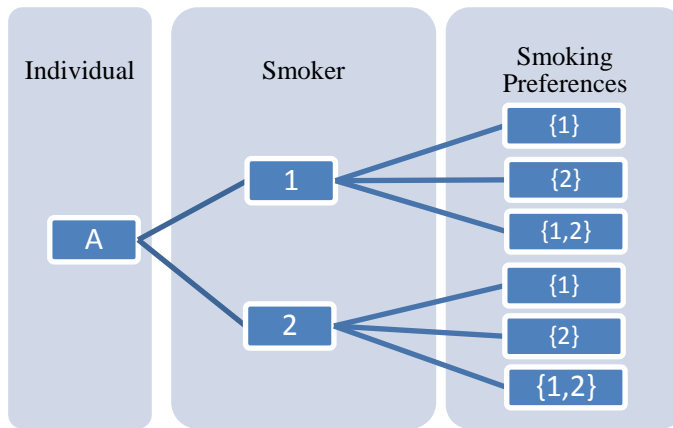


Figure 7: Smoking attitude and preferences classification

Case 1: For example, if there is a smoker passenger i ($s^i = 1$) who prefers sharing the ride with smoking individuals or individuals who have no preference on smoking (i.e., $\Omega^i = \{1,2\}$) and nonsmoker driver j ($s^j = 2$) has no disagreement with smoking on the car (i.e., $\Omega^j = \{1,2\}$), and there is already 1 nonsmoker passenger i' on board ($s^{i'} = 2$) who has no preferences on smoking (i.e., $\Omega^{i'} = \{1,2\}$), then the individual i can share the ride with passenger i' and driver j .

Case 2: For example, if there is a smoker passenger i ($s^i = 1$) who prefers sharing the ride with smoking or no smoking preference individuals (i.e., $\Omega^i = \{1,2\}$) and nonsmoker driver j ($s^j = 2$) disagree with smoking on the car (i.e., $\Omega^j = \{2\}$), and there is already 1 nonsmoker passenger i'

on board ($s^{i'} = 2$) who prefers sharing the ride with nonsmokers(i.e., $\Omega^{i'} = \{1\}$), then the individual i will not share the ride with passenger i' and driver j .

For each passenger i pending to be assigned to driver j , there are four decision checks:

- 1) **Passenger - Driver smoking matching check:** does the smoking preferences of the passenger match with the smoking of the driver. That is,

$$S_{tv}^{ij} = \begin{cases} 1 & ; \text{ smoking preferences for passenger } i \text{ matches with the smoking of driver } j \\ 0 & ; \text{ otherwise} \end{cases} \quad (21)$$

- 2) **Driver - Passenger smoking matching check:** does the smoking preferences of the driver match with the passenger. That is,

$$S_{tv}^{ji} = \begin{cases} 1 & ; \text{ smoking preferences of driver } j \text{ matches with the passenger } i \\ 0 & ; \text{ otherwise} \end{cases} \quad (22)$$

- 3) **Passenger - Passenger onboard smoking matching check:** does the smoking preference of the passenger match with the passengers on board.

$$S_{tv}^{ii'} = \begin{cases} 1 & ; \text{ smoking pref. for pass. } i \text{ matches with pass. on board } i' \\ 0 & ; \text{ otherwise} \end{cases} \quad (23)$$

- 4) **Passenger onboard - Passenger smoking matching check:** do the smoking preferences of the passengers on board match with the passenger.

$$S_{tv}^{i'i} = \begin{cases} 0 \text{ or } 1 & ; \text{ smoking pref. for pass. on board } i' \text{ matches with pass. } i \\ 0 & ; \text{ otherwise} \end{cases} \quad (24)$$

If all the four above mentioned questions are positive, then the passenger is considered to be assigned to the driver with respect to the smoking and smoking preferences criterion.

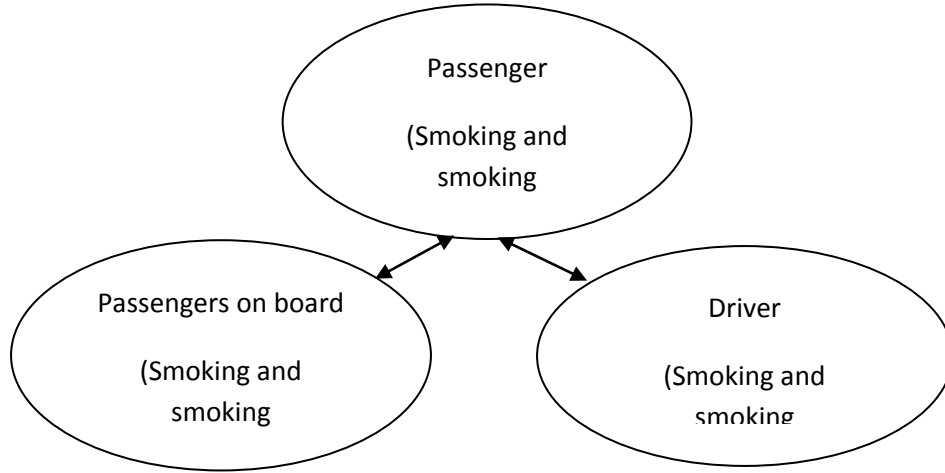


Figure 8: Smoking matching relations

In other words,

$$\rho_{ijtv}^S \leq S_{tv}^{ij} \cdot S_{tv}^{ji} \cdot S_{tv}^{ii'} \cdot S_{tv}^{i'i} \quad (25)$$

Pet restrictions preferences matching formulation

The rideshare system considers matching between drivers and passengers when their pet restriction preferences match. That is,

$$\rho_{ijtv}^P = \begin{cases} 0 \text{ or } 1 & ; \text{ pet restriction pref. of driver and pass. on board match with the pass.} \\ 0 & ; \text{ otherwise} \end{cases} \quad (26)$$

To formulate the constraints, it is supposed that pet policy p is decomposed into 2 classes: Friendly and Unfriendly. Numerical values 1 and 2 are assigned to each class as 1 for Friendly and 2 for Unfriendly. Each person defines his/her own pet policy preferences for the individuals whom he/she will share the trip. The preferences which are denoted by ω would be specified by selecting one of the 3 possible combinations.

