Travel Time Reliability Modeling

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This report includes three papers as follows:

1. Guo F., Rakha H., and Park S. (2010), "A Multi-state Travel Time Reliability Model," Transportation Research Record: Journal of the Transportation Research Board, n 2188, pp. 46-54.
2. Park S., Rakha H., and Guo F. (2010), "Multistate Travel Time Reliability Model: Model Calibration Issues," Transportation Research Record: Journal of the Transportation Research Board, n 2188, pp. 74-84.
3. Park S., Rakha H., and Guo F. (2011), "Multi-state Travel Time Reliability Model: Impact of Incidents on Travel Time Reliability," 14th International IEEE Conference on Intelligent Transportation Systems, Washington D.C., October 5 - 7, 2011.

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A Multi-State Travel Time Reliability Model

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16. Abstract

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ABSTRACT

Travel time unreliability is an important characterization of transportation systems. The appropriate modeling and reporting of travel time reliability is crucial for individual travelers as well as management agencies. In this paper, a novel multi-state model is proposed for travel time modeling and reporting. The model advances travel time modeling in two aspects. First, the multi-state model provides much improved model fitting as compared to single-mode models. Second and more importantly, the proposed model provides a connection between travel time distributions and the underlying travel time state. This connection allows for the quantitative evaluation of probability of each travel time state as well as the uncertainty associated with each state; e.g., the probability of encountering congestion on a given time of day and the expected travel time if congestion were experienced. A simulation study was conducted to demonstrate the performance of the model. The proposed model was applied to field data collected at San Antonio, TX. The variation of the model parameters as a function of time-of-day was also investigated. The simulation study and field data analysis confirmed the superiority of multi-state model over the state-of-practice single-mode travel time reliability models and ease of interpretation.

Key words: travel time reliability, Mixture distribution, Multi-state model; travel time reliability reporting.

INTRODUCTION

The significant variability in travel time has become a concern for both travelers and traffic management agencies. Travel time reliability is related to the uncertainty associated with travel time. The Federal Highway Administration (FHWA) has formally defined travel time reliability as "the consistency or dependability in travel times as measured from day-to-day or across different times of day". With ever-increasing traffic demands and limited resources for expanding highway capacity, traffic congestion and delay has become an unavoidable attribute of the transportation system. There is a need to understand the characteristics of travel time variation, in order to facilitate individual trip decision making and traffic management of the overall transportation system.

Travel time reliability research focuses on quantitatively evaluating travel time uncertainty and its spatial and temporal variations. The majority of existing researches have attempted to use single-mode distributions to model travel time. Emam and Al-Deek *(1)* compared the log-normal, gamma, Weibull, and exponential distributions for the modeling of travel time data and concluded that a log-normal distribution provided the best fit. The standard single-mode distribution assumes travel time is from a single stochastic process. However, this is rarely true due to the complex traffic conditions. For instance, the mean travel time under free-flow conditions and congested conditions can differ substantially. A single-mode distribution cannot sufficient model the large variation associated with this complex situation.

The travel time reliability measures are quantitative measures of travel time variation based on appropriate travel time models. The 1998 California Transportation Plan used percent variation, i.e., the standard deviation divided by the average travel time, as a measure of travel time reliability *(2)*. Other measures include the median and percentile (*3, 4*). Lomax et al. *(5)* compared various measures and recommended three reliability performance measures: 1) percent variation, 2) the misery index (MI), and 3) the buffer time index. van Lint and van Zuylen *(6)* considered both percentile and the skewness of the distribution in evaluating travel time reliability. More recently, Tu *et al (7)* has proposed to model travel time according to the traffic conditions.

This paper proposes a multi-state travel time reliability modeling framework for travel time modeling and reporting. This model is based on the premise that travel time is dominated by the underlying traffic conditions, which is a complex stochastic process and may contain multiple travel time states. Two levels of uncertainty are quantitatively assessed in the proposed model. The first level of uncertainty is the probability of a given traffic condition; and the second level of uncertainty is the variation of travel time for each traffic condition. The proposed model also provides an opportunity for a novel easy-to understand travel time reliability reporting mechanism build upon existing reliability measure. Finally the model shows much improved model fitting to infield travel time data over the state-of-practice models.

The paper is organized as follows. The methodology of the multi-state model framework is introduced first followed by a simulation study demonstrating the performance of the model. Subsequently, the model is applied to field data collected along a section of the I-35 freeway in San Antonio, Texas. Finally, the study summary and discussion is presented.

METHODOLOGY

Model Formulation

Travel time can be affected by various factors and multiple states exist along roadway sections. For instance, traffic could be operating in a congested state, an uncongested state, and a state caused by non-recurrent events such as traffic incidents. It should be noted that the traffic state in this paper with respect to travel time. Different traffic conditions, traditionally defined by speed and traffic density status, can have similar travel speed, thus share same travel time state. Travel time under different state can have distinct characteristics. In a non-congested state, travel speed (and thus travel time) is mainly determined by speed limits and individual driver preference. In a congested state, travel time is dominated by traffic congestion levels and the corresponding variation will be higher than the free-flow state. Similarly, the travel time for traffic state caused by traffic incidents is expected to be longer with larger variation compared to the free-flow and congested states. Because of the stochastic nature of traffic flow, various travel time states are likely to happen randomly and multiple states can exist for a specific time period. The key question of interest is the probability of encounter a specific state and the characteristics of travel time under this state.

In this paper, a multi-state model is proposed to quantitatively assess the probability of a traffic state and the corresponding travel time distribution characteristics. Mathematically, the multi-state model is a mixture of a finite number of component distributions. The component distribution represents the distribution of travel time under the corresponding travel time state. A mixture coefficient is associated with each component distribution representing the probability of the specific state. A finite multi-state model with K component distributions has the following density function,

$$f(T|\boldsymbol{\lambda},\boldsymbol{\theta}) = \sum_{k=1}^{K} \lambda_k f_k(T|\boldsymbol{\theta}_k) \tag{1}$$

where T is the travel time; $f(T|\boldsymbol{\lambda},\boldsymbol{\theta})$ is the density function of the distribution for T; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots \lambda_K)$ is a vector of mixture coefficients and $\sum_{k=1}^{K} \lambda_k = 1$; $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_K)$ is a matrix of model parameters for each component distribution; $\boldsymbol{\theta}_k = (\theta_{k1}, \ldots, \theta_{kI})$ is a vector of model parameters for the kth component distribution; $f_k(T|\boldsymbol{\theta}_k)$ is the density function for the kth component distribution. Depending on the nature of the data, the component distributions $f_k(\cdot)$ can be normal, log-normal, or Weibull distributions.

The kth component $f_k(T|\boldsymbol{\theta}_k)$ in Equation 1 represents the distribution of travel time corresponding to a specific traffic condition, e,g., congested state and free flow state can have their own distinct component distribution. By separating the component distribution, the travel time state can be better represented than a single, uni-mode distribution. The parameter λ_k represents the probability of each state and has a significant implication in travel time reliability reporting.

A special case of the mixture distribution is a 2-component normal distribution; the density function is as follows,

$$f(T|\lambda, \mu_1, \mu_2, \sigma_1, \sigma_2) = \lambda \frac{1}{\sqrt{2\pi}\sigma_1} e^{\frac{(T-\mu_1)^2}{2\sigma_1^2}} + (1-\lambda) \frac{1}{\sqrt{2\pi}\sigma_2} e^{\frac{(T-\mu_2)^2}{2\sigma_2^2}} \tag{2}$$

where λ is the mixture coefficient for the first component distribution, which is a normal distribution with mean $\mu_1$and standard deviation σ_1; the probability for the second component distribution is $1-\lambda$ and the parameters for the second normal distribution are μ_2 and σ_2. By varying the component distribution and the mixture proportions, the mixture model is flexible

to model a wide range of patterns commonly observed for travel time distributions. In theory, the mixture distribution can approximate any density function *(8)*.

Fitting the mixture model involves maximum the likelihood function

$$L = \prod_{i=1}^{N} f(T_i|\boldsymbol{\lambda}, \boldsymbol{\theta}) = \prod_{i=1}^{N} \sum_{k=1}^{K} \lambda_k f_k(T_i|\boldsymbol{\theta}_k), \tag{3}$$

where T_i is the ith observed trip length from a total N trips. Ordinary maximum likelihood estimation (MLE) may not be efficient because of the multiple modes. Instead, the expectation and maximization (EM) algorithm is widely used for estimating parameter estimation and was adopted in this paper *(9)*. The EM algorithm is an iterative procedure involves the following major steps. 1) estimate the probability for each state for every observation based on the current values of $\boldsymbol{\theta}$ and λ_k (the E-step); 2) Find the conditional maximum likelihood estimation of $\boldsymbol{\theta}$ and λ_k for next iteration (the M-step). The E-step and M-step are repeated until convergence has been reached.

Model Interpretation and Travel Time Reliability Reporting

The multi-state model connects the parameters of mixture distribution with the underlying traffic conditions. In particular, the mixture parameter λ_k in Equation 2 represents the probability that a particular travel time follows the kth travel time state and the component distribution, $f_k(T|\boldsymbol{\theta}_k)$, indicate the distribution of travel time under this state. These connections provide an opportunity for a novel two-step travel time reliability reporting mechanism.

The first step is to report the probability of encounter a specific travel time state, e.g., the probability of congestion during morning peak. The second step is to report for a given state, what is the expected travel time under this state. Some well-accepted reliability indexes can be used in the second step, such as the percentile and the misery index, which can be readily calculated from each component distribution. This is method analog to the widely used two-step weather forecasting approach; e.g., 'the probability of rain tomorrow is 80% and, if it does rain, the expected precipitation will be 2 inches/h". The same method can be used in travel reliability forecasting, i.e., 'the probability of encountering a congested state in the morning peak for the study roadway segment is 67% and, if that happens, the expected travel time will be 30 minutes'.

This two-step reporting scheme provides rich information for both travelers and traffic management agencies. By knowing the probability of a congested or incident state and the expected travel time in each state, an individual traveler can make better travel decision based on trip purpose. For traffic management agencies, the proportion of trips in a congested state and the travel time difference between the congested state and the free-flow state can provide critical information on the efficiency of the overall transportation system. This can also provide an opportunity to quantitatively evaluate the effects of congestion alleviation methods.

In practice, the interpretation of the mixture parameter λ_k depending how the travel time samples were collected. Two commonly of sampling schemes, the proportional sampling and the fixed-size sampling scheme, can be used.

In a proportional sampling scheme, the number of trips is proportional to the number of trips for any given time interval. For example, in a 10 percent proportional sampling approach, 10 trips will be selected from every 100 trips during the study period. For the proportional sampling, the probability λ_k can be interpreted from both the macro level and the micro level.

From the macro level, this corresponds to the percentage of vehicles in traffic state, e.g., the percentage of drivers that experience congested traffic conditions. This macro level interpretation can be used to quantitatively assess the system performance by traffic management agencies. The λ_k can also be interpreted from a micro level. Since the relative frequency (percentage) can also be interpreted as the probability for individuals, the probability λ_k also represents the probability that a particular traveler will travel in state k in the given time period. This is most useful for personal trip planning.

In a fixed-size sampling scheme, a fixed number of trips are sampled for a given time period regardless of the total number of trips for this time period. For example, 30 *trips will be sampled every 10 minutes* is a fixed-size sampling. The λ_k under a fixed sample scheme represents the proportion of the total duration where traffic condition is in the kth condition. For example, a λ value of 80% for the congested component implies that out of one hour (60 minutes), the traffic was in a congested state for a total of $0.8 \times 60 \text{ min.} = 48 \text{ min.}$ The fixed sampling scheme also provides useful information for individual travelers; i.e., the proportion of time the traffic will be in a congested condition.

SIMULATION STUDY

To evaluate the model performance and validate the interpretation for the mixture coefficient, a simulation study was conducted. The simulation focused on the model performance under two alternative sampling schemes. The simulation was conducted using the INTEGRATION software *(10)*. The simulation network is a single 16-mile expressway corridor (I-66) in the Northern Virginia Area. The highway has seven on-ramps and seven off-ramps. A high off-ramp traffic demand was observed at an intermediate exit resulting in congestion which form a bottleneck in the network during peak hours. The analyses include trips that traveled the entire 16-mile segment. This network is identical to that used in a simulation validation paper and more details can be found within *(11)*.

The simulation scenarios were generated by varying the traffic demand. An origin-destination (O-D) matrix was developed using field loop detector traffic volume measurements. A congested state and an uncongested state were created by scaling the original O-D matrix (with a peak demand of 16,000 trips/hour) by a constant scaling factor. The total demand for the congested scenario was about two times higher than the demand for the uncongested state. The approximate travel times for the uncongested state and the congested state were 14 minutes and 35 minutes, respectively. To mimic the natural fluctuation of traffic demand, the demands for each scenario were allowed to vary uniformly within 10% of the original O-D matrix. Altogether 500 hours of non-congested and 500 hours of congested states were generated. It should be noted that the hour was a symbolic time unit and the results from the simulation will hold for any time unit. To avoid any potential confusion, the term *time unit* will be used to refer to one of the 1,000 simulation outputs.

The mixture scenarios were generated by sampling from the 1,000 time units of simulation output. The model performance under both proportional sampling and fixed-size sampling were evaluated. The 2-component normal mixture model (Equation 2) was used to fit to the simulated data.

Proportional Sampling

In a proportional sampling scheme, the number of trips sampled in a given time period is proportional to the total number of trips in that time period. In general, the congested state contains more trips than the free-flow state, thus more trips are sampled from the congested state for the same time duration. Analysis based on all trips is a special case of proportional sampling with a sampling rate of 100%. The simulation study was designed to study the effect of the percentage of time units in congested state and the sampling rate on model performance.

A total of 12 scenarios were simulated based on *percentage of times in the congested state* and the *sampling rate*. For each scenario 100 simulations were conducted and, in each simulation, 100 time units were sampled according to the *percentage congested* and *sampling rate*. The percent of trips in the congested state, mean, and standard deviation of the TRUE values of simulated data were calculated. The same set of parameters was also estimated from the 2-component normal mixture model (Equation 2). A comparison of the TRUE parameters to those estimated from the mixture model estimated is shown in TABLE 1.

The performance of the model was evaluated using the mean absolute relative different (MARD). The MARD for λ is defined as $\frac{\sum_{i=1}^{n}|\lambda_i-\hat{\lambda}_i|/n}{\bar{\lambda}}$, where λ_i and $\hat{\lambda}_i$ are the true and estimated mixture proportion parameters respectively; n is the number of simulations ($n = 100$ in this study) and $\bar{\lambda}$ is the average true parameter, i.e., $\bar{\lambda} = \sum_{i=1}^{n} \lambda_i/n$. A smaller MARD indicates that the estimated value is closer to the true value.

