APPENDIX A

APPENDIX A

OREGON STATE HIGHWAY DIVISION
FLEXIBLE PAVEMENT DESIGN PROCEDURE

Background

The Oregon State Highway Division's method of determining the structural thickness of a roadway is based on a soil strength-traffic relationship, and has been in use since 1951. Our system is essentially that of the California Division of Highways, with modifications for Oregon's soil conditions, traffic, and climate. The method described hereafter is based on empirical relationships developed from test roads and from pavement performance of various sections under traffic throughout the State. Modifications of the present procedure are anticipated as test methods are revised and service records of existing roadways obtained and analyzed.

Soils are tested for expansion pressure and resistance value by the AASHO T-190 66 I. Stabilometer R Value specimens are fabricated and tested to bracket both 300 psi exudation and saturation at 95% compaction. A 300 psi exudation design R Value is applied for thickness design for soils containing less than 90% passing the No. 4 sieve. The design R Value for other soils is selected at 95% compaction saturation moisture content; this is more realistic for Oregon, as it approaches the natural moisture contents that have been encountered under existing pavements. Additionally, when pumice, cinders, or silts are encountered, special tests (resilience) are made.

The average 18 kip Equivalent Axle Load (EAL) constant is calculated by the method outlined by Hveem and Sherman in Highway Research Record Number 13, Publication 1110. EAL constants are calculated annually from loadmeter data, and further augmented by a check of weights from 60-plus Truck Scale Sites located at strategic stations throughout the State. One month's weighing is selected for this check of weight constants. To obtain the design traffic coefficient for a particular project, the mean TADT is expanded to an average annual 18 kip EAL from the project average daily truck traffic and an expansion factor, then further expanded to the design period required, through the formula $TC = 9.0\ [(\text{Total 18 kip EAL's}) \div 10^6]^{0.119}$. The average daily truck traffic and expansion factor for each project is provided by the Planning Section.

The required total structural thickness for a roadway section is shown in terms of our standard specification crushed aggregate base. This thickness is referred to as Crushed Base Equivalent (CBE) inches. The total structural thickness requirement is obtained through the formula $CBE = 0.03546\ (TC)\ (100-R)$. Substitutions in the required CBE thickness are made to include treated bases, wearing surfaces, subbases, and treated subgrade materials.

Included is an outline of required laboratory tests, quality requirements, and equivalencies for various materials that satisfy the OSHD Standard Specifications. Also included are various tables and charts needed to select the layer thicknesses, and an example of the traffic analysis sheet.

Explanation of Terminology

CBE	Crushed base equivalent (equal to 1.0 inch aggregate base)
AC (FS)	Asphaltic concrete (final stage)
AC (I)	Asphaltic concrete (immediate)
ACB	Asphaltic concrete base
PMBB	Plant mix bituminous base
CTB	Cement-treated base
ETWS	Emulsion-treated wearing surface
ETB	Emulsion-treated base
CTERM	Cement-treated existing roadway materials
AL	Aggregate level
AB	Aggregate base
CRASB	Crusher-run aggregate subbase
GRASB	Grid-rolled aggregate subbase
LTS	Lime-treated subgrade
CTS	Cement-treated subgrade
WS	Wearing surface
CRPCC	Continuously reinforced portland cement concrete
TB	Treated base (either bituminous or cement)
R Value	A coefficient representing the shearing resistance to plastic deformation of a saturated soil at a given density.
EAL	18,000 pound equivalent axle load
Traffic Coefficient (TC)	A numerical value obtained from converting the magnitude and number of all axle loads into an equivalent number of 18,000 pound axle loads.

Flexible Pavement Design

Design for Frost

1. The requirement for frost protection has been a part of OSHD's surfacing thickness design since at least 1951. Normally, where plus 8% of the subgrade material passes the No. 200 sieve, the total thickness recommended would be equal to one-half the depth of known frost penetration. The depth of frost is based on a survey made in 1953-1954, plus a report by the Regional Geologist for each project. This criteria, as nearly as can be determined, is a minimal requirement in surfacing designs where frost is a problem, and has been established through years of experience and field observation. In certain areas in the State, the need for an even thicker section may be required, and in these instances we use the Corps of Engineers' criteria, which could range from two-thirds to full depth of frost penetration.

Minimum Thickness for Traffic Coefficient

2. In any design procedure it is necessary to consider construction and maintenance problems, if under-design is to be avoided. For these reasons, the OSHD has used minimum thicknesses for various traffic classifications since 1951. The criteria used to establish the varius minimum thicknesses is a result of experience and field observations during the past years. The chart used at present is merely an extension of the original (1956) minimums, and so far has proven valid.

Determination of "R" Value

3. For fine-grained soils the OSHD uses a method of selecting the design moisture content at 95% of the maximum density toward saturation, as this more nearly duplicates the natural moistures found under the existing pavements. This method has been in use intermittently since about 1956, and as an integral part of the design method since 1964. At that time (1964), an extensive program of natural moisture sampling was instigated on a statewide basis. Results from over a thousand tests indicate that with OSHD's compaction requirements (T-99), the method being used is more comparable to the condition of the soil during field compaction than is the more rigid compaction requirement of 300 exudation pressure as applied in the laboratory, which would more or less compare to T-180.

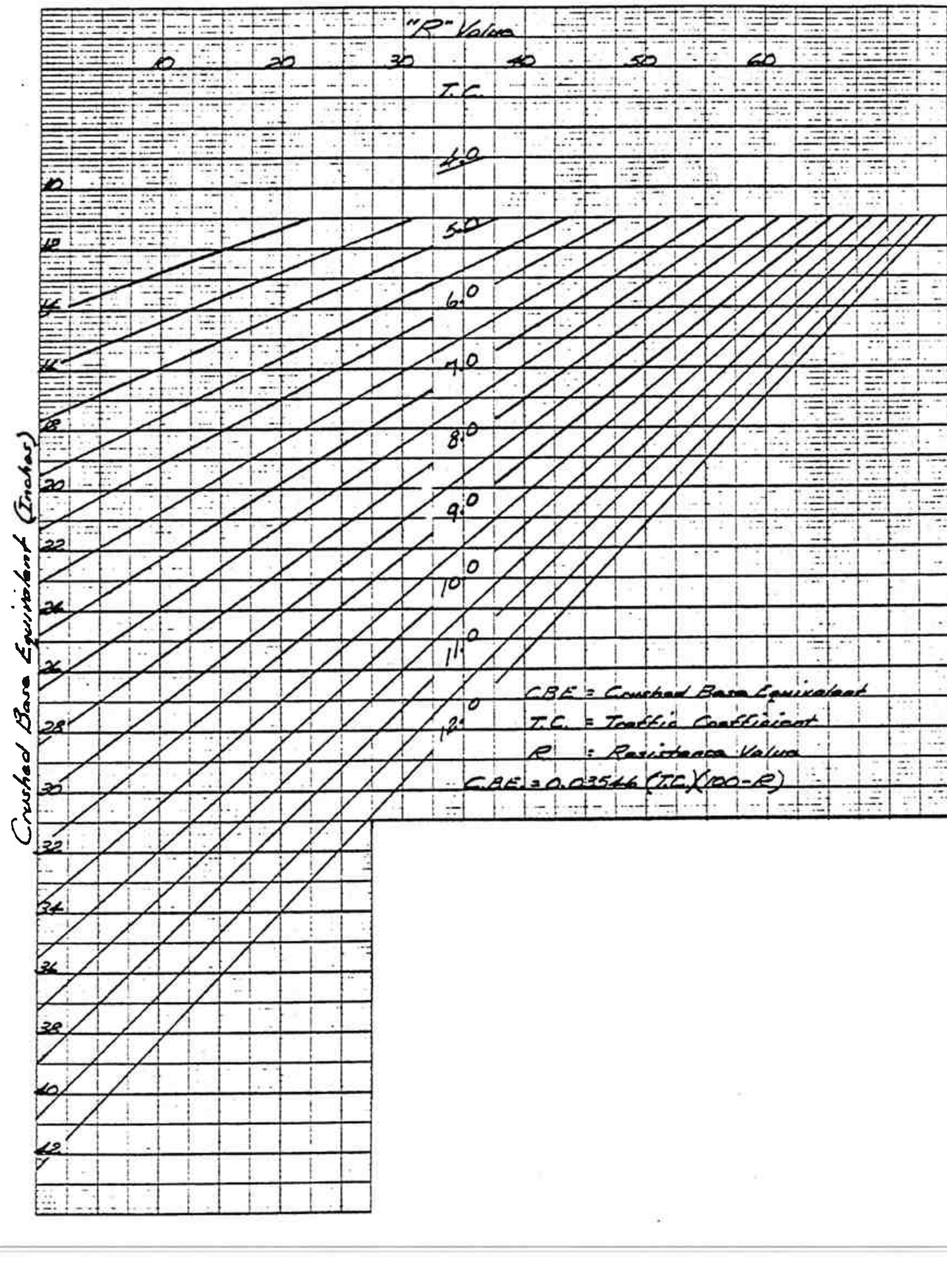


Figure A1 - Chart to Determine Required Crushed Base Equivalent

CBE REQUIREMENT

MINIMUM THICKNESS

ALL ROADS

Traffic Coefficient	18 kip EAL per day	AC Wearing Surface	PMBB or CTB	Aggregate Level	Aggregate Base	Minimum "CBE" Requirement	Actual Thickness
12.0-13.0	1440-2822	4.0"	8.0"	-	7.0"	29.5"	19.0"
11.0-12.0	693-1440	4.0"	8.0"	-	4.5"	27.0"	16.5"
10.0-11.0	311-693	4.0"	6.0"	-	5.5"	24.5"	15.5"
9.0-10.0	128-311	4.0"	5.0"	-	5.0"	22.0"	14.0"
8.0-9.0	48-128	3.5"	4.0"	-	6.0"	20.0"	13.5"
7.0-8.0	16-48	3.5"	3.0"	-	5.0"	17.5"	11.5"
6.0-7.0	4-16	3.0"	-	2.0"	7.0"	15.0"	12.0"
4.8-6.0	1-4	2.0"	-	2.0"	6.5"	12.5"	10.5"
Below 4.8 Cars Only		2.0"	-	2.0"	5.0"	11.0"	9.0"

Where untreated material is used the minimum aggregate base on any project is 4.0".

Crushed Base Equivalent Factors for various materials that comply with the Standard Specifications and Special Provisions are as follows:

1.0" Asphaltic Concrete Wearing Surface & Base	=	2.0" Aggregate Base
1.0" Cement-Treated Base	=	1.8" Aggregate Base
1.0" Plant Mix Bituminous Base	=	1.8" Aggregate Base
1.0" Emulsion-Treated Wearing Surface and Base	=	1.8" Aggregate Base
1.0" Oil Mat	=	1.8" Aggregate Base
1.0" Cement-Treated Existing Roadway Material	=	1.5" Aggregate Base
1.0" Lime or Cement-Treated Subgrade	=	1.0" Aggregate Base
1.0" Aggregate Subbase	=	0.8" Aggregate Base

CONVERSION TABLE - TRAFFIC COEFFICIENT TO 18 KIP STANDARD AXLES

T.C.	18 kip EAL per day	Trucks per day
12.0-13.0	1440-2822	2727-5272
11.0-12.0	693-1440	1308-2727
10.0-11.0	311-693	548-1308
9.0-10.0	128-311	232-548
8.0-9.0	48-128	84-232
7.0-8.0	16-48	30-84
6.0-7.0	4-16	8-30
4.8-6.0	1-4	2-8
Cars only (4.0)		

Use trucks per day for estimating when traffic classification is not available.

TRAFFIC ANALYSIS DATA SHEET

SECTION: ____________________

HIGHWAY: ____________________

COUNTY: ____________________

____________________ Present ADT (19)

____________________ Percent Trucks

AXLES	TWO WAY TRUCK ADT	EXPANSION FACTOR	TOTAL TADT	MEAN TADT	ANNUAL ONE WAY 18 KIP EAL	AVERAGE ANNUAL 18 KIP EAL
2	______	______	____	____	36.5	______
3	______	______	____	____	119.5	______
4	______	______	____	____	157.0	______
5	______	______	____	____	296.0	______
6	______	______	____	____	325.0	______
				____	TOTAL	

18 kip EAL/day ____________________

TOTAL AVERAGE ANNUAL 18 kip EAL ____________________

TWENTY-YEAR 18 kip EAL ____________________

TRAFFIC COEFFICIENT ____________________

OUTLINE OF PAVEMENT STRUCTURE DESIGN AND QUALITY REQUIREMENTS

I. Subgrade

  A. Samples of Native Soil

    1. Frequency

      a. One each 1/4 mile minimum

      b. Changes in soil as evidenced by test or visual examination

    2. Depth

      a. Three to five feet below expected subgrade elevation

  B. Laboratory Tests

1. Mechanical Analysis	Method - AASHO T-88
2. Liquid Limit	Method - AASHO T-89
3. Plasticity Index	Method - AASHO T-90
4. Specific Gravity	Method - Modified AASHO T-100
5. Moisture Density Relation	Method - AASHO T-99 or Miniature Harvard Method
6. Compaction for Stability	Method - AASHO T-190
7. Expansion Pressure	Method - AASHO T-190
8. Resistance to Deformation	Method - AASHO T-190

    9. Natural Moisture. Dried to constant wt. at 220°F.

  C. Required Cover Thickness - INches of Crushed Rock or Crushed Base Equivalent

    1. CBE = (TC) (100-R) (0.03546)

      a. TC = Traffic Coefficient

        (1) Determined as indicated in Form A

      b. R = Resistance Value as determined in I-B-8

        (1) Fine Soils = 90 to 100% pass #4

(a) R Value at moisture content indicated at 95% max. density as determined in I-B-5

(2) Soils less than 90% pass #4

(a) R Value at moisture content indicated at 300 psi exudation pressure

II. Crushed Base Requirements - Quarry Rock or Gravel

A. Specified Limits and Method of Test

1. Percent Crushed - one face fracture on particles larger than 1/4 inch - % by weight

a. Max. size 1-1/2 inch and greater - 50% crushed

b. Max. size 1 inch and less - 70% crushed

2. Abrasion. Method - AASHO T-96

a. 35% max.

3. Liquid Limit and Plasticity Index. Method - AASHO T-89 & T-90

a. Limits vary with quantity passing the #40 screen

4. Sand Equivalent. Method - AASHO T-176

a. 35 minimum

5. Size and Gradation. Method AASHO T-27

a. 3/4" - 0

(1) 0-10% retained on 3/4"

(2) 20-40% pass 3/4" and retained on 3/8"

(3) 40-60% pass 1/4", % of 1/4" - 0 retained on #10 = 40-60

b. 1" - 0

(1) 0-10% retained on 1"

(2) 20-40% pass 1" and retained on 1/2"

(3) 40-55% pass 1/4", % of 1/4" - 0 retained on #10 = 40-60

c. 1-1/2" - 0

(1) 0-5% retained on 1-1/2"

(2) 20-40% pass 1-1/2" and retained on 3/4"

(3) 35-50% pass 1/4", $ of 1/4" - 0 retained on #10 = 40-60

d. 2" - 0

(1) 0-5% retained on 2"

(2) 20-40% pass 2" and retained on 1"

(3) 30-45% pass 1/4", % of 1/4" - 0 retained on #10 = 40-60

e. 2-1/2" - 0

(1) 0-5% retained on 2-1/2"

(2) 20-40% pass 2-1/2" and retained on 1-1/4"

(3) 30-45% pass 1/4", % of 1/4" - 0 retained on #10 = 40-60

6. Degradation. Method - Oregon Highway Department. Attached Description B

a. Materials shown to degrade are required to be upgraded equal to specified crushed rock by means of asphalt, cement or other treatment

7. "R" Value - 80 plus

III. Crushed Base Equivalents

A. Stone Subbase

1. Minimum

a. Abrasion - 45% max.

b. Percent passing #200 - 8% max.

c. Sand equivalent - 25 min.

d. Max. size - 75% compacted thickness, max.

e. Gradation - 10 to 50% pass 1/4"

2. CBE

a. "R" Value 70 to 80 - CBE = 0.8

b. "R" Value 60 to 70 - CBE = 0.5

c. "R" Value below 60 - CBE not applied

B. Plant Mix Bituminous Base

1. Minimum Requirements

a. One inch max. size meeting crushed base requirements

b. Bituminous mixture laboratory designed

(1) "S" Value minimum 35. Method - ASTM D1560

(2) "C" Value minimum 200. Method - ASTM D1560

(3) 70% minimum index of retained strength. Method - AASHO T-165

2. CBE 1.8

C. Cement-Treated Base

1. Minimum requirements

a. One inch maximum size meeting crushed base requirements

b. Cement content is laboratory determined on basis of 1000 psi in 7 days

2. CBE 1.8

D. Asphaltic Concrete

1. Minimum Requirements

a. Aggregate

(1) Crushed base quality graded to maximum density curve

b. Mixture - Laboratory designed

(1) "S" Value minimum 35. Method - ASTM D1560

(2) "C" Value minimum 200. Method - ASTM D1560

(3) 70% minimum index of retained strength test.
Method - AASHO T-165

(4) Proportioned to 3 to 5% void content

c. Production. Inspector controlled

2. CBE 2.0

OREGON STATE HIGHWAY DIVISION

OVERLAY THICKNESS DESIGN

The present system of determining overlay requirements is by deflection measurements. The deflection method used is essentially that of the California Division of Highways, with modifications for Oregon's traffic and Crushed Base Equivalencies. The same test procedure was published by AASHTO in July 1978. The deflection data is further augmented by test pits from which each component of the roadbed structure is sampled and laboratory tested.

Deflection measurements provide a method of nondestructive testing of the strength of the roadway under a given load; this closely duplicates the actual load-carrying capacity of the in-place materials.

Following is a brief description of the equipment required and the test procedure:

The Benkelman Beam and a truck with 11.00 x 22.5 tires, 70 psi pressure, loaded to a single axle weight of 18,000 pounds.

