To: FHWA Virginia Division
P.O. Box 10045
Richmond, Virginia 23240-0045

Submittal Date: October 1982

HPR STUDY PROGRESS REPORT

Title: "Performance Related Specifications for Hydraulic Cement Concrete Used in Construction and Rehabilitation of Ground Transportation Facilities", VHTRC 83-R17

HPR No: 1894 Principal Investigator: W. J. Halstead

INTRODUCTION

This report summarizes the work performed on the subject study from June through September 1982. In accordance with the revised work plan for Task 4 of the project approved May 24, 1982, the new completion date for the project is April 30, 1983.

TECHNICAL ACTIVITIES

1. An analysis was made of selected historical data from concrete tests now stored in the computer. The details of this analysis are shown as Attachment 1 to this report.

2. Guidelines for simulation of the proposed specification were prepared and distributed to all districts. A copy is provided as Attachment 2 to this report. Some data were collected and analyzed.

3. The statistical significance of the proposed requirements was analyzed. The results are included as Attachment 3 to this report.

4. The Data Processing Division was consulted concerning the capabilities of the present Virginia Department of Highways and Transportation system.

FINDINGS

1. Findings from the study of historical data and early results of simulated projects show that revisions are required in the proposed specification before final adoption. Accordingly, the

revision of the Materials Division Manual of Testing cannot be updated until final decisions are made on the revised specification. This initial progress report, therefore, covers progress in all areas rather than those outlined under procedural items 1 and 2 of the work plan.

2. The analysis of information on the 1981 production of 12 selected ready-mix concrete producers has shown that a typical standard deviation associated with a single producer's product could not be established. A single producer's product may vary significantly from project to project, and time elapsed from the beginning to the end of the placement of a lot as now defined casts doubt that all material of the same type on a contract should be considered a single lot. All presently recorded strengths are at 14 days, and thus are not useful in deciding the standard deviation of strength tests at 28 days. Strength levels at 14 days all indicate strengths will be adequate at 28 days. From this study it has been concluded that further analysis of historical data would not be useful for this project.

3. The study of the statistical significance of the proposed specification revealed the following:

   a. The present criterion that 90% of values exceed f'_c may not be adequate for concrete lots with large deviations. Because of this possibility, a second criterion that single low tests not fall below a stated value more than 1 time in 100 is suggested. This would have the effect of requiring a higher average of strength test results than required under criterion 1.

   b. The producer's correction complicates the levels of acceptable averages for strength and introduces very large risks of accepting unsuitable materials. The examination of historical records and research data on new projects provided to date indicate that under present practice this much concession to the producer is not needed, and that it could result in the state accepting lower strengths than generally supplied under the present specifications.

      If a correction is made, a standard deviation indicating reasonably good control (e.g. 550 psi) should be assumed.

   c. The procedure for computing pay factors needs to be modified to be consistent with the adoption of a second criterion.

d. The use of the sample standard deviations for computing the required averages may be questionable. When the number of tests involved is small it may be more realistic to assume a value for the standard deviation based on past experience.

e. The use of sample standard deviations for determining the compliance of air contents is also questionable for small numbers of sampling units. Computations probably should be made on the basis of assuming a value of 0.7 as the standard deviation for air determinations.

4. Consultation with the Data Processing Division of the Department indicates that only minor changes, if any, are needed to provide needed capabilities for determining compliance to new specifications and calculations of reduced pay factors if appropriate.

BUDGET SUMMARY

The 1983 allocated funds for this project are $32,000. As of September 30, 1982, $4,156.58, have been spent, leaving a balance of $27,843.42.

No revision of the amount of funding is anticipated. However, extension of the completion date of the project to June 30, 1983, may be required because of limitations on the amount of time the principal investigator is available.

cc: Mr. Leo E. Busser III
Concrete Research Advisory Committee
Mr. H. H. Newlon, Jr.
Mr. H. E. Brown
Mr. W. J. Halstead
Miss E. L. Knight

1304

Attachment 1

EXAMINATION OF HISTORICAL QUALITY ASSURANCE DATA FOR PORTLAND CEMENT CONCRETE

The Virginia Department of Highways and Transportation maintains computerized records of quality assurance test data for portland cement concretes. These records include the source and amounts of each of the ingredients in the concrete, the ready-mix producer, the project and location on which the concrete was placed, and the results of tests for slump, air content, and the compressive strength of test cylinders. The strength tests are made after 14 days, with the concrete being considered acceptable if the strengths of the test cylinders equal or exceed 0.85 times the 28-day strength requirement.

Even though these data are not suitable for simulating acceptance under the suggested specification, the records for selected producers were examined to determine if a "typical" standard deviation could be estimated for each producer and the extent to which all concrete supplied under a single contract could be treated as a single lot. The selected data are given in Tables 1 and 2 on pages A-1-2 and A-1-4, respectively.

A case-by-case discussion follows.

A-3 Concrete — Table 1

Case 1 (Producer 110)

This producer supplied material on a single contract. Installation began on February 9, 1981, and ended December 31, 1981. During this period 19 samples were taken. The average 14-day strength was 4,853 psi with a standard deviation of 597 psi.

This average was substantially higher than the minimum allowable under the proposed specification, which would have been approximately 3,222 psi assuming that the standard deviation at 28 days would be the same as the 14-day value.

