APPLICATIONS MANUAL FOR LOGIT MODELS OF EXPRESS BUS-FRINGE PARKING CHOICES
by
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(The opinions, findings, and conclusions expressed in this report are those of the author and not necessarily those of the sponsoring agencies.)

Virginia Highway \& Transportation Research Council
(A Cooperative Organization Sponsored Jointly by the Virginia
Department of Highways $\varepsilon$ Transportation and
the University of Virginia)
In Cooperation with the U. S. Department of Transportation Federal Highway Administration

Charlottesville, Virginia
September 1976
VHTRC 77-R22
$2044$

## ABSTRACT

Manual computations and computerized applications of logit models are described. The models demonstrated reflect travel behavior concerning express bus-fringe parking transit. The specific travel issues addressed include the basic automobile vs. express bus transit choice, model transferability between two study areas, submodal split, and $n$-dimentional choice modeling. A series of curves derived from the mathematical models are presented in the appendices to simplify computations. A FORTRAN subroutine for using these models within the UTPS battery of computer programs for transportation planning is provided.
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## PREFACE

This report is one of two which describe the implementation portion of a three-phased study concerning planning procedures for express bus-fringe parking subarea transit. The first two phases concerned the analysis of the application of existing techniques and the development of design guidelines and choice models, respectively.

The implementation of logit models of travel choice behavion estimated in the preceding phase of this study program are the focus of this report. Both manual and computer applications are considered, with emphasis on the former. Another report which was prepared simultaneously with this one describes a planning process wherein these choice models are used. The companion document is titled "A Procedural Method for Express Bus-Fringe Parking Planning" and is available from the Virginia Highway and Transportation Research Council.

The author acknowledges those colleagues who provided significant contributions to this report. Jerry L. Korf, research engineer, developed the FORTRAN subroutine and descriptive information given in Appendix E. Larry Caldwell, graduate assistant, developed the figures given in Appendices A through D, and assisted ${ }^{\circ}$ in preparing the example problems.
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## INTRODUCTION

The purpose of this manual is to demonstrate hand computations and potential computerized uses of logit models of express bus-fringe parking choice behavior that were calibrated in a previous study. (1) A series of plots of the mathematical functions are given in the appendices to simplify use of the models. The implementation of these logit choice models in a practical planning context is demonstrated in another redort. (2)

The specific models described in the text and shown in the appendices are:

1. A binary auto-transit choice model stratified by residential zone accessibility to the fringe lot;
2. binary auto-transit choice models from two study areas (unstratified);
3. park'n ride vs. kiss 'n ride submodal split model; and
4. an n-dimensional choice model (auto to CBD vs. park 'n ride vs. kiss 'n ride).

MANUAL APPLICATIONS

## Direct Application: Auto vs. Transit Choice

The mathematical form of the logistic model is stated by equation (I) for the binary choice case.*

[^0]\[

$$
\begin{equation*}
P_{b}=\frac{e^{G}(X)}{1+e^{G i(X)}} \tag{1}
\end{equation*}
$$

\]

or, alternatively, if the numerator and denominator are divided by

$$
\begin{align*}
& G_{i}(X) \\
& P_{b}=\frac{1}{1+e^{-G_{i}(X)}} \tag{2}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{b}}=\text { the probability of choosing the express bus, } \\
& \mathrm{i}=\text { a model stratification index, and } \\
& G_{i}(X)=a \text { linear function of explanatory variables }
\end{aligned}
$$

specifically,

$$
\begin{equation*}
G_{i}(X)=b_{0}+b_{1} X_{1}+b_{2} x_{2} \ldots+b_{n} X_{n} \tag{3}
\end{equation*}
$$

Calibration of this model requires that values be estimated for the "b" coefficients, which can be done for the logistic model as conveniently as for a linear regression model by using the ULOGIT program in the Urban Transportation Planning System (UTPS) of computer models.

This report focuses on the application of previously calibrated models within the transit planning process. The development of the models used here is described elsewhere. (1) Accordingly, selected models are chosen from reference 1 to show example applications in the forecasting or prediction mode.

A typical set of binary choice models is given in Table 1 for 4 stratification levels relative to zonal accessibility to the fringe lot. The variables and respective strata are defined in Table 2. The $G_{i}(X)$ for the relative vałues model* defined in Table 1 are expressed by equations (4) through (7).

[^1]\[

$$
\begin{align*}
G_{0}(X) & =2.7839+1.0883 X_{2}-3.5738 X_{3}+ \\
& 6.6795 X_{4}+3.3517 X_{5}  \tag{4}\\
G_{1}(X)= & 2.3732-1.3416 X_{1}+1.1430 X_{2}- \\
& 2.3536 X_{3}+4.2932 X_{4}+3.3990 X_{5}  \tag{5}\\
G_{2}(X)= & 4.3230-1.3092 X-3.9319 X_{3}+ \\
& 10.8990 X_{4}+4.7533 X_{5}  \tag{6}\\
G_{3}(X)= & 1.4384 X_{2}-4.7783 X_{3}+8.5377 X_{4}+4.7783 X_{5} \tag{7}
\end{align*}
$$
\]



## Table 2

Variables Used
Independent Variable


Symbol

| Independent Variable | Symbol |
| :--- | :--- |
| Sex $0=$ female; $I=$ male | $X_{l}$ |
| Age $0=(25-44) ; I=$ otherwise | $X_{2}$ |
| No. household autos |  |
| No. licensed drivers |  |
| Total time difference divided by average | $\mathrm{X}_{3}$ |
| total time | $\mathrm{X}_{-4}=\frac{\mathrm{T}_{\mathrm{a}}-\mathrm{T}_{\mathrm{b}}}{\left(\mathrm{T}_{\mathrm{a}}+\mathrm{T}_{\mathrm{b}}\right) / 2}$ |
| Total cost difference divided by average |  |
| total cost |  |

Accessibility Groups
Group 1. Trips from zones adjacent to zone where lot is located.
Group 2. Trips from zones whose minimum time route to the CBD passes through the area where the lot is located.

