

PROBABILISTIC ANALYSIS OF THE  
EIGHT-HOUR-AVERAGED CO IMPACTS OF HIGHWAYS

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(The opinions, findings, and conclusions expressed in this report are those of the author and not necessarily those of the sponsoring agencies.)

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## SUMMARY

This report describes a method for estimating the probability that a highway facility will violate the eight-hour National Ambient Air Quality Standard (NAAQS) for carbon monoxide (CO). The method is predicated on the assumption that overlapping eight-hour time periods which each yield average CO levels in excess of the NAAQS specified maximum level are counted as single rather than multiple pollution episodes.

Air pollution levels are largely subject to the random influences of wind speed, wind direction, and atmospheric turbulence. Thus, with respect to an air quality standard, the acceptability of a proposed facility cannot be established deterministically. Since pollution levels seem best modeled as random variables, it is natural to address potential air quality impacts by estimating the probability of violating the relevant air quality standards.

2550

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BACKGROUND AND INTRODUCTION

In this section previous work on the problem of predicting air quality impacts is examined and the inadequacies of the results obtained are illustrated. Then, the analysis to be used in this report is introduced and justified and its major elements are outlined.

Previous work on the problem of assessing the air quality impact of a proposed facility centered on fitting observed or simulated pollution levels to some distribution function such as the lognormal, and comparing the order statistics of this distribution to the pollution level specified in the relevant air quality standard. In 1965, Zimmer and Larsen (1965) cited empirical evidence from one-year data histories from Chicago, Cincinnati, Los Angeles, New Orleans, Philadelphia, San Francisco, and Washington, D. C., showing that pollution levels for a number of air pollutants were approximately lognormally distributed for all averaging times. In 1967, Larsen et al. (1967) presented empirical results from the analysis of five-year data histories from these same cities (excluding New Orleans) which again indicated that pollutant concentrations were approximately lognormally distributed for all cities for all averaging times. In 1971, the EPA published, as part of its air pollution series, a manual by Larsen (1971) in which he suggests that the expected annual maximum concentration, which he estimates as the exponential of the maximum order statistic from a normal distribution, be used as the design value when determining control strategies and implementation plans.

Subsequently, Patel (1973) noted that in making the above suggestion, Larsen had neglected the sequential dependence of air pollution levels, and had reversed the order of expectation and exponentiation in the determination of the maximum order

statistic. Thus, Patel stated, Larsen's calculation of the expected annual maximum concentration, which was based on the assumption of a sequence of mutually independent, normally distributed random variables, was in error. In his response, Larsen (1973) agreed that pollution levels are sequentially dependent, but he argued that the use of his model was justified because it was an empirical model intended to be an approximation and because it was helpful in calculating a design value. Larsen also pointed out that by using the expected annual maximum rather than the expected annual second-maximum as the design value, his method would tend to minimize the frequency of violating the standards. Larsen also claimed in his response that the interchange of expectation and exponentiation would not produce significant errors. Furthermore, he said that the data suggested his method underpredicted the expected annual maximum about as often it overpredicted it and that, therefore, his method was the recommended one. In commenting on the Larsen-Patel discussion, Neustadter and Sidik (1974) presented the results of a Monte Carlo simulation which indicated that for 24-hour averaged pollution levels (i.e., a yearly sample size of 365), Larsen's expected annual maximum could differ from the observed average annual second-maximum by up to 117%.

In 1976, Kalpasanov and Kurchatova (1976) reported that the lognormal distribution did not fit their air pollution data from Sofia, Bulgaria, as measured by goodness-of-fit tests. This conclusion, as later explained by Mage and Ott (1978), was due to the fact that the "fit" of the lognormal distribution used by Larsen and others was on an "engineering" basis rather than on a "statistical" basis. Air pollution data cannot be said to be lognormally distributed on the basis of goodness-of-fit tests; however, their distribution functions do plot as approximate straight lines on lognormal probability paper, which has led many investigators to assume that the lognormal distribution is an acceptable approximation.

Mage and Ott (1975) and Ott and Mage (1976) hypothesize that air pollution data can be fitted (in an engineering sense) by a censored three-parameter lognormal distribution. They show, using data from ten air quality studies, that the censored three-parameter lognormal distribution produces a better fit, in the squared error sense, to the data than does the standard two-parameter lognormal distribution function. Larsen (1977) subsequently proposed that air pollution data be modeled as variables from a three-parameter lognormal distribution.