Several sections varying from 700 to 1000+ feed per centerline mile are selected as representative of the area. The deflection measurements are made at 50-foot intervals throughout the test sections. Generally test sections are selected every half mile in alternating directions.

After testing is completed, the following procedure is used to obtain the required overlay thickness:

Truck traffic is obtained from the traffic section and converted to a traffic coefficient for the proposed design period.

The deflection measurements are evaluated statistically (temperature corrected if necessary, Figure 1) and reported as the average (mean), the standard deviation, and 80th percentile deflection values. The 80th percentile deflection is equal to the average deflection plus 0.84 times the standard deviation of the deflections.

After the evaluated 80th percentile deflection is calculated, enter Figure A2 with the traffic coefficient and follow this value vertically to the curve corresponding to the deflection (80th percentile) obtained and read the thickness of AC overlay required of the vertical scale.

An outline of common surfacing terminology is given following the design charts.

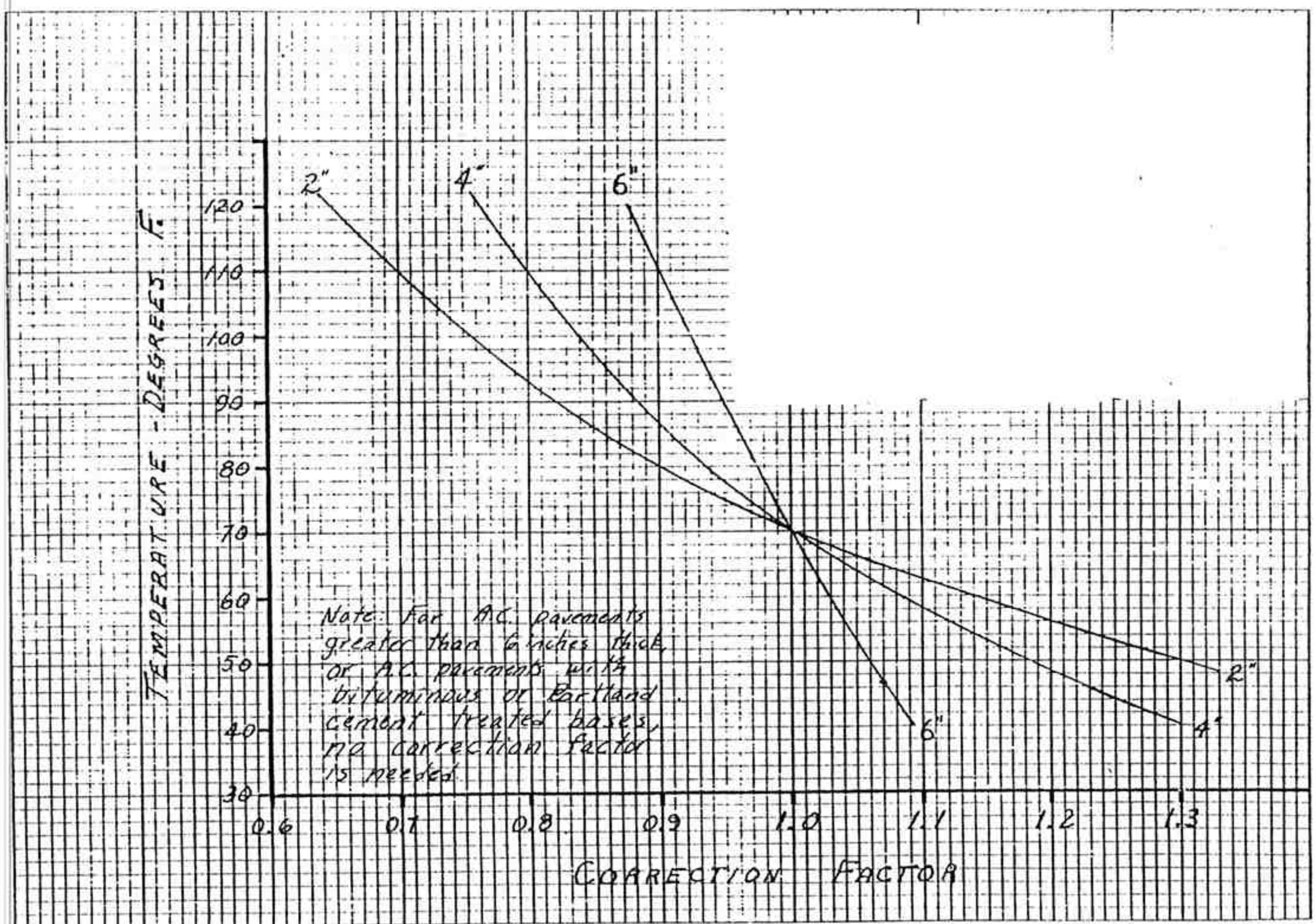


Figure A2 - Temperature Correction Chart (OSHD, 1980)

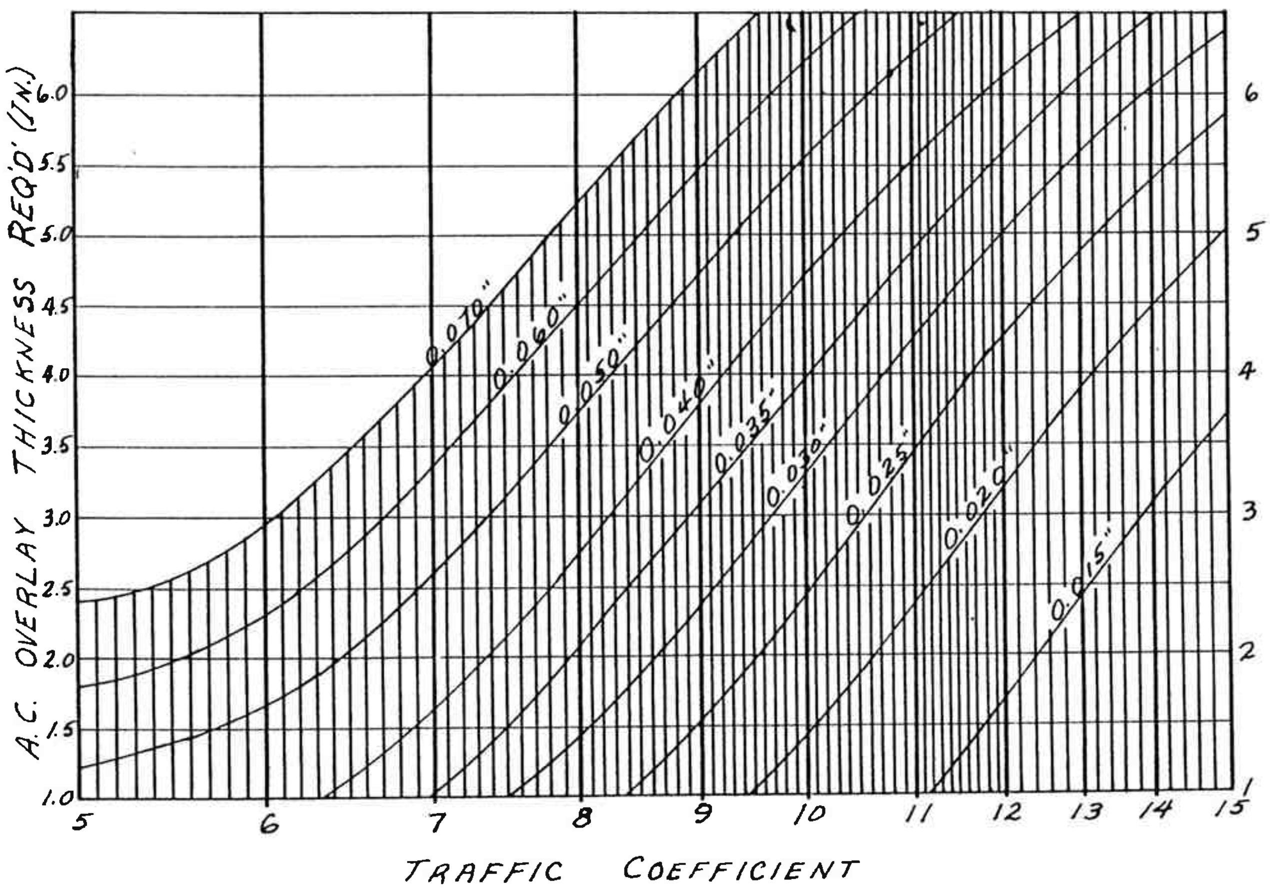


Figure A3 - Asphalt Concrete Overlay Design Chart

Common Surfacing Terminology

Cracks	-	approximately vertical cleavage due to natural causes of traffic action.
Longitudinal crack	-	a crack which follows a course approximately parallel to the centerline.
Transverse crack	-	a crack which follows a course approximately at right angles to the centerline.
Block cracking	-	interconnected cracks forming a series of large blocks.
Hairline cracking	-	a crack barely visible to the eye when pavement is dry, easily seen when pavement is damp.
Raveling	-	the progressive disintegration from the surface downward or edges inward by dislodgement of aggregate particles.
Spalling	-	the breaking away of the pavement along cracks, joints, or edges.
Rutting	-	the formation of longitudinal depressions in the wheel tracks.
Pumping	-	displacement and ejection of water and suspended fine particles at joints, cracks, and edges.
Pot Holes	-	bowl-shaped holes or crater-like depressions of varying sizes in the pavement.
Distortion	-	any deviation of pavement surface from original shape.
Erosion or Scaling	-	displacement of particles of aggregate from pavement surface due to traffic action.
Frost Heave	-	differential upward displacement due to frost.

Bird Bath	-	a depression in the pavement surface that temporarily ponds water.
Disintegration	-	deterioration into small fragments or particles due to any cause.
Patching	-	the correction of pavement defects by maintenance forces, usually the application of bituminous mix.

<u>AC</u>

Alligator Cracking	-	interconnected cracks forming a series of small polygons which resemble an alligator's skin.
Stripping	-	loss of adhesion between binder and aggregate.
Washboarding or Corrugations	-	regular transverse undulations in the surface of the pavement consisting of alternate valleys and crests.
Waves	-	as above, but with a greater distance between valleys and crests.
Flushing or Bleeding	-	upward migration of the bituminous material resulting in a film of free asphalt on the pavement surface.

<u>PCC</u>

Blow-up	-	localized buckling or shattering, usually at a transverse crack of joint, due to excessive longitudinal pressure.
Faulting	-	differential vertical displacement of the slabs adjacent to a crack of joint.
Warping	-	a deviation of the pavement surface from its original slope, caused by temperature and moisture differentials within the slab.

APPENDIX B

APPENDIX B

USE OF LAYERED ELASTIC THEORY

(Including Operating Instructions for ELSYM5)

B.0 INTRODUCTION

This appendix contains detailed information on the background to and use of Layered Systems Analysis. The major portion of this appendix is taken from reference (3) in the body of this report by R.G. Hicks entitled, "Use of Layered Theory in the Design and Evaluation of Pavement Systems."

B.1 LAYERED SYSTEM ANALYSIS

Procedures for prediction of traffic-induced deflections, stresses and strains in pavement systems are based on the principle of continuum mechanics. The essential factors that must be considered in predicting the response of layered pavement systems are: (1) the stress-strain behavior of the materials; (2) the initial and boundary conditions of the problem; and (3) the partial differential equations which govern the problem. The highway engineer, however, need only concern himself with the stress-strain behavior of the material, the physical configuration of the problem, and the general assumptions that have been made or implied in developing solutions to the layered system problem.

Reasonably good predictions of pavement response to load can be obtained provided that carefully selected material properties are used with theories employing realistic assumptions. Unfortunately, the solution of the pavement system problem requires the use of a high-speed digital computer. If an engineer selects a formula that is not applicable to his set of conditions, an incorrect answer is obtained; likewise, if a computer program not suited to the particular problem is used, equally poor results are obtained. Therefore, to properly use the theoretical solutions which are now available, an engineer must thoroughly understand the assumptions and limitations associated with the use of these methods.

Elastic Layered Systems

The response of pavement systems to wheel loadings has been of interest since 1926 when Westergaard (1) used elastic layered theory to predict the response of rigid pavements. Later Burmister (2) solved the problem of elastic multilayered pavement structures (Figure B1) using classical theory of elasticity. The assumptions that Burmister and most others (3,4) have made in developing closed formed solutions are as follows:

1. Each layer acts as a continuous, isotropic, homogeneous, linearly elastic medium infinite in horizontal extent;
2. The surface loading can be represented by a uniformly distributed vertical stress over a circular area;
3. The interface conditions between layers can be represented as either perfectly smooth or perfectly rough;
4. Each layer is continuously supported by the layer beneath;
5. Inertial forces are negligible;
6. Deformations throughout the system are small; and
7. Temperature effects are neglected.

The partial differential equations associated with the boundary value problem can be solved by the use of integral transforms (3,4). The response is then obtained in the form of infinite integrals that must be numerically integrated. If a sufficiently close integration interval spacing is not used, or

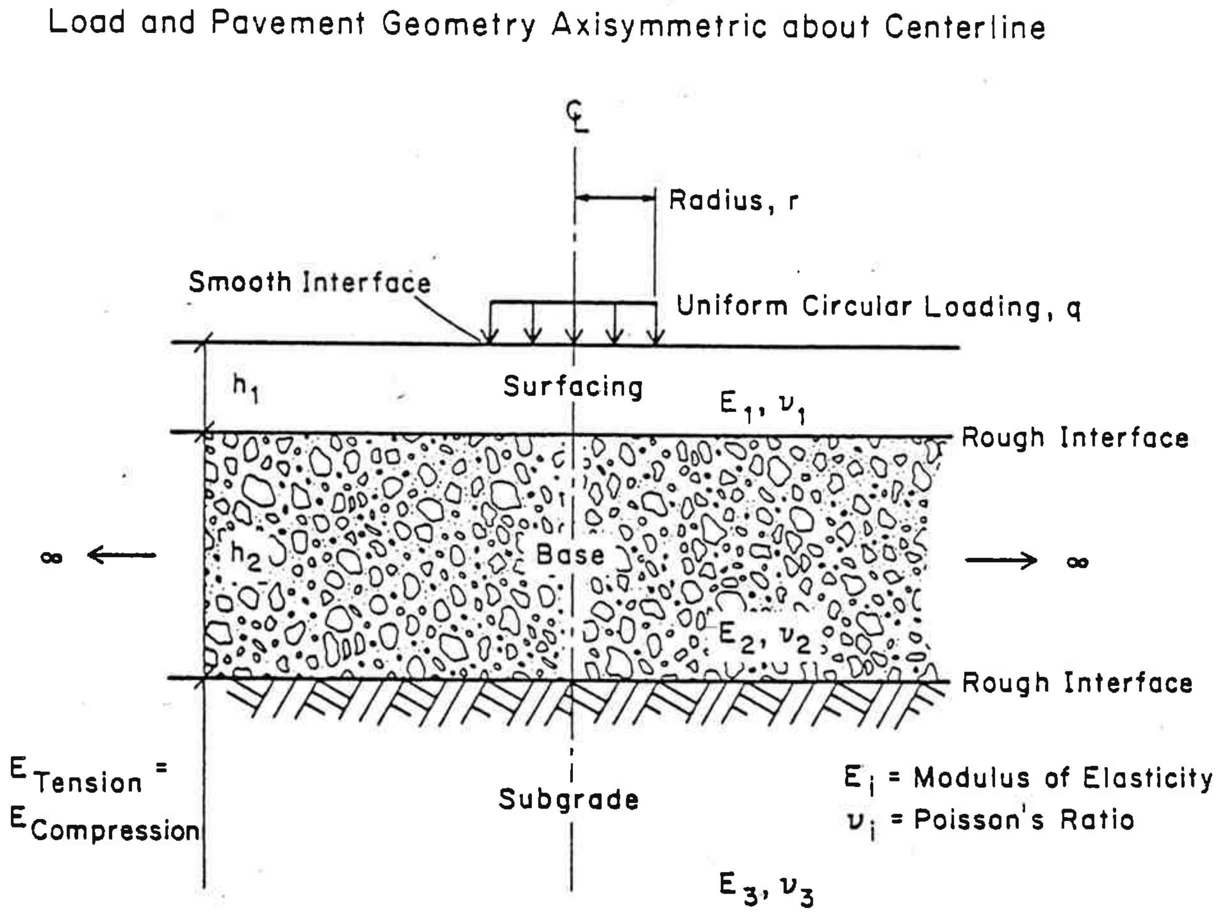


Figure B1 - Classical Linear Elastic Layered Pavement Structure Idealization if the integration is not carried out far enough before "chopping" it off, convergence of the integral to the correct value may not occur.

Comprehensive tables and charts of influence values for 2- and 3-layer systems subject to uniform circular loadings are given in the literature (5,6). The use of these tables and charts can be quite tedious and time-consuming for 3-layer pavement systems, and tabulated solutions for 4-layer systems are not practical. Therefore, for general pavement design applications, the use of a computer is a necessity from a practical standpoint.

Limitations of Layered Theory

In classical layered theories, the pavement structure is normally modeled as an axisymmetric solid. Axisymmetry usually means that both load and pavement geometrics are symmetrical about a common centerline. Unfortunately, the effects of wheel loads applied close to a crack or pavement edge cannot be analyzed by the use of methods which require axisymmetry. Although 3-dimensional solid models could be used with finite-element methods, that representation is not practical for general use because of the large amount of computer time required to solve the model. An extended 2-dimensional finite element program that approximates the loading as a Fourier series has been used to study the effects of edge loadings for multiple rectangular wheel loadings (7). Although this approach could lead to a much better understanding of pavement behavior, it also requires too much computer time for general use in a design method.

Informatino is not available on the conditions of slip which exits at the interface between layers. The assumption of a rough interface condition, which most investigators have used (4,11) appears to be reasonable, although varying degress of slip can be considered (3).

