

# Improved Methods for Operating Public Transportation Services



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**MID ATLANTIC UNIVERSITIES TRANSPORTATION CENTER**

**Improved Methods for Operating Public Transportation Services**

**Final Report**

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<b>16. Abstract</b> <p>In this joint project, West Virginia University and the University of Maryland collaborated in developing improved methods for analyzing and managing public transportation services. Transit travel time data were collected using GPS tracking services and the resulting trends were analyzed to understand the variations in corridor travel time. Special events like football and basketball games were found to increase travel times significantly. Median was found to be a more robust statistic than mean due to the high number of missing values and discrepancies. Analytical models were developed to minimize the total system cost by jointly optimizing the type of bus services (i.e., conventional or flexible service), vehicle sizes, numbers of zones (i.e., route spacings or service areas) for conventional and flexible bus services, headways, and resulting fleet sizes. For the numerical example tested in the study, conventional bus services were found to be economical over flexible services with given input parameters. For the specific instance tested in the study, total costs of conventional bus services were 9.5~10.8 percent lower than the total costs of flexible bus services, by region.</p>			
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## Disclaimer

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## 1.0 Background

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In this joint project, West Virginia University and the University of Maryland collaborated in developing improved methods for analyzing and managing public transportation services. West Virginia University focused on using GPS travel time data from Morgantown, West Virginia, to analyze travel times on transit services; the University of Maryland focused on developing optimization-based models for optimizing the operations of conventional and flexible bus services.

The Mountain Line Transit Authority collects and archives GPS-based location data with the corresponding time stamp on transit buses operating on main corridors in Morgantown. The archived database can be analyzed to obtain travel times between any two points on the bus routes. Since the data collection and archiving has been conducted for a number of years, this provided an opportunity to analyze trends in corridor travel times over multiple years/seasons/special events. It is difficult and expensive to collect corridor travel times over a significant period of time to analyze the variation and trends. Even though buses are normally slower than regular automobiles, bus travel times provide a very good proxy to actual corridor travel times. The database maintained by the Mountain Line Transit Authority provides an opportunity to analyze the relative variation in corridor travel time trends. The analysis of variation of travel time data can be useful in understanding and quantifying (1) relative changes in corridor travel time on special event days compared to normal days and (2) seasonal variations in travel time.

Conventional bus operations are commonly provided in urban mass public transportation. Conventional bus routes and timetables are preset, and buses operate on their fixed routes and fixed schedules. Conventional bus services are relatively economical when carrying many



passengers during peak periods. When ridership is lower, bus operators typically adjust frequencies downward, thus increasing passenger wait times. This may further decrease ridership. Bus operating costs increase to some extent with bus size. Large buses are more economical at high demand densities, as average costs per passenger are relatively low. This leads us to consider the use of mixed bus fleets, consisting of vehicles of different sizes, which may more closely match demand variations. By considering different sizes of buses (expressed in terms of numbers of seats), we may be able to reduce operating costs while also providing more frequent services with small buses.

In urban bus transit networks, it is common to transfer passengers at transfer terminals because it is prohibitively expensive to provide direct trips for passengers among all origins and destinations with conventional bus services. Since transfers are important in public transportation services, it may be beneficial to consider the cost of passenger transfers in the objective functions. However, previous studies did not explore transfers among combinations of conventional and flexible bus services. In this study, the authors sought to minimize total system cost by jointly optimizing various system characteristics while considering transfers within and between services of different types. The jointly optimized characteristics include the type of bus services (i.e., conventional or flexible service), vehicle sizes, and numbers of zones (i.e., route spacings or service areas) for conventional and flexible bus services, headways, and resulting fleet sizes.

### Problem Statement

The objective of this project is to:

1. Statistically analyze bus GPS travel time data from Morgantown, West Virginia in order to identify trends and impacts of various policies.

2. Develop algorithms to jointly optimize system characteristics such as the type of bus services (i.e., conventional or flexible service), vehicle sizes, numbers of zones (i.e., route spacings or service areas) for conventional and flexible bus services, headways, and resulting fleet sizes while considering transfers within and between services of different types.

## **2.0 Bus Travel Time Trends Analysis**

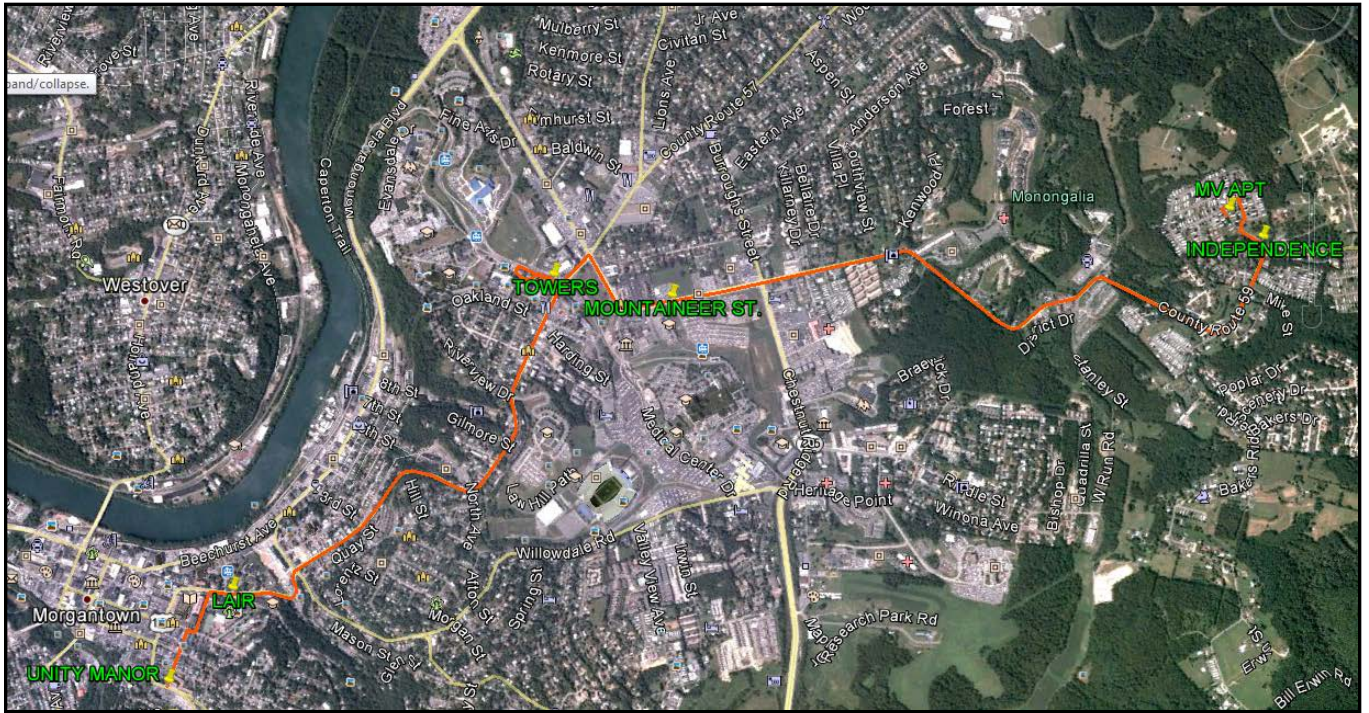
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### **2.1 Introduction**

Transit travel time is a critical issue that influences service attractiveness, operating costs, and system efficiency. Permanent daily travel time data collection during the year is crucial, because it provides important information about system performance. It can help the operation management of the transit systems by providing reliable route planning and scheduling. The objectives of this study were to (1) collect travel time data through a GPS tracker system for the Mountain Line transit service; (2) develop Excel-based software to handle and manage travel time data; and (3) conduct and report statistical analysis of trends associated with travel times and quantify the variation in the selected corridor. Patterns during normal weekdays, weekends, and special events are documented. The selected route is the Gold Line, in which major road segments have been analyzed. The route was chosen based on discussion with the Mountain Line Transit Authority in Morgantown, WV. The analysis can be extended in the future to other routes.

### **2.2 Scope of the Study**

This study attempted to determine factors that affect transit travel time during weekdays, weekends, and special events such as football games, spring breaks, summer, etc. The selected route, the Gold Line, runs from Depot-Downtown-Towers to Mountain Valley Apartments. This route is one of the heaviest routes during the morning and afternoon commuting period. The information provided in this study can help to better understand the factors that affect transit trip time over the year, allowing superior operational decision-making and performance. Figure 1 illustrates the route segment in the study area.



**Figure 1: Gold Line - Segment of Study**

### **2.3 Data collection**

The Mountain Line Transit Authority recently implemented the Shadow Tracker Live system (Advance Tracking Technologies, 2012) as a part of its monitoring control system. The system provides real-time vehicle location information based on differential global positioning system (GPS) technology supported by sensors installed in the vehicle units. The system's software allows users to set up email alerts; hence, at each moment the bus runs over a specific location, an email is sent to the user. The information provides the bus number, date, time, and location. In addition, the systems have the capability to store the complete alert history in their own database, allowing data downloads. According to the ATTI web page, the systems have 10-

second updates while the vehicle is in motion and data storage for up to one year. Data were collected for the Gold Line at the selected road segments described below.

- From Unity Manor to Lair (0.36 miles)
- From Lair to Towers (1.49 miles)
- From Towers to Mountaineer Station (0.66 miles)
- From Mountaineer Station to Independence Apts (1.90 miles)
- From Independence Apt. to Mountaineer Valley Apts (0.24 miles)

The total distance from Unity Manor to Mountaineer Valley Apts is 4.65 miles. Travel time data at those segments were collected through emails alerts. The procedure for obtaining, handling, and managing the data is described in the following section.

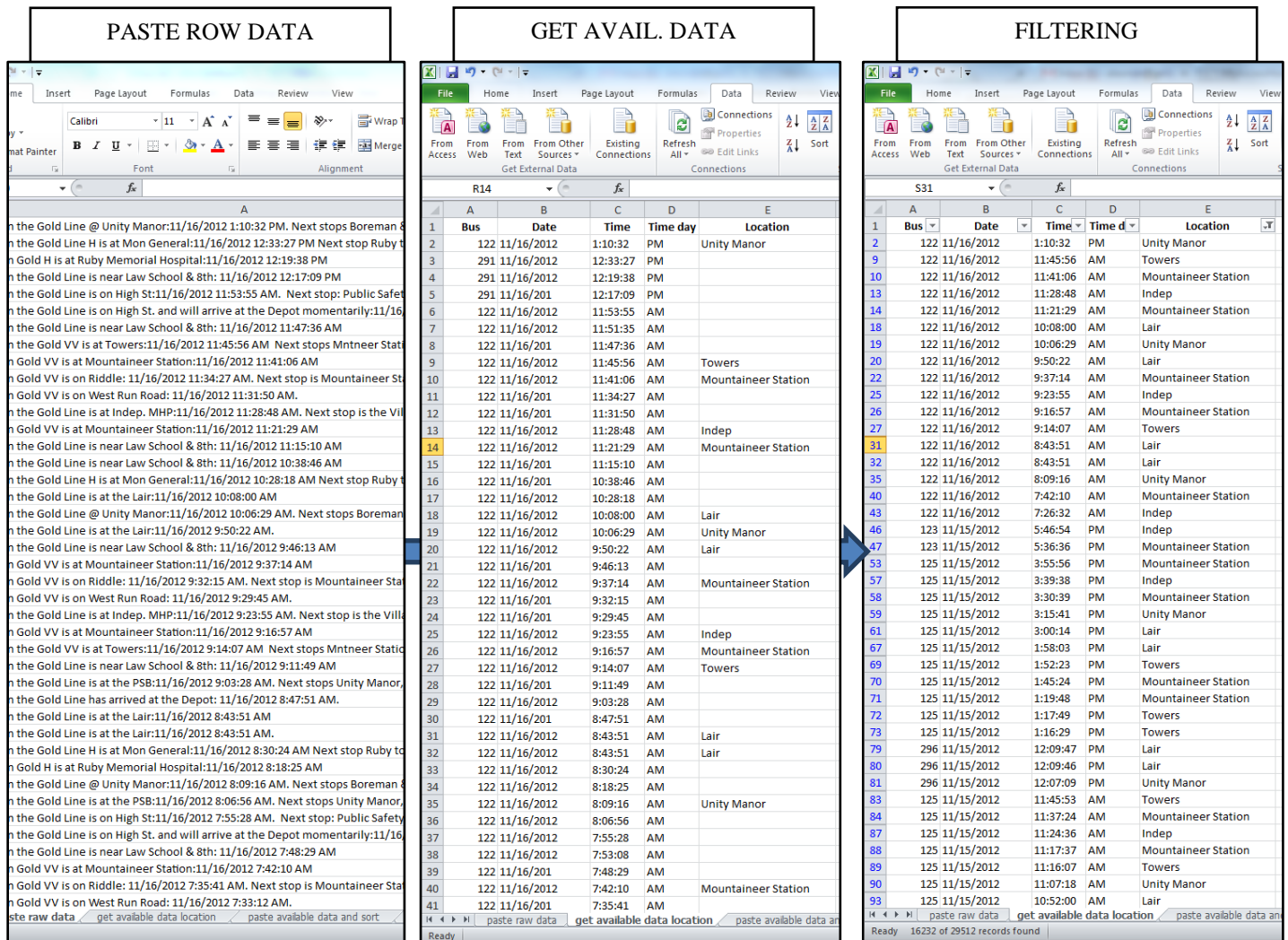
### ***2.3.1 Data Handling Process***

Data were collected through non-commercial email accounts that support POP and IMAP for accessing and storing emails such as Gmail, Hotmail, etc. However, these providers do not allow downloading the complete list of emails in a spreadsheet. Thus, the Mozilla Thunderbird, an email application, was used, since it allows for collecting and downloading the email alerts as CSV files, which are then converted to XLS format. We next clearly outline the Excel spreadsheet-based data cleaning and managing procedure used in this study. The Excel spreadsheets will be made available to anyone for academic purposes if they contact the authors. Figure 2 shows the spreadsheet that stores the email alerts. Column “F” shows the text of the email sent from the device installed in the bus; the email is sent when it reaches a specific location, as previously explained; for example: “Bus 122 on the Gold Line is at the Lair:11/16/2012 9:50:22 AM.” The last text contains important information that needs to be extracted such as bus number, location, date, time of day, hours, etc.

	A	B	C	D	E
1	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 13:10	Bus 122 on the Gold Line @
2	ATTI: #ML06Gold H	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 12:33	Bus 291 on the Gold Line H is
3	ATTI: #ML06Gold H	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 12:19	Bus 291 on Gold H is at Ruby
4	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 12:17	Bus 291 on the Gold Line is r
5	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:54	Bus 122 on the Gold Line is c
6	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:51	Bus 122 on the Gold Line is c
7	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:47	Bus 122 on the Gold Line is r
8	ATTI: #ML06Gold Gold VV	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:46	Bus 122 on the Gold VV is at
9	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:41	Bus 122 on Gold VV is at Mo
10	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:34	Bus 122 on Gold VV is on Rid
11	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:31	Bus 122 on Gold VV is on We
12	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:28	Bus 122 on the Gold Line is a
13	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:21	Bus 122 on Gold VV is at Mo
14	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 11:15	Bus 122 on the Gold Line is r
15	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 10:38	Bus 122 on the Gold Line is r
16	ATTI: #ML06Gold H	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 10:28	Bus 122 on the Gold Line H is
17	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 10:08	Bus 122 on the Gold Line is a
18	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 10:06	Bus 122 on the Gold Line @
19	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:50	Bus 122 on the Gold Line is a
20	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:46	Bus 122 on the Gold Line is r
21	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:37	Bus 122 on Gold VV is at Mo
22	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:32	Bus 122 on Gold VV is on Rid
23	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:29	Bus 122 on Gold VV is on We
24	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:24	Bus 122 on the Gold Line is a
25	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:17	Bus 122 on Gold VV is at Mo
26	ATTI: #ML06Gold Gold VV	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:14	Bus 122 on the Gold VV is at
27	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:11	Bus 122 on the Gold Line is r
28	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 9:03	Bus 122 on the Gold Line is a
29	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 8:47	Bus 122 on the Gold Line has
30	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 8:43	Bus 122 on the Gold Line is a
31	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 8:43	Bus 122 on the Gold Line is a
32	ATTI: #ML06Gold H	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 8:30	Bus 122 on the Gold Line H is
33	ATTI: #ML06Gold H	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 8:18	Bus 122 on Gold H is at Ruby
34	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 8:09	Bus 122 on the Gold Line @
35	ATTI: #ML06gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 8:07	Bus 122 on the Gold Line is a
36	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 7:55	Bus 122 on the Gold Line is c
37	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 7:53	Bus 122 on the Gold Line is c
38	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 7:48	Bus 122 on the Gold Line is r
39	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 7:42	Bus 122 on Gold VV is at Mo
40	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 7:35	Bus 122 on Gold VV is on Rid
41	ATTI: #ML06Gold	alerts@advantrack.com	"ML06gold@gmail.com, wvbusride@hotmail.com"	11/16/2012 7:32	Bus 122 on Gold VV is on We

**Figure 2: Excel Spreadsheet with email alerts**

The next step is to copy column “F,” which contains the email alert text, and paste the information in the worksheet named “PASTE ROW DATA.” This will generate the worksheet “GET AVAILABLE DATA LOCATION” with the clean data. This worksheet is then filtered, and by locking the blank cells, unwanted data are eliminated. Thus, complete data are obtained at the designated locations. Note that rows with blank cells contain information on non-relevant locations to this study. The process is shown in Figure 3.



