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16. Abstract

At the intersections with permissive only signal control, pedestrians will move at the permissive phase with the parallel through vehicular movement and left-turn vehicles, the left-turn vehicles have to yield to both opposing through vehicles and pedestrians at crosswalk. Under such complicated driving conditions, collision risks rise if left-turn drivers make misjudgments and fail to yield to pedestrians. The existing guidelines on left-turn operations mainly focus on the traffic conditions at the intersections and do not give particular considerations to pedestrian safety problems. They are mainly developed based on the left-turn and the opposing through traffic volumes, while the pedestrian volumes and other pedestrian safety related factors, such as design features of the crossroads, direction of sunlight, and the sight distance of the left-turn drivers have not been explicitly taken into account. Thus, the objective of this research is to develop pedestrian safety based warrants for protected left-turn control. In this research, the driving-simulation based experiments will be conducted for identifying and assessing the impacts of the factors that contribute to the crashes between left-turn vehicles and pedestrians during the permissive left-turn phase, and develop pedestrian safety based warrants.

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Development of Pedestrian Safety Based Warrants for Permissive Left-Turn Control

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Research Report SWUTC/10/169302-1

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ABSTRACT

At intersections with the permissive only signal control, pedestrians will move at the permissive phase with the parallel through vehicular movement and left-turn vehicles, the left-turn vehicles will yield to both opposing through vehicles and pedestrians at crosswalk. Under such complicated driving conditions, collision risks rise if left-turn drivers make misjudgments and fail to yield to pedestrians. The existing warrants or guidelines on left-turn operations mainly focus on the traffic conditions at intersections and do not give particular considerations to pedestrian safety problems. They are mainly developed based on the left-turn and the opposing through traffic volumes, while the pedestrian volumes and other pedestrian safety related factors, such as design features of the crossroads, direction of sunlight, and the sight distance of the left-turn drivers have not been explicitly taken into account. Thus, the objective of this research is to develop pedestrian safety based warrants for protected left-turn control. In this research, the driving-simulation based experiments will be conducted for identifying and assessing the impacts of the factors that contribute to the crashes between left-turn vehicles and pedestrians during the permissive left-turn phase, and then develop pedestrian safety based warrants for permissive left-turn control.

Keywords: Permissive Left-Turn Control, Pedestrian Safety, Driving Simulation.

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EXECUTIVE SUMMARY

Pedestrian safety under permissive left-turn signal control is a big problem, because left-turn drivers misjudge the gap and fail to yield to pedestrians under certain conditions. This research investigated the factors that contribute to crashes between left-turn vehicles and pedestrians, and then recommended pedestrian safety based warrants for using permissive left-turn control, including both permissive-only and permissive/protected left-turn (PPLT) signal phasing.

In this study, driving-simulation based experiments were conducted to test drivers' driving performance during permissive left-turns at different types of intersections with different levels of pedestrians. For this purpose, twelve different experimental intersections with permissive left-turn signal control were designed in the driving simulation environment. The design of these intersections considered a wide range of contributing factors including speed limit, number of lanes, intersection type, sight distance, opposing thorough traffic volume and pedestrian volume. To evaluate the risk of collisions between permissive left-turn vehicles and pedestrians, three types of Measure of Effectiveness (MOEs), i.e. maximum deceleration rate, time to collision to pedestrian, and minimum stop distance, were developed based on the data collected from the driving-simulation based experiments. In addition, a post survey was conducted on drivers who completed the driving simulation test in order to obtain subjective evaluation results. The results of this research show that the permissive left-turn phase (either permissive only or PPLT) can be used only if the pedestrian volume is lower than a certain value and the percentage of big size vehicles (such as trucks) in the left-turn traffic is relative low. These results can help traffic engineers better understand pedestrian safety issues related to the left-turn movements of drivers and choose the best left-turn signal time control mode for intersections.

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CHAPTER 1: INTRODUCTION

1.1 Background

For the intersections with permissive only left-turn control, pedestrians will move at the permissive phase with the parallel through vehicular movement. This requires vehicles that turn left to yield to both opposing through vehicles and pedestrians prior to selecting an appropriate gap. Pedestrian accident risks are increased in such complicated driving conditions if left-turn drivers make misjudgments and fail to yield to pedestrians. The 2008 National Highway Traffic Safety Administration's pedestrian crash facts show that, on average, 13 pedestrians die in vehicle crashes each day and pedestrian involved crashes occur far more often with left-turning vehicles than with right-turning and straight through vehicles. Furthermore, the proportion of accidents involving pedestrians and left-turning vehicles varies from 17 to 32 percent of all pedestrian accidents at the intersection (see Table 1). Thus, left-turning movements at signalized intersections represent a considerable safety problem to pedestrians.

Table 1: Pedestrian Accidents and Left-turning Traffic

Studies	Fruin (1973)	Habib (1980)	Zegeer et al.(1982)	Robertson & Carter (1984)	Zaidel& Hocherman (1984)	Almuina (1989)
Country	U.S.	U.S.	U.S.	U.S.	Israel	Canada
Proportion of Left-turning Accidents	31%	25%	22%	17%	13%	32%
Number of Signalized intersections	32	45	1297	62	520	306

The existing signal design guidelines on left-turn operations mainly focus on traffic conditions at the intersections. Few of them give particular considerations to pedestrian safety problems in the determination of the most appropriate left-turn control mode for an intersection. For example, the existing warrants for protected left-turn control are developed mainly based on the left-turn and the opposing through traffic volumes, while the pedestrian volumes and other pedestrian safety related factors, such as the pedestrian volume, the presence of big vehicles, the intersection geometric conditions and the sight distance of the left-turn drivers, have not be explicitly taken into account by these existing signal design guideline.

1.2 Research Objectives

The objective of this research is to develop pedestrian safety based warrants for protected left-turn control. The results of this project will improve the safety at the signalized intersections. To achieve these objectives, the research will:

a. Identify factors that contribute to the crashes between left-turn vehicles and pedestrians during the permissive left-turn phase,

b. Conduct driving-simulation based experiments for assessing the impacts of the indentified influencing factors, and

c. Develop pedestrian safety based warrants for protected left-turn control.

To achieve the above research objectives, different driving scenarios were designed in the driving simulator environment to investigate pedestrian safety under the permissive left-turn signal control. In this study, the driving-simulation based experiments were conducted to identify and assess the impacts of the factors that contribute to the crashes between left-turn vehicles and pedestrians under left-turn permissive control. Later, a survey was completed by the participating drivers to get their subjective assessments of the pedestrian safety at different intersections with various traffic and geometric conditions.

Based on the results of the driving-simulation based experiments and survey, recommendations were that the permissive left-turn signal control, i.e. permissive-only or permissive/protected left-turn (PPLT) signal phasing, should not be used due to the potential risk to pedestrians. The results of this study will help traffic engineers better understand the pedestrian safety issues related to left-turn movements and choose the best left-turn signal time control mode for intersections. Thus, it will reduce casualties and property damages due to the conflicts between crossing pedestrians and left-turn vehicles.

1.3 Organization of the Chapters

This report is organized in the following order. In Chapter 2, literatures in two aspects are reviewed: 1) contributing factors to the collisions between left-turn vehicles and pedestrians under the permissive left-turn control, and 2) existing warrants for protected left-turn control. Chapter 3 describes the experiment procedure and gives detailed description of the devices and tools used in this research. Chapter 4 introduces the experimental design for analyzing pedestrian safety under permissive left-turn signal control. This chapter includes two parts: the testing scenarios design and the measures of effectiveness (MOE) design. Chapter 5 presents the evaluation results of the proposed driving-simulation based experiments. Finally, the conclusions, recommendations, and future directions of this research are given in Chapter 5.

CHAPTER 2: LITERATURE REVIEW

This literature review focuses on the following two major aspects: 1) the contributing factors to the collisions between left-turn vehicles and pedestrians under the permissive left-turn control were reviewed. These factors include traffic signal control methods, pedestrian volume, traffic conditions, intersection geometric features, and others; 2) the existing warrants for protected left-turn control were also introduced and discussed. Following is the detailed discussion about the findings from literatures researched.

2.1 Contributing Factors

2.1.1 Pedestrian Volume

Theoretically, more pedestrians crossing the intersection will cause the left-turn vehicle more difficulty in finding a big enough gap in the pedestrian flow to make a safe left-turn. However, previous researches (Zaidel et al, 1987 and Zegeer et al, 1982) found that it is safer for pedestrians to cross the intersection in a group. It may be due to the fact that a larger group of pedestrians cannot be hidden by the visual impediment of a driver who is making a left turn. In addition, drivers may be more cautious during the left-turn maneuver at intersections where they expect a higher number of pedestrians to be present. Therefore, it is not a simple linear relationship between the pedestrian volume and the likelihood of pedestrian and left-turn vehicle accidents. The most risky situation may occur when pedestrian volume is within a certain range.

2.1.2 Traffic Conditions

Left-turn vehicle volume

Previous research found that left-turn vehicle volume is an important influencing factor on the likelihood of pedestrian accidents. When the left-turn volume is high, drivers become more aggressive and try to make permissive turns even if the time gap of the opposing traffic is relatively small, which will cause the driver to fail to yield to the pedestrians.