Case 2 (Producer 501)

This producer supplied a product that was sampled 68 times during the year. Under the proposed specification, the 11 projects involved would be classed as lots. The number of samples taken for each lot varied from 3 to 18. Standard deviations for the lots varied from 293 psi to 669 psi. The pooled standard deviation was 415 psi and the weighted average was 3,951 psi. Treating all concrete as a single lot indicates an average of 3,897 psi and a standard deviation of 440 psi. All strength values for this producer's concrete were well above the minimum required by the proposed specification.

1306

TABLE 1

Characteristics of A-3 Concrete Furnished to VDHT in 1981 by Selected Producers

Job No.	No. Samples	Time Period Covered	Range of Slump inches	Air Content Average %	Air Content Std. Dev. %	14-Day Strength Average psi	14-Day Strength Std. Dev. psi	Minimum Allowable 14-day strength[1] psi
Producer 110								
1	20	2/09 - 12/23	1.75 - 3.75	6.2	.82	4,892	607	3,242
All 1981[4]	20		3.1 av., .68 σ	6.4	.93	4,853	597	3,222
Producer 501								
1	3	1/15 - 11/16	3.0 - 3.25	6.1	0.36	3,882	645	3,317
2	7	5/14 - 8/27	2.75- 4.0	6.2	0.94	3,756	316	2,894
3	7	7/24 - 10/27	3.0 - 3.75	6.2	1.00	4,115	531	3,128
4	3	9/23 - 10/30	3.25-3.50	6.8	0.40	3,575	313	2,891
5	18	2/24 - 10/16	2.50-4.0	6.5	0.68	3,842	293	2,869
6	5	3/04 - 6/16	3.0 - 3.5	6.5	0.72	3,928	527	3,123
7	6	7/21 - 10/20	2.5 - 3.5	5.6	0.91	3,725	344	2,924
8	5	1/22 - 7/23	1.5 - 3.75	5.8	0.45	4,640	660	3,365
9	5	3/25 - 5/22	2.3 - 4.25	6.1	0.68	4,085	485	3,078
10	5	4/02 - 12/02	3.0 - 3.5	5.9	0.74	4,138	553	3,152
11	4	5/01 - 8/04	3.0 - 3.5	6.2	0.28	3,784	320	2,898
Total	64	1/15 - 12/02	1.5 - 4.25	6.2[2]	0.69[3]	3,951[2]	415[3]	3,002
All 1981[4]	68		3.3 av., .62 σ	6.1	0.77	3,897	440	3,028
Producer 512								
1	1	6/19	4.0	6.2	--	2,880	--	
2	2	2/27 - 5/07	2.5 - 3.5	5.75	--	4,155	--	
3	1	5/11	2.25	6.0	--	3,995	--	
4	1	5/18	2.50	6.1	--	4,060	--	
5	2	6/05 - 7/13	3.0 - 3.75	6.5	--	2,770	--	
6	1	6/08	3.5	5.9	--	2,995	--	
7	1	6/17	3.25	6.1	--	3,115	--	
8	2	8/28 - 9/09	3.5 - 4.0	6.9	--	2,623	--	
9	1	9/29	2.5	6.2	--	3,965	--	
10	1	5/29	3.0	6.5	--	4,265	--	
Total	13			6.1[2]	0.99[3]	3,409[2]	733[3]	
All 1981[4]	13					3,413		3,491
Producer 521								
1	3	7/14 - 8/26	3.5 - 3.5	6.2	0.50	3,621	517	3,112
2	5	2/25 - 3/17	3.25-3.5	6.6	0.08	5,003	686	3,399
3	1	5/26	3.25	6.4	---	3,275	---	
4	2	4/13 - 4/22	3.25-4.50	7.1	---	4,105	---	
5	2	5/08 - 5/22	2.75-3.25	6.0	---	3,530		
6	2	8/27 - 10/19	3.0 -3.5	6.0	---	3,777	---	
All 1981	16	--	3.4 av., .52 σ	6.1	0.56	4,187	867	3,757
Producer 702								
1	32	1/07 - 10/13	2.5 - 4.75	6.0	0.67	4,113	559	3,158
2	25	3/05 - 9/04	3.0 - 4.34	6.1	0.58	4,220	490	3,083
3	7	2/20 - 5/08	3.0 - 3.75	6.4	0.34	3,994	763	3,551
4	3	1/22 - 4/27	3.25 - 4.0	5.7	0.35	3,983	837	3,697
5	3	6/11 - 8/28	2.5 - 3.5	5.6	0.32	3,960	156	2,720
6	4	4/29 - 11/22	3.25- 4.5	6.1	0.52	3,696	397	2,982
7	6	3/23 - 10/22	2.5 - 4.5	5.5	0.65	4,315	363	2,945
Total			2.5	6.0[2]	0.57[3]	4,117[2]	490[2]	3,083
All 1981[4]	98		3.4 av., .44 σ	5.9	.74	4,197	627	3,282
Producer 803								
1	3	7/27 - 9/25	2.0 - 3.5	6.3	0.16	4,446	810	4,154
2	1	1/20	3.0	4.0	---	4,758		
3	1	6/12	4.0	5.0	---	3,324		
All 1981[4]	5			5.6	.39	4,501	968	

[1] Based on proposed specification —
When standard deviation is above 571 = [2,400 + 2.33 x std. dev.] x .85
When standard deviation is below 571 = [3,000 + 1.23 x std. dev.] x .85

[2] Weighted average

[3] Pooled standard deviation

[4] Total 1981 production treated as single population

Case 3 (Producer 512)

This producer's product was sampled 13 times. However, 10 jobs were involved with no job being sampled more than twice. The overall standard deviation for all samples for this producer was 733 psi and the average 14-day strength was 3,413 psi. None of the sampled concrete had 14-day strengths less than 2,400 psi (3,000 x 0.85). Because so few samples were available from a given job a pooled standard deviation could not be determined.

