Research Report
KTC-01-07/SPR 163-95-1F

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RESILIENT MODULUS OF KENTUCKY SOILS

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Research Report KTC-01-07/SPR-163-95-1F
KYSPR-95-163
KYSPR-107

Resilient Modulus of Kentucky Soils

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in cooperation with the
Kentucky Transportation Cabinet
The Commonwealth of Kentucky
and
Federal Highway Administration

June 2001
(Revised) June 30, 2004

1. Report No.
KTC- 01-07/SPR-163-95-1F

2. Government Accession No.

3. Recipients catalog no

4. Title and Subtitle
Study Title: Stresses in Highway Pavement Subgrades and Relationships Among Resilient Modulus and Soil Indices
Report Title: Resilient Modulus of Kentucky Soils

5. Report Date June 30, 2001
(Revised and Reprinted June 2004)

6. Performing Organization Code

7. Author(s)
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Charlie Sun, and Bixian Ni

8. Performing Organization Report No.
KTC—01-07/SPR-163-95-1F
KYSPR-107 (Revised Portion)

9. Performing Organization Name and Address
University of Kentucky Transportation Center
College of Engineering
Lexington, Kentucky 40506-0281

10. Work Unit No. (TRAIS)

11. Contracts or Grant No.
Final---KYSPR-95-163 and KYSPR-107

12. Sponsoring Agency Code
Kentucky Agency name and Agency
200 Mero Street
Frankfort, Ky 40622

13. Type of Report and Period Covered

14. Sponsoring Agency Code

15. Supplementary Notes
Prepared in cooperation with the Kentucky Transportation Cabinet and the United States Department of Transportation, Federal Highway Administration

16. Abstract
In recent years, the American Association of State Highway Transportation Officials (AASHTO) has recommended the use of resilient modulus for characterizing highway materials for pavement design. This recommendation evolved as of a result of a trend in pavement design of using mechanistic models, which are based on the theory of elasticity (layered elastic analysis) or linear and non-linear, finite elements (and finite difference methods), or a combination of both of those theoretical approaches. Although much progress has been made in recent years in developing mathematical, mechanistic pavement design models, results obtained from those models are only as good as the material parameters used in the models. Resilient modulus of the subgrade soil is an important parameter in the mechanistic models and in the 1993 AASHTO pavement design equation. The main goal of this study was to establish a simple and efficient means of predicting the resilient modulus of any given type of Kentucky soil. To accomplish this purpose, 128 tests were performed on several different soil types from various locations of Kentucky. Specimens were remolded to simulate compaction conditions encountered in the field. Tests were performed on soaked and unsoaked specimens so that an assessment could be made of the affect of moisture on resilient modulus values. Vast differences were found between soaked and unsoaked values of resilient modulus.

Based on an analysis of the data, a new mathematical model is proposed which relates resilient modulus to any given selected, or calculated, principal stresses in the subgrade. This model improves the means of obtaining best data "fits" between resilient modulus and stresses. Furthermore, if the AASHTO classification and group index are known, than the resilient modulus of the soil can be predicted from the new model for any known stress condition in the subgrade. Multiple regression analysis was used to obtain relationships between resilient modulus and confining stress and deviator stress. No difficulties were encountered in testing "as-compacted" (unsoaked) samples. Values of R^2 of 91 percent of unsoaked test specimens were greater than, or equal to, 0.87. However, values of R^2 of only 35 percent of tested, soaked samples exceeded 0.87. Difficulties were encountered in testing soaked specimens. More research is needed to test saturated, or nearly saturated, soil specimens—conditions that often exist in the field.

To make the resilient modulus data and the new model readily available to design personnel of the Kentucky Transportation Cabinet, a "windows" computer software application was developed in a client/server environment. This program is embedded in the Kentucky Geotechnical Database, which resides on a Cabinet server in Frankfort, Kentucky. The resilient predictor model and data are readily available to pavement design personnel statewide.

17. Key Words
Highways, Resilient Modulus, Soils, Model, Design, Subgrade

18. Distribution Statement
Unlimited, with the approval of the Kentucky Transportation Cabinet

19. Security Classif. (of this report)--- None

20. Security Classif. (of this page) None

21. No. of Pages 72

22. Price

Form DOT 1700.7 (8-72) **Reproduction of completed page authorized**

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EXECUTIVE SUMMARY

This study developed as a result of a process review by the Federal Highway Administration (FHWA) of the Kentucky Transportation Cabinet's procedure for selecting the strengths of highway soil subgrades for pavement design. FHWA found the Cabinet's procedures to be in accordance with past experience and knowledge. FHWA recommended "that an in-depth assessment be made of the most appropriate strength test to accommodate Kentucky's future needs and that resilient modulus testing be given consideration for informational design values, evaluation of other research efforts, and keeping up with state-of-the-art practices." Moreover, mechanistic pavement design models, which will be published by the American Association of Highway Transportation Officials (AASHTO) in the future, will rely on the resilient modulus of soils as an important soil parameter. The pavement models consist of using the theory of elasticity (layered elastic analysis), or the linear, or non-linear, finite element method, or a combination of those theoretical approaches.

This study was sponsored as a means of responding to the factors cited above and to put the Kentucky Transportation Cabinet in a position to take advantage of the latest highway design technology. Several months were required to purchase and make operational the necessary equipment for performing resilient modulus tests on Kentucky soils.

Resilient modulus tests were performed on a variety of typical Kentucky soils. Soil samples consisted of six bulk samples collected from different physiographical regions of the state and roadway samples generated during roadway studies. Soil types, based on the AASHTO Soil Classification, included A-4, A-6, A-7-5, and A-7-6. The laboratory specimens were compacted to 95 percent of maximum dry density and optimum moisture content obtained from AASHTO T-99. Sixty-eight resilient modulus tests were performed on "as compacted' soil specimens that were not soaked. Sixty resilient modulus tests were performed on compacted, soaked soil specimens. Unsoaked and soaked tests were performed on each soil type. The soaked specimens were allowed to swell for two to three weeks until primary swell ceased.

A new mathematical resilient modulus model, which relates principal stresses and soil resilient modulus, is proposed in the report. This model provides better data "fits" between resilient modulus and stresses than models previously proposed and published by others. Furthermore, if the AASHTO classification and group index of the subgrade soil are known, than the resilient modulus of the soil can be predicted from the new model for any given, or calculated stress condition in the pavement subgrade. Results obtained from the new model are compared to two other published models. Multiple regression analysis is used to obtain regression coefficients of each model. It is shown that results obtained from the new model developed by the authors are generally better than results from the other two proposed models.

Values of R^2 of 91 percent of the "as-compacted" soil specimens were equal to or greater than 0.87. The R^2–value of about 80 percent of those tests was equal to or greater than 0.90. Generally, obtaining acceptable test results on the unsoaked specimens poised no problems. However, difficulties were encountered when soaked, compacted soil specimens were tested. Values of R^2 of 65 percent of soaked specimens ranged from 0.30 to 0.87. Many soaked specimens "bulged" during repeated loading—large excess pore pressures build up during the test. More research is needed to determine the best approach to testing saturated, or nearly saturated, clayey soils. Generally, the mean values of resilient modulus of unsoaked soil specimens were about three times larger than the mean values of resilient modulus of soaked soil specimens. Testing saturated, or nearly saturated, compacted soils to obtain parameters for pavement design is a long-held and well-supported concept.

Repeatability of the resilient modulus test was briefly examined using molded synthetic specimens and soil specimens compacted to nearly identical conditions of dry density and moisture content. Three synthetic specimens were built during the research study. The synthetic specimens were used for calibration and to determine repeatability. In performing all of the resilient modulus tests, the LVDT monitoring devices and the load cell were mounted inside the testing chamber. Both synthetic specimens and soil specimens were tested. Three different synthetic specimens were tested. As the values of resilient modulus increased, the value of the 95-percent confidence level increased. The resilient modulus of the first synthetic specimen ranged from 1,360 ($F_3 = F_d = 2$ psi) to 1,590 ($F_3 = 6$ psi; $F_d = 10$ psi). The 95-percent testing confidence level—based on the authors' model-- ranged from 4.7 to 7.0 percent. For the second synthetic specimen, the resilient modulus ranged from 6,443 ($F_3 = F_d = 2$ psi) to 9,323 ($F_3 = 6$ psi; $F_d = 10$ psi). The 95-percent testing confidence level ranged from 10.4 to 11.6 percent. The resilient modulus of the third synthetic specimen ranged from 15,665 ($F_3 = F_d = 2$ psi) to 32,744 psi ($F_3 = 6$ psi; $F_d = 10$ psi). The 95-percent confidence level ranged from 14.4 to 20.1 percent. The 95-confidence level of the authors' model was better than the 95 percent confidence levels of the other two published models. Resilient modulus repeatability of compacted (unsoaked) soil specimens was also examined. Five soil specimens were compacted to nearly identical conditions. Values of resilient modulus of the five specimens ranged from 24,901 ($F_3 = F_d = 2$) to 32,457 psi (($F_3 = 6$ psi; $F_d = 10$ psi). The 95-percent confidence level ranged from 7 to 23 percent.

Resilient modulus tests were also performed on core specimens obtained from untreated and chemically treated soil subgrades. The subgrade specimens were obtained using a coring technique that uses air as the drilling media, instead of water. Using this technique, good quality subgrade specimens were obtained. Generally, the resilient modulus (computed at the mid-range of testing stresses) of soil-cement core specimens were approximately 1.5 to 4 times larger than resilient modulus values of untreated soil specimens (obtained using a thin-walled sampling tube). The untreated soil specimens were obtained at depths below the top zones of very soft soil of the clayey subgrades. Properties of those soil specimens are very similar to the properties of "as compacted" unsoaked soil specimens. Resilient modulus values of soil-hydrated lime subgrade (core) soil specimens were generally about the same as the unsoaked (and untreated) soil specimens. Resilient modulus tests could not be performed on soil specimens of the top of the subgrade because this thickness of this zone of material was usually too small to obtain specimens. However, it was shown previously in laboratory studies that the resilient modulus of soaked soil specimens was much larger than the resilient modulus of soaked soil specimens.

To make the resilient modulus data and the new model readily available to design personnel of the Kentucky Transportation Cabinet, a "windows" computer program has been programmed in a client/server environment. This program has stored data in the Kentucky Geotechnical Database, which resides on a Cabinet server in Frankfort. Hence, provided proper permission and connections have been made, appropriate highway designers and personnel in the Central Office and Highway District Offices can access the resilient modulus data and model. This makes the predictor model available statewide.

Kentucky Transportation Cabinet is also sponsoring other studies to determine the values of resilient modulus of base aggregates commonly use in Kentucky. The same equipment, except for the soil resilient modulus cell, used to study soils will be used to examine the resilient modulus of aggregates. An additional resilient modulus cell has been acquired for testing the larger aggregate specimens. With completion of those studies, the Kentucky Transportation Cabinet will be in a good position to implement the use of mechanistic pavement design models. Data and predictor models from these studies will provide the parameters necessary for using the mechanistic models.

INTRODUCTION

Resilient modulus has been proposed as a means of characterizing the elastic properties of pavement materials. It is expressed as the ratio of deviator stress applied to the pavement layers (and soil subgrade) and the resilient axial deformation recovered after release of the deviator stress. The assumptions are made tacitly that pavement materials are designed for loading in the elastic range and that the resilient modulus is the only parameter needed to design the thickness of a pavement. Although empirical relations have been used in the past to estimate the resilient modulus of soils, the trend in recent years is to measure the resilient modulus of soils (and other pavement materials) using laboratory tests. Empirical relations attempt to relate the resilient modulus to some type of soil parameter, such as bearing ratio (CBR), or resistance index (R_{value}). A fundamental problem with empirical relations is the models attempt to assign a fixed value of resilient modulus to a given type of soil. However, the value of resilient modulus is stress-strain dependent, that is, the value changes as stress and strain conditions change. AASHTO (American Association of State Highway and Transportation Officials, 1993, 2000) and SHRP (Strategic Highway Research Program, 1989) published a testing standard and protocol, T-294, for performing resilient modulus of soils. Equipment for performing resilient modulus tests of soils and aggregates has steadily evolved and improved over the past few years. Several mathematical expressions are available for modeling the resilient modulus of soils and aggregates. These include such models as proposed by Moossazadeh and Witczak (1981), Dunlap (1963), Seed et al. (1967), May and Witczah (1981) and Uzan (1985). The effectiveness of these models to predict resilient modulus correctly is examined in this report. It is shown that none of those models correctly predict the effects of both confining stress and deviator stress on the resilient modulus of soils. To correctly model the resilient modulus of soils, a new model is offered by the authors. As a means of providing data for evaluating the published models and the new model, 128 resilient modulus tests were performed on a variety of fine-grained soils. Comparisons are made among the various models.

The trend in the design of highway pavements consists of using mechanistic models based on the theory of elasticity (layered elastic analysis) or linear, or non-linear, finite elements, or a combination of both of these theoretical approaches. Although much progress has been made in recent years in developing mathematical, mechanistic pavement design models, results obtained from those models are only as good as the material parameters entered into the models. In 1986 and 1993, the American Association of State Highway Transportation Officials (AASHTO Guides) recommended the use of resilient modulus for characterizing highway materials for pavement design (Mohammad et al., 1994). To promote this concept, the 1962 flexible pavement design equation originally published by the Highway Research Board (1962) was modified in the 1993 AASHTO Guide to include the resilient modulus of soils. This approach attempts to make use of the mechanical properties of the asphalt concrete, base courses, and soil subgrades.

Many state transportation agencies have used, or continue to use, empirical pavement design methods involving soil support values, California Bearing Ratio (CBR), or R-values. Studies have been performed attempting to relate such soil parameters as CBR, R-values, hypothetical soil support values (s), or unconfined compression strengths to resilient modulus (Mohammad et al., 1994). According to Mohammad et al., empirical values and design approaches do not adequately represent the response of pavement to the dynamic loading caused by moving vehicles. The resilient modulus concept arose as a result of efforts to better simulate the loading of pavements by moving vehicles. The resilient modulus test for soils was originally developed by Seed et al. (1967) and was later formulated for highway applications (Claros et al., 1990). Resilient modulus of the subgrade

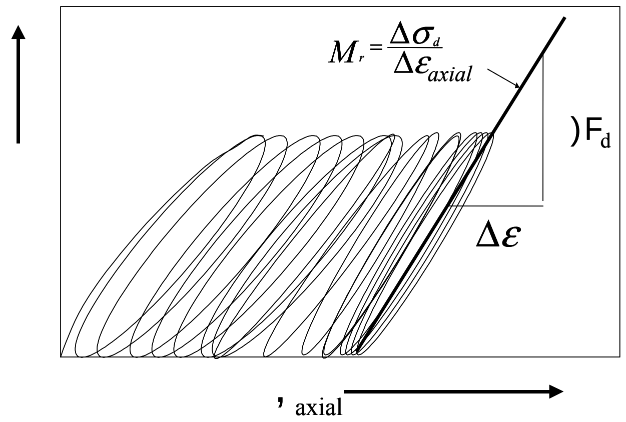


Figure 1. Definition of resilient modulus.

replaces the hypothetical "soil support value" proposed in earlier guides (Highway Research Board, 1962).

The resilient modulus test provides a relationship between deformation (or strain) and stresses in pavement materials, including subgrade soils, subjected to moving vehicular wheels. Hence, it is not necessarily a fixed value but varies according to the applied stresses of moving vehicles and the resulting stress level in the soil subgrade. The test measures the stiffness of a cylindrical specimen of subgrade soil that is subjected to a cyclic or repeated axial load. It provides a means of analyzing different materials and soil conditions, such as moisture and density, and stress states that simulate the loading of actual wheels. For a given deviator stress, the resilient modulus, M_r, is defined as the slope of the deviator-axial strain curve, or simply the ratio of the amplitude of the repeated axial stress to the amplitude of the resultant recoverable axial strain, or (Figure 1):

$$M_r = \frac{\Delta\sigma_d}{\Delta\varepsilon_{axial}} \tag{1}$$

where

ΔF_d = deviator stress = (F_1 - F_3),
F_1 and F_3 = major and minor principle stresses, and
$\Delta ,_{axial}$ = recovered axial strain.

The specimen is subjected to repeated loading at a particular stress level and the recoverable strain is measured. Ideally, the specimen exhibits only elastic strains at the time the resilient modulus is measured. The resilient modulus can, therefore, be thought of as the secant Young's Modulus of a certain material typically different than the initial tangent value (Houston et al., 1993). Resilient modulus is used in many pavement and railroad track designs. This modulus can be used for either the asphalt or subgrade level when the materials are subjected to moving dynamic loads. As shown in Figure 2, the stress level in a subgrade varies with the thickness of the pavement. If the pavement is thin, then the cyclic deviator stresses are high. When the pavement is thick, the cyclic deviator stresses in the subgrade are small. Consequently, the magnitude of the applied cyclic load is varied over a range of anticipated subgrade stress values, as shown in Figure 3, in resilient modulus testing to measure the variation of the resilient modulus, or stiffness.

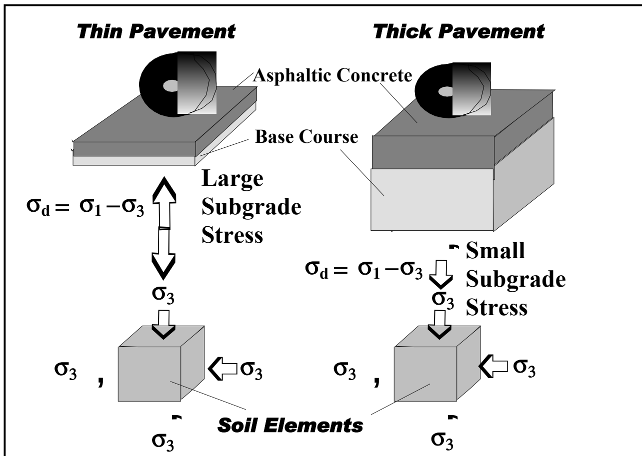


Figure 2. Relative subgrade stress levels for different pavement thickness.

Values of resilient modulus of soils are needed to use in mechanistic pavement design models that will be published by the American Association of State Highway Transportation Officials (AASHTO) in the near future. Those mathematical models will be based on elastic layered theory or the finite element method, or a combination of those methods.

OBJECTIVES

This study had two major objectives:

1. Determine the resilient modulus of the many different types of soils in Kentucky and
2. Develop a new mathematical model for predicting the resilient modulus of any type of soil under typical stresses imposed by traffic loading of wheels.

A third, but a secondary objective, was to examine, or estimate, the subgrade stresses under typical traffic wheel loads for pavements of different thickness. Originally, this objective was to be achieved by installing and monitoring pressure cells at two or three subgrade sites. The intent here was to check the general range of subgrade stresses that typically might be imposed by typical traffic wheel loads. The main reason for this task was to establish that the stress levels specified in the AASHTO testing standard identified as T 294 were realistic. However, engineers of the Kentucky Transportation Cabinet could not find suitable sites for this purpose. Others have verified that stress levels of AASHTO T 294 are reasonable (Drumm et al., for example, 1993; AASHTO 1993) and this task could easily be ignored without harming the main objectives of the study. The study time originally allocated to the third objective was reassigned and devoted to completing objectives 1 and 2.

A major intent of this study was to follow through on a suggestion made by FHWA in 1993 that "that an in-depth assessment be made of the most appropriate strength test to accommodate Kentucky's future needs and that resilient modulus testing be given consideration for informational design values, evaluation of other research efforts, and keeping up with state-of-the-art practices." Another major intent of this study was to put the Kentucky Transportation Cabinet in a position (from a geotechnical point of view) to use the new mechanistic models scheduled to be published by AASHTO in the future. Initially, considerable study time was required and devoted to evaluating

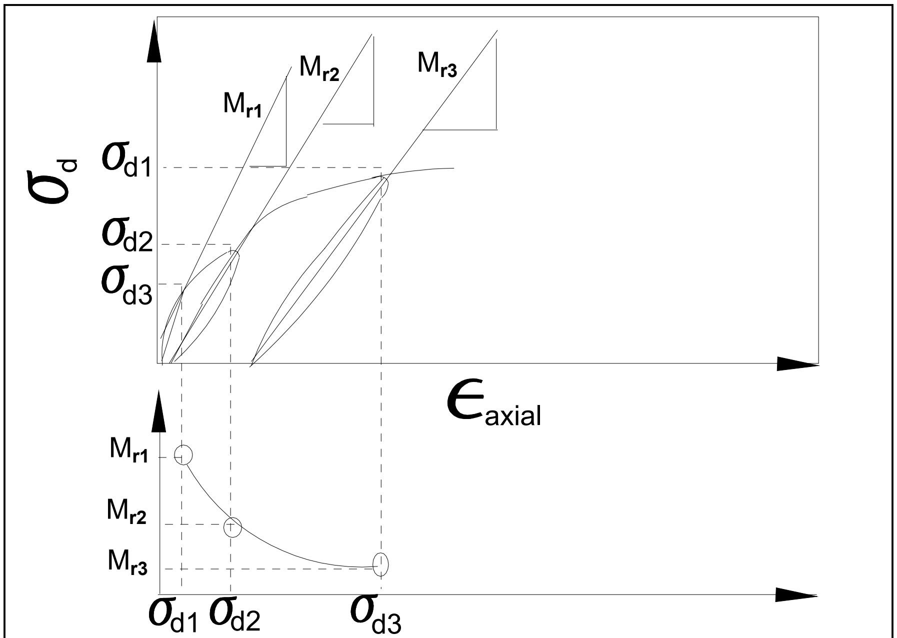


Figure 3. Stress-strain hysterisis loop and resilient modulus determination.

and purchasing resilient modulus testing equipment and to making the equipment operational.

SCOPE OF STUDY

Ironically, few states or agencies have performed a large number of resilient modulus tests mainly because the test requires expensive, specialized testing equipment and software, and it is time consuming. Some efforts have been devoted to estimating and relating, in some fashion, the resilient modulus of soils with selected soil parameters, such as CBR. However, to obtain realistic values of resilient modulus for different types of soils in a given locale, it is essential to perform resilient modulus tests on the soils from that region, or locale. Consequently, 128 resilient modulus tests were performed on roadway samples collected from various locations in Kentucky. A summary of these data is contained in this report and detailed information for each test appears in the Kentucky Geotechnical Database, which is housed on a server of the Kentucky Transportation Cabinet. Resilient modulus equipment used to perform the tests is fully described. The reliability of the testing equipment is evaluated. The resilient modulus tests were performed on compacted soil specimens in unsaturated and saturated states. Compaction and soaking procedures are fully described. Problems that arose in attempting to use the resilient modulus testing procedure, AASHTO T 294, on soaked specimens are described and discussed. Problems that arose in attempting to relate resilient modulus to simpler soil parameters are described. Proposed resilient modulus models that appear in the literature are reviewed and a discussion of deficiencies in the published models is presented.

Mathematical models currently available are cumbersome to use and have some physical admissibility problems. To overcome shortcomings of the published models, the authors propose a new resilient modulus model. Algorithms for performing the necessary regression fit of the new model are fully described. Finally, a "windows-type" computer program was developed in a client/server environment to facilitate use of the new model. Graphical user interfaces have been built which greatly simplify the use of the program. The testing data have been stored in the Kentucky Geotechnical Database of the Kentucky Transportation Cabinet (KyTC) in Frankfort, Kentucky. Hence, the program can be installed to any PC and is accessible to those KyTC officials and engineers who deal directly and indirectly with pavement design. The results of this study are ready to be implemented by KyTC officials, and from a geotechnical viewpoint, the soil resilient modulus can be obtained for most soil types commonly encountered in Kentucky. Resilient modulus obtained from the stored relationships in the database can be used in the new mechanistic models to be published by AASHTO in the future and they can be used in the modified AASHTO equations appearing in the 1993 Pavement Guide.

BACKGROUND

A workshop (Nazarian et al., 1993) held in 1989 at Oregon State University summarized the state of practice at that time in resilient modulus testing. Conclusions were:

- Using resilient modulus as a design parameter would significantly improve pavement design procedures.
- Testing procedures were inadequate.
- Modifications were needed in the testing procedures.
- Constitutive models were incomplete.

Many factors affect the resilient modulus of cohesionless subgrades. Based on numerous laboratory tests, Hardin and Drenevich (1972) suggested that the state of stress, void ratio, and strain amplitude are the main parameters affecting modulus measured in the laboratory. Basically, as void ratio decreases, the modulus of soil increases. Hardin and Drenevich also concluded that a linear logarithmic relationship exists between the modulus and the applied confining pressure.

In the absence of resilient modulus testing devices, the 1986 AASHTO Guide suggested the following relationship between M_r and CBR , or the California Bearing Ratio, (after Heuklelom and Klomp, 1962):

$$M_r = 1500(CBR) \text{ (psi).} \quad (2)$$

The initial expression originated from research by Heukelom and Foster (1960). In a series of dynamic tests performed on a variety of subgrade types, these researchers established a relationship between the dynamic modulus of elasticity and CBR, as shown in Figure 4. Their relationship was

$$M_r = 1565(CBR) \text{ (psi).} \quad (3)$$

Later, Heukelom and Klomp (1962) represent the relationship as shown by Equation 2. Hopkins (1991) reanalyzed the data by Heukelom and Klomp, Figure 4, and suggested the following relationship

$$M_r = 2596(CBR)^{0.874} \text{ (psi).} \quad (4)$$

Many researchers, such as Thomson and Robnett (1976) and Rada and Witczak (1981), suggest that the use of the CBR value for designing pavements is unreliable. Although some engineers have recognized the test as unreliable, the test has been used extensively for designing pavements. Since 1936, the CBR test has been used by many agencies to express the bearing strength of pavement subgrades and in the design of pavements. CBR values have a high coefficient of variance.

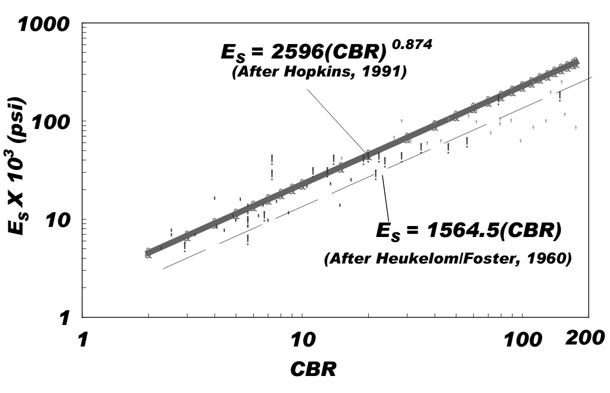


Figure 4. Dynamic modulus of elasticity as a function of CBR (After Heukelom and Foster; 1960; Hopkins, 1991)

As shown by Hopkins (1970, 1991), Beckham and Allen (1990), and Hopkins et al. (1995), the CBR value obtained from the Kentucky procedure (Kentucky Methods, 1995) is larger than the value obtained from the AASHTO CBR procedure (T 193-95), as shown in Figure 5. The higher value of CBR occurs because the Kentucky testing procedure specifies a larger compactive energy than the compactive energy specified by the AASHTO procedure (Hopkins, 1970). In developing the relationship in Figure 5, CBR specimens for the AASHTO CBR procedure were compacted to 95 percent of maximum dry density and optimum moisture content obtained from

AASHTO procedure T-99. Consequently, the Kentucky CBR specimen has a larger dry density than the dry density obtained from the AASHTO procedure. As a result, the Kentucky CBR value is larger than the AASHTO CBR value. The relationship between the AASHTO CBR and the KYCBR may be approximated as

$$CBR_{AASHTO} = 5.29Ln(KYCBR) - 3.91 \tag{5}$$

The mechanics of soil behavior is sometime quite predictable if the surrounding geology is known. Soil reacts to moisture, temperature, handling, and the degree of compaction. Because of these factors limitations exist that make the determination of the actual field modulus in the laboratory very difficult. Uniformity of equipment and testing techniques are essential to obtain comparable data. Factors that cause variations of resilient modulus values have been noted by others, as discussed below.