**Figure 3: Excel spreadsheet-based procedure for filtering and obtaining complete data**

After filtering, clean and complete data are obtained; then, this worksheet is copied (columns A through E) and pasted into the worksheet named “PASTE AVAILABLE DATA AND SORT,” as shown in Figure 4.

Bus	Date	Time	Time day	Location	Month	Day	Year	Hour	Hour	Minutes	Seconds	MOR/AFT	in Sector	From	To	Travel Time	Minutes	
110	2/15/2012	2:46:29	PM	Lair	2	15	2012	2	14	46	29	PM		53189 Lair	Unity Manor	53189	886.48	2/1
291	2/15/2012	3:07:25	PM	Unity Manor	2	15	2012	3	15	7	25	PM		54445 Unity Manor	Lair	140	2.33	2/1
291	2/15/2012	3:09:45	PM	Lair	2	15	2012	3	15	9	45	PM		54585 Lair	Lair	10	0.17	2/1
291	2/15/2012	3:09:55	PM	Lair	2	15	2012	3	15	9	55	PM		54595 Lair	Lair	2554	42.57	2/1
291	2/15/2012	3:52:29	PM	Lair	2	15	2012	3	15	52	29	PM		57149 Lair	Towers	57149	952.48	2/1
293	2/15/2012	1:56:30	PM	Towers	2	15	2012	1	13	56	30	PM		50190 Towers	Indep	589	9.82	2/1
293	2/15/2012	2:06:19	PM	Indep	2	15	2012	2	14	6	19	PM		50779 Indep	MV Apts	389	6.48	2/1
293	2/15/2012	2:12:48	PM	MV Apts	2	15	2012	2	14	6	19	PM		51168 MV Apts	MV Apts	139	2.32	2/1
293	2/15/2012	2:15:07	PM	MV Apts	2	15	2012	2	14	6	19	PM		51307 MV Apts	Lair	1796	29.93	2/1
293	2/15/2012	2:45:03	PM	Lair	2	15	2012	2	14	6	19	PM		51303 Lair	Lair	10	0.17	2/1
293	2/15/2012	2:45:13	PM	Lair	2	15	2012	2	14	6	19	PM		53113 Lair	Mountaineer Station	589	9.82	2/1
293	2/15/2012	2:55:02	PM	Mountaineer Station	2	15	2012	2	14	6	19	PM		53702 Mountaineer Station	Indep	478	7.97	2/1
293	2/15/2012	3:03:00	PM	Indep	2	15	2012	3	15	3	0	PM		54180 Indep	Indep	222	3.70	2/1
293	2/15/2012	3:06:42	PM	Indep	2	15	2012	3	15	6	42	PM		54402 Indep	MV Apts	217	3.62	2/1
293	2/15/2012	3:10:19	PM	MV Apts	2	15	2012	3	15	10	19	PM		54619 MV Apts	Towers	938	15.63	2/1
293	2/15/2012	3:25:57	PM	Towers	2	15	2012	3	15	25	57	PM		55557 Towers	Lair	1447	24.12	2/1
293	2/15/2012	3:50:04	PM	Lair	2	15	2012	3	15	50	4	PM		57004 Lair	Lair	10	0.17	2/1
293	2/15/2012	3:50:14	PM	Lair	2	15	2012	3	15	50	14	PM		57014 Lair	Towers	419	6.98	2/1
293	2/15/2012	3:57:13	PM	Towers	2	15	2012	3	15	57	13	PM		57433 Towers	Mountaineer Station	229	3.82	2/1
293	2/15/2012	4:01:02	PM	Mountaineer Station	2	15	2012	4	16	1	2	PM		57662 Mountaineer Station	Indep	558	9.30	2/1
293	2/15/2012	4:10:20	PM	Indep	2	15	2012	4	16	10	20	PM		58220 Indep	Indep	359	5.98	2/1
293	2/15/2012	4:16:19	PM	Indep	2	15	2012	4	16	16	19	PM		58579 Indep	MV Apts	81	1.35	2/1
293	2/15/2012	4:17:40	PM	MV Apts	2	15	2012	4	16	17	40	PM		58660 MV Apts	Towers	888	14.80	2/1
293	2/15/2012	4:32:28	PM	Towers	2	15	2012	4	16	32	28	PM		59548 Towers	Lair	419	6.98	2/1
293	2/15/2012	4:39:27	PM	Lair	2	15	2012	4	16	39	27	PM		59967 Lair	Towers	1717	28.62	2/1
293	2/15/2012	5:08:04	PM	Towers	2	15	2012	5	17	8	4	PM		61684 Towers	Towers	118	1.97	2/1
293	2/15/2012	5:10:02	PM	Towers	2	15	2012	5	17	10	2	PM		61802 Towers	Mountaineer Station	138	2.30	2/1
293	2/15/2012	5:12:20	PM	Mountaineer Station	2	15	2012	5	17	12	20	PM		61940 Mountaineer Station	Indep	640	10.67	2/1
293	2/15/2012	5:23:00	PM	Indep	2	15	2012	5	17	23	0	PM		62580 Indep	Indep	220	3.67	2/1
293	2/15/2012	5:26:40	PM	Indep	2	15	2012	5	17	26	40	PM		62800 Indep	Mountaineer Station	1915	31.92	2/1
293	2/15/2012	5:58:35	PM	Mountaineer Station	2	15	2012	5	17	58	35	PM		64715 Mountaineer Station	Indep	379	6.32	2/1
293	2/15/2012	6:04:54	PM	Indep	2	15	2012	6	18	4	54	PM		65094 Indep	Indep	181	3.02	2/1
293	2/15/2012	6:07:55	PM	Indep	2	15	2012	6	18	7	55	PM		65275 Indep	MV Apts	78	1.30	2/1
293	2/15/2012	6:09:13	PM	MV Apts	2	15	2012	6	18	9	13	PM		65353 MV Apts	Lair	1247	20.78	2/1
293	2/15/2012	6:30:00	PM	Lair	2	15	2012	6	18	30	0	PM		66600 Lair	Unity Manor	679	11.32	2/1

**Figure 4: Excel Spreadsheet for sorting travel time data**

After pasting the data from the 5 columns, they are marked and sorted by following this sequence: month, day, bus number, time of day (morning/afternoon), hour, minutes, and seconds. It's important to mention that Excel usually saves the sort sequence in the spreadsheet. After data sorting, the data are chronologically organized, and also automatically generate the other columns (with green headings), which consequently generate the worksheet named “DATA FOR ANALYSIS” showed in Figure 5, which is the final data to use in the analysis.



Bus	Date	From	To	Minutes	Time	Time day
110	2/15/2012	Lair	Unity Man	886.48	2:46:29	PM
291	2/15/2012	Unity Man	Lair	2.33	3:07:25	PM
291	2/15/2012	Lair	Lair	0.17	3:09:45	PM
291	2/15/2012	Lair	Lair	42.57	3:09:55	PM
291	2/15/2012	Lair	Towers	952.48	3:52:29	PM
293	2/15/2012	Towers	Indep	9.82	1:56:30	PM
293	2/15/2012	Indep	MV Apts	6.48	2:06:19	PM
293	2/15/2012	MV Apts	MV Apts	2.32	2:12:48	PM
293	2/15/2012	MV Apts	Lair	29.93	2:15:07	PM
293	2/15/2012	Lair	Lair	0.17	2:45:03	PM
293	2/15/2012	Lair	Mountaine	9.82	2:45:13	PM
293	2/15/2012	Mountaine	Indep	7.97	2:55:02	PM
293	2/15/2012	Indep	Indep	3.70	3:03:00	PM
293	2/15/2012	Indep	MV Apts	3.62	3:06:42	PM
293	2/15/2012	MV Apts	Towers	15.63	3:10:19	PM
293	2/15/2012	Towers	Lair	24.12	3:25:57	PM
293	2/15/2012	Lair	Lair	0.17	3:50:04	PM
293	2/15/2012	Lair	Towers	6.98	3:50:14	PM
293	2/15/2012	Towers	Mountaine	3.82	3:57:13	PM
293	2/15/2012	Mountaine	Indep	9.30	4:01:02	PM
293	2/15/2012	Indep	Indep	5.98	4:10:20	PM
293	2/15/2012	Indep	MV Apts	1.35	4:16:19	PM
293	2/15/2012	MV Apts	Towers	14.80	4:17:40	PM
293	2/15/2012	Towers	Lair	6.98	4:32:28	PM
293	2/15/2012	Lair	Towers	28.62	4:39:27	PM
293	2/15/2012	Towers	Towers	1.97	5:08:04	PM
293	2/15/2012	Towers	Mountaine	2.30	5:10:02	PM
293	2/15/2012	Mountaine	Indep	10.67	5:12:20	PM
293	2/15/2012	Indep	Indep	3.67	5:23:00	PM
293	2/15/2012	Indep	Mountaine	31.92	5:26:40	PM
293	2/15/2012	Mountaine	Indep	6.32	5:58:35	PM
293	2/15/2012	Indep	Indep	3.02	6:04:54	PM
293	2/15/2012	Indep	MV Apts	1.30	6:07:55	PM
293	2/15/2012	MV Apts	Lair	20.78	6:09:13	PM
293	2/15/2012	Lair	Unity Man	11.32	6:30:00	PM

**Figure 5: “DATA FOR ANALYSIS” Spreadsheet**

Data stored in “DATA FOR ANALYSIS” are used in another spreadsheet named “GRAPH AND TABLES” to produce the graphs and tables documented in this report. “DATA FOR ANALYSIS” columns A through G are copied and pasted into the spreadsheet “GRAPH AND TABLES” in the worksheet named “ANALYSIS.” This worksheet automatically generates the worksheet “TABLES\_GRAPH,” which displays the travel time for each road segment during each day in the year. The worksheets are shown in Figure 6.

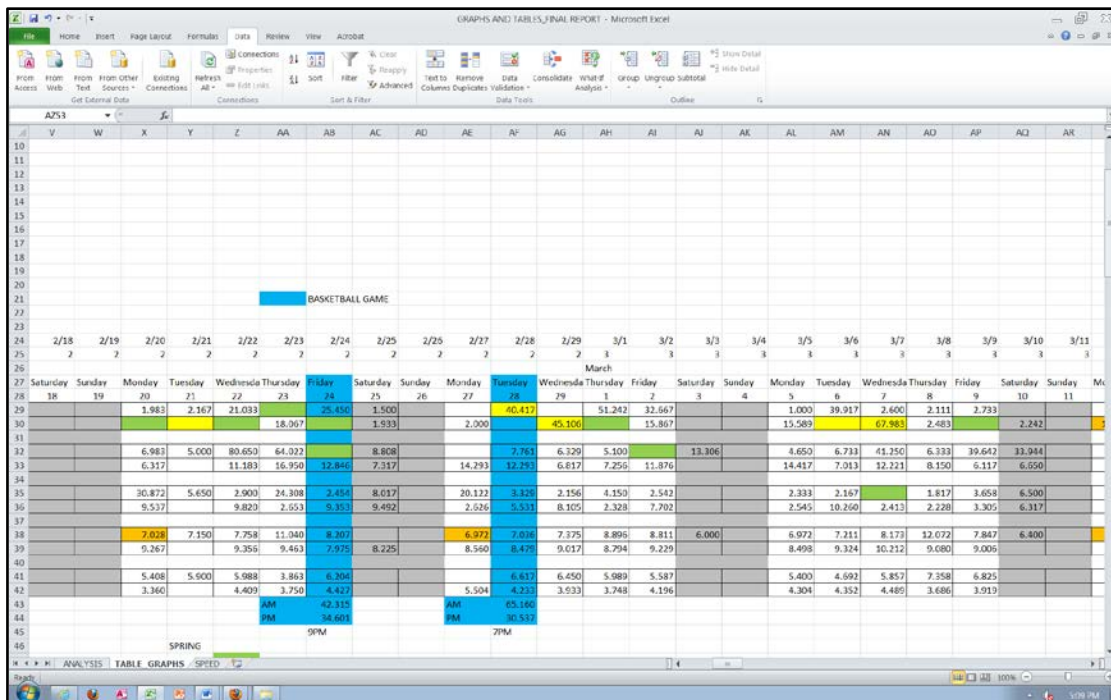
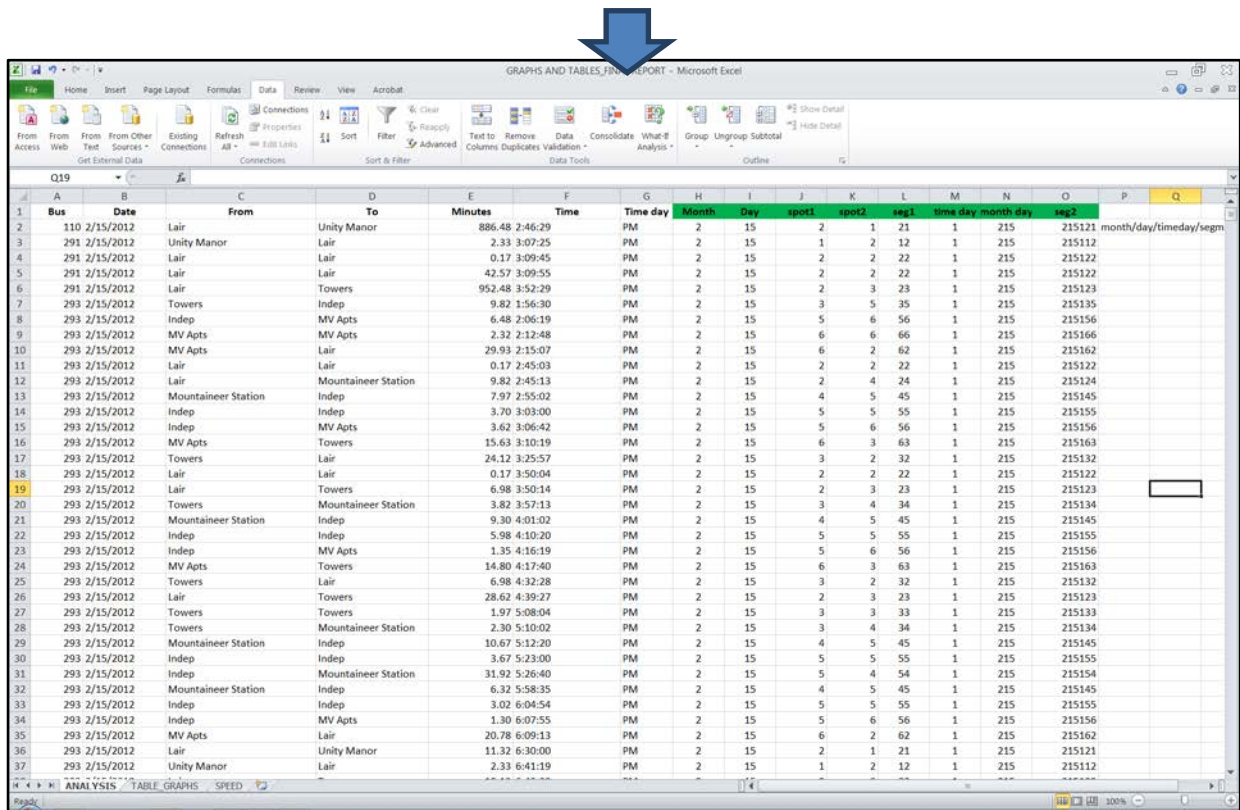


Figure 6: Excel spreadsheet based procedure for generating graphs and tables

Travel time collected by day allows for identifying travel time trends on regular days versus specific days such as special events, basketball games, football games, etc. The data include the following information: bus number, date, origin and destination of the road segment, and travel time for each road segment in minutes (see Table 1).

**Table 1: Data Sample**

<b>Bus</b>	<b>Date</b>	<b>From</b>	<b>To</b>	<b>Travel Time (min)</b>
291	2/15/2012	Unity Manor	Lair	2.333
291	2/15/2012	Lair	Depot	6.316
293	2/15/2012	Tower	MV Apt	16.3
293	2/15/2012	MV Apt	Lair	29.933
293	2/15/2012	Lair	Mountaineer Station	9.816
293	2/15/2012	Mountaineer Station	MV Apt	15.283
293	2/15/2012	MV Apt	Tower	15.633
293	2/15/2012	Tower	Depot	12.466
293	2/15/2012	Depot	Lair	11.65
293	2/15/2012	Lair	Tower	6.983
293	2/15/2012	Tower	Mountaineer Station	3.816
293	2/15/2012	Mountaineer Station	MV Apt	16.633
293	2/15/2012	MV Apt	Tower	14.8

### ***2.3.2 Special Events***

Special events are defined as particular occurrences affecting the regular travel patterns of the bus services. Among these events are: sports events (e.g., basketball and football games); spring break week; summer term. In addition, the travel time during each college semester was

considered (e.g., spring, summer and fall semesters). These special events in the year 2012 for sport events are listed below.

Basketball: Sat., Jan. 7; Sat., Jan. 14; Sat., Jan. 21; Jan. 30; Wed., Feb. 8; Sat., Feb. 11; Fri., Feb. 24; Tue., Feb. 28;

Football Games: Sat., Sept. 1 (12:00 p.m.); Sat., Sept. 15 (4:30 p.m.); Sat., Sept. 22 (12:00 p.m.); Sat., Sept. 29 (12:00 p.m.); Sat., Oct. 20 (7:00 p.m.); Sat., Nov. 3 (3:00 p.m.); Sat., Nov. 17 (7:00 p.m.); Sat., Dec. 1.