In all of the theoretical approaches, inertial forces have been neglected. The inertial force is simply the force on a small element caused by a dynamic loading and is equal to the mass of the element times the acceleration. Also, none of the layered system theories consider the effects of vibrations. Neglecting vibrations is probably not a bad assumption for vehicle speeds lower than 96 km/hr (60 mph) on materials that have cohesion. However, for cohesionless materials compacted to lower relative densities, neglecting vibratory effects may lead to densification that would cause rutting and changes in material properties.

Numerous laboratory tests have indicated that the dynamic modulus of paving materials varies with the confining pressure or deviator stress or both (8,9). The modulus is normally given by the following:

$$(M_R) = \frac{\sigma_d}{\varepsilon_r} \text{, psi} \tag{1}$$

where σ_d = repeated axial stress, psi

ε_r = resilient (or elastic) strain

Because of the variation in stress state that exists in each layer of the pavement system, the dynamic modulus actually changes with both depth and lateral position in each layer. Therefore, uncertainties arise in trying to determine what value of dynamic modulus to use in representing each layer in a linear-elastic layered analysis. Furthermore, elastic layered theory cannot consider variations in the modulus with lateral position. Those limitations for the most part can be overcome by the use of nonlinear finite-element theory (8,10). With this technique the pavement response is initially calculated by using assumed moduli for each layer. The calculated stresses are then used to estimate a new stress-dependent modulus from experimentally measured material properties. Additional stress states are then calculated, and the process is repeated by either an iterative or an incremental procedure. In both cases, the modulus is matched with the stress state in each element. This approach, however, requires considerably more computer time than does a single elastic layered solution.

An excellent alternative approach, which is a practical trade-off, is the use of a nonlinear, iterative elastic layered solution (8,11,12,13). This iterative procedure is analogous to the one used for finite-element theory. In this approach, the base and subgrade can be subdivided into several fictitious layers for better accuracy. The technique uses in each layer a modulus that is dependent on the average stress state which exists in the vicinity beneath the wheel loadings.

Design Implications

Presently several agencies are adopting the use of elastic layered theory in the design and evaluation of pavement systems (14,15,16,17). Shell (14) has incorporated fatigue in the design of highway pavements since 1963. The criterion developed by Shell has also been used extensively since 1963 in the design and evaluation of pavement systems subjected to unusual wheel loads. The Asphalt Institute followed, using similar, yet more sophisticated, tools to develop a procedure to design and evaluate airfield pavements to account for jumbo jet operations (Manual Series 11) and have just this year (1981) issued a new design manual (Manual Series 1) for use in designing highway pavement (15,16).

The Kentucky Highway Department has also developed a design procedure using layered elastic theory (17). The Chevron program (4) was used in this procedure to calculate stresses and deformations in the pavement systems and design criteria were developed based on observed field performance.

More recently, Chevron Research (18) has developed a "Simplified Thickness Design Procedure for Asphalt and Emulsified Asphalt Pavements." The procedure is based on analysis of layered systems (4) and considers only 2-layered systems (full-depth asphalt pavement design plus subgrade). Critical strains in the pavement system are limited to values depending on expected service life. Thicknesses are then determined to minimize the amount of permanent deformation and/or fatigue cracking.

In all these methods, actual stresses or strains in the pavement layers are used to design flexible pavements against the occurrence of cracking (fatigue) or rutting (permanent deformation) (Figure B2). The tensile strain (ε_t) or stress in the case of cement stabilized layers, is normally limited to preclude fatigue-type cracking. The compressive strain (ε_c) on the subgrade is commonly used to preclude rutting. Under a single load, the maximum tensile strain occurs directly beneath the load. In the case of dual or multiple wheels, the maximum strain can occur at other locations. For duals, it could be at points 1, 2 or 3 as shown in Figure B2.

Once the critical strains or stresses are determined, they are normally compared with limiting values such as given in Figure B3. Figure B3a shows a typical relationship of tensile strain in the asphalt layer vs. the number of repetitions to failure; for a given value of strain calculated, one can easily estimate the number of repetitions to cause fatigue cracking.

Fatigue and/or rutting criteria can be developed either from laboratory or field tests. Most fatigue criteria, such as that shown in Figure B3a have been developed from laboratory tests, using either laboratory prepared or field samples. To predict pavement life, these laboratory developed criteria are shifted to the right by a factor ranging from as low as 3 to as great as

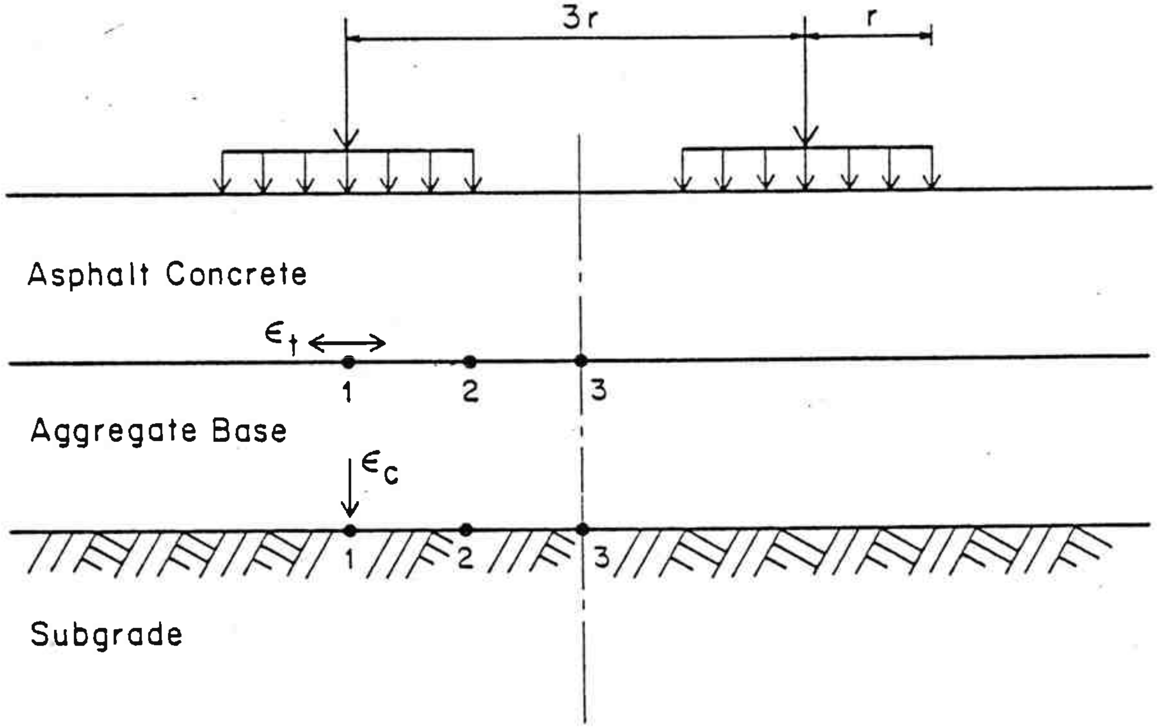


Figure B2. Location of Critical Stresses or Strains in a Layered System

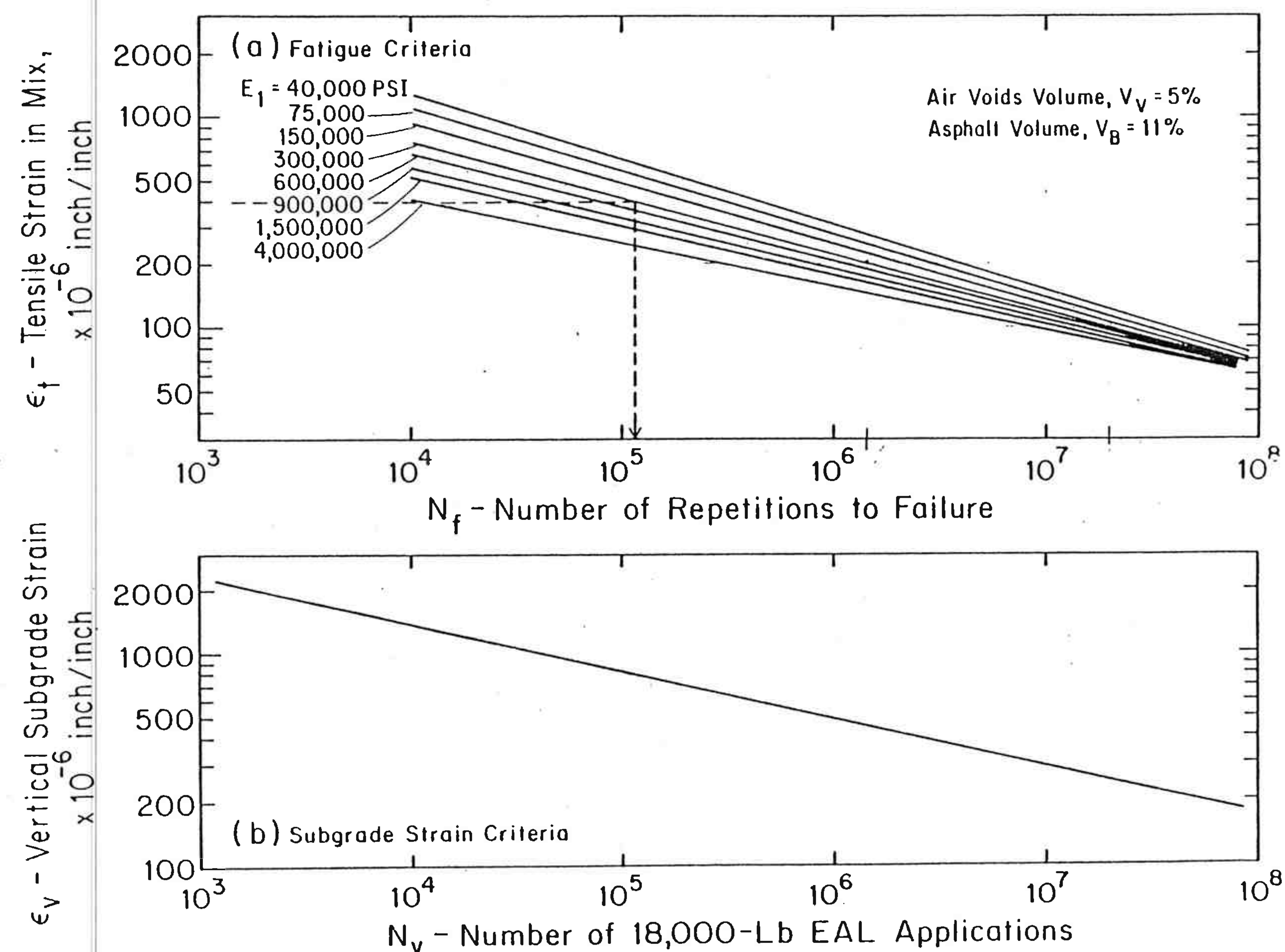


Figure B3 Limiting Strain Criteria to Preclude Fatigue Cracking and Rutting (After reference 18)

20, depending on the test (18,23). This is because the fatigue life measured in the field is always greater than that measured in the laboratory. The most important reason for the difference is the effect of rest periods between successive load applications. The curves shown in Figure B3a have already been shifted to simulate field conditions by a factor of 3 to 5. Most rutting criteria have been developed from field studies. For a given pavement section, the depth of rutting and numbers of repetitions to cause a specific rut depth are recorded. The calculated vertical compressive subgrade strain associated with a given rut depth for the estimated number of load applications can then be established.

In addition to providing users with a capability for better pavement design, layered theory also offers users a method of evaluating pressing problems such as:

1. Impact of increased highway loads; and
2. Effect of using marginal materials on pavement performance.

Many other opportunities may exist for practical use of layered elastic theory. This document should provide engineers with a basic understanding of how to make use of available computer solutions to layered elastic systems.

B.2 COMPUTER PROGRAMS

This chapter presents a detailed discussion of five computer solutions to layered systems. All programs have the capability of solving for stresses, strains and displacements for n-layer systems. The limitations of each program are also included.

Multilayered Elastic System

Description

The Multilayered Elastic System computer program (CHEV5L) will determine the various component stresses and strains in a three-dimensional ideal elastic layered system with a single vertical uniform circular load at the surface of the system (Figure B4). The bottom layer of the system is semi-infinite with all other layers of uniform thickness. All layers extend infinitely in the horizontal direction. The top surface of the system is free of shear and all interfaces between layers have full continuity of stresses and displacements.

With a vertical uniform circular load, the system is axisymmetric with the Z axis perpendicular to the layers and extending through the center of the load. Using cylindrical coordinates, any point in the system may be described by R and Z values. R is the horizontal radial distance out from the center of the load and Z is the depth of the point measured vertically from the surface of the system.

The load is described by the total vertical load in pounds and the contact pressure in psi. The load radius is computed by the program. Each layer of the system is described by modulus of elasticity, Poisson's ratio, and thickness in inches. Each layer is numbered, with the top layer as 1 and each layer below numbered consecutively.

Program Operating Notes

The program operates with the various given R and Z values as follows: For every R value a complete set of characterizing functions is developed for all layers, then the stresses and strains are computed at those points represented by that R and each of the given Z values. The stresses calculated are

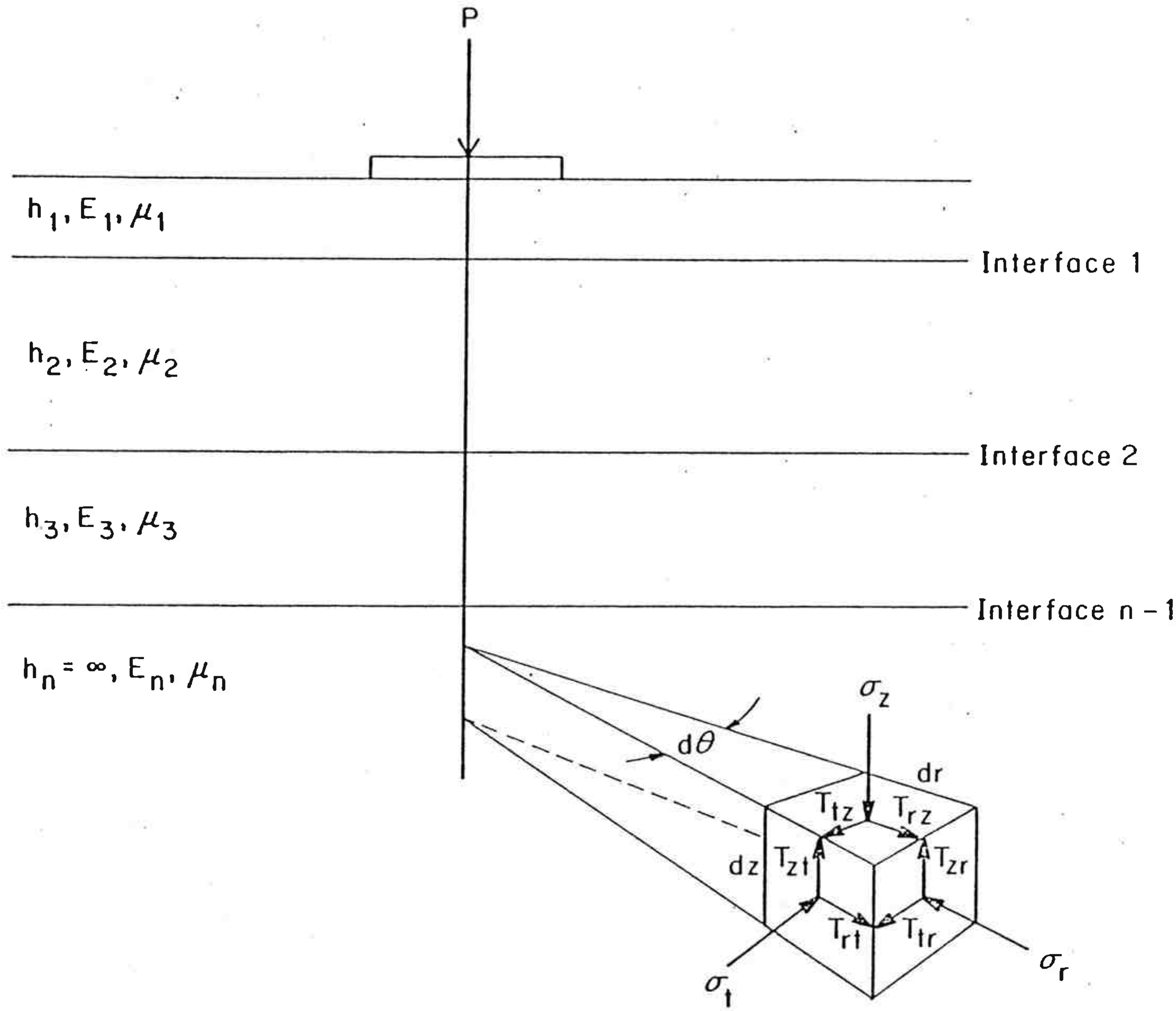


Figure B4 - Generalized Multilayered Elastic System (After Reference 22) shown in Figure B4. The program then steps to the next R value and computes the stresses and strains at those points represented by each of the given Z values and continues until all combinations of R and Z values are used.

When a given Z value is exactly at an interface between two layers, the program will first compute the stresses and strains at this point using the functions for the upper of the two layers, then will recompute the stresses and strains at this same point using the functions from the lower of the two layers. In the output of the program, a negative Z value indicates that the stresses and strains have been computed at an interface and that the characteristics of the upper layer have been used.

Limitations

The following are limitations of the program and/or method.

1. Number of layers in the system: minimum of two and a maximum of five.
2. Number of points in the system where stresses and strains are to be determined: minimum of one (one R and one Z) to a maximum of 121 (maximum of 11 R and 11 Z).
3. All data are positive, no negative values.
4. Poisson's ratio must not have a value of one.
5. Nonlinear behavior of granular bases and subgrade soils cannot be taken into account.
6. Multiple gears cannot be handled directly. Calculations of critical stresses and strains under multiple gears must be done using the principle of superposition (by hand).