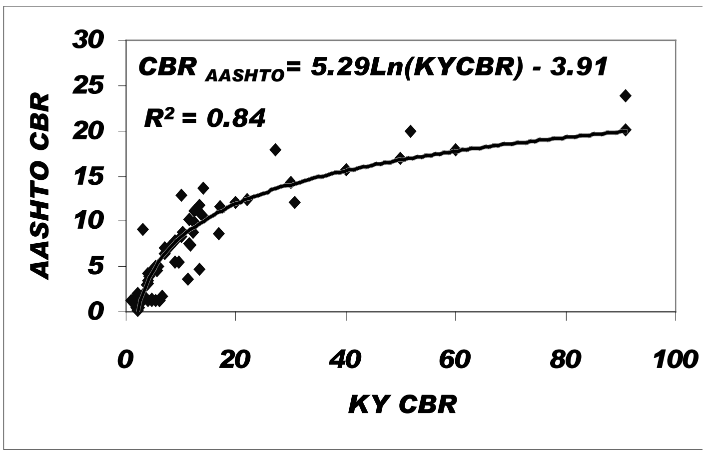


Figure 5. AASHTO CBR as a function of Kentucky CBR(After Hopkins and Beckham, 1995).

The effect of variation in stress level on the magnitude of resilient modulus is very critical because the stress in a subgrade soil depends on pavement thickness (Drumm et al., 1993). Subgrade deviator stresses of thin pavements are higher than the subgrade deviator stresses of thick pavements. Consequently, resilient modulus varies with the magnitude of deviator stress acting in the subgrade. The 1993 AASHTO Guide for Design of Pavement Structures (AASHTO 1993) requires resilient modulus for the subgrade selection. Determination of the subgrade resilient modulus is important for designing pavement thickness. If the selected design resilient modulus value is much higher than the actual field, or in situ, resilient modulus, then thickness of the pavement will be insufficient. If the design value is too low, the design will be too conservative and uneconomical (Ksaibati et al., 1995). The magnitude of resilient modulus is greatly affected when low values of specimen deflection, or strain, occur because of physical difficulties and limitations in measuring very small deflection values. Generally, values of resilient modulus tend to be more

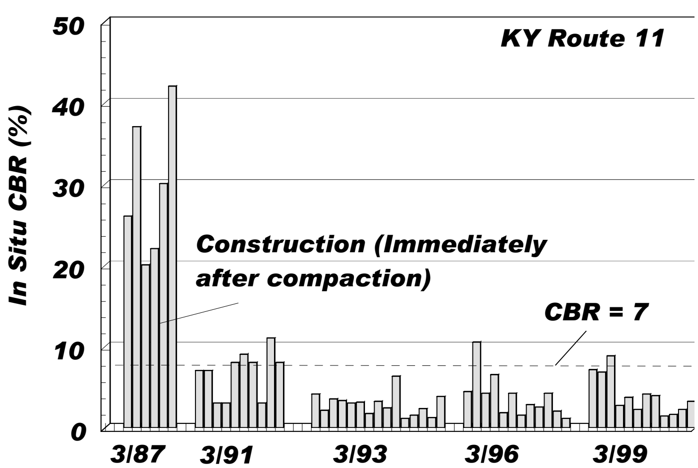


Figure 6. Long-term, in situ CBR strengths of a silty clay subgrade.

accurate as specimen deflections increase and fall within the accuracy range of equipment used to measure deflections.

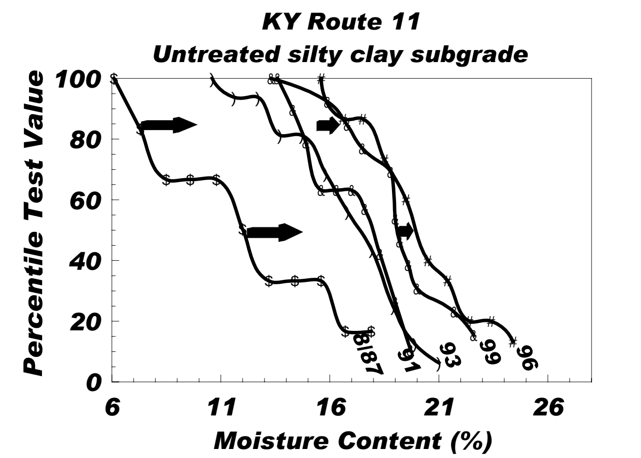


Figure 7. Increase in moisture content of a silty clay subgrade measured over a 12-year period.

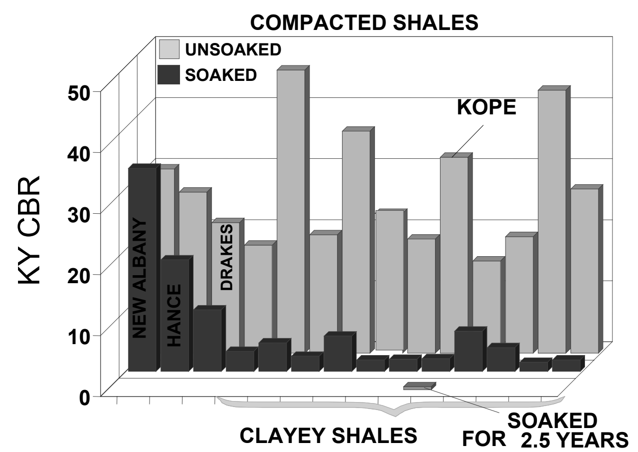


Figure 8. Comparison of laboratory CBR strengths of compacted shales in unsoaked and soaked states.

Subgrade soils in Kentucky are generally constructed near optimum moisture content and at 95 percent (or greater) of maximum dry density obtained from the AASHTO test method designated as T-99. However, environmental and seasonal variations in the weather can greatly alter the design moisture content. Vast differences can exist between unsoaked and soaked bearing strengths of clay subgrades. Past experience [Hopkins and Sharpe, 1985; Hopkins et al. 1988, 1990; Hopkins, 1991; Hopkins, et al. 1994a,b,and c; Hopkins, Beckham, and Hunsucker 1995; Hopkins and Beckham 2000; Hopkins et al. 2002 and 2004] has shown that clayey subgrades usually provide very substantial bearing strength immediately after compaction. However, with increasing time, clayey subgrades tend to increase in moisture content and decrease in bearing strength. For instance, in situ CBR values measured over a 12-year period on a route in Kentucky (Hopkins et al. 2002, 2004) and shown in Figure 6 illustrates this condition. Immediately after construction, in situ CBR values of the silty clay subgrade ranged from about 20 to 42. Between the time after paving in 1987 and 1999, the CBR values decreased to strengths ranging from about 1 to 11. However, about 90 percent of the CBR values were less than or equal to 7. The decrease in bearing strength was attributed to an increase in the moisture content of the subgrade. As illustrated in Figure 7, measurements of moisture contents obtained over the 12-year period are shown as a function of percentile test value. These data clearly show an overall increase in subgrade moisture content. Hence, with an increase in the in situ subgrade moisture and a decrease in bearing stength, the resilient modulus would decrease from some initial value to a lower long-term value.

Laboratory CBR data, shown in Figure 8, also illustrate how the bearing strength of compacted clayey shales and soils may decrease when the specimens are exposed to long soaking periods (greater than two weeks) and allowed to swell (Hopkins and Deen, 1983; Hopkins,1984; Hopkins, 1994a,b; Hopkins, 1988; Hopkins et al. 1995 and and Hopkins and Beckham 2000). CBR values of the unsoaked specimens ranged from about 15 to 42. Soaked CBR values ranged from about 1 to 33. However, if only eleven of the specimens (clayey shales) are considered, then the values range from 1 to 6.5. Soaked and unsoaked CBR values of three of the specimens were greater than about 10 and were durable shales that do not break down into clay-size particles. Consequently, as subgrade clays absorb water and swell, strength decreases and it could be expected that the resilient modulus wouild decrease. Results show moisture content has more influence on clay particles than sand (Mohammad 1995).

Values of resilient modulus are influenced by testing procedures, as shown by Chen (1994), In terms of maximum coefficient of variation, variability between values of resilient modulus obtained from AASHTO T 292-91I and AASHTO T 294-92I testing procedures ranged from 19 to 26 percent. Values obtained from the AASHTO T 294-92I testing method were higher than the values obtained from the AASHTO T 292-91I test method.

The accuracy of devices, such as the Linear Variable Differential Transducers (LVDT), used to measure specimen deflections and the location of the LVDT on the specimens, or at some other location in the testing device, affect the measurements of resilient modulus. Typically, specimen displacements, or deflections, under repeated loads are very small and these two factors greatly influence measurements of resilient modulus values. (Drumm et al., 1993).

SOIL SAMPLING

A view of the types of soils found in Kentucky, and the types of soils that are most likely to be used to construct pavement subgrades in the state, may be obtained by analyzing engineering soils data contained in a geotechnical database (Hopkins et al. 2004). This database contains several thousands of soil records. These data are the result of basic geotechnical tests that have been performed over the past four decades and obtained from various locations throughout Kentucky. Analyses of those data show that, statistically, about 70 percent of the soils in the state classify as clays and silts, as shown in Figure 9. About 16 percent of the soils are fat clays and silts. Some 14 percent of the soils classify as clayey, silty gravel, silty sands and sands, or clayey, silty gravel and gravel. Most silty and sandy soils are located along major rivers and in the extreme western portion—the Jackson Purchase area—of Kentucky. About 86 percent of the soils in the state are materials of poor engineering quality and the likelihood that these poor engineering materials will be used to construct pavement subgrades is very high. The likelihood of pavement construction problems occurring in Kentucky is considerable high. Consequently, this study focused on determining the resilient modulus of fine-grained soils because of the abundance of these types of soils in Kentucky.

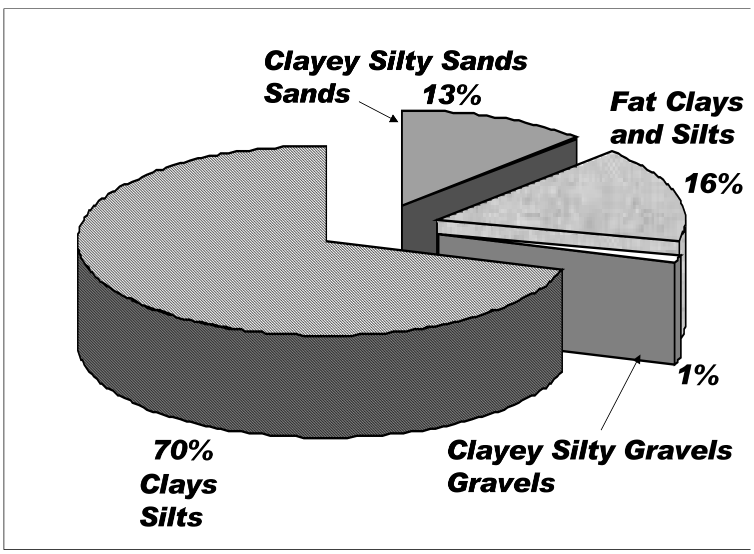


Figure 9. Statistical overview of the types of soils located in Kentucky

Two different sampling approaches were used to collect soil specimens. In the first approach, bulk samples of six different, typical types of soil samples were collected from different physiographical regions of Kentucky. These regions include the Eastern Coal Fields, the Bluegrass Region, the Mississippi Plateaus, the Western Coal Fields, Knobs Region, and the Jackson Purchase.

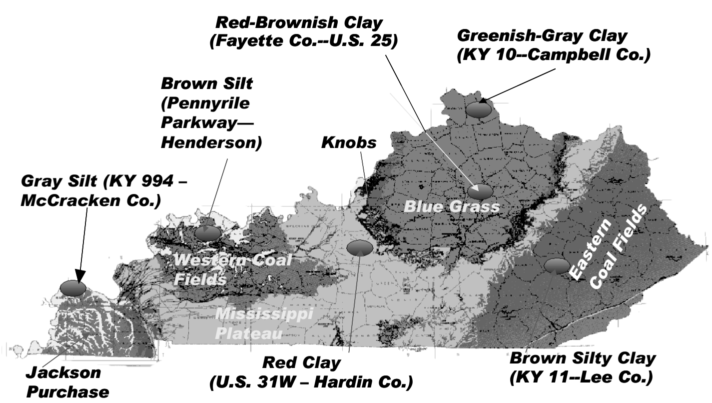


Figure 10. General locations of bulk sampling sites in Kentucky (map from the Kentucky Geological Survey).

General locations of those sampling sites are shown in Figure 10. A quantity of soil that was sufficient to fill two 55-gallon drums was collected of each of the six different soil types. With the exception of the Knobs Region, bulk samples were collected from each physiographical region. The objective of obtaining bulk samples was to have available typical Kentucky soils for testing by others in the future. Also, these soils could be viewed as references soils. Results of resilient modulus tests performed by others in the future could be compared to results tabulated in this report.

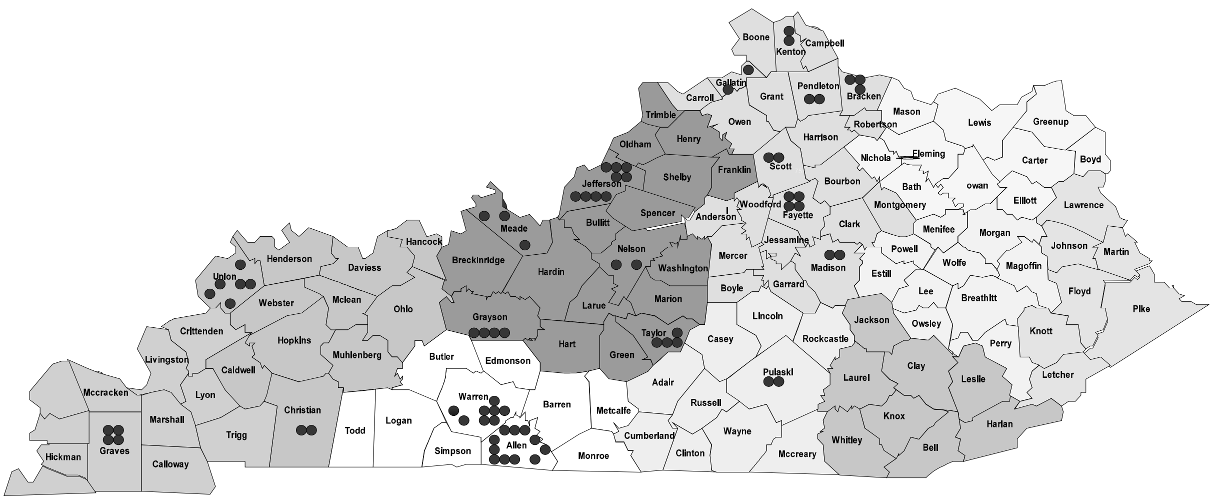


Figure 11. General locations of roadway sampling sites in Kentucky.

Roadway samples supplied by Geotechnical Branch of the Kentucky Transportation Cabinet provided a second group of soil samples for resilient modulus testing. Those samples were collected from different regions of the state, as shown in Figure 11. Also, Cabinet engineers supplied classification and some engineering data. By using roadway samples, classification and some engineering tests did not have to be performed during the study. Also, this approach provided a large number of different types of soils from many regions of the state.

Bulk Samples

Processing

The bulk soil samples were air-dried and processed in a ball mill. The purpose of this procedure was to breakdown the soil clods into individual particles and to produce a uniform material. After processing, each type of bulk sample was stored in drums for immediate testing and for long-term storage and future testing.

Geotechnical Test Methods and Data

Test methods used to determine soil classifications and engineering properties of the six bulk samples are tabulated in Table 1. Standard test methods of AASHTO were generally followed. Classification and engineering properties of the bulk samples are summarized in Table A-1, Appendix A. Based on the AASHTO Classification System, the samples were classified as A-4 (3,4,7) and A-7-6 (17, 18, 22). Using the Unified Classification System, the samples were classified as ML-CL, CL, and CH. Liquid limits and plasticity limits ranged from 26.5 to 52.3 and 5.8 to 26.7 percent, respectively. Clay fractions ranged from about 20 to 53 percent. The relation between the effective stress parameters, N', and c', for the six samples is shown in Figure 12. Triaxial specimens were compacted to 95 percent of maximum dry density and optimum moisture content obtained from AASHTO T-99.

CBR values of unsoaked, or "as compacted", specimens were much higher than CBR values of soaked specimens, as shown in Figures 11 and 12. Values of KY CBR of the "as compacted' specimens were some 2 to 8.5 times greater than the soaked KY CBR values. AASHTO CBR values of the "as compacted" specimens were some 1.4 to 15.4 times greater than the CBR values of the soaked specimens. Soaking the specimens greater reduced the CBR values. Values of CBR of bulk samples mixed with either 5 percent hydrated lime or 10 percent (by dry weight) were some 2 to 18 times greater than soaked CBR values of the untreated samples.

An approximate relation between KY CBR and AASHTO CBR (based on

Table 1. Listing of test methods.

Type of Test	Test Method
Moisture Content	AASHTO T 265-93 (1996)
Liquid Limit	AASHTO T 89-96
Plastic Limit and Plasticity Index	AASHTO T 90-00
Specific Gravity	AASHTO T 100
Particle Analysis of Soils	AASHTO T 88-00
Triaxial Compression Test:	
Unconfined Compressive Strength	AASHTO T 208-96
Consolidated-Undrained Compression With Pore Pressure Measurements	AASHTO T 297-94
Moisture-Density Relations	AASHTO T 180-97
California Bearing Ratio	AASHTO T 193-99
AASHTO	
Kentucky Method	KM – 64-501-95
Resilient Modulus of Soils	AASHTO T 292-91 (1996)

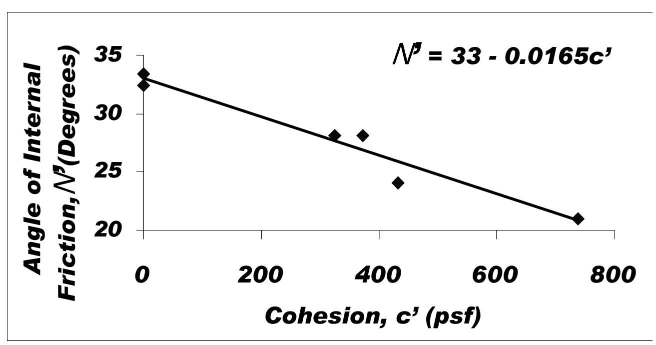


Figure 12. Angle of internal friction, N,' as a function of cohesion, c'.

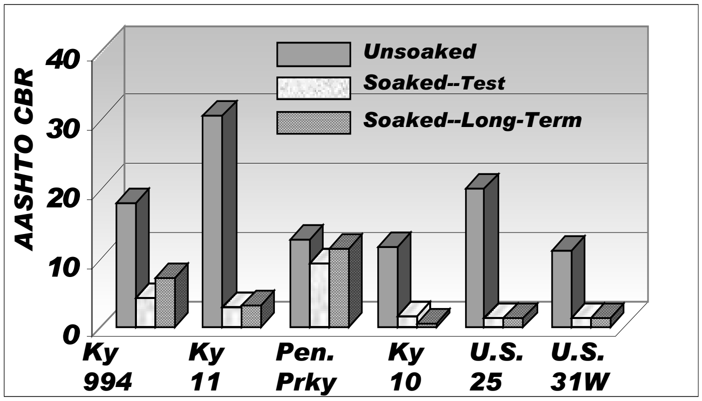


Figure 13. Comparison of soaked and unsoaked values of AASHTO CBR.

specimens remolded to 95 percent of maximum dry density and optimum moisture content obtained from AASHTO T-99) was shown previously in Figure 5. Generally, except for the specimen identified as the "Pennyrile Parkway", AASHTO CBR values of unsoaked specimens were much larger than AASHTO CBR values of soaked specimens, as shown in Figure 13. Soaked CBR values of the specimens treated with either hydrated lime or cement (5 % by dry mass) were much larger than the soaked untreated specimens, as shown in Table A-1 in Appendix A.

Roadway Soil Samples and Geotechnical Properties

Roadway samples collected by the Kentucky Transportation Cabinet during highway corridor studies were provided by that agency for resilient modulus testing. Classification and some engineering data generated during those studies are summarized in Tables A2, A3, and A4 in Appendix A. Testing included liquid and plastic limits, grain-size analysis, specific gravity, ASSHTO and Unified classifications, Kentucky CBR, and moisture-density relations obtained from AASHTO T-99. Soil types, based on the Unified Classification System, included ML, ML-CL, CL, MH, CH, and SC. Based on the AASHTO Classification System, soil types included A-4, A-6, A-7-5, and A-7-6. Liquid and plastic limits ranged from 22 to 70 and 6 to 44, respectively. Clay fraction (percent finer than 0.002 mm-size) of the samples ranged from about 0.3 to 68 percent.

RESILIENT MODULUS TESTING

Testing Equipment

The resilient modulus testing equipment, Figure 14, located at the University of Kentucky Transportation Center is a model RMT-1000, obtained from the Structural Behavior Engineering Laboratories, of Phoenix, Arizona. The system consists of a pressure control panel, plexiglass triaxial cell, a hydraulic power supply, and a computer and software for controlling the testing of a resilient modulus specimen. The system is a complete, closed-loop, servo hydraulic triaxial testing system. The triaxial system has

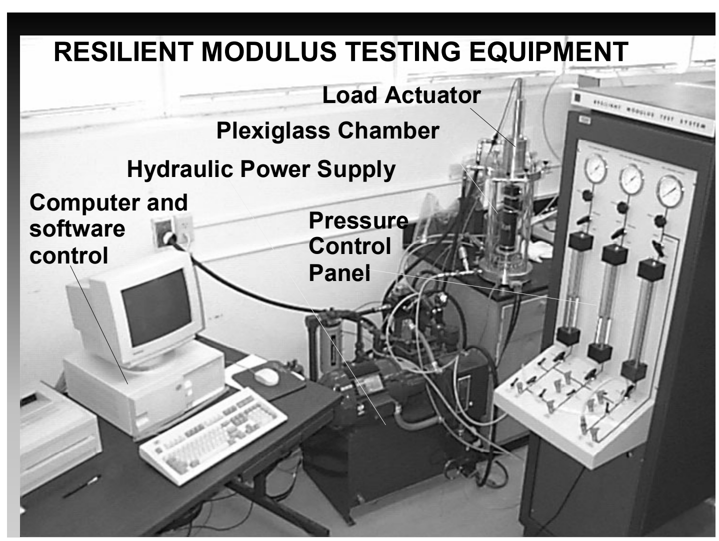


Figure 14. View of resilient modulus testing equipment

self-contained internal transducers. The triaxial cell is constructed with stainless steel and acrylic plastic cell wall. The cell is rated to withstand a confinement stress of 150 psi. A load actuator, Figure 15, applies repeated loads. Various load forms of different shapes are available for applying loading sequences.

System Components

The servo controller is a Model 547-1 with dual AC/DC feedback signal conditioning for load and deformation transfer. The signal conditioning system is a series 5 model 300, 4-channel for 2 internal LVDT's and 2 pressure transducers. A view of the LVDTs mounted internally, on the sides of a specimen, is illustrated in Figure 16. A load cell is mounted at the base of the specimen in the triaxial chamber.

The LVDT Transducer calibrator is a Model 139. It has a 1-inch travel range and a resolution of 0.00005 inches. The load cell, pressure transducer, and pore pressure transducer are calibrated using shunt calibration with preset resistance.

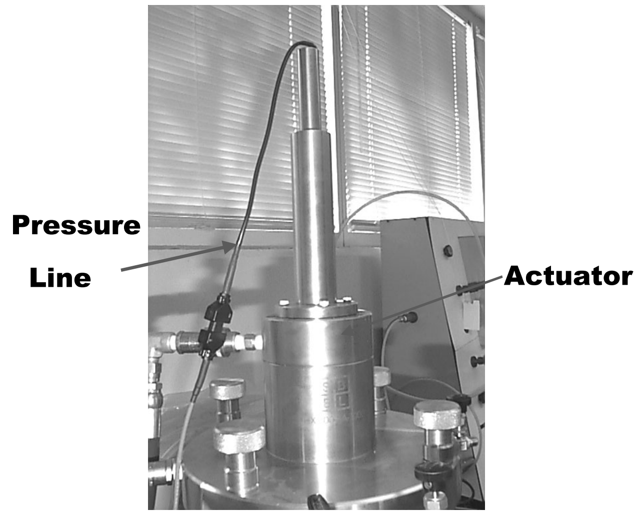


Figure 15. View of loading actuator.

Method of Compacting Resilient Modulus Specimens

The purpose of a compaction procedure is to produce a specimen that has a dry density and moisture content that are near prescribed, or target, values, of dry density and moisture content. For example, if field specifications dictate that a given material must be compacted in the field to 95 percent (or some other selected percentage) of maximum dry density obtained from a standard laboratory compaction procedure, such as AASHTO Designation T-99, or ASTM D 698, then the target values for remolding the laboratory specimen would be selected according to the field specifications. All specimens tested in this study were compacted to 95 percent of maximum dry density and optimum moisture content as determined from AASHTO T-99. This standard was followed because the specifications of the Kentucky Transportation Cabinet require that soil subgrades be compacted to 95 percent, or higher, of maximum dry density and "2 percent of optimum moisture as obtained from the moisture-density standard, AASHTO T-99. A detailed discussion of the compaction

Figure 16. View of LVDTs mounted on the sides of the specimen loading heads inside the triaxial chamber.

procedure used to form resilient modulus specimens has been given elsewhere by Hopkins and Beckham (1993b; 1995). The dry density of specimens remolded according to this procedure is generally within about "0.5 lb/ft^3 of the target dry density. Moisture content of the remolded specimen is usually within about –0.31 percent of optimum moisture content. With some experience in using this procedure, the differences are usually much smaller. The ability to remold specimens to prescribed dry density and moisture content is important in achieving uniformity in resilient modulus testing.

In the specimen remolding process, maximum dry density, ($_{dmax,}$ and optimum moisture content, w_{opt}, of the soil selected for testing is determined from a standard laboratory moisture-density test procedure, such as AASHTO T-99. Target values of moisture content, w, and dry density, ($_{d,}$ are selected from the moisture-density curve obtained from the selected compaction procedure. In this study, a target dry density of 95 percent of maximum dry density and a target moisture content equal to optimum moisture content were used to form the resilient modulus specimens, or

$$\gamma_{dt\arg et} = 0.95\gamma_{d\max} \tag{6}$$

and

$$w_{t\arg et} = w_{opt}. \tag{7}$$

The weight of air-dried soil and the volume of water required to form a specimen of known (or selected) volume may be computed from the following equations:

$$W_{airdried} = \gamma_{dt\arg et}V(1 + w_{hs}) \tag{8}$$

and

$$W_{water} = W_s(w_{t\arg et} - w_{hs}) = \frac{\gamma_{dt\arg et}V}{1 + w_t}(w_t - w_{hs}) \tag{9}$$

where

$W_{airdried}$ = weight of air-dried soil, or the weight of the soil at the time of mixing,

($_{dtarget}$ = target dry density of soil (usually obtained by multiplying a selected value of percent percentage times the maximum dry density obtained from a standard compaction test procedure)

w_{hs}= hygroscopic, or air –dried moisture content, or the moisture content at the time of mixing of the specimen,

W_{water} = weight, or volume, of water,

W_s = weight of oven-dried soil,

w_{target}= target water content, and

V= total volume of specimen (=Br^2h and in this study, r = radius=3.556 cm and h = height = 15.24 cm; V = 192.71cm^3)

In this study, oven-dried soil was not used to form the resilient modulus specimens. Oven drying can change the physical properties of naturally occurring soils. Air-dried soil was used in remolding the resilient modulus specimens. Using Equations 6 through 9, the exact amount of air-dried soil and the volume water needed to form a soil specimen of the selected specimen volume were calculated. However, the soil does not, necessarily, have to be air-dried. The only requirement is that the existing moisture content, w_{hs}, in the material at the time of sampling, must be equal to or less than the selected, target moisture content. After a small sample is obtained to find the existing moisture content of the material, the material may immediately be placed and sealed in a zip-lock plastic bag to prevent any further loss of moisture. The material remains sealed until the time of mixing.