Opposing through vehicle volume

At two-way/two-way intersections, drivers making a left-turn usually have to allow oncoming vehicles from the opposite approach pass before performing the maneuver. Lord (1994) observed that while the driver was waiting for a gap long enough to turn, the majority of pedestrians usually had the time to cross the intersection. As a result, when the driver could finally turn left, there were very few pedestrians or no pedestrians at all in the crosswalk. Therefore, the risk to pedestrians may also be a function of the number of vehicles coming from the opposite approach.

Approaching speed of left-turn vehicles

Vehicle approaching speed can also affect pedestrians' safety. When the speed of opposing vehicles is very high, left-turn vehicles may need to speed up in order to make the left-turn movement and avoid collision with oncoming vehicles. In this case scenario, the risk of pedestrian injury increases because the left-turn vehicle cannot stop immediately.

2.1.3 Intersection Geometric Features

Quaye et al (1993) attempted to develop accident models for predicting pedestrian accidents due to left-turning traffic at signalized intersections. The models showed that

T-intersections were generally more dangerous to pedestrians (for a pedestrian flow above100 ped/hr). This result was also supported by Lord (1996), in which traffic conflict technique was used to evaluate the intersections studied by Quaye et al (1993). It may be because, in these types of intersections, there is no opposing traffic and left-turn drivers are less cautious when they make turns during the permissive phase, which might cause them to fail to yield to the pedestrians who are crossing the intersections during the same phase.

2.1.4 Others

Besides the factors listed above, there are several other factors, such as the lighting and weather conditions, crosswalk marking, pavement surface condition, curb extensions, driver's age, driving experience, and pedestrian's age, that will also affect the risk of accidents between the pedestrian and left-turn vehicles. This study will focus on the impacts of pedestrian volume, traffic conditions and intersection geometric features.

2.2 Existing Warrants for Protected Left-Turn Control

Numerous studies have been conducted for developing warrants for the selection of left-turn signal control mode. In general, the existing warrants can be categorized as being delay based, traffic volume-based, accident/conflict experience based, geometric features-based, and operational speed-based. These existing criteria are summarized in Tables 2 and 3 by categories.

2.2.1 Traffic Volume Based Warrants

Traffic volume-based warrants provide the threshold left-turn volume or products of left-turn and opposing volume for determining the needs of left-turn protection. The summary of left-turn control warrants is presented in Table 2; it can be seen that these existing warrants use constant thresholds for traffic volume to determine the left-turn signal control mode.

2.2.2 Accident/Conflict Experience Based Warrants

Accident experience based warrants are the most important for the selection of the left-turn signal control mode. Some studies developed warrants based on the number of left-turn related accidents. Agent K.R. (1979) found the critical number of left-turn-related accidents was four in one year, six in two years, or eight in three years. Some research also used the number of left-turn conflicts as criteria. Agent K.R. (1979) found that the critical number of conflicts is 14 in a peak hour, and Cottrell (1986) found the critical number of conflicts is four per 100 left-turn vehicles. There is no doubt that both a high number of left-turn related accidents and a high number of left-turn conflicts require a protected left-turn signal phase.

2.2.3 Geometric Features Based Warrants

Geometric features based warrants consist of three types of warrants: 1) sight distance 2) number of left-turn lanes and 3) number of opposing through lanes. Sight distance and opposing speed could be considered together to determine the feasibility of using the permissive phase (either PERM or PPLT). PO mode is the appropriate choice for intersections that have multiple left-turn lanes or more than three opposing through lanes, because it is not safe to make permissive left turns when such complicated geometric conditions exist.

2.2.4 Operational Speed Based Warrants

Operational speed-based warrants are also very important for determining the correct signal control mode for an intersection. Previous research found that PO mode should be provided when opposing speed is greater than 45 mph to avoid collisions between left-turn vehicles and opposing through vehicles that are approaching the intersection at a high rate of speed.

2.2.5 Left-Turn Delay Based Warrants

Compared to protected left-turn signal mode, permissive left-turn signal mode will cause more left-turn delay. Previous research discovered that if average left-turn delay is equal or greater than 35s/vehicle or total left-turn delay is equal or greater than two vehicle-hours, then protected left-turn signal mode should be installed (Agent and Deen 1979, Cottrell 1986, ITE 1991 and Roess et al. 2004)

2.2.6 Others

Other types of warrants, such as cost-benefit analysis, vehicle queue, left-turn storage length, access management conditions, etc. have also been proposed in several previous research studies. These warrants are listed in Table 2 and 3. As seen in these two tables, the existing warrants or guidelines on left-turn operations mainly focus on the traffic conditions at the intersections and do not give particular considerations on the pedestrian safety problems. In fact, pedestrian safety at signalized intersections is a very important factor that should be considered in the signal mode selection. Pedestrians are more vulnerable and many fatal accidents involve left-turn vehicles and pedestrians. Therefore, development of pedestrian safety based warrants for protected left-turn mode is very helpful in the effort to reduce causalities and property damages.

Table 2: Summary of Existing Warrants for Protected Left-Turn Signal Control Mode

Criterion	Warrant		Reference
Traffic Volume	Left-Turn Volume	≥ 2 Veh/Cycle	Agent and Deen 1979, Cottrell 1986, Upchurch 1986, ITE 1991
		≥ 50 vph	
		≥ 50 vph & VCP > 100,000	City of San Diego 2006
		> 200 vph	Roess et al. 2004, Stamatiadis et al. 1997
		> 300 vph	Stamatiadis et al. 1997
		> 320 vph (≥2.5% heavy vehicles)	Roess et al. 2004
	Opposing Through Volume	> 1,000 vph (two opposing lanes)	Agent 1981
		> 1,100 vph (≥2.5% left-turn vehicles)	Roess et al. 2004
	Volume Cross Product (VCP)	> 50,000 (per opposing lane)	Roess et al. 2004
		> 50,000 (one opposing lane)	Agent and Deen 1979, Stamatiadis et al. 1997
		> 100,000 (two opposing lanes)	
		> 50,000 (two-lane approach)	ITE 1991
		> 100,000 (four-lane approach)	
		> 144,000 (two opposing lanes)	Upchurch 1986
		> 100,000 (three opposing lanes)	
		> 100,000 (left-turn volume ≥ 50 vph)	City of San Diego 2006
		> 150,000 (one opposing lane)	Stamatiadis et al. 1997
		> 300,000 (two opposing lanes)	
		200,000 ≥VCP per lane ≥50,000	Cottrell B. 1986

Table 3: Summary of Other Types Criteria for Left-Turn Signal Control Mode

Criterion	Warrant		Reference
Accident Experience	Left-Turn Accidents (on one approach)	$\geq$ 4 in any one year, or $\geq$ 6 in any two consecutive years, or $\geq$ 8 in any three consecutive years	Agent and Deen 1979, Stamatiadis et al. 1997
		$\geq$ 4 in any one year, or $\geq$ 6 in any two consecutive years.	Agent 1987, ITE 1991,
		$\geq$ 5 in any one year.	Cottrell 1986
		$\geq$ 5 in any 12-month period in 3 years	City of San Diego 2006
		> 6 in any one year	ITE Florida Section 1982
	Left-Turn Accidents (on a pair of opposite approaches)	$\geq$ 6 in any one year, or $\geq$ 10 in any two consecutive years.	Agent and Deen 1979, ITE 1991
	Left-Turn Conflicts	$\geq$ 10 basic conflicts in a peak hour	Agent and Deen 1979
		$\geq$ 14 total conflicts in a peak hour	
		$\geq$ 4 per 100 left-turn vehicles	Cottrell 1986
Geometric Characteristic	Sight Distance	$\leq$ 250 ft (opposing speed $\leq$ 35 mph)	ITE Florida Section 1982, Upchurch 1986
		$\leq$ 400 ft (opposing speed $\geq$ 40 mph)	
	Number of Opposing Through Lanes	$\geq$ 3	Cottrell 1986, Agent 1987, City of San Diego 2006
	Number of Left-Turn Lanes	$\geq$ 2	ITE Florida Section 1982, Agent 1987, City of San Diego 2006
Operational Speed	Opposing Through Speed	$\geq$ 45 mph	Agent and Deen 1979, ITE Florida Section 1982 Agent 1987, Upchurch 1986
Other	Left-Turn Delay		Agent and Deen 1979, Cottrell 1986, ITE 1991, Roess et al. 2004
	Cost-Benefit Analysis		Agent and Deen 1979, Cottrell 1986
	High truck or pedestrian volume		City of San Diego 2006
	50 or more school age pedestrian crossing the lane per hour		
	Access Management Condition		Cottrell 1986
	Angle of the two approaches		

CHAPTER 3: METHODOLOGIES

In this study, two methods were used for assessing pedestrian safety at intersections under permissive left-turn signal control: 1) driving simulation-based experiments to test the driving performance of the participants when making permissive left-turns at the designed intersections, and 2) survey to solicit the participants' subjective opinions on the safety of the tested intersections regarding pedestrians and the vehicles making permissive left-turns.

4.1 Driving Simulation-Based Experiments

Participants

A total of 33 drivers participated in the driving simulator experiment. Participants of both genders with different levels of driving experience were recruited for this study. The demographic information of study participants is presented in Table 4.

Table 4: Breakdown of Driver Demographics for all Experiment

Category	Level	Driver Analysis	
		No. of Drivers	% of Total
Gender	Male	15	45.5%
	Female	18	54.5%
Age	Under 25	4	12.1%
	25 to 44	27	81.8%
	Over 44	2	6.1%
Driver Experience	Less than 1 year	6	18.2%
	1 to 3 years	17	51.5%
	More than 3 years	10	30.3%

Practice Scenario

Before the actual experiment, the participants drove in a practice scenario to become familiar with the simulator and the driving simulation environment. When they felt comfortable with driving the simulator, they informed the test administrator, who then started the real test.

