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Wisconsin Highway Research Program

**RAPID BRIDGE CONSTRUCTION TECHNOLOGY:
PRECAST ELEMENTS FOR SUBSTRUCTURES**

Project ID: 0092-07-08

Final Report

By

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June, 2011

Report Summary

The adverse effects of ongoing highway construction on the road users and freight transportation force construction duration to become a vital part of construction planning. Conventional cast in place concrete construction consumes substantial amounts of the construction time by forming, placing and curing of concrete.

The goal of this research is to propose an alternate system of precast bridge substructures which can substitute for conventional cast in place systems in Wisconsin to achieve accelerated construction.

Three types of abutment modules (hollow wall with cap, full height socketed, partial height socketed) are proposed for use in Wisconsin. Design calculation sheets, example designs, and standard type drawings have been provided for the modules. A special design method (in MathCad) and tutorial is provided for detailing reinforcing around the embedded pile in the socket of the abutments.

The “limited length socketed wall with bent cap” is recommended as the preferred precast abutment system for adoption by WisDOT. Two vertical sockets are precast with a thin wall panel of limited width. The design controls the module weight for shipping and required crane capacity for erection. The sockets slip over piles and are subsequently concreted. A precast cap beam is then set atop the wall module to provide support for the bridge slab or girders.

Wisconsin should adopt a standard rectangular section pier/pile/abutment cap module. This multi-purpose structural section could be used as a pile bent cap, a pier cap, and the cap on the socketed abutment modules. To allow repetitive use of formwork, the top of the cap module should not include beam/girder stepped bearings. A standard rectangular section should be used with separate precast pedestals, in varying heights, connected to the top as necessary for providing cross slope through beam seats.

Design calculation sheets, example designs and proposed standard plans are provided for the proposed precast cap module.

In addition to this report, the following materials are included in an Appendix or provided separately to WisDOT as electronic digital software:

- Proposed plan drawings of abutment and cap modules,
- MathCad software for calculation of abutment design loads,
- MathCad software for calculation of cap design loads,
- Example design of solid abutment panel and MathCad software for design,
- Example design of rectangular cap beam in MathCad,
- Example design of socketed abutment panel in MathCad,
- Tutorial MathCad software on design methods for socket region of socketed abutment panel,
- Example design of socket region of abutment panel and production design MathCad software,
- Example design of welded connection between cap pieces,
- Example design of spliced connection between cap pieces,
- Example design of post-tensioned connection between cap pieces.

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1 Problem Definition

The adverse effects of ongoing highway construction on the road users and freight transportation force construction duration to become a vital part of construction planning. Conventional cast in place concrete construction consumes substantial amounts of the construction time by forming, placing and curing of concrete. Delays in concrete pouring on site in case of non-cooperating weather are likely in regions like Wisconsin. Extended construction durations cause inconvenience especially for bridge replacement projects where road closures and detours come into the picture. Bridge inspections in 2007 (1) emphasize the increasing need for bridge replacements as out of 13798 bridges in Wisconsin, 2091 bridges were evaluated to be deficient.

Benefits of prefabricated precast construction have been recognized in Wisconsin for superstructure construction for many years. Standard precast superstructure unit plans are available and frequently used. Precast technology for bridge substructures, however, has not yet been widely utilized in Wisconsin despite the innovative constructions that have taken place in the other states. In many cases substructure construction is the most costly and time consuming part of a bridge construction (2). The existing projects utilizing precast substructures nationwide or internationally showed that precast technology is an efficient and competitive means of reducing the substructure construction time. Other advantages of a precast substructure system can be listed as improved work zone safety, better quality control and less environmental interference. The aim of this research was to review the existing systems and then to develop a system compatible with conditions and current practice in Wisconsin to accelerate substructure construction.

2 Research Objectives

The goal of this research is to propose an alternate system of precast bridge substructures which can substitute for conventional cast in place systems in Wisconsin to achieve accelerated construction. The following items define the objectives of this study used to accomplish this goal.

1. Standardization: Although bridge characteristics vary for each project, a standard system applicable to the widest range of projects is desired. Standardization provides the ability of repetitive use of formwork to manufacturers, familiarity with the system to the contractors, the design engineers and the manufacturers. The construction efficiency will develop faster when a standard system is used since the lessons learnt during one project are immediately applicable to others. The features of the standard system should be picked by analyzing the features of frequently used current substructures in order to be consistent with the demand.
2. Compatibility with the precaster and the DOT needs: The geometry of the units or the features they accommodate is constrained to be easily manufactured with non-complicated, easy to remove formwork systems which minimize fabrication costs for the precasters. The substructures should be chosen and shaped according to the fabrication ability of precasters and highway needs in Wisconsin.
3. Ease in handling: Controlling the weight and size of the components is essential for ease in hauling and erection. The system must have units sized to meet these criteria. On the other hand, limiting the sizes of the components introduces the need for more individual pieces and connections. Simple economical connections are essential wherever they are used.

4. Ease in construction: Reasonable construction tolerances, non complicated practical details that allow rapid construction, and familiarity with proposed construction techniques should be considered.

3 Procedures Followed

3.1 Target Substructure Components

The components included in the substructures group are the foundation, columns, pier or bent caps, and abutments. Based on the frequency of use of these components, need for improvement to current cast-in-place construction, and suitability for precasting, target substructure elements were chosen.

The first down selection was on the foundation type. Deep foundations compose the majority of the bridge foundations in Wisconsin. Therefore, the foundation type to be used with the precast substructures was selected as pile foundations. Steel HP pile sections, HP 10 x 42 and HP 12 x 53, which compose the most frequently used pile types together with cast in place concrete piles, were selected to form the foundation for standard components. Steel HP piles are advantageous for the speed of driving, eliminating cast-in-place concrete, familiarity of practice, and ease in splicing or cutting to adjust the elevations at the pile top.

The second step was to determine the target precast abutment geometry and function. WisDOT has several standard abutment types suitable for different bridge projects: sill, semi retaining, and pile encased abutments. The frequency of use in the last years of these types based on the data given by Wisconsin Highway Information System (3) showed that sill and pile encased abutments were used for most of the bridges. (Figure 1) The common features of these two abutments were the pile foundation encased in the abutment body eliminating the need for a pile cap, no inclined surfaces, no battered piles or no backwall for superstructure. These abutment types have heights ranging from 5 to 10 ft. The standard precast abutment was aimed to fit these characteristics to allow frequent use.

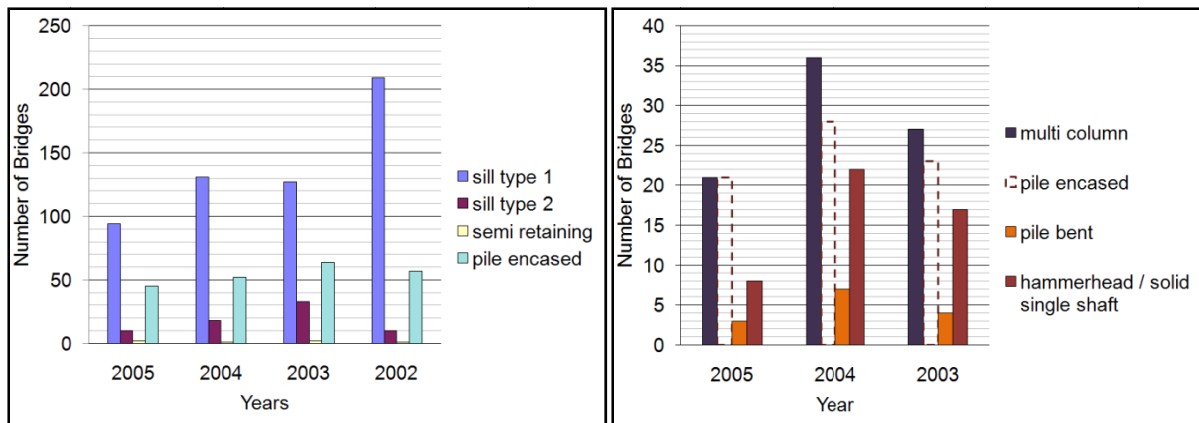


Figure 1 – Frequency of use of various abutment and pier types

Precasting of pier caps has a significant advantage in limiting the time during which construction is performed at heights and thus improves work zone safety. For pier caps, the analysis of frequency of use highlighted multi column piers over single column piers. (Figure 1) The pier cap for the multi column pier type with a rectangular geometry was picked to be part of the standard precast substructure system. Columns were assumed by WisDOT to be cast-in-place for this system due to relatively non complicated forming and lesser need for scaffolding.

3.2 Modularization and Standardization

A module weight limit, to avoid the need for special shipping permits, was assumed at 40 kips for Wisconsin. This weight also falls into the capacity range for a typical crane for erection. Due to the dimensions of the components dictated by bridge geometries, the weight of the abutment and pier cap would become excessive if a single precast piece was used. Therefore modularization was necessary for handling purposes.

Precast Abutments:

Abutments could be broken apart, or modularized, by introducing vertical or horizontal joints. Both alternatives were evaluated.

A configuration where the abutment modules are separated by vertical joints allows for simpler connection details as opposed to using horizontal joints. With long horizontal modules, connected by horizontal joints, the joints have to transfer considerable moment and horizontal shear created by embankment backfill. Using vertical modules, with vertical joints, avoids having joints that need to transfer high forces.

A constraint of a minimum of two piles for each vertical abutment module is selected to make individual vertical modules self supporting and stable. When the settlement is limited and the abutment modules are considered to be non continuous members separated by vertical joints under vertical loading from the superstructure, the loads at the joint are minimized eliminating the need for complicated connections. Under the lateral earth pressure, the modules are anticipated to deflect similarly, minimizing the loads at the vertical joint. The primary function of the joint then is to ensure equal deflection, by providing horizontal shear transfer capacity using a grouted keyed joint.

Standard abutment sections were initially proposed with heights of 3'3", 5', 10' and 38 in thick having joints in the vertical direction. The lengths of the modules was left as a variable. The length could be adjusted to meet the total weight limit of 40 kips (Figure 2). Having only one dimension adjustable makes the use of standard forms possible by precasters. The total abutment length for any specific bridge project is divided into equal length modules without exceeding the 40 kip weight limit, creating identical modules in each bridge project.

If abutment heights other than the 3 standard values are needed, then the standard abutments should be partially buried to achieve the desired elevation.

The modules have a flat top surface and the roadway crown is introduced by using small separate precast pilasters for beam steps or varying the bottom form surface for a cast-in-place slab.

The pile embedment length was selected as 2 ft. This is a desirable depth to allow for easy forming by precasters and in avoiding problems during abutment erection due to pile misalignment. The smaller the pile embedment length, the greater the deviation from vertical that is acceptable in a pile. For the 10 ft high abutment panel, which is under larger lateral earth pressure, a longer pile embedment length might be used where abutment overturning is a concern.

The shallowest abutment height of 3'-3" was determined based on the minimum depth of concrete over the pile which prevents punching of piles through the concrete body. The loading for this situation was taken as the maximum pile axial capacity.

The shallowest section can also be used with thinner wall panels attached to the piles, a system where the pile cap transfers the superstructure load to the pile and the wall panels span between the piles retaining the earth and resisting the lateral earth pressure as shown in Figure 2 at the left. This system may be advantageous due to the lighter nature of the shallowest section and the walls.

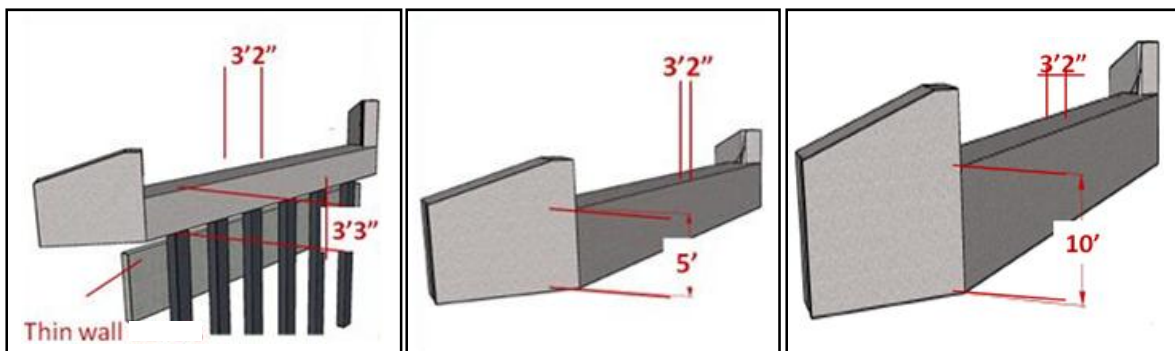


Figure 2 – Summary of the Standard Abutment Module Dimensions

The wingwalls are a similar shape to the abutments, but are generally higher to retain the earth at the superstructure level. The upper portion of the wingwalls is difficult to standardize due to the variability in geometry for different projects. The same standard rectangular precast abutment modules are to be used for wingwalls. A minimum of two piles are placed in each wingwall to make it self-supporting. This avoids the need to transfer moments through the vertical wing wall to front wall joint.

Since the top of the wing wall is usually of a non standard geometry, the top portion will be composed of cast-in-place concrete over the precast module. Completion of this cast-in-place portion is usually not on the critical time path for a project, and can be completed later while construction is already underway on the superstructure. possible details for the wingwall top will be discussed in a following section (Figure 6).

Precast Pier Caps:

A standard pier cap of 3.5 ft x 3.5 ft rectangular cross section was selected by the WisDOT. The 40 kip weight limit target would impose a maximum length of 22 ft. Given that a typical bridge may be 42 ft wide; means of modifying the design of the precast pier caps was needed. Cross sections with reduced concrete area are reviewed in Table 1. The various cross sections and the approximate proportion of their weight compared to the solid rectangular section are shown.

The solid rectangular section is familiar to contractors and designers, it is also easy to manufacture without complicated reinforcement details or formwork. The inverted U section allows easy inspection and easy form disassembling unlike the box sections. Therefore the inverted U section was chosen in addition to the solid rectangular section.

Cross Section	Approximate Weight / solid rectangular
 Solid Rectangular	100 %
 Hollow Rectangular	75 %
 Solid Inverted T	95%
 Hollow Inverted T (4)	80%
 Inverted U (5)	75%
 Tapered Rectangle	90%

Table 1 – Various Cross Sectional Alternatives to Reduce the Weight

Modularizing was also examined for the pier caps. Several possible joint configurations and the load transferring and temporary supporting they would necessitate were studied. (Figure 3) For all the combinations, modularizing the pier cap necessitates major load transferring connection details and temporary supports.

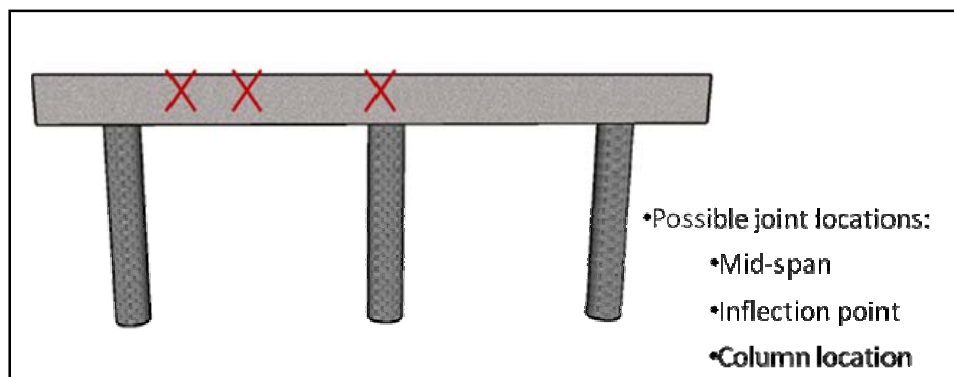


Figure 3 – Possible joint locations marked for a modularized precast pier cap

Considering the multi column piers are used mostly with girder type of superstructures, the intention was to utilize the equipment with larger capacity for girders that already available on the site. Therefore the heaviest precast pier cap that is still constructible was determined to be 80 k. Equipment with 80 k capacity is expected to be used for average length girders, and 40-50 ft long caps can be shipped and lifted as single piece within this limit. A single piece pier cap unit which can cover bridges of 40-50 ft wide is preferred over multiple pieces to eliminate connections. The pier caps are to be manufactured with the beam steps.

3.3 Identifying the Connection Methods

The connection details compiled for this project through literature review were grouped into two according to the use, module to module connections, and module to adjacent member connections. The first group covers the connection details for abutment module to module joints, while the module to adjacent member joint group is composed of abutment module to pile and pier cap to column connection techniques. The connection methods most suitable for the standard system were selected aiming at the simple to construct details with reasonable tolerances, and using the common construction methods familiar to contractors.

Module to Module Connections

Shear keys, lateral post-tensioning, splice sleeve connection, welded connection, bolted connection, and cast in place connections were the methods that were currently in use as identified during the literature review. (Figure 4)



Figure 4 – Abutment module to module connection methods: Spliced Sleeves (top left), Post tensioning (6) (top right), Cast in place closure joints (7) (bottom left), Welding (8) (bottom right)

Vertically separated abutment modules eliminate the need for major load transfers, and continuity and allow for simpler connections. All connection methods were evaluated based on whether they meet the load transfer demand, and whether they are easy to construct without requiring special equipment. Shear key connectors were picked as the connection method of high priority to connect abutment modules together as the load transfer need is minimal and they are simple and common. Shear key abutment module connections were successfully used for the Epping Bridge in NH. (Figure 5) The other options were eliminated due to the disadvantages summarized in Table 2.

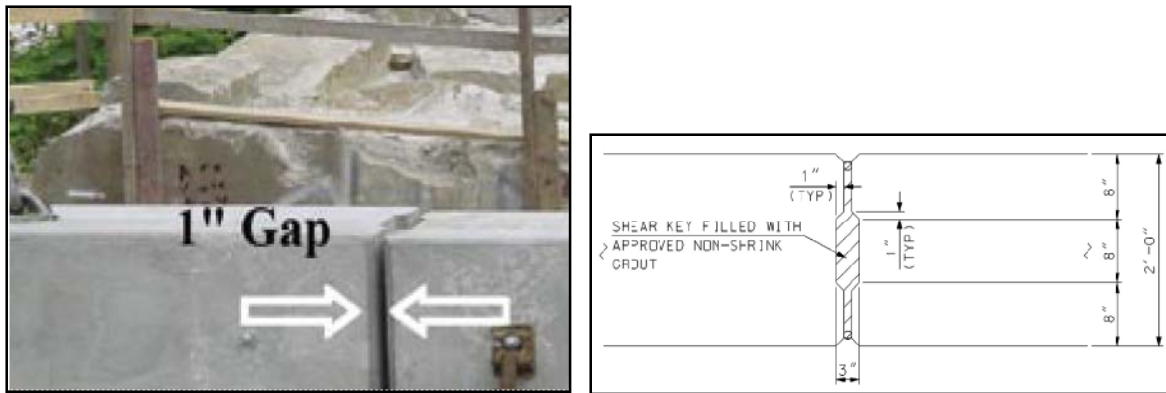


Figure 5 – Example shear key vertical abutment to abutment connection (9)(10)

Connection Type	Reasons for Elimination
Lateral post tensioning	Requires special equipment Unfamiliar to local contractors and may require separate subcontractors Not preferred by the DOT for now Load transfer provided is not needed.
Splice sleeve connection	Tight tolerances Protruding bars from the forms Modules are separated vertically and restrained by piles at the bottom
Welding the modules	Requires high on site quality control Requires a certified welder on site

	Not practiced by the DOT
	Uses steel plates susceptible to corrosion
Bolted connection	No reliable load transfer without welding to be used as a permanent connection
Cast in place connection	Protruding bars from the forms
	Requires forming and reinforcement placement
	Takes longer time

Table 2 – Disadvantages of the eliminated module to module connection methods

The same modules are used for wingwalls and abutments connected together with shear keys. The side surface of the wingwall modules were cast with an angle to fit the abutment module joint, if the wingwall is at an angle to the abutment body. The upper cast-in-place portion of the wingwall is cast on the precast module to embed the vertical bars grouted in the top surface of the bottom module. The cast in place portion also provides another means of connection to the adjacent abutment module by encasing the vertical bars grouted on top of the abutment module (Figure 6).

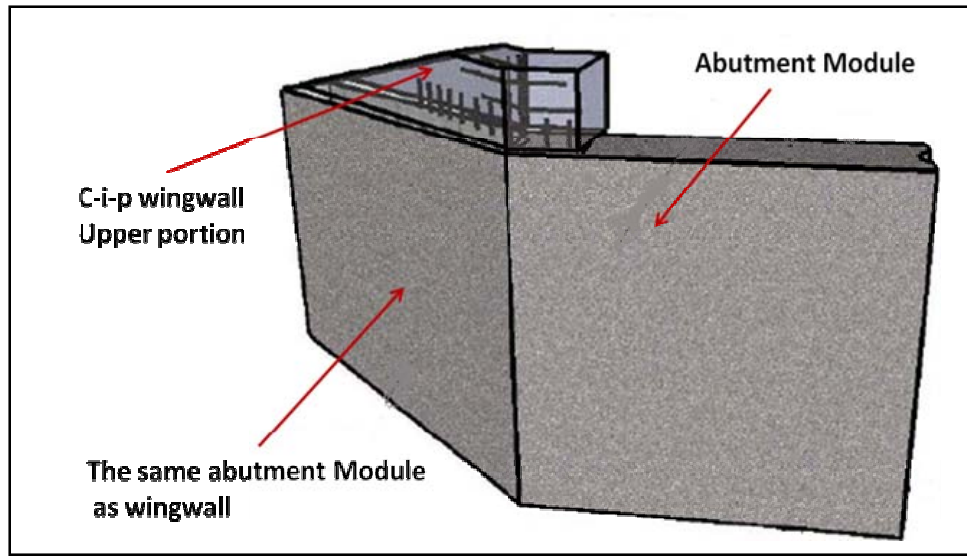


Figure 6 – Wingwall as a combination of the precast abutment module at the bottom and the cast in place portion at the top which also provides the connection

Module to Adjacent Member Connections

These connections include those used for precast abutment module to pile joints and cast in place column to precast pier cap joints.

Pile to Abutment Connections: The two common connection methods for pile to abutment module are embedment of piles in the abutment body and welding the piles to the abutment body (Figure 7). With the welded connections, plates embedded in the precast body are welded to the steel piles at multiple locations. This method was not preferred by WisDOT due to disadvantages similar to those of the welded module to module connection, i.e. welder qualifications and corrosion potential.

The embedded pile connection, which is used in many existing bridge projects with precast abutments and piles, is selected for development in this research. Precast abutment modules need to be cast with internal block out cavities to accommodate the piles. Since

current construction with cast-in-place abutments uses the same embedment technique this method should be easily adopted by contractors.

The block out sizing in the precast abutments was elected as 22 x 22 in x 2 ft (width x thickness x depth). That size accommodates 5-6 in. tolerance on each side for mis-alignment of piles (with the commonly used sizes of steel H piles). The remaining volume after pile embedment is filled with grout using ports from the top or the sides. The inner surface of the block outs are roughened for better bond between the grout and precast concrete.



Figure 7 – Pile to Abutment Connection methods: Embedded connection (11) (left), Welded connection (8) (right)

Column/Pier/Pile to Pier Cap Connections: The main types of existing connections between a pier cap and pier, column or pile include grouted connections, spliced sleeves, or bolted / post tensioned connections (Figure 8). The bolted connection requires tight tolerances in locating the bolts and is only useable in select applications. Post tensioning is an excellent connection method for providing strength without excessive tolerance requirements, but requires specialty work. With splice sleeves, steel dowels exiting from the column/pier must align closely with the splice sleeves embedded in the precast cap

requiring tight tolerances. Grouting in large blockouts provides for large tolerances and is easily accomplished.

Grouted connections can be achieved with grouted block outs, grouted pocket connections, or grouted duct connections (12) which embed the connectors coming from the columns into the block outs, pockets or ducts in the pier caps respectively. The connection is completed by grouting the blockout after the embedded connector is inserted from the column/pier.



Figure 8 – Pier Cap to Column connections: Bolted connection (13) (left), connection by splice sleeves (5) (middle), grouted duct connection (14) (right)

Out of these connections, grouted pockets were defined as the target connection type for this project. Their ability to accommodate adequate tolerances, to create minimal interference with the cap reinforcement and available design guidance made this connection preferable when considered with the disadvantages of other connections listed in Table 3. An example grouted pocket connection is shown in a photo from the construction of the Red Fish Bay Project, Texas (Figure 9).

Connection Type	Reasons for Elimination
Grouted block outs	Interference with the cap reinforcement or protruding cap reinforcement at the block out Requires more grouting
Grouted ducts	Tight tolerances
Splice sleeve	Tight tolerances More costly due
Bolted / Post tensioned	Tight tolerances for bolted connections Special equipment and contractor for post tensioning Not preferred by the DOT

Table 3 – Disadvantages of the eliminated pier cap to column connection methods



Figure 9 – Grouted Pocket Connection (14)

3.4 Initial Analysis and Design of Selected Components

The design of precast modules and traditional cast-in-place components can be similar in many respects. In the present case, however, due to the changes made in the structural geometries of the members, a need for standardization, discontinuities caused by precast joints, changes in recent bridge design code (LRFD) philosophies, and a desire to develop a more efficient system, a different approach was taken in the design of the precast pieces.

Precast Abutments

Abutments are primarily under vertical gravity loads from the superstructure and lateral loads from the retained earth adjacent to the bridge. The amplitude of these loads can vary depending on the structure's geometry. Vertical load is related to the bridge superstructure span and bridge type. The horizontal earth load is a function of the abutment height and backfill material.

To achieve module standardization, three different abutment heights were selected as shown previously in Figure 2. Force calculations for the abutment modules were repeated for those three standard heights.

The abutment was assumed to act as a vertical cantilever beam fixed at its bottom. Lateral earth loads caused lateral bending that induced tension on the abutment wall face in contact with the backfill. The resultant amount of back-face steel reinforcement to resist the bending was not found governing for any of the abutment heights when compared to the minimum required temperature and shrinkage reinforcing.

Design of standard walls to resist specific various vertical loads from superstructures, in order for the standard abutment sections to be used with all the bridges of interest, would be difficult since the load is closely dependent on span length. Alternatively, the abutment modules were designed assuming there was a vertical loading on the abutment creating a resisting force in each pile equal to a pile bearing capacity of 100 kips per pile.

The situation where the loading on the abutment was high enough that the piles reach their structural capacity was also studied. Although this case is applicable to the widest range of bridges, pile axial capacity is usually overdesigned compared to actual bridge vertical forces and the results using this method tend to be conservative for abutment design.

The maximum spacing of piles allowed by WisDOT, 8 ft, was used in the design since it creates the most unfavorable condition – maximum wall span between piles. Even with the maximum pile spacing, the span lengths were small compared to the abutment depths. With the resulting span to depth ratios, strut and tie analysis was preferred over the sectional bending theory for abutment force analysis. Several truss models were formed and reinforcement was proportioned to carry the reactions formed at the tension ties and the compression struts for the abutments with three standard heights, 3'3", 5', 10'.

The regions of the abutment modules around the pile block outs were designed with vertical and tie reinforcement placed around the block outs.

Suggested plan drawings, including reinforcing details, for the solid wall abutments with embedded piles are provided in Appendix A.

Loads used in the abutment design of the panels shown in the standard plans were calculated using the assumptions described above and applied in MathCad sheets provided in Appendix C.

Precast Pier Caps

As in the case of the abutments, the loads applied to bridge pier caps are strongly dependent on the bridge span length. To develop standard pier cap designs, a particular design needed to be able to accommodate a range of possible bridge spans.

Standard pier cap analyses and designs were performed for 4 ranges of bridge spans in 20 ft increments, from 50 ft to 130 ft. The design loads and cap design are provided in MathCad sheets shown in Appendices D and F. The width has little effect since the cap load depends on the girder reactions applied to the cap between piers supporting the cap. With a wider bridge there would just be additional girders but the number of girders between piers does not change significantly.

For each analysis, a bridge with the longest span in the selected range is used to achieve the largest vertical load. Pier frames were analyzed with both fixed and pinned joints between the piers and caps to create the maximum effects at the pier-cap connections and in the cap at midspan between piers respectively. The reactions are calculated for a 42 ft wide bridge with 3 columns of 36 ft height.

The loads applied on the pier frame were dead load of the girders, the deck and the wearing surface, live load including the truck and lane loads creating the worst reactions, loads due

to vehicle braking, wind load in transverse and longitudinal directions of the bridge acting on the superstructure, substructure and vehicles, and loads due to temperature changes. Dead loads were based on the Wisconsin Bridge Manual recommended precast W-girders for the appropriate span length and the recommended deck thickness for the suggested girder spacing. The reactions due to lateral loads in the transverse direction of the bridge dominated relative to the reactions due loads in the longitudinal direction of the bridge.

Since there are pockets for completing the connections to the columns, availability of space for the longitudinal pier cap flexural reinforcement needed to be checked. Pier caps, both solid and inverted U sections designed with mild reinforcement had enough space to accommodate the reinforcement. Prestressing was also considered to decrease the area of reinforcement and was applicable without causing excessive stresses at erection for pier caps of some span ranges.

The column to pier cap connection region was considered to be the extension of the column into the cap. Moment and axial load reactions for design at the top of the columns were maximized in the analysis in which the column to cap joints were fixed. In reality, the precast joint would result in a partially rigid joint and smaller design forces. In order to maximize the reactions at the connection, the reaction at the column top was obtained from the analysis with fixed joints.

Two possible connector configurations were generated, with one or two lines of connectors in each pocket. Connectors were actually vertical rebar dowels extending from the column top into the pocket region. The second configuration (Figure 10) increases the moment arm between the connector bars and is applicable for cases where the first configuration is insufficient to transfer the required bending moment.

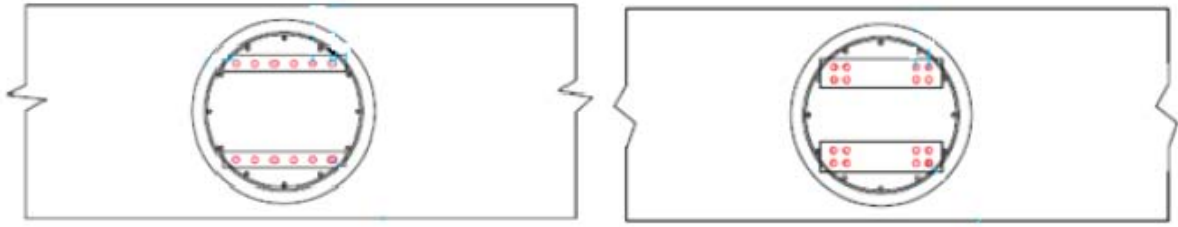


Figure 10 – Connector bar configurations: Configuration I (left), Configuration II (right) with connector dowel bars on the interior (in red) and the normal terminated column reinforcing in a circular pattern.

Moment and axial load interaction diagrams were created for several connector bar sizes and configurations as illustrated in Figure 11. The connection bar pattern satisfying the nearest interaction curve enveloping the reactions was chosen as the design solution.

The reactions belonging to bridges of the various span lengths and the moment axial load capacity diagrams of several connectors are shown in Figure 11 and listed in Table 4. Shear was also checked but did not control the connection design.

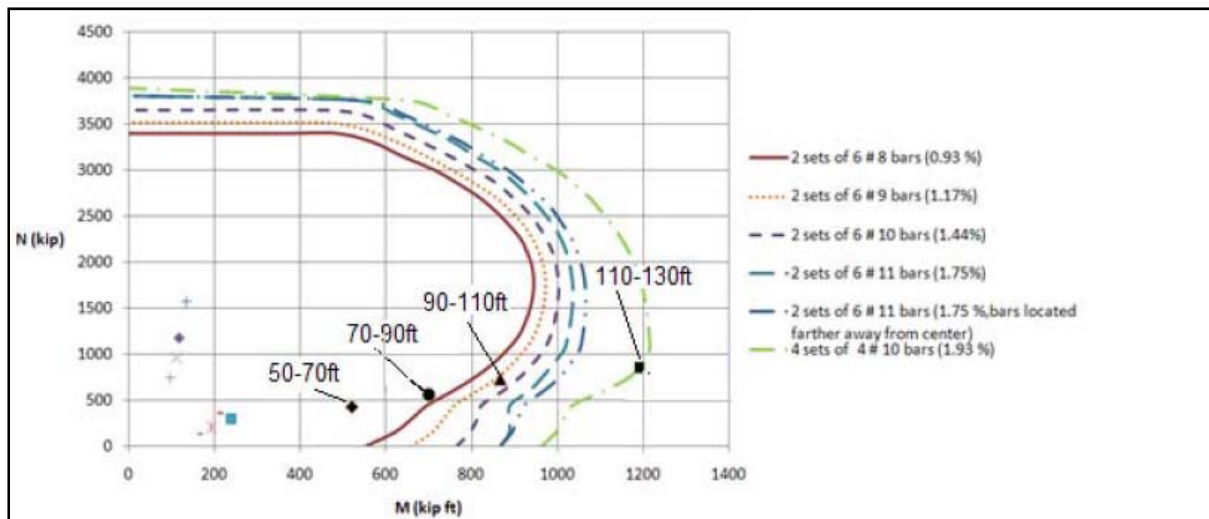


Figure 11 – N-M envelopes for grouted pockets and the reactions enveloped

Span Length (ft)	Connectors	Design Loads (M-N)
50 -70	2 sets of 6 # 9 bars (Configuration I)	520 ft-kip – 420 kip
70-90	2 sets of 6 # 9 bars (Configuration I)	700 ft-kip – 556 kip
90-110	2 sets of 6 # 11 bars (Configuration I)	867 ft-kip – 730 kip
110-130	4 sets of 4 # 11 bars (Configuration II)	1192 ft-kip – 852 kip

Table 4 – Connectors chosen for pier caps with each standard bridge span range

4 Initial Summary

Motivated by the success of existing projects in other states that used precast substructures, this project aimed to generate a system of standard precast substructures for use in Wisconsin. The components of the system to be precast were selected based on demand in Wisconsin. Standard geometries were determined, according to the weight and size limitations described in Section 3-2. Since the size of modules must be limited to control weight, connections between modules are needed and current connection types were briefly examined in Section 3-3. Standard precast abutment sections supported by pile foundations and precast pier cap sections on cast in place columns were proposed in Section 3-4.

Detailed drawings of the designed solid abutments, with required reinforcement, are included in Appendix A. Detailed drawings of the designed pier/pile bent caps, with required reinforcing, are included in Appendix B. The systems are summarized in the figures below (Figure 12) (Figure 13).

Calculations for the design loads on a precast abutment, as described in Section 3-4, were prepared in a MathCad sheet and are shown in Appendix C. Calculations for the design loads on pier/pile bent caps were also done using MathCad as shown in Appendix D.

Calculations for the design of a solid precast abutment with pile pockets as described Section 3-4, in heights of 3.25ft., 5ft., and 10ft., are included in a MathCad sheet of Appendix E. Calculations for design of an abutment with cap beam on piles and separate retaining panels are provided in the MathCad sheets of Appendix F. Calculations for the design of pier/pile bent caps are included in Appendix G. The MathCad sheets are provided electronically to WisDOT.

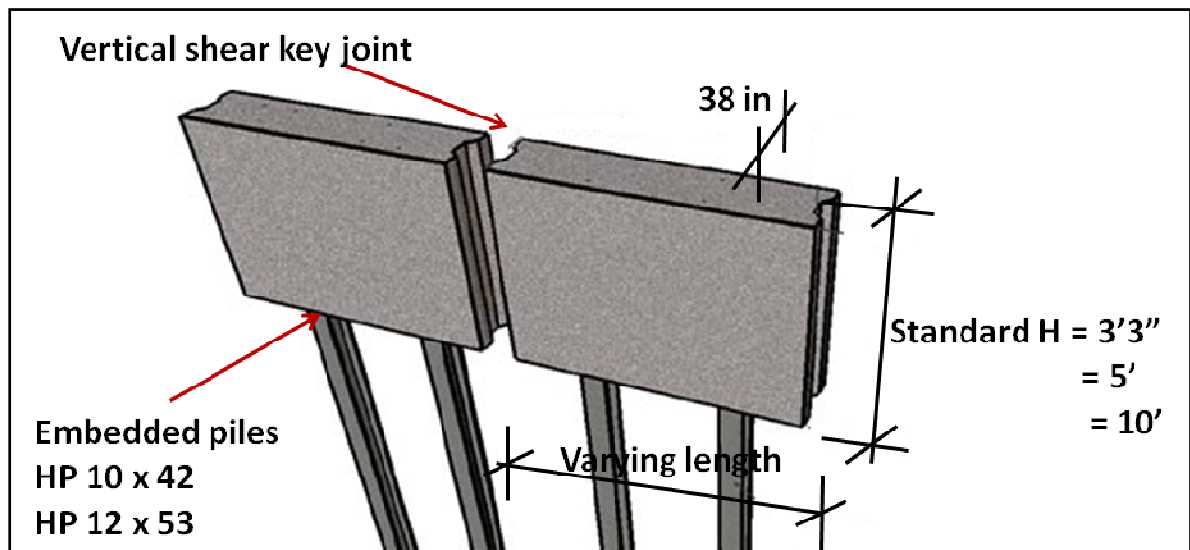


Figure 12 – The Standard Precast Abutment Modules in summary

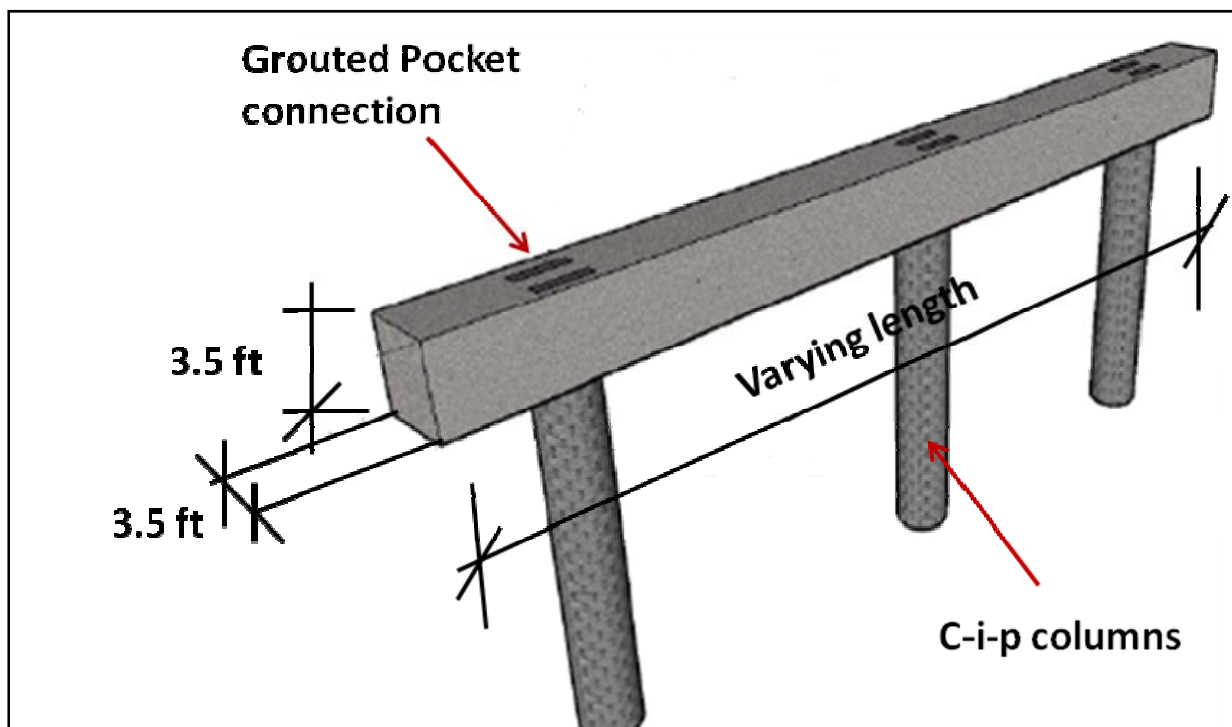


Figure 13 – The Standard Precast Pier cap in summary

Assumptions used in the design of the standardized abutments and pier caps are summarized in the list below.

Abutment design:

- 3 standard abutment heights were pre-selected,
- Pile spacing below abutments assumed as 8 feet,
- Design considered vertical load and lateral backfill pressure,
- The abutments were assumed to be vertical cantilevers,
- Abutments were designed for a vertical load of 100k per pile,
- Design was accomplished with strut-tie modeling.

Pier Cap design:

- Caps were designed for four bridge span ranges (50-70ft, 70-90ft., 90-110ft, 110-130ft),
- Super structure DL, LL, braking, wind, and temperature loading were examined,
- Transverse lateral load effects dominated,
- Piers were assumed to be 36ft. high and 36" diameter,
- Rigid pier to cap connections were assumed,
- Pocketed connections were assumed between pier and caps.

Maintaining similarity between the precast structures to existing cast-in-place structures served as a basis for the selected systems in order to utilize existing familiarity of contractors and designers.

For some cases, however, simulating cast-in-place construction may not be the effective solution and adjustments specific to precast construction should be made. Cast-in-place abutment walls are thick and very heavy, not a suitable design for precast components.

Although certain connection types were selected here, other connection types were also compiled for reference since there are cases where other connection types such as welding or post tensioning can be particularly useful. Welding leads to minimal cast-in-place or

grouting work and is one of the fastest methods of construction. Post tensioning is a secure way of load transfer between members where necessary. These connection types are not currently preferred by the WisDOT, however application in pilot projects may prove their efficiency. Similarly, a precast pile cap combined with precast wall panels as an abutment is an alternative to the solid wall heavy abutment body selected. This option may provide ease in erection and improve constructability.

Aesthetics is not listed as one of the main goals of this research that was dominated by functionality. Precasting, however, can readily incorporate aesthetics by using decorative finishes. Form liners can be used to create texture on the outer surface of elements such as natural stone appearance. Colored concrete, sandblasting for exposed aggregate, and decorative masonry lithes are also options which might be provided with precast concrete to improve the visual image.

For cases where the construction site is a distance from the precast plant and transportation costs are high, on-site casting near the construction location using mobile ready mix plants or locally produced ready mix may be considered.

It is expected that with pilot projects in Wisconsin, the system will be better understood and necessary modifications will be made. Although the initial cost of the system will likely be higher than cast-in-place construction, the indirect cost benefits from reduced construction time and long term durability should also be considered. Stimulating contractor and precaster acceptance by illustrating the benefits and developing familiarity with fabrication will also make the system more desirable.

5 Experience with Baldwin Precast Bridge Abutments

Precast concrete abutments were used on a pilot project for the first time in Wisconsin on the U.S. Highway 63 Bridge over the Rush River just north of Baldwin, WI (B-55-217). The abutment panels used in Baldwin are generally similar to the standard abutment panel plans developed in the first part of this project. Figure 14 is a picture showing the cross section of one of the Baldwin Bridge modules.

The solid slab bridge had a span of 48ft with a 47ft width and 20° skew. The abutment included 45° wing walls at each end. If built as cast-in-place it would have used a WisDOT A5 type abutment with a 10.5ft height. Ten piles would have been needed to support the cast-in-place abutment and wing walls.

Each substitute precast abutment was composed of seven modules, five at the front face and two wing wall pieces. The size of the modules was limited to control their weight. The largest modules, the wingwall pieces, were 7ft-11in high at one end and 13ft high at the other with a width of 14ft- 6in. Front module pieces were typically 11ft-4in wide and 10ft-6in tall. The wingwall modules were 24in thick and the front wall modules were 30in thick. Figure 14 shows an end view of a module and placement is illustrated in Figure 15.

Since two piles were required for each abutment, to make the modules self supporting in resisting loads, a total of 14 piles were required. The voids in the precast panels, to accommodate the 10x42 H-piles, were 26in x 22in in the wing walls and 18in x 22in in the front walls. Full depth voids were used to simplify grouting after placement of the panels. The piles were intended to end at approximately 2ft below the top of the abutment.

Each module had a female keyway formed at the edge. Adjacent modules were connected by grouting the keyways from above.



Figure 14 – Cross section of an abutment segment showing the voids left for pile placement and the female keyways at the edges.

The Baldwin project proved that precast substructure systems can be very effective in attaining the goal of rapid construction. Eight heavy precast panels were set into place at Baldwin, to complete the partial abutments for a staged bridge construction, in six hours. A total lineal length of 96ft of abutment was placed in the first day. On the second day, forming was placed over joints between the panels in two hours with two workers and the joints were grouted in an additional 2.5 hours. The eight panels required a total of 10.5 hours to place and finish with 4 workers on the job. This would be compared to 11 to 14

days for placing reinforcing, forming, pouring and curing on a similar cast-in-place abutment project.

The Baldwin project clearly showed that rapid construction is achievable with precast concrete substructures. The quality of the completed abutment system appeared to be better than a typical cast-in-place abutment.



Figure 15 – Setting the second abutment module in place

Yet, there were several problems identified in the Baldwin Bridge project. Those problems include:

- High cost

The bids on the Baldwin Bridge were excessively high. This is partially due to the fact that contractors in Wisconsin have never had experience with this kind of project. As in a learning curve, after contractors complete initial projects which involve precast elements, more competitive bids are expected. A primary cause of higher than normal cost was assigned to the need for a crane with high lifting capacity. The contractor was also disliked needing a 100 ton crane on the job to set the heavy precast panels. Moving the crane from one side of the bridge to the other created additional problems and would be particularly difficult if staged construction was not used and the bridge was not available for crane crossing of the river. A smaller crane would have been sufficient with lighter panels.

- Contractor concerns

The contractor on the Baldwin job disliked the precast system explicitly because of the size and weight of the precast components being placed. There was particular concern about the safety of workers in moving and placing the large components. It should be recognized, however, that weights of typical precast girders on normal bridges equal or exceed the weight of the abutment pieces, so this concern was probably mis-founded.

- Additional piles

Additional piles were needed on the Baldwin project because the 10 ft. high abutment created panels that were very narrow in width to control the individual panel weight. Thus more panels were needed to achieve the required abutment width. Since the design was predicated on having at least two piles per panel, and the panels were narrow, additional piles were needed compared to the original

design for a cast-in-place abutment. This effect could be avoided if wider panels could be used, but they would have to have a lighter weight per foot of width.

- Pile alignment

The contractor and WisDOT engineers were concerned about the accurate pile placement needed to match the piles with the voids in the precast abutment panels. If a subsurface obstruction had been hit during pile driving, it may have been difficult to meet the pile alignment requirements.

All of the concerns above could be addressed with an improved design of more efficient and lighter panels with shorter pile embedment lengths into the panels.

6 Revised System / Improved Precast

Focussing on the impact of the heavy weight of the Baldwin precast concrete abutments, it is evident that a more efficient precast abutment module design is required. Similar reduction of weight would likely be needed for pier/pile caps. All of the modularity, connections, and standardization conditions explained previously are essential, but with added efficiency in load resistance and weight reduction. The intent of this second phase of precast substructure development was to improve the overall efficiency of the precast substructures. This was accomplished by developing new lightweight efficient abutments and pier/pile caps that could be produced in short pieces and spliced together in the field.

