Research Report
KTC-05-27/SPR 229-01-1F

KENTUCKY TRANSPORTATION CENTER

RESILIENT MODULUS OF COMPACTED CRUSHED STONE AGGREGATE BASES

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College of Engineering

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In all that we do.

Research Report KTC-05-27/SPR-229-01-1F

Resilient Modulus of Compacted Crushed Stone Aggregate Bases

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in cooperation with the
Kentucky Transportation Cabinet
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16. Abstract
In recent years, the American Association of State Highway Transportation Officials (AASHTO) has recommended the use of resilient modulus for characterizing highway materials for pavement design. This recommendation evolved as result of a trend in pavement design of using mechanistic models. Although much progress has been made in recent years in developing mathematical, mechanistic pavement design models, results obtained from those models are only as good as the material parameters used in the models. Resilient modulus of aggregate bases is an important parameter in the mechanistic models. The main goal of this study was to establish a simple and efficient means of predicting the resilient modulus of different types of Kentucky crushed stone aggregate bases. To accomplish this purpose, resilient modulus tests were performed on several different types of aggregate bases commonly used in pavements in Kentucky. Specimens were remolded to simulate compaction conditions typically encountered in the field. Tests were performed on wet and dry specimens. The compacted specimens were 6 inches in diameter and 12 inches in height Crushed limestone base materials included Dense Graded Aggregate (DGA), and Crushed Stone Base (CSB). Number 57s, crushed river gravel, recycled concrete, and asphalt drainage blanket samples were submitted for testing by engineers of the Kentucky Transportation Cabinet.

A new mathematical resilient modulus model, developed in a previous study by researchers of the University of Kentucky Transportation Center (UKTC), was used to relate resilient modulus to any selected, or calculated, principal stresses in the aggregate base. This model improves the means of obtaining best data "fits" between resilient modulus and stresses. Furthermore, the resilient modulus can be predicted, using the UKTC resilient modulus model, when the stress condition and type of Kentucky base aggregate are known. Multiple regression analysis is used to obtain model coefficients, k_1, k_2, and k_3, of the relationships between resilient modulus and confining and deviator stresses used in the testing procedure. Also, multiple regression analysis was performed using other models developed by the National Cooperative Highway Research Program (NCHRP Project 1-37A, 2001) and Uzan (1985) to obtain the model coefficient, k_1, k_2, and k_3.

The resilient modulus data and the UKTC model, as well as models developed by NCHRP and Uzan, are readily available to design personnel of the Kentucky Transportation Cabinet. Computer software was developed in a client/server and Windows environment. This program is embedded in the Kentucky Geotechnical Database, which resides on a Cabinet server in Frankfort, Kentucky.

17. Key Words
Highways, Resilient Modulus, Soils, Model, Design, Subgrade

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EXECUTIVE SUMMARY

This study developed as a result of a process review conducted in the early nineties by the Federal Highway Administration (FHWA) of the Kentucky Transportation Cabinet's procedure for selecting parameters for pavement design. As a result of this study, FHWA recommended "that an in-depth assessment be made of the most appropriate strength test to accommodate Kentucky's future needs and that resilient modulus testing be given consideration for informational design values, evaluation of other research efforts, and keeping up with state-of-the-art practices." Moreover, mechanistic pavement design models, which are under development by the American Association of Highway Transportation Officials (AASHTO), will rely on the resilient modulus of aggregate bases and soils as important model input parameters (ARA, Inc., 2004).

In the design of pavements, resilient modulus has been used for characterizing the non-linear stress-strain behavior of base aggregates and soil subgrades subjected to traffic loadings. The "AASHTO 1993 Guide for Design of Pavement Structures" recommended that highway agencies use a resilient modulus (M_r), obtained from repeated–load triaxial test, for the design of subgrades and bases. In 2004, the National Cooperative Highway Research Program (NCHRP) released the New Mechanistic–Empirical Design Guide for pavement structures. This final report was entitled, "Guide for Mechanistic–Empirical Design of New and Rehabilitated Pavement Structures, NCHRP 1-37A." In the M-E Design Guide, the resilient modulus of unbound materials is required as input to characterize layers for their structural design. As recommended by the guide for design inputs, the resilient modulus of unbound materials may be obtained directly from a resilient modulus test or available correlations. When the resilient modulus is obtained from a resilient modulus test, the guide designates the input as "Level 1," or the highest input level. If the resilient modulus is obtained from correlations, then the guide designates the input as "Level 2."

This study was sponsored as a means of responding to the factors cited above and to put the Kentucky Transportation Cabinet in a position to take advantage of the latest highway design technology. Numerous resilient modulus tests have been performed previously on compacted soils (Hopkins et al, 2001). Several months were required to purchase, develop compaction and testing protocol, and make operational the necessary equipment for performing resilient modulus tests on Kentucky soils and aggregate bases. This current study focused on performing resilient modulus tests on a variety of aggregates commonly used for aggregate base construction in Kentucky. Types of crushed limestone aggregates commonly used in Kentucky include Dense Graded Aggregate (DGA), and Crushed Stone Base (CBS). River gravel is a potential source of aggregate base in the western portion of the state and was included in the study. Other aggregate materials used on occasion include recycled concrete, Number 57 crushed stone, and asphalt drainage blankets, which were also included in the study.

The M-E Design Guide requires the material coefficients k_1, k_2, and k_3. A review was conducted of different mathematical models that have been proposed for relating resilient modulus to principal stresses. Four mathematical models appear to be useful for this purpose, which include those proposed by Seed (1967), Hopkins et al, 2001 and Ni et al, 2002 (UKTC Model), Halin (2001-AASHTO Model), and Uzan (1985). Coefficients, k_1, k_2, and k_3, for those models are obtained using multiple regression analysis of all standard testing stresses and corresponding resilient modulus values. The models provide best data "fits" between resilient modulus and testing stresses. Coefficients for each test are listed in the report. In all tests reported herein (except the asphalt drainage blanket) and for the latter three models cited above, values of R^2 were equal to or greater than 0.96.

Resilient modulus equipment previously used to perform tests on compacted soils was used in the series of tests on aggregates. However, an additional triaxial chamber and load actuator that would accommodate large aggregate specimens had to be purchased. The large triaxial cell obtained accommodates aggregate specimens measuring 6 inches in diameter and 12 inches in height. The design of the triaxial chamber and load actuator permits the placement of the LVDTs (Linear Variable Displacement Transducers) and load cell inside the chamber. This eliminates system strain during the measurement of the resilient modulus testing and load due to piston friction. The equipment is controlled by computer software during all phases of testing. An overhead crane was installed to facilitate the lifting and placement of the heavy testing head that contains the load actuator for loading large specimens.

Particle size analyses were performed on the different types of aggregates. When sufficient fine material was present moisture-density relationships were established. Moisture-density relationships established from test procedure, AASHTO T-99 (2000), were used to remold aggregate specimens for resilient modulus testing. All resilient modulus aggregate specimens measured 6 inches in diameter and 12 inches in height. If sufficient fines were not present in the aggregate to define a moisture-density relationship, then the maximum and minimum values of dry density were determined using a shaker table and large molds. Specimens were molded at different values of relative compactions and tested.

Typical values of resilient modulus obtained from the UKTC resilient modulus model at three, selected stress states of the different aggregates are illustrated and summarized in the table on page xvii.

Based on results reported herein, the following observations, conclusions, and recommendations are made:

- Resilient modulus, by definition, is not a constant value but varies with stress conditions in base aggregates.

- Values of resilient modulus increase as the dry density increases. However, increases of resilient modulus were more noticeable and larger for well-graded aggregates than resilient modulus values of uniformly-graded aggregates. Values of resilient modulus of dense graded aggregate (DGA) generally were larger than values of the resilient modulus of the number 57s, Crushed Stone Base (CSB), river gravel, and recycled concrete

- Resilient modulus tests could not be performed on DGA specimens that represented the upper gradation limit (Kentucky Transportation Cabinet Standard Specifications, 2004) and remolded to about 95 percent of maximum dry density and optimum moisture content (AASHTO T-99). The upper gradation curve allows a maximum of 13 percent particles finer than the U.S. Standard 200 sieve. The combination of a large percentage of fines and a moisture content near optimum created high pore water pressures during cyclic loading, although the test is performed in an undrained state. Consequently, cyclic loading control was a problem. By testing the DGA specimen at moisture contents smaller than optimum moisture content, the test could be performed. The build up of excess pore pressures in the field has been observed indirectly in DGA bases (and subgrade fine-grained soils), as evidenced by the migration of fines to the surfaces of pavements.

- A number of tests were performed to define the resilient modulus of aggregates commonly used in pavement bases in Kentucky. Data that were developed will provide a good means for defining “Level 1,” as well as “Level 2,” resilient modulus input to the mechanistic model developed by AASHTO (American Association of State Highway and Transportation Officials).

Table II. Summary table of typical values of resilient modulus obtained at three selected testing stress conditions.

Aggregate Base Type		Specimen Number	Resilient Modulus, M_r (psi) Selected Stresses $\sigma_3 = 3$, $\sigma_d = 3$ (psi)	$\sigma_3 = 10$, $\sigma_d = 20$ (psi)	$\sigma_3 = 20$, $\sigma_d = 40$ (psi)	Dry Density (lbs/ft^3)	Moisture Content	Percent of Maximum Dry Density	Relative Density (%)
Dense Graded Aggregate (DGA)	As received	DGA-4531-1-3-1	14,657	37,014	65,554	132.7	5.5	93.0	-
		DGA-4531-1-4-1	15,179	36,167	61,089	136.3	5.7	95.6	-
		DGAVULLEX-4531-1-31-1	14,293	40,022	73,704	136.6	5.7	95.8	-
		DGAVULLEX-4531-1-32-1	14,193	34,342	58,239	103.9	6.3	72.1	-
		DGAVULLEX-4531-1-33-1	23,506	53,434	86,299	144.2	5.5	99.9	-
		DGAVULLEX-4531-1-34-1	21,388	41,031	62,434	130.7	5.2	91.7	-
	Blended to Ky Specifications	DGAUPPER-4531-1-60-1	24,271	48,125	73,817	118.9	2.3	83.6	-
		DGACENTER-4531-1-61-1	10,893	35,139	68,128	128.5	4.8	89.2	-
		DGALOWER-4531-1-62-1	13,601	39,502	74,769	117.4	1.9	-	>100
Crushed Stone Base (CSB) (As Received)		CSBUPPER-4531-1-63-1	19,621	46,892	77,313	139.5	4.8	-	-
		CSBCENTER-4531-1-64-1	16,823	42,635	73,319	140.9	3.5	-	-
		CSBLOWER-4531-1-65-1	14,043	34,732	58,817	113.7	2.6	-	≈100
No. 57 Stone	As received	No57VULLEX-4531-1-58-1	23,575	43,738	64,408	90.0	0.9	-	0.1
		No57VULLEX-4531-1-58-2	38,281	47,307	59,887	97.8	0.9	-	100.0
		No57VULLEX-4531-1-58-3	21,749	39,274	58,544	90.8	0.8	-	0.8
		No57VULLEX-4531-1-58-4	24,577	43,882	64,121	97.8	1.0	-	100.0
		No57VULLEX-4531-1-58-5	25,963	42,747	60,707	90.8	0.9	-	1.5
		No57VULLEX-4531-1-58-6	24,041	44,620	65,736	97.9	0.9	-	100.0
	Repeat Tests	No57VULLEX-4531-1-57-1	26,784	49,689	73,255	92.7	0.0	-	36.9
		No57VULLEX-4531-1-57-2	28,189	47,889	69,328	92.7	0.0	-	36.9
		No57VULLEX-4531-1-57-3	30,094	53,889	78,742	92.7	0.0	-	36.9
		No57VULLEX-4531-1-57-4	27,196	47,070	68,428	92.7	0.0	-	36.9
		No57VULLEX-4531-1-57-5	32,714	52,726	74,295	92.7	0.0	-	36.9
Crushed River Gravel (As Received)		RGRAV-4531-1-21-1	14,790	36,839	63,067	126.3	4.9	-	104.3
		RGRAV-4531-1-22-1	12,740	35,663	63,896	103.9	5.3	-	7.0
		RGRAV-4531-1-23-1	14,351	38,163	68,305	126.3	5.6	-	104.3
		RGRAV-4531-1-24-1	13,524	33,826	59,468	103.3	5.8	-	3.8
		RGRAV-4531-1-25-1	12,546	32,353	56,546	103.7	5.5	-	5.9
		RGRAV-4531-1-26-1	16,071	39,046	65,401	121.0	6.0	-	84.5
Recycled Concrete (As Received)		RECON-4531-1-11-1	19,584	43,388	73,043	118.2	11.1	-	164.0
		RECON-4531-1-12-1	17,421	36,892	58,764	109.7	8.5	-	116.0
		RECON-4531-1-13-1	14,372	34,982	60,454	107.2	9.1	-	98.6
		RECON-4531-1-14-1	14,412	33,845	57,051	94.4	8.9	-	-2.7
		RECON-4531-1-15-1	16,306	36,969	61,057	94.7	8.6	-	2.6
		RECON-4531-1-16-1	18,044	40,557	66,453	105.7	10.6	-	88.9
		RECON-4531-1-17-1	16,950	37,380	60,511	120.3	12.2		91.7

- Studies are needed to examine the following areas of research, which may affect the value of resilient modulus of aggregate bases:

  - The effect of different gradations (or particle sizes) of the base materials on the value of resilient modulus needs to be examined. The maximum, or the permissible, percentage of fines (the amount finer than the U. S. Standard No. 200 sieve) for DGA and Crushed Stone Bases should be determined which would not allow excess pore water pressures to build up under cyclic loading of the resilient modulus test. Limited magnitudes of fines and moisture contents could be determined by performing resilient modulus on specimens compacted to different moisture contents and percentages of fines.

  - The effect of migratory subgrade fines (clay-size particles) on the resilient modulus of base materials needs to be examined. During dissipation of excess pore pressures, fine clay-size particles from the subgrade are pushed into the lower portion of the base aggregate. Strengths (and resilient modulus) of the base materials decrease when excess pore pressures occur in the soil subgrade. Secondly, as fines (uncontrolled) enter the bottom of base aggregates from an untreated, fine-grained subgrade, excess pore pressures may build up in the base aggregates due to the increase of fines.

  - The effectiveness of geofabrics (used as grade separators) to prevent migration of fines into the bottom of the aggregate base needs to be studied. Although the migration of fines may be prevented, the geofabric may clog and cease functioning with increasing time. If the material allows fine particles to pass into the base, then the resilient modulus of the base is altered. In either case, the resilient modulus of the base or/and subgrade will be altered.

  - Extensive geotechnical research needs to be performed to examine "filter requirements" between base aggregates and clayey subgrades and how this relationship affects resilient modulus of bases. Findings of this type of research could help redefine and improve the engineering functions of gradations of typical base aggregates commonly used in Kentucky. To prevent migration of subgrade fines into base aggregates, filter criteria must be met between a given type of soil subgrade and a selected type of base aggregate. Moreover, when filter fabric is used as a grade separator to prevent the migration of subgrade fines into the base aggregates, filter criteria must be satisfied between the subgrade soils and the fabric. This novel approach has good potential for improving the function and performance of base aggregates.

  - Tests need to be performed to adequately define the resilient modulus of chemically stabilized subgrades. This study did not address this important determination. In the pavement system, a chemically treated subgrade may function as a base in some cases or as a subbase in others. Chemical stabilization of subgrades in Kentucky is increasingly being used to improve the poor engineering properties of soils. Sufficient testing should be performed to provide "Level 1," as well as "Level 2," resilient modulus data input to the mechanistic model developed by AASHTO. Chemical admixtures to be examined should include hydrated lime, Portland cement, and lime kiln dust. Typical soils found in Kentucky should be included in the study.

With completion of this study on the resilient modulus of aggregates, the Kentucky Transportation Cabinet is in a good position to implement the use of mechanistic pavement design models. A second study, sponsored by the Kentucky Transportation Cabinet, focused on defining the resilient modulus of compacted soils commonly located in Kentucky. Both soaked and unsoaked specimens were tested. Consequently, data for defining the resilient modulus of aggregates and soils are available for use in the mechanistic pavement design model developed for AASHTO. However, a third study is needed to define the resilient modulus of chemically treated subgrades.

INTRODUCTION

Resilient modulus has been proposed as a means of characterizing the elastic properties of pavement materials. It is expressed as the ratio of deviator stress applied to the pavement layers (and the aggregate base layer) and the resilient axial deformation recovered after release of the deviator stress. Assumptions are made tacitly that pavement materials are designed for loading in the elastic range and that the resilient modulus is the only parameter needed to design the thickness of a pavement. Several types of aggregate bases are used in designing and constructing flexible pavements in Kentucky. A structural layer coefficient of 0.14, or a California Bearing Ratio (CBR) of 100 percent, is usually assigned to the aggregate base for design purposes (AASHTO, 1993).

Although empirical relations have been used in the past to estimate the resilient modulus of aggregate bases, the trend in recent years is to measure the resilient modulus of aggregates and soils using laboratory tests. The value of resilient modulus is stress-strain dependent. That is, the value changes as stress and strain conditions change. AASHTO (American Association of State Highway and Transportation Officials, 1993, 2000) and SHRP (Strategic Highway Research Program, 1989) published a testing standard and protocol, T-294, for performing resilient modulus of aggregates. Equipment for performing resilient modulus tests of aggregates and soils aggregates has steadily evolved and improved over the past few years.

Several mathematical expressions are available for modeling the resilient modulus of aggregates and soils. These include such models as proposed by Moossazadeh and Witczak (1981), Dunlap (1963), Seed et al. (1967), May and Witczak (1981) and Uzan (1985), Hopkins et al (2001) and Ni et al, 2001. Effectiveness of those models to predict resilient modulus is discussed in this report. Comparisons are made among the various models.

The trend in the design of highway pavements consists of using mechanistic models (ARA, Inc. 2004). Although much progress has been made in recent years in developing mathematical, mechanistic pavement design models, results obtained from those models are only as good as the material parameters entered into the models. In 1986 and 1993, the American Association of State Highway Transportation Officials (AASHTO Guides) recommended the use of resilient modulus for characterizing highway materials for pavement design (Mohammad et al., 1995). To promote this concept, the 1962 flexible pavement design equation originally published by the Highway Research Board (1962) was modified in the 1993 AASHTO Guide to include the resilient modulus of soils. This approach attempts to make use of the mechanical properties of the asphalt, or concrete, base courses, and soil subgrades.

Many state transportation agencies have used, or continue to use, empirical pavement design methods involving soil support values, California Bearing Ratio (CBR), or R-values. According to Mohammad et al., (1995), empirical values and design approaches do not adequately represent the response of pavement to the dynamic loading caused by moving vehicles. The resilient modulus concept arose as a result of efforts to better simulate the loading of pavements by moving vehicles. The resilient modulus test for soils was originally developed by Seed et al. (1967) and was later formulated for highway applications (Claros et al., 1990).

The resilient modulus test provides a relationship between deformation (or strain) and stresses in pavement materials, including aggregate bases and subgrade soils, subjected to moving vehicular wheels. Hence, it is not necessarily a fixed value but varies according to the applied stresses of moving vehicles and the resulting stress level in the pavement layers. The test measures the stiffness of a cylindrical specimen of aggregate or soil that is subjected to a cyclic or repeated axial load. It provides a means of analyzing different materials and soil conditions, such as moisture and density, and stress states that simulate the loading of actual wheels. For a given deviator stress, the resilient modulus, M_r, is defined as the slope of the deviator-axial strain curve, or simply the ratio of the amplitude of the repeated axial stress to the amplitude of the resultant recoverable axial strain, or (Figure 1):

$$M_r = \frac{\Delta\sigma_d}{\Delta\varepsilon_{axial}} \tag{1}$$

where

$\Delta\sigma_d = \sigma_1 - \sigma_3 =$ deviator stress,

σ_1 and $\sigma_3 =$ major and minor principal stresses, and

$\Delta\varepsilon_{axial} =$ recoverable axial strain.

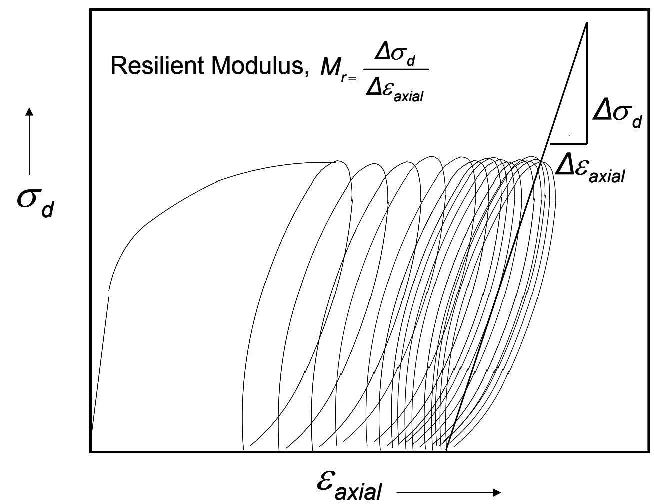

Figure 1. Definition of resilient modulus.

The specimen is subjected to repeated loading at a particular stress level and the recoverable strain is measured. Ideally, the specimen exhibits only elastic strains at the time the resilient modulus is measured. The resilient modulus can, therefore, be thought of as the secant Young's Modulus of a certain material typically different than the initial tangent value (Houston et al., 1993). Resilient modulus is used in many pavement and railroad track designs. This modulus can be used for either the asphalt or subgrade level when the materials are subjected to moving dynamic loads. As shown in Figure 2, the stress level in a subgrade varies with the thickness of the pavement. If the pavement is thin, then the cyclic deviator stresses are large. When the pavement is thick, the cyclic deviator stresses in the subgrade are small. Consequently, the magnitude of the applied cyclic load is varied over a range of anticipated subgrade stress values, as shown in Figure 3, in resilient modulus testing to measure the variation of the resilient modulus, or stiffness.

Values of resilient modulus of aggregate bases are needed to use in mechanistic pavement design models developed by the American Association of State Highway Transportation Officials (AASHTO, Halin, 2001).

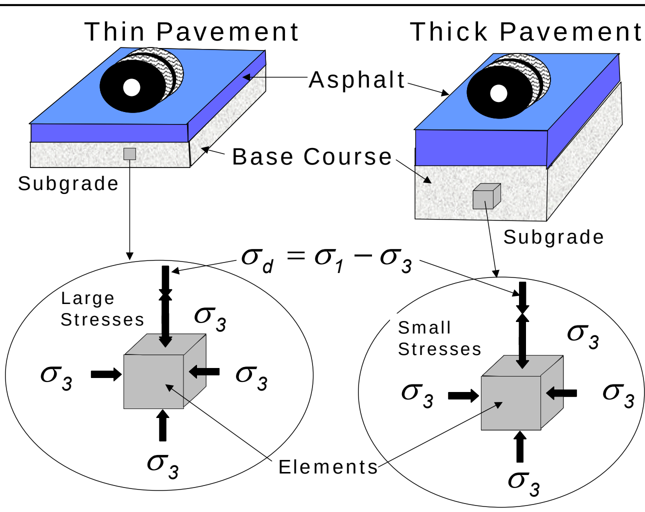

Figure 2. Relative subgrade stress levels for different pavement thickness.

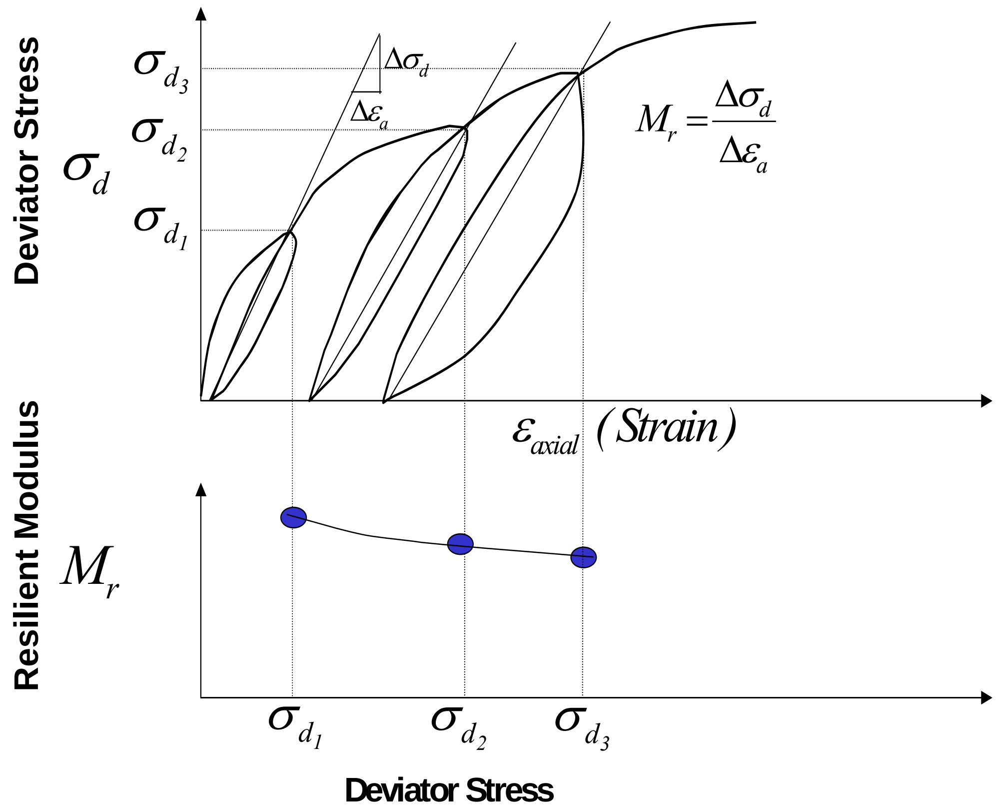

Figure 3. Stress-strain hysteresis loop and resilient modulus determination.

OBJECTIVES

Highways in Kentucky are constructed with various types of aggregate bases. Furthermore the gradation can vary greatly. The objective of this study was to determine values of resilient modulus of different aggregate bases commonly used in pavements in Kentucky.

A major intent of this study was to follow through on a suggestion made by FHWA in 1993 "that an in-depth assessment be made of the most appropriate strength test to accommodate Kentucky's future needs and that resilient modulus testing be given consideration for informational design values, evaluation of other research efforts, and keeping up with state-of-the-art practices." Another major intent of this study was to put the Kentucky Transportation Cabinet in a position (from a design point of view) to use the new mechanistic models developed by AASHTO. Initially, considerable study time was required for purchasing the resilient modulus testing equipment, evaluating the equipment, and making the equipment operational.

SCOPE OF STUDY

Few states or agencies have performed a large number of resilient modulus tests mainly because the test requires expensive, specialized testing equipment and software, the testing procedure is complex, and it is time consuming. The scope of this study mainly included defining values of the resilient modulus of different types of aggregates commonly used in highway pavement bases in Kentucky, examining mathematical expressions, or models, for relating resilient modulus and stresses, and devising an easy means for Cabinet engineers to access the resilient modulus data and mathematical models. Considerable efforts were devoted to devising a compaction protocol for large specimens. This procedure required attention to many details. A summary of the resilient modulus data generated in this study is contained in this report and detailed information for each test appears in the Kentucky Geotechnical Database, which is housed on a server of the Kentucky Transportation Cabinet. Resilient modulus equipment used to perform the tests is fully described. A limited number of tests were performed on aggregate specimens and a synthetic specimen to evaluate the reliability and repeatability of the testing equipment.

BACKGROUND

Values of resilient modulus, M_r, of unbound aggregate, subbase and subgrade are main input parameters in the mechanistic-empirical pavement design procedures developed in the NCHRP (National Cooperative Highway Research Program) Project 1-37A (Halin, 2001). To develop the necessary input data, the Kentucky Transportation Cabinet has sponsored two research studies to generate resilient modulus values. This study represents the second research study sponsored by the Kentucky Transportation Cabinet and it focuses on defining the resilient modulus of aggregates commonly used in Kentucky to construct pavement bases.