Case 4 (Producer 521)

This producer had his product tested for strength 16 times and 6 jobs were involved, only 2 of which were sampled more than twice. There was considerable variation in the average strengths from job to job, which results in a very high apparent standard deviation if all samples were treated as a single population. Strength results for each job were well above requirements.

Case 5 (Producer 702)

This producer's product was tested 98 times. Seven projects were involved, 2 of which were large — one involving 32 samples and the other 25 samples. For the large jobs the standard deviations were 559 psi and 490 psi, which are indicative of good control. However, small lots had standard deviations varying from 156 to 837 psi. Since only 3 samples were taken in each of these cases, the difference in the estimate of standard deviation is most likely a result of the small sample size rather than true differences in variability of the concrete production.

Case 6 (Producer 803)

This producer's product was sampled 5 times and 3 projects were involved. One sample representing a single lot had significantly lower strength than the other two, and this led to an apparently large standard deviation when all the results were treated as a single population.

A-4 Concrete — Table 2

Case 7 (Producer 123)

Only 4 samples of this producer's product were tested, and each was on a different project. Consequently, no computation of the standard deviation could be made on a project basis. Overall the 4 samples had a standard deviation of 816 psi, which is probably not an accurate estimate of the variability of this producer's product.

1008

TABLE 2

Characteristics of A-4 Concrete Furnished to VDHT in 1981 by Selected Producers

Job No.	No. Samples	Time Period Covered	Range of Slump inches	Air Content Average %	Air Content Std. Dev. %	14-Day Strength Average psi	14-Day Strength Std. Dev. psi	Minimum Allowable 14-day strength, psi
Producer 123								
1	1	5/12	4.0	4.1	---	5,720	---	
	1	6/03	3.5	5.2	---	4,150	---	
	1	6/23	4.0	5.6	---	3,335	---	
	1	5/01	3.75	4.2	---	4,625	---	
All 1981	4		Avg. 3.8	4.8	0.64	4,345	816	4,506
Producer 407								
1	5	3/26 - 4/28	2.87 - 3.75	6.5	0.60	3,962	287	3,712
2	4	8/28 - 10/29	3.25 - 3.5	5.7	0.87	3,655*	403	3,838
Avg. Jobs				6.1	0.68	3,826	320	3,673
		* One low value 2,310 not used in average. Coring indicated concrete was satisfactory						
All 1981 4/	9		3.1 avg., .39 σ	6.0	0.78	4,113	1,145	4,645
Producer 512								
1	3	6/18 - 8/10	3.5 - 3.75	7.0	..44	3,972	275	
2	2	5/16 - 7/16	3.0 - 3.0	6.2	---	5,100	---	
3	2	2/26 - 6/03	3.25 - 4.0	5.8	---	4,630	---	
4	3	1/22 - 4/02	2.5 - 3.25	6.3	.91	4,735	579	
5	3	7/02 - 9/09	0.5 - 3.75	6.4	0.80	4,382	912	
All 1981 4/	12		3.2, - 0.36 σ	6.4	0.34	4,685	533	
Producer 703								
1	8	3/13 - 10/21	2.5 - 3.5	6.5	0.95	4,236	441	3,880
2	7	4/08 - 11/19	2.0 - 4.0	6.4	0.43	4,189	543	3,991
3	3	5/21 - 5/21	3.0 - 3.5	6.3	1.03	4,592	88	3,496
4	5	9/03 - 11/171	3.0 - 3.75	6.1	0.76	4,340	92	3,500
5	3	9/15 - 9/15	2.75 - 3.0	6.1	0.43	4,188	407	3,842
6	24	5/07 - 12/30	2.5 - 4.0	6.5	0.76	4,138	450	3,890
Total	50	3/13 - 12/30	2.0 - 4.0	6.4	0.72	4,211	408	3,844
All 1981 4/	52		3.3 avg., .63 σ	6.4	0.83	4,215	487	3,930
Producer 803								
1	3	3/24 - 8/05	3.5 - 4.0	5.7	1.10	3,765	211	3,621
2	1	8/19	3.0	5.3	---	5,688	---	---
3	1	8/07	2.0	5.3	---	3,555	---	---
Producer 811								
1	2	6/04 - 6/16	3.25 - 3.50	6.7	---	3,765	---	
2	1	8/26	2.75	7.0	---	4,376	---	
All 1981 4/	3			6.8	.17	3,969	323	

Case 8 (Producer 407)

This producer's product was sampled 9 times. Two jobs were involved. On one job the strength value was 3,655 psi. This is above the minimum requirement of the present specification but would be below the minimum average strength required by the proposed specification.

Case 9 (Producer 512)

This producer's product was sampled 12 times during the year. Five small jobs were involved. The average was 4,685 psi and the standard deviation, counting all as one lot, was 533 psi. Standard deviations varied from 275 to 912 psi.

Case 10 (Producer 703)

A total of 52 samples were taken from this producer's product. Six jobs were represented, with the minimum standard deviation being 88 psi and the maximum 543 psi.

