EVALUATION OF BENT CAPS IN REINFORCED CONCRETE DECK GIRDER BRIDGES, PART 2

FINAL REPORT

RS 121



Oregon Department of Transportation

EVALUATION OF BENT CAPS IN REINFORCED CONCRETE DECK GIRDER BRIDGES, PART 2

FINAL REPORT

RS 121

by

Christopher Higgins, Ph. D., Professor Ahmet Ekin Senturk

and

Carl C. Koester School of Civil and Construction Engineering Oregon State University Corvallis, OR 97331

for

Oregon Department of Transportation Research Unit 200 Hawthorne Ave. SE, Suite B-240 Salem OR 97301-5192

September 2008

Technical Report Documentation Page					
1. Report No.	2. Government Accessio	n No.	3. Recipient's Catalog	g No.	
OR-RD-08					
4. Title and Subtitle	1		5. Report Date		
Evaluation of Bent Caps in Reinforce	ed Concrete Deck Girder I	Bridges, Part 2	September 2	.008	
			6. Performing Organi	zation Code	
7. Author(s)			8. Performing Organi	zation Report No.	
Christopher Higgins, Ahmet Ekin Se	enturk and Carl C. Koester	•			
9. Performing Organization Name and Add	ress		10. Work Unit No. (T	RAIS)	
School of Civil and Construction En	gineering				
Oregon State University	5		11. Contract or Grant N	No.	
1500 Jefferson Ave SW			RS 121		
Corvallis, OR 97331					
12. Sponsoring Agency Name and Address			13. Type of Report and Period Covered		
Oregon Department of Transportation			Einal Danart	Final Report	
Research Unit			Final Keport	•	
200 Hawthorne Ave. SE, Suite B-240	0				
Salelli, OK 97501-5192			14. Sponsoring Agenc	y Code	
15. Supplementary Notes					
16. Abstract					
This report describes research conducted	to enable evaluation of ex	isting vintage be	ent cap beams in reinforce	ed concrete	
deck girder bridges. The report is organiz	red into two parts: 1) flexu	iral anchorage ca	apacity response and prec	liction of	
reduced development length due to beneficial column axial compression and 2) structural performance of bent cap systems					
and their analytical evaluation. Each of in	The research results from	ptions of the exp	erimental specimens and	l results of n example to	
demonstrate the rating of an actual 1950'	s vintage RCDG bent cap	beam for contin	uous and single trip perm	it loads.	
	1				
17. Key Words 18. Distribution Statement					
Bent caps, anchorage, structural performance, bridge.			ble from NTIS, and onlin	ie at	
http://www.oregon.gov/ODOT/TD/TP_RES/				RES/	
19. Security Classification (of this report)	20. Security Classification	(of this page)	21. No. of Pages	22. Price	
Unclassified	Unclassified	ļ	365		

Technical Report Form DOT F 1700.7 (8-72)

Reproduction of completed page authorized

Rinted on recycled paper

SI* (MODERN METRIC) CONVERSION FACTORS									
APPROXIMATE CONVERSIONS TO SI UNITS APPROXIMATE CONVERSIONS FROM SI UNITS									
Symbol	When You Know	Multiply By	To Find	Symbol	Symbol	When You Know	Multiply	By To Find	Symbol
		LENGTH					LENGTH	L	
in	inches	25.4	millimeters	mm	mm	millimeters	0.039	inches	in
ft	feet	0.305	meters	m	m	meters	3.28	feet	ft
yd	yards	0.914	meters	m	m	meters	1.09	yards	yd
mi	miles	1.61	kilometers	km	km	kilometers	0.621	miles	mi
		AREA					AREA		
in ²	square inches	645.2	millimeters squared	mm ²	mm ²	millimeters squared	0.0016	square inches	in ²
ft^2	square feet	0.093	meters squared	m ²	m ²	meters squared	10.764	square feet	ft^2
yd ²	square yards	0.836	meters squared	m ²	m ²	meters squared	1.196	square yards	yd ²
ac	acres	0.405	hectares	ha	ha	hectares	2.47	acres	ac
mi ²	square miles	2.59	kilometers squared	km ²	km ²	kilometers squared	0.386	square miles	mi ²
		VOLUME					VOLUMI	E	
fl oz	fluid ounces	29.57	milliliters	ml	ml	milliliters	0.034	fluid ounces	fl oz
gal	gallons	3.785	liters	L	L	liters	0.264	gallons	gal
ft ³	cubic feet	0.028	meters cubed	m ³	m ³	meters cubed	35.315	cubic feet	ft^3
yd ³	cubic yards	0.765	meters cubed	m ³	m ³	meters cubed	1.308	cubic yards	yd ³
NO	ΓE: Volumes greater th	an 1000 L shal	l be shown in m ³ .						
		MASS					MASS		
oz	ounces	28.35	grams	g	g	grams	0.035	ounces	OZ
lb	pounds	0.454	kilograms	kg	kg	kilograms	2.205	pounds	lb
Т	short tons (2000 lb)	0.907	megagrams	Mg	Mg	megagrams	1.102	short tons (2000 lb)	Т
	TEMP	ERATURE	<u>(exact)</u>			TEMP	ERATURI	E (exact)	
°F	Fahrenheit	(F-32)/1.8	Celsius	°C	°C	Celsius	1.8C+32	Fahrenheit	°F
*SI is the symbol for the International System of Measurement									

ACKNOWLEDGEMENTS

This research was funded by the Oregon Department of Transportation. The authors would like to thank Mr. Steven M. Soltesz of the Oregon Department of Transportation Research Unit for his assistance in coordinating this research effort. The opinions, findings and conclusions are those of the authors and may not represent those acknowledged.

DISCLAIMER

This document is disseminated under the sponsorship of the Oregon Department of Transportation in the interest of information exchange. The State of Oregon assumes no liability of its contents or use thereof.

The contents of this report reflect the view of the authors who are solely responsible for the facts and accuracy of the material presented. The contents do not necessarily reflect the official views of the Oregon Department of Transportation.

The State of Oregon does not endorse products of manufacturers. Trademarks or manufacturers' names appear herein only because they are considered essential to the object of this document.

This report does not constitute a standard, specification, or regulation.

TABLE OF CONTENTS

1.0	INTRODUCTION	1
2.0	RCDG BRIDGE BENT CAPS	
2.1	OBJECTIVES AND SCOPE OF THE RESEARCH	5
3.0	LITERATURE REVIEW	7
3.1	CODE PROVISIONS	7
3.	.1.1 ACI 318 American Building Code Requirements for Structural Concrete	7
3. S	.1.2 American Association of State Highway and Transportation Officials LRFD Bridge De.	sign 12
3	13 Canadian Hishway Bridse Desisn Code	
3.	.1.4 Eurocode 2: Design of Concrete Structures	
3.	.1.5 Euro-International Committee for Concrete Publications	
3.2	Earlier Work	
3.3	LITERATURE REVIEW SUMMARY	64
3.4	LIMITATIONS OF DESIGN CODES AND THE CURRENT LITERATURE	68
4.0	EXPERIMENTAL SETUP	
11	Ιντροριοτιον	71
4.1 1 2	Ωσεζον Πεσα στμενίτως Τρανςσωστατίων Ιννεντώρυ Ρενιέψ	
4.2 4	2.1 Column Dimensions	
4.	2.2 Bent Cap Heights	
4.	2.3 Bent Cap Widths	
4.	.2.4 Centerline Reinforcement to Anchorage Reinforcement Ratio	
4.	.2.5 Flexural Tension Reinforcement Anchored in Column Section	77
4.	.2.6 Utility Holes	
4.3	SPECIMEN DESIGN	
4.	.3.1 Specimen Dimensions & Reinforcement Arrangement	
4.	.3.2 Specimen Construction	85
4.4	MATERIAL PROPERTIES	
4. 1	4.1 Concrete	8/
4.	I OADING SCHEME	
ч.5 Л	5.1 Static Tests	88
4	5.2 Fatigue Test	
46	ARRANGEMENT OF MEASURING DEVICES	97
4.	.6.1 Internal Sensor Array	
4.	.6.2 External Sensor Array	
5.0	EXPERIMENTAL RESULTS	101
51	Introduction	101
5 2	RESULTS FROM STATIC TESTS	101
5.2	2.1 Global Structural Behavior	
5.	.2.2 Load – Deformation Response of Specimens	
5.	.2.3 Average Shear Stress and Depth of the Shear Compression Block (cs)	
5.	.2.4 Web Reinforcement	117
5.	.2.5 Anchorage Zone	120

5.2	2.6 Crack Growth in Specimens	
5.2	2.7 Summary Observations from Static Tests	
5.3	RESULTS FROM THE FATIGUE TEST	
5.3	2.1 Precracking Test	
5.3	2.2 Fatigue Test	
5.4	Failure Test	
5.4	1.1 Summary observations from the Fatigue Test	
6.0	ANALYTICAL METHODS	
6.1	INTRODUCTION	
6.2	ACI 318 Design Methods	
6.2	2.1 ACI 318-05 Shear Design Method	
6.2	2.2 ACI 318-99 Deep Beam Equations	
6.2	2.3 ACI 318 Prediction Results	
6.3	MCFT SECTIONAL ANALYSIS APPROACH	
6.4	Strut-and-Tie Models	
6.4	Analysis with a Simple Strut-and-Tie Model	
6.4	2.2 Analysis with a Detailed Strut-and-Tie Model	
6.4	2.3 Complex Strut-and-Tie Models	
6.5	MECHANICAL MODELS	
6.5	.1 Zararis Mechanical Model	
6.5	5.2 Modified Zararis Mechanical Model	
6.6	Non-linear Finite Element Analysis	
6.6	5.1 Non-linear Finite Element Analysis Using VecTor2	
6.6	5.2 The Finite Element Model and Trial Analyses	
6.6	5.3 Constitutive Models for Concrete and Steel Reinforcement	
6.6	6.4 Nonlinear Finite Element Analysis Results	
6.7	SUMMARY RESULTS	
7.0	CONCLUSIONS	
7.1	RECOMMENDATIONS FOR FUTURE WORK	
8.0	REFERENCES	

Appendices

APPENDIX A: CONSTITUTIVE MODELS USED IN NLFEA APPENDIX B: EXPERIMENTAL DATA

TABLE OF TABLES

Table 2.1: Characteristics of RCDG Bridge Bent Caps and Girders	5
Table 3.1: Deep Beam Experiments in the Literature	64
Table 3.2: Analytical Methods Proposed in the Literature	65
Table 4.1: Dimensions of test specimens	82
Table 4.2: Reinforcement of test specimens	83
Table 4.3: Day of Test Compressive and Tensile Strengths of Concrete	87
Table 4.4: Reinforcing steel material properties	
Table 4.5: Fatigue Stress Range Derivation	95
Table 5.1: Structural response characteristics of the specimens	111

Table 5.2: Average Shear Stress and Depth of Shear Compression Block	115
Table 5.3: Development lengths and average bond stresses	128
Table 5.4: Specimen D6.A2.G40#4.F Precracking Structural Response Parameters	134
Table 5.5: Measured Crack Widths at the End of the Precracking Test	135
Table 5.6: Specimen D6.A2.G40#4.F Stirrup Mean Stresses	140
Table 5.7: Specimen D6.A2.G40#4.F Stirrup Stress Ranges	142
Table 5.8: Equivalent Constant Amplitude Stress Ranges	144
Table 5.9: Specimen D6.A2.G40#4.F Crack Width Ranges	146
Table 5.10: Specimen D6.A2.G40#4.F failure test structural response parameters	154
Table 5.11: Specimen D6.A2.G40#4.F peak principle stresses and orientations	155
Table 6.1: ACI 318 Shear Design Provisions Prediction Results	160
Table 6.2: Response 2000 Sectional MCFT Analysis Prediction Results	162
Table 6.3: Types and Efficiency Factors of Truss Members for the Simple STM	165
Table 6.4: Simple Strut-and-Tie Model Prediction Results	167
Table 6.5: Types and Efficiency Factors of Truss Members for the Detailed STM	169
Table 6.6: Detailed Strut-and-Tie Model Prediction Results	172
Table 6.7: Zararis Mechanical Model Analysis Prediction Results	184
Table 6.8: Compressive and tensile forces at failure according to the Zararis Mech. Model	185
Table 6.9: Modified Zararis Mechanical Model Analysis Prediction Results	189
Table 6.10: Modified Zararis Mechanical Model Failure Cases	189
Table 6.11: Confidence Intervals and the Corresponding Resistance Factors	190
Table 6.12: Finite Element Analysis Series	198
Table 6.13: VecTor2 Finite Element Analysis Prediction Results	200
Table 6.14: The Effect of Bond Fatigue on Predicted Specimen Capacities	213

TABLE OF FIGURES

Figure 2.1: Bent cap components and the indirect load transfer mechanism	4
Figure 3.1: ACI 318-05 Graphical description of deep and slender beams	9
Figure 3.2: CHBD simple STM example	.15
Figure 3.3: Influence of anchorage conditions on effective cross-sectional area of struts according to CHBD	.16
Figure 3.4: Example CCC and CCT nodes in Eurocode 2 2004	.20
Figure 3.5: Deep Beam Reinforcement detailing for direct (top) and suspended (bottom) loading in CEB-FIP Mod	del
Code 1990	.23
Figure 3.6: Capacity of confined concrete struts	.25
Figure 3.7: Web reinforcement arrangements, Kong, et al. (1970)	.26
Figure 3.8: Direct and indirect loading related to a / d , Fereig and Smith (1977)	.28
Figure 3.9: Stress condition in web of deep beams, Mau and Hsu (1987)	.30
Figure 3.10: Distribution of transverse compressive stresses for various shear span ratios, Mau and Hsu (1987)	.31
Figure 3.11: Effect of shear span ratio, transverse reinforcement, and longitudinal reinforcement to shear strength	,
Mau and Hsu (1987)	.32
Figure 3.12: The simple and refined strut-and-tie models to predict the ultimate shear strength of deep beams, Sia	.0
(1993)	.34
Figure 3.13: Truss action and arch action in a simple beam, Aoyama (1993)	.35
Figure 3.14: Truss action and arch action in a member subjected to bending moment and shear force, Aoyama	
(1993)	.36
Figure 3.15: Specimen details and loading setup, Zielinski and Rigotti (1995)	.38
Figure 3.16: Free body models for horizontally and inclined reinforced corbels, Zielinski and Rigotti (1995)	.39
Figure 3.17: Typical test specimen from high strength concrete deep beam research, Tan, et al. (1995)	.40
Figure 3.18: Specimens for embedment length research Kong, et al. (1996)	.41
Figure 3.19: Specimen types tested by Tan, et al. (1997)	.42
Figure 3.20: Specimen types tested by Tan, et al. (1997)	.44

Figure 3.21: Specimens from the HSC tests, Foster and Gilbert (1998)	45
Figure 3.22: Specimens from the size effect research, Tan and Lu (1999)	46
Figure 3.23: Normalized ultimate stress and diagonal cracking stress versus overall height, Tan and Lu (1999)	47
Figure 3.24: Details of fatigue test deep beam specimens, and the load setup, Teng, et al. (2000)	49
Figure 3.25: Fatigue Strength of Deep Beams, Teng, et al. (2000)	50
Figure 3.26: Specimen and loading details, Aguilar, et al. (2002)	53
Figure 3.27: Dimensions of the nodal zone, and the basic strut-and-tie models used in the analysis, Matamoros a	nd
Wong (2003)	54
Figure 3.28: Fraction of total shear carried by truss elements in direct strut, horizontal truss, vertical truss, and	
statically indeterminate truss. Matamoros and Wong (2003)	55
Figure 3.29: Forces on elements of a deep beam with web reinforcement (a) with critical diagonal crack and (b)	at
failure. Zararis (2003)	57
Figure 3.30: Strut-and-tie model for simply supported deep beams. Tang and Tan (2004).	59
Figure 3.31: (a) Strut-and-tie model for deep beam with web reinforcement: (b) equilibrium of compression force	e in
strut due to web reinforcement Russo (2005)	60
Figure 3 32: Details for test specimens, Ouintero-Febres et al. (2006)	63
Figure 3 33: Overall geometries for deep beam experiments in the literature	67
Figure 3.34. Analytical methods proposed for deep beams in the literature	68
Figure 4.1: Example vintage RCDG bent can in the ODOT inventory	00
Figure 4.2. ODOT inventory bent can column dimensions	73
Figure 4.2: ODOT inventory Bent Cap Overall Heights	75
Figure 4.4: ODOT inventory Bent Cap Widths	,4
Figure 4.5: ODOT inventory Bent Cap Widths for $h = 72$ in (1829 mm)	75
Figure 4.5: ODOT inventory bent cap widths for $h = 48$ in (102) mm)	75
Figure 4.0. ODOT inventory $\Delta_{\rm ev}/\Delta_{\rm ev}$ ratio	70
Figure 4.7. ODOT inventory anchorage types	/ / 78
Figure 4.8. ODOT inventory anchorage types	70
Figure 4.9. ODOT inventory service hole sizes	/ 9
Figure 4.10. ODOT Inventory service note sizes	
Figure 4.11. Specifien name convention	00
Figure 4.12. Dent cap overall specifien unitensions	01 Q/
Figure 4.15. 72 m. (1829 mm) deep specimen reinforcing details	04 Q/
Figure 4.14. 48 III. (1219 IIIII) deep specifien reinforcing details	04
Figure 4.15. Bent cap specimen remnorcing cage	05
Figure 4.10. Bent cap speciment concrete placement.	00
Figure 4.17. Experimental setup for static tests (loading frames shown, bracing frames not shown for clarity)	09
Figure 4.16. Schematic representation of a single increment of the loading protocol for static tests	90
Figure 4.19. Experimental setup for fatigue test (loading frames showin, bracing frames not showin for clarity)	91
Figure 4.20. Simple FEW of MCKenzie River Bildge for dead load analysis	93
Figure 4.21. Sultup strain fanges for P = 60 kips (207 kiv) (sultup #1 fiear column, sultup #7 fiear stud gifaer)	90
Figure 4.22. Typical instrumentation scheme	102
Figure 5.1: Crack pattern for Specimen Do.A4.Gou#5.5	.102
Figure 5.2: Specimen Do.A4.Gou#5.5 shear span at failure	.103
Figure 5.3: Crack pattern for Specimen D6.A4.G40#4.8	.103
Figure 5.4: Specimen D6.A4.G40#4.S snear span at failure	.104
Figure 5.5: Crack pattern for Specimen D6.A2.G50#5.S.	.104
Figure 5.6: Specimen D6.A4.G50 $\#$ 5.8 shear span at failure	.105
Figure 5.7: Crack pattern for Specimen D6.A2.G40#4.S.	.105
Figure 5.8: Specimen D6.A2.G40#4.S shear span at failure	.106
Figure 5.9: Crack pattern for Specimen D4.A2.G40#4.S.	.107
Figure 5.10: Specimen D4.A2.G40#4.S Shear span at failure	.107
Figure 5.11: Centerline load – deformation plot for specimen D6.A4.G60#5.S.	.108
Figure 5.12: Centerline load – deformation plot for specimen D6.A4.G40#4.S.	.109
Figure 5.13: Centerline load – deformation plot for specimen D6.A2.G60#5.S.	.109
Figure 5.14: Centerline load – deformation plot for specimen D6.A2.G40#4.S.	.110
Figure 5.15: Centerline load – deformation plot for specimen D4.A2.G40#4.S	.110

Figure 5.16: Effective volume used in dead weight calculation	111
Figure 5.17: Diagonal displacements for all specimens at the failed shear span (backbone curves are shown for	
clarity)	112
Figure 5.18: Shear compression block (c_s) and flexural compression block (c)	.113
Figure 5.19: (a) Normalized average shear stress – tension reinforcement stress, (b) Normalized average shear st	tress
- vertical web reinforcement stress	
Figure 5.20: Relationships between the tension reinforcement ratio at the column face, web reinforcement ratio,	and
the normalized average shear stress at failure, for statically loaded specimens having a/h ratio of 1.33	
Figure 5.21: Splitting plane of the critical diagonal crack.	.118
Figure 5.22: Stirrup strains – shear force relation for specimen D6.A2.G60#5.S.	.119
Figure 5.25: Change in suffup stresses at different stages of loading	120
Figure 5.24. Flactured suffues at post-failure (specified Do.A2.040#4.5)	120
Figure 5.25: Relation between the column reaction and strains at the embedded reinforcement at the anchorage 2 (Specimen D6.4.4 C40#4.5 NW)	20ne
(Specifield D0.A4.040#4.5 NW)	122
Figure 5.27: Stress distribution in the embedded reinforcement at the anchorage zone (D6.A2.G40#4.S.SE bar).	123
Figure 5.27. Success distribution in the embedded reinforcement at the anchorage zone (D6 Δ 2 G40#4.S SE bar).	124
har)	125
Figure 5 29: Effect of confining stress on hand strength (Kaester Higgins 2008)	129
Figure 5.29: Change in crack growth in Specimen D6 A2, G60#5.8 for various load levels (asterisks denote	.12)
existence of web reinforcement under the crack clin)	130
Figure 5.31. Offset of the concrete surface along the critical diagonal crack	131
Figure 5.32: Load - deformation response of similarly detailed fatigue and static specimens during initial loading	σ134
Figure 5.32: Specimen D6 A2 G40#4 F south shear span instrumentation layout	135
Figure 5 34 [•] Specimen D6 A2 G40#4 F north shear span instrumentation layout	136
Figure 5.35: Specimen D6.A2.G40#4.F stirrup S-S7 shear – strain relation	.136
Figure 5.36: Specimen D6.A2.G40#4.F SE embedded bar shear – strain relation	137
Figure 5.37: Specimen D6.A2.G40#4.F mean shear load and shear load range during fatigue phase	138
Figure 5.38: Specimen D6.A2.G40#4.F south shear span stirrup mean stresses	139
Figure 5.39: Specimen D6.A2.G40#4.F north shear span stirrup mean stresses	139
Figure 5.40: Specimen D6.A2.G40#4.F south shear span stirrup stress ranges	141
Figure 5.41: Specimen D6.A2.G40#4.F north shear span stirrup stress ranges	141
Figure 5.42: Specimen D6.A2.G40#4.F equivalent constant amplitude stress range	143
Figure 5.43: Specimen D6.A2.G40#4.F crack width ranges	147
Figure 5.44: Specimen D6.A2.G40#4.F average crack width range vs. distance from the face of the column	148
Figure 5.45: Specimen D6.A2.G40#4.F stirrup S-2 mean Stress vs. CWR	149
Figure 5.46: Specimen D6.A2.G40#4.F stirrup S-2 stress range vs. CWR	149
Figure 5.47: Specimen D6.A2.G40#4.F Flexural Steel at centerline mean stress and stress range	150
Figure 5.48: Specimen D6.A2.G40#4.F SE anchorage bar mean stress	151
Figure 5.49: Specimen D6.A2.G40#4.F SE load-deformation relation (50 – 150 kips load range)	151
Figure 5.50: Specimen D6.A2.G40#4.F effect of fatigue loading on structural stiffness	152
Figure 5.51: Specimen D6.A2.G40#4.F Effect of fatigue loading on concrete principal stress magnitudes	153
Figure 5.52: Crack pattern for Specimen D6.A2.G40#4.F at failure	154
Figure 5.53: Concrete principal stresses at rosette R-S2 during failure test	155
Figure 6.1: Response 2000 sectional MCFT analysis results	162
Figure 6.2: Proposed simple strut-and-tie model for bent cap specimens	165
Figure 6.3: Failure mode in simple strut-and-tie model	
Figure 6.4: Proposed detailed strut-and-tie model for bent cap specimens	
Figure 6.5: Failure mode in detailed strut-and-tie model	.171
Figure 6.6: Simple and detailed strut-and-tic model prediction results	.172
Figure 6.7. A complex strut-and-the model for bent cap specimens	.1/5
Figure 0.6. Effect of changes in truss geometry to STM ultimate load capacity	.1/4
Figure 6.9. Effect of changes in truss geometry to prediction results	.1/3
Figure 6.10. Forces acting on the element below the oritical diagonal crack (adapted from Zararia 2002)	170
righte 0.11. Porces acting on the crement below the critical diagonal clack (adapted from Zararis, 2003)	.1/0

Figure 6.12: Stress/Force and strain profiles at the load application section	179
Figure 6.13: The free body diagram employed for calculating the ultimate shear capacity of deep beams in the	
Zararis Mechanical Model	182
Figure 6.14: The free body diagram employed for calculating the ultimate shear capacity of deep beams in the	
proposed Modified Zararis Mechanical Model	183
Figure 6.15: The Zararis and the Modified Zararis Mechanical Models analysis results	183
Figure 6.16: Solution procedure for the Modified Zararis Mechanical Model	188
Figure 6.17: Assumed probability density functions for the prediction bias, and the effect of partial safety factor	ors190
Figure 6.18: Finite element models for (a) 6 ft deep specimens and (b) 4 ft deep specimen	194
Figure 6.19: Predicted load-deformation response for different finite element mesh sizes (Specimen	
D4.A2.G40#4.S)	195
Figure 6.20: Computing time for different mesh sizes (Specimen D4.A2.G40#4.S)	195
Figure 6.21: Predicted load-deformation response for different load step sizes (Specimen D4.A2.G40#4.S)	196
Figure 6.22: VecTor2 NLFEA ultimate shear strength prediction results.	199
Figure 6.23: VecTor2 NLFEA ultimate shear strength prediction results	199
Figure 6.24: Predicted and experimental response of specimen D6.A4.G60#5.S.	202
Figure 6.25: Predicted and experimental response of specimen D6.A4.G40#4.S	203
Figure 6.26: Predicted and experimental response of specimen D6.A2.G60#5.S.	203
Figure 6.27: Predicted and experimental response of specimen D6.A2.G40#4.S	204
Figure 6.28: Predicted and experimental response of specimen D4.A2.G40#4.S	204
Figure 6.29: Computing time for monotonic and cyclic analyses (discrete web reinforcement)	205
Figure 6.30: Experimental and VecTor2 predicted crack pattern for specimen D6.A4.G60#5.S	206
Figure 6.31: Experimental and VecTor2 predicted crack pattern for specimen D6.A4.G40#4.S	207
Figure 6.32: Experimental and VecTor2 predicted crack pattern for specimen D6.A2.G60#5.S	207
Figure 6.33: Experimental and VecTor2 predicted crack pattern for specimen D6.A2.G40#4.S	207
Figure 6.34: Experimental and Vector2 predicted crack pattern for specimen D4.A2.G40#4.S	208
Figure 6.35: Concrete principal compressive stresses for specimen D6.A4.G60#5.S	208
Figure 6.36: Concrete principal compressive stresses for specimen D6.A4.G40#4.S	209
Figure 6.37: Concrete principal compressive stresses for specimen D6.A2.G60#5.S	209
Figure 6.38: Concrete principal compressive stresses for specimen D6.A2.G40#4.S	209
Figure 6.39: Concrete principal compressive stresses for specimen D4.A2.G40#4.S	210
Figure 6.40: Steel reinforcement stresses beyond yield (Specimen D6.A4.G60#5.S)	210
Figure 6.41: Finite element model with 80% debonded stirrups at each shear span	211
Figure 6.42: Predicted and experimental response of specimen D6.A2.G40#4.F.	212
Figure 6.43: Change in predicted capacity for fully bonded and 80% debonded stirrups	213
Figure A.1: Hognestad parabolic concrete compression response (Hognestad, 1951)	A-1
Figure A.2: Modified Park – Kent post-peak concrete compression response (Park et al., 1982)	A-2
Figure A.3: Vecchio 1992A concrete compression softening response (Vecchio, 1992)	A-4
Figure A.4: Bentz tension stiffening response (Bentz, 2000).	A-5
Figure A.5: VecTor2 linear tension softening response (Wong and Vecchio, 2002)	A-6
Figure A.6: Kupfer variable Poisson's ratio model for compression (Kupfer et al., 1969)	A-8
Figure A.7: Ductile steel reinforcement stress-strain response	. A-12

1.0 INTRODUCTION

A research program was undertaken at Oregon State University to assess the remaining capacity and life of conventionally reinforced concrete girders. The research program included field and laboratory testing, as well as analytical approaches to develop a methodology for assessment of diagonally-cracked girders, and the results from the research on girders were reported (*Higgins, et al. 2004*). However, bent caps differ from girders in geometric proportions, the manner of loading, support conditions and steel reinforcing details, resulting in a different type of structural behavior, which identifies the need for research solely focusing on conventionally reinforced concrete bent caps.

2.0 RCDG BRIDGE BENT CAPS

Bent caps are non-redundant cross members that support the main girders, and are oriented transverse to the direction of traffic. Typical vintage RCDG bridge bent caps are *deep beams* that support four to five main girders and are in turn typically supported on two columns, which provide relatively small rotational restraint at the ends. Typical bent cap spans range from 21 to 27 ft with main girders, spaced between 7 and 9 ft, framing into the bent caps. Dimensions for bent caps are in the range of 16 in. wide and the depth varies between 4 and 9 ft. Utility holes are commonly located in the bent caps, at upper corners of each shear span.

Vintage reinforcement details typically contain straight bars for the flexural steel and cutoff of these bars where not required by calculation at design. Anchorage of the flexural steel reinforcement is of particular concern since these details would be considered inadequate by modern standards in which the flexural steel is now required to be hooked at the ends for better anchorage. Previous research on vintage girders (*Higgins, et al. 2004*) showed that anchorage of flexural steel plays a key role in assuring the shear capacity of a reinforced concrete member. It was observed that the presence of diagonal tension cracks at the level of the reinforcing steel increases the stresses on the flexural steel and can lead to the rebar pulling out of the concrete if the bar is not adequately anchored. However, anchorage zones in bent caps differ from those in girders, due to active confinement, such as the presence of normal pressure from the column, transverse to the potential splitting plane.

An important characteristic of bent caps is the indirect load transfer mechanism between girders, the bent cap, and the columns. Bent caps are primarily loaded by reaction forces from the main girders. Bent caps "feel" the applied load indirectly, uniformly distributed along the height of the connected girder, instead of concentrated loads directly applied on top of the member. In return, bent caps transfer the load indirectly to the columns, due to the monolithic construction. As a result of the indirect load mechanism, there is little benefit from concrete confinement at the load application regions. The bent cap, its related components, and the load transfer mechanism are illustrated in Figure 2.1.



Figure 2.1: Bent cap components and the indirect load transfer mechanism

As mentioned before, RCDG bridge bent caps are considered in the "deep beam" category of structural components. Deep beams are structural elements which are mainly characterized by a small shear span to depth ratio (a/d ratio). Numerous definitions of deep beams exist in current code provisions, which are discussed in Section 2.1 of this document.

Due to the small a/d ratio the shear resistance mechanism in deep beams is different than that of slender beams. It is widely recognized that in deep beams, shear is resisted by two separate mechanisms: a truss mechanism analogous to the shear resisting mechanism in slender beams, and an arch mechanism unique to deep members, where the load is directly transferred from the load point to the support through the shear span. In contrast to slender beams, due to nonlinear strain distributions in a cross-section. Consequently, approaches based on the conventional beam theory may not be applicable in determining the internal force distribution. Generally, shear deformations are ignored in calculations of slender beams, since they are negligible when compared to flexural deformations. However, shear deformations are characteristically significant in deep beams, and they may have to be accounted for in calculations. The differences in the characteristic properties of bent caps (deep beams), and girders (slender beams) are summarized in Table 2.1.

Characteristic Property	Bent Caps	Girders
Shear span to depth ratio	$a/d < 2.0 \rightarrow Deep Beam$	$a/d > 2.0 \rightarrow$ Slender Beam
Load Transfer Mechanism	Indirect	Direct
Shear Resistance Mechanism	Arch & Truss Mechanisms	Truss Mechanism
Strain Distribution in cross- section	Significantly Nonlinear	Can be assumed as linear in practice
Bernoulli's Hypothesis	Not Valid	Valid (Enables section analysis)
Shear Deformations	Significant	May be neglected in practice
Shear Capacity Prediction Methods	No widely accepted methodology	Generally based on section analysis

Table 2.1: Characteristics of RCDG Bridge Bent Caps and Girders

2.1 OBJECTIVES AND SCOPE OF THE RESEARCH

The main objectives of this research were to improve the understanding of the structural behavior of diagonally cracked RCDG bridge bent caps with 1950s vintage details, and present analytical methods which are suitable for capacity assessment. The research methodology included a detailed review of the literature, and design codes, collection of data describing structural properties of in-field bent caps, construction and laboratory tests of realistic full-scale bent caps specimens to evaluate strength and behavior for a variety of specimen parameters, development of a unique fatigue load protocol using field data to simulate the effects of service exposure to ambient traffic loading, and assessment of capacity using a range of analytical methods.

3.0 LITERATURE REVIEW

3.1 CODE PROVISIONS

In this part, current code provisions, such as American Building Code Requirements for Structural Concrete (2005), American Association of State Highway and Transportation Officials LRFD Bridge Design Specifications (2004), Eurocode 2 Design of Concrete Structures (2004), Canadian Highway Bridge Design Code (2000), Euro-International Committee for Concrete & International Federation for Prestressing Model Code (1990), and International Federation for Prestressing Recommendations on Practical Design of Structural Concrete (1999) were reviewed with regard to design strength and detailing of deep beams.

3.1.1 ACI 318 American Building Code Requirements for Structural Concrete

The 2005 edition of the ACI-318 Building Code Requirements for Structural Concrete (ACI 318-05) defines deep beams in section 10.7.1, as members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports. These members have either a clear span, measured face-to-face of supports, equal to or less than four times the overall member depth, or regions with concentrated loads within twice member depth from the face of the support.

ACI 318-05 permits design of deep beams either by taking into account nonlinear strain distributions, with the effects of cracking on the stress distribution, or by Strut-and-Tie Method, which was recently incorporated into Appendix A in 2002.

ACI 318-05 specifies that nominal shear capacity, V_n , for a deep beam, not to exceed $10\sqrt{f_c}b_wd$, where f_c' (psi) is the specified compressive strength of the concrete, b_w (in.) is the web width, and d (in.) is the distance from the extreme compression fiber to the centroid of the longitudinal reinforcement. There are also limits on the vertical and horizontal shear reinforcement. The area of shear reinforcement perpendicular to the flexural tension reinforcement, A_v (in²), should not be less than 0.0025b_ws, and the web reinforcement spacing, s (in.) should not exceed the smaller of d/5 or 12 in. The minimum area of shear reinforcement parallel to the flexural tension reinforcement, A_{vh} , is calculated as 0.0015b_ws₂, and

 s_2 should not exceed the smaller of d/5 or 12 in. Starting with the 2002 edition of ACI 318, the maximum spacing of web reinforcement was reduced from 18 in. to 12 in. in order to restrain crack widths.

The code commentary recommends that the longitudinal reinforcement in deep beams should be extended to the supports and anchored by embedment, hooks, or welding to special devices.

In the 1999 edition of ACI 318, the nominal shear strength provided by concrete and web reinforcement for deep beams was calculated as:

$$V_{c} = \left(3.5 - 2.5 \frac{M_{u}}{V_{u}d}\right) \left(1.9\sqrt{f_{c}} + 2500\rho_{w}\frac{V_{u}d}{M_{u}}\right) b_{w}d \quad \text{ACI 318-99 11.29 (3.1)}$$
$$V_{s} = \left[\frac{A_{v}}{s} \left(\frac{1 + \frac{l_{n}}{d}}{12}\right) + \frac{A_{vh}}{s_{2}} \left(\frac{11 - \frac{l_{n}}{d}}{12}\right)\right] f_{y}d \quad \text{ACI 318-99 11.30} \quad (3.2)$$

Where V_c and V_s (kips) are the shear resistances of concrete and reinforcement, M_u (kips-in) and V_u (kips) are the factored moment and shear, d (in.) is the effective height, f'_c is the specified compressive strength of concrete (psi), ρ_w is the main tension reinforcement ratio, b_w (in.) is the beam width, A_v and A_{vh} (in²) are the vertical and horizontal web reinforcement cross-sectional areas, s and s_2 (in.) are the spacing's of vertical and horizontal web reinforcement, and f_y is the yield strength of the web reinforcement. However, these provisions were removed due to the severe discontinuities in design strength when clear span to overall member depth ratio was varied. Instead, a new appendix to ACI 318 was introduced in 2002, which provides an alternative methodology for design of deep beams called the Strut-and-Tie Model (STM) approach. This approach represents limit states, utilizing the lower-bound theorem of plasticity, which means the plastic design assures that none of the elements in a STM is loaded beyond its capacity. ACI 318-05 Appendix A presents a limit state design tool, rather than a direct approach to calculate shear and moment capacities of an existing member.

ACI 318-05 Appendix A defines B-regions (beam or Bernoulli regions) as a portion of a member in which the plane sections assumption of flexure theory can be applied. In contrast, D-regions (discontinuity or disturbed regions) are defined as the portion of a member within a distance equal to the height of the member, from a force discontinuity or a geometric discontinuity, where plane sections

assumption is not applicable. According to St. Venant's principle, a local disturbance, such as a concentrated load or reaction dissipates approximately within a distance which is equal to the height of the member. For instance, it can be stated that D-regions extend away from a concentrated load, support reactions, or abrupt changes in geometry a length approximately equal to the height of the member. ACI 318 states that the area between two D-regions can be treated as B-regions, and the traditional shear design procedures can be applied, where beam theory is applicable. There may be cases where two D-regions overlap. In that case, the whole area can be considered as a single D-region for design purposes.

ACI 318-05 Appendix A distinguishes deep beams from slender beams using the shear span to overall height ratio, a_v/h . Accordingly, a deep beam has an a_v/h ratio less than or equal to 2, otherwise it is considered as a slender beam. The ACI distinction between deep/slender beams and beam/discontinuity regions are illustrated in Figure 3.1.



Figure 3.1: ACI 318-05 Graphical description of deep and slender beams

The STM approach is defined as building a truss model of a structural member, or a D-region in such a member, consisting of compressive struts, and tension ties connected at nodes, which are capable of transferring loads between truss members, as well as to the supports or adjacent B-regions. Struts are the prismatic or bottle shaped compression members, which represent the resultant of a parallel or fan-shaped compression field. Ties are tension members which may be prestressed or conventional steel in a reinforced concrete member. The meeting points of struts, ties and concentrated loads are defined as nodes and the concrete surrounding the node is called a nodal zone. At least three forces must act at a node to maintain equilibrium. Depending on forces acting at a node, the nodes are classified as C-C-C, C-C-T, C-T-T, and T-T-T nodes, where C is compressive and T is tensile forces, respectively.

ACI 318-05 Appendix A provides a basic design procedure for D-regions, by means of a STM, having struts, ties, and nodes. The selected truss model must be capable of transferring all concentrated loads to the supports or adjacent B-Regions, and must be in equilibrium under applied loads. The design steps for building a STM involve defining the D-regions in a member, computing resultant forces on D-region boundaries, selecting an appropriate truss model, calculating the internal forces of the struts and ties in the system, and determining the effective widths of the struts and nodal zones according to the internal forces of the struts and ties. Ties are permitted to cross struts, but struts cannot overlap each other. ACI 318-05 also indicates that the minimum angle between the struts and ties should not be less than 25 degrees. The basic design equation for struts, ties and nodal zones is:

$$\phi F_n \ge F_u$$
 ACI 318-05 A-1 (3.3)

Where F_u is the factored force acting in a tie, strut, or nodal zone, F_n is the nominal capacity of a tie, strut, or nodal zone, and ϕ is the strength reduction factor, having a value of 0.75 for all cases.

The nominal strength of a strut, F_{ns} , should be calculated for both ends of a strut, and the minimum value should be taken as:

$$F_{ns} = f_{ce}A_{cs}$$
 ACI 318-05 A-2 (3.4)

Where A_{cs} (in²) is the cross-sectional area of the strut, and f_{ce} (psi) is the smaller of the effective compressive strength of the concrete in the strut or in the nodal zone, which are given as:

$$f_{ce} = 0.85\beta_s f'_c$$
 ACI 318-05 A-3 (3.5)

$$f_{ce} = 0.85\beta_n f'_c$$
 ACI 318-05 A-8 (3.6)

Where β_s and β_n parameters are strut efficiency factors, depending on the strut shapes (uniform, bottle-shaped etc.), and amount of transverse reinforcement crossing the strut. Compressive forces in the strut can be assumed to spread at a slope of 2 longitudinal to 1 transverse relative to the axis of the strut. A verifying equation to determine the efficiency factors for bottle shaped struts, as long as $f_c^{'}$ is less than 6000 psi, is given as:

$$\sum \frac{A_{si}}{b_s s_i} = \sin \alpha_i \ge 0.003 \qquad \text{ACI 318-05 A-4}$$
(3.7)

Where A_{si} is the total area of surface reinforcement at spacing s_i in the *i*-th layer of reinforcement crossing a strut at an angle α_i to the axis of the strut. If a uniform cross-sectional strut is considered, the efficiency factor β_s should be taken as 1.0. For bottle-shaped struts, where the width of the midsection of the strut is larger than the width at the nodes, the efficiency factor should be taken as 0.75, if Equation 3.7 is satisfied, or as 0.60λ , where λ is a modification factor related to the unit weight of the concrete. The efficiency factor for struts in tension members, or the tension flanges of members should be taken as 0.40, which reflects that these struts need to transfer compressive forces across cracks in a tension zone. ACI 318-05 recommends the value 0.60 for all the other cases, such as struts in the web of the beam, where parallel diagonal cracks divide the web into inclined struts.

The nominal strength of a tie, F_{nt}, is calculated as:

$$F_{nt} = A_{ts}f_v + A_{tp}(f_{se} + \Delta f_p)$$
 ACI 318-05 A-6 (3.8)

Where A_{ts} (in²) is the cross-sectional area of the reinforcing bars, f_y (psi) is the yield strength of the reinforcing bars, f_{se} (psi) is the effective stress in prestressing steel, Δf_p (psi) is the increase in prestressing steel stress due to factored loads, and A_{tp} (in²) is the cross-sectional area of the prestressing steel.

The nominal compression strength of a nodal zone, F_{nn} , similar to strut and tie strengths is given as:

$$F_{nn} = f_{ce}A_{nz}$$
 ACI 318-05 A-7 (3.9)

Where f_{ce} (psi) is the effective compressive strength of the concrete in the nodal zone, which was previously defined in Equation 3.5 and Equation 3.6, and A_{nz} (in²) is the smaller of the area of the face of the nodal zone on which F_u acts, taken perpendicular to the line of action of F_u , or the area of a section through the nodal zone, taken perpendicular to the line of action of the resultant force on the section.

3.1.2 American Association of State Highway and Transportation Officials LRFD Bridge Design Specifications

American Association of State Highway and Transportation Officials LRFD Bridge Design Specifications 2004 (AASHTO – LRFD) define deep beams in AAHSTO – LRFD Section 5.2 as deep components, in which the distance from the point of no shear to the face of the support is less than twice the effective depth, or as components in which a load causing more than one-third of the shear at a support is closer than twice the effective depth from the face of the supports. AASHTO – LRFD does not provide a direct assessment method to calculate the shear and moment capacities of existing deep beams. However, it contains provisions for STM approach, as a limit states design tool. Introduced in the 2004 edition of AASHTO – LRFD, use of STM approach is recommended where the conventional methods of design is not applicable due to nonlinear strain distributions. The provisions specify that rather than determining the moment and shear capacities at different sections along the span, the flow of compressive stresses going from the loads to the supports, and the required tension force between the supports must be established.

AASHTO – LRFD provides separate design requirements for transverse reinforcement in deep beams, for both strength and serviceability. The minimum transverse reinforcement area, A_v (in²), within distance s (in.) is given as:

$$A_v \ge 0.0316\sqrt{f_c} \frac{b_v s}{f_y}$$
 AASHTO - LRFD 5.8.2.5-1 (3.10)

Where b_v (in.) is the minimum web width, s (in.) is the spacing of stirrups, $f_c^{'}$ (ksi) is the specified compressive strength of concrete for use in design, and f_y (ksi) is the specified minimum yield strength of the transverse reinforcement.

AASHTO – LRFD 5.6.3.6 has a special requirement, if STM is used. According to the specifications, in order to ensure minimum ductility and control crack growth, an orthogonal grid of reinforcement with a minimum spacing of 12 in. is required near each face. In addition to this, the ratio of the reinforcement area to the gross concrete area should not be less than 0.003 in longitudinal and transverse directions.

For deep beams, AASHTO – LRFD specifies detailing requirements such that the factored tensile resistance, N_R (kips), of transverse reinforcing bars must satisfy:

$$N_{R} = \phi f_{y} A_{s} \ge 0.12 b_{v} s$$
 AASHTO - LRFD 5.13.2.3-1 (3.11)

For both horizontal and vertical components of the orthogonal reinforcing grid, where b_v (in.) is the width of the web, f_y (ksi) is the yield strength of reinforcing steel, A_s (in²) is the area of steel in distance s (in.), which is the spacing of the reinforcement, and ϕ is the resistance factor having a value of 0.90.

