VARIABLE DYNAMIC TESTBED VEHICLE

ANALYSIS OF HANDLING PERFORMANCE WITH AND WITHOUT REAR STEER

245 BROMPTON ROAD. WILLIAMSVILLE. NY 14221 USA

MILLIKEN RESEARCH ASSOCIATES

INCORPORATED

VARIABLE DYNAMIC TESTBED VEHICLE

ANALYSIS OF HANDLING PERFORMANCE WITH AND WITHOUT REAR STEER

By: Hugo S. Radt, Jr. , Milliken Research Associates

January 8, 1997

Performed as subcontract for: Environmental Research Institute of Michigan
David R. McLellan, Program Manager

Prime Contract for Office of Crash Avoidance Research,
National Highway Safety Administration
Technical Representative: Lloyd Emery

Project Monitor: Jet Propulsion Laboratory, California Institute of Technology
Alan Marriott, Program Director

Contents:

1. Introduction

The variable dynamic testbed vehicle (VDTV) was proposed as having the following major subsystems:

- o Front steer by wire
- o Rear steer
- o Front and rear active anti-roll bars
- o Front and rear controllable shock absorbers
- o Steering wheel torque control
- o Throttle by wire
- o Brake by wire
- o Throttle feel (throttle pedal force control)
- o Brake feel (brake pedal force control)
- o Anti-lock brake subsystem
- 0 Yaw control

Because a potential overrun of cost has been predicted, it was decided to consider elimination of several subsystems, including throttle feel, brake feel and rear steer. As an alternative to complete elimination of rear steer, a reduced cost rear steer subsystem was suggested. This alternative subsystem makes use of the available toe change adjustment, therefore is limited to one to two degrees of rear steer. Accordingly, MRA was asked, “What can be done with limited rear steer?” and, “What can be done without rear steer?”

We have reformulated these questions into several parts. Can one meet or achieve the following with limited or without rear steer?

1) Ranges of understeer gradient and lateral acceleration rise time to meet the requirements of Exhibit I (similar to the request for proposals).

2) Goals set for other handling metrics, such as yaw rate overshoot (j-turn maneuver) and sideslip gradient.

3) Emulation of another vehicle.

Questions 1) and 2) are addressed in section 3, question 3) in section 4. First, however, we present the linearized equations of motion for car handling and use these equations to explain how we arrive at which feedback and feedforward gains to vary and by how much.

2. Linearized equations of motion for car handling

It is noted that these linearized equations include effects of tire lateral force and self aligning torque due to slip and camber angles, center of gravity and roll axis locations, camber change with sprung mass roll, roll steer, lateral force and self aligning torque compliance steer and camber change. Application of these equations has been shown to accurately predict car responses for lateral accelerations below about 0.35g.

We note here that these linearized equations have been employed to determine which gains to vary and by how much. However, in evaluating the effect of such gains, we use a computer program that represents the nonlinear equations of motions including lateral and fore and aft load transfers, nonlinear tire data and nonlinear equations for the vehicle dynamics Newton’s and Euler’s equations).

Lateral force equation:

$$\mathbf{mV(d\beta/dt + r) + m_s h(dp/dt) = Y_\beta \beta + Y_r r + Y_\phi \phi + Y_{\delta F}\, \delta_F + Y_{\delta R}\, \delta_R}$$

Yawing moment equation:

$$\mathbf{I_Z(dr/dt) + I_{XZ}(dp/dt) = N_\beta \beta + N_r r + N_\phi \phi + N_{\delta F}\, \delta_F + N_{\delta R}\, \delta_R}$$

Rolling moment equation (about fixed roll axis):

$$\mathbf{I_X(dp/dt) + I_{XZ}(dr/dt) + m_s hV(d\beta/dt + r) = L_\phi \phi + L_P (d\phi/dt)}$$

Definitions:

m = mass of entire vehicle
V = forward velocity (assumed constant)
β = sideslip angle = sideslip velocity / forward velocity
t = time
r = yawing velocity or yaw rate
m_s = sprung mass
h = height of sprung mass center of gravity above the roll axis
p = roll rate (about the roll axis)
ϕ = roll angle
δ_F = front wheel steer angle
δ_R = rear wheel steer angle
I_X = yaw moment of inertia, about vehicle center of gravity
I_{XZ} = product of inertia about roll axis at a point directly below the vehicle center of gravity
I_z = roll moment of inertia about the roll axis
L_ϕ = total roll stiffness
L_P = total roll damping

Stability derivatives are denoted: $M_x = \partial M / \partial x$, where M is lateral force, Y, yawing motion, N, or rolling moment, L, while x is any of the response variable (sideslip, yaw rate, roll angle or roll

Certain of these stability derivatives have special significance for a normal car. For example, N_r is called the yaw damping derivative because it produces a moment proportional to and opposing the yawing velocity. Similarly, Y_β is the lateral damping term because it produces a lateral force that opposes the lateral velocity ($V\beta$). The term, N_β is called the directional stability and is similar in effect to that of a weather vane. That is, when the car is at a positive sideslip angle and has positive N_β the yawing moment is positive, causing the car to turn in the direction of the sideslip, thereby decreasing sideslip. Hence positive directional stability generally produces a stable understeer vehicle.

We make use of this notation with stability derivatives to determine feedback gains to modify the handling responses. An example follows. Suppose we wish to modify the damping of the yaw degree of freedom. To do so, we want to change the term N_r and to do that we add feedback of yaw rate, r, to the front and rear steer angles. That is, we let:

$$\delta_F = KFR * r \text{ and } \delta_R = KRR * r$$

Then: $$N_r{}' r = N_r r + N_{\delta F}\, \delta_F + N_{\delta R}\, \delta_R = (N_r + N_{\delta F} * KFR + N_{\delta R} * KRR) * r$$

and similarly:

$$Y_r{}' r = Y_r r + Y_{\delta F}\, \delta_F + Y_{\delta R}\, \delta_R = (Y_r + Y_{\delta F} * KFR + Y_{\delta R} * KRR) * r$$

Thus we have modified the stability derivatives, N_r and Y_r by using feedback of yaw rate to the front and rear wheels. However, we may not wish to change the lateral force derivative, Y_r. To do so we make:

$$Y_{\delta F} * KFR + Y_{\delta R} * KRR = 0$$

This demonstrates how we can change one stability derivative at a time.

Feedback of yaw rate, r, sideslip angle, β, or roll angle, ϕ, modifies the steady state responses of the VDTV, while feedback of the rate derivatives such as (dr/dt = yaw acceleration), ($d\beta/dt$ = sideslip rate), ($d\phi/dt$ = p = roll rate) or (dp/dt = roll acceleration) change only the transient responses, not the steady state. Accordingly, we can use feedback of r,β and ϕ to modify the steady state gradients and understeer gradient, while using feedback of the higher time derivatives such as (dr/dt) to change the dynamic behavior, such as rise time, while leaving the steady state gradients alone. As an example, we modify the stability derivative, N_r, using the gains, KFR and KRR, as indicated above to change the understeer gradient, then modify gains on the yaw acceleration to change the yaw rate rise time. In this way we are able to vary the understeer gradient and rise time almost independently. (Because the understeer gradient affects the rise time, it must be “set” first, then the rise time adjusted afterwards.)

and $$N_r\mathbf{r} = N_r + N_{\delta F} * KFR + N_{\delta R} * KRR$$

where Y_r' and $N_r\mathbf{r}$ are the corresponding yaw rate derivatives for the car model to be emulated, while Y_r and N_r are the derivatives for VDTV. These equations are then solved simultaneously for KFR and KRR to find the gains.

The lateral acceleration, a_y, (in g-units) effectively "measured" on the roll axis at a point directly below the vehicle center of gravity is also given by:

$$(V/g)*(d\mathbf{b}/dt + r)$$

so that terms in $V *(d\mathbf{b}/dt + r)$ in the equations of motion can also be put in terms of the lateral acceleration. Thus we can also determine effects of feedback of lateral acceleration to the front and rear wheels. In fact, when emulating another car, we must also change the effective mass of the VDTV. To do so we let:

$$m'g = mg + Y_{\delta F} * KFAY + Y_{\delta R} * KRAY$$

where KFAY and KRAY are the front and rear gains between the steer angles and the lateral acceleration, and m' is the total mass of the car to be emulated while m is the mass of VDTV. Similarly, we change the effective inertias of the VDTV by using yaw acceleration feedback to front and rear wheels and roll acceleration feedback to the active anti-roll bars.

3. Analysis of Response Metrics

3.1 UNDERSTEER GRADIENT

By varying KFBETA = $\partial\delta_F / \beta$, and KFR = $\partial\delta_F / r$, we modify the steady state responses, thereby changing the understeer gradient, with the steering ratio held fixed at 16/1. Rear wheel steer is not required to vary the understeer gradient. Figure 1 shows the variation of understeer gradient with lateral acceleration. This curve was obtained for KFBETA = 4.5 and KFR = - 0.338 and shows the understeer gradient increasing rapidly above about 0.8g. The requirement for an understeer gradient of +13 deg/g at 0.15g is demonstrated, although the increase with lateral acceleration is more gradual than indicated on figure 3.5 of exhibit I. The reason is found in the high cornering stiffness and high friction of the tires assumed for the VDTV. Figure 2 shows the friction coefficient of the P275/ZR-17 tires to vary between 1.1 at high load and 1.5 at low load. These values are much higher than found on production car tires. Hence to make the curve of figure 1 rise rapidly at around o.5g it is only necessary to limit the front steer angle as a function of lateral acceleration, despite further increases in steering wheel angle. Thus the upper curve on figure 3.5 of exhibit I can be achieved without rear steer.

Figure 3 shows the variation of understeer gradient for the oversteer case. This curve was obtained for KFBETA = - 3.05 and KFR = 0.366 and becomes more oversteer (i.e., more negative) for increasing lateral acceleration. Hence the lower curve on figure 3-5 of exhibit I is exceeded by our simulation results.

3.2 LATERAL ACCELERATION RISE TIMES

Figure 4 shows the lateral acceleration rise times calculated for fixed gains:

KFR = 0.10 sec and KRRDOT = $\partial\delta_F / (dr/dt)$ = 0.014 sec^2

out to a lateral acceleration of 0.78g. For this set of fixed gains the calculated rise time crosses the required curve of figure 3-6 of exhibit I at around 0.73g. To achieve the smaller "required" value of about 0.07 sec at 0.8g, we changed the yaw acceleration feedback gain to:

KFRDOT = 0.018 sec^2

for this one point. Otherwise the gains remained the same.

