NATIONAL TRANSPORTATION CENTER

RESEARCH REPORT

Mathematical Modeling for Optimizing Skip-Stop Rail Transit Operation Strategy Using Genetic Algorithm

Young-Jae Lee, Ph.D.
Department of Transportation and Urban Infrastructure Studies
Morgan State University

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16. Abstract

With skip-stop rail transit operation, transit agencies can reduce their operating costs and fleet size, and passengers can experience reduced in-transit travel times without extra track and technological improvement. However, since skip-stop operation does not serve all the stations, passengers at exclusive stopping stations can possibly experience increased access time, waiting time, total travel time, and transfer. Only when the stopping stations are carefully coordinated can skip-stop services benefit passengers and transit agencies.

This research developed an optimization model using a Genetic Algorithm that coordinated the stopping stations for skip-stop rail operation. Using the flexibility of the Genetic Algorithm, this model included many realistic conditions, such as different access modes, different stopping scenarios, different collision constraints, different objective functions, and etc.

For this research, the Seoul Metro system's line No. 4 was used as an example. With skip-stop operation, total travel time became about 17-20 percent shorter than with original all-stop operation, depending on the stopping constraints. In-vehicle travel time became about 20-26 percent shorter due to skipping stations, although waiting, transfer, and additional access times increased by 24-38 percent.

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EXECUTIVE SUMMARY

With skip-stop rail transit operation, transit agencies can reduce their operating costs and fleet size, and passengers can experience reduced in-transit travel times without extra track and technological improvement. However, since skip-stop operation does not serve all the stations, passengers at exclusive stopping stations can possibly experience increased access time, waiting time, total travel time, and transfer. Only when the stopping stations are carefully coordinated can skip-stop services benefit passengers and transit agencies.

This research developed an optimization model using a Genetic Algorithm that coordinated the stopping stations for skip-stop rail operation. Using the flexibility of the Genetic Algorithm, this model included many realistic conditions, such as different access modes, different stopping scenarios, different collision constraints, different objective functions, and etc.

For this research, the Seoul Metro system's line No. 4 was used as an example. With skip-stop operation, total travel time became about 17-20 percent shorter than with original all-stop operation, depending on the stopping constraints. In-vehicle travel time became about 20-26 percent shorter due to skipping stations, although waiting, transfer, and additional access times increased by 24-38 percent.

Each train skipped five to nine stations, which reduced five to nine minutes (up to 8 percent) of the operating time. As mentioned, this model was built to minimize the total travel time. If the model's objective was minimizing operating time or minimizing total cost, the model could reduce operating time more.

Although skip-stop operation can be complicated for the operators and can confuse passengers at the beginning of the service, it can reduce passengers' total travel time and operators' investment and operating costs.

INTRODUCTION

Both transit agencies and passengers can benefit from increased transit operating speed. Obviously, passengers can enjoy shortened travel time. Transit agencies can benefit from accelerated rail transit operation's shorter cycle times, which, consequently, lower operating costs and reduce fleet size. If a transit agency decides to keep the same fleet size and the same operating costs, then it can increase service frequencies. Eventually, all these advantages can attract more passengers and increase the agency's revenue.

New technology, new rolling stock, and/or better alignment can increase operating speed; however, they usually require huge investment. In addition to those hardware upgrades, the accelerated operational scheme can increase operating speed by skipping stations. Although accelerated service can increase operating speed, passengers' total travel time may not reduce as shown in Figure 1. Good selection and coordination of skipping stations are necessary to reduce passengers' travel time.

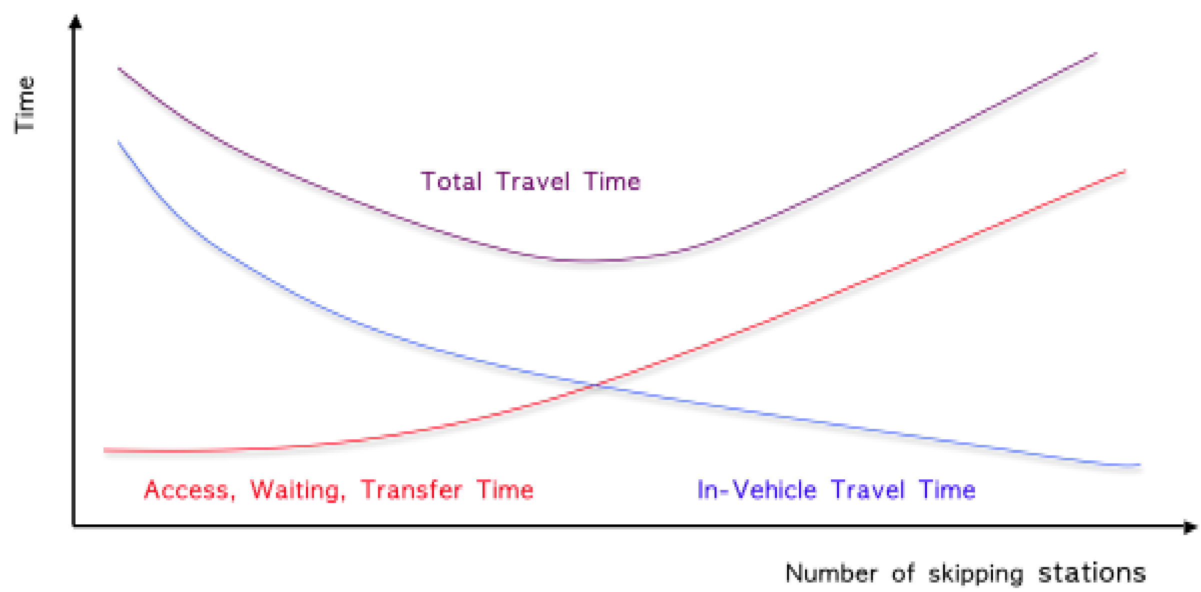


Figure 1. Conceptual Diagram to Show the Advantage of Skipping Stations

Three operational methods can increase rail transit operating speed without requiring technological investment: express/local service, zonal service, and skip-stop service. Despite the potential advantages of these accelerated methods, except for some rail transit lines in New York City and Philadelphia, most current rail transit in the United States uses local or regular service, which is the all-stop operational scheme. This is often due to operational complexity and the lack of methodology in modeling an optimal operational scheme. Nonetheless, there is considerable interest in combining the regular scheme with different accelerated methods in order to improve operating speed and efficiency.

Although both express/local service and zonal service do not require technological investment, they require additional track so that express trains can pass local trains and trains that serve farther zones can pass trains that serve nearer zones (Figures 2a and 2b). Only the skip-stop operation scheme can be implemented without additional track and technological investments, because two different trains, A train and B train, can keep safe separation between trains with proper coordination of stopping stations as shown in Figure 2c. However, since the trains do not stop at all stations, passengers at skipped stations may experience increased access time or waiting time and they may experience transfer.

This research found the optimal coordination of stopping stations mathematically that can increase and improve the overall benefits of the skip-stop operational scheme, and minimize its disadvantages. While the feasible conditions are known conceptually and intuitively, to the best of the author's knowledge, no study has defined them mathematically. This research used the Genetic Algorithm to find the optimal coordination of the stopping stations.

LITERATURE REVIEW

Station-to-Station Travel Time Components (Vuchic, 2007)

Rail transit's station-to-station travel time consists of five components: acceleration, constant speed, coasting, braking, and standing time. Obviously, acceleration and braking take more time than with a constant speed for the same distance traveled. If a train can skip a station, it can maintain a constant speed, avoid breaking and accelerating, skip standing time, and consequently, reduce its travel time.

The following equations were used to compute acceleration time, accelerating distance, braking time, braking distance, travel time with constant speed, and travel distance with constant speed.

$$V = 3.6 \cdot a \cdot t_a \tag{1}$$

$$s_a = \frac{a \cdot t_a^2}{2} \tag{2}$$

$$s_v = \frac{V \cdot t_v}{3.6} \tag{3}$$

$$V = 3.6 \cdot b \cdot t_b \tag{4}$$

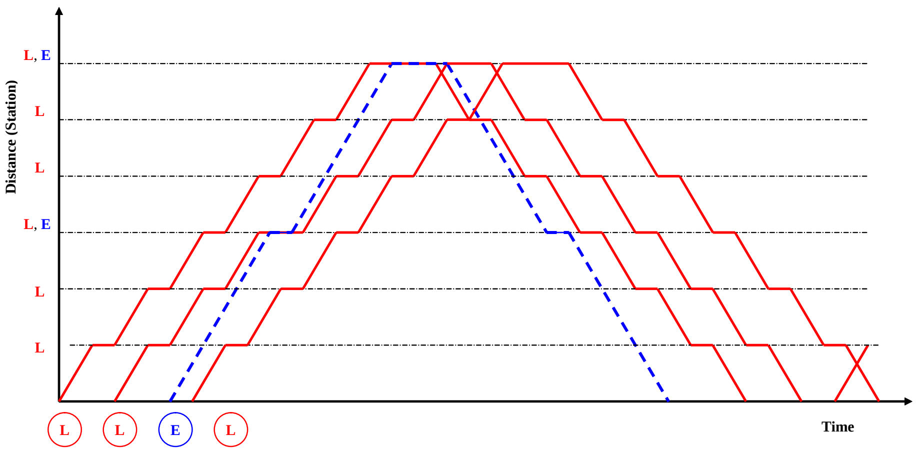
$$s_b = \frac{a \cdot t_b^2}{2} \tag{5}$$

where
V = Speed (km/h)
a = Acceleration rate (m/sec^2)
t_a = Acceleration time (sec)
s_a = Distance traveled with acceleration (m)
t_v = Time traveled with constant speed (sec)
s_v = Distance traveled with constant speed (m)
b = Braking rate (m/sec^2)
t_a = Braking time (sec)
s_a = Distance traveled while braking (m)

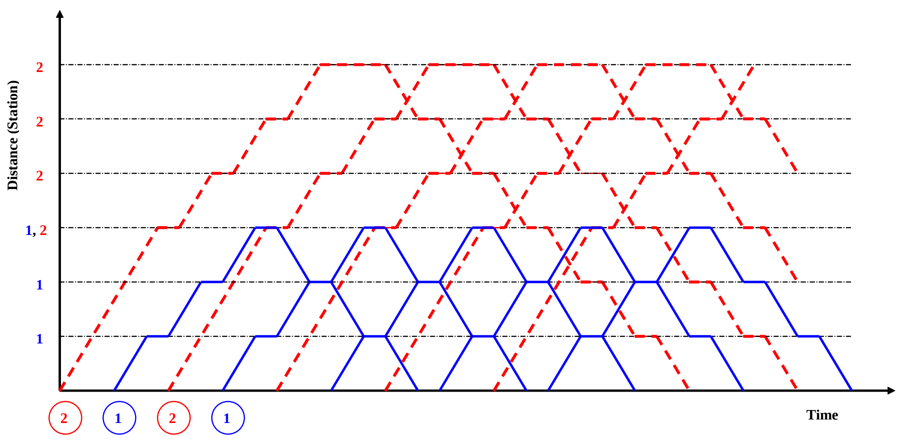
Accelerated Rail Service (Vuchic, 2005)

Although accelerated rail operation provides many benefits to users and operators, accelerated rail transit operational schemes may not be implemented with the existing two tracks. As shown in Figures 2a and 2b, express and zonal service require a third track (at or between the stations) so that the following train can pass the previous train.

