## EMDOT



# High Skew Link Slab Bridge System with Deck Sliding over Backwall or Backwall Sliding over Abutments 

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Appendices


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# High Skew Link Slab Bridge System with Deck Sliding over Backwall or Backwall Sliding over Abutments <br> (Appendices) 

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## APPENDIX A - ACRONYMS AND ABBREVIATIONS

AASHTO - American Association of State Highway and Transportation Officials
AASHTO LRFD - American Association of State Highway and Transportation Officials
Load and Resistant Factor Design
CDP - Cotton duck pads
DOT - Department of Transportation
EPS - Expanded polystyrene
EVA - Ethylene vinyl acetate (commonly known as expanded rubber or foam rubber)
FE - Finite element
FHWA - Federal Highway Administration
FRP - Fiberglass-reinforced pad
MDOT - Michigan Department of Transportation
NCDOT - North Carolina Department of Transportation
NTG - Negative Temperature Gradient
OMOT - Ontario Ministry of Transportation
PC - Prestressed Concrete PCI
PEP - Plain elastomeric pad
PTFE - Polytetrafluorethylene
PTG - Positive Temperature Gradient
ROFP - Random oriented fiber pads
SHA - State Highway Agencies
SREB - Steel-reinforced elastomeric bearings
SREP - Steel-reinforced elastomeric pads
VDOT - Virginia Department of Transportation

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## APPENDIX B

Table B-1. Longitudinal Bearing Translation over South Abutment (in.) - Loading Scenario I

| Girder Label | FE Analysis | Tracker |
| :---: | :---: | :---: |
| A | -0.044 | 0.041 |
| B | -0.042 | - |
| C | -0.034 | 0.027 |
| D | -0.025 | - |
| E | -0.020 | 0.018 |
| F | -0.018 | - |
| G | -0.016 | 0.018 |

Table B-2. Girder Translations - Loading Scenario I

| Measurement <br> Point | FE Analysis (in.) $^{+}$ |  |  | ${\text { Tracker Measurement (in. })^{++}}_{$$}$ Longitudinal $^{\text {L1 }}$ Transverse |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Longitudinal | Transverse | Vertical |  |  |  |
| R1 | -0.016 | 0.024 | -0.004 | 0.008 | -0.007 | 0.001 |
| R2 | -0.019 | 0.047 | -0.091 | 0.012 | -0.031 | 0.060 |
| R3 | -0.026 | 0.062 | -0.306 | 0.023 | -0.040 | 0.194 |
| R4 | -0.015 | 0.026 | -0.018 | 0.009 | -0.010 | 0 |
| R5 | -0.015 | 0.034 | -0.070 | 0.009 | -0.020 | 0.036 |
| R6 | -0.010 | 0.058 | -0.186 | 0.003 | -0.041 | 0.115 |
| R7 | -0.003 | 0.067 | -0.372 | 0.001 | -0.044 | 0.242 |
| R8 | -0.013 | 0.016 | -0.032 | 0.007 | -0.007 | 0 |
| R9 | -0.010 | 0.020 | -0.066 | 0.005 | -0.012 | 0.025 |
| R10 | -0.003 | 0.031 | -0.141 | -0.004 | -0.022 | 0.082 |
| R11 | 0.012 | 0.043 | -0.277 | -0.016 | -0.036 | 0.188 |
| R12 | -0.009 | 0.002 | -0.032 | 0.006 | -0.002 | -0.003 |
| R13 | -0.003 | 0.003 | -0.049 | 0 | -0.003 | 0.016 |
| R14 | 0.013 | 0.006 | -0.110 | -0.021 | -0.009 | 0.084 |

+ Refer FE model coordinates (Figure 3-14)
++ Refer Tracker measurement coordinates (Figure 3-32)

Table B-3. Longitudinal Bearing Translation over South Abutment (in.) - Loading Scenario II

| Girder Label | FE Analysis | Tracker |
| :---: | :---: | :---: |
| A | -0.086 | 0.088 |
| B | -0.083 | - |
| C | -0.071 | 0.065 |
| D | -0.056 | - |
| E | -0.047 | 0.045 |
| F | -0.041 | - |
| G | -0.036 | 0.040 |

Table B-4. Girder Translations - Loading Scenario II

| Measurement <br> Point | FE Analysis (in.) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Longitudinal | Transverse | Vertical | Tongitudinal |  |  |
| Transverse | Vertical |  |  |  |  |  |
| R1 | -0.037 | 0.062 | -0.007 | 0.032 | -0.029 | 0.014 |
| R2 | -0.046 | 0.103 | -0.199 | 0.038 | -0.071 | 0.163 |
| R3 | -0.062 | 0.122 | -0.571 | 0.060 | -0.080 | 0.406 |
| R4 | -0.035 | 0.068 | -0.037 | 0.029 | -0.033 | 0.021 |
| R5 | -0.035 | 0.085 | -0.161 | 0.027 | -0.053 | 0.105 |
| R6 | -0.029 | 0.135 | -0.414 | 0.021 | -0.096 | 0.298 |
| R7 | -0.023 | 0.142 | -0.762 | 0.021 | -0.090 | 0.528 |
| R8 | -0.030 | 0.042 | -0.070 | 0.026 | -0.023 | 0.011 |
| R9 | -0.025 | 0.053 | -0.154 | 0.017 | -0.035 | 0.074 |
| R10 | -0.010 | 0.083 | -0.336 | -0.001 | -0.057 | 0.227 |
| R11 | 0.015 | 0.099 | -0.637 | -0.018 | -0.073 | 0.452 |
| R12 | -0.020 | 0.006 | -0.072 | 0.018 | -0.006 | -0.002 |
| R13 | -0.009 | 0.012 | -0.117 | 0.004 | -0.007 | 0.046 |
| R14 | 0.027 | 0.019 | -0.274 | -0.040 | -0.018 | 0.204 |

+ Refer FE model coordinates (Figure 3-14)
++ Refer Tracker measurement coordinates (Figure 3-32)

Table B-5. Longitudinal Bearing Translation over South Abutment (in.) - Loading Scenario III

| Girder Label | FE Analysis | Tracker |
| :---: | :---: | :---: |
| A | -0.014 | 0.073 |
| B | -0.018 | - |
| C | -0.024 | 0.070 |
| D | -0.033 | - |
| E | -0.046 | 0.068 |
| F | -0.063 | - |
| G | -0.086 | 0.083 |

Table B-6. Girder Translations - Loading Scenario III

| Measurement <br> Point | FE Analysis (in.) ${ }^{+}$ |  |  | Tracker Measurement (in.) ${ }^{++}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Longitudinal | Transverse | Vertical | Longitudinal | Transverse | Vertical |
| R1 | -0.072 | -0.056 | -0.344 | 0.072 | 0.015 | 0.171 |
| R2 | -0.028 | -0.050 | -0.125 | 0.054 | 0.012 | 0.084 |
| R3 | -0.011 | -0.037 | -0.015 | 0.053 | 0.003 | 0.051 |
| R4 | -0.045 | -0.090 | -0.528 | 0.046 | 0.041 | 0.239 |
| R5 | -0.028 | -0.092 | -0.286 | 0.042 | 0.040 | 0.129 |
| R6 | -0.016 | -0.064 | -0.104 | 0.043 | 0.016 | 0.054 |
| R7 | -0.010 | -0.050 | -0.015 | 0.046 | 0.006 | 0.035 |
| R8 | -0.016 | -0.082 | -0.482 | 0.024 | 0.043 | 0.192 |
| R9 | -0.012 | -0.085 | -0.260 | 0.027 | 0.043 | 0.081 |
| R10 | -0.011 | -0.061 | -0.101 | 0.035 | 0.023 | 0.011 |
| R11 | -0.008 | -0.049 | -0.012 | 0.037 | 0.007 | -0.005 |
| R12 | -0.001 | -0.043 | -0.274 | 0.006 | 0.035 | 0.053 |
| R13 | -0.004 | -0.039 | -0.092 | 0.023 | 0.027 | 0 |
| R14 | -0.005 | -0.027 | -0.009 | 0.021 | 0.009 | -0.035 |

+ Refer FE model coordinates (Figure 3-14)
++ Refer Tracker measurement coordinates (Figure 3-32)

Table B-7. Longitudinal Bearing Translation over South Abutment (in.) - Loading Scenario IV

| Girder Label | FE Analysis | Tracker |
| :---: | :---: | :---: |
| A | -0.030 | 0.103 |
| B | -0.037 | - |
| C | -0.048 | 0.102 |
| D | -0.064 | - |
| E | -0.085 | 0.113 |
| F | -0.113 | - |
| G | -0.152 | 0.149 |

