## FINAL CONTRACT REPORT

# THE BEHAVIOR OF INTEGRAL ABUTMENT BRIDGES

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Recommendations for improving the performance of integral bridges are included, and actions for improvement of integral bridge behavior are suggested.						
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(The opinions, findings, and conclusions expressed in this report are those of the authors and not necessarily those of the sponsoring agency)

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#### ABSTRACT

This report presents findings of a literature review, a field trip, and a finite element analysis pertaining to integral bridges. The purpose of the report is to identify problems and uncertainties, and to gain insight into the interactions between the foundation piles, the integral abutment, and the surrounding ground. The field trip included visits to six bridges arranged by Mr. Park Thompson from the Staunton district.

Pertinent literature is reviewed and findings are presented. Important factors identified on the basis of this review are settlement of the approach fill, loads on the abutment piles, the nature of the abutment displacements and the associated earth pressure distribution, secondary loads on the superstructure, and soil structure interaction effects. The causes of approach fill settlement and possible mitigation techniques are discussed

Recommendations for improving the performance of integral bridges are included, and actions for improvement of integral bridge behavior are suggested.

## THE BEHAVIOR OF INTEGRAL ABUTMENT BRIDGES

Sami Arsoy

## **Richard M. Barker**

## J. Michael Duncan

## **INTRODUCTION**

A bridge should be designed such that it is safe, aesthetically pleasing, and economical. Prior to the 1960s, almost every bridge in the U.S. was built with expansion joints. These expansion joints often did not perform as well as intended. They required considerable maintenance, which undermined the economical operation of the bridges. Accident and vehicle damage caused by defective expansion joints raised safety concerns. Starting in the early 1960s, the use of jointless bridges for new bridge construction attracted widespread interest.

Jointless bridges can be classified into four groups (Wolde-Tinsea and Klinger, 1987):

- flexible arch bridges,
- slip joint bridges,
- abutmentless bridges, and
- integral bridges

In the U.S., the term integral bridge usually refers to bridges with short stub-type abutments connected rigidly to the bridge deck without joints. This rigid connection allows the abutment and the superstructure to act as a single structural unit (Figure 1). Typically single rows of piles provide foundation support for the abutments.

The principal advantages of integral bridges include the following:

- Lower construction costs due to elimination of joints (Yang et al., 1985; Greimann et al., 1987; Soltani and Kukreti, 1992).
- Lower maintenance costs due to elimination of joints (Yang et al., 1985; Soltani and Kukreti, 1992; Hoppe and Gomez, 1996). In conventional bridges, much of the cost of maintenance is related to repair of damage at joints.

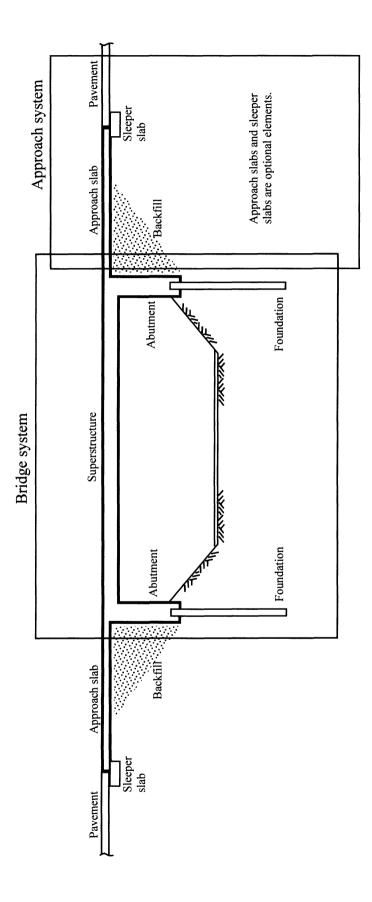


Figure 1. Simplified geometry of an integral abutment bridge.

- Improved seismic performance (Hoppe and Gomez, 1996).
- Fewer piles are required for foundation support (Soltani and Kukreti, 1992; Hoppe and Gomez, 1996). No battered piles are needed (Burke, 1996).
- Construction is simple and rapid (Burke, 1996; GangaRao et al., 1996; Wasserman and Walker, 1996).
- Greater end span ratios are achievable (Burke, 1996; Wasserman and Walker, 1996).
- The smooth, uninterrupted deck of the integral bridge is aesthetically pleasing, and it improves vehicular riding quality (Loveall, 1996; Soltani and Kukreti, 1996).

As temperatures change daily and seasonally, the lengths of integral bridges increase and decrease, pushing the abutment against the approach fill and pulling it away. As a result the bridge superstructure, the abutment, the approach fill, the foundation piles and the foundation soil are all subjected to cyclic loading, and understanding their interactions is important for effective design and satisfactory performance of integral bridges.

## **PURPOSE AND SCOPE**

This report presents findings of a literature review, field inspections, and finite element analysis of integral bridges. The purpose of this report is to present

- findings of a literature review, and field inspections of integral bridges to identify problems and uncertainties,
- results of a finite element analysis performed to gain insight into the interactions between integral abutments, approach fills, foundation piles and foundation soils, and
- practical recommendations based on the literature review conducted.

## **METHODS**

## Literature Review

Articles, research reports, and published journal papers were collected using the resources of the Newman Library of Virginia Tech and the Ozawa Library of the Charles E. Via, Jr. Department of Civil and Environmental Engineering of Virginia Tech. The collected articles, research reports and papers served as the primary source in achieving the purposes of this study. The field inspections and finite element analysis of integral bridges were secondary sources for this report.

#### **Field Inspections of Integral Bridges**

A trip to the Staunton district of VDOT was made on May 25, 1999 to observe the behavior of integral bridges in the area. Mr. Park Thompson from the Staunton district organized the trip. Of the six bridges visited, four are semi-integral, one is fully integral, and one is retrofitted (converted from a jointed bridge to a continuous deck bridge). None of the bridges have approach slabs. Throughout the field trip, Mr. Thompson and Dr. Ed Hoppe from VTRC discussed the problems and short-term needs for integral bridge design, construction, and maintenance. The following bridges were chosen for inspection because they represent typical integral bridges in the Staunton district:

- 1. Bridge No. 6051 on Rt. 635 over Christians Creek: semi-integral bridge.
- 2. Bridge on 2<sup>nd</sup> street over N&W railroad in Staunton: fully integral bridge
- 3. Bridge on Rt. 257 over I 81 in Rockingham County: semi-integral bridge.
- 4. Beaver Creek crossing on US 33: retrofitted to continuous deck bridge.
- 5. Briery Creek crossing on Rt. 731: semi-integral bridge; and
- 6. Bridge on Rt. 257 near George Washington Forest: semi-integral bridge.