As demonstrated in TABLE 1, in scenarios with a small proportion of the congested state, the estimations for all five model parameters ($\lambda, \mu_1, \mu_2, \sigma_1, \sigma_2$) were very close to the true values. For three high-congestion scenarios, (75% of time units in congested state), the model underestimates the true proportion and overestimates the variance of the travel time in the free-flow state. The reason for the bias is that a single normal distribution cannot sufficiently model the travel time in the congested state when the percentage is high. This problem can be resolved by introducing a third component or by using alternative component distributions (e.g. log-normal, gamma, etc.).

The results also confirm that the estimates are robust to the sampling rate. A practical implication is that a relatively small sample will also provide satisfactory estimation and the cost of data collection can be significantly reduced.

Fixed-size Sampling

In a fixed-size sampling scheme, a fixed number of trips would be sampled for any given time period. In this sampling method, the number of trips in a congested state is proportional to the duration of the congested state in the study period. A total of four simulation scenarios were generated; each scenario consisted of 100 simulations. The sample size for each time unit was set to 50. Consequently, the true percentage of trips in the congested state is identical to the proportion of time units in the congested state. The simulation results are shown in TABLE 2. Similar to the proportional sampling scheme, the multi-state model also provides a very accurate estimation for the model parameters. The estimation of λ for a small proportion of the congested state is less than 1 percent. The fixed-size sampling showed more stable estimates for the high congested proportion than did the proportional sampling.

The simulation study indicated that, in general, the multi-state model does reflect the characteristics of travel time under different traffic conditions. The parameters of the

component distribution can be estimated satisfactorily and the interpretation of the mixture parameters will depend on the sampling scheme.

MODEL APPLICATION

The multi-state model was applied to a data set collected from Interstate I-35 near San Antonio, Texas as shown in FIGURE 1. The traffic volume near downtown San Antonio varied between 89,000 and 157,000 vehicles per day (veh/day). The most heavily traveled sections were near the interchange with I-37, with average daily traffic counts between 141,000 and 169,000 vehicles, and between the southern and northern junctions with Loop 410 freeway, with average daily traffic counts between 144,000 and 157,000 veh/day. The analysis covered a 16-km freeway section delimited by the automatic vehicle identification (AVI) stations at New Braunfels Ave. (Station no. 42) and O'Connor Rd. (Station no. 49). Vehicles were tagged with ratio frequency (RF) and the travel times for each tagged vehicle were recorded whenever they passed any pair of AVI stations. The RF tags were randomly assigned to participant vehicles. The trip frequencies for the tagged vehicles can be assumed to be proportional to the total traffic volume; therefore, the interpretation of the mixture component follows that of the *proportional sampling scheme* as introduced above. Within the study corridor, traffic data were gathered at six locations (stations 42 through 49) from June 11, 1998 to December 6, 1998. To investigate the travel time on a typical working day, data from 21 Mondays were selected for modeling.

FIGURE 2 (a) illustrates the variation of travel times and the corresponding time of day the trips occurred (mid-point of a trip). As can be seen, the majority of the trips fall within a relatively narrow interval (more than 90% of the data fall within one standard deviation of the mean). A considerable number of trips have substantially longer travel times. Interestingly, most of these long trips occur in the morning (6AM-10AM) and afternoon (3PM-7PM) peak hours. The congestion caused by traffic demand exceeding capacity could contribute to long travel times and this is consistent with the distribution of number of trips over the time of day, as shown in the histogram in FIGURE 2 (b).

A natural explanation for the patterns shown In FIGURE 2 is that multiple traffic conditions exist, e.g., a free-flow state and a congested stage etc. It is interesting to observe that even during morning peak hours, a large proportion of trip time is still similar to that of the free-flow state. This implies that when the traffic demand is high, the traffic flow can still travel near the speed limit and the trip duration will be similar to the free-flow travel time. However, the stability of traffic flow during high demand is vulnerable and is more likely to transfer to a forced-traffic state. Thus, multiple travel state can co-exist in a given time period.

This multi-traffic-state concept is consistent with the assumption of a mixture distribution. Any trip during the morning peak hours will fall in one of the states with certain probability. The travel time for an individual trip will thus be determined by the probability of falling in each state and the distribution of travel time for the corresponding state. A trip occurring during the morning peak will have a higher probability to fall in a forced-traffic state than a trip during non-peak hours. When a trip falls in a forced-traffic state, its duration is a realization from the distribution for this state, which tends to have a larger mean than the free-flow state.

The multi-state model was applied to the morning peak travel time from Station 49 to Station 42, which includes 521 trips. Three alternative models, a 2-component model (Equation 2), a 3-component model, and a state-of-practice log-normal model were fitted. The

model fitting results for the multi-state models are presented in TABLE 3 and illustrated in FIGURE 3.

To compare the performance of the model, the Akaike information criterion (AIC) was used. The AIC is defined as $-2\log(\mathrm{L}) + 2\mathrm{K}$. where L is the likelihood function and K is the number of parameters. A smaller AIC value indicates better model. As can be seen, the 3-component model provides substantially better model fitting than the 2-component model (AIC 7124 versus 7020). The log-normal model (mean=6.75 sd=0.379) has AIC value of 7236, which is substantially larger than both mixture models. The improved model fitting can also be observed intuitively from FIGURE 3, in which the density functions of the mixture models fit the data histogram more closely. In summary, the multi-state model provides a significant improvement over single model based models.

The third component of the 3-component model is corresponding to a traffic-state caused by non-recurrent congestion with low probability. Interestingly, the parameters of the first component have exactly the same values as the 2-component model, which is expected since they both correspond to the free-flow state.

The 2-component model is composed of two component normal distributions. The first distribution has a smaller mean which corresponds to the free-flow state while second component corresponds to the forced traffic flow state. Following the aforementioned two-step reporting scheme, the probability that a driver encounters a congested traffic state is twice that of encountering a free-flow state (67% versus 33%). The average travel time for a congested state is almost twice as long as that for the free-flow state (1089 versus 588 seconds). The standard deviation for forced-state is substantially larger: 393 versus 38 for the free-flow state.

The standard statistical package R used in this study does not provide the estimation errors. To provide a quantitative evaluation of the estimation errors, a bootstrap method was used *(12)*. The bootstrap consists of the following steps: 1) resample the same number of observations with replacement from the original data (bootstrap samples); 2) estimate the parameters by fitting the bootstrap samples; 3) repeat step 1 and 2 for a large number of times; 4) use the generated parameter estimation to calculate the standard deviation of the model parameters. The ratio between the standard deviation and the point estimation is in generally smaller than 1, which indicates reasonable estimation for this sample size in this analysis.

To evaluate the travel time reliability for a specific state, the 90th percentile travel time was adopted although all the reliability indexes as summarized by Lomax et al. *(5)* can be readily calculated from the component distributions. The 90th percentile travel time for the congested state is calculated as:

90th percentile of $\mathrm{Normal}(\mu = 1089, \sigma = 393) = 1593\ seconds$.

The 90th percentile for free-flow-sate can be calculated similarly.

The report of the travel time follows a two stage procedure. First, a driver has a 67% probability of encountering a congested state, if the congestion does happens, 90% of the time the travel time will be within 1,593 seconds. Alternatively, there is a 33% probability of encounter a non-congested state and if this happens, 90% of the chance the trip length will be within 637 seconds. The interpretation for the 3-component model is similar. This reporting method is similar to the weather forecasting and is easy to be accepted by general public. The method also provides a mechanism to direct the trip decision making. For instance, if a driver has an important trip need to arrive on time, based on the probability of encountering congestion and expected travel time he/she can choose proper starting time to ensure arriving destination on time with high probability.

Variation of Mixture Parameters over Time of Day

The parameters of the multi-state model reflect the characteristics of the corresponding state and the probability to encounter a specific state. It is expected that the probability of encountering a congested state will be higher in peak hours compared to non-peak hours. Thus, the mixture proportion should vary according to the time of day. Furthermore, traffic congestion level will change over time of day thus the distribution of travel time under congested state should not be treated as constant. Therefore, it is important to evaluate how the model parameters vary by time of day.

The full day data from the 21 Monday was used to exam the time of day variation. The travel times were grouped to 24 groups by starting hour. A 2-component mixture model with normal component distribution was fit to each stratum. The parameter estimates for the 24 strata are shown in TABLE 4 and FIGURE 4. The blue lines represent estimated parameters of the first component and the red dotted lines represent those of the second component. FIGURE 4 (a) illustrates the temporal variation in the distribution means, i.e., μ_1 and μ_2, while FIGURE 4 (b) illustrates the temporal variation in the standard deviation, i.e., σ_1 and σ_2 over time of day. FIGURE 4 (c) illustrates the temporal variation in the mixture coefficient λs; and FIGURE 4 (d) shows the 90th travel time percentile for component distributions over time the entire day.

As can be seen, the means and standard deviations of the travel times in the free-flow state, μ_1s and σ_1s, show little fluctuation across the day. The travel time distribution for the forced state does demonstrate some interesting patterns. For non-peak hours, i.e., before 6:00 AM and after 8:00 PM, the two component distributions are very similar, i.e., $\mu_1 \approx \mu_2$ and $\sigma_1 \approx \sigma_2$. In this case, the mixture coefficients are not relevant because the two traffic states are essentially equal.

In the case of the forced traffic state (the second component distribution), the mean and standard deviation show a clear increase starting from 7:00 AM. The upward trend continues until it peaks at 10:00 AM and then starts to drop. The values of μ_2 fluctuate during the daytime and show another surge from 2:00PM to 7:00PM. The pattern of μ_2 is consistent with peak and off-peak traffic patterns, which confirms the existence of a congested state.

The mixture coefficient λ indicates the percentage of trips falling in each state when the two component distributions differ. As can be seen for FIGURE 4 (c), in the morning and afternoon peak hours (7-9 AM, 5-6PM), the probability of the congested state is high (55% and 44% for 7-8AM and 8-9 AM, respectively; and 29% for 5-6PM). There is a relative small probability of encountering the congested state during the day time which is approximately 10%. This trend is consistent with traffic demand patterns in the study area.

FIGURE 4 (d) shows the 90th percentile travel time for each component distribution. As can be seen, the off-peak prediction for both states is very similar. The reliability measure for the free-flow state is very consistent throughout the day. The 90th percentile travel time for the congested states, however, varies significantly over the entire day. The 90th percentile travel time for the morning and afternoon peak hours are much higher than the free-flow state. The results presented in TABLE 4 and FIGURE 4 can provide important information for travelers for trip planning purpose.

SUMMARY AND DISCUSSIONS

Travel time reliability modeling and reporting is a critical component of travel time reliability research. It will provide much needed information for both traffic management agencies and individual travelers. The multi-state travel reliability model proposed in this paper showed several advantages. First, the model is flexible and can provide superior fitting to travel time data comparing to traditional single-mode models such as the log-normal distribution. Second and most importantly, it provides a direct connection between model parameters and the underlying travel time state. This connection will benefit both individual travelers and the management agencies. For individual travelers, a two-state travel-time forecasting method provides a mechanism analogous to the familiar weather forecasting system that should be easily accepted by the general public. For management agencies and researchers, the proportion of travelers in inferior traffic conditions and the corresponding delays as estimated from the multi-state model provides a quantitative way of evaluating traffic system performance.

The simulation study confirmed that the model coefficients do represent the characteristics of underlying traffic conditions and the probability of each state. The simulation study also validated the performance of the multi-state model under fixed-size sampling and proportional sampling.

The application of the mixture model to the San Antonio AVI travel time data indicated the existence of multi-state traffic conditions in the field. The analysis indicated that even during peak periods, there remains a substantial proportion of trips that travel at significantly shorter travel times. The analysis also implies that there might exist another state with much longer travel times and variability but with a small probability of occurrence, which could be the delay caused by traffic incidents. The variation of model parameters over time of day showed how traffic conditions changed temporally and their impacts on the travel time distribution. This analysis also demonstrated how travel time reliability reporting can be structured by time of day.

This paper developed a general framework for multi-state travel time modeling and reporting. There are several issues that still need to be investigated. The simulation indicates that the proportional sampling provides less model fitting for scenarios with a large proportion of congested traffic, which deserves further investigation. Another important question is how many samples are needed to produce a reliable estimation. This paper uses normal component distributions and the appropriateness of alternative distributions such as log-normal or Weibull distributions is worth investigation. The number of component distribution is predefined in this study. To formally determine the optimal number of component distribution is alosworth investigation. Finally, the parameters of the model has only been related to time of day, to connected with other factors such weather condition will be also be of interest.

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TABLE 1 Simulation Results for Proportional Sampling

Proportion of time units in congested stage	Sampling rate	Mean Percent of trips in congested stage (TRUE) $\bar{\lambda}$	Mean estimation for λ (ESTIMATED)	MARD for λ	MARD for μ_1*	MARD for μ_2	MARD for σ_1	MARD for σ_2
10%	100%	17.5%	17.2%	0.5%	0.0%	1.8%	0.4%	9.4%
	50%	17.5%	17.3%	0.4%	0.0%	1.4%	0.4%	7.5%
	10%	17.3%	17.2%	0.4%	0.0%	1.4%	0.5%	7.5%
25%	100%	38.9%	38.6%	0.8%	0.1%	1.2%	1.5%	6.8%
	50%	38.8%	38.4%	0.8%	0.1%	1.2%	1.3%	6.8%
	10%	38.5%	37.9%	0.9%	0.1%	1.5%	1.4%	8.4%
50%	100%	65.6%	64.1%	1.8%	3.1%	1.6%	31.0%	8.1%
	50%	65.5%	61.5%	4.7%	7.8%	2.1%	76.8%	10.4%
	10%	65.3%	62.8%	4.7%	8.4%	2.4%	86.0%	11.4%
75%	100%	85.1%	73.1%	12.1%	28.2%	1.4%	243.8%	12.0%
	50%	85.1%	72.4%	13.1%	32.8%	1.4%	274.4%	12.6%
	10%	84.9%	74.7%	12.0%	30.8%	2.1%	234.0%	13.9%

*μ_1 and σ_1 correspond to free-flow state; μ_2 and σ_2 correspond to congested state.