Table 7: Pet attitude and preferences classification

Individual A (passenger /passenger on board/driver)	
Pet Friendly	Pet policy preferences
1 or 2	{1}, {2}, {1,2}

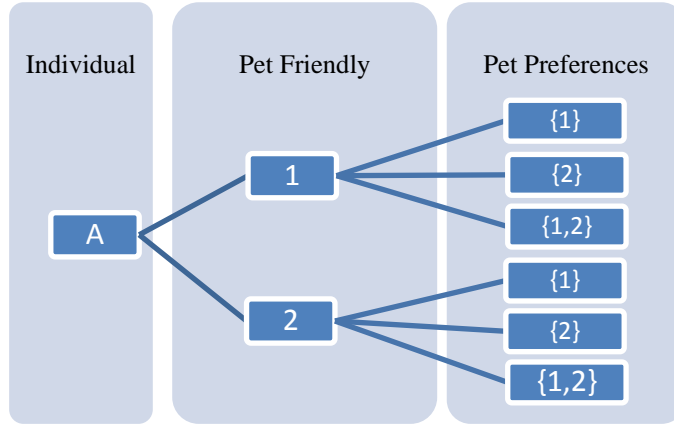


Figure 9: Pet attitude and preferences classification

Case 1: For example, if there is a pet friendly passenger i ($p^i = 1$) who prefers sharing the ride with pet friendly individuals (i.e., $\omega^i = \{1\}$) or individuals who have no preference on pet restriction (i.e., $\omega^i = \{1,2\}$) and there is a pet friendly driver j ($p^j = 1$) who has no disagreement with pet on the car (i.e., $\omega^j = \{1,2\}$), and there is already a pet friendly passenger i' on board ($p^{i'} = 1$) who has no policy for sharing a ride with pets (i.e., $\omega^{i'} = \{1,2\}$), then the individual i can share the ride with passenger i' and driver j .

Case 2: For example, if there is a pet friendly passenger i ($p^i = 1$) who has no preference on pet (i.e., $\omega^i = \{1,2\}$) and a unfriendly driver j ($p^j = 2$) who disagree with pets on board (i.e., $\omega^j = \{2\}$), then the individual i will not share the ride with driver j .

For each passenger i pending to be assigned to driver j , there are four decision checks:

- 1) **Passenger - Driver pet friendliness matching check:** does the pet preferences of the passenger match with the pet policy of the driver. That is,

$$P_{tv}^{ij} = \begin{cases} 1 & ; \text{ pet preferences for passenger } i \text{ matches with the policy of driver } j \\ 0 & ; \text{ otherwise} \end{cases} \quad (27)$$

2) **Driver - Passenger pet friendliness matching check:** does the pet preferences of the driver match with the passenger. That is,

$$P_{tv}^{ji} = \begin{cases} 1 & ; \text{ pet preferences of driver } j \text{ matches with the passenger } i \\ 0 & ; \text{ otherwise} \end{cases} \quad (28)$$

3) **Passenger - Passenger onboard pet friendliness matching check:** does the pet preference of the passenger match with the passengers on board.

$$P_{tv}^{ii'} = \begin{cases} 1 & ; \text{ pet pref. for pass. } i \text{ matches with pass. on board } i' \\ 0 & ; \text{ otherwise} \end{cases} \quad (29)$$

4) **Passenger onboard - Passenger pet friendliness matching check:** do the pet preferences of the passengers on board match with the passenger.

$$P_{tv}^{i'i} = \begin{cases} 0 \text{ or } 1 & ; \text{ pet pref. for pass. on board } i' \text{ matches with pass. } i \\ 0 & ; \text{ otherwise} \end{cases} \quad (30)$$

If all the four above mentioned questions are positive, then the passenger is considered to be assigned to the driver with respect to the pet preferences criterion.

In other words,

$$\rho_{ijtv}^P \leq P_{tv}^{ij} \cdot P_{tv}^{ji} \cdot P_{tv}^{ii'} \cdot P_{tv}^{i'i} \quad (31)$$

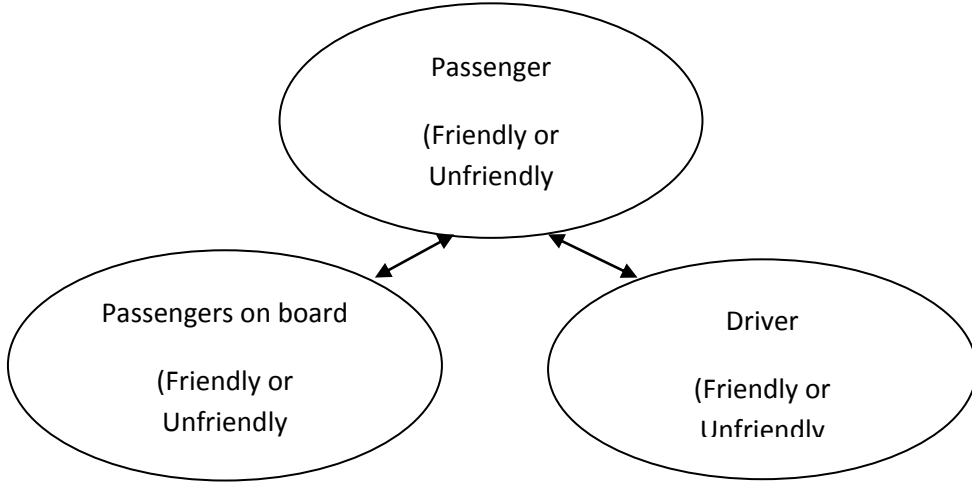


Figure 10: Pet matching relations

Maximum occupancy preferences constraints

The rideshare system considers matching between drivers and passengers when the maximum number of passengers on board policy defined by the driver and passengers are not violated. By definition, passengers can be an Individual travelling alone or a person travelling with his/her pet. In the latter case, a pet is considered to be a passenger.

$$\rho_{ijt}^M = \begin{cases} 0 \text{ or } 1 & ; \text{ \# people pref. of driver and passengers on board match with the passenger} \\ 0 & ; \text{ otherwise} \end{cases}$$