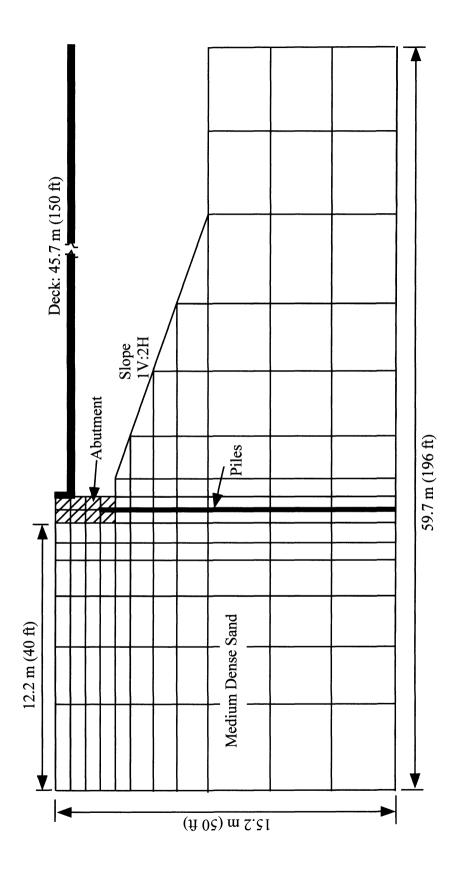
#### **Finite Element Analysis**

The purpose of the analysis was to model expansion of the superstructure due to increasing temperature. The load on the abutment due to expansion of the superstructure was modeled by applying forces at the node where the superstructure is connected to the abutment.

A finite element analysis of a 92-m long, 25-m wide integral bridge was performed to gain insight into the interactions between the superstructure, the abutment, the approach fill, the foundation piles, and the foundation soil. It was assumed that the bridge consists of nine equally spaced W44x285 steel girders and a 23-cm thick concrete deck, resting on 2.6-m high 0.9-m thick abutments, which are supported by equally spaced eighteen HP10x42 steel piles in medium dense sand, as shown in Figure 2. The analyses were performed using the finite element program SAGE 2.03 (Static Analysis of Geotechnical Engineering Problems) developed at Virginia Tech. SAGE is capable, among other features, of analyzing plane strain soil-structure interaction problems such as the integral bridge shown in Figure 2.

The bridge was modeled as a plain strain problem, with symmetry around the centerline of the bridge. The finite element mesh used is shown in Figure 2. An enlarged view of the integral abutment is depicted in Figure 3.

The bridge superstructure and the piles were modeled as beam-bar elements with linear stress-strain properties. The abutment was modeled using four node quadrilateral elements with





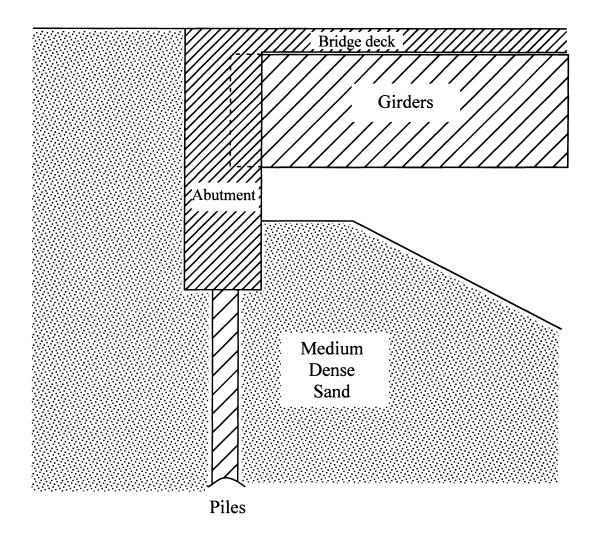


Figure 3. Close-up view of the abutment considered in finite element analysis

linear stress-strain properties. The approach fill and the foundation soil were modeled using four node quadrilateral and three node triangular elements with hyperbolic material properties Tables 1 and 2 summarize the material properties used in the analyses.

The loads applied represent the forces exerted on the abutment by the superstructure (the girders and the deck) as the temperature increases and the superstructure expands. The stiff girders and the attached deck restrain rotation of the abutment as the abutment is loaded, and including the flexural stiffness (EI) of the superstructure in the analysis models this important aspect of the structural behavior. However, because the axial loads in the girders and the deck were not of concern for this analysis, their combined axial stiffness (EA) was set equal (approximately) to zero.

The characteristics of the finite element model and results are thus:

- The applied force corresponds to the axial force generated in the superstructure by temperature increase,
- The resulting displacement of the abutment corresponds to the actual displacement of the bridge due to temperature increase, and
- The forces induced in the abutment, the approach fill, the foundation piles and the foundation soils show how the loads are carried by increased earth pressures in the approach fill and increased shear forces on the foundation piles.

Six load steps were used in the analysis. In the first step, all of the elements except the beam-bar elements (superstructure and pile) were activated and gravity was "turned on." At the end of this step, displacements were set to zero, and the initial stresses due to gravity turn-on were retained. In the second step, the beam-bar elements were activated, and the first of five loads was applied to the abutment. In the third to sixth steps, additional loads, of equal magnitude, were applied.

