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# Evaluating the Use of Transfers for Improving Demand Responsive Systems Adopting Zoning Strategies

## Final Report


**Luca Quadrifoglio, Chung-Wei Shen and Suzie Edrington**

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16. Abstract

Due to widely dispersed population density over large and sparsely suburban/rural areas, conventional fixed route transit services hardly satisfy the travel needs of their residents. As an alternative, demand responsive transit (DRT) systems have flexible routes and schedules that can provide curb-to-curb/door-to-door services to better meet the needs of rural areas. However, rural DRT services are still extremely costly to operate. In this project researchers consider a variation of the regular demand responsive transit system that adopts the transfer practice to reduce operating costs. This practice has been adopted by some agencies with zoning rules for the whole service area or trips that need to cross jurisdictional districts; however, the pros and cons need to be carefully assessed. Researchers evaluated the effect of different transfer operating policies by developing a simulation model of several plausible scenarios. Available data from Houston METRO were used for testing and modeling purpose. The results indicate that zoning with transfer can provide significant benefits to paratransit operations that are managing zoning structure. However, no-zoning strategies, such as the one currently used by Houston METRO performs better overall on average in terms of efficiency. This study provides decision makers and DRT agencies with information for innovative operating practices to improve the performance and cost efficiency of public transportation systems.

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by
Luca Quadrifoglio, Ph.D.
Zachry Department of Civil Engineering
Texas A&M University

Chung-Wei Shen
Zachry Department of Civil Engineering
Texas A&M University

Suzie Edrington
Texas Transportation Institute
Houston, Texas

Final Report UTCM 10-60-59

University Transportation Center for Mobility™
Texas Transportation Institute
The Texas A&M University System
3135 TAMU
College Station, TX 77843-3135

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# EXECUTIVE SUMMARY

This research investigates the different organizational structures of paratransit services that cover large regions. An ADA paratransit service is demand responsive shared-ride transit service using vans or small buses. It is characterized by the use of vehicles that do not operate on a fixed route or a fixed schedule. The paratransit route and schedule are arranged from a user-specified origin to a user-specified destination, and at a user-specified time.

To retain the productivity by focusing on shorter trips within a denser area, some larger systems have outsourced operations to more than one contractor, with each contractor responsible for the service zone to which their vehicles have been assigned. This service design is called a “zonal structure” or “zoning approach.”

The zoning with transfer systems coordinates vehicles’ schedules at the various transfer locations. The schedule coordination of inter-zonal mechanisms of transportation likely reduces trip costs by increasing the ridesharing rate and lowering the number of empty return miles *(1)*. The proper coordination of paratransit services would not only increase efficiency and productivity, but also mobility. Although the operational consolidation of providers appears to achieve economies of scale, the following may impede their coordination: (a) a user may have some concern that the current service levels will decrease; (b) the sponsoring agency may have doubts regarding whether there is a significant cost savings; and (c) the different jurisdictions within which component transportation systems operate may have different operational standards designed particularly to meet the local riders’ needs *(2)*.

Developers of zoning strategies, however, need to decide how to accommodate those trips that cross zones. According to the dealing of inter-zonal trips, the zonal approach can be divided into two variations: (a) zoning without transfer, such as with the service provided in Los Angeles County, and (b) zoning with transfer, such as with the Chicago ADA paratransit service. In zoning without transfer, inter-zonal customers may not need to switch vehicles during their trips. Alternatively, zoning with transfer systems may require inter-zonal customers to switch vehicles.

In the first part of the research, researchers have presented the exact formulation of the coordinated decentralized paratransit system to compare its productivity and service quality with independent decentralized and centralized strategies. The formulation has been proven to work correctly, and the results of the computational experiments of small scale instances demonstrate that the proposed coordinated system is superior to independent decentralized system in terms of passenger miles per vehicle revenue mile. The exact solution approach of the proposed formulation is obviously constrained by problem scale, running time, and computer memory, given its combinatorial nature. However, results in terms of optimal solutions satisfactorily showed a performance comparison between different strategies and the benefits of transfer design.

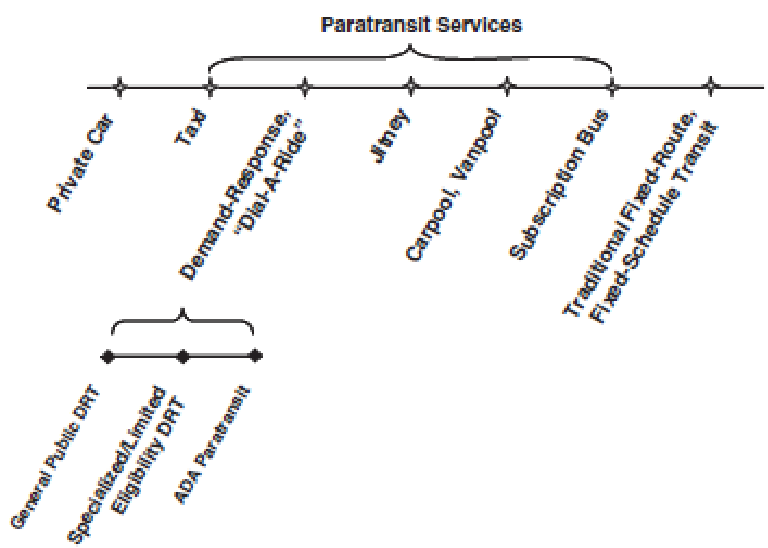
Since the exact method has been limited to relatively small problems, it inspires the second part of the research—develop a heuristic algorithm that is computational practical to solve the larger scale.

In the second part of the report, researchers developed an insertion-based heuristic method in order to compare the performance of different operational designs by a large scale system. In an experiment utilizing Houston's demand responsive service data, we compared the productivity and service levels among three organizational structures: zoning with transfer, zoning without transfer, and a no-zoning design. The zoning without transfer structure also divides its service area into sub-zones, each zone with its own vehicle depot. The zonal service provider can only pick up customers whose pick-up location is within the service area; however, the provider is allowed to drop off customers outside of that area. Each provider is unaware of the state of the system in other zones. Alternatively, the no-zoning control system is a totally centralized system, which is the basic scenario describing paratransit service in general.

The results indicate that zoning with transfer can provide significant benefits to paratransit operations that are managing zoning structure. Our results used the demand data of the paratransit system in Houston, Texas (a relatively low-density region), and we concluded that the zoning with transfer method proved to be a productive organizational structure. Although the exact level of benefit will vary according to the different demand types and different operational standards, this simulation methodology is easily and quickly adaptable to any large-scale paratransit system. Future work should include combining the search of optimal transfer locations or the number of transfer locations to improve the performance of our proposed transfer system.

# CHAPTER 1. INTRODUCTION

ADA Paratransit systems are a demand responsive (DRT) type of service offering disabled customers a better service than fixed route transit systems because they provide curb-to-curb/door-to-door service and flexible schedules (Figure 1)*(3)*. The scheduling/routing of paratransit systems is commonly known as the dial-a-ride problem (DARP). Each passenger is transported by a ridesharing vehicle from a specific origin to a specific destination, at a desired departure or arrival time. DARP is a subclass of vehicle routing problems with pickups and deliveries (VRPPD), which can transport goods or persons. The volume of passengers using paratransit services have tremendously increased from the Americans with Disabilities Act (ADA) in 1990, which requires transit agency to provide disabled customers with a level of service equal to the one offered to regular ones (Figure 2). Paratransit services, however, are extremely costly to operate despite the ridesharing characteristic (Figure 3). In 2008, paratransit ridership made up only 1.8 percent of public transit ridership but 13.3 percent of the total operating cost in the United States *(4)*. The productivity respect to the passenger trips per revenue hour is steadily decreasing (Figure 4).




*From: "Guidebook for Measuring, Assessing, and Improving Performance of Demand-Response Transportation." KFH Group, Transportation Research Board, 2008.*

**Figure 1 ADA paratransit is a type of paratransit services**

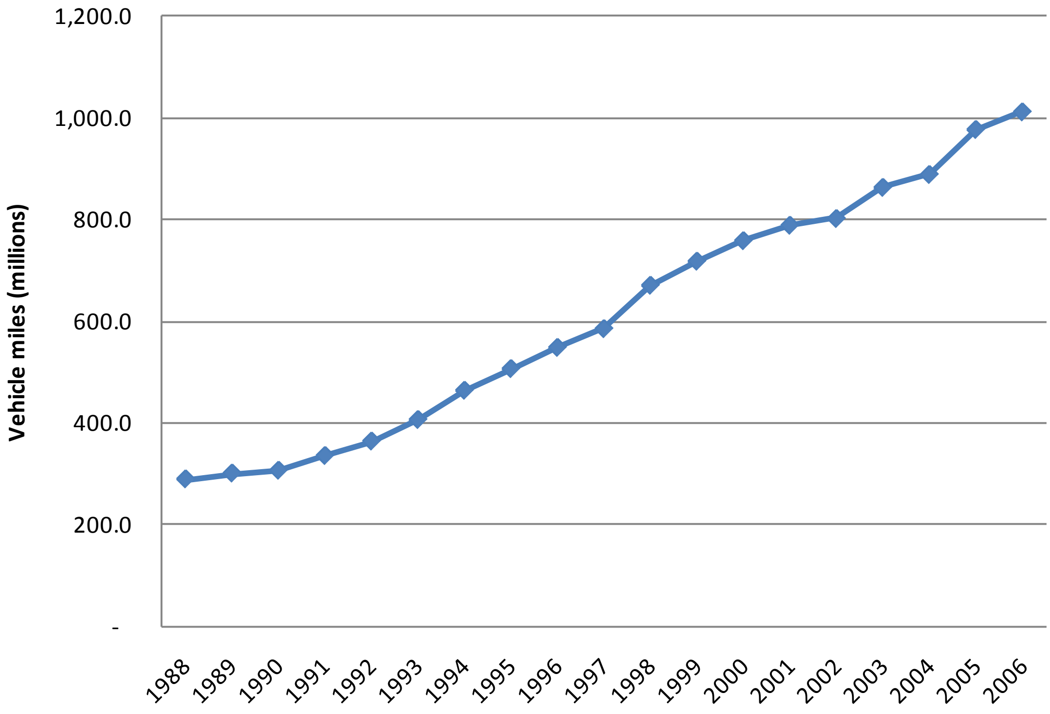



**Figure 2 Vehicle total miles for paratransit system**

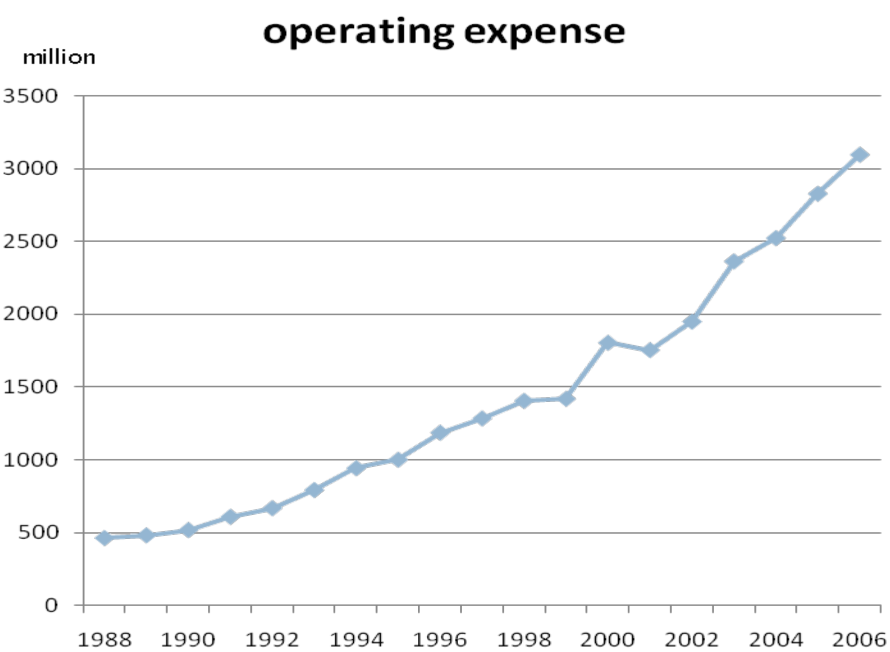



**Figure 3 Operating expense for paratransit system**

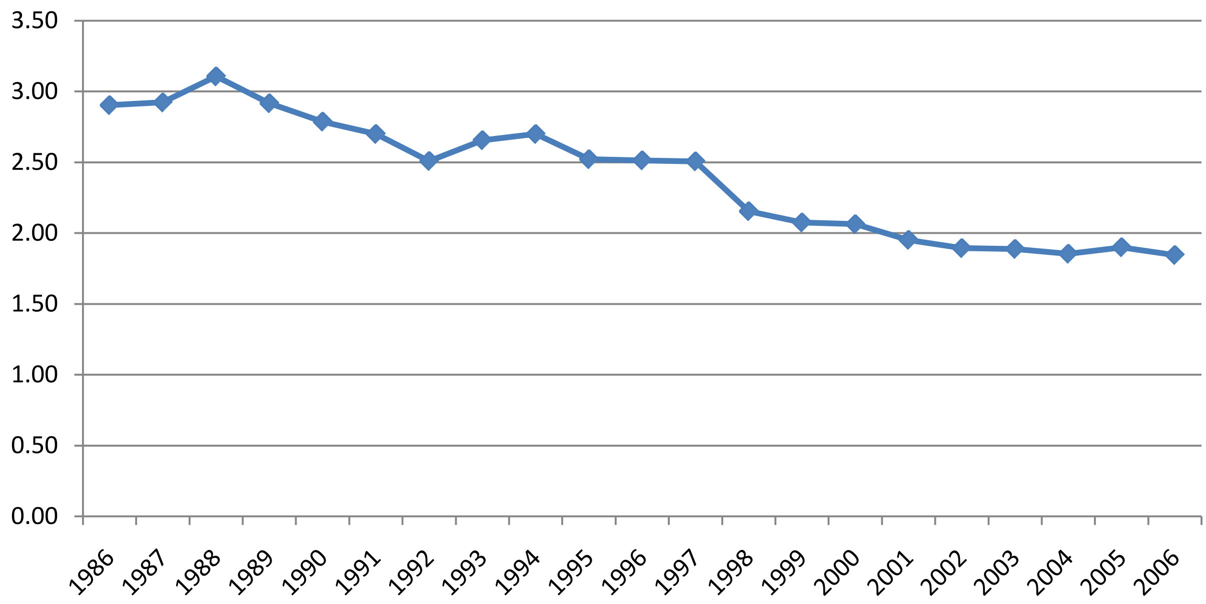



**Figure 4 Passenger trips per vehicle total hour**

For paratransit systems, a decentralized strategy is a more practical method of operating the service due to their ever-sprawling and ever-expanding service areas. Utilizing a decentralized strategy, service providers independently operate within their designated zones and can only cross into other regions to drop off their inter-zonal customers. For example, Metropolitan Transit System in San Diego employs a decentralized strategy and divides its service area into four regions (see Figure 5). In contrast, a centralized strategy considers a whole single region served by one designated provider (in Houston, for instance). A decentralized strategy better fits locations where there is more than one regional center; a centralized strategy better fits places where there is one compact center. Because of increasing urban sprawl, a decentralized strategy becomes more popular even if there is only one regional center.

