THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

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PART 1.—A DESCRIPTION OF THE INVESTIGATION[2]

SINCE 1930, the Bureau of Public Roads has been conducting at the Arlington Experiment Farm, Va., an extensive investigation with the general objective of developing information that will be of assistance in better understanding the structural action of concrete pavement slabs.

More specifically, the research was planned to study the following four main subjects:

1. The effects of loads placed in various ways on pavement slabs of uniform thickness.
2. The "balance of design" or relative economy of typical pavement slab cross-sections.
3. The behavior under load and comparative structural effectiveness of typical longitudinal and transverse joint designs.
4. The effects of temperature conditions and of moisture conditions on the size, shape, and load-carrying ability of pavement slabs.

The study of the effects of loads placed in various ways on slabs of uniform thickness was intended primarily as an experimental verification of the only rational theory of pavement slab stresses thus far advanced, i. e., the Westergaard analysis.[3] The program was accordingly planned in such a way that each of the factors that theoretically might influence the load-stress relation could be examined experimentally and the observed effects compared with those predicted by the theory. In addition, this study was expected to indicate rather definitely what the shape of the slab cross section should be if the design were so balanced that a given load would produce a certain definite maximum stress regardless of the position of the load on the slab.

The study of the balance of design of typical pavement slab cross sections was planned, first, for the purpose of showing the relative economy of the various designs, and second, to provide data upon which to base conclusions as to the proper shape for a perfectly balanced cross section. The data obtained in the load tests on slabs of uniform thickness mentioned in the last part of the preceding paragraph necessarily form an important part of the study of the balance of the cross-section design.

The almost complete lack of data concerning the structural behavior of the various types of longitudinal and transverse joint designs existing at the time this research was planned and the importance of a knowledge of this action in any consideration of the structural design of pavements made a study of the subject imperative. That part of the investigation dealing with joint design was planned to yield data showing the structural effectiveness of most of the commonly used types of joints and also information regarding the effect of dowel spacing and joint width on the structural action of joints.

The fourth part of the investigation, that is, the study of the effects of temperature conditions and of moisture conditions on the size, shape, and load-carrying ability of pavement slabs, was planned to provide information, not heretofore available, on the complex relations created by temperature and moisture variations, and the practical significance of these relations with respect to the design of the pavement slab as a load-carrying structure.

In order to carry out the studies contemplated in this investigation the group of 10 full-size concrete pavement slabs shown on the cover page was constructed. Each of these slabs is 40 feet in length, 20 feet in width, and has a particular cross section. Each slab is divided by a longitudinal and a transverse joint of a particular design and each slab is definitely separated from those adjoining it, in most cases by a 2-inch open joint. The concrete was uniform throughout the group and all slabs were without steel reinforcing. Special efforts were made to obtain subgrade uniformity under the entire group of sections.

The load tests and other studies designed to develop the information desired have been made on these 10 slabs. Some idea of the magnitude of the work of testing may be had when it is realized that, in round numbers, some 30,000 strain measurements, 25,000 deflection observations, 65,000 measurements of slab expansion (or contraction), and 30,000 temperature measurements were made in the course of the investigation. Approximately 10 percent of these were made during the night or early morning hours.

Figure 1 shows the details of the several designs of pavement slab cross section included in the investigation. It will be observed that these include the rather massive edge design suggested a number of years ago by the American Association of State Highway Officials, 3 designs of the conventional thickened-edge type in which the edge thickening is decreased uniformly to zero over a distance of 3 feet, a design in which the upper and lower boundaries of the section are parabolas diverging so as to give a thickened edge, 2 lip-curb sections (with and without the conventional edge thickening), and 4 sections of uniform thickness. The area of the cross section (in square feet) for a 20-foot width pavement of each design is noted in this figure.

The details of the several designs of transverse joint included are shown in figure 2. The ordinary butt-type open joint with 3 different dowel spacings and 2 different widths of joint opening, the continuous steel plate key with 2 widths of joint opening, the thickened slab end (without dowels or other connection), and the "plane of weakness" both with and without dowels

[1] Roscoe Lancaster, Harry D. Cashell, Arthur L. Catudal, and Ernest G. Wiles, junior highway engineers, gave able assistance in carrying on the work reported. They contributed valuable suggestions as to procedure and made observations at all hours and under all weather conditions.

[2] A series of five articles has been planned. The first three will probably be published in consecutive issues. Parts 4 and 5 may not be published in issues consecutive with the rest of the series as they are dependent upon work yet to be completed.

[3] Stresses in Concrete Pavements Computed by Theoretical Analysis, by H. M. Westergaard, PUBLIC ROADS, vol. 7, no. 2, April 1926, and Analytical Tools for Judging Results of Structural Tests of Concrete Pavements, by H. M. Westergaard, PUBLIC ROADS, vol. 14, no. 10, December 1933.

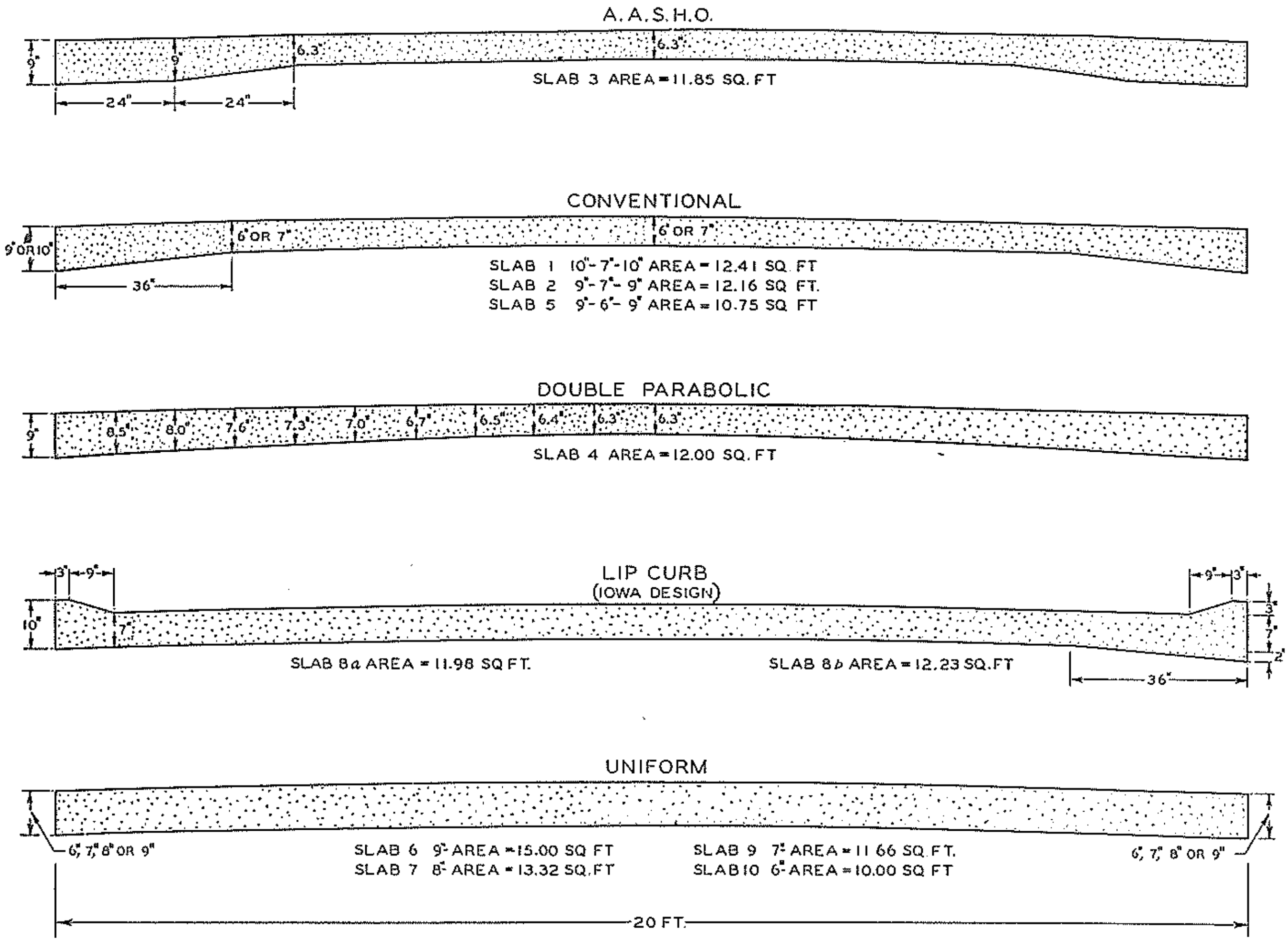


Figure 1.—Designs of Cross Sections Investigated.

comprise the types that have been tested. All of the joints have been kept filled with a typical poured bituminous joint filler.

The structural features of the several types of longitudinal joints used in the 10 slabs are shown in figure 3. In 4 of these, separation of the 2 slabs was accomplished with a deformed plate of heavy sheet metal; in 4 others the slabs were laid half width at a time and bond between the halves of the slab was prevented by a sheet of tarred felt. The other 2 slabs were grooved to create a longitudinal plane of weakness that was intended to crack through and form a separation between the 2 halves of the slab.

TEST SLABS CAREFULLY CONSTRUCTED

The subgrade.—The importance of subgrade uniformity in any test of the structural action of pavement slabs was recognized from the first. The site used was selected with this in mind and a detailed soil survey was made to determine the conditions existing in the area involved, with the result that the soil was classified as a uniform brown silt loam (class A–4). The uniformity of the soil is indicated by the results of tests made in the laboratory on the samples taken during the survey, as shown in table 1.

The original surface of the area having been disturbed by earlier experiments, the subgrade material was entirely removed until the new subgrade was entirely an undisturbed soil formation. On this surface the line of the pavement was laid out. In order to insure proper drainage, deep side ditches with suitable outlets were provided.

Table 1.—*Test data from subgrade samples*

Sample no.	Liquid limit	Plasticity index	Shrinkage		Moisture equivalent	
			Limit	Ratio	Centrifuge	Field
1	24	8	18	1.8	28	20
2	22	5	21	1.7	23	19
3	25	8	19	1.8	26	21
4	24	9	19	1.8	25	20
5	25	9	19	1.8	26	19
6	24	6	18	1.8	28	19
7	23	8	19	1.8	20	18
8	25	9	17	1.8	25	21
9	23	7	18	1.8	19	20
10	32	12	24	1.6	35	28
11	24	7	20	1.8	26	20
12	28	9	24	1.7	34	23
13	24	9	20	1.8	24	20
14	27	12	16	1.8	23	20
15	25	10	17	1.8	25	19
16	25	10	17	1.8	25	19
17	24	8	19	1.8	23	19
18	25	9	18	1.8	24	20
19	24	9	18	1.8	25	20
20	24	7	20	1.8	27	20
21	23	8	18	1.8	23	20
22	24	8	18	1.8	27	19
23	30	14	19	1.8	22	21
24	28	13	18	1.8	27	21
25	29	11	16	1.8	31	23

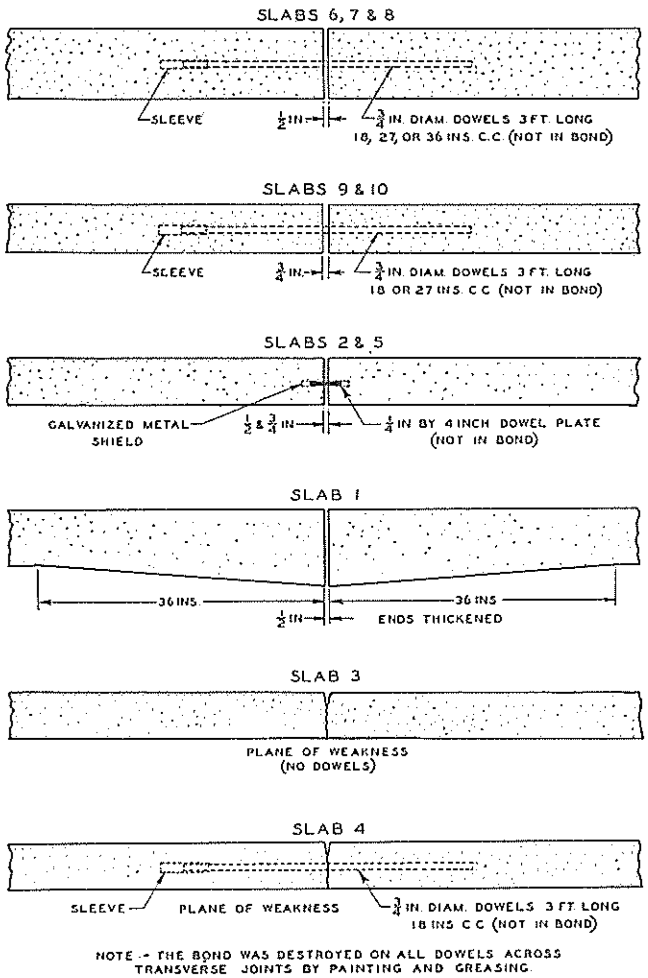


FIGURE 2.—DESIGNS OF TRANSVERSE JOINTS INCLUDED IN THE INVESTIGATION.

The subgrade where the slabs were to be located was next plowed to a depth of approximately 10 inches. It was left in this loose condition for a period of about 4 weeks, during which it was broken up and agitated several times with a disk harrow. The soil was finally compacted, first with a 5-ton tandem roller and then with the wheels of a loaded 5-ton motor truck. The appearance of the subgrade after this manipulation is shown in figure 4. On this compacted soil the forms were set and the final grading completed. Because of the purpose for which the slabs were to be used, great care was taken to have the final subgrade surface exactly to grade and very smooth in order that the thickness of the completed slab would be known definitely. The appearance of the subgrade at the time the concrete was placed is shown in figure 5.

The moisture content of the subgrade was maintained by sprinkling daily and the particular portion on which concrete was to be placed was given an additional light sprinkling immediately prior to placing concrete.

The concrete.—The materials used for the concrete were carefully selected and the mix designed to give high flexural strength. The cement was a standard portland cement of satisfactory quality, and all came from one bin at the plant.

The fine aggregate was a rather coarse, angular quartz sand, containing some grains of chert, feldspar, gneiss and mica. The average fineness modulus of the sand as determined by a number of tests was 3.26. The source of this material is near Fredericksburg, Va.

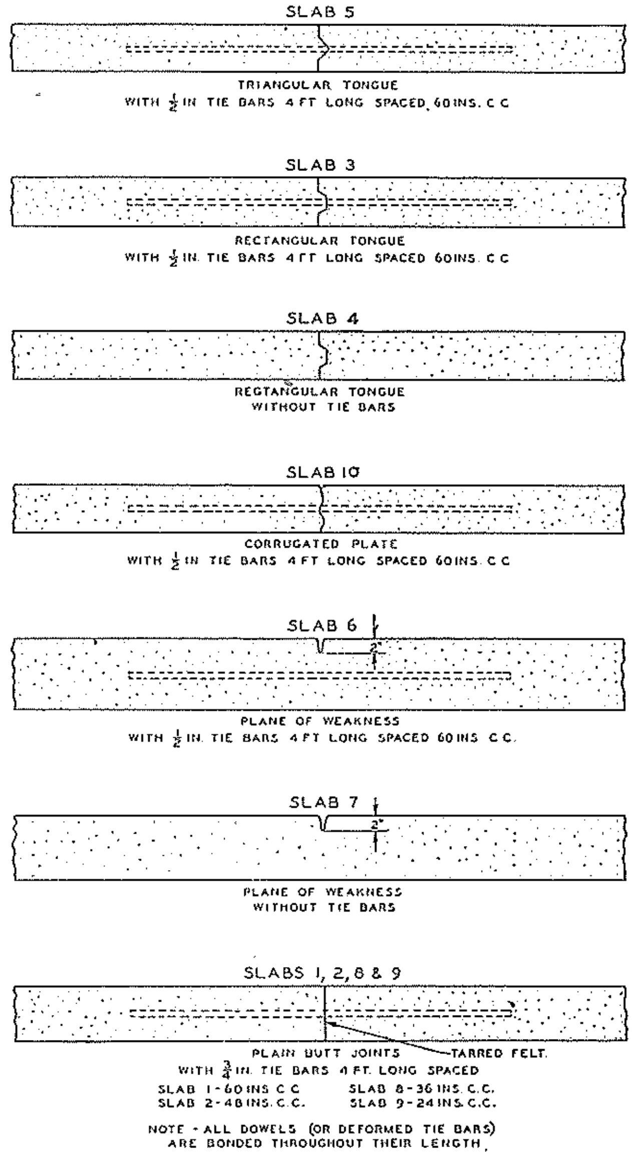


FIGURE 3.—DESIGNS OF LONGITUDINAL JOINTS INCLUDED IN THE INVESTIGATION.

The coarse aggregate was a blue limestone obtained from near Martinsburg, W. Va. It was shipped to the job in three sizes and recombined at the proportioning plant to give the desired grading. The proportions used were:

Size:	Percent
1¼ to 2¼ inches	50
¾ to 1¼ inches	25
¼ to ¾ inch	25

When combined in this way the average fineness modulus of the coarse aggregate was 7.65.

The proportions fixed for the concrete were 1:2:3½, using dry-rodded volume as the basis of measurement. Actually, in batching materials for the mixer, these proportions were controlled by weighing all of the constituents except the water. Figure 6 shows the

FIGURE 4.—APPEARANCE OF THE SUBGRADE AFTER ROLLING HAD BEEN COMPLETED.

proportioning plant used in the construction of the slabs.

Moisture determinations were made on samples from the stock piles each morning and necessary adjustments were made in the batch weights and water content. The water-cement ratio decided upon as a result of trial mixes was 0.85 by volume.

Concrete was mixed for 1½ minutes in a modern paving mixer (size 27–E). At the beginning of each day's run, a preliminary half-size batch was run through the mixer and discarded, the purpose being to coat the interior of the mixer drum and obtain uniformity in subsequent batches. The concrete was dumped on the subgrade and distributed in the usual manner. Compacting and finishing were accomplished with a 2-screed finishing machine, without tamping. The final finish was obtained with a hand belt and edging tools. A double layer of wet burlap was applied immediately after the final belting and this was kept wet for 24 hours, after which it was replaced with a layer of earth 3 to 4 inches thick, which was kept wet for 20 days and then removed. Figure 7 shows the equipment used in mixing and placing the concrete.

In order to have concrete available for such later studies of the physical properties of the concrete as might be necessary, three short extra sections of pavement were cast during the construction of the test slabs.

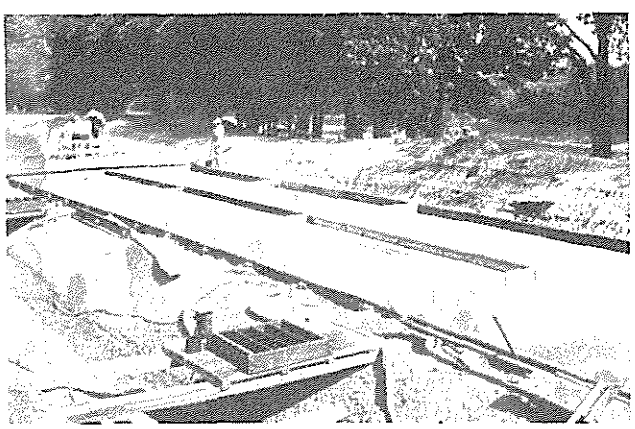


FIGURE 5.—APPEARANCE OF THE SUBGRADE AFTER FORM SETTING AND COMPLETION OF THE FINAL GRADING.

For an early determination of concrete strength, 8 beams and 5 cylinders were made for each of the 10 pavement slabs. These specimens were cast from the concrete after it had been dumped on the subgrade. The beams were 7 by 7 by 30 inches and the cylinders 6 by 12 inches in size. These specimens were protected from moisture loss during the first 24 hours, after which the cylinders were removed to the damp room and the beams were buried in the earth shoulder beside the slab. All of these specimens were tested at the age of 28 days.

The average flexural strength of the beams (80 specimens) was 765 pounds per square inch and the average compressive strength of the cylinders (46 specimens) was 3,525 pounds per square inch.

TEST PROCEDURE DESCRIBED

The tests and observations made in this investigation may be divided into three groups, as follows:

1. Load tests on the pavement slabs, in which definite loadings were applied to the various sections according to a plan and the resulting deflections and strains were measured. These tests form the basis of—

(*a*) The examination of the Westergaard analysis.

(*b*) The study of the pavement cross-sections.

(*c*) The determination of the structural efficiency of the different joint designs.

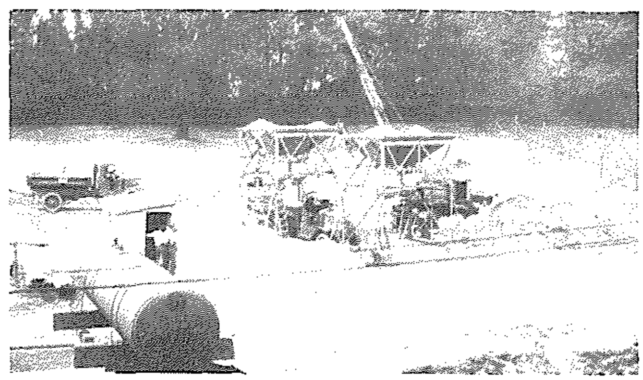


FIGURE 6.—THE PROPORTIONING PLANT.

2. Observations made on the slabs to determine the effects of variations in temperature and moisture conditions on their size, shape, and load-carrying ability. These observations included the determination of—

(*a*) Temperature conditions within and surrounding the pavement.

(*b*) The expansion, contraction, and warping of slabs due to temperature changes and to changes in moisture condition.

(*c*) The strains induced in the concrete through the tendency of the slab to change its size and shape.

3. *Auxiliary tests.*—This group comprises a considerable number of collateral investigations carried out principally in the laboratory, to develop information essential to the interpretation of the data obtained in the tests on the slabs. In this group will be found tests to determine—

(*a*) The physical characteristics of the subgrade.

(*b*) The physical properties of the concrete.

(*c*) The thermal properties of the concrete.

(*d*) The effect of moisture conditions on the strength and stiffness of the concrete.

PROGRAM OF LOAD TESTS BASED ON CAREFUL PRELIMINARY INVESTIGATIONS

Before beginning the general program of load tests, preliminary tests were made on a slab of uniform thickness and on one having a conventional thickened edge. The purpose of these preliminary tests was to determine—

1. The proper points at which to apply the loads for the various studies.
2. The proper position for the strain gages if the critical strain was to be measured for each loading.
3. The extent of the deflections to be measured for each loading.

The information obtained in these tests made possible the detailed planning of the tests that were to follow.

An important development of this preliminary work was the conclusion that relative deflections, as measured in these tests, may not always be a true indication of relative stresses. Since deflection and stress are directly related theoretically, it seems probable that the deflection data, while apparently accurate, are actually quite crude when compared to the strain data. Thus, differences in elastic curvature that are not detectable in the deflection data may cause large differences in stress.

This conclusion made it necessary to depend almost entirely upon the stress data as a basis for the comparisons that it was desired to make.

As soon as the preliminary tests had been completed detailed plans for the load-testing program were developed. Figure 8 shows the plan and elevation of one of the test sections and the points where loads were applied for the different studies and the positions of the strain gages in relation to the load points. Figure 9 is a similar drawing showing the points where loads were applied and the lines along which the deflection curves were determined. This figure also shows the location of the points where the opening and closing of the joints due to temperature were measured.

In the studies of the balance of the designs of slab cross-section the load was applied successively at points 1 foot apart along the transverse axis of one of the quarter-slab panels (points 1 to 10, inclusive, along the line A′—B′ in fig. 8).

For the investigations of joint design, loads were applied at the joint edge, at the center of the slab panel, and at the free edge of each slab, thus permitting a comparison to be made between the maximum stresses developed by a given load acting at the joint edge and those developed by the same load at the other two points, points that represent the extreme limits of slab continuity. These stress data make it possible to set up a rational measure of the structural effectiveness of joint designs.

In the case of the longitudinal joints the load points A, H, and B were used, while for transverse joints the loads were applied at I, H, and G. As the program advanced it was found that some joint designs were not equally effective at all points nor at all times of the year, so that additional loads were applied along lines parallel to the slab axes but displaced from the center line. In all cases, however, the complete data were obtained for the three positions of the load.

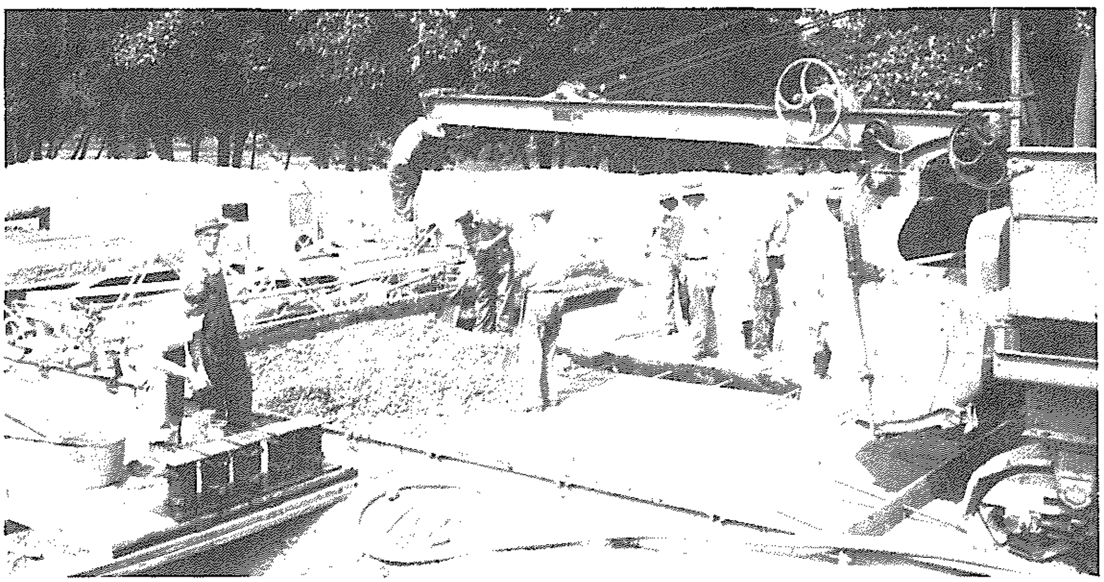


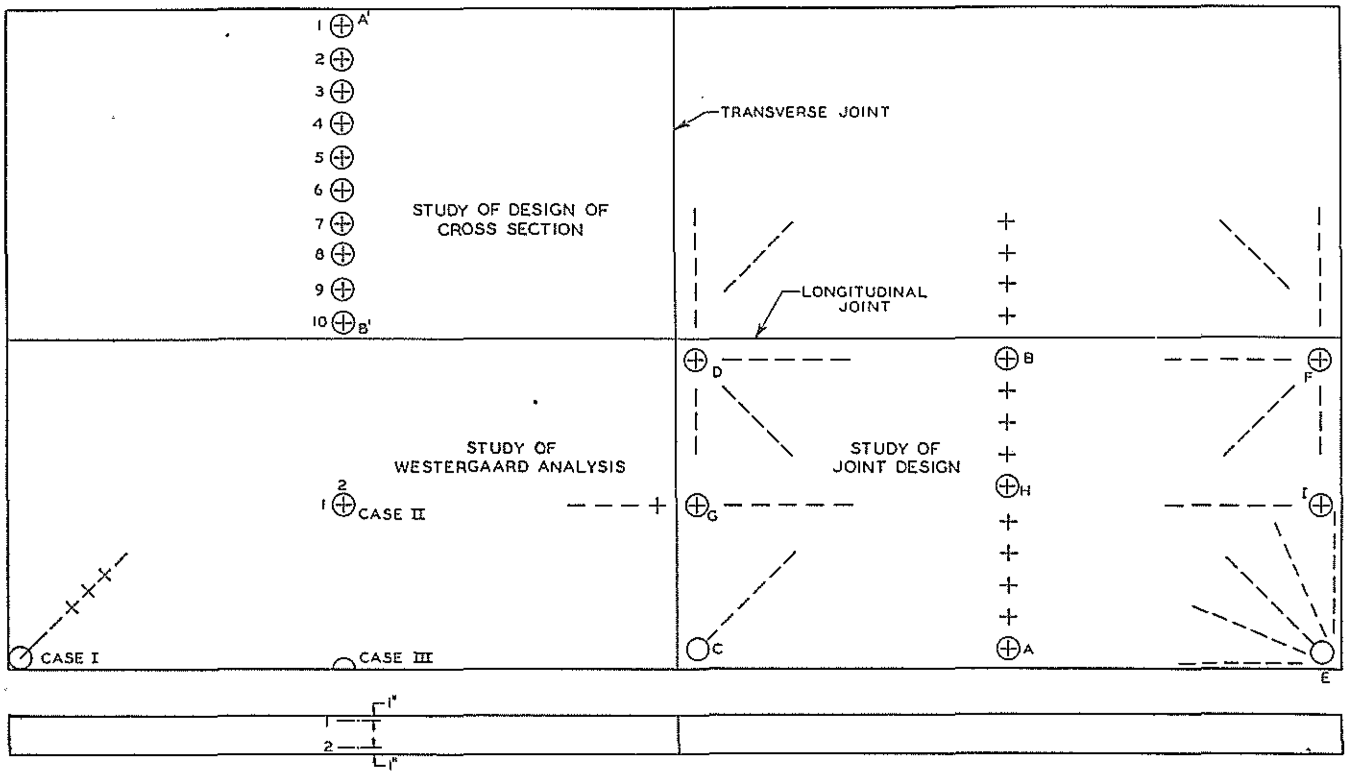
FIGURE 7.—THE MIXING AND PLACING OF THE CONCRETE.

When Westergaard prepared his analysis of the stresses in a concrete pavement slab, he developed a mathematical treatment covering three important cases of loading that were: Case I, a wheel load acting at the free corner of a slab; case II, a wheel load acting at the interior of a slab and at a considerable distance from the edges; case III, a wheel load acting at the edge of a slab and at a considerable distance from a corner.

The loadings applied to the test slabs in the study of the three cases of the Westergaard analysis are shown in figure 8. These tests were made only on the slabs of uniform thickness. For every loading the significant stress and deflection data were obtained. Figure 10 shows the appearance of one of the test sections after the installation of the gage points for the strain and deflection measurements.

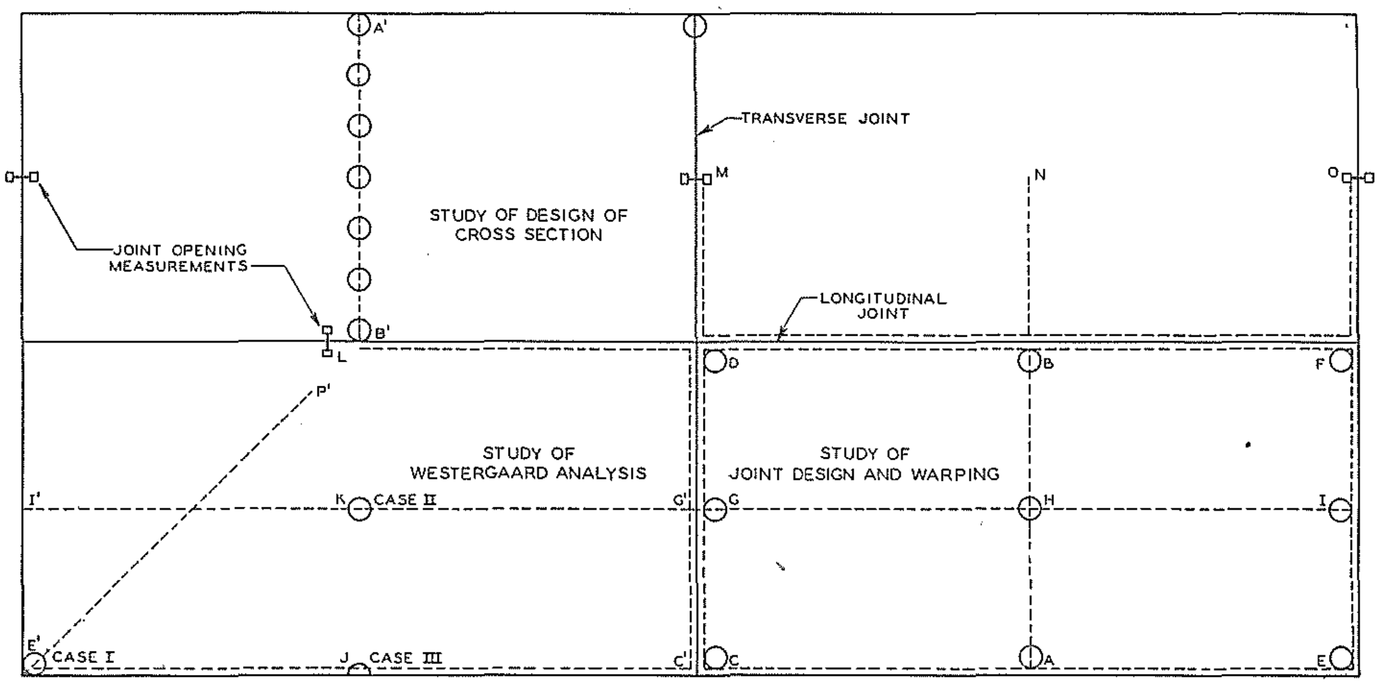
METHOD OF APPLYING LOADS DESCRIBED

It was considered desirable to use loads that would create maximum stresses of approximately one-half of the modulus of rupture of the concrete. Since some of the slabs were of rather heavy design, reactions of some



LEGEND:— CIRCLES SHOW POINTS AT WHICH LOADS WERE APPLIED, SHORT LINES SHOW LOCATION OF STRAIN GAGES.

FIGURE 8.—PLAN AND ELEVATION OF A 20- BY 40-FOOT TEST SECTION, SHOWING THE POINTS WHERE LOADS WERE APPLIED AND THE LOCATIONS OF THE STRAIN GAGES.



REFERENCE POINT (FIXED ELEVATION)

LEGEND — CIRCLES SHOW POINTS AT WHICH LOADS WERE APPLIED, DASH LINES SHOW LOCATION OF CLINOMETER POINTS.

FIGURE 9.—PLAN OF A 20- BY 40-FOOT TEST SECTION SHOWING THE POINTS WHERE LOADS WERE APPLIED AND THE LINES ALONG WHICH DEFLECTIONS WERE MEASURED.

magnitude were necessary to produce such stresses. It was also highly desirable that at the time the test load was being applied no other loads be on the slab, in order that the observed effects could be attributed definitely to a known reaction system. These two considerations and the availability of a large cylindrical steel tank led to the adoption of the loading equipment shown in figure 11. The tank, 30 feet long and 6 feet in diameter, was mounted in a structural steel frame on cradle, supported by two transverse end frames 22 feet apart. Each end frame was provided with a pair of heavy cast-iron wheels of small diameter and these rested upon a railway laid along the earth shoulders parallel to the pavement edge. The tank spanned the slab completely and could be moved longitudinally over the test sections at will.

A heavy wooden bolster or pad was fitted to the lower surface of the tank and so arranged that it could be shifted to any position from one end frame to the other, and thus be placed over any desired point on any of the sections.

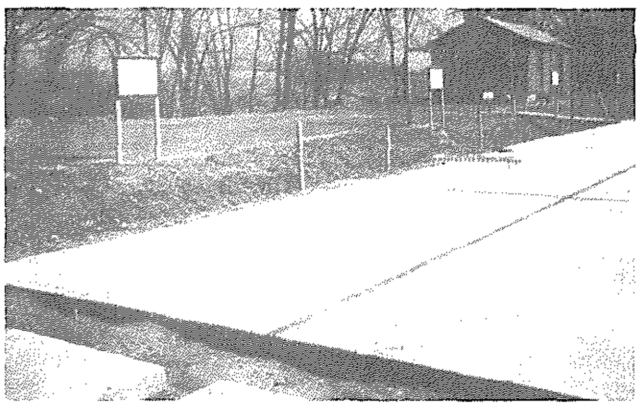


FIGURE 10.—COMPLETE INSTALLATION OF GAGE POINTS FOR STRAIN AND DEFLECTION MEASUREMENTS ON ONE OF THE TEST SECTIONS.

By partially filling the tank with water a reaction in excess of any load required for the loading of the slabs became available. To develop the load on the slab the device shown in figure 12 was constructed. In this figure, A is the wooden bolster that bears against the bottom of the tank, B is a steel facing plate on the lower surface of the bolster, C is a hardened steel knife edge, D is a pair of heat-treated steel beams whose load-deflection rate being known through calibration enables the operator to determine the load on them at any instant by reading the micrometer dial that measures their deflection, E is a ball-bearing screw jack used for developing the thrust, F is a spherical bearing block that prevents eccentricity of loading on the bearing plate G, and H is a sponge-rubber pad to take up surface irregularities on the slab and assure a uniform intensity of load over the entire area of the bearing block.

The capacity of the loading device shown in this figure is approximately 25,000 pounds. One division on the micrometer dial is equivalent to a load increment of about 30 pounds and periodic calibrations have led to the conclusion that the load measurement by this means can be depended upon to be accurate within 100 pounds, which makes the percentage of error small for loads of the magnitudes used in these tests.

The bearing blocks that received the thrust of the jack and applied it to the pavement were of two types and of several sizes. For the study of the Westergaard

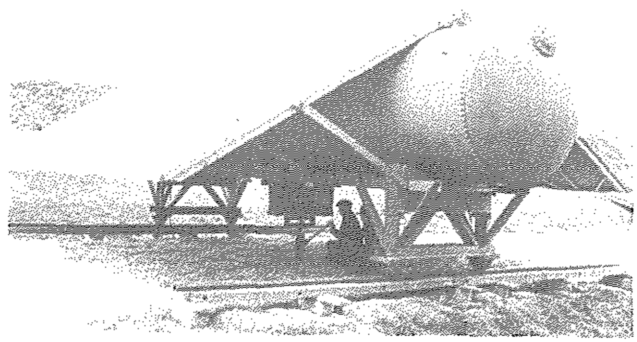


FIGURE 11.—LOADING EQUIPMENT IN PLACE OVER ONE OF THE TEST SECTIONS. THE SLAB IS COVERED WITH STRAW AND SHADED TO PREVENT WARPING.

theory it was necessary to use blocks having both circular (fig. 13 A) and semicircular bearing areas (fig. 13 B), circular for the interior and corner loadings and semicircular for the edge, in order to meet the assumptions of the analysis. Also it was necessary to use several sizes of each in order to investigate the effect of the size of the bearing area on the maximum stress caused by a given load. The diameters of the blocks selected were 6, 8, 12, 16, and 20 inches. The majority of the tests were made with the 8-inch diameter circular block. For the corner loadings the full circular plates were used. When the larger plates were used, distribution of the load was obtained by pyramiding the plates as shown at the right hand side of figure 13 A and also in figure 12.

In a number of the tests, such as those at the interior of the pavement slab, it was necessary to measure the strain in the concrete directly under the bearing plate. For these tests special blocks, provided with a groove across the bottom face large enough to accommodate a

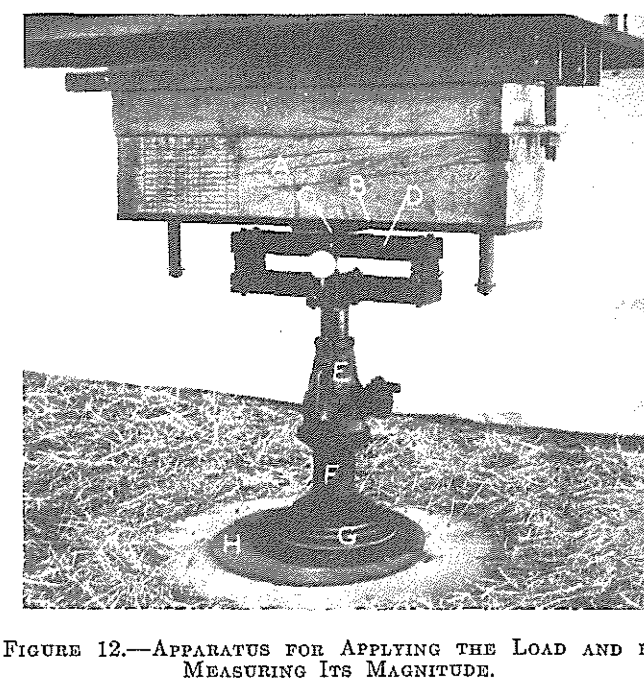


FIGURE 12.—APPARATUS FOR APPLYING THE LOAD AND FOR MEASURING ITS MAGNITUDE.

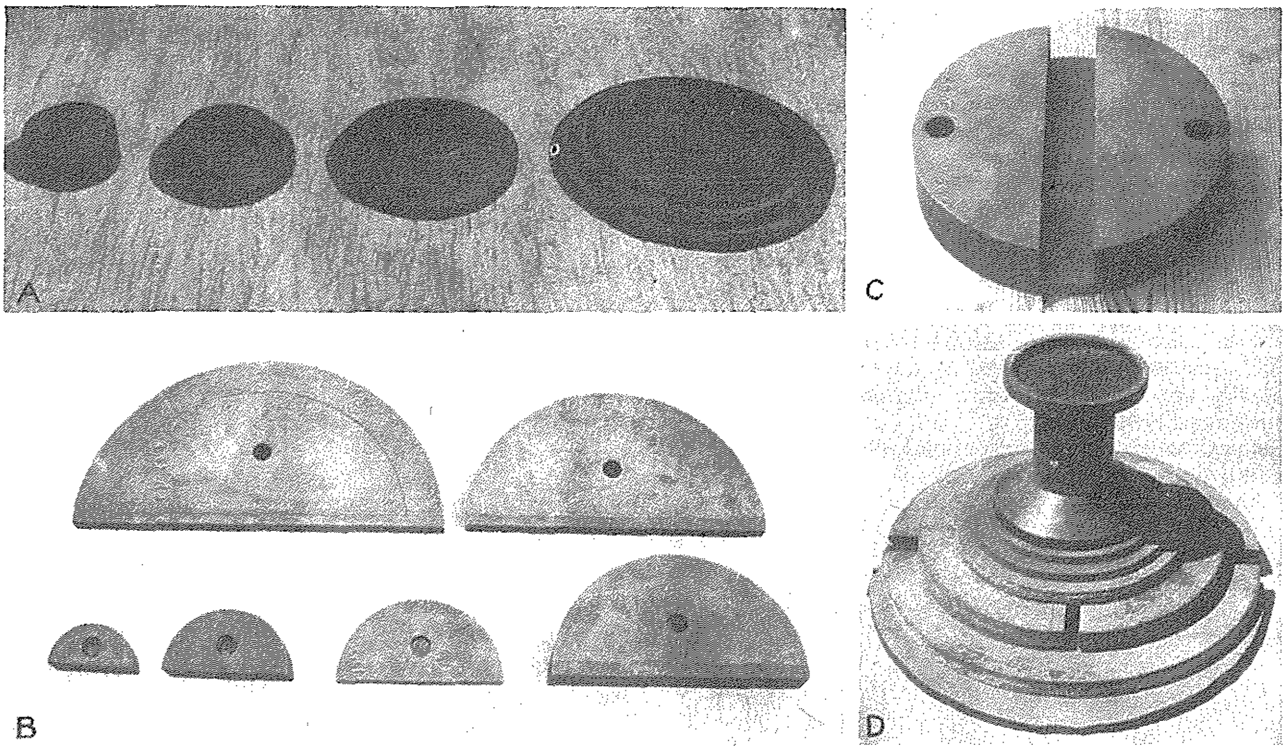


FIGURE 13.—BEARING PLATES: A, CIRCULAR BEARING PLATES USED IN THE CORNER LOADING TESTS; B, SEMICIRCULAR PLATES USED FOR THE EDGE LOADING TESTS; C, SMALL CIRCULAR BEARING BLOCK GROOVED TO PROVIDE SPACE FOR A STRAIN GAGE. THE BLOCK IS SHOWN INVERTED. D, LARGE CIRCULAR BEARING BLOCK WITH SPACE FOR A STRAIN GAGE IN THE LOWER PLATE.

strain gage, were used. The smaller blocks of this type were built as shown in figure 13 C. For the larger areas two segments of the proper size were placed on the sponge-rubber pads on either side of the strain gage and the load was distributed to these by superimposed circular plates as shown in figure 13 D.

Both circular and semicircular plates were used as bearing blocks for the tests at the edge of the pavement slab. The circular plates were the same ones used in the corner loading tests and the semicircular plates were those shown in figure 13 B.

All of the bearing blocks were made of steel and were so designed that the deflection under load produced a negligible effect on the uniformity of load distribution. The effect of the groove in the bearing block on the maximum stress in the slab was investigated and the tests showed that for a given load the grooved block caused the same maximum strain as a block of the same diameter without the groove.

A study was made of the effect of load duration on the magnitude of the strain developed in the concrete. It was found that in some positions, on some of the pavement designs, essentially the maximum strain was developed after the load had been maintained for 1 or 2 minutes, while at other points, 4 or 5 minutes was necessary before this equilibrium was established. As a result of this study the procedure of maintaining the load for 5 minutes before making any strain measurement was adopted for all of the tests. Conversely, 5 minutes was allowed for recovery after the release of each load before the application of the next load. The strains reported in the papers that are to follow are, therefore, maximum strains for the particular loads and are all definitely larger than would be caused by momentary loads of the same magnitude.

ACCURATE MEASUREMENTS OF STRAIN OBTAINED WITH SPECIAL GAGES

Throughout the investigation the strains in the concrete were measured with the recording strain gage shown in figure 14. The gage and its characteristics have been described in detail elsewhere [4] and will be dealt with only briefly here. It consists of a body or frame about 6 inches in length carrying a simple bell crank lever with arms of unequal length. The short arm of this lever is moved by any displacement of the gage points between which the gage is mounted. This motion is transmitted to the long arm of the lever and of course magnified by the ratio of the lengths of the two arms. The long arm of the bell crank carries a stylus point at its free end which makes a trace on the smoked surface of a small glass plate, thus recording a displacement of the end of the arm. The trace on the record slide is thus proportional to the displacement of the gage points and its length is measured, either directly with a comparator or by optical magnification in a projection apparatus. The mechanical magnification in the gage is about 60:1 and ordinarily another magnification of about 30:1 is had in the projection apparatus.

The gages were designed to eliminate ordinary temperature effects. The gage body from tip to tip is made of the alloy "invar", and further compensation is obtained through the use of a pair of dissimilar metals in the long (or stylus) arm of the bell-crank lever.

The accuracy of the gages is sufficient to permit the determination of stress in concrete to within 20 or 25 pounds per square inch, where dependence is placed upon a single observation.

[4] An Improved Recording Strain Gage, by L. W. Teller, PUBLIC ROADS, vol. 14, no. 10, December 1933.

The strain gage is approximately 6 inches in length. Early in the consideration of the program the question was raised as to whether or not a gage of this length would record maximum strains when used under bearing blocks of the sizes that it was desired to use. This matter was investigated rather thoroughly by using special gages of various lengths placed under bearing blocks of a range of sizes, and the data obtained indicated quite conclusively that the gages would record the maximum strain, provided that the entire gage and gage points were within the circumference of the bearing plate and that the axis of the gage lay along one of the diameters of the plate. Theoretically, the stress is not exactly uniform across the area of the slab under the bearing plate, and it is probable that, had it been possible to measure strains with greater precision, the variation due to length of gage would have been detected. With the apparatus described, however, the same maximum unit strain was indicated by at least 3 different lengths of gage under bearing plates of several sizes, so long as the gages were placed in accordance with the 2 provisions mentioned above. As a result of these tests, it was concluded that the

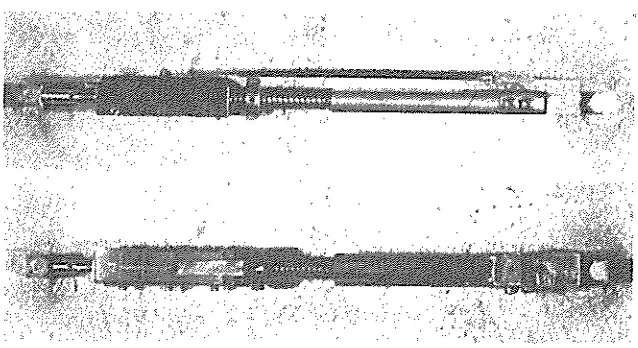


FIGURE 14.—RECORDING STRAIN GAGE OF THE TYPE USED IN THIS INVESTIGATION MOUNTED BETWEEN TWO GAGE POINTS.

6-inch gage would record approximately the maximum strain if used with bearing plates with a diameter of 6 inches or more.

In use the gages were installed between two small brass posts containing drilled and reamed gage holes. These posts, ¼ by ¼ by 1½ inches in size, were set into small holes drilled in the surface of the concrete to a depth of about 1 inch immediately before each test, being held in place with plaster of paris. Various other cementing materials were tried, but it was found that with time the posts tended to work loose with all of them and that, for a temporary setting, plaster of paris was as satisfactory as any of them and considerably more convenient to use.

As usually installed the axis of the strain gage was one-fourth of an inch from the surface of the concrete. This caused the recorded strain to be greater than the strain at the surface of the pavement by an amount that depended upon the relative distances of the gage and that surface from the neutral plane of the pavement slab. In most of the measurements, it was therefore necessary to apply a small correction to the observed strains in order to compensate for the gage position. Figure 15 shows an installation of the gages for a load test at the corner of a slab of constant thickness.

DEFLECTIONS MEASURED WITH CLINOMETERS

The deflection measurements in this investigation were made with the clinometer or "level-bar" shown in

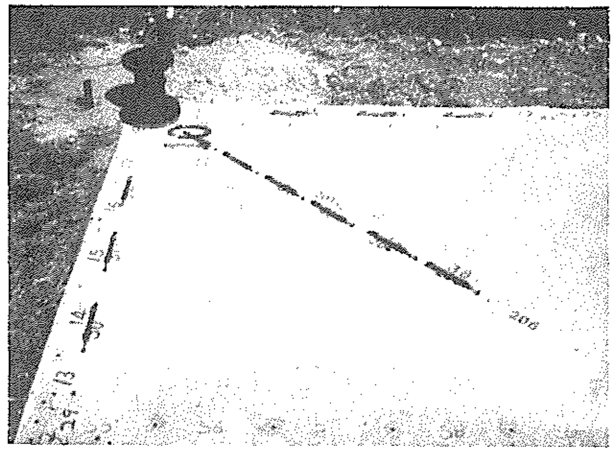


FIGURE 15.—ARRANGEMENT OF STRAIN GAGES FOR A LOAD TEST AT THE CORNER OF A SLAB OF UNIFORM THICKNESS.

figure 16. This instrument was built especially for the project from a design developed by the Bureau in connection with a recent highway bridge research,[5] the principle of the instrument being the same as that of the clinometer loaned by the American Society of Civil Engineers for the tests of the Yadkin River Bridge.[6]

It consists of a rigid, horizontal steel frame carrying a very sensitive spirit level in its upper face and supported by a vertical leg at each end. One of these legs is of fixed length while the length of the other is adjustable by means of a fine pitch screw operated by a knurled hand nut at the top of the instrument. The amount of adjustment made with this nut is indicated in thousandths of an inch by a micrometer dial on the front of the frame. In order that the position assumed by the instrument when it is placed on the clinometer points shall always be the same, a third or steadying leg is provided, projecting at right angles from the center of the frame and turning down at the outer end where it terminates in an adjustable foot.

Small brass cylinders were grouted into holes drilled in the pavement surface at 10-inch intervals along the

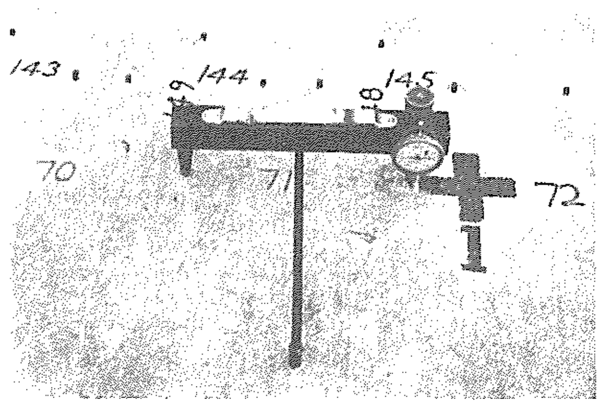


FIGURE 16.—SPECIAL 10-INCH CLINOMETER USED FOR MEASURING SLAB DEFLECTIONS. THE INSTRUMENT IS SHOWN RESTING ON SMALL BRASS CLINOMETER POINTS SET INTO THE SURFACE OF THE PAVEMENT.

lines of desired deflection measurements. The upper or exposed face of each of these contained a small verti-

[5] Application of the Freyssinet Method of Arch Construction to the Rogue River Bridge in Oregon, by Albin L. Gemeny and Conde B. McCullough. Technical Bulletin No. 2 of the Oregon State Highway Commission, Salem, Oreg.

[6] Loading Tests on a Reinforced Concrete Arch, reported by Albin L. Gemeny and W. F. Hunter, PUBLIC ROADS, vol. 9, no. 10, December 1928.

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cally drilled hole and also a narrow horizontal groove or slot with beveled edges. The direction of the slot was made parallel to the long axis of the clinometer frame when the instrument rested on the points. The lower ends of the 2 main clinometer legs are sharp-pointed cones and in setting up the instrument 1 of the legs is set in the drilled hole in the top of 1 of the clinometer points and the other leg is placed in the slot in the adjacent point. Any expansion or contraction of either the instrument or the concrete causes only a very slight horizontal displacement of the leg that rests in the slot and this movement produces no error in the measurement being made.

After the legs of the clinometer are properly set in the gage points the instrument is carefully adjusted to a level position by rotating the knurled hand nut. When level, the micrometer dial is read. The clinometer is then moved 10 inches to the next gage point and the operation repeated. Any deflection of the slab due to load or to warping will change the relative elevation of the clinometer points and this change will be measured by the difference in the adjustments necessary to level the clinometer as indicated by differences in the readings of the micrometer dial before and after the deflection occurred. The operation is simply one of precise leveling along the line of installed points. While the micrometer dial reads directly in thousandths of an inch it was found practicable to estimate ten-thousandths. The design of the adjusting mechanism is such that thread wear and backlash cannot introduce an error in successive measurements.

Benchmarks or reference points completely independent of the pavement were used to fix the datum for the pavement surfaces.

TEMPERATURES MEASURED WITH THERMOCOUPLES

In practically all of the load tests it was necessary to reduce the influence of slab warping to a minimum. It was found that if the slab was kept shaded from all direct sunlight and covered with several inches of dry straw, the temperature differential between the upper and lower surfaces became negligible and the warping of the slab was so small that its influence on stress was not important. Therefore, these precautions were taken in all tests as a matter of regular procedure. The shade and straw covering are shown in figure 11.

Observations to determine the effects of the temperature and moisture conditions within and surrounding the test sections were started soon after the pavement was laid and have been continued to the present time. These observations included extensive temperature measurements, moisture determinations, measurements of the changes in size and shape of the slabs resulting from temperature and moisture variations, and measurements of the strains caused by these variations in various parts of the slab structure.

When the test sections were built a number of resistance coil thermometers were placed in the slabs at selected points to furnish the temperature data then thought necessary. The original installations proved to be inadequate in extent and several of the resistance coils ceased to function for some reason that could not be determined. It was found also that the coils used had a time lag in their operation that was very undesirable for the work to be done.

It became necessary to make other provision for measuring the temperatures in the concrete. The plan adopted was to build two small slabs of concrete of the same materials and proportions as were used in the test sections and to install in these copper-constantan thermocouples for temperature determination. These slabs were each 4 feet square and one was 6 and the other 9 inches in depth. The thermocouples were installed in the center of the slab area. Before placing the concrete two thermocouples were placed in the subgrade under each small slab at depths of 2 inches and ⅛ inch respectively and, as the concrete was being placed, additional thermocouples were placed at 1-inch intervals from the bottom of each slab to the top. With this installation it was possible to determine not only the differential existing between the upper and lower surfaces but also the complete temperature gradient from one to the other.

Thermocouples were also placed at the top and bottom surfaces of the four constant-thickness slabs.

The "average" temperature of the pavement slabs as used in connection with the expansion and contraction measurements was developed from the data obtained with the thermocouple installations in the small slabs. A pavement slab having a thickness of 6 inches or 9 inches was assumed to have an "average" temperature equal to the mean temperature of the small slab of the same thickness and the "average" temperature of sections having a thickness between 6 and 9 inches was obtained by interpolation, assuming a straight-line variation between the mean temperatures of the 6-inch and 9-inch slabs.

MICROMETERS USED TO MEASURE CHANGES IN LENGTH

Measurements were made to determine the extent of both the daily cycle and the annual cycle of dimensional changes in the slabs. These measurements served to show the magnitude of the changes in slab dimensions that were caused by the daily and annual variations in temperature and moisture content, and they also provided a means for determining the relative restraint to expansion and contraction offered by the various joint designs.

To determine the absolute changes in length of the slab sections, the movements of the slab ends with respect to fixed reference points were measured with a micrometer, while the degree of restraint offered by the joint designs was determined by comparing the movement at these joints with that at the free ends of the same slab.

The fixed reference points referred to were installed in concrete posts cast in heavy foundation blocks several feet below the surface of the ground, the posts themselves being completely protected from lateral earth pressure.

Figure 17 shows the 7-inch micrometer built for this purpose, together with the invar reference bar used for a standard of length in these measurements. The guaranteed coefficient of thermal expansion for this material is 0.8×10^{-6} per degree centigrade. Its change in length for air temperature ranges is so small that for the purpose of the tests its length could be considered as being constant throughout the year.

The measurements with this micrometer were made between the tips of conical gage points of stainless steel set horizontally in the upper ends of short steel posts cemented into the slab surface, one on either side of the joint (or slab end) and approximately 7 inches apart. These details can be seen in figure 17.

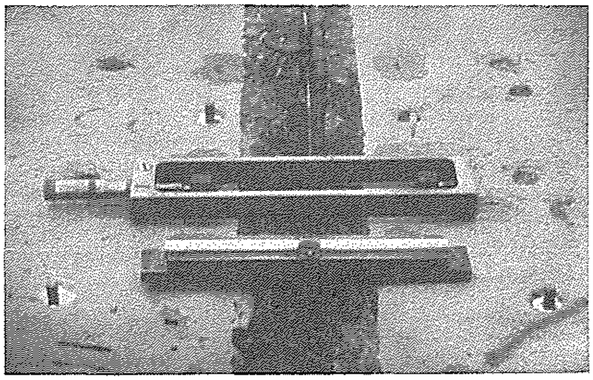


FIGURE 17.—THE SPECIAL 7-INCH MICROMETER FRAME AND THE INVAR REFERENCE BAR USED FOR MEASURING THE EXPANSION AND CONTRACTION OF THE TEST SECTIONS.

Some additional data on the length changes occurring in the pavement slabs were obtained with 3 electric telemeters that were embedded in 3 of the slab panels at the time the concrete was placed. These instruments[7] were installed at mid-depth at the center of the longitudinal axis of three of the 10- by 20-foot panels. They were intended to provide data in connection with one of the designs but, because of certain difficulties that will be discussed later, they failed to do so. They did, however, furnish valuable information regarding elongation caused by both temperature and moisture.

ACTION OF SLABS DURING WARPING STUDIED

The magnitude of the temperature warping in the various sections was determined on numerous occasions over a period of about 3 years. Measurements were made to determine the warped shape of an entire 10- by 20-foot panel. The degree of restraint to free warping caused by the different joint designs was studied at selected points by means of measurements of warping over a limited area near the joint involved.

The necessary temperature data for these studies were obtained from the thermocouple installations and the shape of the warped surface was determined by clinometer measurements along the lines of points shown in figure 9. The measurements of warping with the clinometer were referenced to fixed points or bench marks set into the earth shoulders. Because of the time that was necessary to take readings around the entire perimeter of a 10- by 20-foot slab, frequently the shape of the slab changed sufficiently to develop a considerable error of closure. Care had to be taken to make these long series of measurements at a time when the conditions producing the warping were not changing too rapidly.

In the study of warping some attention was given to the strains in the concrete produced by the forces set up by the warping action of the slab itself and also to the relative strains produced in a slab of given design by a given load when the slab was both warped and unwarped. The procedure for the loading and strain measurement involved only one feature that was different from the rest of the strain measurements. To produce warping the straw cover was removed and the pavement was exposed to the direct rays of the sun for a number of the tests.

[7] For a description see Technologic Paper No. 247, U. S. Bureau of Standards, A New Electric Telemeter, by Burton McCollum and O. S. Peters.

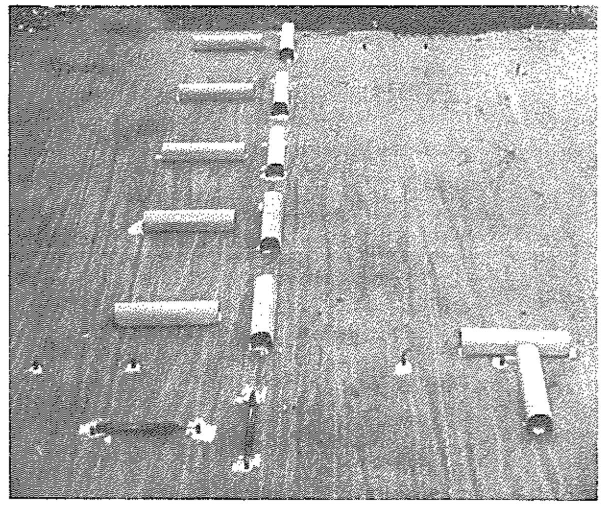


FIGURE 18.—A STRAIN GAGE INSTALLATION FOR MEASURING STRAINS CAUSED BY WARPING.

The very nature of these tests required a wide range of temperature and strain observations that were continuous over the complete warping cycle. It was thought desirable, therefore, to protect the strain gages from direct sunlight with small semicylindrical covers as shown in figure 18. These covers, to be described in a subsequent paper, permitted the free circulation of air around the gages but were so constructed as to resist heat absorption. The purpose of this protection was to keep the gages at the same temperature as the concrete in the pavement.

DIFFICULTY ENCOUNTERED IN MEASURING PRESSURE OF SLABS AGAINST THE SUBGRADE

A group of nine soil pressure cells was placed beneath the 6-inch and 9-inch slabs of constant thickness, arranged in the pattern of a 90° cross. These cells were installed in the subgrade with their diaphragm side down in carefully scraped recesses so that perfect bearing was obtained. The recesses were sufficiently deep to cause the back of the cell to be flush with the general level of the subgrade. The concrete of the slab was placed on these cells but no anchorage was provided to fix the cells to the concrete. The purpose of these installations was to obtain data on the distribution of a load to the subgrade by the two thicknesses of slab. Unfortunately, these data were never obtained as the cells under both slabs failed to record pressure before the load tests were made.

For a short time after the construction of the sections the cells operated and such pressure data as were obtained during this period indicated the normal fluctuations as the slab warped during the day. In the course of a few weeks the cells ceased to record pressure, indicating that a separation between the bottom of the slab and the back of the cells had occurred. Whether this was due to a settlement of the cells or a swelling of the subgrade that raised the slab more than it did the cells is not known. The cells were not embedded because it was desired to maintain the full flexural strength of the slab. Perhaps some anchorage attachment on the backs of the cells that would have held them to the slab without reducing the slab strength would have made a better installation, but this is by no means certain.

AUXILIARY TESTS MADE

A consideration, either theoretical or experimental, of the structural action of a concrete pavement slab lying on an earth subgrade, necessitates either assumptions regarding or a knowledge of the physical properties of the concrete and of the subgrade. Obviously, definite data developed by tests are to be preferred to any assumptions that may be made.

The auxiliary tests made in connection with this investigation were planned to develop information concerning:

(*a*) The concrete—
1. Strength in compression and in flexure.
2. Stress-strain relation in compression and in flexure.
3. Effect of moisture content on the strength and elastic properties.
4. Thermal properties.

(*b*) The subgrade—
1. Resistance offered by the subgrade to horizontal slab movement.
2. Resistance offered by the subgrade to vertical slab movement (or deflection) including an attempt to evaluate the support offered to the slab by the subgrade under the test sections.

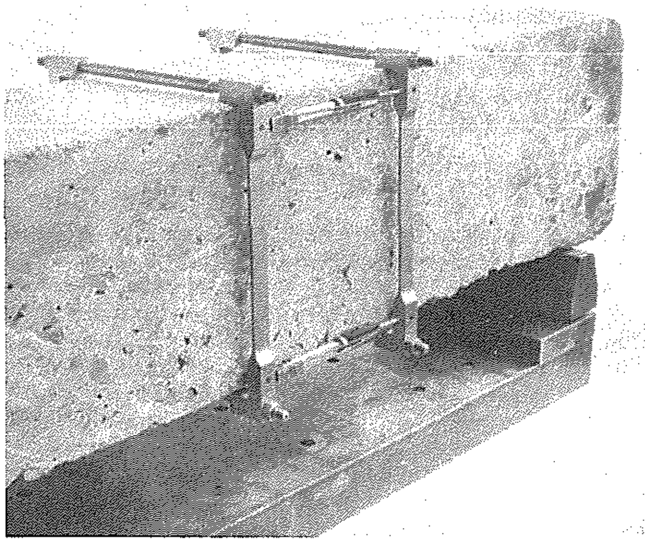


FIGURE 19.—APPARATUS USED FOR DETERMINING THE STRESS-STRAIN RELATION FOR CONCRETE IN FLEXURE.

Although the auxiliary tests are grouped here in the description of test procedure, most of the tests were carried out as separate investigations and the discussions of the data obtained will appear in the particular parts of the report with which they are concerned. In one or two cases these collateral investigations proved to be sufficiently comprehensive and general to warrant a more detailed presentation elsewhere and, in those cases, only the facts that have a direct bearing on the major research will be included in this report, leaving the detailed description of what was done to a separate report.

STRESS-STRAIN RELATION AND COEFFICIENT OF EXPANSION OF CONCRETE DETERMINED

The strength tests of the concrete were made to determine the ultimate strength in compression and in flexure so that safe working stress limits might be fixed.

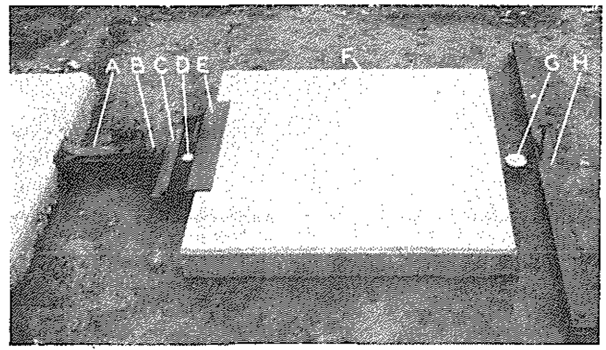


FIGURE 20.—APPARATUS USED FOR DETERMINING THE RESISTANCE OF THE SUBGRADE TO HORIZONTAL MOVEMENT OF THE PAVEMENT.

The procedure followed in making these tests was simply that of good testing practice and included no unusual features. It will not be described therefore. The data obtained have already been given on page 148.

The determination of the stress-strain relations in compression and in flexure was a matter of considerable importance because of the direct application of the data to the analysis of the slab tests. Effort was made to have the tests comprehensive as to scope and precise as to technique to develop thoroughly reliable data. The stress-strain relation in compression was determined from tests on cores, using an extensometer of the Martens' type.[8] These cores were drilled from the small sections provided for the purpose and the program included tests on both wet and dry specimens.

For the determination of the stress-strain relation in flexure on the sawed beams, use was made of equipment designed in the bureau for this particular purpose and shown in figure 19. This apparatus consists of two frames that are clamped around the flexure specimen either side of the midspan and far enough apart to permit the installation of a recording strain gage near the top and bottom on each side of the beam. Each frame makes contact with the specimen at two points on each side of the beam. These points are small chisel-edged studs projecting from the inside of the frame directly opposite the points where the ends of the strain gages make contact with the frame and are held tightly to the specimen by tightening the transverse tie bolts of the frame. When the specimen is flexed the strain gages record the amount of length change that occurs between the frames on either side of the beam and in both tension and compression. The load was applied at the third points of the span and the deflection of the specimen at mid-span was measured with micrometer dials arranged on either side. The equipment for load application and for deflection measurement was omitted in figure 19 so that the details of the strain-measuring apparatus could be seen to better advantage.

Flexure specimens in both the wet and dry states were included in these tests. Since the concrete in the pavement slabs contained moisture and the moisture content of the concrete was assumed to vary, it seemed desirable to develop any information possible relative to the variations in moisture in the pavement slabs and also as to the effect of moisture condition on the physical properties of concrete similar to that used in making the slabs. The moisture content of the slabs was determined by the rather crude but direct

[8] For a description see Handbook of Testing Materials, by Prof. Adolf Martens, 1st ed. 1899 (authorized translation by Gus C. Henning).

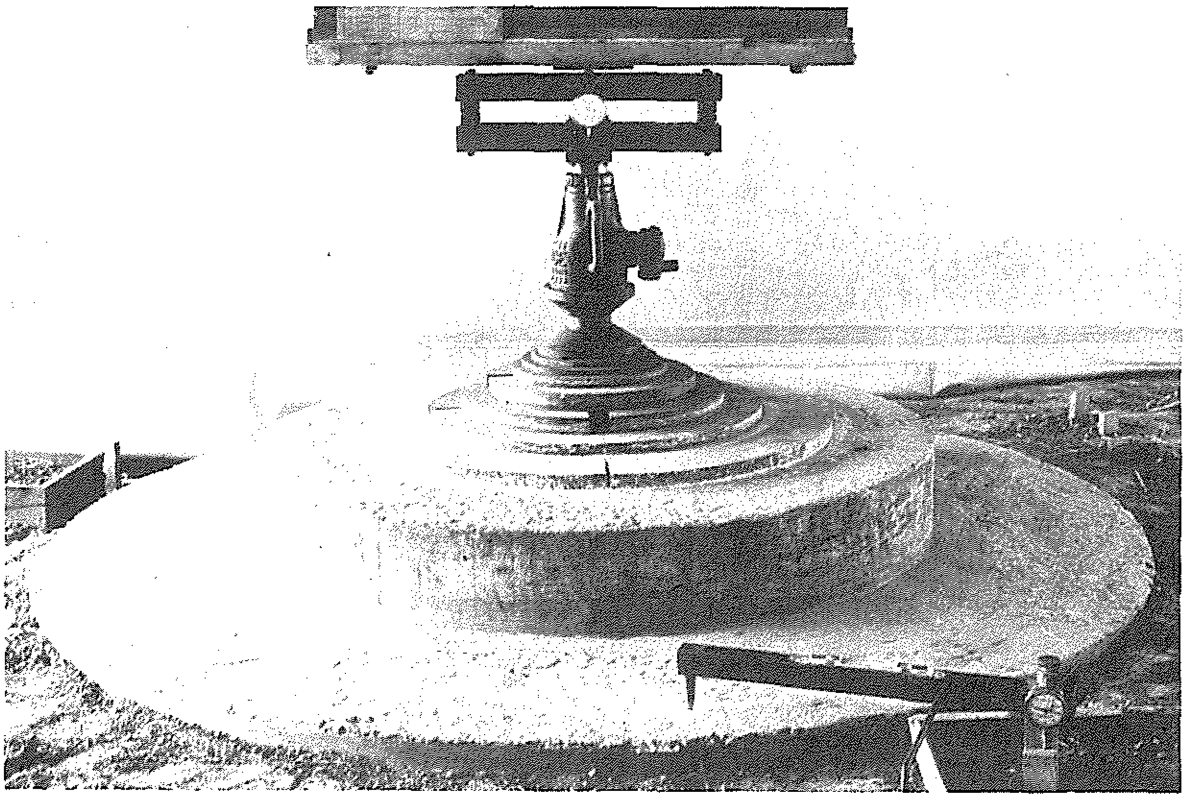


FIGURE 21.—APPARATUS USED FOR DETERMINING THE RESISTANCE OF THE SUBGRADE TO VERTICAL DISPLACEMENT.

method of weighing and drying fragments of concrete broken from the short sections of pavement provided for test specimens of all kinds. The moisture content of the test specimens was also determined by weighing.

The coefficient of thermal expansion of the concrete was determined by a method developed for these tests. Concrete of the same materials and proportions as were used in the pavement sections was placed in a cylindrical mold 12 inches in diameter and 24 inches high and made of a very light-gage sheet copper. This mold was in reality a large can with a watertight bottom and top. An electric telemeter was installed at the midpoint of the longitudinal axis of the cylinder. The concrete was introduced through an opening in the top of the can and as soon as the surface of the concrete was within about half an inch of the top of the mold this opening was sealed off by soldering on a cover plate. This effectively retained within the copper jacket all of the moisture originally in the concrete.

After the concrete had set and the heat of setting had been dissipated, the sealed specimen was placed in an insulated water bath, the temperature of which was placed at different levels within the range of normal air temperatures, and maintained at each until complete temperature equilibrium was established. The unit changes in length accompanying the various changes in temperature were measured with the embedded telemeter. This furnished a simple and apparently satisfactory method for the determination. The mass of concrete was large enough to be representative and the usual difficulties due to moisture changes in the specimen were avoided by using a watertight envelope.

RESISTANCE OF SUBGRADE TO HORIZONTAL AND VERTICAL MOVEMENT STUDIED

The tests made on the subgrade in place were planned with two objects in mind: First, to determine the character of the resistance that the subgrade offered to horizontal movement of the slab; and second, to find out what resistance the subgrade offered to vertical movement of the slab (such as deflection under load) and, if possible, to develop some means for evaluating this resistance in terms that would be applicable to pavement design.

The first tests were made with 4 slabs, each 4 feet square and 6 inches thick, placed on the same subgrade as the large test sections. The general method of test was to move these small slabs horizontally, very slowly, alternately forward and backward by total amounts that equalled approximately the annual cycle of expansion and contraction of the large pavement sections. The thrust necessary to cause horizontal movement and the magnitude of the displacement caused by the thrust were measured from the time that the first detectable movement took place until the total desired displacement had been attained.

Figure 20 shows the apparatus set up for one of these tests. In this figure, A is a jack used to develop the thrust, B is a spherical bearing block that controls the line of the thrust, C is a steel beam whose deflection, as indicated by the micrometer dial D, measures the magnitude of the thrust, E is the frame supporting the steel beam and the dial, F is the small slab, G is the micrometer dial that measures the horizontal displacement of the slab, and H is a rigid member used to support the dial G.

The second group of subgrade tests may be described generally as load-deflection tests on circular bearing plates in intimate contact with the subgrade. The diameters of the plates used were 2, 4, 6, 8, 12, 16, 20, 26, 36, 54, and 84 inches. The two larger sizes were concrete disks cast on the subgrade, the others were steel plates bedded in a ½-inch layer of mortar placed on the subgrade, but separated from it by a layer of waterproofed paper so that the moisture content of the soil would not be altered.

Figure 21 shows the loading equipment set up over the 84-inch bearing plate. It will be observed that the loads were applied on these plates in the same manner as on the pavement sections. For the very small plates a system of dead-load increments was used. Vertical displacements were measured with respect to fixed reference points by means of the clinometer shown in the figure. Generally these measurements were made at three points 120° apart around the periphery of the plates, although in some cases the measurements were made at the midpoint. In general the loads applied were such that a vertical displacement would be produced that approximated in magnitude the observed deflections of the pavement slabs. The deflection of a slab corner, for example, was found to be approximately 0.05 inch, the slab edge 0.02 inch, and the interior of the panel 0.01 inch for a load that did not overstress the concrete.

In making the tests on the subgrade, vertical displacements of 0.005, 0.010, 0.020, 0.035, and 0.050 inch were obtained in nearly all cases. For each displacement value, several loads were applied until a given load produced the same deflection each time it was applied. Each load was applied for 5 minutes and then released for 5 minutes before being applied again. In planning these tests it seemed desirable to arrange the procedure so that the subgrade would be subjected to the same conditions as would obtain under the loaded pavement slab, as nearly as possible. For this reason the rate of loading and the orders of magnitude of subgrade deformation were made to correspond very closely as noted above. The moisture content of the soil was determined before and after each test and protection from sunshine and rainfall was provided.

THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

Reported by L. W. TELLER, Senior Engineer of Tests, and EARL C. SUTHERLAND, Associate Highway Engineer

PART 2.—OBSERVED EFFECTS OF VARIATIONS IN TEMPERATURE AND MOISTURE ON THE SIZE, SHAPE, AND STRESS RESISTANCE OF CONCRETE PAVEMENT SLABS

IT HAS been known for more than half a century that when the temperature or the moisture content of portland cement concrete is changed a corresponding change in volume occurs. Since the pioneer work of Bauschinger [1] in this field many other research workers have contributed to the store of information concerning the extent of these changes and the factors that affect them.

As concrete began to be generally used as a paving material engineers gradually came to appreciate that, under some conditions at least, these volume changes were of such magnitude that provision for them was necessary in the pavement design.

Because of the relative physical proportions of a pavement slab, it was but natural that the first manifestation of volume change to be noticed was expansion and contraction in a longitudinal direction. More than 20 years ago the Bureau studied the expansion and contraction of concrete pavements [2] in an effort to discover the laws that govern such movements.

RESULTS OF EARLY INVESTIGATIONS INADEQUATE FOR PAVEMENT DESIGN

The early investigations were of value, first, for the information that they developed, and, second, because each one served to arouse more wide-spread interest in the subject among highway engineers. As a result of this interest new researches were begun and it was soon discovered that differences in temperature between the upper and lower surfaces of a pavement slab would cause it to warp to a measurable degree,[3] and further, that a similar warping could be brought about by creating a moisture differential between the two surfaces of a concrete slab.[4]

These discoveries were important because the exposure conditions to which pavements are subjected are severe; and expansion, contraction, and warping may be expected daily throughout the life of a concrete pavement slab. If the slab were completely free to move and had no weight it would change in size and shape without restraint and no stress would result. Because of its weight and intimate contact with the subgrade, however, restraint in some degree is always present and every attempt of the slab to change either its size or shape develops stress within the structure.

Other early tests indicated the approximate amount of resistance encountered when the pavement slab expands or contracts longitudinally along the subgrade,[5] and the development of this information made it possible to estimate approximately the tensile or compressive stresses induced in the concrete by this form of restraint.[6]

More recently a careful analysis has been made of the theoretical stress conditions resulting from slab warping caused by temperature differences within the concrete and assumed conditions of restraint.[7] This analysis supplies the means for estimating the stresses that occur under certain conditions of temperature warping.

EXTENSIVE PROGRAM OF STUDY OF MOISTURE AND TEMPERATURE EFFECTS UNDERTAKEN

It will be noted that each of the researches referred to has contributed some information that represents a distinct step toward a better understanding of the reasons for observed slab behavior. However, the engineer who undertakes to apply the information obtained by these previous studies to the design of a pavement finds that there are numerous gaps and uncertainties in the data. When the pavement-design project at Arlington was being planned, it was decided to include a study of the extent of the moisture and temperature changes that occur in pavements in this locality and to determine, as fully as possible, the effect of those changes on the pavement slab. The rather extensive program of observations outlined in part 1 of this series was developed with this object in mind.

The observations that have been made may be grouped according to purpose as follows:

1. A study of the extent of the temperature changes that occur in the various parts of concrete pavement slabs.
2. A study of the longitudinal expansion and contraction of pavement slabs caused by temperature changes and changes in moisture content.
3. A study of the resistance offered by the subgrade to horizontal slab displacement and of the stresses developed in the slab by this resistance.
4. A study of the warping of concrete pavement slabs resulting from variations in temperature and of the stress conditions that result from warping.

The tests and observations reported in this paper were made on the 10 full-size pavement slabs and, in general, the data cover periods of time of from 1 to 3 years. The complete description of the test sections and of the methods of test employed has already been given in part 1 of this series.[8]

[1] Tests of Different Portland Cements, by J. Bauschinger, Mitt. Mech. Tech. Lab., Tech. Hochschule (Munich) v. 8, 1879.
[2] The Expansion and Contraction of Concrete and Concrete Roads, by A. T. Goldbeck and F. H. Jackson, Bull. 532, U. S. Department of Agriculture, October 1917.
[3] The Bates Experimental Road, by Clifford Older, and Highway Researches and What the Results Indicate, by A. T. Goldbeck, papers in the Proc. American Road Builders' Association, 1922.
[4] Effect of Moisture on Concrete, by W. K. Hatt, PUBLIC ROADS, vol. 6, no. 1, March 1925.
[5] Friction Tests of Concrete on Various Subbases, by A. T. Goldbeck, PUBLIC ROADS, vol. 5, no. 5, July 1924.
[6] The Interrelationship of Longitudinal Steel and Transverse Cracks in Concrete Pavements, by A. T. Goldbeck. PUBLIC ROADS, vol. 6, no. 6, August 1925.
[7] Analysis of Stresses in Concrete Roads Caused by Variations in Temperature, by H. M. Westergaard. PUBLIC ROADS, vol. 8, no. 3, May 1927.
[8] The Structural Design of Concrete Pavements, Part 1, by L. W. Teller and Earl C. Sutherland, PUBLIC ROADS, vol. 16, no. 8, October 1935.

TEST PROCEDURE DESCRIBED

Temperature measurements.—The temperature measurements were made in large part on two small slabs that were constructed especially for this purpose some months after the 10 test sections were constructed. These slabs, shown on the cover page, are 4 feet square, one being 6 and the other 9 inches in thickness. They were constructed on the subgrade adjacent to the test sections and the materials and proportions used in the concrete were exactly the same as were used for the large slabs.

The resistance thermometers originally installed in the large slabs were not entirely satisfactory for several reasons, so when the slabs for temperature measurements were built copper-constantan thermocouples were installed. The thermocouples were located at the centers of the slab areas and at 1-inch intervals throughout the depth, and also at both upper and lower slab surfaces. Two thermocouples were also placed in the subgrade beneath each slab, one about one-fourth inch and the other about 2 inches below the surface of the subgrade.

With these installations it was possible to determine quite completely the temperature conditions that existed throughout the depth of the slab. From these data one may estimate with considerable accuracy both the average temperature of the slab and the temperature differential between its upper and lower surfaces. In estimating the temperature conditions for the various pavement sections from the data obtained from the thermocouples, two assumptions were made: First, that the temperatures in the pavement slab were the same as those found in the slab of equal thickness on which temperature measurements were made; and second, that the temperature of slabs having thicknesses intermediate between 6 and 9 inches could be predicted by a straight-line interpolation between the temperatures measured in the 6- and 9-inch slabs.

The average temperature of the slab was obtained by averaging the temperatures measured at the several points throughout its depth. The temperature differential was estimated by drawing a mean curve for the data which were plotted to show the variation in temperature through the slab depth, and from this mean curve taking the indicated temperatures for the upper and lower surfaces of the pavement.

The temperature measurements were usually made in connection with measurements of slab expansion or of slab warping. They cover a period of approximately 2 years and, while not continuous, were planned so as to develop full information concerning both the daily and yearly temperature cycles that occurred during this time.

During the latter part of the investigation it became desirable to obtain data that would show the relation between the temperature differentials in thickened-edge cross sections and those in sections of constant depth. No means for obtaining these data were provided in the original construction and it was necessary to drill small holes to the proper points in the slab cross section, insert thermocouples in the bottom of these holes, and then backfill with cement-sand mortar. Thermocouples were placed in this manner near the upper and lower surfaces of the slab and at the third points of the slab depth, and this arrangement made it possible to estimate very closely the differential in temperature that existed at the several positions on the test sections at which the measurements were made.

Two types of information were desired: First, a knowledge of how the temperature differential varies along the different types of cross section; and second, a comparison of the differential in the edge of a 9-6-9 thickened-edge section with those which occur under the same conditions in the edges of constant-thickness sections of different depths. For the first purpose, the thermocouple installations were made at distances of 2, 18, and 36 inches from the free edge, while for the second only the installation at 2 inches from the edge was used. In each case the thermocouple units were placed at the various depths as described above.

Moisture measurements.—Since moisture variations within the concrete are known to cause physical changes in the size and shape of the pavement slab that are similar to those caused by temperature variations, it was desired to obtain quantitative data concerning the changes. Such data are difficult to obtain, however, because there is no reliable method for determining the moisture content of concrete except by weighing and this is usually not feasible.

In this investigation the moisture content of the concrete was determined by weighing and drying fragments broken from the short slabs originally provided for the purpose of furnishing samples. No means was found for determining differences in moisture content at various depths in the slabs so that no data were obtained to show the nature of the moisture gradient at various seasons of the year. However, measurements were made of the seasonal warping presumably caused by such a moisture gradient. Measurements were also made from which it was possible to determine the direct longitudinal expansion and contraction caused by factors other than temperature change, and from these data it was found possible to estimate the extent and period of the yearly variation in slab length attributable to moisture change alone.

The method of measuring the warping of the slabs under the action of these seasonal moisture changes is described later.

Measurements of the expansion and contraction of the pavement sections.—Practically all of the slab-expansion data were obtained by measurements on the 6-inch constant-thickness section. This section was separated from the other nine by a space of several feet so that there was no possibility of its being affected by the movements of the other slabs. Fixed reference points of the type shown in figure 1 were constructed at each end of the 40-foot section, and the horizontal movement of the slab ends with respect to these reference points was measured with the special micrometer described in the previous paper. The movement at the transverse joint was measured at the same time. The frame of the micrometer is made of soft steel and its length changes considerably with temperature changes. Corrections for this were made by referring all measurements to an invar standard, the length of which changes less than 0.001 inch for a temperature change of 100° F.

The procedure followed was first to place the micrometer and reference bar out of doors and allow them to reach a condition of temperature stability. The measurements at the slab ends were then made and any change in the length of the micrometer, as indicated by the reference reading on the standard bar, applied as a correction to the measured slab displacement.

The expansion and contraction measurements were made once or twice each month over a period of about 4 years. Each set of observations was started in the early hours of the morning, before the minimum slab length was attained, and was continued until late in the evening, long after the maximum expansion had occurred. Measurements were made at 2-hour intervals during this time.

This daily cycle of observations and the related temperature data made possible the determination of the maximum and minimum slab lengths, the range in average slab temperatures that caused the length changes, and from these the length change per degree of temperature change for the slab as a whole.

The series of observations obtained over the yearly periods made possible the determination of not only the annual cycle of length change resulting from temperature but also those resulting from other causes, such as variation in moisture content.

In addition to the measurements at the ends of the 6-inch constant-thickness slab, some data were obtained from telemeters (resistance strain gages) that were installed at the center of one 10 by 20-foot panel in each of two other test sections at the time of construction. These instruments were placed in a longitudinal direction and in the neutral plane of the slab. They were in a position to receive direct longitudinal deformation but not deformation caused by bending of the slab. Each telemeter is provided with an electrical resistance thermometer housed within its shell with which the temperature within the telemeter can be measured at the same time that the deformations are determined.

The schedule of observations with the telemeters was the same as that followed in the measurement of the movement of the slab ends. The telemeter observations covered a period of about 2 years and coincided with the period of the micrometer measurements for only about 8 months, as shown later in figure 14. Failure of the resistance thermometer elements caused the abandonment of the telemeter observations.

Measurements of subgrade resistance to horizontal slab movement.—The measurement of the resistance offered by the subgrade to the horizontal displacement of the pavement slab was determined in tests made with small slabs cast upon the subgrade for this particular purpose, as described in part 1 of this series of papers. (See pt. 1, fig. 20.)

These slabs, each 4 feet square, were moved horizontally with a powerful, smoothly operating mechanical jack. The force required to cause the movement and the amount of the displacement were determined.

Before the tests were begun, a preliminary investigation was made to find a desirable test procedure. As a result of this preliminary work it was decided:

1. That the slabs should be moved at a very slow rate, one comparable with that which obtains in a pavement during the daily cycle of temperature variation.
2. That the slabs should be moved alternately in opposite directions.
3. That the magnitude of the displacement used in the tests should be comparable to the movements that occur at the ends of pavement slabs of moderate length under the influence of daily temperature variations.

This procedure subjected the subgrade under the small test slabs to the same manipulation that it would receive under the end of a pavement slab.

In a number of the tests the horizontal displacement of the subgrade at various depths was measured as the slab was displaced. In order to measure the movement of the subgrade, small recesses were cut into the earth just in front of the slab and micrometer dials were placed in these recesses. The dials were supported on a rigid member that was supported in turn by the subgrade at some distance from the slab. The stems of the dials bore against small wooden plugs inserted in the vertical face of the subgrade at the desired depths. The dials were usually placed at three points across the front or leading edge of the slab.

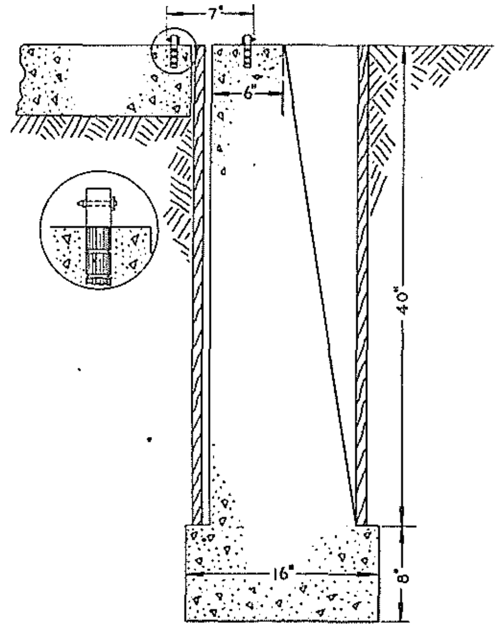


Figure 1.—Reference Point for Measuring Change in Length of Concrete Slab.

A few tests were made to determine the influence of slab thickness (or weight) on the resistance offered to horizontal displacement. For these tests four small slabs of 2-, 4-, 6-, and 8-inch thicknesses were constructed on the regular subgrade. The method followed in testing with these slabs was the same as that used with the 6-inch slabs previously described.

Warping measurements.—Observations have shown that under usual conditions of exposure there is seldom a time when the temperature of a pavement slab is uniform throughout. Either the air is warmer than the subgrade or vice versa, and heat is being conducted by the slab either to the subgrade or away from it. Because concrete is a relatively poor conductor of heat, the transfer of heat through a pavement of ordinary thickness takes time, and a differential in temperature is created within the various parts of the structure. Each element of the concrete attempts to adjust its volume to its own particular temperature.

As previously stated, if the slab were completely free and without weight it would assume a distorted shape without resistance and no stresses would result. Actually, almost no part of the slab is free to adjust itself and, while measurable distortion occurs, restraint caused by the slab weight and by the subgrade reactions is always present. It is apparent that the warping of a pavement is not a simple phenomenon and the distortion that is measured is the result of a very complex system of forces.

Warping has two important effects on the structural action of a pavement slab. In the first place, the distortion that occurs alters the condition of subgrade support and thus affects the magnitude of the stress that will be produced by a given wheel load. In the

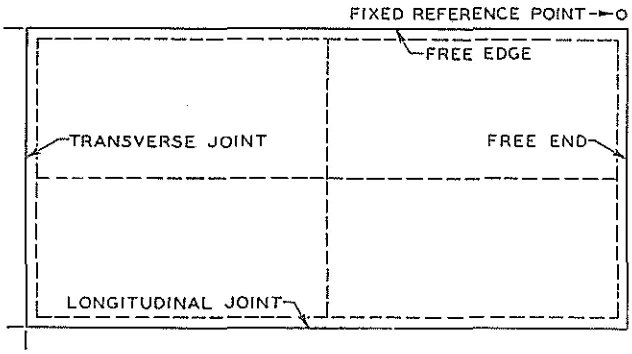


Figure 2.—Plan of 10- by 20-Foot Section Showing the Lines Along Which Deflections Were Measured.

second place, because of the weight of the slab, the warping in itself causes important stresses within the slab structure. Both of these actions place limitations on the maximum wheel load that may be carried by the pavement and, for this reason, any information concerning them is of value.

The studies of warping that were a part of this investigation were divided into three groups, as follows:

1. Measurement of the extent and characteristics of the actual warping that occurs in pavement slabs of conventional designs.
2. Determination of the magnitude of the effect of warping on the critical stresses caused by a given load applied at different points on the pavement slab.
3. Determination of the magnitude and distribution of the stresses caused by restraint to warping in the various parts of the pavement slab.

In order to determine the changes in shape that occur when the pavement warps, it is necessary to measure the vertical displacement of a great many points on its surface. In this investigation the measurements were confined to one quadrant or panel of the test section, and in this quadrant the shapes of the two principal axes and of all of the boundaries were determined. The lines along which the shape determinations were made are shown in figure 2. It will be noted that there is one free edge and one free end to the panel, and that the other edge and end are attached to the other panels of the test section by the longitudinal and transverse joints.

Clinometer points were set at 10-inch intervals along the lines shown in figure 2, and the shape of the slab was determined by measuring successively the slopes of these intervals with the 10-inch clinometer. The measurements were started from a bench mark or reference point located on the shoulder near the pavement, and were carried forward along the line of clinometer points exactly as a line of levels is run.

The measurements of the temperature warping were usually made on days when a large variation in pavement temperature occurred, three complete sets of observations being made. The first measurements were made in the morning at a time when the observed temperatures in the upper and lower surfaces of the slab were equal. Under these conditions the slab was assumed to be flat and all subsequent measured distortions were referred to this plane as a base. The second set of measurements was made during the early afternoon, at the time when the temperature of the upper surface of the pavement was at or near the maximum and the slab was consequently warped downward at the edges. The final measurements were made early the following morning when the temperature of the upper surface of the pavement had reached its minimum value and the maximum upward movement of the edges of the pavement slab had occurred. The three sets of observations were sufficient to determine quite completely the shape of the slab under the conditions of maximum upward and downward temperature warping.

Some additional measurements were made for the purpose of establishing the relative movements of various parts of the slab, in tests that covered the full 24-hour cycle of temperature changes. Because of the frequency of the observations and of the time required to make one set, the measurements were limited to determinations of the transverse curvature of the slab panel along the free end, the center line, and the transverse joint. The vertical displacement at a few points was also measured with micrometer dials supported on long steel stakes driven into the subgrade. Most of these measurements were made at slab corners, and a few at the center of the edge. The measurements were made for the purpose of obtaining additional data on the movement of slab corners during the daily temperature cycle and for comparing, hour by hour, the displacements for slabs of various thicknesses.

Measurements of the warping caused by seasonal changes other than temperature were made with the clinometer in much the same manner as the other warping measurements. These measurements were made only on the 9-inch constant-thickness section. They were made at intervals of about 1-month throughout the year at times when the thermocouples indicated that no temperature differential was present. Because of the relatively short period of time during which this condition usually obtained it was not possible to extend the clinometer measurements over the entire slab. For this reason the observations were limited to what were considered to be the most important regions.

Measurements were started from a bench mark and carried across the two ends and along the longitudinal center line of one of the panels. Since it was necessary to maintain the same datum throughout the year, special bench marks, the details of which are shown in figure 3, were installed near the edges of the slab on which these measurements were made. It will be noted that an electrical resistance thermometer is provided in the air space between the casing wall and the standard that carries the reference point. This permits a correction to be made for variations in the length of the standard resulting from seasonal temperature variations.

Measurements of the effect of temperature warping on load stresses.—The effect of the distortion of the pavement caused by temperature warping on the magnitude of the stresses produced by given wheel loads was studied by determining the critical stresses in the concrete for a given applied load with the slab flat and again in the condition of maximum upward and maximum downward warping. The loads were applied at the corner, the edge, and the interior of each slab upon which these tests were made.

The technique of applying the load and of measuring the strains was essentially the same as that followed in the other tests. The loads were applied to the slab through a bearing block 8 inches in diameter. Since a warped slab was a requirement of the test, it was necessary to arrange the loading tank so as to avoid shading the concrete and thus interfering with normal warping. This was done by moving the tank over the end of the adjoining section and placing a heavy I-beam

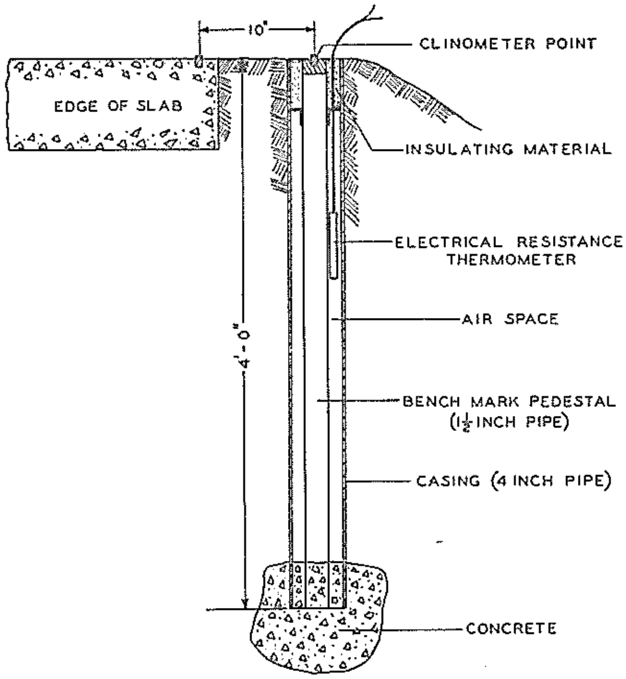


FIGURE 3.—DETAILS OF CLINOMETER BENCH MARK.

beneath and bearing against the tank with one end resting on a fixed support and the other end furnishing a reaction for the jack applying the load. Extension of the jack tended to raise the end of the I-beam and to lift the tank, the I-beam acting as a simple lever. The force exerted downward by the jack supplied the load to the test section. The equipment used for making these tests is shown in figure 4.

STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING DETERMINED

The magnitudes of the stresses created in the various parts of the pavement slab by restrained temperature warping were determined by comparing the deformations measured in the concrete at the points in question with the deformation at a point where there was little or no restraint.

Theoretically, at the edge of a pavement, there should be no warping stress in a direction perpendicular to the edge.[9] Practically, it is not possible to measure these strains at the edge, and in the test section the gages for this determination were placed with their centers 6 inches from the edge. A preliminary study showed that for slabs of these dimensions the stresses at the gage positions were all very small and that the least restraint exists at the mid-point of the free edge in the direction perpendicular to the edge. A strain gage placed at this point, perpendicular to the edge, will record most nearly the full deformation or volume change that the temperature change demands, while similar gages at other places will record different deformations, deformations that are modified by the magnitude of the restraint present at each gage position.

In the discussion that follows, measurements made with a gage in the position just described were considered as resulting from unrestrained warping and were used as a base for determining the amount of restraint existing at other parts of the pavement.

[9] See Analysis of Stresses in Concrete Pavements Due to Variations in Temperature, by H. M. Westergaard, Proc. Sixth Annual Meeting, Highway Research Board, 1927. Also see PUBLIC ROADS, vol. 8, no. 3, May 1927.

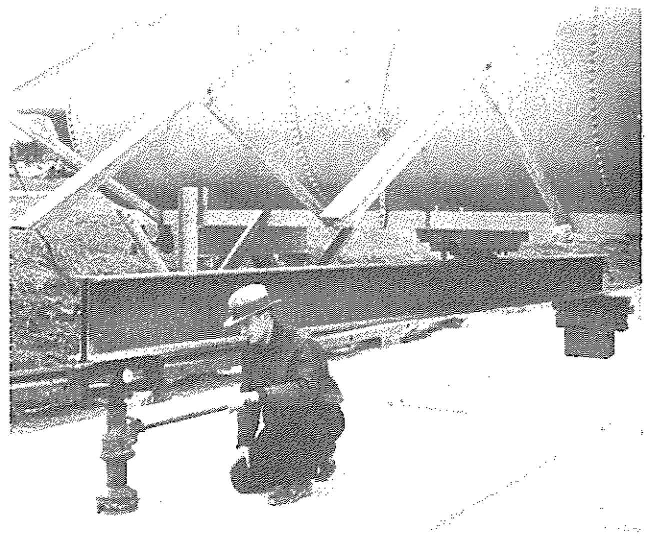


FIGURE 4.—METHOD OF APPLYING LOAD TO SLABS THAT HAD BEEN ALLOWED TO WARP.

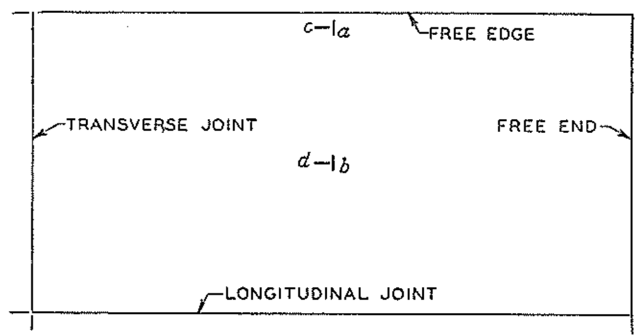


FIGURE 5.—PLAN OF 10- BY 20-FOOT SECTION SHOWING LOCATION OF THE FOUR TYPICAL STRAIN GAGES REFERRED TO IN THE DESCRIPTION OF THE METHOD OF DETERMINING RESTRAINED WARPING STRESS.

For example, in the plan of one of the 10- by 20-foot slabs shown in figure 5, four strain gage positions have been indicated by the letters *a*, *b*, *c*, and *d*. The direction of the line denoting the gage position shows the direction in which the deformation was measured. The deformation measured at *a* was considered to be the full deformation of unrestrained warping and the values measured at the other points were considered to be deformations that were modified by the restraint that existed. Any rise or fall of the temperature of the slab as a whole will cause equal volume changes at all of the gage positions and will not affect the determination of warping stresses to a measurable degree.

The four typical gage positions shown in figure 5 will be used to illustrate the method that was followed in making this study.

Since, at most points on the pavement slab, the concrete is stressed in more than one direction, consideration must be given to the effects of these other stresses on the deformations which are measured with the strain gage before the measured deformations can be converted into stresses. If an isotropic material is subjected to a force acting in a given direction, a certain unit deformation and a corresponding stress in the direction of this force will result. In addition, another deformation of lesser magnitude, of opposite sense and having a line of action perpendicular to that of the applied force, will be produced. The relation between the magnitudes of these two deformations is constant for a given material and is known as "Poisson's ratio." This constant provides the means for determining the effect of combined stresses.

If σ_x represents the unit stress in the direction x, then the unit deformation caused by this stress would be measured by the expression $\frac{\sigma_x}{E}$, in which E is the modulus of elasticity of the material.

Similarly, if σ_y represents the unit stress in the direction y (perpendicular to direction x), the unit deformation in the direction y would be expressed by $\frac{\sigma_y}{E}$. Also a deformation in the direction x would be induced, the magnitude of which would be expressed by $\mu\frac{\sigma_y}{E}$, μ being Poissons' ratio for the material, and this deformation would be opposite in sense to that produced by σ_x.

In case the stresses σ_x and σ_y are acting simultaneously at a point, there will be in each direction a direct and an induced deformation that must be combined. If e represents the unit deformation caused by stress, then

$$e_x=\frac{\sigma_x}{E}-\mu\frac{\sigma_y}{E}$$

and similarly

$$e_y=\frac{\sigma_y}{E}-\mu\frac{\sigma_x}{E}$$

From these equations, the following expressions for unit stress are obtained:

$$\sigma_x=\frac{(e_x+\mu e_y)E}{1-\mu^2} \tag{1}$$

and

$$\sigma_y=\frac{(e_y+\mu e_x)E}{1-\mu^2} \quad (2)^{10}$$

DEFORMATIONS MADE UP OF THREE COMPONENTS

In this investigation the stresses caused by temperature warping were determined by comparing the deformations measured at various points on the slab surface over the period during which the temperature differential was changed and warping developed. The strain gages were generally installed at a time when no temperature differential existed and then allowed to remain until the maximum day or night temperature differential was observed. For these conditions the deformation recorded by each gage was the sum of three components combined algebraically. These component deformations are:

1. A change in length caused by the uniform change in the temperature of the slab as a whole. This change in length extends uniformly through the entire depth of the slab, creates no warping, and affects all strain gages equally.
2. The change in length of the upper surface of the pavement with respect to that of the lower surface, caused by the temperature differential created by the change in air temperature during the day. The differential in length caused by this temperature condition causes warping which, if the slab were weightless, would occur freely and would be unaccompanied by stress.
3. A deformation caused entirely by the bending stresses produced by the efforts of the slab to accommodate itself to the shape demanded by the temperature differential against the resistance of its own weight.

Since the measured deformations are not the result of combined stress alone it is necessary to modify somewhat equations (1) and (2) in order to adapt them to this method of determining the stresses produced by warping. In the following paragraphs the modified stress formulas are developed for each of the four gage positions that were shown in figure 5. The subscripts indicate the particular gage position that is being referred to.

δ=the unit change in the length of the concrete from all causes as actually measured by the strain gage.
ϵ=the unit change in the length of the concrete as measured by the strain gage but with a correction applied for the effect of the stress in a direction perpendicular to that being considered.
e=the unit change in length of the concrete caused by stress (unit strain).
μ=Poisson's ratio.

Referring again to the four gage positions shown in figure 5:

$$\epsilon_a=\delta_a-\mu e_c$$
$$\epsilon_b=\delta_b-\mu e_d$$
$$\epsilon_c=\delta_c$$
$$\epsilon_d=\delta_d-\mu e_b$$

Assuming that the concrete at point a is completely free to deform and using this deformation as a base, the strains at the four positions are:

$$e_a=0$$
$$e_b=\epsilon_a-\epsilon_b$$
$$e_c=\epsilon_a-\epsilon_c$$
$$e_d=\epsilon_a-\epsilon_d$$

If the proper values of ϵ are substituted in these formulas the strains in the concrete can be determined. These formulas multiplied by E, the modulus of elasticity of the concrete, are formulas for stress, expressed in terms of the measured deformations and the elastic constants of the material. They have the following form:

$$\sigma_a=0$$

$$\sigma_b=\frac{\delta_a-\delta_b+\mu(\delta_c-\delta_d)}{1-\mu^2}E$$

$$\sigma_c=\frac{\delta_a-\delta_c}{1+\mu}E$$

$$\sigma_d=\frac{\delta_a-\delta_d+\mu(\delta_c-\delta_b)}{1-\mu^2}E$$

Using the method described the stresses resulting from temperature warping during the day were determined for all of the gage positions shown in figure 6 for the 6- and 9-inch constant-thickness sections. For the conditions of warping that develop at night the stresses determined were those along the lines A–C and D–E and for the 6-inch constant-thickness section only.