In 1974, Turner (1974) first addressed the problem of predicting the impact of a proposed rather than an existing facility. He suggested using simulation to develop a data base from which one could extract the annual maximum pollution level. In 1976, Kumar, Lamb, and Seinfeld (1976) suggested a method for assessing the impact of a proposed facility using the ideas developed by Larsen. Their approach was to predict the annual mean and mean square concentrations and use these to estimate the parameters of the supposed underlying lognormal distribution. They suggested that the annual mean and mean square concentrations could be efficiently calculated using a climatological weighting scheme. (For instance, see Calder [1971].) However, they required the same independence assumption as Larsen required to estimate the expected annual maximum concentration from the supposed underlying lognormal distribution, and their method compares an expected annual maximum to a standard based on the annual second-maximum. Tikvart and Freas (1977) and Venkatram (1979) have proposed other simulation and distribution fitting methods, which again address the problem by finding some sort of order statistic to compare to the appropriate air pollution standard. These methods also rely on the independence assumption made by Larsen.

Hirtzel and Quon (1979), in analyzing data on one-hour CO concentrations in Chicago, found a high degree of persistent correlation among successive values of one-hour concentrations. This sequential dependence, they explain, increases the variability of the error of estimate of the annual mean concentration and other air quality parameters. Thus the sequential dependence of air pollution levels is responsible for increased uncertainty in the parameters of distributions fitted to real or simulated air pollution data, and it is also responsible for the error resulting from the use of results for independent observations to estimate pollution order statistics.

Other methods for estimating the impact of a proposed highway facility are of the so-called worst-case variety such as that proposed by Habegger and Wolsko (1974). These worst-case methods consist of a single estimate of pollution level (generally obtained from a Gaussian dispersion model with source, background, and meteorological input parameters intended to maximize the estimated pollution level) that is compared to the relevant air quality standard. Just as the practice of comparing the maximum order statistic to a standard based on the second-maximum has been questioned, the practice of comparing the worst-case pollution level to such a second-maximum standard is questionable. Furthermore, because the source, background, and meteorological input parameters for worst-case analyses are subjective estimates, such analyses yield only rule-of-thumb estimates of environmental impact.

From the work that has been done in determining the environmental acceptability of a proposed highway facility, two major problems are evident. The first is the sequential independence assumption for pollution levels in light of the high serial correlations which have been documented for atmospheric pollution levels; the second is the general practice of comparing a maximum order statistic to a standard that is based on the annual second-maximum.

The object of this report is to present a method for determining the air quality impact of a proposed highway facility that does not require the dependence assumption, the lognormal (or any other distribution function) assumption, or the comparison of an order statistic to the standard. The method presented is based on the work done by Carpenter (1979).

Air pollution levels are largely subject to the random influences of wind speed, wind direction, and atmospheric turbulence. Thus the air quality acceptability of a proposed facility cannot be established deterministically. Since pollution levels seem best modeled as random variables, it is natural to answer the question of the air quality acceptability of a proposed facility by estimating the probability that such a facility would violate the relevant NAAQS. Estimating the impact of a proposed facility probabilistically would eliminate the need for rule-of-thumb comparisons of order statistics with the standard and the need for the generally applied assumption of sequential independence. This report presents a simulation method for estimating the probability that a proposed highway facility would violate the eight-hour NAAQS for CO.

#### INTERPRETATION OF THE NAAQSs

The NAAQSs cover a variety of pollutants and a variety of averaging times. The principal averaging times considered are one-hour, three-hour, eight-hour, one-day (twenty-four hour), and one-month. When working with one-hour averaging times, the accepted interpretation of the standards is that they apply to nonoverlapping-one-hour, on-the-hour time periods. Similarly, the accepted interpretation of standards which employ one-day averaging times is that they apply to nonoverlapping one-day (generally commencing at midnight) time periods, and the accepted interpretation of standards which employ one-month averaging times is that they apply to calendar months. However, standards which employ either three-hour or eight-hour averaging times have historically been open to questions



of interpretation regarding the overlapping of time periods. The current Virginia State Air Pollution Control Board and EPA policy on such standards is that they apply to all possible three- or eight-hour periods (which implies overlapping), but when counting the number of periods during which the standard was exceeded, overlapping periods are not allowed.\* This condition limits the number of violating periods that may be counted. For instance, the maximum number of violating eight-hour periods in any twenty-four hour period is three. Thus, when working with three- or eight-hour averaging times, one must examine all possible such periods while counting overlapping violations as single violations. This situation of allowing or examining all possible time periods but limiting the count of violations to nonoverlapping periods is what distinguishes the analyses of standards which employ three- or eight-hour averaging times from the analyses of the other standards. In the remainder of this report, the techniques applied by Carpenter (1979) to the analysis of nonoverlapping standards are extended to apply to overlapping standards for which overlapping violations are disallowed.

#### Probability of Eight-Hour Violation

Carpenter (1979) has shown that the probability of violating a nonoverlapping air quality standard may be expressed as a simple function of the relative pairwise and single frequencies of exceeding such a standard. Carpenter's work was predicated on the assumption that a yearly sequence of binary pollution indicators,  $x(t)$ , derived from a sequence of pollution levels,  $\chi(t)$ , constitutes a Markov chain.

