Center for Multimodal Solutions for Congestion Mitigation
University of Florida

Final Report

to the

CENTER FOR MULTIMODAL SOLUTIONS FOR CONGESTION MITIGATION (CMS)

CMS Project Number: **2010-001**

CMS Project Title: **Novel Approaches for Road Congestion Mitigation**

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ABSTRACT

Transportation planning is usually aiming to solve two problems: the traffic assignment and the toll pricing problems. The latter one utilizes information from the first one, in order to find the optimal set of tolls that is the set of tolls that leads to a user equilibrium solution and that would benefit all travelers-users. This problem is particularly hard, so an evolutionary algorithm is proposed based on the work by Buriol et al. (2005) and Ericsson et al. (2002). Computational results are given to depict the success of our approach.

EXECUTIVE SUMMARY

Transportation planning process mainly involves the solution of two important problems, namely, the traffic assignment problem, which minimizes the total travel delay among all travelers, and the toll pricing problem which settles, based on data derived from the first problem, the tolls that would collectively benefit all travelers and would lead to a user equilibrium solution at the same time. These are challenging computational problems for large scale transportation networks. In this research, we proposed an approach to solve the two problems jointly, making use of a Hybrid Genetic Algorithm (HGA) for the optimization of transportation network performance by strategically allocating tolls on some of the links. Since a regular transportation network may have thousands of intersections and hundreds of roads, the proposed algorithm took advantage of a series of mechanisms for speeding up shortest path algorithms.

In this project, we have adapted the evolutionary algorithm from Buriol et al. (2005) and Ericsson et al. (2002) to a transportation problem that deals with multicommodity flow and toll pricing. We have also formulated the equivalent nonlinear programming problem. We have developed several variations of genetic algorithms (GA) specifically targeted to the transportation problem. Initially, we started off with an experimental prototype genetic algorithm designed and developed for this purpose. Our preliminary results were promising to encourage us to pursue research in this direction. By means of computing a solution that minimizes the mean delay of the system, we have dealt with both System Optimum (SO) and User Equilibrium (UE) problems simultaneously. As we apply a heuristic to solve this problem, there is no guarantee that the system optimal solution is achieved. Instead, an efficient solution for the overall transportation system was obtained. We have validated the quality of our solutions from the heuristics through well-known benchmark instances and simulations as shown in Section 4. We had some preliminary results from a simple genetic algorithm that we had developed for this purpose through which we were able to confirm an optimality gap of less than 10%. Solutions for large and real time instances have been tested and presented to show the ability of the GA and HGA algorithms to deal with large instances. Our preliminary ideas and computational results with small instances were presented and awarded the ”Roberto D. Galvao Prize” best paper award (”A hybrid genetic algorithm for road congestion minimization”) in the XLI Symposium of the Brazilian Operational Research Society, September, 2009.

We have established optimal routes for the multicommodity flow problem with user equilibrium constraints and also obtained optimal toll pricing strategies. In addition, we obtained reasonable solutions from the heuristic when compared with the optimal solution. We, finally, designed two algorithms for this problem and

compared the performance of the algorithms and validated the results through simulations.

PROBLEM FORMULATION

Given a transportation network and a series of traffic flow demands, tolls can be levied. The final objective is to reach a User Equilibrium (UE) solution for the least-cost multicommodity flow problem, which is also optimal for the overall system. In order to setup the mathematical framework, it can be assumed that we have a directed network $G=(N,A)$ where N is the set of nodes and A the set of arcs of the network. For each of the arcs of the graph, we define c_a to be the associated capacity and Φ_a the cost, which in turn is depending on the load of the arc, l_a, the time re- quired to traverse the arc, t_a, its power, n_a, and cost, Γ_a. Further, we assume that Φ_a is nonlinear (as has been shown by the functions that usually describe real-world traffic networks), yet convex and strictly increasing.