Multilayered Elastic System - Iterative Method

Description

The Multilayered Elastic System computer program (CHEV5L with iteration) is also used to determine stresses and strains (Figure B4) in a three-dimensional elastic layered system with a single vertical uniform circular load. The program is an extension of CHEV5L and has the capability of accounting for variations in the modulus of each material with depth.

As with CHEV5L, all layers are assumed to extend indefinitely in the horizontal direction. The top surface is free of shear and all interfaces between layers have full continuity of stresses and displacements. A vertical uniform circular load is applied and stresses, strains and displacements are calculated at any point in the system described by R and Z values. The load is described by the total vertical load in pounds and contact pressure in psi.

Program Operating Notes

The basic differences between CHEV5L and CHEV5L with iteration lies in the method of assigning modulus of elasticity and Poisson's ratio to each layer. Examination of materials characterization studies indicates that the modulus of most materials is dependent on the level of stress (or temperature).

In general, the modulus of cohesive soils decreased with increasing repeated stress level σ_d (Figure B5) and is relatively unaffected by small changes in confining pressure. In this program, the modulus of the subgrade (bottom layer) is interpolated from the input modulus-deviator stress relationship. For materials that are not stress-dependent, a horizontal relationship must be input. The variation of Poisson's ratio with stress level is less clear although Hicks and Finn (19) found that it remained

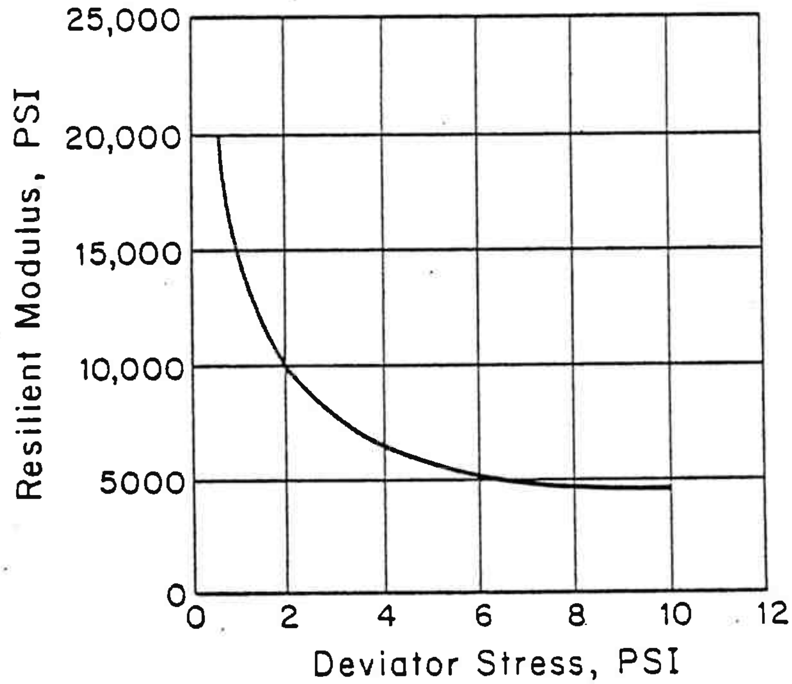


Figure B5 - Resilient Modulus vs. Deviator Stress, Subgrade, Morro Bay (After Reference 11) constant or increased slightly with increasing deviator stress. Poisson's ratio appears not to be significantly affected by confining pressure.

For unstabilized granular materials, the modulus is most affected by confining pressure and slightly affected by the deviator stress or stress frequency. As shown in Figure B6, the modulus of granular materials can be approximated by:

$$M_R = k\sigma_3^n$$

or (2)

$$M_R = \bar{k}\theta^{\bar{n}}$$

where k, $\bar{k}$, n, $\bar{n}$ are constants evaluated from repeated load triaxial test results and σ_3 and θ are confining pressure and the bulk stress ($\theta = \sigma_1 + 2\sigma_3$ in a conventional triaxial test), respectively. Poisson's ratio has been found to remain relatively constant over a range of stress conditions (20).

Results of repeated load triaxial tests on emulsion mixes have indicated that, at early stages of cures, confining pressure most affects the modulus. The behavior of these materials is very similar to that of granular bases (Figure B7). As the curing process progresses, the materials tend to behave more like hot-mix asphalt concrete (Figure B8). Their properties are most affected by temperature and rate (or frequency) of loading as shown in Figure B9 (21). Results indicate that Poisson's ratio may increase with increasing temperature and is affected only slightly by stress level.

Because the modulus of most materials are dependent on the level of stress, an iterative approach (in which the modulus and stress level interaction can be allowed to close on a system having compatible values of each) was developed as follows:

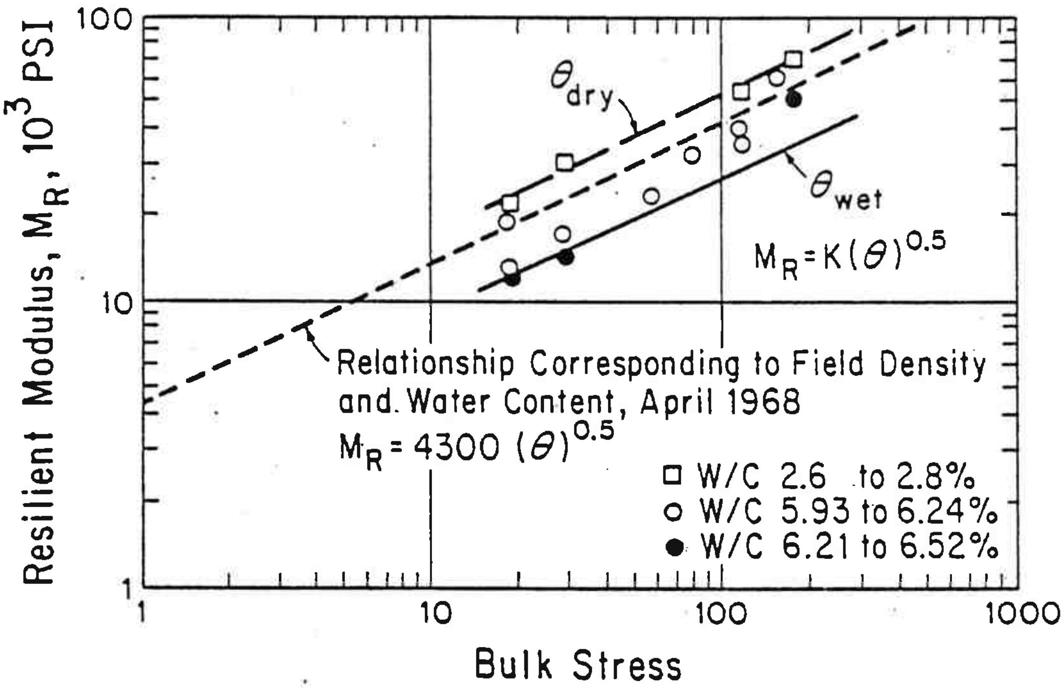


Figure B6 - Resilient Modulus as a Function of Bulk Stress ($\theta = \sigma_1 + 2\sigma_3$); Aggregate Base (After Reference 19)

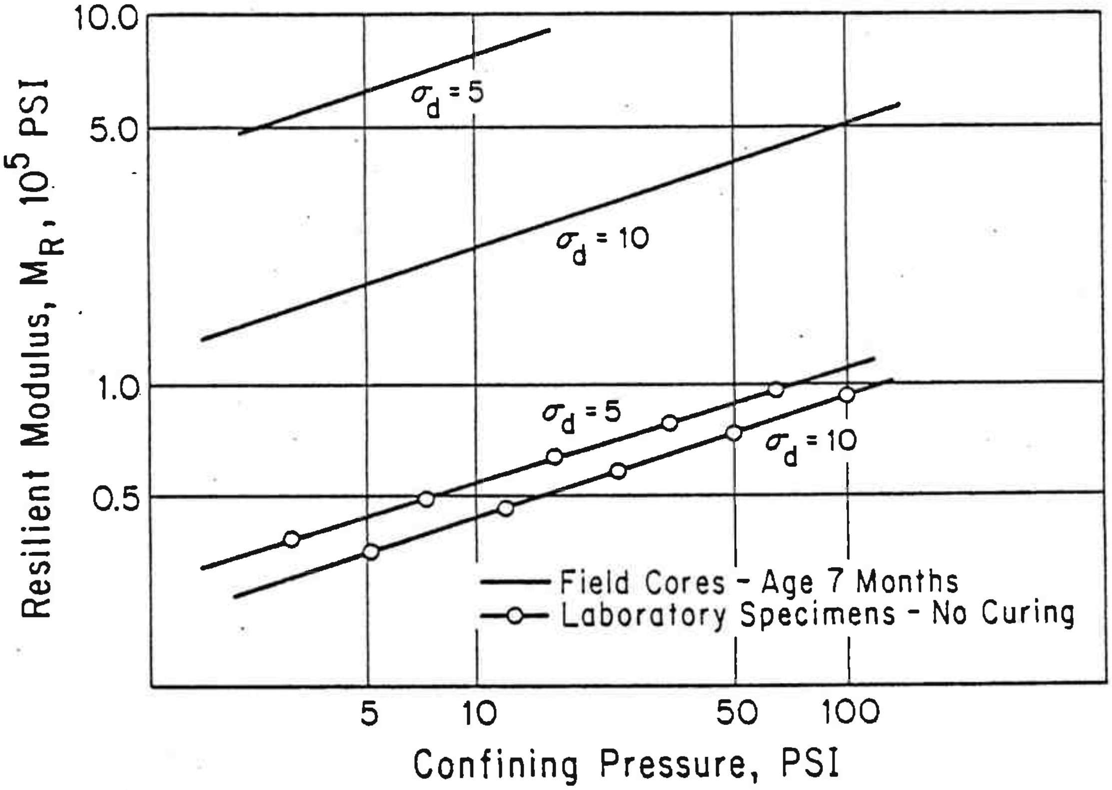


Figure B7 - Comparison of Resilient Modulus vs. Confining Pressure for Specimens of Emulsion Treated Special Aggregate at Different Stages of Curing. Note, σ_d = repeated stress level (After Reference 19)

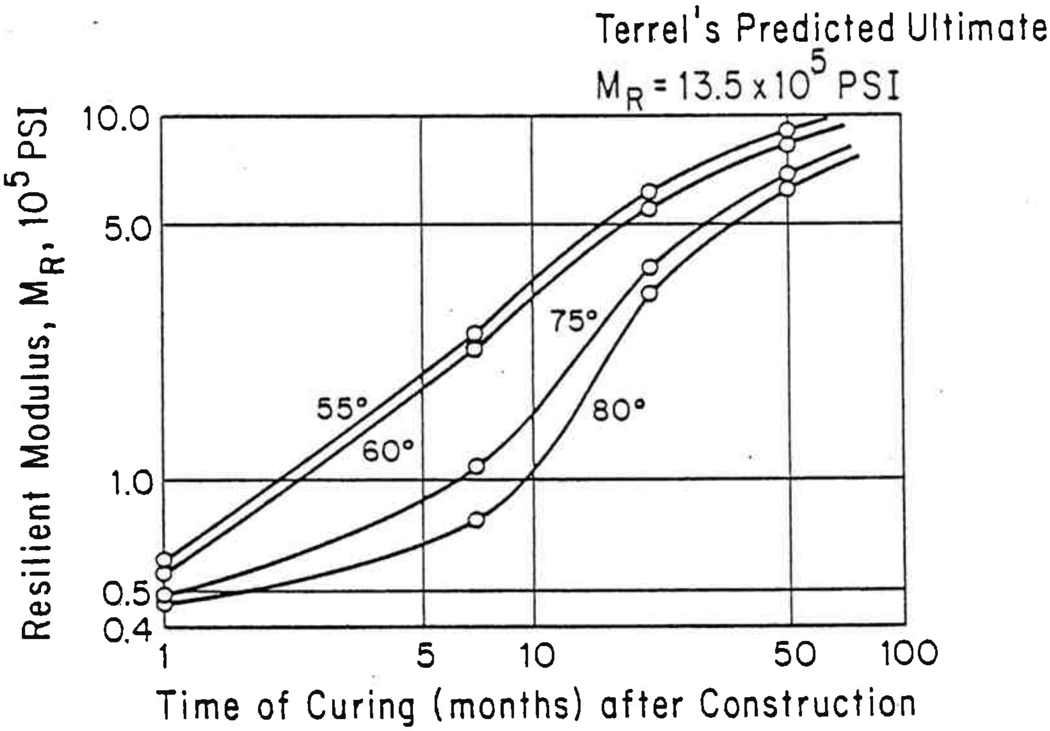


Figure B8 - Resilient Modulus vs. Curing Time for Emulsion Treated Aggregates (After Reference 19)

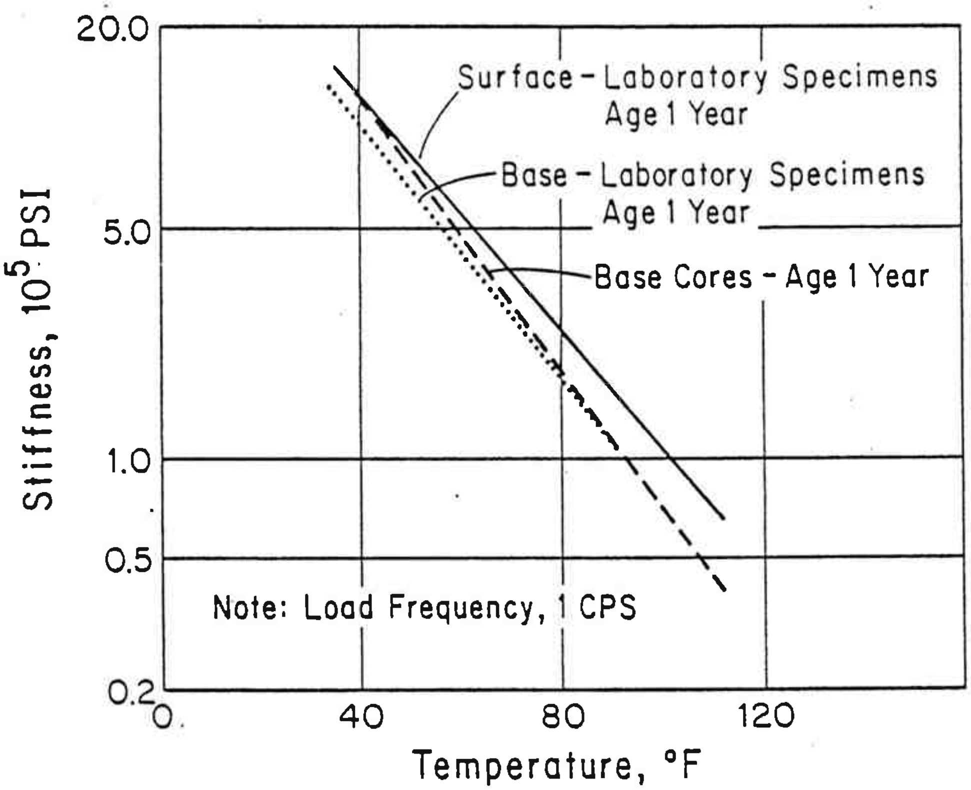


Figure B9 - Stiffness Modulus vs. Temperature for Asphalt Concrete Surface and Base (After Reference 19)

1. The pavement to be analyzed can be represented by a number of layers consistent with the dimensions of the structural section.
2. The modulus value and Poisson's ratio for each of these layers can be estimated with some degree of accuracy based on the known variation of these values with the estimated stress and environmental conditions.
3. The stresses which would occur in this system under the application of the surface load can be calculated using available computer solutions.
4. The pre-existing stress state owing to overburden pressures can be calculated from knowledge of the densities and dimensions of the pavement materials.
5. The resulting stress state can be obtained by superposition of the load-induced and overburden stresses.
6. The modulus which is compatible with the resulting stress state in each layer can be determined from the appropriate modulus-stress relationship for the materials.
7. The modulus of each layer required by the stress state can be compared with the initially assumed value and the process repeated, using the resulting values, until the initial and final modulus values coincide within a specified accuracy.

In CHEV5L with iteration, an average modulus under an arbitrary set of dual wheels (center to center spacing of 3R) is calculated using the procedure outlined above. Calculations must be made a the locations indicated in Figure B10.