Figure 5 shows lateral acceleration rise times on the long side. Values meet or exceed the requirement of figure 3-6 of exhibit I. Again, we were able to satisfy the lateral acceleration rise time requirements without rear steer.

3.3 YAW RATE OVERSHOOT

For the baseline parameters of the VDTV we are able to vary the peak yaw rate response over a wide range without rear steer. Table I shows the effect of KFRDOT on yaw rate percent overshoot.

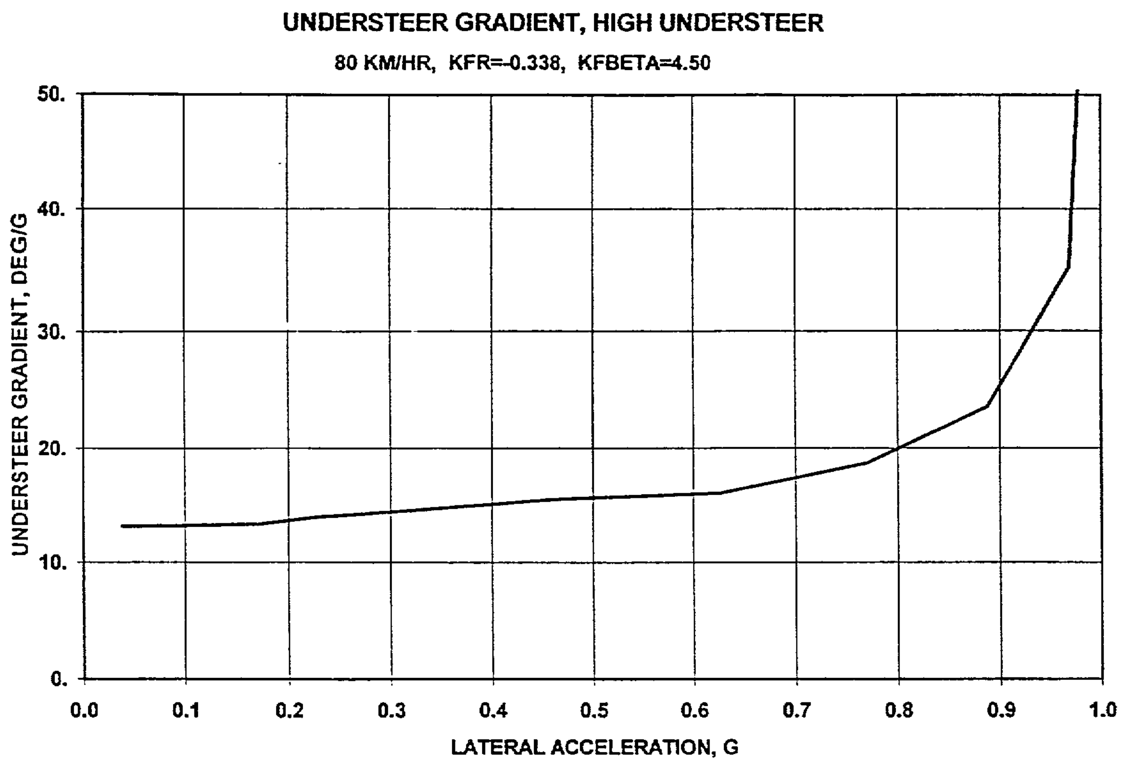

Figure 1 Variation of Understeer Gradient with Lateral Acceleration, High Understeer