Table 1. Properties of structural elements

Property \ Element	Girders and bridge deck	Abutment	Piles
E, Young's modulus (kPa)	200x10 <sup>6</sup>	23.9x10 <sup>6</sup>	200x10 <sup>6</sup>
v, Poisson's ratio	-	0.2	-
EI, Flexural stiffness (kN.m <sup>2</sup> /m)	0.68x10 <sup>6</sup>	-	$4.1 \times 10^3$

Parameter	Approach fill and foundation		
γ, Unit weight (kN/m <sup>3</sup> )	18.2		
K <sub>o</sub> , Coefficient of at rest pressure	0.5		
c, Cohesion	0		
φ, Friction angle (degrees)	36		
$\sigma_t$ , Tensile strength (kPa)	0		
R <sub>f</sub> , Failure ratio	0.7		
K, Young's modulus coefficient	300		
Kur, Unload-reload coefficient	1000		
n, Young's modulus exponent	0.4		
K, Bulk modulus coefficient	75		
m, Bulk modulus exponent	0.2		

Table 2. Hyperbolic stress-strain and strength parameters for approach fill and foundation soils

#### RESULTS

#### **Status of the Bridges Inspected**

Table 3 contains information about the bridges inspected. The lengths of the bridges and the dates of construction are approximate, except for Bridge No. 3. Photographs were taken, and observations concerning the structural integrity of the bridges were recorded. A brief summary of the inspections is presented below.

All of the bridges inspected were well maintained. No performance concerns were noted by Mr. Thompson or Dr. Hoppe for any of the bridges. None of the bridges has approach slabs. As expected, pavement patching at the ends of the bridges constituted most of the maintenance. The longer the bridge, the greater the extent of the pavement patching because of the temperature-induced cyclic movements.

Route 257 overpass on I 81 is the longest semi-integral bridge built in the Staunton district. Therefore, it has experienced more pavement distress at the ends of the bridge than the other bridges. No approach slabs were used on this bridge because the designers believed that

Bridge No.	Date built	Length (m.)	Width	Wing wall orientation	Comments
1	1991	81	2 lanes	Straight	2 spans
2	1983	56	2 lanes	Folded	3 spans, short end span ratio (about 0.4)
3	1993	98	25 m.	Straight	2 spans, longest semi-integral bridge in Staunton district
4	1930s	~ 55	2 lanes	Straight	4 spans, retrofitted in 1994
5	-	~ 60	2 lanes	Folded	On spread footings
6	1937	~ 30	2 lanes	Folded	2 spans, oldest semi-integral bridge in Staunton district

Table 3. Summary statistics of the bridges inspected in the Staunton district.

remedial actions with approach slabs would be more costly and inconvenient to the public than periodically re-grading the settling approach. Figure 4 shows the appearance of the re-graded pavement near the north abutment. Erosion, due probably to poor channeling of the surface water, is a minor problem, as shown in Figure 5. Despite these small problems, the bridge was performing well.

Hoppe and Gomez (1996) monitored the performance of bridge No. 3 from summer of 1993 to January of 1996. During the monitoring period, pavement patching on the approach was needed to cope with settlement. Approximately \$10, 000 for pavement patching was spent in the first two years of operation.

The other five bridges needed pavement work as well. However, the extent of pavement patching was smaller than that of the Rt. 257 overpass on I 81. The ride on all bridges appeared to be comfortable for most drivers.

Pavement repair at approaches to integral bridges that do not have approach slabs is common. Periodic inspection and pavement repair, as practiced in the Staunton district, is likely to be required for maintenance of all integral bridges.

### Problems and Uncertainties Associated with Integral Bridges Discussed in Published Literature

Despite the significant advantages of integral bridges, there are some problems and uncertainties associated with them. These include the following:



Figure 4. Re-graded pavement at the north end of the bridge.

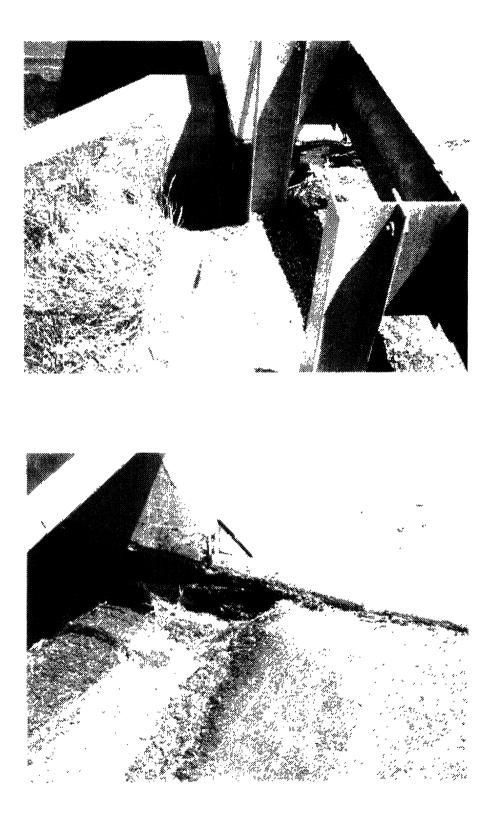


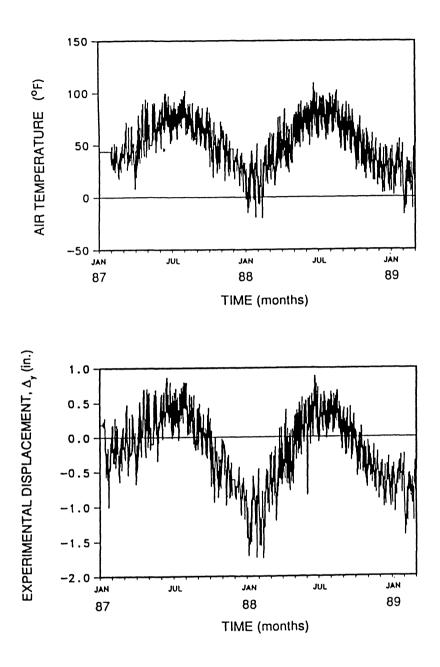
Figure 5. Erosion of backfill near the north abutment