In the first study (Hopkins et al, 2001), sponsored by the Kentucky Transportation Cabinet, resilient modulus tests were performed on several different types of typical soils used in Kentucky to construct subgrades. The tests were performed on specimens compacted to 95 percent of maximum dry density and optimum moisture (AASHTO T-99). Both unsoaked and soaked specimens were tested. Each soil sample was classified according to the AASHTO and Unified Classification Systems. Data are available for determining the resilient modulus of a given soil type when the soil classification is known. Interpretation can be made using the group index of the soil type. In the first study, a new relationship, or mathematical model, was developed that relates the resilient modulus to testing stresses. Multiple regression analysis was used to determine the k-coefficients (so called "k_1, k_2, and k_3") of the new model. All testing stresses are used in the analysis to define the coefficients. Resilient modulus data of numerous soil types are stored in the Kentucky Geotechnical Database (Hopkins, et al 2005). The database is located on a server of the Kentucky Transportation Cabinet.

SAMPLING AND GEOTECHNICAL PROPERTIES

Bulk Samples

Bulk samples of crushed limestone bases most commonly used in Kentucky were collected from actual production runs at selected quarries. These included:

- Dense Graded Aggregate (DGA)
- Crushed Stone Base (CSB).

Other sample types submitted by engineers of the Kentucky Transportation Center for resilient modulus testing included:

- Number 57 crushed limestone
- Crushed river gravel (quartz)

- Recycled crushed concrete
- Asphalt drainage blanket.

Table 1. Listing of geotechnical test methods.

Type of Test	Test Method
Moisture Content	AASHTO T 265-93 (1996)
Maximum Dry Density[1] (Shaker Table)	Relative Density--Method Devised
Minimum Dry Density[1]	Relative Density--Method Devised
Particle Size Analysis	AASHTO T 27-99 AASHTO T –11-91
Moisture-Density Relations	AASHTO T 99 Method D
Resilient Modulus of Aggregates	AASHTO T 292-91 (1996)[2] AASHTO T 307-99 (2003)[3]

1. A way of characterizing the in-place density of a granular material.
2. This standard method permitted internal or external placement of the LVDTs and load cell. The LVDT sensors and load cell were placed internally in the chamber for all tests reported herein. The number of conditioning cycles—repeated load applications-- used in the tests were 200 and not 1000, as specified by AASHTO T 292-91, or 500-1000,as specified by AASHTO 307-99. Load applications used in the tests were 100 for following sequence numbers, as specified by AASHTO 307-99.
3. AASHTO T 307–99 specifies mounting, externally, the LVDT sensors and load cell.

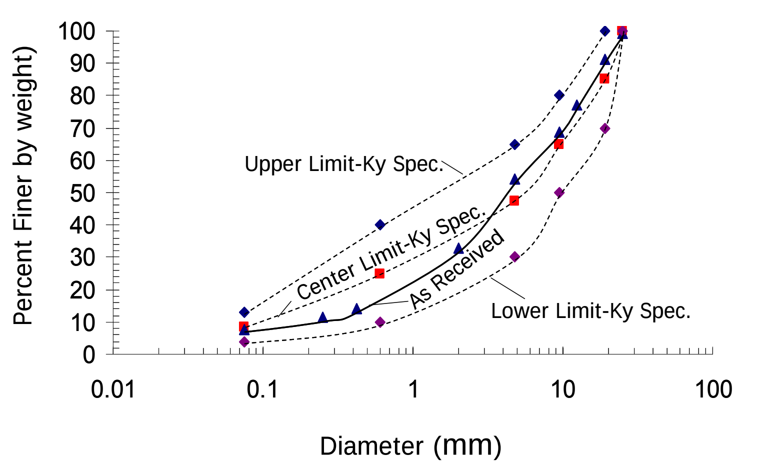

Figure 4. Comparison of upper, center, and lower gradation curves to the gradation curve of DGA sample "as received."

Geotechnical Test Methods and Physical Properties

Test methods used to determine classifications and engineering properties of the bulk samples are tabulated in Table 1. Standard test methods of AASHTO were generally followed.

Gradation and Dry Unit Weights

Two series of resilient modulus tests were performed on specimens of Dense Graded Aggregate (crushed limestone). In the first series, six resilient modulus tests were performed on the DGA sample "as received." Gradation of the DGA sample as it was received from the producer is shown in Figure 4 and compared to the upper, center, and lower gradation specification limits (as specified by the Kentucky Specifications for Road and Bridge Construction (2004)). The Kentucky specifications allow the percentage finer than the U. S. Standard No. 200 sieve to range from 4 to

13. The percent finer than the No. 200 sieve for the "as received" sample was about 8. As shown in Figure 4, and at a particle size below 3 mm, the gradation representing the center of the upper and lower specification gradations contained slightly larger particle sizes than the particle sizes of the "as received" sample. The second series of resilient modulus tests were performed on blended DGA materials representing the upper and lower gradation specifications limits, as well as a gradation curve representing the center of the upper and lower curves (Figure 4).

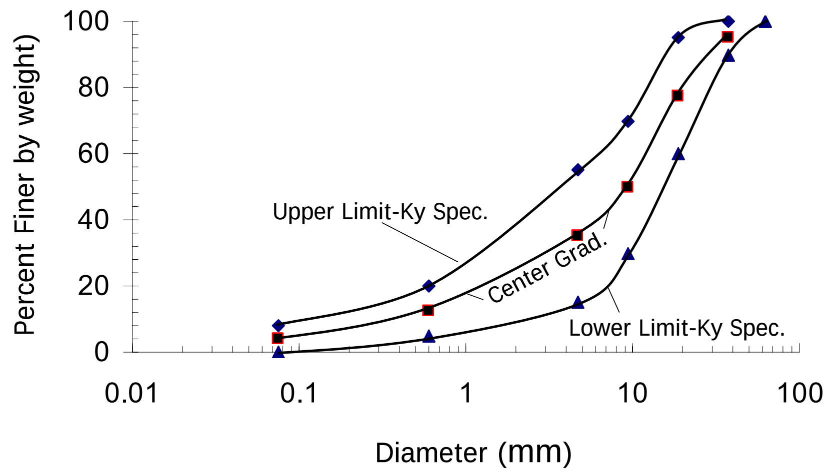

Figure 5. Upper, center, and lower gradation curves of Crushed Stone Base (CSB) specimens blended according to specifications.

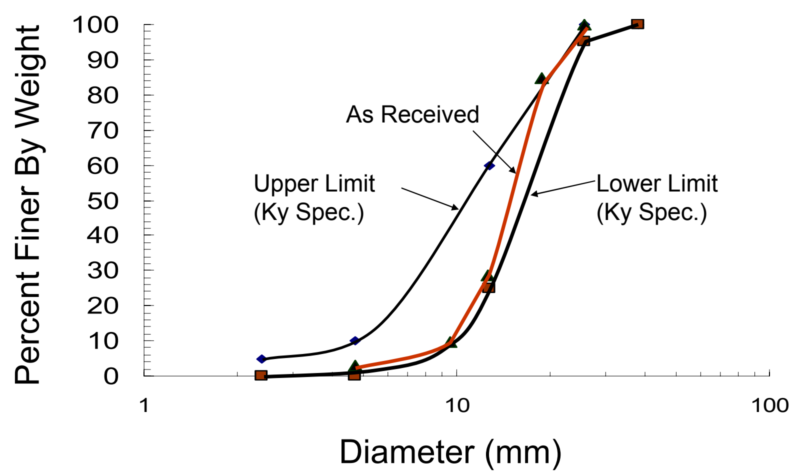

Figure 6. Upper and lower specification gradation curves compared to the gradation curve of No. 57 crushed limestone sample "as received."

Three resilient modulus tests were performed on Crushed Stone Base (CSB). Different particle sizes of the crushed stone aggregate were blended to duplicate the upper and lower specification gradation limits, as shown in Figure 5, and form two specimens for testing. The third blended specimen represented the center gradation curve. The Kentucky specifications allow the percentage finer than the U. S. Standard No. 200 sieve to range from 0 to 8 for CSB material.

Gradation of the Number 57 crushed limestone as received is shown in Figure 6 and compared to the upper and lower gradation limits of the Kentucky specifications. Six resilient modulus tests were performed on this material. Also, resilient modulus tests were performed on the same specimen of the No. 57 stone five times to examine repeatability of the testing equipment and operator. The Kentucky specifications allow the percentage finer than the U. S. Standard No. 8 sieve to range from zero to 5 for this material.

The gradation of the Crushed River Gravel as received is shown in Figure 7. Percentage finer than the U. S. Standard No. 200 sieve of this material was 4. Six resilient modulus tests were performed on this material.

Since recycled concrete is used on occasion as base material, eight tests were performed to characterize the resilient modulus of this material. Gradation of the sample as received is shown in Figure 8. Only 1.7 percent of the particle sizes were finer than the U. S. Standard No. 200.

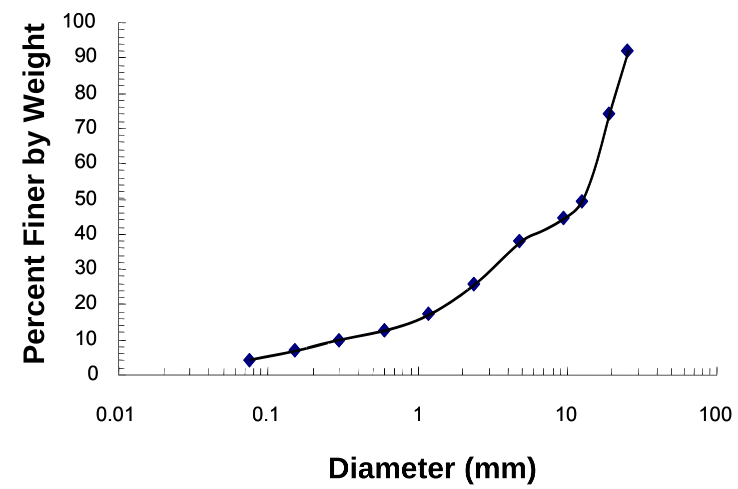

Figure 7. Gradation curve of crushed river gravel (quartz).

Values of gradations for the aggregates included in the testing program for resilient modulus are listed in Tables 2 and 3.

The approach used to form specimens for resilient modulus testing was dependent on whether a moisture-density relationship, as obtained from AASHTO T-99, could be established. When a relationship could be established specimens were remolded to a certain percentage of the maximum dry density and optimum moisture content, or selected "target values."

A moisture-density relationship, as shown in Figure 9, was established for the DGA sample (Figure 4) as received from the quarry. Relationships were also established for the upper and center DGA gradation specification samples (see Figure 4). Those relationships are shown in Figures 10 and 11, respectively. Values of maximum dry density of the three different DGA samples only ranged from 142.2 to 144.1 lbs/ft^3. Optimum moisture contents of the three samples were essentially the same and ranged from 6.7 to 6.9 percent.

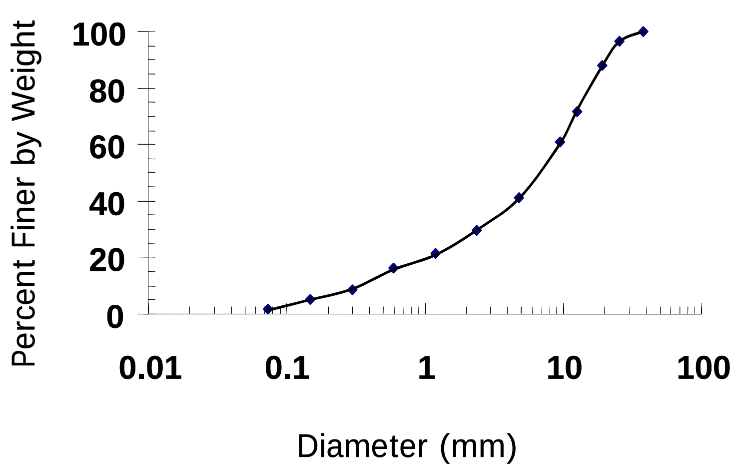

Figure 8. Gradation curve of recycled concrete sample.

Moisture-density relationships for the Crushed Stone Base were established for the upper and center gradation specifications limits. Moisture-density relationships for those samples are shown in Figures 12 and 13, respectively.

Table 2. Gradations of Dense Graded Aggregate, Crushed Stone Base, and No. 57 Stone.

			Sizes of Coarse aggregates Amounts Finer than Each Laboratory Sieve (Square Openings) Percentage by Weight											
U. S. Sieve Size			2 1/2	1 1/2	1	3/4	1/2	3/8	No. 4	No. 10	No. 30	No. 40	No. 60	No. 200
Sieve Opening (mm)			63	37.5	25	19	12.5	9.5	4.75	2.00	0.60	0.425	0.25	0.075
Base Type		Specimen Number												
Dense Grade Aggregate (DGA)	"As Received"	DGA-4531-1-3-1			100	91	77	69	54	33		14	11	8
		DGA-4531-1-4-1			100	91	77	69	54	33		14	11	8
		DGAVULLEX-4531-1-31-1			100	91	77	69	54	33		14	11	8
		DGAVULLEX-4531-1-32-1			100	91	77	69	54	33		14	11	8
		DGAVULLEX-4531-1-33-1			100	91	77	69	54	33		14	11	8
		DGAVULLEX-4531-1-34-1			100	91	77	69	54	33		14	11	8
	Specification Limits													
		DGAUPPER-4531-1-60-1			100	100		80	65		40			13
		DGACENTER-4531-1-61-1			100	85		65	48		25			9
		DGALOWER-4531-1-62-1			100	70		50	30		10			4
Crushed Stone Base Specification Limits		CSBUPPER-4531-1-63-1	100	100		95		70	55		20			8
		CSBCENTER-4531-1-64-1		95		77.5		50	35		12			4
		CSBLOWER-4531-1-65-1	100	90		60		30	15		5			0
Size No. 57 Stone	"As Received"	No57VULLEX-4531-1-58-1			100	85	29	10	3					
		No57VULLEX-4531-1-58-2			100	85	29	10	3					
		No57VULLEX-4531-1-58-3			100	85	29	10	3					
		No57VULLEX-4531-1-58-4			100	85	29	10	3					
		No57VULLEX-4531-1-58-5			100	85	29	10	3					
		No57VULLEX-4531-1-58-6			100	85	29	10	3					
	"As Received" (Repeat Tests)													
		No57VULLEX-4531-1-57-1			100	85	29	10	3					
		No57VULLEX-4531-1-57-2			100	85	29	10	3					
		No57VULLEX-4531-1-57-3			100	85	29	10	3					
		No57VULLEX-4531-1-57-4			100	85	29	10	3					
		No57VULLEX-4531-1-57-5			100	85	29	10	3					

Table 3. Gradations of River Gravel and Recycled Concrete samples.

		Sizes of Coarse aggregates Amounts Finer than Each Laboratory Sieve (Square Openings) Percentage by Weight											
U. S. Sieve Size		1 1/2	1	3/4	1/2	3/8	No. 4	No. 8	No. 16	No. 30	No. 50	No. 100	No. 200
Sieve Opening (mm)		37.5	25	19.0	12.5	9.5	4.75	2.36	1.18	0.6	0.30	0.15	0.075
Base Type	Specimen Number												
River Gravel "As Received"	RGRAV-4531-1-21-1	100	92	74	50	45	38	26	17	13	10	7	4
	RGRAV-4531-1-22-1	100	92	74	50	45	38	26	17	13	10	7	4
	RGRAV-4531-1-23-1	100	92	74	50	45	38	26	17	13	10	7	4
	RGRAV-4531-1-24-1	100	92	74	50	45	38	26	17	13	10	7	4
	RGRAV-4531-1-25-1	100	92	74	50	45	38	26	17	13	10	7	4
	RGRAV-4531-1-26-1	100	92	74	50	45	38	26	17	13	10	7	4
Recycled Concrete "As Received"	RECON-4531-1-11-1	100	96	88	72	61	41	30	22	16	8	5	2
	RECON-4531-1-12-1	100	96	88	72	61	41	30	22	16	8	5	2
	RECON-4531-1-12-2	100	96	88	72	61	41	30	22	16	8	5	2
	RECON-4531-1-13-1	100	96	88	72	61	41	30	22	16	8	5	2
	RECON-4531-1-14-1	100	96	88	72	61	41	30	22	16	8	5	2
	RECON-4531-1-15-1	100	96	88	72	61	41	30	22	16	8	5	2
	RECON-4531-1-16-1	100	96	88	72	61	41	30	22	16	8	5	2
	RECON-4531-1-17-1	100	96	88	72	61	41	30	22	16	8	5	2

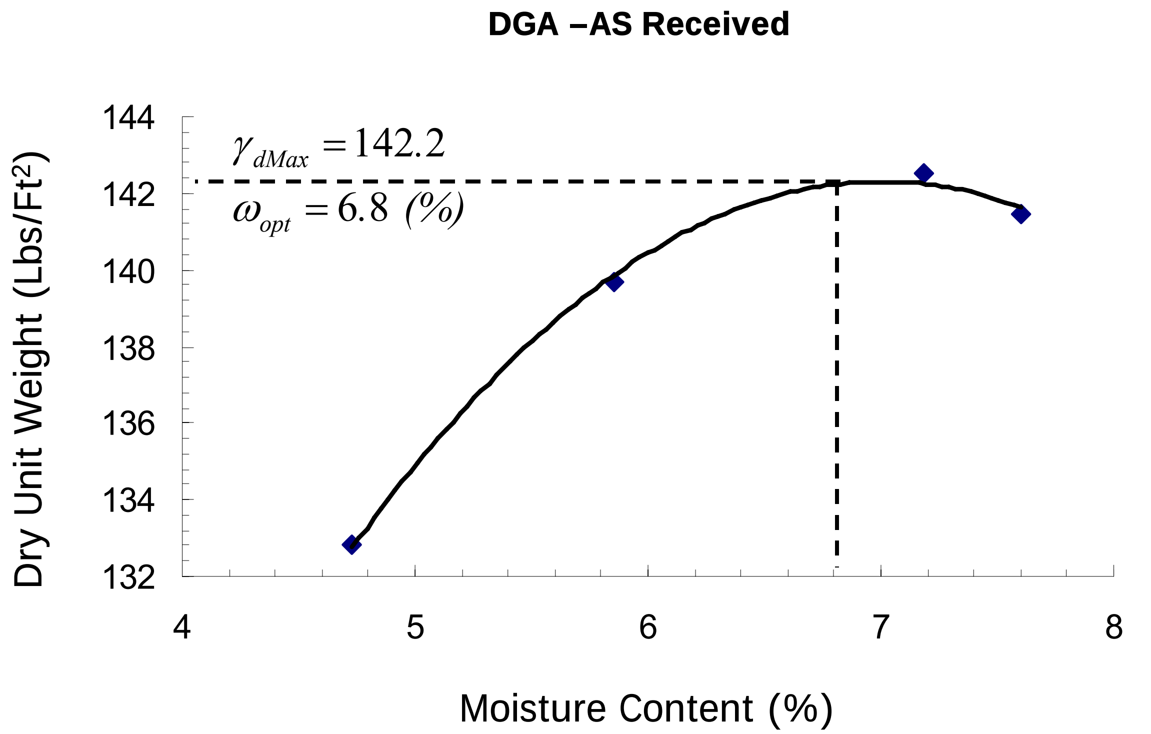

Figure 9. Moisture-density relationship of the DGA specimen as received from quarry.

If moisture-density relationships could not be established from AASHTO T-99, then another approach was adopted to remold resilient modulus specimens. This condition usually occurs when there are insufficient fines (percent finer than the US Standard sieve No. 200) in the aggregate. In those cases, the relative density concept was used.

Relative density is used to characterize the density of granular materials (Lambe, 1969) and it is defined as follows:

$$D_r = \left[\frac{\gamma_{d_{max}}}{\gamma_d} \text{ x } \frac{\gamma_d - \gamma_{d_{min}}}{\gamma_{d_{max}} - \gamma_{d_{min}}} \text{ x } 100\% \right] \tag{4}$$

where

$\gamma_{d_{max}}$ = dry unit weight of aggregate in densest condition,

$\gamma_{d_{min}}$ = dry unit weight of aggregate in loosest conditon, and

γ_d = in-place dry unit weight of aggregate specimen .

When the maximum dry density could not be determined from AASHTO T-99, dry density of the aggregate in the densest state was determined using a shaker table and equipment shown in Figure 14. By weighing the material after shaking and noting the volume of container, the dry density of the aggregate in the densest condition was determined. The dry density of the

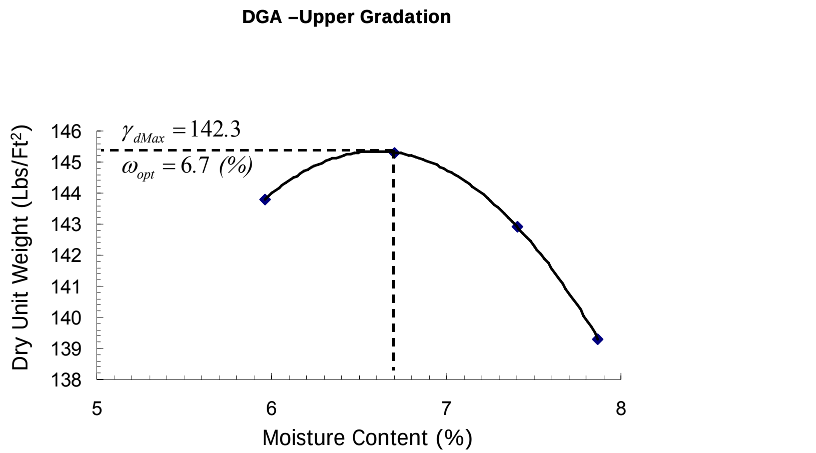

Figure 10. Moisture-density relationship of the DGA specimen representing the upper specification gradation.

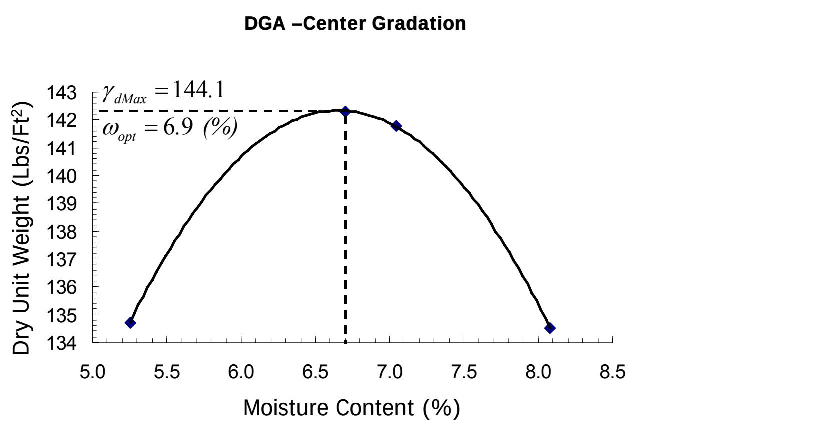

Figure 11. Moisture-density relationship of the DGA specimen representing the center specification gradation.

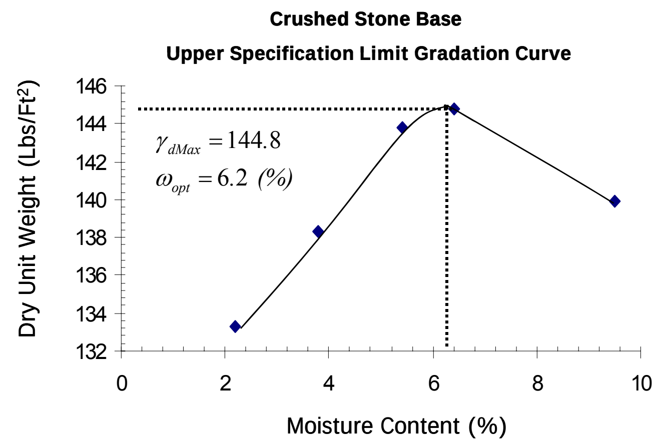

Figure 12. Moisture-density relationship of the upper gradation curve of the Crushed Stone Base.

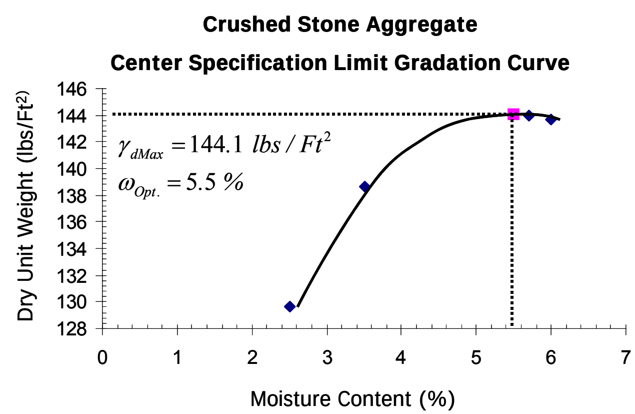

Figure 13. Moisture-density relationship of the center gradation curve of the Crushed Stone Base.

Figure 14. Vibratory equipment and shaker table

aggregate in the loosest condition was determined by loosely placing the aggregate by hand in one of the containers shown in Figure 14. By weighing the material and noting the volume of container, the dry density of the aggregate in the loosest condition was determined.

Descriptive terms that may be used to characterize conveniently the density of granular materials on the basis of relative density are presented in Table 4. For example, when the relative density is some value between zero and 15 percent, the density state is described as "Very Loose." If the relative density ranges from 85 to 100, then the density state may be described as "Very Dense."

Resilient modulus specimens of the following aggregate samples were remolded at selected relative densities:

- Blended DGA material representing the lower specification gradation curve
- Blended Crushed Stone aggregate representing the lower specification gradation curve
- Number 57 aggregate
- River Gravel
- Recycled Concrete.

Values of maximum dry densities and optimum moisture content from AASHTO T-99 for aggregates that contained sufficient fines, maximum and minimum values of dry densities of aggregates that did not contain sufficient fines to perform AASHTO T-99, and target and actual remolding dry densities and moisture contents are given in Tables 5 and 6.

Resilient modulus specimens of DGA as received from the producer were compacted to percents of maximum dry density ranging from 72 to 100. Moisture contents of the specimens ranged from 5.2 to 6.3, which were slightly

Table 4. Characterizing the density of granular materials on the basis of relative density.

Relative Density (%)	Descriptive Term
0-15	Very loose
15-35	Loose
35-65	Medium
65-85	Dense
85-100	Very Dense

Table 5. Dry densities and moisture contents of specimens of Dense Graded Aggregate and Crushed Stone Base.

Specimen Description and Type			Moisture-Density Relationships[1]		Maximum and Minimum Dry Density		Target Values		Actual Values			
Aggregate Base Type		Specimen Number	Maximum Dry Density[1] (lbs/ft^3)	Optimum Moisture Content (%)	Max. (lbs/ft^3)	Min. (lbs/ft^3)	Dry Density[2] (lbs/ft^3)	Optimum Moisture Content (%)	Dry Density (lbs/ft^3)	Moisture Content (%)	Relative. Density, Dr (%)	Percent of Max. Dry Density (%)
DGA	"As Received"	DGA-4531-1-3-1	142.5	6.8			135.4	6.6	132.7	5.5		93.0
		DGA-4531-1-4-1	142.5	6.8			135.4	6.6	136.3	5.7		95.6
		DGAVULLEX-4531-1-31-1	142.5	6.8			135.4	6.6	136.6	5.7		95.8
		DGAVULLEX-4531-1-32-1	142.5	6.8			loose	6.6	103.9	6.3		72.1
		DGAVULLEX-4531-1-33-1	142.5	6.8			142.5	6.6	144.2	5.5		≈100
		DGAVULLEX-4531-1-34-1	142.5	6.8			142.5	6.6	130.7	5.2		91.7
	Specification Limits											
		DGAUPPER-4531-1-60-1	142.3	6.9			-	-	118.9	2.3		83.6
		DGACENTER-4531-1-61-1	144.1	6.9			-	-	128.5	4.8		89.2
		DGALOWER-4531-1-62-1			113.6	107.3	-	-	117.4	1.9	≈100	
Crushed Stone Base Specifications Limits												
		CSBUPPER-4531-1-63-1	144.8	6.2			137.6	6.2	139.5	4.8		96.3
		CSBCENTER-4531-1-64-1	144.1	5.5			136.9	5.5	140.9	3.5		97.8
		CSBLOWER-4531-1-65-1	--	--	114.2	102.2	114.2	2.6	113.7	2.6	≈100	

1. Maximum dry density and optimum moisture content obtained from AASHTO T-99, Method D.

Table 6. Dry densities and moisture contents of specimens of Number 57 stone, River Gravel, and Recycled Concrete.