As earlier defined, Q_{jtv} is the available place on vehicle j at time moment t at point of interest v and Q_j is the available seat on vehicle defined by the driver j that is equal to the seat capacity of the vehicle at its origin point of departure. On the other hand, each passenger defines his/her favorable maximum number of people whom share the ride. Let Q^i , Q^j and $Q^{i'}$ be the favorable maximum number of people sharing the ride defined by passenger i , driver j , and passenger on board i' respectively. That is,

$$\rho_{ijtv}^M = \begin{cases} 0 \text{ or } 1 & ; Q^i \geq Q_j - Q_{jtv} + 1 \text{ and } Q^j \geq Q_j - Q_{jtv} + 1 \text{ and } Q^{i'} \geq Q_j - Q_{jtv} + 1 \\ 0 & ; \text{ otherwise} \end{cases}$$

(32)

equivalently:

$$\rho_{ijtv}^M = \begin{cases} 0 \text{ or } 1 & ; \text{ Min}\{Q^i, Q^j, Q^{i'}\} \geq Q_j - Q_{jtv} + 1 \\ 0 & ; \text{ otherwise} \end{cases} \quad (33)$$

If all the five above mentioned preferences are met, then the passenger is considered to be assigned to the driver, i.e.,

$$5\rho_{ijtv} \leq \rho_{ijtv}^A + \rho_{ijtv}^G + \rho_{ijtv}^S + \rho_{ijtv}^P + \rho_{ijtv}^M \quad \text{for } i \in W_{\square}; j \in D_t; v \in V \quad (34)$$

Formulation of the objective function

Objective Function

As mentioned earlier, DRMO assigns passengers to drivers and identifies the feasible routes for drivers to maximize the total number of matching in a given planning horizon. The objective function for the model is:

$$\begin{aligned} \text{Maximize } & \sum_t^{t+\Delta} \sum_{i \in W_t} \sum_{j \in D_t} \sum_{v_m^j \in R_t^j} (PS_{ijv_m^j}^1 + PS_{ijv_m^j}^2 + PT_{ijv_m^j}^1 + PT_{ijv_m^j}^2 + P_{ijv_m^j}^1 + P_{ijv_m^j}^2 + P_{ijv_m^j}) + \\ & \sum_t^{t+\Delta} \sum_{i \in W_t} \sum_{j \in D_t} \sum_{v_m^j \in R_t^j} (DS_{ijv_m^j}^1 + DS_{ijv_m^j}^2) + \\ & \sum_t^{t+\Delta} \sum_{i \in W_t} \sum_{j \in D_t} \sum_{v_m^j \in R_t^j} (T_{ijv_m^j}^1 + T_{ijv_m^j}^2 + T_{ijv_m^j}^3) + \\ & \sum_t^{t+\Delta} \sum_{i \in W_t} \sum_{j \in D_t} \sum_{j' \in D_t} \sum_{v_m^j \in R_t^j} \sum_{v_k^{j'} \in R_t^{j'}} (CS_{ijv_m^j v_k^{j'}}^1 + CS_{ijv_m^j v_k^{j'}}^2 + CT_{ijv_m^j v_k^{j'}}^1 + CT_{ijv_m^j v_k^{j'}}^2 + C_{ijv_m^j v_k^{j'}}^1 \\ & \quad + C_{ijv_m^j v_k^{j'}}^2 + T_{ijv_m^j v_k^{j'}}^4 + T_{ijv_m^j v_k^{j'}}^5) + \end{aligned}$$

$$\sum_t^{t+\Delta} \sum_{i \in W_t} \sum_{j \in D_t} \sum_{v_m \in R_t^j} \rho_{ijtv} +$$

$$\sum_t^{t+\Delta} \sum_{i \in W_t} \sum_{j \in D_t} \sum_{\partial=1}^5 \rho_{ijt}^{\partial}$$

(35)

The first term in the objective functions maximizes all possible pickups regarding the proximity in time and space, the second term maximizes possible drop-offs, the third term secures continuity of movement for each driver and passenger without need to any reconnection and the fourth term maximizes the number of possible routes and continuity of movement through reconnection, the fifth term seeks to maximize preferences matching and the sixth term maximizes the preference matching for each of the five-fold criteria.

Inputs of the Model

At each time moment t :

P_t : the set of passengers in the system at time moment t ; $i \in P_t$;

O_t : the set of passengers on board at time moment t ; $O_t^j \subset O_t$

O_t^j : the set of passengers on board the vehicle belonging to driver $j \in D_t$

D_t : set of passengers at time moment t

For passenger $i \in P_t$:

v_0^i : origin point of passenger $i \in P_t$; $v_0^i \in V^i$

v_D^i : Destination point of passenger $i \in P_t$; $v_D^i \in V^i$

t_0^i : requested time to start the service by passenger $i \in P_t$ at the origin point

V^i : the set of nodes visiting by passenger $i \in P_t$ en route from origin to destination in the form of successive nodes

For driver $j \in D_t$:

v_m^j : origin/current point of driver $j \in D_t$; $v_m^j \in V^j$

t_0^j : departing time of driver j from his/her origin,

Q^j : the number of places available on the vehicle belonging to driver $j \in D_t$

V^j : the set of nodes visiting by driver $j \in D_t$ en route from origin to destination in the form of successive nodes

For point of interest $v \in V$; $V = V^i \cup V^j$:

$\|D\|$: distance matrix

D_{v_m, v_n} : distance of travel between two successive points $v_m, v_n \in V$

$t_{v_0^i, v_m^j}^j$: the travel time between origin point of the passenger v_0^i and point of interest v_m^j for driver j

$t_{v_m^j, v_k^{j'}}^i$: Walking time for passenger i from v_m^j to $v_k^{j'}$.

Personal information and ridesharing preferences for each passenger $i \in P_t$ and driver $j \in D_t$:

Q^i, Q^j : The favorable maximum number of people sharing the ride defined by passenger i , and driver j .

a^i : Numerical value for the age class of passenger $i \in P_t$ defined as 1 for Young, 2 for Middle age, or 3 for Old.