- Temperature-induced movements of the abutment cause settlement of the approach fill, resulting in a void near the abutment if the bridge has approach slabs (Wolde-Tinsae and Klinger, 1987; Schaefer and Koch, 1992; Ruf et al., 1995; Hoppe and Gomez, 1996; Ng et al., 1998). Traffic loads also contribute to approach fill settlement (Wolde-Tinsae and Klinger 1987).
- Secondary forces (due to shrinkage, creep, settlement, temperature and earth pressure) can cause cracks in concrete bridge abutments (GangaRao et al., 1996; Soltani and Kukreti, 1996). Wing-walls can crack due to rotation and contraction of the superstructure (Wolde-Tinsae and Klinger, 1987).
- Skewed integral bridges tend to rotate under the influence of cyclic changes in earth pressures on the abutment (Hoppe and Gomez, 1996).
- Bridge abutments can be undermined due to water entering into the approach fills at the bridge ends (Wolde-Tinsae and Klinger, 1987).
- The piles that support the abutments may be subjected to high stresses as a result of cyclic expansion and contraction of the bridge superstructure. These stresses can cause formation of plastic hinges in the piles, and may reduce their axial load capacities (Soltani and Kukreti, 1996; Yang et al., 1985; Krauthammer et al. 1994).
- The application of integral bridge concept has limitations. Integral bridges cannot be used with weak embankments or subsoil, and they can only be used for limited lengths, although the maximum length is still somewhat unclear (Burke, 1996; Wasserman and Walker, 1996; Soltani and Kukreti, 1996). Integral bridges are suitable if the expected temperature-induced movement at each abutment is 51 mm (2 in.) or less (GangaRao et al., 1996), and somewhat larger movements may be tolerable.

These limitations are recognized by the agencies that have constructed integral bridges. A number of special features have been employed, involving special detailing practices. Some of these have worked well and are widely accepted, and some are debatable, such as the use of approach slabs to reduce the approach fill settlement. These are highlighted in the following sections.

#### **Bridge Displacement with Temperature**

Both daily and seasonal temperature changes affect integral bridges. Each daily variation in temperature completes a cycle of expansion and contraction, and the cycles repeat over time. The greatest expansion takes place during summer days, while the greatest contraction occurs during winter nights. These extreme temperature variations control the extreme displacements of integral bridges. Figure 6 shows the temperature variation and the measured displaced at the abutment of the Maple River Bridge located in northwest Iowa, which includes some of the most complete and valuable data related to the performance of integral bridges. The Maple River Bridge is 98 m long and 10 m wide with a skew of 30 degrees. The bridge has three spans and



Note: 1 inch = 25.4 mm, and  $^{\circ}C = (^{\circ}F - 32)/1.8$ 

Figure 6. Relationship between air temperature and longitudinal bridge displacement for Maple River Bridge. [From Girton, D. D.; Hawkinson, T.R.; and Greimann, L.F. (1991). Validation of design recommendations for integral-abutment piles, Journal of Structural Engineering, ASCE, Vol. 117, No. 7, July, pp. 2117-2134. Reproduced with permission of ASCE.] consists of a composite concrete deck and steel girders. Two piers of the bridge are located about 30 m from each abutment. As can be seen from Figure 6, the change in ambient temperature completes one cycle for a given day, and longer cycles seasonally. The maximum expansion and the maximum contraction of the bridge coincide with the maximum and minimum ambient temperatures.

The bridge material (steel or concrete) and the geometry of the bridge (curved or skewed) are important factors that affect the displacement of integral bridges.

The resistance provided by the approach fill is so small that it does not have any significant effect on the expansion of the integral bridge (Oesterle et al., 1998). The bridge expands and contracts practically as if the approach fill did not exist. Therefore, the amount of expansion and contraction of integral bridges can be calculated using the method recommended by AASHTO (1996), which does not consider the resistance provided by the approach fills.

#### Loads on Integral Bridge System

Integral bridges are subjected to dead and live loads (primary loads), and additional secondary loads due to creep, shrinkage, thermal gradients, and differential settlements. An adequate design needs to consider both vertical loads [due to dead and live loads] and secondary loads.

Burke (1993) promotes standardization of abutment-superstructure continuity connections such that the abutments need only to be designed for vertical loads for a wide range of applications. Porter et al. (1992) state that simple abutments are better than more complex abutments. Simple abutments, such as stub-type abutments, have been found to perform well in a field survey completed by Oesterle et al. (1998).

#### Shrinkage and Creep

Shrinkage and creep effects can be estimated using the Freyermuth (1969) method. The greatest effect of shrinkage is apparent on the positive moment of single spans and on the continuity connection at abutment of continuous spans (Chen, 1997 and Burke, 1993). Creep effects of continuous single span bridges are greater than shrinkage effects. Both creep and shrinkage are time dependent. Maximum shrinkage moments take place within 30 days of form removal (Burke, 1993 and Mattock, 1961), and creep effects continue for a longer period of time.

#### Temperature Gradient

Temperature gradients through the depth of the bridge beams generate secondary bending moments due to the fact that the centroid of the temperature distribution curve and the centroid of the cross-section of the bridge beams may not coincide. Emerson (1977), Hoffman et al. (1983), Imbsen et al. (1985), and Potgieter and Gamble (1989) studied the temperature distribution through bridge beams. The most important factors are 1) the maximum temperature differential and 2) the distribution of this differential across the depth of the beams (Oesterle et

al., 1998). The moments generated by the thermal gradient are similar to those generated by creep and shrinkage, and can be calculated as prescribed in AASHTO (1996). It appears that in moderate climates, the moments induced by thermal gradients can be ignored (Burke, 1993).

#### Differential Settlement

Differential settlements can also result in secondary bending moments. Barker et al. (1991) and AASHTO (1994) provide simple procedures to estimate differential settlements. If differential settlements are less than 38 mm (1.5 in.), the induced moments can be ignored (Chen, 1997).

#### **Buoyancy** Loads

The bridge superstructure of the integral bridge may also be subject to buoyancy loads. Integral bridges are likely to be subject to uplift when fully submerged. Integral bridges should be limited to areas where the bridge elevation is higher than the highest expected flood level (Burke, 1993), or the buoyancy loads should be considered in design.

#### Pavement Growth

Due to friction between the pavement and its subbase, the pavement does not contract to its original length after expanding. This residual expansion accumulates after repeated temperature cycles and results in pavement growth. Observations indicate that pavement growth at pressure relief joints can be rapid and incremental (Burke, 1993). Designers should consider the pressure generated by pavement growth, since it will be transmitted to the bridge in the form of longitudinal compressive force. James et al. (1991) documented a case of severe abutment damage for a bridge without pressure relief joints. Their numerical stress analysis indicated that the damage was caused by the longitudinal growth of continuous reinforced concrete pavement, causing excessive longitudinal pressures on the abutments.

