2608

RESPONSE OF PLAIN CONCRETE TO A STATE OF BIAXIAL FATIGUE LOADING: EQUIPMENT AND TECHNIQUE

by

Charles E. Smith
Highway Research Engineer

Virginia Highway Research Council
(A Cooperative Organization Sponsored Jointly by the Virginia Department of Highways and the University of Virginia)

Charlottesville, Virginia

June 1974
VHRC 73-R58

SUMMARY

The fatigue strength of concrete is an important factor in the design of certain structures, particularly those associated with transportation functions. A considerable body of research has been developed on fatigue using specimens subjected to uniaxial loadings. The purpose of this project was to study concrete subjected to a state of biaxial fatigue loading.

This report describes the development of equipment to apply such loading and to measure its effect.

RESPONSE OF PLAIN CONCRETE TO A STATE OF BIAXIAL FATIGUE LOADING: EQUIPMENT AND TECHNIQUE

by

Charles E. Smith
Highway Research Engineer

INTRODUCTION

The fatigue strength of concrete is an important factor in the design of certain structures. Numerous research studies have been conducted to evaluate the fatigue characteristics of concrete(1, 2, 3, 4) and to establish mechanical relationships for use in structural designs that will more effectively utilize the material. The major restriction of these past studies has been that the response of concrete was determined under a uniaxial load condition. A uniaxial loading generates only a crude approximation of the actual stress conditions encountered in highway structures such as concrete pavements and bridge decks.

The general behavior of concrete under static multi-axial stresses has been reasonably well established;(5, 6, 7) but, to date, no data are available on the response of concrete to states of biaxial fatigue loading. A knowledge of the response of concrete under combined fatigue stresses would be of considerable practical importance since combined stresses more truly represent the conditions to which a structural member may be exposed. As an initial effort, it was proposed to study the response of concrete to a state of biaxial fatigue loading.

This report deals with the development of the testing equipment and techniques and provides an introduction to the behavior of concrete under fatigue and multi-axial states of stress. It includes a list of Related References and an Annotated Bibliography on fatigue of plain concrete containing approximately sixty papers dated from 1898 to 1972.

PROBLEM STATEMENT

Fatigue is a process of progressive permanent structural change occurring in a material due to the application of repetitive loads, each smaller than the single static load that would cause failure. This process has been observed and verified to occur in concrete.(8) However, the research to date has generally been conducted with a uniaxial load as previously stated. The study of the fatigue response under multi-axial load conditions required the development of the equipment to apply the desired stress as well as the development of the testing techniques. These techniques included the manner in which the test sample was to be prepared and the ways in which information could be obtained from the sample in order to show its behavior under a complex state of stress.

Thus, the first step in the study of the effects of biaxial fatigue stresses on plain concrete was the development of this equipment and test techniques such that the data obtained would be reliable and could be compared with those for the uniaxial cases taken from the literature.

OBJECTIVE AND SCOPE

The major objective of this report is to describe the test equipment, methods of preparing test specimens, materials, and test procedures developed or acquired for the study. A brief review is given of the methods tried and of the major reasons that the methods failed. The techniques which proved to be successful are explained and illustrations are introduced to show important points.

To aid in future research a List of Related References was compiled. This list includes all important papers selected from the literature pertaining to the failure mechanism of concrete under both uniaxial and multi-axial states of stress, to the methods of crack formation and propagation, and to the methods available to detect and present the effects of load on concrete. For the same purpose an Annotated Bibliography of the literature on the fatigue of plain concrete from 1898 to 1972 was prepared. The list of papers from 1898 to 1958 was taken from reference 2, but over half of the literature has been written since 1958. The large number of papers published since then is an indication of the renewed interest in the fatigue of concrete.

SPECIMEN, MATERIALS, AND PREPARATION

Based upon the literature review it was decided that the investigation would be conducted using 3 x 6 inch (76 x 152 mm) concrete cylinders as test specimens. The small size cylinder offered the advantage of working with a large number of specimens from a laboratory batch. A larger number of specimens for each variable would permit a greater statistical reliability. A considerable amount of other published data on the effect of a uniaxial fatigue load on the same size cylinder would be available for comparison of testing.

Since a major portion of the investigation was to be directed toward determining the response of concrete to a state of biaxial fatigue loading, it was proposed to use a standard mixture design for all phases of the experimental work. Due to certain limitations of the testing machine available for the project, a nominal concrete strength of 3,000 psi (20.7 MPa) was selected as a standard. The concrete was made from a Type II cement, locally available sand, and a crushed basalt coarse aggregate. Some specimens were made using quartz gravel as the coarse aggregate, but they were found not to be satisfactory. It was found when studying the crack formation, that due to the internal fracture condition of the quartz, it was difficult to define the failure zone caused by the imposed loads. The basalt performed very well, and did not present such a difficulty. Also, due to its dark appearance, the basalt was much easier to work with under a microscope and showed distinctive crack patterns in photographs.

The investigation required the fabrication of an apparatus that would produce a state of biaxial stress and allow for a fatigue type loading. It was anticipated that the apparatus would be a high pressure container allowing the application of a hydrostatic pressure on the surface of the test cylinder. Thus the specimen had to be prepared in such a way that it would work in such a container and also provide a means for monitoring during testing. Figure 1 shows a specimen before and after it was prepared for testing.

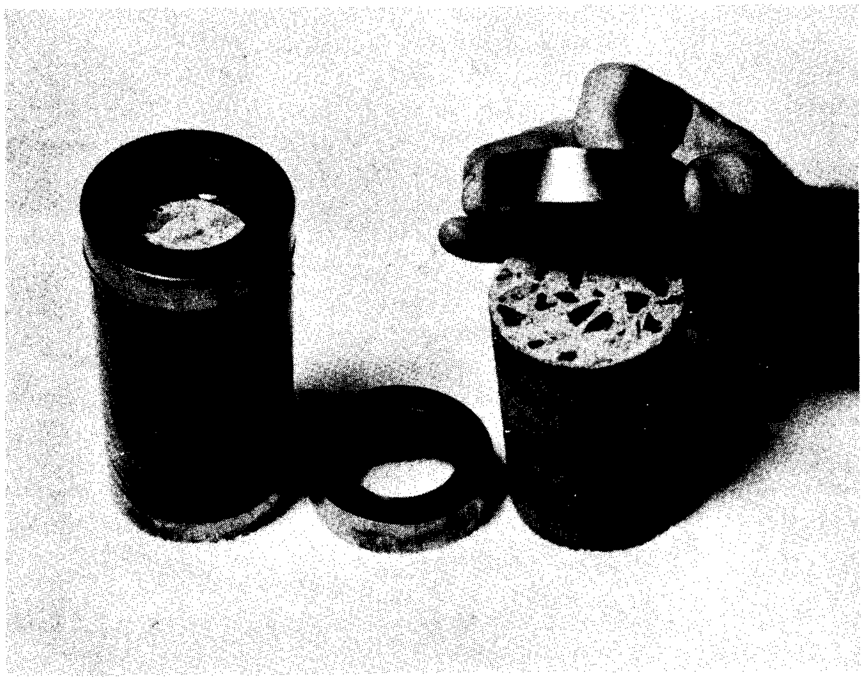

Figure 1. Concrete cylinder and steel rings.

Because of the high pressures used in testing the specimen, a very close tolerance was required between the cylinder and test chamber to ensure that the O-ring would seat. The concrete surface could not be cast or machined to this required tolerance (±.002 in.) (±0.050 mm). To obtain this degree of closeness required that a metal cylinder be machined and then connected to the specimen (see Figure 1). Various methods were tried in order that the ring could be cast into the concrete cylinder, but none of these proved successful. The method which did prove successful was epoxing the rings to the cylinder.

In order to obtain a level surface, the 6-inch (152 mm) cylinder was placed in a holding jig, squared, and 3/4 inch (19 mm) of each end was off. This process left a cylinder $4\frac{1}{2}$ inches (114 mm) tall. On each of these surfaces, a 3/4 inch (19 mm) metal ring was epoxied. The epoxy used for this preparation was from Armstrong Products Company and designated A-12 in their product description. This is a two-part adhesive and has a noncritical mixing ratio of 1:1. The surfaces to be bonded should be clean and dry. It was found that cleaning the surfaces with acetone and then using ethyl alcohol to remove any film remaining from the acetone produced a good bond. Once this cleaning process is complete, the adhesive is applied to both surfaces to be bonded. The surfaces are pressed together and light clamping may be used to keep them in position during curing (approximately 24 hours).

To keep the oil used to apply the hydrostatic pressure from entering the pores of the concrete cylinder, the surface of the cylinder was prepared in the following manner. When the cylinders were removed from the forms, they were placed in the moisture room for one week. At this time they were removed and the surface was brushed with a wire wheel to expose and open all pores. A cement mortar mixture of one part cement and one part sand was applied to the surface, and the cylinder was returned to the moisture room for another week. The cylinder was then removed from the moisture room and the surface sanded to remove all sharp sand particles. At this stage, the cylinder was ready for the metal rings to be applied as described above. With the rings in place and the epoxy cured, the surface was painted with a clean plastic or epoxy that was adsorbed by the cement surface and provided an impermeable layer. A prepared specimen is shown in Figure 1. The preparation of the specimen could usually be completed in two weeks, which allowed it to be tested at 28 days.

EQUIPMENT

V-Scope

The basic instrument used in the nondestructive testing of the cylinders was a V-scope manufactured by James Electronics. This instrument generates ultrasonic energy that supplies pulsed energy to a transducer. Through a second transducer it detects, amplifies, and displays the received signal. The instrument contains an accurate time standard and measuring system for the digital measurement of elapsed time of the sound wave passing through the concrete specimen. Also an auxiliary output is located on the rear of the V-scope. This output can be fed into the vertical input of an oscilloscope so that the frequency and amplitude of the received signal can be studied. The output can also be fed into a peak-to-peak vacuum tube voltmeter for an accurate monitoring of the maximum amplitude of the first received signal and can be used for attenuation measurements. Figure 2 shows the V-scope, transducers, and oscillograph.

In order to detect the small cracks in the concrete specimens, a high frequency transducer had to be used. From the literature, it was found that the 250 kHz transducer would detect microcracks such as would occur in the fatigue studies. Thus, two 250 kHz units were used in the preliminary investigations and were found to be satisfactory. The transducers are $1\frac{1}{4}$ inches (32 mm) in diameter, which allowed them to be easily placed on the end surface of the concrete cylinder through the $1\frac{3}{4}$ inch (44 mm) opening in the metal ring.

