Optimal Traffic Resource Allocation and Management

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University Transportation Center for Alabama
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and The University of Alabama at Huntsville

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UTCA Theme: Management and Safety of Transportation Systems

University Transportation Center for Alabama

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Analysis of an Integrated Maximum Covering and Patrol Routing Problem

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16. Abstract

In this paper, we address the problem of determining the patrol routes of state troopers for maximum coverage of highway spots with high frequencies of crashes (hot spots). We develop a mixed integer linear programming model for this problem under time feasibility and budget limitation. We solve this model using local and tabu-search based heuristics. Via extensive computational experiments using randomly generated data, we test the validity of our solution approaches. Furthermore, using data from the state of Alabama, we provide recommendations for i) critical levels of coverage; ii) factors influencing the service measures; and iii) dynamic changes in routes.

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Executive Summary

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Section 1
Introduction

Traffic accidents pose a great danger to passengers' lives. In 2009, 33,963 people died in traffic crashes in the United States, an 8.9% decline from 2008 and the lowest total since 1954 (NHTSA, 2010). Even though fatality rates continue to drop in the United States, the number of fatalities is still significant.

Furthermore, the economic impact of motor vehicle crashes on U.S. roadways is noteworthy. The NHTSA estimates this cost as $230.6 billion per year (nearly 2.3 percent of the nation's gross domestic product), or an average of $820 per person in the country (Blincoe, *et al*., 2002). Thus, it is of humanitarian and economic importance to reduce traffic accidents.

It is believed that concentrated traffic enforcement has a positive impact in reducing the number of crashes and discouraging dangerous behavior (Steil and Parrish, 2009). One such example, the NHTSA-sponsored "Click it or Ticket" program, uses a combination of publicity and increased law enforcement to educate and motivate the public. Another program, "Targeting Aggressive Cars and Trucks," sponsored by the Federal Motor Carriers Safety Administration (FMSCA, 2008), encourages the participating states to identify additional law enforcement and publicity strategies that will deter aggressive driving. Due to limited resources, a primary concern of public safety officials is the *effective use of patrol cars and state troopers* in reducing traffic accidents. A typical method for state troopers is to patrol "hot spots': certain locations of highways with high frequencies of crashes over a certain time period. These locations are often associated with a particular type of crash (for example, excessive crashes caused by speed or DUI). Furthermore, hot spots vary with respect to the day of week and time of day; that is, a particular highway location may be a hot spot on a particular day and time but not at other times.

With this motivation, given identified hot spots on mile-posted highways, we focus on building effective state trooper patrol routes with maximum hot spot coverage. This problem has similarities to **the orienteering problem (OP)** (Feillet, *et al*., 2005; Tsiligirides, 1984), also known as the selective traveling salesman problem (STSP), which consists of finding a circuit that maximizes collected profit such that travel costs do not exceed a preset value C. For our problem, the service time at a hot spot can be viewed as the "profit" whereas the shift duration is equivalent to setting a value for C. However, due to time windows of hot spots, we have an "expiration time" on the profits. Furthermore, we consider routing multiple cars simultaneously. Therefore, our problem is related to the team orienteering problem with time windows (TOPTW), a variant of OP. In the TOPTW, the goal is to maximize the total profit by a fixed number of routes such that the locations are visited within a time window and the maximum tour length is limited. The main difference between our problem and the TOPTW is that we do not have a fixed profit associated with each location. The collected profit depends on the service time, which could be as short as one minute or as long as the length of the time window (up to 270 minutes).

This property necessitates a novel solution approach to the problem. For this purpose, we develop a mixed integer programming formulation. For real data, unfortunately, the problem is not solvable computationally using a state-of-the-art commercial solver, CPLEX 12.1.* In fact, in the appendix, we prove that our problem belongs to the same class of NP-hard problems as OP (Golden, *et al*., 1987). Therefore, we focus on local search– and tabu search– based heuristic approaches that provide quality solutions in short periods of time. Since this problem needs to be solved over a number of state trooper post regions, several days, and many shifts, having fast and effective heuristic approaches is a requirement for the applicability of the solutions by practitioners. As it is not possible to cover all of the hot spots with given resources, we also provide additional service measures, including the percentage of number of hot spots covered and the percentage of coverage length based on the outcome of the heuristics. These service measures provide additional insights into the solutions and help in evaluating the constraints related to the number of state trooper cars and patrol duration.

To summarize, this paper is unique in that it considers the integrated optimization of strategic crash covering and patrol routing problems while designing an efficient operating plan for state troopers. Its formulation is a methodological contribution to the current literature. Furthermore, the problem-specific heuristic approaches—local and tabu searches—help decision-makers act quickly and rationally to ensure traffic-law enforcement.

The remainder of this paper is structured as follows. In Section , we present the literature review. In Section , we present the general mathematical model, including necessary assumptions and notation. In Section , we present the analysis of the problem and the solution approaches based on the characteristics of the problem. In Section , we discuss the computational results based on randomly generated data and real data. Finally, in Section , we provide our conclusions, recommendations, and future work.

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*CPLEX is a trademark of IBM.

Section 2
Literature Review

Our research builds on the assumption that it is possible to identify hot spots, where accidents are more likely to happen. Most of the literature on accident analysis and prevention focuses on methods identifying hot spots (Anderson, 2006; Cheng and Washington, 2005; Chen and Quddus, 2003; Gatrell, *et al*., 1996; McCullagh, 2006; Miranda-Moreno et al., 2007; Steil and Parrish, 2009). However, our focus is not on hot-spot identification. To identify hot spots, we utilize the data and algorithms of the Critical Analysis Reporting Environment (CARE)—a data-analysis software package developed by researchers at the University of Alabama (Steil and Parrish, 2009). CARE uses advanced analytical and statistical techniques on the crash and citation data for the State of Alabama to generate valuable information, including hot-spot locations, hot-spot times and durations, and hot-spot severity (in terms of number of fatalities). We utilize this information to manage state trooper resources and patrol routes.

Our work mostly borrows from and contributes in two main areas of operations research: state trooper patrolling models and the orienteering problem. Next, we review and summarize the research on these areas.

2.1 State Trooper/Police Patrol Models

The research on police patrols dates back to the early 1970s. The early works were concerned with answering calls for service, mostly related to a police officer servicing a crime call. Hence, mostly queueing models were used (Birge and Pollock, 1989; Chaiken and Dormont, 1978; Green, 1984; Larson, 1973). Other approaches for the patrol routing problem include mathematical modeling (Curtin, *et al*., 2007; Mitchell, 1972; Wolfler-Calvo and Cordone, 2003), heuristic solutions (Lauri and Koukam, 2008; Reis, et al., 2006; Wolfler-Calvo and Cordone, 2003), graph theory (Chawathe, 2007; Duchenne, *et al*., 2005, 2007), and simulation (Machado et al., 2003; Santana et al., 2004). Chawathe (2007), as in our paper, considers a road network with hot spots. By means of graph theory, the road network is translated to an edge-weighted graph to find the patrol routes where the weights are related to the importance of the corresponding locations and the topology of the road network. In this paper, the selection of weights is somewhat arbitrary and influences the selection of routes.

One approach for the mathematical modeling of patrol routing problems is to invoke the *m*-peripatetic salesman problem (*m*-PSP), which consists of determining *m* edge disjoint Hamiltonian cycles of minimum total cost on a complete graph. Wolfler-Calvo and Cordone (2003) introduce *m*-PSP in the design of watchman tours, where it is often important to assign a set of edge-disjoint rounds to the watchman to avoid repeating the same tour and enhancing security. They solve this model via a decomposition heuristic. Duchenne, *et al*. (2005, 2007) improve the formulation of the *m*-PSP by defining new polyhedral properties and cuts and describe exact branch-and-cut solution procedures for the undirected *m*-PSP. The two main differences between this line of work and ours are the time-sensitivity of hot-spot coverage and maximization of coverage benefits instead of minimization of travel costs. Therefore, our model is unprecedented in the patrol-routing literature that addresses the design of patrol routes while covering hot spots within their time limits.

2.2 Orienteering Problem (OP)

The OP is first introduced by Tsiligirides (1984) for the orienteering competition. In this competition, competitors visit as many checkpoints as possible within a time limit where each checkpoint may have different point values depending on difficulty. The competitor with the most points wins the game (Chao, *et al*., 1996a). In a more formal definition, given a weighted graph with profits associated with the vertices, the OP consists of selecting a simple circuit of maximal profit whose length does not exceed a certain pre-specified bound (Feillet, *et al*., 2005). The OP is also known as the selective traveling salesperson problem (Laporte and Martello, 1990) or the maximum collection problem (Butt and Cavalier, 1994). The OP arises in many applications, including the sport game of orienteering (Chao, *et al*., 1996a), the home fuel delivery problem (Golden, *et al*., 1987), athlete recruiting from high schools (Butt and Cavalier, 1994), routing technicians to service customers (Tang and Miller-Hooks, 2005), and the personalized mobile tourist guide (Vansteenwegen et al., 2009).

Some important variants of the orienteering problem include the team orienteering problem (TOP)–where a fixed number of paths is considered, the orienteering problem with time windows (OPTW), and the team orienteering problem with time windows (TOPTW). Since Golden, *et al*. (1987) prove that the OP is NP-hard, for OP and its variants only a few researchers resort to exact algorithms. Righini and Salani (2006) and Righini and Salani (2009) use bi-directional dynamic programming, and Boussier, *et al*. (2007) propose an exact branch-and-price approach coupled with a column generation technique. Most other research on OP and the variants have focused on heuristic approaches, including local search (Vansteenwegen et al., 2009), tabu search (Liang et al., 2002; Schilde et al., 2009; Tang and Miller-Hooks, 2005), path relinking (Schilde et al., 2009; Souffriau, et al., 2010), ant-colony optimization (Ke et al., 2008; Liang et al., 2002; Montemanni and Gambardella, 2009), genetic algorithm (Tasgetiren, 2001), and other metaheuristics (Archetti, *et al*., 2007; Tricoire et al., 2010). A recent review summarizing all of these variants, solution approaches, and benchmark models is presented by Vansteenwegen et al. (2010).