6.1 Abutments

Three new efficient alternatives are proposed for use as precast bridge abutments. WisDOT should select one of these systems as the Wisconsin standard abutment.

6.1.1 Piles with Cap and Separate Retaining Panels

This simple system has frequently been used in construction of railroad bridges where rapid construction is essential to avoid line closures. The vertical load resistance is achieved with a basic pile bent. The precast pile cap, proposed previously, is connected to the piles to quickly create a bent. Then separate thin (8") precast wall panels are placed behind the piles and serve as the backfill retaining wall. An example calculation for the design of this system is provided in Appendix F.

6.1.2 Piles with Cap and Precast Hollow Wall Panels

A second alternative abutment system actually combines the use of a piles embedded in a wall panel with a standard precast pier/pile bent cap at the top. The precast cap is used to support the superstructure and sits on a hollow precast wall forming module as shown in Figure 16. The hollow wall form module provides greatly increased tolerance in position of the driven piles and will be filled with concrete after placement. It is topped with a standard pier/pile cap as used in pile or pier bents. The weakness of this system arises from the need for the concrete fill to harden before placement of the cap and construction continuation.

Reinforcement bars extending from the filled module are embedded in pockets of the bent cap to connect the components both vertically and laterally with the cap acting as a tie strut for the walls. Sufficient development length is required for reinforcement bars both in the cap and hollow panel. Embedment length for piles in the wall module can be variable with a 3ft. minimum length recommended.

The design of this system is basically the same as the design of the solid precast abutment wall with embedded piles as shown previously in Appendix E.

There are explicit advantages and disadvantages for this system. First of all, the design of the bent cap and wall panels is simple. The precast pieces have low weights. Manufacturing bent caps with simple shapes is very easy. The length of the wall panels can be double the center to center distance between two piles and the height of the abutment can be variable for each project.

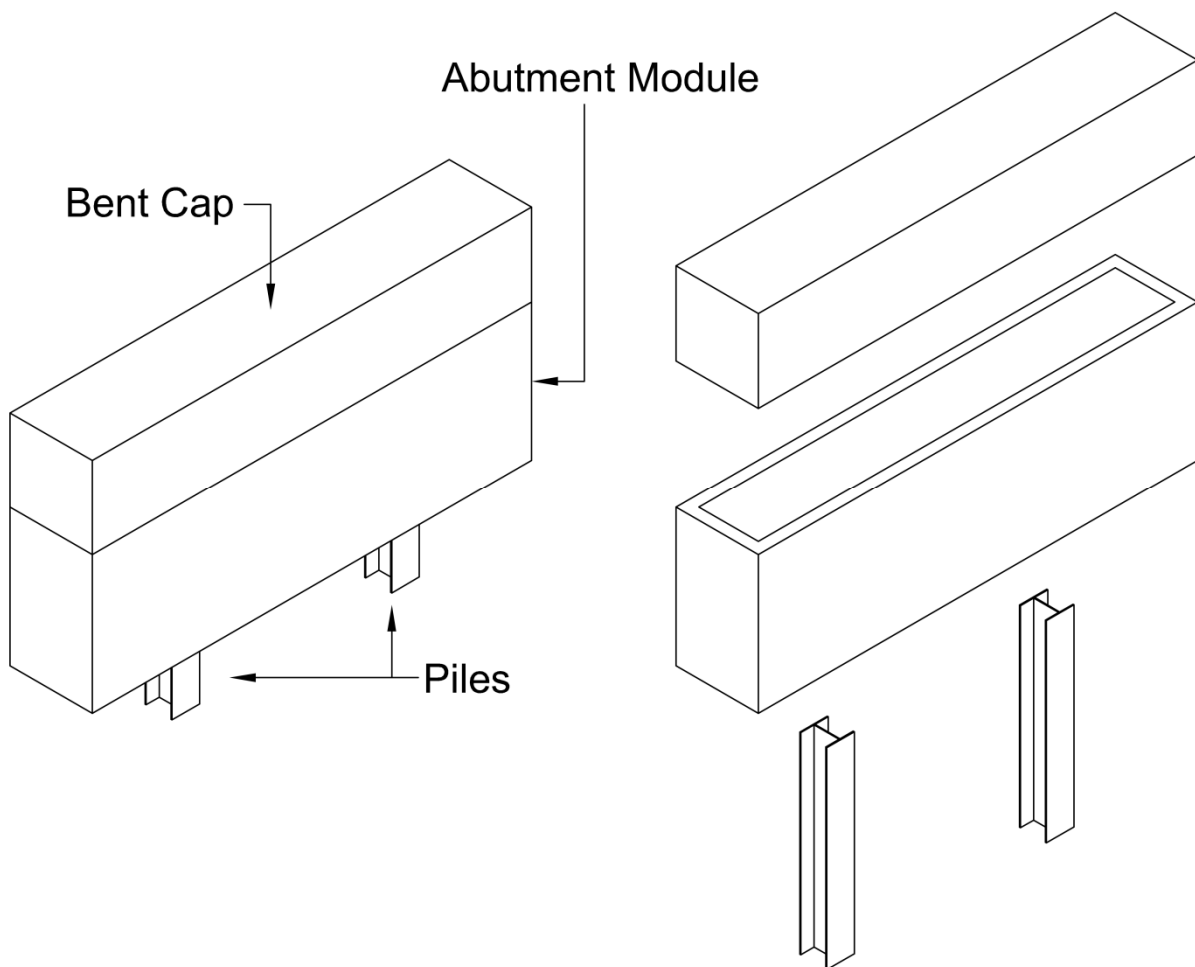


Figure 16 – Pile Bent Cap with Hollow Wall Panel

The system also has disadvantages. A precast wall panel having a full depth hollow core may be difficult to fabricate and may be susceptible to damage during shipping and erection. Cracks are likely to develop in the thin walls, but would not be structurally detrimental. Finally, the amount of concrete/grout is larger than other systems and will lead

to additional casting time and curing. The curing time before following construction activities reduces the “rapid” quality of using precast systems.

6.1.3 Full Length Socketed Wall with Cap

An attractive alternative abutment system uses a full height socketed wall with a standard precast cap as sketched in Figure 17. The wall module is connected to the piles by means of grouted sockets. After driving piles into the sockets or sliding the wall sockets over the piles, the remaining void will be grouted and full bonding will be achieved. With high performance concrete or grout, sufficient strength can be achieved to continue construction above within a day. A standard precast pile/pier cap will be placed above the wall module to support the bridge girders.

The main advantage is the lightweight of the system achieved by eliminating the thick abutment wall. Building the socketed module is a challenge that precasters can meet. The site work will be simple as long as piles are driven with reasonable accuracy. The pile pockets could be placed on either the backfill side or on the outside of the abutment. The aesthetics with the piles on the backside of the abutment may be preferred. None of the connections are complicated.

Reinforcement dowels are placed exiting from the top of the sockets during grouting. The dowels then are embedded in pockets of the cap and again grouted. The cap beam provides a lateral tie for the individual precast wall panels that have grouted keyed vertical joints.

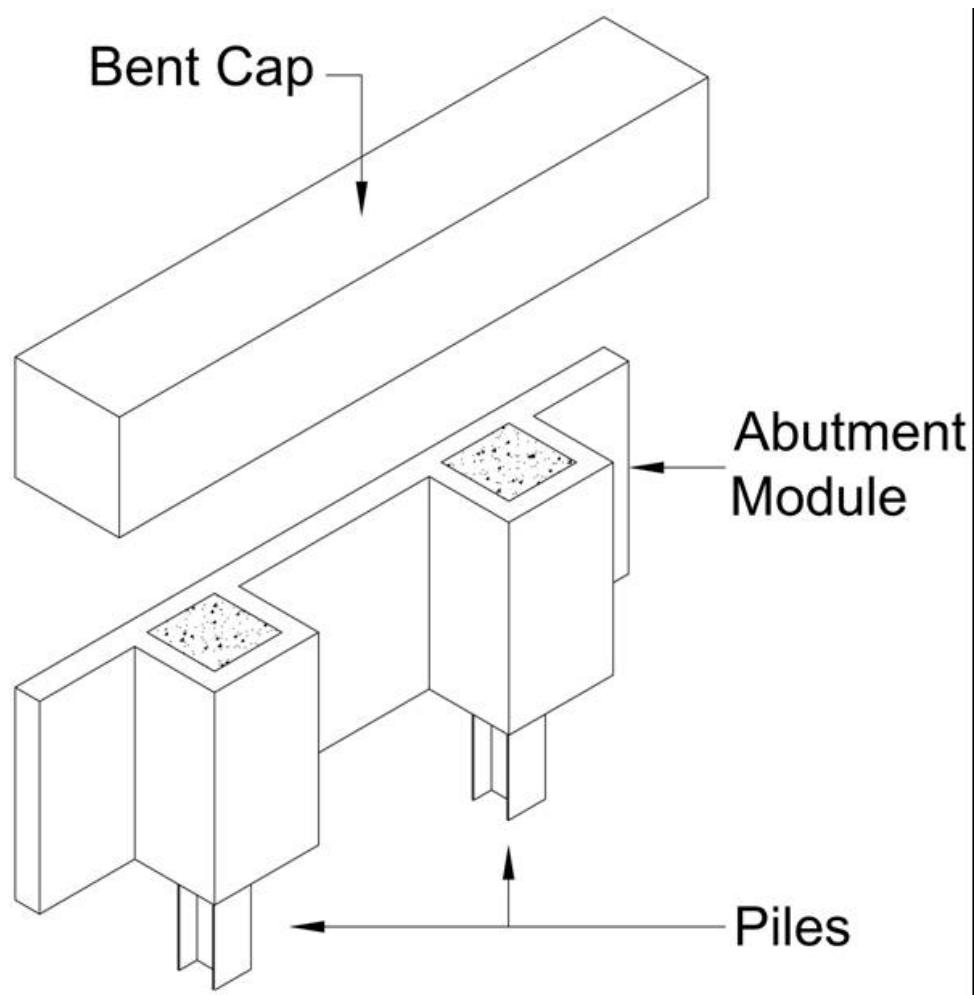


Figure 17 – Full Length Socketed Wall with Bent Cap

6.1.4 Limited Length Socketed Wall with Cap

This system is basically the same as the full embedment type in terms of concept. It has a limited socket length, however, to simplify fabrication and reduce weight, and a concrete strut above the socket (see Figure 18). The embedment length of the pile into the socket should be at least 2 times the width of the HP piles, or 20 to 24 inches for piles commonly used in Wisconsin. The total socket length is 4ft. so this system is most practical for high wall panels. For short height abutments the socket would be full height and the strut is eliminated.

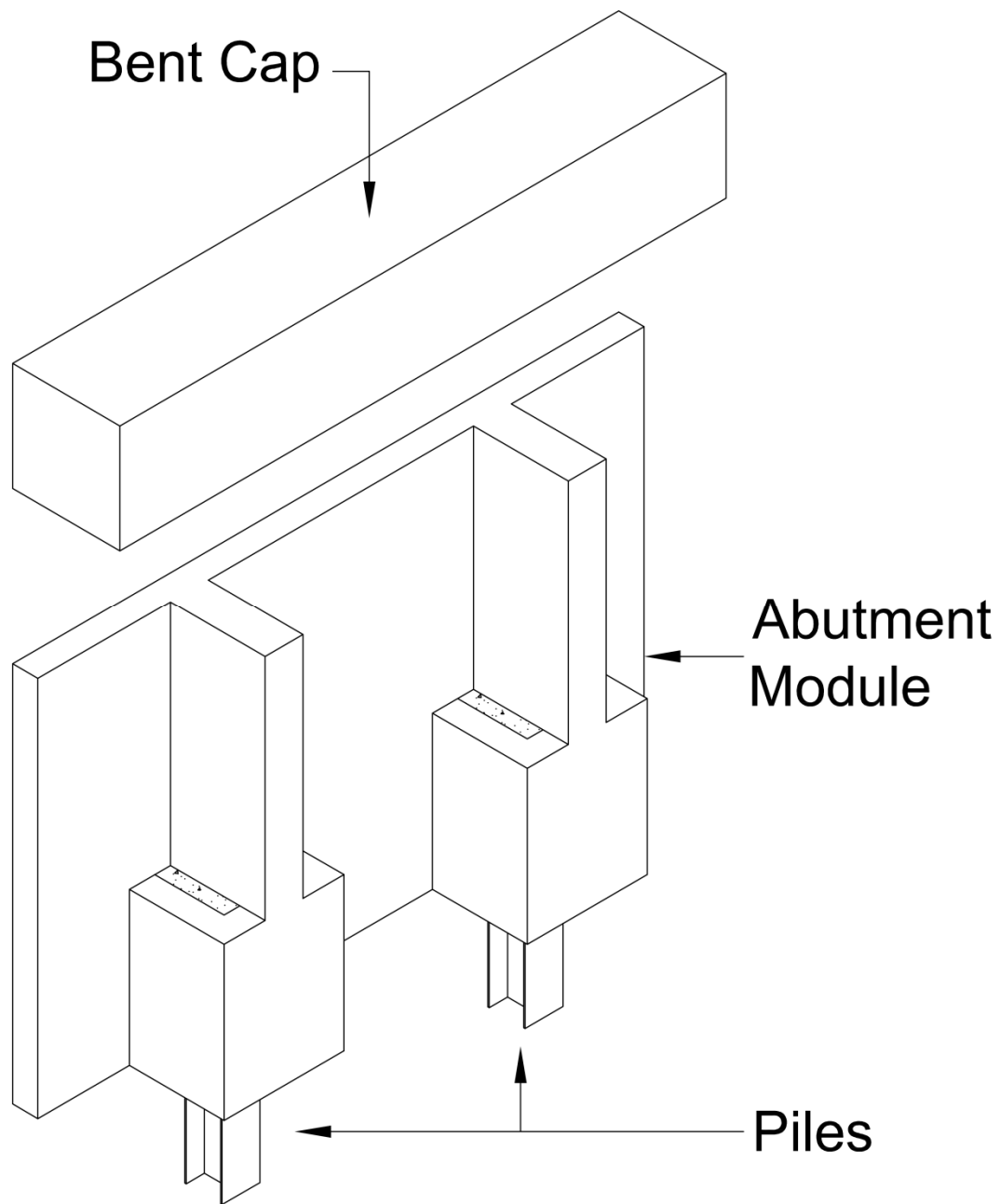


Figure 18 – Limited Length Socketed Wall with Bent Cap

Again, a bent cap will be connected to the abutment wall using a pocketed connection with reinforcement from either the vertical strut (if present) or the socket being embedded in a

grout pocket of the cap. The wall panels have keyed grouted vertical joints and are tied together by the cap.

6.1.5 Design of Socketed Wall Abutments

The force transfer mechanism in the two abutment walls suggested above is particularly complex in the region where the pile is embedded in the socket.

A MathCad design sheet for the upper portion of the “Limited Length Socketed Wall” with the design of the vertical strut is shown in Appendix H with loads as defined previously using the calculations as completed in Appendix C. The portion of the abutment above the socket is assumed to act like a vertical cantilever.

The portion of the wall where the pile is embedded is treated separately. Very little information was found in the literature for design of this type of force transfer system. The most relevant concepts were those used in the design of socketed column foundations as often used in precast building systems in Italy and Turkey. Those systems use a separate precast footing block that is built with an open socket. A column is then placed into this socket and concreted into position.

The design of the socket has been suggested in two alternate approaches. In one approach bond or skin friction is assumed to exist between the surface of the pile and the concrete. The second approach ignores this friction resistance. Both methods were used here to examine the design of the socket region around the embedded pile.

Appendix I provides a tutorial MathCad sheet describing the socket design process and the design equations for concrete wall thickness and reinforcing in the socket region. Appendix J has a production version of the MathCad sheet that can be easily used for socket design. Appendix K includes drawings of the abutments with reinforcing from the design sheets.

6.2 Pier/Pile Cap Connections to Create Long Caps

The cross section for a pier cap or pile cap was previously selected as either solid rectangular or an inverted tee shape. The cap weight again limits the practical length of a single piece cap member. Simple methods are needed for splicing short lightweight cap pieces together to create long cap beams. Three alternatives for connecting adjacent cap segments were considered in this research including welded steel plate connections, spliced reinforcement connections and post-tensioned connections. Connections of pier bent caps could be made over piers, so that there is no need for temporary scaffolding, but that location requires both pier-cap and cap-cap connections to be formed at the same location. It is preferable to locate the splice between piers or piles to obtain a better connection.

One of the main objectives of this research is to modularize and standardize the substructure units. To achieve this goal, the same standard bent cap is intended to be used as a pier cap, a pile cap and a low height abutment module. Adopting this variety of uses would allow repetitive precasting, reducing costs. This standard rectangular bent cap will not have varying height girder seats. Instead, small precast pedestals with different heights will be attached by the precaster at the plant or by the contractor on site. These pedestals will be used to give desired cross slope or super-elevation to the bridge deck by varying the girder heights. This eliminates casting special precast bent caps with different elevated beam seats for each project and further improves standardization and modularity.

6.2.1 Welded Steel Plate Connection

A simple cap-cap joint system involves a welded steel plate connection. Steel plates are embedded and anchored into the top of each cap component during production (see Figure

19). After putting the caps in place on site, the two steel plates will be joined by placing a third steel splice plate overlapping the embedded plates and welding it to the embedded plates. This type of connection is designed for providing an “integrity” tie, but not to transfer flexure or vertical shear forces across the joint.

Design calculations for a welded steel plate connection are provided using a MathCad sheet as shown in Appendix L.

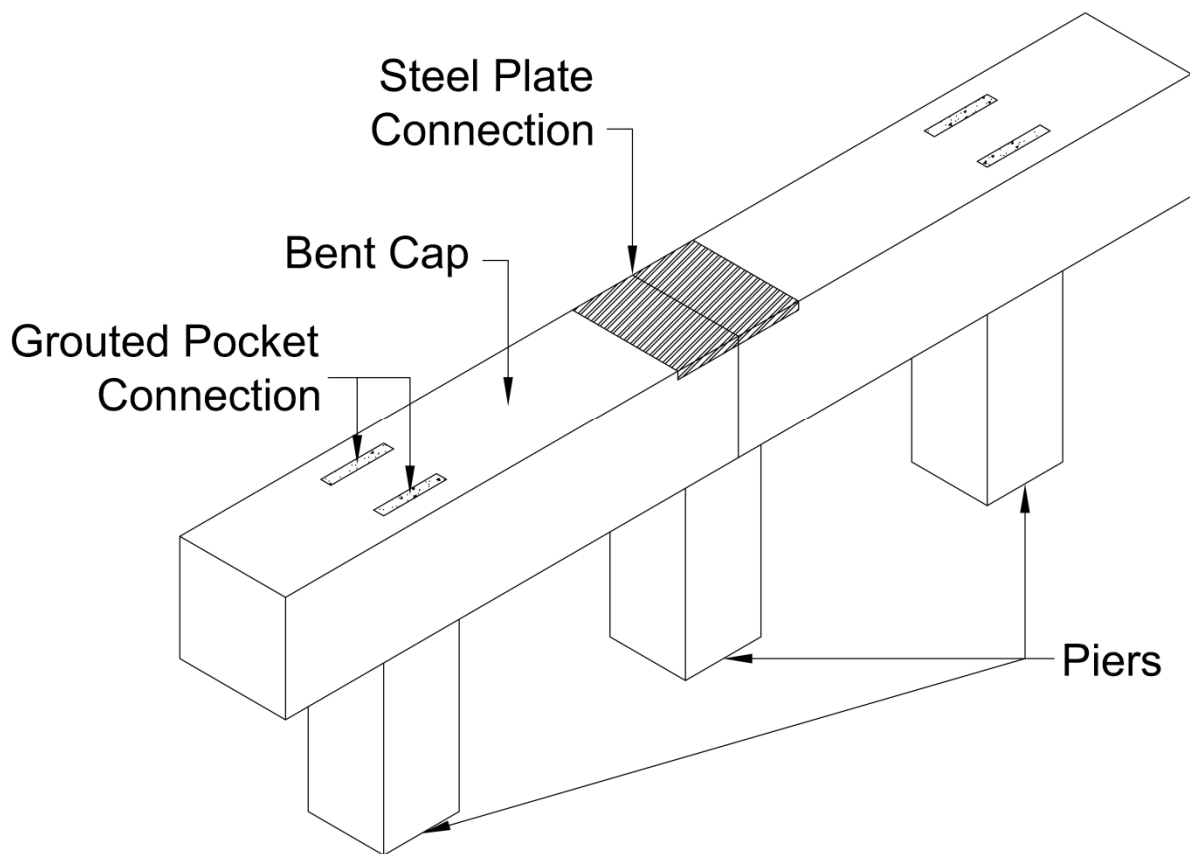


Figure 19 – Isometric View of Welded Steel Plate Connection

This type of connection is very fast to complete, since there is no need for cast in place concrete construction. The caps need to be positioned and leveled, then they can be welded together. For better durability, stainless steel should be used in the connection. If

the connection is placed between piers/piles then the fatigue strength of the weld due to flexural stress, under repeated vehicle loading, needs to be examined.

One weakness of this connection is the fact that there is an eccentricity in transmission of axial forces between cap pieces due to the eccentricity of the steel plate and tension reinforcement inside the cap.

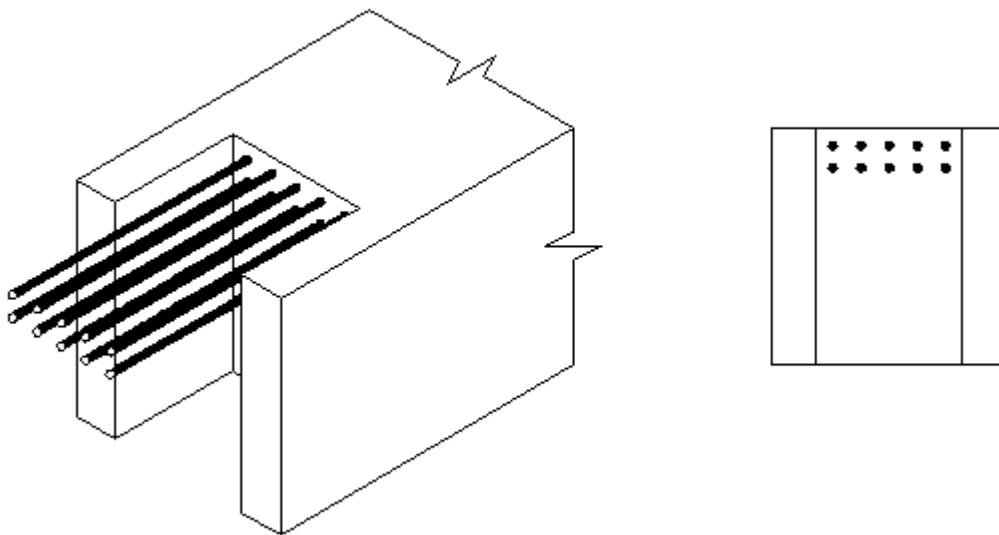


Figure 20 – Cross Section View of Connection

6.2.2 Spliced Reinforcement Connection

A second cap-cap connection option uses spliced reinforcing bars sticking out from both cap modules. Reinforcement will be left protruding out of the end of each cap segment and join inside a gap between the components. The reinforcement bars from each pier cap module can be overlapped or separate splice bars can be dropped in alongside the bars in the caps to serve as splicers. The void around the exposed bars is concreted. Two different options for this connection are shown in Figures 20 and 21, one with a full depth void and the other

is partial depth. These connections can be designed to provide flexural and shear continuity across the joint.

Design calculations for a spliced cap connection, applicable to both options, are provided in the form of MathCad calculations as shown in Appendix M.

Enough development length should be provided within the grouted connection region. Cast-in-place concrete is used with this connection and there is pouring and curing time for concrete to gain its strength, this might delay the construction. Another disadvantage is in the difficulty of forming the bent cap at the connection region with bars sticking out of the formwork. At the region where the cap bars are spliced, other bars coming from the pier or pile should also be accommodated and grouted together with the cap tie bars. Therefore, this region actually makes two connections which are the cap-cap connection and pier-cap connection if the connection is placed above a pier. This connection may also be readily used with an inverted tee bent cap.

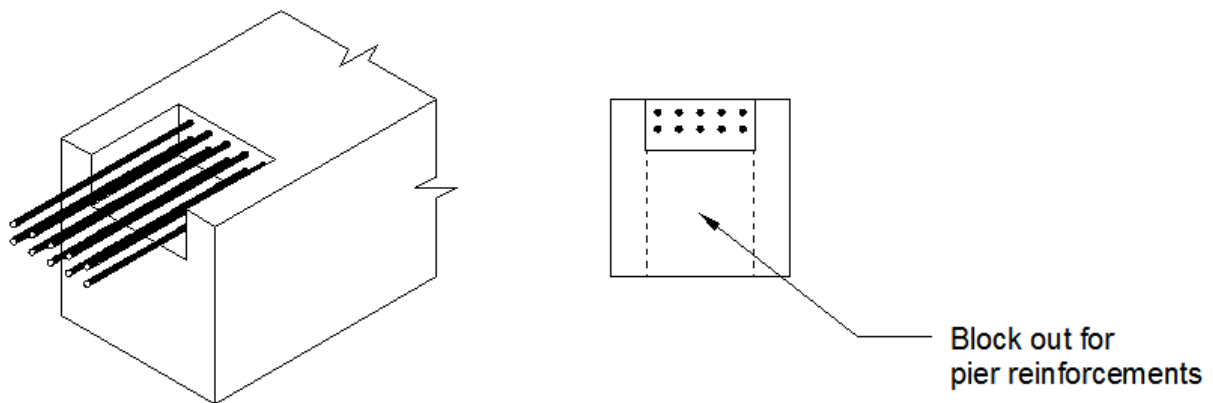


Figure 21 – Second Option for Spliced Reinforcement Connection

6.2.3 Post – Tensioned Connection

The most desirable connection uses post-tensioning of two adjacent cap girders. The cap girders will have ducts for post-tensioning tendons placed during fabrication. Before post-tensioning, the small interface gap between the components will be filled with high performance early strength grout. After post-tensioning, the ducts are also grouted as soon as possible. The post-tensioned joint can easily be designed to transfer flexural and shear forces between cap components. This alternative is fast, durable and reliable. If the connection is made over a pier, girders may actually be set on the cap before the post-tensioning is completed.

Post-tensioned design may include post-tensioning to carry the DL of the girders as well as other loads applied to the caps. If the post-tensioning is completed before the girders are set, then the post-tensioning can be used to resist the internal cap forces from both DL and LL. Calculations for a post-tensioned cap connection are given in a MathCad sheet that is included in Appendix N.

Post-tensioning has not been commonly practiced by WisDOT. For that reason, this method of construction might be costly at first due to lack of familiarity by contractors. But, as in the case of every method, when it is repeated on different projects its cost will definitely decrease. In this system regular reinforcement in the member is only needed to resist dead load (handling & shipping) of the bent cap, unless the girders are to be placed before post-tensioning. Once the post-tensioning is applied the next step in the bridge construction process can proceed immediately.

The greatest advantage of post-tensioning is that the designer can control cracking of the concrete cap, providing very good durability since water and de-icing salts cannot penetrate and reach the reinforcement.

Block outs for the pier to bent cap connection are also required and should be placed to not interfere with the post-tensioning ducts.

7 Recommendations and Summary

A series of design options for precast abutments and pier/pile bent caps have been explored. Use of initial options proved the importance of emphasizing efficiency and reduced weights. The possible systems have been compared with their advantages and disadvantages delimited. The comparison allowed selection of systems that appear to be most suitable for Wisconsin bridge construction.

A pilot bridge construction project using precast abutment panels emphasized how critical appropriate design is to achieving a useable and economical precast substructure. The Baldwin bridge, on US Highway 63, represented the first use of a precast abutment system in Wisconsin.

Construction of the Baldwin bridge was successful in proving that precast substructures can facilitate rapid bridge construction. It was unsuccessful in proving the economy of a precast abutment system because: 1) custom built wood forms were used for the precast components, 2) the concrete substructure panels were 'overdesigned' and so heavy that only one panel could be shipped per truck load, 3) the contractor needed a special high capacity crane to lift and swing the panels into place. The project demonstrated that repetitive use of modular formwork is necessary to reduce precasting costs and lightweight efficient modules are essential to reduce crane costs and erection costs in the field.

Recommendations

1. Wisconsin should use precast substructures: Precast substructures are a wise design alternative for many, but not all, bridge structures. The precast substructure systems, including abutments and pile/pier caps, clearly are proven to facilitate rapid

construction. Precast should be selected when bridge construction, reconstruction or rehabilitation will disrupt the highway and impede public travel. Precast should also be employed where construction conditions reduce the safety for construction workers, such as pier cap construction over water or at high heights. Precast is a particularly desirable alternative in situations where use of cast-in-place substructures will require placement of concrete under water, both from an integrity viewpoint and an environmental viewpoint.

Precast substructures can generally be expected to be more expensive than cast-in-place, but the benefits of rapid construction, improved work zone safety, reduced environmental impact, and long term improved durability balance the higher initial cost.

2. Standard repetitive modules: WisDOT needs to select a small set of standard precast modules that will be used on numerous projects. One standard abutment design and one standard cap design would be best. With standardization, and a commitment to use precast substructures, the cost of precast systems will be reduced. Precast bridge girders would not be economically competitive if the girders were custom formed and built for each bridge. Similar standardization is needed for precast abutments and pier/pile caps.

3. Abutment modules: The “socketed wall with bent cap” or “limited length socketed wall with bent cap” is recommended as the preferred precast abutment system. The abutment module, retaining backfill and mating with piles, would be of a single standard design but the formwork would allow for variable height to accommodate all types of bridge situations. The abutment module would be topped with a standard “cap” module to support the bridge superstructure.

The abutment systems could be divided into two groups with different height ranges of 3ft-5ft and 5ft-10ft. For the first height interval, which is 3ft-5ft, a full length socketed wall is preferred. The limited length socketed should be used for the height range of 5ft-10ft. In that module the total socket length is fixed at 4ft. Therefore, the embedment length of the piles is to be between 2ft-3ft. It is assumed that the precaster can develop a forming system that can be easily switched to produce either type of module from the same basic form. Each selected abutment segment should be produced in the factory with a standard 5ft fixed pile spacing and variable length cantilever walls on each side of the module as shown in Figure 22.

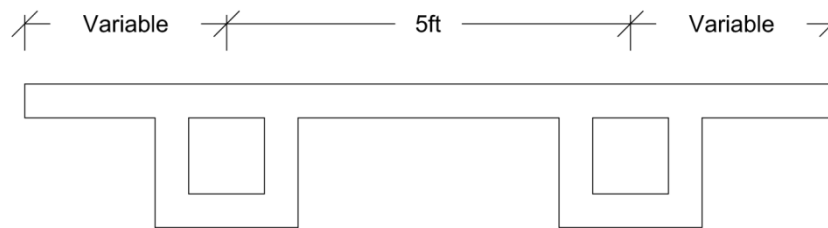


Figure 22 – Cross section view of abutment section which shows the fix length between piles and variable length cantilevers.

4. Abutment module connections: A standard vertical female-female keyed and grouted connection should be used between adjacent walls of the abutment modules. The key should be filled with a rapid cure non-shrink grout to provide out-of-plane integrity between panels. The abutment wall modules should be tied together through connections with the cap module placed above. The cap module should be attached to the abutment wall modules using reinforcing bars extending from the wall sockets or struts into pocket voids in the cap that are subsequently grouted.

5. Cap module: Wisconsin should adopt a standard rectangular section pier/pile/abutment cap module. This multi-purpose structural section could be used as a pile bent cap, a pier cap, and the cap on the socketed abutment modules. To allow repetitive use of formwork, the top of the cap module should not include beam/girder stepped bearings. A standard rectangular section should be used with separate precast pedestals, in varying heights, connected to the top as necessary for providing cross slope through beam seats.

6. Cap module connections: The cap modules should be connected to piers or piles through reinforcing bars extending from the pier or pile and into pockets in the cap module that are subsequently grouted.

When long caps are needed, short pieces should be used in lengths that provide for easy shipping and limit the weight to that appropriate for the crane capacity on site. The short caps should be spliced together in place, on the piers or abutments, using a shiplap joint with steel dowels grouted in vertical pockets to act as ties between the pieces or one of the alternate connection methods (welding, or splicing) described here. Eventually Wisconsin should adopt post-tensioning as the preferred technique for joining the short cap segments together.

Summary:

Three types of abutment modules (hollow wall with cap, full height socketed, partial height socketed) have been developed for use in Wisconsin. Design calculation sheets, example designs, and standard type drawings have been provided for the modules. A special design method (in MathCad) is provided for detailing reinforcing around the embedded pile in the socket of the abutments.

A standardized rectangular section pier cap is proposed for use in Wisconsin. Design calculation sheets, example designs, and standard detail drawings have been developed for the cap members. A standard connection between the cap and piers/pile has been suggested. Example designs are provided in the form of MathCad sheets.

Three new types of connections between standardized pier/pile cap member, to allow long caps while controlling the weight of individual pieces, have been proposed. They are in addition to a shiplap connection that has already been used on Bridge No. B-6-157 on State Highway 25 over the Mississippi slough. Design calculations for the connections are provided.

A series of design calculation methods, example designs, design tutorials and standard drawings have been prepared for WisDOT use. Many of the calculation methods are also provided for WisDOT electronically in the form of MathCad computer software. Selected standard drawings are also provided electronically in .dwg format.

The design aides and proposed detail drawings are in the Appendix to this report and the electronic materials are provided separately in a digital CD format.

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9 Appendices

Appendix A: Standard plans for the precast abutments designed in the initial phase,

Appendix B: Standard plans for the precast pile/pier caps designed in the initial phase,

Appendix C: Calculation of abutment design loads,

Appendix D: Calculation of pier cap design loads,

Appendix E: Solid wall abutment design calculations,

Appendix F: Design forces for abutment with piles and separate wall panels,

Appendix G: Cap beam design calculations,

Appendix H: Design calculations for full height socketed wall,

Appendix I: Tutorial for design of wall sockets,

Appendix J: Design calculations for wall sockets,

Appendix K: Abutments with Full and Limited Height Sockets - Drawings

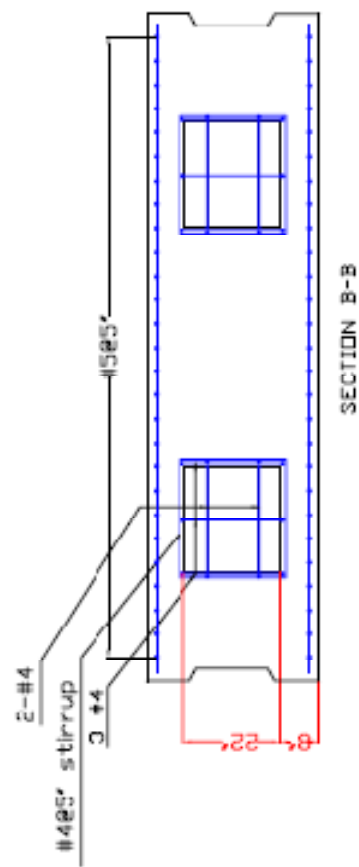
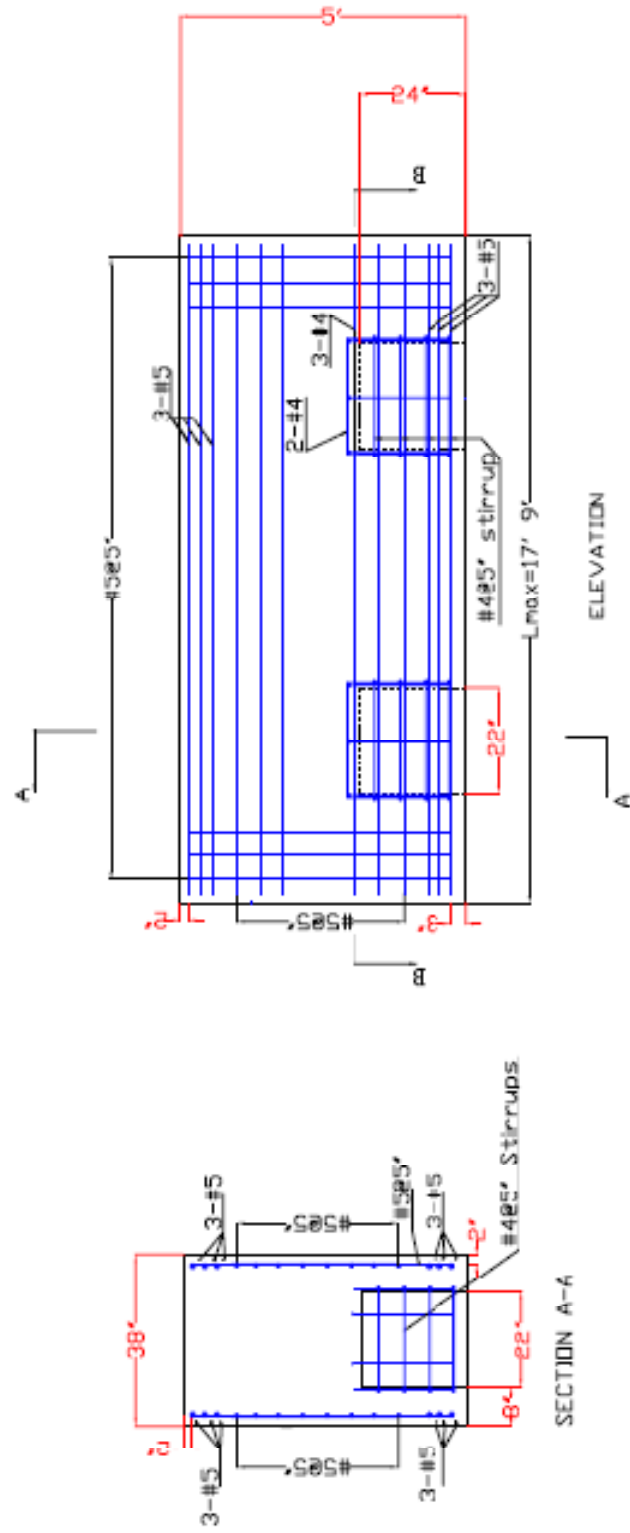
Appendix L: Design calculations for welded cap joint,

Appendix M: Design calculations for spliced cap joint,

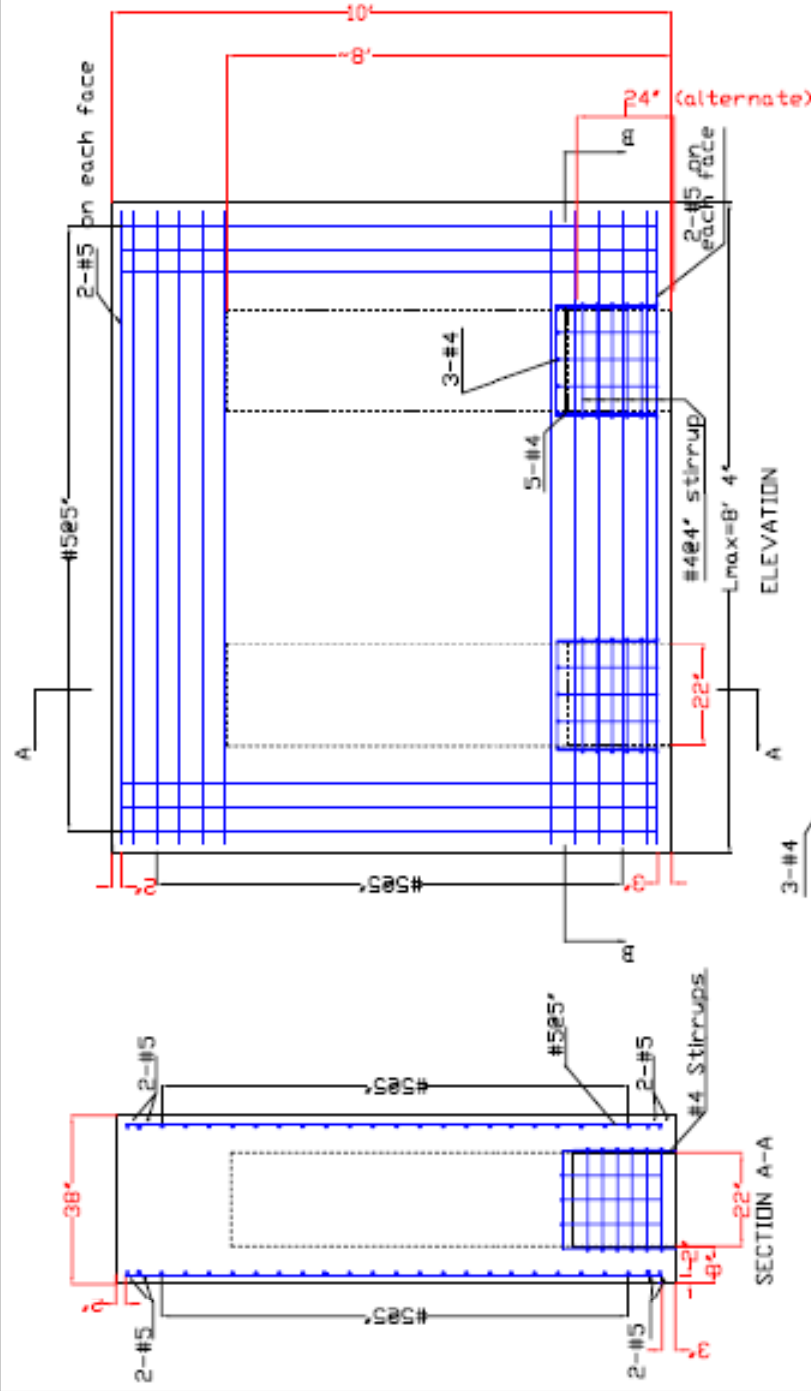
Appendix N: Design calculations for post-tensioned cap joint.

Appendix A: Initial Abutment Design Drawings

Standard plans for the initial precast abutment designs described in Chapter 3.4 of the report follow. Three different height abutments were designed: 3.25ft., 5ft., and 10ft.