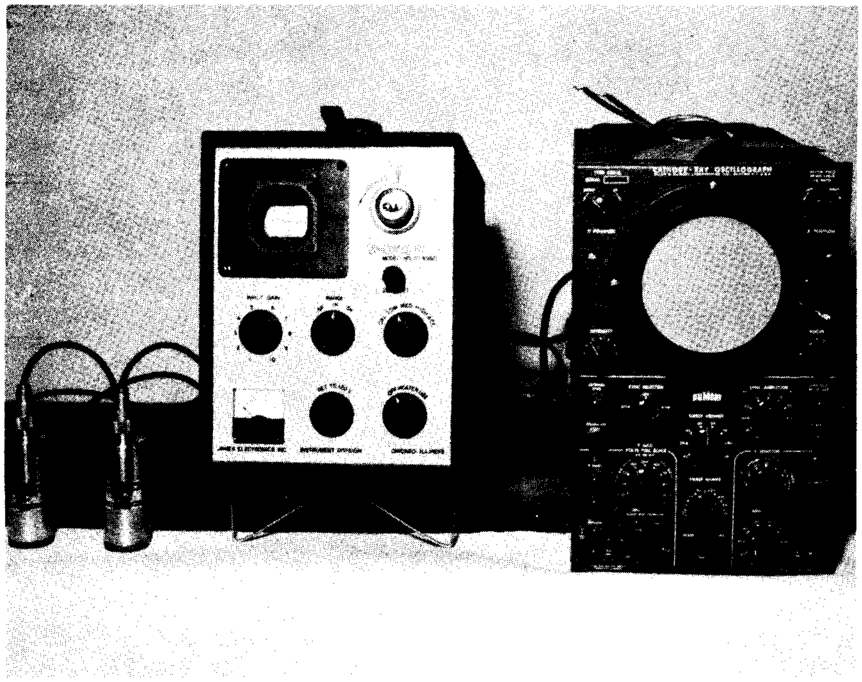

Figure 2. Transducers in combination with the V-scope and oscillograph.

In order to ensure good contact between the cylinder surface and the transducer, a small amount of couplant fluid should be placed on each contact surface. Once this is done the transducers are held to each end of the specimen and the desired reading may be taken. Figure 3 illustrates how the measurements are made. The operating instructions to be followed are outlined in the operation and maintenance manual for the James V-Scope and will not be discussed in this report.

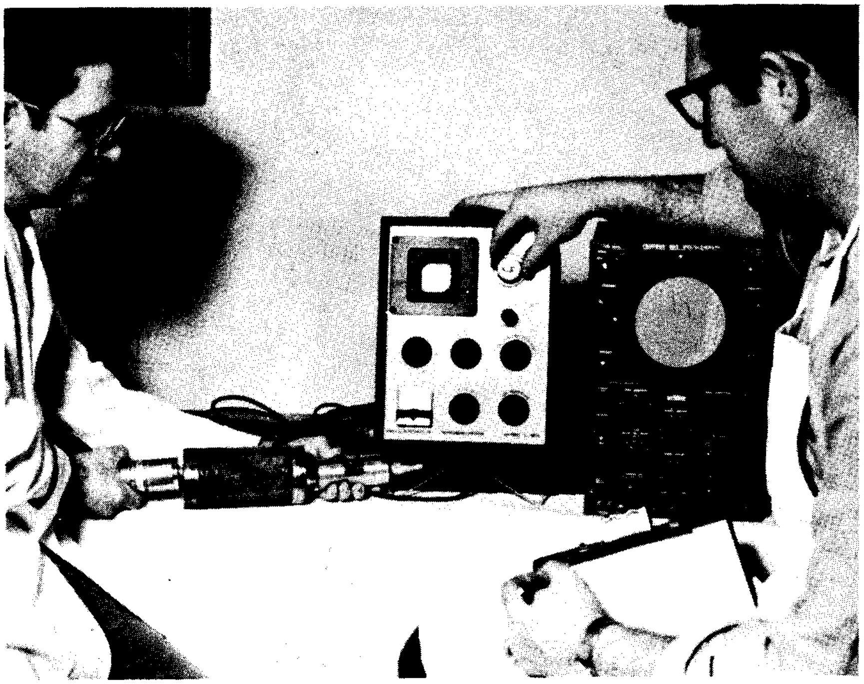

Figure 3. Ultrasonic testing.

Test Chamber

The basic test instrument used to apply the biaxial state of stress to the concrete cylinder by means of a hydrostatic pressure is shown in Figure 4. The instrument is basically a pressure chamber which will allow pressure to be applied to the sides of the test cylinder with the end free of load. The chamber is sealed by means of a rubber O-ring fitting against the sides of the chamber and the sides of the metal rings epoxied to the concrete specimen. There are six main parts: the chamber body, the hose fitting, the drain fitting, the rubber O-ring, the metal fitting ring, and the end plates that are held in place by six bolts each.

The purpose of the drain fitting is to ensure that the chamber is free of air. It is important that the chamber be completely filled with oil and free of any air pockets. This may be achieved by laying the chamber on its side when filling with the drain fitting open and in the up position. The chamber with the test specimen locked inside is filled with oil until it flows evenly out of the drain. At this time the drain fitting is closed and testing may start.

The fitting ring has two purposes. First, it closes the gap between the sides of the chamber and the test specimen. This allows the cylinder to remain centered during testing. Secondly, it acts as a fitting between the chamber and the end plates so that the O-ring might be compressed to some degree when end plates are bolted to the chamber. It was found that by doing this a tight seal could be obtained without any leakage at low pressures.

Much testing was done to find the proper size O-ring that would hold the desired pressures and allow the application of a fatigue load. Based upon the testing a 1/8 inch (3 mm) ring diameter O-ring with a $2\frac{3}{4}$ inch (70 mm) inside diameter was selected. This size O-ring allowed a tight fit and would hold the desired pressure.

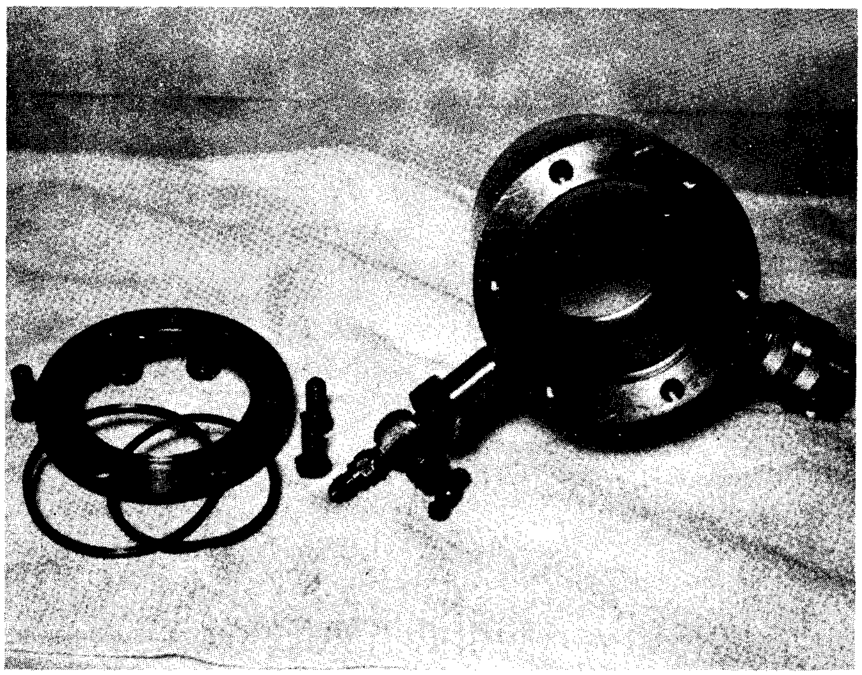

Figure 4. Test chamber with fittings.

TEST PROCEDURES

The procedures used in the biaxial testing of the concrete cylinders were relatively simple. The first step was the preparation of the specimen and taking of the necessary data. The cylinder was then placed in the test chamber as shown in Figure 5, and the chamber connected to the loading machine as shown in Figure 6. Oil was placed in the test chamber (remember that it is important to have all air removed from the chamber as previously discussed) and the specimen and chamber were then ready for testing.

Figure 5. Concrete cylinder in test chamber.

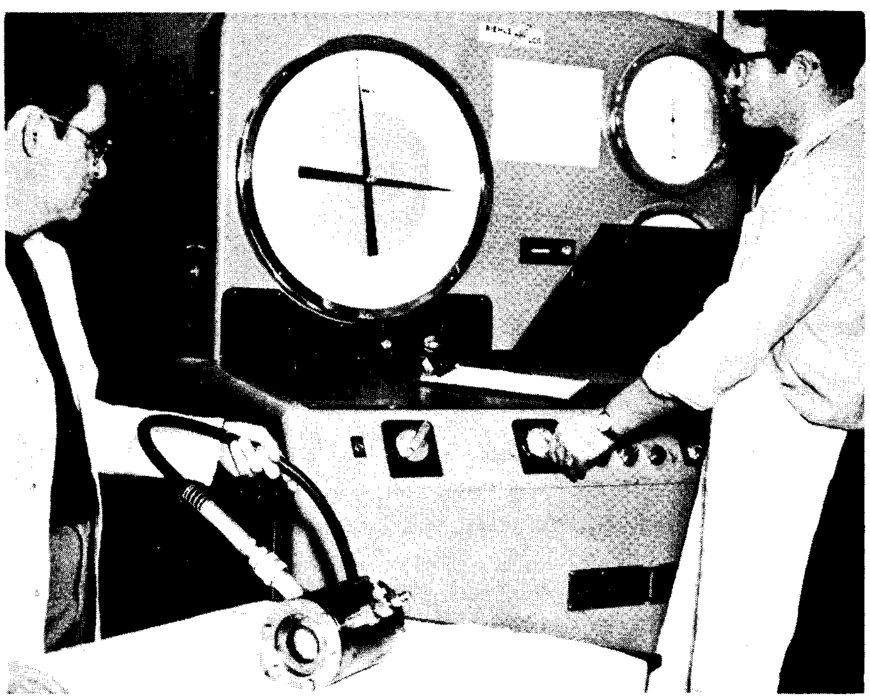

Figure 6. Load testing of concrete cylinder.

Sometimes it was necessary to use a metal specimen as shown in the first cylinder in Figure 7, to obtain the desired fatigue load and load span. In this way the desired loading condition can be obtained without wasting a concrete cylinder

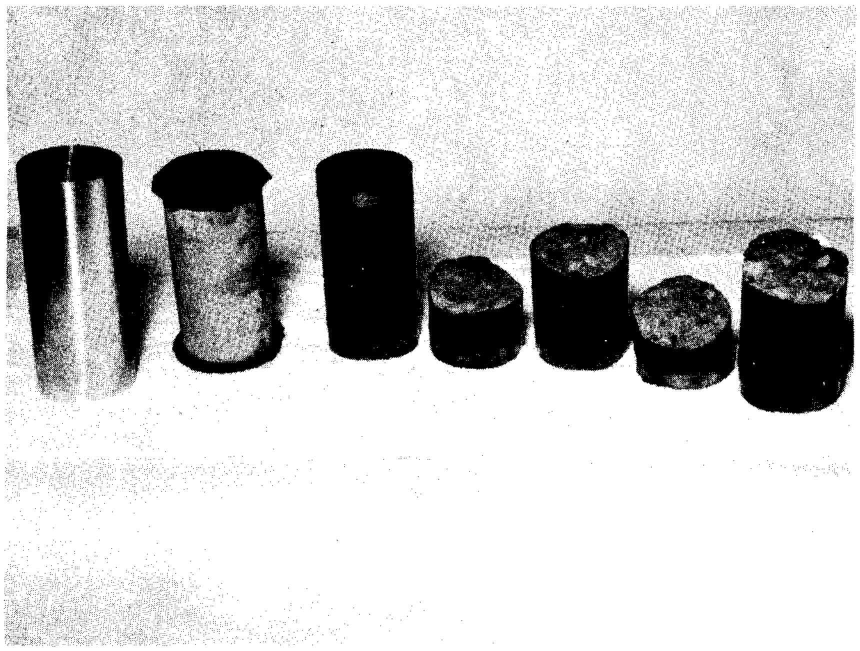

Figure 7. Test cylinders with view of concrete cylinders before and after testing.