The deformations in the concrete were measured with the recording-strain gage that has been previously described.[11] Because of the nature of the tests it was necessary to expose the strain gages to a wide variation in air temperature, varying from complete shade and

[10] For a discussion of the action of combined stresses see Strength of Materials by S. Timoshenko, pt. I, p. 58

[11] See An Improved Recording Strain Gage, PUBLIC ROADS, vol. 14, no. 10, December 1933.

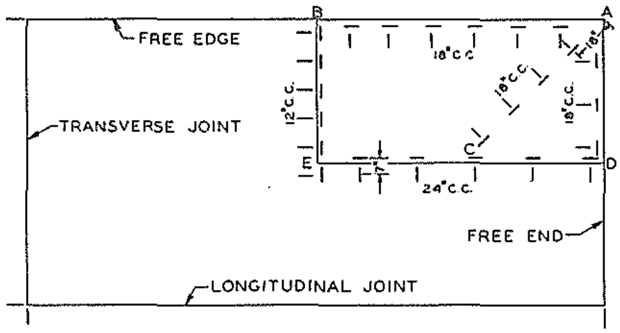


FIGURE 6.—PLAN OF 10- BY 20-FOOT SECTION SHOWING LOCATION OF ALL STRAIN GAGES USED IN DETERMINING THE STRESSES CAUSED BY RESTRAINED WARPING.

low temperature to full exposure to the sun's rays and high temperature, during the period that the deformations were developing. The gages are, by their design, compensated for temperature changes to an extent that makes the effect of ordinary temperature changes negligible. Also, since all gages in the test were exposed to the same temperature conditions, theoretically the comparison of deformations used to determine the stresses would be unaffected by temperature change.

In order to insure the greatest precision possible in these measurements, it was thought advisable to give all of the gages some measure of protection from the extreme temperatures to which they were exposed. Figure 7 shows a close-up view of one of the ventilated covers used for this purpose. These covers were made of sheet metal covered with corrugated paper board as insulation. The cover was painted white on the outside to minimize heat absorption and the ends were left open to allow the air to circulate around the gage. The shelters were made as small as possible in order that the shade afforded the gages would cover no appreciable portion of the slab.

The tests to determine the stresses produced by the restrained temperature warping were made during the spring and summer months for the day condition and during the fall months for the night condition, because the temperature data showed that the maximum temperature differential for each condition occurred at these respective seasons of the year.

STRESSES CAUSED BY RESTRAINED MOISTURE WARPING DIFFICULT TO OBTAIN

At the present time there are two chief obstacles that prevent the development of information concerning the stresses caused by restrained moisture warping comparable in scope to that developed on the subject of restrained temperature warping. The first of these, the inability to determine with any precision the character of the moisture distribution in the concrete, has already been mentioned. The second is the necessity of a very long period over which the strain observations must be continuous. In this investigation it has not yet been possible to make a determination of the stresses resulting from moisture warping. However, later in this report there is some discussion of the subject based on certain observations that have been made.

DATA PRESENTED AND DISCUSSED

To give an idea of the weather conditions that prevail in the area where the tests were made, certain pertinent meteorological data covering the period from November 1, 1931, to November 1, 1932, are given in figure 8. This figure shows the daily and annual temperature variations, the monthly precipitation, and the monthly average relative humidity for a period that is believed to be typical. It will be noted that there is comparatively little freezing weather during the year, so little in fact that the earth beneath concrete pavement sections is rarely frozen. It was not possible to study the structural behavior of the sections for the condition of a frozen subgrade as extensively as was desired. The daily and annual temperature variations are large, however, and during the period of the tests a wide range of temperatures was encountered.

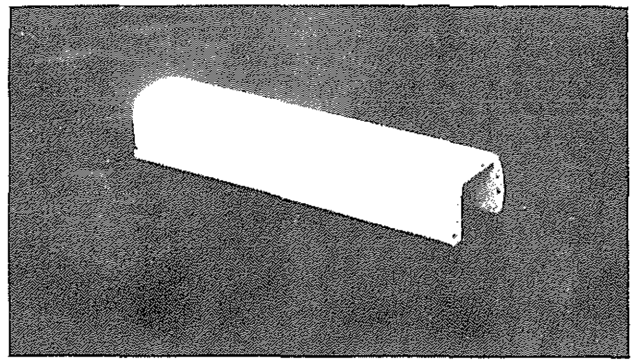


FIGURE 7.—SMALL PROTECTIVE COVER USED OVER THE STRAIN GAGES IN DETERMINING STRESSES CAUSED BY RESTRAINED WARPING.

The temperature of the concrete in the pavement was measured on a number of days during the year for the purpose of studying the daily temperature variations of the concrete at different seasons. In some cases these observations were made at 1 or 2-hour intervals for the complete 24-hour period; in other cases they were started at about 4 a. m. and continued until late in the evening. From the data obtained it is possible to find the critical temperature conditions for each of the days and, inasmuch as the observations were made on days when large changes occurred, it is also possible to form an accurate idea of the occurrence of critical temperature conditions throughout the year. Table 1 contains typical data obtained in this manner from the 6- and 9-inch temperature slabs. This table shows the average temperature of the concrete when it was at the minimum and the maximum values for the day on which the observations were made. It also shows the actual measured temperatures in the upper and lower surfaces of the two slabs.

The tabulated difference between these two temperatures may not be the effective temperature differential, since the temperature gradient between the two surfaces may or may not be uniform. This point will be discussed more fully later.

Several interesting facts are brought out in this table. It is shown that the average slab temperature varies about 75° F. during the year. This figure is important because it is the value that controls the magnitude of the annual change in length of the pavement caused by temperature changes. The maximum surface temperature recorded during this period was 112.5° F. The data show that the temperature of the 6-inch slab is, as a whole, much more responsive to variations in air temperature than is the 9-inch slab, yet the actual temperature differential between the upper and lower surfaces is always greater for the thicker slab. Under certain maximum conditions the value of the tem-

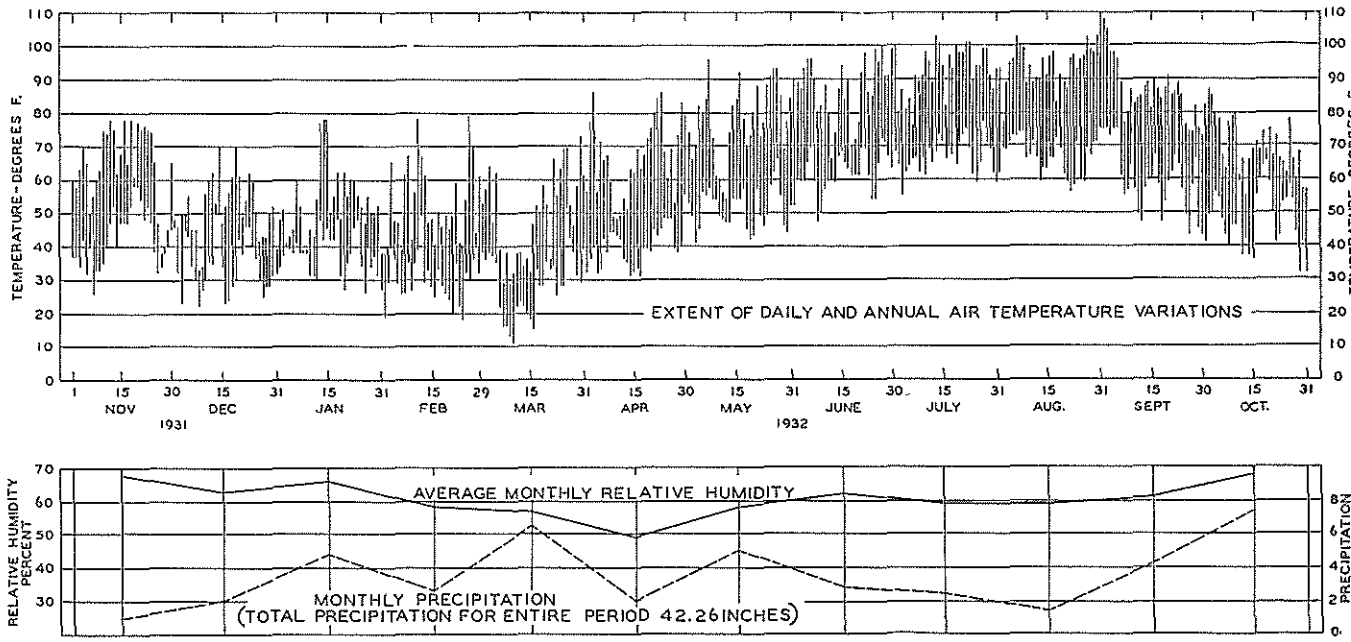


FIGURE 8.—ANNUAL VARIATIONS IN AIR TEMPERATURE, RELATIVE HUMIDITY, AND PRECIPITATION.

TABLE 1.—*Observed temperatures in concrete pavements (degrees F.)*

Date	Average temperature of concrete				Maximum temperature values											
					Night						Day					
	6-inch slab		9-inch slab		6-inch slab			9-inch slab			6-inch slab			9-inch slab		
	Minimum	Maximum	Minimum	Maximum	Bottom	Top	Difference	Bottom	Top	Difference	Bottom	Top	Difference	Bottom	Top	Difference
1931																
Nov. 24		65.1		62.1							59.2	70.0	10.8	55.6	69.1	13.5
Nov. 25	52.2		53.0		56.3	52.3	4.0	57.2	52.0	5.2						
1932																
Feb. 1	26.6	41.2	20.4	40.8	31.1	24.4	6.7	36.1	24.6	11.5	36.7	44.9	8.2	35.8	45.3	9.5
Apr. 14		67.6		63.6							59.7	81.0	21.3	50.0	81.0	31.0
Apr. 15	39.9		42.3		43.0	36.5	6.5	48.9	39.7	9.2	56.8	76.1	19.3	51.6	73.9	22.3
June 8	65.5	91.9	63.5	84.0	70.3	63.9	6.4	68.9	61.3	7.6	80.2	102.7	22.5	69.3	96.1	26.8
July 13	75.4	102.7	76.7	90.8	70.9	75.4	4.5	81.3	75.6	5.7	90.9	112.5	21.6	82.8	111.7	28.9
Aug. 5	70.7	98.4	72.0	93.7	73.8	68.4	5.4	75.4	68.0	7.4	81.3	101.7	20.4	80.2	105.8	25.6
Sept. 1	77.5	100.2			78.8	76.1	2.7				90.7	106.3	15.6			
Oct. 11	60.3	70.0			61.0	59.7	1.3				64.8	76.5	11.7			
Nov. 4	37.4	56.7			40.5	34.9	5.6				46.4	58.3	11.9			
1933																
Jan. 3	28.5	40.6			31.1	27.0	4.1				32.0	40.5	8.5			
Feb. 24	37.0	58.8			39.4	35.1	4.3				49.5	65.1	15.6			
Apr. 13	42.8	69.4			45.9	39.9	6.0				56.8	81.1	24.3			
May 19	59.7	85.6			63.0	56.7	6.3				74.5	96.1	21.6			
June 2	60.1	86.7			63.0	57.4	5.6				72.7	94.1	21.4			
Aug. 15	69.8	92.8			73.2	67.5	5.7				80.6	99.3	18.7			

perature differential in the 9-inch slab is nearly 50 percent greater than the corresponding value in the 6-inch slab. The maximum differential observed at any time in the 6-inch section was 24.3° F., while in the 9-inch slab it was 31° F.

RANGE OF DAILY PAVEMENT TEMPERATURES DETERMINED

From the data that are summarized in table 1, four individual cycles of observations were selected for graphical presentation to illustrate the range and character of the daily and annual variations in temperature that occur in concrete pavements in this locality. Each of the four cycles chosen is typical for the season in which it occurred although, as mentioned before, the observations were made on days when there were large variations in the temperature of the pavement for that particular season. In figures 9, 10, and 11 these data are presented in different ways, each graph being arranged to bring out the significance of the data with respect to some particular point.

The character of the daily variation of temperature throughout the depth of the 6- and 9-inch slabs is shown in figure 9 by means of gradient curves taken periodically throughout the day. It is apparent that the variations are much greater during the day when the sun is shining than at night, and also that the greatest variations occur during the warm seasons of the year. Both of these effects are caused by the absorption of heat from the sun's rays, and the more intense the sunlight the greater will be the effect.

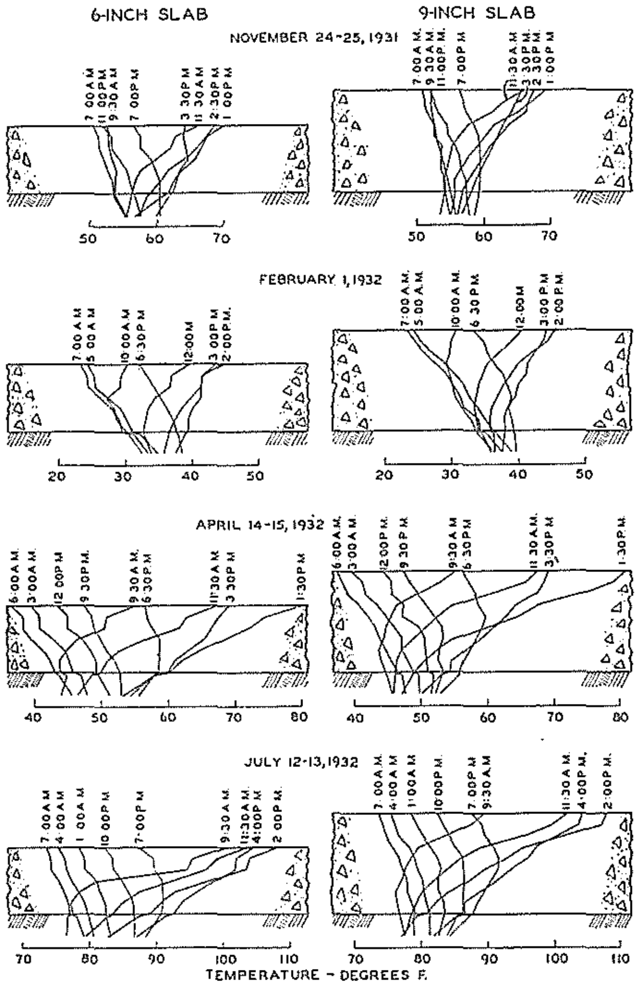


FIGURE 9.—TYPICAL DAILY AND SEASONAL TEMPERATURE VARIATIONS IN CONCRETE PAVEMENT SECTIONS.

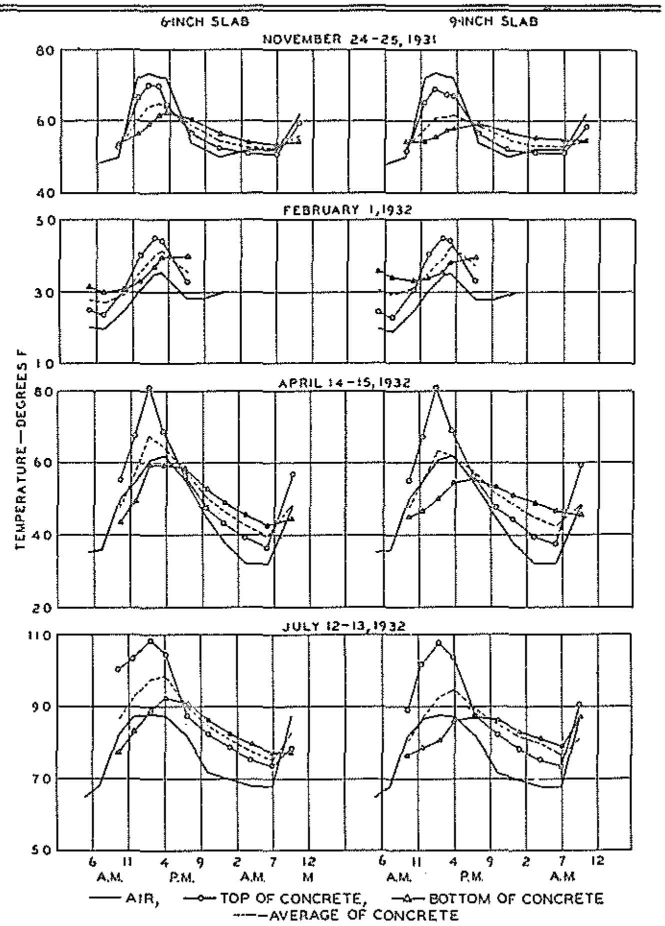


FIGURE 10.—RELATION OF AIR TEMPERATURE TO PAVEMENT TEMPERATURE.

Figure 9 shows why the difference between the temperatures of the two surfaces of a pavement is not necessarily the effective temperature differential. It is probable that an average line drawn through each of the gradient curves shown in this figure would give a better approximation of the effective differential. This is the method that has been used for determining the values of the differential given in this report. It should be noted that in the early morning and in the afternoon when the maximum temperature differentials occur, there is approximately a straight-line gradient in temperature between the upper and lower surfaces of the concrete. These are the two times of the day that are most important in the determination of stresses caused by warping.

It is not unusual at certain seasons of the year to find that the absorption of the heat from the sun has caused the temperature in the upper surface of the slab to be from 10° to 20° F. higher than the air temperature. The effect is greatest when the angle of incidence of the sun's rays to the pavement surface is greatest. In figure 10 the relation between the air temperature, the average slab temperature, and the temperatures of the two surfaces are shown for typical days at four different times during the year.

As previously explained, the average pavement temperature is obtained by averaging the values obtained from all of the thermocouples throughout the slab depth. For this reason the average temperature may not be equal to the mean of the upper and lower surface temperatures. These curves show the rapidity with which the temperature of the upper surface changes during the day and the extent to which it rises above a: temperature during certain parts of the year.

FACTORS AFFECTING PAVEMENT TEMPERATURE DISCUSSED

During the warmer seasons of the year even the average temperature of the concrete rises above the air temperature for considerable periods of time. The temperature of the concrete on any one day is controlled not only by the air temperature on that day but also by several other factors such as the angle of incidence of the sun's rays, the previous temperature conditions, particularly as they affect the temperature of the subgrade, the moisture conditions, and the humidity of the atmosphere.

An example of the effect of these other factors is found in the data in figure 10 for February 1, 1932. It will be observed that the temperatures of both surfaces of the slab are higher than the air temperature on this date. An examination of the temperature data for the period showed that for several days preceding February 1 higher temperatures had prevailed. The heat absorbed by the concrete and the subgrade during the warmer period had not been dissipated when the observations shown in the figure were made.

The observed differences in temperature between the upper and lower surfaces of the 6-inch and 9-inch

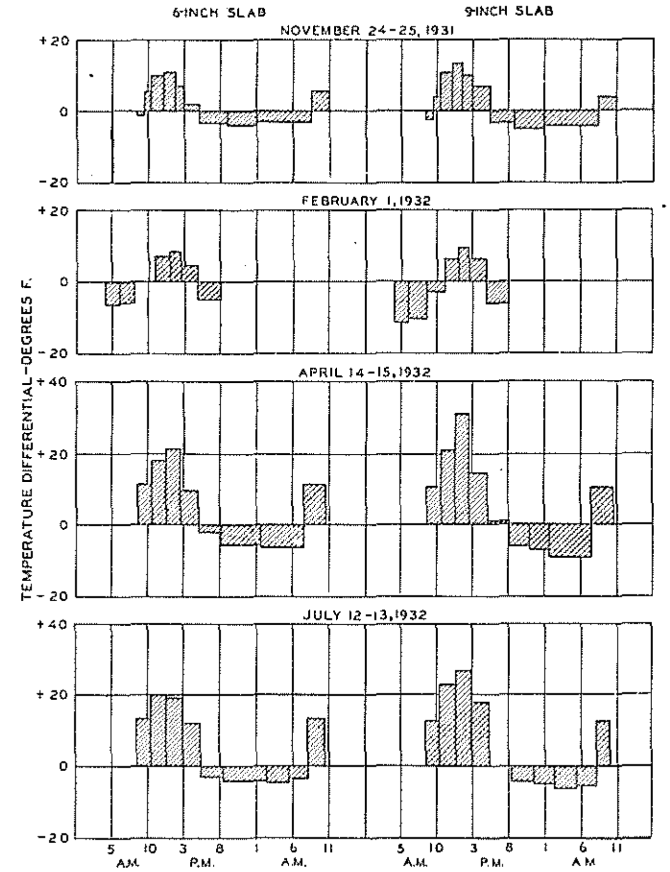


FIGURE 11.—TYPICAL DAILY AND SEASONAL VARIATIONS OF TEMPERATURE DIFFERENTIALS IN CONCRETE PAVEMENT SECTIONS.

slabs, for the 4 typical days, are shown in figure 11. The positive values indicate differences when the upper surface of the pavement is warmer than the lower surface, while the negative values apply to the opposite condition. The principal purpose of this chart is to assist in tracing the variations, during the day and during the year, of the temperature differences that cause warping.

It will be noted that during the spring and summer months when the daily changes in the temperature of the concrete are large, the difference between the maximum positive temperature differentials in the 6- and 9-inch constant-thickness slabs is approximately proportional to the difference in slab depths. During the late fall and winter months, however, when the daily changes in the temperature of the concrete are much smaller, the difference between the positive temperature differentials is much smaller. The maximum difference between the positive temperature differentials of the two slabs appears to be in the spring. At this time, although the intensity of the sunlight is very great, the subgrade is still relatively cold and the subgrade under the 9-inch slab warms up more slowly than that under the 6-inch slab. The negative temperature differentials are so small and vary so much from day to day that it is difficult to find any direct relation between these temperature differentials in slabs of different thicknesses and at different times of the year.

Mention was made previously of thermocouples that were installed especially to determine the effect of a thickened-edge cross section on the magnitude of the temperature differentials developed for various conditions of air and subgrade temperature.

TABLE 2.—*Observed maximum temperature differentials between the upper and lower surfaces of concrete pavement sections*

Date, 1934	Maximum temperature differential				
	At edge of constant-thickness sections		Thickened-edge section, 9–6–9 inch		
	6-inch slab	9-inch slab	Edge	18 inches from edge	36 inches from edge
	°*F.*	°*F.*	°*F.*	°*F.*	°*F.*
Apr. 3	----------	----------	27	25	21
Apr. 5	----------	----------	21	21	20
Apr. 6	----------	20	18	17	15
Apr. 8	----------	25	25	25	22
Apr. 10	15	23	26	24	21
Apr. 15	17	26	25	25	22
Apr. 18	18	27	27	25	22
Apr. 21	18	24	22	21	20
Apr. 22	14	24	21	20	19
Apr. 24	21	31	29	27	20
Apr. 29	15	24	25	23	21
May 1	18	28	28	25	25
May 5	23	32	33	31	27
May 7	24	31	30	30	26
May 8	26	27	28	24	22
May 9	18	25	24	22	20
May 13	21	29	31	28	24
May 14	16	24	27	26	24
May 18	21	30	30	28	25
May 19	20	26	20	27	24
May 20	23	33	32	30	28
May 21	20	31	32	29	26
May 24	19	25	25	23	21
May 28	----------	----------	23	25	21
June 1	21	30	30	28	26
June 3	16	29	28	28	22
June 4	19	27	27	27	23

Table 2 contains data obtained from measurements made on three of the test sections during a 2-month period in the spring of 1934. Each temperature differential shown is the maximum that occurred during the particular day, and the observations were made when there were large temperature variations for the season of the year. The second and third columns of the table show the observed differentials at the edges of the 6-inch and 9-inch constant-thickness sections, respectively, while the last three columns contain comparative data at three points along the cross section of a representative thickened-edge section.

A comparison of the values in the second with those in the fourth column shows the relation between the temperature differential at the edge of a 6-inch slab that is not thickened at the edge and that of one that is thickened to 9 inches. A similar comparison of the data in the third and fourth columns shows the relation between the differential of temperature in the 9-inch edge of a constant-thickness slab and that in the 9-inch edge of a thickened-edge cross section. These comparisons show that the temperature differential that develops at the edge of a 9–6–9 thickened-edge design is approximately equal to that in the edge of a 9-inch constant-thickness section and is approximately 45 percent greater than that in the edge of a 6-inch constant-thickness section. It is indicated by these data, therefore, that increasing the edge thickness of a pavement may be expected to result in a proportionate increase in the temperature differential in the edge region of the slab.

The data in the last three columns show the temperature differentials observed at distances of 2, 18, and 36 inches from the free edge of the 9–6–9 section. The data obtained at a point 36 inches from the edge probably represent very closely those that would be found throughout the 6-inch interior portion of a slab of this design. Comparing the values in the fourth column with those in the sixth, it is indicated that the differential in the edge of this section averaged, for the period of the measurements, about 20 percent greater than that at a point 36 inches from the edge. At a point 18 inches from the edge the increase was approximately 13 percent. The edge depth is 50 percent greater than the depth 36 inches from the edge and, of course, the increase in depth at 18 inches from the edge is one-half of this or 25 percent. Hence, it appears that the increase in the temperature differential near the edges over that in the interior of this thickened-edge design is less than would be expected in view of the relation of center depth to edge depth. It is believed that this is due to the stabilizing influence of the earth shoulder along the vertical face of the slab edge that acts to reduce somewhat the temperature differential. This would apply to both thickened-edge and constant-thickness sections, although the result will probably be less as the edge thickness of the sections is reduced.

EXPANSION AND CONTRACTION OF PAVEMENT SLABS MEASURED

At the same time that the temperature determinations were made the change in length of one of the 40-foot test sections was measured. In figure 12 the variations in length of this section are shown, together with the simultaneous variations in the average temperature of the concrete. These data are plotted to a common base for the same four cycles considered in the discussion of the temperature data. It will be noted that there is a very close phase relation between the temperature and expansion curves, there being little or no lag even at those times of the day when the temperature is changing most rapidly. The lag of the average pavement temperature with respect to air temperature has already been shown in figure 10.

These data show that for a given average temperature the pavement does not always have the same length. For example, at an average temperature of 60° F. in November the change in length, with respect to a certain base, was −0.0105 inch, while in April it was +0.017. This indicates that in the 5 months between November and April the length of the slab has increased 0.0275 inch from some cause other than temperature changes.

In the vicinity of Washington, D. C., the mean monthly precipitation was averaged for the period covered by the observations (1931 to 1934, inclusive) and it was found to be 4.6 inches for the summer months (April to September, inclusive) and 3.1 inches for the winter months (October to March, inclusive). Thus the precipitation was greatest for the period when the slabs were found to be shortest for a given temperature. However, evaporation measurements made in this locality a number of years ago by the Weather Bureau show that during the summer months the loss from a free-water surface averages more than 6 inches per month, while during the winter the loss through evaporation is very small.

This suggests that beginning in the late fall there should be a progressive increase in the moisture content of the soil that continues until the spring temperature rise begins, after which there should be a corresponding progressive decrease. Such soil-moisture determinations as were made confirmed this idea. This being true, it seems reasonable that during the summer months lowered moisture content in the subgrade and a relatively high evaporation rate reduce the moisture content of the concrete and that the opposite conditions in the winter will increase the moisture content of the concrete and thus account for the observed volume changes.

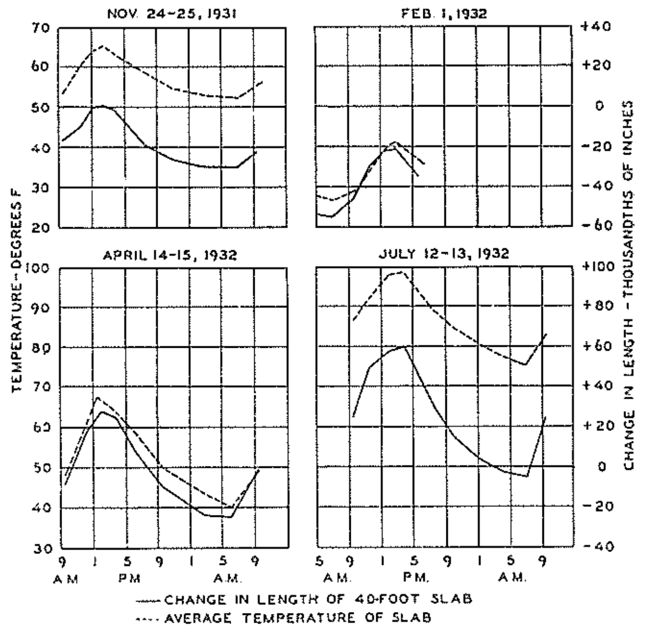


Figure 12.—Variations in Slab Length Compared with Variations in Average Concrete Temperature.

In order to separate the changes in length resulting from these two different causes so that the magnitude of each could be determined, the data obtained during the various daily cycles of observations were grouped on a single sheet and all referred to a common base for comparison in the manner shown in figure 13. The changes in slab length measured on any given day are plotted against the corresponding average concrete temperatures. The plotted points pertaining to a day's observations are averaged with a straight line, the slope of which is the coefficient of thermal expansion for the slab as a whole, as indicated by those particular data. Since all of the data are plotted to a common base, the spread horizontally between these daily average lines is, when taken at a common temperature, a measure of the expansion resulting from causes other than temperature changes.

Figure 13 contains a few typical data plotted in this manner for the purpose of demonstrating the method of analysis. In this graph the coefficient of thermal expansion of the concrete, as determined in the laboratory, is shown by the slope of the dash line through the center. It will be noted that the lines showing the daily averages appear to converge slightly toward the dash line. Throughout the data this convergence varies systematically with the season, indicating that the coefficient of thermal expansion determined from the daily observations of slab expansion undergoes a small annual variation. The cause of this was not determined but it seems likely that the coefficient of thermal expansion effective in the slabs varies slightly with the moisture content of the concrete.

It is possible also that warping in the slab introduces a small error in the determination of the slab length, and such an error, if present, would tend to vary systematically in the same manner as does the extent of the warping. The possibility of subgrade resistance variation being a factor was also considered but calculations indicate that the variations in subgrade resistance caused by variations in subgrade moisture would produce an effect of negligible magnitude.

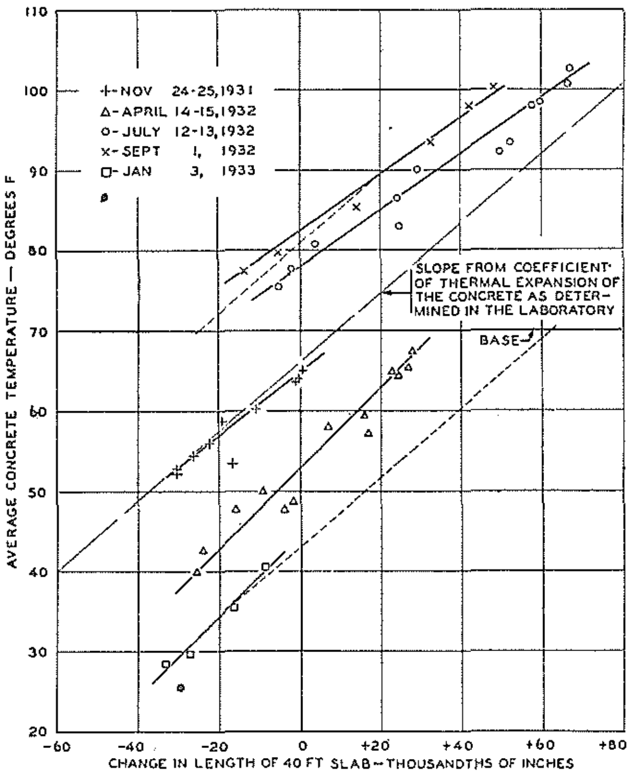


FIGURE 13.—RELATIONS BETWEEN AVERAGE CONCRETE TEMPERATURE AND CHANGE IN LENGTH OF PAVEMENT SLAB FOR VARIOUS PERIODS DURING THE YEAR.

In order to eliminate, to the fullest extent possible, any error caused by this slight convergence, the determination of spread was made by extending auxiliary dotted lines from the center of gravity of each day's observations and parallel to the dash line previously mentioned to an intersection on a common temperature line, chosen arbitrarily at 70° F. in this instance. The distance between intersections is a measure of the length change attributed to moisture.

CHANGES IN PAVEMENT LENGTH CAUSED BY VARIATIONS IN MOISTURE CONTENT DETERMINED

In figure 13 data from only five sets of observations are shown. Actually, observations were made on a great many days and all of the data were analyzed in this way. From this analysis the variations in the length of the 40-foot test section resulting from variations in moisture content were obtained at frequent intervals during a 5-year period. The change in length resulting from variations in moisture content alone during this time is shown in figure 14, plotted with respect to the original length at the time that the concrete took its initial set. The measurements of length change were made on one section with the embedded telemeter during the period between September 1930 and August 1932, and with the micrometer on another section during the period between November 1931 and September 1935. During the 8 months between November 1931 and August 1932, data from both methods are available and there is very close agreement between them.

This graph indicates that there is an annual cyclic variation in length caused by variations in moisture content, the sections being longest (for a given temperature) during the winter and shortest during the summer. The magnitude of this length change is appreciable, corresponding to that which would be caused by a temperature change of 20° to 40° F. for the different years during which the measurements were made. In a 40-foot section this variation in length amounts to about 0.05 to 0.10 inch.

It will be noted that there is a considerable variation in the extent of this length change for the different years, the smallest change occurring in 1930–31 and the largest thus far observed in 1932–33. It is believed that the variation in the magnitude of the length change from year to year is the result of the particular precipitation and evaporation conditions that happened to prevail. The precipitation record is as follows:

	Inches
1930	21.7
1931	33.5
1932	49.5
1933	49.1
1934	51.1

Unfortunately, there is not a similar record of the annual evaporation at the site of the tests. Nor was it practicable to determine the annual variation in the moisture content of the concrete over this period. As previously stated, there was no dependable method available for determining the moisture content of the concrete in the pavement. Using fragments broken from the specimen slabs and determining the moisture content by drying in the laboratory, it was found that the concrete contained 3.5 percent moisture during the summer and 3.8 percent during the winter. These values should be considered as nothing more than an indication of the variation of the moisture content of the pavement.

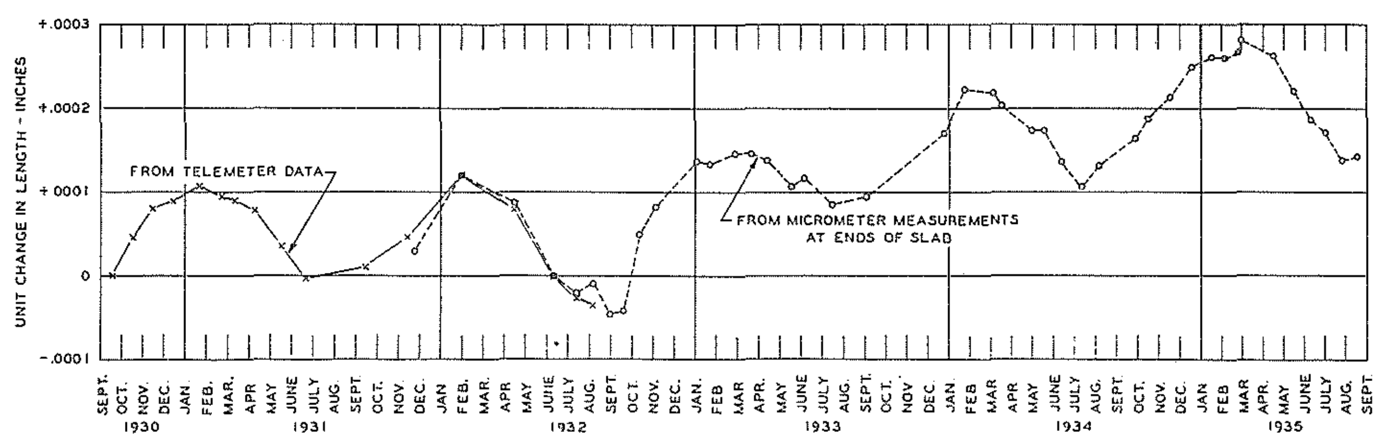


FIGURE 14.—ANNUAL VARIATIONS IN PAVEMENT LENGTH CAUSED BY CHANGES IN MOISTURE CONTENT.

The data presented in figure 14 show a definite progressive increase in the length of the pavement that has become more marked during the fourth and fifth years. The slab, during the summer of 1934, was longer than in the summer of 1931 by 1 part in 10,000. This may seem to be a small amount but it represents a length gain of 6.3 inches in a mile of pavement. Looking at it another way, if the slab ends were completely restrained such a length gain might develop a compressive stress of several hundred pounds per square inch.

Although this tendency for concrete to "grow" in the presence of moisture has long been known and has been the subject of much speculation and experiment, the phenomenon is not well understood and the ultimate extent of the growth cannot be predicted for given materials and conditions of exposure with the information that has thus far been developed.

COEFFICIENTS OF THERMAL EXPANSION FOR CONCRETE DETERMINED BY BOTH LABORATORY AND FIELD TESTS

In figure 13 the variation in slab length was compared with the variation in average concrete temperature for typical cycles of daily observation. Such data can be used to determine the coefficient of thermal expansion that is effective for the slab as a whole. In figure 15 this has been done. The graph was constructed by plotting the maximum change in length for each daily cycle against the corresponding change in the average temperature of the slab. Each daily cycle of observations thus supplied one point for the figure. Through these points a straight line was drawn to average the data. The slope of this average line is the coefficient of thermal expansion that affects the slab as it lies on the subgrade. The value of the coefficient as obtained by the method just described is 0.0000047 inch per inch per degree F. from the telemeter data, and 0.0000049 inch per inch per degree F. from the micrometer measurements at the ends of the slab. These values compare with one of 0.0000048 determined for the same concrete in the laboratory.

The method used in the laboratory determinations was described in part 1 of this series of papers. It consists, briefly, of the casting of a 12 by 24-inch cylindrical specimen in a moisture-tight copper container with a telemeter or recording strain gage of the electrical resistance type embedded in the center of the specimen. As previously noted, each telemeter contains a resistance thermometer, thus permitting simultaneous observations of deformations and temperatures.

After the concrete had hardened and cooled, the sealed cylinder was placed in water baths at 32° F. and 110° F. alternately a number of times, remaining in each until complete temperature equilibrium was attained. Since loss of moisture is prevented by the copper jacket, the difference in length measured by the telemeter under these conditions is the result of temperature change and from it the value of the coefficient of thermal expansion was obtained. The method was later used for determining the thermal coefficient for the concrete in connection with the elaborate test program on the Rogue River bridge.[12]

The close agreement between the coefficients of thermal expansion for the pavement and those obtained with unrestrained concrete in the laboratory indicates that the stresses in the pavement caused by the resistance of the subgrade to horizontal slab movement must be very small.

[12] Application of the Freyssinet Method of Concrete Arch Construction, by Gemeny and McCullough. Tech. Bul. No. 2, Oregon State Highway Department, 1933.

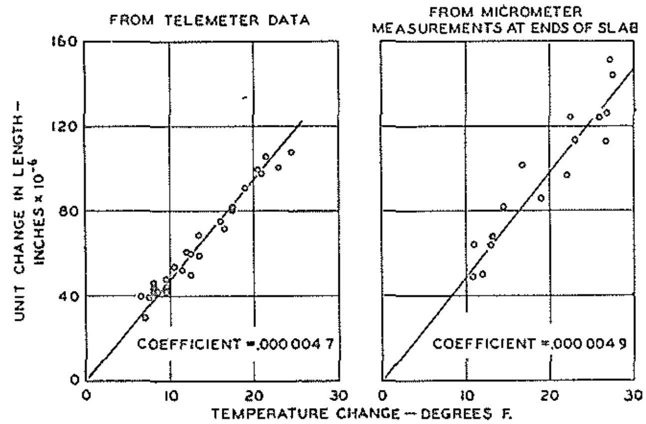


FIGURE 15.—RELATIONS BETWEEN TEMPERATURE CHANGE AND EXPANSION FOR CONCRETE PAVEMENT SLAB.

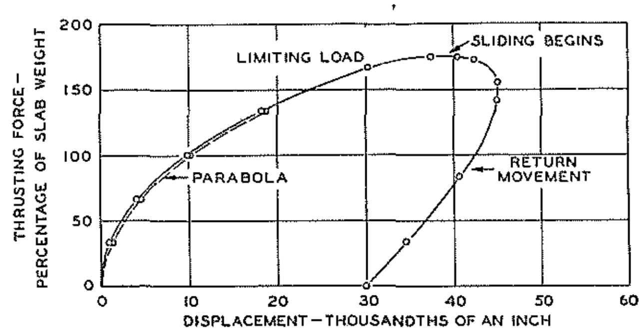


FIGURE 16.—TYPICAL FORCE-DISPLACEMENT CURVE FOR HORIZONTAL SLAB MOVEMENT.

RESISTANCE OF SUBGRADE TO HORIZONTAL SLAB DISPLACEMENT MEASURED

It will be recalled that in the earlier mention of these tests it was stated that the procedure adopted as a result of the preliminary study was one that would subject the subgrade to the same manipulation, as nearly as possible, as that which it receives under a pavement.

The test slabs were moved a distance of approximately 0.040 inch during a period of approximately 6 hours and they were moved in opposite directions on alternate days. The displacement of the slab was measured immediately after the application of each increment of thrust and again just before the next increment was applied.

Figure 16 shows typical data resulting from one of these tests. In this instance the horizontal force, or thrust, was applied at the rate of 50 pounds every 10 minutes until a total of 2,100 pounds caused visible sliding to begin. As soon as this point was reached the reduction of the force was started and continued at the rate of 100 pounds every 10 minutes until all horizontal thrust had been removed.

The curve shows the force-displacement data for the entire test. As the increments of force were applied the successive increments of displacement increased in magnitude in a ratio that closely approximates a parabola as shown by the dotted line adjacent to the curve. During this period there was no visible evidence of the concrete sliding on the subgrade. As the force being applied reached a value of approximately 150

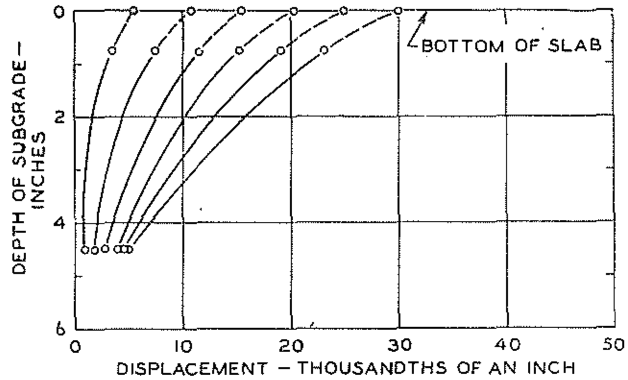


FIGURE 17.—MOVEMENT OF SUBGRADE COMPARED WITH SLAB DISPLACEMENT.

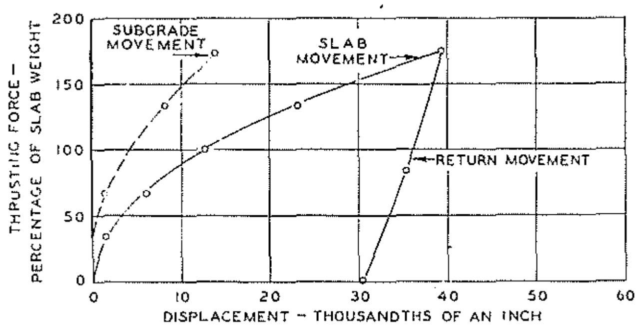


FIGURE 18.—RELATIVE MOVEMENTS OF PAVEMENT AND SUBGRADE FOR A 6-INCH SLAB.

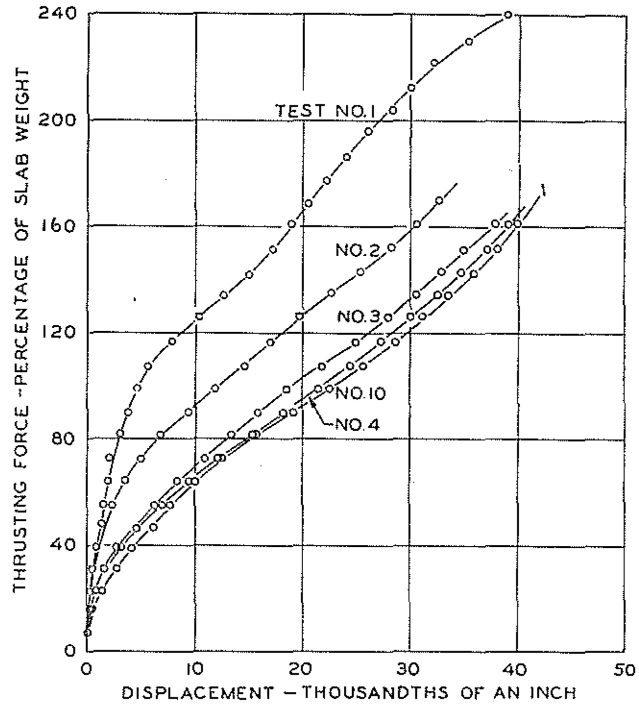


FIGURE 19.—FORCE-DISPLACEMENT CURVES FOR REPEATED TESTS ON 6-INCH SLAB.

percent of the weight of the test slab there is evidence that a condition of actual sliding is impending.

In this test free sliding occurred with a thrust equal to 175 percent of the slab weight and a force greater than this could not be applied. Reduction of this force started a movement of the slab in the opposite or return direction, and in this test it will be observed that complete removal of the thrust caused a recovery of about one third of the total displacement. This recovery is believed to result from elastic deformation of the subgrade caused by the adherence of the earth to the bottom of the slab. When the slab moves there is an actual movement of the subgrade with it.

The character of the horizontal movement that occurs in the subgrade when there is a displacement of the slab is indicated by the test data in figure 17. In these tests micrometer dials measured the horizontal soil movement at depths of three-fourths inch and 4½ inches as the slab was being moved. The particular test for which the data are shown was one in which a slot or groove 5 inches deep had been cut vertically into the subgrade just ahead of the leading edge of the slab. The presence of this groove probably affected somewhat the magnitude of the displacements produced by a given thrust.

Figure 18 shows the average data obtained from a number of tests in which the "bending" of the subgrade was measured without the disturbing influence of the groove just mentioned. The subgrade displacement in this figure is the average of measurements made at both sides and the center of the leading edge of the slab and at a depth of three-fourths inch. In will be observed that at this depth the subgrade movement is about 30 percent of the slab displacement. The return movement of the slab after the release of the thrust is about 25 percent. In many cases the percentage of return was greater than this, ranging up to about 40 percent of the total displacement.

A few tests were made to determine how long the subgrade would maintain this elastic resistance that caused it to move the slab back toward its original position, and it was found that after the horizontal thrust had been kept at a constant value for 8 days and then released, the return movement was practically as great as if the slab had been displaced but momentarily. This indicates that the subgrade tested had a high degree of elastic action for small displacements.

Tests were made in which the slab was displaced a given amount several times, in exactly the same manner each time. The data obtained are shown in figure 19 and it appears that with each successive application there is a reduction in the amount of thrust required to produce a given movement. A condition of practical stability seems to have been reached, however, after a comparatively small number of movements. These data indicate that, for a given subgrade, the resistance to slab movement may be greater for the first movements of the newly constructed pavement than it is later when the concrete has expanded and contracted a number of times.

EFFECT OF SLAB WEIGHT ON RESISTANCE TO HORIZONTAL DISPLACEMENT INVESTIGATED

As previously mentioned, some effort was made to determine the effect of slab weight on the resistance to horizontal displacement by means of tests with slabs of 2, 4, 6, and 8-inch thicknesses.

In making these tests the procedure was first to move the slabs forward and backward through a distance of 0.040 inch several times until it appeared that the subgrade resistance had become stabilized for displacements of this magnitude. The test slabs were then moved through distances of 0.070 and 0.100 inch in the same manner. There was a tendency for all of the slabs to continue to slide under the action of

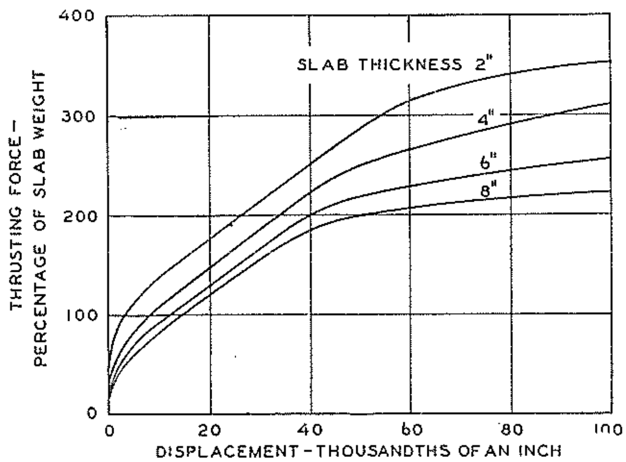


Figure 20.—Force-Displacement Curves for 2-, 4-, 6-, and 8-Inch Slabs.

thrusting forces that would produce the 0.100 inch displacement, and these may be considered as the maximum horizontal resisting forces that could be developed.

Figure 20 shows the relation between thrusting force and displacement for each thickness of slab. The data for displacements of 0.040 inch or less were obtained during the tests in which the maximum displacement was 0.040 inch, and the data for displacements of more than 0.040 inch were obtained from the tests in which the slabs were moved either 0.070 or 0.100 inch.

It is apparent from this figure that the forces necessary to move the slabs of different thicknesses do not bear a constant relation to the respective slab weights for the subgrade in question. In this connection it is well to bear in mind that the total displacement of the slab may be composed of two parts: First, an elastic or semielastic displacement of the soil particles with no sliding of the slab as such; and second, an actual slipping of the slab over the soil surface. The first action necessarily begins as soon as slab displacement starts. Whether or not the second action follows depends upon the nature of the soil and the magnitude of the displacement.

The relation between thrusting force and slab thickness for displacements of several magnitudes is shown in a different manner in figure 21. Again it is apparent that the magnitude of the thrusting forces required is not directly proportional to the respective slab thicknesses (or weights). If it were, the sheaf of curves in this figure would all be straight lines passing through the origin of the graph. To illustrate the variation in another way, if all of the resisting forces were summed up in a coefficient to be applied to the weight of the slab, the value of this coefficient, instead of being constant, would vary with slab weight and displacement as given in table 3.

Table 3.—*Variations of subgrade coefficients of resistance to displacement with slab weight*

Slab thickness	Slab weight	Coefficients of resistance to displacement for displacements of—					
		0.01 inch	0.02 inch	0.03 inch	0.04 inch	0.07 inch	0.10 inch
Inches	*Lbs. per sq. in.*						
8	0.67	0.8	1.2	1.5	1.8	2.1	2.2
6	.50	.9	1.3	1.6	2.0	2.4	2.5
4	.33	1.1	1.5	1.8	2.2	2.8	3.1
2	.17	1.3	1.7	2.1	2.5	3.3	3.5

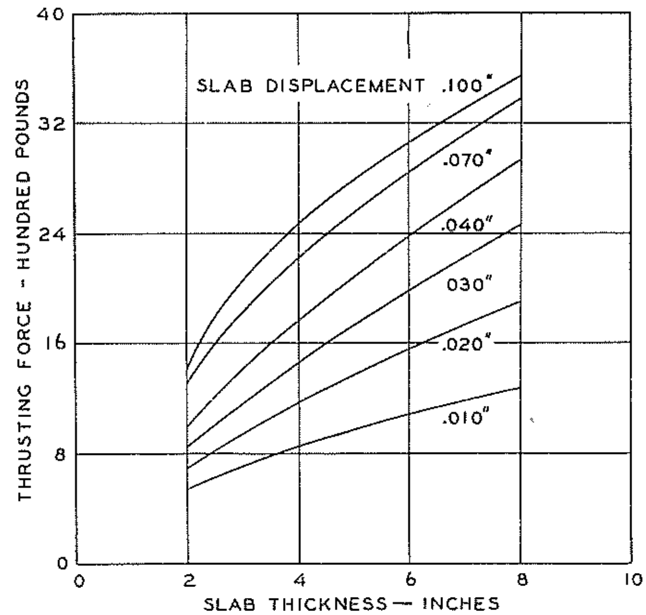


Figure 21.—Relations Between Slab Thickness and Resistance to Displacement.

The data indicate that a weightless slab in intimate contact with the subgrade might, under certain conditions of soil and moisture, develop a very considerable resistance to horizontal displacement.

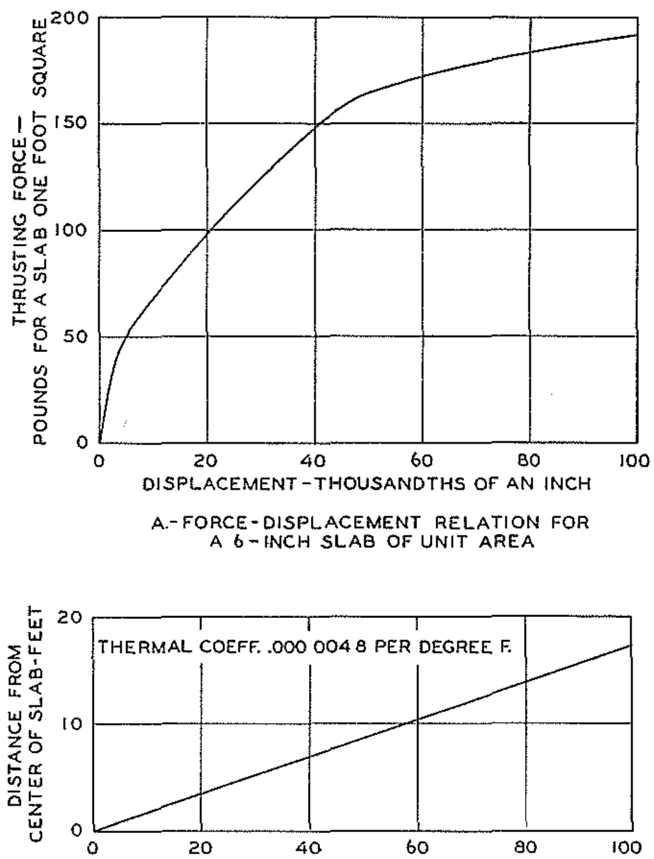
From the results of these studies it seems reasonable to conclude that the resistance offered by the subgrade to the horizontal movement of a pavement slab is composed of two elements: (1) A resistance caused by an elastic or semielastic deformation within the soil; and (2) a resistance that approximates closely that of simple sliding friction. The first of these appears to be independent of slab weight, while the second varies directly with slab weight. It seems quite probable that the relative magnitudes of these two components will vary with different subgrade soils, although no data on this point have been obtained.

METHOD OF CALCULATING PAVEMENT STRESSES DEVELOPED BY SUBGRADE RESISTANCE ILLUSTRATED

It will be apparent from the data and discussion which have been presented that, in any consideration of pavement stresses developed by this subgrade resistance, account should be taken of the extent of the displacement of each part of the slab. A suggested method for utilizing the data for this purpose is outlined in the following example:

In figure 22A the force-displacement curve for the 6-inch slab is repeated from figure 20. In this figure the vertical scale applies to a unit area of one square foot. This scale was chosen simply as a matter of convenience for use in the example. In figure 22B is a curve showing the displacement, resulting from thermal expansion, of the various parts of the slab with respect to its center point, based on the measured thermal coefficient of the concrete and a temperature change of 100° F. Inasmuch as the observed average temperature of the test pavement underwent an annual change of only 75° F., this figure may be too high but it serves the purpose of illustration as well as a lower one.

After having determined the forces necessary to move the unit slab through various displacements and having determined the displacements to which the



A.-FORCE-DISPLACEMENT RELATION FOR A 6-INCH SLAB OF UNIT AREA

B.-DISPLACEMENT AT VARIOUS DISTANCES FROM SLAB CENTER DUE TO A TEMPERATURE CHANGE OF 100° F.

Figure 22.—Relations Between Displacement and Thrusting Force, and Between Displacement and Distance from Center of Slab, for Concrete Pavement Slabs.

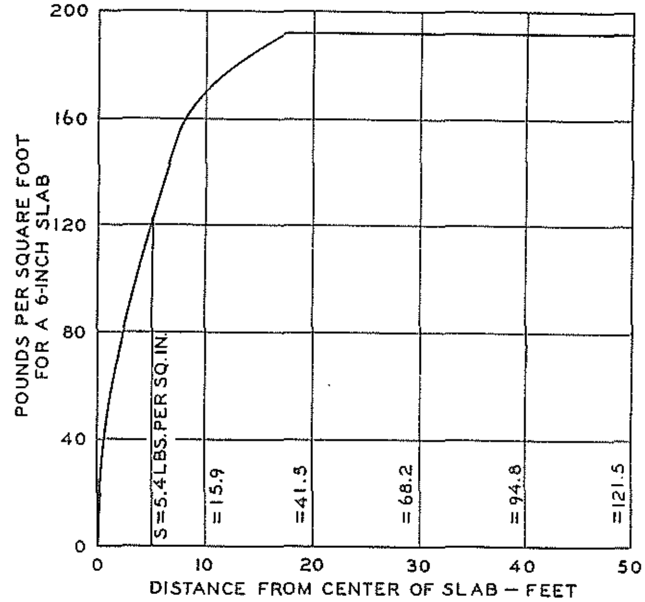


Figure 23.—Variations in Force Needed to Move Unit Areas at Various Points Along the Pavement Slab. The Resultant Stresses are Shown at Intervals.

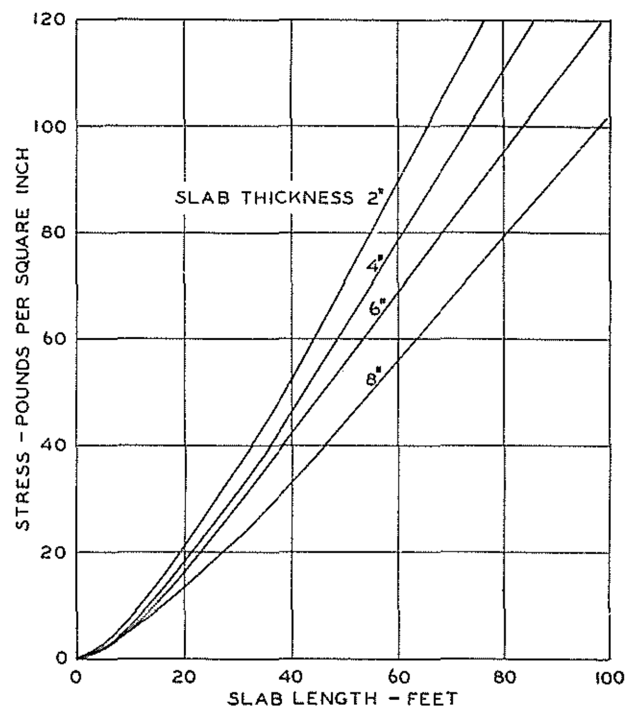


Figure 24.—Effect of Slab Length and Slab Weight on the Stresses Caused by Expansion or Contraction Resulting from a Temperature Change of 100° F.

various unit areas of the pavement may be moved by thermal expansion, the next step is to combine these data on the basis of equal displacements in order to determine the forces of resistance effective at each point throughout the length of the slab. The procedure is as follows:

Assuming a strip of pavement 1 foot in width and of appreciable length, consideration was given to successive sections 1 foot apart beginning at the mid-section of the strip (the section at which no displacement occurs during expansion and contraction). From figure 22B the displacement to be expected at each of the successive sections, caused by the assumed change in temperature, was determined. Then for each displacement the corresponding thrusting force was obtained for each section from the data given in figure 22A. The values of force developed in this way and plotted at the proper distances from the center of the slab determine the curve shown in figure 23. The total force necessary to move a strip of pavement of any given length may be determined from the area under the curve in this figure. The unit stress values to be expected are shown at intervals along the diagram, at the points where they apply.

The stresses caused by subgrade resistance were computed in this manner for the several slab thicknesses from the force-displacement curves shown in figure 20 and the resulting stress distribution diagrams are given in figure 24. These diagrams are applicable only to the particular subgrade conditions that obtained in these tests. It will be observed that the unit stresses developed by the large temperature change that was assumed for the example are small for all moderate slab lengths and also that the unit stresses for a given slab length decrease with increase in slab thickness, in accordance with the theory of subgrade resistance previously discussed.

It is interesting to compare the unit stresses obtained in the above analysis with those which would be obtained by assuming that all of the subgrade resistance developed as the result of simple sliding friction. Referring back to figure 20, it is found that for the 6-inch slab the thrust required to cause sliding was about 250 percent of the slab weight. In other words, a coefficient of friction of 2.5 might be assumed to apply. For a 6-inch slab 50 feet in length the application of this flat coefficient indicates a maximum unit stress of 62.5 pounds. This compares with a unit stress of 56 pounds by the more exact method. The difference is small but it should be remembered that this difference is caused by the elastic action of the soil and will, therefore, vary with the type and condition of the subgrade.

It seems probable that the stresses determined by the two methods would agree best for granular soils, and that for tenacious clays the values obtained by the assumption of a flat coefficient might be considerably in error. For slabs of average length, however, the stresses developed by this type of subgrade resistance are so small that, for average subgrade conditions, stresses computed by the simpler method are probably sufficiently accurate. In the tests at Arlington it was found that there was a tendency for the resistance to horizontal slab displacement to increase as the moisture content of the subgrade increased. Since the coefficient values given in the preceding tabulation were based upon data obtained at a time when the moisture content of the soil was high, it is probable that the values given approximate a maximum for the subgrade in question.

During the period of the observations there was no extended period during which the soil beneath the concrete was frozen, and it was not possible to make a study of the effect of a frozen subgrade upon the stresses being discussed. Temperature measurements in the concrete and subgrade and measurements of length changes during periods of cold weather showed that the changes in average concrete temperature at these times were relatively small. The daily air temperature range is much smaller in winter than in summer because of the decreased intensity of the sunlight. It seems quite possible, therefore, that even if the subgrade is frozen to the slab, the thermal changes in the pavement during such periods will be so small that the stresses in the pavement will not be increased to any important degree by the frozen subgrade.

TEMPERATURE WARPING DISCUSSED

Some difficulty was experienced in determining the shape of a warped slab, as approximately 1½ hours were required to make a complete set of clinometer measurements, and the temperature conditions that produced the warping rarely held constant for this length of time. This was especially true in the early morning when the maximum upward movement at the edges occurred. The data of most value are those made on days when, during the period of the actual measurement, the least change in temperature differential occurred. The occasions were rare when practically constant temperature conditions prevailed during the periods of measurement of both the flat and warped slab. Some measurements were obtained under these conditions, however, and it is believed that these data show very well the shape of the warped surface and the relative movements of its various parts.

Figures 25 and 26 show data obtained on one panel of each of the 6- and 9-inch uniform-thickness sections, respectively, and figure 27 shows similar data from one panel of the 9–7–9 thickened-edge section.

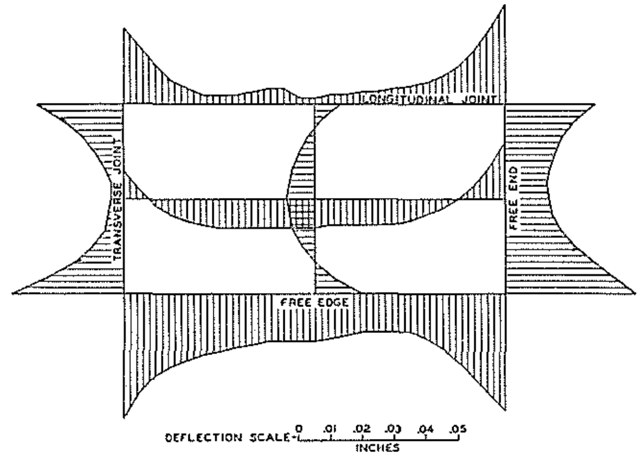


FIGURE 25.—DEFLECTIONS CAUSED BY TEMPERATURE WARPING OF A 10-BY-20-FOOT PANEL OF A 6-INCH UNIFORM-THICKNESS PAVEMENT FROM A 22° F. TEMPERATURE DIFFERENTIAL. ON APRIL 20, 1931: SLAB FLAT AT 8:30 P. M.; EDGES WARPED DOWN AT 1:30 P. M.

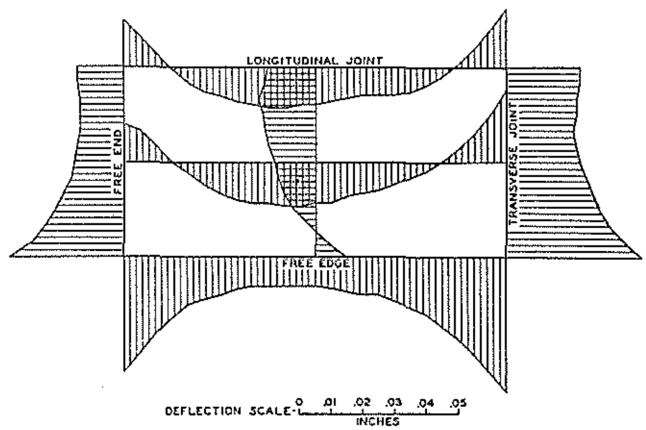


FIGURE 26.—DEFLECTIONS CAUSED BY TEMPERATURE WARPING OF A 10- BY 20-FOOT PANEL OF A 9-INCH UNIFORM-THICKNESS PAVEMENT FROM A 23° F. TEMPERATURE DIFFERENTIAL. ON APRIL 8, 1931; SLAB FLAT AT 10:30 P. M.; EDGES WARPED DOWN AT 2:30 P. M.

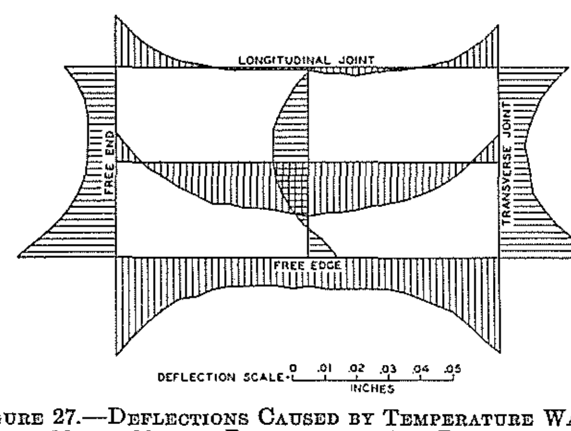


FIGURE 27.—DEFLECTIONS CAUSED BY TEMPERATURE WARPING OF A 10- BY 20-FOOT PANEL OF A 9–7–9 PAVEMENT FROM A 20° F. TEMPERATURE DIFFERENTIAL. ON APRIL 9, 1931; SLAB FLAT AT 9:30 P. M.; EDGES WARPED DOWN AT 2 P. M.

The restraining influence of the attachment to other panels at the longitudinal and transverse joints is evident in all of these diagrams. In the 9-inch uniform-thickness section the longitudinal joint is of the weakened-plane type. At the time of the warping measurements this plane had not cracked, with the result that the section acted almost as a full 20-foot-width slab. The formed groove in the upper surface of the pavement undoubtedly had some effect on the shape of the warped cross section even though the concrete below it had not cracked.

It will be noted that there are slight discrepancies in the deflections recorded at the common corner points when the measurements were made along the different edges of the panel. These are caused by the slight changes in the temperature differential during the period of the measurements.

All of the observations were made during the month of April when there is probably as much warping as at any time during the year, and the data shown are for the condition of downward movement of the edges. The data for the different sections are not directly comparable because there were some small differences in the differential at the time the various measurements were made, as noted on the diagrams.

These diagrams convey a very clear picture of the movements that occur daily in all concrete pavement slabs. In the transverse direction the panels appear to warp quite freely (except in the case of the unbroken longitudinal joint mentioned above). In the longitudinal direction the tendency for the weight of the slab to force the central area to lie flat is evident, being most noticeable if a comparison is made of the central portion of the longitudinal axes in the diagrams for the 6- and 9-inch sections shown in figures 25 and 26, respectively. This is as would be expected since the curvature should be some function of the ratio of length to thickness. The effect of this restraint on the stresses caused by warping will be apparent in data that are presented later.

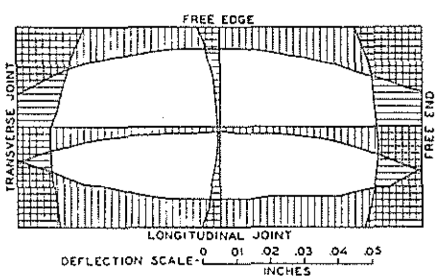


Figure 28.—Deflections Caused by Temperature Warping of a 10- by 20-foot Panel of a 9–6.3–9 Pavement from a 5° F. Temperature Differential. (American Association of State Highway Officials Cross Section). On May 27–28, 1931; Slab Flat at 8 p. m.; Edges Warped up at 5 a. m.

Figure 28 shows data obtained from a thickened-edge section of a different type at a time when the upper surface was at a lower temperature than the lower surface, with a consequent upward movement of the edges. As would be expected from the data on temperature differentials previously presented, the magnitude of the warping in this direction is always much less, since the temperature differential is less than that which occurs when the temperature conditions are reversed. The relation between the magnitude of the temperature differentials and that of the edge movements in the two directions is not a direct one.

In figures 25, 26, and 27 the downward edge movement shown was caused by differentials of about 20° F. The upward movement shown in figure 28, which is about half of that shown in the preceding three figures.

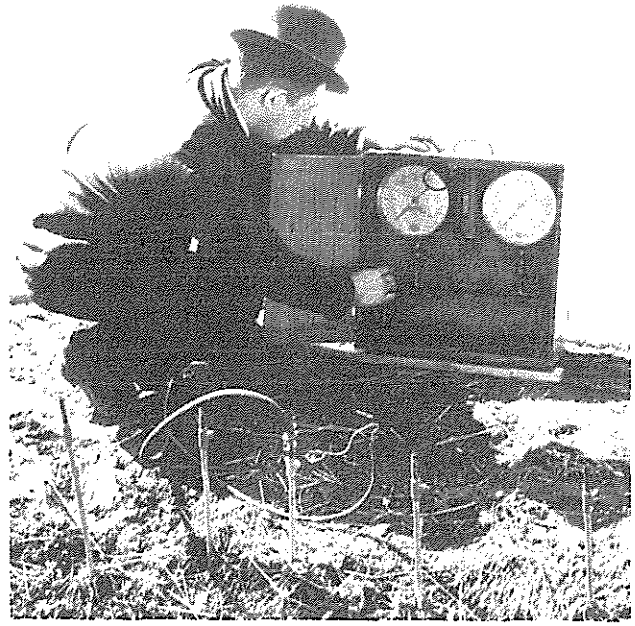


Figure 29.—Measuring the Pressure Between the Pavement Slab and the Subgrade.

is the result of a temperature differential of only 5° F. If the warping diagrams are examined it will be observed that the interior of the panel was actually raised above the flat position as the edges warped downward but that there is little or no depression of the central area as the edges are warped upward. It seems quite probable that the redistribution of subgrade reactions which must attend these changes in shape would account for the difference in the freedom of warping in the two directions that has been noted.

RELATIONS BETWEEN TEMPERATURE WARPING AND SUBGRADE PRESSURE DETERMINED

Some data on the variation in intensity of subgrade pressure resulting from temperature warping were obtained with the pressure cells before the installation ceased functioning. The pressures were measured at several points across the transverse axis of one panel of the 9-inch uniform-thickness section (see fig. 29). The weakened-plane longitudinal joint in this section had not broken at the time and the slab was acting practically as one of a full 20-foot width.

Figure 30 shows the measured unit pressures at five different times during the day. The relation between the warping of the slab and the distribution of reactions is readily apparent. In the morning with the edges of the slab warped upward, the greatest pressure measured was in the interior, the pressure toward the edge being at its minimum at this time. During the day, as the upper surface of the slab expands, there is a complete relief of the high pressure in the interior region and a development of a maximum reaction near the edge, exceeding the maximum previously developed in the interior by at least 50 percent.

Both the warping data and the subgrade pressure data suggest the possibility of an actual lifting of the central area from the subgrade as the edges warp downward. While it is possible that this may actually occur, it seems more probable that the vertical movements measured were all within the range of elastic deformation of this particular subgrade soil. Sub-

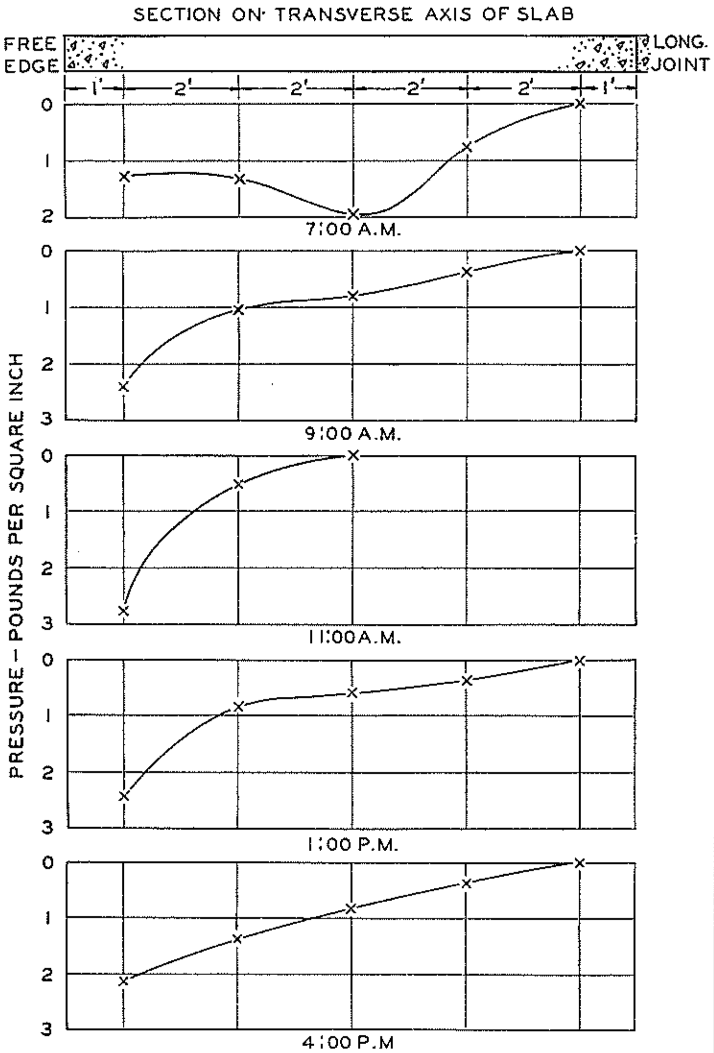


FIGURE 30.—EFFECT OF WARPING UPON THE PRESSURE OF THE SLAB AGAINST THE SUBGRADE FOR A 9-INCH SLAB.

grade bearing tests indicate that the changes of unit pressure shown by the pressure cells might be expected to cause soil deformations approximating in magnitude the vertical movements of the interior of the slab during the daily cycle of warping.

The daily variations in shape of the transverse sections of a pavement slab are well illustrated by the data in figure 31. These data furnish additional evidence on the relative magnitude of the upward and downward temperature warping and show that the maximum warping occurs during the warmest part of the day. Measurements taken at many different times throughout the year show this to be true for all normal days irrespective of the season. This is as would be expected in view of the temperature data that have been obtained.

The daily movements of certain points at the edges of some of the sections measured with micrometer dials are presented in figures 32 and 33. The object of the measurements shown in figure 32 was to compare the movements of sections having different thicknesses and cross sections under exactly the same conditions of air temperature variation. The vertical movement of the corner of a 9-inch uniform-thickness section is 150 percent of that of one 6 inches thick. In other

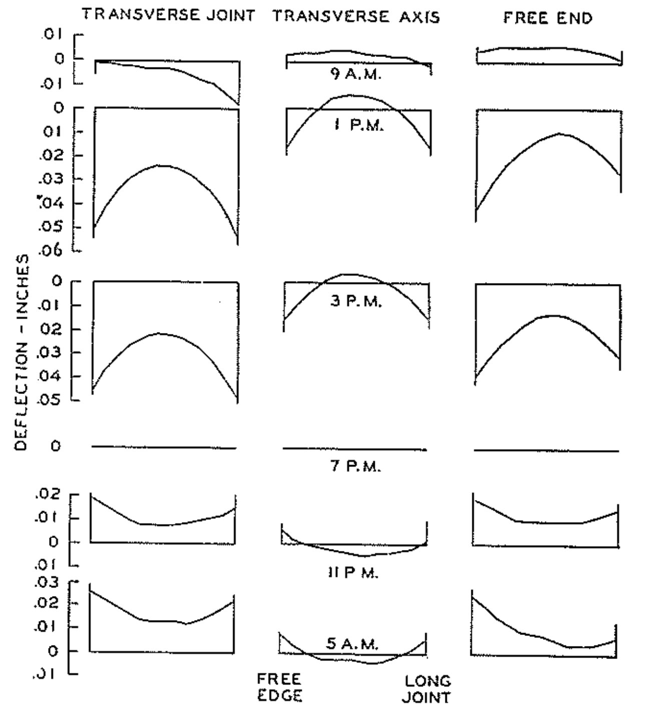


FIGURE 31.—WARPING OF TRANSVERSE ELEMENTS OF 6-INCH UNIFORM-THICKNESS SECTION. MAY 27–28, 1931.

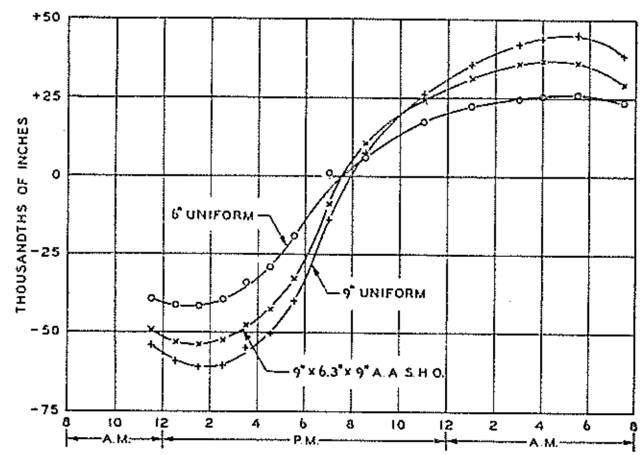


FIGURE 32.—WARPING OF CORNERS OF THREE TEST SECTIONS, MAY 27–28, 1931.

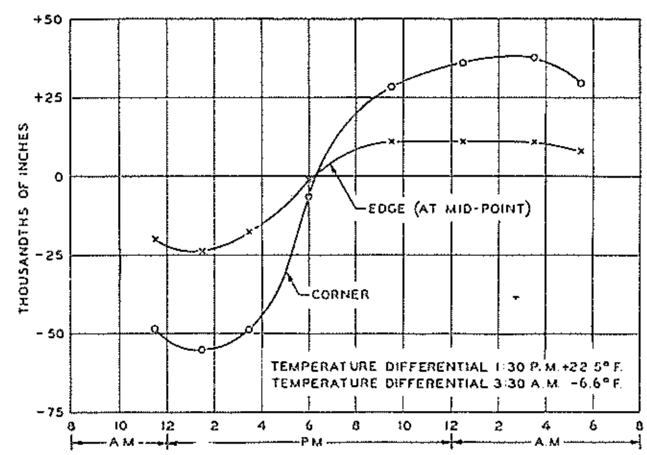


FIGURE 33.—WARPING OF EDGE AND CORNER OF 6-INCH UNIFORM THICKNESS SLAB, JUNE 8–9, 1932.

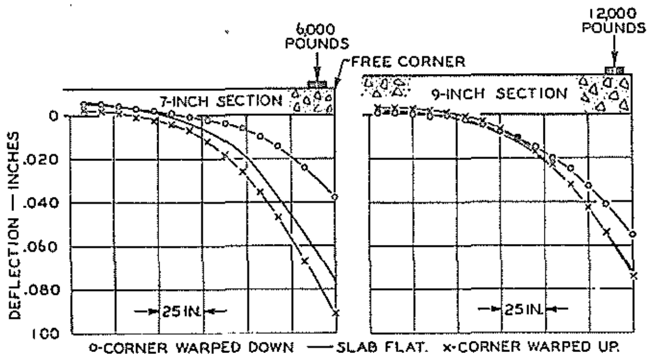


FIGURE 34.—EFFECT OF TEMPERATURE WARPING ON THE CORNER DEFLECTIONS CAUSED BY AN APPLIED LOAD (DEFLECTIONS MEASURED ALONG THE DIAGONAL).

words, direct proportionality apparently exists between the thickness of the section and the magnitude of the corner movement.

Another important indication is that the 9-inch edge of a typical thickened-edge section does not cause warping movements as great as would be found in a section that was uniformly 9 inches thick. In figure 33 the vertical movements of the free corner and the mid-point of the free edge are compared. The maximum temperature differentials observed during this series of observations are noted in the figure and it is of interest to compare these with the vertical movements that they produce at the two points at which the measurements were made. The temperature differential that caused the upward movement of the edges and corners is approximately 30 percent of that which caused downward movement, yet at the corner the upward movement is 70 percent and at the mid-point of the free edge 46 percent of the downward movement. These relations are in general accord with the data previously presented in connection with the discussion of the warping data for the entire slab panel.

EFFECT OF APPLIED LOADS ON WARPED PAVEMENTS INVESTIGATED

It is logical that any redistribution of subgrade reactions, such as those occurring when a pavement warps, must result in a change in the deflections and the stresses that will be produced by a given applied load. As mentioned earlier in the discussion of the investigation, a study was made of the effect of slab shape on the deflections and stresses caused by applied loads. Figure 34 shows the elastic curves of the diagonal at the slab corner under the applied loads noted, for three conditions of warping as they occurred during a single 24-hour period. It is apparent from these curves that the 7-inch slab is affected to a greater degree than the 9-inch slab. Downward warping of the corner reduced the deflection resulting from load by about 50 percent for the 7-inch section and only 25 percent for the 9-inch section. It is believed that this difference is due to the fact that the thinner the slab the more dependent it is on the conditions of local subgrade support.

The data indicate that upward warping has but little effect on the extent of the corner deflection produced by a given load. In the case of the 9-inch slab there is no increase, while for the 7-inch section a slight increase is noted. This condition could be caused by the lack of complete contact with the subgrade with the slab in the flat position, a condition which might

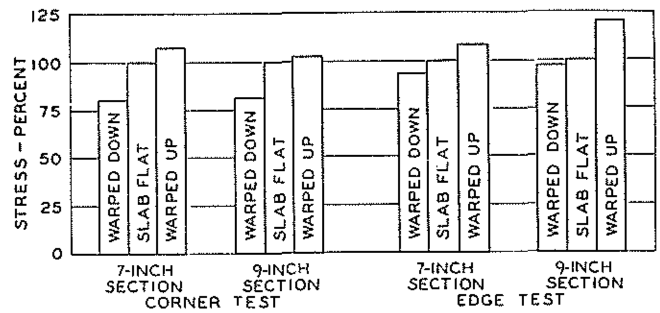


FIGURE 35.—EFFECT OF TEMPERATURE WARPING ON THE MAGNITUDE OF THE STRESS CAUSED BY AN APPLIED LOAD.

easily obtain at the corner of a pavement slab. Complete contact would then be developed only by a downward deflection of the corner.

Some idea of the effect of the condition of warping on the magnitude of the stress that a given load will cause may be had from figure 35. This chart shows the variations in the critical stresses at the corner and edge of two of the sections, referred to the stress produced by the given load on the unwarped slabs as a base. These data were obtained at the same time and under the same conditions as the deflection data given in the preceding figure.

There is a reduction of approximately 20 percent in the critical stress for the corner loading when the corner is warped downward. Since the maximum working stress was about 300 pounds per square inch, this reduction amounted to approximately 60 pounds per square inch. There is also a slight increase in the critical stress if the load is applied at a time when the corner is warped upward.

For loads applied at the edges, it appears that downward warping results in but a slight reduction in the stress produced by a load while upward warping will cause increases that may amount to as much as 20 percent.

Reference to the figures which show the shape of the warped panel suggests a reason for the effects that have just been noted. When the temperature conditions are such that the edges of the pavement warp downward, the longitudinal curvature of the panel is such that the mid-point of the edge tends to move upward at the same time that the transverse curvature is forcing it downward. The result is that this point is not displaced downward to nearly the same degree as is the corner of the slab. So far as subgrade support is concerned the situation is probably but little better than it is for the flat slab condition.

Similar tests were made at the interior of both the 7-inch and 9-inch slabs and it was found that at this point the condition of slab warping has a negligible effect upon the magnitude of the critical stress produced by a given applied load.

These tests have shown quite definitely that even extreme conditions of temperature warping produce variations in the critical stresses caused by applied loads that are considerably smaller than has been generally supposed.

MOISTURE WARPING DISCUSSED

As stated earlier, clinometer measurements on certain critical regions of the 9-inch constant-thickness slab were made periodically over the year at times when no temperature differential could be detected in the concrete. The curves obtained from these measure-

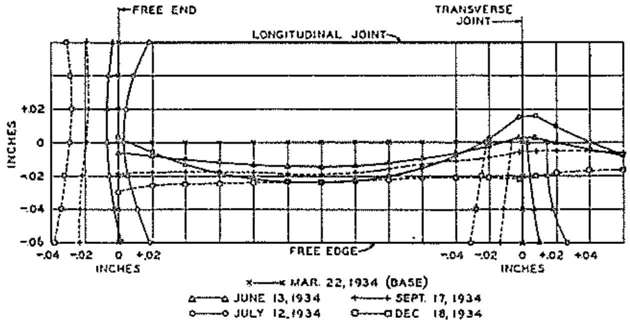


Figure 36.—Variation in Slab Shape at Various Times During the Year from Causes Other Than Temperature Changes, 9-inch Section. Upward Movements Shown as Positive, and Downward Movements Shown as Negative.

ments show the variation in shape of certain parts of the pavement slab at various times during the year from causes other than temperature changes.

Figure 36 shows typical data obtained from these observations of the 9-inch section. While actually a considerable number of sets of such data were obtained, in the figure only a few sets are shown for the sake of clarity. Since no means was available for determining the moisture gradient of the slab, it was not possible to predict with certainty the time when moisture conditions would be such as to cause it to be in a flat condition. In the presentation of the data in figure 36 the slab was assumed to be flat at the time of the March 22 observation on the basis of reasoning that follows.

The measurements of longitudinal expansion and contraction shown in figure 14 indicated that the expansion from moisture reaches a maximum during the winter months (January to March). For the year during which the moisture warping measurements were made (1934), the observed expansion was a maximum in January and by March it had dropped off slightly. This is a period during which the subgrade moisture content reaches a maximum value and during which the rate of evaporation is very low. It seems logical to conclude, therefore, that the moisture content of the concrete would be both high and most nearly constant during these months and that the moisture gradient that causes warping would be a minimum.

If a comparison is made between the curves in figure 36 showing the shape of the longitiudinal center line on March 22 and on December 18, it will be noted that on these dates the shapes are essentially the same, although some vertical movement of the slab as a whole had occurred. This tends to substantiate a conclusion that the slab is warped but little by moisture during the midwinter months. However, it should be remembered that the data shown in figure 36 are referred to the March observations as a base and that the curves indicate the changes in slab shape that occurred between March 22 and the other dates listed in the figure. If the slab was in an unwarped condition at the time of the March measurements, as was assumed, obviously the curves in this figure would indicate the true upward and downward warping of the slab.

Figure 37 shows the effect of moisture on the warping of a free corner and on the vertical displacement of the mid-point of one of the 10- by 20-foot panels, over a period of approximately 1 year. This graph was constructed from the same data and based upon the same measurements as the previous figure, although more

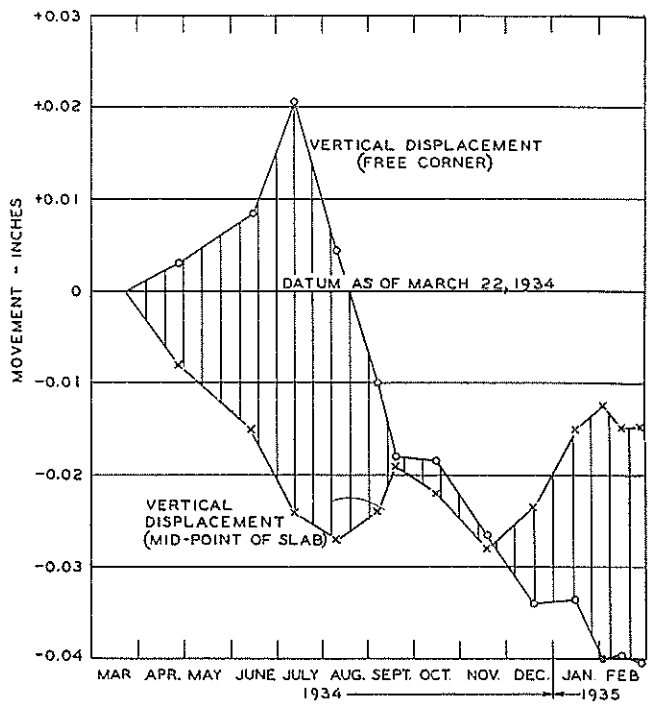


Figure 37.—Seasonal Variation of Warping Caused by Moisture Change, 10- by 20-foot Panel of 9-inch Thickness. The Distance Between Curves (Shaded Area) Shows the Extent of Moisture Warping.

observations were included. In determining the warping at the free corner, it was necessary to make corrections for any vertical displacements of the slab as a whole, as well as for any tilting of the slab that might have occurred. In making the first correction, it was assumed that the vertical displacement of the geometrical center of the panel with respect to the bench mark or reference point at the edge of the slab represented fairly the vertical displacement that took place over the slab as a whole. The correction for tilting was determined by averaging the displacements of three corners of the panel to establish a plane for each observation.

Referring to figure 37, it will be noted that after the middle of July a sudden reversal in the direction of the moisture warping occurred. This is believed to be caused by a marked change in weather conditions that took place at about this time. During August and September of this particular year the precipitation was much above normal (over 17 inches for September alone) and there was an unusually high percentage of hazy and cloudy weather. Normally it would be expected that this change in the direction of moisture warping would occur later, possibly in late August or early September. In this connection it is interesting to compare the relation between moisture warping and time, as shown in this graph, with that between moisture expansion and time as shown in figure 14 and to note the close correlation that exists.

DIFFICULTY ENCOUNTERED IN DETERMINING STRESSES CAUSED BY MOISTURE WARPING

It will be observed from the graph that during July the loss of moisture from the upper surface had caused the free corner to be warped upward approximately 0.045 inch with respect to the mid-point of the panel. From this, an estimate of the stress developed by moisture warping might be attempted, but this is not warranted for the following reasons:

1. The true shape of the warped slab was not determined.
2. Plastic flow undoubtedly enters as a factor and its importance is unknown.
3. Settlement of the slab into the subgrade alters the degree of restraint that exists.

The curve showing the movement of the center of the panel indicates that, as the seasonal warping takes place, the slab settles into the subgrade. This seems probable when one considers that the condition of warping from moisture change develops slowly over a long period of time and that the development is most active during the spring months when the soil of the subgrade contains a considerable amount of moisture. If this is the case, it probably has an important influence upon the amount of restraint that the slab encounters when it warps from moisture changes. If there were complete settlement of the slab into the subgrade so that the subgrade conformed completely to the warped shape, then both edges and interior would have full subgrade support and no restraint would be developed by the weight of the slab.

The extent to which the subgrade adapts itself to the slab as moisture warping develops is no doubt largely dependent upon the type and physical condition of the subgrade material, but it is reasonable to believe that, because of the time element and its effect on both subgrade behavior and on plastic yielding of the concrete itself, the restraint and therefore the stresses developed by moisture warping are not as great as the magnitude of the curvature might lead one to suspect.

The data indicate that the curvature caused by moisture is principally an upward warping of the edges caused by a moisture loss from the upper surface of the pavement. The downward warping of the edges, resulting from a condition in which the moisture content in the upper part of the pavement exceeds that in the lower part, seems to be considerably smaller for the conditions of these tests.

Thus it appears from the data that, at those times when high stresses are developed by temperature warping, as for example, an afternoon in midsummer, the effect of moisture is to cause curvature such that any stresses developed by it will tend to relieve rather than aggravate the stresses caused by the restraint to temperature warping.

STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING DETERMINED

It is believed that one of the most important results of the entire investigation has been the development for the first time of reasonably reliable experimental data showing the magnitude and distribution of the stresses caused by the restrained temperature warping of typical pavement sections. These data, obtained by the methods that were described at the beginning of this paper, are presented in various ways in the figures that follow.

It has been shown that under normal conditions the temperature differential that causes warping is much larger during the day than during the night, and that usually the daily maximum occurs in the early afternoon. Since it was desired principally to determine the magnitude of the warping stresses for the condition of average maximum temperature differential, the greater portion of the measurements were made during the daytime with the upper surface of the pavement at a higher temperature than the lower surface. The warping stresses occurring under this condition are more important also because tension is developed in the bottom of the slab. A sufficient number of night observations was made, however, to give a clear indication of the magnitude and relative importance of stresses developed at night.

The manner in which the stresses produced by restrained temperature warping during the day vary along the two principal axes of two of the test section panels is indicated by the curves in figures 38 and 39. The data in figure 38 apply to the transverse axes, while those in figure 39 apply to the longitudinal axes of the two slabs. At each point along each axis the stresses in both the transverse and longitudinal directions were determined. The values shown in these figures are averages of several sets of measurements made on selected days during the summer and fall.

The data pertaining to the transverse axis were all obtained during the summer but some of those pertaining to the longitudinal axis were obtained during the fall when the temperature differentials were not as large as during the summer. The values shown in figure 38 probably represent the largest that will occur with any frequency in the locality where the tests were made. The maximum values observed at any time were approximately 15 percent greater than the averages shown in this figure.

During a part of the tests the slabs used for the temperature measurements were unavoidably shaded so that complete data on the temperature differentials causing these stresses are not available. From the temperature data obtained it is estimated that the average temperature differentials causing the stresses shown in figure 38 were approximately 18° F. for the 6-inch and 23° F. for the 9-inch test sections.

In the absence of data on sections less than 20 feet in length, it is not possible to predict accurately what the maximum warping stresses on shorter sections would be. However, up to the present time very few concrete pavements have been laid in which the length of the slab units was less than 20 feet. For slab lengths greater than 20 feet, it is believed the data indicate that, for a 6-inch slab thickness, the maximum warping stress will not exceed that developed in the 20-foot slab, while in a 9-inch pavement the maximum warping stress may be somewhat greater than that shown. These conclusions are based on the shape of the longitudinal axis of the two warped slabs as determined with the clinometer.

The relative magnitudes of the stresses from restrained temperature warping at several points near the corners of the 6-inch and 9-inch uniform thickness sections are shown by the stress diagrams in figure 40. The stresses were determined along the lines A—B, A—C, and A—D of figure 6. Along the free edges of the slab the stress in the direction perpendicular to the edge is in all cases negligible and, for this reason, only stresses in the direction parallel to the edge are shown for the lines A—B and A—D (fig. 6). Along the diagonal at the free corner (line A—C, fig. 6), measurable stresses are found in both directions and the stresses perpendicular to and parallel to the diagonal are shown in figure 40.

It is interesting to note that the variation in the magnitude of the stresses measured perpendicular to the diagonal is similar to that of the stresses in the direction parallel to the diagonal. There seems to be

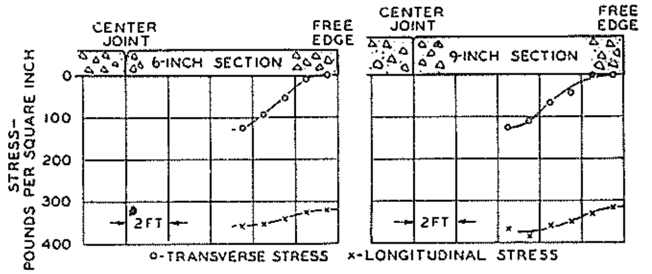


FIGURE 38.—MEASURED WARPING STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING ALONG THE TRANSVERSE AXIS OF TWO TEST SECTION PANELS.

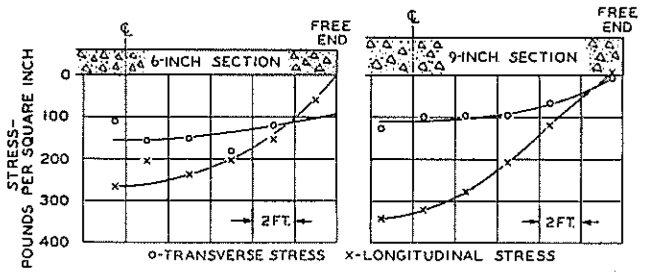


FIGURE 39.—MEASURED WARPING STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING ALONG THE LONGITUDINAL AXIS OF TWO TEST SECTION PANELS.

a consistent lack of uniformity in the variation in stress along the diagonal that might reasonably be attributed to a buckling action across the corner as this portion of the slab attempts to respond to two conflicting sets of forces.

The maximum stress along the free edge as shown in figure 40 is smaller than that shown in figure 38. This is because the stresses at the corner were determined for somewhat smaller temperature differentials.

Figure 41 shows stresses caused by restrained temperature warping at the corner and along the longitudinal axis of the 6-inch constant-thickness section under normal night conditions, i. e., with the upper surface at a lower temperature than the lower surface. The stress values shown are the averages of several sets of observations made on nights when, for night conditions, relatively large temperature differentials developed. The observations were made on 3 of the 4 panels of the test section.

It will be observed that the stresses vary in much the same manner as was found in the daytime measurements, although their magnitude is but about one-fourth as great. This is as would be expected as the observed temperature differentials were in approximately the same ratio. Under night conditions, the stresses developed are tensile stresses in the upper surface and compressive stresses in the lower surface of the pavement. They are, therefore, opposite in sense to the stresses caused by applied loads except for the case of a load applied on the corner of the slab.

STRESSES GREATEST IN LONGITUDINAL DIRECTION

In order to present a general picture of the critical stress conditions that result from restrained temperature warping, the stress diagrams shown in figures 42 to 45, inclusive, were prepared, utilizing the average measured stress curves to determine the shape of the variation curves and adjusting the stress magnitudes to a common value at the interior points of the slab. Figures 42 and 43 show the stresses parallel to the longitudinal axes of the slabs except at the diagonal

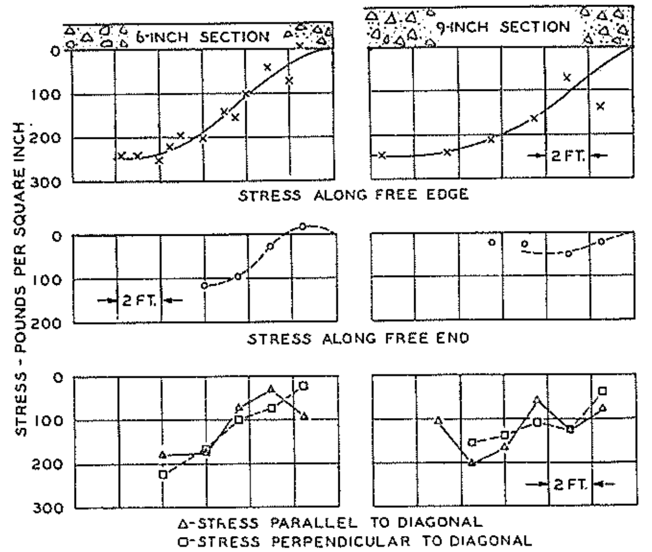


FIGURE 40.—MEASURED WARPING STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING NEAR THE SLAB CORNERS.

at the free corner where the stresses are parallel to the diagonal. Figures 44 and 45 are similar diagrams for the stresses perpendicular to the longitudinal axes or perpendicular to the diagonal.

It was found by numerous measurements that stresses of the magnitudes indicated in these 4 diagrams may be expected to occur frequently in the daytime during the spring and summer months in the locality of Washington, D. C. As previously stated, stresses somewhat exceeding these were found occasionally on days of extreme temperature changes.

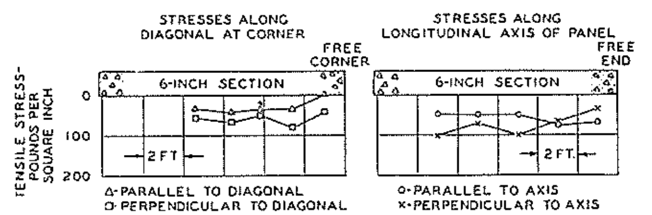


FIGURE 41.—AVERAGE MAXIMUM TENSILE STRESSES OBSERVED IN THE UPPER SURFACE OF THE 6-INCH SECTION CAUSED BY RESTRAINED TEMPERATURE WARPING AT NIGHT.

THEORETICAL AND MEASURED STRESSES COMPARED

Reference has been made to the analysis, by H. M. Westergaard, of the stresses caused by restrained temperature warping from the standpoint of theoretical mechanics. Figure 46 shows theoretical warping stresses computed for the transverse section of a slab of infinite length but finite width and utilizing elastic constants known to apply to the materials in the Arlington tests. The temperature differentials of 18° F. and 25° F. for the 6-inch and 9-inch slabs, respectively, are reasonable in the light of the temperature data obtained in this investigation. Both values are considerably higher than that assumed by Dr. Westergaard in the examples given in his analysis.

Since the Westergaard analysis is based upon the assumption of a slab of infinite length, it is perhaps not permissible to make direct comparisons between the theoretical stresses and those determined experimentally. However, since the length of the experimental slabs is twice the width, it is believed that the sections, particularly the thinner ones, will behave in a

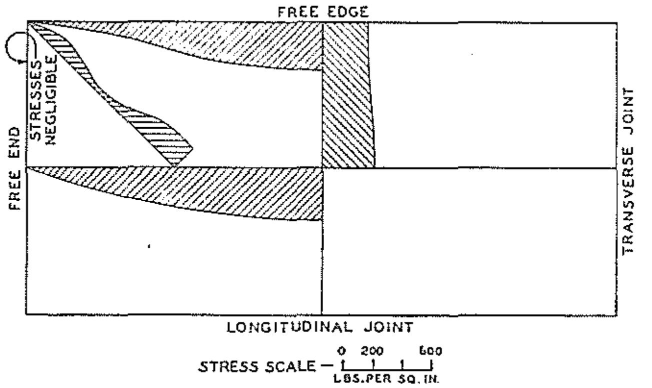


FIGURE 42.—VARIATIONS IN COMPRESSIVE STRESS IN THE UPPER SURFACE OF 10- BY 20-FOOT PANEL OF A 6-INCH PAVEMENT. STRESSES MEASURED IN A LONGITUDINAL DIRECTION OR PARALLEL TO DIAGONAL. STRESSES ARE CAUSED BY RESTRAINED TEMPERATURE WARPING.

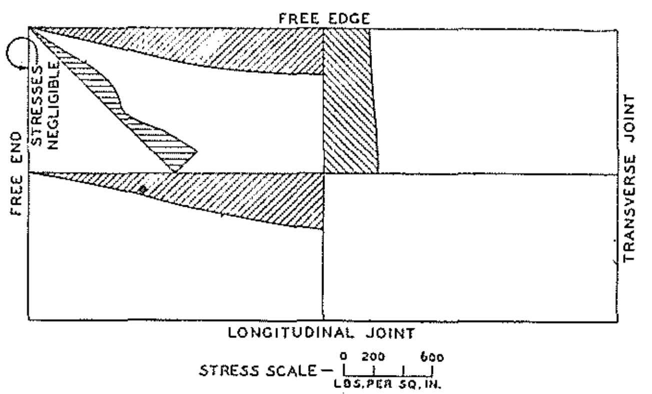


FIGURE 43.—VARIATIONS IN COMPRESSIVE STRESS IN THE UPPER SURFACE OF 10- BY 20-FOOT PANEL OF A 9-INCH PAVEMENT. STRESSES MEASURED IN A LONGITUDINAL DIRECTION OR PARALLEL TO DIAGONAL. STRESSES ARE CAUSED BY RESTRAINED TEMPERATURE WARPING.

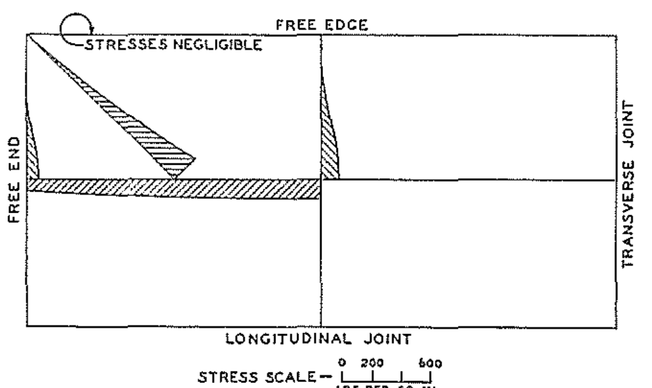


FIGURE 44.—VARIATIONS IN COMPRESSIVE STRESS IN THE UPPER SURFACE OF 10- BY 20-FOOT PANEL OF A 6-INCH PAVEMENT. STRESSES MEASURED IN A TRANSVERSE DIRECTION OR PERPENDICULAR TO DIAGONAL. STRESSES ARE CAUSED BY RESTRAINED TEMPERATURE WARPING.

manner approximating that of longer slabs, and that the stresses in a transverse section near the center are sufficiently like those in the longer slab to make a comparison with the theory of value.