Table B-8. Girder Translations - Loading Scenario IV

| Measurement <br> Point | FE Analysis (in.) ${ }^{+}$ |  |  | Tracker Measurement (in.) ${ }^{++}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Longitudinal | Transverse | Vertical | Longitudinal | Transverse | Vertical |
| R1 | -0.132 | -0.088 | -0.565 | 0.139 | 0.032 | 0.325 |
| R2 | -0.055 | -0.077 | -0.220 | 0.099 | 0.026 | 0.170 |
| R3 | -0.024 | -0.063 | -0.041 | 0.090 | 0.014 | 0.135 |
| R4 | -0.093 | -0.148 | -0.903 | 0.098 | 0.073 | 0.456 |
| R5 | -0.057 | -0.145 | -0.494 | 0.089 | 0.072 | 0.241 |
| R6 | -0.034 | -0.102 | -0.197 | 0.081 | 0.035 | 0.111 |
| R7 | -0.020 | -0.081 | -0.048 | 0.079 | 0.019 | 0.076 |
| R8 | -0.044 | -0.143 | -0.891 | 0.049 | 0.075 | 0.385 |
| R9 | -0.031 | -0.147 | -0.476 | 0.056 | 0.085 | 0.177 |
| R10 | -0.023 | -0.100 | -0.192 | 0.059 | 0.044 | 0.045 |
| R11 | -0.016 | -0.079 | -0.042 | 0.060 | 0.020 | 0.010 |
| R12 | -0.008 | -0.080 | -0.544 | 0.014 | 0.064 | 0.141 |
| R13 | -0.010 | -0.074 | -0.175 | 0.043 | 0.050 | -0.033 |
| R14 | -0.010 | -0.044 | -0.028 | 0.046 | 0.014 | -0.070 |

[^0]++ Refer Tracker measurement coordinates (Figure 3-32)

## APPENDIX C <br> DESIGN PROCEDURE FOR LINK SLABS

## OVERVIEW

AASHTO LRFD (2010) requires combined live and thermal load effects for the service limit state design. The Design Procedure described in the appendix will follow the rationale developed by Ulku et al. (2009). Link slab design moments are calculated using the girder end rotations. HL-93 loading is used to calculate the girder end rotations under live load. Girder end rotations caused by the temperature gradient are calculated using the procedure described by Saadeghvaziri and Hadidi (2002) by ensuring strain and curvature compatibility among sections and reinforcements.

One major improvement in the process presented in this appendix compared to what is given in Ulku et al. (2009) is the inclusion of 3D and skew effects to calculate the resultant link slab design moments and forces.

In order to apply loading, the first step is to establish a composite girder-deck crosssection with an effective width as per AASHTO LRFD (2010) Section 4.6.2.6, the composite moment of inertia, and the modulus of elasticity for concrete.

## Girder End Rotations due to Live Load

AASHTO LRFD (2010) procedures can be followed without considering the effects of the link slab.

- Apply HL-93 loading [HS-20 truck with impact and distribution factor (LRFD section 3.6.2.1 and 4.6.2.2.2) $+0.64 \mathrm{kips} / \mathrm{ft}$ lane loading (LRFD 3.6.1.2.4)] on the simply supported spans to compute maximum girder end rotations.


## Girder End Rotations due to Temperature Gradient

Girder end rotations caused by the temperature gradient are calculated following the procedure described by Saadeghvaziri and Hadidi (2002).

The girder-deck composite cross-section is subjected to the temperature gradient as described in AASHTO LRFD section 3.12.3 (Figure C-1).

Figure C-2 illustrates the compatibility forces and moments developed in the sections and the temperature gradient profile along the cross-section height.


Figure C-1. Temperature profile along cross-section


Figure C-2. Compatibility forces and moments and temperature profile along cross-section height

## Strain Compatibility

For strain compatibility between sections 1 and 2 (ignoring reinforcement contribution);

$$
\begin{align*}
& \varepsilon_{\text {Bottom } 1}=\alpha_{1}\left(T_{2}\right)+\frac{M_{1}}{E_{1} S_{b 1}}+\frac{F_{1}}{E_{1} A_{1}}+\frac{F_{1} d_{b 1}}{E_{1} S_{b 1}}=\varepsilon_{\text {Top } 2} \\
& \varepsilon_{\text {Top } 2}=\alpha_{2}\left(T_{2}\right)+\frac{M_{2}-M_{1}}{E_{2} S_{t 2}}+\frac{F_{2}-F_{1}}{E_{2} A_{2}}+\frac{F_{2} d_{b 2}+F_{1} d_{t 2}}{E_{2} S_{t 2}} \tag{C-1}
\end{align*}
$$

For strain compatibility between sections 2 and 3;

$$
\varepsilon_{\text {Botoom } 2}=\alpha_{2}\left(T_{3}\right)+\frac{M_{2}-M_{1}}{E_{2} S_{b 2}}+\frac{F_{2}-F_{1}}{E_{2} A_{2}}+\frac{F_{2} d_{b 2}+F_{1} d_{t 2}}{E_{2} S_{b 2}}=\varepsilon_{\text {Top } 3}
$$

$$
\begin{equation*}
\varepsilon_{\text {Top } 3}=\alpha_{3}\left(T_{3}\right)+\frac{M_{3}-M_{2}}{E_{3} S_{t 3}}+\frac{F_{3}-F_{2}}{E_{3} A_{3}}+\frac{F_{3} d_{b 3}+F_{2} d_{t 3}}{E_{3} S_{t 3}} \tag{C-2}
\end{equation*}
$$

For strain compatibility between sections 3 and 4;

$$
\begin{align*}
& \varepsilon_{\text {Botoom } 3}=\alpha_{3}\left(T_{4}\right)+\frac{M_{3}-M_{2}}{E_{3} S_{b 3}}+\frac{F_{3}-F_{2}}{E_{3} A_{3}}+\frac{F_{3} d_{b 3}+F_{2} d_{t 3}}{E_{3} S_{b 3}}=\varepsilon_{\text {Top } 4} \\
& \varepsilon_{\text {Top } 4}=\alpha_{4}\left(T_{4}\right)-\frac{M_{3}}{E_{4} S_{t 4}}-\frac{F_{3}}{E_{4} A_{4}}+\frac{F_{3} d_{t 4}}{E_{4} S_{t 4}} \tag{C-3}
\end{align*}
$$

## Curvature Compatibility

For curvature compatibility between sections 1 and 2;

$$
\begin{align*}
& \frac{1}{R_{1}}=\alpha_{1}\left(\frac{T_{2}-T_{1}}{h_{1}}\right)+\frac{M_{1}}{E_{1} I_{1}}+\frac{F_{1} d_{b 1}}{E_{1} I_{1}}=\frac{1}{R_{2}} \\
& \frac{1}{R_{2}}=\alpha_{2}\left(\frac{T_{3}-T_{2}}{h_{2}}\right)+\frac{M_{2}-M_{1}}{E_{2} I_{2}}+\frac{F_{1} d_{t 2}+F_{2} d_{b 2}}{E_{2} I_{2}} \tag{C-4}
\end{align*}
$$

For curvature compatibility between sections 2 and 3;

$$
\begin{align*}
& \frac{1}{R_{2}}=\alpha_{2}\left(\frac{T_{3}-T_{2}}{h_{2}}\right)+\frac{M_{2}-M_{1}}{E_{2} I_{2}}+\frac{F_{1} d_{t 2}+F_{2} d_{b 2}}{E_{2} I_{2}}=\frac{1}{R_{3}} \\
& \frac{1}{R_{3}}=\alpha_{3}\left(\frac{T_{4}-T_{3}}{h_{3}}\right)+\frac{M_{3}-M_{2}}{E_{3} I_{3}}+\frac{F_{2} d_{t 3}+F_{3} d_{b 3}}{E_{3} I_{3}} \tag{C-5}
\end{align*}
$$

For curvature compatibility between sections 3 and 4;

$$
\begin{align*}
& \frac{1}{R_{3}}=\alpha_{3}\left(\frac{T_{4}-T_{3}}{h_{3}}\right)+\frac{M_{3}-M_{2}}{E_{3} I_{3}}+\frac{F_{2} d_{t 3}+F_{3} d_{b 3}}{E_{3} I_{3}}=\frac{1}{R_{4}} \\
& \frac{1}{R_{4}}=\alpha_{3}\left(\frac{T_{5}-T_{4}}{h_{4}}\right)-\frac{M_{3}}{E_{4} I_{4}}+\frac{F_{3} d_{t 4}}{E_{4} I_{4}} \tag{C-6}
\end{align*}
$$

where
$\alpha_{i}$ : Coefficient of thermal expansion for Section i
$T_{i}$ : Girder and deck temperature changes as given in Figure C-1 and Figure C-2
$F_{i}$ : Force resultant of stresses between section i and i+1

$$
\begin{aligned}
& M_{i}: \text { Moment resultant of stresses between section i and } \mathrm{i}+1 \\
& d_{b i}: \text { Distance from centroid to bottom fiber of Section } \mathrm{i} \\
& d_{t i}: \text { Distance from centroid to top fiber of Section } \mathrm{i} \\
& S_{b i}: \text { Bottom section modulus for Section } \mathrm{i} \\
& S_{t i}: \text { Top section modulus for Section } \mathrm{i} \\
& E_{i}: \text { Modulus of elasticity of Section } \mathrm{i} \\
& A_{i}: \text { Cross-sectional area of Section } \mathrm{i} \\
& I_{i}: \text { Moment of inertia of Section } \mathrm{i}
\end{aligned}
$$

Solving the above six simultaneous equations for six unknowns ( $F_{1}, F_{2}, F_{3}, M_{1}, M_{2}, M_{3}$ ), corresponding strain and curvature values can be obtained.