### **2.3.3 Data Issues**

After collection, handling, and review of the data, missing values and discrepancies were encountered. These issues are described below.

- Data collected during the afternoon contained more missing observations, and more discrepancies than morning data. A possible cause was forgetting to turn on the sensor devices in the bus during the afternoon shifts.
- Saturdays presented more missing values. This may be because the devices had not been turned on.
- No information was available on Sundays, as the buses don't operate on Sundays.
- Bus number "1100" had the highest number of discrepancies. It may indicate the time when the buses shift to other buses. In this scenario the last bus is no longer running; thus, data display large values of travel time, because the route was taken by another bus number.
- Missing values are observed for some days, possibly caused by obstruction to the satellite, bushes, trees, etc.

- The analysis was based on the median, because of the large number of missing values and discrepancies affecting the data. Median is more reliable than the average in these circumstances.
- Special events considered included spring break, basketball games, football games, commencement, etc.

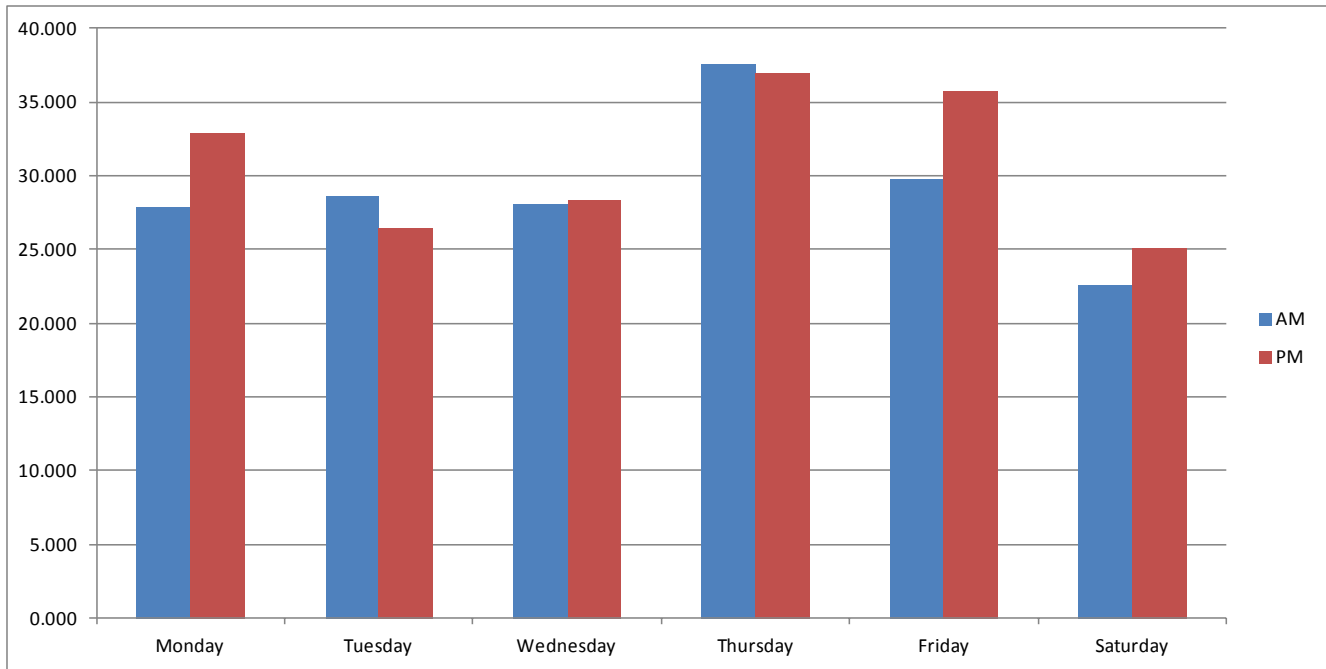
## **2.4 Results**

Data obtained through email alerts have missing and high values of travel times. Therefore, larger values (>100 minutes) of travel time for each segment were removed. After cleaning the data, the median was considered in the analysis. Note that in this work we do not explicitly model the dwell times, as we do not have the required data (Bertinin and El-Geneidy, 2004; Dueker et al., 2004; El-Geneidy and Vijayakumar, 2011). Figures and tables that summarized the results for the most relevant period of time over the year are shown in this section; other figures and tables are documented in the appendix. It is important to highlight that in situations with a large amount of discrepancies in a data set, the median performed better than the mean in terms of accuracy. Table 2 shows the median of a regular week collected over the year (February - November 2012).

**Table 2: Median Travel Time in a Regular Week**

Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	2.33	2.54	2.63	2.87	3.16	1.83
	PM	7.39	2.97	2.74	12.19	11.35	1.83
Lair - Towers	AM	6.65	6.91	6.57	6.16	7.65	6.98
	PM	8.48	6.99	8.57	8.15	7.30	5.45
Towers - Mountaineer Station	AM	2.58	2.66	2.84	2.91	2.67	6.73
	PM	2.98	3.08	3.18	3.09	3.17	6.98
Mountaineer Station - Indep	AM	8.34	8.79	8.35	8.44	8.32	6.98
	PM	9.25	8.92	9.44	9.06	9.01	7.21
Indep - MV APT	AM	7.94	7.65	7.62	17.16	7.94	
	PM	4.80	4.54	4.49	4.45	4.97	3.67
Total Travel Time	AM	<b>27.83</b>	<b>28.54</b>	<b>28.02</b>	<b>37.54</b>	<b>29.74</b>	<b>22.53</b>
	PM	<b>32.90</b>	<b>26.50</b>	<b>28.41</b>	<b>36.94</b>	<b>35.80</b>	<b>25.13</b>

Figure 7 shows the travel time distributions corresponding to a normal week for the gold line bus service. The median travel time during weekdays (Monday-Friday) at the morning time was 30.33 minutes and during the afternoon time was 32.11 minutes. Travel time showed variability over the week; however, during Mondays and Fridays the afternoon travel time was higher than for morning operations. Saturday seems to be the day with the lowest travel time even though a value is missing for the afternoon period. It is expected that this is because there are no college classes during weekends. In addition, the average speed for each road segment was calculated considering the road segment length and travel time for each segment. The average travel speed is illustrated in Table 3.



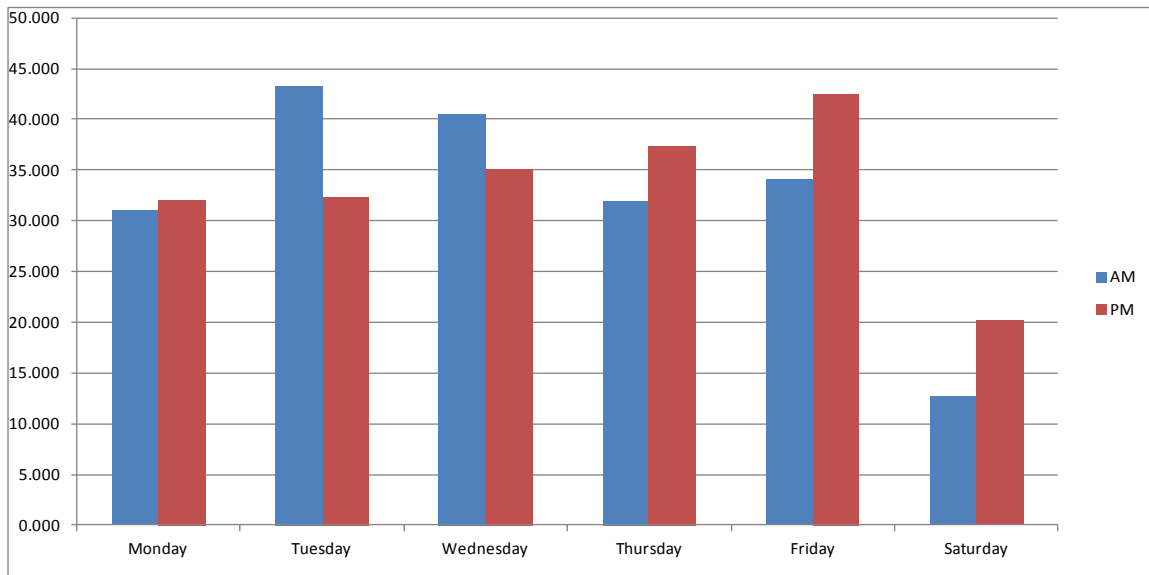
**Figure 7: Regular Week Travel Time**

**Table 3: Average Travel Speed**

Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	9.29	8.51	8.21	7.53	6.83	11.84
	PM	2.14	7.28	3.65	1.77	1.62	11.84
Lair - Towers	AM	13.11	12.94	13.61	14.52	11.69	12.80
	PM	10.54	12.79	10.09	10.97	12.25	16.42
Towers - Mountaineer Station	AM	15.35	14.90	13.94	13.62	14.85	5.88
	PM	13.27	12.86	12.46	12.81	12.48	5.67
Mountaineer Station - Indep	AM	13.67	12.98	13.65	13.50	13.71	16.32
	PM	12.33	12.78	12.08	12.59	12.66	15.81
Indep - MV APT	AM	1.81	1.88	1.89	0.84	1.81	
	PM	3.00	3.17	3.21	3.23	2.90	3.93
<b>Total Travel Time</b>	<b>AM</b>	<b>9.97</b>	<b>9.78</b>	<b>9.96</b>	<b>7.43</b>	<b>9.38</b>	<b>12.39</b>
	<b>PM</b>	<b>7.84</b>	<b>10.53</b>	<b>8.75</b>	<b>7.55</b>	<b>7.38</b>	<b>11.10</b>

Table 3 indicates the average speed for each segment and for the total route for the morning and afternoon period. The average speed was 9.82 miles/hour for the morning and 8.86 miles/hour for the afternoon time. The average speed for the total route is 9.34 miles/hour.

Figure 8 shows the travel time during spring break. It indicates that the travel time is similar to the regular week in the year. A lower travel time was expected during this week; however, it appears that travel time between Independent Apts and Mountain Valley Apts increased during this week, affecting the total travel time. Saturday shows the lowest travel time and lower than a regular week. This week shows patterns contrary to the expected; for this reason, travel time during the spring semester was contrasted with spring break travel time. Figure 4 shows these plots splitting the morning and afternoon times of day.

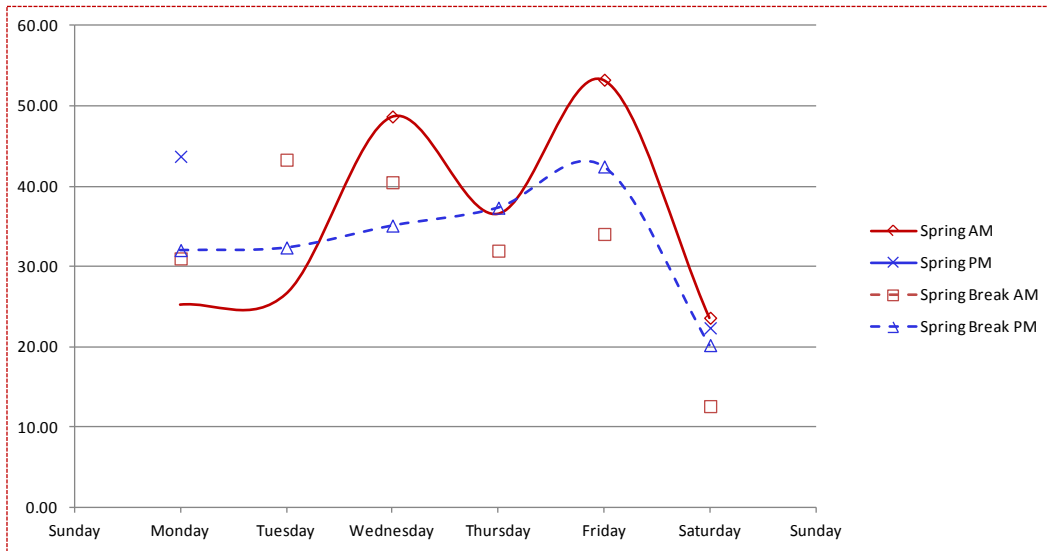


**Figure 8: Spring Break Travel Time**

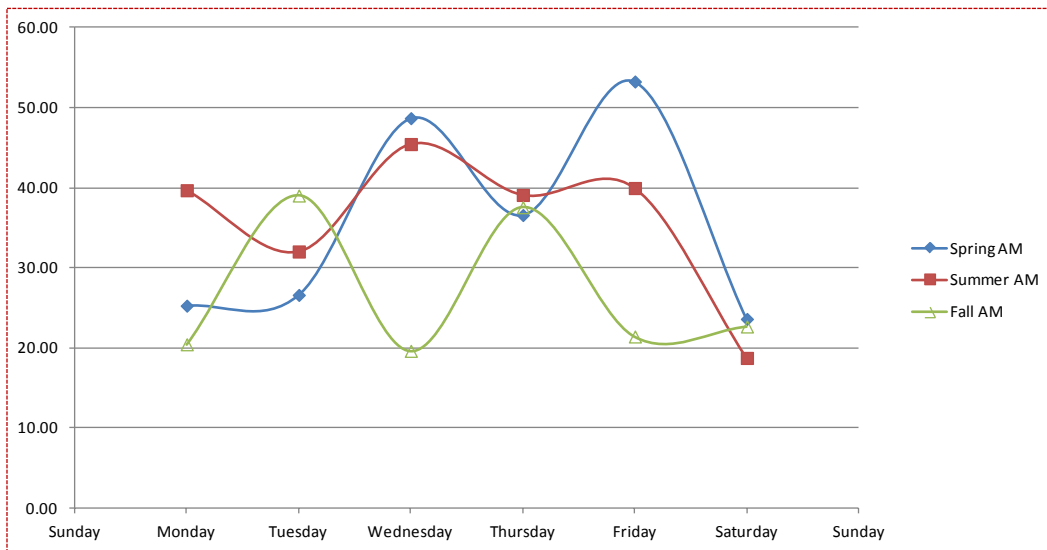
Figure 9 indicates that during spring break, travel times fall below the regular spring week. It indicates that data from the spring break week are reasonable. Figures 10 and 11 illustrate the



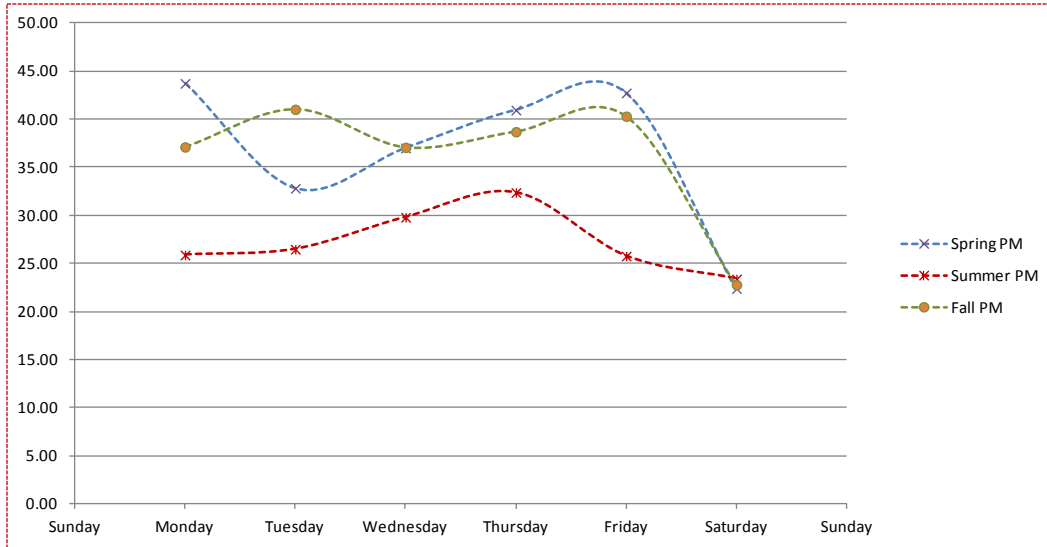
comparison of the travel time for each term season: spring, summer, and fall during morning and afternoon times.



**Figure 9: Spring Break Travel Time**



**Figure 10: Travel Time Comparison (AM)**



**Figure 11: Travel Time Comparison (PM)**

Overall, Figure 10 indicates that fall, summer, and spring exhibit lower, medium, and higher travel times associated with the morning trips (AM), and Figure 11 indicates that spring, fall and summer showed higher, medium, and lower travel time associated with the afternoon trips (PM).

Table 4 displays the travel time during the sports events. A large number of missing information was observed during these days. During football games the bus service was off; thus, on these days data were missed. Basketball games have an impact when comparing with the median travel time. During the AM travel times were 29.031 minutes (median) and 42.32 minutes (basketball), and during the PM they were 30.94 minutes (median) and 34.96 minutes (basketball), indicating a slight increase in travel time during basketball games. A superior comparison of these events is made by comparing the road segment from “Lair to Towers” during basketball games, and the road segment from “Towers to Mountaineer Station” for football games. These segments are closer to the venue of these sports; thus, they can better capture the impact of travel time. The median travel time from “Lair to Tower” during the

afternoon was 7.48 min, and during a basketball games was 12.57 min, which indicates that basketball games’ impact is a 5-minute increase in the travel time. The median travel time from “Tower to Mountaineer Station” was 3.1 minutes, and during football games was 9 minutes, indicating a 6-minute increase in travel time.