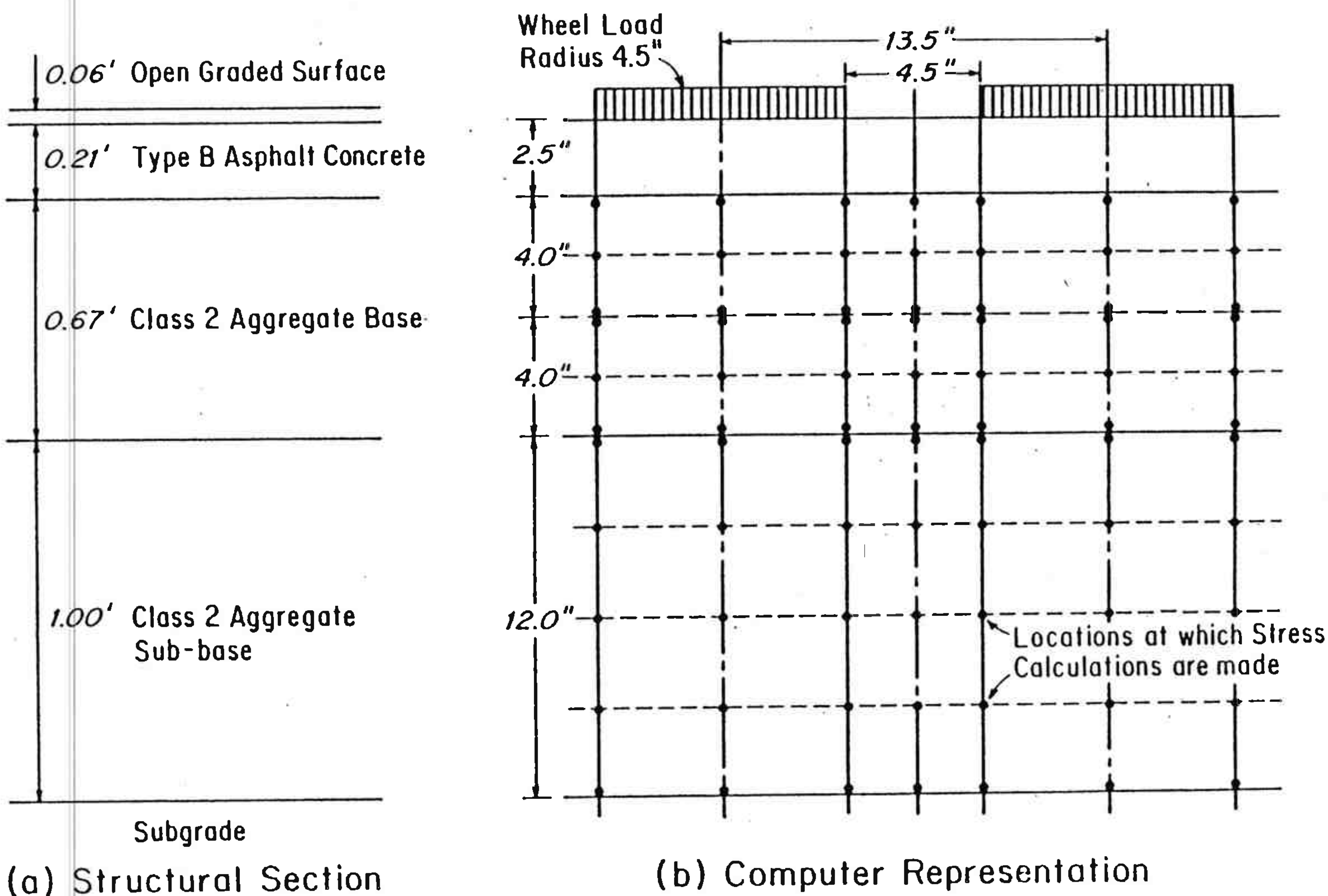


Figure B10 - Morro Bay Pavement, (V-SLO-56-C,0) (After Reference 11)

Once the iteration process has closed, calculations for stresses, strains and displacements in the system are made (for one wheel loading) as in the case of CHEV5L.

Limitations

The following are limitations of the program and/or method:

1. Number of layers in system: 5 must be used. (See example problem in Chapter 4.)
2. Number of points in system where stresses and strains are to be determined: minimum of 6 (0, 1r, 1-1/2r, 2r, 3r, 4r required) to maximum of 11 R values; minimum of eight (top, middle and of each nonlinear layer) to maximum of 11 Z values; all as shown in Figure B10.
3. All data are positive, no negative values.
4. Poisson's ratio must not have a value of one.
5. Multiple gears cannot be handled directly. Calculations of critical stresses and strains under multiple gears must be done (by hand) using the principle of superposition.

Multilayered Elastic System - Multiple Load Option (Shell BISAR)

Description

The BISAR (Bitumen Structures Analysis in Roads) program is a general purpose program for computing stresses, strains and displacements in elastic layered systems subjected to one or more vertical uniform circular loads applied at the surface of the system. Unlike the CHEV5L programs, the surface loads can be combinations of a vertical normal stress and a unidirectional horizontal stress. All layers extend infinitely in the horizontal direction. The top surface of the system is free of shear. All interfaces between layers have an interface friction factor which can vary between zero (full continuity) and one (frictionless slip) between the layers.

Stresses, strains and displacements are calculated in a cylindrical coordinate system for each vertical load. For more than one load, the cylindrical components are transformed to a Cartesian coordinate system and the effect of the multiple load found by summarizing the stresses, strains and displacements of each wheel. Further, the program calculátes only those components which are requested (Table B1)*. If all stresses and strains are calculated, the program calculates the principal stresses and strains and their accompanying directions. The principal directions denote the normals of the planes through the point considered, which are free of shear stress (strain). The highest and lowest of the three principal values give the maximum and minimum normal stresses (strains), and the difference between the principal values divided by two, gives the maximum shear stresses (strains).

For a given problem to be solved using the BISAR program, one needs information regarding:

1. The number of layers;
2. Young's modulus and Poisson's ratio of each layer;
3. The thickness of each layer, except for the bottom one;
4. The interface friction at each interface;
5. The number of loads, the vertical and tangential component of each load, and the position of the loads;
6. The stress, strain and displacement components to be calculated; and

*Any or all of the types of computations may be requested.

Table B1

Stresses, Strains and Displacements Calculated by BISAR*

Displacements	UR	-	Radial Displacement
	UT	-	Tangential Displacement
	UZ	-	Vertical Displacement
Stresses	SRR	-	Radial Stress
	STT	-	Tangential Stress
	SZZ	-	Vertical Stress
	SRT	-	Radial/Tangential
	SRZ	-	Radial/Vertical
	STZ	-	Tangential/Vertical
Strains	ERR	-	Radial Strain
	ETT	-	Tangential Strain
	EZZ	-	Vertical Strain
	ERT	-	Radial/Tangential
	ERZ	-	Radial/Vertical
	ETZ	-	Tangential/Vertical
Total Displacements	UX	-	x-Displacement
	UY	-	y-Displacement
Total Stresses	SXX	-	xx Component of Total Stress
	SXY	-	xy Component of Total Stress
	SXZ	-	xz Component of Total Stress
	SYY	-	yy Component of Total Stress
	SYZ	-	yz Component of Total Stress
Total Strains	EXX	-	xx Component of Total Strain
	EXY	-	xy Component of Total Strain
	EXZ	-	xz Component of Total Strain
	EYY	-	yy Component of Total Strain
	EYZ	-	yz Component of Total Strain

*For additional details, see reference (24).

7. The number of places where calculations are required along with their position (Cartesian coordinates).

Program Operating Notes

BISAR consists of a main program and 24 subprograms. The main program reads all the input data defining the numerical problem and controls the subsequent steps in the calculation of the requested stresses, strains and displacements. The output is partly controlled by the main program and by subprograms SYSTEM, CALC, and OUTPUT. Subprograms MACON1, CONPNT, INGRAL and MATRIX give output only when error messages are generated.

The main program can consider several mulitlayered systems in one run (to a maximum of 99). For each multilayer system, the stresses, strains and displacements due to each load separately and by transforming these to the Cartesian coordinate system. The Cartesian components are added to those of the preceding loads and by the time the last load has been considered, the total stresses, strains and displacements have been calculated. The computed results are printed (separately) for each position requested.

Limitations

The following are limitations of the program and/or method:

1. Number of layers in the system: maximum ot 10, although this can be changed with modifications to the program.
2. Number of systems in one run: maximum of 99.
3. Number of points in the system where stresses and strains can be calculated: maximum of 99.
4. Nonlinear behavior of granular bases and subgrade soils cannot be accounted for.

Multilayered Sustem - Multiple Load Option (ELSYM5)*

Description

The Elastic Layered System computer program (ELSYM5) will determine the various component stresses, strains and displacements along with principal values in a three-dimensional ideal elastic layered system, the layered system being loaded with one or more identical uniform circular loads normal to the surface of the system.

The top surface of the system is free of shear. Each layer is of uniform thickness and extends infinitely in the horizontal direction. All elastic layer interfaces are continuous. The bottom elastic layer may be semi-infinite in thickness or may be given a finite thickness, in which case the program assumes the bottom elastic layer is supported by a rigid base. With a rigid base, the interface between the bottom elastic layer and the base has to be made either fully continuous or slippery.

All locations within the system are described by using the rectangular coordinate system (X,Y,Z) with the XY plane at Z = 0 being the top surface of the elastic system where the loads are applied. The positive Z axis extends vertically down from the surface into the system.

The applied loads are described by any two of the three following items: loads in pounds, stress in pounds per square inch, radius of loading area in inches. The program determines the missing value. Each layer of the system is described by modulus of elasticity, Poisson's ratio and thickness. Each layer is numbered with the top layer as one and numbering each layer consecutively downward.

*This write-up is from text written by Gale Ahlborn, ITTE, University of California at Berkeley, 1972.

Program Operating Notes

The program tests all input data. If any input data is out of range as specified under "Limitations," the problem is terminated for that system with an error message and the program goes on to the next system for operation.

The program uses the convention that compressive stresses are negative and tensile stresses are positive.

The output of the program gives for each depth (Z) all the results for all the XY points. The results for each point are the total results for that point obtained by summing the contribution by each load. When a Z value is determined to be on an interface, the results are determined using the characteristics of the upper of the two layers.

Operating instructions and sample data for ELSYM5 are shown in Figures B11 and B12.

Limitations

Following are the limitations of the program and/or method.

1. Number of different systems for solution: minimum of one, maximum of five.
2. Number of elastic layers in the system: minimum of one, maximum of five.
3. Number of identical uniform circular loads: minimum of one, maximum of 10.
4. Nonlinear behavior of granular bases and subgrade soils cannot be accounted for.
5. Number of points in the system where results are desired: minimum of one (one XY and one Z), maximum of 100 (10 XY and 10 Z).

1)	CC 1- 5	number of systems to be run.	(INTEGER)
		FORMAT (I5)	
2)	CC 1- 3	punch the number 999,	(INTEGER)
	CC 5-60	title to identify problem,	(ALPHA)
		FORMAT (I3,A57)	
3)	CC 1- 5	number of elastic layers in the system,	(INTEGER)
	CC 6-10	number of uniform circular loads to be applied normal to the surface of the system,	(INTEGER)
	CC11-15	number of XY locations where results are desired,	(INTEGER)
	CC16-20	number of Z locations where results are desired,	(INTEGER)
		FORMAT (4I5)	
4)	CC 1- 5	layer number,	(INTEGER)
	CC 6-10	thickness of layer in inches,	(REAL)
	CC11-15	Poisson's ratio of layer,	(REAL)
	CC16-25	modulus of elasticity for layer, in psi.	(REAL)
		FORMAT (I5, 2F5.0,F10.0, 4X,[A2])	
		Note: One card for each elastic layer in the system, leave thickness blank for bottom elastic layer when layer is to be semi-infinite in thickness. If bottom elastic layer is resting on a rigid base, insert the thickness of the bottom elastic layer and CC30-31 (ALPHA) punch FF for full friction rigid base interface <u>or</u> CC30-31 (ALPHA) punch NF for no friction rigid base interface. Cards have to be in sequence from top to bottom elastic layer.	
5)	CC 1-10	load force in pounds,	(REAL)
	CC11-20	load pressure in pounds per square inch,	(REAL)
	CC21-30	load radius in inches.	(REAL)
		Note: Any two of the above items can be input, program determines the third. Only one card required.	
		FORMAT (3F10.0)	
6)	CC 1-10	X position of a load,	(REAL)
	CC11-20	Y position of a load.	(REAL)
		Note: One card per load.	
		FORMAT (2F10.0)	
7)	CC 1-10	X position for evaluation,	(REAL)
	CC11-20	Y position for evaluation.	(REAL)
		Note: One card for each XY position for evaluation.	
		FORMAT (2 F10.0)	
8)	CC 1- 5	first Z value for evaluation,	(REAL)
	CC 6-10	second Z value for evaluation,	(REAL)
	CC11-15	third Z value for evaluation, etc.	(REAL)
		Note: Only one card required, maximum of ten values on the card.	
		FORMAT (10F5.0)	

Cards 2-8 are repeated for each different system to be solved.

Figure B11 - Elsyms Operating Instructions

IBM FORTRAN Coding Form

GX28-7327-6 U/M 050**
Printed in U.S.A.

PROGRAM: INPUT FORMAT - ELSYM5

PROGRAMMER | DATE | PUNCHING INSTRUCTIONS | GRAPHIC | PUNCH | PAGE OF | CARD ELECTRO NUMBER*

	Statement Number	Fortran Statement
1		FORMAT(I5) #SYSTEMS TO BE SOLVED
2	999 IDENTIFICATION	FORMAT(I3,A57)
3		FORMAT (4I5) #LAYERS, #LOADS, # OF XY LOCATIONS WHERE RESULTS DESIRED, # OF Z LOCATIONS WHERE RESULTS DESIRED
4	J H(J) V(J) E(J)	FORMAT(I5,2F5.Ø, F10.Ø) (ONE CARD PER LAYER J)
5	LOAD PRESSURE RADIUS	FORMAT(3F1Ø.Ø) (INPUT ANY TWO)
6	X(J) Y(J)	FORMAT (2F1Ø.Ø) (ONE CARD PER LOAD J)
7	X(I) Y(I)	FORMAT(2F1Ø.Ø) (ONE CARD PER POINT OF INTEREST I)
8	Z(1) Z(2) Z(3) ...	FORMAT(1ØF5.Ø)

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms per pad may vary slightly

Figure B-12 - Input Format - ELSYM5

6. Where there is a rigid base specified, the maximum Z value cannot exceed the depth to the rigid base.
7. All input values except XY positions must be positive.
8. Poisson's ratio must not have a value of one. Poisson's ratio for a bottom elastic layer on a rigid base must not be within the range of 0.748 to 0.752.
9. The program uses a truncated series for the integration process that leads to some approximation for the results at and near the surface and at points out at some distance from the load.

Multilayer Elastic Theory Iterative Method - Dual Wheel Option (PSAD2A)*

Description

PSAD2A is essentially the same as CHEV5L with iteration except that the former has the added capability of printing stresses, strains and displacements due to dual wheel configurations. This feature of PSAD2A is not an option; it is performed automatically.

Program Operating Notes

In the case of dual wheels, PSAD2A allows the distance between loads (from edge to edge) to vary between zero and two load radii for the calculation of an average modulus when iterating, whereas CHEV5L with iteration fixes this distance at one load radii. This can be inferred from the operating notes on CHEV5L with iteration, where it is stated that an average modulus is calculated under an arbitrary set of dual wheels spaced three radii center to center.

*Write-up is based on an excerpt from Report TE 70-5, ITTE, University of California at Berkeley, 1973 (12).

Other than this one difference, the solution method used in PSAD2A is the same as that used by CHEV5L with iteration.

Limitations

1. The number of data sets for the relationship between resilient modulus and deviator stress for the subgrade must be at a minimum of two and at a maximum of 20. A number outside this range will result in an error message and termination of the run.
2. The relationship between resilient modulus and deviator stress for the subgrade may be constant or have negative slope, but the first point must have an abscissa (i.e., stress value) of zero. This may require backward extrapolation of experimental data.
3. Five layers must be used.
4. Consult appropriate limitations for CHEV5L with iteration.

Potential Applications

The question often arises as to the purpose for using layered theory in the design and evaluation of pavement systems. This is particularly so when several presently used pavement design procedures (e.g., AASHO, California, etc.) appear to be very sound. However, many decisions were made in their development (because of lack of necessary tools or better information) that invalidate extrapolation of present design procedures to conditions different from that for which they were developed.

In recent years, considerable emphasis has been placed on the development of a more mechanistic design procedure, one which would allow extrapolation to any set of design conditions. This is particularly important because of ever increasing wheel loads such as those from off-highway vehicles. Layered theory analysis uses actual load data and fundamental material properties and can properly account for rapidly changing design conditions. This does not mean, however, that layered theory analysis will be a practical design tool, because input to the design process requires sophisticated materials testing and computational equipment. What it does mean is that layered analysis techniques can provide the necessary tools to understand and account for:

1. The impact of increased loads on the performance of the pavement system.
2. An evaluation of realistic layer equivalencies for structural materials heretofore not used (e.g., marginal).
3. Verification or modification of load equivalencies in the design of pavement systems and in the assignment of maintenance responsibilities in the case of dual ownership.

To be specific, the following applications are recommended for each of the five programs described herein:

1. CHEV5L. This program is the simplest to operate. It should always be considered first to give a "ball park" solution to a particular problem. The most significant limitation is its inability to handle nonlinear material problems. Therefore, its use should probably be limited to full-depth asphalt pavements over subgrade.
2. CHEV5L with Iteration. This program is slower than CHEV5L but does allow one to account for nonlinear material behavior. This program should be used where a considerable amount of untreated aggregate is present (e.g., unsurfaced roads).
3. Shell BISAR. This program, because of its multiple gear option, would be most useful in the evaluation of off-highway loads.

Further, the additional capability of horizontal stresses and ability to vary friction between layers offer capabilities which none of the others can. However, computational experience of the author with the horizontal stress option indicates that excessive time may be used by the computer to converge to a solution. This option should be used only with extreme caution.

4. ELSYM5. This program, similar to the Shell BISAR model, was found to be the most efficient in terms of computer time. Applications for ELSYM5 are similar to those of BISAR since multiple loads can be considered.

5. PSAD2A. While akin to CHEV5L with iteration, PSAD2A allows evaluation of stresses, strains and displacements due to a dual wheel load configuration. PSAD2A has the capability of handling nonlinear material behavior; in this way it is similar to CHEV5L with iteration.

These five programs (or their equivalent) should provide pavement designers with the necessary tools to evaluate almost any unusual design situation.

B.3 SUMMARY AND RECOMMENDATIONS

This report has presented in simple terms a description of layered elastic analyses and five computer programs which solve the fundamental differential equations. Further, the potential use of layered elastic analysis in the design and evaluation of highway pavements has been discussed.

It is recommended that highway engineers utilize the computer programs to assist in various decision-making processes such as:

1. Establishing more realistic load equivalencies;

B.4 REFERENCES

1. Westergarrd, H.M., "Stresses in Concrete Pavements Computed by Theoretical Analysis," Public Roads, Vol. 7, No. 2, 1926, pp. 25-35.