Specimen Description and Type			Maximum and Minimum Density		Target Values			Actual Values		
Aggregate Base Type		Specimen Number	Max. (lbs/ft^3)	Min. (lbs/ft^3)	Dry Density (lbs/ft^3)		Moisture Content (%)	Dry Density (lbs/ft^3)	Moisture Content (%)	Relative. Density, Dr (%)
No. 57 Stone	“As Received”	No57VULLEX-4531-1-58-1	97.8	90.0		90.0	0	90.1	0.9	0.1
		No57VULLEX-4531-1-58-2	97.8	90.0	97.8		0	97.8	0.9	100.0
		No57VULLEX-4531-1-58-3	97.8	90.0		90.0	0	90.8	0.8	0.8
		No57VULLEX-4531-1-58-4	97.8	90.0	97.8		0	97.8	1.0	100.0
		No57VULLEX-4531-1-58-5	97.8	90.0		90.0	0	90.8	0.9	1.5
		No57VULLEX-4531-1-58-6	97.8	90.0	97.8		0	97.9	0.9	100.0
		CS1-57s	97.8	90.0	-		0	94.9	0.0	64.8
	Repeats	(VULLEX-4531-1-57-1)	97.8	90.0		92.9	0	92.7	0	36.9
		(VULLEX-4531-1-57-2)	97.8	90.0		92.9	0	92.7	0	36.9
		(VULLEX-4531-1-57-3)	97.8	90.0		92.9	0	92.7	0	36.9
		(VULLEX-4531-1-57-4)	97.8	90.0		92.9	0	92.7	0	36.9
		(VULLEX-4531-1-57-5)	97.8	90.0		92.9	0	92.7	0	36.9
River Gravel[1] (Quartz) “As Received”		RGRAV-4531-1-22-1	128.2	105.2		102.6	6.6	103.9	5.3	7.2
		RGRAV-4531-1-24-1	128.2	105.2		102.6	6.6	103.3	5.8	3.7
		RGRAV-4531-1-25-1	128.2	105.2		102.6	6.6	103.7	5.5	6.0
		RGRAV-4531-1-26-1	128.2	105.2	125.1		6.6	121.0	6.0	84.2
		RGRAV-4531-1-21-1	128.2	105.2	125.1		6.6	126.3	4.9	100.0
		RGRAV-4531-1-23-1	128.2	105.2	125.1		6.6	126.3	5.6	100.0
Recycled Concrete[2] “As Received”		RECON-4531-11-1	107.3	94.4	107.3		8.9	117.7	11.1	164.7
		RECON-4531-12-1	107.3	94.4	107.3		8.9	109.7	8.5	116.0
		RECON-4531-13-1	107.3	94.4	107.3		8.9	107.1	9.1	98.6
		RECON-4531-14-1	107.3	94.4		94.4	8.9	94.1	8.9	0.0
		RECON-4531-15-1	107.3	94.4		94.4	8.9	94.7	8.6	2.6
		RECON-4531-16-1	107.3	94.4	107.3		8.9	105.7	10.6	88.9
		RECON-4531-17-1	107.3	94.4	107.3		8.9	106.1	12.2	91.7
Asphalt Drainage Blanket “As Received”		ADB-4531-1-66-1 (tested at Room Temperature $\approx 70^0$F)						108.8		

1. Hydroscopic moisture content of specimen = 2.5 percent. Moisture content of specimen as received and tested = 6.6 percent.
2. Moisture content of specimen as received =8.9 percent.

smaller in value than the optimum moisture content of 6.8 percent. Blended DGA specimens representing the upper and center specification gradation curves were compacted to 84 and 89 percent of maximum dry density, respectively. Blended Crushed Stone specimens representing the upper and center specification gradation curves were compacted to 96 and 98 percent of maximum dry density, respectively.

Blended resilient modulus specimens representing the lower specification curves of DGA and Crushed Stone Base were compacted to relative densities of about 100 percent, or to a "very dense" state. Relative densities of specimens of No. 57 aggregate ranged from about zero to 100 percent. Relative densities of River Gravel specimens ranged from about 4 to 100 percent. The recycled concrete specimens were compacted to relative densities ranging from zero to a value greater than 100 percent. That is, the dry density of one specimen (RECON-4531-12-1) exceeded by about 10.4 lbs/ft^3 the maximum dry density obtained from the shaker table test. For the other specimens of this material, the relative densities ranged from zero to about 100 percent.

RESILIENT MODULUS TESTING

Testing Equipment

The resilient modulus testing equipment, Figure 15, located at the University of Kentucky Transportation Center, is a model RMT-1000, obtained from the Structural Behavior Engineering Laboratories, of Phoenix, Arizona. The system consists of a pressure control panel, plexiglass triaxial cell, a hydraulic power supply, and a computer and software for controlling the testing of a resilient modulus specimen. The system is a complete, closed-loop, servo hydraulic triaxial testing system. The equipment design shown in Figure 16 eliminates the need for a large loading (reaction) frame.

The base and top of the triaxial cell is constructed of stainless steel. The chamber is plexiglass, or acrylic plastic, as shown in Figure 16. The cell is rated to withstand a confinement stress of 150 psi. The triaxial chamber accommodates aggregate specimens measuring 6 inches in diameter and 12 inches in height. A load actuator, as shown in Figures 16 and 17, applies repeated loads. A close-up view of the load actuator is shown in Figure 17. Various load forms of different shapes are available for applying loading sequences. An overhead crane was installed to lift and place the load actuator on the large, specially designed, triaxial cell. The triaxial system has self-contained internal transducers. The triaxial testing cell rests on a massive concrete block.

System Components

The servo controller is a Model 547-1 with dual AC/DC feedback signal conditioning for load and deformation transfer. The signal conditioning system is a series 5 model 300, 4- channel for 2 internal LVDT's and 2 pressure transducers. A view of the LVDTs mounted internally, on the sides of a specimen, is illustrated in Figure 18. A load cell is mounted at the base of the specimen in the triaxial chamber. The porous stone is mounted flush in the base, as shown in Figure 18. The LVDT Transducer calibrator is a Model 139. It has a 1-inch travel range and a resolution of 0.00005 inches. The load cell, pressure transducer, and pore pressure transducer are calibrated using shunt calibration with preset resistance.

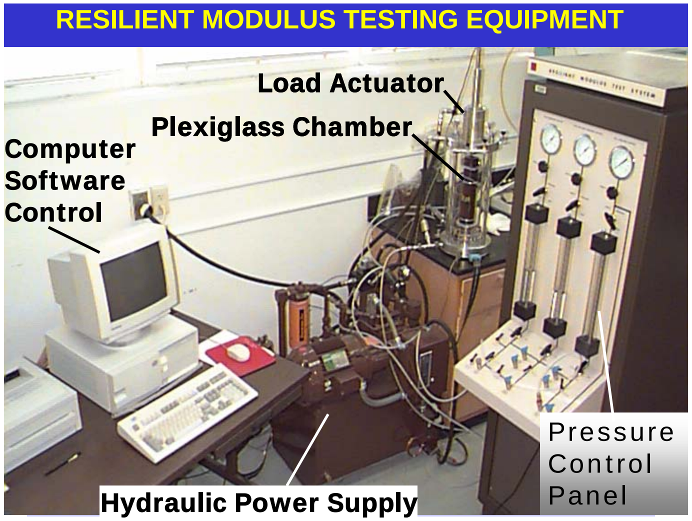

Figure 15. View of resilient modulus testing equipment.

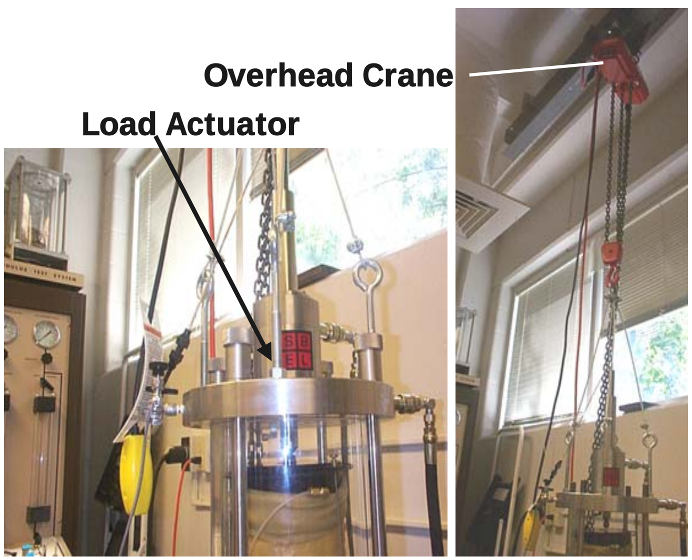

Figure 17. View of loading actuator.

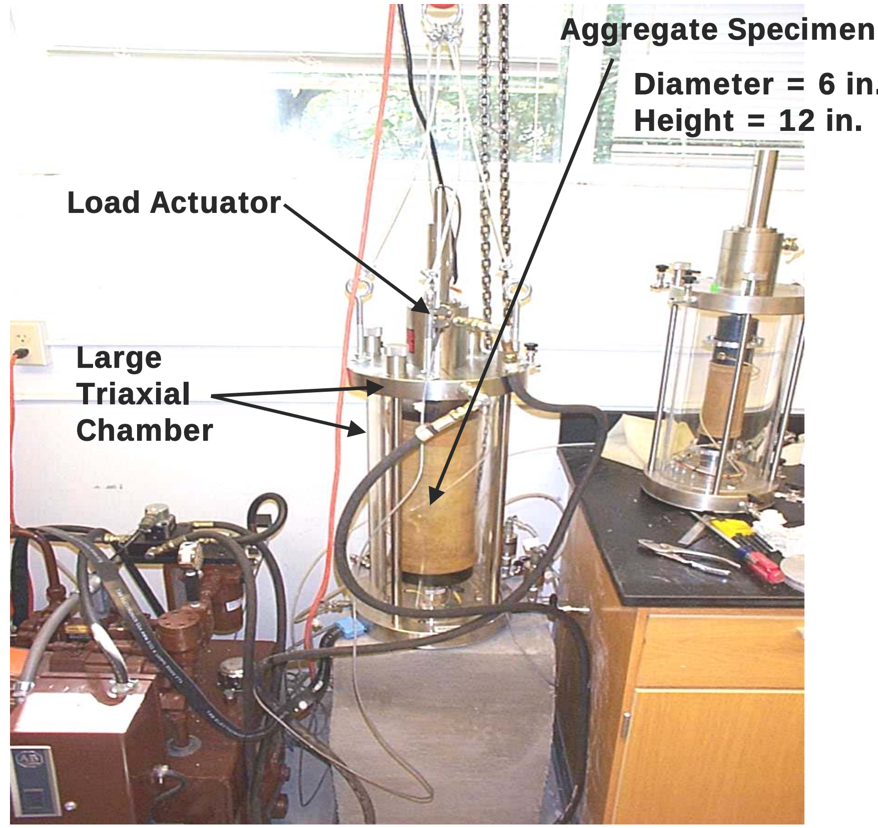

Figure 16. View of aggregate specimen in the triaxial chamber.

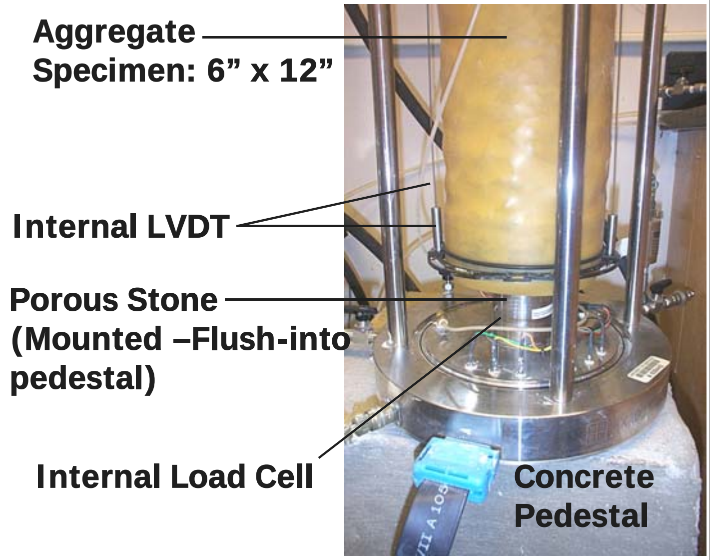

Figure 18. LVDTs and load cell are mounted inside triaxial cell.

Compaction of Aggregate Specimens

After measuring the exact amounts of aggregate and water to form an aggregate specimen of a pre-selected dry density and moisture content (Hopkins and Beckham, 1993; Hopkins et al., 1995; Hopkins et al., 2002), the material was compacted in increments of about 2 to 3 inches in a split mold, as shown in Figure 19. A proctor hammer was used to compact the specimen in small increments. Material that sometimes remained after the specimen mold had been compacted was weighed. On some occasions, a small amount of material remained in the pan. That is, not all of the material could be placed in the mold. However, this did not occur too frequently. The actual dry density and moisture content was based on measuring the weight and moisture content of the material after the test.

Resilient Modulus Testing Protocol

In the resilient modulus testing reported herein, essential elements of AASHTO T292-91 (1996)-20th edition- and T 307-99 (2003)-24th Edition- were followed. However, there were two major exceptions. The load cell and LVDTs were not located *externally,* as shown by the latter standard. Rather the load cell and LVDTs were located *internally,* as shown in Figure 18 and in the former standard above. Considering that extremely small strains are involved in testing aggregate specimens, internal location of the LVDTs aids in eliminating system strains. By locating the load cell internally, errors due to friction of the piston are eliminated.

Both AASHTO T 292-91 and T 307-99 specify a conditioning cycle to be applied to the aggregate specimen. In the former standard, the conditioning sequence consisted of 1000 load applications. The deviator stress and the confining stress are held at 15 psi and 20 psi, respectively, during conditioning. In AASHTO T 307-99, 500-1000 load

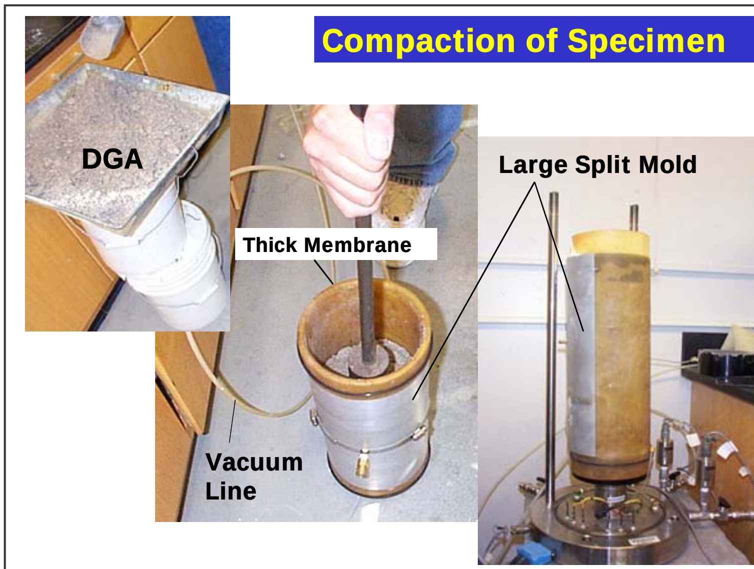

Figure 19. Placing aggregate and compaction of a specimen in a split mold.

applications are specified and the deviator and confining stress are held at 15 psi. Specimen conditioning is intended to eliminate the effects of initial permanent deformation and specimen loading imperfections and not cause permanent plastic deformation.

The other exception in this study consisted of applying only 200 load applications in the conditioning sequence instead of the 500 to 1000 load applications specified by AASHTO T 307-99. The loading sequence is illustrated in Table 7.

Use of 200 load applications was an effort to avoid destroying the integrity of the specimens before applying testing sequences. Using too many load applications in the conditioning stage runs the risk of causing unrecoverable deterioration of the specimens before the actual testing begins because of high stress levels and the lengthy testing cycle of the procedure. Deviator and confining stresses were equal to 15 psi.

After placing the remolded specimen in a triaxial assembly, Figures 16 through 18, repeated loads were applied. In the procedure, 16 load sequences are used. The first test sequence involved the conditioning phase. After the conditioning sequence, 100 load applications were used for each subsequent load sequence. The average recovered deformations for each LVDT are recorded at the last five cycles. The computer data acquisition system records the mean deviator load and the mean recovered deflection. The system then calculates the mean resilient modulus by dividing the mean resilient strain by the applied deviator stress.

The specimen is loaded using a haversine shaped load form. The load pulse is in the form, (1-cos (x))/2, as shown in Figure 20. A Haversine stress pulse was chosen because it better represents the shape of a truck loading on pavement and similar to the load pulse applied by nondestructive testing device, that is, the Falling Weight Deflectometer (FWD). The magnitude of the cyclic load is varied to measure the behavior in aggregate stiffness, or modulus. Before instrumenting the sample, it was visually checked for uniformity and suspected samples were rejected. A view of a resilient modulus test in progress is shown in Figure 21.

REVIEW OF MATHEMATICAL MODELS FOR RELATING RESILIENT MODULUS AND STRESSES

Mathematically, resilient modulus, M_r, has been defined as:

$$M_r = \frac{\sigma_d}{\varepsilon_a}, \tag{5}$$

where

$\sigma_d = \sigma_1 - \sigma_3$ = deviator stress,

σ_1 = major principal stress,

σ_3 = minor principal stress, and

ε_a = axial strain recoverable after release of the deviator stress.

Deformation properties of aggregates are not constant. They are determined by both intrinsic properties of soils and the stresses applied to the soils. A number of mathematical models have been proposed for modeling the resilient modulus of soils and aggregates. Most mathematical expressions relate resilient modulus, the dependent variable, to one independent variable, either the deviator stress, σ_d, or confining stress, σ_3, or the sum of principle stresses, σ_{sum} $(=\sigma_1 + \sigma_2 + \sigma_3)$, or the

Table 7. Testing stresses.

Test Sequence	Confining Stress, or Cell Pressure σ_3 (psi)	Deviator Stress, σ_d (psi)	Major Principle Stress, σ_1 (psi)	Sum of the Principle Stresses, θ (psi)	Number of Cycles[1]
Conditioning	15	15	30	60	200
1	3	3	6	12	100
2	3	6	9	15	100
3	3	9	12	18	100
4	5	5	10	20	100
5	5	10	15	25	100
6	5	15	20	30	100
7	10	10	20	40	100
8	10	20	30	50	100
9	10	30	40	60	100
10	15	10	25	55	100
11	15	15	30	60	100
12	15	30	45	75	100
13	20	15	35	75	100
14	20	20	40	80	100
15	*20*	*40*	*60*	*100*	*100*

M_r is calculated by averaging cycles 96-100 [1]*The number conditioning cycles specified by in AASHTO T 307-99 ranges is 500-1000. In this study 200 conditioning load cycles were used.*

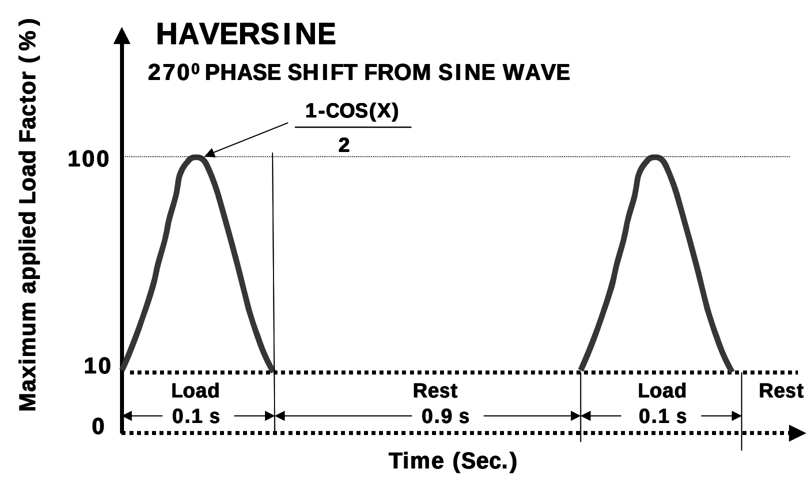

Figure 20. Haversine loading form.

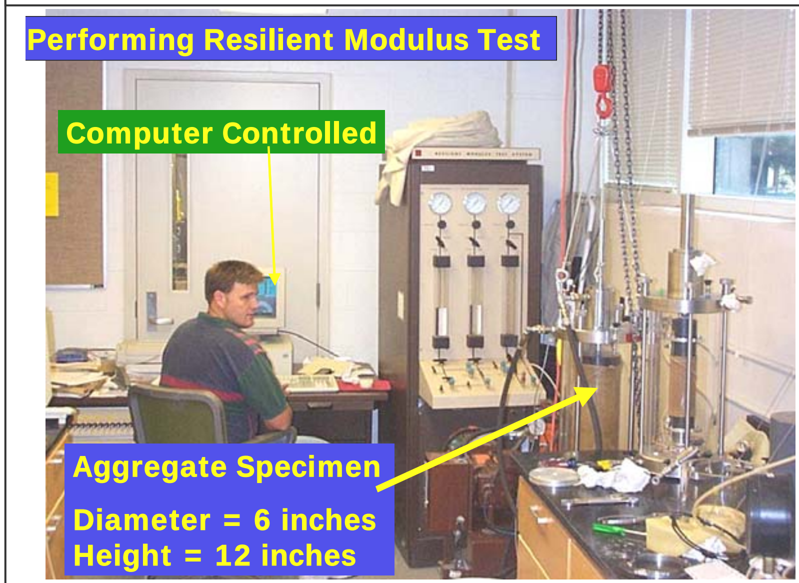

Figure 21. Resilient modulus test in progress.

two independent variables, σ_d and σ_3. Some widely published resilient modulus models are examined below. As shown by this review and analysis of available models, only the later four models are used in the analyses of resilient modulus data reported herein.

Moossazadeh and Witczak (1981)—referred to hereafter as Model 1--proposed the following relationship for presenting resilient modulus data:

$$M_r = k_1 \left(\frac{\sigma_d}{p_a} \right)^{k_2}, \tag{6}$$

where k_1 (y-intercept) and k_2 (slope of the line) are coefficients obtained from a linear regression analysis and p_a is a reference pressure. In this model, the effect of the confining stress is not considered.

Dunlap (1963)--Model 2-- suggests the following relationship:

$$M_r = k_1 \left(\frac{\sigma_3}{p_a} \right)^{k_2}, \tag{7}$$

where k_1 and k_2 are regression coefficients and σ_3 is the confining stress. The influence of the deviator stress is ignored in this relationship.

Seed et al. (1967)--(Model 3)-- suggests that the resilient modulus is a function of the sum of the principle stresses, or

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2}. \tag{8}$$

The term, σ_{sum}, is the sum of principal stresses $(\sigma_1 + \sigma_2 + \sigma_3)$, or for the triaxial compression case, the term is equal to $(\sigma_1 + 2\sigma_3)$. This expression appears in the AASHTO Pavement Design Guide (1993) and in the testing standard, AASHTO T 292-91(2000). Relationships given by Equations 6 and 7 do not consider the effect of shear stress on the resilient modulus of soils.

May and Witczak (1981) and Uzan (1985) --Model 4--proposed another model that considers the effects of shear stress, confining stress, and deviator stress, or

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\sigma_d}{p_a} \right)^{k_3}. \tag{9}$$

The terms, k_1, k_2, and k_3, are correlation regression coefficients. Under identical loading $(\sigma_1 = \sigma_2 = \sigma_3)$, Uzan's model will lead to a value of M_r that either goes to zero when the coefficient, $k_3>0$, or, M_r will become infinite in the case of $k_3<0$. In all of the models cited above, a regression fit can be made for a selected confining stress. However, when the confining stress changes, the coefficients change.

Another resilient modulus model proposed in 2002 (Ni, B., Hopkins, T. C., and Sun) and 2001 (Hopkins, T. C., Beckham, T. L., Sun, L., and Ni, B.), and referred herein as Model 5, is as follows:

$$M_r = k_1\left(\frac{\sigma_3}{p_a}+1\right)^{k_2}\left(\frac{\sigma_d}{p_a}+1\right)^{k_3}. \tag{10}$$

In this model, the coefficients, k_1 and k_2, will always be positive. For most situations the coefficient, k_3, is negative for soils and aggregates. As shown by the relationship given by Equation 10, the resilient modulus increases as the confining stress increases. The modulus will increase or decrease, as in most cases, with the increase of shear stress. When both σ_3 and σ_d approach zero, the value of resilient modulus, M_r, approaches the value of k_1, which is the initial resilient modulus value and a property of the soil. How the resilient modulus of soils changes from its initial value depends on the stress path and the stress state applied to the soil mass. The coefficients, k_1, k_2, and k_3, are derived from test data using multiple correlation regression analysis (See Appendix A).

Another mathematical expression appears in a summary pamphlet prepared by the research team for study NCHRP (National Cooperative Highway Research Program) Project 1-28A (Halin, 2001)—Model 6. This relationship is, as follows:

$$M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}\left(\frac{\tau_{oct}}{p_a}+1\right)^{k_3}, \tag{11}$$

where:

σ_{sum} = sum of all orthogonal normal stresses acting at a given point (or as listed in the summary, σ_{sum} is defined using the symbol, θ, which is defined as the bulk stress).

τ_{oct} = Octahedral shear stress acting on the material, or

$$\tau_{oct} = \frac{\sqrt{2}}{2}(\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}\,. \tag{12}$$

Equation 11 represents the more general case, that is, σ_2 is not equal to σ_3. If σ_2 equals σ_3, then Equation 12 becomes

$$\tau_{oct} = (\sigma_1-\sigma_2) = (\sigma_1-\sigma_3) = \sigma_d = deviator\ \ stress$$

and Equation 11 becomes

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\sigma_d}{p_a} + 1 \right)^{k_3} . \qquad (13)$$

Equations 9 and 10 (Models 4 and 5) are based on the assumption that the normal stresses, σ_2 and σ_3, are equal and represent a specific case (triaxial case). If σ_2 is not equal to σ_3, then Equations 9 and 10 may be written for the more general case, or

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} \right)^{k_3} , \qquad (14)$$

and

$$M_r = k_1 \left(\frac{\sigma_3}{p_a} + 1 \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} . \qquad (15)$$

Consequently, Equations 9 and 10 become Equations 14 and 15.

In the resilient modulus test, the intermediate principal stress, σ_2 is equal to the minor principal stress, or confining stress, σ_3, and the sum of the principal stresses,

$$\theta, \text{ or } \sigma_{\text{sum}} = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_1 + 2\sigma_3 . \qquad (16)$$

The deviator stress is defined as

$$\sigma_d = \sigma_1 - \sigma_3$$

and solving for the major principal stress,

$$\sigma_1 = \sigma_d + \sigma_3 . \qquad (17)$$

Inserting Equation 17 into Equation 16, the sum of the principle stresses may be defined (for the triaxial case)

$$\theta = \sigma_1 + 2\sigma_3 = (\sigma_d + \sigma_3) + 2\sigma_3 = \sigma_d + 3\sigma_3 \quad (18)$$

The sum of the principle stresses appears in the resilient modulus model equations, Equations 9 and 11, proposed by Uzan and NHCRP. Values of the sum of the major principal stresses and the major principal stresses, σ_1, corresponding to testing stresses, the confining stress, σ_3 and the deviator stress, σ_d are shown in Table 7. The various models proposed for characterizing the resilient modulus of granular materials are summarized tin Table 8.