η^i : set of age preferences for passenger $i \in P_t$ which would be specified by selecting one of the 7 possible combinations: $\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

η_m^i : the m th element of the age preference set of passenger $i \in P_t$

α^j : Numerical value for the age class of driver $j \in D_t$ defined as 1 for Young, 2 for Middle age, or 3 for Old.

η^j : set of age preferences for driver $j \in D_t$ which would be specified by selecting one of the 7 possible combinations: {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

η_m^j : the m th element of the age preference set of passenger $j \in P_t$

g^i : Numerical value for the gender of the passenger $i \in P_t$ defined as 1 for Male and 2 for female.

g^j : Numerical value for the gender of driver $j \in D_t$ defined as 1 for Male and 2 for female.

∂^i : set of gender preferences for passenger $i \in P_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

∂_m^i : the m th element of the gender preference set of passenger $i \in P_t$.

∂^j : set of gender preferences for driver $j \in D_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

∂_m^j : the m th element of the gender preference set of driver $j \in D_t$.

s^i : Numerical value for the smoking class of passenger $i \in P_t$ defined as 1 for smoker and 2 for non-smoker.

Ω^i : set of smoking preferences for passenger $i \in P_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

μ_m^i : the m th element of the smoking preference set of passenger $i \in P_t$

s^j : Numerical value for the smoking class of driver $j \in D_t$ defined as 1 for smoker and 2 for non-smoker.

Ω^j : set of smoking preferences for driver $j \in D_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

μ_m^j : the m th element of the smoking preference set of driver $j \in D_t$

p^i : Numerical value for the pet tendency class of passenger $i \in P_t$ defined as 1 for pet friendly and 2 for non- pet friendly.

ω^i : set of pet tendency preferences for passenger $i \in P_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

ζ_m^i : the m th element of the pet preference set of passenger $i \in P_t$

p^j : Numerical value for the pet tendency class of driver $j \in D_t$ defined as 1 for pet friendly and 2 for non- pet friendly.

ω^j : set of pet tendency preferences of driver $j \in D_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

ζ_m^j : the m th element of the pet preference set of driver $j \in D_t$

Other input parameters:

b_{ij} : credit assigned to the driver $j \in D_t$ for picking up and dropping off passenger $i \in P_t$

δ : a predetermined parameter for maximum number of connections per passenger

γ : a predetermined time parameter for maximum allowable waiting time for a passenger

β : a predefined distance parameter for maximum allowable detour distance for a driver

φ : a predefined distance parameter for maximum allowable relocation distance for a passenger

M is a big positive numerical value.

Outputs of the Model

$$\rho_{ijt} = \begin{cases} 1; & \text{Riding preferences of passenger } i \in P_t \text{ matches with driver } j \in D_t \text{ and} \\ & \text{all the passengers on board } i' \in O_t^j \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$A_t^{ij} = \begin{cases} 1; & \text{Age pref. of pass. } i \in P_t \text{ matches with the age of driver } j \in D_t \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$A_t^{ji} = \begin{cases} 1; & \text{age pref. of driver } j \in D_t \text{ matches with the age of pass. } i \in P_t \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$A_t^{ii'} = \begin{cases} 1; & \text{age pref. of pass. } i \in P_t \text{ matches with age of pass. on board } i' \in O_t^j \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$A_t^{i'i} = \begin{cases} 1; & \text{age pref. of pass. on board } i' \in O_t^j \text{ matches with age of pass. } i \in P_t \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$G_t^{ij} = \begin{cases} 1; & \text{gender pref. for passenger } i \in P_t \text{ matches with driver } j \in D_t \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$G_t^{ji} = \begin{cases} 1; & \text{gender pref. of driver } j \in D_t \text{ matches with the pass. } i \in W_t \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$G_t^{ii'} = \begin{cases} 1; & \text{gender pref. for pass. } i \in P_t \text{ matches with pass. on board } i' \in O_t^j \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$G_t^{i'i} = \begin{cases} 1; & \text{gender pref. of pass. on board } i' \in O_t^j \text{ matches with pass. } i \in P_t \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$S_t^{ij} = \begin{cases} 1; & \text{smoking preferences for passenger } i \in P_t \text{ matches with the smoking} \\ & \text{of driver } j \in D_t \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$S_t^{ji} = \begin{cases} 1; & \text{smoking preferences of driver } j \in D_t \text{ matches with the smoking} \\ & \text{of passenger } i \in P_t \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$S_t^{ii'} = \begin{cases} 1; & \text{smoking pref. for pass. } i \in P_t \text{ matches with pass. on board } i' \in O_t^j \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$S_t^{i'i} = \begin{cases} 1; & \text{smoking pref. for pass. on board } i' \in O_t^j \text{ matches with pass. } i \in P_t \\ & \text{at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$P_t^{ij} = \begin{cases} 1; & \text{pet preferences for passenger } i \in P_t \text{ matches with the policy} \\ & \text{of driver } j \in D_t \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$P_t^{ji} = \begin{cases} 1; & \text{pet preferences for driver } j \in D_t \text{ matches with the policy} \\ & \text{of passenger } i \in P_t \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$P_t^{ii'} = \begin{cases} 1; & \text{pet preferences for passenger } i \in P_t \text{ matches with the policy} \\ & \text{of pass. on board } i' \in O_t^j \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$P_t^{i'i} = \begin{cases} 1; & \text{pet preferences for pass. on board } i' \in O_t^j \text{ matches with the policy} \\ & \text{of passenger } i \in P_t \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$\rho_{ijt}^A = \begin{cases} 1; & \text{Age pref. of passenger } i \in P_t \text{ matches with driver } j \in D_t \text{ and} \\ & \text{all the passengers on board } i' \in O_t^j \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$\rho_{ijt}^G = \begin{cases} 1; & \text{Gender pref. of passenger } i \in P_t \text{ matches with driver } j \in D_t \text{ and} \\ & \text{all the passengers on board } i' \in O_t^j \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$\rho_{ijt}^S = \begin{cases} 1; & \text{Sex pref. of passenger } i \in P_t \text{ matches with driver } j \in D_t \text{ and} \\ & \text{all the passengers on board } i' \in O_t^j \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$\rho_{ijt}^P = \begin{cases} 1; & \text{Pet freindly pref. of passenger } i \in P_t \text{ matches with driver } j \in D_t \text{ and} \\ & \text{all the passengers on board } i' \in O_t^i \text{ at time moment } t \\ 0; & \text{Otherwise} \end{cases}$$

$$\rho_{ijt}^M = \begin{cases} 1; & \text{Occupancy pref. of passenger } i \in P_t \text{ matches with driver } j \in D_t \text{ and} \\ & \text{all the passengers on board } i' \in O_t^i \text{ at time momen} \square t \\ 0; & \text{Otherwise} \end{cases}$$

$PS_{ijtv_m^j}^1$: The binary variable that shows passenger i could be picked up by driver j at point v_m^j for the first lag of journey with respect to proximity in space.