TABLE 2 Simulation Results for Fixed-size Sampling Method

Proportion of time units in congested stage	Sample size per time unit	Mean Percent of trips in congested stage (TRUE)	Mean estimation for λ (ESTIMATED)	MARD for λ	MARD for μ_1*	MARD for μ_2	MARD for σ_1	MARD for σ_2
10%	50	10%	10.1%	0.2%	0.0%	0.8%	0.3%	3.5%
25%	50	25%	24.8%	0.6%	0.1%	1.3%	0.7%	7.4%
50%	50	50%	49.4%	1.2%	0.1%	1.4%	1.9%	8.4%
75%	50	75%	72.0%	4.8%	9.0%	1.7%	88.6%	9.7%

*μ_1 and σ_1 correspond to free-flow state; μ_2 and σ_2 correspond to congested state.

TABLE 3 Mixture Normal Model Results for Morning Peak Travel Time

	2-component model						**3-component model**					
	Mixture Proportion λ	*SD**	*Mean* μ	*SD*	σ	*SD*	*Mixture Proportion* λ	*SD*	*Mean* μ	*SD*	σ	*SD*
Comp. 1	33%	0.16	588	71	38	68	0.33	0.04	588	17	38	11
Comp. 2	67%	0.16	1089	177	393	60	0.59	0.09	981	42	230	38
Comp. 3	NA		NA		NA		0.08	0.08	1958	172	223	65
Log likelihood	-3567						-3503					
AIC	7144						7020					
Travel time reliability Reporting	1. The probability of encountering a congested state is 67%; when this happens, 90% of the time the trip will be less than 1,592 seconds.* 2. The probability of encountering a free-flow state is 33%; when this happens, 90% of the time the travel time will be less than 637 seconds.						1. The probability of encountering a congested state is 59%; when this happens, 90% of the time the trip will be less than 1,276 seconds. 2. The probability of encountering a free-flow state is 33%; when this happens, 90% of the time the travel time will be less than 637 seconds. 3. The probability of encountering an incident state is 8%; when this happens, 90% of the time the travel time will be less than 2,244 seconds.					

*: SD is the bootstrap standard deviation for parameters.

TABLE 4 Mixture Model Parameters over Time of Day

	Mixture Coefficient		Mean		Standard Deviation		90th Percentile	
Time	λ_1	λ_2*	μ_1	μ_2	σ_1	σ_2	p1	p2
0:00-1: 00AM	94%	6%	600	651	31	128	640	815
1-2 AM	51%	49%	574	615	26	26	607	648
2-3 AM	48%	52%	581	612	27	23	616	641
3-4 AM	28%	72%	553	613	18	29	576	650
4-5 AM	48%	52%	577	598	46	31	636	638
5-6 AM	79%	21%	567	581	43	19	622	605
6-7 AM	94%	6%	566	695	36	82	612	800
7-8 AM	**44%**	**56%**	**595**	**951**	**39**	**280**	**645**	**1310**
8-9 AM	**56%**	**44%**	**577**	**1220**	**40**	**465**	**628**	**1816**
9-10 AM	**96%**	**4%**	**572**	**1350**	**45**	**291**	**630**	**1723**
10-11 AM	86%	14%	564	679	36	63	610	760
11 AM-12 PM	92%	8%	569	932	35	234	614	1232
12-1 PM	94%	6%	570	667	30	18	608	690
1-2 PM	70%	30%	563	601	30	40	601	652
2-3 PM	89%	11%	578	810	32	199	619	1065
3-4 PM	89%	11%	580	1108	43	454	635	1690
4-5 PM	88%	12%	589	838	45	157	647	1039
5-6 PM	**71%**	**29%**	**583**	**774**	**45**	**122**	**641**	**930**
6-7 PM	97%	3%	564	728	38	17	613	750
7-8 PM	49%	51%	541	593	21	25	568	625
8-9 PM	65%	35%	553	605	29	20	590	631
9-10 PM	52%	48%	560	607	34	25	604	639
10-11 PM	50%	50%	579	615	34	26	623	648
11 PM-Midnight	27%	73%	567	624	44	29	623	661

* The second component distribution corresponding to congested stage

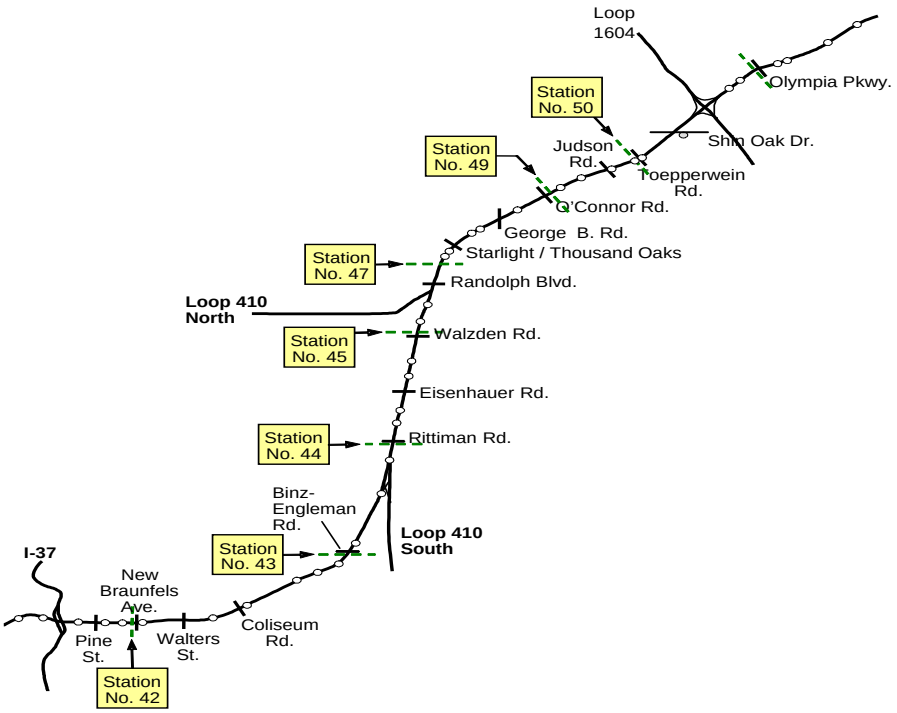
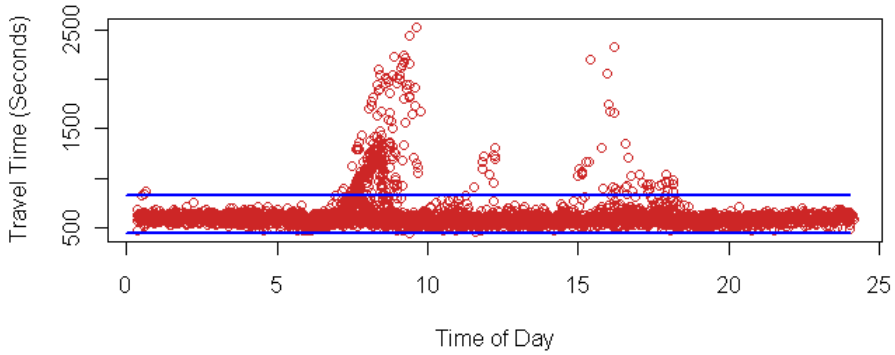
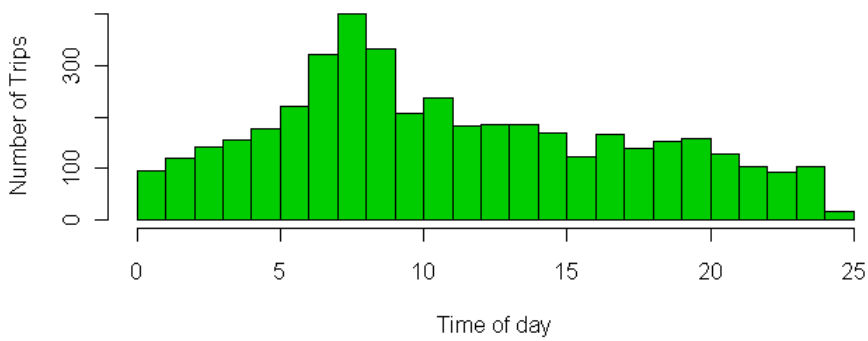


FIGURE 1 San Antonio study corridor.



(a)



(b)

FIGURE 2 The distribution of travel time over time of day.

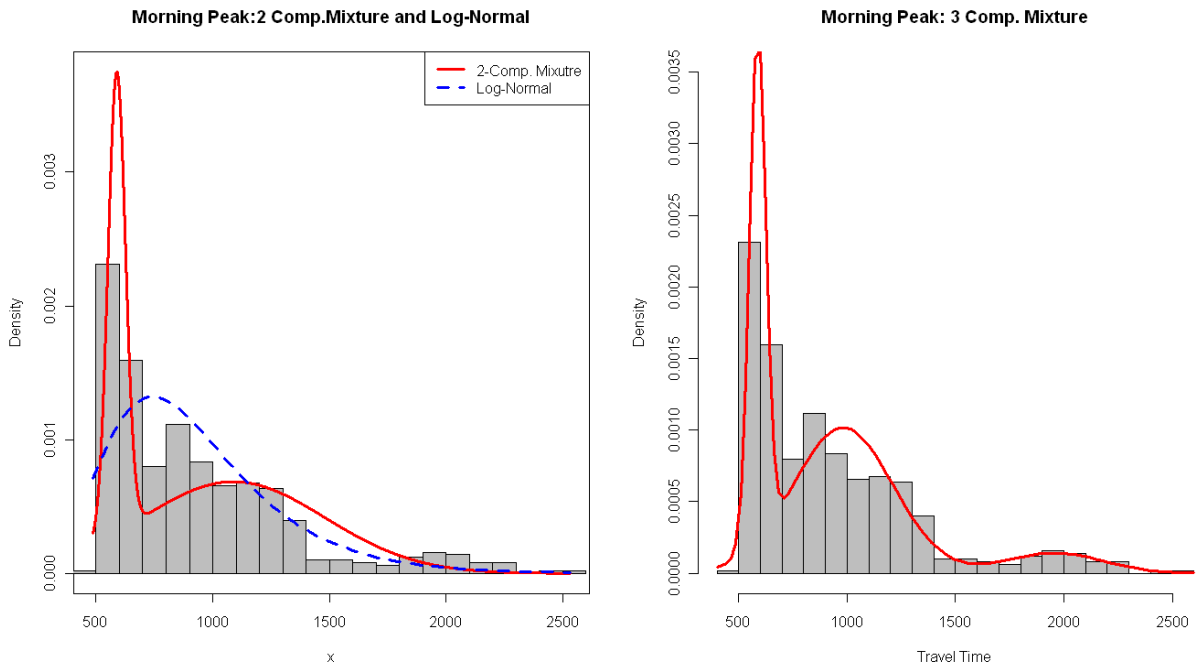


FIGURE 3 The 2- and 3-component mixture-normal model for morning peak travel time.

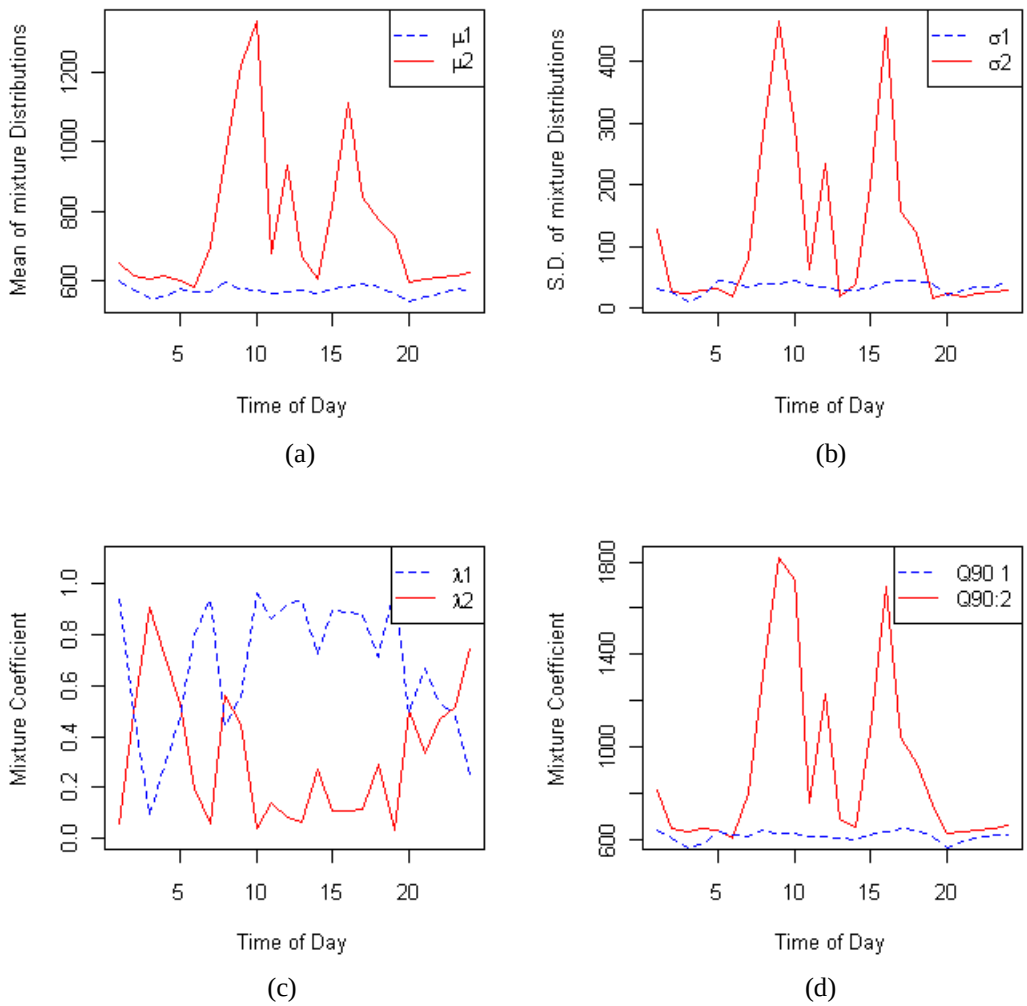


FIGURE 4 The variations of mixture parameters over time of day: (a) Variation of μ over time of day.(b) Variation of σ over time of day; (c) Variation of λ over time of day; (d)) Variation of 90th percentile over time of day.