Similar to ACI 318-05, AASHTO – LRFD has separate provisions for the design strength of compressive struts, tension ties, and nodal zones. The design strength of struts and ties are taken as:

$$P_{r} = \phi P_{n}$$
 AASHTO LRFD 5.6.3.2-1 (3.12)

Where P_r is the factored resistance of struts and ties, P_n is the nominal resistance of strut or tie, and ϕ is the resistance factor for tension or compression, having a value of 0.70 for compression struts, and a value of 0.90 for tension ties in strutand-tie models.

Like ACI 318-05, AASHTO – LRFD limits the effective compressive stresses in the struts and nodal zones. Unlike ACI 318-05, which considers the shape of the struts and the amount of transverse reinforcement crossing a strut, AASHTO – LRFD bases the factors for calculating the effective compressive stresses on Modified Compression Field Theory (MCFT) (*Vecchio and Collins 1986*).

The limiting compressive stress, f_{cu} , in a strut is given as:

$$f_{cu} = \frac{f'_{c}}{0.8 + 170\varepsilon_1} \le 0.85 f'_{c}$$
 AASHTO LRFD 5.6.3.3.3-1 (3.13)

In which:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \alpha_s$$
 AASHTO LRFD 5.6.3.3.3-2 (3.14)

Where ε_s is the tensile strain in the concrete in the direction of the tension tie α_s is the smallest angle between the compressive strut and adjoining tension ties, and f'_c (ksi) is the specified compressive strength. It is assumed that the concrete can withstand compressive stresses up to 0.85 f'_c as long as it is not subjected to principle tensile strains larger than 0.002, and compression struts are not crossed or joined by tension ties. Equation 3.13 takes into account the imposed tensile strains on the concrete due to bonding between the reinforcement and concrete. The expression for ε_1 is based on the assumption that the principal compressive strain in the direction of the strut is 0.002, and the tensile strain in the direction of tension tie equals ε_s , which can be taken as the tensile strain due to factored loads in the reinforcement bars. From Equation 3.13 and Equation 3.14, it is seen that the limiting compressive stress increases as the angle between the compressive strut and the tension tie increases. In AASHTO – LRFD, the strut angle has an upper limit of 65° relative to the tie.

The nominal resistance of a tension tie, P_n , is calculated in a manner similar to ACI 318-05, as:

$$P_n = f_v A_{st} + A_{ps} (f_{pe} + f_v)$$
 AASHTO LRFD 5.6.3.4.1-1 (3.15)

Where f_y (ksi) is the yield strength of the mild steel longitudinal reinforcement, and A_{st} (in²) is the total area of longitudinal mild steel reinforcement in the tie, A_{ps} (in²) is the area of prestressing steel, and f_{pe} (ksi) is the effective stress in the prestressing steel after losses.

For nodal zones, AASHTO – LRFD code limits the concrete compressive stress according to the presence of tension ties. The concrete compressive strength in the node regions of the struts is limited to $0.85\phi f_c^{'}$ for node regions bounded by compressive struts and bearing areas, $0.75\phi f_c^{'}$ for node regions anchoring one tension tie, and $0.65\phi f_c^{'}$ for node regions anchoring tension ties in more than one direction. The resistance factor ϕ has a value of 0.70 for struts and nodes, and 0.90 for ties.

3.1.3 Canadian Highway Bridge Design Code

Canadian Highway Bridge Design Code 2000 (CHBD) defines deep beams as members having a span-to-depth ratio of less than 2.0, where for continuous spans the effective span is taken as the distance between points of contraflexure due to dead load. CHBD presents a sectional analysis method, but warns that that such a methodology for analyzing a deep beam may not be accurate, since this method assumes the reinforcement required at a particular section only depends on the moment, shear, and axial force at the section, regardless of the loading and support conditions. Moreover, the sectional analysis in the Canadian code assumes that the shear flow remains constant through the height, d_v , which is defined as the internal force couple moment arm, and that the longitudinal strain distribution has a linear behavior over the height. For a deep beam these assumptions are not assured. CHBD does not provide a direct assessment method for calculating the moment and shear capacities of existing deep beams, but recommends the flow of forces through the complete structure be considered, rather than a sectional analysis and further recommends use of STM, which permits structural idealization as a truss formed by a series of reinforcing steel tension ties, and concrete compressive struts which are interconnected at nodes capable of transferring the factored loads to the supports. A simple STM of a deep beam from CHBD is shown in Figure 3.2.



Figure 3.2: CHBD simple STM example

CHBD requires the compressive force; F_{cs} (N), in a strut does not exceed a limit value, calculated as:

$$F_{cs} = \phi_c f_{cu} A_{cs}$$
 CAN/CSA-S6-00 Clause 8.10.3.1 (3.16)

Where f_{cu} (MPa) is the limiting compressive stress in the strut, computed from Equation 3.17, A_{cs} (mm²) is the effective cross-sectional area of the strut, which is calculated considering the available concrete area, and the anchorage conditions at the end of strut, as shown in Figure 3.3, and ϕ_c is the resistance factor for concrete having a value of 0.75.



Figure 3.3: Influence of anchorage conditions on effective cross-sectional area of struts according to CHBD

$$f_{cu} = \frac{f'_{c}}{0.8 + 170\varepsilon_1} \le 0.85 f'_{c}$$
 CAN/CSA-S6-00 Clause 8.10.3.3 (3.17)

Where $f_c^{'}$ (MPa) is the specified compressive strength of the concrete, and ϵ_1 is calculated as:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \theta_s$$
 CAN/CSA-S6-00 Clause 8.10.3.3 (3.18)

In which ε_s is the strain in the tensile tie inclined at an angle θ_s to the compressive strut. CHBD uses similar formulation and assumptions as AASHTO – LRFD. However, the influence of anchorage conditions on the effective cross-sectional area of struts and the calculation of the limiting compressive stress in the concrete strut are explained more directly, as illustrated in Figure 3.3. Moreover, CHBD suggests that for a strut anchored by tensile reinforcement, the effective concrete area extends a distance of up to six bar diameters from the anchored bar.

The maximum amount of tensile force, which can be carried by a tie, F_s (N), is calculated as:

$$F_s = \phi_s f_y A_{st} + \phi_p f_{py} A_{ps}$$
 CAN/CSA-S6-00 Clause 8.10.4.1 (3.19)

Where f_y (MPa) and f_{py} (MPa) are the specified yield strengths for reinforcing bars and prestressed tendons, respectively, A_{st} (mm²) and A_{ps} (mm²) are the crosssectional area of the reinforcing bars, and prestressed tendons in the tie, respectively, and ϕ_s , and ϕ_p are the resistance factors for reinforcing bars and prestressed tendons, having values of 0.90 and 0.95, respectively.

CHBD requires that the reinforcement in the tension tie be anchored, so it is capable of carrying the calculated tensile force. If straight reinforcement extends a distance beyond the inner edge of the node region less than the required development length, then the tensile strength of the bar is reduced, assuming the stress decreases linearly over the development length distance.

CHBD has the same provisions for limiting stresses in nodal zones as AASHTO – LRFD. For detailing, CHBD requires an orthogonal grid of reinforcing bars at each face. The maximum spacing for the reinforcing bars in the orthogonal grid should not exceed 300 mm, the reinforcement ratio should be at least 0.3%, and the maximum amount of reinforcing bars at each face should be less than 1500 mm²/m.

3.1.4 Eurocode 2: Design of Concrete Structures

The European Standard EN 1992-1-1:2004 (Eurocode) was published by European Committee for Standardization in 2004. The Eurocode exists in three official languages, which are English, French and German. The English version, which is also the current British Standard, was reviewed in this section.

The Eurocode defines a deep beam as a member for which the span is longer than three times the overall section depth. The Eurocode provides specific detailing requirements for deep beams. Orthogonal reinforcement grid is required at each face, with a minimum reinforcement ratio of 0.1%, but not less than 150 mm²/m in each face, and each direction. The distance between two adjacent bars of the mesh should not exceed the lesser of twice the deep beam thickness or 300 mm. Lastly, the reinforcement, corresponding to the ties in the design model, should be fully anchored for equilibrium in the nodal zone, by bending the bars, by using U-hoops, or by anchorage devices, unless the distance between the node and the end of the beam is greater than the required anchorage length.

The Eurocode presents strut-and-tie models as an analysis option for members where a nonlinear strain distribution exists. The code defines strut-and-tie models as simplified representations of the actual members, consisting of struts representing compressive stress fields, ties representing the reinforcement, and nodes connecting struts and ties, with all the truss members maintaining their equilibrium with the applied loads in the ultimate limit state. Suitable strut-and-tie models must adopt stress trajectories, and distributions from linear-elastic theory, or the load path method.

The design strength for a concrete strut in a region with transverse compressive stress or no transverse stress, $\sigma_{Rd,max}$, is calculated as:

$$\sigma_{\rm Rd,max} = f_{\rm cd}$$
 EN 1992-1-1:2004 6.55 (3.20)

Where f_{cd} (MPa) is the compressive design strength of the concrete, to be calculated as:

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$$
 EN 1992-1-1:2004 3.15 (3.21)

Where f_{ck} (MPa) is the characteristic compressive cylinder strength of concrete at 28 days, γ_c is the partial safety factor for concrete taking a value of 1.5 or 1.2 for persistent/transient and accidental design situations, respectively, and α_{cc} is a coefficient taking into account long term effects on the compressive strength and of

unfavorable effects resulting from the manner in which the load is applied. The Eurocode states that the α_{cc} factor should take a value between 0.8 and 1.0 according to the country in which the code is applied, but recommends a value of 1.0.

The Eurocode requires that the design strength of a concrete strut should be reduced in cracked compression zones, and calculated as:

$$\sigma_{\rm Rd,max} = 0.6 \upsilon' f_{\rm cd}$$
 EN 1992-1-1:2004 6.56 (3.22)

Where the value of υ' is calculated according to the National Annex of the country in which the code is applied, but a recommended formula is also presented in Equation 3.23.

$$\upsilon' = 1 - f_{ck} / 250$$
 EN 1992-1-1:2004 6.57N (3.23)

No specific formula is given for the design strength of transverse ties and reinforcement in the Eurocode. However, the design yield strength f_{yd} can be determined as:

$$f_{yd} = f_{yk} / \gamma_s$$
 EN 1992-1-1:2004 Clause 3.2.7 (3.24)

Where f_{yk} is the characteristic yield strength of reinforcement steel, and γ_s is the partial safety factor for reinforcement with a value of 1.15 or 1.0 for persistent/transient and accidental loading situations, respectively.

The Eurocode provides two cases for calculating the design values for the compressive stresses within nodes, as presented in Figure 3.4. The first is called "in compression nodes" where no ties are anchored at the node, and the latter is called "in compression – tension nodes" with anchored ties provided in one direction. The design stress for in compression nodes is computed as:

$$\sigma_{\rm Rd,max} = k_1 \upsilon' f_{\rm cd}$$
 EN 1992-1-1:2004 6.60 (3.25)

and the design stress for in compression – tension nodes is computed as:

$$\sigma_{\rm Rd,max} = k_2 \upsilon' f_{\rm cd}$$
 EN 1992-1-1:2004 6.61 (3.26)

Where k_1 and k_2 are coefficients recommended to be taken as 1.0, and 0.85 respectively. The υ' factor should be calculated according to Equation 3.23, as explained previously.



Figure 3.4: Example CCC and CCT nodes in Eurocode 2 2004

The Eurocode allows an increase in the design values for the compressive stresses within nodes by up to 10%, if triaxial compression is ensured, all angles between struts are greater than or equal to 55 degrees, the stresses applied at supports or at point loads are uniform, and the node is confined by stirrups, the reinforcement is arranged in multiple layers, or the node is reliably confined by means of bearing arrangement or friction.

3.1.5 Euro-International Committee for Concrete Publications

3.1.5.1 CEB-FIP Model Code 1990

CEB-FIP Model Code 1990 (CEB-FIP) which was published by the Euro-International Committee for Concrete in association with the International Federation for Prestressing, is a general document serving as a basis for the design of buildings and civil engineering works in structural concrete using normal-weight aggregates. CEB-FIP does not cover specific topics such as design of bridges or highways, but it can be considered as a general guide for development of national and international structural design codes. CEB-FIP employs partial safety factors for both demand and resistance. The loading demands are increased, whereas the material strengths are decreased with the use of partial safety factors. CEB-FIP does not parametrically define deep beams, but provides three design methods specifically, for deep beams and walls. The provisions allow design by linear analysis, analysis by statically admissible stress fields, and nonlinear analysis for use with ultimate limit state (ULS) and service limit state (SLS) design. The linear analysis uses the theory of elasticity, assuming Poisson's ratio of 0.0 to 0.2. The provisions indicate that in most cases, only numerical solutions such as finite difference, and finite element methods or boundary element methods are suitable for such linear analysis. Analysis by statically admissible stress fields employs the use of strut-and-tie method. According to CEB-FIP, a member can be simplified as an equivalent truss, consisting of concrete struts as compressive members, steel reinforcement as tensile tie members, and employs nodal regions as connections between truss components. The provisions require that the evaluated stress distribution in the equivalent truss system must closely represent the linear analysis results. For the non-linear analysis, CEB-FIP recommends use of numerical methods, by taking into account the nonlinear material properties, but no detailed information is provided for this method. CEB-FIB provides a generalized design methodology for deep beams. An analysis method for estimating shear and moment capacities of existing deep beams is not presented as it is the case with previously reviewed codes.

The model code provides general allowable stresses which also can be applied to compressive struts, tension ties, and nodal zones. The angle between struts and ties is recommended to be larger than 45 degrees. The design strength of the zones under effectively uniaxial compression (struts) is calculated for uncracked zones as:

$$f_{cd1} = 0.85 \left[1 - \frac{f_{ck}}{250} \right] f_{cd}$$
 CEB-FIP Model Code 6.2-4 (3.27)

and for cracked zones as:

$$\mathbf{f}_{cd2} = 0.60 \left[1 - \frac{\mathbf{f}_{ck}}{250} \right] \mathbf{f}_{cd} \qquad \text{CEB-FIP Model Code 6.2-5} \quad \textbf{(3.28)}$$

Where f_{cd1} (MPa) is the allowable average stress in concrete for uncracked zones, f_{cd2} (MPa) is the allowable average stress in concrete for cracked zones, f_{ck} (MPa) is the characteristic strength of concrete in compression, and f_{cd} (MPa) is the design strength of concrete in compression. The design strength of concrete, f_{cd} , is determined by dividing concrete characteristic strength, f_{ck} , by the partial safety factor γ_c , taking a value of 1.5 or 1.2, depending on whether the design situation addresses persistent/transient or accidental loading.

The design strength of tension ties is computed as:

$$f_{vtd} = f_{vtk} / \gamma_s$$
 CEB-FIP Model Code 6.2-8 (3.29)

Where f_{ytd} (MPa) is the design yield strength of steel in tension, f_{ytk} (MPa) is the characteristic yield strength of steel in tension, and γ_s is the partial safety factor for steel, taking a value of 1.15 or 1.0, depending on whether the design situation addresses persistent/transient of accidental loading.

CEB-FIP requires nodal zones to be checked both for stresses from the compressive struts in the node, and anchorages of ties connecting to the node. Average design stresses in any surface of the node should not exceed f_{cd1} for nodes where only compression nodes meet and f_{cd2} for nodes where main tensile bars are anchored.

CEB-FIP provides detailing instructions for deep beams. For simply supported deep beams, the main longitudinal reinforcement is required to be uniformly distributed over a depth measured from the lower face of the beam, equal to the lesser value of 0.12h, or 0.12l, where h is the total height of the beam, and l is the design span. If the load is applied to the top of the beam, which is called direct loading, an orthogonal reinforcement grid consisting of horizontal layers and stirrups should be arranged. For each direction, a reinforcement ratio of 0.2% is required. If the load is applied at the bottom of the beam, which is called suspended loading, additional stirrups are required to the orthogonal grid described previously. The purpose of these additional stirrups is to transmit the total load from the lower application level to the level corresponding to the lesser of h, or l. The reinforcement detailing for direct and suspended loading are presented in Figure 3.5.



Figure 3.5: Deep Beam Reinforcement detailing for direct (top) and suspended (bottom) loading in CEB-FIP Model Code 1990

3.1.5.2 CEB-FIP Recommendations 1999

The FIP Recommendations for Practical Design of Structural Concrete (CEB-FIP Recommendations) is considered as a state of the art document, which was published by the International Federation of Prestressing in 1999. The CEB-FIP Recommendations is based on the CEB-FIB Model Code 1990, but extends the scope on Strut-and-Tie Models as a supplement, providing much more detail on the design and detailing process.

The CEB-FIP Recommendations document introduces efficiency factors for calculating the capacity of a concrete strut, considering crack widths and geometrical disturbances. The resisting force of a strut, F_{Rcd} , is calculated as:

$$F_{Red} = A_c f_{cd,eff}$$
 CEB-FIP Recommendations 1999 5.6 (3.30)

Where $A_c (mm^2)$ is the area of the strut, and $f_{cd,eff}$ (MPa) is the effective compressive strength of the strut, depending on the state of stress and strain, as well as on the crack widths and geometrical disturbances. The effective compressive strength of the strut is calculated as:
$f_{cd,eff} = v_1 f_{1cd}$ or $f_{cd,eff} = v_2 f_{1cd}$ CEB-FIP Recommendations 1999 5.5

(3.31)

For an uncracked strut, where a rectangular stress block is used instead of a realistic stress distribution, the reduction factor v_1 is used, which is calculated as;

$$v_1 = (1 - f_{ck} / 250)$$
 CEB-FIP Recommendations 1999 5.7 (3.32)

Where f_{ck} (MPa) is the characteristic compressive cylinder strength of concrete at 28 days.

The efficiency factor υ_2 may have different values for different situations. For struts with cracks parallel to the strut and bonded transverse reinforcement, the value is 0.80, since the strength of the strut is reduced due to the transverse tension and disturbances by the reinforcement, and the irregular crack surfaces. For struts transferring compression across cracks with normal crack widths, such as in webs of beams, the value is 0.60. Lastly, for struts transferring compression across large cracks, such as in members with axial tension or flanges in tension the value is 0.45. The CEB-FIP Recommendations also address the presence of confinement around concrete struts. Strut capacity can be increased when transverse reinforcement confines the concrete core. The increase in strut strength depends on a volumetric reinforcement ratio, and the increased capacity can be calculated as:

 $f_{ccd} = (1+1.6 \alpha \omega_w) f_{1cd}$ CEB-FIP Recommendations 1999 5.11(3.33)

Where α and ω_w take values depending on the geometric properties of the section, and the placement of reinforcement, as shown in Figure 3.6.

The strength of nodal zones depends on the stress state at the node. For biaxial compression, an efficiency factor of 1.20 is used. If a triaxial compression stress state is considered (CCC nodes), then the efficiency factor is taken as 3.88.



Figure 3.6: Capacity of confined concrete struts

3.2 EARLIER WORK

In this part, previous experimental and theoretical research in the literature is reviewed, and presented in a chronological manner.

Clark (1951) conducted one of the earliest experimental studies on the shear strength of deep beams. Clark observed that the shear capacity of a beam increased with the strength of concrete when other factors were kept the same. Clark also concluded that the strength in shear was inversely proportional with a/d ratio. In addition, the resistance to shear was found to vary as the square root of web reinforcement and the first power of the ratio of tensile reinforcement.

Ramakrishnan and Ananthanarayana (1968) tested 26 single span rectangular deep beams having a constant span length of 27 in. (69 cm) center to center, and span to depth ratios varying between 0.94 and 2.04. Loads were applied as four point bending, three point bending and uniformly distributed for various specimens. Plain mild steel round bars were used as tensile reinforcement in all of the specimens, while four of the specimens contained web reinforcement, as well. The researchers concluded that in reinforced concrete deep beams, especially with low shear span to depth ratios, the loads were primarily transferred to the supports with direct compression, leading to diagonal-tension shear failures. It was further stated that the failure mechanism in deep beams failing by diagonal tension was similar to that in a concrete cylinder under diametral compression, such as in a tensile splitting test. Ramakrishnan and Ananhanarayana proposed a basic formula for predicting the ultimate shear capacity of a deep beam, using the splitting strength of the concrete, calculated from:

$$P_{c} = \beta K f_{t} b H$$
 (3.34)

Where β is a factor related to the shear span in the beam, having a value of 2 for central concentrated load and symmetrical two-point loading, K is the splitting coefficient having a value of 1.57 for the tensile splitting test, f_t (psi) is the cylinder splitting strength, b (in.) is the beam width, and H (in.) is the overall depth of the beam.

Kong, et al. (1970) investigated the influence of web reinforcement on the behavior of simply supported deep beams. They tested 35 rectangular deep beam specimens of 36 in. (91.5 cm) overall length, width of 3 in. (7.6 cm) and overall span to depth ratios varying between 1 and 3, under four point bending. Undeformed plain round steel was used for tensile reinforcement. The arrangements for web reinforcement are shown in Figure 3.7.



Figure 3.7: Web reinforcement arrangements, Kong, et al. (1970)

During tests, they observed crack patterns starting with flexural cracks at the center at the early stages of loading, followed by diagonal cracks propagating between the loading points and the supports. The failure occurred mostly by splitting of the beam by one of the diagonal cracks between the loading points and the supports. They observed concrete crushing near the loading points or the supports usually as a secondary effect, and a few times the beams failed by crushing of the concrete strut between two parallel diagonal cracks. Kong, et al. concluded that the effectiveness of the web reinforcement to limit deflections and the crack widths highly depends on overall span to depth (L/d) and shear span to depth (a/d) ratios. It was stated that for low L/d and a/d ratios, horizontal web reinforcement placed near the bottom was the most effective, whereas vertical stirrups performed better as L/d and a/d ratios were increased.

Manuel, et al. (1971) tested 12 reinforced concrete deep beam specimens with varying L/d and a/d ratios, and keeping the other parameters constant. They used specimens with a constant cross-section of 4 in. x 18 in. (10 cm x 46 cm), while the L/d and a/d ratios ranged from 1 to 4 and 0.3 to 1.0, respectively. According to their test results, they concluded that the ultimate strength of reinforced concrete deep beams was influenced significantly by the a/d ratio rather than the L/d ratio.

Zsutty (1971) performed a regression analysis to predict the shear capacity of deep beams with and without web reinforcement. The researcher stated that the proposed formula should be used for practical prediction only, since the formula was not meant to convey detailed information concerning the actual or theoretical mechanisms of failure.

Fereig and Smith (1977) studied the effect of direct and indirect loading of beams with short shear spans. They tested a total of 18 beams under direct and indirect loading, with and without web reinforcement, having a/d ratios of 0.5, 1.0 and 2.0. The authors found that the nominal shear stress at failure and the shear span-to-depth ratio became inversely proportional when the a/d ratio was below a limit value which depended on the type of loading. According to the experimental results, the limit value for directly loaded beams without web reinforcement was approximately 2.5, whereas there was a smaller gain in strength for the indirect loading case, and the increase occurred below an a/d ratio of 1.5. They also observed that there was a smaller gain for strength in directly loaded beams when web reinforcement was added; however, the strength contribution was more significant for the indirect loading case. The effect of direct and indirect loading related to a/d ratio is shown in Figure 3.8.



Figure 3.8: Direct and indirect loading related to a / d, Fereig and Smith (1977)

Smith and Vantsiotis (1982) conducted an extensive experimental research program on deep beams to investigate the effect of vertical and horizontal web reinforcement and shear span to effective depth ratio on inclined cracking shear, ultimate shear strength, midspan deflection, tension reinforcement strain, and crack width. They tested 52 simply supported reinforced concrete deep beams, with constant amount of flexural reinforcement, and a constant cross-section of 4 in. x 14 in. (10 cm x 35.5 cm), under four point bending. Shear span to depth ratio for the specimens ranged between 0.77 and 2.01. All of the 52 specimens failed in shear, in a similar way, due to excessive damage to the concrete in the shear span. Vertical flexural cracks were observed at about 20 percent of the ultimate load, followed by inclined cracks that propagated suddenly between 40 and 50 percent of the ultimate load, reducing the stiffness of the beam. The beam failure occurred by crushing of the concrete at the compression zone, or by fracture of concrete along the inclined crack. The failure mode was not affected by the existence of web reinforcement; however, specimens with web reinforcement exhibited considerably smaller crack widths. During tests, it was observed that specimens with higher amounts of web reinforcement but lower concrete strengths, failed at lower loads than specimens with lower amounts of web reinforcement but higher concrete strengths, which indicated that the concrete compressive strength could have a significant effect on ultimate shear strength of reinforced concrete deep beams. A linear regression analysis revealed that the influence of concrete strength on the load capacity was more apparent at lower a/d ratios, but this phenomenon

diminished as the a/d increases, which was explained as a result of the arching action in beams with low a/d ratios. According to test results, the presence of vertical or horizontal web reinforcement had no effect on inclined cracking load. However, it was generally observed that web reinforcement increased the shear strength of the beams. The researchers concluded that horizontal web reinforcement had little influence on ultimate shear strength, which was more noticeable for beams with a/d ratio less than 1. On the other hand, vertical web reinforcement, being more efficient for beams with a/d ratio greater than 1, increased the shear strength.

Collins and Mitchell (1986) summarized the essential features of the 1984 Canadian shear design provisions, which were mainly based on the compression field theory and using concepts from plasticity and truss models developed in Europe at that time. The breakthrough about the 1984 Canadian Concrete Code (CCC) was the introduction of a more rational design model with a better application of the shear behavior, instead of the empirical formulas which were adopted by the 1983 ACI Building Code. The design procedures introduced in the code were based on the assumptions that shear stresses are uniformly distributed over the depth of a section, which cannot be applied to design of disturbed regions near discontinuities. However, the CCC adopted the use of strut and tie truss models to approximate the internal flow of forces in such disturbed regions. The limit stresses in the nodal zones, compressive struts, and tension ties recommended by the 1984 CCC are still used in the latest editions of the Canadian Highway Bridge Design Code (2000) and correspond to those used in the AASHTO LRFD Bridge Design Specifications (2004).

Rogowsky, et al. (1986) conducted an experimental research program on deep beams, and reported results of 7 simply supported and 17 two-span continuous deep beams with varying a/d ratios from 1.0 to 2.5, and different configurations of horizontal and vertical web reinforcement. The depth of the specimens ranged between 15.7 in. (40 cm) and 39.4 in. (100 cm). The width of the specimens was kept constant at 7.9 in. (20 cm). The flexural reinforcement extended through the full span terminating in standard hooks located within the column cages, or kept as straight bars which were cut off 6 in. (15 cm) into the column. For the simply supported specimens, a load setup of three point bending was used. The continuous specimens were loaded via five point bending. The beams were tested until one of the shear spans failed, then that shear span was reinforced with external stirrups. and the specimens were tested again until total failure. The behavior of the simple span beams with light stirrups or no stirrups having hooked flexural steel was that of a truss or a tied arch after diagonal cracking. These beams failed due to crushing of the compressive strut at the shear span. Slip of the anchorage bars was noticed for the specimen with straight flexural reinforcement at 87% of the failure load. The amount of slip increased while inclined cracks propagated at the anchorage

zone. The behavior was almost similar for continuous beams with no web reinforcement, with horizontal web reinforcement, or with vertical stirrups. In the deeper beams with low a/d ratios, the failure was due to crushing of the compressive strut and for the shallower beams with high a/d ratios, failure was due to diagonal tension or opening of the inclined crack. In a companion paper, Rogowsky and MacGregor (*1986*) used the previous test data to propose use of an equilibrium truss model based on the lower-bound theorem of plasticity for the design of reinforced concrete deep beams.

Mau and Hsu (1987) developed a theoretical model for calculating the shear strength of directly loaded deep beams with web reinforcement, extending the softened truss model theory. In the model, they separated the shear span of the beam into three parts, as illustrated in Figure 3.9.



Figure 3.9: Stress condition in web of deep beams, Mau and Hsu (1987)

The top element, consisting of concrete and longitudinal compression steel, resists compression due to sectional moment. The bottom element, only including the longitudinal tension steel, resists tension resulting from the sectional moment. The middle portion, with a height of d_v , including concrete and both compression and tension steel to carry both flexural stresses and horizontal shear stresses, resists shear forces in the section. The authors stated that due to the nature of short shear spans, the top load, and the bottom support reaction create large compressive stresses in the web, causing arch action, which contributes significantly to shear strength of the member. The estimated transverse compression stress distributions for various a/h ratios are shown in Figure 3.10.



Figure 3.10: Distribution of transverse compressive stresses for various shear span ratios, Mau and Hsu (1987)

According to their assumptions, the effective transverse compression stress, p, and the effective shear stress, v, acting on the shear element can be calculated as:

$$p = \frac{V}{ba} \left(\frac{4}{3} - \frac{2}{3}\frac{a}{h}\right) \qquad 0.5 \le a/h \le 2 \qquad (3.35)$$

$$v = \frac{V}{bd_v}$$
(3.36)

The authors used stress-strain relationships for concrete previously proposed by Vecchio and Collins (1986), with a slight modification in the tension stress-strain formula to achieve continuous stress values at cracking. For steel, a linear elastic-perfect plastic relationship was assumed. Using equilibrium and compatibility equations, and the material behavior of the shear element, an iterative solution methodology was proposed. To compare their solution with available experimental results in the literature, 64 specimens were identified which had failed in web shear mode, contained at least minimum web reinforcement by ACI 318-03, had an a/h ratio less than 2, and were simply supported. Their iterative solution yielded results within $\pm 10\%$ error, for 51 of the 64 specimens. The authors also performed a

parametric study, and investigated the effects of three dimensionless parameters within the formulas, namely, the longitudinal reinforcement index, $(\rho_1 f_{ly} / f_c)$, the transverse reinforcement index, $(\rho_t f_{ty} / f_c)$, and the shear span to overall height ratio, a/h, which is shown in Figure 3.11. It was also stated that the transverse reinforcement does not contribute to the shear strength for a/h ratios less than 0.5.



Figure 3.11: Effect of shear span ratio, transverse reinforcement, and longitudinal reinforcement to shear strength, Mau and Hsu (1987)

Cook and Mitchell (1988) performed comparative analyses of disturbed regions using simple strut and tie models and nonlinear finite element analysis which employed compression field theory to account for the strain softening of the cracked concrete. The nonlinear finite element analysis was performed using a computer program, FIELDS, developed by the authors, using two dimensional triangular and quadrilateral plate elements. The reinforcement and the cracks were assumed to be smeared uniformly within the elements, and between the cracks, the reinforcement and the concrete were assumed to have average values of stresses. The simple strut-and-tie models were developed according to the guidelines set by Collins and Mitchell (1986), simplified with a statically determinate truss for simply supported deep beams, and with a statically indeterminate truss for continuous deep beams, accounting for the relative stiffness of members. To compare their solutions with the experimental tests, the authors picked two of the continuous deep beams tested by Rogowsky, MacGregor et al. (1986), concluding that the nonlinear finite element analysis provided a more accurate prediction of the disturbed regions than simple strut and tie models.

Adebar, et al. (1990) conducted an experimental program involving the design of pile caps, which are structural members designed to function as transfer members between the columns and a group of piles. Results showed that the strut-and-tie model approach adopted by the Canadian Concrete Code (1984) was more adept in predicting the capacities, than the ACI Building Code (1983) provisions, which failed to capture the trend of the experimental results.

Adebar and Zhou (1993) investigated the capacity of plain concrete cylinders of varying diameters and heights with single and double punch tests, to study the phenomenon regarding transverse splitting of compression struts, due to spreading of compression within the member, as a follow up of the pile cap tests conducted by Adebar, et al. (1990). The authors stated that the stress limit for concrete subjected to shear depended on the amount of existing reinforcement. The authors suggested a maximum bearing stress formula for design, depending on the aspect ratio of the compression strut, the amount of confinement, and the geometry of the compression stress field.

Wang, et al. (1993) proposed formulas to predict the ultimate shear strength of reinforced concrete deep beams. Their equations were based on limit analysis theorems, and energy methods, assuming perfect plasticity with a modification factor for the material model. According to the authors, the ultimate shear strength of a deep beam can be calculated as:

$$P = \frac{bhf_{t}^{*}}{\cos\beta\cos(\alpha - \beta)} [\sin\alpha + K\cot\alpha\cos\alpha + \psi_{v}\sin\alpha\cos(\alpha - \beta) + \psi_{h}\cos\beta\sin(\alpha - \beta)]$$
(3.37)

Where b is the width of the beam, h is the height of the beam, α is the normal direction of failure criterion, β is the direction of the yield line, K is a constant defined for the failure criterion of concrete, ψ_{ν} and ψ_{h} are the degrees of vertical and horizontal web reinforcement, and f_{t}^{*} is the effective tensile strength of concrete. The authors introduced empirical modification factors with the shear-to-span ratio parameter, based on statistical analyses for the concrete strength, in order to use a perfectly plastic material model. A total of 64 scaled specimens previously reported by Mau and Hsu (1989) were used to calibrate the proposed formula. The ratio of predicted to experimental shear force was computed and the mean of this ratio for the 64 specimens was found to be 1.02, with a coefficient of variation of 12.5%.

Siao (1993) proposed formulas to assess the ultimate shear strength of reinforced concrete deep beams and pile caps based on a refined strut-and-tie model, which can be seen in Figure 3.12.



Figure 3.12: The simple and refined strut-and-tie models to predict the ultimate shear strength of deep beams, Siao (1993)

The author's solution was based on shear failure from diagonal splitting. According to Siao, the ultimate shear strength of a deep beam can be calculated as:

$$V_{u} = 1.8 f_{t} bd$$
 (3.38)

and

$$f_t = 6.96\sqrt{f_c'}[1 + n(p_h \sin^2 \theta + p_v \cos^2 \theta)]$$
 (3.39)

Where n is the modular ratio of steel to concrete, p_h and p_v are the steel ratios of horizontal and vertical web reinforcement ($p_h = A_s/bd$, $p_v = A_{st}/bd$). The proposed formula was compared with 73 test results available in the literature. The ratio of predicted to experimental strength was computed for the 73 specimens, with a mean value of 1.04, and a coefficient of variation of 10.4%.

Aoyama (1993) published a paper explaining the design guideline for shear by the Architectural Institute of Japan (AIJ). The basic design philosophy was based on the recognition of a certain collapse mechanism of a structure, and performing a hinge mechanism design. According to the guideline, the compressive strength of concrete in a simply supported beam is shared by two different mechanisms: the truss mechanism, and the arch mechanism, as shown in Figure 3.13.



Figure 3.13: Truss action and arch action in a simple beam, Aoyama (1993)

The truss mechanism depends on the amount of web reinforcement, and for a beam without web reinforcement, the truss mechanism is non-existent. The diagonal compression members of the truss mechanism and the compression struts in the arch mechanism both consist of concrete. The contributions to the shear strength from the arch mechanism and the truss mechanism were derived using a rectangular section subjected to end moments and shear with or without axial load as shown in Figure 3.14.



Figure 3.14: Truss action and arch action in a member subjected to bending moment and shear force, Aoyama (1993)

According to this approach, shear carried by arch mechanism is calculated from Equation 3.40, assuming the yield strength of flexural reinforcement to be infinitely large, and following the lower bound theorem of plasticity.

$$V_{a} = b \frac{D}{2} (1 - \beta) v_{o} \sigma_{B} \tan \theta$$
 (3.40)

Where b is the width of the section (mm), D is the total depth of the section (mm), $\sigma_{\rm B}$ is the cylinder strength of concrete (MPa), θ is the angle of inclination of the arch with respect to longitudinal axis of the specimen, $v_{\rm o}$ is a strength reduction factor for web concrete in a non-ductile member, calculated from Equation 3.41, and β is the ratio of the truss stress to the effective strength, calculated from Equation 3.42.

$$v_{o} = 0.7 - \frac{\sigma_{B}}{200}$$
 (3.41)

$$\beta = \frac{\sigma_{t}}{\nu_{o}\sigma_{B}} = \frac{\rho_{w}\sigma_{wy}(1 + \cot^{2}\phi)}{\nu_{o}\sigma_{B}}$$
(3.42)

Using the principle of superposition, shear carried by the truss mechanism is calculated as:

$$V_{t} = bj_{t}\rho_{w}\sigma_{wv}\cot\phi \qquad (3.43)$$

Where j_t (mm) is the distance between the centroids of compressive and tensile flexural reinforcement ρ_w is the web reinforcement ratio, σ_{wy} (MPa) is the yield strength of web reinforcement, and ϕ is the angle of struts in the truss mechanism, where the value of $\cot \phi$ is the largest value within the range of:

$$\cot \phi \leq \begin{cases} 2\\ \sqrt{\frac{\nu_{o}\sigma_{B}}{\rho_{w}\sigma_{wy}}} - 1\\ \frac{j_{t}}{D\tan \theta} \end{cases}$$
(3.44)

Combining the contributions from the arch mechanism and the truss mechanism, the ultimate shear strength of the member according to AIJ guidelines can be calculated as:

$$V_{u} = V_{a} + V_{t} = b \frac{D}{2} (1 - \beta) v_{o} \sigma_{B} \tan \theta + b j_{t} \rho_{w} \sigma_{wy} \cot \phi$$
 (3.45)

Goh (1995) studied the feasibility of using neural networks to predict the ultimate shear capacity of deep beams. A neural network application does not use the predefined mathematical relationships between the variables, but instead tries to find a pattern among the input parameters involved in accordance with an output parameter. Goh trained the neural network application using data obtained from previous experimental work in the literature. Input parameters such as p_h and p_v , $f_c^{'}$, d, b, and a/h from 43 deep beam tests were used in training to predict the ultimate shear strength of the beams as the output variable. Results obtained from the neural networks were compared with the previous shear strength predictions by Mau and Hsu (1987) and Siao (1993). Goh stated that the neural network approach produced more reliable results than the previous theoretical work, by comparing the correlation coefficients of predicted to experimental results. However, using limited data from smaller scale test results to train neural networks may not provide similar correspondence for full-scale deep beams.

Zielinski and Rigotti (1995) conducted an experimental research program to determine the maximum shear capacity for structures such as deep beams, corbels, and dapped-end beams. Twenty-eight prismatic beam specimens with a constant cross section of 12 in. (30.5 cm) x 1.75 in. (4.5 cm), and a span of 38 in. (96.5 cm) having only flexural reinforcement (Series H), horizontal (Series S), vertical (Series V), horizontal and vertical (Series C) web reinforcement in addition to flexural reinforcement, and inclined reinforcement (Series B) configurations were tested under direct loading, as shown in Figure 3.15.



Figure 3.15: Specimen details and loading setup, Zielinski and Rigotti (1995)

A high percent of steel was used to achieve a balanced condition where the capacity of concrete struts is equal to that of the steel ties. The authors reported that the specimens with inclined reinforcement had greater ultimate strengths than those with any configuration of horizontal reinforcement. It was also stated that specimen series S and H behaved similar to series C and V, respectively, which yielded to a conclusion that the vertical stirrups did not contribute significantly to the shear capacity of the test samples. The shear strength of the specimens was calculated using strut-and-tie models, adopting a horizontally and inclined reinforced concrete corbel analogy, which are shown in Figure 3.16.



Figure 3.16: Free body models for horizontally and inclined reinforced corbels, Zielinski and Rigotti (1995)

According to these models, the authors concluded that the shear capacity of corbels, dapped-end beams, and deep beams is equal to the capacity of the concrete compression strut, as long as the concrete accompanied by an equal, or greater steel tension capacity, and the capacity can be calculated using the following formulas:

$$\Gamma = A_{hf} f_{y} \quad \text{or} \quad T = A_{wf} f_{y}$$
 (3.46)

$$C = f_{cs} \frac{bh}{\sqrt{2}} = 0.85 f_{c}^{'} \frac{bh}{\sqrt{2}} = 0.6 f_{c}^{'} bh$$
 (3.47)

$$V_u = T$$
 or $V_u = C/\sqrt{2}$ (Horizontal web reinforcement case) (3.48)

$$V_u = \frac{(C+T)}{\sqrt{2}}$$
 (Inclined web reinforcement case) (3.49)

Where A_{hf} is the area of horizontal steel, A_{wf} is the area of steel projected on a line perpendicular to the cracks, b is the width of the beam, and h is the overall height of the beam.

Tan, et al. (1995) tested 19 reinforced concrete deep beams with high-strength concrete (HSC), generally having a compressive strength between 6000 psi (41 MPa) to 8600 psi (59 MPa), under two point direct loading. The specimens had a constant cross-section of 19.7 in. (50 cm) x 4.3 in. (11 cm), with reinforcement ratios of $\rho = 1.23$ and $\rho_v = 0.48$, which were kept constant. The research

investigated the effects of varying shear span-to-depth ($0.27 \le a/d \le 2.70$), and effective span-to-depth ($2.15 \le l_e/d \le 5.38$) ratios to the behavior of the deep beams. A typical test specimen from this research program is shown in Figure 3.17.



Figure 3.17: Typical test specimen from high strength concrete deep beam research, Tan, et al. (1995)

Test results showed that the ultimate and cracking stresses varied slightly for different effective span-to-depth ratios. The ultimate shear stress was independent of l_e/d , but a rapid increase in ultimate shear stress was observed with a decrease in a/d. On the contrary, the ultimate shear stress decreased with an increase in a/d which was attributed to degradation in the effectiveness of the tied arch action. In addition to this, the diagonal cracking stress appeared independent of both l_e/d , and a/d ratios.

Kong, et al. (1996) tested 24 lightweight concrete deep beams to investigate the effect of embedment lengths for the end-anchorage of the main tension reinforcement to deep beam behavior. The tested beams had a cross-section of 30 in. (76 cm) x 4 in. (10 cm), having either an orthogonal grid of web reinforcement, or inclined web reinforcement. Six of the specimens had a span of 60 in. (152 cm), with a clear shear span-to-depth ratio of 0.55. The remaining 18 specimens had a span of 37.5 in. (95 cm), and the clear shear span-to-depth ratio was 0.30. The specimen details are shown in Figure 3.18.



Figure 3.18: Specimens for embedment length research Kong, et al. (1996)

Test results showed that progressive reduction of the embedment length of the tension steel did not result in significant reductions in the ultimate shear strength, maximum crack widths, or deflections, except in cases where the embedded reinforcement was terminated at the centerline of the supports. It was further noted that a standard hook equivalent to an embedment length of 17.5 d_b did not perform better than a straight bar embedment length of 10 d_b. This was attributed to the effect of the clamping force acting on the tension steel, which is created by the support reaction, and the compressive tied-arch strut common for deep beams, to the bond strength, which ensured adequate anchorage even with reduced embedment lengths.

Tan, et al. (1997) continued their experimental research program, this time investigating the effect of web reinforcement on HSC deep beams. They tested 18 specimens under two point direct loading, having a compressive strength between 8000 psi (55 MPa) to 12500 psi (86 MPa). A constant cross-section of 19.7 in (500 mm) x 4.3 in (110 mm) was used, with different arrangements of horizontal and vertical web reinforcements, and varying shear span-to-depth ratios of 0.85, 1.13, and 1.69. The flexural reinforcement ratio was kept constant at $\rho = 2.58$, while the horizontal web reinforcement ratio, ρ_h , was varying between 0 to 3.17, and the

vertical web reinforcement ratio, ρ_v , was between 0 to 2.86. Different types of specimens tested in this research program are shown in Figure 3.19.



Figure 3.19: Specimen types tested by Tan, et al. (1997)

The researchers observed that the contribution of vertical web reinforcements to beam stiffness is more significant when a/d ratio exceeds 1.13. It was also seen that vertical web reinforcement was more effective than horizontal web reinforcement in crack control. However, the best results in terms of serviceability were achieved with web reinforcement placed as an orthogonal grid. It was also noted that the effect of horizontal web reinforcement on shear strength diminished when a/d ratio exceeds 1.13. The researchers concluded that vertical web steel has a greater effect in increasing the ultimate shear resistance than the horizontal web steel of the same steel ratios.

Tan, et al. (1997) investigated the effect of main tension steel in HSC deep beams, by testing 22 specimens having concrete compressive strengths exceeding 8000 psi (55 MPa), with a constant cross-section of 19.5 in. (50 cm) x 4.3 in. (11 cm), and an effective span varying between 48 in. (125 cm) and 136 in. (350 cm). The web reinforcement ratio ρ_v was kept constant at 0.48 percent, while the tensile reinforcement ratio ρ_w was used as a variable, having values of 2.00, 2.58, 4.08 and 5.80. The 4 main types of specimens tested by Tan, et al. are shown in Figure 3.20. According to the authors, the transition point between HSC deep beams and shallow beams was around a/d of 1.5, where this transition occurred at an a/d ratio between 2.0 and 2.5 for medium/low strength concrete deep beams. The failure mode of the HSC deep beams was mainly controlled by the a/d ratio, and the effect of the main tension steel on the failure mode was not significant. Increasing the main tension steel ratio increased the load capacity of the HSC beams, as long as the a/d ratio was less than 1.50.