TEST RESULTS

Considering the variables inherent in fatigue studies and the variation in the strength of concrete, it has been found that all data should be presented in terms of a ratio of the fatigue or load stress level to the static ultimate stress level. Also, specimens should be selected in a random manner such as to minimize the influence of variables in materials and mixing procedures.

Figure 7 shows the way in which the cylinder will break under a biaxial load condition. It is noted that the break is parallel to the direction of the applied stress. This is consistent with results reported in the literature. The direction of the break is important to know, so that the direction of the slice taken for the crack study can be perpendicular to the break. This method was followed in the early stages of this project and found to be satisfactory.

Figure 8 is a slice taken from a specimen as described above. The surface of the slice is polished and washed so it may be studied under a microscope and the crack pattern marked and photographed for study and comparison with other samples. Various dyes were tried, but none were helpful in the location of the microcracks.

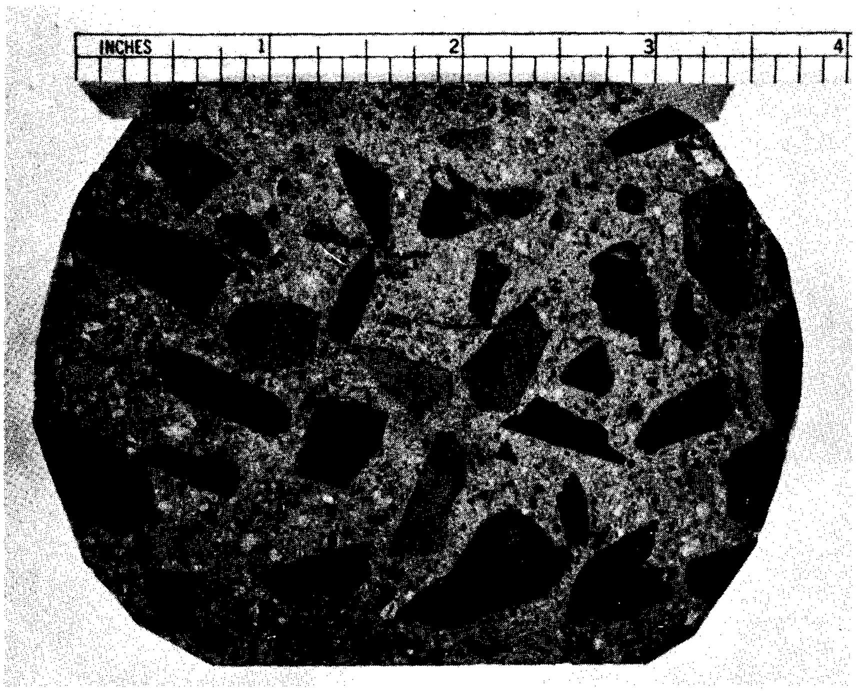

Figure 8. Slice of concrete cylinder after testing with crack pattern marked for investigation.

In order to show how the test results may be presented and also the use of the sonic equipment in obtaining data, Figures 9 and 10 present the results of a uniaxial test. The specimens were loaded to a certain percentage of their ultimate load. In Figure 9 this percentage is plotted against the percent change of the pulse velocity and in Figure 10 against the percent change in attenuation of the received pulse. The pulse velocity involves the determination of the time of travel of a pulse through a certain path in the material. The percent change of the pulse is taken as the ratio of the difference of the pulse velocities through the specimen before and after the loading to the velocity of the loaded specimen. This ratio can be expressed in terms of time traveled since the length of path traveled is the same before and after the loading. In the case of percent change in the attenuation of the received pulse, the voltage readings using an oscilloscope are measured before and after loading the specimens. Both graphs indicate the degree of cracking occurring in the material since this cracking influences the pulse characteristics.

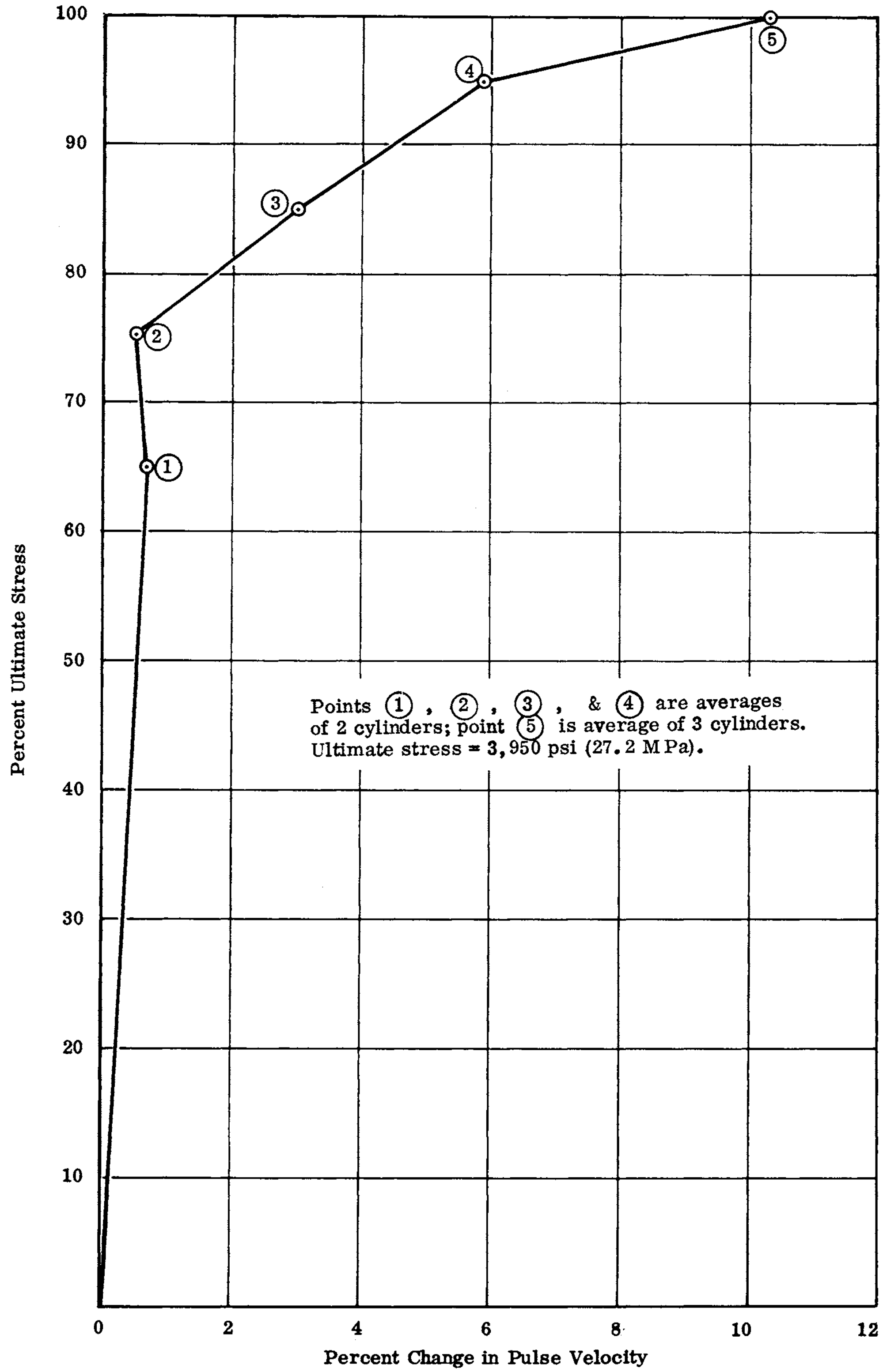

Figure 9. Test uniaxial load.

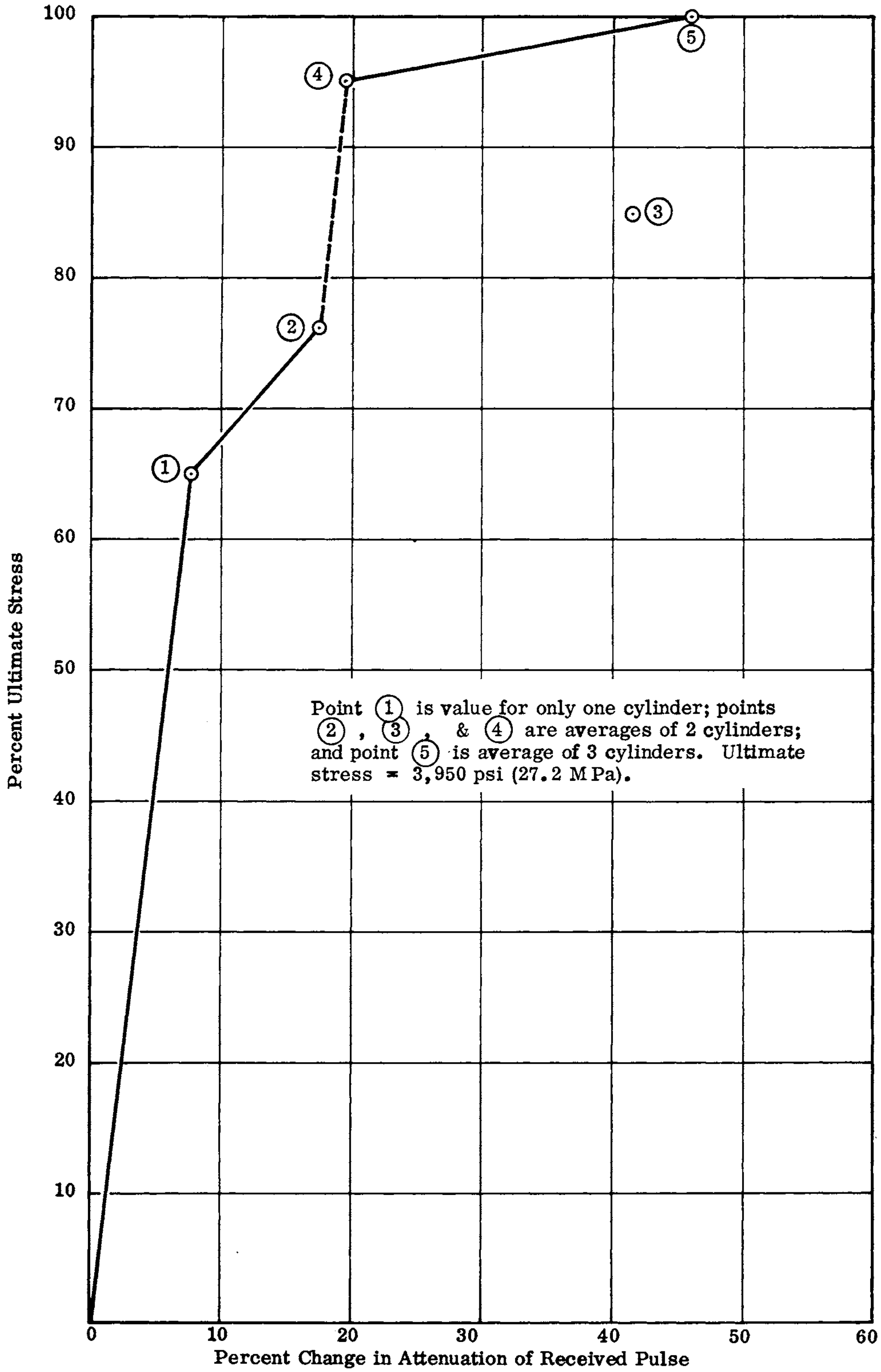

Figure 10. Test uniaxial load.