As our problem bears similarities to the TOPTW, we discuss the TOPTW literature in more detail. The exact branch-and-price algorithm proposed by Boussier, *et al*. (2007) is generic enough to handle different kinds of OP, including the TOPTW. The different branching rules and acceleration techniques introduced in this paper helps solve problem instances with up to 100 nodes. Montemanni and Gambardella (2009) develop local search and ant colony system algorithms based on the solution of a hierarchic generalization of TOPTW. The algorithms are tested effective for OPTW and TOPTW with up to 288 nodes. Last but not the least, Vansteenwegen et al. (2009) present a straightforward and very fast iterated local search heuristic, which combines an insertion step and a shaking step– reverse insertion operation, to escape from local optima. It performs well on the available data sets, ranging from 3-20 routes and 48-288 nodes. The solution quality is slightly worse than that of Boussier, *et al*. (2007) and Montemanni and Gambardella (2009), but the solution approach requires only a few seconds of computation time, more than 100 times faster.

Section 3
General Model

Our problem is formally defined as follows. Within a particular county with an established state trooper post and during a particular shift p, there are historically established hot spots that are more prone to accidents. These hot spots are defined not only with their location on the mile-posted road network, but also with the time they become "hot." We denote the set of hot spots with $\mathcal{N} = \{1, \ldots, n\}$, where each hot spot $i \in \mathcal{N}$ has an earliest e_i and latest time l_i for its hotness. By definition, $e_i < l_i$. We denote $[e_i, l_i]$ as the time window TW_i of hot spot i. Furthermore, we assume that set $\mathcal{N}$ is ordered such that $e_1 \leq e_2 \leq \ldots \leq e_n$. We note that the same location can be labeled with two indices, i and j, and that $l_i < e_j$ indicates two hot spots. Additionally, we define the dummy nodes 0 and $n+1$ to denote the start and end of the shift at the state trooper post respectively. $\mathcal{V} = \{0, n+1\} \cup \mathcal{N}$ denote the set of all hot spots and the state trooper post. For a certain shift p, $e_0 = A_p$ and $l_{n+1} = L_p$, where A_p and L_p are the starting and ending time of the shift p. Given the maximum number of state trooper cars $|\mathcal{K}|$ available, we aim to find the best patrol route for each state trooper car $k \in \mathcal{K}$ with critical stops at hot spots to create a deterrence effect.

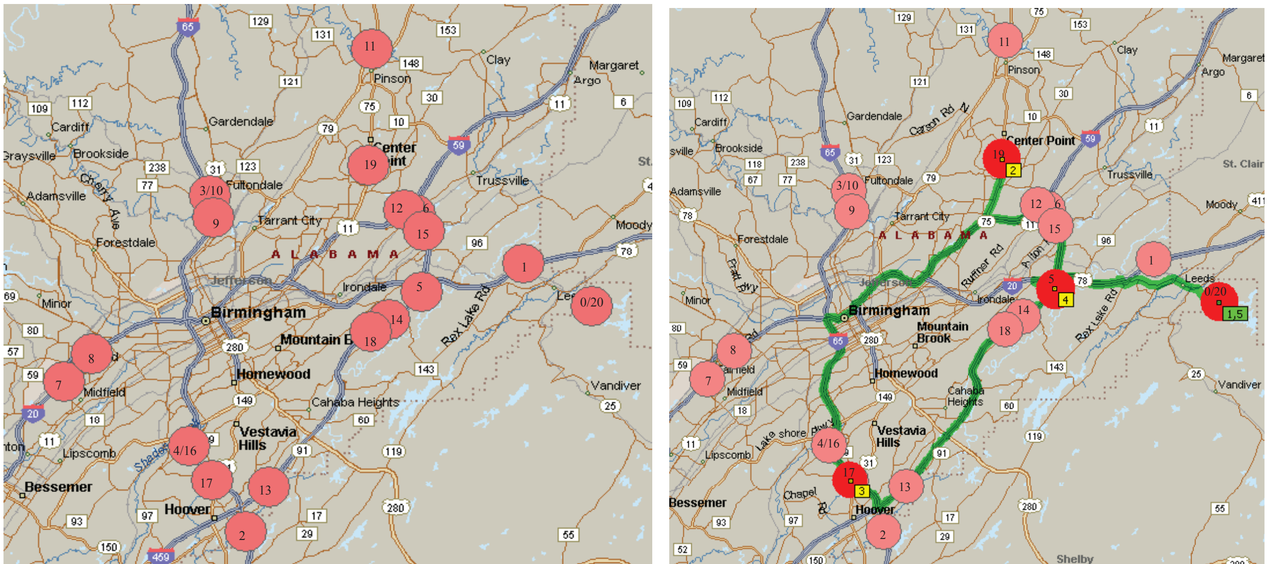
Figure 1 shows an example with 19 hot spots. In this figure, nodes 0 and 20 represent the state trooper post. Furthermore, hot-spot pairs $\{3, 10\}$ and $\{4,16\}$ are at the same location. They are marked as separate hot spots because they have distinct time windows; that is, they become "hot" twice during the shift. For instance, the location marked with hot spots 4 and 16 becomes "hot" between 7:00-8:30am and 11:00am-12:30pm respectively. In Figure 1(b), we show one of the routes of the optimal solution for this example. Even though the state trooper patrol includes hot spots 5, 14, 18, 13, 2, 17, 4, 16, 19, 12, 6, and 15 in that order, only the visits to 5, 17, and 19 fall into their respective time windows, and only these stops count as a deterrent for accidents. Additionally, we let $\mathcal{E} = \{(i, j) : \ i, j \in \mathcal{V}, i \neq j\}$ define the set of edges. The connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represents the underlying road network. We denote the shortest travel time from vertex i to j as $t_{ij} > 0$, $i, j \in \mathcal{V}, i \neq j$. Our objective is to construct the best patrol routes to maximize the total amount of effective service time, which falls within TW_i of hot spot i, $\forall i \in \mathcal{N}$. For this purpose, we define three sets of decision variables: i) $x_{ijk} = 1$ if state trooper car $k \in \mathcal{K}$ travels from vertex i to j, $(i, j) \in \mathcal{E}$, and 0 otherwise. ii) $s_{ik} \geq 0$, the starting time of service for state trooper car $k \in \mathcal{K}$ at vertex $i \in \mathcal{V}$. iii) $f_{ik} \geq 0$, the time state trooper car $k \in \mathcal{K}$ leaves vertex $i \in \mathcal{V}$, that is, the end of service.

Before proceeding with our model development, we summarize the assumptions of the model:

1. There is a one-to-one correspondence between a state trooper car and a state trooper, and all of the state trooper cars are identical.

2. One state trooper car is sufficient to cover each hot spot. That is, having multiple state troopers at the same time at a particular location does not augment their deterrence ability.

3. State troopers travel at a constant speed of 60 miles/hour. Therefore, travel time from one hot spot to another is a calculated constant and does not vary by time of day or day of week.



(a) Hot spots (b) An optimal route

Figure 1: A representative example

4. Refueling is possible from any gas station on the patrol route and is not considered.
5. At the beginning of a shift, all state trooper cars start from the same state trooper post 0 and come back to the same location at the end of the shift.
6. A state trooper car is allowed to arrive before e_i and wait until the start time of the hot spot, but its presence is a deterrent only after e_i.
7. Since roadway traffic accidents have a weekly pattern, we model the problem for a particular day of the week and shift of the day.
8. Each county is divided into several districts, and each district has only one state trooper division. State troopers are only responsible for their own jurisdiction. We conclude that each district is independent from one another, thus each district can be solved independently. The formulation below is for a particular district.

Our objective for the Maximum Covering Patrol Routing Problem (MCPRP) is to maximize the total amount of service time that falls within the time window of a hot spot:

$$\text{Maximize} \sum_{i\in\mathcal{N}}\sum_{k\in\mathcal{K}}(f_{ik}-s_{ik}) \tag{MCPRP}$$

We categorize our constraints under four groups: schedule feasibility, route structuring, visits to hot spots, and integrality and non-negativity constraints.

Constraints Related to Schedule Feasibility

We need to guarantee schedule feasibility with respect to time considerations for each state trooper car k, $k \in \mathcal{K}$. If state trooper car k visits vertex $j \in \mathcal{V}$ after a stop at vertex $i \in \mathcal{V}$—that is,

$x_{ijk} = 1$—then the start time at vertex j should be greater than or equal to the finish time of the current vertex i plus the travel time between i and j; that is, $s_{jk} \geq f_{ik} + t_{ij}$. To ensure schedule feasibility, we need

$$x_{ijk} * (f_{ik} + t_{ij} - s_{jk}) \leq 0$$

for each $(i, j) \in \mathcal{E}$, and $k \in \mathcal{K}$. We linearize these constraints using a big constant value $M_{ij} = \max\{l_i + t_{ij} - e_j, 0\} \geq 0$ as follows:

$$f_{ik} + t_{ij} - s_{jk} \leq (1 - x_{ijk})M_{ij}, \qquad \forall k \in \mathcal{K} \text{ and } \forall (i, j) \in \mathcal{E}. \tag{1}$$

Before we proceed with other constraints, we define $\triangle^+(i) = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}, e_i + t_{ij} \leq l_j\}$ as the set of vertices that are directly reachable from $i \in \mathcal{V}$ within the time window and $\triangle^-(i) = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, e_j + t_{ij} \leq l_i\}$ as the set of vertices from which i is directly reachable. Other schedule feasibility constraints include time window restrictions:

$$e_i \sum_{j \in \triangle^+(i)} x_{ijk} \leq s_{ik}, \qquad \forall k \in \mathcal{K} \text{ and } \forall i \in \mathcal{V}. \tag{2}$$

$$l_i \sum_{j \in \triangle^+(i)} x_{ijk} \geq f_{ik}, \qquad \forall k \in \mathcal{K} \text{ and } \forall i \in \mathcal{V}. \tag{3}$$

$$s_{ik} \leq f_{ik}, \qquad \forall k \in \mathcal{K} \text{ and } \forall i \in \mathcal{V}. \tag{4}$$

Constraint (2) establishes that the effective start time s_{ik} at vertex i by state trooper car k is at least as large as the earliest time window of vertex $i \in \mathcal{V}$. Constraint (3) states that the end of the effective service time f_{ik} must be less than or equal to the latest time window of vertex $i \in \mathcal{V}$. Finally, constraint (4) states that the start time of the service by state trooper car $k \in \mathcal{K}$ at vertex $i \in \mathcal{V}$ is less than or equal to the end of the service.