Ashour (1997) conducted an experimental research program to investigate the behavior of 2 span continuous deep beams. The eight specimens tested had a constant overall length and width of 118 in. (300 cm), and 4.7 in. (12 cm) respectively. The overall height of the beams varied between 16.7 in. (42.5 cm) and 24.6 in. (62.5 cm), to achieve a/d ratios between 0.8 and 1.18. The concrete compressive strengths was between 3625 psi (25.0 MPa), and 5685 psi (39.2 MPa). Ashour observed the same failure mode in all specimens, by a major diagonal crack located in the intermediate shear span, between the edges of the bearing plates located at the support and loading zones. The author also concluded that the vertical web reinforcement had more influence on the shear capacity of the continuous deep beams than the horizontal web reinforcement.



Figure 3.20: Specimen types tested by Tan, et al. (1997)

Foster and Gilbert (*1998*) tested 16 HSC concrete deep beams to examine their behavior, and compared the results with the design methods from the building codes of the era. The concrete strengths ranged from 7250 psi (50 MPa) to 17400 psi (120 MPa), and the shear span-to-depth ratios were varied from 0.5 to 1.32. The beam depth was one of the variables, ranging from 27.5 in. (70 cm) to 47.2 in. (120 cm), with a constant beam width of 4.9 in. (12.5 cm). The test specimens were

tested under either single point or two point direct loading, and the specimen details are shown in Figure 3.21. The researchers observed that the crack formations and the failure modes were similar to those observed in conventional strength concrete deep beams, previously tested in the literature. It was concluded that the ACI deep beam design formulas from the 1989 version of the code yielded conservative results, whereas the plastic truss model derived by Rogowsky and MacGregor (*1986*) was reported to be more reliable and easier to apply.



Figure 3.21: Specimens from the HSC tests, Foster and Gilbert (1998)

Tan and Lu (1999) conducted an experimental program on the size effect in reinforced concrete deep beams. This experimental program was important in terms of contribution to the size effect studies, since data from large scale specimens are very scarce in the literature. Tan and Lu tested 12 specimens, with different depths and effective spans varying between 19.7 in. (50 cm) - 68.9 in. (175 cm), and 59 in. (150 cm) - 178 in. (452 cm), respectively. The beam widths kept constant at 5.5 in (14 cm), in order to maintain two dimensional similarities. The concrete compressive strength for the specimens was 5800 psi (40 MPa), and a constant tensile reinforcement ratio of 2.60% was used. The deep beam specimens from the size effect tests are shown in Figure 3.22.



Figure 3.22: Specimens from the size effect research, Tan and Lu (1999)

The authors compared the normalized ultimate stress and diagonal cracking stress versus overall height, and observed that a size effect was apparent when the overall height increases from 19.7 in. (50 cm) to 39.4 in. (100 cm). However, there was not a significant relative stress reduction as overall height increases beyond 39.4 in. (100 cm), as shown in Figure 3.23, which indicated that there is a critical overall height where the size effect diminishes beyond that point. The size effect in diagonal cracking stress was insignificant. It was also observed that the size effect seemed relatively independent of the a/d ratio.



Figure 3.23: Normalized ultimate stress and diagonal cracking stress versus overall height, Tan and Lu (1999)

Shin, et al. (1999) tested directly loaded and supported high strength concrete deep beams with variable a/d ratios. The researchers stated that the mode of failure was mainly affected by the a/d ratio, rather than the shear reinforcement ratio. ACI 318-95 deep beam equations were found to be overly conservative when compared with test results. Strength prediction formulas were proposed which were based on a regression analysis performed on the test data.

Averbuch and de Buhan (1999) proposed a general framework aimed at the shear design of reinforced concrete deep beams, based on the yield design theory with a "mixed modeling" of the reinforced concrete structure, which depends on the homogenization of shear reinforced concrete zones. The authors stated that the stress distribution in the shear reinforced zone can hardly be approximated by a STM, since the shear reinforcement results in a significant rotation of the compressive stresses towards the vertical direction, as opposed to a shear zone without web reinforcement. Averbuch and de Buhan concluded that the numerical

estimates from the theoretical approach yielded reliable results when compared with the experimental data obtained from previous tests in the literature.

Ashour (2000) developed an analysis method for predicting shear failure of simply supported reinforced concrete deep beams. In this approach, concrete and steel reinforcement were modeled as rigid perfectly plastic materials, and the deep beam was modeled as an assemblage of moving rigid bodies. The strength of deep beams were derived using a location function based on the instantaneous center of relative rotation of moving blocks, which were separated by failure zones of displacement discontinuity. The yield line theory was used to calculate the internal dissipated energy in the concrete and in the steel reinforcement. After a parametric study, Ashour concluded that the shear strength of a deep beam depends on the shear span-to-depth ratio, rather than the effective span-to-depth ratio.

Yun (2000) developed a nonlinear STM approach for analysis and design of reinforced concrete structures. In conventional STM analysis, the struts and ties are placed according to the principle stress flows, generally achieved from a linear elastic finite element analysis. In Yun's approach, the plain concrete was analyzed non-linearly, and the preliminary model was constructed in this manner, to be refined afterwards within an iterative process, taking into account the material non-linearity. The author tested only a single scaled beam specimen, having a cross-section of 20 in. (5.08 cm) x 8 in. (20.3 cm), with an effective span of 10.8 in. (274.3 cm) to demonstrate the proposed nonlinear STM approach. The analytical outcome seemed to satisfactorily match the experimental result

Teng, et al. (2000) conducted an experimental research program on the shear strength of concrete deep beams under fatigue loading. 12 scaled beams, with a concrete compressive strength of 5800 psi (40 MPa), having a constant cross-section of 31.5 in. (80 cm) x 6.9 in. (17.5 cm) were tested under two point direct loading. The specimens had an effective span of 133.8 in. (340 cm) and a shear span-to-an overall height ratio of 1.5. Three beams were either without web reinforcement, with vertical web reinforcement, or with inclined web reinforcement. Each set of specimens had at least one specimen tested under static loading, and the rest was tested under fatigue loading until failure. The specimen details and the load setup are shown in Figure 3.24.



Figure 3.24: Details of fatigue test deep beam specimens, and the load setup, Teng, et al. (2000)

It was observed that cracks widened, and propagated further, especially in the first few cycles of fatigue loading. The specimen without web reinforcement demonstrated the worst performance, and the specimen with inclined web reinforcement was the best in terms of crack control. This was attributed to the efficiency of the inclined web reinforcement, since the stirrups were placed in the tensile principle stress direction. Arrangement of the web reinforcement seemed to have no effect on the formation of first flexural and inclined cracks. The authors reported that some of the specimens failed in flexural mode and concluded that fatigue loading has a more significant impact on flexural strength than the shear strength. According to the test data, the fatigue strength of the deep beams seemed to vary linearly with log N, where N is the number of cycles, as shown in Figure 3.25.



Figure 3.25: Fatigue Strength of Deep Beams, Teng, et al. (2000)

Furthermore, the authors modified the ACI deep beam equations which became obsolete after the 1999 edition of the ACI code, to express the fatigue load range of a deep beam. The fatigue load ranges contributed by the concrete $V_{r,c}$ and by the steel reinforcement $V_{r,s}$ were calculated from Equation 3.50, 3.51, and 3.52, respectively.

$$V_{r,c} = (3.5 - 2.5 \frac{M_u}{V_u d})(1.9(1 - \beta \log N)\sqrt{f_c} + 2500 \frac{1}{N^{\alpha}} \rho \frac{V_u d}{M_u})(1 - \frac{V_{min}}{V_u})bd (3.50)$$

Where

$$V_{r,c} + V_{min} \le 6\sqrt{f_c} bd$$
 (3.51)

$$V_{r,s} = \frac{1}{N^{\alpha}} (1 - \frac{V_{min}}{V_{u}}) \left[\frac{A_{v}}{s_{v}} \left(\frac{1 + \frac{l_{0}}{d}}{12} \right) \right] + \frac{A_{h}}{s_{h}} \left(\frac{11 - \frac{l_{0}}{d}}{12} \right) f_{y} d$$
(3.52)

Finally, the overall fatigue load range was calculated from Equation 3.53 and 3.54.

$$V_{\rm r} = V_{\rm r,c} - V_{\rm r,s}$$
 (3.53)

Where

$$V_{\rm r} = V_{\rm max} - V_{\rm min} \tag{3.54}$$

According to the authors, these formulas can be used to calculate the maximum fatigue load range V_{max} for certain values of N and V_{min} . The constants α and β were calibrated accordingly with the experimental results, and through numerical iterations the suitable values for α and β were found to be 0.12 and 0.059, respectively. However, the deep beam equations used by the authors as a template, were removed from the ACI code due to the severe discontinuities that these equations give when clear span to overall member depth ratio is varied, which limits the proposed formulas.

Hwang, et al. (2000) proposed a softened strut-and-tie model for determining the shear strength of deep beams. Selected test specimens from the literature were compared with prediction results from the proposed method, and also with ACI 318-95 deep beam equations. ACI 318-95 deep beam equations were found to underestimate the contribution of concrete and overestimate the contribution of web reinforcement on the shear strength of deep beams, but still resulting in overly conservative predictions.

Sanad and Saka (2001) used neural networks to predict ultimate shear strength of reinforced concrete deep beams. A collection of experimental data available in the literature consisting of 111 scaled deep beam specimens was used to train the neural network. Ten different input parameters including the effective span, beam width, beam depth, shear span, compressive strength of the concrete, yield strengths of horizontal and vertical web reinforcement, reinforcement ratios of horizontal tensile steel, total horizontal steel, and transverse steel were used. The

neural network predictions were compared with the predictions from ACI 318-95 deep beam equations, and the equations derived by Siao (1993), and by Mau and Hsu (1989). The authors stated that the prediction bias was significantly better than the other methods used for comparison. However, the database used to train the neural networks consisted of data from scaled specimens. In order to get realistic results from neural networks, a large database of full-scale specimens must to be used for training. Acquiring such a database is almost impossible due to high costs of such tests.

Aguilar, et al. (2002) tested four deep beam specimens to evaluate the design procedures for shear strength of deep reinforced concrete beams. The specimens, having a constant overall length of 176 in (4470 mm), and a constant cross-section of 36 in. (91.5 cm) x 12 in. (30.5 cm), were tested directly under monotonic two points loading until failure, as shown in Figure 3.26.

The same amount of tensile flexural steel was used in all of the specimens, whereas the amounts of horizontal and vertical web reinforcement were variable. In addition to this, two different types of anchorages were used, either being mechanical anchorage using bearing plates, or 90-degree hook anchorage. The concrete compressive strength, and the reinforcement yield strength were 4000 psi (28 MPa), and 60 ksi (410 MPa), respectively. The test results were compared with obsolete ACI 318-99 deep beam design equations and with the STM approach presented in Appendix A of the later editions of the ACI specification. The researchers stated that both ACI 318-99 and ACI 318-02 Appendix A provisions were shown to be conservative; however, results from the very basic STMs seemed more reliable with a 25% degree of conservatism.

Specimen ACI-I



Specimen STM-I



Specimen STM-H





Figure 3.26: Specimen and loading details, Aguilar, et al. (2002)

Matamoros and Wong (2003) proposed a procedure and a set of equations for design of simply supported deep beams using STM. Four basic strut and tie models representing basic load transfer mechanisms, as shown in Figure 3.27, were analyzed separately to characterize the contributions from the concrete, vertical web reinforcement, and horizontal web reinforcement to assess the ultimate shear strength of reinforced concrete deep beams.



Figure 3.27: Dimensions of the nodal zone, and the basic strut-and-tie models used in the analysis, Matamoros and Wong (2003)

The first three basic models were statically determinate trusses, including a compression strut mechanism for concrete, a vertical truss mechanism representing the vertical web reinforcement, and a horizontal truss mechanism representing the horizontal web reinforcement. The last model was an internally determinate truss, a combination of the vertical and the horizontal truss mechanisms. Each truss system was solved using principles of static's. When the member responses were compared for different a/d ratios, as shown in Figure 3.28, it was observed that the simple truss models may not be consistent with experimentally observed behavior and may lead to reinforcement configurations that are not appropriate for the particular stress field. The researchers developed correction factors to address the inconsistencies between the experimental results and the magnitude of forces calculated using elastic truss models.



Figure 3.28: Fraction of total shear carried by truss elements in direct strut, horizontal truss, vertical truss, and statically indeterminate truss, Matamoros and Wong (2003)

According to this approach, the ultimate shear strength of a deep beam can be calculated as:

$$V = C_c S_{strut} + C_{wv} S_{tv} + C_{wh} S_{th}$$
(3.55)

Where S_{strut} , S_{tv} , and S_{th} are the nominal strengths of the concrete strut, vertical web reinforcement, and horizontal web reinforcement, respectively, whereas C_c , C_{wv} , and C_{wh} are the corresponding correction factors, calibrated using the experimental test data in the literature. The proposed equations for nominal strengths of the members and the correction factors are

$$S_{\text{strut}} = f_{\text{c}}^{'} b w_{\text{st}}$$
 (3.56)

$$S_{tv} = A_{tv} f_{yv} = \rho_{wv} b \frac{a}{3} f_{yv}$$
 (3.57)

$$S_{th} = A_{th}f_{yh} = \rho_{wh}b\frac{d}{3}f_{yh}$$
 (3.58)

$$C_{c} = \frac{0.3}{a/d} \le 0.85 \sin \theta$$
 (3.59)

$$C_{wv} = 1$$
 (3.60)

$$C_{wh} = 3(1 - \frac{a}{d}) \ge 0$$
 (3.61)

Where, f'_c is the compressive strength of the concrete, a is the shear span, b is the width of the beam, w_{st} is the width of the strut, A_{tv} and A_{th} are the effective areas of horizontal and vertical ties, ρ_{wv} and ρ_{wh} are the reinforcement ratios for vertical and horizontal web reinforcement, and f_{yv} and f_{yh} are the yield stresses for vertical and horizontal web reinforcement. According to the proposed model, the total shear strength is given by

$$V = \frac{0.3}{a/d} f'_{c} bw_{st} + A_{tv} f_{yv} + 3(1 - a/d) A_{th} f_{yh}$$
(3.62)

Zararis (2003) proposed a method to predict shear compression failure in reinforced concrete deep beams having an a/d ratio between 1.0 and 2.5, under two points or a single point direct loading. This study included expressions derived for the restricted depth of compression zone, as well as for the ultimate shear force of deep beams with and without web reinforcement. According to this method, the longitudinal and the vertical steel bars at the crack location undergo not only elongations but shear strains as well. Indicating the directions of the horizontal and vertical reinforcement with x and y respectively, the reinforcement stresses were expressed in matrix form, which implies that the shear forces in reinforcement are caused by a pure shearing deformation of the bars at the crack location regardless of slip at the crack faces, which occurs when dowel forces are produced:

$$\begin{bmatrix} \sigma_{sx} & \tau_{sxy} \\ \tau_{syx} & \sigma_{sy} \end{bmatrix} = E_s \varepsilon_{cr} \begin{bmatrix} \cos^2 \phi & 0.4 \sin \phi \cos \phi \\ 0.4 \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix}$$
(3.63)

Where ε_{cr} the strain is perpendicular to the crack, E_s is the steel modulus of elasticity, and φ is the angle between crack and vertical direction. Zararis used a free body diagram which includes internal and external forces after cracking, and at failure, as shown in Figure 3.29, to calculate the depth of compression zone above the critical diagonal crack, and the ultimate shear strength of the deep beam.



Figure 3.29: Forces on elements of a deep beam with web reinforcement (a) with critical diagonal crack and (b) at failure, Zararis (2003)

According to the proposed theory, the vertical reinforcement is assumed to yield first. Yielding of the vertical reinforcement causes a significant increase of the shear force, V_d , acting on the flexural reinforcement. When the shear force acting on the flexural reinforcement is beyond a limit, a horizontal crack forms at the tension steel level, eventually causing the loss of force V_d . At this point, normal and shear forces of concrete in the compression zone above the diagonal critical crack increase excessively, leading to crushing of the concrete at the top of the crack in deep beams with web reinforcement. From the equilibrium of moments and forces acting on the free body diagram, the ultimate shear strength of a deep beam with and without vertical web reinforcement can be calculated as:

$$V_{u} = \frac{bd}{a/d} \left[\frac{c_{s}}{d} \left(1 - 0.5 \frac{c_{s}}{d} \right) f_{c}' + 0.5 \rho_{v} f_{yv} \left(1 - \frac{c_{s}}{d} \right)^{2} \left(\frac{a}{d} \right)^{2} \right]$$
(3.64)

where a is the shear span, b is the width of the beam, d is the effective depth of the tension reinforcement, ρ_v is the ratio of the horizontal reinforcement, f_{yv} is the yield strength of vertical reinforcement, and c_s is the depth of compression zone above critical diagonal crack, obtained from:

$$\frac{c_{s}}{d} = \frac{1 + 0.27R(a/d)^{2}}{1 + R(a/d)^{2}} \frac{c}{d}$$
(3.65)

Where

$$R = 1 + (\rho_v / \rho)(a / d)^2$$
 (3.66)

and the c/d ratio is the positive root of the following expression:

$$\left(\frac{c}{d}\right)^2 + 600 \frac{\rho}{f'_c} \frac{c}{d} - 600 \frac{\rho}{f'_c} = 0$$
 (3.67)

Zararis checked the validity of the proposed equations for deep beams having vertical web reinforcement with the experimental data in the literature, yielding a prediction bias of 0.996 with a coefficient of variation of 9%.

Tang and Tan (2004) proposed an interactive mechanical model for shear strength of deep beams, using the STM approach, adopting the classical Mohr-Coulomb failure criterion, which provides a linear relationship between principle tensile and compressive stresses. According to the simple STM, shown in Figure 3.30, compressive stresses f_2 are formed in the diagonal strut acting between the load and support zones. The compressive stresses are resisted by the concrete, which may cause failure from crushing. On the other hand, tensile stresses f_1 are formed perpendicular to the diagonal strut, resisted by the main tension reinforcement, web reinforcement, and concrete tensile strength.



Figure 3.30: Strut-and-tie model for simply supported deep beams, Tang and Tan (2004)

According to the Mohr-Coulomb failure criterion, a linear relationship was adopted as:

$$\frac{f_1}{f_{ct}} + \frac{f_2}{f_c'} = 1$$
(3.68)

Where f_{ct} is the tensile strength, and f'_{c} is the cylinder compressive strength of the concrete. Using the identity above, the ultimate shear strength of the deep beam can be calculated as:

$$\frac{1}{V_{n}} = \frac{1}{V_{ds}} + \frac{1}{V_{dc}}$$
(3.69)

Where V_n is the ultimate shear strength of the deep beam, V_{ds} and V_{dc} are the strut capacities against diagonal splitting and crushing of the concrete, respectively, obtained from the following proposed expressions:

$$V_{ds} = \frac{f_{ct}A_{ct} + f_{yw}A_{w}\sin(\theta_{w} + \theta_{s}) + 2f_{y}A_{s}\sin\theta_{s}}{2\cos\theta_{s}}$$
(3.70)

$$V_{dc} = f'_{c} A_{str} \sin \theta_{s}$$
 (3.71)
Where f_{yw} and f_y are the yield strengths of web reinforcement, and main tension reinforcement, A_{ct} is the concrete section along diagonal strut, A_w and A_s are the areas of the web reinforcement, and main tension reinforcement, θ_s is the angle between longitudinal tension reinforcement and diagonal strut, and θ_w is the angle between web reinforcement and horizontal axis of beams at intersection of reinforcement and diagonal strut. Tang and Tan compared the prediction results from the proposed equations with three sets of experimental data in the literature. Comparison with experiments conducted by Tan et al. (1995) where a wide range of a/d ratios was present yielded a prediction bias of 1.10, with a coefficient of variation of 10.5%. Application of the method to data with an emphasis on different configurations of web reinforcement, and varying a/d ratios, provided by Smith and Vantsiotis (1982) produced a prediction bias of 1.30, with a coefficient of variation of 8.6%. The prediction bias and the coefficient of variation from the last set of data for specimens with different configurations of web reinforcement, compiled by Kong et al. (1970, 1972), were 1.32 and 11.6%, respectively.

Russo et al. (2005) proposed a model for determining the shear strength of reinforced concrete deep beams using a STM approach, including the contributions of diagonal concrete strut, longitudinal main reinforcement, and the horizontal/vertical web reinforcement to the shear strength. The simple strut and tie model for a deep beam with web reinforcement and the equilibrium of compression force in the strut due to web reinforcement is shown in Figure 3.31.



Figure 3.31: (a) Strut-and-tie model for deep beam with web reinforcement; (b) equilibrium of compression force in strut due to web reinforcement, Russo (2005)

A simple ultimate shear stress formulation, calibrated according to 240 deep beam tests from the literature, was derived as:

$$v_{n} = 0.76 \left(k \chi f_{c}^{'} \cos \theta + 0.25 \rho_{h} f_{yh} \cot \theta + 0.35 \frac{a}{d} \rho_{v} f_{yv} \right)$$
 (3.72)

where a (mm) is the shear span, d (mm) is the effective depth, θ is the angle of inclined strut and vertical direction, ρ_h and ρ_v are the horizontal and the vertical reinforcement ratios, f'_c (MPa) is the cylindrical concrete compressive strength, f_{yh} (MPa) and f_{yv} (MPa) are the yield strength of the horizontal and vertical web reinforcement, k is the non-dimensional depth of compressive zone with respect to d, calculated as:

$$k = \sqrt{(n\rho_{\rm f})^2 + 2n\rho_{\rm f}} - n\rho_{\rm f}$$
 (3.73)

Where n is the ratio of steel to concrete elastic moduli, and ρ_f is the flexural reinforcement ratio. The parameter χ is a non-dimensional interpolation function calculated as:

$$\chi = \left[0.74 \left(\frac{f_{c}}{105} \right)^{3} - 1.28 \left(\frac{f_{c}}{105} \right)^{2} + 0.22 \left(\frac{f_{c}}{105} \right) + 0.87 \right]$$
(3.74)

Where χ is valid for the range $10 \le f_c \le 105$ (MPa). For 240 tested deep beams in the literature, the proposed shear strength formula produced a prediction bias of 1.0, and a coefficient of variation of 19%.

Tan and Cheng (2006) investigated the size effect phenomenon on shear strength of deep beams, using a STM approach. The researchers stated that the primary cause of size effect is due to the conventional definition of ultimate shear strength of V/(bd), which is adopted from the shear distribution mechanism in steel beams. According to the authors, in reinforced concrete deep beams, after cracking occurs, the loads are mainly transferred via arch action, which is barely a shear transfer mechanism, and arch action does not transmit a tangential force to a nearby parallel plane, as previously stated by ACI-ASCE Committee 426 (1974). Due to the arch action, the effective depth d does not remain constant under increasing load, but reduces with the decreasing depth of the uncracked compression zone, rendering the adaptation of V/bd erroneous. According to the researchers, a secondary cause of size effect occurs from the strut geometry, and boundary conditions. The

proposed ultimate shear strength formula which is based on a simple STM, as shown in Figure 3.30, also taking into account the size effect was computed:

$$V_{n} = \frac{1}{\frac{\sin 2\theta_{s}}{f_{t}A_{c}} + \frac{1}{\nu f_{c}'A_{str}\sin\theta_{s}}}$$
(3.75)

Where θ_s is the angle between longitudinal tension reinforcement and diagonal strut, f_t is the combined tensile strength of reinforcement and concrete, f_c is the compressive strength of concrete, A_c is the area of cross-section, A_{str} is the cross-sectional area of the diagonal strut, and v is the compression efficiency factor calculated from the following expressions

$$\mathbf{v} = \xi \zeta \tag{3.76}$$

$$\xi = 0.8 + \frac{0.4}{\sqrt{1 + (1 - s)/50}}$$
(3.77)

$$\zeta = 0.5 + \sqrt{\frac{kd_s}{l_s}} \le 1.2$$
 (3.78)

Where l and s are the length and the width of the strut, respectively, k is a factor for particular reinforcement grade, d_s is the minimum diameter of main longitudinal steel, and l_s is the maximum spacing of steel bars. The authors compared the predicted shear strengths obtained from the proposed formula with 3 different sets of data compiled from the literature. The prediction bias and the coefficient of variation from each set were 0.92, 6.5%, 0.88, 14.7%, and 0.84, 19.3%, respectively.

Quintero-Febres, et al. (2006), conducted an experimental research program aimed at evaluating the adequacy of the strength factors for concrete struts in strut-and-tie models given in Appendix A of the 2002 ACI Building Code. A total of 12 reinforced concrete deep beam specimens with various shear span-to-depth ratios ranging from 0.82 to 1.57, concrete strengths between 4000 psi (28 MPa) and 8000 psi (55 MPa), and different reinforcement layouts were tested under two point direct loading. All beams had an overall length of 96 in. (244 cm), and a constant cross-section of 18 in. (46 cm) x 6 in. (15 cm), except the high strength concrete specimens, where the beam web width was reduced to 4 in. (10 cm), as shown in Figure 3.32.



Figure 3.32: Details for test specimens, Quintero-Febres, et al. (2006)

The test results compared with ACI 318-02 predictions, and it was observed that the strut strength factors given in Appendix A were adequate for use in normal strength concrete. However, the provisions for required minimum transverse reinforcement in normal strength concrete were found to be inconsistent and needed to be reevaluated.

Brown and Bayrak (2007) conducted an experimental program to examine the effects of load distribution and shear reinforcement on the strength of deep beams. The test results were compared with predictions from simple strut-and-tie models, and the researchers concluded that the application of strut-and-tie models produce conservative values of strength.

3.3 LITERATURE REVIEW SUMMARY

Experimental research on deep beams in the literature is summarized in terms of support conditions, manner of loading, and specimen dimensions, in Table 3.1. Specimen dimensions are also shown in Figure 3.33. Analytical methods proposed in the literature are summarized in terms of the basis of the methodology, number of different data sets and specimens used for calibrating the proposed formulas, the prediction bias and the coefficients of variation, in Table 3.2. Prediction results from available methodologies in the literature are also shown in Figure 3.34, where the error bars represent standard deviations, and the radius of the bubbles indicate the number of specimens used for calibrating the proposed formulas.

Year	Researcher	Support Type	Loading Type	Number of Specimens Tested	b _w [in]	h [in]	a/d
1951	Clark	Direct	Direct	62	6.00	15.0	1.00
						—	—
						18.0	2.00
1968	Ramakrıshnan	Direct	Direct	20	3.00	15.0	0.94
	Ananthanarayan					-	-
1070	a Kana (1	Diment	Diment	25	2.00	30.0	2.00
1970	Kong, <i>et al</i> .	Direct	Direct	35	3.00	10.0	1.00
						- 30	2 00
1071	Manual at al	Direct	Direct	12	4.00	.0	0.30
17/1	Widnuci, et al.	Direct	Direct	12	4.00	10.0	0.50
							1.00
1977	Fereig	Indirect	Indirect	6	6.00	12.0	0.50
	Smith						_
							2.00
1982	Smith	Direct	Direct	52	4.00	14.0	0.77
	Vantsiotis						-
							2.01
1986	Rogowsky, et al.	Direct	Direct	24	7.90	15.7	1.00
						—	—
						39.4	2.00
1995	Tan, <i>et al</i> .	Direct	Direct	19	4.30	19.7	0.27
							- 2 70
1005	Ziolinski	Direct	Direct	24	1 75	12.0	2.70
1993	Rigotti	Direct	Direct	24	1./3	12.0	-

Table 3.1: Deep Beam Experiments in the Literature

1996	Kong, et al.	Direct	Direct	24	4.00	30.0	0.83
							_
							1.33
1997	Tan, et al.	Direct	Direct	53	4.30	19.7	0.85
							-
							1.69
1997	Ashour	Direct	Direct	8	4.70	16.7	0.80
						-	-
						24.6	1.18
1998	Foster	Indirect	Direct	16	4.90	27.5	0.50
	Gilbert					—	-
						47.2	1.32
1999	Tan	Direct	Direct	12	5.50	19.7	0.50
	Lu					—	—
						68.9	1.00
1999	Shin, <i>et al</i> .	Direct	Direct	30	4.90	9.6	1.50
							-
					6.0.0		2.50
2000	Teng, et al.	Direct	Direct	12	6.90	31.5	1.5
2000	Yun	Direct	Direct	1	8.00	18.0	2.0
2002	Aguilar, et al.	Direct	Direct	4	12.00	36.0	1.0
2006	Quintero-Febres,	Direct	Direct	12	6.00	18.0	0.82
	et al.						-
							1.57
2007	Brown	Direct	Direct	10	6.00	30.0	0.50
	Bayrak						-
							1.00

Table 3.2: Analytical Methods Proposed in the Literature

Year	Researcher	Methodology	Number of Data Sets	Number of Specimen s	Bias V _{EXP} /V _P	COV
1968	Ramakrishnan	Mechanical	-	-	-	-
	Ananthanarayan	Approach				
	a					
1971	Zsutty	Regression				
		Analysis				
1986	Collins	Strut-and-Tie	-	-	-	-
	Mitchell					
1986	Rogowsky	Strut-and-Tie	-	-	-	-

	MacGregor					
1987	Mail and Hsu	Softened Truss Model	_	_	_	_
1988	Cook	Strut-and-Tie	1	2	0.97	07
1700	Mitchell	NLFEA	· ·	_	5.21	0.7
1993	Wang, et al.	Energy Method	3	64	1.03	12.8
1993	Siao ¹	Strut-and-Tie	-	-	-	-
1993	Aoyama ²	Mechanical	-	175	1.07	19.0
		Approach				
1995	Goh	Neural	2	20	0.98	8.9
		Networks				
1995	Zielinski	Strut-and-Tie	1	25	1.19	23.8
	Rigotti					
1999	Averbuch	Yield Line	-	-	-	-
	de Buhan	Theory				
1999	Shin, et al.	Regression	-	-	-	-
		Analysis	2	20	1.0.4	11.6
2000	Ashour	Yield Line	2	20	1.04	11.6
2000	X7	Theory	1	1	1.00	
2000	Yun	Strut-and-Tie	1	1	1.00	-
2000	Teng, et al.	Mechanical	-	-	-	-
2000	TT 4 1	Approach	5	100	1.1.7	16.0
2000	Hwang, et al.	Strut-and-Tie	5	123	1.15	16.0
2001	Sanad	Neural	5	10	0.97	7.5
2002	Saka	Networks		175	1.40	22.0
2003	Matamoros	Strut-and-Tie	-	1/5	1.40	22.0
2002	Volig Zororio ³	Maahaniaal	1	50	1.02	12.2
2005	Zararis	Approach	4 2	52	1.02	15.2 9.0
		Approach	$\frac{2}{2}$	28	1.00	9.0
2004	Tano ⁴	Strut-and-Tie	3	19	1.02	10.5
2001	Tan	Strut und The	5	52	1.10	86
				45	1.32	11.6
2005	Russo, et al.	Strut-and-Tie	15	240	1.00	19.0
2006	Tan ⁵	Strut-and-Tie	3	5	1.09	6.6
	Cheng			12	1.15	13.6
	-			19	1.24	22.4

- 1. Proposed methodology by Siao (1993) was only compared with experimental results from shear wall and corbel tests.
- 2. Prediction methodology was later compared with experimental results by Matamoros and Wong (2003).
- 3. Zararis' comparison results are in three separate parts: "deep beams without stirrups", "deep beams with stirrups", and "deep beams with vertical and horizontal web reinforcement".
- 4. Tang and Tan separately compared their theory with three different data sets from Tan *et al.* (1995), Smith and Vantsiotis (1982), and Kong *et al.* (1970).
- 5. Tan and Cheng separately compared their theory with three different data sets from Tan and Cheng (2006), Tan *et al.* (1999), and Kani (1967).



Figure 3.33: Overall geometries for deep beam experiments in the literature



Figure 3.34: Analytical methods proposed for deep beams in the literature

3.4 LIMITATIONS OF DESIGN CODES AND THE CURRENT LITERATURE

Based on the review of the technical literature, the following limitations in current design codes and previous research are identified:

- There is no consensus on a clear definition of deep beams in current design codes.
- None of the current design codes provides a shear strength prediction formula for existing deep beams. The ACI code prior to 1999 included shear strength formulas, but they became obsolete in the later editions.
- Use of shear strength equations for slender beams in current codes is not recommended for analysis of deep beams due to the different internal force transfer mechanisms and nonlinear strain distributions in the cross-section.
- Each design code recommends use of Strut-and-Tie models (STM) in design of deep beams. However, STM analysis provides virtually an infinite number of solutions to a design problem rather than a unique prediction result. In addition, since STM is based on plastic analysis, it is not possible to assess behavior of deep beams at serviceability limit states with this methodology.

- The amount of experimental data available in the literature on shear strength of deep beams is very limited as compared to those on slender beams.
- Due to high costs and lack of adequate equipment, almost all of the laboratory experiments were performed on small-scale specimens. Results from small-scale tests may not reflect the behavior of full-scale deep beams due to the size effect.
- Indirect and direct loading have different effects on shear capacity of deep beams (*Fereig and Smith, 1977*). RCDG bridge bent caps are indirectly loaded members. However, 99% of the deep beam specimens in the literature were tested under direct loading.
- Anchorage of flexural reinforcement, which is thought to be one of the salient parameters for ultimate shear strength of deep beams, has not been studied in detail.
- Deep beam specimens in the literature were mostly (96%) supported directly on bearing plates. Due to significant stress levels at the support locations, local bearing failures were observed frequently. However, RCDG bridge bent caps in the field are supported indirectly via stocky columns, allowing limited rotations at the ends.
- Effect of fatigue loading on deep beam behavior has not been studied in detail. A realistic fatigue load protocol based on actual bridge service performance data needs to be developed.
- A formal theory for deep beam behavior has not been widely accepted. Most of the proposed analysis methods are either based on different modifications of STM, semi-empirical approaches, use of neural networks or regression analysis.
- Proposed strength prediction equations in the literature may not be applicable to full-scale bent caps since related formulas were generally calibrated with data from previous small-scale deep beam tests where specimens were uniformly reinforced (both for flexure and shear) and mostly directly loaded and supported.

4.0 EXPERIMENTAL SETUP

4.1 INTRODUCTION

In order to provide structural response data for evaluation of conventionally reinforced concrete deck-girder (RCDG) bridge bent caps, an experimental program was undertaken in Oregon State University Structural Engineering Department. The experimental program consisted of 6 full-scale bent cap specimens, in which the overall height, number of anchorage bars, web reinforcement size and grade, flexural reinforcement cut-off locations, and loading type (static or fatigue loading) were varied.

In this section, findings from a review of the Oregon Department of Transportation (ODOT) in-service bridge inventory with an emphasis on RCDG bridge bent caps are discussed. Next, the specimens are described in terms of geometry, reinforcement, boundary conditions, and material properties. Finally, the laboratory tests are explained, including detailed information on instrumentation of the specimens.

4.2 OREGON DEPARTMENT OF TRANSPORTATION INVENTORY REVIEW

In order to design full-scale specimens that are realistic and reflective of actual inservice vintage RCDG bridge bent caps, 204 of ODOT's inventory of RCDG bridges constructed from 1947 to 1962 and identified as cracked were reviewed. The bent caps were characterized by span, support conditions, member properties, flexural and transverse steel details, and the size and locations of utility holes, using digital drawings provided by ODOT, as shown in Figure 4.1. A database of these characteristics was developed, and results are summarized below. These data were used to develop full-scale prototype laboratory specimens that are most reflective of the bridges currently in service.



Figure 4.1: Example vintage RCDG bent cap in the ODOT inventory

4.2.1 Column Dimensions

The size and shape of the columns that support the bent caps were characterized. The columns were built integrally with the bent caps and provide connection to the foundation. Both round and square columns were identified in the bridge population considered. The most common were square columns with longitudinal reinforcing bars located in the corners only. The most frequent column type was the 24 in. x 24 in. (610 mm x 610 mm) square column cross-section, having a frequency of 43%. Other common types were 20 in. x 20 in. (508 mm x 508 mm), and 30 in. x 30 in. (762 mm x 762 mm) square columns, with frequencies of 24%, and 11%, respectively. The typical column dimensions and overall column lengths provide little flexural stiffness to the bent caps, and further the columns were not detailed to provide moment restraint, thus can be idealized as providing vertical support alone. The distribution of column section dimensions was grouped together using a bin size of 6 in. (152 mm) and is shown in Figure 4.2.



Figure 4.2: ODOT inventory bent cap column dimensions

4.2.2 Bent Cap Heights

Overall bent cap height in the inventory was identified and categorized using a bin size of 6 in. (152 mm). There was variation among bent cap heights within the inventory. However, bent caps with 72 in. (1829 mm) (8% frequency), and 48 in. (1219 mm) (7% frequency) depths were the most common. The distribution of bent cap heights is shown in Figure 4.3.



Figure 4.3: ODOT inventory Bent Cap Overall Heights

4.2.3 Bent Cap Widths

Bent cap width in the inventory also varied, as was the case with bent cap height. It was observed that a bent cap width of 16.5 in. (419 mm) was the most common, with a frequency of 22%. The distribution of bent cap widths using a bin size of 2 in. (51 mm) is shown in Figure 4.4. In addition, the data were filtered for 72 in. (1829 mm), and 48 in. (1219 mm) bent cap heights, in order to observe the specific width range for these two most common bent cap depths. The width distributions for 72 in. (1829 mm) and 48 in. (1219 mm) deep bent caps are shown in Figure 4.5, and Figure 4.6, respectively, which illustrates that the cap width for both these heights is typically between 16 in. to 18 in.







Figure 4.5: ODOT inventory Bent Cap Widths for h = 72 in. (1829 mm)



Figure 4.6: ODOT inventory bent cap widths for h = 48 in. (1219 mm)

4.2.4 Centerline Reinforcement to Anchorage Reinforcement Ratio

Most bent caps were proportioned for the minimum amount of reinforcement required by calculation. As a result, in some cases flexural bars were terminated where no longer required by design. This results in flexural bar termination within the flexural tension zone, thus not all flexural tension bars extend over the entire length of the cap beams. In this part, a ratio of the area of main flexural tension reinforcement at midspan ($A_{s,mid}$) to the area of reinforcement that is anchored in the columns ($A_{s,end}$) of the bent caps was inspected. In 46% of the bent caps the ratio was determined as 1, where $A_{s,mid}$ equals $A_{s,end}$, which indicates no rebar cutoffs. However, in 54% of the bent caps, cut-offs were observed with various centerline reinforcement to anchorage zone reinforcement ratio. The centerline tension reinforcement to anchorage zone reinforcement ratio distribution is shown in Figure 4.7.



Figure 4.7: ODOT inventory $A_{s,mid}$ / $A_{s,end}$ ratio

4.2.5 Flexural Tension Reinforcement Anchored in Column Section

Three different types of tension reinforcement were identified for the anchorage zones in beam – column connection zone. Anchorage of the flexural reinforcement was provided with straight bars, 90° hooks, or 180° hooks extended into the column section. It was observed that straight bars were used more often (63%) than 90° hooks (10%), or 180° hooks (27%) for anchorage. There were also some cases where multiple anchorage types were used in a single bent cap. Generally these different anchorage types were contained in two different layers of flexural reinforcement. The distribution of flexural rebar anchorage types in the cross-section is shown in Figure 4.8.



Figure 4.8: ODOT inventory anchorage types

4.2.6 Utility Holes

Most of the bent caps in the inventory contained utility holes within the bent cap body. The holes are generally located in the upper stem near the exterior columns. The effect of these service holes on structural performance of bent caps was of interest due to field observations of vertical cracks extending from the deck and crossing through service holes in some of the bent caps. In some bent caps, stirrups that would typically be located at the hole position were placed adjacent to and tangential to the holes, and in other cases no additional stirrups were located around the hole. Three different cases were defined whereas, (1) stirrup present around the hole, (2) no stirrup present around the hole, (3) no holes exist in the bent cap, and results are shown in Figure 4.9. It was observed that 24% of the bent caps had no service holes. Of the bent caps with holes, 66% had no stirrups around the hole, while 34% of the bent caps had stirrups tangential to the service holes. In addition to these data, the utility hole sizes were investigated. Generally, three holes with the diameters of 8 in. (203 mm), 4 in. (102 mm), and 1 in (25 mm) were seen as a group located in the upper corners at each end of the bent caps. There are also some cases where only 8 in. (203 mm) and 4 in. (102 mm) holes were present as a couple. The distribution of hole diameters in the ODOT inventory bent caps is shown in Figure 4.10.



Figure 4.9: ODOT inventory service hole – stirrup relation



Figure 4.10: ODOT inventory service hole sizes

4.3 SPECIMEN DESIGN

4.3.1 Specimen Dimensions & Reinforcement Arrangement

Using the data from the ODOT bent cap inventory, a series of full scale bent cap specimens were designed. The database was used to select the specimen proportions, reinforcement details, and column details, thereby realistically reflecting their in-service counterparts. A total of 6 specimens were constructed, and test variables included bent cap overall height, number of flexural bars anchored in the columns, web reinforcement size and grade, flexural reinforcement cut-off locations, and loading type (static and fatigue loading). The specimen naming convention is illustrated in Figure 4.11.



Figure 4.11: Specimen name convention

A subassemblage test specimen of the pertinent bridge components at the bent cap region was developed. This included the integral columns, cap beam, and portions of the main girders that frame into the cap beam. In indeterminate RCDG bridges, the forces from the main girders are transferred to the supporting elements (bent caps) indirectly, via interface shear. External girders are typically located over the columns and they do not impose shear or bending, whereas loading on the interior girders does produce shear and moment on the bent cap. In order to produce realistic loading conditions, and to avoid creating triaxial compressive stresses, which can significantly increase the shear strength of directly loaded deep beams (*Fereig and Smith, 1977*), the loads were applied indirectly to the bent cap specimen through four stub-girder portions located on each side of the cap beam at the 1/3 span locations.

Five specimens, including the fatigue specimen, had an overall height of 72 in. (1829 mm), and one specimen had an overall height of 48 in. (1219 mm). The locations of the applied loads and the support reactions were kept the same for all of the failure tests. The centerline-to-centerline distance between support reactions and external loads was 96 in. (2438 mm). The shear span-to-overall height ratios were 1.33 and 2.00, for 72 in. (1829 mm) and 48 in. (1219 mm) high specimens, respectively. The overall length of the specimens was 312 in. (7925 mm), including the column portions, and the clear span of the specimens was 264 in. (6706 mm), disregarding the columns. The width of the bent caps was kept constant at 16 in. (406 mm) for all specimens. Each four of the girders were 48 in. (1219 mm) high, with a cross-section of 14 in. x 16 in. (357 mm x 406 mm).

The cap beams were indirectly supported by 96 in. (2438 mm) high integral square columns. The column cross-section was 24 in. x 24 in. (610 mm x 610 mm). Both of the columns were cast on a 1 in. thick, 24 in. x 24 in. (610 mm x 610 mm) A36 steel plate to avoid the crushing of the concrete in the support zones, especially at the roller end. Column reinforcing bars were welded to the plates to ensure anchorage and aid construction. One of the columns was welded to additional A36 steel plates, which were laterally restrained, but moderately free to rotate, simulating the pin-end of a simply supported beam. The other column plate was placed on a high strength roller resting on a slightly curved steel plate, representing the roller-end of a simply supported beam.

Table 4.1 summarizes the dimensions for the six test specimens. The dimensions for the specimens are also shown in Figure 4.12.