The evaluation of sonic data has been thoroughly discussed by Whitehurst.[9] Figures 11 and 12 present the results for the biaxial tests. The test data are again presented as a percentage of the ultimate biaxial load. Again, graphs are used to show the change in the velocity and attenuation measurements.

It should be noted that to obtain useful information a large number of samples should be tested, because of the nature and variability of concrete and the fatigue failure process. As seen from the data in the Appendix, the results are varied; but by using the averaging process with a number of specimens, information can be obtained.

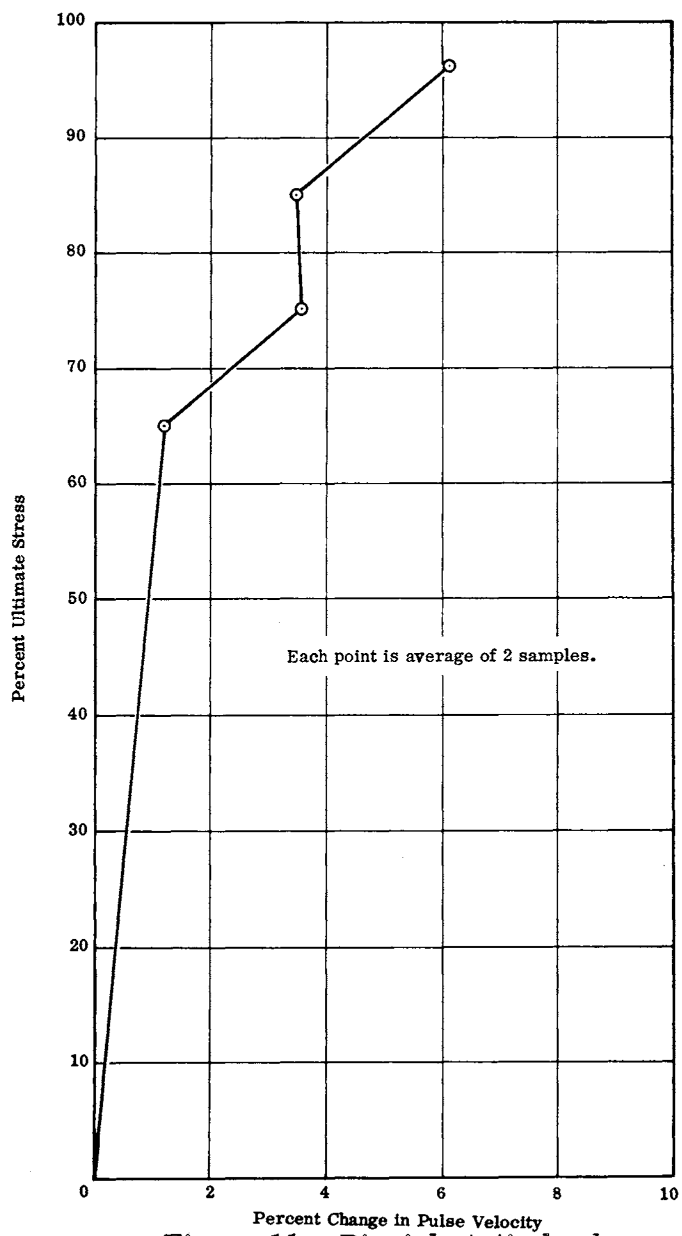

Figure 11. Biaxial static load.

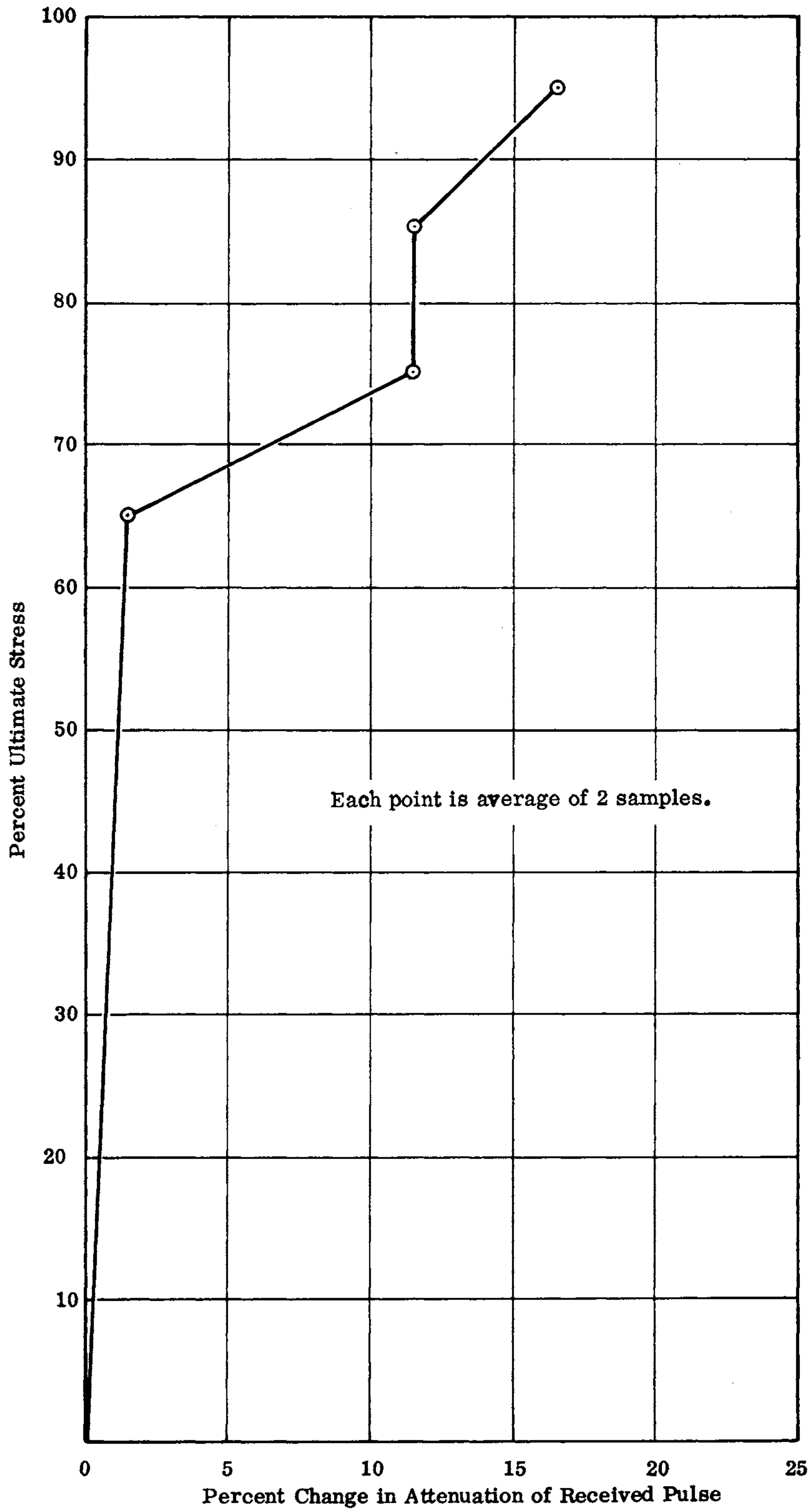

Figure 12. Biaxial static load.

SUMMARY AND RECOMMENDATIONS FOR CONTINUED RESEARCH

From the work completed thus far, it is clear that it is possible to test concrete in a biaxial stress condition. The incomplete nature of the study does not justify definitive conclusions but the equipment and testing techniques described in the report were found to be satisfactory. From the results of the preliminary investigation presented, it is seen that useful information may be obtained on the crack proportions by using the sonic equipment.

Since the purpose of the initial investigation was to develop the testing equipment and techniques, no data were collected on the actual fatigue properties of concrete. During the course of the study to date several areas of the needed research in this area have been identified. These include:

1. A convenient means of measuring fatigue damage.

2. A general failure model to provide easy interpretation of field and laboratory results for efficient use by the designer.

3. Information oriented towards developing a better understanding of crack initiation and propagation in concrete subjected to repeated loads.

4. Further evaluations of the applicability of fracture mechanics concepts to fatigue of concrete.

5. The determination of the effect of random loading on concrete.

6. Studies concerned with determining the cumulated fatigue behavior of concrete.

7. Additional research on the range of stress, particularly in the area of reversal of stress, to determine the maximum values of the various combinations of tensile and compressive stress under 10 million repetitions of load.

8. Studies utilizing repeated loads on concrete in torsion would be of some significance.

9. Further study to develop a new method, extend old methods, or combine nondestructive methods in order to evaluate internal structural damage to concrete in the field.

10. Research to determine the influence of a biaxial state of stress for various combinations of tensile and compressive loads would be a significant area of study.

11. The determination of the influences of conditions of source moisture and temperature gradients, curing conditions, and corrosive environments on the fatigue behavior of concrete.

REFERENCES CITED

(1) Kesler, Clyde E., "Fatigue and Fracture of Concrete", Stanton Walker Lecture Series on the Materials Sciences, Lecture No. 8, Nov. 1970.

(2) "Fatigue of Concrete", ACI Bibliography No. 3, American Concrete Institute, Detroit, Michigan, 1960.

(3) Nordby, Gene M., "Fatigue of Concrete — A Review of Research", ACI Journal, Vol. 30, No. 2, August 1958, pp. 191-220.

(4) Myrdock, John W., "A Critical Review of Research on Fatigue of Plain Concrete", E.E.S. Bulletin 475, University of Illinois 1965.

(5) Wilhelm, W. J., Ed., Proceedings, The First Conference on the Behavior of Structural Concrete Subjected to Combined Loadings, Engineering Experiment Station, West Virginia University, June 1969.