Group 3. Trips from zones whose minimum time routes to the CBS are out of the way from the lot.

Dependent Variable
Calibration: $\quad P_{b}=0$ for auto trips

$$
P_{b}=l \text { for bus trips }
$$

Application:

$$
P_{b}=\text { probability of bus choice }
$$

Note: $a=a u t o$ measure; $b=e x p r e s s ~ b u s ~ m e a s u r e . ~$

An examination of the relationship between Table 1 and equations (4) through (7) clarifies how the tabular form of the models as given in reference $l$ represents the $G_{i}(X)$ for input into equations (1) or (2). The value of $G_{i}(X)$ by itself is meaningless; it must be used in conjunction with equation (1) to provide a value for the real dependent variable, $\mathrm{P}_{\mathrm{b}}$.

## Example Estimates with Desk Calculator

The following example demonstrates estimates of transit choice probabilities that are obtained from logistic models using a hand calculator with an exponential function.

Example 1. Stratified binary choice model
The model used here is that given in Table l. The three socioeconomic variables ( $X_{1}, X_{2}, X_{3}$ ) and accessibility strata permit the specification of a variety of tripmaker groups. The following travel group is specified for the model:

$$
\begin{aligned}
\text { Relative location } & : \text { Accessibility Group } 2 \\
\text { Sex } & : \text { Male } \\
\text { Age } & : 25-44
\end{aligned}
$$

Household [ Automobile $]$ Drivers 0.5

For accessibility group 2, equation (6) applies. For the given values of $X_{1}=1, X_{2}=0$, and $X_{3}=0.5$, equation (3) becomes

$$
\begin{align*}
G_{2}(X)= & 4.3230-1.3092(1)-3.9319(.5)+ \\
& 10.8990 X_{4}+4.7533 X_{5}= \\
& 1.0478+10.8990 X_{4}+4.7533 X_{5} . \tag{1.1}
\end{align*}
$$

Equation (1.1) accounts for all variables except the time and cost characteristics of the respective mode choices. Let us further specify that for a certain residential zone, lot location, and bus service conditions that the travel time for the automobile and bus modes are 20 minutes and 30 minutes,
respectively. With these data, $X_{4}$ becomes

$$
\begin{equation*}
X_{4}=\frac{T_{a}-T_{b}}{\left(T_{a}+T_{b}\right) / 2}=\frac{20-30}{(20+30) / 2}=-0.4 \tag{1.2}
\end{equation*}
$$

Equation (1.1) now is reduced to

$$
\begin{align*}
G_{2}(X) & =1.0478+10.8990(-0.4)+4.7533 X_{5}= \\
& -3.311880+4.7533 X_{5} . \tag{1.3}
\end{align*}
$$

The final measure to be considered relates to the relative costs of the competing modes, whereby inputting a specified value for this term gives an estimate of the probability of bus choice. For example, if the cost by automobile is estimated to be $\$ 1.25$ while the expense incurred via bus is $\$ 0.50$,

$$
\begin{equation*}
. x_{5}=\frac{c_{a}-c_{b}}{\left(c_{a}+c_{b}\right) / 2}=\frac{1.25-0.50}{(1.25+0.50) / 2}=\frac{0.75}{0.875}=0.857 \tag{1.4}
\end{equation*}
$$

Under these cost conditions which give $X_{5}$ a value of 0.8571 , the value of $G_{2}(X)$ is

$$
\begin{equation*}
G_{2}(X)=-3.3118+4.7533(0.8571)=0.7622 . \tag{1.5}
\end{equation*}
$$

Now the value given by equation (1.5) is inserted into equation (1) to provide an estimate of the probability that male travelers between the ages of 25 and 44 who reside in households with 1 car for 2 licensed drivers select a fringe parking-express bus service which takes 30 minutes while auto takes 20 minutes and costs $\$ 0.50$, where the automobile trip is estimated to cost \$1.25.

$$
\begin{equation*}
P_{b}=\frac{e^{G}(X)}{1+e^{G_{2}(X)}}=\frac{e^{0.762 .2}}{1+e^{0.7622}}=0.68 \tag{1.6}
\end{equation*}
$$

Thus, the probability of transit choice for the aforementioned circumstances is computed to be 0.68 .

Table 3 shows the results of a series of calculations from equation (1.3) for different values of $\Delta C$. The computations for Example l are underlined. A plot of this curve is given in Figure 1. If Figure $1(A / D=0.5, \Delta T=-10)$ is entered for $\Delta C=0.75, \mathrm{~Pb}_{\mathrm{b}}$ is directly obtained as 0.68 as in Example 1. Appendix A provides a set of similar curves ( $\mathrm{P}_{\mathrm{b}} \mathrm{vs}$. AC) for variations of the model for accessibility group 2 that result from changes in the explanatory variables. In practice, the planner can use these curves or develop his own as required. Example 2 and subsequent examples demonstrate the direct use of the graphs provided in the Appendices. Each appendix contains a definition of the figures contained therein.

Table 3
Data for Plot of Model Derived in Example 1

$$
P_{b}=\frac{e^{-3.3118+4.7533 X_{5}}}{1+e^{-3.3118+4.7533 X_{5}}}
$$

| $C_{a}$ | $C_{b}$ | $D_{c}$ | $X_{5} *$ | $G_{2}(X)$ | $P_{b}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.00 | 0.50 | 1.50 | 1.200 | 2.3922 | 0.916 |
| 1.75 | 0.50 | 1.26 | 1.111 | 1.9691 | 0.878 |
| 1.50 | 0.50 | 1.00 | 1.000 | 1.4415 | 0.809 |
| 1.25 | 0.50 | 0.75 | 0.857 | 0.7618 | 0.682 |
| 1.00 | 0.50 | 0.50 | 0.667 | -0.1413 | 0.465 |
| 0.75 | 0.50 | 0.25 | 0.400 | -1.4105 | 0.199 |
| 0.50 | 0.50 | .00 | 0.000 | -3.3118 | 0.018 |
| 0.25 | 0.50 | -0.25 | -0.667 | -6.4822 | 0.002 |
| 0.00 | 0.50 | -0.50 | -2.000 | -12.8180 | - |
|  |  | $C_{a}-C_{b}$ |  |  |  |



Figure 1. Example binary choice curve.