Route Structuring Constraints

We characterize the route of a state trooper $k \in \mathcal{K}$ with the following equations:

$$\sum_{j \in \triangle^+(0)} x_{0jk} = 1, \qquad \forall k \in \mathcal{K}. \tag{5}$$

$$\sum_{i \in \triangle^-(j)} x_{ijk} = \sum_{i \in \triangle^+(j)} x_{jik}, \qquad \forall k \in \mathcal{K} \text{ and } \forall j \in \mathcal{N}. \tag{6}$$

$$\sum_{i \in \triangle^-(n+1)} x_{i,n+1,k} = 1, \qquad \forall k \in \mathcal{K}. \tag{7}$$

Constraint (5) ensures all of the state trooper cars leave the state trooper post at the beginning of the shift, and constraints (7) ensures their return to the post at the end of the shift. Finally, constraint (6) states the balance at each hot spot; that is, each state trooper car k that visits hot spot i must leave.

Constraints Related to Visiting Hot Spots

It is possible to have multiple cars visiting the same hot spot as in Figures 2(b) and (c). Therefore, we need to account for any potential double counting if there is overlap during the visits of

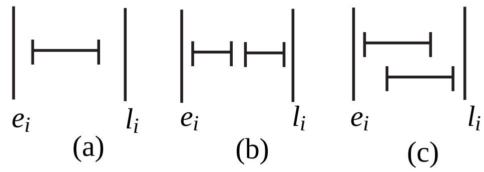


Figure 2: Multiple state troopers at hot spot $i \in \mathcal{N}$

multiple cars, as in Figure 2(c), and eliminate it. The next set of constraints ensure that if multiple cars are at the same hot spot at the same time, they contribute to the objective only once. To establish these constraints, we define the following additional decision variables for $i \in \mathcal{V}$ and $k, g \in \mathcal{K}$, $k \neq g$:

$$y_{ik} = \begin{cases} 1 & \text{if state trooper } k \text{ serves vertex } i; \\ 0 & \text{otherwise.} \end{cases} \qquad \text{and} \qquad u_{ikg} = \begin{cases} 1 & \text{if } s_{ig} \geq f_{ik}; \\ 0 & \text{otherwise.} \end{cases}$$

By definition of y_{ik},

$$\sum_{j \in \triangle^+(i)} x_{ijk} = y_{ik}, \qquad \forall k \in \mathcal{K} \text{ and } \forall i \in \mathcal{N}. \tag{8}$$

$$y_{0,k} = y_{n+1,k} = 1, \qquad \forall k \in \mathcal{K}. \tag{9}$$

Additionally, by definition, u_{ikg} or u_{igk} can only be equal to 1 when both $y_{ik} = 1$ and $y_{ig} = 1$, or else $u_{ikg} = u_{igk} = 0$ for $i \in \mathcal{V}$. The following constraints establish the relationship between y_{ik} and u_{ikg}:

$$u_{ikg} + u_{igk} \leq y_{ik}, \qquad \forall i \in \mathcal{V} \text{ and } k, g \in \mathcal{K},\ g > k. \tag{10}$$

$$u_{ikg} + u_{igk} \leq y_{ig}, \qquad \forall i \in \mathcal{V} \text{ and } k, g \in \mathcal{K},\ g > k. \tag{11}$$

$$u_{ikg} + u_{igk} \geq y_{ik} + y_{ig} - 1, \qquad \forall i \in \mathcal{V} \text{ and } k, g \in \mathcal{K},\ g > k. \tag{12}$$

Now we are ready to present the constraints that eliminate "double counting" if there are two or more cars at the same time window of a certain vertex. That is, for $i \in \mathcal{V}$, if $y_{ik} = 1$ and $y_{ig} = 1$, then $f_{ik} \leq s_{ig}$ or $s_{ik} \geq f_{ig}$, where $k, g \in \mathcal{K}$ and $k \neq g$:

$$f_{ik} - s_{ig} - M * (1 - u_{ikg}) \leq 0, \qquad \forall i \in \mathcal{V} \text{ and } k, g \in \mathcal{K},\ g > k. \tag{13}$$

$$f_{ig} - s_{ik} - M * (1 - u_{igk}) \leq 0, \qquad \forall i \in \mathcal{V} \text{ and } k, g \in \mathcal{K},\ g > k. \tag{14}$$

where M is a large constant.

Integrality and Non-negativity Constraints

Finally, we state continuous and binary variables:

$$s_{ik}, f_{ik} \geq 0 \qquad \text{and} \qquad x_{ijk}, y_{ik}, u_{ikg} \in \{0,1\} \forall i, j \in \mathcal{V} \text{ and } \qquad k, g \in \mathcal{K},\ g > k. \tag{15}$$

Overall Model

The overall model is to maximize the effective service time for MCPRP subject to constraints (1)–(15). We solve this formulation using CPLEX 12.1. However, even for small instances with 40 hot spots and 2 state trooper cars, CPLEX runs out of memory.

Theorem 1 *MCPRP is NP-hard.*

The proof is found in Appendix. Due to Theorem 1, we focus on two two-phase heuristics. These are composed of a construction algorithm and improvements based on local-search and tabu-search. Before we discuss our solution approaches, we note that this model can be used to evaluate other performance measures, including "Percentage of Hot Spots Covered (HS%)" and "Percentage of Coverage Length (TW%)."

HS%: This performance measure calculates, among all the hot spots, the percentage covered as a result of the MCPRP:

$$HS\% = \frac{\sum_{i\in\mathcal{N}}\sum_{k\in\mathcal{K}} y_{ik} - \sum_{i\in\mathcal{N}}\sum_{g\neq k}(u_{igk}+u_{ikg})}{n} * 100,$$

where the numerator represents the total number of visited hot spots.

TW%: This performance measure calculates the percentage of total available time serviced by the MCPRP:

$$TW\% = \frac{\sum_{i\in\mathcal{N}}\sum_{k\in\mathcal{K}}(f_{ik}-s_{ik})}{\sum_{i\in\mathcal{N}}(l_i-e_i)} * 100.$$

In this measure, the numerator is the service time returned by the MCPRP, and the denominator is the total time window length.

Section 4
Solution Approaches

Our solution approaches build on the following characterization of the optimal solution.

Proposition 1 *If the optimal sequences of covered hot spots are known, in the optimal solution, for each state trooper $k \in \mathcal{K}$, for a visited hot spot $i \in \mathcal{N}$,*

$$f_{ik} = \begin{cases} \min\{l_i,\ T - t_{i,n+1}\}, & \text{if } i \text{ is the last hot spot visited on the route of } k; \\ l_i, & \text{otherwise}; \end{cases}$$

where $T = L_p$, the end of the shift p.

This proposition states that, in the optimal solution, the end of service time at a visited hot spot i depends on the order of i in the route. If hot spot $i \in \mathcal{N}$ is the last hot spot on route k, f_{ik} is the minimum of the latest time window of hot spot i and $T - t_{i,n+1}$ (the time required to get back to the post within the shift duration). Otherwise, hot spot $i \in \mathcal{N}$ is an intermittent node in the route and $f_{ik} = l_i$. In other words, state trooper k can stay until the latest time window of each hot spot that is on the route. The complete proof is presented in Appendix.

This proposition states that if there is excess time in a routethe time spent neither in effective coverage nor in travel between hot spotsit does not make a difference for the construction of routes or for the objective value whether a state trooper spends it at the hot spot he just covered or at the hot spot he will cover next. Therefore, by this proposition, we arbitrarily place any excess time at the beginning of the next hot spot without loss of generality. These characteristics are due to two assumptions of the problem: i) the travel time t_{ij} is fixed, as travel speed is constant of 60 miles/hour; and ii) all hot spots have the same priority. If either one of these assumptions is relaxed, then the excess time may not be arbitrarily placed in a route as it influences the order of nodes covered, travel times, and coverage and hence impacts the optimal solution. We report results related to relaxing the hot spots priorities in the computational experiments section.

4.1 Construction Algorithm

Based on Proposition 1, we develop a construction algorithm with two parts involving route initialization and hot-spot insertion.

4.1.1 *Route Initialization Algorithm*

First, we define the following two algorithm parameters H_{limit} and T_{limit}, which help us in building the initial routes:

- H_{limit} provides an upper bound on the number of hot spots to be considered for insertion into a route. Our hot spots are ordered according to the start time of their time windows. To avoid big time gaps between the start times of two consecutive hot spots and hence to eliminate any potential excess waiting, we only consider the next H_{limit} hot spots as the potential next hot spot to be included in this route after a node is inserted into a route. We set H_{limit} as $\lceil n/|\mathcal{K}| \rceil$.

- T_{limit} is a clustering factor where travel time from one hot spot to the next hot spot cannot exceed a certain time span. After preliminary experimentations, we set T_{limit} to 100 minutes, which is reasonable given that for the instances we tested $T = 480$ minutes. If it takes a state trooper more than 100 minutes to travel from her current hot spot to the next, then the algorithm is not going to consider that point.

Hence, H_{limit} provides a temporal limit while T_{limit} provides a spatial restraint on the initial routes.

Algorithm 1 Procedure *RouteInitilization*.