Table 8. Summary of Proposed Resilient Modulus Models.

Model Number	Reference	Independent variable	Equation
1	Moossazadeh and Witczak (1981)	σ_d (Deviator stress)	$M_r = k_1 \left(\frac{\sigma_d}{p_a}\right)^{k_2}$
2	Dunlap (1963)	σ_3 (Confining Stress)	$M_r = k_1 \left(\frac{\sigma_3}{p_a}\right)^{k_2}$
3	Seed, H.B., Mitry, F. G., Monosmith, C. L, and Chan, C. K. (1967)	σ_{sum} (Sum of the Principle Stresses)	$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}$
4	Uzan (1985) May, R.W. and Witczak, M. W.; (1981).	σ_{sum}, σ_d	$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a}\right)^{k_2} \left(\frac{\sigma_d}{p_a}\right)^{k_3}$
5	UKTC (Ni, Hopkins, and Sun, 2002)	σ_3, σ_d	$M_r = k_1 \left(\frac{\sigma_3}{p_a} + 1\right)^{k_2} \left(\frac{\sigma_d}{p_a} + 1\right)^{k_3}$
6	NCHRP (National Cooperative Highway Research Program) Project 1-28A Halin, 2001)	σ_{sum}, τ_{oct} or, if $\sigma_2 = \sigma_3$, then $\sigma_{sum\backslash}, \sigma_d$	$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a}\right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1\right)^{k_3}$ or $M_r = k_1 \left(\frac{\sigma_{sum}}{p_a}\right)^{k_2} \left(\frac{\sigma_d}{p_a} + 1\right)^{k_3}$

TEST RESULTS AND ANALYSIS

Multiple Correlation Analysis

In the relationships expressed by Equations 6, 7 and 8 (Models 1, 2, and 3), respectively, only two variables are involved, as shown in Table 8. The resilient modulus is a dependent variable while either, the deviator stress, confining stress, or the sum of the principle stresses is an independent variable. Consequently, only simple correlation analysis can be performed on those equations. The relation proposed by Model 3 was applied to the experimental data obtained from the different materials because it is a simpler model than the more complex relations expressed by Models 4, 5, and 6 (Equations 9, 10, and 11). Model 3, (Equation 8) is a linear model between the logarithms of the resilient modulus and sum of the principle stresses. Although the equation for Model 3 contains only one independent variable, θ, the confining stress, σ_3, and the deviator stress, σ_d, are included in the θ-term. It can be presented conveniently in a two-dimensional graph, whereas the results of Models 4, 5, an 6 must be presented in a three-dimensional graph, as discussed below. Results of Model 3 analysis were included herein and compared to results obtained from Models 4, 5, and 6

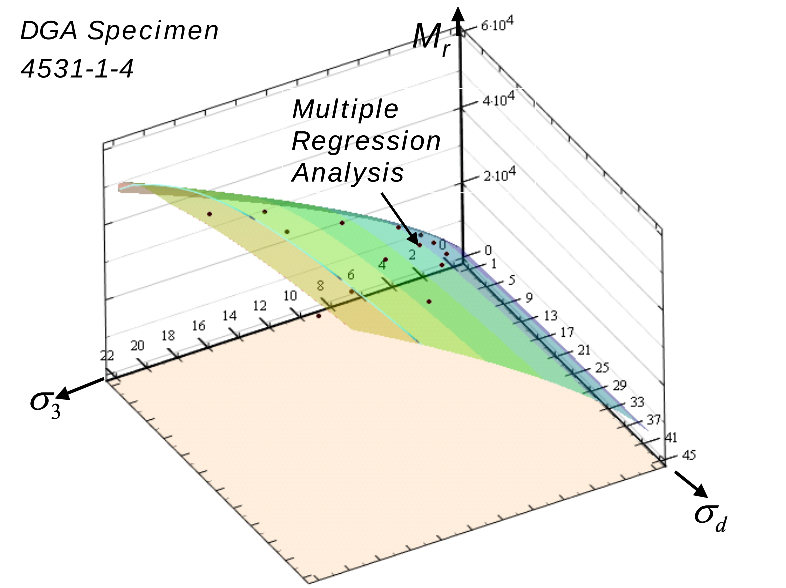

Figure 22. Example illustrating the three-dimensional plane of Model 5 and selected testing stress points.

In the testing procedure, however, the value of resilient modulus is an independent variable and a function of two independent variables, the confining stress, σ_3, and the deviator stress, σ_d. Models 4, 5, and 6, expressed by Equations 9, 10, and 11, respectively, involve two independent variables. The resilient modulus is the dependent variable and the sum of the principle stresses and deviator stress are independent variables in Model 4. In Model 5, the resilient modulus is the dependent variable while the deviator stress and confining stress are independent variables. In Model 6, the resilient modulus is the dependent variable and the sum of the principle stresses and the deviator stresses are the independent variables. Hence, the regression equations of the three models represent a regression plane in a three-dimensional rectangular coordinate system, as illustrated in Figure 22.

Resilient Modulus Test Data and Regression Coefficients of Models 3, 4, 5, and 6

In the multiple regression correlation analysis of Models 4, 5, and 6, all values of M_r obtained at the 15 selected testing stresses (See Table 7) were used, collectively, to obtain the coefficients, k_1, k_2, and k_3 of the multiple regression plane, as illustrated in Figure 22. The coefficient of multiple correlation, R^2, was determined for each of the tests and for each model. This coefficient describes how well the testing points "fit" the regression plane. Multiple regression coefficients, k_1, k_2, and k_3 were determined for the compacted aggregates and all of the materials in this manner. The multiple regression equations used to obtain the coefficients, k_1, k_2, and k_3 are given in Appendix A.

Multiple regression coefficients, k_1, k_2, and k_3 determined from Models 4, 5, and 6, for the compacted aggregates and other materials are summarized in Tables 9 and 10. Coefficients, k_1 and k_2, obtained from linear regression analysis using Model 3 are also included in the summary tables. Dry densities and moisture contents of each specimen were listed in Tables 4 and 5. Values of R^2 of the four models (3, 4, 5, and 6) are also listed. Percentile test values as a function of R^2 for Models 3, 4, 5, and 6 are compared in Figure 23. Excluding R^2-values of the asphalt drainage blanket and PVC tests, the average R^2-values obtained from Models 4, 5, and 6 for the aggregate materials were identical, or numerically equal to 0.992. The average R^2- value for Model 3 was equal to 0.947.

Obtaining large values of R^2 indicates that the testing equipment was very stable, operator error was not pronounced, and the model equations consistently provided a good means of fitting the regression plane.

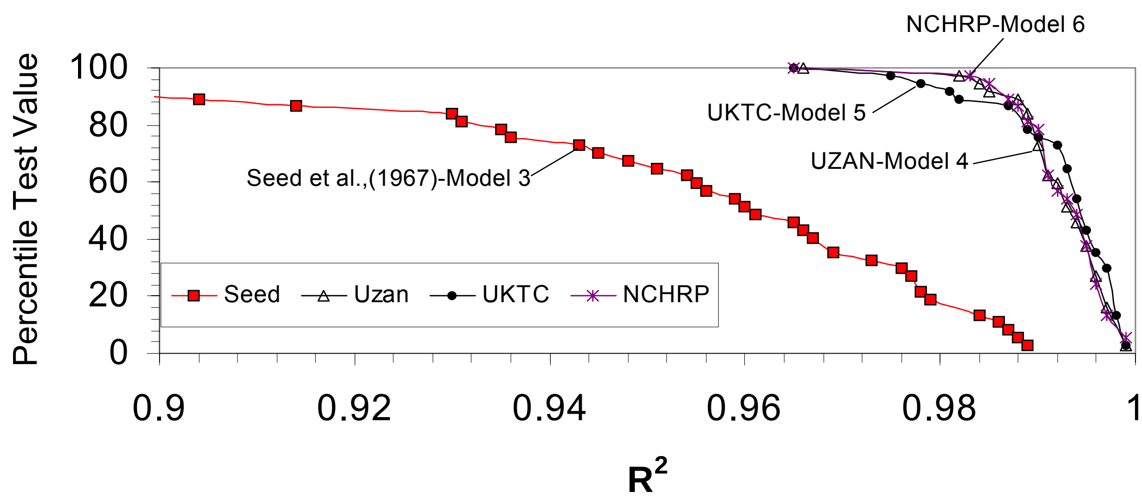

Figure 23. Comparisons of R^2-values obtained from regression analysis using Models 3, 4, 5, and 6.

Resilient modulus models proposed by Uzan, UKTC, and NCHRP are nonlinear material models. Practically, when any nonlinear material model is built into a numerical analyzing program an iteration procedure will be used as the method of solution. Although the average R^2-values of Models 4, 5, and 6 were identical, situations may arise where values of the resilient modulus computed from Models 4 and 6 may diverge. This case may happen on the pavement surface area located away from the loading location. It could represent a potential problem when the models may be applied in nonlinear analyses. For example, whenever the coefficient k_2 or k_3 is negative in Uzan's model (Model 4), and σ_{sum} or σ_d approaches zero, the resilient modulus may diverge, or become very large. This situation is illustrated depicted in Figure 24. In the case of a small value of σ_d, a normal situation for an area located away from the loading area, M_r may become unstable. In the test data shown in Tables 9 and 10 for the granular materials included in the testing program, all of the k_2- coefficients from Uzan's model were positive. However, all of the k_3-coefficients were negative for that model.

Table 9. Coefficients k_i and R^2 of aggregate samples for four different resilient modulus models

Sample	[1]Seed et al. Model 3			Uzan's Model 4				UKTC Model 5				NCHRP Model 6			
	k_1	k_2	R^2	k_1	k_2	k_3	R^2	k_1	k_2	k_3	R^2	k_1	k_2	k_3	R^2
DGA-4531-1-3-1	1759.64	0.7961	0.948	1342.78	1.0930	-0.3222	0.988	4314.77	0.8294	0.0527	0.995	1521.27	1.0797	-0.3396	0.985
DGA-4531-1-4-1	2222.16	0.7268	0.967	1809.49	0.9553	-0.2491	0.997	4952.76	0.7291	0.0788	0.997	1984.06	0.9533	-0.2724	0.997
DGAVULLEX-4531-1-31-1	1571.29	0.8449	0.977	1292.46	1.0486	-0.2241	0.995	3882.75	0.8185	0.1216	0.992	1411.07	1.0497	-0.2511	0.996
DGAVULLEX-4531-1-32-1	2099.46	0.7298	0.969	1754.61	0.9157	-0.2042	0.990	4594.43	0.7199	0.0937	0.995	1903.60	0.9134	-0.2247	0.990
DGAVULLEX-4531-1-33-1	4198.88	0.6609	0.984	3697.82	0.7923	-0.1443	0.997	8458.27	0.6208	0.1165	0.998	3915.90	0.7910	-0.1591	0.997
DGAVULLEX-4531-1-34-1	4349.92	0.5952	0.936	3424.46	0.8414	-0.2695	0.989	8692.36	0.6581	-0.0086	0.989	3805.87	0.8425	-0.3017	0.990
DGAUPPER-4531-1-60-1	4914.80	0.6015	0.943	3988.22	0.8154	-0.2339	0.982	9711.83	0.6357	0.0250	0.982	4368.61	0.8176	-0.2634	0.983
DGACENTER-4531-1-61-1	1039.13	0.9096	0.979	933.00	1.0211	-0.1223	0.984	2662.71	0.7947	0.2215	0.975	972.63	1.0331	-0.1509	0.985
DGALOWER-4531-1-62-1	1310.91	0.8908	0.967	1043.98	1.1260	-0.2578	0.989	3481.78	0.8717	0.1112	0.981	1154.47	1.1275	-0.2891	0.990
CSBUPPER-4531-1-63-1	2120.44	0.7259	0.976	1837.94	0.8716	-0.1591	0.989	4568.77	0.6768	0.1332	0.992	1960.00	0.8685	-0.1735	0.988
CSBCENTER-4531-1-64-1	3359.02	0.6835	0.987	3013.63	0.7973	-0.1253	0.996	6764.21	0.6226	0.1456	0.997	3170.99	0.7939	-0.1354	0.996
CSBLOWER-4531-1-65-1	2388.78	0.7494	0.977	2056.79	0.9032	-0.1683	0.991	5295.43	0.7001	0.1337	0.987	2198.65	0.9030	-0.1874	0.991
No57VULLEX-4531-1-58-1	5513.42	0.5462	0.945	4556.00	0.7426	-0.2152	0.985	10292.80	0.5772	0.0206	0.978	4951.27	0.7456	-0.2435	0.987
No57VULLEX-4531-1-58-2	13149.25	0.3657	0.647	8481.14	0.8151	-0.4916	0.966	22806.48	0.6307	-0.2571	0.965	10323.73	0.8093	-0.5407	0.965
No57VULLEX-4531-1-58-3	4641.68	0.5712	0.914	3466.15	0.8710	-0.3281	0.996	9216.96	0.6742	-0.0549	0.993	3953.28	0.8668	-0.3605	0.995
No57VULLEX-4531-1-58-4	5789.70	0.5388	0.931	4533.73	0.7892	-0.2736	0.996	10887.82	0.6096	-0.0223	0.994	5060.40	0.7851	-0.2999	0.995
No57VULLEX-4531-1-58-5	6469.16	0.5086	0.891	4844.99	0.8057	-0.3252	0.992	12191.57	0.6273	-0.0820	0.987	5501.08	0.8064	-0.3630	0.994
No57VULLEX-4531-1-58-6	5646.00	0.5452	0.954	4636.43	0.7467	-0.2202	0.997	10486.04	0.5784	0.0201	0.998	5070.02	0.7418	-0.2394	0.996
No57VULLEX-4531-1-57-1	6301.08	0.5450	0.951	5173.48	0.7475	-0.2215	0.994	11663.27	0.5822	0.0175	0.998	5654.46	0.7443	-0.2429	0.993
No57VULLEX-4531-1-57-2	6680.77	0.5301	0.893	4933.48	0.8447	-0.3453	0.995	12704.35	0.6547	-0.0798	0.994	5672.02	0.8380	-0.3766	0.993
No57VULLEX-4531-1-57-3	7070.83	0.5396	0.935	5606.04	0.7782	-0.2611	0.996	13335.70	0.6046	-0.0175	0.993	6221.09	0.7752	-0.2872	0.996
No57VULLEX-4531-1-57-4	6345.04	0.5366	0.904	4754.75	0.8294	-0.3192	0.992	12158.70	0.6408	-0.0601	0.994	5409.44	0.8232	-0.3481	0.990
No57VULLEX-4531-1-57-5	8323.90	0.4987	0.874	6100.34	0.8154	-0.3457	0.990	15610.51	0.6305	-0.0968	0.987	7004.44	0.8113	-0.3802	0.990

1. 1967; DGA—Dense Graded Aggregate; No. 57—Number 57 crushed limestone

Table 10. Coefficients k_i and R^2 of aggregate samples for four different Resilient Modulus Models (Continued).

Sample	[1]Seed et al. Model 3			Uzan's Model 4				UKTC Model 5				NCHRP Model 6			
	k_1	k_2	R^2	k_1	k_2	k_3	R^2	k_1	k_2	k_3	R^2	k_1	k_2	k_3	R^2
RGRAV-4531-1-21-1	2121.85	0.7432	0.979	1807.14	0.9087	-0.1815	0.995	4697.10	0.7105	0.1169	0.999	1946.13	0.9034	-0.1957	0.994
RGRAV-4531-1-22-1	1568.11	0.8063	0.989	1429.24	0.9000	-0.1021	0.993	3683.06	0.7038	0.1914	0.993	1484.90	0.9033	-0.1177	0.994
RGRAV-4531-1-23-1	1723.56	0.8084	0.978	1409.65	1.0146	-0.2257	0.999	4141.86	0.7862	0.1102	0.997	1543.80	1.0109	-0.2470	0.999
RGRAV-4531-1-24-1	1709.45	0.7831	0.955	1330.89	1.0420	-0.2848	0.989	4045.26	0.8148	0.0558	0.993	1489.06	1.0408	-0.3157	0.989
RGRAV-4531-1-25-1	1659.99	0.7733	0.973	1408.25	0.9426	-0.1855	0.988	3811.96	0.7386	0.1207	0.992	1517.17	0.9394	-0.2027	0.988
RGRAV-4531-1-26-1	2513.82	0.7124	0.986	2216.76	0.8420	-0.1420	0.997	5352.40	0.6610	0.1321	0.998	2345.37	0.8410	-0.1570	0.997
RECON-4531-1-11-1	2613.47	0.7472	0.930	1875.26	1.0821	-0.3643	0.992	6401.58	0.8383	-0.0317	0.990	2158.78	1.0862	-0.4108	0.995
RECON-4531-1-12-1	3109.75	0.6512	0.956	2498.14	0.8764	-0.2466	0.993	6432.38	0.6827	0.0360	0.997	2761.70	0.8710	-0.2683	0.992
RECON-4531-1-13-1	1890.79	0.7662	0.961	1486.23	1.0115	-0.2681	0.994	4459.98	0.7890	0.0551	0.996	1651.10	1.0120	-0.2993	0.995
RECON-4531-1-14-1	2099.50	0.7285	0.959	1671.54	0.9593	-0.2520	0.990	4718.75	0.7451	0.0603	0.994	1844.57	0.9602	-0.2818	0.991
RECON-4531-1-15-1	2566.12	0.6997	0.960	2070.61	0.9191	-0.2402	0.990	5576.41	0.7190	0.0550	0.997	2278.56	0.9163	-0.2641	0.990
RECON-4531-1-16-1	2941.20	0.6884	0.965	2391.80	0.9007	-0.2323	0.995	6278.59	0.7003	0.0612	0.997	2625.96	0.8974	-0.2549	0.995
RECON-4531-1-17-1	2858.13	0.6739	0.966	2353.13	0.8729	-0.2178	0.994	6052.94	0.6818	0.0610	0.995	2563.94	0.8726	-0.2423	0.994
ADB-4531-1-66-1	277849.82	-0.0758	0.102	212691.92	0.1967	-0.2973	0.545	293226.32	0.1559	-0.2693	0.575	237637.10	0.2076	-0.3445	0.590
PVC-4531-1-41-1	3031.36	0.9074	0.974	2639.12	1.0461	-0.1505	0.982	7992.03	0.8108	0.1940	0.985	2813.54	1.0376	-0.1574	0.981
PVC-4531-1-42-1	3310.05	0.8766	0.965	2998.30	0.9778	-0.1108	0.970	8339.85	0.7725	0.2032	0.976	3153.60	0.9654	-0.1084	0.969
PVC-4531-1-43-1	3341.24	0.8776	0.953	2956.61	0.9996	-0.1323	0.960	8474.36	0.7727	0.2014	0.967	3141.95	0.9843	-0.1289	0.958
PVC-4531-1-44-1	3503.52	0.8637	0.959	3157.70	0.9668	-0.1116	0.964	8586.10	0.7427	0.2214	0.971	3329.91	0.9510	-0.1053	0.962
PVC-4531-1-45-1	3641.73	0.8514	0.960	3378.21	0.9287	-0.0847	0.963	8638.64	0.7290	0.2281	0.972	3518.89	0.9143	-0.0767	0.962

CSB—Crushed Stone Base; RGRAV—River Gravel; RECON—Recycled Concrete; ADB—Asphalt Drainage Blanket; PVC—Polyvinyl Chloride

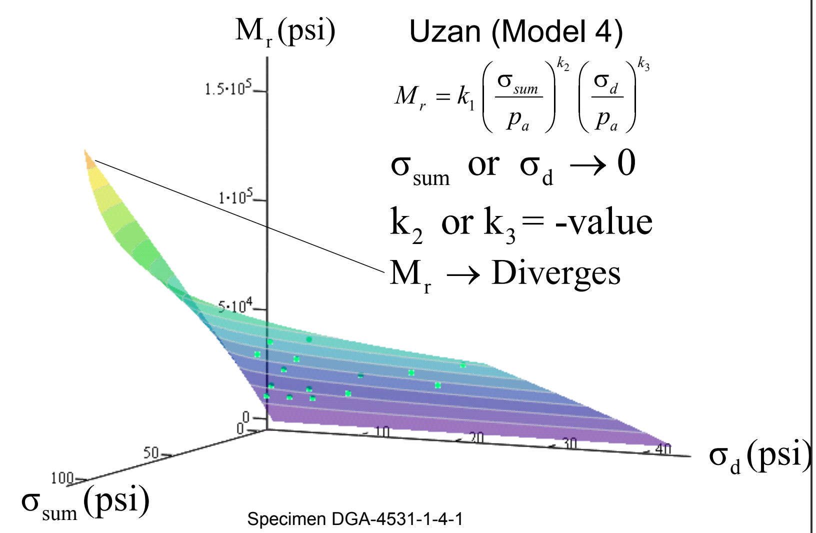

Figure 24. Regression plane of Model 4 and potential divergence problems at small stresses.

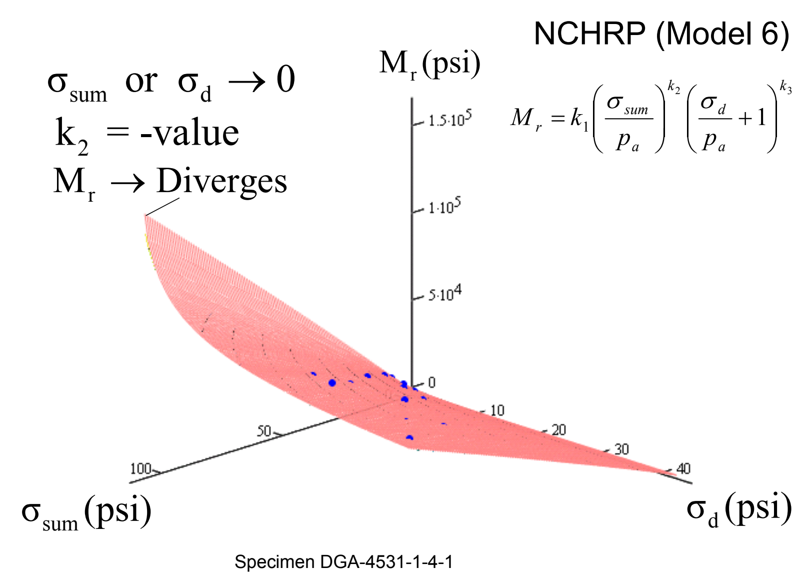

Figure 25. Regression plane of Model 6 and potential divergence problems at small stresses.

When k_2 is negative in the NCHRP model and σ_{sum} becomes small, or approaches zero, M_r may become large and diverge. This potential problem is illustrated in Figure 25. It should be noted, however, that for granular materials included in the testing program all k_2-coefficents[1] were positive, as shown in Tables 8 and 9.

In the UKTC Model, when the values, σ_3 and σ_d become small, or approach zero, the value of M_r approaches the value of k_1. As illustrated in Figure 26, the value of Mr approaches a point in the three-dimensional graph. If σ_3 is not equal to zero and σ_d approaches zero, then M_r is a line in the M_r-σ_3 plane. The value of M_r approaches the term,

$$k_1\left(\frac{\sigma_3}{p_a}+1\right)^{k_2}.$$

If σ_d is not equal to zero and σ_3 approaches zero, then M_r is a line in the M_r-σ_d plane. The value of M_r approaches the term,

$$k_1\left(\frac{\sigma_d}{p_a}+1\right)^{k_3}.$$

In any of the three cases listed above, M_r converges to a value and remains stable.

[1] On rare occasions, negative values of k_2 obtained from the NCHRP model have been observed for resilient modulus tests performed on compacted specimens of soil. Negative k_2-coefficients were obtained for two cases of 68 unsoaked (or "as compacted") specimens that were tested. Negative k_2-coefficients were also obtained for two cases of 60 soaked compacted specimens that were tested. Multiple coefficients of correlations of those four specimens were 0.930, 0.958, 0.650, and 0.993, respectively.

Coefficients of correlation indicate that Models 4, 5, and 6 best describes variation of the resilient modulus. An average value of R^2 obtained from each model was equal to 0.992. If a slightly lower error may be tolerated, then Model 3 could be used, provided divergence of M_r given by the model equation is not a problem. An average value of R^2 was only slightly less than 0.95. Typical graphical relations from Model 3 of M_r as a function of σ_{sum} for DGA specimens are shown in Figures 27 and 28. Similar results may be obtained for the other aggregates. Some data scatter is evident, as shown in the figures. Negative values of k_2 obtained from Model 3, which could cause M_r to diverge, were not observed for the granular materials included in this testing program..

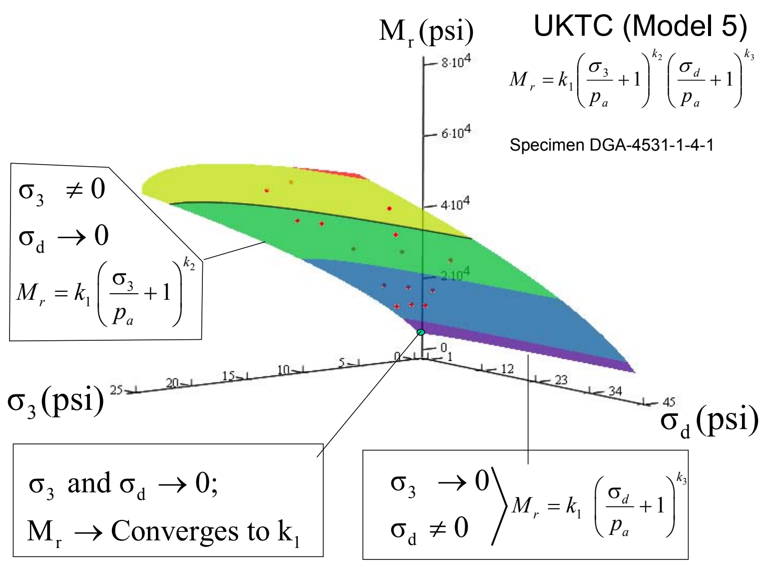

Figure 26. Regression plane of Model 5. As σ_d or σ_3 approaches zero, M_r converges to the constant, k_1.

Numerical values of resilient modulus predicted by Models 3, 4, 5, and 6 are compared in Tables 11, 12, and 13. Average values of resilient modulus obtained for the granular materials for the selected stresses, σ_3 and σ_d are shown in those tables. Values of M_r were computed at three selected stresses of σ_3 and σ_d, as shown in the three tables. The values of stresses selected for comparative purposes represent low, about mid-range, and high values of σ_3 and σ_d listed previously in Table 7. Values of σ_{sum} which appear in Models 4 and 6 were computed from stresses selected for σ_3 and σ_d using Equations 16, 17, and 18.

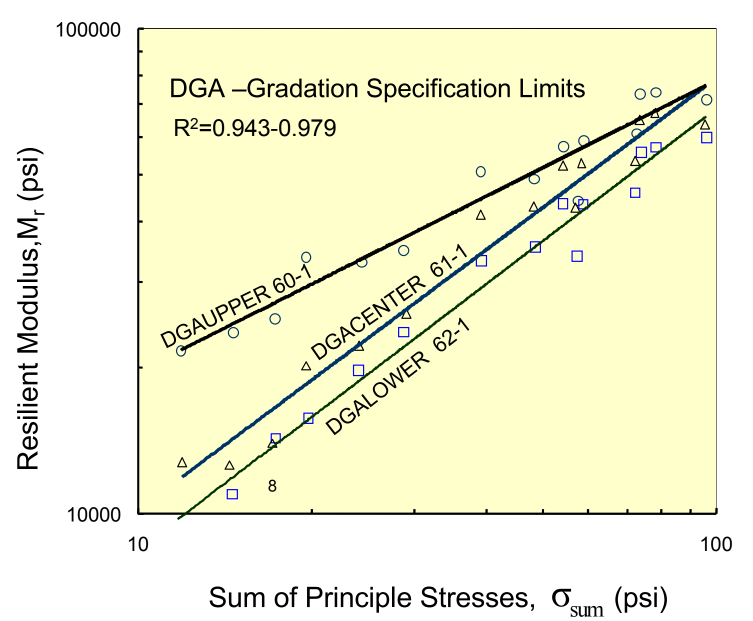

Figure 27. Results of Regression analyses from Model 3 for DGA specimens blended and remolded to gradation specification limits.