Example 2. Stratified binary choice model
a. Given: Male tripmaker, Auto/Drivers = 1

Automobile cost $=\$ 1.25$
Bus cost $=\$ 0.50$
Automobile travel time $=20$ minutes Bus travel time $=30$ minutes

Find the percentage using the express bus for the stated conditions.

$$
\begin{array}{ll}
\text { Solution: } & \text { For } \mathrm{T}_{\mathrm{a}}=20, \mathrm{C}_{\mathrm{b}}=\$ 0.50 \text { use Figure A. } 3 \\
& \text { choose curve for } \mathrm{A} / \mathrm{D}=1, \Delta \mathrm{~T}=-10 \\
& \text { for } \Delta \mathrm{C}=\$ 0.75, \text { obtain } \mathrm{P}_{\mathrm{b}}=0.23
\end{array}
$$

b. What percentage of the travel group defined in part (a) would choose the bus if the bus travel time were reduced to 20 minutes ( $\Delta \mathrm{T}=0$ )?

Solution: Use Figure A.3, $\Delta T=0, A / D=1$
For $\Delta C=0.75, P_{b}=.96$
c. Given: Female tripmaker, $A / D=0.5$

Automobile cost $=\$ 1.25$
Bus cost $=\$ 1.00$
Automobile travel time $=20$ minutes
Bus travel time $=25$ minutes
Find the percentage using the express bus for the stated conditions.

Solution: For $\mathrm{T}_{\mathrm{a}}=20, \mathrm{C}_{\mathrm{b}}=1.00$ use Figure A. 6 .
Choose curve for $A / D=0.5, \Delta T=-5$
For $\Delta C=.25, P_{b}=.73$

Model Transferability: Unstratified Binary Choice Models
The models to be presented here are mathematically similar to those given in the previous section, except that they are calculated with the entire data set, disregarding accessibility stratifications. The particular significance of this application is the contrast between the predictions obtained from the models calibrated in different cities. Accordingly, the analyst must be careful in selecting a model which relates to conditions similar to those in his study area and is, hence, transferable. Example 3 demonstrates a case in which two different models can be applied to separate sections of one city.

In this case the Richmond model was derived for travel from a homogeneous high income area to the central city. The Virginia Beach model was associated with travel from a diverse suburban area to a secondary employment center. The Richmond corridor experiences a high volume of CBD destined traffic, while only a small portion of the vehicles on the Virginia Beach-Norfolk corridor were actually destined for the Norfolk CBD, the destination of the express bus. Using this information as a guideline, it is therefore advisable to apply the Richmond model to situations which experience relatively large volumes of trips from a high income residential area to a dominant employment center. The Virginia Beach model is most applicable in cases where there are multiple employment centers.

In this case, the model given by equation (4) applies for Richmond and the counterpart model for the Virginia Beach-Norfolk area is stated by equation (8). Appendix B provides selected graphs of these models.

$$
\begin{equation*}
G_{0}(X)=1.1625-3.2198 X_{3}+2.9728 X_{4}+1.9312 X_{5} \tag{8}
\end{equation*}
$$

where $X_{3}, X_{4}, X_{5}$ are defined in Table 2.

Example 3. Unstratified binary choice model (alternate models available)
a. Given: Automobiles/Drivers $=0.5$

Automobile cost $=\$ 1.00$
Bus cost $=\$ 0.50$
Automobile travel time $=20$ minutes.
Bus travel time $=30$ minutes
Subarea characteristics: High income, concentrated employment center

Number of CBD work trips generated $=1000$ Number workers between ages 25 and $44=450$ Number workers other ages $=550$

Find the number of expected express bus users.
Solution: For $A / D=0.5, T_{b}=30, T_{a}=20, C_{b}=\$ 0.50$ Use Figure B3. Use "Richmond" curves for a high income area with a concentrated employment center.
$\begin{aligned} & \text { For ages } 25-44 \\ & \text { and } A C=0.50\end{aligned} \quad P_{b}=0.67$
For others and $\} P_{h}=0.857$
$A C=0.50$

Expected number of bus users $=P_{b}^{25-44} N^{25-44}+$

$$
\begin{aligned}
& P_{b}^{\text {other }} N^{\text {other }} \\
= & .67 \times 450+.857 \times 550 \\
= & 302+471=773
\end{aligned}
$$

b.Given: Auto/Drivers $=0.7$

Automobile cost $=\$ 0.76$
Bus cost $=\$ 0.50$
Automobile travel time $=30$ minutes
Bus travel time $=30$ minutes Subarea characteristics: Dispersed employment areas

Find number of expected express bus users for a distribution of 450 workers between the ages of 25 and 44 and 550 workers of others ages destined for the area served by the express bus.

Solution: Figures B7 and B8 apply. Select Virginia Beach model for dispersed employment centers. Interpolating for $A / D=0.7$, the model predicts $43.5 \%$, or 435 , trips by bus. Note that the age distribution is not reflected be the Virginia Beach model and is, hence, irrelevant data.

## Submodel Split Models

Estimates of express bus patronage are not in themselves sufficient for the design of a fringe lot and the related traffic facilities. The actual means by which the users access the service will determine many design requirements. Accordingly, models of the access mode choice can be used to determine the needed parking spaces, pick-up and drop-off lanes, and bicycle storage areas.