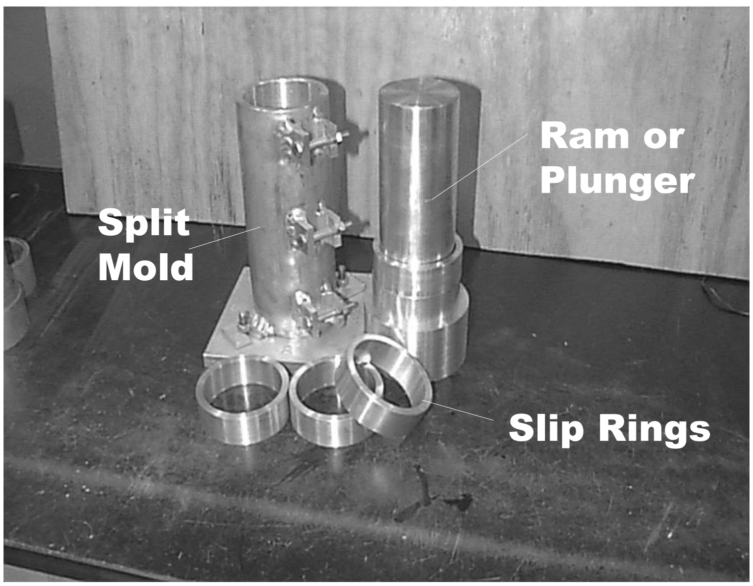


Figure 17. View of split mold, compaction ram, and slip rings

Equipment required to compact a specimen includes some type of apparatus, or other means, for mixing the specimen, an electronic scale with a resolution of 0.01 grams, a split-type mold, and a specially designed compaction ram, and slip rings. Although the split-type mold for compacting the specimens may be designed for any selected dimensions, a type of mold that is convenient for forming specimens for triaxial or permeability testing measures 20.32 cm (8 inches) in height and 7.11 cm (2.8 inches) in diameter. Specimens are compacted to a height of 15.24 cm (6 inches). The inside diameter of this mold is the same as the diameter (7.11 cm) of a commonly used, thin-walled, field sampling tube. By using a split-type mold, the specimen may be removed from the mold conveniently, the need to extrude the compacted specimen from the mold is avoided, and sample disturbance after compaction is reduced. A view of the split mold, slip rings, and compaction ram designed and machined exclusively for forming the resilient modulus specimens is shown in Figure 17.

The function of the ram and rings, which slip over the ram, is to control the height of each layer of the compacted specimen. In the compaction standard, AASHTO T-99, the specimen height is 11.6434 cm (4.584 inches); the specimen is compacted in three layers and each layer is 3.879 cm (1.527 inches) in height. In the proposed compaction procedure, each layer of the specimen is compacted to approximately the same height, or 3.81 cm (1.5 inches). For example, specimens measuring 15.24 cm in height are compacted in four layers but each layer is 3.81 cm in height. A schematic of the ram and slip rings used to compact specimens measuring 15.24 cm in height and 7.11 cm in diameter is illustrated in Figure 18 and 19.

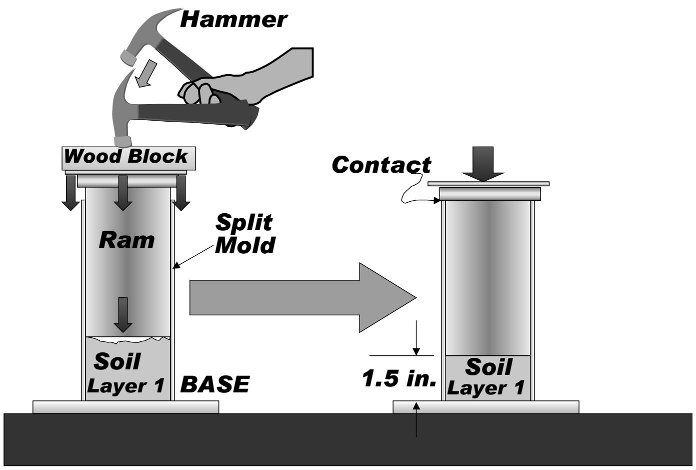


Figure 18. Illustration of compaction procedure.

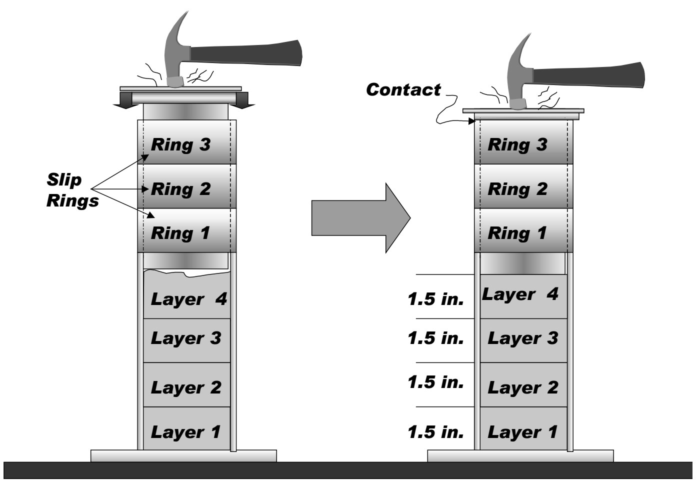


Figure 19. Compaction procedure is repeated for four layers.

To mix the sample, the soil is placed in a mixing bowl and the amount of water, as determined from Equation 51, is added to the material. When the specimen to be formed is 15.24 cm (6 in.) in height and 7.11 cm (2.8 in.) in diameter, the mixed material is divided into four parts of equal weight and stored in plastic (zip-lock) bags to prevent moisture loss. It is imperative that care is exercised in this portion of the procedure to avoid the loss of material when the material is weighed and transferred to the plastic bags. Normally, the material remains sealed in the plastic bags for about 24 hours before remolding to allow an even distribution of moisture.

After the mellowing period, the specimen is compacted as illustrated in Figure 20. The contents of the first bag are placed in the split mold and the ram is hammered down until the collar of the ram rests against the top of the mold. When the collar touches the top of the mold, the first compacted layer is exactly 3.81 cm (1.5 in.) in height. The top of the first layer is scarified and the second bag of material is added to the mold. The first slip ring is slipped over the ram and the second layer is compacted. When the bottom of the first ring touches the top of the mold, the second layer is exactly 3.81 cm (1.5 in.). The procedure is repeated for the third and fourth layers, as shown in Figure 16, respectively. When the last layer is compacted, the specimen is exactly 15.24 cm (6 in.) in height. During the compaction procedure, the number of blows does not have to be counted because the exact amounts of materials and water are used to form the specimen of a selected dry density, water content, and known volume.

For a selected type of soil, resilient modulus tests were performed on both unsoaked, or “as compacted”, and soaked specimens. Specimens to be soaked were initially compacted to 95 percent of maximum dry density and optimum moisture content obtained from AASHTO T-99. After compaction, the split mold containing the compacted specimen was submerged in a water bath, Figure 20. The bottom platen of the split mold was perforated so that water could enter the bottom of the specimen. The top of the specimen was exposed to water. The specimen was allowed to swell until swelling essentially ceased. For soil specimens containing large clay fractions, this procedure generally required some two to four weeks. The time of swelling was much more than the swelling times generally obtained for CBR specimens that are allowed to swell according to criteria of Kentucky Method 64-501-95 (Kentucky Methods 1998), or AASHTO T 193-99.

Resilient Modulus Testing Protocol

The testing procedure, AASHTO T-274, was developed and used to determine the resilient modulus of cohesionless materials. The procedure was lengthy and time-consuming because it required testing of a specimen under numerous stress states and loading conditions. As noted by Nazarian et al. (1993), thirty-three steps were involved in testing a single specimen. Conditioning cycles used in the procedure presumably aids in establishing better contact between the specimen and load platens and in developing a more homogeneous specimen. However, the conditioning procedure ran the risk of causing unrecoverable deterioration of the specimens before the actual testing began because of high stress levels and the lengthy cycle testing of the procedure. To complete a single test, including sample preparation, a testing time of about 4.5 hours was required.

To minimize some of the difficulties of AASHTO T-274, such as sample degradation and disturbance during conditioning, the Strategic Highway Research Program (SHRP) proposed a testing procedure that required only one conditioning step. This procedure reduced the testing time because fewer loading steps and cycles of loading were required. This method required five confining pressures. Deviator stresses ranging from 3 to 40 psi were used in the procedure. However, deviator stresses caused excessive deformation of specimens, especially specimens having a low modulus.

In a cooperative and joint effort, AASHTO and SHRP committees formulated and proposed resilient modulus testing standard, AASHTO TP46-94[1]. In the resilient modulus testing reported herein, AASHTO TP46-94 was followed. In this procedure, a total of 17 load cycles are used. The loading sequence is illustrated in Table 4. Two cycles are used for conditioning of the specimen. Following conditioning, fifteen different stress increments are applied and the resilient modulus values are measured. Each sequence is applied with 100 load cycles at 1 Hz. Deviator stresses applied to the specimen are 2, 4, 6, 8, and 10 psi. No attempt was made at 1 psi because the major dispersion of results experienced was at this stress level. The high dispersion is the product of small strain resilient deformation, which cannot be measured accurately. This new method takes approximately 1.5 hours (including specimen preparation) to complete.

Figure 20. View of water bath and split mold containing compacted specimen.

After placing the remolded specimen in a triaxial assembly, Figures 14 and 16, repeated loads were applied. The specimen is loaded using a haversine shaped load form. The load pulse is in the form, (1-cos)/2, as shown in Figure 21. A Haversine stress pulse was chosen because it better represents the shape of a truck loading on pavement and similar to the load pulse applied by nondestructive testing device, that is the Falling Weight Deflectometer (FWD). The magnitude of the cyclic load is varied to measure the behavior in soil stiffness, or modulus. Before instrumenting the sample, it was visually checked for uniformity and suspected samples were rejected.

[1] The testing standard AASHTO TP46-94 was the forerunner to AASHTO T-292. Essentially, AASSHTO T-292 was followed. The load cell and LVDTs were located inside the testing chamber.

Table 2. Testing Sequence for Subgrade Soils			
Sequence Number	Confining Pressure (psi)	Axial Stress (psi)	Number of Load Applications
1	6	4	100*
2	6	4	100*
3	6	2	100
4	6	4	100
5	6	6	100
6	6	8	100
7	6	10	100
8	4	2	100
9	4	4	100
10	4	6	100
11	4	8	100
12	4	10	100
13	2	2	100
14	2	4	100
15	2	6	100
16	2	8	100
17	2	10	100

* *Conditioning Cycles*

Specimen conditioning is suppose to eliminate the effects of initial permanent deformation and specimen loading imperfections and not cause permanent plastic deformation. There is an issue on the number of repetition of conditioning load cycles that should be applied before suitable conditioning has been achieved (Tennessee, 1993). The AASHTO TP46-94 suggests a number between 500 to 1000 cycles while holding the confining pressure at 6.0 psi and the deviator stress at 4.0 psi. In the testing reported herein, each specimen was conditioned using 200 cycles and 100 cycles were for each load sequence.

The average recovered deformations for each LVDT are recorded at the last five cycles. Table 3 shows the testing sequence for type 2 subgrade soils. Type 2 is composed of path in which both cell pressure and deviator stress are cycled in phase to better represent the real pavement loading.

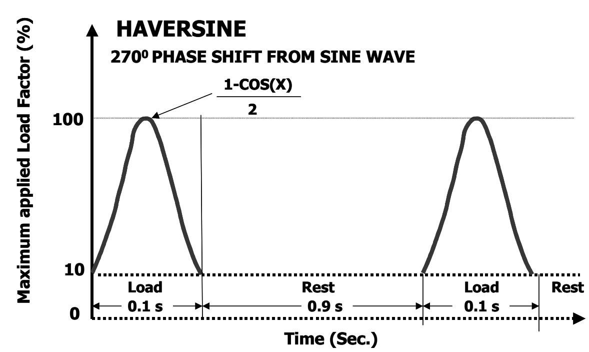


Figure 21. Haversine loading form.

An example data is shown in Appendix B. The computer data acquisition system records the mean deviator load and the mean recovered deflection. The system then calculates the mean resilient modulus by dividing the mean resilient strain by the applied deviator stress.

Review of Mathematical Models for Relating Resilient Modulus and Stresses

Mathematically, resilient modulus, M_r, has been defined as:

$$M_r = \frac{\sigma_d}{\varepsilon_a}, \tag{10}$$

where

F_d = deviator stress = F_1 - F_3 ,
F_1 = major principal stress,
F_3 = minor principal stress, and
g_a = axial strain recoverable after the release of the deviator stress.

Deformation properties of soils are not constant. They are determined by both intrinsic properties of soils and the stresses applied to the soils. A number of mathematical models have been proposed for modeling the resilient modulus of soils and aggregates. Most mathematical expressions relate resilient modulus, the dependent variable, to one independent variable, either the deviator stress, F_d, or confining stress, F_3, or the sum of principle stresses, F_{sum} ($F_1 + F_2 + F_3$), or to two independent variables, F_d and F_3. Some widely published resilient modulus models are examined below. As shown by this review and analysis of available models, only two models are used in the analyses of resilient modulus data reported herein.

Moossazadeh and Witczak (1981) proposed the following relationship for presenting resilient modulus data:

$$M_r = k_1\left(\frac{\sigma_d}{p_a}\right)^{k_2}, \tag{11}$$

where k_1 (y-intercept) and k_2 (slope of the line) are coefficients obtained from a linear regression analysis and p_a is a reference pressure. In this model, the effect of the confining stress is not considered.

Dunlap (1963) suggests the following relationship:

$$M_r = k_1\left(\frac{\sigma_3}{p_a}\right)^{k_2}, \tag{12}$$

where k_1 and k_2 are regression coefficients and F_3 is the confining stress. The influence of the deviator stress is ignored in this relationship.

Seed et al. (1967) suggests that the resilient modulus is a function of the sum of the principle stresses, or

$$M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}. \tag{13}$$

The term, $F_{sum,}$ is the sum of principal stresses ($F_1 + F_2 + F_3$), or for the triaxial compression case, the term is equal to ($F_1 + 2F_3$). This expression appears in the AASHTO Pavement Design Guide (1993) and in the testing standard, AASHTO T 292-91(2000). This relationship does not account for the effect of confining stress on the resilient modulus. Relationships given by Equations 11 and 12 do not consider the effect of shear stress on the resilient modulus of soils.

May and Witczah (1981) and Uzan (1985) propose another model that considers the effects of shear stress and confining stress and deviator stress, or

$$M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}\left(\frac{\sigma_d}{p_a}\right)^{k_3}. \tag{14}$$

The terms, k_1, k_2, and $k_{3,}$ are correlation regression coefficients. Under identical loading ($\sigma_1 = \sigma_2 = \sigma_3$), Uzan's model will lead to a value of M_r that either goes to zero when the coefficient, $k_3>0$, or, M_r will become infinite in the case of $k_3<0$. In all of the models cited above, a regression fit can be made for a selected confining stress. However, when the confining stress changes, the coefficients change.

To correctly model the resilient modulus of soils and aggregates and to account for the influences of confinement stress and deviator stress, a new model is proposed, or

$$M_r = k_1 \left(\frac{\sigma_3}{p_a} + 1 \right)^{k_2} \left(\frac{\sigma_d}{p_a} + 1 \right)^{k_3} . \tag{15}$$

In this model, the coefficients, k_1 and k_2, will always be positive. For most situations the coefficient, k_3, is negative for soils and aggregates. As shown by the relationship given by Equation 15, the resilient modulus increases as the confining stress increases. The modulus will increase or decrease, as in most cases, with the increase of shear stress. When both F_3 and F_d approach zero, the value of resilient modulus, M_r, approaches the value of k_1, which is the initial resilient modulus value and a property of the soil. How the resilient modulus of soils changes from its initial value depends on the stress path and the stress state applied to the soil mass. The coefficients, k_1, k_2, and k_3, are derived from test data using multiple correlation regression analysis.

Another mathematical expression appears in a summary pamphlet prepared by the research team for study NCHRP (National Highway Cooperative Research Program) Project 1-28A (Fall 2001). This relationship is, as follows:

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} , \tag{16}$$

where:

F_{sum} = sum of all orthogonal normal stresses acting at a given point (or as listed in the summary, F_{sum} is defined using the symbol, 2, which is defined as the bulk stress).

J_{oct} =Octahedral shear stress acting on the material, or

$$\tau_{oct} = \frac{\sqrt{2}}{2} (\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} . \tag{17}$$

Equation 16 represents the more general case, that is, F_2 is not equal to F_3. If F_2 equals F_3, then Equation 17 becomes

$$\tau_{oct} = (\sigma_1 - \sigma_2) = (\sigma_1 - \sigma_3) = \sigma_d = deviator \quad stress$$

and Equation 16 becomes

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\sigma_d}{p_a} + 1 \right)^{k_3} . \tag{18}$$

Equations 14 and 15 (Models 4 and 5) are based on the assumption that the normal stresses, F_2 and F_3, are equal and represent a specific case (triaxial case). If F_2 is not equal to F_3, then Equations 14 and 15 may be written for the more general case, or

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} \right)^{k_3}, \tag{19}$$

and

$$M_r = k_1 \left(\frac{\sigma_3}{p_a} + 1 \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3}. \tag{20}$$

Consequently, Equations 14 and 15 become Equations 19 and 20.

Resilient Modulus Test Data

Synthetic Specimens

A series of synthetic specimens were built so that resilient modulus testing equipment could be checked periodically. These specimens were routinely tested during the soils testing program to insure that uniform results were obtained from the resilient modulus testing equipment during production testing. The synthetic specimens provided quality control during testing. The specimens could also be used to compare the results among different resilient modulus testing equipment. The specimens were molded following procedures developed by Stokie II et al. (1990). The specimens were identified as 701, 901, and 961. Dimensions of the specimens were 6.0 inches in height and 2.85 inches in diameter. In molding the specimens, efforts were made to create specimens that had low, medium, and fairly large values of resilient modulus. Actual values of resilient modulus, M_r, obtained from testing the specimens in the equipment shown in Figure 14 ranged from about 1350 to 33,400 psi. The value obtained depends on the type of synthetic specimen tested, the stress level specified, and the model used to analyze the results. Using model 5 (Equation), the value of M_r for specimen 701 ranged from about 1365 to 1575 for the stresses, F_3 equal 2 psi and F_d equal to 2 psi, and F_3 equal 6 psi and F_d equal to 10 psi , respectively. Similarly, using the same stress pairs, the resilient modulus of synthetic specimen 901 ranged from about 6,378 to 9,354, respectively. For specimen 961, the resilient modulus ranged from 15,188 to 33,365 psi. Detailed test results for those specimens are not shown in this report because of the large amount of data. The data resides in files at the University of Kentucky Transportation Center and in the Kentucky Geotechnical Database (Hopkins et al. 2004).

Compacted Soil Specimens

All resilient modulus test data pertaining to the compacted soil specimens resides in the Kentucky Geotechnical Database. In those series of tests, the specimens were compacted to 95 percent of maximum dry density and optimum moisture (AASHTO T-99). The program, using this database, is in a client/server "Windows" environment and the database resides on a production server of the Kentucky Transportation Cabinet. Values of resilient modulus of the unsoaked (or "as compacted") and soaked compacted specimens are stored in the database and are readily available to personnel of the Kentucky Transportation Cabinet statewide. All key district and central office personnel can access the data through the client-server network.

Users have two means of accessing data on the client-server application in the Geotechnical Database. After the user logs on (Figure 22), the graphical user interface (GUI) shown in Figure 23 appears. By clicking on "Engineering Applications", another GUI appears as shown in Figure 24. When the user clicks on "Resilient Modulus," another GUI appears as shown in Figure 25. After clicking on "Resilient Modulus," the GUI screen in Figure 25 appears. By clicking on a soil classification shown in the left-hand portion of the figure, two-dimensional plots of resilient modulus as a function of a selected stress component appears. In the current analytical version, values of resilient modulus for a selected specimen may be plotted as a function of either the confining stress, F_3, the deviator stress, F_d , or the sum of the principle stresses, F_{sum}. In Figure 25, the resilient modulus is shown as a function of the deviator stress. For a selected soil classification, the user may specify data for soaked or unsoaked specimens.

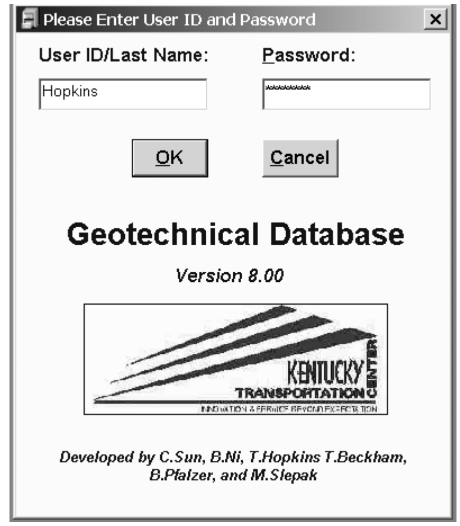


Figure 22. User log-on graphical user interface screen to gain access to the Kentucky Geotechnical Database and resilient modulus data.

The user may also view specimen data as shown in Figure 26. By clicking "Check Model", the

Figure 23. Main menu of the Kentucky Geotechnical Database.

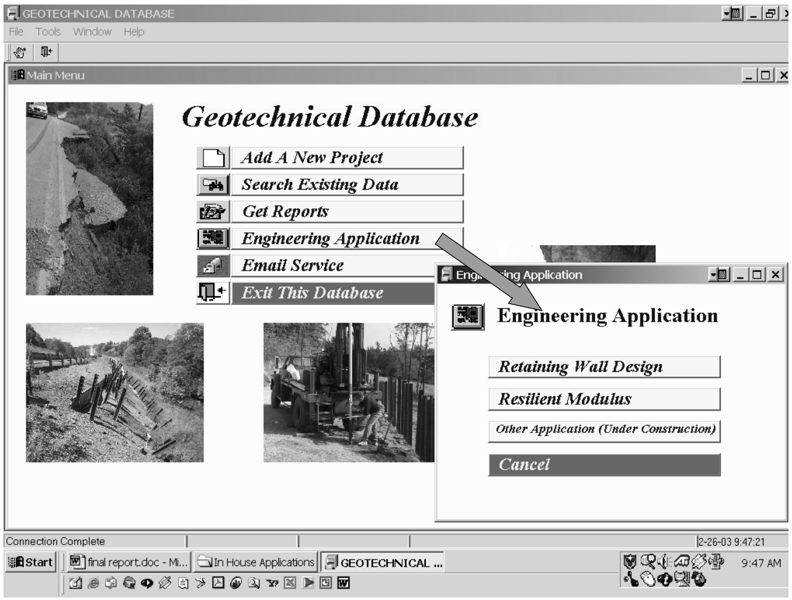


Figure 24. Gaining access to resilient modulus test results for compacted soils and aggregates in the Kentucky Database.

user may select a model number using a dropdown menu and graph the data. Coefficients (k_1, k_2, or k_3) of each model equation are displayed at the bottom of the GUI screen in Figure 25. Each time the user clicks on a classification, multiple regression analysis is automatically performed using the three different models shown in Figure 25. The coefficients of each model are displayed.

The user may also recall and display resilient modulus data by clicking "Data" under the "Show" button dropdown menu, as illustrated in Figure 26. In this case, the data are displayed. If the user chooses to view the entire record of resilient modulus test data of a specimen, then another procedure is available. In the database's main menu, Figure 23, the user clicks on "Search Existing data." When

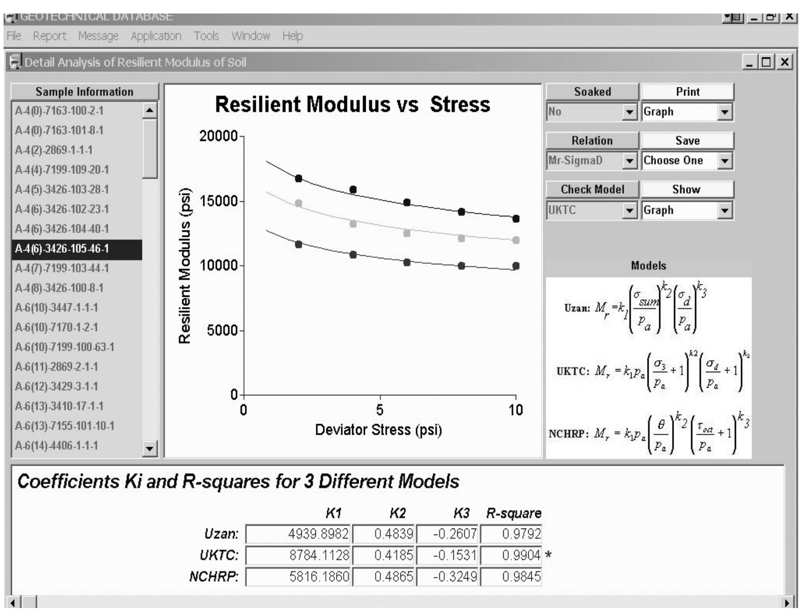


Figure 25. Graphical user interface showing resilient modulus as a function of deviator stress for a selected soil classification.

this event is executed, a GUI screen appears as shown in Figure 27. Using the row number (3426) shown in Figure 26, inserting this number into the box labeled ""Site Row Number" in the GUI shown in Figure 27, and clicking the search button, a GUI screen appears, as shown in Figure 28. Clicking on "Samples" and then on "Tested Soil," the GUI screen appears, as shown in Figure 29.

By clicking on the "properties" tab, a soil menu is displayed, as shown in the right-hand portion of the figure. After clicking on the "Resilient modulus" tab, the complete resilient modulus test file for the selected soil classification, or specimen, appears, as shown in Figure 30. If the resilient modulus tests were performed on both unsoaked and soaked specimens (of the same soil type), then both sets of data appear in the GUI screen.

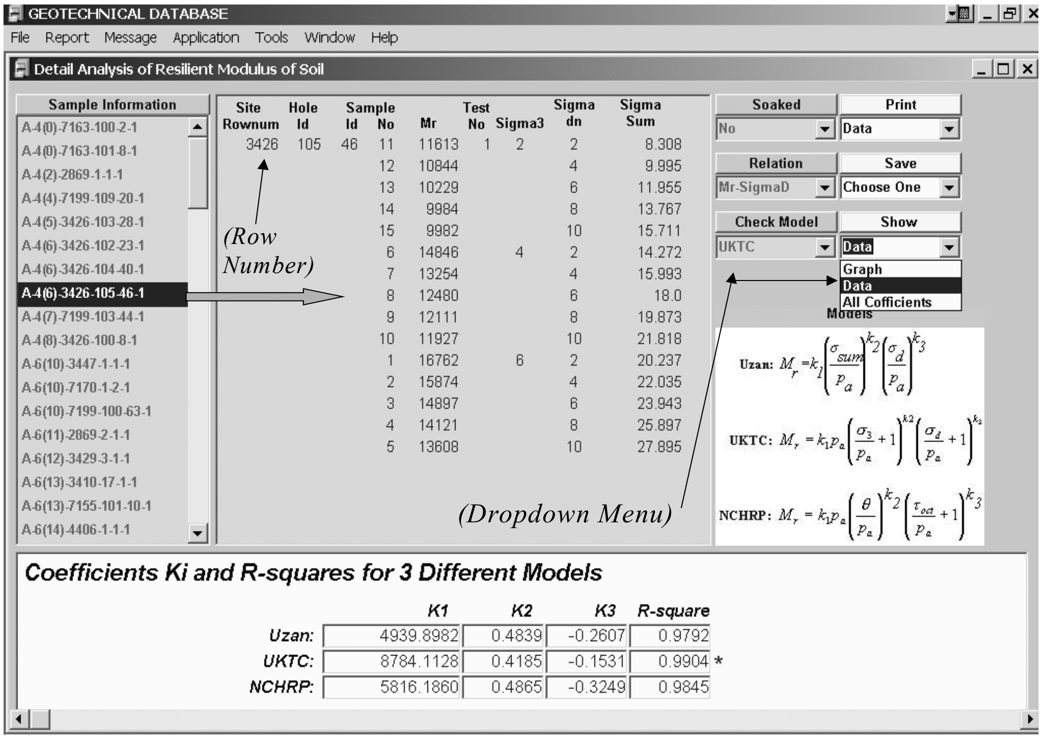


Figure 26. Display of resilient modulus data.