Figure 4.12: Bent cap overall specimen dimensions

	Specimen Name	Beam Dimensions			C Dir	olumn nensions	Stub Girder Dimensions	
#		h [in] (mm)	b [in] (mm)	ln [in] (mm)	h c [in] (mm)	Cross- section [in] (mm)	h g [in] (mm)	Cross- section [in] (mm)
1	D6.A4.G60#5.S	72 (1829)	16 (406)	264 (6706)	96 (2438)	24 x 24 (610 x 610)	48 (1219)	14 x 16 (356 x 406)
2	D6.A4.G40#4.S	72 (1829)	16 (406)	264 (6706)	96 (2438)	24 x 24 (610 x 610)	48 (1219)	14 x 16 (356 x 406)
3	D6.A2.G60#5.S	72 (1829)	16 (406)	264 (6706)	96 (2438)	24 x 24 (610 x 610)	48 (1219)	14 x 16 (356 x 406)
4	D6.A2.G40#4.S	72 (1829)	16 (406)	264 (6706)	96 (2438)	24 x 24 (610 x 610)	48 (1219)	14 x 16 (356 x 406)
5	D4.A2.G40#4.S	48 (1219)	16 (406)	264 (6706)	96 (2438)	24 x 24 (610 x 610)	48 (1219)	14 x 16 (356 x 406)
6	D6.A2.G40#4.F	72 (1829)	16 (406)	264 (6706)	96 (2438)	24 x 24 (610 x 610)	48 (1219)	14 x 16 (356 x 406)

Table 4.1: Dimensions of test specimens

The main tension and compression reinforcement used for the specimens were ASTM 615/615M-05a (2005) Grade 60 (420 MPa) #11 (36 mm) deformed rebar. The main flexural tension reinforcement consisted of 10 or 8 #11 bars, arranged in three layers (4-4-2), with three cut-off locations, for the 72 in. (1829 mm) deep specimens, and in two layers (4-4), with three cut-off locations, for the 48 in. (1219 mm) deep specimen. In two of the specimens, all the bottom layer reinforcement extended into the columns, producing an embedment with four bars. The rest of the specimens had only two bars embedded in the column, having the other two bars of the bottom layer terminating just at the face of the columns. The compression reinforcement consisted of 2 #11 bars for all specimens, which were continuous over the full length of the specimen.

Two different types of stirrups were used for the web reinforcement. Two specimens had ASTM 615/615M-05a (2005) Grade 60 (420 MPa) #5 (16 mm) deformed stirrups, placed at an average spacing of 8.75 in. (222 mm) at the shear spans, and a constant spacing of 8 in. (203 mm) at the constant moment zone. The difference was due to the existence of service holes at the shear spans. The rest of the specimens, had the same spacing arrangement, but with ASTM 615 nominal Grade 40 (276 MPa) #4 (13mm) deformed stirrups, which corresponds more closely to Intermediate Grade (40 ksi, 276 MPa) A305 steel used in the 1950s and 1960s.

Table 4.2 summarizes the main flexural tension reinforcement and vertical web reinforcement ratios at different sections for all of the specimens. The reinforcing details, and the rebar instrumentation for the 72 in. (1829 mm) and 48 in. (1219 mm) specimens are illustrated in Figure 4.13 and Figure 4.14, respectively.

	Specimen Name	Vertical web reinforcement		Longitudinal flexural reinforcement					
#		Grade [ksi] (MPa)	f v [%]	Grade [ksi] (MPa)	CL [%]	f _{cut1} [%]	f _{cut2} [%]	f _{cut3} [%]	
1	D6.A4.G60#5.S	Gr. 60 (420)	0.44	Gr. 60 (420)	1.45	1.14	0.85	0.56	
2	D6.A4.G40#4.S	Gr. 40 (276)	0.29	Gr. 60 (420)	1.45	1.14	0.85	0.56	
3	D6.A2.G60#5.S	Gr. 60 (420)	0.44	Gr. 60 (420)	1.45	1.14	0.85	0.28	
4	D6.A2.G40#4.S	Gr. 40 (276)	0.29	Gr. 60 (420)	1.45	1.14	0.85	0.28	
5	D4.A2.G40#4.S	Gr. 40 (276)	0.29	Gr. 60 (420)	1.76	-	1.31	0.43	
6	D6.A2.G40#4.F	Gr. 40 (276)	0.29	Gr. 60 (420)	1.45	1.14	0.85	0.28	

Table 4.2: Reinforcement of test specimens



Figure 4.13: 72 in. (1829 mm) deep specimen reinforcing details



Figure 4.14: 48 in. (1219 mm) deep specimen reinforcing details

The columns contained four ASTM 615/615M-05a (2005) Grade 60 (420 MPa) #11 (36 mm) bars with one in each corner, and ASTM 615/615M-05a (2005) Grade 60 (420 MPa) #4 (13 mm) ties with a spacing of 4 in. (102 mm) at the lower column section (outside the cap beam), and 8 in. (203mm) through the height of the cap beam.

In order to prevent a premature local failure in the zones of load application, the stub girders were heavily reinforced with a reinforcement cage consisting of ASTM 615/615M-05a (2005) Grade 60 (420 MPa) #6 (19 mm) and #5 (16 mm) closed ties, as shown in Figure 4.15.



Figure 4.15: Bent cap specimen reinforcing cage

4.3.2 Specimen Construction

Due to the large size and self-weight of the specimens, they were constructed in place within the load frame. Steel cages for the column pieces, the cap beam, and the stub girders were assembled. Formwork for the specimens was built using $\frac{3}{4}$ in. (19 mm) thick medium density overlay (MDO) panels. The panels were stiffened by 2x4 (38 mm x 89 mm) and 2x6 (38 mm x 140 mm) wooden studs to maintain geometry, and were assembled together using mechanical connectors. Steel channels were used on each side of the stem, secured by threaded rods, to maintain the web width. The form panels were supported on a steel plate which was further supported by steel post shores, to prevent deformations due to the weight of the

fresh concrete. Steel angles were used to connect the form panels to the steel plate. Furthermore, to control the overall geometry, and provide additional safety during concrete placement, side bracing was used. Concrete cover was provided at all sides using 1.5 in. (38 mm) slab bolsters, which were tied to the web reinforcement, and also placed on the steel plate to maintain bottom cover. PVC pipes with 8 in. and 4 in. diameters were used to form utility holes. Concrete was placed via a concrete boom pump truck, as shown in Figure 4.16, since the specimen was too high for direct placement from the ready mix truck. Concrete was consolidated using a mechanical vibrator, and the top surface was hand trowel finished.



Figure 4.16: Bent cap specimen concrete placement

Cylinders for characterizing the concrete strength were made simultaneously with the specimens in respect to ASTM C192/C192M-05 designation; using ASTM C470/C 470M-05 compatible 6 in. x 12 in. (152 mm x 305 mm) cylinder molds. After concrete placement, specimens were covered with burlap and plastic sheets, and kept moist during the curing process. The formwork was stripped once the specimen gained sufficient strength over one week. The beam surface was whitewashed in order to increase the visibility of cracks. The web reinforcement locations, and the tensile reinforcement cut-offs were marked using a Profometer rebar locator, and a grid of 12 in. x 12. in (305 mm x 305 mm) was placed for crack mapping.

4.4 MATERIAL PROPERTIES

4.4.1 Concrete

Concrete was provided by a local ready-mix supplier, and the same mix design was used for all specimens. The mix design was based on 1950s AASHO Class A concrete used in previous research at OSU (Higgins et al., 2003). The aggregate composition for the mix was reported by the supplier to be: 97% passing the 3/4 in. sieve (19 mm), 82% passing 5/8 in. (16 mm), 57% passing 1/2 in. (12.5 mm), 33% passing 3/8 in. (9.5 mm), 21% passing 5/16 in. (8 mm), 9.3% passing 1/4 in. (6.3 mm), 3.0% passing #4 (4.75 mm), 0.6% passing #8 (2.36 mm) and 0.3% passing the #200 (0.075 mm) sieve. The sand composition of the mix was also reported as: 99.7% passing the 1/4 in. sieve (6.3 mm), 96.8% passing #8 (2.36 mm), 59.4% passing #16 (1.18 mm), 44.9% passing #30 (0.600 mm), 17.9% passing #50 (0.300 mm), 3.7% passing #100 (0.150 mm) and 1.7% passing the #200 (0.075 mm) sieve. The coarse aggregate was from Willamette River bed deposits and consisted of smooth rounded basaltic rock. The specified concrete compressive strength was 3000 psi (21 MPa), which is comparable to the design strength of concrete used in 1950s bridges. Actual concrete compressive strengths were determined from 6 x 12 in. (152 x 305 mm) cylinders which were tested during curing, and on day-of-test in accordance with ASTM C39M/C 39M-05 and ASTM C617-98. Tensile splitting tests were performed on day-of-test to estimate the tensile splitting strength of concrete in accordance with ASTM C496/C496M-04e1. Test day compressive and tensile strengths of concrete are presented in Table 4.3.

#	Specimen Name	f _c ' [psi] (MPa)	f _{ct} [psi] (MPa)
1	D6.A4.G60#5.S	3872 (26.7)	415 (2.9)
2	D6.A4.G40#4.S	3796 (26.2)	386 (2.7)
3	D6.A2.G60#5.S	3995 (27.5)	418 (2.9)
4	D6.A2.G40#4.S	3542 (24.4)	362 (2.5)
5	D4.A2.G40#4.S	3661 (25.2)	376 (2.6)
6 (Precrack)	D6.A2.G40#4.F	3463 (23.9)	353 (2.4)
6 (Failure)	D6.A2.G40#4.F	3828 (26.4)	381 (2.6)

Table 4.3: Day of Test Compressive and Tensile Strengths of Concrete

4.4.2 Reinforcing Steel

All reinforcing steel was fabricated by a local rebar fabricator per OSU approved shop drawings. The Grade 40 (276 MPa) #4 (13 mm) reinforcing bars were taken from the lowest yield-stress heat of steel produced by Cascade Steel Rolling Mills during a production run. Steel samples were cut from Grade 40 (276 MPa) #4 (13 mm) and Grade 60 (420 MPa) #5 (16 mm) bars at a length of 20 in. (508 mm) ready for testing. Steel samples from Grade 60 (420 MPa) #11 (36 mm) bars were machined in accordance with ASTM E8 for the 505 specimen size. All steel specimens were tested in a 110 kip (490 kN) capacity universal testing machine using a 2 in. (51 mm) gage length extensometer to measure strain. The yielding stress (f_v), and the ultimate stress (f_u) were obtained and shown in Table 4.4.

		Fle	exural Ste	eel	Transverse Steel			
#	Specimen Name	f _v [ksi] (MPa)	f _u [ksi] (MPa)	f _u [in/in]	f _v [ksi] (MPa)	f _u [ksi] (MPa)	f _u [in/in]	
1	D6.A4.G60#5.S	71.1	105.5	0.111	62.2	104.8	0.107	
		(490)	(727)		(429)	(723)	ļ	
r	2 D C A A C A 0 H A S	68.2	112.9	0.117	50.5	76.5	0.164	
2	D0.A4.040#4.5	(470)	(778)		(348)	(527)		
3		68.2	112.9	0.117	62.2	104.8	0.107	
5	D0.A2.000#3.5	(470)	(778)		(429)	(723)		
1	D6 A2 C40#4 S	68.2	112.9	0.117	50.2	80.0	0.172	
4	D0.A2.040#4.5	(470)	(778)	0.117	(346)	(552)		
5	5 D4 A2 C40#4 S	69.4	107.1	0.114	50.6	76.2	0.155	
3	D4.A2.040#4.5	(478)	(738)	0.114	(349)	(525)		
6		68.1	103.9	0.100	49.8	78.6	0.163	
6	D6.A2.G40#4.F	(469)	(716)	0.109	(343)	(542)		

Table 4.4: Reinforcing steel material properties

4.5 LOADING SCHEME

4.5.1 Static Tests

Forces were applied to the specimen via two force-controlled closed-loop servohydraulic actuators, at a constant rate of 1.67 kips/sec (7.4 kN/sec) and were measured by 500 kip (2224 kN) capacity load cells. Spreader beams were used to distribute the actuator forces symmetrically to 4 in. (102 mm) wide bearing plates which were placed over centerline of the stub girders in order to achieve indirect loading. The load frame, the position of the specimen, and hydraulic actuators are shown in Figure 4.17. Each main load step consisted of three sub-load steps. Firstly, Actuator 1 was loaded up to the target load level and then unloaded while Actuator 2 maintained a constant load of 5 kips (22 kN). Secondly, Actuator 1 was maintained at a load of 5 kips (22 kN) while Actuator 2 was loaded up to the target load level and then unloaded. Thirdly, Actuator 1 was loaded up to the target load level and, without unloading Actuator 1, Actuator 2 was loaded up to the target load level to achieve the total load for the main load step. In order to reduce the creep effects, the load in Actuator 2 was immediately reduced by 10% once the peak total load was achieved. To complete a main load cycle, Actuator 2 was unloaded, followed by Actuator 1, down to 5 kips (22 kN). Before the unloading phase at the end of each sub-load step, visible cracks were marked and mapped, and pictures were taken. Load magnitudes from each actuator were increased in subsequent cycles by an amount of 50 kips (222 kN) until failure. A schematic representation of the load protocol for a single increment of the static tests is shown in Figure 4.18.



Figure 4.17: Experimental setup for static tests (loading frames shown, bracing frames not shown for clarity)



Figure 4.18: Schematic representation of a single increment of the loading protocol for static tests

4.5.2 Fatigue Test

Fatigue test was performed on specimen D6.A2.G40#4.S. An initial loading, or precrack test, was performed to produce diagonal cracks in the specimen similar to those observed in the field inspections. For the precrack test, the static test load setup and the static load protocol was used, and the specimen was loaded for 5 main cycles up to a total load of 450 kips (2002 kN). Apart from the general static load protocol, in the last cycle Actuator 1 and Actuator 2 was loaded up to 225 kips (1001 kN) each instead of 250 kips (1112 kN). The crack widths measured from the main diagonal cracks varied between 0.030 in. - 0.050 in. (0.76 mm – 1.27 mm), and 0.016 in. – 0.030 in. (0.41 mm – 0.76 mm) for the pinned support and the roller support side shear spans respectively.

Due to the size of the hydraulic cylinders used in the static tests, which were not suitable for high frequency load cycles, and in order to avoid synchronization problems between two independent actuators, it was decided to use a single actuator to perform the high-cycle fatigue test. A 220 kip (979 kN) capacity actuator was placed over the centerline of the deep beam and connected to the reaction frame resulting in the fatigue loads to be applied as a single point loading, thereby applying similar demand on each side. The load was transferred from the actuator to the deep beam via a 15 in. x 15 in. (381 mm x 381 mm) steel plate. As a

precaution against slipping of the actuator clevis pin during long test sessions, a steel saddle was placed between the actuator and the specimen, as shown in Figure 4.19.



Figure 4.19: Experimental setup for fatigue test (loading frames shown, bracing frames not shown for clarity)

The high-cycle fatigue test was performed in 10 increments, and each segment consisted of 100,000 load cycles with a load range of 150 kips -210 kips (667 kN -934 kN) at a frequency of 1.0 Hz. This load range imitates a constant dead load of 150 kips (667 kN), and an ambient traffic load of 60 kips (267 kN) on the specimen. The derivation of this load range is explained in section 3.5.2.1

Before beginning each fatigue segment, a baseline test was performed using the external and internal sensors together which enabled a comparison of relative global behavior of the specimen due to fatigue loading. In order to avoid fatigue damage to the external sensors, due to their mechanical nature, the external sensors were not used during the continuous fatigue cycles. The baseline tests were performed in three phases. In the first phase the specimen loaded up to 210 kips (934 kN), and unloaded to 5 kips (22 kN) for five cycles. In the second phase, a cyclic load ranging between 180 kips (801 kN) and 210 kips (934 kN) was applied at 0.1 Hz, for five cycles. In the last phase, the specimen was loaded up to 215 kips (956 kN), and unloaded to 5 kips (22 kN) for five cycles. During the baseline test, new cracks from the previous fatigue cycle were marked, and the change in crack widths of main diagonal cracks was measured. At the end of each baseline test,

external sensors were removed, except anchorage zone slip sensors and the crack clip array, and a new fatigue cycle increment of 100,000 load cycles was started.

When 1,000,000 cycles of fatigue loading was completed, the fatigue load setup was replaced with the static load setup. The specimen was tested with the static load protocol from the beginning, until failure.

4.5.2.1 Derivation of Fatigue Load Range

Bent caps in RCDG bridges are subjected to repeated loads that range from dead load only to dead load plus superimposed live loads. In order to derive the load ranges applied in the fatigue test, structural analysis tools and field data from a previous research program (*Higgins, et al., 2004*) were used. If a single stirrup in a bent cap is taken into consideration, the maximum stress in the stirrup, under a repeated loading can be expressed as:

$$\sigma_{MAX} = \sigma_{DL} + SR \tag{4.1}$$

Where σ_{MAX} is the total stress in the stirrup, σ_{DL} is the constant stress induced by the dead load, and SR is the live load induced stress range, such as that produced by a truck passing over the bridge.

In order to achieve the dead load component of the induced stress, a finite element analysis was performed on a vintage RCDG bridge model. McKenzie River Bridge in Oregon, which is a 3 span continuous, 4 girder RCDG bridge was used in the model, and the total dead load imposed due to bridge self-weight was calculated. The amount of dead load per unit length on each girder was calculated by dividing the total dead load per unit length to the number of girders, as permitted in AASHTO LRFD. It was assumed that the dead load on external girders was transferred to the foundation directly through the columns and did not affect the stress level in the stirrups between the external and internal girders. A basic finite element model (FEM) of the bridge with line loads applied to the internal girders was constructed in SAP 2000 as shown in Figure 4.20. According to the FEM analysis solution the axial force in a single column connected to the bent cap was 87 kips (387 kN). Regarding the capacity of the fatigue actuator, it was decided to apply a dead load of 150 kips (667 kN), which corresponds to 75 kips (334 kN) of axial force at each column.



Figure 4.20: Simple FEM of McKenzie River Bridge for dead load analysis

The live load component of the induced stress was determined using the available field data. Higgins *et al.* (2004) investigated McKenzie River Bridge, Jasper Bridge, and 15 Mile Creek Bridge in Oregon under ambient traffic and controlled truck loadings. In these series of field tests, strain gages were installed by chipping into the concrete and exposing the embedded stirrups at selected crack locations in both girders and bent caps. Ambient traffic induced variable amplitude stresses in stirrups, and the number of cycles measured was recorded at each bridge over a period of 8 days. The data extracted from the instrumented bent cap stirrups, which were of particular interest, were used in this analysis.

First, the variable amplitude stresses were converted to an equivalent constant amplitude stress range for each instrumented bent cap stirrup using Miner's Rule (1945):

$$SR_{eqv} = \sqrt[m]{\sum \frac{n_i}{N_{tot}} SR_i^m}$$
(4.2)

Where SR_{eqv} is the equivalent constant amplitude stress range from the field, m is a material constant, which is taken as 3 in the case of reinforcing steel, SR_i is the ith stress range, n_i is the number of cycles observed for the ith stress range, and N_{tot} is the total number of cycles at all stress ranges.

Second, the projected number of cycles for the equivalent stress range in 50 years was calculated, since the bridges of interest have been in service over a period of approximately 50 years, by:

$$N_{50 yrs} = \frac{N_{tot}}{n_{test \ days}} \times 365 \times 50$$
(4.3)

Where N_{50yrs} is the projected number of cycles in 50 years, and $n_{test days}$ is the number of days data were collected in the field.

Finally, due to time constraints, the equivalent constant amplitude stress ranges collected from the field were amplified to produce equivalent damage in the laboratory specimen in 1,000,000 cycles. Miner's Rule was rearranged to calculate the stress range for the laboratory test as:

$$SR_{LAB} = \sqrt[m]{\frac{N_{50yrs}}{N_{lab}}}SR_{eqv}^{m}$$
(4.4)

Where SR_{LAB} is the amplified stress range for laboratory tests, m is a material constant, taken as 3 in the case of reinforcing steel, N_{50yrs} is the projected number of cycles in 50 years, N_{LAB} is the number of cycles for laboratory tests, and SR_{eqv} is the equivalent constant amplitude stress range from the field measurements. The results were summarized in Table 4.5. For 1,000,000 cycles of fatigue loading in the laboratory environment, the highest stress range was derived as 8.40 ksi. In order to find the applicable load range to the fatigue specimen, D6.A2.GR40#4.F, strain versus load data compiled from D6.A2.GR40#4.S, which had the same reinforcement detailing with the fatigue specimen, were analyzed. It was found that in order to generate a target stress of 8.40 ksi (58 MPa) in the shear span stirrups, which was the largest laboratory stress range required to produce equivalent damage to the field observed stresses, 30 kips (133 kN) of shear force was needed, which corresponds to a total actuator force of 60 kips (267 kN).

Instrumented Bridge	Sensor Code	Number of Cycles Measured in Field	SR _{eqv} [ksi] (MPa)	Projected Number of Cycles in 50 Yrs	SR _{LAB,100000} ₀ [ksi] (MPa)
McKenzie	CH_11	39814	1.88 (13.0)	89197827	8.40 (57.9)
Bridge	CH_22	17764	0.791 (5.5)	39797815	2.70 (18.6)
	CH_07	2544	1.53 (10.5)	3901513	2.41 (16.6)
	CH_16	13076	1.86 (12.8)	20053529	5.05 (34.8)
Jaspar Bridge	CH_17	13007	1.91 (13.2)	19947710	5.18 (35.7)
Jasper Bridge	CH_18	8142	1.81 (12.5)	12486681	4.20 (29.0)
	CH_19	13811	1.89 (13.0)	21180735	5.23 (36.1)
	CH_32	7432	1.87 (12.9)	11397815	4.21 (29.0)
	CH_06	1241	1.12 (7.7)	2792633	1.58 (10.9)
	CH_07	14644	1.25 (8.6)	32953514	4.01 (27.6)
	CH_14	13154	1.02 (7.0)	29600555	3.16 (21.8)
15 Mile Creek	CH_15	12151	0.94 (6.5)	27343496	2.83 (19.5)
Bridge	CH_16	13162	1.00 (6.9)	29618557	3.09 (21.3)
	CH_17	1284	1.14 (7.9)	2889396	1.62 (11.2)
	CH_18	10279	0.94 (6.5)	23130919	2.68 (18.5)
	CH_19	9129	0.86 (5.9)	20543064	2.36 (16.3)

Table 4.5: Fatigue Stress Range Derivation
A trial run was performed on the fatigue specimen to check the stress ranges using a constant dead load of 150 kips (667 kN) plus a load range of 60 kips (267 kN) resulting in cycles between 150 kips (667 kN) and 210 kips (934 kN). The stirrup stress ranges captured from the trial test corresponding to each of the shear span stirrups are shown in Figure 4.21. Even though stress ranges in some of the stirrups were higher than the target stress range mentioned before, it was decided to use the load range of 60 kips (267 kN) to maintain conservatism. It should also be noted that the stress ranges in all of the stirrups corresponding to the 150 kips – 210 kips (667 kN – 934 kN) load cycles were suitable for investigating the effect of bond fatigue rather than metal fatigue since all of the field measured stress ranges were well below 20 ksi (138 MPa), which is generally taken as the threshold for inducing metal fatigue on reinforcing steel (*McGregor, 1997*).



Figure 4.21: Stirrup strain ranges for P = 60 kips (267 kN) (stirrup #1 near column, stirrup #7 near stub girder)

4.6 ARRANGEMENT OF MEASURING DEVICES

Specimens were instrumented to measure loads, strains, and displacements during testing. Data acquisition was performed by using an Ethernet based IOtech DaqBook/2001 portable data acquisition system with IOtech DBK15 universal current/voltage input cards. Data monitoring and recording was performed using DASYLab 7.0 for subsequent analyses.

4.6.1 Internal Sensor Array

The strains in the longitudinal flexural tension reinforcement bars were measured with Vishay CEA-06-125UN-120 general purpose strain gages with a gage factor of $2.085 \pm 0.5\%$. Two reinforcing bars were instrumented at each reinforcement layer, at the centerline location, by placing strain gages between two rebar deformations. These strain gages were connected to Vishay 2120 series strain gage conditioners running at an excitation voltage of 5V and a gain of x500.

In order the measure the strain in the vertical web reinforcement, the same strain gage model and strain gage conditioning settings were used with those used for the longitudinal tension reinforcement strains. In order to place strain gages, two of the deformations on the stirrups were removed by grinding without decreasing the main steel area. At least 7 stirrups were instrumented at each shear span diagonally where the anticipated main diagonal crack passes through the stirrups. Only for specimen D4.A2.GR40#4.S were the strain gages placed at the mid-height of the stirrups. After the precracking of specimen D6.A2.GR40#4.F, it was observed that most of the stirrup strain gages were damaged in the last phase of the test. In order to measure strains in the vertical web reinforcement, the stirrup locations were marked using a Profometer rebar detector, and the concrete was removed by chipping, thereby exposing stirrups were re-instrumented for data collection through the fatigue test and permitted direct comparison with field measured results.

The demand on the flexural tension bars anchored in the column was measured by instrumenting the bars at three locations within the anchorage zone. This setup allowed monitoring the change in the tension force in the bars through the development length. The same strain gage type and strain gage conditioning settings were used as for the centerline longitudinal tension reinforcement. The first strain gage was placed on the bar at the column face where the column and the beam intersect, the second strain gage was placed on the bar at the centerline of the column, and the third strain gage was placed in between the first two. Only for specimen D6.A4.GR60#5.S were the anchorage bars instrumented at two locations

without using the third gage in the usual setup. For all of the specimens two anchorage bars were instrumented at each side of the specimens.

4.6.2 External Sensor Array

To measure global and local displacements, displacement transducers and string potentiometers were used. A schematic representation of the external sensor locations is illustrated in Figure 4.22.



Figure 4.22: Typical instrumentation scheme

In order to measure centerline and stub girder vertical displacements, 10 in. (254 mm) and 5 in. (127 mm) string potentiometers were used, respectively, at each side of the beam. Support settlements were measured with displacement transducers having 1 in. (25 mm) stroke at the centerline of columns on both sides. The excitation voltage for these sensors was 5V.

The diagonal deformations at each shear span were measured by 2 in. (51 mm) stroke string potentiometers running at an excitation voltage of 25V. Two string potentiometers were placed at the corners of each shear span close to the column

connection. The top string potentiometers measured the positive diagonal displacements, and the bottom string potentiometers measured the negative diagonal displacements, where positive is tension deformation of cracking and negative is compression deformation.

The horizontal displacement of the specimen relative to the strong floor was measured using a 5 in. (127 mm) string potentiometer, which was attached to a stand on the strong floor. The horizontal displacements of columns relative to each other were measured using 5 in. (127 mm) string potentiometers attached to both sides of the roller support columns. The excitation voltage for these sensors was 25V.

An array of displacement sensors were placed to measure characteristic diagonal crack deformations. These sensors were 0.5 in. (13 mm) stroke displacement transducers and were mounted over the pinned side of the shear span across the characteristic diagonal crack. Some of the sensors were mounted to the concrete surface where a stirrup was embedded underneath, and some of them were on plain concrete zones to observe the limiting effect of web reinforcement to the crack width restraint. The specimens with crack clip arrays were D6.A2.GR60#5.S, D6.A2.GR40#4.S, and D6.A2.GR40#4.F.

In order to monitor the possible slip of the flexural tension bars anchored in the columns, the concrete covering the tail of the bars was removed to expose the ends of the bars. The bars were drilled and tapped, and a hook with a threaded end was screwed in. String potentiometers were connected to the hooks and attached to the concrete in order to measure the relative slip of anchorage bars between the reinforcement bar and concrete.

Only for specimen D6.A2/GR40#4.F were Vishay EA-06-015DJ-120/LE strain gages placed on concrete as a 45° strain rosette assembly in order to derive principle strains in the concrete. Two strain rosette assemblies were placed at each shear span on the concrete strut zone. These strain gages were connected to Vishay 2120 series strain gage conditioners running at an excitation voltage of 5V and a gain of x2000, for increased sensitivity.

5.0 EXPERIMENTAL RESULTS

5.1 INTRODUCTION

In this section, experimental results from full-scale bent cap specimen tests are presented. Results from static tests and the fatigue test are discussed separately. All of the specimens tested spanned in a north-south direction. The Figures illustrating the beams show the south end with the pin support at the left end, whereas the north end has the roller support shown at the right end.

In the first section, results from static tests are discussed in terms of global structural behavior, load-deformation response, average shear stress and the depth of compression block reduced due to shear including the effect of vertical web reinforcement, and the effect of embedded reinforcement at the anchorage zone on the ultimate shear strength.

The second section solely focuses on the fatigue test result. The effect of 1,000,000 cycles of fatigue loading on the structural response and other experimental variables are discussed in detail.

A reduced set of data is presented in this section in order to highlight characteristic results and the influence of experimental parameters on the structural behavior. The complete data set is contained in a separate report (*Higgins, et al., 2008*).

5.2 RESULTS FROM STATIC TESTS

5.2.1 Global Structural Behavior

All the specimens were tested to failure as described in Section 3.5.1. Similar to the slender beams, visible flexural cracks at the mid span were initially observed for all 6 ft deep specimens with combined actuator load of 200 kips (890 kN) (100 kip in each cylinder) resulting in a moment of 9600 kips-in (1085 kN-m) in the moment region between the stub girders except specimen D4.A2.G40#4.S where flexural cracking took place at about 100 kips (445 kN) of total load corresponding to a moment of 4800 kips-in (542 kN-m) in the moment region between the stub girders. A characteristic diagonal crack formed in the shear spans suddenly while the specimens were under incremental loading. Since the tests were performed via load control instead of displacement control, the rapid propagation of the critical

diagonal crack was an expected feature. For 72 in. (1.83 m) deep specimens, the magnitude of shear in the shear span varied between 115 kips – 130 kips when the characteristic diagonal crack formed. The diagonal crack was easy to distinguish, initiating at the bottom corner of the beam-column connection penetrating toward the compression block at the load application zones of the stub girders. After the formation of the characteristic diagonal crack, the propagation of the flexural cracks at the mid span diminished, and additional diagonal cracks approximately parallel to the critical diagonal crack formed with further load increases outlining the stress field at each shear span.

Specimen D6.A4.G60#5.S failed while the load was held due to the crushing of concrete in the compression zone. Close to failure, arching diagonal cracks were formed that passed through the service holes. The crack pattern for specimen D6.A4.G60#5.S and a view of the failed shear span are shown in Figure 5.1 and Figure 5.2, respectively. The maximum crack width measured was 0.45 in (11.4 mm) at a peak load of 506.4 kips (2253 kN), and the critical diagonal crack angle was 39.7° with respect to the horizontal direction.



Figure 5.1: Crack pattern for Specimen D6.A4.G60#5.S



Figure 5.2: Specimen D6.A4.G60#5.S shear span at failure

The failure mode for specimen D6.A4.G40#4.S was a shear-compression failure. In this mode, the concrete at the compression block was crushed, and spalling at both ends of the compressive strut was observed. However, there were no cracks visible around service holes. The crack pattern for specimen D6.A4.G40#4.S and a view of the failed shear span are shown in Figure 5.3 and Figure 5.4, respectively. The maximum crack width measured in this specimen was 0.36 in (9.1 mm) at a peak load of 406.6 kips (1809 kN), and the critical diagonal crack angle was 39.5° with respect to the horizontal direction.



Figure 5.3: Crack pattern for Specimen D6.A4.G40#4.S



Figure 5.4: Specimen D6.A4.G40#4.S shear span at failure

Specimen D6.A2.G60#5.S was the only specimen to have a ductile shearcompression failure mode. Spalling of the concrete near the beam-column connection at the main longitudinal reinforcing bar elevation and minor kinking of the embedded reinforcing bars at the beam-column interface was observed post failure. Diagonal cracks crossing service holes were formed similar to those in Specimen D6.A4.G60#5.S. The crack pattern for specimen D6.A4.G40#4.S and a digital photograph of the failed shear span are shown in Figure 5.5 and Figure 5.6, respectively. A maximum crack width of 0.24 in. (6.1 mm) was measured at a peak load of 394.2 kips (1754 kN), and the critical diagonal crack angle was 40.9° with respect to the horizontal direction.



Figure 5.5: Crack pattern for Specimen D6.A2.G50#5.S



Figure 5.6: Specimen D6.A4.G50#5.S shear span at failure

The failure mode for Specimen D6.A2.G40#4.S was a brittle shear-compression failure. Major spalling of the concrete was observed at both ends of the critical diagonal crack. Major deformations occurred in the longitudinal bars both at the compression zone and at the lower beam–column interface. Almost all of the stirrups were fractured at post failure. No diagonal cracks were formed around service holes as was the case with the other specimen having Grade 40 #4 vertical web reinforcement, D6.A4.G40#4.S. The crack pattern for specimen D6.A2.G40#4.S and a digital photograph of the failed shear span are shown in Figure 5.7 and Figure 5.8, respectively. In this test it was not possible to measure the maximum crack width, since the specimen was loaded further after the peak load of 293.8 kips (1307 kN). The critical diagonal crack angle was measured as 40.5° with respect to the horizontal direction.



Figure 5.7: Crack pattern for Specimen D6.A2.G40#4.S



Figure 5.8: Specimen D6.A2.G40#4.S shear span at failure

Specimen D4.A2.G40#4.S was the only specimen with a/d ratio different than the other specimens, having the same shear span length, but with a shallower crosssection. The failure mode of D4.A2.G40#4.S was brittle shear-compression. The crack angle for the critical diagonal crack was shallower than those observed in the 6 ft high specimens. In addition, the critical diagonal crack became almost flat within the compression block connecting to a typical inclined crack propagating from the loading zone at the stub girder. Due to the location of the service holes, the critical diagonal crack passed through the smaller of the service holes, as seen in Figure 5.9. At later stages of loading near failure, inclined cracks were observed over the service holes in the failed shear span, which was unique to this specimen. A horizontal crack around the column circumference at the beam-column connection elevation was also observed. This horizontal crack was not observed in the other specimens, and it was assumed to form due to the bending of the column, which was relatively more slender compared to those in 6ft specimens. Minor spalling of the concrete was observed at the longitudinal reinforcement elevation near the beam-column connection. A digital photograph of the failed shear span is

shown in Figure 5.10. The maximum crack width measured for this specimen was 0.22 in (5.6 mm), at a peak load of 207.3 kips (922 kN), and the critical diagonal crack angle was measured as 37.2° with respect to the horizontal direction.



Figure 5.9: Crack pattern for Specimen D4.A2.G40#4.S



Figure 5.10: Specimen D4.A2.G40#4.S Shear span at failure

5.2.2 Load – Deformation Response of Specimens

Overall load-deformation behavior for each specimen is shown in Figure 5.11–15. The initial stiffness of all the 6 ft high specimens was quite similar, since the reinforcement is ineffective at this phase. As expected, specimen D4.A2.G40#4.S proved to be less stiff than the deeper specimens due to its relatively reduced moment of inertia. Structural response characteristics of the specimens are summarized in Table 5.1. The shear strength of specimen D6.A2.G40#4.F is included for comparison purposes.



Figure 5.11: Centerline load – deformation plot for specimen D6.A4.G60#5.S



Figure 5.12: Centerline load - deformation plot for specimen D6.A4.G40#4.S



Figure 5.13: Centerline load – deformation plot for specimen D6.A2.G60#5.S



Figure 5.14: Centerline load - deformation plot for specimen D6.A2.G40#4.S



Figure 5.15: Centerline load - deformation plot for specimen D4.A2.G40#4.S

#	Specimen Name	V _{APP} [kips] (kN)	V _{DL} [kips] (kN)	V _{TOT} [kips] (kN)	V _{CR} [kips] (kN)	V _{CL} [in] (mm)	V _{Diag} [in] (mm)	w _{cr} [in] (mm)	
1	D6.A4.G60#5.S	495.2	11.2	506.4	130	0.77	0.26	0.45	39.7
_		(2203)	(50)	(2253)	(572)	(19.5)	(6.6)	(11.4)	
2	D6.A4.G40#4.S	395.4	11.2	406.6	125	0.71	0.27	0.36	39.5
_		(1759)	(50)	(1809)	(550)	(18.0)	(6.9)	(9.1)	
3	D6.A2.G60#5.S	383.0	11.2	394.2	113	1.17	0.33	0.24	40.9
		(1704)	(50)	(1754)	(497)	(29.7)	(8.4)	(6.1)	
4	D6.A2.G40#4.S	282.6	11.2	293.8	115	0.54	0.30	-	40.5
		(1257)	(50)	(1307)	(506)	(13.7)	(7.6)		
5	D4.A2.G40#4.S	199.2	8.1	207.3	68	0.68	0.19	0.22	37.2
_		(886)	(36)	(922)	(299)	(17.3)	(4.8)	(5.6)	
6	D6.A2.G40#4.F	271.9	11.2	283.1	-	-	-	-	-
		(1204)	(50)	(1254)					

Table 5.1: Structural response characteristics of the specimens

The ultimate shear load is the combination of two components, expressed as V_{APP} and V_{DL} . V_{APP} is the maximum shear force at the failure shear span applied by the actuators. V_{DL} is the shear force from the beam self-weight acting on the failure plane. In order to calculate the V_{DL} component, the beam self-weight of the effective reinforced concrete volume acting on the diagonally cracked failure plane was computed, as illustrated in Figure 5.16.



Figure 5.16: Effective volume used in dead weight calculation

The unit weight of reinforced concrete was taken as 150 lb/ft³ (2400 kg/m³) and multiplied with half of the effective reinforced concrete volume to calculate the shear force from the beam self-weight acting on the failure plane. V_{TOT} is the total shear force calculated by adding the self weight shear to the maximum shear force applied by the actuators. V_{CR} is the diagonal cracking shear force determined according to the readings from the diagonal displacement sensors measuring the

total diagonal displacement orthogonal to the crack orientation at the shear spans. The relation between the diagonal displacement at the shear spans where failure occurred and the maximum shear force applied by the actuators for all specimens is shown in Figure 5.17. V_{CL} is the centerline displacement corresponding to the failure load calculated by subtracting support settlements from the centerline deflection measurement taken relative to the strong floor. V_{Diag} is the diagonal displacement measured across the tension-field of the failed shear span at the maximum applied load, and is the characteristic diagonal crack angle with respect to the longitudinal axis of the specimen.



Figure 5.17: Diagonal displacements for all specimens at the failed shear span (backbone curves are shown for clarity)

First, a comparison between the specimens having the same anchorage zone steel configuration will be made. When specimens D6.A4.G60#5.S and D6.A4.G40#4.S are compared where the amount of flexural steel at the anchorage zone was the same but the product of the nominal web reinforcement area, A_v , and the measured web reinforcement yield strength, f_{yv} , was reduced approximately by 48%, there was a loss in shear capacity by 99.8 kips (443.5 kN), which corresponds to a 19.7% reduction in capacity. When specimens D6.A2.G60#5.S and D6.A4.G40#4.S are compared, which have the same anchorage zone steel configuration, it is seen that there is a degradation in shear capacity by 100.4 kips (446 kN) corresponding to a 25.5 % decrease in shear capacity when A_v*f_{yv} was reduced by 48%.

A second comparison can be made between specimens having the same web reinforcement configuration but different anchorage zone steel properties. Specimens D6.A4.G60#5.S and D6.A2.G60#5.S both have Grade 60 #5 stirrups as the web reinforcement, but there is a reduction of 52% in the product of the area of embedded flexural steel anchored in the column and the measured tension steel yield strength f_y , which results in a loss of 112.2 kips (498.6 kN) in shear strength corresponding to a 22.2 % reduction in shear capacity. Similarly, when specimens D6.A4.G40#4.S and D6.A2.G40#4.S, both having Grade 40 #4 stirrups as the web reinforcement, are compared, a loss of 112.8 kips in shear strength (501.3 kN) is observed corresponding to a 27.7% reduction in shear capacity when the flexural anchorage capacity was reduced by 50%.

Specimen D4.A2.G40#4.S had the same number of embedded flexural steel anchored in the column and web reinforcement properties as specimen D6.A2.G40#4.S. As a result of reducing the overall height of the specimen by 33.3%, the loss in shear capacity was 86.5 kips.

5.2.3 Average Shear Stress and Depth of the Shear Compression Block (cs)

Depth of the concrete shear compression block, c_s , has been identified as an important variable affecting the overall shear strength of a deep beam since it defines the zone where failure occurs when shear-compression failure mode is observed. The depth of the shear compression block is a portion of the flexural compression block, c, above the flexural cracks as illustrated in Figure 5.18. According to Zararis (2003), the height of the shear compression block depends mainly on the shear span to depth ratio and secondarily on the ratio v/v, where v is the web reinforcement ratio and v/v is the flexural reinforcement ratio.



Figure 5.18: Shear compression block (c_s) and flexural compression block (c)

Table 5.2 shows the approximate shear compression block depth, c_s , measured after failure; the effective depth, d_{vs} , which is the vertical distance between the midheight of the concrete shear compression block and the tension reinforcement at the beam-column connection; flexural reinforcement ratio at the anchorage zone, a, calculated from Equation 5.1; vertical web reinforcement ratio, v, calculated from Equation 5.2; the average shear stress at failure term, average,max, introduced for comparison purposes calculated from Equation 5.3; and the average shear stress at failure, normalized, normalized by the square root of day-of-test cylinder concrete compressive strength in order to eliminate variations in test day concrete strengths for a more direct comparison.

$$\rho_{a} = \frac{A_{s,a}}{b d}$$
(5.1)

Where $A_{s,a}$ is the area of the embedded reinforcement in the anchorage zone, b is the beam width, and d is the effective height, defined as the distance between the centroid of the flexural tension steel bars, and the top compression fiber of the section.

$$\rho_{v} = \frac{A_{v}}{bs}$$
(5.2)

Where A_v is the vertical web reinforcement area, b is the beam width, and s is the vertical web reinforcement spacing.

$$v_{\text{average,max}} = \frac{V_{\text{APP}}}{b \, d_{\text{vs}}}$$
(5.3)

$$v_{\text{normalized}} = \frac{V_{\text{APP}}}{b \, d_{\text{vs}} \sqrt{f_{\text{c}}'}}$$
(5.4)

Where V_{APP} (lbs) is the ultimate shear force including self-weight, b (in) is the beam width, d_{vs} (in) is the vertical distance between the mid height of the concrete shear compression block and the tension reinforcement at the beam-column connection, and f_c ' (psi) is the test day cylinder compressive strength of concrete.

#	Specimen Name	c _s [in] (cm)	d _a [%]	d _v [%]	d _{vs} [in] (cm)	d _{average,max} [ksi] (MPa)	d _{normalized} [-] (NA)
1	D6.A4.G60#5.S	9.500 (24.1)	0.558	0.443	65.05 (165.2)	0.487 (3.32)	7.74
2	D6.A4.G40#4.S	9.125 (23.5)	0.558	0.286	65.23 (165.7)	0.390 (2.69)	6.29
3	D6.A2.G60#5.S	6.250 (15.9)	0.279	0.443	66.67 (169.3)	0.370 (2.55)	5.80
4	D6.A2.G40#4.S	6.250 (15.9)	0.279	0.286	66.67 (169.3)	0.275 (1.90)	4.64
5	D4.A2.G40#4.S	3.750 (9.5)	0.426	0.286	43.92 (111.5)	0.295 (2.03)	4.85
6	D6.A2.G40#4.F	6.500 (16.5)	0.279	0.286	66.55 (169.0)	0.266 (1.83)	4.28

 Table 5.2: Average Shear Stress and Depth of Shear Compression Block

For the first four specimens, where the shear span to depth ratio remains the same, it can be seen that the compression block depth above the critical diagonal crack is primarily affected by the amount of tension reinforcement embedded in the anchorage zone and secondarily affected by the vertical web reinforcement ratio. The depth of the shear compression block above the critical diagonal crack significantly increases with increasing amount of tension reinforcement embedded in the anchorage zone, while it slightly decreases with increasing vertical web reinforcement ratio.

Figure 5.19 illustrates the relation between the reinforcement ratios and the normalized average shear stress at failure for all of the specimens. The dash lines were placed in order to compare specimens having the same tension reinforcement ratio but different web reinforcement ratios or vice versa.



Figure 5.19: (a) Normalized average shear stress – tension reinforcement stress, (b) Normalized average shear stress – vertical web reinforcement stress

By comparing specimens having the same web reinforcement ratios but different tension reinforcement ratios at the column face (D6.A4.G60#5.S – D6.A2.G60#5.S and D6.A4.G40#4.S – D6.A2.G40#4.S, it was observed that the same amount of reduction in the number of bars embedded in the anchorage zone resulted in approximately the same degradation in the normalized average shear stress at failure for both cases. Likewise, by comparing specimens having the same tension reinforcement ratios at the column face but different web reinforcement ratios (D6.A4.G60#5.S - D6.A4.G40#4.S and D6.A2.G60#5.S - D6.A2.G40#4.S), it was seen that the same amount of reduction in the web reinforcement ratio (and also in the yield strength) resulted in approximately the same degradation in the normalized average shear stress at failure for both cases. As expected, the same degradation pattern was not observed when the 4 ft (122 cm) deep specimen D4.A2.G40#4.S was compared with 6 ft (183 cm) deep specimens, which is due to the change in the a/d ratio resulting in a different response behavior. Moreover, when specimens D6.A2.G40#4.S and D6.A2.G40#4.F are compared, the degrading effect of the fatigue loading to the normalized average shear stress at failure can be observed. Detailed results from the fatigue test will be discussed in later sections.