More details including the effect of reinforcement and some other boundary conditions can be found at Saadeghvaziri and Hadidi (2002).

Once the curvature is known, end-slopes can be obtained by integrating curvature along the length;

$$
\begin{equation*}
\frac{d \theta}{d x}=\frac{1}{R_{1}}=\frac{1}{R_{2}}=\frac{1}{R_{3}}=\frac{1}{R_{4}}=\frac{1}{R} \quad \theta(x)=\int \frac{1}{R} d x=\frac{x}{R}+C_{1} \tag{C-7}
\end{equation*}
$$

For a simply supported span with length $L$, since the slope at mid-span will be equal to zero under gradient loading, integration constant $C_{l}$ can be calculated as;

$$
\begin{equation*}
\theta\left(\frac{L}{2}\right)=\frac{L}{2 R}+C_{1}=0 \quad C_{1}=-\frac{L}{2 R} \tag{C-8}
\end{equation*}
$$

Then, the slope equation and the slope at the end will be equal to;

$$
\begin{equation*}
\theta(x)=\frac{x}{R}-\frac{L}{2 R} \quad \theta(L)=\frac{L}{R}-\frac{L}{2 R}=\frac{L}{2 R} \tag{C-9}
\end{equation*}
$$

Link slab moments can be calculated using Eq. C-10 once the girder end rotations are calculated under live and thermal gradient loads.

$$
\begin{equation*}
M_{a}=\frac{2 E_{c} I_{d} \theta}{L_{L}} \tag{C-10}
\end{equation*}
$$

where,
$I_{d}$ : Moment of inertia of the link slab
$L_{L}$ : Length of the link slab (Debond zone length: sum of $5 \%$ of each adjacent girder span + gap between beam ends)

## DESIGN AXIAL FORCE

Axial force for the RHHR support condition can be calculated using a two-spancontinuous model and neglecting the effects of debonding.


Figure C-3. Effect of RHHR type support condition on continuity (Okeil and El-Safty 2005)
For a two-span system with RHHR boundaries, tensile force developed in the link slab would be equal to the horizontal reactions at the interior supports, and this reaction is equal to the continuity moment divided by the distance between the centroid of deck and bearing location (Figure C-3).

## Continuity Moment due to Live Load

Under live load, each span is loaded so as to create maximum negative moment at the interior support (Figure C-4) with composite cross-section properties and neglecting debonding.


Figure C-4. Continuity moment at the interior support under live load

## Continuity Moment due to Temperature Gradient

The continuity moment under temperature gradient loading can be calculated using the superposition concept as given in Saadeghvaziri and Hadidi (2002). For a two-spancontinuous system with constant cross-section in both spans, continuity moment $M_{\text {continuity }}$ can be calculated as;

$$
\begin{equation*}
M_{\text {continuity }}=\frac{\left(F_{2} d_{t g}-M_{3}\right)\left(3 E_{\text {Compositit }} I_{\text {Composite }}\right)}{2 E_{\text {Girder }} I_{\text {Girder }}} \tag{C-11}
\end{equation*}
$$

where
$F_{2}:$ Force resultant of stresses between section 2 and 3 calculated from six
$\quad$ simultaneous equations
$M_{3}:$ Moment resultant of stresses between section 2 and 3 calculated from six
$\quad$ simultaneous equations
$d_{t g}:$ Distance from centroid to top fiber of girder
$E_{\text {Composite }}:$ Modulus of elasticity of composite section
$I_{\text {Composite }}:$ Moment of inertia of composite section
$E_{\text {Girder }}:$ Modulus of elasticity of girder
$I_{\text {Girder }}:$ Moment of inertia of girder

Once the continuity moment is found, tensile force in the link slab is;

$$
\begin{equation*}
T=\frac{M_{\text {continuity }}}{h} \tag{C-12}
\end{equation*}
$$

where, $h$ is the distance between the centroid of deck and bearing location.

## Numerical Example - Skew Link slab Design

## STEP 1: Material and Geometric Properties

Cross-section properties of the girder and the composite section are given in Figure C-5.


Figure C-5. Girder and composite section geometric properties

$$
\begin{array}{ll}
\text { Boundary condition } & \text { RHHR } \\
\text { Skew }(\theta) & 45^{0} \\
\text { Compressive strength of concrete }\left(f_{c}{ }^{\prime}\right) & 4,500 \mathrm{psi} \\
\text { Unit weight of concrete }\left(w_{c}\right) & 0.15 \mathrm{kcf} \\
\text { Concrete modulus of elasticity }\left(E_{c}\right) & 4,067 \mathrm{ksi} \\
\text { (AASHTO LRFD Section } 5.4 .2 .4) & \\
\text { Reinforcement yield strength }\left(f_{y}\right) & 60 \mathrm{ksi} \\
\text { Steel modulus of elasticity }\left(E_{s}\right) & 29,000 \mathrm{ksi} \\
\text { Link slab length }\left(L_{L S}\right)^{+} & 84.4 \mathrm{in} . \\
\text { Effective deck width }(B)^{++} & 66 \mathrm{in} . \\
\text { Link slab thickness } & 9 \mathrm{in} . \\
\text { Moment of inertia of link slab }\left(I_{L S}\right) & 4,009.5 \mathrm{in}^{4} \\
\text { Deck overhang (on either side of the beam) } & 25 \mathrm{in} . \\
\text { Moment of inertia of the composite section } & 375,678 \mathrm{in}^{4} \\
\left(I_{\text {composite }}\right) & \\
\text { + Link slab length }=69.5 \times 12 \times 5 \% \times 2+1 \text { in. gap }=84.4 \text { inches } \\
\text { ++ Link slab section perpendicular to bridge longitudinal axis is considered in the example because design } \\
\text { moments are calculated perpendicular to bridge longitudinal axis. }
\end{array}
$$

## STEP 2: Design Moments

## Step 2.1: Live Load Moment

HL-93 (AASHTO LRFD 2010) loading is applied at a location to create maximum end rotation on the 69.5 ft span of the bridge. The impact factor is taken as 1.33 from Section 3.6.2.1 of AASHTO LRFD (2010). As per Section 3.6.1.3 AASHTO LRFD (2010), a lane load of $0.64 \mathrm{k} / \mathrm{ft}$ is used in addition to the axle loads. Girder end rotation under HL93 loading is $3.47 \times 10^{-3}$ radians. The distribution factor is calculated as 0.508 assuming two or more lanes are loaded from the formulation in AASHTO LRFD (2010) Table 4.6.2.2.2b-1.

The maximum girder-end design rotation is calculated as $1.763 \times 10^{-3}$ radians when the front axle is located 18.4 feet away from the end of the span.

Moment induced by live load =

$$
\mathrm{M}_{\mathrm{a}}=\left(2 \mathrm{E}_{\mathrm{c}} \mathrm{I}_{\mathrm{d}} \theta\right) / \mathrm{L}_{\mathrm{L}}=(2 \times 4067 \times 4009.5 \times 0.001763) /(84.4 \times 12)=-56.77 \mathrm{ft}-\mathrm{kips} \quad \text { OR }
$$

For a 66 in. wide effective section

$$
M_{a}=\frac{2 E_{c I_{d}} \theta}{L_{L}}=\frac{2 \times 4067 \times 4009.5 \times 0.001763}{84.4 \times 12 \times(66 / 12)}=-10.32 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}
$$

Step 2.2: Moment due to Temperature Gradient Loading
Required information, solutions to simultaneous equations, curvature, girder end rotation, and moments due to temperature gradient loads are presented in chapter 4 and Appendix D.

