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Advanced Development and Calibration of the Network Robustness Index to Identify Critical Road Network Links

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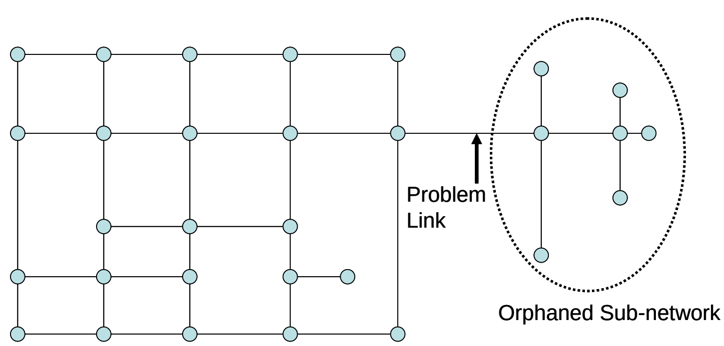
1. Introduction

In this research project, transportation flexibility and reliability concepts are extended and applied to a new method for identifying the most critical links in a road network. Current transportation management practices typically utilize localized level-of-service (LOS) measures such as the volume-to-capacity ratio (V/C) (Bremmer et al. 2004, Dheenadayalu et al. 2004). The solution to congestion planning problems is often to simply add more capacity along existing highway segments. This is a purely localized solution. The localized V/C approach does not necessarily allow planners to identify the most critical highway segments or corridors in terms of maximizing system-wide travel-time benefits, or to assess the robustness of a network. For system-wide objectives, there is a need for coincident consideration of the spatial distribution of traffic demand, the network topology and the network capacity.

While implementing localized solutions may result in localized benefits, these solutions may have limited, negligible or even adverse system-wide effects. Recent studies of induced travel in North America offer strong evidence that this is indeed the case (see Scott 2002). An equal level of investment in another location or locations may provide greater benefits to the overall network. We argue that the localized V/C-based planning approach is inadequate and propose a comprehensive system-wide approach to identifying critical infrastructure and evaluating network performance – the *Network Robustness Index* (NRI) first introduced in Scott et al. (2006) and refined in Sullivan et al. (2010). At the same time, the management approach must complement existing local planning efforts. With the increased focus on ensuring that transportation infrastructure is robust from both a security standpoint as well as for evacuation needs, reliance on localized congestion measures to direct policy is clearly outdated.

Calculation of the original NRI required the sequential removal of individual network links and the iterative application of a user equilibrium traffic assignment model (Scott et al. 2006). One potential problem avoided during the initial application of the NRI to the hypothetical networks used in Scott et al. (2006) was the creation of isolated nodes or multiple isolated sub-networks (see Figure 1).

Figure 1. Illustration of an Isolated Sub-Network



Clearly in a real-world application, the NRI must account for orphaned trips that either start or end within the now isolated sub-network. To incorporate the importance of non-redundant links (links where no alternative routes exist) to the overall system, we reduce the capacity of each link by different percentages such as 50%, 75%, 90%, and 99%. This approach effectively keeps the individual nodes from being isolated from the rest of the network, but restricts throughput on the link so that the majority of traffic using the link must reroute or queue. If links are critical for the network as a whole, system-wide travel time will be substantially impacted in a negative way by the capacity reduction approach. We calculate the system-wide value of a problem link without physically disconnecting the link from the network using this approach.

Most highway capacity research tends to focus on methodologies for estimating or evaluating segment capacities and/or examining very specific capacity improvement projects and solutions. Much of the spatial planning literature addresses the need for increased coordination/cooperation between spatial/geographic planners and transportation planners, and discusses potential benefits if a broader spatial planning context is employed. Overlap is limited between these two literatures. We focus on the linkages between spatial planning and highway capacity needs and develop a new metric for capturing those linkages. The NRI builds on the spatial aspects of existing literature by not only considering spatial location, but also considering network topology.