Multi-state Travel Time Reliability Model: Model Calibration Issues

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ABSTRACT

Travel time reliability has received much attention from researchers and practitioners because it not only affects driver route choice behavior but is also used in the assessment of a transportation system performance. The current state-of-the-art travel time reliability research assumes that roadway travel times follow a uni-modal distribution. Specifically, either a Weibull, exponential, lognormal, or normal distribution is used within the context of travel time reliability modeling. However, field observations demonstrate that roadway travel times are multi-modal especially during peak periods. This paper demonstrates that the multi modes observed in field data are a result of temporal variations in travel times both between and within days. The study evaluates the potential for generating multi-modal travel times using the INTEGRATION software and investigates the underlying traffic states that result in these multi modes. The study validates the two-state model proposed in an earlier study and demonstrates the robustness of model parameter estimates under varying stochastic traffic congestion levels and sampling techniques.

Keywords: travel time, travel time estimation, travel time model, travel time variability, travel time reliability, and mixture model.

INTRODUCTION

Since the provision of accurate travel time estimates is critical in guiding travelers successfully, travel time estimation techniques have been studied extensively to build successful Advanced Traveler Information Systems (ATISs). From a data collection perspective, loop detectors, video detection, and probe vehicles are commonly used in most systems to estimate travel times using algorithmic techniques. The most widely used method is the instantaneous method due to its simplicity and ease of implementation. The travel time of a trip is calculated by summing the travel times of all the links that compose the route at the time the trip begins, under the assumption that traffic conditions remain constant until the trip is completed [1-2]. Consequently, this method does not have the ability to capture the temporal variabilities in travel times.

In the early stages of implementing ATISs, much of the research efforts focused on the estimation of mean travel times. However, given that the provision of accurate travel time information is difficult when heavy congestion occurs on roadways, the concept of travel time reliability has been recognized as important as mean travel time information to reduce travelers' uncertainty. Travel time reliability, which is the opposite of variability, has been accepted as a critical factor for travelers to secure their on-time travels. In addition, travel time reliability is considered as an operational performance measure for traffic operations managers. Consequently, much attention and effort have been devoted to developing reliability and variability measures. The currently used travel time reliability measures include the 90th or 95th percentile travel time, travel time index, buffer index, planning time index, misery index, standard deviation of travel time, and coefficient of variation of travel time considering a single uni-modal distribution of roadway travel times [3-4]. For example, the Washington State Department of Transportation (WSDOT) provides the 95% reliable travel times that are generated from historical data on their website.

Typically travel time reliability studies assume that travel times follow a unimodal normal or lognormal distribution. However, field travel time data were demonstrated to follow a multimodal distribution [5]. A two-state mixture model was proposed to capture the bimodal distribution of travel times. The objectives of this paper are two-fold. First, the paper demonstrates that microscopic traffic simulation can replicate field observed travel time trends and analyzes the causes of these multi modes. Second, the paper validates the two-state model parameter estimation procedures under varying stochastic traffic demand levels.

The study initially introduces the mixture model formulation. Second, the microscopic traffic simulation model is validated by comparing to field observations. Third, the study validates the two-state mixture model parameter estimates using travel time data generated by the simulation tool. Simple scenarios are initially tested followed by testing on more comprehensive and complicated scenarios. Finally, the conclusions and recommendations for future research are presented.

MIXTURE MODEL FORMULATION

A multi-state mixture model is defined as a distribution that convexly combines component distributions that are selected from a parametric family [6]. The following is the formulation of a mixture model

$$f(T|\lambda,\theta) = \sum_{k=1}^{K} \lambda_k f_k(T|\lambda,\theta), \qquad [1]$$

where $\lambda=(\lambda_1, \lambda_2,\ldots, \lambda_n)$ is a vector of mixture coefficients and $\theta_k=(\theta_{k1}, \theta_{k2},\ldots, \theta_{kI})$ is a vector of model parameters for the kth component distribution; $f_k(t|\theta_k)$ is the density function for the kth component distribution.

If the components are all normally distributed, then θ represents the unknown mean and variance of the various component normal distributions. In this study, a mixture of two component normal distributions (k=2) are only considered for ease of interpretation of the results when they are displayed to the general public.

The methodologies used to calibrate the mixture model parameters include the Expectation Maximization (EM) algorithm, Markov-chain Monte Carlo (MCMC) simulation, and spectral methods [6]. This study uses the EM algorithm to estimate the mixture model parameters. The EM is an iterative algorithm that involves two steps in computing the maximum likelihood parameter estimates: an expectation step and a maximization step. In the expectation step, the distribution for the unobserved variables is estimated. In the maximization step, the parameters with maximum likelihood are re-estimated [6-7]. Since the EM algorithm is implemented in most statistical and/or computational software including R, SAS, and MatLab, the calibration of model parameters is fairly straightforward if one is familiar with these software. As such the use of the EM algorithm will not be discussed further in the paper.

The multi-state model provides a convenient travel time reliability analog to the well-accepted weather forecasting example. The general population is familiar with the two-step weather forecasting approach; e.g., "the probability of rain tomorrow is 80% and, in the event that it does rain, the expected precipitation is 2 inches/h". The same method can be used in travel reliability forecasting, i.e., "the probability of encountering congestion in the morning commute is 67% and, in the event that one encounters congestion, the expected travel time is 30 minutes". Furthermore, the travel time under each state can be reported using well-accepted indices such as the percentile and the misery index, which can be readily calculated from each component distribution.

MICROSCOPIC TRAFFIC SIMULATION MODEL VALIDATION

This section demonstrates that microscopic traffic simulation can replicate bimodal travel time distributions that are observed in the field. First, the INTEGRATION and QueensOD software are introduced because they are used in the study. Second, the traffic network and traffic demand are described. Third, the travel times that are generated by the simulation tool are compared with field observed travel times.

INTEGRATION and QueensOD Software

A section of the eastbound lanes of Interstate 66 (I-66) was simulated using the INTEGRATION microscopic traffic simulation software. Travel times along the study section were used to generate travel time distributions for the validation of the mixture models. The INTEGRATION model is a microscopic traffic assignment and simulation software that was developed over the past decade [8-11]. It was conceived as an integrated simulation and traffic assignment model and performs traffic simulations by tracking the movement of individual vehicles every 1/10th of a second. This level of resolution allows for detailed analyses of lane-changing movements and shock wave propagations. It also permits considerable flexibility in representing spatial and temporal variations in traffic conditions. The INTEGRATION model updates vehicle speeds based on a user-specified steady-state speed-spacing relationship and the speed differential between the subject vehicle and the vehicle immediately ahead of it. In

order to ensure realistic vehicle accelerations, the model uses a vehicle dynamics model that estimates the maximum vehicle acceleration level. Specifically, the model utilizes a variable power vehicle dynamics model to estimate the vehicle's tractive force that implicitly accounts for gear-shifting on vehicle acceleration. This model is described in more detail in the literature [12-13].

The calibration of O-D demands to field observed link flows is a problem that has been the focus of extensive research. The most renowned of the approaches is the maximum likelihood approach that was first formulated by Van Zuylen and Willumsen [14] and Willumsen [15] and by Van Aerde *et al*. [16-17]. The traffic demand for a sample day were estimated using a maximum likelihood synthetic O-D estimation software entitled QUEENSOD [16-19]. The QUEENSOD software estimates the maximum likelihood O-D table that replicates the field observed link flows. The numerical solution begins by building a minimum path tree and performing an all-or-nothing traffic assignment of the seed matrix. A relative or absolute link flow error is computed depending on user input. Using the link-flow errors, O-D adjustment factors are computed and utilized to modify the seed O-D matrix. The adjustment of the O-D matrix continues until one of two criteria are met, namely the change in O-D error reaches a user-specified minimum or the number of iterations criterion is met.

Network Construction and Traffic Demand Calibration

The network extends from Manassas (Virginia) west to Vienna east (Exit 46 to Exit 62), and includes seven on-ramps and seven off-ramps along the 16-mile study section, feeding traffic to the Washington D.C. metropolitan area as can be seen in FIGURE 1 [20]. Since I-66 is the only major highway running to and from Washington, D.C. from the west end, traffic from the west to Washington, D.C. is heavy during the morning peak period (5:00 a.m. to 10:00 a.m.). Average daily traffic (ADT) volumes on the section from Route 29 (Lee Highway) to Route 234 (Sundly Road) were estimated at 87,000 vehicles per day in 2005 [21].

There were a total of 31 loop detectors – typically spaced every 0.5 miles – available within the study segment. An earlier study analyzed the loop detector data over a 3-month period, from May 1, 2002 to July 31, 2002 for a total of 44 days [20]. Origin-destination demands were constructed for each 15-minute interval over the 44 days. The analysis period included four Mondays, nine Tuesdays, eight Wednesdays, seven Thursdays, six Fridays, five Saturdays, and five Sundays. The study concluded that the temporal variation in the total traffic demand was similar for each weekday. Mondays and Fridays were different from typical weekdays while Saturdays showed a higher demand when compared to Sundays. For weekdays, the coefficient of variation (CV) remained relatively constant between 5:00 a.m. and 11:00 p.m. (10-20%), reaching higher values during the early morning hours (from 3:00 a.m. to 5:00 a.m., 60%). O-D demands were constructed for a total of 13 typical weekdays (only 13 of the 24 Tuesdays through Thursdays had complete data) to reflect the spectrum of typical weekday traffic conditions along the I-66 corridor. The total demand (sum of all O-D demands) for each day as a function of the time-of-day was computed. The temporal variation in the 13 total demands is similar with stochastic variations, as demonstrated in FIGURE 2 (a). The lowest demand is approximately 75% of the largest demand. Using the calibrated O-D demands a 3-hour window within the morning peak period (6:00 to 9:00 a.m.) was simulated. As can be seen in FIGURE 2 (b) through (d), the simulated morning peak conditions over the 13 days demonstrate a scatter in the data similar to what is typically observed in the field [22]. FIGURE 2 (d) demonstrates that the experienced travel times during the morning peak period over these 13 days can be grouped into two distinct states: congested and uncongested.

Consequently, the proposed two-state model can be used to represent the observed spectrum of traffic conditions, as illustrated in FIGURE 2 (e). In summary, the proposed two-state hypothesis is valid even though the traffic conditions vary over the entire spectrum of the fundamental diagram.

Simulation Validation

The 13 O-D tables were simulated using the INTEGRATION software. The travel time experiences of all trips originating from the west end of the network and traveling to the east end were extracted from the simulation output. The travel times during the morning peak hour from 6 a.m. to 8 a.m. were extracted to only consider congested conditions. The distribution of simulated travel times was compared to field observed travel times gathered along a section of the I-35 Highway in San Antonio, Texas in an attempt to validate the simulated travel time trends. The I-35 corridor (which is a 16-kilometer highway section from New Braunfels Rd. to O'Connor Rd.) connects downtown San Antonio and the northeast neighborhoods of the metropolitan area and has a total of six Automatic Vehicle Identification (AVI) stations [23]. AVI data collected from June 11, 1998 to December 6, 1998 were obtained from the TransGuide system and analyzed. Travel time data during the morning peak hours (7 a.m. to 9 a.m.) were extracted from a total of 21 Mondays of AVI data. It should be noted, that because travel time data were not available along the I-66 corridor the I-35 corridor field observed travel times were analyzed. Furthermore, because loop detector data were not available along the I-35 corridor it was not possible to construct a simulation model of the I-35 corridor.

FIGURE 3 illustrates the travel time histograms and fitted two-state mixture models using the simulated (13 days of I-66) and field (21 days along I-35) data over multiple days. As is evident from the figure, the shape of the simulated travel time histogram resembles that generated from the field data with both datasets producing multiple travel time modes. The results clearly demonstrate that a simulation platform can replicate field observed travel time distributions and that the multiple modes are a result of temporal variations in traffic demands within and between days. In this case the modes are a result of temporal variations between days.

VALIDATION OF MODEL PARAMETER CALIBRATION PROCEDURES

This section tests and validates the two-state model using the traffic simulation platform that was described earlier. As demonstrated in the previous section, the simulation tool was demonstrated to replicate field observed travel time behavior. Furthermore, the simulation tool provides for a controlled environment where data can be gathered on all vehicles within the system without the need for costly field data collection efforts.

Given that the typical variability in weekday traffic demands were in the range of 20%, the O-D demand for a typical day (Tuesday, May 7, 2002) was used together with a lower demand (20% lower). A visual inspection of the INTEGRATION simulation for the typical demand confirmed local bottlenecks along the study section. Specifically, localized congestion that resulted from a high off-ramp traffic demand, formed upstream of the off-ramp while uncongestion was observed downstream of the bottleneck. A reduced traffic demand of 80% the total demand was also simulated for a less congested weekday. A total of 96 O-D tables were constructed considering 15-minute interval time dependent O-D demands. In conducting further analysis only two traffic demands were considered (lowest and highest), as illustrated in FIGURE 4 (a). FIGURE 4 (b) illustrates the macroscopic fundamental diagram (MFD), which is an alternative to the classical flow-density fundamental diagram. The MFD, which was

proposed by Geroliminis and Daganzo, provides a macroscopic measure of performance of a transportation network similar to a fundamental diagram for a roadway segment [24]. The network vehicle accumulation is equivalent to the traffic stream density and the network throughput is equivalent to the segment flow rate. Similar to a standard fundamental diagram, as the vehicle accumulation increases the network throughput also increases until the system reaches its capacity. A further increase in the network accumulation results in the network break down. The maximum throughput represents the system capacity. As can be seen in the diagram, the maximum vehicle accumulation and flow rates on the sample day are higher than those for the 80% demand case. The system is operating at capacity during the peak hour (6 a.m. to 7 a.m.) for the sample day. When the demand is reduced the system is operating slightly below capacity.

Deterministic Model Testing

In demonstrating the two-state model application, a set of simple scenarios were constructed. The model interpretation and travel time reliability reporting issues are documented elsewhere [5] and thus are not discussed further in this paper.

Two sets of O-D tables were modeled using the simulation test bed. These O-D tables included the May 7, 2002 demand from 5 a.m. to 12 p.m. and a scaled demand (80% of the May 7 O-D table). As was discussed earlier, this variability in travel demand is within the typical day-to-day variability that was observed in the field. Given the simulation output results, travel time data were extracted for a single hour (7 a.m. to 8 a.m.) and mixed using three mixture levels, namely: 20:80, 50:50, and 80:20, respectively. In the case of the 20:80 mix, the travel time realizations for the 80% demand was repeated 2 times while that for the 100% demand was repeated 8 times. This mixture is equivalent to having 2 out of 10 days at the 80% demand level and 8 days at the 100% demand level. The travel times for the subject hour were then mixed and modeled using a mixture of two normal distributions.