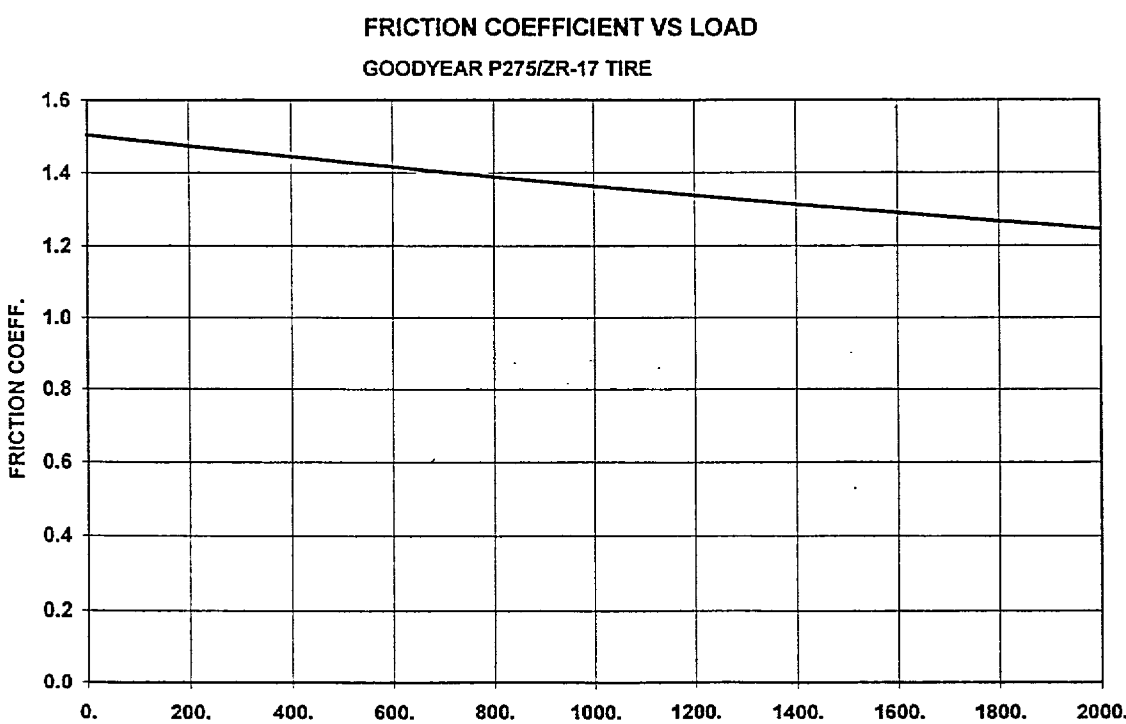

FIG. 9

Figure 2 P275/ZR-17 Tire Friction Coeffcients vs Load

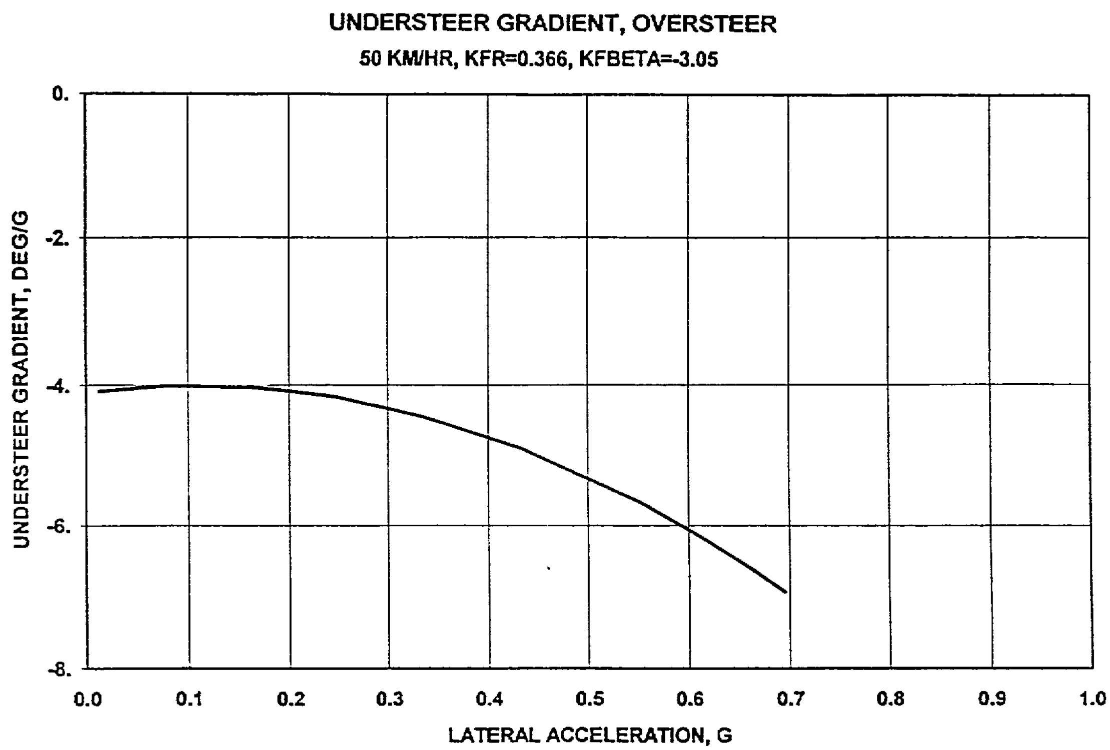

Figure 3 Understeer Gradient Variation with Lateral Acceleration, Oversteer

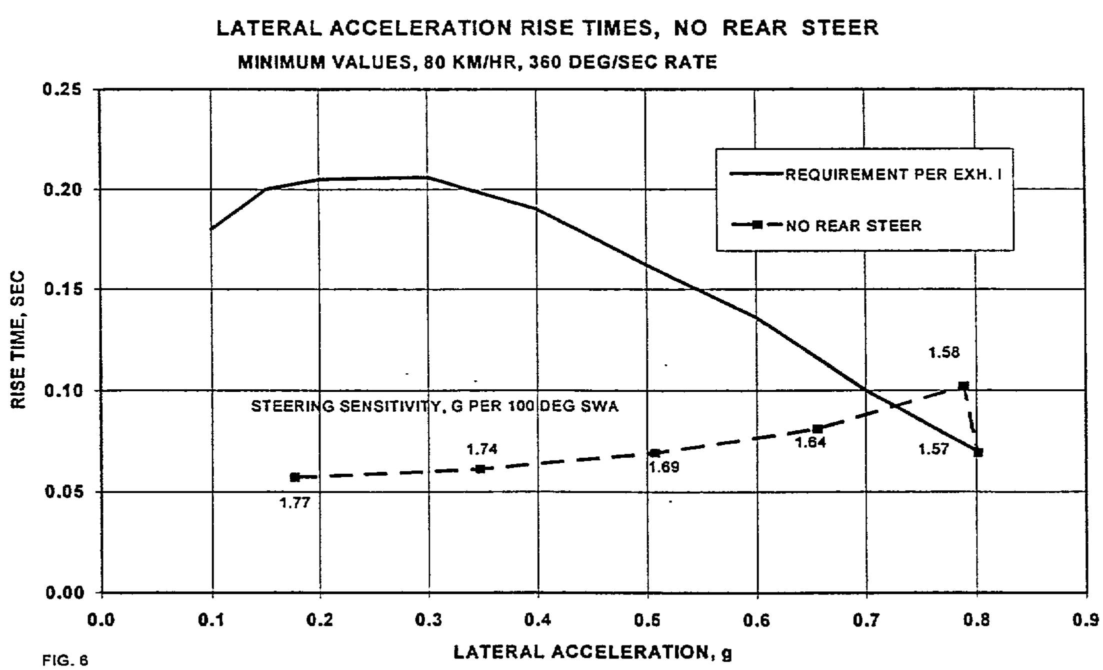

Figure 4 Short Lateral Acceleration Rise Times vs Lateral acceleration

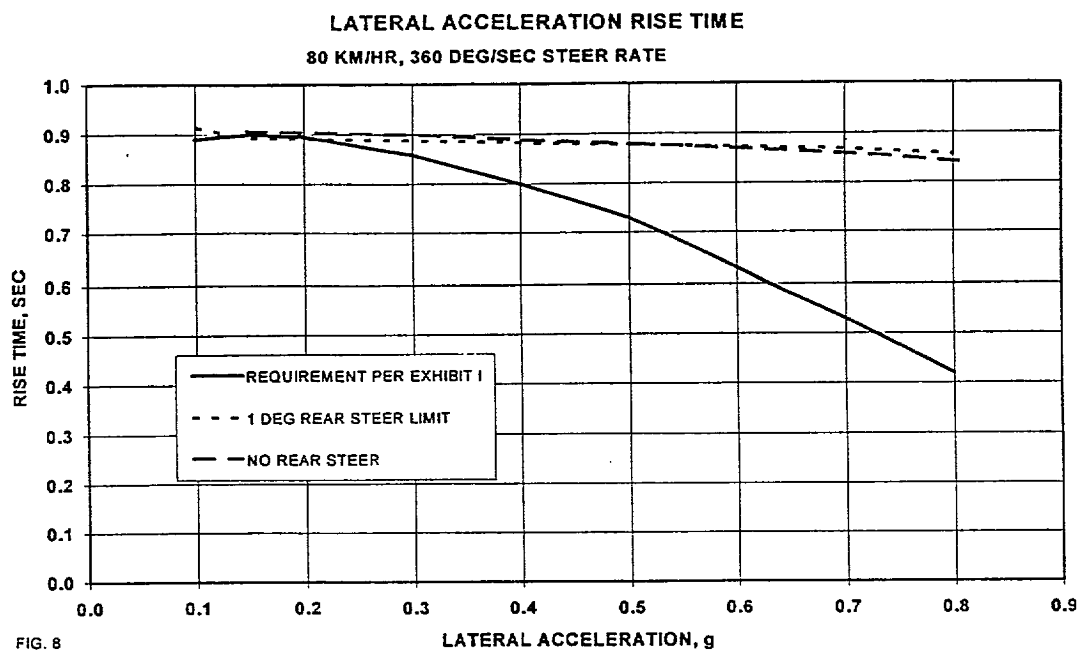

Figure 5 Long Lateral Acceleration Rise Times vs Lateral Acceleration

Table 1
Variation of Yaw Rate Overshoot

KFRDOT	Percent Overshoot
-.02	0.8%
-.01	2.5%
0	9.3%
.01	29.6%
.02	113%

3.4 SIDESLIP GRADIENT

The sideslip gradient (ratio of steady state sideslip angle to lateral acceleration) of the baseline VDTV is about -0.4 deg/g. This small a value at a speed of 80 km/hr occurs because the tires have very high cornering stiffness, making the tangent speed high. The tangent speed is the speed at which the sign of the steady state sideslip angle changes from positive to negative as speed increases. 80 km/hr is only slightly above the tangent speed. Also, at a fixed speed, the sideslip at the vehicle center of gravity depends essentially on the longitudinal center of gravity position, vehicle weight and rear cornering stiffness, therefore cannot be varied significantly with only front steer. Thus a change of rear tires is required to modify the sideslip gradient for fixed speed and no rear steer.

With a limit of one degree of rear steer, and a speed of 120 km/hr, we have demonstrated. a variation of sideslip gradient from +7 to -6 deg/g by varying front and rear gains from both yaw rate and sideslip angle.

The only alternatives to using rear steer to change sideslip gradient are to use different rear tire cornering stiffness or a different speed. Because the driver controls and senses speed directly, it is an inappropriate variable for the investigation of effects of sideslip. Hence a change in tire cornering stiffness is the best alternative. This can be done by changing tires or by changing rear tire pressures.

4. Emulation of a Small Car Model

In order to investigate the accuracy of emulation of another vehicle, we developed a generic small vehicle by changing the basic parameters of the VDTV, while using the same simulation program. Table 2 compares those parameters that were changed.

Table 2 Comparison of Parameters between Small Car Model and Baseline VDTV

Parameter	VDTV	Small Car Model
Center of gravity to front axle, ft	3.46	3.0
Wheelbase, ft	8.83	8.0
Track width, ft	5.1	4.8
Total weight, lbf	4000	2000
Sprung weight, lbf	3560	1800
Yaw moment of inertia, ft-lbf-sec^2	2718	1100
Roll moment of inertia, ft-lbf-sec^2	788	400
Height of car center of gravity, ft	1.76	1.6
Understeer gradient, deg/g	3.1	1.9

As seen from table 2, the model car is about one-half the weight and inertia of the baseline VDTV. The model car has a considerably lower understeer gradient, more typical of a small, “sporty” two seater. Compliances, roll steer and roll camber were left unchanged.

We used data for a P185/70R-14 tire for the small car simulation and a speed of 80 km/hr. Figures 6a thorough 6f show the lateral acceleration, yaw rate, roll angle, sideslip angle, front steer and rear steer time histories for the case of 0.27g steady state lateral acceleration. There is no limit to rear steer and gains were selected as described in section 2. The maximum rear steer angle was 0.6 deg. Agreement between the VDTV emulation and the small car model responses are all good, including the sideslip response. Small deviations occur because the simulated control systems command rack position, not wheel position. Hence suspension and steer compliances cause deviations between the calculated and achieved wheel steer angles. Also, effects of traction on tire forces and moments and on fore and aft load transfer are omitted when determining the stability derivatives for the linear approximation used in estimating appropriate gains.

Figures 7,8 and 9 are similar to figure 6a, except for higher steady state lateral acceleration. The one degree hard limit on rear steer is reached at about 0.6 sec for the case of figure 8 and 0.5 sec for figure 9. The combination of tire force/moment nonlinearities at high lateral acceleration and the one degree limit cause the slightly less damped behavior shown on figures 8 and 9. However, adjustment of gains with lateral acceleration would most likely correct the differences in responses for the higher lateral accelerations. Figure 10 shows the result for unlimited rear steer and compares the lateral acceleration response with those of figure 9. The maximum rear steer angle needed was only 1.6 deg. Again, differences in lateral acceleration between the model and emulation are correctable by programming gains with lateral acceleration.

According to the above discussion, it is most likely that the VDTV can emulate another car up to at least 0.6g, provided rear steer is available up to about two degrees. Even one degree provides reasonable capability for emulation up to about 0.5g. There is also a possibility of accurate emulation up to still higher lateral accelerations with limited rear steer (two degrees, for example) by judiciously programming gains with lateral acceleration.

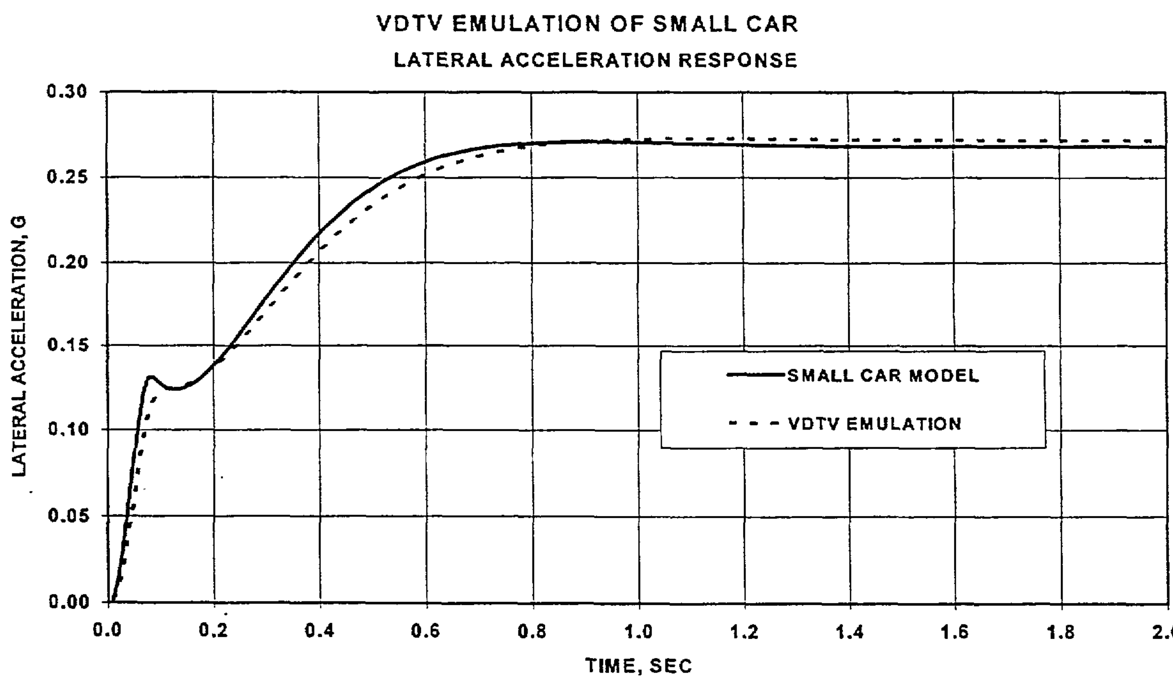

Figure 6 Comparison of Vehicle Responses between Small Car Model and VDTV Emulation
6a Lateral Acceleration