Case 11 (Producer 803)

This producer had only 6 samples tested during the year. Strengths were above minimum, but the variation could not be estimated.

Case 12 (Producer 811)

This producer had 3 samples tested. The standard deviation for the 3 was 323 psi.

Conclusions from Historical Data

The long time span from the beginning of a project to its completion, as indicated by the data for all producers now filed in the computer, indicates that treating all concrete furnished under a contract as a single population might be questionable from a statistical viewpoint. However, even when the total contract is considered a single lot, only a few producers had their products tested a sufficient number of times to provide enough data for good estimates of averages and standard deviations of the lot.

These historical strength data are 14-day strengths, thus they cannot be used to simulate direct application of the proposed specification. It is noted that, based on usual relationships, essentially all strength values would meet the proposed specification. Based on the 14-day strengths, A-3 concretes have strengths well above the minimum specification limits. All tested A-4 concrete also had adequate strength, but the excess over the minimum specification requirements was considerably smaller than for the A-3 concretes.

From this limited study, it was concluded that further evaluation of historical data for the purposes of this project would not provide significant information relating to the standard deviation for a given producer.

ATTACHMENT 2

GUIDELINES FOR SIMULATING STATISTICAL ACCEPTANCE TESTING FOR PORTLAND CEMENT CONCRETE

by

Woodrow J. Halstead
Research Scientist

INTRODUCTION

The proposed revisions of Section 219, Hydraulic Cement Concrete, of the Standard Specifications for Roads and Bridges of the Virginia Department of Highways and Transportation introduce the concept of acceptance on the basis of statistical probabilities, with reduced pay for concrete outside of normal specification limits but not considered to be sufficiently deficient to warrant removal. Under the proposed specification, reduced pay will result, when on a statistical basis, there is more than a 95% probability that more than 10% of the concrete placed in the job is below the minimum requirement for 28-day compressive strength or outside the minimum and maximum limits for the percentage of entrained air.

The system proposed provides for considering all the concrete of a single class made with the same ingredients in a contract as a single lot, except where very large amounts of concrete are involved. The number of samples per lot thus varies with the size of the job. In acceptance a correction is applied to minimize the producer's risk of having acceptable concrete subjected to a reduction in pay. This has the effect of statistically increasing the risk to the state that poorer than indicated concrete will be accepted, but on large lots requiring at least 10 samples the state's risk is reduced. The same risk of accepting poor material exists under present procedures. It is emphasized that under any specification, dependence is placed on good concreting practice and good inspection procedures to assure that inferior concrete is not placed.

Prior to implementation of the new concept, it is desirable to evaluate the proposed revision by its simulated application to projects constructed in the 1982 season. For this simulation several changes in sampling and testing procedures are required. These are explained in the following sections.

INSPECTION AND MONITORING TESTS

Under the revised specifications, there is no significant change in the responsibilities of inspectors with respect to acceptable plant equipment and mixing techniques. Obviously unsatisfactory conditions or improper equipment, materials, and techniques must immediately be called to the attention of the responsible person as in the past. Monitoring of aggregate moisture and water/cement ratios is conducted in the same manner as before. Ultimately, as concrete producers develop improved quality control procedures, it is hoped that the state inspector's involvement in aggregate moisture tests and other quality control tests that are the contractor's responsibility can be minimized.

SAMPLING AND TESTING FOR ACCEPTANCE

A significant change is made in the manner of selecting a truck to be sampled for making strength and other acceptance tests. Such samples will be designated as acceptance samples, and they must be selected by a predetermined system based on random numbers or some other suitable randomizing system. This systematic randomization is extremely important and <u>must</u> be adhered to in order to attain a proper evaluation of the proposed revision. A good procedure to use is described in ASTM D3665. This procedure is published in Part 15 of the ASTM Standards.

Frequency of Sampling

The proposed specification basically establishes the size of a sublot for bridge decks at 50 yd.^{3} and that for structural concrete other than that for bridge decks at 100 yd.^{3}. While some special considerations are included in the proposed specification for small jobs, these are not applicable if the jobs chosen for simulation exceed 150 yd.^{3} for bridge deck concrete or 300 yd.^{3} for other structural concrete.

Randomization Procedures

Randomization should be established to determine the portion of the sublot to be sampled by the use of ASTM D3665, or this can be done by a procedure similar to that used by the Department for other materials in which the percentage of the sublot is determined by drawing numbered discs or washers from a can. The first number drawn represents the first digit of the percentage and the second number drawn represents the second digit of the percentage.

For example, if a 6 and a 4 are drawn, take the acceptance sample from the truck that contains the cubic yard which represents the sixty-fourth percentile of the sublot; that is, if the sublot size is 50 $yd.^3$, take the sample from the truck containing the thirty-second cubic yard. If the sublot size is 100 $yd.^3$, take the sample from the truck containing the sixty-fourth cubic yard of the sublot.

At the end of a job where a full sublot will not be placed, compute the percentage of the sublot in the usual manner. If the percentage determined by the random drawings exceeds the amount of concrete to be placed, do not sample. If it is within the amount to be placed, treat as an additional sublot and make the usual tests.

Example: The random digits drawn are 5 and 4. Thus, the percentage is 54. If the last sublot to be placed represents more than 54 percent of the usual sublot size, take the acceptance sample from the truck containing the fifty-fourth percentile. If less than 54 percent of the normal sublot is to be placed, do not sample.