Percentage differences of average numerical values (shown in Tables 11, 12, and 13) of resilient modulus obtained from Models 3, 4, 5, and 6 are summarized in Table 14.

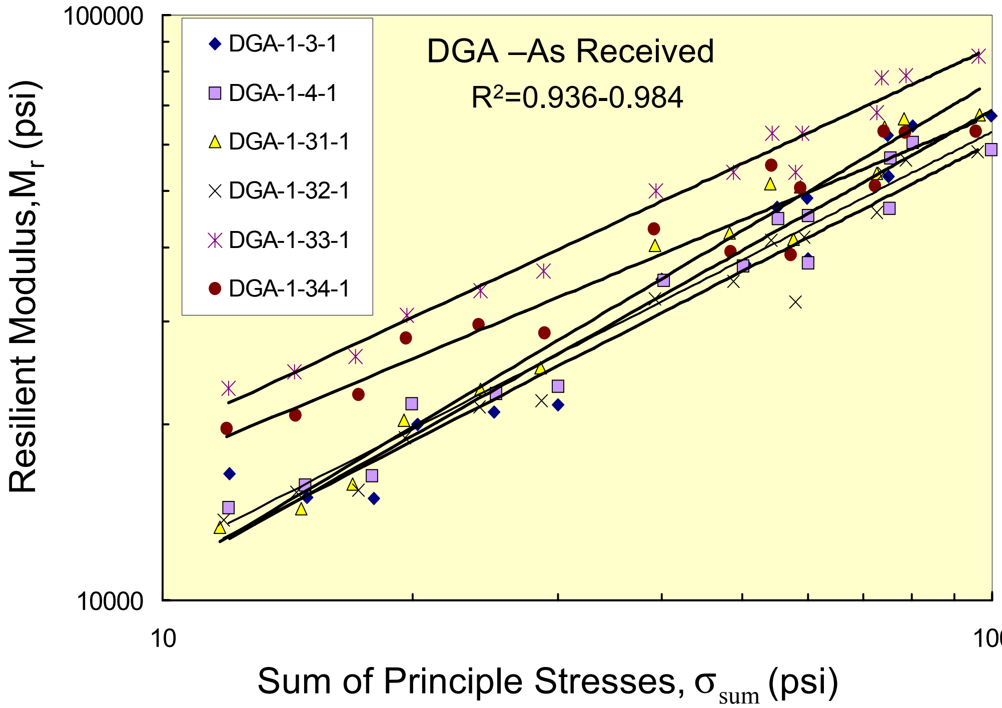

Figure 28. Results of regression analyses from model 3 for remolded DGA specimens of "as received" aggregate.

Percentages differences of the average resilient modulus values (granular materials) obtained from the different models are shown relative to the average value of resilient modulus obtained from Model 5 (UKTC). Average values of resilient modulus from Model 3 ranged from 7.6 percent larger and 10.5 percent smaller than average values of resilient modulus from Model 5. Average values of resilient modulus from model 4 ranged from 0.3 to 3.7 percent smaller than values obtained from Model 5. Average values of resilient modulus from Model 6 ranged from 0.3 to 5 percent smaller than values from Model 6. Models 4, 5, and 6 yielded very similar values of resilient modulus for the range of selected stresses.

Storage and Accessibility of Values of Resilient Modulus of Compacted Aggregates

All resilient modulus test data pertaining to the compacted aggregate specimens resides in the Kentucky Geotechnical Database (Hopkins et al., 2005). The program, using this database, is in a client/server "Windows" environment and the database resides on a production server of the Kentucky Transportation Cabinet. Values of resilient modulus in the database are readily available to personnel of the Kentucky Transportation Cabinet statewide. All key district and central office personnel can access the data through the client-server network.

Users have two means of accessing data on the client-server application in the Geotechnical Database. After the user logs on (Figure 29), the graphical user interface (GUI) shown in Figure 30 appears. By clicking on "Engineering Application", another menu appears as shown in Figure 31. After clicking on "Resilient Modulus," the GUI screen in Figure 32 appears. By clicking on an aggregate type under "Sample Information," shown in the left-hand portion of Figure 32, two-dimensional plots of resilient modulus as a function of a selected stress component appears. In the current analytical version, values of resilient modulus for a selected specimen may be plotted as a

Table 11. Comparison of numerical values of M_r obtained from Models 3, 4, 5, and 6 and calculated at stresses of σ_3 = 3 and σ_d= 3.

Sample Description		Sample Number	Model 3 Seed's Model	Model 4 Uzan's Model	Model 5 UKTC	Model 6 NCHRP
			σ_{sum} = 12 psi	σ_{sum} = 12 psi σ_d = 3 psi	σ_3= 3 psi σ_d= 3 psi	σ_{sum}= 12 psi σ_d= 3 psi
			Resilient Modulus, M_r (psi)			
Dense Graded Aggregate (DGA)	As Received	DGA-4531-1-3-1	12722	14250	14657	13898
		DGA-4531-1-4-1	13523	14779	15179	14532
		DGAVULLEX-4531-1-31-1	12823	13681	14293	13527
		DGAVULLEX-4531-1-32-1	12874	13645	14193	13490
		DGAVULLEX-4531-1-33-1	21695	22601	23506	22422
		DGAVULLEX-4531-1-34-1	19092	20608	21388	20325
	Specification Limits	DGAUPPER-4531-1-60-1	21908	23396	24271	23126
		DGACENTER-4531-1-61-1	9961	10315	10893	10280
		DGALOWER-4531-1-62-1	11991	12908	13601	12738
Crushed Stone Base		CSBUPPER-4531-1-63-1	12878	13460	14043	13337
		CSBCENTER-4531-1-64-1	18358	19043	19621	18899
		CSBLOWER-4531-1-65-1	15379	16129	16823	15989
No. 57 Stone	As Received	No57VULLEX-4531-1-58-1	21422	22767	23575	22530
		No57VULLEX-4531-1-58-2	32622	37458	38281	36449
		No57VULLEX-4531-1-58-3	19189	21051	21749	20670
		No57VULLEX-4531-1-58-4	22085	23856	24577	23490
		No57VULLEX-4531-1-58-5	22894	25097	25963	24669
		No57VULLEX-4531-1-58-6	21884	23278	24041	22983
	Repeats	No57VULLEX-4531-1-57-1	24411	25989	26784	25668
		No57VULLEX-4531-1-57-2	24939	27542	28189	27000
		No57VULLEX-4531-1-57-3	27029	29100	30094	28677
		No57VULLEX-4531-1-57-4	24074	26297	27196	25820
		No57VULLEX-4531-1-57-5	28745	31650	32714	31046
River Gravel		RGRAV-4531-1-21-1	13451	14159	14790	14005
		RGRAV-4531-1-22-1	11630	11958	12740	11903
		RGRAV-4531-1-23-1	12847	13689	14351	13515
		RGRAV-4531-1-24-1	11965	12965	13524	12766
		RGRAV-4531-1-25-1	11342	11951	12546	11824
		RGRAV-4531-1-26-1	14763	15369	16071	15250
Recycled Concrete		RECON-4531-1-11-1	16733	18494	19584	18159
		RECON-4531-1-12-1	15683	16817	17421	16581
		RECON-4531-1-13-1	12691	13670	14372	13481
		RECON-4531-1-14-1	12831	13745	14412	13566
		RECON-4531-1-15-1	14600	15609	16306	15400
		RECON-4531-1-16-1	16271	17374	18044	17151
		RECON-4531-1-17-1	15251	16209	16950	16022
Average Mr-Values (Granular Materials)			17571	18914	19632	18644
Asphalt Drainage Blanket		ADB-4531-1-66-1	230159	250137	250570	246912
PVC Cylinder		PVC-4531-1-42-1	28899	30101	32182	29802
		PVC-4531-1-42-1	29233	30146	32254	29880
		PVC-4531-1-43-1	29579	30649	32702	30327
		PVC-4531-1-44-1	29961	30866	32677	30573
		PVC-4531-1-45-1	30207	30939	32559	30685

Table 12. Comparison of numerical values of M_r obtained from Models 3, 4, 5, and 6 and calculated at stresses of σ_3 = 10 and σ_d= 20.

Sample Description		Sample Number	Model 3 Seed's Model	Model 4 Uzan's Model	Model 5 UKTC	Model 6NCHRP Model 6
			σ_{sum} = 50 psi	σ_{sum} = 50 psi σ_d = 3 psi	σ_3 = 10 psi σ_d = 20 psi	σ_{sum}= 50 psi σ_d = 3 psi
			Resilient Modulus, M_r (psi)			
Dense Graded aggregate (DGA)	As Received	DGA-4531-1-3-1	39625	36795	37014	36945
		DGA-4531-1-4-1	38153	36016	36167	36059
		DGAVULLEX-4531-1-31-1	42819	39939	40022	39898
		DGAVULLEX-4531-1-32-1	36479	34218	34342	34223
		DGAVULLEX-4531-1-33-1	55717	53248	53434	53254
		DGAVULLEX-4531-1-34-1	44645	41066	41031	41013
	Specification Limits	DGAUPPER-4531-1-60-1	51687	48063	48125	47990
		DGACENTER-4531-1-61-1	36480	35122	35139	34965
		DGALOWER-4531-1-62-1	42751	39475	39502	39420
Crushed Stone Base		CSBUPPER-4531-1-63-1	36288	34527	34732	34547
		CSBCENTER-4531-1-64-1	48690	46845	46892	46879
		CSBLOWER-4531-1-65-1	44813	42534	42634	42515
No. 57 Stone	As Received	No57VULLEX-4531-1-58-1	46707	43677	43738	43602
		No57VULLEX-4531-1-58-2	54972	47174	47307	47194
		No57VULLEX-4531-1-58-3	43356	39155	39274	39171
		No57VULLEX-4531-1-58-4	47647	43785	43882	43804
		No57VULLEX-4531-1-58-5	47308	42762	42747	42710
		No57VULLEX-4531-1-58-6	47649	44496	44620	44542
	Repeats	No57VULLEX-4531-1-57-1	53136	49612	49689	49633
		No57VULLEX-4531-1-57-2	53138	47756	47889	47811
		No57VULLEX-4531-1-57-3	58385	53839	53889	53848
		No57VULLEX-4531-1-57-4	51779	46878	47070	46933
		No57VULLEX-4531-1-57-5	58570	52593	52726	52607
River Gravel		RGRAV-4531-1-21-1	38847	36704	36839	36750
		RGRAV-4531-1-22-1	36755	35591	35663	35543
		RGRAV-4531-1-23-1	40722	37953	38163	37974
		RGRAV-4531-1-24-1	36579	33415	33826	33402
		RGRAV-4531-1-25-1	34198	32269	32353	32287
		RGRAV-4531-1-26-1	40807	39040	39046	39035
Recycled Concrete		RECON-4531-1-11-1	48604	43406	43388	43297
		RECON-4531-1-12-1	39720	36793	36892	36832
		RECON-4531-1-13-1	37874	34817	34982	34785
		RECON-4531-1-14-1	36286	33503	33845	33469
		RECON-4531-1-15-1	39626	36738	36969	36748
		RECON-4531-1-16-1	43460	40435	40557	40449
		RECON-4531-1-17-1	39899	37266	37380	37244
Average Mr-Values (Granular Materials)			44283	41042	41160	41038
Asphalt Drainage Blanket		ADB-4531-1-66-1	206566	188427	187708	187549
PVC Cylinder		PVC-4531-1-42-1	102138	100681	100816	100922
		PVC-4531-1-42-1	102138	98622	98698	99006
		PVC-4531-1-43-1	103491	99302	99790	99784
		PVC-4531-1-44-1	102767	99252	99995	99752
		PVC-4531-1-45-1	101810	99157	99362	99623

Table 13. Comparison of numerical values of M_r obtained from Models 3, 4, 5, and 6 calculated at stresses of σ_3 = 20 and σ_d= 40.

Sample Description		Sample Number	Model 3 Seed's Model	Model 4 Uzan's Model	Model 5 UKTC	Model 6NCHRP Model 6
			σ_{sum} = 100 psi	σ_{sum} = 100 psi σ_d = 40 psi	σ_3 = 20 psi σ_d = 40 psi	σ_{sum} = 100 psi σ_d = 40 psi
			Resilient Modulus, M_r (psi)			
Dense Graded aggregate (DGA)	As Received	DGA-4531-1-3-1	*68805*	*62780*	*65554*	*62216*
		DGA-4531-1-4-1	*63139*	*58760*	*61089*	*58188*
		DGAVULLEX-4531-1-31-1	*76906*	*70729*	*73704*	*69820*
		DGAVULLEX-4531-1-32-1	*60497*	*56032*	*58239*	*55460*
		DGAVULLEX-4531-1-33-1	*88093*	*83439*	*86299*	*82840*
		DGAVULLEX-4531-1-34-1	*67447*	*61043*	*62434*	*60100*
	Specification Limits	DGAUPPER-4531-1-60-1	*78423*	*71921*	*73817*	*70915*
		DGACENTER-4531-1-61-1	*68530*	*65486*	*68128*	*64681*
		DGALOWER-4531-1-62-1	*79267*	*72058*	*74769*	*70979*
Crushed Stone Base		CSBUPPER-4531-1-63-1	*60020*	*56577*	*58817*	*56161*
		CSBCENTER-4531-1-64-1	*78197*	*74637*	*77313*	*74238*
		CSBLOWER-4531-1-65-1	*75335*	*70789*	*73319*	*70133*
No. 57 Stone	As Received	No57VULLEX-4531-1-58-1	*68203*	*62954*	*64408*	*62116*
		No57VULLEX-4531-1-58-2	*70830*	*59031*	*59887*	*57598*
		No57VULLEX-4531-1-58-3	*64415*	*57044*	*58545*	*56124*
		No57VULLEX-4531-1-58-4	*69220*	*62594*	*64121*	*61761*
		No57VULLEX-4531-1-58-5	*67304*	*59663*	*60707*	*58588*
		No57VULLEX-4531-1-58-6	*69531*	*64093*	*65736*	*63461*
	Repeats	No57VULLEX-4531-1-57-1	*77528*	*71438*	*73255*	*70672*
		No57VULLEX-4531-1-57-2	*76732*	*67509*	*69328*	*66430*
		No57VULLEX-4531-1-57-3	*84868*	*77047*	*78742*	*76046*
		No57VULLEX-4531-1-57-4	*75109*	*66765*	*68428*	*65787*
		No57VULLEX-4531-1-57-5	*82759*	*72832*	*74295*	*71580*
River Gravel		RGRAV-4531-1-21-1	*65025*	*60760*	*63067*	*60305*
		RGRAV-4531-1-22-1	*64277*	*61878*	*63896*	*61443*
		RGRAV-4531-1-23-1	*71314*	*65573*	*68305*	*64868*
		RGRAV-4531-1-24-1	*62944*	*56478*	*59468*	*55636*
		RGRAV-4531-1-25-1	*58452*	*54538*	*56546*	*54066*
		RGRAV-4531-1-26-1	*66864*	*63420*	*65401*	*62951*
Recycled Concrete		RECON-4531-1-11-1	*81583*	*71389*	*73043*	*69836*
		RECON-4531-1-12-1	*62377*	*56931*	*58764*	*56295*
		RECON-4531-1-13-1	*64415*	*58287*	*60454*	*57421*
		RECON-4531-1-14-1	*60121*	*54701*	*57051*	*53928*
		RECON-4531-1-15-1	*64358*	*58814*	*61057*	*58120*
		RECON-4531-1-16-1	*70035*	*64264*	*66453*	*63532*
		RECON-4531-1-17-1	*63653*	*58684*	*60511*	*57987*
Average Mr-Values (Granular Materials)			70183	64193	66249	63397
Asphalt Drainage Blanket		ADB-4531-1-66-1	*195996*	*175735*	*173386*	*171993*
PVC Cylinder		PVC-4531-1-42-1	*197898*	*187305*	*193907*	*186467*
		PVC-4531-1-42-1	*187533*	*179873*	*186333*	*179796*
		PVC-4531-1-43-1	*190145*	*181151*	*188192*	*181097*
		PVC-4531-1-44-1	*187002*	*179548*	*187447*	*179724*
		PVC-4531-1-45-1	*183690*	*177989*	*185447*	*178363*

Table 14. Comparison of average values of resilient modulus and percent difference relative to Model 5 for granular materials included in the testing program.

Model	σ_3 = 3 psi σ_d = 3pi σ_{sum} = 12 psi M_r (psi)	Percent Difference	σ_3 = 10 psi σ_d = 20 psi σ_{sum} = 50 psi M_r (psi)	Percent Difference	σ_3 = 10 psi σ_d = 20 psi σ_{sum} = 50 psi M_r (psi)	Percent Difference
Seed (Model 3)	17571	-10.5	44283	7.6	70183	5.9
Uzan (Model 4)	18914	-3.7	41042	-0.3	64193	-3.1
UKTC (Model 5)	19632	0	41160	0	66249	0
NCHRP (Model 6)	18644	-5	41038	-0.3	63397	-4.3

function of either the confining stress, σ_3, the deviator stress, σ_d, or the sum of the principle stresses, σ_{sum}. In Figure 32, the resilient modulus is shown as a function of the deviator stress. By clicking "Check Model", the user may select a model type from a dropdown menu and graph the data. Coefficients (k_1, k_2, or k_3) of each model equation are displayed at the bottom of the GUI screen in Figure 32. Each time the user clicks on a specimen number in the left-hand portion of Figure 32, multiple regression analysis is automatically performed using the three different models shown in the right-hand portion of Figure 32. The coefficients of each model are displayed for each three-dimensional plane. Multiple regression equations used in the database are presented in Appendix A.

The user may also recall and display resilient modulus test data by clicking "Data" under the "Show" button on a dropdown menu, as illustrated in Figure 33. In this case, the data are displayed as shown in that figure; an enlarged view of the summary data is displayed in Figure 34. The tabulations show the specimen number, sequence number, values of resilient modulus for each test sequence, the confining stresses, σ_3, the deviator stresses, σ_d, and the sum of the principle stresses, σ_{sum}, for each test sequence. Although the data for each aggregate specimen shown in Figure 34 resides in the Kentucky Geotechnical Database, the summary data for each specimen are also given in Appendices B through H.

If the user chooses to view the entire test data record of resilient modulus of a selected specimen in the database, then the following procedure is available:

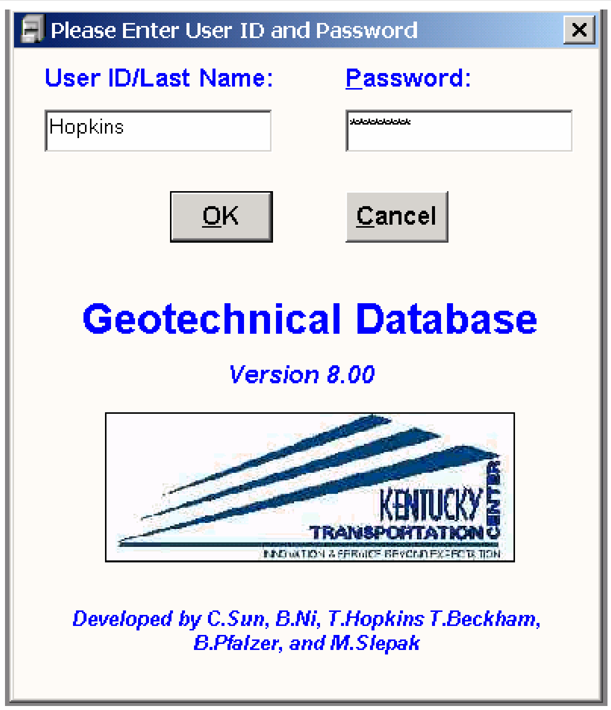

Figure 29. User log-on graphical user interface screen for gaining access to the Kentucky Geotechnical Database and resilient modulus data.

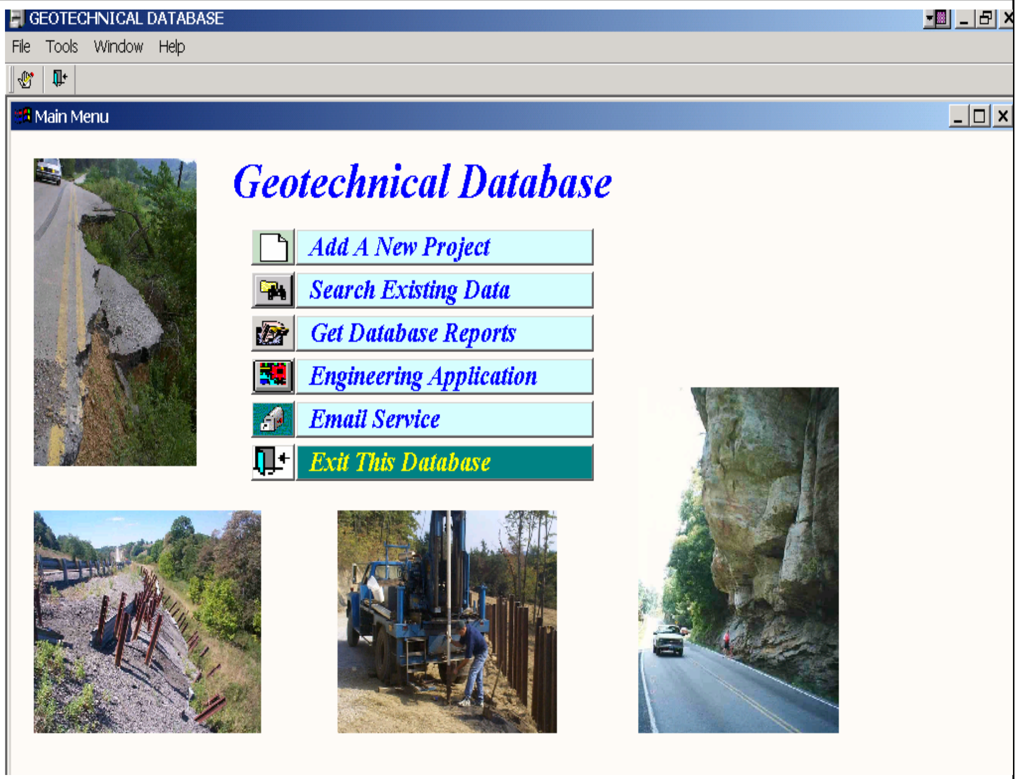

Figure 30. Main menu of the Kentucky Geotechnical Database.

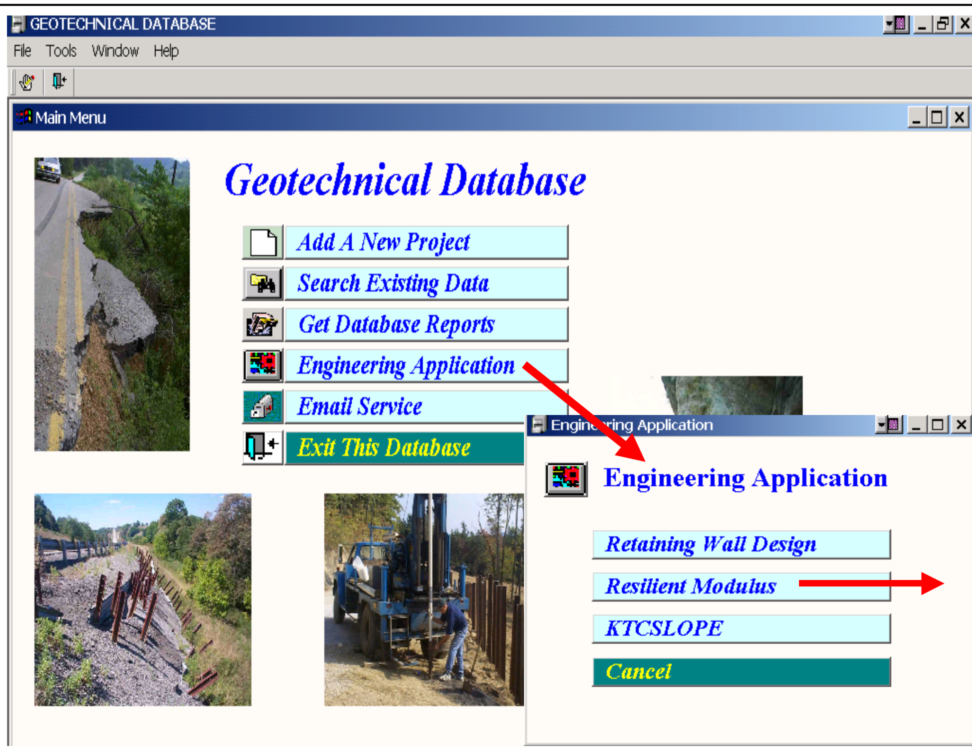

Figure 31. Gaining access to resilient modulus test results for compacted soils and aggregates in the Kentucky Geotechnical Database.

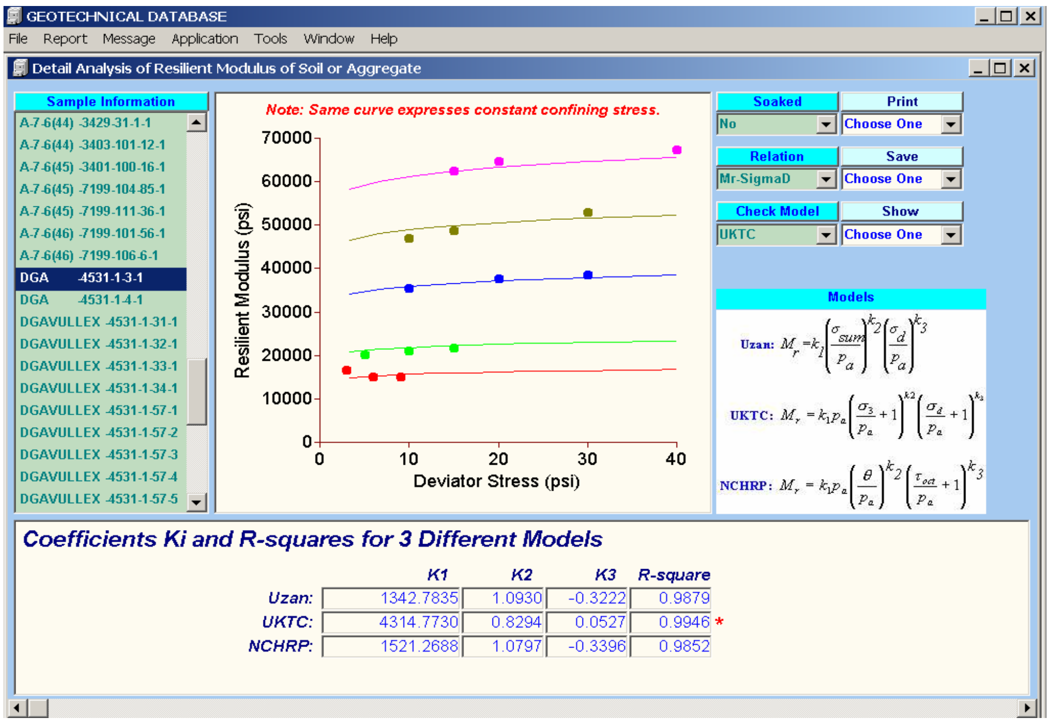

Figure 32. Graphical user interface showing resilient modulus as a function of deviator stress for a selected type of aggregate.