#### **Behavior of Superstructure**

The superstructures of integral bridges are subject to both primary loads (loads acting on the conventional jointed bridges, i.e., dead and live loads, earthquake loads, etc.) and temperature induced secondary loads. Integral bridges must be able to withstand these loads.

Because of the rigid connections between the bridge deck and the abutments, integral bridges have improved seismic resistance compared to jointed bridges (Hoppe and Gomez, 1996).

#### **Behavior of Piers**

Pier connections can be either integral or semi-integral. Field surveys reveal that semiintegral piers experience no distress or noticeable cracking; therefore, the use of semi-integral piers is recommended (Oesterle et al., 1998).

#### **Behavior of Piles Supporting the Abutments**

Integral bridges are often supported on steel H-piles. Other steel piles (pipe piles open-end or filled with concrete) and prestressed concrete piles have also been used. Steel H-piles are often oriented for weak-axis bending to better accommodate bridge displacements. For a given deflection, weak-axis bending generates less stress in the piles than does strong-axis bending.

One of the most important design factors with integral bridges is the ability of the foundation piles to carry the vertical load even when the piles are subjected to temperature-induced displacements. The vertical-load carrying capacities of piles may be reduced due to lateral displacements (Greimann and Wolde-Tinsea, 1988). Piles can fail if the induced lateral loads are higher than the elastic buckling load. Analysis for reduction in vertical load capacity and elastic buckling load under temperature induced displacements can be carried out separately, because they are uncoupled (Greimann and Wolde-Tinsea, 1988).

The ability of piles to accommodate lateral displacements is a significant factor in determining the maximum possible length of integral bridges. In order to build longer integral bridges, pile stresses should be kept low. In addition to weak-axis bending orientation of piles, additional provisions can be made. For example, predrilled oversize holes filled with loose sand after pile driving have emerged as a common alternative (Yang et al., 1985).

Theoretically, predrilling procedure should work if the stiffness of the removed soil is higher than that of the loose sand. In other words, predrilling is a good practice in stiff to very stiff soil conditions. Wolde-Tinsea and Greimann (1985) and Greimann et al. (1986) have investigated this issue through finite element simulations and found that predrilling greatly increases the vertical load carrying capacity of piles. Predrilled length was also a significant factor. For a HP 10x42 steel H-pile, 1.8-3 m (6-10 ft) of predrilled length was necessary in order to take full advantage of predrilling.

The cyclic nature of the temperature changes and pile deflections raises concern about the vulnerability of piles against cyclic loading. A recent study by Oesterle et al. (1998) investigated this issue using full-scale cyclic load tests on one prestressed concrete pile and one H-pile. Both piles were fixed against rotation at the head, and were able to sustain the initially applied vertical load throughout the test. However, the cyclic loading damage to the prestressed concrete pile was unacceptable. This issue appears to require further research because it is likely to control the useful life of the piles.

Abendroth et al. (1989) offer a simplified model to design piles of integral bridges. This method appears to be widely accepted. The method, based on analytical and finite element studies, introduces an equivalent cantilever column to replace the actual pile. In other words, the soil-pile system is reduced down to an equivalent cantilever column. Two alternatives are provided, one involving elastic behavior, and the other involving inelastic behavior of the piles. Finite element simulations indicated that both alternatives were conservative. Both alternatives are concerned with the vertical load carrying capacity of piles under lateral displacements induced by temperature changes. Girton et al. (1991 who evaluated this method experimentally,

concluded that the equivalent cantilever column model is sufficiently accurate for design purposes.

#### Earth Pressures on the Abutment

Depending on the amount of temperature induced displacement of the abutments, earth pressures can be as low as the minimum active or as high as the maximum passive pressures. The mode of displacement of the abutment involves both translation and rotation. Experiments conducted by Thomson and Lutenegger (1998), Fang and Ishibashi (1994), Sherif et al. (1982), Rowe (1954), and Terzaghi (1936) show that both the deformation mode and the magnitude of the deformation affect the magnitude and distribution of the earth pressure.

Approximate solutions to displacement-dependent earth pressures have been developed by Zhang et al. (1998), Clough and Duncan (1991), Chang (1997), and Bang (1985). However, many bridge engineers prefer to use Rankine or Coulomb passive pressure calculations because of their simplicity. Rankine and Coulomb passive pressure calculations are conservative for bridge abutment applications. Oesterle et al.'s (1998) study found that Rankine passive pressures are in good agreement with experiments. Occasionally, the actual passive earth pressures can exceed the design values as in the case of 98-meter long bridge located in Virginia (Hoppe and Gomez, 1996).

For relatively short bridges, neglecting the effects of the passive pressures may be acceptable. Chen (1997) and Burke (1993) recommend that only 2/3 of the full passive pressures be used for most integral bridges. Bridge designers should adopt a conservative approach regarding earth pressures on abutments. It should be borne in mind that the bending moments induced by passive pressures on abutments counteract the dead and live load bending moments in simple spans. Therefore, overestimating passive pressures is not a conservative approach for such bridges.

There is a need to predict earth pressures with reasonable accuracy as a function of abutment displacement. Unfortunately, a simple and reliable method of predicting the relationship between earth pressures and abutment movements does not appear to be available. Therefore, in order to address this issue, the research team [consisting of, in alphabetical order, S. Arsoy, Prof. R. M. Barker, Prof. J. M. Duncan, and R. L. Mokwa] has put significant amount of effort to develop a rational spreadsheet solution. Spreadsheet solutions, based on two of the most promising approaches, were developed as below.

- 1. Rowe's (1954) theory. The theory is based on stress-strain properties of granular materials. It is difficult to understand.
- 2. A method based on elastic and limit equilibrium solutions for both cohesive and cohesionless soils. This method is applicable to both pile caps and abutments, and is easy to understand.

The research team has concluded that the second method would be more useful to practitioners. Details of this method are presented in a separate report by R. L. Mokwa and J. M. Duncan.