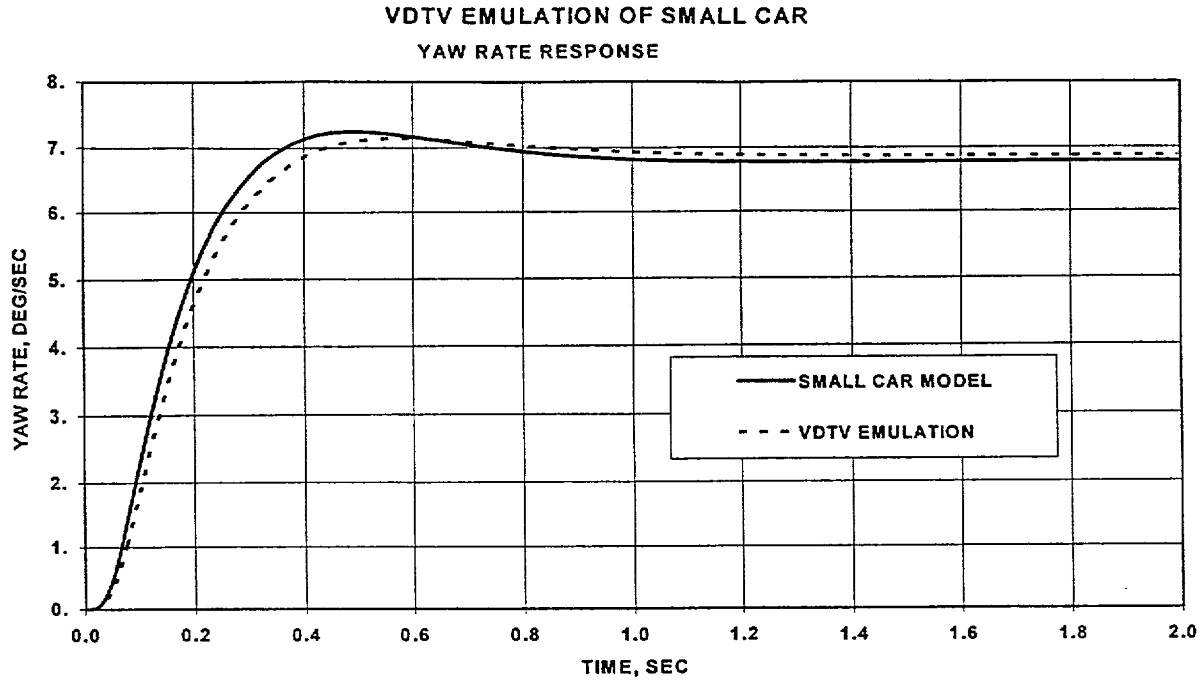

Figure 6b Yaw Rate

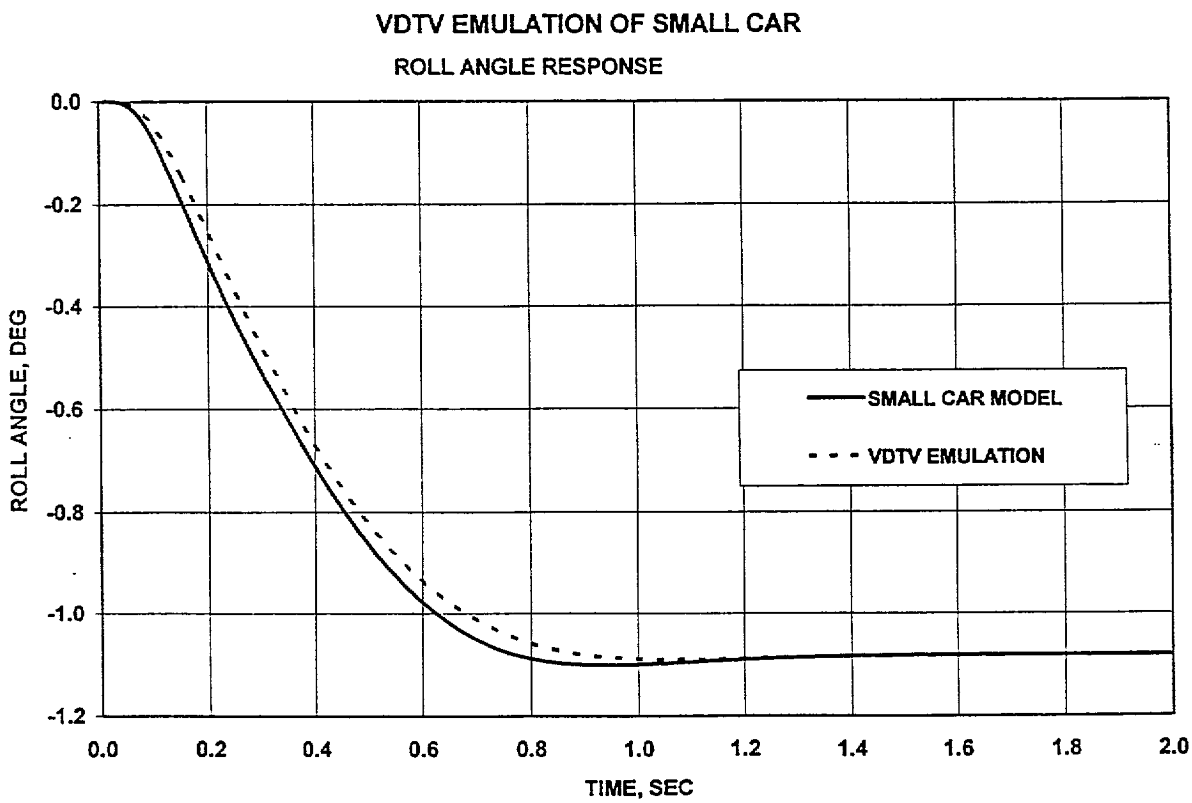

Figure 6c Roll Angle

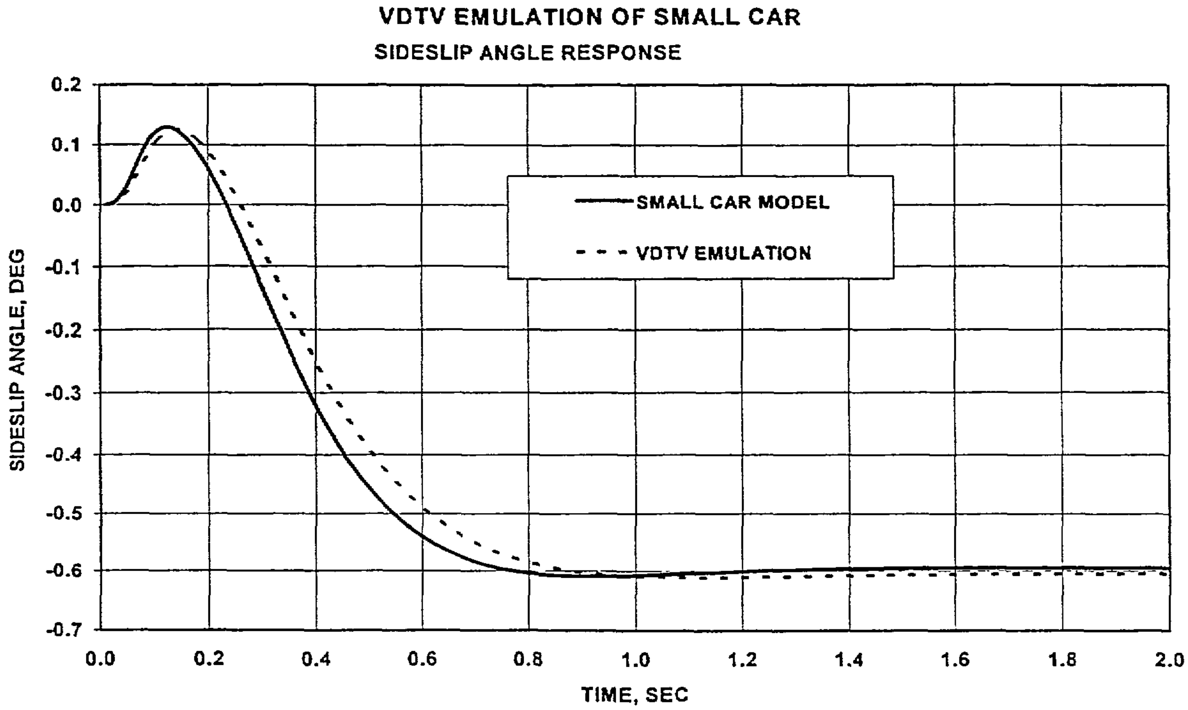

Figure 6d Sideslip Angle

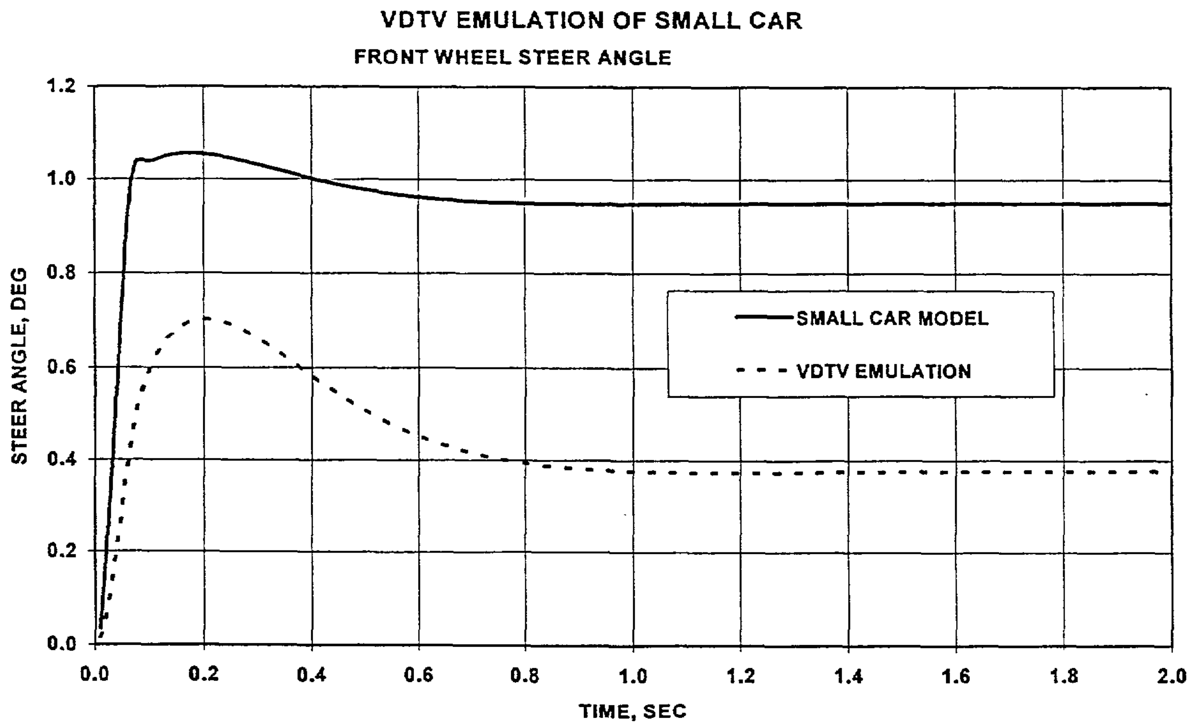

Figure 6e Front Road Wheel Angle

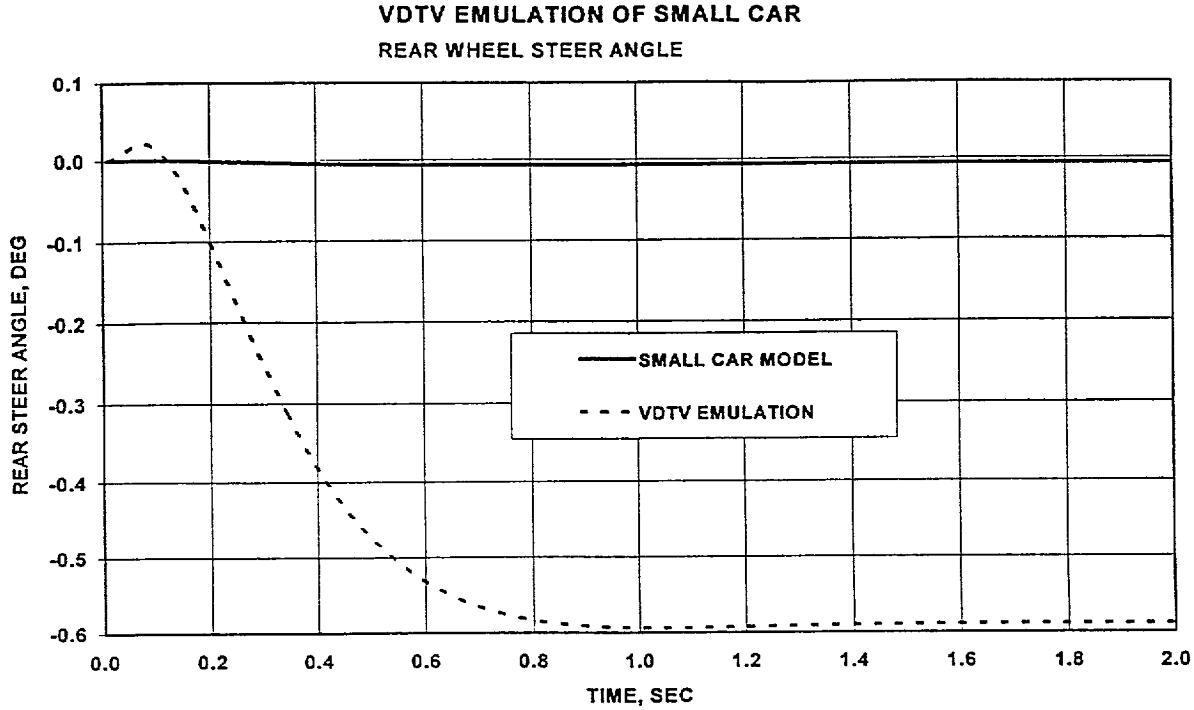