In addition to the above, the set $K \subseteq N \times N$ is defined to represent the set of commodities (origin-destination pairs), having $o(k)$ and $d(k)$ as origin and destination nodes. Each commodity also had an associated flow demand, d_k and let x_a^k to be the contribution of commodity k on arc α. To sum up, the following formulation can be applied:

$$\min \quad \Phi = \frac{\sum_{\alpha \in A} l_\alpha t_\alpha [1 + \Gamma_\alpha (\frac{l_\alpha}{c_\alpha})^{n_\alpha}]}{\sum_{k \in \hat{K}} d_k} \tag{1}$$

$$s.t. \quad l_\alpha = \sum_{k \in \hat{K}} x_\alpha^k \quad \forall \alpha \in A \tag{2}$$

$$\sum_{i:(j,i) \in A} x_{(j,i)}^k - \sum_{i:(i,j) \in A} x_{(i,j)}^k = \begin{cases} -d_k & if\ j = d(k) \\ d_k & if\ j = o(k) \\ 0 & otherwise \end{cases} \tag{3}$$

$$x_\alpha^k \geq 0 \quad \forall \alpha \in A, \quad \forall k \in \hat{K} \tag{4}$$

The objective function describes the mean delay time for the overall system, based on the Bureau of Public Roads function for computing travel costs. Our goal is to obtain a toll allocation scheme such that the final value of Φ is minimized while we obtain a system efficient solution. In the above formulation, the term $\frac{l_a}{c_a}$ represents the overall utilization of arc α.

ALGORITHM IMPLEMENTATIONS

GENETIC ALGORITHM FRAMEWORK

For the solution of the problem defined earlier, the Genetic Algorithm (GA) framework was used. Initially genetic algorithms were successfully applied to a series of network problems. Most notably, the Open Shortest Path First Internet routing problem was tackled by Ericsson et al. (2002) followed by Buriol et al. (2005) who also provided a series of local improvements by employing a Hybrid Genetic Algorithm (HGA).

The idea is to apply the latter algorithm with the proper alterations in order to fully represent a traffic network. In general, let us begin by describing a genetic algorithm. It is a population-based metaheuristic and it is mainly used for combinatorial optimization problems because of the quality of solutions it can provide us with. The population represents the feasible solutions, which are combined through mutations and crossovers to produce a new generation of solutions. When solutions are combined to form a new generation, the higher quality attributes are more likely to be passed along. The algorithm is terminated when no improvement is noted in the offsprings.

Now, let us explore how this framework can be applied to the toll booth problem.Each solution is represented by two arrays w and b. Array w stores the integer arc weights, while b is a binary array indicating the set of tolls. An arc α of the network has weight equal to $w_a$in case $b_a = true$ and zero in case $b_a = false$. Each individual weight belongs to the interval $[1, w_{\max}]$. A solution w defines a total flow $l_a, a \in A$ by means of an equal-cost multipath routing. In OSPF routing, there is no link weight equal to zero, and the shortest path is the one with the shortest distance. In our implementation, non tolled links are considered to have weight zero. Moreover, two paths are considered of equal cost if they have the same total distance and the same number of hops. In case they have the same total distance, but different hop counts, the shortest path is considered the one with less hops. Each demand is routed forward to its destination. Traffic at intermediate nodes is split equally among all outgoing links on shortest paths to the destination. After the flow is defined, the solution is associated with a fitness value defined by the objective function Φ.

Initially, the population is randomly generated. The arc weighs are selected uniformly in $[1, \frac{w_{\max}}{3}]$ and a number of K links is set to have a toll booth. Then, the population can be partitioned in 3 sets A, B, C, where A contains the best solutions, while C the worst ones. Then, the solutions belonging to B are replaced by offsprings produced by crossover between solutions in A and $B \cup C$, using the Bean (1994) random keys crossover scheme. Finally, all solutions in C by new random solutions with arc weights in the interval $[1, w_{\max}]$.

Unlike Bean (1994) we use a biased random-keys scheme, where each pair consists of an elite and a non-elite parent. The elite parent is chosen at random from A, while the non-elite one from the set $B \cup C$. Each weight in the array w is inherited from one of the parents or reset by mutation with mutation probability p_m.

If a mutation does not occur, then the values are inherited by the elite parent with probability $p_A > \frac{1}{2}$. As far as array b is concerned the following rules apply:

- b_i is true if both parents' values are true;
- 50% of the positions b_i, chosen at random where only one of the parents has the position equal to true, are set to true;
- all other positions are set to false.