The relation between the tension reinforcement ratio at the column face, the web reinforcement ratio, and the normalized average shear stress at failure for the statically loaded specimens having the same a/d ratio can also be illustrated as a failure plane as shown in Figure 5.20.



Figure 5.20: Relationships between the tension reinforcement ratio at the column face, web reinforcement ratio, and the normalized average shear stress at failure, for statically loaded specimens having a/h ratio of 1.33

5.2.4 Web Reinforcement

Since strain gages were placed along the critical diagonal crack path, it was possible to measure the maximum demand on stirrups. It was observed that at earlier steps of loading, stirrups strains were insignificant since the demand on stirrups was very low or almost nonexistent prior to cracking. Upon formation of the characteristic diagonal crack, which propagates from the beam soffit–column face upward toward the loading point as shown in Figure 5.21, there was a sudden increase in strain measured in the stirrups crossing diagonal crack. This was observed for all of the bent cap specimens. At later loading stages, the critical diagonal crack continued to propagate and crossed over additional stirrups inducing strains on those which were previously not engaged. It can be seen that stirrups crossing the critical diagonal crack were activated sequentially depending on the progress of the crack propagation.



Figure 5.21: Splitting plane of the critical diagonal crack

The relation between stirrup strains and the shear force on the corresponding shear span for specimen D6.A2.G60#5 is shown in Figure 5.22, as an example. By observing the force–strain relation in the plot, it can be seen that all of the stirrups sequentially achieve yield before failure. Thus, V_s can be reasonably estimated as the yield stress times the area of stirrups crossing the characteristic diagonal crack.



Figure 5.22: Stirrup strains - shear force relation for specimen D6.A2.G60#5.S

The sequential yielding behavior of the vertical web reinforcement can be seen more clearly in Figure 5.23, where the variation of stresses in specimen D6.A2.G60#5.S north shear span stirrups is shown for each increment of 50 kips in applied shear force (see Section 3 for stirrup numbering system), as an example. Stirrup stresses was calculated according to the Hooke's Law, by taking the modulus of elasticity as 29000 ksi (200 GPa) up to the yield point and then assuming perfectly plastic behavior after yielding.



Figure 5.23: Change in stirrup stresses at different stages of loading

In order to assess the post-failure behavior of bent caps, specimen D6.A2.G40#4.S was loaded further after reaching the peak load. Fracture of stirrups was observed as the specimen underwent very large displacements as shown in Figure 5.24. However, this phenomenon is a post-failure event, and the failure of specimens was not controlled by progressive fracture of stirrups.



Figure 5.24: Fractured stirrups at post-failure (specimen D6.A2.G40#4.S)

5.2.5 Anchorage Zone

The flexural tension reinforcement at the bottom of a bent cap, which is anchored into the bent columns, plays a crucial role on the failure mode and the ultimate load capacity of bent caps since a pull-out or anchorage failure may lead to total system collapse of the bridge due to the non-redundant nature of the bent cap member. 1950's vintage RCDG bridge bent caps are usually considered as inadequate due to poorly detailed anchorages when analyzed by conventional methods such as those available in the ACI and AAHSTO specifications. The specimens were designed in

a manner that reasonably reflects their in-field counterparts including the anchorage zone detailing.

As described in Section 3.6, the embedded straight flexural tension reinforcement terminated in the columns. Only a minimum cover was provided at the tail. Reinforcing bars were monitored with strain gages to measure the strain in the bar at distinct locations within the anchorage zone, and the ends of the bars were monitored with displacement transducers to measure the rebar end slip relative to the column surface. Strain gages along the embedment were placed at the column face, ¹/₄ point, and centerline of the column.

Relative displacement measurements revealed that slip of the flexural reinforcement in the column did not occur for 6ft deep bent cap specimens. However, slip of the flexural reinforcement was observed in all the embedded reinforcing bars for specimen D4.A2.G40#4.S with relative displacement ranging between 0.0037 - 0.0049 in. This phenomenon can be explained by the interaction between the beneficial influence of normal pressure from column axial loading on bond strength and the relative bond stress demand on the tension reinforcement. Untrauer and Henry (1965), Malvar (1992), Kong et al. (1996), and Koester and Higgins (2008) studied the influence of normal pressure on bond strength, and their experiments showed that normal pressure applied on the rebar anchorage increases the bond strength. Specimens D6.A2.G40#4.S and D4.A2.G40#4.S had the same properties in terms of web reinforcement and the embedded tension reinforcement. Slip of the reinforcement was measured when approximately 130 kips of shear force was applied to specimen D4.A2.G40#4.S, but no rebar slip event was observed for specimen D6.A2.G40#4.S at the same load level corresponding to the same amount of normal pressure applied on the embedded rebar. However, the demand on the tension reinforcement in specimen D4.A2.G40#4.S at that load was higher than it was in specimen D6.A2.G40#4.S due to the reduced moment arm length in the shallower specimen. When the demand on the tension reinforcement in the deeper specimen reaches the same amount that caused slip of the reinforcement in the shallower specimen, the normal pressure on the rebar also increases with higher applied load increasing the slip resistance.

Embedded tension reinforcement strains were measured with strain gages bonded at three distinct locations along the embedment as explained in Section 3.6.1 for each of the two reinforcing bars at both sides of the specimens except specimen D6.A4.G60#5.S where only two strain gages were bonded for each end of the embedded reinforcing bar. Since the demand on the embedded reinforcing bars at the anchorage zone was very low prior to the appearance of the characteristic diagonal crack, reinforcing bar strains were also almost nonexistent. A sudden increment in strain was observed when the characteristic diagonal crack formed due to the progressive demand on the reinforcing bars as shown in Figure 5.25 where the relation between the column reaction and northwest embedded reinforcement strains are shown for specimen D6.A2.G40#4.S as an example.



Figure 5.25: Relation between the column reaction and strains at the embedded reinforcement at the anchorage zone (Specimen D6.A4.G40#4.S NW)

5.2.5.1 Estimating the Demand on the Tension Reinforcement at Failure

In order to determine the demand on the tension reinforcement at failure, the strain measurement at the column–beam interface is needed. However, it was observed that the strain gage placed at the column face where the column and the beam intersect did not always yield reliable measurements for load steps after the formation of the critical diagonal crack due to extensive cracking at that location and localized bending of the bar at the interface. In order to estimate the demand on the tension reinforcement and the average bond stress at failure, a second order polynomial extrapolation technique was employed. Since the strain and the bond stress must be zero at the tail of the reinforcing bar (taken as the origin), boundary conditions given in Equation 5.5 must to be satisfied.

$$\begin{aligned} \varepsilon(0) &= 0\\ \varepsilon'(0) &= 0 \end{aligned} \tag{5.5}$$

Disturbances due to cracking and bending was minimal at locations of inner two strain gages, and strain readings from those two gages for each new 10 kips increment in shear force were collected to be used in the extrapolation. The strain at the tail of the reinforcing bar was set to zero, and this location was added as a third point for the curve fit. Using a commercially available mathematics package, a second order polynomial curve fit in the form given by Equation 5.6, which satisfies the boundary conditions, was developed from the measured strains in order to achieve a strain distribution function along the anchorage zone. An example plot presenting sequential strain distribution functions for incremental load steps is shown in Figure 5.26.



Figure 5.26: Strain distribution in the embedded reinforcement at the anchorage zone (D6.A2.G40#4.S SE bar)

$$\varepsilon(\mathbf{x}) = \mathbf{A}\mathbf{x}^2 \tag{5.6}$$

The tension stress, A(x), and the tension force, F(x), in the longitudinal reinforcing bar as a function of the distance from the rebar tail is calculated from Equation 5.7, and Equation 5.8, respectively:

$$\sigma(\mathbf{x}) = \begin{cases} \epsilon(\mathbf{x}) \mathbf{E} & \text{for} & \epsilon(\mathbf{x}) < \epsilon_{y} \\ \mathbf{f}_{y} \mathbf{E} & \text{for} & \epsilon(\mathbf{x}) \ge \epsilon_{y} \end{cases}$$
(5.7)

$$F(x) = \begin{cases} \varepsilon(x) A_s E & \text{for} & \varepsilon(x) < \varepsilon_y \\ f_y A_s & \text{for} & \varepsilon(x) \ge \varepsilon_y \end{cases}$$
(5.8)

Where E is the modulus elasticity of steel, taken as 29000 ksi, A_s is the area of the tension reinforcement, and f_y and f_y are the yield stress and yield strain of steel from material tests, respectively. An example plot presenting sequential stress distribution functions for incremental load steps is shown in Figure 5.27.



Figure 5.27: Stress distribution in the embedded reinforcement at the anchorage zone (D6.A2.G40#4.S SE bar)

The bond stress, F(x), between concrete and the reinforcing bar can be defined as the rate of change in the tension force in the bar divided by the nominal perimeter of the bar. Since the force distribution function can be obtained from Equation 5.8, it is possible to calculate the approximate bond stress, F(x) on the reinforcing bar at any location within the anchorage zone from Equation 5.9:

$$\mu(\mathbf{x}) = \frac{1}{\pi d_{b}} \frac{dF(\mathbf{x})}{d\mathbf{x}}$$
(5.9)

Where d_b is the diameter of the reinforcing bar.

The average bond stress, avg, along the anchorage zone can be calculated as the area under the approximate bond stress curve, divided by the distance, which is given in Equation 5.10.

$$\mu_{avg} = \frac{\int_{0}^{x_{y}} \mu(x) dx}{X_{y}} , \quad 0 < X_{y} \le X_{c}$$
 (5.10)

Where x_y is the coordinate of the yield location in the reinforcing bar within the anchorage zone. It should be noted that if the yield location occurs outside of the column anchorage zone, the value of x_y should be taken as the total length of the anchorage zone, which equals the column width less the tail cover, x_c . An example plot presenting bond stress distributions for incrementally larger load steps is shown in Figure 5.28.



Figure 5.28: Bond stress distribution in the embedded reinforcement at the anchorage zone (D6.A2.G40#4.S SE bar)

5.2.5.2 AASHTO LRFD and ACI 318-05 Development Length

In this section development length and average bond stress equations from AASHTO LRFD (2005), and ACI 318-05 (2005) are compared with the experimental data.

Development length, l_d , is defined as the minimum length of the embedment required to fully develop the rebar stress from zero to yield stress, f_{y} , and can be expressed as:

$$l_{d} = \frac{f_{y}d_{b}}{4\mu_{avg}}$$
(5.11)

According to AASHTO-LRFD, the development length of a straight bar only depends on the diameter and the yield strength of the bar and the compressive strength of concrete neglecting all other effects such as passive or active confinement, concrete unit weight, and existence of coatings. AASHTO-LRFD section 5.11.2.1.1 calculates the development length for #11 (36) or smaller straight reinforcing bars as:

$$l_{d \text{ AASHTO LRFD}} = \frac{1.25 A_b f_y}{\sqrt{f'_c}}$$
(5.12)

Where A_b (in²) is the area of the reinforcing bar being developed, f_y (ksi) is the nominal yield strength and f_c ' (ksi) is the concrete cylinder compressive strength. AASHTO-LRFD limits the development length by:

$$l_{d,\min AASHTO LRFD} = 0.4d_b f_v$$
(5.13)

Where d_b (in²) is the diameter of the reinforcing bar, and f_y (ksi) is the nominal yield strength.

ACI 318-05 provides a more complex equation to determine the development length taking into account passive confinement, concrete unit weight, and rebar coating among others, but not considering the effect of active confinement. ACI 318-05 section 12.2.3 calculates the development length as:

$$l_{d ACI} = \left(\frac{3}{40} \frac{f_y}{\sqrt{f_c'}} \frac{\psi_t \psi_e \psi_s \lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right) d_b$$
(5.14)

Where d_b (in) is the diameter of the bar, f_y (psi) is the yield strength of the bar, f_c ' (psi) is the concrete cylinder compressive strength, t, e, and s are coefficients for location of the bar, epoxy coating, and bar size effects,

is a modifier for normal or lightweight concrete, c_b is the least dimension of either the distance from the center of the bar to nearest concrete surface or one-half the center-to-center spacing of the bars being developed, and K_{tr} is a factor that represents the contribution of confining reinforcement, calculated as:

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500 \,\text{sn}}$$
(5.15)

Where A_{tr} (in²) is the area of transverse steel, f_{ty} (psi) is the yield strength of the transverse reinforcement, s (in) is the center to center spacing of the transverse reinforcement, and n is the number of bars being developed in the plane. ACI 318-05 also states that the term $(c_b+K_{tr})/d_b$ cannot be taken as larger than 2.5.

It is possible to determine the average bond stress employed by AASHTO LRFD and ACI 318-05 by equalizing Equation 5.11 to Equation 5.12 and to Equation 5.14, respectively:

$$\mu_{\text{avg AASHTO}} = \frac{\sqrt{f_c'}}{1.25 \pi d_b}$$

$$\mu_{\text{avg ACI}} = \frac{10}{3} \frac{\sqrt{f_c'} \left(\frac{c_b + K_{\text{tr}}}{d_b}\right)}{\psi_t \psi_e \psi_s \lambda}$$
(5.17)

Development lengths and average bond stresses calculated from AASHTO-LRFD and ACI 318-05 expressions are given in Table 5.3. The experimental values, which were derived from the polynomial extrapolation technique described previously, are also included in this table. First, the calculated flexural rebar development lengths and corresponding bond stresses were averaged for south and north ends of specimens. Second, average bond stresses from the north and south ends were compared, and the larger of average values was selected to populate the table. It should be noted that slip of the embedded reinforcement was not observed in any of the specimens, except specimen D4.A2.G40#4.S. Therefore, calculated values from experiments do not represent limiting values such as those from pull-out tests.

#	Specimen Name	l _d aashto [d _b]	l _{ave} AASHTO [psi] (MPa)	l _{d ACI} [d _b]	l _{ave ACI} [psi] (MPa)	l _{d EXP} [d _b]	l _{ave EXP} [psi] (MPa)
1	D6.A4.G60#5.S	50.4	353 (2.43)	44.6	399 (2.75)	-	-
2	D6.A4.G40#4.S	48.8	349 (2.41)	43.2	395 (2.72)	15.0	1122.3
3	D6.A2.G60#5.S	47.6	358 (2.47)	42.1	405 (2.79)	12.3	1370.0
4	D6.A2.G40#4.S	50.5	338 (2.33)	44.7	382 (2.63)	12.7	1324.0
5	D4.A2.G40#4.S	50.6	343 (2.37)	44.7	388 (2.67)	14.7	1162.0

Table 5.3: Development lengths and average bond stresses

As seen in Table 4.3, AASHTO-LRFD provides more conservative results. On the other hand, ACI 318-05 benefits from the additional passive confinement term, and results in relatively less conservative predictions. None of the current North American codes takes into account the beneficial effects of active confinement, such as the presence of normal pressure in the column, which causes a clamping force across the developing flexural reinforcement, thereby improving the bond strength, resulting in reduced development lengths. Koester and Higgins (2008) conducted experimental research to evaluate flexural reinforcing bar anchorages terminating in columns and proposed a modification function for ACI 318-05 development length equation, which introduces a relation between the bond strength and the active confining stress produced in the column. The proposed relation between active confining stress and bond strength within a lower end 95% confidence interval is:

$$\mu_{\text{avgACI,modified}} = (0.8 + \frac{p}{800})\mu_{\text{avgACI}}$$
(5.18)

Where p (psi) is the active confining stress in the column acting transverse to the developing bars.

Even though the bent cap flexural bond stresses from experimental results are not limit values, the data fit very closely with the average results, as shown in Figure 5.29.



Figure 5.29: Effect of confining stress on bond strength (Koester, Higgins 2008)

5.2.6 Crack Growth in Specimens

As explained in Section 3.6.2, an array of crack displacement measurements were taken using, 0.5 in. (13 mm) stroke displacement transducers that were mounted over the pinned side of the shear span along the characteristic diagonal crack. Some of the crack displacements were measured on the concrete surface where a stirrup was embedded underneath, and some of them were on plain concrete zones to observe the limiting effect of web reinforcement to the crack width growth. Specimens with crack clip arrays were D6.A2.GR60#5.S, D6.A2.GR40#4.S, and D6.A2.GR40#4.F.

The crack width changes at increasing loads were observed to be highest near the column-beam interface, and became relatively smaller along the crack path to the

load application point. It was also observed that at lower load levels, the limiting effect of the web reinforcement to restrain the crack width was not very significant. At lower load levels, the average crack width growth was approximately proportional to the applied shear. However, significant restraining effect of the stirrups to the crack width was observed at higher loads, as seen in Figure 5.30.



Figure 5.30: Change in crack growth in Specimen D6.A2. G60#5.S for various load levels (asterisks denote existence of web reinforcement under the crack clip)

Offset of the concrete surface along the critical diagonal crack was observed in all of the specimens, as seen in Figure 5.31. The crack motion was primarily in the direction transverse to the critical diagonal crack, which can also be seen in Figure 5.31.



Figure 5.31: Offset of the concrete surface along the critical diagonal crack

5.2.7 Summary Observations from Static Tests

- The observed failure mode in all of the specimens was shear compression, which is caused by crushing of concrete in the compression zone at the top of a characteristic diagonal crack.
- Failure mode was generally brittle; however, a ductile failure mode was observed in one specimen due to substantial yielding of the flexural anchorages.
- Both anchorage of flexural steel and vertical web reinforcement was key to developing higher ultimate capacity.
- Initial diagonal-tension cracking was observed at an average concrete shear stress approximately corresponding to $v_{cr} = 1.8\sqrt{f'_c}$, with a coefficient of variation of 0.09.
- For specimens having the same a/d ratio, depth of the concrete compression block is mainly controlled by the amount of embedded reinforcement in the anchorage zone. The effect of vertical web reinforcement to the depth of the concrete compression block was minimal.
- Sequential yielding of the vertical web reinforcement was observed in all of the specimens as the characteristic diagonal crack further
propagated into the compression block. Crack widths were sufficiently wide to permit yielding of all stirrups across the crack.

- A polynomial extrapolation method was employed to approximate the tension demand on the embedded reinforcement in the anchorage zone. Results show that the embedded flexural reinforcement was beyond yield at the column face when failure occurred.
- Slip of embedded reinforcement was not observed except for the 4 ft deep specimen, D4.A2.G40#4.S.
- Development length equations provided in AASHTO-LRFD and ACI 318-05 codes were observed to be overly conservative since neither of these provisions takes into account the beneficial effect of active confining stress on bond strength.
- At flexural steel cut-offs, vertical flexural cracks were observed to turn into diagonal cracks near the bar cut-off locations due to the stress concentration at those locations. However, these did not control the strength of specimens.
- The angle of the characteristic diagonal crack appeared to mainly depend on the a/d ratio and secondarily on the amount of embedded reinforcement in the anchorage zone. The effect of vertical web reinforcement on the crack angle was relatively less.
- The heavier web reinforcement configurations resulted in more extensive cracking at the shear span compared to specimens with lighter web reinforcement.
- Offset of the concrete surface was observed most visible along the critical diagonal crack probably as a result of localized stirrup debonding.
- Observed crack motions indicated relatively little motion parallel to the diagonal crack when compared with the motion in the direction transverse to the crack.
- The presence of utility holes did not affect the ultimate shear strength of 6 ft (183 cm) deep specimens considering their location with regard to the characteristic diagonal crack. However their presence might have affected the behavior of the 4 ft (122 cm) deep specimen since the critical diagonal crack passed through one of the holes. This issue is further investigated by comparing experimental and nonlinear finite element analysis results in Section 5.6 of this document.

5.3 **RESULTS FROM THE FATIGUE TEST**

Specimen D6.A2.G40#4.F was subjected to high cycle fatigue to examine the effects of repeated service level loading on structural response and strength. The fatigue experiment was conducted in three phases. In the first phase a precracking test was performed on specimen D6.A2.GR40#4.F using the static test load setup and the previously described loading protocol in order to produce significant diagonal cracks similar to those observed from field inspections. In the second phase the loading scheme was changed to single point loading, and 1,000,000 cycles of fatigue loading were applied to the specimen. During this phase, data were collected from strain gages placed on the transverse reinforcement, flexural reinforcement, and on the concrete surface where mean strain and strain range data were collected continuously. An array of displacement sensors was also placed across the characteristic diagonal crack on the south shear span in order to observe crack motions during fatigue cycles. At intervals of 100,000 cycles, "check" tests were performed to capture additional data from displacement sensors that were detached during fatigue cycling. The last phase of the fatigue experiment was the failure test where the fatigue load setup was replaced with the static load setup. In this final phase, the specimen was tested with the static load protocol from the beginning until failure. More information about material properties, fatigue loading scheme, and instrumentation is given in Section 3.

In the following sections, results and observations from each of the fatigue experiment phases are presented and discussed. Similar to the static test results, a reduced set of data is presented in this section to highlight characteristic results and the influence of experimental parameters on the structural behavior. The complete data set is contained in a separate report (*Higgins, et al., 2008*).

5.3.1 Precracking Test

Specimen D6.A2.G40#4.F was loaded for 5 main static cycles up to a total applied load of 450 kips (2002 kN) resulting in an applied shear load of 225 kips (1001 kN), which corresponds to a total shear load of 236 kips (1050 kN) with the inclusion of dead load on both shear spans. When specimen D6.A2.G40#4.F is compared with specimen D6.A2.G40#4.S, which had the same reinforcement detailing, the total shear load applied for the precracking test corresponds to 83% of the static test specimen's shear capacity. Load–deformation response of the fatigue specimen during the precracking phase is shown in Figure 5.32.



Figure 5.32: Load - deformation response of similarly detailed fatigue and static specimens during initial loading

The initial stiffnesses of specimen D6.A2.G40#4.S and specimen D6.A2.G40#4.F were observed to be similar. The formation of the characteristic diagonal crack in the fatigue specimen took place slightly earlier than its counterpart, which was attributed to different test day concrete strengths (3463 psi vs. 3542 psi). The overall load–deformation response of both specimens was similar, which enables direct comparisons to be made after fatigue cycles. Key structural parameters from the precracking test are given in Table 5.4.

#	Specimen Name	V _{APP} [kips] (kN)	V _{DL} [kips] (kN)	V _{TOT} [kips] (kN)	V _{CR} [kips] (kN)	V _{CL} [in] (mm)	V _{Diag} [in] (mm)
6	D6.A2.G40#4.F	225	11.2	236.2	102	0.33	0.12
		(1001)	(49.8)	(1051)	(454)	(8.4)	(3.1)

 Table 5.4: Specimen D6.A2.G40#4.F Precracking Structural Response Parameters

The crack widths were monitored at six distinct locations on the characteristic diagonal cracks on both shear spans with an Oregon Department of Transportation (ODOT) crack comparator at load and after unloading. A micrometer was used when the crack width was beyond the crack comparator's range. The crack widths measured on the main diagonal cracks when the specimen was unloaded at the end of the precracking test ranged between 0.030 in. - 0.050 in. (0.76 mm – 1.27 mm), and 0.016 in. – 0.030 in. (0.41 mm – 0.76 mm) for the north and the south shear

spans, respectively. Readings from six distinct crack measurement locations at unloaded stages are given in Table 5.5. Crack measurement locations are also shown for the south and north shear spans in Figure 5.33, and Figure 5.34, respectively.

	Crack Width	
Location	[in]	
	(mm)	_
CMI North 1	0.020	
CML North – I	(0.51)	
CMI North 2	0.030	
CML NOT III - 2	(0.76)	_
CMI North 2	0.016	
CML Norui – 5	(0.41)	_
CMI South 1	0.030	
CML South – I	(0.76)	_
CMI South 2	0.050	
CML South – 2	(1.27)	
CMI South 2	0.040	-
Civil South – 3	(1.02)	_

Table 5.5: Measured Crack Widths at the End of the Precracking Test



Figure 5.33: Specimen D6.A2.G40#4.F south shear span instrumentation layout



Figure 5.34: Specimen D6.A2.G40#4.F north shear span instrumentation layout

It was observed that all of the instrumented stirrups reached yielding during the precracking test. As was the case in the previous static tests, stirrups crossing the critical diagonal crack were activated sequentially depending on the progress of the crack propagation. The relation between stirrup strain in Stirrup S-S7 and the applied shear is shown in Figure 5.35 as an example.



Figure 5.35: Specimen D6.A2.G40#4.F stirrup S-S7 shear - strain relation

None of the instrumented flexural bars at mid-span yielded during the precracking test. This result was expected since yielding of the flexural bars at mid-span was not observed in the previous specimens even at much higher loads due to the large amount of reinforcement at mid-span. On the other hand, readings from the strain gages on the embedded reinforcement at the anchorage zone indicated that the bars were either very close to yielding or yielded in the last load cycle of the precracking test. The same polynomial extrapolation method, which was explained previously in Section 5.2.5.1, was employed to approximate the tension demand on the embedded reinforcement at the column face in the anchorage zone. An example plot which shows the relation between anchorage steel strain readings from the SE embedded bar strain gages and the applied shear is shown in Figure 5.36.



Figure 5.36: Specimen D6.A2.G40#4.F SE embedded bar shear - strain relation

5.3.2 Fatigue Test

The fatigue load range was determined from field tests as described in Section 5.3.2.1. The fatigue load was applied with a single hydraulic actuator at the centerline of the specimen for 1,000,000 cycles and ranged between 150 kips and 210 kips. In this case, the shear load at each shear span varied between 75 kips and 105 kips simulating the superimposed dead load on the bridge bent cap plus the live load imposed by vehicles passing through the bridge. The applied fatigue load was monitored directly. Mean shear and the shear range applied to the specimen through the fatigue testing phase is shown in Figure 5.37.



Figure 5.37: Specimen D6.A2.G40#4.F mean shear load and shear load range during fatigue phase

5.3.2.1 Effect of Fatigue Loading on Web Reinforcement

It was observed that in the south shear span the stirrup mean stresses tended to decrease during the fatigue loading as shown in Figure 5.38. The north shear span stirrup mean stresses also decreased in most of the cases as shown in Figure 5.39. However the average reduction in mean stress was less than that observed in the south shear span. Mean stress readings from all north and south stirrups are summarized in Table 5.6 presenting best fit of the results over the entire fatigue cycling and the percentage difference. In general the changes in mean stresses were small. As seen in Figure 5.38 and Figure 5.39, there was a definite temperature influence seen in the strain data. The general declining trend of the mean stresses was very slight and negligible compared with the thermal effects.



Figure 5.38: Specimen D6.A2.G40#4.F south shear span stirrup mean stresses



Figure 5.39: Specimen D6.A2.G40#4.F north shear span stirrup mean stresses

	0-100K	900K-1000K	
	cvcles	cvcles	
Stirrup #	Mean Stress	Mean Stress	% Difference
1	[ksi]	[ksi]	
	(MPa)	(MPa)	
S-S1	27.6	27.1	-1.9
_	(190)	(187)	
S-S2	34.0	32.5	-4.4
	(234)	(224)	
S-S3	34.7	32.8	-5.6
	(239)	(226)	
S-S4	27.6	28.1	1.9
	(190)	(194)	
S-S5	28.0	26.9	-3.7
	(193)	(186)	
S-S6	24.1	21.9	-9.3
	(166)	(151)	
S-S7	26.1	22.8	-12.9
_	(180)	(157)	
S-N1	26.9	26.7	-0.5
	(186)	(184)	
S-N2	36.6	36.4	-0.4
	(252)	(251)	
S-N3	33.2	33.8	1.9
_	(229)	(233)	
S-N4	35.9	34.7	-3.5
	(248)	(239)	
S-N5	32.6	32.3	-0.9
	(225)	(223)	
S-N6	24.8	24.4	-1.6
	(171)	(168)	
S-N7	21.2	20.7	-2.2
	(146)	(143)	

Table 5.6: Specimen D6.A2.G40#4.F Stirrup Mean Stresses

Stress ranges in the first four stirrups from the beam-column interface (except stirrup S-N1) in both shear spans slightly increased during fatigue loading, while strain ranges measured on the remaining stirrups closer to the stub girders tended to slightly decrease as seen in Figure 5.40 and Figure 5.41. The change in the stress ranges occurred mainly in the first half of the fatigue test then became relatively steady and almost constant in the later

part of the test. Stress range readings from all north and south stirrups are summarized in Table 5.7 including best fit line of the data during fatigue cycling and the percentage difference from the initial to final cycles. It was also seen that the two stirrups closest to the stub girders on both shear spans exhibited lower stress ranges than other stirrups in the respective shear spans. Stress ranges in all of the stirrups during fatigue loading were less than 20 ksi (138 MPa), thus the possibility of stirrup fracture due to metal fatigue was not likely.



Figure 5.40: Specimen D6.A2.G40#4.F south shear span stirrup stress ranges



Figure 5.41: Specimen D6.A2.G40#4.F north shear span stirrup stress ranges

	0-100K cycles	900K-1000K cycles	
C4: #	Stress Range	Stress Range	0/ Difference
Surrup #	[ksi]	[ksi]	76 Difference
	(MPa)	(MPa)	
C C1	10.15	10.29	1 29
5-51	(70)	(71)	1.38
5 57	11.43	11.74	2 72
5-52	(79)	(81)	2.12
5-53	10.28	10.51	2.28
6-65	(71)	(72)	2.20
S-S4	10.36	10.99	6.15
5-54	(72)	(76)	0.15
S-S5	9.72	9.65	-0.73
5-55	(67)	(67)	-0.75
S-S6	7.71	7.58	-1 70
0.00	(53)	(52)	1.70
S-S7	6.70	6.34	-5 41
0.07	(46)	(43)	5.11
S-N1	10.24	10.02	-2 10
	(70)	(69)	2.10
S-N2	13.11	13.61	3 84
5112	(90)	(94)	5.01
S-N3	11.61	11.66	0.46
	(80)	(81)	
S-N4	11.13	11.22	0 77
	(77)	(77)	,
S-N5	10.68	10.48	-1.94
	(74)	(72)	
S-N6	7.78	7.73	-0.54
	(54)	(53)	
S-N7	6.40	6.28	-1.78
~ - · ·	(44)	(43)	

Table 5.7: Specimen D6.A2.G40#4.F Stirrup Stress Ranges

Miner's Rule, given in Equation 5.2, was used to calculate equivalent constant amplitude stress ranges for all of the monitored stirrups as shown in Table 5.8. As seen from Figure 5.42, all of the stirrups except two on each side recorded higher equivalent constant amplitude stress ranges than the target stress range, 8.41 ksi (58 MPa), derived from the field tests.



Figure 5.42: Specimen D6.A2.G40#4.F equivalent constant amplitude stress range

	Equivalent Constant
Stimmer #	Amplitude Stress Range
Surrup #	[ksi]
	(MPa)
S-S1	10.26
	(71)
S-S2	11.68
	(81)
S-S3	10.48
	(72)
S-S4	10.86
	(75)
S-S5	9.74
	(67)
S-S6	7.69
	(53)
S-S7	6.53
	(45)
S-N1	10.11
	(70)
S-N2	13.45
	(93)
S-N3	11.61
	(80)
S-N4	11.13
	(77)
S-N5	10.50
	(72)
S-N6	7.69
	(53)
S-N7	6.31
	(43)

Table 5.8: Equivalent Constant Amplitude Stress Ranges

5.3.2.2 Effect of Fatigue Loading on Crack Widths

At earlier stages of fatigue loading (0 - 100,000 cycles), further propagation of existing diagonal cracks was observed as well as formation of a few new diagonal cracks followed by no apparent change in the crack pattern in the later stages. No flexural crack propagation was detected at midspan throughout the fatigue test. A crazing of very fine hairline cracks distributed

across the specimen surface was also observed, but that phenomenon was attributed to drying shrinkage.

Concrete raveling along the cracks and fine particles were observed at locations where concrete cover was removed in order to install strain gages on the stirrup leg. This material was observed at early stages of fatigue loading. In addition, it was possible to visually observe relative slip occurring between the concrete and in the stirrups due to local debonding of the stirrup when the load was cycling.

Crack motions were monitored on the south shear span via an array of displacement transducers as shown in Figure 5.33. All of the sensors were placed at 45 degrees relative to the vertical. The measurements indicated that there were increasing crack width motions measured at different locations along the characteristic diagonal crack as the fatigue loading progressed. It should be noted that the most significant increase in the crack width ranges occurred during the initial 0 - 100,000 fatigue cycle. After these initial cycles, a more steady growth in crack width range was noted at the later stages of the test. One displacement transducer (CClip 11H) was placed at the beam soffit in order to monitor the diagonal crack movement at the beam-column interface. Readings from this particular sensor revealed that there was no change in the crack width range at the beam soffit location throughout the whole fatigue test. Crack width range readings from all displacement transducers on the characteristic diagonal crack were summarized in Table 5.9 including best fit and the percentage difference.

	0-100K cycles	900K-1000K cycles		
Crack	Crack Width Range	Crack Width Range	0/ Difference	
Sensor #	[in]	[in]	76 Difference	
	(mm)	(mm)		
CClip-1	5.814E-03	6.332E-03	8.90	
	(0.148)	(0.161)		
CClip-2	2.081E-03	2.890E-03	38.9	
	(0.053)	(0.073)		
CClip-3	6.318E-03	6.514E-03	3.10	
	(0.160)	(0.165)		
CClip-4	5.195E-03	5.899E-03	13.6	
	(0.132)	(0.150)		
CClip-5	5.052E-03	6.695E-03	32.5	
-	(0.128)	(0.170)		
CClip-6	4.360E-03	6.519E-03	49.5	
	(0.111)	(0.166)		
CClip-7	3.720E-03	5.126E-03	37.8	
	(0.094)	(0.130)		
CClip-8	3.565E-03	4.970E-03	39.4	
-	(0.091)	(0.126)		
CClip-9	3.011E-03	4.447E-03	47.7	
	(0.076)	(0.113)		
CClip-10	3.553E-03	4.217E-03	18.7	
-	(0.090)	(0.107)		
CClip-	4.781E-04	4.810E-04	0.60	
11H	(0.0121)	(0.0122)		

Table 5.9: Specimen D6.A2.G40#4.F Crack Width Ranges

Average crack width ranges for every 100,000 cycles were determined in order to sequentially compare the rate of change in crack width ranges at different locations. As seen in Figure 5.43, change in crack width ranges during the fatigue loading were minimal near the crack tip and at the level of the flexural anchorage but relatively higher near the middle of the shear span. It was also observed that the change in crack width ranges significantly occurred in the first half of the fatigue cycles (0 – 500,000 cycles) then the rate of change diminished at the later stages. At the same time, the readings from the displacement sensor placed at the beam soffit were almost constant throughout the fatigue test.



Figure 5.43: Specimen D6.A2.G40#4.F crack width ranges

Since some of the displacement transducers were placed at locations where the characteristic diagonal crack crosses stirrups, it was possible to inspect the effect of transverse web reinforcement on restraining the crack width ranges and also the relationship between crack width ranges and stirrup stress ranges/mean stresses. The change in crack width range and the mean stress at the associated stirrup at different fatigue load cycles is shown in Figure 5.44 as an example. Decreasing stirrup mean stresses with increasing crack ranges was a characteristic trend which was observed in most of the cases. Figure 5.44 illustrates the change in crack width and the stress range at the related stirrup. From Figure 5.44, it can be observed that approximately in the first half of the fatigue test, the strain range was directly proportional to the crack width range. However, in the second half of the fatigue cycles, the strain range became almost steady while the crack width range kept gradually increasing. It is likely that in the second half of the fatigue test, the effective gage length of the rebar started to increase so even as the crack opens wider, the rebar does not strain by the same proportion, which is an indicator of reduction in bond between concrete and the rebar.



Figure 5.44: Specimen D6.A2.G40#4.F average crack width range vs. distance from the face of the column

5.3.2.3 Effect of Fatigue Loading on Flexural Reinforcement

The change in mean stress and the stress range in the first layer of the flexural reinforcement at the centerline location. The mean stress was observed to be fluctuating with temperature and tended to decrease over cycles. On contrary, the strain range in the flexural reinforcement was nearly constant.

Since anchorage of flexural steel was found to be a key to developing higher ultimate capacity of the specimens, embedded bars in the anchorage zones were also monitored throughout the fatigue cycles. There were not very significant changes in mean stresses measured at three distinct locations at the embedded bar as shown in Figure 5.45. The only reading which showed that the mean stress was decreasing at a very low rate was from the strain gage at the column face location. This phenomenon was not observed at the other two locations, where mean stresses seemed to be constant. A similar behavior was observed concerning strain range readings, where the strain range at the column face location was decreasing gradually and strain ranges at the inner two locations were constant as shown in Figure 5.46.









5.3.2.4 Effect of Fatigue Loading on Structural Stiffness

Fatigue check tests were performed to collect more data from additional displacement sensors and applied to the specimen at intervals of 100,000 cycles. With the inclusion of additional displacement sensors, which were not used during the fatigue cycling in order to avoid fatigue damage on the sensors themselves, it was possible to investigate the effect of fatigue loading on the stiffness of the specimen.

To determine the change in structural stiffness, load-deformation curves were obtained at intervals of 100,000 cycles of fatigue loading. In order to ensure proper data measurement with the reapplied sensors, only data from the last load cycle in a fatigue check test were used for calculations. A first order line fit was employed to determine the slope of a portion of the load-deformation curve for each fatigue check test between 50 kips (223 kN) and 150 kips (668 kN). The slope of the load-deformation curve was taken as the average secant stiffness of the specimen. Figure 5.49 shows the change in the load-deformation response at each 100,000 cycle increment including the baseline precracking test as well. It is seen that the stiffness of the specimen reduced more during the first 100,000 cycles of fatigue loading when compared with the rest of the fatigue cycles as seen in Figure 5.50. This is consistent with the crack extensions and local changes discussed previously. The stiffness still had a slight decline even after 100,000 cycles which was consistent with crack width changes and local stirrup debonding.



Figure 5.47: Specimen D6.A2.G40#4.F Flexural Steel at centerline mean stress and stress range



Figure 5.48: Specimen D6.A2.G40#4.F SE anchorage bar mean stress



Figure 5.49: Specimen D6.A2.G40#4.F SE load-deformation relation (50 - 150 kips load range)



Figure 5.50: Specimen D6.A2.G40#4.F effect of fatigue loading on structural stiffness

5.3.2.5 Effect of Fatigue Loading on Concrete Principal Stresses

The effect of fatigue loading on concrete principal stresses was investigated using 45° rosette assemblies installed in each shear span. Identities from Mohr's circle were employed in order to calculate principle strains, principle stresses, and the directions of principal planes. The principal strains and stresses in a 45° rosette can be calculated from:

$$\varepsilon_{1,2} = \frac{1}{2} \left[\varepsilon_{a} + \varepsilon_{c} \pm \sqrt{\left(\varepsilon_{a} - \varepsilon_{c}\right)^{2} + \left(2\varepsilon_{b} - \varepsilon_{a} - \varepsilon_{c}\right)^{2}} \right]$$
(5.19)

$$\sigma_{1,2} = \frac{E}{2} \left[\frac{\varepsilon_{a} + \varepsilon_{c}}{1 - \upsilon} \pm \frac{1}{1 + \upsilon} \sqrt{\left(\varepsilon_{a} - \varepsilon_{c}\right)^{2} + \left(2\varepsilon_{b} - \varepsilon_{a} - \varepsilon_{c}\right)^{2}} \right]$$
(5.20)

$$\tan 2\theta_{\rm p} = \frac{2\varepsilon_{\rm b} - \varepsilon_{\rm a} - \varepsilon_{\rm c}}{\varepsilon_{\rm a} - \varepsilon_{\rm c}}$$
(5.21)

Where ε_1 and ε_2 are principle strains, ε_a , ε_b , and ε_c are the component strains from the rosette, σ_1 and σ_2 are principle stresses, $2\Theta_p$ is the orientation of the principals, E is the modulus of elasticity, and ε is Poisson's ratio.

In order to compare concrete principle stresses at different stages of fatigue loading, data were collected from the last cycle of the fatigue check tests when the shear load on the shear span was equal to 100 kips (445 kN). It was observed that even though there were minor fluctuations, the concrete principle stresses did not change significantly throughout the fatigue test as shown in Figure 5.51. Fluctuations in the principle stresses were probably due to temperature differences since some of the fatigue check tests were performed at midday while some were conducted at night.



Figure 5.51: Specimen D6.A2.G40#4.F Effect of fatigue loading on concrete principal stress magnitudes

5.4 FAILURE TEST

Once the fatigue phase was completed, the fatigue load setup was replaced with the static load setup, which was used for previous bent cap experiments. The specimen was loaded through the static load protocol which was described in Section 5.3.1.

The failure mode for specimen D6.A2.G40#4.F was shear-compression failure as the concrete at the compression block was crushed. The fatigue loading did not change the failure behavior since specimen D6.A2.G40#4.F failed in a very similar manner to that of the static test counterpart (Specimen D6.A2.G40#4.S). Spalling of concrete at the vicinity of the characteristic diagonal crack was observed. Only a small number of new cracks were detected and marked. No diagonal cracks were

formed around service holes as was the case with the other specimens having Grade 40 #4 transverse web reinforcement. The crack pattern for specimen D6.A4.G40#4.S and a view of the failed shear span are shown in Figure 5.52. The maximum crack width measured in this specimen was 0.48 in (12.2 mm), and the characteristic diagonal crack angle was 40.4° with respect to the horizontal direction. Key structural parameters are listed in Table 5.10. Results from the static test specimen D6.A2.G40#4.S were included for direct comparison. Centerline and diagonal displacement measurements at the failure test were combined with those measured during the precracking test. This was performed by using the final measured displacement at the end of the precracking test as the initial displacement for the failure test. Any offset produced by fatigue loading was neglected.



Figure 5.52: Crack pattern for Specimen D6.A2.G40#4.F at failure

#	Specimen Name	V _{APP} [kips] (kN)	V _{DL} [kips] (kN)	V _{TOT} [kips] (kN)	V _{CL} [in] (mm)	V _{Diag} [in] (mm)
4		282.6	11.2	293.8	0.54	0.30
-	D0.112.040/14.0	(1257.1)	(49.8)	(1306.9)	(13.7)	(7.6)
6	D6 A2 C40#4 E	271.9	11.2	283.1	0.59	0.34
6	D0.A2.G40#4.F	(1210)	(49.8)	(1260)	(15.0)	(8.6)

 Table 5.10: Specimen D6.A2.G40#4.F failure test structural response parameters

Comparison of specimens D6.A2.G40#4.S and D6.A2.G40#4.F shows that fatigue loading did not significantly reduce the ultimate shear strength of the specimen. The reduction in the ultimate shear strength was only 3.6%. By contrast, the centerline and diagonal displacements increased by 9.3% and 13.3%, respectively. This increase in deformations was due to fatigue induced stirrup debonding, which softened the specimen.

As was the case with other bent cap specimens, all of the stirrups yielded before reaching the failure load. In addition, flexural reinforcing bars embedded at the anchorage zone were beyond yield at both north and south ends of the specimen. Unique to this specimen, it was possible to monitor concrete principal strains and stresses and their orientations until failure. Calculations were performed employing the identities in Section 5.3.2.5 for the rosette locations given in Section 5.6.2. Results including peak principle stresses and their orientations are given in Table 5.11 where positive and negative indicate tension and compression, respectively. An example plot showing the relation between applied shear and the concrete principle stresses is given in Figure 5.53. As seen here, the orientations of principle stresses match the characteristic diagonal crack directions very well at related locations.