The majority of the users of the Parham Express in Richmond arrived by either the park 'n ride or kiss 'n ride mode. Hence the data were sufficient to develop only a binary choice model for the two automobile based modes. This model is given by equation (9) and plotted in Appendix C.

$$
\begin{equation*}
P_{d}=-2.2231+5.5835 X_{3} \tag{9}
\end{equation*}
$$

Where

| $P_{d}=$ | The probability of a bus user parking their |
| ---: | :--- |
| car at the lot, and |  |

Example 4. Submodal split.
a. How many of the 773 bus riders in part (a) of example 3 can be expected to park their cars at the lot?
$A / D=0.5$
From Figure C.I, $P_{d}=0.395$ which projects 305 riders
b. How many of the 435 bus travelers in part(b) of example 3 would access the service via the kiss 'n ride mode?

From Figure C.l, $P_{k}=.66$ or 287 riders.

## Multimodal Choice Models

In this section an n-dimensionai choice model which simultaneously considers the basic automobile-transit and transit access decisions is introduced. This model performs a function similar to that which previously had been shown to be accomplished by two binary choice models. Similar models can be calibrated with the ULOGIT program in the UTPS system. Since it is the purpose of this report to demonstrate application, no specific model strategy (i.e., 2 binary models vs. the n-dimensional model) is recommended at this time.

The model applied here is a generalization of the basic. logistic model as stated by equation (l). The computations proceed as follows with the variables defined by. Table 2. First compute equations (10), (11), and (12) using the linear functions defined in equations (13) and (14).

$$
\begin{equation*}
P_{d}=\frac{1}{1+e^{G_{a}(X)}+e^{G_{k}(X)}} \tag{10}
\end{equation*}
$$

$$
\left.\begin{array}{rl}
P_{a}= & \frac{e^{G_{a}(X)}}{1+e^{G_{a}(X)}+e^{G_{k}(X)}} \\
P_{k}= & 1-P_{d}-P_{a} \\
G_{a}(X)= & 1.8503-0.8776 X_{1}-1.9550 X_{2}- \\
& 3.8446 X_{4}-4.9552 X_{5}
\end{array}\right] \quad \begin{aligned}
& G_{k}(X)=2.1623-2.0600 X_{1}-1.9700 X_{2}-3.6987 X_{3}
\end{aligned}
$$

where
d refers to the park 'n ride access mode and express bus,
$k$ refers to the kiss 'n ride access mode and express bus,
a refers to an automobile trip to the CBD, and
$P_{i}$ is the probability of selecting mode $i$.
A fully competitive modification as described in reference 1 is next used to refine the choice probability estimates.

$$
\begin{align*}
& Y_{d}=P_{d}\left(I+Q u_{d}\right)  \tag{15}\\
& Y_{k}=P_{r}\left(I+Q u_{k}\right)  \tag{16}\\
& Y_{a}=P_{a}\left(I+Q u_{a}\right) \tag{17}
\end{align*}
$$

Where

$$
\begin{aligned}
& Q=1 / 3, \\
& u_{i}=P_{i}-\sum_{j=1}^{M} P_{j}^{2}, \\
& i, j=1,2,3, \text { and } \\
& M=3
\end{aligned}
$$

Example 5 shows how direct estimates of the number of tripmakers using each of these alternative travel strategies are obtained with an n-dimensional logit model. Curves developed for this model are given in Appendix D.

Example 5.. n-dimensional logit model (automobile to CBD vs. Express Bus as accessed by park 'n ride or kiss 'n ride)
a. Given: Population $=500$ females

Age $=25-44$
Automobile/Driver $=0.85$
Automobile traviel cost $=\$ 1.25$
Bus travel cost $=\$ 0.50$
Auto travel time $=22$ minutes
Bus travel time $=30$ minutes
Find the number of this subgroup using each modal strategy.

Solution: For female, Age $=25-44, C_{a}=\$ 1.25$
$C_{b}=\$ 0.50, T_{b}=30$ use figures D. 26 and
D. 27 (must interpolate from $A / D=.5$ and $A / D=1.0$ for $A / D=0.85)$.

The values of the $P_{i}$ obtained from Figures D. 26 and D. 27 for $\Delta T=-8$ and the interpolated values for $A / D=0.85$ are shown in Table 4. The following volumes are obtained for each mode.

$$
\begin{aligned}
& \text { Park 'n ride }=N_{\mathrm{C}}=0.593 \times 500=297 \\
& \text { Kiss 'n ride }=\mathrm{N}_{\mathrm{k}}=0.248 \times 500=124 \\
& \text { Automobile to } \mathrm{CBD}=\mathrm{N}_{\mathrm{a}}=0.158 \times 500=\frac{79}{500}
\end{aligned}
$$

b. Given: Population $=500$ males

Age $=25-44$
Automobile/Driver $=0.35$
Automobile travel cost $=\mathbf{~} \$ 1.35$
Bus travel cost $=\$ 0.50$
Automobile travel time $=20$ minutes Bus travel time $=30$ minutes

Find the number of this subgroup using each modal strategy.
Solution: For male, age $=25-44, \mathrm{~T}_{\mathrm{a}}=20, \mathrm{~T}_{\mathrm{b}}=30$, $C_{b}=\$ 0.50$. Use Figures D. 10 and D.ll for $A / D=0$ and 0.5 .