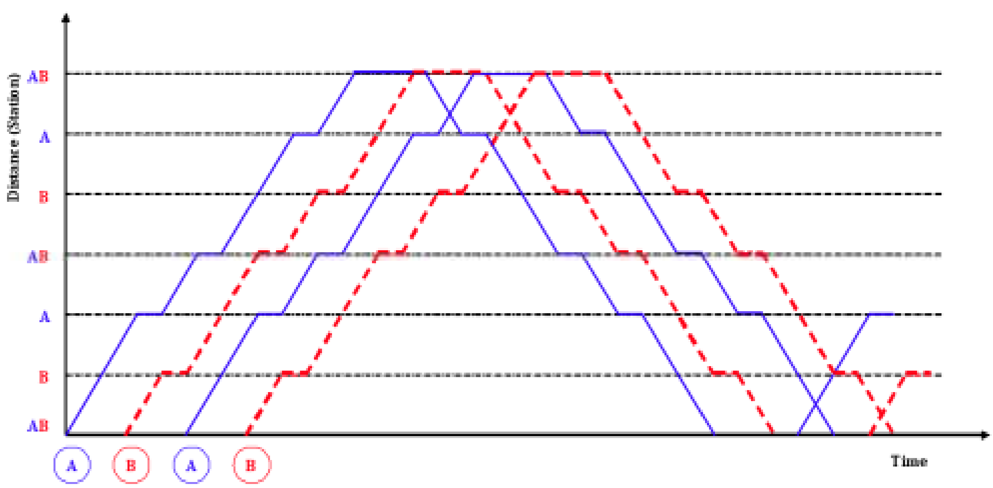
However, as shown in the Figure 2c, skip-stop operation can be implemented without additional track when stopping stations are properly coordinated. Although the distance between two consecutive trains can be narrower at certain sections, they do not collide unless the headway is very short, which is minimum headway plus about one minute.



(a) Operational Time-Distance Diagram for Express Service



(b) Operational Time-Distance Diagram for Zonal Service



(c) Operational Time-Distance Diagram for Alternate Stations Stopping Service

Figure 2. Operational Time-Distance Diagram for Accelerated Rail Operation

Skip-Stop Operation

There are rail systems that use skip-stop operation, such as the SEPTA in Philadelphia and the rail system in Santiago, Chile. However, there have been no rigorous efforts to optimize skip-stop operation, and most research has been based on empirical analysis.

Genetic Algorithm

A Genetic Algorithm (GA) is a heuristic search method that imitates the process of natural evolution. It is motivated by the principles of natural selection and survival of the fittest individuals (Jong, 1998). This method is commonly used to generate useful solutions to optimization problems. There is now considerable evidence that genetic algorithms are useful for global function optimization and NP-hard problems despite continuous arguments.

The common benefit of a GA is its capability to improve the internal knowledge of an environment. This corresponds to a clear understanding of the possible structural changes and the legal operators for selecting and making changes.

In GAs, the problem is treated as the environment and a set of possible solutions is treated as the population. In evolution, a child inherits good features from its parent via gene recombination or mutation. In GAs, recombination and mutation play key roles in the search space (Jong, 1988).

Initially, the process starts by generating random individuals from the entire range of possible solutions (the search space) to form an initial population. The population size depends on the nature of the problem. Each individual in the population is represented by an encoded solution, called a chromosome. The individuals then compete with each other to produce children. In each generation, the fitness of every individual in the population is evaluated. Individuals are selected from the current population based on their fitness, recombined, and randomly mutated to form a new population. The new population is used in the next iteration of the algorithm (Wikipedia; Goldberg, 1989).

The process stops when a terminating condition is reached. This condition could be defined based on the nature of the problem. Some common terminating conditions include the following: a solution is found that satisfies the minimum criteria; a fixed number of generations is reached; the allocated budget (computation time/money) is reached; the highest-ranking solution's fitness is reaching (or has reached) a level such that successive iterations would not produce better results; or a combination of those conditions is achieved (Wikipedia; Goldberg, 1989).

METHODOLOGY

This research considers skip-stop operation as a choice for an accelerated rail transit operational scheme since it does not require additional track. Trains using this scheme increase their speed by skipping stations. However, in order to minimize inconveniences for passengers due to skipping stations, stopping stations must be selected properly and coordinated carefully.

In order to find the best coordination of stopping stations, this research developed the optimization process. The optimization process includes four main components—objective function, constraints, cost estimation, and genetic algorithm—to generate potential solutions. Like a general genetic algorithm, generated solutions are evaluated and compared using the fitness test with the previous optimal solutions. Then, the process keeps searching for the better solution until there is no better solution or until the algorithm reaches the given number of iterations as shown in Figure 3.

Variables—such as origin-destination demand data, station-to-station distances, access modes, access times, etc.—are needed to estimate the user's travel time. The mathematical model in this research will use those variables as inputs for the model, and the results of the model will show the best coordination of stopping stations to minimize the objective function and travel time estimation.

Optimization Process Using Genetic Algorithm

In order to use the Genetic Algorithm for the optimization, the concepts of genes, chromosomes, and their fitness should be defined. In this project, chromosomes are the stations and Matrix S is the gene containing the station types (A, B, or AB chromosomes). The fitness of each gene is estimated based on the objective function. The objective function in this project is the total travel time. It is based on the developed models for calculating the total travel time, which is the fitness of Matrix S. Figure 3 presents the overall view of the optimization process in this project using Genetic Algorithm, and Table 1 shows the operators used to generate the children for the Genetic Algorithm in this research.

For this particular optimization model, in addition to the general operators for the Genetic Algorithm, customized crossover operators were designed, which are types 3-6 in Table 1. Since it is important to keep the better origin and destination pairs together (instead of as a single chromosome), the better 40 percent of chromosome pairs were kept and others were replaced with operators 3 and 4. In addition, the worse 40 percent of chromosome pairs were replaced with operators 5 and 6. The 40 percent comes from experiments with different numbers in this research; however, it will be necessary to find the optimal number in the future study.

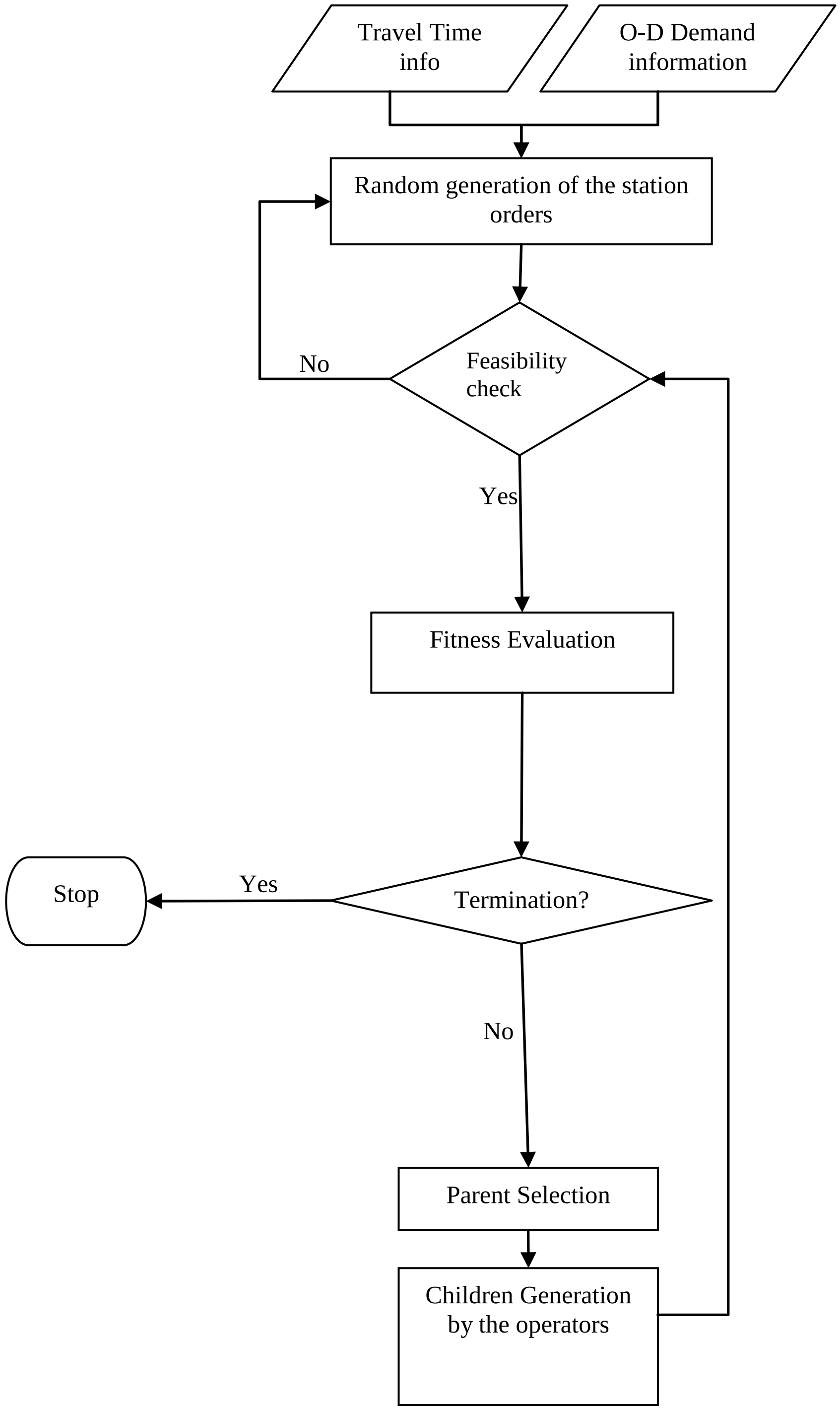
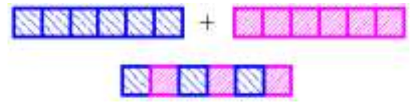
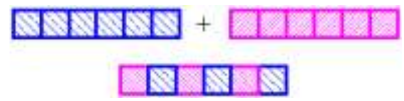
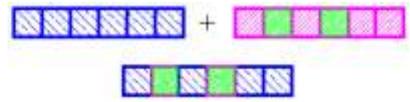
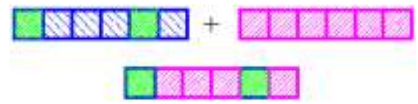
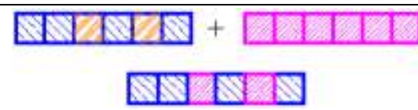
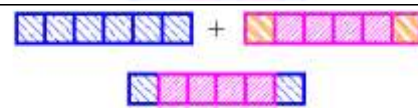
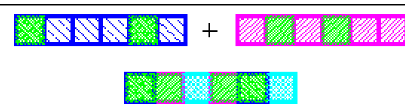
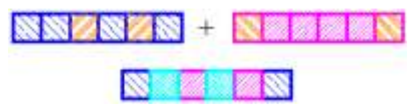
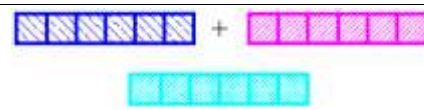
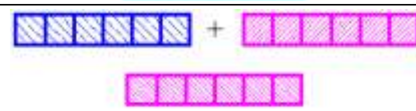
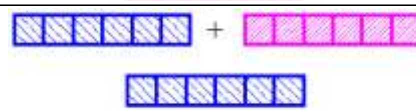