The orientation of the wing-walls (the part of the abutment system designed to retain the approach fill beyond the width of the bridge) can also effect the magnitude of the passive earth pressures. Tests by Thomson and Lutenegger (1998) showed that turn-back (U shaped) wing-walls result in greater earth pressures than transverse wing-walls (Figure 7). This suggests that transverse wing-walls could be used to reduce the magnitude of passive pressures. The turn-back wing-wall orientation has the advantage that it reduces approach fill settlements.

#### **Behavior of Approach System**

The approach system of an integral bridge consists of the backfill, the approach fill, and the foundation soil. An approach slab and a sleeper slab, if used, are also part of the approach system (see Figure 1). Both jointed and integral bridges are vulnerable to differential settlement between the approach system and the bridge abutment. This problem is often referred to as the "bump at the end of the bridge." Causes for the bump problem, in order of importance, include (Briaud et al., 1997):

- 1. Compression of the fill material
- 2. Settlement of the natural soil under the embankment
- 3. Poor construction practices
- 4. High traffic loads
- 5. Poor drainage
- 6. Poor fill material
- 7. Loss of fill by erosion

The "bump" problem is further complicated for integral bridges by the cyclic compression and decompression of the backfill due to temperature cycles. When approach slabs are used, a void between the backfill and the abutment is likely to develop as the abutments move back and forth. The interaction mechanisms between the approach system and the abutments are portrayed in Figure 8. The "bump at the end of the bridge" is pushed out to the end of the approach slabs are used (Schafer and Koch, 1992).

The intended function of approach slabs is (Briaud et al., 1997):

- 1. To span the void that may develop below the slab,
- 2. To provide a ramp for the differential settlement between the abutment and the embankment, and
- 3. To provide a better seal against water percolation and erosion of the backfill material.

It is often argued that the length of the zone of surface deformation extends from the abutment a distance equal to twice the height of the abutment, and that the approach slabs should be made two to three times the height of the abutment. This argument is based on the fact that displacing an abutment causes movement of a wedge of the backfill with a height equal to the height of the abutment and a length equal to tan  $(45+\phi/2)$  times the height of the abutment, which is about twice the abutment height.

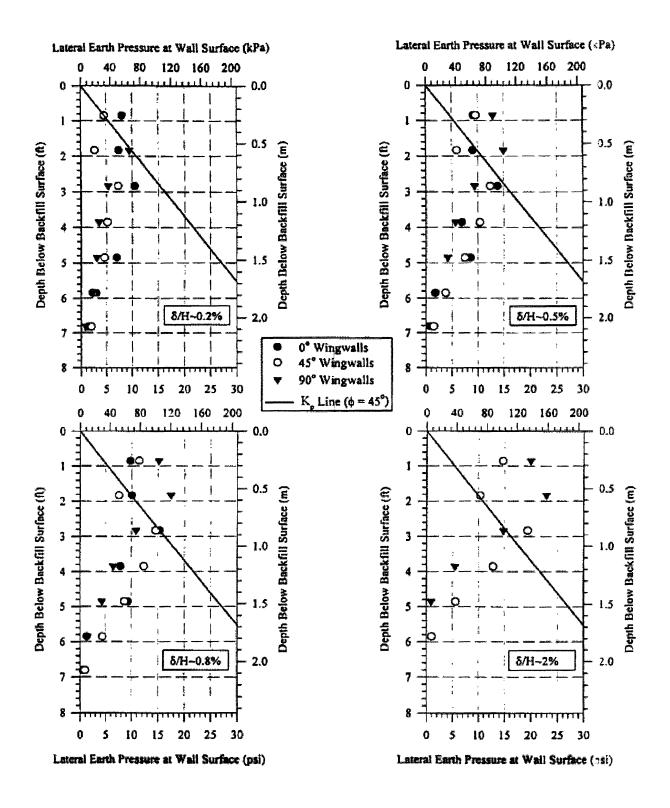
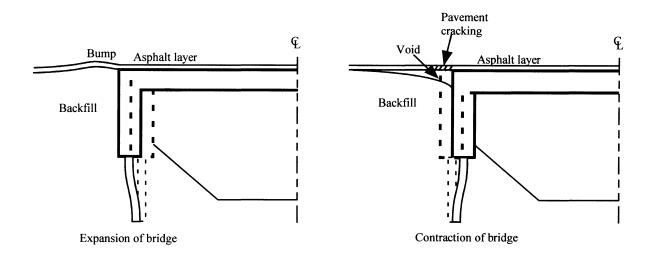
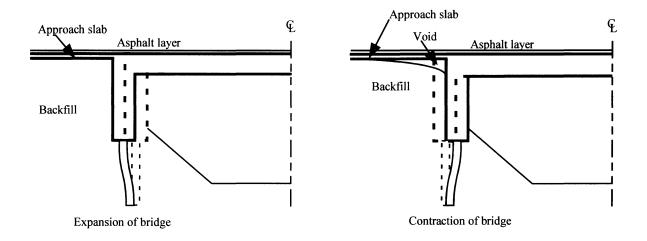


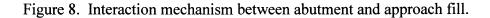
Figure 7. Effect of wing wall orientation and distribution of earth pressures behind abutment as a function of normalized wall displacement (From Thomson and Lutenegger, 1998).



a) Expansion and contraction cycle of a bridge without an approach slab



b) Expansion and contraction cycle of a bridge with an approach slab



## Mitigation of Approach System Settlements

Based on the literature review conducted, it is apparent that no single factor is responsible for approach fill settlement. Several factors contribute. Remedial measures need to be taken to eliminate or mitigate these factors. The most significant factors are the following:

- 1. Settlement of the approach fill, which can be large when compaction and drainage are poor,
- 2. Settlement of the foundation that supports the approach fill, which can be large when the foundation soil is soft and compressible,
- 3. Other factors such as pavement growth and severe traffic loading.

A report prepared for the West Virginia Transportation Department by Bennett et al. (1996) provides a useful discussion about the causes of approach fill settlement, and short-term and long-term remedial actions. Based on this report and other studies, the following measures have been found to be effective in preventing and mitigating approach settlement problem.