The values obtained for the $P_{i}$ from Figures D. 10 and D.ll for $\Delta C=\$ 0.85$ are shown in Table 5. The following volumes are predicted for each mode.
$\begin{aligned} & \text { Park ' } \mathrm{n} \text { ride }=\mathrm{N}_{\mathrm{d}}=0.946 \times 500=473 \\ & \text { Kiss 'n ride }=\mathrm{N}_{\mathrm{k}}=0.042 \times 500=22 \\ & \text { Automobile to } \mathrm{CBD}=\mathrm{N}_{\mathrm{a}}=0.010 \times 500=\underline{5} \\ & \text { TOTAL }=500\end{aligned}$
C. Given: Population $=500$ males, Age $=25-44$ Automobile/Driver $=0.35$ Automobile travel cost $=\$ 1.35$ Bus travel cost $=\$ 0.50$ Automobile travel time $=24$ minutes Bus travel time $=30$ minutes

Find the number using each of the three modes.
Solution: In order to use the figures in Appendix D, we must first interpolate for $A / D$ and then for $\mathrm{T}_{\mathrm{a}}$. This is accomplished by using the results from part b (Table 5) and generating Table 6 from figures D. 22 and D.23. Interpolating between Tables 5 and 6 gives the values required for this problem which are shown in Table 7. The following volumes are estimated for each mode.

Park 'n ride $\quad=N_{d}=0.95 \times 500=475$
Kiss ' n ride $\quad=\mathrm{N}_{\mathrm{k}}=0.042 \times 500=21$
Automobile to $C B D=N_{a}=0.007 \times 500=4$ TOTAL $=500$

Table 4
Values of $\mathrm{P}_{\mathrm{i}}$ for Example 5 a

| Automobile/Driver <br> Model Choice Probabilities | 0.5 <br> (Fig. D.26) | 0.85 | 1.0 <br> (Fig.D.27) |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}_{\mathrm{d}}$ | 0.368 | 0.593 | 0.690 |
| $\mathrm{P}_{\mathrm{k}}$ | 0.529 | 0.248 | 0.127 |
| $\mathrm{P}_{\mathrm{a}}$ | 0.105 | 0.158 | 0.180 |

Table 5
Values of $P_{i}$ for Example $5 b$

| Automobile/Driver <br> Model Choice Probabilities | 0 | 0.35 | 0.5 |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}_{\mathrm{d}}$ | 0.885 | 0.946 | 0.972 |
| $\mathrm{P}_{\mathrm{k}}$ | 0.104 | 0.042 | 0.015 |
| $\mathrm{P}_{\mathrm{a}}$ | 0.010 | 0.010 | 0.010 |

Table 6
Values of $P_{i}$ for $T_{a}=24$ for Example $5 c$
$\begin{array}{llll}\text { Automobile/Driver } & 0 & 0.35 & 0.5\end{array}$

| Model Choice Probabilities | (Fig. 0.22) | (Fig. 0.23) |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}_{\mathrm{d}}$ | 0.893 | 0.9555 | 0.981 |
| $\mathrm{P}_{\mathrm{k}}$ | 0.163 | 0.042 | 0.016 |
| $\mathrm{P}_{\mathrm{a}}$ | 0.004 | 0.003 | 0.002 |

Table 7
Final Values of $P_{i}$ for Example $5 c$

| $\mathrm{T}_{\mathrm{a}}$ | 20 | 24 | 30 |
| :---: | :---: | :---: | :---: |
| Model Choice Probabilities |  |  | 0.946 |
| $\mathrm{P}_{\mathrm{d}}$ | 0.950 | 0.955 |  |
| $\mathrm{P}_{\mathrm{k}}$ | 0.042 | 0.042 | 0.042 |
| $\mathrm{P}_{\mathrm{a}}$ | 0.010 | 0.007 | 0.003 |

## COMPUTER APPLICATIONS

Travel demand models (i.e., separate trip generation, distribution and model choice models or combinations thereof) are implemented within the computerized UTPS through the program UMODEL. Although this UMODEL program can perform a variety of functions, the application of concern here involves the framework. provided for employing user-furnished models. In this discussion of the application of the choice models it is assumed that the user is familiar with the UTPS programs. On this basis, the details of the UTPS procedures are left to. other sources. Accordingly, the discussion here focuses only on the data and FORTRAN subroutine needed to apply the fringe parking choice models within the UTPS system.

Figure 2 shows the data that are processed through a series of UTPS programs for use by demand models. In this case, the dependent variable of interest is the percentage transit (including submodal split). This framework also permits the preparation of trip interchange tables, which when used in conjunction with the choice probabilities produce modal travel volumes between 0-D pairs. Figure 3 shows the processing of the trip end and trip interchange data by program UMODEL into modal travel volumes. This problem application is further exemplified by Figure 4.


TRANTRIPS/ij = PER TRIPS X PROB(TRANSIT/ij)
Figure 4. Example problem.

## FORTRAN Subroutine

To apply a specific mode choice model within the UTPS package, the user must provide his own FORTRAN program. For this purpose, the user has FORTRAN access to program UMODEL through a user coded subroutine (MODE 13). This subroutine has six entry points, each of which is intended to perform specific tasks. The names of these entry points are MODI3A, MODI3B, MODI3C, MODI3D, MODI3E, and MODI3F. MODI3E applies to the problem of concern here; that is, applying a mode choice model to a trip interchange matrix. The user is referred to the UTPS documentation for details on MODI3E.

An example FORTRAN program which implements the $G_{2}(X)$ logit model as defined in equation (6) is given in Appendix E. The variable designations and file names used are intended to confomm with those used in the UTPS training sessions give by Urban Mass Transportation Administration. It is hoped that this feature will simplify the understanding of the program given to those who have attended the training session.

All of the models that were shown in the manual computations section can be utilized within the computerized UTPS planning package. These computer programs can be adapted to follow the planning procedures developed in reference 2 for express busfringe parking subarea transit planning. Planners experienced with the UTPS methodology will intuitively note the significance of this application.

## CONCLUSIONS

This report shows the planning practitioner computations with logit models of estimates of express bus-fringe parking travel behavior. The majority of the material covered focuses on manual planning tools, while their use within a procedural planning framework is described in a complementary document. (2)

The emphasis of this report has been on the manual rather than computerized uses of the models in order to convey a straightforward description of the mechanics of models whose theoretical derivations were introduced in an earlier report. (1) Also because computer applications require a working knowledge of the UTPS planning system, it was felt that the general user of this document would best benefit by material on manual applications. However, those with UTPS experience will find the computer program that has been included to be helpful.