Strain Rosette #	1 [psi] (MPa)	2 [psi] (MPa)	[Degrees]
R-S1	210	-511	40.3
	(1.45)	(3.52)	
R-S2	371	-592	39.6
	(2.56)	(4.09)	
R-N1	332	-546	40.6
	(2.29)	(3.77)	
R-N2	234	-448	35.8
	(1.61)	(3.09)	

Table 5.11: Specimen D6.A2.G40#4.F peak principle stresses and orientations



Figure 5.53: Concrete principal stresses at rosette R-S2 during failure test

5.4.1 Summary observations from the Fatigue Test

A full-scale test of a conventionally-reinforced concrete bent cap designed and constructed to reflect 1950's vintage details and materials was performed under high-cycle fatigue loading. Prior to fatigue loading, the specimen was precracked to achieve desired crack widths corresponding to observed field conditions. High-cycle fatigue loading was applied to the precracked specimen with a constant amplitude load range of 60 kips (267 kN) for 1,000,000 cycles, which was derived based on results from field tests of in-service steel. After fatigue loading, the specimen was tested to failure. Based on the experimental results, the following observations are presented:

- High cycle fatigue leading to metal fatigue of embedded stirrups in a bent cap is unlikely since stress ranges imposed on stirrups are less than 20 ksi (138 MPa), the specified stress range for metal fatigue of reinforcing steel.
- Stirrup mean stresses were observed to be sensitive to thermal changes and overall changes were slight during the fatigue loading.
- Stirrup stress ranges in the first stirrups closer to the beam-column interface slightly increased during fatigue loading, while strain ranges measured on the stirrups closer to the stub girders tended to slightly decrease.
- No significant change in stress ranges were observed for flexural reinforcement at the centerline and at the anchorage zone.
- Concrete principal stress magnitudes and orientations were not significantly affected by fatigue loading.
- The width of the characteristic diagonal crack continuously increased under repeated loading where the highest rate of change observed occurred at the initial cycles (0 – 100,000 cycles).
- During cycling, movement of stirrups relative to the surrounding concrete due to local debonding was observed at locations of applied strain gages where short lengths of reinforcement were exposed. This was also verified by the test data where no change in stress ranges in stirrups were observed even as crack width displacement ranges increased.
- Since metal fatigue of the embedded steel is unlikely, the mechanism for damage from high-cycle fatigue is bond fatigue at the stirrup-concrete interface, which leads to reduced constraint at crack locations increasing crack widths.
- The overall stiffness of the specimen was reduced under repeated loading. The fatigue specimen exhibited larger displacements compared to an otherwise similar specimen not subjected to fatigue.
- High cycle fatigue did not cause a significant degradation in the ultimate capacity for the specimen details and loading considered. However, the increase in crack widths may decrease the resistance of the structure against environmental effects and increase the likelihood of corrosion on embedded reinforcement.

6.0 ANALYTICAL METHODS

6.1 INTRODUCTION

In order to assess the behavior and capacity of the bent cap specimens, various analysis methods were employed, and the results are presented in this section. The experimental results were compared with predicted capacities from the traditional ACI 318-05 shear design method and the obsolete ACI 318-99 deep beam equations. The applicability of a sectional approach of the Modified Compression Field Theory (Vecchio and Collins 1986) was investigated using the computer program Response 2000 (Bentz 2000). Given that current design codes recommend strut-and-tie models for the design of deep beams, simple and detailed strut-and-tie models were constructed, an iterative optimization methodology was introduced, and the sensitivity of outcomes for different levels of complexity was investigated. The formal theory and the mechanical model derived by Zararis (2003) stands out in the literature due to the absence of empirical expressions typically used in past research. In order to model the unique conditions for vintage bent caps, the mechanical model originally proposed by Zararis (2003) was adapted for these conditions, and the related modifications are presented. Finally, non-linear finite element analyses were performed in order to predict the behavior and the shear capacity of the laboratory specimens. Non-linear finite element analyses were carried out with VecTor2 (Vecchio and Pong 2003), a non-commercial finite element analysis package for the analysis of two-dimensional reinforced concrete membrane structures subjected to quasi-static load conditions.

6.2 ACI 318 DESIGN METHODS

6.2.1 ACI 318-05 Shear Design Method

ACI 318-05 design method for shear is based on a parallel truss model with 45° constant inclination diagonals supplemented by an empirical concrete contribution where the nominal shear strength of a conventionally reinforced concrete beam is given as:

$$V_n = V_s + V_c \tag{6.1}$$

Where V_s (lbs) and V_c (lbs) are shear resistances of transverse steel and concrete, respectively.

ACI 318-05 assumes all stirrups yield at failure within the horizontal projection of the 45° inclined crack, which is taken to be equal to the effective depth, d (in.). The shear resistance provided by the transverse steel is determined as:

$$V_{s} = \frac{A_{v}f_{y}d}{s}$$
(6.2)

Where $A_v(in^2)$ is the area of the transverse steel, f_y is the yield strength of the transverse steel (psi), and s (in) is the spacing of the transverse steel.

ACI 318-05 uses a concrete shear resistance, V_c , which is taken to be equal to the shear strength of a beam without transverse reinforcement. Proposed equations are based on a rudimentary statistical analysis and lacks theoretical background such that MacGregor (1984) described the ACI shear provisions by "empirical mumbo jumbo." The method allows either a simplified method given in Equation 6.3 or a detailed method given in Equation 6.4 to compute the concrete shear distribution as:

$$V_{c} = 2\sqrt{f_{c}'b_{w}d}$$
(6.3)

$$V_{c} = \left(1.9\sqrt{f_{c}'} + \frac{2500\rho_{w}V_{u}d}{M_{u}}\right)b_{w}d$$
(6.4)

Where $f_c'(psi)$ is the compressive strength of the concrete, $b_w(in.)$ is the width of the beam stem, w is the flexural reinforcement ratio, $V_u(kip)$ is the factored shear at the design section, and $M_u(kip-in)$ is the factored moment at the design section.

The critical section for shear in ACI 318-05 is taken at a distance "d away from the face of the support." In order to calculate the reinforcement ratio at the critical section, flexural reinforcement areas were adjusted based on the ratio of available reinforcing bar length at the critical section to the development length of the bar calculated from Equation 6.14 while neglecting the contribution of transverse reinforcement.

6.2.2 ACI 318-99 Deep Beam Equations

Currently ACI 318 does not include equations to compute the shear capacity of deep beams. However, earlier editions of the specification contained such equations.

In the 1999 edition of ACI 318, the nominal shear strength for deep beams provided by the web reinforcement and concrete was calculated from two empirical expressions given in Equation 6.5 and Equation 6.6. However, it also allowed use of the simplified concrete shear resistance, given in Equation 6.3.

$$\mathbf{V}_{s} = \left[\frac{\mathbf{A}_{v}}{s}\left(\frac{1+\frac{\mathbf{l}_{n}}{\mathbf{d}}}{12}\right) + \frac{\mathbf{A}_{vh}}{s_{2}}\left(\frac{11-\frac{\mathbf{l}_{n}}{\mathbf{d}}}{12}\right)\right]\mathbf{f}_{y}\mathbf{d}$$
(6.5)

$$V_{c} = \left(3.5 - 2.5 \frac{M_{u}}{V_{u}d}\right) \left(1.9 \sqrt{f_{c}} + 2500 \rho_{w} \frac{V_{u}d}{M_{u}}\right) b_{w}d$$
(6.6)

Where l_n (in.) is the clear span, and A_{vh} (in²) and s_2 (in.) are the area and spacing of the horizontal web reinforcement, respectively.

The critical section in ACI 318-99 shear design provisions for deep beams was specified as half of the distance between concentrated load and the face of the support. These deep beam provisions in ACI 318-99 were removed due to the discontinuities in the estimated design strength when clear span to overall member depth ratio was varied. Deep beams are now designed using the strut-and-tie provisions in ACI 318-02 Appendix A, which is addressed at Section 5.4 of this document.

6.2.3 ACI 318 Prediction Results

Generally, ACI 318 equations produced widely variable outcomes in predicting the ultimate shear strength of bent cap specimens as summarized in Table 6.1. By employing ACI 318-05 shear design equations, sometimes unconservative results were achieved. On the other hand, ACI 318-99 deep beam equations produced very conservative results in all cases (sometimes very conservative up to a factor of 2.02). Surprisingly, the more reasonable results were achieved with ACI 318-05 detailed shear design equations, with a prediction bias of 1.05, but with a coefficient of variation of 15.4%. Overall, ACI 318 equations

both for slender beams and deep beams did not produce consistent predictions of the ultimate shear capacity of bent cap specimens. The most obvious issue with these approaches is their inability to account for changes in the embedded flexural reinforcement configuration at the anchorage zone.

Analysis Series	Specimen	V _{EXP} [kips] (kN)	V _P [kips] (kN)	Bias V _{EXP} /V _P	Mean	STD	COV [%]
	D6.A4.G60#5.S	506.4	440.8	1.14			
ACI 318-05 Slender Beams Simplified Approach	D6.A4.G40#4.S	406.6 (1809)	(1902) 296.2 (1318)	1.37			
	D6.A2.G60#5.S	394.2 (1754)	442.2 (1968)	0.89	1.10	0.179	16.2
	D6.A2.G40#4.S	293.8 (1307)	288.8 (1285)	1.02			
	D4.A2.G40#4.S	207.3 (922)	192.8 (858)	1.08			
	D6.A4.G60#5.S	506.4 (2253)	457.7 (2037)	1.10			
	D6.A4.G40#4.S	406.6 (1809)	314.1 (1398)	1.29			
ACI 318-05 Slender Beams	D6.A2.G60#5.S	394.2 (1754)	457.3 (2035)	0.86	1.05	0.162	15.4
ACI 318-05 Slender Beams Detailed Approach	D6.A2.G40#4.S	293.8 (293.8)	303.0 (1348)	0.96			
	D4.A2.G40#4.S	207.3 (922)	201.1 (895)	Bias Mean STD CO 1.14			
	D6.A4.G60#5.S	506.4 (2253)	259.4 (1154)	1.95			
A CL 210.00	D6.A4.G40#4.S	406.6 (1809)	201.2 (895)	2.02			
ACI 318-99 Deep Beams	D6.A2.G60#5.S	394.2 (1754)	261.3 (1163)	1.50	1.68	0.285	16.9
Simplified Approach	D6.A2.G40#4.S	293.8 (1307)	194.6 (866)	1.51			
	D4.A2.G40#4.S	207.3 (922)	147.5 (656)	1.41			
ACI 318-99 Deep Beams	D6.A4.G60#5.S	506.4 (2253)	359.5 (1600)	1.41	1.35	0.338	25.0

Detailed Approach	D6.A4.G40#4.S	406.6	302.9 (1348)	1.34		
	D6.A2.G60#5.S	394.2 (1754)	359.2	1.10		
	D6.A2.G40#4.S	293.8	286.8	1.02		
	D4.A2.G40#4.S	207.3 (922)	(1270) 110.1 (490)	1.89		

6.3 MCFT SECTIONAL ANALYSIS APPROACH

The modified compression field theory (MCFT) (Vecchio and Collins 1986) is a further development of the compression field theory (CFT) (Collins and Mitchell, 1974) that accounts for the influence of the tensile response of the cracked concrete. MCFT describes the behavior of a single reinforced concrete element via compatibility and equilibrium equations and a set of constitutive relationships for the composite material, which were derived through a series of experiments conducted at the University of Toronto (Vecchio and Collins 1986). MCFT has been implemented in three major ways so far with varying levels of complexity. The simplest method of analysis that uses MCFT is based on the response of a single biaxial element such as in the case of the beam design equations in the AASHTO LRFD code. The second level of complexity is based on a sectional analysis that takes into consideration the variation in the shear stress profile over the depth of the beam. An implementation of the sectional MCFT analysis was proposed by Bentz (2000) accompanied by a computer program, Response 2000. The most rigorous implementation of the MCFT is a membrane based finite element analysis program, which is addressed at Section 5.6 of this document. The most problematic issue with the first two implementations of the MCFT is the assumption of plane sections remain plane after bending, which is not valid for disturbed regions where the strain distribution in a section is significantly nonlinear such as in the case of shear spans in a deep beam. Even though it is known that a MCFT based sectional analysis may not yield reliable results for predicting the capacity of deep beams, a series of analyses with Response-2000 were performed in order to investigate the results obtained from this method.

Based on the experimentally observed cracking patterns, the critical section was chosen as the interface between the column and the bent cap beam since the characteristic diagonal crack propagated from this location. Moreover, regarding the cracking mechanism and the failure mode, it is believed that the rest of the flexural bar layers did not provide significant resistance against shear. The flexural bars embedded at the anchorage zone were assumed to be fully developed at the critical section reflecting the experimental results. Actual material properties from each specimen (see Section 3.4) were used as input for the analyses. The predicted ultimate shear strengths were achieved by using the "sectional response" option in Response 2000. Sectional MCFT ultimate shear strength predictions are summarized in Table 6.2, and also shown in Figure 6.1.



Figure 6.1: Response 2000 sectional MCFT analysis results

Analysis Series	Specimen	V _{EXP} [kips] (kN)	V _P [kips] (kN)	Bias V _{EXP} /V _P	Mean	STD	COV [%]
	D6.A4.G60#5.S	506.4 (2253)	438.9 (1953)	1.15		0.091	7.8
Bernance 2000	D6.A4.G40#4.S	406.6 (1809)	317.1 (1411)	1.28			
Response 2000 Sectional MCFT	D6.A2.G60#5.S	394.2 (1754)	327.4 (1457)	1.20	1.16		
Anarysis	D6.A2.G40#4.S	293.8 (1307)	256.0 (1139)	1.15			
	D4.A2.G40#4.S	207.3 (922)	200.6 (893)	1.03			

Table 6.2: Response 2000 Sectional MCFT Analysis Prediction Results

It can be seen that the sectional MCFT analysis under predicted shear capacities for specimens with the low a/d ratio as previously reported by Potisuk (2004).

However, for specimen D4.A2.G40#4.S, with a/d ratio of 2.1, the prediction bias was 1.03, which demonstrates that sectional MCFT analysis is better suited to predict the shear capacity for slender beams.

6.4 STRUT-AND-TIE MODELS

The use of truss models as conceptual tools in analysis and design of reinforced concrete structures stretches back to early 1900s when Ritter (1899) proposed that a cracked reinforced concrete beam can be idealized as a parallel chord truss with compression struts inclined at 45° with respect to the longitudinal axis of the beam. Mörsch, in consecutive publications, (1909, 1912, 1922) introduced the use of truss models for torsion. As a feasible model for shear and torsion in reinforced concrete and prestressed concrete beams, truss models with diagonals having variable inclination angles were proposed by Kupfer (1964), Lampert and Thürlimann (1971), and Thürliman et al. (1983). Schlaich et al. (1987) discretized structural members into D (discontinuity or disturbed) and B (beam or Bernoulli) regions, generalized the truss analogy, and applied it in the form of the so-called "strut-andtie model" (STM). STM can be referred as a conceptual framework where the stress distribution in a structural member are idealized as a system of compression struts, and tension ties, which are connected to each other by nodes. According to the principle of St. Venant, the localized effect of a disturbance caused by concentrated loads, reactions, or abrupt changes of cross-section creates a complex flow of internal stresses which diminish as the distance away from the disturbance approaches the member-depth. These regions where the distribution of strains is significantly nonlinear are referred as the D-regions. Conversely, the portions of a member where Bernoulli's "plane sections remain plane" hypothesis is applicable due to the linear distribution of strains are referred as the B-regions. Generally, both D and B-regions exist within the same member of the structure. However, in some cases a structural reinforced concrete member may consist entirely of a D-region such as in corbels, pile caps, or deep beams. The conventional theory of flexure and the traditional (V_c+V_s) approach for shear design do not apply to D-regions as previously demonstrated in section 5.2.1 of this document. Instead, all current major design codes such as ACI 318, AASHTO LRFD, CHBD, and CEB-FIP recommend use of strut-and-tie models in design of D-regions. Details on STM provisions from the mentioned design codes are given in Section 2.1 of this document. The proposed STMs in this study are based on the provisions given in ACI 318-05 Appendix A. ACI 318-05 requires a resistance factor of 0.75 to be applied to the calculated nominal capacities when STMs are used in design. No resistance factor was used in this present study.

In order to construct a STM, D and B-regions in a structural member should be identified, and the flow of forces in these regions should be visualized. In a STM,

the flow of concentrated compressive stresses is idealized with compressive struts, and tension ties, required for equilibrium, represent the reinforcing steel.

The Computer Aided Strut-and-Tie (CAST) software, developed by Tjihn and Kuchma (2002) at the University of Illinois at Urbana-Champaign, was used in construction and evaluation of the strut-and-tie models in this study. CAST provides a graphical working environment in which the designer can sketch the boundaries of the D-region, construct an internal load-resisting truss, solve for member forces, and select dimensions of struts and reinforcement for ties. Dimensioning of the nodal zones is performed by the program automatically. Since STM analysis is an iterative design methodology in terms of refining and optimization, repetitive calculations regarding member forces and dimensioning of nodes may be very time consuming when done by hand. In CAST, the position of the nodes can be adjusted at any stage within the design process, and the resulting stress distribution can be reviewed immediately.

In order to construct a STM in CAST, first the boundaries of the D-region are defined. These boundaries do not affect calculations, but provide visual aid in positioning the truss elements. Second, the proposed truss system is drawn as a series of straight lines, the material properties are entered, and the loads are applied. The resulting force distribution can be assessed at this stage. Finally, the estimated reinforcement and strut widths are assigned to the truss members in order to check whether the capacities of the struts and ties as well as the stress on the faces of the nodes are adequate or not. From this point on, minor changes can be applied to the selected STM for optimization purposes. CAST also includes an option for basic capacity prediction. In computing the capacity of a STM, the program locates the weakest truss component using a ratio of applied stress to the stress limit of the member in consideration and determines the capacity by a linear extrapolation of the applied external loads. CAST uses the resistance and efficiency factors from ACI 318-05 Appendix A by default. However, the program also allows user input for resistance and efficiency factors, which enables analyses with other design codes such as AASHTO-LRFD or CEB-FIP.

6.4.1 Analysis with a Simple Strut-and-Tie Model

A simple STM consisting of a main inclined strut, two vertical struts, one horizontal strut, one tension tie, and two nodal zones was constructed for the capacity assessment of bent cap specimens, as shown in Figure 6.2. Only one half of the specimen was modeled to take advantage of symmetry. The types and the efficiency factors of the truss member classifications according to ACI 318-05 Appendix A are summarized in Table 6.3:



Figure 6.2: Proposed simple strut-and-tie model for bent cap specimens

Truss Member	Туре	Efficiency Factor		
S1	Prismatic Strut	0.85		
S2	Bottle-Shaped Strut with Steel	0.64		
S3	Prismatic Strut	0.85		
S4	Prismatic Strut	0.85		
T1	Tension Tie	1.00		
N1	C-C-T Node	0.68		
N2	C-C-C Node	0.85		

 Table 6.3: Types and Efficiency Factors of Truss Members for the Simple STM

The effective width of the top horizontal prismatic strut, S4, was derived from the classical bending theory for a single reinforced beam section:

$$w_c = k d \tag{6.7}$$

Where d (in.) is the effective depth of the beam at the centerline of the beam, and k is calculated from:

$$k = \sqrt{\left(n\rho\right)^2 + 2n\rho - n\rho}$$
 (6.8)

Where n is the modular ratio of steel to concrete, and k is the longitudinal reinforcement ratio. The width of the vertical prismatic strut, S3, was assumed to be equal to the width of the monolithic internal girders that frame into the cap. The tension tie, T1, was placed at the same height with the centroid of the bottom-most reinforcement layer, and the width of the tension tie was calculated as twice the distance between the bottom edge of the cap beam and the centroid of the reinforcing bars at the bottom-most reinforcement layer. The tension tie was distributed into segments with pseudo-nodes at the locations of rebar cut-offs in order to account for different amounts of reinforcing steel. The pseudo-nodes were connected to node N2 with stabilizers, which are zero force members, but required to avoid ill-conditioned structure stiffness matrix in truss analysis. Since yielding of the flexural bars embedded at the anchorage zone was observed during the experiments, the reinforcement in the tie segment connected to node N1 was considered as fully developed. The initial width of the vertical prismatic strut, S1, at the supporting column was taken as two thirds of the column width, and the strut was placed at the centerline of the column for trial analyses. The main inclined strut was assumed to be bottle-shaped since the compressive stresses tend to spread in the shear spans. In order to account for the beneficial effect of web reinforcement the efficiency factor was chosen accordingly. The initial width of the strut main inclined bottle-shaped strut, S2, was determined as:

$$w_{d} = w_{c} \cos \theta + b_{w \text{ girder}} \sin \theta$$
 (6.9)

Where $b_{w,girder}$ is the width of the monolithic internal girders that frame into the cap, and b is the angle between the main inclined strut and x-direction. Since the coordinates of nodes N1 and N2 were already determined, it was possible to calculate b by means of basic geometrical calculations. The dimensioning of the nodal zones was performed by CAST automatically.

Once the initial strut-and-tie models were constructed and solved, it was observed that the ultimate load capacity of the truss system was controlled by the yielding of the tension tie, T1, for all specimens. The initial analyses underestimated the shear strength of bent cap specimens with a prediction bias of 2.27, and a coefficient of variation of 25.4%. Investigating node N1 shows that the demand on the main tension tie depends on the inclination angle of the diagonal strut. In other words,

the predicted capacity by the STM increases when the diagonal strut angle is increased. In order to achieve the most optimal result for the proposed simple STM, an iterative analysis procedure was followed. This procedure involves moving the supporting strut, S1, horizontally, as close as possible to the beam–column interface thus increasing the inclination angle of the diagonal strut S2 while decreasing the width of the supporting strut, S1, as long as the limiting stress capacity of the strut S1 allows. The final results achieved after the iterations produced better but still very conservative prediction results with a prediction bias of 2.03 and a coefficient of variation of 25.3%. As it was the case with the initial analyses, the load capacity of the STM was controlled by the yielding of the tension tie, T1, as illustrated in Figure 6.3. The initial and final shear capacity prediction results are summarized in Table 6.4:



Figure 6.3: Failure mode in simple strut-and-tie model

Analysis Series	Specimen	V _{EXP} [kips] (kN)	V _P [kips] (kN)	Bias V _{EXP} /V _P	Mean	STD	COV [%]
ACI 318-05 Appendix A	D6.A4.G60#5.S	506.4 (2253)	268.1 (1193)	1.89	2.27	0.577	25.4

Table 6.4: Simple Strut-and-Tie Model Prediction Results
Simple STM	D6.A4.G40#4.S	406.6 (1809)	257.1 (1144)	1.58			
Initial Results	D6.A2.G60#5.S	394.2 (1754)	128.6 (572)	3.07			
	D6.A2.G40#4.S	293.8 (1307)	128.6 (572)	2.28			
	D4.A2.G40#4.S	207.3 (922)	82.0 (365)	2.53			
	D6.A4.G60#5.S	506.4 (2253)	299.2 (1331)	1.69			
ACI 318-05	D6.A4.G40#4.S	406.6 (1809)	287.0 (1277)	1.42		0.514	
Appendix A Simple STM Final Results	D6.A2.G60#5.S	394.2 (1754)	143.5 (639)	2.75	2.03		25.3
	D6.A2.G40#4.S	293.8 (1307)	143.5 (639)	2.05			
	D4.A2.G40#4.S	207.3 (922)	91.6 (408)	2.26			

It can be concluded that the most basic STM frequently found in the literature as an example for deep beam design was excessively conservative in predicting the ultimate shear strength of bent cap specimens. This can be attributed to the absence of vertical web reinforcement in the model and to the idealization of compressive forces as a single straight strut, which exerts very large anchorage demand on the tension tie.

6.4.2 Analysis with a Detailed Strut-and-Tie Model

A more detailed STM consisting of three inclined struts, two vertical struts, one horizontal strut, three tension ties, and four nodal zones was also constructed for the capacity assessment of bent cap specimens, as shown in Figure 6.4. Only one half of the specimen was modeled by taking advantage of symmetry.



Figure 6.4: Proposed detailed strut-and-tie model for bent cap specimens

Contrary to the simple STM described in the previous section, the web reinforcement was taken into consideration and idealized as a single tension tie where the total area of the web reinforcement was lumped into a single truss member. In addition, the main inclined compression strut was subdivided into two segments in order to simulate arching of the compressive stresses in the shear span. The types and the efficiency factors of the truss members according to ACI 318-05 Appendix A are summarized in Table 6.5.

Truss Member	Туре	Efficiency Factor
S1	Prismatic Strut	0.85
S2	Bottle-Shaped Strut with Steel	0.64
S3	Bottle-Shaped Strut with Steel	0.64
S4	Prismatic Strut	0.85
S5	Prismatic Strut	0.85
S6	Bottle-Shaped Strut with Steel	0.64
T1	Tension Tie	1.00
Τ2	Tension Tie	1.00

 Table 6.5: Types and Efficiency Factors of Truss Members for the Detailed STM

Т3	Tension Tie	1.00
N1	C-C-T Node	0.68
N2	C-C-T Node	0.68
N3	C-C-C Node	0.85
N4	C-T-T Node	0.51

The effective width of the top horizontal prismatic strut, S5, was calculated from Equation 6.7 and Equation 6.8. The effective width of the vertical prismatic strut, S4, was assumed to be equal to the width of the monolithic internal girders that frame into the cap. The initial width of the vertical prismatic strut, S1, at the supporting column was taken as two thirds of the column width, and the strut was placed at the centerline of the column for trial analyses. The initial effective widths of the bottle shaped struts S2 and S3 were calculated from Equation 6.9. Location of nodes N2 and N4 in x-direction was chosen as the midpoint of the clear shear span. Similar to the simple STM, the tension ties, T1 and T2, were placed at the same height with the centroid of the bottom-most reinforcement layer, and the width of the tension ties was calculated as twice the distance between the bottom edge of the cap beam and the centroid of the reinforcing bars at the bottom-most reinforcement layer. The initial location of node N2 in y-direction was assumed to be 1/3 overall depth away from the top compression fiber. Similar to the simple STM, pseudo-nodes were created to identify tension tie segments with different amounts of reinforcement, and these nodes were connected to node N3 with stabilizers in order to avoid an ill-conditioned stiffness matrix.

Once the initial STMs were constructed and solved, it was observed that the ultimate load capacity of the truss system was controlled either by the yielding of the embedded reinforcement at the anchorage zone (tension tie T2) or yielding of the web reinforcement (tension tie T3). The initial analyses underestimated the strength of bent cap specimens with a prediction bias and a coefficient of variation of 1.71 and 17.6%, respectively. The initial prediction results from the detailed STM better estimated the capacity when compared to the simple STM, but the results were still very conservative.

In order to achieve the most optimal result for the proposed detailed STM, an iterative analysis procedure was followed. According to this model, the demand on the tension tie T2, this represents the embedded flexural steel at the anchorage zone, decreases when the inclination angle of the compressive strut S2 increases. Alternatively, the demands on the tension tie T3, which represents vertical web reinforcement, decreases when the relative angle of orientation between compressive struts S2 and S3 decreases. To achieve the largest predicted capacity, both truss elements should reach the limit state simultaneously. The iterative

optimization procedure involves repositioning the supporting strut, S1, and the node N2. The supporting strut, S1, should be moved horizontally using the same technique explained in the simple STM analysis to relieve the demand on the tension tie T2. In addition, the node N2 should be moved vertically to relieve the demand on the tension tie T3. The iterative analysis ends when yielding of the reinforcement is observed in the flexural steel and the web reinforcement simultaneously as illustrated in Figure 6.5. The final results achieved after the iterations produced the best prediction among the proposed STMs with a bias of 1.24 and a coefficient of variation of 11.0%. This can be attributed to the inclusion of the contribution from vertical web reinforcement and to treatment of the arching strut. The initial and final shear capacity prediction results are summarized in Table 6.6, and shown in Figure 6.6.



Figure 6.5: Failure mode in detailed strut-and-tie model



Figure 6.6: Simple and detailed strut-and-tie model prediction results

Analysis Series	Specimen	V _{EXP} [kips] (kN)	V _P [kips] (kN)	Bias V _{EXP} /V _P	Mean	STD	COV [%]
	D6.A4.G60#5.S	506.4 (2253)	377.9 (1682)	1.34			
ACI 318-05	D6.A4.G40#4.S	406.6 (1809)	243.2 (1082)	1.67			
Detailed STM	D6.A2.G60#5.S	394.2 (1754)	181.3 (807)	2.17	1.71	0.30	17.6
Initial Results	D6.A2.G40#4.S	293.8 (1307)	181.3 (807)	1.62			
	D4.A2.G40#4.S	207.3 (922)	120.3 (535)	1.72			
	D6.A4.G60#5.S	506.4 (2253)	449.7 (2001)	1.13			
ACI 318-05	D6.A4.G40#4.S	406.6 (1809)	365.1 (1625)	1.11			
Detailed STM Final Results	D6.A2.G60#5.S	394.2 (1754)	275.7 (1227)	1.43	1.24	0.14	11.0
	D6.A2.G40#4.S	293.8 (1307)	221.0 (983)	1.33			
	D4.A2.G40#4.S	207.3 (922)	172.2 (766)	1.20			

Table 6.6: Detailed Strut-and-Tie Model Prediction Results

6.4.3 Complex Strut-and-Tie Models

In addition to the proposed STMs, even more sophisticated models were also constructed. The complex STM, shown in Figure 6.7, consisted of five inclined struts, three vertical struts, three horizontal struts, five tension ties, and eight nodal zones.



Figure 6.7: A complex strut-and-tie model for bent cap specimens

Similar to the previous proposed models, only one half of the specimen was modeled taking advantage of symmetry. In this model the web reinforcement was equally distributed to two vertical tension ties. The arching strut was modeled with three inclined strut segments. In order to take into account the effect of indirect loading, the applied load was distributed into three nodes along the height of the compression block, and the distributed load magnitudes were calculated with the assumption of a triangular shear stress distribution. Since the hypothetical truss system was internally and externally determinate, axial stiffnesses of the truss members were calculated accordingly with initial dimensions, and the calculated values were normalized with respect to the member with the highest stiffness. Normalized stiffnesses were applied as relative stiffness factors in CAST. It was observed that performing an iterative analysis on such a complex truss system may be very time consuming since the relative stiffness factors must to be calculated each time a member is resized. However, the most problematic issue was that the

predicted capacities and failure modes were found to be very sensitive to adjustments in the geometry of the truss system, which was not observed to this extent in the previously described strut-and-tie models. In order to observe the effect of changes in truss geometry on ultimate load capacity of the complex strutand-tie model, the locations of nodes N2 and N3 were moved in y-direction alternately. The predicted capacities (V_P) and corresponding lengths (L) of tension ties T4 and T5 were noted for each iteration. The prediction results from the iterative analyses of a specimen were compared with the experimental load capacity of the related specimen (V_{EXP}), and the nearest prediction result and the corresponding tension tie length was determined as VPo and Lo, respectively. The relationship between the ratio of V_P to V_{Po} and the ratio of L to L_o is shown in Figure 6.8. Additionally, the relationship between the prediction bias (V_{EXP}/V_P) from each iteration and the ratio of L to L_0 is shown in Figure 6.9. As seen here, relatively small changes in geometry produce large changes in strength prediction and also change which component controls failure. Therefore, experience is needed to develop and assess these models and results, although the results were consistently conservative.



Figure 6.8: Effect of changes in truss geometry to STM ultimate load capacity



Figure 6.9: Effect of changes in truss geometry to prediction results

Overall, STM analysis based on the provisions from ACI 318-05 Appendix A produced conservative results. In many respects, the predicted strength was excessively conservative. Due to the simple nature of the proposed models, the failure mode of the specimens was not fully captured. The simple STM provided the most conservative results. The detailed STM and the proposed iterative solution predicted higher but still conservative load capacity of the bent cap specimens. Use of a very complex STM is not recommended for capacity prediction since the results were observed to be very sensitive to the location of the truss members and the need to repeatedly calculate relative stiffness factors may be time consuming. It is important to note here that STM analysis is a *design* tool, which provides virtually an infinite number of lower bound solutions for a design situation, rather than a unique prediction result, and it takes experience to determine the most efficient strut-and-tie models for different situations. The excessive conservatism of the STMs at design may be tolerable, but for evaluation of existing structures where the purpose of the analysis is to reveal the best estimate of available capacity may be uneconomical.

6.5 MECHANICAL MODELS

6.5.1 Zararis Mechanical Model

According to Zararis (2003), in simply supported deep beams with an a/d ratio between 1.0 and 2.5, the failure mode is mainly shear compression, which is due to the crushing of the concrete in the compression zone at the top of a critical diagonal crack (referred to as the characteristic diagonal crack in other sections of this document). The critical diagonal crack is governed by shear rather than bending extending from the loading point to the support. The prediction method proposed by Zararis (2003) describes the shear compression failure in deep beams and determines the reduced depth of the compression zone above the critical diagonal crack as well as the ultimate shear capacity of deep beams with or without web reinforcement.

In this model, the crack opening is assumed to be perpendicular to the direction of propagation, implying that the normal and shear forces of the embedded steel bars are the only forces acting on the faces of the diagonal crack. Strains and stresses of reinforcement at diagonal cracks are adopted from a previous study (*Zararis 1988*) where the longitudinal and the vertical steel bars at the crack location undergo not only elongations but shear strains as well. Indicating the directions of the horizontal and vertical reinforcement with x and y, respectively, the reinforcement strains can be expressed in matrix form, which implies that the shear forces in reinforcement are caused by a pure shearing deformation of the bars at the crack location, regardless of slip at the crack faces which occurs when dowel forces are produced:

$$\begin{bmatrix} \varepsilon_{sx} & \gamma_{sxy} \\ \gamma_{syx} & \varepsilon_{sy} \end{bmatrix} = \varepsilon_{cr} \begin{bmatrix} \cos^2 \phi & \sin \phi \cos \phi \\ \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix}$$
(6.10)

Where $_{cr}$ is the strain perpendicular to the crack, $_{sx}$ and $_{sy}$ are the normal strains in x and y directions, respectively, $_{sxy}$ and $_{syx}$ are the shear strains, and is the angle between the critical diagonal crack and the vertical direction.

Constitutive relationships for the reinforcing steel, which are valid prior to yielding, can be written as:

$$\sigma_{\rm s} = {\rm E}_{\rm s} \cdot \varepsilon_{\rm s} \tag{6.11}$$

$$\tau_{\rm s} = G_{\rm s} \cdot \gamma_{\rm s} \tag{6.12}$$

$$G_s = \frac{E_s}{2(1+\upsilon)}$$
(6.13)

Where G_s is the steel normal stress, G_s is the steel shear stress, E_s is the steel modulus of elasticity, and G_s is the steel shear modulus. If the Poisson's ratio for steel, G, is taken as 0.3, resulting in $G_s = 0.4E_s$, the stress tensor for reinforcing bars at a crack can be determined from:

$$\begin{bmatrix} \sigma_{sx} & \tau_{sxy} \\ \tau_{syx} & \sigma_{sy} \end{bmatrix} = E_s \varepsilon_{cr} \begin{bmatrix} \cos^2 \phi & 0.4 \sin \phi \cos \phi \\ 0.4 \sin \phi \cos \phi & \sin^2 \phi \end{bmatrix}$$
(6.14)

It should be noted that the stress tensor implies that on a diagonal crack, in addition to the normal forces, shear forces of steel bars also act.

The critical diagonal crack divides the shear span into two parts by reducing the flexural compression block depth, c, to a depth indicated as c_s , as illustrated in Figure 6.10 and Figure 6.11. The forces acting on the diagonal crack and their equilibrium can be analyzed by constructing two free body diagrams representing the reinforced concrete elements of a deep beam above and below the critical diagonal crack.



Figure 6.10: Forces acting on the element above the critical diagonal crack (*adapted from Zararis* 2003)



Figure 6.11: Forces acting on the element below the critical diagonal crack (*adapted from Zararis*, 2003)

The forces acting on the reinforced concrete element above the critical diagonal crack are the normal force T, the shearing force V_d of longitudinal steel bars, and the force V_s of vertical web reinforcement lumped at the middle of the critical diagonal crack. These forces are in equilibrium with the reaction force P, the compression force C acting at a distance of t_1 away from the top compression fiber, and the shear force at the compression zone V_c as illustrated in Figure 6.10. The arrangement of forces is based on Zararis' original work but adjusted for bent cap specimens. According to Zararis (2003) the force from shearing deformation of the vertical web reinforcement is negligible. In this analysis, the shear span, a, is taken as the distance between the column face and the centerline of stub girders. This was selected based on the observation of the experiments that showed the diagonal crack propagating from the column-beam intersection to the loading points due to indirect loading. The distance t_1 is approximated as $0.4c_s$.

When the forces acting on the reinforced concrete element below the critical diagonal crack are considered, it should be mentioned that the force in the longitudinal steel bars in the region of pure bending, T_{f_2} is greater than the force in the longitudinal bars at the critical diagonal crack, T. This is attributed to the still intact concrete region between the tips of the critical diagonal crack and the flexural crack at the pure bending section, which permit carrying additional compressive forces at a magnitude of C, the resultant acting at a distance t_2 away from the tip of the critical diagonal crack, as illustrated in Figure 6.11. Zararis

(2003) approximated the distance t_2 as 0.15 (c/d)(d-c_s), which was found to be convenient for further simplification at later stages of the analysis. The equilibrium between the tension and compression forces acting in the x direction is given as:

$$T_{f} = C + \Delta C = T + \Delta C = C_{f}$$
(6.15)

Although the plane sections remain plane assumption is not valid within the shear span due to excessive shear deformations, outside this area, as stated by Zararis (2003), it can be assumed that the distribution of strains over a vertical section is linear as illustrated in Figure 6.12.



Figure 6.12: Stress/Force and strain profiles at the load application section

The concrete stress-strain relationship is defined by the Hognestad parabola (*Hognestad 1951*), which is given as:

$$\sigma_{c} = f_{c}' \left[2 \left(\frac{\varepsilon_{c}}{\varepsilon_{co}} \right) - \left(\frac{\varepsilon_{c}}{\varepsilon_{co}} \right)^{2} \right]$$
(6.16)

Where f_c' , and c_o are the peak compressive stress and strain, respectively. In order to calculate C_f and T_f , the value of c_o is taken as 0.002 similar to the Eurocode (1992). If the stress parabola is integrated over the depth of the flexural

compression block with a width of b, the resultant compressive force is calculated as:

$$C_{f} = \frac{2}{3} b c f'_{c}$$
 (6.17)

For the same section, considering the linear distribution of strains, the resultant tensile force can be determined from:

$$T_{f} = A_{s} E_{s} \varepsilon_{s} = \rho b d E_{s} \varepsilon_{c} \frac{(d-c)}{c}$$
(6.18)

Equating T_f and C_f , the depth of the flexural compression block can be obtained from the positive root of the following quadratic equation:

$$\left(\frac{c}{d}\right)^2 + \frac{3}{2}E_s\varepsilon_{co}\frac{\rho}{f_c'}\frac{c}{d} - \frac{3}{2}E_s\varepsilon_{co}\frac{\rho}{f_c'} = 0$$
(6.19)

Once the flexural compression block height, c, is calculated, it is possible to determine the reduced compression block height c_s , by employing moment equilibrium equations for the free body diagrams illustrated in Figure 6.10 and Figure 6.11. Using Equation 6.14, it is possible to express the steel forces at the critical diagonal crack in terms of the normal stress in x direction in order to eliminate the number of unknowns in the moment equilibrium equations:

$$T = \rho b d \sigma_{sx} \tag{6.20}$$

$$V_{d} = 0.4 \rho b d\sigma_{sx} \tan \varphi \tag{6.21}$$

$$V_{s} = \rho_{v} b d\sigma_{sx} (1 - c_{s} / d) \tan^{3} \varphi$$
(6.22)

Where V is the main tension reinforcement ratio and v is the vertical web reinforcement ratio. In bent cap specimens, due to the rebar cut-offs, the main tension reinforcement ratio has different values at the column face and the load application sections. However, according to Zararis (2007), the main tension reinforcement ratio should be calculated using the amount of reinforcing bars that exist at the critical diagonal crack, and it is appropriate to assume steel bars that are not well anchored at the section are not as effective when calculating the ultimate shear strength of a deep beam.

From the equilibrium of moments on the free body diagram above the critical diagonal crack at the point of application of force C, the dowel component V_d is expressed as:

$$V_{d} = \frac{P(a/d) \tan \varphi}{2.5 - c_{s}/d + (1 - c_{s}/d)(1 + 0.5V_{s}/V_{d}) \tan^{2} \varphi}$$
(6.23)

Moreover, from the equilibrium of moments on the free body diagram below the critical diagonal crack at the point of application of force C, while considering $T_f = P(a/z)$, where z is the flexural lever arm, it results in:

$$V_{d} = \frac{P(a/d) \tan \varphi}{\frac{z}{d} \left[2.5 + \frac{(1+0.5V_{s}/V_{d})}{1-0.15c/d} \tan^{2} \varphi \right]}$$
(6.24)

When Equation 6.23 and Equation 6.24 are set equal to one another, the solution for the reduced compression block depth as a function of the flexural compression block depth can be determined as:

$$\frac{c_{s}}{d} = \frac{1 + 0.27R(a/d)^{2}}{1 + R(a/d)^{2}} \frac{c}{d}$$
(6.25)

Where

$$R = 1 + (\rho_v / \rho)(a / d)^2$$
 (6.26)

A number of simplifications and approximations, such as taking the tangent of the crack angle V equal to the a/d ratio and elimination of terms with relatively very insignificant numerical values, were made in order to derive Equation 6.25 and Equation 6.26. The details of these simplifications can be found in Zararis (2003). The proposed equations regarding the reduced compression block indicate that the depth of the reduced compression block mainly depends on the shear span to depth ratio and secondarily on the $_v/$ ratio. Furthermore, it can be said that the depth of compression block increases with increasing amounts of embedded web reinforcement at the anchorage zone, whereas the depth decreases with increasing amounts of vertical web reinforcement, albeit to a lesser degree.

Once the reduced compression depth, c_s , is determined, it is possible to calculate the ultimate shear capacity of the deep beam. According to Zararis (2003), the shear force of the longitudinal reinforcement V_d disappears after stirrup yielding causing an excessive increase in the normal and shear forces of concrete at the reduced compressive zone and eventually resulting in the crushing of the concrete. It is assumed that the concrete compressive stress exceeds the concrete strength, f_c', over the entire depth of the reduced compression zone, thus the concrete compressive force becomes:

$$C = c_{s} b f_{c}'$$
 (6.27)

The ultimate shear capacity of the deep beam can be computed by employing a moment equilibrium equation at the application point of the concrete compressive force C, using the free body diagram illustrated in Figure 6.13, and this analysis results in:

$$V_{p} = \frac{bd}{a/d} \left[\frac{c_{s}}{d} \left(1 - 0.5 \frac{c_{s}}{d} \right) f_{c}' + 0.5 \rho_{v} f_{yv} \left(1 - \frac{c_{s}}{d} \right)^{2} \left(\frac{a}{d} \right)^{2} \right]$$
(6.28)

Where V_p is the predicted ultimate shear capacity of the deep beam.



Figure 6.13: The free body diagram employed for calculating the ultimate shear capacity of deep beams in the Zararis Mechanical Model

Ultimate shear strength predictions for the bent cap specimens using the Zararis Mechanical Model are summarized in Table 6.7 and also shown in Figure 6.15.



Figure 6.14: The free body diagram employed for calculating the ultimate shear capacity of deep beams in the proposed Modified Zararis Mechanical Model



Figure 6.15: The Zararis and the Modified Zararis Mechanical Models analysis results

Analysis Series	Specimen	V _{EXP} [kips]	V _P [kips]	Bias V _{EXP} /V _P	Mean	STD	COV [%]
	D6.A4.G60#5.S	506.4 (2253)	563.3 (2507)	0.90			
	D6.A4.G40#4.S	406.6 (1809)	528.6 (2352)	0.77	0.84	0.072	86
Mechanical	D6.A2.G60#5.S	394.2 (1754)	438.1 (1950)	0.90	0.04	0.072	0.0
Model	D6.A2.G40#4.S	293.8 (1307)	376.8 (1677)	0.78			
	D4.A2.G40#4.S	207.3 (922)	191.5 (852)	1.08			

Table 6.7: Zararis Mechanical Model Analysis Prediction Results

The mean standard deviation and coefficient of variation of the strength prediction bias are shown in Table 6.7 and were calculated only taking into account the first four specimens since it is recognized that better results are achieved using the sectional analysis implementation of MCFT with Response 2000 for beams having an a/d ratio greater than 2.0. Using only the first four specimens, it can be seen that the analysis method as proposed by Zararis (2003) over estimates the ultimate shear strength of the bent cap specimens in all four cases with a mean prediction bias of 0.84, and a coefficient of variation of 8.6%.

In this model it was assumed that there was sufficient main longitudinal steel embedded at the anchorage zone to balance the compressive force generated at the reduced compression zone. However, there may be cases where the maximum compressive forces are limited by the availability of the longitudinal reinforcing steel. The calculated compressive force C, and the maximum tensile force of the steel reinforcement T_{max} , which is the total area of the embedded steel reinforcement at the anchorage zone multiplied by the actual yield strength of steel for the specimens, are listed for each specimen in Table 6.8.

Analysis Series	Specimen	C [kips]	T _{max} [kips]	Ratio C/T _{max}	Mean	STD	COV [%]
	D6.A4.G60#5.S	584.3	443.7	1.32			
Zararis	D6.A4.G40#4.S	614.2	425.6	1.44	1.50	0.283	19.6
Mechanical	D6.A2.G60#5.S	394.2	212.8	1.85	1.32		10.0
Model	D6.A2.G40#4.S	398.4	212.8	1.87			
	D4.A2.G40#4.S	243.5	216.5	1.12			

Table 6.8: Compressive and tensile forces at failure according to the Zararis Mech. Model

Seen from Table 6.8, the compressive force is larger than the maximum tensile force of the steel reinforcement in all cases, thus the horizontal equilibrium in the free body diagram is not satisfied.