**Table 4: Travel Time for Sport Events**

Road Segments	Time of Day	Basketball		Football					
		9:00PM	7:00PM	12:00PM	4:30 PM	12:00PM	12:00PM	7:00PM	3:00PM
Unity Manor - Lair	AM	25.45	40.42			1.98		2.15	
	PM				1.73				
Lair – Towers	AM		7.76		156.30	115.52	7.48	7.65	4.82
	PM	12.85	12.29		6.17			338.97	
Towers - Mountaineer Station	AM	2.45	3.33				8.15	6.82	7.00
	PM	9.35	5.53		8.82			9.07	
Mountaineer Station - Indep	AM	8.21	7.04		7.32	9.98	9.15	7.41	6.98
	PM	7.98	8.48		6.90			8.56	
Indep - MV APT	AM	6.20	6.62						
	PM	4.43	4.23						
Total Travel Time	AM	42.32	65.16		163.62	127.48	24.78	24.03	18.80
	PM	34.60	30.54		23.62			356.59	

## 2.5 Conclusion

This chapter presents a method for collecting, analyzing, and identifying trends in transit travel times collected during special events versus normal working days. In this case, we focus on one main route in Morgantown, WV, but the process can be extended to other routes. The information produced through GPS data has the potential of helping in the planning and managing of a bus route service. For instance, important information provided is the average speed of 9.34 miles/hour for the total segment. This indicates that unless changes are made, the service will not be faster than this speed. Changes in the services or in the roadway network such

as a traffic signal or providing fewer bus stop locations along the segment will increase speed and reduce travel time. The most valuable characteristic of this study is the ability of using data from GPS devices to obtain important information; this allows transit planners to test various strategies for improving transit service at low cost, because collection, handling, and analysis of data are not a complex process, as shown in this study.

### 3.0 Conventional and Flexible Bus Services with Transfer

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#### 3.1 Introduction

In urban bus transit networks, it is common to transfer passengers at transfer terminals because it is prohibitively expensive to provide direct trips for passengers among all origins and destinations with conventional bus services. Since transfers are important in public transportation services, it may be beneficial to coordinate bus arrivals at transfer stations (terminals) so that wait times are minimized and passengers can reliably catch their next bus. Because bus travel times are usually stochastic in urban transit networks, a probabilistic optimization model is needed to find buffer times that help provide reliable connections among buses.

Therefore, this study considered timed transfers coordination. For timed transfers, slack times improve the reliability of connections and smoothly connect buses at transfer terminals. Longer slack times increase bus operating cost and users' waiting cost, but may reduce transfer cost. Therefore, we needed to develop a probabilistic optimization model for timed transfers among conventional and flexible bus services. Uncoordinated and coordinated bus operations were compared through numerical analyses. In order to provide efficient timed transfers, slack times were optimized numerically. Other decision variables such as headways, fleets, vehicle sizes were jointly optimized. Thus, optimization models for improving bus transit services are desirable for better public transportation solutions for users as well as operators.

For a timed transfer analysis, we analyzed a transfer strategy that vehicles do not wait for other vehicles that arrive behind schedule. We also assumed vehicle arrivals in a timed transfer station are probabilistically distributed.

## **3.2 Literature Review**

### ***3.2.1 Relevant Literature***

Kyte et al. (1982) presented a timed-transfer system in Portland, Oregon. They provided the history of planning, implementation, and evaluation of a timed transfer system that had provided services since 1979. This system provided timed transfers to the suburban areas in which demands were low, and provided grid-type bus services to the higher-demand regions. This paper also discusses the performances and results of the implemented system. They use two indicators, which are a successful meet and a successful connection, to analyze the transfer reliabilities. A successful meet is defined as all buses arriving as scheduled at a given time, and a successful connection is a direct transfer connection that results from two routes arriving as scheduled. The authors point out that weekday ridership increased by 40 percent after one year of operation, and local trips using this system increased dramatically. However, the 40% increase in ridership resulted not only from a timed transfer system, but also from new route designs. Bakker et al. (1988) similarly studied a multi-centered time transfer system in Austin, Texas, and confirmed that such a timed transfer system is particularly applicable for low-density cities.

Abkowitz et al. (1987) studied timed transfers between two routes. They compared four policy cases, namely: unscheduled, scheduled transfer without vehicle waiting, scheduled transfers where the lower-frequency bus is held until the higher-frequency vehicle arrives, and scheduled transfers where both buses are held until a transfer event occurs. In other words, this study compared scheduled, waiting/holding, and double-holding transfer strategies. They noted that the effectiveness of timed transfers can vary by route conditions. However, they found that the scheduled transfers are effective (over the unscheduled) when there is incompatibility between headways and the double-holding strategy outperforms the other time transfer strategies

when the headways on intersecting routes are compatible. This study also pointed out that slack time may be better built into the schedule so that vehicle holding does not cause significant delays to passengers.

Domschke (1989) explored a schedule coordination problem with the objective of minimizing waiting times. He provided a mathematical programming formulation which is generally applicable to a public mass transit network such as subways, trains, and/or buses. The formulation is a quadratic assignment problem. With four routes and five transfer stations in a toy network, this study discussed heuristics and a branch and bound algorithm. The heuristics included a starting heuristic, which was based on rigid regret heuristic, and then a heuristic improvement procedure. Lastly, simulated annealing (SA) was applied to improve the solutions. For SA, the quality of the initial solution is important. He found that problems with more than 20 routes could not be solved by exact solution methods.

Knoppers and Muller (1995) provided a theoretical note on transfers in public transportation. Their main concerns were the transfer time needed and the probability of missed connection to minimize passengers' transfer time. They found that when the frequency on the connecting lines increased, the benefit of transfer coordination yield decreased. Muller and Furth (2009) tried to reduce passenger waiting time through transfer scheduling and control. They provided a probabilistic optimization model, and discussed three transfer control types, namely departure punctuality control, attuned departure control, and delayed departure of connecting vehicles. They confirmed that by increasing a buffer (slack), the probability of missing the connection decreases. However, a larger buffer increases the transfer time for people who do not miss their connection. They also found that if the control policy allowed a bus to be held to make a connection, the optimal schedule offset decreased.

Shrivastava et al. (2002) first discuss existing algorithms for solving nonlinear mathematical programming, because transit scheduling problems are often nonlinear. The existing algorithms are generally gradient based, and require at least the first-order derivatives of both objective and constraint functions with respect to the design variables. With the “slope tracking” ability, gradient-based methods can easily identify a relative optimum closest to the initial guess of the optimum design. However, there is no guarantee of locating the global optimum if the design space is known to be non-convex. In such case, exhaustive and random search techniques such as random walk or random walk with direction exploitation are quite useful. The main drawback with these methods is that they often require thousands of function evaluations, even for the simplest functions, to reach the optimum. They also note that genetic algorithms (GAs) are based on exhaustive and random search techniques, and are robust for optimizing nonlinear and non-convex functions. Thus, they apply a GA to schedule coordination problems. The objective function includes waiting time, transfer time, and in-vehicle time for users, and vehicle operating cost for operators. For a scheduling problem, they try to solve routing and scheduling simultaneously. The GA is designed with two substrings, where one represents routes and the other represents frequencies on those routes. By solving benchmark problems, they find that genetic algorithms provide better solutions than other heuristics. They also note that computational times are proportional to the pool size. Cevallos and Zhao (2006) also use a GA to solve a transfer time optimization problem for a fixed route system. Their main focus is efficient computational time.

Lee and Schonfeld (1991) studied optimal slack times for coordinating transfers between rail and bus routes at one terminal. The transfer cost function is formulated as a sum of scheduled delay cost, missed connection cost for bus to train transfer, and missed connection cost for train



to bus transfer. In their paper, the rail transit line was assumed to run on-time (no slack), and slack times for bus routes were to be optimized. Bus arrivals were assumed to vary independently from train arrivals so that the joint probabilities of arrivals could be obtained by simply multiplying the probabilities obtained separately from the bus and train arrivals distributions. Slack times were optimized analytically, and numerical results show that an analytic optimization with simplifying assumptions is limited and difficult to solve for complex situations. Thus, they developed a numerical optimization method to find solutions efficiently.

Ting and Schonfeld (2005) extended Lee and Schofeld (1991)'s study. They explored bus service coordination in multiple hub networks. They analyzed uncoordinated operations and coordinated operations, and compared the results. For uncoordinated operation, the formulation minimizes the total system cost, which is the sum of operating cost, user waiting cost, and user transfer cost. Transfer cost in uncoordinated operation was simply assumed to be the product of the average transfer waiting time and the total number of transfer passengers. For the coordinated operation, the transfer cost consists of slack-time cost, missed connection cost, and dispatching delay cost. Common headway and integer-ratio headway cases were optimized with a heuristic algorithm. Their algorithms and numerical results show when coordinated operations with integer-ratio headways are preferable over uncoordinated operation in terms of total cost. Simplifying assumptions of this work were that: (1) only one dispatching strategy was considered, which means vehicles do not wait for other vehicles that arrive behind schedule; (2) vehicle arrivals on a route were assumed to vary independently from those of other routes, so that the joint probabilities of arrivals may be obtained by simply multiplying the probabilities obtained separately from the two vehicle arrival distributions. A limitation of this work is that it does not ensure integer fleet size.

Chen and Schonfeld (2010) adapted the concept of bus transit coordination methods to freight transportation. They followed the main ideas of joint probabilities and transfer cost components from some previous transit studies (Lee and Schonfeld, 1991; Ting and Schonfeld, 2005). In this study, they proposed two solution approaches, which are a genetic algorithm and sequential quadratic programming (SQP) to find good solutions for frequencies and slack times in intermodal transfers.

Chowdhury and Chien (2002) also studied the coordination of transfers among rail and feeder bus routes. Their objective was to minimize total cost, including supplier and user costs, similarly to other studies. They explored various degrees of coordination such as full coordination, partial coordination, and no coordination. They also followed the assumption of joint probabilities of independent vehicle arrivals, and assumed that trains operate on-time. Recently, Chowdhury and Chien (2011) extended a previous study by jointly optimizing bus size, headway, and slack time for timed transfer. They optimized bus size by assuming maximum allowable bus headways instead of minimum cost headways. Therefore, their optimized bus size may be overestimated. For solving this problem they applied Powell's algorithm (i.e., multi-variable numerical optimization). Unfortunately, they did not present enough details on the methodology that might have helped readers understand how joint variables are optimized and how variables are constrained to be integer. Another limitation of this study is that although it found optimized vehicle size jointly with other decision variables, such as headways and slack times, the vehicle size was optimized for only one time period. Optimizing vehicle size and required fleet size for daily demand or system-wide demand while finding headways and slack times for each time period represents an opportunity for improvement.

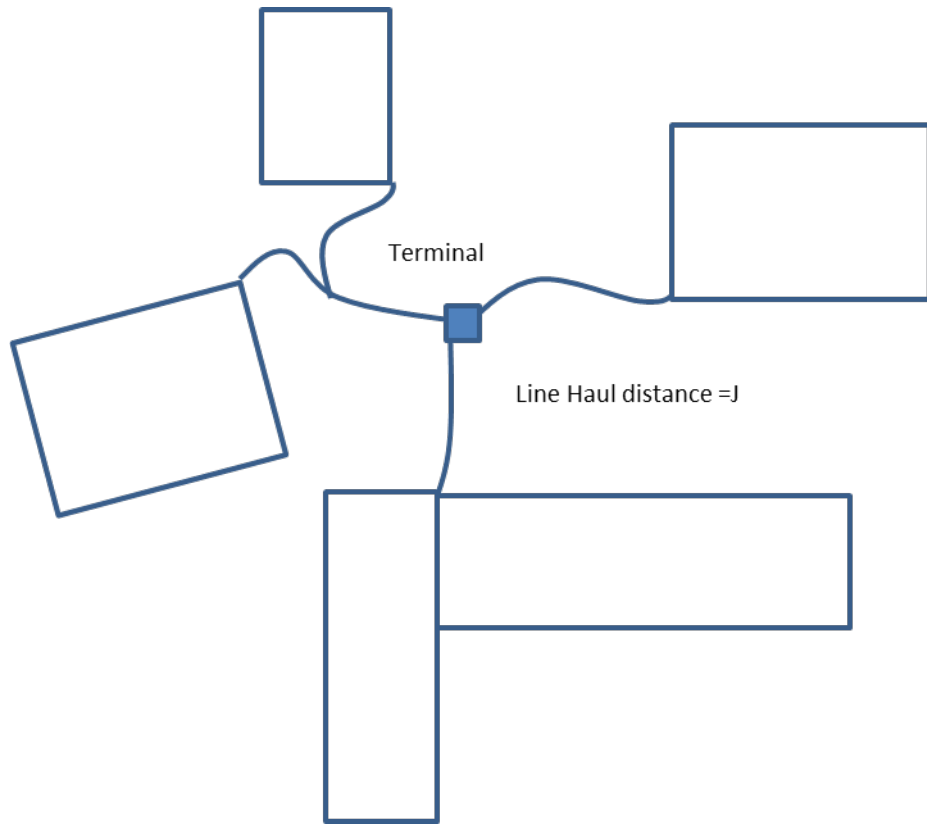
### ***3.2.2 Review Summary***

No study was found for coordinating transfers among conventional and flexible bus services, although there are studies on timed transfer coordination for other modes. In evaluating coordination types, this study explored independent headways and common headways. The formulations developed here also considered multiple regions. The solution method proposed here also ensures integer vehicle size(s) as well as integer fleet size(s). Various analyses of sensitivity to the critical input values should also be conducted.

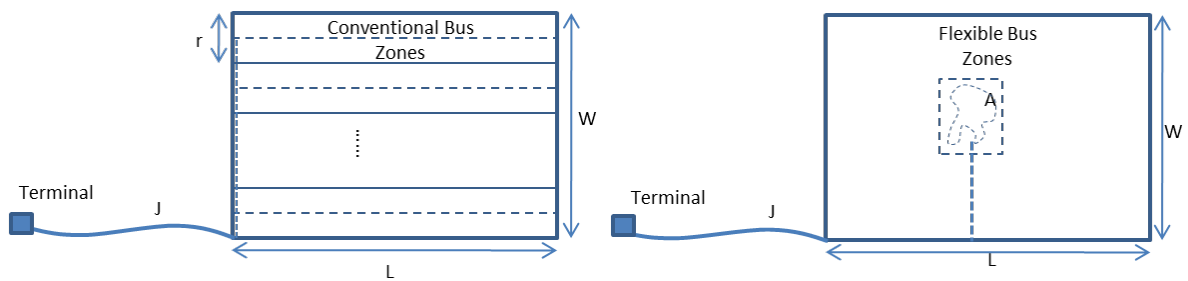
## **3.3 Bus Services and Assumptions**

### ***3.3.1 System Descriptions***

The bus system analyzed here provides bus transit services between a terminal and multiple local regions (shown in Figure 12). For each region, either conventional or flexible bus service is provided, as shown in Figure 13. Detailed descriptions and assumptions of conventional and flexible bus services are provided in the following section.



**Figure 12: Terminal and Local Regions**



**Figure 13: Conventional and Flexible Bus Descriptions**

### 3.3.2 Assumptions

We adopt assumptions from relevant studies (e.g., Kim and Schonfeld, 2012 and 2013) and modify them to be able to consider transfers in the terminal. Superscript  $k$  corresponds to regions, and subscripts  $c$  and  $f$  represent conventional and flexible bus services, respectively. Definitions, units, and default values of variables are presented in Table 5.

All service regions,  $1 \dots k$ , are rectangular, with lengths  $L^k$  and widths  $W^k$ . These regions may have different line haul distances  $J^k$  (miles, in route  $k$ ) connecting a terminal and each region's nearest corner.

#### Assumptions for both conventional and flexible services

- a) The demand is fixed with respect to service quality and price.
- b) The demand is given and uniformly distributed over space within each region and over time within each specified period.
- c) The bus sizes ( $S_c$  for conventional,  $S_f$  for flexible) are optimized based on their service coverage.
- d) Within each local region  $k$ , the average speed ( $V_c$  for conventional bus,  $V_f$  for flexible bus) includes stopping times.
- e) Passenger arrivals at each stop are random and uniformly distributed.
- f) Layover times and external costs are assumed to be negligible.
- g) Transfer coordination is assumed under a no-hold policy.