Moment induced by positive temperature gradient (PTG):

$$
\mathrm{M}_{\mathrm{a}}=\left(2 \mathrm{E}_{\mathrm{c}} \mathrm{I}_{\mathrm{d}} \theta\right) / \mathrm{L}_{\mathrm{L}}=\left(2 \times 4067 \times 4009.5 \times 1.613 \times 10^{-3}\right) /(84.4 \times 12)=51.9 \mathrm{ft}-\mathrm{kips} \quad \mathrm{OR}
$$

For a 66 in. wide effective section

$$
M_{a}=\frac{2 E_{c I_{d}} \theta}{L_{L}}=\frac{2 \times 4067 \times 4009.5 \times 1.613 \times 10^{-3}}{84.4 \times 12 \times(66 / 12)}=9.44 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}
$$

Moment caused by negative thermal gradient (NTG) is -0.3 times the positive gradient loading.

$$
\mathrm{M}_{\mathrm{a}}=51.9 \times-0.3=-15.57 \mathrm{ft}-\mathrm{kips} \text { OR }
$$

For a 66 in. wide effective section
$\mathrm{M}_{\mathrm{a}}=15.57 /(66 / 12)=-2.83 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}$
The following table summarizes the moments calculated in step 2.1 and 2.2.
Table C-1 Summary of Analytical Girder End Rotations and Analytical Design Moments

| Load <br> Case | Analytical Rotation <br> Magnitude (Radians) <br> (a) | Distribution <br> Factor <br> (b) | Analytical Design <br> Rotation Magnitude <br> (Radians) <br> (c) $=(\mathrm{a}) \times(\mathrm{b})$ | Analytical Design <br> Moment $^{+}$ <br> $(\mathrm{k}-\mathrm{ft}) / \mathrm{ft}$ <br> $(\mathrm{d})$ |
| :--- | :---: | :---: | :---: | :---: |
| Live | 0.003470 | 0.508 | 0.001763 | -10.32 |
| PTG | 0.001613 | N/A | 0.001613 | 9.44 |
| NTG | 0.000484 | N/A | 0.000484 | -2.83 |

+ Negative moments cause tension at link slab top fiber. Sign convention is stated in chapter 4
Step 2.3: Moment Reduction due to 3D Effect
AASHTO LRFD (2010) distribution factors are to incorporate 3D effect on load distribution and to find the girder design moments. The following table shows ratios of link slab moments calculated from 3D FE analysis of the specific straight bridge configuration described in chapter 4 of the report to analytical design moments summarized in the above table (i.e., moments calculated in step 2.1 and 2.2). HRRR, RRHR, and RHHR represent different support configurations of a two-span bridge (Hhinge or fixed bearing, R- roller or expansion bearing; HRRR represents expansion bearings underneath the link slab). It is seen that there is a significant reduction in link slab moments based on support configuration and the type of load acting on the bridge. Further, there are no load distribution factors given in AASHTO LRFD (2010) for thermal loads.

Table C-2. Ratios of 3D FE to Analytical Design Moment for a Straight Bridge

| Load Case | HRRR | RRHR | RHHR |
| :--- | :---: | :---: | :---: |
| Live | 0.218 | 0.257 | 0.887 |
| PTG | 0.092 | 0.111 | 0.967 |
| NTG | 0.080 | 0.100 | 0.961 |

Table C-3. Link Slab Design Moment for a Straight Bridge with RHHR

| Load Case | Moment Ratio <br> $(\mathrm{a})$ | Analytical Design Moment <br> $(\mathrm{k}-\mathrm{ft}) / \mathrm{ft}$ <br> $(\mathrm{b})$ | Link Slab Design Moment <br> $(\mathrm{k}-\mathrm{ft}) / \mathrm{ft}$ <br> $(\mathrm{c})=\mathrm{a} \times \mathrm{b}$ |
| :--- | :---: | :---: | :---: |
| Live | 0.887 | -10.32 | -9.2 |
| PTG | 0.967 | 9.44 | 9.1 |
| NTG | 0.961 | -2.83 | -2.7 |

Step 2.4: Moment Reduction due to Skew Effect (Skew Reduction Factors)

Table C-4. Skew Reduction Factors for RHHR

| Skew (Degree) | Ratio of Maximum Link-Slab Effective Moment (Skew/Zero Skew) (Skew Reduction Factors) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lane 1 <br> (a) | Lane 2 <br> (b) | Lane Alt 1 <br> (c) | Lane Alt 2 <br> (d) | NTG <br> (e) | $\begin{gathered} \hline \text { PTG } \\ (\mathbf{f}) \\ \hline \end{gathered}$ |
| 0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 20 | 0.96 | 0.96 | 0.97 | 0.95 | $\approx 1.00$ | $\approx 1.00$ |
| 30 | 0.91 | 0.90 | 0.91 | 0.89 | $\approx 1.00$ | $\approx 1.00$ |
| 45 | 0.77 | 0.74 | 0.76 | 0.72 | $\approx 1.00$ | $\approx 1.00$ |

Analysis results presented in chapter 4 of the report demonstrated that the Lane 2 load is the governing live load case. There is no increase or reduction in moments developed in a skew link slab under NTG or PTG for RHHR support configurations; however, there are skew reduction/amplification factors for other support configurations.

The design example is for a $45^{\circ}$ skew bridge. Hence, live load moment shall be multiplied by 0.74 , and there is no reduction for NTG or PTG moments.

Table C-5. Link Slab Design Moment for Skew Bridge with RHHR

| Load Case | Link Slab Design Moment of a Straight Bridge (k-ft)/ft <br> (a) | Skew Reduction <br> Factor <br> (b) | Link Slab Design Moment of a Skew Bridge $\begin{array}{r} (\mathrm{k}-\mathrm{ft}) / \mathrm{ft} \\ (\mathrm{c})=\mathrm{a} \times \mathrm{b} \\ \hline \end{array}$ |
| :---: | :---: | :---: | :---: |
| Live | -9.2 | 0.74 | -6.8 |
| PTG | 9.1 | 1.00 | 9.1 |
| NTG | -2.7 | 1.00 | -2.7 |

Step 2.5: Resultant Combined Moments
Thermal gradient loading [i.e., NTG and PTG] and live load need to be combined to create critical load combinations. The following load combinations are developed as per AASHTO LRFD (2010) section 3.4. AASHTO LRFD (2010) service 1 load combination requires using load factor of 1.0 for the temperature gradient when the live load is not considered. Exclusion of live load when PTG effect is used in the design yields the critical load combination for positive moment. Hence, it is recommended to use factor of 1.0 for PTG loads.

Service I-Negative Moment: 1.0 Live Load + 0.5 NTG
Service I-Positive Moment: 1.0 PTG
Service I-Negative Moment:

$$
\mathrm{M}_{\mathrm{SI}-\mathrm{N}}=-6.8+0.5 \times-2.7=-8.15 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}
$$

Service I-Positive Moment:
$\mathrm{M}_{\text {SI-P }}=9.1=9.1 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}$

Step 2.6: Cracking Moment
Note: Cracking moment calculated using modulus of rupture of $0.24 \sqrt{f_{c}^{\prime}}, k s i$ is less than both $\mathrm{M}_{\mathrm{SI}-\mathrm{N}}$ and $\mathrm{M}_{\mathrm{SI}-\mathrm{P} \text {. Hence, the links slab cracks and the amount of top and bottom }}$ layer reinforcement should be calculated using $\mathrm{M}_{\text {SI-N }}$ and $\mathrm{M}_{\mathrm{SI}-\mathrm{P}}$, respectively. Detailed example of calculating link slab top and bottom layer reinforcement is provided in Ulku et al. (2009). The amount of reinforcement calculated from these two moments is less than the minimum reinforcement required in AASHTO LRFD section 5.4.2.6. Hence, the minimum reinforcement calculation process as per AASHTO LRFD section 5.4.2.6 is presented here.

Modulus of rupture of 4500 psi strength concrete for calculating the minimum reinforcement

$$
f_{r}=785 \mathrm{psi}\left(0.37 \sqrt{f_{c}^{\prime}}, k s i\right) \text { and }
$$

Cracking moment

$$
M_{c r}=S_{c}\left(f_{r}+f_{c p e}\right)-M_{d n c}\left(\frac{S_{c}}{s_{n c}}-1\right) \geq S_{c} f_{r}
$$

$M_{d n c}$ - Total unfactored dead load moment acting on the link slab that can be eliminated by considering casting sequence of the link slab (e.g., in retrofit applications expansion joint is removed and link slab is replaced).
$f_{\text {cpe }}$ - compressive stress in concrete due to effective prestress forces which is zero in this example because there is no prestress forces in the link slab.
$S_{c}$ - section modulus of the link slab $\left(I_{g} / y_{t}\right)$
$\mathrm{I}_{\mathrm{g}}$ - moment of inertia of the gross section
$y_{t}$ - distance from the neutral axis to the extreme tension fiber
Considering a 9 in . thick, 12 in . wide link slab section;

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{g}}=12 \times 9^{3} / 12=729 \mathrm{in}^{4} \\
& \mathrm{y}_{\mathrm{t}}=4.5 \mathrm{in} .
\end{aligned}
$$

Cracking moment of 9 in. thick, 12 in. wide link slab section;

$$
M_{c r}=S_{c} f_{r}=10.6 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}
$$

Step 2.7: Minimum Flexural Reinforcement
AASHTO LRFD (2010) section 5.7.3.3.2 requires providing adequate steel to develop a factored flexural resistance $\left(\mathrm{M}_{\mathrm{r}}\right)$ equal to the lesser of $1.2 \times \mathrm{M}_{\mathrm{cr}}$ or $1.33 \times$ (factored moment required by the applicable strength load combinations).

$$
1.2 \times \mathrm{M}_{\mathrm{cr}}=1.2 \times 10.6 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}=12.72 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}
$$

AASHTO LRFD (2010) recommends using a zero (0) load factor for the thermal load gradient when a Strength I combination is used. Hence, " $1.33 \times$ (the factored moment required by the applicable strength load combinations)" always yields negative moments. For negative moment at the link slab;

$$
1.33 \times(1.75 \times-6.8+0.0 \times-2.7)=-15.83 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}
$$

When the specification requirements are considered, calculation of amount of minimum negative moment reinforcement (top reinforcement) is governed by $\mathrm{M}_{\mathrm{r}}=1.2 \times \mathrm{M}_{\mathrm{cr}}=12.72$ ft-kips/ft.