Another issue is that the horizontal shear force contribution of the vertical web reinforcement is disregarded in this model. However, considering the rapid rate of change in the diagonal tensile strains in the shear span after yielding of the main longitudinal steel embedded at the anchorage zone and the deformations in the vertical web reinforcement observed during the experiments, the horizontal shear force contribution of the vertical web reinforcement may be significant and necessary to achieve horizontal equilibrium.

6.5.2 Modified Zararis Mechanical Model

In order to better model the unique conditions for vintage bent caps, the proposed model by Zararis was modified in terms of limiting the concrete compressive strength and introducing the horizontal shear contribution of the vertical web reinforcement in order to achieve the equilibrium conditions used in the derivation. In this model the concrete compressive strength is limited to 0.85 f_c ' since it is believed that the existence of tensile stresses and lack of confinement of the compression zone due to the indirect loading mechanism may cause compression softening, which is defined as the reduction of compressive strength and stiffness relative to the uniaxial compressive strength. In addition, by limiting the concrete compression strength to 0.85 f_c ', a degree of familiarity with current design codes such as ACI 318, or AASHTO LRFD is achieved.

Using the stress tensor given in Equation 6.14, the horizontal shear force contribution of the vertical web reinforcement can be calculated as:

$$V_{sd} = 0.4 V_s \tan \varphi \tag{6.29}$$

Since the stirrups are assumed to be at yield while calculating the ultimate shear strength of the deep beam, Equation 6.29 gives the maximum horizontal shear force contribution of the vertical web reinforcement.

In this modified analysis method, first the initial values of c, c_s and P_u are computed according to the original Zararis Mechanical Model but using the reduced concrete strength of 0.85 fc'. Second, the horizontal equilibrium of the forces in the free body diagram illustrated in Figure 6.13 is checked:

$$T_{max} = A_s f_v \ge C = 0.85 f'_c b c_s$$
 (6.30)

Where T_{max} is the tensile load capacity, A_s is the total area, and f_y is the yield strength of the available longitudinal reinforcing steel bars embedded at the anchorage zone and crossing the critical diagonal crack. If Equation 6.30 is satisfied, this indicates that the ultimate shear strength of the deep beam is governed by concrete, and additional contribution from the shear force of the vertical web reinforcement is not needed. Thus, the ultimate shear strength can be directly computed from Equation 6.28 by only replacing the term f_c with 0.85 f_c '.

If Equation 6.30 is not satisfied, then it indicates that an additional contribution from the shear force of the vertical web reinforcement is needed in order to satisfy the horizontal equilibrium equation. The free body diagram constructed for this case is illustrated in Figure 6.14, and the new equilibrium equation becomes:

$$T_{max} + V_{sd} \ge C = 0.85 f'_c b c_s$$
 (6.31)

In case the sum of the tensile load capacity of the longitudinal tensile steel embedded at the anchorage zone and the maximum additional contribution from the horizontal shear force of the vertical web reinforcement exceed the compressive force C, the analysis proceeds with the initial value of c_s as calculated before. However, the contribution from the horizontal shear force of the vertical web reinforcement should be limited to:

$$V_{sd,limited} = C - T_{max} \quad \text{if} \quad T_{max} + V_{sd} > C$$
 (6.32)

Thus, when the moment equilibrium equation at the application point of the compressive force C is employed, the ultimate shear strength of the deep beam can be computed as:

$$V_{\rm p} = \frac{T_{\rm max} \left(1 - 0.5 \frac{c_{\rm s,initial}}{d}\right) + 0.5 V_{\rm s} \left(\frac{a}{d}\right) + 0.5 V_{\rm sd,limited}}{(a/d)}$$
(6.33)

If both Equation 6.30 and Equation 6.31 are not satisfied, this indicates that the ultimate strength of the deep beam is governed by the amount of reinforcing steel, and the reduced compression block depth c_s , should be re-calculated in order to satisfy the horizontal equilibrium. The new reduced compression block depth, $c_{s,limited}$ is given by:

$$c_{s,limited} = \frac{T_{max} + V_{sd}}{0.85 f'_{c} b}$$
 (6.34)

Using the free body diagram illustrated in Figure 6.14 and employing a moment equilibrium equation at the application point of the compressive force C, the ultimate shear strength of the deep beam can be computed from:

$$V_{\rm P} = \frac{T_{\rm max} \left(1 - 0.5 \frac{c_{\rm s, limited}}{d} \right) + 0.5 V_{\rm s} \left(\frac{a}{d} \right) + 0.5 V_{\rm sd}}{(a/d)}$$
(6.35)

The flowchart of the methodology is shown in Figure 6.16. The correlation between the experimental and predicted ultimate shear strengths for the bent cap specimens using the Modified Zararis Mechanical Model are summarized in Table 6.9, and also shown in Figure 6.15.



Figure 6.16: Solution procedure for the Modified Zararis Mechanical Model

Analysis Series	Specimen	V _{EXP} [kips] (kN)	V _P [kips] (kN)	Bias V _{EXP} /V _P	Mean	STD	COV [%]
	D6.A4.G60#5.S	506.4 (2253)	528.8 (2353)	0.96			
Modified	D6.A4.G40#4.S	406.6 (1809)	439.5 (1956)	0.93	0.00	0.059	5.0
Zararis Mechanical	D6.A2.G60#5.S	394.2 (1754)	381.6 (1698)	1.03	0.99		5.9
Model	D6.A2.G40#4.S	293.8 (1307)	280.4 (1248)	1.05			
	D4.A2.G40#4.S	207.3 (922)	192.7 (858)	1.08			

Table 6.9: Modified Zararis Mechanical Model Analysis Prediction Results

As seen here, better correlation was achieved with the Modified Zararis Mechanical Model with a prediction bias of 0.99 and a coefficient of variation of 5.9% based on four bent cap specimens. In addition, the governing parameters at failure are summarized in Table 6.10 for each specimen.

Analysis Series	Specimen	Failure Case	Note
	D6.A4.G60#5.S	2	Strength governed by concrete
Modified	D6.A4.G40#4.S	3	Strength governed by reinforcing steel
Zararıs Mechanical	D6.A2.G60#5.S	2	Strength governed by concrete
Model	D6.A2.G40#4.S	3	Strength governed by reinforcing steel
	D4.A2.G40#4.S	1	Strength governed by concrete ($V_{sd} = 0$)

Table 6.10: Modified Zararis Mechanical Model Failure Cases

According to the analysis results, the strength of bent cap specimens D6.A4.G40#4 and D6.A2.G40#4 is governed by the availability of reinforcing steel. This is due to the fact that the shear force contribution of the vertical web reinforcement is relatively low in specimens with weaker web reinforcement. However, the horizontal shear contribution of the vertical web reinforcement was adequate in specimens D6.A4.G60#5.S and D6.A2.G60#5.S to allow use of the initial reduced compression block depth. Specimen D4.A2.G40#4.S was the only specimen where the shear force contribution of the vertical web reinforcement was not necessary to maintain equilibrium, which may be considered as an indicator of a different behavior caused by the a/d ratio. In order to determine confidence intervals for the proposed methodology, partial safety factors, , were calculated assuming normal distribution probability density functions as shown in Figure 6.17. If a normal distribution is assumed for the analysis method, the probability of over predicting the experimental strength $(V_{EXP}/V_P > 1)$ with the Modified Zararis Mechanical Model varies depending on the selected confidence intervals as seen in Table 6.11. For example, the probability of having a conservative prediction result is 99% if a partial safety of 0.85 is used.



Figure 6.17: Assumed probability density functions for the prediction bias, and the effect of partial safety factors

Confidence Interval	Partial Safety
90%	0.92
95%	0.89
99 %	0.85
99.9%	0.80

Table 6.11: Confidence Intervals and the Corresponding Resistance Factors

An analysis based on the Modified Zararis Model gives a good estimate of the bent cap specimen capacity. The relatively simple nature of the proposed equations allows hand calculations or a spreadsheet program to be used in order to achieve fast results. Such a spreadsheet was constructed with Microsoft Excel and used for the analysis of bent cap specimens in this study.

6.6 NON-LINEAR FINITE ELEMENT ANALYSIS

The advent of computers in the 1940s and the subsequent rapid development of the finite element method (FEM) in the 1950s provided structural engineers with powerful tools for analyzing very complex structures. In order to achieve reliable results with the FEM, realistic modeling of the structure geometry, boundary conditions, and the material behavior are of utmost importance. The latter has proved to be the most challenging part when modeling reinforced concrete structures due to the quasi-brittle and anisotropic nature of concrete, which becomes even more complex with the addition of steel reinforcement. Even though the computing power has grown rapidly resulting in faster computing times even with sophisticated analysis algorithms, the challenge of modeling the behavior of reinforced concrete still remains the subject of many research activities. Quoting from Vecchio (*2001*), "Unfortunately, reinforced concrete is a complex and stubborn material that sometimes refuses to act according to the accepted rules of mechanics".

The two main approaches used in FEM analysis to represent the cracking phenomenon in concrete structures are the "discrete crack approach", and the "smeared crack approach".

In the discrete crack approach, the crack is treated as a geometrical entity where the finite element mesh has to be altered to accommodate propagating cracks. The need to modify the mesh topology as the crack propagates under increasing loads may limit the speed and ease of the analysis when the discrete crack approach is employed. Nilson (1967, 1968) was the first to treat cracks in a FEM of a reinforced concrete beam discretely followed by other researchers such as Ngo and Scordelis (1967), Salah El-Din and El-Adawy Nassef (1975), Hillerborg et al. (1976), Gerstle (1982), Ingraffea et al. (1984), and Gustafsson and Hillerborg (1988).

In the smeared crack approach, the constitutive properties of finite elements in the vicinity of the crack are adjusted rather than changing the topography of the finite element grid. The earliest procedure related to the smeared crack approach (*Rashid 1968*) was based on reducing the material stiffness to zero in the direction of the principal tensile stress once the stress exceeded the tensile capacity of concrete.

More sophisticated material models may be used in the smeared crack approach to take into account the post peak tensile behavior of concrete. In the last decades, the smeared crack approach has become the most widely used approach in practice due to its computational convenience. Naming a few, Cervenka and Grestle (1972), Valliappan and Doolan (1972), Colville and Abbasi (1974), Nam and Salmon (1974), Darwin and Pecknold (1976), Schnobrich (1977), Bazant and Cedolin (1980), Bedard and Kotsovos (1985), Chang et al. (1987), Channakeshava and Iyengar (1988), Vecchio (1989), Gupta and Maestrini (1989), Bhatt and Abdel Kader (1998) and Maekawa et al. (2003) reported that it is possible to achieve reliable results using the smeared crack approach.

There is no consensus on a "better" approach since both smeared and discrete crack models have their own advantages and disadvantages. As mentioned before, the mesh regeneration process in the discrete crack approach may become a tedious and difficult job relegated to the analyst. Smeared crack approach is known to have better handling of the reinforcement (modeled as discrete or smeared) since the behavior at the crack vicinity becomes too complex when the discrete crack approach is used. On the other hand it was reported that the exact failure mode may not be always assessed using smeared crack models, whereas there is better agreement with the discrete crack models and the experimental failure modes (*Jendele et al. 2001*). Another issue with the smeared crack approach is the mesh sensitivity (*Bazant 1976*) where the analyst has to try models with different mesh sizes in order to ensure convergence. However, it should be noted that good results have been achieved on the load capacity prediction using both approaches.

6.6.1 Non-linear Finite Element Analysis Using VecTor2

The finite element code VecTor2 v6.0 was employed in this study to predict the capacity and behavior of the bent cap specimens. VecTor2 is a non-linear finite element analysis program focused on the analysis of two-dimensional reinforced concrete membrane structures under static, cyclic or thermal loads, which has been under constant development at the University of Toronto since early 1990's. The theoretical background of the program is based on the Modified Compression Field Theory (MCFT) (*Vecchio and Collins 1986*) and the Disturbed Stress Field Model (DSFM) (*Vecchio 2000*), which are analytical models for predicting the response of rectangular reinforced concrete elements subjected to in-plane shear and axial stresses.

VecTor2 uses low-order planar triangular, rectangular and quadrilateral elements which model concrete with or without smeared reinforcement. The discrete reinforcement is modeled with linear truss bar elements. Non-dimensional link and contact elements are also supported in order to model bond-slip. VecTor2 employs a rotating-smeared crack model approach to model reinforced concrete. Since the FEM analysis is used as a "tool" in this study, further detail on the derivation of the elements and the solution algorithm is out of the scope of this research and is detailed by Wong and Vecchio (2002).

Although VecTor2 is the core application for the finite element analysis, the software is supported by a preprocessor and a postprocessor. FormWorks v2.0 is the preprocessor software which provides a graphical user interface for developing and visualizing the finite element model and generates input files to be used with VecTor2. FormWorks has auto-meshing capabilities and a bandwidth reduction algorithm to optimize the computing time. Augustus v5.0.6 is the postprocessor of the software suit, which uses the output files generated by VecTor2 and provides a graphical user interface for interpreting the analysis results. Augustus is capable of displaying the deflected shape of the structure, the crack patterns, and the stress and strain distributions in the elements.

6.6.2 The Finite Element Model and Trial Analyses

The specimens were modeled using their full geometry since the support conditions of the test specimens were not exactly symmetric. The roller support in the specimens was simulated by restraining a single node at the bottom of the column in y-direction only. Trial analyses were executed in order to determine the best way to simulate the pin support in the specimens. It was observed that when all bottom nodes were restrained in both x and y directions, the support acted like a moment connection, and extensive cracking occurred in the column due to bending resulting in premature local failure modes, which was not seen during the experiments where the real support condition allowed limited rotations. When the pin support was simulated by restraining a single node at the bottom of the column both in x and y directions, the extensive cracking phenomenon in the column was minimized resulting in limited column cracking similar to that observed during the experiments.

Different out of plane thicknesses were assigned for the column and beam sections. Stub girder portions were not modeled since VecTor2 is a 2D analysis program. Instead, the loads were applied as uniformly distributed along the bent cap beam– girder connection in some of the trial analyses.

Rectangular and triangular elements were used to model plain concrete or concrete with smeared web reinforcement. Truss bar elements were used to model flexural steel in the bent cap, the vertical steel bars in the columns, and the transverse web reinforcement, if stirrups were modeled discretely. Column hoops were always modeled as smeared reinforcement in all of the models. A maximum element aspect ratio of 1.5 was enforced. Example models of 6 ft deep specimens and the 4 ft deep specimen with discrete flexural and transverse reinforcement is shown in Figure 6.18.



Figure 6.18: Finite element models for (a) 6 ft deep specimens and (b) 4 ft deep specimen

Since VecTor2 uses the smeared crack approach, models with different mesh sizes were analyzed in order to ensure convergence and to make a decision on the maximum mesh size that can be used which minimizes the computing time and provides convergence. According to Bedard and Kotsovos (1985) the smallest finite element size should be two to three times greater than the maximum aggregate size, which was $\frac{3}{4}$ in. (20 mm.) in the laboratory specimens. The automesh option was used in FormWorks for different mesh sizes of 1.97 in. (50 mm), 3.94 in. (100 mm), 7.87 in. (200 mm), 11.81 in. (300 mm) in order to check convergence. The initial secant stiffness did not seem to be affected by the mesh size. However, relatively stiffer response under increasing loads and higher ultimate load capacities were achieved in models with larger mesh sizes. It was observed that a mesh size of 3.94 in. (100 mm) was sufficient to satisfy the convergence of the solution while providing reasonable computational time compared to a mesh size of 1.97 in. (50 mm). An example of the convergence trials from specimen D4.A2.G40#4.S is shown in Figure 6.19. For the same specimen, the required computing time as a function of the number of elements in the model

is shown in Figure 6.20. All analyses were performed on an Intel[®] based PC, with a Pentium[®] 4 3.0 GHz CPU, and 1 GB of DDR2 RAM.



Figure 6.19: Predicted load-deformation response for different finite element mesh sizes (Specimen D4.A2.G40#4.S)



Figure 6.20: Computing time for different mesh sizes (Specimen D4.A2.G40#4.S)

For displacement controlled analyses, a load step size of 0.04 in. (1.0 mm) was chosen. For forced controlled analyses, different load step sizes of 1 kip (4.5 kN), 5 kips (22.5 kN), 10 kips (45 kN) and 25 kips (112.5 kN) were tried. Changing the load step size did not affect the behavior until the model was close to failure. Using large load steps such as 10 kips (45 kN) and 25 kips (112.5 kN) yielded artificially higher load capacities. It was observed that a load step size of 5 kips (22.5 kN) was adequate to capture the ultimate load capacity as shown in Figure 6.21.



Figure 6.21: Predicted load-deformation response for different load step sizes (Specimen D4.A2.G40#4.S)

A total of 1757 rectangular and 30 triangular elements were used to represent concrete, and 427 truss elements were used to discretely model the flexural reinforcement and the vertical reinforcement in the columns for the finite element models of the 6 ft. deep specimens with smeared web reinforcement. For the 4 ft. deep beam specimen, a total of 1288 rectangular and 21 triangular elements were used to represent concrete, and 445 truss elements were used to model the discrete flexural reinforcement and the vertical reinforcement in the columns. When web reinforcement was discretely modeled, the total number of elements increased affecting the computing time. A total of 1689 rectangular and 354 triangular elements were used to represent concrete, where 1084 truss elements were used to model the discrete web reinforcement in 6 ft. deep beam models. For the 4 ft deep specimen with discrete web reinforcement, a total of 1225 rectangular and 217

triangular elements were used to represent concrete, and 838 truss elements were used to model the discrete reinforcement.

Results from the material tests were used for the input parameters such as the compressive strength of concrete and the yield and ultimate strengths of steel. The concrete tensile strength, f_t , was calculated from:

$$f'_t = 0.33\sqrt{f'_c}$$
 (MPa) (6.36)

Where f_c ' (MPa) is the cylinder compressive strength of concrete since it was reported that tensile strengths found from the modulus of rupture tests are known to over-estimate the actual tensile strength of concrete when used with VecTor2 (*Saatci 2007*). The initial tangent modulus of concrete, E_c was determined from:

$$E_c = 5500\sqrt{f'_c}$$
 (MPa) (6.37)

The cylinder strain at f_c ', E_o is calculated from:

$$\varepsilon_{0} = 1.8 + 0.0075 f'_{c}$$
 (m ε) (6.38)

The initial Poisson's ratio, E_o , was taken as 0.15. The steel elastic modulus, E_s , was taken as 200,000 MPa (29,000 ksi), the strain hardening modulus, E_{sh} , was assumed as 20,000 MPa (2900 ksi).

6.6.3 Constitutive Models for Concrete and Steel Reinforcement

VecTor2 uses adaptive constitutive models both for concrete and steel reinforcement by taking into account a variety of second-order effects that are unique to reinforced concrete such as compression softening, tension stiffening, tension softening, and tension splitting. The program is also capable of modeling other complex phenomena related to reinforced concrete such as concrete dilation and confinement, bond slip, crack shear deformations, reinforcement dowel action, reinforcement buckling, and crack allocation processes. All material and behavioral models used for concrete and steel reinforcement were the default settings for VecTor2, and brief information about each model used in the finite element analysis section of this study is given in Appendix A of this document. More detail on derivations of these models, and information about other models supported by VecTor2 can be found in the VecTor2 and Formworks Manual (2002), and in the related references.

6.6.4 Nonlinear Finite Element Analysis Results

Once optimal support conditions, mesh sizes, load step sizes, and the material models were determined, further displacement or force based analyses were conducted by using combinations of discrete/smeared web reinforcement, point/distributed loads, and monotonic/cyclic load increments. Varying parameters used in analyses are listed in Table 6.12.

#	Analysis Type	Loading ¹	Loading ²	Flexural Reinforcemen t	Web Reinforcement
1	Displacement Controlled	Monotonic	Point Loads	Discrete	Smeared
2	Force Controlled	Monotonic	Point Loads	Discrete	Smeared
3	Force Controlled	Monotonic	Distributed Loads	Discrete	Smeared
4	Force Controlled	Monotonic	Distributed Loads	Discrete	Discrete
5	Force Controlled	Monotonic	Point Loads	Discrete	Discrete
6	Force Controlled	Cyclic	Point Loads	Discrete	Discrete

Table 6.12: Finite Element Analysis Series

VecTor2 NLFEM analysis ultimate shear strength prediction results are shown in Figure 6.22, where the red circle denotes a premature failure mode observed for specimen D6.A4.G40#4.S only in analysis series 3 and 4 with distributed loads. In the premature failure mode case, the finite element analysis terminated abruptly due to numerical instability.

Generally VecTor2 was capable of accurately predicting the ultimate shear strength in almost all of the cases with low values for the standard deviation and the coefficient of variation. Figure 6.23 shows the prediction results ranked according to the standard deviations. The results that best matched the experimental behavior were achieved in analysis series 5 with a mean prediction bias of 1.02 and a coefficient of variation of 1.46%. Prediction biases, standard deviations, and the coefficients of variation from each analysis series are summarized in Table 6.13.



Figure 6.22: VecTor2 NLFEA ultimate shear strength prediction results



Figure 6.23: VecTor2 NLFEA ultimate shear strength prediction results

It was observed that predicted shear strengths from analyses with smeared reinforcement were slightly unconservative. However, the predicted shear strengths decreased when transverse web reinforcement was modeled discretely. When analysis series 2 and 5 were compared, which were both force controlled and analyzed under monotonic point loading, the difference between the mean prediction biases was 6%. A comparison of analysis series 4 and 5, without taking into account specimen D6.A4.G40#4.S, showed that the ultimate shear capacities decreased insignificantly (1%) when distributed loads simulating indirect loading were applied as opposed to direct point loading. This can be attributed to the two-dimensional analysis approach VecTor2 uses where it was not possible to directly simulate the effect of three dimensional indirect load transfer mechanisms.

Analysis Series	Specimen	V _{EXP}	VP	Rias	Mean	STD	COV
1 mary sis Series	specimen	[kips]	[kips]	$V_{\rm EVD}/V_{\rm D}$	Witan	510	[%]
		(kN)	(kN)	• EXP/ • P			[/0]
		506.4	491.3	1.02			
	D0.A4.000#3.5	(2253)	(2186)	1.05			
		406.6	405.1	1.00			
	D0.A4.040#4.5	(1809)	(1803)	1.00			
VecTor2	D6 A2 G60#5 S	394.2	407.4	0.07	0.07	0.057	5 88
NLFEA #1	D0.A2.000#3.5	(1754)	(1813)	0.97	0.97	0.037	5.00
	$D(\Lambda) C A 0 = 4$	293.8	332.4	0.00			
	D0.A2.G40#4.5	(1307)	(1479)	0.88			
	D_{4} A 2 C $A0$ H_{4} S	207.3	218.6	0.05			
	D4.A2.U40#4.5	(922)	(922) (973) 0.95				
	D6 14 C60#5 S	506.4	505.9	1.00			
	D0.A4.000#3.5	(2253)	(2251)				
	D6.A4.G40#4.S	406.6	419.9	0.97			
		(1809)	(1869)				
VecTor2		394.2	414.6	0.05	0.06	0 028	2 00
NLFEA #2	D0.A2.000#3.5	(1754)	(1845)	0.95	0.90	0.028	2.90
	D6 A 2 C 40 # 4 S	293.8	313.8	0.04			
	D0.A2.040#4.5	(1307)	(1396)	0.94			
	D4 A2 C40#4 S	207.3	222.5	0.02			
	D4.A2.U40#4.S	(922)	(990)	0.93			
VecTor2	D6 14 C60#5 S	506.4	499.5	1.01	1.00	0 078	7 81
NLFEA #3	D0.A4.000#3.5	(2253)	(2223)	1.01	1.00	0.070	7.01
	$D \in A \land C \land 0 \# \land S$	406.6	359.0	1 1 2			
	D6.A4.G40#4.S	(1809)	(1598)	1.13			
	D6 12 C60#5 S	394.2	409.6	0.06			
	D0.A2.000#3.5	(1754)	(1823)	0.90			

Table 6.13: VecTor2 Finite Element Analysis Prediction Results

	D6.A2.G40#4.S	293.8 (1307)	303.4 (1350)	0.97			
	D4.A2.G40#4.S	207.3 (922)	222.4 (990)	0.93			
	D6.A4.G60#5.S	506.4 (2253)	491.3 (2186)	1.03	1		
	D6.A4.G40#4.S	406.6	359.0	1.13			
VecTor2 NLFEA #4	D6.A2.G60#5.S	394.2 (1754)	384.2 (1710)	1.03	1.05	0.045	4.31
	D6.A2.G40#4.S	293.8 (1307)	283.5 (1262)	1.04			
	D4.A2.G40#4.S	207.3 (922)	202.2 (900)	1.02			
	D6.A4.G60#5.S	506.4 (2253)	485.6	1.04			
	D6.A4.G40#4.S	406.6	399.7 (1779)	1.02			
VecTor2 NLFEA #5	D6.A2.G60#5.S	394.2 (1754)	389.5	1.01	1.02	0.015	1.46
	D6.A2.G40#4.S	293.8 (1307)	288.3 (1283)	1.02			
	D4.A2.G40#4.S	207.3 (922)	207.4 (923)	1.00			
	D6.A4.G60#5.S	506.4 (2253)	480.5	1.05			
VecTor2 NLFEA #6	D6.A4.G40#4.S	406.6	404.7	1.00			
	D6.A2.G60#5.S	394.2 (1754)	389.5	1.01	1.02	0.029	2.84
	D6.A2.G40#4.S	293.8	283.2	1.04			
	D4.A2.G40#4.S	207.3 (922)	212.4 (945)	0.98			

6.6.4.1 Load Deflection Response

Load–displacement curves obtained from the experiments and the finite element analyses were compared. The correlation between the experimental and predicted responses from analysis series 5, where best results achieved, is shown in Figure 6.24 – Figure 6.28. It was observed that VecTor2 was capable of capturing the initial stiffness of specimens and the general trend in behavior very well. However, the sudden softening and re-stiffening phenomenon in the response of the experimental specimens around 120 - 150 kips (534 kN - 668 kN), which was due to the sudden propagation of the characteristic diagonal crack, was not seen in the predicted behavior. This issue can be attributed to the smeared crack approach VecTor2 employs and the force-controlled loading protocol which was used in the experiments.



Figure 6.24: Predicted and experimental response of specimen D6.A4.G60#5.S



Figure 6.25: Predicted and experimental response of specimen D6.A4.G40#4.S



Figure 6.26: Predicted and experimental response of specimen D6.A2.G60#5.S


Figure 6.27: Predicted and experimental response of specimen D6.A2.G40#4.S



Figure 6.28: Predicted and experimental response of specimen D4.A2.G40#4.S

Since a cyclic loading protocol was used in the experiments, cyclic analyses of bent cap specimens were also performed. Due to the limitations in VecTor2, the alternating load protocol with sub-load steps explained in Section 3.5.1 could not be directly applied in the finite element analysis. Instead the loads were applied simultaneously simulating the "both lanes loaded" load case while discarding the "one lane loaded" sub-load steps. In the cyclic loading analyses, the loads were increased by 50 kips after each loading–unloading cycle, and a load step size of 5 kips was used similar to the monotonic loading analyses. The major drawback with cyclic loading analyses was the significant increase in the required computing time due to additional load steps. While all of the force controlled monotonic analyses were completed in less than an hour (17 - 45 minutes), the required computing time for cyclic analyses was between 45 to 459 minutes depending on the ultimate load capacity of the specimen as shown in Figure 6.29.



Figure 6.29: Computing time for monotonic and cyclic analyses (discrete web reinforcement)

Applying cyclic loads did not significantly affect the predicted results since the load-displacement curves achieved from the cyclic loading analyses basically followed the backbone curve of the monotonic loading tests as shown in Figure 6.24 – Figure 6.28. The load capacity prediction bias was the same with the monotonic analyses with a standard deviation of 2.88%. The predicted plastic displacement offsets were less than those observed in the experimental tests in all of the cases, but the trend in the response was generally captured. Overall, it can be said that using monotonically increasing loads in analyses of bent cap specimens was sufficient enough and more practical considering the similarities in the predicted behavior and the much longer required computing time for cyclic analyses.

6.6.4.2 Crack Patterns

Crack patterns predicted by VecTor2 were compared with the patterns observed from the tests as shown in Figure 6.30 – Figure 6.34. Generally, the experimental and predicted crack patterns match well. The locations and the heights of vertical cracks that propagate at the early stages of loading were predicted reasonably. The formation of the characteristic diagonal cracks was also correctly predicted. The more extensive cracking pattern observed in bent cap specimens with heavier web reinforcement was also well predicted by VecTor2. However, there were some discrepancies as well. For example, in specimens D6.A2.G40#4.S and D4.A2.G40#4.S the predicted characteristic diagonal cracks had a shallower angle than those seen in the experiments. In addition, the light flexural crack patterns predicted at the column regions were not observed in the tests.



Figure 6.30: Experimental and VecTor2 predicted crack pattern for specimen D6.A4.G60#5.S



Figure 6.31: Experimental and VecTor2 predicted crack pattern for specimen D6.A4.G40#4.S



Figure 6.32: Experimental and VecTor2 predicted crack pattern for specimen D6.A2.G60#5.S



Figure 6.33: Experimental and VecTor2 predicted crack pattern for specimen D6.A2.G40#4.S



Figure 6.34: Experimental and Vector2 predicted crack pattern for specimen D4.A2.G40#4.S

6.6.4.3 Concrete Principal Compressive Stress Distribution

Concrete principal compressive stress trajectories predicted by VecTor2 are shown in Figure 6.35 – Figure 6.39. It can be seen that compressive forces are mainly transferred from the loading zones to the supporting columns through arch action. At later stages of the laboratory tests, a secondary strut outlined by other inclined cracks adjacent to the characteristic diagonal crack was generally observed. A similar response can be seen in the compressive principal stress trajectories predicted by VecTor2. The depth of the flexural compression block at the constant moment zone was also well predicted. Except specimen D4.A2.G40#4.S, the utility holes were mostly outside the path of principle stress trajectories; thus, it can be said that the utility holes do not affect the behavior of 6 ft deep bent cap specimens.



Figure 6.35: Concrete principal compressive stresses for specimen D6.A4.G60#5.S



Figure 6.36: Concrete principal compressive stresses for specimen D6.A4.G40#4.S



Figure 6.37: Concrete principal compressive stresses for specimen D6.A2.G60#5.S



Figure 6.38: Concrete principal compressive stresses for specimen D6.A2.G40#4.S



Figure 6.39: Concrete principal compressive stresses for specimen D4.A2.G40#4.S

6.6.4.4 Steel Reinforcement Stress Distribution

The sequential yielding of stirrups observed in laboratory tests depended on the propagation of the characteristic diagonal crack, and this was also seen in finite element analysis results. VecTor2 predicted yielding of all the transverse web reinforcement in the shear span prior to failure except the stirrup closest to the beam column interface. Yielding of the flexural steel embedded at the anchorage zone near failure was also predicted by VecTor2. Figure 6.40 shows the steel reinforcement stresses at failure for specimen D6.A4.G60#5.S. The smallest bin value (blue) in the Figure corresponds to the yield strength of stirrups, whereas the largest bin value (green) in the Figure corresponds to a stress slightly larger than the yield strength of flexural reinforcement.



Figure 6.40: Steel reinforcement stresses beyond yield (Specimen D6.A4.G60#5.S)

6.6.4.5 Effect of Fatigue Loading on Member Capacity

In order to assess the capacity of the fatigue specimen D6.A2.G40#4.F, a finite element model was constructed with VecTor2 where the web reinforcement was modeled discretely and point loads were applied. The analysis type was chosen as force-controlled. In situations where metal fatigue of the embedded steel is unlikely due to low stress ranges, the mechanism for damage from high-cycle fatigue is bond fatigue at the stirrup-concrete interface, which leads to reduced constraint at crack locations thus increasing crack widths. In order to investigate the possible deteriorating effect of fatigue loading on member strength, first, perfect bond was assumed between web reinforcement and concrete. The nonlinear finite element analysis with perfectly bonded web reinforcement resulted in a prediction bias (V_{EXP}/V_P) of 1.00. Second, a new finite element model was constructed where a portion of the web reinforcement was assumed to be significantly debonded due to fatigue effects in order to simulate the worst conditions. This was accomplished by applying perfect bond to the top and bottom 10% segment of the truss elements, and the "unbonded bars or tensons" bond model was employed for the remaining 80% length in between as illustrated in Figure 6.41. The non-linear finite element analysis with 80% debonded web reinforcement resulted in a prediction bias (V_{EXP}/V_P) of 1.04. When crack patterns of both models were compared, less extensive but wider cracks were observed in the debonded model. The load-deformation response from both models and the experimental data is shown in Figure 6.42. It was verified that the debonded model produced constant stress in the stirrups over the debonded segment.



Figure 6.41: Finite element model with 80% debonded stirrups at each shear span



Figure 6.42: Predicted and experimental response of specimen D6.A2.G40#4.F

According to the non-linear finite element analyses of specimen D6.A2.G40#4.F, an extreme case of 80% debonded stirrups due to bond fatigue resulted in a loss of capacity by 3.6%. In order to investigate the effect of bond fatigue on the strength of other specimens with different reinforcement configurations, a series of non-linear finite element analyses were carried out where the results from models with perfectly bonded and 80% debonded stirrups were compared. The analysis results indicated that while the strength reductions for 6 ft deep specimens with lighter web reinforcement (Grade 40, #4 stirrups) was between 3.6% and 3.9%, severe bond deterioration had a much more significant impact on 6 ft deep specimens with heavier web reinforcement (Grade 60, #5 stirrups). The loss of capacity for 6 ft deep specimens with heavier web reinforcement was observed as 13.0% and 14.0% for specimens D6.A4.G60#5.S and D6.A2.G60#5.S, respectively. For the relatively slender 4 ft deep specimen, D4.A2.G40#4.S, the loss of capacity was only 2.4%. The analysis results are provided in Table 6.14 where the contribution of the stirrups was indicated with a stress term, $A_s f_v / bs$. The deterioration in member capacities due to severe bond fatigue is also shown in Figure 6.43.



Figure 6.43: Change in predicted capacity for fully bonded and 80% debonded stirrups

Analysis Series	Specimen	A_sf_y/bs [psi] (Pa)	V _P Perfectly Bonded [kips] (kN)	V _P 80% Debonded [kips] (kN)	Difference [%]
NLFEA Fatigue Simulation	D6.A4.G60#5.S	275.5 (1900)	485.6 (2161)	422.5 (1880)	13.0
	D6.A4.G40#4.S	144.3 (995)	399.7 (1779)	384.2 (1710)	3.9
	D6.A2.G60#5.S	275.5 (1900)	389.5 (1733)	334.9 (1490)	14.0
	D6.A2.G40#4.S	143.4 (989)	288.3 (1283)	277.9 (1213)	3.7
	D4.A2.G40#4.S	144.6 (997)	207.4 (923)	202.4 (901)	2.4
	D6.A2.G40#4.F	142.3 (981)	283.2 (1260)	273.1 (1215)	3.6

Table 6.14: The Effect of Bond Fatigue on Predicted Specimen Capacities

From the fatigue experiment and the analysis results, debonding of stirrups due to fatigue loading did not seem to extensively affect the capacity of bent caps with light web reinforcement, and this effect tended to diminish further with increasing a/d ratios. However, effects of the fatigue loading appeared to be much more critical for heavily reinforced bent caps when the member capacity relies more on the web reinforcement.

6.7 SUMMARY RESULTS

Six different analytical methods were applied to assess the strength and behavior of the bent cap specimens with 1950s vintage details including ACI 318-05 traditional shear design method, ACI 318-99 deep beam shear design method, MCFT sectional analysis with Response 2000, Strut-and-Tie Models, the Zararis Mechanical Model and its modified adaptation, and non-linear finite element analysis with VecTor2. Based on these analyses and comparisons with experimental results, the following conclusions are presented:

- ACI 318-05 traditional shear design method resulted variable outcomes and sometimes unconservative results. An unconservative prediction bias (V_{EXP}/V_P) of 0.86 was achieved in one of the specimens.
- ACI 318-99 deep beam shear design equations produced conservative results for all specimens, but the results also had coefficients of variation of 16.9% for the simplified approach and 25.0% for the detailed approach.
- ACI 318-05 and ACI 318-99 do not account for changes in the embedded flexural reinforcement configuration at the anchorage zone that was observed in the experiments to affect strength.
- MCFT sectional analysis with Response 2000 produced conservative estimates for the shear strength of bent cap specimens with an a/d ratio of 1.38. However, this method reasonably predicted the capacity of the bent cap specimen with an a/d ratio of 2.1.
- Results from the simple strut-and-tie model, which is frequently seen in the literature, were excessively conservative. The web reinforcement was not taken into account in this model, and the capacity of the member was always limited with the availability of the embedded reinforcement at the anchorage zone.
- A more detailed strut-and-tie model produced still conservative but relatively better prediction results. This model employed a two-piece compression strut and a tension tie representing the web reinforcement. Optimization of the STMs by iteration was necessary to improve the prediction results, which takes time and experience.

- Very sophisticated strut-and-tie models are not recommended for predicting the strength of bent cap specimens since the behavior of the model becomes very sensitive to the location of the nodes, which may result in misleading outcomes.
- The original mechanical model proposed by Zararis (2003) produced unconservative estimates of the strength. This model did not account for the cases where the strength was limited by the availability of the embedded reinforcement at the anchorage zone; thus, the horizontal equilibrium of forces was not always satisfied.
- The Zararis Mechanical Model was modified by limiting the concrete compressive strength to account for the tensile stresses and lack of confinement caused by indirect loading, and the horizontal shear resistance of the vertical web reinforcement was taken into account. The proposed methodology included various checks to ensure horizontal equilibrium.
- The Modified Zararis Mechanical Model reasonably predicted the capacity of the specimens with a prediction bias of 0.99 and a coefficient of variation of 5.9%.
- Considering the available data, a partial safety factor of 0.85 is recommended to account for the method of analysis for conservative estimates with a 99% confidence interval.
- Nonlinear finite element analyses with VecTor2 provided good correlation with experimental results. For a model with discrete reinforcement and point loads, a prediction bias of 1.02 with a coefficient of correlation of 1.5% was achieved.
- The predicted crack patterns and the flow of principle compressive stresses from the nonlinear finite element analyses were in good agreement with the experimental observations.
- It was observed that applying cyclic loads taking into account the hysteretic response of concrete and reinforcing steel did not improve the analysis results while increasing computational time.
- The shear capacity of the fatigue specimen D6.A2.G40#4.F was well predicted by nonlinear finite element analysis, while debonding of the stirrups due to fatigue loads was not considered. Another analysis case where the web reinforcement was assumed to be 80% debonded resulted in a deterioration of capacity by 3.6%.
- According to the nonlinear finite element analyses, strength of specimens with heavy web reinforcement seemed to be much more sensitive to the effects of bond fatigue. The effect of bond fatigue seemed to diminish with decreasing a/d ratios.

7.0 CONCLUSIONS

A research program was undertaken in Oregon State University to assess the remaining capacity and life of in-service RCDG bridge bent caps. Six full-scale bent cap specimens with vintage details were constructed and tested. Pertinent factors that describe the strength of the RCDG bridge bent caps were determined. The experimental results were compared with outcomes from various analytical methods to predict the capacity of the test specimens.

Based on the experimental results, the following conclusions are presented:

- For all bent cap specimens, the failure mode was shear compression due to the crushing of concrete in the compression zone at the top of a characteristic diagonal crack, which is commonly observed in deep beams with an a/d ratio between 1.0 to 2.5, typical of in-field bent caps.
- Failure mode was generally brittle; however, a ductile failure mode was observed in one specimen due to substantial yielding of the flexural anchorages.
- Initial diagonal tension cracking was observed at an average concrete shear stress approximately corresponding to $v_{cr} = 1.8\sqrt{f'_c}$.
- The embedded flexural reinforcement at the anchorage zone, the web reinforcement, and the a/d ratio were all found to be the pertinent parameters that affect the strength of bent cap specimens.
- Sequential yielding of the web reinforcement was observed in all of the specimens as the characteristic diagonal crack further propagated into the compression block.
- A polynomial extrapolation method was introduced to approximate the tension demand on the embedded flexural reinforcement at the anchorage zone. Results showed that the embedded flexural reinforcement was beyond yield at the column face when failure occurs for all specimens.
- Slip of the embedded reinforcement was not observed except for the 4 ft deep specimen. This was attributed to the active confinement provided by the presence of normal pressure in the column that causes a clamping force across the developing flexural reinforcement thereby improving the bond strength resulting in reduced development lengths.
- None of the current North American design codes takes into account the beneficial effects of active confinement on bond resulting in overly conservative development lengths.

- At intermediate steel cut-offs, vertical flexural cracks were observed to turn into diagonal cracks near the bar cut-off locations due to the stress concentration at those locations. However, these did not control the strength of the specimens.
- The heavier web reinforcement configurations resulted in more extensive cracking at the shear span compared to specimens with lighter web reinforcement.
- Offset of the concrete surface was observed most visible along the characteristic diagonal crack probably as a result of localized stirrup debonding.
- A load protocol for fatigue testing of a bent cap was developed based on stress range data from field tests. High cycle fatigue leading to metal fatigue of embedded stirrups in a bent cap is unlikely since stress ranges imposed on stirrups are less than the specified stress range for metal fatigue of reinforcing steel. The mechanism for deterioration from high-cycle fatigue is bond fatigue at the stirrup-concrete interface.
- In the fatigue test, stirrup mean stresses were observed to be sensitive to thermal changes and overall changes from loading were slight. On the other hand stirrup stress ranges tended to increase in the first half of the fatigue test, and then become steady.
- No significant changes in stress ranges and mean stresses were observed for flexural reinforcement at the centerline and at the anchorage zone.
- Concrete principal stress magnitudes and orientations were not observed to be affected by fatigue loading.
- The width of the characteristic diagonal crack continuously increases under repeated loading.
- During cycling, movement of stirrups relative to the surrounding concrete was observed, which indicated local debonding.
- The overall stiffness of the specimen was reduced under repeated loading. In addition, the fatigue specimen exhibited larger displacements compared to an otherwise similar specimen not subjected to fatigue
- High cycle fatigue did not cause a significant degradation in the ultimate capacity for the specimen details and loading considered. However, the continuous increase in crack widths may affect the resistance of the structure against environmental effects such as freeze and thaw effect on concrete and bond and corrosion of the embedded reinforcement.
- Based on the analyses and correlations with experimental results, the following conclusions are presented:
- ACI 318-05 traditional shear design method resulted in variable outcomes and sometimes unconservative results.
- ACI 318-99 deep beam shear design equations generally produced very conservative results.

- ACI methods in general are unable to account for changes in the embedded flexural reinforcement configuration at the anchorage zone, which was observed in the experiments to affect strength.
- MCFT sectional analysis with Response 2000 produced conservative estimates for the shear strength of bent cap specimens with an a/d ratio of 1.38. However, this method reasonably predicted the capacity of the bent cap specimen with an a/d ratio of 2.1.
- The Computer Aided Strut-and-Tie (CAST) software is recommended for faster construction and optimization of strut-and-tie models introduced in this study.
- Results from the simple strut-and-tie model with a single strut were excessively conservative, and the capacity of the member was always limited with the availability of the embedded reinforcement at the anchorage zone.
- A more detailed strut-and-tie model with a two-piece compression strut and a tension tie representing the vertical web reinforcement produced still conservative but relatively better prediction results. However, optimization of the strut-and-tie models by iteration was necessary to improve prediction results.
- Very sophisticated strut-and-tie models are not recommended for predicting the strength of the bent cap specimens considering the time consuming iterative optimization process, the need to frequently re-calculate the stiffness of truss members, and the possible disparate outcomes due to the sensitivity of the results to the location of the nodes.
- The proposed Modified Zararis Mechanical Model reasonably predicted the capacity of the specimens. It required much less time to achieve prediction results with this method compared to strut-and-tie models and non-linear finite element analysis.
- A partial safety factor of 0.85 was recommended for the Modified Zararis Mechanical Model to account for the method of analysis. Additional probabilistic analyses to estimate a partial safety factor that accounts for errors regarding the geometry, material properties, and construction were not carried out since a reliability-based analysis was not within the scope of this study.
- Nonlinear finite element analyses with VecTor2 provided good correlation with experimental results where predicted capacities, crack patterns, and the flow of principle stresses were in good agreement with the experimental results.
- It was observed that applying cyclic loads while taking into account the hysteretic response of concrete and reinforcing steel did not improve the analysis results while significantly increasing the computational time.