Figure 3. Overall Procedure for Finding an Optimal Solution

Table 1. Operators for the Genetic Algorithm Used in this Model

	Type	Explanation	Figure
1	Cross over	Creates a child which inherits odd cells from Father and even cells from Mother	
2	Cross over	Creates a child which inherits even cells from Father and odd cells from Mother	
3	Customized cross over	Creates a child which inherits best genes (40%) from Mother and the others from Father	
4	Customized cross over	Creates a child which inherits best genes (40%) from Father and the others from Mother	
5	Customized cross over	Creates a child in which the worst genes (40%) from Father omitted and replaced by Mother gene	
6	Customized cross over	Creates a child in which the worst genes (40%) from Mother omitted and replaced by Father gene	
7	Combined cross over and mutation	Creates a child which inherits the best Mother and Father genes (40% each) and its other genes are random	
8	Combined cross over and mutation	Creates a child in which the worst genes (40% each) from Mother replaced by Father and worst gene from Father replaced by Mother, the other genes replaced randomly	
9	Whole Non-Uniform Mutation	Creates a child which inherits random genes (maintaining the diversity of genes in the population)	
10	Mother Saver	Creates a child which inherits all of Mother genes (replacing the Mother in the new generation)	
11	Father Saver	Creates a child which inherits all of Father genes (replacing the Father in the new generation)	

Objective Function

There are three types of objective functions: user travel-time minimization, operator's benefit maximization, and total cost minimization.

As mentioned, skip-stop operation reduces in-vehicle travel time for users and increases operating speed for operators. However, some users will experience increased waiting time, access time, egress time, and possibly, transfer time. Thus, there is no guarantee that skip-stop operation will reduce total travel time.

The selection and coordination of stopping stations can be done based on the objective function. If the objective function is user travel-time minimization, the proper selection and coordination of stopping stations will minimize the total travel time of the entire users, including their in-vehicle travel time, access time, egress time, waiting time, and transfer time.

Skip-stop service always increases operating speed, which results in reduced operator costs and fleet size. However, this service does not always produce advantages for operators. If passengers' total travel time increases because of the skip-stop operation, less people will use the transit service and transit agencies will lose revenue. The operator's benefit consists of operator's costs and revenue. Under the objective function of operator's benefit maximization, the selection and coordination of stopping stations will maximize the operator's benefit.

The last possible objective function is minimization of total costs, including user travel time and operator costs. The selection and coordination of stopping stations can be developed to minimize total costs.

In this research, user travel-time minimization was used; however, in future research, other objective functions can be applied, and the results of the different objectives can be compared and evaluated. Also, combination of different objective functions for different times of the day, such as minimizing travel time for the peak hours and maximizing revenue for the off-peak hours, can be examined.

Cost Estimation (Fitness)

The mathematical model estimates passengers' total travel time through each coordination of stopping stations, which is also the objective function of the model. Each passenger's total travel time—which includes access time, waiting time, in-vehicle travel time, transfer time, and egress time—is formulated for each selection of the stopping stations. As a result, the model will suggest the best coordination of the stopping stations for the skip-stop operation strategy.

Three Types of Origin-Destination Pairs

For skip-stop operation, the stations were categorized as Stations A, B, and AB. A trains stop at A stations and AB stations, and B trains stop at B stations and AB stations. Consequently, origin-destination trips are categorized into 9 groups, such as A-A, A-B, A-AB, B-A, B-B, B-AB, AB-A, AB-B and AB-AB. If the origin-destination pair is AB-AB, then passengers have the same headway, access, and egress time, while enjoying reduced in-vehicle travel time (Type I in Table 2).

If an origin-destination pair is other than AB-AB, then passengers' headway will be twice as long as the AB-AB passengers' because they can only take either an A train or B train, not both. If headway becomes twice as long, then waiting time can be up to twice as long. If there is no scheduling information, then waiting time becomes twice as long. However, if scheduling information is available, then average waiting time can be less than half of the headway because passengers can come to stations just before the train arrives.

If the origin-destination pair is either A-B or B-A, then the passenger will need a transfer to reach the destination station or the passenger will need to change the origin or destination station to avoid transfer. Passengers will choose to transfer or change their origin or destination to minimize their travel time depending on their exact origin and destination location (Type III in Table 2). The rest of the cases require headway and waiting time that are up to twice as long (Type II in Table 2).

Table 2. Three Types of Origin–Destination Trips for Nine Origin-Destination Combinations

OD Type	Orig.	Dest.	Decision	Penalty
Type I	AB	AB	Take any coming train	None
Type II	A	A	Take A train	$h^{new}= 2h$, $w^{new}\leq 2w$
	A	AB	Take A train	$h^{new}= 2h$, $w^{new}\leq 2w$
	B	B	Take B train	$h^{new}= 2h$, $w^{new}\leq 2w$
	B	AB	Take B train	$h^{new}= 2h$, $w^{new}\leq 2w$
	AB	A	1. Wait for A train 2. Take B train and walk to A station	Min {($w^{new}\leq 2w$) or (additional egress)}
	AB	B	1. Wait for B train 2. Take A train and walk to B station	Min {($w^{new}\leq 2w$) or (additional egress)}
Type III	A	B	1. Take A train and transfer to B train at AB station 2. Go to B station to take B train 3. Take A train, go to A station, and walk to B station	($w^{new}\leq 2w$) + Min{Min(additional access time or additional egress time) or transfer time}
	B	A	1. Take B train and transfer to A train at AB station 2. Go to A station to take A train 3. Take B train, go to B station, and walk to A station	($w^{new}\leq 2w$) + Min{Min(additional access time or additional egress time) or transfer time}

h = headway, w = waiting time

Reduced Travel Time by Skipping a Station

The amount of reduced travel time depends on acceleration rate, braking rate, maximum constant speed, operation strategy (e.g., whether there is coasting), dwell time, and the distance between stations. Consequently, computation of the saved time can be complicated and has many variables.

According to transit agencies including Washington Metropolitan Area Transit Authority (WMATA) and Seoul Metropolitan Rapid Transit Corporation (SMRT), acceleration rate and braking rate range from 0.75 to 3 mph per second, and maximum speed ranges from 40 to 80 mph.

Assuming 2 mph per second (3.2 km/h/sec) for acceleration and braking rates, to reach the assumed maximum speed of 60 mph, it takes 30 seconds while it travels 0.25 miles by equations 1 and 2. From 60 mph, it also takes 30 seconds to stop while it travels 0.25 miles by equations 4 and 5. Using equation 3, it takes 15 seconds to travel 0.25 miles with a 60 mph constant speed. As a result, not accelerating and not braking can save 15 seconds each. Standing time is about 30 seconds at each station. Under those assumptions, skipping one station can save a total of one minute. Therefore, this research assumed that one minute can be saved each time a train skips one station. However, this computation is based on the above assumptions, so the real time savings by skipping one station vary by station and operation strategy.

Transfer

Type III passengers need to transfer or change their origin or destination station. There are two types of transfers. If the AB station is inside the origin and destination, then passengers can transfer at the AB station and they will spend only additional transfer time, which is headway between two trains, in addition to their original travel time. If there is no transfer station, then passengers need to find a transfer station outside the origin and destination. In this case, passengers not only will spend transfer time, but also will add in-vehicle time (to go to the transfer station and come back) to their original travel time.

Access Modes and Additional Access Time for Changing Origin or Destination Stations

There are some other concerns in the estimation of the total passengers' travel times. The first concern is the access mode to the train station for the Type III passengers. Train users can access the stations by foot, car, bicycle, or feeder bus. Depending on their access mode, their additional access time to their origin or destination stations varies and their decision to transfer or change the origin/destination is based on their additional access time and transfer time.

In this research, two groups—those who walk to the station and those who access it by car or feeder bus—were considered to compute the access time to other origin or destination stations. Since passengers who use Park & Ride, Kiss & Ride, or a feeder bus have a shorter additional access time to the new origin station that does not require transfer to go to their destinations, their average access time to the other origin station will be shorter than passengers who walk to the new origin station. However, the exact amount of additional access time depends on the locations of the origins.

For simplicity, this research assumed that passengers were uniformly distributed throughout the area. When users chose their origin stations, they selected stations that required the shortest total travel time.

Depending on the origin's location, going to the other station increases a different amount of travel time. For the one extreme case, changing an origin station will not increase total travel time at all when passengers' origin is equidistance from the two stations. Thus, passengers arbitrarily choose an origin station, and changing the origin station will not add any additional access time.

When the origin is at the station, changing the origin station requires whole distance traveled from the original station to the new station. The additional access distance is the whole distance between the two stations, and the additional travel time is the additional access time to the other station minus station-to-station, in-vehicle travel time. For example, if it takes 15 minutes to walk to the new origin station and station-to-station, in-vehicle travel time is 3 minutes, then the additional access time is 12 minutes. If the access mode is auto and the additional driving time is 4 minutes, then the additional access time is only one minute.

Therefore, in the above situation, the minimum additional access time is zero minutes, and the maximum additional access time is 12 minutes by foot or one minute by auto. Since uniform distribution of total passengers is assumed, the additional access time is uniformly distributed between minimum additional access time and maximum additional access time. The distribution of walking passengers and driving passengers is a variable for this model, and this distribution rate can be set differently for each origin-destination pair.

Transferring vs. Changing the Origin or Destination Stations

For Type III origin-destination pairs, each pair was examined to find whether passengers would take a transfer or change their origin or destination. Access modes and origins determine the additional access times resulting from an origin or a destination change, so their decision between transfer and changing origin/destination can be made based on their additional access time and transfer time.

In order to estimate the number of passengers who will change their origin station and the number of passengers who will stick to their original station and transfer, the research compared the transfer time and additional access time of Type III passengers. Since transfer time is fixed, regardless of whether the transfer station is inside, outside, or between the origin and destination stations, transfer time is used as a standard. The range of additional access time to the other station that avoids the transfer was estimated based on access modes and passengers' origins. Using transfer time and additional access time, it was assumed that passengers would change their origin if a transfer would result in longer travel time, and passengers would stay at their origin and transfer if a transfer would result in shorter travel time.

Different Weights for Travel Time Components

Total travel time consists of access time, waiting time, in-vehicle travel time, transfer time, and egress time. Although their units are all same, the perception to passengers may vary. Some researchers show that access time, waiting time, transfer time, and egress time can be as costly as three times in-vehicle travel time for the same amount of time (Kittelson & Associates, 2003).

Although this algorithm can handle different weights for different travel time components, the same time value for all travel time components was applied to this example in order to show the absolute time amount for each travel time component and show how the trade-off between travel time components works.