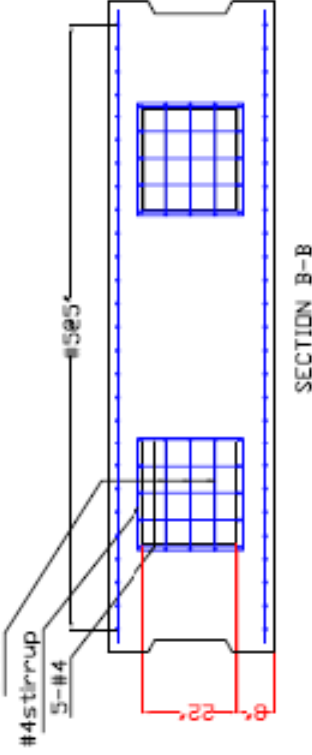
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ABUTMENT: 10' x 3'2" x L

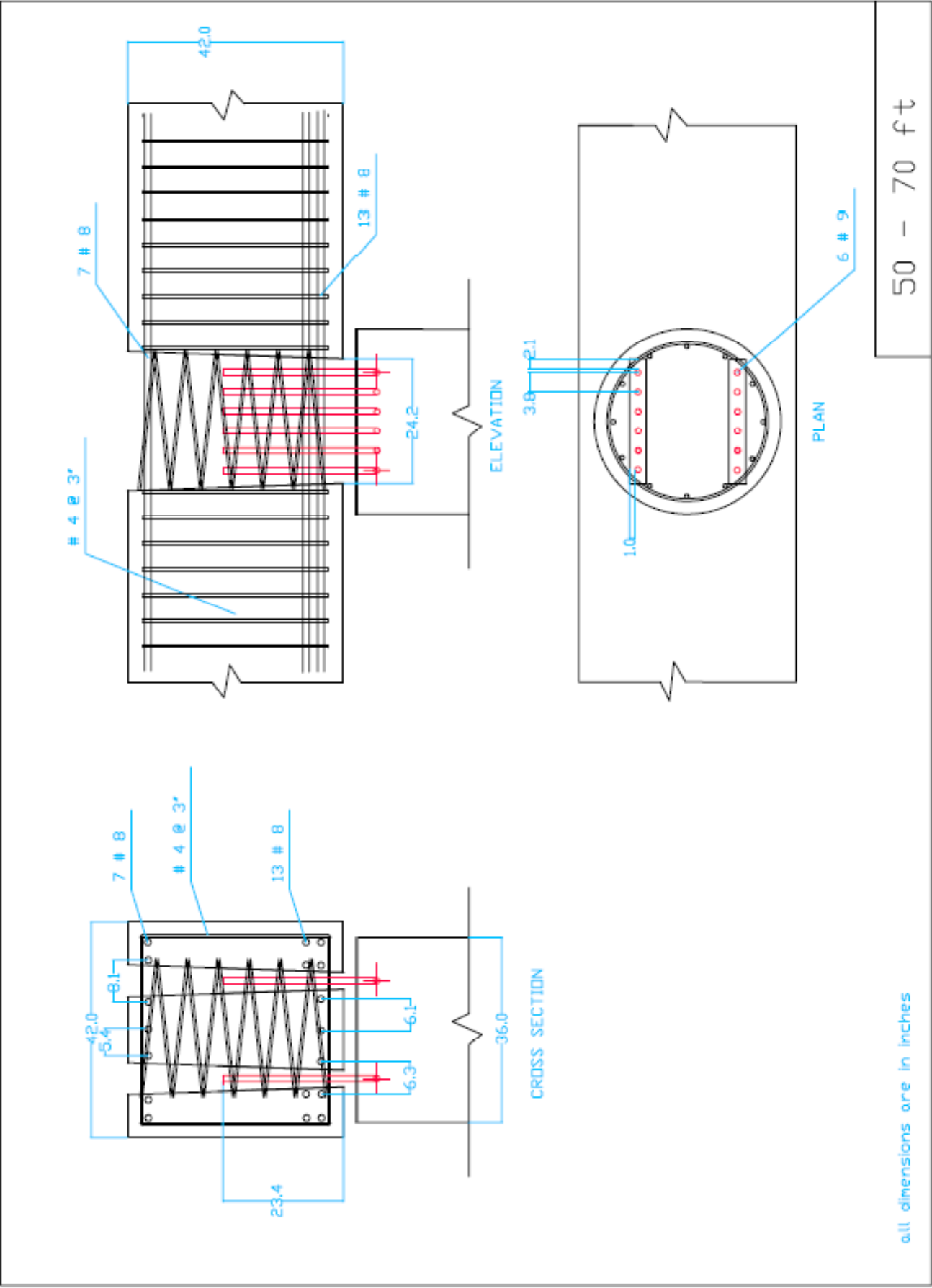
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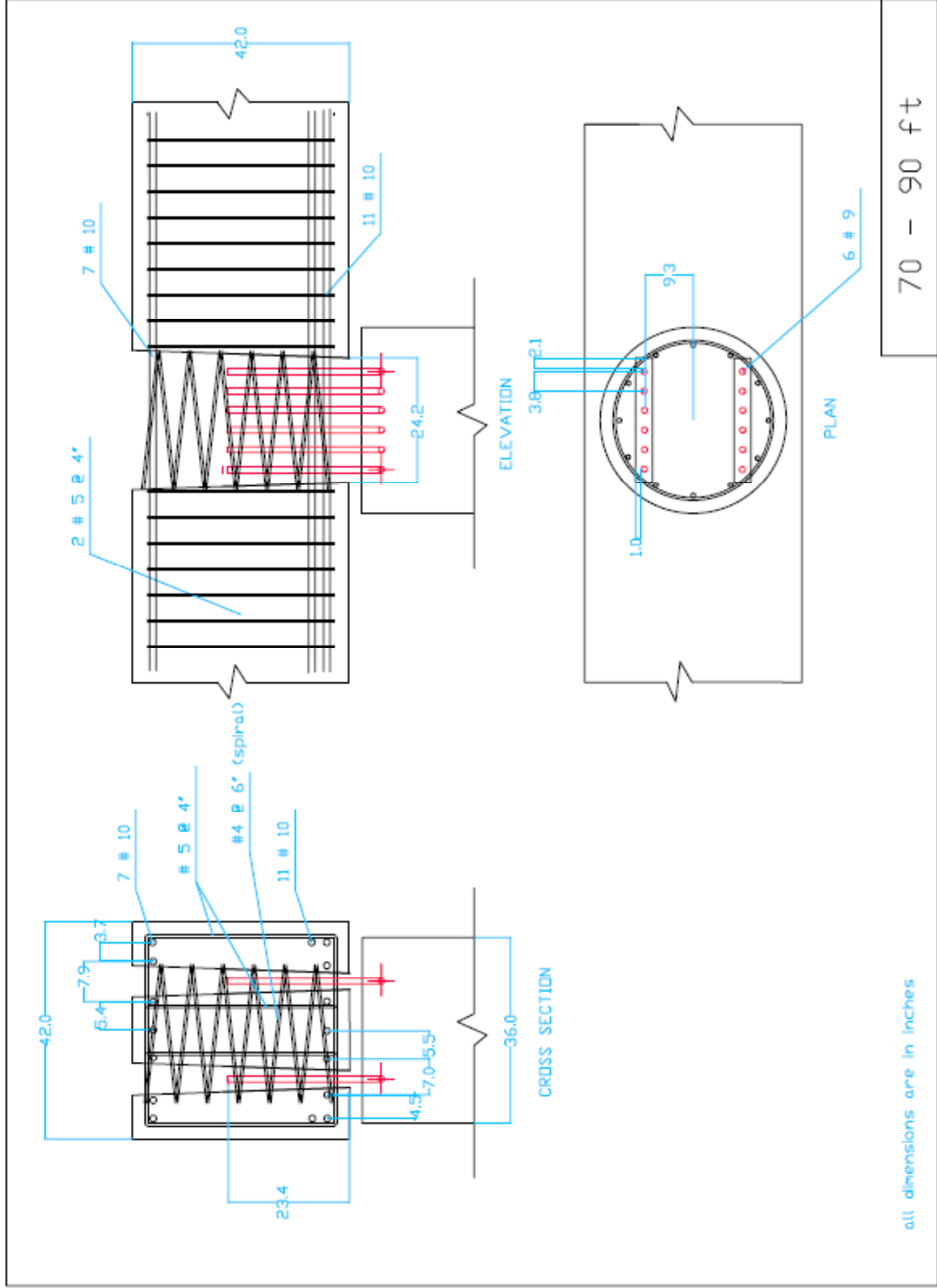
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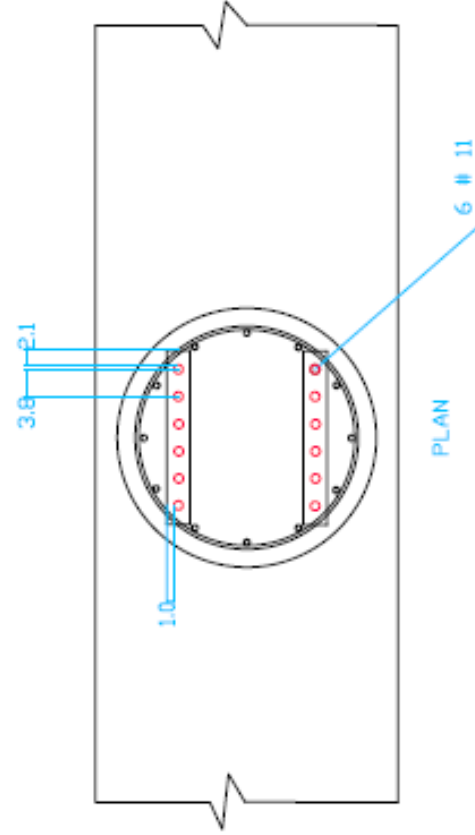
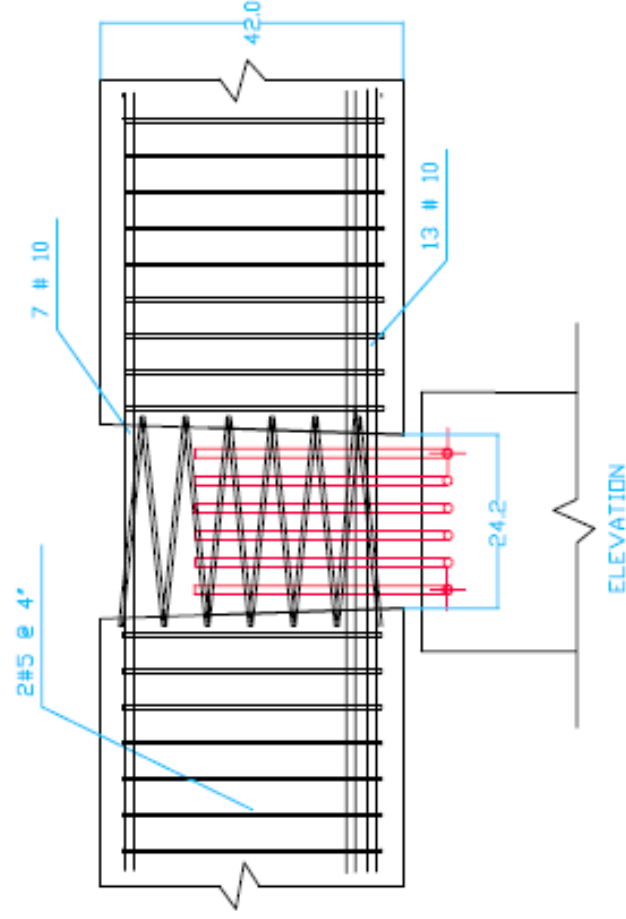
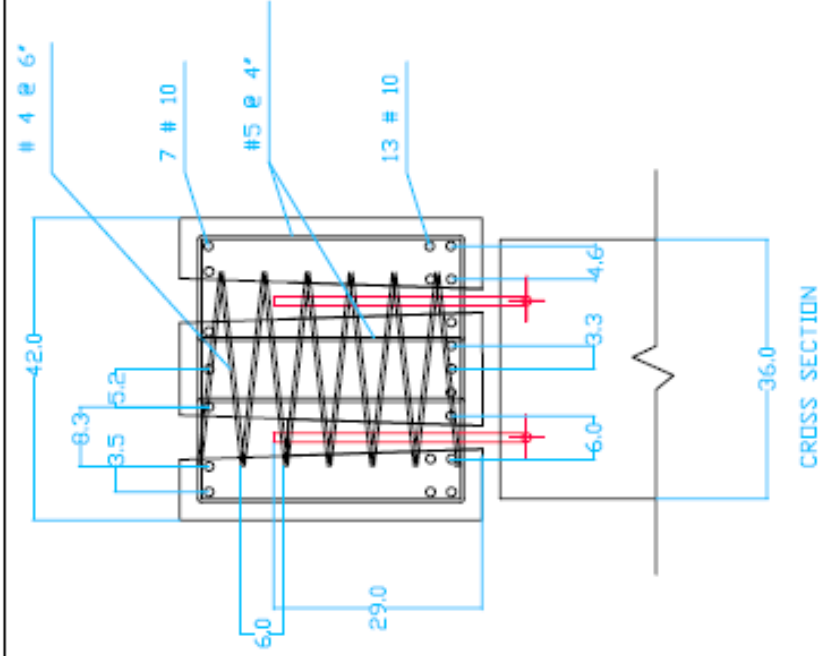


Appendix B: Initial Cap Design Drawings

Standard plans for the initial precast pier/pile cap designs described in Chapter 4 of the report follow. Four different span ranges, controlling the design forces, were considered: 50-70ft., 70-90ft., 90-110ft., and 110-120ft.

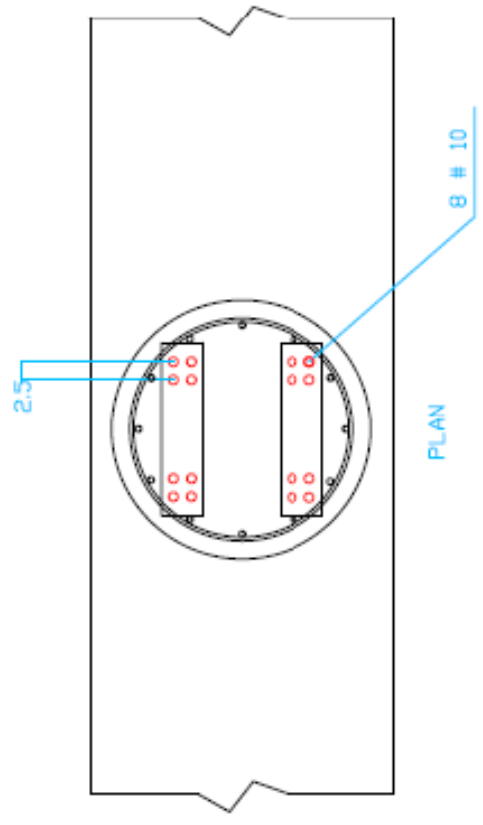
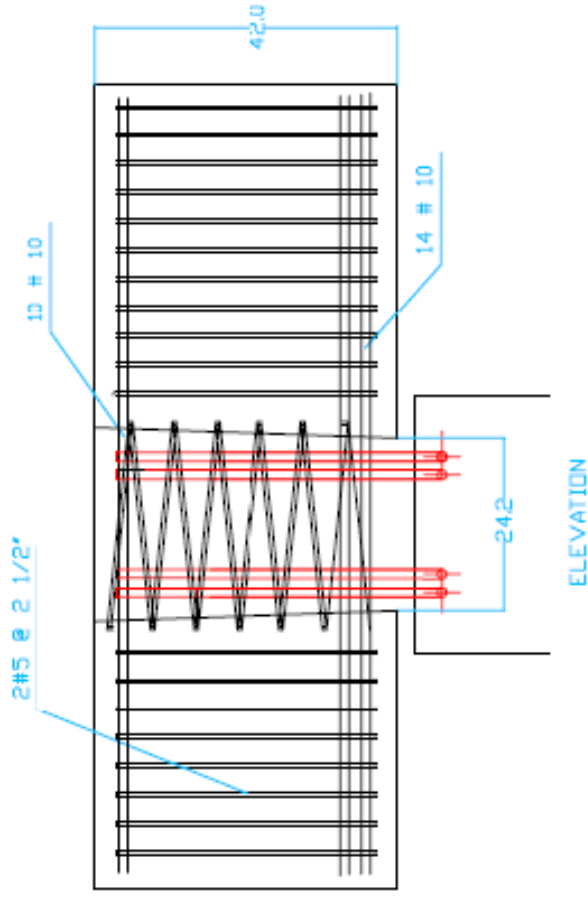
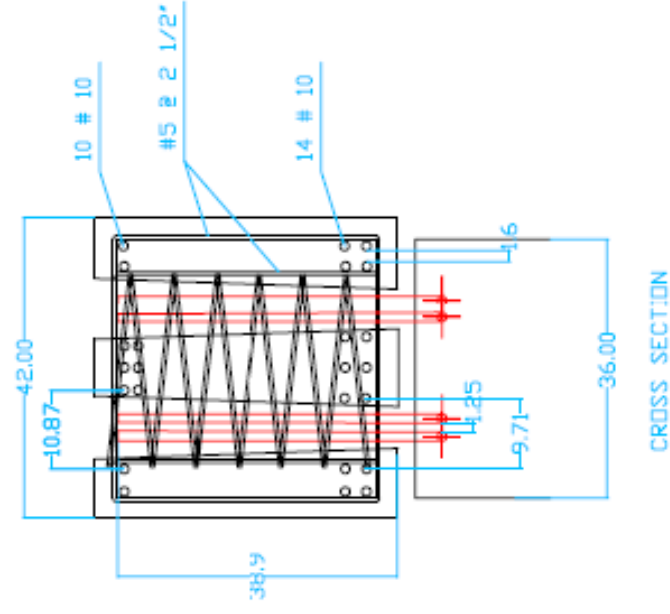






all dimensions are in inches

90 - 110 ft



all dimensions are in inches

110 - 130 ft

Appendix C: Abutment Design Loads

1

Force Calculation Sheet for Abutment Design

Users Guide:

- What does program do?

This design force calculation sheet determines the forces required for the design of a "Limited length socketed wall with bent cap" abutment module. Other wall modules would have a similar approach.

The sheet is designed to calculate forces right above the pile top (see Figure 2), shear force and moment per pile. Vertical forces coming from superstructure is also needed to calculate those forces therefore, user is expected to determine vertical forces per pile formerly and enter them as inputs in this sheet.

- What input is required?

Shape of the abutment module: H : Height of the module
 L : Length of the module
 I_{emb} : Embedment length of the pile, yet socket region

height is fixed as 4ft.

Material properties: f_c : Strength of the concrete
 γ_s : Total unit weight of the soil
 δ : Friction angle between fill and wall
 β : Angle of fill to the horizontal as shown in LRFD

Figure 3.11.5.3-1

shown in LRFD Figure 3.11.5.3-1

θ : Angle of back face of wall to the horizontal as
 Φ_f : Effective angle of integral friction
 wall : Input will be 1 or 0 if active or passive conditions

Load factors: Φ_{DL} : Dead load factor
 Φ_{LL} : Live load factor

Vertical loading: N_{DL} : Vertical dead load on one abutment module
 N_{LL} : Vertical live load on one abutment module
 N_{cap} : Self weight of one cap module
 e : Eccentricity between the centerline of bearing pads

and the centerline of bent cap module as shown on

Figure 3

- What is the output?

Forces due to live load surcharge: p_{LH} : Maximum live load surcharge pressure
 V : Shear force due to LS
 M : Moment due to LS

Forces due to earth pressure: p_{EH} : Maximum earth pressure due to EH
 V : Shear force due to EH
 M : Moment due to EH

Total M & V due to (LS + EH)

Final M & V

Reference Manual:

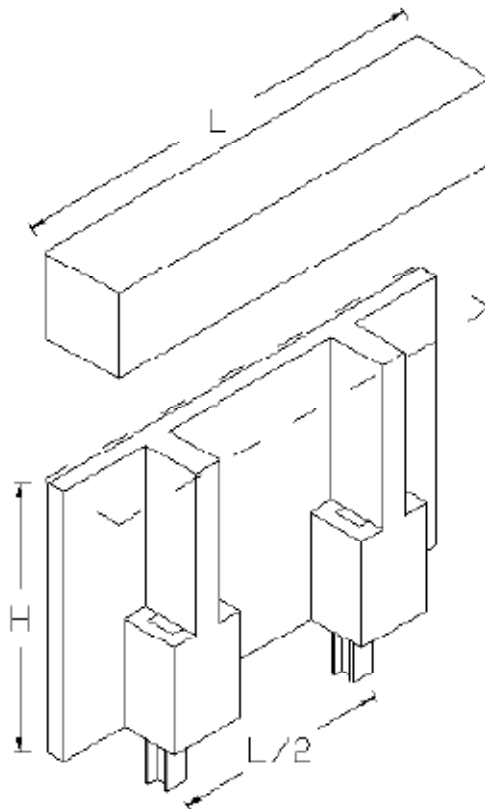
- How calculations are done?
All the calculations are done according to the methods explained in AASHTO LRFD specification. Procedures followed are also described in detail while calculations are done. For example, LRFD 3.11.6.4 is utilized for live load surcharge calculations(LS) and LRFD 3.11.5 is utilized for earth pressure calculations(EH).
- Example design problem is shown below with sample input/output.

3

All inputs are in red

Results are in yellow
colour.

Isometric drawing of the system can be seen below.



Isometric View
Figure 1

Design Constants:

$$H := 10\text{ft}$$

height of the abutment module

$$L := 16\text{ft}$$

width of the abutment module

$$S_{\text{pile}} := \frac{L}{2}$$

pile spacing

$$l_{\text{emb}} := 2\text{ft}$$

embedment length of the pile

Material Properties:Concrete:

$$f_c := 4\text{ksi}$$

ultimate design strength of concrete

$$f_r := 0.37 \sqrt{\frac{f_c}{\text{ksi}}} \text{ ksi} = 0.74 \text{ ksi}$$

modulus of rupture of concrete when it is used for minimum reinforcement calculations, (LRFD 5.4.2.6)

Calculation of Design Loads:

There are two major load types that should be considered in the design of abutment which are:

- 1) Horizontal earth pressure which is caused by fill and live load surcharge from vehicles.
- 2) Vertical load coming from superstructure.

Loads Due to Lateral Earth Pressure:

While designing the abutment under lateral loads, loads acting on half of the abutment is thought to be resisted by one pile region. This is reasonable since each abutment modules is manufactured by two piles symmetrically.

This part of load calculation is divided into two:

- 1) Live load surcharge due to vehicle load
- 2) Load due to lateral earth pressure

1) Live Load Surcharge LS, (LRFD 3.11.6.4):

The procedure used to calculate the wall's horizontal loading due to live load surcharge from vehicular wheel load on the backfill is as follows:

$$\Delta_s = k \cdot \gamma_s \cdot h_{eq}$$

constant horizontal earth pressure due to live load surcharge (ksf)

$$\gamma_s := 0.127 \frac{\text{kip}}{\text{ft}^3}$$

total unit weight of soil (kcf)

k

coefficient of lateral earth pressure

k_o will be used if the wall does not deflect, (LRFD Article 3.11.5.2)

k_a will be used if the wall deflects, (LRFD 3.11.5.3, 3.11.5.6 and 3.11.5.7)

h_{eq}

equivalent height of soil for vehicular load (ft.)

Equivalent height of soil for vehicular loading on abutments perpendicular to traffic is as follows, (LRFD Table 3.11.6.4-1):

$$h_{eq} := \begin{cases} 4\text{ft} & \text{if } H \leq 5\text{ft} \\ 4\text{ft} - \frac{(H - 5\text{ft})}{5} & \text{if } 5\text{ft} < H \leq 10\text{ft} \\ 3\text{ft} - \frac{(H - 10\text{ft})}{10} & \text{if } 10\text{ft} < H < 20\text{ft} \\ 2\text{ft} & \text{if } H \geq 20\text{ft} \end{cases}$$

$$h_{eq} = 3\text{ft}$$

Calculation of k_a according to LRFD 3.11.5.3:

$$\delta := 30\text{deg}$$

friction angle between fill and wall taken as specified in LRFD Table 3.11.5.3-1

$$\beta := 0\text{deg}$$

angle of fill to the horizontal as shown in LRFD Figure 3.11.5.3-1

$$\theta := 90\text{deg}$$

angle of back face of wall to the horizontal as shown in LRFD Figure 3.11.5.3-1

$$\phi_f := 30\text{deg}$$

effective angle of integral friction angle to be determined by laboratory or site investigations

$$\Gamma := \left(1 + \sqrt{\frac{\sin(\phi_f + \delta) \cdot \sin(\phi_f - \beta)}{\sin(\theta - \delta) \cdot \sin(\theta + \beta)}} \right)^2 = 2.914 \quad \text{LRFD equation 3.11.5.3-2}$$

$$k_a := \frac{\sin(\theta + \phi_f)^2}{\Gamma \cdot \sin(\theta)^2 \cdot \sin(\theta - \delta)} = 0.297 \quad \text{LRFD equation 3.11.5.3-1}$$

Calculation of k_o according to LRFD 3.11.5.2:

$$k_o := 1 - \sin(\phi_f) = 0.5 \quad \text{LRFD equation 3.11.5.2-1}$$

Insert the value of wall below as 1 if the wall deflects or 0 if the wall does not deflect:

$$\text{wall} := 1$$

$$k := \begin{cases} k_a & \text{if wall} = 1 \\ k_o & \text{if wall} = 0 \end{cases} \quad k = 0.297$$

First of all, forces just above the pile will be found. Column above pile socket will be designed with those forces. Additionally, applying these forces on the cheeks of socket region, another design will be implemented for socket and pile interaction region.

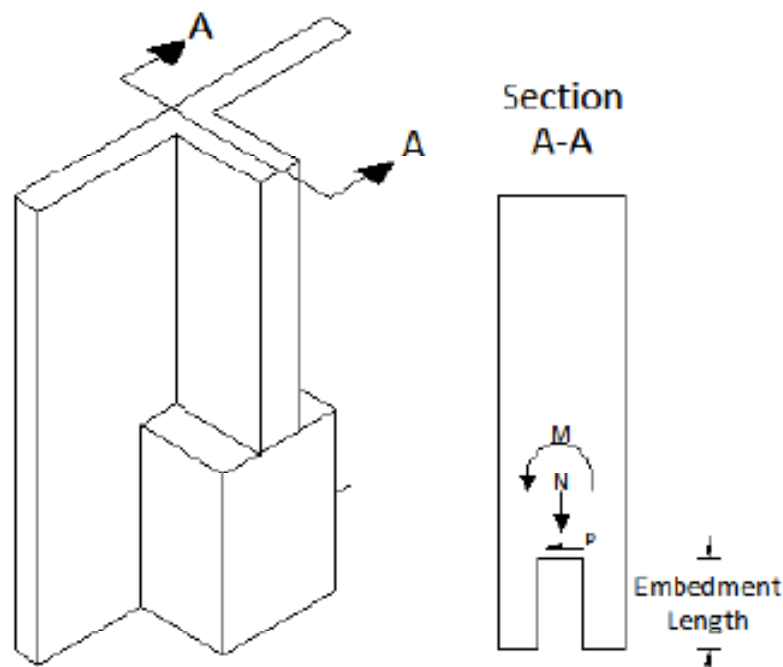


Figure 2

$$\phi_{LL} := 1.75$$

LL load factor, LRFD Table 3.4.1-1

$$p_{LS} := k \cdot h_{eq} \cdot \gamma_s = 0.786 \cdot \text{psi}$$

maximum live load surcharge pressure

$$V_1 := k \cdot h_{eq} \cdot \gamma_s \cdot H \cdot \frac{L}{2} = 9.058 \cdot \text{kip}$$

unfactored shear force per one pile

$$M_1 := V_1 \cdot \left(\frac{H}{2} - l_{emb} \right) = 27.173 \cdot \text{kip} \cdot \text{ft}$$

unfactored moment over pile per pile (since the pressure distribution is constant, moment arm is the half of the pressure distribution height minus embedment length)

$$\phi V_1 := V_1 \cdot \phi_{LL} = 15.851 \cdot \text{kip}$$

factored shear force per pile due to live load surcharge

$$\phi M_1 := M_1 \cdot \phi_{LL} = 47.554 \cdot \text{kip} \cdot \text{ft}$$

factored moment over socket region per pile due to live load surcharge

2) Loads Due to Lateral Earth Pressure E_H , (LRFD 3.11.5):

A second source of load on the wall is earth pressures:

$$\phi_{EH} := \begin{cases} 1.5 & \text{if } k = k_a \\ 1.35 & \text{if } k = k_o \end{cases}$$

load factor is 1.5 if k_a is used, or 1.35 if k_o is used in calculations, LRFD Table 3.4.1-2

$$\phi_{EH} = 1.5$$

While calculating the earth pressure behind the abutment wall, we will use Equivalent-Fluid Method of Estimating Rankine Lateral Earth Pressures method explained in LRFD 3.11.5.5. Please note also that this method can only be applied when we have free draining backfill. Here, we assume that the drainage methods will be applied at the site to get rid of water table behind the wall.

$$p = \gamma_{eq} \cdot z$$

basic earth pressure behind the wall (ksf)

$$\gamma_{eq}$$

equivalent fluid unit weight of soil, not less than 0.03 kcf

$$z$$

depth below surface of soil (ft)

For medium dense sand or gravel, wall height not exceeding 20ft and with a level backfill:

$$\gamma_{eq} := \begin{cases} 0.035 \frac{\text{kip}}{\text{ft}^3} & \text{if } k = k_a \\ 0.05 \frac{\text{kip}}{\text{ft}^3} & \text{if } k = k_o \end{cases}$$

equivalent unit weight of backfill,
LRFD Table 3.11.5.5-1

$$\gamma_{eq} = 0.035 \frac{\text{kip}}{\text{ft}^3}$$

$$p_{EH} := \gamma_{eq} \cdot H = 2.431 \cdot \text{psi}$$

maximum earth pressure at the
bottom of abutment module

$$V_2 := \frac{(\gamma_{eq} \cdot H) \cdot H}{2} \cdot \frac{L}{2} = 14 \cdot \text{kip}$$

unfactored resultant shear per
one pile

$$M_2 := V_2 \cdot \left(\frac{H}{3} - l_{emb} \right) = 18.667 \cdot \text{kip} \cdot \text{ft}$$

unfactored moment at the top of
pile socket region per pile, since
the distribution is triangular,
moment arm is the one third of the
relevant height

$$\phi V_2 := V_2 \cdot \phi_{EH} = 21 \cdot \text{kip}$$

factored resultant shear due to
lateral earth pressure per pile

$$\phi M_2 := M_2 \cdot \phi_{EH} = 28 \cdot \text{kip} \cdot \text{ft}$$

factored moment at the top of
socket region per pile

Total design loads due to earth pressures (LS+EH) per pile:

Unfactored service loads:

Factored loads:

$$V := V_1 + V_2 = 23.058 \cdot \text{kip}$$

$$\phi V := \phi V_1 + \phi V_2 = 36.851 \cdot \text{kip}$$

$$M := M_1 + M_2 = 45.84 \cdot \text{kip} \cdot \text{ft}$$

$$\phi M := \phi M_1 + \phi M_2 = 75.554 \cdot \text{kip} \cdot \text{ft}$$

Vertical Load Coming From Superstructure:

There is an eccentricity of the normal forces coming from superstructure (girder reactions) since the bearing pads of the girders are not placed in the middle of the pilecap (see Figure 3). This eccentricity causes additional moment on the cross section that we are dealing and on the pile also. In this section we will also add those loads to our calculations.

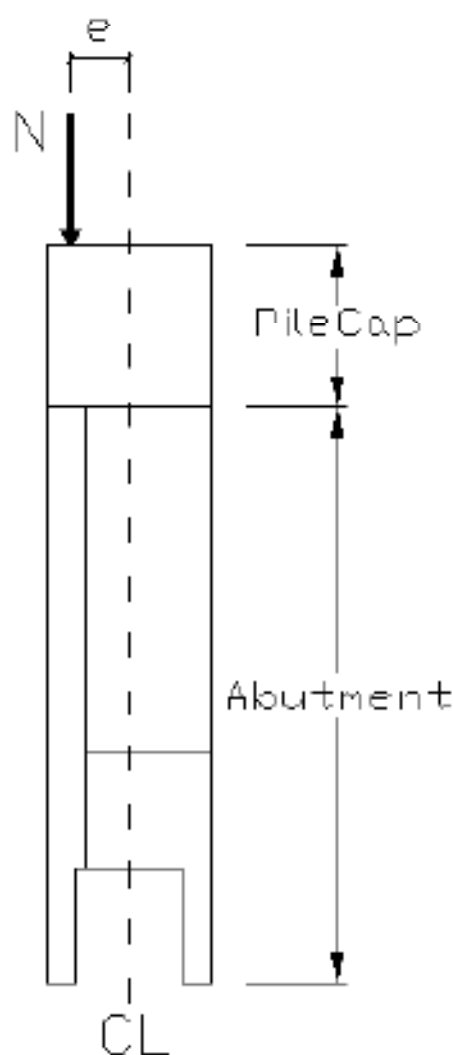


Figure 3

Please insert the sum of normal loads coming from superstructure acting on one abutment module as DL and LL separately.

$N_{DL} := 70\text{kip}$

$N_{LL} := 80\text{kip}$

Please insert the eccentricity between the centerline of bearing pads and the centerline of bent cap module as shown on Figure 3.

$e := 2\text{in}$

Below is the cross section of the abutment part per pile, using those dimensions, we will find the moment of inertia and the section modulus.

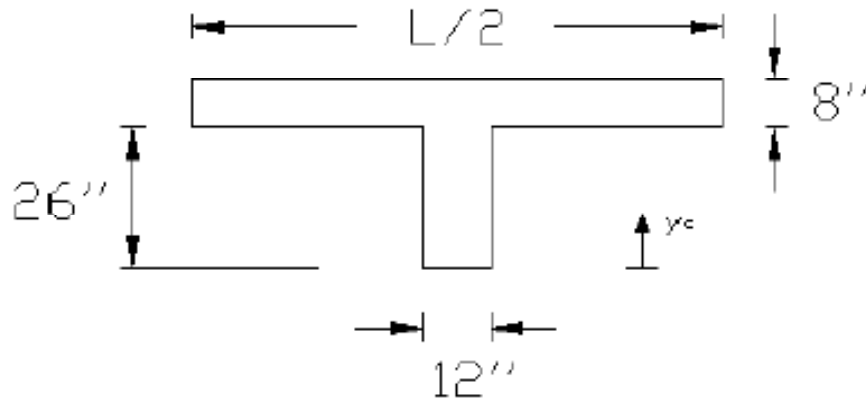


Figure 4

$$y_c := \frac{\frac{L}{2} \cdot 8 \text{ in} \cdot 30 \text{ in} + 12 \text{ in} \cdot 26 \text{ in} \cdot 13 \text{ in}}{\frac{L}{2} \cdot 8 \text{ in} + 12 \text{ in} \cdot 26 \text{ in}} = 25.089 \text{ in}$$

distance between center of gravity of the section and the tension fiber

$$I := \left[\frac{1}{12} \cdot \frac{L}{2} \cdot (8 \text{ in})^3 + \frac{L}{2} \cdot 8 \text{ in} \cdot (30 \text{ in} - y_c)^2 \right] + \left[\frac{1}{12} \cdot 12 \text{ in} \cdot (26 \text{ in})^3 + 12 \text{ in} \cdot 26 \text{ in} \cdot (y_c - 13 \text{ in})^2 \right]$$

$$I = 8.579 \times 10^4 \cdot \text{in}^4$$

moment of inertia of the section per pile

$$\phi_{DL} := 1.25$$

load factor for DL, LRFD Table 3.4.1-2

$$N := \frac{N_{DL} + N_{LL}}{2} = 75 \text{ kip}$$

total unfactored normal load on abutment per pile

$$\phi N := \frac{N_{DL} \cdot \phi_{DL} + N_{LL} \cdot \phi_{LL}}{2}$$

total factored normal load on abutment per pile

$$\phi N = 113.75 \text{ kip}$$

$$M_e := N \cdot e = 12.5 \cdot \text{kip} \cdot \text{ft}$$

unfactored moment due to eccentricity per pile

$$M := M + M_e = 58.34 \cdot \text{kip} \cdot \text{ft}$$

unfactored total moment

$$\phi M_e := \phi N \cdot e = 18.958 \cdot \text{kip} \cdot \text{ft}$$

factored moment due to eccentricity per pile

$$\phi M := \phi M + \phi M_e = 94.512 \cdot \text{kip} \cdot \text{ft}$$

final factored moment

We should also add the self weight of the abutment and bent cap:

$$N_{\text{abutment}} := \frac{(L \cdot 8 \text{ in} + 26 \text{ in} \cdot 12 \text{ in} \cdot 2) \cdot H \cdot 0.15 \frac{\text{kip}}{\text{ft}^3}}{2}$$

$$N_{\text{abutment}} = 11.25 \cdot \text{kip}$$

normal force coming from self weight of the abutment per pile

Please insert the weight of the bent cap:

$$N_{\text{cap}} := 30 \text{ kip}$$

Then, the final normal force and moment are:

$$N := \left(N_{\text{abutment}} + \frac{N_{\text{cap}}}{2} \right) + N = 101.25 \cdot \text{kip}$$

unfactored normal force per pile acting on the center of gravity of the section

$$\phi N := \left(N_{\text{abutment}} + \frac{N_{\text{cap}}}{2} \right) \cdot \phi_{\text{DL}} + \phi N = 146.563 \cdot \text{kip}$$

factored normal force per pile acting on the center of gravity of the section

Further requirements on design moment for flexure, LRFD 5.7.3.3.2:

The design moment can not be lower than the lesser of 1.2 times the cracking moment and 1.33 times the required strength moment (minimum moment usually controls for thick abutments).

$$S_c := \frac{I}{y_c}$$

section modulus of the cross section for the tension fiber per pile

$$M_c := S_c \cdot f_t = 210.869 \cdot \text{kip} \cdot \text{ft}$$

cracking moment of the section per pile

Therefore, our final factored(design) moment is as follows:

$$\phi M := \max(\phi M, \min(1.2 \cdot M_c, 1.33 \cdot \phi M)) = 125.701 \cdot \text{kip} \cdot \text{ft}$$

Total forces over pile socket per pile are as follows:

Unfactored:

$$N = 101.25 \cdot \text{kip}$$

$$V = 23.058 \cdot \text{kip}$$

$$M = 58.34 \cdot \text{kip} \cdot \text{ft}$$

Factored:

$$\phi N = 146.563 \cdot \text{kip}$$

$$\phi V = 36.851 \cdot \text{kip}$$

$$\phi M = 125.701 \cdot \text{kip} \cdot \text{ft}$$

Appendix D: Cap Design Loads

Load Calculation Sheet for Pier Bent Cap

Users Guide:

- What does program do?

This design force calculation sheet determines the unfactored forces required for the design of pier bent cap module for a specific example bridge. Some parts of this example sheet needs manual changes as inputs. Type of forces in example are:

Dead Load
 Wearing Load
 Live Load
 Braking Force
 Thermal Forces
 Wind Load
 On Structures
 On Vehicles
 On Substructures

- What input is required?

Dimensions & Design Parameters:

L	: Bridge span length
W	: Bridge width
Span	: Number of spans
Parapet	: Parapet height
W_{parapet}	: Parapet self weight
H_{girder}	: Girder height
S_g	: Girder spacing
W_{girder}	: Girder self weight
Φ_{pier}	: Diameter of piers
L_{pier}	: Length of piers
N_{pier}	: Number of piers
W_{cap}	: Pile cap width
H_{cap}	: Depth of pile cap
L_{cap}	: Length of pile cap
$R_{1TR}-R_{5Tr}$	: Reaction from axle over pier (controlling case)
μ_{max}	: Maximum coefficient of friction
μ_{min}	: Minimum coefficient of friction
V_B	: Wind base design velocity
P_B	: Wind base pressure

- What is the output?

All forces in the direction of longitudinal and transverse required to design pier cap have been calculated as an output from this sheet.

Reference Manual:

- How calculations are done?
All the calculations are done according to the methods explained in AASHTO LRFD specification. Procedures followed are also described in detail while calculations are done.
- Example design problem is shown below with sample input/output.

All inputs are in red color.

All checks are in yellow color.

Design Constants:

Superstructure:

$$L := 90\text{ft}$$

bridge span length

$$W := 42\text{ft}$$

bridge width

$$\text{Span} := 3$$

number of spans

$$\text{Parapet} := 2\text{ft}$$

parapet height

$$W_{\text{parapet}} := 45 \frac{\text{lbf}}{\text{ft}}$$

parapet self weight

$$N_{\text{lanes}} := \text{floor}\left(\frac{W}{12\text{ft}}\right) = 3$$

number of lanes

Girders:

Using Wisconsin Bridge Manual, we can estimate the beam size and spacing as follows:

From table 19.1a:
For L=90ft

$$H_{\text{girder}} := 45\text{in}$$

girder height

$$S_g := 9\text{ft}$$

girder spacing

$$s := \text{floor}\left(\frac{W}{S_g}\right) = 4$$

number of spacings

$$N_g := s + 1 = 5$$

number of girders

$$W_{\text{girder}} := 583 \frac{\text{lbf}}{\text{ft}}$$

girder self weight

$$t_{\text{deck}} := \frac{S_g + 10\text{ft}}{30} = 7.6\text{in}$$

AASHTO slab thickness,
T2.5.2.6.3-1

$$t_{\text{deck}} := \begin{cases} t_{\text{deck}} & \text{if } t_{\text{deck}} \geq 8\text{in} \\ (8\text{in}) & \text{if } t_{\text{deck}} < 8\text{in} \end{cases}$$

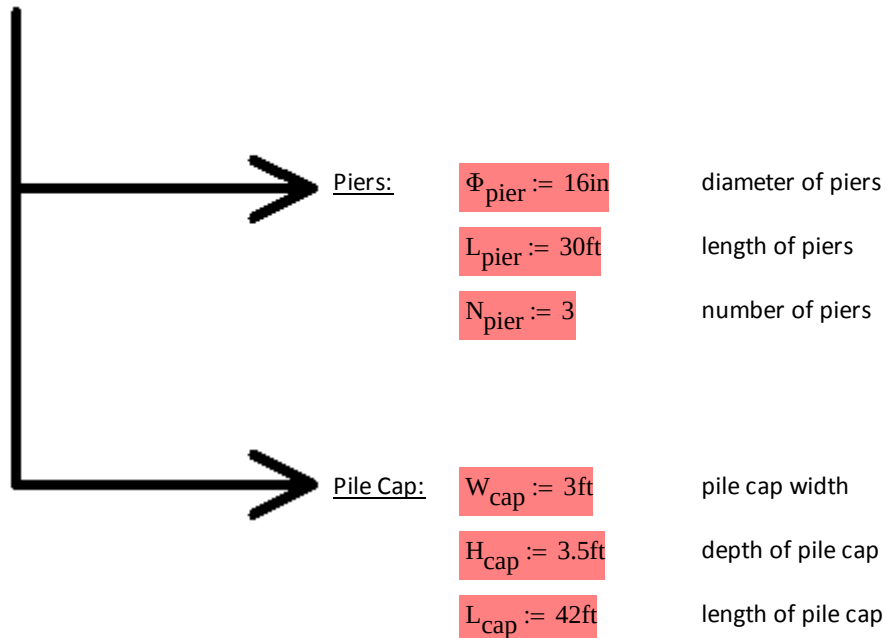
for durability use at least 8 in.

$$t_{\text{deck}} = 8\text{in}$$

$$OH := \frac{[W - S_g \cdot (N_g - 1)]}{2} = 3 \cdot \text{ft}$$

cantilever part of the width of the bridge (overhang)

Substructure:



Dead Load Per Girder Per Linear Foot:

$$DL_{\text{girder}} := W_{\text{girder}} = 583 \cdot \frac{\text{lbf}}{\text{ft}}$$

dead load on girder due to its self weight

$$DL_{\text{deck}} := \left(150 \frac{\text{lbf}}{\text{ft}^3} \right) \cdot t_{\text{deck}} \cdot S_g = 900 \cdot \frac{\text{lbf}}{\text{ft}}$$

dead load on girder due to deck self weight, using the same load for the exterior girders (conservative)

$$DL_{\text{parapet}} := \frac{2 \cdot (W_{\text{parapet}})}{N_g} = 18 \cdot \frac{\text{lbf}}{\text{ft}}$$

dead load on girder due to weight of parapet

$$DL_{\text{per girder}} := DL_{\text{girder}} + DL_{\text{deck}} + DL_{\text{parapet}} = 1.501 \cdot \frac{\text{kip}}{\text{ft}}$$

total dead load plf on girder

Dead Load Reactions On Pier Caps Due To Girders:

Bridge spans will be simply supported beams until deck is poured.

$$P_{\text{intDL}} := 2 \left(\frac{DL_{\text{per_girder}} \cdot L}{2} \right) = 135.09 \cdot \text{kip}$$

point dead load under one
girder support on pier
(interior)

$$P_{\text{extDL}} := \frac{DL_{\text{per_girder}} \cdot L}{2} = 67.545 \cdot \text{kip}$$

point dead load under one
girder support on abutment
(exterior)

Wearing Load:

$$SW_{\text{wearing}} := 20 \text{psf}$$

self weight of wearing surface
per square foot

$$DL_{\text{wearing}} := SW_{\text{wearing}} \cdot S_g = 180 \cdot \text{plf}$$

assuming equal distribution to
the beams

Dead Load Reactions On Pier Caps Due To Weight of Wearing:
Again, spans are simply supported.

$$P_{\text{intWL}} := 2 \cdot \left(\frac{DL_{\text{wearing}} \cdot L}{2} \right) = 16.2 \cdot \text{kip}$$

point wearing load under one
girder support on pier
(interior)

$$P_{\text{extWL}} := \frac{DL_{\text{wearing}} \cdot L}{2} = 8.1 \cdot \text{kip}$$

point wearing load under one
girder support on abutment
(exterior)

Live Load:

Loadings for the pier cap coming from live load will be determined in such a way that it is causing the largest moment and shear in one side of the cap.

We want the loads that exist simultaneously on the structure, for that reason, we will use AASHTO C4.6.2.2d (stiff diaphragms or cross frames). This method would only be applied to axles not over the pier.

And, we will do SAP analysis to calculate the forces coming from axles over the pier.

Axles on span:

The distribution of loads to the beams is calculated as follows:

$$R_j := \frac{N_L}{N_B} + \left(x_j \cdot \frac{\sum e_j}{\sum x_j^2} \right)$$

LRFD C.4.6.2.2d-1

where:

N_L = number of lanes loaded

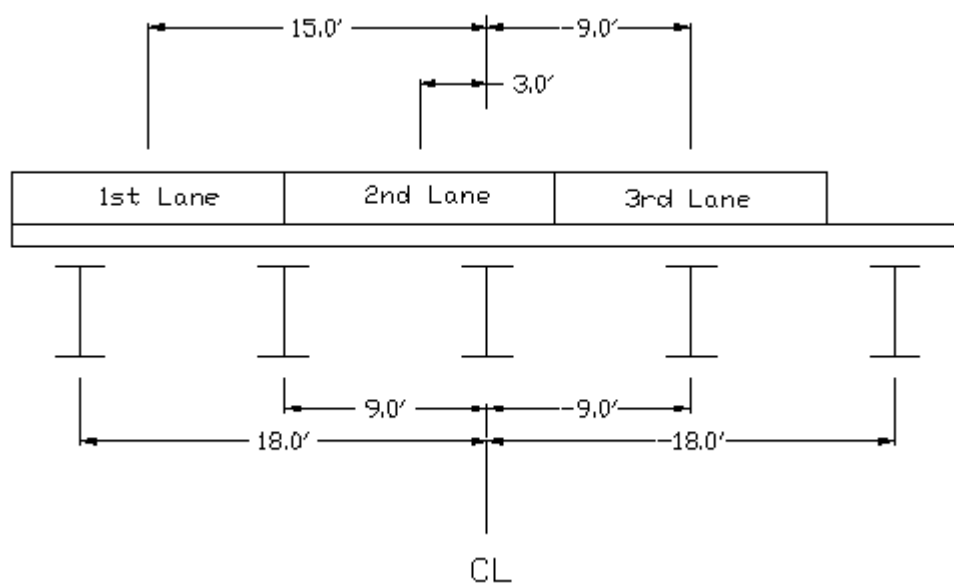
N_B = number of beams

x_j = distance from CL of cap to j'th beam

e_j = eccentricity of lane j from CL of cap

$$N_B := N_g = 5$$

Since lanes can be moved over the span of the cap, we move all lanes adjacent to each other to the left of the cap to create the maximum moment on cap.



Eccentricities:

$$e_1 := 15\text{ft}$$

first lane's eccentricity

$$e_2 := 3\text{ft}$$

second lane's eccentricity

$$e_3 := -9\text{ft}$$

third lane's eccentricity

$$x_1 := \frac{W}{2} - OH = 18\cdot\text{ft}$$

leftmost girder's eccentricity

$$x_2 := x_1 - S_g = 9\cdot\text{ft}$$

$$x_3 := x_2 - S_g = 0\cdot\text{ft}$$

middle girder's eccentricities

$$x_4 := x_3 - S_g = -9\cdot\text{ft}$$

$$x_5 := x_4 - S_g = -18\cdot\text{ft}$$

rightmost girder's eccentricity

$$a := x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

Sum of squares of the girder eccentricities

One lane loaded:

$$R_{11} := \frac{1}{N_B} + \left(x_1 \cdot \frac{e_1}{a} \right) = 0.533$$

leftmost girder's distribution factor

$$R_{12} := \frac{1}{N_B} + \left(x_2 \cdot \frac{e_1}{a} \right) = 0.367$$

$$R_{13} := \frac{1}{N_B} + \left(x_3 \cdot \frac{e_1}{a} \right) = 0.2$$

middle girder's distribution factor

$$R_{14} := \frac{1}{N_B} + \left(x_4 \cdot \frac{e_1}{a} \right) = 0.033$$

$$R_{15} := \frac{1}{N_B} + \left(x_5 \cdot \frac{e_1}{a} \right) = -0.133$$

rightmost girder's distribution factor

$$R_{11} + R_{12} + R_{13} + R_{14} + R_{15} = 1$$

check to see all DF's sum up to 1

Moment created by the girder reactions with respect to the CL of the cap including multi presence factor is:

$$M_1 := 1.2 \left[(R_{11} - R_{15}) \cdot 2S_g + (R_{12} - R_{14})S_g \right] \cdot 32 \text{kip} = 576 \cdot \text{kip} \cdot \text{ft}$$

Two lanes loaded:

In this case, multi presence factor is 1.

$$R_{21} := \frac{2}{N_B} + \left(x_1 \cdot \frac{e_1 + e_2}{a} \right) = 0.8$$

$$R_{22} := \frac{2}{N_B} + \left(x_2 \cdot \frac{e_1 + e_2}{a} \right) = 0.6$$

$$R_{23} := \frac{2}{N_B} + \left(x_3 \cdot \frac{e_1 + e_2}{a} \right) = 0.4$$

$$R_{24} := \frac{2}{N_B} + \left(x_4 \cdot \frac{e_1 + e_2}{a} \right) = 0.2$$

$$R_{25} := \frac{2}{N_B} + \left(x_5 \cdot \frac{e_1 + e_2}{a} \right) = 0$$

$$R_{21} + R_{22} + R_{23} + R_{24} + R_{25} = 2$$

check to see all DF's sum up to 2 since we have loaded two lanes

Moment created by the girder reactions with respect to the CL of the cap including multi presence factor is:

$$M_2 := \left[(R_{21} - R_{25}) \cdot 2S_g + (R_{22} - R_{24})S_g \right] \cdot 64 \text{kip} = 1.152 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Three lanes loaded:

Multi presence factor is 0.85.

$$R_{31} := \frac{3}{N_B} + \left(x_1 \cdot \frac{e_1 + e_2 + e_3}{a} \right)$$

$$R_{32} := \frac{3}{N_B} + \left(x_2 \cdot \frac{e_1 + e_2 + e_3}{a} \right)$$

$$R_{33} := \frac{3}{N_B} + \left(x_3 \cdot \frac{e_1 + e_2 + e_3}{a} \right)$$

$$R_{34} := \frac{3}{N_B} + \left(x_4 \cdot \frac{e_1 + e_2 + e_3}{a} \right)$$

$$R_{35} := \frac{3}{N_B} + \left(x_5 \cdot \frac{e_1 + e_2 + e_3}{a} \right)$$

$$R_{31} + R_{32} + R_{33} + R_{34} + R_{35} = 3$$

check to see all DF's sum up to 3 since we have loaded three lanes

Moment created by the girder reactions with respect to the CL of the cap including multi presence factor is:

$$M_3 := 0.85 \left[(R_{31} - R_{35}) \cdot 2S_g + (R_{32} - R_{34}) S_g \right] \cdot 96 \text{ kip} = 734.4 \cdot \text{kip} \cdot \text{ft}$$

The governing loading case is the one that has the biggest moment in the pier cap, therefore:

$$\max(M_1, M_2, M_3) = 1.152 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Two lanes loaded case governs!