AASHTO LRFD section 5.7.3.3.2 requirement of " $1.33 \times$ (the factored moment required by the applicable strength load combinations)" never yield a positive moment to calculate positive moment reinforcement (i.e., link slab bottom reinforcement). Also, $\mathrm{M}_{\mathrm{SI}-\mathrm{P}}<\mathrm{M}_{\mathrm{cr}}$.

Hence, using $\mathrm{M}_{\mathrm{r}}=1.2 \times \mathrm{M}_{\mathrm{cr}}=12.72 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}$ is recommended for calculating positive moment reinforcement.

Step 2.7.1 Negative Moment Reinforcement (i.e., top fiber in tension)
The minimum amount of steel reinforcement is calculated considering $40 \%$ of the yield strength, $\mathrm{j} \approx 0.9$, and $\mathrm{d}=6.375 \mathrm{in}$.

Effective depth (d) is calculated assuming \#6 bars are used as the transverse reinforcement in the deck and the clear cover to the top transverse bar is 3 in .
$\mathrm{d}=($ link slab thickness $)-($ clear cover to transverse rebar $)+(0.5 \mathrm{x}$ diameter of \#6 bar) $\mathrm{d}=9 \mathrm{in} .-3 \mathrm{in} .+0.5 \mathrm{x} 0.75 \mathrm{in} .=6.375 \mathrm{in}$.

$$
\begin{aligned}
\mathrm{A}_{\text {steel }}=\mathrm{M}_{\mathrm{r}} /\left(0.4 f_{y} \cdot \mathrm{j} \cdot \mathrm{~d}\right) & =(12.72 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}) \times 12 /(0.4 \times 60 \mathrm{ksi} \times 0.9 \times 6.375 \mathrm{in} .) \\
& =1.11 \mathrm{in}^{2} / \mathrm{ft}
\end{aligned}
$$

Use \#6 bars @ $4 \mathrm{in} .=\mathrm{A}_{\text {steel }}=1.32$ in. $^{2}>1.11 \mathrm{in}^{2}$

Step 2.7.2 Positive Moment Reinforcement (i.e., bottom fiber in tension)
The amount of steel reinforcement is calculated considering $40 \%$ of the yield strength, j $\approx 0.9$, and $\mathrm{d}=6.75 \mathrm{in}$.

Effective depth (d) is calculated assuming \#6 bars are used as the transverse reinforcement in the deck and the distance from bottom surface to the centerline of the bottom transverse bar is 1.5 in .
$\mathrm{d}=($ link slab thickness) - (cover to centerline of transverse rebar) - ( diameter of \#6 bar) $\mathrm{d}=9$ in. -1.5 in. -0.75 in. $=6.75$ in.

$$
\begin{aligned}
\mathrm{A}_{\text {steel }}=\mathrm{M}_{\mathrm{r}} /\left(0.4 f_{y .} \mathrm{j} . \mathrm{d}\right) & =(12.72 \mathrm{ft}-\mathrm{kips} / \mathrm{ft}) \times 12 /(0.4 \times 60 \mathrm{ksi} \times 0.9 \times 6.75 \mathrm{in} .) \\
& =1.05 \mathrm{in}^{2} / \mathrm{ft}
\end{aligned}
$$

Use \#6 bars @ 4 in. $=\mathrm{A}_{\text {steel }}=1.32$ in. $^{2}>1.05$ in $^{2}$
Step 2.7.3 Steel Stress and Crack Width Parameter Limits
Section 5.7.3.4 Control of Cracking by Distribution of Reinforcement is not discussed here because the amount of reinforcement provided satisfies crack width limit criterion. Please refer Ulku et al. (2009) for the detailed procedure.

## STEP 3: Design Axial Force

Step 3.1: Axial Force due to Live Load
For an RHHR boundary condition, the axial force in the link slab needs to be calculated using the maximum negative moment at the interior support of a two-span continuous system. HL-93 (AASHTO LRFD 2010) loading is applied at both spans to create a maximum negative moment of -724 ft-kips at the interior support.

Axial force ( F ) acting on the link slab due to HL-93 loading:

$$
F=\frac{M_{\text {continuity }}}{h}=\frac{-724 \times 12}{(54-9 / 2)}=-176 \text { kips or }-27.8 \text { kips } / f t \quad \quad \text { (Tension) }
$$

Step 3.2: Axial Force due to PTG
Axial force acting on the link slab due to positive temperature gradient:

$$
\begin{aligned}
\mathrm{M}_{\text {continuity }} & =\left[\left(\mathrm{F}_{2} \mathrm{~d}_{\mathrm{tg}}-\mathrm{M}_{3}\right)\left(3 \mathrm{E}_{\text {composite }} \mathrm{I}_{\text {composite }}\right)\right] /\left(2 \mathrm{E}_{\text {girder }} \mathrm{I}_{\text {girder }}\right) \\
& =[(25.257 \times 24.73+31.742) \cdot(3 \times 4067 \times 375,678)] /(2 \times 4067 \times 125,390) \\
& =2,950 \text { in-kips }
\end{aligned}
$$

$$
\mathrm{F}=\mathrm{M}_{\text {continuity }} / \mathrm{h}=2950 /(54-9 / 2)=60 \mathrm{kips} \quad \text { or } 11 \mathrm{kips} / \mathrm{ft} \quad \text { (compression) }
$$

Note that $\mathrm{F}_{2}$ is the force at layer $2, \mathrm{~d}_{\mathrm{tg}}$ is the distance from girder top to the girder centroid, and $\mathrm{M}_{3}$ is the moment at layer 3. $\mathrm{F}_{2}$ and $\mathrm{M}_{3}$ calculation is given in MathCAD sheet provided in Appendix D.

Step 3.3: Axial Force due to $N T G$
Axial force acting on the link slab due to negative temperature gradient:

$$
T_{N G}=-0.3 T_{P G}=-0.3 \times 60=18 \text { kips or }-3.2 \text { kips } / \mathrm{ft} \quad \text { (Tension) }
$$

Step 3.4: 3D and Skew Effects on Axial Force
3D and skew effects discussed in Step 2.3 and 2.4 can be directly applied to calculate axial load in a skew link slab due to similarities in moment and force ratios. (See chapter 4 of the report for further details.)

Table C-6. Link Slab Design Force for Straight Bridge with RHHR

| Load <br> Case | Design Force <br> Ratio <br> (a) | Analytical Design Force <br> $($ kips $) / \mathrm{ft}$ <br> $(\mathrm{b})$ | Link Slab Design Force of a Straight Bridge <br> $(\mathrm{kips}) / \mathrm{ft}$ <br> $(\mathrm{c})=\mathrm{a} \times \mathrm{b}$ |
| :--- | :---: | :---: | :---: |
| Live | 0.887 | -27.8 | -24.7 |
| PTG | 0.967 | 11.0 | 10.6 |
| NTG | 0.961 | -3.2 | -3.1 |

Table C-7. Link Slab Design Force for Skew Bridge with RHHR

| Load <br> Case | Link Slab Design Force of a <br> Straight Bridge k/ft <br> (a) | Skew Reduction <br> Factor <br> (b) | Link Slab Design Force of a Skew <br> Bridge $\mathrm{k} / \mathrm{ft}$ <br> (c) $\mathrm{a} \times \mathrm{b}$ |
| :--- | :---: | :---: | :---: |
| Live | -24.7 | 0.74 | -18.3 |
| PTG | 10.6 | 1.00 | 10.6 |
| NTG | -3.1 | 1.00 | -3.1 |

Step 3.4: Resultant Combined Forces
Thermal gradient loading [i.e., NTG and PTG] and live load need to be combined to create critical load combinations.