FIGURE 5 illustrates the travel time distributions for each scenario along with the calibrated distributions. The solid lines depict the calibrated two-state models while the dashed lines illustrate the calibrated uni-modal models. The figure clearly demonstrates that the two-state model provides a better match to the simulated travel times. The various model parameters are summarized in TABLE 1 along with the true values mixture parameters that were simulated. Here λ (proportion of data associated with the first distribution: proportion of 80% demand data), μ_1 (mean of first distribution), and σ_1 (standard deviation of first distribution) are the parameters for the first component distribution, which has a lower mean travel time. The λ parameters play a more important role because the two component distributions (μ_1 and μ_2) are likely to be significantly different during the peak periods. In addition, the λ parameters play an important role because they can be interpreted as the probabilities that a driver's travel times falls into either state. As demonstrated in TABLE 1, the λ parameter estimates are consistent with the actual data mixture rates during the peak hours. For example, the λ parameter for the models is 0.5 when a mix of 50:50 is applied. Similarly, the λ parameters converge to the correct split of 80:20 and 20:80 thus demonstrating the robustness of the EM algorithm.

Stochastic Model Testing

In the previous section, the model application was demonstrated using simple deterministic scenarios as a first test of the calibration procedure. The next step is to test the model

calibration procedures for more complex scenarios. Specifically, in this section more travel demand levels are introduced together with stochastic variations in the demand levels.

Scenario Development

When the two sets of travel times are mixed for the 7 to 8 a.m. peak period, the sample is unbalanced because the traffic demand in the more congested days is typically higher than that in the less congested days. In order to quantify the impact of the number of travel time observations on the parameter estimates, the analysis is conducted twice once with a proportional sample and once with an equal sample size. These sampling schemes have different interpretations as was discussed elsewhere [5].

First, different O-D tables were prepared and supplied to the traffic models to generate travel time data under various congestion levels. Specifically, the O-D tables for May 7, 2002 from 6 a.m. to 9 a.m. were used as the base case. Two sets of random numbers were generated as scale factors that were multiplied by the base case. Each of the sets had a total of 500 realizations following a uniform distribution ranging from 1.0 to 1.1 (for the higher demand set) and from 0.5 and 0.6 (for the lower demand set). The O-D tables were constructed by multiplying the base O-D table by the randomly generated scale factors to construct a total of 1,000 O-D demand scenarios (500 high demand and 500 low demand). These 1,000 O-D demands were simulated using the INTEGRATION software to construct a database of 1,000 simulation runs, as illustrated in STEP1 of FIGURE 6.

Second, in order to vary the mix of high and low demand days, the simulation results were randomly selected from the database of simulation runs. Specifically, a total of 200 simulation runs were randomly drawn from the 1,000 runs. In order to determine the number high and low demand simulation runs to be included in a sample, a binomial distribution with n=200 and p= (0.1, 0.2, 0.5, 0.8, 0.9) was used as summarized in TABLE 2 and FIGURE 6 rather than using fixed values. The travel time data were then extracted from the selected simulation runs and two-state models were then calibrated to the data, as illustrated in STEP2 of FIGURE 6. As described before, two different sets of travel time data were tested; one that included all the travel times and another that included an equal number of observations from the two demand levels (high and low). Prior to calibrating the models, 2.5% of the data were removed from both tails of the travel time distribution to filter out outliers, consequently only 95% of the travel times were used for the model calibration.

Finally, the 200 run sample runs were simulated 1,000 times using a Monte Carlo (MC) simulation to generate a total of 1,000 mixed model parameters.

Proportional Sampling Case

As previously described, if we consider a fixed proportion sample – in this case a 100% sample size – the number of observations for the higher demand levels is larger than that for the lower demand levels and thus the mixture parameter λ is biased towards the longer travel times. Using five mixture distributions a total of 1000 MC simulations were executed and the two-state model parameters were calibrated for each simulation (λ, μ_1, μ_2, σ_1, σ_2).

The calibrated model parameters were then compared to the true parameter values. The true mixture parameter λ was computed as the ratio of the number of samples drawn from the uncongested demand levels which was derived from the binomial distribution to the total number of samples (in this case 200 samples). In addition, the true mean and standard deviation of the component distributions was computed for the high and low demand scenarios separately prior to mixing the data.

After conducting the analysis it was concluded that some parameter estimates were significantly under- or over-estimated. These erroneous model predictions occur when the number of observations in one distribution is very small compared to the other. For example, when the low demand cases constitute only 3% of the days (λ=0.03, μ_1=839, μ_2=2127, σ_1=70, and σ_2=449) the EM algorithm estimates the model parameters as λ=0.29, μ_1=1979, μ_2=2159, σ_1=622, and σ_2=384. Consequently, the EM algorithm introduces more observations into the lower probability distribution. The problem only occurred for 7 percent of the observations.

As demonstrated in TABLE 3 and FIGURE 7 the calibrated parameters are generally consistent with the true parameter values in terms of magnitude. The errors in the component travel time distribution means (μ_1 and μ_2) range from 0% to 5%, as demonstrated in TABLE 3. This is also evident in the μ_1 and μ_2 scatter plots. Since the slopes of the regression lines (1.017 and 0.982) are very close to 1.0, it can be concluded that the mean of the first distribution (μ_1) was overestimated by 1% on average and the mean of the second distribution (μ_2) was underestimated by 3% on average. The proportion of low demand days (λ) was underestimated as expected given that the number of high demand travel times was disproportionally high. The standard deviation estimates for both distributions (σ_1 and σ_2) were also underestimated by 19% and 20%, respectively. It is interesting to note that the error in the λ, μ_1, and σ_1 parameter estimates decreases while that for the μ_2 and σ_2 parameters increases as the proportion of low demand days increases.

In summary, the distribution mean estimates (μ_1 and μ_2) are slightly underestimated while the proportion of days in the first state (λ) and the standard deviation of each state (σ_1, and σ_2) produce a relatively small prediction error (R^2 ranges from 97% to 100%).

Fixed Size Sampling Case

The previous analysis considered the use of proportional sampling. In this section equal sample sizes are considered (i.e. the number of travel time realizations in each state are equal). A total of 20 outliers observations were indentified and excluded from the analysis. These outliers were caused when a very small proportion of observations are within one of the two states.

As demonstrated from the results of TABLE 3, the EM algorithm produces model parameter estimates that are consistent with the true parameter values. Specifically, the estimated proportion of days in low demand state (λ parameter) is consistent with the true values with prediction errors ranging from 2 to 10%. Furthermore, the estimated λ parameter offers a perfect match to the true values (slope 1.022, R^2 1.000). The prediction errors in the state mean estimates (μ_1 and μ_2) range from 0 to 8%. It should be noted, however, that the prediction error for the higher demand state parameters (μ_2 and σ_2) was higher than those produced for the proportional sampling case. The highest prediction errors were observed for the various distribution standard deviations (σ_1 and σ_2) with errors in the range of 14% and 23%, respectively.

In summary, fixed size sampling is more suitable for predicting the probability of encountering a specific state on a typical daily commute.

MODEL APPLICATION IN TRAVEL TIME RELIABILITY MODELING

The paper presented a novice framework for use in measuring the level of unreliability along a roadway segment. The framework is cast as a two-state model that computes the probability a driver will encounter congestion on a typical commute and the associated travel time distribution associated with congestion. The application of such a model would require gathering data over multiple days using vehicle probes and calibrating five parameters, namely:

the mean and standard deviation of each travel time distribution and the probability of encountering congestion (system breakdown). The study demonstrated that the use of equal sample sizes in each state is required in order to achieve a reasonable level of accuracy (estimate of probability of congestion with a margin of error of 10%). Further refinement of the model is required to introduce other factors into the model including weather and incident impacts on the probability of encountering congestion. This framework can be easily integrated in the travel time unreliability model developed by Tu et al. [25]. Specifically, the model estimates the travel time unreliability as the product of the probability of traffic breakdown (congestion) multiplied by some measure of travel time variability after breakdown plus the probability of free-flow conditions (uncongested) multiplied by the travel time variability in free-flow conditions. The parameter λ that is proposed in this paper represents that probability of breakdown. The proposed two-state model would provide an estimate of the parameters required to compute the measure of travel time unreliability.

CONCLUSIONS

A previous study demonstrated that travel times drawn from field data over multiple weekdays are best modeled using mixture distributions as opposed to single distributions. These observed differences in travel time experiences can either be attributed to the fact that traffic conditions from one day to another may differ marginally, however result in major differences in travel time experiences, as was demonstrated in this paper. Alternatively, during the shoulder periods (build-up or decay of congestion) some drivers may experience major congestion while others may not. The study demonstrates that microscopic traffic simulation is capable of producing multimodal distributions and thus can be used to validate statistical tools for the modeling of travel time reliability. Specifically, the study demonstrates how two-state models can evolve and furthermore validates the expectation maximization algorithm for estimating the two-state model parameters.

From a model application standpoint the use of a two-state model can be viewed analogous to a weather forecast. Typically, the weather forecast provides probability of precipitation and corresponding precipitation intensity in the event that precipitation occurs. Similarly, within the context of travel time information the driver will be given a probability of encountering congestion together with some information on the possible duration of the travel time if he/she encounters congestion. If differences in travel time are associated with differing conditions from day to day then the use of equal sample sizes is more appropriate. Alternatively, if differences in travel time are attributed to different experiences within an analysis period then the use of the proportional sampling is more appropriate.

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TABLE 5 Model Calibration Validation (Fixed Mixture)

	λ			μ_1			μ_2			σ_1			σ_2		
Ratio	TRUE	Estimate	Diff.	TRUE	Estimate	Diff.	TRUE	Estimate	Diff.	TRUE	Estimate	Diff.	TRUE	Estimate	Diff.
20:80	0.2	0.18	-8%	821	833	1%	1539	1529	-1%	56	48	-13%	136	128	-6%
50:50	0.5	0.50	0%	821	826	1%	1539	1525	-1%	56	52	-7%	136	125	-8%
80:20	0.8	0.82	2%	821	824	0%	1539	1508	-2%	56	53	-5%	136	114	-16%

TABLE 6 Summary Simulation Scenarios

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Non-congested runs	B(200,0.1)	B(200,0.2)	B(200,0.5)	B(200,0.8)	B(200,0.9)
Congested runs	200- B(200,0.1)	200- B(200,0.2)	200- B(200,0.5)	200- B(200,0.8)	200- B(200,0.9)
Number of repetitions	1000	1000	1000	1000	1000

TABLE 7 Model Estimate Comparison

Classification		Mean λ		Mean Difference				
		True	Estimated	λ	μ_1	μ_2	σ_1	σ_2
Proportional Sample Size	Scenario1	0.101	0.048	53%	4%	1%	43%	14%
	Scenario2	0.200	0.109	46%	2%	1%	29%	14%
	Scenario3	0.502	0.347	31%	1%	1%	11%	15%
	Scenario4	0.799	0.691	14%	1%	3%	6%	22%
	Scenario5	0.899	0.845	6%	0%	5%	4%	29%
Equal Sample Size	Scenario1	0.100	0.094	10%	3%	1%	32%	14%
	Scenario2	0.200	0.196	4%	2%	1%	19%	14%
	Scenario3	0.500	0.508	2%	1%	1%	8%	18%
	Scenario4	0.798	0.817	2%	0%	4%	5%	28%
	Scenario5	0.900	0.923	3%	0%	8%	4%	39%

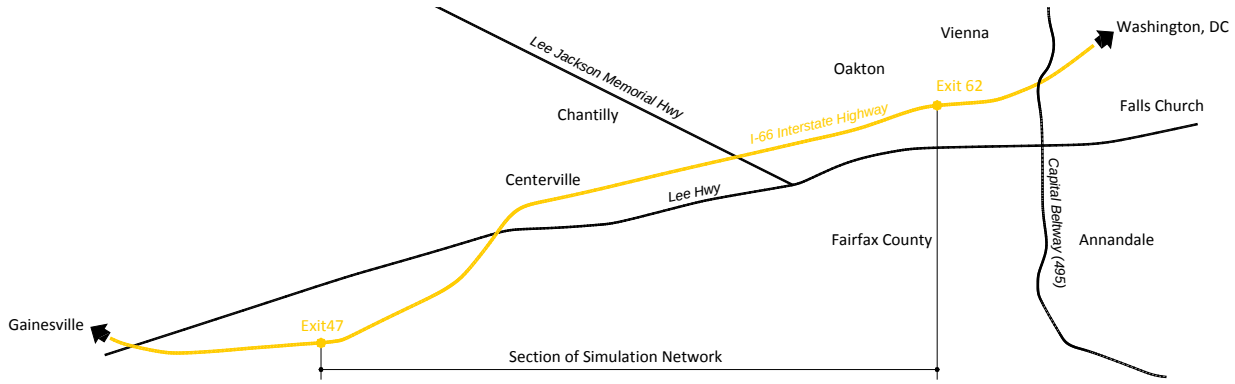

FIGURE 5 Sample I-66 network configuration.

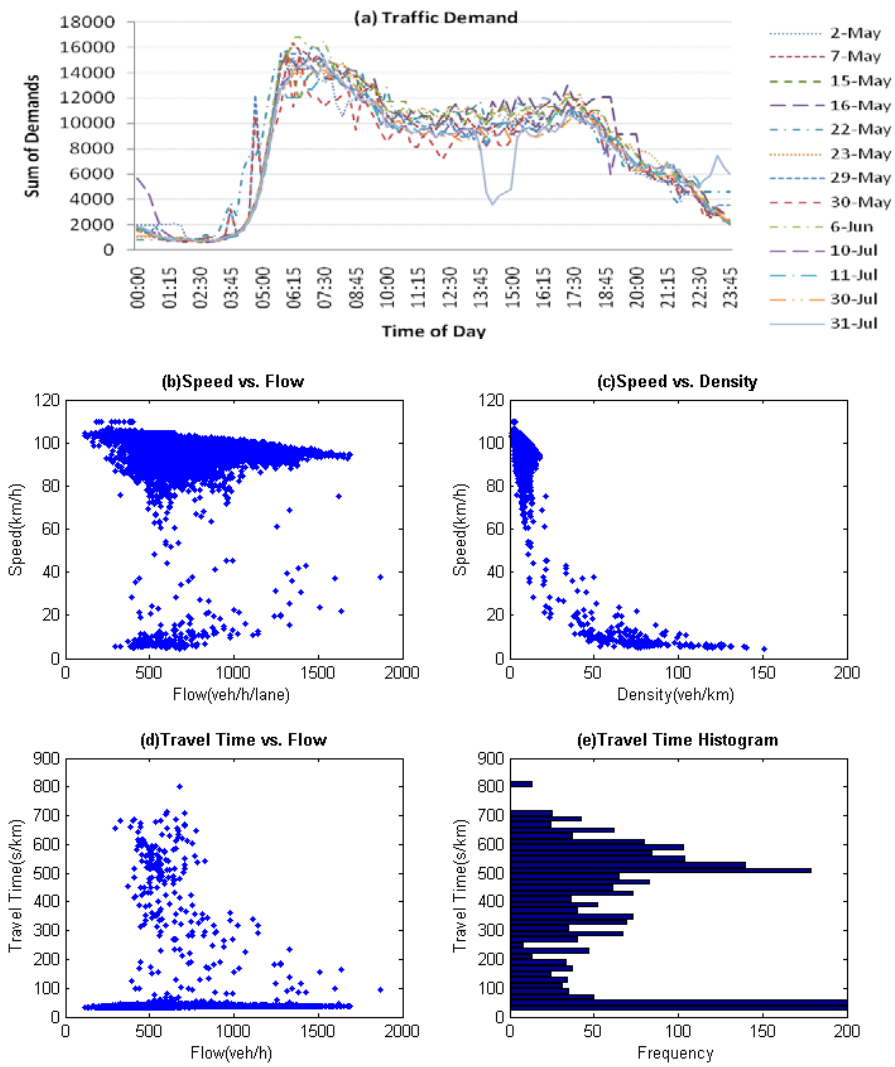

FIGURE 6 Traffic demand, fundamental diagram, and travel time distribution.