We should also find how much percent of the axle load located on span will go to pier. To do that, we can use SAP and proportion the applied load and the result as follows:

Thinking of the second support from the left:

Case 1:

If we put unit load 14 ft to the left and another unit load 4 ft to the right of the support, we get distribution factors for them respectively:

$$DF_1 := 0.99 \quad \text{for the load 14 ft to the left}$$

$$DF_2 := 0.99 \quad \text{for the load 4 ft to the right}$$

Case 2:

If we put unit load 4 ft to the left and another unit load 14 ft to the right of the support, we get distribution factors for them respectively:

$$D_3 := 1.01 \quad \text{for the load 4 ft to the left}$$

$$D_4 := 0.93 \quad \text{for the load 14 ft to the right}$$

Moreover, 14 ft away load is 32 kip and 4 ft away load is 8 kip, therefore, first case is more critical. Putting 32 kip wheel load 14 ft to the left and 8 kip load 4 ft to the right yields more critical result.

For tandem, we can use the highest factor which is for 4 ft away load:

$$DF_{\text{tandem}} := 1.01$$

Axle over pier:

Now, we should find the case of axle over pier using

SAP:

Case 1: Left wheel of the one of the truck is on the leftmost girder

Distances of the wheels from the left side of the deck are:

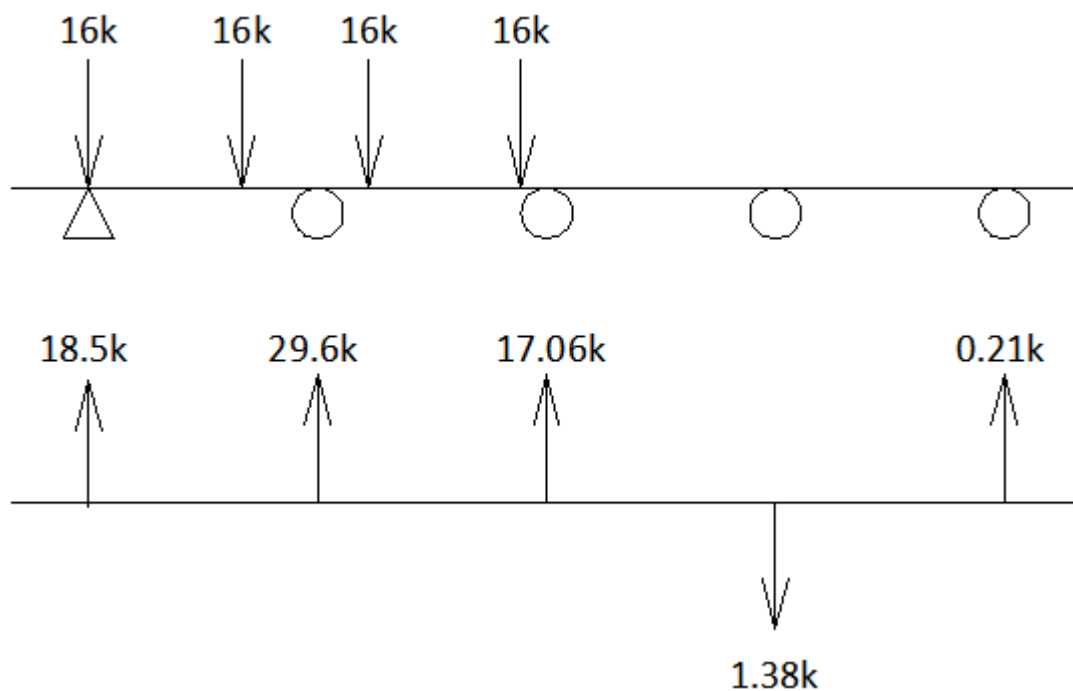
First wheel 3ft

Second wheel 9ft

Third wheel 14ft

Fourth wheel 20ft

Arrangement of loading and results are:



Moment caused by Case-1 is:

$$(18.5\text{kip} - 0.21\text{kip}) \cdot 18\text{ft} + (29.6\text{kip} + 1.38\text{kip}) \cdot 9\text{ft} = 608.04 \cdot \text{kip} \cdot \text{ft}$$

Case 2: Left wheel of the one of the trucks is on 2 ft right to the edge of the slab which is the closest point to the left permitted by AASHTO 3.6.1.3.

Distances of the wheels from the left side of the deck are:

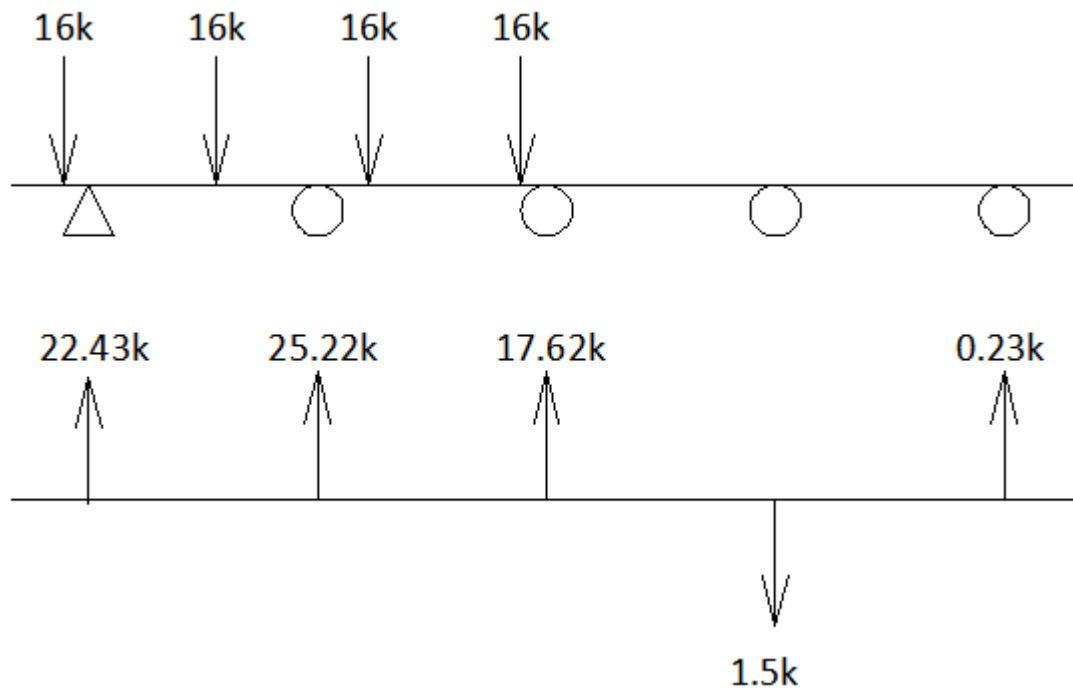
First wheel 2ft

Second wheel 8ft

Third wheel 14ft

Fourth wheel 20ft

Arrangement of loading and results are:



Moment caused by Case-2 is:

$$(22.43\text{kip} - 0.23\text{kip}) \cdot 18\text{ft} + (25.22\text{kip} + 1.5\text{kip}) \cdot 9\text{ft} = 640.08 \cdot \text{kip} \cdot \text{ft}$$

Therefore, Case-2 controls the design!

Total reactions on supports:

Reactions from axle over pier:

Truck:

$$R_{1\text{TR}} := 22.43\text{kip}$$

$$R_{4\text{TR}} := -1.5\text{kip}$$

$$R_{2\text{TR}} := 25.22\text{kip}$$

$$R_{5\text{TR}} := 0.23\text{kip}$$

$$R_{3\text{TR}} := 17.62\text{kip}$$

Tandem:

$$R_{1\text{TA}} := \frac{R_{1\text{TR}}}{64\text{kip}} \cdot 50\text{kip} = 17.523 \cdot \text{kip}$$

$$R_{4\text{TA}} := \frac{R_{4\text{TR}}}{64\text{kip}} \cdot 50\text{kip} = -1.172 \cdot \text{kip}$$

$$R_{2\text{TA}} := \frac{R_{2\text{TR}}}{64\text{kip}} \cdot 50\text{kip} = 19.703 \cdot \text{kip}$$

$$R_{5\text{TA}} := \frac{R_{5\text{TR}}}{64\text{kip}} \cdot 50\text{kip} = 0.18 \cdot \text{kip}$$

$$R_{3\text{TA}} := \frac{R_{3\text{TR}}}{64\text{kip}} \cdot 50\text{kip} = 13.766 \cdot \text{kip}$$

Reactions from axle on span:

Truck:

$$R_{1TRspan} := (32\text{kip} + 8\text{kip})DF_1 \cdot R_{21} = 31.68 \cdot \text{kip}$$

$$R_{2TRspan} := (32\text{kip} + 8\text{kip})DF_1 \cdot R_{22} = 23.76 \cdot \text{kip}$$

$$R_{3TRspan} := (32\text{kip} + 8\text{kip})DF_1 \cdot R_{23} = 15.84 \cdot \text{kip}$$

$$R_{4TRspan} := (32\text{kip} + 8\text{kip})DF_1 \cdot R_{24} = 7.92 \cdot \text{kip}$$

$$R_{5TRspan} := (32\text{kip} + 8\text{kip})DF_1 \cdot R_{25} = 2.198 \times 10^{-15} \cdot \text{kip}$$

Tandem:

$$R_{1TAspan} := (25\text{kip}) \cdot DF_{\text{tandem}} \cdot R_{21} = 20.2 \cdot \text{kip}$$

$$R_{2TAspan} := (25\text{kip}) \cdot DF_{\text{tandem}} \cdot R_{22} = 15.15 \cdot \text{kip}$$

$$R_{3TAspan} := (25\text{kip}) \cdot DF_{\text{tandem}} \cdot R_{23} = 10.1 \cdot \text{kip}$$

$$R_{4TAspan} := (25\text{kip}) \cdot DF_{\text{tandem}} \cdot R_{24} = 5.05 \cdot \text{kip}$$

$$R_{5TAspan} := (25\text{kip}) \cdot DF_{\text{tandem}} \cdot R_{25} = 1.402 \times 10^{-15} \cdot \text{kip}$$

Distribution of lane loading to the girders:

$$R_{1lane} := \left(0.64 \frac{\text{kip}}{\text{ft}} \cdot L \right) \cdot R_{21} = 46.08 \cdot \text{kip}$$

$$R_{2lane} := \left(0.64 \frac{\text{kip}}{\text{ft}} \cdot L \right) \cdot R_{22} = 34.56 \cdot \text{kip}$$

$$R_{3lane} := \left(0.64 \frac{\text{kip}}{\text{ft}} \cdot L \right) \cdot R_{23} = 23.04 \cdot \text{kip}$$

$$R_{4lane} := \left(0.64 \frac{\text{kip}}{\text{ft}} \cdot L \right) \cdot R_{24} = 11.52 \cdot \text{kip}$$

$$R_{5lane} := \left(0.64 \frac{\text{kip}}{\text{ft}} \cdot L \right) \cdot R_{25} = 3.197 \times 10^{-15} \cdot \text{kip}$$

Total loads:

Because of Truck:

$$R_{1TR} := R_{1TR} + R_{1TRspan} + R_{1lane} = 100.19 \cdot \text{kip}$$

$$R_{2TR} := R_{2TR} + R_{2TRspan} + R_{2lane} = 83.54 \cdot \text{kip}$$

$$R_{3TR} := R_{3TR} + R_{3TRspan} + R_{3lane} = 56.5 \cdot \text{kip}$$

$$R_{4TR} := R_{4TR} + R_{4TRspan} + R_{4lane} = 17.94 \cdot \text{kip}$$

$$R_{5TR} := R_{5TR} + R_{5TRspan} + R_{5lane} = 0.23 \cdot \text{kip}$$

Because of Tandem:

$$R_{1TA} := R_{1TA} + R_{1TAspan} + R_{1lane} = 83.803 \cdot \text{kip}$$

$$R_{2TA} := R_{2TA} + R_{2TAspan} + R_{2lane} = 69.413 \cdot \text{kip}$$

$$R_{3TA} := R_{3TA} + R_{3TAspan} + R_{3lane} = 46.906 \cdot \text{kip}$$

$$R_{4TA} := R_{4TA} + R_{4TAspan} + R_{4lane} = 15.398 \cdot \text{kip}$$

$$R_{5TA} := R_{5TA} + R_{5TAspan} + R_{5lane} = 0.18 \cdot \text{kip}$$

As can be seen easily, all loads caused by truck is higher than tandem, therefore our design forces are truck forces!

Braking Force (LRFD 3.6.4):

Braking force is applied in the longitudinal direction of the bridge.
Maximum of the following forces is taken into account and applied to the bridge.

25% of (Truck/Tandem) or 5% of (Truck/Tandem + Lane)

$$F_{0.25xtruck} := 0.25 \cdot (32\text{kip} + 32\text{kip} + 8\text{kip}) = 18 \cdot \text{kip}$$

$$F_{0.25xtandem} := 0.25 \cdot (25\text{kip} + 25\text{kip}) = 12.5 \cdot \text{kip}$$

$$F_{\max 1} := \max(F_{0.25xtruck}, F_{0.25xtandem}) = 18 \cdot \text{kip}$$

$$F_{0.05x(truck+lane)} := 0.05 \cdot \left[(32\text{kip} + 32\text{kip} + 8\text{kip}) + 0.64 \frac{\text{kip}}{\text{ft}} \cdot L \right] = 6.48 \cdot \text{kip}$$

$$F_{0.05x(tandem+lane)} := 0.05 \cdot \left[(25\text{kip} + 25\text{kip}) + 0.64 \frac{\text{kip}}{\text{ft}} \cdot L \right] = 5.38 \cdot \text{kip}$$

$$F_{\max 2} := \max(F_{0.05x(truck+lane)}, F_{0.05x(tandem+lane)}) = 6.48 \cdot \text{kip}$$

$$F_{\max_per_lane} := \max(F_{\max 1}, F_{\max 2}) = 18 \cdot \text{kip}$$

Using multiple presence factor for lanes, we can obtain total braking force required to be applied to the bridge:

MPF = 1.2 for one lane loaded

MPF = 1 for two lanes loaded

MPF = 0.85 for three

MPF = 0.65 for four and higher lanes loaded

The reason why all lanes are loaded is the probability of the bridge to be a one direction bridge in the future.

$$F_{\max_for_bridge} := \max(1.2 \cdot F_{\max_per_lane}, 2 \cdot 1 \cdot F_{\max_per_lane}, 3 \cdot 0.85 \cdot F_{\max_per_lane})$$

$$F_{\max_for_bridge} = 45.9 \cdot \text{kip}$$

$$F_{\max_per_girder} := \frac{F_{\max_for_bridge}}{N_g}$$

$$F_{\max_per_girder} = 9.18 \cdot \text{kip}$$

braking force per girder

Force due to braking force should be applied 6 ft over roadway (LRFD 3.6.4).

$$M_{\text{braking_per_girder}} := F_{\text{max_per_girder}} \cdot \left(6\text{ft} + t_{\text{deck}} + H_{\text{girder}} + \frac{H_{\text{cap}}}{2} \right)$$

$$M_{\text{braking_per_girder}} = 111.69 \cdot \text{kip} \cdot \text{ft}$$

$F_{\text{max_per_girder}}$ and $M_{\text{braking_per_girder}}$ should be applied to pier frame model at each girder location at mid height of the pier cap in the longitudinal direction.

Thermal Forces (WisDOT Bridge Manual 13.4):

Commonly, bridges which are not in earthquake regions have only one fixed pier, other piers and abutments are constructed with expansion joints. While modeling expansion piers, we assume that longitudinal force acting on those piers are dead load times coefficient of friction. Here in this example, we will assume that one pier is fixed and others have expansion joints. We also want to find the biggest force on this fixed pier. Therefore, using longitudinal force equilibrium, we will determine the largest temperature force on fixed pier. The thermal force on a single fixed pier in a bridge is the resultant of the unbalanced forces acting on the substructure units. Maximum friction coefficients are assumed for expansion bearings on one side of the bridge and minimum coefficients on the other side to produce the greatest unbalanced force on the fixed pier.

$$\mu_{\text{max}} := 0.1$$

maximum coefficient of friction

$$\mu_{\text{min}} := 0.06$$

minimum coefficient of friction

$$\Sigma DL_{\text{ext}} := (P_{\text{extDL}} + P_{\text{extWL}}) = 75.645 \cdot \text{kip}$$

dead load on abutment.

$$\Sigma DL_{\text{int}} := (P_{\text{intDL}} + P_{\text{intWL}}) = 151.29 \cdot \text{kip}$$

dead load on interior pier

$$P_{\text{temp}} := \Sigma DL_{\text{ext}} (\mu_{\text{max}} - \mu_{\text{min}}) + \Sigma DL_{\text{int}} \cdot \mu_{\text{max}} = 18.155 \cdot \text{kip}$$

to get the worst force on fixed pier

$$P_{\text{girdertemp}} := \frac{P_{\text{temp}}}{N_g} = 3.631 \cdot \text{kip}$$

thermal force on fixed pier due to one girder.

Wind Load (LRFD 3.8):

In AASHTO LRFD, design velocity of the bridge should be modified with equation LRFD 3.8.1.1-1 accordingly if the bridge height is more than 30 ft. In this example, we will assume our bridge is lower than or equal to 30 ft.

Therefore;

$$V_B := 100\text{mph}$$

for bridges or parts of bridges which are lower than 30 ft.

$$V_{DZ} := V_B$$

design velocity of bridge is equal to base design velocity for our case.

Wind Pressure On Structures, WS (LRFD 3.8.1.2):

$$L_{tributary} := L$$

tributary length of the bridge for one pier cap and set of columns attached to it (ft)

$$P_B := 0.05 \text{ksf}$$

base pressure LRFD Table 3.8.1.2.1-1 for windward direction.

$$P_D := P_B \cdot \left(\frac{V_{DZ}}{V_B} \right)^2 = 0.05 \cdot \frac{\text{kip}}{\text{ft}^2}$$

LRFD Equation 3.8.1.2.1-1 to calculate design wind pressure (ksf)

$$\text{Depth}_{\text{superstructure}} := \text{Parapet} + H_{\text{girder}} + t_{\text{deck}} = 6.417 \cdot \text{ft}$$

depth of superstructure that will be exposed to wind pressure.

$$F_{\text{transwind}} := P_D \cdot \text{Depth}_{\text{superstructure}} \cdot L_{\text{tributary}}$$

wind force on superstructure for the indicated tributary area.

$$F_{\text{transwind}} = 28.875 \cdot \text{kip}$$

$$F_{\text{transwind}} := \begin{cases} F_{\text{transwind}} & \text{if } \frac{F_{\text{transwind}}}{L} \geq 0.3 \text{klf} \\ (0.3 \text{klf} \cdot L) & \text{if } \frac{F_{\text{transwind}}}{L} < 0.3 \text{klf} \end{cases}$$

check for the minimum load per lineal foot that should be applied on bridge superstructure (LRFD 3.8.1.2.1 General)

$$F_{\text{transwind}} = 28.875 \cdot \text{kip}$$

$$F_{\text{pergirder}} := \frac{F_{\text{transwind}}}{N_g} = 5.775 \cdot \text{kip}$$

force at each girder location due to wind on superstructure in transverse direction

$$M_{\text{cap}} := F_{\text{transwind}} \cdot \left(\text{Depth}_{\text{superstructure}} + \frac{H_{\text{cap}}}{2} \right)$$

moment on pier cap due to wind load on superstructure.

$$M_{\text{cap}} = 235.813 \cdot \text{kip} \cdot \text{ft}$$

$$M_{\text{cap_pergirder}} := \frac{M_{\text{cap}}}{N_g} = 47.163 \cdot \text{kip} \cdot \text{ft}$$

moment on each girder location due to wind force on superstructure in transverse direction.

Wind Pressure On Vehicles, WL (LRFD 3.8.1.3):

Wind pressure on vehicles shall be represented by an interruptible, moving force of 0.1 klf acting normal to, and 6 ft above, the roadway and shall be transmitted to the structure.

Moreover, we need to multiply 0.1 kips per lineal foot with the tributary length of the bridge as follows:

$$F_{\text{vehicle}} := (0.1 \text{klf}) \cdot L_{\text{tributary}} = 9 \cdot \text{kip}$$

$$F_{\text{vehicle_pergirder}} := \frac{F_{\text{vehicle}}}{N_g} = 1.8 \cdot \text{kip}$$

force at each girder location in transverse direction due to wind on vehicles

$$M_{\text{capduetovehicle}} := F_{\text{vehicle}} \cdot \left(\text{Depth}_{\text{superstructure}} + \frac{H_{\text{cap}}}{2} + 6 \text{ft} \right)$$

$$M_{\text{capduetovehicle}} = 127.5 \cdot \text{kip} \cdot \text{ft}$$

moment on pier cap due to wind load on vehicle which is applied 6 ft above the roadway.

$$M_{\text{capduetovehicle_pergirder}} := \frac{M_{\text{capduetovehicle}}}{N_g}$$

$$M_{\text{capduetovehicle_pergirder}} = 25.5 \cdot \text{kip} \cdot \text{ft}$$

moment on each girder location due to wind force on vehicle in transverse direction.

Wind On Substructure, (LRFD 3.8.1.2.3):

Assumed base wind pressure pointed out in LRFD is 0.04 ksf for both directions for substructures.

For wind directions taken skewed to the substructure, this force shall be resolved into components perpendicular to the end and front elevations of the substructure.

Wind force on substructure shall be applied simultaneously with the wind force from the superstructure.

$$F_{\text{piercap}} := W_{\text{cap}} \cdot H_{\text{cap}} \cdot 0.04 \text{ksf} = 0.42 \cdot \text{kip}$$

force on piercap that will be applied at the center of the cap.

$$F_{\text{pier}} := \Phi_{\text{pier}} \cdot L_{\text{pier}} \cdot 0.04 \text{ksf} = 1.6 \cdot \text{kip}$$

wind force on one pier.

If the wind is desired to be applied with a skew angle of attack, wind pressure should be separated to its components and applied to structure in longitudinal and transverse directions simultaneously. The components of the pressure with various angles of attack is given in LRFD Table 3.8.1.2.2-1

Appendix E: Initial Abutment Design Calcs

ABUTMENT MODULE: 5' x 3' 2" x L (height x width x length)

- Abutment Dimensions:**

$$H := 5 \cdot \text{ft}$$

height of the abutment module

$$\text{Spile} := 8 \cdot \text{ft}$$

spacing of piles

$$t := 38 \cdot \text{in}$$

thickness of the abutment module

$$L_{\text{module}} := \text{"Variable"}$$

length of the abutment module

- Material Properties:**

Concrete

$$f_c := 3.5 \cdot \text{ksi}$$

ultimate design stress of concrete

$$f_r := 0.37 \cdot \left(\frac{f_c}{\text{ksi}} \right)^{0.5} \cdot \text{ksi}$$

modulus of rupture of concrete

$$f_r = 0.692 \cdot \text{ksi}$$

$$\beta_1 := 0.85$$

Steel

$$f_y := 60 \cdot \text{ksi}$$

yield strength of reinforcing bars

$$E_s := 29000 \cdot \text{ksi}$$

modulus of elasticity of reinforcing bars

$$\epsilon_y := \frac{f_y}{E_s}$$

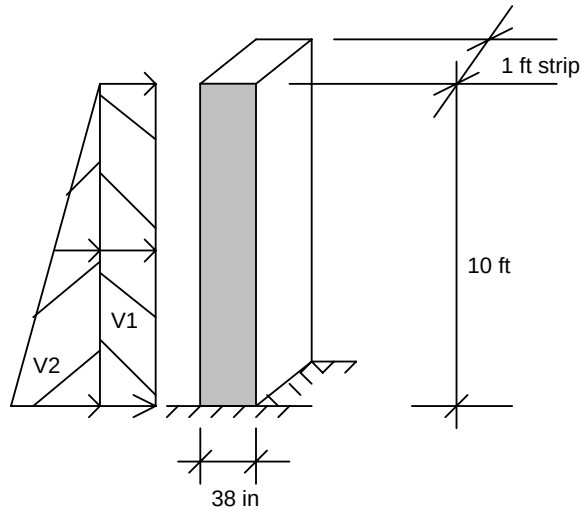
yield strain of reinforcing bars

$$\epsilon_y = 2.069 \times 10^{-3}$$

- Load Cases and Reinforcement Calculations:**

- A. Lateral Load (Earth Pressure)
- B. Vertical Load (Superstructure)
- C. Module - Pile Joint
- D. Vertical Module - Module Joint

A. Lateral Load (Earth Pressure)



Cross Sectional Properties in lateral direction

$$b := 1 \cdot \text{ft}$$

b = width of compression flange(in), unit width

$$h := t$$

depth of the cross section under bending

$$h = 38 \text{ in}$$

$$I := \frac{1}{12} \cdot b \cdot h^3$$

moment of inertia of cross section

$$I = 5.487 \times 10^4 \text{ in}^4$$

$$S_c := \frac{I}{\frac{h}{2}}$$

S_c = section modulus for extreme fiber when tensile stressed are caused by externally applied loads-composite (in^3)

$$\text{cover} := 2 \cdot \text{in}$$

cover of concrete on backface

Reaction Calculations:

1. Live load surcharge:

$$\phi := 1.75$$

load factor

$$h_{ll} := \begin{cases} 4 \cdot \text{ft} & \text{if } H = 5 \cdot \text{ft} \\ 3 \cdot \text{ft} & \text{if } H = 10 \cdot \text{ft} \\ 2 \cdot \text{ft} & \text{if } H = 20 \cdot \text{ft} \end{cases}$$

equivalent height of soil

$$h_{ll} = 4 \text{ ft}$$

$$k_a := 0.33$$

coefficient of active lateral earth pressure, used for typical designs

$$k_o := 0.5$$

coefficient of at rest lateral earth pressure used for conservative designs

$$k := k_a$$

$$\gamma_{\text{soil}} := 0.120 \cdot \frac{\text{kip}}{\text{ft}^3}$$

unit weight of soil

$$V_1 := k \cdot h_{ll} \cdot \gamma_{\text{soil}} \cdot H$$

unfactored resultant shear

$$V_1 = 0.792 \frac{1}{\text{ft}} \text{ kip}$$

$$V_{1\text{fact}} := \phi \cdot V_1$$

factored resultant shear due to live load surcharge

$$V_{1\text{fact}} = 1.386 \frac{1}{\text{ft}} \text{ kip}$$

$$M_1 := V_1 \cdot \frac{H}{2}$$

unfactored moment at the base

$$M_1 = 1.98 \frac{1}{\text{ft}} \text{ kip} \cdot \text{ft}$$

$$M_{1\text{fact}} := \phi \cdot M_1$$

factored moment at the base due to live load surcharge

$$M_{1\text{fact}} = 3.465 \frac{1}{\text{ft}} \text{ kip} \cdot \text{ft}$$

2. Lateral earth Pressure:

$$\phi := 1.5$$

load factor when k_a is used
=1.35 otherwise

$$\gamma_{\text{eq}} := \begin{cases} 0.050 \cdot \frac{\text{kip}}{\text{ft}^3} & \text{if } k = k_o \\ 0.035 \cdot \frac{\text{kip}}{\text{ft}^3} & \text{if } k = k_a \end{cases}$$

equivalent unit weight of backfill

$$\gamma_{\text{eq}} = 0.035 \frac{\text{kip}}{\text{ft}^3}$$

$$V_2 := \frac{1}{2} \cdot \gamma_{\text{eq}} \cdot H^2$$

unfactored resultant shear

$$V_2 = 437.5 \frac{1}{\text{ft}} \text{ lbf}$$

$$V2_{\text{fact}} := \phi \cdot V2$$

factored resultant shear due to lateral earth pressure

$$V2_{\text{fact}} = 0.656 \frac{1}{\text{ft}} \text{ kip}$$

$$M2 := V2 \cdot \frac{H}{3}$$

unfactored moment at the base

$$M2 = 0.729 \frac{1}{\text{ft}} \text{ kip} \cdot \text{ft}$$

$$M2_{\text{fact}} := \phi \cdot M2$$

factored moment at the base due to lateral earth pressure

$$M2_{\text{fact}} = 1.094 \frac{1}{\text{ft}} \text{ kip} \cdot \text{ft}$$

Total earth load (Lateral Earth Pressure+Live Load Surcharge):

Unfactored (service) loads:

$$V := V1 + V2$$

$$V = 1.23 \frac{1}{\text{ft}} \text{ kip}$$

$$M := M1 + M2$$

$$M = 2.709 \frac{1}{\text{ft}} \text{ kip} \cdot \text{ft}$$

Factored Loads:

$$V_{\text{fact}} := V1_{\text{fact}} + V2_{\text{fact}}$$

$$V_{\text{fact}} = 2.042 \frac{1}{\text{ft}} \text{ kip}$$

$$M_{\text{fact}} := M1_{\text{fact}} + M2_{\text{fact}}$$

$$M_{\text{fact}} = 4.559 \frac{1}{\text{ft}} \text{ kip} \cdot \text{ft}$$

Design Moment:

$$M_{\text{cr}} := S_c \cdot f_r$$

cracking moment

$$M_{\text{cr}} = 166.591 \text{ ft kip}$$

$$M_u := M_{\text{fact}} \cdot b$$

$$1.33 \cdot M_u = 6.063 \text{ ft kip}$$

1.33 times factored moment

$$M_{\text{design}} := \max(M_u, \min(1.2 \cdot M_{\text{cr}}, 1.33 \cdot M_u))$$

$$M_{design} = 6.063 \text{ ft kip}$$

Reinforcement Calculations:

a. Flexural Capacity: (ignoring compression reinforcement)

Vertical Backface Reinforcement

$$s := 18 \cdot \text{in}$$

s = spacing of reinforcement

$$No := 3$$

No = bar size

$$Ns := \frac{12 \cdot \text{in}}{s}$$

Ns = Number of bars

$$\text{diameter} := \frac{No}{8} \cdot \text{in}$$

$$A := \left(\frac{\text{diameter}}{2} \right)^2 \cdot \pi$$

$$As := Ns \cdot A$$

As = area of tension rf(in²)

$$As = 0.074 \text{ in}^2$$

$$de := h - \text{cover} - \frac{\text{diameter}}{2}$$

effective depth

$$de = 2.984 \text{ ft}$$

$$c := \frac{As \cdot fy}{0.85 \cdot fc \cdot \beta_1 \cdot b}$$

$$c = 0.146 \text{ in}$$

$$\epsilon_s := 0.003 \cdot \frac{de - c}{c}$$

strain in tension reinforcement

$$\epsilon_s = 0.735$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } \epsilon_y < \epsilon_s \\ \text{"change"} & \text{otherwise} \end{cases}$$

check for tension steel yielding

$$\text{Check} = \text{"ok"}$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } 0.005 < \epsilon_s \\ \text{"change"} & \text{otherwise} \end{cases}$$

check for under / over reinforcement

$$\text{Check} = \text{"ok"}$$

$$a := \beta_1 \cdot c$$

$$M_n := As \cdot fy \cdot \left(de - \frac{a}{2} \right)$$

$$\phi := 0.9$$

$$M_r := \phi \cdot M_n$$

$$M_r = 11.846 \text{ ft kip}$$

$$M_{\text{design}} = 6.063 \text{ ft kip}$$

$$\frac{M_r}{M_{\text{design}}} = 1.954$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } M_{\text{design}} < M_r \\ \text{"change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$

b. Maximum tension reinforcement

$$d_e := \frac{A_s \cdot f_y \cdot d_e}{A_s \cdot f_y}$$

$$\frac{c}{d_e} = 4.065 \times 10^{-3}$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } \frac{c}{d_e} < 0.42 \\ \text{"Overreinforced, change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$

c. Maximum Spacing for crack control:

$$d_c := \text{cover} + \frac{\text{diameter}}{2}$$

$$\beta_s := 1 + \frac{d_c}{0.7 \cdot (h - d_c)}$$

$$\beta_s = 1.087$$

$$\gamma_e := 0.75$$

γ_e = exposure factor
 =1.00 for class 1, cracks can be tolerated
 =0.75 for class 2, concern of corrosion and appearance

$$n := 8$$

$$c_{cr} := 3 \cdot \text{in}$$

$$c_{cr} := \text{root} \left[b \cdot c_{cr} \cdot \frac{c_{cr}}{2} - n \cdot A_s \cdot (d_e - c_{cr}), c_{cr} \right]$$

$$c_{cr} = 1.827 \text{ in}$$

$$I_{cr} := \frac{1}{12} \cdot b \cdot c_{cr}^3 + b \cdot c_{cr} \cdot \left(\frac{c_{cr}}{2} \right)^2 + n \cdot A_s \cdot (d_e - c_{cr})^2$$

$$I_{cr} = 704.753 \text{ in}^4$$

cracked moment of inertia

$$f_s := \frac{M \cdot b \cdot (d_e - c_{cr})}{I_{cr}} \cdot n$$

$$f_s = 12.542 \text{ ksi}$$

$$f_s := \begin{cases} f_s & \text{if } f_s < f_y \\ f_y & \text{otherwise} \end{cases}$$

f_s = tensile stress of rf at service limit state (ksi)

$$f_s = 12.542 \text{ ksi}$$

$$s_{max1} := \left(\frac{700 \cdot \gamma_e}{\beta_s \cdot \frac{f_s}{\text{ksi}}} - 2 \cdot \frac{d_c}{\text{in}} \right) \cdot \text{in}$$

$$s_{max1} = 34.125 \text{ in}$$

$$s_{max2} := \min(1.5 \cdot h, 18 \cdot \text{in})$$

general limitation

$$s_{max2} = 18 \text{ in}$$

$$s_{max} := \min(s_{max1}, s_{max2})$$

$$s_{max} = 18 \text{ in}$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } s \leq s_{max} \\ \text{"change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$



f. Shrinkage and Temperature Reinforcement

Shrinkage and temperature reinforcement

$$s := 6 \cdot \text{in}$$

$$N_o := 4$$

$$N := \frac{12 \cdot \text{in}}{s}$$

$$\text{diameter} := \frac{N_o}{8} \cdot \text{in}$$

$$A := \left(\frac{\text{diameter}}{2} \right)^2 \cdot \pi$$

$$A_{st} := A \cdot N$$

$$A_{st} = 0.393 \text{ in}^2$$

in each direction
on each surface

$$A_g := b \cdot h$$

A_g = gross area of section (in²)

$$A_g = 3.167 \text{ ft}^2$$

$$A_{stmin} := \min \left(0.11 \cdot \frac{\frac{A_g}{\text{in}^2}}{\frac{f_y}{\text{ksi}}}, 0.0015 \cdot \frac{A_g}{\text{in}^2} \right) \cdot \text{in}^2$$

$$A_{stmin} = 0.684 \text{ in}^2$$

Area to be equally distributed for both faces and satisfied in each direction

$$\underline{A_{stmin}} := \frac{A_{stmin}}{2}$$

$$A_{stmin} = 0.342 \text{ in}^2$$

Area to be equally distributed on one face and satisfied in each direction

$$\underline{\text{Check}} := \begin{cases} \text{"ok"} & \text{if } A_{stmin} < A_{st} \\ \text{"change"} & \text{otherwise} \end{cases}$$

Check = "ok"

$$\underline{s_{max}} := \min(12 \cdot \text{in}, 3 \cdot h, 18 \cdot \text{in})$$

$$s_{max} = 12 \text{ in}$$

$$s = 6 \text{ in}$$

$$\underline{\text{Check}} := \begin{cases} \text{"ok"} & \text{if } s < s_{max} \\ \text{"change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$

B. Vertical Load (Superstructure)

Cross Sectional Properties
in vertical direction

$$\underline{b} := t$$

thickness of the abutment

$$\underline{h} := H$$

height of the abutment

$$\underline{I} := \frac{1}{12} \cdot b \cdot h^3$$

moment of inertia of cross section

$$I = 6.84 \times 10^5 \text{ in}^4$$

$$\underline{S_c} := \frac{I}{\frac{h}{2}}$$

S_c = section modulus for extreme fiber when tensile stresses are caused by externally applied loads-composite (in³)

$$\text{cover}_b := 3 \cdot \text{in}$$

concrete cover for bottom reinforcement

coveru := 2·in

concrete cover for top reinforcement

Reaction Calculations:

Pile Properties

$$A_{\text{pile}} := 15.5 \cdot \text{in}^2$$

cross sectional area of HP 12 x53 pile

$$P_{\text{pile}} := A_{\text{pile}} \cdot 36 \cdot \text{ksi} \cdot 0.5$$

HP 12 x 53 pile axial capacity

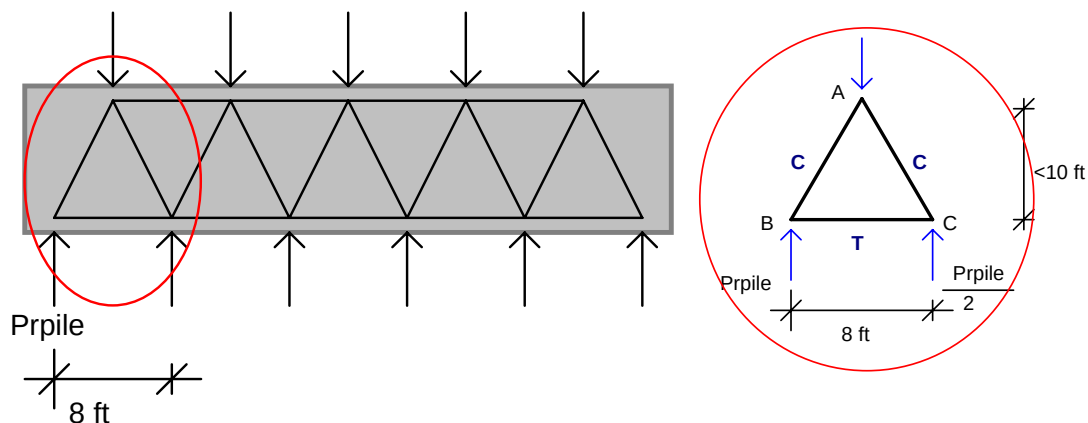
$$P_{\text{pile}} = 279 \text{ kip}$$

$$\text{depth} := 7.44 \text{ in}$$

assumed depth of ties to calculate truss geometry, must be checked after the real value is calculated



Case 2: full pile load is transferred to the truss element at the edge_ Governs other cases



$$\alpha_2 := \text{atan} \left(\frac{H - \text{coveru} - \text{coverb} - \text{depth}}{\frac{S_{\text{pile}}}{2}} \right)$$

$$\alpha_2 = 44.736 \text{ deg}$$

$$AB := \frac{P_{\text{pile}}}{\sin(\alpha_2)}$$

$$C_2 := AB$$

$$C_2 = 396.395 \text{ kip}$$

force in compression strut

$$BC := AB \cdot \cos(\alpha_2)$$

$$T_2 := BC$$

$$T2 = 281.581 \text{ kip}$$

force in tension tie



$$C1 = 198.197 \text{ kip}$$

$$T1 = 140.791 \text{ kip}$$

summary of maximum
compression and tension forces
from 4 cases

$$C2 = 396.395 \text{ kip}$$

$$T2 = 281.581 \text{ kip}$$

$$C3 = 82.763 \text{ kip}$$

$$T3 = 44.155 \text{ kip}$$

$$C4 = 292.547 \text{ kip}$$

$$T4 = 87.994 \text{ kip}$$

$$C := \max(C1, C2, C3, C4)$$

comparison of cases

$$C = 396.395 \text{ kip}$$

$$T := \max(T1, T2, T3, T4)$$

$$T = 281.581 \text{ kip}$$

$$\alpha_s := \alpha_{s2}$$

Reinforcement Calculations:

a. Tension Ties:

Tension tie reinforcement

$$N := 8$$

$$No := 7$$

$$set := 2$$

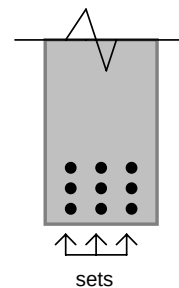
$$diameter := \frac{No}{8} \cdot in$$

$$A := \left(\frac{diameter}{2} \right)^2 \cdot \pi$$

$$As := A \cdot N$$

$$As = 4.811 \text{ in}^2$$

number of
vertical
layers of rf



sets

$$\phi_t := 1$$

$$P_n := f_y \cdot A_{st}$$

$$P_r := \phi_t \cdot P_n$$

$$A_{st} := \frac{T}{\phi_t \cdot f_y}$$

$$A_{st} = 4.693 \text{ in}^2$$

$$As = 4.811 \text{ in}^2$$

load factor for tension

$$Check := \begin{cases} "ok" & \text{if } A_{st} < As \\ "change" & \text{otherwise} \end{cases}$$

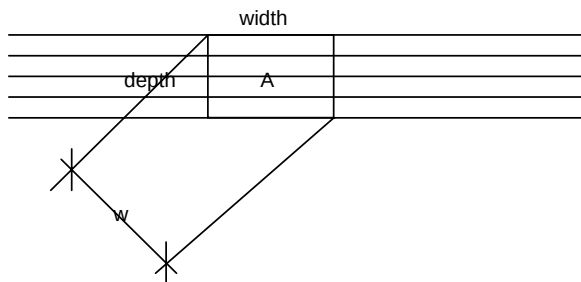
Check = "ok"

$$T_r := \phi_t \cdot f_y \cdot A_s$$

$$T_r = 288.634 \text{ kip}$$

$$\frac{T_r}{T} = 1.025$$

b. Nodal Zones:



$$b_{seat} := 12 \cdot \text{in}$$

assumed beam seat thickness

$$width := 12 \cdot \text{in}$$

assumed beam seat width

$$depth := \frac{N}{set} \cdot \text{diameter} + \left(\frac{N}{set} - 1 \right) \cdot \text{diameter} \cdot 1.5$$

$$depth = 7.438 \text{ in}$$

$$\alpha_s := \alpha_{s2}$$

$$w := depth \cdot \cos(\alpha_s) + width \cdot \sin(\alpha_s)$$

width of compression block

$$A_{cs} := w \cdot b_{seat}$$

$$A_{cs} = 164.753 \text{ in}^2$$

A_{cs} = effective cross sectional area of strut

Anchorage of reinforcement in the node zone:

Tension development length:

$$A_b := A$$

$$d_b := \text{diameter}$$

$$l_{db} := \max \left(\frac{1.25 \cdot \frac{A_b}{\text{in}^2} \cdot \frac{f_y}{\text{ksi}}}{\sqrt{\frac{f_c}{\text{ksi}}}} \cdot \text{in}, 0.4 \cdot \frac{d_b}{\text{in}} \cdot \frac{f_y}{\text{ksi}} \cdot \text{in} \right) \quad \text{for \#11 bar and smaller}$$

$$l_{db} = 24.106 \text{ in}$$

c. Compressive Struts:

Compression strut reinforcement

$$N := 10$$

Number of bars

$$No := 6$$

$$set := 2$$

$$diameter := \frac{No}{8} \cdot in$$

$$A := \left(\frac{diameter}{2} \right)^2 \cdot \pi$$

$$Ass := A \cdot N$$

$$Ass = 4.418 \text{ in}^2$$

$$\phi_c := 0.7$$

$$\epsilon_s := \min \left(\frac{T}{A_s \cdot E_s}, \epsilon_y \right)$$

$$\epsilon_1 := \epsilon_s + (\epsilon_s + 0.002) \cdot \cot(\alpha_s)^2$$

$$f_{cu} := \min \left(\frac{f_c}{0.8 + 170 \cdot \epsilon_1}, 0.85 \cdot f_c \right)$$

$$P_n := f_{cu} \cdot A_{cs} + f_y \cdot Ass$$

$$P_r := \phi_c \cdot P_n$$

$$P_r = 405.046 \text{ kip}$$

$$C = 396.395 \text{ kip}$$

$$Check := \begin{cases} "ok" & \text{if } C < P_r \\ "change" & \text{otherwise} \end{cases}$$

$$Check = "ok"$$

$$\frac{P_r}{C} = 1.022$$

d. Transverse reinforcement:

Transverse reinforcement

$$s := 5 \cdot in$$

$$No := 5$$

$$N := \frac{12 \cdot in}{s}$$

$$diameter := \frac{No}{8} \cdot in$$

Bar Size

Number of sets

Ass = area of reinforcement in the strut

load factor for compression

ϵ_s = tensile strain in the concrete in the direction of the tension tie

α_s = smallest angle between the compressive strut and adjoining tension ties

P_n = strength of strut

spacing

size of the bars

$$A := \left(\frac{\text{diameter}}{2} \right)^2 \cdot \pi$$

$$A_{\text{trans}} := A \cdot N$$

$$A_{\text{trans}} = 0.736 \text{ in}^2$$

$$s_{\text{max}} := 12 \cdot \text{in}$$

$$s = 5 \text{ in}$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } s < s_{\text{max}} \\ \text{"change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$

$$A_g := 1 \cdot \text{ft} \cdot b$$

$$A_{\text{min}} := A_g \cdot 0.003$$

$$A_{\text{min}} = 1.368 \text{ in}^2$$

to be shared by both faces and satisfied in each orthogonal directions

$$A_{\text{min}} := \frac{A_{\text{min}}}{2}$$

to be satisfied in each direction

$$A_{\text{min}} = 0.684 \text{ in}^2$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } A_{\text{min}} < A_{\text{trans}} \\ \text{"change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$

C. Module - Pile Joint

Properties of HP 12 x 53

$$A := 15.5 \cdot \text{in}^2$$

$$z_{xx} := 74 \cdot \text{in}^3$$

$$f_y := 36 \cdot \text{ksi}$$

$$\phi := 0.5$$

$$N := \phi \cdot A \cdot f_y$$

$$N = 279 \text{ kip}$$

structural axial capacity of the pile

$$M := f_y \cdot z_{xx}$$

$$M = 222 \text{ ft kip}$$

structural ultimate moment capacity of piles

Load Calculations:

$$h := 2 \cdot \text{ft}$$

pile embedment length

$$V_{1\text{fact}} = 1.386 \frac{1}{\text{ft}} \text{ kip}$$

live load surcharge resultant acting @ H/2 from bottom

$$V2_{\text{fact}} = 0.656 \frac{1}{\text{ft}} \text{ kip}$$

earth pressure resultant acting @ H/3 from bottom

$$W_{\text{max}} := 40 \cdot \text{kip}$$

maximum transportable weight

$$\gamma_{\text{concrete}} := 150 \cdot \frac{\text{lbf}}{\text{ft}^3}$$

$$L_{\text{max}} := \frac{W_{\text{max}}}{H \cdot t \cdot \gamma_{\text{concrete}}}$$

$$L_{\text{max}} = 16.842 \text{ ft}$$

maximum transportable length of module

$$N_{\text{pilesmin}} := \text{ceil} \left[\frac{L_{\text{max}} - \frac{8}{2} \cdot \text{ft} - \frac{8}{2} \cdot \text{ft}}{(8) \cdot \text{ft}} \right] + 1 \quad \text{minimum number of piles in a module}$$

$$N_{\text{pilesmin}} = 3$$

$$V1_{\text{socket}} := V1_{\text{fact}} \cdot \frac{L_{\text{max}}}{N_{\text{pilesmin}}}$$