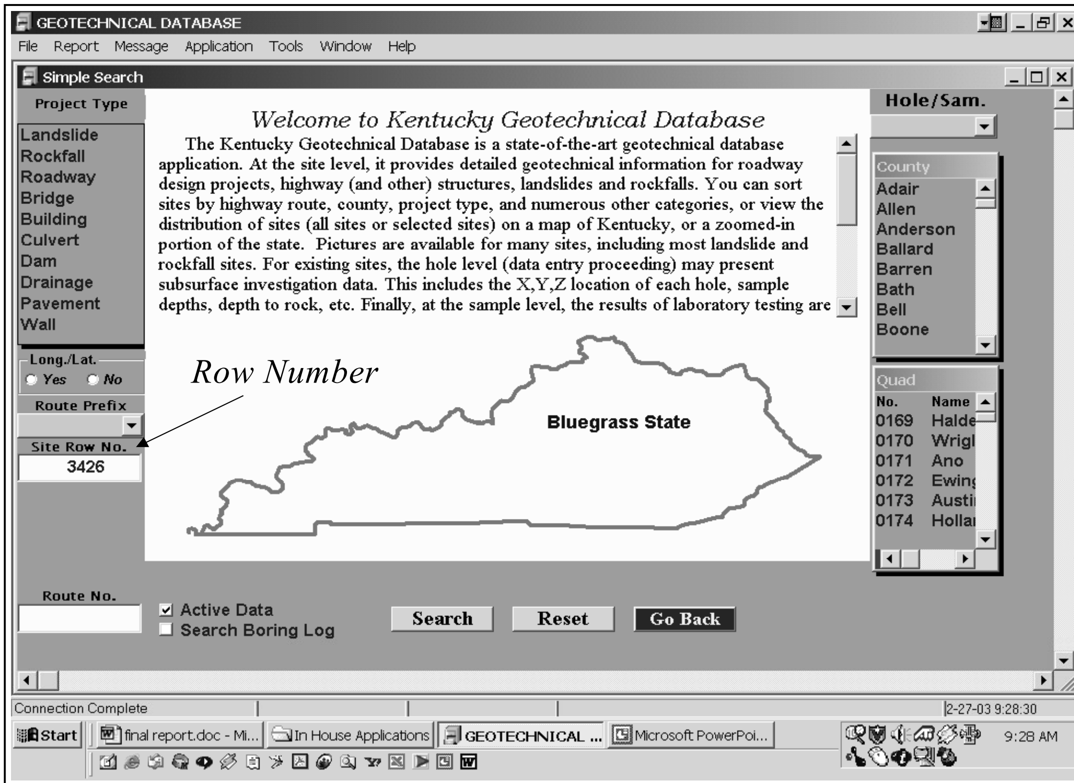


Figure 27. GUI screen for searching data.

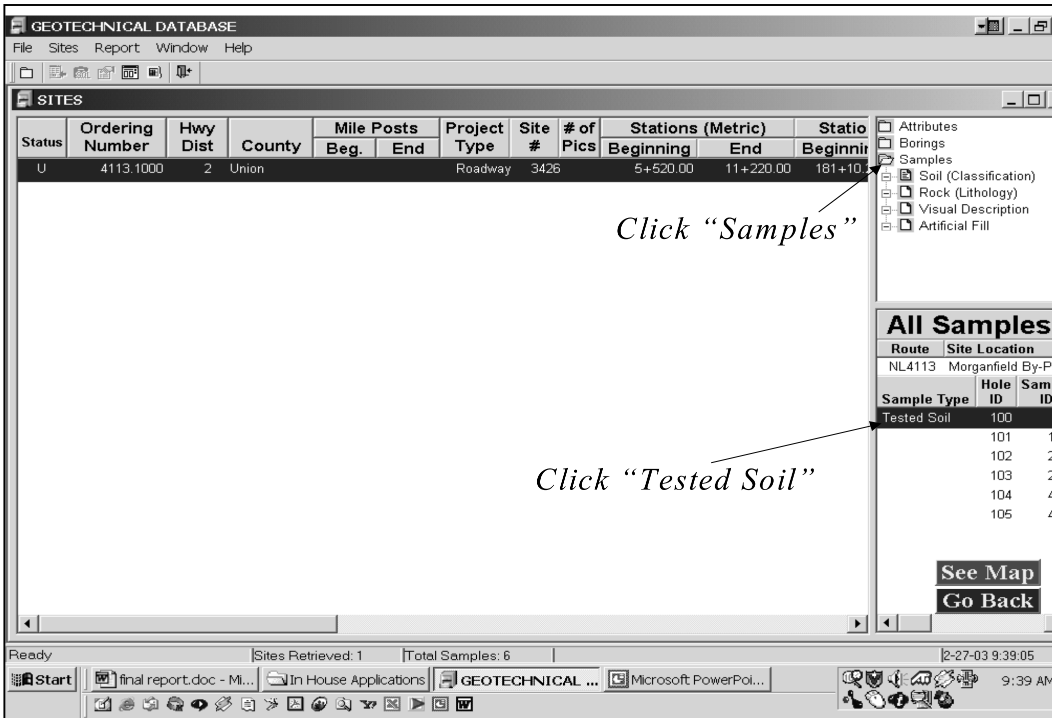


Figure 28. Gaining access in the Kentucky Geotechnical database to the entire resilient modulus test record for a selected specimen.

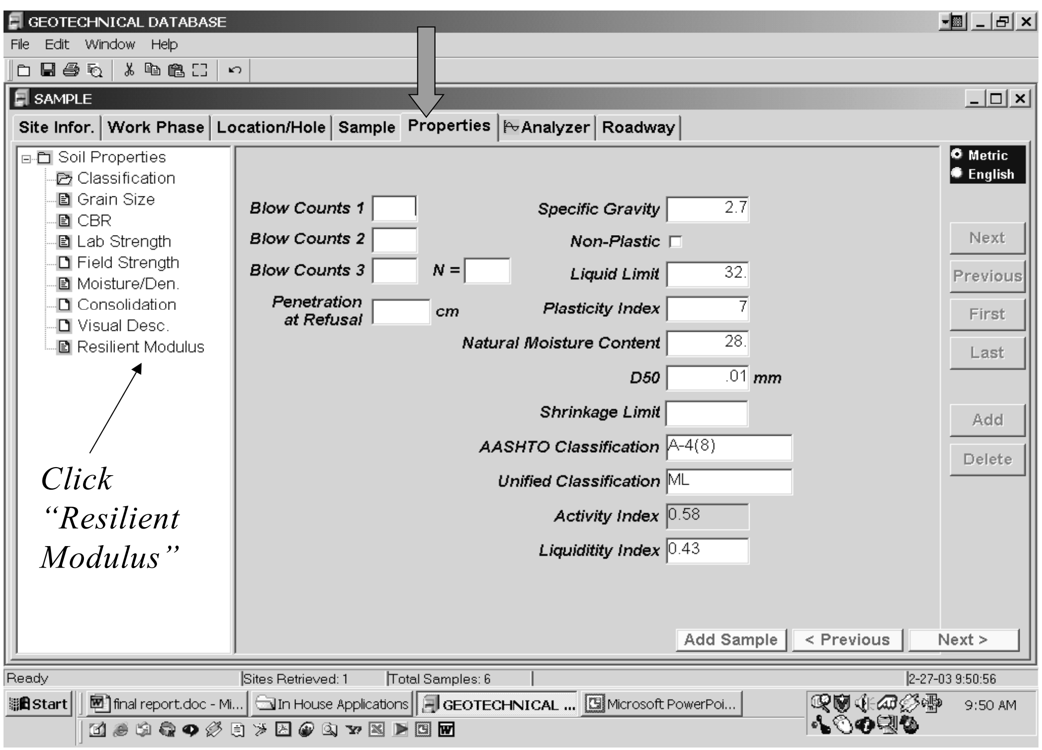


Figure 29. GUI screen for accessing the complete resilient modulus test data.

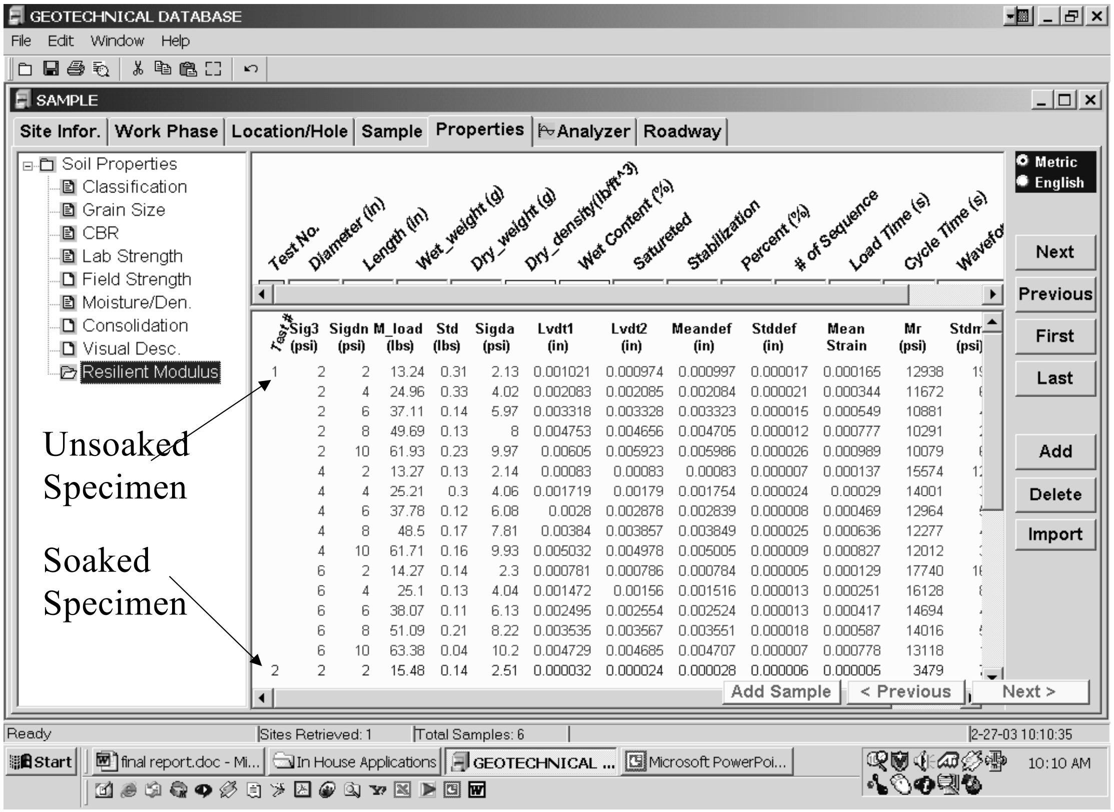


Figure 30. Complete resilient modulus test record for a selected specimen, or soil classification.

ANALYSIS

In performing any type of geotechnical test, the ability to repeat the results of a test is a major concern. To examine this aspect of the resilient modulus test, a limited number of repeatability tests were performed on molded synthetic specimens and a series of compacted soil specimens formed from the same type of soil and to a specified dry density and moisture content. These tests and analyses were very limited in scope because of time constraints. It is recommended that this aspect be studied in much more detail than the tests and analyses shown herein. However, the approach described below may serve as a framework for performing in future testing a much more thorough examination of this important aspect of resilient modulus testing. Regarding the resilient modulus test, a number of different mathematical models have been proposed, as outlined above, to describe the behavior of soils under cyclic loading. These more publicized models were used in analyzing the results of the numerous resilient modulus tests performed during this study. An effort was also made to examine the difference in resilient modulus of unsoaked and soaked compacted specimens.

Accuracy of Remolding Soil Specimens

In constructing soil subgrades, generally specifications require that the subgrade soils be compacted to a selected minimum dry density and mositure content. Usually, in Kentucky, the requirement is that the subgrade soils in the field be compacted to a dry density that is 95 percent, or greater, of some selected value of maximum dry density and to a moisture content that is K 2 percent of optimum moisture content obtained from from AASHTO T-99, or some other moisture-density testing standard. Hence, in performing resilient modulus tests it is important to conform to some type of test laboratory compaction procedure that closely simulates the field compaction of the subgrade soils. In the testing program adopted herein, all laboratory specimens were compacted to 95 percent of maximum dry density and optimum moisture content obtained from AASHTO T-99. In following this rule, a questions arises concerning the the difference between target values of dry density and moisture content and the actual values of dry density and moisture content obtained from the compaction procedure outlined above. In Table 8, the actual values of dry density and moisture content are compared to target values of dry density and mositure contents. Statistical analysis of those data show that at the 95 percent confidence level the differences in the target values and actual dry densities ranged from only 0.41 to a value of –0.17 lbs/ft^3. the mean value was 0.12 lbs/ft^3. The differerences in the target values of moisture content and actual moisture contents obtained for the compacted specimens ranged from 0. to 0.7 percent . The mean value was 0.4 percent. Hence, actual dry densities and mositure contents obtained from the compaction procedure were very close to target values.

Figure 31. View of synthetic specimens.

Repeatability tests

Synthetic Specimens

Five resilient modulus tests were performed on each of the synthetic specimens identified as 701, 901, and 961 to observe the repeatability of the testing equipment and the testing procedure. A view of the specimens is shown in Figure 31. Results of those tests are summarized in Tables 3, 4, and 5, respectively. In this sequence of testing, each specimen was mounted in the chamber, the LVDTs were mounted, the chamber and actuator was put into place, and the test was performed. After completion of the test, the equipment was completed dismounted and the synthetic specimen was removed from the chamber. In a sequent test, the specimen was mounted and the process was repeated. This procedure was performed for each test.

Results of the resilient modulus tests on the synthetic specimens identified as 701, 901, and 961 are summarized in Tables 3, 4, and 5, respectively. The regression coefficients for the resilient modulus models identified as 4, 5, and 6. In this case, multiple regression analysis were performed to obtain a regression plane—all 15 points corresponding to different stresses were used in the multiple regression analyses. An R^2-value of each regression plane for each test was obtained for each model. Values of resilient modulus at low, medium, and high stresses in the testing domain were calculated for each model and compared in the Tables 3, 4, and 5. Values of stresses, F_3 and F_d, are also shown in the tables.

Values of M_r, based on Model 4 and synthetic specimen numbered 701 (Table 3), range from 1360, at stresses of F_3 equal 2 psi and F_d equal to 2 psi, to 1590 psi at stresses of F_3 equal 6 psi and F_d equal to 10 psi . Similarly, values of M_r of the regression plane for Model 5 ranged from 1379 to 1594 psi, respectively. For Model 6, the values of M_r ranged from 1365 to 1589 psi. For specimen

901 (Table 4) and Model 4, the values of M_r ranged from 6167 to 9324 psi. Values of M_r for Model 5 ranged from 6443 to 9523 psi while values of M_r for Model 6 ranged from 6170 to 9323, respectively.

For specimen 961 and Model 4, the values of M_r ranged from 13,304 to 31,304. For Model 5, the Mr-values ranged from 15,665 to 32, 744 psi while for Model 6 the values ranged from 13,290 to 31,595.

The 95 percent confidence testing level was examined for each model and synthetic specimen. Results are summarized in Table 6. For synthetic specimen 701 and Models 4, 5, and 6, the 95 percent confidence level ranged from 4.9 to 7.0, 4.7 to 7.0, and 4.9 to 6.9, respectively. Model 5 in this case appeared to yield slightly better results than Models 4 and 6. Considering synthetic specimen 901, the 95 percent confidence level ranged from 10.1 to 11.9, 10.4 to 11.6, and 10.2 to 11.9, respectively, for Models 4, 5, and 6. For synthetic specimen 961, the 95 percent confidence level ranged from 12.5 to 23.9, 14.4 to 20.1, and 16.7 to 24.0 percent, respectively. Overall, Model 5 appeared to yield slightly better results than Models 4 and 6. For Models 4, 5, and 6, respectively, the 95 percent confidence level for the three specimens ranged from 4.9 to 23.9, 4.7 to 20.1, and 4.9 to 24.0 percent. As the values of resilient modulus increased, the value of the 95 percent confidence level increased.

Using synthetic specimen 701 to check repeatability of the system, Chow (1998) concluded that the system is more accurate at a larger strain and stress. As the deviator stress increased from 2 psi to 10 psi, and for a constant confining stress, the standard deviation decreased from 24 at 2 psi to 12 at 10 psi.

Compacted Soil Specimens

Five soil clayey specimens were also compacted to nearly identical dry densities and optimum moisture contents. Each specimen was compacted to 95 percent of maximum dry density and optimum moisture content as determined by AASHT Test Method T-99. Every effort was made to make each specimen identical. Resilient modulus repeatability tests were performed on the "as compacted" specimens. Comparison of target values and actual dry unit weights and moisture contents obtained for the specimens are shown in Table 8. The actual moisture contents averaged about 0.3 percent below the target (optimum) moisture content while the actual dry unit weights averaged about 0.55 lbs/ft3 above the target dry unit weight. Coefficients of the regression plane for each model are shown in Table 7. The 95 percent testing confidence level for each model using three selected stress levels are summarized in Table 9. For Model 4, the 95 percent confidence level ranges from 7.0 to 32.8 percent. For Model 5, the 95 percent confidence level ranges from 7.2 to 23.3 percent. The 95 percent confidence level for Model 6 ranges from 7.0 to 26.8 percent.. Model 5 proposed by the authors yielded slightly better results than Models 4 and 6. However, a much more detailed study is recommended to confirm this observation.

Comparisons of Resilient Modulus Models Using Unsoaked, "As Compacted" Soil Specimens

Simple Correlation Analysis

To evaluate the different models cited above, 68 laboratory specimens of different types of soils were compacted and resilient modulus tests were performed. Specimens used in this series of tests were compacted to 95 % of maximum dry density and optimum moisture (ASSHTO T-99). Resilient

Table 3. Regression coefficients of Models 4, 5, and 6 and values of resilient modulus at selected stresses of synthetic specimen identified as number 701.

Model Number	Synthetic Specimen	k_1	k_2	k_3	R^2	Low Stresses σ_3	σ_d (psi)	Mr	Mid Stresses σ_3	σ_d (psi)	Mr	High Stresses σ_3	σ_d (psi)	Mr
Model 4[1]	819701A	1220.27	0.075	0.006469419	0.956	2	2	1439	4	6	1533	6	10	1590
	820701A	1164.347	0.075936	0.006074577	0.943	2	2	1375	4	6	1466	6	10	1521
	821701A	1160.079	0.080532	-0.006013488	0.925	2	2	1360	4	6	1448	6	10	1496
	822701A	1175.261	0.076329	-0.005937591	0.884	2	2	1366	4	6	1450	6	10	1495
	823701A	1232.941	0.061403	-0.009605829	0.856	2	2	1382	4	6	1447	6	10	1480
Model 5[3]	819 701A	1296.354	0.063713	0.034593	0.913	2	2	1444	4	6	1536	6	10	1594
	820 701A	1242.008	0.062714	0.034638	0.895	2	2	1382	4	6	1470	6	10	1525
	821 701A	1252.943	0.066484	0.020945	0.853	2	2	1379	4	6	1452	6	10	1499
	822 701A	1263.279	0.063261	0.019656	0.809	2	2	1384	4	6	1453	6	10	1498
	823 701A	1312.542	0.050435	0.009665	0.773	2	2	1402	4	6	1451	6	10	1482
Model 6[3]	819 701A	1215	0.074978	0.008157	0.955	2	2	1428	4	6	1531	6	10	1589
	820 701A	1159.61	0.075663	0.008069	0.944	2	2	1365	4	6	1464	6	10	1520
	821 701A	1164.738	0.08035	-0.00725	0.924	2	2	1370	4	6	1450	6	10	1497
	822 701A	1179.895	0.076039	-0.00697	0.883	2	2	1375	4	6	1452	6	10	1496
	823 701A	1240.857	0.061139	-0.01162	0.851	2	2	1398	4	6	1450	6	10	1481

Table 4. Regression coefficients of Models 4, 5, and 6 and values of resilient modulus at selected stresses of synthetic specimen identified as number 901.

Model Number	Synthetic Specimen	k_1	k_2	k_3	R^2	Low Stresses σ_3	σ_d (psi)	Mr	Mid Stresses σ_3	σ_d (psi)	Mr	High Stresses σ_3	σ_d (psi)	Mr
Model 4[1]	819901A	4531.196	0.211539	0.007272	0.978	2	2	7070	4	6	8461	6	10	9324
	820901A	4200.481	0.208782	0.012144	0.937	2	2	6539	4	6	7849	6	10	8661
	821901A	3678.673	0.24164	0.02041	0.951	2	2	6167	4	6	7672	6	10	8626
	822901A	4454.043	0.180834	0.015515	0.937	2	2	6557	4	6	7724	6	10	8433
	823901A	4190.921	0.196268	0.018159	0.940	2	2	6383	4	6	7635	6	10	8404
Model 5[3]	819901A	5513.788	0.174549	0.086226	0.960	2	2	7343	4	6	8636	6	10	9523
	820901A	5105.857	0.164784	0.095362	0.909	2	2	6795	4	6	8014	6	10	8844
	821901A	4580.421	0.194589	0.115954	0.923	2	2	6443	4	6	7851	6	10	8833
	822901A	5285.694	0.144261	0.084162	0.916	2	2	6793	4	6	7853	6	10	8564
	823901A	5015.116	0.162403	0.089598	0.916	2	2	6615	4	6	7754	6	10	8528
Model 6[3]	819901A	4508.113	0.212367	0.007917	0.978	2	2	7072	4	6	8458	6	10	9323
	820901A	4164.151	0.210056	0.013489	0.935	2	2	6541	4	6	7845	6	10	8661
	821901A	3627.169	0.242843	0.023916	0.949	2	2	6170	4	6	7667	6	10	8628
	822901A	4406.678	0.181671	0.01829	0.936	2	2	6560	4	6	7720	6	10	8435
	823901A	4139.205	0.197461	0.021014	0.938	2	2	6387	4	6	7630	6	10	8405

Table 5. Regression coefficients of Models 4, 5, and 6 and values of resilient modulus at selected stresses of synthetic specimen identified as number 961.

Model Number	Synthetic Specimen	k_1	k_2	k_3	R^2	Low Stresses σ_3 (psi)	Low Stresses σ_d (psi)	Low Stresses Mr (psi)	Mid Stresses σ_3 (psi)	Mid Stresses σ_d (psi)	Mid Stresses Mr (psi)	High Stresses σ_3 (psi)	High Stresses σ_d (psi)	High Stresses Mr (psi)
Model 4[1]	819961A	5570.368	0.501657	4.63062E-05	0.964	2	2	15,810	4	6	23,748	6	10	29,642
	820961B	5800.422	0.498717	-0.01316965	0.988	2	2	16,214	4	6	23,946	6	10	29,649
	821961A	5515.866	0.479433	0.012418277	0.991	2	2	15,077	4	6	22,547	6	10	28,044
	822961A	3672.501	0.635384	-0.049116642	0.973	2	2	13,304	4	6	21,102	6	10	27,249
	823961A	27955.43	0.04859	-0.016885746	0.519	2	2	30,568	4	6	31,212	6	10	31,615
Model 5[3]	819961A	8709.799	0.418819	0.212385	0.917	2	2	17,425	4	6	25,837	6	10	32,744
	820961B	10056.39	0.370362	0.186233	0.935	2	2	18,536	4	6	26,224	6	10	32,313
	821961A	8849.878	0.36786	0.214302	0.959	2	2	16,776	4	6	24,275	6	10	30,268
	822961A	7456.863	0.473823	0.20183	0.928	2	2	15,665	4	6	23,676	6	10	30,421
	823961A	29442.75	0.042145	-0.00026	0.625	2	2	30,829	4	6	31,493	6	10	31,940
Model 6[3]	819961A	5567.259	0.503235	-0.00227	0.964	2	2	15,813	4	6	23,737	6	10	29,617
	820961B	5848.32	0.499343	-0.01725	0.988	2	2	16,209	4	6	23,948	6	10	29,628
	821961A	5470.964	0.479296	0.015742	0.991	2	2	15,081	4	6	22,543	6	10	28,059
	822961A	3787.408	0.637555	-0.06413	0.974	2	2	13,290	4	6	21,108	6	10	27,177
	823961A	28271.13	0.048591	-0.02116	0.514	2	2	30,558	4	6	31,221	6	10	31,595

Table 6. Comparison of the 95 percent confident level for Models 4, 5, and 6.

Synthetic Specimen Number	Stresses (psi)	Model 4[1] Low Stresses Mr (psi)				Model 5[3] Mid Stresses Mr (psi)				Model 6[3] High Stresses Mr (psi)			
		95 % Confidence Range			% Diff.	95 % Confidence Range			% Diff.	95 % Confidence Range			% Diff.
701	$\sigma_3 = 2$ $\sigma_d = 2$	1352	1386	1421	4.9	1365	1398	1432	4.7	1351	1386	1421	4.9
	$\sigma_3 = 4$ $\sigma_d = 6$	1423	1469	1515	6.1	1427	1473	1518	6.0	1424	1469	1515	6.0
	$\sigma_3 = 6$ $\sigma_d = 10$	1462	1516	1572	7.0	1464	1520	1575	7.0	1462	1516	1571	6.9
901	$\sigma_3 = 2$ $\sigma_d = 2$	6,129	6,543	6,958	11.9	6,378	6,798	7,217	11.6	6,133	6,546	6,960	11.9
	$\sigma_3 = 4$ $\sigma_d = 6$	7,444	7,868	8,291	10.2	7,580	8,022	8,464	10.4	7,440	7,864	8,288	10.2
	$\sigma_3 = 6$ $\sigma_d = 10$	8,227	8,690	9,152	10.1	8,362	8,858	9,354	10.6	8,229	8,690	9,151	10.0
961	$\sigma_3 = 2$ $\sigma_d = 2$	13,053	15,101	17,150	23.9	15,188	17,100	19,012	20.1	13,041	15,098	17,156	24.0
	$\sigma_3 = 4$ $\sigma_d = 6$	20,750	22,836	24,922	16.7	23,059	25,004	26,947	14.4	20,755	22,834	24,913	16.7
	$\sigma_3 = 6$ $\sigma_d = 10$	26,738	28,646	30,553	12.5	29,408	31,436	33,465	12.1	26,691	28,620	30,549	12.6

Table 7. Regression coefficients of Models 4, 5, and 6 and values of resilient modulus at selected stresses of compacted soil specimens.

Model Number	Compacted Soil Specimens	k_1	k_2	k_3	R^2	Low Stresses σ_3 (psi)	Low Stresses σ_d (psi)	Low Stresses Mr	Mid Stresses σ_3 (psi)	Mid Stresses σ_d (psi)	Mid Stresses Mr	High Stresses σ_3 (psi)	High Stresses σ_d (psi)	High Stresses Mr
Model 4[1]	1	18902.68	0.277828	-0.237383862	0.965	2	2	28574	4	6	27577	6	10	27619
	2	20263.94	0.215792	-0.164115029	0.953	2	2	28327	4	6	28177	6	10	28503
	3	19677.88	0.239292	-0.177057187	0.949	2	2	28627	4	6	28614	6	10	29055
	4	22091.38	0.17946	-0.173782973	0.872	2	2	28443	4	6	27181	6	10	26924
	5	4751.465	0.796137	-0.381854184	0.881	2	2	19092	4	6	23936	6	10	27997
Model 5[3]	1	30168.7	0.220716	-0.20881	0.948	2	2	30566	4	6	28666	6	10	28095
	2	28481.24	0.176241	-0.13509	0.963	2	2	29798	4	6	29079	6	10	29028
	3	28625.74	0.19488	-0.14528	0.952	2	2	30229	4	6	29526	6	10	29523
	4	29605.67	0.15661	-0.15934	0.925	2	2	29517	4	6	27937	6	10	27403
	5	14493.00	0.601244	-0.16975	0.822	2	2	23283	4	6	27413	6	10	31081
Model 6[3]	1	22149.34	0.279333	-0.30033	0.962	2	2	32152	4	6	28993	6	10	28136
	2	22583.71	0.218546	-0.20979	0.968	2	2	30762	4	6	29166	6	10	28858
	3	22139.25	0.24164	-0.22581	0.960	2	2	31290	4	6	29701	6	10	29447
	4	24717.31	0.18393	-0.22333	0.892	2	2	31037	4	6	28190	6	10	27280
	5	6088.178	0.805286	-0.49035	0.893	2	2	23127	4	6	25928	6	10	28808

Table 8. Comparison of target values and actual values of dry unit weights and moisture contents of remolded soil specimens.