Testing Scenario

After the practice scenarios, the participants drove in the three testing driving scenarios. The order of these three scenarios was decided randomly.

4.2 Post Testing Survey

A survey was conducted by the drivers who participated in the driving simulation test to get the subjective evaluation results. Note that every driver's entire driving process has been recorded by the simulator software, the drivers were asked to review the video replays of their driving and rate the intersections based on a scale from 1 to 5 (1 means very safe, while 5 means very unsafe) to indicate their feeling about the safety level at each intersection when they made a permissive left-turn. Overall, the survey consisted of two parts: Part I-to collect detailed information about the drivers and Part II-to collect driver's rating on the safety level of each intersection by reviewing the video replays of their driving during the test. The survey form is shown in Table 5.

Table 5: Survey Form for Pedestrian Safety under Permissive Left-Turn Control

Date:
First Name: **Middle Name:** **Last Name:**
What is your gender? ☐ Male ☐ Female
What is your age? ☐ Under 25 ☐ Between 25 and 44 ☐ Greater than 44
What is your education level? ☐ High School Diploma or Less ☐ Undergraduate ☐ Graduate
If you have a driver's license, how long have you had the driver's license? ☐ Less than 1 year ☐ 1 to 3 years ☐ More than 3 years

Please watch the video replay of traveling route (1, 2 or 3) and fill out the following part:					
	Very safe(1)	**Safe(2)**	**Average(3)**	**Unsafe(4)**	**Very unsafe(5)**
Intersection1	☐	☐	☐	☐	☐
Intersection2	☐	☐	☐	☐	☐
Intersection3	☐	☐	☐	☐	☐
Intersection4	☐	☐	☐	☐	☐
Comments for this three routes design:					

4.3 Devices, Techniques, and Tools

4.3.1 Hardware

Driving Simulator

The DriveSafety DS-600c driving simulator was used for this study. This simulator is a fully integrated, high-performance, high-fidelity driving simulation system that can effectively approximate real-world driving. Drivers can easily control the steering, accelerating pedal and brake pedal just as they do in a real vehicle. The system has 180-degree visual field view, was projected on three integrated screens by three separate high resolution projectors, and is equipped with a sound system reproducing the sounds of the engine. During testing, the system can collect second by second driving performance data, such as travel time, distance to nearest pedestrian or vehicles, brake rate, and so forth. In addition, the system support of record and playback mode allows the entire driving testing to be recorded and played back at a later time for reviewing purpose.

The detailed introduction of the simulator in both hardware and software sides are presented below:

The driving simulator at Texas Southern University is composed of five hardware components: 1) the cab, 2) the projectors, 3) three large screens, 4) a set of computers for simulation, and 5) a computer for scenario creation (Figure 1 and Figure 2).

- *Cab*. The cab is outfitted with computers, potentiometers, and torque motors that are connected to the accelerator, brakes, and steering. It also features full stereo audio, full instrumentation, and fully interactive vehicle components, all of which provide the realistic feeling of driving.
- *Projectors, Screens and Computers*. The cab is connected to a set of computers for simulation (computers on the rack) that consists of one host computer and six image generation computers. The host computer had the software Vection installed, which are for Backend/Simulator Functionality and runs on Fedora Core 3.0 (Linux system). The six image generation computers generate driving scenes and send them to three high-resolution projectors and the rear and side mirrors in the cab. Through the projectors, the scenes project to three larger screens.
- *Workstation.* The computer for scenario creation is the major workstation for creating various driving scenarios. The software applications, HyperDrive and Dashboard, were installed. HyperDrive creates scenarios and Dashboard transfers scenarios and controls simulation.

The following figures show the principle of all the equipment and their locations at Texas Southern University.

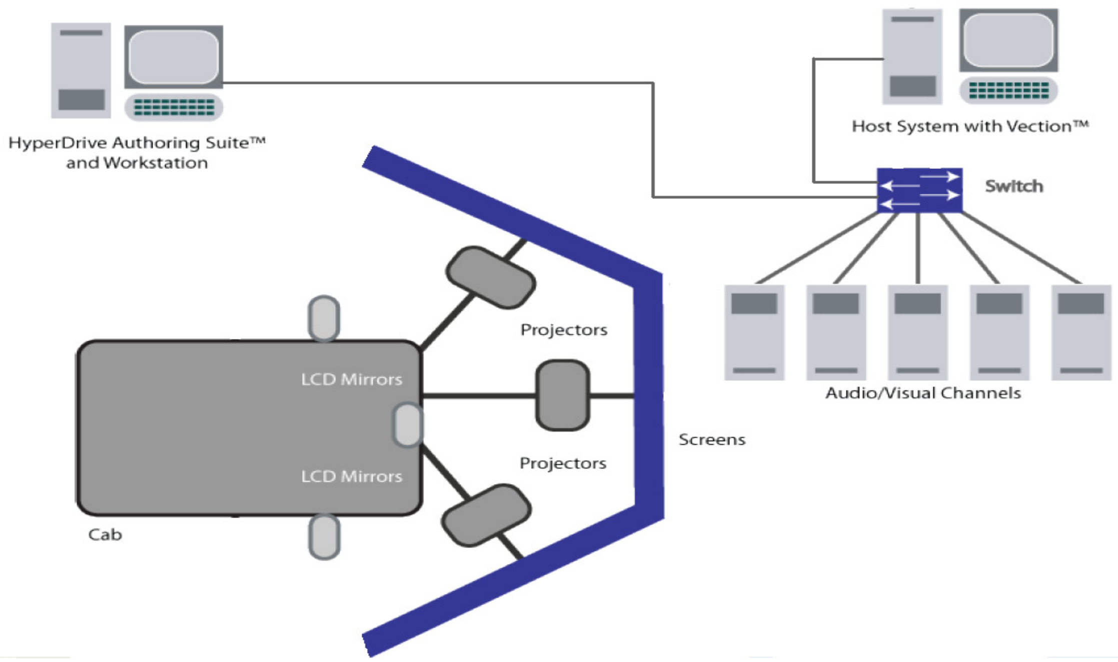

Figure 1: Schematic Diagram of Driver Simulator Components

Figure 2: Driver Simulator Components

4.3.2 Software

To design the driving testing scenarios in the driving simulator environment, three simulator-related software packages are used:

- HyperDrive. This software delivers a Windows-based, drag-n-drop software interface allowing non-technical users the ability to design, build, execute, and analyze driving scenarios without technical or engineering assistance. Driving scenarios include basic autonomous traffic as well as custom-defined scripted vehicle actions and reactions. Figure 3 shows the interface of the HyperDrive software. Figure 4 shows the script interface of HyperDrive.