If the curves in figure 46 are compared with the experimental curves shown in figure 38, it will be noted that the shapes of the theoretical and measured stress

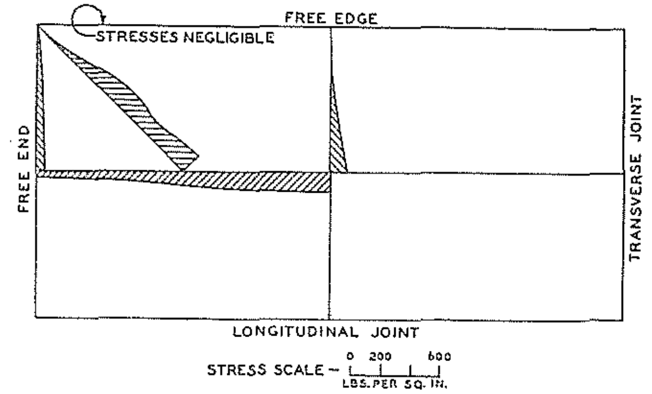


FIGURE 45.—VARIATIONS IN COMPRESSIVE STRESS IN THE UPPER SURFACE OF 10- BY 20-FOOT PANEL OF A 9-INCH PAVEMENT. STRESSES MEASURED IN A TRANSVERSE DIRECTION OR PERPENDICULAR TO DIAGONAL. STRESSES ARE CAUSED BY RESTRAINED TEMPERATURE WARPING.

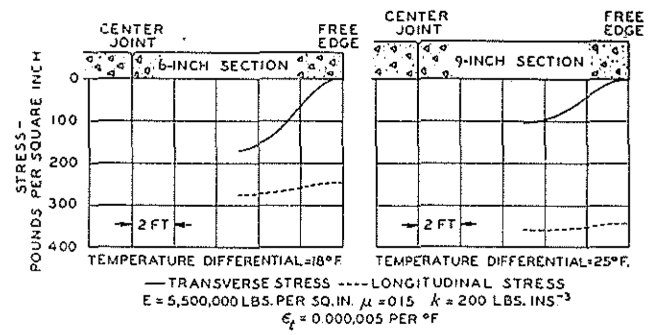


FIGURE 46.—THEORETICAL STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING AT A TRANSVERSE AXIS OF A SLAB OF INFINITE LENGTH.

curves are very similar, and also that the magnitude of the theoretical stresses are of the same order as the values found by measurement. In this connection it should be pointed out again that the average temperature differentials were estimated and may not be exact. From this comparison it seems reasonable to conclude that warping stresses properly calculated with the theoretical formulas will give a fair estimate of the critical warping stresses in pavement slabs 20 feet or more in length.

There is one noticeable difference, however, between the theoretical and measured stress relations. The measured stresses have practically the same magnitudes for both the 6-inch and 9-inch slabs in spite of the 5° F. difference in the temperature differentials. The stresses computed by the theory, on the other hand, reflect this difference in higher stresses for the 9-inch thickness. It is reasonable that the greater temperature differential in the thicker slab should produce higher stresses. Had the length of the 9-inch test section been greater than 20 feet, it is believed that the measured stresses would have been higher and thus more in accord with the theory.

It has been shown previously that, when warped, the 6-inch section tends to remain flat along the central portion of the longitudinal axis, while the 9-inch section showed continued curvature in this region. If the 9-inch slab had been longer it too would have been flattened in the mid-portion of its long axis and this undoubtedly would have increased somewhat the value of the measured stresses.

MERITS OF THICKENED-EDGE DESIGN INVESTIGATED

Data presented earlier in this paper show that when the edge thickness of a paving slab is increased, for the purpose of increasing the load-carrying capacity, there is certain to be a corresponding increase in the temperature differential that develops in this portion of the pavement. For example, it was shown that the temperature differentials observed at the edge of a 9–6–9 section were about 45 percent greater than those observed in the edge of a 6-inch constant-thickness section and approximately the same as those in the edge of a 9-inch constant-thickness section. Since an increase in the temperature differential at any part of a slab that is restrained from warping causes a corresponding increase in the warping stresses at that point, the effect just mentioned is an important consideration in the design of concrete pavements.

The effect of increasing the thickness of the edges of a pavement slab on the magnitude of the critical warping stresses can be illustrated by considering two long and relatively narrow pavement slabs both of the same interior thickness. One is of constant thickness while the other is of a conventional thickened-edge cross section with an edge depth 50 percent greater than the interior depth. The temperature differentials and therefore the warping stresses that develop in the interior of the two slabs will be approximately the same since the slabs have the same thickness in this region and are both sufficiently long to develop complete restraint. At the free edges, however, the differential in temperature for the thickened-edge cross section will be, according to the data obtained in this investigation, some 45 percent greater than in the slab of constant thickness.

The results of the increased temperature differential at the edge of the thickened-edge section will be an increase in the stresses caused by restraint to warping in the edge region of the slab. Since the slabs are relatively narrow in a transverse direction they are relatively free to warp and the warping stresses will be small. Thickening the edge will therefore have but little effect upon the transverse warping stresses. In the direction parallel to the edge of the pavement, the magnitude of the warping stresses varies with the degree of restraint that obtains at the particular point under consideration. The degree of restraint varies, in turn, with the distance from the free end of the slab and with the slab depth. At the extreme end no restraint exists as the slab can warp freely at this point. The rate at which restraint develops with the distance from the extreme end will depend upon the depth of the slab, as will the distance to the point where complete restraint is obtained.

The two slabs considered in the example were long enough to develop complete restraint and consequently maximum warping stresses in their mid-length, and in this region it was found that the temperature differential at the edge of the thickened-edge section was about 45 percent greater than in the edge of the slab having a constant thickness equal to that of the interior of the thickened-edge section. Since, theoretically, there is a direct relation between the magnitude of the stress resulting from completely restrained warping and the temperature differential that exists, this 45 percent increase in the temperature differential would lead one to expect a corresponding increase in the warping stress.

The relations between slab length, slab depth, and restraint to temperature warping were not determined by this investigation. To study this problem thoroughly, a range of slab lengths in each of several thicknesses would have to be constructed and the critical warping stresses determined for each. The data obtained would make it possible to determine what lengths of slab are sufficiently free to warp to make relatively unimportant the increase in warping stresses that results from the increased edge thickness. All of the slabs cast for this study of concrete pavement design were of the same length and only limited data bearing upon the relations could be obtained. These data are presented later in this paper.

WARPING STRESSES IN THICKENED-EDGE AND CONSTANT-THICKNESS SECTIONS COMPARED

Some measurements were made to determine the warping stresses at the edge of a 9–6–9 test section in comparison with those at the same point in the 6-inch and 9-inch constant-thickness sections. The data obtained are given in table 4.

TABLE 4.—*Observed longitudinal warping stresses at the edge of three 20-foot pavement slabs*

Date, 1934	Temperature differential at edge			Observed longitudinal warping stresses			Increase in stress, 9–6–9 slab over—	
	6-inch slab	9-inch slab	9–6–9 slab	6-inch slab	9-inch slab	9–6–9 slab	6-inch slab	9-inch slab
	° F.	*° F.*	*° F.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Percent*	*Percent*
Apr. 18	18	27	27	220	--------	316	44	--------
Apr. 22	14	24	21	186	--------	218	17	--------
Apr. 24	21	31	29	195	--------	291	49	--------
May 9	18	25	24	209	191	245	17	28
May 18	21	30	30	--------	298	380	--------	28
May 19	20	26	29	252	306	380	51	24
May 20	23	33	32	--------	302	361	--------	20
May 21	20	31	32	320	329	409	28	24
May 22	--------	--------	--------	322	252	347	8	38
May 24	19	25	25	266	213	282	6	32
June 2	--------	--------	--------	229	251	336	47	34
June 3	--------	--------	--------	281	273	377	34	38
Average	--------	--------	--------	--------	--------	--------	30	30

These data were obtained from simultaneous measurements at the mid-length of the three 20-foot test slabs—the 6-inch constant-thickness slab (sec. no. 10), the 9-inch constant-thickness slab (sec. no. 6) and the 9–6–9 thickened-edge slab (sec. no. 5). The observations extended over a considerable period of time and it is thought that in spite of some inconsistencies, the data give a very good indication of the relative magnitude of the longitudinal warping stresses in slabs of these thicknesses and this length. The temperature differentials are typical of the highest average values that may be expected to occur frequently in the locality where the tests were made.

It will be noted that the stresses measured at the edge of the 9–6–9 slab are in every instance higher than those measured at the edge of either of the constant-thickness slabs. At times this difference is as much as 100 pounds per square inch for the 6-inch and 80 pounds per square inch for the 9-inch constant-thickness slabs. The last two columns of table 4 show these differences, expressed as a percentage of the stress in the constant-thickness slab. There is a considerable variation in these values for the comparison with the 6-inch slab, the reason for which is not known.

The average increase of warping stress of the thickened-edge section over the 6-inch constant-thickness section is approximately 75 pounds per square inch, or

30 percent. The average difference in corresponding temperature differentials is 47 percent. These data indicate, therefore, that the increase in stress is not as great as might be expected in view of the measured increase in temperature differentials. This may be the result of the thickened-edge slab warping slightly more at the point where the stresses were determined than does the 6-inch slab, because of the difference in the length-depth ratio, and is thus able partially to offset the effect of the increased temperature differential. From these tests it might be concluded that the full effect of increased temperature differential resulting from the increased depth of the slab edge of this 9–6–9 cross-section does not develop in a slab length of 20 feet.

It may seem surprising that the longitudinal warping stress in the thickened-edge section is consistently greater than that in a constant-thickness slab of the same edge depth, particularly in view of the fact that the measured temperature differentials were approximately the same in the two slabs. The explanation is believed to be that in slabs of the same length the 9-inch constant-thickness slab would be expected to warp more freely than the 9–6–9 thickened-edge design.

The data resulting from loading tests that are to be presented in a subsequent report of this series show that the 50-percent increase in edge thickness of the 9–6–9 as compared with the 6-inch constant-thickness slab resulted in a reduction of approximately 28 percent in the critical stress caused by load applied at the edge of the pavement. For example, the reduction in the critical load stress effected by the thickened edge for a 7,000-pound load applied at the edge position amounted to approximately 100 pounds per square inch in these tests.

Figure 39 shows that the average warping stress at the edge of the 6-inch constant-thickness section is approximately 320 pounds per square inch during the summer. It has just been shown that the edge thickening under consideration caused an increase of approximately 30 percent in these stresses. The edge thickening, therefore, causes an increase of approximately 90 pounds per square inch, which is practically equal to the reduction in load stress accomplished by the increased edge thickness. This indicates that, on the basis of the combined load and temperature stresses, at times when the warping stresses are high, the load-carrying capacity of the 20-foot, 9–6–9 section is not increased by the edge thickening. For much shorter slabs this would not be true because the warping stresses would be low, but for slabs of greater length it is indicated that at times when the warping stresses are high, the load-carrying capacity of the edge of a pavement slab may actually be reduced by thickening the slab edge.

IMPORTANCE OF REDUCING WARPING STRESSES DISCUSSED

The data that have just been presented clearly indicate that the stresses arising from restrained temperature warping equal in importance those caused by the heaviest wheel loads. The stresses from this cause are actually large enough to cause failure in concrete of low flexural strength, and since the direction of the stresses is such that they become added to the critical stresses caused by wheel loads, there is little doubt but that warping stress is primarily responsible for much of the cracking in concrete pavements. It must be concluded also that so long as the slabs are of considerable length originally, a thickened-edge design will not reduce the amount of transverse cracking that will occur. This conclusion is in agreement with the observations made on this point in the extensive pavement survey conducted several years ago by the Highway Research Board.[13]

It is evident that either the magnitude of the warping stresses must be reduced by building smaller slab units or by some other means not yet proposed, or the amount of stress resistance available for supporting wheel loads will be greatly curtailed.

The most practical means at present available for reducing warping stresses is through the construction of shorter slabs. As previously mentioned, some data that indicate the possibilities of reducing warping stresses through a decrease in slab length were obtained in this investigation. In the first place, a comparison of the relative magnitudes of the stresses created by warping at the center of the slab in the transverse and longitudinal directions shows that the stress in the direction of the 10-foot dimension is approximately one-third of that in the 20-foot dimension.

Some other data upon this point were obtained during the spring of 1934 when a transverse crack developed at the center of one quadrant of one of the thickened-edge sections (sec. no. 4). This afforded an opportunity to measure the longitudinal warping stress in both the 10- and 20-foot lengths of the same slab and to make a comparison of their magnitudes. The longitudinal stresses were determined at two positions in the mid-lengths of each slab, at one point 6 inches from the edge and at one 5 feet from the edge (center of the panel).

The measurements were made on suitable days during the months of April, May, and June 1934, and all of the data obtained are presented in table 5.

TABLE 5.—*Observed longitudinal warping stresses in 10- and 20-foot slab lengths of sec. no. 4*

Date, 1934	Maximum air temperature	Maximum longitudinal warping stress				Reduction in stress from decreased slab length	
		Interior		Edge			
		20-foot length	10-foot length	20-foot length	10-foot length	Interior	Edge
	°F.	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Percent*	*Percent*
Apr. 26	68	307	132			57	
May 1	74	376	142			62	
May 2	71			121	68		[1] 44
May 13	83	287	81			72	
May 14	90			278	21		92
May 28	83	429	151			65	
June 1	90			354	46		87
June 11	94			235	38		87
June 14	88			313	20		94
June 15	94			252	19		92
June 21	101	451	132			71	
June 22	96	414	130			69	
June 25	97			283	51		82
Average						66	89

[1] Not included in the average.

It is apparent again in these data that the longitudinal warping stress in the 10-foot slab length is consistently much smaller than the corresponding stress in the 20-foot slab. This is especially true for the stresses measured near the edges of the two slabs. The average reduction in the critical warping stress caused by the decreased slab length is approximately 66 percent in the interior of the slab and 89 percent at the point near the free edge. The greater reduction

[13] Economic Value of Reinforcement in Concrete Roads, by C. A. Hogentogler Proc. Fifth Annual Meeting, Highway Research Board, pt. II.

noted in the stresses near the edge of the slab may be a natural condition or it may be caused by the fact that the two 10-foot sections of the slab were restrained from free warping to a certain degree at the longitudinal joint. This restraint results from the fact that the two 10-foot lengths of this section were attached to an uncracked half of the same section by the tongue-and-groove longitudinal joint.

The data and discussion just presented indicate the necessity for short slab units if the stresses produced by restrained warping are to be kept within economical limits. The desirability of slab lengths of approximately 10 feet is indicated and this may, at first thought, seem to be impractical because of the number of transverse joints that would be required. One problem of joint design is to overcome the difficulty of providing satisfactorily for the movements caused by the expansion and contraction of abutting slabs. If very short slabs were used, the very frequency of the joints would largely solve this problem and thus simplify the joint design requirements.

It was shown earlier that the critical temperature-warping stresses in the corner region of a pavement slab occur during the daytime when the sense of the warping stress is opposite to that of the critical stress caused by load. During the night and early morning when the sense of the two stresses is the same, the magnitude of the warping stress is very small. It is evident, therefore, that increasing the thickness of the edge of a pavement slab will be effective in reducing the combined stresses in the corner area and consequently will reduce corner cracking. This conclusion is likewise in agreement with the observations of the survey previously referred to.

SOME EFFECTS OF FREEZING AND THAWING OF SUBGRADE DETERMINED

As mentioned previously, because of the comparatively mild winters in the region where the tests were made there was very little opportunity to study the effects that a frozen subgrade might have on the pavement sections. During the latter part of the winter of 1933–34, however, severe weather caused the subgrade under the test slabs to freeze solidly to a depth of about 2½ inches and frost crystals were found at a slightly greater depth. The earth shoulders were frozen solidly to a depth of about 7 inches and frost crystals were found at depths of 10 or 11 inches.

During this period observations were made with the clinometer to determine the vertical movements that developed in the various parts of the slab, the technic being the same as that used in the measurements of warping. Figure 47 shows the position and shape of the 9-inch constant-thickness section on two different days referred to its position during the preceding September as a base. The first series of observations (Feb. 23) show the position of the slab with the subgrade frozen, while those made on March 6 show its position just after the subgrade had thawed.

These data show that, for the conditions that obtained, the freezing of the subgrade lifted the entire panel almost uniformly to the extent of about one-half inch. It is interesting to note that this lift was produced by a subgrade that was frozen solidly to a depth of only 2½ inches. As soon as the subgrade had thawed completely the measurements showed that the slab settled back about three-fourths of the distance through

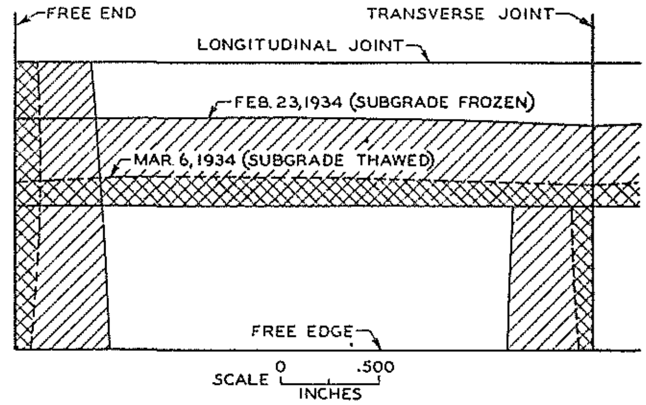


FIGURE 47.—EFFECT OF FROZEN SUBGRADE ON ELEVATION OF 9-INCH UNIFORM-THICKNESS SLAB, BASE ELEVATION AS OF SEPT. 24, 1933.

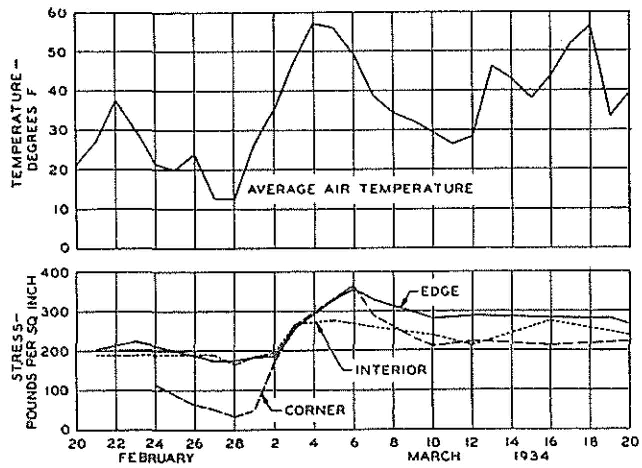


FIGURE 48.—DAILY VARIATION OF STRESSES CAUSED BY A 7,000-POUND LOAD ON A 9-6-9 THICKENED-EDGE SECTION, OVER A PERIOD OF FREEZING AND THAWING OF THE SUBGRADE.

which it had been lifted, and that during the next several days the slab was slowly settling. Traffic on the pavement would probably accelerate this settling during the period of thawing.

Tests were made to determine the effect of the freezing and thawing of the subgrade upon the stresses caused by applied loads. The 9–6–9 thickened-edge section was selected for this purpose and loads were applied at a free corner, at the interior, and at a point 6 inches from a free edge. The 8-inch diameter circular bearing plate was used in all cases. The tests were started during the time when the subgrade was frozen solidly and were repeated frequently until after the subgrade had thawed completely. The variations in the maximum stress produced by a 7,000-pound load applied at each of the three positions previously mentioned during the entire period are shown in figure 48, together with the variations in the average daily air temperature.

From these data it appears that the subgrade was in a fairly normal condition after March 10, or about 10 days after it first started to thaw. The subgrade was no doubt still very wet at this time, but it is known that this particular soil remains very wet during the winter even when not subjected to freezing and thawing.

It will be observed that at all three of the points tested the stresses from the applied load were reduced during the period when the soil was frozen. The effect was much greater at the corner than at the other points, probably because a much greater deflection is required to produce a given stress at this point.

As soon as thawing started there was an immediate increase in the stress at all of the points. As the subgrade became completely thawed the stresses were slightly above normal and they remained at this general level during a period of 6 or 8 days.

It is interesting to note that when the subgrade is in what may be termed its "normal" winter condition, but unfrozen, the stresses produced by a given load at the free corner and interior of this section are of approximately the same magnitude while the stress at the edge is but slightly greater.

It appears from these data that the conditions of freezing and thawing that obtained during the tests had no serious effect upon the magnitude of the stresses developed under the applied load. Had the subgrade been frozen to a greater depth or had the subgrade material not been uniform, it is possible that the effects of freezing would have been more serious.

CONCLUSIONS

In this study of the effects of temperature and of moisture on concrete pavement slabs, it has been found that in the locality where the tests were made (Washington, D. C.):

1. The average pavement temperature undergoes an annual change of about 80° F.
2. The maximum temperature differentials observed at the edges of the test sections were:
   a. For a 6-inch uniform-thickness section, 23° F.
   b. For a 9-inch uniform-thickness section, 33° F.
   c. For a 9-6-9 thickened-edge section, 33° F.

   These maxima occur during the hot afternoons of early summer when the upper surface of the pavement is heated by the intense sunlight and the lower surface is kept cool by a subgrade that is still at a relatively low temperature.
3. In the thickened-edge design (sec. no. 5) the temperature differential in the interior of the slab averaged about 4° F. less than that at the thickened edge during the most critical part of the year.
4. There is a cyclic variation in slab length that is entirely dissociated from temperature changes. The annual variation in the length of the test sections from causes other than temperature changes is approximately equivalent to that caused by a temperature change of 30° F., and the maximum length occurs during the late winter when the ground moisture content is greatest. Conversely, the slab is shortest during the late summer when the ground moisture and, so far as could be determined, the concrete moisture are a minimum.
5. The thermal coefficient of expansion of the concrete as determined in the laboratory is 0.0000048 per degree F. This value agrees almost exactly with that determined by measurement of actual temperature expansion in the test sections, indicating: First, that the movement of a pavement slab from thermal expansion can be predicted accurately from laboratory determinations of the thermal coefficient; and second, that in slabs of moderate length the effect of subgrade restraint on slab expansion is so small as to be negligible.
6. The resistance developed in the subgrade to horizontal slab movement is not merely a matter of sliding friction in the commonly accepted sense of the word. It appears to consist of two elements, one an elastic deformation of the soil horizontally that is present for all displacements of the slab, and the other a frictional resistance that develops only after a certain amount of elastic deformation has occurred. The first element appears to be independent of, while the second varies directly with, the slab weight or thickness. Although only one subgrade material was involved in these tests, it seems probable that the relative importance of the two elements may vary considerably with different types of soils.
7. In pavement slabs of moderate length the tensile stresses resulting from contraction will not be large for subgrade soils of the type used in these tests. The thicker the pavement the lower will be the unit stress from this cause, other conditions being the same.
8. The changes in shape of a pavement slab resulting from restrained temperature warping do not cause large changes in the critical stresses from applied loads. In this investigation, the maximum observed condition of upward warping from temperature was found to increase the critical stress resulting from load by about 5 percent for a corner loading and about 20 percent for an edge loading, as compared with the stresses produced by the given load with the slab in the flat or unwarped condition. Maximum downward warping was found to effect a negligible reduction in the load stress at the edge and a reduction of about 20 percent at the corner.
9. For pavement slabs of the size used in this investigation or larger, certain of the stresses arising from restrained temperature warping are equal in importance to those produced by the heaviest of legal wheel loads. The longitudinal tensile stress in the bottom of the pavement, caused by restrained temperature warping, frequently amounts to as much as 350 pounds per square inch at certain periods of the year and the corresponding stress in the transverse direction is approximately 125 pounds per square inch. These stresses are additive to those produced by wheel loads.
10. In long or even moderately long pavement slabs, when conditions are such as to produce large temperature differentials, thickening the edge of the slab may actually decrease the load-carrying capacity of this part of the pavement. In very short pavement slabs, thickening the edge of the slab

may be expected to increase definitely its load-carrying capacity.

11. Since the critical stresses resulting from restrained warping are opposite in sense to those caused by applied loads in the corner region of a pavement, thickening the edge of the slab may be expected to increase the load-carrying capacity of the slab corner.
12. Because of the facts stated in conclusions 10 and 11, it is evident that thickening the edge of a long pavement slab will not tend to reduce transverse cracking but will tend to reduce corner cracking.
13. The annual cyclic variation in moisture conditions within the concrete produces a warping of the slab surface similar to that caused by temperature. The edges of the slab reach their maximum position of upward warping from this cause during the summer and the maximum position of downward warping during the winter, the extent of the upward movement apparently exceeding that of the downward movement considerably.
14. While sufficient information is not available to permit an estimate to be made of the magnitude of the stresses arising from restrained moisture warping, it appears that at the time of year when the stresses from restrained temperature warping are a maximum (the summer months) any stresses caused by restrained moisture warping will be of opposite sense and will thus tend to reduce rather than to increase the state of stress created by restrained temperature warping.

THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

Reported by L. W. TELLER, Senior Engineer of Tests; and EARL C. SUTHERLAND, Associate Highway Engineer

PART 3.—A STUDY OF CONCRETE PAVEMENT CROSS SECTIONS

DEVELOPMENTS in the design of cross sections for concrete pavements during the past 15 years have produced such marked improvements in the roads constructed that one is likely to feel that a high degree of perfection has been attained. Edge thickening and longitudinal joints have been generally adopted and the present slab lengths are much less than those formerly used.

The most complete study yet made of the stresses in typical pavement slabs resulting from both load and temperature effects confirms the belief that progress has been made in the right direction, but it is clear that better designs are possible with the more complete knowledge now available.

The results of this study are surprising as to the stresses that will exist in concrete pavements with certain combinations of load, temperature conditions, and slab thickness. However, the conclusions are thought to be sufficiently well established for application in current design.

If loads alone are considered, the maximum of economy in the use of material is obtained with a thickened-edge cross section.

While increased edge thickness results in a reduction of the edge stresses from applied load, it also causes an increase in the edge stresses under certain conditions of restrained warping.

Since a balanced cross section should in all cases be designed on the basis of combined load and warping stresses, it is obvious that economy demands that the stresses resulting from warping be limited to low values. The most practical way of doing this is by constructing short pavement slabs.

In short slabs the cross section may be designed on the basis of load alone.

A balanced cross section for load stresses is obtained with a design such as is shown on the cover page.

Edge thickening strengthens slab corners regardless of the length of the slab.

At the present time application of the principles set forth above to the design of pavement slabs involves considerations other than those discussed in this report but necessary to the forming of a correct judgment as to whether or not a completely balanced design should be used or how closely it should be approached.

THE SHAPES of the cross sections used in concrete pavement construction in the United States have gone through an interesting period of development during the last 15 years.

The earliest concrete pavements were laid in slabs that were either thicker in the center than at the edges or else were of uniform thickness at all points. The thick-center-thin-edge design was probably the result of the influence of the distribution of material in those macadam pavements with which engineers were most familiar at the time the early concrete pavements were laid.

Some of the first attempts at a mathematical analysis of the stresses created by wheel loads in the pavement slab treated the transverse section as a beam supported at the ends, and this type of analysis naturally indicated the need for a section which was thicker in the center than at the edges. The uncertainty of assumptions as to subgrade support tended to make engineers hesitant about accepting the suggested theories of design, with the result that a considerable amount of the concrete pavement laid was of a uniform thickness.

Upon the entry of the United States into the World War, the wheel loads on many of our main roads suddenly increased greatly, and instances of edge failure of thin-edge sections began to be reported. These edge failures frequently began with a corner break at a construction joint or transverse crack and often developed into a completely shattered area of considerable size. Many engineers began to suspect that the thick-center-thin-edge pavement was not properly designed.

DEVELOPMENT OF THICKENED-EDGE PAVEMENT DESIGN REVIEWED

On November 12, 1920, in Maricopa County, Ariz., construction was begun under a very extensive paving program, including some 130 miles of concrete pavement. The point of particular interest is that the cross section adopted for the entire project was 3 inches thicker at the edges than at the center, the edge thickening being gradually reduced to zero at a distance of 24 inches from the edge of the slab. The report on this work[1] states that the cross section was "a modified inverted-curb section designed to strengthen the edge and at the same time permit simple construction of the subgrade. The thickened edges add structural strength as the area of load distribution and subgrade resistance decreases, thereby securing a paving slab with a more uniform resisting strength." This is clearly a recognition of the principle of design of thickened-edge slabs as we know it today.

The development of the thickened-edge design created widespread interest among highway engineers and, as the probable worth of the new design began to be appreciated, it was adopted for trial in a number of places.

The test road at Pittsburg, Calif., built during the summer of 1921, contained one section of the new design, and at the conclusion of the test this section was given the highest rating of all of those included in the track.[2]

The sections of the Bates test road in Illinois laid in 1920 and 1921 contained no thickened-edge designs but, fortunately, in the fall of 1922 sections of the new design were added and subjected to heavy traffic during 1923.[3] The result was another early demonstration of the superiority of the new design over sections of uniform thickness when subjected to concentrations of heavy wheel loads. In this test the structural weakness of the edges of slabs of uniform thickness was definitely shown.

[1] Pavement With Thickened Edges Takes Heavy Loads, by C. L. Jenken, Engineering News-Record, Apr. 13, 1922, p. 607.

[2] Report of Highway Research at Pittsburg, Calif., by Lloyd Aldrich and John B. Leonard. California State Printing Office, Sacramento, Calif., 1923.

[3] Highway Research in Illinois, by Clifford Older. Transactions, American Society of Civil Engineers, 1924.

The Bureau of Public Roads at about this time developed a method for determining the stresses in concrete pavement slabs caused by wheel loads and, during 1923 and 1924, made a number of studies of stress.[4] The data obtained from these tests indicated clearly the soundness of the thickened-edge design from a load-carrying standpoint.

The fact that concrete possesses definite elastic properties has led to several attempts to develop some mathematical analysis that would make possible the prediction of the stresses caused by load in a given pavement slab design. The most serious obstacle encountered in these efforts was the difficulty in treating rationally the conditions of subgrade support. It was not until Westergaard presented his analysis of the stresses in a concrete pavement slab in 1925 that there was available an even approximately tenable theory of design.[5] In this analysis it is assumed that the load is applied over a definite area to an elastic slab that rests upon an elastic support. By means of the formulas presented it is possible to calculate the critical stresses resulting from a given load applied at the corner, at the free edge, or in the interior of the slab.

The analysis indicated that for the assumed conditions a thickened-edge design is necessary if the section is to offer uniform resistance to load at all points, thus confirming the evidence obtained from the field tests and stress measurements.

The analysis showed for the first time how important a factor the area of load application is in determining the magnitude of the critical stress. It was not possible however, to determine the correct shape of the slab cross section, nor was the means provided for determining the value of the elastic constant that should be used for a given subgrade in the practical application of the analysis to specific cases of design. In spite of these deficiencies the Westergaard analysis represents one of the most important steps in the development of concrete pavement slab design.

By 1924 several progressive States that were building considerable mileages of concrete pavement had adopted as a standard some form of thickened-edge design, but there was a wide variety of opinion as to what the shape of the cross section should be. Today, a decade later, 41 States are using exclusively some type of thickened-edge design and there still appears to be a considerable divergence of opinion as to the amount and distribution of edge thickening needed, the reason being that neither theory nor experiment has so far supplied the information to enable engineers to design with precision a cross section with equal resistance to applied load at all points.

FACTORS AFFECTING CROSS-SECTION DESIGN INVESTIGATED

The investigation of the structural action of various concrete pavement slab designs which was begun in 1930 by the Bureau at the Arlington Experiment Farm included, as one of its major parts, a study of the behavior of a number of designs of pavement cross section.[6]

While it is usual to think of the design of a cross section as being determined by the variation in the bending moments resulting from applied load, in pavements, as in other structures, other factors must also be considered if a satisfactory design is to result.

Most of the data relating to the effects of temperature variations have already been presented,[7] and in the attending discussion the primary importance of these effects has been brought out. In this study of pavement cross sections particular attention has been given to the following:

1. The effect of the condition of warping on the stresses caused by applied loads.
2. The effect of the changes in the supporting power of the subgrade caused by freezing and thawing or by other causes.
3. The stresses caused by variations in the temperature conditions within the pavement.

While the discussion naturally centers around the load-stress relations developed for the various cross sections, consideration is given to the effects of each of the factors just mentioned as observed in the tests at Arlington.

The sections selected for these tests have already been described in part 1 of this series of papers, but for the convenience of the reader the details of the various cross sections are again shown in figure 1. It will be observed that the sections shown cover fairly well the range of designs in use in this country today. There are four sections of uniform thickness (6, 7, 8, and 9 inches thick). This type of slab is still in use in seven States. There are three thickened-edge slabs of the type in which the edge thickening is reduced at a uniform rate to zero at a short distance from the edge. This general type is used in 28 States at the present time. The parabolic cross section, in which the thickening extends to the center of the slab, is used in 9 States, while the design suggested by the American Association of State Highway Officials (sec. 3 in fig. 1) is used in 1 State.

Although the actual dimensions of the cross sections used by some of the States will be found to differ somewhat from those of the sections shown in figure 1, the differences are not large and it is believed that complete tests on these sections provide adequate data upon which to base a judgment as to the efficiency or balance of almost any of the cross sections in use in the United States at the present time.

PROGRAM OF LOAD TESTING DESCRIBED

The schedule of load testing followed in developing the data on the relative efficiency of the several cross sections can be most readily explained by referring to figure 2, in which the four quarters or quadrants of one of the 20- by 40-foot test sections are shown. In this figure the small circles indicate the points where a load was applied, the load being placed successively on each point beginning at the free edge of the panel; the short lines within the circles in quadrants 1 and 3 show the position and direction of the strain gages; and the broken line A′—B′ in quadrant 4 is the line along which the curvature of the slab was measured. The loading schedule shown in quadrant 1 was followed on at least 1 quadrant of each of the test sections and on more than 1 quadrant if thought desirable. The schedule shown in quadrant 3 was followed on all four quadrants of all of the test sections. Deflection measurements

[4] Stress Measurements in Concrete Pavements, by L. W. Teller, Proc. Fifth Annual Meeting Highway Research Board, Dec. 3-5, 1925.

[5] Stresses in Concrete Pavements Computed by Theoretical Analysis, by H. M. Westergaard. A paper presented before the Highway Research Board, Dec. 3, 1925. Also see Public Roads, vol. 7, no. 2, April 1926.

[6] See The Structural Design of Concrete Pavements, pt. 1, by L. W. Teller and Earl C. Sutherland, Public Roads, vol. 16, no. 8, October 1935.

[7] See The Structural Design of Concrete Pavements, pt. 2, by L. W. Teller and Earl C. Sutherland, Public Roads, vol. 16, no. 9, November 1935.

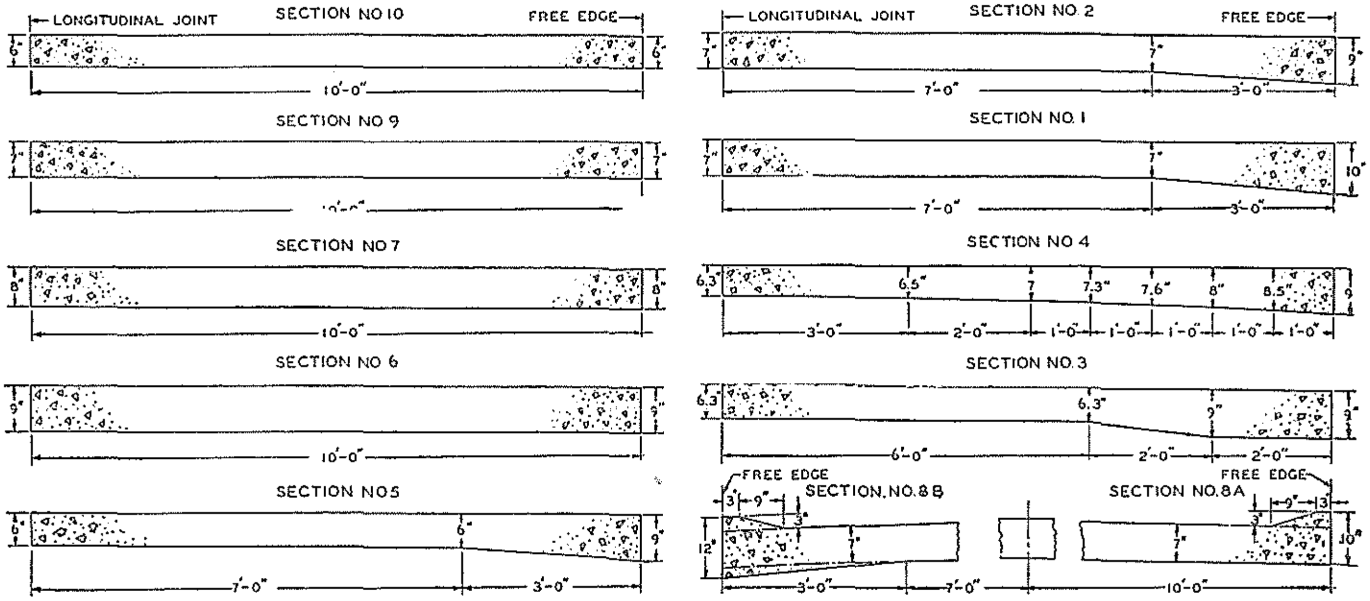


FIGURE 1.—DESIGNS OF PAVEMENT CROSS SECTIONS STUDIED.

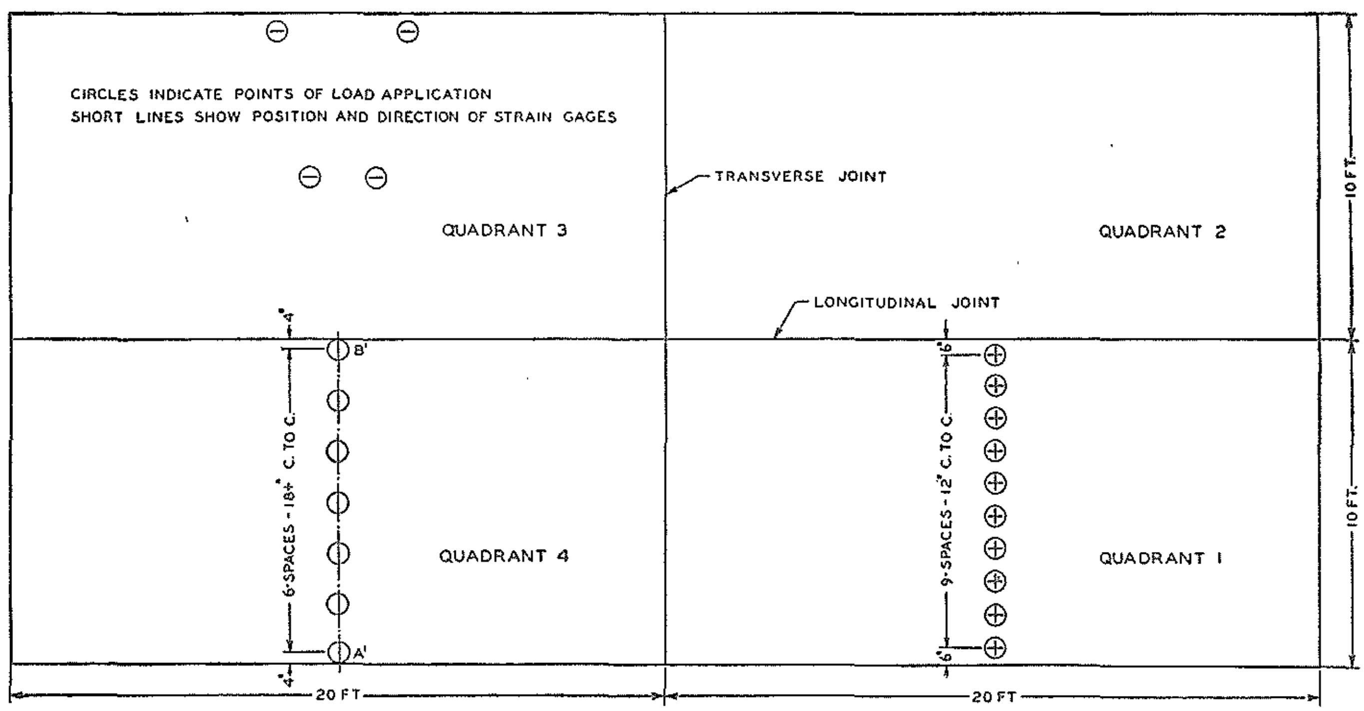


FIGURE 2.—POINTS WHERE LOADS WERE APPLIED AND DEFLECTIONS AND STRAINS WERE MEASURED.

were made on one quadrant of each test section. On a number of the sections load-stress data were obtained during the different seasons of the year, particularly to determine the effect of subgrade condition.

In all of these loadings a bearing block 8 inches in diameter and of the grooved type was used in order that both deflection and strain measurements might be made within the area of load application. Because of the height of the clinometer it was necessary to use a split bearing block approximately 4 inches in height when making deflection measurements. The method of applying the load for deflection and strain measurements is shown in figures 3 and 4, respectively.

For the loading schedule shown in quadrant 1, recording strain gages of the type described in part 1 of this series of papers were installed at each of the 10 positions, being placed either all transversely or all longitudinally in any one test. The selected load was then applied successively at each of the 10 points, the strains in both the longitudinal and transverse directions being recorded at each gage position for each loading.

The tests scheduled in quadrants 1 and 4 were actually performed in the same quadrant of the test section in order that the closest possible relation would exist between the strain and deflection data.

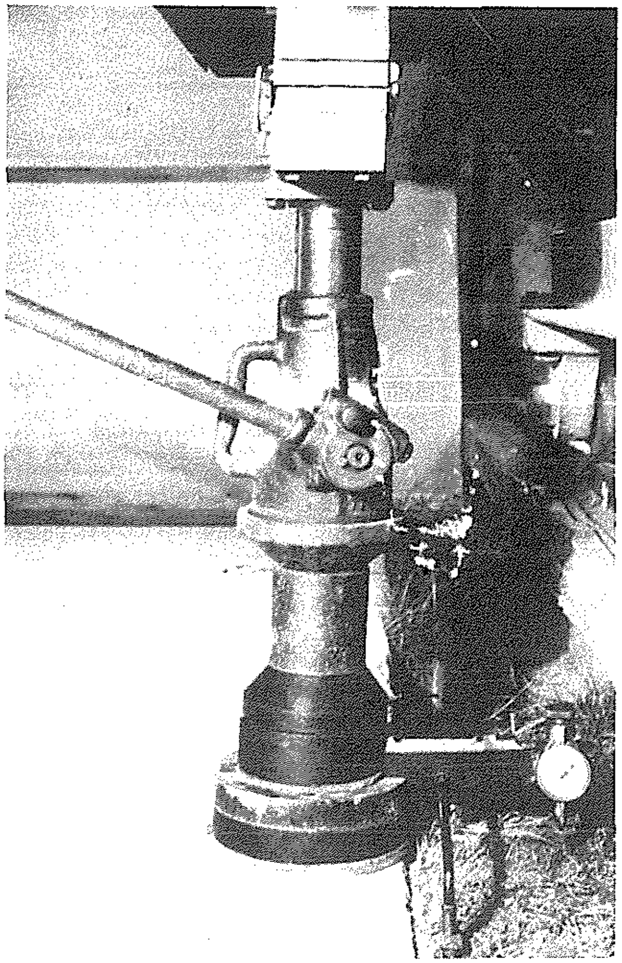


FIGURE 3.—APPLYING A TEST LOAD AT THE EDGE OF A SLAB PANEL. THE DEFLECTIONS WERE REFERRED TO A BENCH MARK LOCATED IN THE SHOULDER.

All of the deflection data were obtained with a 10-inch clinometer using the procedure described in part 1 of this series of papers. The measurements were started at a bench mark set close to the edge of the test section (see fig. 3) and were continued entirely across the slab. The elevation of each clinometer point, with respect to the bench mark, including that directly underneath the load, was thus determined and the true shape of the transverse center line of the slab established. This determination was made first with no load upon the slab and again after the given load had been applied, the difference between the two curves at any point being the vertical deflection of that point caused by the applied load. In this manner the deflection of the slab between points A′ and B′ (see fig. 2) was obtained for a load acting at each of the several loading points.

Only a short period of time was permitted to elapse between the strain and the deflection measurements in order that the condition of the slab might not change.

LARGE DEFLECTIONS AND STRAINS OBTAINED WITHOUT OVERSTRESSING CONCRETE

It was desirable to use loads of sufficient magnitude to cause large deflections and large strains in the concrete since this would reduce the error of measurement. On the other hand, it was necessary to put a limit on the strains produced by loading since it was very important that no injury be done to any of the test

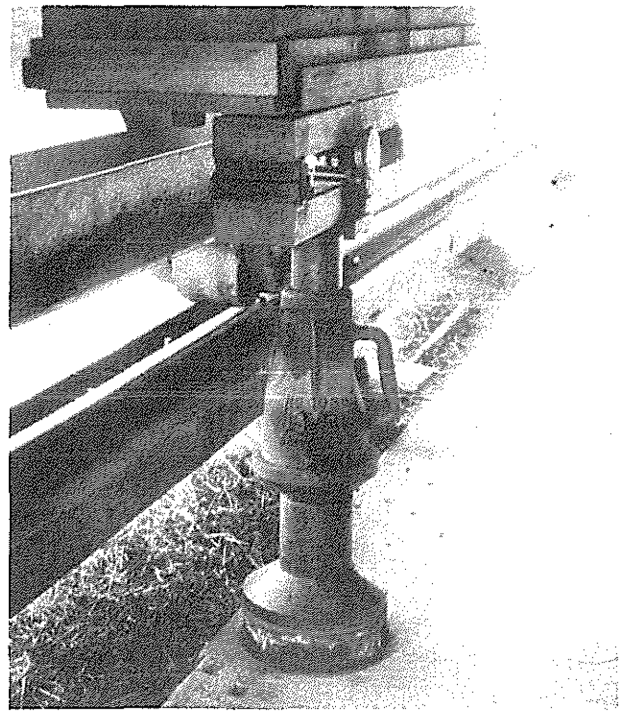


FIGURE 4.—STRAIN MEASUREMENT WITH LOAD PLACED NEAR EDGE OF TEST SECTION. A RECORDING STRAIN GAGE IS INSTALLED DIRECTLY UNDER THE LOAD IN A DIRECTION PERPENDICULAR TO THE EDGE OF THE SLAB.

sections. It was decided to limit the loads applied to the extent that the maximum stresses produced in the concrete would not, in general, exceed one-half of the average modulus of rupture as determined by the strength tests made at the beginning of the investigation. This criterion has proved to be satisfactory. With but one exception, all of the forty 10- by 20-foot test slab panels are apparently intact after 4 years of intensive load testing. The strains and deflections produced by loads of this magnitude were of sufficient size that they could be measured with satisfactory accuracy with the instruments available.

In the preceding paper it was shown that the condition of warping of a pavement slab had a definite effect upon the magnitude of the maximum stress a given load might be expected to produce, particularly if the load were applied at the corner or along the edges of the slab. It has also been established that, for the concrete used in this pavement at least, the moisture condition in the concrete had an important effect upon its elastic properties. These two facts made it necessary to take special precautions to maintain uniform conditions of moisture and of temperature within the concrete of the test sections during any period of load testing.

As previously described, the method adopted for protecting the slabs during these tests consisted of a covering layer of approximately 4 inches of dry straw laid directly on the concrete, with protection from rain, snow, or direct sunlight afforded by a large canvas shelter supported by a framework on the loading tank.

Temperature measurements made within the concrete of the slabs thus protected showed that the differentials in temperature between the upper and lower surfaces of the pavement were always very small, frequently so small as to be unmeasurable. At the time of testing there could have been but little if any temperature warping present in the test sections. It was not possible to determine the variation in moisture throughout the depth of the concrete in the slabs and therefore no positive statement regarding the condition of moisture warping can be made. It seems reasonable, however, that the dead-air spaces in the dry straw layer which provided the thermal insulation would also greatly decrease the rate of moisture evaporation from the surface of the concrete and would reduce correspondingly the tendency for a differential in moisture content to develop. Whatever differential may have existed was held constant during the period of test by this method of protecting the test section.

FORMULAS FOR CALCULATING LOAD STRESSES GIVEN

The strength and elastic properties of the concrete were determined by tests made upon drilled cores and sawed beams obtained from short sections of pavement constructed especially for this purpose at the same time that the test sections were constructed. One of the matters investigated in these collateral tests was the effect of moisture on the stiffness of the concrete, and it was found that the value of the modulus of elasticity varied to some extent with the moisture content, being highest for moist concrete. The details of these tests will be covered in a subsequent paper. It was decided as a result of the tests that the proper value of the modulus of elasticity of the concrete, in bending, and containing as nearly as could be determined the same percentage of moisture as the concrete in the test sections, was 5,500,000 pounds per square inch. This value is used throughout this series of papers in computing stress values from the measured strains.

If a load is applied at a certain point on a pavement slab, the stresses developed at the point of load application can be determined from measured strains by means of the following formulas as described in part 2 of this series of papers:

$$\sigma_x = \frac{E}{1-\mu^2}(e_x + \mu e_y) \qquad (1)$$

$$\sigma_y = \frac{E}{1-\mu^2}(e_y + \mu e_x) \qquad (2)$$

in which σ_x = the stress in the direction of the x–axis.
σ_y = the stress in the direction of the y–axis.
e_x = the unit deformation caused by stress in the direction of the x–axis.
e_y = the unit deformation caused by stress in the direction of the y–axis.
E = the modulus of elasticity of the concrete.
μ = Poisson's ratio for the concrete.

The value of Poisson's ratio was not determined for these tests but was assumed as being 0.15, which seemed to be a fair average value considering such test data as are available.[8]

In carrying out the test schedules described in the preceding paragraphs, a large number of observations were made. In all cases tests were repeated on the same or a different quadrant of the test section until the data obtained were considered to be well established. On some sections more tests were necessary than on others and in certain cases it was deemed desirable to repeat the tests under both summer and winter conditions. It is neither practicable nor desirable to present all of the data which were obtained, but an effort has been made to include all significant data and to show representative data in all cases.

It has been found that the stresses caused by load in the vicinity of the longitudinal joint are affected to a considerable degree by the structural action of the joints. In designing a cross section, therefore, the shape of that portion immediately adjoining the longitudinal joint will be controlled by the design of the joint. Since the characteristics and design of joints are to be discussed in a subsequent paper, it is thought advisable to eliminate from the present discussion detailed consideration of the effect of the joint design upon the cross section. For this reason the data pertaining to that portion of the pavement within 3 feet of the longitudinal joints have been omitted (except in the case of the influence lines). There is every indication that at 3 feet from the longitudinal joint the effect of the joint action is so small as to be unimportant in all cases.

DEFLECTION AND STRESS VARIATION DATA OBTAINED FOR THE VARIOUS CROSS-SECTION DESIGNS

Figure 5 shows deflection curves along a transverse section for a slab of uniform thickness and for one of a conventional thickened-edge design. The magnitudes and positions of the load that produced the deflections are indicated in each case, and attention is called to the difference in the magnitude of the loads used in the two designs. These data show that the slabs are tipped slightly when the load is applied at the free edge, the opposite edge or center of the pavement being actually raised slightly. When the load is applied in the center of the 10-foot section the slab is deflected over practically its entire width. Both of these effects are probably due to the comparative narrowness of the slab. The width, however, is typical of modern pavement construction. It is possible that the narrowness of the panel affects somewhat the elastic curvature and hence the stresses caused by a given load. This point has a bearing on any comparison of measured stresses with stresses calculated by an analysis that assumes an infinitely large panel.

The deflections of the thickened-edge slab are much smaller than those of the slab whose thickness is constant, for loads applied near the free edge. As the load is moved away from the edge, however, this difference decreases, becoming very small at a point 7 feet from the free edge.

Figure 6 shows longitudinal and transverse stresses produced by the same loads on the same two test sections for which deflection data were presented in figure 5. Each curve shows the variation of stress across the transverse section for the magnitude and particular position of load indicated. Longitudinal stress is measured parallel to the long axis of the pavement slab and transverse stress is measured perpendicular to it. These values are based on strains measured in the upper surface of the pavement and hence apply only to that surface. The corresponding stresses in the lower surface of the pavement would be of opposite sense and of very nearly the same magnitude.

AREA OF LOAD INFLUENCE FOUND TO BE RELATIVELY SMALL

It will be observed, from an examination of the stress variation diagrams of figure 6, that a load applied at the free edge of a pavement slab produces two quite different stress conditions in the vicinity of the load. In the transverse direction the section acts somewhat as a cantilever and a fairly high transverse tensile

[8] See Digest of Test Data on Poisson's Ratio for Concrete, by Richart and Roy. Proc. A. S. T. M., vol. 30 (1930), pt. 1, Report of Committee C-9, pp. 661-667.

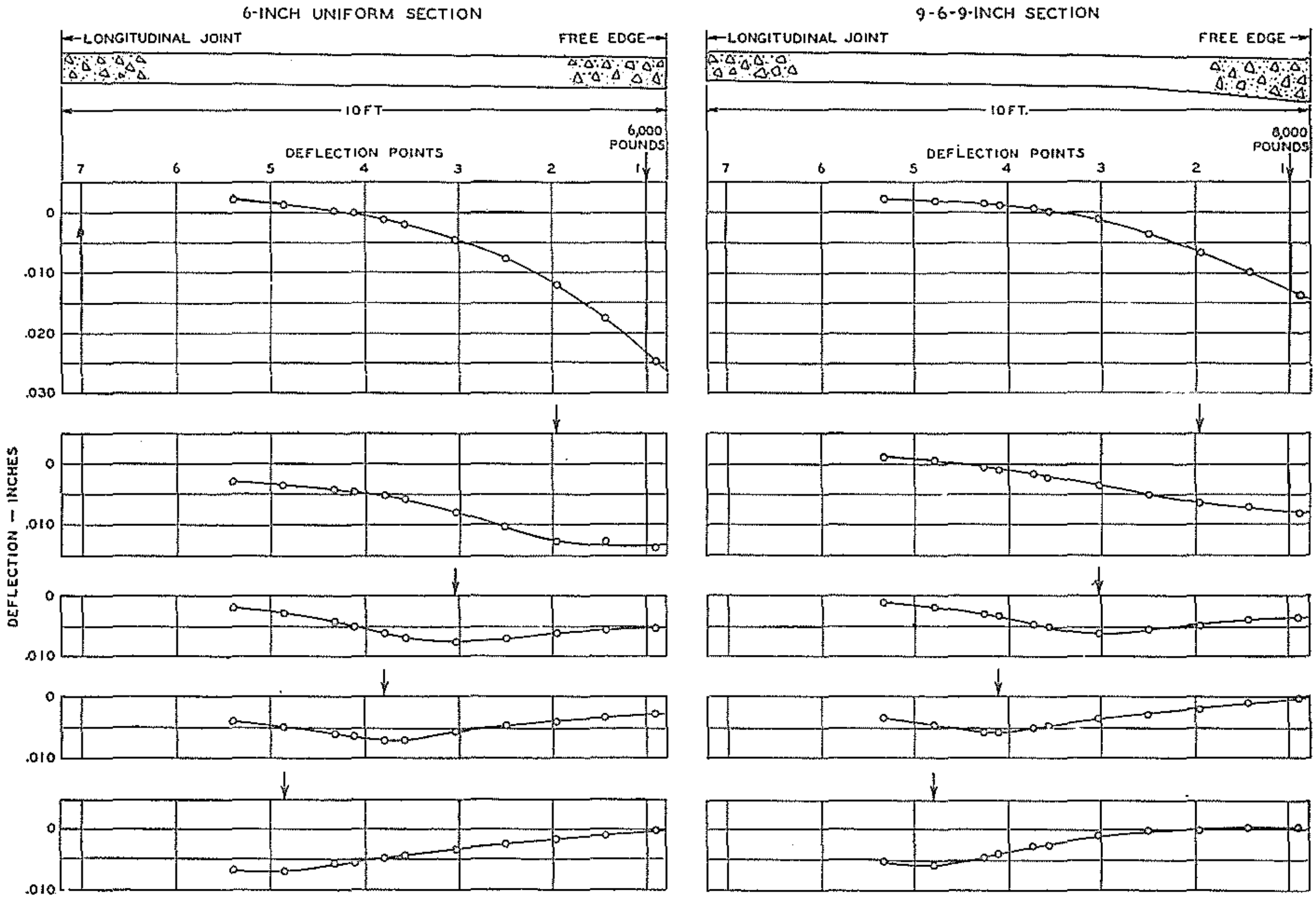


FIGURE 5.—ELASTIC CURVES FOR TWO TYPICAL CROSS SECTIONS FOR VARIOUS POSITIONS OF LOAD.

stress is produced in the upper surface of the pavement, this stress reaching its maximum value at a distance of approximately 2 to 3 feet from the free edge. In the longitudinal direction, the maximum stress produced by the edge loading is a tension in the bottom of the pavement directly underneath the loaded area, with a corresponding compression in the upper surface. For the condition of these tests, in the case of the 6-inch uniform-thickness section, the magnitude of maximum longitudinal stress caused by edge loading is about 3½ times the magnitude of the corresponding maximum stress in the transverse direction and, in the case of the 9–6–9 section, about 6 times that in the transverse direction.

As the position of the load is moved gradually along a transverse section, the stress conditions created change from those just described to those of an interior loading. A load applied at the interior of a panel of infinite extent will cause deflection and stress conditions which do not vary with direction. Even on a panel of the limited dimensions used in these tests, it will be noted that when the loaded area is near the center of the panel the magnitudes of the maximum longitudinal and transverse stresses are practically equal. The variation curves reflect the effect of slab dimension, however, in that for loads applied near the center of the panel the slab is stressed for a somewhat greater distance in the direction of the long dimension than it is in the direction of the short dimension. These curves also show quite definitely that for the interior loading the critical stresses are highly concentrated and that the magnitude of the stress decreases rapidly as the distance from the center of load application increases.

The area of influence of the load, i. e., the area within which appreciable stresses or deflections are produced, will naturally vary with the thickness and design of the cross section and perhaps with other factors. For the designs studied in these tests the data indicate that for the edge loading the area of influence for stresses is roughly a semicircle with a radius of approximately 3 feet, while for the interior loading the area of influence is roughly a circle with a radius of approximately 3 feet. These are only approximate values but the point of interest is the contrast with the areas over which appreciable deflections occur.

The data show that increasing the thickness of the slab near the free edge is an effective method for reducing the maximum stress that can be caused by an applied wheel load. The greatest reduction is thus found in longitudinal tension on the bottom of the slab with the load applied at the free edge of the pavement. There is also a slight reduction in the longitudinal stresses in the interior of the slab which is probably caused by a stiffening effect from the thickened edge.

MAXIMUM DEFLECTION AND MAXIMUM STRESS DIAGRAMS CONSTRUCTED

If a given load is applied successively at a series of positions across a transverse section of a pavement slab, for each position of the load a certain maximum deflection and a certain maximum stress will be pro-

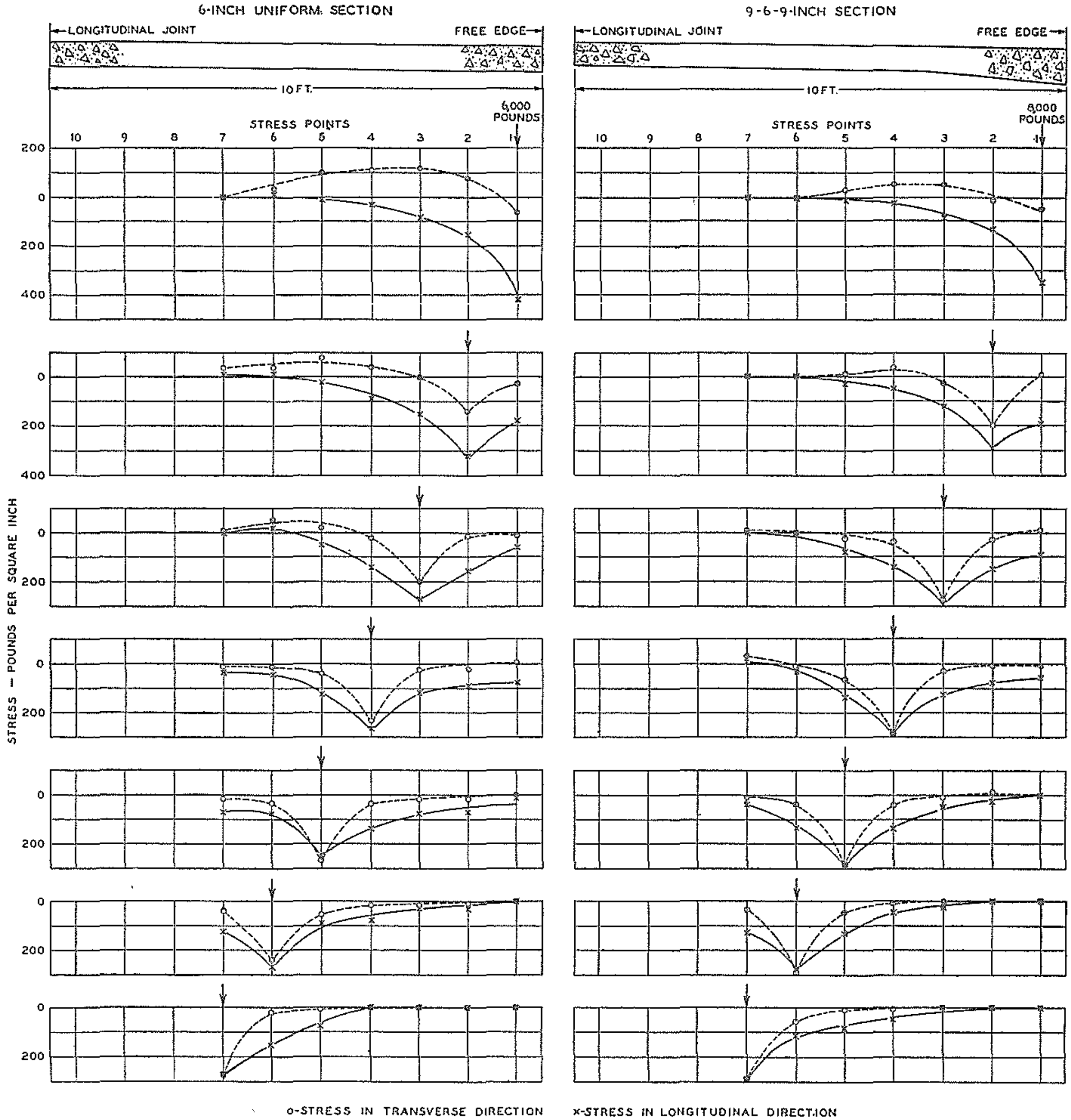


FIGURE 6.—STRESS VARIATIONS IN TWO TYPICAL SECTIONS FOR VARIOUS POSITIONS OF LOAD. VALUES ABOVE THE AXIS INDICATE TENSION AND VALUES BELOW THE AXIS INDICATE COMPRESSION IN THE UPPER SURFACE OF THE SLAB.

duced and the values of these can be determined. If these maximum values are plotted as ordinates at the points on the transverse section at which they were observed, a curve may be drawn through them which is in reality the envelop of all the deflection (or stress) variation curves that might be developed along the section by the load in question. Since this envelop curve shows the maximum deflections (or stresses) that could be developed by the given load regardless of its position, these graphs have been termed maximum deflection (or stress) diagrams in this paper.

Figure 7 shows maximum deflection diagrams for each of the 10 test sections. It should be noted that the diagrams apply only to the case of a single load and that the magnitude of this load varies with the design of the cross section under test.

Sections 10, 9, 7, and 6 are the sections of uniform thickness. The data from these sections show that the maximum deflection caused by a load applied at the free edge of a constant-thickness slab is slightly more than 3 times the maximum deflection caused by a load of the same magnitude applied at an interior point.

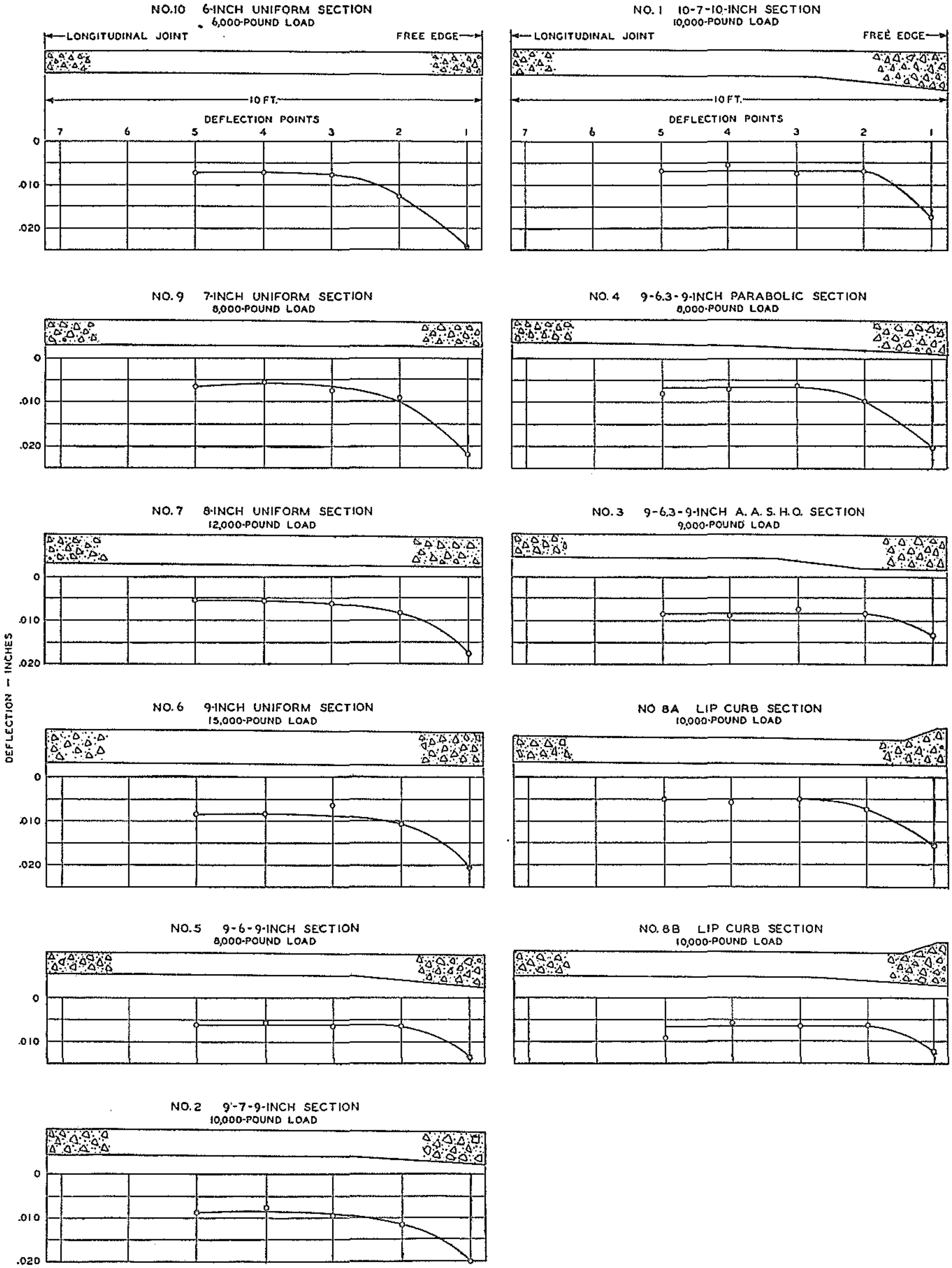


Figure 7.—Maximum Deflection Diagrams for Each of the Various Cross-Section Designs Studied.

As the area of load application is moved from the free edge toward the interior the magnitude of the maximum deflection decreases rapidly until, with the center of load application approximately 3 feet from the edge, the maximum deflection is practically the same as it is for the load at the mid-point of the panel.

Sections 5, 2, and 1 are thickened-edge sections of the type in which the edge thickening is decreased at a uniform rate to zero at a short distance from the edge. The shape of the maximum deflection diagram for this type of section appears to be very similar to that of a section having a constant thickness, although the maximum deflection with the load applied at the edge of the slab is in this case on the average only about 2.35 times the maximum deflection measured for the same load applied at the interior of the panel. It is apparent that the edge thickening of these sections has a relatively small effect on the maximum deflections.

Section 4 is one in which the cross section is bounded by two parabolic curves, the center thickness being seven-tenths of the edge thickness. The edge deflection of this section is also approximately three times the deflection at the interior.

Section 3 has a heavy-edge cross section of a design once suggested by the American Association of State Highway Officials. The deflection of a slab of this design under an edge loading is approximately one and one-half times the deflection under an interior loading.

Section 8 is the section having the lip curb on the upper surface. As previously explained, two panels of this section (called sec. 8A) had the lip curb added to a slab of otherwise constant thickness while the remaining two panels were of the thickened edge type before the addition of the lip curb (sec. 8B in fig. 1). The deflection of the edge of section 8A is a little more than 3 times the deflection of the interior, while in section 8B the deflection under an edge loading is approximately 2 times the deflection under the same load at the interior of the slab.

These data show that in all cases a given load caused greater deflections when applied at the free edge of the pavement than when applied at the interior points. If the magnitude of the deflections were to be used as a criterion for balancing a design, one might conclude that none of the sections has been strengthened sufficiently along the free edge. It will be of interest to keep these curves in mind while examining the indications of the stress data which follow.

The maximum stress diagrams shown in figure 8 were prepared in a manner similar to that used in developing the deflection data just discussed. These curves show the variations in the maximum longitudinal and maximum transverse stresses as the given load is placed successively at the points shown in figure 2, quadrant 1. The stresses are those directly under the load and generally are the maximum in each direction, the one exception being the transverse stress developed by the load applied at the edge of the slab. In this case the maximum transverse stress is developed 2 or more feet from the edge of the pavement. Because of the fact that a greater transverse stress is developed at the same point when the load is placed directly over it, the exception noted above does not affect the accuracy of the maximum stress diagrams of figure 8.

The values plotted in these diagrams are generally the average of two sets of determinations (the data such as are shown in figure 6 being considered as one set) from tests made on one or two quadrants of the test section. In order to provide a general check and to determine more closely the relation between the maximum stresses resulting from edge and interior loadings, the test schedule shown in quadrant 3 of figure 2 was carried out on all 4 quadrants of all the sections, except section 8, the lip-curb design, where only 2 quadrants of each type were available.

MORE UNIFORM LOAD-STRESS DISTRIBUTION FOUND ON THICKENED-EDGE DESIGNS

In general, the average maximum stresses for the edge and interior loadings as determined by these supplementary tests agree closely with corresponding values determined during the measurements mentioned above, i. e., the schedule shown in quadrant 1, figure 2. The measurements that determined the stress variation curves give a good indication of the variation in strength in the various parts of the different cross-section designs. The supplementary tests, although they involve measurements at only the two positions on the cross section (the free edge and the interior), were made at a considerable number of points. For this reason, the average values for each test section (shown as triangular points in figure 8 and some of the other figures that follow) are believed to give the most reliable indication of the relative strengths of the edge and interior of any particular design. Some discrepancies may be noted in this figure, if a comparison is made between the stresses found in different test sections having comparable edge or interior thicknesses. These discrepancies are believed to result in large part from the fact that different sections were tested at different times of the year.

There were variations in the condition of the concrete and of the subgrade that influence these data to some extent so far as the relation between slab thickness and stress caused by a given load is concerned. Other data obtained within a relatively short period of time were used in the comparison of slab thicknesses to be discussed in a subsequent report. Since all of the tests on any one section were made during a short period of time, it is believed that the effect of the seasonal variations in slab and subgrade condition do not enter any of the comparisons that are made in this discussion of pavement cross sections.

If a pavement cross section is to carry loads with a maximum of efficiency, the design, insofar as applied loads are concerned, should theoretically be such that a given load will produce a certain maximum stress regardless of the point of application of the load on the slab. The maximum stress diagram for such a design would have the same ordinates at the free edge, the interior, and all intermediate points. Any variation in the magnitude of the maximum stress at different points across the section is an indication of too much or too little strength at that particular point. The maximum stress diagrams can be used, therefore, as a basis for judging the efficiency or "balance" of the various cross sections that were tested.

Sections 10, 9, 7, and 6 have a uniform thickness at all points. Referring to the stress diagrams for these sections in figure 8, it will be observed that the shape of the curves showing the variation in maximum stress across the slab are very similar for these four sections. An increase in the magnitude of the maximum stress in the longitudinal direction as the position of the load approaches the free edge of the slab is apparent in all of the sections, showing clearly the lower load-carrying capacity of this portion of a slab of uniform thickness

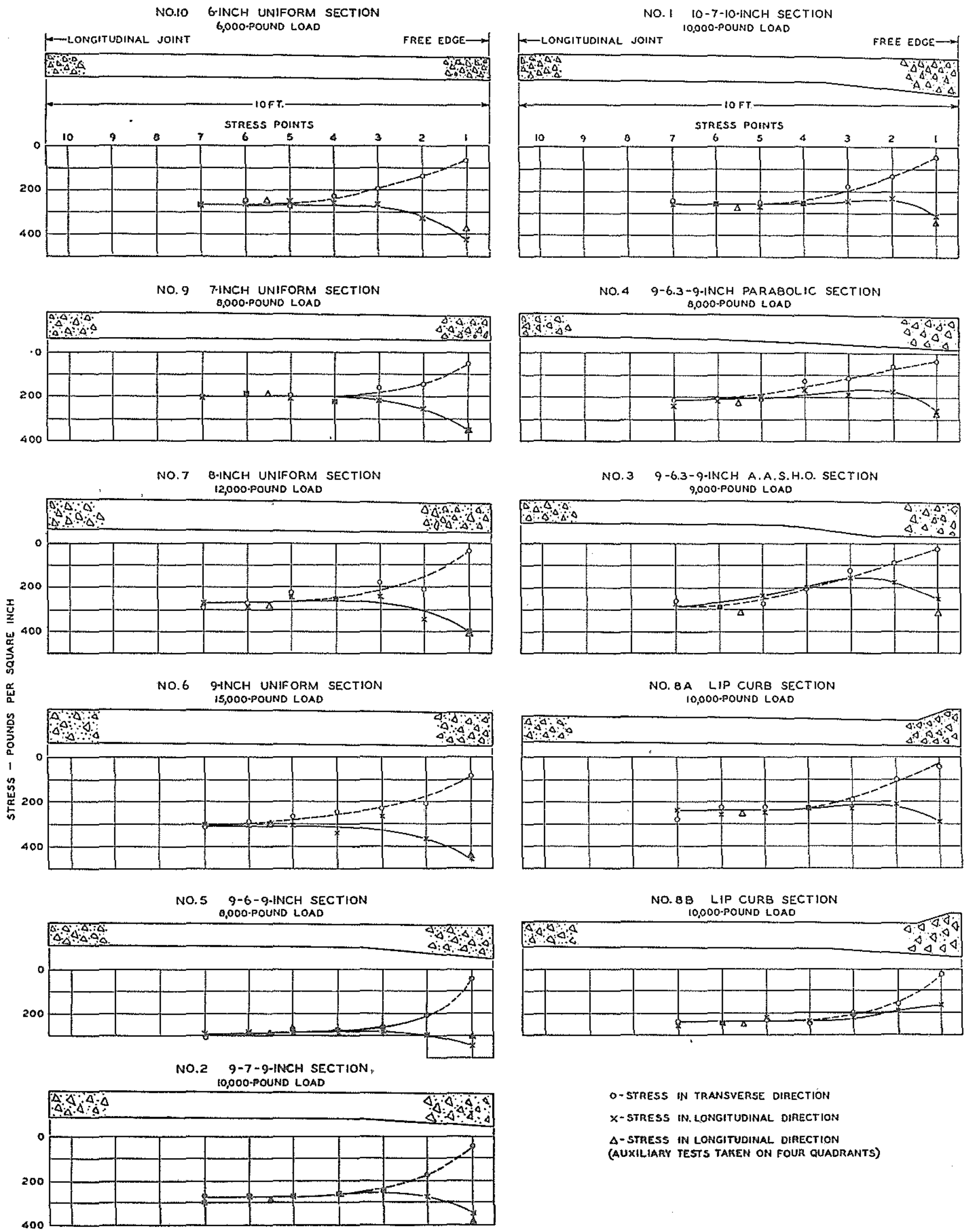


FIGURE 8.—MAXIMUM STRESS DIAGRAMS FOR EACH OF THE VARIOUS CROSS-SECTION DESIGNS STUDIED VALUES INDICATE COMPRESSION IN THE UPPER SURFACE OF THE SLAB.

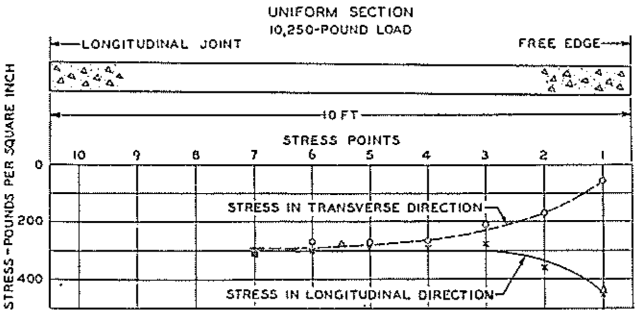


FIGURE 9.—MAXIMUM STRESS DIAGRAMS FOR UNIFORM-THICKNESS SECTIONS DEVELOPED FROM DATA OBTAINED FOR SECTIONS NOS. 10, 9, 7, AND 6.

when load stresses alone are considered. In the transverse direction the stresses are practically zero at the extreme edge of the sections but increase to their maximum value in a distance of approximately 4 feet from the edge. In the case of the longitudinal stresses the maximum value is at the free edge and this value decreases over a distance of approximately 3 feet to a minimum value. The maximum value of the transverse stress is equal to the minimum value of the longitudinal stress throughout the interior region of these sections.

The characteristics of a section of constant thickness are possibly shown better by the average diagram of figure 9. This diagram was constructed by assuming that stress varies directly with load and then averaging the maximum stresses that would be caused in each of the four pavement sections by an average load of 10,250 pounds.

These data indicate that there is a relatively large part of the cross section of a slab of uniform thickness over which a given load will produce practically the same maximum stress and that there is only a relatively small part of the cross section adjacent to a free edge that requires modification to "balance" the cross section. It is indicated that this adjustment of cross section need not extend more than 2½ feet from the free edge of the slab.

The average maximum stress at the extreme edge of these sections of constant thickness is approximately 60 percent greater than the average maximum stress in the interior portions under the given load.

Sections 5, 2, and 1 have thickened-edge cross sections of a similar type, the interior area of each being of a constant thickness. The maximum stress diagrams for these sections (fig. 8) give direct evidence on the reduction in the maximum edge stress resulting from various degrees of edge thickening. Although a definite reduction is apparent for all three of these sections it is indicated that in none of them is the relation between edge and interior thicknesses proper for a constant maximum stress value. It will be noted that the 9–6–9 section is the most nearly balanced, yet even with an edge thickness that is 50 percent greater than the interior thickness, the load placed 6 inches from the extreme edge of this section produced a maximum stress somewhat greater than that caused by the same load applied in the interior of the slab.

EDGE THICKENING FOR BALANCED DESIGN DETERMINED

The relation between edge and interior stresses in the slabs of constant thickness and in the three thickened-edge sections just mentioned provides a means for estimating the extent to which the edge thickness should

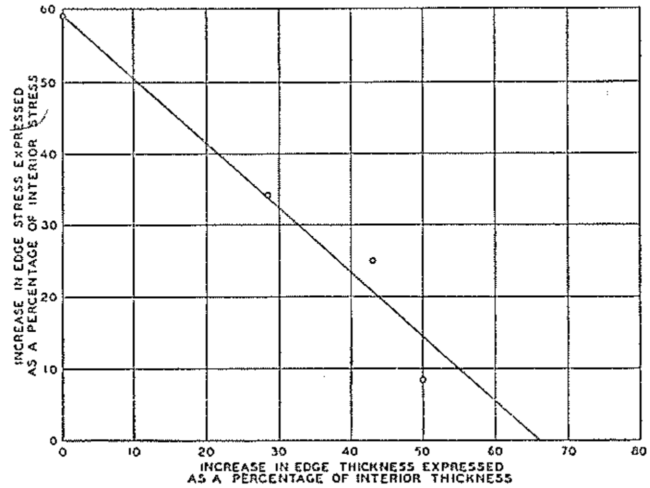


FIGURE 10.—EFFECT OF EDGE THICKENING ON THE REDUCTION OF EDGE STRESS.

be increased if a perfect balance of the cross section is to be attained. In figure 10 the increase of edge stress over interior stress (expressed as a percentage of the latter) has been related to the increase of edge thickness over the interior thickness, similarly expressed, for each of these sections. In the case of the four constant-thickness sections the average value was used. By drawing a straight line through the plotted points and prolonging this line a short distance to the horizontal axis of the graph, the intercept on this axis indicates approximately the percentage of increase in edge thickness that would be required to reduce the edge stress to the same value as the interior stress.

It will be observed that this method of analysis indicates that, for the type of cross section in which the interior of the slab is of constant thickness and the edge thickening is developed at a uniform rate in the outer 2 or 3 feet of the pavement, the thickness of the extreme edge must be about one and two-thirds times that of the interior of the slab, if the section is to be completely balanced for stresses caused by load.

This relation would not necessarily hold for other cross sections of radically different shape and might be affected somewhat by the conditions of subgrade support.

Section 4 has a cross section bounded by two parabolic curves with a thickness of 6.3 inches at the longitudinal joint and 9 inches at the free edge. The maximum stress diagram for this section shows a slightly greater stress at the edge of the slab than at the interior but there is a section at a short distance from the edge in which the stresses are definitely lower than those in the interior of the panel (see fig. 8). The diagram indicates, therefore, that at the extreme edge the section is not of sufficient thickness to balance completely the edge and interior stress values, and that at a short distance from the edge the section is thicker than is necessary. It must be concluded that so far as stress from applied load is concerned this section is not well balanced.

Section 3 has the same center and edge thicknesses as section 4 but the shape of the cross section is quite different. In section 3 the 9-inch edge thickness is maintained in the outer 2 feet and then reduced to the center thickness of 6.3 inches in the next 2 feet of the cross section. The remainder of the slab width is uniformly 6.3 inches thick. The maximum stress diagram for this section shows that the maximum

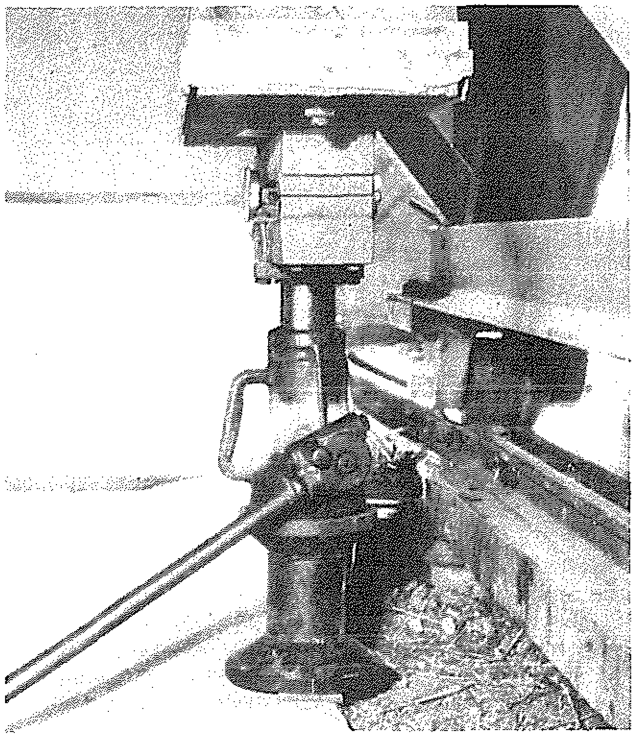


FIGURE 11.—APPLICATION OF A TEST LOAD ON THE EXTREME EDGE OF THE LIP-CURB SECTION.

stress at the free edge is approximately the same as that in the interior of the slab. That part of the cross section at a short distance from the edge is shown to be much stronger than either the edge or the interior and it is this part of the cross section that throws the design out of balance. The diagram also indicates that the heavy edge of this section exerts an influence on the magnitude of the stresses for a short distance beyond the point where the edge thickening is discontinued. The contrast between the deflection diagram (fig. 7) and the stress diagram (fig. 8) for this section is worthy of note. From the deflections it might be concluded that this section was the most nearly balanced of all of those tested, while the stress diagram leads to the conclusion that this section is not well balanced and therefore the most economical use is not made of the material in it.

The stress diagram for section 3 was compared with those for section 4 and the three sections in which the edge thickening was decreased at a uniform rate from the edge of pavement inward (secs. 5, 2, and 1) and it was found that the shape of the cross section has some influence on the relation between the edge and interior thicknesses necessary to balance the maximum stresses for a given load. For example, it has been noted that with the type of cross section represented by sections 5, 2, and 1, an increase in edge thickness of approximately 66 percent over the interior thickness is necessary to balance these stresses, whereas in section 3 an essential balance of edge and interior stresses is obtained with an edge thickness that is only 43 percent greater than that of the interior of the slab. This is undoubtedly due to the greater stiffness of those designs in which the edge thickness is not decreased rapidly from the edge of the pavement, designs which the stress diagrams show to be poorly balanced elsewhere.

SPECIAL METHODS OF LOADING LIP-CURB SECTIONS DESCRIBED

Section 8 was provided with a lip curb on either edge. As mentioned earlier, the two halves of this section, divided longitudinally, are of different design. One half, marked section 8A on figure 1, is of uniform thickness except where the lip curb was added, while on the other half, marked section 8B on figure 1, the lip curb was added to a 9-inch edge, 7-inch interior-thickness cross section. Thus section 8A may be considered as being thickened on the top and section 8B on both bottom and top.

In making tests at the extreme edge of these sections it was necessary to place loads and to measure strains in a manner slightly different from that used on the other sections, because of the presence of the lip curb. The bearing block used was solid, i. e., there was no groove in the bottom, and it was placed on top of the curb, the center of the block being in the center of the level section of the curb, as shown in figure 11. Since this level section is only 3 inches wide, the full area of the bearing block was not in contact with the pavement, and in this position the center of the loaded area was somewhat closer to the edge of the slab than was the case with the other sections. It was considered that this method of loading was justified as it represents the condition of a wheel load "riding" the curb.

The longitudinal strains were measured by installing the two strain gages on the vertical face of the edge of the pavement, one as near the upper and the other as near the lower surface of the pavement as possible.

For reasons that are obvious, it was not practicable to measure the strains in the transverse direction directly under the load when the load was placed at the edge of the lip-curb section. The transverse stress computed from the transverse strain was found to be practically zero at the edges of all of the other sections, so it was assumed to be zero at the edge of the lip-curb section. Consequently, in order to complete the transverse-stress curve for this section, the value plotted represents the transverse effect of the longitudinal stress. All of the other values shown in the stress diagrams are the direct result of strain measurements.

The maximum stress diagram for section 8A shows a slightly higher stress at the edge than at the interior and a slightly lower stress about 18 inches from the edge. Across the remainder of the section the stress is practically constant. It appears that, except for loads applied at the top of the lip curb, this design can be considered as balanced and, since with a lip curb in normal service the application of loads on the top of the curb would probably occur but rarely, it appears that where a lip curb of this design is used no other edge strengthening is necessary. The maximum stress diagram for section 8B confirms this conclusion. It will be seen in figure 8 that, in that part of the cross section which includes the thickening on the bottom of the slab, the stresses are consistently less than in the interior.

It might possibly be argued that, in spite of the fact that the very presence of a lip curb tends to keep the wheel loads away from the edge of the pavement, the section should provide fully for this occasional loading. If such a design is desired, the data indicate that the cross section, exclusive of the lip curb, should be made slightly thicker at the edge than at the interior although not to the same extent as in section 8B. It is probable that an additional inch at the edge, decreased to zero at from 12 to 18 inches from the edge, would accomplish the desired result.

As previously remarked in the discussion of section 3, there is a marked difference between the deductions concerning the relative balance of designs that might be made from the maximum deflection diagrams and those that might be made from the maximum stress diagrams. The deflection data might be taken to indicate that all of the sections are very poorly balanced, while the stress data show that a number of the sections are fairly well balanced. The reason for the difference is that for a given loading the maximum stress is found where the change in the rate of curvature of the elastic curve is greatest and is a highly localized condition, whereas the maximum deflection noted may be the result of deflection over a large area and thus may not be associated with a marked change in the shape of the elastic curve.

This is one of a number of instances in these investigations in which the relations indicated by the deflection data are not in agreement with those shown by the stress data, and in all of the analyses which have been made much more weight has been given to the stress data since the latter are a direct measure of the load-carrying ability of the slab. It is believed that the deflection measurements are, in all cases, as accurate as it is possible to make them by direct methods and that the data show very closely the relative movements which occurred. However, the shapes of the elastic curves of the deflected slabs are not determined by these methods with sufficient precision to warrant any deductions from the deflection data regarding the stress conditions associated with the deflection.

EFFECTS OF MULTIPLE-WHEEL LOADINGS DISCUSSED

The data and discussion thus far presented have related solely to the effects produced by a single load. It has been shown by these and by earlier studies [9] that the area of a pavement slab that is stressed by a single load is relatively small, and the data indicate that if several loads are applied simultaneously, as would occur in the case of a vehicle on the pavement, the critical stress under each load will not be increased by the presence of the other loads, providing the distance between the loads is approximately 3 to 4 feet or more. Thus it is to be expected that 4- and 6-wheel vehicles of the usual types will not have wheel loads so closely spaced that the effect of adjacent wheel loads on stress need be considered. On rare occasions, however, pavements are subjected to heavy loads on closely spaced wheels as, for example, where such cargoes as power shovels, road rollers, electrical transformers, or other concentrated weights are moved over the highway on what are generally known as heavy-duty trailers. These trailers are usually equipped with four wheels at the rear articulated in order to distribute the load to the pavement uniformly.

Figure 12 shows typical wheel spacings (along a plane through the rear axle) for two trailers of the type referred to. It will be noted that the close spacing of the wheels may possibly affect the critical stress in the pavement. Thus, this type of loading might have an influence on the design of the cross section of pavement slabs for certain locations.

No direct tests to determine stresses in the test sections under multiple loadings have been made up to the present time. However, data developed in the

[9] See The Six Wheel Truck and the Pavement, by L. W. Teller, PUBLIC ROADS, vol. 6, no. 8, October 1925. Also see Stresses in Concrete Pavements Computed by Theoretical Analysis, by H. M. Westergaard, PUBLIC ROADS, vol. 7, no. 2, April 1926.

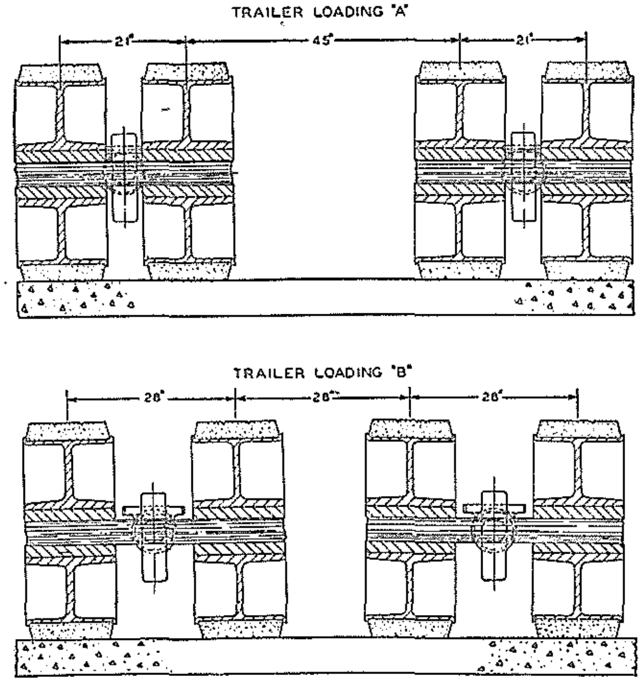


FIGURE 12.—TYPICAL WHEEL ARRANGEMENTS FOR HEAVY-DUTY TRAILERS.

tests which have been described in this report furnish a means for investigating the probable effect of loadings such as are shown in figure 12. It was thought that an analysis of these two cases, from the standpoint of a possible influence on the design of the slab cross section, would be of value in the present discussion.

In the description of the testing procedure it was stated that a load was placed successively at each of the points shown in quadrant 4, figure 2, for deflection measurements, or in quadrant 1 for strain measurements, and that the deflection or strain was measured at each of these points for every position of the load. From the data obtained it is possible to construct diagrams that show the variation in deflection or stress at any one of the points of measurement as the point of loading was moved transversely across the slab. These diagrams are in reality influence lines for deflection or for stress for the several points along the cross section.

Taking again the 6-inch uniform-thickness slab (sec. 10) and the 9–6–9 thickened-edge slab (sec. 5) as representative sections, influence lines for deflection for each of the 7 load positions in quadrant 4 are shown in figure 13, and corresponding influence lines for stress for each of the 10 load positions in quadrant 1 are shown in figure 14. These values are for a 5,000-pound load applied on a circular area 8 inches in diameter.

It was mentioned earlier in this paper that the structural action of the longitudinal joint exerted an influence on the magnitude of the stresses produced by a given load and that the effect might extend as far as 3 feet from the joint. For this reason the data taken within this distance of the joint have been omitted generally from the figures. An exception was made in the case of the influence lines because it is believed that they will be more useful if shown in this way. The values given for points within approximately 3 feet of the longitudinal joint apply strictly only to those designs that include longitudinal joints having the same structural

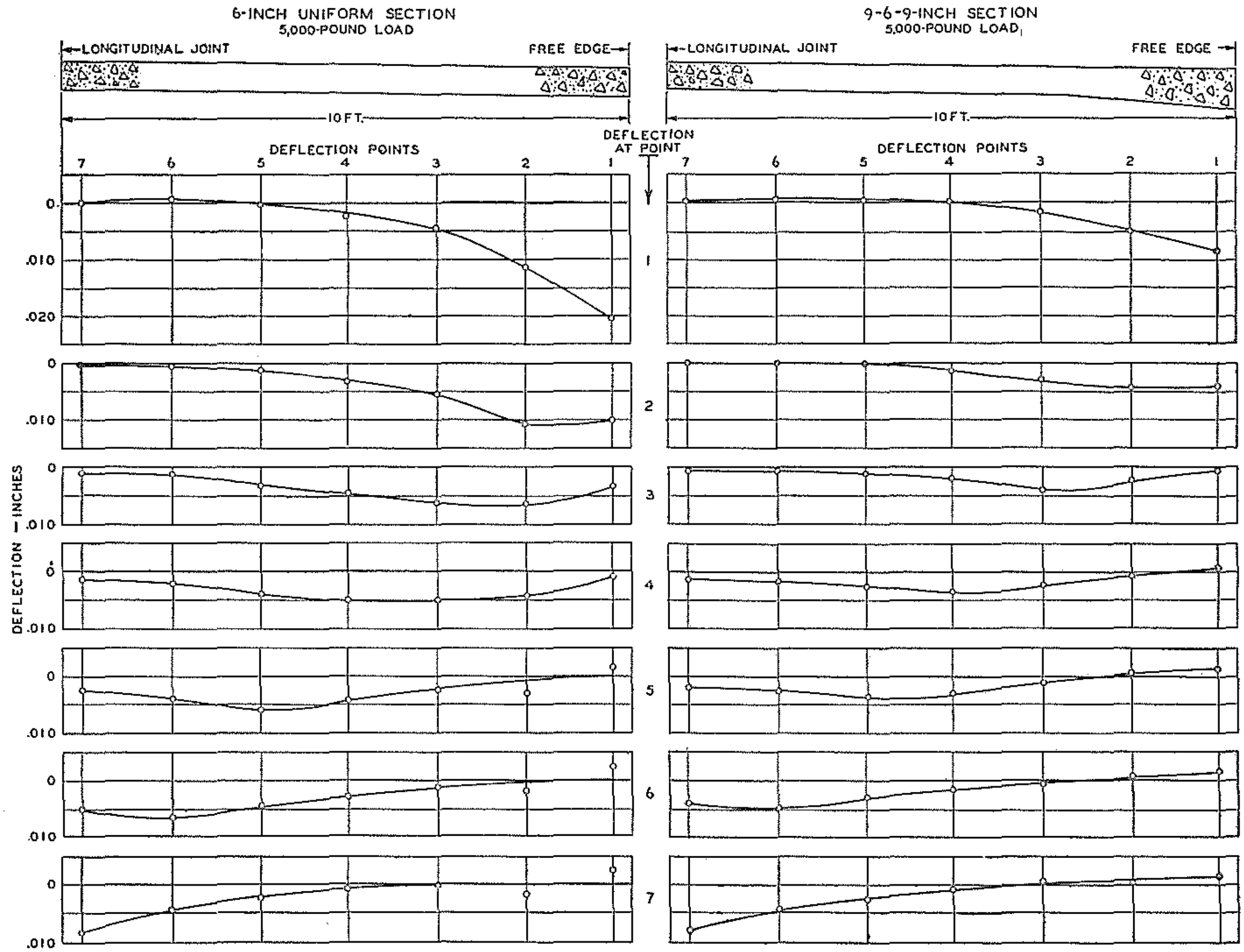


FIGURE 13.—INFLUENCE LINES FOR DEFLECTION ON TWO TYPICAL SECTIONS.

action as those used in the tests, but should apply with reasonable accuracy for other designs of a similar type and degree of joint efficiency.

The longitudinal joint of section 10 is a corrugated steel plate, with ½-inch diameter deformed steel bars in bond and placed transversely at 60-inch intervals. The longitudinal joint for section 5 is a steel plate with a single triangular tongue with steel bars placed in the same manner as in section 10. In tests of the efficiency of the longitudinal joints of the various sections it was found that both of the joints described were of approximately the same efficiency and that both were fairly efficient. It is believed that the data shown will apply with a reasonable degree of accuracy to longitudinal joints of the tongue type where longitudinal continuity is provided. If the longitudinal joint is of a different type then only the values for the outer 7 feet of the cross section apply.

The magnitude of the stresses developed in a concrete pavement slab by a load is affected by the type and character of the subgrade as well as by the elastic properties of the pavement. The absolute values of deflection and stress shown in these diagrams apply only to the particular conditions of the tests. However, as has been said before, the subgrade conditions for the tests at Arlington might well be considered as average and it is believed that the influence lines as developed in these tests should give a good general indication of the effects of different types of loading on pavement slabs.

MAXIMUM STRESS DIAGRAMS FOR MULTIPLE-WHEEL LOADINGS DEVELOPED

From the influence lines for stress, stress-variation curves, similar to those shown in figure 6 but for the particular load positions desired, were constructed for each of the four wheel loads of the two types of trailer. These are shown as light solid lines in figure 15, and since they are all longitudinal stresses they may be combined as in the heavy solid lines to show the stress variation along the pavement cross section under the influence of the four loads applied simultaneously. The light dotted lines indicate the general shape of the enveloping curves, or maximum stress diagrams, that might be expected should this combination of loads be shifted transversely across the section. It should be noted that the curves in this figure are for the 6-inch uniform-thickness section.

For the type of loading designated as "A" (fig. 12) the diagram shows that the interior stress has a constant value to within approximately 3 feet of the free edge of the pavement, and that the maximum edge stress is about 50 percent greater than the interior stress. This differs but very little from the conditions

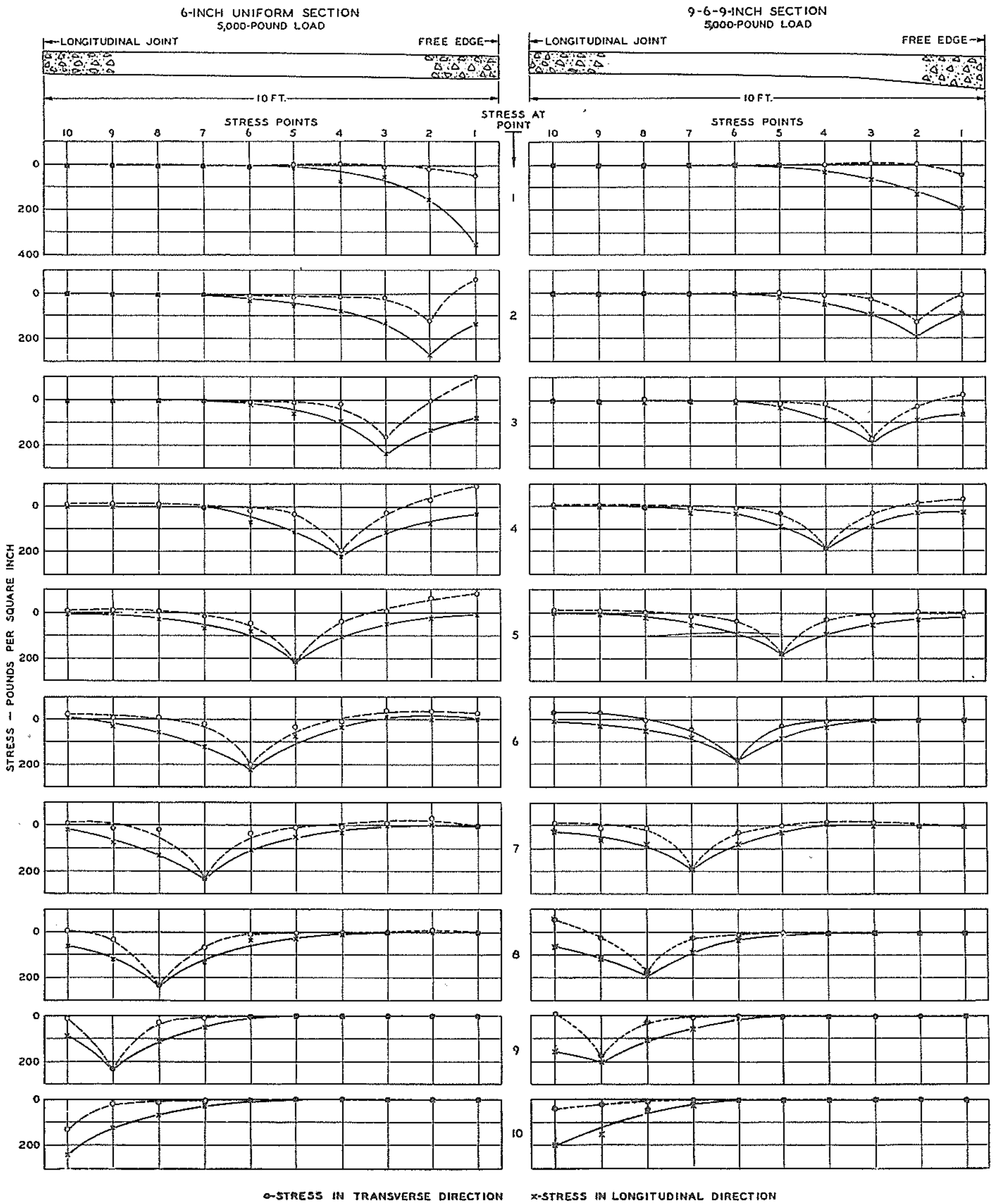


Figure 14.—Influence Lines for Stress on Two Typical Sections. Values Above the Axis Indicate Tension, and Values Below the Axis Indicate Compression in the Upper Surface of the Slab.

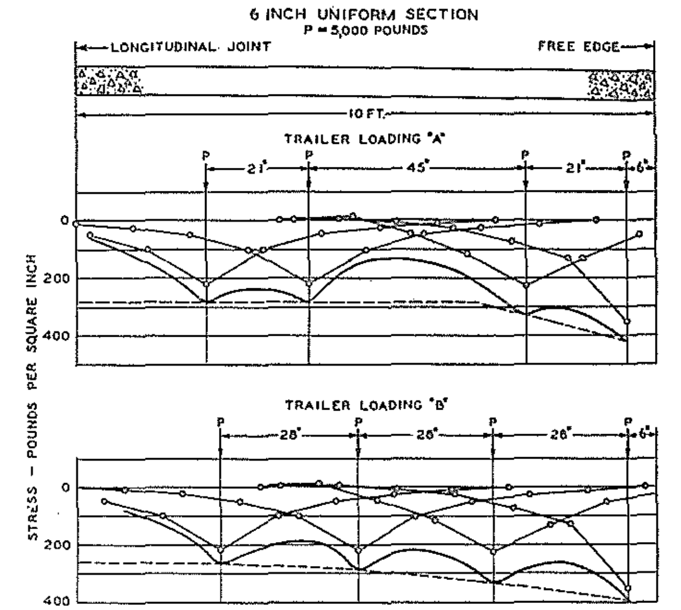


FIGURE 15.—CONSTRUCTION OF MAXIMUM STRESS DIAGRAM FOR TRAILER LOADINGS—OUTER WHEEL OF TRAILER NEAR FREE EDGE OF SLAB.

developed under the single loading, from which it seems reasonable to conclude that a section designed for balanced stresses under single or widely separated loads would prove to be reasonably well balanced under the trailer loading "A."

For a loading of the type designated as "B" the stress conditions are slightly different due to the greater separation and more uniform distribution of the loads. For the same total load the critical stresses are slightly lower than they were for loading "A" and the stress decreases across the full width of the loaded area from the edge toward the center. Again the maximum or edge stress is about 50 percent higher than that developed in the interior of the panel. If a section were to be designed especially for this type of loading, the analysis indicates that it would be economical to extend the edge thickening toward the center of the pavement more than is necessary when designing for conventional types of loading. However, it is indicated again that a pavement balanced for single loadings will also be reasonably well balanced for trailer loads of this type.

In the above analysis the outer load was centered 6 inches from the edge of the pavement. This is an extreme condition and not likely to be encountered in the moving of very heavy loads. It is of interest to make a similar study for the more probable condition of the center of the load group being coincident with the center of the 10-foot slab width. In making this analysis the 9–6–9 thickened-edge section (sec. 5) was used, since this is a fairly well-balanced section and one which is widely used.

The stress variation curves for both trailer loadings on this section are shown in figure 16. These curves were developed in exactly the same manner as those in the preceding figure.

It appears from this diagram that on this cross section the maximum stresses under the four loads are of practically equal magnitude. It is also indicated that at these four critical points the stress developed by the multiple loading is about 30 percent greater than under the single load.