The Markov hypothesis states that the dependence of  $x(t)$  on the history of the indicator process [ $x(1), x(2) \dots x(t-1)$ ] up to time  $t-1$  is totally explained by the dependence of  $x(t)$  on  $x(t-1)$ . This hypothesis thus implies that the entire dependence structure of a sequence of pollution indicators can be explained by the pairwise dependence of consecutive pollution levels. Thus the Markov assumption allows one to write

$$P(x_1, x_2, x_3 \dots x_T) = P(x_1) \left[ \prod_{t=1}^{T-1} P(x_{t+1} | x_t) \right]. \quad \dots (1)$$

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\*R. W. Flournay 1980: personal communication.

Defining  $x(t)=1$  if the pollution level  $\chi(t)$  at time  $t$  exceeds the standard and  $x(t)=0$  otherwise, and defining  $P_i=P(x_t=i)$  and  $P_{ij}=P(x_t=j|x_{t-1}=i)$ , we may use equation 1 to write

$$\begin{aligned}
 P(V) &= 1 - P(x_1=x_2=\dots x_T=0) - \sum_{t=1}^T P(x_t=1 \text{ and } x_k=0 \text{ for } k \neq t), \\
 &= 1 - P_0 P_{00}^{T-1} - P_0 P_{00}^{T-2} P_{01} - P_1 P_{10} P_{00}^{T-2} - (T-2) P_0 P_{01} P_{10} P_{00}^{T-3}, \\
 &\dots(2)
 \end{aligned}$$

where  $P(V)$  is the probability of violating a nonoverlapping standard and  $T$  is the total number of nonoverlapping time periods in a year.

Unhappily, the result given by equation 2, which is a direct result of the Markov assumption, cannot be applied to standards that employ overlapping time periods. The Markov assumption states that a knowledge of the entire history of a process  $[x(1), x(2), \dots, x(t-1)]$  up to time  $t-1$  will supply no more information about  $x(t)$  than a knowledge of  $x(t-1)$  alone will supply. To see that the Markov assumption cannot apply to overlapping time averages, let us first define

$$\bar{\chi}(t) = [\chi(t) + \chi(t+1) + \chi(t+2) + \dots + \chi(t+N)] / (N+1). \quad \dots(3)$$

From this definition we see that  $\bar{\chi}(t)$  is the  $(N+1)$  - hour average pollution level at time  $t$ . From equation 3 we can write the relation

$$\bar{\chi}(t) = [N+1)\bar{\chi}(t-1) - \chi(t-1) + \chi(t+N)] / (N+1), \quad \dots(4)$$

which relates  $\bar{\chi}(t)$  and  $\bar{\chi}(t-1)$  and demonstrates that  $\bar{\chi}(t)$  depends on  $\bar{\chi}(t-1)$ . However, from equations 3 and 4, we can write

$$\chi(t-1) = (N+1)\bar{\chi}(t-2) - \chi(t-2) - \sum_{i=0}^{N-2} \chi(t+i), \quad \dots(5)$$

which allows us to rewrite equation 4 as

$$\bar{\chi}(t) = [(N+1)\bar{\chi}(t-1) - (N+1)\bar{\chi}(t-2) + \chi(t-2) + \sum_{i=0}^{N-2} \chi(t+i)] / (N+1). \quad \dots(6)$$

Thus we can see from equation 6 that  $\bar{\chi}(t)$  depends on both  $\bar{\chi}(t-1)$  and  $\bar{\chi}(t-2)$ . This process can be carried out again to reveal that  $\bar{\chi}(t)$  depended on  $\bar{\chi}(t-3)$ ,  $\bar{\chi}(t-2)$ , and  $\bar{\chi}(t-1)$ . In fact, consideration of equations 3, 4, 5, and 6 reveals that  $\bar{\chi}(t)$ , depends not on only the previous value  $\bar{\chi}(t-1)$ , but rather on the entire history [ $\bar{\chi}(1), \bar{\chi}(2) \dots \bar{\chi}(t-1)$ ]. Thus equations 3, 4, 5, and 6 demonstrate that the Markov property does not apply to sequences of overlapping averages; i.e., for sequences of overlapping time averages, the next time average in the sequence will depend on the entire history of the process rather than on just the previous time average.

Since the Markov assumption is not valid for overlapping time averages, we find that there is no way to simplify the expression for the probability of violating an overlapping standard. In particular, equation 2 is not valid for overlapping standards and, furthermore, the expression for evaluating the probability of violating an overlapping standard will involve estimates of probabilities involving  $T = 8,760$  terms, each of which is the probability of a sequence of up to  $T = 8,760$  individual elements. Such a problem cannot be solved analytically because there is no feasible way of obtaining estimates of probabilities of sequences of such length. Fortunately, simulation procedures coupled with the binomial probability function can still be applied to the problem.