$PS_{ijtv_m^j}^2$: The binary variable that shows passenger i could be picked up by driver j at his point of origin v_0^i for the first lag of journey when driver j makes a detour on the way from point v_m^j to point v_{m+1}^j with respect to proximity in space.

$PT_{ijtv_m^j}^1$: The binary variable that shows passenger i could be picked up by driver j at point v_m^j for the first lag of journey with respect to proximity in time.

$PT_{ijtv_m^j}^2$: the binary variable that shows passenger i could be picked up by driver j at his point of origin v_0^i for the first lag of journey with respect to proximity in time when driver j makes a detour on the way from point v_m^j to point v_{m+1}^j .

$P_{ijtv_m^j}$: The binary variable that equals 1 when at least one of the two binary variables $P_{ijtv_m^j}^1$ and $P_{ijtv_m^j}^2$ equals 1.

$DS_{ijtv_m^j}^1$: The binary variable that shows passenger i could be dropped off by driver j at point v_m^j for the last lag of journey.

$DS_{ijtv_m^j}^2$: The binary variable that shows passenger i could be dropped off by driver j at his point of destination v_D^i for the last lag of journey when driver j makes a detour on the way from point v_m^j to point v_{m+1}^j .

DS_{ijt} : the binary variable that shows passengter i may get a ride from driver j for his last lag of journey, with respect to proximity in space.

$CS_{jv_k^{j'}jrv_m^j t}^1$: The binary variable that shows passenger i will leave driver j at point v_m^j to have a walk to reach to the point of connection with driver j' .

$CS_{jv_k^{j'}jrv_m^j t}^2$: The binary variable that shows driver j makes a detour at point v_m^j to pickup/drop off passenger i at point $v_k^{j'}$ which is en route for driver j' .

$C_{ijv_m^j jrv_k^{j'} t}$: The binary variable that equals 1 when at least one of the two binary variables

$C_{ijv_m^j jrv_k^{j'} t}^1$ and $C_{ijv_m^j jrv_k^{j'} t}^2$ equals 1.

$t_{v_m^j}^j$: the time moment that driver j meets point of interest v_m^j .

$RF_{ijv_m^j}$: the binary variable that shows passenger i could be picked up by driver j at point v_m^j or when driver j makes a detour at point v_m^j to pick up the passenger at his point of origin.

$RL_{ijv_m^j}$: the binary variable that shows passenger i could be dropped off by driver j at point v_m^j or when driver j makes a detour at point v_m^j to drop off the passenger at his point of destination.

RFL_{ij} : the binary variable that shows passenger i could be picked up and dropped off by driver j .

$RFCL_{ijj'}$: the binary variable that shows passenger i could be picked up by driver j and dropped off by driver j' .

$T_{ijv_m^j t}^1$: added time to travel time of driver j between points of interest v_m^j and v_{m+1}^j due to making a detour to pick up passenger i at his/her point of origin.

$T_{ijv_m^j t}^2$: added time to travel time of driver j between points of interest v_m^j and v_{m+1}^j due to making a detour to drop off passenger i at his/her destination.

$T_{ijv_m^j t}^3$: added time to travel time of driver j between points of interest v_m^j and v_{m+1}^j due to making a detour to pick up and drop off passenger i at his/her destination.

$T_{ijv_m^j jrv_k^{j'} t}^4$: added time to travel time of driver j between points of interest v_m^j and v_{m+1}^j due to making a detour to pick up passenger i at connection point $v_k^{j'}$ belonging to driver j' .

$T_{ijv_m^j j'v_k^{j'}}^5$: added time to travel time of driver j between points of interest v_m^j and v_{m+1}^j due to making a detour to pick up passenger i at connection point $v_k^{j'}$ belonging to driver j' and then drop off the passenger at his/her destination.

Numerical Example

In time interval 9:30 AM to 9:45 AM, there is a request for rideshare from an individual. The time requested for service is 9:45 with maximum 10 minutes waiting time. This person is: Young ($a^i = 1$) willing to share the ride only with Young individuals (i.e., $\eta^i = \{1\}$), Male ($g^i = 1$) willing to share the ride only with male individuals (i.e., $\partial^i = \{1\}$), Smoker ($s^i = 1$) and prefers sharing the ride with smoking individuals or individuals who have no preference on smoking (i.e., $\Omega^i = \{1,2\}$), Pet friendly passenger ($p^i = 1$) and prefers sharing the ride with pet friendly individuals (i.e., $\omega^i = \{1\}$) or individuals who have no preference on pet restriction (i.e., $\omega^i = \{1,2\}$). Also, the maximum favorable number of people sharing the ride defined by this passenger is 3 ($Q^i = 3$). Also, v_0^i with coordinates (10,15) and v_D^i with coordinates (15,15) is the current points of origin and destination for passenger i with acceptable relocating distance, φ , 0.5 miles. In addition, there are two drivers available in the system: Driver j who is already started his journey and has a passenger on board is: Young ($a^j = 1$) willing to give a ride to young and middle age individuals (i.e., $\eta^j = \{1,2\}$), Male ($g^j = 1$) and has no gender preferences (i.e., $\partial^j = \{1,2\}$), Nonsmoker ($s^j = 2$) and has no disagreement with smoking on the car (i.e., $\Omega^j = \{1,2\}$), Pet friendly driver j ($p^j = 1$) who has no disagreement with pet on the car (i.e., $\omega^j = \{1,2\}$). Also, the maximum favorable number of people sharing the ride defined by the driver is 4 ($Q^j = 4$). At 9:30 a.m. he is in point (5,8) and his path is (7,9), (10,14), (12,12), (15,16). Acceptable detour distance for driver j , β_j , is 1.5 miles. The passenger on board i' is: Young ($a^{i'} = 1$) and has no age preferences (i.e., $\eta^{i'} = \{1,2,3\}$), Male ($g^{i'} = 1$) and has no gender preference $\partial^{i'} = \{1,2\}$), Nonsmoker ($s^{i'} = 2$) and has no preferences on smoking (i.e., $\Omega^{i'} = \{1,2\}$), pet friendly passenger who has no policy for sharing a ride with pets (i.e., $\omega^{i'} = \{1,2\}$). Also, the maximum favorable number of people sharing the ride defined by this passenger on board is 4 ($Q^{i'} = 4$). Driver j' is: Young person ($a^{j'} = 1$) willing to give a ride to young and middle age individuals (i.e., $\eta^{j'} = \{1,2\}$), Male ($g^{j'} = 1$) and has no gender