Recent work recognizes the need to move toward a system-wide approach that incorporates travel demand and associated costs in metrics of network performance. For example, Sullivan et al. (2009) provides a comprehensive review of existing transportation network disruption literature and organizes papers according to methodological approach and research objective. Sullivan et al. (2010) advances the development of a system-wide methodology for evaluating network robustness and identifying and ranking the most critical links in a transportation network. Scott et al. (2006) introduce a new measure for identifying the most critical links in a transportation network and evaluating network performance, and test the measure against the V/C – the NRI. Jenelius et al. (2006) derive link importance and site exposure indices based on increased travel costs when specific links are closed. The authors use unweighted increases in travel costs to convey an “equal opportunity” perspective on importance and exposure, and increases in travel costs weighted with travel demand to capture a “social efficiency” perspective on importance and exposure. The measures are calculated for the road network of northern Sweden. Wilson (2006) investigates the impact of a transportation disruption on supply chain management using system dynamics simulation, comparing a vendor-managed inventory system and a traditional supply chain for a five-echelon supply chain. Clark and Watling (2005) propose a technique for estimating the probability distribution for total travel time on a road network assuming daily variations in the travel demand matrix. Morlok and Chang (2004) describe techniques used to measure the flexibility of a transportation system to accommodate changes in traffic. The authors employ a system-wide approach for measuring flexibility based first on a traditional capacity modeling approach with fixed spatial patterns of traffic and then based on a dynamic approach where spatial patterns and cargo can vary. Chen et al. (2002) also incorporate variability in their reliability evaluation framework, which combines reliability and uncertainty analysis, network equilibrium models, and sensitivity analysis to perform a probabilistic assessment of capacity and travel-time reliability.

Iida (1999) outlines basic concepts, remaining problems, and future directions of road network reliability analysis. The author addresses the need to first analyze the reliability of the links that comprise the network prior to analyzing the reliability of the network itself. Taylor (1999) outlines the use of dense network modeling and network reliability in the planning and design of traffic management. The author applies reliability indices for the study of different trip movements within a network. These studies point to the need for alternative methods for evaluating networks. In meeting this challenge, Bell (2000) uses a game theory approach, not common to the transportation community, to evaluate the performance reliability of a transportation network.

The NRI is implemented as an algorithm, an approach that others have used for difficult problems involving various infrastructure networks. For example, Ball and Golden (1989) build on previous work and show that both the most vital arcs problem (MVAP) and the most important arcs problem (MIAP) cannot be solved with numerical programming techniques (i.e., they are NP-hard). The authors provide a solvable approximation for MVAP. These problems involve identifying arcs whose removal from the network result in the greatest increase in the length of the shortest path between two specified nodes. Corley and Sha (1982) do not examine network reliability per se, but also propose an algorithm for identifying the most vital links or nodes in a network whose removal results in the greatest increase in the shortest distance between two specific nodes. Ball (1979) presents an algorithm to compute reliability measures on a stochastic communications network. The reliability concepts discussed are not transportation related, but have a wide application pertaining to network performance in general.

In this research project we test a reduced link capacity approach for addressing the problem of isolated sub-networks so that these links can be included in the Network Robustness Index (NRI) for real-world scenarios. This research expands the methodological basis for the use of a specific capacity-disruption in lieu of link-removal in determining network robustness described in Sullivan et al., 2010.