1: Uncovered hot spot set $\mathcal{U} \leftarrow \mathcal{N}$. For $k \in \mathcal{K}$, initialize $Route_k \leftarrow \emptyset$.
2: **for** $\forall k \in \mathcal{K}$ **do**
3:     $Route_k \leftarrow Route_k \cup \{0\}$.
4:     $i^* \leftarrow \arg\max_{i \in \mathcal{U}} \{l_i - \max(e_i, t_{0i}) : i \leq H_{limit}, t_{0i} \leq T_{limit}, t_{0i} \leq l_i\}$.
5:     $s_{i^*,k} \leftarrow \max\{e_{i^*}, t_{0,i^*}\}$ and $f_{i^*,k} \leftarrow l_{i^*}$. $Route_k \leftarrow Route_k \cup \{i^*\}$. $\mathcal{U} \leftarrow \mathcal{U} \setminus \{i^*\}$.
6: **end for**
7: **for** $\forall k \in \mathcal{K}$ **do**
8:     $i \leftarrow Route_k.lastHotSpot$.
9:     **for** $\forall j \in \mathcal{U}$ such that $i < j \leq (i + H_{limit})$, $t_{ij} \leq T_{limit}$, and $l_i + t_{ij} < l_j$ **do**
10:         **if** $l_j + t_{j,n+1} < T$ **then**
11:             $i^* \leftarrow \arg\max_{j \in \mathcal{U}} \{l_j - \max(e_j, l_i + t_{ij})\}$.
12:             $s_{i^*,k} \leftarrow \max\{e_{i^*}, l_i + t_{i,i^*}\}$ and $f_{i^*,k} \leftarrow l_{i^*}$. $Route_k \leftarrow Route_k \cup \{i^*\}$.
13:             **if** $e_{i^*} < l_i + t_{i,i^*}$ **then**
14:                 $l_{i^*} \leftarrow l_i + t_{i,i^*}$.
15:             **else**
16:                 $\mathcal{U} \leftarrow \mathcal{U} \setminus \{i^*\}$.
17:             **end if**
18:         **else**
19:             **if** $l_i + t_{ij} < T - t_{j,n+1}$ **then**
20:                 $i^* \leftarrow \arg\max_{j \in \mathcal{U}} \{T - t_{j,n+1} - \max(e_j, l_i + t_{ij})\}$;
21:                 $s_{i^*,k} \leftarrow \max\{e_{i^*}, l_i + t_{i,i^*}\}$ and $f_{i^*,k} \leftarrow T - t_{i^*,n+1}$. $Route_k \leftarrow Route_k \cup \{i^*\}$.
22:                 Repeat Steps 13 to 17.
23:             **end if**
24:         **end if**
25:     **end for**
26: **end for**

The *RouteInitialization* heuristic, for which the pseudo-code is given in Algorithm 1, builds on a greedy principle. Each state trooper car starts from the state trooper post at the beginning of the shift. Among all the hot spots within the distance range T_{limit} and time range H_{limit}, if the arrival time of state trooper k from hot spot i at one of these hot spots—say hot spot j—comes before the end of the time window ($l_i + t_{ij} < l_j$), the heuristic picks the hot spot that maximizes the objective as the next place to visit (i^*). The maximum contribution is calculated as $\max_j\{l_j - \max(e_j, l_i + t_{ij})\}$. Then the start and finish times of service at i^* are calculated by comparisons between the arrival time at i^* and the earliest and latest time windows respectively, as in line 12. After the next hot spot is selected, the algorithm is divided into two cases as described in steps 10 and 19: whether there is enough time for the state trooper to *fully* service the next hot spot and be back at the state trooper post before the end of the shift. In the first case, there exist hot spots where the coverage and travel-to-post times are within the shift duration. Among these hot spots, the hot spot i^* with the maximum coverage potential is added to the route. Steps 13 through 17 check for potential multi-car visits. Specifically, if a state trooper arrives at or before e_{i^*}, the hot spot i^* is covered fully from $[e_{i^*}, l_{i^*}]$ and is removed from $\mathcal{U}$. Otherwise, hot spot i^* is split into uncovered $[e_{i^*}, s_{i^*,k}]$ and covered $[s_{i^*,k}, f_{i^*,k}]$ parts. In this situation, i^* with an updated l_{i^*} stays in $\mathcal{U}$. For the second case, starting with Step 19, it is not feasible for a state trooper to stay until the end of the time window of hot spot j due to the approaching end of the shift. Therefore, by factoring in the travel time from hot spot j to the state trooper post $n+1$, the state trooper can stay until $T - t_{j,n+1}$. Among all the partially coverable hot spots, the one with the maximum coverage gain i^* is selected. Again, to ensure multi-car visits, steps 13 through 17 are repeated. In this way, initial $|\mathcal{K}|$ routes are created in parallel.

4.1.2 *Insertion Algorithm*

After route initialization, to cover the hot spots that are not covered yet, we proceed with the following insertion algorithm. To insert an uncovered hot spot $\bar{i} \in \mathcal{U}$ before a hot spot i in a certain route $k \in \mathcal{K}$, we first check the time-window feasibility of hot spot i; that is, the arrival time at hot spot i is less than the latest time window of the hot spot: $l_{\bar{i}} + t_{\bar{i},i} < l_i$. In this algorithm, starting with the first hot spot of the first route, we check if we can insert any more hot spots until no longer feasible. The search ends when all of the $|\mathcal{K}|$ routes are checked.

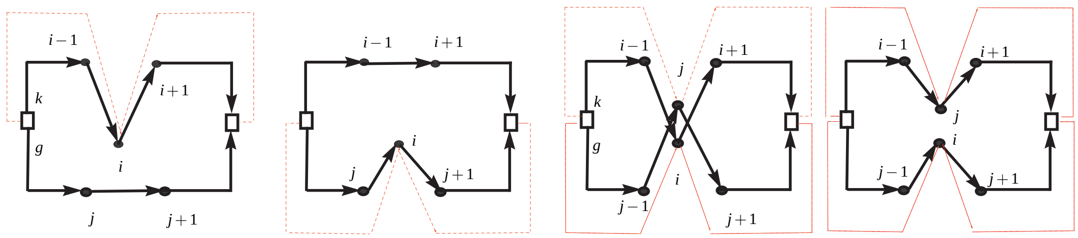
If it is feasible (in terms of travel and coverage times) to insert a new hot spot $\bar{i}$ *right before* hot spot i on route k, this insertion will not influence the start or finish times of hot spots on this route prior to hot spot $i-1$. Insertion of $\bar{i}$ will only shift the starting time of the hot spot i, s_{ik}, to $s_{i'k}$. Hot spots after i will not be affected since the finishing time at i remains unchanged; that is, $f_{ik} = l_i$. The additional coverage of hot spot $\bar{i}$ benefits the objective function by as much as $f_{\bar{i},k} - s_{\bar{i},k}$, where $f_{\bar{i},k} = l_{\bar{i}}$ and $s_{\bar{i},k} = \max(e_{\bar{i}}, l_{i-1} + t_{i-1,\bar{i}})$. On the other hand, the coverage of hot spot i may potentially be reduced due to the late start $s'_{i,k}$ at hot spot i. The change in the objective due to insertion of $\bar{i}$ *right before* hot spot i is given as:

$$\begin{aligned}\delta &= \text{Benefit After } \bar{i} \text{ Insertion} - \text{Original Benefit}\\ &= \{f_{\bar{i},k} - s_{\bar{i},k}\} + \{f_{ik} - s'_{ik}\} - \{f_{ik} - s_{ik}\}\\ &= l_{\bar{i}} - s_{\bar{i},k} - (s'_{i,k} - s_{i,k}).\end{aligned} \tag{16}$$

When $\delta > 0$, there is value in including $\bar{i}$ between hot spots $i-1$ and i; or otherwise, we continue to check the next uncovered hot spot.

4.2 Improvement Algorithms

As mentioned above, hot spots are inserted sequentially. The construction algorithm is affected by the selection and order of the subsequently inserted hot spots. The improvement algorithms address this issue by utilizing modified versions of relocate and exchange operators introduced originally for the vehicle-routing problem with time windows (Braysy and Gendreau, 2005a,b). The relocate operator finds improvements by moving a hot spot from one route to another, whereas the exchange operator exchanges hot spots between two routes. The modification step involves revoking the insertion algorithm after each move.



(a) Relocate Operator (b) Exchange Operator

Figure 3: Neighborhood search operators

4.2.1 *Relocate Operator*

In Figure 3(a), we present the relocate operator, where hot spot i from the origin route k is moved into the destination route g, $k \neq g$. In the figure, we also represent the other routes visiting i—due to the possible visits by multiple cars—in dotted red lines. We let (s_{ik}, f_{ik}) and (s_{ig}, f_{ig}) as well as $(s_{i+1,k}, f_{i+1,k})$ and $(s'_{i+1,k}, f'_{i+1,k})$ denote the start and finish times at hot spots i and $i+1$ before and after the move respectively. Hot spots j and $j+1$ follow a similar notation. After the move, the change in the objective is

$$\begin{aligned}\Delta &= (f_{ig} - s_{ig}) - (f_{ik} - s_{ik}) + (f'_{i+1,k} - s'_{i+1,k}) - (f_{i+1,k} - s_{i+1,k}) + (f'_{j+1,g} - s'_{j+1,g}) - (f_{j+1,g} - s_{j+1,g}) \\ &= (s_{ik} - s_{ig}) + (s_{i+1,k} - s'_{i+1,k}) + (s_{j+1,g} - s'_{j+1,g}),\end{aligned}$$

as finishing times before and after the move are the same. However, modification of the start times of the coverage is more complicated due to the possibility of covering a hot spot with multiple cars. If hot spot i is only visited by route k or k is the first of multiple visits to hot spot i, the start time after the move is obtained by comparing the arrival time at hot spot i from a visit at j with the earliest time window hot spot i; that is, $s_{ig} = \max\{f_{jg} + t_{ij}, e_i\}$. Otherwise, hot spot i is visited by multiple cars and route/car k is an intermittent car. That is, the hot spot i is covered by some other car(s) until s_{ik}. Therefore, the start time after the move is obtained by comparing the arrival time at hot spot i from j and s_{ik}; that is, $s_{ig} = \max\{f_{jg} + t_{ij}, s_{ik}\}$. A similar check takes place for updating $s'_{i+1,k}$ and $s'_{j+1,g}$.