**Table 5: Notations**

Variable	Definition	Baseline Value
$a$	hourly fixed cost coefficient for operating bus (\$/bus hr)	30.0
$A^k$	service zone area(mile <sup>2</sup> )= $L^k W^k / N'$	-
$b$	hourly variable cost coefficient for bus operation (\$/seat hr)	0.2
$d$	bus stop spacing (miles)	0.2
$D_c^k$	distance of one flexible bus tour in local region k (miles)	-
$D_f^k$	equivalent line haul distance for flexible bus on region k (= $(L^k + W^k) / z + 2J^k / y$ ), (miles)	-
$D^k$	equivalent average bus round trip distance for conventional bus on region k (= $2J^k / y + W^k / z + 2L^k$ ), (miles)	-
$f$	directional demand split factor	1.0
$F_c^k, F_f^k$	fleet size for route k (buses) subscript corresponds to (c = conventional, f=flexible)	-
$h_c, h_c^k$	headway for conventional bus; for region k (hours/bus)	-
$h_f, h_f^k$	headway for flexible bus; for region k (hours/bus)	-
$h_c^{k,max}, h_f^{k,max}$	maximum allowable headway for region k; subscript: c = conventional, f=flexible	-
$h_c^{k,min}, h_f^{k,min}$	minimum cost headway for region k; subscript: c = conventional, f=flexible	-
$k$	index (k: regions)	-
$J^k$	line haul distance of region k (miles)	-
$l_c, l_f$	load factor for conventional and flexible bus (passengers/seat)	1.0
$L^k, W^k$	length and width of local region k (miles)	-
$M^k$	equivalent average trip distance for region k (= $J^k / y_c + W^k / 2z_c + L^k / 2$ )	-
$n$	number of passengers in one flexible bus tour	-
$N, N'$	number of zones in local region for conventional and flexible bus	-
$Q^k$	round trip demand density (trips/mile <sup>2</sup> /hr)	-
$Q_t^k$	threshold demand density between conventional and flexible service (trips/mile <sup>2</sup> /hr)	-
$r^k$	route spacing for conventional bus at region k (miles)	-
$R_c^k$	round trip time of conventional bus for region k (hours)	-
$R_f^k$	round trip time of flexible bus for region k (hours)	-
$S_c^k, S_f^k$	conventional and flexible bus sizes for region k (seats/bus)	{7,10,16,25,35,40,50}
$TC_c^{u,k}, TC_f^{u,k}$	uncoordinated total service cost over region k subscript: c = conventional, f=flexible	-
$OC_c^k, OC_f^k$	operating cost for region k subscript: c = conventional, f=flexible	-
$IVC_c^k, IVC_f^k$	in-vehicle cost for region k subscript: c = conventional, f=flexible	-
$WC_c^k, WC_f^k$	waiting cost for region k subscript: c = conventional, f=flexible	-
$TFC_c^{u,k}, TFC_f^{u,k}$	uncoordinated transfer for region k subscript: c = conventional, f=flexible	-
$XC_c^k$	access cost for region k subscript: c = conventional, f=flexible	-
$t$	substitution variable	-
$u$	average number of passengers per stop for flexible bus	1.2
$V_c$	local service speed for conventional bus (miles/hr)	30
$V_f$	local service speed for flexible bus (miles/hr)	25
$V_x$	average passenger access speed (mile/hr)	2.5
$v_v, v_w, v_x, v_f$	value of in-vehicle time, wait time, access and transfer time (\$/passenger hr)	10,15,15,12
$y$	express speed/local speed ratio for conventional bus	conventional bus = 1.8 flexible bus = 2.0
$z$	non-stop ratio = local non-stop speed/local speed; same values as y	-
$\emptyset$	constant in the flexible bus tour equation (Daganzo, 1984) for flexible bus	1.15
$\sigma$	standard deviation of travel times	0.25, 0.4, 0.15, 0.2
*	superscript indicating optimal value; subscript: c = conventional, f=flexible	-

Assumptions for conventional services

- a) The region  $k$  is divided into  $N^k$  parallel zones with a width  $r^k = W^k/N^k$  for conventional bus, as shown in Figure 2. Local routes branch from the line haul route segment to run along the middle of each zone, at a route spacing  $r^k = W^k/N^k$ .
- b)  $Q^k$  trips/hour, entirely channeled to (or through) the single terminal, are uniformly distributed over the service area.
- c) In each round trip, as shown in Figure 2, buses travel from the terminal a line haul distance  $J^k$  at non-stop speed  $yV_c$  to a corner of the local regions, then travel an average of  $W^k/2$  miles at local non-stop speed  $zV_c$  from the corner to the assigned zone, then run a local route of length  $L^k$  at local speed  $V_c$  along the central axis of the zone while stopping for passengers every  $d$  miles, and then reverse the above process in returning to the terminal.

Assumptions for flexible services

- a) To simplify the flexible bus formulation, region  $k$  is divided into  $N^k$  equal zones, each having an optimizable zone area  $A^k = L^k W^k / N^k$ . The zones should be “fairly compact and fairly convex” (Stein, 1978).
- b) Buses travel from the terminal line haul distance  $J^k$  at non-stop speed  $yV_f$  and an average distance  $(L^k + W^k)/2$  miles at local non-stop speed  $zV_f$  to the center of each zone. They collect (or distribute) passengers at their door steps through an efficiently routed tour of  $n$  stops and length  $D_c^k$  at local speed  $V_f$ .  $D_c^k$  is approximated according to Stein (1978), in which  $D_c^k = \phi \sqrt{n A^k}$ , and  $\phi = 1.15$  for the rectilinear space assumed here (Daganzo, 1984). The values of  $n$  and  $D_c^k$  are endogenously determined. To return to their starting

point the buses retrace an average of  $(L^k + W^k)/2$  miles at  $zV_f$  miles per hour and  $J^k$  miles at  $yV_f$  miles per hour.

- c) Buses operate on preset schedules with flexible routing designed to minimize each tour distance  $D_c^{ki}$ .
- d) Tour departure headways are equal for all zones in the region and uniform within each period.

### 3.4 Uncoordinated Bus Operations

In this section, we explore uncoordinated bus operations. Conventional and flexible bus cost functions are formulated, with consideration of transfer cost functions. For both conventional and flexible bus services, total cost functions consist of bus operating cost, user in-vehicle cost, user waiting cost, user transfer cost. For conventional bus service, access cost term is additionally considered.

In the following subsections, we construct cost functions of conventional and flexible bus services, and analyze them to find optimal values of decision variables such as vehicle sizes, the number of zones in each region, headways, fleet sizes. In uncoordinated bus operations, we optimize them analytically.

#### 3.4.1 Conventional Bus Formulation and Analytic Optimization

In the conventional bus cost formulation, we consider bus operating cost, user in-vehicle cost, user waiting cost, user access cost, and user transfer cost.

$$TC_c^u = \sum_{k=1}^K \{OC_c^{u,k} + IV C_c^{u,k} + WC_c^{u,k} + XC_c^{u,k} + TFC_c^{u,k}\} \quad (1)$$

Conventional bus operating cost,  $OC_c^{u,k}$ , can be formulated by multiplying unit bus operating cost,  $B$ , and the number of zones in region  $k$ ,  $N_c^{u,k}$ , and fleet size,  $F_c^{u,k}$ .



$$OC_c^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} \quad (2)$$

Fleet size,  $F_c^{u,k}$ , can be formulated by

$$F_c^{u,k} = \frac{D^k}{V_c \cdot h_c^k} \quad (3)$$

Conventional bus user in-vehicle cost is then formulated by

$$IVC_c^{u,k} = v_v Q^k \frac{M^k}{V_c} \quad (4)$$

Since we assume that passengers arrive at the stop randomly and uniformly over time, the waiting time may be estimated as in Welding (1957), Osuna and Newell (1972), and Ting and Schonfeld (2005).

$$w^k = \frac{E(h^k)}{2} \left( 1 + \frac{(\sigma^k)^2}{[E(h^k)]^2} \right) \quad (5)$$

Thus, the waiting cost of uncoordinated conventional bus service is

$$WC_c^{u,k} = v_w Q^k w^k = v_w Q^k \frac{E(h_c^k)}{2} \left( 1 + \frac{(\sigma^k)^2}{[E(h_c^k)]^2} \right) \quad (6)$$

As mentioned in Kim and Schonfeld (2012), because the spacing between adjacent branches of local bus service is  $r^k$ , and because service trip origins (or destinations) are uniformly distributed over the area, the average access distance to the nearest route is one-fourth of route spacings,  $r^k/4$ . Similarly, the access distance alongside the route to the nearest transit stop is one-fourth of the bus stop spacing,  $d/4$ . Therefore, the access cost for the conventional bus system,  $XC_c^{u,k}$ , is

$$XC_c^{u,k} = \frac{v_x \cdot Q^k (r^k + d)}{4V_x} = \frac{v_x \cdot Q^k \left( \frac{W^k}{N_c^{u,k}} + d \right)}{4V_x} \quad (7)$$

Transfer cost,  $TFC_c^{u,k}$ , can be similarly formulated to waiting cost function. The only difference is that the transfer cost considers only transfer demand.

$$TFC_c^{u,k} = v_f Q_t^k w^k = v_f Q_t^k \left( \frac{E(h_c^k)}{2} + \frac{(\sigma^k)^2}{2E(h_c^k)} \right) \quad (8)$$

For the sake of simplicity, the expected value of headway,  $E(h^k)$ , and the headway,  $h^k$ , are interchangeable here. The total cost function of conventional bus service is then

$$TC_c^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} + v_v Q^k \frac{M^k}{V_c} + v_w Q^k \frac{E(h^k)}{2} \left( 1 + \frac{(\sigma^k)^2}{[E(h^k)]^2} \right) + \frac{v_x \cdot Q^k \left( \frac{W^k}{N_c^{u,k}} + d \right)}{4V_x} + v_f Q_t^k \left( \frac{E(h^k)}{2} + \frac{(\sigma^k)^2}{2E(h^k)} \right) \quad (9)$$

By rearranging Equation 9, we get:

$$TC_c^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} + v_v Q^k \frac{M^k}{V_c} + \frac{h_c^k}{2} (v_w Q^k + v_f Q_t^k) + \frac{\sigma^2}{2h_c^k} (v_w Q^k + v_f Q_t^k) + \frac{v_x \cdot Q^k \left( \frac{W^k}{N_c^{u,k}} + d \right)}{4V_x} \quad (10)$$

Then, we substitute the headway to the fleet size using Equation 3, which is  $h_c^k = \frac{D^k}{V_c \cdot F_c^{u,k}}$

$$TC_c^{u,k} = B^k \cdot N_c^{u,k} \cdot F_c^{u,k} + \frac{v_v Q^k M^k}{V_c} + \frac{D^k (v_w Q^k + v_f Q_t^k)}{2V_c \cdot F_c^{u,k}} + \frac{\sigma^2 V_c \cdot F_c^{u,k} (v_w Q^k + v_f Q_t^k)}{2D^k} + \frac{v_x \cdot Q^k \left( \frac{W^k}{N_c^{u,k}} + d \right)}{4V_x} \quad (11)$$

Equation 11 is now used for optimizing decision variables, namely the number of zones,  $N_c^{u,k}$ , and fleet sizes,  $F_c^{u,k}$ .

By taking the partial derivative of  $TC_c^{u,k}$  with respect to the number of zones,  $N_c^{u,k}$ :

$$\frac{\partial TC_c^{u,k}}{\partial N_c^{u,k}} = B^k F_c^{u,k} - \frac{v_x \cdot Q^k W^k}{4V_x (N_c^{u,k})^2} = 0 \quad (12)$$

To guarantee the global minimum solution, we take the second-order derivative of  $TC_c^{u,k}$  with respect to the number of zones,  $N_c^{u,k}$ . As shown in Equation 13, the value of second-order derivation is positive, so that the optimal value of the number of zones results in the global minimum.

$$\frac{\partial^2 T C_c^{u,k}}{\partial (N_c^{u,k})^2} = \frac{v_x \cdot Q^k W^k}{2V_x (N_c^{u,k})^3} > 0 \quad (13)$$

Rearranging Equation 12, we get

$$(N_c^{u,k})^2 = -\frac{v_x \cdot Q^k W^k}{4B^k V_x F_c^{u,k}} \quad (14)$$

Then, we find the optimal fleet size by an analytic optimization. Similarly to the number of zones, we take the partial derivation of Equation 11 regarding the fleet size,  $F_c^{u,k}$ , as shown in Equation 15.

$$\frac{\partial T C_c^{u,k}}{\partial F_c^{u,k}} = B^k \cdot N_c^{u,k} - \frac{D^k (v_w Q^k + v_f Q_t^k)}{2V_c (F_c^{u,k})^2} + \frac{\sigma^2 V_c \cdot (v_w Q^k + v_f Q_t^k)}{2D^k} = 0 \quad (15)$$

Second-order derivation of Equation 11 with respect to the fleet size,  $F_c^{u,k}$ , is then

$$\frac{\partial^2 T C_c^{u,k}}{\partial (F_c^{u,k})^2} = \frac{D^k (v_w Q^k + v_f Q_t^k)}{V_c \cdot (F_c^{u,k})^3} > 0 \quad (16)$$

We notice that Equations 14 and 16 have positive values in all possible input values; thus, optimized values of the number of zones,  $N_c^{u,k}$  and fleet size,  $F_c^{u,k}$  guarantee globally optimum of the objective function (i.e., Equation 11).

By rearranging Equation 15,

$$(F_c^{u,k})^2 = \frac{(D^k)^2 (v_w Q^k + v_f Q_t^k)}{V_c \{ + \sigma^2 V_c \cdot (v_w Q^k + v_f Q_t^k) \}} \quad (17)$$

We now take a square to Equation 14 and arrange them as follows.

$$(F_c^{u,k})^2 = \frac{(v_x)^2 \cdot (Q^k)^2 (W^k)^2}{16(B^k)^2 (V_x)^2 (N_c^{u,k})^4} \quad (18)$$

Now, set Equations 17 and 18 to be equal, and arrange it as the function of the number of zones,  $N_c^{u,k}$ .

$$16(B^k)^2 (V_x)^2 (N_c^{u,k})^4 - 2(v_x)^2 D^k B^k (Q^k)^2 (W^k)^2 V_c N_c^{u,k} - \sigma^2 (v_x)^2 (Q^k)^2 (W^k)^2 (V_c)^2 (v_w Q^k + v_f Q_t^k) = 0 \quad (19)$$

The Equation 19 is a fourth-order polynomial equation; the values of the number of zones,  $N_c^{u,k}$ , can be found with a MATLAB function called “roots.” This “roots” function solves polynomial equations with eigenvalues of companion matrix. (Mathworks, 2013). Once values are obtained, we take only positive and real numbers.

Then, we substitute the value of the number of zones,  $N_c^{u,k}$ , into Equation 14 to find the optimized fleet size,  $F_c^{u,k}$ . However, these two variables have to be integers, thus we select their nearest integer from optimized values.

The minimum cost headway can be obtained from optimized fleet size,  $F_c^{u,k}$ , with Equation 3, which can be rewritten as follows.

$$h_c^{k,min} = \frac{D^k}{V_c \cdot F_c^{u,k}} \quad (20)$$

The resulting headway should not exceed the maximum allowable headway. The maximum allowable headway is expressed in Equation 21.

$$h_c^{k,max} = \frac{S^k l^k N_c^{u,k}}{f^k \cdot Q^k} \quad (21)$$

In conventional bus service formulations, Equations 1 through 21, we do not optimize bus sizes, which also affects system performance such as via bus operating cost and maximum allowable headways. Here, we only consider an exhaustive set of bus sizes such as {7, 15, 25, 35,45 seats}. Therefore, by simply analyzing equations multiple times, which is the same as the number of vehicle size cases, we jointly find the optimized vehicle size in each region.

### ***3.4.2 Flexible Bus Formulation and Analytic Optimization***

In the flexible bus cost formulation, we consider bus operating cost, user in-vehicle cost, user waiting cost, and user transfer cost. Unlike conventional bus formulation, user access cost is

not included because flexible bus services pick up passengers from their homes to their destinations, or vice versa.

$$TC_f^u = \sum_{k=1}^K \{OC_f^{u,k} + IV C_f^{u,k} + WC_f^{u,k} + TFC_f^{u,k}\} \quad (22)$$

Flexible bus operating cost in region k,  $OC_f^{u,k}$ , is formulated by multiplying bus operating cost,  $B^k$ , and the number of flexible service zones,  $N_f^{u,k}$ , and the fleet size,  $F_f^{u,k}$ .

$$OC_f^{u,k} = B^k \cdot N_f^{u,k} \cdot F_f^{u,k} \quad (23)$$

The fleet size,  $F_f^{u,k}$  is expressed as terms of the equivalent line-haul distance,  $D_f^k$ , the flexible bus tour distance in region k,  $D_c^k$ .

$$F_f^{u,k} = \frac{(D_f^k + D_c^k)}{V_f h_f^k} \quad (24)$$

The flexible bus tour distance,  $D_c^k$ , is then formulated by

$$D_c^k = \phi \sqrt{n A^k} \quad (25)$$

The number of passengers in one tour,  $n$ , is shown in Equation 26.

$$n = \frac{A^k Q^k h_f^k}{u L^k W^k} \quad (26)$$

By substituting Equation 26 and  $A^k = L^k W^k / N_f^{u,k}$  into Equation 25,

$$D_c^k = \frac{\phi}{N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k h_f^k}{u}} \quad (27)$$

By substituting Equations 24 and 27 into Equation 23, the flexible bus operating cost is formulated as follows.

$$OC_f^{u,k} = \frac{D_f^k B^k N_f^{u,k}}{V_f h_f^k} + \frac{\phi B^k}{V_f} \sqrt{\frac{L^k W^k Q^k}{u h_f^k}} \quad (28)$$

The flexible bus in-vehicle cost is then formulated by

$$IVC_f^{u,k} = \frac{v_v Q^k (D_f^k + D_c^k)}{2V_f} = \frac{v_v Q^k D_f^k}{2V_f} + \frac{\phi v_v Q^k}{2V_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k h_f^k}{u}} \quad (29)$$

As mentioned in Equation 5 earlier, there is an assumption that passengers come to the terminal randomly and uniformly over time. Therefore, the waiting time in Equation 5 is still applicable to flexible bus services. Thus the waiting cost of flexible bus service is

$$WC_f^{u,k} = v_w Q^k W^k = v_w Q^k \frac{E(h_f^k)}{2} \left( 1 + \frac{(\sigma^k)^2}{[E(h_f^k)]^2} \right) \quad (30)$$

The flexible bus transfer cost function is also formulated similarly

$$TFC_f^{u,k} = v_f Q_t^k W^k = v_w Q_t^k \frac{E(h_f^k)}{2} \left( 1 + \frac{(\sigma^k)^2}{[E(h_f^k)]^2} \right) \quad (31)$$

Similar to conventional bus formulations, the expected value of headway,  $E(h_f^k)$ , and the headway,  $h_f^k$ , are assumed to be interchangeable in this paper.