2. Research Methodology

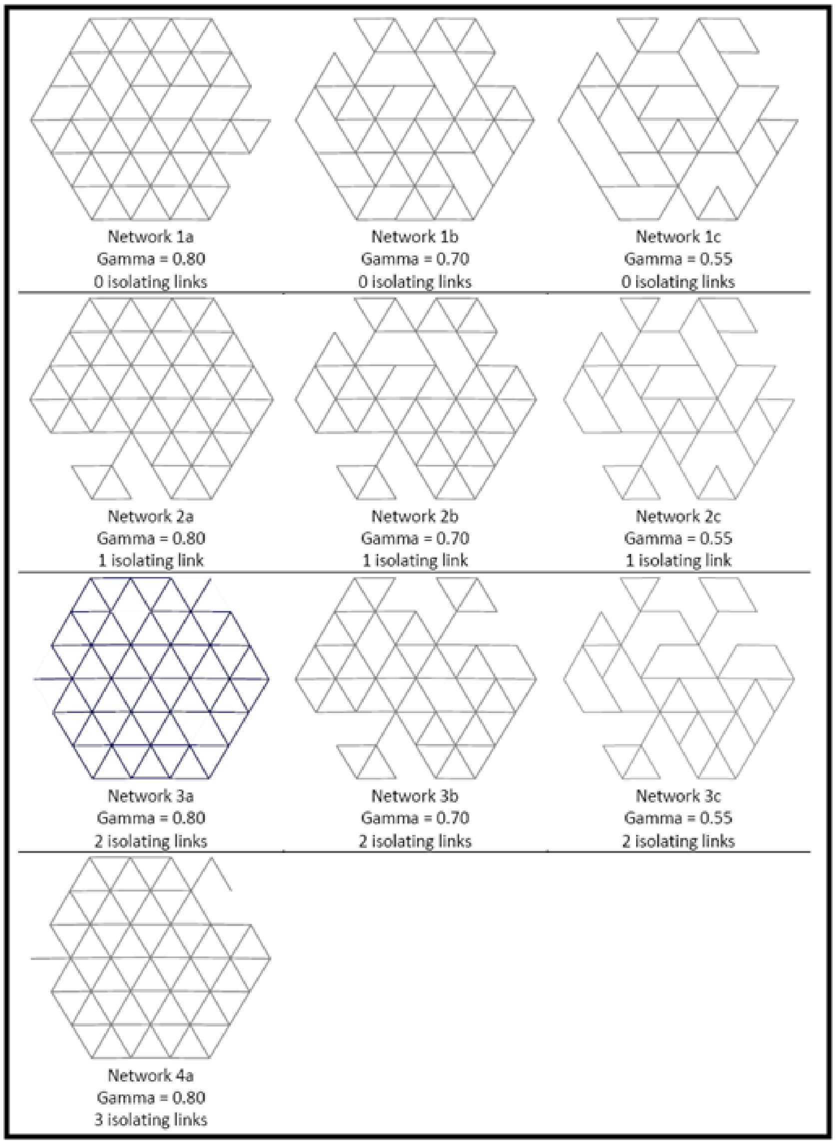
A sub-set of studies on network disruption have acknowledged the inherent problem encountered when modeling the effects of link-removal on a network with one or more isolated sub-networks. Studies that quantify only the instantaneous impact of link or node removal do not have a problem with isolating links, but usually acknowledge that the removal of an isolating link creates a more serious threat than the removal of a non-isolating link. The problem occurs when re-routing is included in the model. Since the trips that had previously used the isolating link can no longer be completed, their travel time cannot be quantified. Consequently, the effects of these trips are not included when, in fact, these lost or delayed trips may be the most significant contributors to the adverse effect of a link-removal. Therefore, it is imperative that these trips be included in the models.

A potential solution to the problem caused by isolated sub-networks is to use a link capacity-disruption lower than 100% to calculate the NRI, which essentially allows the capacity to be reduced on each isolating link instead of removing the link altogether. We present a methodological basis for the use of a specific capacity-disruption for in determining network robustness. We utilize various capacity-disruption percentages in lieu of complete link-removal in assessing transportation network robustness. A procedure that utilizes capacity-disruption will be immune to the effects of poor connectivity and isolating links in real-world transportation networks. A range of optimal capacity-disruption levels was found by analyzing the NRI values for each of three hypothetical networks with varying levels of connectivity and capacity loss between 30% and 100% (Sullivan et al., 2010). For the networks with isolating links, evenly-spaced link disruption levels between 30% and 99% (30%, 40%, 50%, 60%, 70%, 75%, 80%, 85%, 90%, 95%, and 99%) were modeled. As the individual link-capacities are reduced, a portion of the traffic normally assigned to the disrupted link re-routes in the same way that it had when the link was completely removed. These higher travel times contribute to larger NRI values in the same way that they did when complete link-removals were used.