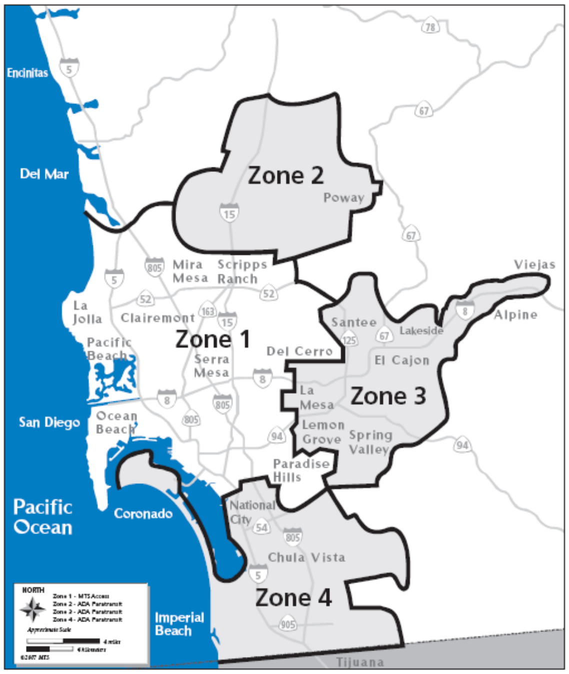



**Figure 5 Service regions in the San Diego**

Utilizing a decentralized strategy, however, will likely reduce the productivity of the system; in fact, additional geographical constraints (zone boundaries) are added to the system and the scheduling solution intuitively cannot be improved, as the set of feasible routing solutions is reduced. This is because the total vehicles empty backhaul miles (defined as the miles driven by a vehicle with no customers onboard, excluding the first/last trip segments to/from the depot) increase compared to the centralized strategy. Quadrifoglio et al. *(5)* found the operating choices of a decentralized strategy to have a significant impact on the performance of demand responsive transit services. Utilizing paratransit data in Houston, Shen et al. *(6)* showed that adopting at decentralized strategy increases the total vehicles used and empty backhaul miles driven against the centralized strategy.

A coordinated decentralized system makes use of transfer points and is the operating practice adopted by some agencies (see paratransit in Chicago and the Twin Cities and rural agencies around Dallas, for example) requiring customers to switch vehicles at particular locations to complete their inter-zonal trips. This practice is attracting more and more attention from transit providers because of its perceived potential to significantly reduce operating costs mainly by reducing empty backhaul miles and by increasing rideshare rates. However, it requires synchronization between operators and increases customers' discomfort, which might be quite undesirable for elderly or disabled riders (but certainly more tolerable for healthy customers, thus increasing the potential benefit of adopting this operating practice for regular demand responsive service within service areas). Coordination in this research refers to switching inter-zonal passengers through the arrangement of the vehicle routes and schedules at specific transfer locations.

There is a need to quantify the trade-offs between the pros and cons when adopting the coordination strategy as opposed to not adopting it. As reviewed in the next section, coordination or integration between demand responsive transit systems and fixed-route transit systems has been investigated in the literature; however, the study of the coordination among independent decentralized paratransit systems, especially for exact formulation and solutions approaches, is regarded with little attention.

The rest of the contents are organized into two major sections. In the first part, researchers focused on the method to: (a) provide an innovative formulation of a generalization of the classical static DARP adding the flexibility of considering the transfer option for customers; and (b) quantify the productivity and service quality of decentralized paratransit system with transfers as opposed to alternative strategies used in practice.

Chapter 2 reviews the relevant literature on DARP. Chapter 3 introduces a description of the problem and key assumptions regarding it. Chapter 4 introduces the model formulation. Chapter 5 describes the performance analysis of simulated instances. Chapter 6 ends with summary on the formulation.

In the second section, we used a heuristic-based simulation as a study method in order to better understand the effects of zoning and zoning with transfers on paratransit services in large scale. Chapter 7 explains motivations and the background. Chapter 8 reviews the transfer system. Chapter 9 details the computational experiment. Chapter 10 presents the performance and results analysis and Chapter 11 contains the conclusion.

## CHAPTER 2. LITERATURE REVIEW

The scheduling and routing of classic paratransit systems is known as the Dial-A-Ride Problem (DARP), in the common terminology used for the study of Vehicle Routing Problems (VRP); the DARP without ride time constraints is denoted by the term Pickup and Delivery Problem (PDP). After Wilson and Sussman *(7)* first introduced the real-time algorithms for DARP, this problem has been studied considerably over the past few decades and has been mainly focused on developing an optimization algorithm because of its combinatorial characteristic.

The most recent surveys on DARP and PDP were presented by Cordeau and Laporte *(8)*, and Berbeglia *(9)*, respectively. The paratransit services using a transfer system is a generalization of the DARP. The transfer of passengers will always require more than one vehicle to fulfill a trip; therefore, the spatial and temporal synchronization constraints will, by necessity, be imposed on more than one vehicle. A schedule delay in one vehicle route may necessitate a change in all other routes. Therefore, it is computationally difficult even when simply trying to develop a heuristic algorithm. Shang and Cuff *(10)* provide a concurrent heuristic approach to solve the PDP with transfer issue, using as an example a Health Maintenance Organization. They show that their proposed heuristic performs better than the HMO’s scheduling heuristic, according to the overall lower number of delays, total travel hours, and total number of vehicles. However, this paper considered neither excess passenger travel times nor vehicle capacity constraints. Cortes et al. *(11)* studied a PDP with transfers through the process of Mixed Integer Programming (MIP). They found that the transfers permitted a higher level of efficiency, over the total vehicle travel time. Due to the complexity of the problem, this solution can only handle very small instances, which are maximized at six customers. They suggested further developments of the transfer application on the strategic design and planning of paratransit system. Unlike DARP, PDP does not have travel times constraints to reduce customer dissatisfaction.

In comparison, the performance evaluation of practical operation strategies such as decentralized strategy on DARP has received meager attention. McKnight and Pagano *(12)* explored the service quality of DARP by investigating 42 service providers in the United States. It was found that the quality of special transportation services for elderly and disable persons tends to increase as the ridership of the provider increases. Wilson and Hendrickson *(13)* summarized the earlier models that predicted the performance of flexible routed transportation system. Paquette et al. *(14)* concluded that further study is needed for better understanding the trade-offs among costs and quality of different operational policies in dial-a-ride systems.

Coordination of paratransit services increases not only efficiency and productivity but mobility. From the evaluation of Burkhardt *(15)*, around $700 million per year to transportation providers in the United States could be generated after implementing successful coordination. Consolidation of inter-zonal transportation will likely reduce trip costs by higher ridesharing rate and lower empty return miles *(1)*. Malucelli et al. *(16)* presented a flexible collective transportation system. They suggested a future study should deal with allowing the passenger to transfer from one vehicle to another. Häll et al. *(17)* introduced the integrated DARP, where some part of journey may be carried out by a fixed route service. Aldaihani and Dessouky *(18)* proposed a system that integrates fixed routes within a pickup and delivery problem (PDP). An integer programming formulation of cooperative PDP with time windows was analyzed by Lin *(19)*. It concluded that the cooperative strategy may achieve savings in both total cost and vehicles used, under the assumptions of all delivery locations are identical, the transfer is only allowed at the last pickup location of the returning vehicle, and the vehicle capacity is unlimited.

The size of the service area is one of the key factors that affect the productivity of Demand Responsive Transit (DRT). In general, the larger the service area, the longer the trip length, and thus DRT will not always be able to consistently serve a given number of passengers in a specified amount of time *(3)*. The impact on the productivity of the different area sizes was first studied by Wilson et al. *(20)*. They demonstrated that the number of vehicles used is proportional to the size of the service area. Chira-Chavala and Venter *(21)* adopt the data provided by the Outreach Paratransit Service in Santa Clara County, California, and observe that longer trip lengths contribute to an increase in empty trip miles in an expanding service area.

In addition, a large area usually comes with more dispersed trips. Large service areas with dispersed trip patterns, which translate to a lower demand density, make it difficult to achieve the most beneficial of effects of ride-sharing. On average, larger service areas mean more dispersed origin and destination points than those enjoyed by more compact service areas. In low-density areas, DRT systems have a lower productivity level than those systems that function in a municipal area *(22)*.

Quadrifoglio et al. *(5)* performed a simulation study to test the productivity of zoning without transfer, comparing the performance of that strategy with a centralized, no-zoning case based on data obtained from Los Angeles. To retain the productivity by focusing on shorter trips within a denser area, some larger systems have outsourced operations to more than one contractor, with each contractor responsible for the service zone to which their vehicles have been assigned. This service design is called a zonal structure or zoning approach. Adjacent zones would have no overlapping or have shared buffer areas. The zoning approach is attractive not only because it creates more manageable pieces of work, but more importantly because it establishes an ongoing spirit of competition throughout the contract term *(2)*. Zonal demand responsive service is also used for dispatching, as well as for fare determination purposes *(23)*.

Based on the above review, the effect due to the addition of transfers between independent decentralized systems has not been studied. In the following, the first part provides a formulation and performance comparisons with alternative strategies. The second part investigates the different organizational structures of paratransit services that cover large regions.

# SECTION 1: MODEL FORMULATION

## CHAPTER 3. PROBLEM DESCRIPTION AND KEY ASSUMPTIONS FORMULATION

In this section, researchers formally introduce the problem and key assumptions of the coordinated decentralized paratransit system, adopted from actual operating paratransit services. These assumptions identify the problem scope and provide the basis for the following formulation.

Within the service area, a number of requests are given; each request has a specific pick-up location and a drop-off location, as well as a time window specified for both of these locations. According to the pick-up and drop-off locations, requests will be categorized into two sets: inter-zonal requests and intra-zonal requests. The requests, whose pick-up and drop-off locations respectively belong to different zones, are inter-zonal requests; the requests, whose pick-up and drop-off locations belong to the same zone, are intra-zonal requests. Figure 6 illustrates the coordinated decentralized paratransit system. Two zones are generated by boundaries; transfer points are located on the boundaries between contiguous zones. Vehicles with limited capacity return to the depots where they start out. Under the restriction of decentralized strategy considering transfers, vehicles can only travel within their designated service zone. Vehicles picking up inter-zonal passengers will need to stop at a transfer location to drop them off. Vehicle dropping off inter-zonal passengers will need to stop at a transfer location to pick them up.

At every pick-up or drop-off location, researchers identify only one operation, which is either loading or unloading of passengers. At transfer nodes, vehicles may either load or unload passengers or both. To capture the difference between these operations, researchers generate two corresponding nodes (load node and unload node) at each transfer location for each transfer request.

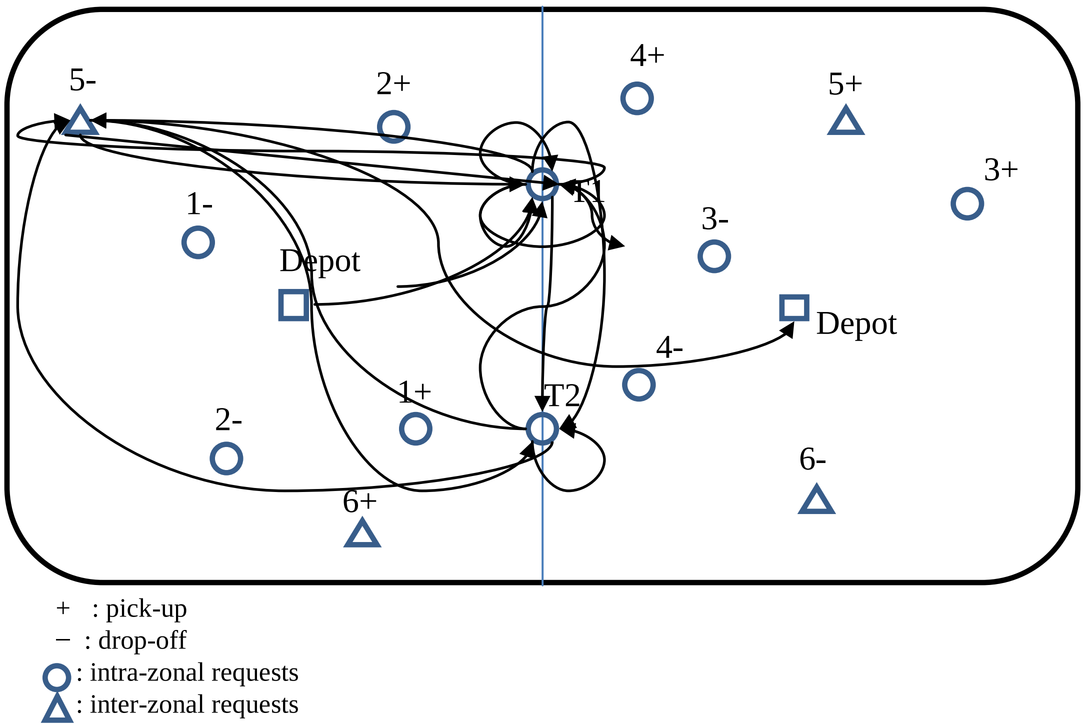



**Figure 6 Example of coordinated decentralized system**

When a vehicle visits the transfer node, it will either load or unload passengers according to the node's characteristic. Figure 7 shows how vehicles enter and leave the transfer location. A pair of pick-up and drop-off nodes at each transfer location will be generated.