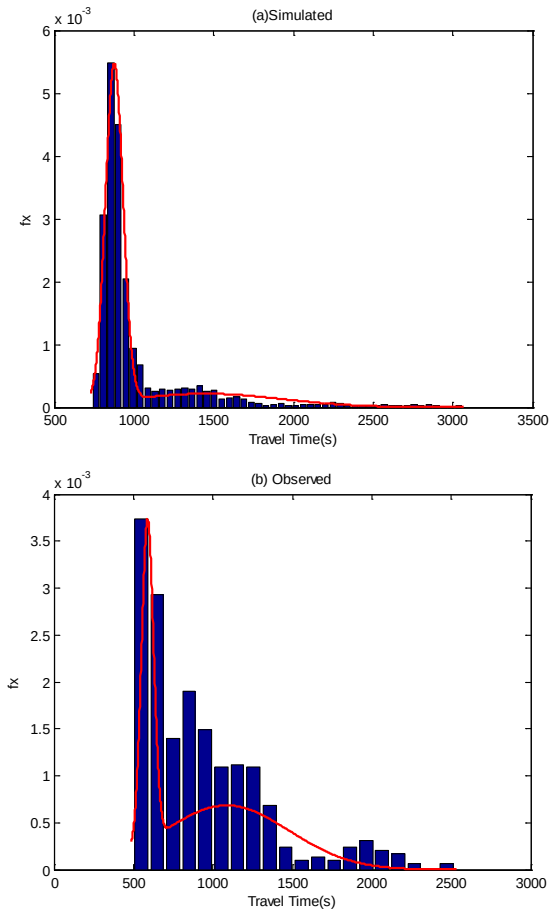

FIGURE 7 Travel time distributions.

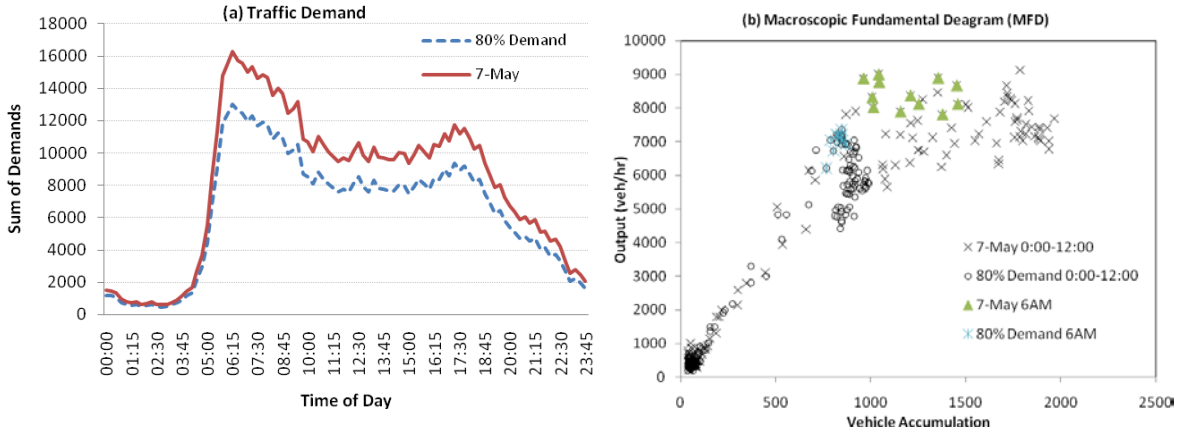

FIGURE 8 Traffic demands and MFD.

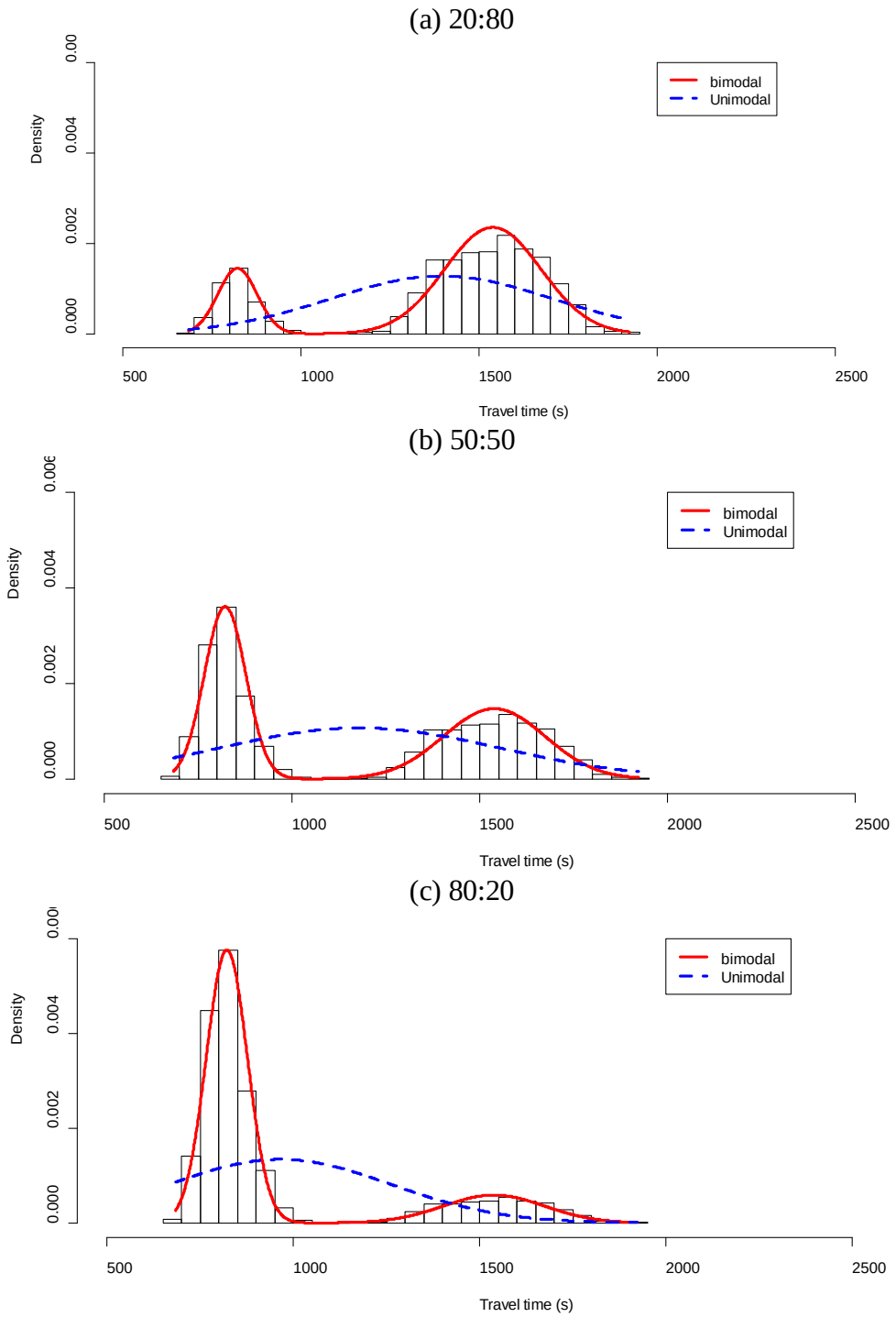

FIGURE 9 Simulated Travel Time Distributions and Model Fits

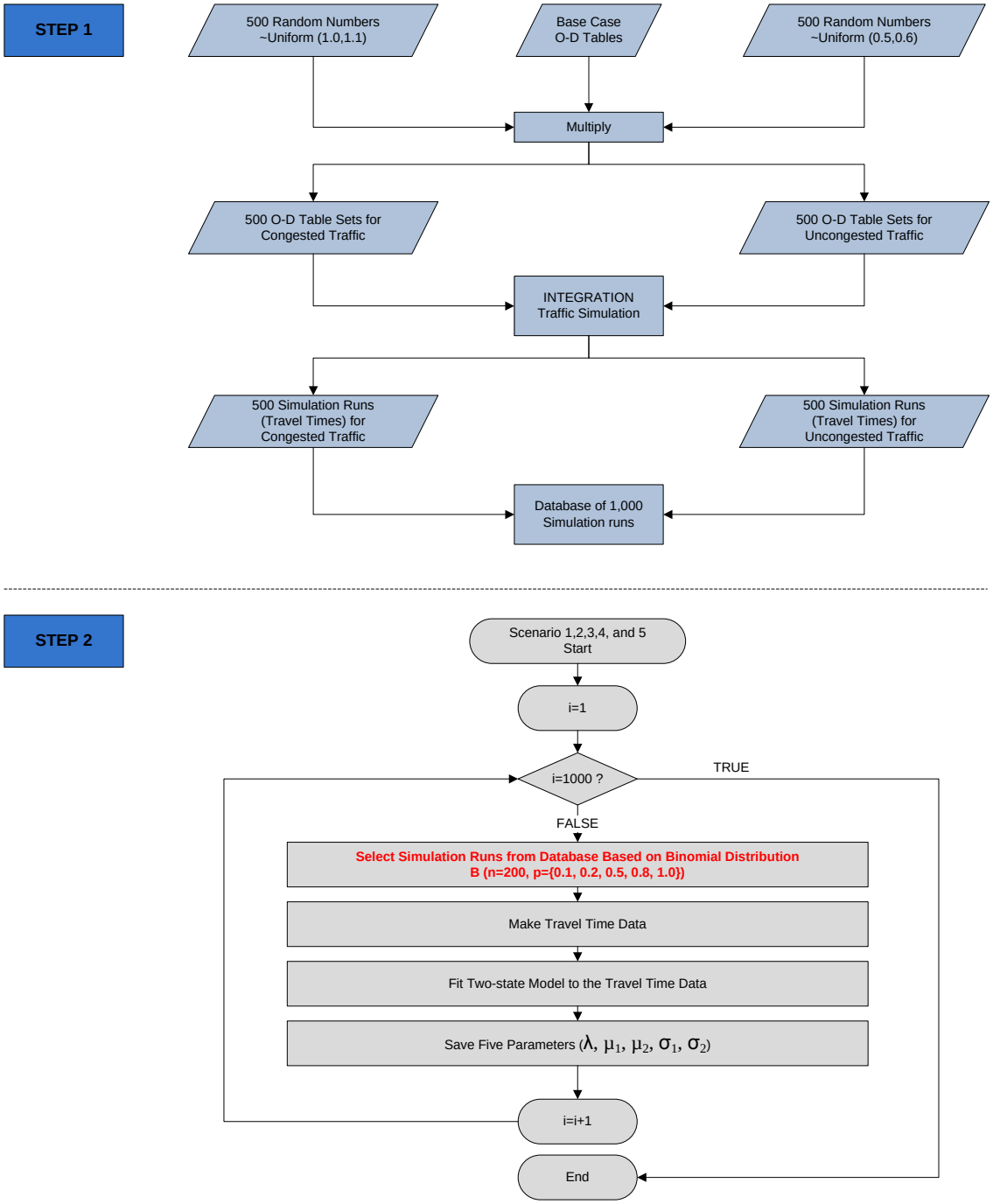

FIGURE 10 Stochastic Scenario Development Framework

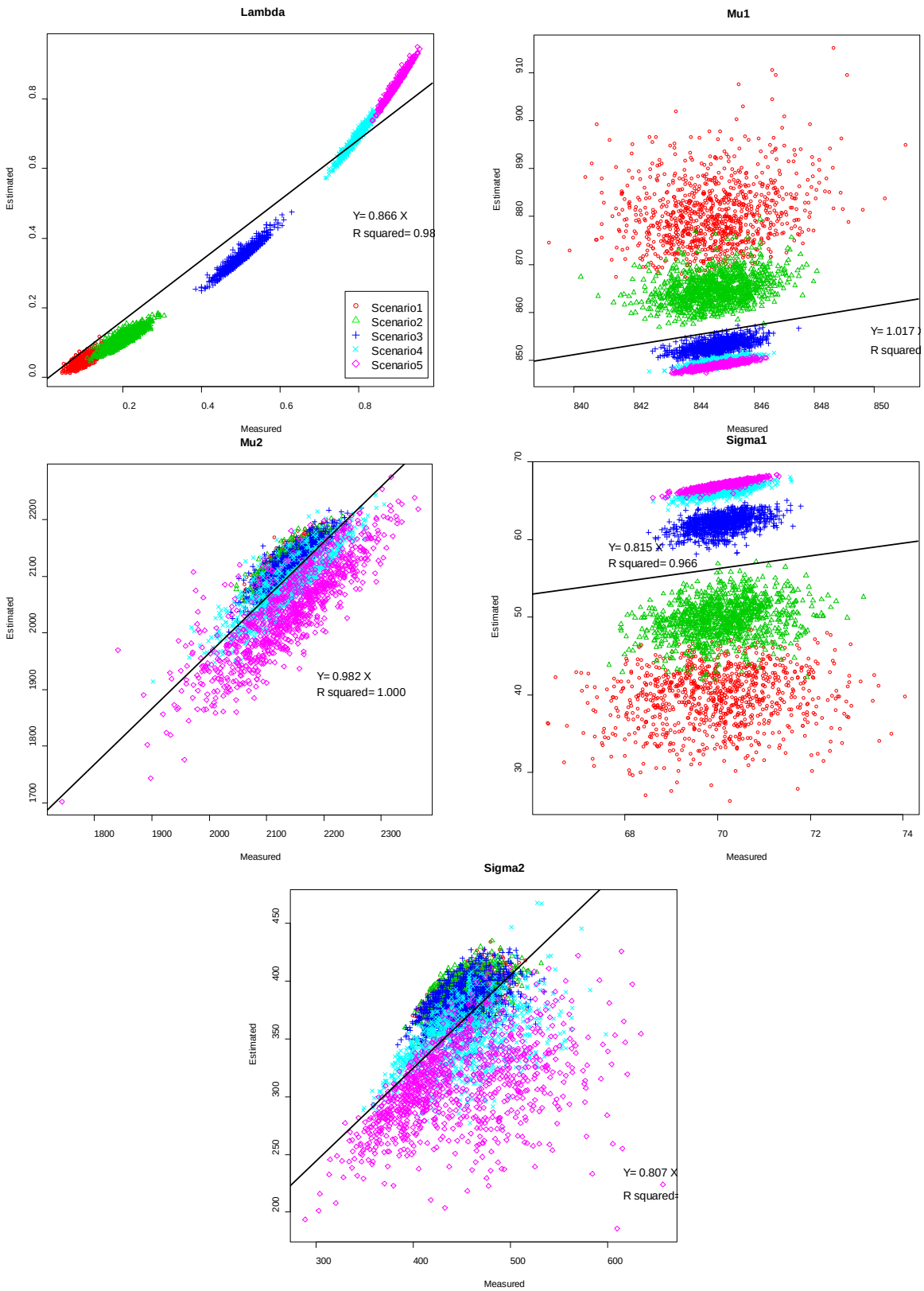

FIGURE 11 Estimated vs. True Parameter Estimates (Proportional Sampling Case)