2. Burmister, D.M., "The General Theory of Stresses and Displacements in Layered Systems," Journal Applied Physics, 1945.

3. Peutz, M.G.F., Van kempen, H.P.M., and Jones, A., "Layered Systems Under Normal Surface Loads," Highway Research Record 228, 1968, pp. 34-45.

4. Warren, H. and Dieckman, W.L., "Numerical Computation of Stresses and Strains in a Multiple Layer Asphalt Pavement System," Chevron Research Corp., Unpublished internal report, 1963.

5. Jones, A., "Tables of Stresses in Three-Layer Elastic Systems," HRB Bulletin 342, 1962, pp. 176-214.

6. Peattie, K.R., "Stress and Strain Factors for Three-Layer Elastic Systems," HRB Bulletin 342, 1962, pp. 315-353.

7. Pichumani, R., "Theoretical Analysis of Airfield Pavement Structures," Air Force Weapons Lab., Kirtland Air Force Base, Tech. Report AFWL-TR-71-26, July 1971.

8. Hicks, R.G., "Factors Influencing the Resilient Properties of Granular Materials," Institute of Transportation and Traffic Engineering, University of California, Berkeley, Dissertation Series, May 1970.

9. Pell, P.S. and Brown, S.F., "The Characteristics of Materials for the Design of Flexible Pavement Structures," Proc., Third International Conference on Structural Design of Asphalt Pavements, 1972.

10. Duncan, J.M., Monismith, C.L. and Wilson, E.L., "Finite Element Analysis of Pavements," Highway Research Board Record 228, 1968, pp. 18-33.

11. Kasianchuk, D.A., "Fatigue Considerations in the Design of Asphalt Concrete Pavements," University of California, Berkeley, Ph.D. Dissertation, 1968.

12. Monismith, C.L., Epps, J.A., Kasianchuk, D.A., and McLean, D.B., "Asphalt Mixture Behavior in Repeated Flexure," Report No. TE 70-5, University of California, Berkeley, December 1970, 303 pps.

13. Barksdale, R.D., "Analysis of Layered Systems," School of Civil Engineering, Georgia Institute of Technology, July 1969.

14. Shell International Petroleum Co. Ltd., "1963 Design Charts for Flexible Pavements," 1963 and "Shell Pavement Design Manual," 1978.

15. The Asphalt Institute, "Full-Depth Asphalt Pavements on Air Carrier Airports," Manual Series No. 11, January 1973.

16. The Asphalt Institute, "Thickness Design - Asphalt Pavements for Highways and Streets," Manual Series No. 1, September 1981.

17. Havins, J.H., Deen, R.C. and Southgate, H.F., "Pavement Design Scheme," Highway Research Board Record 140, Washington, D.C., 1973.

18. Santucci, L.E., "Thickness Design Procedure for Asphalt and Emulsified Asphalt Mix," Proc., Fourth International Conference on the Structural Design of Asphalt Pavements, 1977.

19. Hicks, R.G. and Finn, F.N., "Prediction of Pavement Performance from Calculated Stresses and Strains at the San Diego Test Road," Proc., Association of Asphalt Paving Technologists, Williamsburg, VA, 1974.

20. Allen, J.J., "The Effects on Non-Constant Lateral Pressures on the Resilient Response of Granular Materials," Ph.D. Dissertation, Dept. of Civil Engineering, University of Illinois, May 1973.

21. Terrel, Ronald L., "Factors Influencing the Resilient Characteristics of Asphalt Treated Aggregates," Institute of Transportation and Traffic Engineering, University of California, 1967.

22. Yoder, E.J. and Witczak, M.W., Principals of Pavement Design, 2nd Edition, Wiley, 1975.

23. Brown, S.F. and Pell, P.S., "A Fundamental Structural Design Procedure for Flexible Pavements," Proc., Third International Conference on Structural Design of Asphalt Pavements, 1972.

24. DeJong, D.L., Peutz, M.G.F. and Kornswagen, A.R., "Computer Program BISAR, Layered Systems Under Normal and Tangential Surface Loads," Koninklijke/Shell-Laboratorium, Amsterdam, Report AMSR 0006.73, 1973.

APPENDIX C

APPENDIX C

APPROXIMATE ANALYSIS OF PAVEMENTS USING A MODIFIED BOUSSINESQ APPROACH

Boussinesq Equations

Boussinesq (3) determined solutions for a point load at the surface of an elastic half space. These can be used to determine the vertical and horizontal stresses and strains at any point in the half space. Figure C1 shows this loading situation, together with the geometrical descriptions required for solution of the equations. The equations utilized for pavement analysis are as follows:

$$\sigma_z = \frac{3P}{2\pi R^2} \cos \theta^3 \tag{C-1}$$

$$\sigma_r = \frac{P}{2\pi R^2} (3\cos\theta\sin^2\theta - \frac{1-2\nu}{1+\cos\theta}) \tag{C-2}$$

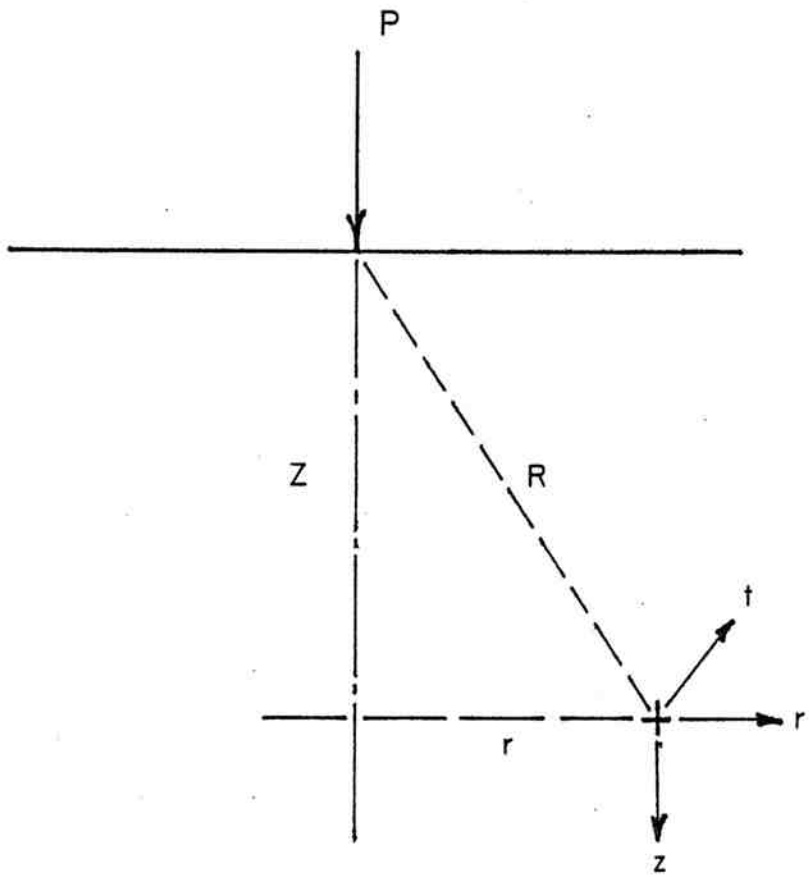
$$\sigma_t = \frac{P}{2\pi R^2} (1 - 2\nu)(-\cos\theta + \frac{1}{(1+\cos\theta)}) \tag{C-3}$$

$$\varepsilon_z = \frac{(1+\nu)P}{2\pi R^2 E} (3 \cos^3\theta - 2\nu \cos\theta) \tag{C-4}$$

where σ_z, σ_r and σ_t are the stresses in the vertical, radial and tangential directions at the point in consideration, ε_z is the vertical strain, and E and ν are the modulus of elasticity and Poisson's ratio of the material.

For the simple case of determination of the stresses and strains on the axis of a circular uniformly distributed load, as shown in Figure C2, a conglomeration of smaller point loads can be used, and the equations integrated either analytically or numerically. The analytical solutions for σ_z and ε_z yield:

$$\sigma_z = \sigma_o\left[1 - \frac{1}{(1+(a/z)^2)^{3/2}}\right] \tag{C-5}$$



$$R = \sqrt{z^2 + r^2}$$

$$\cos \theta = z/R$$

Figure C1 - Loading Geometry For a Point Load On The Surface of An Elastic Half-Space

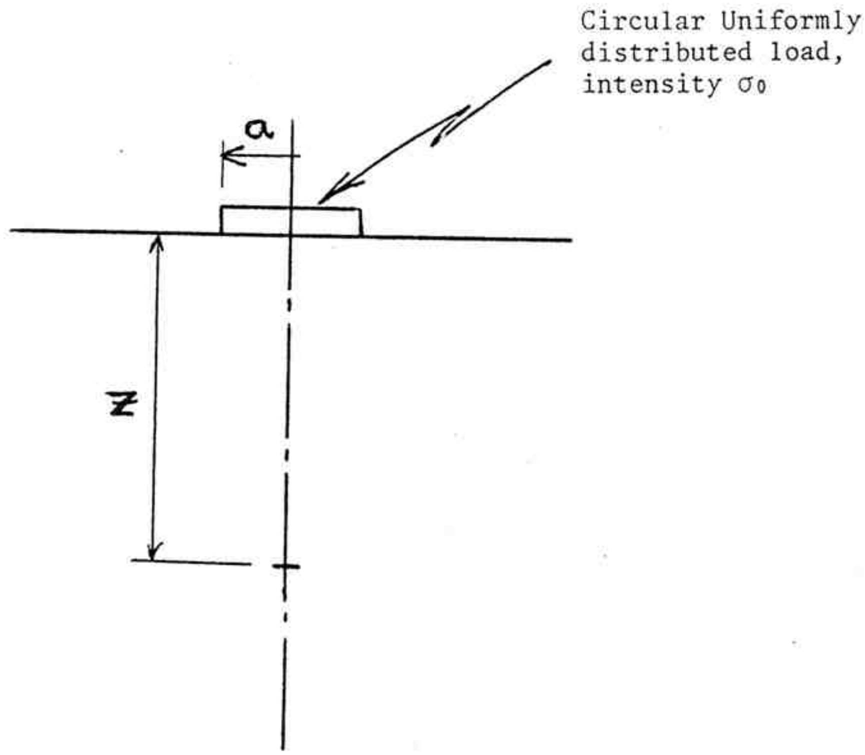


Figure C2 - Loading Geometry for a Circular Uniformly Distributed Load on the Surface of an Elastic Half-Space

$$\varepsilon_z = \frac{\sigma_0(1-\nu)}{E} \left[\frac{(\frac{z}{a})}{(1 + (\frac{z}{a})^2)^{3/2}} - (1 - 2\nu) \left\{ \frac{(\frac{z}{a})}{(1 + (\frac{z}{a})^2)^{1/2}} - 1 \right\} \right] \qquad \text{(C-6)}$$

For an off-axis location, solution for a uniformly distributed load can only be obtained numerically. However, unless such a location is close to the point of contact of load, the point load equations can be used without serious error.

The simplest application of the Boussinesq equations is for the on-axis situation utilizing Eqs. (C-5) and (C-6). Because of the axisymmetric loading situation, the stresses and strains in the radial and tangential directions are identical and only determination of σ_r and ε_r remains to completely describe the stress/strain regime at any point. However, this simple application cannot deal with a dual wheel loading arrangement, which requires the use of the point load Eqs. (C-1) to (C-4) and application of the principle of superposition. For such an arrangement, calculations are required for off-axis locations, where three stress and three strain parameters are required to completely describe the stress/strain regime.

For either the simple on-axis solution, or the more complex off-axis situation, the stresses or strains which are not determined by Eqs. (C-1) to (C-6) are determined by the generalized Hooke law:

$$\varepsilon_z = \frac{1}{E} (\sigma_z - \nu(\sigma_r + \sigma_t)) \qquad \text{(C-7)}$$

$$\varepsilon_r = \frac{1}{E} (\sigma_r - \nu(\sigma_z + \sigma_t)) \qquad \text{(C-8)}$$

$$\varepsilon_t = \frac{1}{E} (\sigma_t - \nu(\sigma_z + \sigma_r)) \qquad \text{(C-9)}$$

The question still remains of how to use these equations for a layered elastic structure such as exists in a pavement rather than for an elastic half space. This is achieved by use of the method of equivalent thicknesses developed by Odemark (4), which has been described by Ullidtz (1) and Ullidtz and Peattie (2).

Method of Equivalent Thicknesses

This is based on the assumption that the load distributing ability of a layer of material depends only on its structural stiffness:

$$D_i = \frac{E_i \, h_i^3}{12(1-\nu_i^2)} \tag{C-10}$$

where D_i is the structural stiffness of the ith layer of a multilayer system and E_i, ν_i, and h_i are the properties of that layer. So, for two layers of material with different properties, provided the structural stiffnesses are the same, they will distribute loads in the same way. For example, if:

$$D_i = D_{i+1}$$

then

$$\frac{E_i \, h_i^3}{12(1-\nu_i^2)} = \frac{E_{i+1} \, (h_{i+1})^3}{12(1-\nu_{i+1}^2)} \tag{C-11}$$

and by rearranging, the thickness of layer i+1 material required to produce the same stiffness in that layer as in layer i is:

$$h_{i+1} = h_i \times 3\sqrt{\frac{E_i}{E_{i+1}} \times \frac{(1-(\nu_{i+1})^2)}{(1-\nu_i^2)}} \tag{C-12}$$

This equation forms the basis for the "equivalent thickness" concept. For any layer of material, with properties E_i, ν_i, and h_i, the equivalent thickness he_i, in terms of material with properties E_{i+1} and ν_{i+1}, is given by:

$$he_{i,i+1} = h_i \times 3\sqrt{\frac{E_i}{E_{i+1}} \times \frac{(1-(\nu_{i+1})^2)}{(1-\nu_i^2)}} \tag{C-13}$$

In this equation, the notation for he is that the first subscript describes the layer which the equivalent thickness is calculated for, and the second subscript is the layer whose properties are being used for the transformation. This concept is usually applied to adjacent layers but it could be applied between any two layers in a multilayer system. Its use for a multilayer pavement is best illustrated by the simple example presented in Figure C3 for a three-layer pavement system. It should be noted that in this simple case where $\nu_1 = \nu_2 = \nu_3$, the calculations are simplified such that only moduli and thicknesses are involved. This example also shows how the method can be applied to successive layers until they are all expressed in terms of the properties of one layer. The general equation for transforming (n-1) layers of a pavement with n layers to an equivalent thickness of material of properties E_n and ν_n is:

$$He_{\Sigma n-1,n} = \sum_{i=1}^{n-1} h_i \times 3\sqrt{\frac{E_i}{E_n} \times \frac{(1-\nu_n^2)}{(1-\nu_i^2)}} \tag{C-14}$$

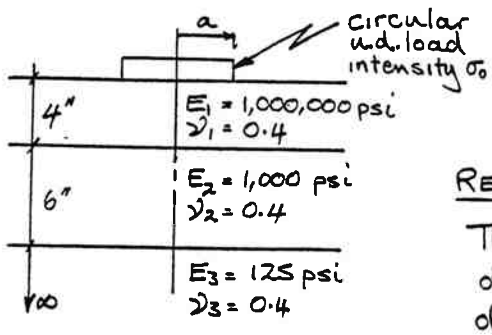
This is the same as:

$$he_{\Sigma n-1,n} = ((((h_i \times 3\sqrt{\frac{E_1}{E_2} \times \frac{(1-\nu_2^2)}{(1-\nu_1^2)}} + h_2) \times 3\sqrt{\frac{E_2}{E_3} \times \frac{(1-\nu_3^2)}{(1-\nu_2^2)}} + h_3)$$

$$\times 3\sqrt{\frac{E_3}{E_4} \times \frac{(1-\nu_4^2)}{(1-\nu_3^2)}} + h_{n-1}) \times 3\sqrt{\frac{E_{n-1}}{E_n} \times \frac{(1-\nu_n^2)}{(1-\nu_{n-1}^2)}})$$

Calculation of Stresses and Strains

The underlying concept of the calculation procedure is that, when any point in the system is considered, the layers of material above and below that point have the same properties, viz, that the condition of homogeneity, for which the Boussinesq equations are applicable, is obeyed. For the example considered in Figure C3, the stresses and strains immediately below the interface between the top layer and the layer immediately below, location (2), can



REQUIRED

The equivalent thicknesses of layers 1 & 2 in terms of the properties of layer 3

SOLUTION

The values of he_1 and he_2 are found by application of equation C-13:

$$he_{i,i+1} = h_i \times \sqrt[3]{\frac{E_i}{E_{i+1}} \times \frac{1-\nu_{i+1}^2}{1-\nu_i^2}}$$

Since $\nu_1 = \nu_3$, $he_{1,3} = 4 \times \sqrt[3]{1{,}000{,}000/125} = 4 \times \sqrt[3]{8{,}000}$

i.e. $he_{1,3} = 80''$

Since $\nu_2 = \nu_3$, $he_{2,3} = 6 \times \sqrt[3]{1{,}000/125} = 6 \times \sqrt[3]{8}$

ie $he_{2,3} = 12''$

So, the total equivalent thickness of both pavement layers in terms of the subgrade is:

$$he_{1+2,3} = he_{1,3} + he_{2,3} = 80 + 12 = 92''$$

ie $he_{1+2,3} = 92''$

Figure C3 - Simple Application of the Method of Equivalent Thicknesses be calculated, taking full account of the stiffer top layer, by converting the top layer to one with thickness he_1 and properties E_2 and ν_2.