Algorithm Duration

The other concern is continuous transit operation during the day. The optimal solution for a certain period, such as morning peak, is not necessarily the optimal solution for a whole day or whole week. To make the precise evaluation, O-D demand for each hour and each hour's headway must be available. However, for simplicity, this research used one peak-hour demand and headway to find the optimal coordination of the stopping stations. Consequently, the result is optimal for that period only. Once the data is available for a whole day, it will not be difficult to find an optimal solution for a day or week.

This research developed a mathematical model that suggested optimal stopping stations for the skip-stop operational scheme. The model considers the nine aforementioned cases to minimize total passenger travel time in the route. Obviously, some passengers will have longer travel times due to the longer headway and transfers, but good selection of the alternate stopping stations can save travel time for more passengers. If a particular route is not suited for the skip-stop operation, then the results will show all stations as AB, which means that all trains must stop at all stations. The all-cost estimation process is shown in Figure 4.

Constraints

The most important constraint in this algorithm is the avoidance of collisions between the two trains. Unlike regular service, in which trains stop at all stations, skip-stop operation allows trains to skip stations. Once a train skips a station, its distance from the preceding train, which stops and skips at different stations, becomes shorter. Since two trains should not collide, this constraint is critical. In this study, while two trains avoid collision, four scenarios were suggested based on the rules for stopping stations and initial headways in Table 3.

Scenario I has the most restricted constraint, as it provides the least amount of feasible solutions. The constraint does not allow two consecutive types of exclusive stations, even if there is a general station between two same types of exclusive stations. For example, A-AB-A combination is not allowed. There must be a B station between A stations, even if there is an AB station in between them. In other words, after an A station there must be a B station before an A station is located. Since this scenario requires a different exclusive stopping station between other kinds of stopping stations, users can accept this scenario relatively easily because the distribution of the stations looks uniform. Because no train skips two more stations than the other type of train skips, two different types of trains will not collide as long as the headway is longer than the sum of safety distance between the two trains and the time saved by skipping one station.

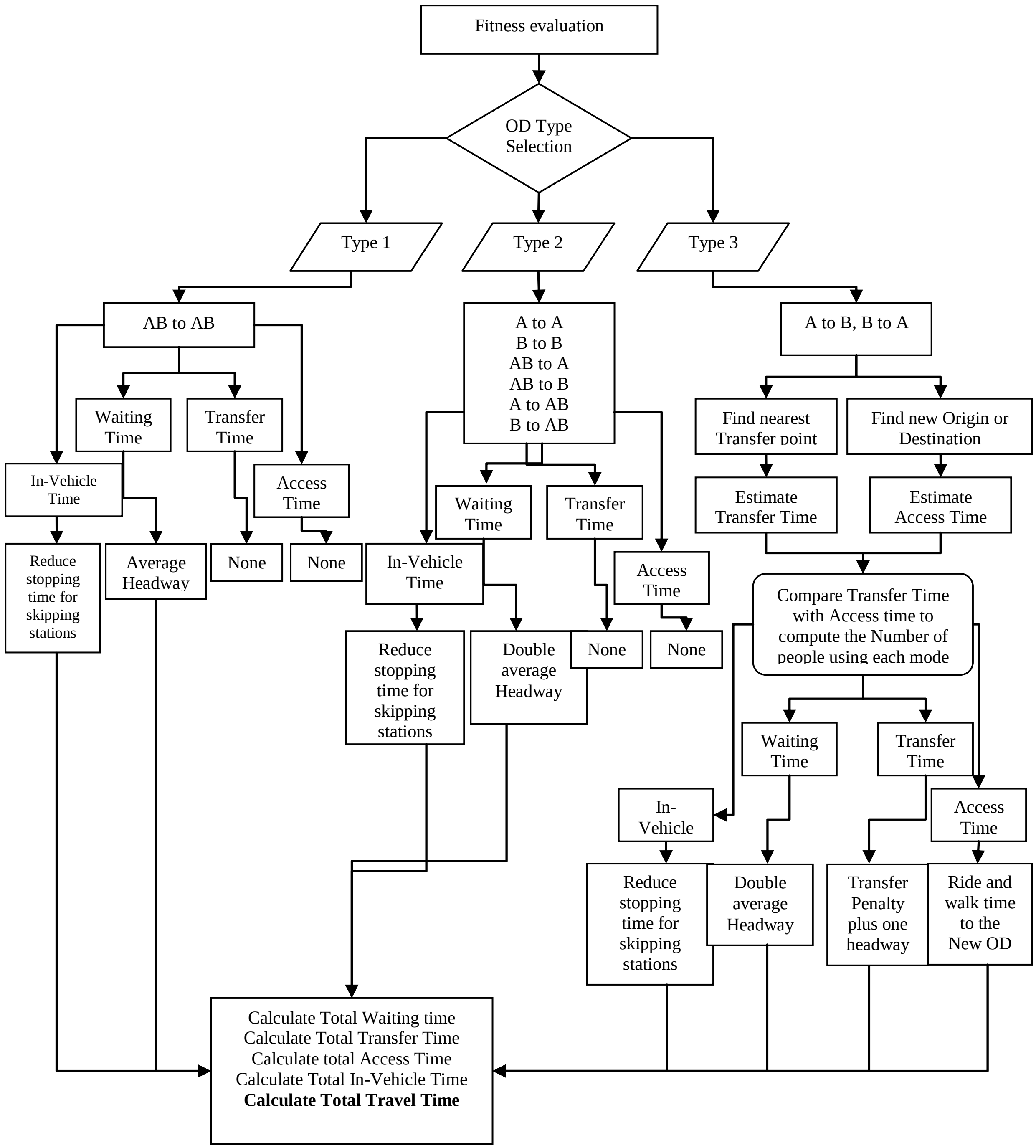


Figure 4. Fitness Evaluation Process

Table 3. Three Types of Collision Constraints

Collision Constraints	Definition	Constraints
Scenario I	Different type of exclusive stopping station between exclusive stopping stations	0 ≤ (CIS x saving time by skipping) ≤ 1 or -1 ≤ (CIS x saving time by skipping) ≤ 0
Scenario II	No consecutive same type of exclusive stopping stations	-(Headway – Safety) ≤ (CIS x Saving time by skipping) ≤ (Headway – Safety) & $-2 < (CIS_i – CIS_{i-1}) + (CIS_{i+1} – CIS_i) < 2$
Scenario III	Uniform headway between different types of trains	-(Headway – Safety) ≤ (CIS x Saving time by skipping) ≤ (Headway – Safety)
Scenario IV	Uniform headway between same type of trains	Min ≤ (CIS x Saving time by skipping) ≤Max, Max ≤ 2 x (Headway – Safety), Min ≥- 2 x (Headway – Safety), (Max – Min) ≤2 x (Headway – Safety)

CIS: Collision Index Score

However, if the same type of exclusive stopping stations are allowed to repeat whether there is a general station in between them or not, it is necessary to check whether two trains will collide at every station. To check whether the two trains will collide, the arrival times of two different, consecutive trains (A train and B train) must be checked. If the difference is greater than safety time and standing time, then the two trains can operate without collision. If the difference is less than safety time and standing time, then the two trains will collide and the selection of stopping stations is infeasible.

Scenario II does not require a different kind of exclusive stopping station between two of the same kind of stopping stations as long as there is a general stopping station between them. As long as there are not two consecutive, same types of stopping stations, this scenario is acceptable. For example, this scenario will allow A-AB-A combination. Since this scenario relaxes the constraint of Scenario I, Scenario II provides more feasible solutions and better results.

Although it does not distribute the stopping stations as uniformly as Scenario I does, users can comfortably accept Scenario II because they can easily access different type of stations. For example, if users want to go to the other type of station to catch the other type of train because the original station requires transfer, they can go to the adjacent station, which is either the other type of station or a general stopping station.

Scenarios III and VI do not restrict the distribution and coordination of stations as long as the coordination of stopping stations prevents collision. The difference between Scenarios III and IV is the initial headway. Scenario III keeps the initial headway between the two different types of trains. Scenario IV keeps the initial headway between the two same types of trains, but not necessarily between two different types of trains (i.e., A-A trains should have the same headway as the B-B trains, but headway between A-B trains and B-A trains does not need to be the same).

In order to consider those four scenarios, “1” was assigned to A stations, “0” was assigned to AB stations, and “-1” was assigned to B stations. Then, at each station, the cumulative score from the terminal, which is called “Collision Index Score,” was used to compute the separation between the two trains. If the number becomes bigger, positively or negatively, one train is going much faster than the other and the chance of collision becomes higher. If headway, safety distance between trains, and time saved by skipping are given, then the feasible area for each collision constraint can be defined (Table 3).

Obviously, Scenario I has fewer feasible solutions than Scenario II, Scenario II has fewer feasible solutions than Scenario III, and Scenario III has fewer feasible solutions than Scenario IV. In this research, the results and time savings from all four scenarios will be presented and discussed.

EXAMPLE AND RESULTS

After the mathematical model was developed, the real data from Seoul Metro in Korea was applied to see if accelerated service was feasible for that rail transit line. This research selected Seoul Metro's Line No. 4 as an example, which includes the Gwacheon-Ansan line. Korea Railroad serves Line No. 4 (Figure 5). Line No. 4 has 48 stations, and the total travel time between two terminals during the morning peak is 1 hour and 52 minutes, with 2.5 to 3 minutes of headway. Line No. 4 currently provides not only local service, but also zonal service and express service. Because there was limited data for this model, this example only tested the applicability (or functionality) of skip-stop operation for the metro line, not the actual feasibility.