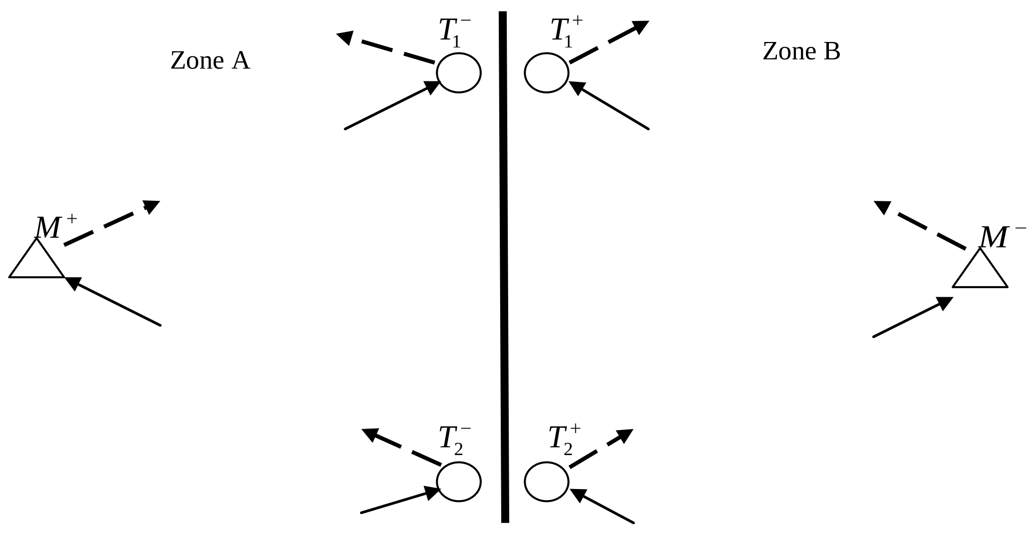



**Figure 7 Transfer mechanism representation**

Manhattan distances are used to calculate the symmetrical travel distance between any pairs of point. This estimated travel distance was verified to be close to the actual travel distance by Quadrifoglio et al. *(5)*. For example, A $(x_1, y_1)$ and B $(x_2, y_2)$ represent either the pick-up or drop-off point, respectively. The travel distance can be calculated as $|x_1 - x_2| + |y_1 - y_2|$. This calculation implies that the network is arranged in a grid pattern. Researchers also assume no traffic jams on the system, and the travel time between two points is only a matter of travel distance and vehicle speed. This assumption may not allow considering precise travel time between points, but they do not alter the results of our following performance comparison. The link distances/speeds are input of the model and can be easily updated with more accurate values when available.

Researchers assume that each inter-zonal request can only switch vehicles at a particular transfer location once, and each intra-zonal request does not switch vehicles to complete its trip. Concerning the customers' discomfort, more than one transfer might be quite undesirable other than quite unreasonable *(24)*. In fact, in practice, the passengers of paratransit are assured to transfer at most once in Chicago and Boston (both using a coordinated decentralized system).

All requests are known in advance, which means our problem is in static mode. This is also quite reasonable, as nearly 90 percent of the paratransit customers book their ride at least a day in advance allowing for static scheduling of the service (generally performed during the night before operations). This research focus on the productivity and service quality of transfers design on decentralized strategy rather than investigates the feasibility to accommodate all unexpected events such as the absence of customers, breakdowns of vehicles, cancellations of requests, and so on. The last assumption is that vehicles are allowed to wait at a pick-up node when they arrive before the earliest pick-up time even with passengers onboard.

## CHAPTER 4. MODEL FORMULATION

This problem is formulated as a mixed integer programming (MIP) problem. For the clarity of index and notation, here we assume only two zones, and otherwise the numbers of intra-zonal and inter-zonal requests are the same between two zones in the following context. The formulation is applicable to multiple zones' case by adding corresponding sets under the assumption that exact one transfer is allowed for every inter-zonal customer. Every request consists of pick-up/drop-off locations and the corresponding time windows. Within each zone, there are two types of requests shown below:

- $Z = \{1, \cdots, z\}$: Set of zones.
- $N_z = \{1, \cdots, n_z\}$: Set of intra-zonal requests in zone $z$.
- $M_z = \{1, \cdots, m_z\}$: Set of inter-zonal requests whose pick-up nodes are in zone $z$.

The node sets within each zone include:

- $N_z^+$: Set of pick-up nodes for requests in $N_z$.
- $N_z^-$: Set of drop-off nodes for requests in $N_z$.

- $M_z^+$: Set of pick-up nodes for requests in $M_z$.
- $M_z^-$: Set of drop-off nodes for requests in $M_z$.
- $V(i)$: Set of nodes that within the same zone as node $i$.
- $T = \{1, \cdots, t\}$: Set of transfer locations for $M_z$.
- $S_z$: Set of all nodes within zone $z$.

For each request $r$ belonging to $M_z$, the set of drop-off nodes will be generated at each transfer location in the same zone with node $i \in M_z^+$; for each request $r$ belongs to $M_z$, the set of pick-up nodes will be generated at each transfer location in the same zone with node $i \in M_z^-$. The corresponding notations are:

- $Z(i)$: Zone of node $i$.
- $d(r)$: Set of generated drop-off nodes at transfer location for request $r \in M_z$.
- $p(r)$: Set of generated pick-up nodes at transfer location for request $r \in M_z$.
- $t(r)$: Set of generated paired pick-up and drop-off nodes at transfer locations for request $r \in M_z$.

Let $K_z$ be the set of vehicles in zone $z$. Every vehicle leaves from and arrives to the same depot and has a capacity $Q_k$, and each node $i \in S_z$ is associated with a load $q_i$. Every arc $(i, j)$ is associated to a routing cost $c_{ij}^k$ and a travel time $t_{ij}^k$ for vehicle $k$. For each node $i \in S_z$, $e_i$ and $l_i$ represent the earliest and latest time at which service may begin, and $d_i$ is the service duration at node $i$. $F$ denotes the fix cost of assigning each extra vehicle. $W$ represents the maximum passenger waiting times at transfer location. $G$ is a sufficiently large constant (usually noted as '*M*', which would, however, conflict with other notation in this research). Three decision variables are introduced in our formulation. First, the binary variable $x_{ij}^k$ equals to 1 if vehicle $k$ uses link $(i, j)$, and 0 otherwise. Second, for each node $i$ and each vehicle $k \in K_z$, let $B_i^k$ be the time variable at which vehicle $k$ begins service (pick-up or drop-off) at node $i$. Third, $Q_i^k$ is the load variable of vehicle $k$ after visiting node $i$. The following is the formulation of the coordinated decentralized paratransit system:

$$Min \sum_{z \in Z} \sum_{i,j \in S_z} \sum_{k \in K_z} c_{ij}^k x_{ij}^k + F \sum_{z \in Z} \sum_{k \in K_z} (1 - x_{0,2n_z+2m_z+2tm_z+1}^k) \tag{1}$$

Subject to

$$\sum_{k \in K_z} \sum_{j \in V(i)} x_{ij}^k = 1 \quad \forall z \in Z,\ i \in (N_z^+ \cup N_z^- \cup M_z^+ \cup M_z^-) \tag{2}$$

$$\sum_{k \in K_z} \sum_{j \in V(i)} x_{ji}^k - \sum_{k \in K_z} \sum_{j \in V(i)} x_{ij}^k = 0 \quad \forall z \in Z, i \in S_z \tag{3}$$

$$\sum_{j \in S_z} x_{0j}^k = 1 \quad \forall z \in Z, k \in K_z \tag{4}$$

$$\sum_{i \in S_z} x_{i,2n_z+2m_z+2tm_z+1}^k = 1 \quad \forall z \in Z, k \in K_z \tag{5}$$

$$\sum_{k\in K_{z(j)}} \sum_{i\in(V(j)\backslash d(r))} \sum_{j\in d(r)} x_{ij}^k = 1 \quad \forall r \in M_z \tag{6}$$

$$\sum_{k\in K_{z(j)}} \sum_{i\in(V(j)\backslash p(r))} \sum_{j\in p(r)} x_{ij}^k = 1 \quad \forall r \in M_z \tag{7}$$

$$\sum_{k\in K_{z(i)}} \sum_{a\in V(i)} x_{a,i}^k = \sum_{k\in K_{z(j)}} \sum_{b\in V(j)} x_{j,b}^k \qquad \forall r \in M_z, \forall\, i, j \in t(r) \text{ and } i \in d(r),\, j \in p(r) \tag{8}$$

$$\sum_{j\in V(i)} x_{ij}^k - \sum_{j\in V(i)} x_{j,i+n_z}^k = 0 \quad \forall z \in Z,\, i \in N_z^+,\, \forall k \in K_z \tag{9}$$

$$\sum_{j\in V(i)} x_{ij}^k - \sum_{j\in V(i)} \sum_{t=1\cdots|T|} x_{j,i+2tm_z}^k = 0 \quad \forall z \in Z, i \in M_z^+, k \in K_z \tag{10}$$

$$\sum_{j\in V(i)} x_{ij}^k - \sum_{j\in V(i)} \sum_{t=1\cdots|T|} x_{j,i+2tm_z}^k = 0 \quad \forall z \in Z, i \in M_z^-, k \in K_z \tag{11}$$

$$B_j^k \ge B_i^k + d_i + t_{ij} - G(1 - x_{ij}^k) \quad \forall z \in Z,\, i, j \in S_z,\, k \in K_z \tag{12}$$

$$e_i \le B_i^k \le l_i \quad \forall z \in Z, i \in S_z, k \in K_z \tag{13}$$

$$B_i^k + d_i \le B_j^{k'} \le B_i^k + d_i + W \quad \forall r \in M,\ \forall i, j \in t(r) \text{ and } i \in d(r), j \in p(r), k \in K_{z(i)}, k' \in K_{z(j)} \tag{14}$$

$$B_{i+n_z}^k - (B_i^k + d_i + t_{i,i+n_z}) \ge 0 \quad \forall z \in Z, i \in N_z^+, k \in K_z \tag{15}$$

$$G(1 - \sum_{j\in S_z} \sum_{k\in K_z} x_{j,i+2tm_z}^k) + B_{i+2tm_z}^k - (B_i^k + d_i + t_{i,i+2tm_z}^k) \ge 0 \quad \forall z \in Z, i \in M_z^+, t = 1\cdots|T|, k \in K_z \tag{16}$$

$$G(1 - \sum_{j\in S_z} \sum_{k\in K_z} x_{j,i+2tm_z}^k) + B_i^k - (B_{i+2tm_z}^k + d_i + t_{i,i+2tm_z}^k) \ge 0 \quad \forall z \in Z, i \in M_z^-, t = 1\cdots|T|, k \in K_z \tag{17}$$

$$Q_j^k \ge Q_i^k + q_j - G(1 - x_{ij}^k) \quad \forall z \in Z,\, i, j \in S_z,\, k \in K_z \tag{18}$$

$$\max\{0, q_i\} \le Q_i^k \le \min\{Q_k, Q_k + q_i\} \quad \forall z \in Z,\, i \in S_z,\, k \in K_z \tag{19}$$

$$x_{ij}^k = \{0,1\} \tag{20}$$

$$B_i^k, Q_i^k \ge 0 \quad \forall z \in Z, i \in S_z,\, k \in K_z \tag{21}$$

The objective function (1) minimizes the total traveling cost plus the total vehicle fixed cost, where 0 and $2n_z + 2m_z + 2tm_z + 1$ represents the origin and destination depot in each zone, respectively. For example, if $x_{0,2n_z+2m_z+2tm_z+1}^k$ equals to 1, it means vehicle $k$ goes to destination depot directly from origin depot, thus will not incur the fixed cost $F$. Constraints (2) guarantee that all pick-up and drop-off nodes, except those at transfer locations, must be visited exactly once, and constraints (3) ensure the flow conservation for all nodes. Constraints (4) and (5) guarantee that each vehicle route starts out and returns to depot, respectively. Constraints (6) and (7) ensure that exactly one node is chosen from all possible transfer nodes for each inter-zonal request. For inter-zonal requests, constraints (8) define the flow conservation of each pair of pick-up and drop-off nodes at transfer locations. These constraints assure that the vehicles can only pick-up inter-zonal customers at the transfer locations where the customers are delivered to. Constraints (9) to (11) are pairing constraints: each paired request must be served by the same vehicle, where $t$ is the number of transfer locations. Constraints (12) and (13) guarantee time consistency and the key constraints (14) ensure the transfer times cannot exceed the maximum passenger waiting times at transfer locations, for an acceptable service level.

Constraints (15) to (17) are precedence consistency for inter-zonal and intra-zonal requests. The "Big $G$" part in constraints (16) and (17) is needed and crucial, because exactly one drop-off (pick-up) node at transfer locations will be chosen for inter-zonal requests. The precedence constraints do not apply to those transfer nodes that are not chosen. In Figure 8, the solid arrows are the optimal solution and the dotted arrows are the feasible but not optimal solution. The number next to arrows are corresponding travel times. The paired numbers in the parentheses are the arrival and departure time. The waiting time at transfer nodes is 10. Without the Big *G* part, the arrival time at final destination would be wrongly decided by the longer number (which is 60 in this case). However, the correct arrival time of $M^-$ would be 50 as $T_2$ is not used for the transfer of this example's customer. Constraints (18) to (19) are capacity constraints.