For the simple case of a uniformly distributed load and determination of the stresses and strains on the axis of the load, application of Eqs. (C-5) and (C-6), setting $z_1 = he_1$, yields σ_{z2} and ε_{z2} at a location in a homogeneous material equivalent to the top of layer 2 in the three-layer pavement, as shown in Figure C3. Subsequent application of Hooke's law (Eqs. (C-7) and (C-8), remembering that for the axisymmetric case, $\sigma_r = \sigma_t$, yields:

$$\sigma_{r2} = (\sigma_{z2} - E_2\, \varepsilon_{z2})/2\nu_2 \quad \text{(C-16)}$$

and

$$\varepsilon_{r2} = \frac{1}{E_2}\,(\sigma_{r2} - \nu_2(\sigma_{r2} + \sigma_{z2})) \quad \text{(C-17)}$$

Hence, the full set of normal stresses and strains at the top of the second layer are obtained. Those at the bottom of the top layer can be determined, taking full account of the modular ratio between it and the second layer, since by compatibility and equilibrium, for a rough interface:

$$\varepsilon_{ri} = \varepsilon_{ri+1} \quad \text{(C-18)}$$

$$\sigma_{zi} = \sigma_{zi+1} \quad \text{(C-19)}$$

And applying Hooke's law again

$$\varepsilon_{z1} = \frac{1}{E_1}\,(\sigma_{z1} - 2\nu\ \sigma_{r1}) \quad \text{(C-20)}$$

$$\sigma_{r1} = (\varepsilon_{r1} E_1 + \nu_1\, \sigma_{z1})/(1 - \nu_1) \quad \text{(C-21)}$$

A procedure similar to that described above is applied at the interface between the second and third layers, setting $z_2 = he_2$, to obtain a full set of normal stresses at that interface. Figure C4 shows the complete set of calculations for the pavement illustrated in Figure C3, with appropriate loading conditions. To distinguish between calculations for the top and bottom of a layer, the additional subscripts t and b are used. Hence, σ_{zt2} is the vertical stress at the top of layer 2.

A slightly more complex but similar procedure is used for determination of the stresses and strains for an off-axis location, using the point load Eqs. (C-1) to (C-3) and Hooke's law (C-7) to (C-9). In this case, at each interface, the relationships shown in Eq. (C-18) and (C-19) are used, with the additional compatibility relationship:

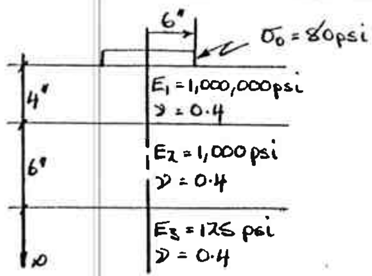
$$\varepsilon_{ti} = \varepsilon_{ti+1} \qquad \text{(C-22)}$$

Ullidtz and Peattie (2) suggest that in order to obtain good agreement between the stresses and strains calculated by the modified Boussinesq approach and by exact elastic theory, correction factors should be applied to the equivalent thicknesses. For the simple case of calculations on the axis of a uniformly distributed load, Eq. (C-14) is modified as follows:

$$he'_{\Sigma n-1,n} = f \times \sum_{i=1}^{n-1} h_i \times \sqrt[3]{\frac{E_i}{E_n} \times \frac{(1-\nu_n^2)}{(1-\nu_i^2)}} \qquad \text{(C-23)}$$

Where, the correction factor f, is 0.8 except for the first interface where it is 1.0, or for a two-layer system where it is 0.9.

Additional correction factors are required when using the point load equations for more general analyses, since the assumption that the uniformly distributed load can be approximated by a point load produces inaccuracies near the surface of the pavement. These corrections are as follows:



REQUIRED

Complete stress/strain calculations for the pavement shown using Bousinesq equations for a circular u.d. load

SOLUTION

Step 1: Calculate the stresses & strains at the top of layer 2 by first transforming layer 1 to an equivalent thickness in terms of layer 2 material.

$$he_{1,2} = 4 \times \sqrt[3]{1{,}000} = 40''$$

Calculation of σ_z & ε_z at depth 40" for a homogeneous material with properties E_2 & ν_2 then yields values similar to those at the top of layer 2, since the overlying layer has the same stiffness as 40" of layer 2. Applying the Bousinesq equations

$$\sigma_{zt2} = \sigma_0\left[1 - \frac{1}{(1+(a/z)^2)^{3/2}}\right]$$

$$= 80\left[1 - 1/(1+(6/40)^2)^{3/2}\right] = \underline{2.626\text{psi}}$$

$$\varepsilon_{zt2} = \frac{\sigma_0(1+\nu)}{E}\left[\frac{(z/a)}{(1+(z/a)^2)^{3/2}} - (1-2\nu)\left\{\frac{(z/a)}{(1+(z/a)^2)^{1/2}} - 1\right\}\right]$$

$$= \frac{80(1.4)}{1000}\left[\frac{40/6}{(1+(40/6)^2)^{3/2}} - 0.2\left\{\frac{(40/6)}{(1+(40/6)^2)^{1/2}} - 1\right\}\right]$$

$= \underline{2685 \text{ microstrain}}$ (μst)

Applying Hookes law (equations C-16 & C-17):

$$\sigma_{rt2} = (\sigma_{zt2} - E_2\varepsilon_{zt2})/2\nu_2$$

$= \underline{-.074 \text{ psi}}$

$$\varepsilon_{rt2} = \frac{1}{E_2}(\sigma_{rt2} - \nu_2(\sigma_{rt2} + \sigma_{zt2}))$$

$= \underline{-1095 \text{ microstrain}}$

Step 2: Calculate the stresses and strains at the bottom of layer 1.

By equilibrium, $\sigma_{zb1} = \sigma_{zt2} = 2.626\text{psi}$

By compatibility, $\varepsilon_{rb1} = \varepsilon_{rt2} = -1095\ \mu st$

Using Hookes laws equations 2.20 and 2.21:

$$\varepsilon_{zb1} = \frac{1}{E_1}[\sigma_{zb1} - 2\nu\sigma_{rb1}]$$

$$\sigma_{rb1} = (\varepsilon_{rb1}E_1 + \nu_1\sigma_{zb1})/1-\nu_1$$

$\therefore\ \sigma_{rb1} = \underline{-1823 \text{ psi}}$

& $\varepsilon_{zb1} = \underline{1461\ \mu st}$

Step 3: Calculate the stresses and strains at the top of layer 2. This is identical to step 1, but uses the equivalent thickness of the top 2 layers and the material parameters for layer 3. The calculations give:

$\sigma_{zt3} = \underline{0.508\text{psi}}$ $\varepsilon_{zt3} = \underline{4167\ \mu st}$

$\sigma_{rt3} = \underline{-0.016\text{psi}}$ $\varepsilon_{rt3} = \underline{-1702\ \mu st}$

Step 4: Calculate the stresses and strains at the bottom of layer 2. This is identical to step 2, but uses the properties of layer 2 in the calculations, which give the following results:

$\sigma_{zb2} = \underline{0.508\text{psi}}$ $\varepsilon_{rb2} = \underline{-1702\ \mu st}$

$\sigma_{rb2} = \underline{-2.498\text{psi}}$ $\varepsilon_{zb2} = \underline{2506\ \mu st}$

Figure C4 - Example of Stress/Strain Calculations on the Centerline of a Single Uniformly distributed Load

$$\text{for } z_i < a: \quad z_i' = \frac{1.5\ a}{2(1-\nu_i^2)-(2(1-\nu_i^2)-0.7)x(z_i/2a)} \tag{C-24}$$

$$\text{for } z_i > a: \quad z_i' = z_i + 0.6\ a^2/z_i \tag{C-25}$$

where

$$z_i = he_i'$$

The modifications to the previous example due to the appropriate corrections are shown in a reworked example in Figure C5. For the same pavement and loading the example is reworked in Figure C6 for both the on-axis and off-axis cases, by applying the point load equations and using appropriate corrections.

Summary

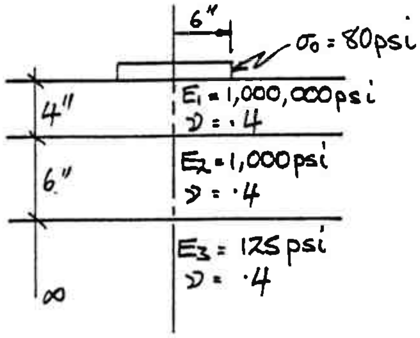
The procedures for applying the Boussinesq equations, in conjunction with the method of equivalent thicknesses, to calculate stresses and strains for both on- and off-axis locations in a multilayer pavement, have been presented. Figure C7 gives a flow chart representing the order of the calculations.

ANALYSIS OF DEFLECTIONS IN A MULTILAYER ELASTIC STRUCTURE BY BOUSSINESQ EQUATIONS AND METHOD OF EQUIVALENT THICKNESSES

Boussinesq Equations

The general equation for deflections at a depth z and radius r, in an elastic half space, due to a point load, as shown in Figure C3, is:

$$d_{(z)(r)} = \frac{(1+\nu)\cdot P}{2\pi R\cdot E}[2(1-\nu) + \cos^2\theta] \tag{C-26}$$



REQUIRED

Complete stress/strain calculations, as in Fig 4 but applying correction factors

SOLUTION

Step 1 : The calculation of stresses at the top of layer 2 is identical to those given in Fig 4, since there is no correction applied to the first interface of a 3-layer system (i.e f = 1.0 in Eqn C-23)

Step 2 : Similar to step 1, there is no difference in the calculations for the bottom of layer 1.

Step 3 : To calculate the stresses/strains at the top of layer 3, the equivalent thickness of the first 2 layers must be corrected by a factor, f, of 0.8.

$\therefore \quad h'_{e_{1+2,3}} = .8 \times 92 = 73.6''$

Equations C-1 and C-2 are used to calcutate σ_{zt_3} & ε_{zt_3}, then equations C-16 & C-17 to calcutate σ_{rt_3} and ε_{rt_3}:

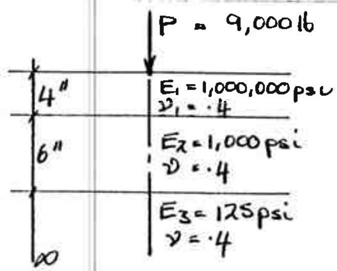
$\sigma_{zt_3} = 0.79$ psi , $\varepsilon_{zt_3} = 6488\,\mu st$

$\sigma_{rt_3} = -0.03$ psi , $\varepsilon_{rt_3} = -2562\,\mu st$

Step 4 Equilibrium & compatibility yield σ_{zb_2} & ε_{rb_2}, then application of equations C-20 & C-21 give σ_{rb_2} & ε_{zb_2}:

$\sigma_{rb_2} = -3.89$psi , $\varepsilon_{zb_2} = 6488$

Figure C5 - Reworking of Figure CA Example with Correction Factors



REQUIRED

Complete stress/strain calculations using the point load equations for locations immediately below the centre of the load, and under the edge of the load (1st interface only)

SOLUTION – On axis

Step 1: The stresses/strains at the top of layer 2 are calculated by equations C-1 to C-3 and C-7 to C-9, after correction of the equivalent thickess, by eqnation C-25:

$$h_{e1,2} = 40' = h'_{e1,2} = z_1 > a$$

$$\therefore \quad z'_1 = 40 + 0.6 \times 6^2/40 = \underline{40.54"}$$

$$\sigma_{zt2} = \frac{3P}{2\pi R^2} \cos^3\theta$$

For the on axis case $\cos^3\theta = 1$ and $R = z'$

$$P = \sigma_0 . \pi a^2 = 80 \times \pi \times 6^2 \simeq 9{,}000\ \text{lb.}$$

$$\therefore \quad \sigma_{zt2} = 3 \times 9{,}000/(2\pi \times 40.54^2)$$

$$= \underline{2.615\ \text{psi}}$$

$$\sigma_{rt2} = \frac{P}{2\pi R^2}\left(3\cos\theta\sin^2\theta - \frac{1-2\nu}{1+\cos\theta}\right)$$

For the on axis case $\cos\theta = 1$ & $\sin\theta = 0$

$$\therefore \quad \sigma_{rt2} = 9000/(2\pi R^2) \times (-.2/2)$$

$$= \underline{-.087\ \text{psi}}$$

$$\underline{\sigma_{tt2}} = -.087\ \text{psi (due to symmetry} = \sigma_{rt2})$$

$$\varepsilon_{zt2} = \frac{1}{E_2}\left(\sigma_{zt2} - \nu(\sigma_{rt2} + \sigma_{tt2})\right)$$

$$= \frac{1}{1000}\left(2.615 - .4(2 \times -.087)\right)$$

$$= \underline{2686\ \mu st}$$

$$\varepsilon_{rt2} = \frac{1}{1000}\left(-.087 - .4(2.615 - .087)\right)$$

$$= \underline{-1098\ \mu st} = \underline{\varepsilon_{tt2}}$$

Step 2: Calculate the stresses and strains at the bottom of layer 1:

By equilibrium, $\sigma_{zb1} = \sigma_{zt2} = \underline{2.615\ \text{psi}}$

By compatibility, $\varepsilon_{rb1} = \varepsilon_{rt2} = \underline{-1098\ \mu st}$

and, $\varepsilon_{tb1} = \varepsilon_{tt2} = \underline{-1098\ \mu st}$

By Hookes law, $\sigma_{rb1} = (\varepsilon_{rb1}E_1 + \nu_1\sigma_{zb1})/(1-\nu_1)$

$= \underline{-1832\ \text{psi} = \sigma_{tb1}}$

and $\varepsilon_{z1} = (\sigma_{zb1} - 2\nu\sigma_{rb1})/E_1$

$= \underline{1468\ \mu st}$

Step 3: The stresses and strains at the top of layer 3 are calculated after correction of the equivalent thicknesses of layers 1 & 2, in the same way as in step 1, but using the properties of layer 3:

$$h'_{e1+2,3} = 73.6 = z_{1\&2} > a$$

$$\therefore \quad z'_{1\&2} = 73.6 + 0.6 \times 6^3/73.6 = 73.9"$$

Use of this value in equations C-1 to C-3 gives:

$\underline{\sigma_{zt3} = .79\ \text{psi}, \quad \sigma_{rt3} = \sigma_{tt3} = -.03\ \text{psi}}$

Figure C6 - Example of the Use of Boussinesq Point Load Equations

Step 3 (cont): Application of Hookes law (equations C-7 – C-8):

$\varepsilon_{zt3} = 6498\,\mu st$, $\varepsilon_{rt3} = \varepsilon_{tt3} = -2658\,\mu st$

Step 4: Calculation of the stresses & strains at the bottom of layer 2 is similar to those for step 2, but using the properties of layer 2:

$\sigma_{zb2} = .79$ psi, $\varepsilon_{rb2} = \varepsilon_{tb2} = -2658\,\mu st$

$\sigma_{rb2} = -3.90$ psi, $\sigma_{tb2} = -3.90$ psi, $\varepsilon_{zb2} = 3914\,\mu st$

COMPARISON OF ALL THE ABOVE CALCULATIONS WITH THOSE PRODUCED USING THE EQUATIONS FOR A UNIFORMLY DISTRIBUTED LOAD (Figure 5)

SOLUTION for off axis calculations, $r = 6''$

Step 1: Calculation of stresses and strains at the top of layer 2, utilises $z_1' = 40.54''$ as before. The values of R, cos θ & sin θ are calculated before application of eqns C-1 – C-3:

$$R = \sqrt{z^2+r^2} = \sqrt{40.54^2+6^2} = 40.98''$$

$$\cos\theta = z/R = .9892, \quad \sin\theta = r/R = .1464$$

$$\sigma_{zt2} = \frac{3P}{2\pi R^2}\cdot\cos^3\theta = 2.49\text{ psi}$$

$$\sigma_{rt2} = \frac{P}{2\pi R^2}\left(3\cos\theta\cdot\sin^2\theta - \frac{1-2\nu_1}{1+\cos\theta}\right) = -.032\text{ psi}$$

$$\sigma_{tt2} = \frac{P}{2\pi R^2}(1-2\nu_1)\left(-\cos\theta + \frac{1}{1+\cos\theta}\right) = -.082\text{ psi}$$

By Hookes law:

$$\varepsilon_{zt2} = \frac{1}{E_2}(\sigma_{zt2} - \nu_2(\sigma_{rt2}+\sigma_{tt2})) = 2536\,\mu st$$

$$\varepsilon_{rt2} = \frac{1}{E_2}(\sigma_{rt2} - \nu_2(\sigma_{zt2}+\sigma_{tt2})) = -994\,\mu st$$

$$\varepsilon_{tt2} = \frac{1}{E_2}(\sigma_{tt2} - \nu_2(\sigma_{zt2}+\sigma_{rt2})) = -1067\,\mu st$$

SOLUTION for off axis calculations, $r = 6''$

Step 2: The stresses and strains at the bottom of layer 1 are calculated as follows:

By equilibrium, $\sigma_{zb1} = \sigma_{zt2} = 2.49$ psi

By compatibility, $\varepsilon_{rb1} = \varepsilon_{rt2} = -994\,\mu st$

" " , $\varepsilon_{tb1} = \varepsilon_{tt2} = -1067\,\mu st$

By Hookes law: $\varepsilon_{zb1} = \frac{1}{E_1}(\sigma_{zb1} - \nu_1(\sigma_{rb1}+\sigma_{tb1}))$ — (1)

but σ_{rb1} & σ_{tb1} are unknown

similarly, since $\varepsilon_{rb1} = \frac{1}{E_1}(\sigma_{rb1} - \nu_1(\sigma_{zb1}+\sigma_{tb1}))$ — (2)

and $\varepsilon_{tb1} = \frac{1}{E_1}(\sigma_{tb1} - \nu_1(\sigma_{zb1}+\sigma_{rb1}))$ — (3)

rearrangement of these equations is needed to get ε_{zb1}, σ_{rb1} and σ_{tb1}

multiplying (3) × ν_1 and adding to (2) gives:

$$\sigma_{rb1} = \frac{E_1(\varepsilon_{rb1}+\nu_1\varepsilon_{tb1}) + 2\nu_1^2\sigma_{zb1}}{1-\nu^2} \quad (4)$$

from (2)

$$\sigma_{tb1} = E_1\varepsilon_{tb1} + \nu_1(\sigma_{rb1}+\sigma_{zb1}) \quad (5)$$

Application of equations (4), (5) and (1) gives:

$\sigma_{rb1} = -1744.5$ psi

$\sigma_{tb1} = -1741.7$ psi

$\varepsilon_{zb1} = 1375\,\mu st$

Figure C6 - (continued)

1. Determine Properties of Pavement, and Loading Arrangement:

   No. of Layers
   Modulus of each layer (E)
   Poisson's ratio of each layer (ν)
   Depth of each layer (h)
   Tyre pressure (σ_o)
   Radius of load area (a)

2. Select the uppermost layer in the pavement where calculations are needed, and the interface concerned.

3. Calculate the equivalent thickness, he, for all layers above the interface in terms of the layer below, correct to he' and apply the point load correction factors if necessary to determine z'.