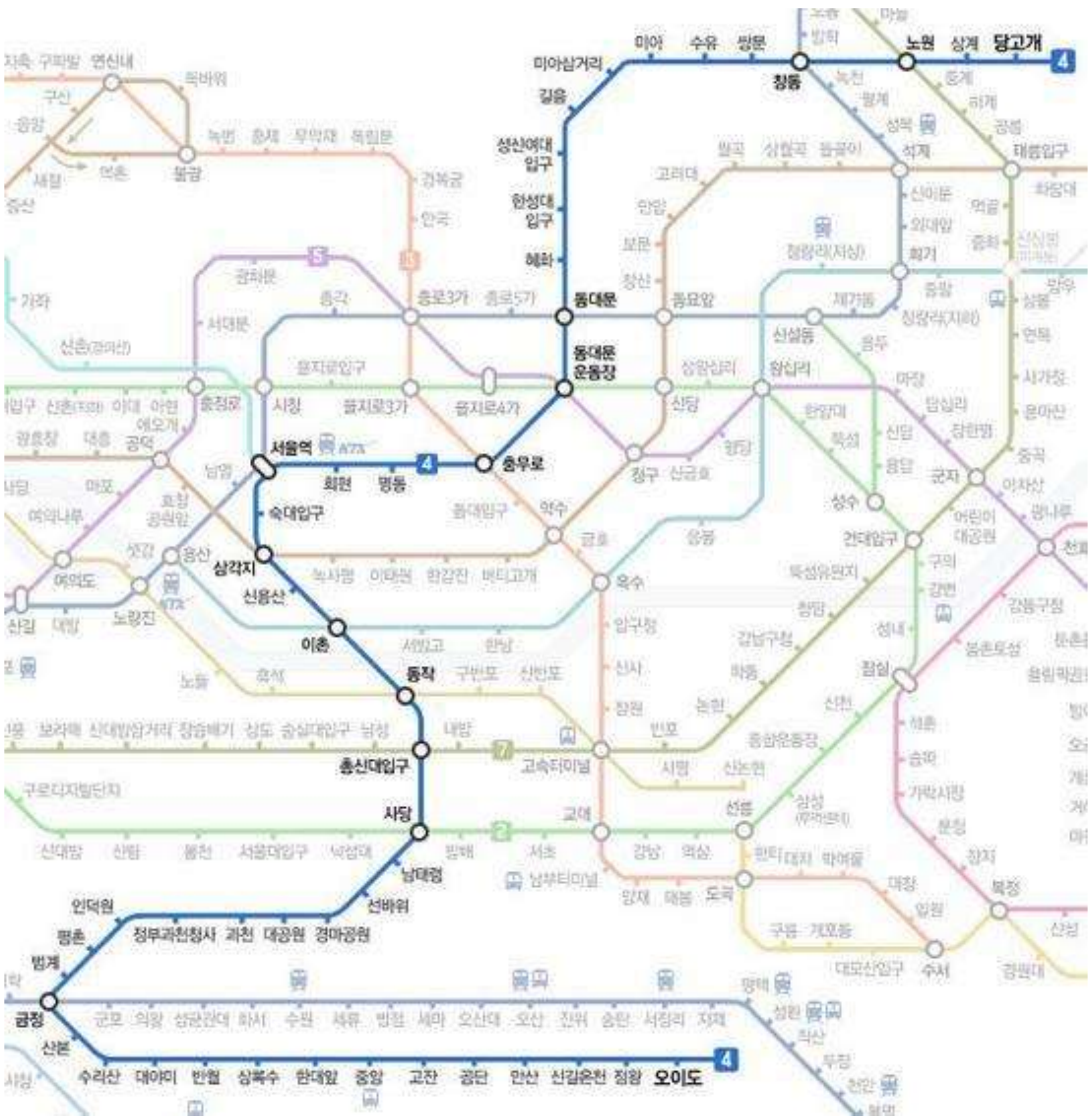


Figure 5. The Seoul Metro System's Line No. 4

In order to run this model, hourly origin-destination demand was essential. However, the only available data was monthly O-D demand and each hour's number of boarding and alighting passengers. As a result, it was necessary to manipulate the data to get the hourly O-D demand from the monthly O-D demand and each station's morning peak hourly boarding and alighting ratio. This analysis used the O-D demand from October 2008 (Table 4). The Geum Jung station was missing from the O-D data, so O-D data for only 47 stations was used with three-minute headway.

The assumptions for the examples were as follows:

- Because standing time at the station is 30 seconds from the schedule, a train can save one minute (including acceleration time, deceleration time, and standing time) if it skips one station.
- Safety distance between two trains is one minute. Table 5 shows the O-D travel time schedule.
- Access time to the other station by foot is six times longer than rail's in-vehicle travel time.
- Access time by auto or feeder bus is 1.5 times longer than rail's in-vehicle travel time.
- Total passengers are uniformly distributed; accordingly, their additional access time is distributed uniformly between minimum additional access time and maximum additional access time.
- Seventy percent of passengers walked to the station, and 30 percent of passengers arrived at the origin station via a car or a feeder bus.

Table 6 shows the four optimal coordinations of stopping stations after 5,000 iterations, in addition to the original all-stop scenario, using different feasibility constraints to avoid collision.

As programmed, Scenario I always has B stations after A stations, even when there are AB stations in between the A and B stations. Scenario II has A station (19^{th} station) after A station (16^{th} station) with no B station between them because there are AB stations (17^{th} and 18^{th} stations) between A stations.

In Scenario III, since skipping a station saves one minute and safety distance between two trains is one minute, skipping two stations is allowed when two different types of trains have uniform headway under the three-minute headway assumption. If one type of train skipped two stations, then the other type of train can skip as many as four stations before the previous train skips another station. In this example, stations from 12^{th} station to 14^{th} station were allocated as A stations, because before 12^{th} station, there was one more B station than A station from the terminal.

Table 4. Origin-Destination Demand (One Hour during the Morning Peak)

O/D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	113	192	145	75	139	53	97	52	39	28	55	58	30	1721	44
2	130	0	121	199	165	309	117	202	155	110	78	172	169	84	14144	143
3	123	57	0	133	156	264	96	170	125	85	58	136	83	51	4193	102
4	124	142	202	0	138	275	120	204	137	97	70	156	98	60	5888	118
5	89	172	328	205	0	160	129	228	152	155	121	285	231	143	33129	237
6	122	232	408	305	115	0	145	267	133	153	119	225	198	131	25823	192
7	48	94	158	142	105	165	0	183	71	72	66	94	94	53	4976	70
8	84	161	270	233	184	270	162	0	118	152	173	221	176	112	19686	183
9	40	106	176	131	102	133	61	105	0	83	91	137	113	68	7716	126
10	20	49	87	68	68	95	36	89	50	0	34	90	72	48	3441	82
11	12	29	47	39	42	58	25	75	43	26	0	46	43	25	1055	48
12	6	16	28	23	24	28	9	24	15	13	7	0	11	11	127	22
13	3	7	8	7	9	12	4	10	7	6	4	5	0	1	0	6
14	2	4	6	5	7	9	3	7	5	4	3	5	1	0	0	8
15	2	5	8	6	9	11	4	10	7	5	4	6	2	3	0	5
16	2	6	12	9	11	14	4	12	8	7	5	11	6	9	49	0
17	2	6	8	7	9	11	4	11	7	6	4	6	8	5	42	4
18	2	5	8	5	8	11	4	9	9	6	4	13	3	7	24	16
19	5	13	22	16	22	29	10	23	19	18	14	35	11	14	145	33
20	1	2	4	3	4	5	2	4	3	2	2	5	1	2	3	6
21	3	7	10	8	13	18	6	16	11	8	6	15	3	7	23	14
22	1	3	6	4	5	8	3	6	5	5	4	11	4	4	17	18
23	1	1	2	2	2	4	2	3	3	3	2	7	3	3	7	8
24	5	8	10	14	18	30	10	25	25	24	19	56	30	31	951	81
25	6	11	17	17	22	33	9	26	27	24	17	53	22	25	560	63
26	0	0	1	0	1	1	0	1	1	0	0	1	0	1	0	1
27	2	3	4	5	5	9	3	7	6	4	4	8	3	3	5	8
28	1	2	3	3	4	7	2	8	5	4	4	6	4	4	7	10
29	2	7	14	15	17	26	7	18	9	10	8	13	6	6	5	21
30	1	1	2	3	3	5	2	4	5	6	5	21	9	9	13	24
31	3	6	13	10	12	15	6	15	12	9	9	25	9	10	17	31
32	4	9	15	12	16	26	9	21	21	17	14	41	24	26	43	57
33	2	4	7	6	7	14	4	9	12	10	8	28	14	16	28	46
34	6	10	19	15	20	35	10	23	33	26	20	77	34	39	61	98
35	2	4	7	5	7	16	4	12	13	12	9	34	20	19	31	52
36	0	1	1	1	2	3	1	2	2	2	1	8	3	4	8	8
37	0	0	1	1	1	2	1	1	1	1	1	3	2	2	3	4
38	0	1	2	1	2	4	1	2	3	2	2	7	5	5	6	7
39	4	5	8	6	9	17	6	12	16	12	8	33	24	26	26	45
40	1	3	5	3	5	8	3	5	7	6	4	19	10	13	13	24
41	2	4	8	6	7	14	4	10	12	10	8	32	15	16	20	36
42	2	3	5	3	4	10	3	7	8	8	6	19	12	11	14	25
43	1	1	2	1	2	4	1	3	3	3	2	7	6	6	5	8
44	3	3	6	5	6	12	5	10	9	6	5	16	15	19	11	21
45	0	0	0	1	1	1	0	1	1	1	0	2	1	2	2	3
46	2	3	6	5	6	9	4	6	6	5	3	9	7	8	7	13

47	4	7	7	6	10	16	6	15	11	7	6	9	8	6	7	11
O/D	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	47	16	18	6	23	5	3	10	9	0	2	1	3	1	3	4
2	146	46	64	19	60	15	4	20	25	1	3	3	9	1	10	11
3	83	33	40	10	32	10	3	10	15	1	2	2	6	1	6	8
4	104	30	45	10	41	10	3	21	22	0	2	2	13	2	8	10
5	214	60	91	25	106	20	7	42	43	2	5	5	19	3	13	18
6	167	74	80	21	98	22	9	52	51	1	5	5	22	4	11	21
7	68	28	33	9	40	8	4	20	17	1	2	2	7	2	6	8
8	168	54	67	16	87	16	6	44	37	1	4	6	15	3	12	17
9	107	45	50	12	60	13	6	36	38	1	3	4	9	4	10	15
10	57	31	33	8	32	9	5	28	26	1	1	2	5	3	6	11
11	37	15	21	4	21	7	2	17	15	0	1	2	4	2	4	6
12	10	18	10	2	9	4	2	13	12	0	0	0	1	2	2	4
13	9	2	2	0	1	1	0	4	2	0	0	0	0	0	1	1
14	5	4	2	1	2	1	0	4	3	0	0	0	0	0	1	1
15	5	4	5	1	4	1	1	5	3	0	0	0	0	1	1	2
16	4	10	6	2	4	3	1	10	9	0	0	0	0	1	1	3
17	0	3	4	1	4	2	1	9	5	0	0	0	0	1	1	2
18	7	0	6	3	9	4	2	12	11	0	1	2	2	1	3	4
19	18	12	0	7	21	18	6	45	32	1	3	4	5	5	7	10
20	6	5	7	0	8	3	1	8	6	0	1	0	1	1	2	3
21	9	8	12	4	0	7	4	28	18	1	2	1	2	2	4	6
22	15	11	19	6	14	0	4	24	13	0	1	1	4	2	4	4
23	9	5	8	2	9	6	0	15	6	0	1	1	5	1	2	2
24	80	40	83	19	80	41	19	0	34	6	18	29	36	11	19	26
25	45	41	54	16	51	23	7	35	0	5	15	26	34	10	15	21
26	2	1	2	1	3	1	1	4	3	0	2	2	2	0	1	1
27	9	9	15	5	73	6	3	16	42	4	0	23	26	6	14	17
28	7	11	13	3	46	4	3	15	60	3	15	0	11	19	46	125
29	8	25	19	6	109	13	9	110	81	3	24	12	0	14	42	79
30	23	12	23	5	120	9	4	34	22	1	7	23	15	0	36	30
31	21	32	26	9	242	15	5	75	35	2	12	62	39	24	0	67
32	46	33	44	14	612	16	7	110	37	2	14	123	88	32	81	0
33	39	23	39	8	318	11	5	55	38	2	16	23	34	39	128	75
34	88	39	77	19	1478	27	8	214	68	3	24	93	138	69	199	167
35	46	19	35	13	439	12	4	49	30	2	17	26	47	24	102	83
36	11	3	6	2	13	2	1	3	8	0	7	6	7	5	27	21
37	3	2	3	2	6	1	0	0	4	0	2	4	3	2	13	16
38	8	4	5	2	8	2	1	2	6	1	6	5	10	5	16	21
39	41	20	30	11	339	11	5	50	43	2	21	24	46	15	62	105
40	17	10	15	5	69	6	3	16	21	1	8	7	18	8	30	42
41	25	14	23	7	167	8	3	26	31	1	8	8	31	10	36	52
42	21	12	15	6	86	7	3	19	20	1	8	5	22	9	28	36
43	6	5	6	1	8	2	1	2	7	0	2	3	8	2	9	14
44	17	15	11	5	53	7	2	10	21	1	8	8	18	7	20	50
45	3	1	2	1	1	1	0	0	3	0	1	1	3	1	4	5
46	10	9	8	3	23	4	2	7	13	0	5	3	11	3	11	22
47	9	9	10	3	25	7	2	15	14	1	6	2	17	4	13	19