The truck load to be sampled for each sublot should be established prior to beginning the concrete placement. However, this information should be kept confidential until the load to be sampled arrives on the job.

While the particular randomizing method to use is a matter of convenience or judgement, it is emphasized that arbitrarily selecting a load for sampling other than the one selected by the randomizing procedure must not be permitted. Only when a load designated as an acceptance sample is rejected and removed from the job should a change be made. In this case the next load placed automatically becomes the load for acceptance sampling.

START-UP AND MONITORING PROCEDURES

Start-up procedures and requirements concerning proper moisture determinations in the aggregate, mixing temperatures, etc., remain unchanged. Although under both the old and revised specifications it is the contractor's responsibility to control all the properties of the concrete within the specification limits, monitoring of air content by state personnel at the beginning of a placement is desirable. The state's final acceptance for air content, however, is to be based on the statistical analysis of the test results on the randomly selected acceptance samples.

A special procedure has been introduced for monitoring the air content of bridge deck concrete. Under the new procedure, at the beginning of each day, when 3 consecutive loads of concrete show that the amount of entrained air is within the required specification limits and the average of the three tests is within ± 0.8% of the target value (mid-point of range for air), reduced monitoring can be instituted on the basis of 1 randomly selected sample for each 5 loads.

Monitoring tests may be made using the Chace air indicator. However, a determination must be made by the air pressure meter or the volumetric method before a load of concrete is rejected. Any load for which the air content determined by the last two procedures is outside the specification limits should be rejected and removed from the job. When this occurs for a load that would normally be sampled for acceptance tests, all data and specimens, if already made, will be discarded, and the next load to be placed in the job will be so sampled. This is consistent with the randomizing procedure, since the characteristics of the concrete placed in the structure and not the concrete produced are desired.

ADDITIONAL TESTS REQUIRED FOR SIMULATED APPLICATION OF REVISED SPECIFICATION

Acceptance for strength under the new specification is to be based on 28-day tests on the randomly selected acceptance samples required for the revised specification. These can also be used as the acceptance tests for the present specification. However, if the usual 14-day breaks as well as the 28-day breaks are desired, make 2 sets of 3 each of 4 in. x 8 in. cylinders for the projects selected for the simulated application of the revised specification. Test one set at 14 days and the other at 28 days. The additional data for small cylinders at 14 days may permit subsequent revision of the specification to permit statistical acceptance on the basis of 14-day tests. A test for entrained air by the air pressure meter must be made for each acceptance sample.

DATA REPORTING

Field personnel will continue to record all data on the same forms as in the past and send in reports to the Materials Division, except that the reports for the simulated projects should so indicate. These data will be used to compute acceptance and simulated pay factors. For any placements in which the amount of retarders, water-reducing agents, or water have been intentionally varied, the records submitted should also so indicate.

PAY FACTORS

Recent analyses of the statistical significance of the criterion for acceptance, and for computing pay factors, have raised some doubts that the producer's correction as now provided for in the proposed revision is valid in all cases and further study is needed. However, the data needed would not be affected by any changes in the manner of computing pay factors. Thus, for the time being, field personnel need not estimate simulated pay factors.

-1406

ATTACHMENT 3

STATISTICAL SIGNIFICANCE OF PROPOSED REVISIONS

The proposed acceptance plan for portland cement concrete was examined to determine the effect of the statistical procedures on the acceptance levels for concrete and to assess their impact from an engineering point of view. This study revealed the need for reconsideration of some of the concepts involved. The first problem is with the concept that all material is acceptable when less than 10% of the population is below f'_c. For large standard deviations, assuming normal distribution, there is a statistical chance that some results for single strength test specimens could be lower than design standards permit. With good inspection there is a good probability that the actual strength of the concrete in the structure will not be as low as that calculated, but to protect against this possibility a second criterion is needed; that is, there should be a requirement that no single test result be below a stated strength in pounds per sq. in.

Guidelines of the American Concrete Institute suggest that "good" quality control procedures for concrete should give standard deviations in the 500 to 600 psi range, and this range is generally supported by the quality assurance data for concrete supplied to Virginia Department of Highways and Transportation projects. Consequently, if the central value, 550 psi, is assumed to be typical, concrete with 90% of the value above 4,000 psi would have a probability of having 1% of its value below 3,423 psi. Thus, for consistency and added incentive for good control, there should be a second criterion to require that no more than a 1% chance that single strength values would fall below 3,400 psi. This means that the first criterion would be the controlling factor for all lots having standard deviations below 571 psi and the second criterion would be the controlling factor for all lots having standard deviations above 571 psi. This is determined by the following computation.

If $\overline{X}_1$ is the average required by Criterion 1 and $\overline{X}_2$ is the average required by Criterion 2, the change-over occurs when $\overline{X}_1 = \overline{X}_2 =$

$$4{,}000 + 1.28\ \sigma = 3{,}400 + 2.33\ \sigma.$$

$$600 = 1.05,$$

$$\sigma = 5.71.$$

For values of σ less than 571, $\bar{X}_1$ will be the higher, but for values of σ greater than 571, $\bar{X}_2$ will be higher. The higher average would be considered the minimum average of the concrete production for 100% acceptance and payment. It is noted that the number of tests made does not affect this target average. However, the number of tests made does affect the probability that unsuitable material will be accepted or that suitable material will be rejected. Thus, a concrete producer should set his target values higher than this value to minimize his risk that testing variations would lead to reduced payment.