Multi-state Travel Time Reliability Model: Impact of Incidents on Travel Time Reliability

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Abstract— This paper attempts to quantify the impact of traffic incidents on travel time reliability using a newly proposed multi-state travel time reliability model. Given that the multi-state travel time reliability model provides significantly better fits when compared to using a single-mode density function, it is possible to quantify the incident impacts more accurately. In order to obtain travel times, the study simulates weekday traffic on a section of I-66 over 17 days, once with incidents and once without them, using the INTEGRATION microscopic traffic simulation software. The simulated travel time data sets are then used to fit a three-state travel time reliability model (three normal distributions) to calibrate the parameters of the density function using the Expectation Maximization (EM) algorithm. The study demonstrates that incidents do not introduce an additional component distribution when congestion has already onset; instead they increase the mean travel time and variability in travel times for the congested conditions. For instance, the 90th percentile travel time of the second component distribution increases by up to 93 percent. Additionally, the study addresses technical issues related to the calibration and interpretation of the model from a practical standpoint.

INTRODUCTION

Traffic incidents are one of the major factors contributing to severe non-recurring congestion on U.S. highways. The Highway Capacity Manual (HCM) reported that traffic incidents are responsible for more than 50% of freeway congestion [1]. A better understanding of the impact of traffic incidents on travel time reliability is critical to the development of efficient traffic control strategies. There have been many research efforts in the area of traffic incident modeling. In general terms, the previous studies can be divided in two categories. The first category is to develop traffic incident detection algorithms for the implementation of automatic incident detection into traffic management systems. The other category is to quantify the impacts of traffic incidents with regard to economic, safety, delay, and other perspectives. Specifically, the delay estimation under incidents has been studied extensively because it is the essential component of a successful and reliable Advanced Traveler Information System (ATIS).

The provision of reliable travel time information to roadway users is the main function of ATIS. However, it is hard to maintain system reliability because errors in travel time estimation increase as congestion gets heavier due to increased travel time variability. Consequently, travel time reliability is being provided to travelers and system managers to help make better decisions. In other words, travel time reliability information helps drivers secure their on-time travels by reducing uncertainty. It can also be used as an operational performance measure by traffic operations managers.

Given that travel time reliability is very useful from a practical standpoint, diverse approaches to travel time modeling have been suggested. A common methodology is to use a probability density function such as a normal, log-normal, or Weibull distribution. However, a single-mode density function frequently does not fit well to field-measured travel time data. This is clearer if travel times are collected in transition periods such as a time window changing from a congested state to an uncongested one, or vice versa.

In order to bridge this gap, a multi-state travel time reliability model was proposed to provide more good fits by allowing the use of a combination of multiple probability distribution components [2, 3]. Consequently, the objective of the research presented in this paper is to quantify the impacts of traffic incidents on travel time reliability more accurately using the multi-state travel time reliability model. In addition, the study aims to answer technical questions that potentially arise during the calibration and interpretation of the model from a practical standpoint.

The paper is organized as follow. First, previous studies on traffic incidents and the proposed multi-state travel time reliability model are reviewed. After that, the details on the modeling of traffic incidents are followed. Specifically, the construction of simulation network, origin-destination (O-D) traffic demands, and design of traffic incident scenarios are explained. The travel time reliability under incident-free conditions is then quantified using the multi-state travel time reliability model. Next, the impacts of the traffic incidents are

analyzed quantitatively and graphically. Finally, the research conclusions are presented.

STATE-OF-THE-ART MODELING OF TRAVEL TIME RELIABILITY

Traffic Incidents

Traffic incidents are defined as any non-recurring events that result in a roadway capacity reduction or irregular high- traffic demand, such as traffic crashes, disabled vehicles, spilled cargo, highway maintenance and reconstruction projects, and special non-emergency events [4]. It has been reported that traffic incidents are responsible for a significant amount of freeway congestion. For instance, it was addressed in the 2000 Traffic Incident Management Handbook that they account for between 50 and 60 percent of total congestion delays in most metropolitan areas [4].

Since traffic incidents induce substantial negative impacts on traffic flow, many publications have attempted to address measures related to traffic incidents. For example, the Federal Highway Administration (FHWA) continuously updates handbooks on how to manage traffic incidents efficiently [4-7]. The FHWA reports provide guidance on the following applications: incident detection, verification, motorist information, response, site management, traffic management, clearance, and recovery. On the other hand, there were several studies in the area of developing efficient algorithms for the estimation of traffic incident duration and induced delay. For example, an incident-related congestion estimation algorithm that uses occupancy data from loop detectors was developed by the Washington State Transportation Center (TRAC) with the aim of implementing the algorithm within the system [8].

Traffic incident management has been emphasized for the mitigation of negative impacts of incidents. However, literature on the characteristics of traffic incidents, such as duration, incident rate, and queue length, are limited. From the literature that does exist, Prevedouros et al. conducted a study that summarized real-world incident statistics in the United States, the United Kingdom, and Greece, and developed models that predict incident duration and delays [9]. Specifically, the average incident duration, incident rate, and other characteristics were summarized in the study. The average incident duration was variable depending on the time and location of data collection: London M25 – 69 min., British Motorways – 46 min., Los Angeles I-10 – 20.7 min., Portland Metropolitan Area – 33 min., SP California F I-880 – 25 min., and Attica Tollway – 33.3 min.

Smith and Smith (2001) developed a model to estimate the clearance time of freeway accidents using traffic data received from the Virginia Department of Transportation's (VDOT's) Smart Traffic Center. The study investigated the performance of three models: a stochastic model, a nonparametric regression model, and a classification tree model [10]. In the course of investigation, the average clearance time from all traffic incidents detected in the state of Virginia was modeled using a Weibull distribution. The clearance time calculated from the distribution parameters was 40 min. This clearance time was adopted as the duration of traffic incidents for our study because a section of Interstate 66 (I-66) in Virginia was used for the simulation network in this study.

Multi-state Travel Time Reliability Model

A multi-state travel time reliability model was proposed for the quantification of travel time reliability [2, 3]. It provides a better fit to field-observed travel time distributions when compared to the state-of-practice single-mode travel time reliability models. It generates traveler-friendly travel time reliability information that provides the probabilities of encountering congested and uncongested states over a time period of interest as well as the mean and variance of the travel time distributions; e.g., "the probability of encountering a congested state is 95% and, if it happens, the expected travel time will be 55 minutes." Specifically, mixture models, which are defined as a distribution that convexly combines component distributions as formulated in Equation 1, are used.

$$f(T|\lambda,\theta)=\sum_{k=1}^{K}\lambda_k f_k(T|\lambda,\theta) \tag{1}$$

where $\lambda=(\lambda_1, \lambda_2,\ldots, \lambda_n)$ is a vector of mixture coefficients and $\theta_k=(\theta_{k1}, \theta_{k2},\ldots, \theta_{kl})$ is a vector of model parameters for the kth component distribution; $f_k(t|\theta_k)$ is the density function for the kth component distribution.

For instance, field-observed morning peak travel times collected in San Antonio, TX are fitted to a three-component model and a log-normal distribution, as illustrated in Fig. 1. Specifically, the three-component model is composed of three-component normal distributions. The figure clearly shows the advantages of the multi-state model over the single-mode model. There are several

methodologies available to estimate the mixture model parameters such as the Expectation Maximization (EM) algorithm, Markov-chain Monte Carlo (MCMC) simulation, and spectral methods [11]. This study uses the EM algorithm to estimate the model parameters. More details on the model are available in the previous studies [2, 3].

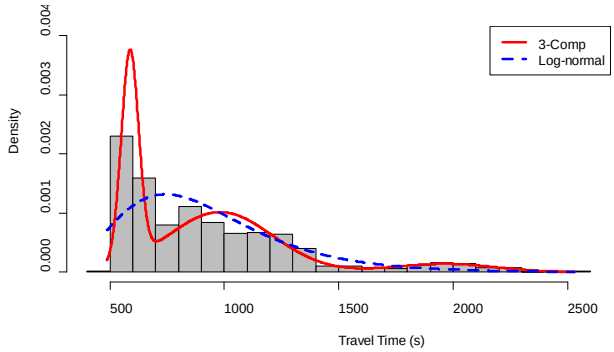


Fig. 1. Three-component mixture normal and log-normal density function.

MODELING OF INCIDENT TRAVEL TIMES

Since field-measured travel time data sets under incident conditions were very limited, this study used microscopic simulation models to obtain travel times under typical traffic conditions and under incident conditions. Specifically, a section of I-66 eastbound was used as the study network and the INTEGRATION software was used to build simulation models. The details on the modeling procedures are provided in the next section.

Network Construction

The network used in the study is a section of I-66 eastbound that feeds commuting traffic from the west to the Washington, D.C. metropolitan area [12]. Specifically, the network runs from Manassas (Virginia) on the west end to Vienna (Virginia) on the east end (Exit 46 to Exit 62). Seven on-ramps and seven off-ramps are included in the 16-mile study section.

The number of lanes along the section varies from three to six including acceleration and deceleration lanes. The posted speed limit was 65 mph. Due to the commuting vehicles from the west to Washington, D.C., traffic is heavy in the morning peak period from 5:00 a.m. to 10:00 a.m. The estimated average daily traffic (ADT) volume on the section from Route 29 (Lee Highway) to Route 234 (Sundly Road) was 87,000 vehicles per day in 2005 [13].

Origin-Destination Traffic Demands

Traffic demand is one of the most essential pieces of data for a reliable traffic simulation model. This study calculated O-D traffic demand tables from the loop detector data collected over a 3-month period between May 1 and July 31, 2002. Specifically, the study segment had a total of 31 loop detectors spaced over 0.5 miles. A total of 44 days of 15-minute O-D demands was estimated by the maximum likelihood approach using the QueensOD software [12]. From among the 44 days, 17 days of traffic demands were selected and used as the demand input data for this study; three Mondays, four Tuesdays, four Wednesdays, two Thursdays, and four Fridays. In other words, a total of 17 days was simulated with the 17 days of demands instead of using a single-day demand with different random number seeds. Consequently, each of the simulations can be considered as a realization of typical weekdays.

July 10, 2002 (a Wednesday) was the highest demand day among the 17 days, while July 22, 2002 (a Monday) had the lowest demand. The O-D demand for July 22 was 16% lower than July 10. The variations in the O-D demands are illustrated in Fig. 2: plot (a) shows the total demand (sum of all O-D demands) for the two days and plot (b) shows one of the O-D demands of interest, which includes the O-D demand traveling from the west end to the east end of the network.

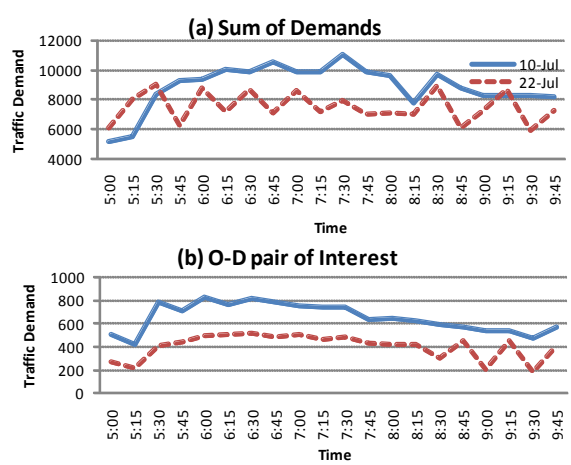


Fig. 2. Traffic Demand.

Incident Modeling

There are numerous factors that categorize traffic incidents such as type, location, severity, and duration. Traffic incidents include disablements, accidents, or others. They are also categorized based on the horizontal and longitudinal locations where

they occur. Severity can be defined as shoulder disablement, shoulder accident, one-, two-, or three-lane blocked.

Three traffic incident scenarios were designed in this study by varying the severity of traffic incidents. The duration of the traffic incidents was fixed at 40 minutes based on the mean duration of all the traffic incidents in Virginia [10]. The longitudinal location was set to upstream of an off-ramp (Exit 52) where localized congestion occurs due to the high off-ramp traffic demand. The details of the traffic incident scenarios are described in TABLE I.