SOLUTION EVALUATION

The procedure for evaluating a solution is presented in Figure 1. For the system update as far as new loads are concerned, the shortest paths are computed using the well known Djikstra's shortest path algorithm described in Ahuja et al. (1993) with a small alteration. With the alteration, two paths are considered of equal cost if they have the same total distance/cost and the same number of hops.

```
procedure EvaluateSolution(w, l, f)
    forall a ∈ A do μ_a = 1;
    forall t ∈ T do
        π^t ← ReverseDijkstra(w);
        g^t ← ComputeSPG(w, π^t);
        δ^t ← ComputeDelta(g^t);
        L^t ← ComputePartialLoads(μ, δ, π, g^t);
    end forall
    l ← ComputeTotalLoads(L);
    S ← UpdateMultiandDelta();
    if |S| > 0 UpdateSolution();
    forall a ∈ A if l_a = 0 then μ_a = 0;
    f ← Σ_{a∈A} μ_a;
    return(f, μ);
end procedure
```

Figure 1: Pseudo-code for solution evaluation

LOCAL IMPROVEMENT

Beginning from a given solution, we analyze solutions in the neighborhood of the solution w searching for an improving solution, i.e. with a smaller cost. If there exists such a solution then it is replaced. Otherwise the current solution is considered to be a local minimum. We incorporate the local improvement procedure in the Genetic Algorithm framework in order to further enhance it and be able to detect better quality solutions with less computational effort. Local improvement is more specifically applied to all solutions obtained by crossover so as to further improve them. However, using the whole neighborhood for our search can be computationally demanding and not as efficient. So, we devised a reduced neighborhood search scheme which is described below.

Once more, let lα denote the total arc load on $a \in A$ in the solution w. Note that $\Phi_a(l_a)$ is the total routing cost on arc α. Then, the local improvement scheme only deals with a subset of the arc weights, which have the highest routing cost while their weight remains less than $w_{\max}$. If the selected arc has a toll booth installed then we increase its weight in an attempt to reduce the load. If there is not booth installed, then we proceed to introduce one with an initial weight equal to one, while a toll is removed from another arc. For more details, please see Figure 2.

```
procedure LocalImprovement(g, w, b)
    i ← 1;
    while i ≤ g do
        Renumber the arc indexes such that
            Φ_a(l_a) ≥ Φ_{a+1}(l_{a+1}); ∀a = 1, ..., |A| − 1;
        a' ← 1; flag ← F;
        if b_{a'} = F
            b_{a'} ← T
            w_{a'} ← 1
            flag ← V
        end if
        for ŵ = w_{a'} + 1, ..., w_{a'} + ⌈(w_max − w_{a'})/4⌉ do
            w'_a ← w_a, ∀a ∈ A, a ≠ a';
            w'_{a'} ← ŵ;
            if Ψ_{w',b'} < Ψ_{w,b} then
                w ← w';
            end if
        end for
        if flag then
            for ten a'' ∈ R do
                b_{a''} ← F;
                if Ψ_{w',b'} < Ψ_{w,b} then break
            end for
        end if
        if notImproved(w, w'') then restoreOriginalSol(a', a'', w'')
        i ← i + 1;
    end while
end LocalImprovement.
```

Figure 2: Pseudo-code for the local improvement scheme

UPDATING SCHEME

Let us denote now by $G^t = (N, A^t)$ the graph of all the shortest paths associated with each destination node $t \in T$. The weight of each arc α′ is altered and that causes the graph to be recomputed from the start. In our case, we only update the part of it which was mostly affected by the weight change. We only have to worry for nonnegative weights.

Hence, when a toll is levied in an arc or a toll is removed from an arc, we use the dynamic shortest paths shown in Buriol et al. (2005) to dynamically update the shortest paths graph instead of reconstructing it from scratch. In order to avoid having cycles of zero cost, we alter the distance computed by adding $\frac{1}{|E|}$ to each arc traversed. Then, we have a clear view of which path has the least hops just by comparing the distance values. Finally, if more than one paths have the same cost, only the one that uses the least number of hops will remain.

The loads are also updated in their turn, instead of being recomputed, using the same algorithmic approach. Only the paths whose loads were affected are updated at each iteration.

COMPUTATIONAL RESULTS

THE NINE-NODE EXAMPLE

In this section, to prove the efficiency of the algorithm devised, we compare it to the MINTB approach in the nine node problem discussed in Hearn and Ramana (1988). Figure 3 presents the optimality gap obtained for this problem. The optimal objective function value as obtained by Hearn and Ramana (1988) is 22.59314. Clearly, our approach does not produce the optimal solution. However this is to be expected, since in small networks the system optimal solution obtained by our Hybrid Genetic Algorithm (and any Genetic Algorithm, in fact) can deviate significantly.