The Producer's Risk Correction

A second problem is with the proposed application of a correction to the average of the test results to reduce the producer's risk of having acceptable material subjected to a reduction in payment.

The correction is based on establishing the 95% confidence interval for the average of the test results. That is, given an average of n results with a given standard deviation, it can be stated that there is 95% probability that the true average will be between a limit of

$$\bar{X} - \frac{1.65\ \sigma}{\sqrt{n}} \quad \text{and} \quad \bar{X} + \frac{1.65\ \sigma}{\sqrt{n}}.$$

Thus, to avoid unfairly applying a penalty to a producer's product based on a limited number of tests, the producer's correction equal to $\frac{1.65\ \sigma}{\sqrt{n}}$ is added to the average to establish the adjusted quality level that is the basis for computing the pay factor. Since one of the purposes of the overall proposed specification is to establish a situation whereby the producer with good quality control would have an advantage, the use of the actual standard deviation of the test results for computing this factor would have an effect opposite to that desired since the correction increases with increased variability. This lowers the test average needed for 100% payment. It is recommended that this situation be avoided by the use of an assumed standard deviation of 550 psi. This value is the midpoint of the range, 500-600 psi, for standard deviation considered by ACI to indicate "good" quality control. On this basis the correction to be applied becomes a function of only the number of tests made. A producer whose product has a standard deviation less than 550 psi benefits by a lower than 5% risk of having acceptable material rejected, and a producer whose product has a larger standard deviation than 550 psi has a greater than 5% risk of having material subjected to reduced payment factors. This is as it should be.

Table 1 illustrates the decreasing value of this correction as the number of tests made increases from 3 to 30.

Table 1

The Producer's Correction Assuming a Standard Deviation of 550 psi

n	C_{pr}*
3	524 psi
5	406 psi
10	287 psi
30	166 psi

$$*C_{pr} = \frac{1.65 \times \sigma}{\sqrt{n}}, \ (\sigma = 550).$$

The Acceptable Average for Strength

Under the proposed specification, the acceptable average of a product is dependent on the standard deviation of the population of test results represented by that average. A difficulty is encountered in determining a proper estimate of the standard deviation. In the presently proposed specification the estimate of the standard deviation indicated by the test results themselves is used. However, such an estimate is subject to large uncertainties when the numbers of test results are low, and the assumption that the sample standard deviation accurately represents the population standard deviation may not be valid.

There are several alternatives for establishing the applicable standard deviation for computing the target average for the production. One of these is to base the standard deviation on the concrete producer's production for the 30 most recent tests on the same grade of concrete using the same ingredients. For this computation the test made for the contract in question should be included, and if fewer than 30 results are available (as is likely) a pooled standard deviation would be calculated by determining the standard deviation for jobs completed within the previous 60 days, treating each job as a separate lot. Certified data from other than state contracts could be made acceptable for this computation. When more than 10 but fewer than 30 results for the last 60 days are available from a producer's records, the computation would be made on the basis of the available data. When fewer than 10 results are available, the estimate of standard deviation based on the data from the lot could be subject to such high uncertainties that an assumed standard deviation based on experience would be preferable. This would normally be a high value; for example, 900 psi.

Required Averages for 100% Payment

To show the effects of various standard deviations and various numbers of tests, computations of the average required for 100% payment under various conditions have been computed.

Table 2 shows the desired minimum true average ($\overline{X}_{SL}$) for A4 concrete (f'_c = 4,000 psi) for various standard deviations. The averages shown in Table 2 are true averages and are independent of the number of tests made.

Table 2

True Average for Acceptance of A4 Concrete at Full Bid Price for Different Standard Deviations

Standard Deviation	Minimum Average*
400	4,512**
500	4,640**
600	4,798
700	5,031
800	5,264
900	5,497
1,000	5,633

*Average shown is desired true minimum average of population.

**Established by Criterion 1, all other values established by Criterion 2.

Under the revised proposal to apply a producer's risk correction that would vary only with the number of tests conducted, the adjusted averages below which partial payment would be made for various selected standard deviations and numbers of tests would be as indicated in Table 3.

TABLE 3

Minimum Adjusted Averages for Full Payment at Various Standard Deviations and for Different Numbers of Tests

n	$\overline{X}_{SL}$	C_{pr}	$\overline{X}_{adj}$*
σ = 400			
3	4512	524	3988
5	4512	406	4106
10	4512	287	4225
30	4512	166	4326
σ = 500			
3	4640	524	4116
5	4640	406	4234
10	4640	287	4353
30	4640	166	4474
σ = 600	4798	524	4274
3	4798	524	4274
5	4798	406	4392
10	4798	287	4511
30	4798	166	4632
σ = 700			
3	5031	425	4507
5	5031	406	4625
10	5031	287	4744
30	5031	166	4865
σ = 800			
3	5264	524	4740
5	5264	406	4858
10	5264	287	4977
30	5264	166	5098
σ = 900			
3	5497	524	4973
5	5497	406	5091
10	5497	287	5210
30	5497	166	5331
σ = 1000			
3	5730	524	5206
5	5730	406	5324
10	5730	287	5443
30	5730	166	5564

*Lowest average of test results acceptable without applying partial payments.