The project team initially used a set of hypothetical transportation networks and origin-destination (OD) trip matrix. These networks were first used by Scott et al., 2006. The three hypothetical networks each have a different level of connectivity as measured by the gamma index. The gamma index is a reasonably simple connectivity index that measures the actual number of links divided by the maximum possible number of links for the network. The index values range from 0 (a completely disconnected network) to 1 (a completely connected network). The OD matrix was generated by assuming that the central node in the network is a Level 1 population center (550,000 – 600,000 people), the 6 surrounding nodes are Level 2 population centers (200,000 – 300,000 people), and the remaining 30 nodes are Level 3 population centers (50,000 – 200,000 people)). Each person is assumed to generate 0.6 trips per day, and a production-constrained gravity model is used to derive the trip matrix. The total number of trips is 3,439,490.

For this research project we develop an additional seven hypothetical networks that contain one, two, or three isolating links at connectivity levels paralleling the original three networks in Scott et al., 2006. We examine the measures of overall network robustness when varying levels of link capacity-disruption are used in the calculation of the NRI. The additional seven networks are shown in Figure 2 along with the original three networks. The original networks are Networks 1a, 1b, and 1c.

Figure 2. Hypothetical Test Networks Used in the Study



The networks in Group 2 have one isolating link and those in Group 3 have two isolating links. The single network in Group 4 has three isolating links (two in succession). Additional networks with different numbers of isolating links were created because it was hypothesized that the number of isolating links will have an effect on the distribution of NRI values. To check that the level of connectivity of the network was not the sole predictor of robustness, varying levels of connectivity were used and held constant across all groups. To maintain the integrity of the central-place aspect of the original three networks the isolating links were located on the outer edges of the networks, as they would appear in real-world transportation networks. Decreasing levels of connectivity were created by randomly dropping links until the desired gamma index (a measure of the actual number of links in the network over the maximum possible number of links) was reached.

3. Results

The primary conclusion from this investigation is that the use of complete link-removal to model network robustness is not only infeasible for networks with isolating links, but also does not yield unique results due to the influence of Braess' Paradox. *Braess' Paradox* states that adding a small capacity link to a transportation network may actually decrease the performance of the network instead of improving it. This is due to large amounts of congestion on the small capacity link. It appears that the Braess' Paradox phenomenon is more pronounced on networks with less connectivity. This seems reasonable as the choice of alternative routes is limited on networks with low connectivity and traffic is more likely to queue on disrupted links.

Our analysis suggests an upper limit of 99% on disruption levels to be used in robustness analysis. However, this upper limit may fall as low as 95%, depending on network connectivity. A link rank-order analysis of the NRI values indicates that the most stable range for capacity loss will vary with the level of connectivity of the network, but is likely to fall between 75% and 99%. This analysis yielded 8,784 separate NRI values – one for each link, on each network, under each of 12 capacity-disruption levels between 30% and 99%. A summary of the NRI values yielded by this analysis can be found in Table 1.