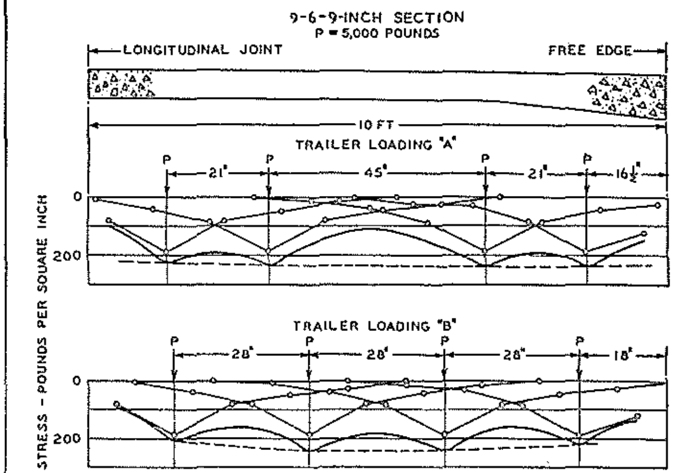


FIGURE 16.—CONSTRUCTION OF MAXIMUM STRESS DIAGRAM FOR TRAILER LOADINGS—CENTER OF TRAILER OVER CENTER LINE OF SLAB PANEL.

This means that 5,000-pound wheel loads distributed in this manner would cause critical stresses some 30 percent higher than would single or more widely separated wheel loads of the same magnitude.

As stated at the beginning of this discussion of multiple-wheel effects no direct tests of such loadings have yet been made nor have the relations between stresses developed under an 8-inch diameter circular bearing block and those under various types and sizes of motor-vehicle tires been established. For these reasons the data should not be extended too far and the analysis should be considered indicative rather than conclusive.

There is another factor involved in an evaluation of multiple-wheel loading concerning which no data are available. When a single load is placed on a pavement slab and a certain maximum stress is developed, this maximum stress extends over a distance approximately equal to the diameter of the bearing area and then diminishes rapidly. For trailer loadings of the type described the maximum stress value obtains over a much greater percentage of the pavement width, and whether or not for a given stress value its effect on the slab is more severe has not been established.

In the preceding paper the importance of the stresses caused by restrained warping was pointed out. If heavy loads are to be moved on trailers having closely spaced wheels, consideration should be given to the possibility of over-stressing the pavement through a combination of warping stresses and load stresses. It may be desirable to confine the movement of certain types of heavy vehicles to those parts of the day when warping stresses are lowest, i. e., at night or in the early morning.

CRITICAL STRESSES RESULTING FROM LOAD NOT GREATLY AFFECTED BY SLAB WARPING

In the first part of this paper attention was called to certain effects of temperature that may influence the load-stress relation in concrete pavements, and it was stated that in this investigation consideration was given to the following:

1. The effect of the condition of warping on the stresses caused by applied loads.
2. The effect of the changes in the supporting power of the subgrade caused by freezing and thawing or by other causes.

3. The stresses caused by variations in the temperature of the pavement.

In the following paragraphs the significance of the data obtained with respect to the design of the cross section will be discussed.

One of the important effects of temperature changes on a concrete pavement is that temperature differentials are developed which cause marked changes in the shape of the slab. Since this results in a modification of the condition of support, it affects the magnitude of the stresses caused by a given applied load, as was brought out in part 2 of this series of papers.

In studying the design of a pavement cross section some consideration should be given to the changes in the load-stress relation caused by slab warping in order to determine what correction, if any, should be applied to basic data obtained from pavement sections tested in a flat or unwarped condition.

A study was made of the effect of both upward and downward warping on the load-stress relation and these data were presented and analyzed in part 2. It was shown that the effect of slab warping on the magnitude of the stresses caused by load was negligible in the interior portion of the slab panel. In figure 35 of part 2 it was shown that:

1. Maximum downward warping reduced the critical stresses caused by load at the edge of the pavement by about 6 percent in the 7-inch slab (sec. 9) and about 2 percent in the 9-inch slab (sec. 6).

2. Maximum upward warping increased the critical stresses caused by load at the edge of the pavement by about 8 percent in the 7-inch slab and about 20 percent in the 9-inch-slab.

From these data it would be concluded that it is not necessary to make any correction to the load-stress relation at the interior of the section, but that small adjustments of the critical stresses at the outer end of the cross section may be desirable to compensate for the effects of slab warping.

FREEZING AND THAWING OF THE SUBGRADE PRODUCES CONSIDERABLE EFFECT ON STRESSES

In part 2 there was included a description of certain load tests conducted during a period of freezing and thawing of the subgrade and a rather full discussion of the effect of the condition of the subgrade on the load-stress relation was given. This is a matter having possible bearing on the design of pavement cross sections and it is pertinent to reexamine the data with this in mind.

It will be recalled that at the time of these particular tests the subgrade was at first frozen solidly to a depth of 2½ inches under the pavement and to a depth of 7 inches below the surface of the earth shoulder, and that later it thawed rapidly until near the surface it became very wet and completely plastic. During the time that the earth was frozen, the pavement sections were displaced upward or heaved about one-half inch.

The conditions of pavement support are probably more uncertain during the period immediately following a thawing of the subgrade than at any other time. For this reason stress data obtained after a thaw are of more importance than those obtained with the subgrade in the frozen state.

Figure 17 shows the maximum stress diagram for section 5 (9–6–9 cross section) under a 7,000-pound load with the subgrade in a softened condition immediately after a thaw. The values shown are the averages of two sets of strain measurements.

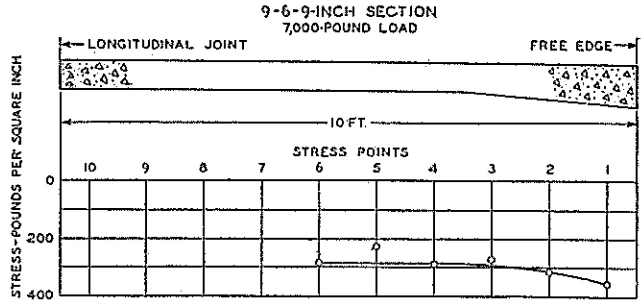


Figure 17.—Maximum Stress Diagram for One of the Test Sections With the Subgrade in a Softened Condition Immediately After a Thaw.

The maximum stresses at the various points in the interior of the slab are much less uniform than they were found to be with the subgrade in its normal condition. (See fig. 8.) The lack of uniformity in the stresses is probably the reflection of a similar lack of uniformity in the physical condition of the subgrade. Also the stresses for a 7,000-pound load (fig. 17) are of approximately the same magnitude as those shown for the 8,000-pound load (fig. 8).

It will be noted that the curve showing the variation of maximum stress across the section has been drawn through the maximum stress values rather than as an average of all the values. This was done deliberately so that the relation between edge and interior stresses as shown by the diagram for the condition of a thawing subgrade would represent the most critical value developed by these tests. As the curve is drawn the maximum edge stress is approximately 25 percent above the value of the maximum stress at the interior. This is practically the same relation as was found in other tests during the winter months when the subgrade is wet, so it appears that, for this section at least, it might safely be assumed to be representative of winter conditions. For the remainder of the year the relation between edge and interior stress is as shown in figure 8.

COMBINED LOAD AND TEMPERATURE WARPING STRESSES DISCUSSED

When the temperature of the pavement changes two effects are produced. First, there is a change in the average temperature of the concrete; and second, there is almost certain to be a change in the relation between the temperatures of the upper and lower surfaces of the slab.

The first effect changes the dimensions of the slab and, because there is a resistance to horizontal movement developed in the earth of the subgrade, direct tensile or compressive stresses (depending upon the direction of movement) are created, which attain their maximum value in the mid-section of the slab. The unit stresses developed in this manner are normally of the same magnitude across the entire width of any given section and for this reason do not enter as a factor in the design of the cross section. Their principal influence is a general increase or decrease in the amount of stress resistance that is available for carrying wheel loads.

The stresses developed as a result of restrained warping under the action of a temperature differential (the second effect mentioned above) are of more importance than the direct stresses described in the preceding paragraph, because at certain times they may be of considerably greater magnitude, but more particularly because they vary in magnitude from point to point

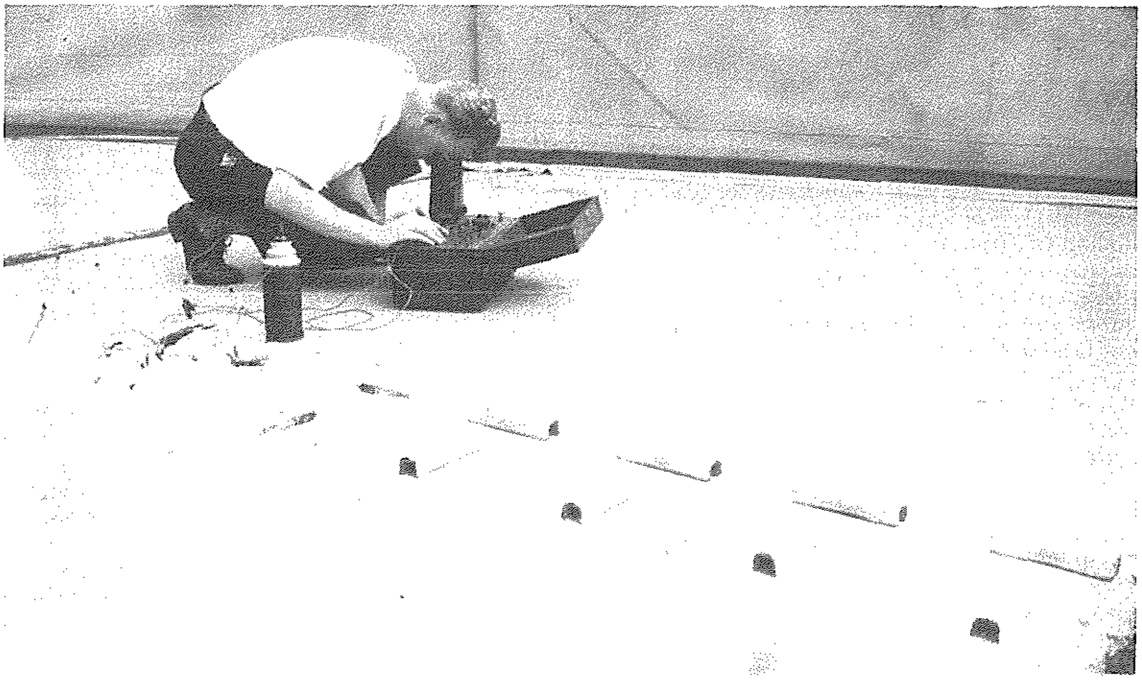


FIGURE 18.—ARRANGEMENT OF RECORDING STRAIN GAGES AND THERMOCOUPLE EQUIPMENT FOR STUDYING THE STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING.

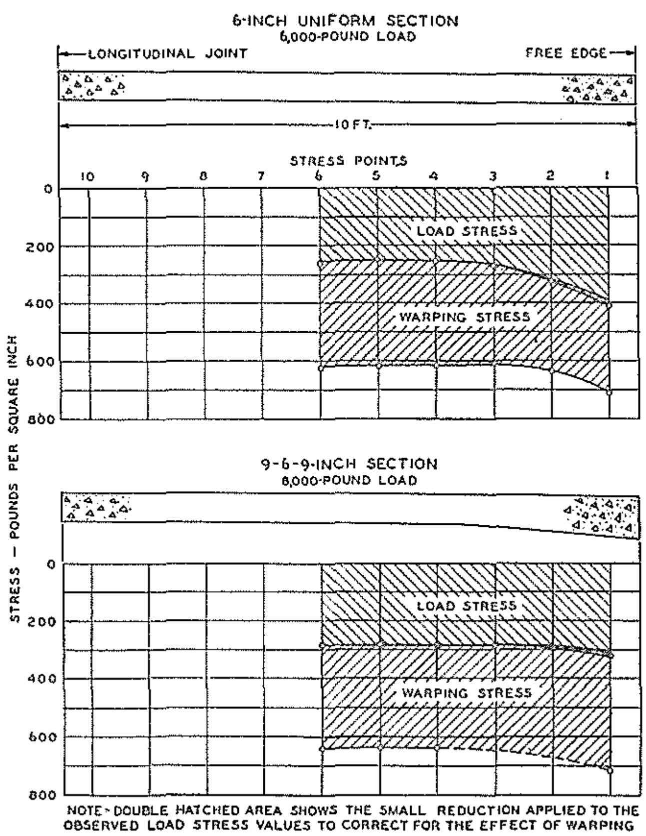


FIGURE 19.—MAXIMUM STRESS DIAGRAMS FOR COMBINED LOAD AND WARPING STRESSES FOR TWO TYPICAL CROSS SECTIONS. THE DOUBLE-HATCHED AREA SHOWS THE SMALL REDUCTION APPLIED TO THE OBSERVED LOAD STRESS VALUES TO CORRECT FOR THE EFFECT OF WARPING.

across a given section. It is desirable, therefore, to examine these stresses as they combine with the stresses caused by applied loads in order to determine their influence on the design of the cross section.

The stresses resulting from restrained warping were not determined for all of the test sections, but the data for one constant-thickness section and one of the thickened-edge sections will be given as typical examples. The two sections selected were the 6-inch constant-thickness section (sec. 10) and the 9–6–9 thickened-edge section (sec. 5). The technique employed in these tests has been described in part 2 and a typical installation of the apparatus is shown in figure 18.

Figure 19 shows the maximum stress diagrams for these two sections and the stress values shown are the result of restrained warping and applied load combined.

The load stresses are those shown in figure 8, while the warping stresses are average maximum values as determined by measurement during the spring and summer months when the temperature warping is at a maximum, as shown in figure 35 of part 2.

It was not possible to measure the warping stresses on the sections having thickened edges in the usual manner, due to the difficulty of measuring the change in length which should occur in the interior of the slab for the condition of free warping. It is probable, however, that the warping stresses for the 9–6–9 section approximate in magnitude those of the 6-inch uniform-thickness section except in the vicinity of the edges. It was shown in part 2 that the average warping stresses at the edge of the 9–6–9 section are approximately 30 percent above those at the edges of the 6-inch section of constant thickness, and this relation has been used to establish the warping stress at the edge of the 9–6–9 section in figure 19.

Since the load stresses near the edge are reduced slightly by the same downward warping of the edges that produces the critical warping stresses, a slight correction has been applied to the load stress values in this diagram. The correction was made in accordance with the experimental data on this point previously referred to.

The curves in figure 19 for the maximum combined stress across the two typical pavement sections show a very uniform stress condition throughout the interior area of both sections. Since the interior of both of these sections is of constant thickness, it is indicated that for a balanced cross section the interior of the slab should be of a constant thickness. This was shown to be the requirement when only the stresses developed by applied loads were considered. The shape of the interior portion of the cross section is the same, therefore, whether determined by load stresses alone or by combined stresses.

Referring to the diagram for the 6-inch constant-thickness section in figure 19, it will be seen that the combined stress at the edge is about 90 pounds per square inch, or 15 percent above that in the interior. This compares with an increase of 165 pounds per square inch or 60 percent where load stresses only are considered (see fig. 8), and the comparison indicates that when the combination of the stresses from restrained warping and from applied loads is considered, the pavement of constant thickness is more nearly balanced than it is for the stresses from applied load alone. In order to balance the combined stresses for this section it would be necessary to reduce the edge stresses by slightly more than one half of the amount that would be necessary to balance the load stresses only.

For the 9–6–9 section, the diagram in figure 19 shows that combined stress at the edge of the pavement is about 85 pounds per square inch or 13 percent above the interior stress. This is practically the same relation as was found to exist in the slab of constant thickness and indicates that so far as maximum combined stresses across a mid-slab section are concerned, the 9–6–9 section is no better balanced than the section of constant thickness. This somewhat startling result is explained by the fact that the increase in the warping stress at the edge of the thickened-edge slab, caused by the increased temperature differential which results from the greater depth, is approximately equal to the reduction in load stress at this point accomplished by the increased edge thickness.

SHORT SLAB LENGTHS NEEDED TO MINIMIZE STRESSES CAUSED BY TEMPERATURE WARPING

It has been shown, during the discussion of warping stresses in part 2, that the magnitude of the critical warping stress that most directly affects the load capacity of the pavement is a function of slab length. The data show that in a slab length of 10 feet the maximum warping stresses are small but that they increase with length until the length is such that complete restraint to warping exists in the interior of the slab. The length necessary for complete restraint in slabs of different thicknesses has not been determined in this investigation. It is indicated that practically complete restraint is developed by a 20-foot length in a slab of 6-inch constant thickness, and also that this length in a 9–6–9 thickened-edge section is not sufficient to develop full restraint at the mid-length of the panel.

Because of the effect of slab length, it is apparent that the analysis of combined stresses given above can be applied exactly only to slab lengths of 20 feet.

It is not known whether the stresses caused by load in the test sections are larger or smaller than those developed in actual pavements by traffic, so that the comparison between the combined stresses observed in the test sections and the combined stresses that occur in pavement slabs in service is not certain. It is apparent, however, that in pavement slabs as they are designed today the factor of safety against breaking must be very small at times when conditions are such as to produce high warping stresses. The relatively frequent transverse cracking in our more heavily traveled concrete pavements is also an indication that the combined stresses in them often exceed the flexural strength of the concrete.

At first thought one might expect immediate cracking when the combined stresses exceeded the strength of the concrete, and this would probably occur if the high stresses extended completely across the slab. When the critical stress is highly localized, as it appears to be under an isolated load, a single application of an excessive stress may produce no immediate effect. In cases where load stresses are responsible either wholly or in part for the cracking of a pavement, it is but natural to expect the transverse cracking to develop gradually and to continue over a period of years.

Consider, for example, the case of a heavy wheel load moving longitudinally on the pavement at a time when high warping stresses are present in the slab. Being of the same sense in the mid-section of the slab, the stresses combine and may exceed the ultimate flexural strength of the concrete. These tests indicate that the high combined stress will be found to exist only for a very short distance directly under each wheel. The remainder of the cross section will not be overstressed. It is not probable that such a stress condition would cause a full-length transverse crack immediately, but it is reasonable to believe that a great many repetitions of the condition would cause such a crack.

If the length and width of a pavement slab were equal, the warping stresses would be the same in the two directions and it would be reasonable to expect the added stress caused by wheel loads to cause longitudinal cracking to precede transverse cracking, because the wheel load traverses the complete length of the slab, stressing every point in the longitudinal section, yet this wheel stresses to the same degree only a comparatively small part of any traverse section. This may explain the early appearance of longitudinal cracks in some of the older pavements that were built without a longitudinal joint.

In concrete slabs where the combination of the length, depth, and flexural strength are such that warping stresses alone may cause the slab to crack, it is to be expected that such cracks will develop suddenly, since the high stress condition extends over the entire cross section.

SHORT SLABS MAY BE DESIGNED ON THE BASIS OF LOAD STRESSES ALONE

Before proceeding with a discussion of the application of the results of the investigation to the design of a pavement cross section, it may be well to review briefly some of the points developed in parts 1 and 2.

It has been demonstrated conclusively that variations in the temperature of the concrete in the pavement slab frequently cause large stresses in certain parts of the slab. These stresses combine with those caused by applied loads, and the pavement design, to be adequate, must be based upon combined stresses rather than upon load stresses alone.

It will be recalled that loads were applied with the test sections in the flat condition and in the warped condition and that tests were made when the subgrade was in its normal condition of moisture and also at a time of excessive moisture immediately after a thaw. Data were presented to show the influence of concrete temperature on (1) the direct stresses resulting from the resistance offered by the subgrade to horizontal slab movement, and (2) the bending stresses caused by restraint to free warping which exist in practically every part of the slab.

Since the magnitude of the stresses caused by a given applied load is affected by the warping of the slab to a certain degree at the edge of the slab and to a different degree at the interior, it is evident that this is a factor that must be considered.

It was shown in figure 17 that the relation between edge and interior stresses is practically the same during the very wet conditions that prevail immediately after a thaw as during the balance of the winter. Many of the tests that supplied the data for the maximum stress diagrams were conducted during normal winter conditions. It is believed to be unnecessary, therefore, to make an allowance for subgrade condition in setting the ratio of edge to interior thickness.

The usual assumption concerning the direct stresses of tension or compression, caused by the resistance to horizontal movement offered by the subgrade, is that they are uniform over the full cross section of a slab of constant thickness. It is probable that the same assumption could be applied to the thickened-edge sections of this investigation with no great error. If this is assumed to be the case, the stresses referred to will not affect the shape of the cross section.

The effect of closely grouped wheel loads was investigated to a limited extent, and the maximum stress diagrams which were developed indicate that the shapes of the load-stress curves for the multiple loads are approximately the same as for single loads. This leads to the tentative conclusion that a cross section designed for single loadings will not need to be modified because of multiple loadings of the type described.

In order to design a cross section from the maximum stress diagrams shown in figure 19 it will be necessary to make allowance for the following:

1. The effect of the condition of warping on the stresses caused by loads applied along the cross section of the slab.

2. The stresses caused by the restrained warping resulting from temperature differentials within the slab.

The stresses caused by restrained warping are dependent in magnitude upon the length and the thickness of the pavement slab and this must be considered in a discussion of the design of the cross section. It was shown in table 5 of part 2 that if the length of slabs of the usual pavement thicknesses is reduced to approximately 10 feet, the maximum stresses caused by restrained warping are quite small. The difference between the warping stress at the edge and that at the interior points is reduced to such an extent as to be unimportant in its effect upon the design of the cross section. If a pavement slab were to be deliberately designed with a length of 10 feet, it would be sufficient to design the cross section on the basis of load stresses alone.

The maximum stress diagrams shown in figures 8 and 9 will be used to design a balanced pavement cross section with the slab length limited to 10 feet. The principal decisions to be made are:

1. What should be the shape of the interior portion of the cross section?

2. What should be the relation between the depth of the slab at the interior where the conditions of support are most favorable and the depth at the free edge where they are the least favorable?

3. What should be the shape of the cross section near the free edge?

It is definitely indicated by all of the data and shown more clearly by the average curve for the four constant-thickness sections that, except in the immediate vicinity of the slab edge, the pavement slab should be of uniform thickness (see fig. 9).

A load applied at an unsupported edge causes a maximum stress considerably higher than that produced by the same load placed at interior points of the slab. Designing for balanced load stresses therefore requires a strengthening of free slab edges. The degree of thickening required for this strengthening can be determined from figure 10, which indicates that the depth of the slab at the free edge should be one and two-thirds times the depth in the interior region. As mentioned before, this ratio applies only to cross sections in which the additional or edge thickening is reduced to zero approximately uniformly between the edge and a point a short distance from the edge.

The distance in which this reduction of edge thickness should be accomplished is determined by the variation in the stresses in the longitudinal direction since these are the critical load stresses in the vicinity of the edge. The average curve for stress for the four constant-thickness sections (see fig. 9) shows that the interior stress is practically uniform from the center out to a point approximately 2½ feet from the edge, and indicates that for this area no additional depth would be required. The data also indicate that the effect of the thickened edge on the maximum stress from load is felt even beyond the point at which the thickening ends. Considering the stress diagrams for all of the sections, it seems reasonable to conclude that the edge thickening need not extend more than approximately 2 feet from the edge. Again this applies only to the type of cross section in which the edge thickness is reduced to the interior thickness at an essentially uniform rate, as in section 5. While the data obtained with the test sections of this type indicate that this is probably the minimum distance over which the edge thickening should be reduced, as a practical matter in shaping the subgrade the tendency will be to increase rather than decrease this dimension. In balancing the cross section for load, it is probably permissible to adopt the minimum distance, or 2 feet, as this dimension.

TEST RESULTS APPLIED TO THE DESIGN OF A BALANCED CROSS SECTION

It is not to be expected that the data from field tests, no matter how carefully the tests were conducted, would justify fine distinctions as to the shape of the cross section where the edge thickening is introduced. The data indicate quite definitely that the desired balance of stresses is accomplished to a satisfactory degree by a cross section in which the increase in slab thickness is developed at a uniform rate as the edge is approached, as in sections 1, 2, and 5. None of these three sections had an edge depth sufficient to balance fully the load stresses, yet the advantage of this type of edge thickening over that provided in sections 3 and 4 is readily apparent in the maximum stress diagrams.

The pavement cross section shown in figure 20 has been designed in accordance with the principles developed by this investigation, and it represents a section that is completely balanced so far as the stresses from applied loads are concerned. The interior of the cross section is of constant depth and the relation between this depth and that at the extreme edge is 3 : 5 and is constant for the usual range of pavement thicknesses. The difference between the depth at the edge and that at the interior is reduced to zero in a distance of 2 feet.

It is apparent that the design based on these principles is not radically different from that used by a number of States today. However, attention is called to the fact that this design does not take into account temperature stresses and for this reason can be effective only when conditions are such that large temperature stresses will not develop. Climatic conditions so constant as to accomplish this are rare indeed, but as mentioned previously, a very large reduction in the magnitude of the critical temperature stresses can be obtained by using short slabs.

It is indicated by the data presented in part 2 that even with a 10-foot slab length the maximum stresses caused by restrained warping are greater in magnitude than the increase in load stress caused by upward warping of the edges of the pavement. Therefore, the critical combined stresses to be considered in the design of the cross sections are those occuring during the daytime when the edges of the slab are warped downward.

It was assumed earlier that the small difference between the warping stresses at the edge and those at the interior of a 10-foot slab would not cause the relation for combined stresses to be materially different from that which obtained for load stresses alone. In proceeding from the unwarped condition, for which the cross section shown in figure 20 was designed, to that of the combined stresses of the warped slab of the 10-foot length, it is only necessary, therefore, to modify the load stresses slightly due to the effect of the condition of downward warping. It has already been shown that this decrease for a load applied at the edge of the 7- and the 9-inch sections was about 5 percent.

If the edge stresses shown in figure 9 are decreased by 5 percent, it is found that the edge thickness of a balanced slab should be approximately 1.6 times the interior thickness instead of 1.66 as was indicated for the unwarped slab condition. This is a small difference and is perhaps beyond the accuracy of this method for determining the relation between edge and interior thickness. The data do lead definitely to the conclusion that in slab lengths of approximately 10 feet a balanced design requires a maximum edge thickness that is slightly more than one and one-half times the interior thickness.

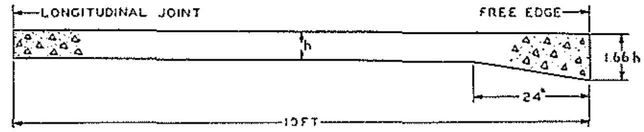


FIGURE 20.—SLAB PANEL CROSS-SECTION COMPLETELY BALANCED FOR LOAD STRESSES.

Increasing either slab length or pavement thickness increases the warping stress up to the point at which the length for a given thickness is such that complete restraint to warping exists in the center of the slab. For this reason, increasing the edge thickness of a long slab may not increase the load-carrying capacity of the slab edge as much as would be expected. As shown in the maximum stress diagrams for combined stress in the two 20-foot slabs of figure 19, the benefits derived from the thickened edge from the load-stress standpoint are practically offset in the 20-foot section by the increased warping stresses which unavoidably result. It is conceivable that there may be cases of slabs so long that complete restraint to warping is developed in which the thickening of the slab edge may actually lower the load-carrying ability of the edge.

It may seem from the above discussion that the thickening of slab edges should be eliminated in slabs with lengths of 20 feet or more. So far as strengthening the edge in the mid-length of the slab this is probably true. The high combined stresses in long slabs that carry heavy loads eventually lead to transverse cracking which shortens the slab length and thus reduces the warping stresses and consequently the combined stresses at the edges of the slab.

While the design of slab corners is not discussed in this paper, edge thickening is so intimately connected with corner design that some mention of the effect of edge thickening on the stress conditions in the region adjacent to the slab corner should be included. Thickening the edges of a pavement slab does not increase the combined stresses in the corner because the critical warping stresses are opposite in sense to those caused by load. Edge thickening is effective therefore in reducing the combined stresses at the corner.

This study leads inevitably to the conclusion that a condition of balanced stresses in a pavement cross section is possible only when the critical stresses arising from restraint to warping are definitely limited to low values. The most practical method of insuring this seems to be through the construction of short pavement slabs. Application of this conclusion to the design of pavement slabs involves considerations other than those discussed in this report but necessary to the forming of a correct judgment as to whether or not a completely balanced design should be used or how closely it should be approached.

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THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

Reported by L. W. TELLER, Senior Engineer of Tests, and EARL C. SUTHERLAND, Associate Highway Engineer

PART 4.—A STUDY OF THE STRUCTURAL ACTION OF SEVERAL TYPES OF TRANSVERSE AND LONGITUDINAL JOINT DESIGNS[1]

PRECEDING REPORTS on parts of this investigation have presented: (1) A general description of the entire project and of the methods employed in making the tests (part 1); (2) a discussion of the effects of temperature and moisture variations on the size, shape, and load-carrying ability of pavement slabs as observed during the course of these studies (part 2); and (3) a discussion of the results of tests on various pavement cross-sections (part 3).

This report contains a description of the studies that were made of the structural action of the several transverse and longitudinal joints included in the investigation. In presenting this material, certain descriptive matter will be repeated from the preceding reports for the purpose of amplification together with such data from parts 2 and 3 as are necessary for an adequate treatment of the subject.

In dealing with the subject of the design and use of joints in concrete pavements, it is of considerable interest to look backward over the period of concrete pavement construction and trace the development of theory and practice in regard to joint construction. This development will be sketched rather briefly.

A number of concrete pavements were built in Europe and in the United States long before the beginning of the present century. There is mention of one constructed in Inverness, Scotland, as early as 1865,[2] while in this country one of the earliest of which there is an authentic record is that constructed in Bellefontaine, Ohio, in 1892. So little information is given in the accounts of these early concrete pavements that in most cases no details of the spacing and design of the joints are available. It appears, however, that the joints were simply small spaces left between adjacent slabs and were intended to be filled with earth, although as far back as 1871 a patent was granted that gave the inventor rights covering the use of gum, tar, rubber, or other water-repellent substances as a filler for joints in pavements made of concrete blocks.[3] Some mention is made in engineering literature of the use of pitch and of creosote oil for the same purpose at about the same time.

EARLY JOINTS DESIGNED TO PROTECT SLAB EDGES FROM DAMAGE BY STEEL-TIRED WHEELS

The Bellefontaine pavement was laid in small slabs or blocks 5 or 6 feet square and tarred paper was placed between the blocks to allow for expansion.[4] It is interesting to note that with this small-slab construction practically no cracking has occurred in this pavement during more than 40 years of service. At the time it was constructed all traffic was carried on steel-tired wheels and much damage was done to the edges of the slabs by teamsters who drove so that their wagon wheels followed the joints.[5] Efforts to protect slab edges from the damaging action of steel-tired wheels seem to have been the dominant throught in the early consideration of joint design.

Figure 1 shows a drawing of what is one of the first, if not the first, joint designs for concrete pavements patented in this country.[6] The object of this design, as stated in the patent, was to allow adjacent blocks to heave without injury to their edges. Direct expansion apparently was not a consideration. It was specified that the metal forming the joint should be stiff enough to permit tamping the concrete around it, yet light enough to crush in the event that heaving occurred.

The decade between 1900 and 1910 might well be considered as the early formative period in concrete pavement history. A new type of pavement was developing and the literature of this period contains many inquiries as to how concrete roads should be built, with little or no information available to supply the answers. The joint, intended to provide for expansion and to control cracking, made its appearance, although it continued to be a simple opening between slabs. Breaking of the slab edges under the action of the steel-tired wheels was still a serious problem and its effect is reflected in the staggered and oblique joint designs of the period. Some pavements of which there is a record of the joint construction are described briefly as follows:

Grand Rapids, Mich., 1901–02: In a two-course pavement, joints were placed along the curbs and transversely at intervals of 25 feet in the base course. The width of these joints is not recorded. The top course was laid in alternate blocks 6 feet square with expansion joints one-fourth inch in width between blocks. These joints were filled with asphalt.

Toronto, Canada, 1902: This pavement was laid in blocks 20 feet square with ¾-inch expansion joints between the blocks. The joints were filled with "paving pitch." It is reported that under heavy traffic the edges of the slabs shattered badly.

Richmond, Ind., 1903–04: The earliest concrete pavements in this city date from 1896. Of the early pavements no information about the joint design was found, although it appears that small slabs were used. The pavements laid in 1903–4 were in large slabs with expansion joints 1 inch wide. It was reported that these wide joints were troublesome because of chipping at the slab edges. Mention of temperature cracking in connection with this pavement appeared in the early reports.

[1] A series of five articles has been planned. Parts 1, 2, and 3 have appeared in PUBLIC ROADS, vol. 16, nos. 8, 9, and 10, October, November, and December 1935, respectively. Because of its length, Part 4 will be presented in two issues of PUBLIC ROADS. The second installment will appear in the October issue.

[2] Cement and Concrete—A general refrence book, 1929. Portland Cement Association, p. 49.

[3] United States Patent no. 120208 granted Oct. 24, 1871, to H. A. Gunther.

[4] Portland Cement Pavement, by G. W. Bartholomew, Jr., Engineering News, vol. 33, no. 1, Jan. 3, 1895, p. 5.

[5] The Concrete Pavements of Bellefontaine, Ohio, by Prof. F. H. Eno, Engineering News, vol. 51, no. 1, Jan. 7, 1904, p. 15.

[6] United States Patent no. 312897 granted Feb. 24, 1885, to C. F. Rapp.

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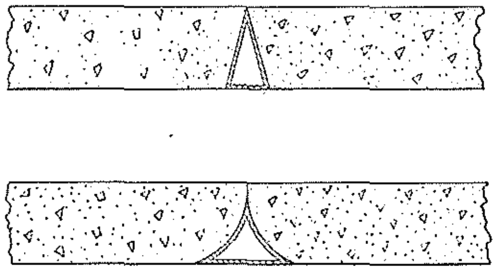


PATENT GRANTED 1885
NO.312,897—C.F.RAPP

Figure 1.—One of the First Joint Designs for Concrete Pavements Patented in the United States.

Washington, D. C., 1906: This pavement was laid in slabs 100 feet long separated by 1-inch joints filled with a bituminous material.

City of Panama, 1906–7: It is recorded that the first concrete paving in this city consisted of slabs 10 feet in length. On wide streets the pavement was divided longitudinally at the center and the slabs were staggered on either side of this joint.

Because of difficulties with chipping and spalling along the joints, commercial companies specializing in concrete pavement construction began gradually to increase the spacing between joints. This practice continued for many years and culminated in the construction of hundreds of miles of concrete pavements in which the only joints constructed were at places where the paving operation was stopped for some reason.

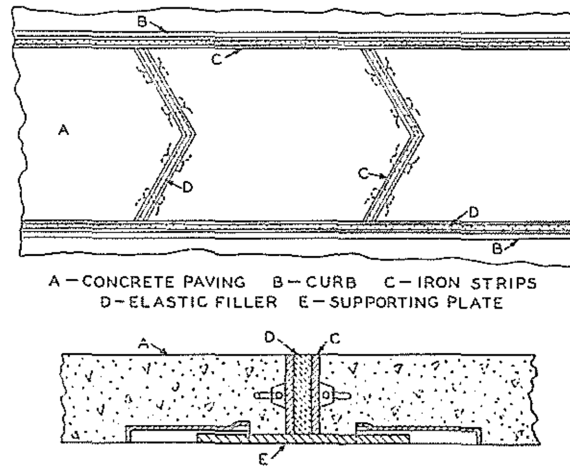
That some engineers at this time appreciated the advantages of crack control in concrete pavements is evidenced by the following quotation from a patent for a joint design granted to Mr. R. Kieserling, a German citizen, by the United States Patent Office in 1906: [7]

"As it is well known, irregularly running cracks appear after a short time in paving made from concrete or other cement mixtures, which lead to the destruction of the pavement. * * * I avoid this irregular formation of cracks by providing for the occurrence of cracks at definite places and causing them to run in a direction previously determined upon." Mr. Kieserling's design for accomplishing this control of cracking is shown in figure 2.

WIDE DIFFERENCES FOUND IN STRUCTURAL DETAILS OF EARLY JOINTS

By 1910 the automobile had demonstrated itself to be a practical machine and with its increasing use came the demand for more and better highways, particularly interstate highways. The stimulus thus given to road building is reflected in the increased discussion of concrete pavement design. Spalling and chipping at the joints and separation of adjacent slabs, both vertically and horizontally, were troubles which gave a great deal of concern in these early pavements. To overcome chipping, diagonal joints and edges armored with metal were frequently recommended. For the displacement at the joints, particularly the longitudinal joints, numerous suggestions appeared—more attention to drainage, reinforcement to prevent longitudinal cracks, dished subgrade to provide a thicker pavement at the center of the road, and rolled stone subbases—each had its advocates.

[7] United States Patent No. 839600, granted Dec. 25, 1906, to R. Kieserling, Germany.



PATENT GRANTED 1906
NO. 839,600—R.KIESERLING

Figure 2.—An Early Patented Joint Design for Concrete Pavements.

Speaking editorially, one of the leading engineering journals of the country said of the concrete pavement joint designs of this period: [8]

> Practice exhibits a heterogeneous array of expansion joint details, spacings, and arrangements. This is most true of transverse joint practice. The plan is general of placing joints between pavement edge and curb and, when railway tracks occupy the streets, of placing joints on each side of the tracks just outside the tie ends. There is no similar uniformity in transverse expansion joint practice. They are spaced 25, 30, 37½, 50, 60, and 100 feet apart, and the most common spacings are perhaps 25 and 30 feet. Usually they are square across the roadway but various diagonal arrangements are employed. Structurally the differences are wide. Joints with metal guard plates, joints with rounded edges only, joints of all widths from ¼ to 1 inch, joints with fillers of a dozen characters are employed.

As mentioned previously, some of the State highway departments adopted the practice of laying concrete pavements without joints except at points where the construction operation was stopped. By 1915 a number of States were building their pavements in this manner. The reasons prompting this policy were described by one State highway engineer,[9] who stated that the occurrence of transverse cracks had been almost as erratic in pavements with the joints spaced 50 feet apart as in those in which the spacing was 100 feet. The difficulty of constructing smooth surfaces in the vicinity of the joints and the chipping of the slab edges at the joints under the action of traffic were also important considerations.

At the tenth annual convention of the American Concrete Institute (1914) certain recommended specifications for concrete pavement construction were adopted, and the recommendations relative to joint construction probably reflect the thought as to the best practice at that time.

In these specifications it was recommended that transverse joints should be not less than one-fourth nor more than three-eighths inch in width and should be placed across the pavement perpendicular to the center line, not more than 35 feet apart. It was further recommended that a longitudinal joint not less than one-

[8] Engineering and Contracting, vol. 40, no. 2, July 9, 1913.

[9] H. E. Bilger, State road engineer of Illinois, in a paper delivered before the Illinois Society of Engineers and Surveyors 1915. Also Engineering and Contracting, vol. 43, no. 11, Mar. 17, 1915, pp. 254–5.

fourth inch in width should be constructed between the curb and the pavement and that all joints should extend completely through the pavement and be perpendicular to its surface. Also, the concrete at transverse joints should be protected with soft-steel, joint-protection plates rigidly attached to the concrete, the surface edges of the metal plates to conform to the surfaces of the concrete. All joints found to be more than one-fourth inch too high or one-half inch too low were to be removed. It was specified further that all joints were to be formed by inserting, during construction, and leaving in place the required thickness of joint filler, this filler to extend through the entire thickness of the pavement.

IN 1917 LOAD TRANSFER APPEARED AS A FACTOR IN JOINT DESIGN

It will be observed that provision for expansion and protection of the joint edges are dominant considerations and that mutual support through transfer of load is not mentioned as a joint requirement. The smoothness tolerances are of interest in contrast with the specifications of today.

Load transfer as a factor in joint design was soon to appear, however. In the design of a concrete pavement constructed between two Army camps near Newport News, Va., during the winter of 1917–18, steel dowels were placed across all transverse joints for the stated purpose of transmitting load across the joint by shear.[10] The joints were three-eighths inch in width and four three-quarter inch diameter steel dowels were used in the 20-foot pavement width. It was recommended that eight rather than four dowels be used. Heavy truck traffic during the World War period apparently failed to damage these joints.

Following the World War the use of steel dowels spread rapidly wherever concrete pavement was being laid and has continued up to the present time.

Although the principle had been well known for many years, one of the earliest references to the use of the weakened-plane contraction joint for crack control appears in connection with a pavement laid in West Virginia in 1919.[11] It was constructed by grooving the bottom of the slab by setting a thin board on edge on the subgrade, the width of the board being approximately one-half the thickness of the pavement. The concrete was then cast over the board.

Soon after this the recommendation appeared that this type of joint be formed by a board one-fourth inch thick and 6 inches wide so cupped or warped as to give a tongue-and-groove effect to adjoining slabs, thus preventing uneven settlement of the abutting edges.

Figure 3 shows typical joint designs for which patents were granted in this country during the decade following 1910. The essential feature of the design shown in figure 3–A was the use of steel protection plates at the joint edges, tied in to a general system of reinforcement. The object of the design shown in figure 3–B was to permit the placing of the joint filler in advance of the concreting operation. It will be noted that an air chamber was provided to take care of the filler material during expansion. Figure 3–C shows a design intended to protect the edges of the slabs and at the same time serve as a container for the filler.

The use of a steel T-section embedded in the plastic filler material was proposed in the design shown in figure 3–D, the T-section presumably serving to protect the edges of the concrete. While the design shown in figure 3–E was apparently intended primarily to provide a sliding key or bridge in order to hold the filler material, both the design and the claim contain the germ of an idea that appears in many of the joint designs being promoted today. Figure 3–F shows a heavily armored expansion joint in some respects quite similar to designs recently proposed although the idea of load transfer does not appear in the claims.

The design shown in figure 3–G is definitely intended to provide "an interlocking engagement of the adjacent concrete sections" although the compressible material which is interposed between the corrugated plates, together with separation caused by contraction, would probably completely defeat the purpose. Figure 3–H shows a design that includes the use of dowels which are not bonded to the concrete, and are installed for the stated purpose of maintaining the engaged sections of concrete in the proper relation to each other and at the same time permitting independent expansion and contraction.

NEED FOR BETTER EXPANSION AND CONTRACTION JOINTS RECOGNIZED

The disappearance of steel-tired vehicles from the highway, a change which accelerated rapidly during the period following the World War, eliminated what had been one of the worst problems in joint design, i. e., chipping of the slab edges. The result was the general omission of the steel, edge-protection plates from joint designs. A new trouble appeared, however, with the increased use of concrete pavements. Expansion failures known as "blow-ups" began to appear 3 or 4 years after the laying of the pavement, and the seriousness of some of these created a renewed interest in joints providing relief for expansion.

The desire to improve the appearance of concrete pavements by control of cracking led to the more widespread use of the so-called "contraction joints." As already noted, the earliest joints of this type were constructed by grooving the bottom of the slab. The irregularity of the crack on the slab surface, coupled with the difficulty in sealing these joints effectively, led to an unfavorable reaction which resulted in the general abandonment of this design. Shortly after 1920 a weakened-plane joint appeared in which the upper surface of the slab was grooved. While more difficult to construct, it obviated the difficulties just noted and, with the development of mechanical methods for grooving the concrete at the time of construction, this type of contraction joint came into rather widespread use.

The decade following 1920 also saw the general adoption of longitudinal joints that divide the pavement into slabs approximately 10 feet wide. Experience showed that such joints practically eliminated longitudinal cracking and, since this width is about what is required for a single lane of traffic, the practice of building pavements in slabs about 10 feet wide has developed naturally and has resulted in effective control of longitudinal cracking.

During the early part of this decade researches such as the test road at Pittsburg, Calif., the Bates road tests in Illinois, and experiments of the Bureau of Public Roads at Arlington, Va., developed certain basic facts concerning the effect of loads on pavement slabs of various designs. In all of these researches the need for strengthening slab edges was definitely indicated. Free edges of slabs can be strengthened most simply by increasing the slab depth, but where the slab adjoins others the possibility for inter-slab support as a means

[10] Engineering News-Record, vol. 88, no. 9, Mar. 2, 1922, pp. 357–8.
[11] Engineering News-Record, vol. 85, no. 7, Aug. 12, 1920, p. 305.

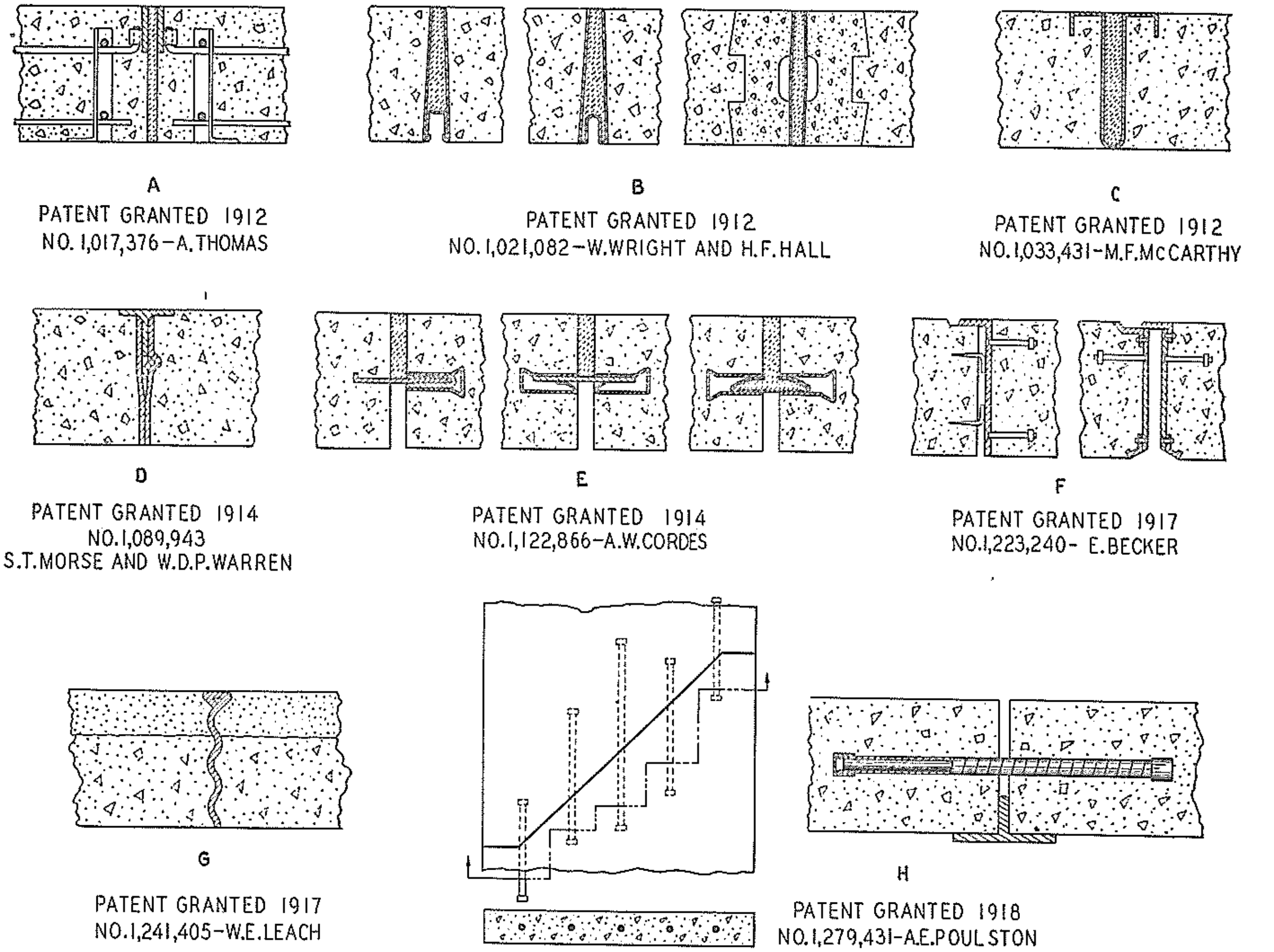


Figure 3.—Some Joint Designs That Were Patented in the United States Between 1910 and 1919.

for strengthening the edges has long been recognized and has led to many proposals for joint designs in which varying degrees of interlocking action are developed. The use of transverse joint designs in which some form of load-transfer mechanism is incorporated has become quite general, the round, steel dowel bar being the most common.

Efforts were also made to strengthen structurally, by systems of steel reinforcement, certain parts of the slab, usually the edges and corners. Some of these proposed systems were very simple; others were quite extensive and complicated.

In figure 4 are shown a number of typical joint designs for which patents were granted between 1919 and 1929. It is of interest to compare this group with that shown in figure 3 and note how the changes in ideas about joint design that have just been discussed are reflected in these two groups of designs. The idea of edge protection disappears and the idea of load transfer appears as the most important factor in the design.

Figure 4–A shows a method for the control of cracking by means of a transverse, steel parting strip so deformed as to create corrugations of various shapes to provide for an interlocking action of the two slab edges. Figure 4–B is similar except that complete separation is provided without cracking and a single approximately rectangular tongue and groove is formed. Figure 4–C shows a deformed metal plate intended for longitudinal joints and forming a triangular tongue and groove similar to that used in one of the test sections. Figure 4–D shows a doweled joint with a short cap to provide for end freedom of the dowel during expansion. In figure 4–E are shown several designs incorporating various methods of load transfer together with a collapsible metal box intended to form the opening between the slabs at the time of construction and to remain in place as a seal afterward.

The use on one slab of rounded projections that engage sockets of the same shape on the adjoining slab is proposed for load transfer in the longitudinal joint shown in figure 4–F. The use of bonded dowels is contemplated in this design. Figure 4–G shows an expansion joint in which inter-slab action is obtained by both a concrete tongue and groove and by steel bars which pass from slab to slab.

CLOSER SPACING OF TRANSVERSE JOINTS GRADUALLY ADOPTED

The great differences of opinion as to how far apart joints should be placed, which were remarked in the 1915 editorial, persisted for many years. In 1931 three States used expansion joints only at bridge approaches while several others employed them only under special conditions (which frequently meant only at bridge approaches). The remainder, with the exception of one State, installed expansion joints at intervals of

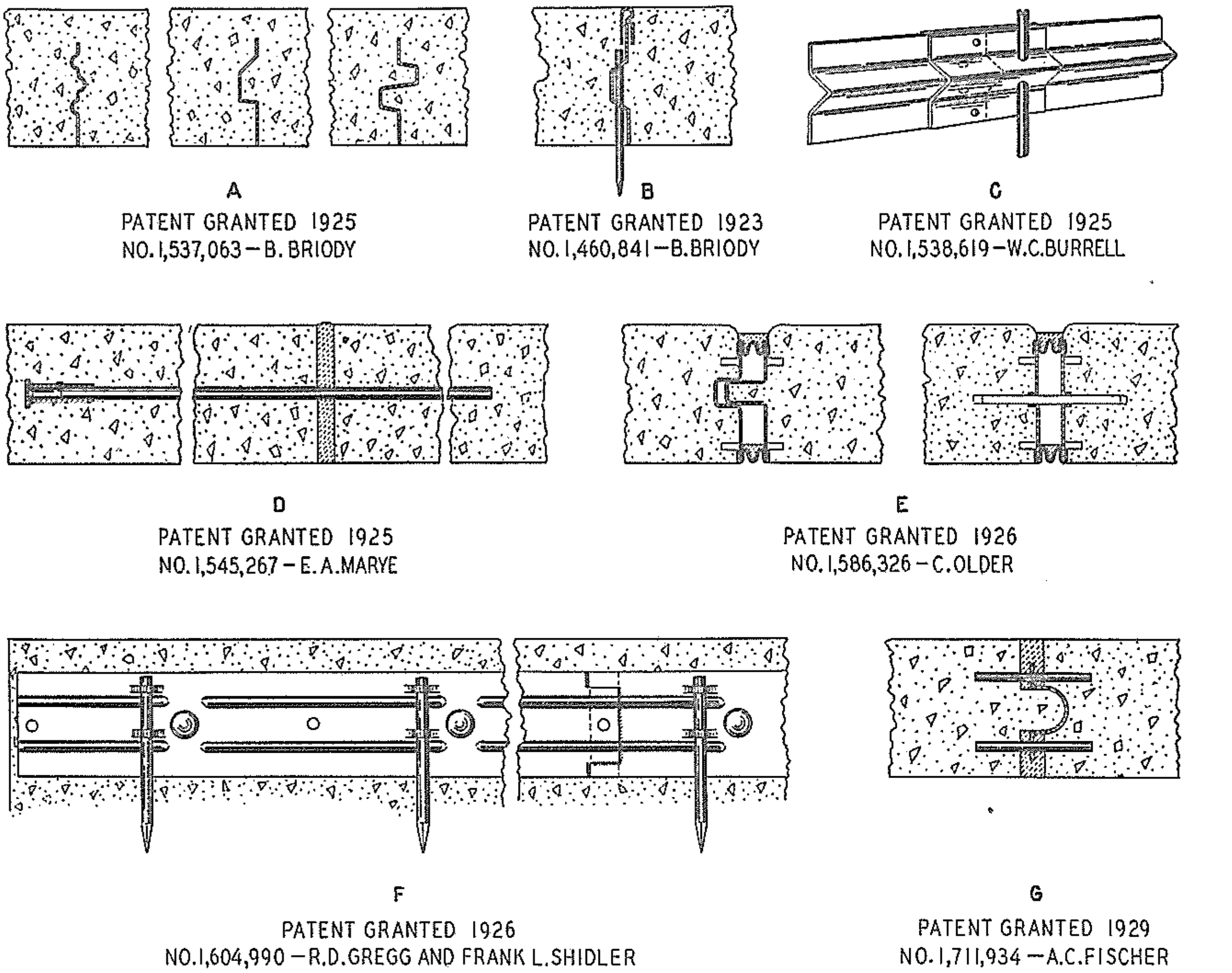


Figure 4.—Some Joint Designs That Were Patented in the United States Between 1919 and 1929.

approximately 100 feet or less. This one State constructed a 4-inch expansion joint at approximately 800-foot intervals and used no other transverse joints in concrete pavements.

By 1934 all of the States, with but one exception, were installing expansion joints at intervals of 100 feet or less (and in this one the interval used was 150 feet). Also, the adoption by many States of the policy of using contraction joints between the expansion joints resulted in a still further reduction of the interval between transverse joints. During the early part of 1934 the Bureau of Public Roads made the requirement that on Federal-aid road construction expansion joints should be provided at intervals of not more than 100 feet and that in plain-concrete slabs transverse joints should be placed at intervals not exceeding 30 feet. It was required also that the width of expansion joints should be not less than three-fourths nor more than 1 inch and that some provision for load transfer should be made in all transverse joint installations. These requirements for Federal-aid construction have probably accelerated the trend toward a shorter distance between joints, a trend that has been discernible for a number of years in spite of the wide variation of opinion which has existed.

Although there is widespread acceptance of the desirability of inter-slab load support at transverse joints, there is both a wide divergence of opinion as to how it should be accomplished and a decided lack of agreement on the fundamental structural requirements of a satisfactory joint design. This condition is caused principally by a lack of conclusive evidence from tests or other sources as to what these requirements should be.

In 1927 Westergaard published an analytical treatment of the action of a doweled joint under load.[12] This valuable contribution to the general subject of joint design has apparently not been given the attention it deserves. The analysis showed the effect of dowel spacing on the stresses directly under a load acting at a doweled edge of a pavement slab, throwing new light on the critical stresses in the vicinity of joints of this type. It is indicated that dowels, even under the ideal conditions that were assumed, must be placed very close together if they are to be reasonably effective as a means for transferring load.

Aside from Westergaard's analysis there has been little information available except for occasional reports of the observed service behavior of certain joint installations.

An examination of the designs shown in figure 5 will reveal how widely opinions vary as to what is required structurally in joint action. It will be noted that some believe that a joint should be shear resistant

[12] Analysis of Stresses in Concrete Roads Caused by Variations of Temperature. Public Roads, vol. 8, no. 3, May 1927.

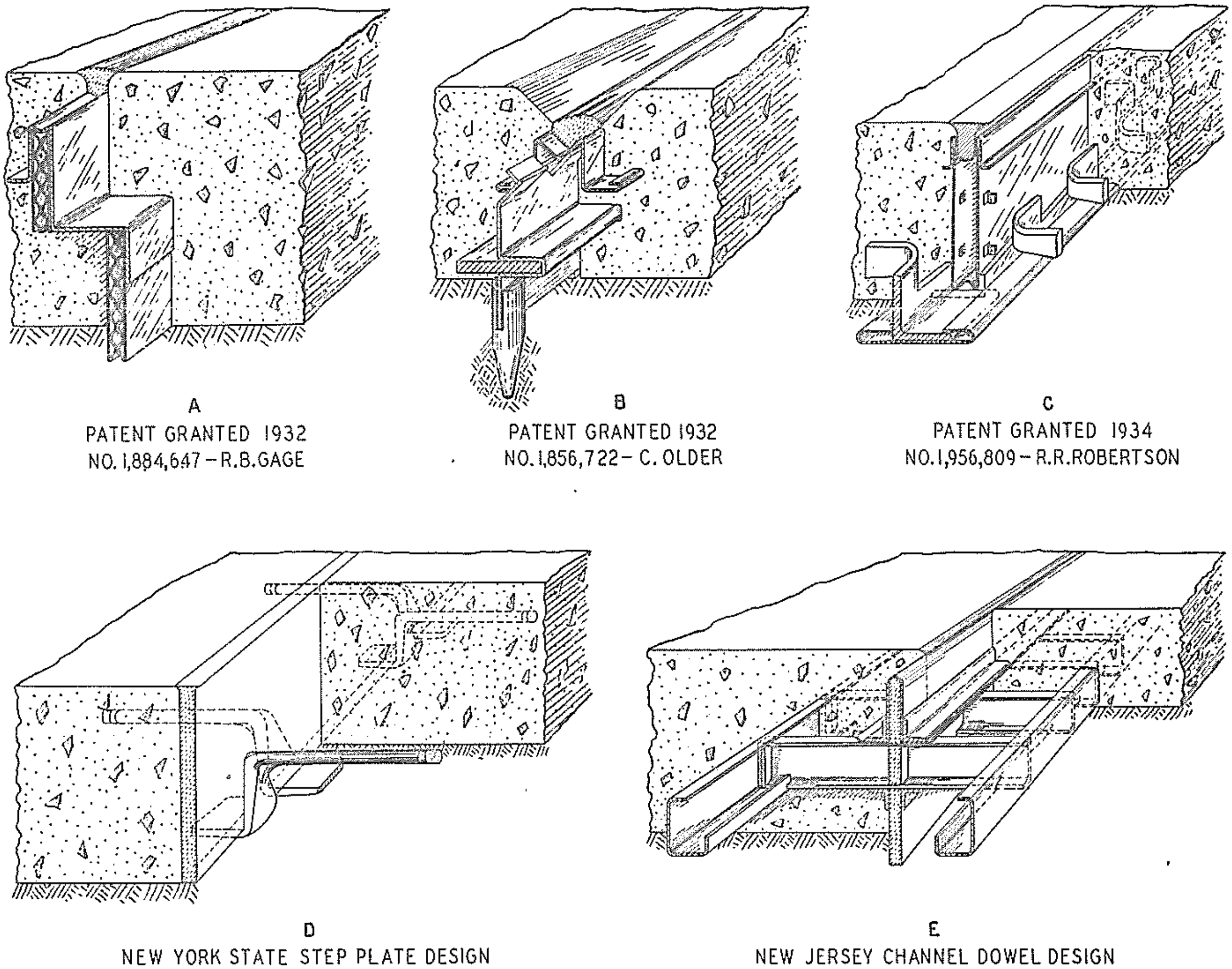


FIGURE 5.—JOINT DESIGNS THAT TYPIFY VARIOUS OPINIONS AS TO WHAT IS REQUIRED STRUCTURALLY IN JOINT ACTION.

but should be without stiffness so far as vertically applied loads are concerned. Others are equally convinced that an effort should be made in designing the joint to develop the same resistance to bending at the joint as is found in the interior of the slab.

Figure 5–A shows a design in which no effort is made to develop bending resistance in the joint structure itself. If the load approaches the joint from one direction there is direct transfer to the adjacent slab through the reaction developed on the ledge or shelf on the adjacent slab. The joint in this case acts somewhat as a free hinge. If the load approaches from the opposite direction there can be no transfer of load.

One of the designs, shown in figure 5–B, shows a steel plate running the length of the joint and fitting into grooves or recesses formed into the two opposing slab ends. The plate acts as a key or spline and by its stiffness transfers part of the load across the joint. The flexibility of the plate permits a certain amount of hinge action to occur. Figure 5–C shows another design in which one slab rests on a shelf on a slab opposite. The shelf or ledge in this case is of steel and is anchored into the concrete of the slab end. In order to obtain the same support for each slab, the shelf angles are cut into short sections, half of the projections extending from each slab and so staggered that they intermesh, giving a typical hinge construction.

Another joint identical in principle but differing in the details of its design is that being used in New York State and shown in figure 5–D. In this case the shelves are individual castings anchored into concrete as shown. In neither of these is there any attempt to develop resistance to bending in the joint structure.

A design differing radically in principle is that used by the State of New Jersey and shown in figure 5–E. The theory behind this design is that the same resistance to bending should be provided at the joint as is found at the other points along the slab, and the series of stiff members which span the joint in this design are for this purpose.

JOINTS MAY ACT TO RELIEVE STRESSES RESULTING FROM EXPANSION, CONTRACTION, OR WARPING

A feature of joint design that has given considerable concern and that has been and is still being given a great deal of study is the filling and sealing of expansion joints. It presents a related but separate problem and was not a part of the investigation that is being reported in this series of papers.

In this brief review it has been noted that joints appeared with the first use of concrete for paving, probably the division of the early pavements into small units being as much for convenience in construction as for any other reason. Later, expansion joints as such appeared with the expressed idea that their use would control the cracking which inevitably occurred. Difficulty in the construction of joints and their apparent ineffectiveness as a means of crack control led to a reaction that resulted in a decreased use of joints for a period. The results obtained by this policy were not altogether satisfactory, however, and the continued urge for smoother and better appearing pavements led to the introduction of what have been called contraction joints placed between expansion joints, the length of the slab units being gradually decreased. Load transfer as a recognized factor in joint design appeared after the World War and is now quite generally considered to be an essential requirement.

The importance of freely acting joints as a means for the relief of stresses developed by restrained temperature (and possibly moisture) warping is as yet not generally appreciated, although the results obtained with the longitudinal center joint have been evident for years and both the theoretical and experimental indications of the importance of warping stresses were pointed out before the Highway Research Board nearly a decade ago.[13] [14]

As shown in the preceding papers of this series, the present investigation has developed a considerable amount of information about the magnitude and the distribution of warping stresses. This information, much of which is new, emphasizes the necessity for controlling these stresses in concrete pavements if adequate wheel-load resistance is to be provided. The data that have been presented relative to warping stresses bear directly, therefore, on one important function that a joint should be designed to perform.

Thus it appears that joints in concrete pavements may be classified according to their intended function, as follows:

1. Those designed to provide space in which unrestrained expansion can occur.

2. Those designed for the relief or control of the direct tensile stresses caused by restrained contraction.

3. Those designed to permit warping to occur, thus reducing restraint and controlling the magnitude of the bending stresses developed by restrained warping.

Obviously a joint may and frequently does perform all three of these functions. An expansion joint, for example, may permit unrestrained expansion, contraction, and warping, while a joint of the so-called contraction type may actually benefit the pavement more by its ability to relieve warping stresses than by its intended function of relieving direct tensile stresses caused by contraction.

It should be kept in mind that joints are needed in concrete pavements for the one purpose of reducing as much as possible the stresses resulting from causes other than applied loads in order that the natural stress resistance of the pavement may be conserved to the greatest possible extent for carrying the loads of traffic.

A joint is potentially a point of structural weakness and may limit the load-carrying capacity of the entire pavement so that it is important to examine joint designs from this standpoint as well as for their ability to permit unrestrained expansion, contraction, and warping.

[13] Analysis of Stresses in Concrete Pavements Due to Variations of Temperature, by H. M. Westergaard, Proceedings, Sixth Annual Meeting, Highway Research Board, December 1926, pp. 201-215.

[14] Progress Report on the Experimental Curing Slabs at Arlington, Virginia, by J. T. Pauls, Proceedings, Sixth Annual Meeting, Highway Research Board, December 1926, pp. 192-201.

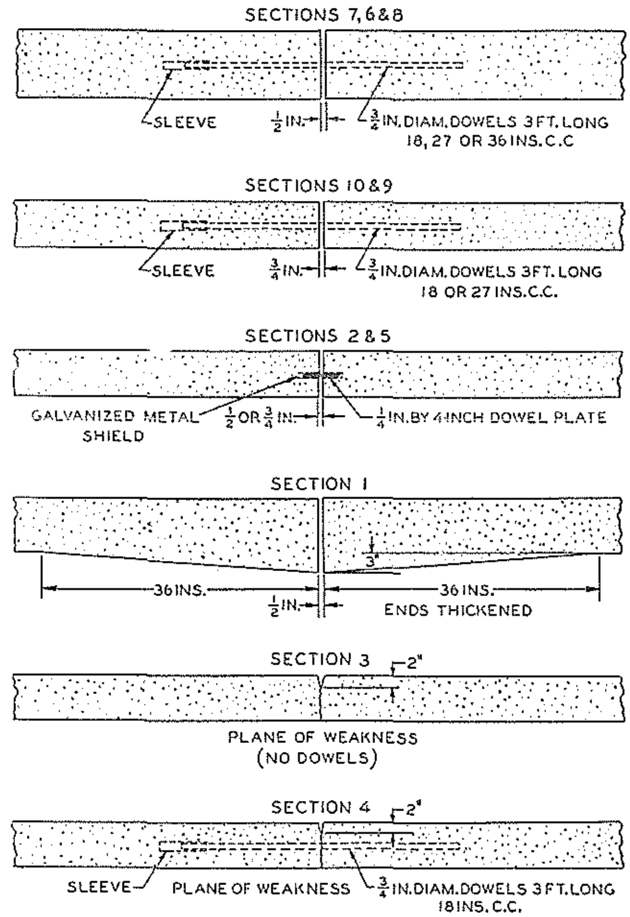


Figure 6.—Designs of Transverse Joints Included in the Investigation. The Bond was Destroyed on all Dowels Across Transverse Joints by Painting and Greasing.

In studying the structural action of joints in this investigation, each joint was subjected to tests to determine its relative effectiveness for:

1. Permitting unrestrained expansion and contraction.
2. Permitting unrestrained warping at the joint.
3. Reducing the structural weakness created by the break in the slab continuity at the joint.

INSTALLATION AND DETAILS OF TRANSVERSE JOINTS DESCRIBED

In the first paper of this series there was given a brief description of the 10 transverse and the 10 longitudinal joints that were included in the pavement sections built for this investigation. Before beginning the description of the tests and the discussion of the results, it is desirable to refer again to the details of these joints.

The details of the several types of transverse joints studied are shown in figure 6. The joints are all classed as expansion and contraction joints with the exception of the two transverse plane-of-weakness or dummy joints that were incorporated in sections 3 and 4. These two are primarily contraction joints. The transverse joints are divided into four groups, according to type.

The first group comprises the dowelled expansion and contraction joints found in sections 6, 7, 8, 9, and 10. In this group the dowels were round, rolled-steel bars three-fourths inch in diameter and 3 feet in length in all cases but both the spacing of the dowels and the distance between the abutting slab ends (or joint opening) were varied, as shown in table 1, in order to determine the

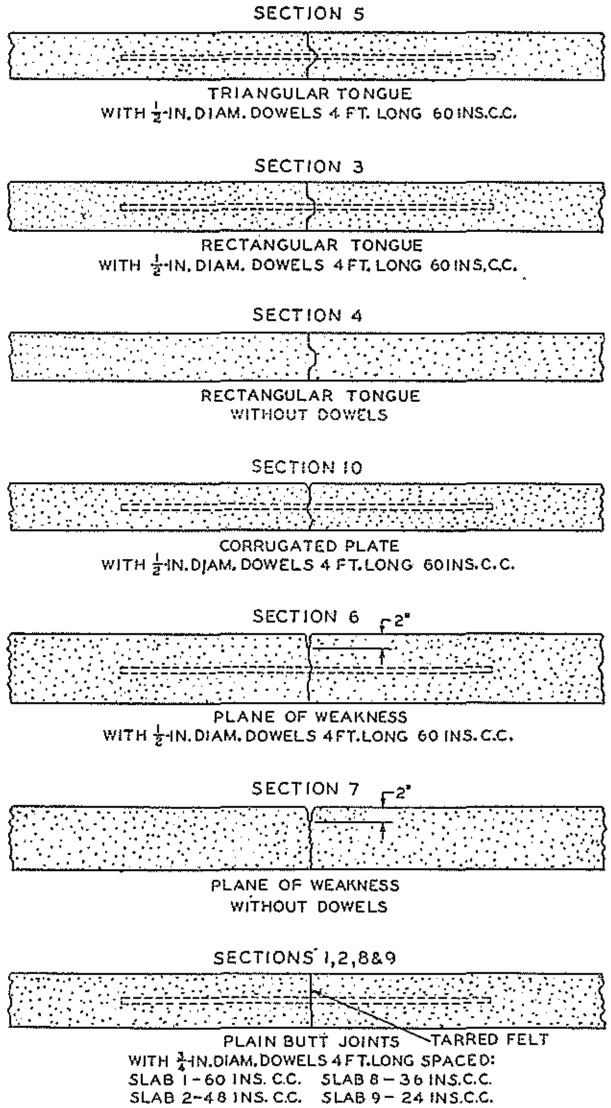


FIGURE 7.—DESIGNS OF LONGITUDINAL JOINTS INCLUDED IN THE INVESTIGATION. ALL DOWELS (OR DEFORMED TIE BARS) WERE BONDED THROUGHOUT THEIR LENGTH.

effect of these variables on the structural action and general efficiency of joints of this type.

TABLE 1.—*Details of doweled expansion and contraction joints*

Section no.	Joint opening	Dowel spacing
	Inch	*Inches*
6	½	27
7	½	18
8	½	36
9	¾	27
10	¾	18

At the time of installation the dowels were carefully painted and coated with grease to prevent bonding with the concrete, and special pains were taken to insure that all of the dowels were parallel to the subgrade and to the longitudinal axis of the pavement section. As will be noted in the drawings (fig. 6) a short cap or sleeve on one end of each dowel permits free longitudinal movement of the dowel within the concrete.

In the second group of transverse joints are the two plate-dowel designs found in sections 2 and 5, the only difference between the two being in the width of the joint opening. In each the steel dowel plate is one-fourth by 4 inches in section and is continuous for the full 10-foot width of the pavement slab. The bonding of the dowel plates to the concrete was prevented by an encasing shield of sheet metal which extends beyond the edges of the dowel plate in such a manner as to provide for free movement of the dowel plate during expansion and contraction of the pavement. The widths of the joint openings employed in these two joints are one-half inch (sec. 2) and three-fourths inch (sec. 5).

The third type of transverse joint is that in which the thickness of the slab ends abutting the joint has been increased above the thickness of the interior of the slab for the purpose of strengthening the transverse slab edges. In this design no load transfer is attempted since no inter-support is necessary; hence there is no direct connection between adjacent slabs. Such a joint was placed in section 1. The ends of this section at the open joints were also thickened.

The fourth and last type of transverse joint included in this investigation is the weakened-plane or dummy joint found in sections 3 and 4. These transverse joints were constructed in the same manner as the longitudinal joint of the same type except that bonding of the dowels was prevented in one (section 4), while in the other (section 3) all dowels were omitted. The spacing of the dowels in section 4 is 18 inches.

INSTALLATION AND DETAILS OF LONGITUDINAL JOINTS DESCRIBED

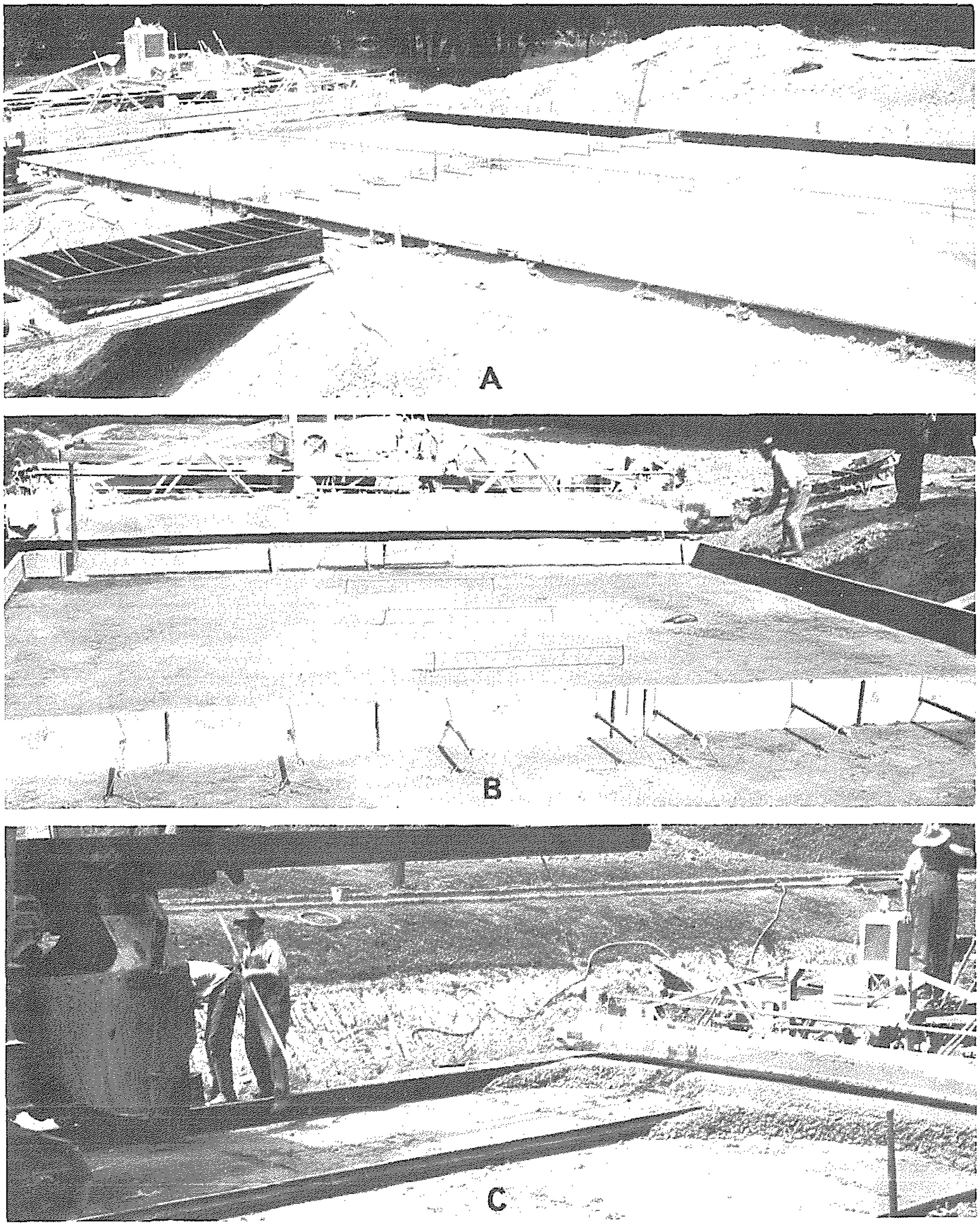
Details of the designs of the 10 longitudinal joints are shown in figure 7. With the exception of those found in sections 4 and 7, where no steel crosses the joint and free contraction is permitted, all of the designs are primarily warping joints.

By definition a dowel may or may not be bonded to the two pieces which it joins. In woodworking practice, dowels are more often bonded than not. In concrete pavement construction the dowels that cross the longitudinal joint are nearly always bonded to the concrete and are usually called tie-bars, although they are more exactly described as dowels in bond when they are called upon to withstand shearing forces. In this report the steel bars used for joining two abutting slab edges are generally referred to as dowels if the bond has been broken and dowels in bond if the bond still exists.

The longitudinal joint designs included in this investigation can also be grouped according to type.

The first group consists of four sections (sections 5, 3, 4, and 10) in which a tongue-and-groove construction was obtained by casting the concrete around a preformed, steel joint plate. The rectangular- and triangular-shaped tongue and groove and the sinusoidal tongue and groove (sections 3, 5, and 10, respectively) are held together with dowel bars in bond at 60-inch intervals.

In the second group are the longitudinal plane-of-weakness or dummy joints in which the surface of the slab was grooved to a depth of approximately one-third of the slab thickness at the time of construction, it being intended that an irregular fracture would subsequently develop extending from the bottom of the groove downward to the bottom of the slab. One of these sections (section 6) has dowel bars in bond placed



Longitudinal and Transverse Joints in Place Ready for the Concrete to Be Cast. A, Corrugated Metal Plate Used to Form the Longitudinal Joint in Section 10. B, Dowels in Place for a Doweled Transverse Joint and for a Longitudinal Weakened-Plane Joint With Dowel Bars in Bond. The Wooden Header Was Left in Place Until the Concrete Had Hardened and Was Then Removed. C, Rectangular Tongue-and-Groove Longitudinal Joint.

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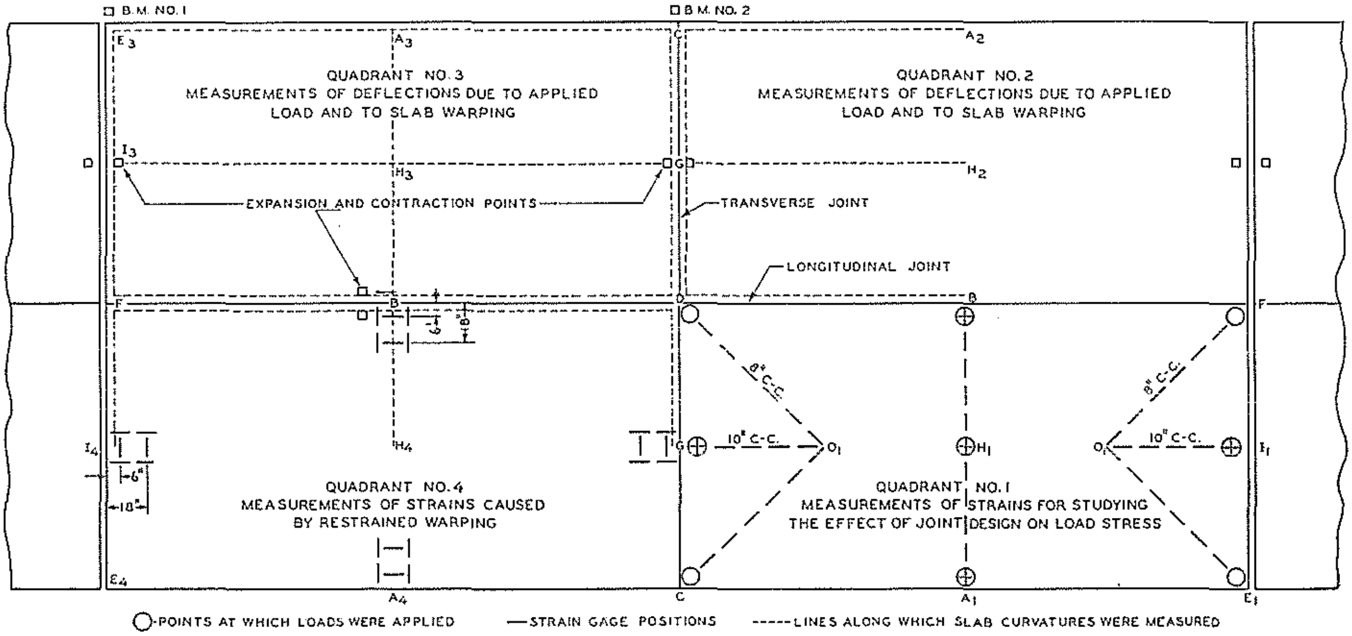

FIGURE 8.—PLAN OF A 20 BY 40-FOOT TEST SECTION SHOWING THE POSITIONS OF THE APPLIED LOADS AND OF THE MEASURING INSTRUMENTS FOR THE STUDY OF JOINT BEHAVIOR. MEASUREMENTS OF DEFLECTIONS IN QUADRANTS 2 AND 3 WERE MADE WITH LOADS APPLIED TO QUADRANT 3 AT LOAD POSITIONS CORRESPONDING TO THOSE INDICATED BY CIRCLES IN QUADRANT 1.

across the joint at 60-inch intervals, while in the other (section 7) no dowels were used.

The third group embraces four sections (sections 1, 2, 8, and 9) in which the vertical faces of the abutting edges of the two 10-foot slabs were separated by a single thickness of tarred felt but held together by dowels in bond. These dowels were deformed bars of steel ¾ inch in diameter and 4 feet in length. They were spaced at intervals of 60, 48, 36, and 24 inches, respectively, in the four sections listed above.

As stated previously, in making the study of the structural behavior of these joint designs, tests were made on each to determine:

1. How freely expansion and contraction occurred.
2. To what degree unrestrained warping of the slab edges was permitted.
3. Their relative effectiveness in reducing the natural weakness of the joint edge by transferring load or by other means.

Measurements of expansion and contraction, of slab shape, and of slab deflection and strain under load were necessary to make these determinations. The location of the points at which the various measurements were made are shown on the plan of one of the test sections in figure 8. In this figure a letter is assigned to a definite point on a test slab while the subscript indicates the quadrant number. For example, the letter "E" is assigned to the free corner, and "H" the center point of the test panel and the subscripts 1, 2, 3, and 4 indicate the four quadrants of the test section. As in earlier descriptions, the various tests are described as having been made on the different quadrants of the test section for the sake of clarity in presentation. Actually certain tests were frequently made on more than one quadrant of a given test section.

SCHEDULE OF DEFLECTION AND STRAIN MEASUREMENTS OUTLINED

The points at which the expansion and contraction of the slab as a whole were measured are shown in the free ends and at the transverse joint in quadrants 2 and 3. The measurements were made with the specially constructed micrometer described in the first report of this series, the normal distance between the gage points being approximately 7 inches. The movements at the transverse test joint were compared with those at a joint which was known to be free to expand and contract and this comparison served as a basis for estimating the relative restraint offered by the various designs to longitudinal slab movements. These measurements covered both the daily and annual cycles of changes in slab length.

The study of the annual variations was made from measurements made twice daily over a period of about 2 years. The measurements were made at about 9 a. m. and about 3.30 p. m.

In the study of the daily variation in slab lengths, measurements were made on selected days at 2-hour intervals for a complete 24-hour period. The days were selected so as to give as wide a temperature range as possible for the particular season of the year.

From the data obtained it is possible to estimate very closely the extent of both of these cycles of change in slab length and joint width.

The relative restraint to free warping developed by the various joints was determined by comparing the magnitude of the deflection at the joint in question with that at a free edge under a given temperature condition and also by comparing the strains resulting from warping restraint at corresponding points at the free edge and at the joint under test. The shape of the deflected slab was determined with the clinometer and the movements of the extreme corners of the slabs were also measured with micrometer dials on fixed supports. The tests were usually started very early in the morning and readings were taken at 1-hour intervals until the maximum warping in each direction had occurred. In making the comparison for a transverse joint, deflections at the free corner (point E) were compared with those at the transverse joint corner (point C). For a longitudinal joint the deflections at the free corner (point E) were compared with those at a longitudinal joint corner (point F).

The lines along which the clinometer points were installed for the warping studies are shown in the third quadrant of figure 8. The restraint to warping offered by a transverse joint was indicated by a comparison of the curvature along the line E_3—A_3 with that along the line C_3—A_3. For the same study of the longitudinal joint action the curvature along the line E_3—I_3 was compared with that along the line F_3—I_3.

In measuring the curvature with the clinometer two sets of readings were made for each comparison, the first in the early morning at a time when the upper surface of the pavement was at a lower temperature than the lower surface and a second set in the early afternoon when the temperature of the upper surface of the slab was above that of the lower surface.

Since any tendency of the joint to restrain the slab edge from warping freely would be reflected by an increase in the magnitude of the warping stresses, a comparison was made in each case between the warping stresses at a free edge and those at the joint in question. The method of arriving at the values of the warping stresses from measured strains was described in part 2 of this series of reports. The location of the strain gages for these comparisons is shown in the fourth quadrant of the test section in figure 8. In the study of the transverse joint the stresses indicated by the group of gages at I_4 were compared with those measured by corresponding gages at G_4. Similarly, for the study of the longitudinal joint the stresses indicated by the group of gages at A_4 were compared with those found at the corresponding positions at point B_4.

As remarked before, every joint is potentially a structural weak spot and some means for strengthening this part of the slab is usually a part of the joint design. The common method is by transferring part of the load to the adjoining slab through the shear resistance of the joint. In this investigation the relative effectiveness of the various joints from the standpoint of their ability to strengthen the slab edge was determined by comparing the critical strains and deflections caused by a load acting near a joint edge with those produced by the same load at other points on the slab.

Loads were applied at the four corner points C, D, E, and F (fig. 8) to determine how effectively the joint functioned in reducing the critical deformations caused by a load acting at the corner of the slab. Similarly, the effectiveness of the design under the action of loads applied at the joint edge (but away from the slab corner) was determined from data obtained with loads applied at points A, B, G, I, and H. The lines along which the curvature of the slab was measured under the action of the applied loads are shown in the second, third, and fourth quadrants, while the strain-gage locations that were used in this part of the study are shown in the first quadrant of this figure. The detailed schedule of the deflection measurements follows:

Load applied at point:	Deflection measured along line:
CORNER TEST	
E_3.	E_3—A_3 and E_3—F_3.
C_3.	A_3—A_2 and C_3—D_3.
F_3.	E_3—I_4.
D_3.	D_3—B_3.
EDGE TEST	
A_3.	A_3—B_3.
B_3.	A_3—H_4.
I_3.	I_3—H_3.
G_2.	H_3—H_2.

For each test the shape of the unloaded slab was determined, the load was applied and the shape of the loaded slab measured, the change in shape being taken as the deflection caused by loading.

CRITERION OF JOINT EFFICIENCY ADOPTED

A somewhat similar schedule was followed in making the strain measurements. For the load applied at the slab corners the strains were measured in the upper surface of the slab along the bisector of the corner angle. For example, with a load applied at point E_1 the strains were measured along the line E_1—O_1 and similarly for the other corners of the slab.

For loads acting at the edge points the strains were measured both parallel and perpendicular to the slab edge at the point of load application and for a sufficient distance along a line perpendicular to the edge to insure the finding of the critical tensile stress in the upper surface of the slab. For example, with a load acting at point A_1 the strains were measured in both directions directly under the load and along the line A_1—H_1.

Loads were applied at point H solely for the purpose of obtaining a comparison of the critical stresses caused by a load at this point with those caused by the same load applied at certain other points. Since the critical stresses occur directly under the load in the case of a load acting in the interior of the slab, only the strains developed in the upper surface directly under the load were measured. These strains at point H were measured in both the longitudinal and transverse directions.

Before making a comparison of the relative effectiveness of various joints for accomplishing any certain purpose, it is necessary to establish some rational basis of comparison. If it is desired to compare joints on the basis of their ability to reduce the deflection of the slab edge at some particular point, then deflection measurements at that point may be used as a means for estimating the effectiveness of the joint. However, if the purpose of the joint design is to reduce the stresses from applied loads so as to, in effect, increase the load-carrying capacity of the edge of the slab, then it is necessary to arrive at the basis of comparison through the measurements of strains and not deflections, for it has been clearly demonstrated in these tests that the precision of the deflection data is not sufficient to warrant any conclusions relative to attendant stress conditions. The question then arises as to how stresses determined from strain measurements may best be used as a basis for judging the relative structural effectiveness of various joint designs.

It has been established that if a given load is applied at various points on the surface of a concrete pavement slab of uniform thickness the critical stress will be a minimum when the load is applied at an interior point and that the critical stress will reach its maximum value when the load is applied at the free edge.

If a joint operated with a maximum amount of structural efficiency, it would reduce the critical load stress at the joint edge to a value equal to that found in the interior of the slab. If, on the other hand, it was completely ineffective the critical stress for a load at the joint would equal the critical stress for the same load acting at a free edge.

These two values therefore, delimit the range of joint efficiency so far as the ability to reduce load stresses is concerned and suggest a stress formula which will furnish a rational measure of joint efficiency.

If, for a given applied load on a slab of uniform thickness,

σ_j is the critical stress for the load applied at the joint edge,

σ_f is the critical stress for the load applied at the free edge,

and σ_i is the critical stress for the load applied at an interior point.

Then the joint efficiency, E, may be expressed as follows:

$$E=\frac{\sigma_f-\sigma_j}{\sigma_f-\sigma_i}$$

In other words, the reduction in edge stress accomplished by the joint under consideration is compared to that accomplished by the complete continuity of the interior condition, as a measure of efficiency.

In making the stress determinations upon which the joint efficiency values were based, it was not considered desirable to depend entirely upon stress values obtained at a single point no matter how well established the value might be.

For the determination at each load point, therefore, eight tests were made, each at a somewhat different location. For example, to establish a stress value for the free edge (point A) eight tests were made altogether, and in no two was the bearing p ate in the same spot on the same quadrant of the test section, although in all cases it was at the free edge and close to the midpoint of the slab length.

Tests were made also at all of the longitudinal joint corners on the four constant-thickness slabs but were not made at these corners on the thickened-edge slabs because there would be no basis for comparing strains measured at corners of different thicknesses.

Deflection and strain data that indicate the strengthening effect of thickened edges at slab corners appear later in this report.

DATA ON VARIATIONS IN JOINT WIDTH PRESENTED

The annual variation in the widths of the various transverse joint openings is indicated by the data shown in figure 9 in which the ordinates are the variations in the measured joint width when compared to a set of initial measurements made in November 1930, shortly after the pavement was constructed. The morning measurements were made between 9 and 9:30 and the afternoon measurements between 3:30 and 4 o'clock.

The joint designations used in this figure are as follows: The transverse joint in the center of the test section is given the same number as the test section in which it is located. For example, joint 3–3 refers to the transverse joint across the center of test section 3. The open joint between two adjoining test sections is given the numbers of the two sections between which it is found, as for example, joint 2–3 is the open joint between test sections 2 and 3. It will be recalled that these open joints between the test sections were all 2 inches wide and were kept open, preventing any connection between the slab ends.

It was mentioned earlier that the expansion and contraction measurements were of necessity made at the level of the upper surface of the pavement. The observed horizontal movements were therefore the results of the direct expansion and contraction of the slab combined algebraically with the horizontal component (at the plane of the measurements) of any tilting of the slab ends caused by warping. To determine the changes in joint width caused solely by expansion or contraction of the slab, it is necessary to correct the observed changes for the effect of the warping present at the time of observation.

Figure 9 shows the seasonal variations in observed joint width. The correction for warping mentioned in the preceding paragraph is considerably more important in a consideration of the daily variations than it is when seasonal changes are involved for the following reasons:

The afternoon observations were made at the time of maximum downward warping of the slab edges. During the summer when the pavement was expanded to its greatest length the slab edges were warped upward by moisture conditions within the slab to the maximum degree. Data were presented in the second report of this series to show the relative degrees of the daily and seasonal warping found in the test sections at different seasons of the year. The data presented in that report on the effect of seasonal moisture changes indicated that the moisture condition was rather unstable during the summer months. This resulted from abnormal weather conditions during the summer of the year in which the observations were made. Moisture warping observations that have been made since that report was written and the data showing the change in length of the pavement caused by moisture changes both indicated that, under normal summer weather conditions, the moisture state of the pavement is quite stable during the summer months.

It is indicated by the data just mentioned that the downward warping of the slab edges on critical days during the summer at the time of the afternoon observations is approximately equal to the upward warping caused by the seasonal change in its moisture condition. This conclusion is based upon the assumption that all of the seasonal moisture warping is upward; in other words, that there is at no time more moisture in the pores of the concrete in the upper part of the pavement slab than is present in those near the subgrade. This assumption seems to be supported by all of the data available from these tests.

It appears therefore that, at the time the measurements indicating the maximum closing of the joints were made, the slabs were probably in nearly a flat condition. For the morning observations in winter all evidence indicates little warping from either moisture or temperature. If these assumptions are correct, data such as are shown in figure 9 indicate the seasonal variations in the widths of the joints with sufficient accuracy without corrections for the effects of warping. The indicated daily variations in joint width are not true measures of expansion and contraction and should not be used without correction. The method used for determining the magnitude of the warping correction is described a little farther on in this report in connection with the discussion of daily variations in joint width.

SLABS DID NOT EXPAND AND CONTRACT SYMMETRICALLY

If a comparison is made between the annual variations in width of the several joints, it will be observed that the movements at the joints formed by the free ends of the slabs are, in general, greater than those at the regular transverse joints. The observed difference between the opening of the transverse test joints and that of the open joints between the test sections indicates that the resistance of the former to expansion and contraction movements causes either a deformation that alters the length of the slab or a shifting of the center of the slab panel longitudinally over the subgrade.

That there is no appreciable stress deformation that changes the slab length is shown by the numerous

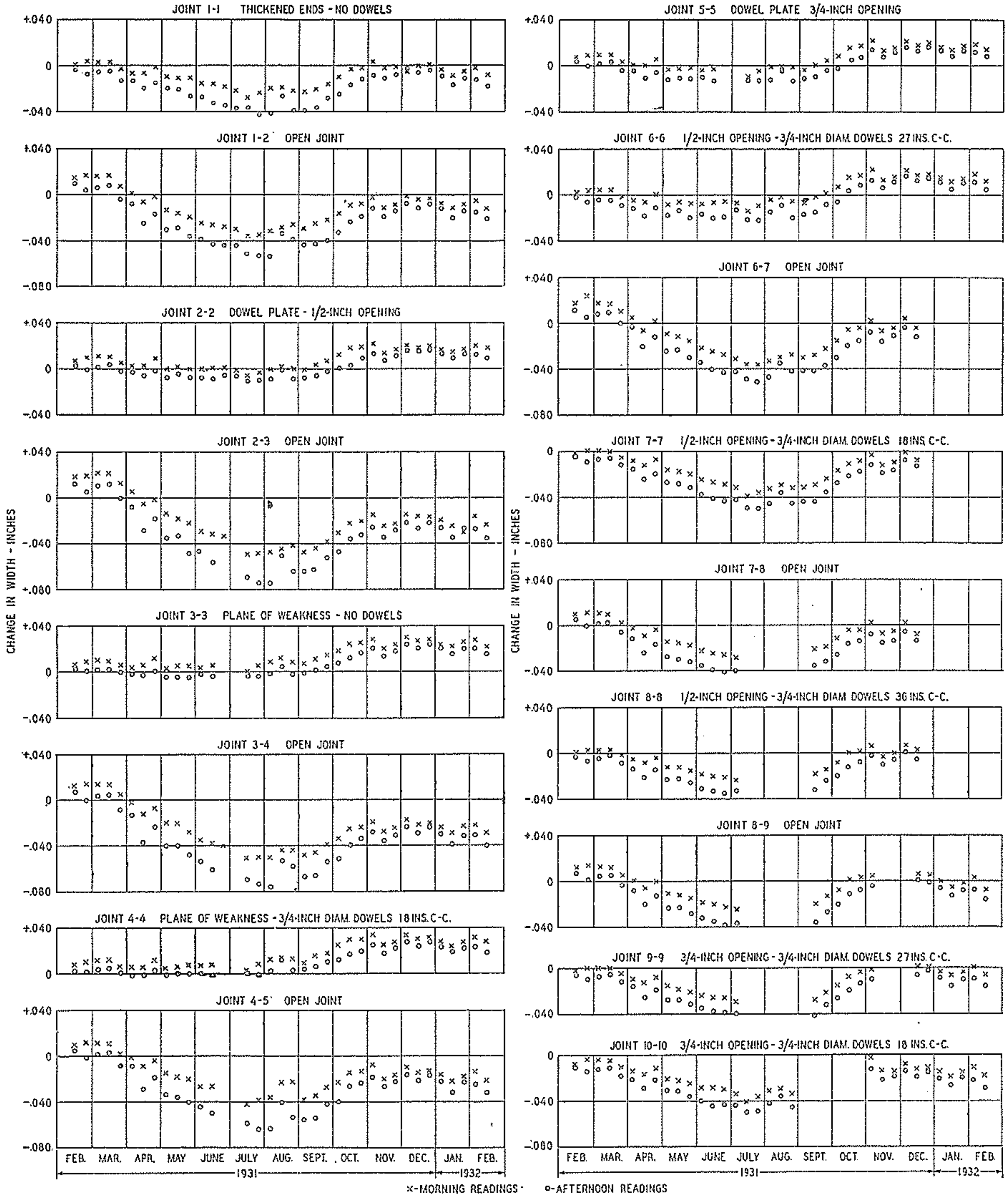

FIGURE 9.—SEASONAL VARIATIONS IN WIDTH OF EACH OF THE TRANSVERSE JOINTS OVER A TYPICAL 1-YEAR PERIOD. JOINT OPENING SHOWN AS POSITIVE, AND JOINT CLOSING SHOWN AS NEGATIVE. EACH VALUE IS A 10-DAY AVERAGE.

measurements of the variation in slab length with temperature changes that were presented in the second paper of this series. It will be remembered that these indicated that the deformation or change in slab length caused by the subgrade resistance during expansion or contraction is negligible in slabs of this length. It must be concluded, therefore, that the 10- by 20-foot slabs do not expand and contract symmetrically with respect to the subgrade at their midpoints. This eccentricity of movement is evident in all of the sections, although it is more noticeable in some than in others. It will be discussed in more detail a little farther on in this report.

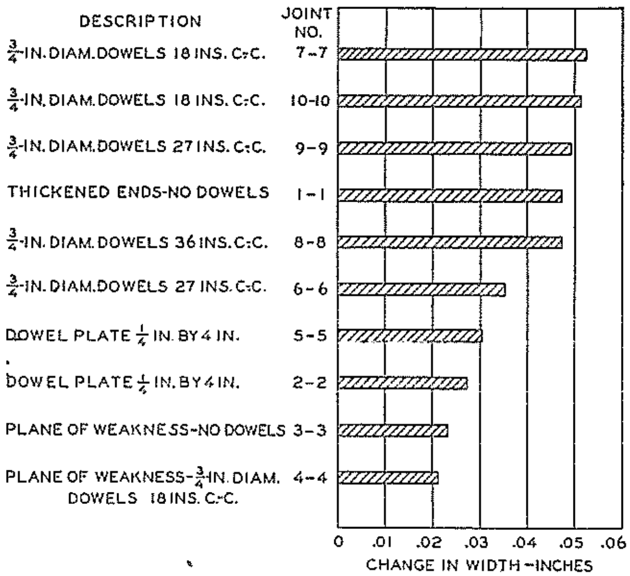

FIGURE 10.—MAXIMUM SEASONAL CHANGES IN WIDTH OF THE TRANSVERSE JOINTS. MAXIMUM RANGE FEBRUARY 16 TO JUNE 22, 1931. THE RANGES SHOWN ARE FROM THE AVERAGE MINIMUM AS SHOWN BY THE MORNING MEASUREMENTS ON COLD DAYS IN FEBRUARY TO THE AVERAGE MAXIMUM AS SHOWN BY THE AFTERNOON MEASUREMENTS ON HOT DAYS IN JUNE.

It will be observed further that several of the transverse joints opened more during the winter of 1932 than during the winter of 1931 and that consistently the adjacent open joints opened less during this same period and each by approximately the same amount. This condition is most noticeable in the two transverse planes of weakness (joint 3–3 and joint 4–4) and for the two joints containing the dowel plates (joints 2–2 and 5–5), and also, for some reason that is not apparent, in the dowelled joint 6–6. During 1931 these joints closed very little if any after March.

Figure 10 was constructed, from the same basic data that were presented in the preceding figure, for the purpose of showing the relative freedom of the different transverse joints to expand and contract. Selecting arbitrarily the period between February 16 and June 22 as giving a wide range in temperature, the average maximum range of movement for each of the transverse joints during this period was determined. In the preparation of figure 10, the daily values rather than 10-day averages such as are shown in figure 9 were used to obtain the average maximum and average minimum values. The indicated seasonal movements are therefore greater in figure 10 than in figure 9. The data for the joints are arranged in this figure in the order of descending values of the observed maximum seasonal movement. Since the sections are all of the same length and each is completely separated from its neighbors, the amount of movement which occurs at each test joint during a given period of time may be assumed to be a measure of the relative freedom of action so far as expansion and contraction are concerned.

Joint 1–1 was constructed as a clear opening one-half inch wide between two thickened-end slabs. It was filled with a bituminous joint filler shortly after construction. So far as the joint filler is concerned, it should offer relatively little resistance to expansion and contraction movements. The opinion has been expressed that this type of slab end exercises a restraining action that prevents the slab from contracting

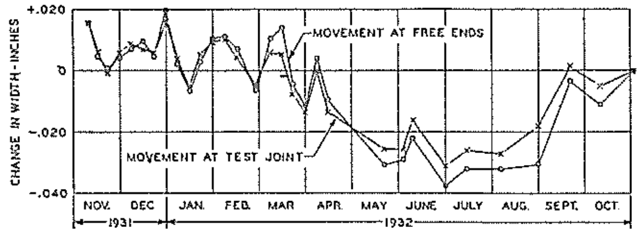

FIGURE 11.—COMPARATIVE MOVEMENTS AT THE TRANSVERSE JOINT AND AT THE FREE ENDS OF ONE OF THE TEST SECTIONS. POSITIVE VALUE INDICATES OPENING OF THE JOINT AND NEGATIVE VALUE INDICATES CLOSING OF THE JOINT.

freely and causes a corresponding stress in the concrete, due to the inclined surface of the subgrade over which it presumably has to move.

In the second report of this series it was shown that, for the subgrade material and slab lengths concerned in these tests, the earth of the subgrade adhered to the bottom of the slab, "bending" or moving forward with the slab. Under such conditions the incline of the lower surface of the slab would not increase the resistance over that of a flat slab. It is indicated by the data in figure 10 that the thickened-end slab joint 1–1 permits the slabs to expand and contract as freely as any of the other transverse joints tested in this investigation.

Joints 7–7, 10–10, 9–9, 8–8, and 6–6 contain the unbonded, round, steel dowel bars. The movements at all of these are of approximately the same magnitude and about the same as that at joint 1–1, with the one exception of the seasonal movement of joint 6–6. There is no apparent reason why the seasonal movement of this one joint should be appreciably different from those of the other joints of the same type. The data indicate a high degree of relative freedom for the dowel joints with little or no variation in the restraint with the number of dowels per joint.

Other data, which supplement those shown in figure 9, were obtained in the measurements on section 10. These data are given in figure 11 in which the amount of movement at the test joint 10–10 and at the free ends of the section are shown at frequent intervals over a period of about 1 year. This section was separated from the one adjoining by an open space of considerable width. The expansion and contraction measurements were therefore made to fixed reference points at each end of the section. Since both ends were completely free the difference between the movements at the free ends when compared with the corresponding movements at the transverse test joint, as shown in figure 11, give a good idea of the degree of restraint developed in joint 10–10. In figure 10 this joint is among the group compared, hence a basis is furnished for estimating the order of restraint to expansion and contraction offered by each of the joints.

DOWEL-PLATE JOINTS OFFERED MORE RESISTANCE TO SLAB MOVEMENT THAN DID UNBONDED DOWEL BARS

For example, if the maximum movement found during the year shown in figure 11, i. e., November 1931 to November 1932, for joint 10–10 is expressed as a percentage of the movement measured at the completely free slab ends, it will be found that the movement at the joint was, in round numbers, 80 percent of that at the free ends of the test section. If this percentage is applied to the movement shown in figure 10 for joint 10–10, the analysis indicates that a movement for a completely unrestrained joint would be of the order of 0.065 inch. With this value as a basis, the restraining action of each of the 10 joint designs can be estimated.

It is indicated that joint 1–1, constructed as an open joint and filled with a poured joint filler, is not completely free and it seems likely that during the expansion of the concrete the compression of the filler material in the joint required an amount of force approximating that required to overcome both the resistance of the filler and the resistance of the dowels in each of joints 7–7, 10–10, 9–9, and 8–8. If this is so, then the force required to cause the dowels to slide in these joints must be very small, because the resistance of the joint filler to compression must be about the same in each of the joints in this group.

Joints 5–5 and 2–2 contained the one-fourth by 4-inch steel dowel plates. The measurements show that the seasonal movement of these two joints was approximately three-fifths of that of the dowelled-joint group (7–7, 10–10, 9–9, 8–8, and 6–6). It is indicated, therefore, that the plate-dowel joints offer a greater resistance to expansion and contraction than do joints containing properly installed round steel dowel bars which are not in bond with the concrete.

Joints 3–3 and 4–4 are the two transverse plane-of-weakness or dummy-type joints; the latter contain three-fourths-inch diameter dowel bars spaced 18 inches between centers. The seasonal movements of these joints are the smallest for any of the transverse joints. That this is caused by the complete closing of these joints in the early summer is clearly shown by the data already presented in figure 9. In examining this figure, attention is called particularly to the large movements of joint 3–4 lying between the two dummy joints. The closing of the dummy joints throws any subsequent expansion into slab displacement or slab deformation under stress. In such slabs the short slab length and the adjacent open joints provide the necessary relief from compression. The plane of weakness joints contract freely and relieve tension. Also, joints of this type undoubtedly reduce greatly the warping stresses. They do not appear to relieve slab expansion to any great extent, however.

Figure 12 shows data obtained during a cycle of measurements of width made at each of the transverse joints at approximately 2-hour intervals over a period of 24 hours. As noted in the figure, these particular observations were made during the latter part of June 1931, the time of year when large temperature differentials are developed in the test slabs.