(6) Newman, K., "Criteria for the Behavior of Plain Concrete Under Complex States of Stress", Proceedings, International Conference on the Structure of Concrete and its Behavior Under Load, Imperial College and Cement and Concrete Association, London, 1968, pp. 255-287.

(7) Kupfer, H., Hilsdorf, H. K., and Rusch, H., "Behavior of Concrete Under Biaxial Stresses", ACI Journal, No. 8, Proceedings, V. 66, August 1969, pp. 656-666.

(8) Neville, A. M., "Fatigue Strength", Hardened Concrete: Physical and Mechanical Aspects, ACI Monograph No. 6, 1971, pp. 79-87.

(9) Whitehurst, E. A., Evaluation of Concrete Properties from Sonic Tests, ACI Monograph No. 2, 1966.

RELATED REFERENCES

(1) Akroyd, T. N. W., "Concrete under Triaxial Stress", Magazine of Concrete Research, Vol. 13, No. 39, November 1961, pp. 111-118.

(2) Awad, M. M., "Strength and Deformation Characteristics of Plain Concrete Subjected to High Repeated and Sustained Loads", Ph.D. Thesis, University of Illinois, January 1971.

(3) Bobbin, J. E., and Malin, W. J., "Automatic Ultrasonic Velocity Measurements", Prepared for Presentation at the 1970 Spring Conference, American Society for Nondestructive Testing, Los Angeles, California, March 1970.

(4) Bremner, T. W., and Moavenzaheh, F., "Fracture of Portland Cement Concrete", Structures, Solid Mechanics and Engineering Design, Ed. by M. Te'eni, Wiley-Interscience, 1971, pp. 997-1,008.

(5) Clark, Jr., W. G., "Fracture Mechanics in Fatigue", Experimental Mechanics, No. 9, Sept. 1971, pp. 421-428.

(6) Dougill, J. W., "Further Consideration of the Mathematical Model for Progressive Fracture of a Heterogeneous Material", Magazine of Concrete Research, Vol. 23, No. 74, March 1971, pp. 5-10.

(7) Dougill, J. W., "A Mathematical Model for the Failure of Cement Paste and Mortars", Magazine of Concrete Research, Vol. 19, No. 60, Sept. 1967, pp. 135-142.

(8) "Evaluation of Concrete Properties from Sonic Tests", American Concrete Institute Monograph No. 2, ACI, Detroit, Michigan, 1966.

(9) Freudenthal, A. M., "Planning and Interpretation of Fatigue Tests", Special Technical Publication, No. 121, ASTM, June 1951, pp. 3-13.

(10) Gillespie, H., "Concrete Failure-Theory, Mechanism, and Concept," Ph.D. Thesis, University of Arizona, June 1968.

(11) Glucklich, J., "Fracture of Plain Concrete", Journal of the Engineering Mechanics Division, Proceedings, ASCE, Vol. 89, No. EM6, December 1963, pp. 127-137.

(12) Hanna, S. J., "Fracture of Hardened Cement Paste", (Progress Report), Joint Highway Research Project, Purdue University, June 1967.

(13) Helmy, M. A., Goode, C. D., "The Strength of Concrete under Combined Shear and Direct Stress", Magazine of Concrete Research, Vol. 19, No. 59, June 1967, pp. 105-112.

(14) Hobbs, D. W., "Strength of Concrete under Combined Stress", Cement and Concrete Research, Vol. 1, Pergamon Press, 1971, pp. 41-56.

(15) Hobbs, D. W., "Strength and Deformation Properties of Plain Concrete Subject to Combined Stress, Part 1: Strength Results Obtained on One Concrete", Technical Report 42.451, Cement and Concrete Association, London, November 1970.

(16) Hobbs, D. W., "Strength and Deformation Properties of Plain Concrete Subject to Combined Stress, Part 2: Strength in Multiaxial Compression," Technical Report 42.463, Cement and Concrete Association, London, March 1972.

(17) Hruban, I., and Vitek, B., "Failure Theory of Concrete — Biaxial State of Stress", Structures, Solid Mechanics and Engineering Design, Ed. by M. Te'eni, Wiley-Interscience, 1971, pp. 1,053-1,060.

(18) Hsu, T. T. C., Slate, F. O., Strurman, G. M., and Winter, G., "Microcracking of Plain Concrete and the Shape of the Stress-Strain Curve," Proceedings, ACI Vol. 60, No. 2, February 1963, pp. 209-223.

(19) Illston, J. M., and Stevens, R. F., "Internal Cracking", Concrete, Vol. 6, No. 7, London, July 1972, pp. 28-31.

(20) Jones, R., and Facaoaru, I., "Recommendations for Testing Concrete by the Ultrasonic Pulse Method", Materials and Structures, RILEM, Vol. 2, No. 10, July-August 1969, pp. 275-284.

(21) Kaplan, M. F., "Crack Propagation and the Fracture of Concrete", Proceedings, ACI, Vol. 58, No. 11, November 1961, pp. 591-609.

(22) Kesler, C. E., Naus, D. S., and Lott, J. L., "Fracture Mechanics — Its Applicability to Concrete", Presented at the International Conference on Mechanical Behavior of Materials, Kyoto, Japan, August 1971.

(23) Krishnaswamy, K. T., "Strength and Microcracking of Plain Concrete Under Triaxial Compression", Proceedings, ACI, Vol. 65, No. 10, October 1968, pp. 856-862.

(24) Krishnaswamy, K. T., Chandrashekhara, K., and Sundara Raja Irengar, K. T., "Strength of Concrete Under Biaxial Compression", Proceedings, ACI, Vol. 62, No. 1, February 1965, pp. 239-449.

(25) Johnson, R. P., and Lowe, P. G., "Behavior of Concrete Under Biaxial and Triaxial Stress", Structures, Solid Mechanics and Engineering Design, Ed. by M. Te'eni, Wiley-Interscience, 1971, pp. 1,039-1,051.

(26) Liu, T. C. Y., Nilson, A. H. and Slate, F. O., "Biaxial Stress-Strain Relations for Concrete", Journal of the Structural Division, Proceedings, ASCE, Vol. 98, No. ST5, May 1972, pp. 1,025-1,034.

(27) Liu, T. C. Y., Nilson, A. H. and Slate, F. O., "Stress-Strain Response and Fracture of Concrete in Uniaxial and Biaxial Compression," Proceedings, ACI, Vol. 69, No. 5, May 1972, pp. 29-295.

(28) Liu, T. C. Y., "Stress-Strain Response and Fracture of Concrete in Biaxial Compression", Ph.D. Thesis, Cornell University, February 1971.

(29) Lott, J. L., and Kesler, C. E., "Crack Propagation in Plain Concrete", Highway Research Board, Special Report 90, 1966, pp. 204-218.

(30) Mills, L. L., and Zimmerman, R. M., "Compressive Strength of Plain Concrete Under Multiaxial Conditions", Proceedings, ACI, Vol. 67, No. 10, October 1970, pp. 802-807.

(31) Moavenzadeh, F., and Kuguel, R., "Fracture of Concrete", Journal of Materials, JMLSA, Vol. 4, No. 3, September 1969, pp. 497-519.

(32) Mielenz, R. C., "Diagnosing Concrete Failures", Stanton Walker Lecture Series on the Materials Sciences Lecture No. 2, November 1964.

(33) Newman, K., Newman, J. B., and McCreath, D. R., "The Influence of Aggregate Particles on the Local Strain Distribution and Fracture Mechanism of Cement Paste During Drying Shrinkage and Loading to Failure", Materials and Structures, RILEM, Vol. 2, No. 7, January and February 1969, pp. 73-84.

(34) Newman, K., and Newman, J. B., "Failure Theories and Design Criteria for Plain Concrete", Structures, Solid Mechanics and Engineering Design, Ed. by M. Te'eni, Wiley-Interscience, 1971, pp. 963-995.

(35) Naus, J. D., "Applicability of Linear-Elastic Fracture Mechanics to Portland Cement Concretes", Ph.D. Thesis, University of Illinois, June 1971.

(36) Neunow, R. A., "Research and Applications for the "V" Scope as Related to Ultrasonic Nondestructive Testing of Wood, Concrete, and Asphalt", James Electronics, Inc., Chicago, Illinois.

(37) Naus, D., Rejali, H. M., Lott, J. L., and Kesler, C. E., "Cracking of Concrete", Engineering Experiment Station, Bulletin 504, University of Illinois, 1970.

(38) Pochtovik, G., and Kashkin, S. K., "Ultrasonic Measurements in Concrete Surfaces", Translated from the Russian: Automobil NYE Dorogi, Vol. 25, No. 7, pp. 13-14, Office of Engineering Reference, Denver, Colorado, March 1965.

(39) Popovics, P., "Fracture Mechanism in Concrete: How Much Do We Know?", Journal of the Engineering Mechanics Division, Proceedings, ASCE, Vol. 95, No. EM3, June 1969, pp. 531-544.

(40) Paris, P. C., "The Fracture Mechanic Approach to Fatigue", Proceedings, 10th Sagmose Army Materials Research Conference, Syracuse University, 1964, pp. 107-132.

(41) Robinson, G. S., "Methods of Detecting the Formation and Propagation of Microcracks in Concrete", Proceedings, International Conference on the Structure of Concrete and its Behavior under Load, Imperial College and Cement and Concrete Association, London, 1968, pp. 131-145.

(42) Robinson, G. S., "Behavior of Concrete in Biaxial Compression", Journal of of the Structural Division, Proceedings, ASCE, Vol. 93, No. ST1, Feb. 1967, pp. 71-86.

(43) Rosenthal, I., and Glucklich, J., "Strength of Plain Concrete under Biaxial Stress", Proceedings, ACI, Vol. 67, No. 11, Nov. 1970, pp. 903-914.

(44) Richart, F. E., Brandtzaeg, A., and Brown, R. L., "A Study of the Failure of Concrete under Combined Compression Stresses", Engineering Experimental Station, Bulletin No. 185, University of Illinois, 1928.

(45) Raju, N. K., "Small Concrete Specimens under Repeated Compressive Loads by Pulse Velocity Technique", Journal of Materials, JMLSA, Vol. 5, No. 2, June 1970, pp. 262-272.

(46) Slate, F. O., and Darwin, D., "Effect of Paste-Aggregate Bond Strength on Behavior of Concrete", Journal of Materials, JMLSA, Vol. 5, No. 1, March 1970, pp. 86-98.

(47) Shah, S. P., and Chandra, S., "Mechanical Behavior of Concrete Examined by Ultrasonic Measurements", Journal of Materials, JMLSA, Vol. 5, No. 3, Sept. 1970, pp. 550-563.

(48) Shah, S. P., and Chandra, S., "Critical Stress, Volume Change, and Microcracking of Concrete", Proceedings, ACI, Vol. 65, No. 9, Sept. 1968, pp. 770-781.

(49) Slate, F. O., Nilson, A. H., and Buyukozturk, O., "Stress-Strain Response and Fracture of a Concrete Model in Biaxial Loading", Proceedings, ACI, Vol. 68, No. 8, August 1971, pp. 590-599.