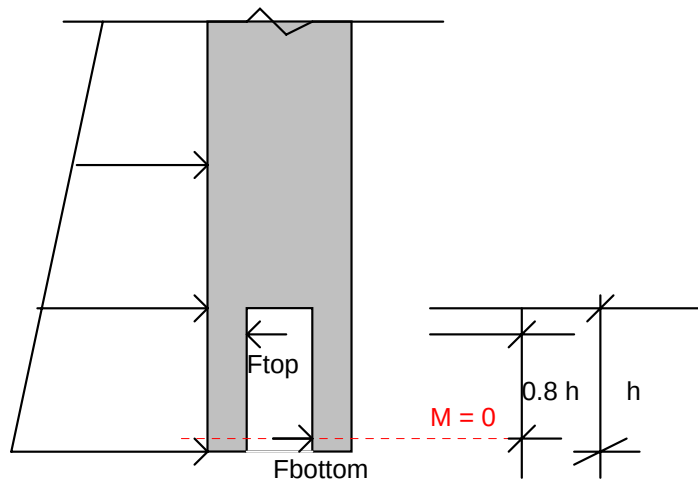
shear due to live load surcharge per one block out

$$V1_{\text{socket}} = 7.781 \text{ kip}$$

$$V2_{\text{socket}} := V2_{\text{fact}} \cdot \frac{L_{\text{max}}}{N_{\text{pilesmin}}}$$

shear due to earth pressure per one block out

$$V2_{\text{socket}} = 3.684 \text{ kip}$$



$$M_{\text{bottom}} := 0$$

$$F_{top} := \frac{V1_{socket} \cdot \left(\frac{H}{2} - 0.1 \cdot h \right) + V2_{socket} \cdot \left(\frac{H}{3} - 0.1 \cdot h \right)}{h - 0.1 \cdot h - 0.1 \cdot h}$$

$$F_{top} = 14.562 \text{ kip}$$

Force acting close to top surface of breakout

$$F_{bot} := F_{top} - (V1_{socket} + V2_{socket})$$

$$F_{bot} = 3.097 \text{ kip}$$

Force acting close to bottom surface of breakout

Reinforcement Calculations:

a. Shear around the breakout:

Tie reinforcement around breakout

$$N_s := 2$$

number of ties

$$N_o := 4$$

size of the tie

$$\text{diameter} := \frac{N_o}{8} \cdot \text{in}$$

$$A := \left(\frac{\text{diameter}}{2} \right)^2 \cdot \pi$$

$$A_s := N_s \cdot A$$

$$A_s = 0.393 \text{ in}^2$$

$$A_s := 2 \cdot A_s$$

with two legs of tie reinforcement

$$A_s = 0.785 \text{ in}^2$$

$$f_y := 60 \cdot \text{ksi}$$

$$A_{tie} := \frac{\max(F_{top}, F_{bot})}{f_y}$$

$$A_{tie} = 0.243 \text{ in}^2$$

tie area through the middle height

$$A_{tie} := 2 \cdot A_{tie}$$

tie area through full height

$$A_{tie} = 0.485 \text{ in}^2$$

$$A_s = 0.785 \text{ in}^2$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } A_{tie} < A_s \\ \text{"change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$

b. Vertical reinforcement:

Vertical reinforcement around breakout

$$N_s := 2$$

number of vertical bars

$$N_o := 4$$

size of the vertical bars

$$s := \frac{(12 + 4 + 4) \cdot \text{in}}{N_s}$$

spacing of the bars

$$s = 10 \text{ in}$$

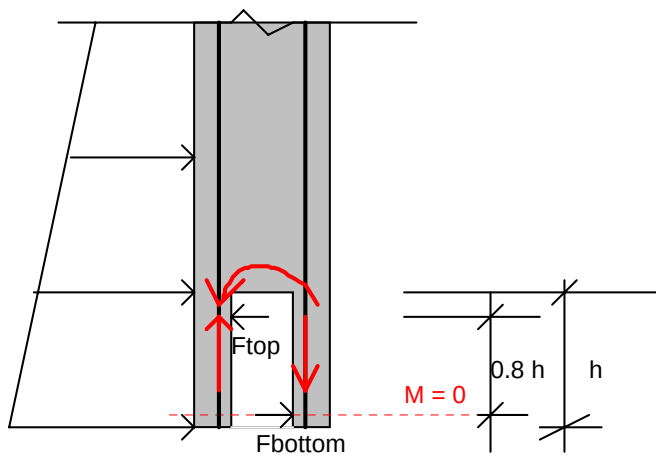
$$\text{diameter} := \frac{N_o}{8} \cdot \text{in}$$

$$A := \left(\frac{\text{diameter}}{2} \right)^2 \cdot \pi$$

$$A_s := N_s \cdot A$$

$$A_s = 0.393 \text{ in}^2$$

on each side (U shape for anchorage)



$$M := F_{\text{bot}} \cdot h$$

$$M = 6.194 \text{ ft kip}$$

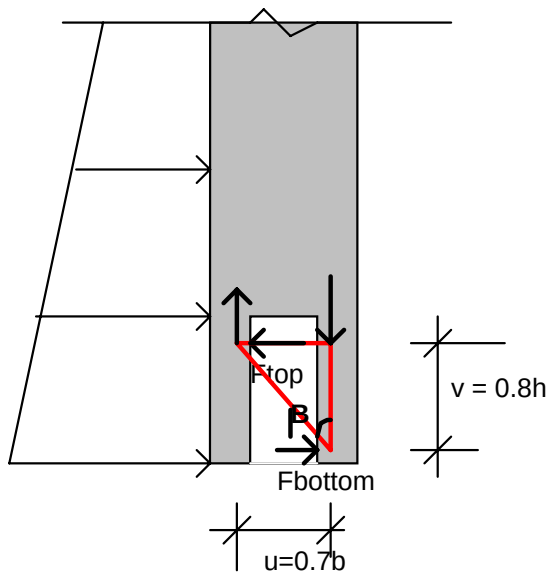
$$A_{\text{vertical}} := \frac{M}{0.7 \cdot t \cdot f_y}$$

$$A_{\text{vertical}} = 0.047 \text{ in}^2$$

$$\text{Check} := \begin{cases} \text{"ok"} & \text{if } A_{\text{vertical}} < A_s \\ \text{"change"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"ok"}$$

c. Alternatively by truss analogy at the block-out



$$u := 0.7 \cdot b$$

$$v := 0.8 \cdot h$$

$$\beta := \text{atan}\left(\frac{u}{v}\right)$$

$$\beta = 54.178 \text{ deg}$$

$$T := \frac{F_{\text{bot}}}{\sin(\beta)}$$

$$T = 3.82 \text{ kip}$$

Tension tie reinforcement (diagonal)

$$N := 1$$

number of diagonal bars

$$N_o := 4$$

size of bars

$$\text{diameter} := \frac{N_o}{8} \cdot \text{in}$$

$$A := \left(\frac{\text{diameter}}{2}\right)^2 \cdot \pi$$

$$A_s := A \cdot N$$

$$A_{s2} := 2 \cdot A_s$$

two legs for tie reinforcement

$$A_s = 0.393 \text{ in}^2$$

$$\phi_t := 1$$

$$A_{st} := \frac{T}{\phi_t \cdot f_y}$$

$$A_{st} = 0.064 \text{ in}^2$$

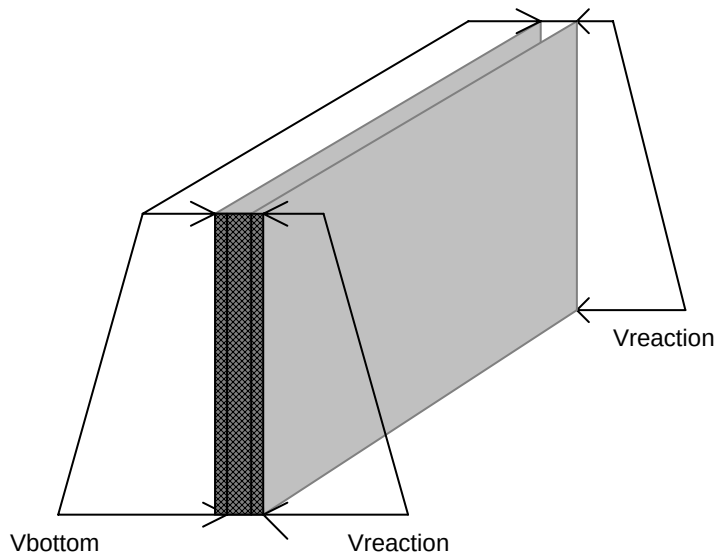
$$A_s = 0.393 \text{ in}^2$$

```

Check := | "ok"   if Ast < As
         | "change" otherwise
Check = "ok"

```

D. Vertical Module-Module Joint



The shear force at the joint is calculated ignoring any contribution of piles and maximizing the difference in loading of two adjacent modules.

$$V_{\text{bottom}} := 1.73 \cdot k \cdot h_{ll} \cdot \gamma_{\text{soil}} + 1.5 \cdot \gamma_{\text{eq}} \cdot H$$

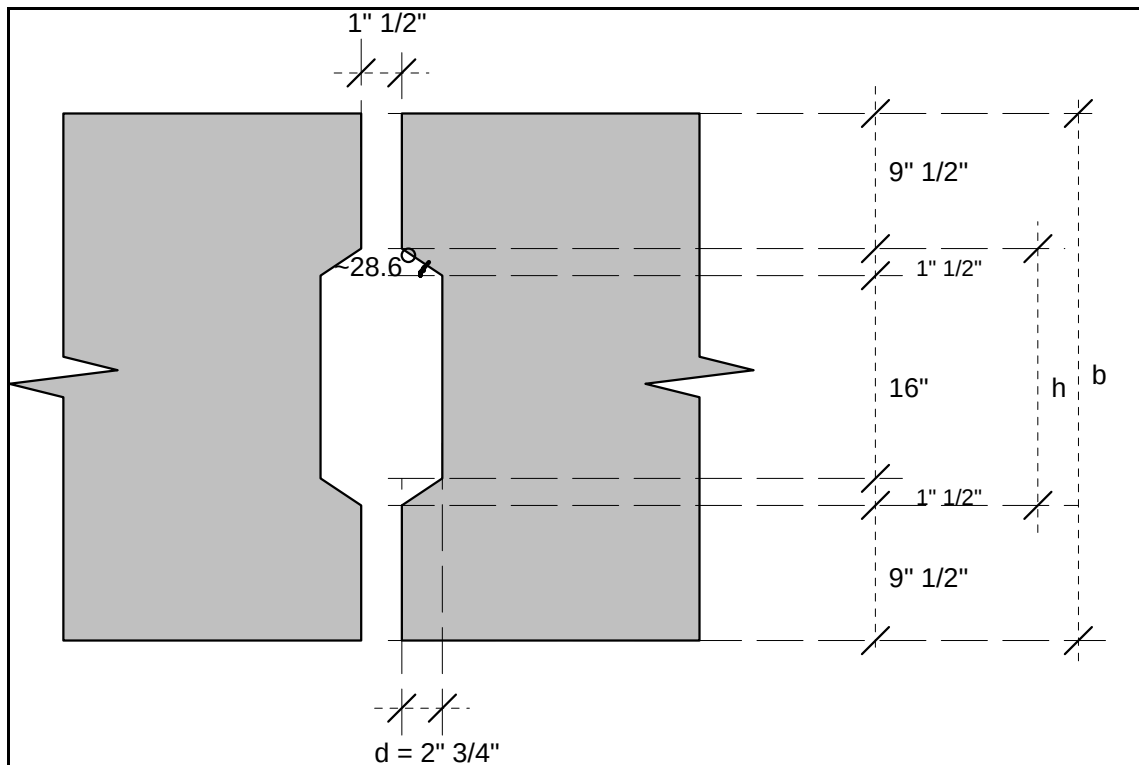
$$V_{\text{bottom}} = 0.537 \frac{\text{kip}}{\text{ft}^2}$$

$$V_{\text{reaction}} := V_{\text{bottom}} \cdot \frac{L_{\text{max}}}{2}$$

$$V_{\text{reaction}} = 4.518 \frac{\text{kip}}{\text{ft}}$$

Detailing the shear key:

When a geometry is created using the recommendations given in Chapter 5



$$d := 2 \cdot \text{in} + \frac{3}{4} \cdot \text{in}$$

$$h := 16 \cdot \text{in} + 2 \cdot \left(1 \cdot \text{in} + \frac{1}{2} \cdot \text{in} \right)$$

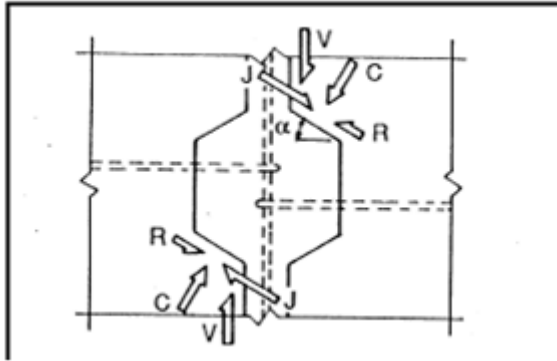
$$\frac{h}{b - h} = 1$$

$$\frac{h}{d} = 6.909$$

$$\alpha := \text{atan} \left(\frac{1 \cdot \text{in} + \frac{1}{2} \cdot \text{in}}{d} \right)$$

$$\alpha = 28.61 \text{ deg}$$

Strength Check:



$$J := V_{\text{reaction}} \cdot \sin(\alpha)$$

$$J = 2.164 \frac{\text{kip}}{\text{ft}}$$

$$C := V_{\text{reaction}} \cdot \cos(\alpha)$$

$$C = 3.966 \frac{\text{kip}}{\text{ft}}$$

$$R := C \cdot \tan(\Phi)$$

$$R := C \cdot 0.6$$

$$R = 2.38 \frac{\text{kip}}{\text{ft}}$$

$$\text{Check} := \begin{cases} \text{"OK"} & \text{if } J \leq R \\ \text{"Change Geometry"} & \text{otherwise} \end{cases}$$

$$\text{Check} = \text{"OK"}$$

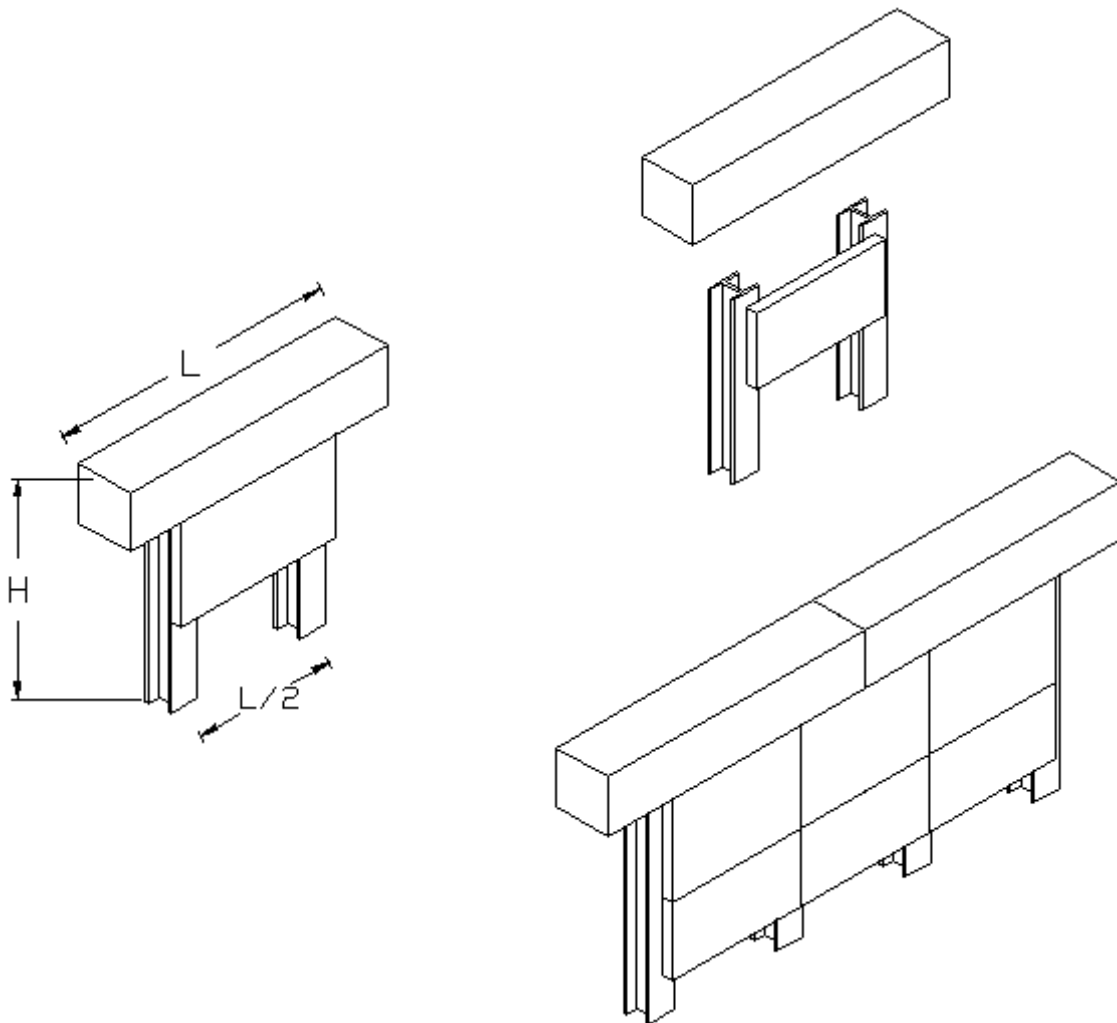
Appendix F: Design Loads- Abutment Back Panels

Design Force Calculation Example for Abutment with Separate Wall Panels

All inputs are in red colour.

This design force calculation sheet determines the forces at the bottom of abutment for the 1st abutment type which is thin wall panels with pile bent cap at the top.

Isometric drawings of the system can be seen below.



Design Constants:

$$H := 10\text{ft}$$

Height of the abutment module.

$$L := 16\text{ft}$$

Width of the abutment module.

$$S_{\text{pile}} := \frac{L}{2}$$

Pile spacing.

Calculation of Design Loads:

There are two major load types that should be considered in the design of abutment which are:

- 1) Horizontal earth pressure which is caused by fill and live load surcharge from vehicles.
- 2) Vertical load coming from superstructure.

Loads Due To Lateral Earth Pressure:

While designing the abutment under lateral loads, loads acting on half of the abutment is thought to be resisted by one pile region. This is reasonable since each abutment modules is manufactured by two piles symmetrically.

This part of load calculation is divided into two:

- 1) Live load surcharge due to vehicle load
- 2) Load due to lateral earth pressure

1) Live Load Surcharge LS, (LRFD 3.11.6.4):

The procedure used to calculate the wall's horizontal loading due to live load surcharge from vehicular wheel load on the backfill is as follows:

$$\Delta_s = k \cdot \gamma_s \cdot h_{eq}$$

constant horizontal earth pressure due to live load surcharge (ksf)

$$\gamma_s$$

total unit weight of soil (kcf)

$$\gamma_s := 0.127 \frac{\text{kip}}{\text{ft}^3}$$

$$k$$

coefficient of lateral earth pressure

k_o will be used if the wall does not deflect, (LRFD Article 3.11.5.2)

k_a will be used if the wall deflects, (LRFD 3.11.5.3, 3.11.5.6 and 3.11.5.7)

$$h_{eq}$$

equivalent height of soil for vehicular load (ft.)

Equivalent height of soil for vehicular loading on abutments perpendicular to traffic is as follows:

(LRFD Table 3.11.6.4-1)

$$h_{eq} := \begin{cases} 4\text{ft} & \text{if } H \leq 5\text{ft} \\ 4\text{ft} - \frac{(H - 5\text{ft})}{5} & \text{if } 5\text{ft} < H \leq 10\text{ft} \\ 3\text{ft} - \frac{(H - 10\text{ft})}{10} & \text{if } 10\text{ft} < H < 20\text{ft} \\ 2\text{ft} & \text{if } H \geq 20\text{ft} \end{cases} = 3 \cdot \text{ft}$$

Calculation of k_a according to LRFD 3.11.5.3:

$$\delta := 30\text{deg}$$

friction angle between fill and wall taken as specified in LRFD Table 3.11.5.3-1

$$\beta := 0\text{deg}$$

angle of fill to the horizontal as shown in LRFD Figure 3.11.5.3-1

$$\theta := 90\text{deg}$$

angle of back face of wall to the horizontal as shown in LRFD Figure 3.11.5.3-1

$$\phi_f := 30\text{deg}$$

effective angle of internal friction angle to be determined by laboratory or site investigations

$$\Gamma := \left(1 + \sqrt{\frac{\sin(\phi_f + \delta) \cdot \sin(\phi_f - \beta)}{\sin(\theta - \delta) \cdot \sin(\theta + \beta)}} \right)^2 \quad \text{LRFD equation 3.11.5.3-2}$$

$$\Gamma = 2.914$$

$$k_a := \frac{\sin(\theta + \phi_f)^2}{\Gamma \cdot \sin(\theta)^2 \cdot \sin(\theta - \delta)} = 0.297 \quad \text{LRFD equation 3.11.5.3-1}$$

Calculation of k_o according to LRFD 3.11.5.2:

$$k_o := 1 - \sin(\phi_f) = 0.5$$

LRFD equation 3.11.5.2-1

Insert the value of wall below as 1 if the wall deflects or 0 if the wall does not deflect:

$$\text{wall} := 1$$

$$k := \begin{cases} k_a & \text{if wall} = 1 \\ k_o & \text{if wall} = 0 \end{cases} = 0.297$$

Forces, in this type of abutment, are resisted by the piles. Therefore, we will determine them at the bottom of the abutment.

$$\phi_{LL} := 1.75$$

LL load factor, LRFD Table 3.4.1-1

$$V_1 := k \cdot h_{eq} \cdot \gamma_s \cdot H \cdot \frac{L}{2} = 9.058 \cdot \text{kip}$$

unfactored shear force per one pile

$$\phi V_1 := V_1 \cdot \phi_{LL} = 15.851 \cdot \text{kip}$$

factored shear force per pile due to live load surcharge

$$M_1 := V_1 \cdot \frac{H}{2} = 45.289 \cdot \text{kip} \cdot \text{ft}$$

unfactored moment over socket region per pile
(since the pressure distribution is constant,
moment arm is the half of the pressure distribution
height)

$$\phi M_1 := M_1 \cdot \phi_{LL} = 79.256 \cdot \text{kip} \cdot \text{ft}$$

factored moment over socket region per pile due to
live load surcharge

2) Loads Due to Lateral Earth Pressure EH, (LRFD 3.11.5):

A second source of load on the wall is earth pressures and it has a triangular distribution on the abutment back face, zero at the top and the largest at the bottom:

$$\phi_{EH} := \begin{cases} 1.5 & \text{if } k = k_a \\ 1.35 & \text{if } k = k_o \end{cases} = 1.5$$

load factor is 1.5 if k_a is used, or 1.35 if k_o is used
in calculations, LRFD Table 3.4.1-2

While calculating the earth pressure behind the abutment wall, we will use Equivalent-Fluid Method of Estimating Rankine Lateral Earth Pressures method explained in LRFD 3.11.5.5. Please note also that this method can only be applied when we have free draining backfill. Here, we assume that the drainage methods will be applied at the site to get rid of water table behind the wall and this will keep soil behind the wall free of water all the time.

$p = \gamma_{eq} \cdot z$	basic earth pressure behind the wall (ksf)
γ_{eq}	equivalent fluid unit weight of soil, not less than 0.03 kcf
z	depth below surface of soil (ft)

For medium dense sand or gravel, wall height not exceeding 20ft and with a level backfill:

$$\gamma_{eq} := \begin{cases} 0.035 \frac{\text{kip}}{\text{ft}^3} & \text{if } k = k_a \\ 0.05 \frac{\text{kip}}{\text{ft}^3} & \text{if } k = k_o \end{cases}$$

equivalent unit weight of backfill, LRFD Table 3.11.5.5-1

$$\gamma_{eq} = 0.035 \cdot \frac{\text{kip}}{\text{ft}^3}$$

$$V_2 := 0.5 \cdot \gamma_{eq} \cdot H^2 \cdot \frac{L}{2} = 14 \cdot \text{kip}$$

unfactored resultant shear per one pile

$$\phi V_2 := V_2 \cdot \phi_{EH} = 21 \cdot \text{kip}$$

factored resultant shear due to lateral earth pressure per pile

$$M_2 := V_2 \cdot \frac{H}{3} = 46.667 \cdot \text{kip} \cdot \text{ft}$$

unfactored moment at the top of pile socket region per pile, since the distribution is triangular, moment arm is the one third of the relevant height

$$\phi M_2 := M_2 \cdot \phi_{EH} = 70 \cdot \text{kip} \cdot \text{ft}$$

factored moment at the top of socket region per pile

Total design loads due to earth pressures (LS+EH) per pile:

Unfactored service loads:

Factored loads:

$$V := V_1 + V_2 = 23.058 \cdot \text{kip}$$

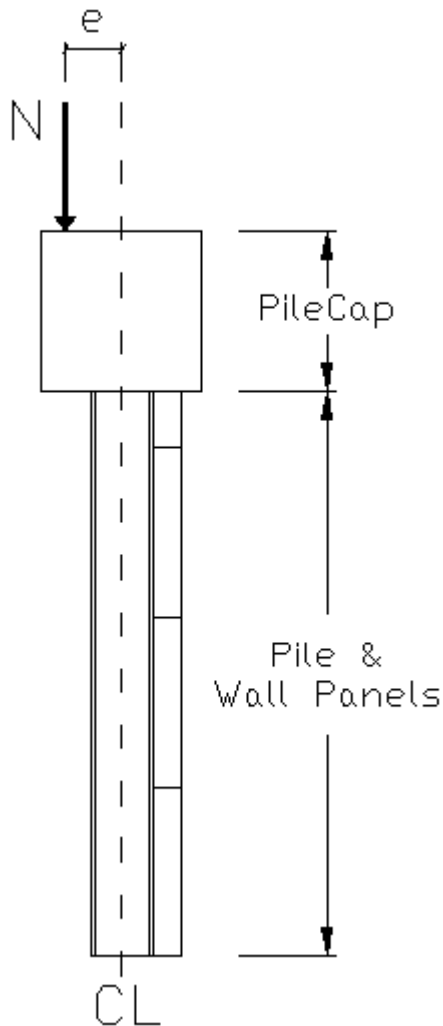
$$\phi V := \phi V_1 + \phi V_2 = 36.851 \cdot \text{kip}$$

$$M := M_1 + M_2 = 91.956 \cdot \text{kip} \cdot \text{ft}$$

$$\phi M := \phi M_1 + \phi M_2 = 149.256 \cdot \text{kip} \cdot \text{ft}$$

Vertical Load Coming From Superstructure:

There is an eccentricity of the normal forces coming from superstructure since the bearing pads of the girders are not place at the middle part of the pilecap. This eccentricity causes additional moment on the cross section that we are dealing and on the pile also. In this section we will also add those loads to our calculations.



Please insert the total sum of normal loads coming from superstructure acting on one abutment module as DL and LL separately.

$$N_{DL} := 115 \text{kip}$$

$$N_{LL} := 200 \text{kip}$$

Please insert the eccentricity between the centerline of bearing pads and the centerline of pile bent cap.

$$e := 10 \text{in}$$

$$\phi_{DL} := 1.25$$

load factor for DL, LRFD Table 3.4.1-2

$$N := \frac{N_{DL} + N_{LL}}{2} = 157.5 \cdot \text{kip}$$

total unfactored normal load on abutment per pile

$$\phi N := \frac{N_{DL} \cdot \phi_{DL} + N_{LL} \cdot \phi_{LL}}{2} = 246.875 \cdot \text{kip}$$

total factored normal load on abutment per pile

$$M_e := N \cdot e = 131.25 \cdot \text{kip} \cdot \text{ft}$$

unfactored moment due to eccentricity per pile

$$\phi M_e := \phi N \cdot e = 205.729 \cdot \text{kip} \cdot \text{ft}$$

factored moment due to eccentricity per pile

We should also add the self weight of the pile cap and the pile, please enter their total weight as force per pile below:

$$N_{\text{self}} := 11 \cdot \text{kip}$$

normal force due to self weight of the pile and pile bent cap per pile

Then, the final normal force is:

$$N := N_{\text{self}} + N = 168.5 \cdot \text{kip}$$

unfactored normal force per pile acting on the center of gravity of the section

$$\phi N := N_{\text{self}} \cdot \phi_{DL} + \phi N = 260.625 \cdot \text{kip}$$

factored normal force per pile acting on the center of gravity of the section

Total forces over pile socket per pile are as follows:

Unfactored:

Factored:

$$N = 168.5 \cdot \text{kip}$$

$$\phi N = 260.625 \cdot \text{kip}$$

$$V = 23.058 \cdot \text{kip}$$

$$\phi V = 36.851 \cdot \text{kip}$$

$$M := M + M_e = 223.206 \cdot \text{kip} \cdot \text{ft}$$

$$\phi M := \phi M + \phi M_e = 354.985 \cdot \text{kip} \cdot \text{ft}$$

Design Force for Wall Panels

Wall panels running between two piles are subjected to bending due to earth pressure at the back. Panels will be designed to behave like simply supported since no moment resisting joints will be provided at the site which will be cheap and easy to implement. Only simple connections that will hold the panels in place until the soil is filled and compressed at the back.

The most critical panel is at the bottom, which resists the largest pressure. In order to distribute reinforcement uniformly, it will be designed with the largest pressure at the bottom.

$$\sigma_{\text{panel}} := k \cdot \gamma_s \cdot h_{\text{eq}} + H \cdot \gamma_{\text{eq}} = 3.217 \cdot \text{psi}$$

This pressure will be assumed to be applied all over the panel surface.

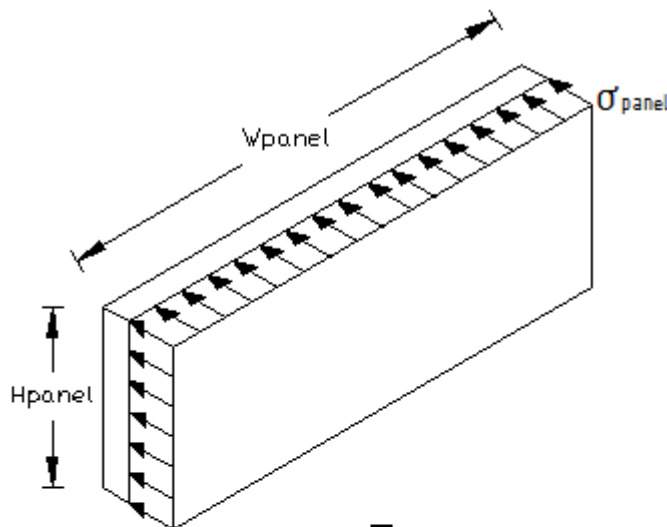
Please insert the dimensions of the wall panel below:

$$H_{\text{panel}} := 3\text{ft}$$

height of the wall panel

$$W_{\text{panel}} := S_{\text{pile}}$$

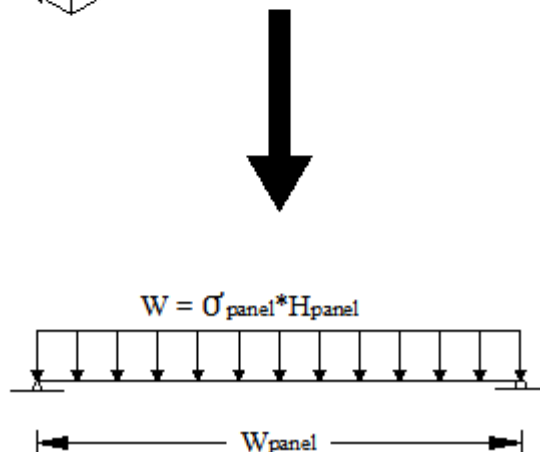
width of the wall panel



$$M_{\text{max}} := \frac{(\sigma_{\text{panel}} \cdot H_{\text{panel}}) \cdot W_{\text{panel}}^2}{8}$$

$$M_{\text{max}} = 11.117 \cdot \text{kip} \cdot \text{ft}$$

Enough tension reinforcement to resist the above maximum moment should be provided in wall panel.



Appendix G: Pier Cap Design Calcs

Pier Bent Cap Design Sheet

Users Guide:

- What does program do?

This design sheet is intended to show how to determine the amount and the location of the reinforcements other than in the pocket region of a pier cap.

- What input is required?

Shape of the abutment module:	b	: Width of pier cap cross section
	h	: height of pier cap cross section
	L	: Length of the bent cap
	cover	: Clear cover
Material properties:	f_c	: Strength of the concrete
	f_y	: Yield strength of steel
	E_s	: Modulus of elasticity of steel
Loads:	M_{d_pos}	: Maximum factored positive design moment
	M_{d_neg}	: Maximum factored negative design moment
	V_d	: Maximum factored design shear force
Resistance factors:	Φ	: Resistance factor for tension controlled RC sections
sections	Φ_c	: Resistance factor for compression controlled RC sections
Design parameters:	N	: Bar size
	n	: Number of bars
	n_{layer}	: Number of horizontal layers of longitudinal rebars
	s_{layer}	: center to center distance between different layer reinforcements
	γ_e	: Exposure factor
	$c_{service}$	: Initial guess for neutral axis depth under service loads
	s	: Spacing of reinforcement
	n_{leg}	: Number of stirrup legs in one cross section

- What is the output?

The checks which verify whether the entered amount of reinforcement is enough or not are the outputs of this sheet. When all the checks are okay, we make sure that our design is completed.

Reference Manual:

- How calculations are done?

All the calculations are done according to the methods explained in AASHTO LRFD specification.

Procedures followed are also described in detail while calculations are done.

- Example design problem is shown below with sample input/output.

All inputs are in red color.

All checks are in yellow color.

Pier Cap Dimensions:

$$b := 3.5\text{ft}$$

width of the x-section

$$h := 3.5\text{ft}$$

height of the x-section

$$L := 42\text{ft}$$

length of the cap

$$\text{cover} := 2.5\text{in}$$

clear cover

$$I_g := \frac{1}{12} \cdot b \cdot h^3 = 12.505 \cdot \text{ft}^4$$

gross moment of inertia of the section

Material Properties:

Concrete:

$$f_c := 4.0 \cdot \text{ksi}$$

design strength of concrete

$$f_r := 0.37 \cdot \left(\frac{f_c}{\text{ksi}} \right)^{0.5} \text{ ksi} = 0.74 \cdot \text{ksi}$$

modulus of rupture of concrete (LRFD 5.4.2.6) to be used in minimum reinforcement check

$$\beta_1 := \min \left[0.85, 0.85 - 0.05 \cdot \left(\frac{f_c}{\text{ksi}} - 4 \right) \right] = 0.85$$

neutral axis multiplier, LRFD 5.7.2.2

Steel:

$$f_y := 60 \cdot \text{ksi}$$

yield strength of reinforcing bars

$$E_s := 29000 \cdot \text{ksi}$$

modulus of elasticity of reinforcing bars (LRFD 5.4.3.2)

$$\epsilon_y := \frac{f_y}{E_s} = 2.069 \times 10^{-3}$$

yield strain of reinforcing bars

Resistance Factors, LRFD 5.5.4.2:

$$\phi := 0.9$$

for tension controlled reinforced concrete sections

$$\phi_c := 0.75$$

for compression controlled reinforced concrete sections

Design Forces:

Design forces are for transverse direction of the bridge. Please enter design moments and shear obtained from analysis under factored loads below:

$$M_{d_pos} := 1915 \cdot \text{kip} \cdot \text{ft}$$

maximum positive design moment

$$M_{d_neg} := 1187 \cdot \text{kip} \cdot \text{ft}$$

maximum negative design moment

$$V_d := 550 \cdot \text{kip}$$

maximum design shear force

Reinforcement Calculations:Positive Moment Reinforcement, (ignoring compression steel's contribution):

Please change the amount of steel below until you reach design moment:

$$N := 10$$

bar size

$$n := 11$$

number of longitudinal bars

$$n_{\text{layer}} := 2$$

number of horizontal layers of longitudinal reinforcements, it can be entered up to 4 layers for this sheet

$$s_{\text{layer}} := 3 \text{ in}$$

center to center distance between different layer reinforcements

$$\phi_{\text{long}} := \frac{N}{8} \cdot \text{in} = 1.25 \cdot \text{in}$$

diameter of longitudinal reinforcement

$$A_{\text{pos}} := n \cdot \left[\pi \cdot \left(\frac{\phi_{\text{long}}}{2} \right)^2 \right] = 13.499 \cdot \text{in}^2$$

area of positive moment reinforcement

$$N_{\text{stir}} := 5$$

rebar size for stirrup, change if not enough for shear, shear calculations are below flexure calculations

$$\phi_{\text{stir}} := \frac{N_{\text{stir}}}{8} \cdot \text{in} = 0.625 \cdot \text{in}$$

diameter of stirrups

$$s_{\text{long}} := \frac{b - 2 \cdot \text{cover} - 2 \cdot \phi_{\text{stir}} - \phi_{\text{long}}}{n - 1} = 3.45 \cdot \text{in}$$

spacing between longitudinal reinforcement (center to center)

$$d_{e1} := h - \text{cover} - \phi_{\text{stir}} - \frac{\phi_{\text{long}}}{2} = 38.25 \cdot \text{in}$$

effective depth of the first line of longitudinal reinforcement

$$d_{e2} := d_{e1} - s_{\text{layer}} = 35.25 \cdot \text{in}$$

for second layer

$$d_{e3} := d_{e2} - s_{\text{layer}} = 32.25 \cdot \text{in}$$

for third layer

$$d_{e4} := d_{e3} - s_{\text{layer}} = 29.25 \cdot \text{in}$$

for fourth layer

$$d_e := \begin{cases} d_{e1} & \text{if } n_{\text{layer}} = 1 \\ \frac{d_{e1} + d_{e2}}{2} & \text{if } n_{\text{layer}} = 2 \\ \frac{d_{e1} + d_{e2} + d_{e3}}{3} & \text{if } n_{\text{layer}} = 3 \\ \frac{d_{e1} + d_{e2} + d_{e3} + d_{e4}}{4} & \text{if } n_{\text{layer}} = 4 \end{cases} = 36.75 \cdot \text{in}$$

effective depth

$$c := \frac{A_{\text{pos}} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 6.673 \cdot \text{in}$$

neutral axis depth from the top

$$\epsilon_{s1} := 0.003 \cdot \frac{d_{e1} - c}{c} = 0.014$$

strain in tension reinforcement

Check if the section is tension controlled:

section_is := $\begin{cases} \text{"tension controlled"} & \text{if } \epsilon_{s1} \geq 0.005 \\ \text{"compression controlled"} & \text{otherwise} \end{cases}$

if section is compression controlled,
either make it tension controlled or
use $\phi_c = 0.75$ for resistance factor

section_is = "tension controlled"

$$a := \beta_1 \cdot c = 5.672 \cdot \text{in}$$

$$M_n := A_{\text{pos}} \cdot f_y \cdot \left(d_e - \frac{a}{2} \right) = 2.289 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

$$M_r := \phi \cdot M_n = 2.06 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

factored moment capacity

moment_capacity := $\begin{cases} \text{"is enough"} & \text{if } M_{d_pos} < M_r \\ \text{"is not enough"} & \text{otherwise} \end{cases}$

check for moment capacity

moment_capacity = "is enough"

Control of Cracking by Distribution of Reinforcement, LRFD 5.7.3.4:

$$s \leq \frac{700 \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

in which:

$$\beta_s := 1 + \frac{d_c}{0.7 \cdot (h - d_c)}$$

where:

γ_e = exposure factor
= 1 for Class 1 exposure
condition
= 0.75 for Class 2 exposure
condition
(use Class 2 if the element is
exposed to water)

d_c = thickness of concrete cover
from tension fiber to center of
closest reinforcement (in)

f_{ss} = tensile stress in steel at the
service limit state (ksi)

h = overall thickness or depth of
component (in)

$$\gamma_e := 0.75$$

$$d_c := \text{cover} + \phi_{\text{stir}} + \frac{\phi_{\text{long}}}{2} = 3.75 \cdot \text{in}$$

$$\beta_s := 1 + \frac{d_c}{0.7 \cdot (h - d_c)} = 1.14$$

$$M_{\text{service}} := \frac{M_{d_{\text{pos}}}}{1.5} = 1.277 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

assuming that the service moment
is design moment over a factor of
1.5

ignore compression steel!

$$n_{\text{modular}} := 8$$

modular ratio

$$x := 3 \text{ in}$$

initial guess for neutral axis depth

$$x := \text{root} \left[b \cdot x \cdot \frac{x}{2} - A_{\text{pos}} \cdot n_{\text{modular}} \cdot (d_e - x), x \right]$$

$$x = 11.414 \cdot \text{in}$$

$$I_{\text{cr}} := \frac{1}{3} \cdot b \cdot x^3 + A_{\text{pos}} \cdot n_{\text{modular}} \cdot (d_e - x)^2 = 9.014 \times 10^4 \cdot \text{in}^4$$

$$f_{ss} := \frac{M_{\text{service}} \cdot (d_e - x)}{I_{\text{cr}}} \cdot n_{\text{modular}} = 34.448 \cdot \text{ksi}$$

$$s_{\text{max}} := \frac{700 \gamma_e}{\beta_s \cdot \frac{f_{ss}}{\text{ksi}}} \text{in} - 2 \cdot d_c = 5.868 \cdot \text{in}$$

$$s_{\max 1} := \text{floor}\left(\frac{s_{\max}}{\text{in}}\right) \text{ in} = 5 \cdot \text{in}$$

Minimum and maximum spacing of reinforcement shall also comply with the provisions of Articles 5.10.3.1 and 5.10.3.2 respectively.

Which are:

$$s_{\max 2} := \min(1.5 \cdot h, 18 \cdot \text{in})$$

$$s_{\max 2} = 18 \cdot \text{in}$$

s can not be greater than 1.5 times the thickness of the member or 18 in.

$$s_{\max} := \min(s_{\max 1}, s_{\max 2})$$

$$s_{\max} = 5 \cdot \text{in}$$

$$\text{spacing_is} := \begin{cases} \text{"ok"} & \text{if } s_{\text{long}} \leq s_{\max} \\ \text{"not ok"} & \text{otherwise} \end{cases}$$

$$\text{spacing_is} = \text{"ok"}$$

Negative Moment Reinforcement, (ignoring compression steel's contribution):

Please change the amount of steel below until you reach design moment:

$$N := 9$$

bar size

$$n := 8$$

number of longitudinal bars

$$n_{\text{layer}} := 1$$

number of horizontal layers of longitudinal reinforcements, it can be entered up to 4 layers for this sheet

$$s_{\text{layer}} := 3 \text{ in}$$

center to center distance between different layer reinforcements

$$\phi_{\text{long}} := \frac{N}{8} \cdot \text{in} = 1.125 \cdot \text{in}$$

diameter of longitudinal reinforcement

$$A_{\text{neg}} := n \cdot \left[\pi \cdot \left(\frac{\phi_{\text{long}}}{2} \right)^2 \right] = 7.952 \cdot \text{in}^2$$

area of positive moment
reinforcement

$$s_{\text{long}} := \frac{b - 2 \cdot \text{cover} - 2 \cdot \phi_{\text{stir}} - \phi_{\text{long}}}{n - 1} = 4.946 \cdot \text{in}$$

spacing between longitudinal
reinforcement (center to center)

$$d_{e1} := h - \text{cover} - \phi_{\text{stir}} - \frac{\phi_{\text{long}}}{2} = 38.313 \cdot \text{in}$$

effective depth of the first line of
longitudinal reinforcement

$$d_{e2} := d_{e1} - s_{\text{layer}} = 35.313 \cdot \text{in}$$

for second layer

$$d_{e3} := d_{e2} - s_{\text{layer}} = 32.313 \cdot \text{in}$$

for third layer

$$d_{e4} := d_{e3} - s_{\text{layer}} = 29.312 \cdot \text{in}$$

for fourth layer

$$d_e := \begin{cases} d_{e1} & \text{if } n_{\text{layer}} = 1 \\ \frac{d_{e1} + d_{e2}}{2} & \text{if } n_{\text{layer}} = 2 \\ \frac{d_{e1} + d_{e2} + d_{e3}}{3} & \text{if } n_{\text{layer}} = 3 \\ \frac{d_{e1} + d_{e2} + d_{e3} + d_{e4}}{4} & \text{if } n_{\text{layer}} = 4 \end{cases} = 38.313 \cdot \text{in}$$

effective depth

$$c := \frac{A_{\text{neg}} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 3.931 \cdot \text{in}$$

neutral axis depth from the top

$$\epsilon_{s1} := 0.003 \cdot \frac{d_{e1} - c}{c} = 0.026$$

strain in tension reinforcement

Check if the section is tension controlled:

$$\text{section_is} := \begin{cases} \text{"tension controlled"} & \text{if } \epsilon_{s1} \geq 0.005 \\ \text{"compression controlled"} & \text{otherwise} \end{cases}$$

if section is compression controlled,
either make it tension controlled or
use $\phi_c = 0.75$ for resistance factor

$$\text{section_is} = \text{"tension controlled"}$$

$$a := \beta_1 \cdot c = 3.341 \cdot \text{in}$$

$$M_n := A_{\text{neg}} \cdot f_y \cdot \left(d_e - \frac{a}{2} \right) = 1.457 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

$$M_r := \phi \cdot M_n = 1.311 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

factored moment capacity

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{d_neg} < M_r \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

check for moment capacity

$$\text{moment_capacity} = \text{"is enough"}$$

Control of Cracking by Distribution of Reinforcement, LRFD 5.7.3.4:

$$s \leq \frac{700 \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

in which:

$$\beta_s := 1 + \frac{d_c}{0.7 \cdot (h - d_c)}$$

where:

γ_e = exposure factor
 = 1 for Class 1 exposure condition
 = 0.75 for Class 2 exposure condition
 (use Class 2 if the element is exposed to water)

d_c = thickness of concrete cover from tension fiber to center of closest reinforcement (in)

f_{ss} = tensile stress in steel at the service limit state (ksi)

h = overall thickness or depth of component (in)

$$\gamma_e := 0.75$$

$$d_c := \text{cover} + \phi_{\text{stir}} + \frac{\phi_{\text{long}}}{2} = 3.688 \cdot \text{in}$$

$$\beta_s := 1 + \frac{d_c}{0.7 \cdot (h - d_c)} = 1.137$$

$$M_{\text{service}} := \frac{M_{d_{\text{neg}}}}{1.5} = 791.333 \cdot \text{kip} \cdot \text{ft}$$

assuming that the service moment is design moment over a factor of 1.5

ignore compression steel!

$$x := 3 \text{ in}$$

initial guess for neutral axis depth

$$x := \text{root} \left[b \cdot x \cdot \frac{x}{2} - A_{\text{neg}} \cdot n_{\text{modular}} \cdot (d_e - x), x \right]$$

$$x = 9.365 \cdot \text{in}$$

$$I_{\text{cr}} := \frac{1}{3} \cdot b \cdot x^3 + A_{\text{neg}} \cdot n_{\text{modular}} \cdot (d_e - x)^2 = 6.481 \times 10^4 \cdot \text{in}^4$$

$$f_{\text{ss}} := \frac{M_{\text{service}} \cdot (d_e - x)}{I_{\text{cr}}} \cdot n_{\text{modular}} = 33.933 \cdot \text{ksi}$$

$$s_{\text{max}} := \frac{700 \gamma_e}{\beta_s \cdot \frac{f_{\text{ss}}}{\text{ksi}}} \text{in} - 2 \cdot d_c = 6.226 \cdot \text{in}$$

$$s_{\text{max1}} := \text{floor} \left(\frac{s_{\text{max}}}{\text{in}} \right) \text{in} = 6 \cdot \text{in}$$

Minimum and maximum spacing of reinforcement shall also comply with the provisions of Articles 5.10.3.1 and 5.10.3.2 respectively.