Table 1. Summary of NRI Values

Network	Maximum NRI		Minimum NRI		Average NRI across all disruption levels (minutes)
	Value (minutes)	Disruption Level	Value (minutes)	Disruption Level	
1a	12.3 million	100%	-121,360	50%	1.3 million
1b	124.5 million	99%	-156,000	70%	5.8 million
1c	3.3 billion	99%	-6.0 million	80%	159.4 million
2a	1.1E+15	99%	-656,210	75%	1.1E+12
2b	1.1E+15	99%	-1.1 million	75%	1.2E+12
2c	1.1E+15	99%	-10.1 million	75%	1.6E+12
3a	9.5E+14	99%	-149,326	50%	1.1E+12
3b	1.1E+15	99%	-193,297	30%	1.9E+12
3c	1.1E+15	99%	-27.3 million	90%	2.4E+12
4a	2.1E+15	99%	-390,816	50%	3.1E+12

Notes:
1. Minima do not include the NRI values returned for 100% disruption of isolating links. For Networks 2a → 4a, the NRI value for the isolating links at the 100% capacity disruption scenario were all lower than the minimum NRI shown.
2. All of the maxima for Networks 2a → 4a occurred for an isolating link

The table includes the maximum and minimum values of the link NRIs for each network, across all of the disruption levels analyzed. Next to each maximum or minimum NRI value, the disruption level where the value occurred is also given. The average NRI value across all of the disruption levels is also provided for comparison purposes, since the average NRI was used to evaluate the results presented in Scott et al., (2006) and Sullivan et al., (2010).

Our results indicate that the 100% capacity-disruption does not necessarily represent the worst-case network capacity-disruption scenario. Higher travel time values occur at the 99% disruption level when isolating links are present (see Note 2 in Table 4). For modeling purposes, a capacity-disruption other than 100% may better serve the derivation of robustness indices for networks. It is important to identify a disruption level that results in the most stable rank order. The results suggest that a range of disruption levels below 100% will be most appropriate.

3.1. Stability of NRI Rank-Orders

The rank-order of the links in each network is the primary tool to be used for intra-network comparisons. It is critical to have an index that results in stable and consistent rank orders. With 12 separate disruption-levels on 10 different networks, we obtained a total of 120 different rank orders. For each network, the change in the rank of each NRI value for each link was recorded as the difference between the rank at the X% disruption level and the rank at the Y% disruption level where X and Y are consecutive capacity disruption levels on link *a*.

$$\Delta R_a(X \rightarrow Y) = |\mathrm{Rank}(NRI_a^{X\%}) - \mathrm{Rank}(NRI_a^{Y\%})|$$

The average change in rank across all links is determined for the $X \rightarrow Y$ transition Where I is the set of links in the network and N is the number of links in the network.

$$\Delta R_{ave}(X \rightarrow Y) = (\textstyle\sum_{a \in I} \Delta R_a) / N$$

These results were evaluated to find a consistent and stable rank-order as it is hypothesized that this will point to an appropriate capacity-disruption level for intra-network robustness analyses. A stable rank-ordering is expected to result in a more useful rank-order. The location of the minimum value for the average change in rank is assumed to correspond with the range of disruption where the rank-order is most stable.

$$S_n = \mathrm{Min}(\Delta R_{ave}(X \rightarrow Y), \text{ over all intervals})$$

A summary of all 10 networks is provided in Table 2. The values provided in the table are the ΔR_{ave} values. The $X \rightarrow Y$ intervals are given at the left, and the minimum value is shaded for each network. As shown in the table, the rank-order stability is achieved between 99% and 70%, depending on the network. This finding indicates that the use of capacity-disruption levels lower than 70% does not result in a stable rank-order for the links. In addition, the rank-order provided by the 100% capacity-disruption level is not appropriate, providing further evidence for the rejection of the link- or node-removal approach to assessing network robustness. The number of isolating links in a network does not seem to have a significant effect on the specific capacity-disruption level where stability is achieved.