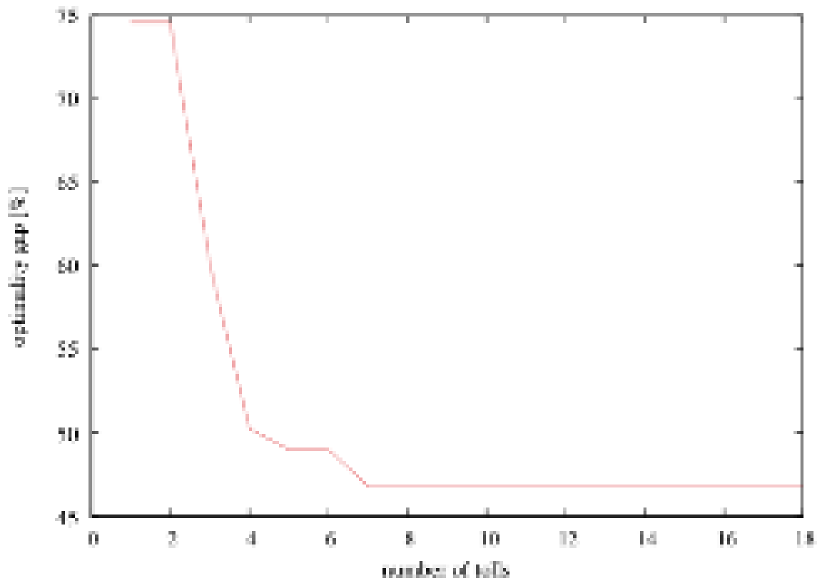

Figure 3: Number of tolls vs optimality gap for the nine node example

LARGE SCALE REALISTIC APPLICATIONS

More examples in realistic problems were tackled with our approach including the well known benchmark problems Sioux Falls, North Dakota (LeBlanc et al., 1975). In this case the delay function in each of the arcs is

$$\Phi_a = \sum_{a \in A} l_a t_a [1 + \beta_a (\frac{l_a}{c_a})^4]$$

More instances were also studied, such as Stockholm, Winnipeg and Barcelona, with similar delay functions. All the attributes (which are known) are presented below in Table 1. Also, in Figures 4-7, we present the optimality gap of our heuristic approach. At first, we implemented a solver for the problems based on cvxopt (Dahl and Vandeberghe, 2005) and obtained the solutions in Table 2.

Table 1: Attributes of the benchmark problems

INSTANCE	VERTICES	ARCS	OD PAIRS	DESTINATIONS
Sioux Falls	24	76	528	24
Stockholm	416	962	1623	45
Barcelona	1020	2522	7922	108
Winnipeg	1052	2836	4345	138

Table 2: Solutions obtained with cvxopt

INSTANCE	OPTIMAL OBJECTIVE FUNCTION VALUE	TIME (in seconds)
Nine Node Problem	22.539181	<1
Sioux Falls	19.950794	22
Stockholm	N/A	86400

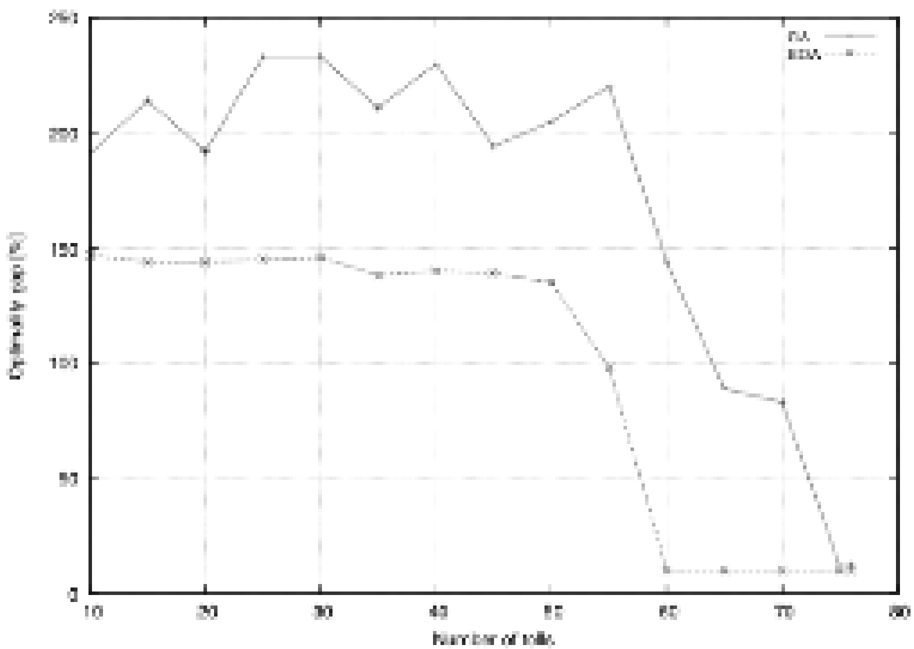

Figure 4: Quality of solutions obtained by GA and HGA in the *Sioux Falls* problem