Maximum Dry Unit Weight (Lbs/Ft^3)	Optimum Moisture Content (%)	Target Values		Actual Values		Deviations	
		Dry Unit Weight (Lbs/Ft^3)	Optimum Moisture Content (%)	Dry Unit Weight (Lbs/Ft^3)	Optimum Moisture Content (%)	Dry Unit Weight (Lbs/Ft^3	Moisture Content (%)
97.2	21.5	92.34	21.5	91.72	21.66	0.62	-0.16
97.2	21.5	92.34	21.5	92.84	21.98	-0.50	-0.48
97.2	21.5	92.34	21.5	91.43	21.64	0.91	-0.14
97.2	21.5	92.34	21.5	91.09	21.94	1.25	-0.44
97.2	21.5	92.34	21.5	91.88	21.62	0.46	-0.12
			Average Values	91.79	21.76	0.55	-0.26

Table 9. The 95 percent confidence level for compacted soil specimens.

Model Number	Stresses (psi)	Resilient Modulus, M_r (psi) 95 % Confidence Range Low	Mean	High	Percentage Difference
Model 4 (Uzan)	$\sigma_3 = 2 \quad \sigma_d = 2$	21,391	26,613	31,835	32.8
	$\sigma_3 = 4 \quad \sigma_d = 6$	24,749	27,097	29,395	15.8
	$\sigma_3 = 6 \quad \sigma_d = 10$	27,006	28,020	29,034	7.0
Model 5 (UKTC)	$\sigma_3 = 2 \quad \sigma_d = 2$	24,901	28,679	32,457	23.3
	$\sigma_3 = 4 \quad \sigma_d = 6$	27,465	28,524	29,583	7.2
	$\sigma_3 = 6 \quad \sigma_d = 10$	27,273	29,026	30,779	11.4
Model 6 (AASHTO)	$\sigma_3 = 2 \quad \sigma_d = 2$	25,084	29,674	34,264	26.8
	$\sigma_3 = 4 \quad \sigma_d = 6$	26,556	28,396	30,236	12.2
	$\sigma_3 = 6 \quad \sigma_d = 10$	27,478	28,506	29,534	7.0

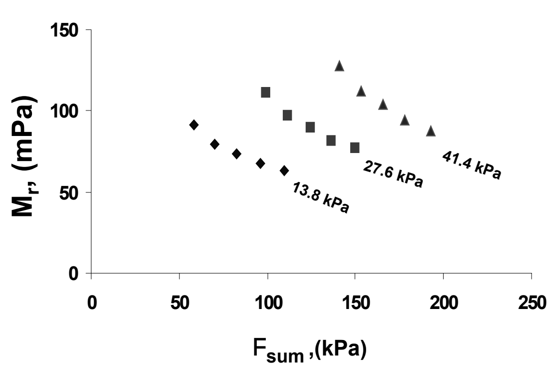


Figure 32. Resilient modulus, M_r, as a function of the sum of the principal stresses, F_{sum}.

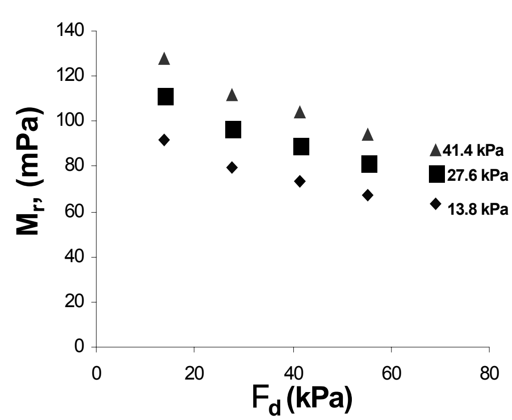


Figure 33. Relationship between deviator stress, F_d, and resilient modulus, M_r.

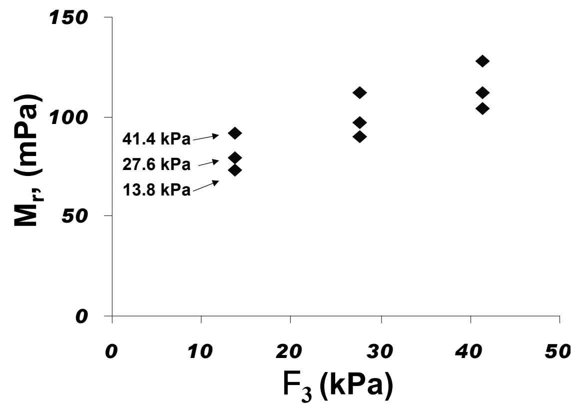


Figure 34. Relationship between confining stress, F_3, and resilient modulus, M_r.

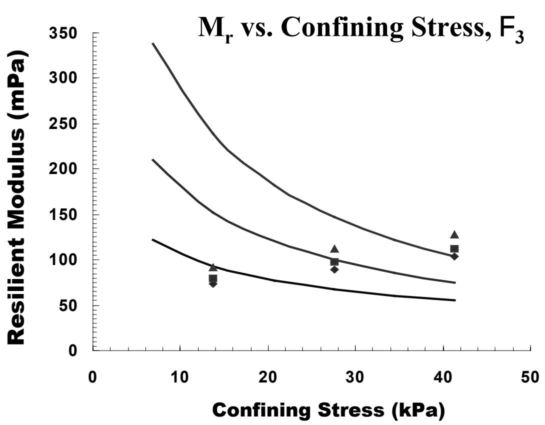


Figure 35. Prediction of relationship between resilient modulus, M_r, and confining stress, F_3, using Model 3.

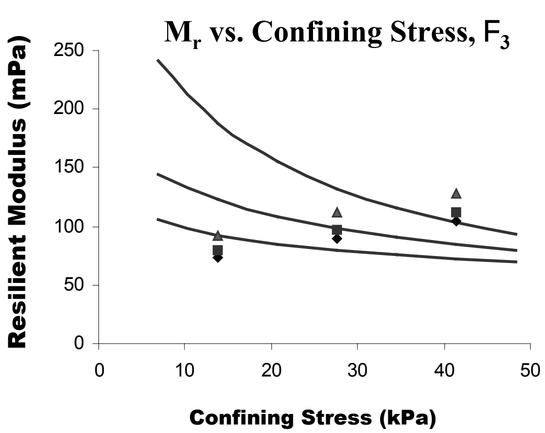


Figure 36. Prediction of relationship between resilient modulus, M_r, and confining stress, F_3, using Model 4.

modulus data generated from those tests have been published elsewhere (Hopkins et al. and Ni et al., 2002). Resilient modulus data shown in Figures 32 through 34 are typical of the type of data obtained from the resilient modulus tests. In Figure 32, the relationship between resilient modulus and the sum of the principal stresses is shown. Three data sets shown in this figure correspond to confining stresses of 13.8, 27.6, and 41.4 kPa (2, 4, and 6 psi, respectively). The relationship between deviator stress and resilient modulus is shown in Figure 33 and the three data sets correspond to confining stresses of 13.8, 27.6, and 41.4 kPa (2, 4, and 6 psi). Similarly, in Figure 34, the relationship between confining stress and resilient modulus is shown. The three data sets correspond to confining stresses of 13.8, 27.6, and 41.4 kPa. The data curves depicted in Figures 32 through 34 illustrate that confining and deviator stresses have different effects on the resilient modulus of soils. Under a constant confining stress, the resilient modulus of soils decreases as the deviator stress increases, as shown in Figure 33. If the deviator stress is held constant, then the resilient modulus increases as the confining stress increases.

Model 1 ($M_r = k_1(\sigma_d / p_a)^{k_2}$) does not consider the effect of the confining stress on resilient modulus of soils while Model 2 ($M_r = k_1(\sigma_3 / p_a)^{k_2}$) does not consider the effect of deviator stress on resilient modulus. Therefore, these two models have limited use. Although Model 3 ($M_r = k_1(\sigma_{sum} / p_a)^{k_2}$) includes the sum of principle stresses, and $F_{sum} = F_1 + F_2 + F_3 = 3F_3 + F_d$, the model only contains one independent variable, F_{sum}. The effects of both confining stress and deviator stress of this model are not considered as independent variables. Although Model 4 ($M_r = k_1(\sigma_{sum} / p_a)^{k_2}(\sigma_d / p_a)^{k_3}$) does consider the effects of both the sum of the principle stresses and deviator stress on the resilient modulus, the coefficients k_1, k_2, and k_3 vary significantly when simple regression analysis is performed for each confining stress. However, as shown below, when multiple

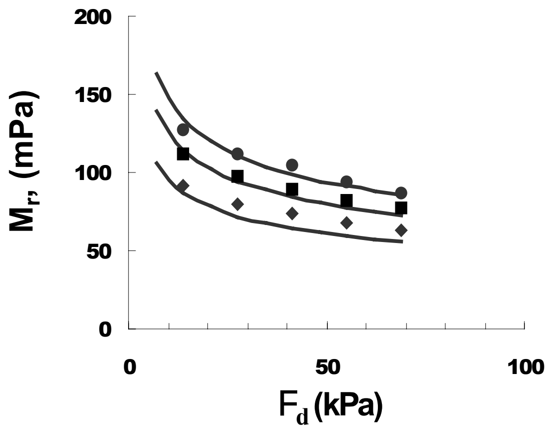


Figure 37. Prediction of the relationship between resilient modulus, M_r, and deviator stress, F_d, from the new model 5.

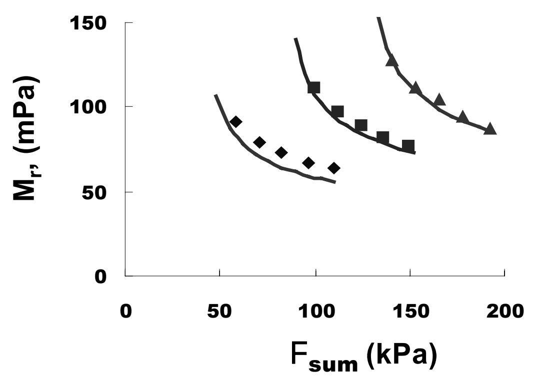


Figure 38. Prediction of the relationship between resilient modulus, M_r, and the sum of the principal stresses, F_{sum}, from the new Model 5.

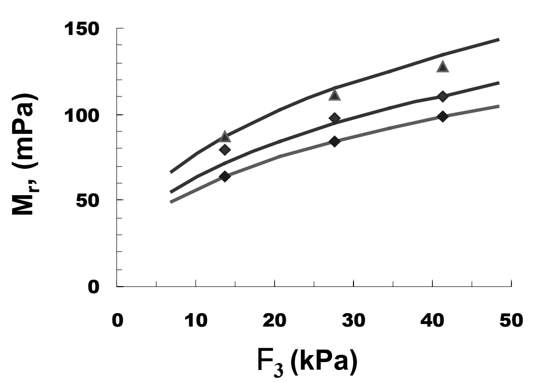


Figure 39. Prediction of the relationship between resilient modulus, M_r, and confining stress, F_3, from the new model 5.

regression analysis is performed on all data points the relationship for Model 4 improves.

Resilient modulus test data indicate that as the deviator stress increases the resilient modulus decreases, but as the confining stress increases, the resilient modulus tends to increase. Any one of the three data sets in Figure 33 could be used to obtain the correlation coefficients, k_1 and k_2, from a simple regression analysis. If Model 3 correctly represents the relationship between resilient modulus and stress state, then the values of k_1 and k_2 should be nearly the same for each curve. As shown in Table 8, the value of k_1 ranges from 305,213 to 4,739,146 while k_2 varies from –0.572 to –1.202. Figure 35 shows the results of using Model 3 to predict the relationship between resilient modulus and confining stress using the three sets of k_1 and k_2 values obtained from the simple regression analysis. Model 3 does not correctly include the effects of confining stress on resilient modulus. In Figure 36, regression results from Model 4 are shown. The three sets of correlation coefficients, k_1, k_2, and k_3, obtained from regression analysis are shown in Table 9. The correlation coefficients (k_1, k_2, and k_3) of Model 4 vary significantly.

To model the relationship between resilient modulus of soils (and aggregates) and stress state correctly, the following model (Equation 15, or 20) has been proposed:

$$M_r = k_1\left(\frac{\sigma_3}{p_a}+1\right)^{k_2}\left(\frac{\sigma_d}{p_a}+1\right)^{k_3}.$$

This model considers separately the effects of deviator stress and confining stress on the resilient modulus. When F_3 and F_d approach zero, M_r approaches the coefficient k_1. Therefore, k_1 is the initial resilient modulus of the soil before any load is applied. Test data appearing in Figures 33 and 34 are used in a simple regression analysis to obtain the coefficients, k_1, k_2, and k_3, of the new model. Results are shown in Table 10. Although the confining stress changes, the value of the each coefficient, k_1, k_2, or k_3, is nearly the same. For instance the three different values of the

Table 10. Correlation coefficients of Models 3, 4, and 5.

Confining Stress, F_3 (kPa)	Model 3 $M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}$		Model 4 $M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}\left(\frac{\sigma_d}{p_a}\right)^{k_3}$			Model 5 $M_r = k_1\left(\frac{\sigma_3}{p_a}+1\right)^{k2}\left(\frac{\sigma_d}{p_a}+1\right)^{k_3}$		
	k_1	k_2	k_1	k_2	k_3	k_1	k_2	k_3
13.8	305,213	-0.572	176,657	-0.121	-0.270	80,844	0.392	-0.281
27.6	1,209,923	-0.899	419,437	-0.112	-0.467	80,479	0.404	-0.284
41.4	4,739,146	-1.202	1,834,656	-0.066	-0.869	80,765	0.415	-0.286

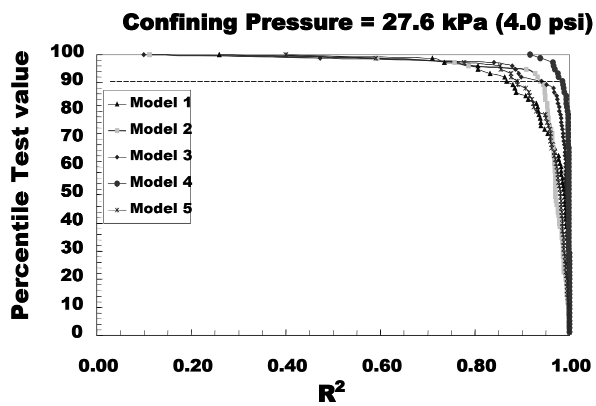


Figure 40. Percentile test value as a function of R^2 obtained for models 1 through 5.

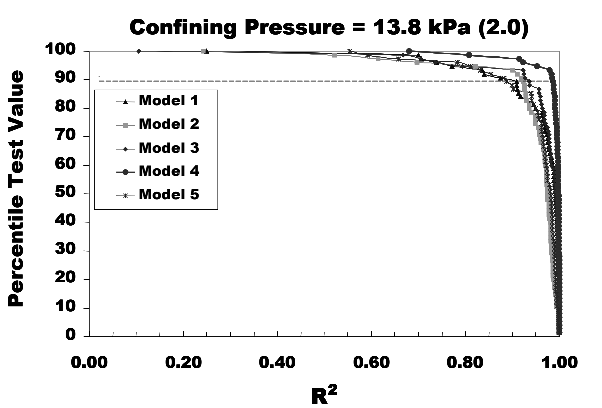


Figure 41. Percentile test value as a function of R^2 obtained for models 1 through 5.

coefficient, k_1, range only from 80,479 to 80,844, or a difference of less than 1 percent.

Values of the coefficients, k_2 and k_3, range only from 0.392 to 0.415 and –0.281 to –0.286, or a difference of about 5 and 1.7 percent, respectively. As shown in Table 8, any set of constants could be used to predict the relationships between resilient modulus of soils and stress state. For example, the values, k_1 = 80,844, k_2 = 0.392, and k_3 = -0.281, from Table 9 are used in the proposed Model 5 to predict the relationships of the resilient modulus to confining stress, deviator stress, and the sum of the principal stresses. The predicted relationships are compared to the actual test data in Figures 37, 38, and 39, respectively. The results show that the new model predicts the various relationships very well. Moreover, the results also prove that the new model correctly includes the effects of both confining stress and deviator stress on the resilient modulus of soils. Each of the five models provide a reasonable correlation when the confining stress is held constant in the simple correlation analysis, as illustrated in Figures 40, 41, and 42. In each of those figures, the percentile test value is shown as a function of R^2 for confining stresses of 13.8, 27.6, and 41.4 kPa (2, 4, and 6 psi), respectively. Values of R^2 at the 90th percentile test value are summarized in Table 11. Generally, the value of R^2 was equal to or exceeded 0.90. Although Models 3 and 4 yielded slightly better regression curves than Model 5 for a constant confining pressure, there was much greater variation in the coefficients when all confining

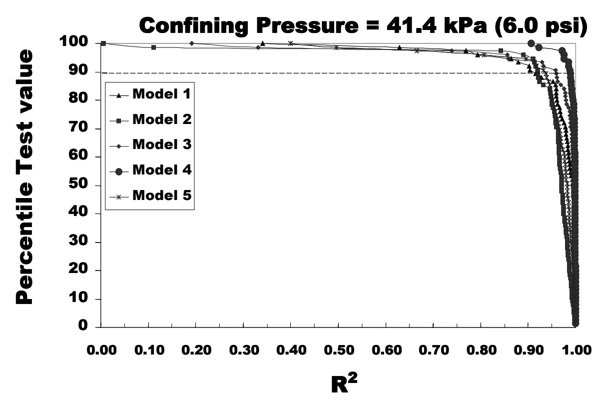


Figure 42. Percentile test value as a function of R^2 obtained from models 1 through 5.

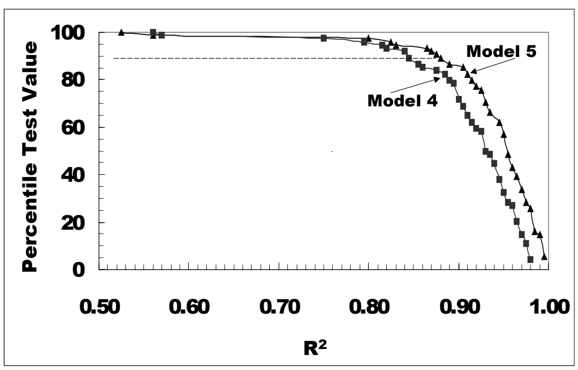


Figure 43. Comparison of R^2 results of Models 4 and 5.

Table 11. Summary of R^2-values at the 90th percentage test value obtained for the five models--simple correlation analysis.

Model Number	Confining Pressure (kPa, psi) 13.8 (2.0)	27.6 (4.0)	41.4 (6.0)
	R^2		
1	0.91	0.87	0.92
2	0.93	0.94	0.92
3	0.94	0.94	0.96
4	0.98	0.98	0.98
5	0.90	0.90	0.94

curves were considered than the coefficients for Model 5, as illustrated in Table 9. Models 1 and 2 can only be used to determine a regression curve for a constant confining stress or deviator stress. Hence, these two models cannot be used in a general sense and their uses are limited.

Multiple Correlation Analysis

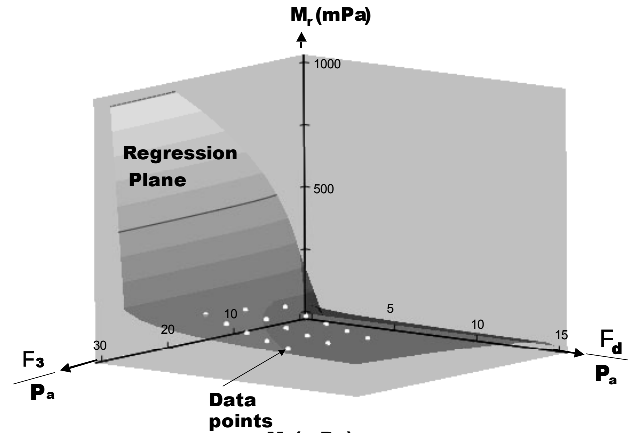
In the relationships expressed by Equations 7, 8, and 9 (Models 1, 2, and 3), respectively, only two variables are involved. The resilient modulus is a dependent variable while either the deviator stress, confining stress, or sum of principle stresses is an independent variable. Consequently, only simple correlation analysis can be performed on those equations.

However, Models 4, 5, and 6, expressed by Equations 14, 15, and 16, respectively, involve three variables. The resilient modulus is the dependent variable and the sum of the principle stresses and deviator stress are independent variables in Model 4. In Model 5, the resilient modulus is the dependent variable while the deviator stress and confining stress are independent variables. In Model 6, the resilient modulus is the dependent variable and the sum of the principle stresses and the deviator stresses are the independent variables. Hence, the regression equations of the three models represent a regression plane in a three-dimensional rectangular coordinate system. In the multiple correlation analysis, all 15 data points were used collectively to obtain the coefficients k_1, k_2, and k_3. The coefficient of multiple correlation, R^2, was determined for each of the tests and for each model. Multiple regression coefficients determined for "as compacted," unsoaked specimens using each of the three models (4, 5, and 6) are tabulated in Appendix C. Results of coefficients for soaked, compacted specimens are tabulated in Appendix D.

Percentile test values as a function of the coefficient of multiple correlation for models 4 and 5 are shown in Figure 43. At the 90th percentile test value the value of R^2 obtained from model 5 is about 0.88. For Model 4, the corresponding value is 0.84. At the 67th percentile test value, the values of R^2 are 0.94 and 0.88, respectively. Model 5 provides a slightly better "fit" of the relationship between resilient modulus and stresses than Model 4 for the domain of stresses used in the test.

Typical views of the least square regression planes of Models 4 and 5 are shown in Figures 44 and 45, respectively. Actual data points are shown plotted on the regression planes of both models. In both cases, the points lie close to the regression planes. However, as shown in Figure 44, the regression plane, or the value of resilient modulus, of Model 4 approaches infinity as the values of stress become small, or as the values of stress approach zero (This situation may also occur for Model 6). Figure 46 provides another view of this situation. However, as the stresses approach zero in Model 5, the resilient modulus does not approach infinity, as illustrated in Figure 45. The resilient modulus of the regression plane of Model 5 approaches the coefficient k_1, or the resilient modulus approaches the initial resilient modulus of the specimen as the stresses approach zero. Consequently, Model 5 appears to provide a better correlation plane than Model 4 and it does not diverge toward infinity at low stresses.

Mutiple regression analyses were also performed on unsoaked, compacted specimens using Equation 16, or Model 6 See Appendix C). The R^2-values obtained for Model 6 are compared to the R^2-values obtained for Models 4 and 5 in Figure 47. In performing the resilient modulus testing, the question arises concerning the acceptance of test results. The results shown in Figure 48 suggest, perhaps tentatively, that test results should only be accepted when the value of R^2 of the regression plane is equal to, or greater than about 0.90 for Models 5 and 6. This value corresponds to the 85th percentile test value, or greater. When the value of R^2 is below 0.90, consideration should be given to redoing the



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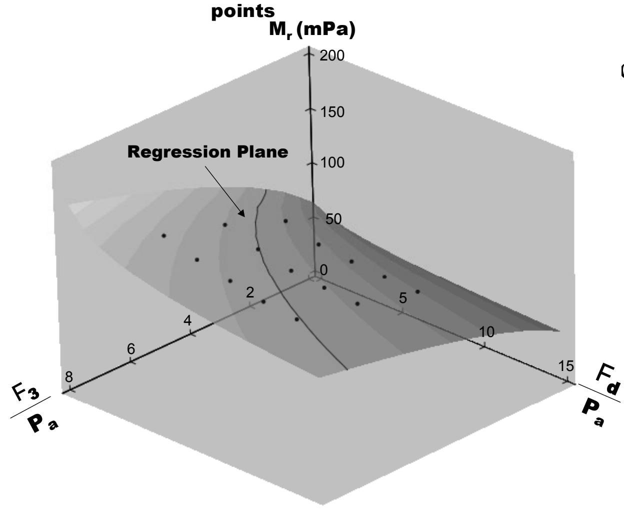

Figure 45. Least square regression plane of Model 5.

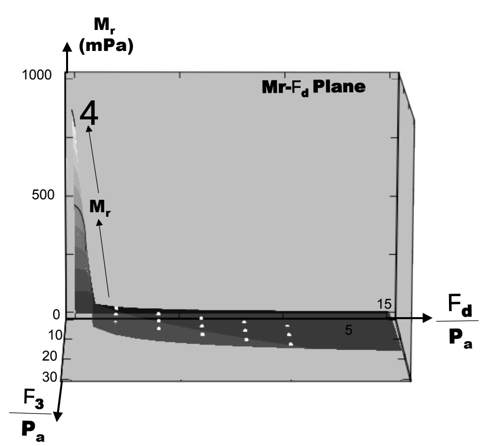

Figure 46. View of the regression plane of Model 4 in the direction of the. M_r-F_d plane.

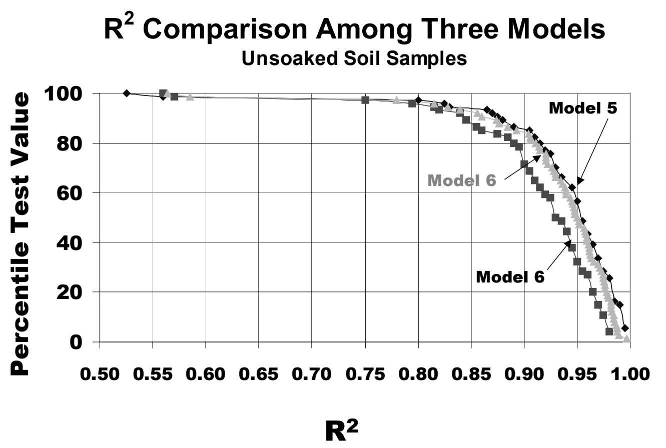

Figure 47. Comparison of R^2 values obtained from Models 4, 5, and 6.

test. An R^2-value of 0.90 corresponds to a percentile test value of about 71 for Model 4. More research concerning repeatability and R^2 acceptance criteria is needed to establish good acceptance testing criteria.