O/D	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
1	2	6	2	0	0	0	4	1	2	2	1	3	0	2	4
2	5	12	4	1	0	1	6	3	4	3	1	4	1	4	6
3	3	9	3	0	1	1	4	2	4	2	1	3	0	3	4
4	5	10	4	0	1	1	4	3	5	3	1	3	0	3	4
5	7	21	7	1	1	2	9	4	7	5	2	6	1	6	9
6	10	27	11	1	1	2	13	6	10	7	3	8	1	7	11
7	4	8	4	1	1	1	5	2	3	3	1	4	0	3	5
8	8	18	8	1	1	1	10	4	7	5	3	7	1	5	11
9	8	20	8	1	1	1	10	4	8	5	2	6	0	4	7
10	5	14	6	1	0	1	6	3	5	4	1	3	1	3	3
11	3	8	3	0	0	1	3	1	3	2	1	2	0	1	2
12	3	8	4	0	0	1	3	1	3	2	1	1	0	1	1
13	1	2	1	0	0	0	1	0	1	1	0	1	0	0	0
14	1	2	1	0	0	0	2	1	1	1	0	1	0	0	0
15	1	3	2	0	0	0	1	0	1	1	0	1	0	0	0
16	2	5	3	0	0	0	3	1	2	1	0	1	0	1	1
17	2	5	2	0	0	0	2	1	1	1	0	1	0	0	0
18	3	6	3	0	0	0	3	1	2	1	1	2	0	1	1
19	7	17	8	1	1	1	7	3	5	4	1	3	0	2	2
20	2	3	2	0	0	0	2	1	1	1	0	1	0	1	0
21	4	8	4	1	0	1	5	2	3	2	1	2	0	1	1
22	3	7	3	1	0	1	3	1	2	2	0	2	0	1	2
23	2	3	2	0	0	0	2	1	1	1	0	1	0	1	1
24	24	39	22	4	3	4	31	13	17	13	6	13	2	8	10
25	21	37	20	4	3	4	26	11	17	12	4	11	2	7	9
26	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0
27	23	34	28	6	3	6	20	8	10	9	2	8	1	6	6
28	25	101	28	3	3	4	22	6	7	4	2	6	1	3	3
29	34	143	50	5	2	8	39	14	35	17	6	15	2	9	15
30	44	87	28	4	2	4	15	9	11	10	3	8	1	4	5
31	121	211	98	19	11	14	56	26	36	25	7	20	3	10	11
32	84	199	93	17	14	20	106	41	55	39	14	54	5	26	22
33	0	55	109	21	12	20	76	32	42	33	11	32	3	19	13
34	60	0	325	56	53	70	279	122	170	115	43	112	15	83	56
35	112	294	0	51	107	134	269	104	152	101	42	121	12	67	61
36	29	71	57	0	50	31	66	28	36	24	11	33	3	17	13
37	16	53	126	52	0	23	58	22	30	18	9	33	3	12	9
38	23	70	125	29	22	0	58	38	72	41	21	83	4	24	13
39	81	271	269	64	57	56	0	32	156	99	52	282	25	147	109
40	33	129	112	23	21	40	32	0	65	40	23	126	11	63	53
41	42	164	156	35	28	73	146	65	0	34	30	172	29	164	117
42	30	108	97	20	14	41	87	38	36	0	13	99	13	83	59
43	11	41	42	10	8	18	48	22	39	13	0	14	4	24	17
44	31	115	131	25	32	80	323	152	252	128	15	0	9	101	48
45	3	14	11	3	2	4	26	13	34	17	7	12	0	12	8
46	17	71	62	12	10	21	121	61	164	79	23	109	10	0	31
47	12	54	56	13	8	13	101	51	119	60	18	56	6	33	0

Table 5. Origin-Destination Travel Time (One Hour during the Morning Peak)

O/D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0.0	1.5	3.5	6.0	8.0	11.0	12.5	15.0	17.0	19.5	21.5	23.5	26.0	27.5	30.0	31.5
2	2.0	0.0	2.0	4.5	6.5	9.5	11.0	13.5	15.5	18.0	20.0	22.0	24.5	26.0	28.5	30.0
3	4.0	2.0	0.0	2.5	4.5	7.5	9.0	11.5	13.5	16.0	18.0	20.0	22.5	24.0	26.5	28.0
4	6.5	4.5	2.5	0.0	2.0	5.0	6.5	9.0	11.0	13.5	15.5	17.5	20.0	21.5	24.0	25.5
5	8.5	6.5	4.5	2.0	0.0	3.0	4.5	7.0	9.0	11.5	13.5	15.5	18.0	19.5	22.0	23.5
6	11.0	9.0	7.0	4.5	2.5	0.0	1.5	4.0	6.0	8.5	10.5	12.5	15.0	16.5	19.0	20.5
7	13.0	11.0	9.0	6.5	4.5	2.0	0.0	2.5	4.5	7.0	9.0	11.0	13.5	15.0	17.5	19.0
8	15.5	13.5	11.5	9.0	7.0	4.5	2.5	0.0	2.0	4.5	6.5	8.5	11.0	12.5	15.0	16.5
9	17.5	15.5	13.5	11.0	9.0	6.5	4.5	2.0	0.0	2.5	4.5	6.5	9.0	10.5	13.0	14.5
10	20.0	18.0	16.0	13.5	11.5	9.0	7.0	4.5	2.5	0.0	2.0	4.0	6.5	8.0	10.5	12.0
11	22.0	20.0	18.0	15.5	13.5	11.0	9.0	6.5	4.5	2.0	0.0	2.0	4.5	6.0	8.5	10.0
12	24.0	22.0	20.0	17.5	15.5	13.0	11.0	8.5	6.5	4.0	2.0	0.0	2.5	4.0	6.5	8.0
13	26.5	24.5	22.5	20.0	18.0	15.5	13.5	11.0	9.0	6.5	4.5	2.5	0.0	1.5	4.0	5.5
14	28.0	26.0	24.0	21.5	19.5	17.0	15.0	12.5	10.5	8.0	6.0	4.0	1.5	0.0	2.5	4.0
15	30.5	28.5	26.5	24.0	22.0	19.5	17.5	15.0	13.0	10.5	8.5	6.5	4.0	2.5	0.0	1.5
16	32.0	30.0	28.0	25.5	23.5	21.0	19.0	16.5	14.5	12.0	10.0	8.0	5.5	4.0	1.5	0.0
17	33.5	31.5	29.5	27.0	25.0	22.5	20.5	18.0	16.0	13.5	11.5	9.5	7.0	5.5	3.0	1.5
18	35.5	33.5	31.5	29.0	27.0	24.5	22.5	20.0	18.0	15.5	13.5	11.5	9.0	7.5	5.0	3.5
19	37.5	35.5	33.5	31.0	29.0	26.5	24.5	22.0	20.0	17.5	15.5	13.5	11.0	9.5	7.0	5.5
20	39.5	37.5	35.5	33.0	31.0	28.5	26.5	24.0	22.0	19.5	17.5	15.5	13.0	11.5	9.0	7.5
21	41.0	39.0	37.0	34.5	32.5	30.0	28.0	25.5	23.5	21.0	19.0	17.0	14.5	13.0	10.5	9.0
22	43.0	41.0	39.0	36.5	34.5	32.0	30.0	27.5	25.5	23.0	21.0	19.0	16.5	15.0	12.5	11.0
23	46.5	44.5	42.5	40.0	38.0	35.5	33.5	31.0	29.0	26.5	24.5	22.5	20.0	18.5	16.0	14.5
24	49.5	47.5	45.5	43.0	41.0	38.5	36.5	34.0	32.0	29.5	27.5	25.5	23.0	21.5	19.0	17.5
25	51.5	49.5	47.5	45.0	43.0	40.5	38.5	36.0	34.0	31.5	29.5	27.5	25.0	23.5	21.0	19.5
26	53.5	51.5	49.5	47.0	45.0	42.5	40.5	38.0	36.0	33.5	31.5	29.5	27.0	25.5	23.0	21.5
27	56.5	54.5	52.5	50.0	48.0	45.5	43.5	41.0	39.0	36.5	34.5	32.5	30.0	28.5	26.0	24.5
28	58.5	56.5	54.5	52.0	50.0	47.5	45.5	43.0	41.0	38.5	36.5	34.5	32.0	30.5	28.0	26.5
29	60.5	58.5	56.5	54.0	52.0	49.5	47.5	45.0	43.0	40.5	38.5	36.5	34.0	32.5	30.0	28.5
30	62.5	60.5	58.5	56.0	54.0	51.5	49.5	47.0	45.0	42.5	40.5	38.5	36.0	34.5	32.0	30.5
31	64.5	62.5	60.5	58.0	56.0	53.5	51.5	49.0	47.0	44.5	42.5	40.5	38.0	36.5	34.0	32.5
32	68.5	66.5	64.5	62.0	60.0	57.5	55.5	53.0	51.0	48.5	46.5	44.5	42.0	40.5	38.0	36.5
33	71.0	69.0	67.0	64.5	62.5	60.0	58.0	55.5	53.5	51.0	49.0	47.0	44.5	43.0	40.5	39.0
34	73.0	71.0	69.0	66.5	64.5	62.0	60.0	57.5	55.5	53.0	51.0	49.0	46.5	45.0	42.5	41.0
35	81.0	79.0	77.0	74.5	72.5	70.0	68.0	65.5	63.5	61.0	59.0	57.0	54.5	53.0	50.5	49.0
36	82.5	80.5	78.5	76.0	74.0	71.5	69.5	67.0	65.0	62.5	60.5	58.5	56.0	54.5	52.0	50.5
37	85.5	83.5	81.5	79.0	77.0	74.5	72.5	70.0	68.0	65.5	63.5	61.5	59.0	57.5	55.0	53.5
38	88.0	86.0	84.0	81.5	79.5	77.0	75.0	72.5	70.5	68.0	66.0	64.0	61.5	60.0	57.5	56.0
39	92.0	90.0	88.0	85.5	83.5	81.0	79.0	76.5	74.5	72.0	70.0	68.0	65.5	64.0	61.5	60.0
40	94.0	92.0	90.0	87.5	85.5	83.0	81.0	78.5	76.5	74.0	72.0	70.0	67.5	66.0	63.5	62.0
41	96.0	94.0	92.0	89.5	87.5	85.0	83.0	80.5	78.5	76.0	74.0	72.0	69.5	68.0	65.5	64.0
42	98.0	96.0	94.0	91.5	89.5	87.0	85.0	82.5	80.5	78.0	76.0	74.0	71.5	70.0	67.5	66.0
43	100.5	98.5	96.5	94.0	92.0	89.5	87.5	85.0	83.0	80.5	78.5	76.5	74.0	72.5	70.0	68.5
44	103.5	101.5	99.5	97.0	95.0	92.5	90.5	88.0	86.0	83.5	81.5	79.5	77.0	75.5	73.0	71.5
45	106.5	104.5	102.5	100.0	98.0	95.5	93.5	91.0	89.0	86.5	84.5	82.5	80.0	78.5	76.0	74.5
46	110.0	108.0	106.0	103.5	101.5	99.0	97.0	94.5	92.5	90.0	88.0	86.0	83.5	82.0	79.5	78.0