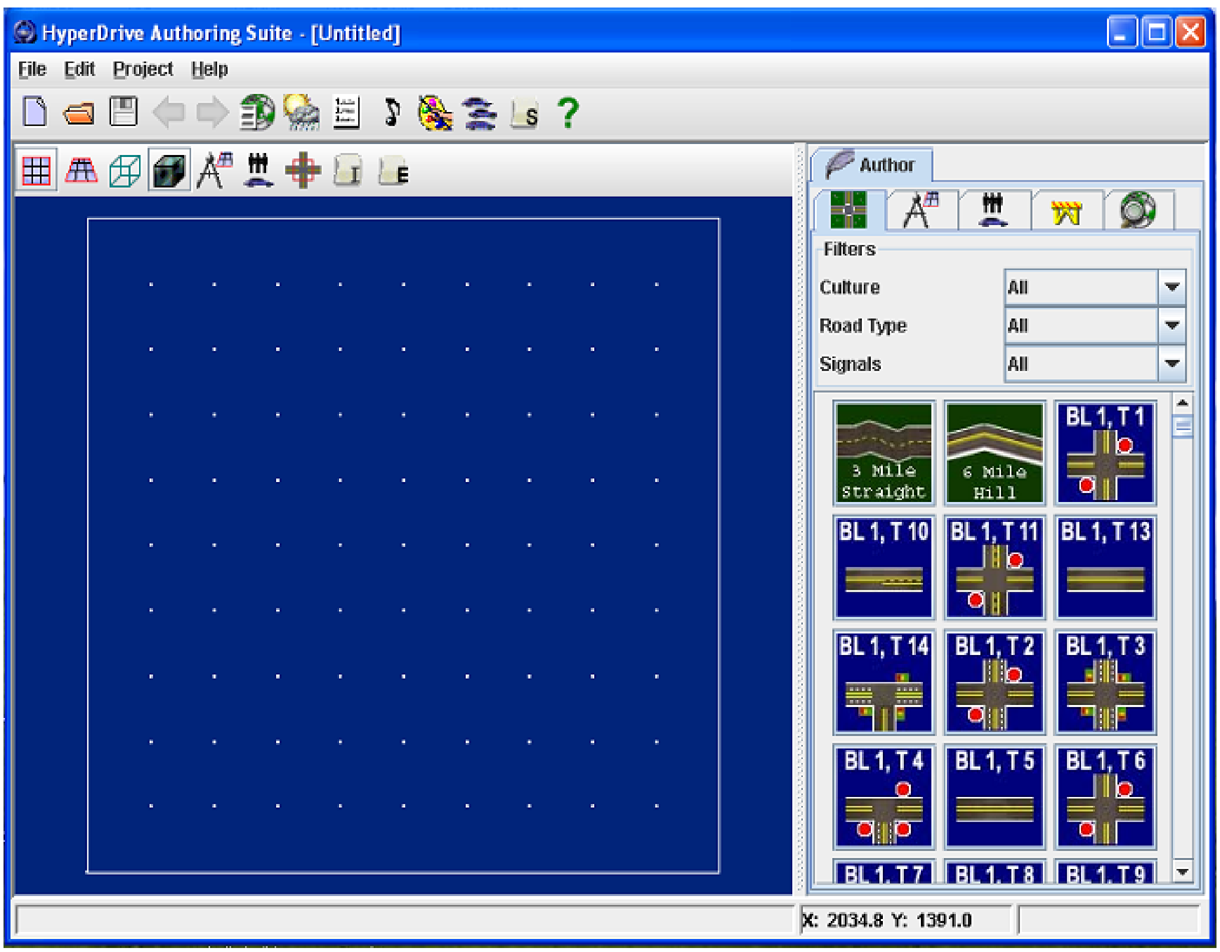

Figure 3: HyperDrive Interface

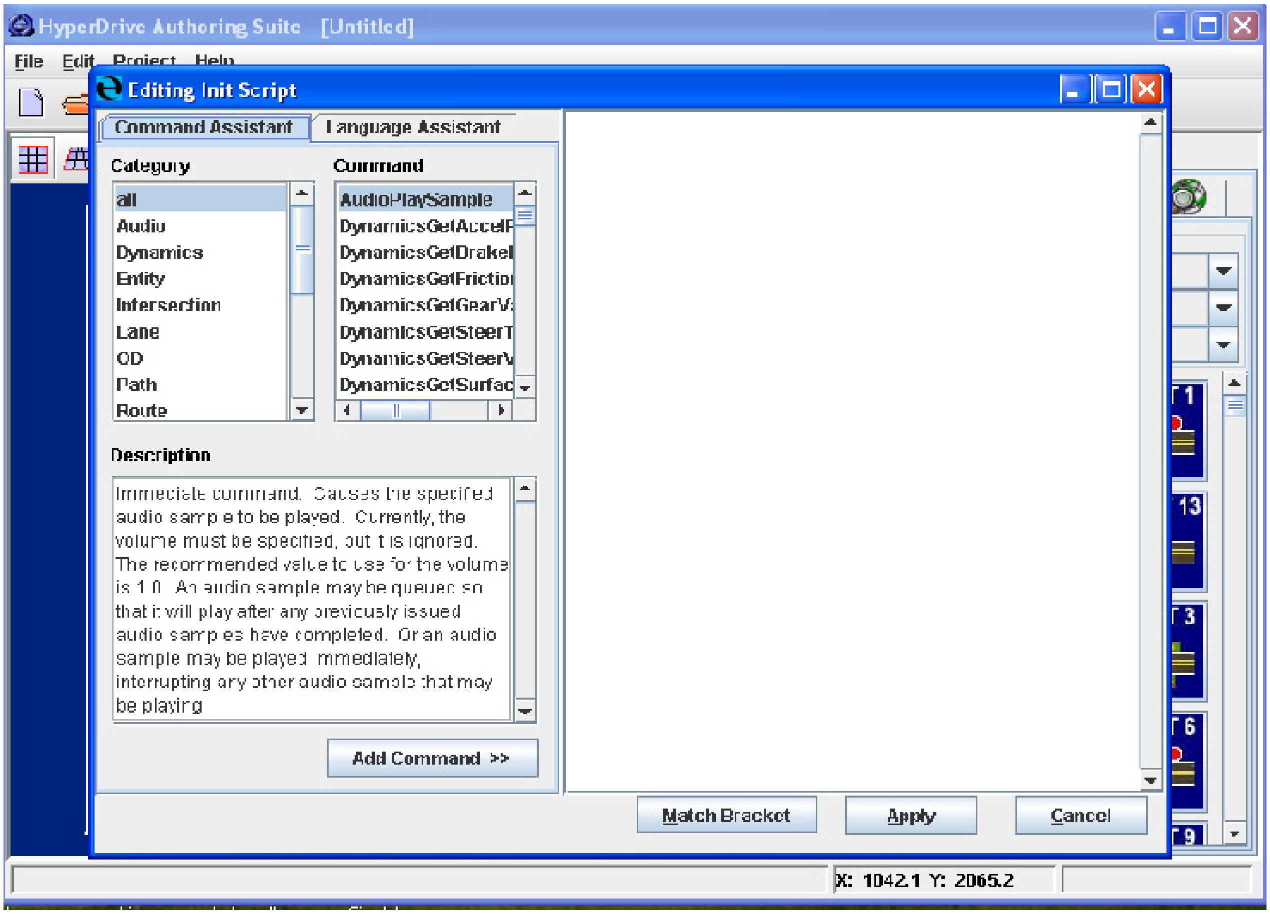

Figure 4: Script Frame in HyperDriver

- Vection. This software is a deterministic real-time simulation system. It is a run-time software package that includes advanced vehicle dynamics, scenarios control with both scripted and autonomous traffic simulation, flexible data collection, audio and visual subsystems, and integrated support for cab instrumentation, control loading, and motion platform control. Vection provides an extremely powerful environment for providing realistic driving simulation experiences and measuring the desired results.
- Dashboard. This software acts as a connector between HyperDrive and Vection. It is comprised of several different sections - a simulator section, a component section, a project management section, and a simulator control section. Figure 5 shows the interface of the Dashboard software.

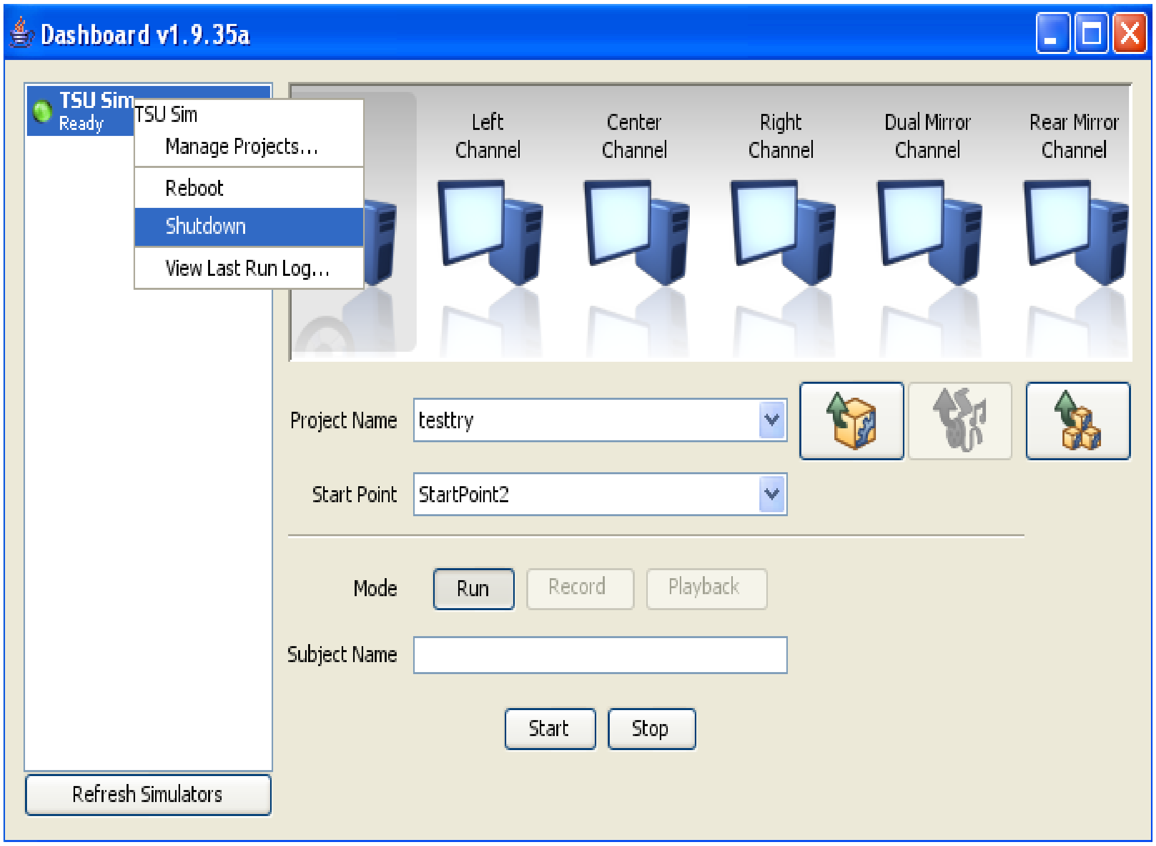

Figure 5: Dashboard Interface

Besides the "Run" test mode, Dashboard also supports record and playback mode, which can allow the entire drive to be recorded and played back at a later time for review purposes.

In addition, this research also used several mathematical and statistical techniques as well as some computing and coding tools. These included Microsoft Excel, SPSS, and Tcl Language, etc.

- Microsoft Excel is one of the most widely used computer software in data analysis. This software was used to process the outputs of the tests and calculate the measures of effectiveness (MOEs);
- SPSS is a computer statistics program for data management and analysis. SPSS was used to compare the means of MOES for two groups of outputs collected from the tests;

- Tcl Language: The word “Tcl” is originally from “Tool Command Language”. It is a scripting language created by John Ousterhout. This scripting language was used in the HyperDrive to collect the different types of dynamic data: Deceleration rate, Time to collision to pedestrian, distance to the nearest pedestrian.