The model presented above significantly grows in size with the number of inter-zonal and intra-zonal requests. Each inter-zonal request adds $2t$ nodes, where $t$ is the number of transfer locations, in its both pick-up and drop-off zone and also adds the corresponding arcs to the network resulting in the rapidly increasing solving time.

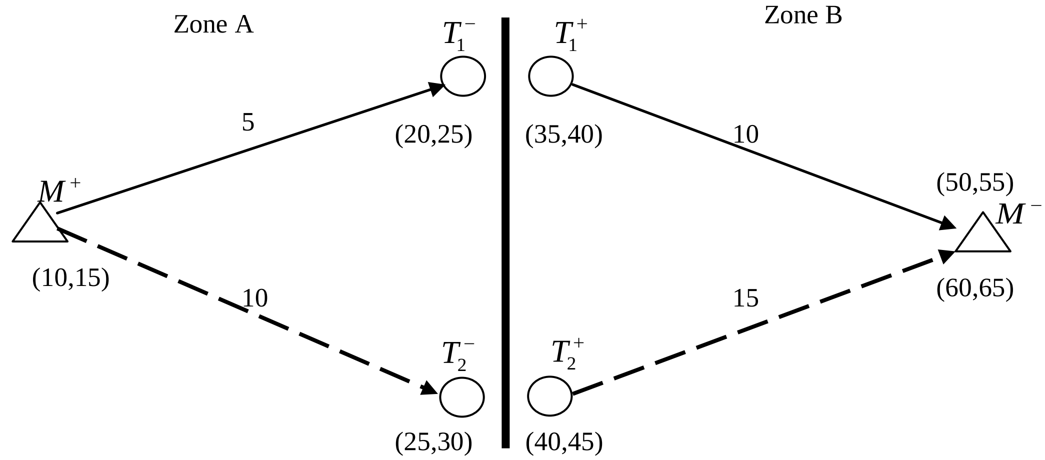



**Figure 8 An example of inter-zonal request**

Researchers applied the following straightforward arc elimination rules to reduce the network size as these arcs cannot belong to a feasible solution:

- No arc can go from the destination depot (node) to any node.
- No arc can go to the origin depot (node) from any node.
- No arc can go from any pick-up point to the destination depot.
- No arc can go from the origin depot to any drop-off node.
- No arc can go from the drop-off node to its corresponding pick-up node.

No arc can go between each pair of generated drop-off (pick-up) nodes at transfer location for the same request.

### 4.1. Parameters Setting

The optimization model will use the following system parameters:

- Vehicles’ speed: 30 miles/hour. (This converts the travel distance into travel time.)
- Passenger load or unload time: 5 minutes.
- Time-windows: 20 minutes plus the requested pickup time.
- Maximum travel time factor: 2.5 (the ratio of maximum travel times divided by direct travel times).
- Maximum passenger waiting times at transfer point: 10 minutes.
- Number of available vehicles: unlimited.
- Vehicle capacity: 5 persons.

# CHAPTER 5. NUMERICAL RESULTS ANALYSIS

Researchers have developed a strict formulation of a coordinated decentralized paratransit system, assuming transfer positions are fixed and known in advance. In this section, researchers compare the paratransit system performance among independent decentralized, coordinated decentralized, and centralized strategies. The question then is whether the proposed transfer mechanism can improve the productivity of paratransit systems. The formulation was implemented by using ILOG OPL 6.3 and CPLEX 12.1. It was run on a 2.33 GHz Core2 Duo with 2 GB of memory. Experimental results conducted with CPLEX demonstrate the validation of the proposed innovative formulation.

## 5.1 Performance Measurements

Researchers investigate the performance of various centralized and decentralized strategies from the productivity and service quality perspectives. For productivity perspectives, the number of vehicles used is the most direct indicator to compare the efficiency of alternative strategies for DARP. Vehicle revenue miles are another measurement defined as the summation of travel miles from first pick-up location to last drop-off location for all used vehicles. The vehicle revenue miles without passengers onboard are known as “empty backhaul miles.”

Passenger miles (traveled) are the summation of travel miles multiplied by number of customers on board for each travel segment. Passenger miles per vehicle revenue mile is one of the performance categories to measure the productivity of the transit system which is adopted by Safe, Accountable, Flexible, Efficient Transportation Equity Act: A Legacy for Users (SAFETEA-LU) to establish a provision in the FTA Urbanized Area Formula program to distribute funds to urbanized areas under a population of 200,000. For the operator, the smaller empty backhaul miles are better because the passenger miles per vehicle revenue mile would increase with smaller empty backhaul miles.

Except the performance measurements of productivity perspectives, researchers analyze the performance measurements of service quality for different strategies. From the service quality viewpoint, the waiting times and scheduled travel times of customers are the major concerns except the fare level. The waiting times are the time difference between requested pick-up time and actual pick-up time. Again, the actual travel times of customers cannot exceed 2.5 times of direct travel times because of maximum travel time factor for both intra-zonal and inter-zonal requests.

## 5.2 Simulated Instances and Comparison of Strategies

Each test case includes six requests in each zone, and half of them are transfer requests. In all instances, the coordinates of six pick-up nodes for each sub-zone are generated in the 10 by 20 (the expected direct travel time for trips within this area is 20 minutes) square mile area; the whole service area would be 20 by 20 square mile (the expected direct travel time for trips within this area is 26.7 minutes). For the drop-off nodes of six requests, three nodes are chosen within the same zone, and the other three nodes are chosen from the adjacent zone. The simulated duration of the generated pick-up time is 150 minutes. For the independent decentralized scenario, transfers are not allowed and vehicles can only drop-off customers outside their designated service zone. There is one depot in each zone for decentralized cases. For centralized strategy, no additional transfer and pick-up restrictions are added to the system and one depot is provided. The above two strategies adopt the formulations developed by *(25)*. For coordinated scenario, two transfer locations are available between two zones. Suppose we want to minimize both the total vehicle revenue miles and number of vehicle used in this study. The location of depot, therefore, does not affect the results of either vehicle revenue miles or passenger miles since deadhead miles (i.e., leaving or returning to the depot) are not included in vehicle revenue miles.

After the preliminary results of test cases, researchers found the two-way passenger exchange at transfer points simultaneous can largely decrease the empty backhaul miles compared with only one-way passenger feeding. Therefore, researchers compared the performance of alternative centralized and decentralized scenarios through 15 instances that exchange passengers at transfer locations. To increase the experiment's statistical efficiency and validation, this research applies the variance-reduction technique to synchronize the random number across the different configurations on each particular replication. This procedure can help to obtain greater precision through fewer runs. Tables 1 and 2 show the computational results of 15 instances for each strategy. Here, the numbers from 1 to 3 represent three scenarios: Independent Decentralized $(i=1)$, Coordinated Decentralized $(i=2)$ and Centralized strategy $(i=3)$.

Researchers observed that the centralized strategy has the smallest number of total vehicles used. This result does not beyond our speculation because the centralized strategy can be seen as pursuing global optimization (since no additional transfer and geographical constraints are added) under the objective to minimize the total vehicle used along with vehicle revenue miles.

**Table 1 Productivity for the 15 replications.**

| Run | Independent Decentralized | | | | Coordinated Decentralized | | | | Centralized | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Vehicles used | Vehicle revenue miles | Passenger miles | Passenger miles/vehicle revenue mile | Vehicles used | Vehicle revenue miles | Passenger miles | Passenger miles/vehicle revenue mile | Vehicles used | Vehicle revenue miles | Passenger miles | Passenger miles/vehicle revenue mile |
| 1 | 4 | 178.30 | 199.19 | 1.1171 | 4 | 161.90 | 207.62 | 1.2823 | 3 | 153.87 | 219.62 | 1.4273 |
| 2 | 4 | 190.37 | 171.43 | 0.9005 | 4 | 183.67 | 176.59 | 0.9615 | 3 | 217.70 | 229.86 | 1.0559 |
| 3 | 4 | 180.75 | 213.51 | 1.1812 | 4 | 170.82 | 218.68 | 1.2802 | 3 | 184.21 | 217.81 | 1.1824 |
| 4 | 5 | 170.06 | 206.47 | 1.2141 | 4 | 188.94 | 218.14 | 1.1545 | 3 | 201.35 | 270.47 | 1.3433 |
| 5 | 4 | 190.85 | 205.39 | 1.0762 | 4 | 184.68 | 223.54 | 1.2104 | 3 | 211.30 | 249.74 | 1.1819 |
| 6 | 4 | 223.69 | 273.69 | 1.2235 | 4 | 196.66 | 245.31 | 1.2474 | 4 | 190.81 | 239.46 | 1.2549 |
| 7 | 4 | 185.40 | 238.84 | 1.2883 | 4 | 175.23 | 231.59 | 1.3217 | 3 | 197.47 | 284.95 | 1.4430 |
| 8 | 5 | 210.89 | 240.92 | 1.1424 | 4 | 234.23 | 296.85 | 1.2673 | 4 | 206.67 | 248.26 | 1.2013 |
| 9 | 4 | 209.97 | 248.13 | 1.1818 | 4 | 194.43 | 235.87 | 1.2132 | 4 | 168.41 | 225.84 | 1.3410 |
| 10 | 4 | 221.44 | 262.14 | 1.1838 | 4 | 209.50 | 291.35 | 1.3907 | 4 | 190.75 | 254.06 | 1.3319 |
| 11 | 5 | 164.89 | 201.08 | 1.2195 | 4 | 170.10 | 225.75 | 1.3272 | 4 | 157.10 | 219.57 | 1.3976 |
| 12 | 5 | 177.86 | 203.52 | 1.1443 | 4 | 184.47 | 245.09 | 1.3286 | 3 | 183.91 | 225.10 | 1.2240 |
| 13 | 5 | 220.84 | 228.24 | 1.0336 | 5 | 203.60 | 234.07 | 1.1496 | 4 | 207.86 | 241.93 | 1.1639 |
| 14 | 4 | 186.15 | 211.03 | 1.1336 | 4 | 183.07 | 247.53 | 1.3521 | 4 | 153.80 | 213.57 | 1.3886 |
| 15 | 4 | 219.39 | 255.30 | 1.1637 | 4 | 199.59 | 251.68 | 1.2610 | 4 | 184.40 | 240.86 | 1.3061 |
| Avg | 4.33 | 195.39 | 223.93 | 1.1469 | 4.07 | 189.39 | 236.64 | 1.2498 | 3.53 | 187.31 | 238.74 | 1.2829 |

**Table 2 Service quality for the 15 replications.**

| Run | Independent Decentralized | | Coordinated Decentralized | | Centralized | |
|---|---|---|---|---|---|---|
| | Total waiting times | Average scheduled travel times | Total waiting times | Average scheduled travel times | Total waiting times | Average scheduled travel times |
| 1 | 58.80 | 43.38 | 90.23 | 58.33 | 70.82 | 52.10 |
| 2 | 120.47 | 30.24 | 89.38 | 45.44 | 84.25 | 44.69 |
| 3 | 105.86 | 46.74 | 50.93 | 59.55 | 80.76 | 47.23 |
| 4 | 65.95 | 41.84 | 153.05 | 54.34 | 66.93 | 60.76 |
| 5 | 63.36 | 45.85 | 107.29 | 61.62 | 107.5 | 51.21 |
| 6 | 99.23 | 58.20 | 72.12 | 59.46 | 91.08 | 59.43 |
| 7 | 31.56 | 49.25 | 54.55 | 67.05 | 105.91 | 60.15 |
| 8 | 121.00 | 47.57 | 123.05 | 64.06 | 122.73 | 48.74 |
| 9 | 81.92 | 50.18 | 106.18 | 53.45 | 40.26 | 46.70 |
| 10 | 49.87 | 55.65 | 95.88 | 71.83 | 53.37 | 57.20 |
| 11 | 96.24 | 48.05 | 56.24 | 51.70 | 66.45 | 51.71 |
| 12 | 91.95 | 46.81 | 37.47 | 61.46 | 56.17 | 43.34 |
| 13 | 42.93 | 41.00 | 86.06 | 64.88 | 73.40 | 49.06 |
| 14 | 53.21 | 46.70 | 65.53 | 62.32 | 106.46 | 51.17 |
| 15 | 20.12 | 52.96 | 86.84 | 66.74 | 50.55 | 55.73 |
| Avg | 73.50 | 46.96 | 84.99 | 60.15 | 78.44 | 51.95 |

Unit: Minute

In order to examine whether the measurements are significantly different among the three strategies, researchers constructed the all pair-wise confidence intervals for six measurements: number of vehicles used, vehicle revenue miles, passenger miles, passenger miles per vehicle revenue mile, customers average waiting times, and scheduled travel times. Because there are three pairs of comparison among three strategies, we must make each individual interval at level 98.33 percent (1-0.05/3) to achieve 95 percent overall confidence level according to the Bonferroni correction. In Table 3, the number represents the confidence intervals of differences $\mu_{i_2} - \mu_{i_1}$ for each measurement, for all $i_1$ and $i_2$ between 1 and 2, with $i_1 < i_2$. The numbers with asterisks in Table 3 indicate those intervals missing zero, i.e., those pairs of measurements are significantly different under the corresponding strategies.