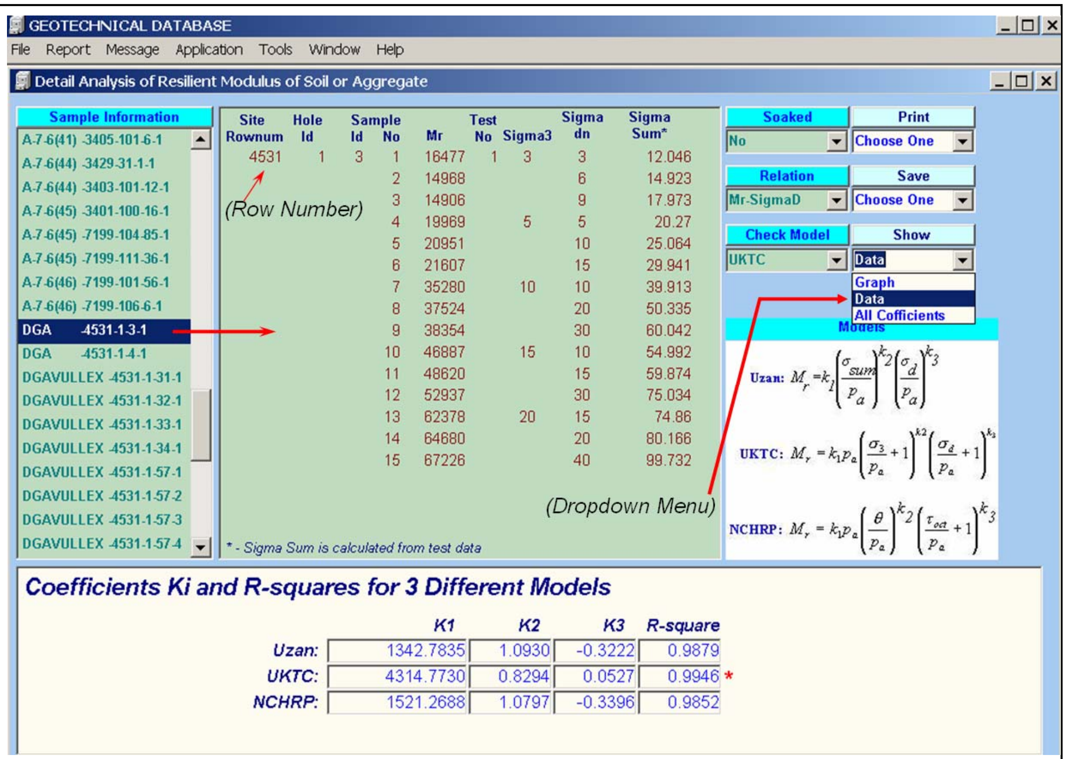

Figure 33. Display of resilient modulus summary data in the database.

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum*
4531	1	3	1	16477	1	3	3	12.046
			2	14968			6	14.923
			3	14906			9	17.973
			4	19969		5	5	20.27
			5	20951			10	25.064
			6	21607			15	29.941
			7	35280		10	10	39.913
			8	37524			20	50.335
			9	38354			30	60.042
			10	46887		15	10	54.992
			11	48620			15	59.874
			12	52937			30	75.034
			13	62378		20	15	74.86
			14	64680			20	80.166
			15	67226			40	99.732

** - Sigma Sum is calculated from test data*

Figure 34. View of resilient modulus test data for a selected aggregate specimen.

- In the database's main menu, Figure 30, the user clicks on "Search Existing data." When this event is executed, a GUI (Graphical User Interface) screen appears as shown in Figure 35. All resilient modulus data of aggregates are stored in a site labeled "4531". Consequently, the number "4531" will appear in all aggregate specimen identification numbers, as shown in Figure 32.

- Using the row number (or site number) shown in Figure 34 ("4531"), and inserting this number into the box labeled "Site Row Number" in the GUI shown in Figure 35, and clicking the search button, a GUI screen appears, as shown in Figure 36. Clicking on "Samples" in the upper right-hand corner of Figure 36, listings of specimen numbers appear in the lower right-hand corner of this figure. By double-clicking on any desired specimen number in the lower right-hand corner of Figure 36, the GUI screen shown in Figure 37 appears.

- By clicking the "Properties" tab in Figure 37, the GUI screen shown in Figure 38 appears. Clicking on the folder labeled "Resilient Modulus" obtains detailed resilient modulus data for a selected specimen number, as illustrated in Figure 39.

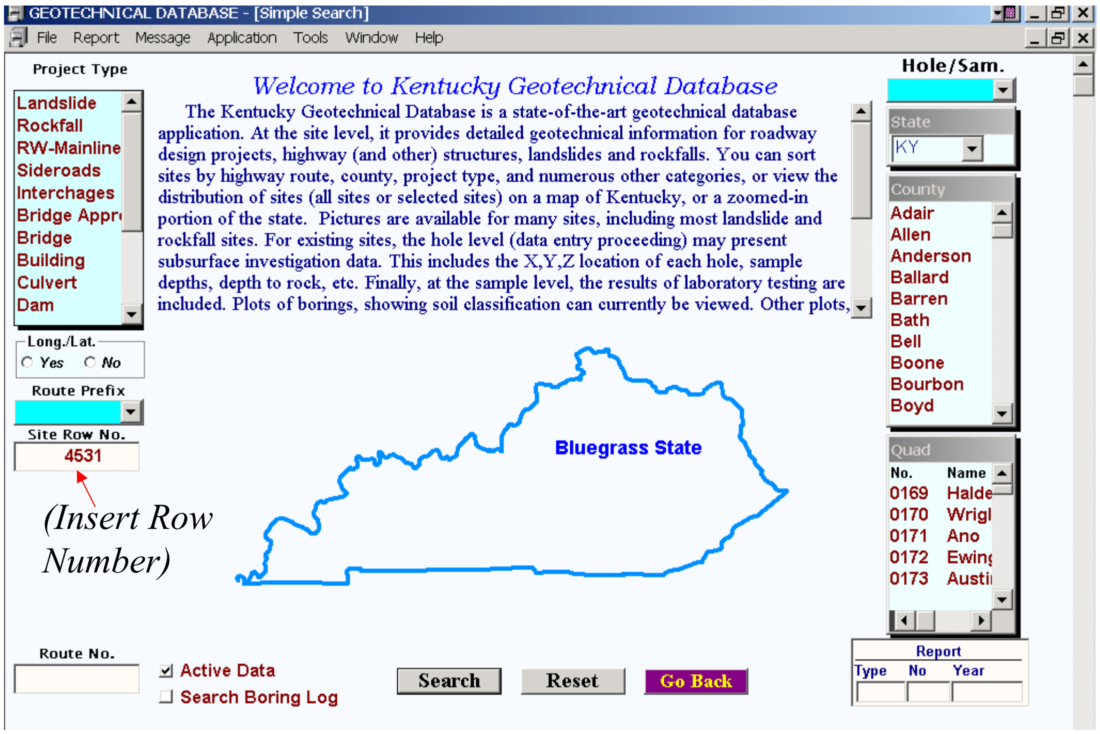

Figure 35. GUI screen for searching data.

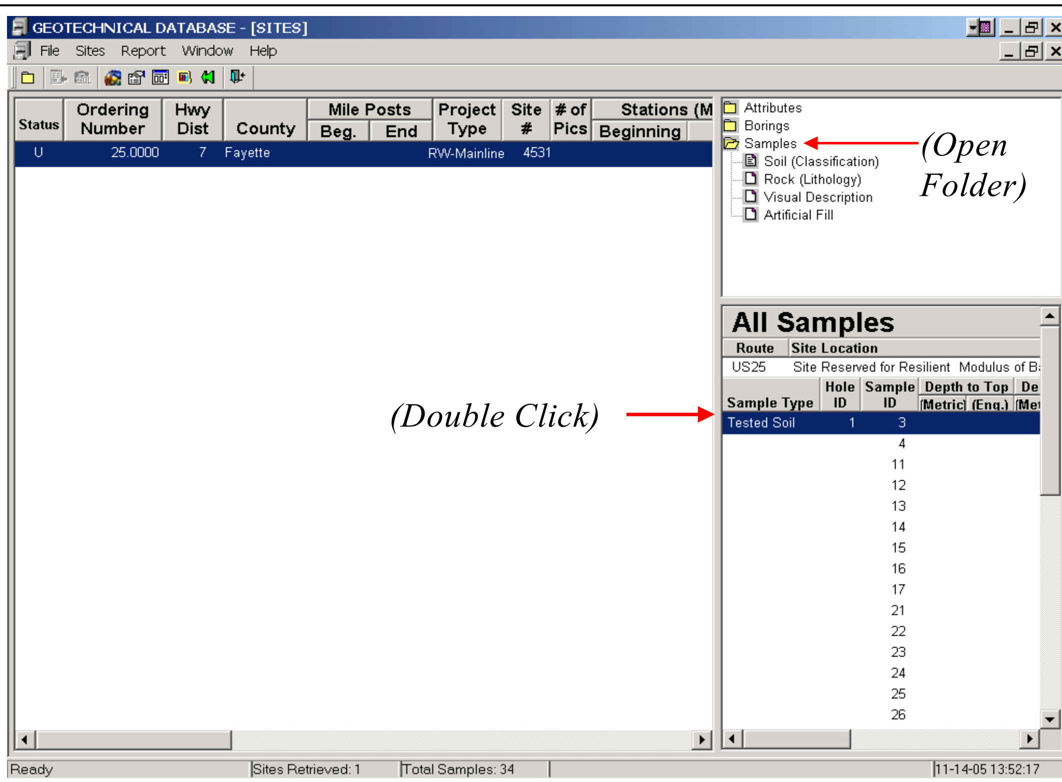

Figure 36. Gaining access to resilient modulus test record for a selected specimen in the Kentucky Geotechnical Database.

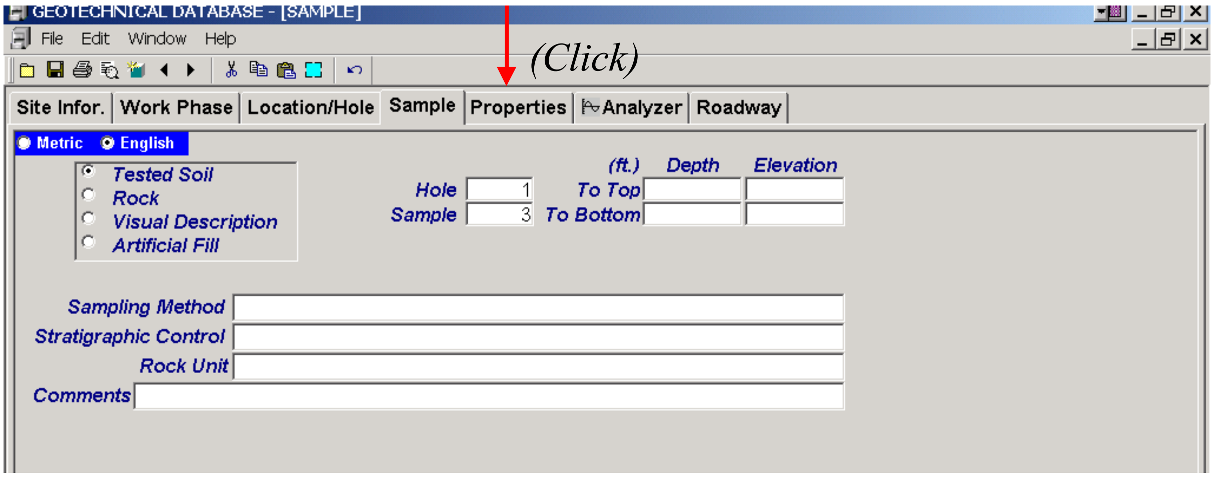

Figure 37. GUI screen for accessing the complete resilient modulus test data.

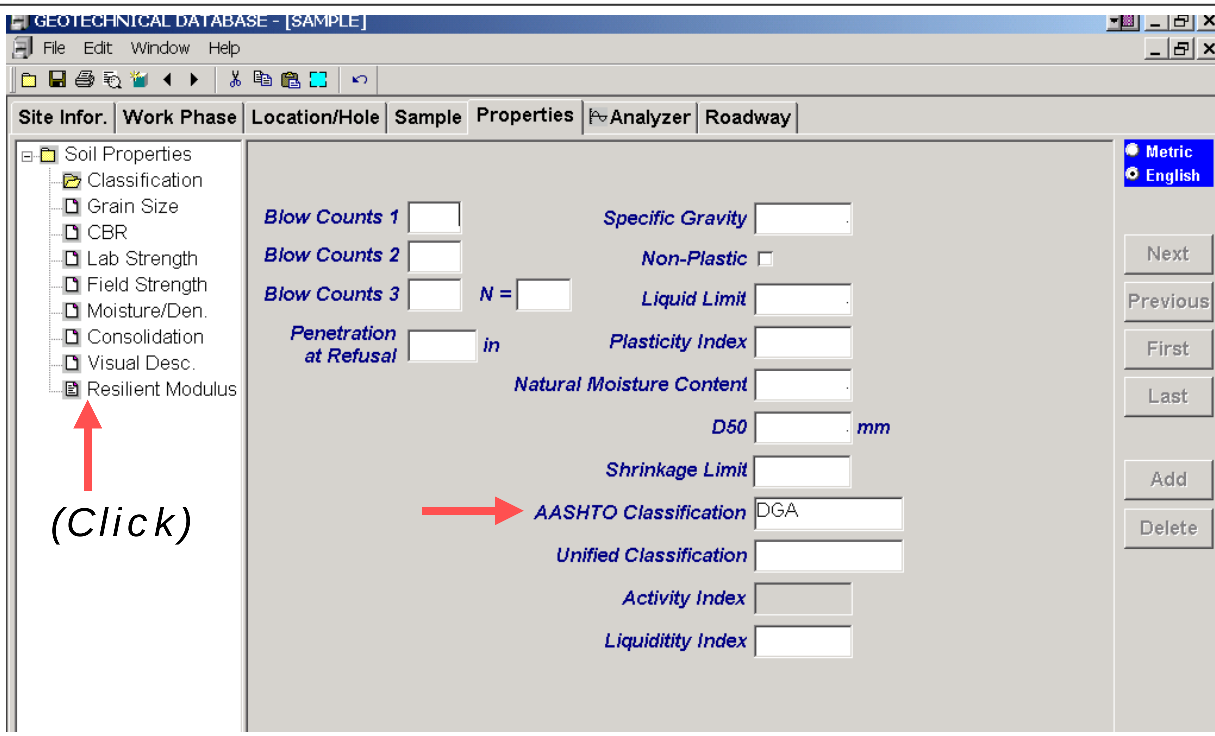

Figure 38. GUI screen for obtaining detailed resilient modulus test data.

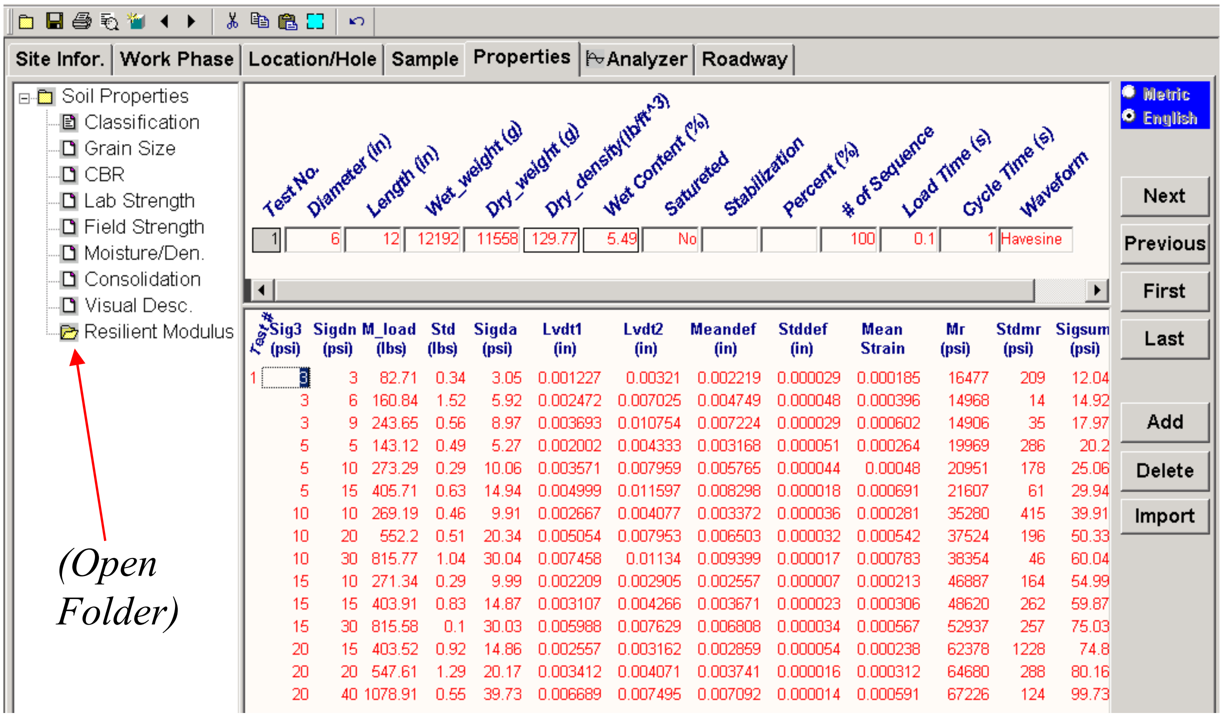

Figure 39. Complete resilient modulus test record for a selected aggregate specimen.

Variation of Resilient Modulus and Dry Density

Because dry densities of different types of aggregates vary in the field, numerous resilient modulus tests were performed on selected aggregate types to determine the effect of the variation of dry density on the values of resilient modulus. Aggregate types included Dense Graded Aggregate, Crushed Stone Base, No. 57 Stones, River Gravel, and Recycled Concrete.

Resilient modulus test results obtained for specimens of the dense graded aggregate (as received from the producer) compacted at different dry densities are shown in Figure 40. Values of resilient modulus shown in Figure 40 are those obtained from Model 5 (UKTC), which are listed Tables 11, 12, and 13. The maximum dry density and optimum moisture content (Figure 9) of this “well graded” material were 142.2 lbs/ft^3 and 6.8 percent, respectively. In this plot, resilient modulus values of DGA specimens were calculated at three selected stress conditions and graphed as a function of dry density. The percent of maximum dry density of this series of tests ranged from 72 to 100. For example, the highest and lowest values of M_r observed for specimen DGA-4531-1-3-1 were 65,554 and 14,657 psi, respectively. Dry density and moisture content of this specimen were 132.6 and 5.9 percent, respectively. The specimen had been compacted at 93 percent of maximum dry density.

As shown in Figure 40, the resilient modulus values changed slightly as the percent of maximum dry density increased from 72 percent (103.6 lbs/ft^3) to about 96 percent (136.6 lbs/ft^3). However from about 96 percent of maximum dry density to 100 percent of maximum dry density (142.2 lbs/ft^3) a sharp increase occurred in the values of resilient modulus. For example, from 72 percent to about 96 percent, the resilient modulus of the lower curve in Figure 40 increases only approximately 4 percent. From about 96 percent to 100 percent of maximum dry density the M_r-value increases about 37 percent. The M_r-value of the upper curve increases about 7 percent when the percent of

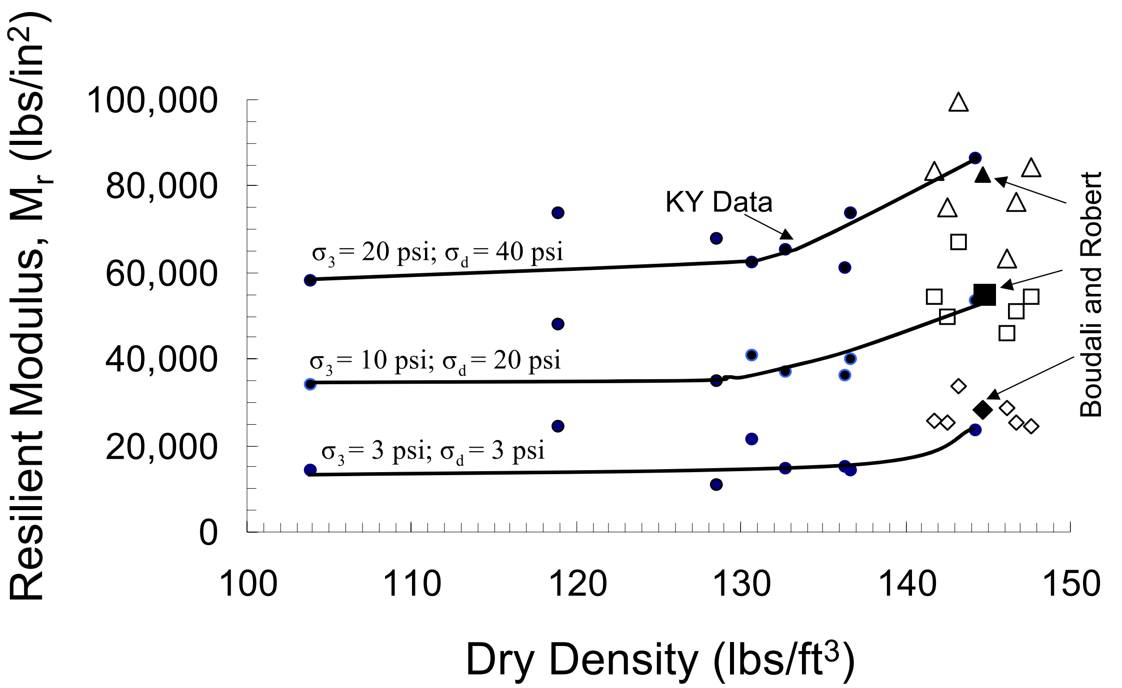

Figure 40. Variation of resilient modulus, M_r, as a function of dry density of Dense Graded Aggregate specimens

maximum dry density increases from 72 to about 92. From 92 to 100 percent of maximum dry density the value of M_r increases about 37 percent.

Average values for crushed (similar to DGA) limestone of residual modulus published by Boudali and Roberts (1998) are compared in Figure 40 to M_r-values obtained in this testing program. They performed resilient modulus tests on six different (approximate) gradations occurring between the gradation limits shown in Figure 41. Those tests were performed on compacted specimens. Dry densities of their specimens were not varied too much and ranched from only about 141.7 to 147.5—a variation of only about 4 percent. The dry densities averaged about 144.7 lbs/ft^3. Values of resilient of their tests ranged from 24,548 (lowest value observed for the six tests) to 99,475 psi (highest value observed for the six tests). The "open" data points in Figure 40 represent the values obtained by Boudali and Roberts for the six tests using Uzan's model. Average observed values (large filled points) for the six tests ranged from 28,131 to 82,711 psi. The later values are compared to values obtained from tests performed on the "as received" DGA specimens. The resilient modulus of specimen DGAVullex 4531-1-33-1 ranged from a low value of resilient modulus of 23,506 to a high value of 86,299 psi. This specimen had been compacted to a value of about 100 percent of maximum dry density.

Resilient modulus tests were performed on DGA specimens composed of blended gradations that represented the upper and lower specifications limits, as well as a gradation representing the center of those specification limits (see Figure 4). Resilient modulus results obtained from the UKTC model are shown in Table 15 for three selected, testing stress states. Theses stresses represent a lower, center, and higher testing states of stress. Dry density as a function of resilient modulus values shown in Table 15 are shown and compared to the DGA test results in Figure 40.

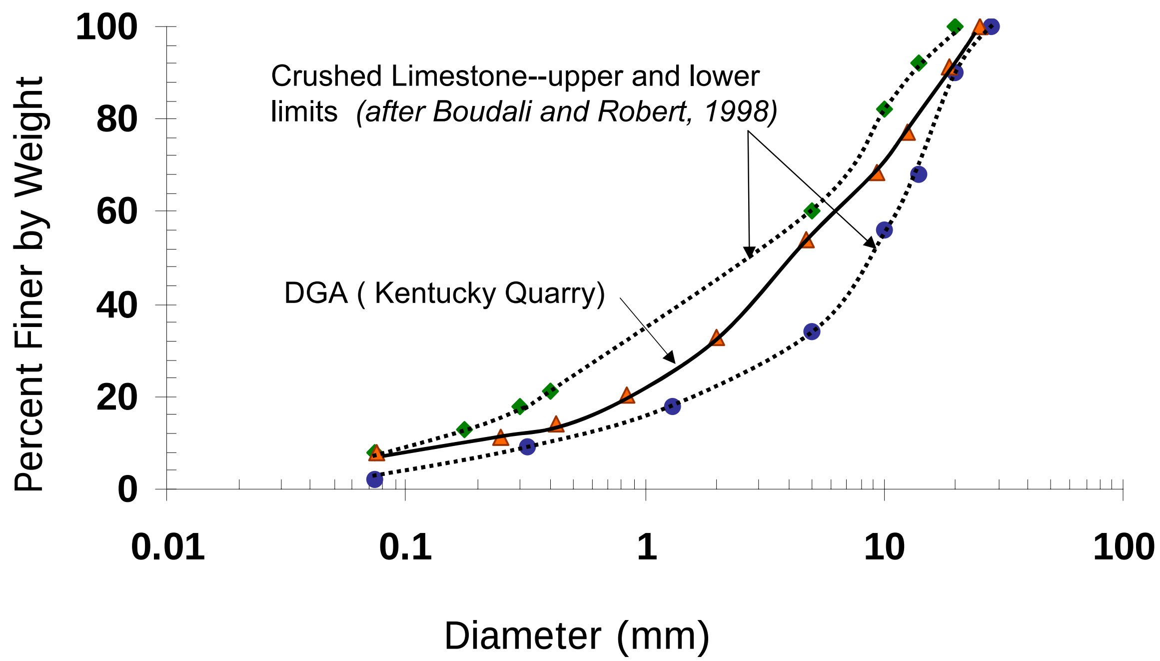

Figure 41. Comparison of DGA gradation as received and tested from a producer in Kentucky and the approximate upper and lower gradation limits of crushed limestone reported by Boudali and Robert, (1998).

Table 15. Minimum and maximum resilient values observed for different specification gradation limits of DGA.

Gradation Curve[1]	Dry Density (lbs/ft^3)	Moisture Content (%)	Percent of Maximum Dry Density	Relative Density, Dr (Percent)	Resilient Modulus (psi) (Selected Stress States)		
					$\sigma_3 = 3$ psi $\sigma_d = 3$ psi	$\sigma_3 = 10$ psi $\sigma_d = 20$ psi	$\sigma_3 = 20$ psi $\sigma_d = 40$ psi
Upper Specification Limit[2]	118.9	2.3[3]	83.6	--	24,271	48,125	73,817
Center[2]	128.5	4.8	89.2	--	10,893	35,139	68,128
Lower Specification Limit	117.4	1.9	---	103.6[4]	13,601	39,502	74,769

1. Gradation curves of tested specimens are shown in Figures 4 and 6.
2. Maximum dry density and optimum moisture of the upper and center gradation materials were 142.5 lbs/ft^3 and 6.6 percent, respectively.
3. Specimen could not be tested at 6.6 percent because excess pore pressures built-up during cyclic loading. Moisture content of specimen reduced to avoid excess pore pressure built-up.
4. Minimum and maximum dry densities were 107.4 and 113.6 lbs/ft^3.

Results of resilient modulus tests performed on specimens of Crushed Stone Base are summarized in Table 16. The blended specimens represent the upper and lower specifications limits, as well as a gradation representing the center of those specification gradation limits (see Figure 6). Values of resilient modulus shown in the table were obtained from the UKTC model at three selected, testing stress states. Theses stresses represent a lower, center, and higher testing states of stress. Resilient modulus values ranged from a low value of 14,043 psi to a high value of 77,313 psi..

Table 16. Minimum and maximum resilient values observed for different specification gradation limits of Crushed Stone Base.