- 1. Settlements should receive prime attention during design. Settlement analysis should be performed to estimate settlements of the bridge and its approaches. In order to achieve this, sufficient geotechnical data should be obtained.
- 2. An efficient drainage system should be incorporated in the design. In general, keeping the water away from the soil is a simple yet significant factor in reducing the settlement of the soil.
- 3. Adequate compaction specifications and procedures should be employed. The denser the soil, the less vulnerable it is for settlement. One exception to this is the soil within the close proximity of the abutment. The cyclic nature of the abutment movement will loosen dense backfill and densify the loose backfill. In other words, deformations induced by the abutment results in a density that is independent of the initial density of the backfill material. Therefore, using very dense backfill is not likely to help reduce settlement associated with moving abutments. This should be recognized and either approach slabs with sleeper slabs, or continuous pavement patching is required to compensate for the inevitable approach fill settlements.
- 4. If the foundation soil is likely to settle significantly, soil improvement such as preloading, vertical drains, and other stabilization techniques should be considered. Removal and replacement of the unsuitable material may be a viable alternative. To reduce the loads on the foundation soil, the embankment can be constructed of lightweight materials.
- 5. It should be recognized that integral bridges require continuous, yet reduced, maintenance. Depending on the circumstances, the maintenance comprises asphalt overlays, slab jacking, and approach slab adjustment or replacement.

#### Soil-Structure Interaction Effects, as Shown by Finite Element Analysis

#### **Abutment Displacement and Forces**

As load is applied to the abutment, it deflects and rotates as illustrated by the finite element results shown in Figure 9. These movements generate reactions through increased earth pressures from the approach fill, and increased shear forces at the tops of the piles which support the abutment.

#### **Pile Displacement and Forces**

As the abutment deflects under load, the piles that support it also deflect, as shown in Figure 10. The results of the finite element analyses show that the ground around the piles moves significantly as the load is applied to the abutment. The moving ground does not provide as great resistance to movements of the pile as would non-moving ground, because the resistance provided by ground is governed by the relative movement of the pile with respect to the ground. If the ground around the piles moved the same amount as the piles, the ground would provide no resistance to pile displacement. The piles would deflect as if they were surrounded by air.

The results of the analyses showed that the shear forces at the tops of the piles were only 12% to 26% of the total load, as shown in Table 4. This is due to the fact that the relative displacement between the pile and the surrounding ground is small, which results in low shear forces at the top of the pile. Computed deflections of the piles are shown in Figure 10.

Temperature-driven expansion and contraction of the bridge deck induce forces and movements in the abutment, the approach fill, the foundation piles and the foundation soil. These interaction effects can be illustrated by free body diagrams of the abutment and the piles shown in Figure 11. Table 4 summarizes the force applied to the abutment, the corresponding displacement at the point of load application, the displacement of the pile top, the resistance provided by backfill as percentage of the applied load, and the shear force at the top of the pile as percentage of the applied load.

Force on abutment (kN/m)	Displacement at point of load application. Point A (mm)	Displacement at top of pile. Point B (mm)	Resistance provided by abutment, % of applied load	Shear force at top of pile, % of applied load
45	11	7	88	12
90	26	17	83	17
135	46	29	79	21
170	61	45	76	24
225	82	62	74	26

Table 4. Displacements and forces induced by superstructure expansion.

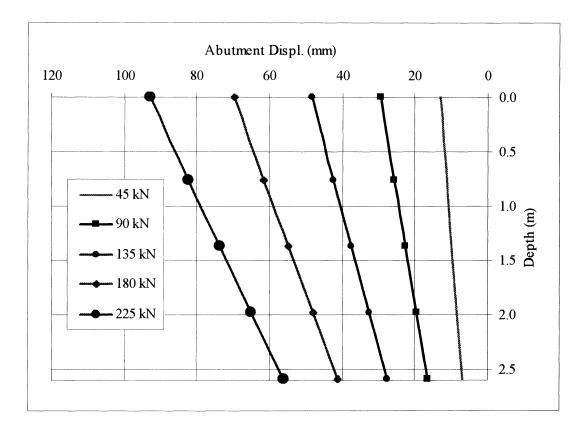


Figure 9. Relationship between abutment displacement and applied load.

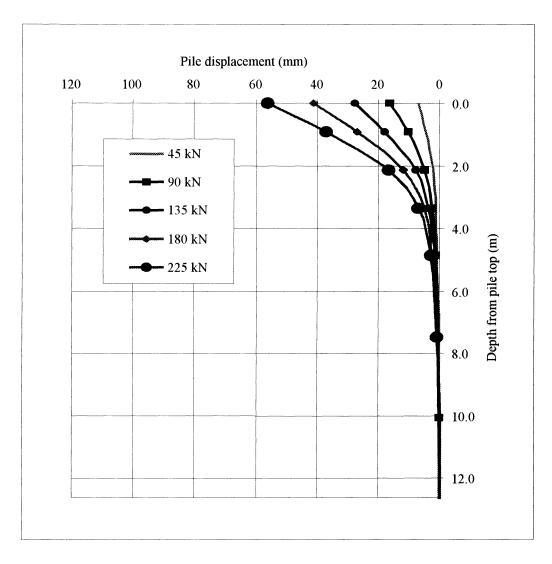


Figure 10. Relationship between pile displacement and applied load at abutment.

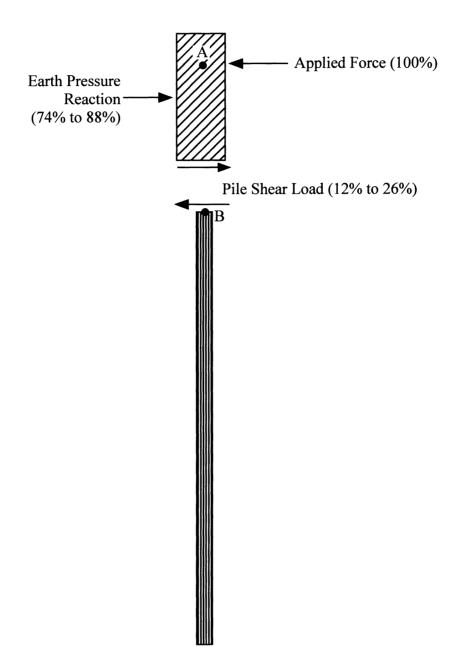


Figure 11. Free body diagram of the abutment and piles

## Settlement of Approach System from Finite Element Analysis

The vertical surface deformation of the approach embankment due to horizontal displacement of the abutment in the finite element simulation conducted is shown in Figure 12. The figure indicates that the lateral extent of the movement is about 3.5 times the 2.5-meter height of the abutment.