- The shear capacity of the fatigue specimen D6.A2.G40#4.F was well predicted by nonlinear finite element analysis, while the debonding of the stirrups due to fatigue loads was not considered. Another analysis case where the web reinforcement was assumed to be 80% debonded resulted in a deterioration of capacity by only 3.6% for this specimen.
- Further finite element analyses showed that strength of specimens with heavy web reinforcement seemed to be much more sensitive to the effects of bond fatigue. The effect of bond fatigue seemed to diminish with increasing a/d ratios.

7.1 RECOMMENDATIONS FOR FUTURE WORK

Following recommendations are presented regarding future research:

- A partial safety factor was estimated for the Modified Zararis Mechanical Model to account for the analysis method in this study. In order to determine a resistance factor for the proposed approach, another partial safety factor which accounts for errors regarding the geometry, material properties, and construction must be evaluated. This can be accomplished by use of statistical data on "as-build" errors available in the literature. The "as-build" partial safety factor can be calibrated for the proposed method by populating necessary data using random sampling techniques such as the Monte Carlo simulations.
- It was possible to accurately predict the strength of bent cap specimens with two dimensional nonlinear finite element analyses without considering the effects of indirect loading. As mentioned in Section 5.6.4, the significance of indirect loading may be related to scaling effects. Further experimental research in collaboration with nonlinear finite element analyses is recommended. Deep beam specimens with different scales may be constructed and tested under various direct and indirect load conditions.
- This research demonstrated that debonding of the web reinforcement due to bond fatigue may be of some importance for heavily reinforced bent caps. Nonlinear finite element analyses showed that the effect of bond fatigue on the capacity of bent cap specimens becomes significant for members with low a/d ratios and heavy web reinforcement. Further experimental research in collaboration with nonlinear finite element analyses is recommended. Deep beam specimens with varying a/d ratios and initially debonded stirrups may be constructed and tested to simulate a condition of serious bond deterioration.

8.0 REFERENCES

- AASHO, "Standart Specifications for Highway Brisges, 6th Edition," American Association of State Highway Officials, Washington, D.C., 1953.
- AASHTO, "AASHTO LRFD Bridge Design Specificatons, 4th Edition," American Association of State Highway Officials, Washington, D.C., 2007.
- ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary (318R-99)," American Concrete Institute, Farmington Hills, MI, 1999.
- ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (318R-05)," American Concrete Institute, Farmington Hills, MI, 2002.
- Adebar, P., Kuchma, D., Collins, M. P., "Strut-and-Tie Models for the Design of Pile Caps: An Experimental Study," ACI Structural Journal, Vol. 87, No. 1, January-February 1990, pp. 81-92.
- Adebar, P., Zhou, Z., "Bearing Strength of Compressive Struts Confined by Plain Concrete," ACI Structural Journal, Vol. 90. No. 5, September-October, 1993. pp. 534-541.
- Aguilar, G., Matamoros, A. B., Parra-Montesinos G. J., Ramirez, J. A., and Wight, J. K., "Experimental Evaluation of Design Procedure for Shear Strength of Deep Reinforced Concrete Beams," ACI Structural Journal, Vol. 99, No. 4, July-August, 2002, pp. 539-548.
- Aoyama, H., "Design Philosophy for Shear in Earthquake Resistance in Japan," Earthquake Resistance of Reinforced Concrete Structures – A Volume Honoring Hiroyuki Aoyama, University of Tokyo Press, Tokyo, Japan, 1993, pp. 407-418.
- Ashour, A. F., "Tests of Reinforced Concrete Continuous Deep Beams," ACI Structural Journal, Vol. 94, No. 1, January-February, 1997, pp. 3-12.
- Ashour, A. F., "Shear Capacity of Reinforced Concrete Deep Beams," *ASCE Journal of Structural Engineering*, Vol. 126, No. 9, September, 2000, pp. 1045-1052.

- Asin, M., "The Behavior of Reinforced Concrete Continuous Deep Beams," PhD thesis, Department of Civil Engineering, Delft University of Technology, Delft, The netherlands, 1999, 168 pp.
- ASTM, "Standard Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement," *ASTM 615/615M-05A*, West Conshohocken, PA, 2005.
- ASTM, "Standard Practice for Making and Curing Concrete Test Specimens in the Laboratory," *ASTM C192/C192M-05*, West Conshohocken, PA, 2005.
- ASTM, "Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens," *ASTM C39/C39M-05*, West Conshohocken, PA, 2005.
- ASTM, "Standard Practice for Capping Cylindrical Concrete Specimens," *ASTM* C617-98, West Conshohocken, PA, 2003.
- ASTM, "Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens," *ASTM C496/C496M-04e1*, West Conshohocken, PA, 2004.
- Averbuch, D., and de Buhan, P., "Shear Design of Reinforced Concrete Deep Beams: A Numerical Approach," ASCE Journal of Structural Engineering, Vol. 125, No. 3, March, 1999, pp. 309-318.
- Bazant, Z. P., "Instability, Ductility and Size Effect in Strain-Softening Concrete," ASCE Journal of Engineering Mechanics Division, V. 102, No. EM2, April, 1976, pp. 331-344.
- Bazant, Z. P., and Cedolin, L., "Fracture Mechanics of Reinforced Concrete," ASCE Journal of Engineering Mechanics, V. 106, No. 6, December, 1980, pp. 1287-1306.
- Bedard, C., and Kotsovos, M. D., "Application of NLFEA to Concrete Structures," ASCE Journal of Structural Mechanics, V. 111, No., 12, December, 1985, pp. 2691-2707.
- Bhatt, P., Abdel Kader, M. "Prediction of Shear Strength of Reinforced Concrete Beams by Non-linear Finite Element Analysis", *Computers and Structures*, V. 68, No. 1/3, July-August, 1998, pp. 139-155.
- Bentz, E.C., "Sectional Analysis of Reinforced Concrete Members," PhD thesis, Department of Civil Engineering, University of Toronto, Toronto, Ontario, Canada, 2000, 198 pp.

- Brown, M. D., and Bayrak, O., "Investigation of Deep Beams with Various Load Configurations", ACI Structural Journal, Vol. 104. No.5, September, 2006, pp. 611-620.
- CEB-FIP, "Model Code 1990," Comité Euro-International du Béton, Lausanne, Switzerland, 1993.
- CEN, "Eurocode 2: Design of Concrete Structures Part 1-1: General Rules and Rules for Buildings (BS EN 1992-1-1:2004)," Comité Européen de Normalisation, London, United Kingdom, 2004.
- Cervenka, V., and Grestle, K.H, "Inelastic Analysis of Reinforced Concrete Panels, Part II: Experimental Verification and Application," *IABSE Publications*, V. 32-II, 1972, pp. 25-39.
- Chang, T. Y., Taniguchi, H., and Chen, W. F., "Nonlinear Finite Element Analysis of Reinforced Concrete Panels," ASCE Journal of Structural Engineering, V. 113, No. 1, January, 1987, pp. 122-140.
- Channakeshava, C., and Iyengar, K. T. S. R., "Elasto-Plastic Cracking Analysis of Reinforced Concrete," ASCE Journal of Structural Engineering, V. 114, No. 11, November, 1988, pp. 2421-2438.
- Clark, A.P., "Diagonal Tension in Reinforced Concrete Beams," *ACI Journal*, Vol. 48, No. 10, October, 1951, pp. 145-156.
- Collins, M. P., and Mitchell, D., "A Rational Approach to Shear Design The 1984 Canadian Code Provisions," *ACI Journal*, Vol. 83, No. 6, November, 1986, pp. 925-933.
- Colville, J., and Abbasi, J., "Plane Stress Reinforced Concrete Finite Elements," ASCE Journal of Structural Division, V. 100, No. ST5, May, 1974, pp. 1067-1083.
- Cook, W. D., and Mitchell, D., "Studies of Disturbed Regions near Discontinuities in Reinforced Concrete Members," ACI Structural Journal, Vol. 85, No. 2, March, 1988, pp. 206-216.
- CSA International, "Canadian Highway Bridge Design Code (CAN/CSA-S6-00)," Canadian Standards Association, Toronto, Ontario, 2000.

- Darwin, D., and Pecknold, D. A. W., "Analysis of RC Shear Panels Under Cyclic Loading," ASCE Journal of Structural Division, V.102, No. ST2, February, 1976, pp. 355-369.
- Fereig, S. M., and Smith, K.N., "Indirect Loading on Beams with Short Shear Spans," *ACI Journal*, Vol. 74, No. 5, May, 1977, pp. 220-222.
- FIP-Commission 3 "Practical Design", "FIP Recommendations: Practical Design of Structural Concrete," London, United Kingdom, 1999.
- Foster, S. J., and Gilbert, R. I., "Experimental Studies on High-Strength Concrete Deep Beams," ACI Structural Journal, Vol. 95, No. 4, July-August, 1998, pp. 382-390.
- Grestle, W. H., "The Fracture Mechanics of Bond in Reinforced Concrete," MS Thesis, Department of Structural Engineering, Cornell University, Ithaca, NY, 1982.
- Goh, A. T. C., "Prediction of Ultimate Shear Strength of Deep Beams Using Neural Networks," ACI Structural Journal, Vol. 92, No. 1, January-February, 1995, pp. 28-32.
- Gupta, A. K., and Maestrini, S. R., "Post-Cracking Behavior of Membrane Reinforced Concrete Elements Including Tension Stiffening," ASCE Journal of Structural Engineering, V. 115, No. 4, April, 1989, pp. 957-976.
- Gustafsson, P. J., and Hillerborg, A., 1984, "Improvements in Concrete Design Achieved through the Application of Fracture Mechanics," *Application of Fracture Mechanics to Cementitious Materials*, NATO Advanced Research Workshop, Northwestern University, Evanston, IL, 1984.
- Higgins, C., Yim, S.C., Miller, T. H., Robelo, M. J., and Potisuk, T., "Remaining Life of Reinforced Concrete Beams with Diagonal-Tension Cracks." *Rep. No. FHWA-OR-RD-04-12*, Federal Highway Administration, Washington D.C, 2004.
- Higgins, C., Miller, T.H., Rosowsky, D. V., Yim, S. C., Potisuk, T., Daniels, T.K., Nicholas, B.S., Robelo, M.J., Lee, A.-Y., and Forrest, R. W., "Assessment Methodology for Diagonally Cracked Reinforced Concrete Deck Girders.", *Rep. No. FHWA-OR-RD-05-04*, Federal Highway Administration, Washington, D.C., 2004.

- Hillerborg, A., Modeer, M., and Petrsson, P. E., "Analysis of Crack Formation and Crack Growth in Concrete by Means of Fracture Mechanics and Finite Elements," *Cement and Concrete Research*, V. 6, 773-782, 1976.
- Hognestad, E., "A Study of Combined Bending and Axial Load in Reinforced Concrete Members," *Bulletin No. 399*, University of Illinois Engineering Experiment Station, Urbana, November, 1951, 128 pp.
- Hwang, S. J., Lu, W. Y., and Lee, H. J., "Shear Strength Prediction for Deep Beams," ACI Structural Journal, Vol. 97, No. 3, May-June, 2000, pp. 367-376.
- Ingraffea, A. R., and Saouma, V., "Numerical Modeling of Discrete Crack Propagarion in Reinforced and Plain Concrete," *Fracture Mechanics of Concrete: Structural Application and Numerical Calculation*, edited by G. Sih and A. DiTomasso, Martinus Nijhoff, Hingham, MA, pp. 171-252, 1984.
- Jendele, L., Cervenka, J., Saouma, V., Pukl, R., "On the Choice between Discrete or Smeared Approach in Practical Structural FE Analyses of Concrete Structures," *Proceedings of the 4th International Conference on Analysis of Discontinuous Deformations (ICADD-4)*, Glasgow, Scotland, 2001.
- Kupfer, H., "Erweiterung der Mörsch'schen Fachwerkanalogie bei der Berechnung der Schubfestigkeit von Stahlbetonbalken" (On the Limitations of Applicability of the Truss Analogy for the Shear Design of Reinforced Concrete Beams), Festschrift F. Campus "Amici et Alumni," Université de Liége, 1964.
- Koester, C. C., and Higgins, C., "Testing and Evaluation of Flexural Straight-Bar Anchorages in Columns", submitted to *ACI Structural Journal*, 2008.
- Kong, F., Robins, P. J., and Cole, D. F., "Web Reinforcement Effects on Deep Beams," ACI Journal, Vol. 67, No. 12, December, 1970, pp. 1010-1018.
- Kong, F., Teng, S., Singh, A., and Tan, K. H., "Effect of Embedment Length of Tension Reinforcement on the Behavior of Lightweight Concrete Deep Beams," ACI Structural Journal, Vol. 93, No. 1, January-February, 1996, pp. 21-29.
- Lampert, P., and Thürlimann, B., "Ultimate Strength and Design of Reinforced Concrete Beams in Torsion and Bending," *Publication No. 31-I*, IABSE, Zurich, 107-131.

- MacGregor, J. G., "Challenges and Changes in the Design of Concrete Structures," *Concrete International*, V. 6, No. 2, Feb. 1984, pp. 48-52.
- Maekawa, K., Pimanmas, A., and Okamura, H., "Nonlinear Mechanics of Reinforced Concrete," Spon Press, New York, NY, 2003, pp. 721.
- Malvar, L. J., "Bond of Reinforcement under Controlled Confinement," ACI Materials Journal, Vol. 89, No. 6, November, 1992, pp. 593-601.
- Manuel, R. F., Slight, B. W., and Suter. G. T., "Deep Beam Behavior Affected by Length and Shear Span Variations," *ACI Journal*, Vol. 68, No. 12, December, 1971, pp. 954-958.
- Matamoros, A. B., and Wong, K. H., "Design of Simply Supported Deep Beams Using Strut-and-Tie Models," *ACI Structural Journal*, Vol. 100, No. 6, November-December, 2003, pp. 704-712.
- Mau, S. T., and Hsu, C. T., "Shear Strength Prediction for Deep Beams with Web Reinforcement," ACI Structural Journal, Vol. 84, No. 6, November, 1987, pp. 513-523.
- Miner, M. A., "Cumulative Damage in Fatigue," ASME Trans. Journal of Applied Mechanics, Vol. 12, 1945, pp. A159-A164.
- Mörsch, E., "Concrete Steel Construction", McGraw-Hill, New York, 1909, pp.368.
- Mörsch, E., "Der Eisenbetonbau" (Reinforced Concrete Construction), 4. Aufl., K. Wittwer, Stuttgart, 1912.
- Mörsch, E., "Der Eisenbetonbau Seine Theorie und Anwendung" (Reinforced Concrete Construction – Theory and Application), 5th Edition, Vol.1, Part 2, K. Wittwer, Stuttgart, 1922.
- Nam, C.-H., and Salmon, C. G., "Finite Element Analysis of Concrete Beams," ASCE Journal of Structural Division, V. 100, No. ST12, December, 1974, pp. 2419-2432.
- Ngo, D., and Scordelis, A. C., "Finite Element Analysis of Concrete Beams," ACI Journal, V. 64, No. 3, March, 1967, pp. 152-163.

- Nilson, A. H., "Finite Element Analysis of Reinforced Concrete," PhD Thesis, Department of Civil Engineering, University of California, Berkeley, CA, 1967.
- Nilson, A. H., "Nonlinear Analysis of Reinforced Concrete by Finite Element Method," *ACI Journal*, V. 65, No. 9, September, 1968, pp. 757-766.
- Potisuk, T., "Analysis of Conventionally Reinforced Concrete Bridges for Shear," PhD thesis, Department of Civil, Construction and Environmental Engineering, Oregon State University, Corvallis, Oregon, 2004, pp. 153.
- Quintero-Febres, C. G., Parra-Montesinos, G., and Wight, J. K., "Strength of Struts in Deep Concrete Members Designed Using Strut-and-Tie Method," ACI Structural Journal, Vol. 103, No.4, July-August, 2006, pp. 577-586.
- Ramakrishnan, V., and Ananthanarayana Y., "Ultimate Strength of Deep Beams in Shear," *ACI Journal*, Vol. 65, No. 2, February, 1968, pp. 87-98.
- Rashid, Y. R., "Ultimate Strength Analysis of Prestressed Concrete Pressure Vessels," *Nuclear Engineering and Design*, V. 7, 1968, pp. 334-344.
- Ritter, W., "Die Bauweise Hennebique" (Construction According to Hennebique), Schweizerische Bauzeitung, V. XXXIII, No.7, Zurich, Switzerland, 1899.
- Rogowsky, D. M., MacGregor, J.G., and Ong, S. Y., "Tests of Reinforced Concrete Deep Beams," *ACI Journal*, Vol. 83, No. 4, July, 1986, pp. 614-623.
- Rogowsky, D. M., MacGregor, J.G., "Design of Reinforced Concrete Deep Beams," *Concrete International : Design and Construction*, Vol. 8, No. 8, August, 1986, pp. 49-58.
- Russo, G., Venir, R., and Pauletta, M., "Reinforced Concrete Deep Beams Shear Strength Model and Design Formula," ACI Structural Journal, Vol. 102, No. 3, May-June, 2005, pp. 429-437.
- Saatci, S., "Behavior and Modeling of Reinforced Concrete Structures Subjected to Impact Loads," PhD thesis, Department of Civil Engineering, University of Toronto, Toronto, Ontario, Canada, 2007, 288 pp.
- Salah El-Din, A. S., and El-Adawy Nassef, M. M., "A Modified Approach for Estimating the Cracking Moment of Reinforced Concrete Beams," ACI Journal, V. 72, No. 7, July, 1975, pp. 356-360.

- Sanad, A., and Saka, M. P., "Prediction of Ultimate Shear Strength of Reinforced-Concrete Deep Beams Using Neural Networks," ASCE Journal of Structural Engineering, Vol. 127, No. 7, July, 2001, pp. 818-828.
- Schnobrich, W. C., "Behavior of Reinforced Concrete Structures Predicted by the Finite Element Method," *Computers and Structures*, V. 7, 1977, pp.265-376.
- Shin, S. W., Lee, K. S., Moon, J., and Ghosh S. K., "Shear Strength of Reinforced High-Strength Concrete Beams with Shear Span-to-Depth Ratios between 1.5 and 2.5," ACI Structural Journal, Vol. 96, No. 4, July-August, 1993, pp. 549-557.
- Siao, W. B., "Strut-and-Tie Model for Shear Behavior in Deep Beams and Pile Caps Failing in Diagonal Splitting," ACI Structural Journal, Vol. 90, No. 4, July-August, 1993, pp. 356-363.
- Smith, K. N., and Vantsiotis, A. S., "Shear Strength of Deep Beams," *ACI Journal*, Vol. 79, No. 3, May, 1982, pp. 201-213.
- Tan, K. H., Kong, F., Teng, S., and Guan, L., "High-Strength Concrete Deep Beams with Effective Span and Shear Span Variations," ACI Structural Journal, Vol. 92, No. 4, July-August, 1995, pp. 1-11.
- Tan, K. H., Teng, S., Kong, F., and Lu, H. Y., "Main Tension Steel in High Strength Concrete Deep and Short Beams," ACI Structural Journal, Vol. 94, No. 6, November-December, 1997, pp. 752-768.
- Tan, K. H., Kong, F., Teng, S., and Weng, L., "Effect of Web Reinforcement on High-Strength Concrete Deep Beams," ACI Structural Journal, Vol. 94, No. 5, September-October, 1997, pp. 572-582.
- Tan, K. H., and Lu, H. Y., "Shear Behavior of Large Reinforced Concrete Deep Beams and Code Comparisons," ACI Structural Journal, Vol. 96, No.5, September-October, 1999, pp. 836-846.
- Tan, K. H., and Cheng, G. H., "Size Effect on Shear Strength of Deep Beams: Investigating with Strut-and-Tie Model," ASCE Journal of Structural Engineering, Vol. 132, No. 5, May, 2006, pp. 673-685.
- Tang, C. Y., "Interactive Mechanical Model for Shear Strength of Deep Beams," ASCE Journal of Structural Engineering, Vol. 130, No. 10, October, 2004, pp. 1534-1544.

- Tang, C. Y., and Tan, K. H., "Interactive Mechanical Model for Shear Strength of Deep Beams," ASCE Journal of Structural Engineering, Vol. 130, No. 10, October, 2004, pp. 1534-1544.
- Thürlimann, B., Marti, P., Pralong, J., Ritz, P., and Zimmerli, B., "Anwendung der Plastizitätstheorie auf Sthalbeton" (Application of the Plasticity Theory to Reinforced Concrete), Institut für Baustatik and Konstruktion, ETH, Zurich, 1983.
- Untrauer, R. E., and Henry, R. L., "Influence of Normal Pressure on Bond Strength," *ACI Journal*, Vol. 62, No. 5, May, 1965, pp. 577-586.
- Valliappan,S., and Doolan, T., "Nonlinear Stress Analysis of Reinforced Concrete," ASCE Journal of Structural Division, V. 98, No. ST4, April, 1972, pp. 885-898.
- Vecchio, F. J., and Collins, M. P., "The modified compression field theory for reinforced concrete elements subjected to shear," *ACI Journal*, V. 83, No. 2, March, 1986, pp. 219-231.
- Vecchio, F. J., "Non-linear Finite Element Analysis of Reinforced Concrete Membranes," ACI Structural Journal, V. 86, No. 1, January, 1989, pp. 26-35.
- Vecchio, F. J., "Disturbed Stress Field Model for Reinforced Concrete: Formulation," ASCE Journal of Structural Engineering, Vol. 126, No. 9, pp. 1070-1077.
- Vecchio, F. J., "Non-linear Finite Element Analysis of Reinforced Concrete: At the Crossroads?," *Structural Concrete*, V.2, No.4, December, 2001, pp. 201-212.
- Vecchio, F. J., and Wong, P., "VecTor2 and Formworks Manual," Department of Civil Engineering, University of Toronto, Toronto, Ontario, Canada, 2002.
- Wang, W., Jiang D., and Hsu, C. T., "Shear Strength of Reinforced Concrete Deep Beams," ASCE Journal of Structural Engineering, Vol. 119, No. 8, August, 1993, pp. 2294-2312.
- Yun, Y. M., "Nonlinear Strut-and-Tie Model Approach for Structural Concrete," ACI Structural Journal, Vol. 97, No. 4, July-August, 2000, pp. 581-590.

- Zararis, P.D., "Failure Mechanisms in RC Plates Carrying in-plane Forces", *ASCE Journal of Structural Engineering*, Vol. 114, No. 3, March, 1988, pp. 553-574.
- Zararis, P.D., "Shear Compression Failure in Reinforced Concrete Deep Beams," ASCE Journal of Structural Engineering, Vol. 129, No. 4, April, 2003, pp. 544-553.
- Zararis, P.D., "RE: Deep Beam Experiments in Oregon State University," E-mail to the author, 5 November, 2007.
- Zielinski, Z. A., and Rigotti, M., "Tests on Shear Capacity of Reinforced Concrete," ASCE Journal of Structural Engineering, Vol. 121, No. 11, November, 1995, 1660-1666.
- Zsutty, T., "Shear Strength Prediction for Separate Categories of Simple Beam Tests," ACI Journal, Vol. 68, No. 2, February, 1971, pp. 183-143.

APPENDIX A: CONSTITUTIVE MODELS USED IN NLFEA

In this section, brief information is given about the material and behavioral models employed in the non-linear finite element analyses portion of this study.

CONCRETE COMPRESSION PRE-PEAK RESPONSE

For the pre-peak response of concrete, the Hognestad parabola (*Hognestad, 1951*) was used which is suitable for normal strength concrete. The stress-strain curve for the Hognestad parabola, shown in Figure A.1, is described as:

$$\mathbf{f}_{ci} = -\mathbf{f}_{p} \left[2 \left(\frac{\varepsilon_{ci}}{\varepsilon_{p}} \right) - \left(\frac{\varepsilon_{ci}}{\varepsilon_{p}} \right)^{2} \right] < 0 \quad \text{for} \quad \varepsilon_{ci} < 0$$
 (A1.1)





Figure A.1: Hognestad parabolic concrete compression response (Hognestad, 1951)

CONCRETE COMPRESSION POST-PEAK RESPONSE

The Modified Kent – Park model (*Park et al., 1982*), which was adapted for VecTor2 by Vecchio, was chosen for the post-peak compression response of concrete. This model accounts for the enhancement of concrete strength and ductility due to confinement provided by the transverse reinforcement. The stress-strain curve for the Modified Park – Kent model, which is shown in Figure A.2, is described as:

$$f_{ci} = -\left[f_p + Z_m f_p(\varepsilon_{ci} - \varepsilon_p)\right] \text{ or } -0.2f_p \text{ for } \varepsilon_{ci} < \varepsilon_p < 0 \quad (A1.2)$$

Where

$$Z_{\rm m} = \frac{0.5}{\frac{3+0.29|f_{\rm c}'|}{145|f_{\rm c}'|-1000} \times \frac{\varepsilon_{\rm o}}{-0.002} + \left(\frac{|f_{\rm lat}|}{170}\right)^{0.9} + \varepsilon_{\rm p}}$$
(A1.3)

Where $_{o}$ is the concrete compressive strain corresponding to f_{c} (MPa), and f_{lat} (MPa) is the summation of principal stresses, acting transversely to the direction under consideration:



Figure A.2: Modified Park – Kent post-peak concrete compression response (Park et al., 1982)

CONCRETE COMPRESSION SOFTENING

Compression softening in cracked concrete is defined as the reduction of compressive strength and stiffness, relative to the uniaxial compressive strength, due to coexisting transverse cracking and tensile straining. The Vecchio 1992-A model, shown in Figure A.3, was used to simulate the effect of concrete compression softening in this analysis. This strength-and-strain softened model, where both the uniaxial compressive strength, and the corresponding strain is reduced by a factor, C_d, in order to determine the peak compressive strength, and corresponding strain, is based on a statistical analysis of panel and shell element experiments which were conducted at the University of Toronto (*Vecchio and Collins, 1992*). The formulation of the model is given as:

$$\beta_{d} = \frac{1}{1 + C_{s} \cdot C_{d}} \le 1$$
(A1.5)

$$C_{d} = \begin{cases} 0 & \text{if } r < 0.28 \\ 0.35(r - 0.28)^{0.80} & \text{if } r > 0.28 \end{cases}$$
(A1.6)

$$r = \frac{-\varepsilon_{c1}}{\varepsilon_{c2}} \le 400$$
 (A1.7)

$$C_{s} = \begin{cases} 0 & \text{if shear slip is not considered} \\ 0.55 & \text{if shear slip is considered} \end{cases}$$
(A1.8)

$$\mathbf{f}_{\mathrm{p}} = \beta_{\mathrm{d}} \mathbf{f}_{\mathrm{c}}^{\prime} \tag{A1.9}$$

$$\varepsilon_{\rm p} = \beta_{\rm d} \varepsilon_{\rm o} \tag{A1.10}$$

Where C_d is the strain softening factor, C_s is the shear slip factor, $_{c1}$ is the principal tensile strain, and $_{c2}$ is the principal compression strain.



Figure A.3: Vecchio 1992A concrete compression softening response (Vecchio, 1992)

CONCRETE TENSION STIFFENING

Due to the brittle nature of concrete response under tension, the behavior should be differentiated into two parts, as the uncracked response and the cracked response. Prior to cracking, concrete tension response can be assumed as linear elastic:

$$f_{c1} = E_c \varepsilon_{c1}$$
 for $0 < \varepsilon_{c1} < \varepsilon_{cr}$ (A1.11)

$$\varepsilon_{\rm cr} = \frac{f_{\rm cr}}{E_{\rm c}}$$
(A1.12)

Where $_{cr}$ is the cracking strain, E_c is the initial tangent stiffness of concrete, $_{c1}$ is the principal tensile strain, and f_{cr} is the cracking stress of concrete.

After cracking of reinforced concrete, a short length of reinforcement steel debonds from concrete at the crack locations, and the concrete tensile stresses diminish to zero at the free surface of the cracks, where the applied loads must be carried only by the reinforcement steel locally. However, the concrete can still carry tension stresses between the cracks through the action of bond. Although tension stresses carried by still intact portions of concrete cannot exceed the cracking stress, f_{cr},

they act over a relatively large cross-section, resulting in a greater stiffness that that of a bare reinforcement bar. This phenomenon is referred as "tension stiffening".

The tension stiffening model proposed by Sato and Vecchio (2003), which accounts for the bond characteristics of the reinforcement, was used in this model. The average concrete tensile stress-strain response curve, based on a statistical analysis of experimental data given in Figure A.4, is determined as:

$$f_{c1}^{a} = \frac{f_{cr}}{1 + \sqrt{\frac{2.2 \cdot \varepsilon_{c1}}{\sum_{i=1}^{m} \frac{\rho_{i}}{d_{bi}} |\cos \theta_{i}|}}} \quad \text{for} \quad 0 < \varepsilon_{cr} < \varepsilon_{c1}$$
(A1.13)

Where "i" indicates a reinforcement component (i = 1 to m), where $_i$ is the reinforcement ratio, d_{bi} is the reinforcement bar diameter, and $_i$ is the inclination of reinforcement component.



Figure A.4: Bentz tension stiffening response (Bentz, 2000)

CONCRETE TENSION SOFTENING

Since concrete is not a perfectly brittle material, post-cracking tensile stresses may exist in plane concrete. After cracking, the tensile stresses gradually diminish to zero under increasing tensile strains, rather than abruptly disappearing. This phenomenon is referred as "tension softening". VecTor2 compares the average concrete tensile stress due to tension softening, f_{c1}^{a} , and the average concrete tensile stress due to tension stiffening, f_{c1}^{b} , and assumes the larger of the two tensile stresses to be the average post-cracking tensile stress.

The linear tension softening base curve, shown in Figure A.5, was used in this analysis, which is determined from:

$$\mathbf{f}_{cl}^{b} = \mathbf{f}_{cr} \left[1 - \frac{\left(\boldsymbol{\varepsilon}_{cl} - \boldsymbol{\varepsilon}_{cr} \right)}{\left(\boldsymbol{\varepsilon}_{ch} - \boldsymbol{\varepsilon}_{cr} \right)} \right] \ge 0 \quad \text{for} \quad \boldsymbol{\varepsilon}_{cr} < \boldsymbol{\varepsilon}_{cl}$$
(A1.14)

$$\varepsilon_{\rm ch} = \frac{2G_{\rm f}}{L_{\rm r} \cdot f_{\rm cr}} \quad \text{for} \quad 1.1\varepsilon_{\rm cr} < \varepsilon_{\rm ch} < 10\varepsilon_{\rm cr}$$
(A1.15)

Where $_{ch}$ is the characteristic strain, G_f is the fracture energy with an assigned value of 75 N/m in VecTor2, and L_r is the distance over which the crack is assumed to be uniformly distributed, and assigned a value of half the crack spacing.



Figure A.5: VecTor2 linear tension softening response (Wong and Vecchio, 2002)

CONCRETE TENSION SPLITTING

Tension splitting is referred as the formation of splitting cracks parallel to the reinforcing bars in tension, due to the prying action of bar deformations. Since horizontal cracks at the flexural reinforcement elevation were not observed in the bent cap experiments, the effect of tension splitting was not considered in the present analysis.

CONCRETE CONFINEMENT STRENGTH

VecTor2 uses a strength enhancement factor, 1, to take into account the enhanced strength and ductility of confined concrete in compression. With the inclusion of the concrete confinement parameter, the peak compressive stress and strain become:

$$f_{p} = \beta_{d}\beta_{l}f_{c}'$$
 (A1.16)

$$\varepsilon_{\rm p} = \beta_{\rm d} \beta_{\rm l} \varepsilon_{\rm o} \tag{A1.17}$$

In this analysis, an adaptation of the relationship proposed by Kupfer *et al.* (1969) was used in order to determine the strength of concrete subjected to biaxial compression, which is given as:

$$\beta_{1} = \left[1 + 0.92 \left(\frac{f_{c2}}{f_{c}'} \right) - 0.76 \left(\frac{f_{c2}}{f_{c}'} \right)^{2} \right] \text{ for } f_{c3} < f_{c2} < 0 \text{ and } f_{c1} = 0 \text{ (A1.18)}$$

Where f_{c1} , f_{c2} and f_{c3} are the compressive stresses in acting in their respective directions.

CONCRETE DILATATION

For concrete in tension, VecTor2 calculates the Poisson's ratio as follows:

$$v_{12} = v_{21} = \begin{cases} v_{o} & \text{for } 0 < \varepsilon_{c1} < \varepsilon_{cr} \\ v_{o} \left(1 - \frac{\varepsilon_{c1}}{2\varepsilon_{cr}} \right) \ge 0 & \text{for } \varepsilon_{cr} < \varepsilon_{c1} \end{cases}$$
(A1.19)

Where V_o is the initial Poisson's ratio.

For concrete in compression, a variable Poisson's ratio based on the study by Kupfer *et al.* (1969), which increases nonlinearly as compressive strain increases, was used in this analysis. The Poisson's ratio in this model is shown in Figure A.6, and determined as:



Figure A.6: Kupfer variable Poisson's ratio model for compression (*Kupfer et al., 1969*)

CONCRETE CRACKING

Concrete cracking strength, f_{cr} , is generally different from the value of tensile concrete strength, f'_t , due to existing stress states, e.g. the cracking strength generally decreases as transversely acting compressive stresses increase. In this analysis, the Mohr-Coulomb criterion is used to determine the concrete cracking strength, which is determined as:
$$\mathbf{f}_{\rm cr} = \mathbf{f}_{\rm cru} \left(\frac{1 + \mathbf{f}_{\rm c3}}{\mathbf{f}_{\rm c}'} \right) \quad \text{for} \quad 0.20 \mathbf{f}_{\rm t}' \le \mathbf{f}_{\rm cr} \le \mathbf{f}_{\rm t}' \tag{A1.21}$$

Where

$$f_{c3} = \begin{cases} -f_c' \left[2 \left(\frac{\varepsilon_{c3}}{\varepsilon_o} \right) - \left(\frac{\varepsilon_{c3}}{\varepsilon_o} \right)^2 \right] & \text{for} \quad \varepsilon_{c3} < \varepsilon_o < 0 \\ -f_c' & \text{for} \quad \varepsilon_o < \varepsilon_{c3} < 0 \\ 0 & \text{for} \quad 0 < \varepsilon_{c3} \end{cases}$$
(A1.22)

$$f_{cru} = f'_{c} \frac{2c \cdot \cos \phi}{2\cos \phi}$$
 (A1.23)

$$c = f'_c \frac{1 - \sin \phi}{2 \cos \phi}$$
(A1.24)

Where c is the cohesion and f is the angle of friction with an assigned value of 37°.

The local shear stress on the crack surface is limited using the crack check equation from the Modified Compression Field Theory by Vecchio and Collins (1986). The maximum local shear stress is based on the work of Walraven (1981), and determined as:

$$v_{ci}^{max} = \frac{\sqrt{f'_c}}{0.3 + 24 w / (a + 26)}$$
 (A1.25)

Where w is the crack spacing (mm), and a is the maximum aggregate size (mm).

CONCRETE SLIP DISTORTIONS

Crack shear slip deformations are taken into account in this analysis, and determined by the Lai-Vecchio model (2002), which is a combination of the Walraven (1981) and Maekawa (1991) slip distortion models. The slip along a crack, s, is computed as

$$\delta_{s} = \delta_{2} \sqrt{\frac{\Psi}{1 - \Psi}} \le 2w$$
(A1.26)

Where

$$\delta_2 = \frac{0.5 v_{ci}^{max} + v_{co}}{1.8 w^{-0.8} + (0.234 w^{-0.707} - 0.20) \cdot f_{cc}}$$
(A1.27)

$$\psi = \frac{v_{ci}}{v_{ci}^{max}}$$
(A1.28)

$$v_{ci}^{max} = \frac{\sqrt{f_c'}}{0.31 + 24 w / (a + 16)}$$
 (A1.29)

$$v_{co} = \frac{f_{cc}}{30} \tag{A1.30}$$

Where f_{cc} (MPa) is the concrete cube strength, taken as 1.2f'_c.

CONCRETE HYSTERETIC RESPONSE

The stress-strain response curves of concrete under loading, unloading, and reloading are generally non-coincident due to plastic offset strains caused by internal damage. In order to simulate the behavior of laboratory specimens which were tested under cyclic loading, the concrete hysteresis model proposed by Vecchio (1999) was used. This model yields a nonlinear response of loading and unloading with plastic offsets.

The resulting concrete stress f_c when unloading in compression to a strain of E_c is determined by:

$$f_{c} = f_{cm} + E_{c}(\varepsilon_{c} - \varepsilon_{cm}) + \frac{E_{c}(\varepsilon_{c} - \varepsilon_{cm})^{N_{c}}}{N_{c}(\varepsilon_{c}^{p} - \varepsilon_{cm})^{N_{c}-1}} \quad \text{for} \quad 1 \le N_{c} \le 20 \quad \textbf{(A1.31)}$$

Where $_{c}^{p}$ is the current plastic offset strain, $_{cm}$ is the maximum previously attained compressive strain, f_{cm} is the corresponding stress, N_{c} is the Ramsberg-Osgood power term representing the deviation from linear elasticity, calculated as:

$$N_{c} = \frac{E_{c} \cdot \left(\varepsilon_{c}^{p} - \varepsilon_{cm}\right)}{f_{cm} + E_{c} \left(\varepsilon_{c}^{p} - \varepsilon_{cm}\right)}$$
(A1.32)

When unloading in tension to a strain of E_c , the resulting concrete stress is determined by:

$$\mathbf{f}_{c} = \mathbf{f}_{tm} - \mathbf{E}_{c}(\varepsilon_{tm} - \varepsilon_{c}) + \frac{\mathbf{E}_{c}(\varepsilon_{tm} - \varepsilon_{c})^{N_{t}}}{N_{c}(\varepsilon_{tm} - \varepsilon_{c}^{p})^{N_{t}-1}} \quad \text{for} \quad 1 \le N_{t} \le 20 \quad \textbf{(A1.33)}$$

Where E_c^{p} is the current plastic offset strain, t_{tm} is the maximum previously attained tensile strain, f_{tm} is the corresponding stress, N_t is the Ramsberg-Osgood power term representing the deviation from linear elasticity, calculated as:

$$N_{t} = \frac{E_{c} \cdot \left(\varepsilon_{tm} - \varepsilon_{c}^{p}\right)}{E_{c} \left(\varepsilon_{tm} - \varepsilon_{c}^{p}\right) - f_{tm}}$$
(A1.34)

STEEL REINFORCEMENT RESPONSE

VecTor2 uses a trilinear reinforcement stress-strain response curve, consisting of an initial linear-elastic response, a yield plateau, and a linear strain-hardening phase until rupture, as shown in Figure A.7. The trilinear stress strain curve is derived as:

$$\mathbf{f}_{s} = \begin{cases} \mathbf{E}_{s} \boldsymbol{\varepsilon}_{s} & \text{for } |\boldsymbol{\varepsilon}_{s}| \leq \boldsymbol{\varepsilon}_{y} \\ \mathbf{f}_{y} & \text{for } \boldsymbol{\varepsilon}_{y} < |\boldsymbol{\varepsilon}_{s}| \leq \boldsymbol{\varepsilon}_{sh} \\ \mathbf{f}_{y} + \mathbf{E}_{sh} \left(\boldsymbol{\varepsilon}_{s} - \boldsymbol{\varepsilon}_{sh}\right) & \text{for } \boldsymbol{\varepsilon}_{sh} < |\boldsymbol{\varepsilon}_{s}| \leq \boldsymbol{\varepsilon}_{u} \\ 0 & \text{for } \boldsymbol{\varepsilon}_{u} < |\boldsymbol{\varepsilon}_{s}| \end{cases}$$
(A1.35)

$$\varepsilon_{u} = \varepsilon_{sh} + \frac{\left(f_{u} - f_{y}\right)}{E_{sh}}$$
(A1.36)

where $_{s}$ is the reinforcement strain, $_{y}$ is the yield strain, $_{sh}$ is the strain at the onset of strain hardening, $_{u}$ is the ultimate strain, E_{s} is the elastic modulus, E_{sh} is the strain hardening modulus, f_{y} is the yield strength, and f_{u} is the ultimate strength.



Figure A.7: Ductile steel reinforcement stress-strain response

STEEL REINFORCEMENT DOWEL ACTION

Dowel action is a phenomenon which occurs when a steel bar in a crack is subjected to a shear displacement, and defined as the shear resistance offered by reinforcing bars crossing a crack as the crack slips transversely to the axis of the reinforcement. Especially for beams with small amounts of transverse reinforcement, dowel action may contribute significantly to the shear strength. The Tassios Model for dowel action is used in this analysis. The dowel force is computed as:

$$V_{d} = E_{s}I_{z}\lambda^{3}\delta_{s} \le V_{du}$$
 (A1.37)

$$I_z = \frac{\pi d_b^4}{64}$$
 (A1.38)

$$\lambda = \sqrt[4]{\frac{k_c d_b}{4E_s I_z}}$$
(A1.39)

$$k_{c} = \frac{127c\sqrt{f_{c}'}}{d_{b}^{2/3}}$$
(A1.40)

$$c = 0.8$$
 (A1.41)

$$V_{du} = 1.27 d_b^2 \sqrt{f'_c f_y}$$
 (A1.42)

Where I_z is the area moment of inertia of the reinforcement. The parameter V compares the stiffness of concrete to that of the reinforcing bar. The parameter k_c is the stiffness of notional concrete foundation, where c is an experimentally based coefficient to reflect the bar spacing. The ultimate dowel force V_{du} corresponds to plastic hinging of the reinforcement and crushing of the surrounding concrete in multiaxial compression.

BOND MODELS

Perfect bond between concrete and the reinforcement is assumed in this analysis, except for the models with debonded stirrups. In order to model the unbounded stirrup lengths, "Unbonded Bars or Tendons" option was selected.

APPENDIX B: EXPERIMENTAL DATA

D6.A4.G60#5.S





D6.A4.G60#5.S South Shear Span Load - Diagonal Displacement





D6.A4.G60#5.S North Shear Span Load - Diagonal Displacement





D6.A4.G60#5.S North Shear Span Load - Diagonal Displacement





¹⁵⁰⁰ 2000 με [in/in x 10⁻⁶]

2500 3000 3500 4000

-500 0







B-7



D6.A4.G40#4.S





D6.A4.G40#4.S South Shear Span Load - Diagonal Displacement





D6.A4.G40#4.S North Shear Span Load - Diagonal Displacement

















B-14











D6.A2.G60#5.S





D6.A2.G60#5.S South Shear Span Load - Diagonal Displacement





D6.A2.G60#5.S South Shear Span Load - Diagonal Displacement



-500

 με [in/in x 10⁻⁶]

D6.A2.G60#5.S South Shear Span Load - Diagonal Displacement

4000







B-22



1500 2000 2500 με [in/in 10⁻⁶]

3000

3500 4000

1000

0 500

0 -500

B-23

0 5000

4500



D6.A2.G40#4.S





D6.A2.G40#4.S South Shear Span Load - Diagonal Displacement





D6.A2.G40#4.S North Shear Span Load - Diagonal Displacement





B-29














D4.A2.G40#4.S





D4.A2.G40#4.S South Shear Span Load - Diagonal Displacement











2000 2500 µɛ [in/in 10⁻⁶] -500

0 500 1000 1500



D6.A2.G40#4.F









D6.A2.G40#4.F North Shear Span Load - Diagonal Displacement Precrack











D6.A2.G40#4.F Flexural Steel Layer 2b Total Load - Strain

































0.25 0.23









D6.A2.G40#4.F CClip11H Crack Width Range (Horizontal Crack Displacement @ Beam - Column Interface)

0.01



D6.A2.G40#4.F Average Crack Width Range on the Shear Span



).25























D6.A2.G40#4.F South Stirrup 1 Mean Stress - Crack Width Range









0.005 0.0055 Crack Width Range [in]

10

9.75 0.0035

0.004

0.0045

D6.A2.G40#4.F South Stirrup 3 Mean Stress - Crack Width Range

69

67 0.007

0.0065

0.006





















B-66






























Shear Load - Anchorage Zone Embedded Steel Strains 300K Cycles Check Test













Shear Load - Anchorage Zone Embedded Steel Strains 700K Cycles Check Test









Shear Load - Anchorage Zone Embedded Steel Strains 1000K Cycles Check Test



























Summary of Concrete Principal Stresses from Check Tests















400000 600000 Cycles [-]

0.00

0

200000

.13

---0.00 1000000

800000









400000 600000 Cycles [-]





D6.A2.G40#4.F South Shear Span Load - Diagonal Displacement Failure







D6.A2.G40#4.F North Shear Span Load - Diagonal Displacement Failure















B-99





με [in/in x 10⁻⁶]
















2348 µE

1000 1500 2000 2500 με [in/in 10⁻⁶]

-500

B-105

3000 3500 4000 4500 5000