CHAPTER 4: EXPERIMENTAL DESIGN

This chapter is to present the design of the proposed study. It includes two parts: 1) Testing Scenarios Design and 2) Measures of Effectiveness Design

3.1 Testing Scenarios Design

Based on the literature review results, six important contributing factors to the collisions between the pedestrian and left-turn vehicles were considered in the design of the testing scenarios: 1) speed limit, 2) number of lane, 3) intersection type, 4) sight distance, 5) opposing through traffic volume and 6) pedestrian volume. As a result, three different testing routes were built in the driving simulator environment, including 12 different experimental intersections with permissive left-turn signal controls and different traffic and geometric conditions. Following are the detailed descriptions of these three routes:

Route1: It is a four-lane roadway and the speed limit is 50mph. Drivers will pass through four permissive left-turn intersections with different pedestrian and opposing volumes (see Figure 6a for the detailed traffic volume information). Among these intersections, two of them are three-leg intersections and others are four-leg intersections.

Route2: It is a four-lane road way and the speed limit is 35mph. Drivers will pass through four permissive left-turn intersections with different levels of pedestrian and opposing volumes (see Figure 6b for the detailed traffic volume information). Among these intersections, two of them are three-leg intersections and others are four-leg intersections.

Route3: It is a two-lane road way and the speed limit is 25mph. Drivers will pass through four permissive left-turn intersections with different levels of pedestrian and opposing volumes (see Figure 6c for the detailed traffic volume information), and all of them are four-leg intersections.

Note that, all these testing routes are designed in the urban area. The schematic maps of these three driving testing routes are presented in Figure 6.

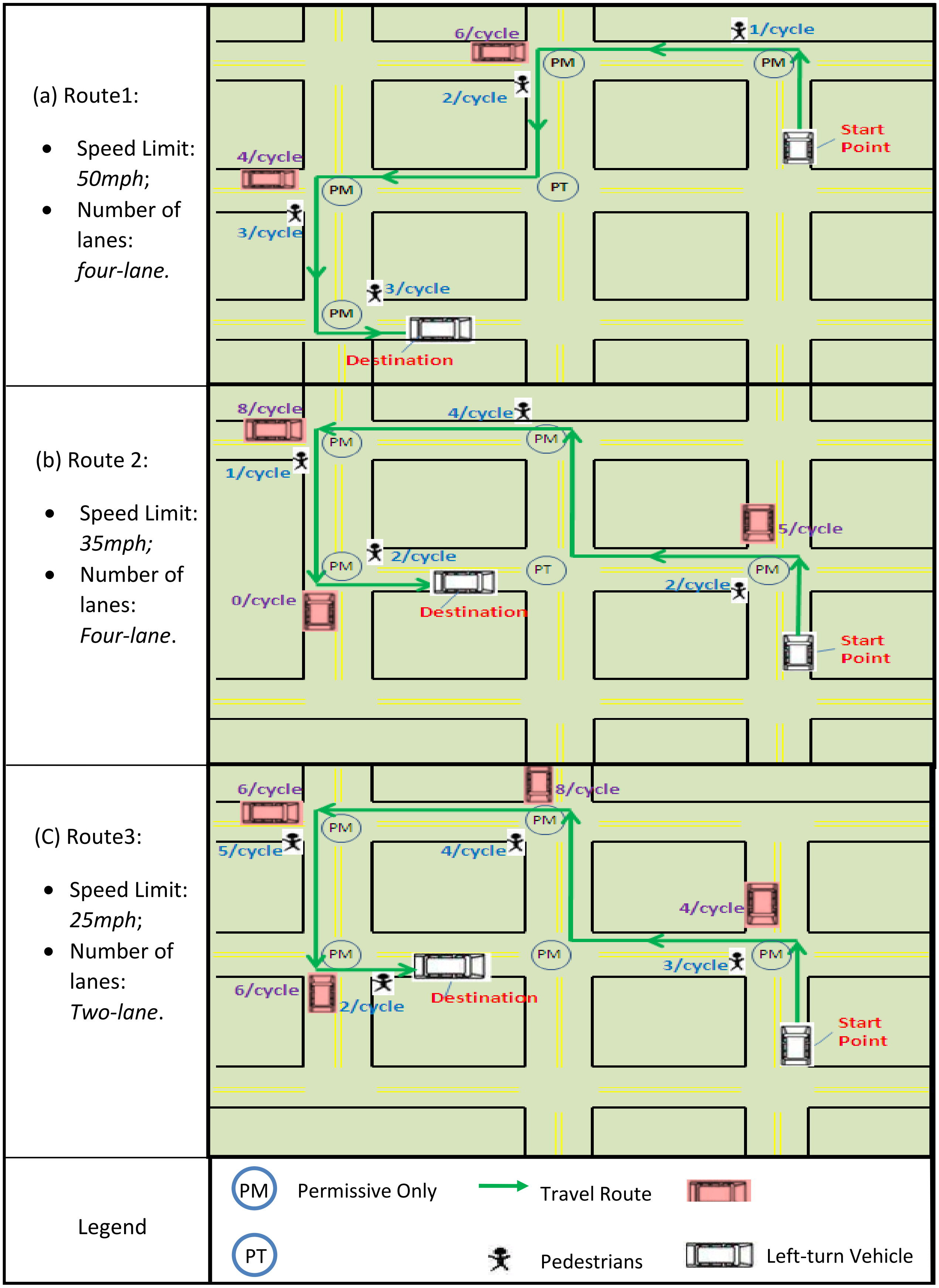

Figure 6: Schematic Map of the Testing Driving Routes Design

3.2 Measures of Effectiveness Design

To evaluate the risk of collisions between permissive left-turn vehicles and pedestrians, f three types of MOEs were designed for the driving simulation based experiments: 1) maximum deceleration rate, 2) time to collision to pedestrian, and 3) minimum stop distance. Based on these MOEs, three different types of critical events were identified. Following are the detailed descriptions of these three types of critical events.

- *Maximum Deceleration Rate Based Critical Events*

Deceleration is a good surrogate measure for safety research. It can indicate the potential severity of the conflict events. During the testing, the deceleration rate of the subject vehicle was collected every second. If the deceleration rate is greater than 0.5G ($4.9m^2/s$, Muttart, J.W., at.el, 2007), which is equivalent to the rate for the full braking on wet pavement and is approximately the point at which skid marks begin to appear in most cases, it indicates that the subjective vehicle had a hard brake and may be involved in a conflict with either pedestrian or opposing traffic during its permissive left-turn. Such events were marked as a maximum deceleration rate based critical event. Figure 7 is an example that shows how this type of critical event can been identified based on the second-by-second deceleration data collected from the driving simulation experiments.

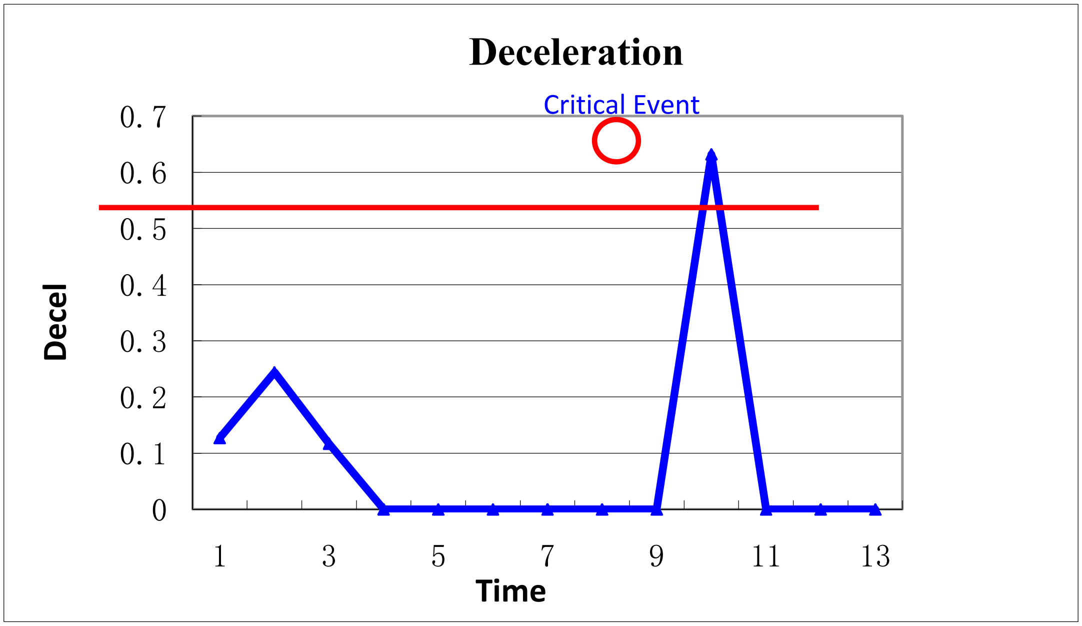

Figure 7: Maximum Deceleration Rate Based Critical Events

- *Time to Collision to Pedestrian (TTCTP) based Critical Events*

TTCTP is time to collision to the nearest pedestrian for left-turn vehicles. Basically, a lower value of TTCTP indicates a greater likelihood of a collision. It can be determined by the distance from vehicle to pedestrian (D) and the portion of the speed of both vehicle and pedestrian in the direction of a collision (see Figure 8). Equation (1) shows how to calculate the TTCTP value, and Figure 8 shows the variables in Equation (1). If TTCTP value is less than a threshold, it indicates a critical event between the left-turn vehicle and pedestrian.