If $\Delta \leq 0$, the relocate operator is not successful in generating a better solution and is not pursued any further. We move onto the next route and/or hot spot. Otherwise $\Delta > 0$ and we invoke the insertion algorithm again as relocation may open additional possibilities to insert an uncovered hot spot. We check if an uncovered hot spot can be inserted between the nodes defined by the modified arcs one by one: $(i-1,i+1)$, (j,i), and $(i,j+1)$. We let δ_1, δ_2, δ_3 be the benefits of inserting an uncovered hot spot before $i+1$, i, and $j+1$ respectively. Each of these benefits is calculated as in Equation 16. If $\delta_1 > 0$, the insertion before $i+1$ is accepted and updated benefit $\hat{\Delta}$ is set as $\Delta+\delta_1$. Otherwise, if $\delta_2 > 0$, the insertion before i is accepted and $\hat{\Delta}$ is set as $\Delta+\delta_2$. Finally, if $\delta_3 > 0$, $\hat{\Delta}$ is set as $\Delta+\delta_3$. If none of the insertions are favorable—that is, $\delta_a < 0$ for $a = 1,2,3$—the $\hat{\Delta}$ is the same as Δ. Among the positive $\hat{\Delta}$ obtained through the whole relocate neighborhood, we pick the one that provides the maximum benefit and implement the relocate (and if there is one, insertion) associated with that maximum $\hat{\Delta}$. That is, we use the Global Best (GB) acceptance rule.

4.2.2 *Exchange Operator*

In Figure 3(b), we present the exchange operator, where two hot spots i and j swap routes simultaneously. As in Figure 3(a), the dotted red lines represent the possibility of other state trooper car(s) covering hot spots i and j. After the swap, the start times of the hot spots i, $i+1$, j, and $j+1$ will be modified. The corresponding change in the objective is

$$\begin{aligned}\Delta &= (f_{ig}-s_{ig})-(f_{ik}-s_{ik})+(f'_{i+1,k}-s'_{i+1,k})-(f_{i+1,k}-s_{i+1,k})\\&\quad+(f_{jk}-s_{jk})-(f_{jg}-s_{jg})+(f'_{j+1,g}-s'_{j+1,g})-(f_{j+1,g}-s_{j+1,g})\\&=(s_{ik}-s_{ig})+(s_{jg}-s_{jk})+(s_{i+1,k}-s'_{i+1,k})+(s_{j+1,g}-s'_{j+1,g}).\end{aligned}$$

Similar to the relocate operator, these start times are influenced by the number of state trooper cars visiting the hot spot and the order of the cars. In particular,

$$s_{ig} = \begin{cases}\max\{f_{j-1,g}+t_{j-1,i},\ e_i\} & \text{if } k \text{ is the 1st visit;}\\ \max\{f_{j-1,g}+t_{j-1,i},\ s_{ik}\} & \text{O/W.}\end{cases} \qquad s_{jk} = \begin{cases}\max\{f_{i-1,k}+t_{i-1,j},\ e_j\} & \text{if } g \text{ is the 1st visit;}\\ \max\{f_{i-1,k}+t_{i-1,j},\ s_{jg}\} & \text{O/W.}\end{cases}$$

The start times $s'_{i+1,k}$ and $s'_{j+1,g}$ are calculated in a similar manner.

If $\Delta > 0$, the exchange is a candidate to be accepted. As with the relocate operator, the exchange may provide a possibility to insert an uncovered hot spot between $(i-1,j)$, $(j,i+1)$, $(j-1,i)$, and $(i,j+1)$. The benefits of insertion on these arcs are calculated as δ_1, δ_2, δ_3, and δ_4 respectively, as in Equation 16. The insertion is evaluated in that order, and the first insertion with a positive benefit—that is, $\delta_a > 0$ for $a = 1,2,3,4$—is accepted. The total benefit $\hat{\Delta}$ is updated as $\Delta+\delta_a$. If none of the insertions return a benefit, then $\hat{\Delta}$ is just set to Δ. Similar to the relocate operator, the exchange operator is implemented using the GB criteria. The exchange (and potential insertion) associated with the largest $\hat{\Delta}$ in the neighborhood is accepted. After the exchange (and the potential insertion), the routes and the uncovered hot spot set $\mathcal{U}$ are updated accordingly.

4.2.3 *Local Search*

After introducing the neighborhood search components, Figure 4 depicts how these play a role in our local search implementation. In the first stage of improvement, the algorithm loops through the relocate operator embedded with the insertion step until no improvement is found. Note that after the relocate operator is embedded with insertion step, the insertion algorithm is called again because if there is any move, the $\mathcal{U}$ set and routes are updated. Thus, there is a chance to insert an uncovered hot spot into any of the existing routes. In the third stage of improvement, the exchange operator embedded with insertion step keeps searching until no further improvement can be found, followed by the insertion step for the same reason as the first stage of improvement. The local search terminates when no further improvement is available.

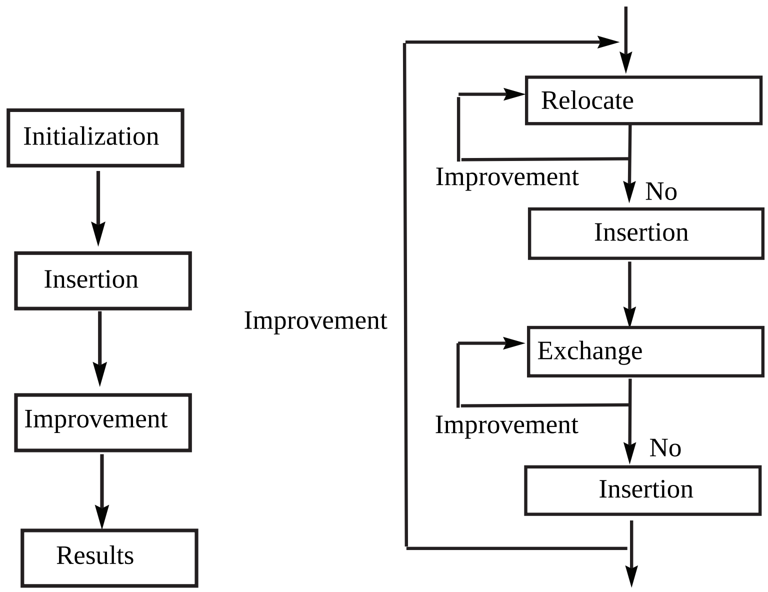


Figure 4: Local search and improvement flow charts

4.2.4 *Tabu Search*

Based on the fact that local search can be trapped at a local optimum, we also apply a tabu-search algorithm as a part of the improvement step.

In our implementation, the tabu list consists of two attributes: state trooper car index and hot-spot identification. Specifically, if the most recent solution includes covering hot spot i by state trooper k, then the (i,k) pair is marked as tabu. The tabu list length and tabu tenure are set to $5 \times \lfloor \sqrt{n} \rfloor$, directly correlated with the total number of hot spots n. In the neighborhood, the relocate operator is followed by the exchange operator. Each operation is conducted over all of the routes and visited hot spots. Random numbers determine the starting hot spot and the starting route number for each operator. Once the search starts, it sweeps through all of the hot spots and routes exhaustively.

If it is feasible to carry out a particular operation, both state trooper car and visited hot spot indices are added to the tabu list. With the relocate operator, only the relocated hot spot and its corresponding state trooper indices are added to the tabu list. On the other hand, with the exchange operator, both of the exchanged hot spots and their corresponding route indices are added to the tabu list. As an aspiration criteria, tabu is only overridden when the newly obtained objective is better than the best one found thus far.

Section 5
Computational Experiments

5.1 Performance-based Experiments

To test the performance and effectiveness of the model and heuristic approaches, we conduct a series of numerical studies on randomly generated problems ranging from small to moderately large as well as on real-life data captured from CARE (see Section).

To benchmark the quality and runtime of our heuristics, we also run CPLEX 12.1 for all of the instances. We implement and run the algorithms using C++ on a Dell Poweredge 6850 with four dual-core 3.66GHz Xeon processors and 8GB of memory.

5.1.1 *Experiment with Randomly Generated Data*

We randomly pick 10, 20, and 40 locations on the highway and corresponding earliest and latest time windows from a pool of real-life data, with 20 instances in each data set. Both algorithms are tested when there are up to 8 state trooper cars available; that is, a total of 480 ($3 \times 20 \times 8$) instances.

We compare the solutions returned by local search (LS) and tabu search (TS) with the ones obtained from CPLEX, as shown in Table 1. Unfortunately, CPLEX runs out of memory for even relatively small instances, such as when 2 state trooper cars are available for 40 hot spots. We evaluate our heuristics by calculating the percentage of the gap between the objective returned by our heuristics and lower bound (LB) of CPLEX, which is defined as **Gap** $= (Objective - LB)/LB * 100$. Note that since we have a maximization problem, the lower bound returns the best feasible solution that CPLEX can obtain and a positive gap indicates that the heuristics outperform the best feasible solution returned by CPLEX.

In Table 1, we report both average (Avg.) and maximum (Max.) gaps that demonstrate the best performance of the heuristics. We also report the number of times that CPLEX is able to find optimal solution out of all 20 instances, contained in the column “No. opt.,” and the number of times that LS/TS is at least as good as the LB returned by CPLEX, contained in the column of “No. best.”

In Table 1, we observe that CPLEX has a deteriorating performance as the number of hot spots and state trooper cars increases. On the contrary, for these instances where CPLEX is struggling, the frequency of finding a solution at least as good as the CPLEX LB (“No. best”) is increasing for our heuristics. Specifically, our heuristics are able to find a solution at least as good as the CPLEX LB for 10 HS and 20 HS most of the time and for 40 HS some of the time, especially with a higher number of cars. In fact, the heuristics return slightly better solutions when there are a higher number of hot spots and state trooper cars. With respect to the performance comparison between LS and TS, even though there is not much gap difference for LS and TS, LS still performs slightly better than TS especially for higher number of hot spots.