Service I-Negative Force: 1.0 Live Load + 0.5 NTG
Service I-Positive Force: 1.0 PTG
Service I-Negative force:
$F_{S I-N}=-18.3+0.5 \times-3.1=-19.85 \mathrm{kips} / \mathrm{ft}$
Service I-Positive Force:
$\mathrm{F}_{\text {SI-P }}=10.6=10.6 \mathrm{kips} / \mathrm{ft}$
Step 3.5: Check for Axial Load Capacity

Steel area provided in the link-slab $=0.88 \mathrm{in}^{2}+0.88 \mathrm{in}^{2}=1.76 \mathrm{in}^{2} / \mathrm{ft}$
Assuming steel carries the total axial load
$f_{\text {steel }}=(19.45 \mathrm{kips} / \mathrm{ft}) /\left(1.76 \mathrm{in}^{2} / \mathrm{ft}\right)=11.05 \mathrm{ksi}<f_{s a}=0.6 \times 60 \mathrm{ksi}=36 \mathrm{ksi} \mathrm{OK}$.

## STEP 4: Moment-Force Interaction

| Load Combination | Moment (from Step 2) <br> $\mathrm{ft}-\mathrm{kips} / \mathrm{ft}$ | Axial Force (from Step 3) <br> $\mathrm{kips} / \mathrm{ft}$ |
| :---: | :---: | :---: |
| Service I - Positive | 9.1 (i.e., top fiber compression) | 10.60 (Compression) |
| Service I - Negative | 8.15 (i.e., top fiber tension) | 19.85 (Tension) |



Figure C-1. Moment and Interaction Diagram under Service Loads for unit link slab width

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## APPENDIX D - LINK SLAB MOMENT DUE TO THERMAL GRADIENT (MathCAD)

Temperature profile through the deck-girder composite section for Postive Temeperature Gradient (PTG)
$\mathrm{T}_{1}:=41 \quad \mathrm{~T}_{2}:=11 \quad \mathrm{~T}_{3}:=6.42 \quad \mathrm{~T}_{4}:=0 \quad \mathrm{~T}_{5}:=0$
Material properties

Concrete modulus $\quad \mathrm{E}_{\mathrm{c}}:=4067 \mathrm{ksi}$
Thermal expansion coefficients, in/in/F

$$
\alpha_{1}:=6 \cdot 10^{-6}
$$

$\alpha_{2}:=6 \cdot 10^{-6}$
$\alpha_{3}:=6 \cdot 10^{-6}$
$\alpha_{4}:=6 \cdot 10^{-6}$
Section properties (length in inches; area in inches ${ }^{2}$ )
$\mathrm{b}_{1}:=66 \quad \mathrm{~b}_{2}:=16 \quad \mathrm{~h}_{1}:=4 \quad \mathrm{~h}_{2}:=5 \quad \mathrm{~h}_{3}:=7 \quad \mathrm{~h}_{4}:=38$
$\mathrm{d}_{\mathrm{b} 1}:=2 \quad \mathrm{~d}_{\mathrm{t} 1}:=2 \quad \mathrm{~d}_{\mathrm{b} 2}:=2.5 \quad \mathrm{~d}_{\mathrm{t} 2}:=2.5 \quad \mathrm{~d}_{\mathrm{b} 3}:=3.5 \quad \mathrm{~d}_{\mathrm{t} 3}:=3.5 \quad \mathrm{~d}_{\mathrm{b} 4}:=14.91 \quad \mathrm{~d}_{\mathrm{t} 4}:=23.09$
$\mathrm{A}_{1}:=\mathrm{h}_{1} \cdot \mathrm{~b}_{1} \quad \mathrm{~A}_{2}:=\mathrm{h}_{2} \cdot \mathrm{~b}_{1} \quad \mathrm{~A}_{3}:=\mathrm{h}_{3} \cdot \mathrm{~b}_{2} \quad \mathrm{~A}_{4}:=447.5$
$\mathrm{A}_{1}=264 \quad \mathrm{~A}_{2}=330 \quad \mathrm{~A}_{3}=112 \quad \mathrm{~A}_{4}=447.5$
Length of bridge span (in.)

$$
L:=834
$$

Length of link slab (in.)

$$
\mathrm{L}_{\mathrm{L}}:=\mathrm{L} \cdot 0.05 \cdot 2+1 \quad \mathrm{~L}_{\mathrm{L}}=84.4
$$

Moment of inertia of each layer (in. ${ }^{4}$ ).

$$
\begin{aligned}
& \mathrm{I}_{1}:=\mathrm{b}_{1} \cdot \frac{\mathrm{~h}_{1}{ }^{3}}{12} \quad \mathrm{I}_{2}:=\mathrm{b}_{1} \cdot \frac{\mathrm{~h}_{2}{ }^{3}}{12} \quad \mathrm{I}_{3}:=\mathrm{b}_{2} \cdot \frac{\mathrm{~h}_{3}{ }^{3}}{12} \quad \mathrm{I}_{4}:=61 \\
& \mathrm{I}_{1}=352
\end{aligned} \mathrm{I}_{2}=687.5 \quad \mathrm{I}_{3}=457.333 \quad \mathrm{I}_{4}=6.189 \times 10^{4}
$$

Section modulus (in. ${ }^{3}$ ).

$$
\begin{aligned}
& \mathrm{S}_{\mathrm{b} 1}:=\frac{\mathrm{I}_{1}}{\mathrm{~d}_{\mathrm{b} 1}} \quad \mathrm{~S}_{\mathrm{b} 2}:=\frac{\mathrm{I}_{2}}{\mathrm{~d}_{\mathrm{b} 2}} \quad \mathrm{~S}_{\mathrm{b} 3}:=\frac{\mathrm{I}_{3}}{\mathrm{~d}_{\mathrm{b} 3}} \quad \mathrm{~S}_{\mathrm{b} 4}:=\frac{\mathrm{I}_{4}}{\mathrm{~d}_{\mathrm{b} 4}} \\
& \mathrm{~S}_{\mathrm{t} 1}:=\frac{\mathrm{I}_{1}}{\mathrm{~d}_{\mathrm{t} 1}}
\end{aligned} \quad \mathrm{~S}_{\mathrm{t} 2}:=\frac{\mathrm{I}_{2}}{\mathrm{~d}_{\mathrm{t} 2}} \quad \mathrm{~S}_{\mathrm{t} 3}:=\frac{\mathrm{I}_{3}}{\mathrm{~d}_{\mathrm{t} 3}} \quad \mathrm{~S}_{\mathrm{t} 4}:=\frac{\mathrm{I}_{4}}{\mathrm{~d}_{\mathrm{t} 4}} .
$$

Moment of inertia of the link slab (in ${ }^{4}$ )

$$
\mathrm{I}_{\mathrm{d}}:=\mathrm{b}_{1} \cdot \frac{\left(\mathrm{~h}_{1}+\mathrm{h}_{2}\right)^{3}}{12} \quad \mathrm{I}_{\mathrm{d}}=4.01 \times 10^{3}
$$

Solution process of six simultaneous equatons
Initial estimates

$$
\begin{array}{llll}
\mathrm{M}_{1}:=100 & \mathrm{M}_{2}:=100 & \mathrm{M}_{3}:=100 & \mathrm{M}_{4}:=100 \\
\mathrm{~F}_{1}:=100 & \mathrm{~F}_{2}:=100 & \mathrm{~F}_{3}:=100 & \mathrm{~F}_{4}:=100
\end{array}
$$