Which are:

$$s_{\max 2} := \min(1.5 \cdot h, 18 \cdot \text{in})$$

$$s_{\max 2} = 18 \cdot \text{in}$$

$$s_{\max} := \min(s_{\max 1}, s_{\max 2})$$

$$s_{\max} = 6 \cdot \text{in}$$

s can not be greater than 1.5 times the thickness of the member or 18 in.

$$\text{spacing_is} := \begin{cases} \text{"ok"} & \text{if } s_{\text{long}} \leq s_{\max} \\ \text{"not ok"} & \text{otherwise} \end{cases}$$

$$\text{spacing_is} = \text{"ok"}$$

Shear Check, LRFD 5.8.3:

We need transverse reinforcement if:

$$V_u > 0.5 * \phi * V_c$$

where:

V_u = factored shear force

V_c = nominal shear resistance of the concrete

ϕ = resistance factor specified in 5.5.4.2

Shear strength of concrete, LRFD 5.8.3.3:

$$V_c = 0.0316 * \beta * (f_c)^{0.5} * b_v * d_v$$

$$b_v := b = 42 \cdot \text{in}$$

effective web width, LRFD 5.8.2.9

$$d_v := \max(0.72 \cdot h, 0.9 \cdot d_e) = 34.481 \cdot \text{in}$$

effective depth, LRFD 5.8.2.9

Since we will put at least minimum transverse reinforcement, we can use $\beta = 2$, LRFD 5.8.3.4.1

$$\beta := 2$$

$$V_c := \left(0.0316 \cdot \beta \cdot \sqrt{\frac{f_c}{\text{ksi}}} \cdot \frac{b_v}{\text{in}} \cdot \frac{d_v}{\text{in}} \right) \cdot \text{kip} = 183.054 \cdot \text{kip}$$

Now check if we need transverse reinforcement:

$$\text{transverse_reinforcement} := \begin{cases} \text{"is not needed"} & \text{if } V_d \leq 0.5 \cdot \phi \cdot V_c \\ \text{"is needed"} & \text{otherwise} \end{cases}$$

$$\text{transverse_reinforcement} = \text{"is needed"}$$

We will put minimum amount of transverse reinforcement and check the strength, LRFD 5.8.2.5:

Shear stress on concrete, LRFD 5.8.2.9:

$$v_u := \frac{V_d}{\phi \cdot b_v \cdot d_v} = 0.422 \cdot \text{ksi}$$

Maximum spacing of transverse reinforcement, LRFD 5.8.2.7:

$$s_{\max} := \begin{cases} \min(0.8 \cdot d_v, 24\text{in}) & \text{if } v_u < 0.125 \cdot f_c \\ \min(0.4 \cdot d_v, 12\text{in}) & \text{if } v_u \geq 0.125 \cdot f_c \end{cases} = 24 \cdot \text{in}$$

$$s_{\max} := \text{floor}\left(\frac{s_{\max}}{\text{in}}\right) \cdot \text{in} = 23 \cdot \text{in}$$

Please enter the desired transverse reinforcement number and spacing below:

$$s := 5\text{in}$$

should be lesser than the maximum value that is found above

$$n_{\text{leg}} := 4$$

number of stirrup legs put in one cross section

$$A_v := n_{\text{leg}} \cdot \pi \cdot \left(\frac{N_{\text{stir}}}{8} \cdot \text{in} \cdot \frac{1}{2} \right)^2 = 1.227 \cdot \text{in}^2$$

$$A_{v_min} := 0.0316 \cdot \sqrt{\frac{f_c}{\text{ksi}}} \cdot \frac{b_v \cdot s}{\frac{f_y}{\text{ksi}}} = 0.221 \cdot \text{in}^2$$

$$\text{transverse_reinforcement} := \begin{cases} \text{"is enough for minimum amount"} & \text{if } \frac{A_v}{A_{v_min}} \geq 1 \\ \text{"is not enough for minimum amount"} & \text{otherwise} \end{cases}$$

$$\text{transverse_reinforcement} = \text{"is enough for minimum amount"}$$

Strength contribution of transverse reinforcement, LRFD 5.8.3.3:

Since the transverse reinforcement is inclined with 90 degrees to longitudinal reinforcement, equation reduces to:

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s}$$

where:

θ = angle of inclination of diagonal compressive stresses as in LRFD 5.8.3.4

Since we have at least minimum transverse reinforcement, we can use $\theta = 45$ degree

$$\theta := 45\text{deg}$$

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s} = 507.778 \cdot \text{kip}$$

$$V_n := \min(V_c + V_s, 0.25 \cdot f_c \cdot b_v \cdot d_v)$$

nominal shear strength of the section, LRFD 5.8.3.3

$$V_n = 690.832 \cdot \text{kip}$$

$$\text{shear_strength} := \begin{cases} \text{"is enough"} & \text{if } V_d \leq \phi \cdot V_n \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{shear_strength} = \text{"is enough"}$$

Shrinkage and Temperature Reinforcement, LRFD 5.10.8:

The area of S&T reinforcement per foot, on each face and in each direction shall not be less than:

$$A_{ST} := \left[\frac{1.3 \cdot \frac{b}{\text{in}} \cdot \frac{h}{\text{in}}}{2 \cdot \frac{(b+h)}{\text{in}} \cdot \frac{f_y}{\text{ksi}}} \right] \frac{\text{in}^2}{\text{ft}} = 0.228 \cdot \frac{\text{in}^2}{\text{ft}}$$

Furthermore, A_s should satisfy following conditions:

$$A_{ST} := \begin{cases} 0.11 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST} \leq 0.11 \frac{\text{in}^2}{\text{ft}} = 0.228 \cdot \frac{\text{in}^2}{\text{ft}} \\ 0.6 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST} \geq 0.6 \frac{\text{in}^2}{\text{ft}} \\ A_{ST} & \text{otherwise} \end{cases} \quad \text{defines the limits for reinforcement}$$

$$A_{ST} := (2 \cdot b + 2 \cdot h) \cdot A_{ST} = 3.185 \cdot \text{in}^2 \quad \text{total S\&T reinforcement required for the cross section}$$

Now check if the provided negative and positive reinforcement is enough for S&T reinforcement:

$$\text{check} := \begin{cases} \text{"ok"} & \text{if } (A_{\text{neg}} + A_{\text{pos}}) \geq A_{ST} \\ \text{"add reinforcement"} & \text{otherwise} \end{cases}$$

$$\text{check} = \text{"ok"}$$

Maximum spacing can not exceed 3 times the wall thickness or 12in:

$$s_{\max 1} := \min(3 \cdot b, 12\text{in}) = 12 \cdot \text{in}$$

Appendix H: Design of Full Height Socketed Abutment

Abutment Design Sheet: outside socket region

Users Guide:

- What does program do?

This design sheet is intended to show how to determine the amount and the location of the reinforcements other than socket region reinforcement.

- What input is required?

Shape of the abutment module:	H	: Height of the module
	L	: Length of the module
	t	: Thickness of the wall
Material properties:	f_c	: Strength of the concrete
	f_y	: Yield strength of steel
	E_s	: Modulus of elasticity of steel
Loads:	M	: Factored design moment acting above per pile
	N	: Factored axial design force acting above per pile
	V	: Factored design shear force acting above per pile
	p_{LS}	: Unfactored maximum pressure at the bottom of abutment module due to live load surcharge
	p_{EH}	: Unfactored maximum pressure at the bottom of abutment module due to earth pressure
Load factors:	Φ_{EH}	: Dead load factor (earth pressure)
	Φ_{LL}	: Live load factor
Resistance factors:	Φ_F	: Resistance factor for flexure
	Φ_V	: Resistance factor for shear
	Φ_N	: Resistance factor for normal force
Design parameters:	N	: Bar number
	s	: Bar spacing
	γ_e	: Exposure factor
	$c_{service}$	: Trial value of depth of neutral axis to calculate steel stress under service loads

- What is the output?

The checks which verify whether the entered amount of reinforcement is enough or not are the outputs of this sheet. When all the checks are okay, we make sure that our design is completed.

Reference Manual:

- How calculations are done?

All the calculations are done according to the methods explained in AASHTO LRFD specification.

Procedures followed are also described in detail while calculations are done.

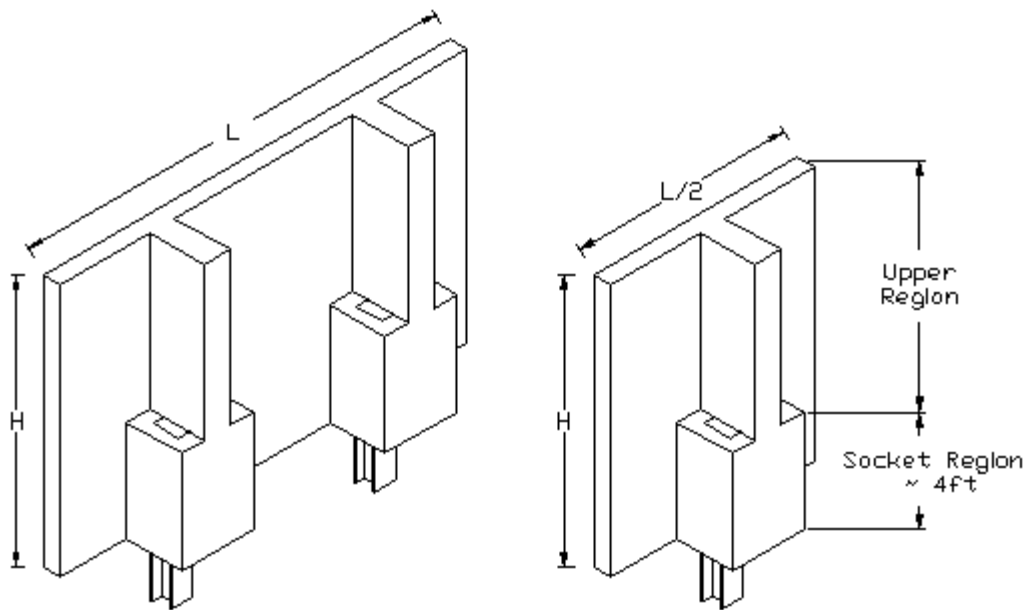
Shrinkage and temperature reinforcement is calculated for wall for each direction (LRFD 4.10.8) and then safety of wall in flexure and shear has been checked as cantilever and fixed-fixed wall parts. There is also restriction over spacing of reinforcement close to tension face dictated by crack control (LRFD 5.7.3.4). Shear capacity of the elements have been calculated according to LRFD 5.8. For flexure of column region, only the minimum limits dictated by LRFD have been given.. Shear check of column is also implemented (LRFD 5.8).

- Example design problem is shown below with sample input/output.

All inputs are in red
colour

All checks are in yellow colour

This sheet is created to design the type #3 abutment system. Design will be done in two stages. Firstly, the body above socket region, which is from 4ft to the top of abutment, will be designed and then socket region will be analyzed and designed separately. Moreover, design will be carried out for half of the system which includes only one pile and the other half will have same design symmetrically (see Figure 1).



Isometric View
Figure 1

Design Constants:

$$H := 10\text{ft}$$

height of the abutment module.

$$L := 16\text{ft}$$

width of the abutment module.

$$S_{\text{pile}} := \frac{L}{2}$$

pile spacing.

$$t := 8\text{in}$$

thickness of the thin wall

Material Properties:**Concrete:**

$$f_c := 4\text{ksi}$$

ultimate design strength of concrete

$$f_r := 0.37 \sqrt{\frac{f_c}{\text{ksi}}} \text{ ksi} = 0.74 \cdot \text{ksi}$$

modulus of rupture of concrete when it is used for minimum reinforcement calculations, (LRFD 5.4.2.6)

$$\beta_1 := \max \left[0.85 - 0.05 \cdot \frac{(f_c - 4\text{ksi})}{\text{ksi}}, 0.65 \right] = 0.85$$

neutral axis multiplier, (LRFD 5.7.2.2)

Steel:

$$f_y := 60\text{ksi}$$

yield strength of reinforcing bars.

$$E_s := 29000\text{ksi}$$

modulus of elasticity of reinforcing bars, (LRFD 5.4.3.2).

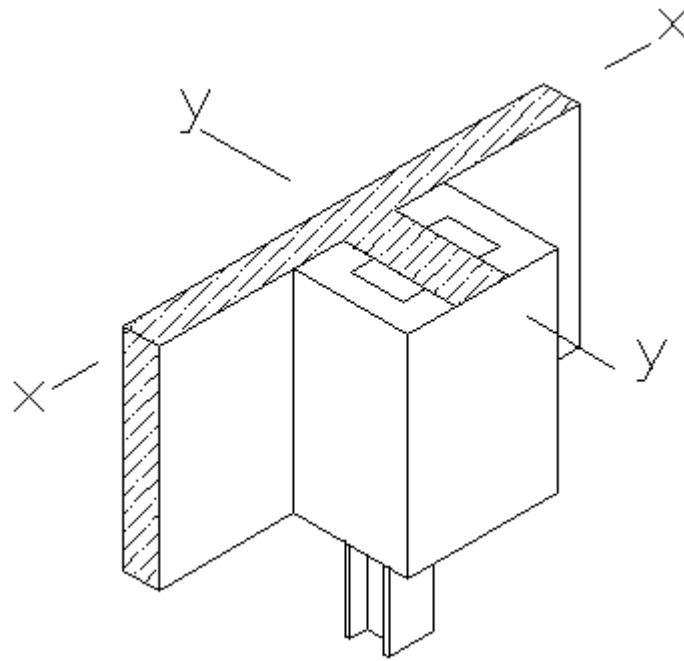
$$\epsilon_y := \frac{f_y}{E_s} = 2.069 \times 10^{-3}$$

yield strain of reinforcing bars.

Design of Upper Region:

Flexural Design:

Hatched cross section is the one that we will be designing for this part.



**Isometric View of Socket Region
Figure 2**

Moreover
,

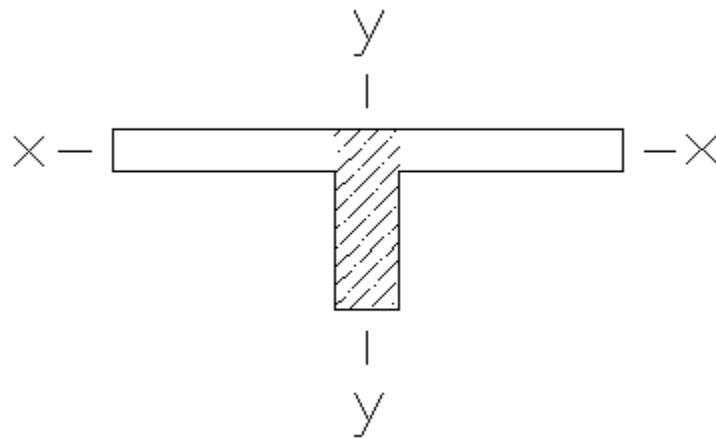


Figure 3

- Hatched region will be designed as column
- Shrinkage and temperature reinforcement will be put in thin wall in both directions and be checked if it has the required capacity for moment due to earth pressure

Please insert the factored forces acting right above the pile top:

$$M := 125 \text{ kip}\cdot\text{ft}$$

factored forces acting on the members from structural analysis

$$N := 150 \text{ kip}$$

$$V := 37 \text{ kip}$$

$$\phi_F := 0.9$$

conventional construction resistance factors for concrete design from LRFD 5.5.4.2.1

$$\phi_V := 0.9$$

$$\phi_N := 0.75$$

resistance factor assuming the section is compression controlled (it is 0.9 for tensioned controlled section). This will be changed based on the section tension reinforcement strain later in design.

Strain in tension reinforcement is 0.002 or less
Strain in tension reinforcement is 0.005 or higher
In between, make linear interpolation

= compression controlled
= tension controlled

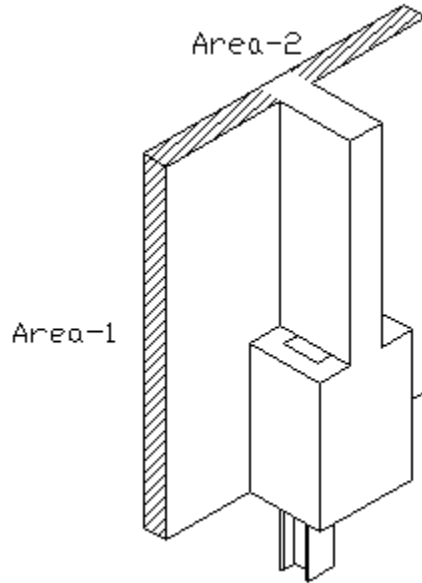
$$M_d := \frac{M}{\phi_F} = 138.889 \cdot \text{kip}\cdot\text{ft}$$

$$N_d := \frac{N}{\phi_N} = 200 \cdot \text{kip}$$

$$V_d := \frac{V}{\phi_V} = 41.111 \cdot \text{kip}$$

Shrinkage and Temperature Reinforcement, LRFD 5.10.8:

The area of S&T reinforcement per foot, on each face and in each direction shall not be less than:



Area 1 is the lateral cross section of the abutment module, whereas Area 2 is the vertical cross section of module.

Figure 4

For Area-1:

$$b := t = 8 \cdot \text{in}$$

$$h := H = 120 \cdot \text{in}$$

$$A_{ST1} := \left[\frac{1.3 \cdot \frac{b}{\text{in}} \cdot \frac{h}{\text{in}}}{2 \cdot \frac{(b+h)}{\text{in}} \cdot \frac{f_y}{\text{ksi}}} \right] \frac{\text{in}^2}{\text{ft}} = 0.081 \cdot \frac{\text{in}^2}{\text{ft}}$$

Furthermore, A_s should satisfy following conditions:

$$A_{ST1} := \begin{cases} 0.11 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST1} \leq 0.11 \frac{\text{in}^2}{\text{ft}} \\ 0.6 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST1} \geq 0.6 \frac{\text{in}^2}{\text{ft}} \\ A_{ST1} & \text{otherwise} \end{cases} \quad A_{ST1} = 0.11 \cdot \frac{\text{in}^2}{\text{ft}}$$

Maximum spacing can not exceed 3 times the wall thickness or 18in:

$$s_{\max 1} := \min(3 \cdot t, 18 \text{ in}) = 18 \cdot \text{in}$$

Please enter below the rebar number and spacing to get the required area of steel:

$$N := 3$$

$$s := 12 \text{ in}$$

$$A := 2 \left[\frac{12 \text{ in}}{s} \cdot \pi \cdot \left(\frac{N}{16} \text{ in} \right)^2 \right] = 0.221 \cdot \text{in}^2$$

Since we have two faces, we multiplied the reinforcement by two

Therefore, 2#3 @ 12in for this direction of wall is enough.

For Area-2:

$$b := t = 8 \cdot \text{in}$$

$$h := \frac{L}{2} = 96 \cdot \text{in}$$

$$A_{ST1} := \left[\frac{1.3 \cdot \frac{b}{\text{in}} \cdot \frac{h}{\text{in}}}{2 \cdot \frac{(b + h)}{\text{in}} \cdot \frac{f_y}{\text{ksi}}} \right] \frac{\text{in}^2}{\text{ft}} = 0.08 \cdot \frac{\text{in}^2}{\text{ft}}$$

Furthermore, A_s should satisfy following conditions:

$$A_{ST1} := \begin{cases} 0.11 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST1} \leq 0.11 \frac{\text{in}^2}{\text{ft}} \\ 0.6 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST1} \geq 0.6 \frac{\text{in}^2}{\text{ft}} \\ A_{ST1} & \text{otherwise} \end{cases} \quad A_{ST1} = 0.11 \cdot \frac{\text{in}^2}{\text{ft}}$$

Maximum spacing can not exceed 3 times the wall thickness or 18in:

$$s_{\max 1} := \min(3 \cdot t, 18\text{in}) = 18 \cdot \text{in}$$

Please enter below the rebar number and spacing to get the required area of steel:

$$N := 3$$

$$s := 12\text{in}$$

$$A := 2 \left[\frac{12\text{in}}{s} \cdot \pi \cdot \left(\frac{N}{16} \text{in} \right)^2 \right] = 0.221 \cdot \text{in}^2$$

Since we have two faces, we multiplied the reinforcement by two

Therefore, 2#3 @ 12in for this direction of wall is enough.

Finally, the required area of steel for shrinkage and temperature reinforcement is 2#3 @ 12in for both direction of the wall.

Now, we should check to verify that this amount of reinforcement in the wall is enough for flexure due to earth pressure.

Flexural capacity of the wall will be checked with the maximum pressure assuming it is being applied throughout the wall and clear dimensions will be used.

We have two regions to check the flexural capacity, one is the either side of one module which is acting as cantilever and the other is the middle region which is acting as a beam having two fixed end conditions.

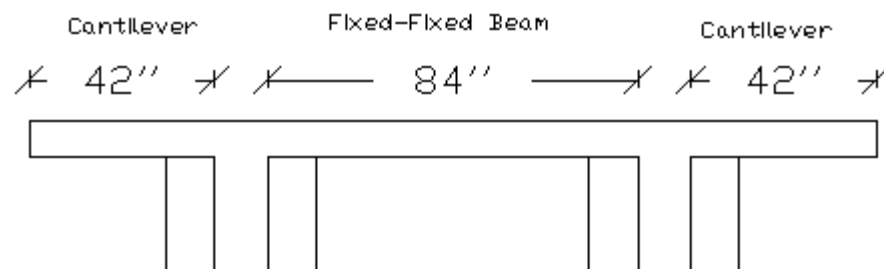


Figure 5

Please enter the unfactored pressure at the bottom of the abutment module (maximum pressure) due to both live load surcharge and lateral earth pressure.

$$p_{LS} := 0.786\text{psi}$$

$$p_{EH} := 2.431\text{psi}$$

$$\phi_{LL} := 1.75$$

live load factor

$$\phi_{EH} := 1.5$$

dead load factor

$$p := \phi_{EH} \cdot p_{EH} + \phi_{LL} \cdot p_{LS} = 5.022 \cdot \text{psi}$$

maximum factored pressure at the bottom of the abutment module

Cantilever wall:

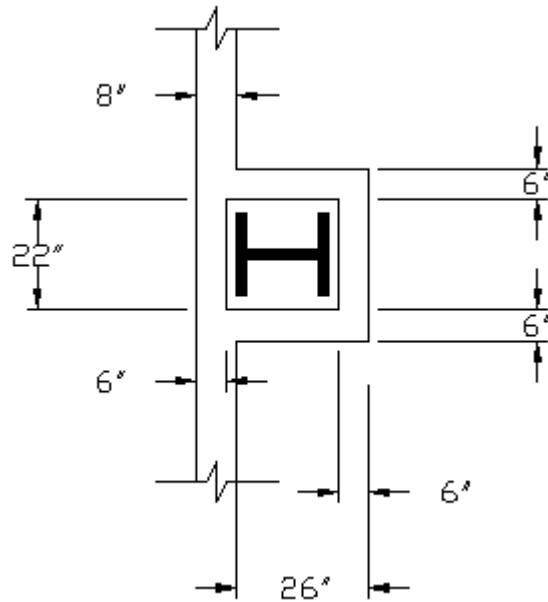
$$M_{\text{cantilever}} := \frac{\left[\left(\frac{L}{4} - \frac{12\text{in}}{2} \right) \cdot H \cdot p \right] \cdot \left[\left(\frac{L}{4} - \frac{12\text{in}}{2} \right) \cdot \frac{1}{2} \right]}{\phi_F} = 49.216 \cdot \text{kip} \cdot \text{ft}$$

$$V_{\text{cantilever}} := \frac{p \cdot \left(\frac{L}{4} - \frac{12\text{in}}{2} \right) \cdot H}{\phi_V} = 28.123 \cdot \text{kip}$$

Flexural check:

Capacity of the wall with S&T reinforcement is:

Pile socket is 2 in inside to the wall, therefore effective thickness of the wall is actually 6 in.



Cross Section View of Bottom of Abutment
Figure 6

$$\text{cover} := 1.5\text{in}$$

clear cover for the reinforcements

$$d := 6\text{in} - \left(\frac{N}{8} \text{in} \cdot \frac{1}{2} \right) = 5.812 \cdot \text{in}$$

$$N_s := \text{floor}\left(\frac{H - 2 \cdot \text{cover}}{s}\right) + 1 = 10$$

number of tension steel (bottom steel)

$$N_{st} := \text{floor}\left(\frac{H - 2 \cdot \text{cover}}{s}\right) + 1 = 10$$

number of compression steel (top steel)

$$A_s := N_s \cdot \pi \cdot \left(\frac{N}{8} \text{ in} \cdot \frac{1}{2}\right)^2 = 1.104 \cdot \text{in}^2$$

bottom steel area (tension steel)

$$A_{st} := N_{st} \cdot \pi \cdot \left(\frac{N}{8} \text{ in} \cdot \frac{1}{2}\right)^2 = 1.104 \cdot \text{in}^2$$

top steel area (compression steel)

$$c_{st} := \text{cover} + \frac{N}{8} \text{ in} \cdot \frac{1}{2} = 1.688 \cdot \text{in}$$

top steel distance from compression fiber

Assume that compression steel is yielded and in tension:

set sum of the forces to zero and get the neutral axis depth:

$$c := 1 \text{ in}$$

initial guess

$$c := \text{root}\left[\left(0.85 f_c \cdot H \cdot \beta_1 \cdot c - A_s \cdot f_y - A_{st} \cdot f_y\right), c\right] = 0.382 \cdot \text{in}$$

Now, we should check if the compression steel is in tension and it has yielded as we have assumed:

$$\text{check} := \begin{cases} \text{"Assumption is true"} & \text{if } \left(c < c_{st} - \frac{N}{16} \text{ in}\right) \wedge \left[\frac{(c_{st} - c) \cdot 0.003}{c} > \epsilon_y\right] \\ \text{"Assumption is not true, revise it"} & \text{otherwise} \end{cases}$$

$$\text{check} = \text{"Assumption is true"}$$

$$a := \beta_1 \cdot c = 0.325 \cdot \text{in}$$

$$T_1 := A_s \cdot f_y = 66.268 \cdot \text{kip}$$

force in bottom steel

$$T_2 := A_{st} \cdot f_y = 66.268 \cdot \text{kip}$$

force in top steel

$$M_{\text{capacity}} := T_1 \cdot \left(d - \frac{a}{2} \right) + T_2 \cdot \left(c_{\text{st}} - \frac{a}{2} \right) = 39.624 \cdot \text{kip} \cdot \text{ft}$$

$$\text{capacity} := \begin{cases} \text{"is enough"} & \text{if } \frac{M_{\text{capacity}}}{M_{\text{cantilever}}} \geq 1 \\ \text{"is not enough, change design values "} & \text{otherwise} \end{cases}$$

$$\text{capacity} = \text{"is not enough, change design values "}$$

Since we have used the maximum pressure at the bottom for all over the wall, we got a high moment. Actually, our capacity is enough!

Actual moment for the wall is:

$$p_1 := \phi_{LL} \cdot p_{LS} = 1.376 \cdot \text{psi}$$

pressure at the top

$$p_2 := \phi_{EH} \cdot p_{EH} + \phi_{LL} \cdot p_{LS} = 5.022 \cdot \text{psi}$$

pressure at the bottom

$$Q := \frac{p_1 + p_2}{2} \cdot H \cdot \left(\frac{L}{4} - \frac{12\text{in}}{2} \right) = 16.122 \cdot \text{kip}$$

total force due to pressure distribution

$$M_{\text{actual}} := \frac{Q \cdot \left[\left(\frac{L}{4} - \frac{12\text{in}}{2} \right) \cdot \frac{1}{2} \right]}{\phi_F} = 31.348 \cdot \text{kip} \cdot \text{ft}$$

actual total moment that wall should resist

There is also a restriction over spacing of reinforcement close to tension face which states that spacing can not be larger than the value dictated by crack control, LRFD 5.7.3.4:

$$s \leq \frac{700\gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

in which:

$$\beta_s := 1 + \frac{d_c}{0.7 \cdot (h - d_c)}$$

where:

γ_e = exposure factor
 = 1 for Class 1 exposure condition
 = 0.75 for Class 2 exposure condition
 (use Class 2 if the element is exposed to water)

d_c = thickness of concrete cover from tension fiber to center of closest reinforcement (in)

f_{ss} = tensile stress in steel at the service limit state (ksi)

h = overall thickness or depth of component (in)

$$\gamma_e := 1$$

$$d_c := 2\text{in} + \frac{N}{8}\text{in} \cdot \frac{1}{2} = 2.188 \cdot \text{in}$$

$$h := t = 8 \cdot \text{in}$$

$$\beta_s := 1 + \frac{d_c}{0.7 \cdot (h - d_c)} = 1.538$$

$$M_{\text{service}} := \frac{[P_{\text{LS}} + (P_{\text{EH}} + P_{\text{LS}})]}{2} \cdot H \cdot \frac{\left(\frac{L}{4} - \frac{12\text{in}}{2}\right)^2}{2} = 17.653 \cdot \text{kip} \cdot \text{ft}$$

We will try different neutral axis depths to get the moment of the section same as service moment that we have calculated above:

ignore compression steel!

$$n := 8$$

modular ratio

$$x := 3\text{in}$$

initial guess for neutral axis depth

$$x := \text{root}\left[H \cdot x \cdot \frac{x}{2} - A_s \cdot n \cdot (d - x), x\right]$$

$$x = 0.854 \cdot \text{in}$$

$$I_{\text{cr}} := \frac{1}{3} \cdot H \cdot x^3 + A_s \cdot n \cdot (d - x)^2 = 242.155 \cdot \text{in}^4$$

$$f_{\text{ss}} := \frac{M_{\text{service}} \cdot (d - x)}{I_{\text{cr}}} \cdot n = 34.698 \cdot \text{ksi}$$

$$s_{\text{max}} := \frac{700 \gamma_e}{\beta_s \cdot \frac{f_{\text{ss}}}{\text{ksi}}} \text{in} - 2 \cdot d_c = 8.745 \cdot \text{in}$$

$$s_{\text{max}} := \text{floor}\left(\frac{s_{\text{max}}}{\text{in}}\right) \text{in} = 8 \cdot \text{in}$$

Therefore, the spacing will be 8in which will further increase the flexural capacity!

Shear check, LRFD 5.8:

We need transverse reinforcement if:

$$V_u > 0.5 \cdot \phi \cdot V_c$$

where:

V_u = factored shear force

V_c = nominal shear resistance of the concrete

ϕ = resistance factor specified in 5.5.4.2

Shear strength of concrete, LRFD 5.8.3.3:

$$V_c = 0.0316 \cdot \beta \cdot (f_c)^{0.5} \cdot b_v \cdot d_v$$

$$b_v := H = 10 \cdot \text{ft}$$

effective web width, LRFD 5.8.2.9

$$d_v := \max(0.72 \cdot t, 0.9 \cdot d) = 5.76 \cdot \text{in}$$

effective depth, LRFD 5.8.2.9

since the depth of our wall is less than 16 in, we can use $\beta = 2$, LRFD 5.8.3.4.1

$$\beta := 2$$

$$V_c := \left(0.0316 \cdot \beta \cdot \sqrt{\frac{f_c}{\text{ksi}}} \cdot \frac{b_v}{\text{in}} \cdot \frac{d_v}{\text{in}} \right) \cdot \text{kip} = 87.368 \cdot \text{kip}$$

Now check if we need transverse reinforcement:

$$\text{transverse_reinforcement} := \begin{cases} \text{"is not needed"} & \text{if } V_{\text{cantilever}} \leq 0.5 \cdot \phi_V \cdot V_c \\ \text{"is needed"} & \text{otherwise} \end{cases}$$

$$\text{transverse_reinforcement} = \text{"is not needed"}$$



In case we need transverse reinforcement:

Shear stress on concrete, LRFD 5.8.2.9:

$$V_u := V_{\text{cantilever}} \cdot \phi_V = 25.311 \cdot \text{kip}$$

$$v_u := \frac{V_u}{\phi_V \cdot b_V \cdot d_V} = 0.041 \cdot \text{ksi}$$

Maximum spacing of transverse reinforcement, LRFD 5.8.2.7:

$$s_{\max} := \begin{cases} \min(0.8 \cdot d_V, 24\text{in}) & \text{if } v_u < 0.125 \cdot f_c \\ \min(0.4 \cdot d_V, 12\text{in}) & \text{if } v_u \geq 0.125 \cdot f_c \end{cases}$$

$$s_{\max} := \text{floor}\left(\frac{s_{\max}}{\text{in}}\right) \text{in} = 4 \cdot \text{in}$$

Minimum transverse reinforcement, LRFD 5.8.2.5:

We should satisfy at least this amount of transverse reinforcement:

$$A_V > 0.0316 \cdot (f_c)^{0.5} \cdot b_V \cdot s / f_y$$

where:

A_V = area of reinforcement within distance s

b_V = web thickness

s = spacing of reinforcement

$$A_V := 0.0316 \sqrt{\frac{f_c}{\text{ksi}}} \cdot \frac{b_V \cdot s_{\max}}{\frac{f_y}{\text{ksi}}} = 0.506 \cdot \text{in}^2$$



Fixed beam:

$$w := p \cdot H = 7.232 \cdot \frac{\text{kip}}{\text{ft}} \quad \text{distributed load over beam}$$

$$l := \frac{L}{2} - 12\text{in} = 84\text{in} \quad \text{clear length of the beam}$$

$$V_{\text{fixed}} := \frac{w \cdot l}{2} = 25.311 \cdot \text{kip} \quad \text{shear force at the support}$$

$$M_{\text{end}} := \frac{\frac{w \cdot l^2}{12}}{\phi_F} = 32.81 \cdot \text{kip} \cdot \text{ft} \quad \text{moment at the ends}$$

$$M_{\text{center}} := \frac{\frac{w \cdot l^2}{24}}{\phi_F} = 16.405 \cdot \text{kip} \cdot \text{ft} \quad \text{moment at the center}$$

Negative moment resistance at the ends are the same as the resistance that we got for cantilever part and for positive moment resistance our d is slightly increasing which will also increase our capacity. Therefore, since those moments are far lower than our capacity, using the same reinforcement all over the wall region is enough and safe.

$$\text{fixed_wall_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{\text{capacity}} \geq M_{\text{end}} \wedge M_{\text{center}} \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{fixed_wall_capacity} = \text{"is enough"}$$

For shear design, since the factored shear is the same as the cantilever part, we do not need any transverse reinforcement. The capacity of concrete is enough to carry it.

Design of column region 12in x 34in:

Limits for reinforcement by LRFD 5.7.4.2

$$A_{s_max} := 0.08 \cdot (12\text{in} \cdot 34\text{in}) = 32.64 \cdot \text{in}^2$$

maximum area of reinforcement

$$A_{s_min} := 0.135 \cdot (12\text{in} \cdot 34\text{in}) \cdot \frac{f_c}{f_y} = 3.672 \cdot \text{in}^2$$

minimum area of reinforcement

Minimum number is #5 for longitudinal reinforcement.

Taking above limits into account, column should be designed conventionally.

Shear check for column, LRFD 5.8:

We need transverse reinforcement if:

$$V_u > 0.5 * \phi * V_c$$

where:

V_u = factored shear force

V_c = nominal shear resistance of the concrete

ϕ = resistance factor specified in 5.5.4.2

Shear strength of concrete, LRFD 5.8.3.3:

$$V_c = 0.0316 * \beta * (f_c)^{0.5} * b_v * d_v$$

$$b_v := 12\text{in}$$

effective web width, LRFD 5.8.2.9

$$d := 34\text{in} - 1.5\text{in} - \frac{6}{8}\text{in} \cdot \frac{1}{2} = 32.125\text{in}$$

$$d_v := \max[(0.72 \cdot 34\text{in}), (0.9 \cdot d)] = 28.912\text{in}$$

effective depth, LRFD 5.8.2.9

since we will have at least minimum amount of transverse reinforcement, we can use $\beta = 2$, LRFD 5.8.3.4.1

$$\beta := 2$$

$$V_c := \left(0.0316 \cdot \beta \cdot \sqrt{\frac{f_c}{\text{ksi}}} \cdot \frac{b_v}{\text{in}} \cdot \frac{d_v}{\text{in}} \right) \cdot \text{kip} = 43.854 \cdot \text{kip}$$

$$V_u := V = 37 \cdot \text{kip}$$

shear_check := $\begin{cases} \text{"put minimum transverse reinf"} & \text{if } 0.5 \cdot \phi_V \cdot V_c \geq V_u \\ \text{"recalculate the strength putting transverse reinf"} & \text{otherwise} \end{cases}$

shear_check = "recalculate the strength putting transverse reinf"

We will put minimum amount of transverse reinforcement into column and check the strength, LRFD 5.8.2.5:

Shear stress on concrete, LRFD 5.8.2.9:

$$v_u := \frac{V_u}{\phi_V \cdot b_v \cdot d_v} = 0.118 \cdot \text{ksi}$$

Maximum spacing of transverse reinforcement, LRFD 5.8.2.7:

$$s_{\max} := \begin{cases} \min(0.8 \cdot d_v, 24\text{in}) & \text{if } v_u < 0.125 \cdot f_c \\ \min(0.4 \cdot d_v, 12\text{in}) & \text{if } v_u \geq 0.125 \cdot f_c \end{cases} = 23.13 \cdot \text{in}$$

$$s_{\max} := \text{floor}\left(\frac{s_{\max}}{\text{in}}\right) \text{in} = 23 \cdot \text{in}$$

Please enter the desired transverse reinforcement number and spacing below:

N := 3

s := 10in

should be lesser than the maximum value that is found above

$$A_v := 2 \cdot \pi \cdot \left(\frac{N}{8} \text{ in} \cdot \frac{1}{2} \right)^2 = 0.221 \cdot \text{in}^2$$

$$A_{v_min} := 0.0316 \cdot \sqrt{\frac{f_c}{\text{ksi}}} \cdot \frac{b_v \cdot s}{\frac{f_y}{\text{ksi}}} = 0.126 \cdot \text{in}^2$$

transverse_reinforcement :=	"is enough for minimum amount" if $\frac{A_v}{A_{v_min}} \geq 1$ "is not enough for minimum amount" otherwise
-----------------------------	--

transverse_reinforcement = "is enough for minimum amount"

Strength contribution of transverse reinforcement, LRFD 5.8.3.3:

Since the transverse reinforcement is inclined with 90 degrees to longitudinal reinforcement, equation reduces to:

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s}$$

where:

θ = angle of inclination of diagonal compressive stresses as in LRFD 5.8.3.4

Since we have at least minimum transverse reinforcement, we can use $\theta = 45$ degree

$$\theta := 45 \text{ deg}$$

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s} = 38.319 \cdot \text{kip}$$

$$V_n := \min(V_c + V_s, 0.25 \cdot f_c \cdot b_v \cdot d_v)$$

nominal shear strength of the section, LRFD 5.8.3.3

$$V_n = 82.174 \cdot \text{kip}$$

shear_strength :=	"is enough" if $V_u \leq \phi_V \cdot V_n$ "is not enough" otherwise
-------------------	--

shear_strength = "is enough"

Appendix I: Tutorial – Abutment Socket Design

TUTORIAL: Socket Region Design Sheet

Users Guide:

- What does program do?

This design sheet shows the design of socket region in which steel HP pile is inserted.

- What input is required?

Dimensions:	l_{emb}	: Embedment length of pile in socket
	e	: Eccentricity of axial force on pile
	y	: Assumed eccentricity for C_1
	y_1	: Assumed eccentricity for C_2
		(see Figure 2 for eccentricities e , y and y_1)
	h	: Depth of steel HP pile
	b_{int}	: Inner dimension of the socket
	h_w	: Thickness of the socket wall
Material properties:	f_c	: Strength of the concrete
	f_y	: Yield strength of steel
	E_s	: Modulus of elasticity of steel
Loads:	M	: Factored design moment at top of pile
	N	: Factored axial design force at top of pile
	P	: Factored design shear force at top of pile
Resistance factors:	Φ_F	: Resistance factor for flexure
	Φ_{punc}	: Resistance factor for shear from ACI
	Φ_T	: Resistance factor for strut-tie tension member
	Φ_c	: Resistance factor for strut-tie compression member
Design parameters:	N	: Reinforcement bar size
	n	: Number of reinforcement bars
	α	: Coefficient of friction between steel and concrete
	$cover_{int}$	: Socket region interior cover to main reinforcing
	$cover_{ext}$	: Socket region exterior cover to main reinforcing

- What is the output?

The checks which verify whether the entered amount of reinforcement is enough or not are the outputs of this sheet. When all the checks are okay, we make sure that our design is completed.

Reference Manual:

- How calculations are done?
Procedures followed are described step by step in detail while calculations are done.
- Example design problem is shown below with sample input/output.

Description of Forces:

There are two primary loads applied on abutment bodies, which are horizontal and vertical forces. These loads must be estimated prior to design. Horizontal loads are divided into horizontal earth pressure caused by fill and live load surcharge from vehicles. Earth pressure loads from fill have a triangular distribution on the back face, zero at the top and largest at the bottom assuming active conditions. While calculating this type of pressure behind the abutment wall, we use Equivalent-Fluid Method of Estimating Rankine Lateral Earth Pressures explained in LRFD 3.11.5.5. Please note also that this method can only be applied when we have free draining backfill. Here, we assume that drainage methods will be applied at the site to lower the water table behind the wall and this will keep the soil behind the wall drained all the time. Horizontal loading due to live load surcharge from vehicular wheel load is assumed to create a constant horizontal earth pressure. The constant pressure is calculated as the multiplication of the coefficient of lateral earth pressure (active conditions apply), unit weight of soil and equivalent height of soil for vehicular load. This equivalent height has been tabulated for designers in AASHTO to pick up the right value for a case. Linear interpolation between heights is also allowed.

In addition to horizontal loads, we have vertical loads which are coming from the superstructure as dead load and live load. In the design examples, vertical loads are assumed. In order to be precise, all the dimensions of the bridge should be known so that the exact dead load can be calculated and the total factored dead load is calculated.

Discussion of Two Methods:

The abutment is connected to piles through sockets. There are two concepts that can be used to determine the forces inside of the socket region. They are models with friction and without friction. The most common method is to find resisting forces ignoring friction inside of the socket region. By doing so, we ignore the contribution of the friction forces to resisting moment and we increase the compressive design forces. On the other hand, when we include the presence of the friction forces in our calculations, we have the extra resisting moment coming from those frictions which, in turn, reduces other design forces. That is because the required moment capacity from compressive forces is lower than the case when we ignore frictions. Total model is divided into two submodels and then analyzed separately to get the forces. This method is similar to Osanai, Y., Watanabe, F., Okamoto, S. (1996). Stress transfer mechanism of socket base connections with precast concrete columns. ACI Structural Journal, v 93, n 3, p 266-276

Figure 1 shows a model without friction and Figure 2 shows the model with friction and how it is divided into two submodels.

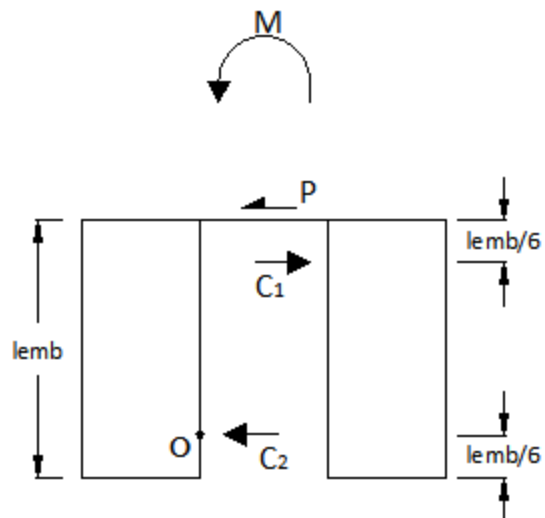


Figure 1
Ignoring Friction

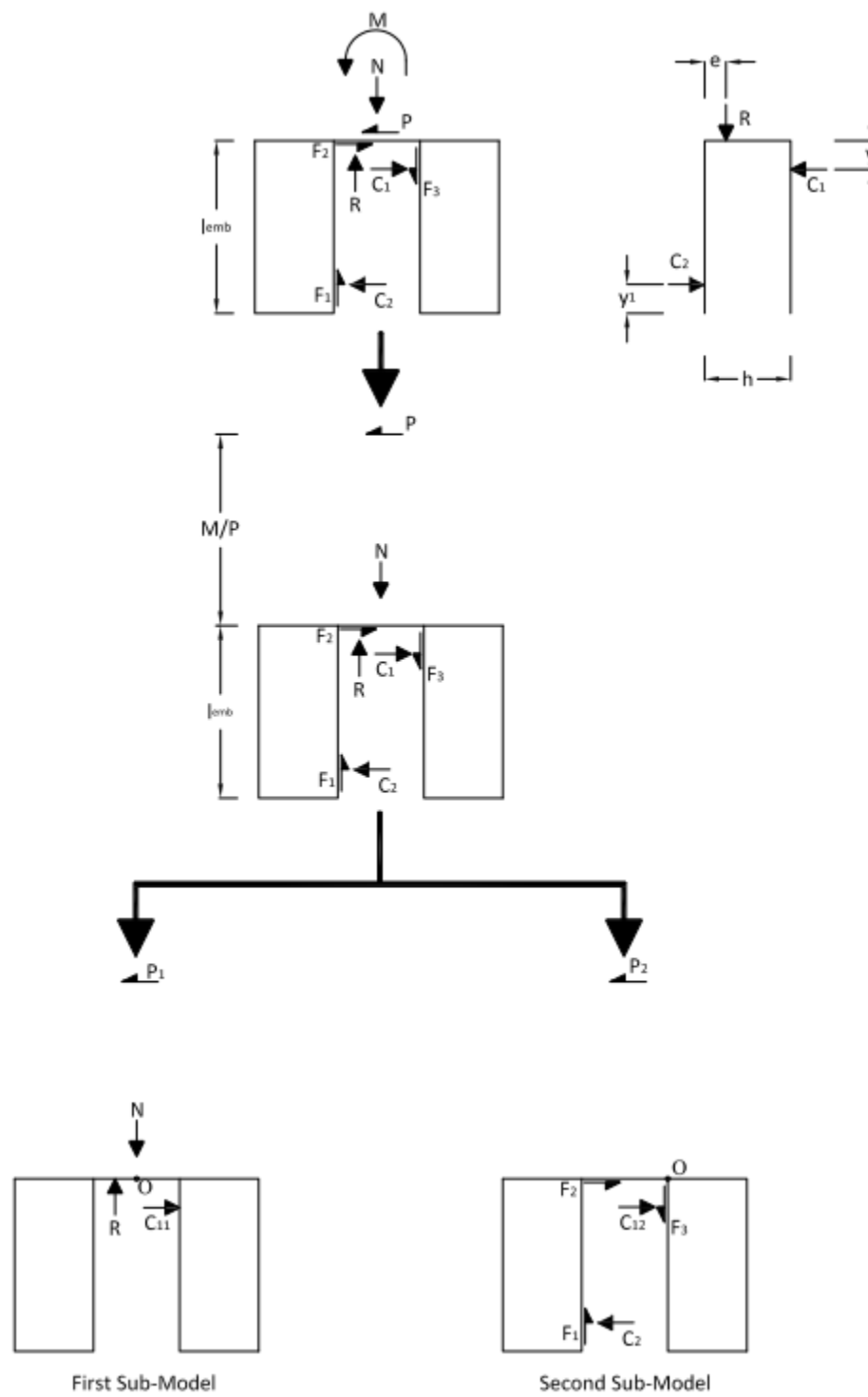


Figure 2
Including Friction
Resistance

All inputs are in red color.