Table 1: Performance of LS and TS for random data

Data Set	No.	No.	CPLEX	LS			TS		
	Cars	Instances	No. opt.	Avg.	Max.	No. best	Avg.	Max.	No. best
10HS	3	20	20	-1.4	0.0	18	-1.4	0.0	18
	4	20	3	0.0	0.0	20	-2.1	0.0	19
	5	20	1	0.1	3.4	20	0.1	3.4	20
	6	20	1	0.1	3.4	20	0.1	3.4	20
	7	20	0	0.1	3.4	20	0.1	3.4	20
	8	20	0	0.1	3.4	20	0.1	3.4	20
20HS	3	20	2	-1.3	0.0	5	-1.5	0.0	4
	4	20	2	-1.0	0.0	8	-1.0	0.0	5
	5	20	2	-0.8	0.0	12	-0.9	0.0	15
	6	20	0	-0.3	0.0	16	-0.8	0.0	16
	7	20	0	-0.5	0.0	17	-0.5	0.0	17
	8	20	0	-0.1	0.0	17	-0.1	0.0	17
40HS	3	20	0	-4.9	0.0	0	-5.7	0.0	0
	4	20	0	-2.6	0.0	0	-3.3	0.0	0
	5	20	0	-0.5	4.1	4	-1.3	1.9	2
	6	20	0	-0.9	1.4	8	-1.3	1.1	3
	7	20	0	0.0	4.4	12	-0.4	4.4	8
	8	20	0	0.1	3.3	14	-0.1	2.6	12

From the perspective of runtime of local search or tabu-based improvement, both are less than 15 seconds even for instances with 40 hot spots. On the contrary, the more cars there are and the bigger the road network is, the longer it takes CPLEX to find an optimal solution. For instance, it typically takes around 1 – 2 hours for CPLEX to find an optimal solution (for smaller instances) or just an LB (for larger instances). Thus, we conclude that our heuristic approaches are more practical since state troopers need to respond to road condition changes relatively frequently.

5.1.2 ***Experiment with Real-Life Data***

We also solve the real instances obtained from the CARE database and optimize covering and routing for state troopers on the highways by work shift, by day of week, and by region. Due to the large number of tests, we select three representative areas with a large number of hot spots: Jefferson County rural area (Jeff), the Mobile area (Mob), and Tuscaloosa County rural area (Tus). The most representative days and times for the experiment are Monday, Friday, and Saturday with three shifts: a morning shift from 7:00am to 3:00pm, an afternoon shift from 3:00pm to 11:00pm, and an evening shift from 11:00pm to 7:00am. As the other weekdays (Tuesday through Thursday) mimic Monday and Sunday mimics Saturday, we do not report the results for these days.

In Table 2, we present the results for local and tabu search respectively. Note that the data instances are referred to using the first letter representing the day (Monday) and the second letter representing the shift. For instance, MM refers to the Monday morning shift. With three work days and three shifts, there are a total of nine instances in every county. Each instance is tested with various state trooper cars from 3 to 8. At the last row of each county, we summarize the number of optimal solutions CPLEX returned. For each instance with a particular number of state troopers, we report the gap between objective returned by local and tabu search and LB of CPLEX respectively. A positive gap refers to a better objective value by our heuristics, whereas a negative gap indicates that the best feasible solution returned by the CPLEX is better.

Table 2: Performance of LS and TS for real data

Instances		LS						TS					
		3	**4**	**5**	**6**	**7**	**8**	**3**	**4**	**5**	**6**	**7**	**8**
Jeff	MM	-1.5	-7.1	0.0	0.0	0.0	0.6	-1.5	-7.1	0.0	0.0	0.0	0.6
	MA	-5.8	-6.1	-7.4	-0.2	-2.8	-1.0	-7.6	-7.4	-8.6	-0.5	-2.3	-0.3
	ME	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	FM	-2.6	-2.9	0.0	0.0	0.0	0.0	-2.6	-2.9	0.0	0.0	0.0	0.0
	FA	-1.2	-2.3	0.1	0.0	0.0	-0.6	-1.2	-2.3	-2.1	0.0	0.0	-0.6
	FE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SM	0.0	**3.2**	0.0	0.0	0.0	0.0	0.0	**3.2**	0.0	0.0	0.0	0.0
	SA	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	No. CPX Opt.	1	1	0	0	0	0	1	1	0	0	0	0
Mob	MM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	MA	-3.2	-2.8	0.0	-2.5	0.0	0.0	-3.2	-2.8	0.0	-0.3	0.0	0.0
	ME	-1.2	0.0	0.0	0.0	0.0	0.0	-1.2	0.0	0.0	0.0	0.0	0.0
	FM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	FA	-1.7	0.0	0.0	0.0	0.0	0.0	-1.7	-0.8	0.0	0.0	0.0	0.0
	FE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-4.1	0.0	0.0	0.0	0.0
	SM	-4.7	-0.2	-0.7	0.0	0.0	0.0	-4.7	-0.2	-0.3	0.0	0.0	0.0
	SA	**6.6**	**19.5**	0.0	0.0	0.0	0.0	**6.6**	**19.5**	0.0	0.0	0.0	0.0
	SE	-1.8	0.0	0.0	0.0	0.0	0.0	-1.8	0.0	0.0	0.0	0.0	0.0
	No. CPX Opt.	5	2	0	0	0	0	5	2	0	0	0	0
Tus	MM	-0.2	0.0	0.0	0.0	0.0	0.0	-0.2	0.0	0.0	0.0	0.0	0.0
	MA	-0.2	-0.8	0.0	0.5	0.7	-0.3	-0.2	-0.8	0.0	0.5	0.7	-0.3
	ME	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	FM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	FA	0.0	0.0	0.0	0.0	0.0	0.0	-2.8	0.0	0.0	0.0	0.0	0.0
	FE	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-4.6	0.0	0.0	0.0	0.0
	SA	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	SE	-1.4	0.0	0.0	0.0	0.0	0.0	-1.4	0.0	0.0	0.0	0.0	0.0
	No. CPX Opt.	5	0	0	0	0	0	5	0	0	0	0	0

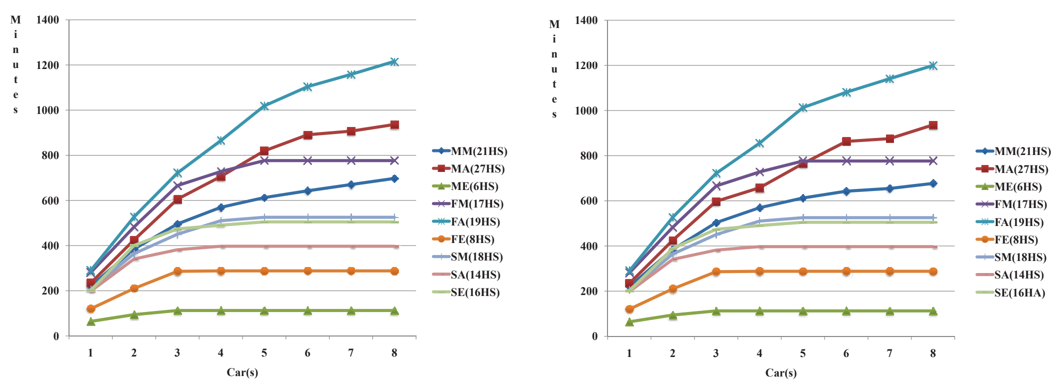
Typically the gap between the heuristics and CPLEX is nonnegative since the solution quality is as good as or better than that of the CPLEX LB. Most gaps fall between -1% and 1%, with very few outliers. Some of these extremes are the negative gaps of -5.8%, -6.1%, and -7.4% for Jefferson during the Monday afternoon shift with three, four, and five state trooper cars respectively. In this particular instance, the number of hot spots is 27 with varying durations. With a limited number of state trooper cars and an excess number of hot spots to cover, the heuristics tend to not perform as well since, in general, they do depend on the improvements (relocate or exchange) among a number of routes.

On the other extreme, there is a positive gap of 19.5% for Mobile during Saturday afternoon shift with four state trooper cars. This is attributed to the poor performance of CPLEX; it is not due to our formulation or the gap. More specifically, for this instance as well as the instances marked in bold, CPLEX claims to reach the optimum with the lower bound equal to the upper bound. However, our heuristics *return a better solution* than the claimed CPLEX optimum. We double checked these solutions with manual calculations and found that the solutions returned by the heuristics are indeed feasible and optimal. We reported our model and these problematic instances to ILOG technical support group. They confirmed that there is an internal failure in the CPLEX engine when solving these instances. These instances have been added to their test bed to improve the CPLEX engine.

In summary, as the size of the problem grows, CPLEX has a harder time in obtaining reasonable solutions. In comparison between LS and TS, LS outperforms TS slightly most times. Again, for the computational time, our heuristics provide results within seconds; while CPLEX takes at least couple of hours to find a relatively good feasible solution.