On the same day that the daily variations in transverse joint width shown in figure 12 were obtained, the warped shapes of the slabs were measured at intervals of approximately 2 hours with the clinometer. From these clinometer data, curves similar in character to those shown in figures 25–28 and 31 of the second paper of this series were obtained. These curves were plotted to suitable scales and the slope of the upper surface at the extreme end of the pavement slab was estimated graphically. From this the change in slope of the vertical end face of the slab was determined for the rather extreme conditions of temperature warping which obtained on the particular day that the measurements were made. From this change in slope the effect upon the measurement of joint width was calculated and applied as a correction to the expansion

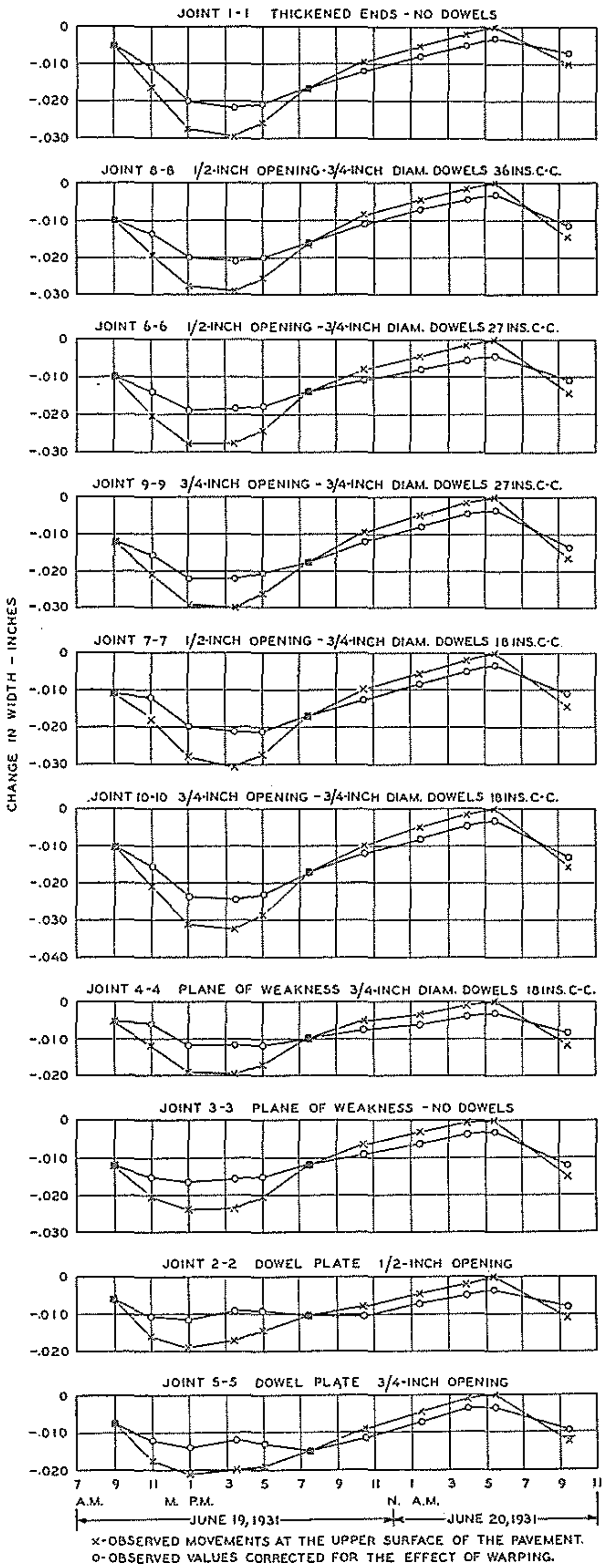

Figure 12.—Measured Changes in Width of the Various Transverse Joints Caused by Daily Changes in Temperature of the Pavement. Negative Values Indicate Closing of the Joint.

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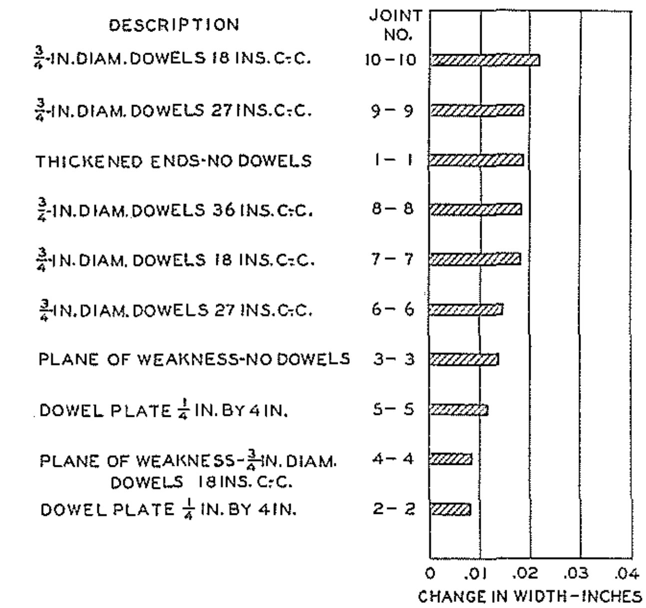


FIGURE 13.—MAXIMUM DAILY CHANGES IN WIDTH OF THE TRANSVERSE JOINTS.

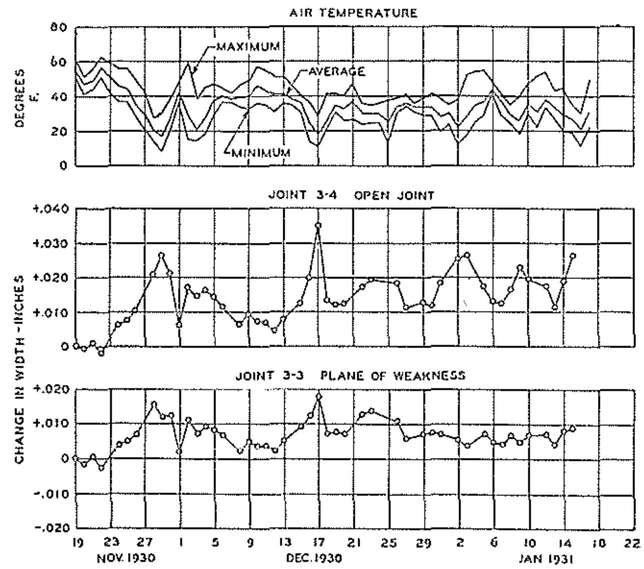


FIGURE 14.—CHANGES IN THE WIDTH OF ADJACENT TRANSVERSE AND OPEN JOINTS DURING A PERIOD WHEN ECCENTRIC MOVEMENTS WERE NOTED. OBSERVATIONS WERE MADE AT 2 P. M. POSITIVF VALUE INDICATES OPENING OF THE JOINT AND NEGATIVE VALUE INDICATES CLOSING OF THE JOINT.

and contraction measurements obtained at the transverse joints on the same day. Both the uncorrected and the corrected values are shown in figure 12.

It will be noted that the apparent change in joint width caused by the rather extreme temperature warping amounts to a closing of 0.007 to 0.009 inch during the day and an opening of 0.003 to 0.004 inch during the night. From the corrected curves of daily variations in joint width, figure 13 was constructed to show the relative extent of the true expansion and contraction that occurred at each transverse test joint for this particular June day.

In general the indications of figure 13 as to relative joint freedom are the same as those shown in figure 10 for the slower seasonal movements. The restraint to expansion and contraction offered by the dowel-plate joints is apparent in figure 13 and it appears that this restraint is greatest at the time when the slab edges are warped downward to the greatest degree. It is possible that slab warping causes an increase in the friction between the plate and its sockets and that the irregularity in these curves is caused by the variation in the frictional resistance as the extent of the slab warping varies.

ECCENTRICITY OF SLAB MOVEMENT DURING EXPANSION AND CONTRACTION STUDIED

In connection with the discussion of figure 9, mention was made of a tendency for the measured movement at the transverse test joints to differ in magnitude from that observed at the open joints at certain times and under certain conditions of temperature. A study of this relation has been made throughout the period of the investigation and much has been learned of its nature although the causes for the observed behavior are not entirely clear.

Figure 14 shows the movements at joint 3–3, a transverse plane of weakness without dowels. Figure 14 also shows the corresponding movements at the open joint 3–4 and maximum, minimum, and average air temperatures for a period of about two months during the winter of 1930–31.

It will be noted in this figure that frequently the movement at the test joint is of about the same magnitude as that at the free end, and yet whenever a marked temperature change occurs there is a tendency for the movement at the free end greatly to exceed that at the test joint. This effect is particularly noticeable during a sudden drop in temperature such as that which occurred between December 15 and 17, 1930. Whenever the observed movement at the free end exceeds that at the transverse joint, the slab is not expanding or contracting symmetrically with respect to the midpoint between the two, the slab panel being shifted as a whole toward the end at which the smallest change of position was observed.

This phenomenon was noticed on all of the test sections, being greater on some than on others, and being noticeably greater during the winter than in the summer. Also, it was greater during the first winter following the laying of the pavement than during the second winter, as can be seen by an examination of the data in figure 9.

To bring out the variation in the degree of eccentricity during the year, figure 15 was prepared from observations on section 7. The transverse joint in this section contained round dowels at 18-inch intervals. The movements of the transverse joint, expressed as percentages of that at the free end of the slab are shown as ordinates to the curve. It is interesting to note how this variation follows in a general way the annual variation in average daily air temperature, definitely indicating that the phenomenon is caused primarily by temperature. During the summer months the movements at the two points of measurement are nearly the same, but during cold weather the movement at the transverse joint is from 5 to 15 percent less than that at the open joint at the free end of the slab. As already stated, the difference was found to be greatest for any particular season, at times of sudden change in temperature.

The observations were continued over a period of nearly 5 years in order to determine whether the movements tended either to open or to close the transverse joints or whether they were compensating in their effects.

The net change in width of each of the transverse joints and of each of the open joints on which measurements were made is shown, for the period between November 1930 and August 1935, in table 2.

TABLE 2.—*Changes in transverse joint width* [1] *(November 1930 to August 1935)*

Test joints		Open joints	
Joint no.	Net change in width	Joint no.	Net change in width
	Inches		*Inches*
1-1	+0.010	1-2	−0.074
2-2	+.006	2-3	−.089
3-3	+.003	3-4	−.108
4-4	+.110	4-5	−.090
5-5	+.072	5-6	----------
6-6	+.039	6-7	−.026
7-7	+.003	7-8	−.034
8-8	+.018	8-9	−.023
9-9	+.005	9-10	----------
10-10	−.024		

[1] The values shown are the net changes in joint width after the observed widths had been corrected for the estimated effects of pavement temperature difference and moisture difference, averaged for the two halves of the pavement (on either side of the longitudinal center joint) and expressed in inches per 20 feet of slab length. A positive sign indicates an opening and a negative sign a closing of the joint.

The data indicate that while all of the transverse joints, except that in section 10, opened to some degree, the two weakened-plane joints (3-3 and 4-4) and the two containing the dowel plates (2-2 and 5-5) apparently opened by about one-tenth inch during the period covered by the observations. In all of the other sections the opening has been very much less and in section 10 a slight closing has apparently occurred.

A comparison of the movement at the transverse joints with that at the open joints will show that the change in width has been accompanied by a corresponding change in the open joints. This indicates that some resistance to expansion and contraction existing at the transverse joint caused a permanent displacement of the pavement slab away from the transverse joint and toward the open joint. The magnitude of the displacement is probably a rough measure of the relative freedom of the various transverse joints to expand and contract.

In figure 20 of the second paper of this series data were presented to show the approximate magnitude of the force required to move a slab a given distance on this particular subgrade. An estimate of the restraining force developed by the different transverse joints may be made by finding the distance through which any particular joint causes the abutting slab to be displaced over the subgrade and then calculating the force required to cause this movement.

LONGITUDINAL JOINTS TENDED TO OPEN IN WARM WEATHER

The restraining forces were calculated in this manner for the four sections in figure 10 that show the least seasonal movement at the transverse joint (secs. 2, 5, 3, and 4), and the estimated unit stress developed by joint restraint in each is shown in table 3 with the data upon which the calculations were based. It is indicated that the dowel-plate joints caused restraint which may develop either tensile or compressive stresses of approximately 30 pounds per square inch, and the plane-of-weakness or dummy joint may cause compressive stresses of approximately the same magnitude. It is probable that if the slabs had been forced to expand and contract against the full resistance of their joints, instead of being comparatively free to shift their position as they were in these tests, a much more serious stress would have resulted.

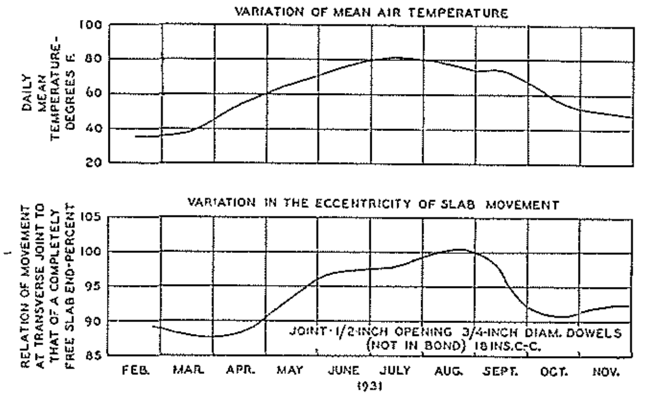


FIGURE 15.—VARIATIONS IN THE ECCENTRICITY OF SLAB MOVEMENT COMPARED WITH THE VARIATIONS OF MEAN AIR TEMPERATURE. THE CURVES SHOW THE GENERAL TREND OF THE TEMPERATURE AND OF THE ECCENTRIC MOVEMENTS OF THE JOINTS BUT DO NOT INDICATE CHANGES THAT TAKE PLACE OVER SHORT PERIODS OF TIME.

The tendency for the joints to increase in width with time is of particular interest in connection with the weakened-plane joints when no dowels or other positive means for load transfer has been provided. The increase, though small, is a large percentage of the width of the original crack and it makes the load-transfer action of such joints problematical. It also creates a joint opening that is difficult to seal against moisture and solid matter.

TABLE 3.—*Estimated stresses resulting from joint restraint during expansion and contraction* [1]

Test section no.	Type of slab cross section	Type of transverse joint	Weight of slab, 10-foot width	Displacement of slab	Total computed thrusting force	Area of cross section	Estimated unit stress [2]
			Pounds	*Inches*	*Pounds*	*Sq. in.*	*Lbs. per sq. in.*
2	9-7-9	Dowel plate	18,250	0.025	25,550	875	±29
5	9-6-9	do	16,125	.022	21,350	774	±28
3	9-6.3-9 (A. A. S. H. O.).	Plane of weakness (no dowels).	17,775	.029	27,100	853	−32
4	9-6.3-9 (parabolic).	Plane of weakness (doweled).	18,000	.031	28,800	804	−33

[1] The period covered is the same as that shown in fig. 10.
[2] A positive sign indicates tensile stress and a negative sign indicates compressive stress.

The daily measurements of width of the longitudinal center joints of 6 of the 10 test sections over a period of about 1 year are shown in figure 16. Joints C-6 and C-7 are not shown, as they were constructed as planes of weakness and had not cracked through at the time of the measurements. Joints C-3 and C-10 had not been equipped with measuring points at the time of the measurements.

The measurements were made at the same time of day as those at the transverse joints, so that this graph shows the same relation as was brought out in figure 9, i. e., the maximum seasonal movements. In studying this graph it is well to bear in mind that the effective slab length in this direction is 10 feet and that all joints except C-4 and C-7 were crossed by bonded steel bars 4 feet in length. As noted above, joint C-7 had not cracked through at the time of the measurements but joint C-4 affords a good example of a free joint for purposes of comparison. It will be observed that the movement of this joint is approximately twice as great as that measured at the bonded joints.

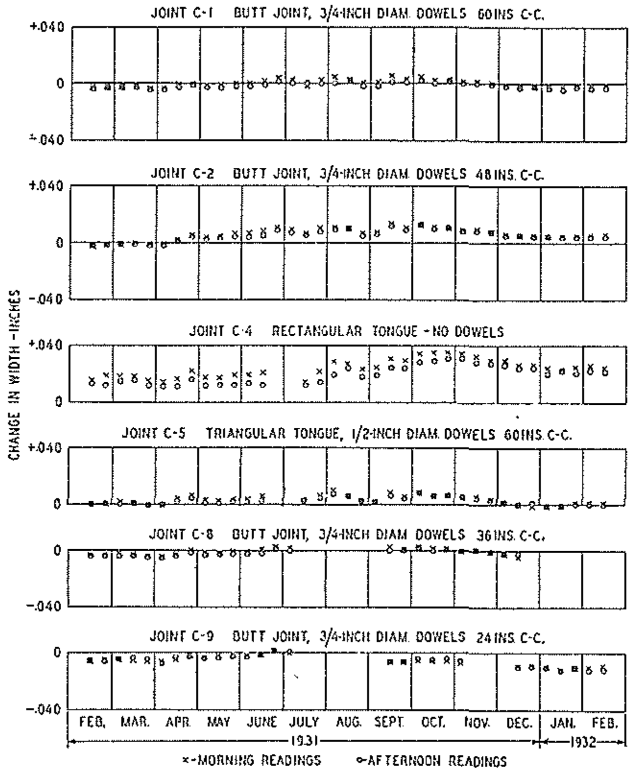


Figure 16.—Seasonal Variations in Width of Each of the Longitudinal Joints Over a Typical 1-Year Period. Joint Opening Shown as Positive and Joint Closing Shown as Negative. Each Value is a 10-Day Average.

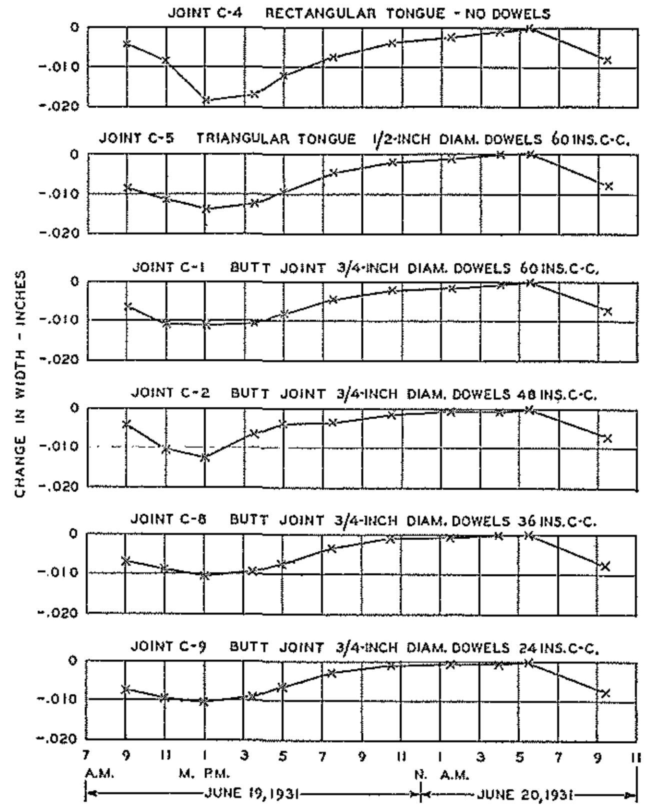


Figure 17.—Apparent Changes in Width of the Various Longitudical Joints Caused by Daily Changes In Temperature of the Pavement. A Negative Value Indicates Closing of the Joints.

These data indicated that in all cases the longitudinal joints tend to open as warm weather comes on, although a slight closing occurs each day as the slab expands with temperature. The possibility that this rather curious behavior was the result of temperature and moisture warping was investigated in the same manner as that described in the discussion of transverse joint movements. It was found that the change in slope of the slab edge at the longitudinal joint under extreme conditions of daily temperature warping might be as great as 10/10,000 for maximum downward warping (afternoon condition) and 2/10,000 for maximum upward warping (night condition).

The effect of moisture change was also investigated and it was found that from extreme upward warping from this cause (summer condition) to extreme downward warping (winter condition) a change in slope of the slab edge of about 12/10,000 occurred. The normal or unwarped condition of the slab, representing the condition where no moisture gradient was present, was not determined so that it is not possible to break down this total change in slope into its two components of upward and downward warping. The evidence indicates, however, that the slab is practically unwarped from this cause during the winter.

CERTAIN TYPES OF TRANSVERSE JOINT OFFERED LITTLE OR NO RESTRAINT TO SLAB WARPING

Assuming that as warping occurs, joints containing dowel bars in bond either widen or close at the top of the slab and that at the plane of the bars the joint width remains constant, it is found that the total temperature warping would cause an apparent daily change in width of the longitudinal joint of a 7-inch slab (with the bonded dowel bars at mid-depth) of 0.008 inch. Since the total change in slope for extreme daily temperature warping equals that for total warping caused by annual moisture changes, it is probable that moisture change will produce an annual change in width of approximately the same magnitude.

The effect of the daily cycle of temperature changes is an apparent closing of the joint during the day and a corresponding opening during the night. The seasonal cycle of moisture variations should cause an apparent opening of the joints during the summer months and a closing during the winter.

This is in agreement with the observed behavior, as will be seen by referring to figures 16 and 17, which show the uncorrected measurements of longitudinal joint width during a 24-hour cycle of temperature changes. It is found that the magnitude of the observed variations agrees closely with what might be expected from the warping that was known to occur.

While the method of computing the effect of warping on joint width is necessarily an approximate one, it is believed that these computations show quite definitely that a large part of the apparent variation of longitudinal joint width shown in figures 16 and 17 is caused by slab warping and that, in the case of the joints containing bonded steel, there was in reality little or no opening and closing caused by expansion and contraction.

Joint C–4 is a rectangular tongue-and-groove joint without bonded steel. There is a tendency for this joint to increase in width with time. It is not known how long this progressive opening will continue but the fact that it has opened is of particular interest as indicating the probable movement of the longitudinal

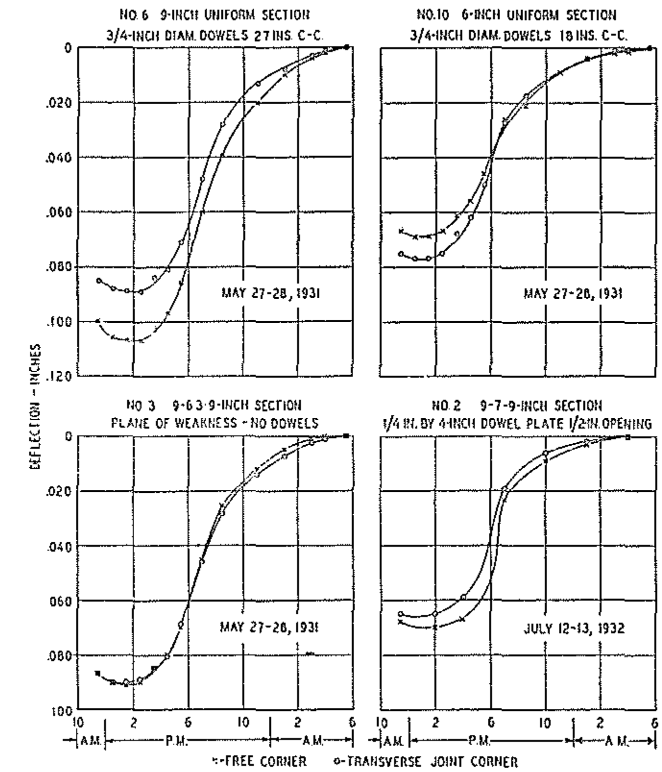


Figure 18.—Comparison of the Cycles of Temperature Warping Movements at the Transverse and Free Joint Corners of Typical Slabs.

plane-of-weakness joint (C–7) had this joint been cracked through at the time.

A study was made of the relative vertical deflections at the free and the joint edges of the various slabs to determine the magnitude of the restraint offered by the various transverse and longitudinal joints. The deflected or warped shapes of the slab axes were measured with a clinometer and the cycles of vertical displacement of slab corners were measured with micrometer dials. Typical deflection data obtained from these measurements are shown in figures 18 to 21, inclusive. Since there were no thermocouple installations in the majority of the slabs, it was not possible to determine exactly the time at which the different slabs were of constant temperature throughout their depth and hence were in the unwarped condition. Therefore, for these particular comparisons, the total change that occurred during a full daily cycle is used in each case.

The extent of the vertical displacements caused by temperature warping at both the free and the transverse joint corners of several of the test sections are shown in figure 18. Since the deflection resulting from warping is greater at the corner than at any other point along the edge, it seems reasonable that any restraining effect of the joint would be most apparent in deflection data obtained at the corners. For this reason any indications of restraint in this figure probably represent approximately maximum conditions.

Data for two doweled joints, a dummy joint, and a dowel plate are included in this figure. Data for the doweled joint with the dowels at 27-inch intervals indicate some resistance to warping, while those for the other doweled joints in which dowels are installed at 18-inch intervals actually show a greater deflection at the joint corner than at the free corner. This is rather typical of much of the data obtained during these particular measurements. At times the free corner would show a slightly greater range of deflections than the joint corner and again, at some other time, the reverse would be true. The differences were generally so small as to be of no great importance in interpreting the data. The data for the transverse weakened-plane or dummy joint and for the dowel-plate joint indicate quite definitely that little or no restraint to warping is offered by either design.

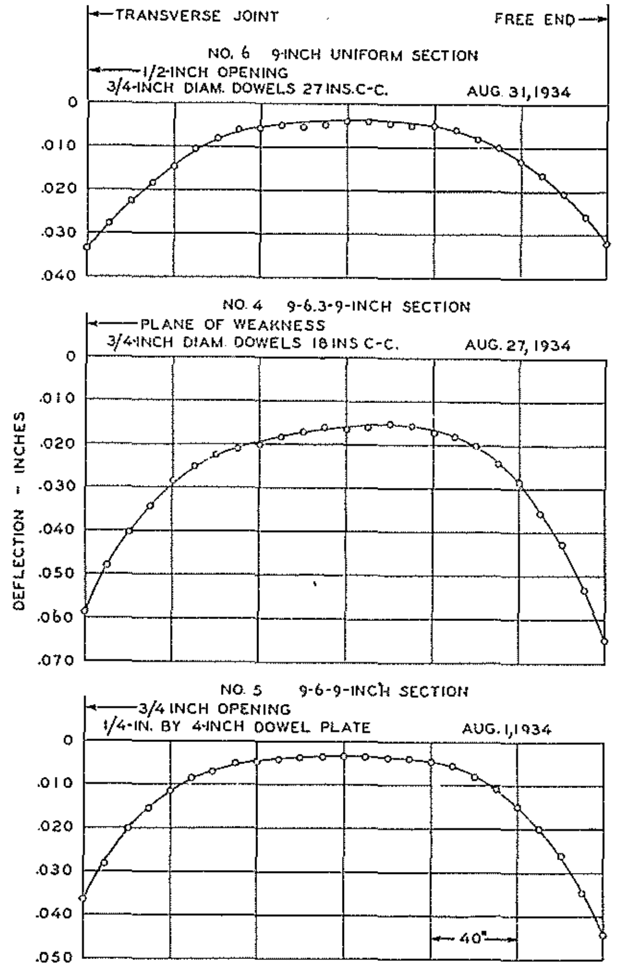


Figure 19.—Temperature Warping Deflections Along Free Edge of Typical Sections.

In figure 19 the effects of temperature warping along the free edge of several of the sections are shown for days on which relatively large temperature variations occurred. The zero or base values for these curves were observations made at approximately 6 a. m. in each case, while the other set was made in the early afternoon at the time of maximum downward warping of the slab edges. An indication of the degree of restraint to warping offered by several transverse joints is obtained by comparing the warping at the transverse joint with that at the free end of the slab.

The joint types covered by figure 19 are the same as in the preceding figure, except that the dummy joint shown in figure 19 contains dowels and the dowel-plate joint has a different joint opening. The indications of this graph are in general agreement with those of figure 18 and, so far as these deflection data go, one would conclude that none of the designs of transverse joints included in these observations show any indi-

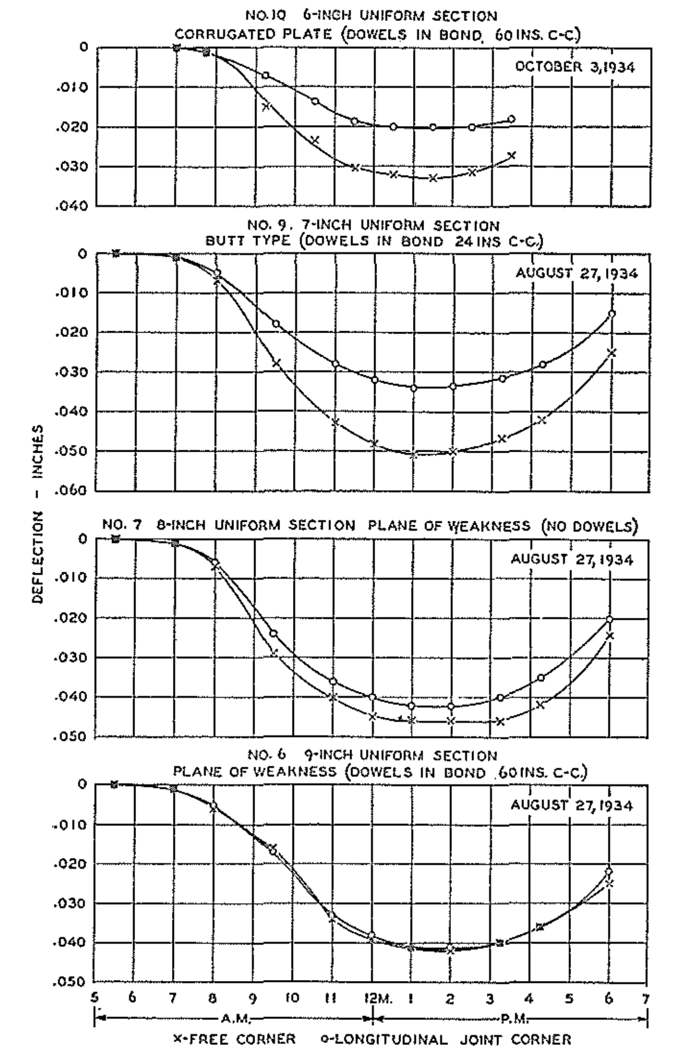


Figure 20.—Comparison of the Cycles of Temperature Warping Movements at the Longitudinal and Free Joint Corners of the Slabs of Uniform Thickness.

cation of offering serious resistance to warping. Final judgment as to the efficiency of the various joints in permitting warping should be reserved until certain stress data to be presented later are examined.

CERTAIN TYPES OF LONGITUDINAL JOINT OFFERED NOTICEABLE RESTRAINT TO SLAB WARPING

In the case of the longitudinal joints the study of the effect of the joint design on the restraint to warping, based upon the deflection data, was restricted to the four sections having a constant slab thickness; in all of the others the thickening of the free edges prevented a direct comparison of deflection data obtained at the joint with those obtained at the free edge of the slab. Fortunately, the more important types of longitudinal joint are represented in these four sections, as shown in the following summary:

Types of longitudinal joint:	*Slab thickness Inches*
Corrugated plate with bars at 60-inch intervals	6
Butt joint (tarred felt) with bars at 24-inch intervals	7
Plane of weakness. No bars	8
Plane of weakness with bars at 24-inch intervals	9

Figure 20 shows the relative magnitudes of the vertical displacements measured at the free and longitudinal joint corners of each of these four sections

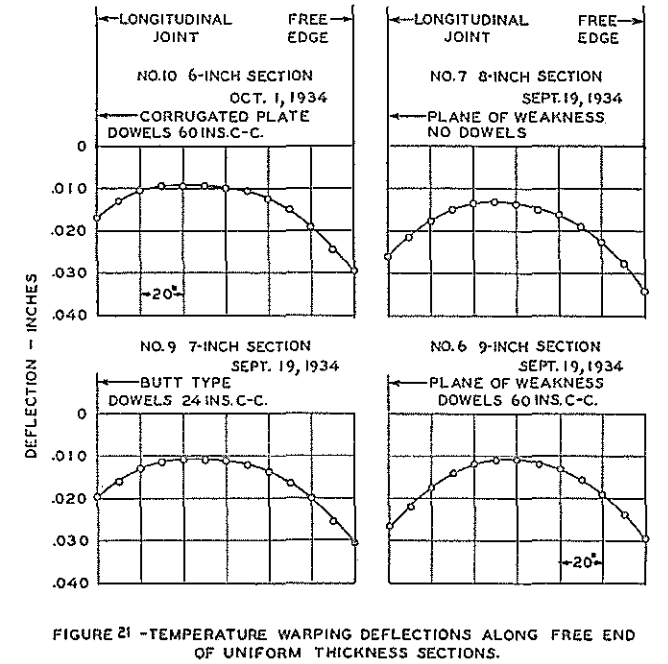


Figure 21.—Temperature Warping Deflections Along Free End of Uniform-Thickness Sections.

over a full daily cycle. The method of measurement was the same as that described in connection with the discussion of transverse joint action.

These curves indicate that the weakened-plane type of joint, which provides a greatly reduced section at the joint, caused very little restraint to temperature warping even when crossed by bonded steel. The butt-type joint and the type that contains a deformed, metal separating plate both appear to offer noticeable resistance to warping action.

Restraint to warping in joints that are crossed by bonded steel results from a resisting moment which develops between the tension of the bonded steel bars and the compression in the abutting surfaces of concrete in the two slab edges as the joint closes when the slab begins to warp. The length of the moment arm depends upon the distance to which the effective section extends either above or below the plane containing the steel. With the position of this plane fixed and at some point typical of general practice as, for example, halfway between the two surfaces of the slab, a relatively deep groove in the upper surface such as is present in weakened-plane joints greatly reduces the moment arm for the condition of downward warping and effects a corresponding reduction in the restraint. Restraint to upward warping would not be relieved by such a groove, however. Joints that contain little or no reduction in section provide a greater moment arm and, with other conditions the same, will develop more restraint to warping.

Curves showing the change in slope and the extent of the warping that occurred across the free end of each of the constant-thickness slabs during some particular day are shown in figure 21. The data were obtained with the clinometer in the manner previously described. The evidences of restraint shown by these data are in accord with what was shown by figure 20.

Additional information on the relative restraint to warping of the various transverse joints was obtained as a result of stress determinations based on strain measurements made at the positions shown in quadrant

4 of figure 8. The two gages placed perpendicular to and 6 inches from the free edge were used as index gages and the average deformation measured at these points, when corrected by means of Poisson's ratio for the strain measured by the gage parallel to the slab edge, was used as a base for computing the strains caused by restrained warping at other points.[16]

Stress values obtained in this manner for typical cases of warping for certain of the transverse joints are given in table 4 and for the longitudinal joints in table 5. A comparison between the stresses found at corresponding points near the joint edge and free edge of a slab panel, for a given temperature condition, brings out the effect of the joint on the stresses caused by warping. For example, the stresses at a point 18 inches from the joint edge should be compared with those found at the same distance from the free edge.

TABLE 4.—*Warping stresses caused by the various transverse joints*

Date tested (1934)	Test section no.	Type of joint	Spacing of dowels	Longitudinal stress[1]			Transverse stress[1]	
				6 inches from transverse joint	18 inches from transverse joint	18 inches from free end	6 inches from transverse joint	6 inches from free end
			Inches	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*
July 10	7	Doweled	18	+5	+32	0		
July 26	7	do	18	+20	+5	−50		
June 28	4	Plane of weakness.	18	−68	−25	−35	−12	+59
June 29	4	do	18	−93	−35	−30	−14	+22
June 30	3	do	None	+69	+80	−10	−5	−65
July 3	3	do	do	+97	+100	+15	+42	−43
Sept. 20	2	Dowel plate	Continuous	−25	−22	+32	+56	+30
Sept. 28	2	do	do	−9	−42	+10	−14	−61
Sept. 10	4	Crack	None	−140	−140	−25		

[1] A positive sign indicates tensile stress and a negative sign compressive stress in the upper surface of the slab.

TABLE 5.—*Warping stresses caused by the various longitudinal joints*

Date tested (1934)	Test section no.	Type of joint	Dowel spacing	Transverse stress[1]		
				6 inches from joint	18 inches from joint	18 inches from free edge
			Inches	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*
July 7	[2] 3	Tongue	60	−53	−23	+40
July 11	3	do	60	−15	−10	+68
Aug. 22	[2] 5	do	60	−6	−57	−40
Oct. 3	10	do	60	−38	−60	−18
Sept. 18	9	Butt	24	+6	+28	+48
Sept. 19	9	do	24	−20	+8	+38
Sept. 26	[2] 2	do	48	−68	−80	−18
Oct. 2	2	do	48	−59	−90	−27
July 21	6	Plane of weakness	60	−41	−65	+2
July 24	6	do	60	−7	−42	+55
July 16	7	do	None	−20	−25	−15
July 17	7	do	None	−17	−32	+10

[1] A positive sign indicates tensile stress and a negative sign compressive stress in the upper surface of the slab.
[2] Thickened-edge slab.

DATA SHOW THAT JOINTS SHOULD OFFER A MINIMUM OF RESTRAINT TO WARPING

Since the deformations were all measured in the upper surface of the pavement for a condition in which the edges of the slab were warping downward, restraint to warping would be expected to cause an increase in the compressive stresses at the positions near the joint edge over those found at corresponding positions near the free edge.

A number of tests were made on thickened-edge slabs. The fact that a slab has thickened edges should not affect the stress comparisons so far as the transverse joints are concerned but the comparisons for the longitudinal joints are affected because the index gages, being at the thicker, outside edge of the slab, will record a larger deformation for a given degree of bending than will the gage at a corresponding position near the thinner, longitudinal joint edge. In the case of transverse stresses, the stresses themselves are small and the effect at the thickened edge does not appear to be important. In the case of the longitudinal stresses, however, the effect is more important and these data have been omitted from table 5 for this reason. It is indicated by both the deflection and stress data that none of the longitudinal joints causes any serious restraint to warping in the direction parallel to the joint.

It will be noted that the stress values given in tables 4 and 5 are generally small and that they tend to be erratic. It is believed that the tendency for the stress values to be erratic is caused in part by the variable behavior of the slabs, which was mentioned previously in connection with the deflection data, and in part by the small deformations and relatively few measurements involved. In most of the other stress determinations in these investigations it was possible to average a considerable number of observations of deformations which in themselves were of much greater magnitude than those being considered here. It may be said, however, that the stress data show no indications of serious restraint to warping in any of the designs tested.

Attention is called to the fact that the tests on the weakened-plane joints were made during the summer, when the joints were closed, a condition which should produce the maximum restraint to warping in slabs of the length used in these tests.

The crack in one of the slabs of section 4 was tested as a joint and, as shown by the data in table 4, appeared to exert a greater restraint to warping than any of the joint designs. There are probably two related causes for this. The first, and probably the most important, is the firm connection of the cracked panel with the adjoining uncracked panel by a tongue-and-groove type of longitudinal joint which exerted a stiffening effect on the cracked panel. The second is that the two broken edges of the cracked panel appeared to be tightly in contact at all times.

For warping to take place it is necessary to displace the two slabs abutting the crack longitudinally a slight amount. Except for some frictional resistance in the longitudinal tongue-and-groove joint the only force resisting this longitudinal movement is the resistance to deformation of the subgrade material, the magnitude of which varies with the length of the slab. In this particular case the resistance must have been quite small because the slab length was only 10 feet. In longer slabs, forces of considerable magnitude might easily be created at times when the concrete was in an expanded condition and the cracks tightly closed, although the presence of the crack tends to ameliorate the warping stress conditions in its vicinity.

The deflection and stress data just presented to show the relative degree of the restraint to warping offered by the various joints tested in this investigation are but a part of the data obtained although they are typical

[16] For a discussion of the formulas and methods of computing these stresses the reader is referred to the second report of this series published in PUBLIC ROADS, vol. 16, no. 9, November 1935.

in all instances. To some extent they are erratic but it is believed that in spite of this the deflection and stress data both indicate that none of the joint designs which were tested are sufficiently resistant to bending to offer serious restraint to warping. The data do point to the danger of designing joints which are resistant to bending because of the warping stresses that such designs are likely to cause to be developed at times when warping of the slab occurs. It is indicated that a fundamental structural requirement in joint designs should be that the resistance to bending in both directions, but particularly in a plane perpendicular to the direction of the joint, be a minimum in order that the stresses caused by warping restraint will be as small as possible.

STUDY MADE OF THE RELATIVE ABILITY OF VARIOUS JOINTS TO STRENGTHEN THE JOINT EDGE OF THE SLAB

The third group of tests of joints was planned to develop data that would show the relative effectiveness of the different designs from the standpoint of their ability to reduce the natural weakness of the slab at the joint edge. With the one exception of the thickened ends used at the transverse joint in section 1, this strengthening of the slab edge was accomplished through a transfer to the adjacent slab of a part of any load applied near the joint. The influence of the several designs on the deflections and on the stresses in the vicinity of the applied loads will be brought out in the discussion which follows.

Figures 22 to 25 contain data showing how the various parts of the different slabs deflect under the influence of loads applied at the points designated in the figures. The curves show the measured deflections for the two slabs abutting the joint under test, and, for comparison, the deflection of a corresponding edge or corner under the same load but lacking the support of a connecting joint. For example, in figure 22, which shows the deflections of the outside edge of the test section, a load applied on one side of the transverse joint deflected the two abutting slab ends in the manner shown by the crosses, while the same load placed near the free end of the outside edge produced the deflection of the free end which is indicated by the circles. Referring back to figure 8, which shows the location of the lines of clinometer points in the four quadrants, the crosses in figure 22 show deflections at points along the line A_3—A_2 for a load at C_3 while the circles show the deflections at points along the line E_3—A_3 for the same load at E_3.

In making an estimate of the ability of the various joints to reduce the deflection of the slab edge on which the load is applied, certain assumptions have been made:

1. If the joint design has a maximum of effectiveness in performing this function, a load placed on one side of but close to the joint edge should cause equal deflection of the two slab ends that abut the joint.
2. If the joint design is completely ineffective in this respect, a load on one side of, but close to, the joint should produce no deflection of the slab end on the opposite side of the joint.

The first and second assumptions serve as the basis for the first method of estimating joint effectiveness from the deflection data. The method will be described in a succeeding paragraph.

3. If the joint design has a maximum of effectiveness, the application of a given load at the joint should produce a deflection having a magnitude one-half as great as that which would be produced by the same load acting at an unsupported edge.
4. If the joint design is completely ineffective, a given load will cause the same deflection when applied at the joint edge and at a corresponding point of the free edge of the test slab.

The third and fourth assumptions are the basis of the second method of estimating joint effectiveness from the deflection data.

It is believed that all of the assumptions are correct for the condition of complete and uniform contact between the slab and the subgrade, because, for such a condition, the load-deflection relation is practically linear for loads within the safe stress range. It is indicated by these tests that this condition rarely prevails and that the extreme edges of the slab are in full contact with the subgrade only when they are warped downward.

Since the joint tests were made with the slabs in an unwarped condition brought about by the protective coverings described in the first report, the edges of the panels were not in full contact with the subgrade at the time the loads were applied, with the result that the load-deflection relation is not linear. A typical example of the character of this relation for a load applied at the free corner of one of the test sections is given in figure 26. It will be noted that for equal increments of load, each succeeding increment causes a somewhat smaller increment of deflection. This condition affects the third assumption because, for a given load, the free edges and corners are generally deflected more than the joint edges and corners. The third assumption therefore does not apply strictly to the conditions under which the tests were made and this fact should be considered in the application of these data.

On the basis of these four assumptions it is possible to estimate the extent to which the various joints are effective in reducing the deflection of the loaded joint edge, by making use of the deflection data given in figures 22 to 25, inclusive.

DEFLECTION OF LOADED SIDE OF JOINT ALWAYS EXCEEDED THAT OF ADJACENT SIDE

Two methods are available for making such an estimate. As noted previously, the first method makes use of the first and second assumptions, while the second method makes use of the third and fourth assumptions. Figure 27 shows the comparisons that are involved in each method of analysis.

While it is believed that the first method is probably the better measure of the ability of the joint design to reduce deflection because it does not involve the third assumption, both methods are of some value and will be used in the comparisons which follow.

Figure 22 shows the deflections along the outside edge of the various slabs caused by a load at points E_3 and C_3 and gives some indication of the effectiveness of the joint constructions in reducing the deflections caused by loads applied at the joint corners. For a load at point E_3 the shape of the deflected slab is shown between points E_3 and A_3, while for a load at point C_3 the shape is shown between points A_3 and A_2.

Figure 23 shows the extent of the deflections along the centerline of a 10- by 20-foot panel caused by loads applied either at point I_3 or G_3 and indicates the effect of the various transverse joints on the deflection caused by a load acting at a transverse joint at a point away from a longitudinal edge. For a load at point I_3 the shape of the deflected centerline is shown between

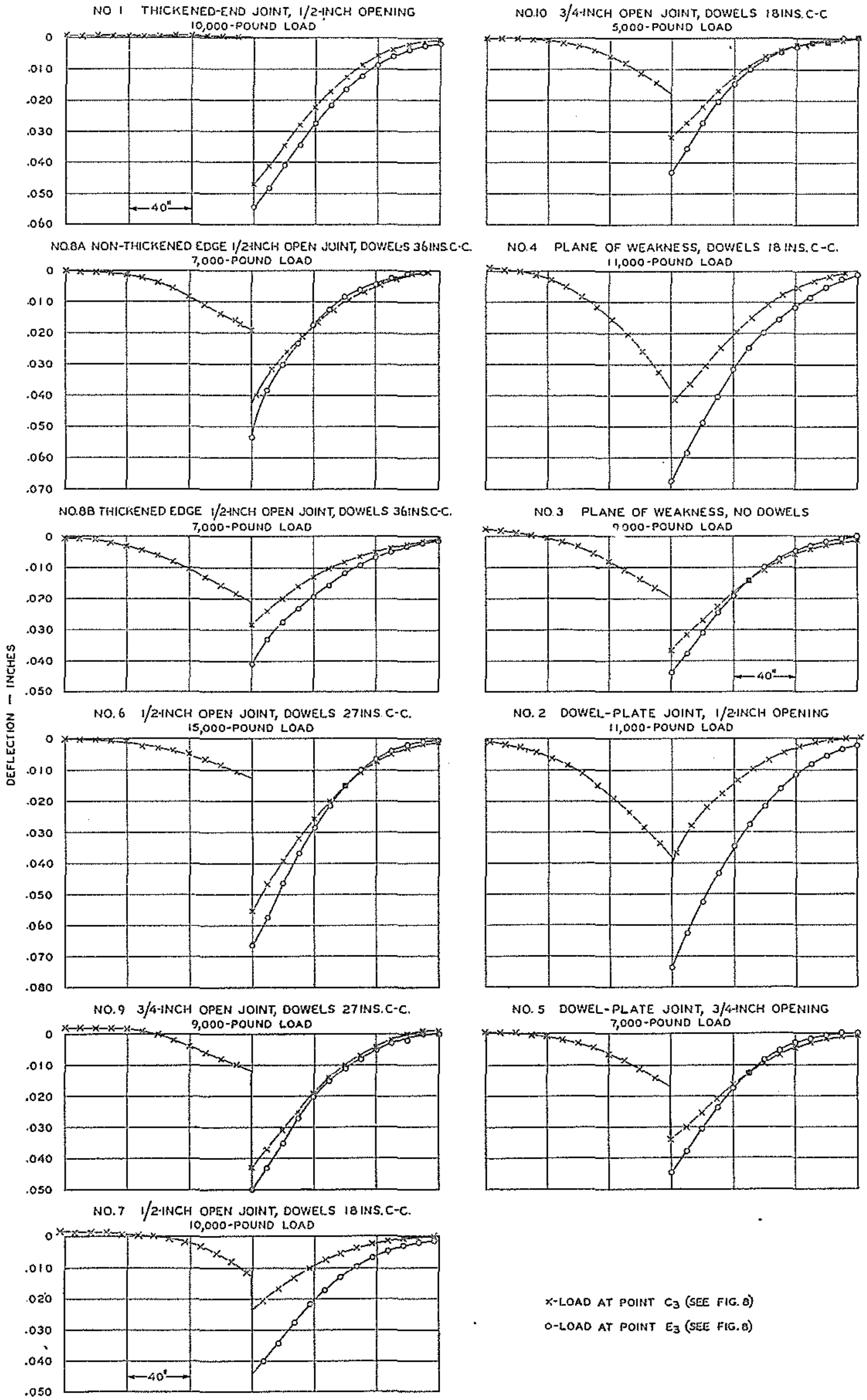


FIGURE 22.—COMPARISON OF DEFLECTIONS ALONG FREE EDGE OF TEST PANELS FOR LOADS PLACED AT FREE CORNER AND AT CORRESPONDING TRANSVERSE JOINT CORNER. ALL DOWELS USED IN TRANSVERSE JOINTS WERE ¾ INCH IN DIAMETER AND 36 INCHES LONG.

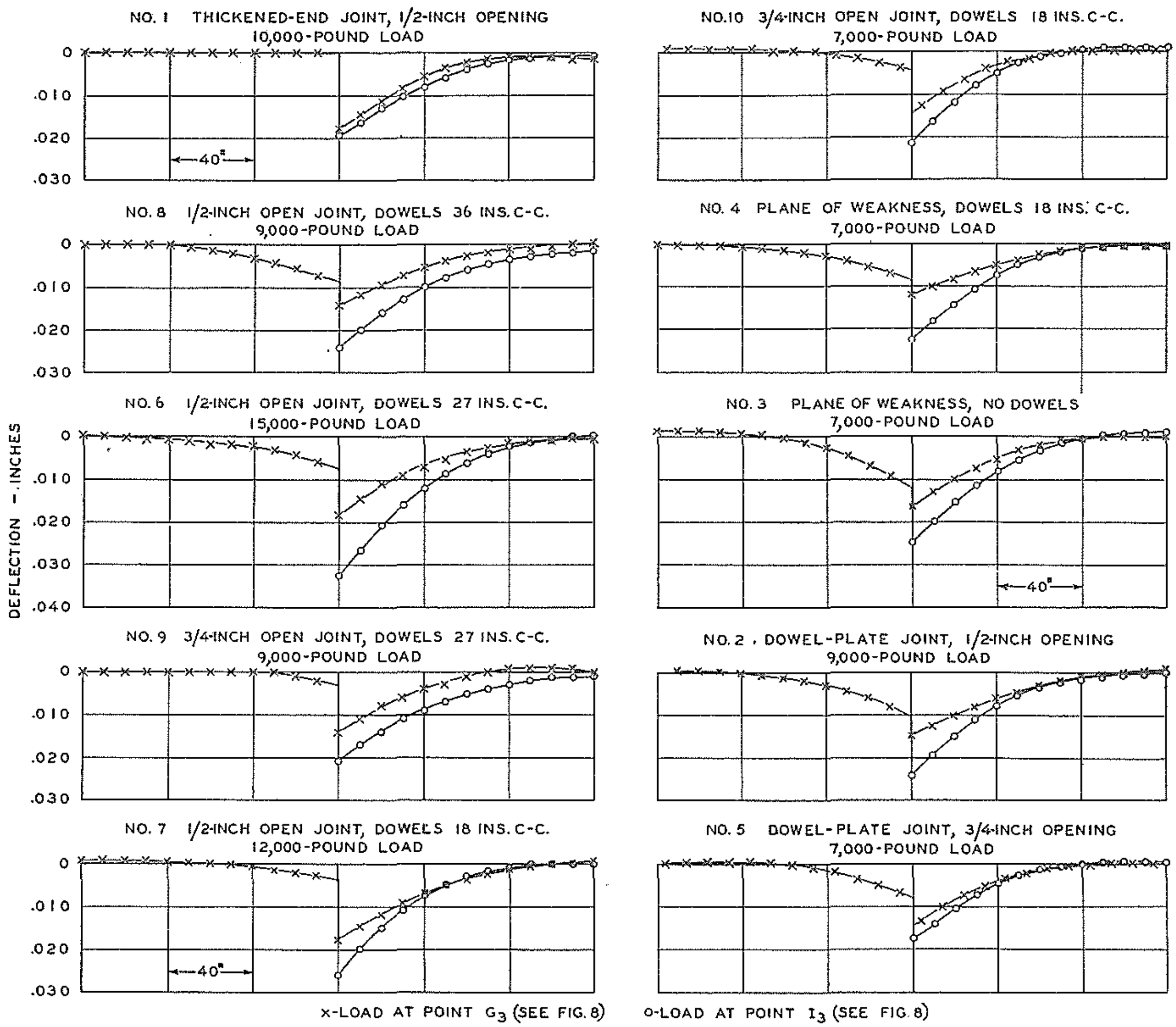


FIGURE 23.—COMPARISON OF DEFLECTIONS ALONG LONGITUDINAL CENTERLINE OF TEST PANELS FOR LOADS PLACED AT MIDPOINT OF FREE END AND AT CORRESPONDING POINT AT TRANSVERSE JOINT. ALL DOWELS USED IN TRANSVERSE JOINTS WERE ¾ INCH IN DIAMETER AND 36 INCHES LONG.

points I_3 and H_3 while for a load at G_3 the deflections are shown between points H_3 and H_2.

It will be observed that the data for transverse joint 1–1 (thickened slab ends) are included in these figures. Since there is a complete separation of slab ends in this construction and no load transfer is intended, these particular load-deflection data do not have the same significance as do those that apply to the other sections. As would be expected from the design, the two ends of the 10- by 20-foot panel behave in identical fashion and a load applied on one side of the transverse joint causes no deflection of the slab end on the other side of the joint.

Figure 24 shows the deflections along the end of the slabs caused by loads applied either at points E_3 or F_3 and indicates the effectiveness of the various longitudinal joints in reducing the deflections caused by loads applied at the joint corners of the slab. The shape of the slab is shown along the line E_3—F_3 for a load applied at E_3 and, similarly, with a load at F_3, the deflections between the points E_3 and I_4 are shown.

Figure 25 shows the deflections along the transverse centerline of a slab panel caused by loads applied at points A_3 and B_3, and these data indicate the ability of the various longitudinal joints to reduce deflection for loads applied near the longitudinal joint and at some distance from a transverse joint. For a load acting at point A_3 the deflections are shown between points A_3 and B_3, while for a load acting at B_3 the deflections are shown between points A_3 and H_4.

In the case of the thickened-edge slabs it is not possible to compare directly the deflections at the free and the longitudinal joint edges of the slabs. For this reason the comparisons in figures 24 and 25 are restricted to the constant-thickness sections.

In the data just presented for the transverse joints, it will be observed that the loaded side of the joint always deflects more than the adjacent side to which load is transmitted by the joint structure. The difference is greater for the joints containing the round dowel bars than for those which contain the dowel plates. There is also a variation in the magnitude of this differ-

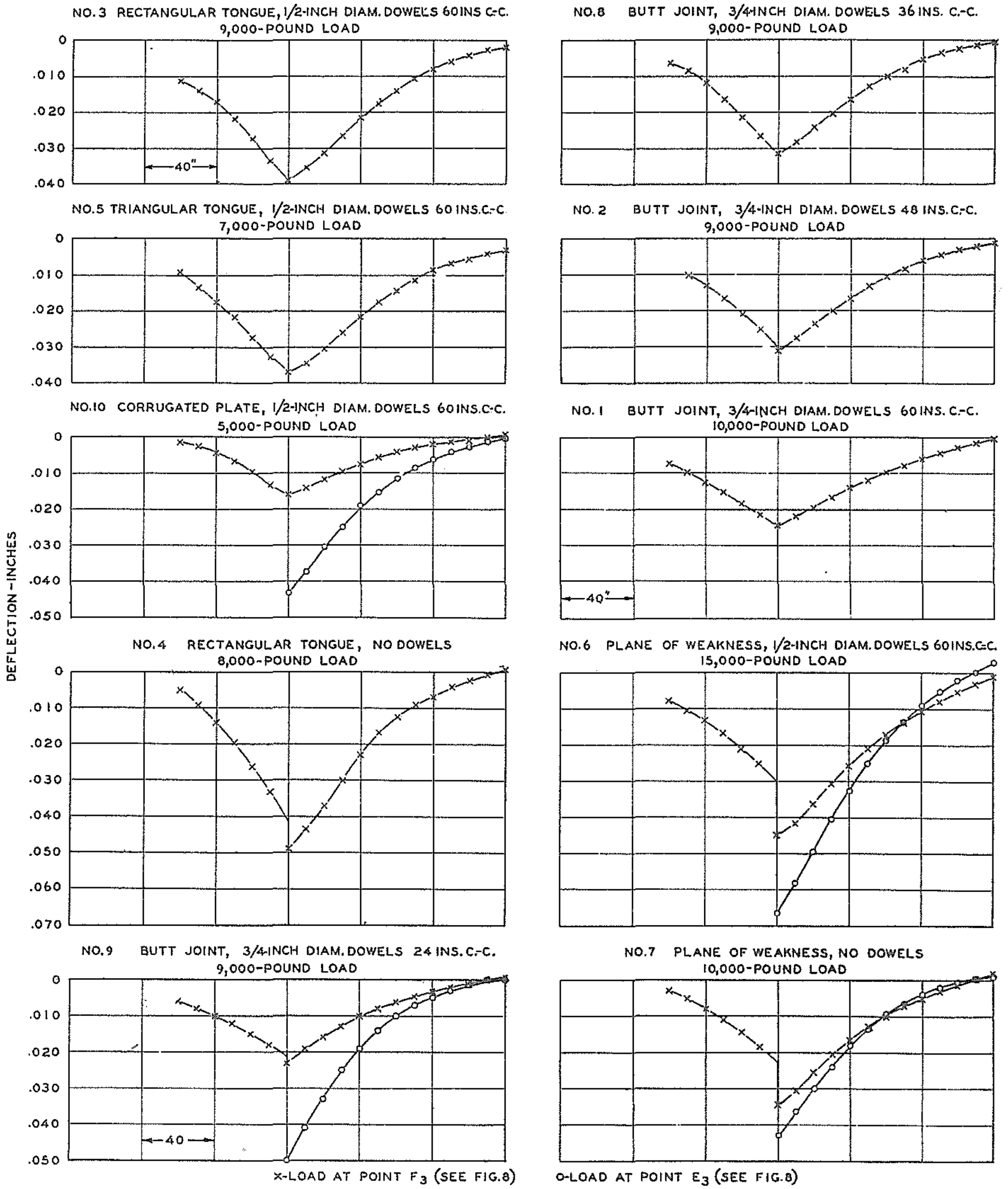


Figure 24.—Comparison of Deflections Along Free End of Test Panels for Loads Placed at Free Corner and at Corresponding Longitudinal Joint Corner. Dowels ¾ Inch in Diameter Used in Joints C-1, C-2, C-8, and C-9; Dowels ½ Inch in Diameter Used in Joints C-3, C-5, C-6, and C-10. All Dowels in Bond.

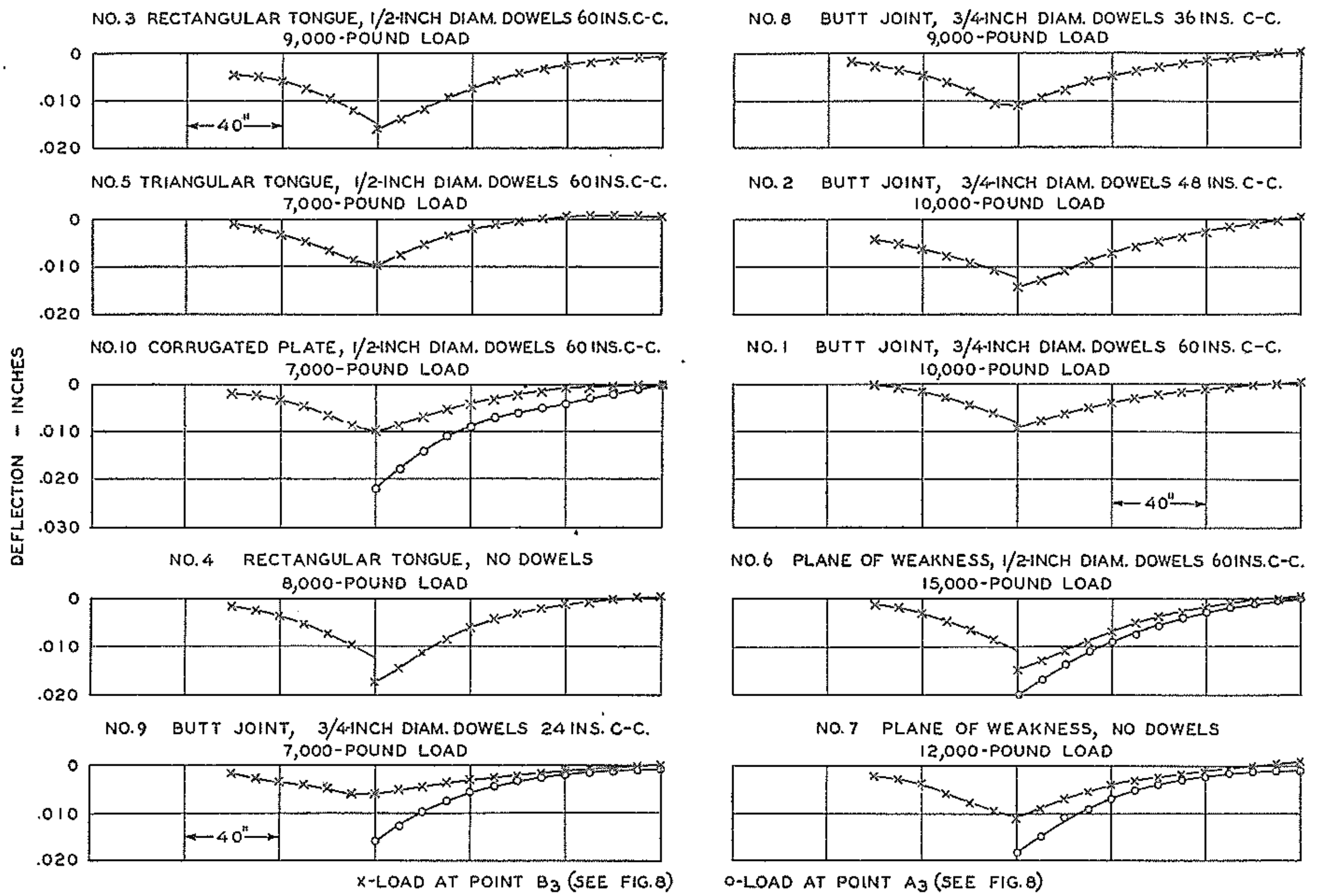


Figure 25.—Comparison of Deflections Along Transverse Centerline of Test Panels for Loads Placed at Midpoint of Free Edge and at the Corresponding Point at Longitudinal Joint. Dowels ¾ Inch in Diameter Used in Joints C–1, C–2, C–8, and C–9. Dowels ½ Inch in Diameter Used in Joints C–3, C–5, C–6, and C–10. All Dowels in Bond.

ence among the joints that contain the dowel bars. This variation might possibly result from a lack of stiffness in the dowel bars or from a looseness of the dowels in the concrete, or from a combination of the two. In order to develop information regarding the relative importance of each of these factors, a series of loads was applied on each of the four quadrants of two of the sections at a joint corner (point C) and at several points directly over dowel bars at some distance from the corners (near point G).

For the purpose of these comparisons it is assumed that, for a load applied on one side of a doweled joint, so long as the dowels are firmly in bearing on each side of the joint, the deflection rate of the adjacent slab end will bear a constant relation to the deflection rate of the loaded slab end. The ratio of the one to the other will be a constant as each load increment is applied. When the loaded edge begins to deflect, the adjacent edge will not follow immediately if any deficiency in dowel seating is present, and this lag will be evident as a variation in the ratio between the load deflection rates for those increments of load applied while the seating deficiency exists.

DATA ON LOAD-DEFLECTION MEASUREMENTS AT JOINT EDGES PRESENTED

Figure 28–A shows the load-deflection relation for both the loaded and unloaded joint ends at the corner of the panel, for sections 6 and 7, for a series of uniform increments of applied load. From the corresponding deflection increments "k" and "p", the mean slope ratio, $\frac{p}{k}$, was calculated for each increment of load and these values are plotted as ordinates to the curves shown in figure 28–B.

This graph indicates that the dowels at the corner of section 6 were slightly loose, because the ratio is smaller for the first load increments than for the later ones. Apparently the 6,000-pound load was sufficient to take up the dowel looseness completely, and it is noted that this load caused a maximum deflection of the loaded corner of about 0.016 inch. In the case of section 7 there is no indication of dowel looseness, the value of the ratio being practically constant for all load increments.

The flexibility of the dowels is indicated by the fact that for the upper increments of load the deflection of the adjacent slab corner is but 50 or 60 percent of that of the corner on which the load was applied.

The deflections of the loaded corners of the two sections are approximately the same for a given load despite the difference in slab thickness. The amount of support received from the adjacent slab corner is not the same in the two cases, however, and this probably accounts to a large extent for the effect noted.

Figure 29 contains data obtained in similar tests on section 7 in which the loads were applied directly over the dowels but at some distance from the longitudinal edges of the slab (near point G). These are average curves from tests made at several points on the same

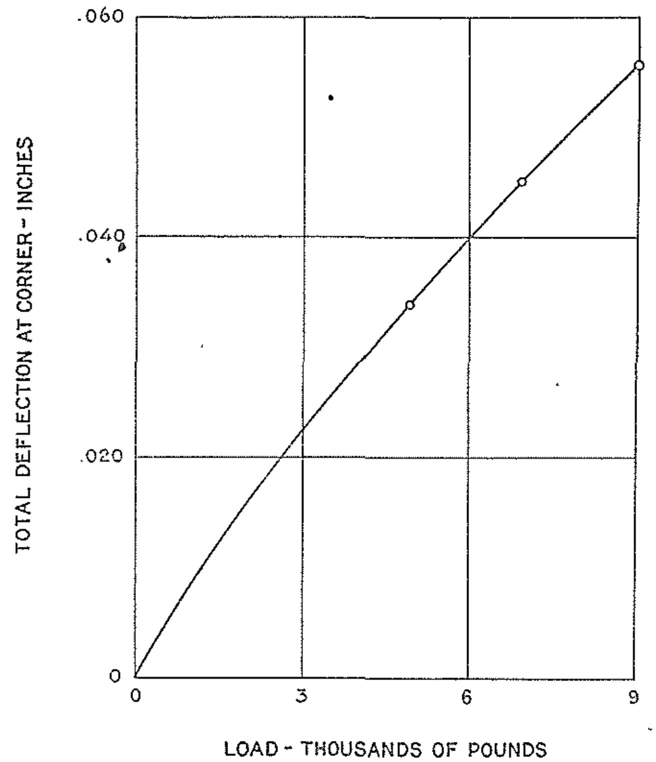


FIGURE 26.—TYPICAL LOAD-DEFLECTION RELATION FOR A LOAD ACTING AT A FREE CORNER.

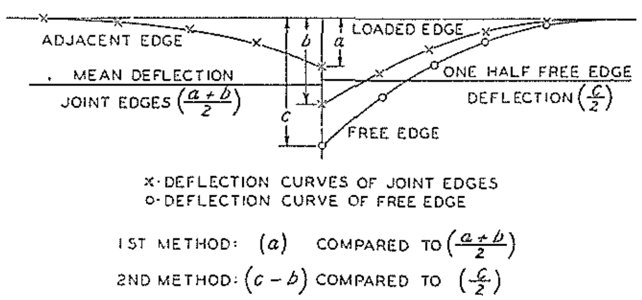


FIGURE 27.—METHODS OF ESTIMATING THE EFFECTIVENESS OF JOINTS FOR REDUCING SLAB DEFLECTION ON THE BASIS OF LOAD-DEFLECTION DATA.

slab and are plotted in the same manner as the data shown in the preceding figure. In this case it appears that the differences between the deflections of the two slab ends should be attributed entirely to dowel flexure, there being no indication of dowel looseness.

A comparison of the values of the slope ratios shown in the last two figures shows that the value is smaller for loads applied near the joint at points remote from the corner than for loads applied near the slab corner. This indicates that to obtain the same percentage of deflection across a doweled joint at all points a stiffer transfer medium is required at points that are away from the slab corner than is required in the immediate vicinity of the corner. Also a stiffer medium is required for thick slabs than for thin slabs.

It is thought that the data shown in the last two figures furnish an explanation for most of the differences found in the deflection data presented in figures 22 to 25, inclusive. In general, it was found that only a very few of the dowels were sufficiently loose to produce any apparent detrimental effect on the ability of the joint to transfer load. This is rather surprising when consideration is given to the very small deflec-

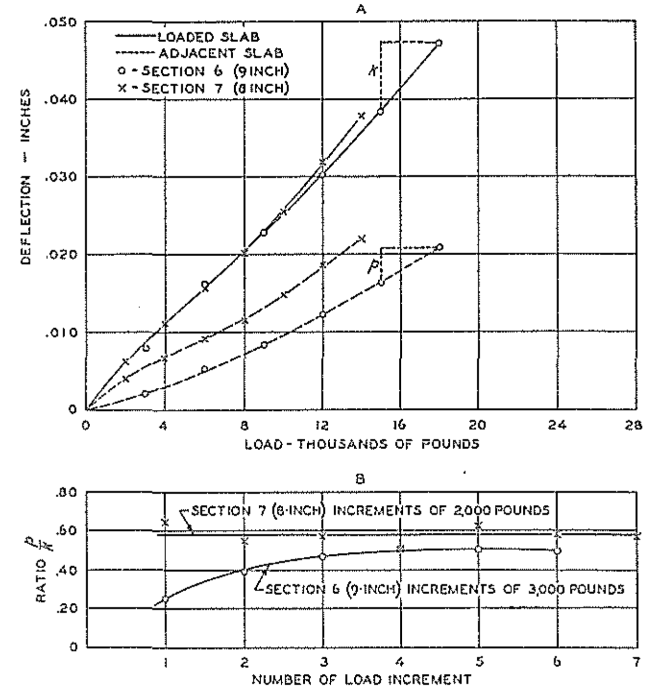


FIGURE 28.—DATA ON STUDY OF DOWEL STIFFNESS AND DOWEL LOOSENESS, TRANSVERSE JOINT CORNERS.

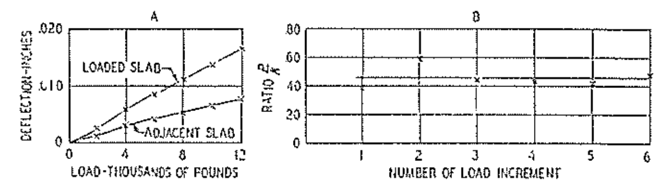


FIGURE 29.—DATA ON STUDY OF DOWEL STIFFNESS AND DOWEL LOOSENESS, TRANSVERSE JOINT EDGES (SECTION 7).

tions involved in this action. During construction considerable care was taken to place the concrete around the dowels properly, but typical field methods were employed in all of the operations and the condition of these dowels is considered representative of what can be obtained where dowels are carefully installed in first-class construction.

Somewhat similar studies of the deflections of concrete pavement slabs caused by loads acting near joints have been reported by the State highway organizations of Georgia [16] and Michigan.[17] The data given in these two reports are in general agreement with those which have been presented in figures 22 to 25, inclusive.

TWO METHODS USED TO MEASURE EFFECTIVENESS OF JOINTS IN REDUCING DEFLECTION OF THE LOADED JOINT EDGE

Tables 6 and 7 were developed from the data shown in figures 22 to 25, inclusive, for the purpose of bringing out more directly the comparisons that appear to be significant. Table 6 applies to the transverse joints and table 7 to the longitudinal joints.

In columns 5 and 8 of these tables a comparison is made between the sum of the maximum deflections of the two slab edges abutting the joint in question and the deflection of a corresponding point at a free edge of the same slab, the load being the same in both cases. Referring to figure 27, the comparison of deflections

[16] Tests of Load Transmission across Joints in Concrete Pavements, by Searcy B. Slack, Engineering News-Record, vol. 107, no. 2, July 9, 1931, p. 53.
[17] Tests of Aggregate Interlock at Joints and Cracks, by A. C. Benkelman, Engineering News-Record, vol. 111, no. 8, Aug. 24, 1933, pp. 227-232.

involved in the computation of the values shown in columns 5 and 8 is expressed as a percentage of the free edge deflection by the formula

$$\frac{(a+b)-c}{c}$$

Columns 6 and 9 show the effectiveness of the various joints in reducing the deflection of the loaded edge by means of a comparison of the deflection of the unloaded joint edge with the mean deflection at the joint. This is the first method previously mentioned and the values are computed by the formula

$$\frac{a}{\left(\frac{a+b}{2}\right)}$$

Columns 7 and 10 give similar comparative values developed by the second method, the reduction in deflection effected by the joint being compared to one-half of the deflection of the free edge (a reduction of one-half of the free edge deflection would mean perfect joint action). This relation is expressed by the formula

$$\frac{c-b}{\left(\frac{c}{2}\right)}$$

TABLE 6.—*Various relations between the deflections caused by loads placed at the free and the transverse joint edges of slabs*

Test section no.	Type of joint	Spacing of dowels	Joint opening	Tests at slab corners: Sum of deflections at joint corner exceeds that at free corner by [1]	Tests at slab corners: Joint effectiveness estimated by first method	Tests at slab corners: Joint effectiveness estimated by second method	Tests at midpoint of slab end: Sum of deflections at midpoint of joint end exceeds that at midpoint of free end by [1]	Tests at midpoint of slab end: Joint effectiveness estimated by first method	Tests at midpoint of slab end: Joint effectiveness estimated by second method
1	2	3	4	5	6	7	8	9	10
		Inches	*Inches*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
8	Doweled	36	½	18	73	50	−5	76	83
6	do	27	½	3	37	33	−22	67	88
9	do	27	¾	10	44	28	−19	35	67
7	do	18	½	−18	70	94	−19	33	65
10	do	18	¾	14	71	53	−14	47	67
4	Plane of weakness	18		19	95	74	−10	84	96
3	do	None		28	70	32	14	84	68
2	Dowel plate		½	5	97	92	4	82	77
5	do		¾	13	66	48	27	70	34

[1] A negative sign indicates that the sum of the deflections of the two edges abutting the joint under test is smaller than the deflection of the corresponding point at the free edge of the slab.

The values in table 6 indicate that for a given load the sum of the deflections at the corner of a transverse joint is nearly always greater than the deflection of the corresponding free joint corner. At the transverse joint edge at a distance from the corner, in some cases the sum of the deflections at the joint edge exceeds the corresponding deflections at the free edge and in other cases it does not.

Earlier in the discussion it was shown that the load-deflection relation at a slab corner is not linear because the slab is not completely in contact with the subgrade when the deflection begins and the conditions of support change gradually as the deflection progresses. It was shown in figure 26 that for equal increments of load the resulting increments of deflection gradually

TABLE 7.—*Various relations between the deflections caused by loads placed at the free and longitudinal joint edges of the slabs*

Test section no.	Type of joint	Type of tongue	Spacing of dowels [1]	Tests at slab corners: Sum of deflections at joint corner exceeds that at free corner by [2]	Tests at slab corners: Joint effectiveness estimated by first method	Tests at slab corners: Joint effectiveness estimated by second method	Tests at midpoint of slab end: Sum of deflections at midpoint of joint end exceeds that at midpoint of free end by [2]	Tests at midpoint of slab end: Joint effectiveness estimated by first method	Tests at midpoint of slab end: Joint effectiveness estimated by second method
1	2	3	4	5	6	7	8	9	10
			Inches	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
3	Tongue	Rectangle	60		100			97	
5	do	Triangle	60		100			100	
10	do	Corrugated	60	−26	100	126	−12	100	111
4	do	Rectangle	None		91			83	
9	Butt		24	−12	96	108	−24	100	124
8	do		36		100			100	
2	do		48		93			92	
1	do		60		100			94	
6	Plane of weakness.		60	12	80	65	28	85	52
7	do		None	33	79	40	21	100	80

[1] All dowels across longitudinal joint were fully bonded.
[2] A negative sign indicates that the sum of the deflections of the two edges abutting the joint under test is smaller than the deflection of the corresponding point at the free edge of the slab.

decrease as the magnitude of the deflection increases. It is thought that this is the reason that the combined deflections at a transverse joint corner nearly always exceed the corresponding deflection at a free corner. The free corner, on account of its relatively greater deflection, is able to offer more resistance to deflection. The values given in columns 5 and 8 of these tables are significant because they illustrate the point just discussed and help to explain differences in the values of joint effectiveness as estimated by the two methods of analysis.

When the loads are applied at the slab edges at some distance from a corner, the deflections are all small, relatively. The effect of varying subgrade support is therefore much less.

It is apparent from table 6 that the two methods of analyzing the deflection data to determine the relative effectiveness of the transverse joint designs in reducing the deflection of the loaded edge of a joint give different values for the same joint design. For the corner loading the first method of calculation, with one exception, yields higher values, while for the interior edge condition the relation between the values obtained by the two methods of comparison is quite irregular. The most probable cause for this irregularity has already been mentioned. The values obtained by the first method probably give the better idea of the ability of the joints to transfer load, while those obtained by the second probably give a better indication of the relative ability of the joints to reduce the maximum deflection of the loaded edge. Neither method gives a complete measure of the structural efficiency of the joint to which it is applied, however.

WEAKENED-PLANE JOINTS CONTAINING DOWELS WERE EFFECTIVE IN REDUCING SLAB DEFLECTION

From table 6 it appears that the transverse joints that depend upon round dowels for their connection are not as effective in reducing maximum deflections as might reasonably be expected. The data for the joints having dowels spaced 18 and 27 inches apart, respectively, indicate that the effectiveness of the joint in reducing corner deflection is increased as the dowel spacing is decreased. The joint containing the dowel bars with a 36-inch spacing is not in line with the others in respect to this relation. Why this should be is not known, although the fact that this particular transverse joint was installed in the section having the lip-curb cross section may have been at least partly responsible. The two halves of this section (longitudinally) were different, so that there was opportunity for but one-half as many comparisons as on the other sections. Also, because of the presence of the lip curb on the upper surface, certain difficulties in duplicating loading conditions were always present in the tests on this section.

So far as the values computed from the deflection data indicate, dowel spacing within the limits of the tests did not influence the effectiveness of the doweled joints in reducing the deflection of the loaded edge. Neither do they show any consistent difference attributable to the differences in the width of the joint opening used in these tests.

The tests on the transverse plane-of-weakness or dummy joints were made in November when the slabs were contracted and the joints were opened slightly. The comparative values for the weakened-plane joints containing dowels (particularly those computed by the first method) indicate a very effective construction so far as ability to reduce slab deflection is concerned. The joint without the dowels is also indicated as being quite effective by these values. It should be remembered in this connection that the slab length in which the joints were used is but 20 feet.

It is indicated further by the values in table 6 that the two joints containing the one-fourth by 4-inch dowel plates are effective in reducing slab deflection. When used with a joint opening of one-half inch the effectiveness of a dowel plate of this thickness seems to be noticeably greater than when used with a three-fourths-inch joint opening.

Table 7 contains similar comparative values for the longitudinal joints computed by both methods wherever possible. One is struck immediately by the generally higher order of values in this table when compared to those given in the preceding table for the transverse joints. This is a direct reflection of the better structural connection obtained in the longitudinal joints where little or no change in joint width occurs and no provision for slab expansion was necessary.

It will be noted in table 7 that the values computed for the corners of the longitudinal joints in sections 9 and 10 show that the combined deflections at the joint are less than the deflection at the corresponding free corners. This relation is the reverse of that generally shown in table 6 for the transverse joints.

As stated earlier, it was possible to make comparisons between the deflections of free and longitudinal joint edges of slabs only for sections of constant thickness and even then only two of the four constant-thickness sections (sections 9 and 10) were suitable for the desired comparison. The other two constant-thickness sections were of the plane-of-weakness type and only one of these contained bonded steel across the joint. It is believed, however, that the relations indicated for sections 9 and 10 will probably be found in any constant-thickness section in which the dummy-joint construction is not used and the slab edges are

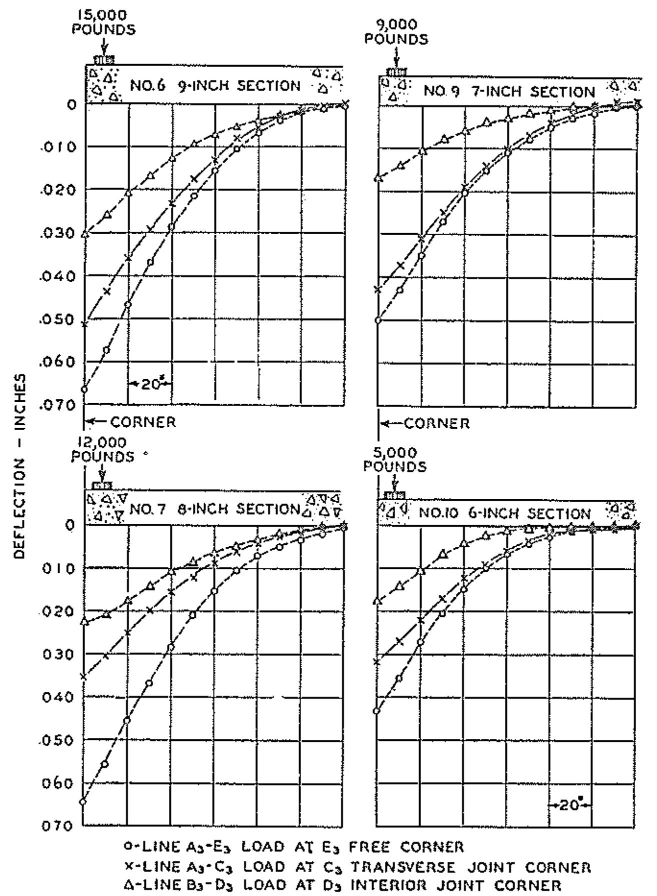


FIGURE 30.—COMPARISON OF DEFLECTIONS OF SLAB EDGES AT THE FREE TRANSVERSE JOINT AND INTERIOR JOINT CORNERS FOR THE UNIFORM-THICKNESS SECTIONS.

held in contact by bonded steel members across the joint. That such a design permits the development of a resisting moment during deflection has already been pointed out. The effect of this moment is to stiffen the joint against deflection under load and this would produce the relation indicated by the values in table 7. The magnitude of the moment that it is possible to develop will depend upon the effective depth of the slab at the joint, the amount and position of the steel capable of taking tension, any opening of the joint, and other factors.

Table 7 shows that the sum of the deflections at the longitudinal joint exceeds that at a corresponding point at the free edge in the two plane-of-weakness joints. For the other two sections of constant thickness (sections 9 and 10), the sum of the deflections at the joint is less than that at the free edge because of the effect of resisting moments due to the design of the joints. This has been discussed earlier in this report. The effect of the deep groove in the plane-of-weakness joint is apparent if the data for section 6 are compared with those for section 10.

Figure 30 shows a comparison of the deflections of the different corners of the four constant-thickness sections under a given load. The greatest deflection occurs at a completely free or unattached corner. Some reduction in deflection is brought about when the corner is attached on one side by a transverse joint. A still greater reduction is effected when the corner is supported along the other side by a longitudinal joint capable of transferring load. The joints and combinations of joints are different in each of the four sections.

THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

Reported by L. W. TELLER, Senior Engineer of Tests, and EARL C. SUTHERLAND, Associate Highway Engineer

PART 4.—A STUDY OF THE STRUCTURAL ACTION OF SEVERAL TYPES OF TRANSVERSE AND LONGITUDINAL JOINT DESIGNS—Concluded [1]

THE SIGNIFICANCE of deflection data in connection with tests of the structural action of joints is a matter about which there seems to be a difference of opinion. A brief discussion of it at this point is pertinent.

The successive changes in curvature of the deflection (or elastic) curve of the slab are values of slope which, if determined with sufficient frequency and precision, may be used to form a slope curve. The changes in curvature of this slope curve, in turn, if determined with sufficient precision, give values of moment, a direct measure of stress. However, the determination of second differences, if these differences are to be significant, must be based upon a precise knowledge of the shape of the basic curve and accurate methods of determination of the changes in curvature.

It has not been found possible in this investigation to measure slab curvature with sufficient precision to permit the use of the deflection data as a basis for estimating absolute or even relative stresses at critical points. A comparison of the relative deflections and of the relative stresses in the vicinity of a load applied on one edge of two typical doweled joints will be shown later in this report and the data presented illustrate the point which has just been made. It is felt that the deflection data have definite value for certain purposes and complete deflection data were obtained in practically all of the tests. Main reliance has been placed upon the strain data, however, for comparisons that would show the relative structural efficiency of the various joints.

> JOINTS are needed in concrete pavements for the one purpose of reducing as much as possible the stresses resulting from causes other than applied loads in order that the natural stress resistance of the pavement may be conserved to the greatest possible extent for carrying the loads of traffic.
>
> A joint is potentially a point of structural weakness and may limit the load-carrying capacity of the entire pavement.
>
> Joints are classified by function as:
>
> 1. Those designed to provide space in which unrestrained expansion can occur.
> 2. Those designed for the relief or control of the direct tensile stresses caused by restrained contraction.
> 3. Those designed to permit warping to occur, thus reducing restraint and controlling the magnitude of the bending stresses developed by restrained warping.
>
> Expansion joints should be provided at no greater intervals than about 100 feet in order to keep the joint openings from becoming excessive.
>
> The spacing of contraction joints will be determined by the permissible unit stress in the concrete. If this is restricted to a low value, which is desirable, contraction joints should be provided at intervals of about 30 feet.
>
> It is indicated that joints to control warping should be spaced at intervals of about 10 feet.
>
> A free edge is a structural weak spot in a slab of uniform thicknesses, and it is necessary to strengthen the joint edges by thickening the slab at this point or by the introduction of some mechanism for transferring part of the applied load across the joint to the adjacent slab.
>
> The doweled transverse joints investigated were quite effective in relieving stresses caused by expansion, contraction, and warping, but they were not particularly effective in controlling load stresses near the joint edge.
>
> The dowel-plate joint tested had merit as a means for load transfer, though it offered more resistance to expansion and contraction than is desirable.
>
> Aggregate interlock as it occurs in weakened-plane joints cannot be depended upon to control load stresses. Even when joints of this type are held closely by bonded steel bars there is wide variation in the critical stress value caused by a given load; therefore, it appears necessary to provide independent means for load transfer in plane-of-weakness joints.
>
> Tongue-and-groove joints held together by bonded steel bars were found to be the most efficient structurally of any of the joints studied. However, modifications of the designs might improve their action.

Before presenting the results of the strain measurements made in connection with the joint tests, it is desired to call attention to two conditions that affect directly the precision of the efficiency values which appear later in this discussion. In the first place, it should be remembered that the tests were made on specimens that were built and tested under field conditions. Certain unexpected variations in the deflection and strain data have consistently appeared when certain sections or certain panels of a given section were tested. These indicate that variations in the strength of the specimen or in the condition of support are present in spite of all the precautions taken to guard against them.

In the second place, the criterion that has been set up as a measure of joint efficiency on the basis of the stress data, while sound in principle, has one practical weakness that should be recognized. Although the critical stress values as determined from the strain measurements are of appreciable magnitude, being generally of the order of 250 to 350 pounds per square inch, when these values are used in the application of the efficiency formula the significant ratio is developed from differences in stress values, both in the numerator and in the denominator of the expression. The differences naturally are of much less magnitude that the stress values themselves and the result is that the ratio of differences is very sensitive to small changes in the stress values from which the differences were obtained. Thus variations in stress determination that are quite unimportant, insofar as the total value of the stress is concerned,

[1] Because of its length, Part 4 is presented in two issues of PUBLIC ROADS. The first installment appeared in the September 1936 issue.

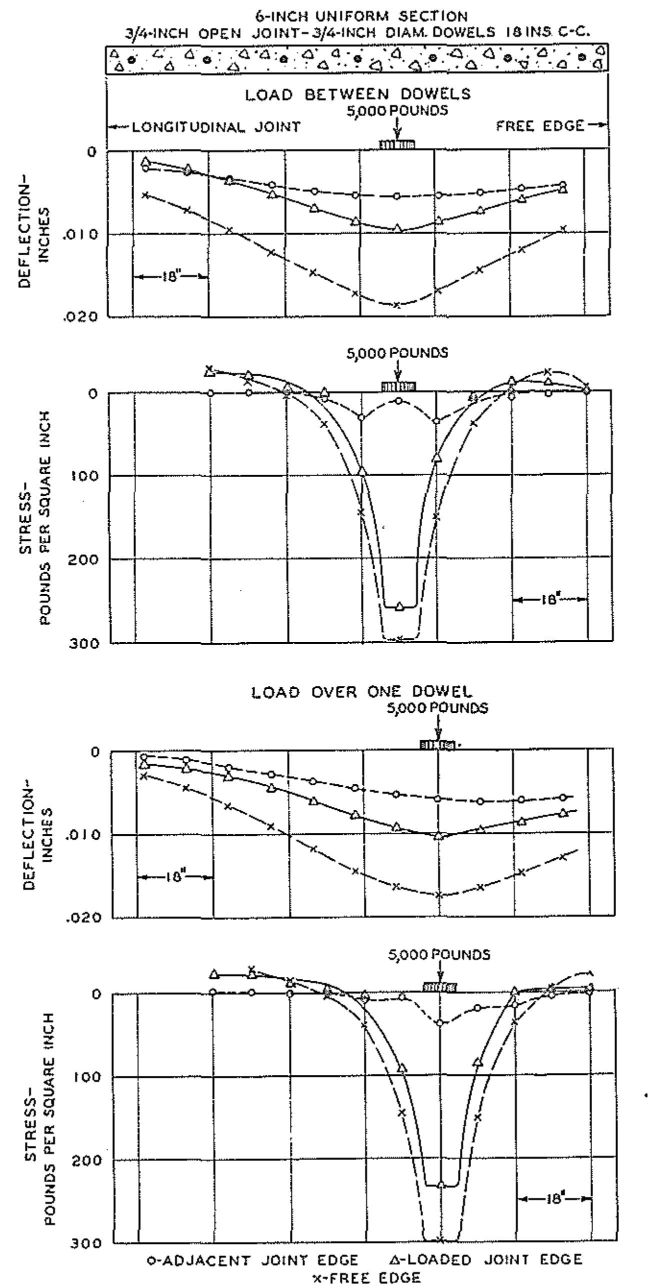


Figure 31.—Deflection and Stress Variation Curves at the Free and Joint Edges of a Typical Transverse Dowel Joint. Stress Values Above the Axis Indicate Tension, and Stress Values Below the Axis Indicate Compression in the Upper Surface of the Slab.

may be sufficient to cause appreciable variations in the ratio by which the structural efficiency is measured.

The stress values used in computing the efficiency values to be presented later were based on averages from tests made at not less than eight comparable points in order to minimize the effect of individual variations in the strain data and are believed to be quite well established. Still, a realization of the manner in which these values were derived will show the necessity for care in the use of individual figures, and will indicate the reasons for certain apparent inconsistencies in the test data.

The tests to determine the effectiveness of the various joints in relieving slab stress were divided into four general groups for convenience in presentation, as follows:

1. Tests to show the character of the stress and deflection variations parallel to the joint.
2. Tests to determine the effect of the transverse joint design on the critical stresses caused by a load acting near a transverse joint, but at a distance from a corner.
3. Tests to determine the effect of the longitudinal joint design on the critical stresses caused by a load acting near a longitudinal joint but at a distance from a corner.
4. Tests to determine the effect of the different joint designs, both transverse and longitudinal, on the critical stresses developed by a load acting on a slab corner.

Mention has already been made of the fact that, with a load acting at the edge of a pavement slab, it has been determined that the highest stress will be found directly under the load in a direction parallel to the edge of the slab. In making the stress measurements for loads applied at joint edges, the stress just mentioned is the critical stress, all others being of less significance. This critical stress was determined for each test at a joint edge and in addition the stress-variation curves were determined through the load position in a direction perpendicular to the joint and for some distance back on each slab.

REDUCTION IN DEFLECTION EXCEEDED REDUCTION IN STRESS

While the stress variation along the edge of the slab is of interest for the comparisons to be made, it was not considered sufficiently important to justify the amount of work that would be involved if these data were to be obtained for every joint. Stress-variation data along the edge were obtained only for one transverse joint with the 18-inch dowel spacing (section 10) and for the longitudinal joint with the 24-inch dowel spacing (section 9). For these two joints data were obtained for a load applied midway between dowels, directly over a dowel, and at a free end. The variations in stress on both the loaded panel and adjacent panel were determined in each case.

The deflection variation and stress variation along the free edge and the two edges of the transverse joint in section 10 are shown in figure 31, while similar data for the longitudinal joint in section 9 are shown in figure 32. The method of grouping makes it possible more easily to make comparisons between the influence of the design on deflection and that on stresses, comparisons that are of particular interest because they show why strain measurements furnish a better basis than deflection measurements for judging the ability of joints to perform their intended function of stress reduction.

If the relations between free-edge deflection and loaded joint-edge deflection are compared and if a similar comparison is made between free-edge stress and that developed at the loaded joint edge, for each of the two joints, it will be found that reductions in deflection and reductions in critical stress are as shown in table 8.

From these values it is apparent that the reduction in load deflection that is obtained with either of these joint designs is not a measure of the reduction to be expected in corresponding critical stress values.

If a similar study is made of the relative deflections and the relative stresses on the two sides of the joint when a load is applied on one of the sides, and ratios

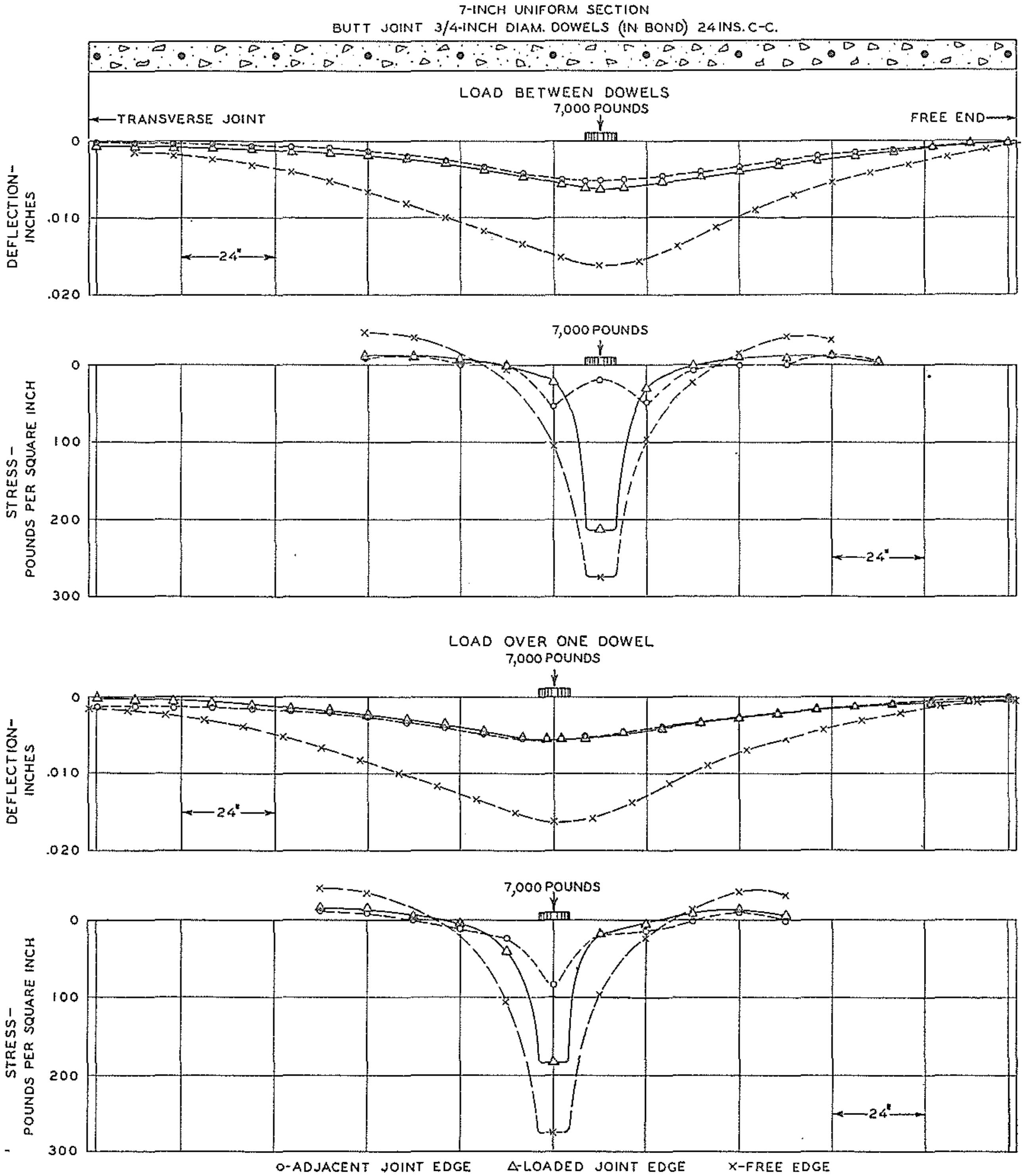


FIGURE 32.—DEFLECTION AND STRESS VARIATION CURVES AT THE FREE AND JOINT EDGES OF A TYPICAL LONGITUDINAL DOWEL JOINT. STRESS VALUES ABOVE THE AXIS INDICATE TENSION, AND STRESS VALUES BELOW THE AXIS INDICATE COMPRESSION IN THE UPPER SURFACE OF THE SLAB.

are calculated which express the maximum deflection, or stress, in the adjacent edge as a percentage of that found in the edge on which the load was applied, the ratios will have the values shown in table 9.

Again it is evident that the deflection relations are not a usable measure of the stress conditions that accompany them. The point is well illustrated in the case of the longitudinal joint with the load applied directly over a dowel. The deflection curves of the two slab edges are closely comparable, as nearly as can be judged by visual examination (see fig. 32), and the maximum deflection of each is identical. Yet the maximum stress in the loaded edge is more than twice that of the adjacent edge. This is direct evidence of the presence of changes in curvature that are not apparent in the deflection data and for which there is no dependable measure except strain data. Since the reduction of the critical edge stress is one of the chief functions of the joint, this is a very important fact and it has a direct bearing on methods of testing joints for structural efficiency. It emphasizes the impossibility of forming sound judgments regarding the effect of joint designs on stress from deflection data alone. The reasons for this apparent anomaly have already been discussed.

TABLE 8.—*Comparison of deflection reductions and stress reductions*

	Transverse joint (section 10)	Longitudinal joint (section 9)
	Percent	*Percent*
Load midway between dowels:		
Reduction in maximum deflection	49	60
Reduction in maximum stress	12	23
Load directly over a dowel:		
Reduction in maximum deflection	40	65
Reduction in maximum stress	22	33

TABLE 9.—*Comparison of deflection ratios and stress ratios between the loaded and adjacent joint edges*

	Transverse joint (section 10)	Longitudinal joint (section 9)
Load midway between dowels:		
Ratio of deflections (adjacent vs. loaded edge)	0.58	0.82
Ratio of stresses (adjacent vs. loaded edge)	.14	.23
Load directly over a dowel:		
Ratio of deflections (adjacent vs. loaded edge)	.58	1.00
Ratio of stresses (adjacent vs. loaded edge)	.16	.45

EFFICIENCIES OF VARIOUS TRANSVERSE JOINTS COMPARED FOR LOADS NEAR JOINT EDGES

There are other interesting points brought out in figures 31 and 32. The concentration of the critical stress along these joints is very clearly shown by the stress-variation curves. It will be noted that for these spacings practically all edge stress of any magnitude occurs within a distance of two dowel spacings in the case of a load applied over a dowel and within three dowel spacings for a load applied between dowels. The distribution of the deflection is much greater.

The position of the load with respect to the dowel not only affects the distribution of the stress but also the magnitude of the critical stress, the highest value being observed when the load was midway between the dowels, in each of the joints tested.

In comparing the data in figure 31 with the comparable data in figure 32 the greater stiffness of the longitudinal joint is evident both in the deflection and the stress relations. Because of the presence of the bonded bars a resisting moment is developed during the deflection of the longitudinal joint which accounts for the fact that a deflection reduction of more than 50 percent is obtained. The data indicate that the presence of this resisting moment has no important effect on the stresses in a direction parallel to the joint edge although it does affect the stresses in a direction perpendicular to the joint edge. The effect of the close proximity of the two slab edges in this joint is to make the steel bars more effective as shear units and this causes greater stress reduction, particularly when the load is applied over a dowel. This is shown by the comparative values in tables 8 and 9.

Finally, it is to be noted that for joints such as these, the stresses developed parallel to the joint in the edge of the adjacent slab are relatively quite low in magnitude.

The data which have just been presented serve two important purposes; first, they illustrate the necessity for stress determinations in a study of joint action; and second, they give a general picture of the stress conditions along the edge of the slab which is helpful in connection with the discussion of the other stress data which follow.

Figure 33 shows stress values, as determined from strain measurements in the vicinity of the load applied at the edge of a slab, either at a transverse joint (point G) or at a free edge (point I), for the purpose of studying the structural efficiency of the various joints from the standpoint of their ability to control critical load stresses. The method of placing the loads and of measuring the strains was previously explained in connection with figure 8. The curves connecting the circles show the stress variation along a line perpendicular to the joint and passing through the center of load application. The single values shown by the crosses indicate the maximum values of the stress at the edge of the slab in a direction parallel to the joint. These stresses reach a maximum at the point of load application in those cases where the load is at some distance from a corner.

In figure 8 points G and I are shown on the longitudinal centerline of the panel. Tests were made at these points and at many other points along the transverse joint or free edge and it was found that the edge condition shown by the typical data in figure 33 applies at all points along the edge except within a distance of approximately 3 feet of a corner. Within this distance there is a gradual transition from the edge to the corner condition. For the corner condition the bending stress under the load is negligible and the critical stress is found to be a tensile stress at some distance from the load and along the bisector of the corner angle.

The data in figure 33 show that the most important stress to be controlled by a transverse joint for a loading such as that at point G is that occurring directly under the load and in a direction parallel to the slab edge. Using the method of calculating structural efficiency from stress values that was described earlier in this report, the average values for each of the joints were determined. These values as given in table 10 are not based on the data shown in figure 33 together with the corresponding data for the center of the slab, but upon similar and much more extensive tests in which only the strains occurring directly under the load were measured.

TABLE 10.—*Efficiencies of the various transverse joints for controlling the stresses caused by loads placed near the joint edges*

Test section no.	Type of joint	Spacing of dowels	Joint opening	Joint efficiency: Winter	Summer	Average (various seasons)	Over dowels	Between dowels
		Inches	*Inches*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
1	Thickened end	None	½			57		
8	Dowel	36	½				46	8
6	do	27	½				31	6
9	do	27	¾				16	20
7	do	18	½				23	8
10	do	18	¾				40	28
4	Plane of weakness	18		71	66			
3	do	None		4	41			
2	Dowel plate		½			59		
5	do		¾			66		

The joint in section 1 differs from the others in that there is no connection between the two ends of the

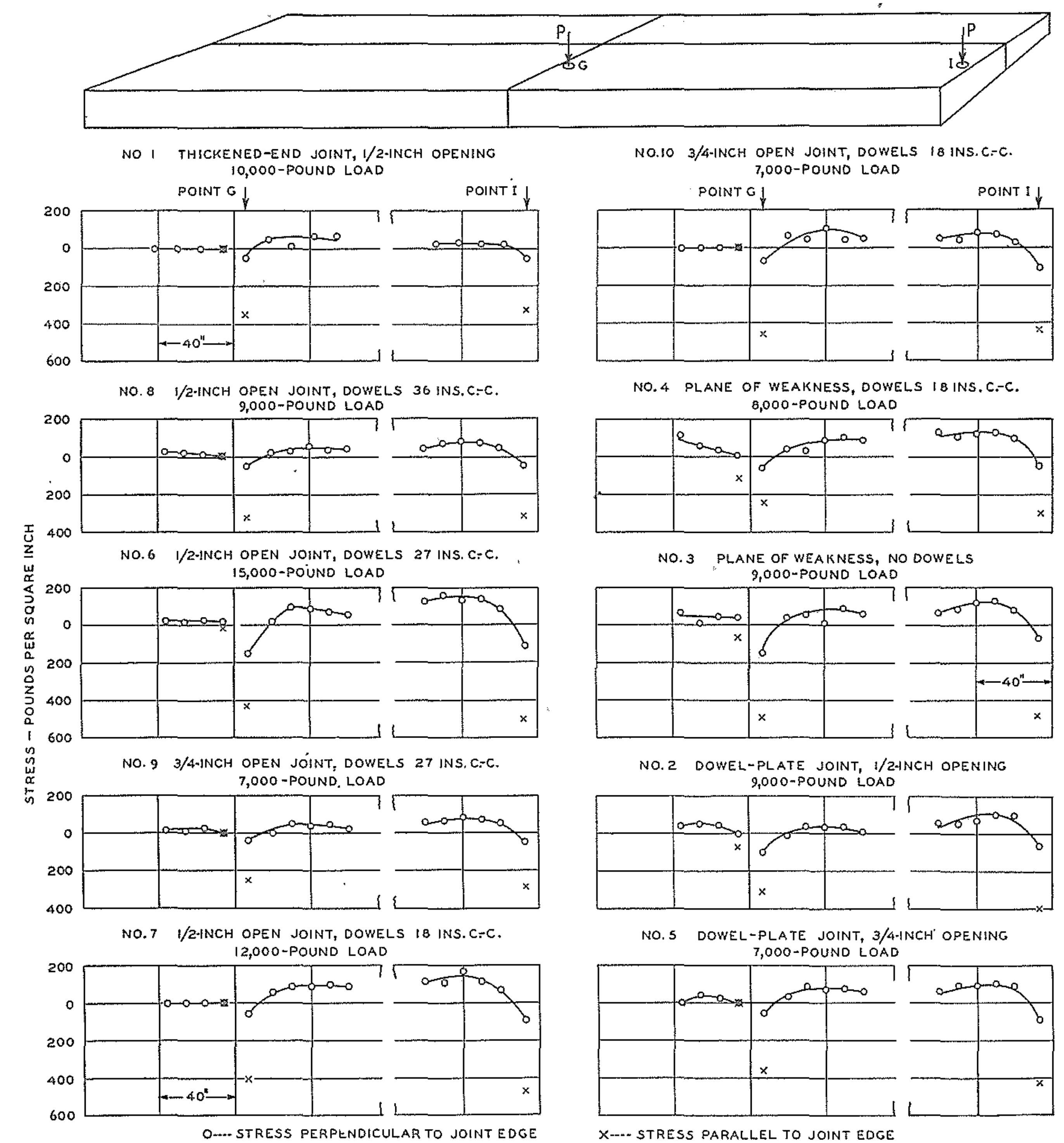


FIGURE 33.—COMPARISON OF LOAD STRESSES MEASURED AT THE FREE EDGES AND AT THE TRANSVERSE JOINT EDGES OF EACH OF THE SECTIONS. VALUES ABOVE THE AXIS INDICATE TENSION, AND VALUES BELOW THE AXIS INDICATE COMPRESSION IN THE UPPER SURFACE OF THE SLAB.

slabs that form the joint, the edges being strengthened by edge thickening. To make these edges as strong as the center area it is necessary to design the transverse joint edges according to the principles that were described in part 3 of this series of papers.