(50) Shah, S. P., and Slate, F. O., "Internal Microcracking Mortar-Aggregate Bond and the Stress-Strain Curve of Concrete", Proceedings, International Conference on the Structure of Concrete and Its Behavior Under Load, Imperial College and Cement and Concrete Association, London, 1968, pp. 82-92.

(51) Shah, S. P., "Micromechanics of Concrete and Fibre Reinforced Concrete", Structures, Solid Mechanics and Engineering Design, Ed. by M. Te'eni, Wiley-Interscience, 1971, pp. 367-376.

(52) Taylor, M. A., "General Behavior Theory for Cement Pastes, Mortars, and Concretes", Proceedings, ACI, Vol. 68, No. 10, October 1971, pp. 756-762.

(53) "Triaxial Testing of Plain Concrete", Report of Task Group I, Subcommittee C09.02.03, Elastic and Inelastic Properties of Concrete of ASTM Committee C-9 on Concrete and Concrete Aggregates, June 1971.

(54) Vile, W. D. G., "The Strength of Concrete Under Short-Term Static Biaxial Stress", Proceedings, International Conference on the Structure of Concrete and Its Behavior Under Load, Imperial College and Cement and Concrete Association, London, 1968, pp. 275-288.

(55) Weibull, W., Fatigue Testing and Analysis of Results, Pergamon Press, New York, New York, 1961.

(56) Yoshimoto, A., Ogino, S., and Kawakami, M., "Microcracking Effect on Flexural Strength of Concrete after Repeated Loading," Proceedings, ACI, Vol. 69, No. 4, pp. 233-240.

(57) Zimmerman, R. M., "The Effect of Various Types of Lateral Restraint on the Triaxial Behavior of Plain Concrete", Ph.D. Thesis, University of Colorado, 1965.

(58) Slate, F. O., Buyukozturk, O., and Nilson, A. H., "Deformation and Fracture of Particulate Composite", Journal of the Engineering Mechanics Division, Proceedings, ASCE, Vol. 98, No. EM3, June 1972, pp. 581-593.

ANNOTATED BIBLIOGRAPHY

(1) De Joly, "The Strength and Elasticity of Portland Cements" (La Resistance et L'Elasticite des Ciments Portland), Annales, Ponts et Chaussees, V. 16, Series 7, pp. 198-244. (in French). 1898.

Tests were made on tension briquets aged from 2 to 20 days at frequencies ranging from 26 to 92 cycles per min. A fatigue limit of approximately 50 percent of static ultimate strength was obtained. An increase in the frequency of applications reduced the required repetitions for failure. Rest periods appeared to permit recovery from fatigue effects.

(2) Van Ornum, J. L., "Fatigue of Cement Products", Transactions, ASCE, V. 51, p. 443. 1903.

Tests made on 4-week old neat 2-in. cement cubes at a frequency of 4 cycles per min. indicate a fatigue limit of approximately 55 percent of the static ultimate strength.

(3) Falk, M. S., Cements, Mortars, and Concrete, M. C. Clark, New York, pp. 66-69. 1904.

Summary of tests by De Joly and Van Ornum (1903).

(4) Van Ornum, J. L., "Fatigue of Concrete", Transactions, ASCE, V. 58, pp. 294-320. 1907.

Compression tests of 5 x 5 x 12-in. prisms at 4 to 8 cycles per min. and flexural tests of 4 x 6 x 72 in. reinforced concrete beams at 2 to 4 cycles per min. are given. The beams were reinforced with $2\frac{1}{2}$ percent of steel. Tests were made at 1, 6, and 12 months. A fatigue limit of 50 percent of static ultimate strength was determined with the required repetitions to failure being rapidly reduced at stresses above 55 percent of the static ultimate. Some data are given on stress-strain relationships under repeated loading.

(5) Berry, H. C., "Apparatus for Repeated Loads on Concrete Cylinders and a Typical Result", Proceedings, ASTM, V. 10, pp. 581-586, 1910.

A machine to apply repeated load on 8 x 16 in. cylinders is described. Initial tests were made on 6-month old concrete. Permanent and elastic deformations for a maximum of 150,000 cycles on specimens loaded at $0.52\ f'_c$ were measured.

(6) Graft, Otto, "The Compressive and Tensile Elasticity of Concrete" (Die Druckelastizitaet und Zugelastizitaet des Betons) Forschungsarbeiten auf dem Gebiete des Ingenieur Wesens, No. 227, Verlag Julius Springer, Berlin. (in German). 1920.

A study to determine a "real" modulus of elasticity for concrete by repeated loading. Number of cycles was between two and ten. Studies were made of the influence of level of load, velocity of loading and unloading, and different concrete properties, not true fatigue.

(7) Williams, G. M., "Some Determinations of the Stress-Deformation Relations for Concretes under Repeated and Continuous Loadings," Proceedings, ASTM, V. 20, Part II, p. 238.

Tests were made of 8 x 16 in. cylinders at 28 days at 6 to 8 cycles of load per day. Limited data show a slight increase in modulus of elasticity with repetitions. The data are in conflict with those of Van Ornum but may have been influenced by concrete aging.

(8) Clemmer, H. F., "Fatigue of Concrete", Proceedings, ASTM, V. 22, Part II, pp. 408-419. 1922.

Illinois fatigue test 1922-1924; see also References 54, and 21. For annotation see Reference 21.

(9) Hatt, W. K., "Note on Fatigue of Mortar", Proceedings, ACI, V. 18, pp. 167-173.

Purdue fatigue tests 1922-1928; see also References 10, 11, 12, 34, and 17. for annotation see Reference 17.

(10) Creppes, R. B., "Fatigue of Mortar", Proceedings, ASTM, V. 23, Part II, pp. 329-340; Discussion by A. N. Johnson, H. F. Clemmer, and A. T. Goldbeck. 1923.

(11) Hatt, W. K., "Fatigue of Concrete", Proceedings, Highway Research Board, V. 4, pp. 47-60. 1924.

Purdue fatigue tests 1922-1928; see also References 9, 10, 12, 17, and 34. For annotation see Reference 17.

(12) Hatt, W. K., "Researches in Concrete", Bulletin No. 24, Purdue University, pp. 44-55, 1925.

Purdue fatigue tests 1922-1928; see also References 9, 10, 11, 17, and 34. For annotation see Reference 17.

(13) Probst, E., "Investigations on the Influence of Repeated Loading on the Elasticity and the Strength of Plain and Reinforced Concrete", (Untersuchungen ueber den Einfluss wiederholter Belastungen auf Elastizitaet und Festigkeit von Beton und Eisenbeton), Zeitschrift, Technische Hochschule (Karlsruhe) (Published by Verlag Julius Springer, Berlin); Also Bauingenieur (Berlin), V. 6, No. 33, Nov., pp. 931-935. (in German). 1925.

Earlier tests made by other investigators and a description of a machine for fatigue tests are discussed. A brief report is made of tests on plain concrete specimens, 2.8 x 2.8 x 11.2 in. loaded in compression. Tests are reported in more detail by Mehmel, Reference 15.

(14) Ros, M., "The Elasticity of Mortar and Concrete in Compression", (Die Druckelastizitaet des Moertels und des Betons) Diskussionbesricht No. 8, Eidgenoessische Materialprusfungsanstalt an der Technischen Hochschule (EMPA), Zurich, Dec., pp. 3-11. (in German). 1925.

The author offers general conclusions about fatigue strength by considering some new tests and the results of Ornum, Mehmel, and Probst.

(15) Mehmel, A., "Investigations on the Effect of Frequently Repeated Stress on the Elasticity under Compression and the Compressive Strength of Concrete" (Untersuchungen ueber den Einfluss haeufig wiederholter Druckbeanspruchungen auf Cruckelastizitaet und Druckfestigkeit von Beton), Mitteilungen, Institut fuer Beton und Eisenbeton an der Technischen Hochschule, Karlsruhe, 74 pp. (Published by Verlag Julius Springer, Berlin) (in German). 1926.

Compression tests on plain concrete specimens 2.8 x 2.8 x 11.2 in. are reported. The maximum load, range of load, and speed of loading were varied. Measurements of elastic and plastic deformations were taken. The duration of tests was up to 1,500,000 cycles. The stress-strain line, which is at first concave to the strain axis, became a straight line, but the modulus of elasticity dropped as repeated load was applied. As long as the upper load is below the fatigue strength the line will remain straight and the deformation approximately constant. Statically loaded specimens, which were first exposed to repeated loading, showed lower elastic and plastic deformations than concrete which was loaded for the first time. The compressive strength was not influenced by repeated load below the fatigue strength. But a specimen whose upper fatigue load is beyond the fatigue strength never becomes stabilized with regard to deformations. The stress-strain line becomes convex to the axis after a number of cycles. The magnitude of the deformations increases until the concrete is broken. The criteria for fatigue strength is dated as the deformation after a distinct number of cycles will just stabilize.

Variation of loading between 30 and 90 cycles per min. does not influence the fatigue strength. Increasing the range of loading decreases the fatigue strength. Only a few specimens were tested.

(16) Mills, R. E., and Dawson, R. F., "Fatigue of Concrete", Proceedings, Highway Research Board, V. 7, pp. 160-172. 1927.

The work of De Joly, Van Ornum, Berry, Illinois Division of Highways, Purdue University, and Technical College, Karlsruhe (Probst), is summarized.

(17) Hatt, W. K., and Mills, R. E., "Physical and Mechanical Properties of Portland Cements and Concretes", Bulletin No. 34, Purdue University, pp. 34-53, 94-95, 1928.

Purdue fatigue tests 1922-1928; see also References 9, 10, 11, 12, and 34.

Tests of plain concrete and mortar beams of 4 x 4 in. test section subjected to completely reversed bending couples are reported. The specimens were tested at a frequency of 10 cycles per minute with overnight and weekend rests. Some specimens were afforded an extended 5-week rest during repairs of the test equipment.

An indicated endurance limit was observed at approximately 55 percent of the static modulus of rupture for specimens sufficiently aged. The limit was found to decrease for specimens insufficiently aged although the scatter in the data precluded an exact estimate of the amount of reduction.

Observations were made on the progressive deformation of the specimens together with notations of the apparent beneficial effects of both the periods of rest and a previous loading history at a stress level below the endurance limit. The latter observation is in agreement with that made by Clemmer (Reference 8). The apparent endurance limit was in close agreement with that of Clemmer's tests although the latter were subjected to loads which varied from approximately zero to a tensile maximum.

A brief notation of observed progressive bond failure was made.

(18) Probst, E., "The Formation of Cracks in Plain and Reinforced Concrete Structures with Special Reference to the Effect of Repeated Loading" (Die Rissbildung bei Beton und Eisenbetonkonstrucktionen unter besonderer Beruecksichtigung des Einflusses wiederholter Belastung), Proceedings, Second International Congress for Bridge and Structural Engineering, (Vienna 1928), pp. 492-497 (Published by Verlag Julius Springer, Berlin) (in German). 1928.