By substituting Equations 28 through 31 into Equation 22, the total flexible bus cost in region  $k$  becomes

$$TC_f^{u,k} = \frac{D_f^k B^k N_f^{u,k}}{V_f h_f^k} + \frac{\phi B^k}{V_f} \sqrt{\frac{L^k W^k Q^k}{u h_f^k}} + \frac{v_v Q^k D_f^k}{2V_f} + \frac{\phi v_v Q^k}{2V_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k h_f^k}{u}} + \frac{v_w Q^k h_f^k}{2} + \frac{v_w Q^k (\sigma^k)^2}{2h_f^k} + \frac{v_f Q_t^k h_f^k}{2} + \frac{v_f Q_t^k (\sigma^k)^2}{2h_f^k} \quad (32)$$

Now, Equation 32 is a function of two decision variables, namely the number of zones,  $N_f^{u,k}$ , and the headway,  $h_f^k$ . We analytically solve this total cost function. By taking the partial derivation of Equation 32 with respect to the number of zones,  $N_f^{u,k}$ , we get Equation 33.

$$\frac{\partial TC_f^{u,k}}{\partial N_f^{u,k}} = \frac{B^k D_f^k}{V_f h_f^k} - \frac{\phi v_v Q^k}{2V_f} \sqrt{\frac{L^k W^k Q^k h_f^k}{u}} \frac{1}{(N_f^{u,k})^2} = 0 \quad (33)$$

Second-order derivation in Equation 34 shows that the total cost function is convex.

$$\frac{\partial^2 TC_f^{u,k}}{\partial (N_f^{u,k})^2} = \frac{\phi v_v Q^k}{2V_f} \sqrt{\frac{L^k W^k Q^k h_f^k}{u}} \frac{1}{(N_f^{u,k})^3} > 0 \quad (34)$$

By re-writing Equation 33, the number of zones,  $N_f^{u,k}$ , then becomes

$$N_f^{u,k} = \sqrt[4]{\frac{(\emptyset v_v)^2 L^k W^k (Q^k h_f^k)^3}{4(B^k D_f^k)^2 u}} \quad (35)$$

To find the optimized headway, we take the partial derivative of the total cost function in Equation 32 with respect to the headway,  $h_f^k$ .

$$\frac{\partial TC_f^{u,k}}{\partial h_f^k} = -\frac{D_f^k B^k N_f^{u,k}}{v_f} h_f^{k-2} - \frac{\emptyset B^k}{2v_f} \sqrt{\frac{L^k W^k Q^k}{u}} h_f^{k-\frac{3}{2}} + \frac{\emptyset v_v Q^k}{4v_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u}} h_f^{k-\frac{1}{2}} + \frac{v_w Q^k + v_f Q_t^k}{2} - \frac{(\sigma^k)^2 (v_w Q^k + v_f Q_t^k)}{2} h_f^{k-2} = 0 \quad (36)$$

Second-order derivation is shown in Equation 37 as follows.

$$\frac{\partial^2 TC_f^{u,k}}{\partial (h_f^k)^2} = \frac{2D_f^k B^k N_f^{u,k}}{v_f} h_f^{k-3} + \frac{3\emptyset B^k}{4v_f} \sqrt{\frac{L^k W^k Q^k}{u}} h_f^{k-\frac{5}{2}} - \frac{\emptyset v_v Q^k}{8v_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u}} h_f^{k-\frac{3}{2}} + (\sigma^k)^2 (v_w Q^k + v_f Q_t^k) h_f^{k-3} \quad (37)$$

To be a globally convex function, Equation 37 has to be positive with whichever values of headways. By rearranging Equation 37, we get

$$\left\{ \frac{2D_f^k B^k N_f^{u,k}}{v_f} + (\sigma^k)^2 (v_w Q^k + v_f Q_t^k) \right\} \frac{1}{\sqrt{h_f^{k^3}}} + \left\{ \frac{3\emptyset B^k}{4v_f} \sqrt{\frac{L^k W^k Q^k}{u}} \right\} \frac{1}{h_f^k} - \left\{ \frac{\emptyset v_v Q^k}{8v_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u}} \right\} > 0 \quad (38)$$

Once we get values of headways, Equation 38 has to be checked.

By rewriting Equation 36,

$$\left\{ -\frac{D_f^k B^k N_f^{u,k}}{v_f} - \frac{(\sigma^k)^2 (v_w Q^k + v_f Q_t^k)}{2} \right\} - \frac{\emptyset B^k}{2v_f} \sqrt{\frac{L^k W^k Q^k}{u}} \sqrt{h_f^k} + \frac{\emptyset v_v Q^k}{4v_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u}} \sqrt{h_f^{k^3}} + \frac{v_w Q^k + v_f Q_t^k}{2} h_f^{k^2} = 0 \quad (39)$$

We can solve Equations 35 and 39 simultaneously to find the optimized number of zones,  $N_f^{u,k}$ , and the headway,  $h_f^k$ . By substituting Equation 35 into Equation 39, we get

$$\frac{(v_w Q^k + v_f Q_f^k)}{2} h_f^{\frac{8}{4}} - \frac{D_f^k B^k}{V_f} \sqrt{\frac{(\emptyset v_v)^2 L^k W^k (Q^k)^3}{4(B^k D_f^k)^2 u}} h_f^{\frac{3}{4}} + \frac{\emptyset v_v Q^k}{4V_f N_f^{u,k}} \sqrt{\frac{4(B^k D_f^k)^2 L^k W^k}{(\emptyset v_v)^2 Q^k u}} h_f^{\frac{3}{4}} - \frac{\emptyset B^k}{2V_f} \sqrt{\frac{L^k W^k Q^k}{u}} h_f^{\frac{2}{4}} - \frac{(\sigma^k)^2 (v_w Q^k + v_f Q_f^k)}{2} = 0 \quad (40)$$

By substituting  $h_f^{\frac{1}{4}}$  into t, we get the 8<sup>th</sup> order polynomial equation that is the function of t.

$$\frac{(v_w Q^k + v_f Q_f^k)}{2} t^8 - \left\{ \frac{D_f^k B^k}{V_f} \sqrt{\frac{(\emptyset v_v)^2 L^k W^k (Q^k)^3}{4(B^k D_f^k)^2 u}} + \frac{\emptyset v_v Q^k}{4V_f N_f^{u,k}} \sqrt{\frac{4(B^k D_f^k)^2 L^k W^k}{(\emptyset v_v)^2 Q^k u}} \right\} t^3 - \frac{\emptyset B^k}{2V_f} \sqrt{\frac{L^k W^k Q^k}{u}} t^2 - \frac{(\sigma^k)^2 (v_w Q^k + v_f Q_f^k)}{2} = 0 \quad (41)$$

Equation 41, which is the function of t, is now solvable with the function called ‘‘roots’’ as we used in the conventional bus formulation. Once we obtain values of t, we select only positive and real values. Then, optimal headways are obtained (i.e.,  $h_f^k = t^4$ ).

In order to find the number of zones,  $N_f^{u,k}$ , we put the optimized headway into Equation 35. The fleet size,  $F_f^{u,k}$ , is also found by using Equation 24. When we obtain the optimized number of zones,  $N_f^{u,k}$ , and the fleet size,  $F_f^{u,k}$ , we need to ensure their integer values, which means that the optimized headways have to be modified.

The resulting minimum cost headway can be found by substituting Equation 27 into Equation 24, obtaining

$$F_f^{u,k} = \frac{D_f^k}{V_f h_f^k} + \frac{\emptyset}{V_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u h_f^k}} \quad (42)$$

After we rearrange Equation 42 to the function of the headway by setting t equal to  $\sqrt{\frac{1}{h_f^k}}$ ,

$$\frac{D_f^k}{V_f} t^2 + \frac{\emptyset}{V_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u}} t - F_f^{u,k} = 0 \quad (43)$$



Thus,  $t$  can have two values to satisfy Equation 43, however, we take the higher value of  $t$ , which is shown in Equation 44, because the value of  $t$  must be positive.

$$t = \left\{ -\frac{\phi}{V_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u}} + \sqrt{\left(\frac{\phi}{V_f N_f^{u,k}}\right)^2 \frac{L^k W^k Q^k}{u} + 4F_f^{u,k} \frac{D_f^k}{V_f}} \right\} / \frac{2D_f^k}{V_f} \quad (44)$$

The minimum cost headway that ensures an integer number of zones,  $N_f^{u,k}$ , and the fleet size,  $F_f^{u,k}$ , is now found in Equation 45.

$$h_f^{k,min} = \frac{4(D_f^k)^2}{(V_f)^2 \left\{ -\frac{\phi}{V_f N_f^{u,k}} \sqrt{\frac{L^k W^k Q^k}{u}} + \sqrt{\left(\frac{\phi}{V_f N_f^{u,k}}\right)^2 \frac{L^k W^k Q^k}{u} + 4F_f^{u,k} \frac{D_f^k}{V_f}} \right\}^2} \quad (45)$$

As mentioned in conventional bus service formulations, we only consider an exhaustive set of bus sizes such as {7, 15, 25, 35, 45 seats}. Therefore, by simply analyzing equations multiple times, which is the same as the number of vehicle size cases, we jointly find the optimized vehicle size in each region.

### 3.5 Numerical Evaluations

#### 3.5.1 Base Case Study

##### Input values

We consider four different regions that are connected to a terminal. The transfer demand, non-transfer demand, and total demand by route and the size of each region are provided in Table 6.

**Table 6: Input Values for Base Case Example**

Region	A	B	C	D
Total demand by route (trips/hour)	402.5	339.5	319.5	83
Transfer demand (trips/hour)	315	244.5	264.5	61
Non transfer demand (trips/hour)	87.5	95	55	22
Line-haul Distance (miles)	5	10	4	2
Length of Region (miles)	3	4	2	2
Width of Region (miles)	4	3	3	2

***Optimization Results***

With input values given in Tables 1 and 2, optimization results of conventional bus services are provided in Table 7. As mentioned earlier and provided in Table 1, we use exhaustive lists of vehicle sizes, which is a set of {7, 10, 16, 25, 35, 40, 50}. The optimized bus sizes for regions A, B, C, and D are 10, 16, 10, and 7 seats/bus, respectively. Route spacings, headways, and fleet sizes are jointly optimized. Total costs that consist of operating cost, in-vehicle cost, waiting cost, access cost, and transfer cost are also obtained.

Optimization results of flexible bus services are shown in Table 8. The optimized bus size of region C is changed to 7 seats, compared to 10 seats in conventional bus service results. Flexible bus service areas range from 0.6667 mi<sup>2</sup> to 1.0 mi<sup>2</sup> by region. Headways and fleet sizes are also shown in Table 8.

**Table 7: Optimization Results of Conventional Service**

Region	A	B	C	D
Bus Size (# of seats)	10	16	10	7
The number of zones (route spacings)	7(0.5714)	5(0.6)	5(0.6)	3(0.6667)
Headway (hours)	0.2294	0.3463	0.1685	0.2444
Fleet size (buses)	14	10	10	3
Operating Cost (\$)	448.00	332.00	320.00	94.20
In-Vehicle Cost (\$)	723.00	949.34	431.92	73.78
Waiting Cost (\$)	1514.80	2058.20	723.75	254.03
Access Cost (\$)	465.75	407.40	383.40	107.90
Transfer Cost (\$)	948.42	1185.80	479.33	149.36
Total Cost (\$)	4100.00	4932.77	2338.40	679.27

**Table 8: Optimization Results of Flexible Bus Service**

Region	A	B	C	D
Bus Size (# of seats)	10	16	7	7
The number of zones (service areas)	13(0.9231)	12(1.0)	9(0.6667)	4(1.0)
Headway (hours)	0.3001	0.4239	0.1972	0.3373
Fleet size (buses)	26	24	18	4
Operating Cost (\$)	800.00	796.80	647.73	147.10
In-Vehicle Cost (\$)	1207.99	143797	722.00	163.96
Waiting Cost (\$)	1534.64	2040.42	745.93	283.81
Transfer Cost (\$)	960.82	1175.57	494.02	166.88
Total Cost (\$)	4535.44	5451.76	2609.68	761.74

In the base case study, we notice that conventional bus services are economical over flexible services in all regions. The total cost differences are shown in Table 5. As shown in

Table 9, conventional bus services are cheaper than conventional bus services, with 9.5~10.8 % cost differences.

**Table 9: Total Cost Variation between Conventional and Flexible Bus Services**

Region	A	B	C	D
Conventional Service Total Cost (\$)	4100.00	4932.77	2338.40	679.27
Flexible Service Total Cost	4535.44	5451.76	2609.68	761.74
Difference (%)*	9.6	9.5	10.4	10.8

\* Difference (%) is computed by (Flexible – Conventional) / Flexible

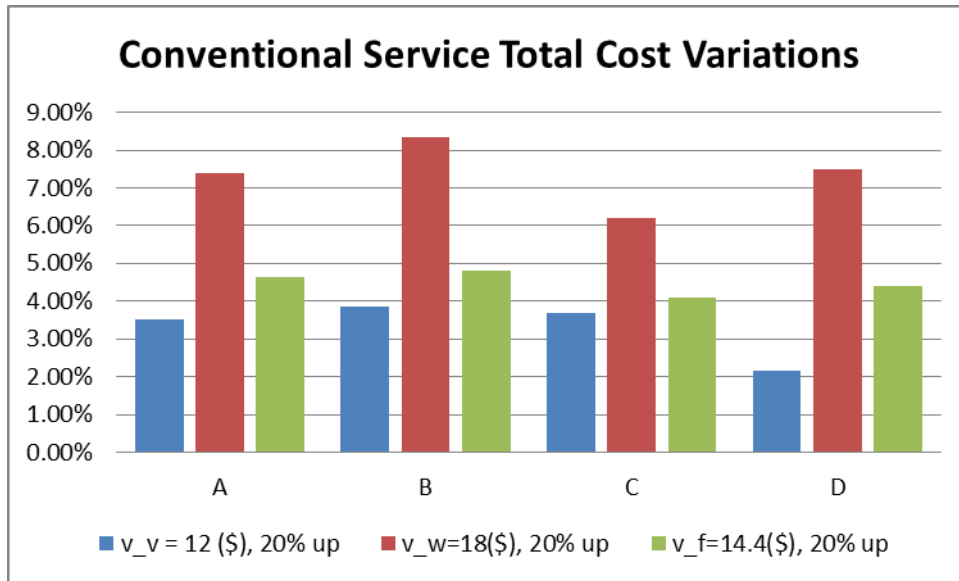
### 3.5.2 Sensitivity Analysis

In this sensitivity analysis, we consider variation of time values, namely in-vehicle time, waiting time, transfer time. Here, we increase time values by 20%, thus values of in-vehicle time, waiting time, and transfer time are 12, 18, and 14.4 dollars per hour (see Table 10).

**Table 10: Total Cost Variations with respect to Sensitivity Cases**

	Conventional Services				Flexible Services			
	A	B	C	D	A	B	C	D
Base Case (\$)	4100.0	4932.8	2338.4	679.3	4535.4	5451.8	2609.7	761.7
$v_v = 12$ (\$), 20% up	4244.6	5122.6	2424.8	694.0	4762.7	5698.1	2754.1	792.8
$v_w = 18$ (\$), 20% up	4403.0	5344.4	2483.1	730.1	4841.5	5859.8	2760.0	811.2
$v_f = 14.4$ (\$), 20% up	4289.7	5169.9	2434.3	709.1	4727.5	5686.9	2710.9	790.7

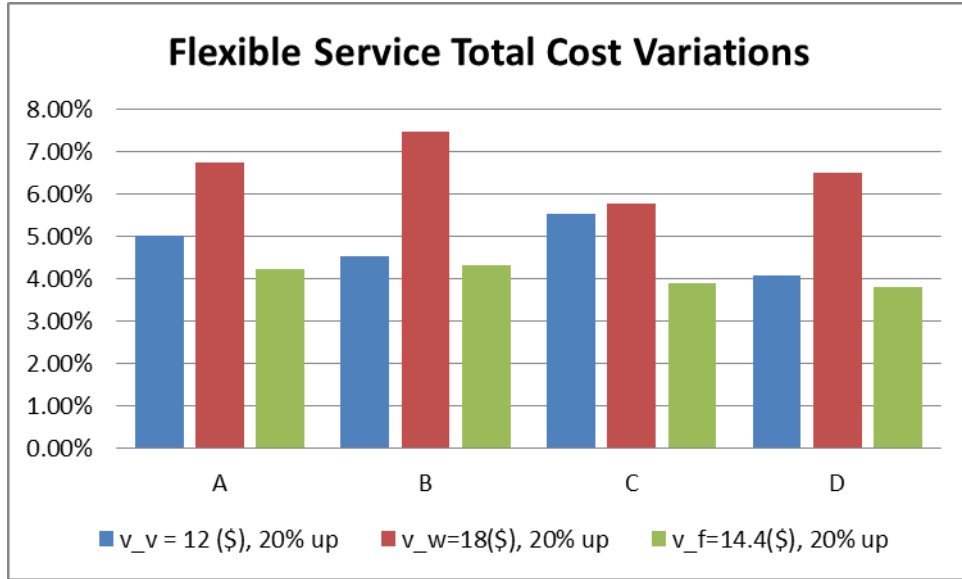
Total cost variations in conventional bus services are shown in Figure 14. By increasing the in-vehicle time value by 20 percent, total cost variations range from 2.17~3.53 % increases compared to the base case. When increasing the waiting time value by 20 percent, total costs increase by 6.19~8.34 % over the base case. Here, we notice that cost variations from waiting time values are larger than sensitivity cases of in-vehicle time and transfer time values. When we increase transfer time values by 20 percent, total costs of conventional bus services are increased around 4.1~4.8 %.