47	112.0	110.0	108.0	105.5	103.5	101.0	99.0	96.5	94.5	92.0	90.0	88.0	85.5	84.0	81.5	80.0

O/D	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	33.0	35.0	37.0	39.0	40.5	42.5	46.0	49.0	51.0	53.0	56.0	58.0	60.0	62.0	64.0	68.0
2	31.5	33.5	35.5	37.5	39.0	41.0	44.5	47.5	49.5	51.5	54.5	56.5	58.5	60.5	62.5	66.5
3	29.5	31.5	33.5	35.5	37.0	39.0	42.5	45.5	47.5	49.5	52.5	54.5	56.5	58.5	60.5	64.5
4	27.0	29.0	31.0	33.0	34.5	36.5	40.0	43.0	45.0	47.0	50.0	52.0	54.0	56.0	58.0	62.0
5	25.0	27.0	29.0	31.0	32.5	34.5	38.0	41.0	43.0	45.0	48.0	50.0	52.0	54.0	56.0	60.0
6	22.0	24.0	26.0	28.0	29.5	31.5	35.0	38.0	40.0	42.0	45.0	47.0	49.0	51.0	53.0	57.0
7	20.5	22.5	24.5	26.5	28.0	30.0	33.5	36.5	38.5	40.5	43.5	45.5	47.5	49.5	51.5	55.5
8	18.0	20.0	22.0	24.0	25.5	27.5	31.0	34.0	36.0	38.0	41.0	43.0	45.0	47.0	49.0	53.0
9	16.0	18.0	20.0	22.0	23.5	25.5	29.0	32.0	34.0	36.0	39.0	41.0	43.0	45.0	47.0	51.0
10	13.5	15.5	17.5	19.5	21.0	23.0	26.5	29.5	31.5	33.5	36.5	38.5	40.5	42.5	44.5	48.5
11	11.5	13.5	15.5	17.5	19.0	21.0	24.5	27.5	29.5	31.5	34.5	36.5	38.5	40.5	42.5	46.5
12	9.5	11.5	13.5	15.5	17.0	19.0	22.5	25.5	27.5	29.5	32.5	34.5	36.5	38.5	40.5	44.5
13	7.0	9.0	11.0	13.0	14.5	16.5	20.0	23.0	25.0	27.0	30.0	32.0	34.0	36.0	38.0	42.0
14	5.5	7.5	9.5	11.5	13.0	15.0	18.5	21.5	23.5	25.5	28.5	30.5	32.5	34.5	36.5	40.5
15	3.0	5.0	7.0	9.0	10.5	12.5	16.0	19.0	21.0	23.0	26.0	28.0	30.0	32.0	34.0	38.0
16	1.5	3.5	5.5	7.5	9.0	11.0	14.5	17.5	19.5	21.5	24.5	26.5	28.5	30.5	32.5	36.5
17	0.0	2.0	4.0	6.0	7.5	9.5	13.0	16.0	18.0	20.0	23.0	25.0	27.0	29.0	31.0	35.0
18	2.0	0.0	2.0	4.0	5.5	7.5	11.0	14.0	16.0	18.0	21.0	23.0	25.0	27.0	29.0	33.0
19	4.0	2.0	0.0	2.0	3.5	5.5	9.0	12.0	14.0	16.0	19.0	21.0	23.0	25.0	27.0	31.0
20	6.0	4.0	2.0	0.0	1.5	3.5	7.0	10.0	12.0	14.0	17.0	19.0	21.0	23.0	25.0	29.0
21	7.5	5.5	3.5	1.5	0.0	2.0	5.5	8.5	10.5	12.5	15.5	17.5	19.5	21.5	23.5	27.5
22	9.5	7.5	5.5	3.5	2.0	0.0	3.5	6.5	8.5	10.5	13.5	15.5	17.5	19.5	21.5	25.5
23	13.0	11.0	9.0	7.0	5.5	3.5	0.0	3.0	5.0	7.0	10.0	12.0	14.0	16.0	18.0	22.0
24	16.0	14.0	12.0	10.0	8.5	6.5	3.0	0.0	2.0	4.0	7.0	9.0	11.0	13.0	15.0	19.0
25	18.0	16.0	14.0	12.0	10.5	8.5	5.0	2.0	0.0	2.0	5.0	7.0	9.0	11.0	13.0	17.0
26	20.0	18.0	16.0	14.0	12.5	10.5	7.0	4.0	2.0	0.0	3.0	5.0	7.0	9.0	11.0	15.0
27	23.0	21.0	19.0	17.0	15.5	13.5	10.0	7.0	5.0	3.0	0.0	2.0	4.0	6.0	8.0	12.0
28	25.0	23.0	21.0	19.0	17.5	15.5	12.0	9.0	7.0	5.0	2.0	0.0	2.0	4.0	6.0	10.0
29	27.0	25.0	23.0	21.0	19.5	17.5	14.0	11.0	9.0	7.0	4.0	2.0	0.0	2.0	4.0	8.0
30	29.0	27.0	25.0	23.0	21.5	19.5	16.0	13.0	11.0	9.0	6.0	4.0	2.0	0.0	2.0	6.0
31	31.0	29.0	27.0	25.0	23.5	21.5	18.0	15.0	13.0	11.0	8.0	6.0	4.0	2.0	0.0	4.0
32	35.0	33.0	31.0	29.0	27.5	25.5	22.0	19.0	17.0	15.0	12.0	10.0	8.0	6.0	4.0	0.0
33	37.5	35.5	33.5	31.5	30.0	28.0	24.5	21.5	19.5	17.5	14.5	12.5	10.5	8.5	6.5	2.5
34	39.5	37.5	35.5	33.5	32.0	30.0	26.5	23.5	21.5	19.5	16.5	14.5	12.5	10.5	8.5	4.5
35	47.5	45.5	43.5	41.5	40.0	38.0	34.5	31.5	29.5	27.5	24.5	22.5	20.5	18.5	16.5	12.5
36	49.0	47.0	45.0	43.0	41.5	39.5	36.0	33.0	31.0	29.0	26.0	24.0	22.0	20.0	18.0	14.0
37	52.0	50.0	48.0	46.0	44.5	42.5	39.0	36.0	34.0	32.0	29.0	27.0	25.0	23.0	21.0	17.0
38	54.5	52.5	50.5	48.5	47.0	45.0	41.5	38.5	36.5	34.5	31.5	29.5	27.5	25.5	23.5	19.5
39	58.5	56.5	54.5	52.5	51.0	49.0	45.5	42.5	40.5	38.5	35.5	33.5	31.5	29.5	27.5	23.5
40	60.5	58.5	56.5	54.5	53.0	51.0	47.5	44.5	42.5	40.5	37.5	35.5	33.5	31.5	29.5	25.5
41	62.5	60.5	58.5	56.5	55.0	53.0	49.5	46.5	44.5	42.5	39.5	37.5	35.5	33.5	31.5	27.5
42	64.5	62.5	60.5	58.5	57.0	55.0	51.5	48.5	46.5	44.5	41.5	39.5	37.5	35.5	33.5	29.5
43	67.0	65.0	63.0	61.0	59.5	57.5	54.0	51.0	49.0	47.0	44.0	42.0	40.0	38.0	36.0	32.0
44	70.0	68.0	66.0	64.0	62.5	60.5	57.0	54.0	52.0	50.0	47.0	45.0	43.0	41.0	39.0	35.0
45	73.0	71.0	69.0	67.0	65.5	63.5	60.0	57.0	55.0	53.0	50.0	48.0	46.0	44.0	42.0	38.0
46	76.5	74.5	72.5	70.5	69.0	67.0	63.5	60.5	58.5	56.5	53.5	51.5	49.5	47.5	45.5	41.5
47	78.5	76.5	74.5	72.5	71.0	69.0	65.5	62.5	60.5	58.5	55.5	53.5	51.5	49.5	47.5	43.5