Work done by Mehmel and Heim under Probst is reviewed and discussed.

(19) Probst, E., "Discussion of Question VI on Concrete", Final Report, First Congress, International Association for Bridge and Structural Engineering (Paris, 1932), Zurich, pp. 447-456.

Fatigue tests of plain concrete prisms and reinforced concrete beams are described. Earlier tests described by Heim, Treiber, and Yoshida are reviewed.

(20) Ban, Shizuo, "The Fatigue Phenomenon in Concrete", (Der Ermuedungsvorgang von Beton), Bauingenieur (Berlin), Vol. 14, No. 13-14, pp. 188-192. (in German). 1933.

Tests were made at Karlsruhe under Probst. Included were fatigue tests of concrete prisms (four only) to determine the relation between the shape of the stress-strain curve and the course of fatigue. As opposed to Mehmel, Ban studied loading the arm of the curve and the hysteresis loop.

(21) "Fatigue of Concrete", Engineering Report No. 34-1, Illinois Division of Highways, March, 149 pp. 1934.

Illinois fatigue test 1922-1924, review and summary; see also References 8, and 54.

Tests of plain concrete beams subjected to bending loads varying from approximately zero to a tensile maximum are reported. The test specimens were 6 x 6 in. cantilever beams to which wheel loads were applied at 40 cycles per min. No rest periods were introduced other than those which occurred when the loading apparatus was stopped on failure of a test specimen.

The behavior of concretes of various mixes was observed. An apparent endurance limit of approximately 55 percent of the static modulus of rupture was observed for concretes of normal mixes although this apparent limit was diminished for leaner mixes. No specimens were tested beyond 3,000,000 repetitions of load.

Beneficial effects of a previous loading history at a stress level below the endurance limit was noted. No mention of the mode of failure was made.

(22) Graf, O., and Brenner, E., "Experiments for Investigating the Resistance of Concrete under Often Repeated Compression Loads" (Versuche zur Ermittlung der Wilderstandsfaehigkeit) Deutscher Ausschuss fuer Eisenbeton (Berlin), No. 76, pp. 1-13 (Published by Wilhelm Ernst und Sohn) (in German). 1934.

Compressive tests were made on 60 specimens, 5.2 inches square and 16 in. high, loaded at a frequency of 4.3 cycles per sec. Studies were made on specimens of different compressive strength. Observation was made of the cracks and some measurements of modulus of elasticity were taken.

The tests indicate that fatigue strength changes with the range of stress and is influenced by the loading frequency. Specimens which were loaded below their fatigue strength still exhibited some compressive cracks.

(23) Treiber, F., "Behavior under the Static and Frequently Repeated Loads" (Das Verhalten unter dem Einfluss dauernd ruhender und haeufig wiederholter Belastung), Dissertation at Karlsruhe under Probst. 1934.

See also "Results of Tests on the Behavior of Reinforced Concrete T-Beams Under Static and Repeated Loading", (Das Verhalten von Eisenbeton-T-Balken unter dem Einfluss dauernd ruhender und haeufig wiederholter Belastung), Bauingenieur (Berlin), V. 15, No. 13, March, pp. 131-133; No. 14, April, pp. 178-182. (in German).

Tests were made on 14 specimens loaded below their fatigue strength on a 6-ft. span. Different amounts of reinforcement were used. It was noted during the repeated loading that new cracks appeared, the width of cracks increased, the stiffness of the beams decreased, but the deformations become more and more stabilized. The ultimate strength was not found to be influenced by repeated loading as long as the upper load is less than the fatigue strength.

(24) Graf, O., and Brenner, E., "Experiments for Investigating the Resistance of Concrete under Often Repeated Compression Loads" (Versuche zur Ermittlung der Widerstandsfaehigkeit von Beton gegen oftmals wiederholte Druckbelastung), (Deutscher Ausschuss fuer Eisenbeton (Berlin), No. 83, Part 2, pp. 1-12 (Published by Wilhelm Ernst und Sohn) (in German). 1936.

This is a sequel to Reference 22 and discusses compression fatigue tests of concrete prisms. Concretes of various mixes were investigated. Also studied were effect of range of stress, frequency of loading, and load-time characteristics of two different hydraulic fatigue testing machines. Tests indicated that the fatigue strength ratio decreases slightly with increasing water-cement ratio and that it increases with decreasing cement content. Kind and grading of aggregate apparently did not influence the fatigue strength ratio to any extent.

(25) Cassie, W. F., "The Fatigue of Concrete", Journal, Institution of Civil Engineers (London), V. 11, No. 4, pp. 165-167. 1939.

Published in Abstract, this is an excellent review of the information on fatigue of concrete which was then available.

(26) Williams, H. A., "Fatigue Tests of Lightweight Aggregate Concrete Beams", ACI JOURNAL, V. 14, No. 5, Apr. (Proceedings V. 39), pp. 441-447. 1943.

Tests were made on lightweight aggregate concrete beams subjected to completely reversed cycles of stress. No indication of endurance limit was exhibited by specimens. The apparatus used was similar to that in the Purdue University tests.

(27) LeCamus, B., "Research on the Behavior of Concrete and Reinforced Concrete Subjected to Repeated Loads" (Recherches sur le Comportement du Beton et du Beton Arme soumis a des efforts Repetes), Compte Rendu des Recherches Effectuees en 1945-1946, Laboratoires du Batiment et des Travaux Publics, Paris pp. 25-45. (in French). 1946.

An extensive study was made of fatigue behavior of plain and reinforced concrete loaded in compression, tension, and flexure. Twenty-two specimens of rectangular cross section were loaded in compression up to 1,000,000 cycles at 500 cycles per min. A fatigue strength of 0.62 was indicated. Results indicated an increase by about 10 percent in the static strength of specimens which did not fail in fatigue. Six plain beams loaded in flexure gave a fatigue ratio after 1,000,000 cycles of 0.59. The fatigue ratio for bond strength, which was determined from pull-out tests, was 0.69. Five reinforced beams designed to fail by rupture of tensile steel gave a fatigue strength of 0.55 and five beams which failed by crushing of the concrete in compression had a fatigue strength of 0.60.

Fifteen reinforced specimens with either 45 deg inclined, vertical, or no stirrups gave fatigue strengths of 0.41, 0.35, and 0.42, respectively, and in all cases shear failures occurred.

(28) Kesler, Clyde E., "Effect of Speed of Testing on Flexural Fatigue Strength of Plain Concrete", Proceedings, Highway Research Board, V. 32, pp. 251-258.

The research investigated the relation between fatigue strengths of medium and high strength concretes under repeated loadings applied at 70, 230, and 440 cycles per sec. The conclusion was that loading frequency, within the range investigated, has little or no effect on fatigue strength.

(29) McCall, John T., "Probability of Fatigue Failure of Plain Concrete", ACI JOURNAL, V. 30, No. 2, Aug. (Proceedings, V. 55), pp. 233-244. 1958.

Fatigue tests were run on concrete beams and the data were analyzed in an attempt to determine the relationship for concrete between stress, number of cycles to failure, and probability of failure. The relationship determined is presented graphically.

(30) Murdock, John W., and Kesler, Clyde E., "Effect of Range of Stress on Fatigue Strength of Plain Concrete Beams", ACI JOURNAL, V. 30, No. 2, Aug. (Proceedings V. 55), pp. 222-231. 1959.

From the results of tests of 175 plain concrete beams subjected to repeated flexural loading it was found that plain concrete exhibits no fatigue limit when subjected to loads which produce no reversals of stress. Fatigue strengths at 10,000,000 repetitions of stress were determined for each of several ranges of stress investigated, and these strengths were found to be dependent on the range of stress to which the specimen is subjected. A modified Goodman diagram is plotted.

(31) Nordby, Gene M., "Fatigue of Concrete -- A Review of Research", ACI JOURNAL, V. 30, No. 2, Aug. (Proceedings V. 55), pp. 191-220, 1959.

Investigations of fatigue of concrete, starting from 1898, are reviewed. The most important studies are summarized. The work has been divided into fatigue in compression, fatigue in flexure, fatigue in tension, fatigue in bond, and fatigue of prestressed concrete. An excellent review of the knowledge to date.

(32) Assimacopoulos, B. A., Warner, R. F., and Ekberg, C. E., "High Speed Fatigue Tests on Small Specimens of Plain Concrete", Journal, Prestressed Concrete Institute, Vol. 4, 1959, pp. 53-70.

(33) "Fatigue of Concrete", ACI Bibliography No. 3, American Concrete Institute, Detroit, Michigan, 1960.

An annotated bibliography on the papers on fatigue of plain, reinforced and prestressed concrete through mid-1958.

(34) Murdock, J. W., "The Mechanism of Fatigue Failure in Concrete", Ph.D., Thesis, University of Illinois, September 1960.

This investigation reports the results of approximately 85 flexural fatigue tests to determine the mechanism of fatigue failure in concrete. All specimens were simply supported on a 40 in. span and subjected to reported symmetrical two-point loading.

The tests included specimens of sand-cement mortar using idealized specimens in which single pre-shaped natural aggregates were included in the tension zone of the mortar specimens.

It was found that the inclusion of the aggregate in the matrix reduced the static flexural capacity in every case. No parameters were found which could consistently or accurately describe both progressive and incipient failure.

(35) Antrim, S. D., "A Study of the Mechanism of Fatigue in Cement Paste and Plain Concrete", Joint Highway Research Project No. 6, Purdue University, May 1965.

This investigation was concerned with the determination of the mechanism of fatigue in cement paste and plain concrete. Water-cement ratios of 0.70 and 0.45 were used for both paste and concrete specimens. Paste specimens were cylinders 2 inches by 4 inches and the concrete samples were 3 x 6 inch cylinders. It was found that the fatigue behavior of cement paste is sensitive to changes in the water-cement ratio of the paste and to changes in the moisture content of the paste.

The fatigue mechanisms proposed for cement paste and plain concrete are basically the same in that small cracks form and propagate in the cement paste under repeated loads.

(36) Murdock, J. W., "A Critical Review of Research on Fatigue of Plain Concrete", Engineering Experiment Station, Bulletin 475, University of Illinois, 1965.

This bulletin presents a concise and orderly evaluation of previous investigations of the fatigue behavior of plain concrete. It assesses the present state and limitations of the knowledge in this field, and indicates areas of potentially fruitful research.

Investigations of plain concrete fatigue behavior under both axial and bending loads are reported, and both strengths and weaknesses of the investigations are noted. Only those details of testing procedure which may have affected test results are described.

(37) Glucklich, J., "Static and Fatigue Fracture of Portland Cement Mortar in Flexure", Proceedings, 1st International Conference on Fracture, Sendai, Japan, 1965, pp. 1343-1383.

(38) Shah, S. P., and Winter, G., "Response of Concrete to Repeated Loading", Symposium on the Effects of Repeated Loading of Materials and Structures, RILEM, Vol. III, Mexico, 1966, 26 pp.