**Figure 14: Conventional Service Cost Variations**

Figure 15 shows total cost changes on flexible bus services with different sensitivity inputs. The sensitivity case of in-vehicle time value shows that total costs increase from 4.047~5.53%, which are bigger increases compared to conventional bus services, while cost increments by waiting time values are smaller than those of conventional bus services. When we increase the transfer time value from \$12 to \$14.40 per hour, the total cost of flexible bus

services increases by 3.8~4.31 percent, which are similar to the variations observed for conventional bus services.



**Figure 15: Flexible Service Cost Variations**

### 3.6 Conclusion

Bus transit services, which consider transfer costs, are formulated for a terminal connecting multiple local regions. By considering different types of services, which are conventional and flexible bus services, demand variations better fit to the type of bus services. Besides the type of bus services, sizes of conventional and flexible buses, the number of zones, headways, and resulting fleet sizes for uncoordinated bus operations are jointly optimized analytically. Probabilistically distributed travel times are considered in the modeling. Numerical examples and its sensitivity analysis are discussed. As an extension, coordinated operations are also discussed.

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## 5.0: Appendix

### A.1 Travel Time by Route Segment

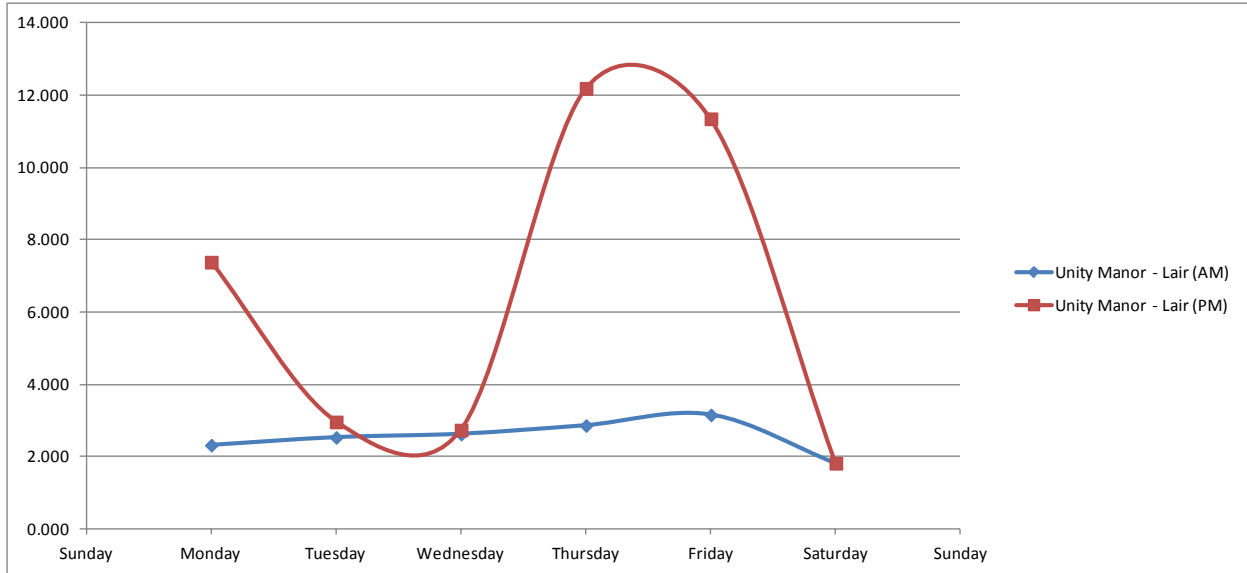


Figure A.1.1: Unity Manor - Lair

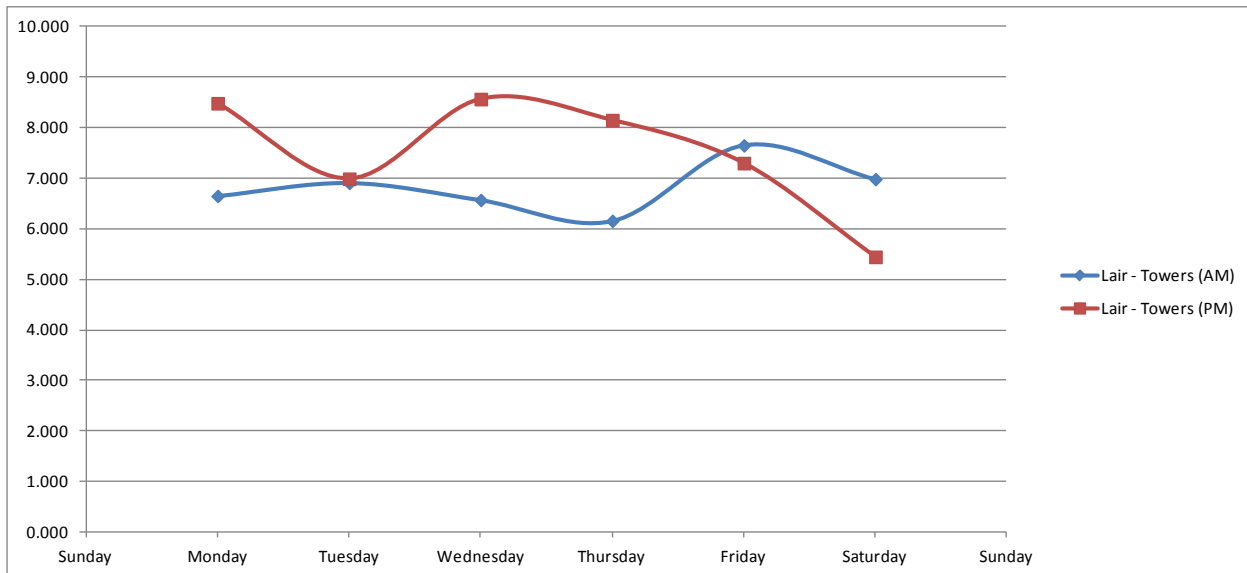
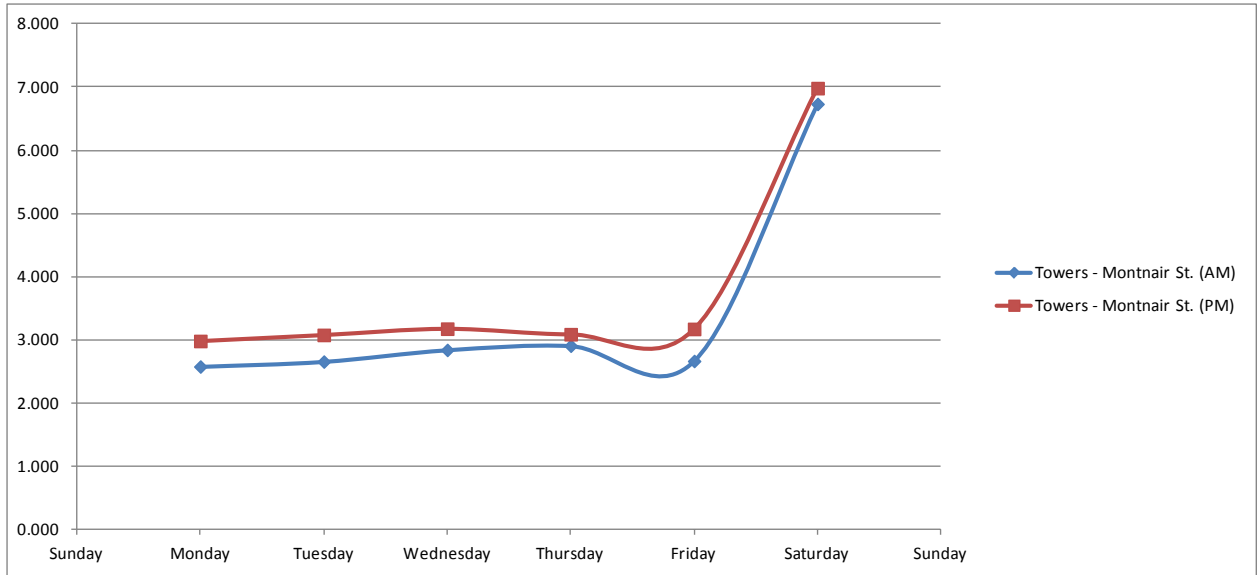
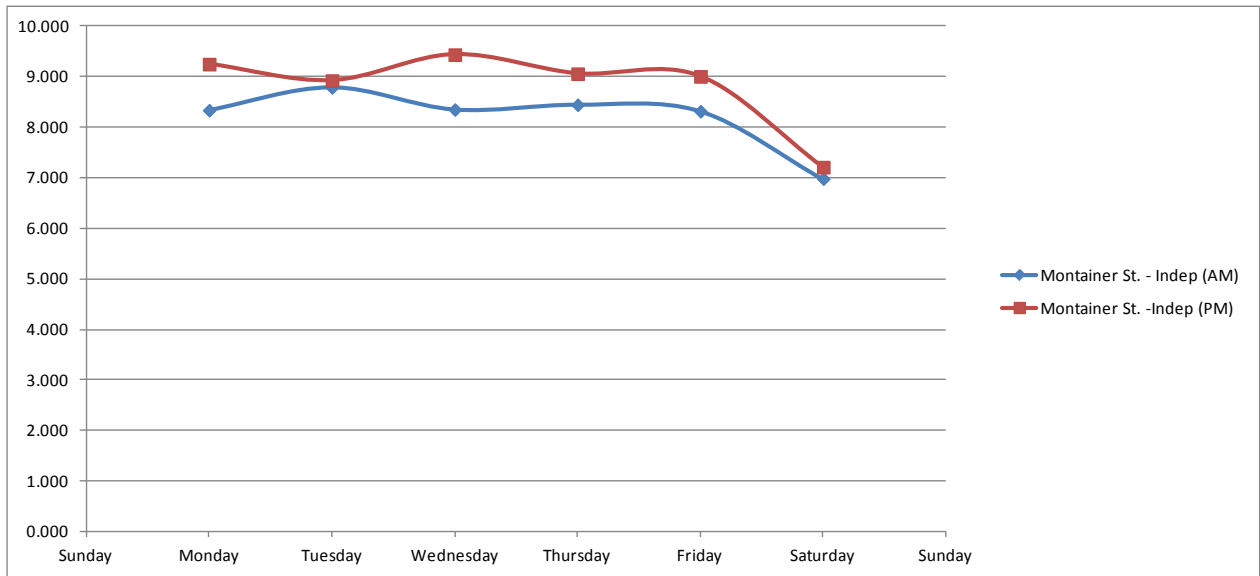


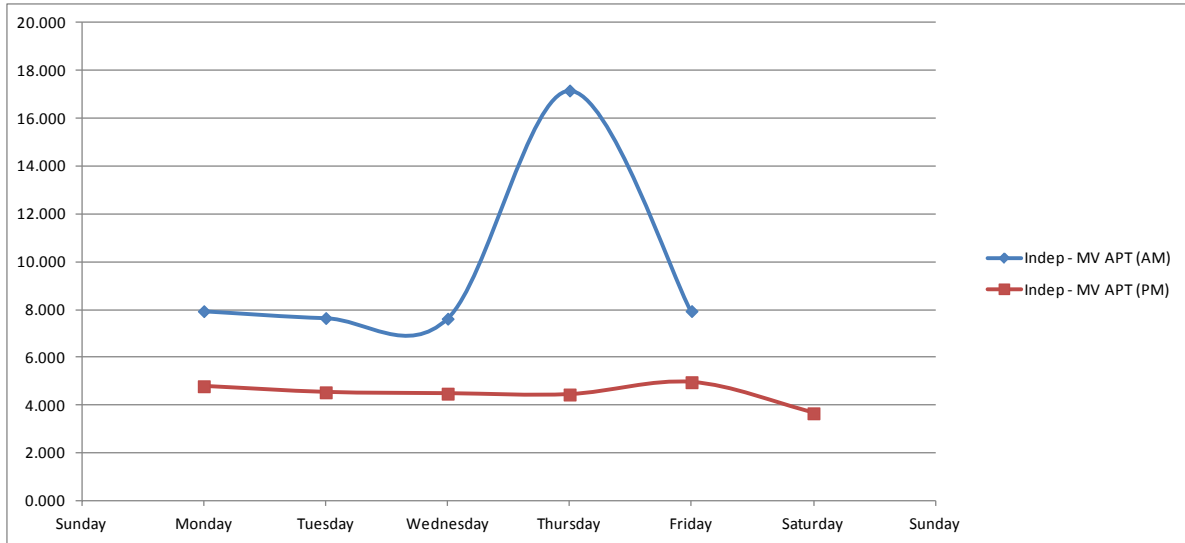
Figure A.1.2: Lair - Towers



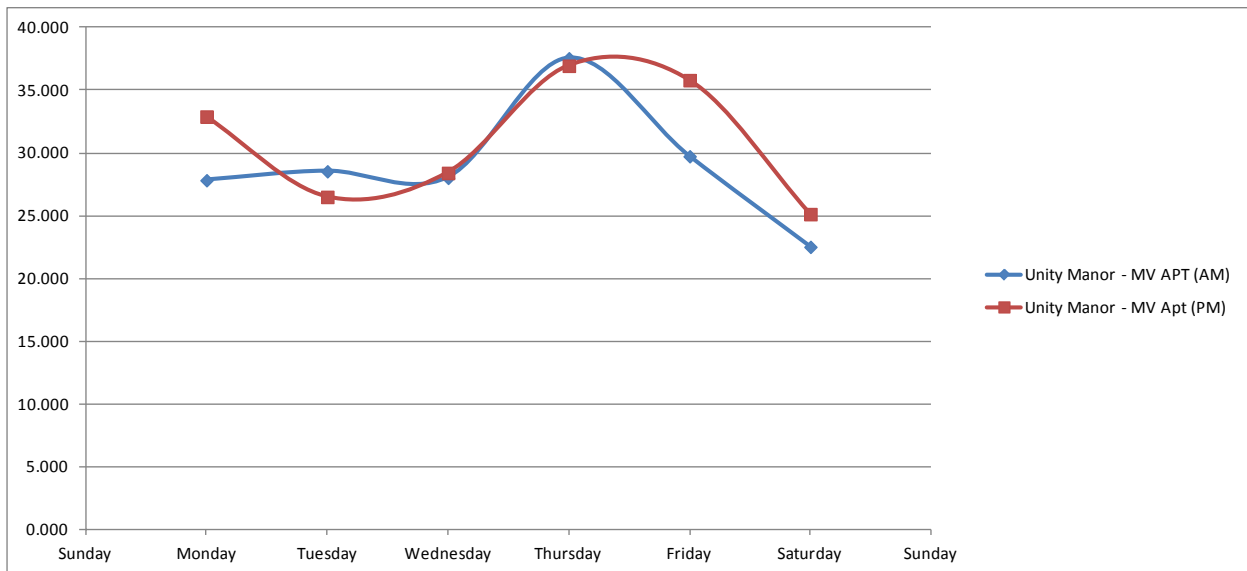
**Figure A.1.3: Towers - Mountaineer Station**



**Figure A.1.4: Mountaineer Station - Independence**



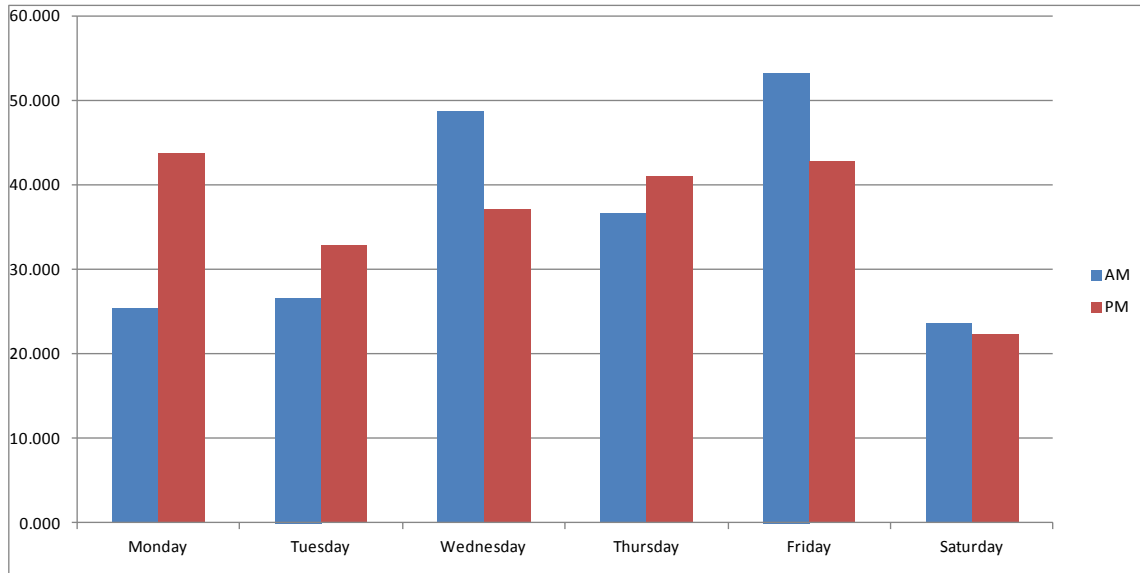
**Figure A.1.5: Independence - Mountain Valley Apt.**