O/D	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
1	70.5	72.5	80.0	82.0	85.0	87.5	91.5	93.5	95.5	97.5	100.0	102.5	106.0	109.5	112.0
2	69.0	71.0	78.5	80.5	83.5	86.0	90.0	92.0	94.0	96.0	98.5	101.0	104.5	108.0	110.5
3	67.0	69.0	76.5	78.5	81.5	84.0	88.0	90.0	92.0	94.0	96.5	99.0	102.5	106.0	108.5
4	64.5	66.5	74.0	76.0	79.0	81.5	85.5	87.5	89.5	91.5	94.0	96.5	100.0	103.5	106.0
5	62.5	64.5	72.0	74.0	77.0	79.5	83.5	85.5	87.5	89.5	92.0	94.5	98.0	101.5	104.0
6	59.5	61.5	69.0	71.0	74.0	76.5	80.5	82.5	84.5	86.5	89.0	91.5	95.0	98.5	101.0
7	58.0	60.0	67.5	69.5	72.5	75.0	79.0	81.0	83.0	85.0	87.5	90.0	93.5	97.0	99.5
8	55.5	57.5	65.0	67.0	70.0	72.5	76.5	78.5	80.5	82.5	85.0	87.5	91.0	94.5	97.0
9	53.5	55.5	63.0	65.0	68.0	70.5	74.5	76.5	78.5	80.5	83.0	85.5	89.0	92.5	95.0
10	51.0	53.0	60.5	62.5	65.5	68.0	72.0	74.0	76.0	78.0	80.5	83.0	86.5	90.0	92.5
11	49.0	51.0	58.5	60.5	63.5	66.0	70.0	72.0	74.0	76.0	78.5	81.0	84.5	88.0	90.5
12	47.0	49.0	56.5	58.5	61.5	64.0	68.0	70.0	72.0	74.0	76.5	79.0	82.5	86.0	88.5
13	44.5	46.5	54.0	56.0	59.0	61.5	65.5	67.5	69.5	71.5	74.0	76.5	80.0	83.5	86.0
14	43.0	45.0	52.5	54.5	57.5	60.0	64.0	66.0	68.0	70.0	72.5	75.0	78.5	82.0	84.5
15	40.5	42.5	50.0	52.0	55.0	57.5	61.5	63.5	65.5	67.5	70.0	72.5	76.0	79.5	82.0
16	39.0	41.0	48.5	50.5	53.5	56.0	60.0	62.0	64.0	66.0	68.5	71.0	74.5	78.0	80.5
17	37.5	39.5	47.0	49.0	52.0	54.5	58.5	60.5	62.5	64.5	67.0	69.5	73.0	76.5	79.0
18	35.5	37.5	45.0	47.0	50.0	52.5	56.5	58.5	60.5	62.5	65.0	67.5	71.0	74.5	77.0
19	33.5	35.5	43.0	45.0	48.0	50.5	54.5	56.5	58.5	60.5	63.0	65.5	69.0	72.5	75.0
20	31.5	33.5	41.0	43.0	46.0	48.5	52.5	54.5	56.5	58.5	61.0	63.5	67.0	70.5	73.0
21	30.0	32.0	39.5	41.5	44.5	47.0	51.0	53.0	55.0	57.0	59.5	62.0	65.5	69.0	71.5
22	28.0	30.0	37.5	39.5	42.5	45.0	49.0	51.0	53.0	55.0	57.5	60.0	63.5	67.0	69.5
23	24.5	26.5	34.0	36.0	39.0	41.5	45.5	47.5	49.5	51.5	54.0	56.5	60.0	63.5	66.0
24	21.5	23.5	31.0	33.0	36.0	38.5	42.5	44.5	46.5	48.5	51.0	53.5	57.0	60.5	63.0
25	19.5	21.5	29.0	31.0	34.0	36.5	40.5	42.5	44.5	46.5	49.0	51.5	55.0	58.5	61.0
26	17.5	19.5	27.0	29.0	32.0	34.5	38.5	40.5	42.5	44.5	47.0	49.5	53.0	56.5	59.0
27	14.5	16.5	24.0	26.0	29.0	31.5	35.5	37.5	39.5	41.5	44.0	46.5	50.0	53.5	56.0
28	12.5	14.5	22.0	24.0	27.0	29.5	33.5	35.5	37.5	39.5	42.0	44.5	48.0	51.5	54.0
29	10.5	12.5	20.0	22.0	25.0	27.5	31.5	33.5	35.5	37.5	40.0	42.5	46.0	49.5	52.0
30	8.5	10.5	18.0	20.0	23.0	25.5	29.5	31.5	33.5	35.5	38.0	40.5	44.0	47.5	50.0
31	6.5	8.5	16.0	18.0	21.0	23.5	27.5	29.5	31.5	33.5	36.0	38.5	42.0	45.5	48.0
32	2.5	4.5	12.0	14.0	17.0	19.5	23.5	25.5	27.5	29.5	32.0	34.5	38.0	41.5	44.0
33	0.0	2.0	9.5	11.5	14.5	17.0	21.0	23.0	25.0	27.0	29.5	32.0	35.5	39.0	41.5
34	2.0	0.0	7.5	9.5	12.5	15.0	19.0	21.0	23.0	25.0	27.5	30.0	33.5	37.0	39.5
35	10.0	8.0	0.0	2.0	5.0	7.5	11.5	13.5	15.5	17.5	20.0	22.5	26.0	29.5	32.0
36	11.5	9.5	1.5	0.0	3.0	5.5	9.5	11.5	13.5	15.5	18.0	20.5	24.0	27.5	30.0
37	14.5	12.5	4.5	3.0	0.0	2.5	6.5	8.5	10.5	12.5	15.0	17.5	21.0	24.5	27.0
38	17.0	15.0	7.0	5.5	2.5	0.0	4.0	6.0	8.0	10.0	12.5	15.0	18.5	22.0	24.5
39	21.0	19.0	11.0	9.5	6.5	4.0	0.0	2.0	4.0	6.0	8.5	11.0	14.5	18.0	20.5
40	23.0	21.0	13.0	11.5	8.5	6.0	2.0	0.0	2.0	4.0	6.5	9.0	12.5	16.0	18.5
41	25.0	23.0	15.0	13.5	10.5	8.0	4.0	2.0	0.0	2.0	4.5	7.0	10.5	14.0	16.5
42	27.0	25.0	17.0	15.5	12.5	10.0	6.0	4.0	2.0	0.0	2.5	5.0	8.5	12.0	14.5
43	29.5	27.5	19.5	18.0	15.0	12.5	8.5	6.5	4.5	2.5	0.0	2.5	6.0	9.5	12.0
44	32.5	30.5	22.5	21.0	18.0	15.5	11.5	9.5	7.5	5.5	3.0	0.0	3.5	7.0	9.5
45	35.5	33.5	25.5	24.0	21.0	18.5	14.5	12.5	10.5	8.5	6.0	3.0	0.0	3.5	6.0
46	39.0	37.0	29.0	27.5	24.5	22.0	18.0	16.0	14.0	12.0	9.5	6.5	3.5	0.0	2.5
47	41.0	39.0	31.0	29.5	26.5	24.0	20.0	18.0	16.0	14.0	11.5	8.5	5.5	2.0	0.0

Table 6. Coordination of Skipping and Stopping Stations

Scenario	Original	I	II	III	IV
1	AB	AB	AB	AB	AB
2	AB	AB	AB	AB	AB
3	AB	AB	AB	AB	AB
4	AB	AB	AB	B	AB
5	AB	AB	AB	AB	AB
6	AB	AB	AB	AB	AB
7	AB	B	B	A	AB
8	AB	AB	AB	AB	AB
9	AB	A	A	A	B
10	AB	B	B	B	B
11	AB	A	A	B	B
12	AB	B	B	A	B
13	AB	A	A	A	A
14	AB	B	B	A	B
15	AB	AB	AB	AB	AB
16	AB	AB	A	AB	AB
17	AB	AB	AB	AB	A
18	AB	AB	AB	AB	AB
19	AB	AB	A	AB	AB
20	AB	AB	B	AB	AB
21	AB	AB	AB	AB	AB
22	AB	AB	B	AB	A
23	AB	AB	AB	B	B
24	AB	A	AB	AB	A
25	AB	AB	A	AB	B
26	AB	AB	B	AB	B
27	AB	B	AB	AB	A
28	AB	A	AB	AB	B
29	AB	AB	AB	AB	AB
30	AB	AB	A	B	AB
31	AB	AB	AB	AB	AB
32	AB	AB	AB	B	AB
33	AB	AB	AB	AB	AB
34	AB	AB	AB	AB	AB
35	AB	AB	AB	A	AB
36	AB	AB	AB	AB	AB
37	AB	AB	AB	AB	AB
38	AB	B	A	B	AB
39	AB	AB	AB	AB	AB
40	AB	AB	AB	B	AB
41	AB	AB	AB	AB	AB
42	AB	AB	B	A	AB
43	AB	AB	AB	B	AB
44	AB	AB	AB	AB	AB
45	AB	A	AB	AB	AB
46	AB	AB	AB	A	AB
47	AB	AB	AB	AB	AB

Scenario IV does not require uniform headway between the A and B trains. As a result, the headway between A and B trains can be as long as five minutes. (The five-minute figure is based on the six-minute headway between two A trains minus the one-minute safety distance between A and B trains.) Since the maximum allowable headway between A and B trains is five minutes, the maximum number of consecutive A stations is four, which is five-minute headway minus one minute of the safety distance. In this example, stations from 9th station and 12th station are all B stations. In order to make this service safe and feasible, the headway between the B and A trains should be 5 minutes, and headway between A and B trains should be one minute. For feasibility and safety, the cumulative number of A stations at any station will not be more than that of B stations.

In this example, the results of all four scenarios met the programming constraints.

Table 7 shows the total in-vehicle travel time, total waiting time, total transfer time, and total additional access and egress time for all four cases with the original all-stop case. Obviously, total travel time becomes shorter with more relaxed constraints.

In this example, 181,830 passengers travelled during the one-hour morning peak, and their average total travel time with the original all-stop operation was 21.78 minutes. With skip-stop operation, depending on the stopping constraints, their total travel time became 17-20 percent shorter than that with original all-stop operation. While in-vehicle travel time became 20-26 percent shorter due to skipping stations, waiting time, transfer time, and additional access time were 24-38 percent longer.

Each train skipped five to nine stations, which reduced five to nine minutes (up to 8 percent) of the agency's operating time. As mentioned, this model was built to minimize the total travel time. If the objective of the model was minimization of operating time or total cost, the model could reduce operating time further.

Table 7. Travel Time Characteristics of Skip-Stop Operation

Scenario	Original	I	II	III	IV
Number of AB stations	47	35	31	30	33
Number of A stations	0	6	8	8	5
Number of B stations	0	6	8	9	9
In-Vehicle Time	3,688,169.62	2,946,771.66	2,929,213.27	2,847,704.26	2,811,122.13
Waiting Time	272,744.43	334,290.85	342,502.14	367,530.57	327,646.25
Transfer Time	0.00	1,935.99	3,185.69	6,032.21	2,144.89
Additional Access Time	0.00	1,256.20	1,955.11	3,486.26	1,268.45
Total Travel Time	3,960,914.05	3,284,254.71	3,276,856.22	3,224,753.29	3,142,181.72
Average Total Travel Time	21.78	18.06	18.02	17.73	17.28
Travel Time Reduction	-	-17.08%	-17.26%	-18.60%	-20.66%

Unit: minutes

CONCLUSIONS

Well-coordinated skip-stop service can reduce passengers' total travel time and improve overall service, since this service can increase operating speed. However, the selection and coordination of stopping and skipping stations requires big efforts since it is a huge combinatorial problem.

This research showed how the optimization process for the skip-stop service and the selection and coordination of the stopping and skipping stations could be pursued. As discussed, this model used the Genetic Algorithm, which can handle different objective functions and include different constraints for preventing a collision. This model also considered different access modes, as well as passengers' different options and choices (including access modes) when the same train does not serve the origin and destination stations. Since this model considered those components, the results were more realistic.

In this example, 181,830 passengers travelled in one-hour during the morning peak, and their average total travel time with the original all-stop operation was 21.78 minutes. With skip-stop operation, depending on the stopping constraints, their total travel time became about 17-20 percent shorter than that with the original all-stop operation. While in-vehicle travel time became about 20-26 percent shorter due to skipping stations, waiting time, transfer time, and additional access time became 24-38 percent longer.

Each train skipped five to nine stations, which reduced five to nine minutes (up to 8 percent) of the operating time. That reduces up to eight percent of the transit agency's operating time. As mentioned, this model was built to minimize the total travel time. If the model's objective was minimization of operating time or total cost, the model could reduce operating time more.

Although skip-stop operation can be complicated for the operators and it can confuse passengers at the beginning of the service, it can certainly reduce passengers' total travel time and the operator's investment and operating costs.

This research concentrated on the modeling and solution processes. In the future, this research can be utilized in many different ways by changing input values to create the general conditions for skip-stop operation. For example, this research categorized the passengers into two groups, those who walk to stations and those who ride to stations. In the future, the research team can show whether skip-stop operation is more suitable for a walking-oriented environment or driving-oriented environment. In addition, this research team can show the difference between the results of skip-stop operation with total cost minimization and with passenger travel time minimization.

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