4. Calculate σ_z & ε_z for a uniformly distributed load (or, σ_z, σ_r & σ_t for a point load) below the interface, inputting E & ν for the layer below interface and z' to the Boussinesq Equations.

5. Calculate the remaining 2 or 3 stress/strain parameters below the interface with Hooke's Law & the same E & ν.

6. ε_t & σ_z above the interface are equal to those below. The remaining 2 or 3 stress/strain parameters, if required, are calculated from Hooke's Law.

7. Repeat 3-6 for subsequent locations.

Figure C-7 - Flow Diagram for Boussinesq Analysis of a Layered System

For a uniformly distributed load (Figure C4), integration of this equation yields:

$$d_z = \frac{(1+\nu)\sigma_{oa}}{E}\left[\frac{1}{(1+(a/z)^2)^{1/2}} + (1-2\nu)\{(1+(z/a)^2)^{1/2} - z/a\}\right]$$

This equation is only valid for calculation of deflections on the load axis.

Calculation of Deflections

The majority of the deflections in a pavement occurs in the subgrade, and use of either Eq. (C-26) or (C-27) will give an estimate of the deflection at the surface of the pavement. But this will be too large, since the stiffer pavement layers reduce the vertical strains in the subgrade and, hence, reduce the deflections although they deflect a significant amount themselves which contributes to the surface deflection.

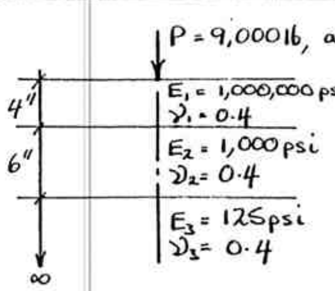
The application of the method of equivalent thicknesses is different when calculating deflections to that for calculation of stresses and strains. The calculations start with the top layer and work down through the pavement until the subgrade is reached. For each pavement layer, the calculation procedure is as follows:

1. Determine the equivalent thicknesses of the overlying layers, using Eq. (C-23) with an appropriate correction factor, and if the point load Eq. (C-26) is being used, correct again by either Eq. (C-24) or (C-25).
2. Determine the deflection at the surface of the layer, and the deflection at the bottom of the layer, by either Eq. (C-26) or (C-27). The difference is the deflection in that layer.

For the subgrade, only the deflection at the surface is required, having calculated the equivalent thickness of all the pavement layers. Summation of this deflection with the deflections in each pavement layer gives the total deflection which occurs at the pavement surface.

If only the maximum deflection is required, use of Eq. (C-27) for a uniformly distributed load is best, but if the shape of the deflection bowl is required, then the equation for a point load (C-26) is required, and the process is repeated for each radial distance (r). It should be noted that the calculations for the top layer involve the special case of z = 0; this should be corrected by Eq. (C-24) to compensate for the approximation of a uniformly distributed load by a point load.

Since there is close agreement between the use of either Eqs. (C-26) or (C-27) to determine the on-axis (maximum) deflecton, only the use of more general point load equation is presented herein. Figure C8 shows the deflection calculations for the maximum deflection and for radial distance, r = a for the pavement used in the examples for stress and strain calculations.



REQUIRED

Complete deflection calculations using the Boussinesq equation for a point load, for locations immediately below the centre of the load and, under the edge of the load.

SOLUTION – On axis case

Step 1: Calculate the surface deflection assuming all material is layer 1:

a) Although the depth is 0, calculate a corrected depth:

$$\text{for } z < a, \quad z_1' = \frac{1.5a}{2(1-\nu^2)-(2(1-\nu^2)-0.7)\times(z/2a)}$$

$$\therefore \quad z_1' = (1.5\times 6)/2(1-\nu^2) = 9/1.68 = \underline{\underline{5.357''}}$$

b) $$d_{(0)(0)} = \frac{(1+\nu)\sigma_0 a^2}{2.R.E}\left[2(1-\nu)+\cos^3\theta\right] = \frac{(1.4)(80)(36)}{2(5.357)(10^6)}\left[2(0.6)+(1)\right] = \underline{\underline{0.0008279''}}$$

Step 2: Calculate the deflection at depth 4" in a pavement consisting entirely of layer 1 material

a) $$\text{for } z<a, \quad z_1' = \frac{(1.5)(6)}{2(.84)-(2(.84)-.7)\times(4/12)} = \underline{\underline{6.65''}}$$

b) $$d_{(4)(0)} = \frac{(1.4)(80)(36)}{2(6.65)(10^6)}\left[2(0.6)+(1)\right] = \underline{\underline{0.0006669''}}$$

Step 3: Deflection in layer 1:

a) $$d_1 = d_{(0)(0)} - d_{(4)(0)} = 0.0008279 - 0.0006669 = \underline{\underline{0.0001610''}}$$

Step 4: Calculate the deflection at the surface of layer 2, assuming all the pavement is layer 2 material:

a) $he_{1,2} = 40''$ & $he_{1,2}' = z_1 > a$

$$\therefore \quad z_1' = 40 + 0.6\times 6^2/40 = \underline{40.54''}$$

b) $$d_{(4)(0)} = \frac{(1.4)(80)(36)}{(2)(40.54)(1000)}\left[2(0.6)+(1)\right] = \underline{\underline{0.10940''}}$$

Step 5: Calculate the deflection at the bottom of layer 2, assuming all the pavement is layer 2 material:

a) $z = 40 + 6 = 46''$

$$z' = 46 + 0.6\times 6^2/46 = \underline{46.47''}$$

b) $$d_{(10)(0)} = \frac{(1.4)(80)(36)}{2(46.47)(1000)}\left[2(0.6)+1\right] = \underline{\underline{0.09544''}}$$

Step 6: Deflection in layer 2 = $d_2 = d_{(4)(0)} - d_{(10)(0)}$

$$\therefore \quad d_2 = 0.10940 - 0.09544 = \underline{\underline{0.01396''}}$$

Step 7: Calculate the deflection at the surface of the subgrade, assuming all the pavement is subgrade:

a) $he_{1+2,3} = 92''$ & $he_{1+2,3}' = 73.6'' = z_{1+2} > a$

$$\therefore z_{1+2}' = 73.6 + 0.6\times 6^2/73.6 = \underline{73.89''}$$

b) $$d_{(10)(0)} = \frac{(1.4)(80)(36)}{2(73.89)(125)}\left[2(0.6)+(1)\right] = \underline{\underline{d_3 = 0.48017''}}$$

Step 8: SUMMARY OF DEFLECTIONS:

In Layer 1, $d_1 = 0.000161''$

In Layer 2, $d_2 = 0.01396''$

In Layer 3, $d_3 = 0.48017''$

∴ Total (Surface) Deflection = 0.49429"

Say, $\underline{\underline{\delta = 0.49''}}$

Figure C8 - Example Illustrating Calculation of Deflections

SOLUTION — Off axis

Step 1: The solutions are required for the edge of the load, i.e referring to Figure 3, $r = 6''$

a) $z_0' = 5.357''$ (as for the on-axis sol^n)

$R = [(z_0')^2 + (r)^2]^{1/2} = 8.043$

$\cos\theta = z_0'/R = 0.666$

b) $d_{(0)(6)} = \frac{(1.4)(80)(36)}{(2)(8.043)(10^6)}[2(1-2) + (.666)^2]$

$= 0.000412$

Step 2: Deflection at depth 4" and radius 6":

a) $z < a$ & $z_1' = 6.65''$ (as before)

$R = [(6.65)^2 + (6)^2]^{1/2} = 8.957$

$\cos\theta = z_1'/R = 0.742$

b) $d_{(4)(6)} = \frac{(1.4)(80)(36)}{(2)(8.957)(10^6)}[2(0.6) + (.742)^2]$

$= 0.000394$

Step 3: Deflection in layer 1 at radius 6":

a) $d_1 = d_{(0)(6)} - d_{(4)(6)}$

$= 0.000412 - 0.000394$

$= 0.000018''$

Step 4; 5 & 6

Proceed as in steps 4, 5 & 6 for the on-axis solution, and obtain the deflection in layer 2.
The only difference is in the calculation of R and $\cos\theta$

$d_2 = d_{(4)(6)} - d_{(10)(6)}$

$= 0.013218''$

Step 7: Calculating the deflection for the subgrade surface:

$d_3 = 0.47717$

Step 8: Summary of Deflections:

layer 1:	d_1 =	0.000018
layer 2:	d_2 =	0.01322
layer 3:	d_3 =	0.47717
∴ Total deflection	=	0.49041

Say 0.49"

NOTE The calculated deflection for $r = 6''$ is only slightly smaller than that for $r = 0$ (on axis).
This is due to the very weak subgrade and very stiff top layer.
However, the very high proportion of deflection in the subgrade is typical.
For more realistic structures, the further away from the load centre, the greater the proportion of deflection in the lower layers.

Figure C8 - (continued)

REFERENCES

1. Ullidtz, Per, "A Fundamental Method for Prediction of Roughness, Rutting and Cracking of Pavements," Proceedings, Association of Asphalt Paving Technologists, Vol. 48, 1979.

2. Ullidtz, Per, and Peattie, K.R., "Pavement Analysis by Programmable Calculators," Transportation Journal, American Society of Civil Engineers, September 1980.

3. Boussinesq, J., "Application des potentiels a l'etude de equilibre et du mouvement des solides elastique," Gauthier-Villars, Paris, France, 1885.

4. Odemark, N., "Undersokning av elasticitetsegenskarperna hos olika Jordartur samt teorie for berakning av belagningar enligt elasticitetsteorien," Statens Vaginstitute, Meddelande 77, 1949.

APPENDIX D

APPENDIX D

ESTIMATION OF RESILIENT MODULUS AND POISSON'S RATIO

Selection of the Elastic Properties of the Pavement Layers

Values of modulus, E, and Poisson's ratio, ν, may be assigned to the layers by any appropriate procedure. The most simple approach, according to the procedures outlined by Brown (1) is given below, viz:

$$C_v = \frac{M_A/G_a}{M_A/G_a + M_B/G_b} \qquad (1)$$

where

M_A, M_B = percentages by mass of aggregate and binder, respectively.

G_a, G_b = specific gravities of aggregate and binder, respectively.

If the void content is greater than 3%, a corrected value of C_v (C_v') is required:

$$C_v' = \frac{C_v}{1 + (0.01\ V_v - 0.03)} \qquad (2)$$

The mix modulus is obtained from:

$$E = S_{bit}[1 + \frac{2.5}{n} \cdot \frac{C_v'}{1 - C_v'}]^n \qquad (3)$$

where

$$n = 0.83 \log_{10} (\frac{4 \times 10^4}{S_{bit}}) \quad (S_{bit} \text{ in } MN/m^2)$$

This procedure is only valid if $S_{bit} \geqslant 5\ MN/m^2$ and $C_v \geqslant 0.9$ and $\leqslant 0.7$.

The Poisson's ratio of bituminous materials is generally accepted to range from 0.30 to 0.50 and is largely temperature-dependent. For average temperature conditions (10°C to 30°C) a value of 0.40 is reasonable, and was used in this case.

Soils (Subgrade Materials) and Unbound Granular Materials

The modulus of the subgrade may be estimated from its California Bearing Ratio (CBR) value using the relationship:

$$E_{subgrade} = 10 \cdot CBR \tag{4}$$

where E is in MN/m^2 and the CBR in percent.

The modulus of a granular subbase is usually taken to be 2.5 times that of the subgrade, i.e.:

$$E_{subbase} = 2.5 \cdot E_{subgrade} \tag{5}$$

For cohesive soils, Poisson's ratio usually lies in the range 0.4 to 0.5 and is primarily dependent on moisture content and level of compaction. For sands and other granular materials, typical values of Poisson's ratio are 0.25 to 0.4. This type of material is the least understood of paving materials, since the stress-strain behavior is highly complex. Poisson's ratio is dependent on moisture condition, level of compaction and stress regime. A value of 0.4 is commonly adapted.

REFERENCES

1. Brown, S.F., "The Analytical Design of Bituminous Pavements," Department of Civil Engineering, University of Nottingham, 1980.

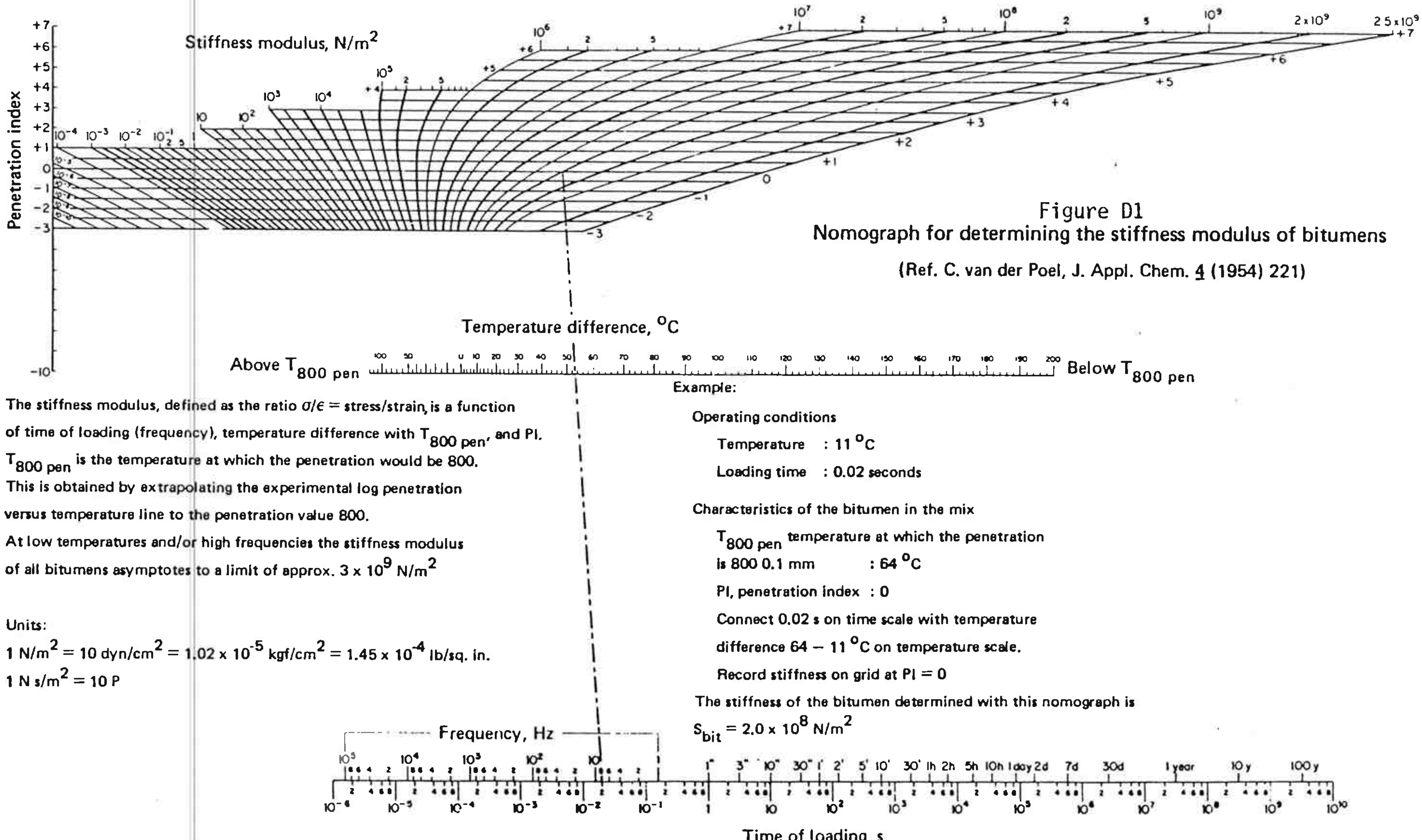


Figure D1
Nomograph for determining the stiffness modulus of bitumens

(Ref. C. van der Poel, J. Appl. Chem. 4 (1954) 221)

The stiffness modulus, defined as the ratio σ/ϵ = stress/strain, is a function of time of loading (frequency), temperature difference with $T_{800\ pen}$, and PI.

$T_{800\ pen}$ is the temperature at which the penetration would be 800.
This is obtained by extrapolating the experimental log penetration versus temperature line to the penetration value 800.
At low temperatures and/or high frequencies the stiffness modulus of all bitumens asymptotes to a limit of approx. 3×10^9 N/m^2

Units:
1 N/m^2 = 10 dyn/cm^2 = 1.02×10^{-5} kgf/cm^2 = 1.45×10^{-4} lb/sq. in.
1 N s/m^2 = 10 P

Example:

Operating conditions

Temperature : 11 °C
Loading time : 0.02 seconds

Characteristics of the bitumen in the mix

$T_{800\ pen}$ temperature at which the penetration is 800 0.1 mm : 64 °C
PI, penetration index : 0
Connect 0.02 s on time scale with temperature difference 64 – 11 °C on temperature scale.
Record stiffness on grid at PI = 0

The stiffness of the bitumen determined with this nomograph is
$S_{bit} = 2.0 \times 10^8$ N/m^2