**Table 3 All pair-wise confidence intervals of measurements**

(a) Vehicles Used

| Paired-t | | $i_2$ = 2 | $i_2$ = 3 |
|---|---|---|---|
| $i_1$ | 1 | -0.27 ± 0.31 | - 0.80 ± 0.46* |
| | 2 | | - 0.53 ± 0.35* |

(b) Vehicle Revenue Miles

| Paired-t | | $i_2$ = 2 | $i_2$ = 3 |
|---|---|---|---|
| $i_1$ | 1 | - 6.00 ± 9.92 | - 8.08 ± 17.01 |
| | 2 | | -2.08 ± 14.27 |

(c) Passenger Miles

| Paired-t | | $i_2$ = 2 | $i_2$ = 3 |
|---|---|---|---|
| $i_1$ | 1 | 12.72 ± 15.54 | 14.81 ± 20.32 |
| | 2 | | 2.09 ± 22.85 |

(d) Passenger Miles per Vehicle Revenue Mile

| Paired-t | | $i_2$ = 2 | $i_2$ = 3 |
|---|---|---|---|
| $i_1$ | 1 | 0.1029 ± 0.0533* | 0.1360 ± 0.0553* |
| | 2 | | 0.0330 ± 0.0644 |

(e) Waiting Times

| Paired-t | | $i_2$ = 2 | $i_2$ = 3 |
|---|---|---|---|
| $i_1$ | 1 | 11.49 ± 31.11 | 4.94 ± 25.07 |
| | 2 | | - 6.54 ± 27.06 |

(f) Scheduled Travel Time

| Paired-t | | $i_2$ = 2 | $i_2$ = 3 |
|---|---|---|---|
| $i_1$ | 1 | 13.19 ± 4.24* | 4.99 ± 4.41* |
| | 2 | | - 8.20 ± 4.96* |

In Table 3(a), the total number of vehicles used in coordinated decentralized strategy is not smaller than the number in independent decentralized strategy. The number in centralized strategy, however, is significant different from other two strategies. From Table 3(b) and 3(c), no evidence can show that which strategy is better or worse than the other two strategies in terms of vehicle revenue miles and passenger miles. Researchers observed that the vehicle revenue miles and the vehicles used in each run among three strategies are negative correlated (see run 4). In the runs that the total vehicles used are equal among three strategies, the coordinated decentralized strategy has lower vehicle revenue miles but higher passenger miles as oppose to independent decentralized strategy.

For passenger miles per vehicle revenue mile, Table 3(d) indicates that the value of the coordinated strategy is higher than the one of the independent strategy; the value of the centralized strategy is higher than the value of the independent case either. However, the value between coordinated and centralized strategies is not significantly different. On average, the coordinated strategy can improve 9.0 percent of passenger miles per vehicle revenue mile compared with independent strategy. The centralized strategy can improve 10.9 percent of passenger miles per vehicle revenue mile compared with independent strategy. It is shown that the transfer design can significantly increase the passenger miles per vehicle revenue mile due to the decrease of empty backhaul miles compared with independent decentralized strategy. The expected benefit of transfer mechanism is higher productivity, but close coordination is required among vehicles in different zones.

From the service quality perspective, the waiting times are desired to be as small as possible. The coordinated strategy presents a slightly higher average value. Table 3(e) shows the all pair-wise comparisons for passenger waiting times of three strategies. Another indicator is the scheduled travel times of each passenger (Table 3[f]). The values among three strategies are significantly different from each other. Researchers conclude that the coordinated strategy has the highest value, which is 28.1 percent higher than the value of independent strategy and is 15.8 percent higher than the value of centralized strategy on average. This is expected, as, for the coordinated strategy, the inter-zonal passengers have to transfer vehicles at specific location, which would add extra travel time. The total scheduled travel time, however, does not violate the maximum allowed ratio (travel times divided by direct travel times). Of course, inter-zonal passengers have to transfer, which could be a real burden in itself (especially for the disabled), regardless of the potential additional travel/waiting time.

# CHAPTER 6. SUMMARY ON RESULTS FOR FORMULATION

In the first part, researchers have presented the exact formulation of the coordinated decentralized paratransit system to compare its productivity and service quality with independent decentralized and centralized strategies. The formulation has been proven to work correctly, and the results of the computational experiments of small scale instances demonstrate that the proposed coordinated system is superior to independent decentralized system in terms of passenger miles per vehicle revenue mile. The higher productivity over the coordinated strategy is achieved by the reduction of empty backhaul miles through close coordination among service providers Based on the service quality, the proposed coordinated system increases the average scheduled travel time of passengers compared with other two strategies. This outcome results from the inter-zonal passengers increase extra travel distance and take extra service waiting times for the transfer restriction. However, the maximum scheduled travel times are bounded by the acceptable maximum travel time factor. The passenger miles, vehicle revenue miles and passenger waiting times do not have difference among three strategies.

The exact solution approach of the proposed formulation is obviously constrained by problem scale, running time, and computer memory, given its combinatorial nature. However, results in terms of optimal solutions satisfactorily showed a performance comparison between different strategies. In second part, researchers developed heuristic methods for solving more realistic medium to large scale problems and eventually carry out performance comparisons again. Developed heuristics need to pay particular attention to the coordination at transfer points.

# SECTION 2: HEURISTIC METHOD

## CHAPTER 7. MOTIVATIONS AND BACKGROUND

This part investigated the different organizational structures of paratransit services that cover large regions using real data from Houston.

Due to the complexity of our problem, researchers have shown that the exact method can only solve quite a small size of data limiting the validity for the improvement of transfer design. Therefore, researchers developed an insertion-based heuristic so that researchers can evaluate the performance of real size case. As we mentioned in the first part, developers of zoning strategies need to decide how to accommodate those trips crossing zones. According the dealing of inter-zonal trips, the zonal approach can be divided into two variations: (a) zoning without transfer, such as with the service provided in Los Angeles County, and (b) zoning with transfer, such as with the Chicago ADA paratransit service. In zoning without transfer, inter-zonal customers may not need to switch vehicles during their trips. Alternatively, zoning with transfer systems may require inter-zonal customers to switch vehicles.

Although the operational consolidation of providers appears to achieve economies of scale, the following may impede their coordination: (a) a user may have some concern that the current service levels will decrease; (b) the sponsoring agency may have doubts regarding whether there is a significant cost savings; and (c) the different jurisdictions within which component transportation systems operate may have different operational standards designed particularly to meet the local riders' needs *(2)*. To the best of our knowledge, there is no quantitative evidence to demonstrate the benefits and concurrent costs that occur from adopting a zoning with transfer design for a large-scale paratransit system.

In an experiment utilizing Houston's demand responsive service data, researchers compared the productivity and service levels among three organizational structures: zoning with transfer, zoning without transfer, and a no-zoning design. The zoning without transfer structure also divides its service area into sub-zones, each zone with its own vehicle depot. The zonal service provider can only pick up customers whose pick-up location is within the service area; however, the provider is allowed to drop-off customers outside of that area. Each provider is unaware of the state of the system in other zones. Alternatively, the no-zoning control system is a totally centralized system, which is the basic scenario describing paratransit service in general.

The rest of this part is organized into three sections. Researchers first define the paratransit services of the zoning with transfers system, followed by a description of the demand data used. The computational experience of the algorithm is then outlined. Finally, researchers summarize the results of the simulation.

## CHAPTER 8. TRANSFER SYSTEM

In this section we provide a description of the zoning without and with transfers strategies and the detail of the scheduling procedure used.

Within a demand response service area, the service provider may subdivide the service area into zones. A zone is a geographical boundary. A list of customers will request a certain number of trips, defined by their location and time. In practice, each trip has a specific pair of scheduled pick-up and drop-off locations, as well as a desired pick-up (or drop-off) time for each the pick-up (or drop-off) location. Each pick-up and drop-off is considered a node in the system. All trips can be categorized as either inter-zonal trips or intra-zonal trips, as determined by the pick-up and drop-off locations. Trips with pick-up and drop-off locations in different zones are inter-zonal trips, and trips with pick-up and drop-off locations within the same zone are intra-zonal trips (see Figure 9).

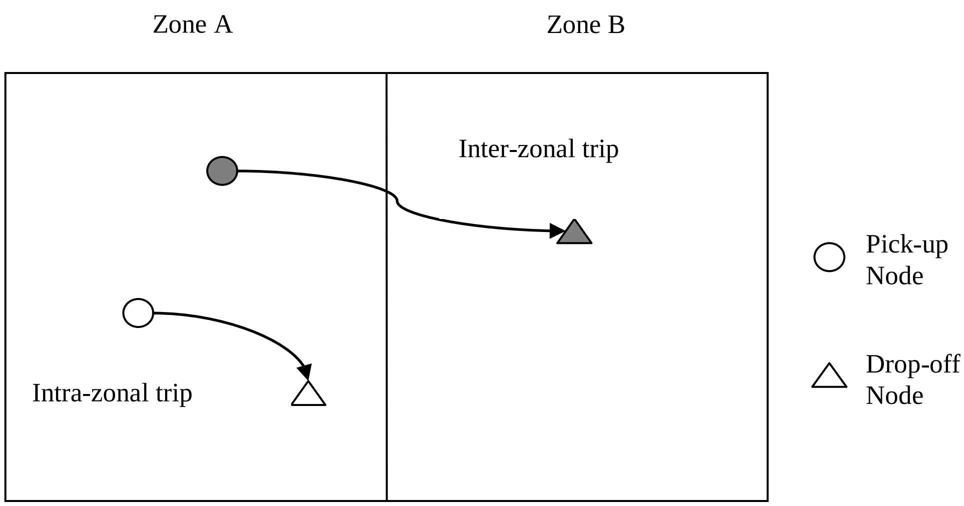



**Figure 9 Categories of trip by zonal structure**

For the zoning without transfer policy, zones are served and would be independently operated by different carriers. Figure 10 illustrates the characteristics of this policy. The pick-up location of each customer determines the zone and its service provider. Vehicles are, however, allowed to traverse zone boundaries in order to drop-off inter-zonal customers.

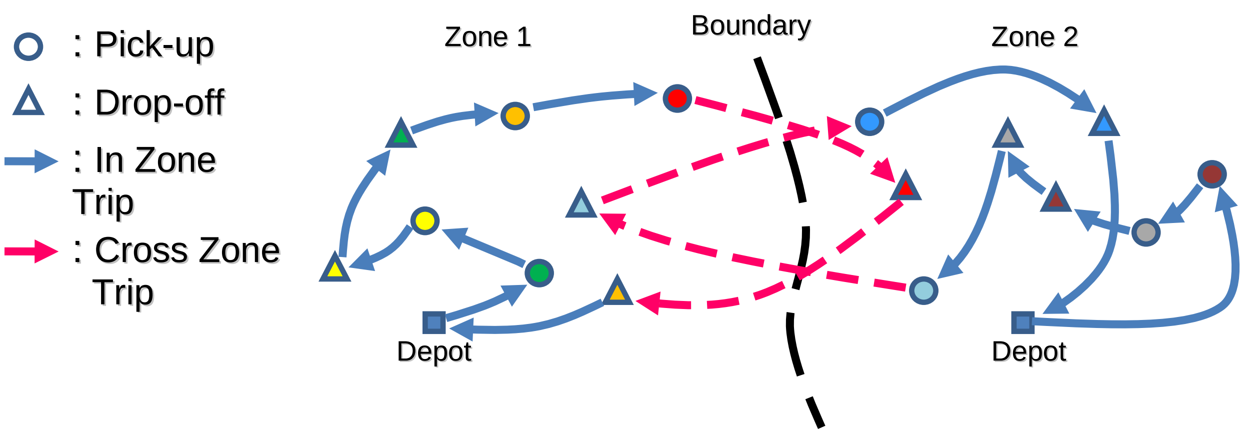



**Figure 10 Zoning without transfer policy**

In zoning with transfer control, inter-zonal passengers must transfer from one vehicle to another through given transfer locations in order to reach their final destination, while intra-zonal passengers do not switch vehicles to complete their trip. To highlight the loadings and unloadings at transfer locations, we generated two corresponding nodes (a load node and an unload node) at each transfer location, for each inter-zonal trip (i.e., when a vehicle visits the transfer node, it will either load or unload passengers according to the node's characteristic). Thus, the trip can be treated as two intra-zonal trips when schedules are coordinated, such that inter-zonal customers can switch vehicles at specific transfer locations (Figure 11b).

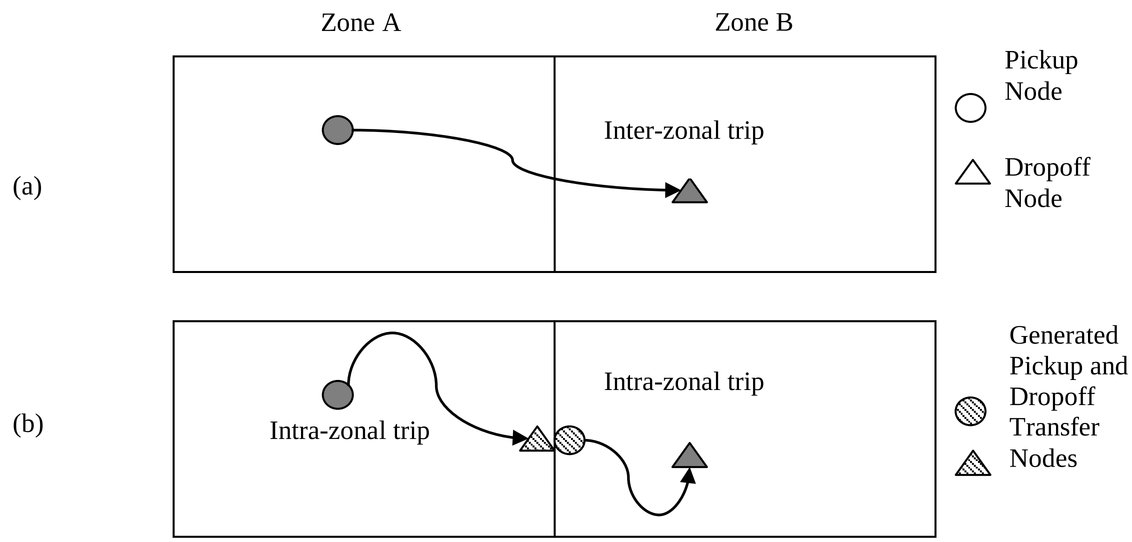



**Figure 11 Example of generated intra-zonal travel trip**

Researchers assumed that each inter-zonal trip could only switch vehicles at particular transfer locations, and only once. The transfer locations at which a vehicle might stop are typically on the boundaries of subzones. If passengers need to travel between zones that do not border one another, the transfer locations can be located within a shared buffer zone at a distance between the two zones. Concerning customer discomfort, more than one transfer might be undesirable, and in certain circumstances, quite unreasonable. In practice, the passengers of the paratransit systems in both Chicago and Boston are assured, at most, one transfer (both systems use a coordinated zoning system).