$$TTCTP = \frac{D}{V_A \times \cos\alpha + V_P \times \cos\beta} \qquad (1)$$

Where D is the distance to pedestrian

V_A is the speed of the left-turn vehicle

V_P is the speed of the closest pedestrian

α is the angle between vehicle heading and the direction of collision

β is the angle between pedestrian heading and the direction of collision

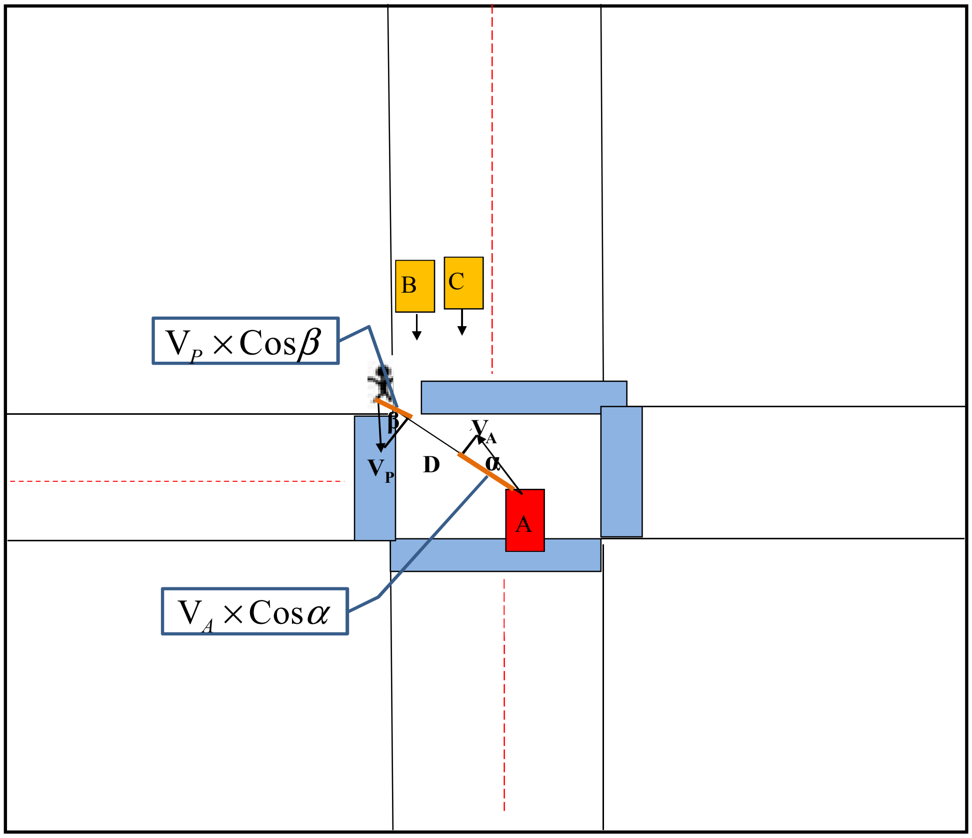

Figure 8: Time to Collision to Pedestrian (TTCTP)

By reviewing the videos recorded during the driving testing, it is found that, once the collected TTCTP value is less than 0.5s, there will be a near collision event between left-turn vehicle and pedestrian. Therefore, 0.5 seconds was chosen as the threshold of TTCTP value to indentify this type of critical event.

- *Minimum Stop Distance based Critical Events*

Once the left-turn drivers are aware of the risk of the collision with pedestrian, they will hit the brake to stop the vehicle. However, because of the inertia, a minimum stop distance is needed to allow the left-turn vehicle to avoid a collision. Thus, if the distance from the left-turn vehicle to the nearest pedestrian is less than this minimum stop distance, it indicates a critical event between the left-turn vehicle and pedestrian. The minimum stop distance depends on the vehicle's velocity and the coefficient of kinetic friction and it can be calculated by Equation (2):

$$d = \frac{v^2}{2ug} \tag{2}$$

Where *d is the stop distance*
u is the coefficient of kinetic friction (Assume u=0.8)
g is the acceleration of gravity
v is the initial speed of the vehicle

When the actual distance between the left-turn vehicle and pedestrian is less than the safe distance given by Equation (2), a critical event will be identified. Figure 9 is an example that shows how this type of critical event can be identified based on the second-by- second velocity and distance to pedestrian data collected from the driving simulation experiments, the red line in this figure is the minimum stop distance which estimated by Equation 2 and the green line is the actual distance between left-turn vehicle and pedestrian, therefore, if any points on the green line are below the red line, it indicates that the actual distance between left-turn vehicle and pedestrian is less than the minimum stop distance, and it is identified as a critical event. In additional, for this type of critical event, the collected TTCTP value must be positive because the negative TTCTP value indicates that the vehicle has already passed pedestrian and there is no chance to have a collision. Therefore, at this time, even if the distance between the left-turn vehicle and pedestrian is less than the minimum stop distance, there will be no accident risk for the pedestrian and left-turn vehicle.

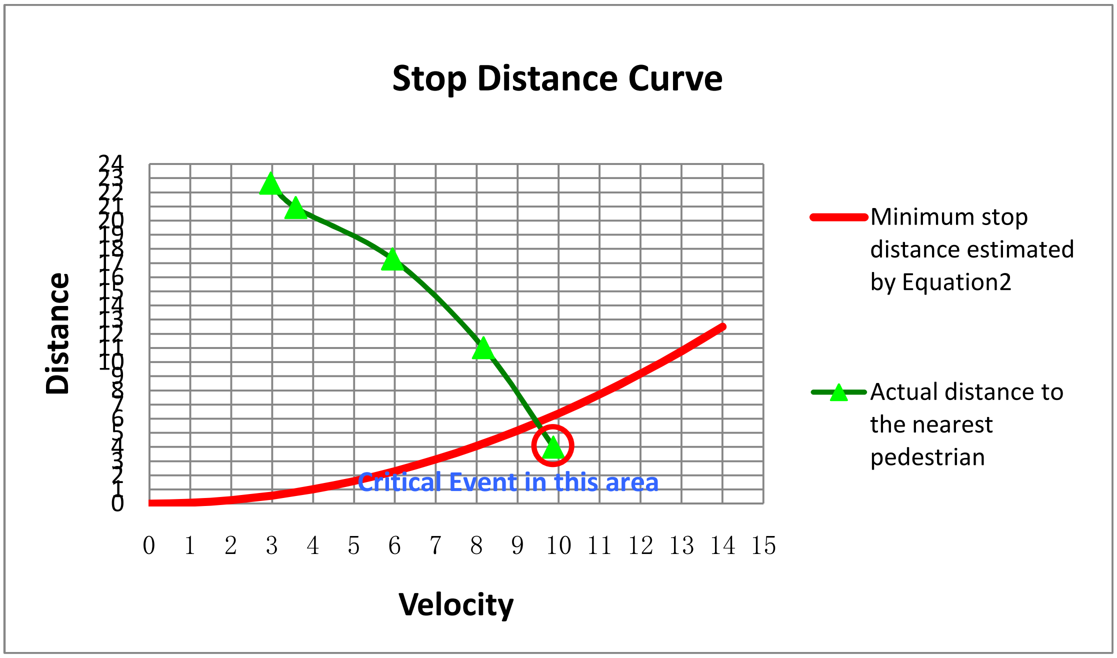

Figure 9: Minimum Stop Distance based Critical Events

In this study, the pedestrian safety level at each testing interaction was evaluated based on the total number of these three types of critical events collected during the driving simulator test. Intersections with a large number of total critical events indicate high risk of the collision between pedestrians and left-turn vehicles, and a small number of critical events indicate the low risk of collision. In this study, some statistical methods will be used to analyze the collected critical event data.

CHAPTER 5: RESULTS

In this chapter, the results of both driving simulator-based experiments and post testing survey for the proposed driving scenarios are presented and discussed.

Basically, there are two sets of data collected from the driving simulator based experiments: 1) the objective data: the total number of critical events identified based on the collected MOEs during the driving testing and 2) the subjective data: the score of intersection safety level collected during the post testing survey. These two results were compared in Figure 10. From Figure 10, it can be seen that the collected subjective and objective evaluation results are quite correlated (R^2=0.6375). Thus, in the data analysis, we will focus on the analysis of the collected objective data, which is the total number of critical events identified based on the three MOEs.