Gradation Curve[1]	Dry Density	Moisture Content	Percent of Maximum Dry Density	Relative Density, Dr (Percent)	Resilient Modulus (psi) (Selected Stress States) σ_3 = 3 psi, σ_d = 3 psi	σ_3 = 10 psi, σ_d = 20 psi	σ_3 = 20 psi, σ_d = 40 psi
Upper Specification Limit[2]	139.5	4.8	96.3	--	14,043	34,732	58,817
Center[2]	140.9	3.5	97.8	--	19,621	46,893	77,313
Lower Specification Limit	113.7	2.6	--	≈100[4]	16,823	42,635	73,319

1. Gradation curves of tested specimens are shown in Figure 6.
2. Maximum dry density and optimum moisture content of the upper and center gradation materials were 142.5 lbs/ft^3 and 6.6 percent, respectively.
3. Specimen could not be tested at 6.6 percent because excess pore pressures built-up during cyclic loading. Moisture content reduced to avoid excess pore pressure built-up.
4. Minimum and maximum dry densities were 102.2 and 114.2 lbs/ft^3

As shown in Figure 42, variation of resilient modulus values as a function of dry density was also determined for the No. 57 crushed stone. Resilient modulus results (obtained from the UKTC model) are shown in Tables 11, 12, and 13 for three selected, testing stress states. Theses stresses represent a lower, center, and higher testing states of stress. Since this material contained no fines, the specimens were compacted at different relative densities, $D_{r.}$ Minimum and maximum dry densities of this material were 90.0 and 97.3 lbs/ft^3, respectively. At the lowest selected stress state , resiient modulus ranged from an average value of 23,762 psi to 28,966 psi for average values of relative compaction of one and 100 percent, repectivley (lower curve in Figure 42). As the dry density increased, the M_r-value of the lower curve increased approximately 18 percent. At the larger stress state (upper curve in Figure 42), the M_r-value only increased about 3 percent as the dry density increased from 90 to 98 lbs/ft^3 , or the relative density increased from about one to 100 percent. The average M_r-value increased from about 61,220 to 63,248 psi, repectivley. Overall, the resilient modulus of the the "uniform-graded" (see Figure 6) No. 57 Stone did not increase significant as the dry density increased.

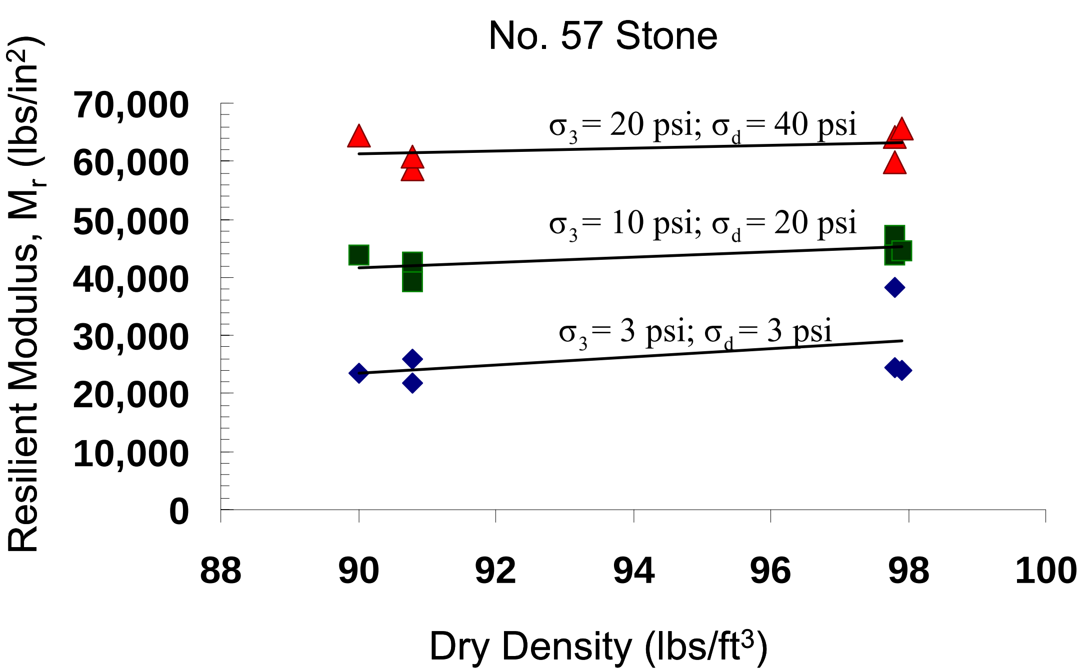

Figure 42. Variation of resilient modulus, M_r, as a function of dry density of No. 57 crushed stone specimens.

Attempts to establish a moisture density relation for the Crushed River Gravel were unsuccessful because the material lacked sufficient fines. The percent finer than the US Standard Sieve No. 200 was only 4 percent. The maximum and minimum dry densities of this material were established using the equipment in Figure 14 and were equal to 105.2 and 128.2 lbs/ft^3, respectively. Relative densities of the specimens ranged from about 4 to 100 percent.

Variation of the resilient modulus of river gravel with increasing dry density is illustrated in Figure 43. At the lower stress state (lower curve), the average resilient modulus ranged from 13,960 psi at a relative compaction of about 4 percent to 14,047 psi at a ralative compaction of about 100 percent. The change in M_r was essentially unchanged. At the higher stress state (upper curve), the M_r-value decreased slightly about 7 percent as the dry density increased. The average resilient modulus ranged from 65,089 psi at a relative compaction near 4 percent to 60,472 psi at a relative compaction of about 100 percent. Essentially, the value of M_r did not change significantly with increasing density for this material.

The percent finer than the US Standard No. 200 Sieve of the "as received" Recycled Concrete sample was only 1.7 percent, as shown in Figure 8. Consequently, the maximum and minimum values of dry density were determined because of insufficient fines needed to establish a moisture-density relation. Maximum and minimum dry densities were 107.3 and 94.4 lbs/ft^3**,** respectivley. Relative densities of the resiilient modulus test specimens ranged from zero to values above 100 percent. Apparently, some crushing of the concrete pieces may have occurred when attempting to compact specimens to relative densities of 100 percent. In attempting to determine the maximum dry density, the shaker table was used and the specimen was not compacted with a Proctor hammer. Consequently, the compaction of specimens may have created some fine material during compaction which resulted in a dry density greater than the value obtained from the shaker table. Resilent modulus values obtained from the UKTC model using selected stress conditons are shown as a function of dry density in Figure 44. In each case, the reslient increases with increasing dry density.

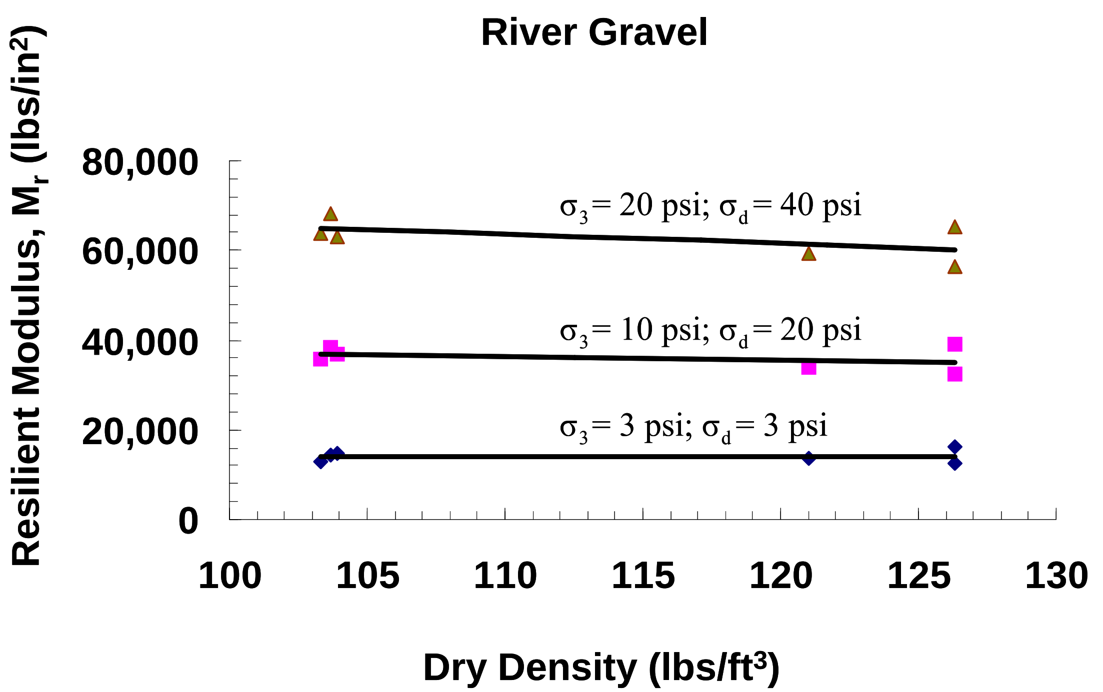

Figure 43. Variation of resilient modulus, M_r, as a function of dry density of crushed river gravel.

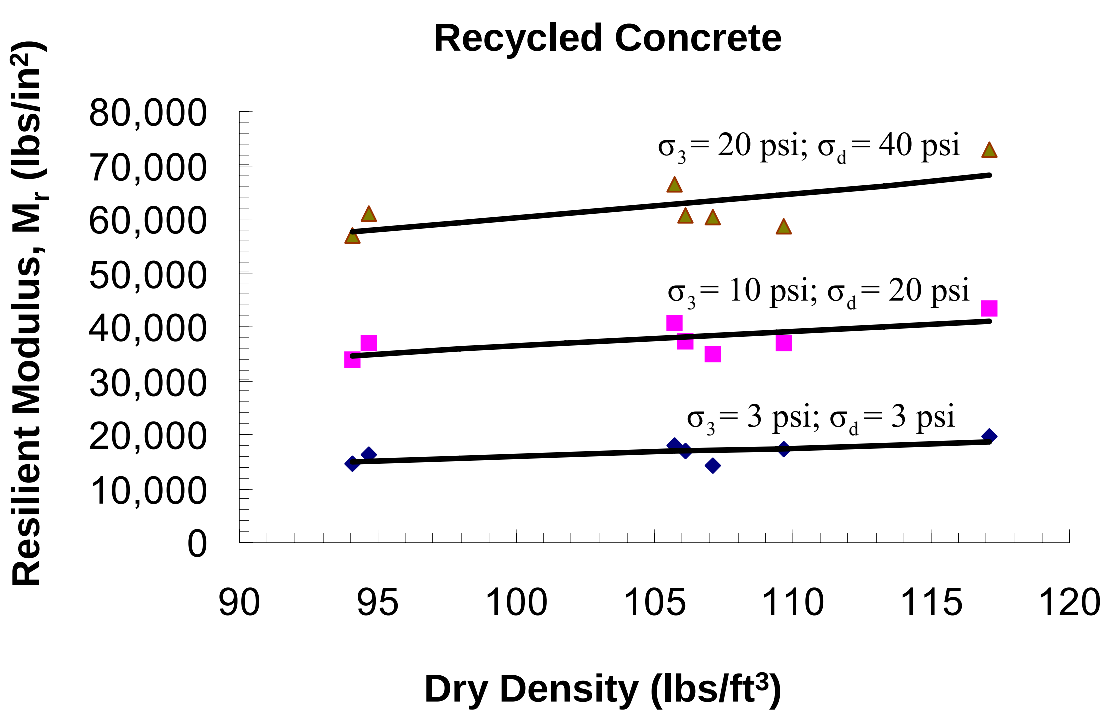

Figure 44. Variation of resilient modulus, M_r, as a function of dry density of recycled concrete aggregate.

As the dry density increases from about 94.4 to 117.7 lbs/ft^3, the reslient modulus of the lower curve at selected stresses shown in the figure increase approximately 15 percent. At the stresses shown in the figure, the reslient modulus of the upper curve increases about 19 percent.

Repeatability of Resilient Modulus Testing Equipment

Synthetic Specimen of PVC

Repeatability of the resilient modulus test is dependent on the testing equipment, skills of the operator, and the composition of the testing specimen. To be viable, however, testing equipment should reproduce results that are acceptable within certain limits. To check the ability of the resilient modulus testing equipment used in this study, tests were performed on a very uniform cylindrical specimen of PVC plastic (polyvinyl chloride or vinyl) measuring 6 inches in diameter and 12 inches in height—the same dimensions used in remolding specimens of aggregate. The synthetic specimen provided quality control during testing. This specimen was routinely tested during the aggregate resilient modulus-testing program to insure that uniform results were obtained from the resilient modulus testing equipment during production testing.

Five resilient modulus tests were performed on the PVC cylinder to examine the repeatability of the test. Each time the test was completely broken down and the cylinder was removed from the chamber and reset. LVDTs were reset. Correlation regression coefficients obtained from Models 4, 5, 6, and 6 for the PVC tests were summarized in Table 10. Resilient modulus values based on those coefficients for each model are given in the top portion of Table 17. Values of R^2 for the five repeated tests obtained from the four models ranged from 0.953 to 0.985. Resilient modulus values were computed for three different, selected states of stress, as shown in the table. Minimum and maximum values obtained from each model at the 95 percent confidence level are also shown. At the 95 percent confidence level, the percentage differences in minimum and maximum computed values of resilient modulus ranged from 1.4 to 7.5 percent. The percentage differences of Seed's model ranged from 3.6 to 7.2 percent while Uzan's model yielded percentage values that ranged from 2.0 to 5.0. The percentage differences for the UKTC model and the NCHRP model ranged from 1.6 to 4.4 and 1.9 to 4.3, respectively. These results indicated that the test could be repeated with a good degree of confidence.

No. 57 Aggregate

As another means of examining the repeatability of the resilient modulus test, a second series of repeated resilient modulus tests were performed on aggregate specimens molded from No. 57 Stone. Specimens were remolded to nearly identical dry densities and at a relative compaction of about 8 percent. Results of this testing scenario are summarized in the bottom portion of Table 17. R^2-values from the Seed model ranged from 0.874 to 0.951. Values of R^2 from the other three models ranged from 0.990 to 0.998. At the 95 percent confidence level, percentage differences observed between maximum and minimum values computed from the correlation regression coefficients for three selected stress conditions ranged from 11.5 to 18.1. Based on those test results, the resilient modulus test data could be repeated with reasonable confidence.

Table 17. Ninety-five percent confidence levels of resilient tests repeated on a PVC cylinder and No. 57 stone aggregate.

Specimen	Model											
	Seed	Uzan	UKTC	NCHRP	Seed	Uzan	UKTC	NCHRP	Seed	Uzan	UKTC	NCHRP
	Stress State											
	$\sigma_3 = 3$ ps; $\sigma_d = 3$ psi $\sigma_{sum} = 12$ psi				$\sigma_3 = 10$ psi; $\sigma_d = 20$ psi $\sigma_{sum} = 50$ psi				$\sigma_3 = 10$ psi; $\sigma_d = 20$ psi $\sigma_{sum} = 100$ psi			
	M_r (psi)											
PVC-4531-1-41-1	28899	30101	32182	29802	105508	100681	100816	100922	197897	187305	193907	186467
PVC-4531-1-42-1	29233	30146	32254	29880	102138	98622	98698	99006	187533	179873	186333	179796
PVC-4531-1-43-1	29579	30649	32702	30327	103491	99302	99790	99784	190145	181151	188192	181097
PVC-4531-1-44-1	29961	30866	32677	30573	102767	99252	99995	99752	187002	179548	187447	179724
PVC-4531-1-45-1	30207	30939	32559	30685	101810	99157	99362	99623	183690	177989	185447	178363
Minimum	28899	30101	32182	29802	101810	98622	98698	99006	183690	177989	185447	178363
Maximum	30207	30939	32702	30685	105508	100681	100816	100922	197897	187305	193907	186467
Percentage Difference at the 95 percent confidence level	4.5	2.7	1.6	2.9	3.6	2.0	2.1	1.9	7.2	5.0	4.4	4.3
No57VULLEX-4531-1-57-1	24411	25989	26784	25668	53136	49612	49689	49633	77528	71438	73255	70672
No57VULLEX-4531-1-57-2	24939	27542	28189	27000	53138	47756	47889	47811	76732	67509	69328	66430
No57VULLEX-4531-1-57-3	27029	29100	30094	28677	58385	53839	53889	53848	84868	77047	78742	76046
No57VULLEX-4531-1-57-4	24074	26297	27196	25820	51779	46878	47070	46933	75109	66765	68428	65787
No57VULLEX-4531-1-57-5	28745	31650	32714	31046	58570	52593	52726	52607	82759	72832	74295	71580
Minimum	24074	25989	26784	25668	51779	46878	47070	46933	75109	66765	68428	65787
Maximum	28745	31650	32714	31046	58570	53839	53889	53848	84868	77047	78742	76046
Percentage Difference at the 95 percent confidence level	16.2	17.9	18.1	17.3	11.6	12.9	12.7	12.8	11.5	13.3	13.1	13.5

CONCLUSIONS

The following conclusions were made:

- Resilient modulus, by definition, is not a constant value but varies with the stress condition.

- Values of resilient modulus of well-graded aggregate increase as the dry density increases. Increases of resilient modulus were more noticeable and larger for well-graded aggregates than resilient modulus values of uniformly-graded aggregates. Values of resilient modulus of dense graded aggregate (DGA) generally were greater than values of the resilient modulus of the number 57 stone, crushed stone base, river gravel, and recycled concrete.

- Resilient modulus tests could not be performed on DGA specimens representing the upper gradation limit (Kentucky Transportation Cabinet Standard Specifications, 2002) and remolded to maximum dry density and optimum moisture content (AASHTO T-99). The specification allows a maximum of 13 percent of material finer than the US No. 200 sieve. The combination of a large percentage of fines and a moisture content near optimum creates high pore water pressures during cyclic loading, although the test is performed in an undrained state. By reducing the water content, the test could be performed. The build up of excess pore pressures has been observed indirectly in base materials as evidenced by the migration of fines to the surfaces of pavements.

- A number of tests were performed to define the resilient modulus of aggregates commonly used in pavement bases in Kentucky. Data that was developed will provide a good means for defining "Level 1," as well as "Level 2," resilient modulus input to the mechanistic model developed by AASHTO (American Association of State Highway and Transportation Officials).

- Based on resilient modulus repeatability tests, test results could be repeated with reasonable confidence.

RECOMMENDATIONS

- Studies are recommended to examine the following areas of research:

  - ❑ The effect of different gradations (or particle sizes) of base materials on the value of resilient modulus needs to be examined. The maximum, or the permissible, percentage of fines for DGA and Crushed Stone Bases should be determined which would not allow excess pore water pressures to build-up under cyclic loading of the resilient modulus test. Fines (controlled) are added to the matrix when the limestone materials are blended at the quarry.

  - ❑ The effect of migratory subgrade fines (clay-size particles) on the resilient modulus of base materials needs to be examined. During dissipation of excess pore pressures, fine clayey-size particles from the subgrade are pushed into the lower portion of the base aggregate. Strengths (and resilient modulus) of the base materials decrease when excess pore pressures occur in the soil subgrade. Secondly, as fines (uncontrolled) enter the bottom of base aggregates from an untreated (fine-grained) subgrade, excess pore pressures may build up in the base aggregates due to the increase of fines.

- The effectiveness of geofabrics (used as grade separators) to prevent migration of fines into the bottom of the aggregate base needs to be studied. Although the migration of fines may be prevented, the geofabric may clog and cease functioning with increasing time. If the material allows fine particles to pass into the base, than the resilient modulus of the base is altered. In either case, the resilient modulus of the base or/and subgrade will be altered.

- Extensive geotechnical research needs to be performed to examine "filter requirements" between base aggregates and clayey subgrades. Findings of this type of research could help redefine and improve the engineering functions of gradations of typical base aggregates commonly used in Kentucky. To prevent migration of subgrade fines into base aggregates, filter criteria must be met between a given type of soil subgrade and a selected type of base aggregate. Moreover, when filter fabric is used as a grade separator to prevent the migration of subgrade fines into the base aggregates, filter criteria must be satisfied between the subgrade soils and the fabric. This is a novel approach to improving the performance and function of base aggregates.

- Resilient modulus tests should be performed on laboratory and field specimens (Hopkins et al, 1995, 2002) of chemically stabilized subgrades. Values of resilient modulus will be needed as input in the new AASHTO mechanistic model. This study did not address this important determination since it was not within the scope of the study. In the pavement system, a chemically treated subgrade may function as a base in some cases or as a subbase in others. Chemical stabilization of subgrades in Kentucky is increasingly being used to improve the poor engineering properties of soils. Sufficient testing should be performed to provide "Level 1," as well as "Level 2," resilient modulus data input to the mechanistic model developed by AASHTO. Chemical admixtures to be examined should include hydrated lime, Portland cement, and lime kiln dust. Typical soils found in Kentucky should be included in the study.

With completion of this study on the resilient modulus of aggregates, the Kentucky Transportation Cabinet is in a good position to implement the use of mechanistic pavement design models. A second study, sponsored by the Kentucky Transportation Cabinet, focused on defining the resilient modulus of compacted soils commonly located in Kentucky. Both soaked and unsoaked specimens were tested. Consequently, data for defining the resilient modulus of aggregates and soils are available for use in the mechanistic pavement design model developed by AASHTO. However, a third study is needed to define the resilient modulus of chemically treated subgrades.

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Appendix A

Determination of Coefficients for Resilient Modulus Models Using Simple/Multiple Regression Analysis

This appendix presents the general method of simple/multiple linear regression and how to use this method to determine the coefficients of models for resilient modulus of soils or aggregate materials.

General Linear Regression

Assume we have a linear function as follows:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + ... + a_mx_m \tag{1}$$

and we have n set of data:

$$\begin{aligned}
&(y_1, x_{11}, x_{21}, x_{31}, ..., x_{m1})\\
&(y_2, x_{12}, x_{22}, x_{32}, ..., x_{m2})\\
&(\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots)\\
&(y_n, x_{1n}, x_{2n}, x_{3n}, ..., x_{mn})
\end{aligned} \tag{2}$$

The purpose of linear regression analysis is to obtain the coefficients $a_0, a_1, a_2, ..., a_m$, which make the overall differences between the tested y_i values and predicted y values to minimum. That is, to make

$$Q = \sum_{j=1}^{n}[y_j - (a_0 + a_1x_{1j} + a_2x_{2j} + ... + a_mx_{mj})]^2 \tag{3}$$

to minimum. By calculus, there has to be

$$\frac{\partial Q}{\partial a_i} \equiv 0 \qquad i = 0, 1, 2, \ldots m \tag{4}$$

That is,

$$\begin{aligned}
&\frac{\partial Q}{\partial a_0} = \sum_{j=1}^{n} 2*[y_j - (a_0 + a_1x_{1j} + a_2x_{2j} + ... + a_mx_{mj})]*(-1) \equiv 0\\
&\frac{\partial Q}{\partial a_1} = \sum_{j=1}^{n} 2*[y_j - (a_0 + a_1x_{1j} + a_2x_{2j} + ... + a_mx_{mj})]*(-x_{1j}) \equiv 0\\
&\ldots\ldots\\
&\frac{\partial Q}{\partial a_i} = \sum_{j=1}^{n} 2*[y_j - (a_0 + a_1x_{1j} + a_2x_{2j} + ... + a_mx_{mj})]*(-x_{ij}) \equiv 0\\
&\ldots\ldots\\
&\frac{\partial Q}{\partial a_m} = \sum_{j=1}^{n} 2*[y_j - (a_0 + a_1x_{1j} + a_2x_{2j} + ... + a_mx_{mj})]*(-x_{mj}) \equiv 0
\end{aligned} \tag{4a}$$

Simplify (4a) and express those in tensor format; the coefficients are determined by solving the following equations:

$$C'C\begin{pmatrix} a_0 \\ a_1 \\ : \\ : \\ a_m \end{pmatrix} = C'\begin{pmatrix} y_1 \\ y_2 \\ : \\ : \\ y_n \end{pmatrix} \tag{4b}$$

where

$$C = \begin{pmatrix} 1 \; x_{11} \;\; x_{21} \; ....x_{m1} \\ 1 \; x_{12} \;\; x_{22} \; ....x_{m2} \\ .... \\ 1 \; x_{1n} \;\; x_{2n} \; ....x_{mn} \end{pmatrix} \tag{5}$$

C' = Transpose of *C*

The confidence in the coefficients obtained from the above linear regression is determined by R^2 defined as follows:

$$R^2 = 1 - \frac{Q}{S_{yy}} \tag{6}$$

where Q is already defined in equation (3),

$$S_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 \tag{7}$$

and

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \;, \tag{8}$$

the mean of tested *y* values.

Determine the Coefficients for Resilient Modulus of Aggregate Materials

Dunlap (Model 1):

$$M_r = k_1\left(\frac{\sigma_3}{p_a}\right)^{k_2} \tag{9}$$

Moossazadeh and Witczak (Model 2):

$$M_r = k_1\left(\frac{\sigma_d}{p_a}\right)^{k_2} \tag{10}$$

Seed et al. (Model 3):

$$M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2} \tag{11}$$

Uzan (Model 4):

$$M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}\left(\frac{\sigma_d}{p_a}\right)^{k_3} \tag{12}$$

UKTC (Model 5):

$$M_r = k_1\left(\frac{\sigma_3}{p_a}+1\right)^{k_2}\left(\frac{\sigma_d}{p_a}+1\right)^{k_3} \tag{13}$$

NCHRP (Model 6):

$$M_r = k\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}\left(\frac{\tau_{oct}}{p_a}+1\right)^{k_3}. \tag{14}$$

In the above models,

M_r = Resilient modulus,
p_a = Reference pressure (used to normalize M_r units),
σ_3 = Minimum effective principal stress,
σ_d = Deviator stress, σ_{sum} = Sum of three principal stresses,
σ_{sum} = sum of three principal stresses, and
τ_{oct} = Octahedral shear stress acting on the material,
k_1, k_2 and k_3 are coefficients need to be determined.

There are one variable and two coefficients in first three Models, and two variables and three coefficients in other three models. All models are not linear equations and have to be transferred into a linear equation in order to apply a linear regression analysis.

All six models can be linearized as following:

$$Log(M_r) = Log(k_1 p_a) + k_2 Log(X_1) + k_3 Log(X_2) \tag{15}$$

where

X_1 stands for σ_3/p_a and σ_d/p_a in Models 1 and 2 respectively; for σ_{sum}/p_a in Models 3, 4, and 6 respectively; and for $(\sigma_3/p_a + 1)$ in model 5. X_2 stands for σ_d/p_a, $(\sigma_d/p_a + 1)$, and $(\tau_{oct}/p_a + 1)$ in models 4, 5, and 6 respectively.

Table A-1. Original Test Data

M_r	σ_3 *(psi)*	σ_d *(psi)*
14408	3	3
15757	3	6
16285	3	9
21642	5	5
22519	5	10
23169	5	15
35154	10	10
37279	10	20
37680	10	30
44883	15	10
45506	15	15
46817	15	30
56864	20	15
60505	20	20
58820	20	40

Let

$$y = Log(M_r),\ a_0 = Log(k_1 p_a), a_1 = k_2, a_2 = k_3, x_1 = Log(X_1), x_2 = Log($$
;

we have a simple linear equation:

$$y = a_0 + a_1 x_1 + a_2 x_2$$

Assume we have n set of data

(y_1, x_{11}, x_{21})
(y_2, x_{12}, x_{22})
(………...)
(y_n, x_{1n}, x_{2n})

where

$y_i = Log(M_{ri}),\ x_{1i} = Log(X_{1i}),\ x_{2i} = Log(X_{2i})\ \ for\ \ i = 1\ to\ n$. Coefficients a_0, a_1 and a_2 are solved from the following equation:

$$C'C \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = C' \begin{pmatrix} y_1 \\ y_2 \\ : \\ : \\ y_n \end{pmatrix} \tag{17}$$

where
$$C = \begin{pmatrix} 1\ x_{11}\ \ x_{21}\ \ ....x_{m1} \\ 1\ x_{12}\ \ x_{22}\ \ ....x_{m2} \\ .... \\ 1\ x_{1n}\ \ x_{2n}\ \ ....x_{mn} \end{pmatrix} \tag{18}$$

The task turns to solving a 3 by 3 linear equation for a_0, a_1 and a_2. And

$$k_1 = \frac{e^{a_0}}{p_a},\ \ k_2 = a_1,\ \ k_3 = a_2. \tag{19}$$

The value of R^2 is still determined by Equation 6.