5.2 Managerial Insights

In this section, we provide managerial insights for decision makers based on our solutions using real data. In Figures 5 and 6, we plot the objective value of *MCPRP* returned by LS and TS with respect to different state trooper cars respectively. From the plotted charts, we can determine how many state trooper cars are needed for each data set. Intuitively, as the number of state troopers on patrol increases, hot-spot coverage improves. However, there are diminishing returns with the addition of each state trooper. One interesting observation is that, as there are more hot spots, the

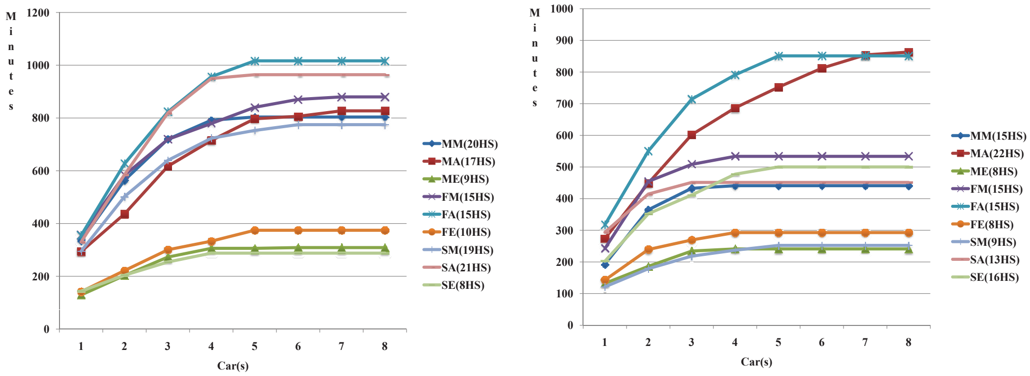


(a) Local Search (b) Tabu Search

Figure 5: The coverage with LS and TS due to different state trooper cars in Jefferson County

objective is higher. This is due to higher potential coverage. However, in Jefferson County, the top line corresponds to Friday afternoon with 19 hot spots. This particular instance returned a higher objective compared to, say, Monday afternoon with 27 hot spots. Investigating this phenomenon further, we found that the hot-spot time windows are not equal. In the data set with 19 hot spots, most hot spots are "hot" for more than an hour, whereas in the data set with 27 hot spots, most of the hot spots are only "hot" for half an hour. Hence, the objective not only depends on the total number of hot spots available but also length of each hot spot.

Investigating Figures 5 and 6, we can help identify how many state troopers are needed in each



(a) Mobile Local Search

(b) Tuscaloosa Local Search

Figure 6: The coverage with LS in the city of Mobile and Tuscaloosa County

shift on each day. For instance, for Jefferson County on Monday and Friday evenings, three state trooper cars suffice. However, for Saturday evening at least five cars are needed. Furthermore, for Monday and Friday afternoons, even eight cars may not be enough. This analysis not only provides a good basis for how to allocate resources; it also demonstrates how the adverse effects of lack of resources (that is, potential budget and personnel cuts) can be alleviated.

Note, theoretically speaking, that all lines should be concave; however, in part (b) of Figure 5, the objective for Monday afternoon is not concave, since they are returned by our heuristics.

Table 3: Service measure performances by incremental state troopers

Data Set		MM	MA	ME	FM	FA	FE	SM	SA	SE
Jeff	Cars	8	8	3	5	8	3	5	4	5
	HS	21	27	6	17	19	8	18	14	16
	HS%	90	93	67	100	100	100	89	100	100
	TW	810	1110	179	960	1410	299	600	449	570
	TW%	86	84	63	81	86	96	88	88	89
Mob	Cars	5	7	4	6	5	5	6	5	4
	HS	20	17	9	15	15	10	19	21	8
	HS%	100	100	100	100	100	100	100	100	100
	TW	840	870	330	930	1050	420	910	1020	299
	TW%	96	95	93	94	97	89	85	95	96
Tus	Cars	4	7	4	4	5	4	5	3	3
	HS	15	22	8	15	15	8	9	13	16
	HS%	100	100	100	100	100	100	100	100	94
	TW	480	870	270	600	900	330	270	480	539
	TW%	92	98	89	89	95	89	93	94	76

For these instances, we also compute performance measures for our suggested covering plan: how many hot spots we will cover and how long the hot spots will be covered. In Table 3, we present a detailed plan with respect to how many state troopers are needed per shift, per day, and per region, shown in row “Cars” and performance measures shown in rows “HS%” and “TW%” for the Jefferson, Mobile, and Tuscaloosa areas. From these results, we observe that hot-spot coverage percentages are quite close to 100% for our suggested plan. Furthermore, the objective coverage percentage is above 85%, except for three instances. For example, the “TW%” is 63% for the Jefferson ME shift and 76% for the Tuscaloosa SE shift. This is due to the start time of these hot spots and the travel time required to reach these hot spots. For these instances, even with unlimited resources, it is not possible to fully cover the total hot times, unless the state troopers are allowed to start patrolling from locations other than the state trooper post.

In a final experiment, we evaluate the impact of having hot spots with varying weights. Until this last experiment, all of the experiments assume equally weighted hot spots. However, in real life, some hot spots are more important than others due to the potential severity of the accidents at those locations. We represent these severity levels by attaching different weights to hot spots. We use two arbitrary weight schemes for testing purposes: high variance with weights of 1, 1.5, and 2; and low variance with weights of 1, 1.1, and 1.2. In Table 4, we report the performance of LS with 2, 4, 6, and 8 cars using these two weight schemes. On the bottom row of the table, we calculate the average and maximum gap over all of the instances given a particular resource level. Since TS has similar performance as LS, for the sake of the brevity, we do not report the results. The results of weighted schemes demonstrate the benefit of heuristics, as the heuristics beat the LB of the CPLEX with high percentages, especially for instances with high number of hot spots such as Mobile SA (21 HS), Jefferson MA (27 HS), Jefferson MM (21 HS), and Tuscaloosa MA (22 HS). The benefits are more pronounced with high variance weight scheme. Even though Proposition 1 does not hold for hot spots with varying weights and the heuristics are based on this proposition, the performance of the heuristics is very robust.

Table 4: LS performance for real data with different weights

	Instances	High Weights (1,1.5,2)				Low Weights (1,1.1,1.2)			
		2	4	6	8	2	4	6	8
Jeff	MM	0%	9%	24%	0%	-2%	0%	1%	8%
	MA	22%	-6%	26%	30%	6%	8%	13%	7%
	ME	10%	0%	0%	10%	0%	1%	1%	2%
	FM	-9%	24%	24%	1%	-13%	-4%	-3%	-3%
	FA	0%	-6%	23%	11%	-4%	-3%	-1%	-3%
	FE	13%	0%	56%	14%	3%	0%	4%	4%
	SM	-4%	6%	-19%	-5%	-10%	-5%	-15%	-15%
	SA	-7%	-26%	-20%	-15%	-1%	6%	-12%	-12%
	SE	5%	-1%	1%	4%	-2%	12%	-3%	-3%
Mob	MM	10%	29%	1%	3%	-4%	2%	4%	4%
	MA	6%	17%	0%	21%	-13%	23%	2%	2%
	ME	0%	-8%	2%	8%	-8%	-1%	-4%	-4%
	FM	-10%	-7%	-33%	-24%	-6%	5%	-3%	-3%
	FA	2%	10%	-2%	4%	-6%	6%	0%	0%
	FE	20%	2%	2%	5%	-5%	-4%	-11%	-11%
	SM	8%	-8%	-5%	-10%	4%	13%	11%	11%
	SA	17%	33%	15%	15%	-2%	7%	2%	2%
	SE	16%	0%	0%	17%	1%	0%	2%	2%
Tus	MM	-5%	-14%	15%	-3%	-12%	10%	-13%	-13%
	MA	25%	10%	18%	8%	0%	23%	6%	5%
	ME	-1%	31%	18%	13%	-9%	3%	0%	0%
	FM	-4%	-12%	-12%	-2%	-12%	-9%	-11%	-11%
	FA	30%	38%	41%	35%	3%	5%	7%	7%
	FE	21%	39%	39%	24%	0%	4%	4%	4%
	SM	-13%	-1%	-1%	-10%	-12%	-11%	-11%	-11%
	SA	3%	6%	-11%	4%	-8%	-7%	-9%	-9%
	SE	7%	7%	39%	9%	-11%	2%	3%	3%
	Avg.	6%	6%	9%	6%	-5%	3%	-1%	-1%
	Max.	30%	39%	56%	35%	6%	23%	13%	11%

Section 6
Conclusions and Future Work

To maximize the effectiveness of state trooper patrols by covering hot spots, we develop a novel model. In this model, we determine whether a state trooper visits a hot spot and their arrival and departure times at the hot spots. As the large instances of the problem are beyond the capability of any off-the-shelf optimization software, we design algorithms based on local and tabu search using different neighborhoods. Then we test our model and solution approaches by using sets of random and real data. Compared with the CPLEX LB, in most instances our solutions are at least as good as or better than CPLEX and have short runtimes. Furthermore, we have found several instances where CPLEX failed to solve the problem.

The computational testing results are particularly useful for decision-makers in determining the optimal number of state troopers.This is important as better coverage is believed to lead to fewer accidents, lower economic impact, and better road safety for everybody. On the other hand, the model also shows the best coverage given a particular resource level. This analysis would be valuable to determine how to reallocate resources in the event of a potential budget cut or increase.

The contributions of the paper to the literature are threefold. First, the literature on TOPTW focuses on benefit collection of fixed values given a priori, whereas MCPRP treats profits as a set of “continuous decision variables” and allows multiple visits to the same hot spot. Second, the solution approaches developed can solve even real-life instances of the problem within seconds. Finally, this paper departs significantly from the TOPTW literature by introducing *effective patrolling measures* (HS% and TW%), which are useful for decision-makers to determine the optimal levels of coverage for a given resource level.

There are several potential extensions. First, in this paper, we assumed constant travel speed for state troopers traveling from one hot spot to another. Instead of constant travel speed, generalizing the problem where travel speed is correlated with time of day or day of week would be practical and interesting. Second, the model could be extended to consider multiple state trooper posts or the ability of the troopers to take their work cars home instead of returning to the state trooper post. This problem would be analogous to the multi-depot vehicle routing problem with time windows. Thirdly, we are interested in incorporating an on-call response into the model, especially to utilize coverage for accidents immediately using dynamic crash information. Finally, the mission statements of many of the highway patrol departments in the United States reflect the belief that issuing citations is an effective auto-crash countermeasure (Steil and Parrish, 2009). Hence, the results of this paper can be extended into an application focused on revenue management.