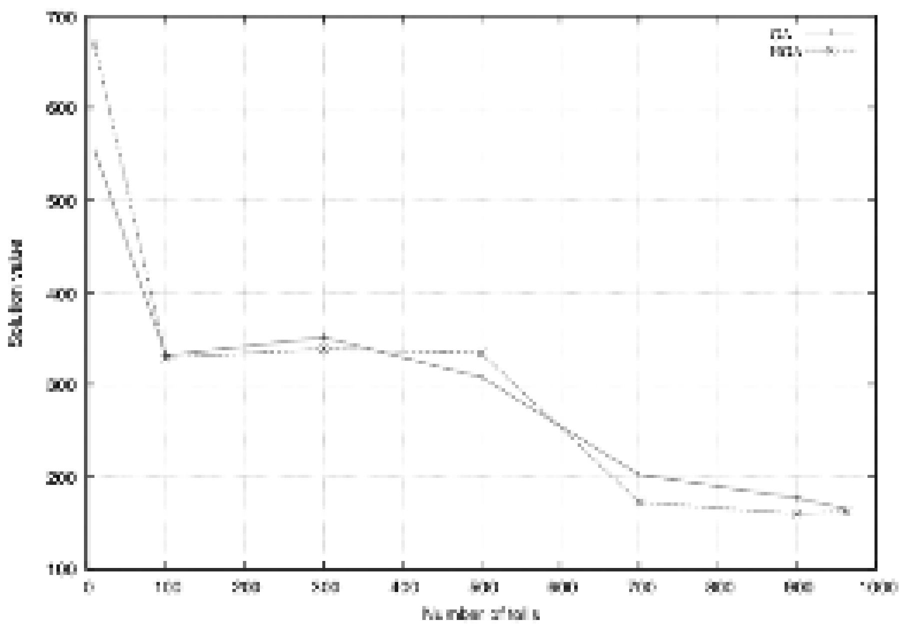

Figure 5: Quality of solutions obtained by GA and HGA for the *Stockholm* problem