Given
$\alpha_{1} \cdot T_{2}+\frac{M_{1}}{E_{c} \cdot S_{b 1}}+\frac{F_{1}}{E_{c} \cdot A_{1}}+F_{1} \cdot \frac{d_{b 1}}{E_{c} \cdot S_{b 1}}-\alpha_{2} \cdot T_{2}-\frac{\left(M_{2}-M_{1}\right)}{E_{c} \cdot S_{t} 2}-\frac{\left(F_{2}-F_{1}\right)}{E_{c} \cdot A_{2}}+\frac{\left(F_{2} \cdot d_{b 2}+F_{1} \cdot d_{t 2}\right) \cdot-1}{E_{c} \cdot S_{t 2}}=0$
$\alpha_{2} \cdot T_{3}+\frac{\left(M_{2}-M_{1}\right)}{E_{c} \cdot S_{b 2}}+\frac{\left(F_{2}-F_{1}\right)}{E_{c} \cdot A_{2}}+\frac{\left(F_{2} \cdot d_{b 2}+F_{1} \cdot d_{t 2}\right)}{E_{c} \cdot S_{b 2}}-\alpha_{3} \cdot T_{3}-\frac{\left(M_{3}-M_{2}\right)}{E_{c} \cdot S_{t 3}}-\frac{\left(F_{3}-F_{2}\right)}{E_{c} \cdot A_{3}}-\frac{\left(F_{3} \cdot d_{b 3}+F_{2} \cdot d_{t 3}\right)}{E_{c} \cdot S_{t 3}}=0$
$\alpha_{3} \cdot T_{4}+\frac{\left(M_{3}-M_{2}\right)}{E_{c} \cdot S_{b 3}}+\frac{\left(F_{3}-F_{2}\right)}{E_{c} \cdot A_{3}}+\frac{\left(F_{3} \cdot d_{b 3}+F_{2} \cdot d_{t 3}\right)}{E_{c} \cdot S_{b 3}}-\alpha_{4} \cdot T_{4}-\frac{\left(M_{3}\right) \cdot-1}{E_{c} \cdot S_{t 4}}-\frac{\left(F_{3}\right) \cdot-1}{E_{c} \cdot A_{4}}-\frac{\left(F_{3} \cdot d_{44}\right)}{E_{c} \cdot S_{t 4}}=0$
$\alpha_{1} \cdot \frac{\left(T_{2}-T_{1}\right)}{h_{1}}+\frac{M_{1}}{E_{c} \cdot I_{1}}+F_{1} \cdot \frac{d_{b 1}}{E_{c} \cdot I_{1}}-\alpha_{2} \cdot \frac{\left(T_{3}-T_{2}\right)}{h_{2}}-\frac{\left(M_{2}-M_{1}\right)}{E_{c} \cdot I_{2}}-\frac{\left(F_{1} \cdot d_{2}+F_{2} \cdot d_{b 2}\right)}{E_{c} \cdot I_{2}}=0$
$\alpha_{2} \cdot \frac{\left(\mathrm{~T}_{3}-\mathrm{T}_{2}\right)}{\mathrm{h}_{2}}+\frac{\left(\mathrm{M}_{2}-\mathrm{M}_{1}\right)}{\mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{2}}+\frac{\left(\mathrm{F}_{1} \cdot \mathrm{~d}_{2}+\mathrm{F}_{2} \cdot \mathrm{~d}_{\mathrm{b} 2}\right)}{\mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{2}}-\alpha_{3} \cdot \frac{\left(\mathrm{~T}_{4}-\mathrm{T}_{3}\right)}{\mathrm{h}_{3}}-\frac{\left(\mathrm{M}_{3}-\mathrm{M}_{2}\right)}{\mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{3}}-\frac{\left(\mathrm{F}_{2} \cdot \mathrm{~d}_{\mathrm{t}}+\mathrm{F}_{3} \cdot \mathrm{~d}_{\mathrm{b} 3}\right)}{\mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{3}}=0$
$\alpha_{3} \cdot \frac{\left(T_{4}-T_{3}\right)}{h_{3}}+\frac{\left(M_{3}-M_{2}\right)}{E_{c} \cdot I_{3}}+\frac{\left(\mathrm{F}_{2} \cdot \mathrm{~d}_{43}+\mathrm{F}_{3} \cdot \mathrm{~d}_{\mathrm{b} 3}\right)}{\mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{3}}-\alpha_{3} \cdot \frac{\left(\mathrm{~T}_{5}-\mathrm{T}_{4}\right)}{\mathrm{h}_{4}}-\frac{\left(\mathrm{M}_{3} \cdot-1\right)}{\mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{4}}-\frac{\left(\mathrm{F}_{3} \cdot \mathrm{~d}_{44}\right)}{\mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{4}}=0$

$$
\left[\begin{array}{l}
F_{1} \\
F_{2} \\
F_{3} \\
M_{1} \\
M_{1} \\
M_{1}
\end{array}\right]:=\operatorname{Find}\left(F_{1}, \quad F_{2}, \quad F_{3}, \quad M_{1}, \quad M_{2}, \quad M_{3}\right)
$$

$F_{1}=-33.11$
$\mathrm{F}_{2}=25.257$
$\mathrm{F}_{3}=40.79 \quad \mathrm{kips}$
$\mathrm{M}_{1}=136.178$
$\mathrm{M}_{2}=181.992$
$\mathrm{M}_{3}=-31.742 \quad$ kip-in

Curvature $:=\alpha_{3} \cdot \frac{\left(T_{4}-T_{3}\right)}{h_{3}}+\frac{\left(M_{3}-M_{2}\right)}{E_{c} \cdot I_{3}}+\frac{\left(F_{2} \cdot d_{t 3}+F_{3} \cdot d_{b 3}\right)}{E_{c} \cdot I_{3}}$
Curvature $=3.868 \times 10^{-6}$

$$
\begin{array}{lll}
{ }^{{ }_{\mathrm{PTG}}} & =\text { Curvature }-\frac{\mathrm{L}}{2} & { }_{\mathrm{PTG}}=1.613 \times 10^{-3} \\
{ }^{\mathrm{rad}} \\
{ }_{\mathrm{NTG}}:={ }_{\mathrm{PTG}}--0.3 & \theta_{\mathrm{NTG}}=-4.839 \times 10^{-4} & \mathrm{rad}
\end{array}
$$

Moment calculations

$$
\begin{array}{ll}
\text { Moment }_{\mathrm{PTG}}:=2 \cdot \mathrm{E}_{\mathrm{c}} \cdot \mathrm{I}_{\mathrm{d}} \cdot \frac{\theta_{\mathrm{PTG}}}{\mathrm{~L}_{\mathrm{L}} \cdot 12} & \text { Moment }_{\mathrm{NTG}}:=\text { Moment }_{\mathrm{PTG}} \cdot-0.3 \\
\text { Moment }_{\mathrm{PTG}}=51.938 \mathrm{ft}-\mathrm{kips} & \text { Moment }_{\mathrm{NTG}}=-15.581 \quad \mathrm{ft}-\mathrm{kips}
\end{array}
$$

Design moment for 66 in. wide effective section

$$
\begin{array}{ll}
\text { Des }_{\text {M.PTG }}:=\text { Moment }_{\text {PTG }} \cdot \frac{12}{66} & \text { Des }_{\text {M.NTG }}:=\text { Moment }_{\text {NTG }} \cdot \frac{12}{66} \\
\text { Des }_{\text {M.PTG }}=9.443 & \frac{\mathrm{ft}-\mathrm{kips}}{\mathrm{ft}}
\end{array} \quad \text { Des }_{\text {M.NTG }}=-2.833 \quad \frac{\mathrm{ft}-\mathrm{kips}}{\mathrm{ft}}
$$

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## APPENDIX E

Proposed Design Details in MDOT Design Guide Format

- Skew Link Slab


DRAWN BY:
APPROVED BY:
CHECKED BY:

FIGURE E-2: PROPOSED DETAIL
SKEW LINK SLAB DETAIL

ISSUED:
SUPEREDES:


## SECTION A-A (LONGITUDINAL SECTION THRU LINK SLAB)



SECTION B-B (TRANSVERSE SECTION THRU LINK SLAB)

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## APPENDIX F

Proposed Design Details in MDOT Design Guide Format

- Deck Sliding over Backwall System

| DRAWN BY: <br> APPROVED BY: <br> CHECKED BY: | FIGURE F-1: PROPOSED DETAIL |  | ISSUED BY: SUPEREDES |  |
| :---: | :---: | :---: | :---: | :---: |
|  | INDEPENDENT BACKWALL |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |  |  |



| DRAWN BY: <br> APPROVED BY: <br> CHECKED BY: | FIGURE F-3: PROPOSED DETAILS | ISSUED BY: <br> SUPEREDES: |
| :--- | :---: | :--- |
|  | TYPICAL BEARING DETAIL |  |




(C) TYPICAL SLOT DETAILS


## FIXED BEARINGS

NOTE:

* SEE DETAILS OF GIRDER END RESTRAIN

L: LENGTH OF THE DIAGNAL BETWEEN ACUTE CORNERS (SEE RUB PLATE DESIGN EXAMPLE)
W: DECK WIDTH
$\beta$ : ANGLE (SEE RUB PLATE DESIGN EXAMPLE)
$\phi$ : DIAMETER OF THE POSITION DOWEL
$\Delta$ T: |MAXIMUM TEMPERATURE| + |MINIMUM TEMPERATURE| FROM TABLE 5-2

Intentionally left blank

## APPENDIX G

Rub Plate Design Procedure

| DRAWN BY: | FIGURE G-1: PROPOSED RUB PLATE DESIGN PROCEDURE | ISSUED: |
| :---: | :---: | :---: |
| CHECKED BY: | RUB PLATE DESIGN GIRDER END RESTRAIN | SUPEREDES: |