preferences (i.e., $\partial^{j'} = \{1,2\}$), Nonsmoker ($s^{j'} = 2$) and has no disagreement with smoking on the car (i.e., $\Omega^{j'} = \{1,2\}$), Pet friendly driver j' ($p^{j'} = 1$) who has no disagreement with pet on the car (i.e., $\omega^{j'} = \{1,2\}$). Also, the maximum favorable number of people sharing the ride defined by the driver is 4 ($Q^{j'} = 4$). This driver will start his journey at 9:40 and his path from his origin to his destination is (9, 9.5), (10.1,15.2), (14, 16). Acceptable detour distance for driver j' , $\beta_{j'}$, is 1.5 miles. Figure 11 shows the map for this numerical example.

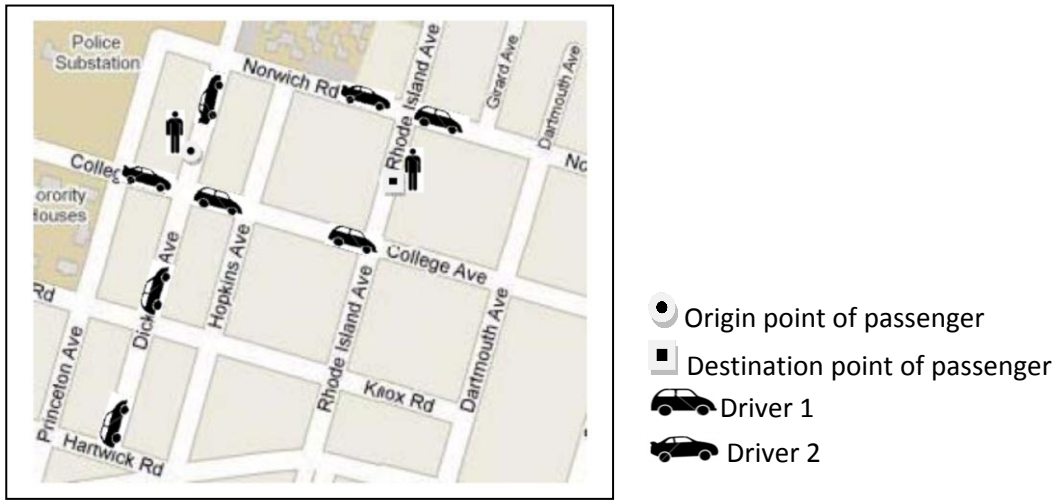


Figure 11: The map for the numerical example.

Age preference of the passenger matches with driver j' and his passenger on board ($Ap1d2= 1$, $Ap1p1=1$), and age preferences of the driver and his passenger on board matches with the passenger ($Ad2p1= 1$, $Apo1p1=1$), so the passenger matches the vehicle with respect to age criterion ($RAp1d2=1$), Gender preference of the passenger matches with driver j' and his passenger on board ($Gp1d2= 1$, $Gp1p1=1$), and gender preferences of the driver and his passenger on board matches with the passenger ($Gd2p1= 1$, $Gpo1p1=1$), so the passenger matches the vehicle with respect to gender criterion ($RGp1d2=1$), Smoking preference of the passenger matches with driver j' and his passenger on board ($Sp1d2= 1$, $Sp1p1=1$), and smoking preferences of the driver and his passenger on board matches with the passenger ($Sd2p1= 1$, $Spo1p1=1$), so the passenger matches the vehicle with respect to smoke criterion ($RSp1d2=1$), Pet restriction preference of the passenger matches with driver j' and his passenger on board ($Pp1d2= 1$, $Pp1p1=1$), and pet restriction preferences of the driver and his passenger

on board matches with the passenger ($Pd2p1= 1$, $Ppo1p1=1$), so the passenger matches the vehicle with respect to pet restriction criterion ($RPp1d2=1$), and occupancy preferences of the driver, his passenger on board and the passenger matches ($RMp1d2=1$), therefore the passenger matches with vehicle j' with respect to rideshare preferences ($Rp1d2=1$). Likewise, Age preference of the passenger matches with driver j' ($Ap1d2= 1$), and age preferences of the driver matches with the passenger ($Ad2p1= 1$), so the passenger matches the vehicle with respect to age criterion ($RAp1d2=1$), Gender preference of the passenger matches with driver ($Gp1d2= 1$), and gender preferences of the driver matches with the passenger ($Gd2p1= 1$), so the passenger matches the vehicle j' with respect to gender criterion ($RGp1d2=1$), Smoking preference of the passenger matches with the driver ($Sp1d2= 1$), and smoking preferences of the driver matches with the passenger ($Sd2p1= 1$), so the passenger matches the vehicle j' with respect to smoke criterion ($RSp1d2=1$), Pet restriction preference of the passenger matches with driver j' ($Pp1d2= 1$), and pet restriction preferences of the driver matches with the passenger ($Pd2p1= 1$), so the passenger matches the vehicle j' with respect to pet restriction criterion ($RPp1d2=1$), and occupancy preferences of the driver, his passenger on board and the passenger matches ($RMp1d2=1$), therefore the passenger matches with vehicle j' with respect to rideshare preferences ($Rp1d2=1$). This situation is shown in Figure 12.