Figure 6f Rear Road Wheel Angle

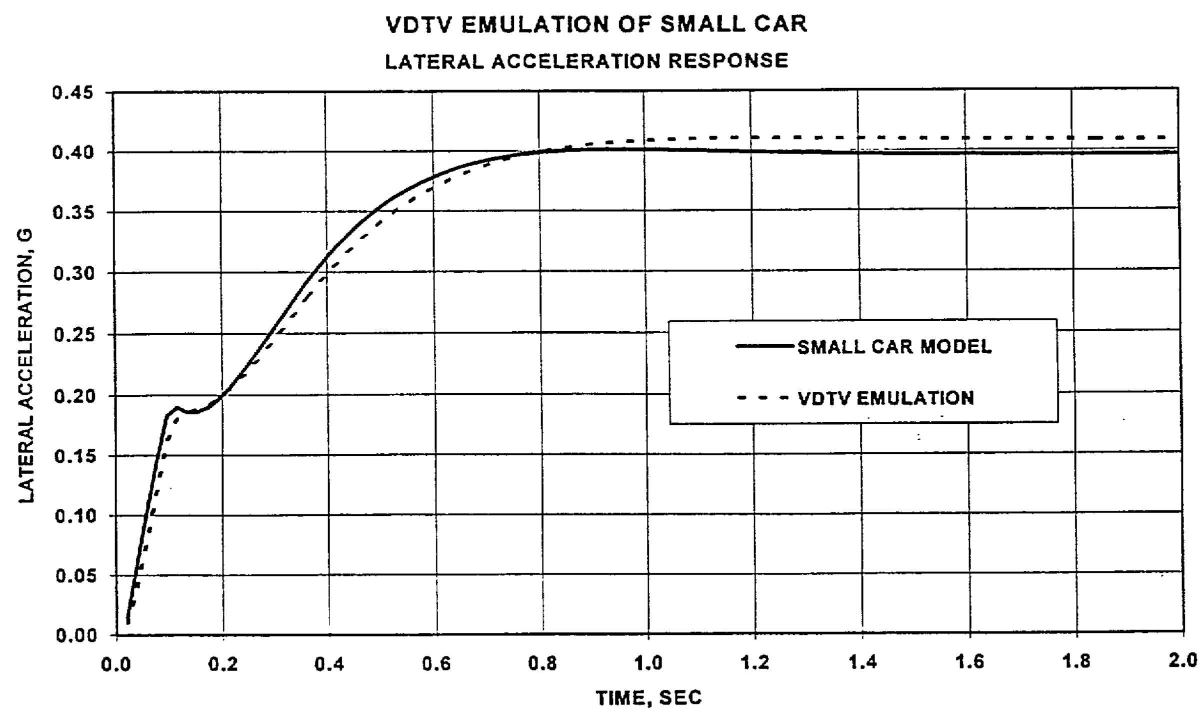

Figure 7 Small Car Emulation Comparison for 0.4g Steady State Lateral Acceleration

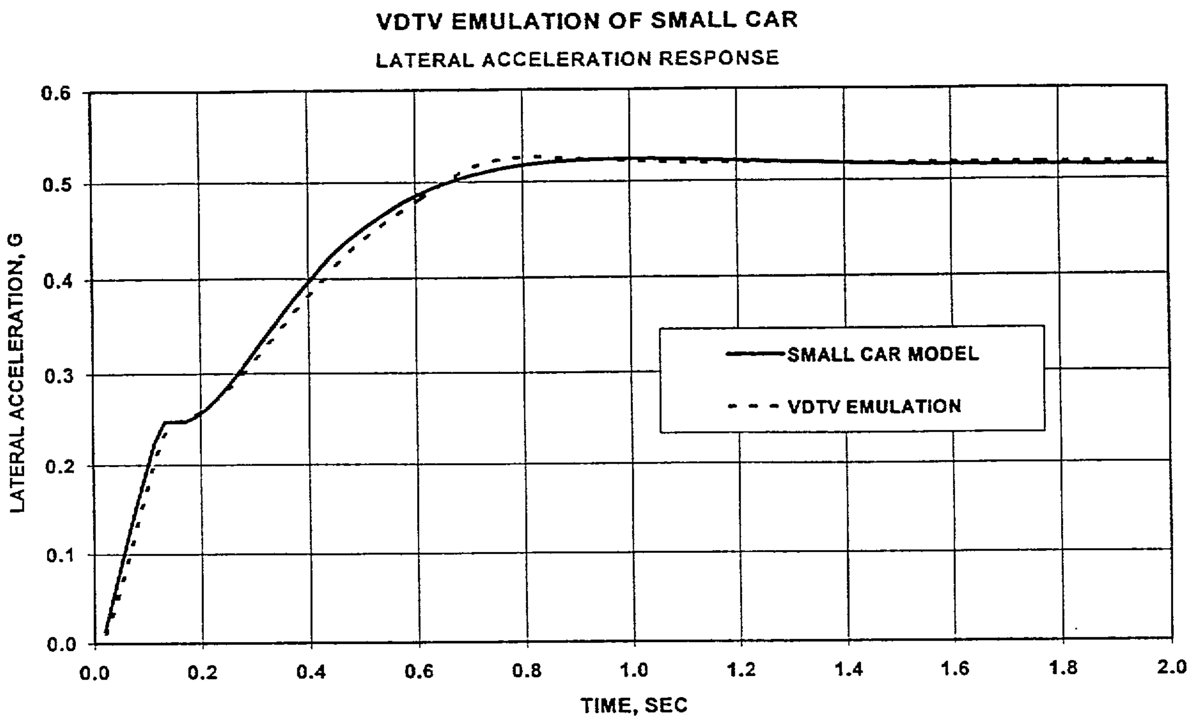

Figure 8 Small Car Emulation Comparison for 0.5g Steady State Lateral Acceleration