## FIGURE A

## RUB PLATE DESIGN

TWO GIRDER ENDS ARE RESTRAINED
FORCES ON THE RUB PLATE $\left(R_{p}\right)=(10 \% \times$ TOTAL VERTICAL BEARING REACTION $) / 2$

$$
=(10 \% \times 1000 \mathrm{kips}) / 2=50 \mathrm{kips}
$$

DETERMINE SIZE OF RUB PLATES:
$\Delta T=\mid$ MAXIMUM TEMPERATURE + |MINIMUM TEMPERATURE $=115^{\circ} \mathrm{F}$ (TABLE 5-2)
$\mathrm{L}_{\text {th }}=$ EFFECTIVE LENGTH OF THERMAL MOVEMENT, 123 ft
EXTIMATED MAXIMUM MOVEMENT IN ONE DIRECTION AT THE ABUTMENT.
$\left(\Delta \mathrm{L}_{\text {rub }}\right)_{\mathrm{c}}=\mathrm{L}_{\text {th }} \times \alpha \times 1.2 \times \Delta \mathrm{T}_{\mathrm{c}}=123 \mathrm{ft}(12 \mathrm{ir} / \mathrm{tt})\left(6 \times 10^{-6} /{ }^{\circ} \mathrm{F}\right) \times\left(1.2 \times 72.7^{\circ} \mathrm{F}\right)=0.8$ inch
$\left(\Delta \mathrm{L}_{\text {rub }}\right)_{\mathrm{e}}=\mathrm{L}_{\mathrm{th}} \times \alpha \times 1.2 \times \Delta \mathrm{T}_{\mathrm{e}}=123 \mathrm{ft}(12 \mathrm{in} / \mathrm{tt})\left(6 \times 10^{-6} /{ }^{\circ} \mathrm{F}\right) \times\left(1.2 \times 42.3^{\circ} \mathrm{F}\right)=0.5$ inch
WHERE $\quad \Delta \mathrm{T}_{\mathrm{c}}=$ CONTRACTION THERMAL LOAD
$\Delta \mathrm{T}_{\mathrm{e}}=$ EXPANSION THERMAL LOAD
$\left(\Delta \mathrm{L}_{\text {rub }}\right)_{c}>\left(\Delta \mathrm{L}_{\text {rub }}\right)_{\mathrm{e}}$, SO CONSIDER $\left(\Delta \mathrm{L}_{\text {rub }}\right)_{\mathrm{c}}$ FOR FURTHER CALCULATIONS
HEIGHT OF RUB PLATE:
$h_{\text {rp }}=\mathrm{T}_{\text {bottom flange }}-2$ inch $=7$ inch -2 inch $=5$ inch
THIS EXAMPLE CONSIDERS AASHTO TYPE III GIRDER, OF WHICH BOTTOM FLANGE THICKNESS IS 7 in. CLEARANCE FROM TOP AND BOTTOM IS 1 inch.

MAXIMUM "GALLING STRESS" FOR ASTM A276 TYPE 316 STEEL, OF WHICH THE RUB PLATES
ARE CONSTRUCTED:
$\mathrm{F}_{\mathrm{g}}=2000 \mathrm{psi}$
ALLOWABLE GALLING STRESS:
$\mathrm{f}_{\mathrm{g}}=0.55 \mathrm{~F}_{\mathrm{g}}=1100 \mathrm{psi}$
MINIMUM RUB PLATE WIDTH:
$\mathrm{w}_{\text {min }}=\mathrm{Rp} / \mathrm{hrp}(\mathrm{fg})=50 \mathrm{kip}(1000 \mathrm{lbs} / 1 \mathrm{kip}) /[5 \mathrm{in}(1100 \mathrm{psi})] \mathrm{inch}$
ENSURE THE MINIMUM RUB PLATE WIDTH IS MAINTAINED DURING EXTREMES OF THE TEMPERATURE CYCLE
$\mathrm{w}=\mathrm{w}_{\text {min }}+\left(\Delta \mathrm{L}_{\text {rub }}\right)_{\mathrm{c}}=9$ inch +0.8 inch $=9.8$ inch $\approx 10$ inch
USE 5 inch $\times 10$ inch $\times 0.5$ inch in rub plate
NOTE: LENGTH OF THE CONCRETE KEY ALONG GIRDER CENTER LINE SHOULD BE MINIMUM OF 10 INCH.

## APPENDIX H

Proposed Design Details in MDOT Design Guide Format - Semi - Integral Abutments

FIGURE H-1: PROPOSED DETAILS
SEMI INTEGRAL ABUTMENT EMPIRICAL APPROACH SLAB DETAILS FOR LINK SLAB BRIDGES

ISSUED: SUPEREDES:


## PLAN OF APPROACH



NOTES:
ATTACH APPROACH CURB AND GUTTER TO THE APPROACH SLAB WITH BOTTOM MAT TRANSVERSE REINFORCEMENT AND TO THE BRIDGE DECK WITH BOTTOM MAT LONGITUDINAL REINFORCEMENT

POUR APPROACH SLABS FROM EXPANSION LOCATION TOWARD REFERENCE LINE.

## APPROACH SLABS SHOULD BE CAST AT NIGHT WITH NIGHT TIME CASTING

 OF SUPERSTRUCTURE CONCRETEEJ3 OR EJ4 JOINT WIDTH TO ACCOMMODATE THERMAL MOVEMENT
$=\mathrm{L}_{\mathrm{th}} \times 1.2\left(\Delta \mathrm{~T}_{\mathrm{e}}\right) \cos \beta$
REFER RUB PLATE DESIGN SHEET FOR $\mathrm{L}_{\mathrm{th}}, \Delta \mathrm{T}_{\mathrm{e}}$ AND $\beta$ DEFINITIONS
USE SLEEPER SLAB WITH ALL APPROACH SLABS INCLUDING HMA ROADWAY

DRAWN BY:
APPROVED BY: CHECKED BY:

FIGURE H-2 - PROPOSED DETAIL
SEMI - INTEGRAL ABUTMENT DETAILS WITH BACKWALL IN-LINE WITH ABUTMENT

ISSUED:
SUPEREDES:


NOTE:

* THICKNESS = BEARING VERTICAL DEFORMATION PLUS 1"
** OPT. CONSTRUCTION JOINT (IF CONSTRUCTION JOINT IS USED, CAST LOWER PORTION OF BACKWALL PRIOR TO PLACING DECK REINFORCEMENT
*** EA06 BARS


THE THICKNESS OF THE EPS LAYER (Th EPs ) SHALL BE DETERMINED USING THE FOLLOWING FORMULA:
h = HEIGHT OF BACKWALL IN INCHES
$\Delta L=$ THERMAL MOVEMENT FOR THE ENTIRE TEMPERATURE RANGE IN INCHES (EXPANSION +CONTRACTION )
$T h_{\text {EPS }}=$ EPS THICKNESS IN INCHES (SHALL NOT BE $<10^{\circ}$ )
$T h_{\text {EPS }}=10[0.01 \mathrm{~h}+0.67(\Delta \mathrm{~L})]$

DRAWN BY:
APPROVED BY:
CHECKED BY:

FIGURE H-3: PROPOSED DETAIL
SEMI - INTEGRAL ABUTMENT DETAILS WITH BACKWALL OFFSET FROM ABUTMENT

ISSUED:
SUPEREDES:


NOTE:
THE THICKNESS OF THE EPS LAYER (Th EPs ) SHALL BE DETERMINED USING THE FOLLOWING FORMULA:
h = HEIGHT OF BACKWALL IN INCHES
$\Delta L=$ THERMAL MOVEMENT FOR THE ENTIRE TEMPERATURE RANGE IN INCHES (EXPANSION +CONTRACTION )
$\mathrm{Th}_{\text {EPS }}=$ EPS THICKNESS IN INCHES (SHALL NOT BE <10 ${ }^{\circ}$ )
$\mathrm{Th}_{\text {EPS }}=10[0.01 \mathrm{~h}+0.67(\Delta \mathrm{~L})]$




DRAWN BY:
APPROVED BY: CHECKED BY:

FIGURE H-7: PROPOSED DETAIL
DETAIL A - DIMENSION OF THE SLOT AND CONCRETE KEY WITH RUB PLATES FOR SEMI-INTEGRAL ABUTMENT

ISSUED BY: SUPEREDES:



DETAIL A

## GIRDER END RESTRAIN DETAILS FOR BACKWALL OFFSET FROM ABUTMENT CONFIGURATION

NOTE:

* SEE DETAILS OF GIRDER END RESTRAIN
$L_{\mathrm{th}}$ : LENGTH OF THE DIAGNAL BETWEEN ACUTE CORNERS (SEE RUB PLATE DESIGN EXAMPLE)
W: DECK WIDTH
$\beta$ : ANGLE (SEE RUB PLATE DESIGN EXAMPLE)
$\phi$ : DIAMETER OF THE POSITION DOWEL
$\Delta T$ : |MAXIMUM TEMPERATURE| + |MINIMUM TEMPERATURE| FROM TABLE 5-2


[^0]:    + Refer FE model coordinates (Figure 3-14)