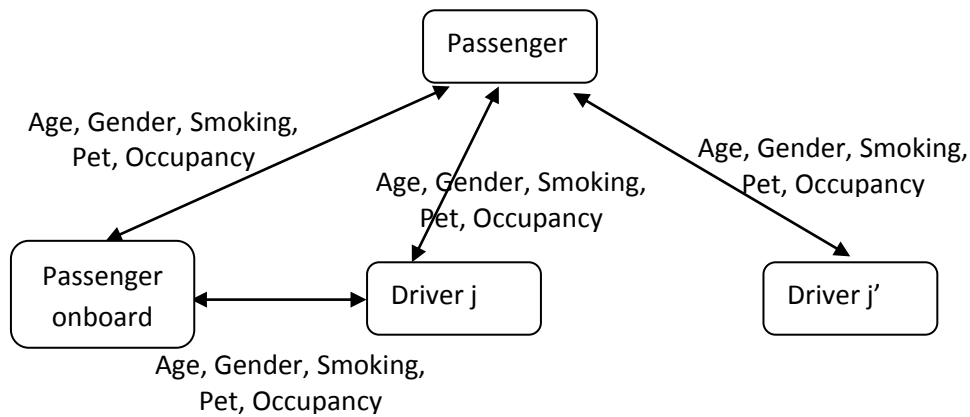


Figure 12: Rideshare preference matching relationships

Driver j' can pick up the passenger with respect to proximity in space if the passenger walk to the second visiting point of this driver ($PS1p1d2v2=1$), or when the driver make a detour from

his first visiting point to the origin point of the passenger ($PS2p1d2v1=1$) or from his second visiting point to the origin point of the passenger ($PS2p1d2v2=1$). This driver can also pick up the passenger with respect to proximity in time when he makes a detour from his first visiting point to the origin point of the passenger ($PT2p1d2v1=1$) or making a detour from his second visiting point to the origin point of the passenger ($PT2p1d2v2=1$).

Driver j can pick up the passenger with respect to proximity in time if he makes a detour from one of his second, third or fourth visiting nodes to the point of origin of the passenger ($PT2p1d1v2=1$, $PT2p1d1v3=1$, $PT2p1d1v4=1$). Also, the passenger can reach to his destination when only driver 1 makes a detour from his fourth visiting node to drop off the passenger at his point of destination ($DS2p1d1v4=1$). Figure 13 shows the relationships.

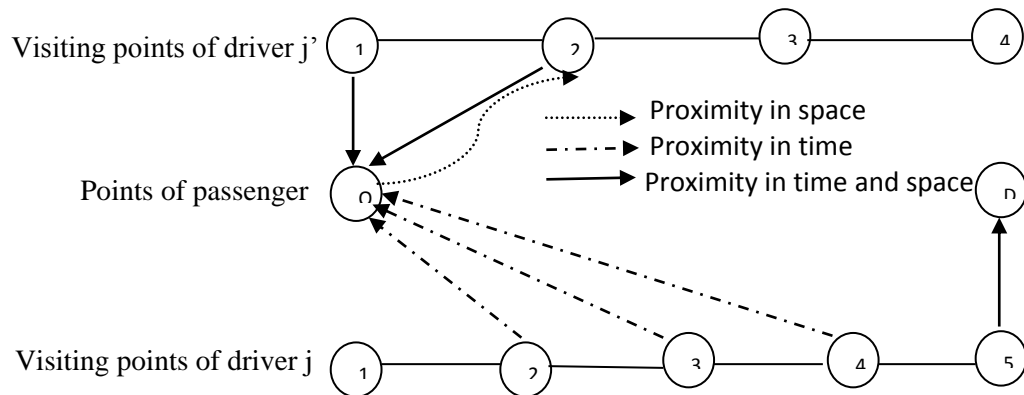


Figure 13: Proximity in time and space relationships for pick up and drop off the passenger

According to what has discussed so far, driver j' can pick up the passenger ($P2p1d2v1=1$) and driver j can drop him off ($DS2p1d1v4=1$). To have a feasible solution, there should be a connection between the two drivers with respect to proximity in time and space. With respect to proximity in space, there is a connection between the drivers if driver j makes a detour from his second visiting node to drop off the passenger at the first visiting node of driver j' ($CS2p1d1v2d2v1=1$) or when the passenger leaves driver j' who makes a detour from his first visiting node to drop off the passenger at the third visiting node of driver j ($CS2p1d2v1d1v3=1$). With respect to time, driver j can drop off the passenger when he makes a detour from his first visiting node to drop off the passenger at the first or second visiting node of driver j' ($CT2p1d1v1d2v1=1$, $CT2p1d1v1d2v2=1$), or from his second visiting node to drop off the

passenger at the first or second visiting node of driver j' ($CT2p1d1v2d2v1=1$, $CT2p1d1v2d2v2=1$), or from his third visiting node to drop off the passenger at the second visiting node of driver j' ($CT2p1d1v3d2v2=1$). Also, with respect to time Driver j' can drop off the passenger at the fourth visiting node of driver j when he makes a detour at his first visiting node ($CT2p1d2v1d1v4=1$). The result shows that the only feasible connection with respect to both of the proximity in time and space is when driver j makes a detour at his second visiting node to drop off the passenger at the first visiting node of driver j' ($C2p1d1v2d2v1=1$) (See Figure 14).

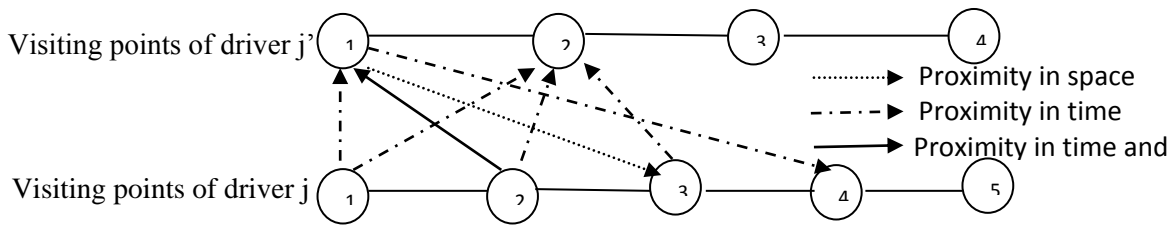


Figure 14: Proximity in time and space relationships for pick up and drop off the passenger

Therefore, the passenger starts his first leg of journey with driver j' ($RFp1d2v1=RFp1d2=1$) and ends his last leg with driver j ($RLp1d1v4=RLp1d1=1$), but since there is no feasible connection with respect to proximity in time and space to connect driver j' to driver j , it is concluded that there is no rideshare for the passenger.