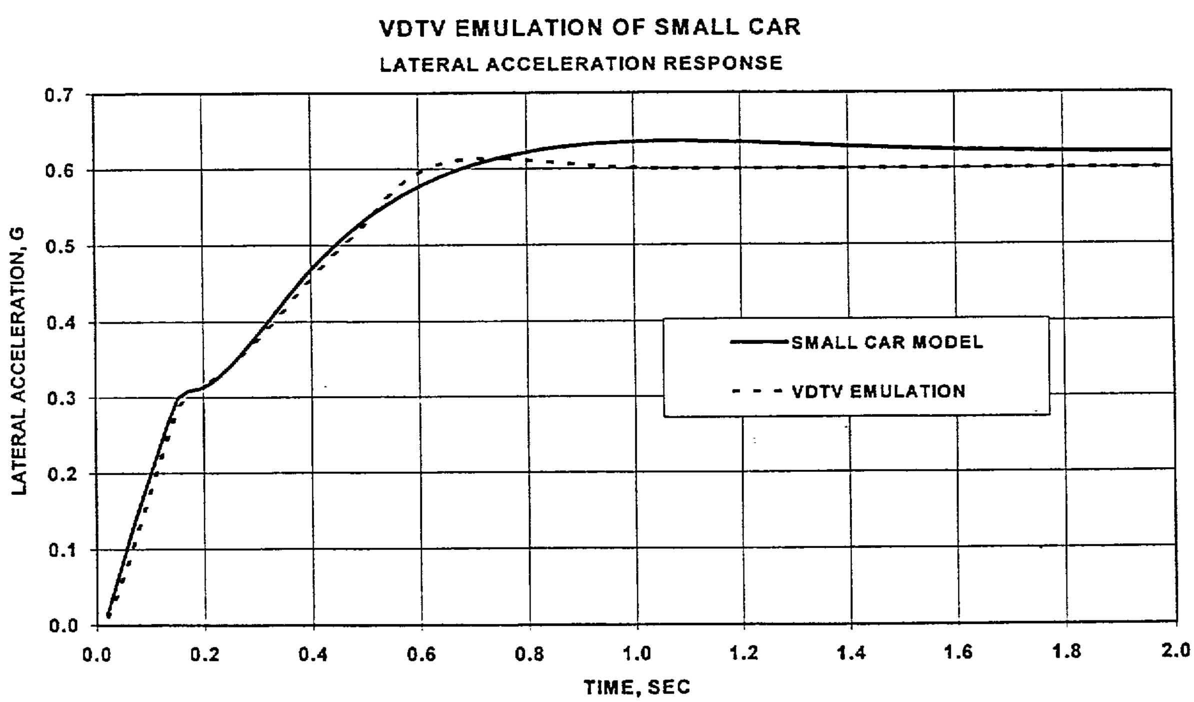

Figure 9 Small Car Emulation Comparison for 0.6g Steady State Lateral Acceleration

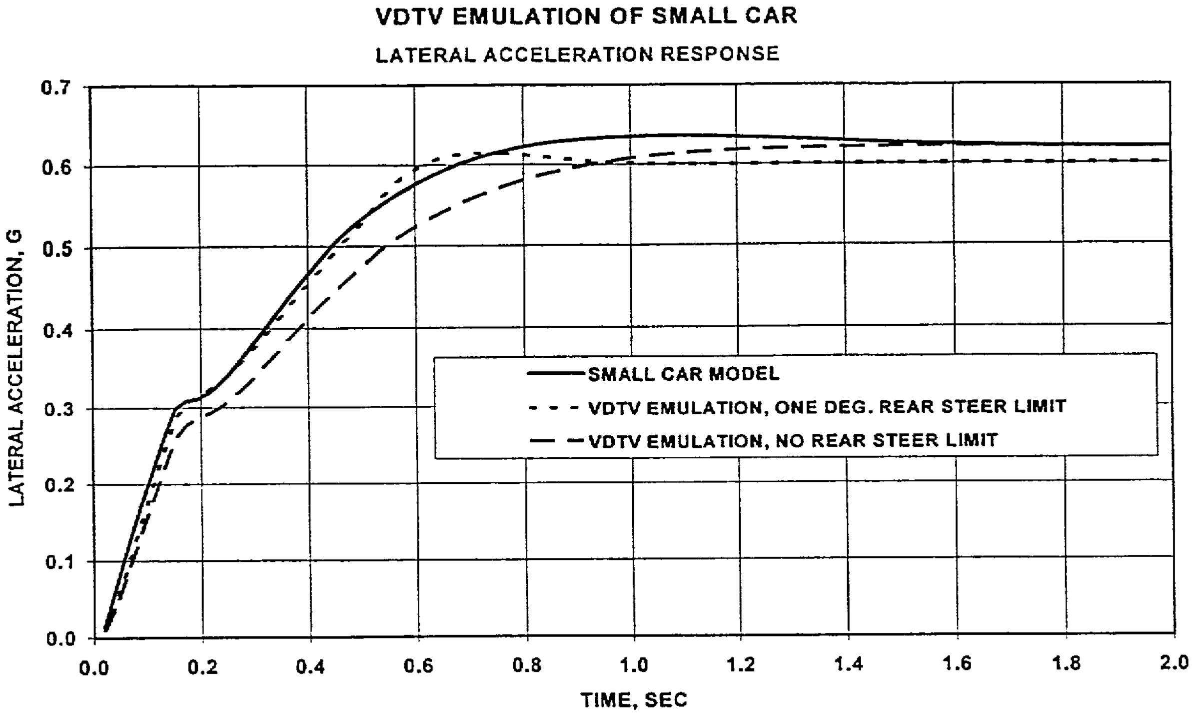

Figure 10 Comparison of Small Car Emulation with Limited and Unlimited Rear Steer

When no rear steer is available the sideslip response cannot be emulated, so that the relationship between lateral acceleration and yaw rate is not matched. As shown on page 6,

$$a_y = (V/g)^* \, (d\beta dt + r)$$

so that loss of sideslip emulation means that either the yaw rate or the lateral acceleration responses can be matched, but not both. Figure 11a shows a case where we came close to matching the yaw rate response without rear steer, but figure 11b shows that the corresponding lateral acceleration is not matched, nor is the sideslip angle, shown on figure 11c. Figure 12a shows a similar case where we nearly matched the lateral acceleration response, but figure 12b shows that the yaw rate response is then mismatched.

In order to obtain better agreement between the model and the emulation without rear steer, it is necessary to match the vehicle sideslip response (β). To do so requires a physical change to the car, not just front steer gains. We assumed different tires on the VDTV by multiplying the front cornering stiffness by 2 and dividing the rear cornering stiffness by 2, then varying the front steer gains to match the lateral acceleration response. Figure 13a shows that the emulation is close, while figures 13b and 13c show that the yaw rate and sideslip angle are also closely matched. It is noted that roll angles are emulated closely, by adjustment of the active anti-roll bar gains, provided the lateral acceleration responses are matched. We have demonstrated roll response emulation, although the figures are not presented here.