As indicated in table 10 the doweled joints were tested at points both directly over and midway between dowels. The efficiencies of these joints are generally low at all points, but for loads applied over a dowel the efficiency is often much higher than for loads applied midway between dowels.

EFFICIENCIES OF VARIOUS LONGITUDINAL JOINTS COMPARED FOR LOADS NEAR JOINT EDGES

The dowel spacing was varied in the different sections for the purpose of bringing out the effect of dowel spacing on structural efficiency. It has been learned during the testing of the sections that for edge tests the deflections are so small that the stiffness of the two slab ends, as determined by slab thickness, and the stiffness of the structural connection as determined by the width of the joint opening and by the spacing of the units in the case of the doweled joints, are two very important factors which affect the structural action of the joint. To study the matter of dowel spacing properly, the slabs should be of the same thickness and the joint openings should be the same throughout, leaving the single variable of dowel spacing. This does not mean that the data obtained are of no value but it does explain the apparently inconsistent relations which appear when the data are examined from the standpoint of dowel spacing alone.

The effect of the spacing of the dowels is largely eliminated in the data for loads applied directly over a dowel. The low efficiency for this loading indicates the inadequacy of a ¾-inch dowel installed in this manner for transmitting load across joint openings such as were used in this investigation. The efficiencies are somewhat higher for the ½-inch opening than for the ¾-inch opening although the variation in slab stiffness complicates the data. Some looseness of the dowels may have been present and deflection of the dowels certainly occurred, both of which would lower the efficiency of the joint and to the greatest degree in thick slabs. The matter of dowel spacing will be discussed on a theoretical basis later in this report.

The data in table 10 show a great difference in the efficiency of the weakened-plane joint, without dowels, in winter as compared with summer. The very low efficiency of this joint during the cold season results from opening of the joint as the pavement contracts. It would appear that aggregate interlock cannot be depended upon to transfer load effectively when the pavement is in a contracted condition even on relatively short slabs such as these. For longer slabs the reduction might be still greater, while for shorter slabs it might be expected to be less.

The efficiency of the weakened-plane joint with ¾-inch dowel bars at intervals of 18 inches was found to be high at all seasons of the year.

With the dowel plates, joint openings of one-half inch and three-fourths inch were used to determine the effect of this variable. It will be noted that the joint with the wider opening shows a slightly higher efficiency, contrary to what might be expected. The plate in this case was called upon to deflect a 6-inch slab across a ¾-inch opening, while in the other case a plate of the same size had to deflect a 7-inch slab across a ½-inch opening. The effect of the difference in joint opening is thus obscured by the complicating variable of slab thickness. Both joints appear to be quite efficient in slab edges of this general thickness.

Figure 34 shows typical stress data corresponding to those shown in figure 33 but obtained in tests at longitudinal joints, the loads being applied at points A and B. As stated previously, the stress conditions shown were found to apply at all points along the joint except within approximately 3 feet of a slab corner. The data in this figure indicate again that the critical stress for a load acting near a joint is found directly under the load and in a direction parallel to the slab edge.

The stress data in this figure are shown for both the constant-thickness and the thickened-edge slab. Since the stresses for loads applied at point A are affected by the slab thickness at this point, direct comparison of the stresses at points A and B does not give a true indication of joint efficiency for the thickened-edge slab.

Table 11 contains efficiency values for the longitudinal joints calculated in the same manner as those in table 10 for transverse joints. The stress values used in these computations were average values obtained in tests at a great many points. The loads were placed arbitrarily at points over and at various points between dowels in order that the final averages might be representative of average conditions along a joint of the particular type being tested. The difficulty mentioned in connection with thickened-edge slabs was overcome in the following manner. An average empirical relation was established between the interior and edge stresses on the constant-thickness slabs. This relation was applied to the interior stress of the thickened-edge slabs to determine what the edge stress would have been had the free edge been of the same thickness as the longitudinal joint edge, and this calculated value for free-edge stress was used in the efficiency formula.

TABLE 11.—*Efficiencies of the various longitudinal joints for controlling the stresses caused by loads placed near the joint edges*

Test section no.	Type of joint	Type of tongue	Spacing of dowels [1]	Joint efficiency
			Inches	*Percent*
3	Tongue	Rectangular	60	78
5	do	Triangular	60	75
10	do	Corrugated	60	72
4	do	Rectangular	None	50
9	Butt		24	52
8	do		36	42
2	do		48	51
1	do		60	47
6	Plane of weakness		60	44
7	do		None	39

[1] All dowels across longitudinal joints were fully bonded.

EFFECT OF DOWEL SPACING ON JOINT EFFICIENCY DISCUSSED

All of the tongue-and-groove joints that are held closed by the bonded bars appear to have relatively high efficiencies. The tongue-and-groove joint in section 4 has no dowel bars to hold it together. It was tested in a slightly open condition and it will be noted that, although it has a substantial tongue that is roughly rectangular in shape, a marked reduction in efficiency occurs when the bonded steel is omitted. It appears from this table that the shape of the tongue is of little importance in controlling load stresses so long as the joint edges are held together with bonded steel. The highest efficiency value found was with the joint containing the rectangular tongue and groove, however.

In the four butt-type longitudinal joints, the slab thickness at the joint edge was 7 inches in each case, each joint was separated by tarred felt, and ¾-inch dowels were used throughout. The dowels were deformed bars in bond but their function was to transfer load through shear. In all, 59 load tests were made on these four joints at various times and the loads were applied at various distances from a dowel bar.

From the strain data efficiency values were calculated for each of these tests. These efficiency values were grouped according to the distance between the center of the load and the nearest dowel and each group was averaged, from 4 to 16 values constituting a group. These average group values are shown in figure 35 plotted against the space between the load and the dowel and a curve has been drawn through the values. There is considerable dispersion among the values and the curve as drawn may not be correct as to shape. In

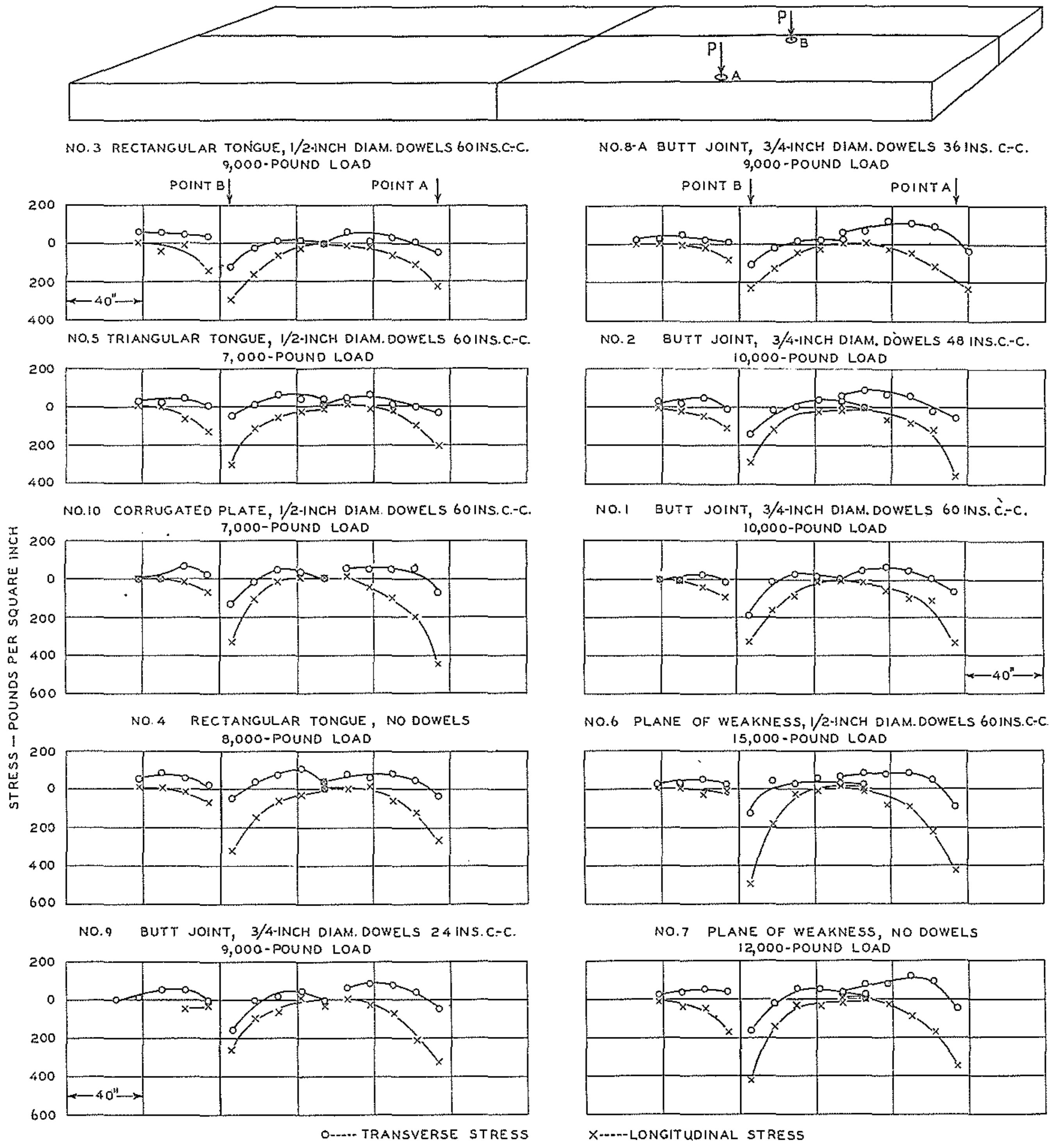


FIGURE 34.—COMPARISON OF LOAD STRESSES MEASURED AT THE FREE EDGES AND AT THE LONGITUDINAL JOINT EDGES OF EACH OF THE SECTIONS. VALUES ABOVE THE AXIS INDICATE TENSION, AND VALUES BELOW THE AXIS INDICATE COMPRESSION IN THE UPPER SURFACE OF THE SLAB.

spite of these deficiencies, it is believed that these data show a useful indication of the effect of dowel spacing on structural efficiency for a joint of such construction that little or no deflection of the load-transfer units can occur.

A comparison of the relative efficiency of the closed, longitudinal, butt joints with the open, transverse, expansion joints having the same dowel spacing shows that the efficiency of a given longitudinal joint is much higher than that of the corresponding transverse joint, particularly for loads applied between dowels. It is obvious that the conditions for load transfer through the dowels in these longitudinal joints are much more favorable than they are in any of the doweled transverse joints.

Neither the butt-type longitudinal joints as a group nor the weakened-plane longitudinal joints were found to have efficiencies comparable to the tongue-and-

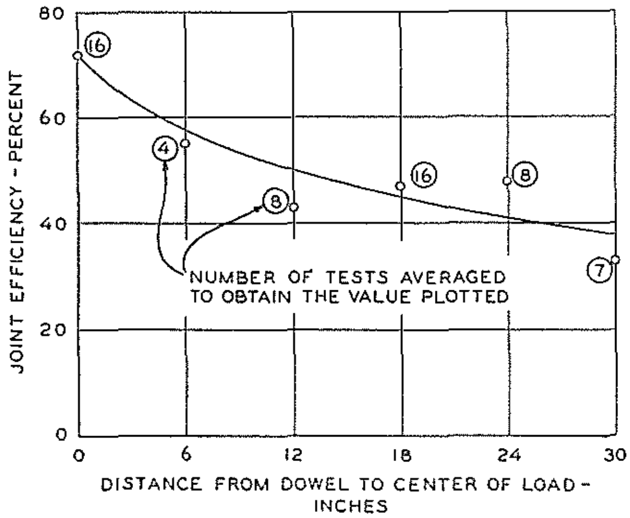


FIGURE 35.—VARIATION IN JOINT EFFICIENCY WITH DISTANCE BETWEEN LOAD AND NEAREST DOWEL (FROM TESTS OF LONGITUDINAL BUTT JOINTS, SECTIONS 1, 2, 8, AND 9).

groove joints in controlling the stresses that occur directly under a load. It is perhaps surprising that the weakened-plane joint that is held closed by bonded steel bars (section 6) should show such low efficiency. It was found in testing these joints that, for loads at certain positions, the indicated joint efficiency was very high, while at other load positions the efficiency was practically zero. It was frequently noted that at a certain point this joint would be efficient when the load was placed on one side of the joint and inefficient when the load was placed directly opposite on the other side of the joint.

The load stresses that occur directly under a load are of a critical magnitude only over a small area and if the stresses are to be controlled by the action of the joint it is necessary that the joint be effective in transferring load in the immediate vicinity of the load. If the functioning of such a joint is dependent upon the interlocking of the broken edges, then the efficiency will depend upon the tightness of the contact and upon the peculiar form of the fractured face directly under the load. If these are favorable the efficiency may be quite high; if they are not then the joint will not reduce the critical stresses. It will be recalled in this connection that the bars in the longitudinal joint were 60 inches apart. In the transverse joint of the same type in which ¾-inch dowels at 18-inch intervals were used, the indicated efficiency was high under all conditions.

EFFECT OF JOINT DESIGN ON CONTROL OF CORNER STRESSES STUDIED

The discussion of the stress data has thus far been confined to the effectiveness of the different joint designs in controlling or reducing the critical stresses that occur when a load is applied at a joint edge but at a distance of 3 feet or more from any corner. With certain slab designs, as, for example, those of constant thickness, a critical stress may also be developed when a heavy load is applied at an unsupported corner. In this case the critical tensile stress is no longer found directly under the load but appears along the bisector of the corner angle in the upper surface of the slab and at some distance from the center of load application. The stress-reducing function of a joint design should

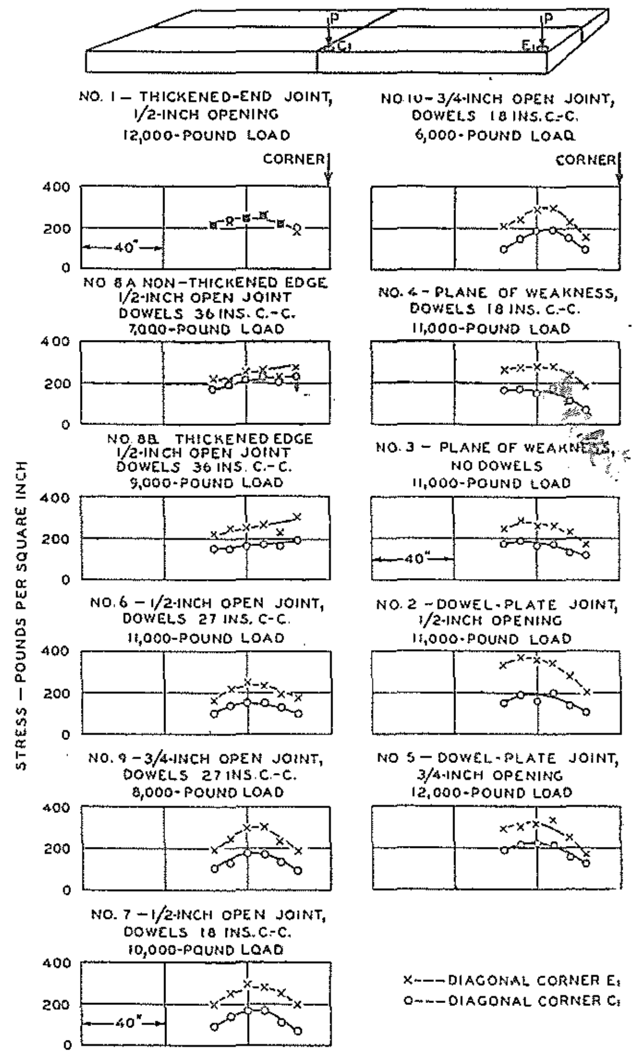


FIGURE 36.—COMPARISON OF THE CRITICAL LOAD STRESSES MEASURED AT THE FREE AND TRANSVERSE JOINT CORNERS. VALUES INDICATE TENSION IN UPPER SURFACE OF SLAB.

extend to the relief of these corner stresses. Since under a given load the slab corner tends to deflect more than the edge, joints that are effective for the edge condition are quite likely to be effective near the slab corner also, but a joint that is quite effective in the corner region may be considerably less effective when the load is applied at an edge but away from a corner.

Figure 36 shows stress data obtained from the load tests that were made for the purpose of determining the efficiency of the various transverse joints in controlling the critical corner stresses. Figure 37 shows similar data obtained in the same way in tests at the four longitudinal joints that were used in the constant-thickness sections. The reason that data are not given for the other sections has already been discussed.

The stress values shown in these two figures are the averages obtained from tests at four corners in each section. From them stress-reduction values were calculated for each of the joints tested and these are given in table 12. It should be kept in mind that the stress-reduction values shown in the table are not a measure of the general structural efficiency of the different joints but only an indication of the relative ability of the joints to control the critical stresses caused by a load acting at a slab corner. The values are simply the percentage of reduction of the free-corner stress obtained through the use of the various joint constructions.

It will be noted that there are no values in this table for sections 1 and 8. In section 1 the free and joint ends of the slab are of identical construction and the stresses in the free and test joint corners should be of the same magnitude for a given load. Section 8 has a lip-curb design and because of the difficulties in testing caused by the shape of the cross section and the fact that the number of corners available for comparisons are very limited, comparisons were not made.

TABLE 12.—*Reduction in corner stress caused by transverse and longitudinal joint action*

TRANSVERSE JOINTS

Test section no.	Type of joint	Spacing of dowels	Joint opening	Stress: At free corner	Stress: At joint corner	Stress: Difference	Reduction in corner stress
		Inches	*Inches*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Percent*
1	Thickened end	None	½				
8	Dowel	36	½				
6	do	27	½	247	154	93	38
9	do	27	¾	302	176	126	42
7	do	18	½	295	168	127	43
10	do	18	¾	299	195	104	35
4	Plane of weakness	18		280	172	108	39
3	do	None		283	186	97	34
2	Dowel plate		½	370	203	167	45
5	do		¾	336	225	111	33

LONGITUDINAL JOINTS

Test section no.	Type of joint	Spacing of dowels	Joint opening	Stress: At free corner	Stress: At joint corner	Stress: Difference	Reduction in corner stress
10	Corrugated tongue	60		298	150	148	50
9	Butt	24		302	136	166	55
6	Plane of weakness	60		247	134	113	46
7	do	None		295	180	115	39

Theoretically the maximum amount of load which can be transferred by a joint design can never quite equal 50 percent of that applied to the one side of the joint because of the eccentricity of the point of load application with respect to the joint. Under ideal conditions a transfer of approximately one half of the load to the adjoining slab should result in a corresponding reduction of approximately 50 percent in the critical stress. In the case of a corner this should apply also and as a matter of fact, because of the distributed nature of the bending that accompanies corner deflection, in practice it would be expected to apply even more to corners than to edges. It is probable, therefore, that the actual efficiency of the joints in reducing the critical stresses at corners is approximately double the values listed in table 12.

It was shown previously by the deflection data that it is not possible for a joint to have an indicated efficiency of 100 percent (based upon a comparison of deflections at the free and joint edges) unless the slab is in perfect contact with the subgrade. Since the slabs were unwarped when the corner loadings were applied and thus perfect contact with the subgrade did not exist, it is probable that the percentage of load actually transferred is somewhat more than one might assume from the stress reduction values given in table 12. The reasons for this have been discussed previously in connection with the application of the second method of analysis to the deflection data.

Considering all of the evidence regarding the ability of the various transverse and longitudinal joints to reduce or control the critical stresses resulting from a load applied near a slab corner, it is indicated that practically all of the joints have a relatively high degree of effectiveness.

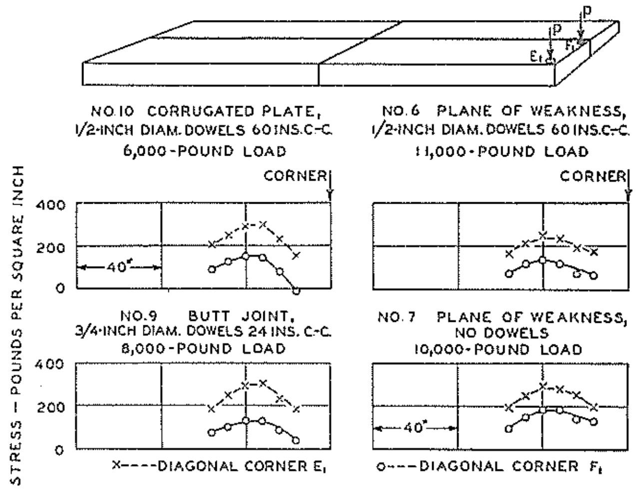


FIGURE 37.—COMPARISON OF THE CRITICAL LOAD STRESSES MEASURED AT THE FREE AND LONGITUDINAL JOINT CORNERS. VALUES INDICATE TENSION IN UPPER SURFACE OF SLAB.

EFFECT OF DOWEL SPACING ON JOINT EFFICIENCY DISCUSSED FROM A THEORETICAL STANDPOINT

The transverse joints of the weakened-plane type were tested during the winter when they were in the opened condition. However, the amount of opening resulting from temperature contraction was not large in slabs of this length. This probably explains the fairly high degree of effectiveness shown by the undoweled joint.

The dowel-plate joint having the ½-inch joint opening appears to be somewhat more effective in controlling corner stresses than does the similar joint with the ¾-inch opening. In the case of the doweled joints containing the ¾-inch diameter round bars the effect of joint opening is not definite, probably for the reasons previously discussed. The same is true for indications as to the effect of dowel spacing.

It will be noted that two of the longitudinal joints, on the basis of the corner stress-reduction data, appear to transfer a full half of the load across the joint to the adjacent slab (sections 9 and 10). Section 9 is a 7-inch uniform-thickness slab having a longitudinal joint of the butt type crossed by ¾-inch bonded dowels at 24-inch centers, while section 10 is a 6-inch uniform-thickness section having a longitudinal joint consisting of a corrugated, steel dividing plate and held together with ½-inch bars at 60-inch intervals. Thus, in each, conditions are favorable for the development of a resisting moment and a high degree of load transfer.

The effect of edge thickening in reducing the corner stresses of the thickened-edge slabs is of interest in connection with joint design even though it may not be considered an actual joint design problem. An indication of this effect may be obtained from the stress curves in figure 36 by comparing the stresses at the free corners of thickened-edge slabs with those at the corresponding point of comparable slabs of uniform thickness.

It has been shown that, for a number of reasons, it has not been possible during this investigation to develop from the test data as complete information regarding the proper dowel spacing to control efficiently the stresses that occur directly under a load applied near a

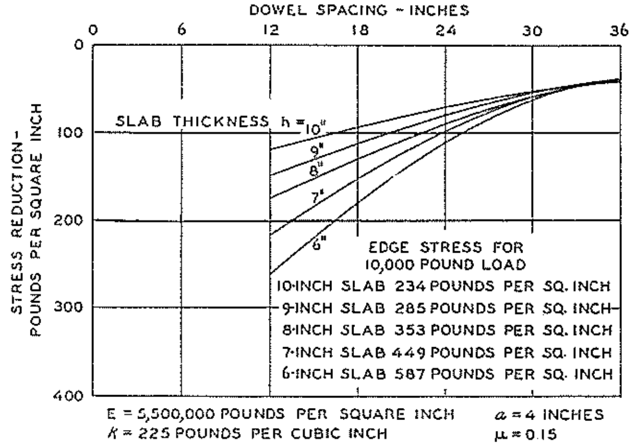


FIGURE 38.—EFFECT OF DOWEL SPACING ON REDUCTION OF EDGE STRESS COMPUTED BY WESTERGAARD'S EXACT METHOD. RELATIONS SHOWN FOR A COMMON LOAD.

joint as it is desirable to obtain. This was caused in part by the fact that most of the dowel spacings were too great to be effective and in part by the presence of other complicating variables in a number of the tests.

It is believed that a short discussion of the subject from a theoretical standpoint will help to clarify the general relations between load, stress, slab thickness, and dowel spacing.

Making use of the more exact formulas developed by Westergaard in his analysis of this subject,[2] that is, the formulas in which the reactions of the four dowels nearest the load are taken into account, the values that determine the sets of curves shown in figures 38 and 39 were computed. The constants used in the calculations were appropriate to the conditions of the tests at Arlington. In the analysis by Westergaard it was assumed that the dowels were of sufficient stiffness to cause the two sides of the joint to deflect equally. Since dowels do not perform in this ideal manner, it is to be expected that the theoretical stress reductions for given conditions will be greater than those that will be obtained in practice, with joints as they are constructed at the present time.

The stress reductions shown in figure 38 are for a constant load of 10,000 pounds applied on slabs of 6, 7, 8, 9, and 10-inch thicknesses. The stresses theoretically developed in the free edge of each slab by this same load are tabulated in the lower part of this figure.

In figure 39 similar relations are shown, but in this case the magnitude of the applied load was varied in order that the edge stress in each of the various thicknesses of slab would have a constant value of 300 pounds per square inch.

Both of these figures show very clearly that both the amount and the rate of stress reduction increase as the dowel spacing decreases. It is indicated that, even for the ideal condition represented by the basic specification of the analysis, dowels spaced 3 feet or more apart are of little value in reducing slab stresses. When the dowel spacing is 2 feet or less, the dowel reactions become more effective in reducing stress and the analysis shows that if dowels are to be of appreciable value in reducing edge stresses, they must be closely spaced, even when complete rigidity exists in the dowel.

[2] Spacing of Dowels, by H. M. Westergaard. Proceedings, Eighth Annual Meeting of the Highway Research Board, 1928, pp. 154–158. [Footnote 12 in the first installment of this paper (PUBLIC ROADS, September 1936, p. 147) is incorrect, and should refer to the above article.]

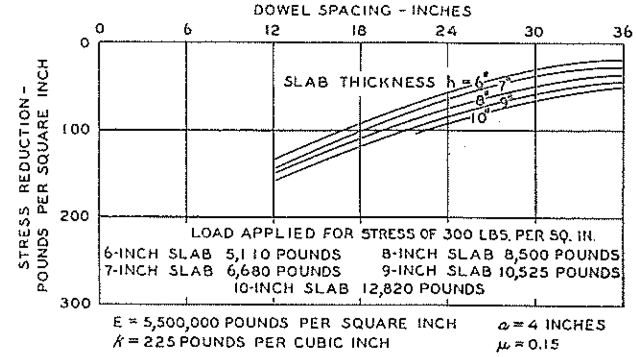


FIGURE 39.—EFFECT OF DOWEL SPACING ON REDUCTION OF EDGE STRESS COMPUTED BY WESTERGAARD'S EXACT METHOD. RELATIONS SHOWN FOR A COMMON EDGE STRESS.

Both theory and experiment show that a load that will produce a critical stress of 300 pounds per square inch in the free edge of a slab will cause a critical stress of slightly less than 200 pounds per square inch when applied in the interior of the slab (provided the slab is of uniform thickness). Thus in this type of slab complete continuity will effect a stress reduction of a little more than 100 pounds per square inch. There are a number of other factors that affect this relation somewhat but the above general statement is approximately true.

Figure 39 shows that, theoretically, in order to accomplish the same reduction with dowels that is obtained by the continuity of the slab (about 100 pounds per square inch), a dowel spacing of approximately 21 inches would be required with a slab 9 inches thick and a spacing of approximately 17 inches with a slab 6 inches thick.

The data presented in this report show that in joints of the types tested, the stress reductions to be expected in joints as actually constructed will fall considerably short of that theoretically possible.

If a doweled joint is to bring about a satisfactory control of edge stresses it would appear that the dowel units will have to provide more shear resistance individually and be spaced much nearer to each other than has been the practice in the past.

MEASUREMENTS MADE OF COMBINED STRESSES AT JOINTS

A pavement slab to be perfectly designed structurally should be so proportioned that a given load, wherever applied, would at all times produce no more than a selected maximum stress at any point. Such a design would make the most economical use of the material and would have no weak spots at which overstressing could occur and failure begin.

The load-carrying capacity of any pavement slab should be indicated by the most critical combinations of load and warping stresses at the different parts of the slab, and the more nearly the ideal design is approached the more readily will these combined stresses attain a common value. There is, however, one consideration that should be mentioned because it affects the generality of application of the statement that was just made. For a load placed at the edge of a slab or at an interior point the load stress is highly localized as has been shown in a number of the figures of this and the preceding papers. When a load is applied on the corner of a slab the distribution of the high stress values is considerably greater. It seems probable, therefore, that a combined stress having a magnitude that would cause a structural failure in the case of a corner loading would not necessarily produce a structural failure when developed under the load in the case of an edge or interior loading. If this is true, a pavement slab should be so designed that the combined load and warping stress at the corner is less than at other parts of the slab.

Since the interior portion of the slab is inherently the strongest and comprises the greatest area, the object of the design should be to increase the load-carrying capacity of the free and joint edges and of the corners to equal that of the interior of the slab. With this thought in mind table 13 was prepared. The values in the four columns headed "Maximum load stresses" show the magnitudes of the maximum stresses for various positions of a given load, expressed as a percentage of those found for an interior position of this load (point H). The values shown in the columns headed "Warping stresses" are expressed in the same way and are taken from the maximum average warping stresses measured on the two test sections concerned and published in the second report of this series.

The first two columns of each group show data obtained from the thickened-edge slab (sec. 5) while the third and fourth columns contain similar values which apply to three points along the free edge of the 6-inch constant-thickness slab (sec. 10). Other factors being constant, these are the three points where the relations for a constant-thickness slab might be expected to be different from those for a thickened-edge slab.

The warping stresses are applicable only to slabs of the general dimensions tested in this investigation.

TABLE 13.—*Relation of both critical load and warping stresses at points near the free and joint edges of typical slabs compared to the stresses at the interior*

Load at point	Maximum load stresses (upper surface of slab) (percentage of that at point H on each section)				Warping stresses (percentage of that at point H on each section) [1]			
	Section 5		Section 10		Section 5		Section 10	
	Compression	Tension	Compression	Tension	Compression	Tension [2]	Compression	Tension
	Percent	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
E		71		130	0	11		11
A	109	19	159	45	115		80	
C		48		88	0	11		11
I	159	48			22			
H	100	0			100			
G	120	34			22			
F	84	67			0	11		
B	115	26			80			
D	110	34			0	11		

[1] Warping stresses are for the same points in the upper surface of the slab for which load stresses are given and are for conditions of average maximum warping.
[2] Values are for stresses parallel to the bisector of the corner angle since they are to be combined with the load stresses in that direction.

It will be noted that two load stresses are shown for each point of load application on these two slabs except for points E and C. The values shown in the first column in each case are stresses in the top of the slab directly under the load. These are not shown for points E and C because of their very small magnitude. The second column in each case contains the maximum stresses occurring in the top of the slab at some distance from the area of load application. The efficiency of the longitudinal and the transverse joints naturally affects some of the values given so that relations shown in the table apply to pavements of equivalent cross section and joint efficiency.

The effect of edge thickening on the load and warping stresses at the various points is apparent if the values pertaining to them for section 5 are compared to those for the same points on section 10. Except in those cases where the edge thickening affects the relation, the values shown for points at the free edges and free ends of the sections would apply equally well for joints having little or no structural effectiveness. The effect of the joint design in section 10 in reducing load stresses is reflected in the comparative magnitude of the load-stress values at the corresponding points on the free and joint corners of this constant-thickness slab.

IMPORTANCE OF CONTROLLING LONGITUDINAL WARPING STRESSES EMPHASIZED

There is a small variation in the relation between the load-stress values at the various parts of a pavement slab at different seasons of the year. This will be discussed more thoroughly in the next paper of this series. The relations shown in table 13 for the critical stresses directly under the load are based on data obtained from a great many tests made during the winter months. The values shown for the less critical stresses (those not directly under the load) are based upon less extensive data obtained in tests made at various seasons of the year, although wherever possible the relations shown are averages from several tests.

In order to emphasize the importance of the data just shown and to present them in a more easily assimilated form, table 14 was prepared. In this table the critical combined stresses are given in absolute units and represent stresses that might reasonably be expected to develop in each of the two slabs under the action of a 7,000-pound load and temperature warping of average maximum intensity as determined during the course of this investigation. The values follow directly from the percentages given in the preceding table. In all cases except at the corners the stresses apply to afternoon conditions. For the corners the warping is that which occurs during the night. It was explained in a previous report that it was not possible to determine the corner warping stresses for a thickened-edge slab. For this reason it was necessary to apply to the thickened-edge slab the corner warping

TABLE 14.—*Combined critical load and warping stresses* [1] *at the midpoint and at points near the free and joint edges of panels of two of the test sections*

Load at point	Load stress [2]		Warping stress		Combined stress		Combined stress (percentage of that at point H)	
	Compression	Tension	Compression	Tension	Compression	Tension	Compression	Tension
	Lbs. per sq. in.	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Percent*	*Percent*
Section 5:								
E	0	176	0	40	0	216	0	36
A	268		413		681		112	
C		119	0	40		159		26
I	393		80		473		78	
H	247		360		607		100	
G	297		80		377		62	
F	207	165	0	40	207	205	34	34
B	285		320		605		100	
D	272	84	0	40	272	124	45	20
Section 10:								
E		321		40		361		60
A	393		320		713		117	
C		217		40		257		42

[1] Maximum stresses in upper surface of slab.
[2] The load stresses in each section were produced with a 7,000-pound load.

stresses determined from measurements on the corner of a constant-thickness slab. The magnitude of these stresses is so small that this method should introduce no error of consequence.

The warping stresses shown in this table are for slabs 10 feet wide and 20 feet long. It was brought out in the discussion of warping stresses in the second report that, for slabs of the thicknesses used, the maximum warping stresses are approximately as large as they would be for much longer slabs of the same width. The combined stresses shown in table 14 should therefore represent the condition where effective control of warping stress has not been provided.

There was no opportunity in this investigation to make an extensive study of warping stresses on short slabs, but the work that was done indicated that the magnitude of the critical warping stresses would be greatly reduced as the length of the slab was reduced below the 20-foot length used in this series of tests. For short slabs the values of the combined stress will tend to approach the value of the load stress alone.

It is obvious from table 14 that the most important step in the effort to balance the combined stress values is to reduce the warping stress at points A, H, and B. The most effective means for doing this seems to be by shortening the length of the slab. This has already been discussed in connection with cross-section design in the preceding paper and needs no further discussion here. The effect of edge thickening on the load and warping stresses was also discussed in a previous report.

One of the most difficult problems in connection with concrete pavement construction is the control of transverse cracking. It is important to control the critical load stress along a slab edge abutting a longitudinal joint because this stress combines directly with a warping stress that tends naturally to be high. The combined stress, being a longitudinal stress, is in a position to start the formation of a transverse crack if its value becomes excessive. Longitudinal joint designs of high structural efficiency are desirable therefore as an aid in controlling transverse cracking.

JOINTS SHOULD PERMIT FREE FLEXURE OF SLAB EDGES

The longitudinal joints in both sections 5 and 10 were very effective in reducing the stresses under the load and it is apparent from table 14 that, where the warping stresses are controlled, the cross section of a slab having a thickened edge and an efficient longitudinal joint is very well balanced. Because the width of the slab was but one half of its length, the warping stresses at points I and G are much smaller than those at points A and B. This probably explains why longitudinal cracking is seldom observed in slabs having a width of approximately 10 feet.

It is apparently unnecessary, in order to balance the general design of a pavement slab, to reduce the combined stresses at points I and G unless the warping stresses in the longitudinal direction are controlled. Where these stresses are controlled, leaving practically all of the flexural strength of the slab available for carrying load, then it becomes necessary to provide transverse joints that are effective in reducing the stresses directly under the load, when the load is near the joint, in order to make the load-carrying capacity of the slab at point G comparable to that at the interior point H.

The effect of edge thickening and of the joint construction on the stress conditions of the corners E, C, F, and D are well shown by table 14. The warping stresses at the corners are so low that, on the basis of combined stresses, the corners do not appear to be critical points. Because of the distribution of the maximum stress from a load applied at a corner and because of the greater likelihood of impact, weakened subgrade support resulting from the infiltration of water, and possibly other factors, it appears desirable to make the corners of the slab somewhat stronger in relation to the other parts of the slab than would appear to be necessary from the combined stress values in the table.

A comparison of the load stresses occurring along the bisector of the corner angle, for a load acting at point E on each of the two slabs, shows that edge thickening is very effective in reducing these stresses. The effectiveness of joints in controlling these stresses at joint corners was discussed earlier in this paper in connection with table 12.

It is interesting to note that at the inside corners, where the load stresses along the bisector of the corner angle are very low, the stresses directly under the load become relatively high. This is due to the action of the joints causing the slab at point D to behave more in the manner of the interior of the slab. One joint acting effectively will cause the stresses at this point to be distributed as at a free edge, while with both joints effective a stress distribution more like that which exists in the case of an interior loading is created. Thus the position and magnitude of the critical stress at a slab corner depend upon the action of the joint or joints at that corner. Joints that are very effective in controlling the stresses along the bisector of the corner angle may cause a critical stress condition under a load acting near the corner.

It has already been shown that, from the standpoint of reducing warping stresses, free action of the corners at point D is desirable. Such construction would likewise reduce the load stress just discussed and increase slightly the load stress along the bisector of the corner angle of the slab. Therefore, as far as both warping and load stresses are concerned, the joints should be so designed that resisting moments that prevent free flexure are not developed in the joint.

Earlier in this paper it was stated that joints are introduced into concrete pavements for the purpose of controlling certain stresses that are present from causes other than load, and that joints may be classified according to the stresses they are intended to relieve as follows:

1. Expansion joints to control the direct compression stress caused by expansion of the concrete.
2. Contraction joints to control the direct tensile stresses caused by contraction of the concrete.
3. Warping joints to control the bending stresses resulting from restrained warping.

Data developed during the course of this investigation and reported in this and the two preceding papers of this series permit certain general observations to be made and also suggest certain ways in which the joint designs that were tested may be improved.

SPACING OF EXPANSION, CONTRACTION, AND WARPING JOINTS SHOULD BE APPROXIMATELY 100, 30, AND 10 FEET, RESPECTIVELY

The proper spacing of joints is a matter concerning which there has frequently been a wide difference of opinion. The trend of thought as reflected in construction practice during the past was brought out in the historical review at the beginning of this paper. As recently as the December 1932 meeting of the Highway

Research Board, in a paper on the design of joints,[3] R. D. Bradbury stated that: "The proper spacing of transverse joints is largely a matter of judgment based upon experience". In other words, there was available no rational method by which the proper spacing of joints could be determined.

Since joints cost money it has frequently been the policy to install as few joints as possible and to have these of the cheapest type. It is well to remember, however, that the stress reductions accomplished by the introduction of the joint may be worth more in added load-carrying capacity than the cost of the joint installation. This study indicates that frequency of joints can increase the safe load-carrying capacity of a pavement without any increase in slab thickness. Also, frequent joints and the resulting short slab lengths simplify somewhat the structural requirements of transverse joint designs.

It is not the intention to suggest that as a result of this investigation it is now possible to determine rationally the proper spacing of joints under all conditions. Much additional information is needed before this desirable objective can be attained; particularly needed are data on the effects of radically different subgrade conditions and on a number of factors that affect warping stresses. However, the data already obtained make possible several useful generalizations relative to joints. The tests have shown that the distance between expansion joints will not be determined so much by the magnitude of the compressive stresses during expansion as it will by a consideration of the amount of horizontal movement that it is desirable to permit at any one joint. The data presented in the second report[4] show that for ordinary slab lengths the compressive stresses during expansion are relatively quite small, provided no restraint is offered at the slab ends.

Figure 40 has been prepared from data obtained during these studies to give an idea of the average changes in length that occur annually in concrete pavements from changes in the moisture state of the concrete and in the average temperature of the slab (fig. 40–A), and of those that occur daily from temperature change alone (fig. 40–B). The length changes are in inches and apply to a slab 100 feet in length. This graph shows that the rise in temperature from winter to summer caused an expansion of about 0.45 inch in this length of slab. During this same period a loss of moisture occurred which caused a contraction of about 0.15 inch. The net result of the combined annual volume changes was an expansion of about 0.30 inch from winter to summer. The daily changes in length are approximately 0.03 inch in winter and 0.08 inch in summer. The values apply exactly only to climatic conditions and to concrete having volume-change characteristics such as those which existed in these tests. However, there is nothing unusual about either.

It will be recalled that data presented in the second report showed that the test slabs at Arlington are gradually increasing in length. The ultimate extent of this growth cannot be predicted, but after four annual cycles of length change it amounted to approximately 0.17 inch in a 100-foot slab. Such a change when present will have to be cared for in the expansion-joint design.

[3] Design of Joints in Concrete Pavements, by R. D. Bradbury, Proceedings 12th Annual Meeting, Highway Research Board, 1932, part I, pp. 105–136.
[4] Structural Design of Concrete Pavements, (see fig. 23 and attendant discussion) Public Roads, vol. 16, no. 9, November 1935.

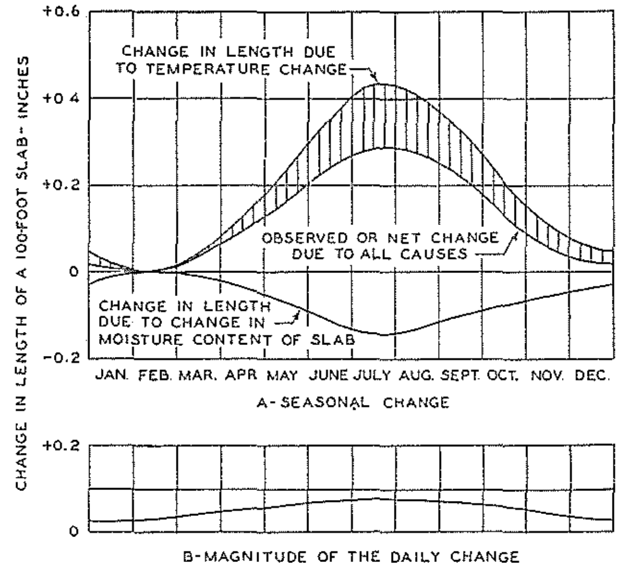


Figure 40.—Average Seasonal and Daily Changes in Length of a 100-Foot Slab Caused by Variations in Temperature and Moisture Content. (Based on Observed Movements.)

In view of the present knowledge on the subject, it seems reasonable to conclude that expansion joints should be provided at no greater than 100-foot intervals in order to keep the joint openings from becoming excessive.

The spacing of contraction joints, unlike that of expansion joints, will be determined by the permissible unit stress in the concrete. If this is restricted to a low value, as is most desirable because of its direct effect on load-carrying capacity, the test data indicate that the contraction-joint interval should be kept quite small, possibly of the general order of 30 feet.

In the second and third papers of the series it was shown that, if the stresses caused by restrained temperature warping are to be properly controlled, the length and width of the slab panels must be kept quite small. Although additional studies should be made to determine what the maximum dimensions should be for various slab thicknesses, the present data indicate that a satisfactory control of warping stresses would ordinarily be obtained if the maximum dimensions of the slab were 10 or 12 feet, indicating that the interval between warping joints should be of the same general order.

EDGE THICKENING AT JOINTS EFFECTIVE ONLY FOR SHORT SLABS

The joint tests in this investigation as originally planned did not include provision for a study of types and arrangement of joints to control warping stresses, and it has not yet been possible to conduct such a study. There are three arrangements that might be considered:

1. Placing joints that will both provide for expansion and relieve contraction stresses at intervals sufficiently small to control warping stress effectively.
2. Placing expansion joints at intervals sufficiently small to relieve contraction stresses and, between the expansion joints, placing joints intended to relieve warping stresses only.
3. Placing expansion joints at the proper intervals, between these placing the contraction joints at the intervals necessary to control tensile stress and, finally, between the contraction joints placing warping joints as frequently as necessary.

In deciding which of these different arrangements should be used and to what extent the ideal installation should be approached there are several factors to be taken into consideration:

1. The effectiveness of the proposed joints in reducing the stresses caused by restrainted warping;
2. The efficiency of the joints in reducing the critical stresses caused by a load acting near the joint:
3. The difficulty of maintaining the joints in a properly sealed and smooth condition;
4. Installation difficulties; and
5. Cost.[5]

The strengthening of slab edges at joints has been applied, in practice, to longitudinal joints and to a more limited extent to transverse joints. The application to longitudinal joints appears to have been successful but there has been some criticism of the attempts to use edge thickening at transverse joints because, on certain projects at least, it is reported that transverse cracks have formed within 3 or 4 feet of the transverse joints. The formation of these cracks is attributed in various ways to the presence of the thickened slab end.

In this investigation one of the sections was constructed with thickened ends and this section has been carefully studied over the entire period of the test. It was found that the thickened ends did not increase the resistance of the subgrade to horizontal slab movement because in slabs 20 feet long the subgrade adhered to the concrete and there was little or no sliding of the slab ends. The data obtained indicated no greater tensile stress in this slab, during contraction, than in one built without the thickened ends.

The design of edge thickening for balancing load stresses was described in the third paper of the series, and it was pointed out that edge thickening to be most effective should be limited to relatively short slabs because of the increased warping stresses that tend to develop under certain conditions. These considerations apply with equal force to both longitudinal and transverse joint edges. It is necessary that special care should be taken in the early curing period of such designs to insulate the slabs and prevent the formation of large temperature differentials. In the report of the curing experiments at Arlington[6] some years ago mention was made of transverse cracking which occurred close to the ends of several of the sections that were not protected from the sun's rays during the first 24 hours after placing. This cracking, which was similar in location to that reported on some of the thickened-end pavements, was attributed to high warping stress during the early period of strength development, pointing to the desirability of insulative coverings for curing concrete pavements.

With thickened-end slabs, blocking of the lower portion of the transverse joint with concrete spilled during construction or with solid matter entering after construction is likely to be a serious matter because of the eccentricity of thrust and consequent greatly increased bending moments that may develop near the joint during expansion of the slabs. It is especially necessary, therefore, that, where thickened ends are to be used at transverse joints, care should be taken to insure that there is space for free expansion at all times.

[5] For a discussion of current costs and other considerations, the reader is referred to a paper entitled "Developments in Transverse Joints and Fillers in Concrete Pavements and Bases" by R. E. Toms, presented before a meeting of the Association of State Highway Officials of the North Atlantic States, Baltimore, Md., Feb. 14, 1935. See also American Highways, Vol. 14, No. 2, April 1935, for a similar discussion by the same author.

[6] The Arlington Curing Experiments, by L. W. Teller and H. L. Bosley, PUBLIC ROADS, Vol. 10, No. 12, February 1930. pp. 218-219.

IMPROVEMENTS IN DESIGN OF DOWELED JOINTS RECOMMENDED

The doweled transverse joints tested were found to be effective in the two functions of permitting unrestrained expansion and contraction and in allowing the slab ends to warp freely. These joints as constructed in this investigation were not satisfactory, however, so far as their ability to reduce the stresses caused by load is concerned. For loads acting at joint corners fair reductions in the critical stress were obtained, and the same is true for loads applied directly over dowels, but for other conditions of loading the stress reductions generally were much smaller than is desirable.

Attempts to improve the doweled joint designs should begin with efforts to increase their effectiveness in reducing the critical stresses caused by a load placed near the joint but at a distance from a corner.

It is indicated that the doweled transverse joints as built and tested in this investigation have the following weaknesses:

1. The individual units were too widely spaced.
2. The individual units were not stiff enough effectively to transfer loads of the magnitude and under the conditions involved.
3. It is difficult to obtain complete and perfect embedment of a dowel bar.
4. Even if perfect embedment were obtained the unit bearing stress on the concrete is apt to be excessive when heavy loads are applied on one side of the joint.

The closest dowel spacing tested was 18 inches and it is evident from the data that, for dowel size, joint openings, slab thicknesses, and loads of the same general order as were used in the tests, this spacing is too great. It is not possible to state, from the test data, what the proper spacing should be in order to make this joint highly effective in relieving the important edge stresses. The minimum spacing of dowels will be determined by the magnitude of the critical stress caused by a load applied at the joint edge at a distance from a corner. If the spacing is close enough to control this stress satisfactorily, the stress conditions for a load acting at the slab corner will also be satisfactorily controlled, so long as no resisting moment is allowed to develop in the joint itself.

It has been shown previously by some of the load-deflection measurements that one very important cause of the low efficiency of the doweled joints in controlling load stresses is the lack of stiffness in the dowel itself This suggests that the size or shape of the dowel should be changed, that the joint opening should be decreased, or that the bearing conditions of the dowel in the slab should be improved in order to increase the resistance to bending of the unit. Any great increase in the bending resistance of the joint is undesirable because it reduces the ability of the joint to relieve warping stress, one of its most important functions. It is necessary, therefore, to proceed cautiously with any changes tending to increase joint stiffness.

Tests made for the purpose indicated that, when the concrete around the dowel is placed with great care, little or no play between the dowel and the concrete existed. It is difficult to be certain that this condition will always be obtained in construction. Indeed, it is to be expected that it will not, unless unusual attention is given to it. Furthermore, although no thorough study has been made of the effect of continued service on the seating of dowels, there is good reason to believe that such usage tends to develop looseness.

Under the small deflections of pavement slabs, continued good bearing is essential if the dowels are to maintain their original effectiveness. This suggests that some bearing other than that of the concrete should be provided in order to make the bearing conditions more effective and permanent. What the best form for such a device should be cannot be determined without more tests. Certainly there are problems connected with its design which will have to be worked out, and this is true also for the other possibilities that have been discussed.

The doweled joint is not an ideal type and probably will never approach closely to its theoretical efficiency, but there is little doubt that it can be improved considerably by correcting its recognized weaknesses. From the information at present available it seems probable that the greatest all-around effectiveness in a joint of this type will be had with dowel members that are not too stiff, that are spaced closely in a joint that is opened as little as possible, and with good bearing of the dowels in the slabs insured through the installation of an effective dowel seat.

FURTHER INFORMATION NEEDED ON ACTION OF VARIOUS JOINTS

The tests made with the limited number of dowel-plate joints included in this investigation indicate that this type is quite effective in relieving warping stress and in reducing the critical stresses caused by loads acting near the joints. The continuous plate, as used in these tests, appears to control the stresses directly under a load more effectively than round dowels at any of the spacings tested.

The tests showed that the dowel-plate joints offer more resistance to expansion and contraction of the slab than do the joints containing the round dowels regardless of their spacing. The concrete was carefully placed around the dowel-plate covers at the time of construction. Because of the small space between the plate and the subgrade special manipulation was necessary but a satisfactory installation was obtained. There can be little doubt that the same tight gripping of the plate in its socket, which caused the resistance to slab movement just mentioned, is responsible for the effectiveness of the construction in reducing the edge stress.

Only two dowel-plate joints were studied and the information developed leaves unanswered a number of questions. For example, it is desirable to know what width and thickness of dowel plate will be most generally effective in slabs of different thickness. Also it is desirable that means be developed for effectively sealing the joint or by other means reducing corrosion of the dowel plate to a minimum.

The data indicate that the dowel-plate joint has considerable merit and that a more thorough study of its possibilities is warranted. Determination of its effectiveness after having been in service for some time would seem to be particularly important.

This investigation revealed that the weakened-plane tranverse joint without dowels is not effective in reducing the stresses directly under a load acting near the joint when the joint is open and may not be effective when the joint is tightly closed. It appears to be fairly effective in reducing corner stresses when closed but may become very ineffective when open. The fact that these joints sometimes do not function effectively though tightly closed is apparent due to an inclined fracture. The character of the support varies from side to side of the joint and from point to point along each side, being effective in some places and quite ineffective in others.

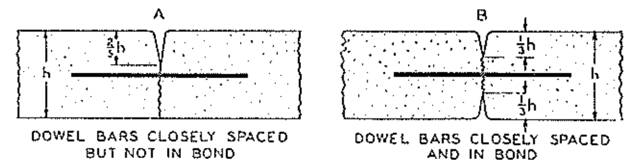


FIGURE 41.—PLANE-OF-WEAKNESS JOINTS DESIGNED TO PERMIT FREE WARPING.

The weakened-plane transverse joint with dowel bars spaced 18 inches apart was found to be much more consistent in its behavior and fairly efficient in reducing corner stresses and stresses directly under the load. There is little to indicate that aggregate interlock can be depended upon to control the critical stresses caused by load under any conditions and this applies to the longitudinal as well as the transverse plane-of-weakness joints. It appears that to control the stresses effectively and thus strengthen the joint edge, the same type and character of edge support will be necessary with a weakened plane of the type tested as would be required with butt joints.

The weakened-plane joint will control warping stresses effectively if it is so designed that a resisting moment within the joint cannot be developed. In a warping joint, prevention of the development of a resisting moment may be accomplished in any one of three ways: (1) By preventing the steel dowels from taking tension through a destruction of bond on one or both halves of the dowel; (2) by preventing the concrete from developing compression by separating the two slab ends; or (3) by greatly reducing the length of the moment arm so that for a given joint deflection the magnitude of the resisting moment is greatly reduced even though the steel dowels take tension and the concrete surfaces are tightly interlocked.

Weakened-plane joints designed to prevent the development of large resisting moments during warping are shown in figure 41. It should be recalled that the downward warping of the slab edges normally exceeds the upward warping by a considerable degree and, further, that under the conditions that cause upward warping of the slab edges, the concrete is in a contracted state and the joints are opened, the dowels being without bond.

In this class only the longitudinal joints of sections 3, 4, 5, and 10 are considered. None of these was intended as an expansion joint and none of the designs included in this group could be expected to function satisfactorily as an expansion joint because the shape of the interlocking elements is such that separation horizontally is in each case accompanied by a separation vertically that would prevent effective load transfer by the joint.

Of the four joints considered, only that in section 4 could be expected to relieve direct tensile stress caused by slab contraction. This joint, it will be recalled, had a trapezoidal tongue roughly rectangular in shape although there is appreciable slope to the upper and lower faces. No dowels or tie bars cross the joint. At the time the load tests were made the joint was opened slightly so that the stress values obtained probably indicate the efficiency under critical conditions.

The joint was found to be fairly effective in reducing the critical corner stress, but, for loads applied at the joint edge at a distance from the corner, the efficiency

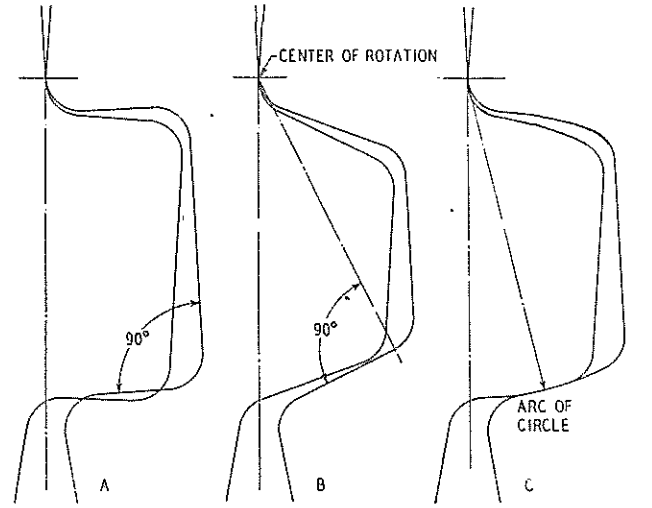


FIGURE 42.—RELATIVE DISPLACEMENTS OF THE VARIOUS PARTS OF TONGUE-AND-GROOVE JOINTS DURING DOWNWARD WARPING.

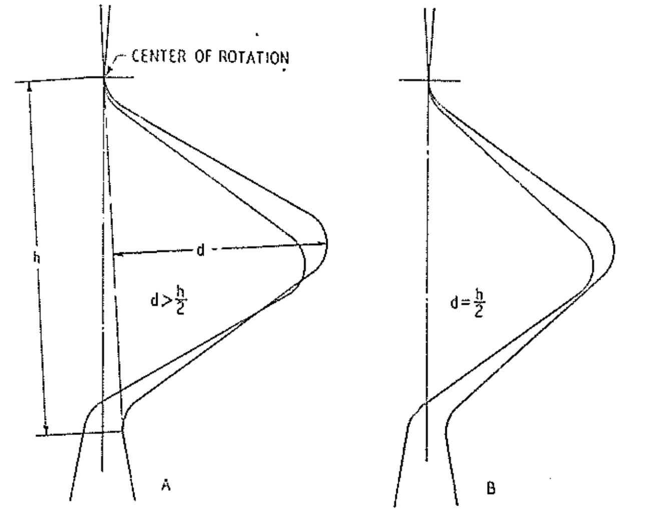


FIGURE 43.—RELATIVE DISPLACEMENTS OF THE VARIOUS PARTS OF TWO TRIANGULAR TONGUE-AND-GROOVE JOINTS DURING DOWNWARD WARPING.

in reducing the critical stress is much less than it was found to be for the same type of joint held closed by bonded steel. This undoubtedly results from the tendency for the tongue to loosen as it is withdrawn from the groove and indicates the necessity for designing a different shape of tongue if this general type is to be considered as a contraction joint. Although a perfectly rectangular tongue section would probably be the most effective design for controlling load stresses, it would restrain warping and, probably to a lesser extent, free horizontal movement. It appears necessary, therefore, that the shape of the tongue and groove should depart from the perfectly rectangular form.

ACTION OF TONGUE-AND-GROOVE JOINTS DURING SLAB WARPING DESCRIBED

Figure 42 illustrates a simple method for determining graphically the relative movements of the two sides of three designs of tongue-and-groove joints during warping of the slab ends. It is assumed that in each design the ends of the two slabs both above and below the tongue and groove have been relieved by inclining the face of the edge slightly as shown in the section. The point of contact and probable center of rotation would be just above the tongue during downward warping and just below the tongue during upward warping, approximately as shown in the figure.

In the first design (fig. 42–A) the upper and lower faces of the tongue are parallel. It is apparent that as warping occurs the tongue will bind in the groove and will not be able to take the position that it would assume if unrestrained warping were to be permitted. Restraint is developed that will cause undesirable warping stress in the slabs near the joints and high local stresses in the elements of the joint itself.

In the second design (fig. 42–B) there is considerable slope to both the upper and lower faces of the tongue. When warping occurs there is a tendency for these faces of the tongue and groove to separate, depriving the joint of its ability to transfer load during small deflections.

Figure 42–C shows a section modified in accordance with the preceding discussion. The upper and lower surfaces of the tongue have been shaped so that neither excessive bearing pressures nor loss of contact should occur during slab warping. It is emphasized that this design is only a suggested application of the results of these tests and should be given an experimental verification before being recommended as a design suitable for contraction and warping joints.

Figure 43 is a similar study of the triangular shape for tongue-and-groove joints. For the assumed conditions it appears that if the depth of the tongue "*d*" is greater than approximately one-half of its height "*h*", warping will cause high local bearing stress near the end of the tongue (fig. 43–A), while if the depth is less than about one-half the height, separation will occur as the slabs bend. This analysis indicates and the test data show that the triangular tongue and groove is likely to be less satisfactory during contraction or warping than the modified rectangular forms.

For the reasons just discussed in connection with the joints shown in figures 42 and 43, it is apparent that the corrugated plate used in the longitudinal joint in section 10 would be unsatisfactory as a contraction joint and would be less satisfactory than the modified rectangular tongue as a joint for the relief of warping stress. It is probable, however, that because of the many possible points of contact and lack of sharp corners, it will not be so likely to develop high local bearing stress as either the perfectly rectangular tongue or the deep triangular shape during warping.

The tongue-and-groove types, as a class, have been shown to be quite effective when constructed and tested in the manner described. The preceding discussion was intended to bring out the weak points of the designs in order that means may be found for improvements that will add to both the structural effectiveness and the durability of the joints.

The doweled transverse joints are not considered to be butt-type joints because of the wide joint opening. Only the four longitudinal butt-type joints found in sections 1, 2, 8, and 9 will be discussed. They are primarily joints for the relief of warping stress, being unable to function either as expansion or contraction joints because of the bonded dowels. Four different dowel spacings were used as was shown in figure 7. As stated earlier, it was not possible to determine the effectiveness of all of these joints in reducing corner stresses because a number of them were in thickened-edge slabs, but all of them were tested to determine their efficiency in reducing the stresses directly under the load when the load was applied along the edge but away from a corner. The results of these tests have been shown in table 11 and in figure 34 of this paper. The one joint of this type on which it was possible to make such determinations was found to be effective in reducing critical stresses for the corner loading.

PREVENTION OF RESISTING MOMENT NECESSARY IN DESIGN OF BUTT-TYPE JOINTS

So far as dowel spacing is concerned, butt-type longitudinal joints can be made more effective in their function of controlling load stress by the close spacing of dowels in the same manner as expansion joints. In the matter of dowel stiffness the situation is different, however, because the small opening between slabs greatly reduces dowel flexure, as was shown by all of the deflection data for these joints. It is probable that in longitudinal joints of the butt type the need for better bearing for the dowels is as great as in the doweled expansion joint and that the same general type of bearing should be provided.

If restraint to warping is to be eliminated, it is necessary to make some provision for preventing the edges of the abutting slabs from being pressed together during warping, particularly near the upper and lower surfaces of the slabs. This can be accomplished by the introduction of a compressible layer between the edges during construction, as was done in the case of the test slabs, or perhaps better by so shaping the slab edges that the necessary clearance will be provided in a manner similar to that suggested in connection with the plane-of-weakness joints.

The amount of steel that must be placed in a warping joint in order to hold the slab edges together depends primarily upon the amount of resistance to horizontal movement to be overcome and upon the unit stress permissible in the steel.

In joints that contain bonded steel the use of designs that do not permit large resisting moments to develop in the joint is desirable for two reasons. In the first place, the prevention of these moments relieves the concrete of the stresses arising from warping restraint and thus conserves its strength for load-carrying purposes. In the second place, the prevention of these moments will further protect the pavement structure by preventing the steel in the bonded dowels or tie-bars from being overstressed in tension.

The amount of bonded steel likely to be used across a longitudinal joint will be sufficient to prevent large separations of the two slab edges during contraction, but will be insufficient to prevent some separation of the slab edges resulting from angular change during warping. Indeed it is desirable that the amount of restraint to these rotational movements during warping be kept as small as possible. A given temperature differential in the pavement tends to cause a given rotational movement of the abutting faces at the joint. If this rotation brings the concrete into tight contact, it develops compression in the concrete and this tends to separate the slab edges by a certain amount at the plane of the steel. For a given percentage of steel taking tension, the magnitude of the tensile stress developed in the steel when this given separation occurs will depend directly upon the effective length of the steel that is yielding under the tension.

If the bond is deliberately prevented for a few inches in the center of the bar, as for example, with a coating of bitumen, more bar length would be available to yield under the given force. The unit deformation in the critical section of the bar would be smaller, and the stress would be correspondingly reduced. The net result would be less restraint in the joint for a given temperature differential and a given percentage of steel. Furthermore, such a coating would protect from corrosion the most vulnerable part of the bar.[7]

With designs that will permit resisting moments to develop during warping it is not possible to calculate the amount of steel required, but with these moments eliminated the calculation becomes a relatively simple matter.

In the discussion of joints in this article there has been presented (1) a brief history of joint development up to the time at which this investigation was planned; (2) a description of the joints that were studied and of the manner in which they were tested; (3) a presentation and discussion of all pertinent data bearing upon the ability of the various joint designs to relieve the stresses caused by expansion, contraction, restrained warping, and applied load; and (4) a discussion of certain improvements in design suggested by the results of the tests.

CONCLUSIONS

The following statements give what are believed to be the most important conclusions to be drawn as a result of this study:

1. Joints are installed in concrete pavements for the purpose of conserving the natural flexural strength of the slab for its primary function of carrying loads. This is accomplished through the relief and control of the stresses caused by expansion, contraction, and restrained warping. Joints in concrete pavements should therefore be so designed and so spaced as to permit the entire pavement to expand, contract, and warp with a minimum of restraint.

2. While the proper spacing of joints to accomplish this end was not definitely determined by this investigation, it is indicated that joints to control warping should be spaced at intervals of the general order of 10 feet, that expansion will be satisfactorily cared for by suitable joints at intervals of approximately 100 feet, and that contraction joints should be installed at some lesser interval, the length of which must be such that the direct tensile stresses in the concrete are definitely limited to low values. Data presented in the second report of this series indicate that under the conditions of these tests a slab length of the order of 30 feet would accomplish this.

3. Since a free edge is a structural weak spot in a slab of uniform thickness, it is necessary to strengthen the joint edges by thickening the slab at this point or by the introduction of some mechanism for transferring a part of the applied load across the joint to the adjacent slab. Otherwise, the strength of the joint edge will determine the load-carrying capacity of the pavement.

4. The structural effectiveness of a joint design is measured by its ability to reduce the critical edge stress to a value equal to the critical stress which exists in the interior area of the slab.

5. The most critical stress caused by a load applied at a joint but away from a corner is that directly under the load in a direction parallel to the joint. It is especially desirable to control these stresses along a

[7] The idea of coating the midsection of bonded bars with bitumen was suggested by Mr. Bengt Friberg.

longitudinal joint so as to limit the combined load and warping stress to a value that will be unlikely to cause transverse cracking.

6. The most critical stress caused by a load applied at the free corner of a slab of constant thickness is a tensile stress along the bisector of the corner angle and at some distance from the center of load application. Edge thickening reduces this critical stress considerably and at interior corners the action of the longitudinal and transverse joints frequently reduces the critical corner stress to relatively low values.

7. There is nothing in the results of these tests to indicate that edge thickening cannot be applied to the transverse edges of concrete pavement slabs with as much success as to the longitudinal edges. If the full benefits of edge thickening are to be obtained in either case, the slabs must be short.

8. The doweled transverse joints tested in this investigation were found to be quite effective in relieving the stresses caused by expansion, contraction, and warping. They were not particularly effective, however, in controlling the critical stress caused by a load applied near the joint edge.

9. The tests indicate that doweled joints as they are usually designed are deficient in two important respects:

a. The individual units are not sufficiently close together to control effectively the stress developed directly under the load.

b. For joint openings such as are usually employed in expansion joints, the individual dowels are not sufficiently stiff to transfer load effectively. Increasing the stiffness of the dowels will result in an undesirable increase in the restraint to warping offered by the joint and for this reason should not be carried too far.

10. The continuous plate key or dowel plate as used in these tests appears to have considerable merit as a means for load transfer. The joint as built for the tests offers more resistance to expansion and contraction than is desirable and for this and other reasons it is believed that a further study of the type should be made.

11. Aggregate interlock as it occurs in the weakened-plane joints cannot be depended upon to control load stresses. Even when joints of this type are held closed by bonded steel bars there is a wide variation in the value of the critical stress caused by a given load, from side to side of the joint and from point to point along it. For this reason it appears necessary to provide independent means for load transfer in plane-of-weakness joints.

12. The joints of the tongue-and-groove type that were held closed by bonded steel bars were found to be the most efficient structurally of any of those tested. It appears, however, that certain modifications of the designs might improve their action by permitting the slabs to warp more freely and at the same time maintaining the bearing between the tongue and the groove.

13. It was shown in the second report of this series that over a considerable period of time there may be a permanent increase in the length of the pavement slab. In designing the expansion joints for a pavement, consideration should be given to this possibility and some allowance made for it.

THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, PUBLIC ROADS ADMINISTRATION

Reported by L. W. TELLER, Principal Engineer of Tests
and
EARL C. SUTHERLAND, Senior Highway Engineer

PART 5.—AN EXPERIMENTAL STUDY OF THE WESTERGAARD ANALYSIS OF STRESS CONDITIONS IN CONCRETE PAVEMENT SLABS OF UNIFORM THICKNESS

THIS IS the last of a series of reports of an extensive investigation, undertaken by the Public Roads Administration in 1930 with the general objective of developing information that would be of assistance in better understanding the structural action of concrete pavement slabs. Much of the work has been described in four previous reports published in this same journal, as follows:

PART 1.—A Description of the Investigation, vol. 16, No. 8, October 1935.

PART 2.—Observed Effects of Variations in Temperature and Moisture on the Size, Shape and Stress Resistance of Concrete Pavement Slabs, vol. 16, No. 9, November 1935.

PART 3.—A Study of Concrete Pavement Cross Sections, vol. 16, No. 10, December 1935.

PART 4.—A Study of the Structural Action of Several Types of Transverse and Longitudinal Joint Designs, vol. 17, Nos. 7 and 8, September and October 1936.

Since concrete is a material with recognized elastic properties, engineers concerned with the design of concrete pavement have long searched for a theory that would adequately express the relations between applied forces and the resultant stresses in pavement slabs of this material.

A reliable general theory of slab stresses would serve at least three important purposes. In the first place it would enable the designer to determine the thickness and the form that the slab should have to function without failure under specified conditions of loading and support; second, it would make possible accurate estimates of the loads which might be imposed with safety on existing pavements; and third, it would provide a useful tool for judging the relative effects of vehicle loads of various magnitudes in studies of the costs of providing facilities for vehicles of different types and weights.

From time to time, over a period of many years, theoretical treatments of the load-stress relation in concrete pavements have been offered (*9, 4, 10, 1*).[1] None of the early analyses was general in scope. Rather, each was concerned with some special situation that was assumed to be critical and for which rather broad assumptions were sometimes proposed. The principal weakness consistently seemed to lie in the assumptions that were made regarding the conditions of support, although other assumptions open to serious question were sometimes present.

ELASTIC ACTION OF SUBGRADE AN IMPORTANT FACTOR IN WESTERGAARD ANALYSIS

It was not until the original Westergaard analysis was published that a rational theory of general application became available (*23*). In this treatment, it is assumed that the slab acts as a homogeneous, isotropic, elastic solid in equilibrium and that the reactions of the soil are vertical only and are proportional to the deflections of the slab. The relations between applied loads and critical stresses are then developed on the basis of elastic theory for the three cases of a wheel load applied on the surface of the slab at a free corner, at an interior point and at a free edge (at some distance from a corner) respectively.

In the first paper of this series (*18*) mention was made of the Westergaard analysis and it was stated that one of the objects of the investigation being described was to study the elements and relationships of the analysis by means of load tests on full-size pavement slabs of constant thickness together with such collateral tests as might be found necessary. The investigation was planned in 1929, the sections constructed in 1930 and a considerable amount of testing under load was done during 1931 and 1932. These early tests were referred to by Westergaard in his supplementary paper (*25*) in which he says "The tests suggest certain adaptations of the theory. There will be needed some restatements of analytical results and some supplementing and modification of the theory".

The extension of the original analysis that is contained in this second paper permitted a more comprehensive study to be made and, as a result, much additional information has been developed. The present paper contains a description of the work that was done on this part of the general project, with a presentation of the data obtained and a discussion of their significance.

If one studies the Westergaard analysis he is at once impressed by the importance of the assumption which relates to subgrade support. In the original theory (*23*) it was assumed that the reactions of the subgrade are vertical only and are proportional to the deflections of the slab, the reaction per unit of area at a given point being the product of the deflection at that point and a coefficient of subgrade stiffness, k, which was termed the modulus of subgrade reaction. This modulus is normally expressed in pounds per square inch per inch of deflection (or pounds per inch cube).

[1] Numbers in italics refer to bibliography at end of article. The above references are in chronological order.

In the supplementary theory (*25*) Westergaard proposed a new coefficient, K, defined by the relation

$$K=kl$$

in which l is a linear dimension, termed the radius of relative stiffness, which appeared in the original theory.

The reason for the proposed new coefficient was an expectation that K would be less dependent on the stiffness of the pavement slab than k. Westergaard stated that "the truth may lie between the two extreme cases of a constant k and a constant K."

To make practical use of the analysis one must be able to assign a value to the modulus of subgrade reaction for the particular soil structure with which he is concerned. At the time the investigation was undertaken no determinations of the value of such a soil coefficient had been made, so there was no background of experience in testing that would indicate either the probable range in values of the coefficient or a procedure by which values might be obtained. Therefore, it was necessary to devise a test procedure that would indicate how the soil of the subgrade beneath the test sections behaved when subjected to pressure intensities and vertical deformations of the same order as occur under pavement slabs in service.

Because of its importance as a part of the study of the Westergaard analysis and because of its current general interest, this study of the elastic action of the subgrade is reported in some detail in this paper, being presented as a separate section, preceding the discussion of the results of the studies of the other relationships expressed in the analysis.

DETERMINATION OF THE MODULUS OF SUBGRADE REACTION

The ideal subgrade assumed by Westergaard is perfectly elastic, has uniform elastic properties at all points and its vertical deformation varies as a linear function of the vertical pressure exerted on its surface. Such a subgrade probably does not exist and the problem becomes one of determining by some test procedure, how nearly the soil under a given pavement approaches the ideal and what stiffness coefficient, if any, can reasonably be assigned to it for the purpose of applying the method of analysis to a particular problem.

While it is a fact that stress values as computed with the Westergaard equations are not particularly sensitive to variations in the value of the subgrade modulus k, if comparisons are to be made between computed stresses and those determined experimentally, the value of the coefficient k used in the theoretical computations must be determined with at least a fair degree of accuracy before dependable values for the computed stresses can be had.

The elastic deflections of a concrete pavement slab of usual design under the action of normal highway loadings are quite small, probably of the order of 0.05 inch or less, depending upon the position of the load, the details of the slab design and other factors that influence deflection. The area over which deflections occur is, on the other hand, relatively large, as may be seen by referring to load-deflection data presented in Part 3 of this series (*20*). In developing a test to be applied to the soil in place for determining the modulus of subgrade reaction, it seemed only reasonable to study the behavior of the soil when subjected to deformations of the same general order of magnitude as would obtain under a pavement slab deflected by a motor vehicle wheel load and, at the same time, to deform the soil over a relatively large area.

THREE METHODS OF MEASURING LOAD SUSTAINING ABILITY OF SOIL DISCUSSED

There appear to be at least three methods or procedures by which the load sustaining ability of the soil can be measured under field conditions. These may be described briefly, as follows:

1. Load-displacement tests in which loads are applied at the center of rigid circular plates of relatively small size, the pressure intensity on the soil being uniform over the entire area of the plate. In these tests the applied load, the mean vertical plate displacement and usually the time intervals are measured.

2. Load-displacement or load-deflection tests in which the load is applied at the center of slightly flexible rectangular or circular plates of relatively large dimensions. In this case some bending of the plate (or slab) occurs and the pressure intensity under the plate is not uniform throughout the area of its contact with the soil. The load, the vertical displacement of various points throughout the area of the plate and possibly time intervals are measured.

3. Load-deflection tests on full-size pavement slabs in which the load-deflection data are obtained by measurement and used in the Westergaard deflection formulas to provide a value for the soil stiffness coefficient or "modulus of subgrade reaction."

If all three methods were equally satisfactory the first procedure offers the practical advantage of requiring test loads of lesser magnitude with a corresponding reduction in the size of the equipment. Also the number of measurements is less since no plate deflections are involved. Its use is complicated, however, by two conditions. The first is that the ability of a soil to sustain a given unit pressure varies within limits with the area over which the pressure is applied to the soil. This variation may be quite marked and this makes it necessary to determine the effect of size of plate in order to avoid error from using a bearing plate that is too small. The second complication is that the supporting ability of a soil varies with its moisture state and it is necessary, therefore, to take special precautions to insure that the soil on which the bearing plate is placed is in the same physical state and moisture condition as that which will obtain or does obtain under the pavement to be considered.

The second procedure has a certain theoretical appeal but offers considerable practical difficulty as a method of test. In this procedure the plate is deflected by the centrally applied load much as the pavement slab deflects under the action of a wheel load. The shape of the deflected plate must be determined precisely and its vertical displacement measured in order to be able to estimate accurately the volumetric displacement of the soil that is effected by the application of the test load on the plate. The modulus of subgrade reaction is then computed by dividing the load (in pounds) by the volume of displaced soil (in cubic inches). Some use has been made of this procedure in England (*12*). The possibilities of the method should be more thoroughly explored.

When using the third method, the procedure is to apply test loads at the free corner, free edge or interior point of a pavement slab of uniform thickness and of normal size. If the elastic modulus of the concrete in the slab is known, for the moisture and other conditions that obtain, it is possible to determine the value of the effective modulus of subgrade reaction from the maximum slab deflection under the applied load by means of the deflection formulas given by Westergaard in his supplementary paper (*25*). This method for determining the soil stiffness coefficient will be discussed further in a later part of this report in connection with the presentation of deflection data obtained in such tests. The remaining part of this section of the report will be devoted primarily to a discussion of work done with the first method, i. e., load-displacement tests with rigid plates of relatively small size.

Load-displacement tests with rigid plates have been made many times in the past in studies of the bearing capacity of soil for foundations. The data obtained in such tests are not applicable to the problem of pavement support, however, because of the conditions surrounding the tests and the extent to which the soil deformations were carried.

LOAD-DISPLACEMENT TESTS WITH RIGID PLATES

When a rigid plate, supported by soil, is subjected to a vertical force or load applied at its center, the soil deforms and the plate moves downward. This downward movement of the plate under load has been variously termed deflection, displacement, penetration, settlement or subsidence, and tests which make use of this action have been termed accordingly load-deflection—or load-subsidence tests. In this discussion the terms displacement and load-displacement tests will be used.

The load-displacement tests which were a part of this general investigation comprise four series, as follows:

Series 1.—Five circular bearing plates and one square bearing plate were used. The diameters of the circular plates were 8, 12, 16, 20, and 36 inches respectively and the square plate had 48-inch sides. The magnitudes of the loads applied to each plate were such as would cause displacements within the range 0.01 to 0.05 inch, this being the approximate range of concrete pavement slab deflections under the action of legal maximum wheel loads.

The tests of this series were divided into two parts. In the first part, all plates were placed successively, in the descending order of size, at the same location on the subgrade while in the second part of the series the test with each plate was made at a different location but within the same general area.

Series 2.—Eleven circular plates, having diameters of 2, 4, 6, 8, 12, 16, 20, 26, 36, 54 and 84 inches respectively, were used. As in series 1, the applied loads were such as to cause displacements within the range 0.01 to 0.05 inch. In series 2 the tests with each size of plate were made at a separate location on the subgrade.

Series 3.—These tests were the same as those of series 2 except that the maximum plate displacement was increased to approximately 0.25 inch, and a somewhat different loading procedure was followed.

Series 4.—Only the 54-inch diameter plate was used. The plate remained at one location and the displacements were kept within the range 0.01 to 0.05 inch as in series 2. The loads were applied, however, in June and in January.

The first tests made in this study were those of series 1. They were intended to explore the effect of size of bearing plate on the load-displacement relation and also to compare data obtained in a series of tests at one location with those from tests that were identical except that each test of the series was made at a different location in the same general area. Obviously it would be preferable to have each test made in an area undisturbed by previous loading, provided there was sufficient general uniformity in the soil structure to eliminate the possibility of local variations in structure affecting certain tests (particularly those in which small bearing plates were used).

The tests of series 2 were designed to extend considerably the data from series 1 on the effect of size of the load-displacement relation.

The tests of series 3 were to provide information on the comparative supporting ability of the soil when subjected to deformations greater than those which usually occur under rigid pavements. These data extended the range of the information obtained and permitted certain comparisons with other data to be made.

Series 4 was simply a study of the effect of the annual change in the physical state of the soil directly under the slab on its ability to support load. For this reason only the 54-inch diameter bearing plate was used and the displacements were limited to the 0.01 to .005 inch range.

METHODS OF TESTING DESCRIBED

All of the soil tests of the four series were made on a part of the originally prepared subgrade that was reserved for this purpose. The soil was described as a uniform brown silt loam (classification A–4) and detailed information concerning it is contained in the first report of this series (*18*).

When making tests such as those described it is important that the soil on which the bearing plates are placed be in the same physical state as that under the pavement to which the tests are to be related. An effort was made to accomplish this by casting the larger bearing plates (which were of concrete) on the subgrade several months in advance of the first scheduled tests and by casting, at the same time, a number of concrete slabs, each 4 feet square, at those locations where tests with the smaller bearing plates were to be made later. By this means the soil was given the same protection and the same opportunity for moisture equilibrium was afforded as obtained with the test pavement itself.

The larger plates of concrete were left in place and loads were applied at the scheduled time. The smaller plates were of steel and with these the procedure was different. At the proper time the small concrete protecting slab was removed and waterproof paper spread over the area where it had been. A thin layer of portland cement mortar or of plaster of Paris was then spread over the waterproof membrane and the bearing plate was bedded in this mortar. This procedure gave a uniform contact between the soil and the bearing plate yet prevented moisture from the mortar from entering the soil. Figure 1 shows one of these smaller

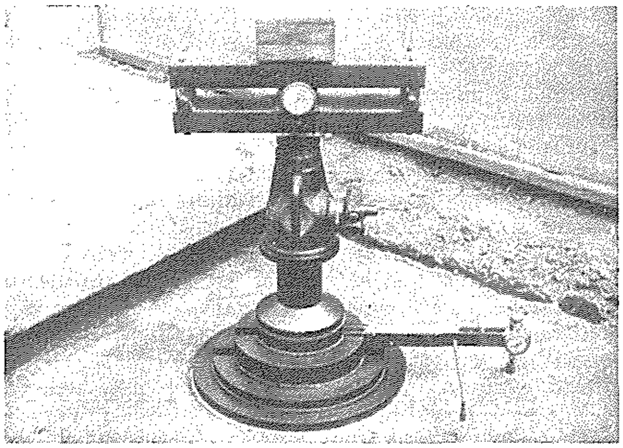


FIGURE 1.—DETERMINING THE MODULUS OF SUBGRADE REACTION WITH A SMALL BEARING PLATE. THE SMALL CONCRETE SLAB THAT COVERED THE SUBGRADE HAS BEEN TURNED BACK. DISPLACEMENT IS MEASURED WITH A CLINOMETER.

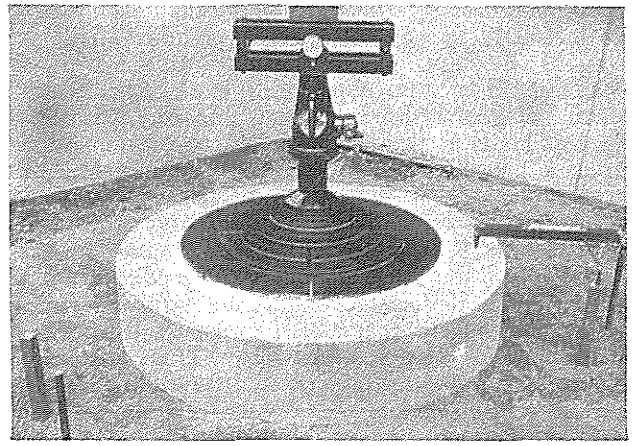


FIGURE 2.—DETERMINING THE MODULUS OF SUBGRADE REACTION WITH A LARGE BEARING PLATE. THE BEARING PLATE WAS CAST ON THE SUBGRADE SOME TIME BEFORE TESTING. DISPLACEMENT IS MEASURED WITH A CLINOMETER.

plates in place while figure 2 shows a test with one of the larger plates.

The vertical loading force was applied by a jack reacting against a large horizontal cylindrical tank mounted on a dolly frame and filled with water. The magnitude of the force was determined with a dynamometer. All of this equipment is described in the first report (*18*). The vertical movements of the bearing plate were measured either with dial micrometers supported on a bridge or by a series of clinometer measurements in the manner described in the first report (*18*). When using the clinometer it was necessary to place the reference point at some distance from the bearing plate and to carry the level line over a series of intermediate points to the bearing plate so that soil movements in the vicinity of the bearing plate would not cause errors in the displacement data. With the smaller bearing plates the mean displacement was obtained from a single point in the center of the plate while for the large plates the displacements of three points symmetrically spaced along the perimeter were measured and averaged. All of the plates were sufficiently rigid to prevent appreciable bending as used.

In the tests of series 1, 2 and 4 an effort was made to deform the soil in a manner simliar to that which might be expected under a concrete pavement. A comparatively large number of loadings were applied in each test and the magnitudes of the plate displacements were kept within the normal load-deflection limits of a concrete pavement slab.

For each size of bearing plate a series of 3 to 5 ascending load values was selected, such that the series would give a good spread of displacement values and the maximum would not produce a displacement greater than the desired limit. With the smallest load value selected for a plate of a given size, the load was applied and removed several times. The number of applications was not constant but was determined by the character of the data, it being desired to reach a condition such that each succeeding application of a given load would produce the same vertical displacement of the bearing plate. This might be termed a state of approximate elastic equilibrium. The number of loadings required to develop this condition, with the soil on which tests were made, usually varied from about 5 to 10. When a satisfactory load-displacement relation had been determined for the lowest load value, the procedure was repeated with the next higher load value and so on until the displacement limit was reached. As stated previously, when the tests with plates of various diameters were to be made at one location, the procedure was to test with the largest plate first then with the next largest and so on.

When a load is applied to a bearing plate a displacement of the bearing plate begins and, under some conditions, may continue for a long time before a state of complete equilibrium is reached. Similarly when the load is removed a certain amount of elastic recovery occurs and this too may continue for some time. As a practical matter it is not possible to continue each test cycle until the last vestiges of either displacement or recovery have disappeared before proceeding with the next loading cycle. After some experimentation it was decided that for the soil condition, load intensities and sizes of bearing plates in this investigation, a condition of essentially complete equilibrium would be reached if each load was maintained for 5 minutes after reaching its full magnitude and if after the complete removal of that load 5 minutes elapsed before the application of the next load.

The procedure followed in the tests of series 3 was somewhat different, as mentioned previously. As the data from this series were, in part, to be compared with data developed in tests of series 2, the tests of series 3 were made immediately after the completion of those of series 2 so that no change in the condition of the subgrade soil could occur between the two series. In order to obtain load-deformation data for the soil in question which might be compared directly with those obtained by other agencies for other soils, the soil deformation limit was increased to approximately 0.25 inch for this series and the loading procedure was modified to conform more closely to that followed in the tests made by others. In the tests of series 3 only one load of each magnitude was applied. This load was left on for 5 minutes, removed completely and a period of 5 minutes allowed to elapse before the next larger load was applied. The maximum displacement limit of approximately 0.25 inch applied only to the smaller sizes of bearing plates since the maximum reaction possible with the loading equipment used was only about 50,000 pounds and this was insufficient to cause a dis-

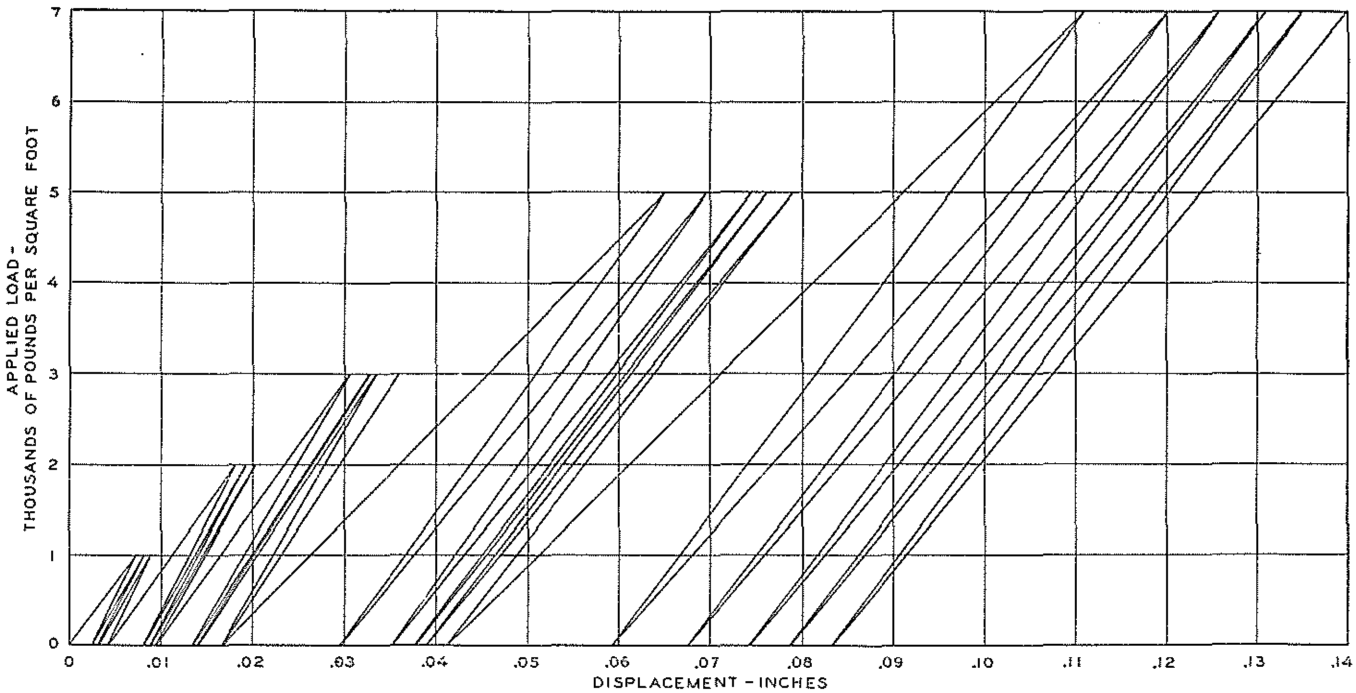


FIGURE 3.—TYPICAL LOAD-DISPLACEMENT DATA FOR A 26-INCH DIAMETER BEARING PLATE PLOTTED WITH RESPECT TO THE INITIAL POSITION.

placement of the desired magnitude with the larger plates.

Information regarding the moisture condition of the soil was obtained from samples taken from under a 4- by 4-foot concrete slab immediately adjacent to the point at which the bearing test was being made. The soil samples were taken just before and just after the bearing test. The moisture content was determined by weighing, drying, and reweighing.

SOILS TESTED SHOW HIGH PERCENTAGE OF RECOVERY AFTER REMOVAL OF TEST LOAD

When a bearing plate is subjected to a sequence of loads in the manner described for series 1, 2, and 4, data of the type shown in figure 3 result. This graph shows a log of the progressive displacements of a 26-inch diameter plate caused by a series of loadings of five magnitudes, the load of each magnitude being repeated several times, as described earlier in the report. The mean plate displacements are shown throughout both with respect to the original plate position and to the position just before the particular load was applied. The graph shows also the magnitude of the recovery during the 5 minute period following the removal of each load.

The data obtained in these load-displacement tests showed that, for a given plate size and load intensity, the magnitude of the displacement usually decreased somewhat with each load application until several loads have been applied, after which the load-displacement relation remained fairly stable. As soon as the pressure intensity was increased by the application of a greater force to the plate, the same repetition of loadings was necessary to again bring about the condition of approximate elastic equilibrium. Data of this type and extent were obtained for each of the various sizes of bearing plate listed earlier.

It is apparent in the graph that for the conditions of soil, pressure intensity, soil deformation, and time which obtained in this test, the action was never completely elastic. This is evidenced by the residual deformation after each loading. A study was made of the data to determine the percentage of recovery that followed the complete removal of the load and the result of this study is summarized in table 1.

TABLE 1.—*Average recovery of soil after removal of test load*

Displacement	Diameter of bearing plate in inches—			
	4	20	54	84
Inches	*Percent*	*Percent*	*Percent*	*Percent*
0.010	93	90	93	89
.020	90	90	95	97
.050	90	91	95	99

The percentages of deformation recovered upon removal of the load, as shown in this table, are values determined by averaging the movements after preliminary loadings had developed a state of approximate elastic stability.

The data indicate that the percentage of recovery was rather high in all cases, that it was greater with large plates than with small ones and that it tended to be greater for the larger plate displacements than for the smaller ones.

EFFECT OF SIZE OF BEARING AREA STUDIED

Since the recovery is not complete, there is a cumulative residual or permanent displacement as the test proceeds. For the soil on which these tests were made, the magnitude of this cumulative deformation appears to vary with the number of load applications and with the pressure intensity used.

This gradual settlement of the bearing plates under the succession of test loads raised the interesting question as to what effect such a yielding of the soil might have on the load-strain relation of a concrete pavement slab. To obtain information on this point, tests were made on a full-size pavement slab on the same subgrade that was used for the soil bearing tests. In the tests on the pavement slab a load was applied a relatively large number of times at the corner, the edge and the interior and the critical strain measured for each load application. It was found that at the corner there was a progressive increase in the strain caused by a given load, but at a diminishing rate, until there had been about 70 repetitions of the load. After this there was apparently no increase in the critical strain. The maximum increase in strain from the initial to the final loading was approximately 10 percent. Repetition of load at the free edge and at an interior point caused no appreciable change in the critical strains. The data from these load strain tests are presented in graph form in a later section of this report.

It will be recalled that in series 1 tests were made with various sizes of bearing plates, both at a common location and at individual locations. The tests with each size of plate provided data of the type shown in figure 3. The data from all of the individual tests of series 1 are summarized in figure 4 which shows load-displacement relations for each of the several sizes of bearing plate, both for the tests at a common location and for those at individual locations. The displacement values in this figure are with respect to the position of the plate immediately before the application of the particular load, being in each case the average of 2 or 3 determinations made after the subgrade had attained a condition of approximate elastic equilibrium for the particular displacement. No tests were made at the common location with the 36-inch round plate and the 48-inch square plate.

The data obtained in this series of tests afforded two significant comparisons. They showed first the important effect of plate area on the pressure intensity required to produce a given plate displacement on the soil in question. A similar effect has been observed by other investigators in tests where much larger plate displacements were used. This effect will be discussed in more detail later in this report. The second comparison was between the tests with a series of plate sizes at one location and a similar series at individual locations. For this comparison the data show that for a given plate size (within the 8- to 20-inch diameter range) essentially the same load intensity-plate displacement relation obtained. It was considered better procedure, however, to make the tests with each plate size at individual locations in the subsequent program of series 2.

In figure 5a and 5b the same data are arranged in a different manner. In figure 5a are shown the relations between pressure intensity and the diameter of the bearing plate for three magnitudes of plate displacement, 0.01, 0.02, and 0.05 inch, respectively. This graph brings out forcibly the importance of both the area and the degree of the displacement in determining the load supporting ability of the soil. The shape of the curves, shown in this graph, is generally similar to that found in other tests, both in this country and in Europe, in which loads were applied to rigid plates of various sizes that were supported by soils not strictly granular in character (*3*, *5*, *6*, *8*, *2*). Generally speaking, the other tests cited differed from those described in this report in two important particulars. In the first place, the soil deformations were carried considerably farther than in the present study and, in the second place, the maximum dimension of the bearing plate, i. e., the diameter of the circular plate or the side of a square plate was, with one exception, less than 42 inches.

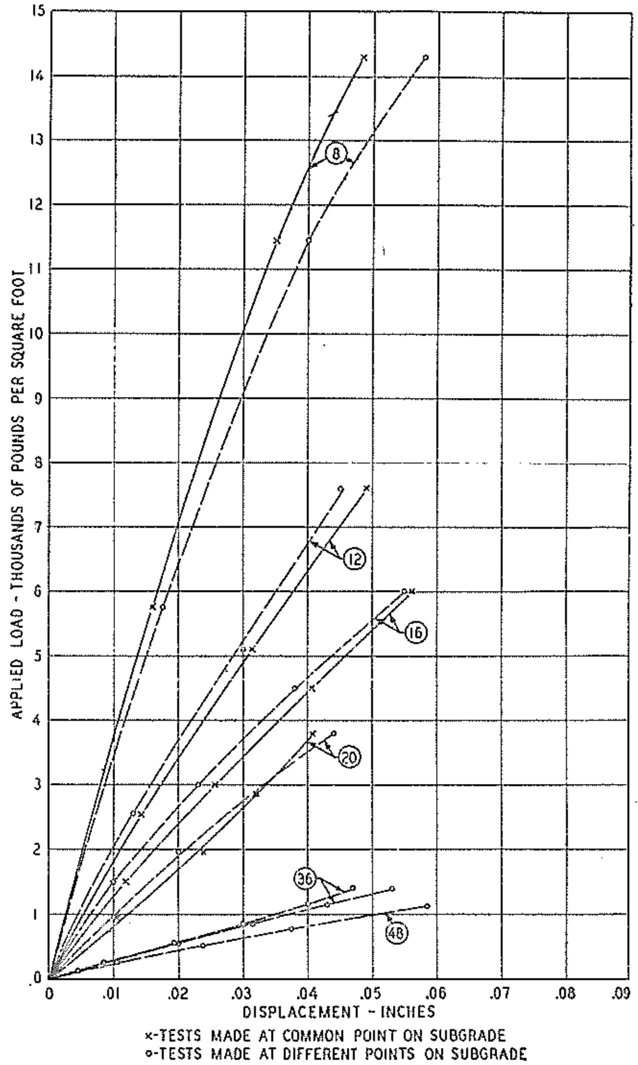


FIGURE 4.—LOAD-DISPLACEMENT RELATIONS FOR THE SEVERAL BEARING AREAS OF THE FIRST SERIES OF TESTS. FIGURES IN CIRCLES INDICATE THE DIAMETER OF THE BEARING PLATE IN INCHES.

In figure 5b the data from series 1 are again shown but, in this case, with the pressure intensity related to the perimeter-area ratio, an inverse function of the plate size, in the manner suggested by Housel (*6*). Three curves, one for each of the three magnitudes of plate displacement used in the tests, are given in the graph.

In both figures 5a and 5b the data obtained with the 48-inch square plate are included, being plotted on the basis of area, the shape factor being ignored. Thus there may be reason to question the accuracy of the points which are plotted on the 54-inch diameter and 0.083-inch^{-1} perimeter-area ratio ordinates respectively in the two parts of this particular figure. Data obtained in the subsequent series of tests support the relations shown in figures 5a and 5b, however.

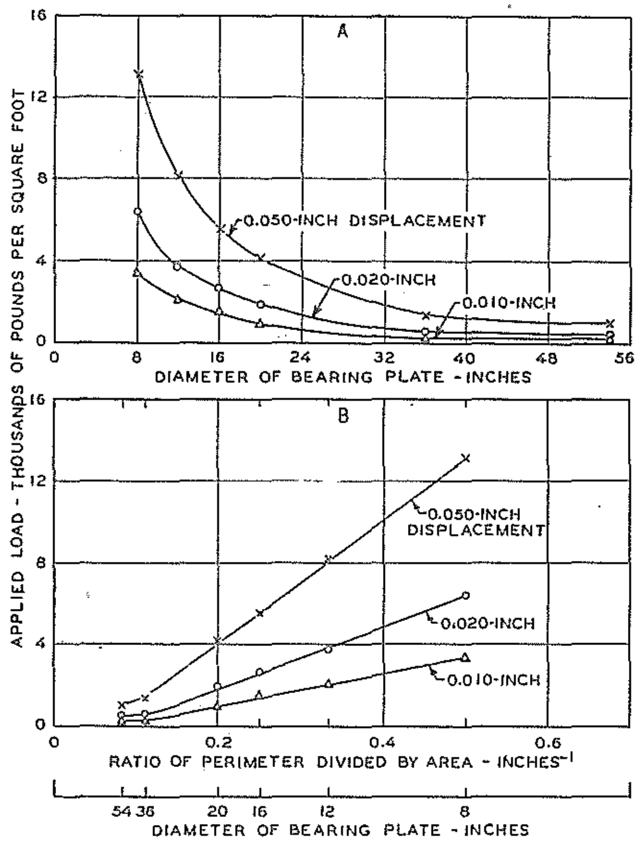


FIGURE 5.—EFFECT OF BEARING PLATE SIZE ON THE LOAD REQUIRED TO PRODUCE A GIVEN PLATE DISPLACEMENT. DATA FROM SERIES 1.

It is of interest to note that, when the data are plotted in the manner shown in figure 5b, an essentially straight line relation exists between the pressure intensity and the ratio of perimeter to area for plates of 20-inch diameter and less while for the larger plates used in this series, it appears that the linear relation does not hold. The use of the inverse function of plate size $\left(\frac{2}{\text{radius}}\right)$ for the abscissas results in such a convergence of this scale near the origin that it is difficult to determine at what point on these curves the departure from linearity begins but it appears to be in the vicinity of 30 inches on the diameter of bearing plate scale.

SMALL PLATES FOUND UNSUITABLE FOR USE IN DETERMINING RELATION BETWEEN PRESSURE INTENSITY AND PLATE DISPLACEMENT

It will be recalled that, in series 2 and series 3, tests were made with eleven sizes of circular bearing plates ranging from 2 to 84 inches in diameter. In series 2 the loading procedure and displacement ranges were the same as in series 1, while in series 3 the loading procedure was different and the displacement range was increased consistently.

In figures 6 and 7 are summarized the load displacement data obtained in the tests of series 2 and series 3 respectively. These graphs correspond to figure 4 which contains similar data from the tests of series 1. Attention is called to the difference in the horizontal scales used in the two figures, this being necessary because of the difference in the displacement range of the two series of tests. It will be noted that in series 3 the displacements are limited to values less than the maximum in the case of the large bearing plates. With the large areas, the available load reaction of approximately 50,000 pounds limited the pressure intensity (and the plate displacement) which could be obtained.

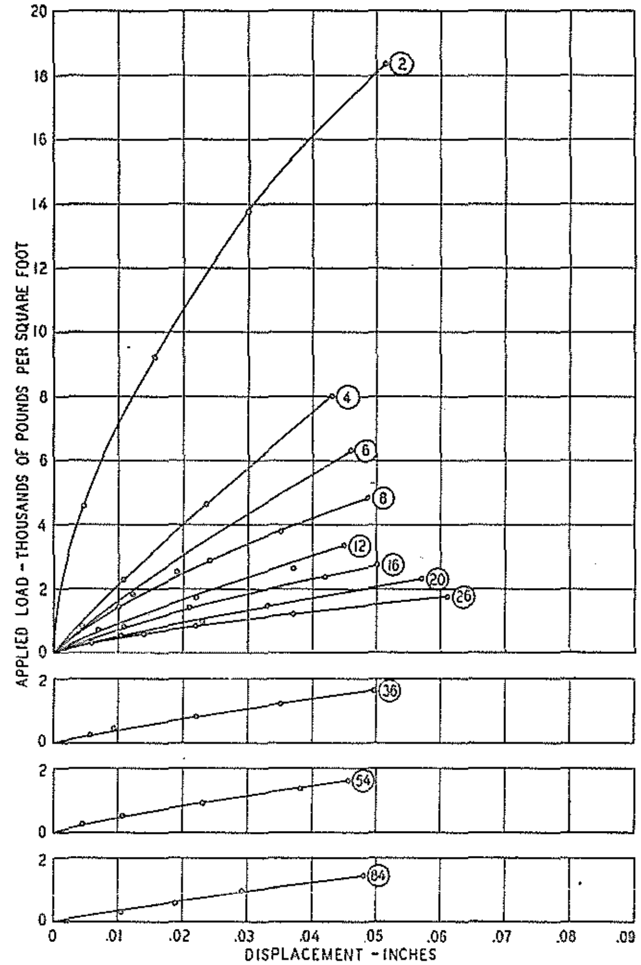


FIGURE 6.—LOAD-DISPLACEMENT RELATIONS FOR THE SEVERAL BEARING AREAS OF THE SECOND SERIES OF TESTS. FIGURES IN CIRCLES INDICATE THE DIAMETER OF THE BEARING PLATE IN INCHES.

Figure 8 shows the load-displacement data from the tests of series 2, plotted in the same manner as was used in figure 5a for series 1 in order to bring out the effect of the size of the bearing plate on the ability of the soil to resist deformation. As in the corresponding earlier graph the relation is shown for each of the three displacement magnitudes, 0.01, 0.02 and 0.05 inch respectively.

Figure 9 is similar to figure 8 but contains data from the tests of series 3. Because of the larger displacements in the tests of series 3, the action of the soil was somewhat different and this difference was most marked when the 4-, 6-, and 8-inch diameter bearing plates were employed. Generally speaking, however, the relations between plate size and the pressure intensity necessary to produce a given displacement were similar to those developed by the tests in which the displacements were limited to the small values of 0.01, 0.02 and 0.05 inch, as shown in figures 5a and 8.

The data consistently show that, for the conditions that existed in the present tests, the effect of plate size on the pressure intensity-plate displacement rela-

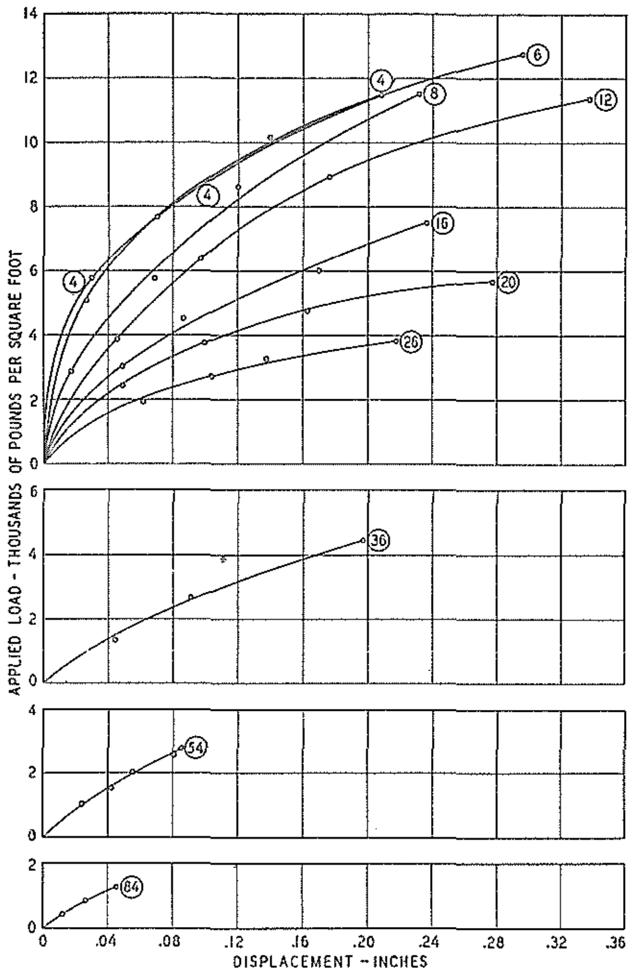


Figure 7.—Load-Displacement Relations for the Several Bearing Areas of the Third Series of Tests. Figures in Circles Indicate the Diameter of the Bearing Plate in Inches.