Now, if the passenger accepts for .91 more miles walking distance relocation, the model results in $DS1p1d2v3=1$ which means the passenger can reach to his destination when he leaves driver j' at his third (final) visiting node and walks for 1.41 miles to reach to his destination. Likewise, if driver j accepts to detour for more .36 miles to pick up the passenger, he can pick up the passenger at his point of origin ($P2p1d1v2=1$) and drop him off at his destination after a detour ($DS2p1d1v4=1$). And finally if driver j' accepts to detour for more .5 miles, he can connect the passenger to driver j ($C2p1d2v1d1v3=1$) who will drop the passenger at his destination after making a detour ($DS2p1d1v4=1$). Figure 15 shows the compromise solutions.

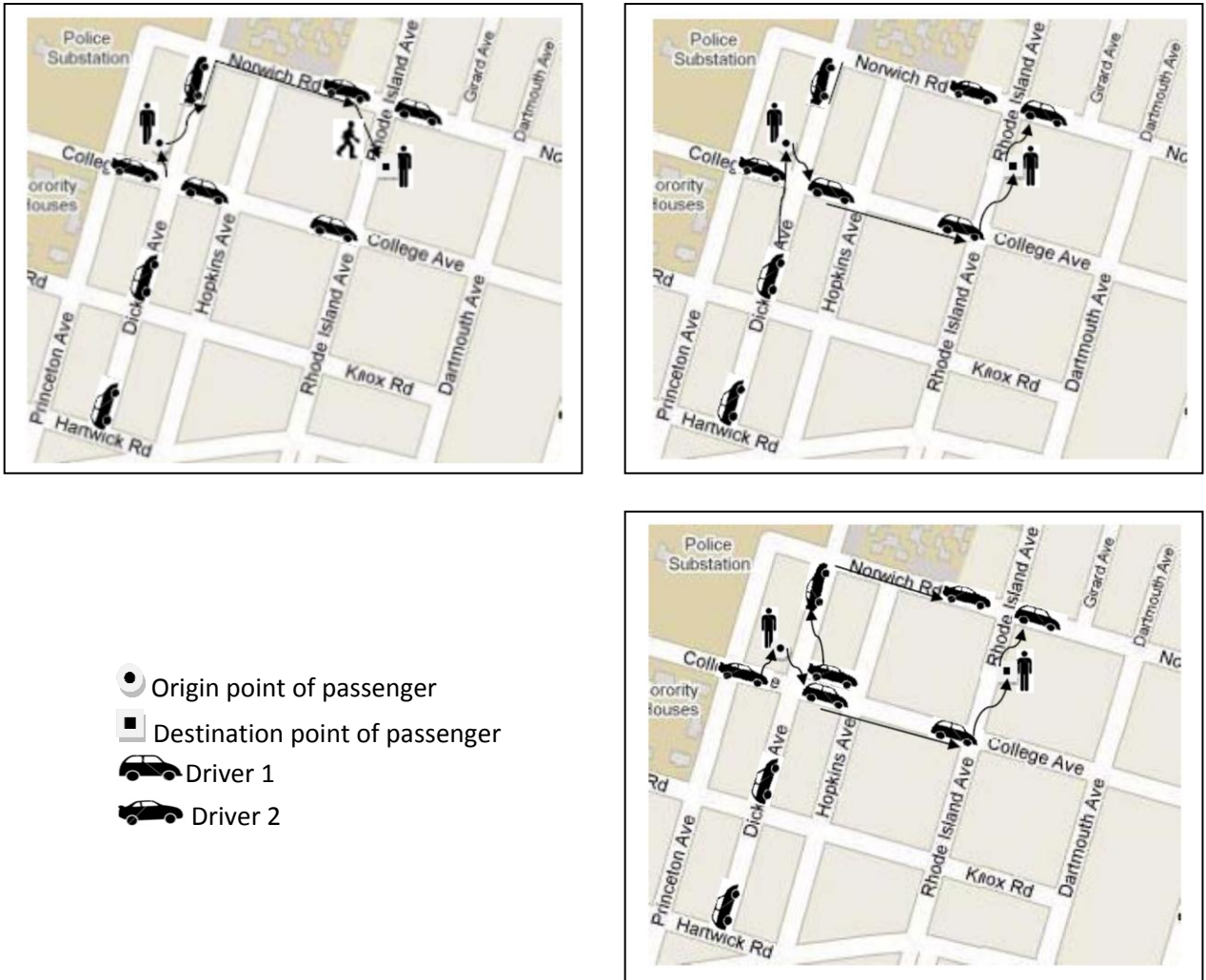


Figure 15: Compromise solutions.

Upper left: The passenger accepts to walk for more .91 miles. Upper right: Driver j accepts to detour for more .36 miles. Lower right: Driver j' accepts to detour for more .5 miles.

Conclusion

This research project presented a Dynamic Rideshare Matching Optimization model that is aimed at identifying suitable matches between passengers requesting rideshare services with appropriate drivers available to carpool for credits and HOV lane privileges. DRMO receives passengers and drivers information and preferences continuously over time and assigns passengers to drivers with respect to proximity in time and space and compatibility of characteristics and preferences among the passengers, drivers and passengers onboard. DRMO maximizes total number of assignments in a given planning horizon and secures that all the constraint for vehicle occupancy, waiting time to pickup, number of connections, detour distance

for vehicles and relocation distance for passenger are satisfied. The ridesharing preferences and characteristic considered in the model are: age, gender, smoke, and pet restrictions as well as the maximum number of people sharing a ride. To better understand the model, a numerical example with compromise solutions were presented and discussed. The authors currently are working on developing solution algorithms for solving the optimization model proposed in this paper for large scale real-world problems.

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