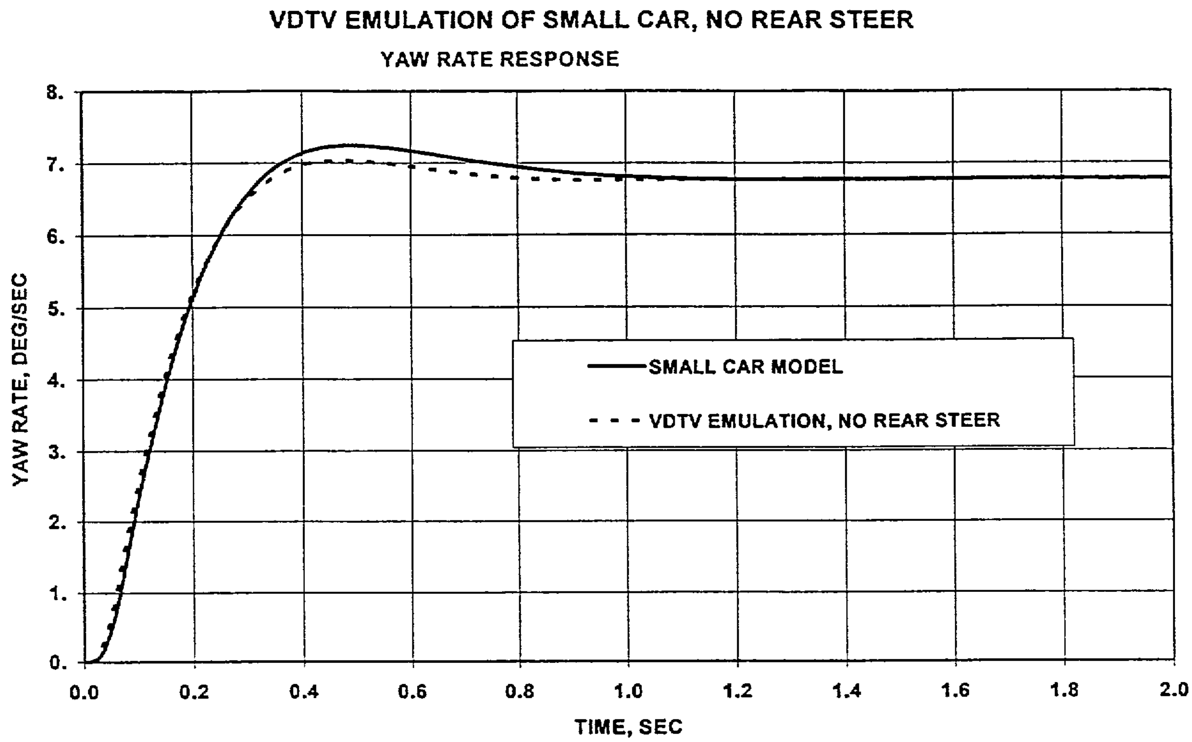

Figure 11a Emulation of Yaw Rate Response without Rear Steer

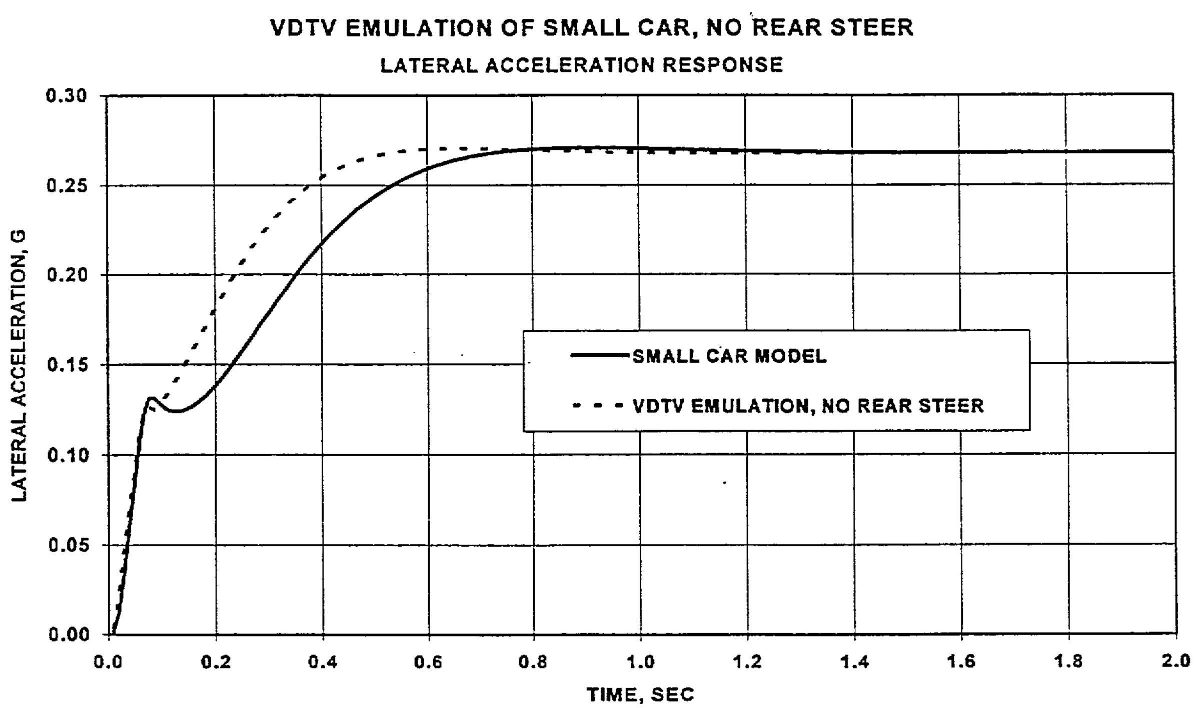

Figure 11b Yaw Rate Response (Lateral Acceleration Matched for No Rear Steer)

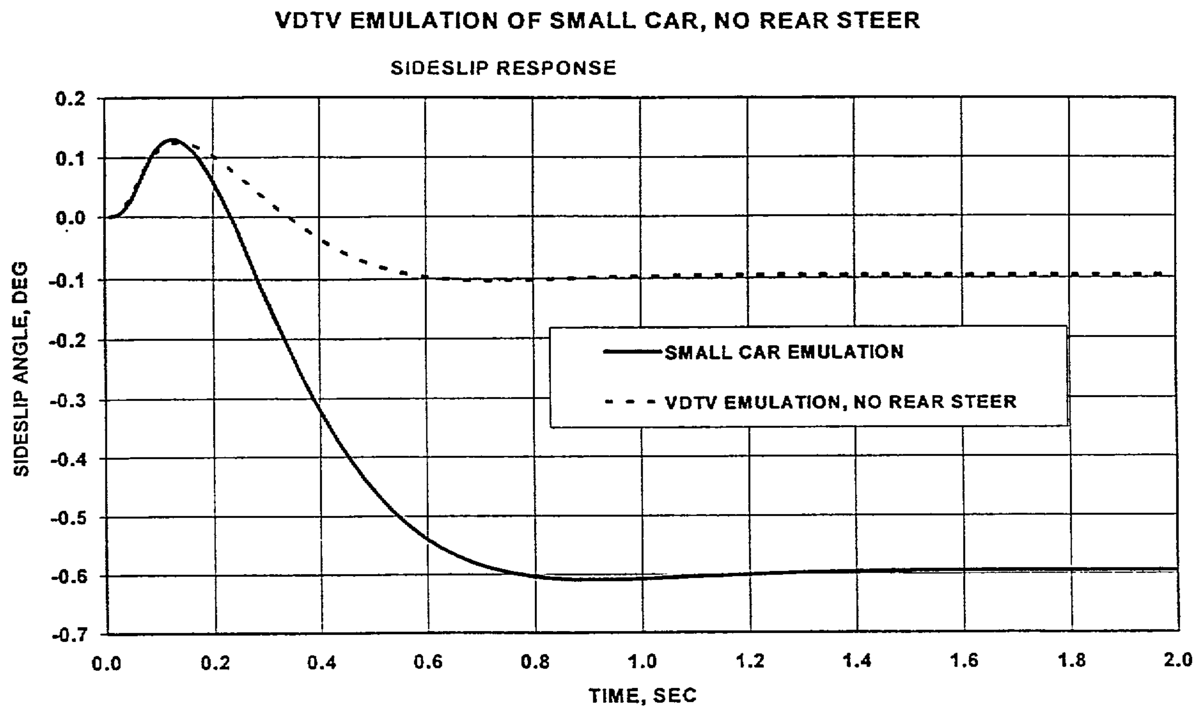

Figure 11c Sideslip Response (Lateral Acceleration Matched for No Rear Steer)

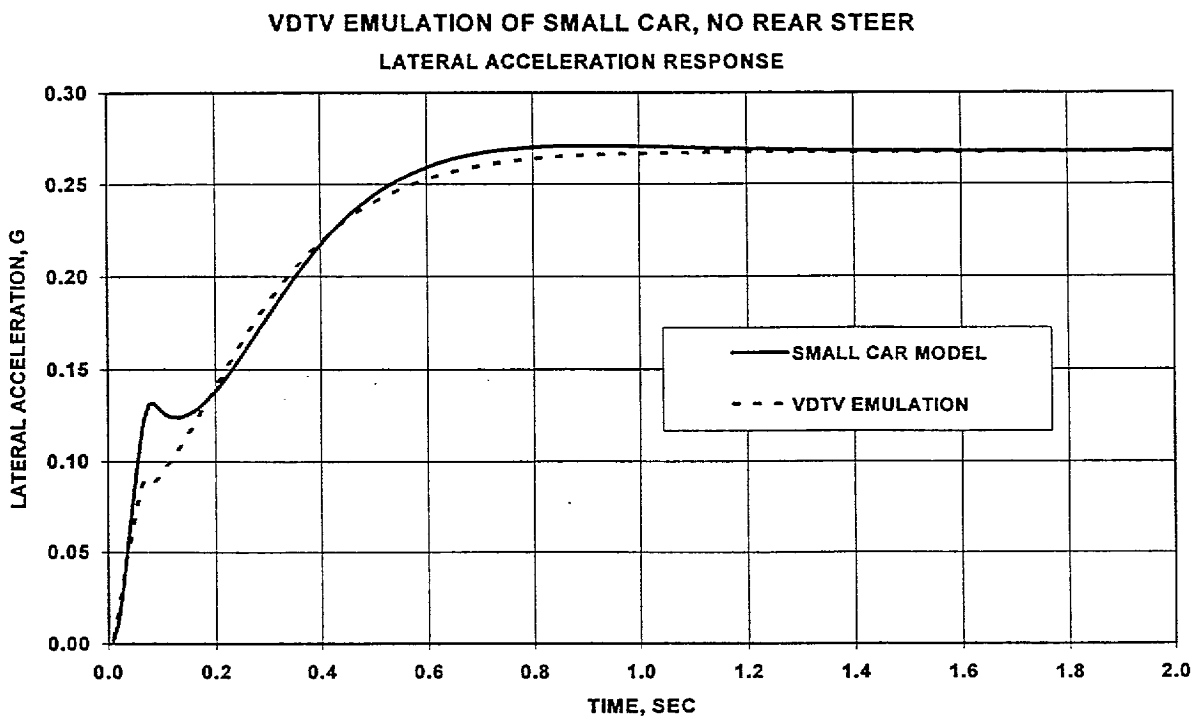

Figure 12a Lateral Acceleration Response (Matched)

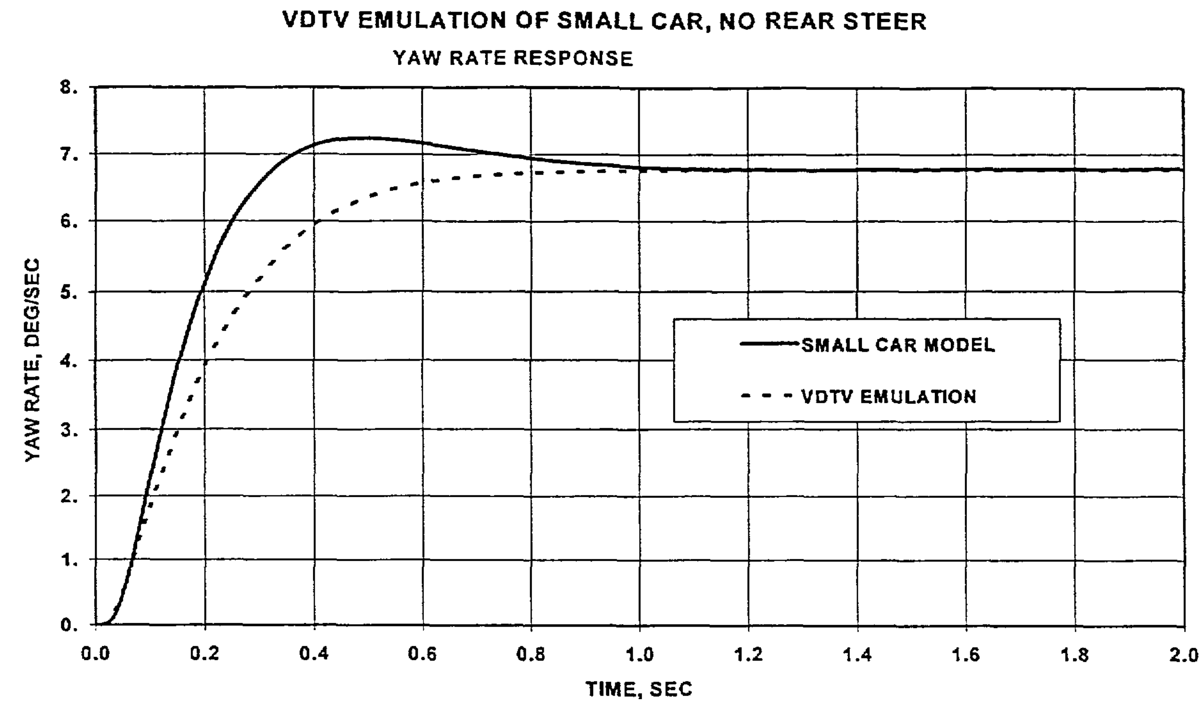

Figure 12b Yaw Rate Response (Lateral Acceleration Response Matched)