Since any year-long sequence of eight-hour averaged pollution levels can be determined to either violate or not violate the eight-hour air quality standard, the binomial probability distribution, which applies to binary (i.e., "yes", "no") data, can be employed to estimate  $P(V_8)$ , the probability of violating the eight-hour standard. In particular, for  $N$  independent years of data (each year of data commencing with hour  $\tau$ , where  $\tau=1$  is defined as a calendar year, and  $1 \leq \tau \leq T$ ), let us define  $Y_\tau(i)$ ,  $1 \leq i \leq N$ , as  $Y_\tau(i)=1$ , if the data from year  $i$  indicate a violation of the relevant standard and  $Y_\tau(i)=0$  otherwise. From this definition of  $Y_\tau(i)$ , we can employ the binomial probability distribution to find  $\hat{P}_\tau(V_8)$ , the estimate of  $P_\tau(V_8)$ , the conditional probability of violating the eight-hour standard for a year, given that the year commenced with hour  $\tau$ , as

$$\hat{P}_\tau(V_8) = [\sum_{i=1}^N Y_\tau(i)]/N, \quad \dots(7)$$

and we can estimate the variance of this estimate as

$$\text{Var}[\hat{P}_\tau(V_8)] = \hat{P}_\tau(V_8)[1 - \hat{P}_\tau(V_8)]/N. \quad \dots(8)$$

Furthermore, since each possible  $\tau$  could be chosen (assuming no bias) with probability  $1/T$ , we can use equation 7 to find  $\hat{P}(V_8)$ , the estimate of the probability of violating the eight-hour standard for any one-year period without regard to the starting time as

$$\begin{aligned} \hat{P}(V_8) &= \sum_{\tau=1}^T \hat{P}_\tau(V_8)P(\tau) \\ &= \sum_{\tau=1}^T \hat{P}_\tau(V_8)/T. \end{aligned} \quad \dots(9)$$

Since the NAAQSs do not specify starting times for the various standards, the probability of violating such standards must be independent of starting time. Thus  $P(V_8)$ , which from equation 9 is independent of  $\tau$  (since it is a sum over all possible values of  $\tau$ ), must be the desired estimate of the probability of violating an eight-hour NAAQS.

The variance of  $\hat{P}(V_8)$  is defined by the following relationship, which gives the variance of a sum of products,

$$\begin{aligned} \text{Var} [\hat{P}(V_8)] &= \text{Var} \left[ \sum_{\tau=1}^T P_{\tau}(V_8)/T \right] \\ &= \text{Var} \left[ \sum_{\tau=1}^T \hat{P}_{\tau}(V_8) \right] / T^2 \\ &= \sum_{\tau=1}^T \text{Var}[\hat{P}_{\tau}(V_8)] / T^2 \\ &\quad + \sum_{i=1}^T \sum_{j=1}^T \text{Cov}[\hat{P}_i(V_8), \hat{P}_j(V_8)] / T^2. \quad \dots(10) \end{aligned}$$

While equation 8 can be employed to determine  $\text{Var}[\hat{P}_{\tau}(V_8)]$  in equation 10, the covariance terms are not evaluable. However, the covariance terms can be bounded. Since  $\hat{P}_i(V_8)$  and  $\hat{P}_j(V_8)$  are determined from N-year long sequences of data that overlap (for  $i$  assumed  $> j$ ) by  $T+1-j$  individual data items,  $\hat{P}_i(V_8)$  and  $\hat{P}_j(V_8)$  should be positively correlated and should indeed have a correlation of almost 1.0 for  $i \approx j$  that decreases to a correlation of almost 0.0 for  $i-j \approx T$ . Thus, we can bound  $\text{Var}[\hat{P}(V_8)]$  by noting that

$$0 \leq \text{Cov}[\hat{P}_i(V_8), \hat{P}_j(V_8)] \leq \text{Var}[\hat{P}_i(V_8)]$$

to obtain

$$\sum_{\tau=1}^T \text{Var}[\hat{P}_{\tau}(V_8)]/T^2 \leq \text{Var}[\hat{P}(V_8)] \leq \sum_{\tau=1}^T \text{Var}[\hat{P}_{\tau}(V_8)]/T. \quad \dots(11)$$

#### CONCLUSIONS AND RECOMMENDATIONS

The primary results of this report are contained in equations 7, 8, 9, and 11. Equations 7 and 9 estimate the probability of violating an eight-hour air quality standard, and equations 8 and 11 serve to bound the variance of this estimate. The only assumption used in the development of these results is that violations of eight-hour air quality standards will be determined in accordance with the present guidelines of the Virginia State Air Pollution Control Board and those of the EPA. Thus the results of this report are immediately implementable.

The author recommends that a project be undertaken to develop the computer software necessary to implement the method described in this report.

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2568



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2570