For this study, researchers set the hard time windows as follows: the earliest arrival time was $ET_i$ and latest departure time was $LT_i$ $(i = 1, 2, \cdots, N)$, for both the pick-up and drop-off nodes. In the following context, "$+i$"("$-i$") denotes the point of pick-up (or drop-off) of customer $i$. The earliest vehicle arrival time is denoted as $AT_i$ and the earliest vehicle departure time is denoted as $DT_i$. At pick-up nodes, the time gap between $ET_i$ and $LT_i$ denotes the width of a predefined pick-up time window. For example, one node may be a pick-up home address scheduled within a half hour window of time, between 6:45 a.m. and 7:15 a.m.

In many demand response scheduling systems' insertion algorithms, the objective is to minimize the vehicle travel distance while maintaining an acceptable level of service. In order to maintain such a service level, the ratio of maximum ride time $(MRT_i)$ to direct ride time $(DRT_i)$ needs to be within a specified value $R$, called maximum ride time factor, for every customer. Therefore, the $ET_{-i}$ and $LT_{-i}$ of the drop-off node would be decided by the corresponding $ET_{+i}$ and $LT_{+i}$ of the pick-up node and $R$:

$ET_{-i} = ET_{+i} + DRT_i$
$LT_{-i} = ET_{+i} + R \times DRT_i$
if $LT_{-i} < LT_{+i}$, then
$LT_{-i} = LT_{+i} + DRT_i$

*R* can be a constant (such as in Los Angeles County) or an inverse function of the direct trip length (such as Houston), in order to avoid extremely long maximum trips for already long direct journeys. Except in a case where the pick-up and drop-off vehicles arrive at the transfer location at the exactly same time, the earlier arriving vehicle must wait until another vehicle arrives (i.e., we would not allow customers to wait alone at transfer locations). Researchers calculated the node distances based on the Manhattan distances used to calculate the symmetrical travel distances between any two pairs of nodes. This estimated travel distance was verified to be close to the actual travel distance by Quadrifoglio et al. (*8*). The distance calculation implied that the network was arranged in a rectilinear grid pattern. Researchers also assumed there were no traffic jams on the system, and the travel time between any two points was only a matter of the travel distance and vehicle speed. This assumption might not allow for a precise calculation of the travel time between two points, but it does not alter the results of our following performance comparison. The link distances and speeds were input into the model, and can easily be updated with more accurate values, if and when those values become available.

### 8.1 Scheduling Algorithm

The new insertion-based heuristic makes use of the generic insertion framework of Solomon's sequential approach. This algorithm processes ride requests sequentially, inserting one customer at a time into the vehicle schedule until all requests have been serviced.

After sorting all customers by requested pick-up times, one empty route is generated in each service zone. Each empty route starts from and ends at the same depot. Every inter-zonal trip generates a drop-off and pick-up node at a transfer location. According to the designated zone of each trip, the schedule is reviewed for possible insertions of unassigned trips, sequentially by their earliest pick-up times. In this study, the insertion review, from the first to the last unassigned trip, is called one round. A more detailed description of the procedure used for insertion review is described below. Those trips that cannot be inserted into the schedule during a round will be copied to the unassigned list of trips. This insertion procedure requires that we maintain one route in each zone during each round.

During the search procedure, four constraints are taken into consideration. First, the arrival time, $AT_i$, of a vehicle at the pick-up (or drop-off) location should be no later than $LT_{+i}$ $(LT_{-i})$.

Second, for each passenger the drop-off time should be later than the pick-up time; this is also known as a precedence constraint. Third, after inserting the new trip, we must check whether the insertion will violate the assigned customers' successive time windows. Finally, the capacity of each vehicle is also necessary in order to consider the proper process for inserting the unassigned trips. Figure 12 illustrates the algorithm procedure in a diagram.

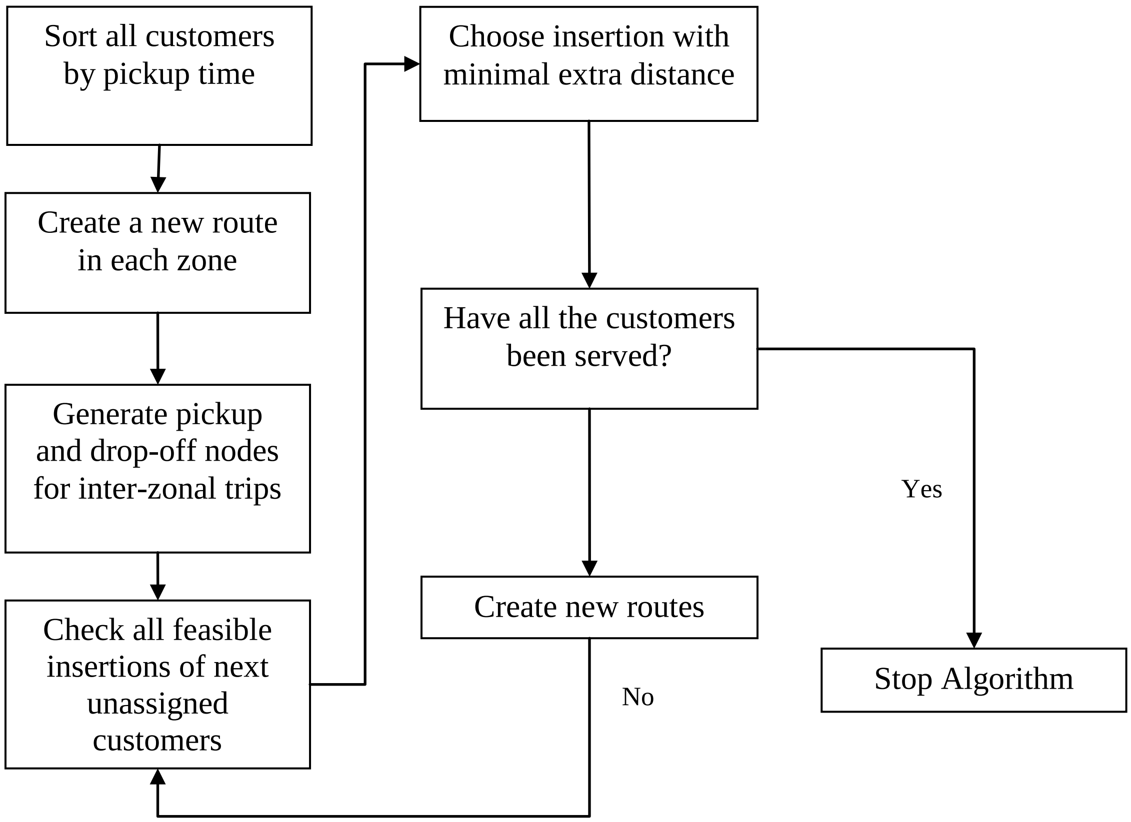



**Figure 12 Flow chart of the algorithm**

In our proposed zoning with transfer service, the heuristics will need to pay particular attention to the coordination at transfer nodes for inter-zonal passengers. In addition to the previous four constraints, vehicles with passengers onboard cannot wait at transfer nodes longer than a specific maximum vehicle idle time $(IT_{max})$; vehicles without passengers onboard do not have restrictions on idle time spent at transfer nodes. The restriction of maximum idle time at a transfer node helps to maintain an acceptable level of service for inter-zonal customers. For operators, the allowance of vehicle idle time can increase the possibility of feasible insertions, and thus increase the productivity of the service.

After inserting a new node into the vehicle schedules, the schedule of the nodes after the newly inserted node needs to be reviewed. Due to the transferring of inter-zonal customers between routes, a change in one route may make all other routes infeasible. This interdependence problem complicates the use of the standard insertion method. In order to ease the computational effort of reviewing the feasibility of inserting new nodes, researchers maintain two quantities for each assigned node: the vehicle waiting time, $WT_i$, and the vehicle slack time, $ST_i$, both of which are updated by a bottom-up procedure. Vehicle waiting time is denoted by the time difference between $AT_i$ and $ET_i$ if $AT_i < ET_i$. Vehicle slack time is the time difference between $AT_i$ and $LT_i$. Consider the case in which there is one node in the route, if $AT_i < ET_i$ at node $i$, $WT_i = ET_i - AT_i$ and $ST_i = LT_i - ET_i$. If $AT_i \geq ET_i$, $WT_i = 0$ and $ST_i = LT_i - AT_i$. Figures 13(a)–(b) give a graphic illustration of the above two situations. The summation of $WT_i$ and $ST_i$ is denoted by $PT_i$, the maximal postponed time, which is the maximal time interval that can be used for inserting new customers before this node. At the non-transfer node, the maximal slack time at each location is determined by the minimum of its maximal $ST_i$, or the $PT$ of the next scheduled node. Figure 13(c) illustrates the calculation of $PT_i$ for two consecutive non-transfer nodes, applying a bottom-up procedure. Considering the coordination of transfers, the maximum slack time of corresponding pick-up and drop-off nodes at the transfer location will be the minimum $PT$ of its following node and the slack time of the connecting vehicle at the transfer point. We may assume, without loss of generalization, that $v_i$ represents the node for the transferring drop-off location $i$, and $v_j$ represents the node for the transferring pick-up location $j$, and $AT_i < AT_j$.

Except in a case where the pick-up and drop-off vehicles arrive at the transfer location at the exactly same time, the earlier arriving vehicle must wait until another vehicle arrives (i.e., we would not allow customers to wait alone at transfer locations). Therefore, at transferring nodes, the maximal postponed time would be the sum of $WT$, $ST$, and $IT$. After the passengers disembarked, the drop-off vehicle is allow to depart; the pick-up vehicle departs after the transfer passengers board. The above requirements can be summarized as:

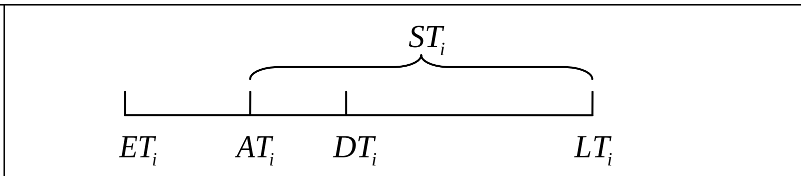
$$AT_j - AT_i = IT_i \leq IT_{\max}$$
$$DT_j = AT_j + \text{passenger boarding service time}$$
$$DT_i = AT_i + IT_i + \text{passenger disembarking service time}$$

Researchers chose the minimum to fall between $ST_i$ and $ST_j$, as the maximum slack time for both $i$ and $j$. Using Figure 13(d) as an example, in the first step we calculate the $ST$ of $v_i$, and $v_j$, using separate bottom-up updated procedures. We obtain the vehicle idle time at the transfer location as follows:

$$ST_i = \min\left\{LT_i - \max(AT_i, AT_j), PT_1\right\}$$
$$ST_j = \min\left\{LT_j - \max(AT_i, AT_j), PT_2\right\}$$

In the second step, the $ST$ of two transferring nodes will be updated at the minimum level between $ST_i$ and $ST_j$. and $ST_i = ST_j = \min(ST_i, ST_j)$.

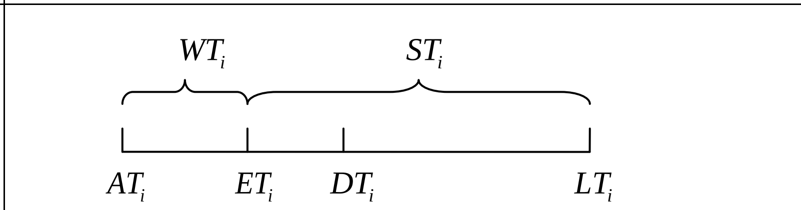
(a)




$AT_i \geq ET_i$:

$PT_i = WT_i + ST_i = LT_i - AT_i$

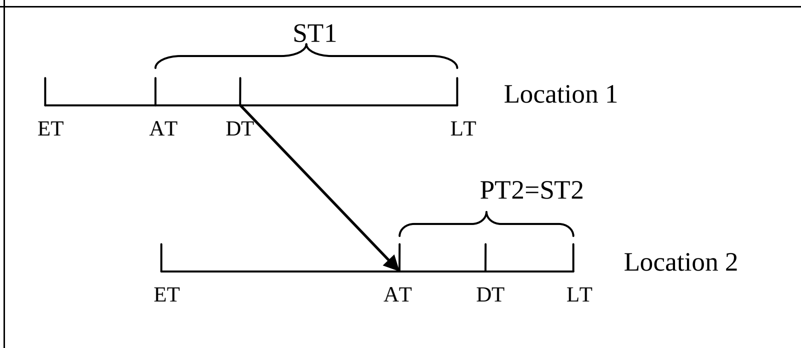
(b)




$AT_i < ET_i$:

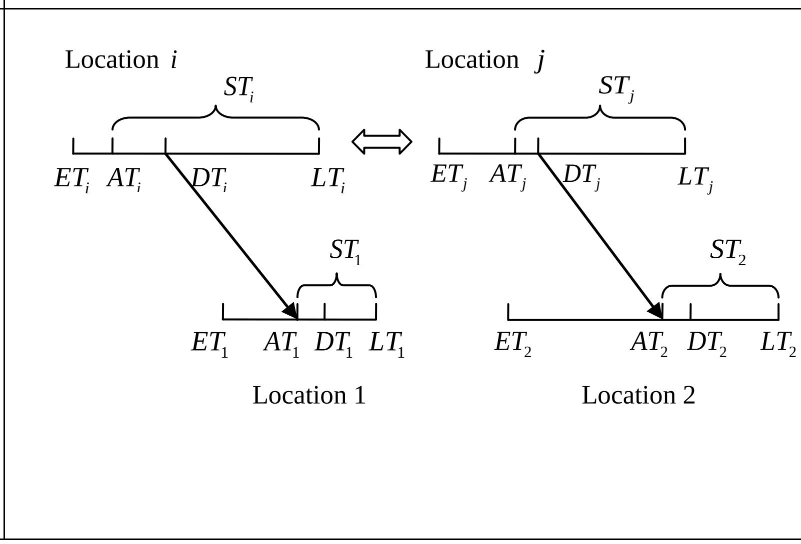
$PT_i = WT_i + ST_i = LT_i - AT_i$

(c)




Location 2:
PT= ST2
Location 1:
ST1= min{ST1, PT2=ST2}= ST2

(d)




Location 1: $PT_1 = ST_1$

Location 2: $PT_2 = ST_2$

Location $i$:

$ST_i = \min\left\{LT_i - \max(AT_i, AT_j), PT_1\right\}$

$ST_i = ST_j = \min(ST_i, ST_j)$

Location $j$:

$ST_j = \min\left\{LT_j - \max(AT_i, AT_j), PT_2\right\}$

$ST_i = ST_j = \min(ST_i, ST_j)$

**Figure 13 Examples of calculations of postponed times in different situations**

## CHAPTER 9. COMPUTATIONAL EXPERIMENT

In order to demonstrate the productivity and level of service provided by the proposed zoning with transfer paratransit system, researchers compared the results of zoning without transfer and no-zoning with the same sequential insertion algorithm proposed in the previous section. Below, we present the real demand data provided by the Metropolitan Transit Authority of Harris County, which was used to generate the random samples. Then, we describe the configurations of three organizational structures. Finally, an analysis of the simulation results is provided including the sensitivity analysis on maximum ride time factor $R$.