Probability of Acceptance or Rejection of Concrete at Different Strength Levels

To illustrate the effects of different standard deviations and different numbers of tests, calculations were made to show the probability of acceptance (at full payment) of lots of concrete at different (true) average strengths assuming the estimate of standard deviation also represents the true variability. These are shown in Table 4. These calculations are made as follows:

$$\frac{\overline{X}_{(true)} - \overline{X}_{(adj.)}}{\sigma / \sqrt{n}} = Z$$ (number of standard deviations the true average is above the adjusted acceptance limit).

The probability of acceptance is read from the quality level table, assuming a normal distribution.

Example: $\sigma = 400$, $n = 3$, $\overline{X}_{adj.} = 3{,}988$,

$4900 = \overline{X}_t$, and

$$\frac{4900 = 3988}{400/\ 3} = 3.95\ (Z),$$

when $Z = 3.95$.

Probability of acceptance = 99.9+.

The Rejectable Quality

The above discussion provides for establishing the lowest average for which full payment will be made. However, it is necessary to establish a level below which concrete should be removed or further investigation made by coring.

Under Criterion 1, the presently proposed specification sets the adjusted quality level at which concrete should be rejected at 73.7%. This is determined as follows: the specification stipulates that the reduced pay factor shall not be less than 0.70 when determined by the equation.

$$R_{PF} = (AQL + 10)^2/10{,}000,$$
$$0.7 = (AQL + 10)^2/10{,}000,$$
$$AQL = \sqrt{7{,}000} - 10,$$
$$AQL = 73.7.$$

Table 4

True Average of Population psi	Probability of Acceptance (Expressed as Percentage) n = 3	n = 5	n = 10	n = 30	n = 3	n = 5	n = 10	n = 30
	Std. Deviation = 400 psi				Std. Deviation = 600 psi			
4900	99.9+	99.9+	99.9+	99.9+	96.4	97.1	98.0	99.3
4800	99.9+	99.9+	99.9+	99.9+	93.3	93.6	93.6	93.7
4700	99.9+	99.9+	99.9+	99.9+	89.1	87.5	83.9	73.2
4600	99.6	99.7	99.8	99.9	82.6	78.2	68.1	38.6
4500	98.3	98.6	98.5	98.3	74.2	65.5	47.6	11.5
4400	93.8	95.0	91.6	77.0	64.1	51.2	29.1	1.7
4300	91.2	86.0	72.2	36.4	52.8	36.7	13.3	0.3
4200	82.1	70.0	42.1	3.3	41.7	23.6	5.0	nil
4100	68.4	51.0	16.4	0.2	31.8	13.8	1.5	nil
4000	52.0	27.8	4.8	nil	21.5	7.2	0.4	nil
	n = 3	n = 5	n = 10	n = 30	n = 3	n = 5	n = 10	n = 30
	Std. Deviation = 700 psi				Std. Deviation = 800 psi			
4900	83.4	81.1	76.9	60.6	64.1	54.7	38.2	8.7
4800	74.6	71.8	59.9	40.5	55.2	43.4	24.2	2.1
4700	68.6	59.5	42.1	10.0	46.4	33.0	15.8	0.3
4600	59.1	46.8	25.8	1.9	38.2	23.6	6.8	nil
4500	50.8	34.5	13.6	0.2	30.2	15.9	2.9	nil
4400	39.7	23.6	5.9	nil	23.0	10.0	1.1	nil
4300	30.5	14.9	2.3	nil	17.1	6.0	0.4	nil
4200	22.4	8.7	0.7	nil	12.1	3.3	nil	nil
4100	15.6	4.6	0.2	nil	8.4	1.7	nil	nil
4000	10.6	3.3	nil	nil	6.4	0.8	nil	nil

When the standard deviation is 571 psi, the average of the concrete with a quality level of 0.737 must be 4,363 psi (4,000 + .636 σ). On this basis a requirement that single test results should not be below 3,000 psi more than 1% of the time is indicated as a reasonable limit for rejection under Criterion 2 (4,363 - 2.33 σ). Table 5 shows the minimum test averages for different standard deviations and different numbers of tests on the basis of the 3,000 psi lower limit for a single test (1% level).

It is important to realize that there is a high statistical probability that concrete of lower strength than the rejectable level will be accepted when n is low. Table 6 shows the probabilities of acceptance for various standard deviations and numbers of test.

TABLE 5

Minimum Test Averages for Acceptance

Standard Deviation, psi	Min. Avg.* Adjusted, psi	Minimum test averages for reduced payment when:** n = 3, C_{pr} = 524	n = 5, C_{pr} = 406	n = 10, C_{pr} = 287	n = 30, C_{pr} = 166
400	4254	3730	3848	3867	4088
500	4318	3794	3912	4031	4152
600	4398	3874	3992	4111	4232
700	4631	4107	4225	4344	4465
800	4864	4340	4458	4577	4698
900	5097	4573	4691	4810	4931

*The minimum adjusted average is equivalent to the minimum "true" average required to provide an approximate 5% producer's risk.

**When the averages of tests made are lower than the indicated values, the product would be rejected or an investigation of the concrete quality in place would be conducted.

TABLE 6

Probabilities of Accepting Concrete with Strengths Equal to the Rejectable Average

Standard Dev., psi	Probability of Acceptance (Expressed as a %) n = 3	n = 5	n = 10	n = 30
400	87	80	59	16
600	64	51	28	1.6
700	62	51	31	3.4
800	61	51	33	5.4

This table demonstrates that, statistically, when adjustments are made to assure that the producer's risk is low (approximately 5%) with a small number of samples there is very little assurance that all concrete at the rejectable quality level is being rejected. Approximately 30 samples are required to reduce the risk of accepting rejectable quality to 5% or below. It is emphasized that dependence must be placed on proper mix design, good quality control, and good placement procedures to assure an acceptable product. Consequently, good inspection must be maintained.