All checks are in yellow color.

Friction Forces Included:

Design Constants:

$$M := 125 \text{ kip}\cdot\text{ft}$$

factored moment at top of pile
from applied loads

$$N := 145 \text{ kip}$$

factored axial force at top of pile
from applied loads

$$P := 37 \text{ kip}$$

factored shear force at top of pile
(from lateral soil pressure)

$$h := 12 \text{ in}$$

depth of steel HP pile

$$l_{\text{emb}} := 2 \text{ ft}$$

embedment length of pile in socket

$$e := \frac{h}{4} = 3 \cdot \text{in}$$

eccentricity of axial force on pile

$$y := \frac{l_{\text{emb}}}{6} = 4 \cdot \text{in}$$

assumed eccentricity for C_1 ,
resultant compression resistance
on socket side

$$y_1 := \frac{l_{\text{emb}}}{6} = 4 \cdot \text{in}$$

eccentricity for C_2 , compression
resultant on other socket side

The eccentricities (e , y and y_1) are
assumed as resultant locations of
stress resultants between socket
grout and pile. Please look at
Figure 2 to see where these
eccentricities occur.

$$\phi_F := 0.9$$

LRFD resistance reduction factor
for flexure

$$f_y := 60 \text{ ksi}$$

yield strength of reinforcement

$$E_s := 29000 \text{ ksi}$$

modulus of elasticity of steel

$$\phi_T := 1$$

resistance reduction factor for
strut-tie tension member LRFD
5.5.4.2.1

$$f_c := 4 \text{ ksi}$$

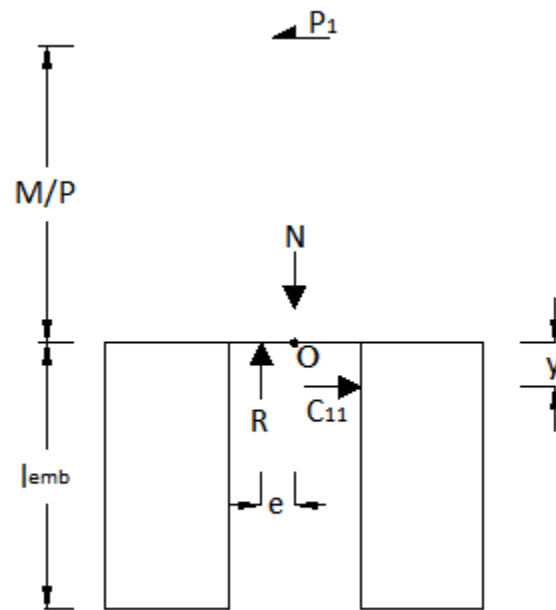
concrete strength

$$\beta_1 := \max \left[0.85 - 0.05 \cdot \frac{(f_c - 4 \text{ ksi})}{\text{ksi}}, 0.65 \right] = 0.85$$

neutral axis multiplier for depth
of concrete stress block, LRFD
5.7.2.2

From first sub-model:

The first analysis sub-model is used to represent a subset of the forces between the pile and socket without any friction. The remaining forces are included in a second sub-model. The total condition is a sum of the two separate sub-models.



First Sub-Model

Figure 3

Equilibrium in the vertical direction:

$$R := N = 145 \cdot \text{kip}$$

vertical force equilibrium

Equilibrium in the horizontal direction:

$$C_{11}(P_1) := P_1$$

C_{11} is equal to P_1 and here we define C_{11} as a function of P_1 , P_1 is a portion of the total lateral force "P"

$\Sigma M_0 = 0$, Moment equilibrium:

$$P_1 := \text{root} \left[\left[P_1 \cdot \left(\frac{M}{P} \right) + C_{11}(P_1) \cdot y - R \cdot e \right], P_1 \right]$$

root of this equation gives us the value of P_1 equating the total moment to zero with respect to point 0

$$P_1 = 9.766 \cdot \text{kip}$$

$$C_{11}(P_1) = 9.766 \cdot \text{kip}$$

From second sub-model:

The remaining socket-pile forces are resolved using Figure 4.

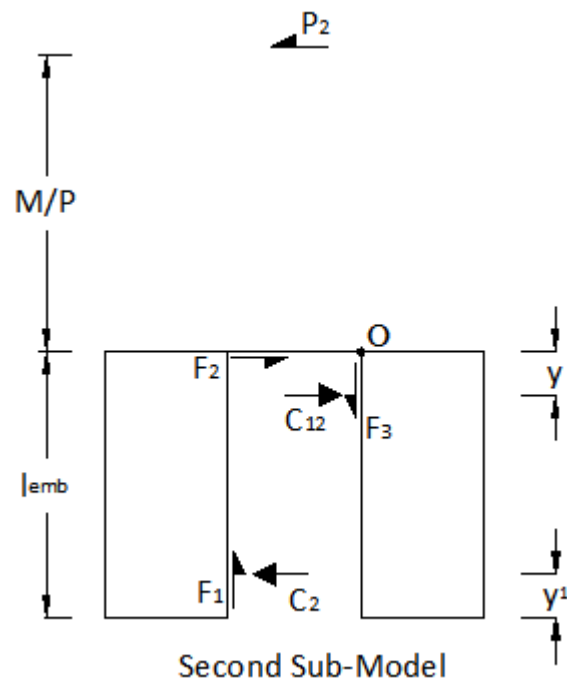


Figure 4

$$P_2 := P - P_1 = 27.234 \cdot \text{kip}$$

P_2 is the remaining lateral load that needs to be resisted by the connection

Equilibrium in the vertical direction:

Assuming α is same for both F_1 and F_3 , where α = coefficient of friction between steel and concrete:

$$\alpha := 0.25$$

$$F_1 = F_3$$

Then considering lateral forces:

C_2 is taken as a function of C_{12}

$$C_2(C_{12}) := C_{12}$$

$$\begin{aligned} \text{since } F_1 &= C_2 \cdot \alpha \\ F_3 &= C_{12} \cdot \alpha \\ F_1 &= F_3, \text{ then} \\ C_{12} &= C_2 \end{aligned}$$

where F_1 and F_3 are functions of C_{12}

$$F_1(C_{12}) := \alpha \cdot C_{12}$$

$$F_3(C_{12}) := \alpha \cdot C_{12}$$

Equilibrium in the horizontal direction:

$$F_2 := P_2 = 27.234 \cdot \text{kip}$$

since $C_{12} = C_2$, only F_2 remains to resist remaining lateral load P_2

Considering moment equilibrium about point "O", $\Sigma M_0 = 0$:

$$C_{12} := \text{root} \left[\left[P_2 \cdot \left(\frac{M}{P} \right) + C_{12} \cdot y - F_1(C_{12}) \cdot h - C_2(C_{12}) \cdot (l_{\text{emb}} - y_1) \right], C_{12} \right]$$

$$C_{12} = 58.109 \cdot \text{kip}$$

$$C_1 := C_{11}(P_1) + C_{12} = 67.875 \cdot \text{kip}$$

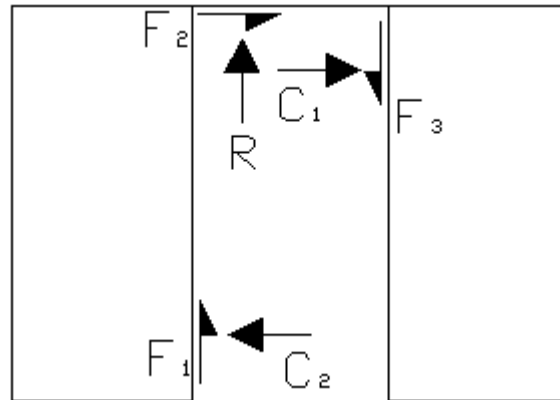
where C_1 is the total resultant compression at top of the pile

$$C_2 := C_{12} = 58.109 \cdot \text{kip}$$

Summary of design forces:

$$F_2 = 27.234 \cdot \text{kip}$$

$$R = 145 \cdot \text{kip}$$



$$C_1 = 67.875 \cdot \text{kip}$$

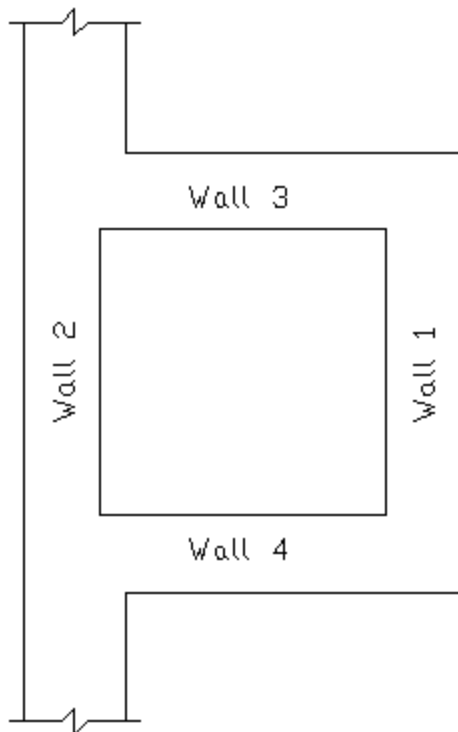
$$F_3(C_{12}) = 14.527 \cdot \text{kip}$$

$$F_1(C_{12}) = 14.527 \cdot \text{kip}$$

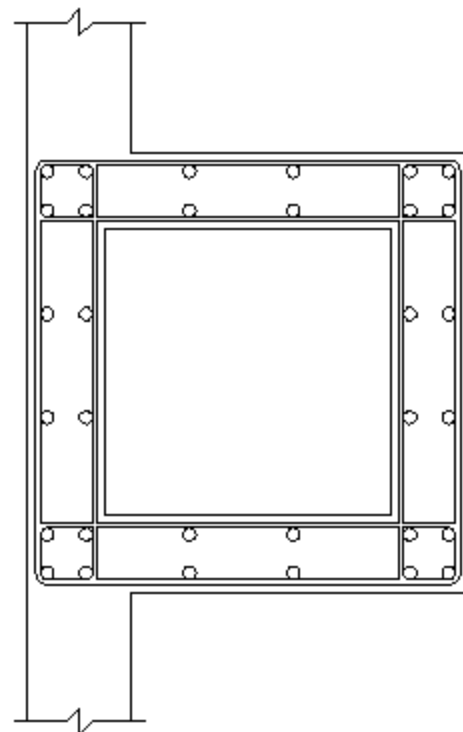
$$C_2 = 58.109 \cdot \text{kip}$$

**Side View
Figure 5**

Reinforcement Calculation:



**Plan View of Socket
Figure 6**



**Plan View of Socket Reinforcing
Figure 7**

Wall 1 and 2 are the transverse walls whereas wall 3 and 4 are parallel walls to applied forces. Wall 2 is part of the abutment wall and wall 1 is the outside of the socket. Symmetric reinforcements from side-side and top-bottom are named as below.

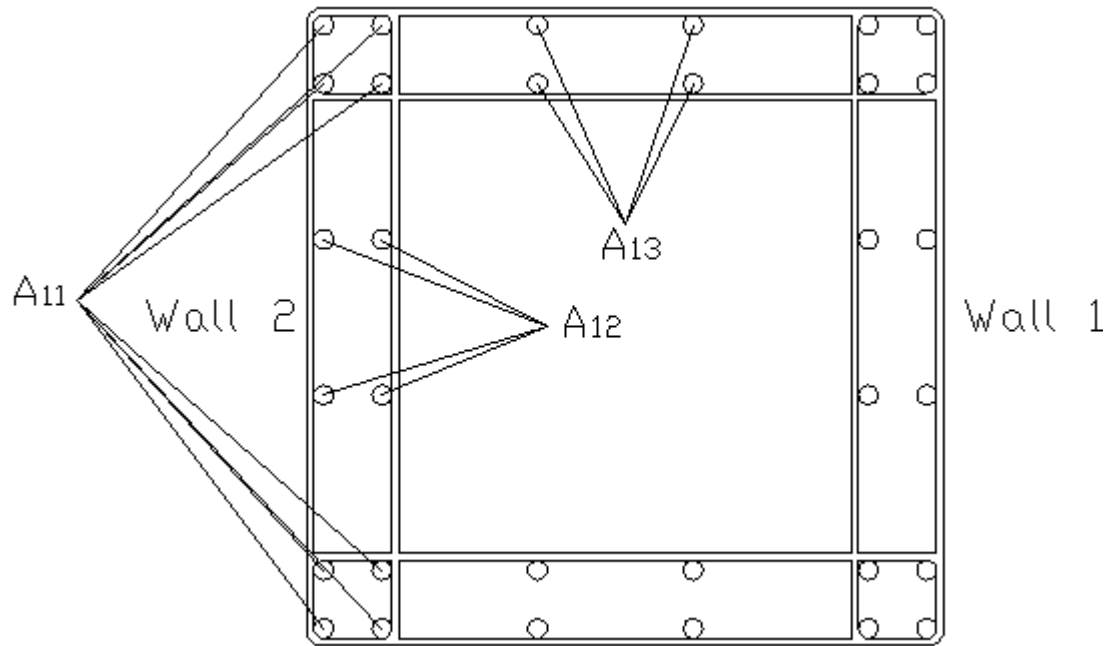


Figure 8

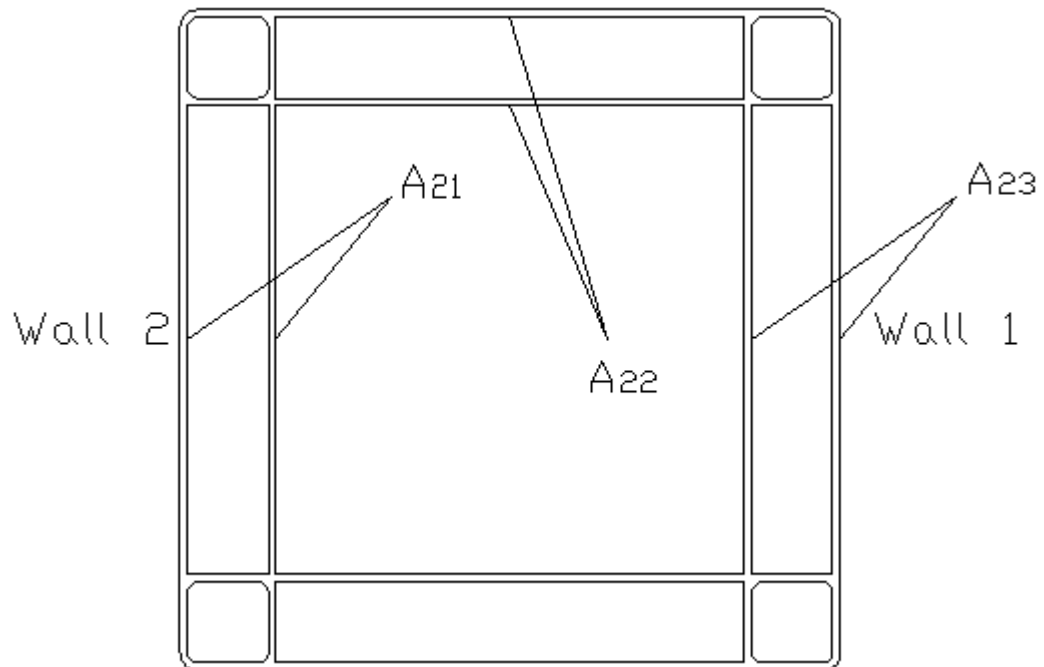


Figure 9

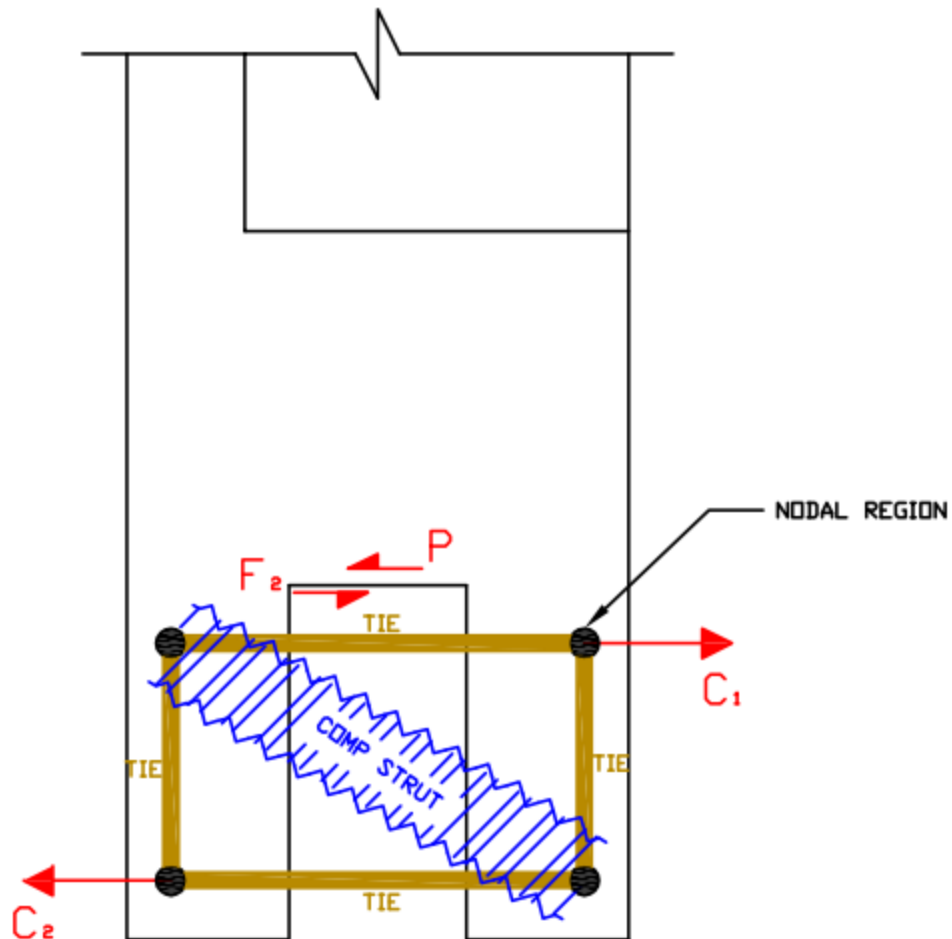
Reinforcement labeled as A_{12} is column reinforcement and it will be coming from that analysis

Epoxy coated steel bars will be used for the socket region. For that reason, concrete cover to main reinforcing will be 1.5" for exterior rebars and 1" for interior rebars. Moreover, cover to ties and stirrups will not be less than 1" (interior rebars are assumed on inside surfaces of socket, where concrete will be placed later to complete the connection).

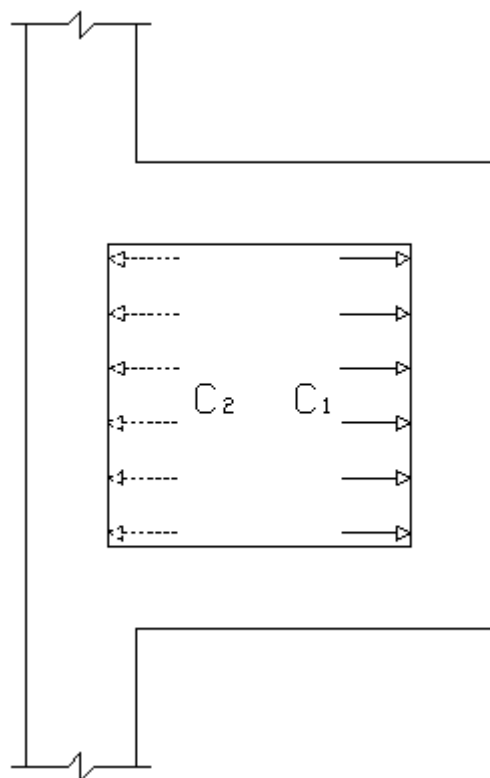
$$\text{cover}_{\text{int}} := 1\text{in}$$

$$\text{cover}_{\text{ext}} := 1.5\text{in}$$

A resistance concept is needed in the socket design to resist the forces just found. Truss action is assumed in the "disturbed" socket region as recommended by AASHTO (LRFD 5.6.3.1). Figure 10 shows a cross section through the wall with a suggested truss mechanism.



Side View
Figure 10



Plan View
Figure 11

For parallel walls(3&4) the reinforcement is needed (Figure 10):

Tie at top of socket:

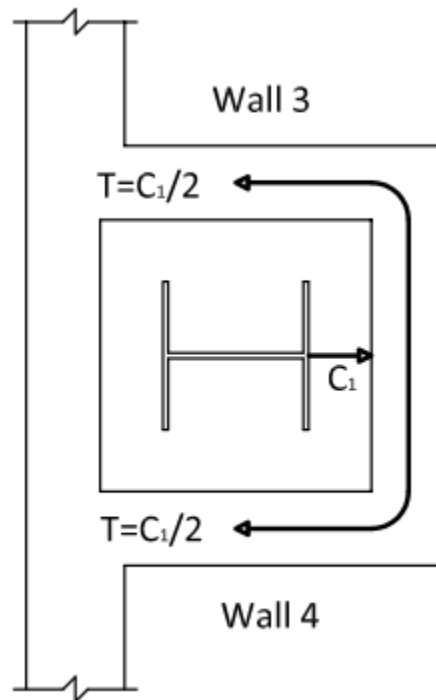


Figure 12

C_1 is applied to wall 1 (Figure 11) and is resisted by tension ties in walls 3&4 as shown in Figure 10.

Try different size and number of reinforcements until $T_{provided} > T_{req}$ and $T = C_1/2$, since one tie is in each of walls 3&4 as in Figure 12.

For region where C_1 is applied (top) (reinforcement A_{22}):

Bars are assumed and capacity is checked.

$$N_{barT1} := 5$$

rebar size

$$n_{barT1} := 2$$

number of rebars

$$T_{1req} := \frac{C_1}{2} = 33.938 \cdot \text{kip} \quad (\text{see Figure 12})$$

$$T_{1provided} := \left[n_{barT1} \cdot \frac{\pi \cdot \left(\frac{N_{barT1}}{8} \text{in} \right)^2}{4} \cdot f_y \right] \cdot \phi_T = 36.816 \cdot \text{kip} \quad \text{tension capacity of bars}$$

$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } T_{1provided} > T_{1req} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

Tie at bottom of socket:

For region where C_2 is applied (bottom) (reinforcement A_{22}), the approach is similar to top tie:

$$N_{barT2} := 5 \quad \text{rebar size}$$

$$n_{barT2} := 2 \quad \text{number of rebars}$$

$$T_{2req} := \frac{C_2}{2} = 29.054 \cdot \text{kip}$$

$$T_{2provided} := \left[n_{barT2} \cdot \frac{\pi \cdot \left(\frac{N_{barT2}}{8} \text{in} \right)^2}{4} \cdot f_y \right] \cdot \phi_T = 36.816 \cdot \text{kip}$$

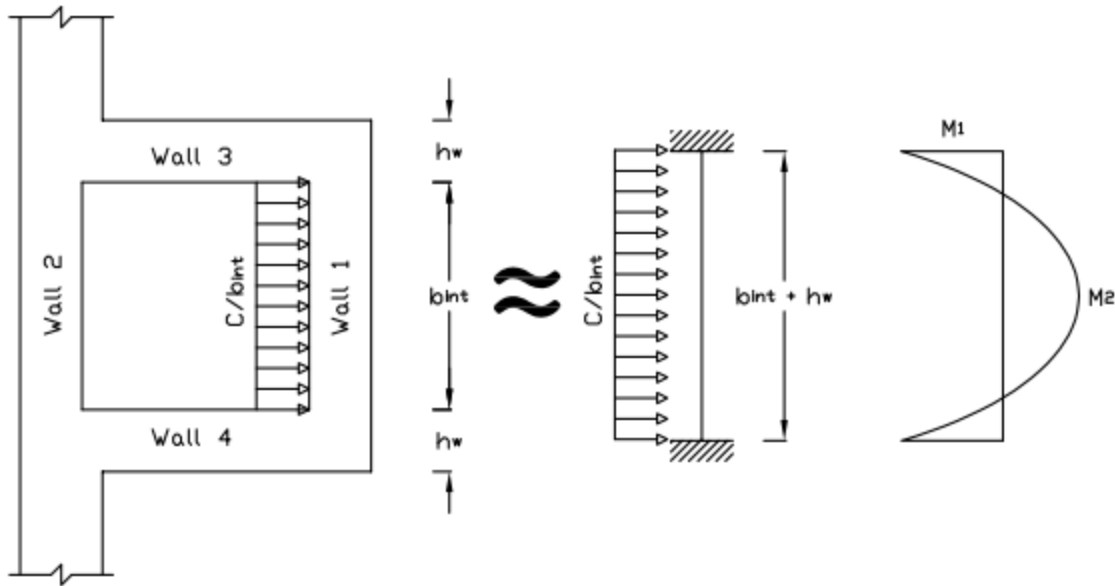
$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } T_{2provided} > T_{2req} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

For bending of transverse walls 1&2, flexural reinforcing:

$$b_{int} := 22 \text{in} \quad \text{inner dimension of the socket}$$

$$h_w := 6 \text{in} \quad \text{thickness of the socket wall}$$



Plan View
Figure 13

$$M_1 = M_2 = (C / b_{int}) \cdot (b_{int} + h_w)^2 / 16$$

As shown in Figures 11&13, the load intensity is calculated from the internal socket width but the moments are conservatively taken for a longer span. The bending moments in the socket wall are assumed to be "partially fix ended" since the walls 3 & 4 apply some bending restraint to walls 1 & 2. The assumed moment diagram is also shown in Figure 13.

1) Flexure - For region where C_1 is applied (top) (reinforcement A_{23} - see Figure 9):

$$w_1 := \frac{C_1}{b_{int}} = 37.023 \cdot \frac{\text{kip}}{\text{ft}}$$

distributed load on inner face of wall due to C_1 force

$$M_d := \frac{w_1 \cdot (b_{int} + h_w)^2}{16} = 12.598 \cdot \text{kip} \cdot \text{ft}$$

conservative estimate of design moment for both negative and positive region with "supports" assumed at middle of walls 3 & 4

Assuming the effective concrete width (vertical) for wall 1 acting as a beam is $l_{emb} / 4$:

Please change the amount of steel until sufficient capacity is achieved, the amount of reinforcement which is entered below is same for both positive and negative reinforcement:

$$N_{\text{barFL1}} := 5$$

rebar size

$$n_{\text{barFL1}} := 3$$

number of rebars

$$A_s := n_{\text{barFL1}} \cdot \frac{\pi \cdot \left(\frac{N_{\text{barFL1}}}{8} \text{in} \right)^2}{4} = 0.92 \cdot \text{in}^2$$

amount of reinforcement

Assuming compression steel is in compression and has not yielded:

$$\epsilon_s(x) := 0.003 \cdot \left[\frac{x - \text{cover}_{\text{int}} - \left(\frac{N_{\text{barFL1}}}{8} \text{in} \right) \cdot \frac{1}{2}}{x} \right]$$

strain in compression steel

$$f(x) := 0.85 \cdot f_c \cdot \frac{l_{\text{emb}}}{4} \cdot \beta_1 \cdot x + A_s \cdot (E_s \cdot \epsilon_s(x) - 0.85 f_c) - A_s \cdot f_y$$

axial force equilibrium in beam

$$x := 1 \text{ in}$$

initial guess for neutral axis depth

$$x := \text{root}(f(x), x) = 1.914 \cdot \text{in}$$

neutral axis depth

$$C := 0.85 \cdot f_c \cdot \left(\frac{l_{\text{emb}}}{4} \cdot \beta_1 \cdot x - A_s \right) = 30.059 \cdot \text{kip}$$

compression force in concrete

$$C_s := A_s \cdot E_s \cdot \epsilon_s(x) = 25.164 \cdot \text{kip}$$

force in compression steel
(positive is compression)

$$d := h_w - \text{cover}_{\text{ext}} - \frac{N_{\text{barFL1}}}{8} \text{in} \cdot \frac{1}{2} = 4.188 \cdot \text{in}$$

effective depth of the section

Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{\text{int}} + \frac{N_{\text{barFL1}}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$$\text{check_1} = \text{"Ok"}$$

Check if compression steel has yielded:

$$\text{check_2} := \begin{cases} \text{"Ok"} & \text{if } \left| \epsilon_s(x) \right| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then separate calculations are required

$$\text{check_2} = \text{"Ok"}$$

Moment capacity is:

$$M_{\text{provided}} := \phi_F \cdot \left[C \cdot \left(d - \frac{\beta_1 \cdot x}{2} \right) + C_s \cdot \left[d - \text{cover}_{\text{int}} - \left(\frac{N_{\text{barFL1}}}{8} \text{in} \right) \cdot \frac{1}{2} \right] \right] = 13.033 \cdot \text{kip} \cdot \text{ft}$$

Check if the capacity is enough:

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{\text{provided}} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{moment_capacity} = \text{"is enough"}$$

2) Flexure - For region where C_2 is applied (bottom) (reinforcement A_{21}):

$$w_2 := \frac{C_2}{b_{\text{int}}} = 31.696 \cdot \frac{\text{kip}}{\text{ft}}$$

distributed load on inner face of wall at bottom

$$M_d := \frac{w_2 \cdot (b_{\text{int}} + h_w)^2}{16} = 10.785 \cdot \text{kip} \cdot \text{ft}$$

for both negative and positive moment

Assuming the effective concrete width is $l_{\text{emb}} / 4$:

Please change the amount of steel until enough moment capacity is achieved, the amount of reinforcement which is entered below is same for both positive and negative reinforcement:

$$N_{\text{barFL2}} := 5$$

rebar size

$$n_{\text{barFL2}} := 3$$

number of rebars

$$A_s := n_{\text{barFL2}} \cdot \frac{\pi \cdot \left(\frac{N_{\text{barFL2}}}{8} \text{in} \right)^2}{4} = 0.92 \cdot \text{in}^2 \quad \text{amount of reinforcement}$$

Assuming compression steel is in compression and has not yielded:

$$\epsilon_s(x) := 0.003 \cdot \left[\frac{\left[x - \text{cover}_{\text{int}} - \left(\frac{N_{\text{barFL2}}}{8} \text{in} \right) \cdot \frac{1}{2} \right]}{x} \right] \quad \text{strain in compression steel}$$

$$f(x) := 0.85 \cdot f_c \cdot \frac{l_{\text{emb}}}{4} \cdot \beta_1 \cdot x + A_s \cdot (E_s \cdot \epsilon_s(x) - 0.85 f_c) - A_s \cdot f_y \quad \text{force equilibrium}$$

$$x := 1 \text{in} \quad \text{initial guess for neutral axis depth}$$

$$x := \text{root}(f(x), x) = 1.914 \cdot \text{in} \quad \text{neutral axis depth}$$

$$C := 0.85 \cdot f_c \cdot \frac{l_{\text{emb}}}{4} \cdot \beta_1 \cdot x - 0.85 f_c \cdot A_s = 30.059 \cdot \text{kip} \quad \text{compression force in concrete}$$

$$C_s := A_s \cdot E_s \cdot \epsilon_s(x) = 25.164 \cdot \text{kip} \quad \begin{array}{l} \text{force in compression steel} \\ \text{(positive is compression)} \end{array}$$

$$d := h_w - \text{cover}_{\text{ext}} - \frac{N_{\text{barFL2}}}{8} \text{in} \cdot \frac{1}{2} = 4.188 \cdot \text{in} \quad \text{effective depth of the section}$$

Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{\text{int}} + \frac{N_{\text{barFL2}}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$$\text{check_1} = \text{"Ok"}$$

Check if compression steel has yielded:

$$\text{check_2} := \begin{cases} \text{"Ok"} & \text{if } \left| \epsilon_s(x) \right| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then separate calculations are required

$$\text{check_2} = \text{"Ok"}$$

Moment capacity is:

$$M_{\text{provided}} := \phi_F \cdot \left[C \cdot \left(d - \frac{\beta_1 \cdot x}{2} \right) + C_s \cdot \left[d - \text{cover}_{\text{int}} - \left(\frac{N_{\text{barFL1}}}{8} \text{in} \right) \cdot \frac{1}{2} \right] \right] = 13.033 \cdot \text{kip} \cdot \text{ft}$$

Check if the capacity is enough:

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{\text{provided}} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{moment_capacity} = \text{"is enough"}$$

Punching Failure Check for Wall 1, ACI 318-08 Section 11.11:

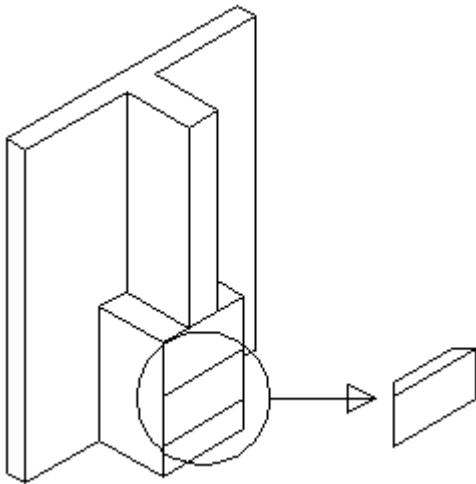


Figure 14
Isometric View of Punching
Shear Region

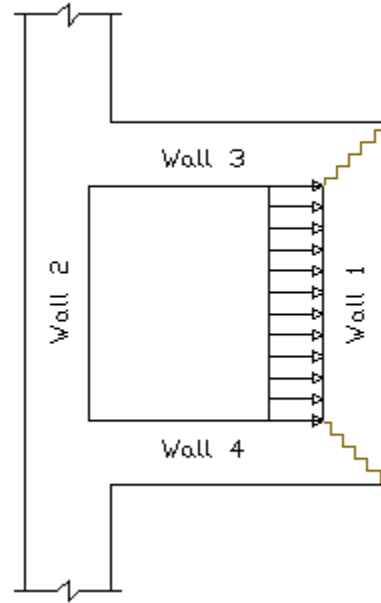


Figure 15
Punching Failure

$$\phi_{\text{punc}} := 0.75$$

strength reduction factor for shear
from ACI

$$b_o := \begin{cases} 2 \cdot \left(\frac{l_{\text{emb}}}{3} + d \right) + 2(b_{\text{int}} + d) & \text{if } l_{\text{emb}} \leq (4\text{ft} - d) \\ 2 \cdot \left(\frac{l_{\text{emb}}}{3} + d \right) + (b_{\text{int}} + d) & \text{otherwise} \end{cases}$$

critical perimeter, taken $d/2$ away
from concentrated load area,
assuming that crack occurs with a
strip depth of $l_{\text{emb}}/3$

$$b_o = 76.75 \cdot \text{in}$$

Above equation for b_o indicates that if the embedment of the pile is same or pretty close to socket length (4ft), the critical perimeter reduces.

$$\text{constant} := \min \left[\left[2 + \frac{4}{\frac{b_{\text{int}}}{\left(\frac{l_{\text{emb}}}{3} \right)}} \right], \left[2 + \frac{20}{\frac{b_o}{d}} \right], 2 \right] = 2$$

minimum of those constants will
be used per ACI code

$$V_{c_punc} := \text{constant} \cdot \left[\left(\sqrt{\frac{f_c}{\text{psi}}} \right) \cdot b_o \cdot d \right] = 40.653 \cdot \text{kip}$$

Strength coming from rebars:

$$V_{s_punc} := \left[2 \cdot n_{\text{barT1}} \cdot \pi \cdot \left(\frac{N_{\text{barT1}}}{8} \text{in} \cdot \frac{1}{2} \right)^2 \right] \cdot f_y = 73.631 \cdot \text{kip}$$

the reinforcement crossing the shear crack, A_{22} , was calculated previously to resist C_1

$$V_{n1} := V_{c_punc} + V_{s_punc} = 114.284 \cdot \text{kip} \quad \text{ACI Eq. (11-2)}$$

$$V_{n2} := 6 \cdot \left(\sqrt{\frac{f_c}{\text{psi}}} \right) \cdot b_o \cdot d = 121.959 \cdot \text{kip} \quad \text{ACI 11.11.3.2}$$

$$V_n := \min(V_{n1}, V_{n2}) = 114.284 \cdot \text{kip}$$

$$C_1 = 67.875 \cdot \text{kip}$$

$$\text{punching_strength} := \begin{cases} \text{"is enough"} & \text{if } \phi_{\text{punc}} \cdot V_n \geq C_1 \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{punching_strength} = \text{"is enough"}$$

Shrinkage and Temperature Reinforcement in Socket Walls, LRFD 5.10.8 (reinforcement A_{13} - see Figure 8):

The area of S&T reinforcement per foot, on each face and in each direction shall not be less than:

$$A_{ST} := \left[\frac{1.3 \cdot \frac{(b_{\text{int}} + 2h_w)^2 - b_{\text{int}}^2}{\text{in}^2}}{\frac{4(b_{\text{int}} + 2h_w) + 4b_{\text{int}}}{\text{in}} \cdot \frac{f_y}{\text{ksi}}} \right] \frac{\text{in}^2}{\text{ft}} = 0.065 \cdot \frac{\text{in}^2}{\text{ft}}$$

This amount of reinforcement is 1.3 times area of gross concrete divided by perimeter exposed to air times yield strength of steel.

Furthermore, A_s should satisfy following conditions:

$$A_{ST} := \begin{cases} 0.11 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST} \leq 0.11 \frac{\text{in}^2}{\text{ft}} \\ 0.6 \frac{\text{in}^2}{\text{ft}} & \text{if } A_{ST} \geq 0.6 \frac{\text{in}^2}{\text{ft}} \\ A_{ST} & \text{otherwise} \end{cases}$$

defines the limits for reinforcement

$$A_{ST} := b_{\text{int}} \cdot A_{ST} = 0.202 \cdot \text{in}^2$$

total S&T reinforcement required for the cross section

Now please enter reinforcement amount below to get the required S&T reinforcement:

$$N_{\text{barTS}} := 3$$

rebar size

$$A_{13} := 4 \cdot \pi \cdot \left(\frac{N_{\text{barTS}}}{8} \text{in} \frac{1}{2} \right)^2 = 0.442 \cdot \text{in}^2$$

number of reinforcements are 4 as can be seen from the drawings, but it can be increased checking the spacing limitations

$$\text{check} := \begin{cases} \text{"ok"} & \text{if } A_{13} \geq A_{ST} \\ \text{"add reinforcement"} & \text{otherwise} \end{cases}$$

$$\text{check} = \text{"ok"}$$

Maximum spacing can not exceed 3 times the wall thickness or 12in:

$$s_{\text{max1}} := \min(3 \cdot h_w, 12\text{in}) = 12 \cdot \text{in}$$

Longitudinal vertical reinforcement at the corners of walls (reinforcement A_{11}):

At the corner nodes the tension tie forces (C_1 and C_2) are balanced by the horizontal component of the diagonal. The vertical component of the diagonal needs to be resisted by vertical (longitudinal) reinforcing at the wall corners. After calculating the force (F) needed for this balance, we will determine the required area of reinforcement.

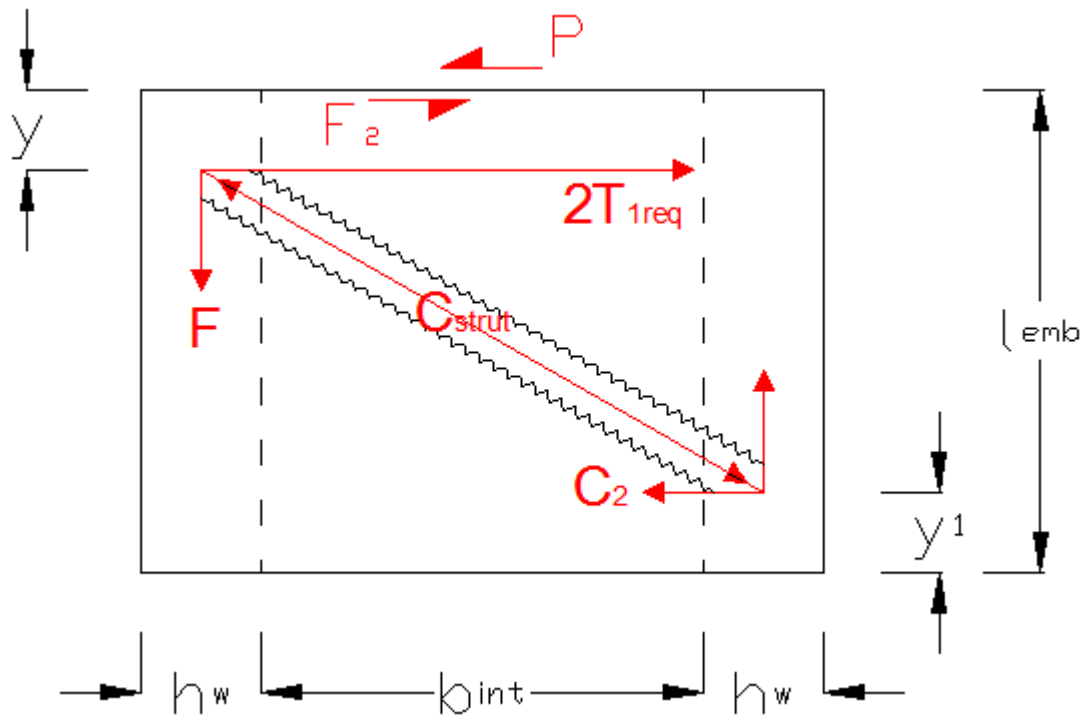


Figure 16
Ties & Struts in Socket
Elevation View

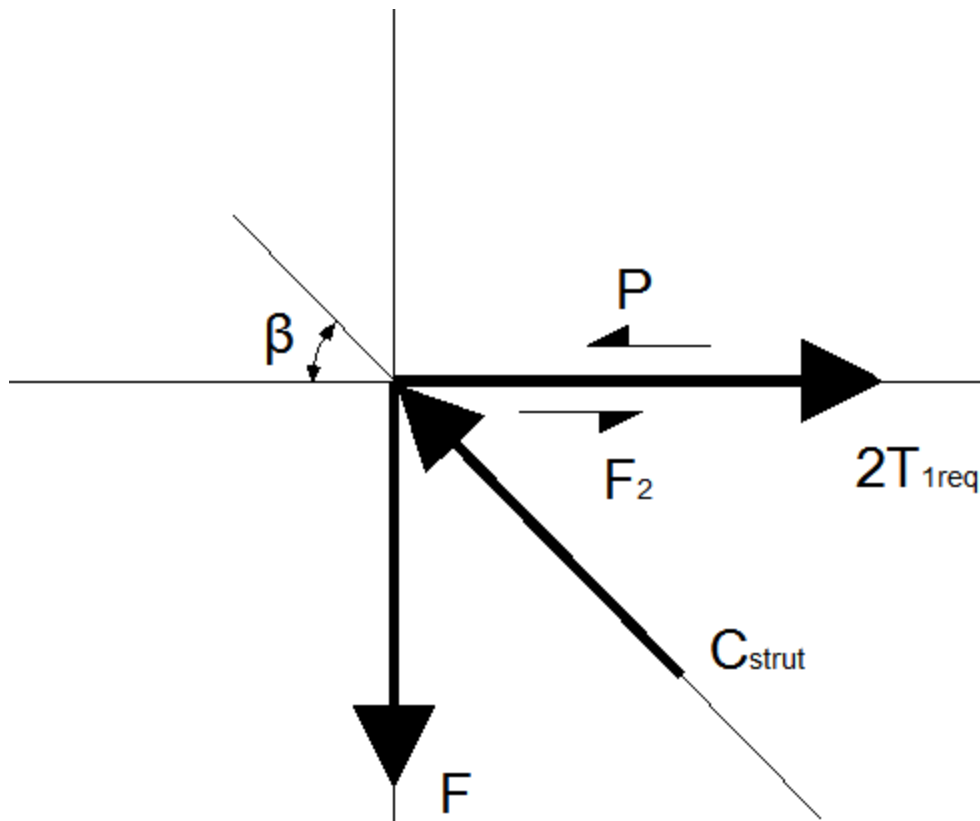


Figure 17

$$\beta := \operatorname{atan}\left(\frac{l_{\text{emb}} - y - y_1}{b_{\text{int}} + h_w}\right) = 29.745 \cdot \text{deg}$$

the strut angle depends on socket size and pile embedment length, strut-tie joint is assumed at middle of wall thickness

$$C_{\text{strut}} \cdot \cos(\beta) = 2 \cdot T_{1\text{req}} - (P - F_2)$$

from horizontal equilibrium at top (see Figures 16 & 17) with the lateral load P and the top friction F_2

$$C_{\text{strut}} := \frac{2T_{1\text{req}} - (P - F_2)}{\cos(\beta)} = 66.927 \cdot \text{kip}$$

$$F_{\text{req}} := C_{\text{strut}} \cdot \sin(\beta) = 33.205 \cdot \text{kip}$$

Try different size of reinforcements until $F_{\text{provided}} > F_{\text{req}}$.

$$N_{\text{barF}} := 3$$

rebar size

We will have 4 rebars at each corner, therefore, for one side we have 8 bars!

$$F_{\text{provided}} := 8 \cdot \left[\frac{\pi \cdot \left(\frac{N_{\text{barF}}}{8} \text{in} \right)^2}{4} \right] \cdot f_y \cdot \phi_T = 53.014 \cdot \text{kip}$$

$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } F_{\text{provided}} > F_{\text{req}} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

Check also whether or not concrete is safe in strut, LRFD 5.6.3:

$$\phi_c := 0.7$$

for compression in strut, LRFD 5.5.4.2

$$\epsilon_{T1} := \frac{T_{1\text{req}}}{n_{\text{barT1}} \cdot \left[\frac{\pi \cdot \left(\frac{N_{\text{barT1}}}{8} \text{in} \right)^2}{4} \right] \cdot E_s} = 1.907 \times 10^{-3}$$

strain in tension ties

$$\alpha_s := \beta$$

the smallest angle between the compressive strut and adjoining tension ties (deg)

$$\epsilon_l := \epsilon_{T1} + (\epsilon_{T1} + 0.002) \cdot \cot(\alpha_s)^2 = 0.014$$

$$f_{cu} := \min \left(\frac{f_c}{0.8 + 170 \cdot \epsilon_l}, 0.85 f_c \right) = 1.266 \cdot \text{ksi}$$

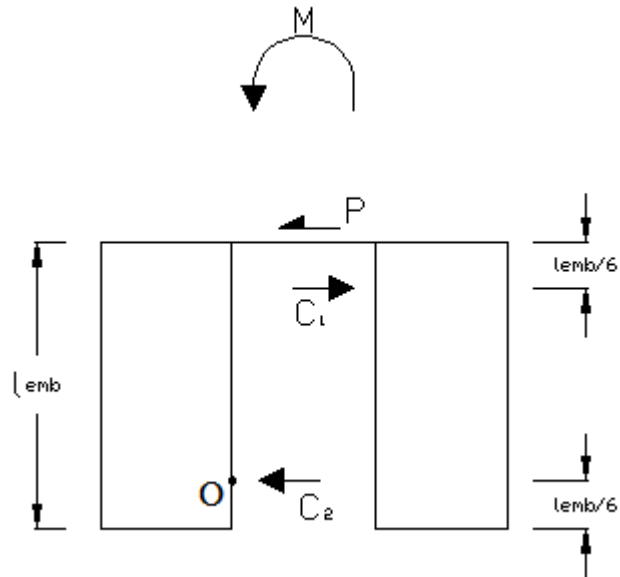
$$C_{\text{provided}} := \phi_c \cdot \left(2 \cdot f_{cu} \cdot h_w^2 \right) = 63.829 \cdot \text{kip}$$

provided concrete strength in strut
assuming strut size is $h_w \times h_w$

concrete_in_strut_is :=	"safe" if $C_{\text{provided}} > C_{\text{strut}}$ "not safe" otherwise
-------------------------	--

concrete_in_strut_is = "not safe"

It seems that concrete in compression does not have adequate strength to resist strut force therefore, the thickness of the side cheeks (or the thickness of the complete socket region) can be increased by 1/2" or more.