This report therefore provides engineers and planners concerned with urban transportation planning a ready reference for interpreting research results concerning logistic models of travel choice behavior. The specific models demonstrated here can be applied during the sketch planning and design phases of fringe parking-express bus services as described in reference 2.
$2072$

## REFERENCES

1. Kavak, F. C., and M. J. Demetsky, Demand Estimation for Express Bus-Fringe Parking Services, Virginia Highway and Transportation Research Council, VFTRC 75-R60, Charlottesville, Virginia, 1975.
2. Wester, K. W., and M. J. Demetsky, A Procedural Method for Express Bus-Fringe Parking Planning, Virginia Highway and Transportation Research Council, Charlottesville, Virginia, -1976.
$2074$

## APPENDIX A

## PARHAM EXPRESS BINARY CHOICE MODEL CURVES

Appendix A contains a representative set of curves derived from the logit choice models of auto to CBD vs. express bus choice behavior for the Parham Express in Richmond, Virginia. The purpose of these charts is to provide the planner a means for estimating express bus-fringe parking usage without having to master the mathematics of the model. A representative sample of the curves. that can be developed for accessibility group 2 is given. If additional curves are needed (for example, for the other accessibility groups) the procedures given in the text of this report can be easily applied.

The figures provided are first classified according to the primary independent variable, and then according to variations of the variables which specify the constant for the model. For example, if the curve represents the probability of bus choice $\left(P_{b}\right)$ vs. the relative costs of the competing modes ( $\Delta C$ ); then the variables which are assumed constant include sex, age, households, and the relative travel times of the two modes. A further drivers assumption regarding each curve is that the value of cost for one mode must also be held constant (e.g. $C_{b}=0.50, C_{a}$ varies).

The basic model plotted here is

$$
\begin{equation*}
P_{b}=\frac{e^{G(X)}}{1+e^{G(X)}} \tag{A-I}
\end{equation*}
$$

where

$$
\begin{aligned}
& G(X)=4.3230-1.3092 X_{1}-3.9319 X_{3}+10.8990 X_{4}+4.7533 X_{5}, \\
& X_{1}=0,1 \text { (male, female), } \\
& X_{3}=A / D=\text { number household autos/ number drivers, } \\
& X_{4}=\frac{T_{a}-T_{b}}{\left(T_{a}+T_{b}\right) / 2}
\end{aligned}
$$

$x_{5}=\frac{C_{a}-C_{b}}{\left(C_{a}+C_{b}\right) / 2}$
a = auto mode, and
$\mathrm{b}=\mathrm{bus}$ mode

The curves provided for accessibility group 2 are as follows:
A.I $\quad P_{b}$ vs. $\Delta C$ ( 8 curves)

Constants: Male, $C_{b}=\$ 0.25 \mathrm{~T}_{\mathrm{a}}=20$
Variables: $A / D=0.5,1\} 2 \times 4=8$ Curves
$\Delta T=0 .-5,-10,-15$
A. 2 Same as A.l except for females
A. 3 Same as A.l except $C_{b}=\$ 0.50$ (Parham Express Cost)
A. 4 Same as A. 3 except females
A. 5 Same as A.l except $C_{b}=. \$ 1.00$
A. 6 Same as A. 5 except females
A. $7 \quad \mathrm{P}_{\mathrm{b}}$ vs. $\Delta \mathrm{C}$ (8 curves)

Constants: Male, $\mathrm{C}_{\mathrm{a}}=\$ 1.25, \mathrm{~T}_{\mathrm{a}}=20$
Variables: $A / D=0.5, l^{2} .2 \times 4=8$ curves
$\Delta T=0,-5,-10,-15$
A. 8 Same as A. 7 except female
A. 9 Same as A. 7 except $C_{a}=\$ 2.50$
A. 10 Same as A. 9 except female
A. $11 \quad P_{b}$ vs. $\Delta T$ ( 4 curves)

Constants: Male, $\mathrm{T}_{\mathrm{b}}=20, \mathrm{C}_{\mathrm{b}}=\$ 0.50$
Variables: $A / D=0.5,1$ \} $2 \times 2=4$ curves
$C_{a}=\$ 1.25, \$ 2.50$
A. 12 Same as A. 11 except female
A.13 Same as A.ll except $T_{b}=30$
A. 14 Same as A.l3 except female
A. 15 Same as A. 11 except $T_{b}=40$
A. 16 Same as A. 15 except female



Cost Difference



Cost Difference


Cost Difference
$\mathrm{C}_{\mathrm{a}}-\mathrm{C}_{\mathrm{b}}$
Figure $\mathrm{A}-\mathrm{S}$



Travel Difference




Travel Difference



Travel Difference




Travel Difference
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## APPENDIX B

UNSTRATIFIED BINARY CURVES: RICHMOND AND VIRGINIA BEACH
The curves in Appendix B are derived from logit models calibrated from all automobile and bus trip observations in each of two study areas. The purpose is similar to that stated for the models shown in Appendix A; and the reader is referred to the introductory material of that section.

The curves provided are derived from logistic models with the following linear functions.