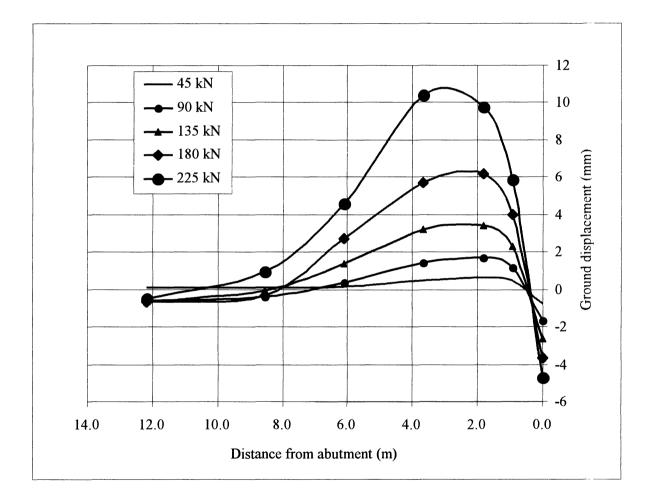


Figure 12. Settlement profile of the ground behind the abutment.

#### **SUMMARY OF FINDINGS**

#### **Use of Approach Slabs**

Most states construct approach slabs as part of integral bridges, with the expectation that they will mitigate the settlement of the backfill near the abutment. In order for approach slabs to meet this expectation, they should be on supports free from excessive displacement. Without settlement analyses, one cannot be sure that the abutment and the sleeper slab will provide adequate supports for the approach slab.

As the abutments move back and forth due to fluctuating temperature, the backfill behind the abutments will be subjected to considerable deformations. The finite element method provides a tool for estimating approach slab performance. The effectiveness of such finite element analyses depends on using a realistic stress-strain model for the approach fill and foundation soils.

Recommendations found in the literature concerning the length of the approach slabs are based on experience, finite element simulation, and approximate calculation methods.

#### **Cyclic Loading Damage to Piles**

As the abutment moves back and forth under the influence of changing temperature, the piles that support the abutment are subject to lateral loads and bending. Being sure that these repeated movements will not cause damage to the piles is an important design issue that needs further study.

Integral bridges are often supported by steel H-piles. Pipe piles and prestressed concrete piles have also been used. Pipe piles filled with concrete have high resistance to local buckling. They also have large moment and shear capacity (Coduto, 1994).

Pipe piles filled with concrete are more ductile than prestressed concrete piles, and have greater resistance to local buckling than steel H-piles. For these reasons concrete filled steel tubular columns are gaining popularity in high rise buildings, especially where seismic risk is a concern (Hooper et al., 1999).

Sites most suited for pipe piles include soft clays, silts, and loose-to-medium dense sands underlain by dense-bearing granular material (Prakash and Sharma, 1990). Pipe piles can be driven closed-end or open-end. When driven open-end, the soil within the pipe pile can be removed. According to Prakash and Sharma, pipe piles are generally economical in the range of 21-24 m (40-80 ft), and can carry loads as high as 1100 kN (250 kips). Research is needed to determine the type of pile that is the most economical and durable for cyclic loading.

## CONCLUSIONS

Based on the results of a literature review, field inspections, and a finite element analysis, the following conclusions are drawn concerning the behavior of integral abutment bridges.

- 1. Integral abutment bridges perform well with fewer maintenance problems than conventional bridges. Without joints in the bridge deck, the usual damage to the girders and piers caused by water and contaminants from the roadway is not observed.
- 2. With jointless bridges, all of the movement due to temperature changes takes place at the abutments and this approach system area requires special attention to avoid development of a severe "bump at the end of the bridge." Finite element analyses show that the zone of surface deformation extends from the back of the abutment a distance equal to about three to four times the height of the abutment.
- 3. The movement of the abutment into the approach fill develops passive earth pressure that is displacement-dependent. Using full passive pressure regardless of displacement is not conservative because it reduces the flexural effects of dead and live load in the bridge girders.
- 4. The ground around the piles moves along with the movement of the abutment. The relative movement between the pile and ground is therefore reduced, resulting in relatively low shear forces at the top of the pile.
- 5. The total lateral movement of the top of the pile relative to the end embedded in the ground is important because it reduces the axial load capacity of the pile. This lateral movement is one of the key variables in assessing the maximum design length of integral abutment bridges. The cyclic nature of these movements raises concern about the vulnerability of piles to cyclic loading.
- 6. Settlement of the approach fill will occur with time. It can be mitigated by using a properly compacted well-drained backfill, but it cannot be eliminated.
- The literature review and field inspections indicate that the maximum lengths of integral abutment bridges have not been reached in Virginia. Jointless bridges over 180 meters in total length have been built in other states and have performed satisfactorily.

#### RECOMMENDATIONS

Based on the results of this study, the following are the recommendations of the authors for improving the performance and behavior of integral abutment bridges.

- As part of the design process, approach fill settlement should be estimated. For approach fills over clay, consolidation testing and conventional consolidation settlement analysis is appropriate. For approach fills over sandy and gravelly subsoils, the magnitudes of time dependent settlements can be estimated using the procedures outlined in the Engineering Manual for Settlement Studies by Duncan and Buchignani (1976). If the settlement predicted is relatively large, use of an approach slab and a U-shaped abutment is appropriate. If the predicted settlement is relatively small, periodic maintenance consisting of asphalt overlays of the approach fill pavement at the end of the bridge is sufficient and cost effective. The final decision [whether an approach slab should be used] should be based on the combined knowledge of geotechnical, structural, maintenance, pavement, and construction engineers.
- Displacement-dependent earth pressures can be calculated by the method presented in a separate report by R. L. Mokwa and J. M. Duncan. The reader should consult this report [to be published by VTRC about the same time as this report] for further information.
- The limitations on axial load capacity of piles under cyclic lateral movement need to be determined. Laboratory tests are recommended to determine the pile type that is most economical and durable for cyclic loading.
- Increased lengths of integral abutment bridges should be considered in future new construction and retrofit projects.

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