**Figure A.1.6: Total Travel Time**

## A.2. WVU Spring Semester

Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	2.12	2.17	2.72	9.42	21.06	1.50
	PM	15.59	6.98	11.52	15.25	15.87	1.82
Lair - Towers	AM	5.37	7.03	28.53	8.27	13.83	8.81
	PM	12.10	9.96	9.42	9.80	10.41	5.20
Towers - Mountaineer Station	AM	2.64	3.16	2.98	3.02	2.79	6.34
	PM	3.03	3.11	3.27	2.83	3.30	7.71
Mountaineer Station - Indep	AM	7.75	7.62	7.88	8.81	7.85	6.98
	PM	8.56	8.51	8.95	8.63	8.73	7.65
Indep - MV APT	AM	7.40	6.65	6.58	7.06	7.73	
	PM	4.45	4.26	3.85	4.45	4.43	
Total Travel Time	AM	25.29	26.63	48.68	36.60	53.26	23.63
	PM	43.73	32.82	37.00	40.95	42.73	22.38



**Figure A.2: WVU Spring Semester**

### A.3 WVU Summer Season

Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	2.50	2.54	7.66	2.32	9.18	2.00
	PM	2.50	2.49	2.58	9.78	2.64	2.04
Lair - Towers	AM	6.98	6.32	5.98	5.82	6.92	4.00
	PM	6.72	6.65	7.50	6.03	5.59	4.50
Towers - Mountaineer Station	AM	2.53	2.44	2.71	3.03	2.50	6.32
	PM	2.83	3.36	3.13	3.33	2.99	6.40
Mountaineer Station - Indep	AM	8.36	8.40	8.61	7.74	8.56	6.45
	PM	8.54	8.74	10.07	8.30	8.58	6.78
Indep - MV APT	AM	19.32	12.36	20.49	20.21	12.81	
	PM	5.30	5.25	6.53	4.93	5.97	3.67
Total Travel Time	AM	39.69	32.05	45.45	39.11	39.97	18.77
	PM	25.88	26.48	29.80	32.37	25.77	23.39

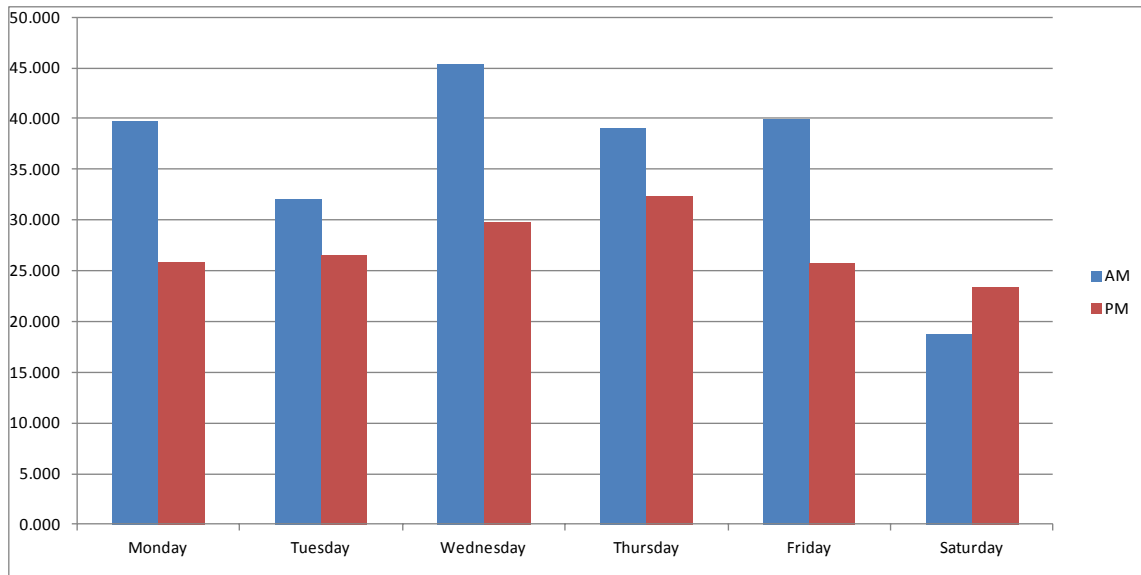


Figure A.3: WVU Summer Semester

### A.4 WVU Fall Season

Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	2.67	20.08	2.65	10.98	2.15	2.15
	PM	15.75	21.27	23.53	9.44	3.70	2.12
Lair - Towers	AM	6.82	6.91	5.32	15.56	7.82	6.23
	PM	6.98	6.67		14.80	20.94	5.98
Towers - Mountaineer Station	AM	2.78	2.13	2.29	2.83	2.83	7.08
	PM	2.98	2.74	2.99	3.11	3.53	7.48
Mountaineer Station - Indep	AM	8.21	9.91	9.39	8.19	8.64	7.23
	PM	11.39	10.35	10.55	11.36	12.13	7.21
Indep - MV APT	AM						
	PM						
Total Travel Time	AM	20.47	39.03	19.65	37.57	21.44	22.69
	PM	37.11	41.03	37.07	38.70	40.31	22.79

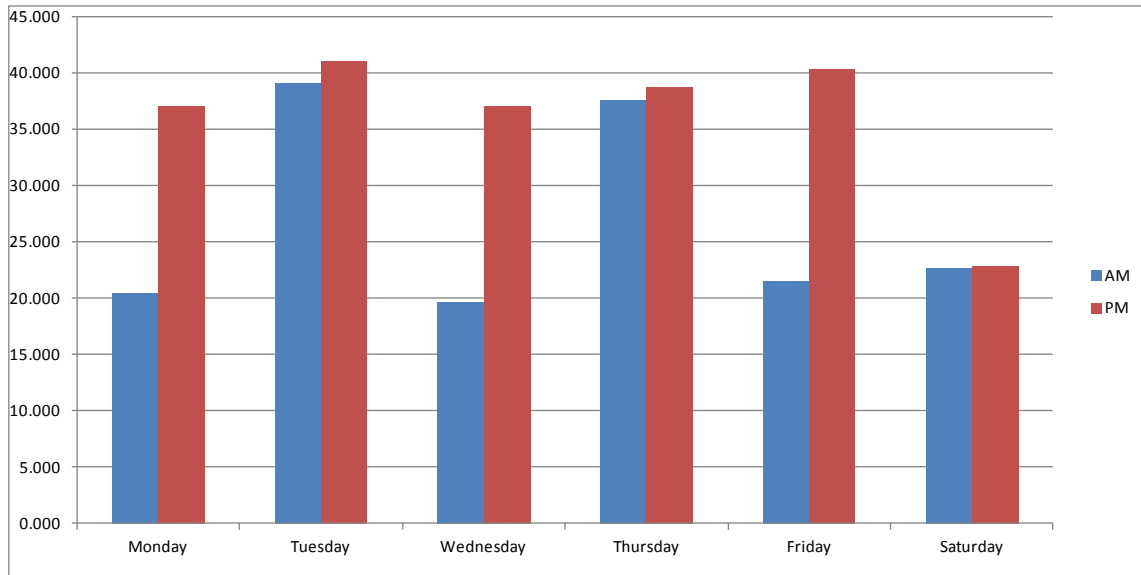
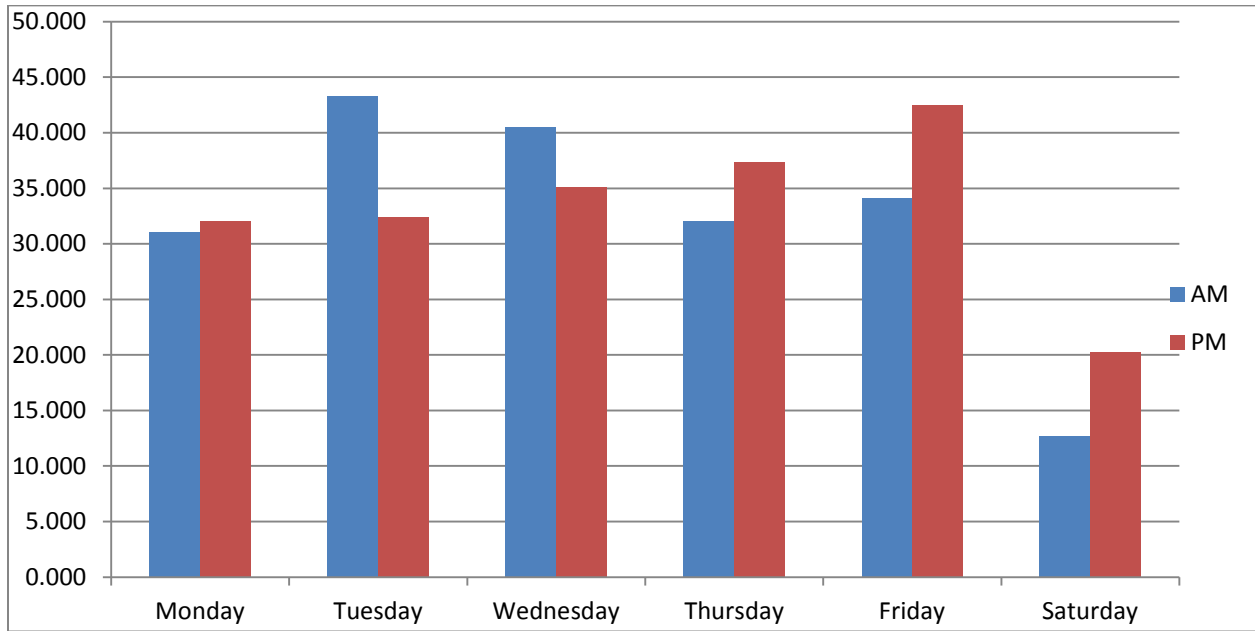


Figure A.4.: WVU Fall Semester

### A.5 Spring Break Week

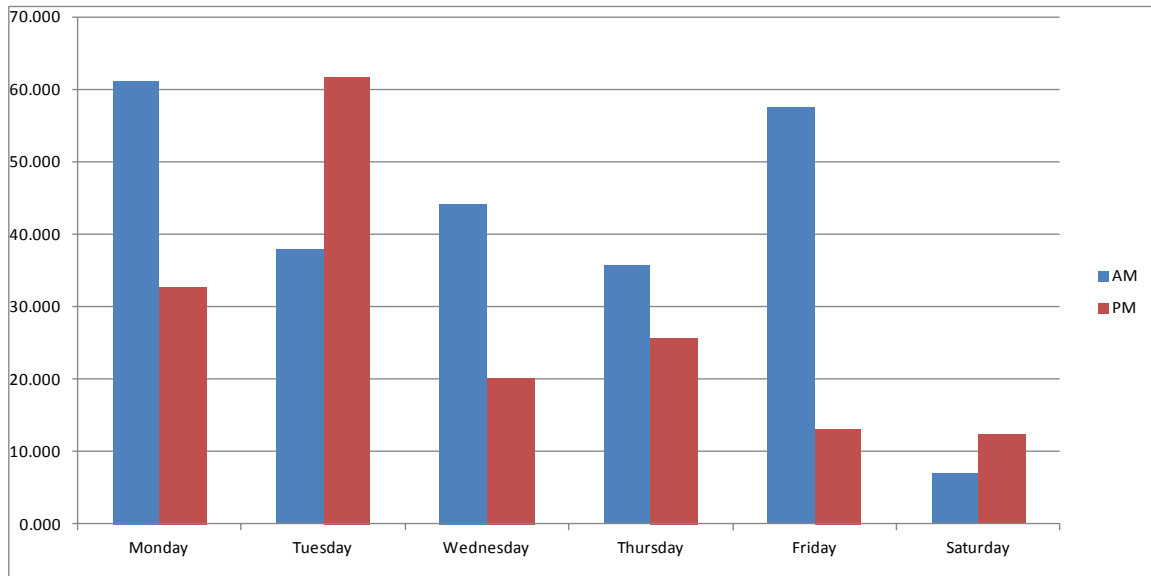
Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	1.98	1.87	1.69	2.87	1.66	1.67
	PM	1.92	9.34	11.85	9.03	18.62	
Lair - Towers	AM		9.71	11.20		5.24	
	PM	11.56	6.65	5.08	10.48	4.95	4.00
Towers - Mountaineer Station	AM	2.27	5.93	3.48	2.78	2.55	5.07
	PM	3.26	2.97	3.13	3.30	4.03	9.42
Mountaineer Station - Indep	AM	8.62	8.15	8.00	9.18	6.91	5.91
	PM	8.17	8.18	7.67	8.75	7.43	6.82
Indep - MV APT	AM	18.15	17.65	16.13	17.16	17.71	
	PM	7.15	5.24	7.38	5.78	7.44	
Total Travel Time	AM	31.02	43.31	40.50	31.99	34.06	12.64
	PM	32.06	32.38	35.11	37.34	42.46	20.23



**Figure A.5: Travel Time over Spring Break**

### A.6. WVU - Final Week

Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	2.00	2.05	15.96	20.73	36.43	
	PM		35.83		14.74		
Lair - Towers	AM	5.37	7.03			13.83	
	PM	17.08	9.96				
Towers - Mountaineer Station	AM	24.02	2.79				
	PM	2.27	2.19	7.17	2.87		
Mountaineer Station - Indep	AM	8.61	7.62	21.55	8.78		6.98
	PM	8.18	8.54	8.51	8.01	9.05	12.32
Indep - MV APT	AM	21.06	18.32	6.57	6.07	7.14	
	PM	5.15	5.15	4.41		4.05	
Total Travel Time	AM	61.06	37.81	44.08	35.58	57.41	6.98
	PM	32.67	61.68	20.09	25.61	13.09	12.32

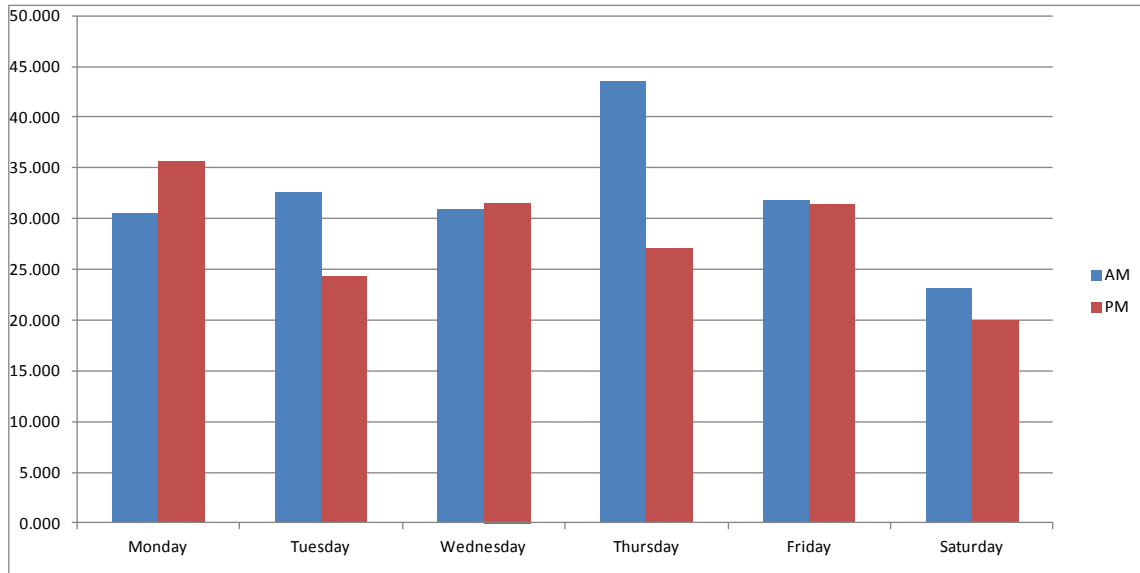


**Figure A.6: Final Week Travel Time**



### A.7. Weekday After Finals

Road Segments	Time of Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Unity Manor - Lair	AM	2.01	1.97	2.53	2.42	2.56	1.67
	PM	7.39	2.04	6.94	1.88	1.86	1.50
Lair - Towers	AM	5.33	6.35	5.58	10.47	4.41	6.48
	PM	11.76	6.88	7.62	7.92	12.33	3.32
Towers - Mountaineer Station	AM	2.58	2.76	2.28	3.28	4.44	7.07
	PM	3.12	2.87	2.57	2.63	2.80	7.15
Mountaineer Station - Indep	AM	8.57	8.53	7.62	7.77	7.00	7.90
	PM	8.09	7.79	8.08	8.39	9.00	8.01
Indep - MV APT	AM	12.01	12.96	12.86	19.64	13.46	
	PM	5.24	4.74	6.28	6.28	5.44	
Total Travel Time	AM	30.50	32.57	30.86	43.58	31.86	23.12
	PM	35.61	24.31	31.49	27.09	31.42	19.98



**Figure A.7: After Final Week Travel Time**