## Richmond

$$
\begin{aligned}
& \text { 1. Age }=25-44 \text { years } \\
& G_{0}(X)=2.7839-3.5738 X_{3}+6.6795 X_{4}+3.5717 X_{5}(B-1) \\
& \text { 2. } \text { Age }=\text { other } \\
& G_{0}(X)=1.6956-3.5738 X_{3}+6.6795 X_{4}+3.5717 X_{5}(B-2)
\end{aligned}
$$

Virginia Beach

$$
\begin{equation*}
G_{0}(X)=1.1625-3.2198 X_{3}+2.9728 X_{4}+1.9312 X_{5} \tag{B-3}
\end{equation*}
$$

The curves given in this section are Identified as follows:
B.I $\quad P_{b}$ vs. $\Delta C$ (3 curves)

Constants: $A / D=0.5, T_{b}=30, T_{a}=15, C_{b}=\$ 0.50$
B. 2 Same as B.l except $A / D=1$
B. 3 Same as B.l except $T_{a}=20$
B. 4 Same as B. 3 except $A / D=1$
B. 5 Same as B. 1 except $T_{a}=25$
B. 6 Same as B. 5 except A/D $=1$
B. 7 Same as B. 1 except $T_{a}=30$
B. 8 Same as B. 7 except $A / D=1$
B. $9 \quad P_{b}$ vs. $\Delta T$ (3 curves)

Constants: $A / D=0.5, C_{a}=1.25, C_{b}=0.5, T_{b}=30$
B. 10 Same as B. 9 except $A / D=1$
B. 11 Same as B. 9 except $C_{a}=\$ 2.50$
B. 12 Same as B.ll except $A / D=1$













## APPENDIX C

## SUBMODAL SPLIT CURVES

Appendix $C$ gives a curve for estimating the proportions of express bus users who access the lot by park 'n ride and kiss'n ride. The model was calibrated for the Parham Express in Richmond and uses the ratio of household automobiles to drivers as the only explanatory variable.

$$
\begin{align*}
& G_{k}=3.2231-5.5835 X_{3}, \text { and } \\
& G_{p}=1-P_{k}=-2.2231+5.5835 X_{3} \tag{C-2}
\end{align*}
$$

where,

$$
\begin{aligned}
G_{k}= & \text { Linear function to estimate the probability of kiss ' } n \\
& \text { ride access, } P_{a} \text {, and } \\
G_{p}= & \text { Linear function to estimate the probability of park ' } n \\
& \text { ride access, } P_{p} .
\end{aligned}
$$



Figure C-1

## APPENDIX D

## MULTIMODAL CHOICE MODELS

An n-dimensional choice model was calibrated with the Parham Express data which simultaneously computes the probability for either of three modes: automobile to CBD, park 'n ride, and kiss 'n ride. This represents an alternative approach to using the basic binary choice model and the submodal choice model in a sequential manner.

The models which are charted in this Appendix are defined as

$$
\begin{align*}
& Y_{d}=P_{d}\left(I+Q u_{d}\right)  \tag{D-1}\\
& Y_{a}=P_{a}\left(I+Q u_{a}\right)  \tag{D-2}\\
& Y_{k}=P_{i}\left(I+Q u_{k}\right) \tag{D-3}
\end{align*}
$$

where $Y_{d}, Y_{a}$, and $Y_{k}$ are the estimated probabilities for a fully competitive model for the park 'n ride (d), automobile to CBD (a), and kiss:'n ride (k) modes, respectively.

$$
\begin{align*}
Q & =1 / 3 \\
u_{i} & =P_{i}-\sum_{j=1}^{M} P_{j}^{2}
\end{align*}
$$

where

$$
\begin{aligned}
& i, j=1,2, \text { or } 3 \\
& M=3, \\
& P_{d}=\frac{1}{I+e^{G_{a}(X)}+e^{G_{k}(X)}} \\
& P_{a}=\frac{e^{G_{a}(X)}}{I+e^{G_{a}(X)}+e^{G_{k}(X)}} \\
& P_{k}=1-P_{d}-P_{a}
\end{aligned}
$$

where
$G_{a}(X)=1.8503-0.8776 X_{1}-1.9550 X_{2}-3.8446 X_{4}-4.9552 X_{5}$
$G_{k}(X)=2.1623-2.0600 X_{1}-1.9900 X_{2}-3.6907 X_{3}$
$X_{1}=$ sex; $0=$ female, $l=$ male
$X_{2}=$ age; $0=(25-44)$, 1 other
$X_{3}=$ autos/drivers
$\mathrm{X}_{4}=\left(\mathrm{T}_{\mathrm{a}}-\mathrm{T}_{\mathrm{b}}\right) /\left(\mathrm{T}_{\mathrm{a}}+\mathrm{T}_{\mathrm{b}}\right) / 2$
$x_{5}=\left(c_{a}-c_{b}\right) /\left(c_{a}+c_{b}\right) / 2$
The curves provided in this Appendix are as follows:
D.I $P_{i}$ vs. $\Delta C$

Constants: Female, Age $=25-44, \mathrm{~A} / \mathrm{D}=0, \mathrm{~T}_{\mathrm{a}}=20$,

$$
T_{b}=30, C_{b}=0.50
$$

D. 2 Same as D.1 except $A / D=0.5$
D. 3 Same as D. 1 except $A / D=1.0$
D. 4 Same as D.1 except Age $\neq 25-44$
D. 5 Same as D. 4 except $A / D=0.5$
D. 6 Same as D. 4 except $A / D=1.0$
D. 7 Same as D.l except Male
D. 8 Same as D. 7 except $A / D=0.5$
D. 9 Same as D. 7 except $A / D=1.0$
D. 10 Same as D. 7 except Age $\neq 25-44$
D. 11 Same as D. 10 except $A / D=0.5$
D. 12 Same as D. 10 except $A / D=1.0$
D. 13 Same as D. 1 except $T_{a}=30$
D. 14 Same as D. 13 except $A / D=0.5$
D. 15 Same as.D.13 except $A / D=1.0$
D. 16 Same as D. 13 except Age $\neq 25-44$
D. 17 Same as D. 16 except $A / D=0.5$
D. 18 Same as D. 16 except $A / D=1.0$
D. 19 Same as D. 13 except Male
D. 20 Same as D. 19 except $A / D=0.5$
D. 21 Same as D. 19 except $A / D=1.0$
D. 22 Same as D. 19 except Age $\neq 25$-44
D. 23 Same as D. 22 except $A / D=0.5$
D. 24 Same as D. 22 except $A / D=1.0$
D. $25 \quad P_{b}$ vs. $\Delta T$