Resilient Modulus of Laboratory Compacted Soaked Soil Specimens

Results of resilient modulus tests performed on soaked, compacted soil specimens (Appendix D) were oftentimes erratic. Typical curves—M_r as a function of deviator stress--obtained from this series of tests are shown in Figure 48. Data for this test is shown in Figure 49. Values of M_r for the test shown ranged from about 1,800 to a value sligthly less than 6,000. For this same soil type, or classification, the resilient modulus value of an unsoaked specimen ranged from about 10,000 to 16,000 psi. As shown in figure 49, the resilient modulus increases with an increase in deviator stress. This behavior may be caused by an increase in pore pressures during the cyclic loading of the specimen. The behavior could also be caused by the increased stretching of the rubber membrane when a specimen starts bulges (see Figure 50). Frequently, deflections observed during the test exceeded the capacity of the deflection range of the LVDTs. During cylic loadings, a bulge usually developed in the soaked, compacted clayey specimens, as shown in Figure 50. In some cases, the

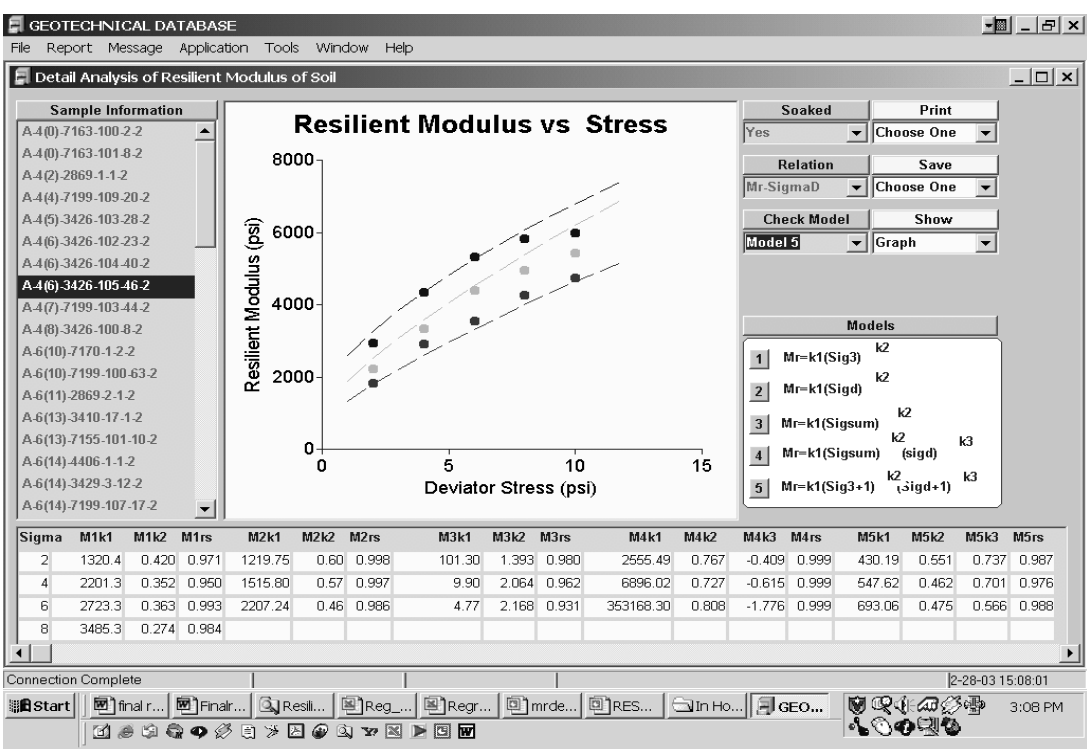

Figure 48. Typical results obtained for soaked specimens.

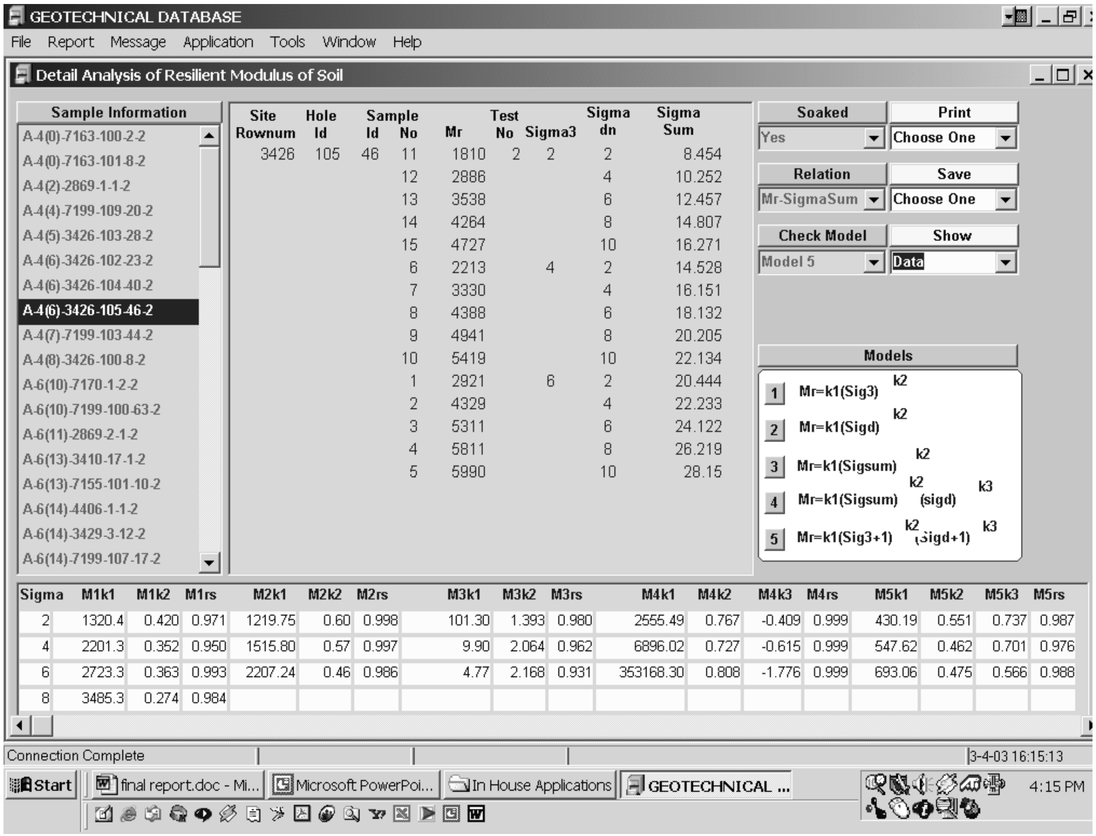

Figure 49. Typical resilient modulus data for a soaked compacted specimen.

deflections were so large that the test could not be completed. As shown in Appendix C, values of R^2 of approximately 91 perecent of the unsoaked ("as compacted") test specimens were greater than or equal to 0.87 (based on the author's model--UKTC). Values of R^2 of only 4 percent of the unsoaked specimens were equal to or less than 0.80. As shown in Appendix D, values of R^2 of only about 32 percent of the soaked specimens were equal to or greater than 0.90. Values of R^2 of 59 percent of the unsoaked specimens were egual to or less than 0.80. For all testing stresses, average values of resilient modulus of the unsoaked test specimens ranged from about 18,500 to 20,407 psi. Average values of the resilient modulus of soaked specimens ranged from 2600 to 3800 psi. Resilient modulus values of the unsoaked specimes were some 5 to 7 times larger than reslient modulus values of soaked specimens. Hence, significant difference exists between the quality of resilient modulus results obtained for unsoaked and soaked specimens.

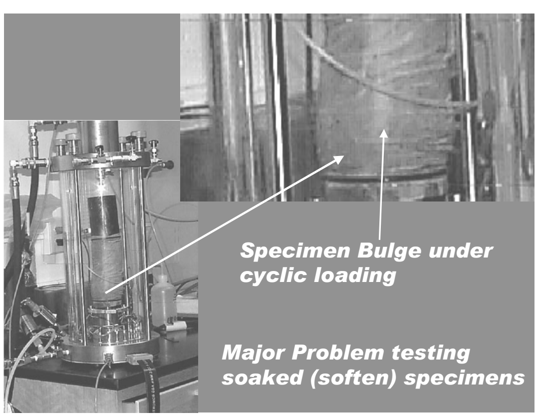

Figure 50. Bulge of soaked specimen occurring during cyclic loading.

Resilient modulus values of soaked specimes have been included in the the Kentucky Geotechnical Database. However, theses data should be used with caution It has been included at this time only for informational purposes. Much more research needs to be performed on saturated,

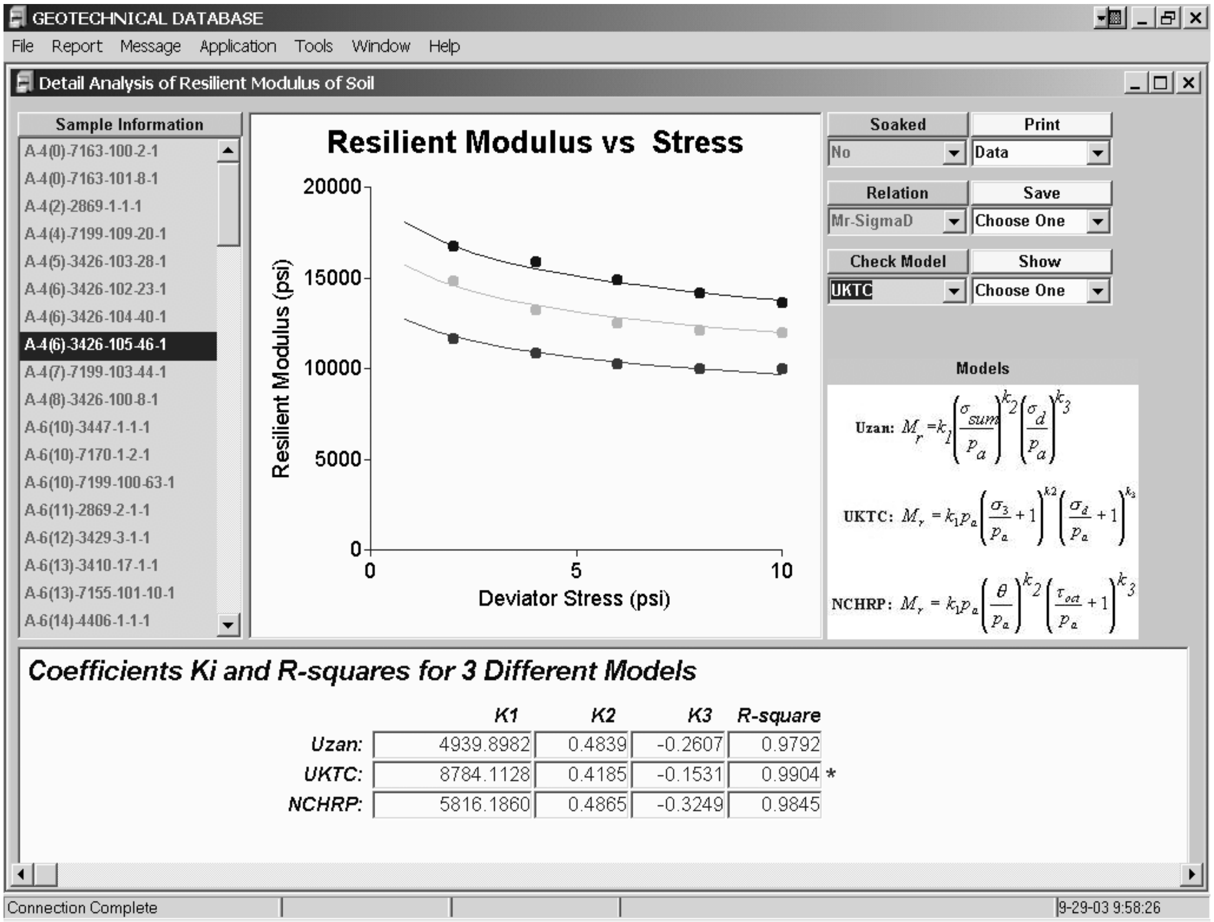

Figure 51. Graphical user interface showing the coefficients of correlation, k1, k2, k3, obtained from a of a multi-regression analysis (of a plane of testing stresses) of resilient modulus data for a selected soil classification.

or nearly saturated specimens to define a good testing procedure.

Based on the review of the different models (described above) that have been proposed for relating resilient modulus to stresses, Models 1, 2, and 3 are not in the database because of the shortcomings of those models. Initially, those three models were included in the database. Moreover, it is evident that multiple regression analysis should be performed to simulate testing conditions. Consequently, the method of analyzing the resilient modulus data was revised, as shown in the GUI screen in Figure 51. Whenever the soil classification is clicked at the right-hand side of the GUI screen, the curves shown in the center of the figure appear and the multiple regression coefficients for the Uzan, UKTC (University of Kentucky Transportation Center), and NHRCP (National Highway Research Cooperative Program) models are displayed at the bottom of the GUI. Also, the value of R^2 of the multiple regression analysis for each model is shown. Hence, the user can select the model for each data set that best expresses the relationship between the resilient modulus and the deviator and confining stresses. The user may graph the resilient modulus as a function of the deviator stress or the confining stress. In the future, the user will have the option of viewing the three-dimensional plot, as illustrated in Figure 45, of the selected model, that is, the regression plane and data points will be displayed in future applications.

Results of Multiple Regression Analysis of Resilient Modulus Tests Performed on Untreated and Treated Subgrade Specimens

In a recent study (Hopkins et al. 2002), undisturbed core specimens of treated and untreated highway subgrades were obtained and resilient modulus tests were performed on the core specimens. The subgrade specimens were obtained using a coring technique, developed by T.L Beckham (one of the principle authors of the above study), which used air as the drilling media, instead of water. Using this technique, good quality subgrade specimens were obtained.

An example of the regression planes obtained from multiple regression analyses using Model 5, Equation 15, is shown in Figure 52. In this figure, the regression planes obtained for the a soil-cement subgrade specimen and the untreated subgrade specimen are compared. Both specimens were obtained at the same location. Variation of the resilient modulus with deviator stress and confining stress is illustrated in this three-dimensional graph. Actual M_r-F_d-F_3 data points obtained from the resilient modulus tests are compared to each regression plane predicted from the Model 5 analyses. The upper plane is the resilient modulus regression plane of a soil-cement specimen while the lower plane is the regression plane of an untreated soil specimen obtained at the same location as the soil-cement core. Values of resilient modulus of the soil-cement cores were much larger than resilient modulus values of the untreated specimens.

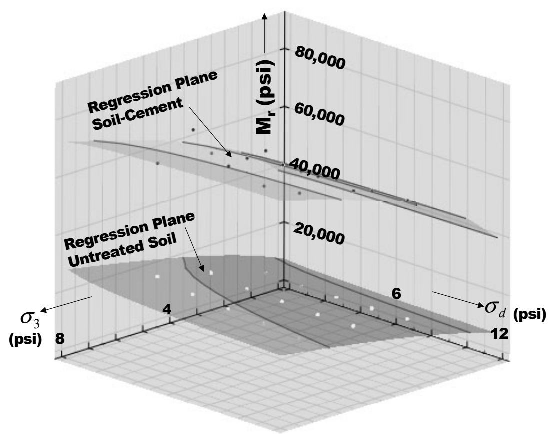

Figure 52. Examples of least square regression planes from Model 5 for soil-cement and untreated soil specimens.

As one means of comparing values of resilient modulus of chemically treated and untreated specimens, resilient modulus values were calculated using the coefficients, k_1, k_2, and k_3, from Model 5, Equation 15. Deviator and confining stresses equal to 41.4 kPa (6 psi) and 27.6 kPa (4 psi), respectively, were assumed in the calculations. Those stresses are located at about the midpoint of the domain of testing stresses (and regression planes shown in Figure 52). Values of resilient modulus obtained for the untreated and soil-cement specimens are compared in Figure 53. Percentile test value is shown as a function of the resilient modulus. In all cases, the resilient modulus of the soil-cement specimens are larger than resilient modulus of the untreated specimens. Values of resilient modulus of the untreated subgrade specimens range from 6 ksi (41.36 mPa) at the 100th percentile test value to 22 ksi (151.65 mPa) at the 15th percentile test value. However, at the 100th and 15th percentile values, the resilient modulus values of the soil-cement field specimens range from about 9 to 90 ksi (62.05 to 620.46 mPa), respectivley. Values of resilient modulus of the soil-cement specimens are about 1.5 to 4.1 times

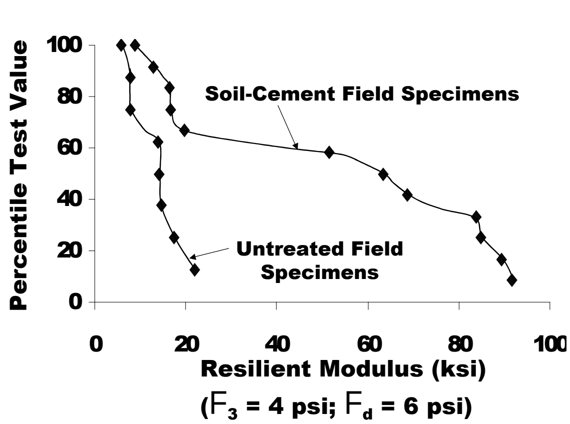

Figure 53. Percentile test value as function of resilient modulus of untreated field specimens and soil-cement field specimens.

larger than the resilient modulus of the unsoaked and untreated field specimens.

Values of resilient modulus of soil-hydrated lime specimens and untreated, unsoaked field specimens, as a function of percentile test value, are compared in Figure 54. In both series of specimens, the values of resilient modulus are fairly large. Basically, values of resilient modulus of the two different series of specimens are nearly equal from about the 95th to 20th percentile test value and range from about 6 ksi to 22 ksi (41.36 to 151.65 mPa). Values of resilient modulus of the soil-hydrated lime specimens ranged from 22 to 60 ksi (151.65 to 413.58 mPa) between the 20th and 5$^{th.}$ percentile test values. Past testing (Hopkins et al., 1985) has shown that clayey soils, when first compacted and not subjected to soaking, have CBR values that range from about 10 to 45. However, when the same clayey soils are soaked, the CBR values generally range from about 1 to 6. Accordingly, it could be expected that values of unsoaked specimens would be larger than values of resilient modulus of soaked specimens.

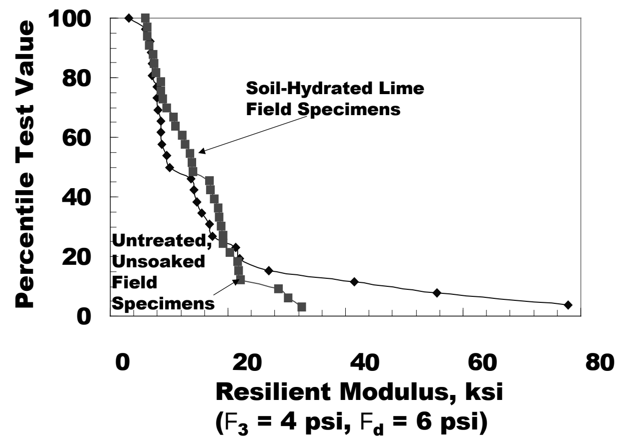

Figure 54. Percentile test value as function of resilient modulus of untreated and soil-hydrated lime field specimens.

The untreated field specimens were obtained below the " soft zone" of untreated soil. These specimens were unsaturated (or unsoaked) and their resilient modulus behavior is similar to the resilient modulus behavior of "as compacted" (unsaturated) specimens. To illustrate, the resilient modulus of field specimens are compared in Figure 55 to resilient modulus of recompacted (Kentucky) clayey soils of all types (Hopkins et al,, 2002). Assuming deviator and confining stresses equal to 6 psi and 4 psi (41.4 to 27.5 kPa), respectivley, values of resilient modulus were computed using the regression coefficients of Model 5 (Equation 15). The laboratory data in this figure represent the results of about 72 resilient modulus tests that were performed on unsoaked, or "as compacted," and untreated specimens (Hopkins et al 2002). Values of resilient modulus of the laboratory specimens ranged from about 9.4 to 26 ksi (64.79 to 179.22 mPa) at the 100th and 10th percentile test values, respectively. Values of resilient modulus of the field specimens were only slightly lower than the resilient modulus values of the laboratory (unsoaked) compacted specimens, as illustrated in Figure 55. Values of resilient modulus of the field specimens ranged from about 6 ksi to 26 ksi (41.35 to 179.22 mPa) at the 100th and 10th percentile test values, respectively. However, values of resilient modulus of soaked specimens were much smaller than values of resilient modulus of unsoaked specimens.

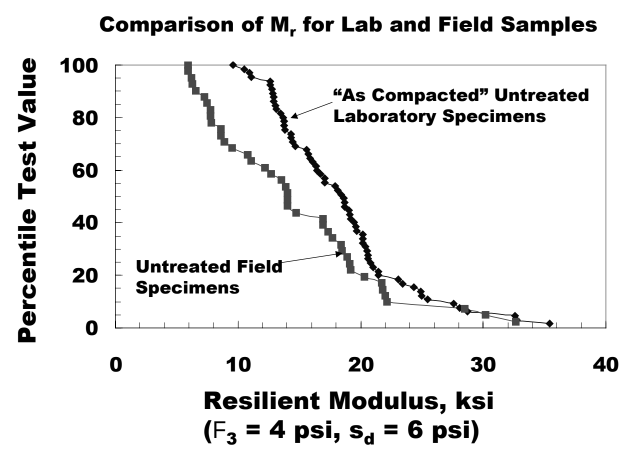

Figure 55. Percentile test value as function of resilient modulus of untreated field specimens and "as-compacted" laboratory specimens.

Regression Analyses--Resilient Modulus As Function of Selected Geotechnical Test Parameters

Efforts to correlate resilient modulus to other engineering geotechnical test parameters were unsuccessful (Chow, 1998). Geotechnical variables used in the regression analyses included unconfined compressive strength, liquid limit, plasticity index, Ky CBR, and the percent finer than the 0.002-mm sieve size. In the attempted correlations, the average value of confining stress was used and the deviator stress was held constant. Regression analyses were performed for each deviator stress, that is, 2,4,6,8, and 10 psi. A linear regression with 95 percent confidence level was drawn through the plots to determine the R^2 (R-Square) fit using the lease square method. In all cases, considerable scatter was encountered and R^2 values were very low. No well-defined relationships were found.

SUMMARY AND CONCLUSIONS

Resilient modulus tests were performed on a variety of Kentucky soils. Soil samples consisted of six bulk samples collected from different physiographical regions of the state and roadway samples generated during roadway studies. Soil types, based on the AASHTO Soil Classification, included A-4, A-6, A-7-5, and A-7-6. The laboratory specimens were compacted to 95 percent of maximum dry density and optimum moisture content obtained from AASHTO T-99. About 150 resilient modulus tests were performed on the compacted specimens. Resilient modulus tests were also performed on core specimens obtained from untreated and chemically treated soil subgrades. Based on the results of this testing program, the following observations, conclusions, and recommendations are offered:

- Various mathematical models appear in the literature for expressing the relationship between resilient modulus and stress state. In this report, mathematical models that are oftentimes cited in the literature are examined and evaluated for their ability to relate resilient modulus and stress. To generate resilient modulus data for making the evaluation and correlation analysis, over 150 resilient tests were performed on remolded clayey soils. Mathematical models that have been proposed in the literature and relate resilient modulus and stress conditions were used to analyze the resilient modulus data generated during this study. Multiple regression analysis showed that the mathematical model (identified as Model 5) proposed by the authors,

$$M_r = k_1\left(\frac{\sigma_3}{p_a}+1\right)^{k_2}\left(\frac{\tau_{oct}}{p_a}+1\right)^{k_3},$$

provided a better data fit than other models. However, it only provided a slightly better data fit than the model (identified as Model 6 herein) proposed in the NCHRP Project 1-28A, or

$$(M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}\left(\frac{\tau_{oct}}{p_a}+1\right)^{k_3}.$$

- Repeatability of the resilient modulus test was briefly examined using molded synthetic specimens and soil specimens compacted to nearly identical conditions of dry density and moisture content. For repeatability resilient modulus tests performed on the three synthetic specimens (identified herein as 701, 901, and 961), percentage differences in resilient modulus defined by the 95 confidence levels ranged from 4.9 to 7.0, 10.1 to 11.9, and 12.5 to 23.9, respectively. The 95-confidence level of authors' model was better than the 95 percent confidence levels of other models. As the resilient modulus of the synthetic specimens increase, the 95 percent confidence levels increase, or repeatability was not as certain at high values of resilient modulus when compared to low values of resilient modulus. Percentage differences in resilient modulus values defined by the 95 percent confidence levels of soil specimens compacted to nearly identical dry densities and moisture contents ranged from 7.0 to 32.8, 7.2 to 23.3, and 7.0 to 26.8 for Models identified as 4 ($M_r = k_1\left(\frac{\sigma_{sum}}{p_a}\right)^{k_2}\left(\frac{\sigma_d}{p_a}\right)^{k_3}$), 5 (authors' model), and the 6 (NHCRP model, respectively. It is recommended that future resilient modulus testing should study the repeatability issue in much greater depth than the brief examination described herein. The repeatability of different resilient modulus testing equipment should be studied.
- Performing resilient modulus testing on "as compacted", or unsaturated compacted soil specimens posed no problem following AASHTO T-294. However, significant problems were encountered when specimens were soaked for long-periods of time. The saturated, or nearly saturated specimens suffered large deformations under cyclic loads and oftentimes bulged, or experienced large deformations. Soaking compacted soil specimens before testing to simulate in situ conditions is a well-established engineering approach. It is recommended that the resilient modulus testing procedure be revised to accomplish this task. Resilient modulus testing should be performed on saturated, or nearly saturated, compacted soil specimens to define an appropriate testing procedure.

RECOMMENDATIONS

In the design of highway pavements, testing soaked, compacted soil specimens is a well-founded principle in efforts to simulate likely field conditions. For example, performing CBR tests on soaked soil specimens has been an accepted practice for many decades. While no problems were encountered in performing resilient modulus tests on unsoaked ("as compacted") soil specimens, resilient modulus testing of soaked soil specimens produced erratic results. It is recommended that resilient modulus testing protocol be revised, or developed, so that resilient modulus tests can be performed on saturated, or nearly saturated specimens. Research is needed to develop the new, or revised, testing protocol.

To insure good quality testing results, research is needed to examine the repeatability of the resilient modulus test using current testing protocol. Research should also focus on developing resilient modulus acceptance testing criteria. More research is needed to define differences that may exist among different resilient modulus testing equipment.

REFERENCES

American Association of State Highway and Transportation Officials (1993). "*AASHTO Guide for Design of Pavement Structures,"* Washington, D.C., USA.

American Association of State Highway and Transportation Officials (2000). *Standard Specifications for Transportation Materials and Methods of Sampling and Testing, Part II-Test*s, II-1015-1029, 20th edition, Washington, D.C., USA.

Beckham, T.L. and Allen, D.L.; (1990), "*Correlation of ASTM and Kentucky CBR Methods for Fayette County*," Research Report KTC-90-17, University of Kentucky Transportation Center, College of Engineering, Lexington, Kentucky.

Chen D., Zaman M. M., and Laguros J. G. (1994). *"Resilient Moduli of Aggregate Materials: Variability Due to Testing Procedure and Aggregate Type."* Transportation Research Record TRB No. 1462, Transportation Research Board National Research Council, Washington, D.C.

Chow, N. (1998). "Analyzing the Validity and Limitations of Using resilient Modulus Testing for Subgrade Soils," University of Kentucky, College of Engineering, Civil Engineering, Master's Thesis, Lexington, Kentucky.

Claros G., Hudson W. R., and Stokoe II K. H. (1990). "Modifications of Resilient Modulus Testing Procedure for Equipment Calibration". Transportation Research Record TRB No. 1278, Transportation Research Board National Research Council, Washington, D.C.

Drumm E. C., Hudson J., Li Z., Madgett M., and Baker D. (1993). "*Resilient Modulus of Tennessee Subgrades."* Laboratory Testing, Modeling and Alternative Test Method. Tennessee Department of Transportation, University of Tennessee Knoxville, Volume 1.

Dunlap, W.S. (1963), *"A Report on a Mathematical Model Describing the Deformation Characteristics of Granular Materials,"* Technical Report 1, Project 2-8-62-27, TTI, Texas A & M University.

Kentucky Methods. (1998). Issued by the Commonwealth of Kentucky, Transportation Cabinet, Frankfort, Kentucky.

Hardin B., and Drenevich V. (1972). *Shear Modulus and Damping in Soils: Measurement and Parameter Effects.* Soil Mechanics and Foundations Division, Journal of the Proceedings of the American Society of Civil Engineers (ASCE), Vol 98, No. SM6, June.

Heukelom W. and Foster (1960). "Dynamic Testing of Pavements." ASCE ;Journal of the Structural Division, No. SM 1, 86.

Heukelom W. and Klomp A. J. G. (1962). "*Dynamic Testing as a Means of Controlling Pavements During and After Construction."* Proceedings, International Conference on the Structural Design of Asphalt Pavements, University of Michigan.

Highway Research Board (1962). "The AASHO Road Test," Special reports 61A, 61B, 61C, 61D, 61E, 61F, and 61G, National Academy of sciences—National Research Council, Washington 25, D. C.

Hopkins, T.C. (1970). *"Relationship Between Soil Support Value and Kentucky CBR,"* Research Report 297, Division of Research, Kentucky Department of Highways, Lexington, Kentucky.

Hopkins, T. C. and Deen, R.C. (March-December 1983). "*Identification of Shales,"* Geotechnical Testing Journal, Vol. 7, American Society for Testing Materials.

Hopkins, T.C. (August 1984). *"Relationship Between Kentucky CBR and Slake Durability"*, University of Kentucky Transportation Center, College of Engineering, Research Report UKTRP 84-24.