### 9.1 Demand Data Description

METROLift is a paratransit service in Harris County, Texas, currently in compliance with the Americans with Disabilities Act (ADA). On average, over 5,000 trips are made daily, from 3:45 a.m. to 1:30 a.m. the following day. Test samples were generated according to the locations (pick-up and drop-off) and time distribution. Using GIS software, researchers counted the number of pick-up and drop-off locations for every square mile area (see Figure 14, 15). Figure 16 shows the actual pick-up time distribution. Because the pick-up and drop-off locations are independently generated, the pick-up and drop-off point might unrealistically be generated within the same square mile area. In this case, a new drop-off location will be generated.

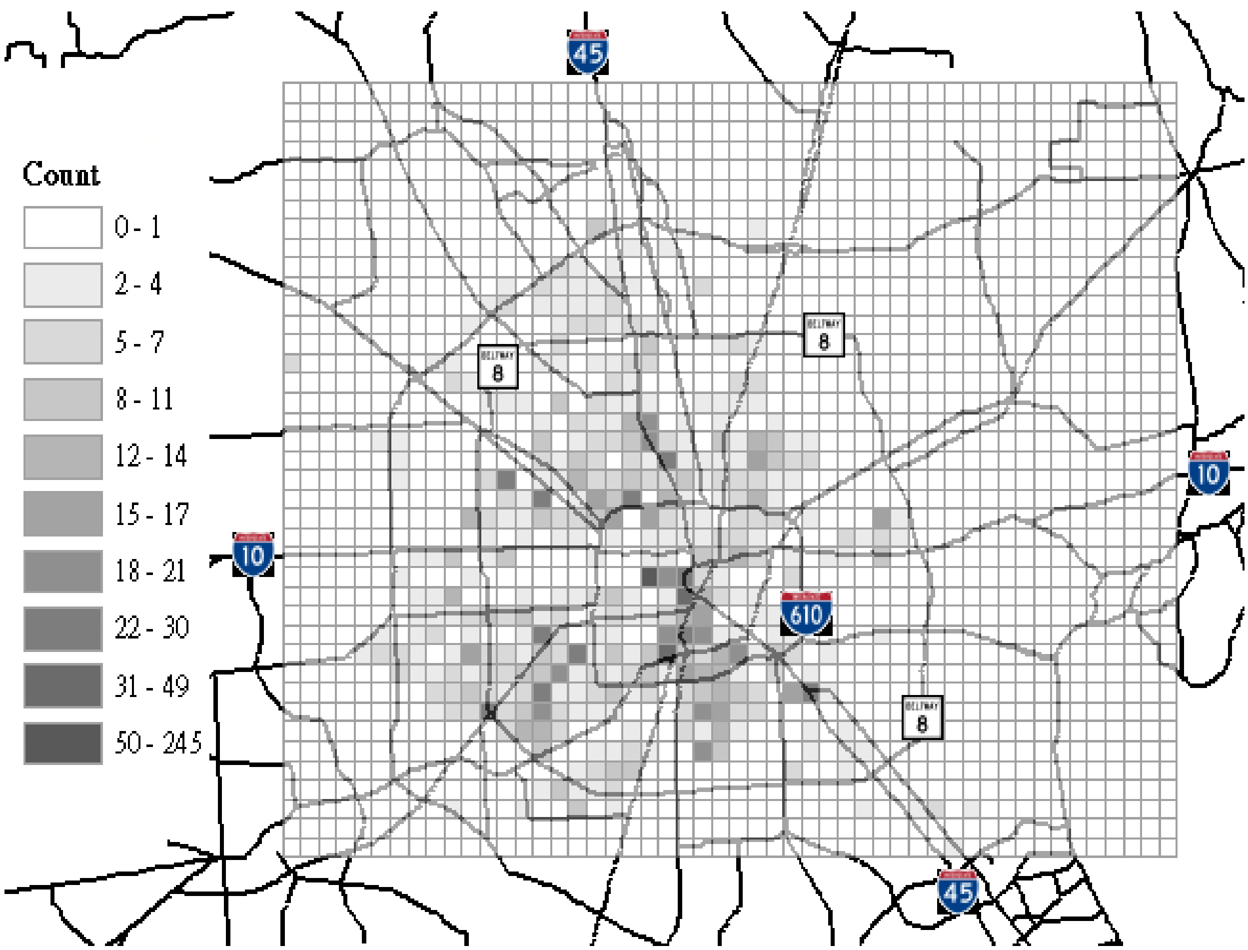



**Figure 14 Distribution of pick-up locations**

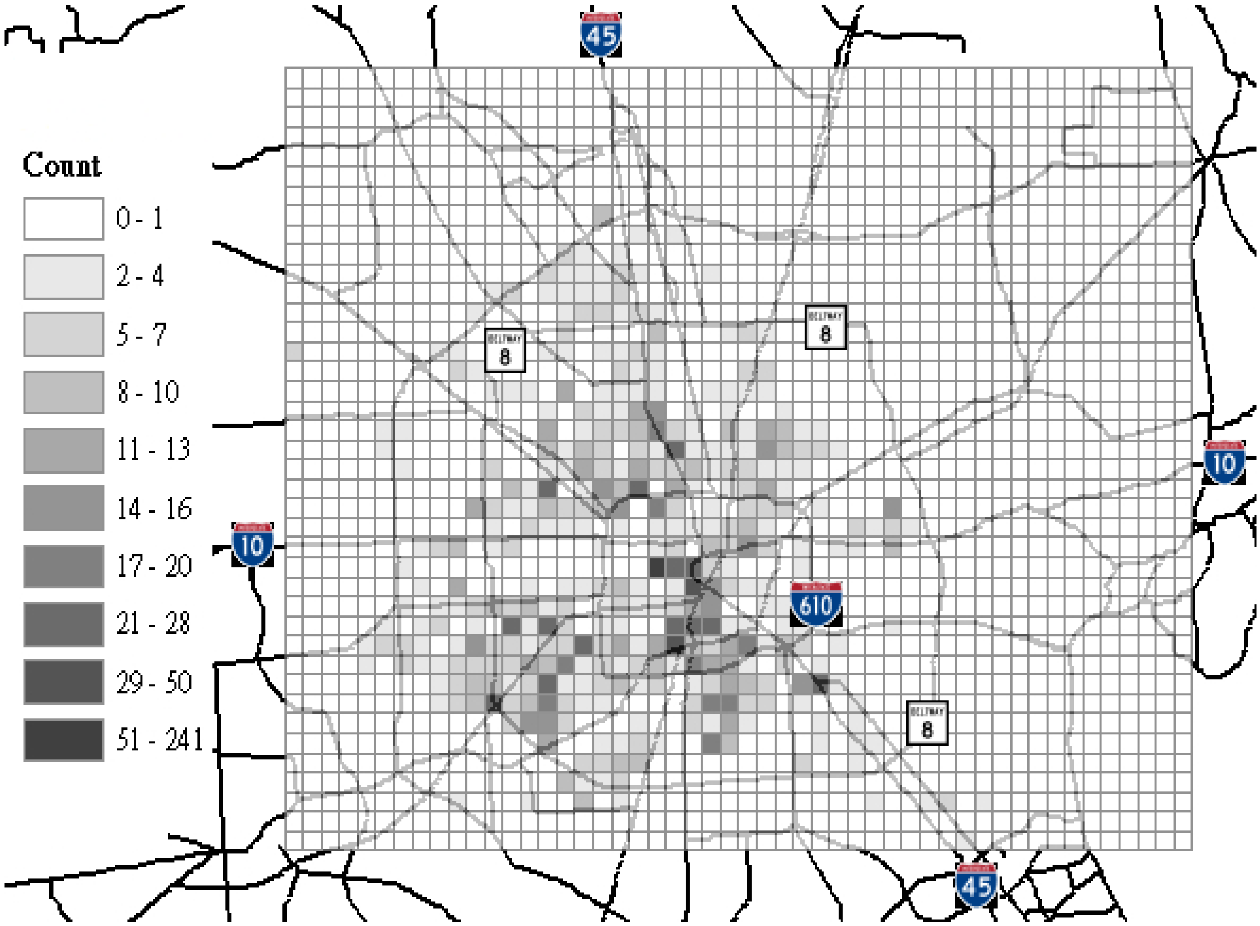



**Figure 15 Distribution of drop-off locations**

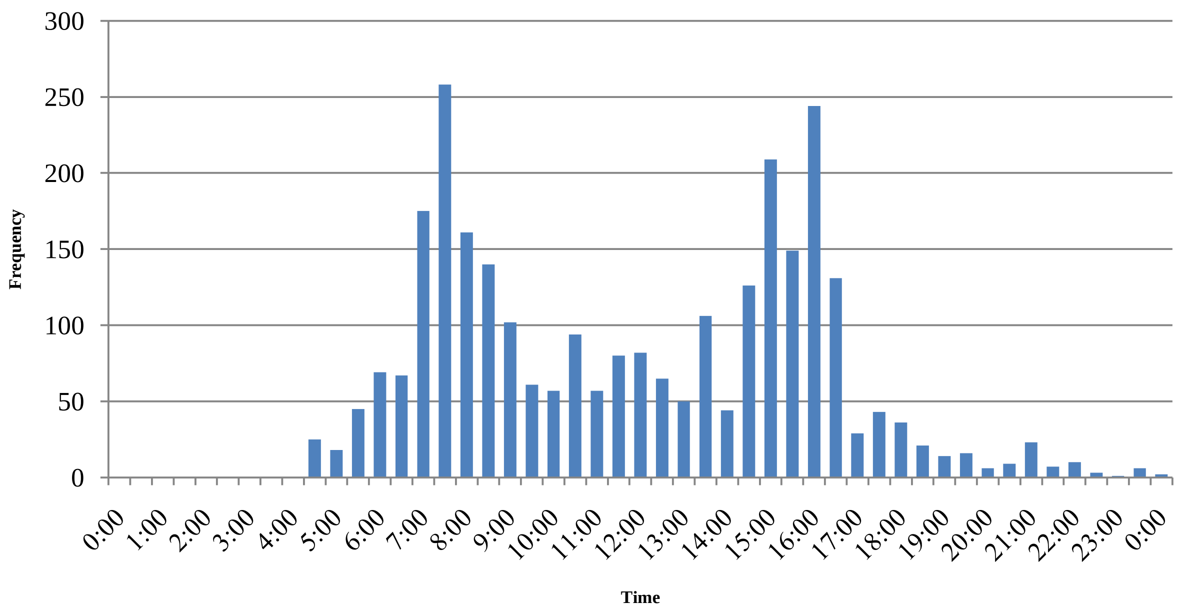


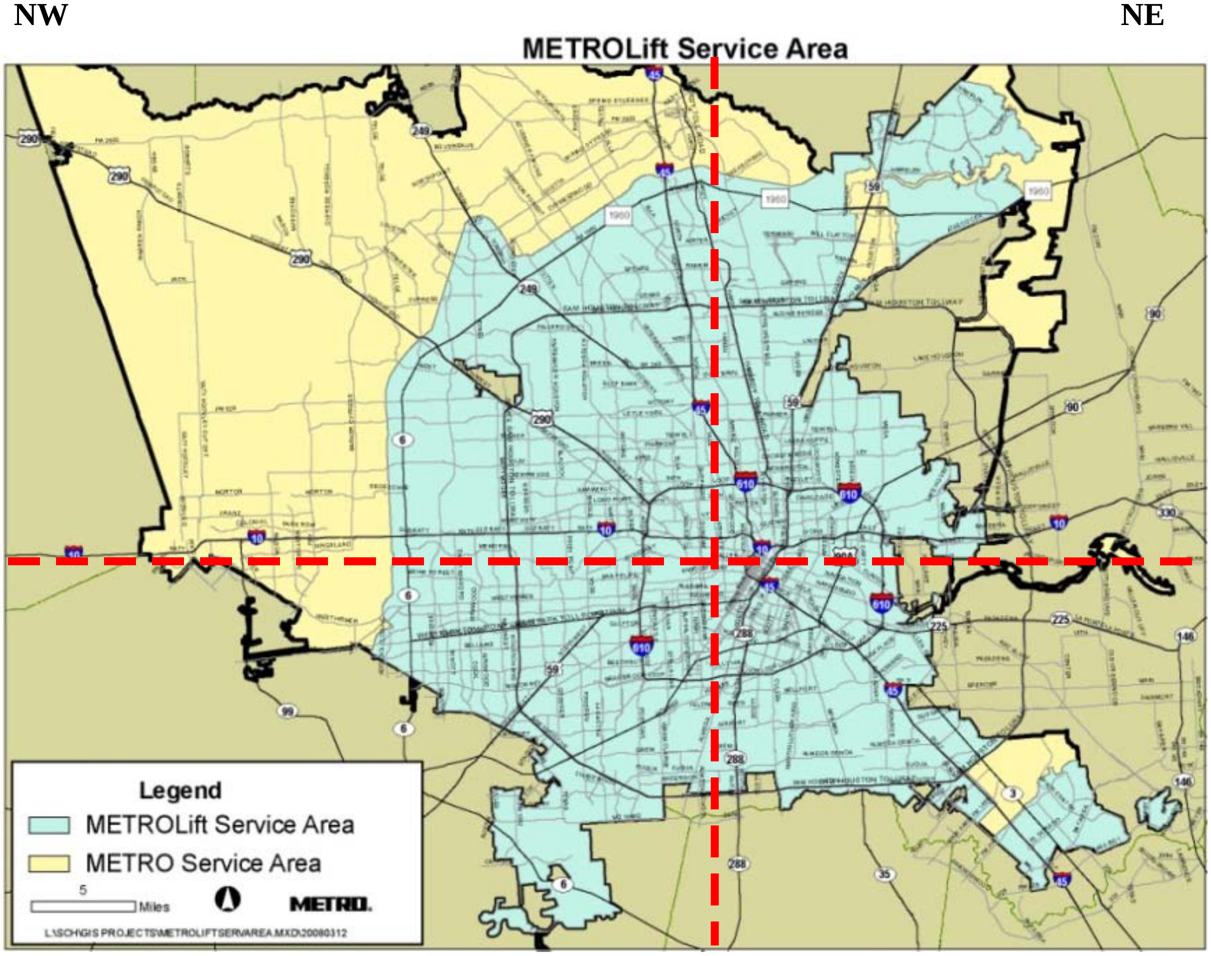

**Figure 16 Distribution of requested pick-up time and cumulative percentage**

### 9.2 Zoning Configurations

The configuration of a zoning structure is defined by its boundaries; transfer locations are often located at a zone boundary. Researchers used the following four rules to build the sub-zones, as shown in Figure 17:

1. It is better not to situate a popular destination or high demand density area in one exclusive zone.
2. Each zone should accommodate a certain volume of trips originating from it.
3. The percentage and number of inter-zonal trips attached to each zone should be close.
4. Zones should be mutually adjacent, so that more than one transfer can be avoided.

NW NE




SW SE

**Figure 17 Zones built in Houston region**

By checking the pick-up/drop-off location distribution, researchers located a one square mile area with an extremely high demand density (250 pick-ups per day). This spot sits roughly in the lower center section of the service area. The origins of the trips leading to this spot and the destinations desired from this spot are both uniformly scattered throughout the area. Therefore, this made an idea center from which we formed zones. If we included this spot in one specific zone, other zones would have had to make more inter-zonal trips, which in turn would have decreased the overall service quality. Based on the selection of this spot, the service area was administratively divided into four geographical quadrants of unequal size: the northwest (NW), the northeast (NE), the southeast (SE), and the southwest (SW). Trips in each zone were observed to be large enough to maintain a minimum level of operational scale, although individual trips from each zone were not equal in length. Table 4 shows the percentage of inter-zonal trips and the number of drop-off locations beyond a designated pick-up zone. In practice, passengers do not usually require a transfer if their destinations are just one or two blocks beyond a particular zone boundary. Therefore, researchers set a one mile-wide buffer area along each zone boundary.

**Table 4 Pick-up and drop-off percentage between each zone**

| Pick-up | Number of pick-up | Drop-off | | | |
|---|---|---|---|---|---|
| | | Northwest | Northeast | Southwest | Southeast |
| Northwest | 1528 | 60% | 17% | 13% | 10% |
| Northeast | 1316 | 18% | 58% | 9% | 15% |
| Southwest | 897 | 9% | 4% | 65% | 22% |
| Southeast | 1259 | 9% | 10% | 19% | 62% |

For the zones that we generated, five locations provided for transfer needs. The center of the four quadrants was selected to be the transfer location for all inter-zonal trips traveling between NW and SE, or NE and SW. Our research found that transfer locations were best located at the edges of the zones nearest the major inter-zonal corridor.

The default parameters used in the simulation are as follows:

- Vehicle speed: 25 miles per hour.
- Average boarding or disembarkation time: 1 minute.
- Maximum ride time factor $R$: customers have different number according to their direct travel distance. $R$ =1.5 if $DRT > 72$ minutes; $R$ =2 if 48 minutes $< DRT \leq 72$ minutes; $R$ =2.5 if $DRT \leq 48$ minutes.
- Pick-up time windows: 40 minutes from the desired pick-up time.

The three scenarios listed below were tested on the randomly generated instances, and 10 replications were run to deal with the randomness of the simulation.

1. Zoning without transfer: The region was divided into four service zones, and each zone had its own carrier. Customers were zoned by their pick-up locations and served by their designated service carrier. Vehicles in each zone could cross boundaries only to drop-off inter-zonal customers.

2. Zoning with transfer: This scenario respected the same geographical zones and carrier design as in scenario 1. Vehicles in this system, however, were always within each single zone. Customers needed to transfer at the zone boundary.

3. No-zoning: The region was served by a single carrier. Current Houston paratransit service adheres to this scenario.

The statistics reported are the averages taken of 10 replications. The heuristic was implemented via computer program C and run on a on a 2.33 GHz Core2 Duo Computer with 2 GB of memory.

## CHAPTER 10. PERFORMANCE AND RESULTS ANALYSIS

Researchers investigated the performance of scenarios from the aspects of system efficiency and service quality. For system efficiency, the number of vehicles used was the most straightforward indicator for comparison of alternative scenarios. Deadhead miles were the miles that a vehicle traveled from its home depot to its first pick-up node, and from its last drop-off node to its home depot. Vehicle revenue miles were the defined as the summation of travelled miles from first pick-up location to last drop-off location, for all vehicles. Vehicle revenue miles with no passengers on board were defined as empty miles. Total miles included revenue miles and deadhead miles.

Passenger trips per vehicle revenue hour was an important performance measure for capturing the productivity of a particular demand responsive system. Higher passenger trips per vehicle hour usually means more trips can be scheduled within a given time period.

Passenger miles (traveled) was calculated as the summation of traveled miles multiplied by the number of customers on board for each travel segment. Passenger miles per vehicle revenue mile was another performance measurement used to calculate the productivity of the demand responsive system. It captured the difference between the systems with longer or shorter trips, on average. Vehicle idle time is the time gap between the vehicle arrival time and earliest pick-up time at the pick-up location.

Except for in terms of efficiency, we thoroughly analyzed the service quality of various different strategies. From the service quality point of view, deviation from the desired pick-up time and passenger ride time were the major passenger concerns (besides fare amount). Passenger wait time was calculated as the time difference between the requested pick-up time and the scheduled pick-up time. Passenger ride time was the actual drop-off time minus the actual pick-up time. Again, the passenger ride time could not exceed the maximum ride time factor for both intra-zonal and inter-zonal requests.