Example of Calculating k_1, k_2, and k_3 from the Test Data for UKTC Model (Model 5)

Equations 13, 15 –18, 6 – 8 are used to calculate k_1, k_2, and k_3, and to evaluate R^2. Assume p_a = 1 *psi*. Test data are shown in Table A-1.

Consider UKTC model:

$$M_r = k_1 p_a (\frac{\sigma_3}{p_a} + 1)^{k_2} (\frac{\sigma_d}{p_a} + 1)^{k_3}$$

Note that p_a = 1 *psi* and linearize UKTC model as:

$$Log(M_r) = Log(k_1) + k_2 Log(\sigma_3 + 1) + k_3 Log(\sigma_d + 1)$$

Let

$$y = Log(M_r),\ a_0 = Log(k_1), a_1 = k_2, a_2 = k_3, x_1 = Log(\sigma_3 + 1), x_2 = Log(\sigma_d + 1).$$

We have a simple linear equation:

$$y = a_0 + a_1 x_1 + a_2 x_2$$

Convert original test data to linear item data, as shown in Table A-2.

From Equation 18, C and C' will be:

$$C = \begin{bmatrix} 1 & 1.386294361 & 1.386294361 \\ 1 & 1.386294361 & 1.945910149 \\ 1 & 1.386294361 & 2.302585093 \\ 1 & 1.791759469 & 1.791759469 \\ 1 & 1.791759469 & 2.397895273 \\ 1 & 1.791759469 & 2.772588722 \\ 1 & 2.397895273 & 2.397895273 \\ 1 & 2.397895273 & 3.044522438 \\ 1 & 2.397895273 & 3.433987204 \\ 1 & 2.772588722 & 2.397895273 \\ 1 & 2.772588722 & 2.772588722 \\ 1 & 2.772588722 & 3.433987204 \\ 1 & 3.044522438 & 2.772588722 \\ 1 & 3.044522438 & 3.044522438 \\ 1 & 3.044522438 & 3.713572067 \end{bmatrix}$$

Table A-2. Converted Data

Log(Mr)	**Log(σ_3+1)**	**Lg(σ_d+1)**
9.575539	1.386294361	1.386294361
9.66504	1.386294361	1.945910149
9.698	1.386294361	2.302585093
9.982391	1.791759469	1.791759469
10.02211	1.791759469	2.397895273
10.05057	1.791759469	2.772588722
10.46749	2.397895273	2.397895273
10.52619	2.397895273	3.044522438
10.53688	2.397895273	3.433987204
10.71181	2.772588722	2.397895273
10.7256	2.772588722	2.772588722
10.754	2.772588722	3.433987204
10.94842	3.044522438	2.772588722
11.01048	3.044522438	3.044522438
10.98224	3.044522438	3.713572067

C' = Transpose of C

$$C'C = \begin{bmatrix} 15 & 34.179181 & 39.608592 \\ 34.179181 & 83.515443 & 94.443871 \\ 39.608592 & 94.443871 & 110.445516 \end{bmatrix}$$

$$C'\begin{pmatrix} y_1 \\ y_2 \\ : \\ : \\ y_n \end{pmatrix} = C'\begin{pmatrix} 9.57553889 \\ 9.66503999 \\ 9.69799972 \\ 9.98239115 \\ 10.02211468 \\ 10.05057046 \\ 10.46749369 \\ 10.52618544 \\ 10.53688473 \\ 10.71181438 \\ 10.72559946 \\ 10.75400166 \\ 10.94841773 \\ 11.01048129 \\ 10.98223721 \end{pmatrix} = \begin{bmatrix} 155.656770 \\ 359.119430 \\ 414.540132 \end{bmatrix}$$

Substituting to Equation 4b, then

$$\begin{bmatrix} 15 & 34.179181 & 39.608592 \\ 34.179181 & 83.515443 & 94.443871 \\ 39.608592 & 94.443871 & 110.445516 \end{bmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 155.656770 \\ 359.119430 \\ 414.540132 \end{pmatrix}$$

Solving this equation, we get:

$$\boxed{\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 8.507814 \\ 0.729068 \\ 0.078787 \end{pmatrix}}$$:

From Equation 19, k_1, k_2, and k_3 will be:

$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 4953.323026 \\ 0.729068 \\ 0.078787 \end{pmatrix}$$

$$Q = \sum_{j=1}^{15} [y_j - (a_0 + a_1 x_{1j} + a_2 x_{2j})]^2 = 0.011645256$$

$$S_{yy} = \sum_{i=1}^{15} (y_i - \bar{y})^2 = 3.524374018$$

$$R^2 = 1 - \frac{Q}{S_{yy}} = 0.996695794$$

$$M_r = 4953.323026(\sigma_3 + 1)^{0.729068} (\sigma_d + 1)^{0.078787}$$

That is, the function can be used to predict resilient modulus, M_r, from test data.

Appendix B

Values of Testing Stresses and Resilient Modulus Recorded During Testing of Compacted Specimens of Dense Graded Aggregates As Received From the Producer

Table B-1. Dense Graded Aggregate (DGA-4531-1-3-1)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	3	1	16477	1	3	3	12.046
			2	14968			6	14.923
			3	14906			9	17.973
			4	19969		5	5	20.27
			5	20951			10	25.064
			6	21607			15	29.941
			7	35280		10	10	39.913
			8	37524			20	50.335
			9	38354			30	60.042
			10	46887		15	10	54.992
			11	48620			15	59.874
			12	52937			30	75.034
			13	62378		20	15	74.86
			14	64680			20	80.166
			15	67226			40	99.732

Table B-2. Dense Graded Aggregate (DGA-4531-1-4-1)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	4	1	14408	1	3	3	12.015
			2	15757			6	14.867
			3	16285			9	17.87
			4	21642		5	5	19.975
			5	22519			10	25.205
			6	23169			15	29.936
			7	35154		10	10	40.165
			8	37279			20	50.155
			9	37680			30	59.955
			10	44883		15	10	55.25
			11	45506			15	59.952
			12	46817			30	75.193
			13	56864		20	15	75.358
			14	60505			20	80.263
			15	58820			40	99.67

* Sigma sum is calculated from test data.

Table B-3. Dense Graded Aggregate (DGA-4531-1-31-1)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	31	1	13304	1	3	2.7	11.732
			2	14310			5.4	14.672
			3	15779			8.1	16.96
			4	20292		5	4.5	19.563
			5	22960			9	24.157
			6	24954			13.5	28.567
			7	40410		10	9	39.237
			8	42392			18	48.192
			9	41360			27	57.638
			10	51572		15	9	53.968
			11	50894			13.5	58.669
			12	53569			27	72.545
			13	64410		20	13.5	74.186
			14	66561			18	78.325
			15	67486			36	96.444

Table B-4. Dense Graded Aggregate (DGA-4531-1-32-1)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	32	1	13688	1	3	2.7	11.836
			2	15266			5.4	14.509
			3	15414			8.1	17.216
			4	18982		5	4.5	19.606
			5	21399			9	24.085
			6	21926			13.5	28.635
			7	32793		10	9	39.175
			8	35066			18	48.824
			9	32336			27	57.877
			10	41147		15	9	54.125
			11	41689			13.5	59.306
			12	45978			27	72.628
			13	53610		20	13.5	74.046
			14	56568			18	78.629
			15	58373			36	96.103

* Sigma sum is calculated from test data.

Table B-5. Dense Graded Aggregate (DGA-4531-1-33-1)

Site Rownum	Hole Id	Sample Id	No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	33	1	22977	1	3	2.7	12.021
			2	24545			5.4	14.409
			3	26114			8.1	17.075
			4	30665		5	4.5	19.707
			5	33827			9	24.147
			6	36518			13.5	28.793
			7	50106		10	9	39.351
			8	53815			18	48.765
			9	53831			27	57.937
			10	62727		15	9	54.301
			11	62788			13.5	59.094
			12	68072			27	72.604
			13	78138		20	13.5	73.686
			14	78740			18	78.674
			15	85093			36	96.412

Table B-6. Dense Graded Aggregate (DGA-4531-1-34-1)

Site Rownum	Hole Id	Sample Id	No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	34	1	19667	1	3	2.7	11.935
			2	20706			5.4	14.474
			3	22425			8.1	17.22
			4	28042		5	4.5	19.646
			5	29615			9	24.061
			6	28632			13.5	28.852
			7	43023		10	9	39.123
			8	39350			18	48.389
			9	38894			27	57.146
			10	55296		15	9	54.09
			11	50573			13.5	58.738
			12	51000			27	72.187
			13	63391		20	13.5	73.98
			14	63169			18	78.564
			15	63231			36	95.642

* Sigma sum is calculated from test data

Appendix C

Values of Testing Stresses and Resilient Modulus Recorded During Testing of Compacted Specimens Representing the Upper, Center, and Lower Specification Limit Gradations of Dense Graded Aggregate

Table C-1. Dense Graded Aggregate (DGA-4531-1-60-1: Upper Gradation)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	60	1	21735	1	3	2.7	11.86
			2	23729			5.4	14.626
			3	25331			8.1	17.234
			4	33895		5	4.5	19.51
			5	33093			9	24.332
			6	34951			13.5	28.726
			7	50926		10	9	39.08
			8	49129			18	48.304
			9	44299			27	57.473
			10	57330		15	9	54.265
			11	58904			13.5	58.885
			12	60952			27	72.553
			13	73598		20	13.5	73.606
			14	74114			18	78.376
			15	71716			36	95.988

Table C-2. Dense Graded Aggregate (DGA-4531-1-61-1: Center Gradation)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	61	1	8875	1	3	2.7	11.71
			2	11017			5.4	14.525
			3	14363			8.1	17.284
			4	15749		5	4.5	19.667
			5	19790			9	24.043
			6	23761			13.5	28.718
			7	33296		10	9	39.139
			8	35628			18	48.612
			9	34094			27	57.244
			10	43657		15	9	54.161
			11	43625			13.5	58.722
			12	46029			27	72.153
			13	55771		20	13.5	73.901
			14	57198			18	78.506
			15	59906			36	95.884

[*] Sigma sum is calculated from test data.

Table C-3. Dense Graded Aggregate (DGA-4531-1-62-1: Lower Gradation)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	62	1	12795	1	3	2.7	11.92
			2	12605			5.4	14.387
			3	13978			8.1	17.084
			4	20197		5	4.5	19.557
			5	22200			9	24.121
			6	25833			13.5	29.098
			7	41346		10	9	39.11
			8	43071			18	48.295
			9	42803			27	56.999
			10	52207		15	9	54.365
			11	52940			13.5	58.418
			12	53371			27	72.429
			13	64931		20	13.5	73.735
			14	66976			18	78.193
			15	63643			36	95.621

* Sigma sum is calculated from test data.

Appendix D

Values of Testing Stresses and Resilient Modulus Recorded During Testing of Compacted Specimens Representing the Upper, Center, and Lower Specification Limit Gradations of Crushed Stone Base Aggregates

Table D-1. Crushed Stone Base—Upper Gradation Curve

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	63	1	13383	1	3	2.7	11.683
			2	14603			5.4	14.677
			3	15963			8.1	17.434
			4	19873		5	4.5	19.632
			5	20935			9	24.416
			6	23140			13.5	28.99
			7	33667		10	9	39.301
			8	34524			18	48.511
			9	33021			27	57.648
			10	37673		15	9	54.32
			11	41981			13.5	58.684
			12	45656			27	72.268
			13	51901		20	13.5	73.974
			14	55039			18	78.608
			15	59589			36	96.109

Table D-2. Crushed Stone Base—Center Gradation Curve

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	64	1	19491	1	3	2.7	11.531
			2	20161			5.4	14.391
			3	23072			8.1	17.178
			4	25337		5	4.5	19.615
			5	28464			9	24.237
			6	31220			13.5	28.56
			7	44654		10	9	39.037
			8	45827			18	48.163
			9	49029			27	57.427
			10	54591		15	9	54.077
			11	54831			13.5	58.74
			12	60672			27	72.139
			13	66448		20	13.5	73.934
			14	68941			18	78.118
			15	75366			36	95.891

[*] Sigma sum is calculated from test data.

Table D-3. Crushed Stone Base—Lower Gradation Curve

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	65	1	15908	1	3	2.7	11.776
			2	16578			5.4	14.4
			3	18642			8.1	17.535
			4	23095		5	4.5	19.532
			5	25971			9	24.216
			6	28787			13.5	28.568
			7	45192		10	9	39.171
			8	43150			18	48.167
			9	43541			27	57.308
			10	51108		15	9	54.107
			11	49955			13.5	58.728
			12	56293			27	72.244
			13	63324		20	13.5	73.847
			14	64066			18	78.495
			15	69733			36	96.029

* Sigma sum is calculated from test data.

Appendix E

Values of Testing Stresses and Resilient Modulus Recorded During Testing of Specimens of Number 57s Crushed Limestone As Received From the Producer and Compacted At Different Relative Densities

Table E-1. Number 57s (NoVullex-4531-1-58-1)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	58	1	21447	1	3	2.7	11.803
			2	22440			5.4	14.667
			3	24795			8.1	17.195
			4	30097		5	4.5	19.742
			5	31419			9	24.16
			6	33149			13.5	28.66
			7	47134		10	9	39.255
			8	44008			18	48.329
			9	42674			27	57.368
			10	57205		15	9	54.363
			11	53932			13.5	59.022
			12	51928			27	72.651
			13	63729		20	13.5	73.713
			14	58304			18	78.448
			15	63524			36	95.906

Table E-2. Number 57s (NoVullex-4531-1-58-2)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	58	1	38186	2	3	2.7	11.885
			2	31092			5.4	14.564
			3	30141			8.1	17.294
			4	48855		5	4.5	19.66
			5	39479			9	24.197
			6	37134			13.5	28.787
			7	63628		10	9	39.267
			8	51468			18	48.044
			9	43085			27	57.511
			10	73085		15	9	54.14
			11	64650			13.5	58.786
			12	51665			27	72.525
			13	73439		20	13.5	74.005
			14	69759			18	78.212
			15	65356			36	96.287

[*] Sigma sum is calculated from test data.

Table E-3. Number 57s (NoVullex-4531-1-58-3)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	58	1	21023	3	3	2.7	11.862
			2	20391			5.4	14.5
			3	20573			8.1	17.275
			4	29734		5	4.5	19.652
			5	26900			9	24.266
			6	27840			13.5	28.698
			7	42705		10	9	39.131
			8	38672			18	47.972
			9	39121			27	57.848
			10	55112		15	9	54.189
			11	51119			13.5	58.909
			12	49697			27	72.291
			13	59712		20	13.5	73.661
			14	58684			18	78.328
			15	59177			36	96.1

Table E-4. Number 57s (NoVullex-4531-1-58-4)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	58	1	23984	4	3	2.7	11.787
			2	23096			5.4	14.598
			3	24105			8.1	17.293
			4	32834		5	4.5	19.656
			5	31002			9	24.179
			6	31565			13.5	28.57
			7	46082		10	9	39.012
			8	43086			18	48.341
			9	44121			27	57.592
			10	58183		15	9	54.014
			11	55244			13.5	58.64
			12	54416			27	72.162
			13	63174		20	13.5	73.842
			14	63638			18	77.923
			15	64703			36	96.181

* Sigma sum is calculated from test data.

Table E-5. Number 57s (NoVullex-4531-1-58-5)

Site Rownum	Hole Id	Sample Id	No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	58	1	24391	5	3	2.7	12.156
			2	23562			5.4	14.536
			3	24496			8.1	17.126
			4	34476		5	4.5	19.544
			5	31631			9	23.931
			6	31510			13.5	28.68
			7	46383		10	9	39.152
			8	43203			18	48.377
			9	42538			27	57.389
			10	60956		15	9	54.178
			11	56518			13.5	58.61
			12	52912			27	72.539
			13	65045		20	13.5	73.781
			14	61553			18	78.484
			15	58141			36	96.03

Table E-6. Number 57s (NoVullex-4531-1-58-6)

Site Rownum	Hole Id	Sample Id	No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	58	1	24288	6	3	2.7	11.941
			2	23300			5.4	14.531
			3	24678			8.1	17.138
			4	30804		5	4.5	19.678
			5	30721			9	23.964
			6	31193			13.5	29.349
			7	44902		10	9	39.185
			8	45175			18	48.171
			9	45377			27	57.44
			10	55964		15	9	54.288
			11	54334			13.5	58.802
			12	55469			27	72.348
			13	63579		20	13.5	73.51
			14	62666			18	78.079
			15	66260			36	96.158

*Sigma sum is calculated from test data.

Appendix F

Values of Testing Stresses and Resilient Modulus Recorded For Five Repeat Tests of Number 57s Crushed Limestone Compacted To the Same Relative Density

Table F-1. Number 57s (NoVullex-4531-1-57-1)

Site Rownum	Hole Id	Sample Id	No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	57	1	26162	1	3	2.7	11.924
			2	26912			5.4	14.589
			3	27487			8.1	17.133
			4	34865		5	4.5	19.578
			5	35472			9	23.961
			6	33523			13.5	28.943
			7	48950		10	9	39.151
			8	49677			18	48.319
			9	49683			27	57.634
			10	61147		15	9	54.111
			11	62272			13.5	58.148
			12	62504			27	72.641
			13	70240		20	13.5	73.695
			14	72353			18	78.409
			15	73371			36	96.141

Table F-2. Number 57s (NoVullex-4531-1-57-2)

Site Rownum	Hole Id	Sample Id	No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	57	1	28410	2	3	2.7	11.636
			2	25374			5.4	14.515
			3	26612			8.1	17.188
			4	37352		5	4.5	19.694
			5	34824			9	24.032
			6	33834			13.5	28.7
			7	49644		10	9	39.164
			8	46936			18	48.276
			9	47565			27	57.697
			10	67491		15	9	54.129
			11	64565			13.5	58.508
			12	60184			27	72.322
			13	73205		20	13.5	73.682
			14	71383			18	78.319
			15	70292			36	96.048

* Sigma sum is calculated from test data.

Table F-3. Number 57s (NoVullex-4531-1-57-3)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	57	1	29446	3	3	2.7	11.943
			2	27947			5.4	14.568
			3	29939			8.1	17.104
			4	39833		5	4.5	19.712
			5	37851			9	24.148
			6	38199			13.5	28.954
			7	57377		10	9	39.214
			8	55067			18	48.06
			9	54513			27	57.334
			10	70343		15	9	54.236
			11	67859			13.5	58.641
			12	66938			27	72.14
			13	78290		20	13.5	73.763
			14	76415			18	78.518
			15	77669			36	96.018

Table F-4. Number 57s (NoVullex-4531-1-57-4)

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	57	1	26885	4	3	2.7	11.997
			2	26405			5.4	14.531
			3	24960			8.1	17.33
			4	37181		5	4.5	19.669
			5	33453			9	24.219
			6	32214			13.5	28.924
			7	49096		10	9	39.197
			8	46837			18	48.179
			9	47469			27	57.867
			10	63297		15	9	54.11
			11	58681			13.5	58.918
			12	61750			27	72.386
			13	71596		20	13.5	73.613
			14	70929			18	78.144
			15	68179			36	96.262

* Sigma sum is calculated from test data.

Table F-5. Number 57s (NoVullex-4531-1-57-5)

Site Rownum	Hole Id	Sample Id	No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	57	1	32211	5	3	2.7	11.936
			2	29198			5.4	14.599
			3	30304			8.1	17.211
			4	42546		5	4.5	19.835
			5	38490			9	24.281
			6	38518			13.5	28.716
			7	62396		10	9	39.222
			8	53183			18	48.288
			9	51559			27	57.572
			10	74391		15	9	54.049
			11	67273			13.5	58.964
			12	64112			27	72.449
			13	78373		20	13.5	73.671
			14	76722			18	78.177
			15	76278			36	95.988

* Sigma sum is calculated from test data.

Appendix G

Values of Testing Stresses and Resilient Modulus Recorded During Testing of River Gravel Specimens Compacted to Different Relative Compaction Values

Table G-1. River Gravel, RGRAV-4531-1-21-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	21	1	14863	1	3	2.7	11.953
			2	15882			5.4	14.492
			3	16025			8.1	17.233
			4	20142		5	4.5	19.68
			5	21866			9	24.377
			6	22847			13.5	28.581
			7	34949		10	9	39.299
			8	37154			18	48.393
			9	37473			27	57.623
			10	43142		15	9	54.121
			11	44345			13.5	58.727
			12	49530			27	72.686
			13	56059		20	13.5	74.042
			14	58526			18	78.608
			15	63377			36	96.429

Table G-2. River Gravel, RGRAV-4531-1-22-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	22	1	11930	1	3	2.7	12.447
			2	13543			5.4	14.624
			3	15235			8.1	17.223
			4	17497		5	4.5	19.746
			5	20430			9	24.183
			6	22551			13.5	28.653
			7	33130		10	9	39.307
			8	36730			18	48.696
			9	37123			27	57.788
			10	42473		15	9	54.108
			11	43920			13.5	58.64
			12	46246			27	72.31
			13	48360		20	13.5	74.173
			14	57661			18	78.85
			15	60152			36	96.713

[*] Sigma sum is calculated from test data.

Table G-3. River Gravel, RGRAV-4531-1-23-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	23	1	14137	1	3	2.7	11.897
			2	14429			5.4	14.47
			3	15957			8.1	17.399
			4	20251		5	4.5	19.644
			5	21214			9	24.017
			6	23690			13.5	28.84
			7	36816		10	9	39.507
			8	38762			18	48.248
			9	40181			27	57.754
			10	48358		15	9	53.952
			11	49036			13.5	58.757
			12	51634			27	72.664
			13	61293		20	13.5	74.072
			14	61010			18	78.507
			15	65389			36	96.167

Table G-4. River Gravel, RGRAV-4531-1-24-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	24	1	13359	1	3	2.7	11.96
			2	14043			5.4	14.631
			3	14197			8.1	17.142
			4	18022		5	4.5	19.737
			5	20541			9	24.336
			6	20770			13.5	28.789
			7	33470		10	9	39.181
			8	35642			18	48.468
			9	32655			27	57.863
			10	42027		15	9	54.069
			11	42635			13.5	59.125
			12	44979			27	72.197
			13	58499		20	13.5	74.042
			14	61976			18	78.629
			15	57113			36	101.468

* Sigma sum is calculated from test data.

Table G-5. River Gravel, RGRAV-4531-1-25-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	25	1	12346	1	3	2.7	11.954
			2	13440			5.4	14.62
			3	13440			8.1	17.154
			4	17102		5	4.5	19.739
			5	19223			9	24.078
			6	20576			13.5	28.878
			7	30861		10	9	39.375
			8	35021			18	48.48
			9	31056			27	57.486
			10	39015		15	9	54.307
			11	36660			13.5	58.923
			12	43754			27	72.677
			13	50343		20	13.5	74.258
			14	53929			18	78.084
			15	55886			36	96.113

Table G-6. River Gravel, RGRAV-4531-1-26-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	26	1	15550	1	3	2.7	12.035
			2	16773			5.4	14.536
			3	18363			8.1	17.182
			4	21610		5	4.5	19.592
			5	23808			9	24.006
			6	25995			13.5	28.536
			7	36170		10	9	39.318
			8	39032			18	48.816
			9	39057			27	57.319
			10	45321		15	9	54.327
			11	47035			13.5	58.691
			12	50626			27	72.246
			13	57529		20	13.5	74.06
			14	60645			18	78.488
			15	63965			36	96.02

[*] Sigma sum is calculated from test data.

Appendix H

Values of Testing Stresses and Resilient Modulus Recorded During Testing of Crushed Recycled Concrete Specimens Compacted to Different Relative Compaction Values

Table H-1. Recycled Concrete, RECON-4531-1-11-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	11	1	17538	1	3	2.7	12.172
			2	18534			5.4	14.654
			3	19940			8.1	17.31
			4	27602		5	4.5	19.741
			5	27172			9	24.065
			6	28486			13.5	28.835
			7	47002		10	9	39.019
			8	45323			18	47.911
			9	42424			27	57.5
			10	64329		15	9	54.013
			11	59291			13.5	58.483
			12	55181			27	72.396
			13	76319		20	13.5	73.8
			14	74839			18	78.275
			15	68369			36	96.013

Table H-2. Recycled Concrete, RECON-4531-1-12-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	12	1	17393	1	3	2.7	11.741
			2	17425			5.4	14.694
			3	18225			8.1	17.201
			4	23376		5	4.5	19.697
			5	23744			9	24.14
			6	24702			13.5	28.712
			7	36007		10	9	39.029
			8	37497			18	48.228
			9	35253			27	57.44
			10	45067		15	9	54.094
			11	45295			13.5	58.85
			12	48018			27	72.573
			13	57907		20	13.5	73.658
			14	58915			18	78.544
			15	59546			36	96.138

* Sigma sum is calculated from test data.

Table H-3. Recycled Concrete, RECON-4531-1-13-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	13	1	13221	1	3	2.7	12.066
			2	15056			5.4	14.6
			3	15421			8.1	17.256
			4	20559		5	4.5	19.63
			5	21104			9	24.138
			6	21937			13.5	28.765
			7	33376		10	9	39.233
			8	35683			18	48.35
			9	33793			27	58.202
			10	45984		15	9	54.345
			11	44755			13.5	58.977
			12	47055			27	72.939
			13	58378		20	13.5	73.828
			14	59331			18	78.266
			15	58828			36	96.447

Table H-4. Recycled Concrete, RECON-4531-1-14-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	14	1	13176	1	3	2.7	12.004
			2	14997			5.4	14.656
			3	15701			8.1	17.281
			4	20352		5	4.5	19.73
			5	21044			9	24.316
			6	21410			13.5	29.05
			7	33436		10	9	39.366
			8	34988			18	48.5
			9	31871			27	59.081
			10	42814		15	9	54.171
			11	42252			13.5	59.125
			12	45356			27	72.857
			13	53928		20	13.5	73.967
			14	55889			18	78.357
			15	56049			36	97.125

* Sigma sum is calculated from test data.

Table H-5. Recycled Concrete, RECON-4531-1-15-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	15	1	15699	1	3	2.7	12.042
			2	17057			5.4	14.515
			3	17707			8.1	17.291
			4	22373		5	4.5	19.568
			5	23582			9	24.373
			6	23163			13.5	28.722
			7	34615		10	9	39.349
			8	36614			18	48.639
			9	35491			27	58.476
			10	46123		15	9	54.422
			11	45862			13.5	59.214
			12	49159			27	73.051
			13	58724		20	13.5	74.01
			14	61042			18	78.324
			15	61984			36	96.122

Table H-6. Recycled Concrete, RECON-4531-1-16-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	16	1	17433	1	3	2.7	11.795
			2	17877			5.4	14.544
			3	19027			8.1	17.377
			4	25297		5	4.5	19.556
			5	25685			9	24.093
			6	26647			13.5	28.561
			7	40332		10	9	39.248
			8	41454			18	48.41
			9	40169			27	57.934
			10	49474		15	9	54.461
			11	49876			13.5	58.919
			12	54118			27	72.305
			13	62558		20	13.5	73.526
			14	64329			18	77.741
			15	64422			36	96.241

* Sigma sum is calculated from test data.

Table H-7. Recycled Concrete, RECON-4531-1-17-1

Site Rownum	Hole Id	Sample Id	Sample No	Mr	Test No	Sigma3	Sigma dn	Sigma Sum
4531	1	17	1	15917	1	3	2.7	12.054
			2	17061			5.4	14.599
			3	17910			8.1	17.149
			4	23521		5	4.5	19.614
			5	24171			9	24.168
			6	25050			13.5	28.961
			7	37710		10	9	39.438
			8	38357			18	48.668
			9	36616			27	57.526
			10	45097		15	9	54.297
			11	45657			13.5	59.132
			12	48467			27	72.52
			13	57398		20	13.5	73.925
			14	59060			18	78.214
			15	58549			36	96.337

* Sigma sum is calculated from test data.

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