Hopkins, T.C. and Sharpe, G.W. (March 1985). *"Unstable Subgrade I 65, Hardin County,"* (I 65-5(17) 92); FSP 047-0065-091-094-0396", University of Kentucky Transportation Center, College of Engineering, Research Report UKTRP 85-9.

Hopkins, T.C. Hunsucker, D. and Sharpe, G.W.; (1988). "*Highway Field Trials of Chemically Stabilized Soil Subgrades"*, Proceedings of the Ohio River Valley Soils Seminar XIX, Lexington, Kentucky.

Hopkins, T.C., (January 1988). *"Shear Strength of Compacted Shales,"* University of Kentucky Transportation Center, College of Engineering, Research Report UKTRP 88-1, January 1988.

Hopkins, T. C. Allen, D.L. Deen, R. C.; and Grayson, C. G.; (December 1988). "*Slope Maintenance and Slide Restoration*," Report FHWA-RT-88-040, Kentucky Transportation Center, College of Engineering, Lexington, Kentucky.

Hopkins, T.C. and Hunsucker, D.; (November 1990), "*Interim Design Guidelines for Modified Pavements,"* University of Kentucky Transportation Center, College of Engineering (Interim unpublished Report).

Hopkins, T. C. (June 1991). "*Bearing Capacity Analyses of Pavements*," Research Report KTC-91-8, University of Kentucky Transportation Center, College of Engineering.

Hopkins, T.C. and Beckham, T. L. (September 1993b). "*Proposed Procedure for Compacting Laboratory Specimens for Physical Properties Testing*," Proceedings, Tenth Annual International Pittsburgh Coal Conference, Pittsburgh, Pennsylvania.

Hopkins, T.C., Hunsucker, D.Q., and Beckham, T.L.; (1994a). "*Selection of Design Strengths of Untreated Soil Subgrades and Subgrades Treated with Cement and Hydrated Lime*," Transportation Research Record No. 1440, pp 37-44.

Hopkins, T.C., Hunsucker, D.Q., and Beckham, T.L. (1994b). *"Long-term Performance of Untreated Soil Subgrades and Subgrades Treated with Cement and Hydrated Lime*," Transportation Research Record No. 1440, pp 45-52.

Hopkins, T.C., Hunsucker, D. Q., and Beckham, T.L. (October 1994c). "*Residue By-Product From An Atmospheric Fluidized Bed Combustion Process Used in Highway Subgrade Modification,*" Symposium Proceedings, Conference sponsored by Federal Highway Administration and the Environmental Protection Agency, Denver, Colorado.

Hopkins, T.C. (July 1994a). "*Minimum Bearing Strength of Soil Subgrades Required to Construct Flexible Pavements*," Proceedings, The 4th International Conference on the Bearing Capacity of Roads and Airfields, Vol.1, Minneapolis, Minnesota.

Hopkins, T.C. (July 1994b). "*Case Studies of Flexible Pavement Failures During Construction*," Proceedings, The 4th International Conference on the Bearing Capacity of Roads and Airfields, Vol.1, Minneapolis, Minnesota.

Hopkins, T. C., Beckham, T. L., and Hunsucker, D. Q. (June 1995); "*Modification of Highway Soil Subgrades,*" Research Report KTC-94-11, University of Kentucky Transportation Center, College of Engineering.

Hopkins, T. C. and Slepak, M. E.; (July 1998). "*Estimated Factors of Safety of the AASHO Road Test Flexible Pavement Sections Based on Limiting Equilibrium Methods,"* Proceedings, Fifth International Conference on the Bearing Capacity of Roads, Railroads, and Airfields, Trondheim, Norway.

Hopkins, T. C. and Beckham, T.L. (2000). "*Influence of Clay Fraction and Moisture on the Behavior of Soil-Aggregate Mixtures,"* Proceedings of the Fifth International Symposium on Unbound Aggregates in Roads, UNBAR 5, University of Nottingham, United Kingdom, A. A. Balkema/Rotterdam/ Brookfield.

Hopkins, T. C., Beckham, T. L., Sun, L., Ni, B., and Butcher, B. (June 2002). *"Long-Term Benefits of Stabilizing Soil Subgrades,"* Research Report KTC-02-19/SPR-196-99-1F, University of Kentucky Transportation Center, college of Engineering, Lexington, Kentucky.

Hopkins, T. C., Slepak, M. E., and Sun, L. (2002). *"Limiting Equilibrium Methods in Bearing Capacity Analysis of Flexible Pavements Reinforced With Geosynthetics,"* Proceedings, Sixth International Conference on the Bearing Capacity of Roads, Railways (BCRA'02) and Airfields, Lisbon, Portugal.

Hopkins, T. C.,Beckham, T. L., Sun, L., and Pfalzer, B. (2004). "*Kentucky Geotechnical Database,"* Research Report KTC-03-06/SPR-177-98-1F, University of Kentucky Transportation Center, college of Engineering, Lexington, Kentucky.

Houston W. N., Houston S. L., and Anderson T. W. (1993). "*Stress State Considerations for Resilient Modulus Testing of Pavement Subgrade."* Transportation Research Record TRB No. 1406, Transportation Research Board National Research Council, Washington, D.C.

Ksaibati K., Burczyk J. M., and Whelan M. L. (1995). *"Effect of Selecting Subgrade Resilient Modulus Values on Asphalt on Asphalt Overlay Design Thicknesses."* Transportation Research Record TRB No. 1473, Transportation Research Board National Research Council, Washington, D.C.

Ni, B., Hopkins, T. C., and Sun, L. (2002). *"Modeling the Resilient Modulus of Soils,"* Proceedings, Sixth International Conference on the Bearing Capacity of Roads, Railways and Airfields (BCRA'02), Lisbon, Portugal.

May, R.W. and Witczah, M. W.; (1981). "*Effective Granular Modulus to Model Pavement Response,"* T*ransportation Research Record 810*, Transportation Research Board, National Research Council, Washington, D.C.

Mohammad L. N. and Puppala A. J., Alavilli P. (1994). "*Effect of Strain Measurements on Resilient Modulus of Sands".* Dynamic Geotechnical Testing II, ASTM Special Technical Publication 1213 (R. J. Ebelhar, V. P. Drenvich, and B. L. Kutter, eds.). ASTM Philadelphia, Pa.

Mohammad L. N., Puppala A. J., and Alavilli P. (1995). *"Resilient Properties of Laboratory Compacted Subgrade Soils."* Transportation Research Record TRB No. 1504, Transportation Research Board National Research Council, Washington, D.C.

Moossazadeh, J. M., and Witczak, W. (1981). *"Prediction of Subgrade Moduli for Soil That Exhibits Nonlinear Behavior,"* Transportation Research Record. Transportation Research Board, National Research Council, Washington, D.C.

Nazarian, S. and Feliberti M. (1993). "*Methodology for Resilient Modulus Testing of Cohesionless Subgrades."* Transportation Research Record TRB No. 1406, Transportation Research Board National Research Council, Washington, D.C.

Rada, C. and Witczak W. M. (1981). "*Comprehensive Evaluation of Laboratory Resilient Moduli Results for Granular Material."* Transportation Research Record TRB No. 810, Transportation Research Board National Research Council, Washington, D.C.

Strategic Highway Research Program. (1989). "Resilient Modulus of Unbound Granular Base/Subbase Materials and Subgrade Soils," SHRP Protocol P-46, UGO7, SSO7.

Seed, H.B., Mitry, F. G., Monosmith, C. L, and Chan, C. K. 1967. "*Prediction of Pavement Deflection from Laboratory Repeated Load tests,"* NCHRP Report 35.

Stokoe II, K. H., Kim, D. S., and Andrus, R. D., (1990), *"Development of Synthetic Specimens for Calibration and Evaluation of M_r Equipment*," Transportation Research Record 1278.

Thomson M. R., and Robnett Q. L. (1976*). "Resilient Properties of Subgrade Soils."* Final Report – Data Summary. Transportation Engineering Series 14, Illinois Cooperative

Highway Research and Transportation Program Series 160, University of Illinois at Urbana-Champaign.

Uzan, J. 1985. *"Characterization of Granular Materials,"* Transportation Research Record 1022, Transportation Research Board, National Research Council, Washington, D.C.

Appendix A

Index and Engineering Properties of Bulk and Roadway Soil Samples

Table A-1. Engineering properties of bulk soil samples

Site	Ky 994	Ky 11	Pennyrile Parkway	Ky 10	U.S. 25	U.S.31W
Liquid Limit (%)	26.5	34.4	28.2	41.1	47.7	52.3
Plasticity Index (%)	5.8	7.6	8.5	18.9	19.0	26.7
Specific Gravity	2.64	2.76	2.69	2.76		2.73
Percent Finer (%):						
No. 10 sieve	98.5	90.6	100.0	95.9	96.4	95.1
No. 200 sieve	80.3	70.1	99.4	91.4	83.7	79.7
0.002mm	21.8	21.2	20.0	40.8	50.5	52.8
Classification:						
AASHTO	A-4(3)	A-4(4)	A-4(7)	A-7-6(18)	A-7-6(17)	A-7-6(22)
Unified	CL-ML	ML	CL	CL	CL	CH
CUw/PP[1]: Effective Stress Parameter,N'(deg.)	32.4	28.1	33.4	24.1	28.1	21.0
Effective Stress Parameter, c' (psf)	0	372.8	0	431.2	324.4	737.3
KYCBR[2]-as compacted	27.3	12.1	22.1	17.1	51.8	12.3
KYCBR[3]—soaked according to standard	3.9	3.6	10.8	2.0	6.6	4.6
KYCBR[4]—soaked until swell ceases	6.9	3.9	8.2	1.9	1.0	5.4
AASHTO[2]-as compacted	17.9	30.6	12.5	11.6	20.0	11.1
AASHTO[3]-soaked according to standard	4.2	2.9	9.1	1.6	1.3	1.2
AASHTO[4]-soaked until swell ceases	7.1	3.1	11.2	0.4	1.2	1.3
AASHTO--5 % Hydrated Lime		21.0		9.6	22.5	30.3
AAHTO—10 % Cement	71.5		18.6			

1. Consolidated-undrained triaxial compression tests with pore pressure measurements. Specimens compacted to 95 % of maximum dry density and optimum moisture content obtained from AAHTO T-99.
2. CBR test performed on the "as compacted" specimen.
3. CBR specimen allowed to soak and swell according to criteria specified by the Ky CBR method and the AASHTO method.
4. CBR specimens allowed to swell until swelling ceased.

Table A-2. Engineering Properties of Roadway Samples.

Sample identification	LL (%)	PI (%)	Max Dry Density (pcf)	Opt. M. C. (%)	Q_u (psi) As comp	KY CBR (%)	AASHTO CBR (%)	- No. 0.002 mm (%)	-No. 200 (%)	AASHTO CLASS.	GI	Unified Class.
Union (S-8) 009703555	32	7	109.0	17.0	27.4	10.3	8.9	12.1	99.8	A-4	8	ML
Union (S-23) 009703570	29	6	112.0	15.0	26.17	6.9		17.1	99.2	A-4	6	ML
Union (S-40) 009703877	29	6	111.0	15.0	30.02	12.1		23.8	98.8	A-4	6	ML
Union (S-28) 009703865	28	6	112.0	15.0	27.33	11.8	7.4	18.9	97.4	A-4	5	CL-ML
Union (S-46) 009703883	30	6	110.0	16.0	27.24	10.0	12.9	17.8	99.8	A-4	6	ML
Allen (S-44) 009705767	33	9	109.0	16	40.82	13.3	11.8	11.5	81.9	A-4	7	ML
Allen (S-20) 009705542	30	8	108.0	17	19.58	17.3		15.6	72.4	A-4	4	CL
Jefferson (S-16) 009705232	31	9	107.0	16	32.66	8.8	5.5	23.9	98.2	A-4	9	CL
Taylor (S-8) 009710848	25	9	115.0	15	17.09	9.3		22.6	37.6	A-4	0	SC
Taylor (S-2) 009710842	22	13	121.0	13	18.66	13.7	5.6	16.5	37.8	A-4	0	SC
Graves (S-33)00971159	32	10	111.0	15	24.23	9.5	3.9	18.1	46.7	A-4	2	SC
Fayette (S-22) 009701432	38	16	109.0	19.0	32.45	4.9		31.5	87.3	A-6	13	CL
Scott (S-7) 009701346	40	17	104.0	20.0	33.15	12.1	8.8	40.7	93.3	A-6	17	CL
Meade (S-11) 009700498	33	13	113.0	14.0	40.65	11.4	7.5	23.3	74.9	A-6	8	CL
Union (S-17) 009703564	37	14	110.0	16.0	33.96	9.7		23.9	98.8	A-6	15	CL
Pendleton (S-4) 009702763	29	12	116.0	14.0	33.37	12.3	9.9	24.5	73.6	A-6	7	CL
Warren (S-49) 009705183	39	19	109.0	17.0	36.87	9.5	5.5	31.5	94.6	A-6	19	CL
Warren (S-12) 009704334	37	14	106.0	17	30.36	11.4	10.2	31.3	94.2	A-6	14	CL
Allen (S-1) 009704875	34	13	106.0	16	25.4	8.6		8.6	74.7	A-6	9	CL
Allen (S-17) 009705168	40	16	107.0	18	41.7	14.0	13.7	36.3	82.9	A-6	14	CL
Allen (S-93) 009706341	34	13	110.0	17	26.92	8.3		23.9	60.9	A-6	6	CL
Allen (S-63) 009705787	39	17	104.0	20	17.87	5.7		22.2	68.1	A-6	10	CH
Allen (S-93) 009706341	34	13	110.0	17	26.92	8.3		23.9	60.9	A-6	6	CL
Allen (S-63) 009705787	39	17	104.0	20	17.87	5.7		22.2	68.1	A-6	10	CH
Jefferson (S-1) 009708852	33	16	115.0	14	44.92	7.9		29.3	72.2	A-6	10	CL

Table A-3. Engineering Properties of Roadway Samples.

Sample identification	LL (%)	PI (%)	Max Dry Density (pcf)	Opt. M. C. (%)	Q_u (psi) As comp	KY CBR (%)	AASHTO CBR (%)	- No. 0.002 mm (%)	-No. 200 (%)	AASHTO CLASS.	GI	Unified Class.
Jefferson (S-3) 009708858	32	15	116.0	15	44.55	5.9	4.2	24.4	73.8	A-6	9	CL
Christian (S-5) 009711063	35	16	102.0	18	15.6	6.4		19.1	98.4	A-6	16	CL
Christian (S-10) 009711068	33	13	111.0	15	58.25	8.6		28.1	99	A-6	13	CL
Taylor (S-32) 009610873	32	20	113.0	16	28.33	11.9	6.5	29.6	68	A-6	6	CL
Graves (S-2)009711136	33	17	116.0	14	29.94	11.5		26.2	69.8	A-6	10	CL
Graves (S-9)009711144	33	24	114.0	14	28.16	5.2	5.9	16.7	73.2	A-6	9	CL
Graves (S-36)009711162	32	15	110.0	15	26.57	7.2	2.0	30.1	84.9	A-6	11	CL
Jefferson (S-1)1998GT00-01890	34	15	108.3	17	35.71	6.4		0.3	92.9	A-6	14	CL
Jefferson (S-2)1998GT00-01891	33	19	106.4	20	52.47	7.4		25.2	62.9	A-6	6	CL
Kenton (S-3)1998GT00-00876	40	21	107.2	19	29.59	4.1		27	90.6	A-6	19	CL
Madison (S-4)1998GT00-01436	37	15	102.8	19	40.38	6.0		32.5	97.8	A-6	16	CL
Madison (S-7)1998GT00-01439	33	15	112.3	14	50.12	8.8		27.4	79.1	A-6	10	CL
Warren (S-5) 009700306	40	19	107.3	16.0	57.42	11.2	3.6	34.9	98.4	A-6	20	CL
Gallatin (S-18) 009701203	58	28	99.0	23.0	31.51	2.9		50.1	96.9	A-7-5	33	CH
Taylor (S-15) 009710856	48	32	99.0	23	38.43	6.8	4.8	48.5	74	A-7-5	13	ML
Grayson (S-16) 009708587	82	39	92.0	25	30.61	1.1	0.7	68.2	96.5	A-7-5	52	MH
Grayson (S-30) 009708601	62	31	101.0	22	27.95	1.0		57.6	96	A-7-5	36	CH
Nelson (S-1) 009700082	49	26	107.0	18.0	57.04	7.4		39.6	93.7	A-7-6	27	CL
Nelson (S-12) 009700093	66	39	92.0	24.0	119.05	2.1		65.1	95.9	A-7-6	44	CH
Warren (S-16) 009700317	67	43	98.0	25.0	62.68	5.5		63.9	93.9	A-7-6	45	CH

Table A-4. Engineering Properties of Roadway Samples.

Sample identification	LL (%)	PI (%)	Max Dry Density (pcf)	Opt. M. C. (%)	Q_u (psi) As comp	KY CBR (%)	AASHTO CBR (%)	- No. 0.002 mm (%)	-No. 200 (%)	AASHTO CLASS.	GI	Unified Class.
Fayette (S-16) 009701426	48	23	103.0	21.0	37.46	10.3		36.8	88.3	A-7-6	22	CL
Fayette (S-3) 009701333	43	18	101.0	21.0	42.69	5.0		35.5	85.7	A-7-6	17	CL
Fayette (S-9) 009701419	41	16	104.0	20.0	37.54	13.9	10.7	38.3	86	A-7-6	15	CL
Scott (S-1) 009701340	63	34	97.0	26.0	40.38	3.1		50.9	90.4	A-7-6	36	CH
Meade (S-1) 009700488	68	43	92.0	24.0	41.58	3.0		54.2	86.7	A-7-6	41	CH
Meade (S-6) 009700493	62	40	102.0	22.0	35.22	3.0		48.8	92.2	A-7-6	41	CH
Gallatin (S-11) 009701048	41	19	115.0	15.0	49.18	6.2		22.9	61.6	A-7-6	10	CL
Pendleton (S-7) 009702766	58	30	101.0	22.0	30.13	2.1		53.1	97.5	A-7-6	35	CH
Warren (S-21) 009704343	44	21	108.0	18.0	33.32	9.1		37.6	93.8	A-7-6	22	CL
Warren (S-42) 009705176	66	43	97.0	23	39.59	5.5		57.9	85.4	A-7-6	40	CH
Allen (S-36) 009705558	70	44	98.0	21	55.87	4.3		55.8	90.6	A-7-6	45	CH
Allen (S-29) 009705551	42	18	104.0	19	24.84	13.4	4.7	29.1	76.9	A-7-6	14	CL
Jefferson (S-6) 009705222	54	33	102.0	21	27.68	5.4	5.0	42	75.2	A-7-6	29	CH
Jefferson (S-11) 009705227	74	46	86.0	32	19.79	2.8		52.2	94.7	A-7-6	51	CH
Pulaski (S-12) 009706980	59	33	100.0	22	47.1	5.6	4.6	55.7	85	A-7-6	31	CH
Pulaski (S-16) 009706984	57	36	100.0	22	26.46	3.5		52.6	81.6	A-7-6	31	CH
Grayson (S-5) 009708576	45	20	100.0	22	18.7	4.6		36.1	95.6	A-7-6	26	CL
Grayson (S-18) 009708589	50	23	101.0	22	23.85	2.2		43.1	78.3	A-7-6	22	CH
Bracken (S-6)1998GT00-01147	47	25	105.1	19	32.44	4.2		44	93.6	A-7-6	26	CL
Bracken (S-14)1998GT00-01155	46	22	98.0	23	36.27	3.1		45.9	93.5	A-7-6	23	CL
Bracken (S-15)1998GT00-01156	51	28	102.9	19	36.55	2.8		52.9	96	A-7-6	30	CH
Jefferson (S-6)1998GT00-1895	42	22	105.6	18	26.69	3.3		46.8	97.3	A-7-6	23	CL
Kenton (S-4)1998GT00-00877	45	23	98.7	24	29.63	2.0		36.4	88.2	A-7-6	21	CL

Appendix B

Determination of Coefficients for Resilient Modulus Models Using Simple/Multiple Regression Analysis

This appendix presents the general method of simple/multiple linear regression and how to use this method to determine the coefficients of models for resilient modulus of soils or aggregate materials.

General linear regression

Assume we have a linear function as follows:

$$y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + ... + a_m x_m \tag{1}$$

and we have *n* set of data:

$$\begin{aligned} &(y_1, x_{11}, x_{21}, x_{31}, ..., x_{m1}) \\ &(y_2, x_{12}, x_{22}, x_{32}, ..., x_{m2}) \\ &(\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots) \\ &(y_n, x_{1n}, x_{2n}, x_{3n}, ..., x_{mn}) \end{aligned} \tag{2}$$

The purpose of linear regression analysis is to obtain the coefficients $a_0, a_1, a_2, ..., a_m$, which make the overall differences between the tested y_i values and predicted y values to minimum. That is, to make

$$Q = \sum_{j=1}^{n} [y_j - (a_0 + a_1 x_{1j} + a_2 x_{2j} + ... + a_m x_{mj})]^2 \tag{3}$$

to minimum. By calculus, there has to be

$$\frac{\partial Q}{\partial a_i} \equiv 0 \qquad i = 0, 1, 2, \ldots m \tag{4}$$

That is,

$$\begin{aligned} &\frac{\partial Q}{\partial a_0} = \sum_{j=1}^{n} 2 * [y_j - (a_0 + a_1 x_{1j} + a_2 x_{2j} + ... + a_m x_{mj})] * (-1) \equiv 0 \\ &\frac{\partial Q}{\partial a_1} = \sum_{j=1}^{n} 2 * [y_j - (a_0 + a_1 x_{1j} + a_2 x_{2j} + ... + a_m x_{mj})] * (-x_{1j}) \equiv 0 \\ &\ldots\ldots \end{aligned} \tag{4a}$$

$$\frac{\partial Q}{\partial a_i} = \sum_{j=1}^{n} 2 * [y_j - (a_0 + a_1 x_{1j} + a_2 x_{2j} + ... + a_m x_{mj})] * (-x_{ij}) \equiv 0$$

$$\ldots\ldots$$

$$\frac{\partial Q}{\partial a_m} = \sum_{j=1}^{n} 2 * [y_j - (a_0 + a_1 x_{1j} + a_2 x_{2j} + ... + a_m x_{mj})] * (-x_{mj}) \equiv 0$$

Simplify (4a) and express those in tensor format; the coefficients are determined by solving the following equations:

$$C'C \begin{pmatrix} a_0 \\ a_1 \\ : \\ : \\ a_m \end{pmatrix} = C' \begin{pmatrix} y_1 \\ y_2 \\ : \\ : \\ y_n \end{pmatrix} \tag{4b}$$

where

$$C = \begin{pmatrix} 1\ x_{11}\ \ x_{21}\ \ ....x_{m1} \\ 1\ x_{12}\ \ x_{22}\ \ ....x_{m2} \\ .... \\ 1\ x_{1n}\ \ x_{2n}\ \ ....x_{mn} \end{pmatrix} \tag{5}$$

C' = Transpose of C

The confidence in the coefficients obtained from the above linear regression is determined by R^2 defined as follows:

$$R^2 = 1 - \frac{Q}{S_{yy}} \tag{6}$$

where Q is already defined in equation (3),

$$S_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2 \tag{7}$$

and

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \ , \tag{8}$$

the mean of tested y values.

Determine the Coefficients in the Six Models for Resilient Modulus of Soils or aggregate materials:

Dunlap (Model 1):

$$M_r = k_1 \left(\frac{\sigma_3}{p_a} \right)^{k_2} \tag{9}$$

Moossazadeh and Witczak (Model 2):

$$M_r = k_1 \left(\frac{\sigma_d}{p_a} \right)^{k_2} \tag{10}$$

Seed et al. (Model 3):

$$M_r = k_1 \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \tag{11}$$

Uzan (Model 4):

$$M_r = k_1 p_a \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\sigma_d}{p_a} \right)^{k_3} \tag{12}$$

UKTC (Model 5):

$$M_r = k_1 p_a \left(\frac{\sigma_3}{p_a} + 1 \right)^{k_2} \left(\frac{\sigma_d}{p_a} + 1 \right)^{k_3} \tag{13}$$

NCHRP (Model 6):

$$M_r = k_1 p_a \left(\frac{\sigma_{sum}}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} . \tag{14}$$

In the above models,

M_r = Resilient modulus,
p_a = Reference pressure (used to normalize M_r units),
σ_3 = Minimum effective principal stress,
σ_d = Deviator stress, σ_{sum} = Sum of three principal stresses,
σ_{sum} = sum of three principal stresses, and
τ_{oct} = Octahedral shear stress acting on the material,
k_1, k_2 and k_3 are coefficients need to be determined.

There are one variable and two coefficients in first three Models, and two variables and three coefficients in other three models. All models are not linear equations and have to be transferred into a linear equation in order to apply a linear regression analysis.

All six models can be linearized as following:

$$Log(M_r) = Log(k_1 p_a) + k_2 Log(X_1) + k_3 Log(X_2) \tag{15}$$

where

X_1 stands for σ_3/p_a and σ_d/p_a in Models 1 and 2 respectively; for σ_{sum}/p_a in Models 3, 4, and 6 respectively; and for $(\sigma_3/p_a + 1)$ in model 5. X_2 stands for σ_d/p_a, $(\sigma_d/p_a + 1)$, and $(\tau_{oct}/p_a + 1)$ in models 4, 5, and 6 respectively.

Let $y = Log(M_r)$, $a_0 = Log(k_1 p_a), a_1 = k_2, a_2 = k_3, x_1 = Log(X_1), x_2 = Log(X_2)$; we have a simple linear equation:

$$y = a_0 + a_1 x_1 + a_2 x_2 \tag{16}$$

Assume we have n set of data

$$\begin{array}{l} (y_1, x_{11}, x_{21}) \\ (y_2, x_{12}, x_{22}) \\ (\ldots\ldots\ldots..) \\ (y_n, x_{1n}, x_{2n}) \end{array}$$

where

$y_i = Log(M_{ri})$, $x_{1i} = Log(X_{1i})$, $x_{2i} = Log(X_{2i})$ $for\ \ i = 1\ to\ n$. Coefficients a_0, a_1 and a_2 are solved from following equation:

$$C'C \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = C' \begin{pmatrix} y_1 \\ y_2 \\ : \\ : \\ y_n \end{pmatrix} \tag{17}$$

where $$C = \begin{pmatrix} 1\ x_{11}\ \ x_{21}\ \ \ldots\ldots\ldots. x_{m1} \\ 1\ x_{12}\ \ x_{22}\ \ \ldots\ldots\ldots. x_{m2} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 1\ x_{1n}\ \ x_{2n}\ \ \ldots\ldots\ldots.. x_{mn} \end{pmatrix} \tag{18}$$

Table B-2. Converted Data

Log(Mr)	Log(σ_3+1)	Lg(σ_d+1)
9.575539	1.386294361	1.386294361
9.66504	1.386294361	1.945910149
9.698	1.386294361	2.302585093
9.982391	1.791759469	1.791759469
10.02211	1.791759469	2.397895273
10.05057	1.791759469	2.772588722
10.46749	2.397895273	2.397895273
10.52619	2.397895273	3.044522438
10.53688	2.397895273	3.433987204
10.71181	2.772588722	2.397895273
10.7256	2.772588722	2.772588722
10.754	2.772588722	3.433987204
10.94842	3.044522438	2.772588722
11.01048	3.044522438	3.044522438
10.98224	3.044522438	3.713572067

Table B-1. Original Test Data

M_r	σ_3 *(psi)*	σ_d *(psi)*
14408	3	3
15757	3	6
16285	3	9
21642	5	5
22519	5	10
23169	5	15
35154	10	10
37279	10	20
37680	10	30
44883	15	10
45506	15	15
46817	15	30
56864	20	15
60505	20	20
58820	20	40

The task turns to solving a 3 by 3 linear equation for a_0, a_1 and a_2. And

$$k_1 = \frac{e^{a_0}}{p_a}, \; k_2 = a_1 , \quad k_3 = a_2 . \tag{19}$$

The value of R^2 is still determined by Equation 6.