Table 5 shows the results generated by the three test scenarios. First, researchers observed that the no-zone system had the smallest number of vehicles, while the zoning with transfer and zoning without transfer policies had larger numbers. This may be attributed to the above two reasons. The no-zoning system had no restriction regarding choice of the next unassigned trip; thus, the probability of finding a better insertion was higher. In addition, in favor of the sequential insertion method, the number of trips in earlier build routes was higher than in the latter build route. Therefore, if the latter build route had only one or two inter-zonal trips, it might possibly have been served by one vehicle in a no-zoning system or in a zoning without transfer system; it must be served by two vehicles in a zoning with transfer case.

**Table 5 Comparison of performance for three zoning scenarios, $R$ varied according to trip length**

| Scenario | # of vehicles | Total miles | Deadhead miles | Empty miles | Passenger miles | Passenger miles/total mile | Passenger trips/revenue hour | Vehicle Idle time | Average Passenger waiting time* | Average Passenger Ride Time* |
|---|---|---|---|---|---|---|---|---|---|---|
| Zoning with transfer | 254 | 49,170 | 4,205 | 9,236 | 80,377 | 1.63 | 2.32 | 35,656 | 22.9 | 41.8 |
| No-zoning | 208 | 46,124 | 5,473 | 7,427 | 71,518 | 1.55 | 2.13 | 21,128 | 24.1 | 34.9 |
| Zoning without transfer | 266 | 48,907 | 5,839 | 16,149 | 71,251 | 1.46 | 1.74 | 25,586 | 22.8 | 34.7 |
| * Time in minute | | | | | | | | | | |

By allowing transfers for zoning policy, the deadhead miles and empty miles decreased, as compared to the zoning without transfer policy. For the operator, the smaller number of empty miles is a better result, because the passenger miles per vehicle revenue mile increases as the empty miles decreases. Zoning with transfer showed a significant improvement in passenger miles over both the no-zoning and zoning without transfer policy. The higher number of passenger miles could contribute to the longer trip's travel length or the higher rideshare rate. Since we use the same data set to run the simulation, it concludes that the zoning with transfer has higher rideshared rate. Although the zoning constrains the likelihood of finding a better insertion, researchers can see from the results that the transfer policy not only recovered the deficit from the no transfer case but significant increased the number. Due to the highest number of passenger miles among the three cases, the no-zoning policy showed the highest number of passenger miles per total mile.

Zoning with transfer improved the passenger trips per revenue hour, significantly. With the schedule and route coordination of inter-zonal customers at a particular transfer point, this strategy demonstrates that a zonal service that acts as a feeder and distributor would increase productivity. Such a transfer policy would increase the vehicle idle time, partially due to the vehicle's time spent idling at transfer point to pick-up inter-zonal customers for the latter build route. As for the level of service, the coordination at the transfer locations slightly increased the passenger waiting time, as compared to the waiting time of a zoning without transfer system. However, the passenger waiting time was still significantly lower when compared to a no-zoning case. A zoning with transfer policy showed the highest passenger ride time among the three scenarios. This was due to the inter-zonal trips having to switch vehicles at various transfer locations; thus, the system required some extra travel distance and additional waiting time. The passengers usually can endure longer travel time than waiting time.

Researchers further investigated the performance of three strategies if we use constant maximum ride time factor where $R$ is 2.5 for all trips (like done in other cities, such as Los Angeles). Table 6 shows the results generated by the three test scenarios.

**Table 6 Comparison of performance for three zoning scenarios, $R = 2.5$**

| Scenario | # of vehicles | Total miles | Deadhead miles | Empty miles | Passenger miles | Passenger miles/total mile | Passenger trips/revenue hour | Vehicle Idle time* | Average Passenger waiting time* | Average Passenger Ride Time* |
|---|---|---|---|---|---|---|---|---|---|---|
| Zoning with transfer | 238 | 55,531 | 3,899 | 7,770 | 85,980 | 1.55 | 2.47 | 30,626 | 23.5 | 44.9 |
| No-zoning | 191 | 50,085 | 4,952 | 5,255 | 85,698 | 1.71 | 2.30 | 17,990 | 24.4 | 42.1 |
| Zoning without transfer | 240 | 59,909 | 5,219 | 11,443 | 84,875 | 1.42 | 1.91 | 21,190 | 23.0 | 41.6 |
| * Time in minute | | | | | | | | | | |

First, researchers observe that, in this case, all scenarios have lower number of vehicle and higher passenger miles. By allowing larger maximum ride time factor for longer trips, the longer trips have larger drop-off time windows increasing the possibility to be inserted into an existing route. The differences in the total miles between the zoning without transfer and the zoning with transfer policies were nearly equal to the summation of the differences between the deadhead miles and the empty miles. Due to the lowest number of deadhead miles and the lowest number of empty miles among the three cases, the no-zoning policy showed the highest number of passenger miles per total mile. In this case ( $R = 2.5$), it inherently favors the no-zoning and zoning without transfer policies by the increase of passenger miles; however, the zoning with transfer system still has the highest passenger trips per revenue hour. As for the level of service, the coordination at the transfer locations also increased the passenger waiting time, as compared to the waiting time of a no-zoning system. The cost of higher efficiency is the decrease of service level. All three scenarios increased passenger waiting time and ride time, especially for the zoning without transfer and no-zoning policies.

## CHAPTER 11. CONCLUSION

The effects of including transfers between service zones were examined in-depth in the ADA paratransit system design. First, researchers constructed a mixed-integer linear formulation to prove the potential benefit of transfer design in a strict method. Second, we proposed an insertion-based heuristic that is computationally practical to solve realistically-sized problems thus help to draw the apparent benefits and cost of transfer design.

The results indicate that zoning with transfer can provide significant benefits to paratransit operations that are managing zoning structure. Our results used the demand data of the paratransit system in Houston, Texas (a relatively low-density region) for modeling and simulation purpose, and concluded that the zoning with transfer method proved to be a more productive organizational structure than zoning without transfer. It is worth emphasizing that no-zoning case adopted by Houston METRO still performs better than zoning cases on average in terms of efficiency. The transfer design in this research enabled the system to increase the passenger trips per revenue hour significantly without excessively increasing in-vehicle ride times for passengers. Furthermore, researchers consider the simulations of the three zoning scenarios indicative of their relative performances, in general. Although the exact level of benefit will vary according to the different demand types and different operational standards, this simulation methodology is easily and quickly adaptable to any large-scale or rural paratransit system. Future work should include combining the search of optimal transfer locations or the number of transfer locations to improve the performance of our proposed transfer system.

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University Transportation Center for Mobility™

Texas Transportation Institute

The Texas A&M University System

College Station, TX 77843-3135

Tel: 979.845.2538     Fax: 979.845.9761

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