Ignoring Friction Forces:

$$\Sigma M_0 = 0$$

$$C_1 := \frac{M + P \cdot \left(\frac{5}{6} l_{emb} \right)}{\left(\frac{2}{3} l_{emb} \right)} = 140 \cdot \text{kip}$$

Equilibrium in the horizontal direction:

$$C_2 := C_1 - P = 103 \cdot \text{kip}$$

Reinforcement Calculation:

For parallel walls(3&4) the reinforcement is needed (Figure 10):

Tie at top of socket:

Try different size and number of reinforcements until $T_{provided} > T_{req}$ and $T = C_1/2$ since one tie is in each of walls 3&4 as in Figure 12.

For region where C_1 is applied (top) (reinforcement A_{22}):

Bars are assumed and capacity is checked.

$$N_{barT1} := 5$$

rebar size

$$n_{barT1} := 4$$

number of rebars

$$T_{1req} := \frac{C_1}{2} = 70 \cdot \text{kip}$$

(see Figure 12)

$$T_{1provided} := \left[n_{barT1} \cdot \frac{\pi \cdot \left(\frac{N_{barT1}}{8} \text{ in} \right)^2}{4} \cdot f_y \right] \cdot \phi_T = 73.631 \cdot \text{kip}$$

tension capacity of bars

$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } T_{1provided} > T_{1req} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

Tie at bottom of socket:

For region where C_2 is applied (bottom) (reinforcement A_{22}), the approach is similar to top tie:

$$N_{barT2} := 5$$

rebar size

$$n_{barT2} := 3$$

number of rebars

$$T_{2req} := \frac{C_2}{2} = 51.5 \cdot \text{kip}$$

$$T_{2provided} := \left[n_{barT2} \cdot \frac{\pi \cdot \left(\frac{N_{barT2}}{8} \text{ in} \right)^2}{4} \cdot f_y \right] \cdot \phi_T = 55.223 \cdot \text{kip}$$

$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } T_{2\text{provided}} > T_{2\text{req}} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

For bending of transverse walls 1&2, flexural reinforcing:

1) Flexure - For region where C_1 is applied (top) (reinforcement A_{23} - see Figure 9):

$$w_1 := \frac{C_1}{b_{\text{int}}} = 76.364 \cdot \frac{\text{kip}}{\text{ft}}$$

$$M_d := \frac{w_1 \cdot (b_{\text{int}} + h_w)^2}{16} = 25.985 \cdot \text{kip} \cdot \text{ft}$$

Please change the amount of steel until sufficient capacity is achieved, the amount of reinforcement which is entered below is same for both positive and negative reinforcement:

$$N_{\text{barFL1}} := 7$$

rebar size

$$n_{\text{barFL1}} := 4$$

number of rebars

$$A_s := n_{\text{barFL1}} \cdot \frac{\pi \cdot \left(\frac{N_{\text{barFL1}}}{8} \text{in} \right)^2}{4} = 2.405 \cdot \text{in}^2$$

amount of reinforcement

Assuming compression steel is in compression and has not yielded:

$$\epsilon_s(x) := 0.003 \cdot \left[\frac{x - \text{cover}_{\text{int}} - \left(\frac{N_{\text{barFL1}}}{8} \text{in} \right) \cdot \frac{1}{2}}{x} \right]$$

strain in compression steel

$$f(x) := 0.85 \cdot f_c \cdot \frac{l_{\text{emb}}}{4} \cdot \beta_1 \cdot x + A_s \cdot (E_s \cdot \epsilon_s(x) - 0.85 f_c) - A_s \cdot f_y$$

axial force equilibrium in beam

$$x := 1 \text{ in}$$

initial guess for neutral axis depth

$$x := \text{root}(f(x), x) = 2.838 \cdot \text{in}$$

neutral axis depth

$$C := 0.85 \cdot f_c \cdot \frac{l_{emb}}{4} \cdot \beta_1 \cdot x - 0.85 f_c \cdot A_s = 41.039 \cdot \text{kip}$$

compression force in concrete

$$C_s := A_s \cdot E_s \cdot \epsilon_s(x) = 103.278 \cdot \text{kip}$$

force in compression steel
(positive is compression)

$$d := h_w - \text{cover}_{ext} - \frac{N_{barFL1}}{8} \text{in} \cdot \frac{1}{2} = 4.063 \cdot \text{in}$$

effective depth of the section

Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{int} + \frac{N_{barFL1}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$$\text{check_1} = \text{"Ok"}$$

Check if compression steel has yielded:

$$\text{check_2} := \begin{cases} \text{"Ok"} & \text{if } |\epsilon_s(x)| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then
separate calculations are required

$$\text{check_2} = \text{"Ok"}$$

Moment capacity is:

$$M_{provided} := \phi_F \cdot \left[C \cdot \left(d - \frac{\beta_1 \cdot x}{2} \right) + C_s \cdot \left[d - \text{cover}_{int} - \left(\frac{N_{barFL1}}{8} \text{in} \right) \cdot \frac{1}{2} \right] \right] = 29.124 \cdot \text{kip} \cdot \text{ft}$$

Check if the capacity is enough:

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{provided} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{moment_capacity} = \text{"is enough"}$$

2) Flexure - For region where C_2 is applied (bottom) (reinforcement A_{21}):

$$w_2 := \frac{C_2}{b_{\text{int}}} = 56.182 \cdot \frac{\text{kip}}{\text{ft}}$$

$$M_d := \frac{w_2 \cdot (b_{\text{int}} + h_w)^2}{16} = 19.117 \cdot \text{kip} \cdot \text{ft}$$

Please change the amount of steel until enough moment capacity is achieved, the amount of reinforcement which is entered below is same for both positive and negative reinforcement:

$$N_{\text{barFL2}} := 6$$

rebar size

$$n_{\text{barFL2}} := 4$$

number of rebars

$$A_s := n_{\text{barFL2}} \cdot \frac{\pi \cdot \left(\frac{N_{\text{barFL2}}}{8} \text{in} \right)^2}{4} = 1.767 \cdot \text{in}^2$$

amount of reinforcement

Assuming compression steel is in compression and has not yielded:

$$\epsilon_s(x) := 0.003 \cdot \left[\frac{x - \text{cover}_{\text{int}} - \left(\frac{N_{\text{barFL2}}}{8} \text{in} \right) \cdot \frac{1}{2}}{x} \right]$$

strain in compression steel

$$f(x) := 0.85 \cdot f_c \cdot \frac{l_{\text{emb}}}{4} \cdot \beta_1 \cdot x + A_s \cdot (E_s \cdot \epsilon_s(x) - 0.85 f_c) - A_s \cdot f_y$$

force equilibrium

$$x := 1 \text{in}$$

initial guess for neutral axis depth

$$x := \text{root}(f(x), x) = 2.49 \cdot \text{in}$$

neutral axis depth

$$C := 0.85 \cdot f_c \cdot \frac{l_{\text{emb}}}{4} \cdot \beta_1 \cdot x - 0.85 f_c \cdot A_s = 37.174 \cdot \text{kip}$$

compression force in concrete

$$C_s := A_s \cdot E_s \cdot \epsilon_s(x) = 68.855 \cdot \text{kip}$$

force in compression steel
(positive is compression)

$$d := h_w - \text{cover}_{\text{ext}} - \frac{N_{\text{barFL2}}}{8} \text{in} \cdot \frac{1}{2} = 4.125 \cdot \text{in}$$

effective depth of the section

Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{\text{int}} + \frac{N_{\text{barFL2}}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$$\text{check_1} = \text{"Ok"}$$

Check if compression steel has yielded:

$$\text{check_2} := \begin{cases} \text{"Ok"} & \text{if } |\epsilon_s(x)| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then
separate calculations are required

$$\text{check_2} = \text{"Ok"}$$

Moment capacity is:

$$M_{\text{provided}} := \phi_F \cdot \left[C \cdot \left(d - \frac{\beta_1 \cdot x}{2} \right) + C_s \cdot \left[d - \text{cover}_{\text{int}} - \left(\frac{N_{\text{barFL1}}}{8} \text{in} \right) \cdot \frac{1}{2} \right] \right] = 22.428 \cdot \text{kip} \cdot \text{ft}$$

Check if the capacity is enough:

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{\text{provided}} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{moment_capacity} = \text{"is enough"}$$

Punching Failure Check for Wall 1, ACI 318-08 Section 11.11:

$$b_o := \begin{cases} 2 \cdot \left(\frac{l_{emb}}{3} + d \right) + 2(b_{int} + d) & \text{if } l_{emb} \leq (4ft - d) \\ 2 \cdot \left(\frac{l_{emb}}{3} + d \right) + (b_{int} + d) & \text{otherwise} \end{cases}$$

critical perimeter, taken $d/2$ away from concentrated load area, assuming that crack occurs with a strip depth of $l_{emb}/3$

$$b_o = 76.5 \cdot \text{in}$$

$$\text{constant} := \min \left[\left[2 + \frac{4}{\frac{b_{int}}{\left(\frac{l_{emb}}{3} \right)}} \right], \left[2 + \frac{20}{\frac{b_o}{d}} \right], 2 \right] = 2$$

minimum of those constants will be used per ACI code

$$V_{c_punc} := \text{constant} \cdot \left[\left(\sqrt{\frac{f_c}{\text{psi}}} \text{ psi} \right) \cdot b_o \cdot d \right] = 39.916 \cdot \text{kip}$$

Strength coming from rebars:

$$V_{s_punc} := \left[2 \cdot n_{barT1} \cdot \pi \cdot \left(\frac{N_{barT1}}{8} \text{ in} \cdot \frac{1}{2} \right)^2 \right] \cdot f_y = 147.262 \cdot \text{kip}$$

the reinforcement crossing the shear crack, A_{22} , was calculated previously to resist C_1

$$V_{n1} := V_{c_punc} + V_{s_punc} = 187.178 \cdot \text{kip}$$

ACI Eq. (11-2)

$$V_{n2} := 6 \cdot \left(\sqrt{\frac{f_c}{\text{psi}}} \cdot \text{psi} \right) \cdot b_o \cdot d = 119.748 \cdot \text{kip}$$

ACI 11.11.3.2

$$V_n := \min(V_{n1}, V_{n2}) = 119.748 \cdot \text{kip}$$

$$C_1 = 140 \cdot \text{kip}$$

$$\text{punching_strength} := \begin{cases} \text{"is enough"} & \text{if } \phi_{punc} \cdot V_n \geq C_1 \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{punching_strength} = \text{"is not enough"}$$

Shrinkage and Temperature Reinforcement in Socket Walls, LRFD 5.10.8 (reinforcement A_{13} - see Figure 8):

The area of S&T reinforcement per foot, on each face and in each direction shall not be less than:

$$A_{ST} := \left[\frac{1.3 \cdot \frac{(b_{int} + 2h_w)^2 - b_{int}^2}{in^2}}{\frac{4(b_{int} + 2h_w) + 4b_{int}}{in} \cdot \frac{f_y}{ksi}} \right] \frac{in^2}{ft} = 0.065 \cdot \frac{in^2}{ft}$$

This amount of reinforcement is 1.3 times area of gross concrete divided by perimeter exposed to air times yield strength of steel.

Furthermore, A_s should satisfy following conditions:

$$A_{ST} := \begin{cases} 0.11 \frac{in^2}{ft} & \text{if } A_{ST} \leq 0.11 \frac{in^2}{ft} \\ 0.6 \frac{in^2}{ft} & \text{if } A_{ST} \geq 0.6 \frac{in^2}{ft} \\ A_{ST} & \text{otherwise} \end{cases} \quad \text{defines the limits for reinforcement}$$

$$A_{ST} := b_{int} \cdot A_{ST} = 0.202 \cdot in^2 \quad \text{total S\&T reinforcement required for the cross section}$$

Now please enter reinforcement amount below to get the required S&T reinforcement:

$$N_{barTS} := 3 \quad \text{rebar size}$$

$$A_{13} := 4 \cdot \pi \cdot \left(\frac{N_{barTS}}{8} \cdot in \frac{1}{2} \right)^2 = 0.442 \cdot in^2 \quad \text{number of reinforcements are 4 as can be seen from the drawings, but it can be increased checking the spacing limitations}$$

$$check := \begin{cases} "ok" & \text{if } A_{13} \geq A_{ST} \\ "add reinforcement" & \text{otherwise} \end{cases}$$

$$check = "ok"$$

Maximum spacing can not exceed 3 times the wall thickness or 12in:

$$s_{max1} := \min(3 \cdot h_w, 12in) = 12 \cdot in$$

Longitudinal reinforcement at the corners of walls (reinforcement A_{11}):

Tension forces in the top and bottom ties (equal to C_1 and C_2) are balanced with force in vertical reinforcement and a diagonal compression strut that has been formed in concrete. After calculating the force (F) needed for this balance, we will determine the required area of reinforcement.

$$\beta := \operatorname{atan}\left(\frac{l_{\text{emb}} - y - y_1}{b_{\text{int}} + h_w}\right) = 29.745 \cdot \text{deg}$$

the strut angle depends on socket size and pile embedment length, strut-tie joint is assumed at middle of wall thickness

$$C_{\text{strut}} \cdot \cos(\beta) = 2 \cdot T_{1\text{req}} - (P - F_2)$$

from horizontal equilibrium at top (see Figures 16 & 17) with the lateral load P and the top friction F_2

$$C_{\text{strut}} := \frac{2T_{1\text{req}} - (P - F_2)}{\cos(\beta)} = 149.997 \cdot \text{kip}$$

$$F_{\text{req}} := C_{\text{strut}} \cdot \sin(\beta) = 74.419 \cdot \text{kip}$$

Try different size of reinforcements until $F_{\text{provided}} > F_{\text{req}}$.

$$N_{\text{barF}} := 4$$

rebar size

We will have 4 rebars at each corner, therefore, for one side we have 8 bars!

$$F_{\text{provided}} := 8 \cdot \left[\frac{\pi \cdot \left(\frac{N_{\text{barF}}}{8} \right)^2}{4} \right] \cdot f_y \cdot \phi_T = 94.248 \cdot \text{kip}$$

$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } F_{\text{provided}} > F_{\text{req}} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

Check also whether or not concrete is safe in strut, LRFD 5.6.3:

$$\varepsilon_{T1} := \frac{T_{1req}}{n_{barT1} \cdot \left[\frac{\pi \cdot \left(\frac{N_{barT1}}{8} \right)^2}{4} \right] \cdot E_s} = 1.967 \times 10^{-3}$$

strain in tension ties

$$\alpha_s := \beta$$

the smallest angle between the compressive strut and adjoining tension ties (deg)

$$\varepsilon_l := \varepsilon_{T1} + (\varepsilon_{T1} + 0.002) \cdot \cot(\alpha_s)^2 = 0.014$$

$$f_{cu} := \min\left(\frac{f_c}{0.8 + 170 \cdot \varepsilon_l}, 0.85f_c\right) = 1.25 \cdot \text{ksi}$$

$$C_{provided} := \phi_c \cdot (2 \cdot f_{cu} \cdot h_w^2) = 63.007 \cdot \text{kip}$$

provided concrete strength in strut
assuming strut size is $h_w \times h_w$

$$\text{concrete_in_strut_is} := \begin{cases} \text{"safe"} & \text{if } C_{provided} > C_{strut} \\ \text{"not safe"} & \text{otherwise} \end{cases}$$

$$\text{concrete_in_strut_is} = \text{"not safe"}$$

Both punching shear strength and strength of strut are not enough, therefore:

To increase the strut strength and punching strength, thickness of the socket wall and/or concrete strength can be increased.

Increasing the thickness to 9.5" is enough for both strengths!

Appendix J: Abutment Socket Design

Socket Region Design Sheet - Production Version

All inputs are in red color.

All checks are in yellow color.

Design Constants:

$$M := 125 \text{ kip}\cdot\text{ft}$$

factored moment at top of pile
from applied loads

$$N := 145 \text{ kip}$$

factored axial force at top of pile
from applied loads

$$P := 37 \text{ kip}$$

factored shear force at top of pile
(from lateral soil pressure)

$$h := 12 \text{ in}$$

depth of steel HP pile

$$l_{\text{emb}} := 2 \text{ ft}$$

embedment length of pile in socket

$$e := \frac{h}{4} = 3 \cdot \text{in}$$

eccentricity of axial force on pile

$$y := \frac{l_{\text{emb}}}{6} = 4 \cdot \text{in}$$

assumed eccentricity for C_1 ,
resultant compression resistance
on socket side

$$y_1 := \frac{l_{\text{emb}}}{6} = 4 \cdot \text{in}$$

eccentricity for C_2 , compression
resultant on other socket side

The eccentricities (e , y and y_1) are
assumed as resultant locations of
stress resultants between socket
grout and pile. Please look at
Figure 2 to see where these
eccentricities occur.

$$\alpha := 0.25$$

coefficient of friction between pile
and concrete

$$\phi_F := 0.9$$

LRFD resistance reduction factor
for flexure

$$f_y := 60 \text{ ksi}$$

yield strength of reinforcement

$$E_s := 29000 \text{ ksi}$$

modulus of elasticity of steel

$$\phi_T := 1$$

resistance reduction factor for
strut-tie tension member LRFD
5.5.4.2.1

$$f_c := 4\text{ksi}$$

concrete strength

$$\beta_1 := \max \left[0.85 - 0.05 \cdot \frac{(f_c - 4\text{ksi})}{\text{ksi}}, 0.65 \right] = 0.85$$

neutral axis multiplier for depth of concrete stress block, LRFD 5.7.2.2

$$b_{\text{int}} := 22\text{in}$$

inner dimension of the socket

$$h_w := 6\text{in}$$

thickness of the socket wall

Friction Forces Included:



Reinforcement Calculation:



$$\text{cover}_{\text{int}} := 1\text{in}$$

interior cover for socket

$$\text{cover}_{\text{ext}} := 1.5\text{in}$$

exterior cover for socket



For parallel walls(3&4) the reinforcement is needed (Figure 10):

Tie at top of socket:



$$N_{\text{barT1}} := 5$$

rebar size

$$n_{\text{barT1}} := 2$$

number of rebars



$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } T_{1\text{provided}} > T_{1\text{req}} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

Tie at bottom of socket:

$$N_{\text{barT2}} := 5$$

rebar size

$$n_{\text{barT2}} := 2$$

number of rebars



$$\text{reinforcement_is} := \begin{cases} \text{"enough"} & \text{if } T_{2\text{provided}} > T_{2\text{req}} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

$$\text{reinforcement_is} = \text{"enough"}$$

For bending of transverse walls 1&2, flexural reinforcing:



1) Flexure - For region where C_1 is applied (top) (reinforcement A_{23} - see Figure 9):



$$N_{\text{barFL1}} := 5$$

rebar size

$$n_{\text{barFL1}} := 3$$

number of rebars



Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{\text{int}} + \frac{N_{\text{barFL1}}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$$\text{check_1} = \text{"Ok"}$$

Check if compression steel has yielded:

$$\text{check_2} := \begin{cases} \text{"Ok"} & \text{if } |\epsilon_s(x)| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then separate calculations are required

$$\text{check_2} = \text{"Ok"}$$



Check if the capacity is enough:

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{\text{provided}} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{moment_capacity} = \text{"is enough"}$$

2) Flexure - For region where C_2 is applied (bottom) (reinforcement A_{21}):



$$N_{\text{barFL2}} := 5$$

rebar size

$$n_{\text{barFL2}} := 3$$

number of rebars



Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{\text{int}} + \frac{N_{\text{barFL2}}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$$\text{check_1} = \text{"Ok"}$$

Check if compression steel has yielded:

$$\text{check_2} := \begin{cases} \text{"Ok"} & \text{if } \left| \epsilon_s(x) \right| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then separate calculations are required

$$\text{check_2} = \text{"Ok"}$$



Check if the capacity is enough:

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{\text{provided}} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{moment_capacity} = \text{"is enough"}$$

Punching Failure Check for Wall 1, ACI 318-08 Section 11.11:

$$\phi_{\text{punc}} := 0.75$$

strength reduction factor for shear from ACI



$$\text{punching_strength} := \begin{cases} \text{"is enough"} & \text{if } \phi_{\text{punc}} \cdot V_n \geq C_1 \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$$\text{punching_strength} = \text{"is enough"}$$

Shrinkage and Temperature Reinforcement in Socket Walls, LRFD 5.10.8 (reinforcement A₁₃ - see Figure 8):



$$N_{\text{barTS}} := 3$$

rebar size



$$\text{check} := \begin{cases} \text{"ok"} & \text{if } A_{13} \geq A_{ST} \\ \text{"add reinforcement"} & \text{otherwise} \end{cases}$$

$$\text{check} = \text{"ok"}$$

Maximum spacing can not exceed 3 times the wall thickness or 12in:

$$s_{\text{max1}} := \min(3 \cdot h_w, 12\text{in}) = 12\text{in}$$

Longitudinal vertical reinforcement at the corners of walls (reinforcement A₁₁):



$$N_{\text{barF}} := 3$$

rebar size



```

reinforcement_is := | "enough"  if  $F_{\text{provided}} > F_{\text{req}}$ 
                    | "not enough" otherwise

```

```
reinforcement_is = "enough"
```

Check also whether or not concrete is safe in strut, LRFD 5.6.3:

```
 $\phi_c := 0.7$ 
```

for compression in strut, LRFD
5.5.4.2



```

concrete_in_strut_is := | "safe"  if  $C_{\text{provided}} > C_{\text{strut}}$ 
                       | "not safe" otherwise

```

```
concrete_in_strut_is = "not safe"
```

Ignoring Friction Forces:



Reinforcement Calculation:

For parallel walls(3&4) the reinforcement is needed (Figure 10):

Tie at top of socket:



```
 $N_{\text{barT1}} := 5$ 
```

rebar size

```
 $n_{\text{barT1}} := 4$ 
```

number of rebars



```

reinforcement_is := | "enough"  if  $T_{1\text{provided}} > T_{1\text{req}}$ 
                    | "not enough" otherwise

```

```
reinforcement_is = "enough"
```

Tie at bottom of socket:

For region where C_2 is applied (bottom) (reinforcement A_{22}), the approach is similar to top tie:

```
 $N_{\text{barT2}} := 5$ 
```

rebar size

```
 $n_{\text{barT2}} := 4$ 
```

number of rebars



```

reinforcement_is := | "enough"  if  $T_{2\text{provided}} > T_{2\text{req}}$ 
                    | "not enough" otherwise

```

```
reinforcement_is = "enough"
```

For bending of transverse walls 1&2, flexural reinforcing:

1) Flexure - For region where C_1 is applied (top) (reinforcement A_{23} - see Figure 9):



$N_{\text{barFL1}} := 7$

rebar size

$n_{\text{barFL1}} := 4$

number of rebars



Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{\text{int}} + \frac{N_{\text{barFL1}}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$\text{check_1} = \text{"Ok"}$

Check if compression steel has yielded:

$$\text{check_2} := \begin{cases} \text{"Ok"} & \text{if } |\epsilon_s(x)| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then separate calculations are required

$\text{check_2} = \text{"Ok"}$



Check if the capacity is enough:

$$\text{moment_capacity} := \begin{cases} \text{"is enough"} & \text{if } M_{\text{provided}} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

$\text{moment_capacity} = \text{"is enough"}$

2) Flexure - For region where C_2 is applied (bottom) (reinforcement A_{21}):



$N_{\text{barFL2}} := 6$

rebar size

$n_{\text{barFL2}} := 4$

number of rebars



Check if compression steel is in compression:

$$\text{check_1} := \begin{cases} \text{"Ok"} & \text{if } x > \left(\text{cover}_{\text{int}} + \frac{N_{\text{barFL2}}}{8} \text{in} \cdot \frac{1}{2} \right) \\ \text{"tension"} & \text{otherwise} \end{cases}$$

$\text{check_1} = \text{"Ok"}$

Check if compression steel has yielded:

check_2 :=
$$\begin{cases} \text{"Ok"} & \text{if } |\epsilon_s(x)| < \frac{f_y}{E_s} \\ \text{"not Ok"} & \text{otherwise} \end{cases}$$

"Ok" if not yielded, if yielded then separate calculations are required

check_2 = "Ok"



Check if the capacity is enough:

moment_capacity :=
$$\begin{cases} \text{"is enough"} & \text{if } M_{\text{provided}} \geq M_d \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

moment_capacity = "is enough"

Punching Failure Check for Wall 1, ACI 318-08 Section 11.11:



punching_strength :=
$$\begin{cases} \text{"is enough"} & \text{if } \phi_{\text{punc}} \cdot V_n \geq C_1 \\ \text{"is not enough"} & \text{otherwise} \end{cases}$$

punching_strength = "is not enough"

Shrinkage and Temperature Reinforcement in Socket Walls, LRFD 5.10.8 (reinforcement A₁₃ - see Figure 8):



N_{barTS} := 3

rebar size



check :=
$$\begin{cases} \text{"ok"} & \text{if } A_{13} \geq A_{ST} \\ \text{"add reinforcement"} & \text{otherwise} \end{cases}$$

check = "ok"

Maximum spacing can not exceed 3 times the wall thickness or 12in:

s_{max1} := min(3·h_w, 12in) = 12·in

Longitudinal reinforcement at the corners of walls (reinforcement A₁₁):



N_{barF} := 4

rebar size



reinforcement_is :=
$$\begin{cases} \text{"enough"} & \text{if } F_{\text{provided}} > F_{\text{req}} \\ \text{"not enough"} & \text{otherwise} \end{cases}$$

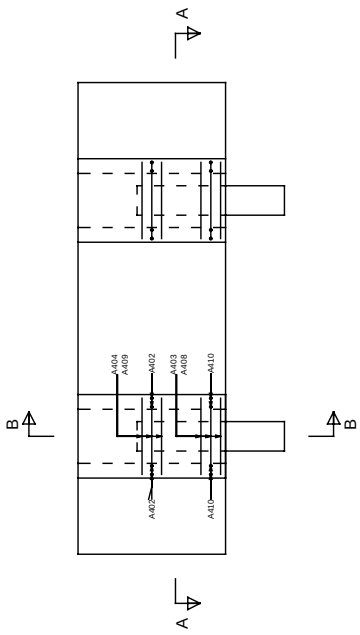
reinforcement_is = "enough"



concrete_in_strut_is :=
$$\begin{cases} \text{"safe"} & \text{if } C_{\text{provided}} > C_{\text{strut}} \\ \text{"not safe"} & \text{otherwise} \end{cases}$$

concrete_in_strut_is = "not safe"

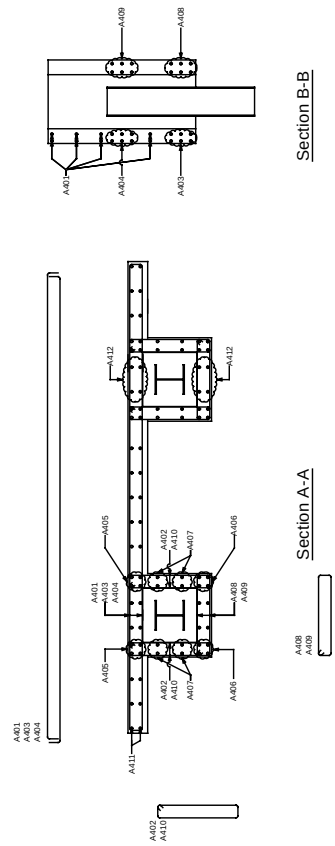
Appendix K: Abutments with Full and Limited Height Sockets - Drawings



Elevation View

Reinforcements are labeled differently in drawings and in design example sheet as:

Drawing	Design Sheet
A403 & A404	A ₂₁
A408 & A409	A ₂₃
A402 & A410	A ₂₂
A407	A ₁₃
A412	A ₁₂
A405 & A406	A ₁₁



NEW ABUTMENT SYSTEM 3'-5" REINFORCEMENT DETAIL	
UNIVERSITY OF WISCONSIN-MADISON PROJECT: EFFICIENT DESIGN OF PRECAST CONCRETE SUBSTRUCTURES FOR BRIDGES	
DATE:	December 2010

Appendix L: Welded Connections in Long Caps

Welded Steel Plate Connection for Caps

The connection between two steel plates will be welded with complete penetration groove welding. AASHTO LRFD section 6.13.3.2.2a states that "The factored resistance of complete penetration groove welded connections subjected to tension or compression normal to the effective area or parallel to the axis of the weld shall be taken as the factored resistance of the base metal."

As understood from the above statement, strength of the steel plate governs the design of the connection.

$b := 3.5\text{ft}$	width of pier cap
$h := 3.5\text{ft}$	depth of pier cap
$f_c := 4\text{ksi}$	concrete strength
$\beta_1 := \max\left[0.85 - 0.05 \cdot \frac{(f_c - 4\text{ksi})}{\text{ksi}}, 0.65\right] = 0.85$	neutral axis multiplier, (LRFD 5.7.2.2)
$f_y := 60\text{ksi}$	yield strength of steel plate
$E_s := 29000\text{ksi}$	modulus of elasticity of steel LRFD 5.4.3.2
$M_{\max} := 1187\text{kip}\cdot\text{ft}$	maximum negative moment at the location
$m := 4\text{in}$	width of the block outs (grouted pockets) for pier reinforcement
$t := 0.25\text{in}$	thickness of the steel plate
Ignoring the contribution from bottom reinforcement:	
$x := 1\text{in}$	initial guess for neutral axis depth
$x := \text{root}\left[0.85f_c \cdot b \cdot \beta_1 \cdot x - (b - 2m) \cdot t \cdot f_y, x\right]$	
$x = 4.202 \cdot \text{in}$	neutral axis depth

Check if the section is tension controlled:

$$\text{section_is} := \begin{cases} \text{"tension controlled"} & \text{if } 0.003 \cdot \frac{\left(b - \frac{t}{2} - x\right)}{x} \geq 0.005 \\ \text{"compression controlled"} & \text{otherwise} \end{cases}$$

section_is = "tension controlled"

Therefore, the strength reduction factor is 0.9

$$M_n := 0.9 \cdot f_y \cdot (b - 2m) \cdot t \cdot \left(b - \frac{t}{2} - \frac{\beta_1 \cdot x}{2} \right) = 1.533 \times 10^3 \cdot \text{kip} \cdot \text{ft}$$

Even 0.25" thick steel plate is enough to resist the maximum moment at the connection region.

The strength of the weld between tension reinforcement and steel plate is not checked here because the length of the reinforcement that will be welded throughout the steel plate is thought to be enough to transfer the force from plate to reinforcements.

The fatigue of the reinforcement welding is not checked in this design example either.

Appendix M: Splice Connections in Long Caps

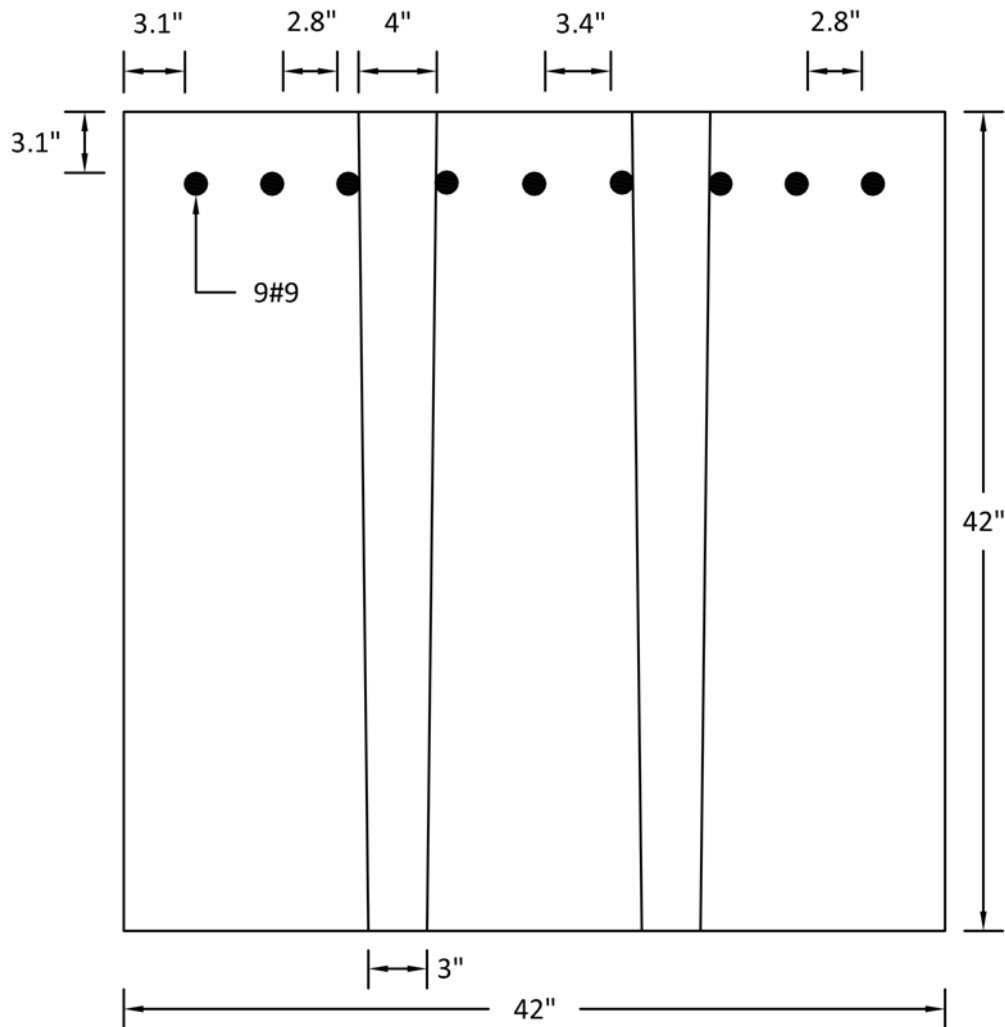
Spliced Reinforcement Connection for Caps

AASHTO 5.11.5.3.1 Lap Splices in Tension

Find tension development length AASHTO 5.11.2:

$N_o := 9$	bar number, should be less than #11 bar restricted by AASHTO 5.11.5.2.1
$n := 9$	number of bars
$\phi := \frac{N_o}{8} \text{ in} = 1.125 \cdot \text{in}$	diameter of the bars
$\phi_{\text{stir}} := 0.625 \text{ in}$	diameter of stirrups
$m := 4 \text{ in}$	width of the block outs (grouted pockets) for pier reinforcement
$b := 3.5 \text{ ft}$	width of bent cap
$\text{cover} := 2.5 \text{ in}$	clear cover for bent cap
$f_c := 4 \text{ ksi}$	concrete strength at 28 days
$f_y := 60 \text{ ksi}$	steel yield strength
$d_b := \frac{N_o}{8} \text{ in}$	diameter of bar
$A_b := \frac{\pi \cdot d_b^2}{4} = 0.994 \cdot \text{in}^2$	area of bar
$A_{s_required} := 7.167 \text{ in}^2$	required amount of reinforcement area
$A_{s_provided} := n \cdot A_b = 8.946 \cdot \text{in}^2$	provided reinforcement area
$\%A_{s_spliced} := 100$	percent of A_s spliced

Check to see if there is enough space between two reinforcements from the drawing below:



Therefore, there is enough space between two reinforcements for splicing!

$$l_{db} := \max \left[\left(\frac{1.25 \cdot \frac{A_b \cdot f_y}{\text{in}^2 \cdot \text{ksi}}}{\sqrt{\frac{f_c}{\text{ksi}}}} \right) \text{in}, 0.4d_b \cdot \frac{f_y}{\text{ksi}} \right] = 37.276 \cdot \text{in}$$

basic tension development length
for bars equal or lesser than #11
AASHTO 5.11.2.1.1

$$l_{db} := \max(l_{db}, 12\text{in}) = 37.276 \cdot \text{in}$$

basic tension development length
can not be less than 12in

Modification factors for basic development length:

Condition₁: For top horizontal or nearly horizontal reinforcement, so placed that more than 12in of fresh concrete is cast below the reinforcement

Condition₁ := "no"

$$A_1 := \begin{cases} 1.4 & \text{if Condition}_1 = \text{"yes"} \\ 1 & \text{if Condition}_1 = \text{"no"} \end{cases}$$

Condition₂: For lightweight aggregate concrete where $f_{ct}(\text{ksi})$ is specified.

Condition₂ := "no"

$f_{ct} := 3\text{ksi}$

$$A_2 := \begin{cases} \max\left(\frac{0.22 \sqrt{\frac{f_c}{\text{ksi}}}}{\frac{f_{ct}}{\text{ksi}}}, 1\right) & \text{if Condition}_2 = \text{"yes"} \\ 1 & \text{if Condition}_2 = \text{"no"} \end{cases}$$

Condition₃: For all-lightweight concrete where f_{ct} is not specified.

Condition₃ := "no"

$$A_3 := \begin{cases} 1.3 & \text{if Condition}_3 = \text{"yes"} \\ 1 & \text{if Condition}_3 = \text{"no"} \end{cases}$$

Condition₄: For sand-lightweight concrete where f_{ct} is not specified.

Condition₄ := "no"

$$A_4 := \begin{cases} 1.2 & \text{if Condition}_4 = \text{"yes"} \\ 1 & \text{if Condition}_4 = \text{"no"} \end{cases}$$

Condition₅: For epoxy coated bars with cover less than $3d_b$ or with clear spacing between bars less than $6d_b$.

Condition₅ := "no"

$$A_5 := \begin{cases} 1.5 & \text{if Condition}_5 = \text{"yes"} \\ 1 & \text{if Condition}_5 = \text{"no"} \end{cases}$$

Condition₆: For epoxy coated bars not covered above.

Condition₆ := "no"

$$A_6 := \begin{cases} 1.2 & \text{if Condition}_6 = \text{"yes"} \\ 1 & \text{if Condition}_6 = \text{"no"} \end{cases}$$

Condition₇: Reinforcement being developed in the length under consideration is spaced laterally not less than 6in center to center, with not less than 3in clear cover measured in the direction of the spacing.

Condition₇ := "no"

$$A_7 := \begin{cases} 0.8 & \text{if Condition}_7 = \text{"yes"} \\ 1 & \text{if Condition}_7 = \text{"no"} \end{cases}$$

$$A_8 := \frac{A_{s_required}}{A_{s_provided}}$$

Condition₉: Reinforcement is enclosed within a spiral composed of bars of not less than 0.25in in diameter and spaced at not more than a 4in pitch.

Condition₉ := "no"

$$A_9 := \begin{cases} 0.75 & \text{if Condition}_9 = \text{"yes"} \\ 1 & \text{if Condition}_9 = \text{"no"} \end{cases}$$

$$l_d := \begin{cases} l_{db} \cdot 1.7 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_7 \cdot A_8 \cdot A_9 & \text{if } A_1 \cdot A_5 \geq 1.7 \vee A_1 \cdot A_6 \geq 1.7 \\ l_{db} \cdot A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 \cdot A_7 \cdot A_8 \cdot A_9 & \text{otherwise} \end{cases}$$

$$l_d = 29.862 \cdot \text{in}$$

tension development length after
factors applied

$$\text{splice} := \begin{cases} \text{if } \frac{A_{s_provided}}{A_{s_required}} \geq 2 \\ \quad \begin{cases} \text{"Class A"} & \text{if } \%A_{s_spliced} = 50 \vee 75 \\ \text{"Class B"} & \text{if } \%A_{s_spliced} = 100 \end{cases} \\ \text{if } \frac{A_{s_provided}}{A_{s_required}} < 2 \\ \quad \begin{cases} \text{"Class B"} & \text{if } \%A_{s_spliced} = 50 \\ \text{"Class C"} & \text{if } \%A_{s_spliced} = 75 \vee 100 \end{cases} \end{cases}$$

splice condition

$$l_{splice} := \begin{cases} l_d & \text{if splice = "Class A"} \\ 1.3l_d & \text{if splice = "Class B"} \\ 1.7l_d & \text{if splice = "Class C"} \end{cases}$$

splice length

$$l_{splice} := \max(l_{splice}, 12\text{in}) = 50.766 \cdot \text{in}$$

splice length can not be less than
12 in

Using smaller diameter but more reinforcement bars will decrease the splice length!

Appendix N: Post-Tensioned Connections in Long Caps

Post-tensioned Connection for Caps

Design Constants:

$f_c := 8\text{ksi}$	concrete ultimate strength
$f_{ci} := 6.2\text{ksi}$	concrete initial strength
$f_{pu} := 270\text{ksi}$	ultimate strand stress
$\text{loss} := 30\text{ksi}$	assumed loss
$E_{\text{strand}} := 28500\text{ksi}$	
$A_s := 0.153\text{in}^2$	area of one strand (0.5" diameter)

Section Properties:

$h := 3.5\text{ft}$	$b := 3.5\text{ft}$
$A := h \cdot b = 1.764 \times 10^3 \cdot \text{in}^2$	$I := \frac{1}{12} \cdot b \cdot h^3 = 2.593 \times 10^5 \cdot \text{in}^4$
$c_b := \frac{-h}{2} = -21 \cdot \text{in}$	$c_t := \frac{h}{2} = 21 \cdot \text{in}$
$z_b := \frac{I}{c_b} = -1.235 \times 10^4 \cdot \text{in}^3$	$z_t := \frac{I}{c_t} = 1.235 \times 10^4 \cdot \text{in}^3$
$e := \frac{h}{2} - 2.5\text{in} - \frac{3.31\text{in}}{2} = 16.845 \cdot \text{in}$	strand location at the middle support with 2.5" clear cover and 3.31" diameter duct which has a capacity of 12 strands

Post tensioning duct information from:

<http://www.dsiamerica.com/products/post-tensioning/strand-post-tensioning-system/installation/ducts.html>

$M_{\text{max}} := -1187\text{kip}\cdot\text{ft}$	maximum moment negative moment in the cap
$f_{\text{tens}} := \frac{M_{\text{max}}}{z_t} = -1.154 \cdot \text{ksi}$	maximum tension stress at the top extreme fiber due to maximum negative moment

ACI Allowable Concrete Stresses

Initial:

$$f_{c_i} := 0.6 \cdot f_{ci} = 3.72 \cdot \text{ksi}$$

$$f_{ti} := -3 \sqrt{\frac{f_{ci}}{\text{psi}}} \text{ psi} = -0.236 \cdot \text{ksi}$$

Service:

$$f_{cs} := 0.6 \cdot f_c = 4.8 \cdot \text{ksi}$$

$$f_{ts} := -7.5 \cdot \sqrt{\frac{f_c}{\text{psi}}} \text{ psi} = -0.671 \cdot \text{ksi}$$

$$f_{ts} := -0.6 \text{ ksi}$$

reducing the tensile stress of concrete a little bit to be on the safe side

$$T_e := - \left[\frac{f_{tens} - f_{ts}}{\left(\frac{1}{A} + \frac{e}{z_t} \right)} \right] = 286.651 \cdot \text{kip}$$

required minimum effective force after 75 years to get the limiting tensile stress at the top fiber

$$T_{e_req} := h \cdot b \cdot 0.125 \text{ ksi} = 220.5 \cdot \text{kip}$$

minimum limit set by ACI

$$T_e := \max(T_e, T_{e_req}) = 286.651 \cdot \text{kip}$$

$$f_{\text{aftertransfer}} := 0.74 \cdot f_{pu} = 199.8 \cdot \text{ksi}$$

$$\Delta f_{\text{jack}} := \frac{0.25 \text{ in}}{42 \text{ ft}} \cdot E_{\text{strand}} = 14.137 \cdot \text{ksi}$$

loss due to slip of anchorage (assuming a 1/4in slip)

$$f_{\text{jacking}} := f_{\text{aftertransfer}} + \Delta f_{\text{jack}} = 213.937 \cdot \text{ksi}$$

$$\text{jacking_stress} := \begin{cases} \text{"is OK"} & \text{if } f_{\text{jacking}} \leq 0.8 \cdot f_{\text{pu}} \\ \text{"is not OK"} & \text{otherwise} \end{cases}$$

$$\text{jacking_stress} = \text{"is OK"}$$

$$f_{\text{eff}} := f_{\text{aftertransfer}} - \text{loss} = 169.8 \cdot \text{ksi}$$

effective stress after 75 years

$$\text{ratio} := \frac{f_{\text{aftertransfer}}}{f_{\text{eff}}} = 1.177$$

$$T_0 := T_e \cdot \text{ratio} = 337.296 \cdot \text{kip}$$

initial prestress force without loss

$$n := \frac{\frac{T_e}{f_{\text{eff}}}}{A_s} = 11.034$$

number of strands

Therefore, 11 strands are required! The cross sectional drawing can be seen below.