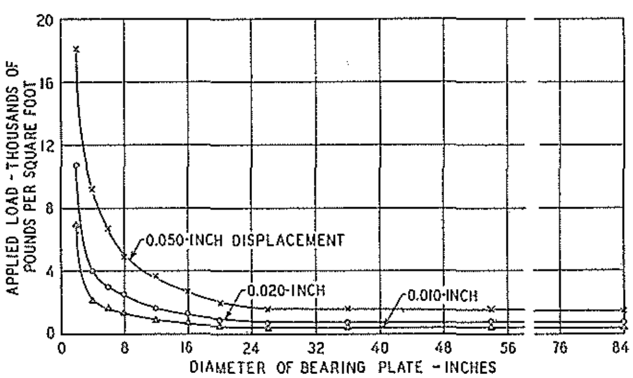


Figure 8.—Effect of Bearing Plate Size on the Load Required to Produce a Given Plate Displacement. Data From Series 2.

tion is not important for plate diameters of about 26 inches or larger, but for plates of less than the 26-inch diameter there is an effect which becomes increasingly large as the plate size diminishes.

Certain earlier investigators, (*3*) and (*5*), concluded that, for soils of the cohesive type, a given pressure intensity applied to bearing plates of various sizes might be expected to produce vertical soil deformations which would be directly proportional to the square root of the areas of the plates. An analysis of the present data indicates that the relation just stated applies quite well for bearing plates having diameters of 26 inches or less, but that for larger plates it does not apply, the magnitude of the divergence increasing rapidly as the size of the plate is increased. For a plate 84 inches in diameter the measured displacement was approximately one half of that which would be predicted if the above-mentioned relation held true.

It should not be assumed that the relations found in the bearing tests at Arlington are applicable generally. Further tests are needed on other soils in place, particularly tests that include large bearing areas, before any attempt can be made to generalize on the relations. It should be remembered also that the studies at Arlington were made primarily to determine the behavior of soil undergoing relatively small deformations and the magnitude of the deformation is shown to be an important variable in the test.

The data obtained are important, however, in showing the size of plates which must be considered in studies of the soil condition of the Arlington pavement research and furthermore because they indicate the necessity for a knowledge of the effect of plate size when making bearing tests with other soil conditions.

SEASONAL VARIATION IN SUBGRADE SUPPORT STUDIED

It will be recalled that the modulus of subgrade reaction is a stiffness coefficient which expresses the resistance of the soil structure to deformation under load in pounds per square inch of pressure per inch of deformation (in the direction of the loading force). Certain pertinent facts have been brought out. It has been shown that, (1) the soil structure is imperfectly elastic; (2) the elastic behavior of the soil is affected by its moisture state; (3) the load resistance of the soil structure, i. e., the pressure intensity required to produce a certain deformation, depends upon the magnitude of the deformation and, within certain limits, upon the area over which the pressure is applied to the soil structure. It is evident that these conditions place limitations on the manner in which the stiffness coefficient can be determined and on the extent to which it can be applied. However, it is believed that, for soils of cohesive character at least, it is possible to obtain approximate but usable values for the coefficient from load-displacement tests with rigid bearing plates on the soil in place, provided certain precautions are taken to minimize the effect of the disturbing influences mentioned above. A study of the data suggests the nature of the precautions to be taken.

If values of the stiffness coefficient k are calculated from load-deformation data, such as those shown in figure 3, it will be found that the value of the coefficient varies with the size of the plate used in the test and with the magnitude of the soil deformation. This has been mentioned previously and is indicated by the relations shown in figures 8 and 9. An analysis of the data obtained in the more comprehensive tests of series 2 was made to determine the extent and characteristics of the variations in the value of the coefficient for the conditions that obtained in this series of tests. Values were calculated for each size of bearing plate used and for soil deformations of 0.01, 0.02, and 0.05 inch for each plate size. The variation in the value of the coefficient with plate size and with the magnitude of the plate displacement or soil deformation is shown in figure 10. From this figure it appears that when making tests to determine the value of the soil stiffness coefficient k

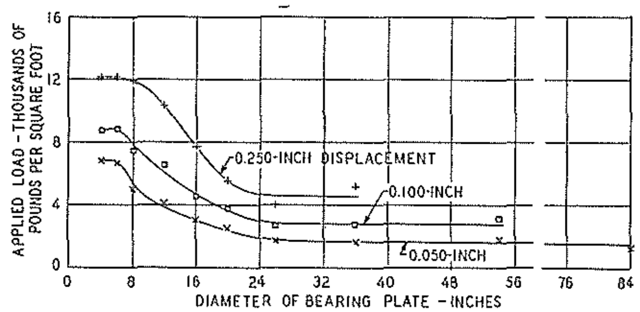


Figure 9.—Effect of Bearing Plate Size on the Load Required to Produce a Given Plate Displacement. Data From Series 3.

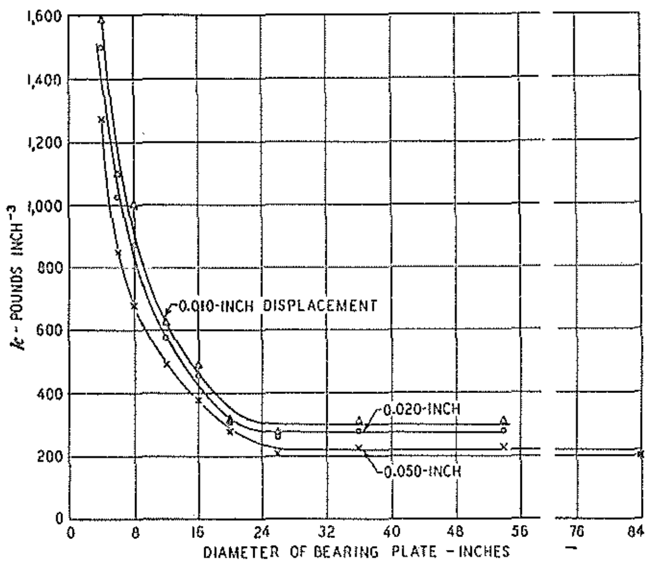


Figure 10.—Values of k Determined From Bearing Tests Made on Circular Plates of Various Diameters. Data From Series 2.

it is necessary to limit the deformation to a magnitude within the range of pavement deflection and that it is of great importance to use a bearing plate of adequate size. For the conditions of the tests of series 2 the minimum plate diameter is indicated as being about 26[illegible] inches. It should not be concluded, however, that this size of bearing plate would necessarily be adequate for other soil conditions. There is, in fact, evidence that it would not be. This point will be referred to again later.

A similar analysis was made of the data obtained in the tests of series 3. It will be recalled that in this series larger displacements were included and only one load of each magnitude was applied. The values of the coefficient for the various conditions of the tests of series 3 are shown in figure 11.

A comparison of figure 11 with figure 10 shows that the general indications, mentioned above, as to the effect on the value of the stiffness coefficient of the magnitude of soil deformation and of the size of the bearing plate are supported by the data from the tests of series 3 as well as those of series 2. Furthermore, in the one case where the two series are most nearly comparable, i. e., the tests with a limiting soil deformation of 0.05 inch, the agreement between the values obtained in series 2 and series 3 is rather good.

The shapes of the curves shown in figure 11 are different from those in figure 10, particularly in the region of the small bearing plates. This is attributed to the

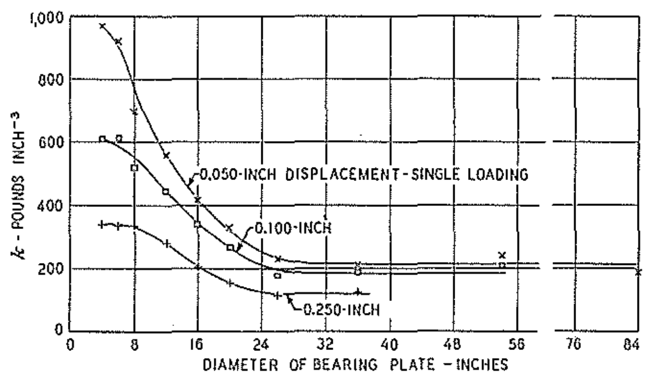


Figure 11.—Values of k Determined From Bearing Tests Made on Circular Plates of Various Diameters. Data From Series 3.

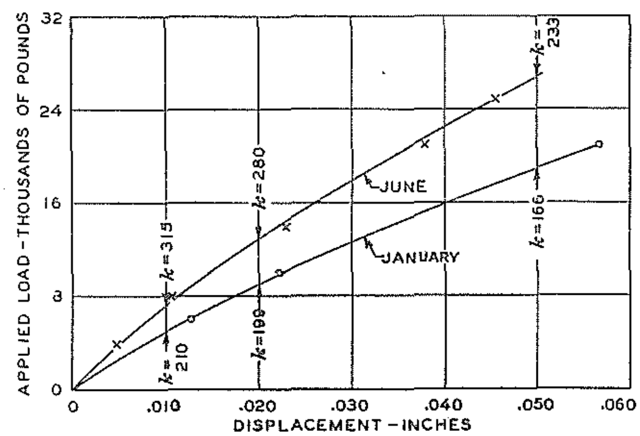


Figure 12.—Effect of Seasonal Moisture Condition and of Soil Deformation on the Load Supporting Ability of the Subgrade. Data From Series 4.

difference in test procedure used in the two series of tests as mentioned in connection with the discussion of figures 8 and 9.

The tests of series 2 and series 3, from which the data shown in figures 10 and 11 were obtained, were made during the summer. The moisture content of the soil was determined to a depth of about 12 inches and was found to be quite uniform, averaging 17 percent.

Because the moisture content of the soil beneath the pavement was known to vary from summer to winter, load-displacement tests with the 54-inch diameter bearing plate were made under both normal summer and normal winter conditions. This made possible some study of the effect of seasonal moisture change on the value of the soil stiffness coefficient.

In these tests, which were designated series 4, the bearing plate remained in position on the subgrade for some time before the test so that moisture conditions were stabilized at the time of test. For the summer condition the moisture content of the soil was uniformly 17 percent to a depth of about 12 inches while for the winter condition it was about 25 percent in the 0- to 6-inch zone and about 19 percent in the 6- to 12-inch zone.

The loading procedure in series 4 was the same as in series 2.

Load-displacement data from the tests of series 4 are shown in figure 12 for both the summer and the winter condition. On the graph are shown also values of the modulus k calculated from data obtained in the bearing plate tests for soil deformations of 0.01, 0.02 and 0.05 inch, respectively. It is apparent from this figure that the soil structure in question was much more resistant to load deformation under the conditions which prevailed when the summer tests were made than it was at the time of the winter tests, the increase in stiffness from winter to summer being roughly 40 to 50 percent. Undoubtedly the magnitude of the seasonal change in subgrade support will vary with the characteristics of the soil material and with the temperature and moisture changes that occur. Under favorable conditions the variation may be much smaller than that cited. It is obviously important, however, to make sure that the soil at the site of the bearing test is of the same character and density and is in the same moisture state as the soil supporting the pavement under consideration. The important influence which the magnitude of the soil deformation has on the value of the modulus k is further emphasized in this graph.

CAREFUL PROCEDURE NECESSARY IN DETERMINING MODULUS OF SUBGRADE REACTION FROM LOAD-DISPLACEMENT TESTS

The experiments with rigid bearing plates have led to the conclusion that approximate but usable values for the modulus of subgrade reaction as used in the Westergaard equations can be obtained by means of load-displacement tests with rigid bearing plates. The criterion by which the usability of the modulus values determined in this manner is judged has been a comparison of calculated and measured strains in full-size pavement slabs. Since the modulus of subgrade reaction k is but one of several coefficients that must be determined before the comparison can be made the discussion of this comparison will appear in the more general section to follow.

The studies have indicated that there are certain limitations which must be recognized, certain precautions which must be taken and certain procedures which are desirable when determining values for the modulus of subgrade reaction by means of load-displacement tests with rigid bearing plates. These will be mentioned briefly under appropriate headings.

Condition of the Soil at the Site of the Test.—It is obvious that if the data are to reflect the supporting properties of the subgrade, the soil at the site of the test should be truly representative of that subgrade. Not only should the soil materials be the same, but the structure, density and moisture state must likewise be duplicated. It appears that this can be accomplished best by giving the soil to be tested the same compaction as that given the subgrade, by covering the soil to be tested with a concrete slab and by permitting sufficient time to elapse for the development of a stable condition of soil mositure before testing. Unless these precautions are taken, there is no assurance that the soil structure tested was representative of that under the pavement. In the case of an existing pavement these conditions can be satisfied by removing a small section of the pavement and making the bearing plate test on the subgrade itself. Both procedures have been used in the Arlington tests and both are believed to be satisfactory.

Bearing Plate.—Certain physical characteristics of the bearing plate must be considered. It was shown that, within limits, the area of the plate had a very marked effect on the value of the modulus k as determined by the bearing plate test and furthermore that the value which was obtained with data from tests with relatively large plates could not be predicted from similar data obtained in tests with small plates. Also there is evidence that the minimum size of plate that will give satisfactory data depends upon the soil structure being tested. It is important, therefore, to determine by tests with plates of several sizes the minimum size that will be satisfactory for a given soil condition. If this is not possible an expedient would be to test with a relatively large plate, a 48- to 60-inch diameter being suggested by the data at present available.

The rigidity of the plate is apparently an important factor. The use of slightly flexible plates may be feasible but the degree of flexibility is another influence to be considered and sufficient data are not yet available to permit comparisons to be made of the relative value of rigid and slightly flexible bearing plates. Until further studies are made of the use of slightly flexible plates it is believed that rigid plates should be used exclusively, a rigid plate being arbitrarily defined as one which, under the conditions of the load-displacement test, does not deflect or "dish" from perimeter to midpoint by more than 0.004 inch. (NOTE.—The 54-inch diameter bearing plate used in these tests was made of concrete and was 12 inches in thickness. Under a 15,000-pound total applied load, the maximum measured deflection of a diameter of this disc was 0.0034 inch.)

The effect of the shape of the bearing plate is another element which has not been studied adequately. Until more information is available it seems advisable to make all bearing tests with circular plates.

Range of Plate Displacement.—It has been shown that the magnitude of the soil deformation (or plate displacement) had an important effect on the soil stiffness coefficient in the tests at Arlington. How great this effect might be with soil structures radically different from the subgrade under the experimental pavement is not known. It is believed, however, that the maximum displacement of the bearing plate in tests to determine the modulus of subgrade reaction k should not exceed the average deflection of the pavement slab under the expected wheel load. In the absence of measured values, it is probably safe to assume a value within the range 0.02–0.03 inch for this purpose.

Bedding the Bearing Plate.—Because of the small maximum displacement it is especially important that the bearing plate be carefully bedded on the soil under test. The procedure followed in the tests previously described appears to be one way by which a satisfactory initial contact can be obtained.

Loading Procedure.—It seems desirable to have the soil structure in as nearly an elastic condition as possible when the modulus of subgrade reaction is determined and it appears that in the tests at Arlington this end was substantially attained by the procedure adopted. It will be recalled that in this procedure a sequence of loads of equal magnitude was applied to the bearing plate until the soil structure reached a condition such that each successive load produced essentially the same plate displacement and elastic recovery in a given time interval.

Other Tests.—In conjunction with a subsequent program of studies of the structural action of certain types of transverse joints, there was afforded an opportunity to make tests that provide limited comparisons between values of the modulus of subgrade reaction as determined by pavement slab deflections and values obtained by bearing plate tests for a somewhat different soil structure. These comparisons will be discussed later in the more general section of this report.

LOAD-DEFLECTION AND LOAD-STRESS RELATIONS FOR PAVEMENT SLABS OF UNIFORM THICKNESS

WESTERGAARD EQUATIONS FOR DEFLECTION AND STRESS

The Westergaard equations for deflection and for stress in concrete pavement slabs are developed for three cases of load position, as follows:

Case I in which "a wheel load acts close to a rectangular corner of a large panel of the slab" (*23*), referred to as a corner loading.

Case II in which "the wheel load is at a considerable distance from the edges," referred to as an interior loading.

Case III in which "the wheel load is at the edge but at a considerable distance from any corner," referred to as an edge loading.

In the original analysis it was assumed that the slab acts as a homogeneous, isotropic, elastic solid in equilibrium and that the reactions of the subgrade are vertical only and are proportional to the deflection of the slab.

On the basis of these assumptions the following equations for maximum deflection and for critical stress were presented:

Maximum deflections:

$$z_c=\left(1.1-0.88\,\frac{a_1}{l}\right)\frac{P}{kl^2}\text{------------}(1)$$

$$z_i=\frac{P}{8kl^2}\text{-----------------}(2)$$

$$z_e=\frac{1}{\sqrt{6}}\,(1+0.4\,\mu)\,\frac{P}{kl^2}\text{-------------}(3)$$

Critical stresses:

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{Eh^3}{12(1-\mu^2)k}\right)^{-0.15}(a\sqrt{2})^{0.6}\right]\text{------}(4)$$

$$\sigma_i=0.3162\frac{P}{h^2}[\log_{10}\,(h^3)-4\,\log_{10}(\sqrt{1.6a^2+h^2}-0.675h)-\log_{10}k+6.478]\text{----------}(5)$$

$$\sigma_e=0.572\frac{P}{h^2}[\log_{10}\,(h^3)-4\,\log_{10}(\sqrt{1.6a^2+h^2}-0.675h)-\log_{10}k+5.767]\text{----------}(6)$$

In the above equations, the following notation is used:

z_c, z_i, z_e, maximum deflection for corner, interior, and edge loadings respectively.

σ_c, σ_i, σ_e, maximum tensile stress for corner, interior, and edge loadings respectively.

P=applied load, in pounds.

h=thickness of slab, in inches.

a=radius, in inches of the circular area (cases I and II) or the semicircular area (case III) over which the load P is assumed to be uniformly distributed.

$a_1=a\sqrt{2}$ in the case of the corner loading, the distance from the extreme slab corner to the center of the area of load application when the slab edges are tangent to that area.

$l=\sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$ a dimension, termed the radius of relative stiffness, measured in inches.

k=modulus of subgrade reaction, in pounds inch^{-3}.

E=modulus of elasticity of the concrete, in pounds inch^{-2}.

μ=Poisson's ratio for concrete.

It will be noted that the elastic constants E and μ do not appear in equations (5) and (6). In the original presentation these equations were limited to the case where E=3,000,000 lbs. in.$^{-2}$ and μ=0.15. Subsequently Westergaard generalized these equations and transcribed equation (4). These equations as restated are:

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right]\text{------}(7)$$

$$\sigma_i=0.275(1+\mu)\frac{P}{h^2}\log_{10}\left(\frac{Eh^3}{kb^4}\right)\text{---------}(8)$$

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-0.71\right]\text{---}(9)$$

the term b being an equivalent radius dependent upon a and h and expressed in inches.

In this subsequent paper (*25*) Westergaard proposed a new coefficient, K, defined by the relation

$$K=kl\ (\text{measured in lbs. in.}^{-2}).$$

This new coefficient K, like k, is a measure of the resistance of the subgrade. The reason given for the proposal was the expectation that K would be less dependent on the stiffness of the slab than is k and Westergaard stated his belief that "the truth may lie between the two extremes of a constant k and a constant K."

Westergaard in this subsequent discussion introduced another coefficient D which he called the deflection modulus of the pavement. This was defined as

$$D=kl^2=Kl$$

He restated equations (7), (8) and (9) in terms of the new coefficient of subgrade stiffness and gave a new equation for the maximum stress for the interior case of loading based upon the conception that, for this case, the reactions of the subgrade will be more closely concentrated around the load than are the deflections. The modified equation for maximum stress for the interior case of loading is

$$\sigma''_i=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-54.54\left(\frac{l}{L}\right)^2Z\right]\text{--}(10)$$

in which,

L is the maximum radial distance from the center of load application within which a redistribution of subgrade reactions is made, in inches.

Z is a ratio of reduction of the maximum deflections which Westergaard states may be expected to vary between 0 and 0.39. If $Z=0$, equation (10) reduces to equation (8).

It is with the equations given above that the discussion which follows will be concerned. For a more complete explanation of the terms used, reference to the Westergaard papers (*23*) and (*25*) is suggested.

EXPERIMENTAL PROCEDURE DESCRIBED

In the original planning of the investigation at Arlington four sections of uniform thickness were provided. Each test section was 20 feet wide by 40 feet long divided into four equal quadrants by a longitudinal joint and a transverse joint. The thicknesses of the four sections of uniform depth were 6, 7, 8 and 9 inches respectively. Except in the case of a few special studies, the test sections were protected in order to maintain them in a uniform moisture condition and to prevent warping from variations in temperature while load testing was in progress. The method used for this protection is described in detail in the first of this series of reports (*18*).

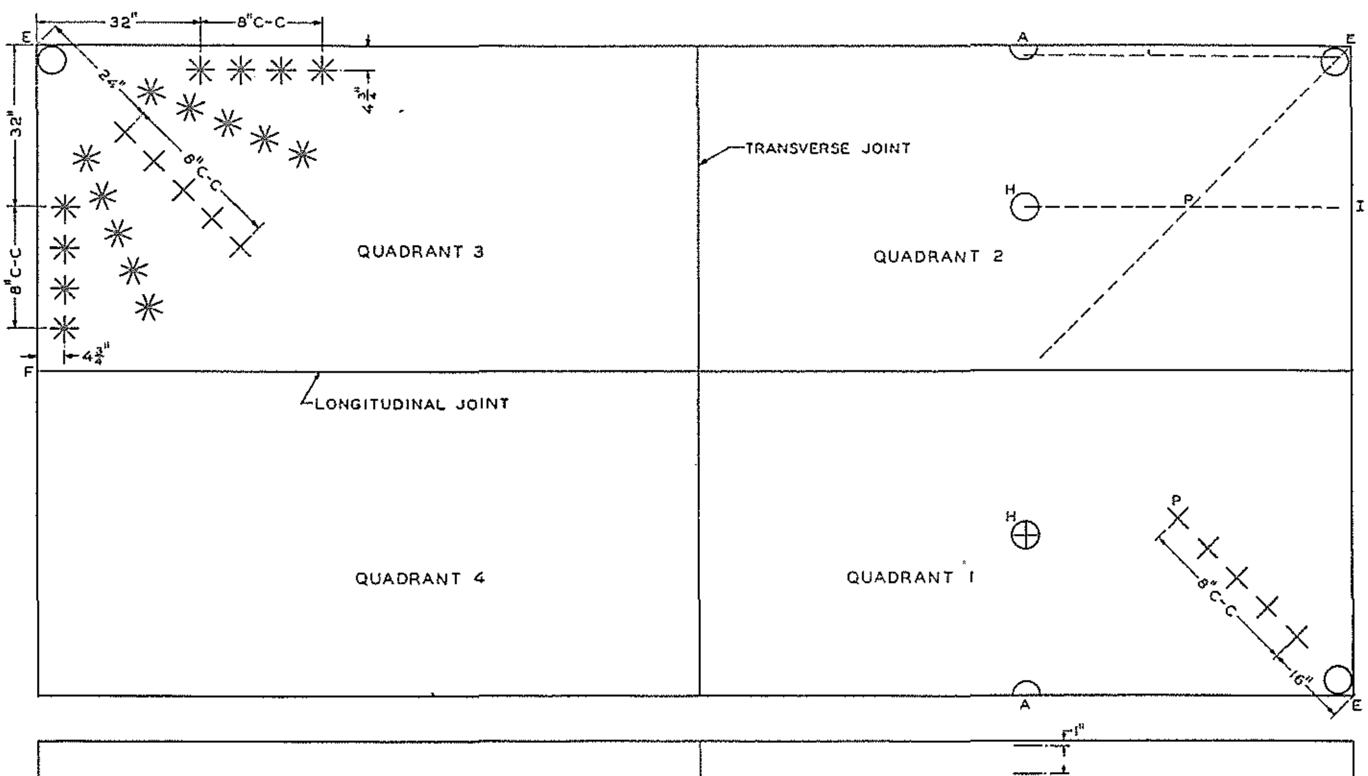
It was stated in the first report that the program of tests for these sections of uniform thickness was planned in such a way that each of the factors which theoretically might influence the load-stress relation could be examined experimentally and the observed effects compared with those predicted by the theory. Thus originally it was planned to study the effect on the load-stress relation of slab thickness, of load position and of size of bearing area. As the work proceeded the study was extended considerably beyond the limits originally contemplated and an examination was made of various assumptions used in the development of the equations and of the several coefficients that appear in them.

In general, loads were applied to the test sections at the three positions assumed in the development of the Westergaard analysis, i. e., the corner, an interior point and a free edge. These positions are shown at E, H, and A on the plan of a typical test section shown in figure 13. The positions of the strain gages installed for each case of loading are shown by the short solid lines in quadrant 1 while the dash lines in quadrant 2 show where the slab deflections were measured.

In certain studies of the structural action of slab corners to be described later, strains were measured in four directions in the corner area. The positions of the strain gages used in this part of the investigation are shown in quadrant 3.

It will be noted that for case I, with the load acting at E, both the strains and the deflections were measured along the bisector of the corner angle E–P. For case II, with the load acting at H, strains were measured in two directions immediately under the load and deflections were measured along the line I–H. For case III, with the load acting at A, the strains were measured on the slab edge 1 inch below the top surface and 1 inch above the bottom surface directly opposite the center of the load A while the deflections were measured along the line E–A.

The apparatus used for obtaining and measuring test loads, the various plates used in applying these loads to the pavement, and the instruments used in



CIRCLES AND SEMI-CIRCLES SHOW POSITIONS AT WHICH LOADS WERE APPLIED.
CROSSES (QUADRANTS 1 AND 3) AND ROSETTES (QUADRANT 3) SHOW STRAIN GAGE POSITIONS.
DASH LINES (QUADRANT 2) SHOW LINES ALONG WHICH DEFLECTIONS WERE MEASURED.

FIGURE 13.—PLAN AND ELEVATION OF A TYPICAL 20- BY 40-FOOT TEST SECTION.

he strain and deflection measurements were completely described in the first report of this series (*18*) and the details will not be repeated here.

The bearing plates used for the corner and interior cases of loading were circular. For the case of edge loading Westergaard assumed the loaded area to be semicircular in shape and in the major part of the tests being reported semicircular plates were used for loads applied at the edge. In certain of the edge tests, however, circular plates were used and in such cases the fact is noted either in the text or on the figures.

The loading tests on sections of uniform thickness considered in this report can be roughly divided into four groups. First, there were a number of what may be called auxiliary or collateral tests designed to settle questions of instrumentation or to supply information needed for the proper execution of the main program or for the interpretation of the data obtained in it. Second, there were certain preliminary or exploratory tests on the experimental sections themselves designed to develop satisfactory methods for obtaining the desired information. Third, there was the main program which comprised the determination of the load-deflection and load-strain relations for the four uniform thickness test sections under the conditions prescribed for the tests together with the determination of values for the several coefficients that appear in the theoretical equations. Fourth, there were a few supplementary studies made because, with test sections available and a testing technique developed, it seemed desirable to obtain data that would throw light on certain problems which were not a part of the original program. If these objectives are kept in mind, it is believed that the purpose for which given tests were made will be clear in the subsequent discussion.

METHOD OF STRESS DETERMINATION OUTLINED

In general the method of arriving at experimental stress values was the same as that described in the earlier reports of this investigation. However, there were some unusual conditions involved in the program being described which made special studies of the strain measuring technique desirable.

It will be recalled that the method used generally throughout this investigation was to measure strains with a temperature compensated, recording type of gage (*17*) installed between metal points set in the surface of the pavement slab. From the strains recorded by these gages, stress values were obtained by means of the equations

$$\sigma_x=\frac{E}{1-\mu^2}(e_x+\mu e_y) \qquad (11)$$

$$\sigma_y=\frac{E}{1-\mu^2}(e_y+\mu e_x) \qquad (12)$$

in which,

σ_x is the stress in the direction of the x-axis.
σ_y is the stress in the direction of the y-axis.
e_x is the unit deformation in the direction of the x-axis caused by stress in that direction.
e_y is the unit deformation in the direction of the y-axis caused by stress in that direction.
E is the modulus of elasticity of the concrete.
μ is Poisson's ratio for the concrete.

In applying these equations to measured strains, values for E were determined experimentally because of the importance of the term in the equations. Poisson's ratio, on the other hand, was assumed and the value 0.15 was used in all computations in which the term entered. There were two reasons for this, first, such experimental values of the ratio as are available lie generally between 0.10 and 0.20 with the majority in the upper half of the range and second, small changes in the value of the ratio have an unimportant effect on the computed stress values.

It is obvious that, in any comparison of theoretical stresses with values determined experimentally, it is essential that the strain measuring technique be such as to yield representative values. In this particular investigation the question of gage length was of unusual importance because stress values are the basis for the principal comparison and the test conditions are such as to produce rather abrupt stress variations in the area where strain measurements were to be made. For theoretical reasons, therefore, a short gage length is indicated. Experience has shown, however, that because concrete in small masses may lack the homogeneity that is assumed, the use of too short a gage length is likely to cause trouble by giving strain values that are not representative. A short gage mounted directly over a large piece of aggregate may show quite a different strain indication than would a similar gage mounted between two such pieces because the modulus of elasticity of the stone is different from that of the mortar. A fairly long gage tends to average these deformations and to indicate unit strains which, when combined with the "average" modulus of elasticity, will give stress values that are nearly correct.

In this investigation a collateral study was made to determine the length of gage necessary to give representative strain values in the tests that were scheduled. It was found that a gage length of about 6 inches was sufficient to average out local effects of aggregate in the concrete being used. This was evidenced by the fact that when this gage length was used a given load applied at homologous points on a given test slab always produced essentially the same maximum strain.

Another study was made to determine the effect of gage length on indicated unit strains when the measurements were being made within the area over which the tests loads were applied. For this study a special gage having an effective gage length of 2.2 inches was constructed and this gage was used to determine strain variation under the bearing plates of 6-, 8-, 12-, 16- and 20-inch diameters. Figure 14 shows typical data obtained in these tests. The curve marked A shows the maximum unit strain values recorded in a 2.2-inch gage length at the middle of a diameter of the bearing area for a given applied load and each size of bearing area listed above. Curve B shows, for the same load and bearing areas, unit strain values which are averaged from three 2.2-inch gage lengths arranged end to end along one diameter of the bearing area. These average unit strains, therefore, are those which would be shown by a single 6.6-inch gage centrally located along the same diameter. The location of the gage lengths with respect to the bearing area is shown in the legend of figure 14.

Because of possible local aggregate effects with a gage of this length the actual procedure was to leave the gage in one position and to shift the position of the loaded areas with respect to the gage length. The data indicate that for the conditions of these tests, the stress variation along a diameter of a bearing area was not sufficient to show different unit strains for gage lengths

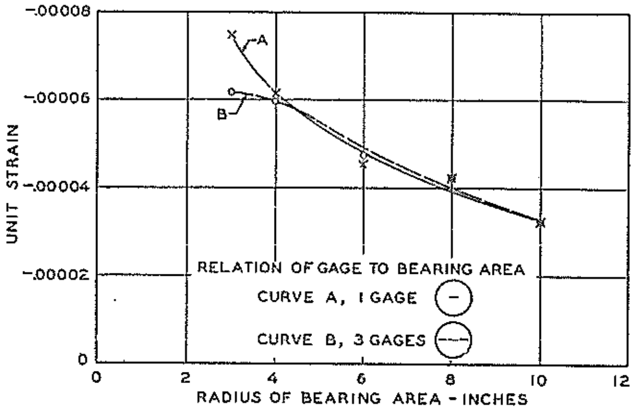


Figure 14.—Comparison of Average Unit Strains Measured Within Bearing Areas of Different Sizes. The Strains Were Measured Over A Distance of 2.2 Inches for Curve A and Over a Distance of 6.6 Inches for Curve B.

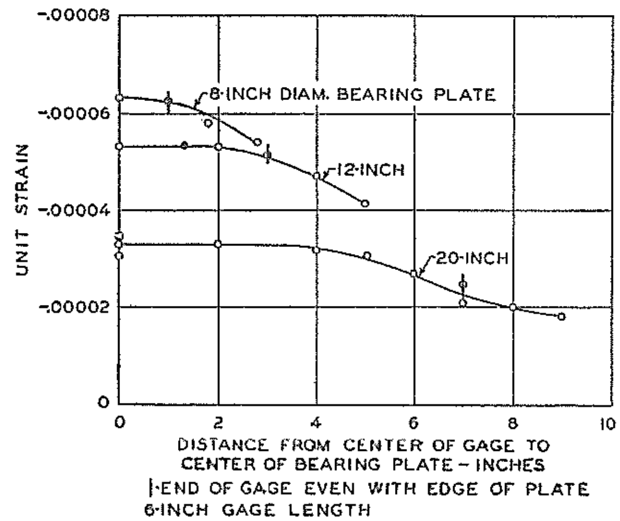


Figure 15.—Variation in the Magnitude of Indicated Unit Strain Under Bearing Plates of Three Sizes.

of 2.2 inches and 6.6 inches respectively so long as the plate diameter was 8 inches or greater and the gage was positioned along a diameter of the area.

In this same group of tests some study was made of the strain variations within the bearing areas when a gage length of about 6 inches was employed. Bearing areas of 8-, 12- and 20-inch diameters were used and the tests were made by shifting the bearing area with respect to the gage length in the manner previously described. However, in these tests, the strain determinations were made over the entire length of the diameter in each case. The data obtained are shown in figure 15. It appears from these data that the maximum unit strain occurs under the center of the bearing area, as would be expected, and that for bearing areas of these sizes there is no appreciable reduction in the strain for some distance toward the edge of the bearing area. In other words the strain variation curve is rather flat in this region. The reduction in strain magnitude near the edge of the bearing area is evident in these data.

In tests such as those just described the precision with which unit strain values are measured is a very important matter. In the published description of the strain gage used so extensively in this general investigation (*17*) it was stated that the accuracy of the gage is sufficient to permit the determination of stress in concrete to within 20 or 25 pounds per square inch, where dependence is placed upon a single observation. Subsequent extensive use of these gages has shown this to be a very conservative estimate. Stress determination is affected by precision in determining the effective modulus of elasticity as well as by precision in measuring strains in the material so that any lack of precision in stress determination may be the combined result of several influences. In this general investigation every effort was made to keep the gages functioning at maximum efficiency and usually dependence was not placed on a single observation unless that observation was supported by other data. In order to find out what consistency could be expected in the load-strain measurements when the same operation was repeated, an 8,000-pound load was applied 80 times at the edge of the 7-inch uniform-thickness test section and the strain recorded for each application. Of the 80 individual determinations only one differed from the average by more than 3½ percent, and some idea of the consistency of the combined load and strain measuring operation may be had from the following figures.

Edge loading, 8,000-pound load, 7-inch slab thickness

Average unit strain	0.00006675
Standard deviation	.00000146
Coefficient of variation 2.18.	

DETERMINATIONS MADE OF MODULUS OF ELASTICITY OF THE CONCRETE

Two methods were used for studying the values of the modulus of elasticity of the concrete, first, the testing of laboratory specimens in compression or bending and, second, by deflection measurements on the test sections themselves.

Stress determinations from measured strains require a knowledge of the value of the modulus and since the exposed pavement sections were tested over a long period of time it was necessary to know also how much the value varied from summer to winter. It was shown in the second report of this series (*19*) that there is an annual change in the moisture state of the concrete in the test sections sufficient to cause rather large volume changes from summer to winter. It is well established that changes in moisture state cause changes in the stiffness of concrete. It seemed probable, therefore, that this change in moisture content from its lowest value in summer to a higher value in winter was such as to cause changes in the modulus of elasticity that would have to be taken into account.

When the test sections were originally laid, extra slabs were provided from which laboratory test specimens could be taken. From these slabs cores were drilled and prisms for flexure testing were sawed and used in laboratory studies of the modulus of elasticity. It was found by flexure tests on prisms that those dried for 12 months by storage in the normal atmosphere of the laboratory had an average modulus of elasticity of 4,500,000 pounds per square inch while a comparable group that had been immersed in water at laboratory temperature had an average modulus of 6,000,000 pounds per square inch.

An attempt was made to determine the moisture content of the concrete in the test pavement in order that some comparison might be made between it and the mositure content of the concrete in the test prisms as an indirect measure of the modulus of elasticity of the concrete in the pavement sections. Fragments broken from the extra slabs on the subgrade were weighed and dried. The effort was not entirely satisfactory, however, because of variation in moisture content from top to bottom of the pavement and because of difficulty in getting a representative sample for the moisture determination. As nearly as could be determined the moisture content of the concrete that had been immersed for 10 months was slightly greater than that of the pavement during the wet winter months. It seems reasonable to conclude, therefore, that the modulus of elasticity of the pavement concrete in winter would be somewhat less than 6,000,000 pounds per square inch. While it was not possible to establish a reliable comparison between the moisture content of the air dry prisms and that of the pavement during the dry summer condition it is believed that the difference in the corresponding modulus values would not be large and that the value of 4,500,000 pounds per square inch determined by the laboratory tests might be used in conjunction with the strains measured during the summer tests.

Values for the modulus of elasticity determined from slab deflections will be discussed later and comparisons will be made between them and the values determined from the flexure tests of prisms.

POSITION AND DIRECTION OF CRITICAL STRESSES INVESTIGATED

Before attempting to make direct comparisons between computed and observed stresses it was considered desirable to make certain exploratory tests on the test sections to determine the position and direction of the critical stress for each position of loading, to ascertain the effect of repetition of load on stress magnitude and to establish the relation between load magnitude and stress magnitude for each case.

In the discussion of the comparisons of computed and observed deformations which is presented later it was found convenient to discuss the cases of corner, interior and edge loading separately and in that order. Because of this fact, the discussion of the work that comprised the preliminary load tests has been arranged in the same order.

A study was made of the strain variations in the corner area of a typical section for case I (the corner loading) in order to determine the location and direction of maximum strain. Strains were measured in two directions at various positions along the bisector of the corner angle and in four directions at comparable positions along two other lines radiating from the slab corner and making an angle of 22.5° with the corner bisector. The maximum strain at each position along the three radial lines for the particular test conditions that obtained is shown in figure 16. Along the bisector of the corner angle the direction of the maximum stress was known to be parallel to the bisector but along the other two radial lines the direction of the maximum strain was found to make an angle of about 15° with the radial line in each case. This matter of the direction of stresses in the corner area will be discussed much more fully later in the report.

The strains in the vicinity of the slab edges are not shown in figure 16 but it has been well established that for a corner loading on a concrete pavement slab of uniform thickness the strains along the edges that form the corner are definitely of less magnitude than those along and in proximity to the corner bisector. The data shown in figure 16 are typical of those obtained in

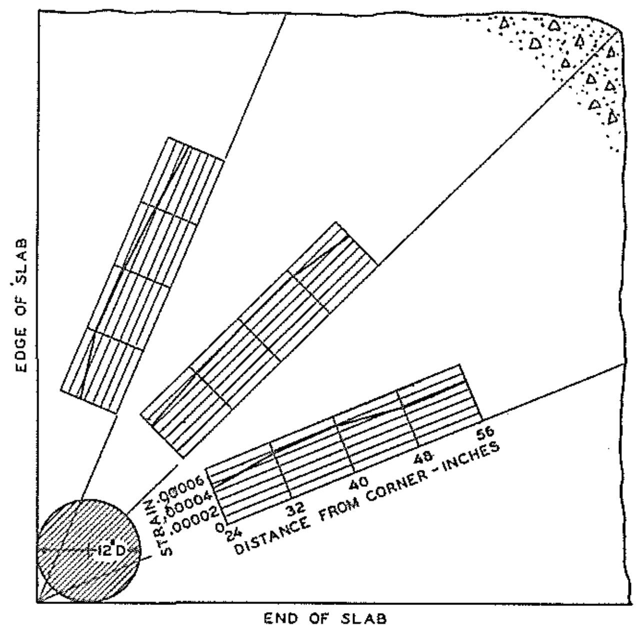


FIGURE 16.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—7-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING.

a number of similar corner tests and these data consistently indicated that the strain measured along the corner bisector at the proper location would be at least as large as, if not larger than any other strain to be found in the corner region. This being true it is only necessary to locate the position along the bisector at which the radial strain reaches a maximum and at this position measure the radial strain and the strain at right angles to it. The combination of these two strains determines the critical stress for the case of the corner loading.

In his analysis of this case Westergaard gives an equation for finding the distance from the corner along the corner bisector at which the maximum stress theoretically develops. This equation is

$$x_1=2\sqrt{a_1 l}$$

in which

x_1 is the distance from the corner to the point of maximum stress measured along the bisector of the corner angle.

a_1 and l are as previously defined.

The distance x_1 was determined experimentally for each of the slab thicknesses, for a range in loads, after various numbers of load applications and for different conditions of temperature warping.

In table 2 the experimentally determined values of this dimension are given for the four slab thicknesses, a range of loads and after various numbers of loadings had been applied. The tests covered a considerable period of time so that the soil moisture was not the same at the time the different sections were tested. For example, when the 6-inch and 7-inch sections were tested the subgrade soil was in a relatively dry state and a slight separation between the lower surface of the slab was noted after the application of a number of loads. In contrast, the subgrade was in a wet condition after severe freezing and thawing when the comparable tests were made on the 8-inch and 9-inch sections.

TABLE 2.—*Experimentally determined values of the distance, x_1, along the bisector of the corner angle at which the maximum stress was found to occur. These values are averages from the stress curves of the four quadrants. Loads were applied on 12-inch diameter bearing plates*

Slab thickness	Load	Distance, x_1		
		After first load	After 12 load applications	After 35 load applications
Inches	*Pounds*	*Inches*	*Inches*	*Inches*
6	4,000	34	35	35
	5,000	34	33	35
	6,000	33	33	33
7	4,000	35	34	38
	5,000	35	34	37
	7,000	33	33	34
	9,000	33	34	33
8	3,000	33	43	41
	6,000	33	33	39
	8,000	33	33	32
	10,000	33	33	31
9	3,000	46	48	46
	6,000	41	47	43
	8,000	41	40	42
	10,000	41	39	40

The data of table 2 indicate a tendency for the distance x_1 to increase, for the low loads, with the number of load applications. For the higher loads the distance remained fairly constant as the loads were repeated. There seems also to be a tendency for the distance to decrease as the magnitude of the load is increased, other conditions remaining the same.

In table 3 are shown experimentally determined values for x_1 for three conditions of temperature warping. Each value in this table was from an average stress curve, each point of which was the average of eight separate stress determinations. The values in table 3 tend to decrease as the degree of contact between the slab corners and the subgrade is increased. This same effect was noted in the data of table 2 as the magnitude of the load was increased.

TABLE 3.—*Experimentally determined values of the distance, x_1, along the bisector of the corner angle at which the maximum stress was found to occur. These values are averages from the stress curves of the four quadrants for each condition of warping. Loads were applied on 12-inch diameter bearing plates*

Slab thickness	Distance, x_1		
	Edges warped down	Slab flat	Edges warped up
Inches	*Inches*	*Inches*	*Inches*
6	31	36	37
7	30	34	36
9	31	33	38

With the possible exception of those for the 9-inch slab the experimentally determined values for x_1 were found to be in good agreement with computed values. This is especially true for the condition of the unwarped or the downward warped slab corners. When the corner was warped upward an increase in the value of the distance x_1 was noted.

In figure 17 average values of x_1 from a considerable number of tests with the pavement in an unwarped condition are compared with values computed with the equation mentioned above. It will be noted that the comparisons include three sizes of bearing area and four slab thicknesses.

Generally there is good agreement between the computed and observed values of this dimension. While there is some tendency for the experimentally determined values to be greater than the computed values, particularly with the larger bearing areas, the difference is not great and when the flat character of the stress variation curve along the corner bisector is considered, it is apparent that small errors in determining the value of x_1 would be of very little importance. It is concluded, therefore, that the theoretical equation for computing this distance is sufficiently accurate for all practical purposes.

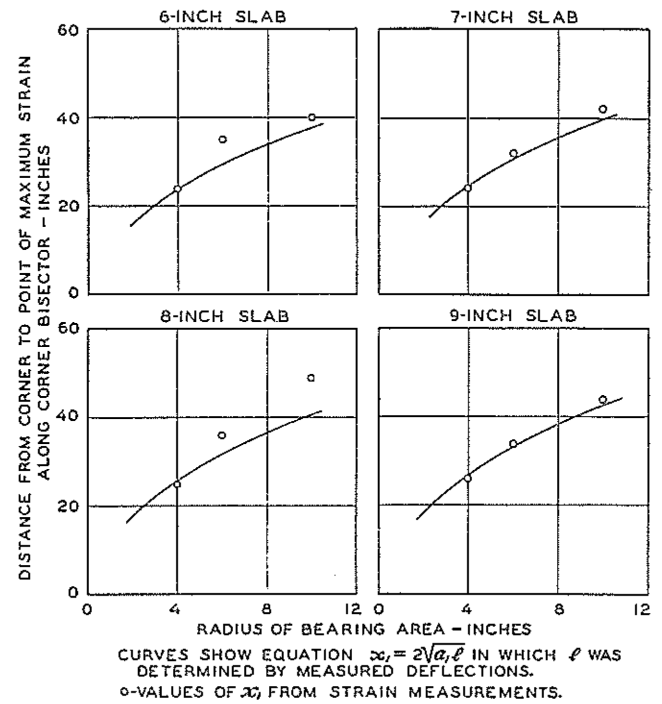


FIGURE 17.—COMPARISON OF THE OBSERVED AND THEORETICAL DISTANCES FROM THE CORNER TO THE POINT OF MAXIMUM TENSILE STRAIN ALONG THE BISECTOR OF THE CORNER ANGLE.

EFFECTS OF REPEATED CORNER LOADING STUDIED

The behavior of a slab corner under repeated loading was first studied in a series of tests on the 7-inch uniform-depth test section made during the late fall when the subgrade soil was in a normal state. A load of 8,000 pounds was applied 80 times to the free corner of each of the four quadrants of the test section. Each load was maintained at full value for 1 minute and after the removal of the load 1 minute was allowed to elapse before the next loading was begun. The loads were applied in groups of ten and the maximum strains for each group of 10 loadings were averaged. The data obtained are shown in figure 18. Since, in this graph the data from all four quadrants are combined, each point on the curve is an average of 10 tests in each of four quadrants or 40 observed values.

The data show a distinct tendency for the strain to increase with repetition of the test load, the average strain for the last group of ten loadings being approximately 10 percent greater than the average strain for the first group. The rate of increase is not constant, decreasing as the number of load applications increased. The increase after 60 repetitions of the load was very small.

Following these experiments it was decided to investigate the relation for all four thicknesses of pavement. In carrying out this subsequent program the procedure followed with each test section was the same and comprised the following schedule:

1. Two applications of a test load of each of three or four different magnitudes were made on each of the four free corners of each of the four sections. For each load application the maximum deflection and the maximum strain were measured.
2. A series of 12 loads of the maximum magnitude used in (1), above, was applied but no measurements of deflection or strain were made.
3. The same loadings and measurements made under (1), above, were repeated.
4. A series of 50 to 70 loadings of the maximum magnitude used in (1), (above), was applied, no measurements of deflection or strain being made.
5. The same loadings and measurements made under (1), above, were again repeated.

The load-deflection and load-strain relations developed during the tests of this schedule are shown in figure 19 while the strain variation along the bisector of the corner angle, for the maximum load magnitude is shown for each slab thickness in figure 20. Each point shown in these two figures is an average of two measurements in each of four quadrants or eight observations in all.

The tests extended over a winter period so that the subgrade was not in the same condition when all of the sections were tested.

The 7-inch section was tested in early December a short time after the data shown in figure 18 were obtained. The earlier series of tests may have affected the soil under the slab corners. The 6-inch section was tested in early January. Although the soil beneath the pavement was not frozen when the tests were made it probably had been frozen slightly a short time before. The tests on the 8-inch section were made during March shortly after the subgrade had thawed after having been frozen to a depth of about 12 inches. During the time that the soil was frozen the test section was heaved upward approximately 1 inch but at the time of load testing it had resumed approximately its original position. The soil was in a wet condition but there is reason to believe that the pavement was in good contact with the soil over its entire area when the load tests were made. The 9-inch section was tested about the first of April. The subgrade had been frozen

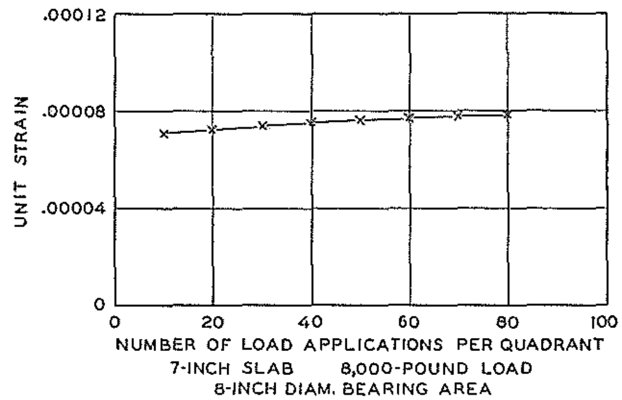


FIGURE 18.—EFFECT ON STRAIN OF REPEATED LOADS APPLIED AT THE CORNER OF A CONCRETE PAVEMENT.

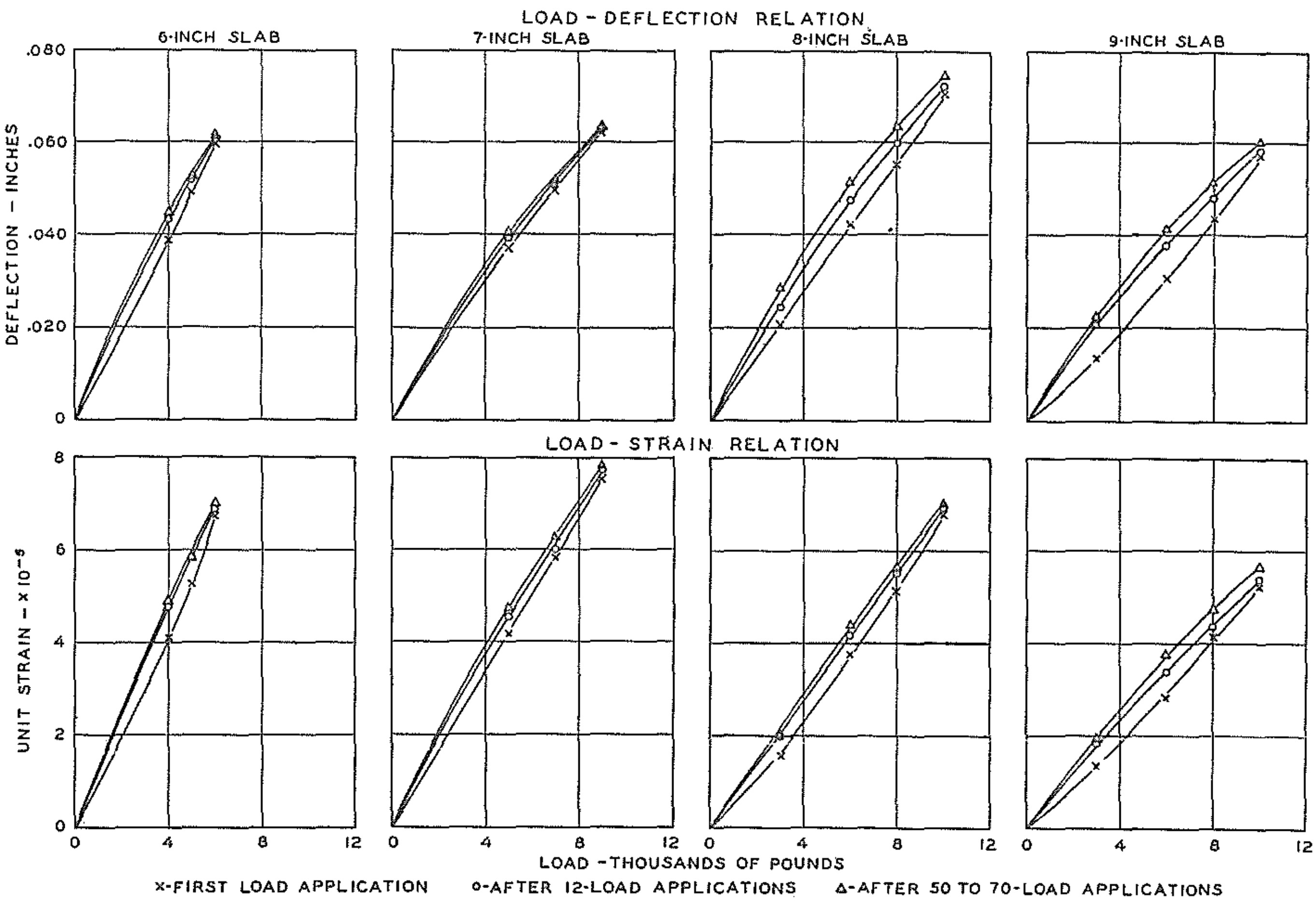


FIGURE 19.—EFFECT OF REPEATED LOADING ON THE LOAD-DEFLECTION AND LOAD-STRAIN RELATIONS AT A FREE CORNER.

as described above and, in addition, the section had been inundated for about 30 hours by flood backwater. The soil was, therefore, quite soft at the time the tests were made on this section. The swelling of the soil from moisture probably improved the uniformity of its contact with the lower surface of the pavement, however.

It is indicated by the data of figure 19 that the deflections and the stresses caused by a given load increased with the repetition of the load for all thicknesses in the same manner as found earlier with the 7-inch section. It will be noted also that when the load magnitude was greatest the effect of repeated loading on deflection and stress was least. The data consistently indicate that the change in deflection and strain under repeated loading is the result of an adjustment of the conditions of soil support under the slab corner as the repetition of loading proceeds. This adjustment continues until a condition of equilibrium is established.

The lack of linearity in the load-deflection and load-strain data for these slab corners is further evidence that as the slab corner is deflected the conditions of soil reaction under it change.

Similar tests to determine the load-deflection and load-strain characteristics of the interior region of the test sections were made and representative data from these are shown in figures 21, 22 and 23.

Figure 21 is a typical strain variation graph for an interior loading. Each point on the curves is the average of two strain measurements in each of four quadrants, eight observations in all. The radial strains were measured along a radius from the center of the loaded area and at distances from that center as shown. The tangential strains were measured at the same positions but in a direction normal to the radius. The shape of the curves under the bearing plate was drawn in accordance with data discussed earlier in this report and shown in figure 15.

These data indicate that directly under the bearing area the two strains have essentially the same magnitude but at points not within the bearing area the tangential strain is greater than the radial strain. The radial strains become zero at a distance from the center of the bearing area approximately equal to l while the tangential strains approach zero at a distance of about $1.6l$. The data in figure 21 were obtained on the section of 9-inch thickness but tests on the sections of other thicknesses showed the relation to be essentially the same when expressed in terms of l. This means that the critical stress from a given wheel load will not be increased by the presence of another equal load so long as the two loads are separated by a distance of $1.6l$ or more. For most highway pavements this would correspond to a distance of 40 to 60 inches.

It is interesting to note how closely this is in accord with experimental strain data obtained in studies of the effect of 6-wheel trucks on concrete pavements made by the Public Roads Administration some 17 years ago (*16*).

PRELIMINARY TESTS YIELDED SIGNIFICANT RESULTS

Repeated load tests were made at the interior position (case II) following the same procedure as that used for the corner case. Typical data, obtained in the tests on the 7-inch thickness section, are shown in figure 22. The data from the tests at the interior consistently show that repeated loading causes no increase in the magnitude of the critical strain, within the limits of the tests of this program.

A large number of tests were made in studies of the load-deflection and load-strain relations for the interior loading. Typical data from these tests are given in figure 23. Both the load-deflection and the load-strain relation were consistently linear for the interior loading, within deflection and strain limits such as those shown in this figure.

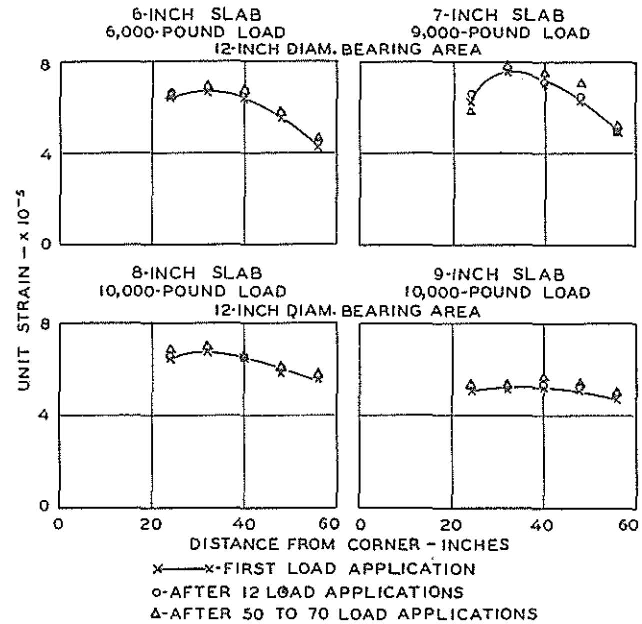


FIGURE 20.—EFFECT OF REPEATED LOADING ON THE STRAIN DISTRIBUTION ALONG THE BISECTOR OF THE CORNER ANGLE.

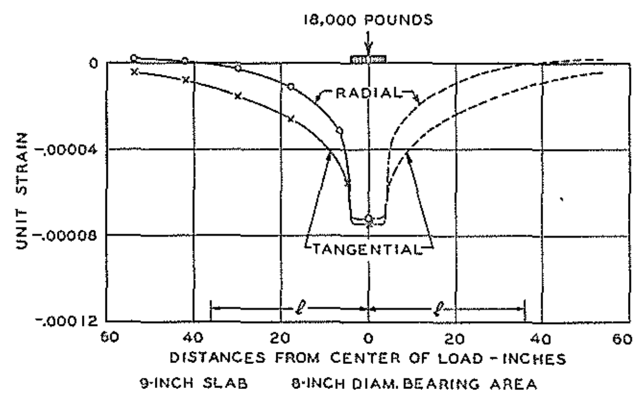


FIGURE 21.—VARIATION IN TANGENTIAL AND RADIAL STRAINS IN THE VICINITY OF A LOADED AREA AT THE INTERIOR OF A CONCRETE PAVEMENT SLAB.

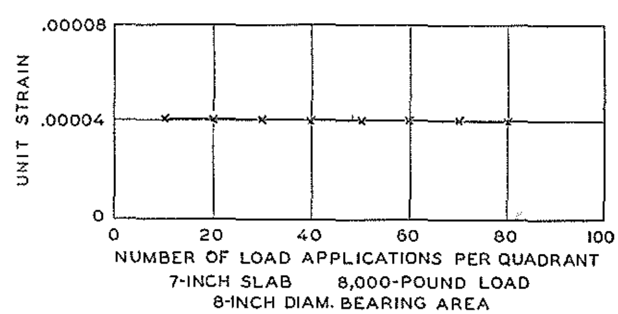


FIGURE 22.—EFFECT ON STRAIN OF REPEATED LOADS APPLIED AT THE INTERIOR OF A CONCRETE PAVEMENT.

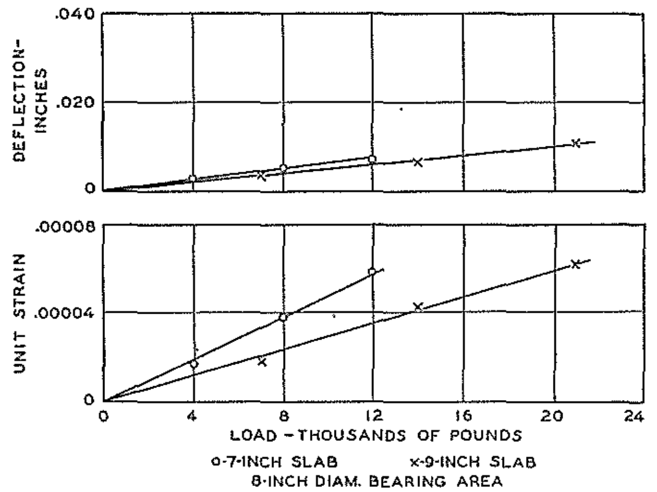


FIGURE 23.—TYPICAL LOAD-DEFLECTION AND LOAD-STRAIN RELATIONS AT THE INTERIOR OF A CONCRETE PAVEMENT.

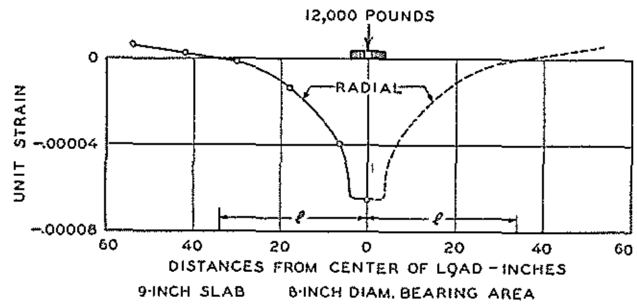


FIGURE 24.—VARIATION IN RADIAL STRAINS ALONG THE EDGE OF A PAVEMENT FOR A LOAD APPLIED ON A CIRCULAR AREA AT THE EDGE.

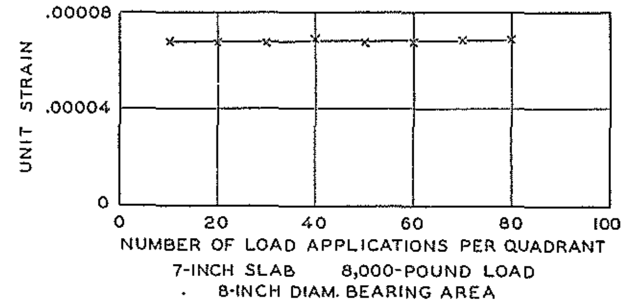


FIGURE 25.—EFFECT ON STRAIN OF REPEATED LOADS APPLIED AT THE EDGE OF A CONCRETE PAVEMENT.

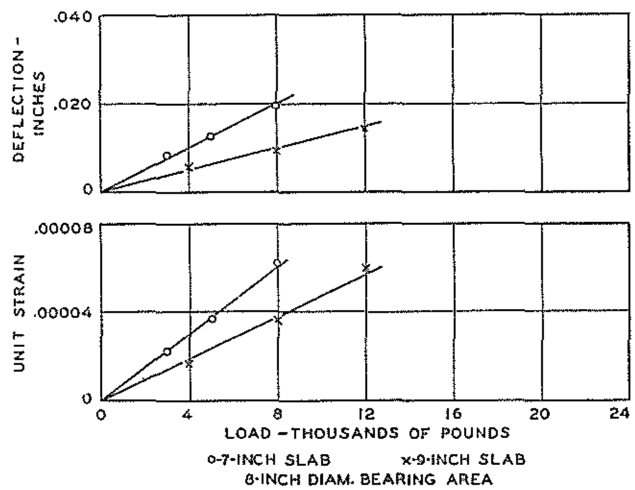


FIGURE 26.—TYPICAL LOAD-DEFLECTION AND LOAD-STRAIN RELATIONS AT THE EDGE OF A CONCRETE PAVEMENT.

At the free edge, tests were made to determine the shape of the strain variation curve along a line through the center of the bearing area and parallel to the edge of the pavement; to determine the effect of repeated loads; and, to determine the load-deflection and load-strain relations using circular bearing areas centered 6 inches from the slab edge. Typical data from these tests are shown in figures 24, 25 and 26.

The strain variation curve (fig. 24) shows radial strains only since tangential strains at the points of measurement shown are negligible. Each point shown on the curve is an average of eight observations, two in each of the four quadrants of the test section. The measurements of strain were made on the upper surface of the slab, hence the maximum strain shown is a compression in the upper surface of the pavement. It will be noted that the distance from the center of the loaded area to the point where the strain changes from compression to tension is approximately equal to l. The tensile strain in the upper surface of the pavement is much smaller than that in the bottom surface directly under the area of load application. It is this maximum tensile deformation in the bottom of the slab directly under the load that is the critical strain. Numerous tests have shown that the maximum compressive strain measured under the center of a circular bearing plate of 8- to 12-inch diameter when the plate is tangent to the slab edge is essentially equal to the maximum tensile strain measured at the bottom of the slab edge. In other words the maximum strain shown in the variation diagram may be considered as equal to the critical strain.

The data from repeated loading are given in figure 25. The procedure was the same as that followed for case I (corner) and case II (interior). It is indicated that the critical strain for the edge loading is not increased by a repetition of load application. In this respect it is like the interior case. This difference in behavior between the corner on the one hand and the interior and edge on the other is attributed to two factors, first, there is more deflection and hence more soil deformation when the free corner of a slab of uniform thickness is loaded and, second, for corner loading there is less confinement for the soil than is present with either the edge or interior loadings.

In figure 26 are given some typical load-deflection and load-strain data for the edge loading. It was found that these relations were linear within the limits of the tests.

To summarize, the preliminary load testing program gave data as to the strain distribution which located the position of the critical strain for each of the three cases of loading considered in the Westergaard analysis. It verified the equation for locating the position of maximum stress for corner loads as given by Westergaard. The application of repeated loads was found to affect the magnitude of the critical strain to a measurable extent with corner loading but not for the interior or the edge loadings. Finally the preliminary load testing program showed that the load-deflection and load-strain relations were nearly linear for corner loading and linear for the interior and edge loadings, within the limits of the tests.

LOAD-DEFLECTION DATA AND THEIR SIGNIFICANCE

The methods used for determining the deflected shape of the test sections under various load and other conditions were described and discussed in the earlier reports of this series.

The principal use made of the deflection data in the present study is for evaluating the constants that appear in the various equations of the Westergaard analysis.

In connection with the presentation of his generalized equations (*25*) Westergaard suggests methods for determining the value of the quantities D, l, k, K, E, L and Z from observed values of deflections and stresses. These will be discussed briefly.

The deflection modulus, D, was defined as being equal to kl^2 and values for it may be obtained by substituting the observed maximum deflection value in the appropriate equation for maximum deflection (1), (2) or (3).

The value of l is obtained by adjusting the scales (both horizontal and vertical) of a theoretical deflection diagram, in which the ordinates are deflection units and the abscissas are distances expressed in terms of l, until the theoretical elastic curve coincides as nearly as possible with the observed shape of the deflected slab.

For the corner loading the theoretical deflection diagram may be constructed, once D is determined, by means of the generalized equation for corner deflection

$$z=\frac{P}{D}\left(1.1e^{-\frac{x}{l}}-\frac{a_1}{l}0.88e^{-\frac{2x}{l}}\right) \qquad (13)$$

z being the deflection at any distance x from the slab corner, measured along the bisector of the corner angle, while e is the Naperian base. Values of l are assumed until coincidence is obtained.

For the interior and edge cases, the theoretical deflection diagrams were constructed by means of coefficients of deflection obtained from the diagrams given by Westergaard in the original analysis (*23*) (figures 4 and 8). Values of l were assumed as in the case of the corner loading until the best degree of coincidence was attained.

With values for D and l determined in the manner just described k, K, and E may be evaluated by means of the following equations:

$$k=\frac{D}{l^2} \qquad (14)$$

$$K=\frac{D}{l} \qquad (15)$$

$$E=\frac{12(1-\mu^2)Dl^2}{h^3} \qquad (16)$$

There appears to be no direct method for the determination of values for the quantities L and Z from deflection data. Westergaard suggests that these terms be evaluated from the theoretical equations for stress. Discussion of this question will accordingly be deferred until after the introduction of the stress data. Except where stated otherwise, the value of Z will be taken as zero, however.

Observed deflection data for each of the four test sections of uniform thickness for the three cases of corner, interior and edge loading are shown in figures 27, 28, and 29, respectively. On these diagrams the observed shape of the deflected slab is shown by curves drawn through crosses. Each cross is the average of 12 observed values, three measurements in each of four quadrants. The small circles are computed values which indicate the position of the matched theoretical diagram constructed for the purpose of determing l.

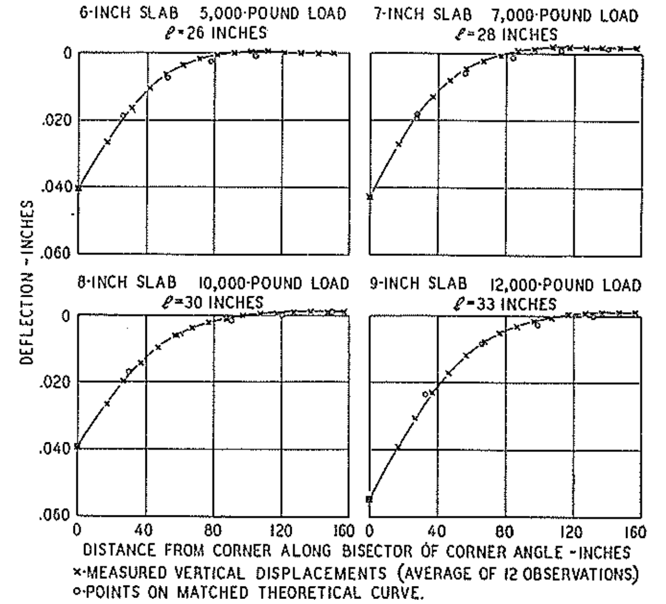


FIGURE 27.—DEFLECTION OF THE BISECTOR OF THE CORNER ANGLE USED FOR EVALUATING l.

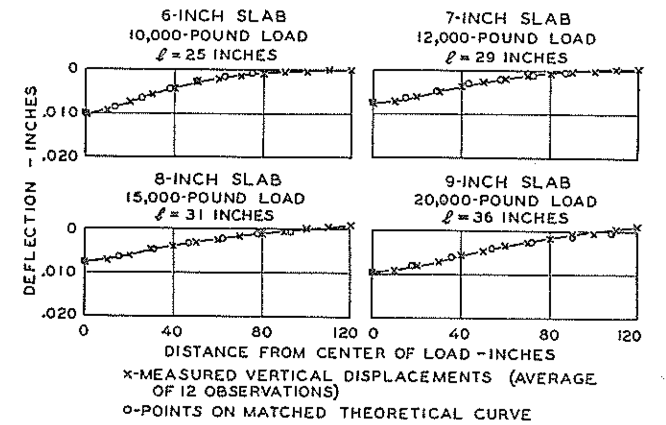


FIGURE 28.—DEFLECTION OF SLAB FROM INTERIOR LOADING USED FOR EVALUATING l.

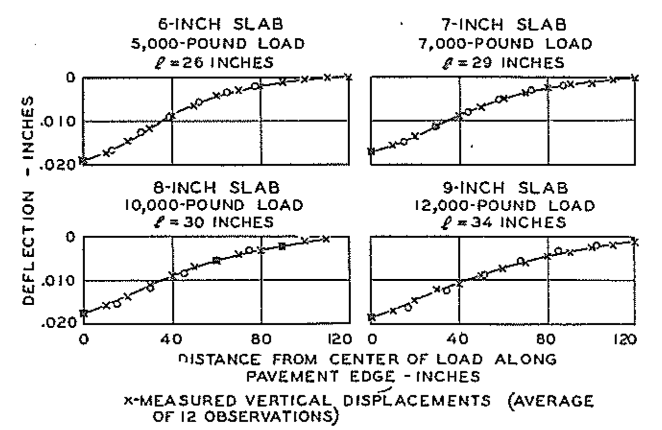


FIGURE 29.—DEFLECTIONS ALONG FREE EDGE OF PAVEMENT USED FOR EVALUATING l.

Values of the several coefficients, derived by the methods just described from the deflection data shown in figures 27 to 29, inclusive, are given in table 4.

TABLE 4.—*Values for various coefficients, used in the Westergaard equations, determined from measured deflections*

Position of load	Time of testing	Slab thickness	l	k	K	D	E
		Inches	*Inches*	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-1}$	*Lbs. in.*$^{-2}$
Corner	Late summer	6	26	143	3,708	96,400	3,540,000
	Winter	7	28	161	4,515	126,400	3,390,000
	Winter	8	30	227	6,825	204,700	4,220,000
	Late fall	9	33	168	5,535	182,600	3,200,000
Interior	Late summer	6	25	195	4,880	122,000	4,140,000
	Winter	7	29	238	6,895	200,000	5,750,000
	Summer	7	28	222	6,230	174,400	4,670,000
	Winter	8	31	260	8,065	250,000	5,500,000
	Late fall	9	36	203	7,315	263,200	5,490,000
	Summer	9	33	220	7,290	240,500	4,210,000
Edge	Late summer	6	26	171	4,440	115,400	4,235,000
	Winter	7	29	212	6,145	178,200	5,125,000
	Winter	8	30	279	8,365	251,000	5,175,000
	Late fall	9	34	243	8,260	280,800	5,220,000

The deflection tests on the four sections were not made at the same time but covered a period of about 4 months, as follows:

Test section:	*Month tested*
6-inch	September.
7-inch	December.
8-inch	November.
9-inch	October.

During the period when a particular section was under test it was insulated against air temperature fluctuations with a layer of straw and was protected from precipitation and direct sunlight with a temporary canvas shelter. However, during the 4-month period between late summer and winter there are important changes in temperature, evaporation rate, ground moisture, etc., and it seems reasonable that some effects of this seasonal change are present in the data. If so, the effect would be most noticeable in comparisons between different test sections.

For a given slab thickness, values of the radius of relative stiffness, l, are in good agreement for the three cases of loading. For conditions that are comparable there is rather good agreement also between the values of modulus of subgrade reaction, k, as determined by pavement deflection, for the interior and edge loadings but the value for the corner loading is consistently lower. This is believed to be the result of incomplete contact between the corner area and the subgrade and is in accord with the evidence of the strain data previously discussed. It will be noted that the values of k, as determined from the deflections of the 6-inch section of uniform thickness are somewhat lower than as determined by the deflection data from the three other test sections.

SEASONAL CHANGES AFFECT CERTAIN COEFFICIENTS

In the earlier discussion of the modulus of subgrade reaction, k, in the first section of this report, the data from the bearing plate tests of series 4 showed that for a given plate displacement the value of k was appreciably greater in summer than in winter. For example, at a displacement of 0.02 inch, the summer value was 280 lbs. in.$^{-3}$ and the winter value was 199 lbs. in^{-3}. This same trend is not evident in the limited data on the values of k, determined from the deflections of the interior of the 7-inch and 9-inch sections shown in table 4. It will be noted that the value of k is nearly the same for summer conditions as it is for late fall or winter conditions in the two comparisons available in this table. It may be recalled that in the study of seasonal effects on the value of k with the bearing plate tests, the plate was left continuously in place on the subgrade. While the plate used was relatively large, it actually was much smaller than a full-size pavement slab. It is conceivable, therefore, that a larger change in soil moisture may have occurred under the bearing plate than was possible under the test section and that this larger change in soil moisture might account for the larger variation in the value of k as determined from the bearing plate test.

The general level of values of k from the bearing plate tests is in reasonably good agreement with that determined from pavement deflections.

The coefficients D and K being dependent on l and k are affected directly by any conditions that influence the value of either l or k. The values of the modulus of elasticity for the concrete, E, as determined from the slab deflections are in the same general range as the values that were obtained from the tests of the laboratory specimens. Values obtained from corner deflections are distinctly lower than those determined by deflection data from interior or edge loading. This is believed to be a direct reflection of the imperfect subgrade support that obtained under the unwarped slab corners and of certain rotational movements that will be discussed presently. For this reason the value of E, determined from corner deflections, is considered a less reliable index of slab stiffness than those determined from interior or edge deflection data. It is noted also that there is a tendency for the values determined from the deflections of the 6-inch section to be slightly lower than those for the other three sections. A comparison of the data obtained in summer with those obtained in winter in tests at the interior of the 7-inch section show a much higher value of E for the winter condition. The same trend is evident in the tests made in summer and in late fall at the interior of the 9-inch section.

Since the value of the modulus of elasticity of the concrete varies with the moisture state of the material and since the moisture state is known to vary but no good measure of its range or rate of variation was available, an uncertainty as to the exact value of the modulus E was always present in any consideration of data that required its use, particularly where long time periods were involved in the testing.

Assuming that values of E=5,500,000 lbs. in.$^{-2}$ and k=200 lbs. in.$^{-3}$ were fairly representative of the winter condition and that values of E=4,500,000 lbs. in.$^{-2}$ and k=280 lbs. in.$^{-3}$ are equally representative of the summer condition of the test sections, values of the coefficients l, D and K were computed for each of the four thicknesses of test section. These computed values are given in table 5. A comparison can be made between these computed values and certain of the values derived from the measured deflections which were shown in table 4. In making such comparisons it is well to bear in mind, first, that the computed values are only as good as the selected values of E and k on which the computations are based, and second, that the computations presuppose the ideal subgrade reaction to obtain. The agreement between the computed values and those derived from the measured deflections is, in general, better for the winter observations than for the summer and better for the interior and edge loadings than for the corner. For the winter condition several comparisons are available and for the interior and edge cases the agreement may be considered good. For the corner case the values of D and K derived from measured deflections tend to be lower than the computed values.

TABLE 5.—*Computed values of coefficients used in the Westergaard equations*

WINTER CONDITION

[E=5,500,000 lbs. in.$^{-2}$, k=200 lbs. in.$^{-3}$]

Slab thickness	l	D	K
Inches	*Inches*	*Lbs. in.$^{-1}$*	*Lbs. in.$^{-2}$*
6	26.7	142,300	5,340
7	29.9	179,300	5,990
8	33.1	219,100	6,620
9	36.2	261,500	7,230

SUMMER CONDITION

[E=4,500,000 lbs. in.$^{-2}$, k=280 lbs. in.$^{-3}$]

Slab thickness	l	D	K
6	23.3	152,300	6,530
7	26.2	191,900	7,330
8	28.9	234,500	8,100
9	31.6	279,800	8,850

For the summer condition only two comparisons are available, the interior case for the 7-inch and 9-inch sections. In these the values of D and K derived from the measured deflections tend to be 10 to 15 percent lower than the corresponding computed values of table 5. There is generally good agreement in the values of l for the four pavement thicknesses and the three cases of loading.

Further studies of the corner case were made in an effort to establish the reason for the differences between the computed values of the coefficients and those obtained from the measured corner deflections. Tests were made with the 7-inch and 9-inch sections, loads being applied when the corners were flat, warped up and warped down. Measurements were made not only of the deflection but also of the rotational movement of the slab about the longitudinal joint as the corner loads were applied. These rotational movements were measured by placing two recording strain gages on the slab end spanning the joint, one gage near the top and one near the bottom surface of the pavement.

The vertical displacement of the free corner resulting from rotational movement at the longitudinal joint was calculated from the movements recorded by the two strain gages. This displacement is an apparent deflection of the slab corner that must be subtracted from the measured maximum deflection at the corner if the displacement caused by slab flexure alone is to be obtained. Displacement values calculated in this manner for the 7,000-pound load on the corner of the 7-inch test section and the 10,000-pound load on the 9-inch test section are shown in table 6. It will be noted that the values are about the same for the two slab thicknesses but that they vary considerably with the degree of subgrade support afforded the loaded corner, being approximately three times as great for the upward warped corner as for the same corner brought into better contact with the subgrade by downward warping.

The measured total vertical displacements along the bisectors of the corner angle for the 7-inch and 9-inch sections, for the three conditions of temperature warping are shown in figure 30. Each value on the apparent deflection curves in this figure is the total displacement from a fixed datum. The calculated movement of the corner resulting from rotation of the slab about the longitudinal joint is indicated as a correction to be subtracted from the measured total downward movement of the corner point. Because of the rotational movement of the slab about the longitudinal joint it is apparent that the measured load-deflection relation for the corner case cannot be expected to compare directly with the theoretical relation. As nearly as could be determined in this investigation, the measured maximum corner deflection for the downward warped condition when corrected for slab rotation is in reasonably good agreement with the theoretical value. The measured maximum values for the other two conditions of warping are approximately equal after correction and both are approximately 25 percent greater than the corrected value for the downward warped condition.

TABLE 6.—*Displacements caused by rotational movement of the slabs about the longitudinal joint*

Condition of edges of slab	Corner displacement	
	7-in. slab	9-in. slab
	Inches	*Inches*
Warped down	.006	.008
Flat	.015	.016
Warped up	.021	.020

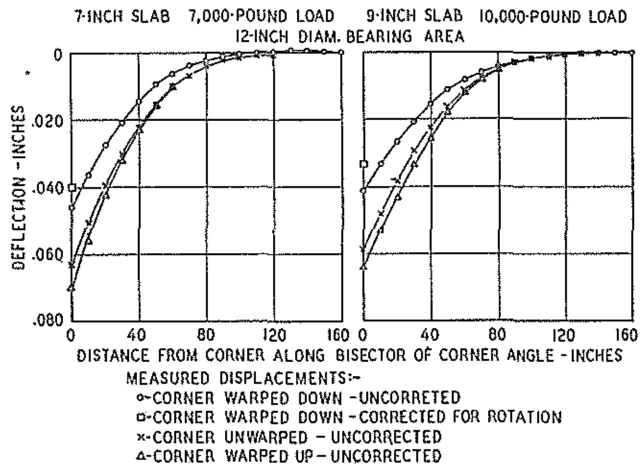


FIGURE 30.—DEFLECTION OF THE BISECTOR OF THE CORNER ANGLE FOR THREE CONDITIONS OF TEMPERATURE WARPING.

It is believed that this special study of the deflection of the corners of the 7-inch and 9-inch test sections indicates that, because the measured apparent deflections of the slab corner contain displacements from causes other than flexure, such measurements are not suitable for use in determining the value of the several coefficients previously discussed. One possible exception is the value of the radius of relative stiffness, l, which is determined more by the coincidence of curve shapes than by absolute deflection magnitude. Because of this, values of l determined from corner loadings are in good agreement with values determined from interior and edge loadings for all four slab thicknesses.

LOAD-STRESS RELATIONS ANALYZED

One of the early steps in the testing program was a study of the effect of the size of the bearing area over which the load was distributed on the amount of strain produced by a given load for test sections of various thicknesses and for the three cases of loading considered by Westergaard.

In testing the corners bearing plates having radii, a, of 3, 4, 6, 8, and 10 inches were used. In testing the the interior and edge positions, however, the 6-inch

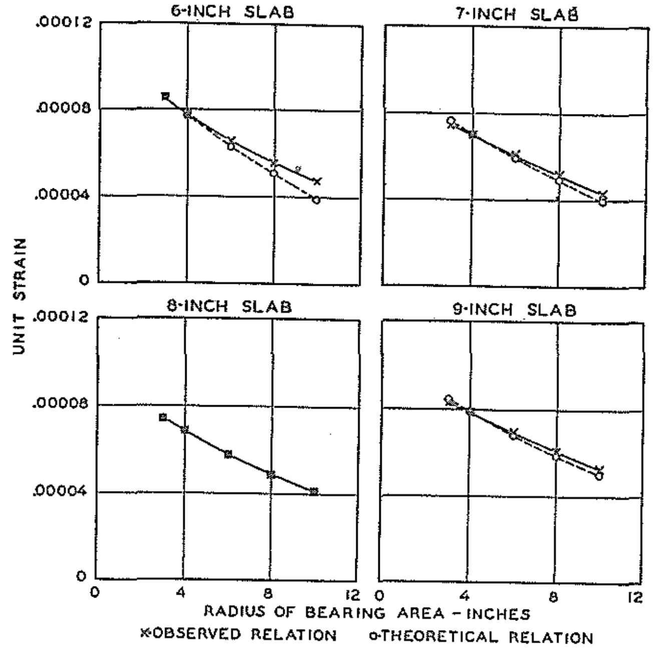


FIGURE 31.—EFFECT OF SIZE OF BEARING AREA ON STRAIN—CORNER LOADING.

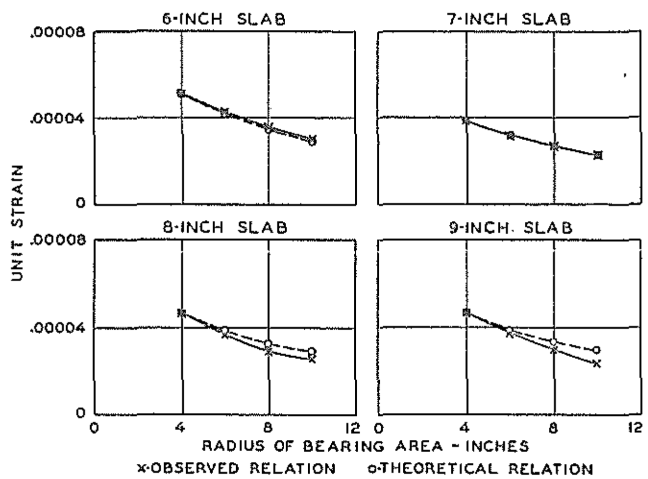


FIGURE 32.—EFFECT OF SIZE OF BEARING AREA ON STRAIN—INTERIOR LOADING.

diameter plate (a=3 inches) was omitted because of having to measure strains under the bearing plate with a gage length exceeding the plate diameter. The measured critical strains for each size of bearing area and each slab thickness are shown for the corner, interior and edge loadings in figures 31, 32 and 33, respectively. In order to compare the observed effect of size of bearing area with that indicated by theory, strain values were computed for each value of the radius, a, and these theoretical relations are shown by the broken line curves in the three figures. For purposes of comparison the two curves in each graph were made to coincide for the case of the bearing area with 4-inch radius. For this reason they do not indicate the relative magnitude of observed and computed strains for a given test condition.

In the case of the corner loading (fig. 31) the observed effect of size of bearing area on strain is almost exactly the same as that indicated by theory. The two

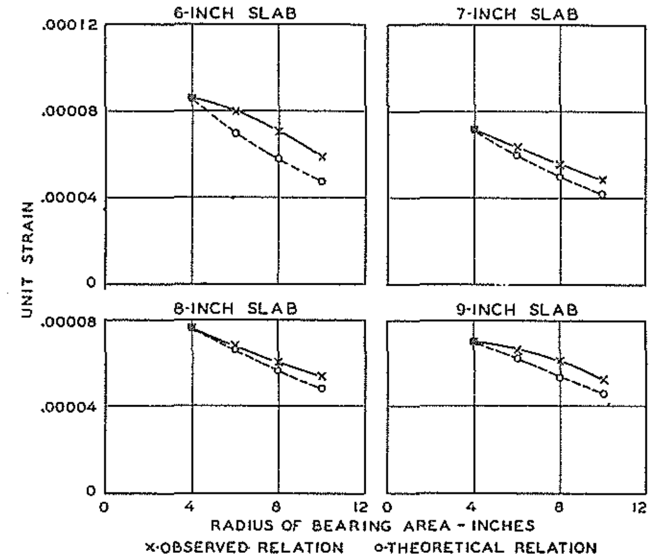


FIGURE 33.—EFFECT OF SIZE OF BEARING AREA ON STRAIN—EDGE LOADING.

curves coincide exactly in the comparison for the 8-inch slab while for the other three sections the observed influence of the size of bearing area is only slightly less than the theoretical.

For loads applied at the interior (fig. 32) the agreement between the observed and theoretical relations is very close in all cases. Such divergences as appear are so small as to be without significance in interpreting the results.

The comparison between the observed and theoretical relations for the edge loading (fig. 33) is the only one in which a significant difference exists. It will be recalled that in this edge loading (case III) a semicircular bearing area was used in the original analysis. In the test program at Arlington a semicircular bearing plate was used and the effort was made to center the load over the center of area of the plate so as to have uniform load distribution to the slab surface. On the 7-inch and 8-inch test sections the curve showing the observed relation has essentially the same shape as the theoretical curve, although lying somewhat flatter with respect to the x-axis. The curves showing the observed relation for the 6-inch and 9-inch sections on the other hand are of somewhat different shape, being concave with respect to the x-axis instead of convex. The apparent divergence between the observed and theoretical curves for the 6-inch and 9-inch sections is emphasized by the fact that it is the data for the smallest bearing area which appear to be out of line and it so happened that it was at this point that the curves were made to coincide, principally because the 8-inch diameter bearing area was used extensively in the load-strain studies. Even with the discrepancies that have been pointed out for the edge loading there is fair agreement between the experimental data and the theory.

As a general statement it is believed that the observed effect of size of bearing area on strain is so similar to the theoretical relation as to indicate that the latter can be used with confidence in its accuracy.

In subsequent tests where the size of bearing area entered as a variable only the 8-, 12- and 20-inch diameter plates were used.

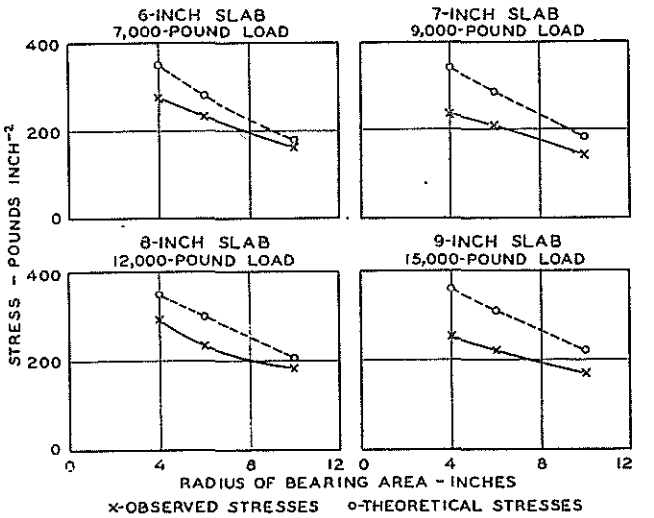


Figure 34.—Comparison of Observed and Theoretical Stresses for Corner Loading Using Coefficients Determined From Deflections Measured at the Corner.

LOAD-STRESS RELATION FOR CORNER LOADING

The first direct comparison between observed and theoretical stresses is shown in figure 34. The term observed stresses used hereafter refers to the critical stress values derived from measured strains, in the manner described earlier in the report. The observed stress values in figure 34 are compared with theoretical values computed with coefficients obtained from corner deflections measured at the same time as the strain measurements. Each observed stress value in this and subsequent figures is based on an average of eight measured strains (2 tests in each quadrant of the test section).

It will be observed in figure 34 that, in every case, the observed stress value is smaller than the theoretical value. This difference is due in large part to the fact that the apparent modulus of elasticity of the concrete, as determined from corner deflection data, is much smaller than the value of the modulus of elasticity determined by other methods. This was brought out in table 4 and the attendant discussion.

When values of the modulus of elasticity obtained from the measured deflections of the interior and edge were averaged and applied to the corner strains for a given test section a better agreement between observed stresses and theoretical stresses resulted. This comparison is shown in figure 35. It is believed that this is a better comparison than that of figure 34 because all evidence indicates that the modulus of elasticity determined from deflection data obtained in tests at the interior and edge is approximately correct. In figure 35 the agreement between the observed and theoretical stress values is very close for the 6-inch and 8-inch sections while for the 7-inch and 9-inch sections the observed stresses are higher than the theoretical stresses by 14 and 18 percent, respectively. The observed influence of the size of bearing area is essentially that indicated by theory.

Some further tests were made to discover, if possible, the reason for the difference between the agreement for the 6- and 8-inch sections and that for the 7- and 9-inch sections as shown in figure 35. The first of these tests were made on the 7- and 9-inch sections during the summer under the condition of maximum downward

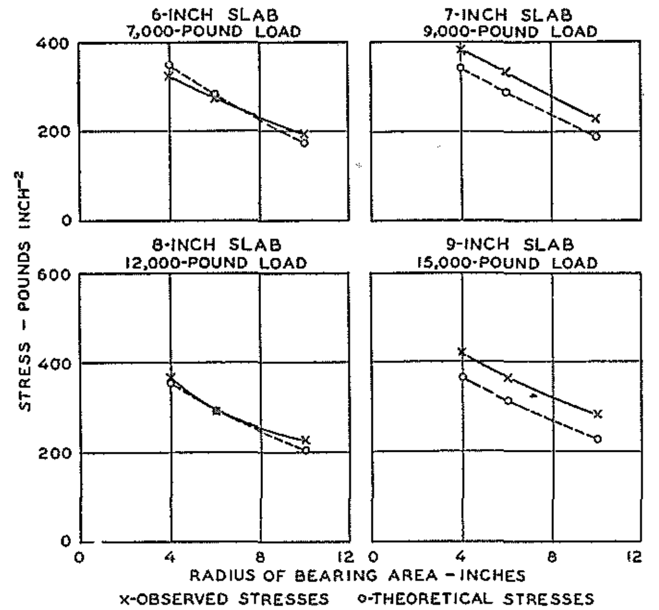


Figure 35.—Comparison of Observed and Theoretical Stresses for Corner Loading Using Average Coefficients Determined From Deflections Measured at the Interior and Edge.

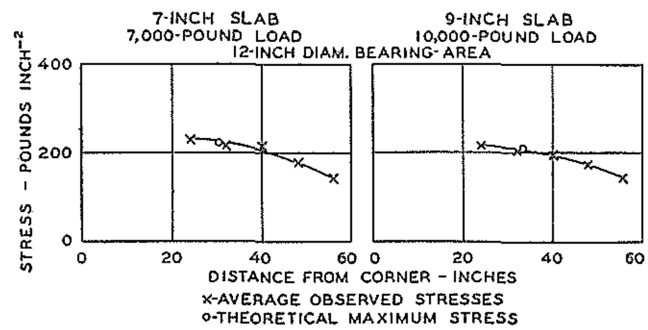


Figure 36.—Comparison of Theoretical Maximum Stress for Corner Loading With Observed Stresses Along the Bisector of the Corner Angle With the Corner Warped Downward.

warping of the slab corners. It is reasonable to assume, therefore, that the pavement corner was in good contact with the subgrade at the time of test. While only one size of bearing plate was used, tests were made on all four quadrants of the test sections and the tests were repeated in two different years. The data may, therefore, be considered representative.

The average observed stresses along the corner bisector for both the 7-inch and the 9-inch test sections, together with the theoretical maximum stress, are shown in figure 36. Each observed value is an average based on 16 strain measurements. The agreement between the observed and theoretical stress values from these tests indicates that, if the conditions are such that the corner is receiving full subgrade support, values of critical stress computed with the Westergaard equation for critical stress for corner loading (case I) can be used with confidence. When full subgrade support does not exist the computed stresses will be too low.

Further study was given to the effect of the condition of subgrade support on critical stress for corner loading in a series of tests on the 6-inch, 7-inch, and 9-inch sections. In this study loads were applied to the slab corners when those corners were warped upward, unwarped and warped downward. The slabs were as-

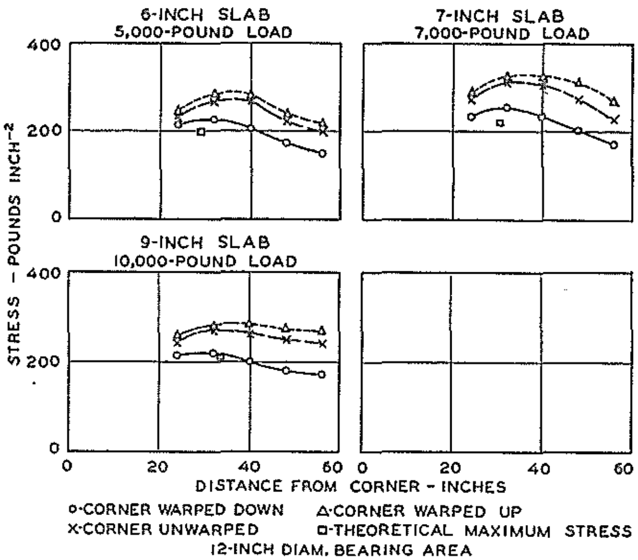


Figure 37.—Average Observed Stresses Along the Bisector of the Corner Angle for Three Conditions of Warping.

sumed to be flat when the temperature of the upper surface and of the lower surface were the same. The 8-inch section was not included because it was necessary to keep the program within practical limits. Conditions for maximum upward warping occur only at certain times in the night or early morning hours in winter and with only three sections included the program extended over nearly one entire winter before the desired data could be obtained. The data from this study are shown in figure 37. Each observed stress value is the result of eight separate strain measurements. The maximum theoretical corner stress is shown on each graph.

Table 7 shows the observed maximum stresses for each condition of warping and for each slab thickness, expressed as a percentage of the theoretical stress. It will be noted that the observed stresses are slightly greater than the theoretical stresses for the condition of downward warping. It is quite possible that even when the corners were warped downward full contact with the subgrade was not established. The observed stresses for the flat and upward warped conditions, i. e., for the conditions of least subgrade support, are higher than the theoretical values by 30 to 50 percent. Thus it is evident that where the assumed condition of full subgrade support does not obtain some correction must be applied to the stress values computed by the theoretical equation for maximum stress from corner loading. Westergaard analyzed the load-deflection and the load-stress relations for slab corners that are not fully supported by introducing a modified condition of subgrade reaction that was assumed to represent the support given a slab corner when that corner was warped upward (*26*).

This analysis provides corrections to be applied to values computed with the equations of the original analysis. In order to compute the corrections it is necessary to determine experimentally certain quantities that appear in the correction equations. A serious effort was made to determine the quantities and the corrections in connection with the investigation being described but for various reasons it was not

Table 7.—*Comparison of theoretical corner stresses with maximum observed stresses for three conditions of warping*

Slab thickness	Observed stresses expressed as percentages of theoretical values		
	Corner warped downward	No warping	Corner warped upward
Inches	*Percent*	*Percent*	*Percent*
6	114	137	144
7	116	141	149
9	105	132	138

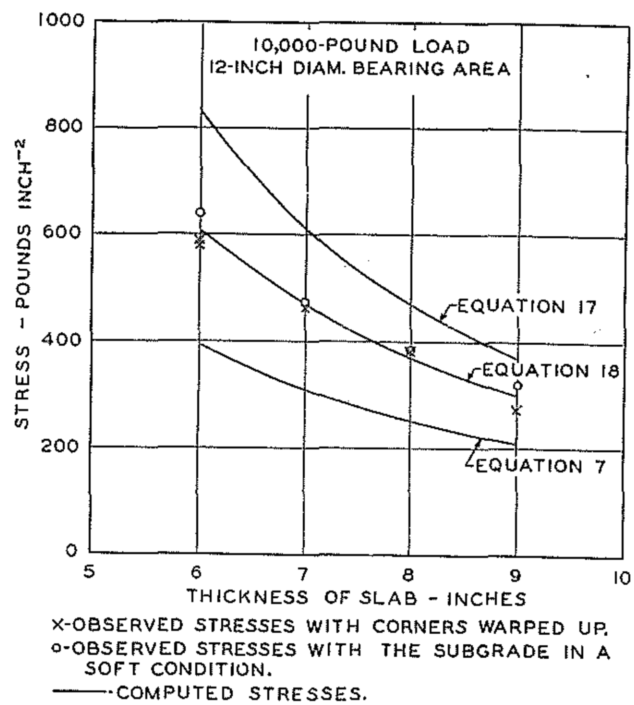


Figure 38.—Comparison of Observed Maximum Corner Stresses With Those Computed by Three Stress Equations for Four Slab Thicknesses.

possible to do so in a satisfactory manner. As a general statement it may be said that the corrections obtained in the manner suggested were too small, at least for the conditions of this investigation.

OBSERVED STRESSES COMPARED WITH VALUES COMPUTED BY VARIOUS STRESS EQUATIONS

In figures 38 and 39 comparisons are made between observed maximum corner stresses and those computed with three different equations which will be described presently. The comparison in figure 38 is for four slab thicknesses, the load and size of loaded area being constant while those for figure 39 are for different sizes of loaded area, with a constant load and slab thickness. As in most of the tests, each observed stress value is the result of eight individual strain measurements. The stress values in figure 38 were either for the condition of upward warping (crosses) or for a very soft subgrade condition (circles) hence are representative of a low corner support condition. Those in figure 39 are for the upward warped condition only. The values of E and k used were selected as representative for the conditions that obtained at the time of the tests.

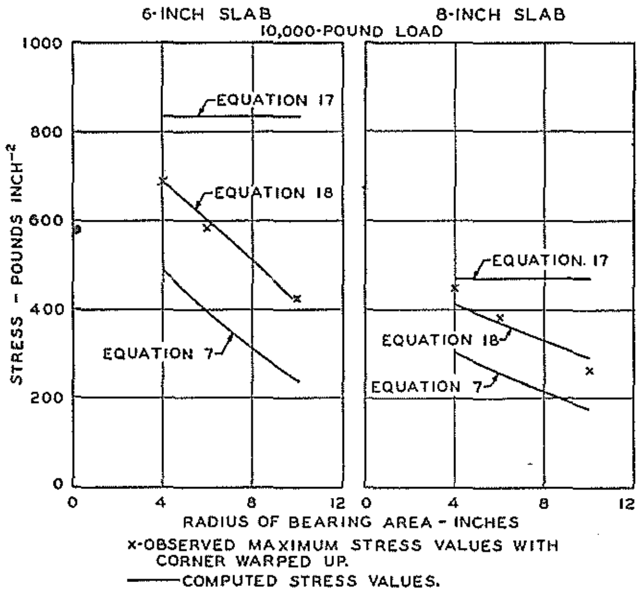


Figure 39.—Comparison of Maximum Observed Corner Stress Values With Those Computed by Three Stress Equations to Show Effect of Size of Bearing Area on Stress.

The three equations referred to above for computing the maximum corner stress values shown in figures 38 and 39 are

$$\sigma_c=\frac{3P}{h^2} \tag{17}$$

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right] \tag{7}$$

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.3}(a\sqrt{2})^{1.2}\right] \tag{18}$$

Equation 17 was suggested by Goldbeck (*4*) in 1919 as an approximate formula for computing the stress in concrete slabs. In various forms it came to be known as the "corner formula." It assumes that the slab corner receives no support from the subgrade and that the load is applied at a point at the extreme corner of the slab. In general, stress values computed with this equation are very much higher than those observed in this investigation. In figure 38 the difference will be observed to vary with the pavement thickness, the computed stress being 38, 30, 22, and 24 percent greater respectively than the average observed maximum stress for the corners of the 6-, 7-, 8-, and 9-inch test sections. Since in equation 17 the load is assumed to be concentrated at a point it is obvious that the computed stresses would not be affected by the size of the loaded area. This is apparent in the comparisons of figure 39. For the 6-inch section the computed stresses are greater than those observed for the bearing areas with radii of 4, 6, and 10 inches by 21, 43, and 97 percent respectively. For the 8-inch section the percentages are 4, 23, and 79 respectively for the same three sizes of bearing area.

Equation 7 is the original Westergaard equation for maximum stress for a load applied on the pavement corner over a circular area of radius, a, with full subgrade support measured by the reaction modulus k. (Note.—It is apparent that equation 17 is a special case of equation 7 for the condition of $k=0$ with a infinitely small).

The data presented in figures 38 and 39 support those shown previously in demonstrating that the maximum stresses for corner loading observed in this investigation are considerably greater than the theoretical maxima. If the comparative data from these three figures are averaged it is found that the theoretical values are about one-third less than the observed values and as evidenced by the graphs the difference is fairly constant for various pavement thicknesses and sizes of bearing area.

Equation 18 is an empirical equation developed by modification of the exponents in the Westergaard equation in such a manner as to cause the computed values to coincide more nearly with the observed data for the case of the incompletely supported slab corners shown in figures 38 and 39. The equation thus has no theoretical foundation. It was published earlier in slightly different form (*7*) but the comparisons with observed data were not presented at that time. The data in figure 38 show that for the poorly supported corners to which the observed stress values apply, stress values computed by the empirical equation are in good agreement. The data shown in this figure were obtained in tests using a 12-inch diameter bearing plate. Comparisons with test data obtained with bearing plates of other sizes are shown in figure 39 and the agreement between the observed and computed stress values is generally good.

In addition to the comparisons just shown, it is of interest to note that in a recent report of a laboratory investigation of stress conditions in the corner region of concrete slabs at the Iowa Engineering Experiment Station (*13*) it is stated that the critical corner stresses determined experimentally in that investigation are in general agreement with values calculated by the empirical stress equation developed from the data obtained at Arlington (equation 18).

To summarize, it has been shown that for the corner loading (case I) the conditions of soil support for which the Westergaard equation for maximum stress was developed existed only for short periods of extreme downward warping and that for these periods only was there agreement between the observed stresses and those computed by the Westergaard equation. For the rest of the time the corners were not fully supported and the computed stresses were much lower than the observed stresses.

An empirical equation has been developed that seems to fit the observed data reasonably well and, for conditions of poor corner support, its use is suggested, at least until more comprehensive information is available. It is emphasized, however, that the equation is without theoretical foundation and may not be applicable to all conditions.