(39) Hilsdorf, H. K., and Kesler, C. E., "Fatigue Strength of Concrete Under Varying Flexural Stresses", Proceedings, ACI, Vol. 63, No. 10, Oct. 1966, pp. 1059-1076.

In this investigation plain concrete specimens were subjected to repeated flexural stresses according to various load histories; the maximum load within a test was varied between two limits, or rest periods were introduced. The results were interpreted according to various physical models and compared to the Miner rule. This hypothesis may give conservative or unsafe predictions of the fatigue strength depending on the load program. An improved design method is suggested.

(40) Linger, D. A., and Gillespie, H. A., "A Study of the Mechanism of Concrete Fatigue and Fracture", Highway Research News No. 22, Highway Research Board, February 1966, pp. 40-51.

The purpose of this study was to define the applicability of the Griffith Theory of Failure to the case of concrete fatigue. As a result, a brief discussion of the Griffith Theory and its application to concrete fracture is made and is followed with a background discussion of fatigue of concrete.

(41) Ople, J. R., F. S., and Hulsbos, C. L., "Probable Fatigue Life of Plain Concrete with Stress Gradient", Proceedings, ACI, Vol. 67, No. 1, January 1966, pp. 59-80.

The results of constant load cycle tests conducted on plain concrete specimens to study the effect of compressive stress gradients on fatigue life are presented and discussed.

Application of the results of the study for estimating beam fatigue life as limited by fatigue failure of the concrete in compression is briefly discussed. An approximate design check against the possibility of concrete failure in beams subjected to repeated flexural loads is formulated for a specified fatigue life $N = 2,000,000$ cycles and probability "design limit" $P \leq 0.00001$.

(42) Bennett, E. W., and Muir, S. E., "Some Fatigue Tests on High-Strength Concrete in Axial Compression", Magazine of Concrete Research, Vol. 19, No. 59, June 1967, pp. 113-117.

A programme of tests was undertaken of concrete prisms in axial compression to study the influence of static strength and the maximum size of coarse aggregate upon the fatigue strength at one million cycles.

Four types of concrete were tested having average static strengths of about 6,000 and 8,500 lb/in^2, each made with aggregate of 3/4 and 3/8 in. maximum size. The fatigue strength varied between 66 and 71 percent of the static strength and, although the percentage values were significantly lower with the higher-strength concrete and with the 3/8 in. aggregate, the actual magnitude of the differences was small. The general strain history was typical of previous fatigue tests, and the 'run-out' specimens showed a marked increase in static strength over similar specimens that had not been loaded.

(43) Lloyd, J. P., Lott, J. L., and Kesler, C. E., "Fatigue of Concrete", Engineering Experiment Station, Bulletin 499, University of Illinois, 1968.

Investigations of repeating loads, static loads, fracture mechanics, drying shrinkage, and effect of time rate of applied stress modulus of rupture are correlated with a proposed failure mechanism. Fatigue failure of plain concrete is related to the presence of discontinuities, the presence of stresses, some of a repeating nature, and the resistance of concrete to fracture or growth of discontinuities.

The present state of knowledge of flexural fatigue is reviewed with emphasis on those aspects which have practical significance on concrete pavement design.

(44) Raithby, K. D., and Whiffin, A. C., "Failure of Plain Concrete under Fatigue Loading, A Review of Current Knowledge", Road Research Laboratory, Report LR 231, England, 1968.

This report reviews the current state of the art in fatigue of concrete and gives a brief description of the work in fatigue proposed by the Road Research Laboratory.

(45) Raju, N. K., "Fatigue of High Strength Concrete in Compression", Ph.D. Thesis, University of Leeds, United Kingdom, August 1968.

This thesis deals with the crack initiation and propagation under a unidirectional compressive stress, and the nature of fatigue damage in plain concrete under this type loading.

(46) Neal, J. A., and Kesler, C. E., "The Fatigue of Plain Concrete", Proceedings, International Conference on the Structure of Concrete and Its Behavior under Load, Imperial College and Cement and Concrete Association, London, 1968, pp. 226-237.

This paper presents the different levels at which the problem of fatigue in plain concrete has been researched. It summarizes the information known about fatigue of plain concrete as of 1968 and makes recommendations for further studies.

(47) Bennett, E. W., and Raju, N. K., "Cumulative Fatigue Damage of Plain Concrete in Compression", International Conference of Structure, Solid Mechanics and Engineering Design in Civil Engineering Materials, University of Southampton, Paper 94, April 1969, pp. 1-14.

This paper reports an investigation of the nature of fatigue damage under unidirectional compressive stress. Crack initiation and propagation are examined by special tests, and an analysis is made of the position and orientation of the cracks developed in a prism under static and repeated loading. This is compared with the results obtained by pulse velocity measurements and with the strain behavior observed in fatigue tests.

(48) Chandra, S., "Fracture of Concrete under Monotonically Increasing, Cyclic, and Sustained Loading", Ph.D. Thesis, University of Colorado, June 1969.

All specimens were prismatic in shape. Paste and mortar specimens were 2" x 2" in cross section and 6" longitudinal; concrete specimens were 4" x 4" x 12" and stone specimens were 1" x 1" x 3". The purpose of this study was to investigate the failure phenomenon of concrete subjected to monotonically increasing, cyclic, and sustained loading.

It was found that the inelastic behavior of mortars and concrete is indicated by the existence of initiation stress and a critical stress where both Poisson's ratio and the volume of concrete start to increase. It was found that all things being equal, cyclic load was more damaging than sustained loading.

(49) Raju, N. K., "Small Concrete Specimens under Repeated Compressive Loads by Pulse Velocity Technique", Journal of Materials, JMLSA, Vol. 5, No. 2, June 1970, pp. 262-272.

The paper deals with the use of an ultrasonic pulse velocity technique for the study of microcracks formed in high strength concrete in a state of uniaxial compression under static and repeated loads.

Tests conducted on prismatic concrete specimens have revealed significant differences in the magnitude of pulse velocity decrease under static- and repeated-load systems. The progressive nature of the failure in concrete under repeated loads is studied by the parameter percentage decrease in pulse velocity in relation to the percentage of fatigue life. An empirical relation between the parameters was established from the test data which could be used to predict the remaining fatigue life of a partially fatigued specimen.

(50) Shah, S. P., and Chandra, S., "Fracture of Concrete Subjected to Cyclic and Sustained Loading", Proceedings, ACI, Vol. 67, No. 10, October 1970, pp. 816-825.

To study the mechanics of failure of concrete subjected to slowly applied cyclic or sustained stresses, sealed specimens of

paste and concrete were tested under the maximum cyclic or sustained compressive stresses of 60 to 90 percent of the ultimate stress. Volumetric strains, ultrasonic velocity and attenuation, and internal microcrack propagation were examined. Sustained or cyclic loading resulted in progressive crack propagation. For stresses lower than 70 percent of the ultimate, when failure did not occur within the test duration, cracks propagated at a relatively slow rate. Crack growth under sustained stresses appeared to result from the phenomenon of stress corrosion while under cyclic loading the process of load repetition seemed to play an important role. Sustained loading also had a strengthening effect on concrete, probably because of the consolidation of the hardened cement paste.

(51) Ballinger, C. A., "The Cumulative Fatigue Damage Characteristics of Plain Concrete", Office of Research, Federal Highway Administration, U. S. Department of Transportation, August 1970.

A laboratory study was conducted to investigate the fatigue characteristics of plain concrete. The primary goal was to evaluate the effects of variable loads on the fatigue life and to determine whether the Miner hypothesis adequately represents cumulative damage.

It is concluded that the Miner hypothesis represents the cumulative damage characteristics of plain concrete in a reasonable manner. Also the accuracy of the S/N Diagram is very dependent on the accuracy of the prediction of the normalizing strength values.

(52) Raju, N. K., "Effect of Stress Concentrations on the Static and Fatigue Strength of Concrete in Compression", Proceedings, Materiaux et Construction, Vol. 3, No. 14, 1970, pp. 85-89.

The effect of stress concentrations produced by a circular hole on the static and fatigue strength of plain concrete in uniaxial compression was investigated by experiments. Tests conducted on prismatic specimens showed that the ultimate static compressive strength and the fatigue strength are not significantly altered by the introduction of a stress concentrating hole.

The air cavities and microcracks inherently present in the concrete affect the ultimate strength and deformation characteristics of the material to such an extent that the effect of the artificially introduced stress concentrations is negligibly small.

(53) Awad, M. M., "Strength and Deformation Characteristics of Plain Concrete Subjected to High Repeated and Sustained Loads", Ph.D. Thesis, University of Illinois, January 1971.

This investigation dealt with the study of the strength and deformation characteristics of plain concrete when subjected to repeatedly applied high compressive loads. It was found that the static stress-strain relationship was highly dependent on the strain

rate. It was reported that the failure strains under repeated loads are higher, the lower the level of the maximum stress, the smaller the stress range, or the smaller the stress rate. A model to predict the failure strain is proposed.

Tests were conducted on 4 in. by 4 in. by 12 in. plain concrete prisms.

(54) Kesler, C. E., "Fatigue and Fracture of Concrete", Stanton Walker Lecture Series on the Materials Sciences, Lecture No. 8, Nov. 1970.

This paper summarizes the information then known concerning the fatigue characteristics of plain concrete. It also covers areas of needed research as of 1970.

(55) Raju, N. K., "Comparative Study of the Fatigue Behavior of Concrete, Mortar, and Paste in Uniaxial Compression", Proceedings, ACI, Vol. 67, No. 6, June 1970, pp. 461-463.

The effect of repeated loads on the fatigue strength of plain concrete, cement mortar and paste in uniaxial compression was investigated by experiments. Fatigue tests on prismatic specimens of concrete and its constituents exhibited essentially the same fatigue characteristics. The deformation studies indicated that the instantaneously recoverable elastic strains, with a limiting value towards failure in static and fatigue tests, could serve as a macroscopic failure criterion.

(56) Raithby, K. D., "Failure of Concrete under Repeated Traffic Loading", Concrete, Vol. 4, No. 10, London, October 1970, pp. 403-407.

The Road Research Laboratory embarked on an extensive programme of fatigue tests on road materials. In this article, details of that portion of the programme which deals with concrete are given. This is preceded by a historical survey of previous research, the results of which have formed the basis of the present investigation, and been used to draw valid conclusions about the effects of variables which may influence the fatigue life of concrete roads.

(57) Neville, A. M., "Fatigue Strength", Hardened Concrete: Physical and Mechanical Aspects, ACI Monograph No. 6, 1971. pp. 79-87.

This paper presents information taken from previous research concerning the fatigue characteristics of plain concrete.

(58) Yoshimoto, A., Ogino, S., and Kawakami, M., "Microcracking Effect on Flexural Strength of Concrete after Repeated Loading", Proceedings, ACI, Vol. 69, No. 4, pp. 233-240.

The purpose of this investigation was to study the flexural strength of concrete beam specimens after repeated loading. The test results showed that some specimens produced an increase in flexural strength after repeated loading. An attempt is made to explain this strength increase. It appears likely that microcrack propagation is responsible for this increase in flexural strength.