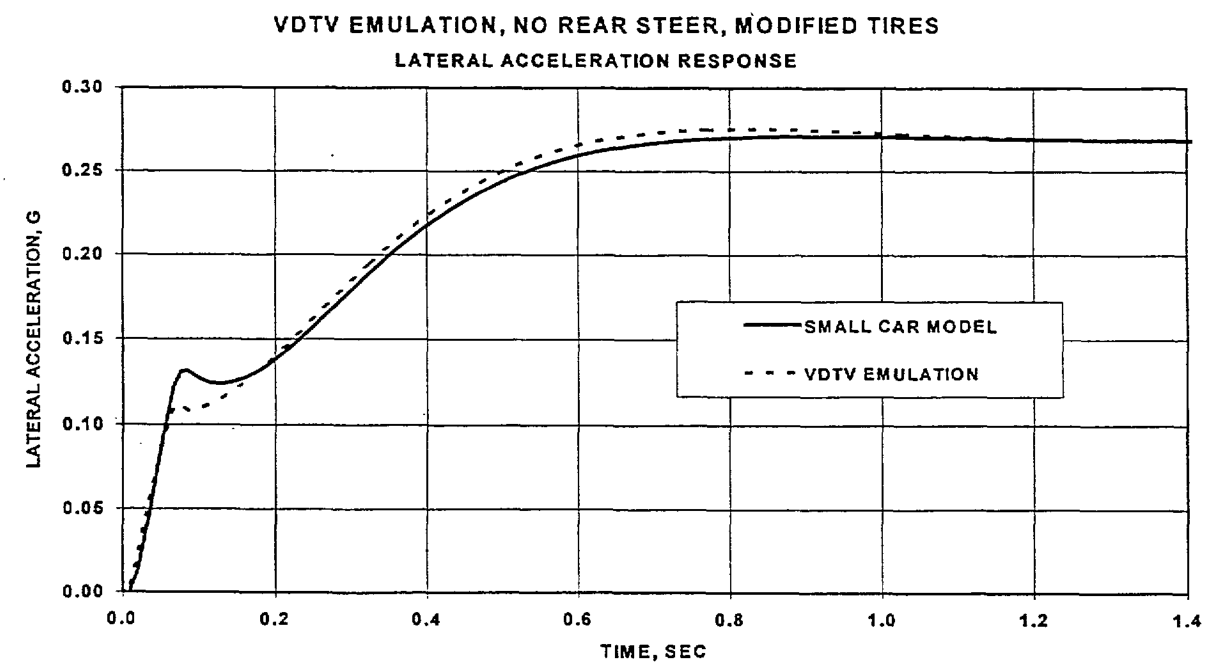

Figure 13a Lateral Acceleration Response, Modified tires

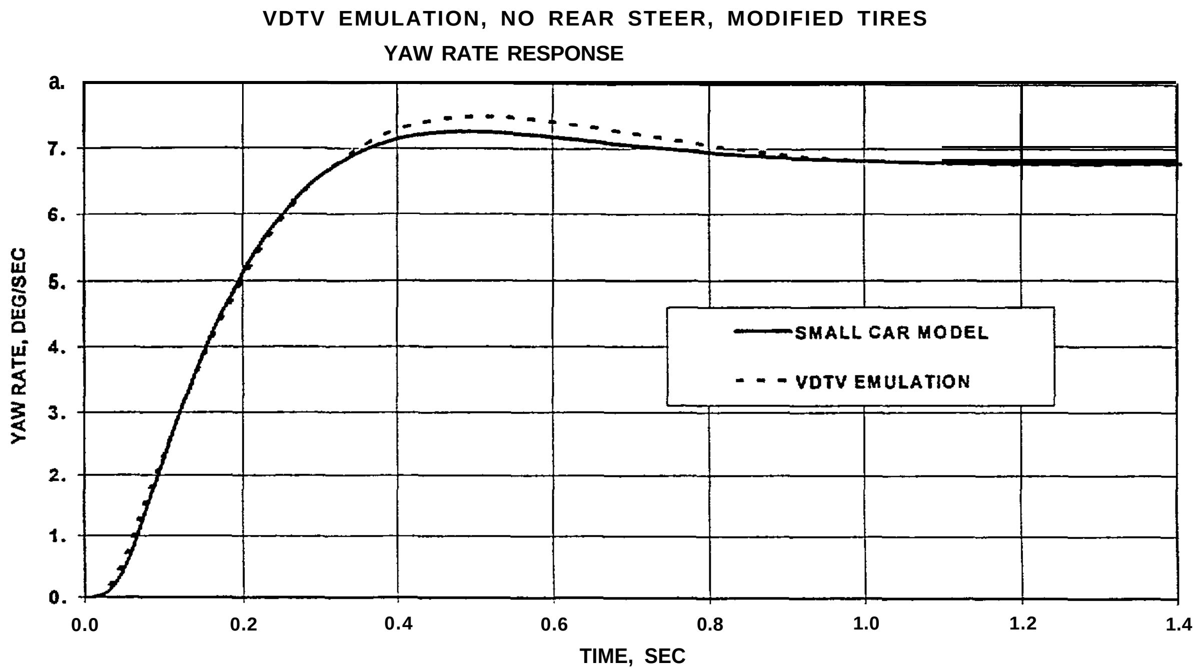

Figure 13b Yaw Rate Response (Modified Tires)

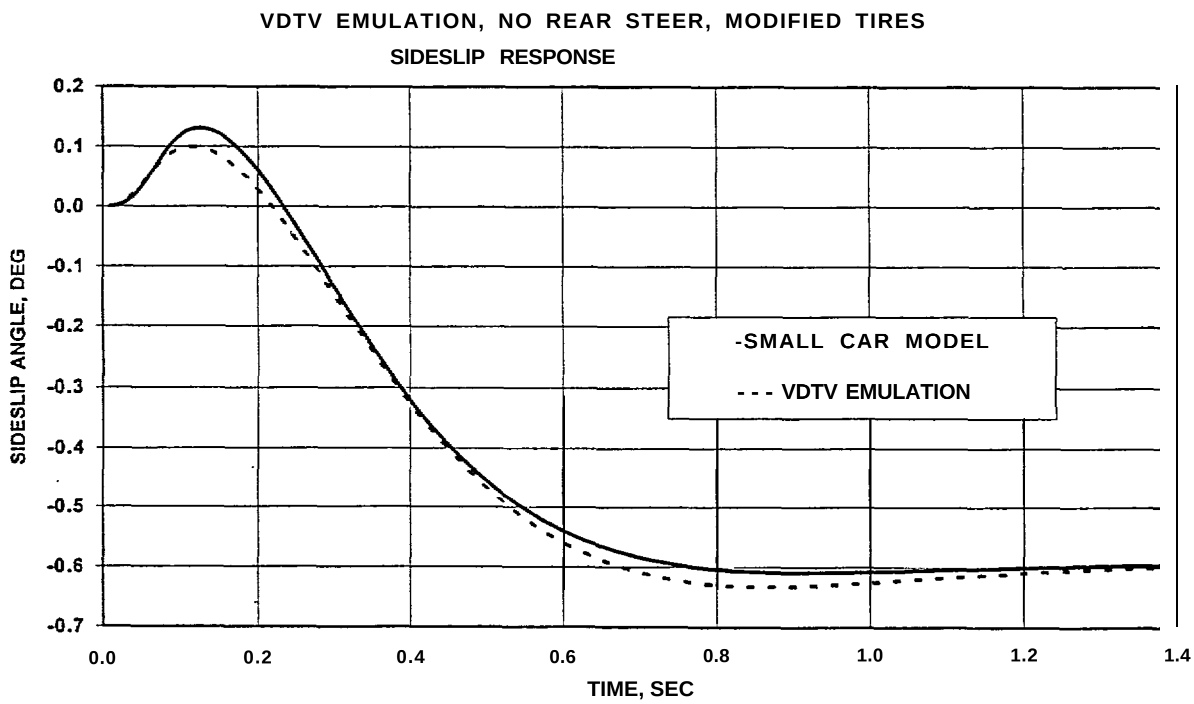

Figure 13c Sideslip Angle Response (Modified Tires)

4. Conclusions

1. We have been able to demonstrate achievement of the requirements on understeer gradient and lateral acceleration rise time specified in exhibit I without using rear steer. Understeer gradient and lateral acceleration rise time can be independently varied, within limits, using only front steer by wire.

2. Yaw rate overshoot can also be varied without using rear steer, but sideslip gradient cannot be varied because it depends on the force/moment characteristics of the rear tires and on vehicle weight and center of gravity position. It cannot be varied by changing front steer gains. However, sideslip gradient can be varied between +7 and +6deg/g using rear steer limited to one degree. The sideslip gradient can be varied by changing the rear tires or by changing rear tire pressures.

3. We have demonstrated that a small car model can be emulated using both front and rear steer. Limited rear steer yields degrades emulation at higher lateral accelerations unless one compensates by programming gains with lateral acceleration. One degree of rear steer provides useful emulation results, up to about 0.5g, while two degrees of rear steer provides emulation up to about 0.6g, both without compensation by gain programming with lateral acceleration.

4. Without rear steer one can emulate either the yaw rate or lateral acceleration responses, but not both simultaneously because the sideslip is not matched. Roll angle is matched only if the lateral acceleration response is matched.

5. Modification of the tires, by changing tires or by changing tire pressures, can produce a match of the sideslip angle response, so that both yaw rate and lateral acceleration can be emulated simultaneously. However, control of tire pressures via laptop computer may be costly and changing of tires obviously cannot be done via laptop computer. However, with a willingness to physically change tires or tire pressures, one could emulate other cars using VDTV without rear steer, provided a certain amount of trial and error gain changes or “calibration” is acceptable.