Section 7
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Appendix

Proof of Theorem 1:

The maximal covering location problem (MCLP) establishes a set of m facilities to maximize the total weight of "covered" customers, where a customer is considered covered if she is located at most a distance r from the closest facility. The problem was originally introduced by Church and ReVelle (1974) and is NP-hard (Marianov and ReVelle, 1995). To prove that MCPRP is NP-hard, we need to show that the MCLP is polynomially reducible to MCPRP.

Suppose we had a polynomial algorithm for solving the decision version of the MCPRP. Given $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, time windows associated with all hot spots, $|\mathcal{K}|$ state trooper cars, shift duration $\mathcal{T}$, and a positive number $\mathcal{B}$, our algorithm would produce a "yes" or "no" answer in polynomial time to the decision question of MCPRP: are there $|\mathcal{K}|$ routes that satisfy the time-window restrictions of all hot spots and that take less than $\mathcal{T}$ such that the total coverage time is at least $\mathcal{B}$? Now construct an instance of MCPRP as follows: $[e_i,\ l_i] = [e_i,\ e_i + \alpha_i]$, where α_i is an arbitrary small number, say 1 minute, such that the stop at hot spot i can only collect α_i.

Now consider the following notation for the MCLP:

I	Set of hot spots.
$\mathcal{J}$	Set of all of the routes that satisfy the time windows and shift duration restrictions.
a_i	Coverage benefit, that is for any $k \in \mathcal{K}$, $a_i = f_{ik} - s_{ik} = l_i - e_i = \alpha_i$.
N_i	Set of routes that include hot spot i.

Decision Variables

X_j	1, if route j is selected as a part of patrolling plan, 0, otherwise.
Y_i	1, if hot spot i is covered, 0, otherwise.

The mathematical formulation is presented as

$$\max \sum_{i \in I} a_i Y_i \tag{17}$$

$$s.t.$$

$$\sum_{j \in N_i} X_j \geq Y_i, \qquad \forall i \in I. \tag{18}$$

$$\sum_{j \in \mathcal{J}} X_j = |\mathcal{K}|. \tag{19}$$

$$Y_i \in \{0,1\} \text{ and } X_j \in \{0,1\}, \qquad \forall i \in I \text{ and } j \in \mathcal{J}. \tag{20}$$

Constraint 18 allows the coverage Y_i to equal 1 only when one or more routes in set N_i are chosen. The number of routes is restricted to $|\mathcal{K}|$ in constraint 19. The solution to this problem specifies not only the maximal hot spot coverage but also the $|\mathcal{K}|$ routes that achieve this maximal coverage.

The transformation to MCLP is polynomial since all of the problem parameters can be obtained in polynomial time, including the set $\mathcal{J}$. Note that the size and construction of the routes are limited by the time windows of hot spots and the shift duration. If a hot spot is chosen for a route, there are only $(n-p_1)$ choices where $p_1 \geq 1$ due to the time window restrictions, and every time a hot spot is included in a route, the available choices decrease super-linearly. Then, a route can be constructed by evaluating $n \times (n-p_1) \times (n-p_2) \times \ldots \times (n-p_k)$, where $p_k > p_{k-1} > \ldots > p_2 > p_1$ and $p_k < n$ due to $\mathcal{T}$ and time windows. Thus, the set $\mathcal{J}$ can be constructed by an algorithm with $O(n^{p_k})$ complexity.

Overall, the optimal solution to MCPRP provides an answer (yes/no) to the decision version of the MCLP whether there exists $|\mathcal{K}|$ "facilities" (routes) to cover the "customers" (hot spots) to obtain a benefit that is at least $\mathcal{B}$. Therefore, the proof is complete. ■

Proof of Proposition 1:

The proof covers two cases. The first case considers the situation where pushing the end of service time at one hot spot does not eliminate visits to the future hot spots. The second case covers the possibility of reduction in the number of hot spots visited in the remainder of the coverage due to incrementing the service time at one hot spot.

Case 1: No Hot-Spot Elimination
First, let S^* be an optimal solution with the objective function value $v(S^*)$. For route $k \in \mathcal{K}$ in S^*, let i be the last hot spot where $f_{ik} < \min(l_i,\ 480 - t_{i,n+1})$.

For a state trooper to get back to the state trooper post by the end of the shift, the finish time at the last hot spot of his route should satisfy $f_{ik} + t_{i,n+1} \leq 480$. Now let us create a new solution S' from S^* where everything is kept the same except $f'_{ik} = \min(l_i,\ 480 - t_{i,n+1})$. Thus, $f'_{ik} > f_{ik}$. Hence, the objective value of S', $v(S')$, is larger than $v(S^*)$, which contradicts that S^* is optimal. Hence, if i is the last hot spot visited on route k, $f_{ik} = \min(l_i,\ 480 - t_{i,n+1})$.

Consider now the situation where i is not the last hot spot on route k. Suppose S^* is an optimal solution such that there is at least one hot spot i satisfying $f_{ik} < l_i$. We again create a new solution S' from S^* where everything is kept the same except $f_{ik} = l_i$. The difference between $v(S^*) - v(S') = f_{ik} - l_i - s_{i+1,k} + s'_{i+1,k}$, where $s'_{i+1,k}$ is the start time at hot spot $i+1$ on route k in solution S'. Now $s'_{i+1,k} - s_{i+1,k} = \max(l_i + t_{i,i+1},\ e_{i+1}) - \max(f_{ik} + t_{i,i+1},\ e_{i+1})$

$$= \begin{cases} l_i - f_{ik} & \text{if } e_{i+1} \leq f_{ik} + t_{i,i+1}; \\ l_i + t_{i,i+1} - e_{i+1} & \text{if } f_{ik} + t_{i,i+1} < e_{i+1} \leq l_i + t_{i,i+1}; \\ 0 & \text{if } e_{i+1} > l_i + t_{i,i+1}. \end{cases}$$

Note that in all cases $s'_{i+1,k} - s_{i+1,k} \leq l_i - f_{ik}$. Therefore, $v(S^*) - v(S') < 0$, which contradicts that S^* is the optimal solution. Since i is an arbitrary hot spot, in the optimal solution $f_{ik} = l_i$ on a route k.

Case 2: Possible Hot-Spot Elimination

In this case, in the newly created solution S', the adjustment at the previous hot spot makes it infeasible to reach the next hot spot(s) on the original route. So, state trooper k needs to skip some hot spot(s) on the original route to go to the next reachable hot spot. We prove this case by induction.

Case 2a: Base Step The increment of service time at hot spot i only eliminates the next hot spot $i+1$ on the route. We assume that the triangular inequality holds; that is, $t_{i,i+2} \leq t_{i,i+1} + t_{i+1,i+2}$. Then, for route k, the difference in the objective functions $v(S^*)$ and $v(S')$ comes from the changes of contributions of hot spots i, $i+1$, and $i+2$. These contributions are

- $\Delta_i = f_{ik} - s_{ik}$ and $\Delta'_i = l_i - s_{ik}$;
- $\Delta_{i+1} = l_{i+1} - \max(e_{i+1}, f_{ik} + t_{i,i+1})$ and $\Delta'_{i+1} = 0$; and
- $\Delta_{i+2} = l_{i+2} - \max(e_{i+2}, l_{i+1} + t_{i+1,i+2})$ and $\Delta'_{i+2} = l_{i+2} - \max(e_{i+2}, l_i + t_{i,i+2})$.

Then $v(S') - v(S^*) = \sum_{j=i}^{i+2} \Delta'_j - \sum_{j=i}^{i+2} \Delta_j =$
$l_i - \max(e_{i+2}, l_i + t_{i,i+2}) - f_{ik} - l_{i+1} + \max(e_{i+1}, f_{ik} + t_{i,i+1}) + \max(e_{i+2}, l_{i+1} + t_{i+1,i+2})$. Based on different cases of $\max(e_j, l_{i+1} + t_{i+1,j}) - \max(e_j, l_i + t_{ij})$, we simplify this statement and observe that $v(S') \geq v(S^*)$ for every case. Even though one fewer hot spot is covered, the coverage time is not shortened. Hence, this objective value is at least as good as the original objective value.

Case 2b: Induction Step Now we assume that the increment in the service time eliminates the next consecutive $b > 1$ hot spots. In this case, let $v(S'_b)$ denote the objective function for the modified solution S'_b. We assume that $v(S'_b) - v(S^*) \geq 0$. We need to prove that if $b+1$ hot spots are eliminated, $v(S'_{b+1}) \geq v(S_{b+1})$ holds. From the triangular inequality, we know $t_{i,i+b+1} \leq t_{i,i+b+1} + t_{i+b+1,i+b+2} \leq t_{i,i+1} + t_{i+1,i+2} + \ldots + t_{i+b-1,i+b} + t_{i+b,i+b+1} + t_{i+b+1,i+b+2}$. In addition, for $j = 1, \ldots, b+1$, $\Delta_{i+j} = l_{i+j} - \max(e_{i+j}, f_{ik} + t_{i,i+j})$ and $\Delta'_{i+j} = 0$; and $\Delta_{i+b+2} = l_{i+b+2} - \max(e_{i+b+2}, l_{i+b+1} + t_{i+b+1,i+b+2})$ and $\Delta'_{i+2} = l_{i+b+2} - \max(e_{i+b+2}, l_i + t_{i,i+b+2})$. Then

$$
\begin{aligned}
v(S'_{b+1}) - v(S^*) &= \sum_{j=i}^{i+b+2} \Delta'_j - \sum_{j=i}^{i+b+2} \Delta_j \\
&= v(S'_b) - v(S^*) - (l_{i+b+1} - \max(e_{i+b+1}, l_i + t_{i,i+b+1})) \\
&\quad - \max(e_{i+b+2}, l_i + t_{i,i+b+2}) + \max(e_{i+b+2}, l_{i+b+1} + t_{i+b+1,i+b+2})
\end{aligned}
$$

Based on the cases of $\max(e_{j+1}, l_j + t_{j,j+1}) - \max(e_{j+1}, l_i + t_{i,j+1})$ and the induction step, $v(S'_{b+1}) \geq v(S^*)$. Hence, the modified solution is as good as S^*.

This concludes the proof. ■

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