Example of calculating k_1, k_2 and k_3 from the test data for UKTC model (model 5)

Equations 13, 15 –18, 6 – 8 are used to calculate k_1, k_2, and k_3, and to evaluate R^2. Assume p_a = 1 *psi*. Test data are shown in Table D-1.
Consider UKTC model:

$$M_r = k_1 p_a (\frac{\sigma_3}{p_a} + 1)^{k_2} (\frac{\sigma_d}{p_a} + 1)^{k_3}$$

Note that p_a = 1 *psi* and linearize UKTC model as:

$$Log(M_r) = Log(k_1) + k_2 Log(\sigma_3 + 1) + k_3 Log(\sigma_d + 1)$$

Let

$$y = Log(M_r), \; a_0 = Log(k_1), a_1 = k_2, a_2 = k_3, x_1 = Log(\sigma_3 + 1), x_2 = Log(\sigma_d + 1) .$$

We have a simple linear equation:

$$y = a_0 + a_1 x_1 + a_2 x_2$$

Convert original test data to linear item data, as shown in Table D-2.

From Equation 18, C and C' will be:

$$C = \begin{bmatrix} 1 & 1.386294361 & 1.386294361 \\ 1 & 1.386294361 & 1.945910149 \\ 1 & 1.386294361 & 2.302585093 \\ 1 & 1.791759469 & 1.791759469 \\ 1 & 1.791759469 & 2.397895273 \\ 1 & 1.791759469 & 2.772588722 \\ 1 & 2.397895273 & 2.397895273 \\ 1 & 2.397895273 & 3.044522438 \\ 1 & 2.397895273 & 3.433987204 \\ 1 & 2.772588722 & 2.397895273 \\ 1 & 2.772588722 & 2.772588722 \\ 1 & 2.772588722 & 3.433987204 \\ 1 & 3.044522438 & 2.772588722 \\ 1 & 3.044522438 & 3.044522438 \\ 1 & 3.044522438 & 3.713572067 \end{bmatrix}$$

C' = Transpose of *C*

$$C'C = \begin{bmatrix} 15 & 34.179181 & 39.608592 \\ 34.179181 & 83.515443 & 94.443871 \\ 39.608592 & 94.443871 & 110.445516 \end{bmatrix}$$

$$C'\begin{pmatrix} y_1 \\ y_2 \\ : \\ : \\ y_n \end{pmatrix} = C'\begin{pmatrix} 9.57553889 \\ 9.66503999 \\ 9.69799972 \\ 9.98239115 \\ 10.02211468 \\ 10.05057046 \\ 10.46749369 \\ 10.52618544 \\ 10.53688473 \\ 10.71181438 \\ 10.72559946 \\ 10.75400166 \\ 10.94841773 \\ 11.01048129 \\ 10.98223721 \end{pmatrix} = \begin{bmatrix} 155.656770 \\ 359.119430 \\ 414.540132 \end{bmatrix}$$

Substituting to Equation 4b, then:

$$\begin{bmatrix} 15 & 34.179181 & 39.608592 \\ 34.179181 & 83.515443 & 94.443871 \\ 39.608592 & 94.443871 & 110.445516 \end{bmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 155.656770 \\ 359.119430 \\ 414.540132 \end{pmatrix}$$

Solving this equation, we get:

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 8.507814 \\ 0.729068 \\ 0.078787 \end{pmatrix}$$

From Equation 19, k_1, k_2, and k_3 will be:

$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 4953.323026 \\ 0.729068 \\ 0.078787 \end{pmatrix}$$

$$Q = \sum_{j=1}^{15} [y_j - (a_0 + a_1 x_{1j} + a_2 x_{2j})]^2 = 0.011645256$$

$$S_{yy} = \sum_{i=1}^{15} (y_i - \overline{y})^2 = 3.524374018$$

$$R^2 = 1 - \frac{Q}{S_{yy}} = 0.996695794$$

$$M_r = 4953.323026(\sigma_3 + 1)^{0.729068} (\sigma_d + 1)^{0.078787}$$

That is, the function can be used to predict resilient modulus, M_r, from test data.

Appendix C

Comparison of Resilient Modulus Coefficients Obtained from Multiple Regression Analysis of Laboratory "As–Compacted" Soil Specimens Using Models 4, 5, and 6.

Table C-1. Values of Resilient Modulus Obtained from Multiple Regression Analysis of Laboratory "As–Compacted" Soil Specimens Using Models 4, 5, and 6.

Sample	Uzan's Model				UKTC Model				NCHRP Model			
	K_1	K_2	K_3	R^2	K_1	K_2	K_3	$*R^2$	K_1	K_2	K_3	R^2
A-4(0)	2818.612	0.7058	-0.3419	0.9835	6374.112	0.6078	-0.1705	0.9804	3492.245	0.7076	-0.4237	0.9817
A-4(0)	4512.47	0.5681	-0.3449	0.9738	9074.286	0.4912	-0.224	0.9961	5594.84	0.5721	-0.4301	0.9797
A-4(2)	17415.22	0.1615	-0.1854	0.895	22804.2	0.1435	-0.1821	0.9394	19563.09	0.1643	-0.2326	0.915
A-4(4)	5671.45	0.4872	-0.3524	0.9804	10788.11	0.4232	-0.2677	0.9955	7067.061	0.4911	-0.4394	0.9876
A-4(5)	5766.958	0.4844	-0.2842	0.9744	10347.5	0.4208	-0.1798	0.9907	6892.575	0.4868	-0.3537	0.9778
A-4(6)	4772.855	0.5097	-0.2709	0.9664	8580.093	0.4447	-0.1503	0.991	5651.634	0.5131	-0.3383	0.9731
A-4(6)	5702.158	0.4641	-0.2684	0.9837	10013.61	0.4011	-0.1688	0.9937	6747.316	0.4665	-0.3342	0.9878
A-4(6)	4939.898	0.4839	-0.2607	0.9792	8784.113	0.4185	-0.1531	0.9904	5816.186	0.4865	-0.3249	0.9845
A-4(7)	10292.8	0.3132	-0.2537	0.9629	15750.07	0.275	-0.206	0.9881	12055.79	0.3168	-0.3174	0.976
A-4(8)	6658.17	0.4063	-0.2905	0.9772	11113.32	0.3563	-0.2115	0.9959	7988.833	0.4092	-0.3622	0.9845
A-6(10)	19389.75	0.2149	-0.2492	0.9368	27367.19	0.1893	-0.2368	0.9686	22654.19	0.219	-0.3128	0.9571
A-6(10)	15852.78	0.1726	-0.1941	0.9057	21179.73	0.1513	-0.1893	0.9274	17931.25	0.1737	-0.2415	0.9112
A-6(10)	6834.92	0.4316	-0.4705	0.9788	13387.81	0.3766	-0.4361	0.9936	9173.737	0.437	-0.5872	0.9892
A-6(11)	5250.612	0.3318	-0.1539	0.5739	7932.313	0.2764	-0.0814	0.56	5779.081	0.3352	-0.1944	0.585
A-6(12)	14244.13	0.2316	-0.2147	0.9421	20004.25	0.2018	-0.1891	0.9633	16299.67	0.2347	-0.2692	0.961
A-6(13)	18453.33	0.2019	-0.2972	0.9272	26215.34	0.1835	-0.2992	0.9592	22221.15	0.207	-0.3735	0.9499
A-6(13)	13644.61	0.3002	-0.1444	0.9517	19657.22	0.2523	-0.0755	0.953	14932.57	0.3024	-0.181	0.963
A-6(14)	24050.23	0.0813	-0.1996	0.9434	29199.3	0.0755	-0.2216	0.9674	27257.94	0.0843	-0.2504	0.9623
A-6(14)	2525.767	0.3471	0.054	0.5652	3428.918	0.2799	0.1828	0.5279	2442.555	0.3481	0.0648	0.5636
A-6(14)	41668.47	-0.0323	-0.1704	0.9142	44895.82	-0.0215	-0.224	0.9282	46369.63	-0.0296	-0.214	0.93
A-6(15)	8891.046	0.2707	-0.237	0.9761	12887.33	0.2365	-0.1947	0.9929	10310.31	0.2736	-0.296	0.9868
A-6(16)	10934.74	0.2188	-0.2034	0.9554	14947.51	0.1942	-0.1771	0.9831	12417.97	0.2215	-0.2544	0.9697
A-6(17)	23292.81	0.0816	-0.205	0.9082	28254.27	0.0775	-0.2272	0.9365	26484.1	0.0851	-0.2577	0.9298
A-6(19)	14569.71	0.2364	-0.1897	0.9198	20332.99	0.2012	-0.1552	0.9299	16410.88	0.2394	-0.2382	0.9391
A-6(19)	17834.68	0.1217	-0.2003	0.8887	22744.99	0.1093	-0.2136	0.9186	20219.44	0.1245	-0.2509	0.9056

Table C-2. Values of Resilient Modulus Obtained from Multiple Regression Analysis of Laboratory "As–Compacted" Soil Specimens Using Models 4, 5, and 6.

Sample	Uzan's Model				UKTC Model				NCHRP Model			
	K_1	K_2	K_2	R^2	K_1	K_2	K_2	R^2	K_1	K_2	K_2	R^2
A-6(20)	2326.453	0.7883	-0.1919	0.9816	5305.504	0.6695	0.0311	0.9485	2625.168	0.7895	-0.2384	0.982
A-6(6)	13977.44	0.2769	-0.2407	0.941	21330.64	0.2336	-0.2119	0.9534	16271.98	0.2786	-0.2996	0.9469
A-6(6)	10432.7	0.3171	-0.2568	0.9707	16224.86	0.2725	-0.2089	0.981	12257.58	0.3201	-0.3209	0.9827
A-6(6)	10013.61	0.3816	-0.3298	0.9597	17281.64	0.3273	-0.2766	0.9808	12300.56	0.3864	-0.4129	0.9746
A-6(7)	13472.42	0.2854	-0.289	0.9425	20623.78	0.2505	-0.2601	0.9743	16147.17	0.2897	-0.3624	0.9613
A-6(8)	19876.63	0.1929	-0.2348	0.9697	27056.97	0.1687	-0.2216	0.9849	23021.87	0.1955	-0.2929	0.9786
A-6(9)	21894.7	0.1758	-0.2167	0.9141	29231.44	0.1565	-0.2106	0.949	25069.32	0.1795	-0.2723	0.936
A-6(9)	12605.64	0.2747	-0.2427	0.9669	18794.13	0.2363	-0.2062	0.9847	14674.99	0.2772	-0.3026	0.975
A-6(9)	6333.448	0.4556	-0.3538	0.966	11768.71	0.3969	-0.2835	0.9965	7902.228	0.4598	-0.4418	0.9765
A-7-5(13)	7827.512	0.4893	-0.1517	0.8984	13662.36	0.4041	-0.0223	0.8826	8607.593	0.4922	-0.1912	0.9062
A-7-5(33)	12657.43	0.212	-0.3017	0.9099	18333.77	0.1873	-0.3012	0.939	15286.11	0.2174	-0.3795	0.9343
A-7-5(36)	11253.11	0.1294	-0.3236	0.9462	15362.73	0.1177	-0.3571	0.9678	13773.48	0.1346	-0.406	0.9646
A-7-5(52)	11306.13	0.1469	-0.1825	0.6035	14536.24	0.1241	-0.177	0.6198	12670.1	0.1522	-0.2326	0.6358
A-7-6(10)	31489.2	0.0854	-0.2159	0.9718	38700.2	0.0769	-0.2375	0.9838	36054.97	0.0877	-0.2694	0.9818
A-7-6(14)	13761.09	0.2268	-0.3035	0.9303	20118.6	0.2042	-0.2991	0.9632	16635.6	0.2319	-0.3812	0.9518
A-7-6(15)	22647.39	0.1474	-0.189	0.8437	28535.38	0.1321	-0.179	0.8692	25499.14	0.1498	-0.2364	0.856
A-7-6(16)	15946.59	0.1852	-0.2899	0.9262	22350.41	0.1694	-0.2992	0.9565	19127.84	0.1898	-0.3641	0.948
A-7-6(17)	13544.01	0.2542	-0.2111	0.8995	19129.75	0.2216	-0.171	0.9259	15450.55	0.2582	-0.2657	0.9222
A-7-6(20)	15569.98	0.1915	-0.169	0.8976	20605.23	0.1632	-0.1449	0.9147	17307.58	0.1945	-0.2126	0.9204
A-7-6(21)	27173.57	-0.0375	-0.1397	0.9467	28526.82	-0.0273	-0.1859	0.9554	29667.28	-0.0357	-0.1749	0.958
A-7-6(22)	24081.52	0.0989	-0.1567	0.948	28871.2	0.0871	-0.1598	0.9716	26592.91	0.1007	-0.1962	0.9659
A-7-6(22)	15569.98	0.1915	-0.169	0.8976	20605.23	0.1632	-0.1449	0.9147	17307.58	0.1945	-0.2126	0.9204
A-7-6(22)	16537.73	0.1315	-0.2453	0.9402	21478.33	0.1198	-0.2584	0.9652	19275.69	0.1354	-0.3077	0.9587
A-7-6(23)	13498.04	0.1899	-0.2669	0.9461	19139.32	0.1657	-0.273	0.9727	15972.12	0.1928	-0.3336	0.9611
A-7-6(23)	13070.33	0.2253	-0.2234	0.8658	18786.62	0.1953	-0.2111	0.9068	15035.96	0.2296	-0.2817	0.8931

Table C-3. Values of Resilient Modulus Obtained from Multiple Regression Analysis of Laboratory "As–Compacted" Soil Specimens Using Models 4, 5, and 6.

Sample	Uzan's Model				UKTC Model				NCHRP Model			
	K_1	K_2	K_2	R^2	K_1	K_2	K_2	R^2	K_1	K_2	K_2	R^2
A-7-6(26)	11081.14	0.1548	-0.2639	0.9382	14809.14	0.1442	-0.2751	0.9698	13069.02	0.1589	-0.331	0.9569
A-7-6(26)	15521.79	0.216	-0.2043	0.8975	21936.34	0.1834	-0.1894	0.9208	17660.75	0.219	-0.2567	0.9207
A-7-6(27)	26486.75	0.1399	-0.1102	0.8607	31574.34	0.13	-0.0887	0.9341	28364.68	0.1422	-0.1389	0.8842
A-7-6(30)	16330.67	0.1725	-0.1879	0.8436	21450.43	0.1582	-0.1812	0.8848	18381.5	0.175	-0.2355	0.8599
A-7-6(31)	17106.26	0.0932	-0.1887	0.9056	20906.18	0.0836	-0.2038	0.9301	19250.65	0.0965	-0.2373	0.9283
A-7-6(35)	11021.46	0.2738	-0.3308	0.9193	17026.05	0.245	-0.3159	0.95	13565.7	0.2787	-0.415	0.9392
A-7-6(36)	21920.99	0.0567	-0.114	0.7543	24469.92	0.0575	-0.1223	0.8002	23536.32	0.0592	-0.144	0.7796
A-7-6(37)	23775.24	0.1426	-0.1295	0.8497	29138.05	0.1263	-0.1122	0.8916	25770.87	0.1453	-0.1633	0.8741
A-7-6(40)	15745.34	0.229	-0.1301	0.8245	21006.77	0.1947	-0.0833	0.8279	17080.62	0.2312	-0.1635	0.8396
A-7-6(41)	18227.74	0.1441	-0.0995	0.8164	21842.22	0.1263	-0.0714	0.8318	19399.45	0.1455	-0.1246	0.828
A-7-6(41)	20255.87	0.0977	-0.1534	0.9269	24253.11	0.0867	-0.1571	0.9493	22307.99	0.0999	-0.1924	0.9462
A-7-6(44)	29132.22	0.1796	-0.1856	0.944	37873.24	0.1571	-0.1647	0.9527	32712.09	0.1806	-0.2298	0.9387
A-7-6(44)	20623.78	0.0294	-0.1957	0.8495	23863.37	0.0324	-0.2355	0.8785	23299.8	0.0339	-0.2475	0.8765
A-7-6(45)	17586.73	0.2648	-0.089	0.8033	22897.89	0.2359	-0.0139	0.8722	18584.81	0.2673	-0.1131	0.8149
A-7-6(45)	19327.8	0.0937	-0.3483	0.9017	25887.1	0.0943	-0.4039	0.9309	24040.62	0.1003	-0.4389	0.9261
A-7-6(45)	10316.5	0.3401	-0.2131	0.9302	15739.05	0.299	-0.147	0.979	11784.02	0.3435	-0.2673	0.9455
A-7-6(46)	14227.04	0.1796	-0.3266	0.9263	20314.7	0.1645	-0.3459	0.9561	17448.34	0.1851	-0.4102	0.9474
A-7-6(46)	14983.43	0.1385	-0.3993	0.9224	21585.99	0.1326	-0.4523	0.949	19256.42	0.1448	-0.5016	0.9434

Shaded area: **$R^2$$0.87 in 91 percent of tests

Appendix D

Comparison of Resilient Modulus Results Obtained from Multiple Regression Analysis of Laboratory "Soaked," Compacted Soil Specimens Using Models 4, 5, and 6.

Table D-1. Values of Resilient Modulus Obtained from Multiple Regression Analysis of Laboratory, Soaked Compacted Soil Specimens Using Models 4, 5, and 6.

Sample	Uzan's Model				UKTC Model				NCHRP Model			
	K_1	K_2	K_2	R^2	K_1	K_2	K_2	$**R^2$	K_1	K_2	K_2	R^2
A-4(0)	518.998	1.0212	0.0423	0.5337	1220.603	0.909	0.3846	0.5584	505.4757	1.0386	0.0252	0.5322
A-4(0)	253.4585	1.3192	0.008	0.4877	855.7686	1.1359	0.4398	0.5214	251.5395	1.3358	-0.0145	0.4877
A-4(2)	185.2488	1.1125	-0.3931	0.8029	650.6032	0.9349	-0.105	0.7798	237.5077	1.1181	-0.4937	0.8099
A-4(4)	1762.168	0.3356	0.3944	0.8228	1721.24	0.3304	0.6032	0.8362	1374.713	0.3402	0.4791	0.8078
A-4(5)	458.5182	0.8336	0.0869	0.9762	909.7774	0.7011	0.4014	0.9669	434.328	0.8347	0.1052	0.9753
A-4(6)	528.9003	0.6042	0.2395	0.9826	784.4635	0.493	0.5173	0.9637	455.1832	0.6032	0.2964	0.9817
A-4(6)	405.8972	0.7218	0.1872	0.9562	699.8738	0.5922	0.4883	0.9376	361.0802	0.7229	0.2286	0.9532
A-4(6)	472.0099	0.5132	0.3926	0.9769	587.1616	0.4251	0.668	0.97	369.2596	0.514	0.4818	0.9695
A-4(7)	1043.463	0.599	0.0792	0.9484	1633.859	0.5236	0.3098	0.9689	993.1682	0.5997	0.0963	0.9472
A-4(8)	565.3246	0.8062	0.0745	0.9637	1099.598	0.6862	0.3718	0.97	539.6388	0.8076	0.0895	0.9627
A-6(10)	638.4862	0.7441	-0.3553	0.8371	1588.746	0.5905	-0.1598	0.7626	797.7538	0.746	-0.4404	0.8354
A-6(10)	618.5636	0.7971	-0.0965	0.617	1217.92	0.7426	0.1517	0.7382	656.4195	0.8087	-0.1361	0.6238
A-6(11)	17.6247	1.793	-0.6801	0.9014	152.9024	1.5851	-0.3761	0.8602	25.3328	1.8289	-0.8633	0.9124
A-6(13)	729.0921	0.863	-0.5242	0.7703	2166.569	0.7102	-0.3266	0.7412	1014.245	0.8628	-0.6459	0.762
A-6(13)	465.2152	1.0732	-0.1187	0.8923	1407.964	0.8719	0.2167	0.8374	500.9969	1.0769	-0.1517	0.894
A-6(14)	1043.358	0.5677	0.4795	0.8861	1181.335	0.5358	0.7858	0.9085	773.0162	0.5727	0.5823	0.8723
A-6(14)	231.8754	1.1503	-0.3987	0.7608	801.6724	0.9719	-0.0753	0.7638	297.4363	1.1602	-0.5056	0.7719
A-6(14)	3311.313	0.0613	0.6957	0.9376	2164.187	0.0787	0.8791	0.9308	2143.296	0.0601	0.8578	0.9282
A-6(15)	1091.6	0.3735	0.0446	0.8927	1506.885	0.2968	0.1919	0.8385	1061.884	0.3715	0.0578	0.8947
A-6(16)	838.991	0.5493	0.0501	0.9415	1296.858	0.4649	0.2658	0.9119	813.056	0.5473	0.0648	0.9427

Table D-2. Values of Resilient Modulus Obtained from Multiple Regression Analysis of Laboratory, Soaked Compacted Soil Specimens Using Models 4, 5, and 6.

Sample	Uzan's Model				UKTC Model				NCHRP Model			
	K_1	K_2	K_2	R^2	K_1	K_2	K_2	$**R^2$	K_1	K_2	K_2	R^2
A-6(17)	1194.759	0.6143	0.3809	0.6186	1478.968	0.5976	0.6652	0.6393	941.0535	0.6282	0.4476	0.6009
A-6(19)	760.062	1.3084	-0.563	0.72	3965.154	1.0541	-0.2705	0.6648	1081.495	1.3199	-0.7107	0.7322
A-6(19)	688.2829	0.9485	-0.4927	0.7948	2289.755	0.7616	-0.2722	0.7315	937.0157	0.9529	-0.6132	0.7976
A-6(20)	748.5964	0.7801	-0.1798	0.8218	1653.914	0.6707	0.0443	0.8203	838.4039	0.7835	-0.2269	0.8259
A-6(6)	97.0183	1.5984	-0.7575	0.7428	708.3937	1.2489	-0.3437	0.6607	156.0537	1.6044	-0.9421	0.7445
A-6(6)	1014.144	0.7746	0.1503	0.4814	1723.479	0.7267	0.429	0.5286	923.25	0.7896	0.1618	0.4741
A-6(8)	2139.655	0.5007	-0.0338	0.3261	3013.628	0.5121	0.1251	0.4437	2184.189	0.5115	-0.0579	0.3293
A-6(9)	191.579	1.3556	-0.4422	0.895	900.6349	1.0923	-0.0631	0.84	252.9774	1.3612	-0.5535	0.8999
A-6(9)	663.6136	0.6019	0.0966	0.9252	1045.656	0.513	0.3355	0.9323	624.5304	0.6023	0.1184	0.9242
A-7-5(13)	6455.579	0.0241	0.1049	0.4275	6065.062	0.0288	0.1403	0.4455	6047.499	0.0215	0.1329	0.4391
A-7-5(33)	1542.099	0.5339	-0.6512	0.9074	3890.916	0.4266	-0.6176	0.88	2314.155	0.5384	-0.8074	0.9045
A-7-5(36)	0.1594	3.2052	-0.7747	0.9796	82.253	1.375	0	0.98	0.043	3.8839	-1.151	0.9823
A-7-5(52)	16262.22	-0.3528	-0.5007	0.9972	3855.284	0.6667	-1	0.8602	8792.901	0.0031	-0.7296	0.9985
A-7-6(10)	979.2628	0.541	-0.2458	0.6011	1836.836	0.4488	-0.1029	0.5858	1143.215	0.541	-0.303	0.5968
A-7-6(14)	1592.085	0.4599	0.4939	0.8515	1570.58	0.4658	0.7628	0.8774	1168.294	0.4646	0.6008	0.8372
A-7-6(15)	2193.163	0.2841	0.528	0.9046	1897.894	0.2798	0.7503	0.9076	1575.456	0.2857	0.6475	0.8926
A-7-6(16)	846.0682	0.7185	0.3158	0.8049	1172.742	0.6877	0.6319	0.8554	694.2277	0.7258	0.3775	0.7939
A-7-6(17)	309.4511	1.1762	-0.3015	0.502	1008.985	1.048	0.0159	0.5409	374.3533	1.1968	-0.4035	0.5171
A-7-6(21)	10648.77	0.0872	-0.5344	0.938	16302.93	0.0736	-0.6348	0.9466	14855.12	0.0944	-0.6679	0.9481
A-7-6(22)	741.4443	0.6307	0.364	0.921	946.7168	0.5836	0.6711	0.9467	590.1637	0.6331	0.4445	0.9135

Table D-3. Values of Resilient Modulus Obtained from Multiple Regression Analysis of Laboratory, Soaked Compacted Soil Specimens Using Models 4, 5, and 6.

Sample	Uzan's Model				UKTC Model				NCHRP Model			
	K_1	K_2	K_2	R^2	K_1	K_2	K_2	$**R^2$	K_1	K_2	K_2	R^2
A-7-6(22)	1554.486	0.4469	0.3106	0.8161	1776.855	0.4316	0.529	0.8446	1279.852	0.4537	0.3718	0.7985
A-7-6(22)	5137.902	0.0884	-0.3626	0.7028	5117.391	0.3348	-0.4605	0.8726	6397.1	0.0934	-0.4501	0.7067
A-7-6(23)	1918.886	0.5766	-0.7187	0.9339	5268.495	0.4601	-0.6882	0.9073	2998.598	0.582	-0.8909	0.9302
A-7-6(23)	1747.253	0.8153	-0.9356	0.9513	6617.017	0.6805	-0.8749	0.9344	3132.855	0.8219	-1.1606	0.9495
A-7-6(26)	551.3666	0.9353	-0.4213	0.8732	1692.734	0.7765	-0.1994	0.8512	718.1655	0.9397	-0.5256	0.8781
A-7-6(26)	962.5635	0.8864	-0.8615	0.868	3837.207	0.7148	-0.7556	0.8291	1646.653	0.8917	-1.0671	0.8638
A-7-6(27)	976.8177	0.6646	0.0068	0.5698	1694.258	0.5753	0.2447	0.5769	972.5291	0.6682	0.0029	0.5697
A-7-6(30)	9.044	1.9492	-0.7153	0.8253	74.9109	1.875	-0.4	0.7062	3.6002	2.4642	-1.0264	0.8093
A-7-6(31)	4782.411	0.2944	-0.2983	0.3641	6399.659	0.3475	-0.2645	0.4809	5777.926	0.301	-0.3785	0.3808
A-7-6(35)	2046.119	0.1646	-0.3189	0.9476	2892.568	0.1408	-0.3372	0.9404	2496.887	0.1659	-0.3942	0.9392
A-7-6(36)	2228.09	0.3664	-0.0613	0.2981	3060.397	0.3497	0.0425	0.3749	2315.081	0.3718	-0.0837	0.303
A-7-6(40)	1206.646	0.5843	0.1551	0.5674	1619.544	0.5816	0.3885	0.6511	1094.442	0.5919	0.1795	0.5604
A-7-6(41)	1858.639	0.5339	-0.3618	0.3205	3549.99	0.5271	-0.294	0.4203	2330.178	0.5554	-0.4781	0.3564
A-7-6(41)	19106.81	-0.2575	-0.3233	0.634	17026.05	-0.1478	-0.4975	0.6182	23376.82	-0.2482	-0.4118	0.6496
A-7-6(44)	457.0076	0.8732	-0.5301	0.7942	1275.763	0.9149	-0.4916	0.7952	647.0347	0.8717	-0.6587	0.7975
A-7-6(44)	1207.008	0.5399	0.1315	0.6416	1645.007	0.5194	0.3419	0.7011	1111.649	0.5495	0.1471	0.6317
A-7-6(45)	3792.573	0.4295	-0.1546	0.9638	6162.267	0.3603	-0.0439	0.9484	4176.797	0.4319	-0.1938	0.9707
A-7-6(45)	45963.37	-0.8081	-0.192	0.994	1250.877	0.75	-0.4	0.9825	30742.46	-0.6151	-0.2854	0.9928
A-7-6(45)	262.828	1.1551	-0.8064	0.999	2468.584	0.625	-0.6	0.9651	7.8987	2.634	-1.3701	0.9979
A-7-6(46)	4484.131	0.0155	-0.3538	0.8456	5666.347	0.0116	-0.4281	0.8266	5592.602	0.0141	-0.433	0.8266

Shaded area: $**R^2$$0.87 in 35 percent of tests