Constants: Female, Age $=25-44, A / D=0, C_{a}=\$ 1.25$, $C_{b}=\$ 0.50, T_{b}=30$
D. 26 Same as D. 25 except $A / D=0.5$
D. 27 Same as D. 25 except $A / D=1.0$
D. 28 . Same as D. 25 except Age $\neq 25-44$
D. 29 Same as D. 25 except $A / D=0.5$
D. 30 Same as D. 28 except $A / D=1.0$
D. 31 Same as D. 25 except Male
D. 32 Same as D.31 except $A / D=0.5$
D. 33 Same as D.31 except $A / D=1.0$
D. 34 Same as D. 31 except Age $\neq 25-44$

2100
D. 35 Same as D. 34 except $A / D=0.5$
D. 36 Same as D. 34 except $A / D=1.0$


ost Difference
$C_{a}-C_{b}$
Figure $D-1$



Cost Difference

D-6








Cost Difference
$C_{a}-C_{b}$
Figure $D-9$







Cost Difference
$C_{a}-C_{b}$
Figure $D-5$



Cost Difference
əoṭ०५ sng fo K7!t!qeqoud
D-13



Cost Difference



$$
\begin{aligned}
& \text { Cost Difference } \\
& C_{a}-C_{b} \\
& \text { Figure } D-22
\end{aligned}
$$



D-15



Cost Difference
$C_{a}-C_{b}$
Figure $D-23$





Travel Difference
$o$
$m$
1
0
0
0
3
0
0
$i$


Travel Difference








## APPENDIX E

FORTRAN SUBROUTINE FOR UMODEL
The discussion that follows is directed toward those planners who are familiar with UTPS procedures. It refers to an example deck configuration (Figure E-I) of a UTPS computer implementation of the logit model referred to in the text of this report as equation (6).

The UTPS computer package provides a set of cataloged Job Control Language (JCL) procedures that facilitate the insertion of the user provided FORTRAN code into the UMODEL program. This JCL procedure is named USERCODE and its use is illustrated. It should be noted that those cards beginning with // are JCL cards used to invoke and, in some cases, modify the cataloged procedures Cards beginning with //* are JCL comment cards which serve to explain these control cards. The cards that contain /. in the first two columns are command cards for the IBM system utility program IEBUPDTE which does the actual insertion of the fortran code.

The cards following the /. cards and terminated by. the next set of JCL cards constitute the FORTRAN code necessary to use the logit model for modal split forecasting. The execution of this code within the framework of the UMODEL program is initiated by the JCL cards shown immediately following the code.

For purposes of illustration, three data files are assumed to exist. The first file, named RCO.ZONEDATA, is a table of statistics on certain zonal characteristics. This can be viewed as equivalent to a deck of data cards, one for each zone, that are sorted in ascending zonal order. Columns $l$ through 3 of these cards contain the zone number; columns. 5 through 8 contain the decimal fraction of the zone's work force that is male; columns 10 through 13 contain the number of vehicles registered in the zone; and columns 14 through 16 contain the number of licensed drivers in the zone. These production data are made known to the UMODEL program by assigning the file to any 1 of 7 possible special names (i.e., Al through A7, with Al chosen for this example).

The second file, named RCO.XCIMPEDS, contains four tables in standard UTPS interleaved format. These are tables of interchange impedances in terms of time and cost. Tables one and two are interchange transit travel times and auto travel times, respectively. Tables three and four contain transit travel costs and auto travel costs, respectively. These tables are made available to the UMODEL program by assigning the file the special name "J1".

The third file, named RCO.WORKTRPS, contains a trip table of CBD work trips. This table is assigned to the UMODEL program by giving it the special local name of "J2".

The execution of the UMODEL program produces a fourth file, named RCO.CBDTRIPS, that contains two tables of CBD work trips in the standard UTPS interleaved format. The first table reflects the logit model development of CBD transit trips while the second table is its complement, that of auto person trips. The special local name for this file is "J8". The JCL card immediately following the "J8" assignment card is used to specify that the output file be saved for future use. The FORTRAN code contains comments that clarify the purpose of individual statements.

The algorithm calculates separate $G_{2}(X)$ fuimu_uıs for male and female tripmakers. The probability of choosing transit is then determined for each sex on a zone by zone basis. These two sex groups are then summed to produce the first output table, that of forecasted CBD transit work trips for the zone. The auto work trips are established by simple subtraction to produce the second table to be outputed.

C G2OFXF $=4.3230+3.9319$ ORATIO 10.8990 THATIO

- G2OFXM $=$ G2OFXF $-1.3092 \quad 4.7533$ CHATIO
S
REAL\#4 4 ORATIO, TRATIO, CRATIO, GZOFXM, G2OFXF, EXOFGM, EXOFGF,
NMMALS, NMFMLS, PROBTM, PROBTF, TRNTRP
EXOFGF $=$ EXPON (GZOFXF)
PROBTM $=$ EXOFGM $/(1.0+$ EXOFGM $)$
PROBTF $=$ EXOFGF $/(1.0+$ EXOFGF $)$
STRATIFY THE NUMBER OF CBD WORK TRIPS BY SEX
NMMALS $=x(2)-x(9)$
NMFMLS $=x(9)-$ NMMALS




[^0]:    *The binary choice is a special case of an $n$-dimensional set of choices (any number) where the computations are less complex than for the general case. An n-dimensional choice model which considers the automobile, bus (park 'n ride) and bus (kiss 'n ride) is described in a subsequent section.

[^1]:    *The term "relative values model" refers to the manner by which the alternative model characteristics are specified, i.e., the travel time or cost of a certain mode divided by the average of the automobile and bus times or costs for the journey.