TABLE VIII
Proposed Traffic Incident Scenarios

Index	Length	Location	Time		Severity
			Begin	End	
Scenario-1	250 m (four lanes)	250 m upstream from Exit 52	6:00 a.m.	6:40 a.m.	One lane blocked from shoulder
Scenario-2					Two lanes blocked from shoulder
Scenario-3					Three lanes blocked from shoulder

TRAVEL TIME RELIABILITY UNDER INCIDENT-FREE CONDITIONS

Travel Time Data Set

Although each of the 17 days of demands consisted of the entire single-day demands (24 hours) in every 15-minute interval, morning peak hours were only simulated from 5:00 a.m. to 10:00 a.m. The simulated travel times for the 17 days were then mixed together to take the travel time variation over the 17 days into account. This mixed travel time data set can be thought of as a historical travel time data set under incident-free conditions aggregated over 17 days.

Since there were many O-D pairs in the demand tables, the study selected one O-D pair for the analysis, which includes the vehicles originating from the west end (Manassas, VA) and traveling to the east end (Vienna, VA). Consequently, all the study results are based on the selected O-D pair if any specific descriptions are not given in the text.

In general, the longest travel times were observed from 7:00 a.m. to 8:00 a.m. and the demand of interest was higher from 6:00 a.m. to 8:00 a.m. compared to other time periods during the morning peak hours. In order to quantify the travel time reliability over time-of-the-day, 1-hour travel time data sets were extracted from the mixed travel time data set for every 15 minutes from 5:00 a.m. to 9:00 a.m. Consequently, a total of 17 travel time data sets were generated and used to fit a multi-state model.

Calibration of Multi-State Model

Initially, a two-component mixture model that has two normal component distributions was used to fit the travel times but it seemed that there were three components in the travel time distributions: first, an uncongested state; second, a medium-level congested state; and third, a heavily congested state. Fig. 3 clearly shows that the three-component model provided a better fit to the travel time distribution than did the two-component model.

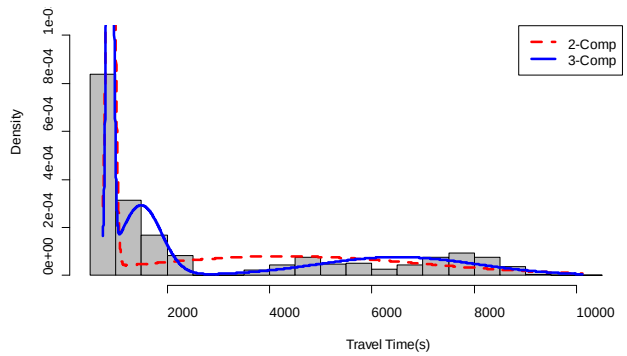


Fig. 3. Two- and Three-state Mixture Model for Without Incident

Another technical issue which arose through the calibration procedures was that a multi-state model should be calibrated multiple times in order to ensure that the calibrated parameters are stable from a practical standpoint. This is because using the EM algorithm generates a unique parameter set every time it calibrates a model, even though the same travel time data set is used. For instance, the variation in the mixture coefficient is illustrated in Fig. 4. Fig. 4 shows the calibrated mixture coefficients as the results of 10 calibrations using the EM algorithm with the same travel time data set. Consequently, there is a need to study how to determine a representative parameter set effectively from a practical standpoint. In this study, 10 calibrations were carried out for a single 1-hour travel time data set, and the average of 10 parameters was then used as a representative parameter.

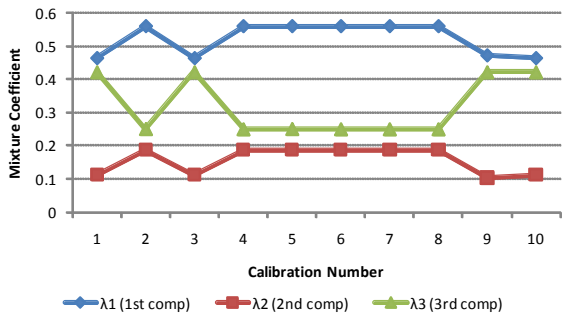


Fig. 4. Variation in Mixture Coefficient When using the EM

Calibration Results

The calibration results showed that the congestion level became severer from 5:00 a.m. until around 8:00 a.m., as the traffic demand increased. In terms of the shape of the travel time distribution, the third component distribution showed growth on the right hand side and shrinkage on the left hand side during the morning peak hours. The model parameters of the third component distribution clearly demonstrated the changes in travel time reliability.

As can be seen in TABLE II, the mixture coefficient, mean travel time, and standard deviation of the third component distribution increased and then decreased. For instance, the third component accounted for 30 percent of the travel time distribution and had a mean travel time of 108 minutes (6503 seconds) in the time window from 7:00 a.m. to 8:00 a.m. Furthermore, the variability increase can be identified from Fig. 5. The difference between the mean travel time and the 90th percentile became bigger around 6:00 a.m. to 8:00 a.m.

TABLE IX
Mixture Model Parameters for Travel Times without Incident

Time	Mixture Coefficient			Mean Travel Time (s)			Standard Deviation		
	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
5:00	69%	19%	12%	864	1350	2372	60	304	659
5:15	60%	20%	20%	872	1415	2831	55	328	926
5:30	52%	16%	32%	874	1288	3095	49	238	1319
5:45	51%	16%	33%	885	1554	4010	55	421	1489
6:00	45%	19%	36%	885	1431	4543	53	386	1614
6:15	39%	23%	37%	888	1427	5385	54	378	1742
6:30	38%	26%	36%	884	1435	5851	52	388	1704
6:45	39%	29%	32%	882	1476	6356	51	397	1619
7:00	41%	29%	30%	884	1493	6503	54	394	1561
7:15	43%	34%	23%	878	1474	6444	54	375	1401
7:30	47%	35%	18%	880	1715	6058	57	571	1368
7:45	47%	38%	15%	870	1503	6070	55	319	991
8:00	50%	33%	17%	863	1442	5306	52	261	1017
8:15	58%	37%	6%	864	1663	6022	55	263	786
8:30	51%	27%	22%	852	1268	3691	47	173	760
8:45	58%	19%	23%	851	1141	1790	49	149	257

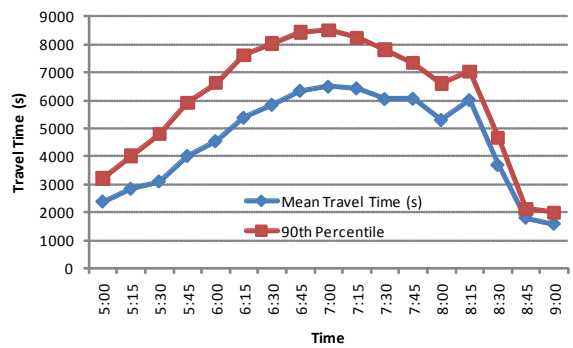


Fig. 5. 90th Percentile and Mean Travel Time of Third Component

The calibrated mixture model parameters can be interpreted in a two-step reporting procedure. The first step is reporting the probability of encountering each traffic state. In the three-state mixture model, the first state will be an uncongested state and the second and third states will be congested states. The second step is reporting the expected travel time under a given traffic state. For example, the travelers who plan to leave around 6:00 a.m. can be informed of the probability of encountering each state and the expected travel time from the calibrated parameters in TABLE II: Uncongested state – 45% and 885 seconds; medium congested state – 19% and 1,431 seconds; and heavily congested state – 36% and 4,543 seconds.

IMPACTS OF TRAFFIC INCIDENTS ON TRAVEL TIME RELIABILITY

Travel Time Data Set under Incident Conditions

Given the 17 days of O-D demands, each of the days was simulated three times with one of the traffic incident scenarios. In other words, each of the traffic incident scenarios was simulated with the 17 days of O-D demands. Consequently, four travel time data sets were prepared and used to mix the travel times to represent different levels of traffic conditions under the traffic incidents: 1) Without-incident, 2) Incident scenario-1, 3) Incident scenario-2, and 4) Incident scenario-3.

First of all, the number of days that had the traffic incidents should be determined. In this study, an assumption was made that the number of incident days would range from 1 to 7. Specifically, 1 day up to 7 days (41% of incident days: 7/17) were randomly selected out of the 17 days as the incident days. The other days (16 - 10 days) were used as the incident-free days. Based on the selection of the incident days, a new travel time data set was made by extracting the travel times from the without-incident data set and one of the incident data sets and mixing them together. Since the incident days were randomly selected, the selection procedures were repeated 10 times. Consequently, a total of 210 data sets (3 incident scenarios * 7 numbers of incident days * 10 repetitions = 210 data sets) were prepared.

Incident Impacts and Multi-State Model Issues

As explained earlier, each of the data sets was used to fit a three-component mixture model 10 times using the EM algorithm, and then the 10 parameter sets were averaged. The mixture model parameter calibration was conducted only for the travel times of the vehicles leaving their origin (the west end of the network) between 6:00 a.m. and 7:00 a.m. because the incidents occur at 6:00 a.m. and last till 6:40 a.m. in all the traffic incident scenarios.

The calibrated parameters are shown in TABLE III along with the parameters for the travel times without incidents for an easy comparison. In general, all the parameters of the first component remained almost constant for all the incident scenarios as the number of incident days increased. This implies that the probability of falling in an uncongested traffic state remained constant even though traffic incidents occurred on the roadway. In contrast, the parameters of the second component distribution increased as more incidents were introduced while the mixture coefficient of the third component decreased. These results clearly demonstrate that the second component distribution has more weight, as illustrated in Fig. 6.

TABLE X
Mixture Model Parameters under Incident Conditions

Classification			Number of incident days							
			0	1	2	3	4	5	6	7
Scenario 1	Mix. Coef.	1st	45%	45%	45%	45%	46%	45%	45%	46%
		2nd	19%	20%	20%	20%	21%	19%	23%	25%
		3rd	36%	34%	35%	35%	33%	36%	31%	29%
	Mean Travel Time	1st	885	885	885	885	886	884	885	887
		2nd	1431	1535	1574	1632	1792	1604	2007	2317
		3rd	4543	4724	4798	4898	5088	5022	5386	5596
	Std. Dev.	1st	53	54	54	54	55	53	55	57
		2nd	386	436	446	489	590	457	687	860
		3rd	1614	1595	1589	1608	1604	1647	1472	1466
Scenario 2	Mix. Coef.	1st	45%	45%	45%	46%	45%	45%	47%	47%
		2nd	19%	20%	19%	23%	21%	20%	28%	29%
		3rd	36%	35%	36%	32%	34%	35%	25%	24%
	Mean Travel Time	1st	885	885	884	886	885	884	890	890
		2nd	1431	1531	1498	1920	1625	1656	2588	2738
		3rd	4543	4746	4742	5141	5106	5136	5934	6102
	Std. Dev.	1st	53	54	53	56	54	54	61	61
		2nd	386	437	401	592	489	485	925	995
		3rd	1614	1608	1656	1537	1681	1693	1385	1336
Scenario 3	Mix. Coef.	1st	45%	45%	45%	46%	45%	47%	46%	47%
		2nd	19%	21%	20%	22%	19%	27%	24%	26%
		3rd	36%	34%	35%	32%	36%	25%	30%	27%
	Mean Travel Time	1st	885	885	885	886	884	890	887	890
		2nd	1431	1571	1454	1866	1646	2561	1998	2642
		3rd	4543	4792	4816	5119	5013	5841	5677	6006
	Std. Dev.	1st	53	54	53	56	53	60	56	61
		2nd	386	462	400	555	447	924	636	836
		3rd	1614	1601	1726	1546	1676	1421	1630	1487

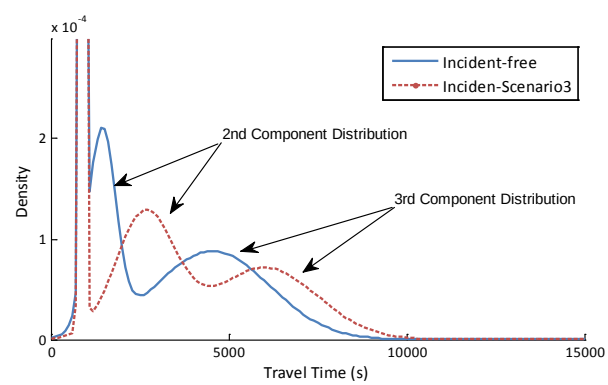
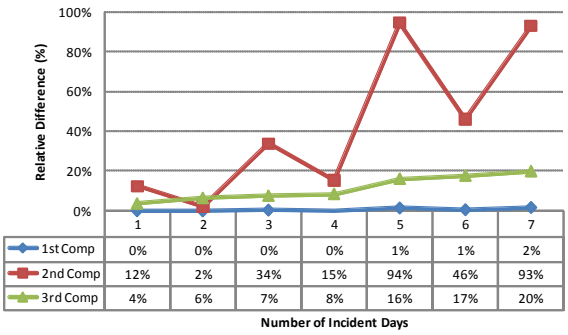


Fig. 6. Comparison of the Incident-free Model to the Incident

The calibration results clearly demonstrated that the travel time unreliability decreased as the traffic incidents were introduced. The 90th percentile travel times increased significantly as more incidents were introduced, as illustrated in Fig. 7. Specifically, the 90th percentile travel time of the second component distribution increased by up to 93 percent when compared to the incident-free calibration results.



	1	2	3	4	5	6	7
1st Comp	0%	0%	0%	0%	1%	1%	2%
2nd Comp	12%	2%	34%	15%	94%	46%	93%
3rd Comp	4%	6%	7%	8%	16%	17%	20%

Fig. 7. Relative difference of the travel times of the incident days to those of the incident-free day in the 90th Percentile Travel Times.

A technical question arose during the calibration procedures for the incident scenarios; that is, does a four- component mixture model need to be used in order to have better fits? Since the incidents might induce significant congestion, we examined if an additional component distribution, which might have the longest travel times, was introduced. From visual inspection of all the travel time histograms, it was concluded that no additional component distributions was reqyured because severe congestion had already onset. Consequently, it can be concluded that a two-component or three-component mixture model is generally sufficient to represent diverse traffic conditions found on the network.

CONCLUSION

This study quantified the impacts of traffic incidents on travel time reliability using the multi-state travel time reliability model. The study results demonstrated that the introduction of the traffic incidents decreased the travel time reliability significantly. Specifically, the congested state became more dominant and the travel time distribution expanded wider. For instance, the 90th percentile travel time of the second component distribution increased by up to 93 percent. The study also demonstrated that the multi-state travel time reliability model provided a better fit than a single-mode distribution model when analyzing the reliability of the travel times under traffic incident conditions.

The study addressed two practical issues related to the calibration of the multi-state travel time reliability model. First, the study recommended multiple repetitions of calibrations when using the EM algorithm to ensure the stability of the model parameters. Second, the study concluded that a two- or three-component mixture model is reasonable to use to cover diverse traffic congestion levels.

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