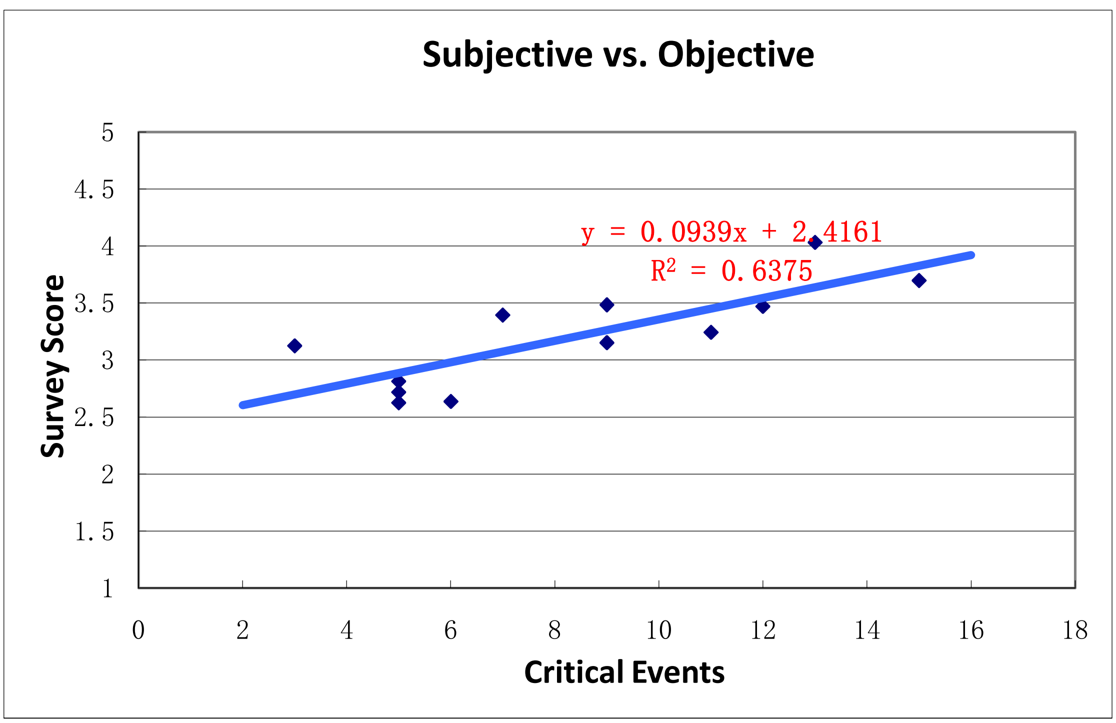

Figure 10: Subjective Evaluation versus Objective Evaluation

Based on the data collected during the driving simulator test, three different types of MOEs are derived. These MOEs are maximum deceleration rate, minimum time to collision and minimum stop distance. Based on these three MOEs, the total number of critical events are counted to indicate the possibility of collisions between the left-turn vehicles and pedestrians under permissive left-turn signal control at each intersection. Two statistical methods, Poisson Regression and independent T-test, were used to identify factors that contribute to the collisions between left-turn vehicles and pedestrians during the permissive left-turn phase. These factors include intersection type, speed limit, number of lanes, sight distance, opposing through traffic volume and pedestrian volume. Note that, among these factors, intersection type is highly correlated with opposing through traffic volume. It is because the opposing through traffic volume would all be zero for the three-leg “T” intersections. Thus, in this study, the intersection type was not included in the Poisson model and the independent T-test was used to test if the means of number of critical events between “T” and “X” intersection are significantly different and which intersection type is more dangerous to pedestrians and vehicles making permissive left-turns.

Poisson regression model is a classical model for counted data, it was considered in this study for modeling the critical events between left-turn vehicles and pedestrians because critical events are randomly distributed and its frequency is discrete and positive numbers. The statistic software package SPSS was used for developing this model. The relationship between the expected number of critical events occurring at intersection approach i (dependent variable Y_i) and a set of explanatory variables $X_{i1}, X_{i2}, \ldots X_{in}$ that represent the features of intersections (i.e., intersection geometric, speed limit, traffic volume conditions) could be modeled by Equation (3):

$$Function(Y_i) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_n X_{in} \quad (3)$$

Where $\beta_0, \beta_1, \beta_2, \ldots \beta_n$ are the coefficients of the independent variables $X_{i1}, X_{i2}, \ldots X_{in}$. The regression procedure is to estimate model parameters and the coefficient parameter vector $\beta(\beta_0, \beta_1, \beta_2, \ldots \beta_n)$.

The Poisson regression model is suitable for modeling collision frequency, according to the literature. This model assumes that, given the value of X_i (a vector of possible explanatory variables i.e.$X_{i1,}$ $X_{i2}, \dots X_{in}$), the dependent variable y_i follows a Poisson distribution, which can be expressed as:

$$P(y_i|X_i) = \frac{\exp(-\mu_i)\mu_i^{y_i}}{y_i!} \quad , \tag{4}$$

where y_i denotes the total number of critical events that occurred at intersection approach pair i, and μ_i is the conditional mean of y_i, which is a non-linear function of X_i and can be expressed as follows :

$$In\, \mu_i = \beta X_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_n X_{in} \quad . \tag{5}$$

Then, the expected number of critical events at intersection approach pair i can be estimated by:

$$E(y_i|X_i) = \mu_i = e^{\beta X_i} \quad , \tag{6}$$

where β is the vector of regression coefficients that can be estimated by the standard maximum likelihood method with the likelihood function given by:

$$L(\beta) = \prod_i \frac{\exp[-\exp(\beta X_i)][\exp(\beta X_i)]^{y_i}}{y_i!} \quad , \tag{7}$$

The dependent variables and independent variables considered in the model development are described in Table 6.

Table 6: Dependent and Independent Variables Used in Poisson Model

Variables	Description
Dependent Variables	
Critical events	Number of critical events at each intersection
Speed Limit	
Greater than 40mph or less	1 if greater than 40mph, 0 otherwise
No. Lane	
Number of lane on two approaches	1 if four-lane, 0 if two-lane
Truck Present on Left-turn Lane*	
Truck present on left-turn lane or not	1 if truck present on the left-turn lane, 0 otherwise
Opposing Through Traffic Volume	
Opposing through traffic volume	Number of opposing through vehicles per cycle averagely
Pedestrian Volume	
Pedestrian volume	Number of Pedestrians per cycle averagely

**Note: truck present on left-turn lane means that the left-turn vehicle is following a truck that is also intending to turn left and block the sight of the following left-turn vehicles.*

The results of Poisson regression are presented in Table 7.

Table 7: Poisson Regression Results

Model		Poisson - Dependent Variable: Critical Events	
	Variables	**Estimated Coefficients**	**p-value**
Regression Results	**Speed Limit**	-0.040	0.907
	Number of Lanes	-0.146	0.712
	Trucks Present on Left-turn Lane	1.031	0.001
	Opposing Through Traffic	0.030	0.710
	Pedestrian Volume	0.160	0.033
	Log likelihood	272.0502	
	Sample Size	12	

As seen in these results, only two factors, i.e. trucks present on left-turn lanes and pedestrian volume, have significant impacts on the critical evens between left-turn vehicles and pedestrians (p-value less than 0.05) and other three factors are quite insignificant. The following is the detailed analysis of the impacts of different influencing factors on the pedestrian safety under permissive left-turn signal control:

1. Truck Present on Left-turn Lane

The results in Table 7 show that the trucks present on left-turn lane is the most significant influencing factor on the pedestrian safety under permissive left-turn signal control (p-value=0.001). In addition, the results of post testing survey also revealed that drivers felt unsafe to make permissive left-turn if there is a truck in front of their vehicles. In general, the presence of a big truck on the left-turn lane will reduce the sight distance of the following left-turn vehicles, which will significantly increase the pedestrians' risk. Therefore, if the intersection has high percentage of left turn trucks, it would be unsafe to allow permissive left-turns.

2. Pedestrian Volume

According to the Poisson regression model results given in Table 7, pedestrian volume is also a significant factor in pedestrian safety under permissive left-turn signal control (p-value=0.033). Figure 11 shows the relationship between the number of critical events and pedestrian volume.

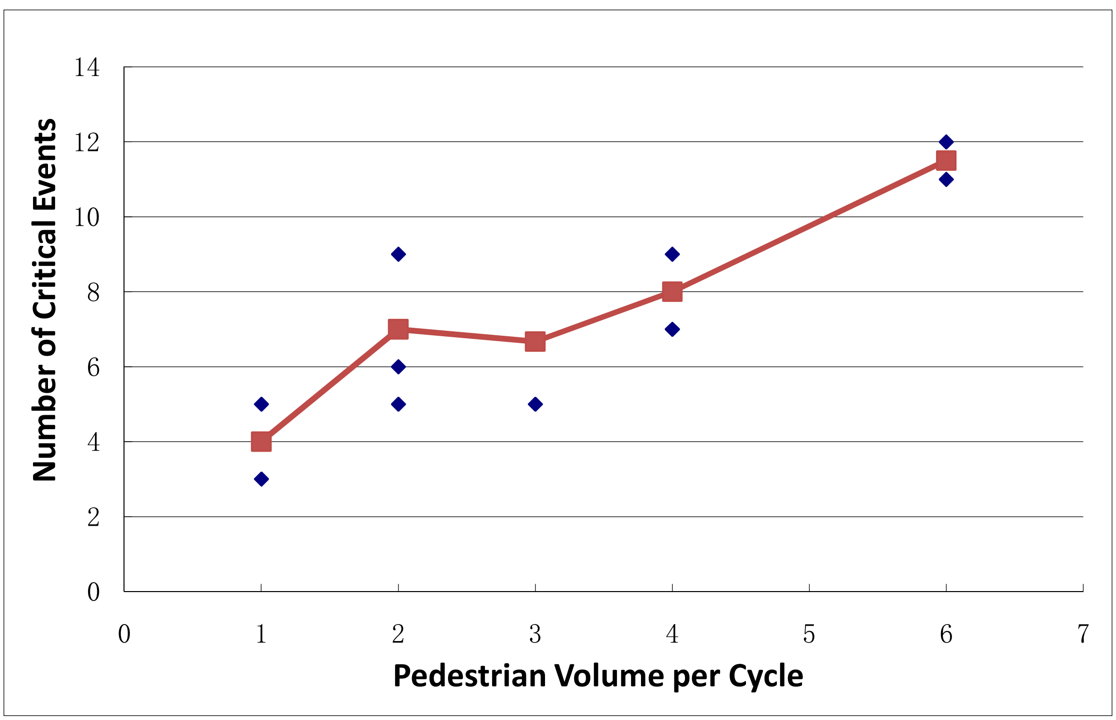

Figure 11: Relationship between Pedestrian and Critical Events

The red line in Figure 11 is the linkage of the average number of total critical events at different levels of pedestrian volume (pedestrians per cycle). As shown in this figure, in general, the more pedestrians crossing the intersection, the more critical events will be caused. The critical events will sharply and continually increase when the number of pedestrians per cycle is greater than four. Thus, four pedestrians per cycle (PPC) will be the critical point for determining if it is safe to use permissive left-turn signal control or not. This critical pedestrian volume in PPC can be converted to the hourly pedestrian volume by the following equation:

$$\text{Critical pedestrian hourly volume} = \frac{3600}{\textit{Cycle Length}} \times \textit{critial pedestrians per cycle}$$

$$= \frac{14400}{\text{Cycle Length}} \tag{8}$$

This result indicates that, if the pedestrian volume crossing intersection is greater than 14400/Cycle Length per hour, it will be not be safe to allow left-turn vehicles to make permissive turns due to the pedestrians' safety.

3. Opposing Through Traffic Volume

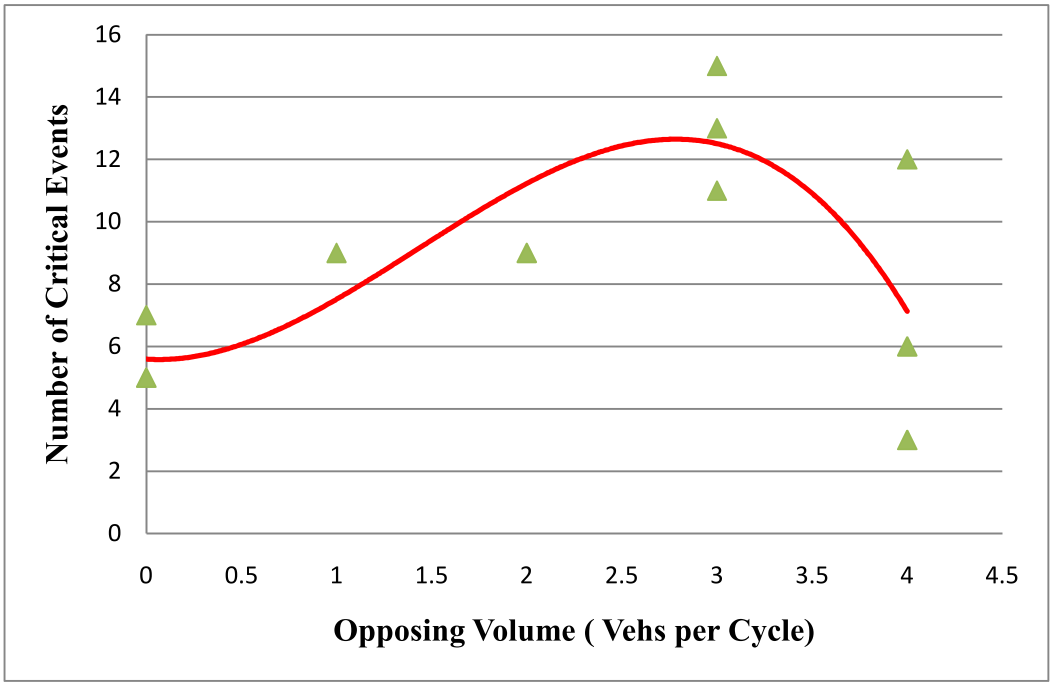

Figure 12: Relationship between Opposing Through Traffic Volume and Critical Events

Although opposing through traffic volume is not a significant factor according to the Poisson regression model results given in Table 5 (p-value=0.710), literatures (Lord, 1994) show that opposing through traffic volume is also a very important factor on the pedestrian's safety under the permissive left-turn signal control. In order to analyze the impact of this factor, the relationship between critical events and the opposing through traffic volume are plotted in Figure 12. The red line in Figure 12 is the linkage of the average number of critical events at different levels of opposing through vehicles per cycle. It can be seen that, in general, the number of critical events increases with the increase of the opposing through traffic. However, after the value increase to a certain degree, the critical events decrease. This result is reasonable because with the opposing through traffic volume increase, the number of available gaps in the opposing traffic flow will reduce. Thus, the chance that left-turn vehicles will take the small gaps to make a risky permissive left turn will increase, which cause the critical events to pedestrians increase. However, with the opposing through traffic volume continually increasing, the left-turn vehicle will wait and have more time to yield to opposing through vehicles in the permissive phase, which will give more time to the pedestrians to cross the intersection safely. Thus, the critical events will decrease after the opposing through traffic volume reach to a certain degree. This result is also consistent with the findings in Lord (1994).

4. Speed Limit and number of lanes

Compare with other factors, speed limit and number of lane does not show significant impacts on the critical events between the pedestrian and left-turn vehicles under permissive left-turn signal control. It may be due to the limits of simulator based study because the impact of speed is not as sensitive as other factors in the driving simulator environment. Thus, the impacts of these factors need to be further investigated by the field study.

5. Intersection Type

The studied intersections were divided into two independent groups by intersection type, i.e. three-leg intersections and four-leg intersections. The independent T-test was used to test if the mean of critical events between these two groups were significantly different.

T-test results show that the Intersection Type is a significant influencing factor on the safety of pedestrians at intersections with permissive left-turn signal control (P-value = 0.075). Since the Mean value of the four-leg intersection (9.75) is greater than the mean value of the three-leg intersection (5.50), the four-leg intersection usually will have more collisions between left-turn vehicles and pedestrians. This result is different with the findings in some literatures. However, it is a reasonable result because the drivers at the T-intersections just need to yield to pedestrians, but they also have to yield to both opposing through traffic and pedestrians at the four-leg intersections. The more complicated traffic conditions in the four-leg intersections will cause more risk to pedestrians. In addition, the post testing survey results show that the average score for the four-leg intersection is 3.35, while the average score for the three-leg intersection is 2.89. Therefore, the subjective evaluation also indicates that four-leg intersections will be more risky than three-leg intersections.

Based on the results discussed above, the following key findings were obtained:

1) Left-turn driver's sight distance is extremely important to the safety of pedestrians under the permissive left-turn signal control. If there is a truck on the left-turn lane block the view of other left-turn vehicles, it increases the risk of collisions between permissive left-turn vehicles and pedestrians significantly.

2) The probability of collisions between permissive left-turn vehicles and pedestrians will increase with the increase of pedestrian volume. If the pedestrian volume is above a certain value (14400/Cycle Length) per hour, the risk to pedestrians will increase sharply and continually.

3) Opposing through traffic volume is also an important factor to the safety of pedestrians at intersections with permissive left-turn signal control. The collision risk will increase with the increase of the opposing through traffic and then decrease after the opposing through traffic reaches a certain degree.

4) Both driving simulator based experiments results and the post testing survey results show that the four-leg intersections will cause more collisions between pedestrians and permissive left-turn vehicles compared to the three-leg intersections.

Based on these findings, the following new pedestrian safety based warrants on using permissive left-turn phases were proposed:

The permissive left-turn phase (either permissive only or PPLT) can be used only at intersections which meet the following criteria

1) Pedestrian volume is lower than 14400/Cycle Length per hour, and
2) The percentage of big size vehicles (such as trucks) in the left-turn traffic volume is relatively low.

CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

In this study, pedestrian safety under permissive left-turn signal control was investigated by driving simulator based experiments and the post testing survey conducted by participating drivers. Based on the results of the simulator based experiments and the survey, two pedestrian safety based warrants for selecting protected and permissive left-turn signal control modes were proposed. The key findings of this study include:

1). If there is a truck in the left-turn lane blocking the view of other left-turn vehicles, it increases the risk of collisions between permissive left-turn vehicles and pedestrians significantly.

2) The probability of collisions between permissive left-turn vehicles and pedestrians will increase with increased pedestrian volume. If pedestrian volume is at a certain value (14400/Cycle Length) per hour, the risk to pedestrians will increase sharply and continually.

3) Opposing through traffic volume is also an important factor to the safety of pedestrians at intersections with permissive left-turn signal control. The collision risk will increase with the increase of opposing through traffic and then decrease after the opposing through traffic reaches a certain degree.

4) Both driving simulator based experiments results and the post testing survey results show that the four-leg intersections cause more collisions between pedestrians and permissive left-turn vehicles compared to the three-leg intersections.

Researchers will continue their efforts on improving this study in the future using the following aspects: First, driving simulators cannot simulate pedestrian's behavior and cannot accurately test the impacts of certain factors (such as speed limit), which will affect the results of the experiments. Thus, in the future, field study will be conducted to verify and further improve the results of this study. Secondly, since drivers feel uncomfortable driving more than ten minutes in the driving simulator environment, researchers will need to cut down the number of experimental intersections in each testing scenario. As a result, this study has a quite small sample size (just 12 tested intersections) and the tested intersections are not enough to cover all combinations of the various influencing factors. In the future, more testing scenarios need to be designed and tested to further improve the results of this study.

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