THE REDUCED PAY FACTOR

For standard deviations below 571 psi, Criterion 1 establishes the reduced pay factor by the equation

$$R_{pf} = (AQL + 10)^2/10{,}000, \qquad (1)$$

where

R_{pf} - is the reduced pay factor, and

AQL - adjusted quality level (determined by adding a producer's correction to the average of test results; calculating z from the adjusted average and reading AQL from the normal distribution tables.

With this formula all concrete with an adjusted level of 90% or above is paid for at the full bid price and the 0.70 pay factor is reached for an adjusted quality level of 73.7%. However, when Criterion 2 is applicable (standard deviation above 571 psi), an average higher than that required under Criterion 1 is needed and more than 90% of the population must be higher than f'_c for 100% payment. Similarly, the quality level for the lowest acceptable average for a pay factor of 0.70 will be higher than 73.7%.

Accordingly, it's recommended that the method of determining the pay factor be revised as follows:

Criterion 1

AQL for 100% pay = 90%

AQL for 70% pay = 73.7%

Determine adjusted quality level based on adjusted average and determine the proportionate amount of total difference between QL for 100% and QL for 70% pay. Multiply this proportion by 30 (100% - 70%) to determine percentage above 70.

Example:

An average of 5 tests on A4 concrete is equal to 4,050 psi and the standard deviation is 400.

Add a producer's correction of 406:

Adjusted average is 4,456.

Adjusted quality level

$$Z = \frac{4,456 - 4,000}{400} = 1.14 \text{ AQL} = 87.3, \text{ and}$$

$$\frac{87.3 - 73.7}{90 - 73.7} \times 0.30 = 0.250;$$

$\therefore$ Pay factor = 0.70 + 0.25 = 0.95.

Criterion 2

Under Criterion 2 the AQL for 100% pay and the AQL for 70% pay both will vary with the standard deviation and must be determined for the standard deviation indicated. The pay factor will be based on the proportionate amount of the difference between 70% pay and 100% pay that the AQL exceeds the lower limit.

For Example:

An average of 5 tests on A4 concrete is equal to

4,500 psi and the standard deviation is 800.

Add a producer's correction of 406;

$\therefore$ Adjusted average = 4,906.

Adjusted quality level

$$Z = \frac{4,906 - 4,000}{800} = 1.13 \text{ AQL} = 87.1.$$

AQL for 70% pay =

$\overline{X}$ = 3,000 + 2.33 σ = 1,864 = 4,864, and

$Z = \frac{4,864 - 4,000}{800}$ = 1.08 AQL (70%) = 86%;

$\therefore$ AQL for 70% pay = 86%.

AQL for 100% pay =

$\overline{X}$ = 3,400 + 2.33 σ = 5,264,

Z = 5,264 - 4,000 = 1.58 AQL (100%) = 94.3%, and

$\frac{8.71 - 86.0}{94.3 - 86.0}$ x .30 = 0.40.

Pay factor = 0.70 + 0.040 = 0.740.

The practical effect of these requirements can best be illustrated by computing the acceptability and, where applicable, the reduced pay factor that would be applied for actual averages of test results at 28 days for concrete requiring an f'_c of 4,000 psi at 28 days (Class A4 by Virginia Department of Highways and Transportation specifications). Table 7 shows such pay factors for a range of 28-day strength values between 3,700 psi and 5,100 psi and for various standard deviations. When R is shown in the table, the computed pay factor is less than 0.700. When the computed pay factor is below this value, the concrete is not acceptable under the present proposals. This table demonstrates the effects of both making a small number of tests and the standard deviation. For example, if only 3 tests are made, test results averaging 4,000 psi are accepted at full pay for a standard deviation of 400 psi, but when the standard deviation is 800 psi the average must be 4,340 psi before the concrete is acceptable. Full price would not be paid unless the test average is about 4,800 psi. If as many as 30 test results are available, concrete with average strengths of 4,000 psi are rejected for all standard deviations of 400 psi or greater. For a standard deviation of 800 psi, an average of 5,100 psi is required in order to assure full payment with 30 tests. This demonstrates the lower risks to the consumer associated with making a large number of tests.

Table 7

Pay Factors for Concrete Established at Different Standard Deviations and for Different Numbers of Tests (n)

Average of 28-Day Test Results, psi	n = 3			n = 5			n = 10			n = 30		
	σ 400	600	800	400	600	800	400	600	800	400	600	800
3700	R*											
3800	.799	R		R								
3900	.917	.729		.777								
4000	1.0	.816		.899	R		.750			R		
4100		.895		.993	.801		.879	R		.722		
4200		.961		1.0	.882		.980	.785		.855	R	
4300		1.0	R		.950		1.0	.867		.957	.768	
4400			.781		1.0	R		.936		1.0	.849	R
4500			.866			.740		.994	R		.923	R
4600			.913			.823		1.0	.722		.985	R
4700			.978			.902			.812		1.0	R
4800			1.0			.971			.892			.798
4900						1.0			.957			.873
5000									1.0			.945
5100												1.0

*"R" indicates computed pay factor is below 0.70 and concrete should be rejected or further investigation of quality made.