Table 2. Rank-Order Analysis Summary

Capacity Disruption Interval	Network 1a Average ΔRank	Network 1b Average ΔRank	Network 1c Average ΔRank	Network 2a Average ΔRank	Network 2b Average ΔRank	Network 2c Average ΔRank	Network 3a Average ΔRank	Network 3b Average ΔRank	Network 3c Average ΔRank	Network 4a Average ΔRank
100% - 99%	4.43	2.54	2.55	7.81	4.35	3.76	6.81	6.08	7.76	9.40
99% - 95%	3.55	2.57	1.93	5.98	3.00	2.86	5.29	3.11	7.24	5.12
95% - 90%	4.45	2.41	1.59	5.48	2.65	2.03	5.78	4.24	3.83	6.55
90% - 85%	5.52	3.05	2.07	7.36	3.16	2.28	5.33	4.00	2.97	7.12
85% - 80%	5.60	3.11	1.48	7.07	3.92	1.76	5.62	3.95	5.00	8.29
80% - 75%	5.38	3.73	1.28	8.07	4.86	2.38	6.29	3.16	4.45	9.76
75% - 70%	6.36	4.59	1.79	9.29	4.11	1.59	6.36	4.16	4.72	8.55
70% - 60%	6.45	5.32	2.48	10.36	5.65	2.69	6.29	5.46	6.41	12.60
60% - 50%	8.31	5.73	2.62	9.95	7.30	2.48	7.02	5.38	7.03	12.83
50% - 40%	8.31	7.38	3.41	10.07	8.19	3.38	6.36	5.92	5.97	13.43
40% - 30%	9.33	11.46	4.72	12.17	12.54	4.17	7.93	8.11	5.93	14.98

As the connectivity decreases for a given network (from a to b to c), the capacity-disruption level where the rank order stability is maximized decreases as well. In addition, the overall stability of the rank orders improves (as indicated by a decreasing S_n). This finding is significant because the low-connectivity networks modeled in this study may be the best approximations of real-world transportation networks. Real-world networks tend to have lower gamma indices (Vinod et al., 2003). Based on rank-order stability, analysis of robustness by metropolitan planning organizations may be optimal with capacity-disruption levels between 70% and 90%. Also significant is the fact that the average change in rank-order for all disruption levels for our least-connected networks (1c, 2c, and 3c) have tended to be the lowest overall.

4. Conclusions

Previously researchers have tended to use complete link removal to measure network robustness (100% capacity reduction). The use of a capacity reduction value less than 100% allows networks with isolating links to be evaluated using the NRI methodology. Our research has shows that capacity-disruption levels lower than about 50% are not likely to result in stable, accurate NRI values and should not be considered for analyses of this type. A more realistic lower limit of about 70% is recommended. Our analysis indicates that, due to the presence of Braess' Paradox, a realistic upper limit of 99% on the disruption levels should be used. However, it should be acknowledged that, as the level of connectivity of the network increases, the upper limit on the range of desirable disruption levels decreases, and may fall as low as 95%. Therefore, a practical capacity-disruption range for robustness analysis is between 70% and 99%, but a more realistic range of between 70% and 95% is recommended.

Within this practical range of disruption levels, the stability of the rank-orders of the NRI values is important for the determination of the most critical individual link(s) in the network. Those links whose disruption results in the highest NRI values are the ones that are most critical in improving the robustness of the overall network. Therefore, it is important to find a capacity disruption level where the rank ordering is stable. Our analysis indicates that the most stable range for the rank-orders is network and demand dependent and varies with the level of connectivity of the network, but is likely to fall between 75% and 99%. Based on these three lines of evidence, the recommended range of capacity disruption levels for robustness analysis is between 75% and 95%.

A significant finding is that the use of complete link-removal, or 100% capacity-disruption, to model network robustness, is not only infeasible for networks with isolating links, but also does not yield unique results due to the influence of Braess' paradox in the successive traffic assignments involved with this computational procedure. Therefore, an alternative capacity disruption level should be sought for determining network robustness when this sequential link-disruption procedure is being used. This capacity-disruption level is likely to fall between 75% and 99%, and it depends on the level of network connectivity and the presence of isolating links. Further research should be conducted to narrow the range of appropriate capacity-disruption levels to an optimum.

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