LOAD-STRESS RELATION FOR INTERIOR LOADING

A direct comparison between observed and computed stresses for interior loading (case II) is shown in figure 40. The values of observed stress shown in this graph are the critical values for each size of loaded area as determined from strain measurements in the manner previously described and each is the average of eight individual observations. The theoretical stress values in this figure were computed with the equation of the original Westergaard analysis, as restated in generalized

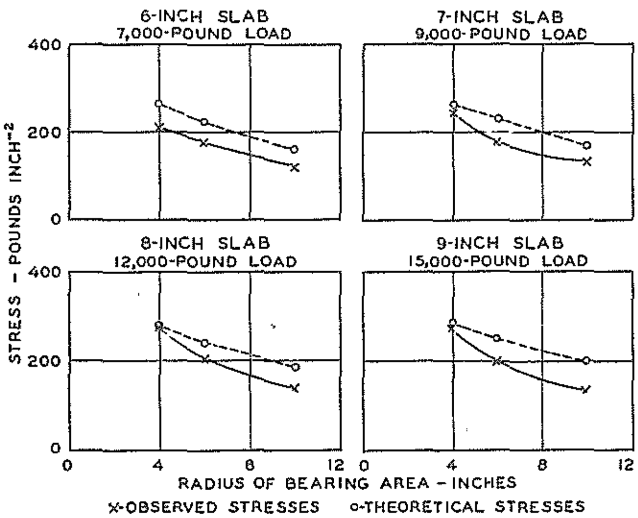


Figure 40.—Comparison of Observed and Theoretical Stresses for Interior Case of Loading $Z=0$.

form (equation 8), the coefficients being determined from deflection measurements of the pavement slab made at the time the strains were measured.

In this comparison the theoretical values are appreciably higher than the observed stresses. The opinion has been expressed by Westergaard that for interior loading the subgrade reactions may be more closely concentrated around the load than are the deflections and that for a given subgrade this redistribution of subgrade reactions results in more support than was contemplated in the original analysis. Based on this conception his "supplementary theory" for interior loading (*25*) was developed. In this supplementary analysis the equation for maximum load stress was modified to the form shown as equation 10 earlier in this discussion. Two new quantities, Z and L, appeared in this equation. Values for these must be determined from experimental data. Westergaard suggests that these constants be established by adjusting their values until fair agreement exists between observed and computed deflection and stresses.

In the present investigation for the subgrade as originally constructed this procedure resulted in the following values: $Z=0.05$ and $L=1.75l$, for the four sections of uniform thickness.

The values of the various coefficients determined from deflections using a value of $Z=0.05$ are shown in table 8. These were developed from the same deflection data as were used for those of table 4 but differ slightly because of the influence of the quantity Z. The values of the modulus of elasticity E, shown in table 8, are in good agreement with those determined by other methods. The values of k, shown in table 8 for the late fall and winter conditions, are in good agreement with the values determined by the bearing tests with rigid plates. For the summer condition the agreement is not as good and a possible explanation for this was offered earlier in the report.

A direct comparison between observed maximum stresses for the interior loading and maximum stress values computed by the Westergaard equation based on a redistribution of subgrade reactions is given in figure 41. Except for the value of L, which as stated above was found to be equal to $1.75l$, the coefficients used in the computation were those given in table 8.

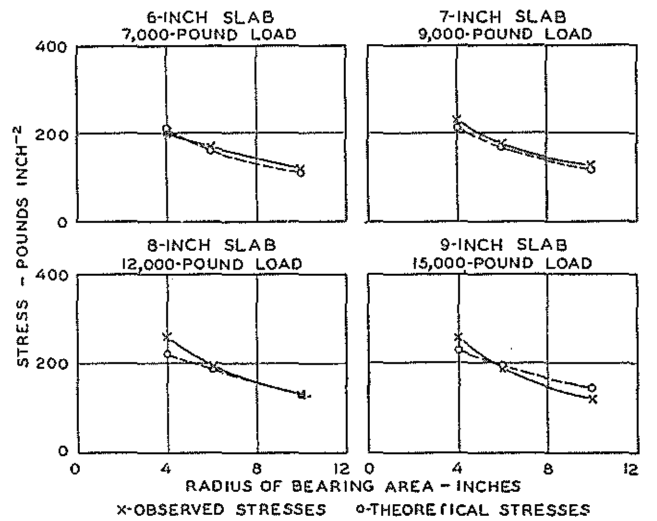


Figure 41.—Comparison of Observed and Theoretical Stresses for Interior Case of Loading $Z = 0.05$ and $L = 1.75l$.

Table 8.—*Coefficients determined from the deflections for the interior case of loading $Z=0.05$*

Time of testing	Slab thickness	l	k	K	D	E
	Inches	*Inches*	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-1}$	*Lbs. in.*$^{-2}$
Late summer	6	25	185	4,640	116,000	3,930,000
Winter	7	29	226	6,550	190,000	5,460,000
Summer	7	28	211	5,920	165,700	4,430,000
Winter	8	31	247	7,640	237,000	5,220,000
Late fall	9	36	103	6,940	250,000	5,220,000
Summer	9	33	210	6,920	228,500	4,000,000

In general, there is good agreement between the observed and the computed values for all sizes of bearing areas and for all thicknesses of pavement even though only one value of Z and one of L were used in the computation.

So far there has been little or no opportunity to study experimentally the range of variation of these constants to be expected with various concretes and subgrade conditions. Also while the range in size of loaded areas for which data are shown are probably adequate for the usual conditions of highway loading the areas of contact of some airplane tires are quite outside the range. In this connection one of the supplementary studies which is described at the end of this report was made to determine the effect of large bearing areas and of bearing areas of other than circular shape on the load-stress relation and experimental data obtained with larger areas for interior loading are presented in connection with the description of that work.

LOAD-STRESS RELATION FOR EDGE LOADING

A direct comparison between observed stresses and computed stresses for the edge loading is shown in figure 42. The observed stresses are based on measured critical strains while the computed values were obtained with the Westergaard equation for maximum stress for the edge loading, case III, given earlier in this report as equation 9. The coefficients used in the computations were those obtained from deflections made at the time of the strain measurements and are shown in table 4.

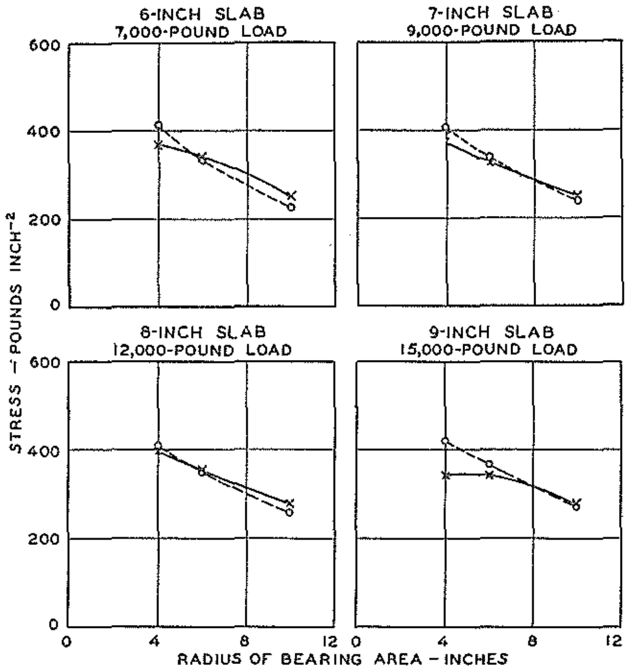


FIGURE 42.—COMPARISON OF OBSERVED AND THEORETICAL STRESSES FOR THE EDGE CASE OF LOADING.

It is apparent from these data that theoretically the effect of size of bearing area is somewhat more pronounced than that observed in these tests although for the 7-inch and 8-inch sections the difference is quite small and the general agreement between the observed and the computed values is quite good. In the case of the 6-inch and 9-inch sections, the agreement is good for the 12-inch and 20-inch diameter bearing areas but for the 8-inch diameter plate the observed stress is definitely lower for both test sections than theory indicates it should be. It will be recalled that this same anomaly was apparent in the strain data shown in figure 33. Just why the stress values for the 8-inch semicircular plate should be low on these sections is not clear. The tests made on the several quadrants of the test sections show no more than the usual spread between individual values so that the average value is not adversely affected by erratic data. Furthermore, repeat tests made 2 years later on the same sections showed essentially the same load-strain values. Even with these values included the general agreement between the observed and computed values may be considered good and it seems reasonable to conclude that the Westergaard equation for maximum stress for edge loading gives an accurate indication when the assumed condition for the loaded area exists and the slab is in an unwarped condition.

Some additional study of the case of an edge load was made to determine the effect of upward warping on the load-stress relation. Some of the tests were made during the late fall with the soil of the subgrade in a moderately wet condition while others were made when the subgrade was soft from thawing after having been frozen to a depth of several inches. The observed stress values from these tests are shown in figures 43 and 44 together with relations computed by two equations for edge stress. The first is the theoretical equation developed by Westergaard for edge loading (case III) the generalized form of which has been

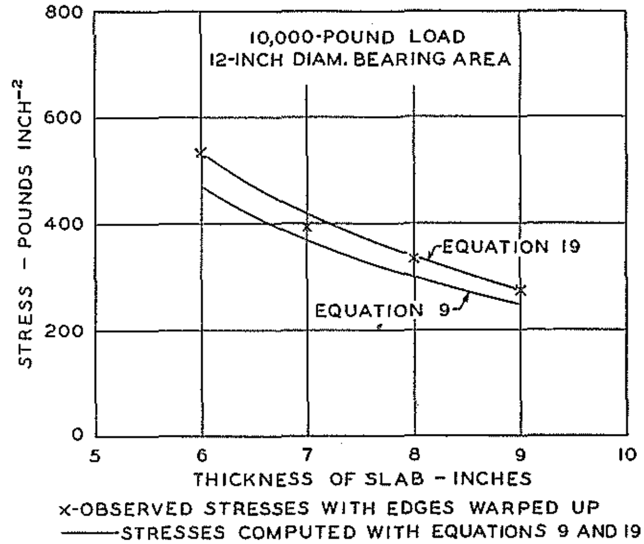


FIGURE 43.—COMPARISON OF OBSERVED MAXIMUM EDGE STRESSES AND EDGE STRESSES COMPUTED BY THEORETICAL EQUATION 9 AND EMPIRICAL EQUATION 19 FOR FOUR SLAB THICKNESSES.

given as equation 9. The second equation is as follows

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)+\log_{10}\left(\frac{b}{1-\mu^2}\right)-1.0792\right]\quad(19)$$

and is an empirical equation developed to fit the observed stress values obtained in load tests on the upward warped slab edges for conditions of subgrade support such as prevailed when these particular tests were made. The same empirical equation, in slightly different form, has been published previously (7) but the data from which it was developed were not presented at that time.

Figure 43 shows the effect of slab thickness on the maximum stress from edge loading while in figure 44 the effect on stress magnitude of the size of the loaded area is brought out. As in the graphs shown previously each observed stress value is an average derived from eight individual strain measurements, two on each of the four quadrants of the test section. From the data in this figure it is apparent that the observed stresses, for the conditions of the test, exceed the values computed by the Westergaard equation for maximum edge stress by about 10 percent. In making this comparison it is well to keep in mind that upward warping of slab edges develops only when the bottom surface of the slab is warmer than the upper surface. Appreciable upward warping occurs relatively infrequently in the region where these tests were made. The tests for which the data are shown in figure 43 were made before sunrise on winter mornings when the cycle of temperature changes happened to be favorable. At the time of year during which the observed data were obtained the supporting value of the subgrade is apt to be lessened by a high moisture content. In view of the critical conditions represented it is rather surprising that the observed stress values do not exceed those computed by equation 9 by a greater percentage.

The observed stress values are in good agreement with the values computed by the empirical equation for maximum edge stress, equation 19, for all four pavement thicknesses represented in the investigation.

In figure 44 the observed data indicate that, when the slab edge is warped up the effect of the size of loaded area, as measured by the radius, *a*, is slightly less pronounced than theory indicates. The values are higher than those computed by the theoretical equation (equation 9) also, as would be expected in the light of the preceding discussion. Stress values computed by the empirical equation (equation 19) agree closely with the observed stresses for the 12-inch and 20-inch diameter bearing areas but are somewhat higher than the stresses observed in the tests with the 8-inch diameter area, the average difference being of the order of 10 percent.

The empirical equation was derived by adjusting the theoretical relation expressed by the Westergaard equation for maximum edge stress until the empirical relation fitted the observed data as closely as possible. The empirical equation is, therefore, specific rather than general in its application. However, it yields stress values that are somewhat higher, for given conditions, than those computed with the theoretical equation, a difference that would be expected to exist generally when the edge of the slab is warped upward. Therefore, if this extreme condition is being considered, the use of the empirical equation is suggested, at least until such time as more comprehensive data are available.

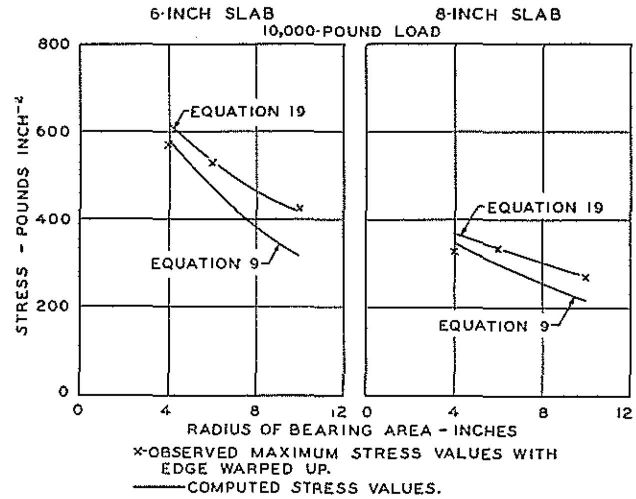


FIGURE 44.—COMPARISON OF OBSERVED MAXIMUM EDGE STRESSES AND EDGE STRESSES COMPUTED BY THEORETICAL EQUATION 9 AND EMPIRICAL EQUATION 19 FOR THREE SIZES OF BEARING AREA.

RESULTS OF INVESTIGATION REVIEWED

Before presenting the discussion of certain supplementary studies of pavement stresses that were made as a part of this investigation it is desirable to review briefly and to discuss the general aspects of the material which has been presented up to this point.

The Westergaard analyses of the load-deflection and load-stress relations in concrete pavement slabs of uniform thickness, resting on an elastic subgrade, have been studied experimentally. Comparisons have been made between computed and observed deflections and between computed and observed stresses on full-size pavement sections of 6-, 7-, 8- and 9-inch uniform thicknesses. Studies have been made of methods for determining the various coefficients and other quantities that appear in the Westergaard equations.

As a result of this study the general conclusion is drawn that the Westergaard analysis expresses quite accurately the relations between load and deflection and between load and critical stress for various thicknesses of pavement and for various sizes of bearing area provided the slab is in full contact with the subgrade. The subgrade stiffness coefficient and other quantities must of course be determined for the particular conditions that exist.

It has been found possible to determine usable values of the modulus of subgrade reaction, *k*, by means of load-displacement tests with rigid bearing plates. In making such tests it is necessary to use plates of adequate size and to limit the displacement to a value that approximates the pavement deflection under load.

The experimental determination of the other quantities that appear in the equations has been found to be feasible although the facility with which the determinations may be made and the precision of the values obtained is not the same for all quantities. An uncertain factor is the effect of moisture variation in the concrete of the pavement. Moisture variation affects the stiffness of the concrete and if a moisture differential exists it may affect the state of stress through restrained warping. The influence of variation in concrete stiffness is relatively unimportant so far as the use of the theoretical equations is concerned but has a direct effect when measured strains are being converted into stresses. No satisfactory means has yet been found for indicating the moisture conditions that exist in a concrete pavement slab in place on the subgrade.

Unless the values for the quantities that appear in the stress equations have been determined with great care too great reliance should not be placed on the absolute value of computed deflections or stresses.

For conditions where the slab corners or edges are not in full contact with the subgrade because of temperature warping or other cause, empirical equations have been developed that fit the data obtained in tests reported herein. These equations are not offered as a replacement for the theoretical equations of Westergaard but it is thought they may prove useful in certain studies where incomplete subgrade support exists or is assumed.

USE OF THE EQUATIONS

In the second report of this series (*19*) it was shown that the stresses caused by restrained temperature warping can be large and that the magnitude of these stresses depends not only on the temperature differential between the upper and lower surfaces of the pavement but also on the length and thickness of the pavement unit or slab. The greatest temperature differentials develop during the afternoon of clear days in late spring and early summer, the temperature of the upper surface being as much as 20° to 30° Fahrenheit or more above that of the lower surface at such times (for the vicinity of Washington, D. C.). This condition causes the edges of the slab to tend to warp downward and, as this tendency is restrained, a tensile stress is developed in the bottom of the slab. When the upper surface of the pavement is at a temperature lower than that of the bottom the tendency for an upward warping of the slab edges is created. This condition may develop during the night and early morning hours from time to time throughout the year. The magnitude of the maximum temperature differential developed at night usually does not exceed about one-third of that which may develop during the day. The restraint to the upward warping of the slab edges caused by the weight of the slab or by the joint construction produces tensile stress in the upper surface of the slab.

These stresses which develop as a result of restraint to warping combine with the maximum stresses developed by wheel loads. An adequate slab design, therefore, is one which will satisfactorily withstand the most critical combination of load and warping stresses. It has been shown (*19*) that the most effective means for controlling the magnitude of the temperature warping stresses in concrete pavements is by limiting the length and width of the panels or slab units and maximum values of the order of 10 to 15 feet were suggested. Even with slab units limited to dimensions of 10 or 15 feet, however, the stresses from restrained temperature warping are not eliminated. Their magnitude is simply controlled to a reasonable maximum.

This brief discussion of temperature warping has been included because it is believed that it is a subject which must be considered in most computations of pavement slab stresses. For a more thorough discussion the reader is referred to published reports (*24*, *19*, *7*).

For computing the maximum stress for the corner loading Westergaard developed the generalized theoretical equation given earlier in the report and for convenience repeated here.

Theoretical Equations (*Case I*)

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right]\text{------}(7)$$

This can be restated in terms of l as follows:

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

These equations give the most accurate indication of maximum load stress when the pavement corner is in full contact with the subgrade. In this investigation the condition was attained only when the corner was warped downward. If these equations are used for computing load stress the condition of corner warping due to temperature would be such as to create a moderate compressive stress in the upper surface of the slab in the region where the load would create the maximum tensile stress. Thus the combined stress would tend to be slightly lower than the load stress alone.

When the slab corner is not in complete bearing on the subgrade, due to upward warping, the theoretical equation will give load-stress values somewhat lower than those that will be developed. For this condition the Arlington experiments indicate the empirical equation, repeated below, will give computed values that are more nearly in accord with those observed.

Empirical Equations (*Case I*)

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.3}(a\sqrt{2})^{1.2}\right]\text{------}(18)$$

or expressed in terms of l

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right]$$

As a general rule the most critical condition for the corner loading is at night when the corner tends to warp upward. The subgrade support is least effective at that time and any warping stress present in the corner is additive to the load stress.

The case of interior loading is covered by the original Westergaard stress equation in generalized form, given as equation 8 earlier in the paper, or by the equation which was developed for the modified condition of subgrade support in the supplementary paper (*25*) and given in this report as equation 10.

Theoretical Equations (*Case II*)

$$\sigma_i''=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-54.54\left(\frac{l}{L}\right)^2Z\right]\text{--}(10)$$

If $\mu=0.15$ and $Z=0$ this equation can be simply expressed in terms of l, as follows:

$$\sigma_i=0.316\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+1.069\right]$$

The load stresses at the interior of a slab are not appreciably affected by the condition of warping of the slab and the same equation may be used for either upward or downward warping. Temperature warping stresses are highest in the interior region of the slab, however, and the combined stress value may vary widely from night to day.

There is little information at the present time to indicate what values of Z and L should be used in the general equation for various conditions of pavement and subgrade stiffness. It will be recalled that the data presented in figure 41 showed good agreement between observed and computed stress values when $Z=0.05$ and $L=1.75l$. These values were determined from the deflections and stresses observed on all of the four different thicknesses of pavement and there was no apparent difference in the value of either coefficient for the various thicknesses of slab. As stated previously, however, there is no information to indicate what values of the coefficients should be applied to other concretes and other subgrades. When dealing with conditions that are not known the values assigned to the coefficients Z and L should be such as to result in conservative load stresses. For such use the values $Z=0.2$ and $L=5l$ have been suggested (*7*) pending the development of more comprehensive data.

The stresses resulting from edge loading, like those from corner loading are affected by the degree of warping present at the time the load is applied. The theoretical equation for edge stress as given in generalized form by Westergaard appeared as equation 9 earlier in the report, as follows:

Theoretical Equations (*Case III*)

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-0.71\right]\text{----}(9)$$

If $\mu=0.15$ it can be expressed simply in terms of l:

$$\sigma_e=0.572\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+0.360\right]$$

In the Arlington tests it was found that under critical conditions of subgrade support and upward warping of the slab edges, the values computed by this equation were somewhat low and an empirical equation was developed which fitted the observed stress values more closely.

Empirical Equations (Case III)

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)+\log_{10}\left(\frac{b}{1-\mu^2}\right)-1.079\right] \quad (19)$$

This equation also can be expressed in terms of l, when $\mu=0.15$, as follows:

$$\sigma_e=0.572\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+\log_{10}b\right]$$

The critical stress from edge loading is a tension in the bottom edge of the pavement directly under the loaded area in a direction parallel to the pavement edge. Temperature warping stress in this direction can at times be high in the region of the slab edge and rather wide variations in combined stress from night to day may occur. The most critical condition for edge loading is that which occurs when the stress created during downward warping of the pavement edge (during the day) is combined with load stress. For this case the load stress would be computed with the theoretical equation (equation 9).

TABLE 9.—*Stresses computed by the Westergaard equation for the case of a corner loading and full subgrade support*

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right] \quad (7)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches — 2	4	6	8	10	E=4,000,000, a in inches — 2	4	6	8	10	E=5,000,000, a in inches — 2	4	6	8	10	E=6,000,000, a in inches — 2	4	6	8	10
Inches	*Lbs. in.$^{-3}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*
6	50	641	541	461	390	327	649	553	476	409	348	655	563	488	423	364	660	570	497	434	377
	100	619	500	420	342	271	628	523	437	362	295	635	533	450	378	313	641	540	461	390	327
	200	598	474	375	288	210	606	489	394	311	236	613	500	408	328	257	619	509	420	342	271
	300	581	451	346	254	171	592	467	366	278	199	600	479	382	296	219	606	489	394	311	236
7	50	480	412	357	309	265	486	420	367	321	280	490	427	376	331	201	493	432	382	330	299
	100	466	390	320	275	227	472	399	341	289	243	476	406	350	300	255	479	412	357	309	265
	200	450	366	298	238	185	456	376	311	254	203	462	384	321	266	216	465	390	329	275	228
	300	439	350	278	215	158	447	361	292	232	177	452	369	303	244	191	453	376	311	254	204
8	50	373	325	285	250	218	377	330	292	250	220	380	335	298	266	237	383	339	303	271	243
	100	363	309	265	226	191	367	315	273	236	203	371	320	279	244	211	373	324	285	250	218
	200	352	291	242	199	161	356	298	251	211	174	360	304	259	219	183	363	309	264	226	191
	300	344	280	228	182	141	349	288	238	194	155	353	294	245	203	165	356	298	252	211	174
9	50	299	262	233	206	183	302	267	238	213	191	304	270	242	218	197	305	273	246	223	201
	100	291	250	217	188	162	294	255	224	196	171	297	259	228	202	178	298	262	232	206	183
	200	282	237	201	168	139	286	243	208	177	149	289	247	213	183	156	290	250	217	188	162
	300	277	229	190	156	125	281	235	197	165	135	284	230	203	172	143	285	243	208	177	149

TABLE 10.—*Stresses computed by a modified equation for the case of corner loading and deficient subgrade support*

[NOTE: This is a purely empirical modification of the Westergaard corner equation, developed on the basis of experimental stress determinations, for the case of a slab corner which does not receive full subgrade support.]

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.3}(a\sqrt{2})^{1.2}\right] \quad (18)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches— 2	4	6	8	10	E=4,000,000, a in inches— 2	4	6	8	10	E=5,000,000, a in inches— 2	4	6	8	10	E=6,000,000, a in inches— 2	4	6	8	10
Inches	*Lbs. in.$^{-3}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*
6	50	780	731	667	598	525	792	739	680	617	551	795	745	690	631	569	797	750	698	642	583
	100	778	707	628	543	454	783	718	645	567	480	786	725	657	586	508	789	731	667	598	525
	200	766	678	581	476	367	771	691	601	506	405	775	700	616	527	433	778	707	628	543	454
	300	757	658	548	430	306	763	672	571	463	350	768	683	588	487	381	771	691	601	506	405
7	50	584	547	506	462	415	586	552	514	474	432	588	556	521	483	443	589	550	526	490	452
	100	577	532	481	427	370	580	538	492	442	390	582	543	500	453	404	584	547	506	462	415
	200	569	513	451	384	314	573	521	464	403	339	575	527	474	416	356	577	532	481	427	370
	300	563	500	430	354	275	567	509	445	376	303	570	516	456	391	323	573	521	464	403	338
8	50	449	424	396	367	335	451	428	402	375	346	452	431	407	381	354	453	433	410	386	360
	100	445	414	380	343	304	447	419	387	353	318	448	422	392	361	328	449	424	396	367	335
	200	439	401	359	314	266	442	407	368	327	283	444	411	375	336	295	445	414	380	343	304
	300	436	393	345	294	240	438	399	355	308	259	440	403	362	319	272	442	407	368	327	283
9	50	357	339	319	298	275	358	341	323	304	283	359	343	326	308	289	359	345	329	311	293
	100	353	331	307	281	253	355	335	312	288	263	356	337	316	294	270	357	339	319	298	275
	200	350	322	292	260	226	351	326	299	269	238	353	329	303	276	247	353	331	307	281	253
	300	347	316	282	246	208	349	321	290	256	221	350	324	295	264	231	351	326	299	269	238

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TABLES OF COMPUTED STRESSES PREPARED TO SHOW EFFECT OF VARIABLES IN EQUATIONS

In order to show the manner in which the various quantities that appear in the equations for maximum stress affect the computed values tables 9 to 16 have been prepared. Tables 9 and 10 apply to corner loading, case I, and show stress values for full and partial subgrade support respectively. Tables 11 and 12 show stress values for the interior loading, case II, for two assumptions regarding the distribution of the subgrade reactions within the deflected area. The stress values of table 11 are based on the original Westergaard assumptions (Z being 0) while those of table 12 were computed for values of Z and L determined experimentally in this investigation. Tables 13 and 14 give computed stress values for edge loading, case III, for a fully supported edge and for one that has warped upward and has only partial subgrade support. The equations used for computing the stress values in tables 10 and 14 are empirical, developed to fit the experimental data of this investigation. The stress values of table 12 although computed with a theoretical equation are based on constants determined experimentally for the conditions that existed at Arlington. The stress values of these three tables (10, 12, and 14) may be limited in their application, therefore.

Tables 15 and 16, computed by equations 8 and 9 respectively, show the effect of varying the value of several of the coefficients on the computed stress magnitude for interior loading, case II, and edge loading, case III.

TABLE 11.—*Stresses computed by the Westergaard equation for the case of interior loading for the case $Z=0$*

$$\sigma_i=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)\right] \quad (8)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches —					E=4,000,000, a in inches —					E=5,000,000, a in inches —					E=6,000,000, a in inches —				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.$^{-3}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*
6	50	487	421	361	313	274	498	432	372	324	285	507	441	381	333	294	514	448	388	340	301
	100	461	395	335	287	248	472	406	346	298	259	480	414	354	306	267	487	421	361	313	274
	200	435	368	308	260	221	446	379	319	271	232	454	388	328	280	241	461	395	335	287	248
	300	419	353	293	245	206	430	364	304	256	217	439	372	312	264	225	446	370	319	271	232
7	50	360	310	279	245	217	368	327	287	253	225	374	333	293	259	231	379	339	298	264	236
	100	341	300	260	226	197	349	308	268	234	206	355	314	274	240	212	360	319	279	245	217
	200	321	280	240	206	178	329	288	248	214	186	335	295	254	220	192	341	300	260	226	198
	300	310	269	229	195	167	318	277	237	203	175	324	283	243	209	181	329	288	248	214	186
8	50	276	250	222	197	176	282	256	228	203	182	287	261	233	208	187	291	265	237	212	191
	100	261	235	207	182	161	267	241	213	188	168	272	246	218	193	172	276	250	222	197	176
	200	246	220	192	167	146	252	226	198	174	153	257	231	203	178	157	261	235	207	182	161
	300	238	211	183	159	138	244	217	189	165	144	249	222	194	170	149	252	226	198	174	153
9	50	218	200	180	162	146	223	205	185	167	151	227	209	189	171	155	230	212	192	174	158
	100	206	188	169	150	134	211	193	173	155	130	215	197	177	159	143	218	200	180	162	146
	200	194	177	157	138	123	199	182	162	143	127	203	185	166	147	131	206	188	169	150	134
	300	188	170	150	132	116	192	175	155	137	121	196	179	159	140	124	199	182	162	143	127

TABLE 12.—*Stresses computed by the Westergaard equation for the case of interior loading*

$$\sigma_i''=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-54.54\left(\frac{l}{L}\right)^2 Z\right] \quad (10)$$

[P=10,000 pounds. μ=0.15. Z=0.05. L=1.75l]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches—					E=4,000,000, a in inches—					E=5,000,000, a in inches—					E=6,000,000, a in inches—				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.$^{-3}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-2}$*
6	50	409	343	283	235	196	420	354	294	246	207	429	362	303	254	216	436	369	310	261	222
	100	383	317	257	209	170	304	327	268	219	181	402	336	276	228	189	409	343	283	235	196
	200	357	290	230	182	143	367	301	241	193	154	376	310	250	201	163	383	317	257	208	170
	300	341	275	215	166	128	352	286	226	177	139	360	294	234	186	147	367	301	241	193	154
7	50	303	262	222	188	160	311	270	229	196	168	317	276	236	202	174	322	281	241	207	179
	100	284	243	203	169	140	291	250	210	176	148	297	257	216	182	154	303	262	221	188	159
	200	264	223	183	149	121	272	231	191	157	129	278	237	197	163	135	283	242	202	168	140
	300	252	211	171	137	109	260	220	179	145	117	267	226	186	152	124	272	231	191	157	129
8	50	232	206	178	153	132	238	212	184	159	138	243	217	189	164	143	247	221	193	168	147
	100	217	191	163	138	117	223	197	169	144	124	228	202	174	149	128	232	206	178	153	132
	200	202	176	148	123	102	208	182	154	130	109	213	187	159	134	113	217	191	163	138	117
	300	194	167	139	115	94	200	173	145	121	100	205	178	150	126	105	208	182	154	130	109
9	50	183	165	145	127	111	188	170	150	132	116	192	174	154	136	120	195	177	157	139	123
	100	171	153	134	115	99	176	159	139	120	104	180	162	142	124	108	183	165	146	127	111
	200	159	142	122	103	88	165	147	127	109	93	168	151	131	112	96	171	154	134	115	99
	300	153	135	115	97	81	158	140	120	102	86	161	144	124	105	90	165	147	127	109	93

TABLE 13.—*Stresses computed by the Westergaard equation for the case of edge loading and full subgrade support*

$$\sigma_e = 0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right) - 0.71\right] \qquad (9)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches—					E=4,000,000 a in inches—					E=5,000,000 a in inches—					E=6,000,000 a in inches—				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	769	649	541	453	383	789	669	561	473	403	804	694	576	489	410	817	697	589	501	431
	100	721	601	493	406	335	741	621	513	426	355	756	637	528	441	371	769	649	541	454	383
	200	673	553	445	358	287	693	573	465	378	307	708	589	480	393	323	721	601	493	406	336
	300	645	525	417	330	260	665	545	437	350	279	680	561	452	365	295	693	573	465	378	307
7	50	568	494	422	360	309	583	509	436	375	324	594	520	448	386	335	603	530	457	395	345
	100	533	459	386	326	274	548	474	401	340	289	559	485	412	351	300	568	494	422	360	310
	200	498	424	351	290	239	513	439	366	305	254	524	450	377	316	265	533	459	386	325	274
	300	477	404	331	269	219	492	418	345	284	233	503	429	357	295	245	513	439	366	305	254
8	50	436	388	337	293	255	447	399	340	304	266	456	408	357	313	275	463	415	365	320	282
	100	409	361	311	266	228	420	372	322	277	239	429	381	331	286	248	436	388	338	293	255
	200	382	334	284	239	201	393	345	295	250	213	402	354	304	259	221	409	361	311	266	228
	300	366	319	268	223	186	377	330	279	235	197	386	338	288	243	205	393	345	295	250	213
9	50	344	312	276	243	214	353	321	285	252	223	360	328	292	259	230	365	333	297	264	235
	100	323	291	255	222	193	332	300	264	230	202	338	306	270	237	208	344	312	276	243	214
	200	301	269	233	200	171	310	278	242	209	180	317	285	249	216	187	323	291	255	222	193
	300	280	257	221	188	150	298	266	230	197	168	305	273	237	204	175	310	278	242	209	180

TABLE 14.—*Stresses computed by a modified equation for the case of edge loading and deficient subgrade support*

[NOTE: This is a purely empirical modi fication of the Westergaard edge equation, developed on the basis of experimental stress determinations, for the case of a slab edge which does not receive full subgrade support.]

$$\sigma_e = 0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right) + \log_{10}\left(\frac{b}{1-\mu^2}\right) - 1.0792\right] \qquad (19)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches—					E=4,000,000 a in inches—					E=5,000,000 a in inches—					E=6,000,000 a in inches—				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	774	684	603	537	485	793	704	622	557	504	809	719	638	572	520	821	732	650	585	532
	100	726	636	555	489	437	746	656	575	509	457	761	671	590	525	472	774	684	603	537	485
	200	678	588	507	442	389	698	608	527	461	409	713	623	542	477	424	726	636	555	489	437
	300	650	560	479	414	361	670	580	499	433	381	685	595	514	449	396	698	608	527	461	409
7	50	577	521	467	421	383	591	536	481	435	397	603	547	493	447	409	612	556	502	456	418
	100	542	486	432	386	348	556	501	446	400	362	567	512	458	412	374	577	521	467	421	383
	200	506	451	397	351	313	521	466	411	365	327	532	477	422	376	338	542	486	432	386	348
	300	486	431	376	330	292	500	445	391	345	307	512	456	402	356	318	521	466	411	365	327
8	50	446	410	372	339	311	457	421	384	350	322	466	430	392	359	330	473	437	399	366	338
	100	419	383	346	312	284	430	395	357	323	295	439	403	366	332	304	446	410	372	339	311
	200	392	356	310	285	257	403	368	330	296	268	412	376	338	305	277	419	383	346	312	284
	300	376	341	303	269	241	388	352	314	281	252	396	361	323	289	261	403	368	330	296	268
9	50	355	331	304	279	257	364	340	313	288	266	371	347	320	295	273	376	352	325	300	279
	100	334	310	283	258	236	342	318	292	267	245	349	325	298	273	252	355	331	304	279	257
	200	312	288	261	237	215	321	297	270	245	224	328	304	277	252	231	334	310	283	258	236
	300	300	276	249	224	202	309	285	258	233	211	316	292	265	240	218	321	297	270	245	224

As will be noted the computed stress values in table 11 are for a value of Z=0 (and L=0). The values may be corrected for other values of Z and L, rather simply, by adding a stress correction, σ_i', computed with the following equation:

$$\sigma_i' = -\frac{15(1+\mu)ZP}{h^2}\left(\frac{l}{L}\right)^2$$

The stress values in tables 9 to 14, inclusive, were computed for a value of μ=0.15. The maximum stress for corner loading is affected very little by changes in μ within the normal range of variation of this ratio. The effect of changes in the value of μ for the interior and edge cases of loading is shown in tables 15 and 16 respectively. The effect is not large and if it is desired to obtain computed stress values for intermediate values of the ratio, direct interpolation is sufficiently accurate for all practical purposes.

TABLE 15.—*Stresses computed for the case of interior loading to show the effects of variations in the values of h, k, a, E and μ*

[P=10,000 pounds. Z=0]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches— 2	6	10	E=5,000,000, a in inches— 2	6	10	E=6,000,000, a in inches— 2	6	10
				μ=0.05						
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	100	421	306	226	439	324	244	445	330	250
	200	397	282	202	415	299	220	421	306	226
	300	383	268	188	400	285	206	407	292	212
8	100	238	189	147	248	190	157	252	202	161
	200	225	175	134	235	185	144	238	189	147
	300	217	167	126	227	177	136	230	181	139
9	100	188	154	123	196	162	131	199	165	133
	200	178	143	112	185	151	120	188	154	123
	300	171	137	106	179	145	114	182	148	116
				μ=0.15						
6	100	461	335	248	480	354	267	487	361	274
	200	435	308	221	454	328	241	461	335	248
	300	419	293	206	439	312	225	446	319	232
8	100	261	207	161	272	218	172	276	222	176
	200	246	192	146	257	203	157	261	207	161
	300	238	183	138	249	194	149	252	198	153
9	100	206	169	134	215	177	143	218	180	146
	200	194	157	123	203	166	131	206	169	134
	300	188	150	116	196	159	124	199	162	127
				μ=0.25						
6	100	501	364	269	522	385	291	530	393	298
	200	472	335	241	493	356	262	501	364	269
	300	455	318	224	477	340	245	484	347	253
8	100	284	225	175	296	237	187	300	241	191
	200	268	209	159	280	221	171	284	225	175
	300	258	199	150	270	211	162	274	215	166
9	100	224	183	146	234	193	155	237	196	159
	200	211	170	133	221	180	143	224	183	146
	300	204	163	126	213	172	135	217	176	138

TABLE 16.—*Stresses computed for the case of edge loading to show the effects of variations in the values of h, k, a, E and μ*

[P=10,000 pounds]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches— 2	6	10	E=5,000,000, a in inches— 2	6	10	E=6,000,000, a in inches— 2	6	10
				μ=0.05						
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	100	685	468	319	719	502	352	730	514	364
	200	640	423	273	673	456	307	685	468	319
	300	613	396	247	646	430	280	658	442	292
8	100	389	295	217	407	314	236	414	321	242
	200	363	270	191	382	288	210	389	295	217
	300	348	255	176	367	274	195	374	280	202
9	100	307	242	183	322	257	198	327	262	203
	200	286	222	163	301	237	178	307	242	183
	300	275	210	151	290	225	166	295	230	171
				μ=0.15						
6	100	721	493	335	756	528	371	769	541	383
	200	673	445	287	708	480	323	721	493	336
	300	645	417	260	680	452	295	693	465	307
8	100	409	311	228	429	331	248	436	338	255
	200	382	284	201	402	304	221	409	311	228
	300	366	268	186	386	288	205	393	295	213
9	100	323	255	193	338	270	208	344	276	214
	200	301	233	171	317	249	187	323	255	193
	300	289	221	159	305	237	175	310	242	180
				μ=0.25						
6	100	757	518	352	794	555	389	807	568	402
	200	707	467	302	744	504	339	757	518	352
	300	677	438	273	714	475	310	728	488	323
8	100	429	326	240	450	347	261	458	354	268
	200	401	298	211	422	319	232	429	326	240
	300	385	281	195	405	302	216	413	310	223
9	100	339	268	202	355	284	219	361	290	225
	200	317	245	180	333	262	196	339	268	202
	300	304	232	167	320	249	183	326	254	189

SUPPLEMENTARY STUDIES

Mention was made early in this report of certain supplementary studies that were not a part of the originally scheduled program but which, because of circumstances, it seemed desirable to make and to report in conjunction with the work already described in this report. The supplementary studies comprise two distinct investigations; one, of the deflection and stress conditions in the vicinity of loaded slab corners and, two, of the stress reductions effected when the area over which the load is applied is considerably larger than any used in the tests described previously.

THE EFFECT OF LOADS ON SLAB CORNERS STUDIED

In the original manuscript of his paper "Theory of Stresses in Concrete Roads" Westergaard lists certain questions relating to the structural action of slab corners suggesting them as desirable subjects for further investigation. Among these were the following:

1. The influence of some nonuniformity in the distribution of the bending moment over the section of width $2x$; this influence may be expected to have the greater relative importance, the larger the value of the distance a_1, from the corner to the load.
2. The influence of twisting due to an eccentric position of the load, with the load closer to one edge than to the other.
3. The influence of an uneven thickness of slab, especially in the case of a thickened edge.

A fourth question, concerning impact loads, was mentioned but will not be discussed here. Certain data obtained in this investigation have a bearing on the three items that are listed, however.

The shape assumed by a slab corner when deflected by a load applied at or near the corner was determined by measuring the vertical displacements along the two edges of the slab corner and along the bisector of the corner angle.

In figure 45 deflection contours are shown for the 1st and the 60th application of a test load of 9,000 pounds on the corner of a slab of 7-inch uniform thickness, the loaded area being 8 inches in diameter and so placed that the slab edges were tangent to it. These data indicate that, while repeated loading caused a slight increase in the magnitude of the deflection at a given point, it produced no important change in the shape of the deflected corner area.[2]

Deflection contours for a similar symmetrical corner loading but of somewhat lesser magnitude are shown, for the 7-inch uniform-thickness and the 9–7–9-inch sections, in figure 46. It will be observed that within a distance of about 80 inches from the slab corner the

[2] In this and in subsequent similar graphs the deflection contours are the result of averaging the measurements on the four free corners of the test section.

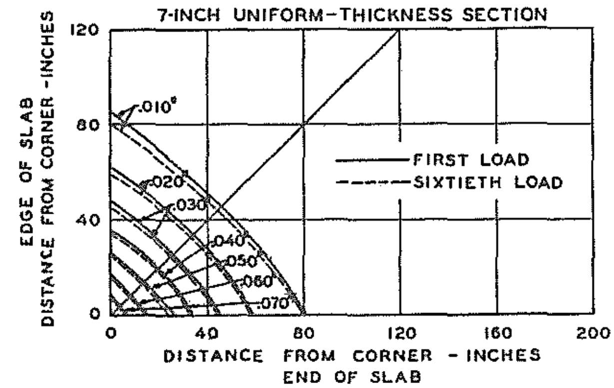


Figure 45.—Effect of Repeated Loading on the Deflected Shape of a Slab Corner—Symmetrical Loading.

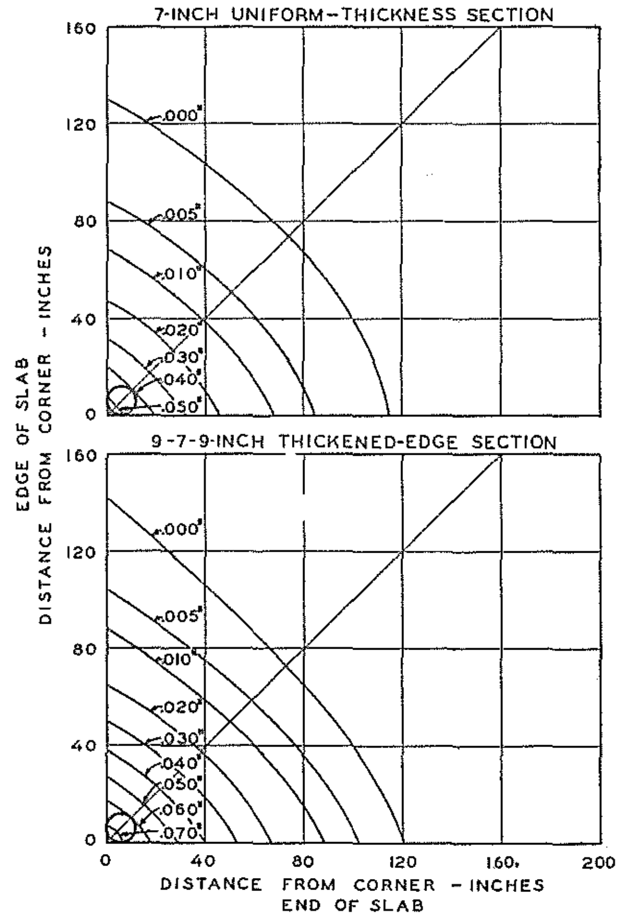


Figure 46.—Deflection Contours for Symmetrical Corner Loading.

deflections appear to be symmetrical on either side of the corner bisector for both the uniform-thickness and the thickened-edge cross sections. Beyond this distance the deflection of the slab end is slightly less than that of the side edge. The difference is small but appears in the data for both sections. It is believed to be due to the difference in the panel dimensions. Along the side edge there is a continuous structure 240 inches in length while along the end edge are two units each 120 inches in length hinged together at the longitudinal joint. It was stated earlier in the report that when the free corner is loaded there is a slight rotation about the longitudinal

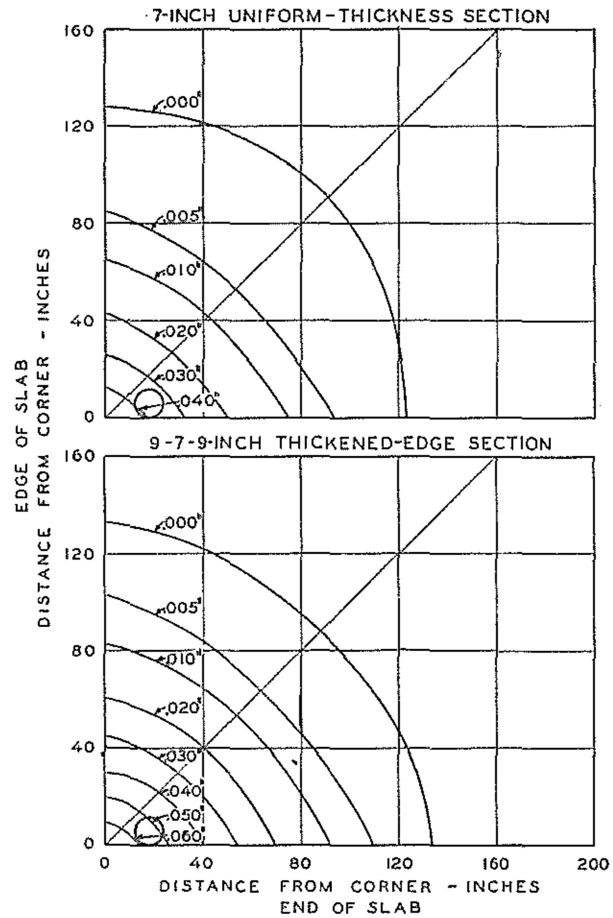


Figure 47.—Deflection Contours for Eccentric Corner Loading.

joint. Obviously there is no corresponding movement along the side edge of the slab.

In figure 47 are deflection contours for the same two corners under the action of a load of the same magnitude but displaced 18 inches toward the center line of the pavement. Such an eccentricity of load would tend to create a twisting moment in the slab corner. It is apparent from the data that with this loading the deflection, at a given distance from the corner, is greater at the end than at the side of the slab. A comparison of the data in figure 46 with those for the same test section in figure 47 shows that the maximum deflection for the given load is greater when that load is applied on the slab corner, which is as would be expected.

In order to determine both the direction and magnitude of the stresses at various locations in the corner region, measurements of strain were made in either three or four directions at each location. From these "rosette" strain measurements the direction and magnitude of the principal strains may be obtained by a method described by Osgood and Strum (*11*).

The location of the various gage lines used in this study were shown in figure 13 (quadrant 3). Where strains are measured in four directions instead of three, the fourth strain value serves as a useful check.

The installation of each strain rosette necessitated the drilling of eight small holes into which the gage points were cemented. Since these were installed in the tension face of the slab the tendency would be to weaken somewhat the flexural resistance of the corner and to cause some variation in relative strength at different distances from the corner. Because of this possibility certain of the tests were made with fewer installations than are shown in figure 13.

MAXIMUM STRAIN UNDER ECCENTRIC LOADING LOCATED

It will be noted that the strains were measured along the two slab edges and/or along three rays 22½° apart which quadrisect the corner angle, the center ray being the bisector of that angle. These rays will be spoken of as the corner bisector and the 22½° rays in the subsequent discussion. The positions of the loaded area in these special corner tests were described in connection with the deflection data just presented.

For the symmetrically placed corner loading on the 7-inch and 9-inch uniform-thickness sections, strains were measured with rosette installations along the 22½° rays and in two directions at intervals along the corner bisector. No strains were measured along the slab edges. For the same position of loading in testing the 8-inch uniform-thickness and 9–6–9-inch thickened-edge sections, strains were measured with rosette installations along both slab edges and the two 22½° rays and in two directions at intervals along the corner bisector. For this position of load, the direction of the principal strain in the region near the bisector of the corner angle was known to be parallel to that bisector.

For the eccentric corner loading, in which the position of the loaded area was moved along the end edge of the slab toward the pavement center line, for a distance of 18 inches, rosette installations were used along the two edges, the two 22½° rays and along the corner bisector as well.

The data obtained in these load-strain studies are shown in figures 48 to 66 inclusive.

Four of the test sections had loads applied symmetrically with respect to the slab corner, two had the load applied eccentrically. This made six combinations of load position and test section. For each of these six combinations there are three graphs.[3] The first graph in each group shows the direction of maximum strain for each of the 15 or more locations at which strains were measured. The second graph in each group shows the magnitudes of both the maximum strain and the minimum strain at each of these gage locations. The third graph of each group shows how the magnitude of the maximum strain varies with distance from the slab corner along the various rays on which gages were installed.

The slab corners used for the symmetrical loading were those of the 7-, 8-, and 9-inch uniform-thickness and the 9–6–9-inch thickened-edge sections. Those used for the eccentric loading were of the 8-inch uniform-thickness and the 9–6–9-inch thickened-edge sections.

The six graphs showing the direction of the maximum strains for the six slab-load combinations are figures 48, 51, 54, 57, 60, and 63, respectively.

The six graphs that show the magnitude of the maximum strains and the magnitude of the minimum strains (or those in the perpendicular direction at each gage location) are figures 49, 52, 55, 58, 61, and 64, respectively.

[3] In these figures and the attendant discussion maximum strain denotes the maximum strain for the given gage location and minimum strain is the strain measured at the same location in a direction perpendicular to that of the maximum strain. The maximum strains are approximately radial and the minimum strains approximately tangential in direction with respect to the center of load application.

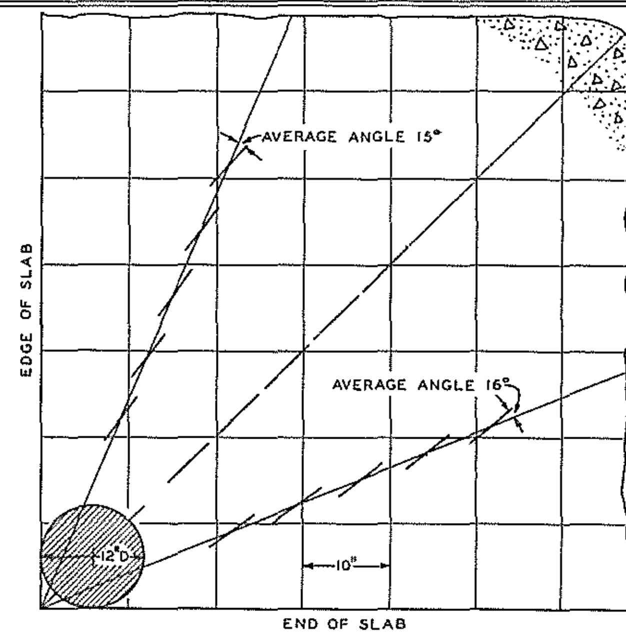


FIGURE 48.—DIRECTION OF MAXIMUM STRAINS—7-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING.

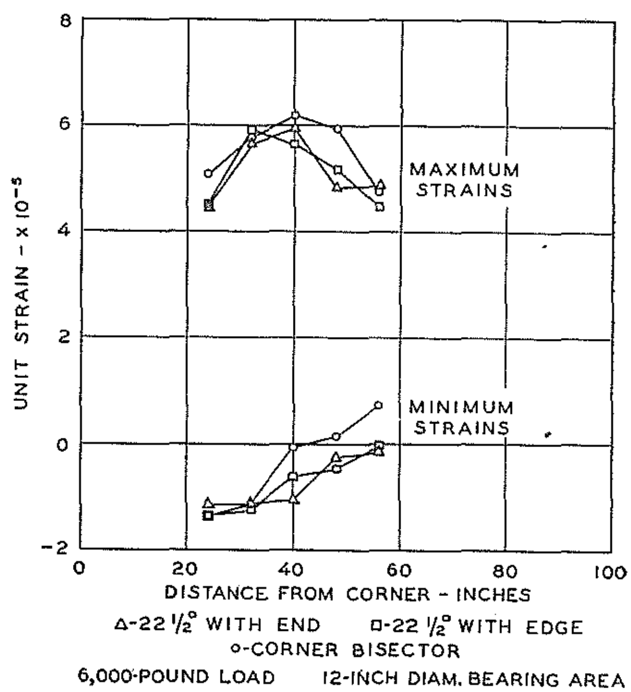


FIGURE 49.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—7-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON FOUR QUADRANTS.

The six graphs showing the strain variation for various points along the rays on which the gages were installed are figures 50, 53, 56, 59, 62, and 65, respectively.

All of these figures are largely self-explanatory.

Referring to figures 48, 51, and 54 which show the direction of maximum strains for the symmetrical corner loading on the three sections of uniform thickness, it is apparent that while the maximum strain is along the central ray (the corner bisector) the maximum strains along the two 22½° rays are not parallel to the respective rays but have a direction which makes an angle of approximately 15° with these rays. The strains along the edges of the slab corner were not measured in the

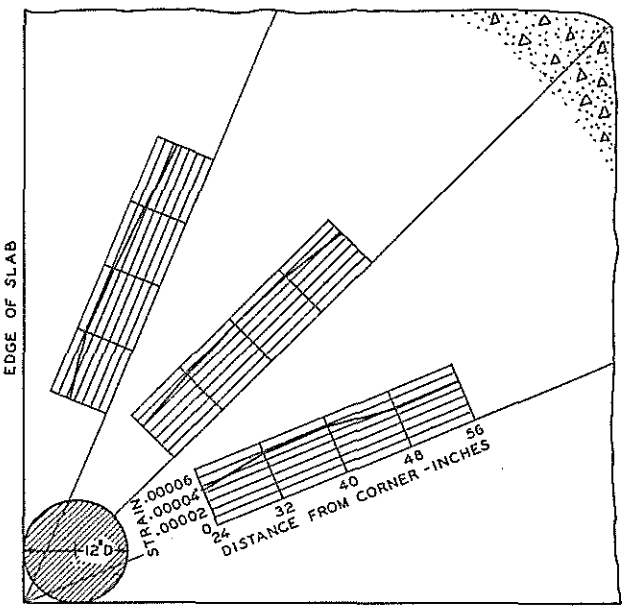


FIGURE 50.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—7-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING.

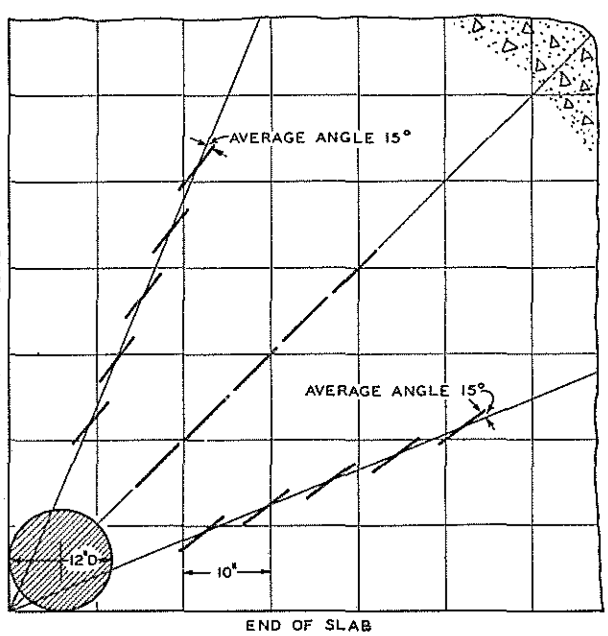


FIGURE 51.—DIRECTION OF MAXIMUM STRAINS—9-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

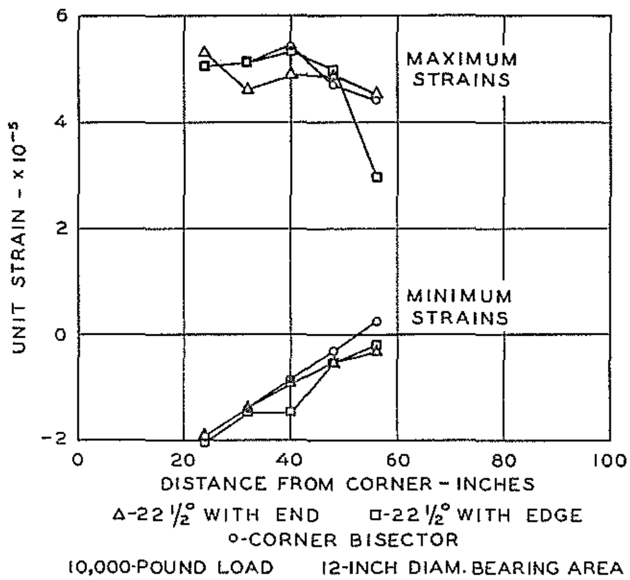


FIGURE 52.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—9-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

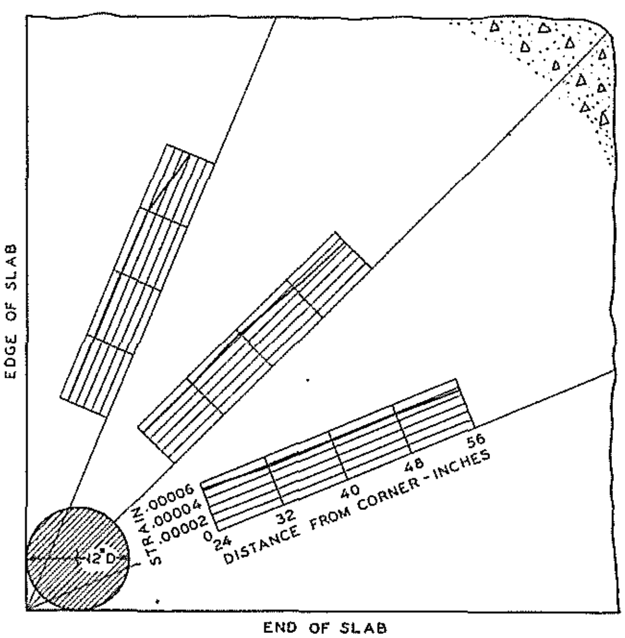


FIGURE 53.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—9-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

cases of the 7-inch and 9-inch sections, figures 48 and 51. These strains were measured, however, in testing the corner of the 8-inch uniform-thickness section and, from figure 54, it is evident that the maximum strain makes an angle of approximately 28° with the slab edge.

For the symmetrical loading the directions of the maximum strains in the corner area of the 9–6–9-inch thickened-edge section were found to be essentially the same as for the uniform-thickness section (see figure 57). Apparently the thickening of the edge of this section was so distributed that it did not appreciably affect the stress trajectories.

The direction of the maximum strains changed appreciably, however, when the eccentric load, previously described, was used. The effect of this eccentricity can be seen by comparing figure 54 with figure 60 and figure 57 with figure 63. Because of the variation in the angle along some of the rays the range in the angle between the direction of the maximum stress and the ray is indicated by the small figures beside the rays in figures 57, 60, and 63.

SECTION OF MAXIMUM MOMENT NOT A STRAIGHT LINE

It was stated earlier that strains were not measured along the two slab edges in the cases of the 7-inch and 9-inch test sections and there was reason to believe that the installation of a large number of gage points in the tension face of the slab might affect the test data.

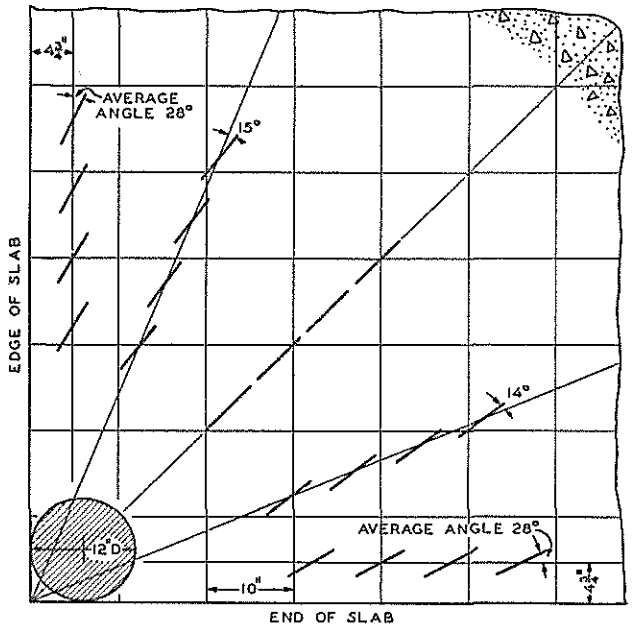


FIGURE 54.—DIRECTION OF MAXIMUM STRAINS—8-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

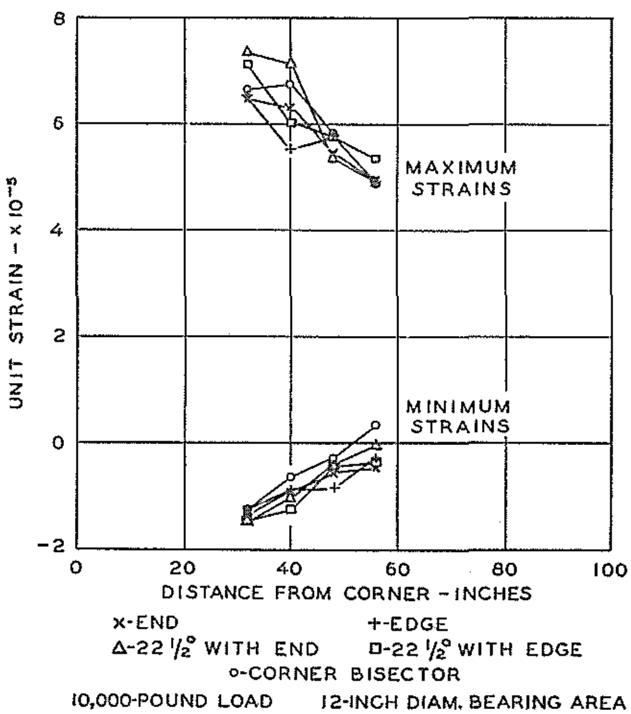


FIGURE 55.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—8-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

There is evidence to this effect in the set of graphs showing the strain variations along the several rays. The data for the 7-inch and 9-inch corners, figures 49, 50, 52, and 53, indicate the maximum measured strain to be on the corner bisector and at a distance of about 40 inches from the corner, for the conditions of the tests, while the maximum strain values on the two 22½° rays were of slightly lesser magnitude and at approximately the same distance from the slab corner. When, however, the strain gage points were installed along the two

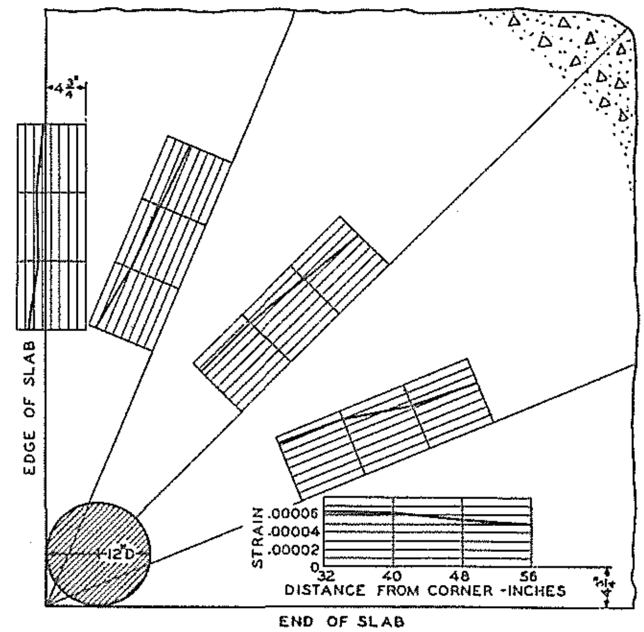


FIGURE 56.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—8-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

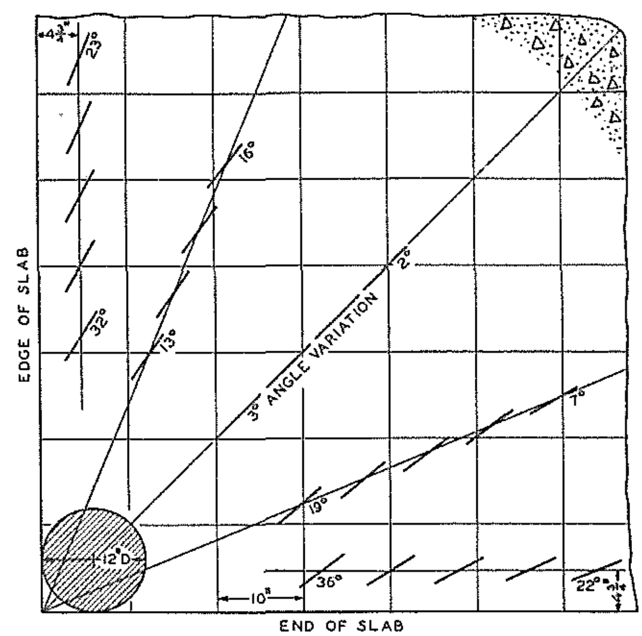


FIGURE 57.—DIRECTION OF MAXIMUM STRAINS—9-6-9-INCH THICKENED-EDGE SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

slab edges as well as on the rays mentioned it was found that the entire section of maximum strain was appreciably nearer the slab corner and the maximum strain no longer appeared on the corner bisector.

It is concluded, therefore, that the strain data for the 7-inch and 9-inch sections give a somewhat more reliable indication of the strain distribution in the corner of a section of uniform thickness than those obtained with the 8-inch-section and shown in figures 55 and 56.

Similar data for the eccentric loading applied to the 8-inch uniform-thickness and the 9-6-9-inch thickened-edge sections are shown in figures 61 and 62 and in figures 64 and 65, respectively.

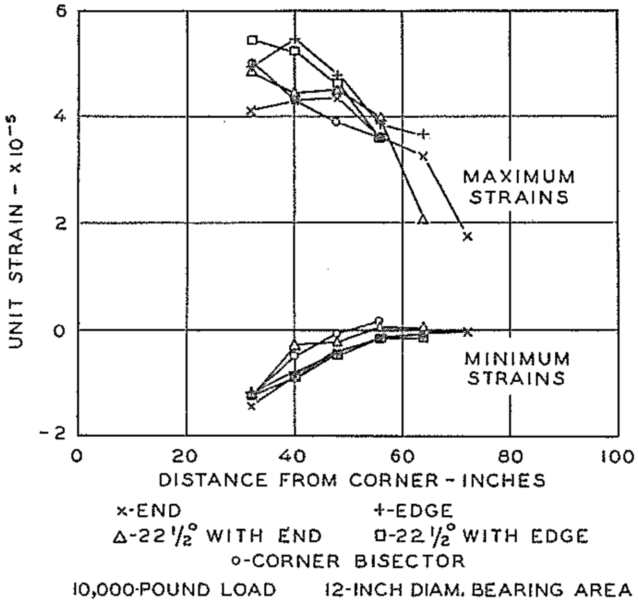


FIGURE 58.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—9–6–9-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

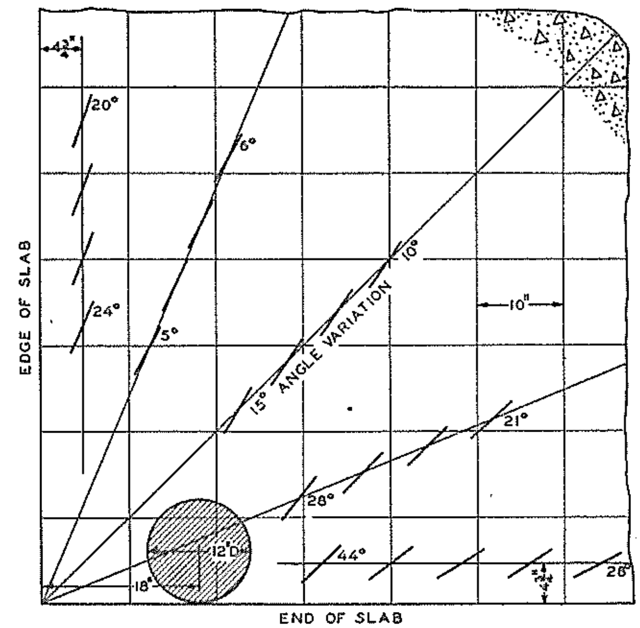


FIGURE 60.—DIRECTION OF MAXIMUM STRAINS—8-INCH UNIFORM-THICKNESS SECTION—ECCENTRIC CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

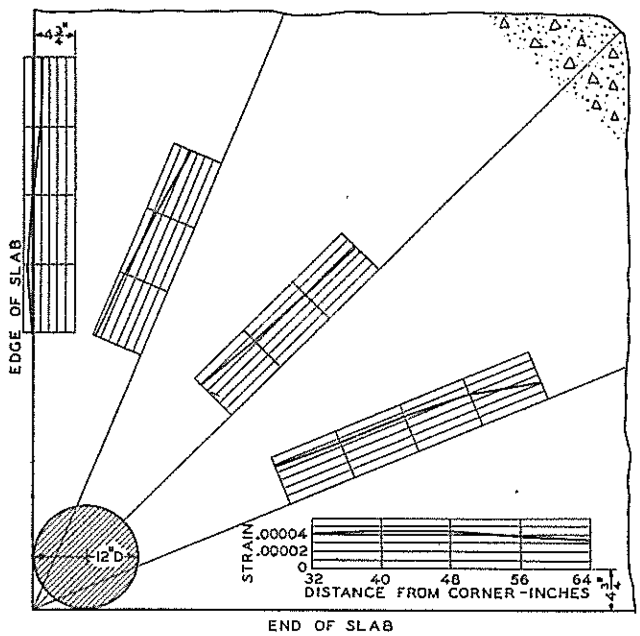


FIGURE 59.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—9–6–9-INCH THICKENED-EDGE SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

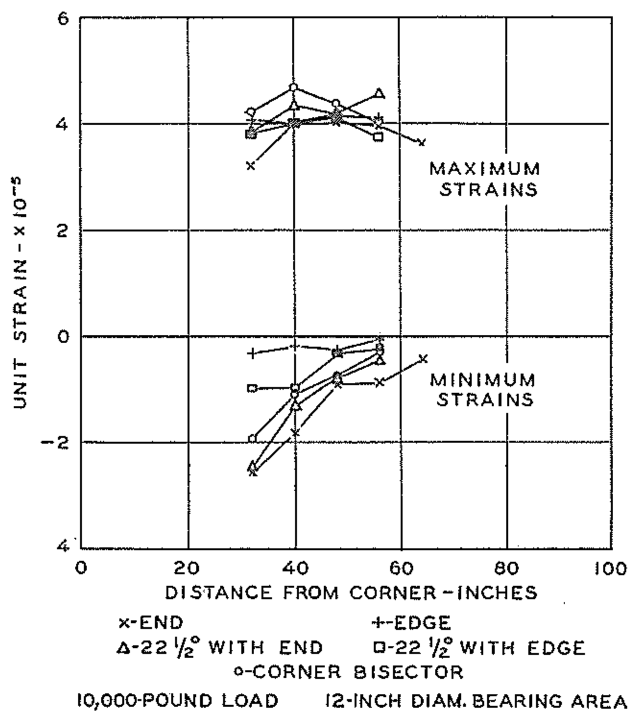


FIGURE 61.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—8-INCH SECTION—ECCENTRIC LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

For a given test section there is a striking difference in the stress magnitudes resulting from the symmetrical and eccentric load positions used in this investigation. The maximum tensile strain caused by the symmetrical loading was approximately 50 percent greater than that caused by the eccentric loading and this applies to both the 8-inch uniform and the 9–6–9-inch test sections. Compressive strains, on the other hand, were higher for the eccentric loading.

It is apparent from the strain data which have been presented that there is a relatively large area across the slab corner within which the magnitude of the maximum stress, as previously defined, does not vary appreciably. This is in concordance with data obtained by Spangler and Lightburn (*14*) in carefully controlled laboratory studies of the strain distribution in the corner of a concrete slab.

In his original analysis of pavement slab stresses Westergaard states that in the case of corner loading when the distance x_1 (from the corner to the section of maximum moment) is not too large, the bending moment will be approximately uniformly distributed over the width, $2x_1$, of the cross section.

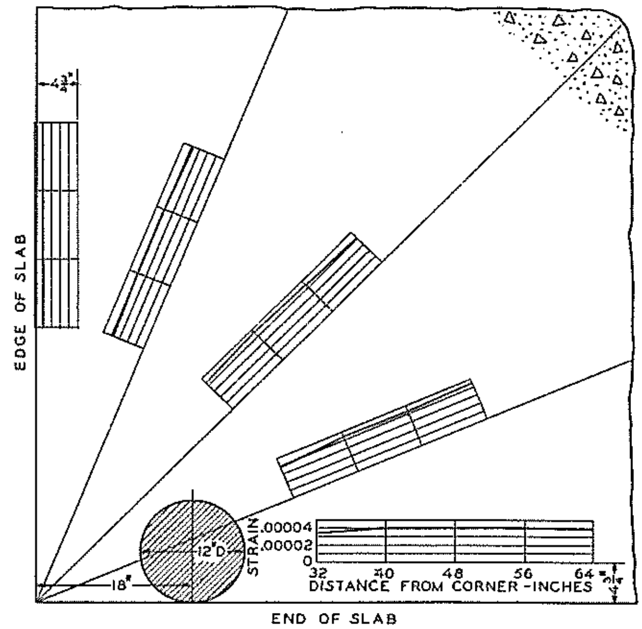


FIGURE 62.—VARIATION IN MAGNITUDE OF MAXIUMM STRAINS—8-INCH UNIFORM-THICKNESS SECTION—ECCENTRIC CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

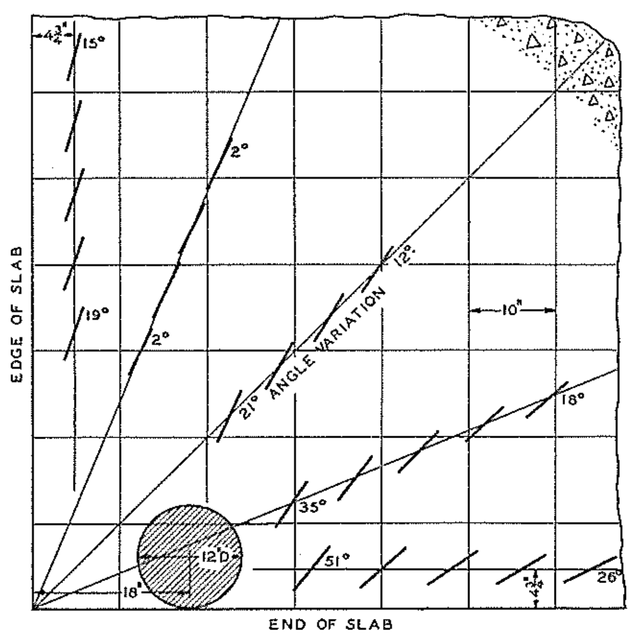


FIGURE 63.—DIRECTION OF MAXIMUM STRAINS—9-6-9-INCH THICKENED-EDGE SECTION—ECCENTRIC CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

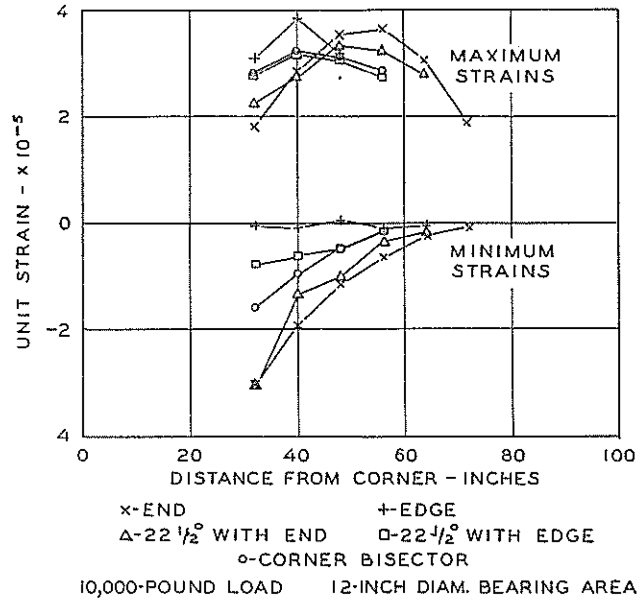


FIGURE 64.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—9-6-9-INCH SECTION—ECCENTRIC LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

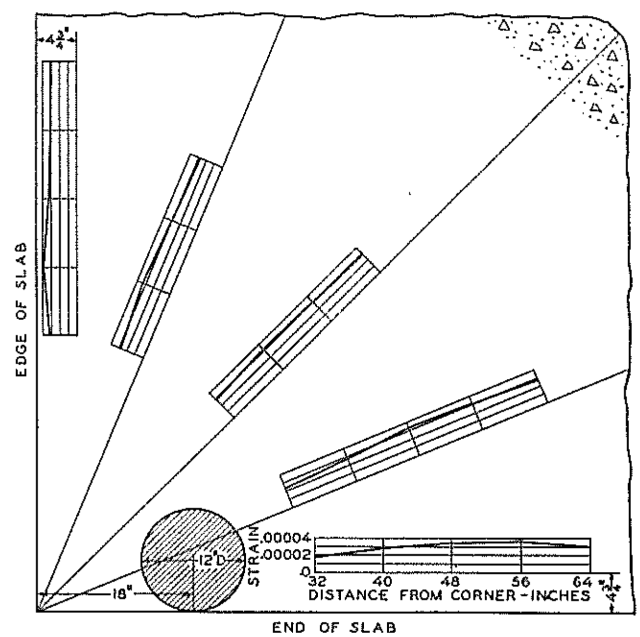


FIGURE 65.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—9-6-9-INCH THICKENED-EDGE SECTION—ECCENTRIC CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

Both observation of corner failures in the field and failures deliberately produced by corner loading during tests (*15, 22, 14*) have shown a tendency for the line of fracture resulting from a corner loading to be curved rather than straight although the curvature usually is not pronounced. The strain data that have been presented indicate that for the conditions of these tests the section of maximum moment is not a straight line normal to the bisector of the corner angle but follows a curved path similar to the deflection contours. In figure 66 are shown data obtained from the tests of the 8-inch uniform-thickness section. In this figure maximum strain values, as determined from strains measured in four directions, have been resolved to show the component in the direction parallel to the corner bisector. These are the strain values on the straight line section across the corner perpendicular to the corner bisector. In the figure the section shown is at a calculated distance, $x_1 = 32.2$ inches, from the corner. In addition to the plotted values of the strains observed on the five lines radiating from the slab corner the value of the average normal strain is shown by the horizontal dash line. It is apparent that in this test the component of the maximum strain observed on one of the 22½° rays was approximately 10 percent greater than the average

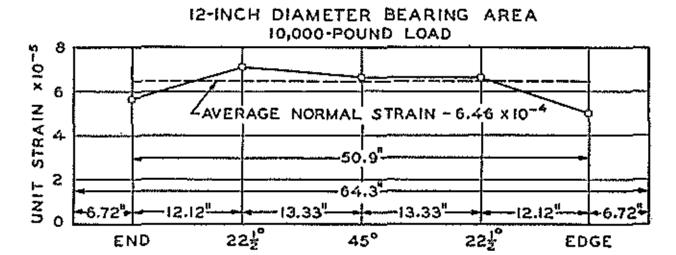


FIGURE 66.—DISTRIBUTION OF OBSERVED STRAINS ACROSS PLANE SECTION PERPENDICULAR TO CORNER BISECTOR AT A COMPUTED DISTANCE, X_1, FROM THE CORNER—8-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

strain normal to the section. The variation in strain across the assumed plane section is evident.

Since strains were not measured along the slab edges at the corners of the 7-inch and 9-inch sections it is not possible to analyze the data from those sections in the manner shown in figure 66.

LOAD TESTS MADE WITH LARGE CONTACT AREAS

Originally the Westergaard analysis of the stresses in concrete pavement slabs *(23, 25)* was a discussion of the effects of such loadings as were being encountered in highway practice and, for purposes of simplicity, it was assumed that the load was applied to the pavement over areas that were either circular or semicircular in shape.

In planning the program of tests previously discussed in this report, provision was made for a range of circular and semicircular areas of contact sufficient to represent the tire contact areas normally met with in highway service.

Subsequently Westergaard supplemented his original analysis with two papers which discussed respectively the effect of (1) larger wheel loads on correspondingly larger loaded areas *(27)* and (2) loaded areas that are of other than circular shape *(28)*.

In the first paper *(27)* the analysis for the case of interior loading is reexamined to determine its suitability for determining stresses when the areas of contact are much larger than those found in highway service. It was concluded by the author that the analysis is applicable to the conditions of loading found in airport runway service except that under some conditions certain small corrections are desirable. The means for making these corrections are given in the paper. Only the interior case of loading is discussed.

In the second paper *(28)* the effect of shape of loaded area is one of the subjects discussed. This is a valuable addition to the analysis since it furnishes for the first time the means for estimating the importance of variations in shape of loaded area which are known to exist. Both of these subjects are of great current interest because of their direct bearing on the design of concrete pavement slabs for airport service.

The original program of the investigation being reported contained no provision for tests with very large areas of contact since it was intended to cover the conditions of highway service. However, before it became necessary to vacate the test site there was an opportunity to make a limited number of tests that yielded data which afford significant comparisons between theory and observed performance for concrete pavements loaded over relatively large areas of both circular and elliptical shape.

The concrete pavement available for the load tests with the large contact areas was one which had been constructed in 1940 for use in another investigation. It was 20 feet wide divided at the center by a tongue and groove joint with ⅝-inch diameter tie-bars at 60-inch intervals. The sections were 30 feet in length between transverse expansion joints and were divided into 15-foot slab lengths with contraction joints of the plane of weakness type. The slab units were thus 10 by 15 feet. The pavement was of 8-inch uniform depth.

The concrete used in the pavement was mixed in the following proportions (dry weight):

	Pounds
Cement	94
Fine aggregate (sand)	200
Coarse aggregate (siliceous gravel)	330

The concrete had a 2½-inch slump at the time of placing. At 28 days the average crushing strength was 4,600 pounds per square inch and the average modulus of rupture was 550 pounds per square inch. The average modulus of elasticity at the time of the actual load testing on the pavement was 4,800,000 pounds per square inch.

These test sections had been laid on the site of some of the sections of the original investigation *(18)*. However, the grade had been changed slightly and the character of the subgrade immediately under the new sections had been altered by mixing sand with the original silty loam to a depth of several inches and recompacting at optimum moisture to maximum density. This resulted in an average dry density of 136 pounds per cubic foot within the treated depth. The effect of this treatment will be discussed later.

The equipment used for applying the loads was essentially the same as that described in the first report of this series except that for the larger loads a hydraulic jack and test gage were substituted for the mechanical jack and steel beam dynamometer. The hydraulic jack and test gage were calibrated prior to use in the tests.

Loads were applied to the pavement through rigid bearing plates that were either circular or elliptical in shape. The elliptical plates were of the proportions shown in figure 67. This is a conventionalized average obtained from actual tire impressions which seem to conform reasonably well to the contact areas of large-size, low-pressure pneumatic tires at recommended inflation pressure and capacity load. Dimensional data for both the circular and the elliptical areas of contact are given in table 17.

TABLE 17.—*Dimensions of contact areas*

Circular contact areas		Elliptical contact areas		
Diameter	Area	Length of major axis	Length of minor axis	Area
Inches	*Sq. ins.*	*Inches*	*Inches*	*Sq. ins.*
8	50	12	6.86	72
12	113	24	13.71	289
20	314	36	20.57	649
25	491	48	27.43	1,155
30	707	--------	-------	-------
36	1,018	--------	-------	-------

Considerable care was taken in seating the bearing plates to insure uniform distribution of the load over the area of contact. A thin layer of plaster of Paris was first placed on the concrete test slab and formed to

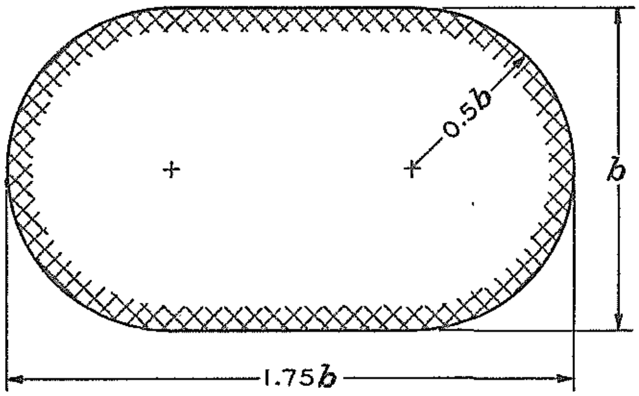


FIGURE 67.—PROPORTIONS OF ELLIPTICALLY SHAPED BEARING AREAS.

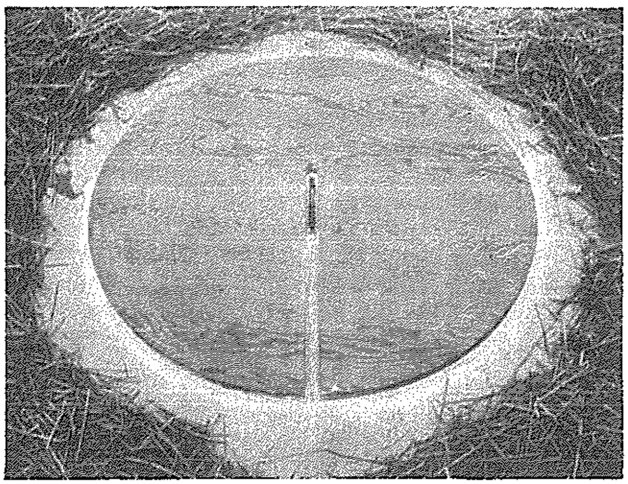


FIGURE 68.—CIRCULAR PLYWOOD BEARING PLATE SEATED ON THE PAVEMENT.

a smooth plane surface. When this had hardened, a sponge rubber mat ½ inch in thickness was placed on the plaster surface. A plate of plywood, 1 inch in thickness, cut to the exact shape of the desired area of contact was then laid on the rubber mat and on this plywood plate an assembly of stiff plates was built up in such a manner as to insure rigidity. These back-up plates were of steel and of concrete and were bedded individually in plaster of Paris. Typical circular and elliptical plywood plates are shown in figures 68 and 69 respectively, while the complete assembly for a test with a circular area is shown on the cover page.

TEST PROGRAM PLANNED TO YIELD DATA FOR AIRPORT DESIGN

Strains were measured with the recording type of strain gage used throughout the investigation except that for these tests the glass slide on which the movement of the stylus is traced was fitted into a small carriage so arranged that it could slide longitudinally on the gage body. This carriage was moved by means of a fine wire attached to it and extending from the gage position to the edge of the loaded area in a groove provided in the plywood plate. The strain gage, groove, and wire can be seen in figures 68 and 69. This arrangement permitted a series of strain measurements to be made without disturbing the bearing plate assembly.

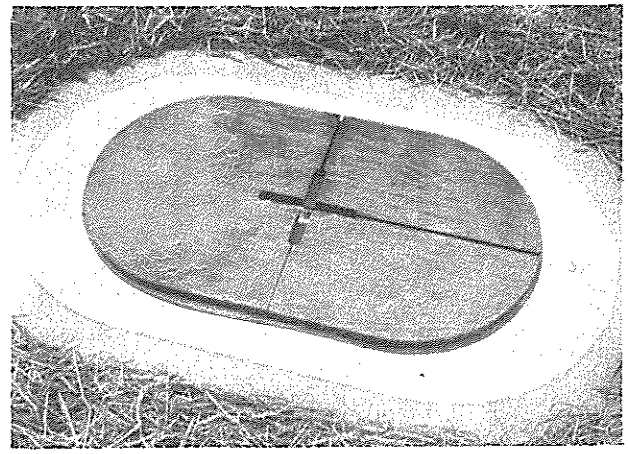
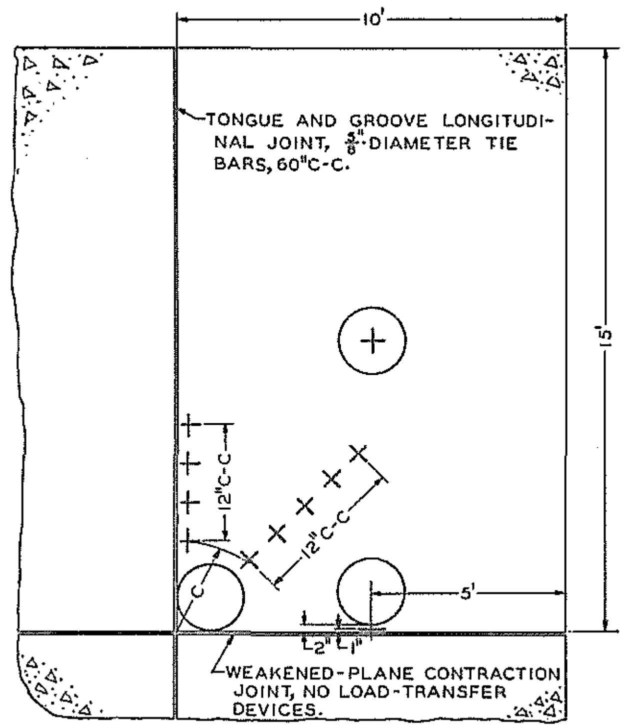


FIGURE 69.—ELLIPTICAL PLYWOOD BEARING PLATE SEATED ON THE PAVEMENT.



THE DISTANCE, C, RANGED BETWEEN 16-INCHES FOR THE 8-INCH DIAMETER BEARING AREA AND 40-INCHES FOR THE 36-INCH DIAMETER BEARING AREA.

FIGURE 70.—LOCATIONS AT WHICH LOADS WERE APPLIED AND THE POSITIONS AT WHICH GAGES WERE PLACED TO MEASURE THE CRITICAL STRAINS.

Because of a limit on the time during which the pavement sections were available it was necessary to restrict the study to a comparatively small number of tests. It was decided, therefore, to apply loads at the locations that seemed most important from the standpoint of airport pavement service. The locations selected were:

1. An interior corner, i. e., one formed by the intersection of a transverse contraction joint and the longitudinal joint.
2. An interior point, as far as possible from slab edges.
3. A transverse contraction joint edge at some distance from a slab corner.

These locations together with the strain gage positions used with each are shown in figure 70.

No tests were made at free edges or free corners and no tests were made along the longitudinal joint edge because it had been found in earlier tests (*21*) that the type of joint in these slabs is highly effective in controlling the critical stress caused by a load acting in the region along the joint.

A study was made of the strain distribution within these large bearing areas for the interior and edge loadings and it was found that strains measured at the positions shown would be as high as any within the area. In the case of the corner loading where strains were being measured outside the bearing area it was necessary to make preliminary tests to locate the position at which the strain along the bisector of the corner angle would be a maximum for each of the several bearing plate sizes. In this connection it may be recalled that the studies of joint behavior described in the fourth report of this series (*21*) showed that at interior slab corners, where corner support is obtained from adjoining panels, the critical stress may develop directly under the loaded area or at some point along the edges of the loaded panel rather than along the bisector of the corner angle as is the case with a free corner.

The protection given the test section and other details of procedure were the same as those already described for the main part of the investigation.

SUBGRADE TESTS GIVE SURPRISING RESULTS

Since the character of the subgrade immediately beneath the pavement of the test section had been altered in the manner previously described it was necessary to determine a value for the effective modulus of subgrade reaction before any comparisons could be made between theoretical and observed stresses. These subgrade tests developed data that are of considerable interest.

It was found in the determination of values for k on the original subgrade that a rigid circular plate 30 inches or more in diameter was of sufficient size to eliminate the effect of plate size. Because of this finding the first tests on the modified subgrade were made with a rigid circular plate of 36-inch diameter in the manner described in the first section of this report. The data obtained indicated a value of approximately 400 lbs. in.$^{-3}$ for the modulus of subgrade reaction and, contrary to the data from the earlier tests (see fig. 12), showed about the same values for plate displacements of 0.01, 0.02, and 0.05 inch. While it appeared reasonable to expect that the treatment given the subgrade soil would cause an increase in its resistance to deformation under load, it had not been anticipated that the rather marked effect of the magnitude of the plate displacement on the modulus of subgrade reaction which had been evident in the bearing tests on the original subgrade would be so greatly reduced in the tests on the modified subgrade.

The value of the modulus, k, as determined from load deflection tests on the pavement on one quadrant of the test section was found to be 285 lbs. in.$^{-3}$, considerably lower than that indicated by the bearing tests with the 36-inch diameter rigid plate but essentially the same as that found for the original unmodified subgrade under similar summer conditions.

This rather surprising result has several interesting implications. It is indicated that in modifying the character of the upper layer of the subgrade, in the manner previously described, its load supporting ability within a given deformation limit was considerably increased so long as the given unit load was applied over a relatively small area. When the loaded area was relatively large, as with the slab deflection test, on the other hand, the influence of the strengthened upper layer on the load support offered by the subgrade as a whole tended to disappear. This suggests that there is a relation between the size of a loaded area on a pavement and the strengthening effect to be expected from a given base course beneath that pavement.

Unfortunately there was no opportunity to study this matter thoroughly in connection with the present investigation. It was possible, however, to make determinations of the value of k from slab deflection tests on other quadrants of the test section and also to make one determination with a rigid bearing plate of greater size. This latter test was made with a 54-inch diameter plate on the subgrade in an opening cut through the pavement in one quadrant of the test section. This is shown in figure 71. The data obtained from the bearing test indicated a value of $k = 315$ lbs. in.$^{-3}$ at a displacement of 0.02 inch, about 10 percent greater than the value obtained from the slab deflection in this particular quadrant. This difference in the values of the modulus of subgrade reaction would have a negligible effect on the magnitude of computed stresses for the conditions that obtained in this study.

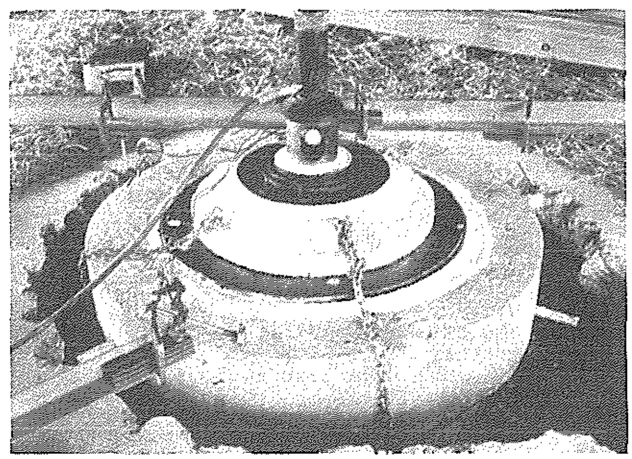


FIGURE 71.—DETERMINING THE MODULUS OF SUBGRADE REACTION WITH A LARGE BEARING PLATE PLACED ON THE SUBGRADE THROUGH AN OPENING CUT IN THE PAVEMENT.

It was not possible to make bearing tests with the 54-inch diameter rigid plate through the pavement on each quadrant of the test section. An average value for k determined in this manner is, therefore, not available.

The average value obtained from the data obtained in the load-deflection tests on the pavement in three of the four quadrants of the test section was 250 lbs. in.$^{-3}$. This value was used in computing certain of the stress values that will be shown in one of the subsequent graphs.

In order to obtain data that would permit strain data to be handled more flexibly in the analysis, a study was made of the linearity of the load-strain relation for a range of areas of contact which included both those used in the general investigation and the larger areas used in this supplementary study. These tests were made for interior loading only. Data obtained with circular contact areas with diameters ranging from 8 to

36 inches are shown in figure 72. The maximum load magnitude was sufficient in each case to produce a stress of at least 300 pounds per square inch. Each value shown in this figure is the average of eight observations, two in each of the four quadrants of the test section.

From these data it can be concluded that the load-stress relation is linear within the stress range shown for all the contact areas used.

UNCERTAIN NATURE OF EDGE SUPPORT AT FRACTURED FACES DEMONSTRATED

In figure 73 there is shown a comparison of stress values computed by means of the Westergaard equation with stress values derived from the strains observed in the loading tests, for each of the six sizes of circular contact area included in the tests.

For concrete slabs lying on a subgrade and acted upon by vertical static loads applied in the interior region, both theory and test show that the maximum flexural stress is closely proportional to the applied load. This fact makes it possible to extend load-stress data beyond the range of observed values without the risk of appreciable error. In deriving the observed stress values of figure 73 the data were extended somewhat beyond the observed range in the cases of the two smaller bearing areas in order to obtain stress values corresponding to the assumed load of 20,000 pounds. While these two stress values may be slightly above the usual working stress range for concrete in flexure the procedure is sound and the comparison between the theoretical and observed values valid throughout the stress range shown.

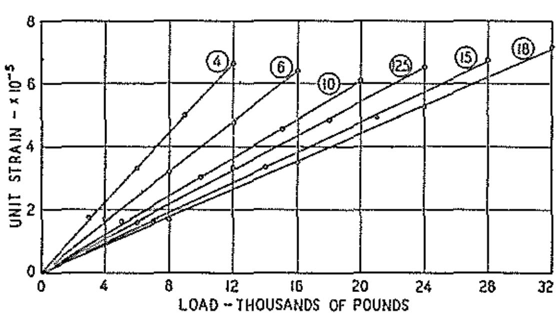


FIGURE 72.—LOAD-STRAIN RELATION FOR A RANGE OF SIZE OF LOADED CIRCULAR AREAS—INTERIOR CASE OF LOADING—FIGURES IN CIRCLES SHOW RADII OF LOADED AREAS.

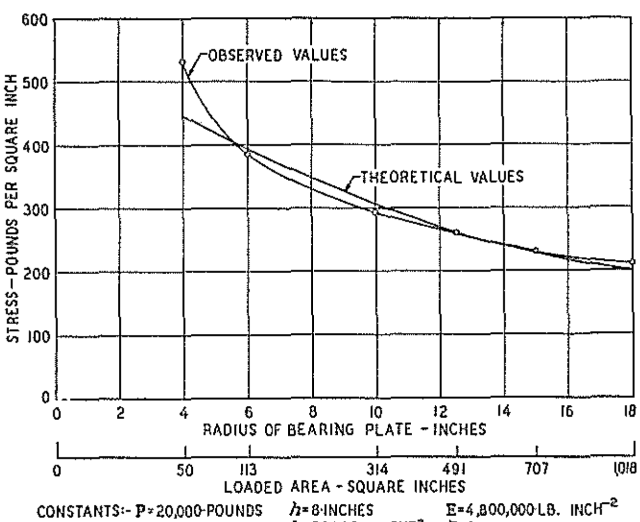


FIGURE 73.—COMPARISON OF THEORETICAL AND OBSERVED STRESSES FOR A RANGE OF SIZE OF LOADED CIRCULAR AREAS—INTERIOR CASE OF LOADING.

It will be noted in the legend that, for calculating the theoretical stresses, it was assumed the quantity Z (termed the ratio of reduction) equals 0. When this value is assumed the general agreement between the theoretical stress values and those obtained from the measured strains is remarkably close and it is evident that the observed effect of size of loaded area on the stress developed by a given load is essentially the same as the theoretical relation. The one exception is the value obtained with the 8-inch diameter area where the observed value exceeds the theoretical by approximately 19 percent. The reason for this difference is not known. Referring back to similar data from the tests on the original pavement sections, shown in figure 40, it will be observed that the same tendency is evident where the 8-inch diameter area was used on the 8-inch and 9-inch sections of uniform thickness. In this connection it is pointed out that a contact area of this size is usually associated with a wheel load of about 3,500 pounds which, with pavement slabs of usual thicknesses, would cause a stress of less than 100 pounds per square inch. Thus the disparity occurs under conditions that are relatively unimportant.

In the computations for the theoretical stress values of figure 73 a value of $k=250$ lbs. in.$^{-3}$ was used since this represented an average from tests on three quadrants. While this value may appear somewhat low in view of the data from the bearing tests, it is worth noting that the use of a value of $k=300$ lbs. in.$^{-3}$ would cause a decrease in the computed stress values of only about 3 percent.

From these data it may be concluded that for the conditions of these tests there is good agreement between the observed stresses and those computed by means of the Westergaard equation for bearing areas of circular shape over a range of diameters from 8 to 36 inches.

Tests were made also with loaded areas of elliptical shape. On the basis of strain distribution studies the gages were located at the center of both the major and minor axes of the area as shown in figure 69. In figure 74 the stress values derived from the strains measured along each axis are shown for all of the elliptical areas listed in table 17, i. e., for a range of areas from 72 to 1,155 square inches. The observed maximum stresses for corresponding circular areas are shown on the same graph. The graph affords several useful comparisons. In the first place it is apparent that the stress along the major axis of the elliptical area is less than that along the minor axis, the difference being as much as 20 percent for the largest area. The maximum stress observed under each corresponding circular area was somewhat greater than that found along the major axis and slightly less than that found along the minor axis of the elliptical area.

In the past, for purposes of stress computation for highway loadings, it has often been assumed that the actual tire contact area could be represented by the circle of equivalent area. It is interesting to note that for areas such as are found in highway service, the data indicate the error resulting from this assumption would be small, probably 5 percent or less. For the larger areas the percentage would be slightly more.

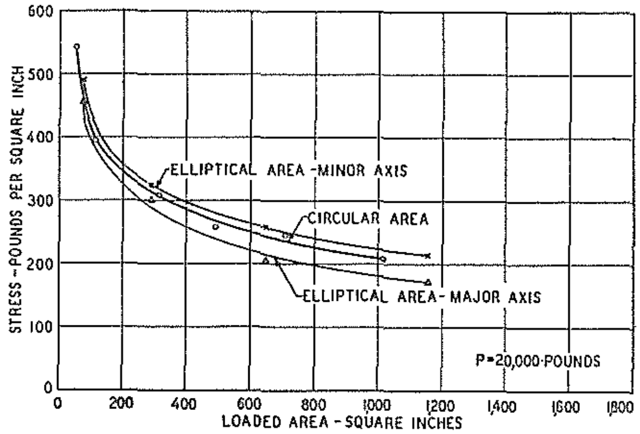


FIGURE 74.—COMPARISON BETWEEN MAXIMUM STRESSES OBSERVED UNDER CIRCULAR AND ELLIPTICAL BEARING PLATES OF EQUAL AREA—INTERIOR CASE OF LOADING.

It was mentioned earlier that Westergaard has, in a recent paper, extended his analysis to include loaded areas of other than circular shape (*28*). In a discussion of a part of this paper by the authors (*28*) comparisons were made between observed stresses and those computed by the equations developed by Westergaard for elliptically shaped loaded areas. It was shown that there was good agreement between the observed data and the theoretical relations.

Figure 75 shows observed stress values for a load of 20,000 pounds applied on circular contact areas ranging from 8 to 36 inches in diameter placed at each of three load positions shown in figure 70. When the load was applied at the mid-point of the slab end, i. e., along the weakened plane contraction joint midway between the slab edge and the longitudinal joint, tests were made with the contraction joint open and with it closed by a horizontal force of 60,000 pounds (over the 10-foot lane width) applied at the ends of the test section. When the load was applied at the interior corner the contraction joint was open.

The strain gages for the interior corner loading were located along the bisector of the corner angle at the position to measure the maximum strain developed in this direction. It is evident from the relatively low stress values that the slab corner is being strongly supported along the longitudinal joint and does not act as a free corner. Rather it approaches the condition of a free edge and the maximum stress for this case would be within the loaded area and more nearly equal in magnitude to that found in the test at the mid-point of the slab end, with the joint open.

The stress values for the interior loading serve as a reference in analyzing the other stress values.

The stresses observed for the load applied at the mid-point of the slab end bear a normal relation to those for the interior loading so long as the contraction joint is open. When, however, the joint is closed by pressure applied at the opposite ends of the test section, it is observed that the stresses for this edge loading are reduced appreciably for all sizes of bearing area and that for the larger areas the stresses become essentially equal to those found in the tests at the interior of the slab. For convenience the stresses, expressed as the percentages by which they exceed the stress for interior loading, are shown at three points along the graph. This comparison shows very clearly the uncertain nature of the edge support furnished by the fractured

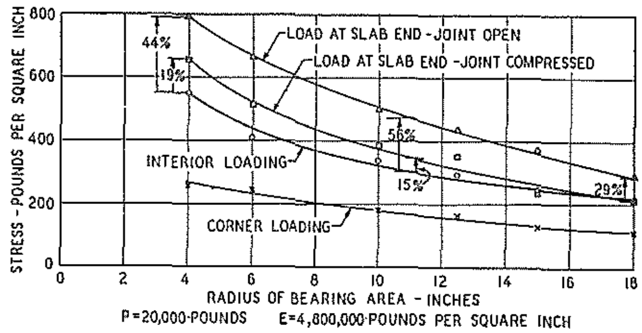


FIGURE 75.—EFFECT OF VARIATION IN POSITION OF LOAD AND SIZE OF BEARING AREA ON THE STRESS CAUSED BY A GIVEN LOAD.

faces alone in a joint of the plane of weakness type, and indicates the need for edge strengthening if a balanced design is to be obtained.

CONCLUSION

In the presentation and discussion of the data obtained in this rather extended study, an effort has been made to simplify the presentation by dividing the material into sections, each more or less complete in itself, with occasional summary statements in which the significant developments are pointed out and such conclusions as seem warranted are given. These detailed statements need not be repeated here.

The broad purpose of all of the studies described in this report has been to compare the observed structural behavior of pavement slabs of uniform thickness supported by a subgrade and subjected to vertically applied static loads with the behavior indicated by the Westergaard analysis (*23*, *25*, *27*, *28*). On this point it is concluded that, within the limits of the investigation and so long as the basic conditions assumed for the analysis are approximated, the Westergaard theory describes quite accurately the action of the pavement.

There is need for further experimental study of some of the quantities that appear in the analysis, however. It is particularly desirable that information be developed which will permit the equations to be applied more readily and with greater certainty to problems that require the computation of values of absolute stress.

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