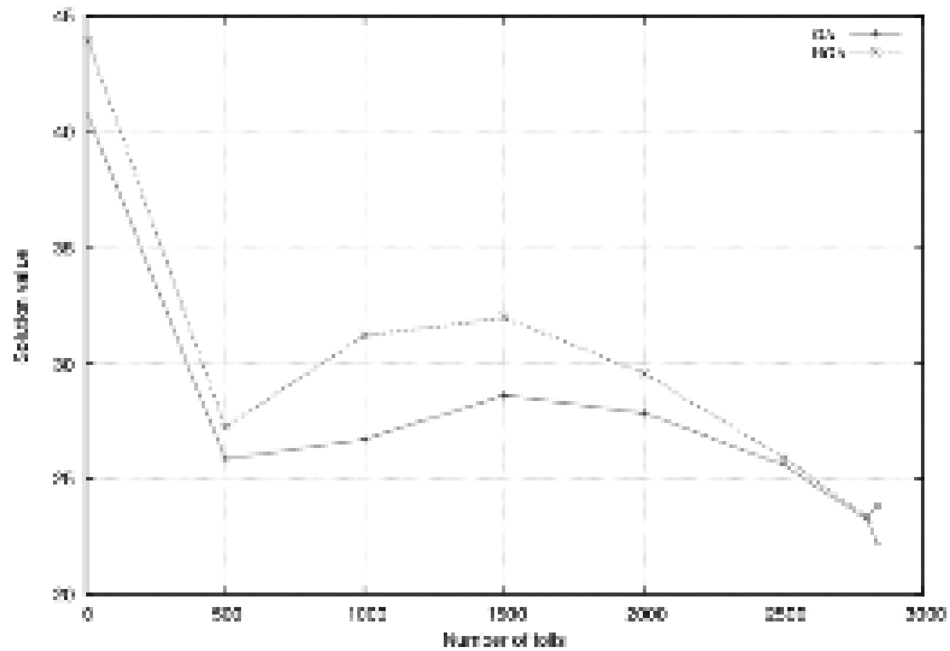

Figure 6: Quality of solutions obtained by GA and HGA for the *Winnipeg* problem

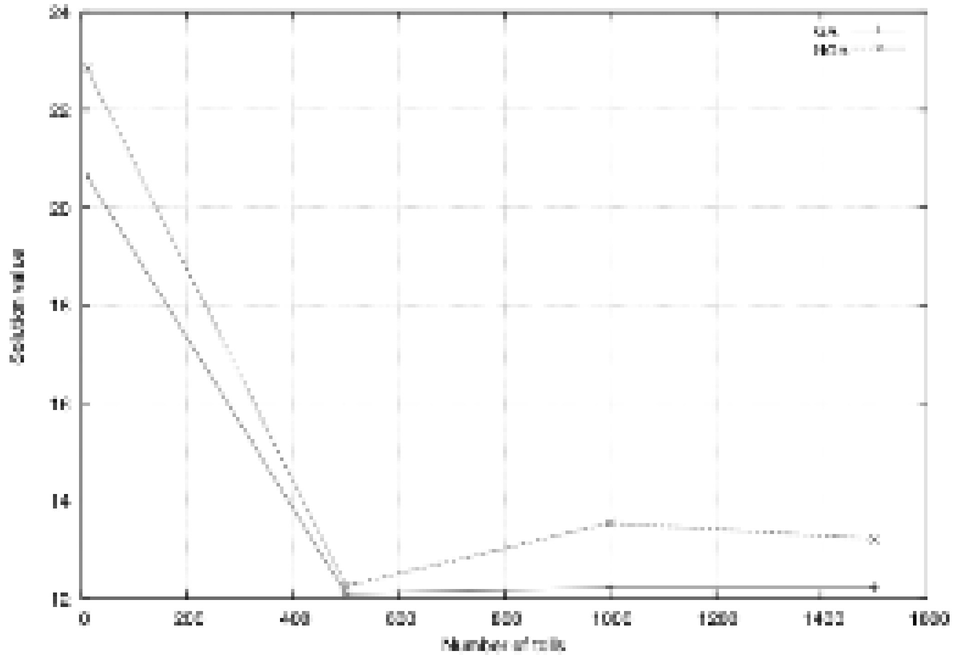

Figure 7: Quality of solutions obtained by GA and HGA for the *Barcelona* problem

PUBLICATIONS AND SUCCESS INDICATORS

PUBLICATIONS

The project awarded has led to a series of publications and presentations. The following is a list of the most important ones:

1. Publications

    - *A hybrid genetic algorithm for road congestion minimization,* Buriol, L.S., Hirsch, M.J., Pardalos, P.M., Querido, T., Resende, M.G.C., and Ritt, M. Proceedings of the XLI Simposio Brasileiro de Pesquisa Operacional, pp 2515—2526, Citeseer (2009). **Awarded the Roberto D. Galvao Prize for best paper.**
    - *A biased random-key genetic algorithm for road congestion minimization,* Buriol, L.S., Hirsch, M.J., Pardalos, P.M., Querido, T., Resende, M.G.C., and Ritt, M. Optimization Letters, pp 1—15, Springer (2010).

2. Presentations

    - *A hybrid genetic algortihm for road congestion minimization,* Instituto de Informatica, Universidade Federal do Rio Grande do Sul, Buriol, L.S., Hirsch, M.J., Pardalos, P.M., Querido, T., Resende, M.G.C., and Ritt, M., Citeseer (2010).

STUDENTS

During the project, two students from the Center for Applied Optimization were involved in CMS funded projects.

1. **Qipeng P. Zheng**
   Graduated with a PhD in Summer 2010. Current Affiliation:
   Assistant Professor, Dept. of Industrial and Management Systems Engineering, University of West Virginia
2. **Chrysafis Vogiatzis**
   PhD Student, Dept. of Industrial and Systems Engineering, University of Florida. Expected graduation date: Fall 2013.

CONCLUSIONS AND FUTURE RESEARCH

At the moment, further refinement of the code of the algorithm is being performed. It has been deemed necessary to further enhance the programming framework in order to improve efficiency and, consequently, make the algorithmic approach less computationally expensive.

In addition to that, a new publication is under way, soon to appear in the Journal Optimization Letters by Springer. In this publication, the computational results have been improved along with the theoretical aspects of the genetic algorithm framework used, which are investigated in more depth.

As for future research, the project still remains an interesting and fascinating field with numerous approaches that can be adopted and implemented. It is our goal to attempt an approach using the Continuous Genetic Random Adaptive Search Procedure (C-GRASP) (Hirsch et al., 2007), which has been noted to offer a series of good results after its enhancements in speed (Hirsch et al., 2009) such as the toll booth problem tackled herein.

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